CHAPTER 1 DISCONTINUITY REGIONS AND STRUT-AND-TIE MODELS

CHAPTER 2 COLUMNS: 2ND ORDER MOMENTS IN SWAY AND NON-SWAY FRAMES

CHAPTER 3 STRAIN COMPATIBILITY FOR ANALYSIS OF MORE COMPLEX SECTIONS

- Moment Curvature Relationships
- Deflection Calculation Aids

CHAPTER 4 ANALYSIS AND DESIGN OF TWO-WAY SLABS

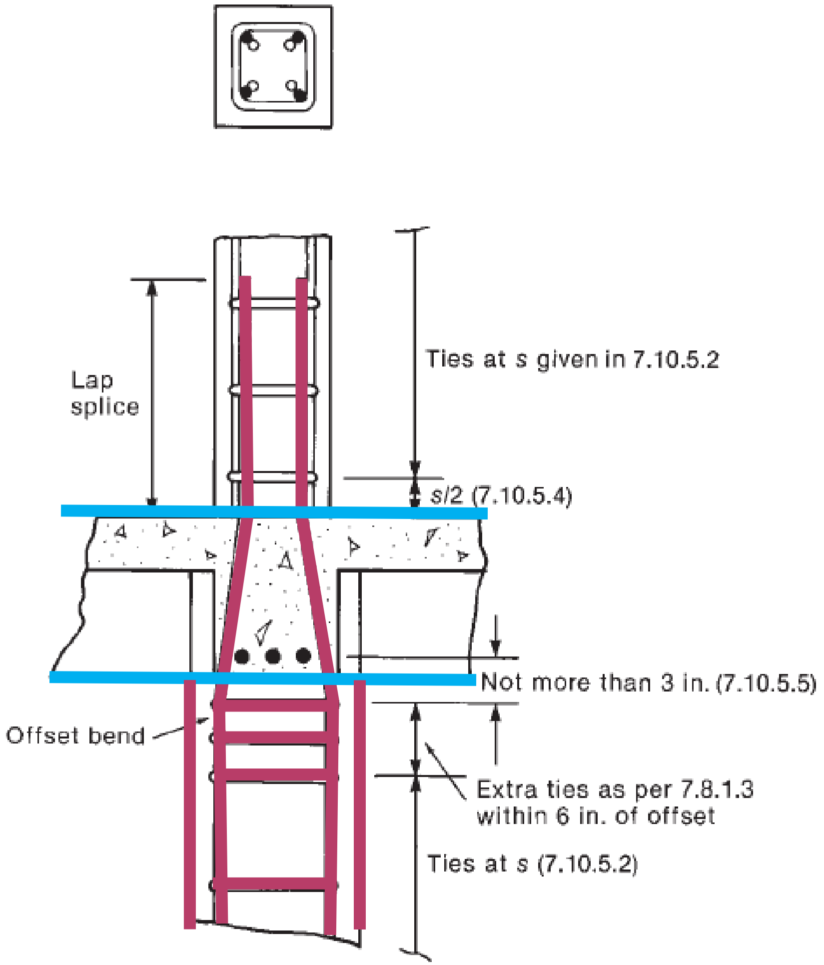
CHAPTER 5 DESIGN FOR EARTHQUAKE RESISTANCE

CHAPTER 2 COLUMNS: 2ND ORDER MOMENTS IN SWAY AND NON-SWAY FRAMES

2.0 INTRODUCTION

- A column is a vertical structural member supporting axial compressive loads, with or w/o moments.
- The x-sectional dimensions of a column are generally considerably less than its height.
- Columns support vertical loads from the roof and transmit these loads to the foundations.
- In a typical construction cycle, the reinforcement and concrete for the beams and slabs in a floor are placed first(discussion with NS).

2.0 INTRODUCTION



(a) Tie spacing at interior column–beam joint.

2.0 INTRODUCTION

- Once this concrete has hardened, the reinforcement and concrete for the columns over that floor are placed. The process is illustrated in Figs 11-1, and 11-2
- Fig. 11-1 shows a completed column prior to construction of the formwork for the next floor.
- This is a tied column, so called because the longitudinal bars are tied together with smaller bars at intervals up the column.

2.0 INTRODUCTION

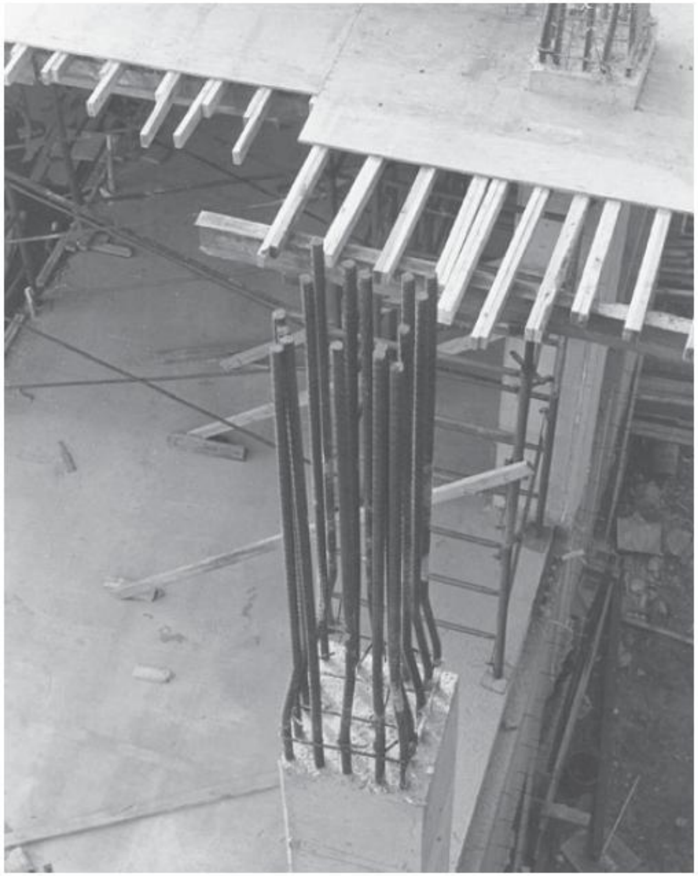
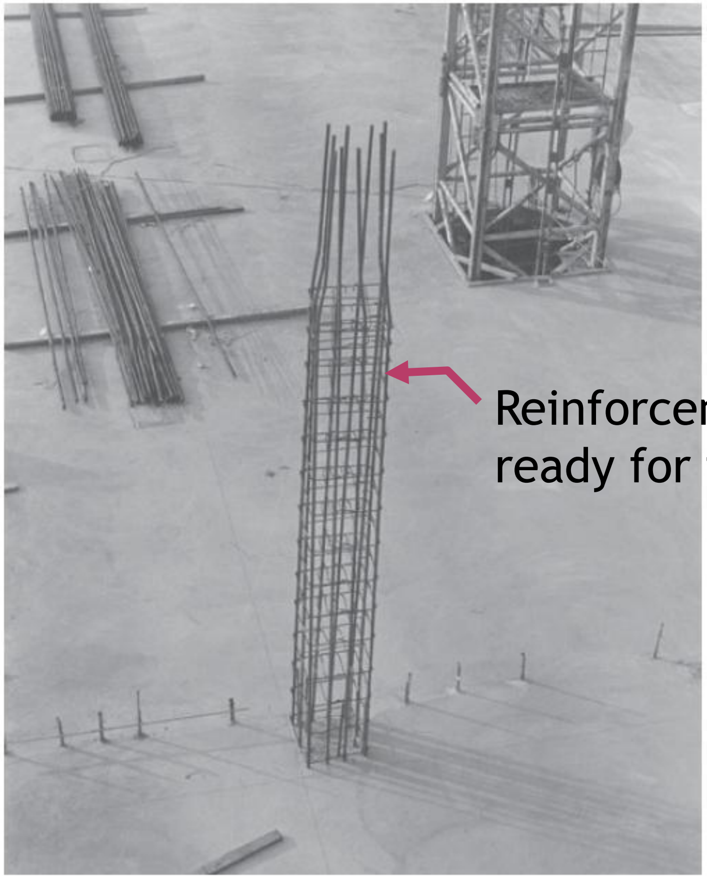


Fig. 11-1
Tied column under construction. (Photograph courtesy of J. G. MacGregor.)

2.0 INTRODUCTION



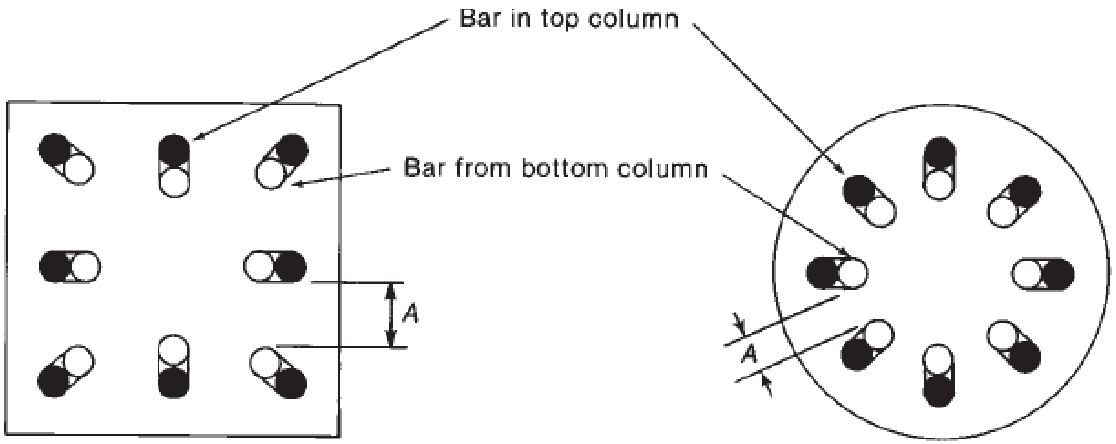
Reinforcement cage
ready for formwork

Fig. 11-2
Reinforcement cage for a tied column. (Photograph courtesy of J. G. MacGregor.)

2.0 INTRODUCTION

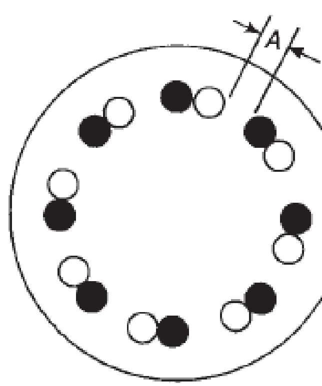
- One set of ties is visible just above the concrete. The longitudinal bars protruding from the column will extend through the floor into the next-higher column and will be lap spliced with the bars in that column.
- The longitudinal bars are bent inward to fit inside the cage of bars for the next-higher column. (other splice details are sometimes used: SNS)

2.0 INTRODUCTION



(a) Radial splice—Rectangular column.

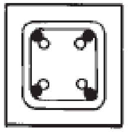
(b) Radial splice—Circular column.

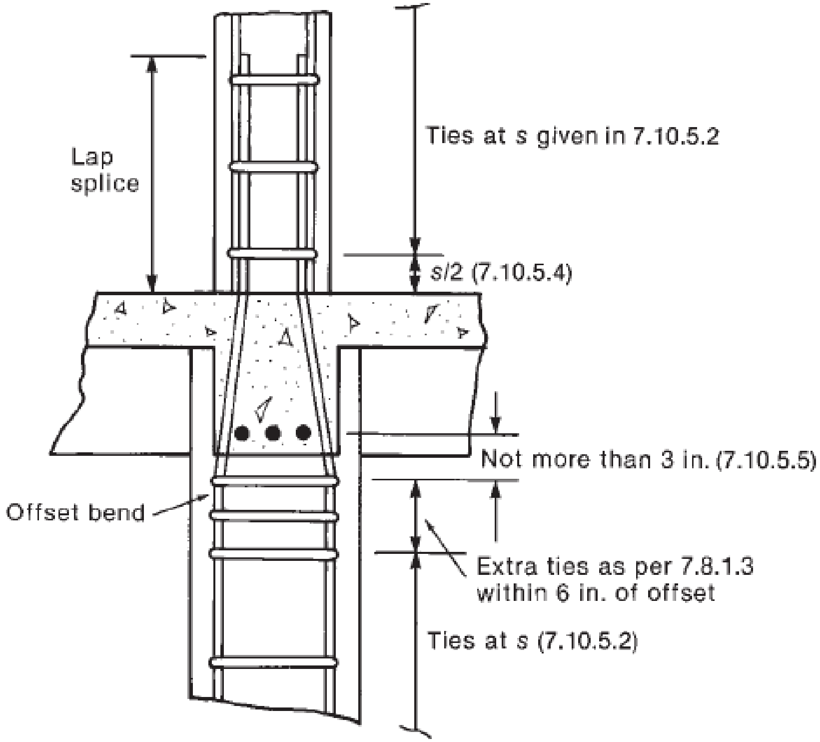


(c) Side-by-side lap splice.

A shall not be less than $1\frac{1}{2}$ in., $1\frac{1}{2}$ bar diameters or $1\frac{1}{3}$ times aggregate size.

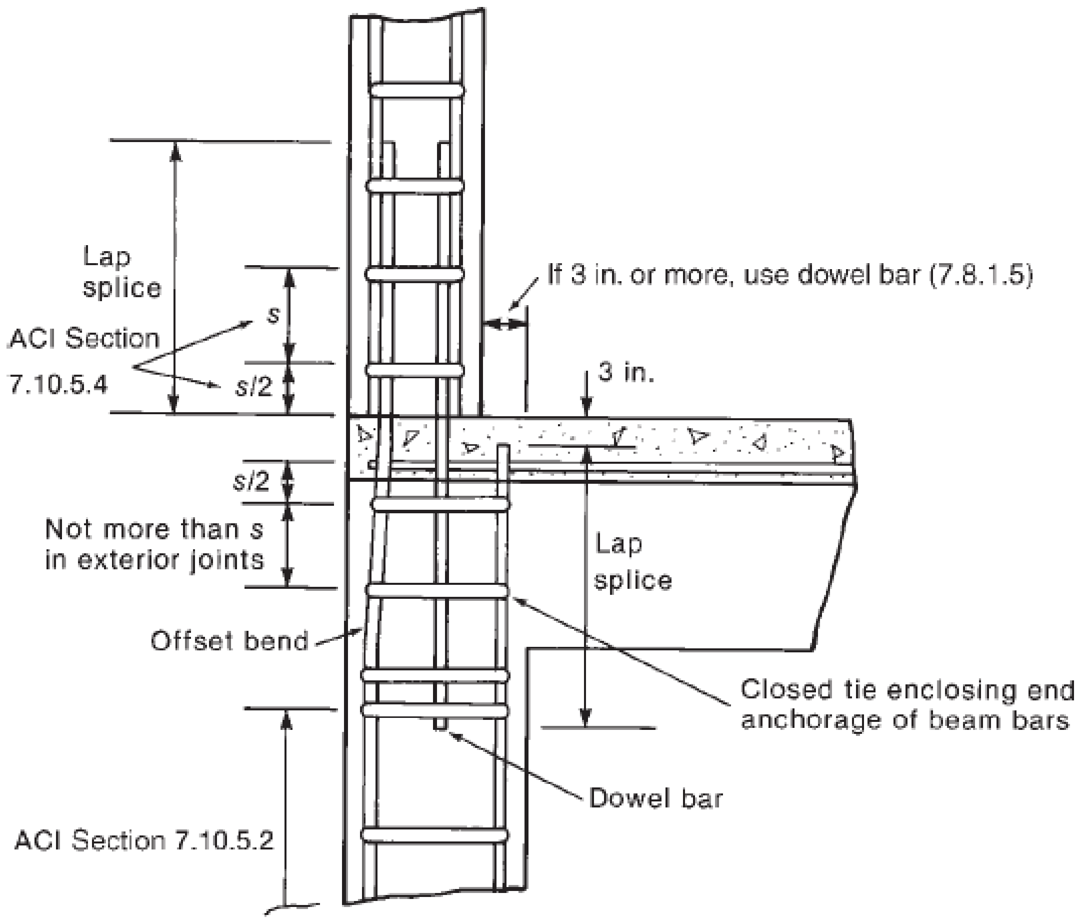
2.0 INTRODUCTION





(a) Tie spacing at interior column–beam joint.

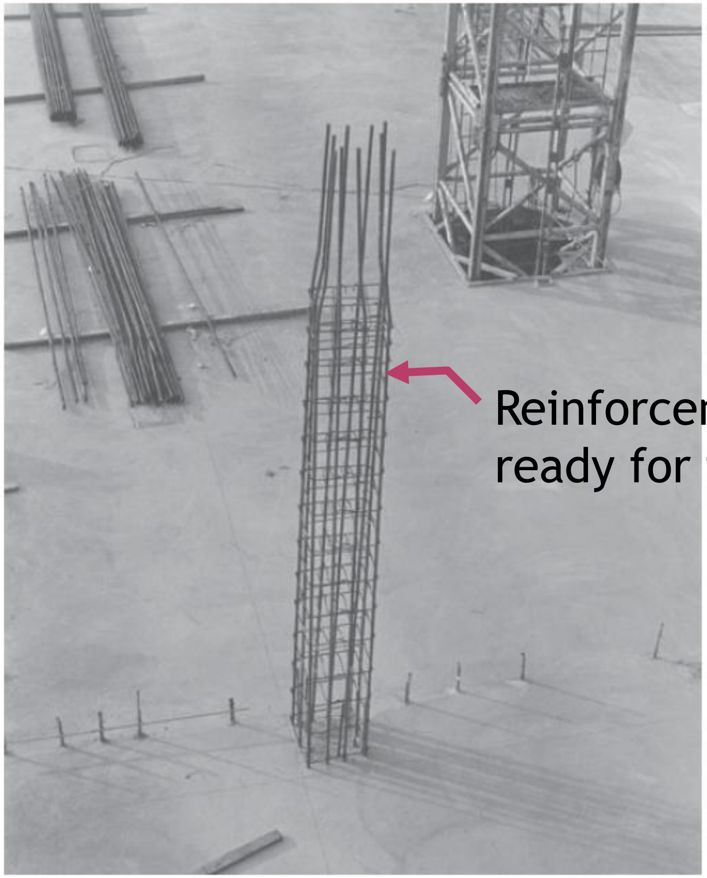
2.0 INTRODUCTION



(b) Ties at exterior column–beam joint.

2.0 INTRODUCTION

- A reinforcement cage that is ready for the column forms is shown in Fig 11-2. the lap splice at the bottom of the column and the ties can be seen in this photograph.
- The more general terms compression members and members subjected to combined axial loads and bending are used to refer to columns, walls, and members in concrete trusses and frames. These may be vertical, inclined, or horizontal. A column is a special case of a compression member that is vertical



Reinforcement cage ready for formwork

Fig. 11-2
Reinforcement cage for a tied column. (Photograph courtesy of J. G. MacGregor.)

2.0 INTRODUCTION

- Stability effects must be considered in the design of compression members. If the moments induced by slenderness effects weaken a column appreciably, it is referred to as a slender column or long column.
- The great majority of concrete columns are sufficiently stocky that slenderness can be ignored. Such columns are referred to as short columns.
- Although the theory developed in this chapter applies to columns in seismic regions, such columns require special detailing to resist the shear forces and repeated cyclic loading from the EQ. In seismic regions the ties are heavier and much more closely spaced than shown in Figs above.

2.1 TIED AND SPIRAL COLUMNS

- Most of the columns in buildings in nonseismic regions are tied columns. They may be square, circular, rectangular, L-shaped or any other required shape.
- Occasionally, when high strength and/or ductility are required, the bars are placed in a circle, and the ties are replaced by a bar bent into a helix or a spiral, with a pitch from 35 to 85 mm.
- Such a column, called a spiral column, is shown in Fig. 11-3 (SNS)

2.1 TIED AND SPIRAL COLUMNS

Fig. 11-3
Spiral column. (Photograph courtesy of J. G. MacGregor.)

2.1 TIED AND SPIRAL COLUMNS

- The spiral acts to restrain the lateral expansion of the column core under axial loads causing crushing and, in doing so, delays the failure of the core, making the column more ductile.
- **2.1.1 Behavior of Tied and Spiral Columns**
- Fig. 11-4a shows a portion of the core of a spiral column enclosed by one and a half turns of a spiral. Under a compressive load, the concrete in this column shortens longitudinally under the stress f_1 and so, to satisfy the Poisson's ratio, it expands laterally.

2.1 TIED AND SPIRAL COLUMNS

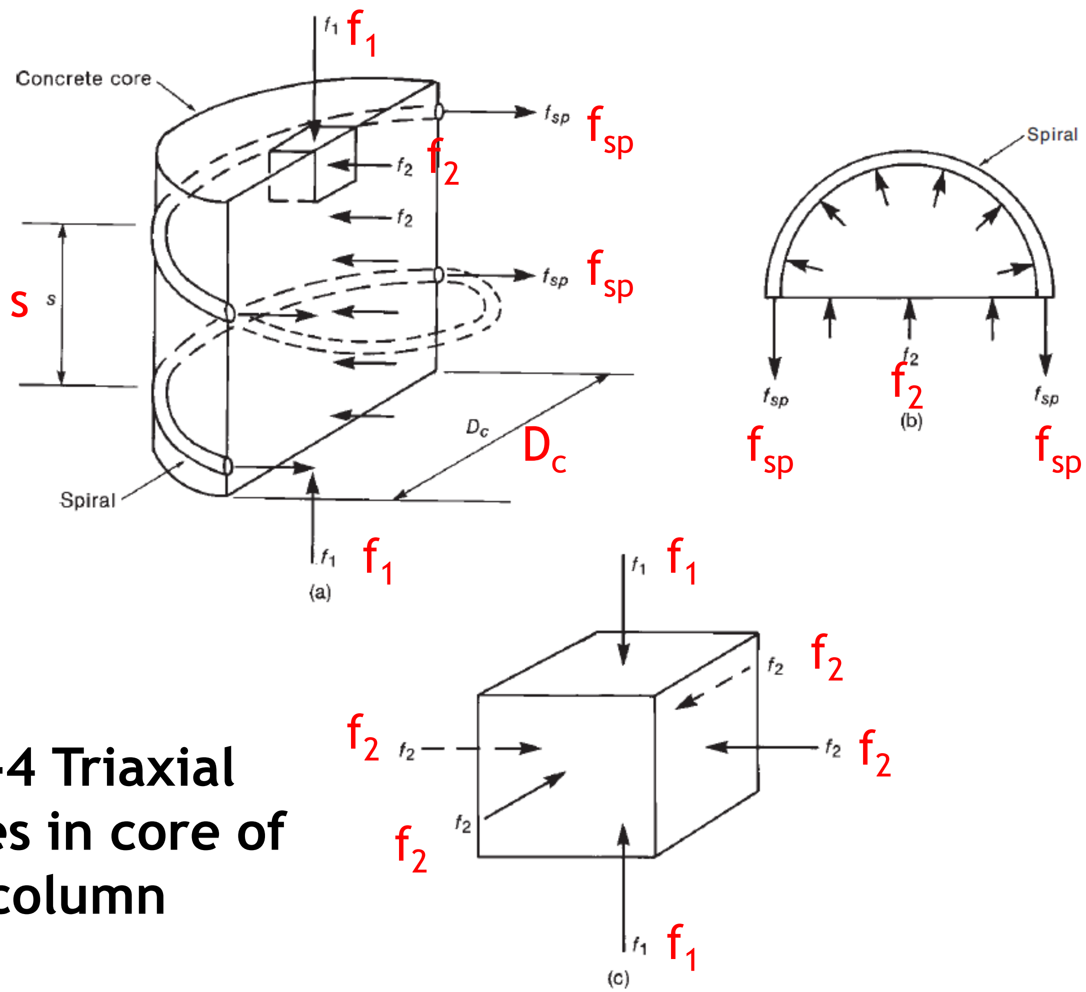


Fig 11-4 Triaxial stresses in core of spiral column

2.1 TIED AND SPIRAL COLUMNS

- This lateral expansion is especially pronounced at stresses in excess of 70% of the cylinder strength.
- In spiral column, the lateral expansion of the concrete inside the spiral (the core) is restrained by the spiral.
- This stresses the spiral in tension (see fig). For equilibrium the concrete is subjected to lateral compressive stresses f_2.
- An element taken out of the core (see fig) is subjected to triaxial compression which increases the strength of concrete: $f_1 = f_c' + 2.1 f_2$.

2.1 TIED AND SPIRAL COLUMNS

- In a tied column in a nonseismic region, the ties are spaced roughly the width of the column apart and, as a result, provide relatively little lateral restraint to the core.
- Hence, normal ties have little effect on the strength of the core in a tied column. They do, however, act to reduce the unsupported length of the longitudinal bars, thus reducing the danger of buckling of those bars as the bar stresses approach yield.

2.1 TIED AND SPIRAL COLUMNS

- Fig 11-5 presents load-deflection diagrams for a tied column and a spiral column subjected to axial loads. The initial parts of these diagrams are similar. As the maximum load is reached, vertical cracks and crushing develop in the concrete shell outside the ties or spiral, and this concrete spalls off. (SNS)
- When this occurs in a tied column, the capacity of the core that remains is less than the load on the column. The concrete core is crushed, and the reinforcements buckles outward b/n ties. This occurs suddenly, w/o warning, in a brittle manner.

2.1 TIED AND SPIRAL COLUMNS

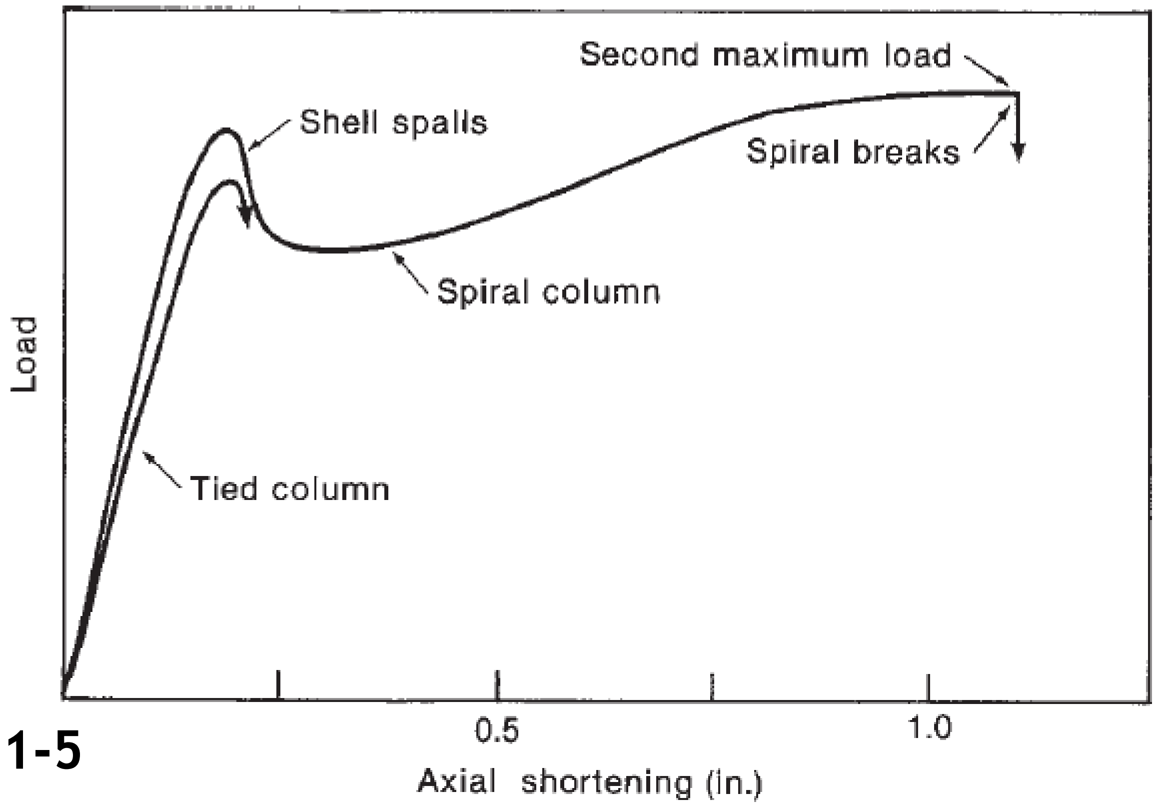


Fig 11-5

(a) Axially loaded columns.

2.1 TIED AND SPIRAL COLUMNS

- When the shell spalls off a spiral column, the column does not fail immediately because the strength of the cores has been enhanced by the triaxial stresses.
- As a result, the column can undergo large deformations, eventually reaching a 2nd maximum load, when the spirals yield and the column finally collapses.
- Such a failure is much more ductile and gives warning of impending failure (spalling of the concrete cover), along with possible load redistribution to other members

2.1 TIED AND SPIRAL COLUMNS

- Fig 11-6 and 11-7 show tied and spiral columns, respectively, after an EQ. Both columns are in the same building and have undergone the same deformations. The tied column has failed completely, while the spiral column, although badly damaged, is still supporting a load.
- The very minimal ties in Fig 11-6 were inadequate to confine the core concrete. Had the column been detailed according to ACI Section 21.4, the column would have performed much better.

2.1 TIED AND SPIRAL COLUMNS

Fig. 11-6
Tied column destroyed in 1971 San Fernando earth-quake. (Photograph courtesy of National Bureau of Standards.)

2.1 TIED AND SPIRAL COLUMNS

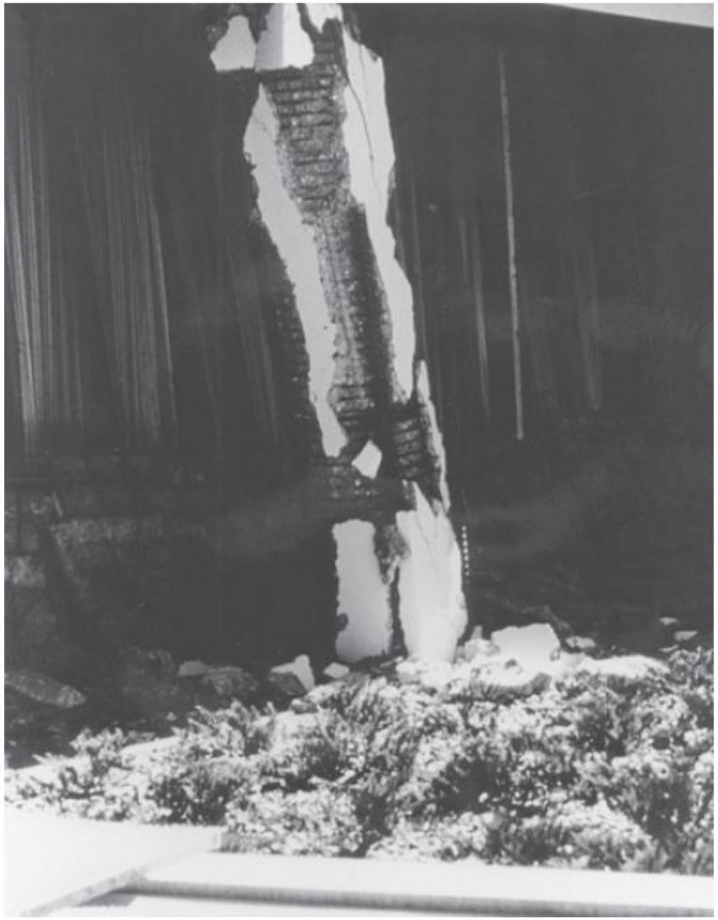


Fig. 11-7
Spiral column damaged by 1971 San Fernando earthquake. Although this column has been deflected sideways 20 in., it is still carrying load. (Photograph courtesy of National Bureau of Standards.)

2.2 STRENGTH OF AXIALLY LOADED COLUMNS

- When a symmetrical column is subjected to a concentric axial load, P, longitudinal strains ε, develop uniformly across the section as shown in Fig 11-8a.
- Because the steel and concrete are bonded together, the strains in the concrete and steel are equal. For any given strain, it is possible to compute the stresses in the concrete and steel using the stress-strain curves for the two materials.
- Failure occurs when P_o reaches a maximum: $P_o = f_{cd}A_c + f_{yd}A_{s,tot}$ (where $A_c = A_g - A_{s,tot}$)

2.2 STRENGTH OF AXIALLY LOADED COLUMNS

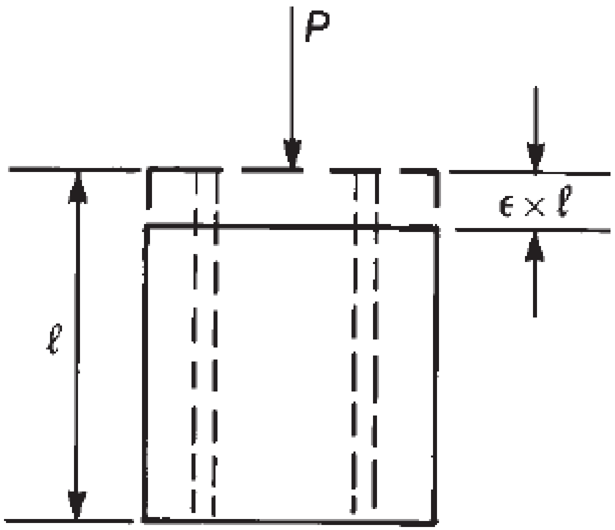


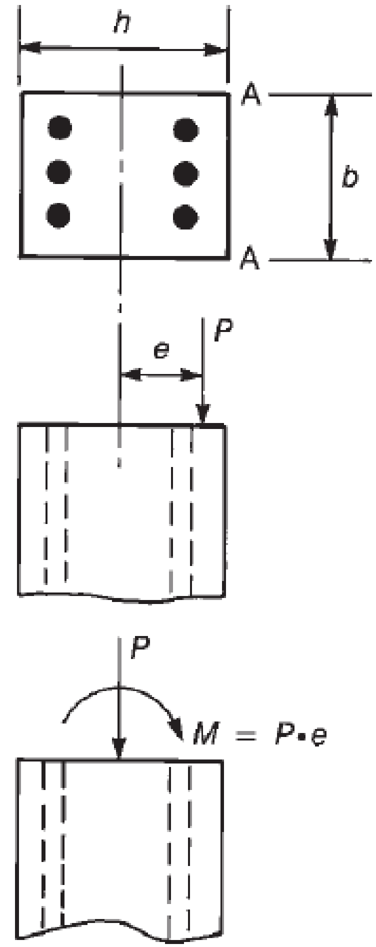
Fig 11-8

(a) Strains in column.

2.3 STRENGTH OF COLUMNS UNDER AXIAL LOAD AND BENDING

- Almost all compression members in concrete structures are subjected to moments in addition to axial loads. These may be due to misalignment of the load on the column, as shown in Fig 11-9b, or may result from the column resisting a portion of the unbalanced moments at the ends of the beams supported by the columns (Fig 11-9c). (SNS)
- The distance e is referred to as the eccentricity of load. These 2 cases are the same, because the eccentric load can be replaced by an axial load P plus a moment $M=P\times e$ about the centroid.

2.3 STRENGTH OF COLUMNS UNDER AXIAL LOAD AND BENDING



(a) Cross section.

(b) Eccentric load.

(c) Axial load and moment.

Fig. 11-9
Load and moment on column.

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- **Interaction Diagrams**
- Interaction diagrams for columns are generally computed by assuming a series of strain distributions at the ULS (SNS), each corresponding to a particular point on the interaction diagram, and computing the corresponding values of P and M.
- M is determined w.r.t the centroid of the x-section because analysis results are referred to the centroidal axis
- Once enough such points have been computed, the results are summarized in an interaction diagram (see Fig 11-13)discussion

STRAIN DISTRIBUTION IN THE ULTIMATE LIMIT STATE (OLD)

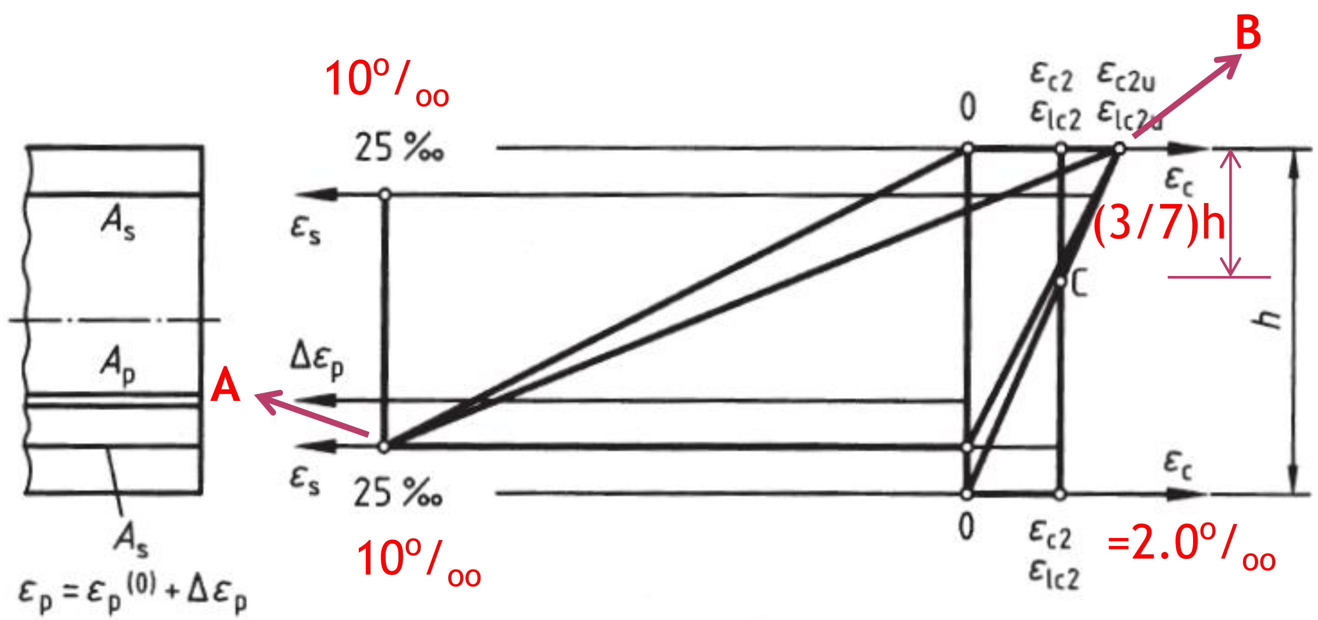
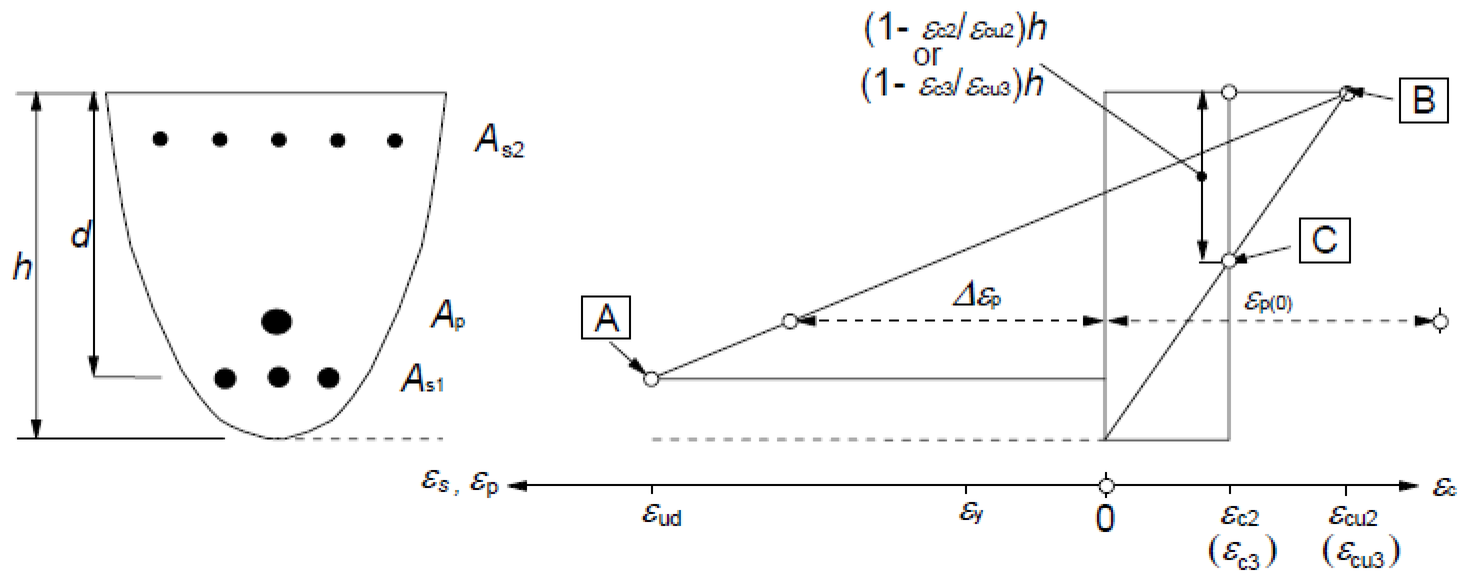


Figure 30: Possible distribution of strain in reinforcement and prestressing steel at the ultimate limit state

STRAIN DISTRIBUTION IN THE ULTIMATE LIMIT STATE (NEW)



A - reinforcing steel tension strain limit

B - concrete compression strain limit

C - concrete pure compression strain limit

Figure 6.1: Possible strain distributions in the ultimate limit state

STRESS-STRAIN RELATIONS OF CONCRETE FOR THE DESIGN OF CROSS SECTIONS

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \quad \text{for} \quad 0 \le \varepsilon_c \le \varepsilon_{c2}$$

$$\sigma_c = f_{cd} \quad \textit{for} \quad \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$$

where:

- n is the exponent according to Table 3.1
- ε_{c2} is the strain at reaching the maximum strength according to Table 3.1
- ε_{cu2} is the ultimate strain according to Table 3.1

TABLE 3.1(EN 1992-1-1:2004)

Table 3.1 Strength and deformation characteristics for concrete

Strength classes for concrete															Analytical relation / Explanation
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90	
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105	
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{cm} = f_{ck}+8$(MPa)
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0	$f_{ctm}=0,30\times f_{ck}^{(2/3)} \leq$ C50/60 $f_{ctm}=2,12\cdot\ln(1+(f_{cm}/10))$ > C50/60
$f_{ctk,0,05}$ (MPa)	1,1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5	$f_{ctk;0,05}=0,7\times f_{ctm}$ 5% fractile
$f_{ctk,0,95}$ (MPa)	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6	$f_{ctk;0,95}=1,3\times f_{ctm}$ 95% fractile
E_{cm} (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44	$E_{cm}=22[(f_{cm})/10]^{0,3}$ (f_{cm} in MPa)
ε_{c1} (‰)	1,8	1,9	2,0	2,1	2,2	2,25	2,3	2,4	2,45	2,5	2,6	2,7	2,8	2,8	see Figure 3.2 ε_{c1}(‰) $=0,7\,f_{cm}^{0,31} < 2,8$
ε_{cu1} (‰)	3,5									3,2	3,0	2,8	2,8	2,8	see Figure 3.2 for $f_{ck} \geq 50$ Mpa ε_{cu1}(‰)$=2,8+27[(98-f_{cm})/100]^4$
ε_{c2} (‰)	2,0									2,2	2,3	2,4	2,5	2,6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa ε_{c2}(‰)$=2,0+0,085(f_{ck}-50)^{0,53}$
ε_{cu2} (‰)	3,5									3,1	2,9	2,7	2,6	2,6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa ε_{cu2}(‰)$=2,6+35[(90-f_{ck})/100]^4$
n	2,0									1,75	1,6	1,45	1,4	1,4	for $f_{ck} \geq 50$ Mpa $n=1,4+23,4[(90-f_{ck})/100]^4$
ε_{c3} (‰)	1,75									1,8	1,9	2,0	2,2	2,3	see Figure 3.4 for $f_{ck} \geq 50$ Mpa ε_{c3}(‰)$=1,75+0,55[(f_{ck}-50)/40]$
ε_{cu3} (‰)	3,5									3,1	2,9	2,7	2,6	2,6	see Figure 3.4 for $f_{ck} \geq 50$ Mpa ε_{cu3}(‰)$=2,6+35[(90-f_{ck})/100]^4$

STRESS-STRAIN RELATIONS OF CONCRETE FOR THE DESIGN OF CROSS SECTIONS

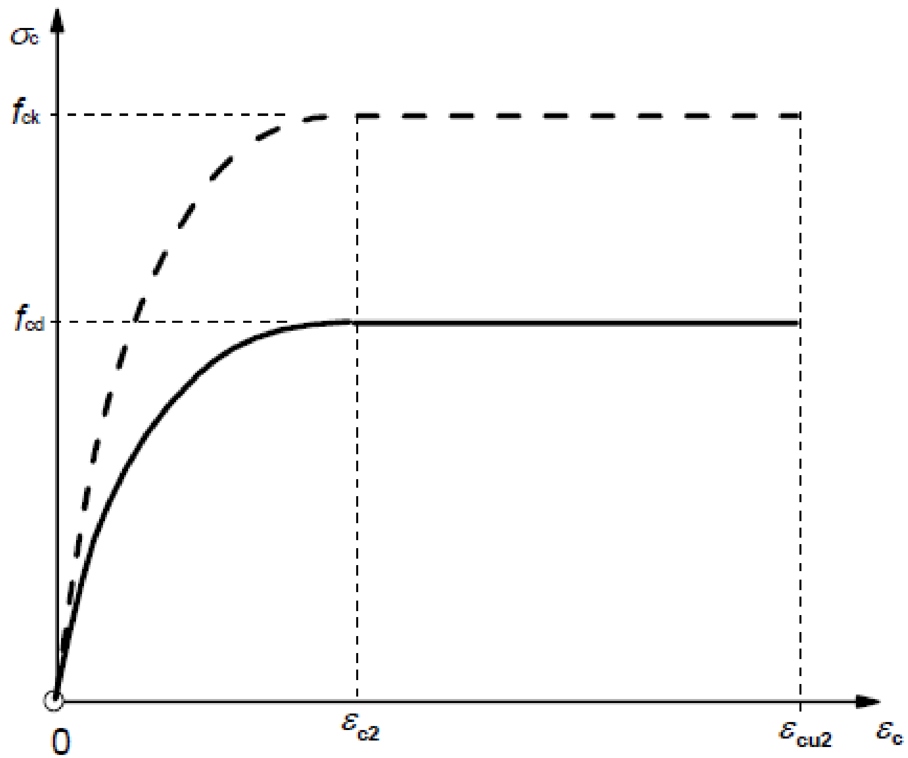


Figure 3.3: Parabola-rectangle diagram for concrete under compression.

STRESS-STRAIN RELATIONS OF STEEL FOR THE DESIGN OF CROSS SECTIONS

3.2.7 Design assumptions

(1) Design should be based on the nominal cross-section area of the reinforcement and the design values derived from the characteristic values given in 3.2.2.

(2) For normal design, either of the following assumptions may be made (see Figure 3.8):

a) an inclined top branch with a strain limit of ε_{ud} and a maximum stress of kf_{yk}/γ_s at ε_{uk}, where $k = (f_t/f_y)_k$,

b) a horizontal top branch without the need to check the strain limit.

Note 1: The value of ε_{ud} for use in a Country may be found in its National Annex. The recommended value is $0,9\varepsilon_{uk}$

Note 2: The value of $(f_t/f_y)_k$ is given in Annex C.

STRESS-STRAIN RELATIONS OF STEEL FOR THE DESIGN OF CROSS SECTIONS

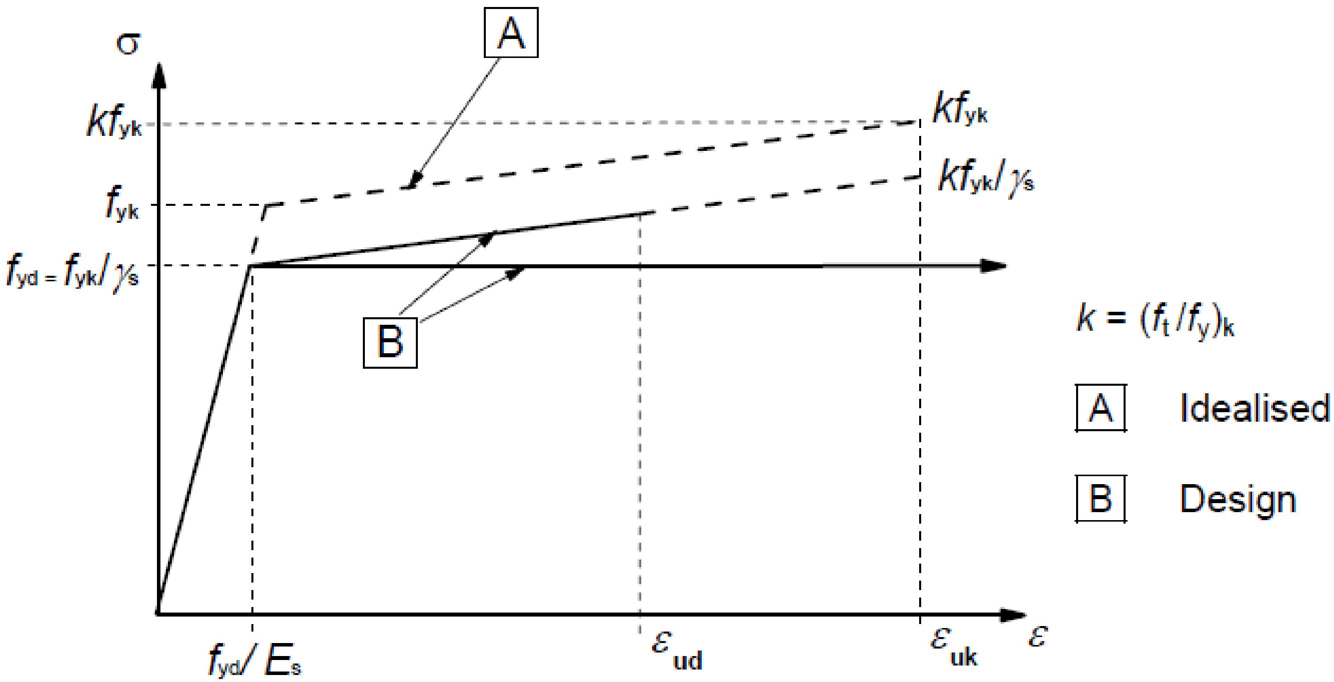
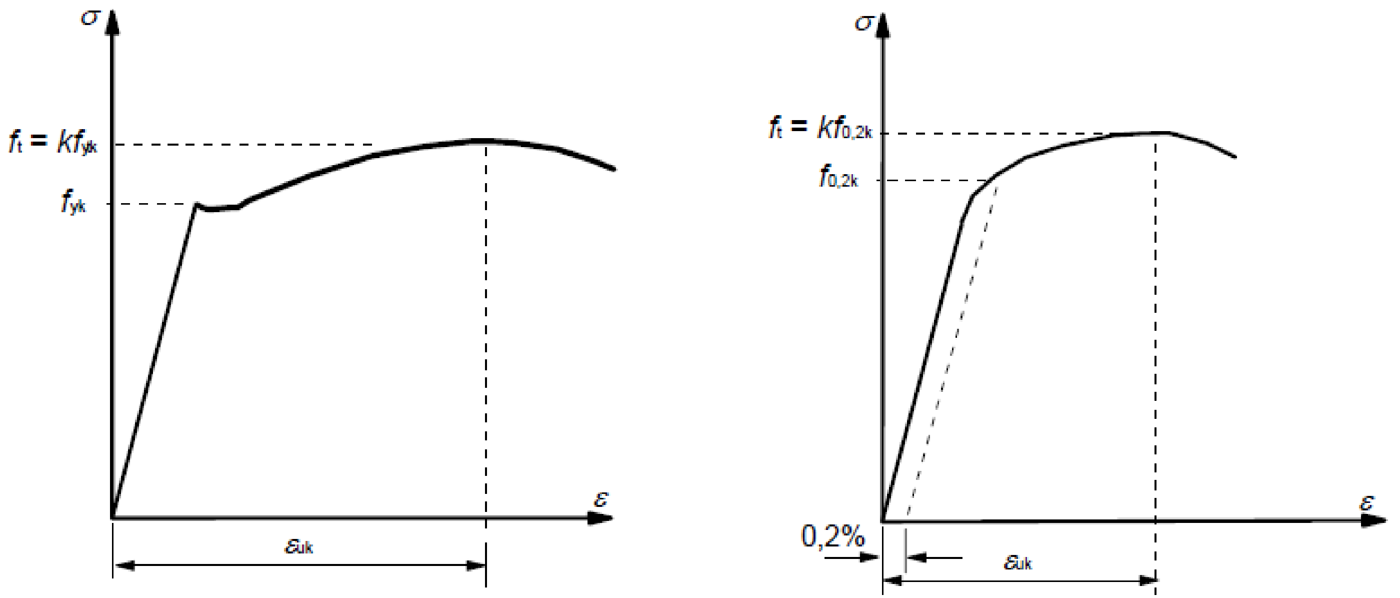


Figure 3.8: Idealised and design stress-strain diagrams for reinforcing steel (for tension and compression)

STRESS-STRAIN DIAGRAMS OF REINFORCING STEEL



a) Hot rolled steel **b) Cold worked steel**

Figure 3.7: Stress-strain diagrams of typical reinforcing steel (absolute values are shown for tensile stress and strain)

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

Uniform compression

Onset of cracking

Axial load resistance

Balanced failure

Moment resistance

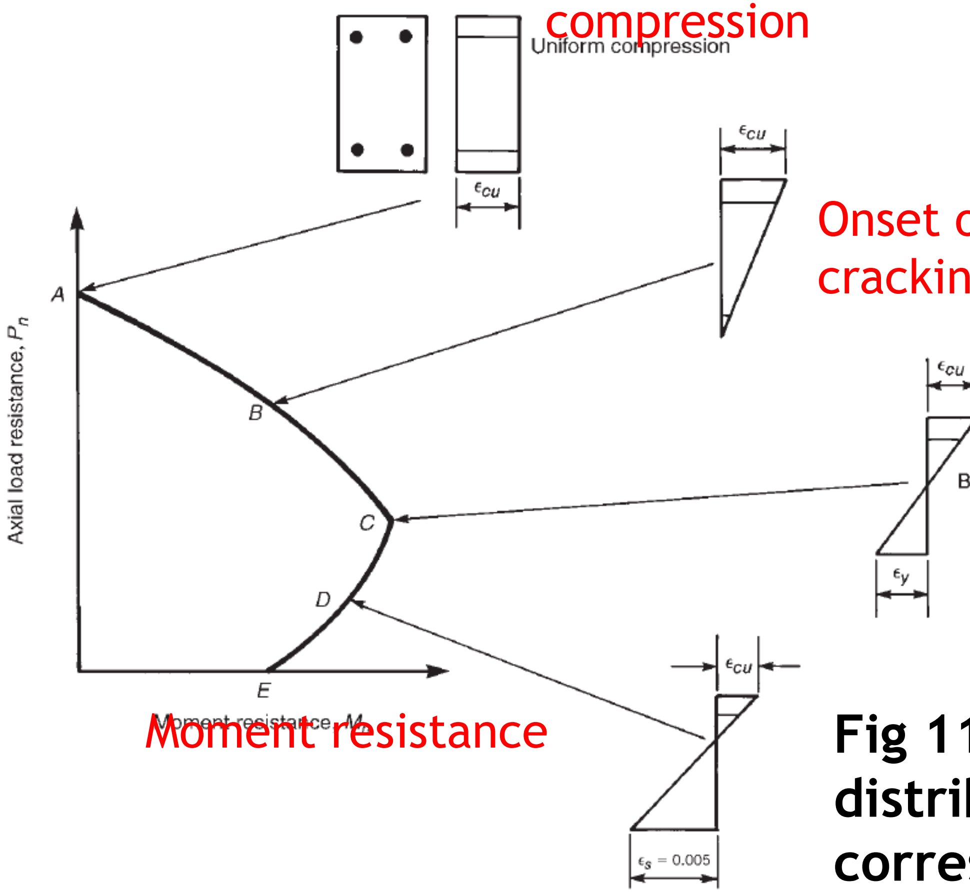


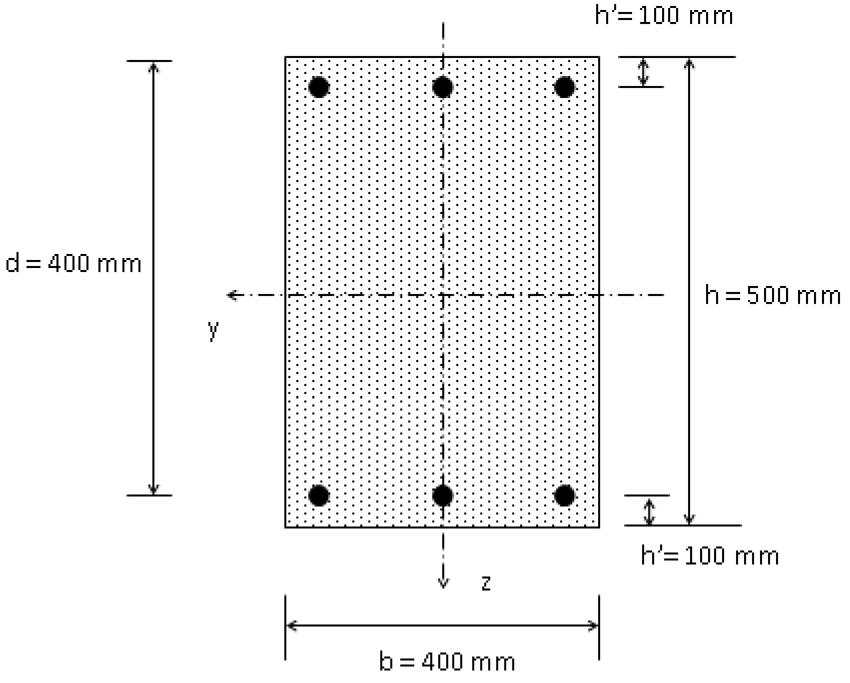
Fig. 11-13
Strain distributions corresponding to points on the interaction diagram.

Fig 11-13 Strain distributions corresponding to points on the interaction diagram

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- Draw the interaction diagram for the column cross section (SNS). Use class of concrete C-30 and grade of reinforcing steel, S-460.
- Show a minimum of 6 points on the interaction diagram corresponding to
  - 1. Pure axial compression
  - 2. Balanced failure
  - 3. Zero tension (Onset of cracking)
  - 2. Pure flexure
  - 5. A point b/n balanced failure and pure flexure
  - 6. A point b/n pure axial compression and zero tension

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS



C-30 concrete

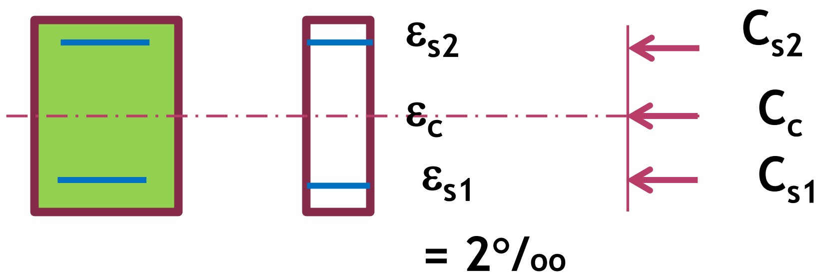
S-460 reinforcing steel

$A_{s,tot} = 6800\ mm^2$

Fig. Column cross section

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- Solution
  - 1. Pure axial compression

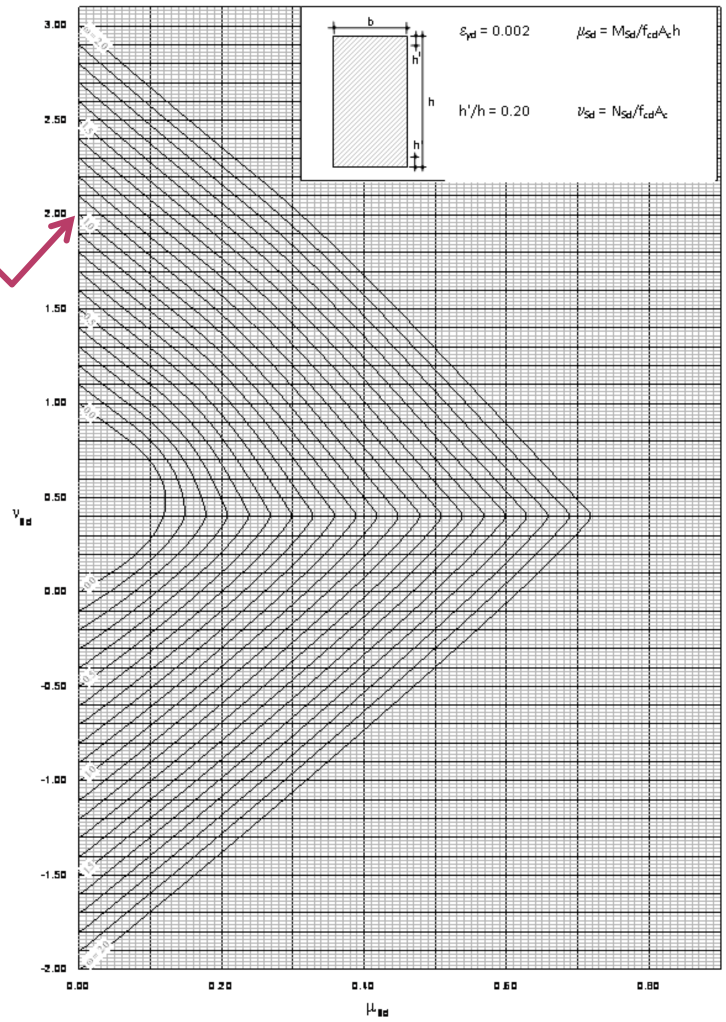


Cross section strain distribution(ULS) stress resultant

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

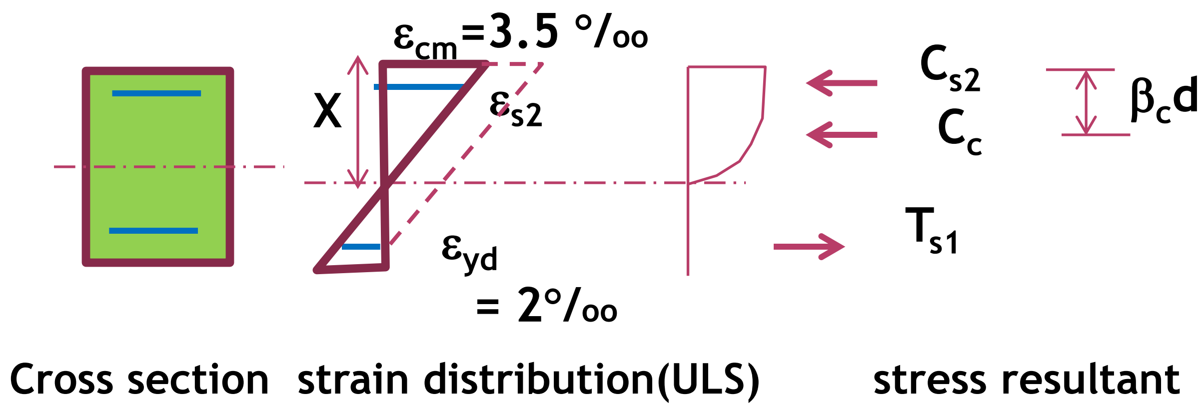
- Cont'd
- $P_u = C_{s2} + C_{s1} + C_c = \sigma_{s2}A_{s2} + \sigma_{s1}A_{s1} + f_{cd}bh$
- $\varepsilon_{yd} = f_{yd}/E_s = 400/200000 = 0.002 \rightarrow$ reinforcement has yielded
- $\rightarrow P_u = f_{yd}A_{s,tot}/2 + f_{yd}A_{s,tot}/2 + f_{cd}bh = f_{yd}A_{s,tot} + f_{cd}bh$
- $\nu_u = P_u / f_{cd}bh = (f_{yd}A_{s,tot})/(f_{cd}bh) + (f_{cd}bh)/(f_{cd}bh) = \rho f_{yd}/f_{cd} + 1 =$ **$\omega + 1$; where ω is called the mechanical reinforcement ratio and equal to $(6800/(400\times500))\times(400/13.6) = 1.0 \rightarrow$** $\nu_u = 1 + 1 = 2.0$

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS



2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- 2. balanced failure



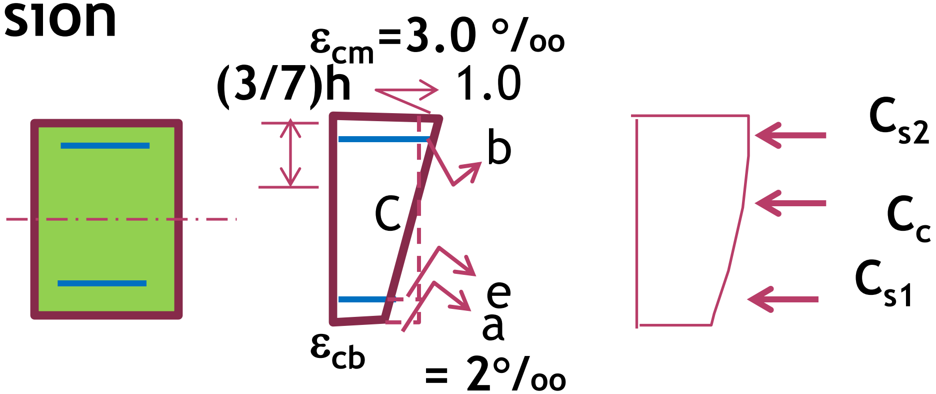
- $x/3.5 = d/(3.5+2) \rightarrow x = (400/5.5)\times 3.5 = 254.5454$ mm $\rightarrow \varepsilon_{s2}/(254.54-100)=3.5/254.54 \rightarrow \varepsilon_{s2} = 2.125°/oo > 2$ °/oo $\rightarrow$ reinforcement has yielded
- $\rightarrow C_{s2} = T_{s1} = 3400\times 400 = 1360000$ N

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- Cont'd
- $\varepsilon_{cm} > \varepsilon_o$ and NA is within the section $\rightarrow \alpha_c = k_x(3\varepsilon_{cm} - 2)/3\varepsilon_{cm} = (254.54/400)(3\times3.5\text{-}2)/(3\times3.5) = 0.5151 \rightarrow C_c = \alpha_c f_{cd} bd = 0.51\times13.6\times400\times400 =$ 1120967.7 N
- $\beta_c = k_x(\varepsilon_{cm}(3\varepsilon_{cm}\text{-}4)+2)/(2\varepsilon_{cm}(3\varepsilon_{cm}\text{-}2)) = (254.54/400)(3.5(3\times3.5\text{-}4)+2)/(2\times3.5(3\times3.5\text{-}2)) = 0.2647 \rightarrow \beta_c d = 0.2647\times400 = 105.882$ mm $\rightarrow$
- $M_u = C_c(h/2\text{-}\ \beta_c d) + C_{s2}(h/2\text{-}h') + T_{s1}(h/2\text{-}h') = 1120969.7(250\text{-}105.882) + 1360000(250\text{-}100) + 1360000(250\text{-}100) = 569551912.2$ Nmm
- $P_u = C_c + T_{s1} + C_{s2} = C_c = 1120969.7 \rightarrow$

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- Cont'd
- ν_u = P/(f_{cd}bh) = 1120969.7/(13.6×400×500) = **0.412**
- μ_u = M_u/(f_{cd}bh^2) = 569551912.2/(13.6×400×500^2) = **0.419**
- **6. A point b/n pure axial compression and zero tension**



Cross section strain distribution(ULS) stress resultant

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- Choose ε_{cm} = 3 °/₀₀ (strain profile passes also thru C)
- Strain in the bottom concrete fiber ε_{cb}: from a=((4/7)/(3/7))×1 = 4/3 = 1.33 → ε_{cb} = 2 -1.33 = 0.667 °/₀₀ → (entire cross section under compression as assumed)
- Determine strain in reinforcement: from b/114.286 = 1/214.286 → b = 0.533 °/₀₀ → ε_{s2} = 2 + 0.533 = 2.533 °/₀₀ > 2 °/₀₀ → reinforcement has yielded and from e/185.714 = 1.33/285.714 → e = 0.867 °/₀₀ → ε_{s1} = 2-0.867 = 1.133 °/₀₀ < 2 °/₀₀ → reinforcement has not yielded

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

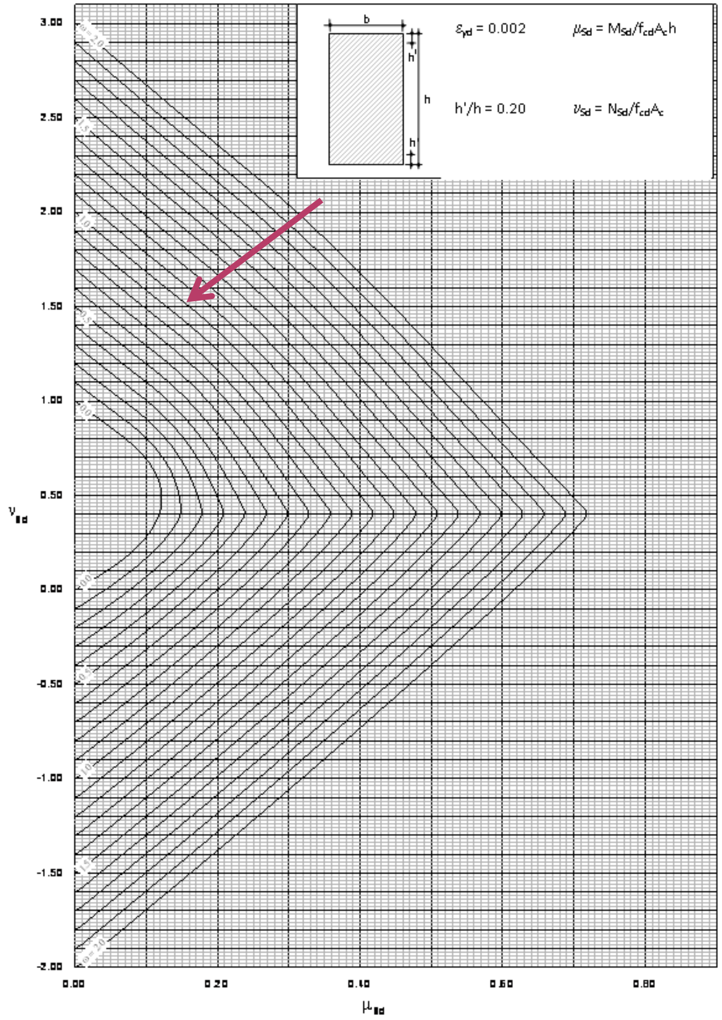
- Cont'd
- $\varepsilon_{cm} > \varepsilon_o$ and NA outside of the section$\rightarrow$ α_c = $(1/189)(125+64\varepsilon_{cm}-16\varepsilon_{cm}^2) = (1/189)(125+64(3)-16(3)^2) = 0.915344 \rightarrow$ $C_c = \alpha_c f_{cd} bd = 0.915344\times13.6\times400\times400 = 1991788.4$ N; $C_{s2} = (A_{s,tot}/2)\times f_{yd} = 3400\times400 = 1360000$ N; $C_{s1} = (A_{s,tot}/2)\times\sigma_{s1} = 3400\times(1.133/1000)\times200000 = 770666.67$ N $\rightarrow$
- $\beta_c = 0.5-(40/7)(\varepsilon_{cm}-2)^2/(125+64\varepsilon_{cm}-16\varepsilon_{cm}^2) = 0.5-(40/7(3-2)^2/(125+64\times3-16\times3^2) = 0.467 \rightarrow \beta_c d = 0.467\times400 = 186.788$ mm
- $P_u=C_c+C_{s1}+C_{s2} = 4122455.1$ N
- $M_u = 1991788.4(250-186.788)+1360000(250-100)-770666.67(250-100) = 214304927.8$ Nmm

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- Cont'd
- ν_u = $P_u/(f_{cd}bh)$ = $4122455.1/(13.6\times400\times500)$ = **1.5156**
- μ_u = $M_u/(f_{cd}bh^2)$ = $214304927.8/(13.6\times400\times500^2)$ = **0.1576**

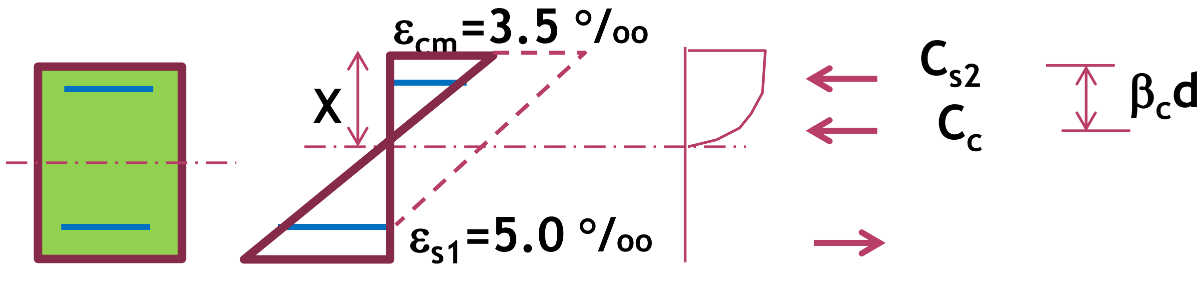
2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

0.2 0.4 0.6 0.8



2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- **2. Pure flexure**
- Start with $\varepsilon_{cm} / \varepsilon_{s1}$ = 3.5 °/₀₀ / **5 °/₀₀ and repeat until $|\nu_u| \approx 0.00$**



Cross section strain distribution stress resultant

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- After some trials$\rightarrow$
- Use $\varepsilon_{cm}/\varepsilon_{s1}$ = 3.5 °/₀₀ / **6.227 °/₀₀**
- k_x= 3.5/(3.5+6.227)=0.359823 $\rightarrow$x=143.9293mm
- ε_{s2} = ((x-100)/x)×3.5 = **1.06825 °/₀₀**$\rightarrow$ **C_{s2}=3400×(1.06825/1000)×200000=726410N**
- $\alpha_c = k_x(3\varepsilon_{cm} - 2)/3\varepsilon_{cm}$ = (143.9293/400)(3×3.5-2)/(3×3.5) = 0.291285 $\rightarrow$ $C_c = \alpha_c f_{cd} bd$ = 0.291285 ×13.6×400×400 = 633837.1 N
- $P_u=C_c+T_{s1}+C_{s2}$ = 633837.1-1360000+726410 = 247.1N
- $\rightarrow$ $\nu_u = P_u/(f_{cd}\times b\times h)$ = 247.1/(13.6×400×500) = 0.00

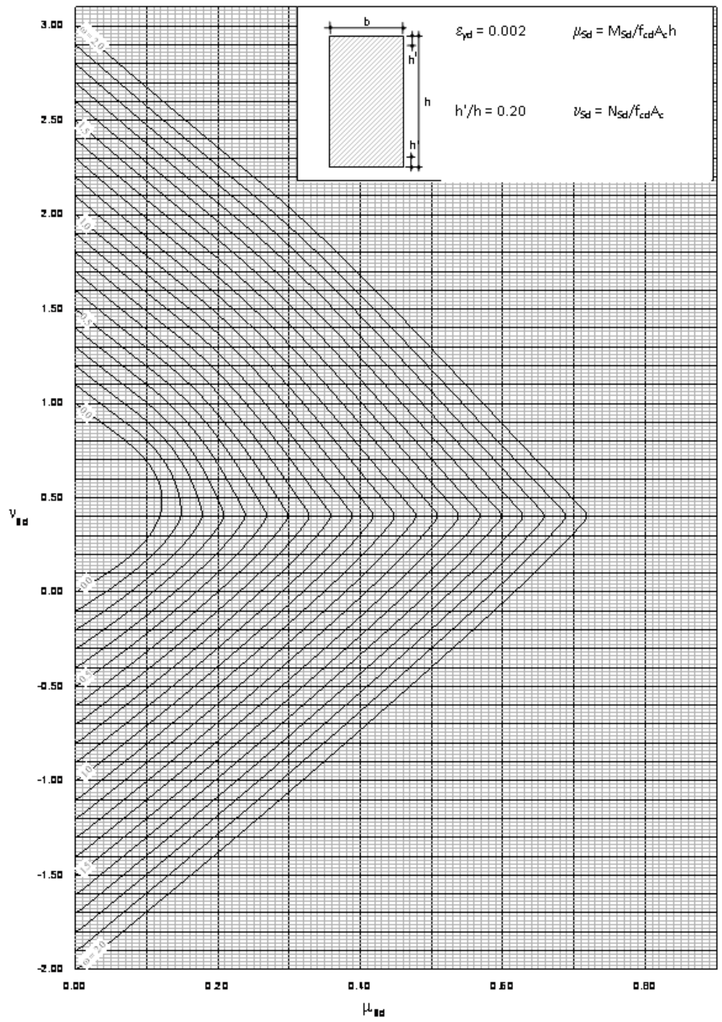
2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- $\beta_c = k_x(\varepsilon_{cm}(3\varepsilon_{cm}-4)+2)/(2\varepsilon_{cm}(3\varepsilon_{cm}-2)) = (143.9293/400)(3.5(3\times3.5-4)+2)/(2\times3.5(3\times3.5-2)) = 0.149674 \rightarrow \beta_c d = 0.149674\times400 = 59.8696$ mm $\rightarrow$
- $M_u = C_c(h/2- \beta_c d) + C_{s2}(h/2-h') + T_{s1}(h/2-h') = 633837.1(250-59.8696) + 726410(250-100) + 1360000(250-100) = 433473111$ Nmm
- $\mu_u = M_u/(f_{cd}bh^2) = 433473111 /(13.6\times400\times500^2) = 0.319$

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS

- Insert Uchart1 with cover ratio h'/h = 0.2 (discussion about the systematic production of such uniaxial interaction diagrams)
- SNS or project from the original

2.4 INTERACTION DIAGRAMS FOR CONCRETE COLUMNS



2.5 BIAXIALLY LOADED COLUMNS

- Up to this point we have dealt with columns subjected to axial loads accompanied by bending about one axis. It is not unusual for columns to support axial forces and bending about two ⊥ axes (corner columns under gravity loads or other columns for LC: gravity plus lateral loading)
- For a given cross section and reinforcing pattern, one can draw an interaction diagram for axial load and bending about either axis. These interaction diagram form the two edges of an interaction surface for axial load and bending about 2 axes (SNS)

2.5 BIAXIALLY LOADED COLUMNS

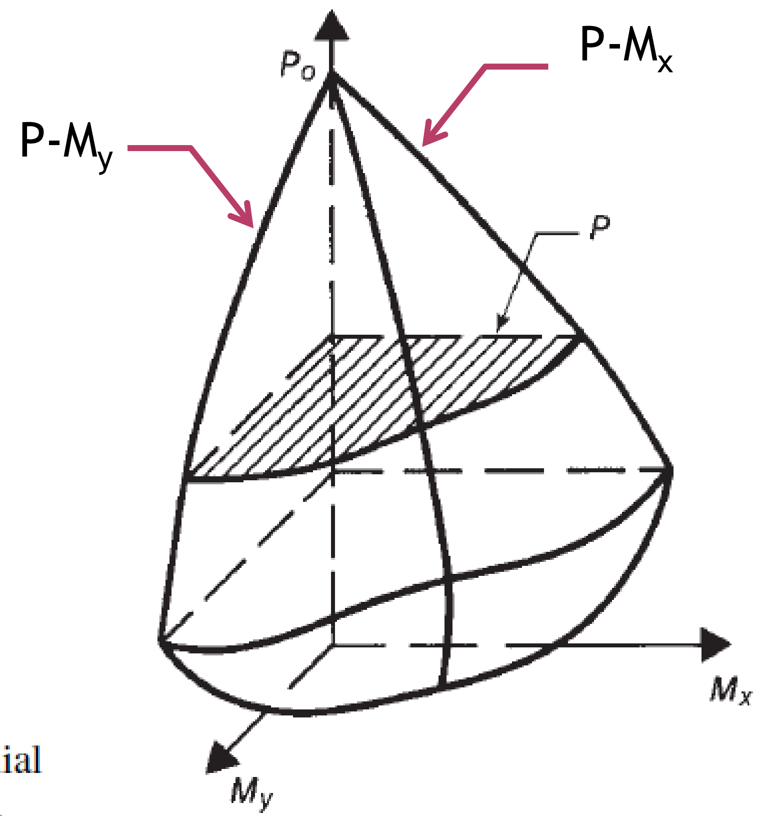
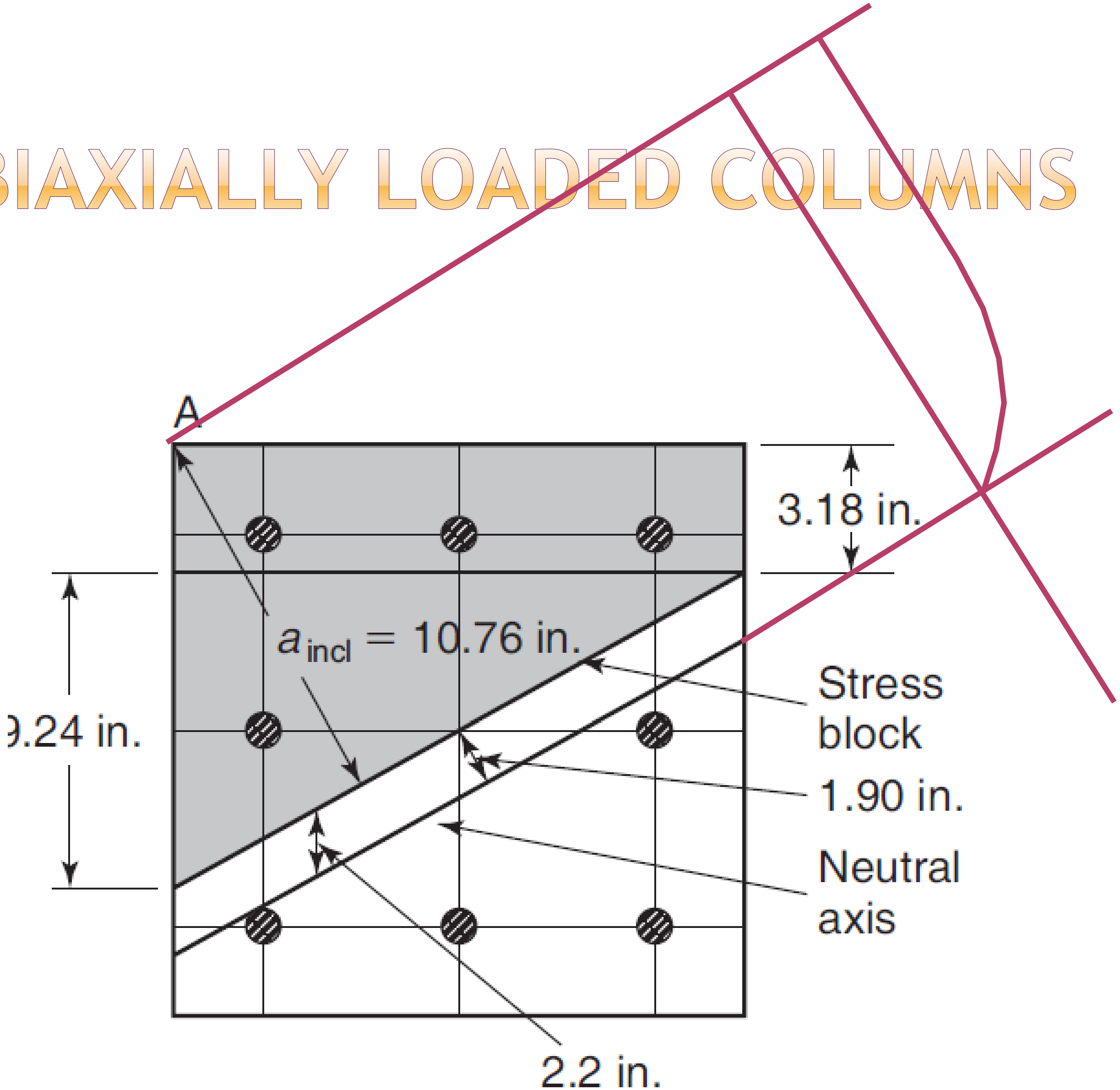


Fig. 11-32
Interaction surface for axial load and biaxial bending.

2.5 BIAXIALLY LOADED COLUMNS

- For a given cross section and reinforcement arrangement as shown in the NS, there exists a unique associated interaction surface. The stress resultant of a strain distribution in the ULS represents one point on the interaction surface. However this is not useful as 3D representation is not suitable for design aid calculations
- biaxial interaction diagrams calculated and prepared as load contours or P-M diagrams drawn on planes of constant angles relating the magnitudes of the biaxial moments are more suitable for design (but difficult to derive)
- More discussion in Chapter3

2.5 BIAXIALLY LOADED COLUMNS



2.5 BIAXIALLY LOADED COLUMNS

- Therefore approximate solutions are sought to solve biaxial bending problems. $\rightarrow$ seek approximate solutions
- The most common approximation common across different codes are:
  - Approximate equations for load contours (EBCS EN 1992-1-1, ACI, British code, etc)(refer publications on evaluation of the different approximate methods)

2.5 BIAXIALLY LOADED COLUMNS

- Rigorously derived biaxial interaction diagrams for EBCS-2: Part 1 and DIN 1045, refer to EBCS-2:Part 2 and Interaction diagrams for biaxial bending to DIN 1045
- They are prepared as load contours for biaxially loaded columns with different reinforcement arrangement (4-corner reinforcement, 8-rebar arrangement, uniformly distributed reinforcement on 2-edges, uniformly distributed reinforcement on 4-edges and so on.

2.5 BIAXIALLY LOADED COLUMNS

- Finally, how is this, i.e. cross section capacities relevant to column strengths? Ans: A column (short or slender is said to have reached an ULS when the critical cross section has reached an ultimate limit state). So the design of a column is reduced to the design of the critical cross section located somewhere along the length of the column unless the column is very slender and reaches the ULS of instability
- Project biaxial charts from original document, i.e. EBCS-2:Part 2 and DIN 1045

2.6 SLENDER COLUMNS

- In this chapter we will deal with many topics outlined in the course content. They include:
  - ULS of buckling (discussion)/ " EBCS EN 1992-1-1 $\rightarrow$ Analysis of 2nd order effects with axial load"/ " German Lit $\rightarrow$ULS Induced by Lateral Deflection of Columns"/ "ACI $\rightarrow$Slender Columns"
  - PΔ analysis
  - Rigorous 2nd order analysis
  - Strain compatibility principles for cross section analysis (see more discussion in chapter 3)
  - Moment-Curvature relationship
  - Assignments

2.6 SLENDER COLUMNS

- **2.6.1 Introduction**
- An eccentrically loaded, pin end columns is shown in figure (SNS). The moments at the ends of the column are:
  - $M_e = P \times e$
- When the loads P are applied the column deflects laterally by an amount δ as shown. For equilibrium, the internal moment at mid-height must be:
  - $M_c = P(e + \delta)$, i.e. the deflection increases the moments for which the columns must be designed (Note: 2^{nd} order analysis!)

2.6 SLENDER COLUMNS

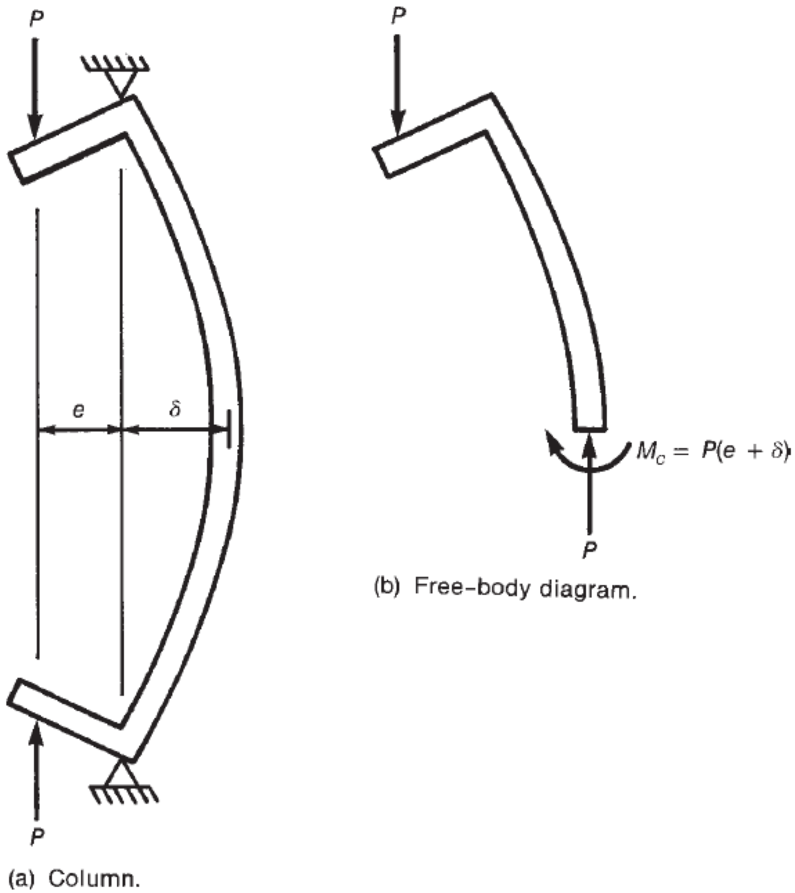


Fig. 12-1
Forces in a deflected column.

2.6 SLENDER COLUMNS

- The load-moment curves for the end moments at the support and the maximum moment at mid-height are drawn on the interaction diagram of the column (SNS)
  - OA is the load-moment curve for the end moment
  - OB is the load-moment curve for the maximum column moment
- Failure occurs when the load moment curve OB for the critical section intersects the interaction diagram for the cross-section.

2.6 SLENDER COLUMNS

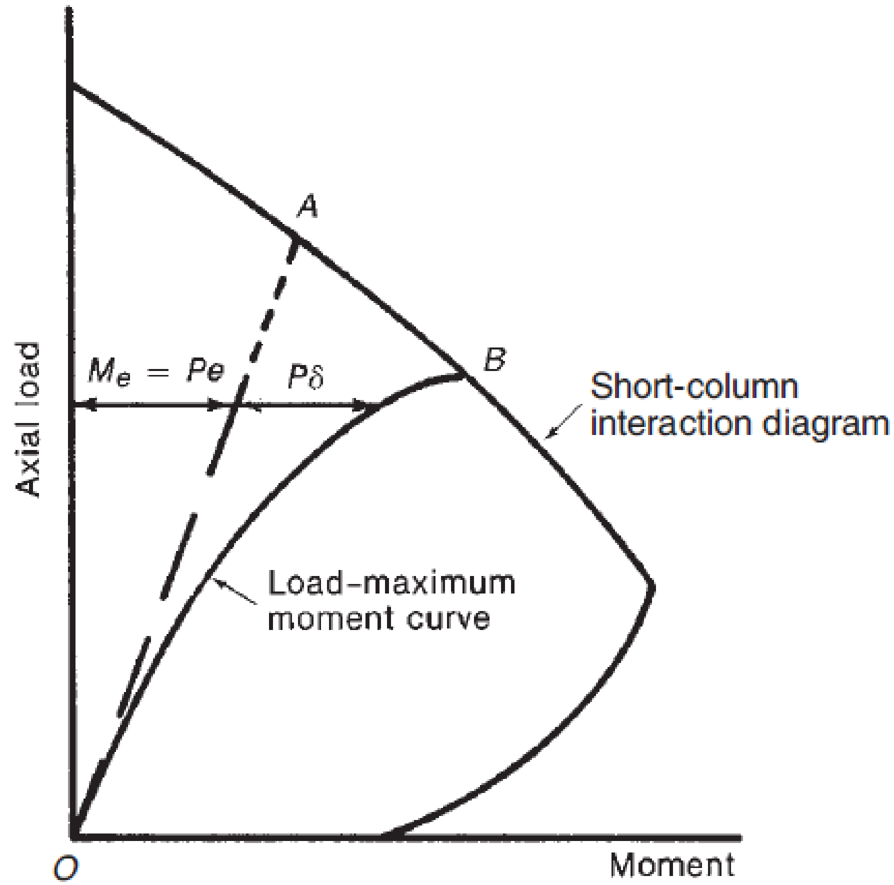


Fig. 12-2
Load and moment in a column.

2.6 SLENDER COLUMNS

- Thus the load and moment at failure are denoted by point B in the figure.
- Because of the increase in the maximum moment due to deflections, the axial load capacity is reduced from A to B. This reduction in axial load capacity results from what are referred to as slenderness effects.
- A slender column is defined as a column that has a significant reduction in its axial load capacity, due to moments that result from lateral deflections of the column. In the ACI a “significant reduction” was taken as $\geq$ 5%.

2.6 SLENDER COLUMNS

- **2.6.2 Buckling of axially loaded elastic columns**
- Figure (SNS) illustrates 3 states of equilibrium. If the ball in Fig a is displaced laterally and released, it will return to its original position. This is a **stable equilibrium**. If the ball in Fig c is displaced and released, it will roll off the hill. This is **unstable equilibrium** The transition b/n stable and unstable equilibrium is **neutral equilibrium** illustrated in Fig b. Here the ball will remain in the displaced position.

2.6 SLENDER COLUMNS

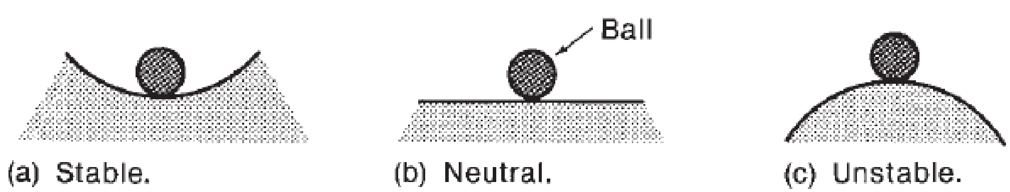
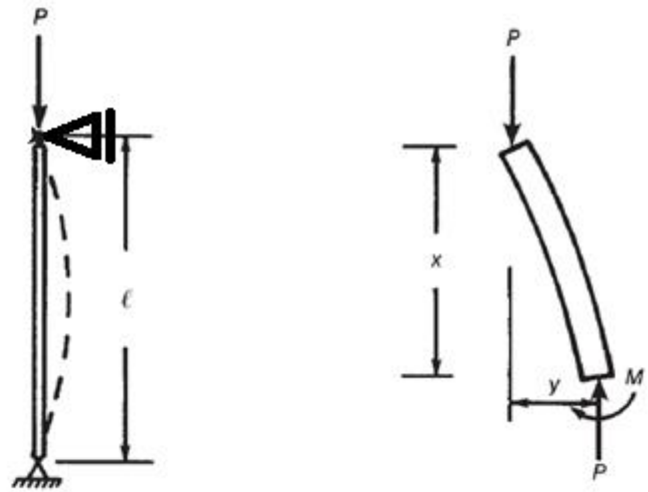


Fig. 12-3
States of equilibrium.

2.6 SLENDER COLUMNS

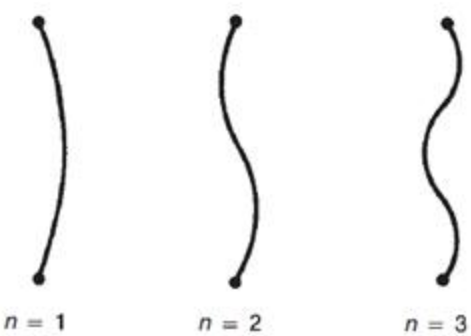
- Similar states of equilibrium exist for the axially loaded column in the figure (SNS) provided that the following conditions are fulfilled
  - The column is made of a linearly elastic material that follows Hook's law
  - The column is perfectly straight
  - The column is loaded by a vertical load P that is applied through the centroid of the cross-section and aligned with the longitudinal axis of the column
  - Deflections are small so that the approximate formula for curvature can be used
- Such a column is called an ideal column

2.6 SLENDER COLUMNS



(a) Column.

(b) Free-body diagram.



(c) Number of half-sine waves.

Fig. 12-4
Buckling of a pin-ended column.

2.6 SLENDER COLUMNS

- Such a column remains straight and undergoes only axial compression when the axial load $P < P_{cr}$. The **straight form of the equilibrium is stable**, which means that the column returns to the straight position if it is disturbed.
- As the load is gradually increased, we reach a condition of neutral equilibrium and the corresponding value of the load is the **critical load P_{cr}**. The critical load can maintain the column in static equilibrium **either in the straight position or in a slightly bent position**.

2.6 SLENDER COLUMNS

- **This equilibrium state is called neutral state and the governing DE for the column in neutral equilibrium is:**
- **$v'' + k^2 v = 0$ (v = the lateral deflection and $k^2 = P/EI$**
- At higher values of the load, the column is **unstable** and will collapse by bending. For the ideal case at hand, the **column is in equilibrium in the straight position even when $p > p_{cr}$**. However, the **equilibrium is unstable**, and the smallest imaginable disturbance will cause the column to deflect sideways; the deflections increase immediately and the column will collapse

2.6 SLENDER COLUMNS

- The buckling of the ideal column is associated with bifurcation of equilibrium at P_{cr} (column is in neutral equilibrium in either the straight or a slightly bent position) as shown in the load-deflection diagram (SNS). These kind of analysis constitute stability **problem with bifurcation of equilibrium**
- Of course, actual columns do not behave in this idealized manner because imperfections always exist. Nevertheless it is instructive to study ideal columns because they provide insight into the behavior of real columns.
- That explains the statement in EC2, Section 5.8, pp 64

2.6 SLENDER COLUMNS

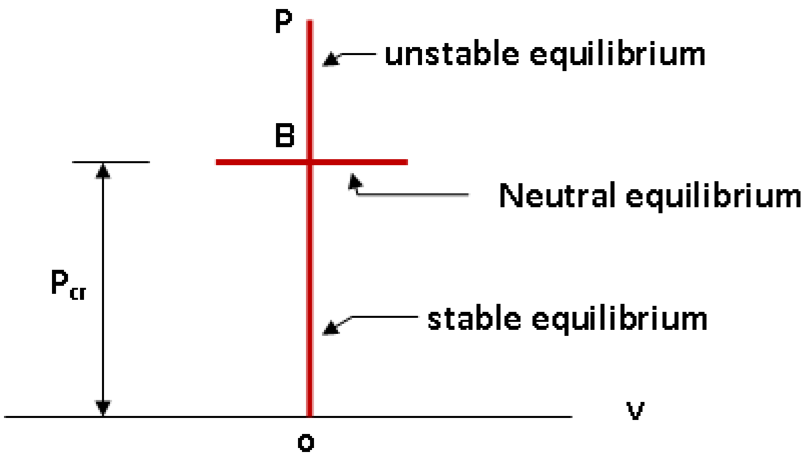


Fig: Load-deflection diagram for an ideal column (solution of the governing DE for stability analysis with bifurcation of equilibrium)

2.6 SLENDER COLUMNS

- For the simply supported ideal column, the general solution of the DE is:

  $v = C_1 \sin kx + C_2 \cos kx$

  From the boundary conditions $C_2 = 0$ and

  $C_1 \sin kL = 0$ (a)

  $\rightarrow C_1 = 0$ or $\sin kL = 0$

  $\rightarrow$ If $C_1 = 0$, the deflection v is zero and the column is straight. In that case Eq. (a) is satisfied for any value of the quantity kL. $\rightarrow$ The axial load P may also have any value even greater than P_{cr} **(Note: $k^2 = P/EI$).**

2.6 SLENDER COLUMNS

- This solution of the DE (often called the trivial solution) is represented by the vertical axis of the load-deflection diagram shown above. This solution corresponds to an ideal column that is in equilibrium (either stable or unstable) under the action of the compressive load P.
- The other possibility for satisfying Eq. (a) is to meet the following condition:
- $\sin kL = 0 \rightarrow kL = n\pi \rightarrow P = (n^2\pi^2EI/L^2)$ and the smallest critical load for the column is obtained when n = 1
- $\rightarrow P_{cr} = \pi^2EI/L^2$.

2.6 SLENDER COLUMNS

- The critical load for an ideal elastic column is also known as the **Euler load** after the famous mathematician Leonhard (1707-1783) that determined the critical load for an ideal column
- The corresponding buckled shape (sometimes called a mode shape) is

$$v = C_1 \sin(\pi x/L)$$

- The constant C_1 represents the deflection at midpoint ($x = L/2$), of the column and may be positive or negative. Therefore the part of the load-deflection diagram corresponding to P_{cr} is a horizontal straight line as shown in the figure above. The deflection at this load is undefined, although it must remain small for our equations to be valid

2.6 SLENDER COLUMNS

- The bifurcation point B is at the critical load; above point B the equilibrium is unstable, and below it stable
- **Effects of large deflections, imperfections, and inelastic behavior**
  - The equation for the critical load was derived for an ideal column in which the **deflections are small, the construction is perfect, and the material follows Hooke's law.**
  - As a consequence, we found that the **magnitudes of the deflections at buckling were undefined** (linear eigenvalue problem). Thus at $P = P_{cr}$, the column may have any small deflection, a condition represented by the horizontal line A in the load-deflection diagram shown below (only the right hand half is shown) (SNS).

2.6 SLENDER COLUMNS

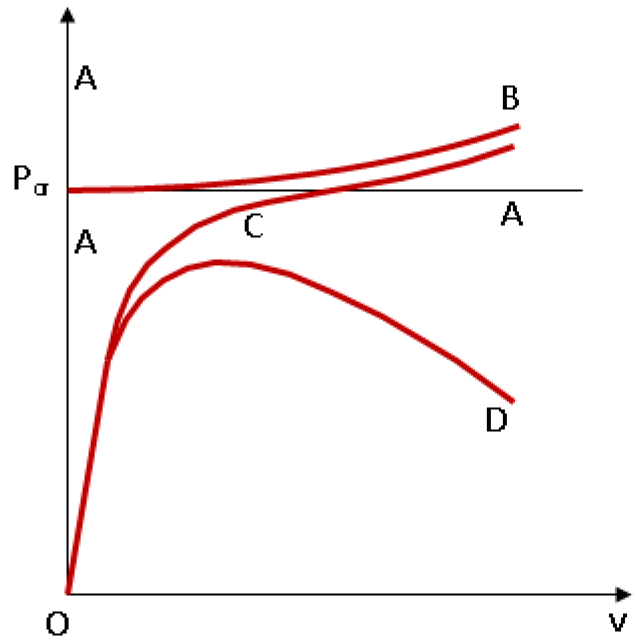


Fig. Load-deflection diagram for columns: Lines A, ideal elastic column with small deflections; curve B, ideal elastic column with large deflections; Curve C, elastic column with imperfections; and curve D, inelastic column with imperfections

2.6 SLENDER COLUMNS

- Four idealized cases are shown in Figure (SNS), together with the corresponding values of the effective length, $k\ell$.
- Effective length of a column is defined as the length of a pin ended column having the same stiffness and the same buckling load as the original column.
- Frames a and b are prevented against deflecting laterally when they buckle. They are said to be braced against sidesway.
- Frames c and d are free to sway laterally when they buckle. They are called unbraced or sway frames. The critical loads of the columns in Fig 12-6 are in the ratio 1:4:1:1/4

2.6 SLENDER COLUMNS

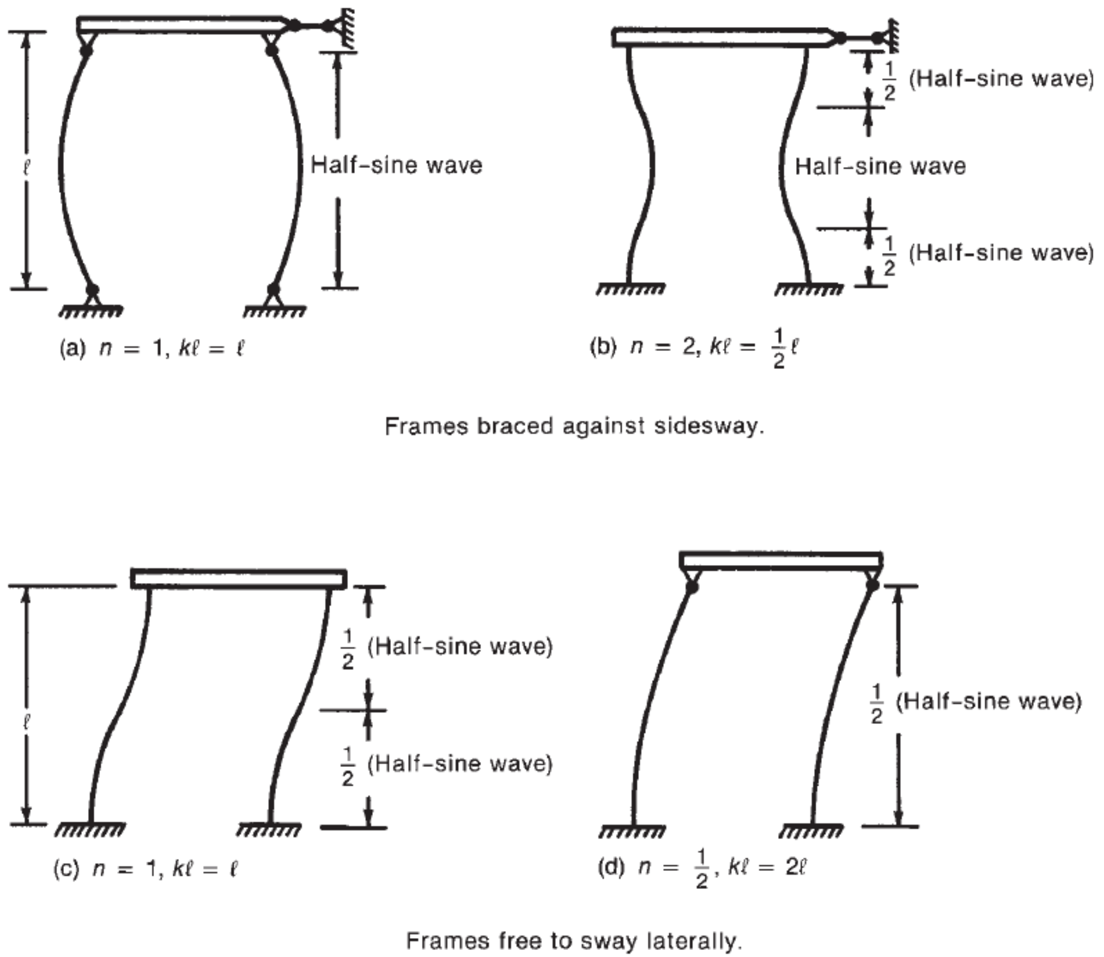


Fig. 12-6
Effective lengths of idealized columns.

2.6 SLENDER COLUMNS

- Thus it is seen that the restraints against end rotation and lateral translation have a major effect on the buckling load of axially loaded elastic columns.
- In actual structures fully fixed ends, such as in Fig 12-6 b to d, rarely, if ever, exist.
- In the following are discussed, the behavior and design of pin ended columns, as in Fig 12-6 a; restrained columns in frames that are braced against lateral displacement (braced or nonsway frames), Fig 12-6 b; and restrained columns in frames free to translate sideways (unbraced frames or sway frames), Fig 12-6c and d

2.6 SLENDER COLUMNS

- Pin-ended columns are rare in cast-in-place concrete construction, but do occur in precast construction. Occasionally, these will be slender, as, for example, the columns supporting the back of a precast grandstand.
- Most concrete building structures are braced frames, with the bracing provided by shear walls, stairwells, or elevator shafts that are considerably stiffer than the column themselves.

2.6 SLENDER COLUMNS

- **2.6.3 Slender columns in structures**
- Occasionally, unbraced frames are encountered near the tops of tall buildings, where the stiff elevator core may be discontinued before the top of the building, or in industrial buildings where an open bay exists to accommodate a travelling crane.
- Most building columns fall in the short column category. Exceptions occur in industrial buildings and in buildings that have a high main floor story for architectural or functional reasons. An extreme example(SNS)

2.6 SLENDER COLUMNS

Fig. 12-7
Bank of Brazil building, Porto Alegre, Brazil. Each floor extends out over the floor below it. (Photograph courtesy of J. G. MacGregor.)

2.6 SLENDER COLUMNS

- The presentation of slender columns is divided into 4 progressively more complex parts. In the 1st 2 sections slender pin-ended and restrained columns are discussed. These sections deal with Pδ effects. In section 3 columns in sway frames are discussed and finally column designs based on rigorous 2nd order analysis with nonlinear material behavior will be introduced.

2.6 SLENDER COLUMNS

- **2.6.4 Behavior and analysis of pin-ended columns**
- Lateral deflections of a slender column cause an increase in the column moments (SNS). These increased moments cause an increase in the deflections, which in turn lead to an increase in moments. As a result, the load-moment line O-B is non-linear. If the axial load is below the critical load, the process will converge to a stable situation. If the load is $\geq$ the "critical load", it will not. $\rightarrow$ "critical load" needs qualification (SSAN)

2.6 SLENDER COLUMNS

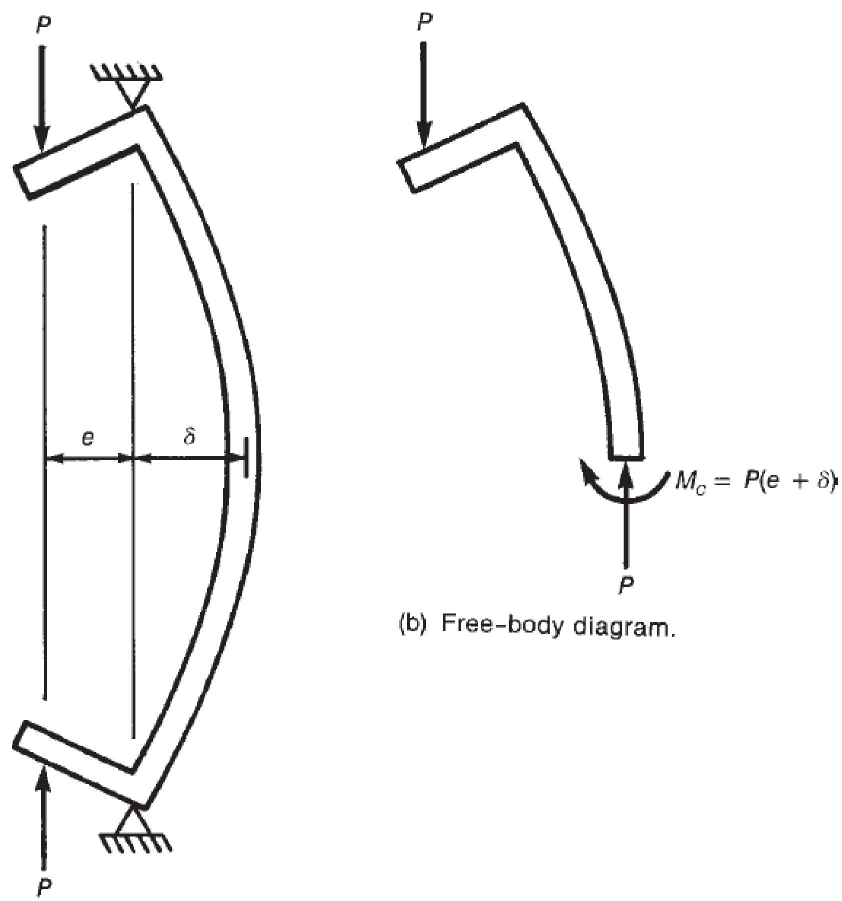


Fig. 12-1
Forces in a deflected column.

(a) Column.

(b) Free-body diagram.

2.6 SLENDER COLUMNS

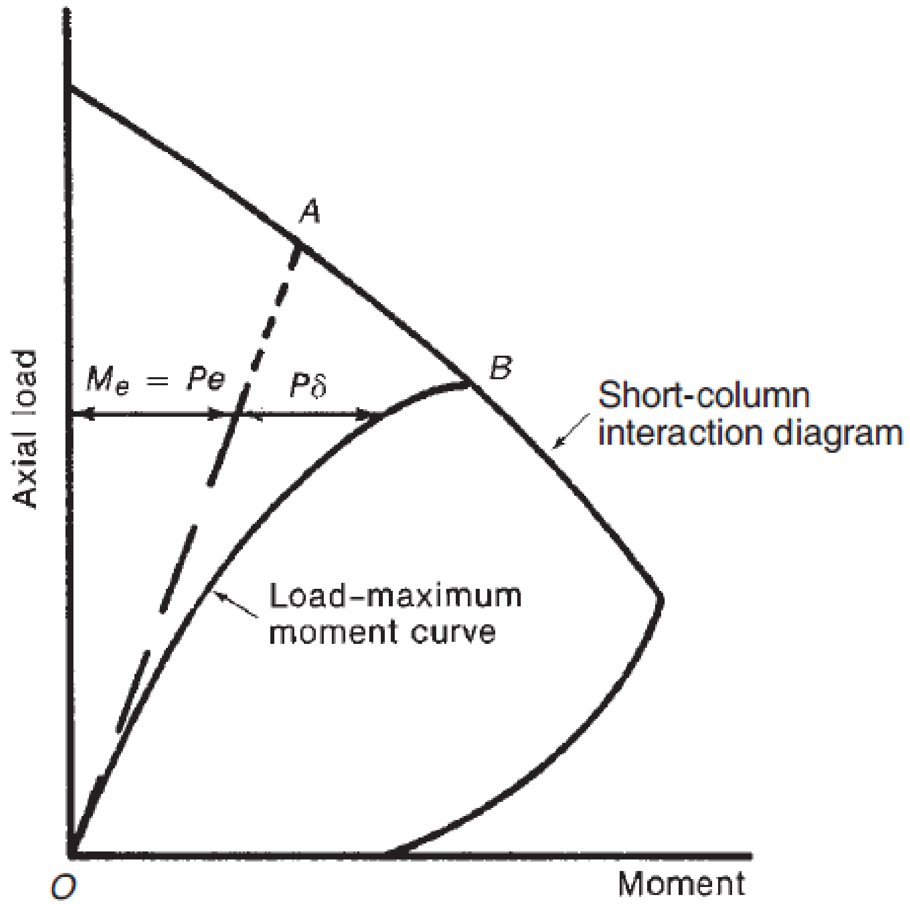


Fig. 12-2
Load and moment in a column.

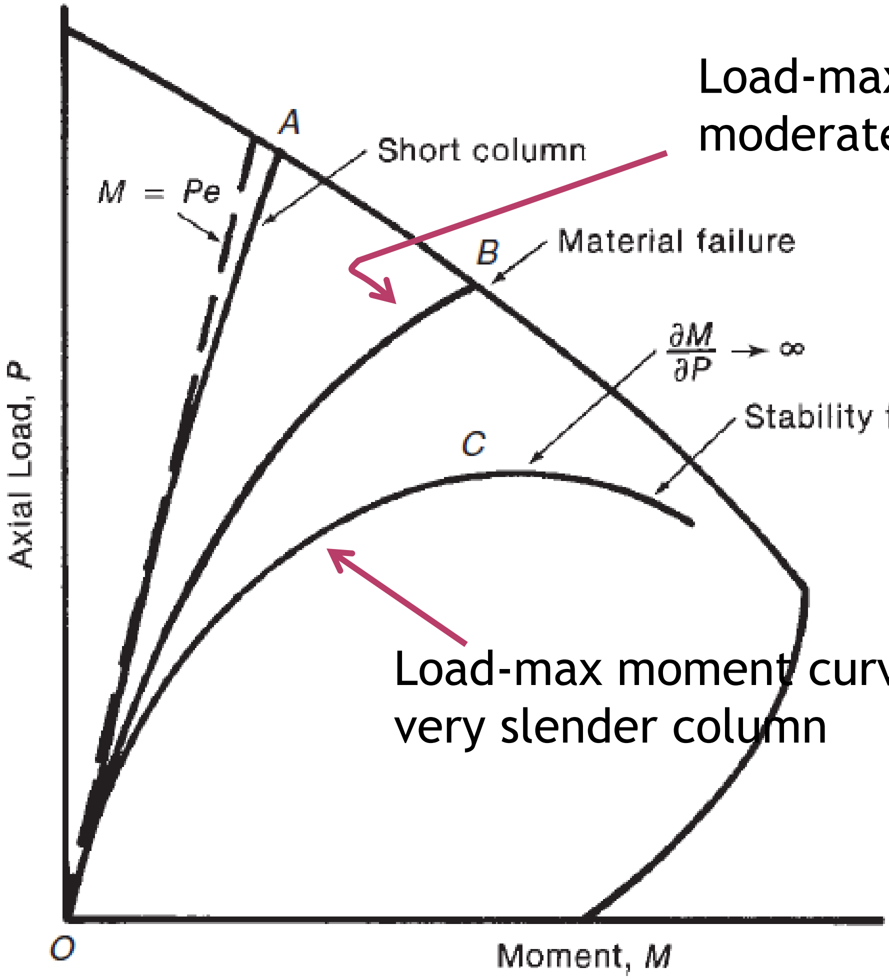
2.6 SLENDER COLUMNS

- Cont'd
- The "critical load", discussed here can not be the elastic buckling load (stability problems with bifurcation of equilibrium) for the reasons we have seen above.
- The present situation with increased moment causing increased lateral deflection, causing in turn the moment to increase constitutes a stability problem, if the cycle fails to converge (possible scenario for vey slender columns).

2.6 SLENDER COLUMNS

- This kind of instability problem w/o bifurcation of equilibrium can be solved by carrying out a 2nd order analysis, taking into account the non-linear behavior of the constituent materials.
- If the result of such analysis (or experiment) is presented as load versus deflection or load versus moment, the relationship will be a non-linear curve concave downwards (SNS). Drawing a parallel with ideal elastic buckling, the region with positive slope, i.e., to the left of the maximum pt on the curve represents the stable situation. → i.e.

2.6 SLENDER COLUMNS



Load-max moment curve for moderately slender column

Load-max moment curve for very slender column

2.6 SLENDER COLUMNS

- Cont'd
- The simply supported column with end eccentricity is in a state of stable equilibrium as long as the axial load and moment are less than the values corresponding to the peak point in the load-max moment curve OC.
- At the peak point the state of equilibrium is neutral
- Beyond the peak point, the slope is negative and axial load must be reduced to sustain the deflection or the increased moment. The column is in a state of unstable equilibrium in this region (failure is catastrophic in this range)(Project Gondar building collapse, Staircase collapse).

2.6 SLENDER COLUMNS

- In a 1st order analysis, the equations of equilibrium are derived by assuming that the deflections have a negligible effect on the internal forces in the members.
- In a 2nd order analysis, the equations of equilibrium consider the deformed shape of the structure. **Instability can be investigated only via a second-order-analysis, because it is the loss of equilibrium (divergence of the deflection iteration) of the deformed structure that causes instability**.
- However because many engineering calculations and computer programs are based on 1st order analysis, methods have been derived to modify the results of a 1st order analysis to approximate the 2nd order effects.

2.6 SLENDER COLUMNS

- **2.6.5 Material Failures and Stability Failures**
- Load-moment curves are plotted in Figure (SNS) for pin ended columns of 3 different lengths, all loaded with the same end eccentricity, e. The load-moment curve O-A for a **relatively short column** is practically the same as line Pe. For a column of moderate length, line O-B, the deflections become significant, reducing the failure load. This column fails when the load-moment curve intersects the interaction diagram at point B. This is called **material failure** and is the type of failure expected in most practical columns in braced frames.

2.6 SLENDER COLUMNS

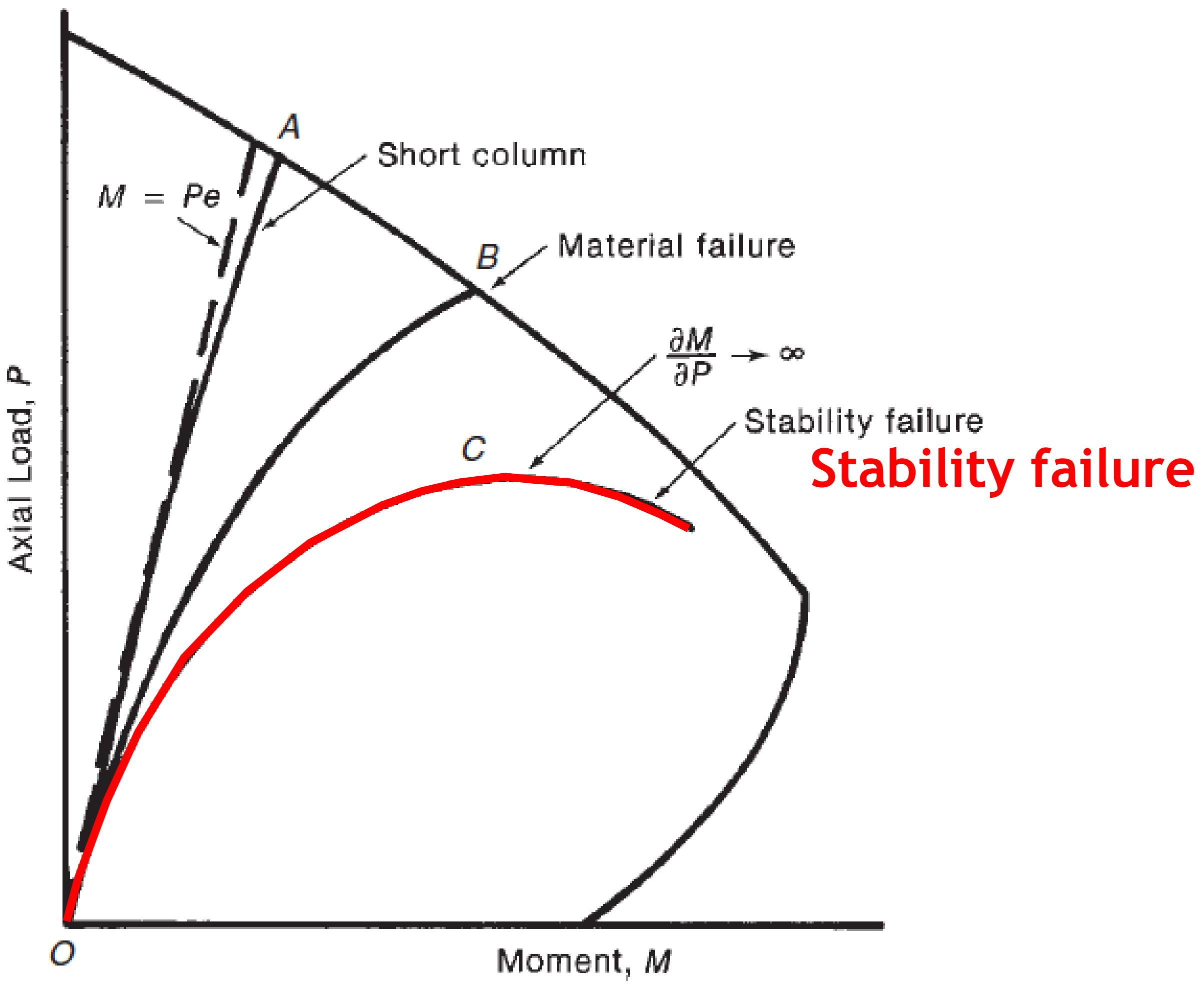


Fig Material and Stability failure

2.6 SLENDER COLUMNS

- Cont'd
- If a very slender column is loaded with increasing axial load, P, applied at a constant end eccentricity, e, it may reach a defection δ at which the value of **∂M/∂P approaches infinity or becomes negative** (refer earlier discussion). When this occurs, the column becomes unstable, since with further deflections, the axial load capacity will drop. This type of failure is known as stability failure and occurs only with very slender braced columns or with slender columns in sway frames

2.6 SLENDER COLUMNS

- **2.6.6 Moment Magnifier for Symmetrically Loaded Pin-Ended Column**
- Project EBCS EN 1992-1-1 to show relevance of 2.6.6. (S. 70-71)
- The column in Figure (SSAN) deflects an amount δ_o (1st order deflection) under the action of the end moment, M_o. When the axial loads P are applied, the deflection increases by the amount δ_a. The final deflection at midspan is $\delta = \delta_o + \delta_a$. The total deflection is called 2nd order deflection. Assuming that the deflected shape approaches half a sin wave, the P-δ moment diagram is also a sin wave.

2.6 SLENDER COLUMNS

- Cont'd
- Observe that the P-δ moments, (with the maximum value equal to $P(\delta_o + \delta_a)$ at the middle) are the causes for the additional deflections, such as the maximum additional deflection δ_a in the middle
- Using the moment area method, the deflection δ_a is the moment about the support of the M/EI diagram b/n the support and the midspan shown shaded in The figure (where M is the P-δ moments) . The area of this portion is:
- Area = $((P/EI)(\delta_0 + \delta_a))(\ell/2)\times(2/\pi)$

2.6 SLENDER COLUMNS

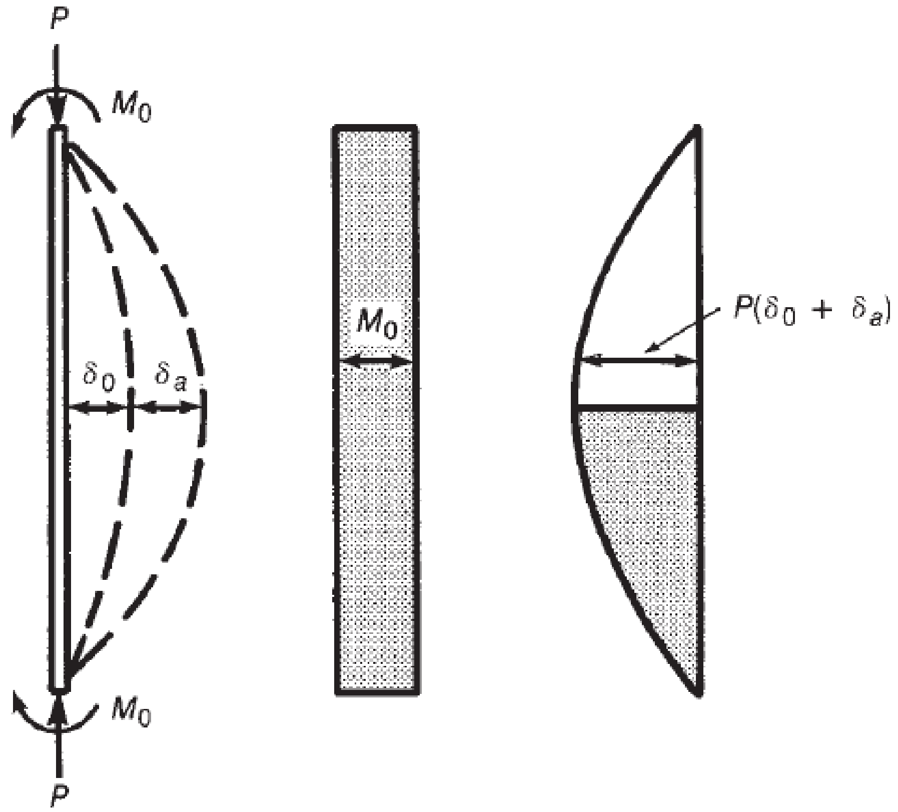


Fig. 12-10
Moments in a deflected column.

(a) Deflected column.

(b) M_0.

(c) $P\text{–}\delta$ Moments.

2.6 SLENDER COLUMNS

- The centroid of the $P\delta$ moment diagram is located at ℓ/π from the support. $\rightarrow \delta_a = [(P/EI(\delta_0 + \delta_a))(\ell/2)\times(2/\pi)](\ell/\pi) = (P\ell^2/\pi^2EI)(\delta_o + \delta_a) = (\delta_o + \delta_a)P/P_E$; where P_E is the ideal elastic buckling load, $P_E = \pi^2EI/\ell^2$
- Rearranging $\rightarrow \delta_a = \delta_o\ ((P/P_E)/(1- P/P_E))$
- Since the final deflection δ is the sum of δ_o and δ_a, $\rightarrow \delta= \delta_o+ \delta_o((P/P_E)/(1- P/P_E)) \rightarrow$ **$\delta = \delta_o/(1- P/P_E)$**
- This equation shows that the 2nd order deflection, δ, increases as P/P_E increases reaching infinity when $p = P_E$

2.6 SLENDER COLUMNS

- The maximum 2nd order bending moment is: $M_c = M_0 + P\delta \rightarrow M_c = M_0 + P\,\delta_0/(1 - P/P_E)$
- For the 1st order moment diagram corresponding to equal end eccentricities $\rightarrow \delta_0 = M_0\ell^2/(8EI)$. Substituting this and $(P/P_E)\pi^2 EI/\ell^2$ into the expression for $M_c \rightarrow$ **$M_c = (M_0(1 + 0.23\,P/P_E)/(1-P/P_E))$. See NS for comparison b/n ACI and EBCS EN 1992-1-1 for a constant 1st order moment and specific value of $P/P_E = 0.5$.**
- The coefficient 0.23 is a function of the shape of M_0 diagram. For example, it becomes -0.38, for a triangular moment diagram with M_0 at one end of the column and zero moment at the other and -0.18 for columns with equal and opposite end moments

2.6 SLENDER COLUMNS

- ACI $\rightarrow$

$$M_c = M_0\left(\frac{1+0.23\,P/P_E}{1-P/P_E}\right) = M_0\left(\frac{1+0.23\times 0.5}{1-0.5}\right) = 2.23\times M_0$$

- EBCS EN 1992-1-1 $\rightarrow$

$$M_{Ed} = M_{0E,d}\left(1+\frac{\beta}{(N_B/N_{Ed})-1}\right) = M_{0E,d}\left(1+\frac{\pi^2}{8\times((N_B/N_{Ed})-1)}\right) =$$

$$M_{0E,d}\left(1+\frac{\pi^2}{8\times(2-1)}\right) = 2.23\times M_{0E,d}$$

2.6 SLENDER COLUMNS

- In the ACI Code, the $(1 + 0.23\ P/P_E)$ term is omitted because the factor 0.23 varies as a function of the moment diagram, for P/P_E = 0.25 to -0.18, the term $(1 + C\ P/P_E)$ varies from 1.06 to 0.96 and the magnified moment M_c is given essentially as:
- $\rightarrow$ $\mathbf{M_c = \delta_{ns} M_0}$; where δ_{ns} is called the **nonsway-moment magnifier** and is given by:
- $\rightarrow$ $\mathbf{\delta_{ns} = 1/(1 - P/P_c)}$; where $P_c = P_E$ (discussion on what EI to use to determine the elastic buckling load)

2.6 SLENDER COLUMNS

- **2.6.7 P-δ Moments and P-Δ Moments**
- Two different types of 2nd order moments act on the column in a frame:
  - 1. **P-δ Moments**. These result from deflections, δ, of the axis of the bent column away from the chord joining the ends of the column, (SPS). The slenderness effects in pin-ended columns and in nonsway frames result from P-δ moments.
  - 2. **P-Δ Moments**. These result from lateral deflections, Δ, of the column from their original undeflected locations (SCS).

2.6 SLENDER COLUMNS

- **2.6.8 Effect of Unequal End Moments on the Strength of a Slender Pin-ended Columns**
- Up to now, we have considered only pin-ended columns subjected to equal moments at the two ends. This is a very special case, for which the maximum 2nd order moment, $P\delta$, occurs at a section where the **1st order moment**, Pe, is also **a maximum**. As a result these quantities can be added directly as shown in previous slides.
- In the usual case, the end eccentricities, $e_1=M_1/P$ and $e_2=M_2/P$ are not equal and give 1st order moment diagrams as shown shaded in Fig (SNS)

2.6 SLENDER COLUMNS

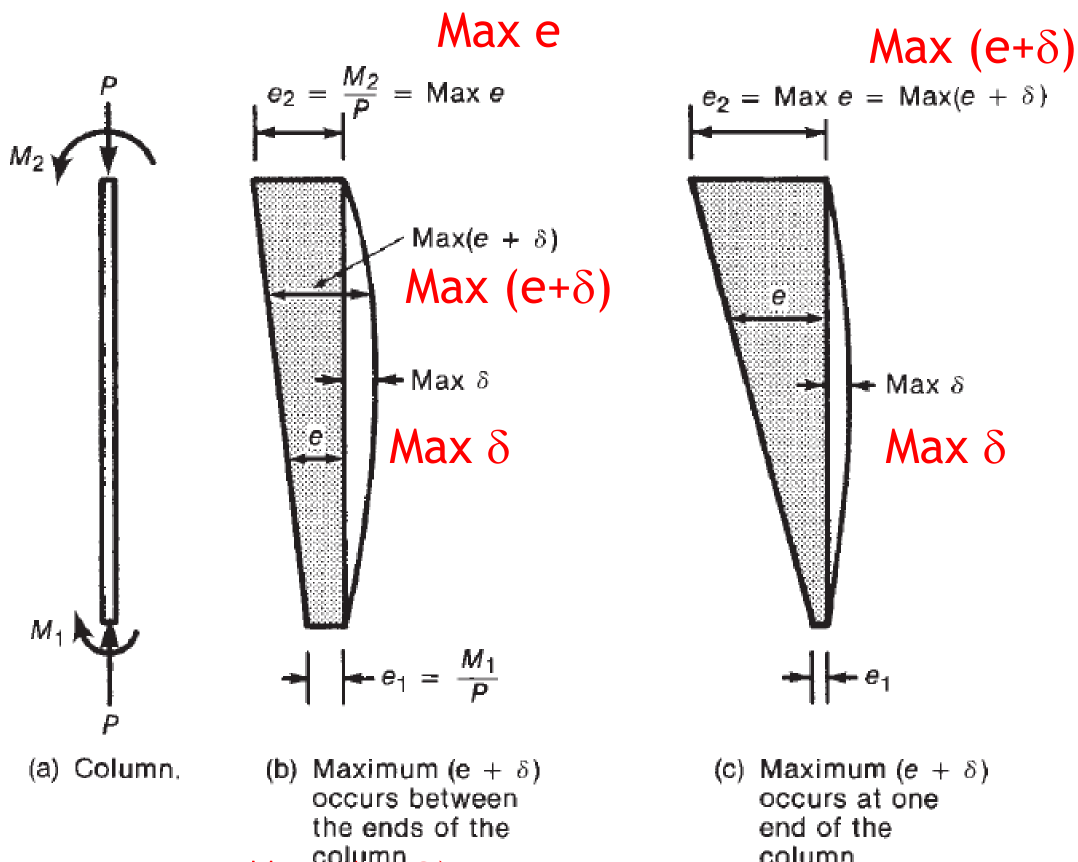


Fig. 12-11
Moments in columns with unequal end moments.

Moments in columns with unequal end moments

Max (e+δ) occurs b/n the ends of the column

Max (e+δ) occurs at one end of the column

2.6 SLENDER COLUMNS

- The max value of δ and max e do not occur at the same location. As a result e_{max} and δ_{max} cannot be added directly.
- In the moment-magnifier design procedure, the column subjected to unequal end moments in Fig (SNS), is replaced with a similar column subjected to equal moments of $c_m M_2$ at both ends as shown in Fig (SNS).
- The moments $c_m M_2$ are chosen so that the maximum magnified moment is the same in both columns. The expression for the equivalent moment factor C_m was originally derived for use in the design of steel “beam-columns”.

2.6 SLENDER COLUMNS

M_2

Max M

$C_m M_2$

Max M

(a) Actual moments at failure

(b) Equivalent moments at failure

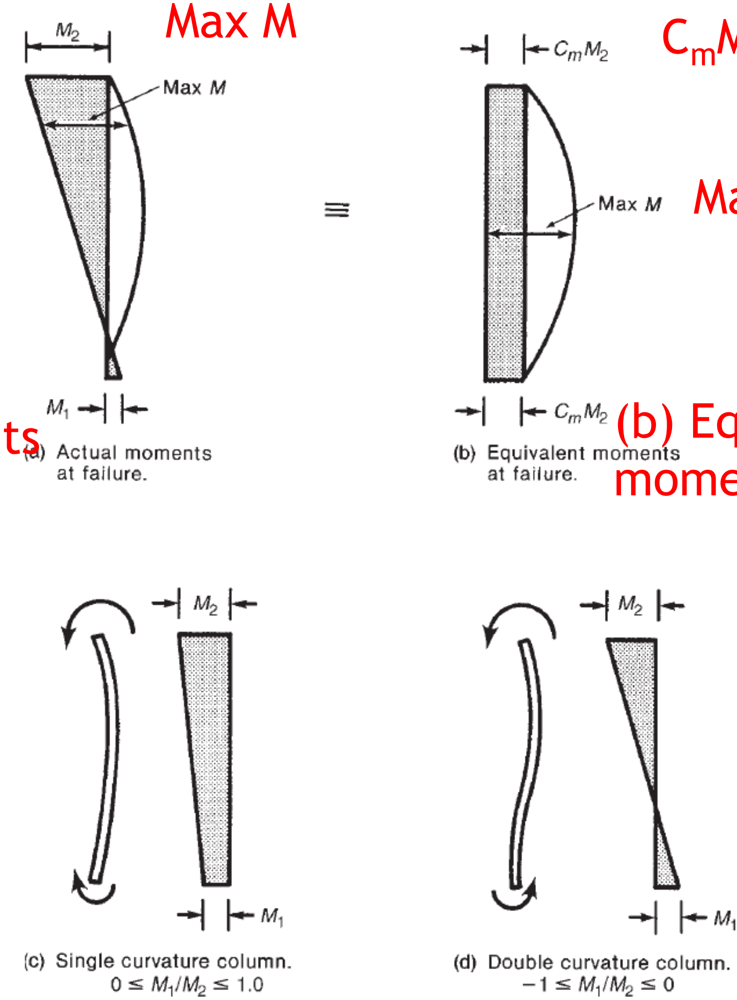


Fig: Equivalent moment factor, C_m

Fig. 12-13
Equivalent moment factor, C_m.

2.6 SLENDER COLUMNS

- $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$
- In the expression, M_1 and M_2 are the smaller and larger 1st order column end moments. The sign convention for the ratio M_1/M_2 is illustrated in Fig 12-13c and d. If the moments M_1 and M_2 cause single curvature bending, M_1/M_2 is positive. If the moments M_1 and M_2 bend the column in double curvature with a point of contraflexure b/n the two ends , M_1/M_2 is negative (SPS). GOTO S134

2.6 SLENDER COLUMNS

- **2.6.9 Column Stiffness, EI**
- The calculation of the critical load P_c (SPS it is ideal elastic buckling load) involves the use of the flexural stiffness, EI, of the column.
- The value of EI chosen for a given column section, must approximate the EI of the column at the time of failure, taking into account the type of failure (material failure or stability failure) and the effects of cracking, creep, and nonlinearity of the stress-strain curves at the time of failure
- Fig (SNS) shows **moment-curvature diagrams** for 3 different load levels for a typical cross section
- **Assignment # 1: Draw moment-curvature diagrams for 3 different load levels as shown in the next slide (b/h = 400/400; h'/h = 0.1; Rebar 4ϕ20)**

2.6 SLENDER COLUMNS

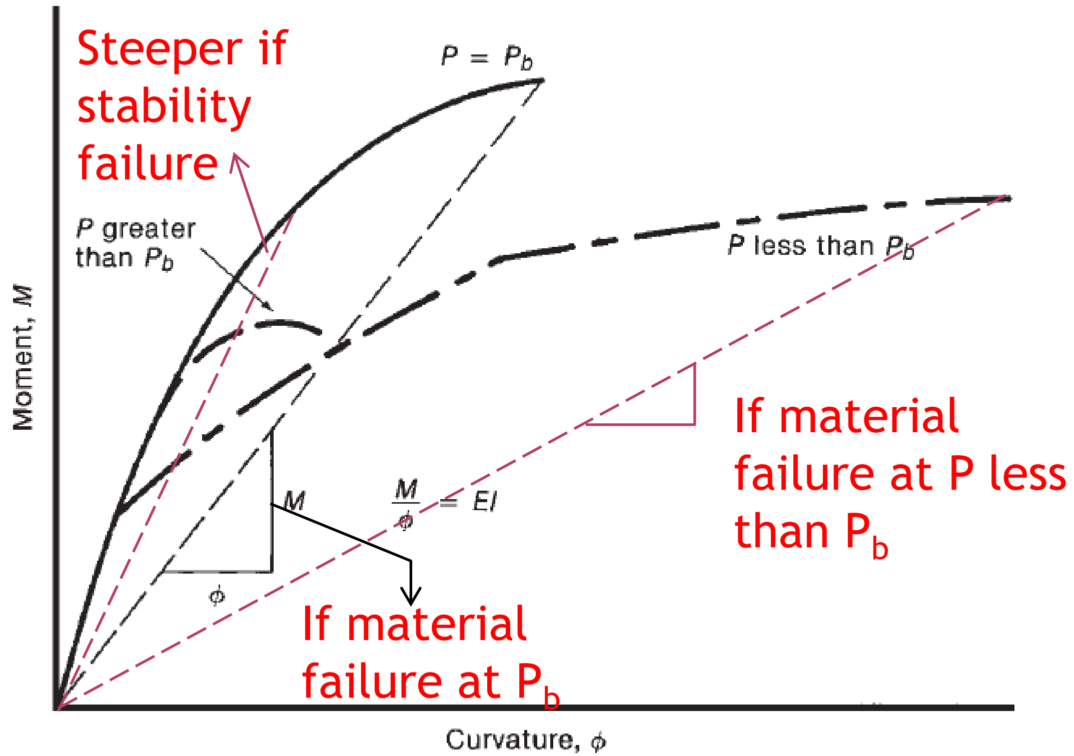


Fig. 12-14
Moment–curvature diagrams for a column cross section.

2.6 SLENDER COLUMNS

- A radial line in such a diagram has slope $M/\phi = EI$. The value of EI depends on the particular radial line selected. In a material failure, failure occurs when the most highly stressed section fails (point B SNS). For such a case, the appropriate radial line should intercept the end of the moment-curvature diagram, as shown for the $P = P_b$ (balanced-failure load). On the other hand, a stability failure occurs before the cross section fails (point C). This corresponds to a steeper line in the M-ϕ diagrams. $\rightarrow$

2.6 SLENDER COLUMNS

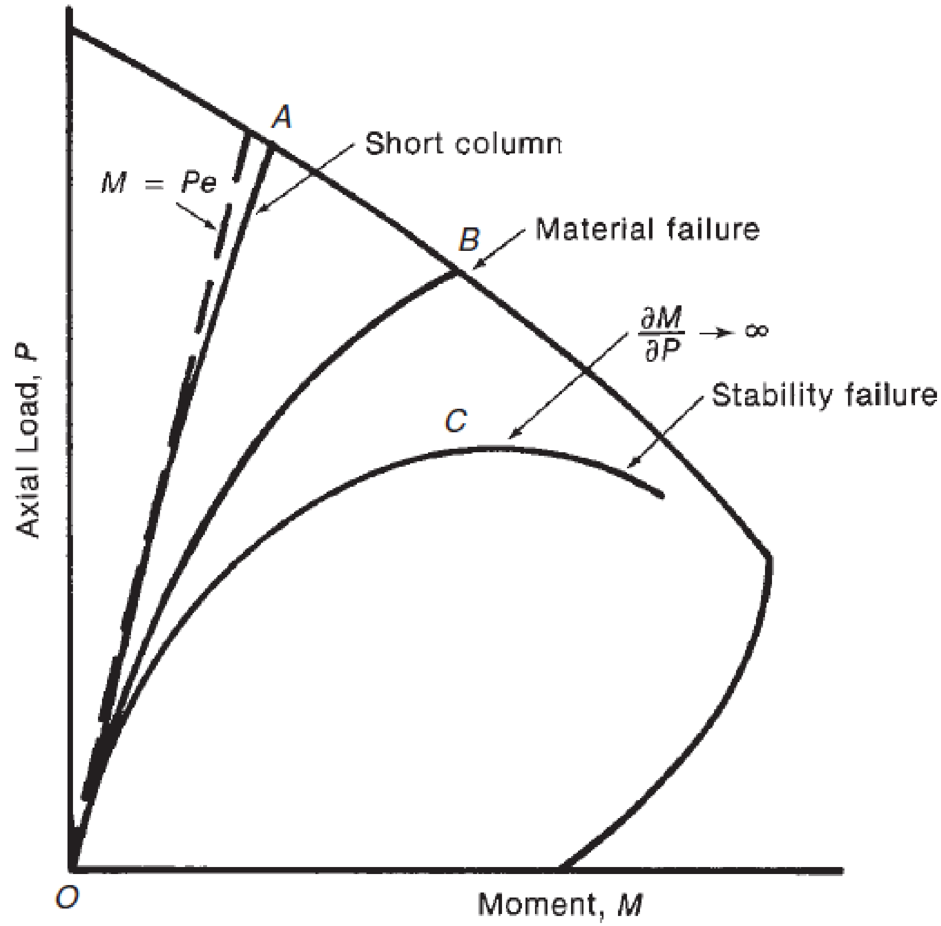


Fig. 12-8
Material and stability failures.

2.6 SLENDER COLUMNS

- Cont'd
- The multitude of radial lines that can be drawn in the M-ϕ diagrams suggests that there is no all-encompassing value of EI for slender concrete columns.
- The following two different sets of stiffness values, EI, are given in the ACI to calculate P_c. (Assignment #2: How do they compare with the result you get from Assignment #1 and assuming material failure. Compare with EI-values according to EBCS EN 1992-1-1, Section 5.8.7.2 Eqn. 5.21)
  - 1. $EI = (0.2E_cI_g + E_sI_{se})/(1 + \beta_{dns})$ (more accurate but requires knowledge of required amount of reinforcement
  - 2. $EI = 0.40E_cI_g/(1 + \beta_{dns})$ (I_{se}= moment of inertia of reinforcement about the centroidal axis)

2.6 SLENDER COLUMNS

- **2.6.10 EI for the computation of Frame Deflections and for Second-Order Analysis**
- The above expressions for EI are only for use in ACI Eq. (10-10) to compute P_c when one is using the moment-magnifier method. These represent the behavior of a single, highly loaded column.
- ACI Section 10.11.1 gives a different set of values of the moment of inertia, I, for use
  - (a) in elastic frame analysis, to compute the moments in beams and columns and the lateral deflections of frames, and
  - To compute Ψ used in computing the effective length factor, k. $\rightarrow$

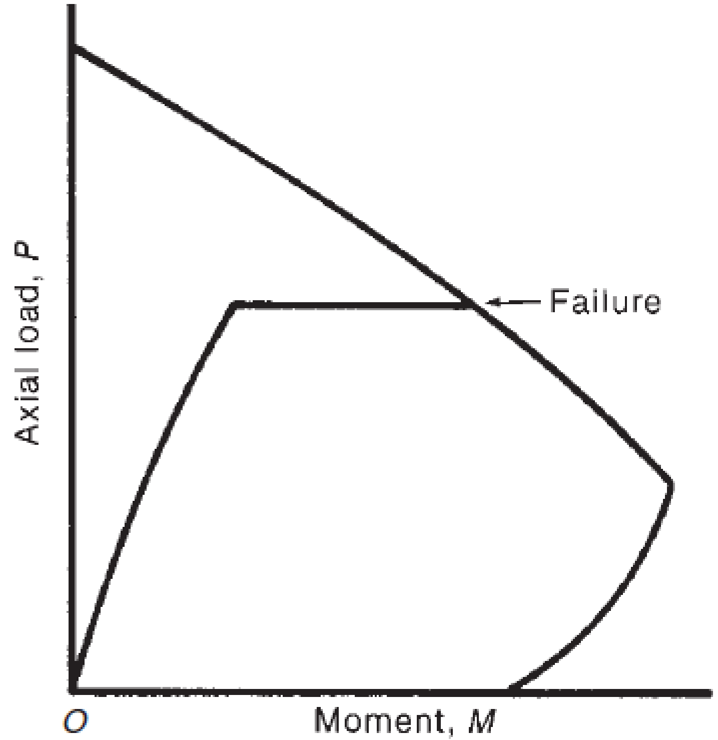
2.6 SLENDER COLUMNS

- Cont'd
- The lateral deflection of a frame is affected by the stiffnesses of all the beams and columns in the frame. For this reason, the moment of inertias in ACI Section 10.11.1 are intended to represent an overall average of the moment of inertia values of EI for each type of member in a frame.
- Similarly the effective lengths of a column is affected by the flexural stiffnesses of the number of beams and columns.$\rightarrow$ Use EI= $0.7E_CI_C$ for columns and EI= $0.35E_CI_C$ for beams.

2.6 SLENDER COLUMNS

- **2.6.11 Effect of sustained Loads on Pin-Ended Columns**
- Up to now, the discussion has been limited to columns failing under short-time loadings. Columns in structures, on the other hand, are subjected to sustained dead loads and sometimes to sustained live loads. The creep of the concrete under sustained loads increases the column deflections, increasing the moment $M = P(e + \delta)$ and thus weakening the column (SNS).

2.6 SLENDER COLUMNS



(b) Creep buckling.

Fig. 12-15
Load–moment behavior for hinged columns subjected to sustained loads.

2.6 SLENDER COLUMNS

- The ACI Code moment-magnifier procedure uses the reduced-modulus procedure. The value of EI is reduced by dividing by $(1 + \beta_{dns})$ (SPS for expressions of EI), where for hinged columns and columns in restrained frames, β_{dns} is defined as the ratio of the factored axial load due to dead load to the total factored axial load.

2.6 SLENDER COLUMNS

- **2.6.12 limiting Slenderness Ratios for Slender Columns**
- Most columns in structures are sufficiently short and stocky to be unaffected by slenderness effects.
- To avoid checking slenderness effects for all columns, ACI Section 10.12.2 allows slenderness effects to be ignored in the case of hinged columns and of columns in nonsway frames if:
  - $k\ell_u/r < 34 - 12(M_1/M_2)$ in nonsway frames
  - and $k\ell_u/r < 22$ in sway frames (r = radius of gyration)
  - Compare with EBCS EN 1992-1-1 provisions for limiting slenderness ratios for slender columns

2.6 SLENDER COLUMNS

- ACI Section 10.11, “Magnified Moments-General,” gives general requirements for the design of slender columns in both nonsway and sway columns.
- If a column is in a nonsway frame, design involves ACI Sections 10.11 and 10.12, “Magnified moments –Nonsway Frames”.
- If a column is in a sway frame, design involves ACI Sections 10.11 and 10.13, “magnified Moments-Sway Frames”

2.6 SLENDER COLUMNS

- **2.6.13 Definition of nonsway and sway Frames**
- The preceding discussions were based on the assumption that frames could be separated into nonsway (braced) or sway (unbarced).
- In actuality, there is no such thing as "completely braced" frames and no clear cut boundary exists b/n nonsway and sway frames. Some frames are clearly unbraced. Other frames are connected to shear walls, elevator shafts, and so on, which clearly restrict the lateral movement of the frame. Because no wall is completely rigid, however, there will always be some lateral movement of a braced frame, and hence some PΔ moments result from the lateral movements

2.6 SLENDER COLUMNS

- For the **purpose of design**, a story or a frame may be considered “nonsway,” if horizontal displacements do not significantly reduce the load carrying capacity of the structure. This criterion could be restated as follows: a frame can be considered “nonsway”, if the PΔ moments due to lateral deflections are small compared to the 1st order moments due to lateral loads. ACI Code Section 10.10.5.1 allows designers to assume that a frame is nonsway if the increase in column end moments due to 2nd order effects does not exceed 5% of the 1st order moments

2.6 SLENDER COLUMNS

- Alternatively ACI Code Section 10.10.5.2 allows designers to assume that a story in a frame is nonsway if:
  - **$Q = \Sigma P_u \Delta_o / (V_{us} \ell_c) \leq 0.05$**
  - Where Q is the stability index; ΣP_u= total vertical load in all columns and walls in the story; V_{us} is the story shear due to factored lateral loads; Δ_o is the 1st order story drift (relative deflection b/n the top and bottom of that story)due to V_{us} ; ℓ_c is the story height measured from center to center of the joint

2.6 SLENDER COLUMNS

- **2.6.14 Design of Columns in Nonsway Frames (See restrained column in Fig SNS)**
- **In all modern concrete and steel design codes, the empirical assumption (effective length method) is made that ℓ_i can be taken equal to the effective length for elastic buckling, $k\ell$.**
- The effective length of a column, $k\ell_u$, is defined as the length of an equivalent pin-ended column having the same buckling load as the real column in the frame

2.6 SLENDER COLUMNS

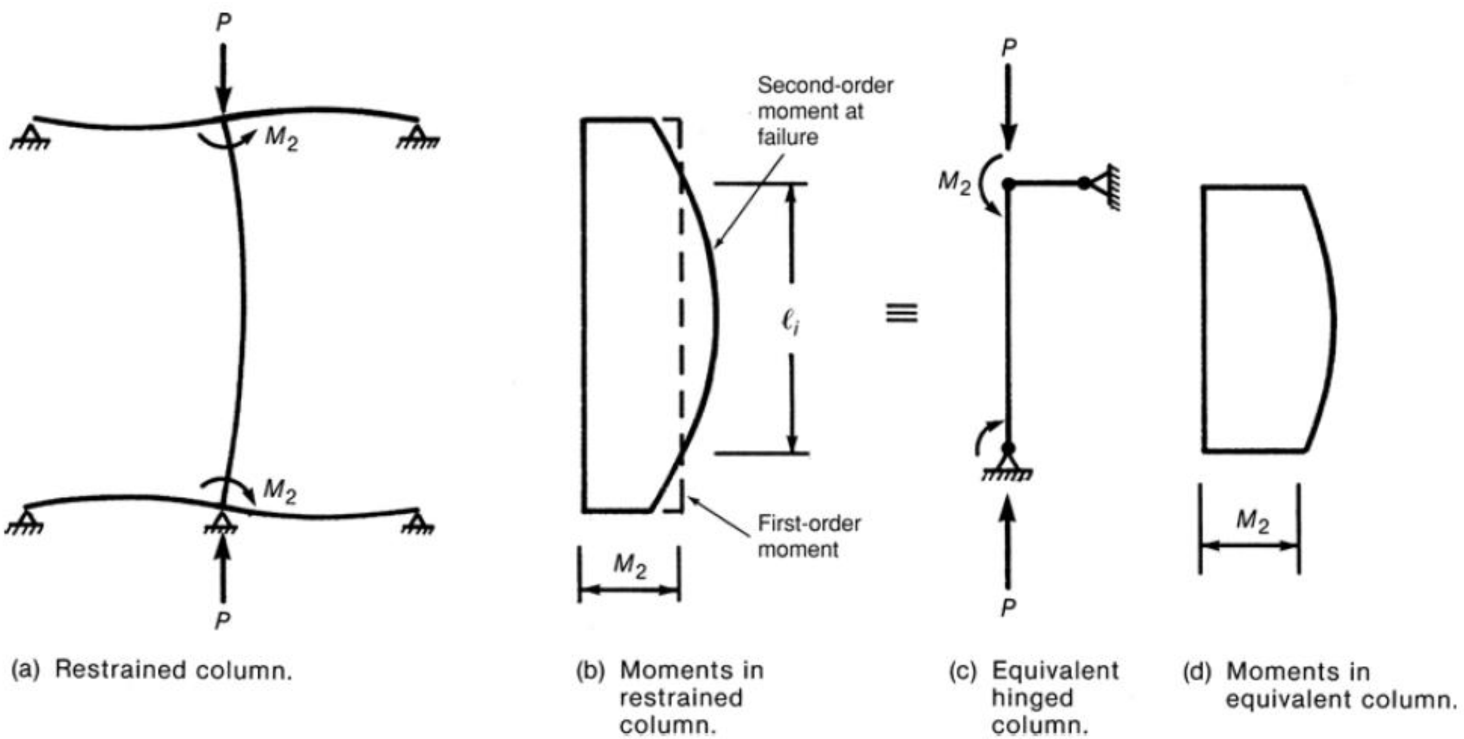
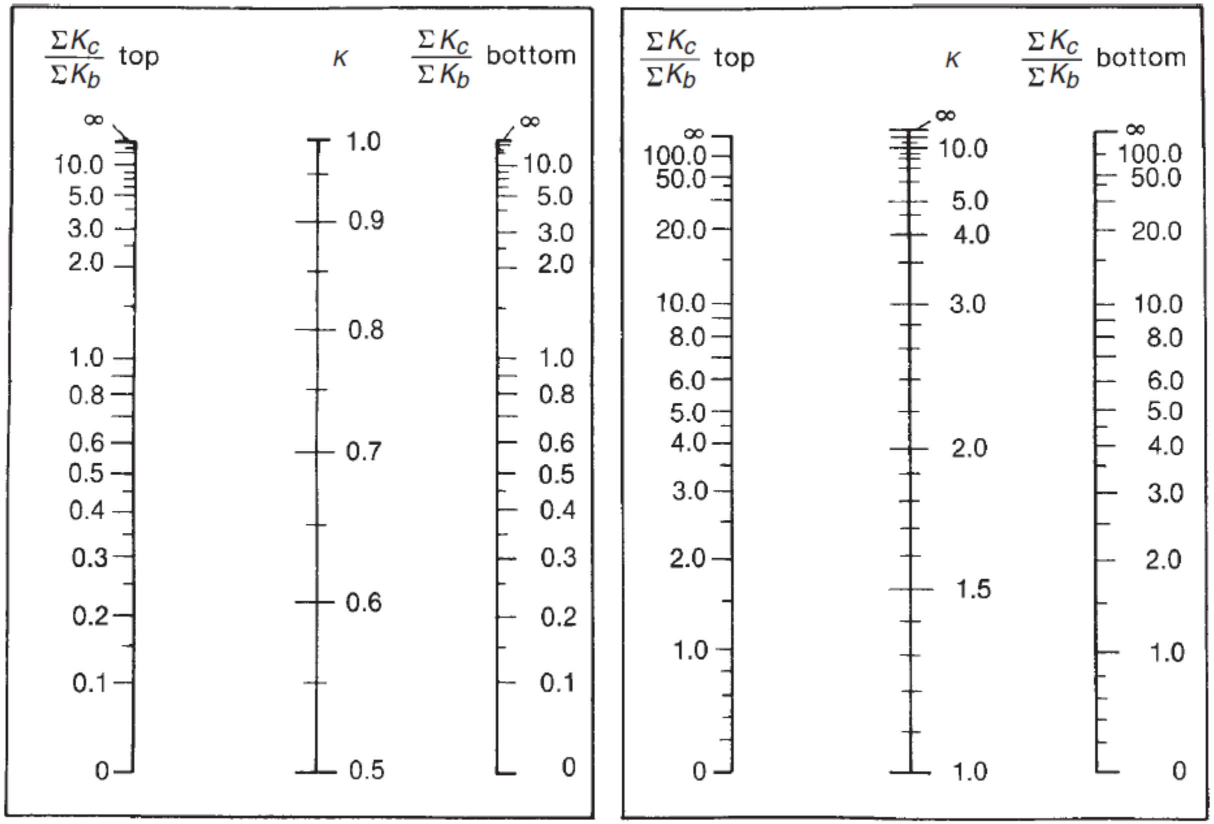


Fig. 12-24
Replacement of restrained column with an equivalent hinged column for design.

2.6 SLENDER COLUMNS

- Cont'd
- The value of the effective length coefficient is a function of the relative stiffnesses, ψ, of the beams and columns at each end of the column, where ψ is:
  - $\psi = \Sigma(E_c I_c/\ell_c)/\Sigma(E_b I_b/\ell_b)$; where b and c refer to beams and columns and ℓ_b and ℓ_c are measured center to center of joints.
- Further reading $\rightarrow$ Refer Macgregor

2.6 SLENDER COLUMNS



(a) Nonsway frames. (b) Sway frames.

Nonsway frames **Sway frames**

Fig. 12-26
Nomographs for effective length factors.

Nomographs for effective length factors

2.6 SLENDER COLUMNS

- Cont'd
- **Summary of Moment-Magnifier Design Procedure for Slender Columns in Braced Frames**
  - 1. Length of column ℓ_u
  - 2. Effective length with reduced bending stiffness for beams (0.35 EI) and columns (0.70EI) as in elastic frame analysis to compute internal forces and deflections
  - 3. Evaluation of whether the frame is braced: **$Q = \Sigma P_u \Delta_o / (V_{us} \ell_c) \leq 0.05$ (Δ_o is the story drift)**

2.6 SLENDER COLUMNS

- Cont'd
  - 4. Consideration of slenderness effect: No if $k\ell_u/r<34-12M_1/M_2$ in Non-sway frame
  - 5. Minimum moment: $M_{2,min} = P_u(15+0.03h)$; where 15 and h are in mm
  - 6. Moment-magnifier equation: $M_c = \delta_{ns}M_2$; where $\delta_{ns} = C_m/(1-P_u/0.75P_c) \geq 1.0$ with $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$; where M_1/M_2 is positive for single curvature bending, $P_c = \pi^2EI/(kl_u)^2$ and $EI = (0.2E_cI_g+ E_sI_{se})/(1 + \beta_d)$ or $EI = 0.40E_cI_g/(1 +\beta_{dns})$
  - If P_u exceeds $0.75\ P_c$, δ_{ns} will be negative. Such a column would be unstable. Even when δ_{ns} exceeds 2.0 $\rightarrow$ consider enlarging the section

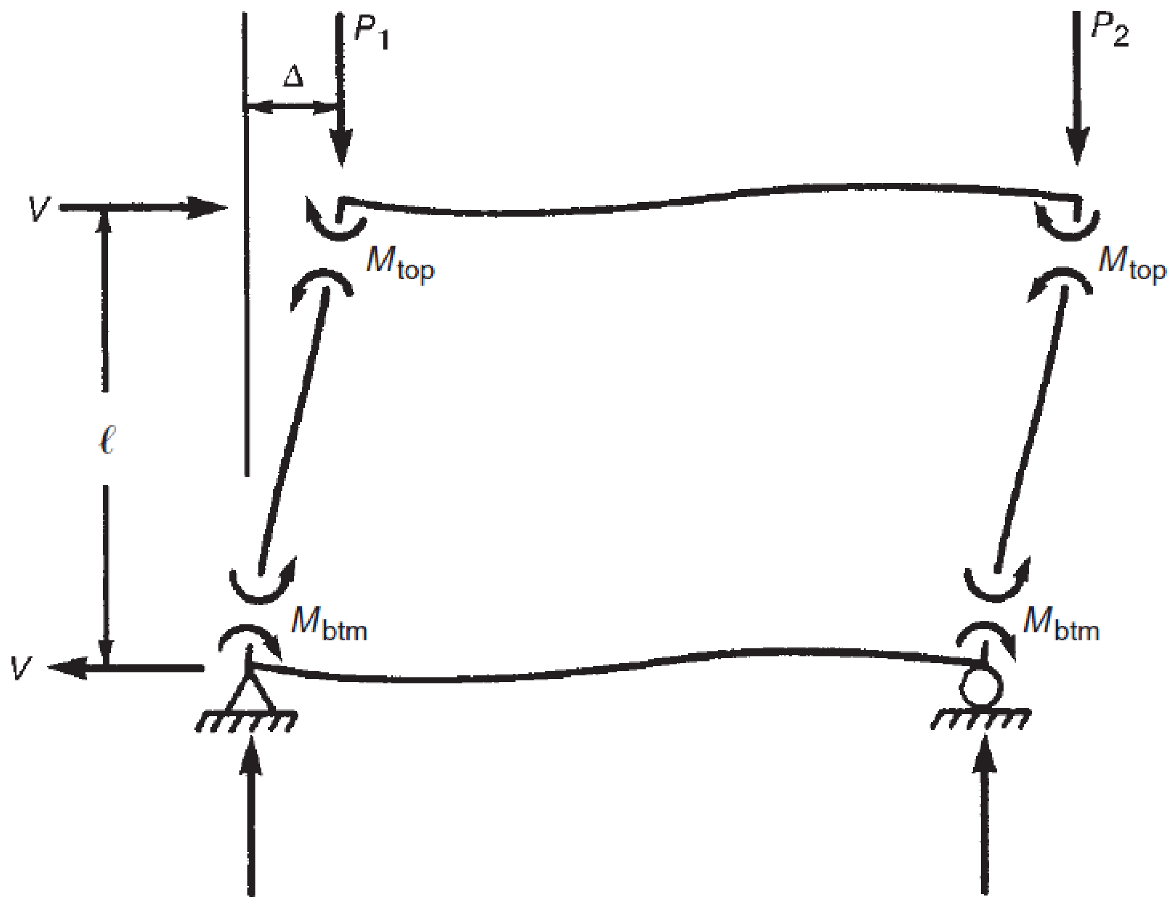
2.6 SLENDER COLUMNS

- **Exercise: Compare steps with the moment magnifier method for slender columns according to EBCS EN 1992-1-1**
- Example:
- Refer to example for a slender pin-ended column in Macgregor.
- **Carry out the design using the Revised Ethiopian Building Code (EBCS EN 1992-1-1:2013 /EC2:2004 (Assignment No. 2)**

2.6 SLENDER COLUMNS

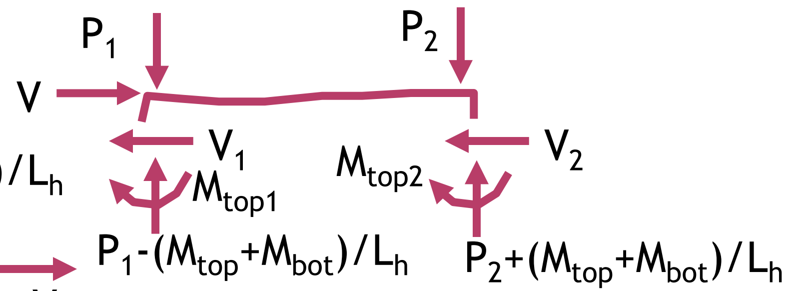
- **2.6.15 Behavior of Restrained Columns in Sway Frame**
  - Statics of Sway Frames
  - An unbraced frame is one that depends on moments in the columns to resist lateral loads and lateral deflections. Such a frame is shown in Figure(SNS). The sum of the moments at the tops and bottoms of all the columns must equilibrate the applied lateral-load moment, $V\ell$, plus the moment due to the vertical loads, $\Sigma P\Delta$. Thus
    - $\Sigma(M_{top} + M_{btm}) = V\ell + \Sigma P\Delta$

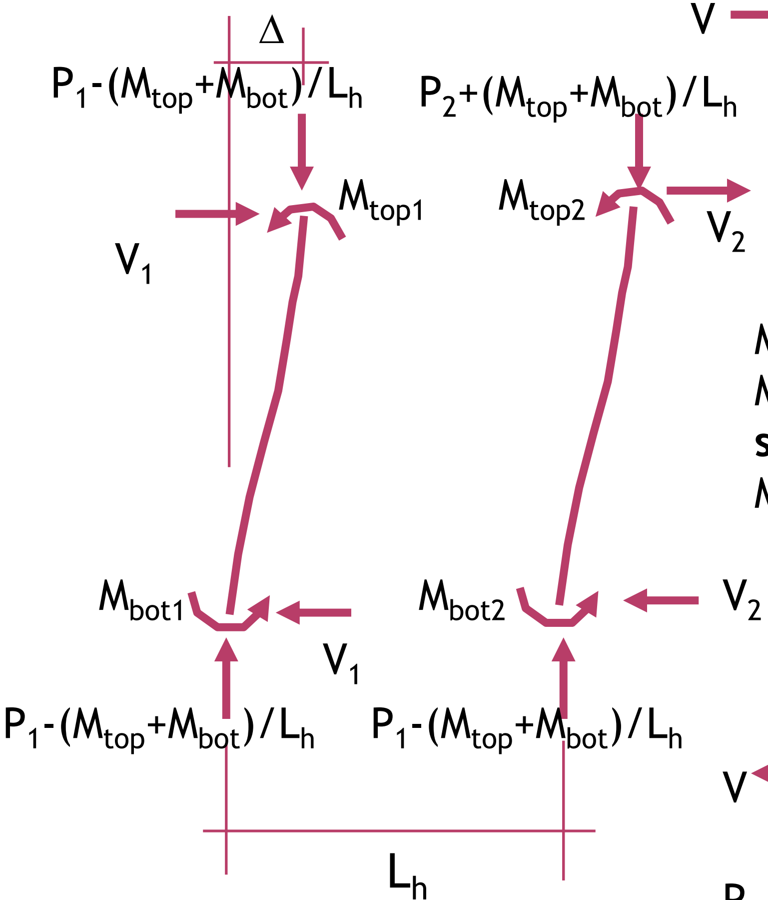
2.6 SLENDER COLUMNS



(a) Column moments in a sway frame.

2.6 SLENDER COLUMNS



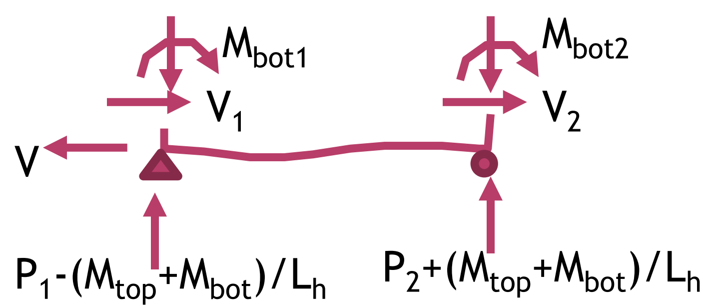


$M_{top1} + M_{bot1} = V_1 l + P_1\Delta - ((M_{top} + M_{bot})/L_h)\ \Delta$

$M_{top2} + M_{bot2} = V_2 l + P_2\Delta + \ ((M_{top} + M_{bot})/L_h)\ \Delta$

summing

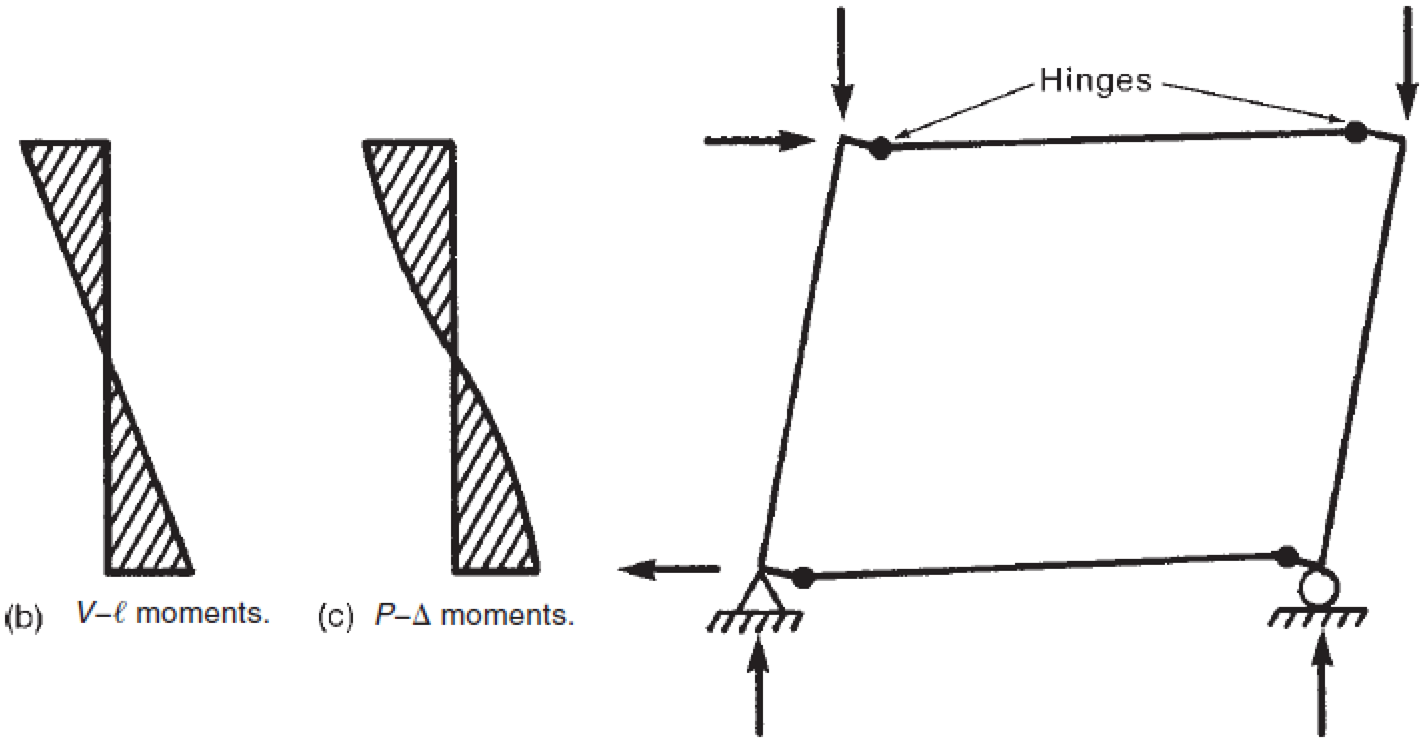
$M_{top} + M_{bot} = Vl + \Sigma P\Delta$



2.6 SLENDER COLUMNS

- It should be noted that both columns have deflected laterally by the same amount Δ. For this reason, it is not possible to consider columns independently in an unbraced frame.
- If a sway frame includes some pin-ended columns (e.g. precast concrete building), the vertical loads in the pin-ended columns are included in ΣP above. Such columns are referred to as leaning columns, because they depend on the frame for their stability.
- The V-ℓ moment diagram due to the lateral load and the P-Δ moment due to story drift is shown in Figure (SNS). It can be seen that they are directly additive because the maximum for both occur at the same point, i.e. at the ends of the column.

2.6 SLENDER COLUMNS



(b) V–ℓ moments. (c) P–Δ moments.

(d) Effect of beam hinges.

Fig: Column moments in a sway frame

2.6 SLENDER COLUMNS

- Because the maximum lateral load moments and the P-Δ moments both occur at the ends of the columns, and hence can be added directly, the equivalent moment factor, C_m, does not apply for sway frames.
- The magnified moment M_c in sway frames is given by: (this is used as method no 3 for sway moment magnification discussed later)
  - $M_c = M_0(1-0.18P/P_E)/(1-P/P_E)$; the term in bracket is left out by the ACI because the resulting change in the magnification does not vary significantly. **Compare with EBCS EN 1992-1-1**

2.6 SLENDER COLUMNS

- Cont’d
  - It is also important to note that if hinges were to form at the ends of the beams in the frame as shown in Figure (SPS), the frame would be unstable. Thus the beams must resist the full magnified end moment from the columns for the frame to remain stable (ACI Section 10.13.7).
  - Loads causing sway are seldom sustained (exceptions are frames supporting reaction from an arch roof or earth loads). If a sustained load acts on an unbraced frame, the deflections increase with time, leading directly to an increase in the P-Δ moment. $\rightarrow$

2.6 SLENDER COLUMNS

- Cont'd
  - This process is very sensitive to small variations in material properties and loadings. As a result, structures subjected to sustained lateral loads should always be braced. According to Macgregor/ Wight (previous ACI President), braced frames should be used wherever possible, regardless of whether the lateral loads are short time or sustained

2.6 SLENDER COLUMNS

- M_{ns} and M_s Moments
  - Two different types of moments occur in frames:
    - 1. moments due to loads not causing appreciable sway, M_{ns}
    - 2. moments due to loads causing appreciable sway, M_s
  - The slenderness effects of these two kinds of moments are considered separately in the ACI Code design process because each is **magnified differently as the individual columns deflect and as the entire frame deflects**.
  - Column moments that cause no appreciable sway are magnified when the column deflects by an amount δ relative to its original straight axis **such that the moments at points along the length of the column exceed those at the ends** (P-δ effect). $\rightarrow$

2.6 SLENDER COLUMNS

- Cont'd
  - On the other hand, the column moments due to lateral loads can cause appreciable sway. They are magnified by the P-Δ moments resulting from the sway deflections, Δ, at joints in the frame. This is referred to as the P- Δ effect or the lateral drift effect.
  - ACI Section 10.0, defines the nonsway moment, M_{ns}, as the factored end moment on a column due to loads that cause no appreciable sway, as computed by a 1st order elastic analysis. These moments result from gravity loads. $\rightarrow$

2.6 SLENDER COLUMNS

- Cont'd
  - The sway moment, M_s, is defined as the factored end moment on a column due to loads which cause appreciable sway, calculated by a 1st order elastic frame analysis. These moments result from lateral loads or in some cases from large unsymmetrical gravity loads or from gravity loads on highly unsymmetrical frames.
  - Treating the P-δ and P-Δ moments separately simplifies design. The nonsway moments frequently result from a series of pattern loads (see chapter 10). The pattern loads can lead to a moment envelope for the nonsway moments $\rightarrow$

2.6 SLENDER COLUMNS

- Cont'd

  →**The maximum end moments from the moment envelope are then combined with the magnified sway moments from a 2nd order analysis or from a sway moment-magnifier analysis.**

  - NB: This approach completely excludes the P- δ contribution of the gravity loads. That is why it is checked whether the maximum moment occurs b/n the ends of the column.

2.6 SLENDER COLUMNS

- **2.6.16 Calculation of Moments in Sway Frames by Using Second-Order Analysis**
  - **First-order and Second-Order Analysis**: A 1st order analysis is one in which the effect of lateral deflections on action effects is ignored. In a 2nd order analysis the effect of deflections on action effects is included. Because the moments are directly affected by the lateral deflections, it is important that the stiffness, EI, used in the analysis be representative of the stage immediately prior to ultimate.

2.6 SLENDER COLUMNS

- Cont'd
  - **Second-Order Analysis**
  - In a 2nd order analysis, column moments and lateral frame deflections increase more rapidly than do loads. (Recall the load-moment curves for the critical section including 2nd order deflection. Two consecutive equal load increments ΔP, do not result in corresponding constant moment increments ΔM. This would have been the case if the load eccentricities ($e_o + \delta$) are a constant in both load steps. While e_o is a constant, δ for the second load step is larger. Therefore the moment increases more rapidly than the load. $\rightarrow$ (P-δ relationship is nonlinear) $\rightarrow$ Thus it is necessary to calculate the 2nd order effects at the factored load level (Check EBCS EN 1992-1-1)

2.6 SLENDER COLUMNS

- Cont'd
  - Stiffness of the Members (supplementary to previously discussed)
    - (1) ULS: The stiffnesses appropriate for strength calculations must estimate the lateral deflections accurately at the factored load level. ACI Section 10.11.1 recommends that the beam stiffness be taken as **$0.35E_cI_g$**. Two levels of behavior must be distinguished in selecting the EI of columns. The lateral deflections of the frame are influenced by the stiffness of all the members in the frame and by the variable degree of cracking of these members. Thus , the EI used in the frame analysis should be an average value, ACI recommends $0.7E_cI_g$.
    - The value of EI for shear walls is the same as for beams where cracked and columns where uncracked

2.6 SLENDER COLUMNS

- Cont'd
  - On the other hand, in designing an individual column in a non-sway frame in accordance with Equation … ($\delta_{ns}=C_m/(1-P_u/0.75P_c)$, the EI used in calculating δ_{ns}, must be for that column. This EI must reflect the greater chance that a particular column will be more cracked, or weaker, than the overall average; hence, this EI will tend to be smaller than the average EI for all the columns acting together. Thus EI=…(recall the two Equations for EI (SPS)).

2.6 SLENDER COLUMNS

- Cont'd
  - **(2) SLS**: The moments of inertia given in ACI Section 10.11.1 (i.e. $0.35I_g$ for beams and $0.7I_g$ for columns) is for the ULS. At service loads, the members are cracked less. In computing deflections or vibrations, the values of I should be representative of the degree of cracking at service loads. The Commentary R10.11.1 suggests that I at service loads be taken as 1/0.7 = 1.43 times those for ULS. (Improved in 6th edition)
- Effects of Sustained Loads: Loads causing appreciable sidesway are generally short-duration loads, such as wind and EQ, as a result do not cause creep. If they are sustained, divide stiffness by $(1+\beta_{ds})$ for frame analysis

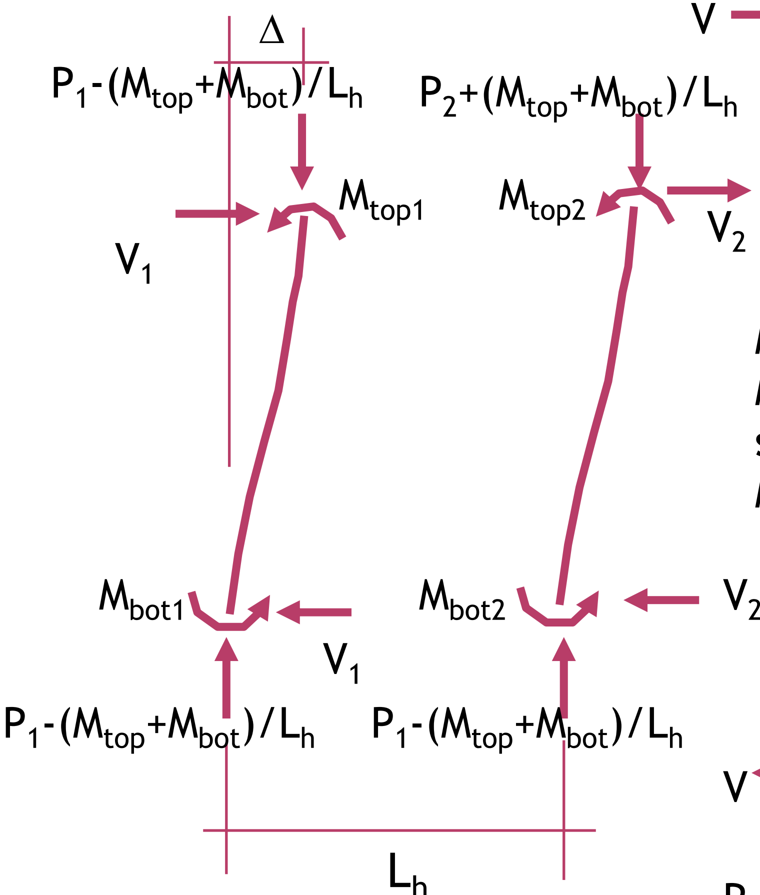
2.6 SLENDER COLUMNS

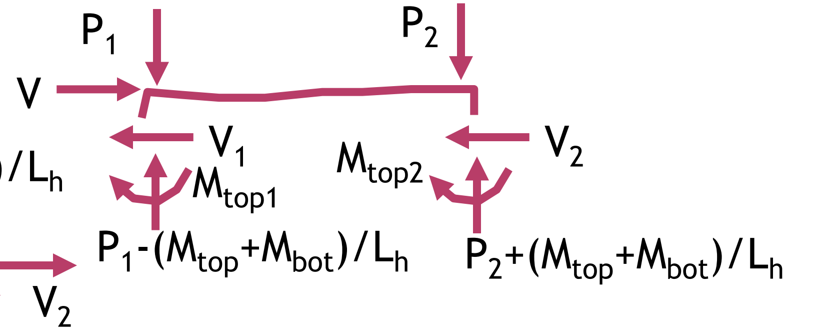
- Methods of Second-Order Analysis
  - **(1) Iterative P-Δ Analysis**: When a frame is displaced sideways under the action of lateral and vertical loads as shown in Figure, (SNS), the column end moments must equilibrate the lateral loads and a moment equal to $(\Sigma P)\Delta$
    - $\Sigma(M_{top} + M_{btm}) = V\ell_c + \Sigma P\Delta$;where Δ is the lateral deflection of the top of the story relative to the bottom of the story (story displacement)
    - The moment $\Sigma P\Delta$ in a given story can be represented by statically equivalent shear forces, $\Sigma P\Delta/\ell_c$. These shears give an overturning moment of $(\Sigma P\Delta/\ell_c) \times (\ell_c) = \Sigma P\Delta$.

2.6 SLENDER COLUMNS

- The figures in the following slides show FBDs of the individual columns and beams for the two statically equivalent Frames, the original with the external gravity loads and PΔ moments and the statically equivalent frame without both but with equivalent horizontal loads at the joints
- These horizontal external forces or their derivatives give rise to the sway forces that will be used in the iterative PΔ analysis in the coming section

2.6 SLENDER COLUMNS



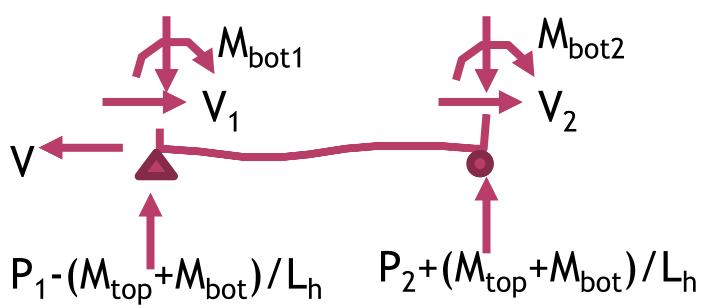


$M_{top1} + M_{bot1} = V_1 l + P_1 \Delta - ((M_{top} + M_{bot})/L_h)\ \Delta$

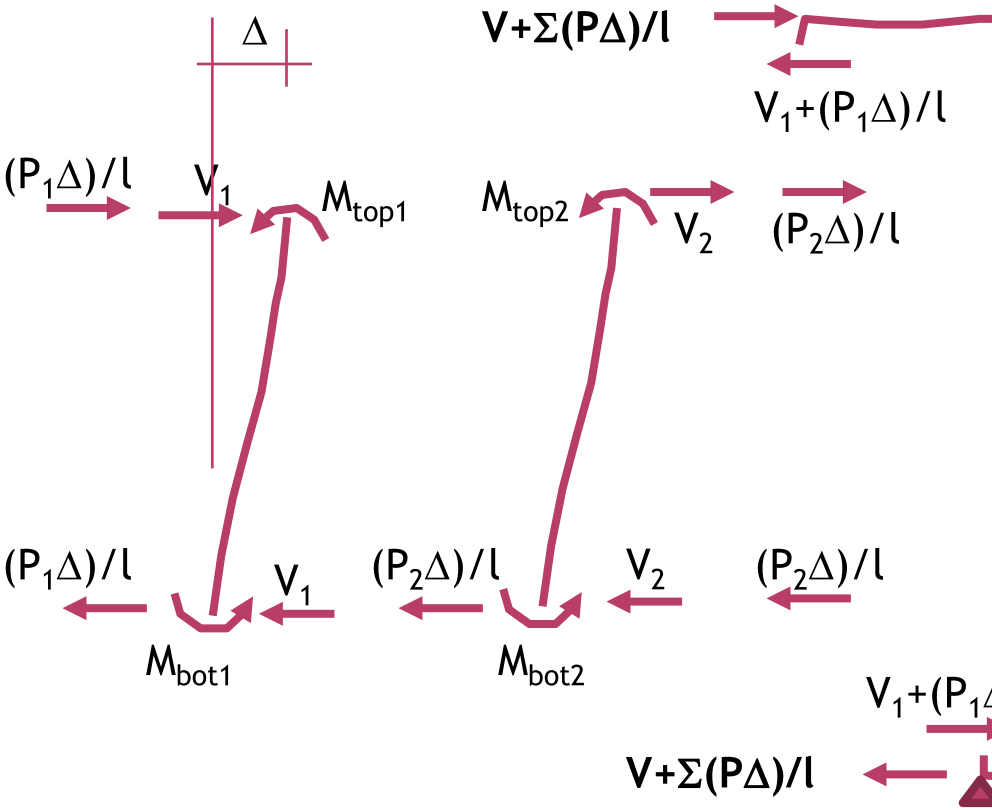
$M_{top2} + M_{bot2} = V_2 l + P_2 \Delta + ((M_{top} + M_{bot})/L_h)\ \Delta$

summing

$M_{top} + M_{bot} = V l + \Sigma P \Delta$



PΔ MOMENT REPLACED BY STATICALLY EQUIVALENT STORY SHEAR (→SWAY FORCE) Σ(PΔ)/L



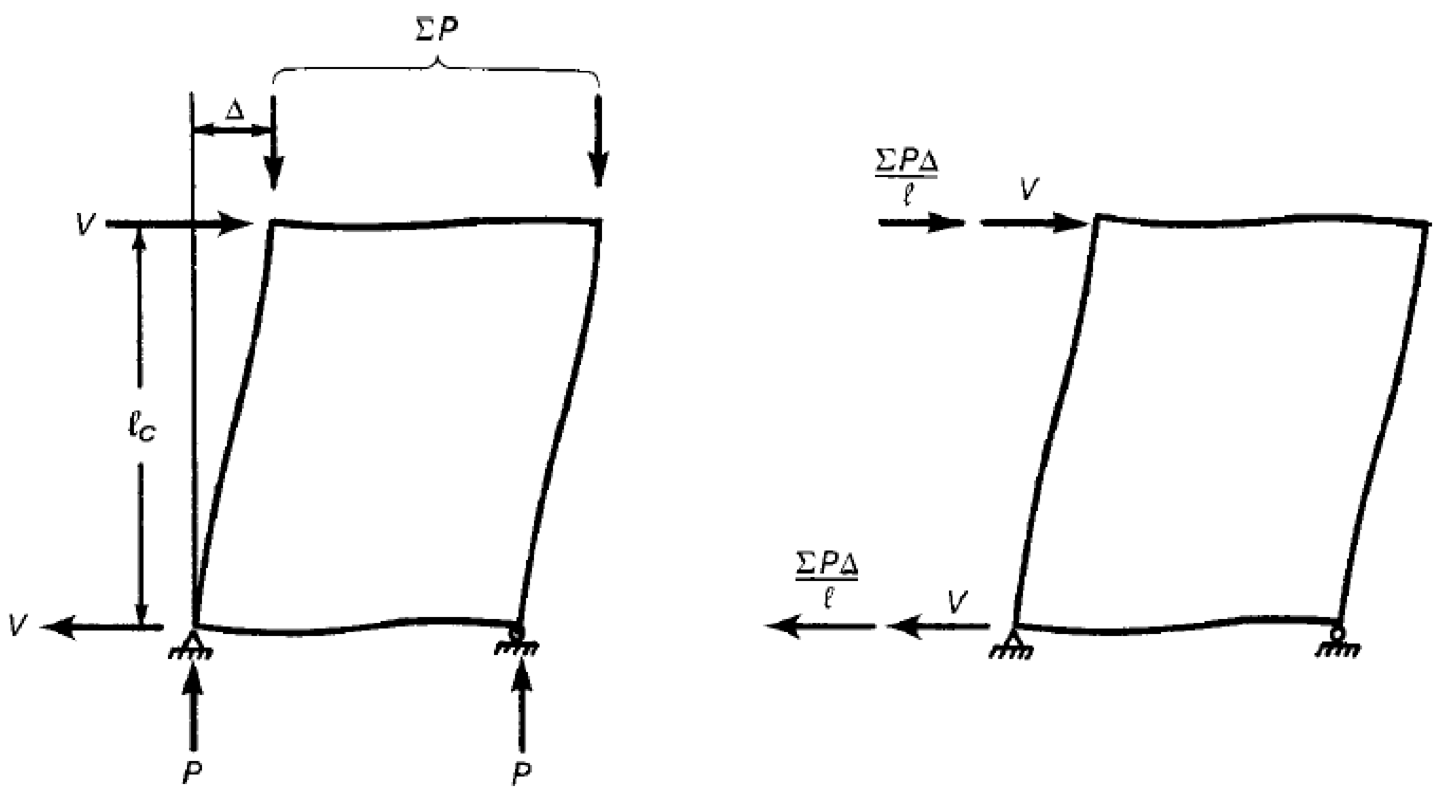
$M_{top1} + M_{bot1} = V_1 l + P_1\Delta$

$M_{top2} + M_{bot2} = V_2 l + P_2\Delta$

summing

$M_{top} + M_{bot} = Vl + \Sigma P\Delta$

2.6 SLENDER COLUMNS



(a) Loads on a sway frame.

(b) $\Sigma P\Delta/\ell$ shears in a story.

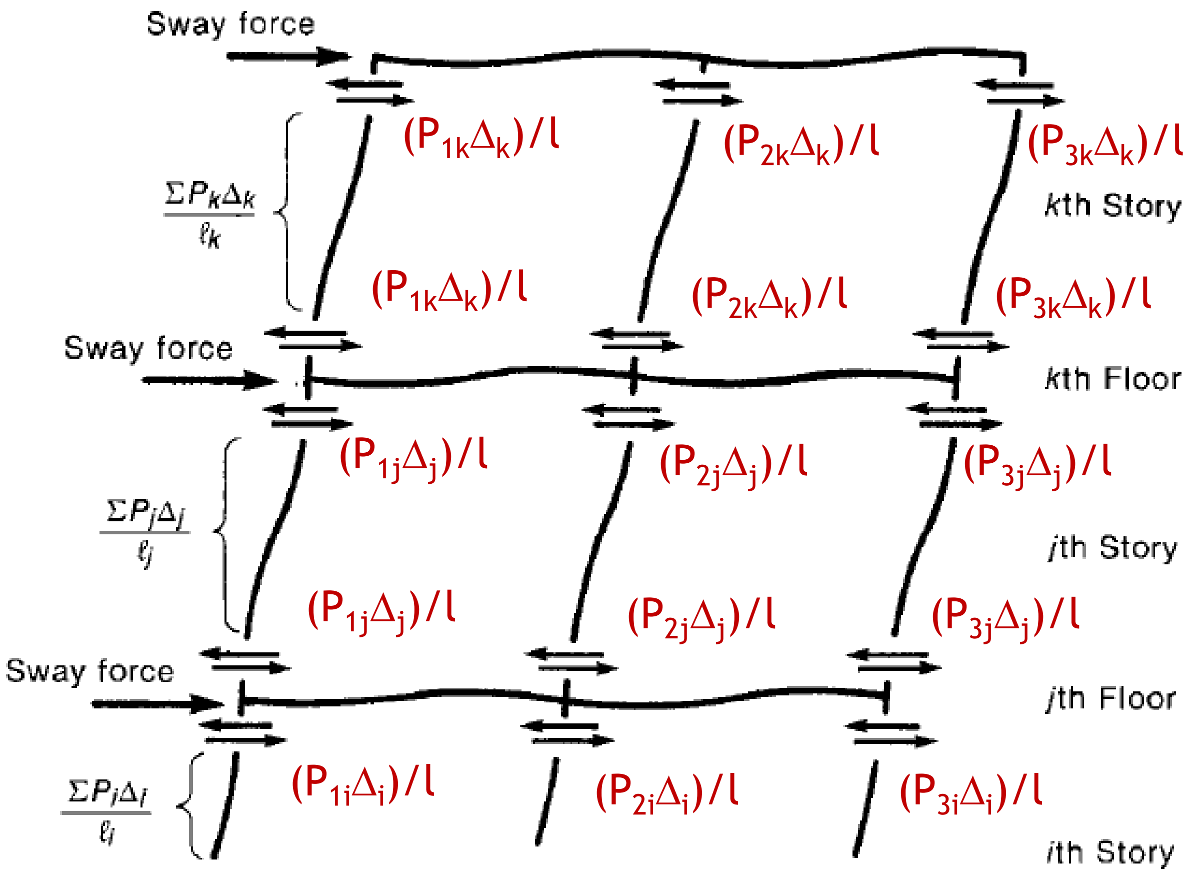
Fig: Iterative P-Δ analysis

2.6 SLENDER COLUMNS

- **Figure shows the story shears in two different stories.**
- Story $P\Delta$ moments in the k^{th} and j^{th} stories are $\Sigma P_k \Delta_k$ and $\Sigma P_j \Delta_j$ respectively
- The algebraic sum of the story shears from the columns above and below a given floor gives rise to a **sway force** acting on that floor. **At the j^{th} floor**, the **sway force** is:

- Sway force $_j$ = $(\Sigma P_i \Delta_i / \ell_i)$ - $(\Sigma P_j \Delta_j / \ell_j)$ (sign: positive $P\Delta/\ell$ and positive sway force correspond to forces that would overturn the structure in the same direction as the wind load would.

2.6 SLENDER COLUMNS



(c) Calculation of sway forces.

Fig: Iterative P-Δ analysis

2.6 SLENDER COLUMNS

- the sway forces are added to the applied lateral loads at each floor level, and the structure is reanalyzed, giving new lateral deflections and larger column moments. This process is continued until convergence is obtained (deflection iteration).

- **the iterative P-Δ analysis is used to derive** the direct **P-Δ analysis for sway frames described in next section.**

- Ideally a correction is made to this process using the flexibility factor γ applied to the deflection as $\gamma \times \Sigma P\Delta/\ell$ **(refer MacGregor)**

2.6 SLENDER COLUMNS

- **(2) Direct P-Δ Analysis for Sway Frames**
  - The iterative calculation procedure described in the preceding section can be described mathematically as an infinite series. The sum of the terms in this series gives the 2nd order deflection Δ (refer Macgregor):

$$\Delta = \frac{\Delta_0}{\left(1 - \gamma \frac{\Sigma P_u \Delta_0}{V_{us} l_c}\right)}$$

  - where V_{us} = story shear due to lateral loads; ℓ_c = story height; ΣP_u = the total axial load in all columns in the story; $\gamma \cong 1.15$; Δ_0= 1st order story displacement due to the story shear, V_{us}; Δ = 2nd order deflection

2.6 SLENDER COLUMNS

- Cont'd
  - Since the moments in the frame are directly proportional to the deflections, the 2nd order moments are:
  - $M=\delta_s M_s=M_0/(1-\gamma(\Sigma P_u\Delta_0)/(V_u\ell_c))$
  - ACI Section 10.11.2.2.defines the **stability index** for a story as:
  - $Q= \Sigma P_u\Delta_0/(V_{us}\ell_c)$
  - **Substituting this into the above equation→** $\delta_s M_s=M_s/(1- Q) \geq M_s$
  - ACI limits the use of above equation to $Q \leq 1/3$.

2.6 SLENDER COLUMNS

- 2.6.17 design of Columns in Sway Frames
- The ACI Code design procedure for slender columns in sway frames consists of five steps:
  - (1) The unmagnified moments, M_{ns}, due to loads not causing appreciable sway are computed (via regular 1st order elastic-frame analysis)
  - (2) The magnified sway moments, $\delta_s M_s$, are computed (e.g. direct P-Δ analysis)
  - (3) The magnified sway moments, $\delta_s M_s$, are added to the unmagnified nonsway moments. M_{ns}.

2.6 SLENDER COLUMNS

- Cont'd
  - (4) A check is made whether the maximum moment occurs b/n the ends of the column
    - Normally, the maximum moment in the column will be the P-Δ moment at one end, and the column is designed for this moment. However, if the axial loads on the column are high and the slenderness exceeds the limits given in the ACI, it is necessary to check whether the P-δ moment at some section b/n the ends of the column exceeds the maximum end moment. This is done by using the braced-frame magnifier. If the magnified moment is greater than the P-Δ moment, then the column will be designed with the magnified moment (occurs rarely)

2.6 SLENDER COLUMNS

- Cont'd
  - (5) a check is made whether sidesway buckling can occur under gravity loads alone. (Again rarely a problem assuming the design is carried out by professionals ready for designing RC structures)
- **Each of the steps are discussed as follows**
- **(1) Computation of $\delta_s M_s$ by Using Second-Order Analysis**
  - ACI Section 10.13.2.1.allows the use of 2nd order analysis to compute $\delta_s M_s$. If torsional displacements of the frame are significant, a 3-D 2nd order analysis should be used.
- **(2) Computation of $\delta_s M_s$ by Using Direct P-Δ Analysis for Sway Frames**
  - $\delta_s M_s = M_s/(1-Q)$; where $Q = \Sigma P_u \Delta_0/(V_u \ell_c)$):

2.6 SLENDER COLUMNS

- Cont'd
- (3) Computation of $\delta_s M_s$ by Using Sway-Frame Moment Magnifier
  - $\delta_s M_s = M_s/(1-\Sigma P_u/(0.75\Sigma P_c))$: In this case, the values of P_c are calculated by using the effective lengths, $k\ell_u$, evaluated for columns in a sway frame, with β_d defined as β_d = max factored sustained shear in a story/total factored shear in the story. In most sway frames, the story shear is due to wind or seismic loads and hence is not sustained, resulting in β_d=0.
  - The use of the summation terms accounts for the fact that sway instability involves all the columns and bracing members in the story. →

2.6 SLENDER COLUMNS

- If $1/(1-\Sigma P_u/(0.75\Sigma P_c)$ is negative, the load in a story or more stories in the frame, ΣP_u, exceeds the buckling load of the story ΣP_c, indicating that the frame is unstable. A stiffer frame is required.

(4) Moments at the Ends of the Columns

- The unmagnified nonsway moments, M_{ns}, are added to the magnified sway moments, $\delta_s M_s$, at each end of the columns.
- $M_1=M_{1ns} + \delta_s M_{1s}$; $M_2=M_{2ns} + \delta_s M_{2s}$
- The addition is carried out for the moments at the top and bottom of each column. The larger absolute sum is called M_2, and the smaller M_1. By definition, M_2 is always +ve, and M_1 is taken as –ve if the column is bent in double curvature

2.6 SLENDER COLUMNS

- (5) Maximum Moments b/n the Ends of the Column
  - If ℓ_u/r exceeds the value given by $\ell_u/r > \sqrt{(P_u/f_c'A_g)}$, there is a chance that the maximum moment on the column will exceed the larger end moment, M_2. This would occur if M_c, computed for the braced column, was larger than the end moments M_1 and M_2.
- (6) Sidesway Buckling Under Gravity Load
  - Classical case of sidesway buckling under gravity load alone

2.6 SLENDER COLUMNS

- Cont'd
  - U = 1.4D + 1.7L. Since there are 3 methods to calculate $\delta_s M_s$, 3 corresponding methods are given to check sidesway buckling (previous edition of ACI)
  - 2008 Edition:ACI Code Section 10.10.2.1 guards against this by requiring that the secondary-to-primary moment ratio shall not exceed 1.4.
- (7) Minimum moment
  - The ACI Code specifies a minimum moment $M_{2,min}$ to be considered in the design of columns in nonsway frames, but not for columns in sway frames.

2.6 SLENDER COLUMNS

- Example: Design of Columns in a Sway Frame
  - Refer MacGregor
  - **Assignment →Design the columns using EBCS EN 1992-1-1**