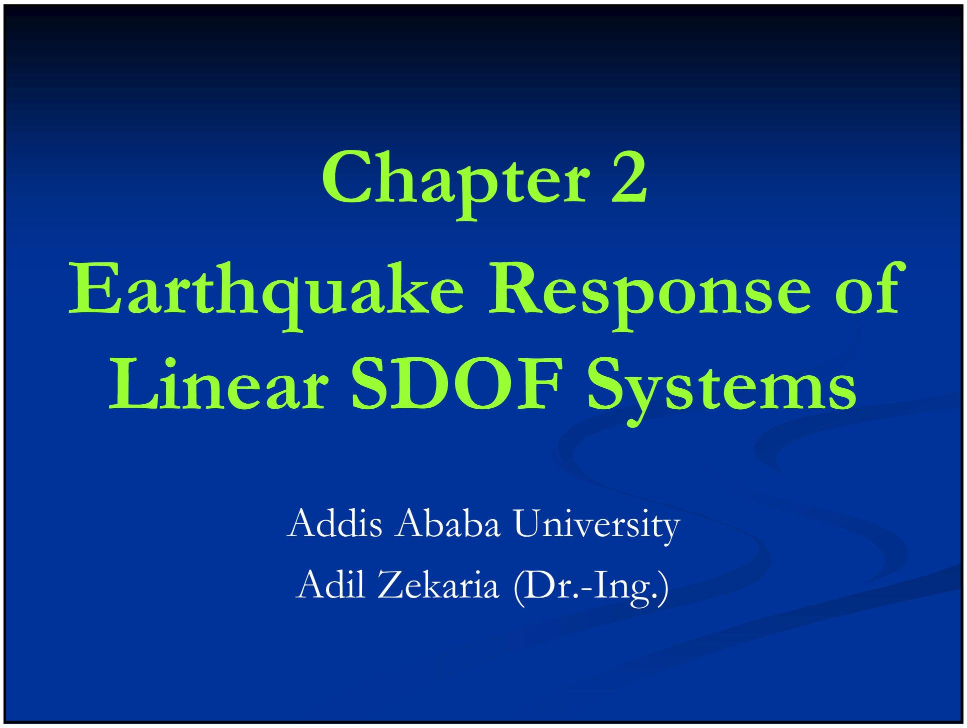
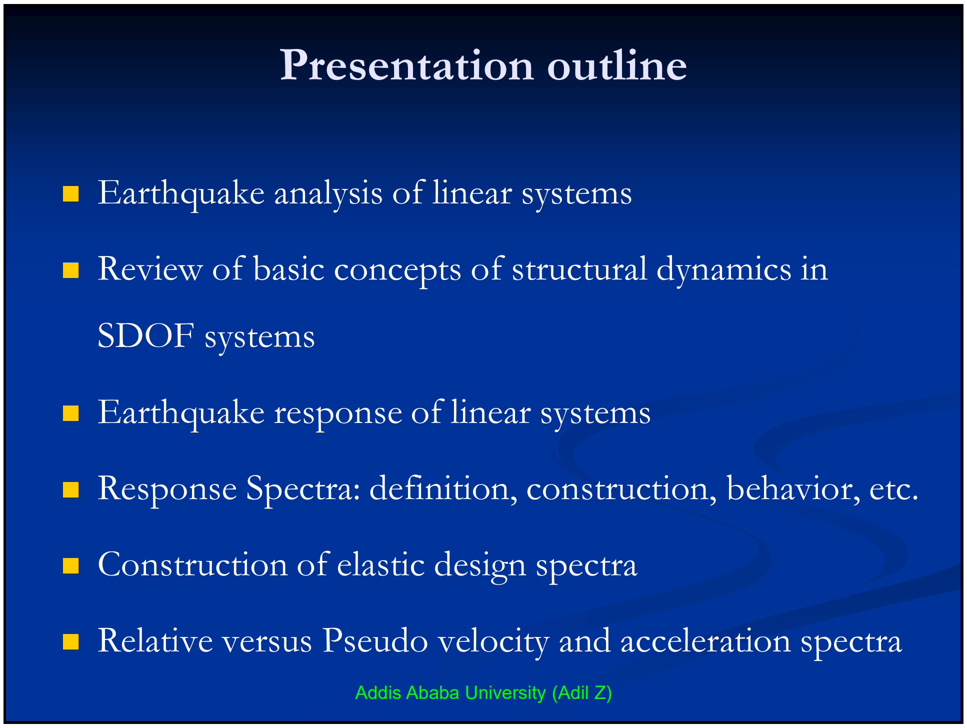


Chapter 2

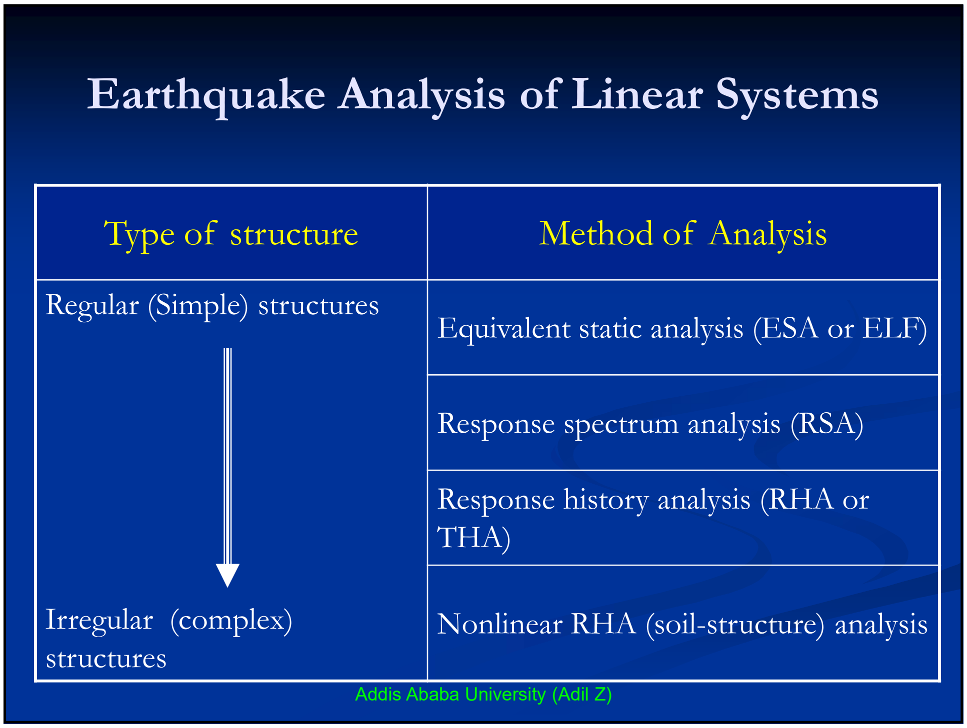
Earthquake Response of Linear SDOF Systems

Addis Ababa University
Adil Zekaria (Dr.-Ing.)



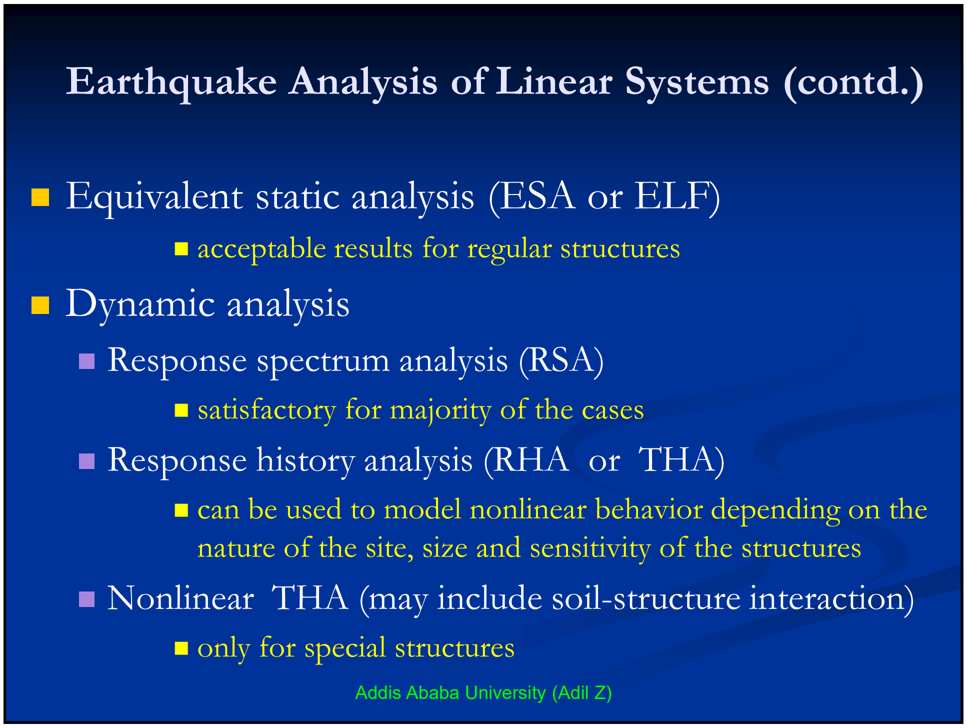
Presentation outline

- Earthquake analysis of linear systems
- Review of basic concepts of structural dynamics in SDOF systems
- Earthquake response of linear systems
- Response Spectra: definition, construction, behavior, etc.
- Construction of elastic design spectra
- Relative versus Pseudo velocity and acceleration spectra



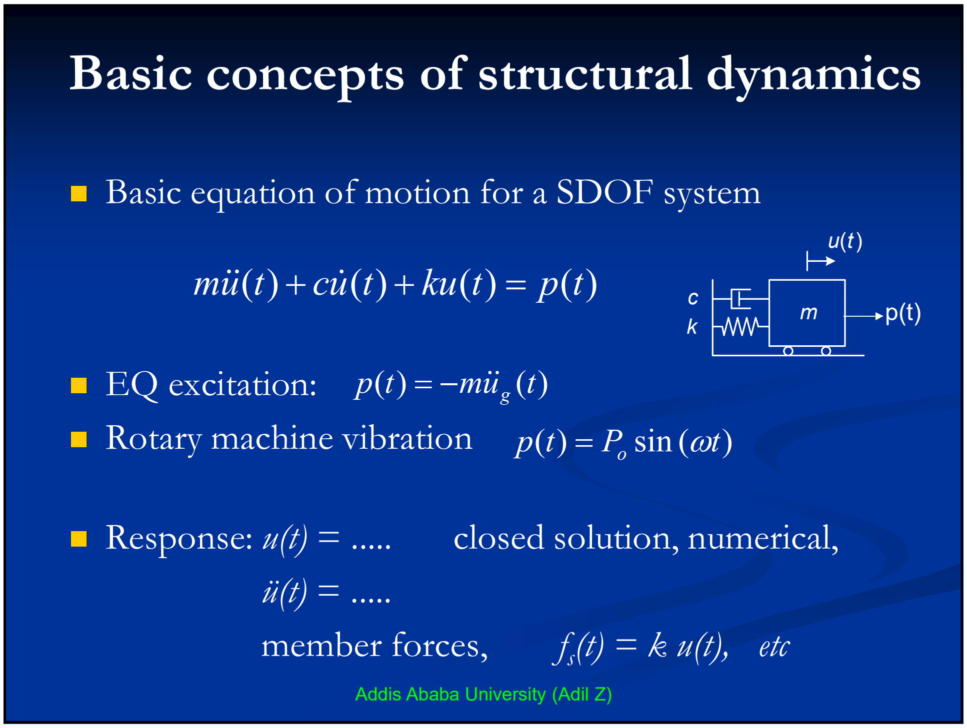
Earthquake Analysis of Linear Systems

Type of structure	Method of Analysis
Regular (Simple) structures	Equivalent static analysis (ESA or ELF)
↓	Response spectrum analysis (RSA)
↓	Response history analysis (RHA or THA)
Irregular (complex) structures	Nonlinear RHA (soil-structure) analysis



Earthquake Analysis of Linear Systems (contd.)

- Equivalent static analysis (ESA or ELF)
    - acceptable results for regular structures
- Dynamic analysis
    - Response spectrum analysis (RSA)
        - satisfactory for majority of the cases
    - Response history analysis (RHA or THA)
        - can be used to model nonlinear behavior depending on the nature of the site, size and sensitivity of the structures
    - Nonlinear THA (may include soil-structure interaction)
        - only for special structures



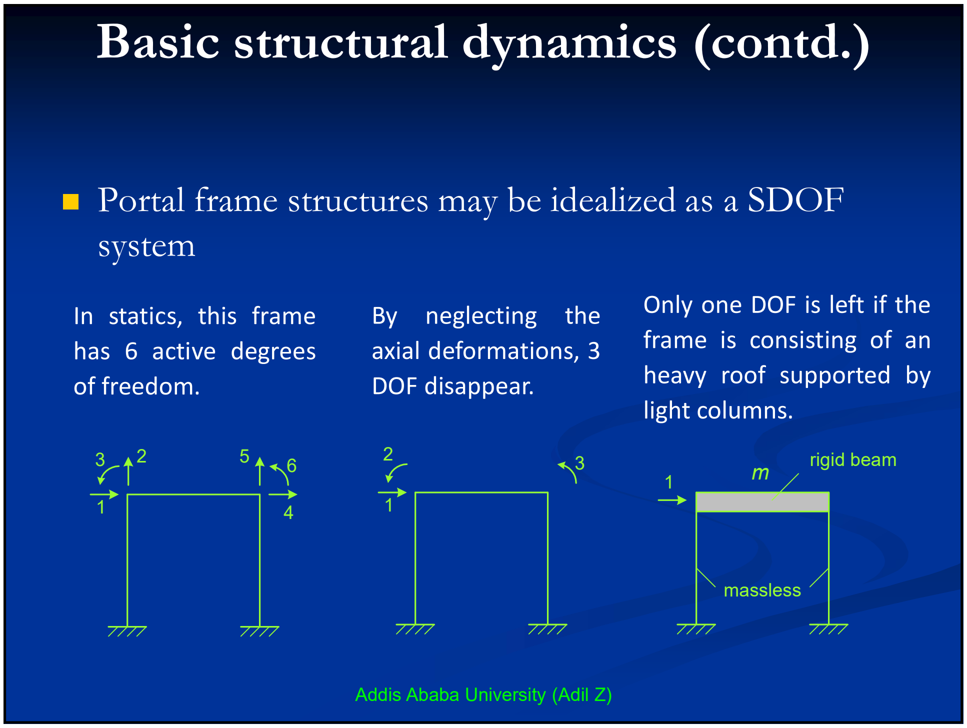
Basic concepts of structural dynamics

- Basic equation of motion for a SDOF system

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=p(t)$$

- EQ excitation: $p(t)=-m\ddot{u}_g(t)$
- Rotary machine vibration $p(t)=P_o\sin(\omega t)$

- Response: $u(t)$ = ….. closed solution, numerical,
  $\ddot{u}(t)$ = …..
  member forces, $f_s(t) = k\,u(t)$, *etc*



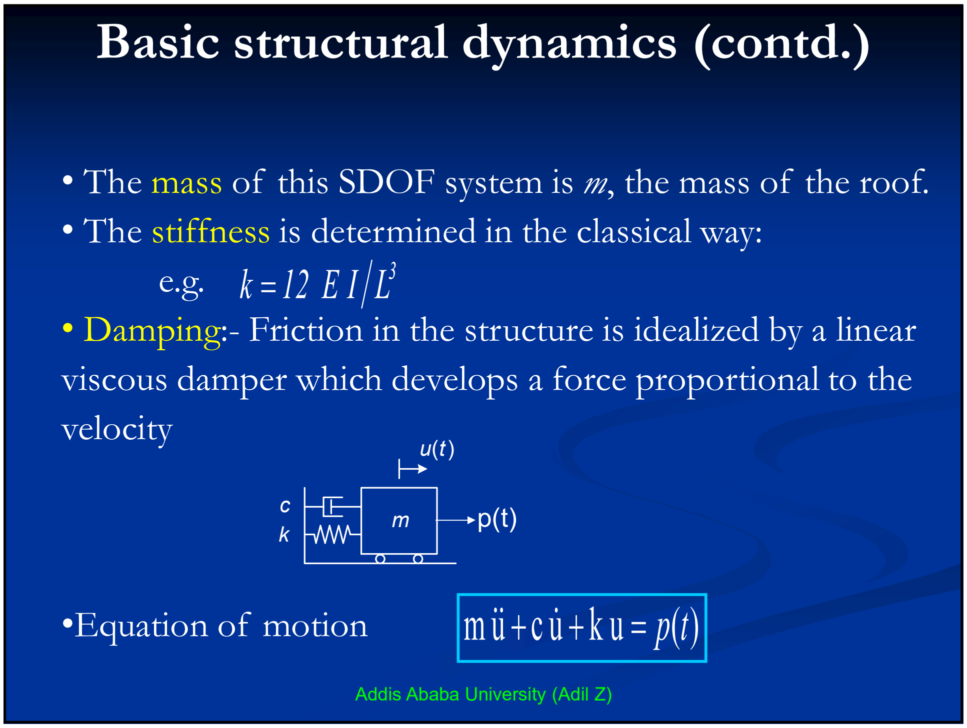
Basic structural dynamics (contd.)

- Portal frame structures may be idealized as a SDOF system

In statics, this frame has 6 active degrees of freedom.

By neglecting the axial deformations, 3 DOF disappear.

Only one DOF is left if the frame is consisting of an heavy roof supported by light columns.



Basic structural dynamics (contd.)

- The mass of this SDOF system is m, the mass of the roof.
- The stiffness is determined in the classical way:

  e.g. $k = 12\,EI/L^3$
- Damping:- Friction in the structure is idealized by a linear viscous damper which develops a force proportional to the velocity

•Equation of motion $\boxed{m\ddot{u} + c\dot{u} + ku = p(t)}$

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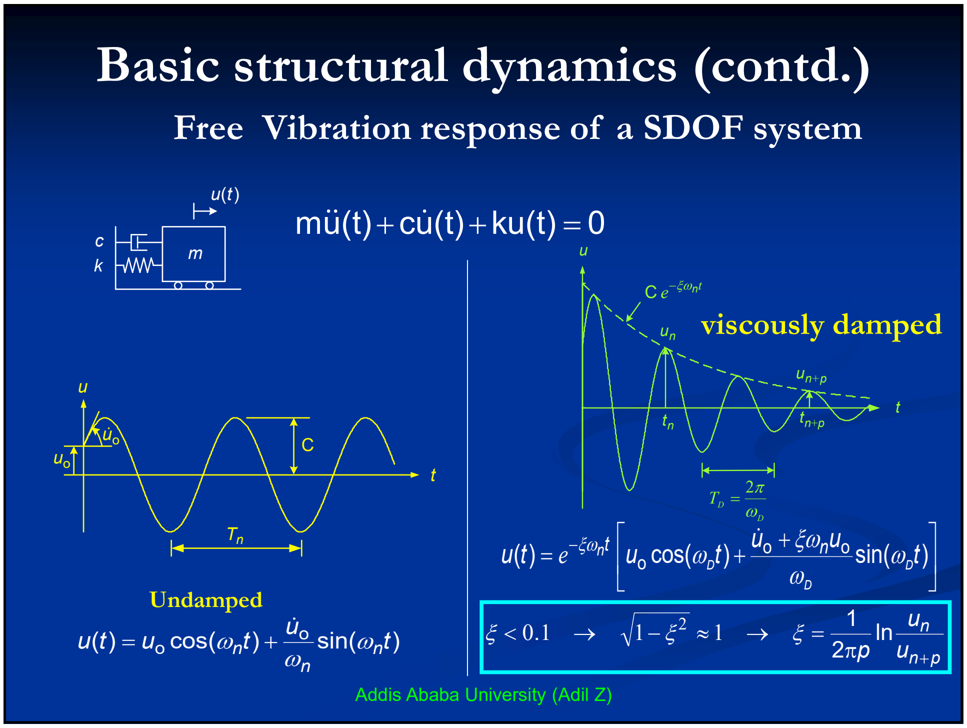

Basic structural dynamics (contd.)

Free Vibration response of a SDOF system

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = 0$$

viscously damped

$$u(t) = e^{-\xi\omega_n t}\left[u_o\cos(\omega_D t) + \frac{\dot{u}_o + \xi\omega_n u_o}{\omega_D}\sin(\omega_D t)\right]$$

$$\boxed{\xi < 0.1 \quad \rightarrow \quad \sqrt{1-\xi^2} \approx 1 \quad \rightarrow \quad \xi = \frac{1}{2\pi p}\ln\frac{u_n}{u_{n+p}}}$$

Undamped

$$u(t) = u_o\cos(\omega_n t) + \frac{\dot{u}_o}{\omega_n}\sin(\omega_n t)$$

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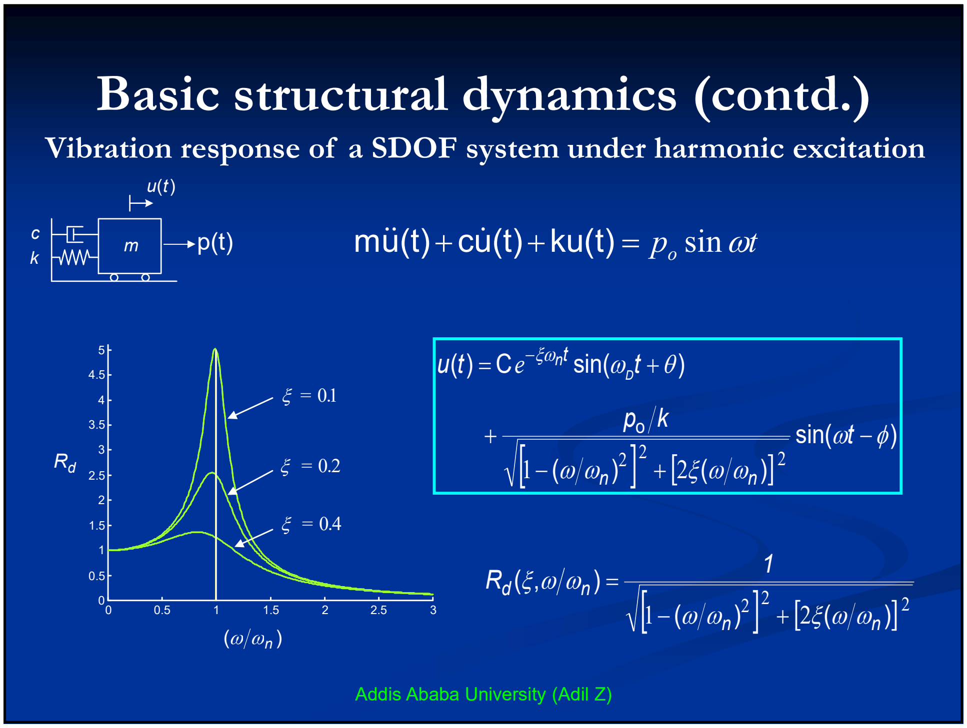


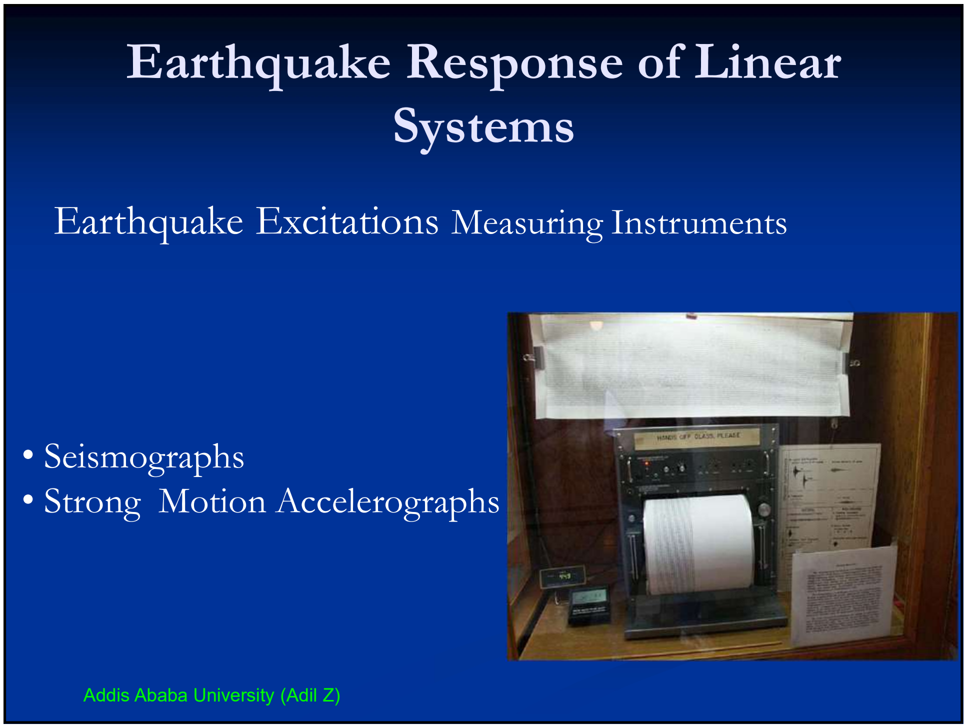
Basic structural dynamics (contd.)

Vibration response of a SDOF system under harmonic excitation

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=p_o\sin\omega t$$

$$\boxed{u(t)=C\,e^{-\xi\omega_n t}\sin(\omega_D t+\theta)+\frac{p_o/k}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2+\left[2\xi(\omega/\omega_n)\right]^2}}\sin(\omega t-\phi)}$$

$$R_d(\xi,\omega/\omega_n)=\frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2+\left[2\xi(\omega/\omega_n)\right]^2}}$$

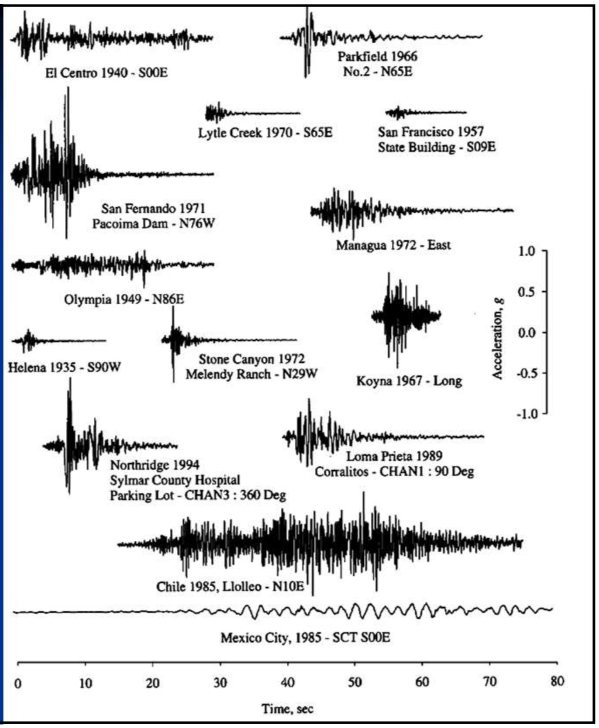


Earthquake Response of Linear Systems

Earthquake Excitations Measuring Instruments

- Seismographs
- Strong Motion Accelerographs

Ground Motion Records During Several Earthquakes



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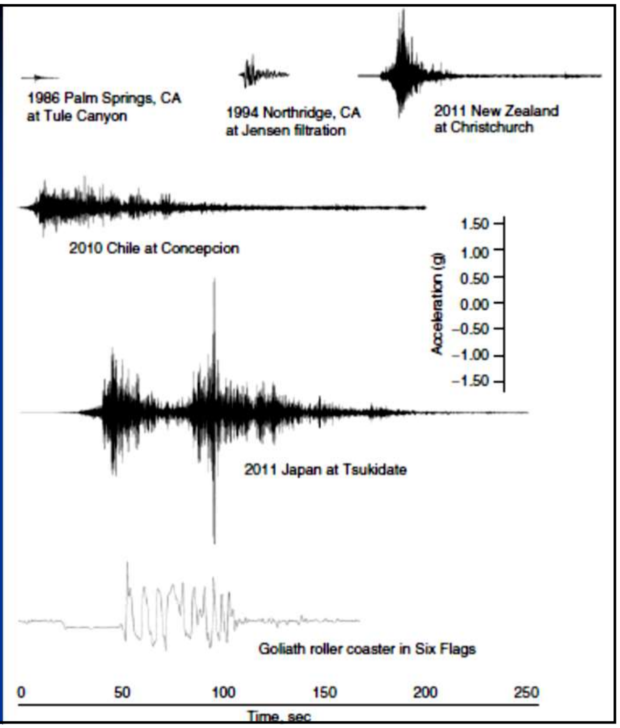
Ground Motion Records During Several Earthquakes



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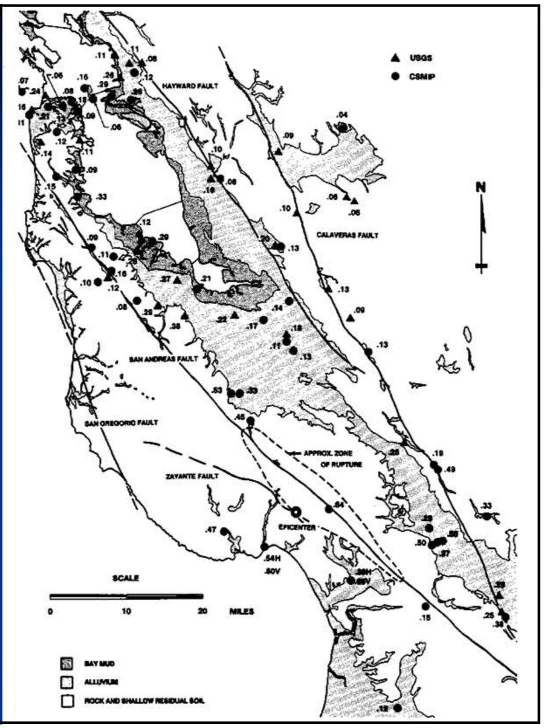
Peak Ground Acceleration Records During Loma Prieta Earthquakes (1989)



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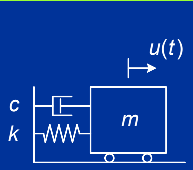
Introduction to Response Spectra

As an introduction to earthquake analysis, the concept of response spectra is first presented.

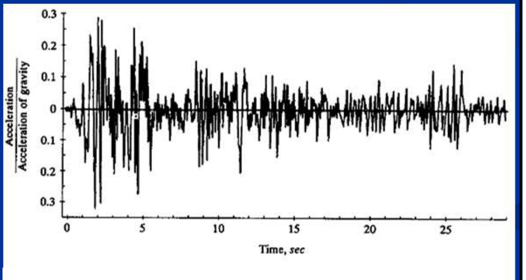
An idealized SDOF system is subjected to a ground motion $\ddot{u}_g(t)$. The deformation response $u(t)$ is to be calculated.

$$m(\ddot{u}_g + \ddot{u}) + c\,\dot{u} + k\,u = 0$$

$$\ddot{u} + 2\xi\omega_n\,\dot{u} + \omega_n^2 u = -\ddot{u}_g(t)$$



As an example , consider the ground acceleration registered during Elcentro EQ of 1940.



Accelerogram from El Centro earthquake, May 18, 1940 (*NS* component).

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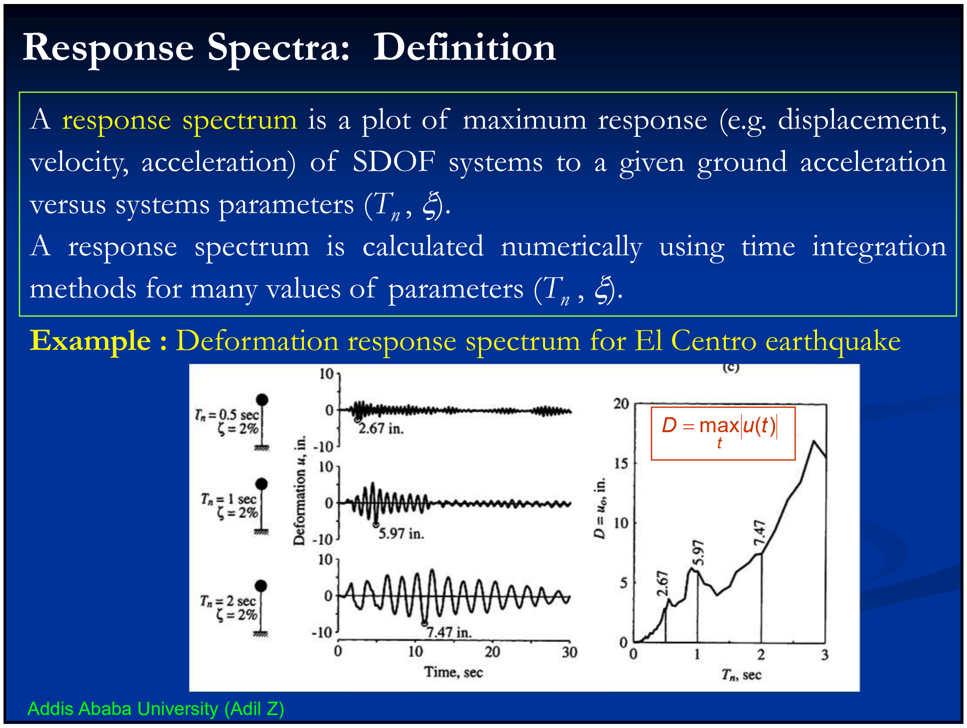
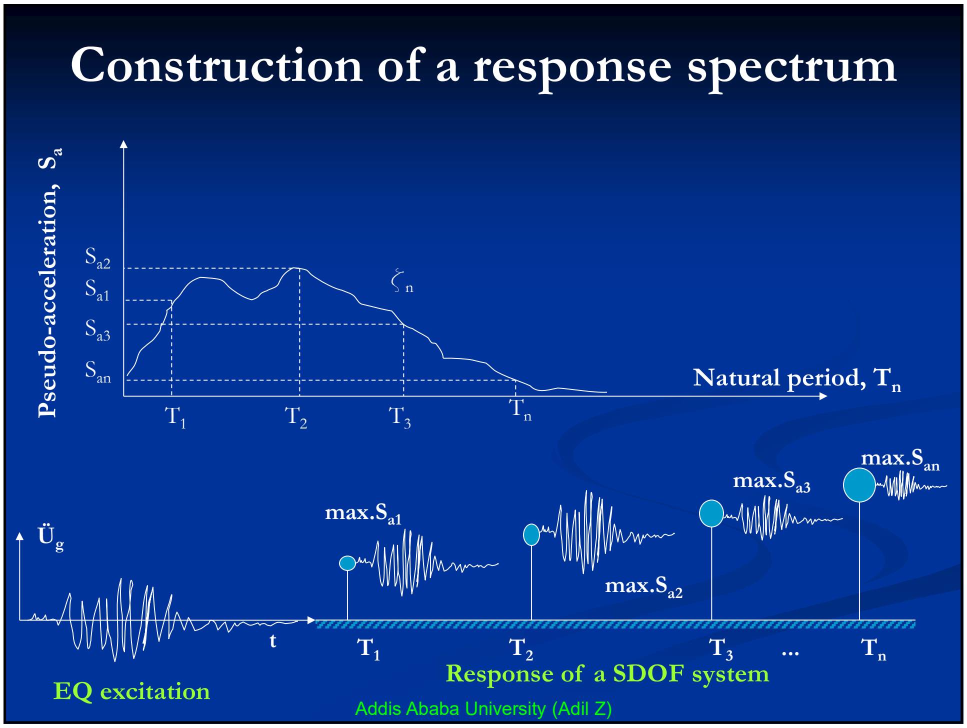


Response Spectra: Definition

A response spectrum is a plot of maximum response (e.g. displacement, velocity, acceleration) of SDOF systems to a given ground acceleration versus systems parameters (T_n, ξ).

A response spectrum is calculated numerically using time integration methods for many values of parameters (T_n , ξ).

Example : Deformation response spectrum for El Centro earthquake



Construction of a response spectrum

EQ excitation

Response of a SDOF system

Construction of a response spectrum

1. Numerically define ground acceleration $\ddot{u}_g(t)$
2. Select natural vibration period T_n and ζ of SDOF
3. Compute deformation response $u(t)$
4. Determine u_0 peak value of $u(t)$
5. The spectral ordinates are $D=u_0$, $V=\omega D$ and $A=\omega^2 D$
6. Repeat step 2 to 5 for a range of T_n and ζ
7. present steps 2 to 6 graphically

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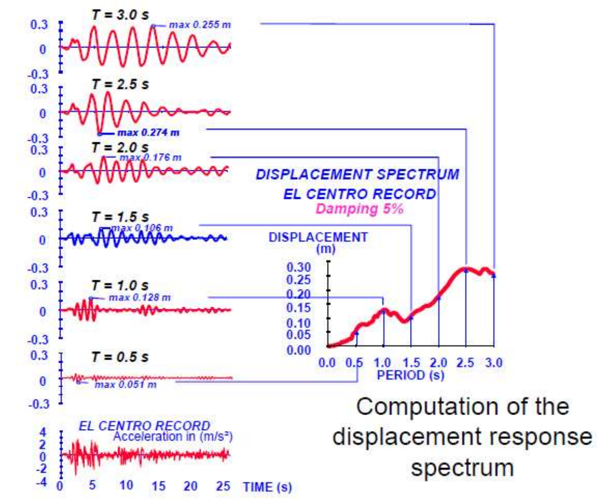
Displacement Response Spectrum



Computation of the displacement response spectrum

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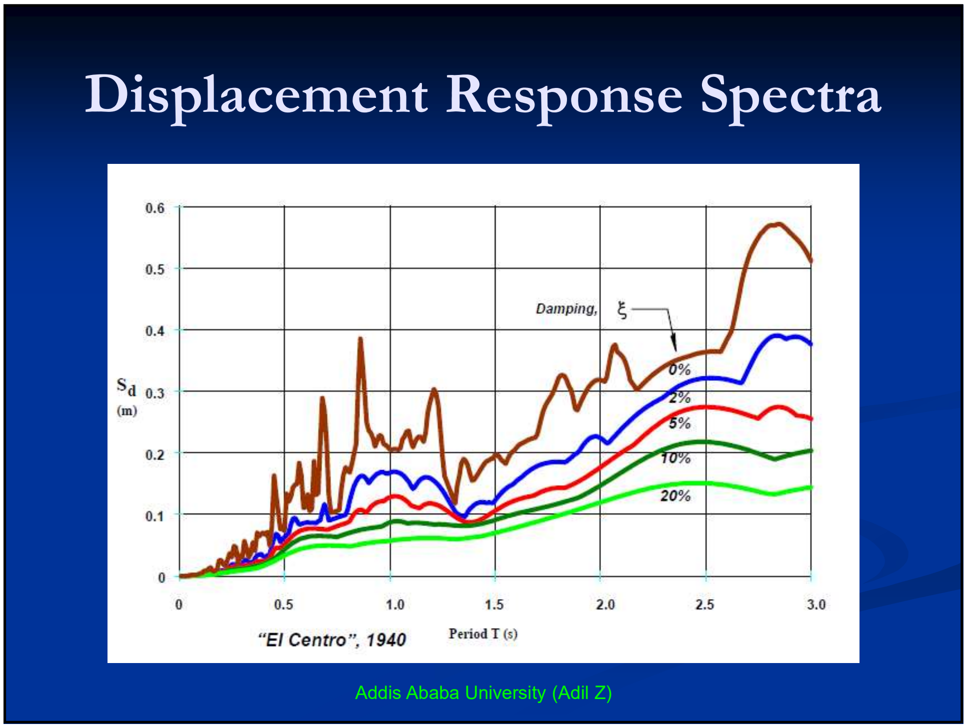

Displacement Response Spectra

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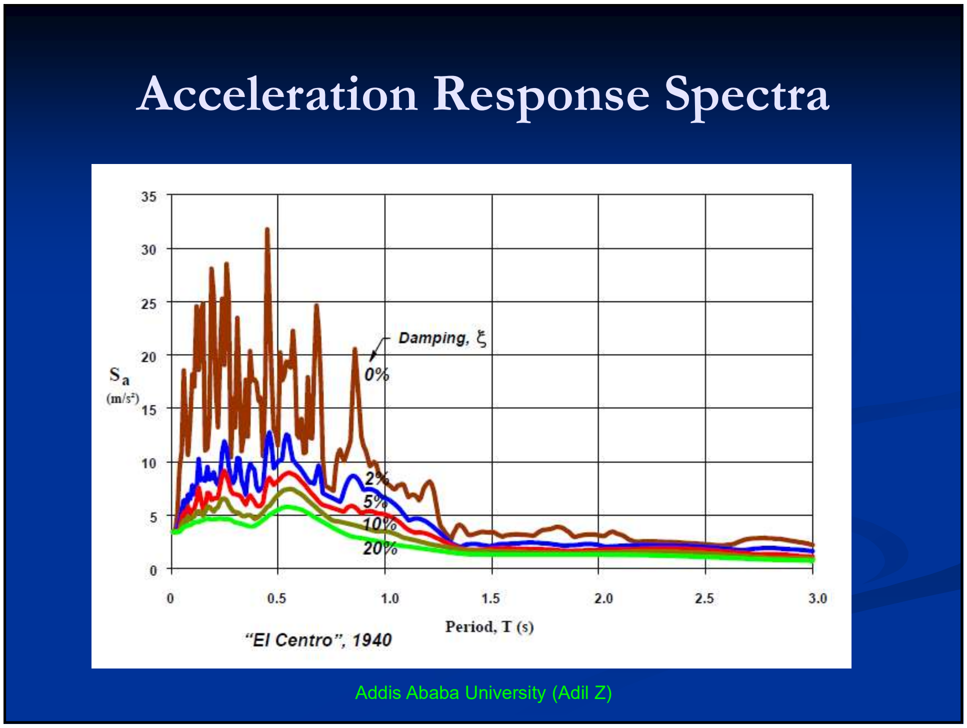

Acceleration Response Spectra

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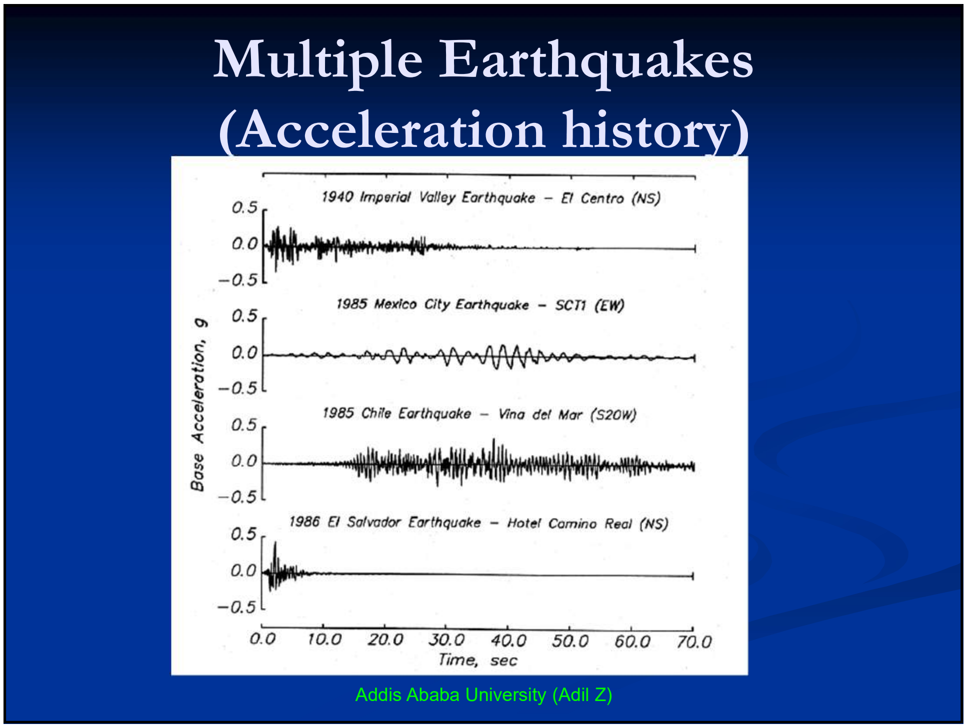

Multiple Earthquakes (Acceleration history)

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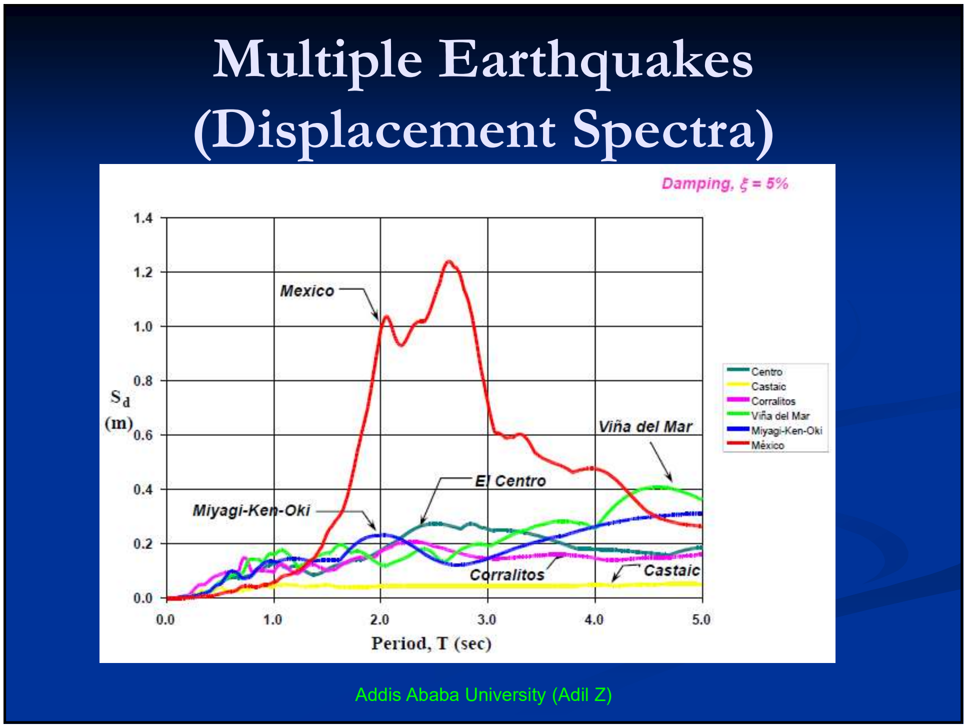

Multiple Earthquakes (Displacement Spectra)

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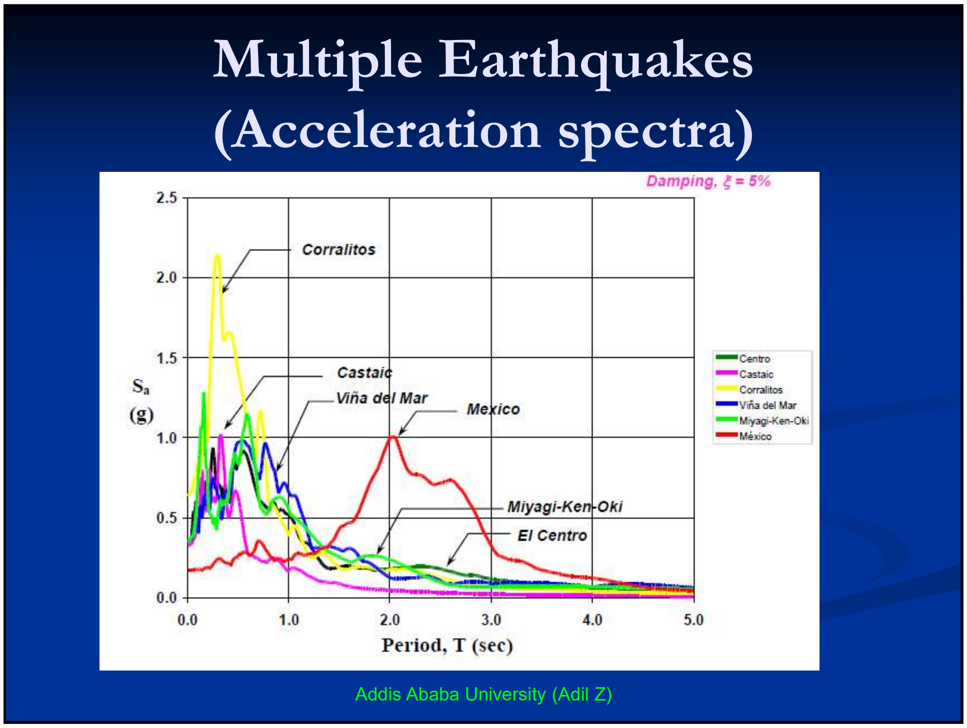

Multiple Earthquakes (Acceleration spectra)

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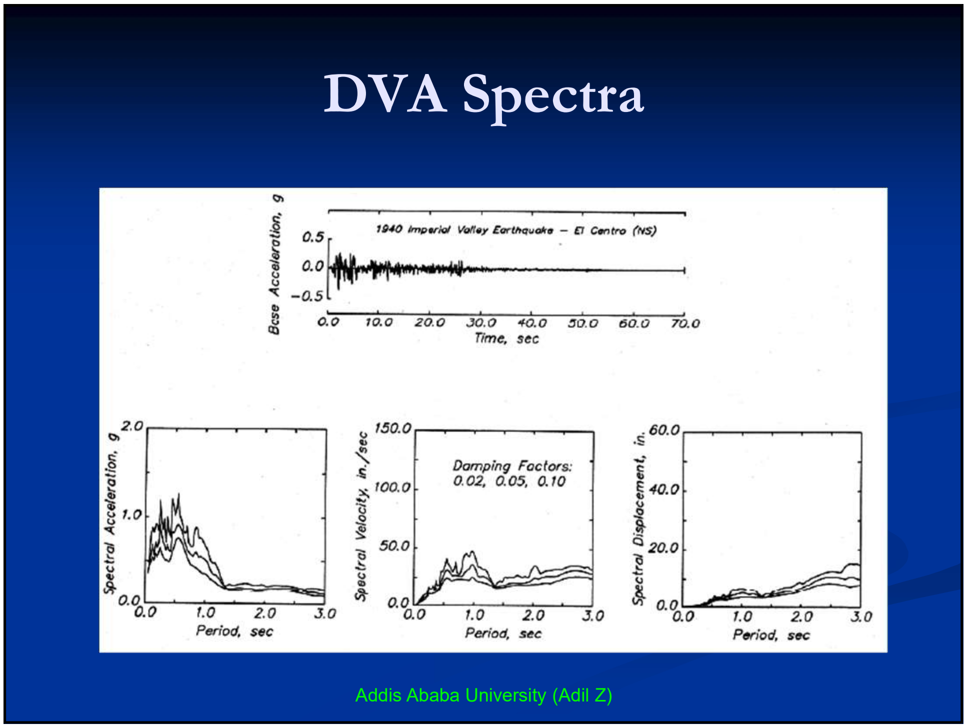

DVA Spectra

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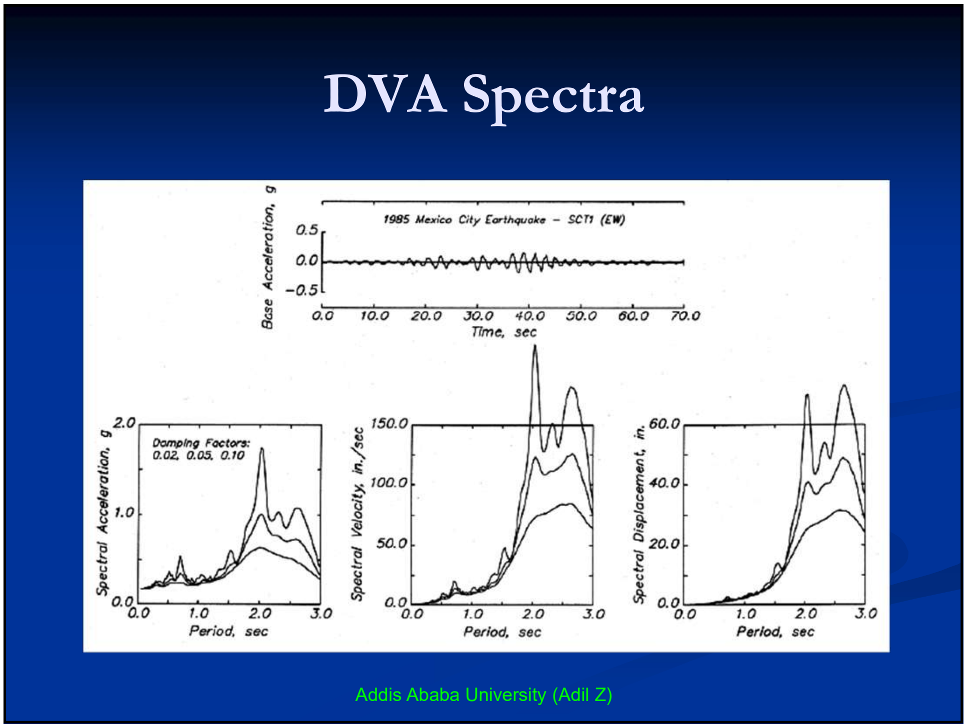
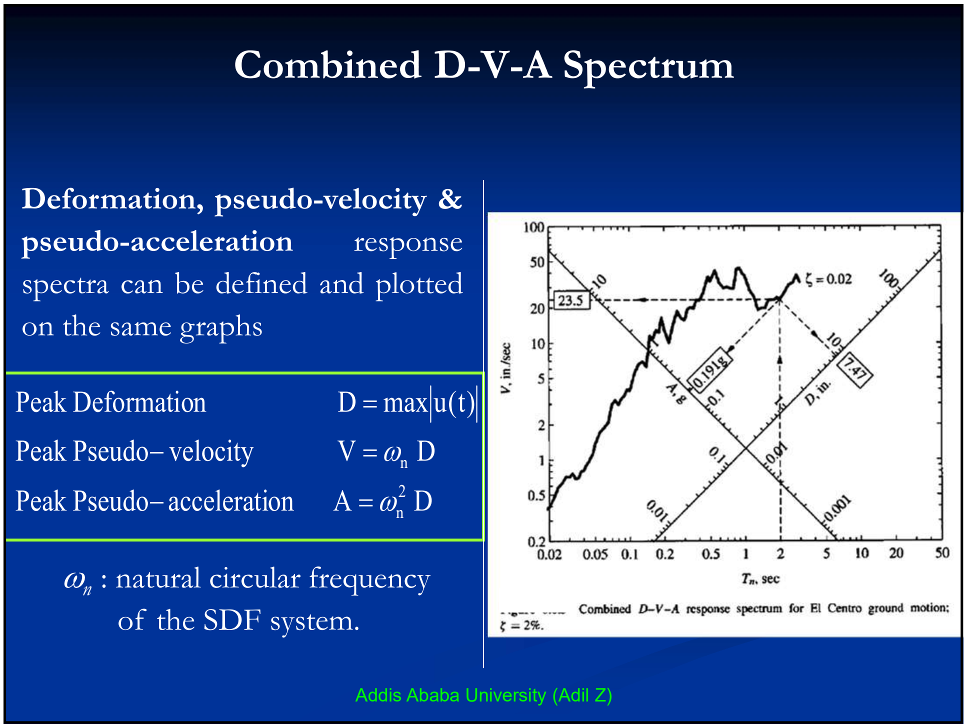


DVA Spectra



Combined D-V-A Spectrum

Deformation, pseudo-velocity & pseudo-acceleration response spectra can be defined and plotted on the same graphs

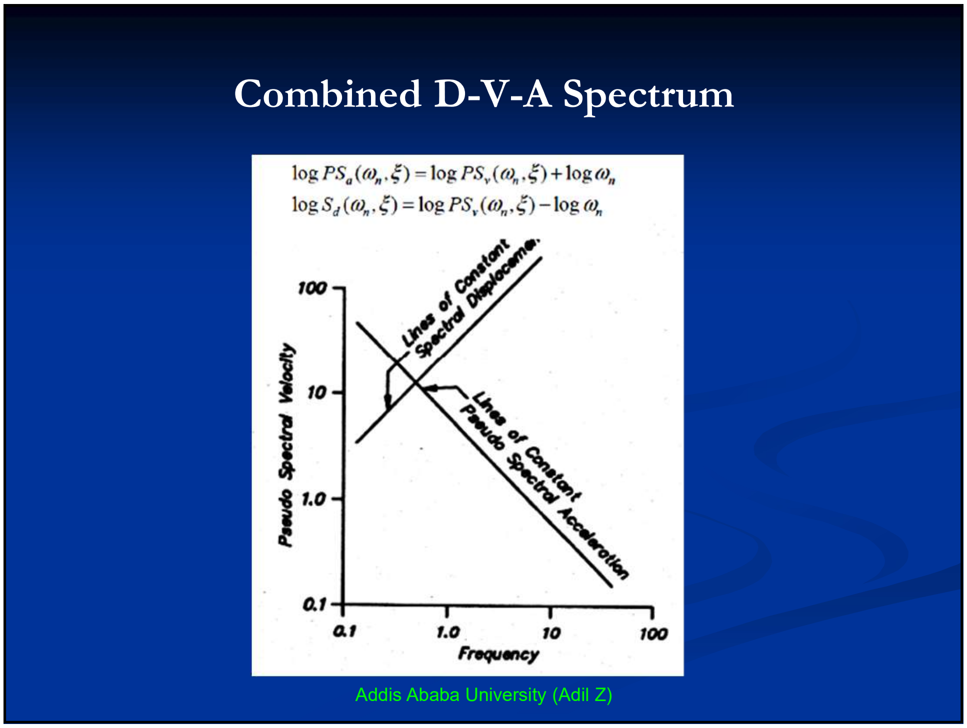
Peak Deformation $D = \max|u(t)|$

Peak Pseudo−velocity $V = \omega_n D$

Peak Pseudo−acceleration $A = \omega_n^2 D$

ω_n : natural circular frequency of the SDF system.

Combined D–V–A response spectrum for El Centro ground motion; $\zeta = 2\%$.



Combined D-V-A Spectrum

$$\log PS_a(\omega_n,\xi) = \log PS_v(\omega_n,\xi) + \log \omega_n$$

$$\log S_d(\omega_n,\xi) = \log PS_v(\omega_n,\xi) - \log \omega_n$$

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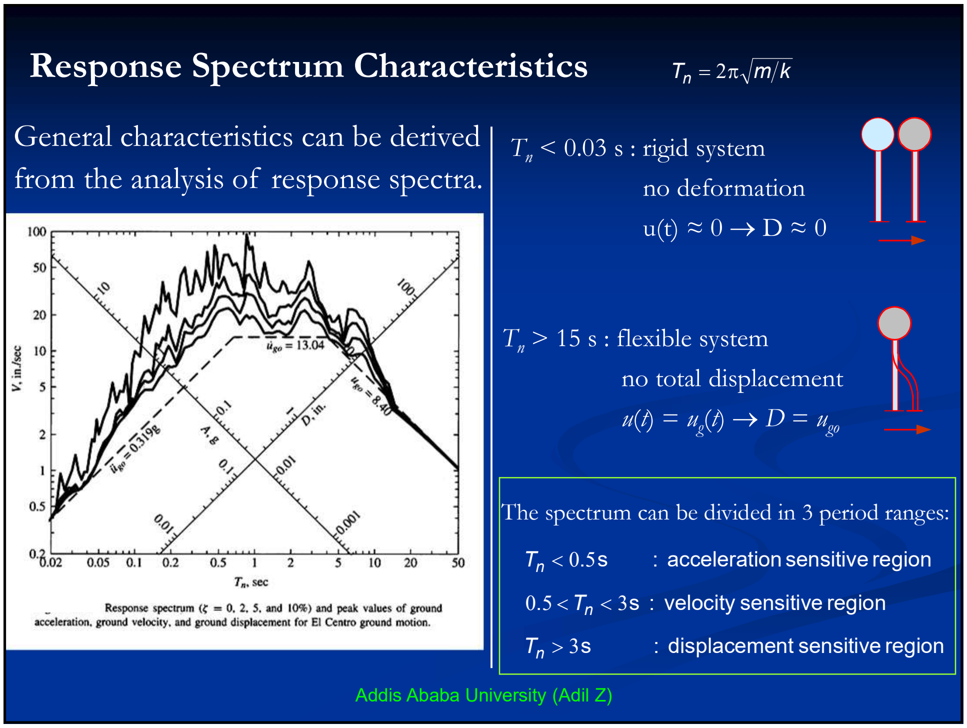

Response Spectrum Characteristics

$T_n = 2\pi\sqrt{m/k}$

General characteristics can be derived from the analysis of response spectra.

Response spectrum (ζ = 0, 2, 5, and 10%) and peak values of ground acceleration, ground velocity, and ground displacement for El Centro ground motion.

$T_n < 0.03$ s : rigid system
no deformation
$u(t) \approx 0 \rightarrow D \approx 0$

$T_n > 15$ s : flexible system
no total displacement
$u(t) = u_g(t) \rightarrow D = u_{go}$

The spectrum can be divided in 3 period ranges:

$T_n < 0.5$s : acceleration sensitive region

$0.5 < T_n < 3$s : velocity sensitive region

$T_n > 3$s : displacement sensitive region

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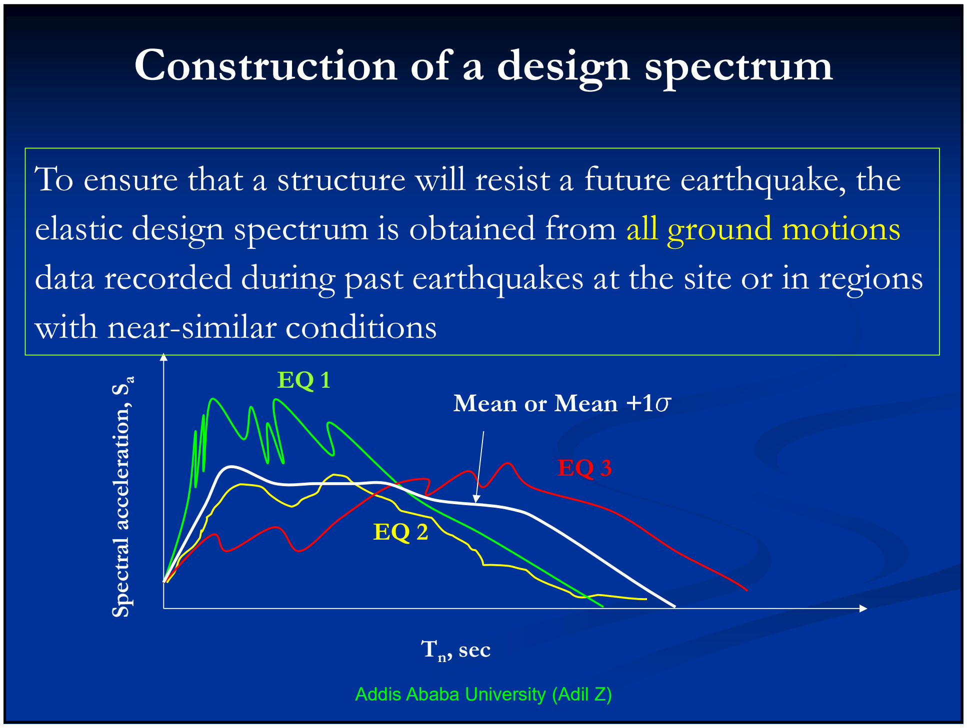
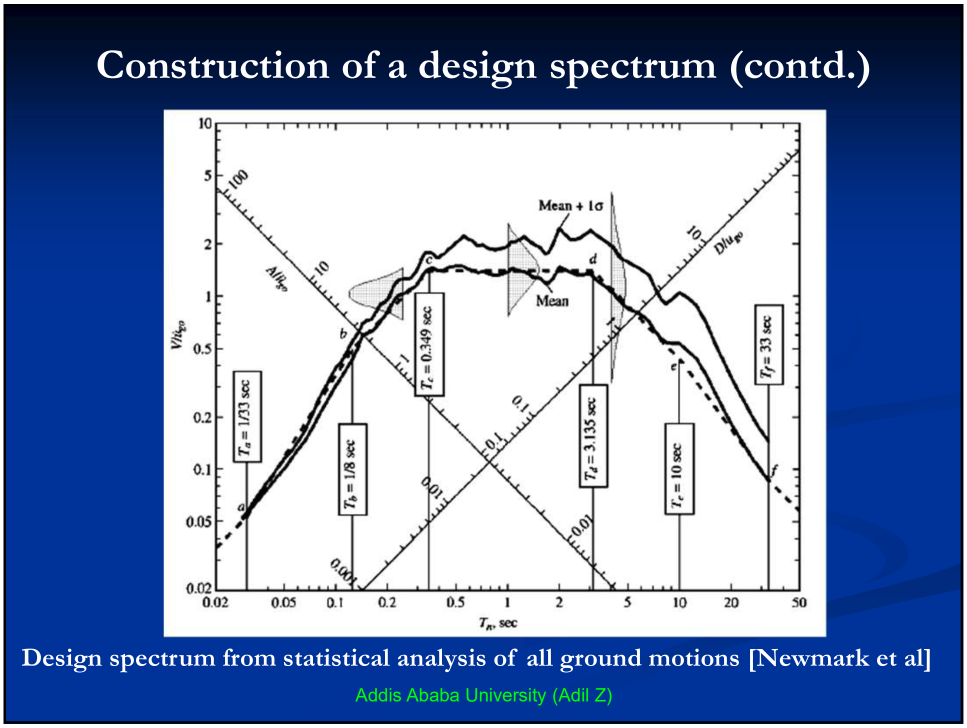


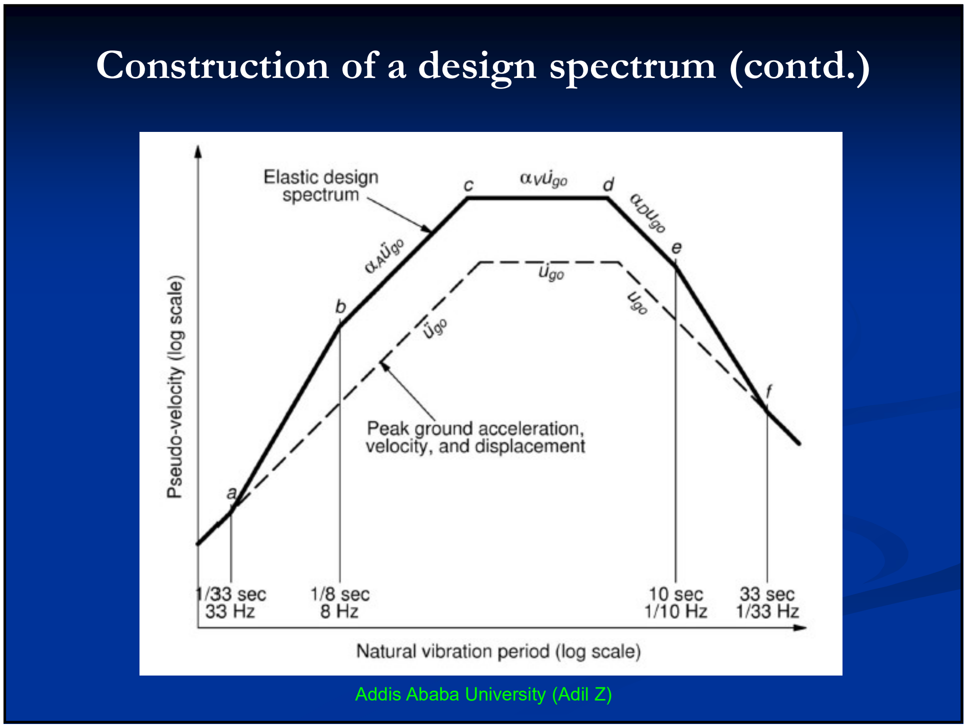
Construction of a design spectrum

To ensure that a structure will resist a future earthquake, the elastic design spectrum is obtained from all ground motions data recorded during past earthquakes at the site or in regions with near-similar conditions

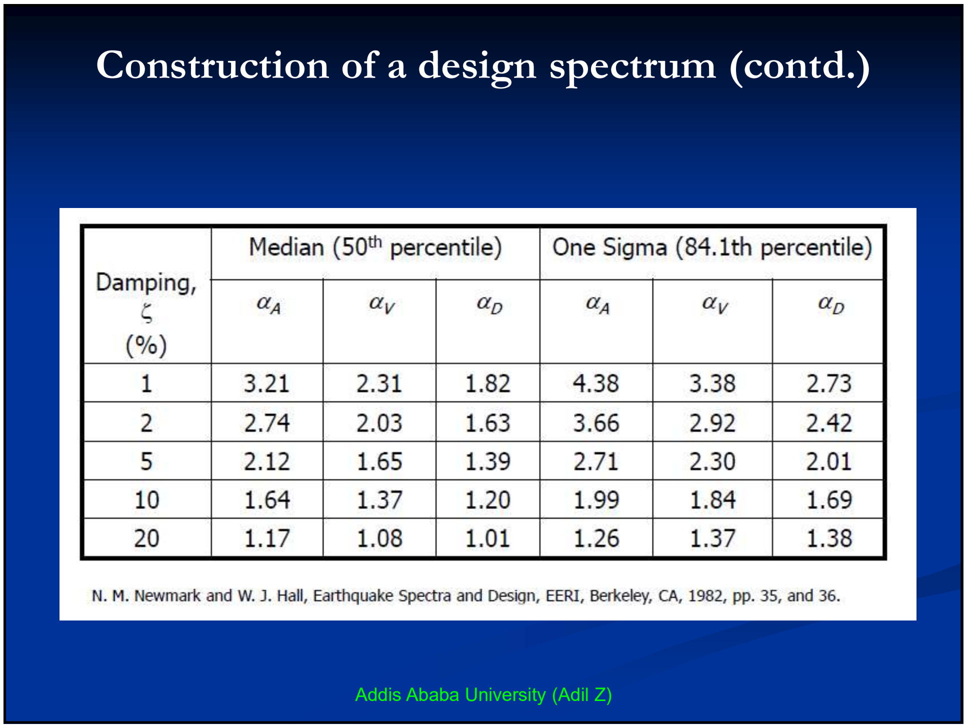


Construction of a design spectrum (contd.)

Design spectrum from statistical analysis of all ground motions [Newmark et al]



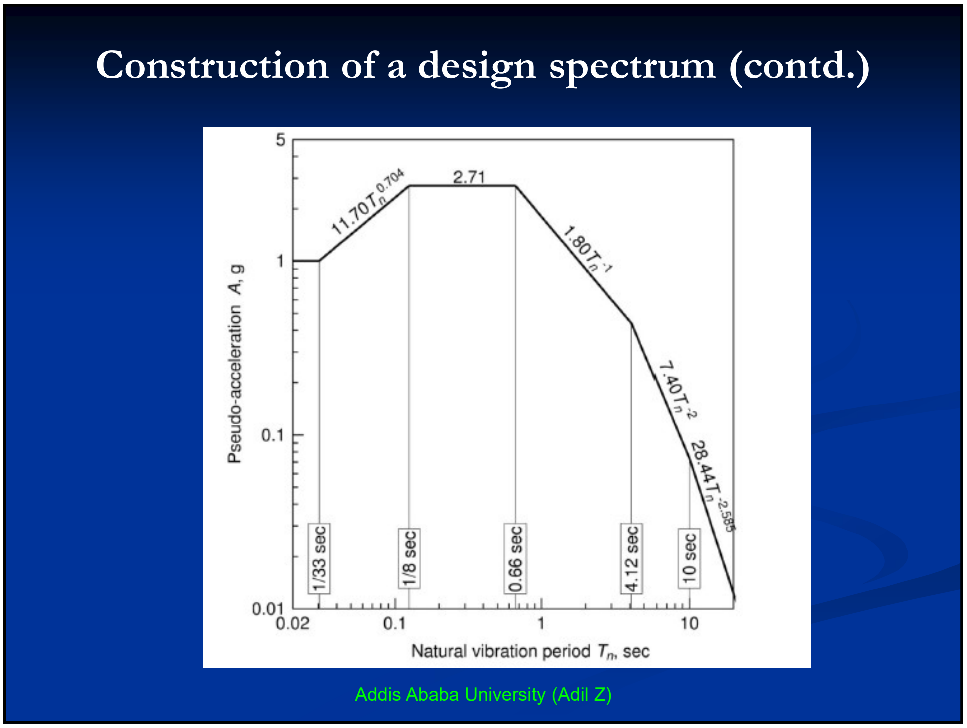
Construction of a design spectrum (contd.)



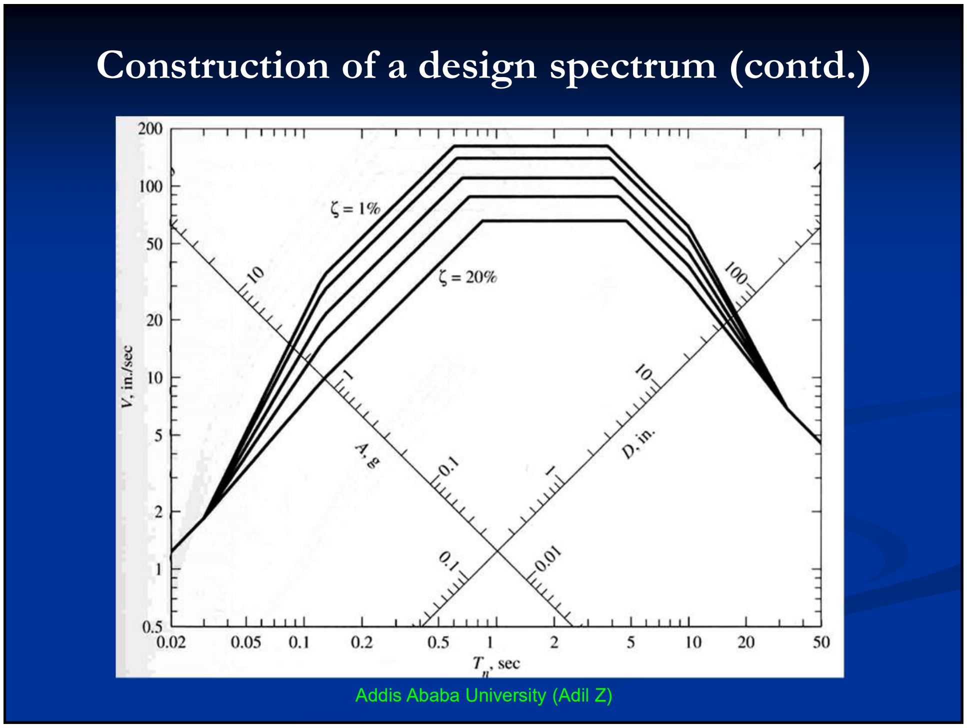
Construction of a design spectrum (contd.)

Damping, ζ (%)	Median (50^{th} percentile)			One Sigma (84.1th percentile)		
	α_A	α_V	α_D	α_A	α_V	α_D
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.39	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

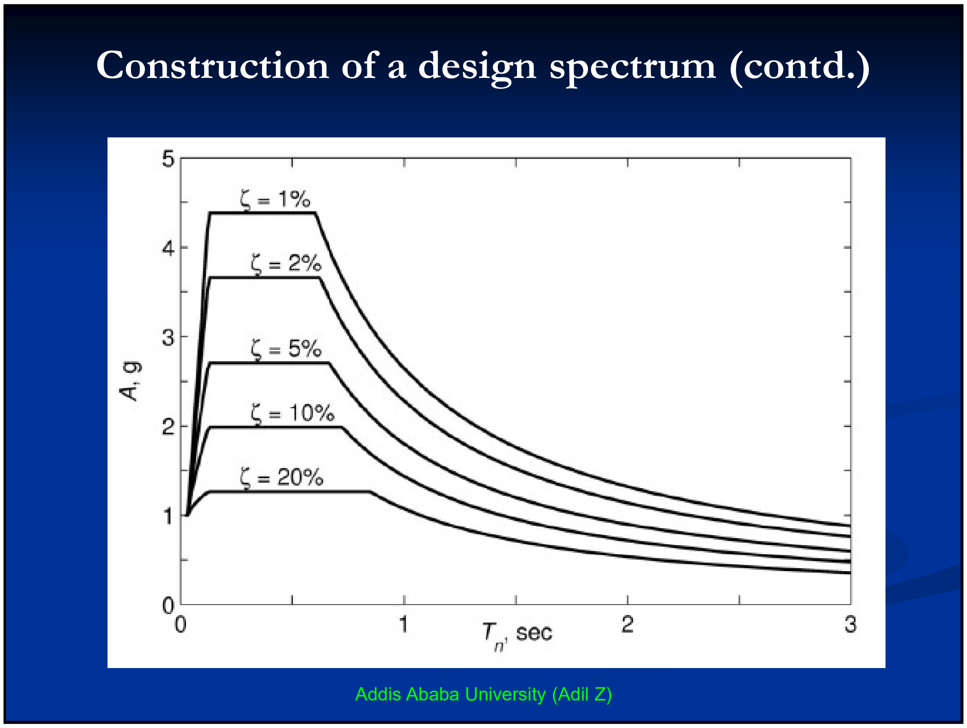
N. M. Newmark and W. J. Hall, Earthquake Spectra and Design, EERI, Berkeley, CA, 1982, pp. 35, and 36.



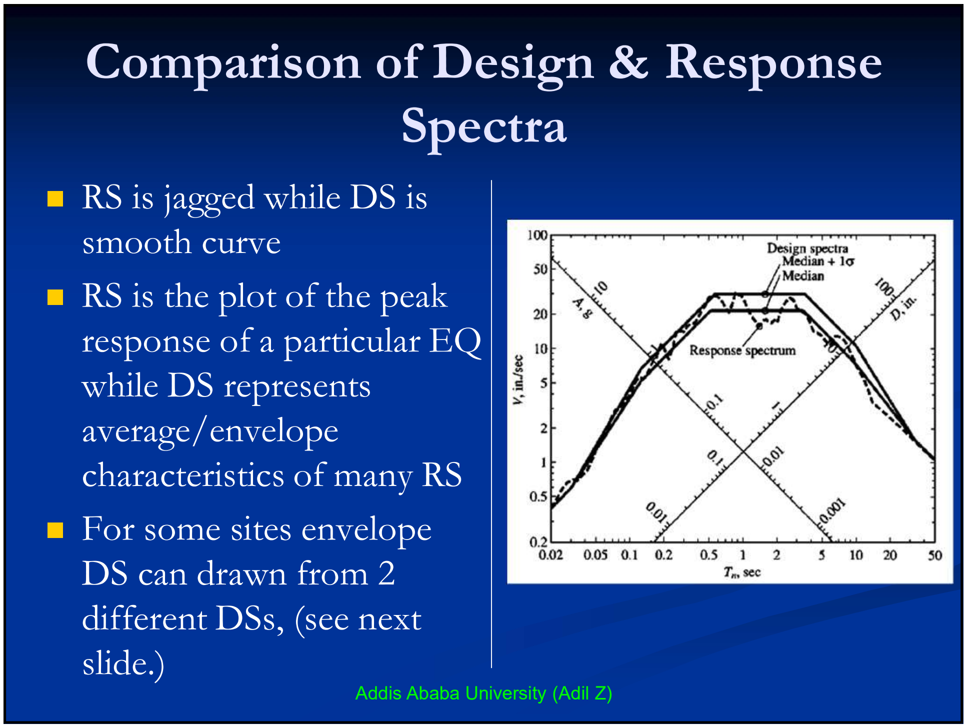
Construction of a design spectrum (contd.)



Construction of a design spectrum (contd.)

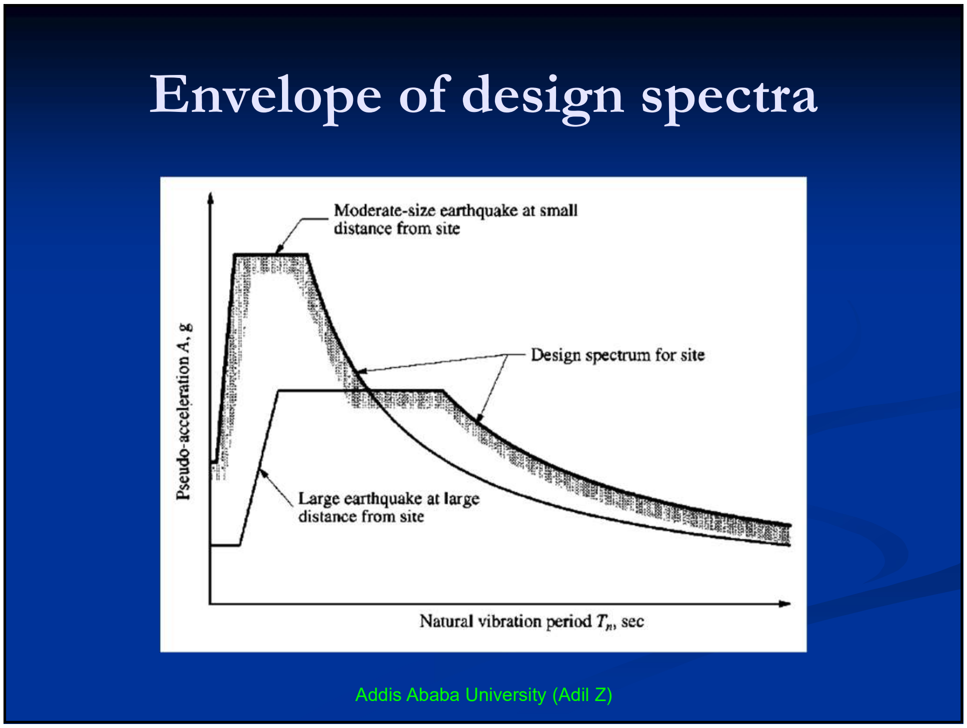


Construction of a design spectrum (contd.)



Comparison of Design & Response Spectra

- RS is jagged while DS is smooth curve
- RS is the plot of the peak response of a particular EQ while DS represents average/envelope characteristics of many RS
- For some sites envelope DS can drawn from 2 different DSs, (see next slide.)



Envelope of design spectra

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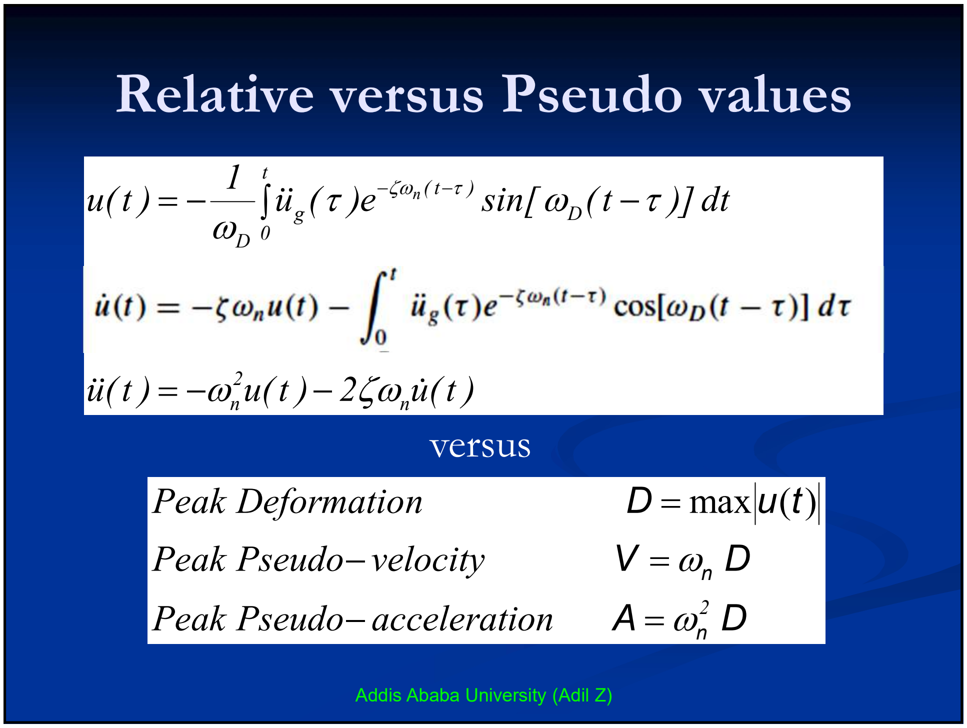

Relative versus Pseudo values

$$u(t) = -\frac{1}{\omega_D}\int_0^t \ddot{u}_g(\tau)e^{-\zeta\omega_n(t-\tau)}\sin[\omega_D(t-\tau)]\,dt$$

$$\dot{u}(t) = -\zeta\omega_n u(t) - \int_0^t \ddot{u}_g(\tau)e^{-\zeta\omega_n(t-\tau)}\cos[\omega_D(t-\tau)]\,d\tau$$

$$\ddot{u}(t) = -\omega_n^2 u(t) - 2\zeta\omega_n\dot{u}(t)$$

versus

Peak Deformation	$D = \max\lvert u(t)\rvert$
Peak Pseudo−velocity	$V = \omega_n D$
Peak Pseudo−acceleration	$A = \omega_n^2 D$

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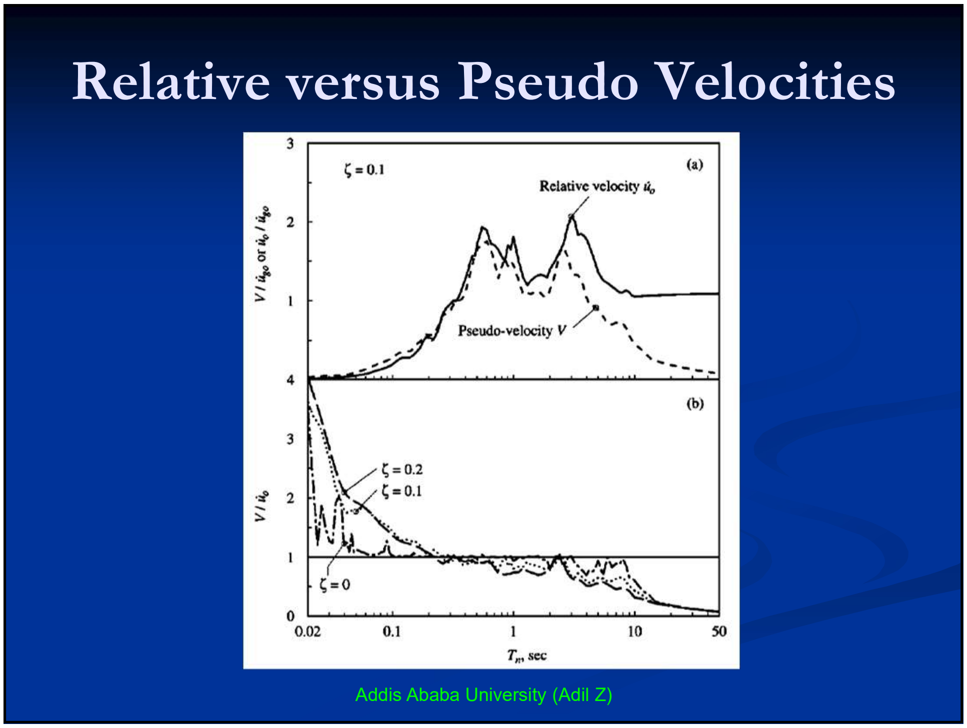

Relative versus Pseudo Velocities

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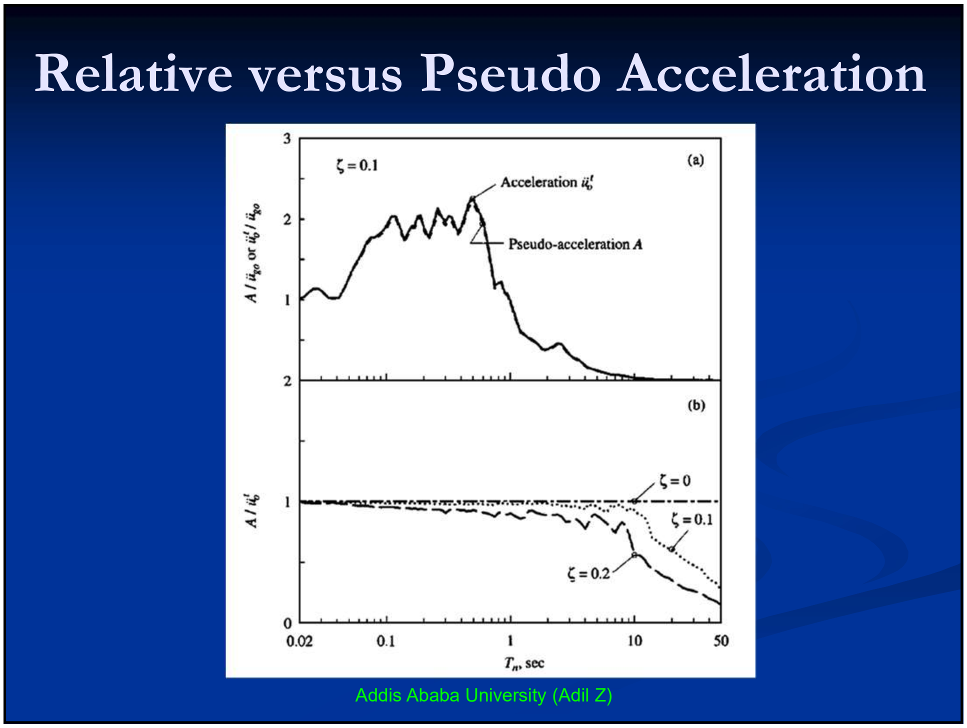

Relative versus Pseudo Acceleration

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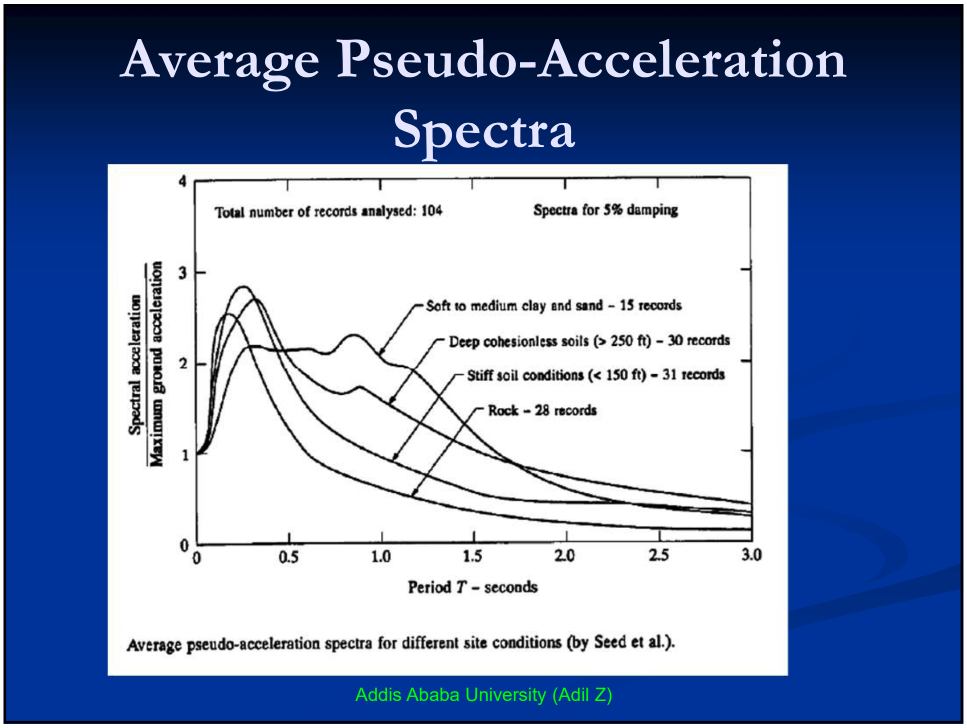

Average Pseudo-Acceleration Spectra

Average pseudo-acceleration spectra for different site conditions (by Seed et al.).

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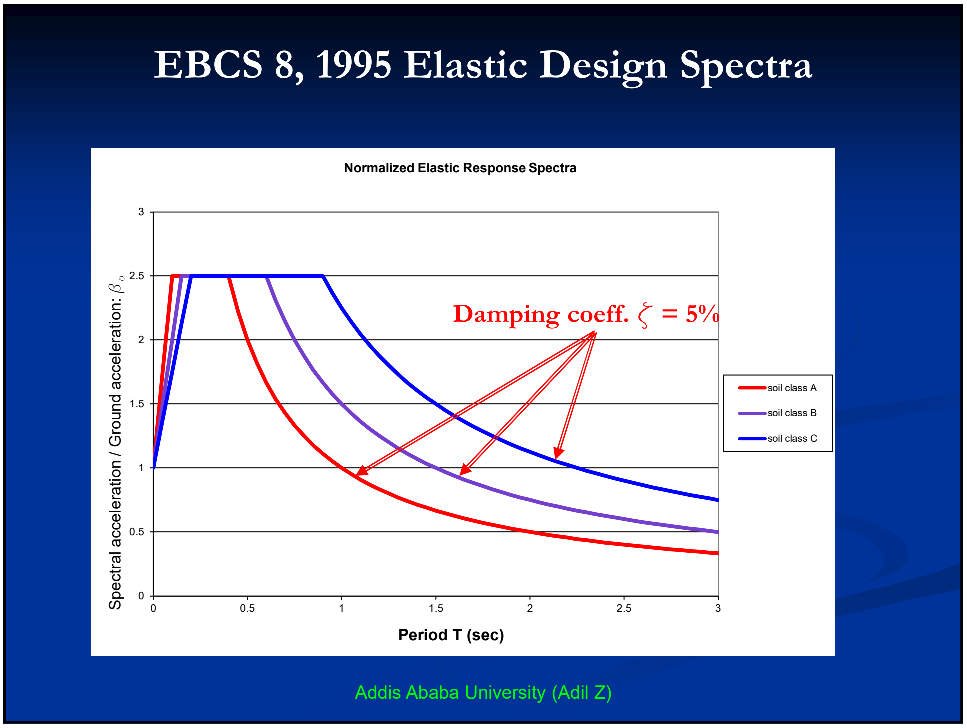

EBCS 8, 1995 Elastic Design Spectra

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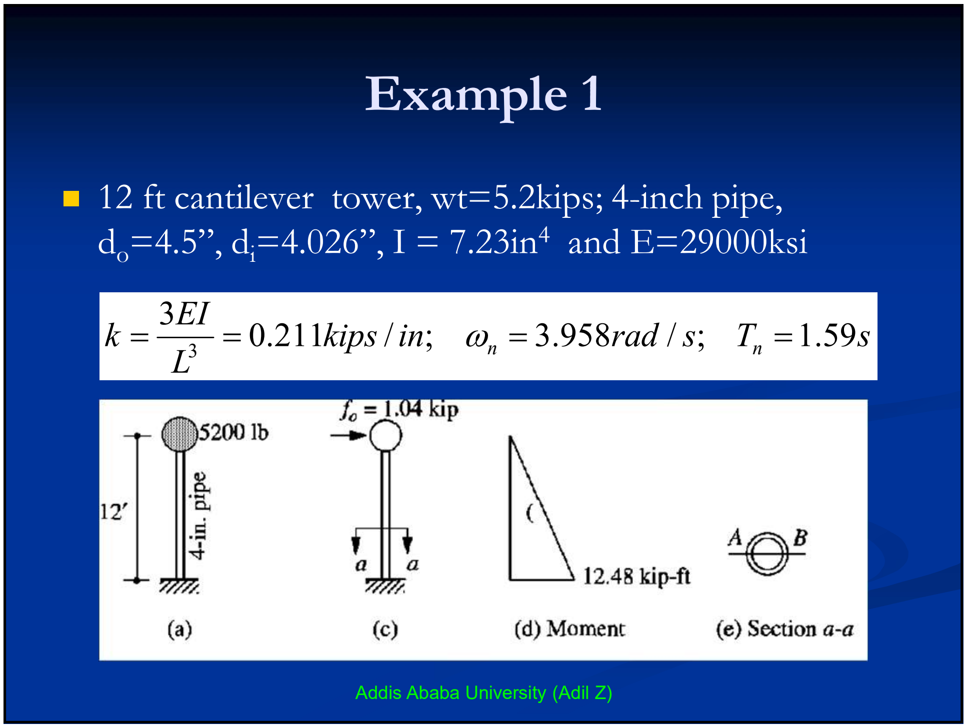

Example 1

- 12 ft cantilever tower, wt=5.2kips; 4-inch pipe, d_o=4.5”, d_i=4.026”, I = 7.23in^4 and E=29000ksi

$$k = \frac{3EI}{L^3} = 0.211 kips/in; \quad \omega_n = 3.958 rad/s; \quad T_n = 1.59 s$$

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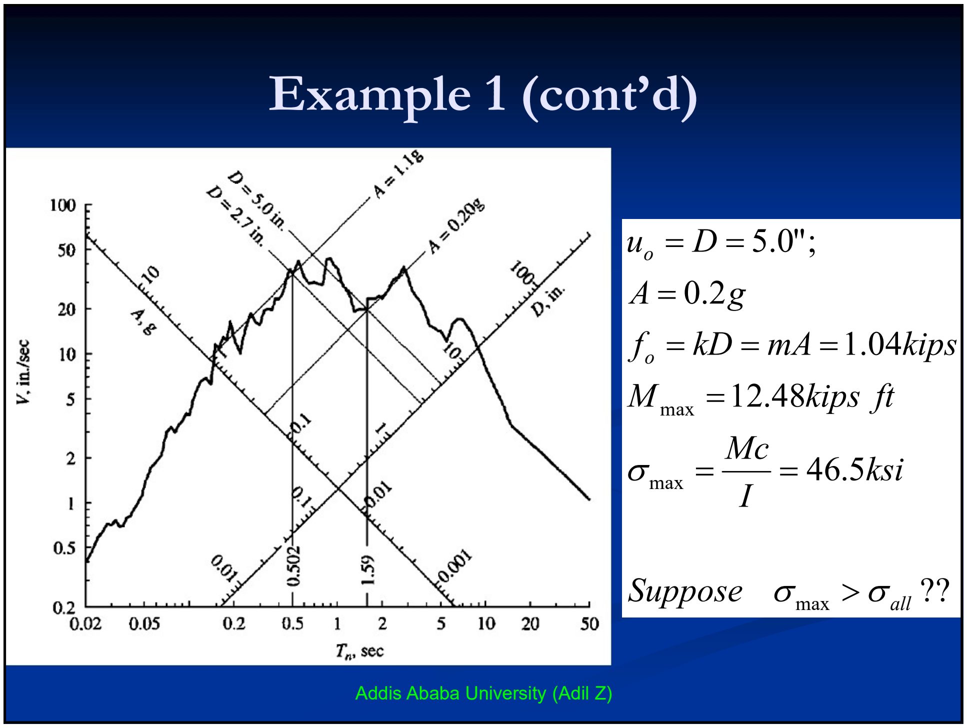

Example 1 (cont’d)

$$u_o = D = 5.0";$$

$$A = 0.2g$$

$$f_o = kD = mA = 1.04 kips$$

$$M_{\max} = 12.48 kips\ ft$$

$$\sigma_{\max} = \frac{Mc}{I} = 46.5 ksi$$

$$Suppose \quad \sigma_{\max} > \sigma_{all} ??$$

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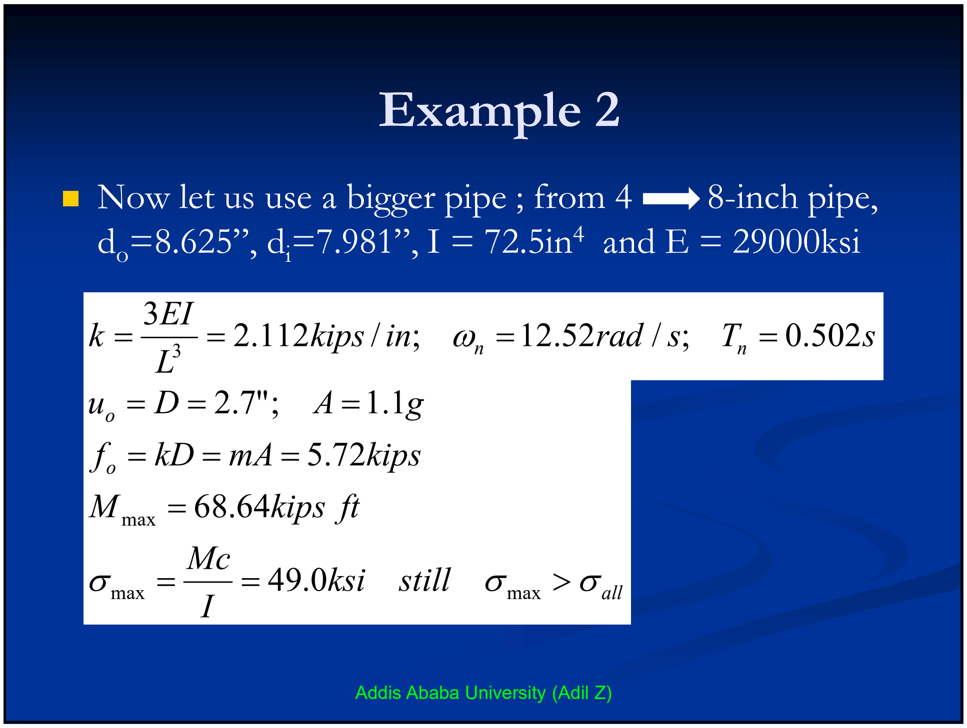

Example 2

- Now let us use a bigger pipe ; from 4 ⟶ 8-inch pipe, d_o=8.625”, d_i=7.981”, I = 72.5in^4 and E = 29000ksi

$$k = \frac{3EI}{L^3} = 2.112 kips/in; \quad \omega_n = 12.52 rad/s; \quad T_n = 0.502s$$

$$u_o = D = 2.7"; \quad A = 1.1g$$

$$f_o = kD = mA = 5.72 kips$$

$$M_{\max} = 68.64 kips\ ft$$

$$\sigma_{\max} = \frac{Mc}{I} = 49.0 ksi \quad still \quad \sigma_{\max} > \sigma_{all}$$