Analysis of RC walls and square cross section of unit length side for calculation of interaction diagrams

Analysis of RC Sections based on Strain Compatibility Principles

- **Background**
  - The design moment resistances of singly or doubly reinforced sections with rectangular compression zones can be easily determined using the General Design Chart in EBCS2:Part2. The Design Chart is prepared on the basis of parabolic-rectangular stress-strain (σ–ε) curves for concrete and elastic-plastic behavior with a horizontal or an inclined top branch for reinforcing bars.
  - If the above conditions are not fulfilled, a trial-and-error solution based on strain compatibility can be used.

Analysis of RC Sections based on Strain Compatibility Principles

- **Steps for Determining the Design Axial Load and Uniaxial Bending Resistance**
  - 1. Assume the strain distribution in the ULS
  - 2. for non-rectangular compression zone, the use of the more accurate stress-strain curve of concrete tends to be involved. So use the equivalent rectangular stress block, with depth $a = \lambda x$ (EBCS EN 1992-1-1: 2013 pp35 SNS)
  - 3. compute the strains in each layer of reinforcement from the assumed strain distribution
  - 4. from the stress-strain curve for the reinforcement and the strains from step 3, determine the stress in each layer of reinforcement

EBCS EN 1992-1-1:2013

(3) A rectangular stress distribution (as given in Fig. 3.5) may be assumed. The factor λ,

defining the effective height of the compression zone and the factor η, defining the effective strength, follow from:

$\lambda = 0{,}8$ for $f_{ck} \leq 50$ MPa (3.19)

$\lambda = 0{,}8 - (f_{ck} - 50)/400$ for $50 < f_{ck} \leq 90$ MPa (3.20)

and

$\eta = 1{,}0$ for $f_{ck} \leq 50$ MPa (3.21)

$\eta = 1{,}0 - (f_{ck} - 50)/200$ for $50 < f_{ck} \leq 90$ MPa (3.22)

Note: If the width of the compression zone decreases in the direction of the extreme compression fibre, the value ηf_{cd} should be reduced by 10%.

EBCS EN 1992-1-1:2013

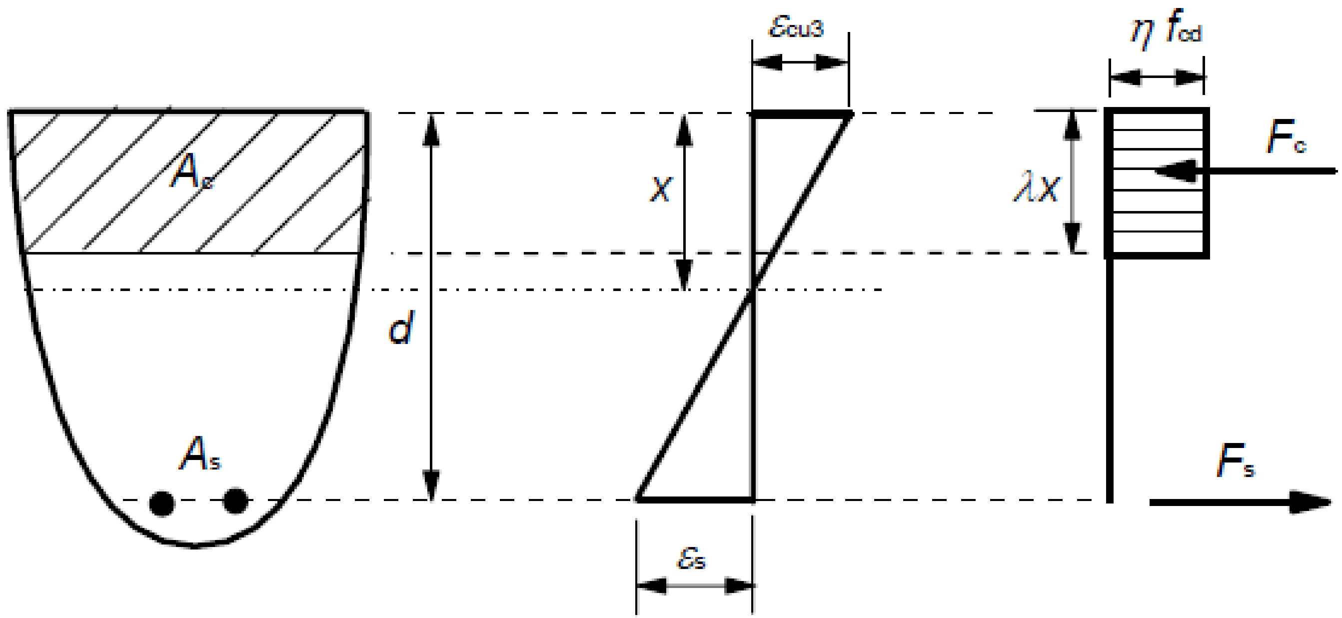


Figure 3.5: Rectangular stress distribution

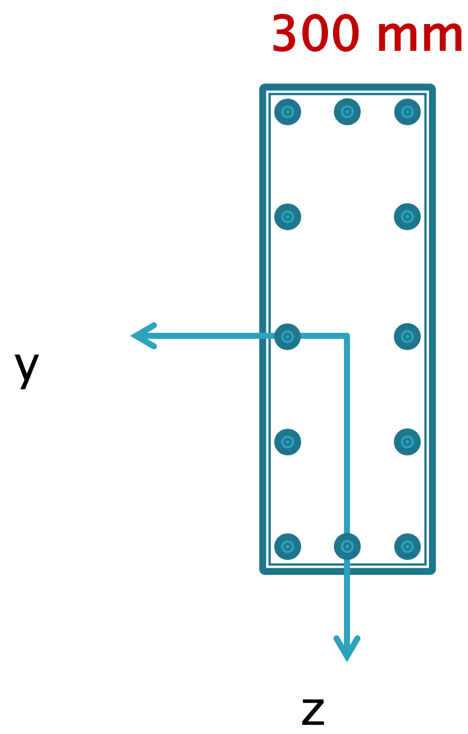
Analysis of RC Sections based on Strain Compatibility Principles

- **Steps for Axial Load plus Uniaxial Bending (Cont'd)**
  - 5. compute the force in the compression zone and in each layer of reinforcement
  - 6. compute P = C – T. For pure flexure, P equals zero. If calculated value of P is not equal to zero, adjust the strain distribution (move the NA under conditions of ultimate limit strain) and repeat steps 1 to 6 until P is as close to zero as desired. The imbalance should not exceed 0.1 to 0.5 percent of C (Macgregor).

Analysis of RC Sections based on Strain Compatibility Principles

- **Steps for Axial Load pus Uniaxial Bending (Cont'd)**
  - 7. Sum moments of the internal forces. If P=0, this can be about any convenient axis. We shall sum the moments about the centroid of the cross section.

Example



Cover up to centroid of bars = 50 mm. C/C of bars on the longer side = 200 mm

C20/25

S400

Rebar dia = 20 mm

Example

- P=0.0 (Pure uniaxial bending resistance, M_{yu})
- M_{yu} = 482.16 kNm; z_o = −0.2523 m; ε_o = 3.8676; ε_{1b} = −3.0313; ε_{2b} = 10.7665; ε_{2s} = 10.00

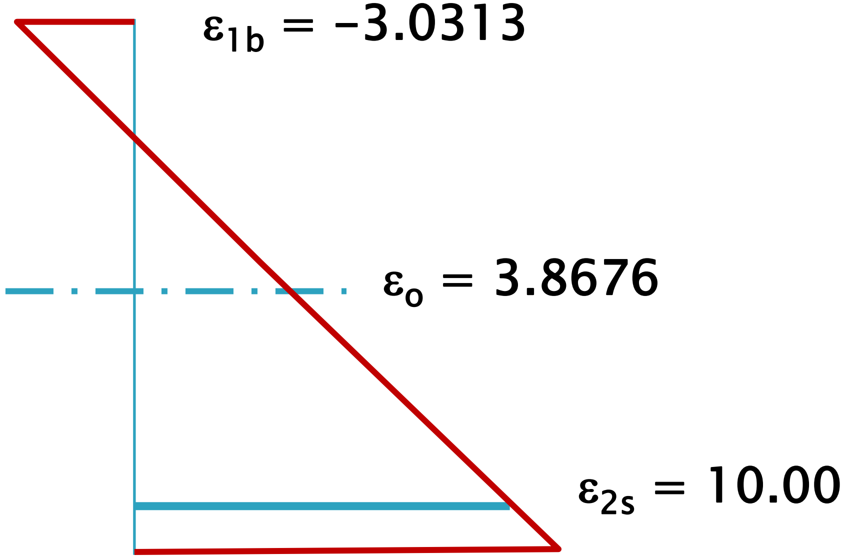


Fig. Strain Distribution in the ULS

Example (Assignment: check the values with hand calculation)

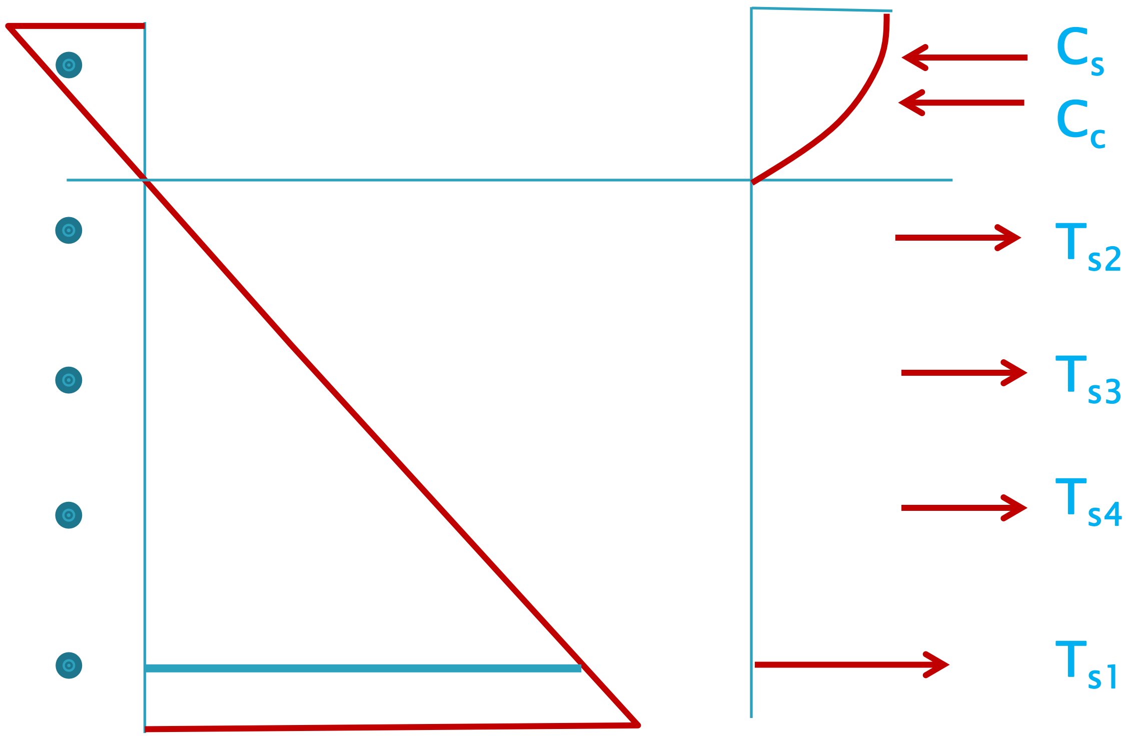


Fig. Strain Distribution in the ULS

Example

- $P_u = -690.02$ kN; $M_{yu} = 614.131$ kNm; $z_o = -0.1109$ m; $\varepsilon_o = 1.1448$; $\varepsilon_{1b} = -3.5$; $\varepsilon_{2b} = 5.7895$; $\varepsilon_{2s} = 5.2734$

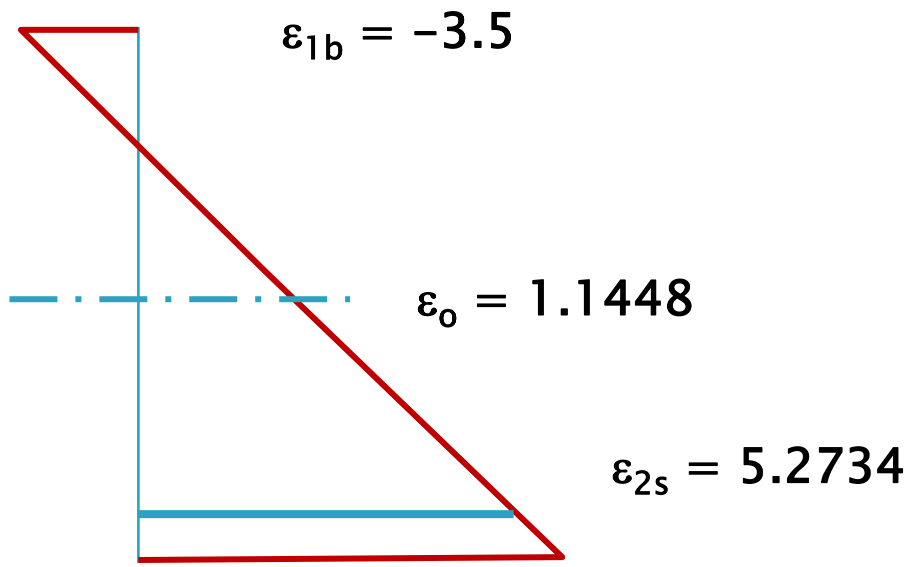
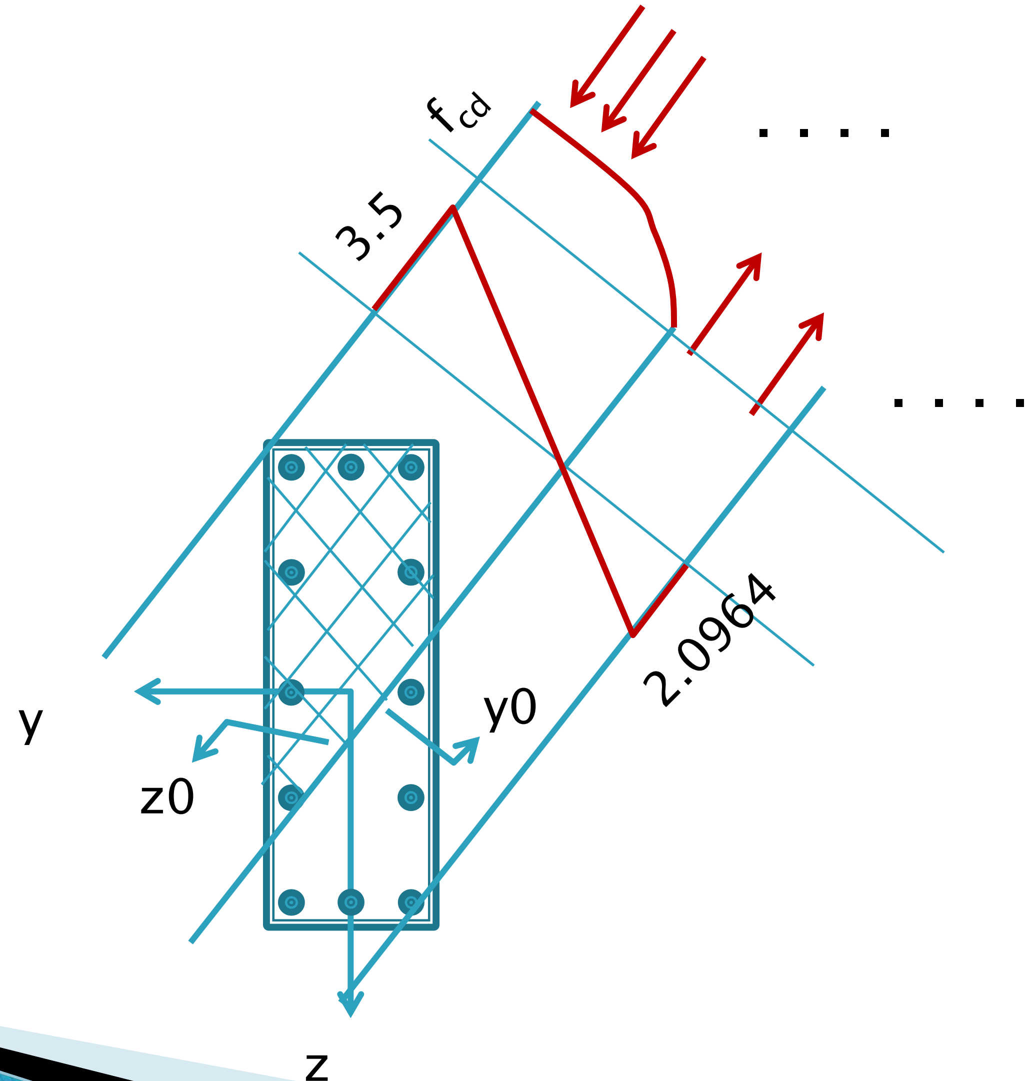


Fig. Strain Distribution in the ULS

Example

- P_u = –1571.41kN; M_{yu} = 559.359 kNm; M_{zu} = 62.8592 kNm; y_0 = –0.0674m; z_0 = 0.0901m; ε_o =–0.4257%o; ε_{1b} = –3.500%o; ε_{2b} = 2.6487%o; ε_{2s} = 2.0964%o
- NA, stress distribution, strain distribution in the ULS, etc. (See next slide)

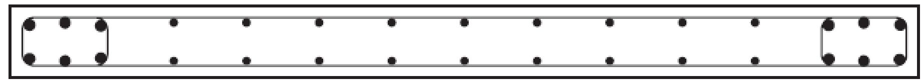
Example (Assignment: check the values with hand calculation)



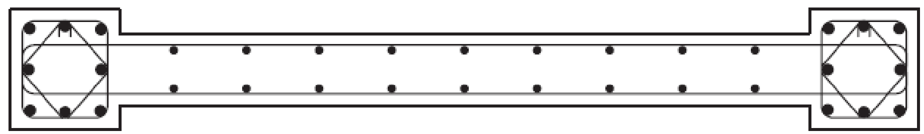
Analysis of RC Sections based on Strain Compatibility Principles

- Applications (Cont'd)
  - **Moment Resistance of Wall Assemblies, Walls with Flanges, and Walls with Boundary Elements**
  - Frequently, shear walls have webs and flanges that act together to form H-, C-, T-, and L-shaped wall cross sections referred to as wall assemblies. The effective flange widths can be taken from ACI Code Sections 8.12.2 and 8.12.3 (or ES EN 1998-1-1). In regions subject to earthquakes, ACI Code Section 21.9.5.2 limits the flange widths to the smaller of (Refer to provisions by EN 1998-1)
    - (a) half the distance to an adjacent web or
    - 25 percent of the total height of wall

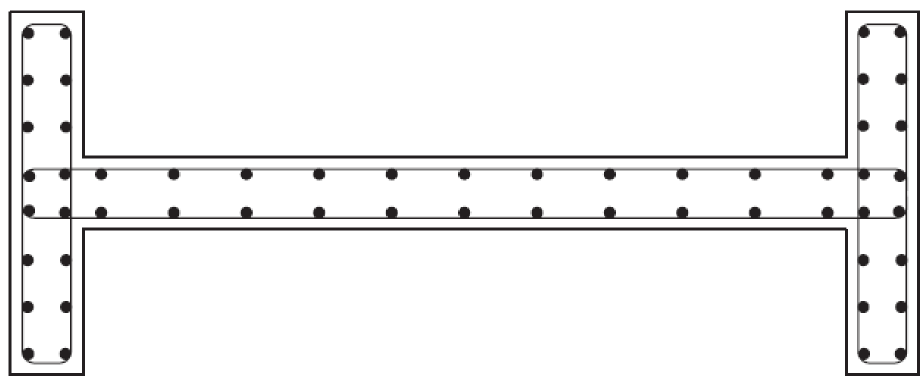
Analysis of RC Sections based on Strain Compatibility Principles



(a) Boundary element within dimensions of wall.



(b) Wall with enlarged boundary element.



(c) Wall with reinforcement concentrated in flanges.

Fig. 18-16
Structural walls with concentrated reinforcement at their edges.

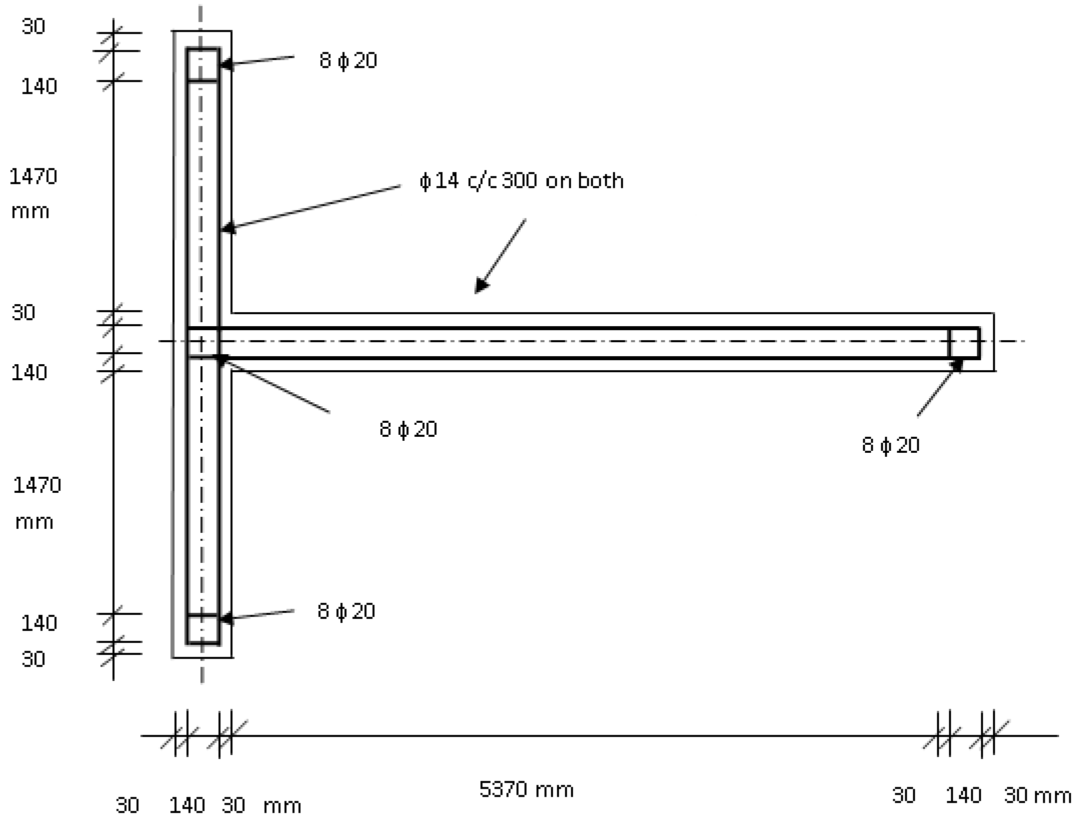
Analysis of RC Sections based on Strain Compatibility Principles

- **Applications (Cont'd)**
  - **Biaxially Loaded Walls**: A wall is said to be biaxially loaded if it resists axial load plus moments about two axes. One method of computing the strength of such walls is the equivalent eccentricity method. In this method a fraction b/n 0.4 and 0.8 times the weak-axis moment is added to the strong-axis moment. The wall is then designed for the axial load and the combined biaxial moment treated as a case of uniaxial bending and compression

Analysis of RC Sections based on Strain Compatibility Principles

- Examples
  - **1. Reinforced Concrete T-wall Section**
  - The structural wall is reinforced with ϕ 14 c/c 300 along all faces and 8 ϕ 20 as boundary elements, at the intersection and far ends of the flange and web (SNS). The steel grade and concrete class used are S-460 and C-30 respectively.
  - Required is the design normal and biaxial bending resistance of the T-wall section

Analysis of RC Sections based on Strain Compatibility Principles



Analysis of RC Sections based on Strain Compatibility Principles

- **Input data**

- T-Wall Example   March 07, 2006
- 2
- 1.3600E+04 -2.000 -3.500 0.0000 0.0000 2.000
- 4.0000E+05 -2.000 10.00 0.0000 2.000 1.000
- 2 1 1
- 4 1 4
- 5.740 1.740 .0000
- 5.540 1.740 .0000
- 5.540 .1000 .0000
- .0000 .1000 .0000

Analysis of RC Sections based on Strain Compatibility Principles

- 43 2 4
- .0300 .0000 3.14e-04
- .0300 .0700 3.14e-04
- .1000 .0700 3.14e-04
- .1700 .0700 3.14e-04
- .1700 .0000 3.14e-04
- .4700 .0700 1.54e-04
- .7700 .0700 1.54e-04
- 1.070 .0700 1.54e-04
- 1.370 .0700 1.54e-04
- 1.670 .0700 1.54e-04
- 1.970 .0700 1.54e-04
- 2.270 .0700 1.54e-04

Analysis of RC Sections based on Strain Compatibility Principles

- 2.570 .0700 1.54e-04
- 2.870 .0700 1.54e-04
- 3.170 .0700 1.54e-04
- 3.470 .0700 1.54e-04
- 3.770 .0700 1.54e-04
- 4.070 .0700 1.54e-04
- 4.370 .0700 1.54e-04
- 4.670 .0700 1.54e-04
- 4.970 .0700 1.54e-04
- 5.270 .0700 1.54e-04
- 5.570 .0700 3.14e-04
- 5.570 .0000 3.14e-04
- 5.640 .0700 3.14e-04
- 5.710 .0700 3.14e-04
- 5.710 .0000 3.14e-04
- 5.570 .3700 1.54e-04
- 5.570 .6700 1.54e-04

Analysis of RC Sections based on Strain Compatibility Principles

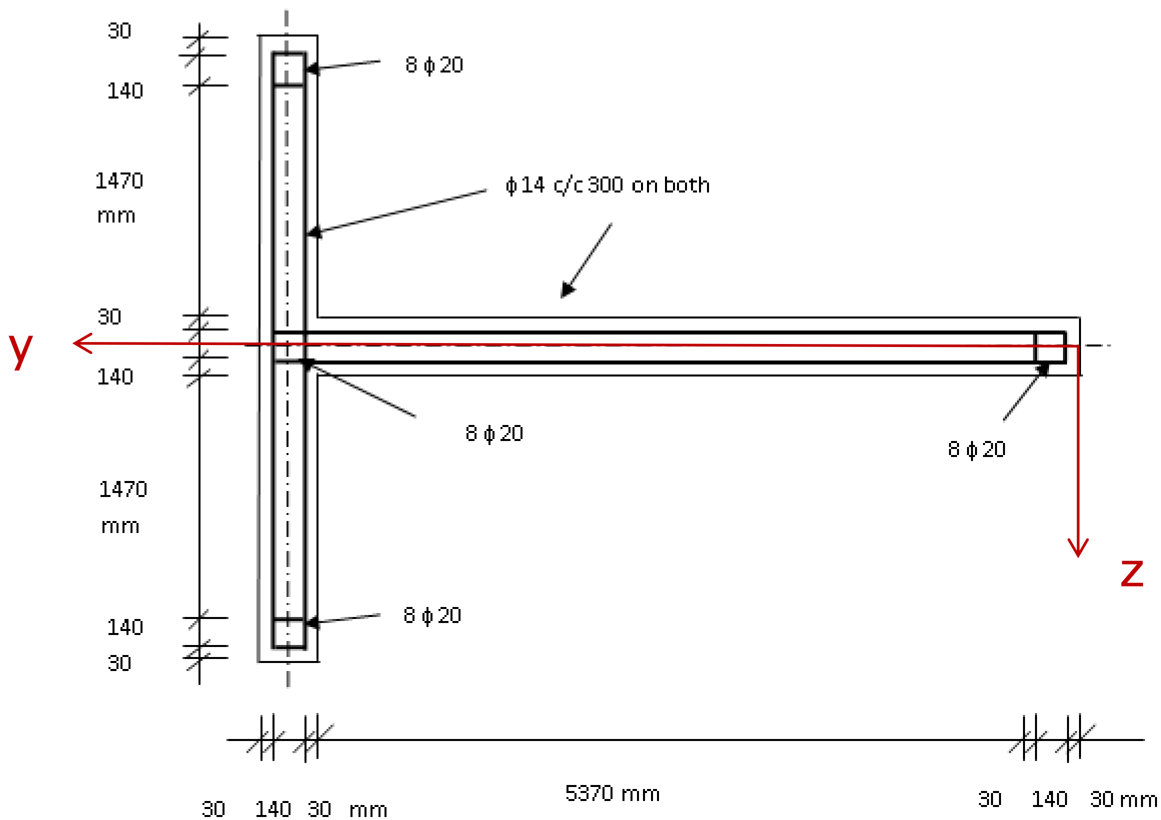
- 5.570 .9700 1.54e-04
- 5.570 1.270 1.54e-04
- 5.570 1.570 3.14e-04
- 5.640 1.570 3.14e-04
- 5.710 1.570 3.14e-04
- 5.570 1.640 3.14e-04
- 5.710 1.640 3.14e-04
- 5.570 1.710 3.14e-04
- 5.640 1.710 3.14e-04
- 5.710 1.710 3.14e-04
- 5.710 0.370 1.54e-04
- 5.710 0.670 1.54e-04
- 5.710 0.970 1.54e-04
- 5.710 1.270 1.54e-04
- 3
- -2500. 7500.0 6500.0 0 0
- 0

Analysis of RC Sections based on Strain Compatibility Principles

- --
- Factor of Safety to ULS
- --
- N : -2500.00 M.y : 7500.00 M.z : 6500.00
- N.u = -2592.93 M.yu = 7777.17 M.zu = 6740.05
- R.u/R = 1.0370 alf.M = 40.9137 alf.k = 90.0844
- alf.0 = 0.0844 y.0 _ = 117.662 z.0 = -0.1734
- eps.0 = 0.3853 deps/dy = -0.0033 deps/dz = 2.2221
- eps.1b = -3.5000 eps.2b = 4.2337 eps.2s = 4.1669
- ---
- Input Data: twall.dat Output Data: twall.out

- Internal forces w.r.t centroid of concrete section:
- ySP = 3.8773 zSP = 0.0000

Analysis of RC Sections based on Strain Compatibility Principles

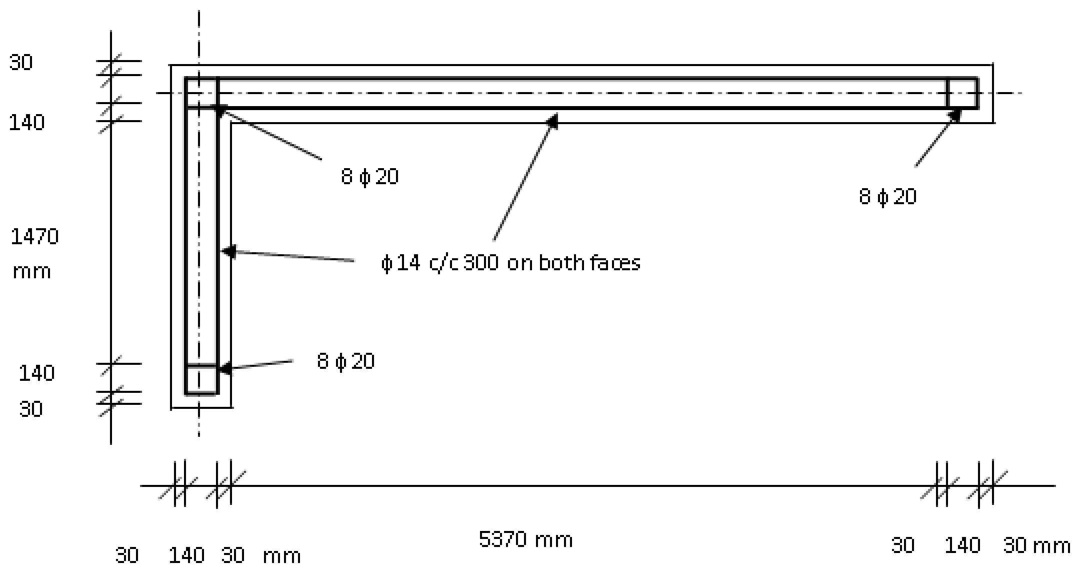


Analysis of RC Sections based on Strain Compatibility Principles

- 2. **Reinforced Concrete L-wall Section**

- The reinforcement, material strength and design action effects are similar to the T-wall example in section 4.2. The factor of safety of **0.32** indicates that the design has to be revised.

Analysis of RC Sections based on Strain Compatibility Principles



Analysis of RC Sections based on Strain Compatibility Principles

- **A.6 Input Data**
-
- L-Wall Example March 07, 2006
- 2
- 1.3600E+04 -2.000 -3.500 0.0000 0.0000 2.000
- 4.0000E+05 -2.000 10.00 0.0000 2.000 1.000
- 2 1 1
- 6 1 1
- 5.740 1.740 .0000
- 5.540 1.740 .0000
- 5.540 .1000 .0000
- .0000 .1000 .0000
- .0000 -.1000 .0000
- 5.740 -.1000 .0000

Analysis of RC Sections based on Strain Compatibility Principles

- 66 2 1
- .0300 .0000 3.14e-04
- .0300 .0700 3.14e-04
- .1000 .0700 3.14e-04
- .1700 .0700 3.14e-04
- .1700 .0000 3.14e-04
- .0300 -.0700 3.14e-04
- .1000 -.0700 3.14e-04
- .1700 -.0700 3.14e-04
- .4700 .0700 1.54e-04
- .7700 .0700 1.54e-04
- 1.070 .0700 1.54e-04
- 1.370 .0700 1.54e-04
- 1.670 .0700 1.54e-04
- 1.970 .0700 1.54e-04
- 2.270 .0700 1.54e-04
- 2.570 .0700 1.54e-04

Analysis of RC Sections based on Strain Compatibility Principles

2.870	.0700	1.54e-04
3.170	.0700	1.54e-04
3.470	.0700	1.54e-04
3.770	.0700	1.54e-04
4.070	.0700	1.54e-04
4.370	.0700	1.54e-04
4.670	.0700	1.54e-04
4.970	.0700	1.54e-04
5.270	.0700	1.54e-04
.4700	-.0700	1.54e-04
.7700	-.0700	1.54e-04

Analysis of RC Sections based on Strain Compatibility Principles

- 1.070 -.0700 1.54e-04
- 1.370 -.0700 1.54e-04
- 1.670 -.0700 1.54e-04
- 1.970 -.0700 1.54e-04
- 2.270 -.0700 1.54e-04
- 2.570 -.0700 1.54e-04
- 2.870 -.0700 1.54e-04
- 3.170 -.0700 1.54e-04
- 3.470 -.0700 1.54e-04
- 3.770 -.0700 1.54e-04
- 4.070 -.0700 1.54e-04
- 4.370 -.0700 1.54e-04
- 4.670 -.0700 1.54e-04
- 4.970 -.0700 1.54e-04
- 5.270 -.0700 1.54e-04
- 5.570 .0700 3.14e-04
- 5.570 .0000 3.14e-04
- 5.640 .0700 3.14e-04
- 5.710 .0700 3.14e-04
- 5.710 .0000 3.14e-04
- 5.570 -.0700 3.14e-04
- 5.640 -.0700 3.14e-04

Analysis of RC Sections based on Strain Compatibility Principles

- 5.710 -.0700 3.14e-04
- 5.570 .3700 1.54e-04
- 5.570 .6700 1.54e-04
- 5.570 .9700 1.54e-04
- 5.570 1.270 1.54e-04
- 5.570 1.570 3.14e-04
- 5.640 1.570 3.14e-04
- 5.710 1.570 3.14e-04
- 5.570 1.640 3.14e-04
- 5.710 1.640 3.14e-04
- 5.570 1.710 3.14e-04
- 5.640 1.710 3.14e-04
- 5.710 1.710 3.14e-04
- 5.710 0.370 1.54e-04
- 5.710 0.670 1.54e-04
- 5.710 0.970 1.54e-04
- 5.710 1.270 1.54e-04
- 3
- -2500. 7500.0 6500.0 0 0
- 0

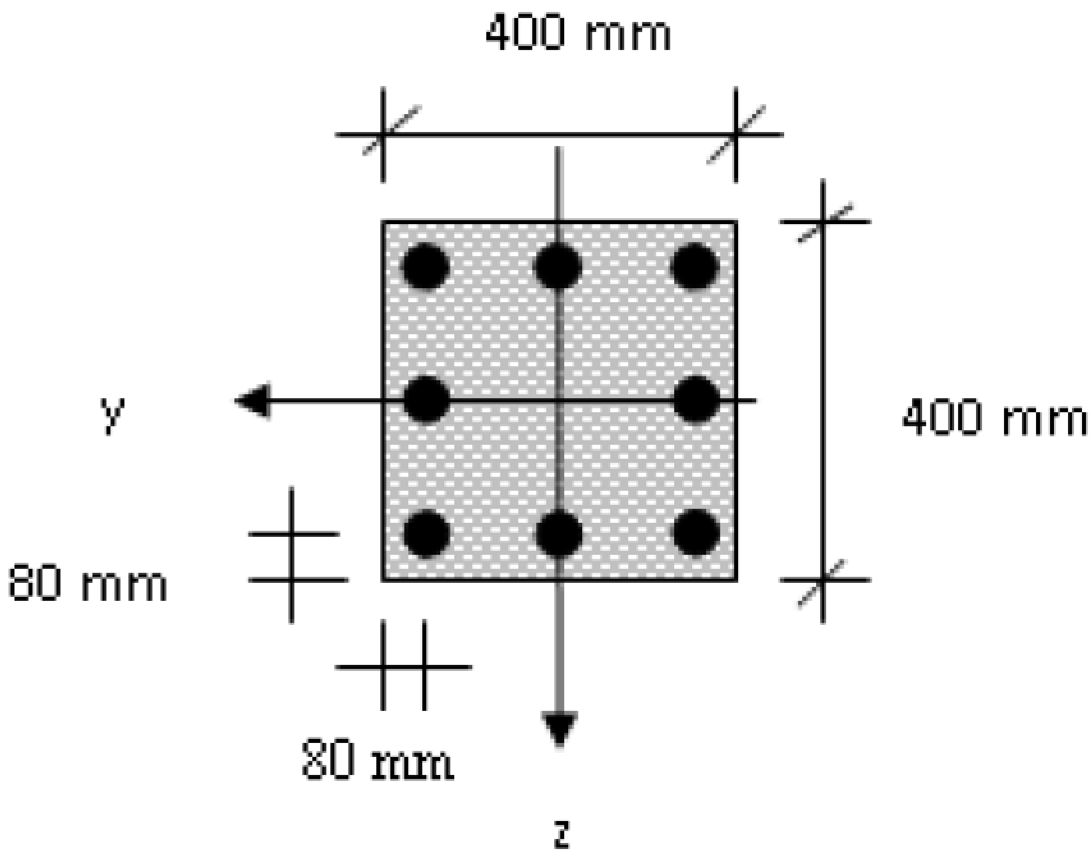
Analysis of RC Sections based on Strain Compatibility Principles

- Internal forces w.r.t centroid of concrete section:
- ySP = 3.4856 zSP = 0.2044
-
- --
- Factor of Safety to ULS
- --
- N : -2500.00 M.y : 7500.00 M.z : 6500.00
- N.u = -791.386 M.yu = 2374.38 M.zu = 2057.97
- **R.u/R = 0.3166** alf.M = 40.9168 alf.k = 92.4560
- alf.0 = 2.4560 y.0 _ = 3.2306 z.0 = -0.1386
- eps.0 = 0.8608 deps/dy = -0.2665 deps/dz = 6.2125
- eps.1b = -1.2899 eps.2b = 10.1944 eps.2s = 10.0000
- --
- Input Data: lwall.dat Output Data: lwall.out
-

Analysis of RC Sections based on Strain Compatibility Principles

- 3. Column sections under biaxial bending (preparation of interaction diagrams)

Analysis of RC Sections based on Strain Compatibility Principles



Analysis of RC Sections based on Strain Compatibility Principles

- **A.4.1 Input Data**
-
- Biaxial- Rectangular X-Section with 8-rebars 26.02.06
- 2
- 1.3600E+04 -2.000 -3.500 0.0000 0.0000 2.000
- 4.0000E+05 -2.000 10.00 0.0000 2.000 1.000
- 2 3 1
- 1 1 5
- 0.2000 0.2000 0.0000
- 3 2 5
- 0.1200 0.1200 3.4000E-04
- 0.1200 0.0000 3.4000E-04
- 0.0000 0.1200 3.4000E-04
- 3
- -1740.8 139.26 52.22 0 0
- 0

Analysis of RC Sections based on Strain Compatibility Principles

- -------------------------------------
- Factor of Safety to ULS
- -------------------------------------
- N : -1740.80 M.y : 139.260 M.z : 52.2200
- N.u = -1737.18 M.yu = 138.972 M.zu = 52.1115
- R.u/R = **0.9979** alf.M = 20.5549 alf.k = 114.243
- alf.0 = 24.2430 y.0 _ = -0.2370 z.0 = 0.1067
- eps.0 = -0.9414 deps/dy = -3.9722 deps/dz = 8.8207
- eps.1b = -3.5000 eps.2b = 1.6171 eps.2s = 0.5937
-
- ---
- Input Data: bar8mod.dat Output Data: bar8mod.out
-

Moment-Curvature relationships and Use of M-κ relations to Assess Ductility

- Ductility is an important property of RC structures. Why?
- Concrete is a brittle material. However RC can display significant ductility if properly designed (normal RC structures and those designed for EQ resistance(displacement ductility and curvature ductility demands in EQ design))
- Definition of ductility. How do we measure it? (displacement and/ or curvature ductility ratio μ_ϕ)
- See extract in next slide

Moment-Curvature relationships and Use of M-κ relations to Assess Ductility

5.2.3.4 Local ductility condition

(1)P For the required overall ductility of the structure to be achieved, the potential regions for plastic hinge formation, to be defined later for each type of building element, shall possess high plastic rotational capacities.

(2) Paragraph **(1)**P is deemed to be satisfied if the following conditions are met:

a) a sufficient curvature ductility is provided in all critical regions of primary seismic elements, including column ends (depending on the potential for plastic hinge formation in columns) (see **(3)** of this subclause);

Moment-Curvature relationships and Use of M-κ relations to Assess Ductility

(3) Unless more precise data are available and except when **(4)** of this subclause applies, **(2)**a) of this subclause is deemed to be satisfied if the curvature ductility factor μ_ϕ of these regions (defined as the ratio of the post-ultimate strength curvature at 85% of the moment of resistance, to the curvature at yield, provided that the limiting strains of concrete and steel ε_{cu} and $\varepsilon_{su,k}$ are not exceeded) is at least equal to the following values:

$$\mu_\phi = 2q_o - 1 \qquad \text{if } T_1 \geq T_C \qquad (5.4)$$

$$\mu_\phi = 1+2(q_o - 1)T_C/T_1 \qquad \text{if } T_1 < T_C \qquad (5.5)$$

where q_o is the corresponding basic value of the behaviour factor from Table 5.1 and T_1 is the fundamental period of the building, both taken within the vertical plane in which bending takes place, and T_C is the period at the upper limit of the constant acceleration region of the spectrum, according to **3.2.2.2(2)**P.

Moment-Curvature relationships and Use of M-κ relations to Assess Ductility

- The ductility provisions to ensure local and global ductility in the design of structures for earthquake resistance are in addition to the provisions in EBCS EN 1992-1-1 for ductile design of flexural members.
- Discuss the provisions in terms of limitation of the NA depth and the role of compression reinforcements to improve ductility
- Use the worksheet for calculation of moment-curvature

Moment-Curvature relationships and Use of M-κ relations to Assess Ductility

- Define ductility with the help of the moment-curvature diagrams
- Ductility decreases with increase in longitudinal tension reinforcement.
- Ductility is enhanced with compression reinforcement. Discuss with the help of the example in Chapter 5.

ULS in flexure non-ductile members

- Sudden w/o warning

ULS in shear in all RC members

- Discuss ULS in shear (diagonal tension and diagonal compression)
- ULS in shear is brittle and should be avoided. We will see how in Chapter 5

ULS in bond

- ULS in bond is also brittle and should be avoided. How?
- By providing sufficient anchorage length, hooks, anchorage plates, etc.
- Definition of ductility. Let us answer the question of how do we measure it? (displacement and/ or curvature ductility ratio)
- Discuss moment-curvature using the worksheet example (Continue with structure failures for COTM)

Structure Failures

- Structure as a whole may collapse w/o warning as a result of individual member failures
- Example is Omo steel bridge failure
- Structure could also collapse w/o warning as a result of a particular story or more than one story becoming unstable.
- Examples are collapse of Gondar building and staircase in Addis.

Structure Failures

Structure Failures

- Project staircase. It has collapsed suddenly w/o warning. Give dimensions and material strengths (C20/25 and S300 reinforcement)
- Is it a stability failure? assignment: Using simple hand calculations determine the probable cause of collapse of the staircase in Addis. Make simplifying assumptions.
- Such types of premature failures should be avoided

Structure Failures

Structure Failures

Structure Failures

Structure Failures

Structure Failures

Structure Failures

Structure Failures