Earthquake Response of Inelastic SDOF Systems

Addis Ababa University
Adil Zekaria (Dr.-Ing.)

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Presentation outline

- Introduction
- Typical force-deformation (hysteresis) relation
- Ductility: definition and determination
- Relationship between strength and ductility
- Ductility demand and ductility capacity
- Normalized yield strength, yield reduction factor and ductility factor
- Effect of yielding
- Relative effect yielding and damping
- Construction of constant ductility response spectra
- Inelastic design spectra

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Introduction

- In Earthquake Engineering it is common practice to design against a large earthquake, that has a given mean period of return (say 500 years), quite larger than the expected life of the construction.
- Most buildings are designed, however, for base shear smaller than the elastic base shear associated with the strongest shaking that can occur at the site.
- It should not be surprising that buildings suffer damage during intense ground shaking.

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Introduction

The response of structures deforming into their inelastic range during intense ground shaking is of central importance in earthquake engineering

Six-story Imperial County Service Building after Imperial valley EQ of 1979 (Magnitude 6.5 & PGA near building of 0.23g). Columns shattered at and dropped 15 cm at ground level

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Introduction

Olive View Hospital, Psychiatric Unit, San Fernando, California. 1971 San Fernando Earthquake of Magnitude 6.4. This unit was a 2-story reinforced concrete building.

Lightweight concrete was used in the construction of this building. Note that the building collapsed completely at the first (soft) story and the second floor dropped to the ground after moving laterally about 2 meters

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Introduction

Olive View Hospital, San Fernando, California. 1971 San Fernando Earthquake of Magnitude 6.4.

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Introduction

- If you know the peak ground acceleration associated with the design earthquake, you can derive elastic design spectra and then, from the ordinates of the pseudo-acceleration spectrum, derive equivalent static forces to be used in the member design procedure.
- However, in the almost totality of cases the structural engineer does not design the structures considering the ordinates of the elastic spectrum of the maximum earthquake, the preferred procedure is to reduce these ordinates by factors that can be as high as 5 or more.
- This, of course, leads to a large reduction in the cost of the structure.

Introduction

- If we design for forces smaller than the forces likely to occur during a large earthquake, our structures will be damaged, or even destroyed.
- The reasoning behind such design procedure is that, for the unlikely occurrence of a large earthquake, a large damage in the construction is acceptable as far as no human lives are taken in a complete structural collapse and that, in the mean, the costs for repairing a damaged building are not disproportionate to its value.

Introduction

- To ascertain the amount of acceptable reduction of earthquake loads it is necessary to study:
  - the behavior of structural members and systems subjected to cyclic loading outside the elastic range, to understand the amount of plastic deformation and cumulated plastic deformation that can be sustained before collapse and
  - the global structural behavior for inelastic response, so that we can relate the reduction in design ordinates to the increase in members' plastic deformation.

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Typical force-deformation (hysteresis) relations
Structural steel, RC & Masonry

- Investigation of the cyclic behavior of structural members, sub-assemblages and scaled or real sized building model, either in labs or via numerical simulations, constitutes the bulk of Earthquake Engineering.
- What is important, at the moment, is the understanding of how different these behaviors can be, due to different materials or structural configurations, with instability playing an important role.
- *We will see 3 different diagrams, force vs deformation, for a clamped steel beam subjected to flexure, a reinforced concrete sub-assemblage an a masonry wall.*

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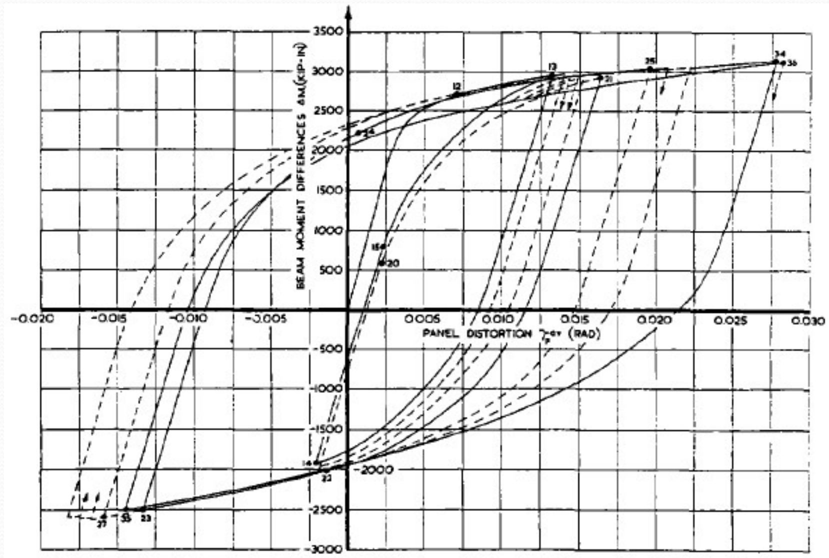
Typical force-deformation (hysteresis) relations

Structural steel



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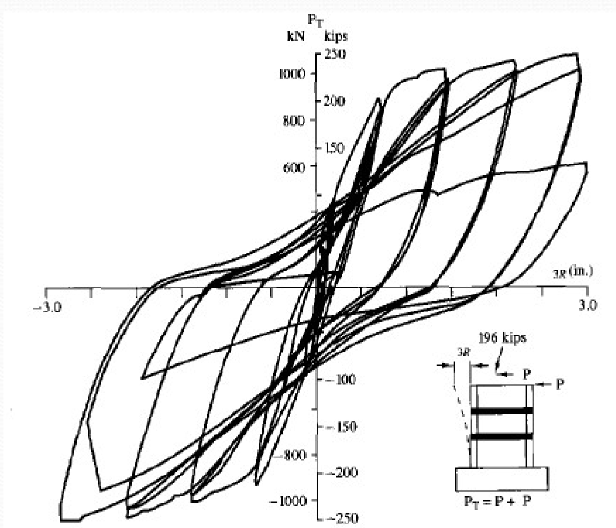
Typical force-deformation (hysteresis) relations

Reinforced Concrete



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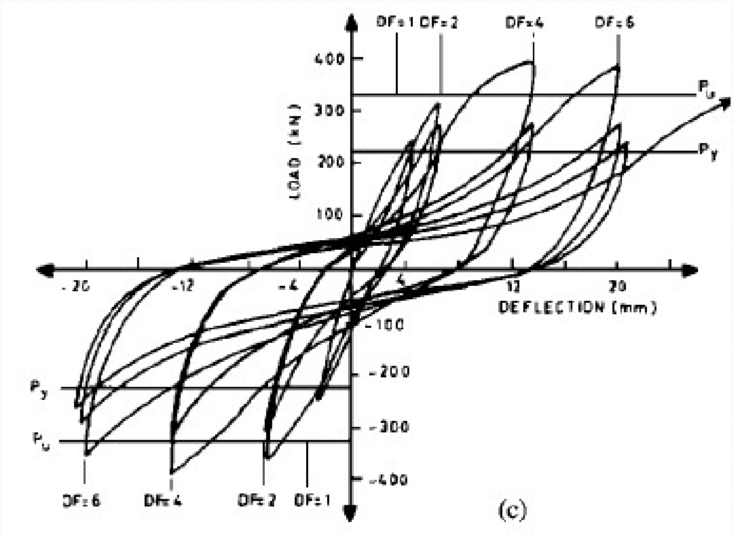
Typical force-deformation (hysteresis) relations

Masonry



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Ductility: Definition & Determination
Effect of Confinement

- The following slides present:
  - Different levels of ductility
  - Determination of ductility
  - Relationship between curvature ductility and member ductility
  - Relationship between confinement and ductility of a section
  - Different tie configuration for confinement
  - Relationship between strength and ductility
  - Definition of ductility demand and ductility capacity

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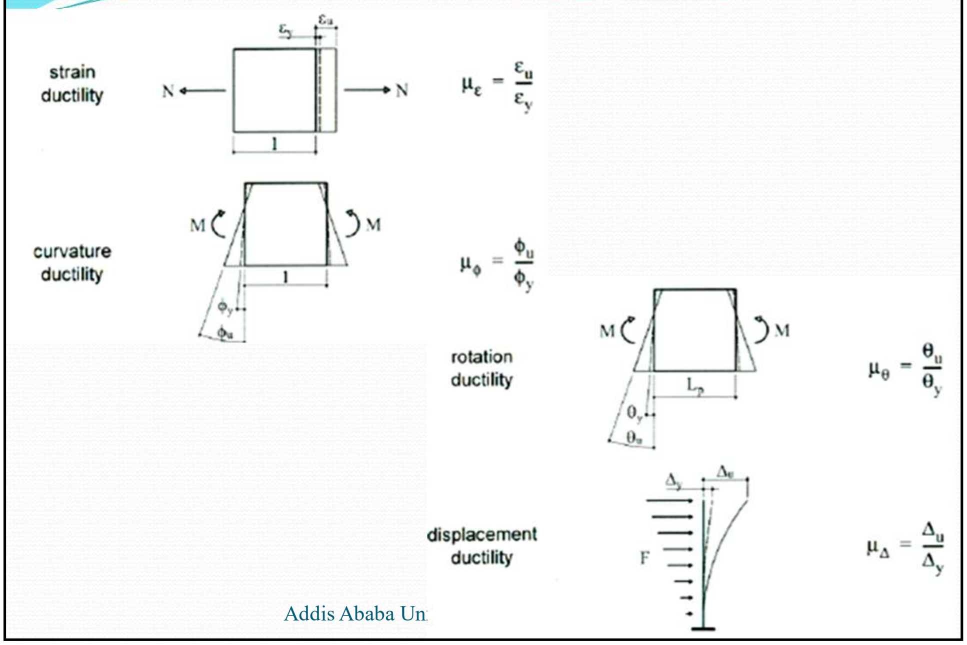
Ductility – Definition (Types of ductilities)



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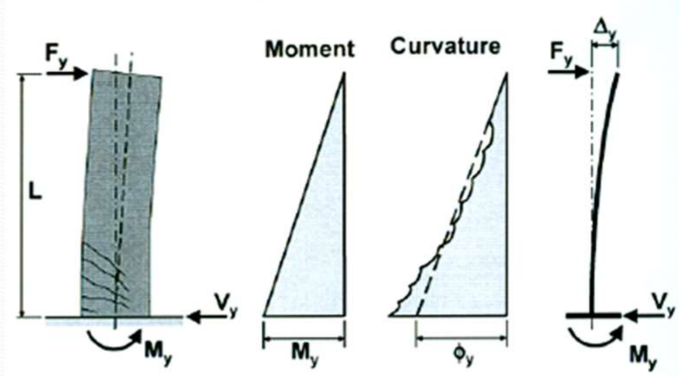
Ductility – Definition (cont'd)

- Elastic deformation Δ_y



The elastic deformation capacity of the cantilever is estimated as:

$$\Delta_y = \phi_y \frac{L^2}{3} \quad \cdots(2)$$

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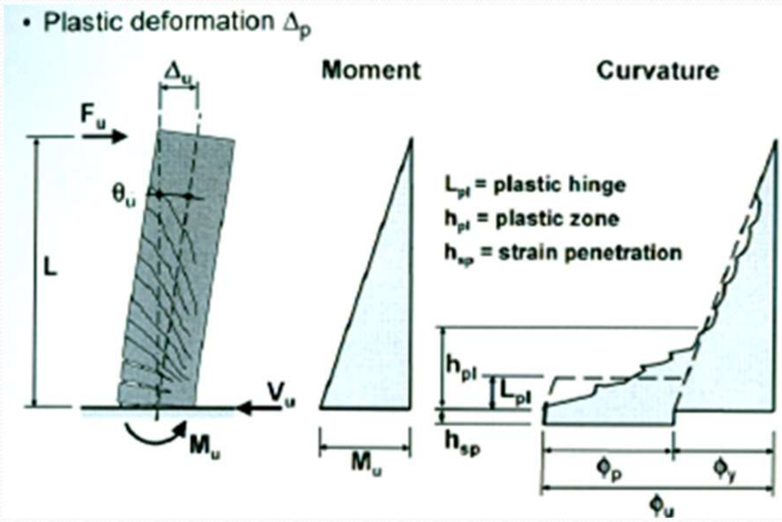
Ductility – Determination



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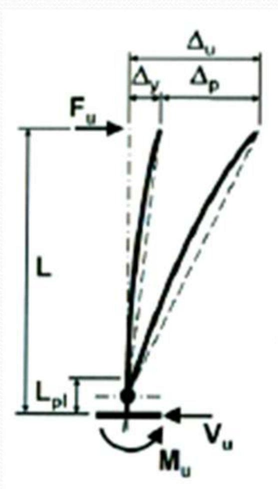
Ductility – Determination (cont'd)



The plastic hinge model neglects

- Tension shifts
- Shear deformation
- Anchorage deformation
- Strength degradation

- From the geometry

$$\Delta_p = (\phi_u - \phi_y) L_{pl} \left(L - \frac{L_{pl}}{2} \right) \quad \cdots (3)$$

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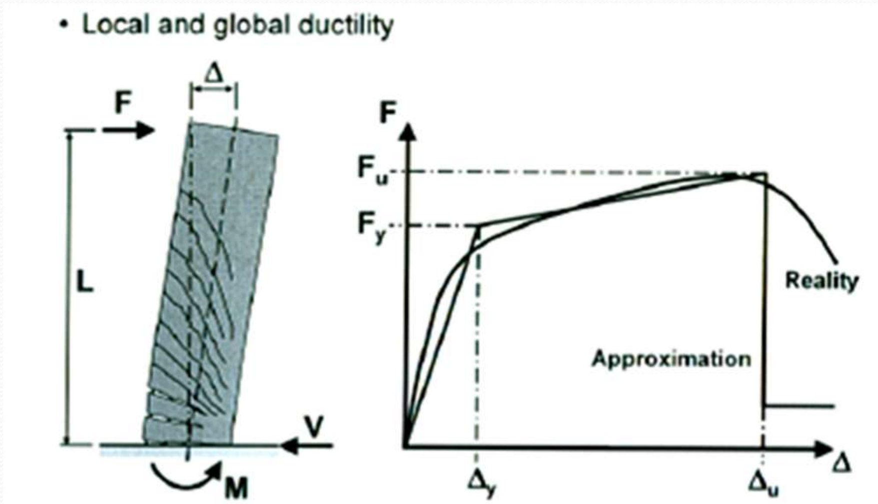
Ductility – Determination (cont'd)



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Ductility – Determination (cont'd)

Adding equations (2) and (3) yield the total deformation

$$\Delta_u = \Delta_y + \Delta_p = \phi_y \frac{L^2}{3} + (\phi_u - \phi_y) L_{pl} \left(L - \frac{L_{pl}}{2} \right)$$

Hence, the resulting displacement ductility (global) is:

$$\mu_\Delta = \frac{\Delta_u}{\Delta_y} = 1 + \frac{(\phi_u - \phi_y) L_{pl} (L - L_{pl}/2)}{(\phi_y L^2)/3} = 1 + \frac{(\mu_\phi - 1) L_{pl} (L - L_{pl}/2)}{L^2/3}$$

The relationship between local and global ductility is:

$$\mu_\phi = \frac{\phi_u}{\phi_y} = 1 + \frac{\mu_\Delta - 1}{3 \frac{L_{pl}}{L} \left(1 - 0.5 \frac{L_{pl}}{L} \right)}$$

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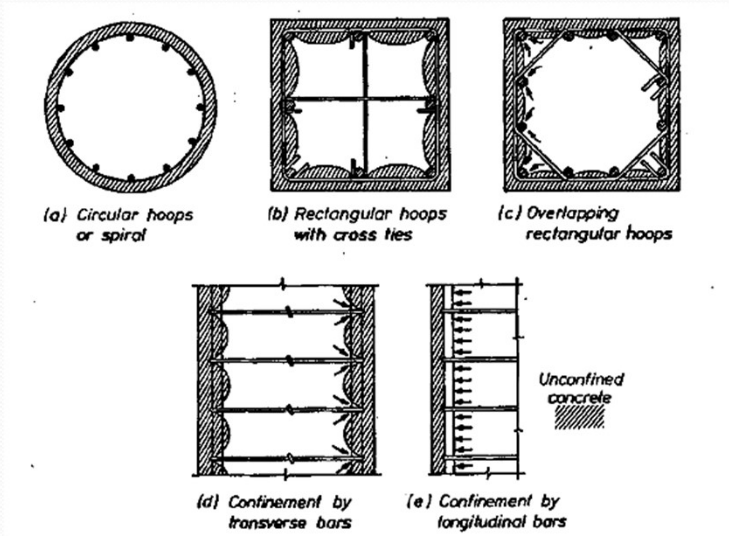
Relationship between Confinement & Ductility



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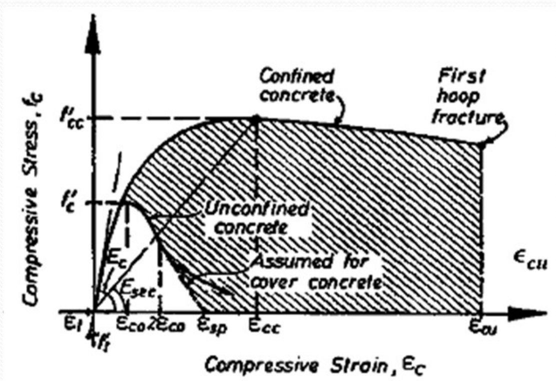
Relationship between Confinement & Ductility



$$\epsilon_{cu} = 0.004 + 1.4\rho_s f_{yh}\epsilon_{sm}/f'_{cc}$$

$$K = \frac{f'_{cc}}{f'_c} = \left(-1.254 + 2.254\sqrt{1 + \frac{7.94 f'_l}{f'_c}} - \frac{2 f'_l}{f'_c}\right) \qquad \epsilon_{cc} = 0.002[1 + 5(f'_{cc}/f'_c - 1)]$$

Typical values for ϵ_{cu} range from 0.012 to 0.05, a 4- to 16-fold increase over the traditionally assumed value for unconfined concrete.

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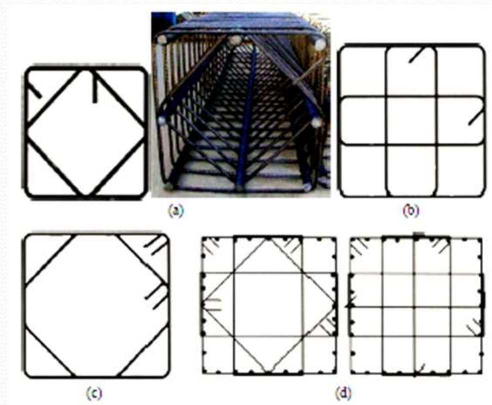
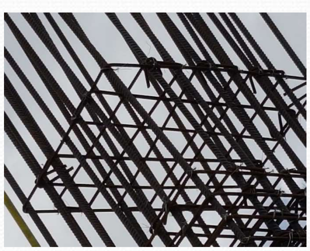
Different tie configurations for confinement

- As shown in the previous slide's stress-strain relationship, confined sections have larger strength, ductility and energy absorption capacity than the unconfined sections.
- Different hoop and tie configurations are used with varying degree of confinement.

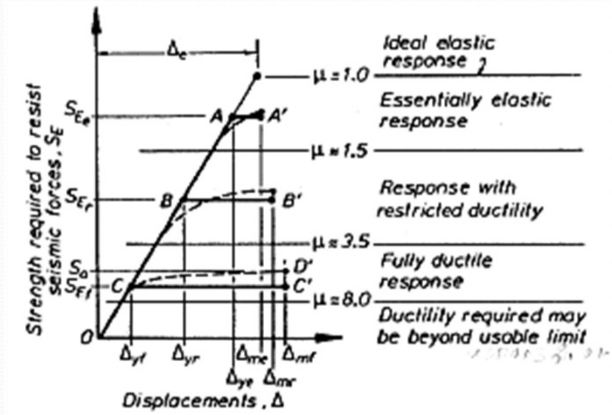


Different tie configurations for confinement



Relationship between strength & Ductility

- For seismic collapse prevention, the following approximation hold
  - "quality" of seismic behavior = strength x ductility
- To survive an EQ different combination of strength and ductility is possible



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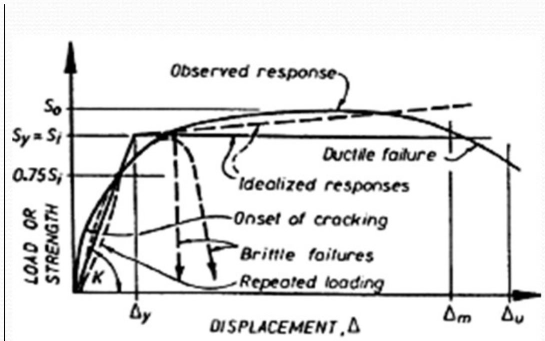
Ductility demand and ductility capacity

- Ductility demand is the required ductility to be imposed on the system
  - Ductility demand = Δ_m/Δ_y
- Ductility capacity is the ability of the structure to deform beyond the elastic limit and sustain the plastic deformation demand without collapsing
  - Ductility capacity = Δ_u/Δ_y



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Elastoplastic Idealization

- A more complex behavior may be represented with an elastoplastic (i.e. elastic-perfectly plastic) bilinear idealization, as shown in the figure next slide, where two important requirements are obeyed:
  - the initial stiffness of the idealized elastoplastic system is the same of the real system, so that the natural frequencies of vibration for small deformation are equal,
  - the yielding strength is chosen so that the sum of stored and dissipated energy in the elastoplastic system is the same as the energy stored and dissipated in the real system.

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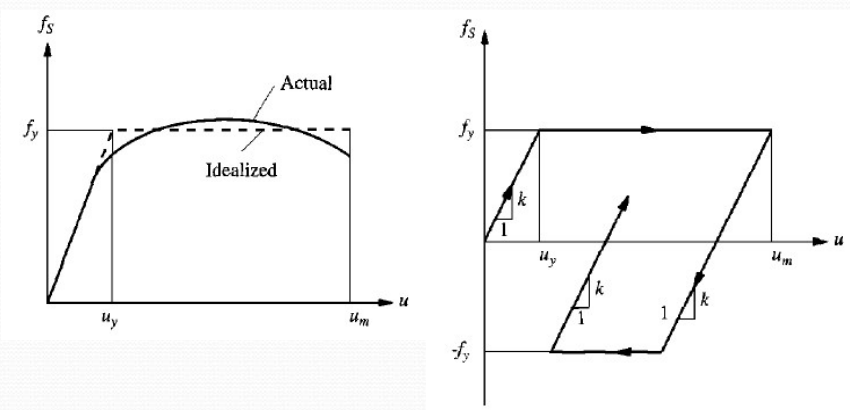
Elastoplastic Idealization



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Elastoplastic Idealization

- In perfect plasticity, when yielding
  - the force is constant, $f_s = f_y$ and
  - the stiffness is null, $k_y = 0$. The force f_y is the yielding force, the displacement $u_y = f_y/k$ is the yield deformation.
- In the right part of the figure, you can see that at unloading ($u_x = 0$) the stiffness is equal to the initial stiffness, and we have $f_s = k(u-u_{ptot})$ where u_{ptot} is the total plastic deformation.

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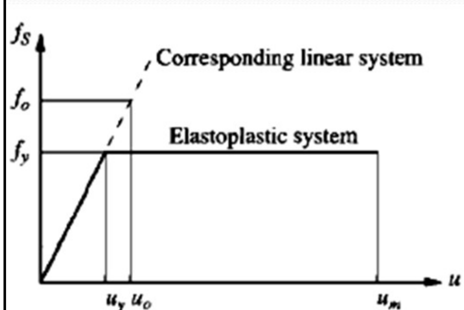
Normalized yield strength, yield reduction factor and ductility factor



The normalized yield strength $\bar{f}_y$

$$\bar{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o}$$

Yield reduction factor R_y

$$R_y = \frac{f_o}{f_y} = \frac{u_o}{u_y}$$

Ductility factor $\mu = \frac{u_m}{u_y}$

$$\frac{u_m}{u_o} = \mu \bar{f}_y = \frac{\mu}{R_y}$$

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Normalized yield strength, yield reduction factor and ductility factor (cont'd)

Governing equation for an inelastic system is:

$$m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = -m\ddot{u}_g(t)$$

$$\Rightarrow \ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u_y \tilde{f}_s(u,\dot{u}) = -\ddot{u}_g(t)$$

$$where\ \omega_n = \sqrt{\frac{k}{m}} \quad \xi = \frac{c}{2m\omega_n} \quad \tilde{f}_s(u,\dot{u}) = \frac{f_s(u,\dot{u})}{f_y}$$

Note that : $\frac{f_s}{m} = \frac{1}{m}\frac{f_y}{f_y} f_s = \frac{1}{m} k u_y \frac{f_s}{f_y} = \omega_n^2 u_y \tilde{f}_{s\,bar}$

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Normalized yield strength, yield reduction factor and ductility factor (cont'd)

$$From \quad \ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u_y \tilde{f}_s(u,\dot{u}) = -\ddot{u}_g(t)$$

$$Letting \quad \mu(t) = \frac{u(t)}{u_y} \quad and \quad a_y = \frac{f_y}{m} = \omega_n^2 u_o \bar{f}_y$$

$$u(t) = u_y\mu(t), \quad \dot{u}(t) = u_y\dot{\mu}(t), \quad \ddot{u}(t) = u_y\ddot{\mu}(t)$$

$$\Rightarrow \ddot{\mu} + 2\xi\omega_n\dot{\mu} + \omega_n^2\tilde{f}_s(\mu,\dot{\mu}) = -\omega_n^2\frac{\ddot{u}_g}{a_y}$$

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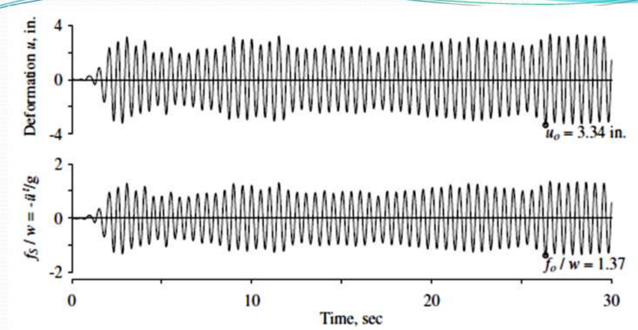
Elastic response



- In the figure above, the elastic response of an undamped, $T_n = 0.5$ s system to El Centro 1940 NS ground motion.
- Top, the deformations, bottom the elastic force normalized with respect to weight, from the latter peak value we know that all elastoplastic systems with $f_y < 1.37$w will experience plastic deformations during the EC1940NS ground motion.

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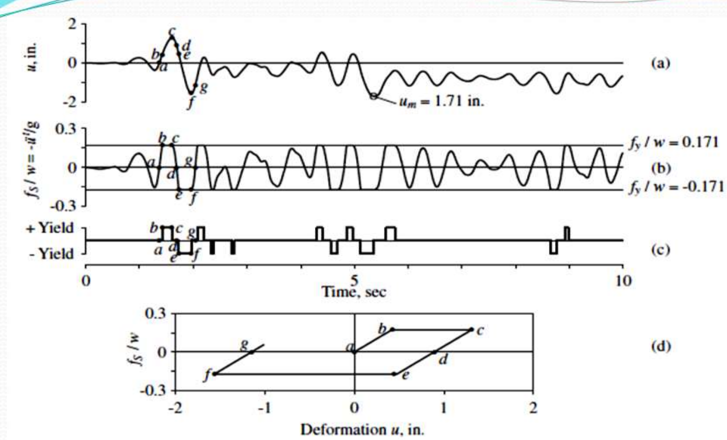
Inelastic response, fy = 1/8 = 0.125



- The various response graphs above were computed for $f_{ybar} = 0.125$. (i.e., $R_y = 8$ and $f_y = 1.37/8\text{w} = 0.171\text{w}$) and $\xi = 0$, $T_n = 0.5$ s.

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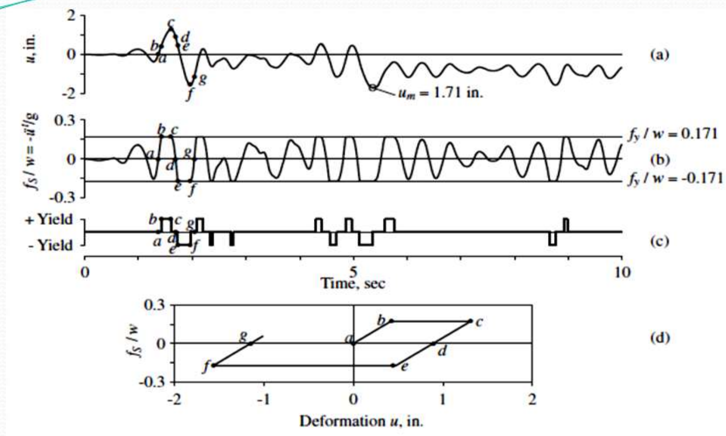
Inelastic response, fy = 1/8



- Top, the deformation response, note that the peak response is u_m = 1.71 in, different from u_0 = 3.34 in; it is $\mu = R_y \frac{u_m}{u_0}$ = 4.09

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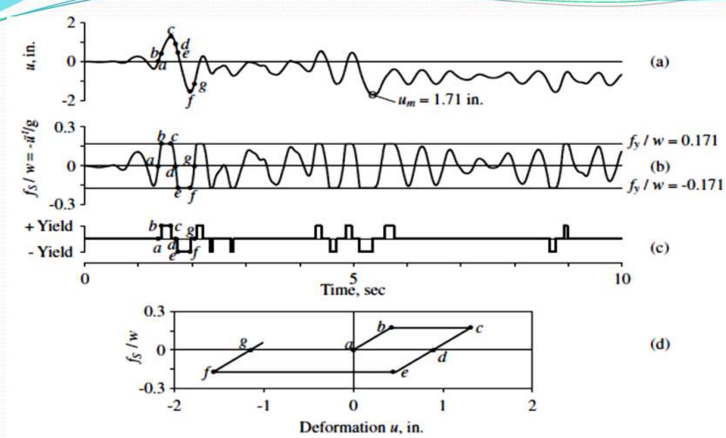
Inelastic response, fy = 1/8



- Second row, normalized force f_S/w, note that the response is clipped at $f_S = f_y = 1.171w$
- Third row, response in terms of yielding state, positive or negative depending on the sign of velocity

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Inelastic response, fy = 1/8

- The force-deformation diagram for the first two excursions in plastic domain, the time points *a*, *b*, *c*, *d*, *e*, *f* and *g* are the same in all 4 graphs:
  - until $t = b$ we have an elastic behavior,
  - until $t = c$ the velocity is positive and the system accumulates positive plastic deformations,
  - until $t = e$ we have an elastic unloading (note that for $t = d$ the force is zero, the deformation is equal to the total plastic deformation),
  - until $t = f$ we have another plastic excursion, cumulating negative plastic deformations
  - until at $t = g$ we have an inversion of the velocity and an elastic reloading.

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Response for different fy 's

$\bar{f}_y$	u_m	u_p	μ
1.000	2.25	0.00	1.00
0.500	1.62	0.17	1.44
0.250	1.75	1.10	3.11
0.125	2.07	1.13	7.36

- This table was computed for $T_n = 0.5$ s and $\xi = 5\%$ for the EC1940NS excitation.
- Elastic response was computed first, with peak response $u_0 = 2.25$ in and peak force $f_0 = 0.919$w, later the computation was repeated for $\bar{f}_y = 0.5, 0.25, 0.125$.
- In our example, all the peak values of the e-p responses are smaller than the elastic one, but this is not always true, and shouldn't be generalized.
- The permanent displacements u_p increase for decreasing yield strengths, and also this fact shouldn't be generalized.
- Last, the ductility ratios increase for decreasing yield strengths, for our example it is $\mu \approx R_y$.

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Effect of Yielding

Elastic system

- Oscillation about initial equilibrium position
- There will be no permanent deformation

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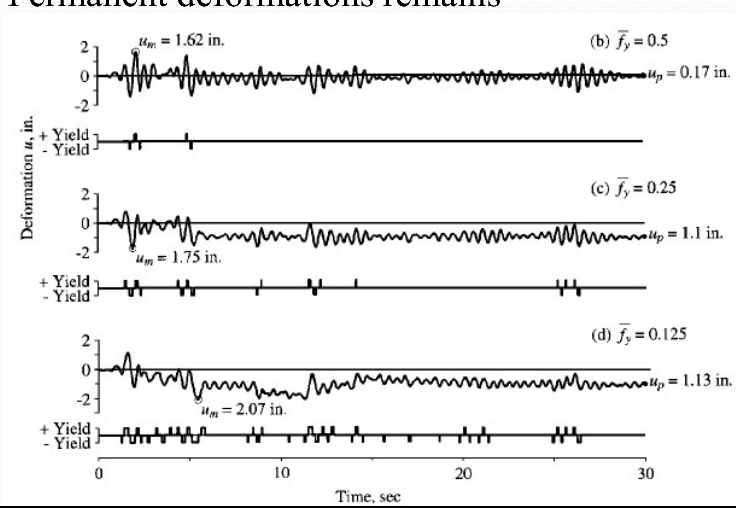
Effect of Yielding (cont'd)

Inelastic Systems

- Oscillation not about initial equilibrium position
- Permanent deformations remains



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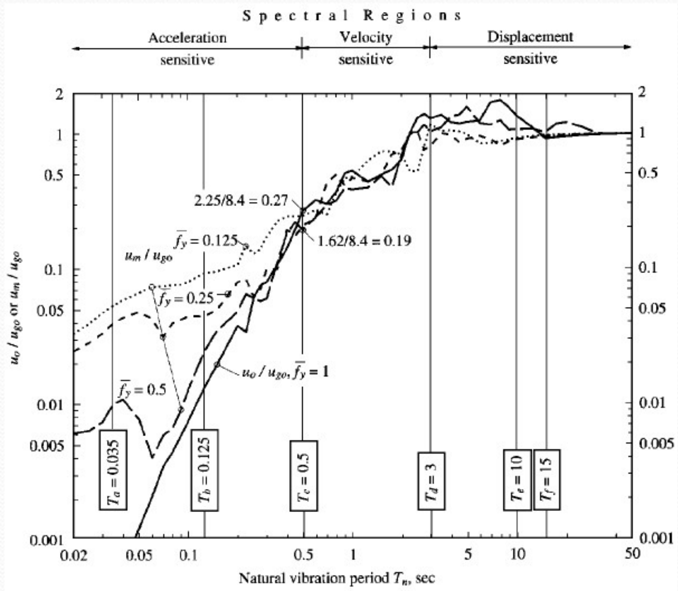
Peak deformation & ductility demand of elastoplastic system (cont'd)



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Peak deformation & ductility demand of elastoplastic system (cont'd)

- In the previous slide, for EC1940NS, for ξ = 5%, for different values of T_n and for f_{ybar} = 1.0, 0.5, 0.25, 0.125 the peak response u_0 of the equivalent system and the peak responses of the 3 inelastic systems has been computed
- There are two distinct zones: left (acceleration sensitive zone) there is a strong dependency on f_{ybar}, the peak responses grow with R_y; right (displacement sensitive zone) the 4 curves intersects with each other and there is no clear dependency on f_{ybar}.

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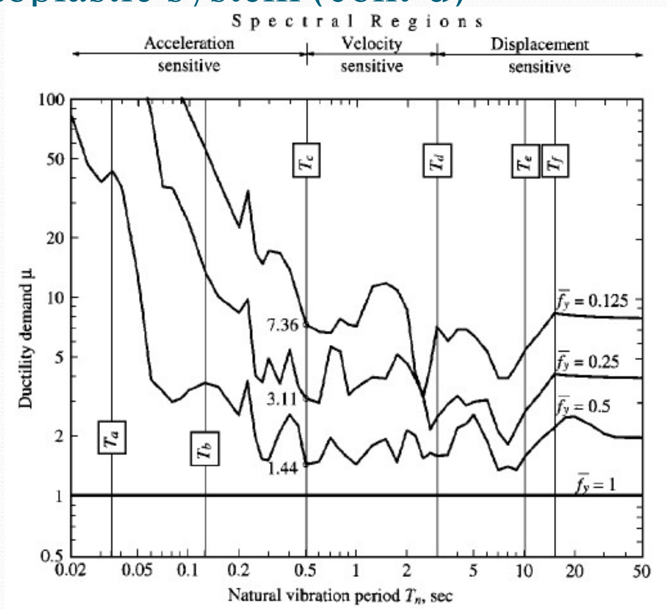
Peak deformation & ductility demand of elastoplastic system (cont'd)



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Peak deformation & ductility demand of elastoplastic system (cont'd)

- With the same setup as before, in graph of previous slide are reported the values of the ductility factor μ.
- The values of μ are almost equal to R_y for large values of T_n, and in the limit, for $T_n \rightarrow \infty$, there is a strict equality. An even more interesting observation regard the interval $T_c \leq T_n \leq T_f$, where the values of μ oscillate near the value of R_y.
- On the other hand, the behavior is completely different in the acceleration sensitive zone, where μ grows very fast, and the ductility demand is very high even for low values (0.5) of the yield strength reduction factor.

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Construction of constant ductility spectrum

1. Numerically define ground acceleration $\ddot{u}_g(t)$

   From $$\ddot{\mu} + 2\xi\omega_n\dot{\mu} + \omega_n^2 \tilde{f}_s(\mu, \dot{\mu}) = -\omega_n^2 \frac{\ddot{u}_g}{a_y}$$

2. Select and fix the damping ratio ξ and period T_n
3. Compute deformation response u(t) of linear system
4. Determine u_0 peak value of u(t) and peak force $f_0 = ku_0$

- These steps are the same as that of the construction of elastic response spectrum, but the following steps are different

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Construction of constant ductility spectrum (cont'd)

5. Determine u(t) of elastoplastic system with same ξ and T_n and yield force $f_y = [f_{ybar} * f_0]$ with $f_{ybar} < 1$. From u(t) determine u_m and corresponding μ for enough values of f_{ybar} to get as many data pts of (f_{ybar}, μ) as required
6. For a selected μ interpolate f_{ybar} (if necessary) and determine spectral ordinates corresponding to f_{ybar}
7. Repeat step 3 to 6 for a range of T_n and μ selected in 6
8. present steps 3 to 7 for several values of μ

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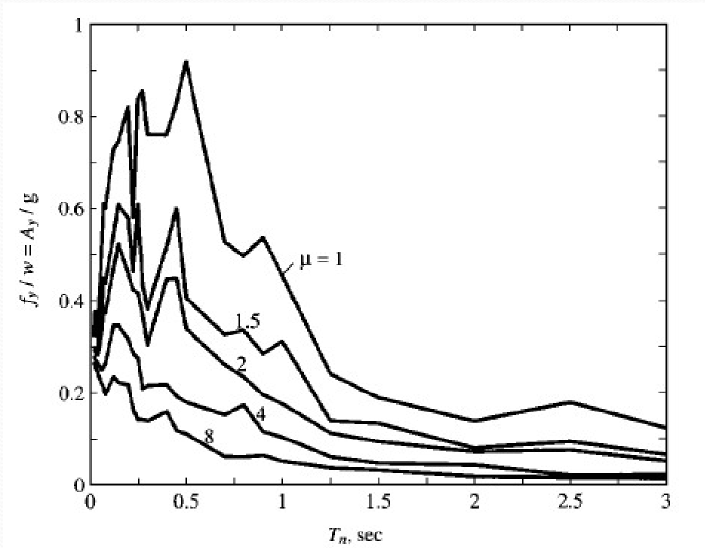
Constant ductility response spectrum



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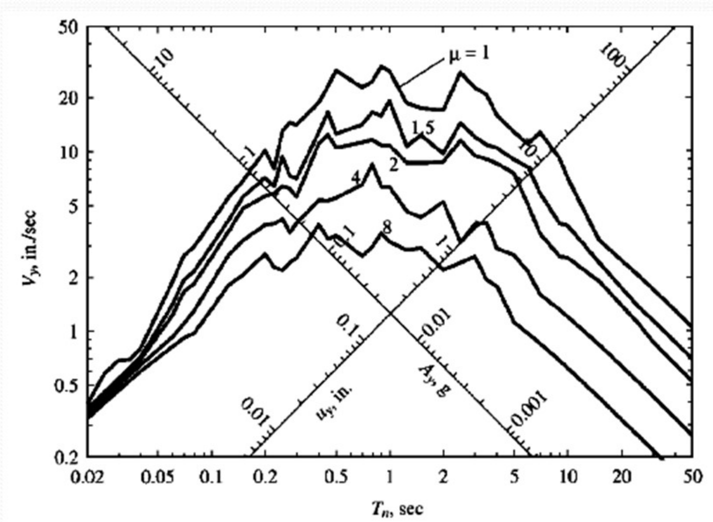
Constant ductility response spectrum (cont'd)



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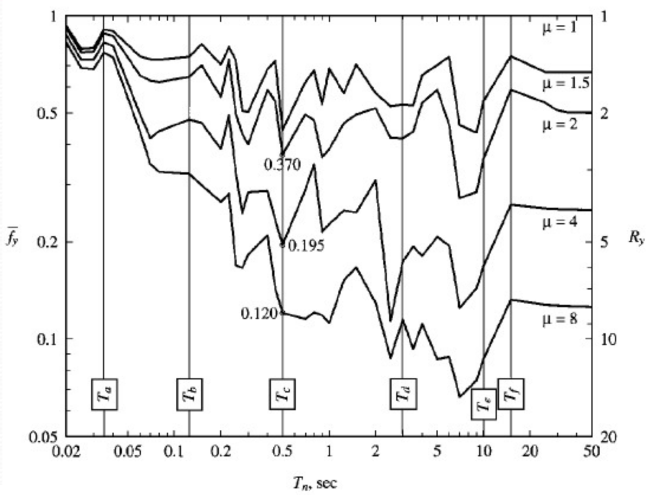
Design yield strength & allowable ductility



Short period systems ($T_n < T_a$) $f_{ybar} = 1$;
Long period systems ($T_n > T_f$) $f_{ybar} = 1/\mu$;
Medium period ($T_b < T_n < T_d$) f_{ybar} is irregular

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Relative effects of Yielding & Damping

- Damping has negligible influence but effect of yielding on design force is very important in displacement sensitive region $T_n > T_f$
- Damping has negligible influence but effect of yielding on peak deformation & ductility demand is very important in acceleration sensitive region $T_n < T_a$
- Damping is most effective in velocity sensitive region where yielding is even more effective

❖ The effectiveness of damping is smaller for inelastic systems and decreases as inelastic deformation increases

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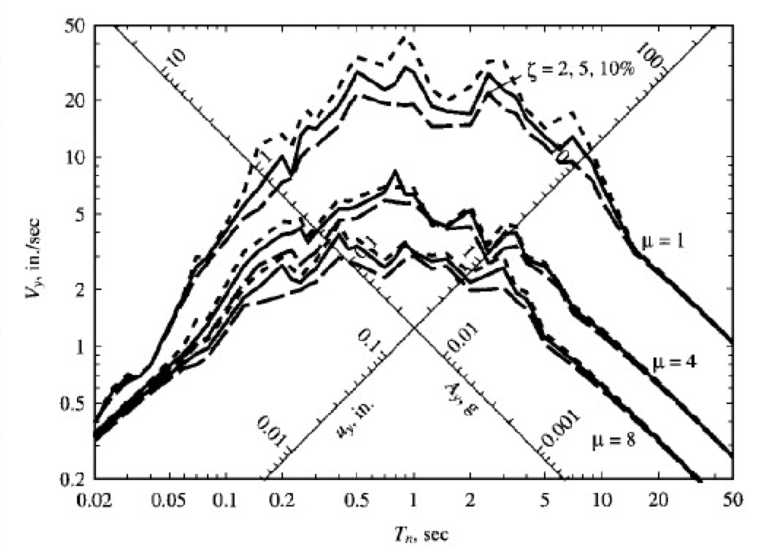
Relative effects of Yielding & Damping (Cont'd)



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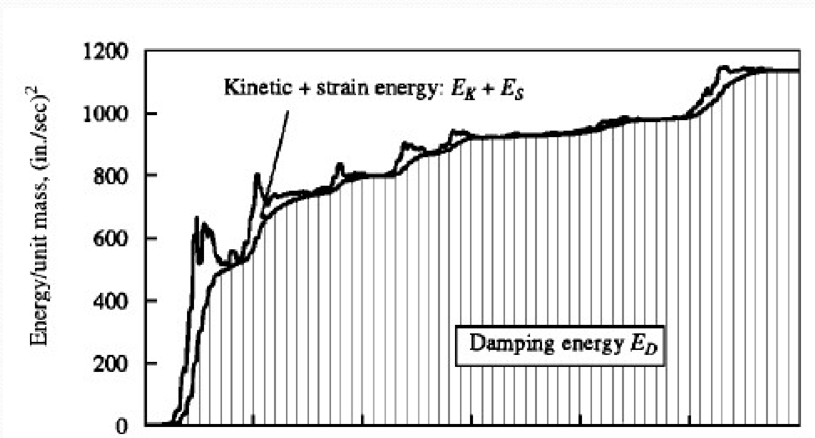
Relative effects of Yielding & Damping Energy dissipation



Energy dissipation of linear system $T_n = 0.5$ sec, $\zeta = 5\%$

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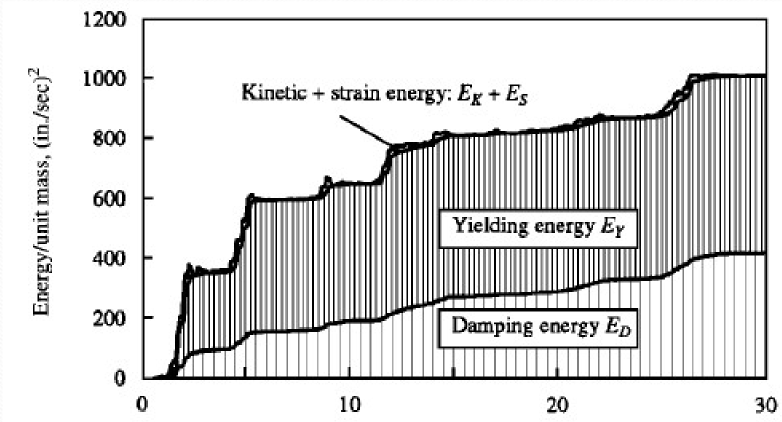
Relative effects of Yielding & Damping Energy dissipation (cont'd)



Energy dissipation of elastoplastic system

T_n = 0.5 sec, ζ = 5% , f_{ybar} =0.25

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Inelastic Design Spectra

Simpler approach of constructing constant-ductility design spectrum is by multiplying elastic design spectrum by normalized strength $f_{y,bar}$

$$\bar{f}_y = \begin{cases} 1 & T_n < T_a \\ (2\mu - 1)^{-1/2} & T_b < T_n < T_c \quad \textit{Equal energy principle} \\ \mu^{-1} & T_n > T_c \quad \textit{Equal displacement principle} \end{cases}$$

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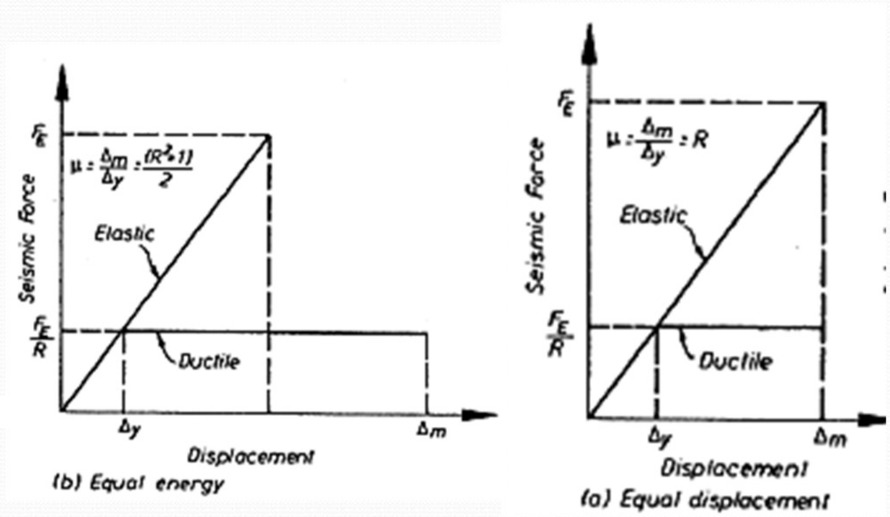
Inelastic Design Spectra (cont'd)



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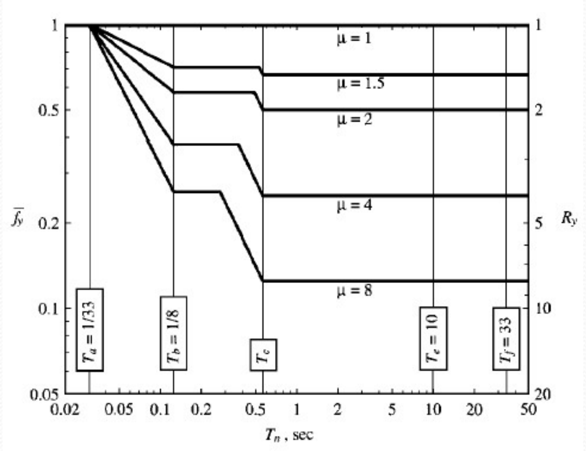
Inelastic Design Spectra (cont'd)

$$\bar{f}_y = \begin{cases} 1 & T_n < T_a \\ (2\mu - 1)^{-1/2} & T_b < T_n < T_c \\ \mu^{-1} & T_n > T_c \end{cases}$$



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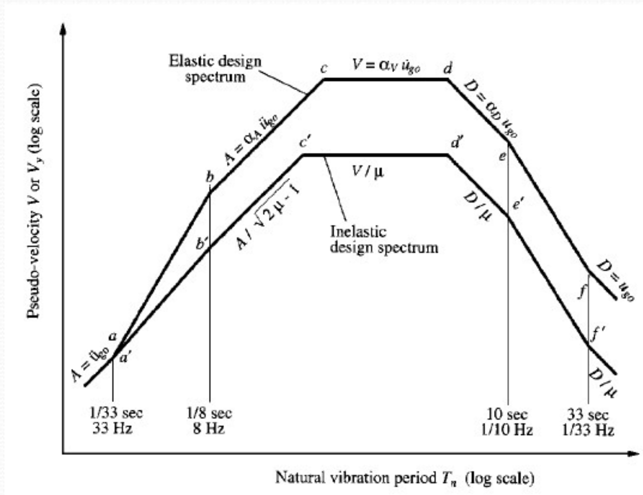
Inelastic Design Spectra (cont'd)



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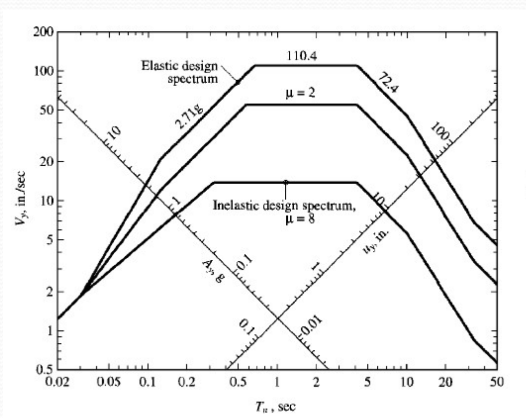
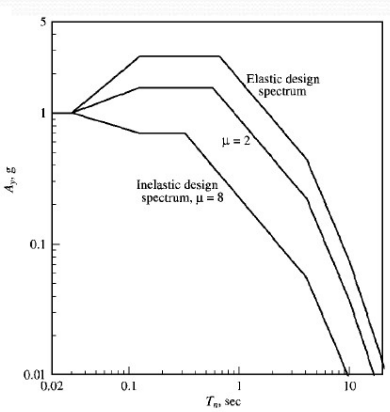
Inelastic Design Spectra (cont'd)





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Application: design of a SDOF system

- Decide the available ductility level μ (type of structure, materials, details. etc).
- Preliminary design, m, k, ξ; ω_n; T_n.
- From an inelastic design spectrum, for known values of ξ, T_n and μ read A_y .
- The design yield strength is

$$f_y = mA_y$$

- The design peak deformation,

$$u_m = \frac{\mu D_y}{R_y} = \frac{\mu}{R_y(\mu, T_n)} \frac{A_y}{\omega_n^2}$$

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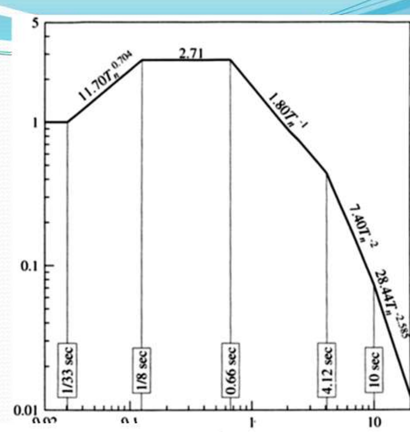
Example

- One story frame, weight w, period is $\boldsymbol{T_n}$ = 0.25 s, damping ratio is $\boldsymbol{\xi}$ = 5%, peak ground acceleration is $\ddot{u}_{g0}$ = 0.5g. Find design forces for
  1) system remains elastic, $\boldsymbol{\mu}$ = 1
  2) $\boldsymbol{\mu}$ = 4 and
  3) $\boldsymbol{\mu}$ = 8.
- In the figure, a reference elastic spectrum for $\ddot{u}_{g0}$ **= 1g,** A_y**(0.25) = 2.71g;** for $\ddot{u}_{g0}$ **= 0.5g** it is f_0 **=1.355w.**



For $T_n = 0.25$ s, $R_y = \sqrt{2\mu - 1}$, hence

$$f_y = \frac{1.355w}{\sqrt{2\mu-1}}, \quad u_m = \frac{\mu}{\sqrt{2\mu-1}} \frac{A_y}{\omega_n^2} = \frac{\mu}{\sqrt{2\mu-1}} \frac{1.355\text{g}\, T_n^2}{4\pi^2}.$$

$$\mu = 1: \quad f_y = 1.355w,\ u_m = 2.104\,\text{cm},$$
$$\mu = 4: \quad f_y = 0.512w,\ u_m = 3.182\,\text{cm},$$
$$\mu = 8: \quad f_y = 0.350w,\ u_m = 4.347\,\text{cm}.$$

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Any questions?

- Refer to

Chapter 7 – Earthquake response of Inelastic systems
In Chopra's Structural Dynamics book

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