# Groundwater Hydraulics

Chapter 1 – Introduction

CENG 6606

# Contents

# What is groundwater?

- Equivalent terminologies: subsurface water, ground water
- **Definitions**
  - Water occupying all the voids within a geologic stratum (Todd, 1980)
  - All the water found beneath the surface of the ground (Bear and Verruijt, 1987)
  - Practically, all the water beneath the water table (i.e., in the saturated zone) and above the water table (i.e., in the unsaturated zone, vadose zone, zone of aeration) are called groundwater.
- Groundwater velocity is very small and depends on local hydrogeologic conditions , 2 m/year to 2 m/day are normal (Todd, 1980).

# 1. Definitions of aquifers

- Aquifer, Aquitards, Aquicludes, and Aquifuges are a geological formation or a group of formations that can/cannot contain water, and that water can/cannot move within the formation

| | Formation nature | Store water? | Transmit water? |
|---|---|---|---|
| Aquifer | Pervious | Yes | Yes |
| Aquitard | Semi pervious | Yes | Yes but slower than that in an aquifer |
| Aquiclude | Semi pervious | Yes | No |
| Aquifuge | Impervious | No | No |

Latin: Aqui ≡ water; -fer ≡ "to bear", aquifer ≡ "water bearer"
-tard ≡ "slow";
-clude ≡ "to shut or close";
-fuge ≡ "to drive away" (Todd, 1980)

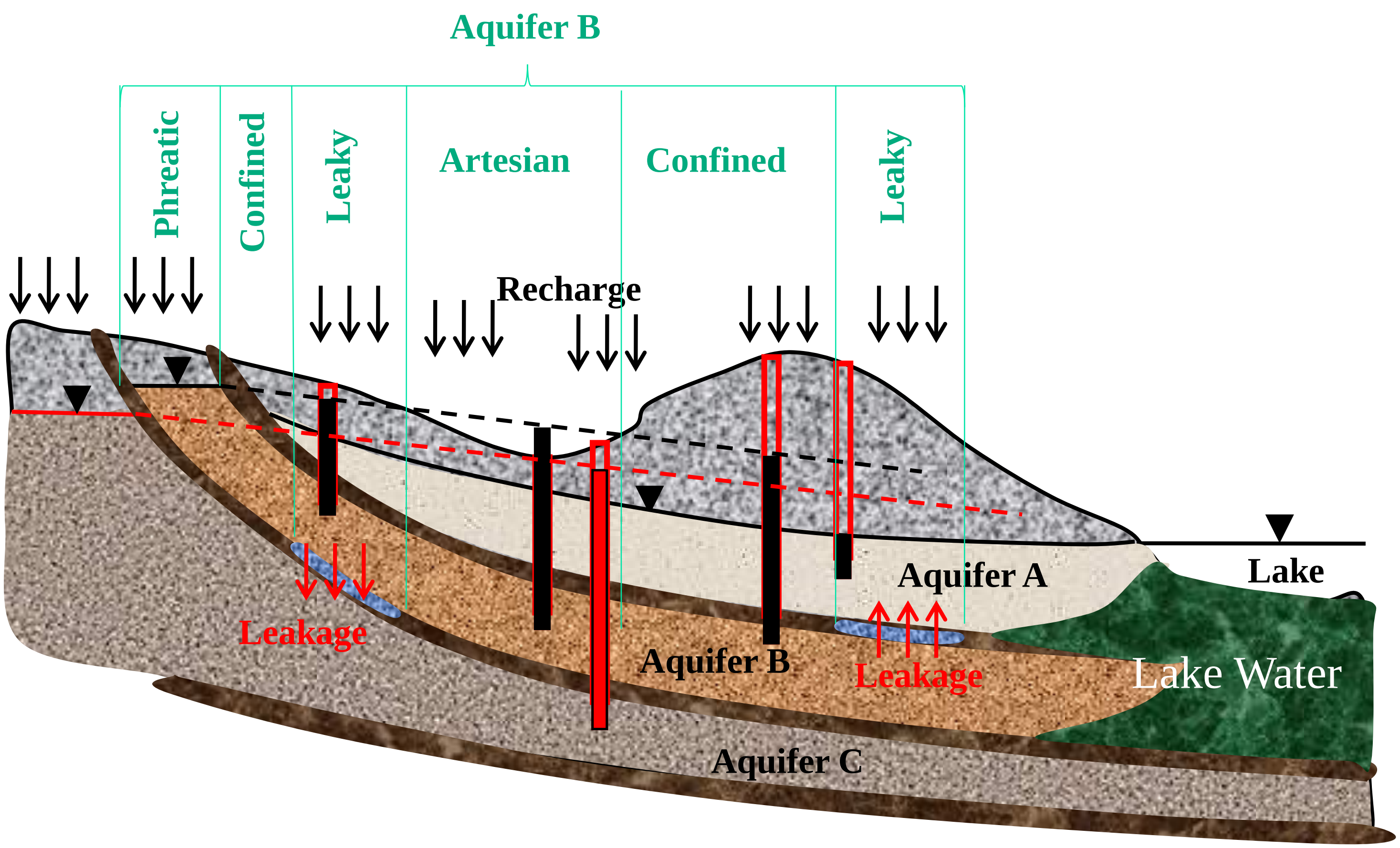
# 1. Definitions …

- **Confined aquifer**: an aquifer bounded from above and from below by impervious formations (aquiclude or aquifuge)
- **Unconfined aquifer** (phreatic aquifer or water table aquifer): an aquifer in which water table serves as its upper boundary
- **Perched aquifer**: An unconfined aquifer which has an impervious layer of limited areal extent located between the ground surface and the water table (of the unconfined aquifer)
- **Confining layer**: a geologic formation that is impervious to water, e.g., unconsolidated soils such as silt and clay; consolidated bed rock such as limestone, sandstone, siltstone, basalt, granite, …, etc. The latter rock formations can also be aquifers when only consolidated formations are considered.

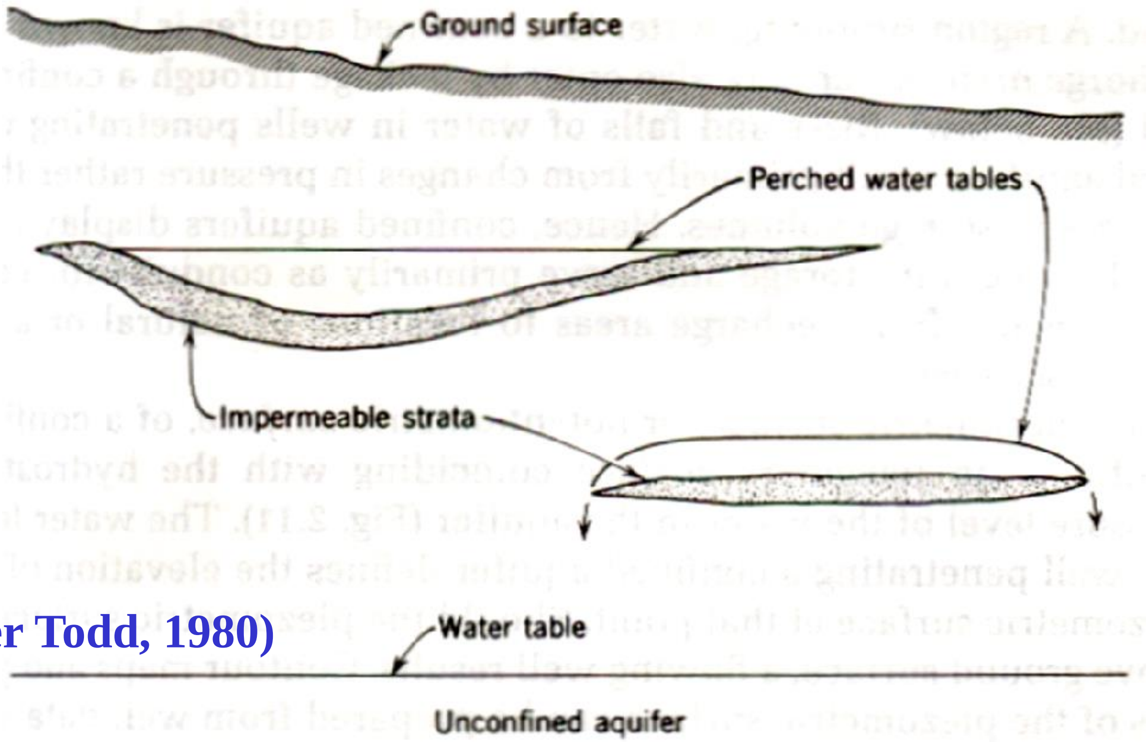
# 1. Definitions …

- **Piezometric surface or Potentiometric surface**: an imaginary surface connecting the water levels of a number of observation wells tapping into a confined aquifer.
  - **Note**: use water table instead of piezometric surface for unconfined aquifers
- **Artesian aquifer**: a confined aquifer whose piezometric surface is above the ground surface (i.e., water comes out automatically from a well in an artesian aquifer)
- **Double porosity aquifer**
  - For fractured rocks
    - Matrix blocks: low permeability, high storativity
    - Fractures: high permeability, low sotrativity

# Schematic of aquifers



# Schematic of perched aquifers




**(After Todd, 1980)**

**Fig. 2.12** Sketch of perched water tables.

- **Saturated zone**: Aquifers in which pore space is completely filled with water.
- **Unsaturated zone**: Aquifers in which the pore space is filled partially with liquid phase (water), and partially with gas phase.

# 2. Fundamental physical properties

- Six fundamental physical properties for describing hydraulic aspects of saturated groundwater flow in aquifers
  - Three fluid properties
    - Density, $\rho$ ($M/L^3$)
    - Dynamic viscosity, $\mu$ ($M/L \cdot T$) (or kinematic viscosity, $\nu = \mu/\rho$, $L^2/T$)
    - Fluid compressibility, $\beta$ ($LT^2/M$) (or 1/Pa)
  - Three medium properties
    - Porosity, n
    - Permeability, k ($L^2$)
    - Matrix compressibility, $\alpha$ ($LT^2/M$) (or 1/Pa)

# 3. Porosity (pore space)

- **Pore space** (voids, pores, or interstices): The portion of a geologic formation that is not occupied by solid matter (e.g., soil grain or rock matrix).
- **Effective or interconnected pore space**: Pores that form a continuous phase through which water or solute can move
- **Isolated or non-inter connected pore space**: Pores that are dispersed (scattered) over the medium. These pores cannot contribute to transport of matter across the porous medium. Also known as **dead-end pores**.
- Only the connected pores (gray) can transmit water; the unconnected pores (white) are not a part of the effective porosity

# Schematic of various pore space

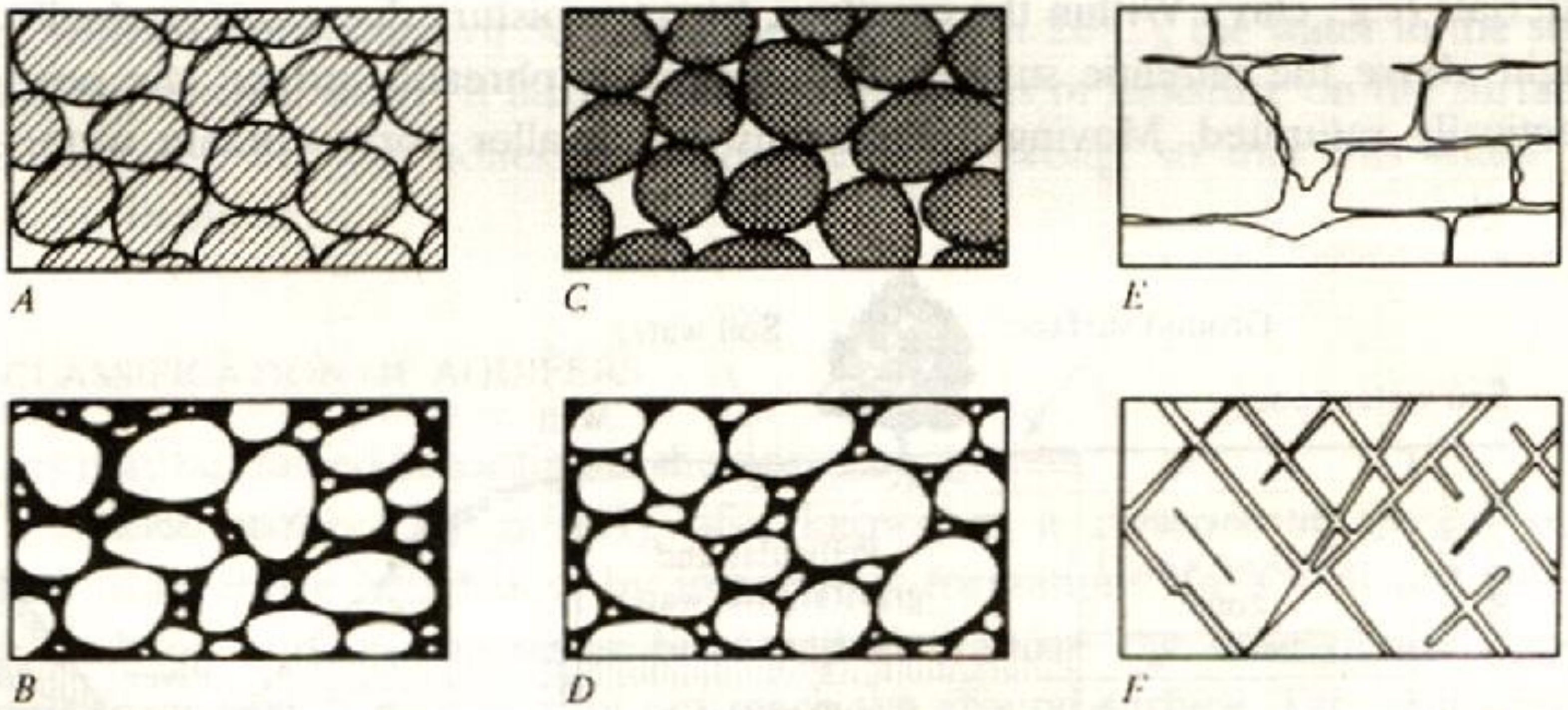



Fig. 1.2. Diagram showing several types of rock interstices. A Well-sorted sedimentary deposit having high porosity; B. Poorly sorted sedimentary deposit having low porosity; C. Well-sorted sedimentary deposit consisting of pebbles that are themselves porous, so that the deposit as a whole has a very high porosity; D. Well-sorted sedimentary deposit whose porosity has been diminished by the deposition of mineral matter in the interstices; E. Rock rendered porous by solution; F. Rock rendered porous by fracturing (*after Meinzer, 1942*).

**(After Bear and Verruijt, 1987)**

n ($d_{50}$, shape, arrangement, clay)

# 3. Porosity …

- **Porosity is a function of**

(a) **Grain size distribution**: soils with uniformly distributed grains have larger porosities than soils with un-uniformly distributed grains

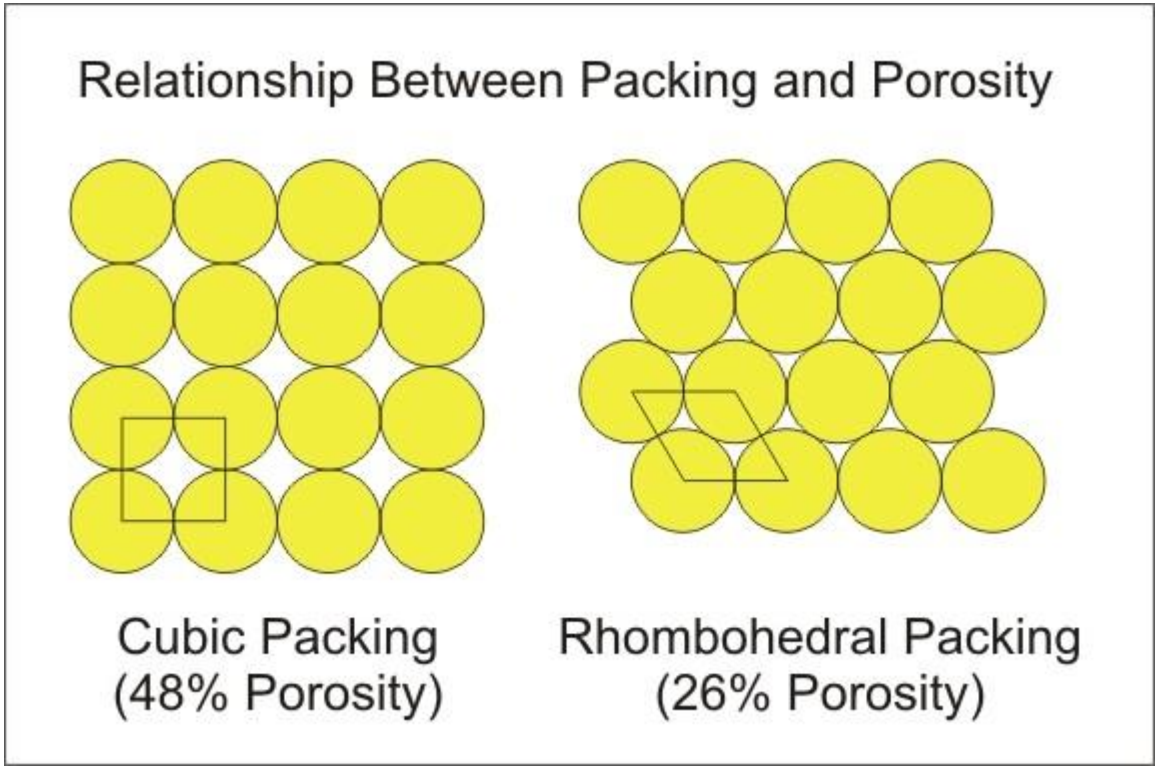
(b) **Grain shape:** Sphere-shaped grains will pack more tightly and have less porosity than particles of other shapes

(c) **Grain arrangement:** Porosities of well-rounded sediments range from 26% (rhombohedral packing) to 48% (cubic packing)

(d) **Clays**, clay-rich or organic soils have very high porosities because:

  - Grain shapes are highly irregular
  - Dispersive effect of the electrostatic charge on the surfaces of certain book-shaped clay minerals causes clay particles to be repelled by each other, resulting in high porosity.

# Cubic packing



# 3. Porosity …

| Unconsolidated deposits (sediments) | Porosity (%) |
|---|---|
| Gravel | 25-40 |
| Sand | 25-50 |
| Silt | 35-50 |
| Clay | 40-70 |
| **Rocks** | |
| Fractured basalt | 5-50 |
| Karst limestone | 5-50 |
| Sandstone | 5-30 |
| Limestone, dolomite | 0-20 |
| Shale | 0-10 |
| Fractured crystalline rock | 0-10 |
| Dense crystalline rock | 0-5 |

(After Freeze and Cherry, 1979)

# 3. Porosity …

- Porosity of rocks (Two porosities)
  - **Primary:** pore space between grains
  - **Secondary:** pore space caused by fracturing
- **Sedimentary rocks (Fetter, 1994):** Formed by sediments by digenesis. Sediments are products of weathering of rocks or chemically precipitated materials. Changes in sediments due to overlying materials and physiochemical reactions with fluid in the pore space result in pore space variation.
- **Compaction:** porosity is reduced
- **Dissolution:** porosity is increased
- **Precipitation** (e.g., cementing materials such as calcite, dolomite, or silica): porosity is reduced

- **Limestone and dolomites**: Formed respectively by calcium carbonate and calcium-magnesium carbonate, which were originally part of an aqueous solution. These rocks may be dissolved in a zone of circulating groundwater, resulting in huge caverns that have sizes as large as a building. For example, the caverns at Carlsbad, New Mexico.

Bottomless Pit

Hall of Giants

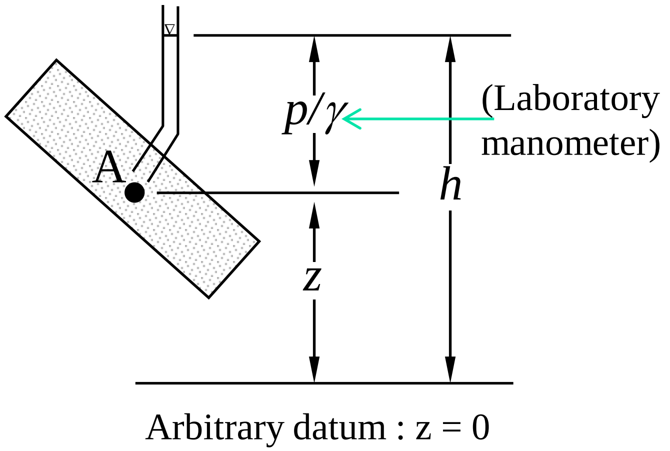
The Big Room

# 4. Hydraulic head

- Hydraulic head, h (L): For incompressible fluids (density is a constant) it is the sum of potential energy and pressure energy **per unit weight** of water. Sum of elevation head (z) and pressure head (p/γ)

- $[h] = \frac{energy}{weightofwater} = \frac{Nm}{N} = m$
- $\frac{p}{\gamma} = \frac{p}{\rho g} = \frac{N/m^2}{(Kg/m^3)(m/s^2)} = \frac{Nm}{N} = m$
- $[z] = \frac{PotentialEnergy}{WeightofWater} = \frac{Nm}{N} = m$

$$h = \frac{p}{\gamma} + z \qquad (2)$$




Schematic of hydraulic head, pressure head, and elevation (potential) head

# 5. Hydraulic conductivity

- Fluid conductivity, K [L/T]: It is measurement of the ease of a particular fluid passing through the pore space of a porous medium (i.e., conductive properties of a porous medium for a particular fluid) (Hubbert, 1956). The proportionality constant in Darcy's law, which depends on medium and fluid properties i.e. grain size, density and viscosity of fluid.
- If the fluid is water K is hydraulic conductivity: $K = \frac{Cd^3\rho g}{\mu}$
- Where C: shape factor, a medium property, d: representative grain diameter, a medium property and μ: fluid density, g: gravity
- $\gamma = \rho g$: specific weight of fluid, driving force exerted by gravity on a unit volume of the fluid ($[\gamma] = kg/m^3 \cdot m/s^2 = (kg \cdot m/s^2)/m^3 = N/m^3$ = force per unit volume)

μ: Dynamic viscosity (resistance of the fluid to shearing)

# 5. Hydraulic conductivity

Table 2.1. Typical values of hydraulic conductivity and permeability[a]

| $-\log_{10} \cdot K$ (cm/sec) | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Permeability | Pervious | | | | Semipervious | | | | Impervious | | | | | |
| Aquifer | Good (Aquifer) | | | | | Poor (Aquitard, aquiclude) | | | | None (Aquifuge) | | | | |
| Soils | Clean gravel | Clean sand or sand and gravel | | | | Very fine sand, silt, loess, loam, solonetz | | | | | | | | |
| Soils | | | | | Peat | | Stratified clay | | | Unweathered clay | | | | |
| Rocks | | | | | Oil rocks | | | Sandstone | | Good limestone, dolomite | | Breccia, granite | | |
| $-\log_{10} \cdot k$ (cm²) | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $\log_{10} k$ (md) | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | −1 | −2 | −3 | −4 | −5 |

[a] From Bear *et al.* (1968).

(After Bear and Verruijt, 1987)

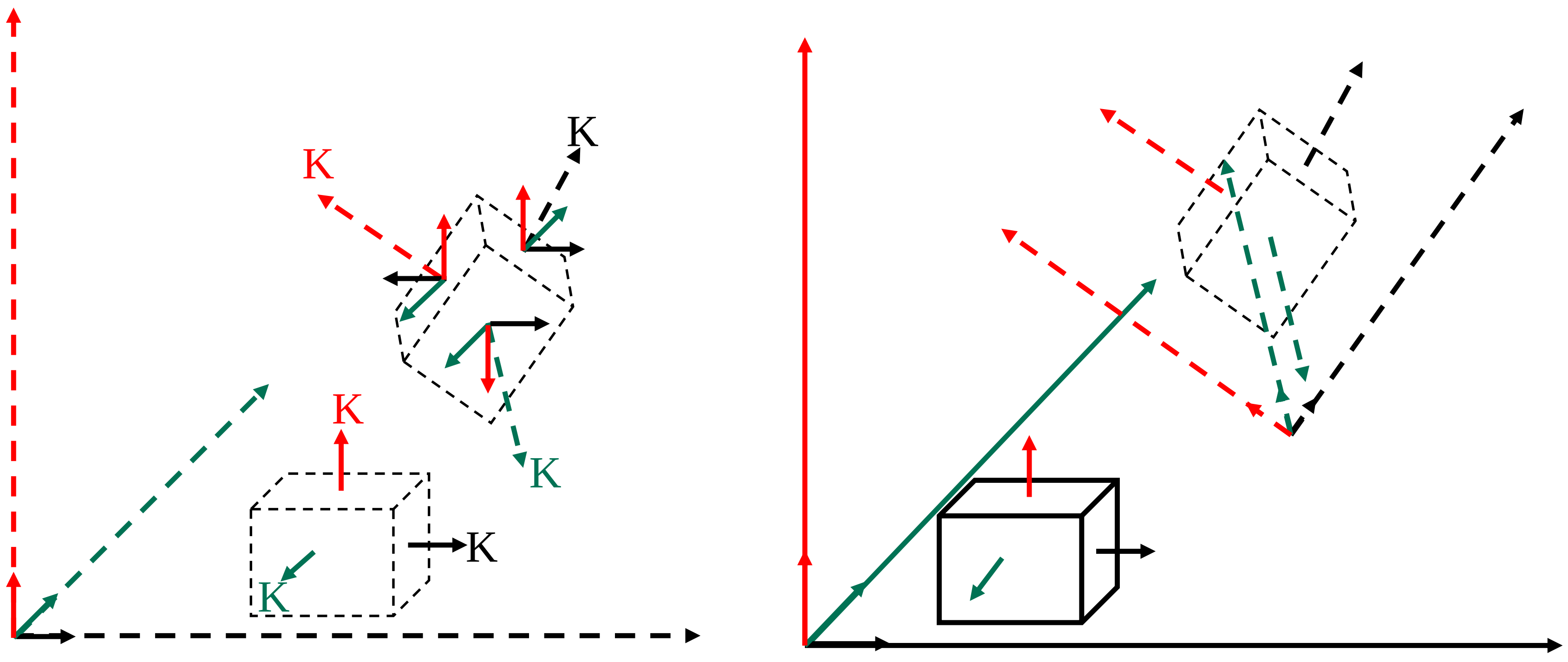
# 5. Hydraulic conductivity

- Hydraulic conductivity as a tensor
- Tensor
  - Zero-order: a scalar such as hydraulic head, a single-valued quantity
  - First-order: a vector such as velocity, having 3 components
  - Second-order: a tensor such as hydraulic conductivity, having 9 components

$$\mathbf{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

- Symmetric hydraulic conductivity : $K_{ij} = K_{ji}$ $(i \neq j)$
- Principal hydraulic conductivity: the coordinate system aligns along the principal axes of **K** such that $K_{ij} = 0$ $(i \neq j)$
- Isotropic hydraulic conductivity: the value of **K** does not depend on direction, i.e., $K_{ij} = 0$ $(i \neq j)$, and $K_{ii} = K$

# Hydraulic conductivity Tensor




- Contravariant and covariant tensors

# 5. Hydraulic conductivity

- **Determination of hydraulic conductivity**
  - Theoretical calculation
    - $k = Cd^2 \Rightarrow K = k\rho g/\mu$
    - Few estimates are reliable because of the difficulty of including all possible variables in porous media
  - Laboratory measurements
    - Permeameter (Fetter, 1994)
      - **Constant head experiment**: For non cohesive sediments, such as sand and rocks because of the required duration for experiment is short
      - **Falling head experiment**: For cohesive sediments with a low permeability, such as clay and silt
  - Field measurements
    - Pumping test, slug test
    - Tracer test

# 5. Hydraulic conductivity

- Heterogeneity and anisotropy
  - Heterogeneity: $K(\mathbf{x}_1) \neq K(\mathbf{x}_2)$, $\mathbf{x}_1 \neq \mathbf{x}_2$
    - Layered sediments, e.g., sedimentary rocks and unconsolidated marine deposits
    - Spatial discontinuity, e.g., the presence of faults or large-scale stratigraphic features
  - Evidences from stochastic studies
    - Hydraulic conductivity tends to be log-normally distributed
    - $Y = \log_{10}K$ has standard deviation in the range 0.5 ~1.5, meaning K values in most geological formations show variations of 1 – 2 orders of magnitude (Freeze, 1975)

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Freeze, R. A., A stochastic-Conceptual Analysis of One-Dimensional Groundwater Flow in Nonuniform Homogeneous *Media*, *Water Resources Research*, Vol. 11, No. 5, pp.725-741, 1975.

# 5. Hydraulic conductivity

**Example:** (Fetter, 1994) Find the geometric mean of the following set of hydraulic conductivity values and compare it to the arithmetic mean : ***K (cm/s)*** *= 2.17 × 10^-2^ , 2.58 × 10^-2^ , 2.55 × 10^-3^ , 1.67 × 10^-1^ , 9.50 × 10^-4^ ;* ***Sum of K (cm/s)*** *= 2.18 × 10^-1^*

Solution :

$$\text{Geometric mean} = K_G = \left(\prod_{i=1}^{n} K_i\right)^{1/n} \Rightarrow \ln K_G = \frac{1}{n}\sum_{i=1}^{n}\ln\left(K_i\right)$$

$$\frac{1}{5}\sum_{i=1}^{5}\ln\left(K_i\right) = -4.44 \Rightarrow K_G = \exp\left(\sum_{i=1}^{5}\ln\left(K_i\right)\right) = \exp\left(-4.44\right) = 1.18\times 10^{-2}\ \text{cm/s}$$

$$\text{Arithmetic mean} = \frac{1}{5}\sum_{i=1}^{5}K_i = \frac{2.18\times 10^{-1}}{5} = 4.36\times 10^{-2}\ \text{cm/s} > 1.18\times 10^{-2}\ \text{cm/s}$$

**Which is the best estimate and why? (think)**

# Answer

- $1.67 \times 10^{-1}$ ,
- $2.58 \times 10^{-2}$ ,
- $2.17 \times 10^{-2}$ ,
- $2.55 \times 10^{-3}$ ,
- $9.50 \times 10^{-4}$

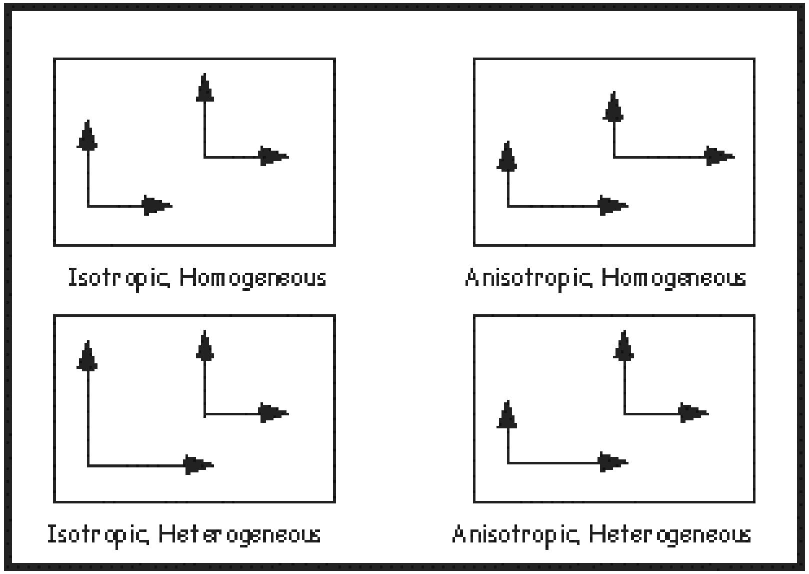
***$4.36 \times 10^{-2}$ Arithmetic mean***

***$1.18 \times 10^{-2}$ Geometric mean***

- Note that :
  - arithmetic mean of hydraulic conductivity is dominated by the largest value of K.
  - If observed values of hydraulic conductivity have orders of magnitude differences then the arithmetic mean would be significantly different from the geometric one.
  - The locations with small values of hydraulic conductivity have significant impacts to solute transport than to groundwater flow estimations. Thus, it is more favorable to use the geometric mean instead of an arithmetic one in groundwater hydrology.

# 5. Hydraulic conductivity

- Anisotropy
  - Hydraulic conductivity depends on the direction of measurement, e.g., $K_x \neq K_z$
  - Principal directions of hydraulic conductivity: the directions at which hydraulic conductivity attains its maximum and minimum values, which are always perpendicular to one another.
- Four cases



# Transmissivity (T)

- **Definition:** The rate (Q) at which water is transmitted through a unit width ($\Delta y = 1$) of aquifer under a unit hydraulic gradient ( $\nabla h = 1$).

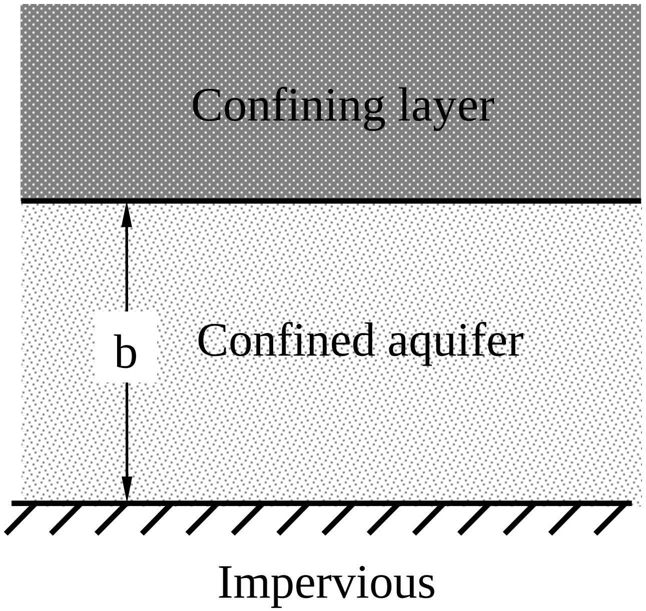
$$Q = vA = (-K\nabla h)A = (-K\nabla h)b\Delta z$$

$$\Rightarrow \frac{Q}{\Delta z} = -Kb\nabla h = -T\nabla h$$

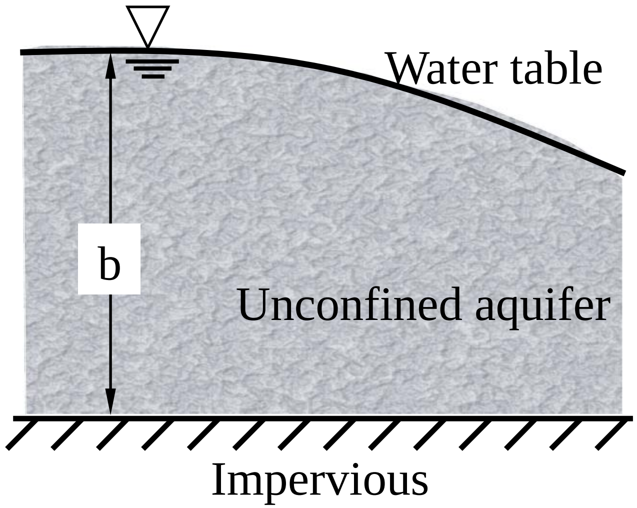
  - in which b is the "saturated thickness" of aquifer
- Transmissivity is well defined for the analysis of well hydraulics in a confined aquifer in which the flow field is essentially horizontal and two-dimensional, in which b is the (average) thickness of the aquifer between upper and lower confining layers
- It is, however, not well defined in unconfined aquifer but is still commonly used. In this case, the saturated thickness is the height of the water table above the top of the underlying aquitard (impervious layer) that bounds the aquifer

# Transmissivity (T)

Schematic representation of the definition of transmissivity




$$T = Kb$$




(Adapted from Freeze and Cherry, 1979)

Note that transmissivities greater than 0.015 $m^2/s$ represent good aquifers for water well exploitation (Freeze and Cherry, 1979)

# 6. Storage capacity

- Specific storage (specific storativity), $S_s$ (1/L)
- **Definition:** the amount of water released from (or added to) storage per unit decline (or unit rise) in hydraulic head from unit volume of *saturated* aquifer .

$$S_s = \frac{\Delta V_{water}}{V_{aquifer} \times \Delta h}$$

- **Storativity (Storage coefficient), S**: is the amount of water released from (or added to) storage per unit decline (or unit rise) in hydraulic head normal to the unit surface area of saturated aquifer
- Similar to transmissivity, storativity is developed primarily for the analysis of well hydraulics in a confined aquifer

$$S = \frac{\Delta V_{water}}{A_{aquifer} \times \Delta h} = S_s b$$

b is the saturated thickness of the aquifer

- **Storativity of a confined aquifer:** Water is released from a confined aquifer via
  - Expansion of water due to decline of hydraulic head
  - Release of pore water due to compaction of soil skeleton that is again induced by the decline of hydraulic head
- In general, storage coefficients for a confined aquifer are small, in the range of 0.005 to 0.00005 (Freeze and Cherry, 1979)
- **Storativity of an unconfined aquifer:** Water is released in unconfined aquifer via
  - Primary release: storage from the decline of water table, which is generally known as specific yield, $S_y$
  - Secondary release: the expansion of water and expel of water from aquifer compaction
  - $S = S_y + hS_s$, h is the saturated thickness of the unconfined aquifer . The usual range of $S_y$ is 0.01 ~ 0.30.

# 6. Storage capacity

- It is customarily to approximate the storativity of an unconfined aquifer by its specific yield.
- **Specific yield, $S_y$:** The ratio of the volume of water that drains from a saturated aquifer due to the attraction of gravity to the total volume of the aquifer. This is also called gravity drainage.
- **Specific retention ($S_r$):** the volume of water retained in an aquifer per unit area per unit drop of the water table after drainage has stopped, which is hold between soil particles by surface tension. Hence, the smaller the particle size, the larger the surface tension and the larger the specific retention
  - Specific retention is responsible for the volume of water a soil can retain against gravity drainage.
  - Maximum specific yield occurs in sediments in the medium-to-coarse sand-size range (0.5 to 1.0 mm).

# 6. Storage capacity

**Schematic representation of storativity in confined and unconfined aquifers**

Aquifer remains saturated in spite of the decline of piezometric surface

Aquifer above the original water table becomes unsaturated as the water table declines to a new level

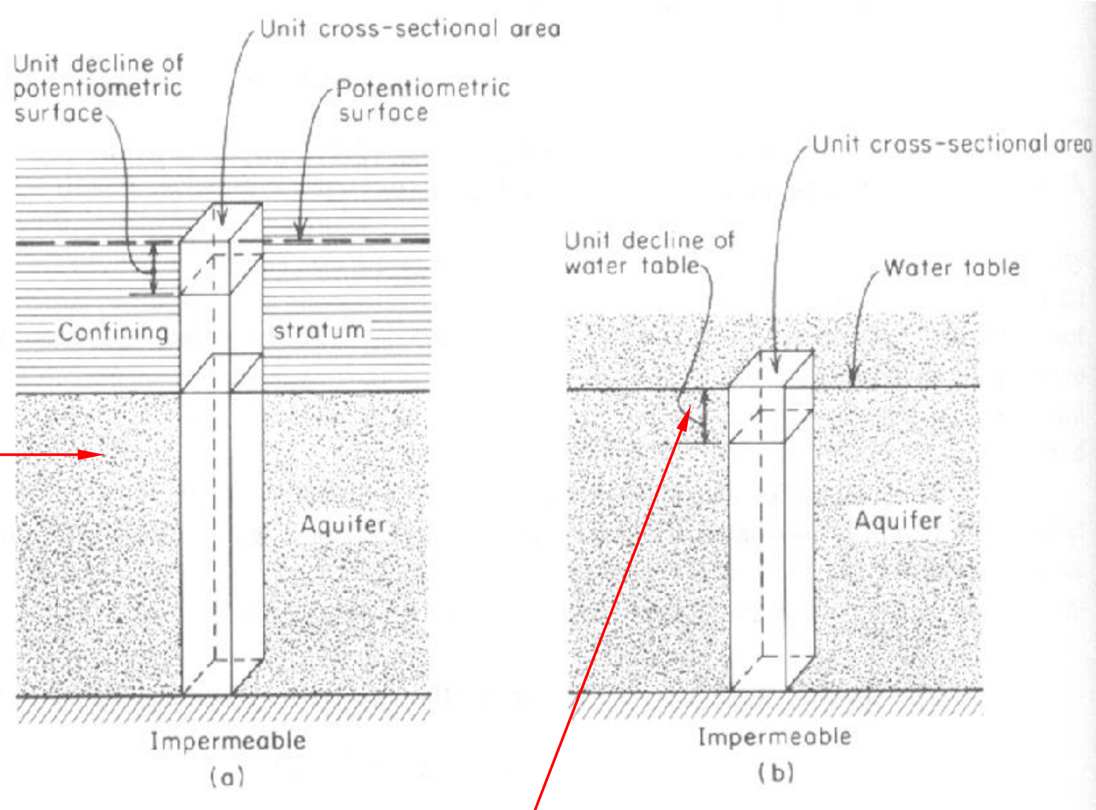



Figure 2.22 Schematic representation of the storativity in (a) confined and (b) unconfined aquifers (after Ferris et al., 1962).

**(After Freeze and Cherry, 1979)**

# Compressibility (Freeze and Cherry):

- The physical phenomena related to the reduction of aquifer volume due to a stress applied to a unit mass of saturated medium
  - Compression of the water in the pores
  - Compression of the individual soil grains (negligible in practice)
  - A rearrangement of soil grains into a more tightly packed configuration
- Definition : ratio of strain to stress
  - **Physical meaning** : the change of volume of a material due to the change of stress applied to that material, with a unit of 1/[stress] (or 1/[pressure])
- Two compressibility in groundwater Hydraulics
  - Water compressibility
  - Aquifer compressibility

# Aquifer storativity

- **Storativity:** The amount of water per unit surface area of saturated aquifer released from (or added to) storage per unit decline (or unit rise) in hydraulic head normal to that surface
- Mechanisms
  - Elastic expansion of water
  - Compaction of solid matrix
  - Drainage from pore space between the initial and final water tables (primarily for unconfined aquifers), i.e., specific yield $S_y$
- Used in two-dimensional, transient groundwater flow equations
- Definition (for both confined and unconfined aquifers)

$$S = \frac{\Delta V_{water}}{A_{aquifer} \times \Delta h} = S_s b \qquad \text{(for constant aquifer thickness)}$$

$$= \int_{b_1(x,y)}^{b_2(x,y)} S_s(x,y,z)\,dz \qquad \text{(for variable aquifer thickness)}$$

(Note again that storativity is dimensionless)

# Aquifer storativity/Hydraulic Diffusivity

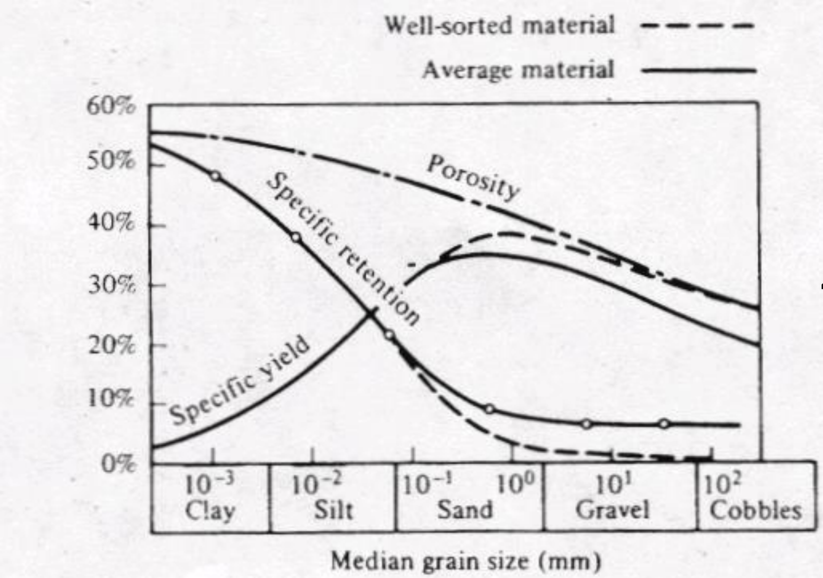



Fig. 4.2. Relationship between specific yield and grain size (*from Conkling et al., 1934, as modified by Davis and De Wiest, 1966*).

(After Bear and Verruijt, 1987)

1. Specific retention increases with decreasing grain size
2. Maximum $S_y$ occurs in medium-to-coarse sand soils.

- Hydraulic diffusivity: D ($L^2/T$)
  - Represent the diffusive characteristics of groundwater

$$D = \frac{T}{S} = \frac{K}{S_s}$$

# 7. Porous Vs Fractured media

- Porous media
  - Continuum media
  - A multiphase medium with at least one solid phase and one fluid phase (Greenkorn, 1983; Yeh, 1999)
    - Soils
    - Wood, asphalts
    - Skin, hair, feathers, teeth, and lungs
    - ceramics, contact lenses, membranes
- Fractured media
  - Discrete media
    - Fractured rocks (sedimentary, crystalline, argillaceous)
  - Fractured porous media: discrete media with porous solid matrix

# Porous Vs fractured media

- **Soil** (Jury, Gardner, and Gardner, 1991[a])
  - A granular material that is a heterogeneous mixture of solid, liquid, and gaseous material
- **Solid phase:** it has **mineral portion** containing particles of varying sizes, shapes, and chemical composition and **organic portion** containing a diverse population of live, active organisms as well as plant and animal residues in different stages of decomposition
- **Liquid phase:** It consists of **the soil water** held by forces in the soil matrix and varies significantly in mobility depending on its location and **Solute** materials contained in soil water, coming from dissolution of soil mineral phase or from the soil surface.

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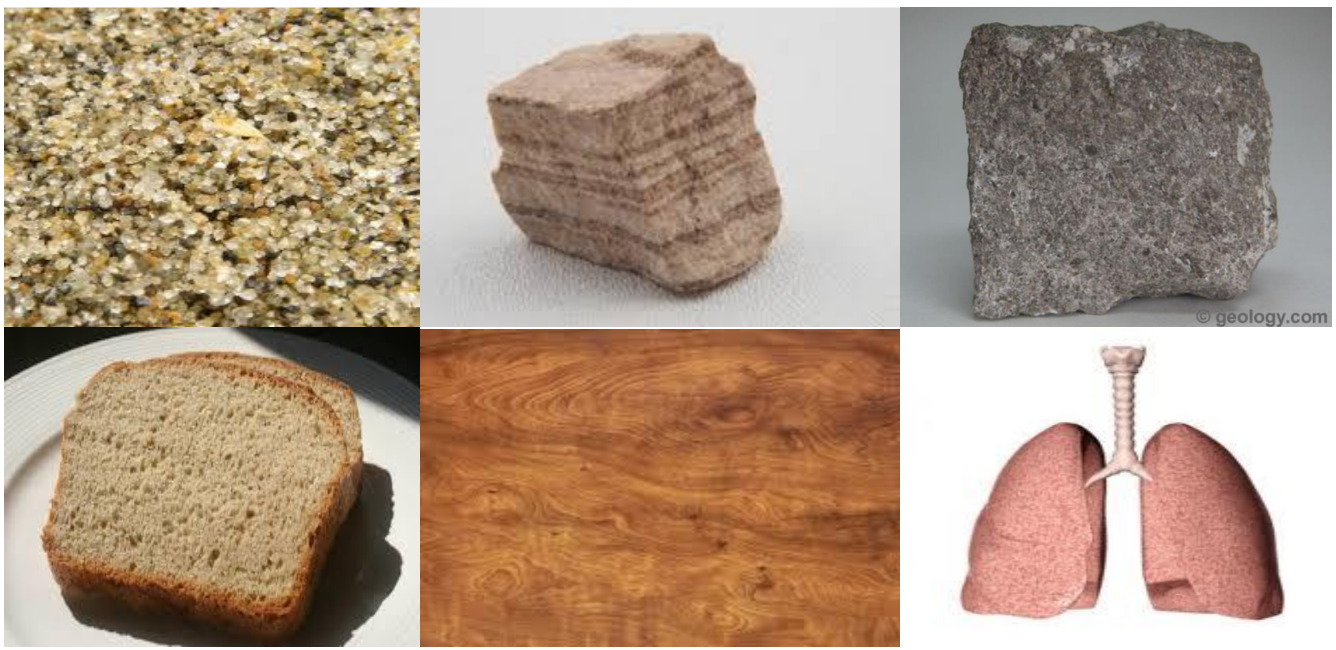
[a]Jury, W. A., W. R. Gardner, and W. H. Gardner, Soil Physics, 5th ed., John Wiley & Sons, New York, 1991

# Porous Vs Fractured media

**Gaseous (vapor) phase**

- $CO_2$ and $O_2$: mutually complementary gases in soil, depending on plant respiration and biological activity
- Water vapor : formed by the evaporation process in unsaturated soil
- Volatile organic compounds (VOC) : gas phase of VOC’s




**Examples of porous media: Beach sand; Sand stone; Lime stone; Dry bread; Wood; Lung**

# Porous Vs Fractured media

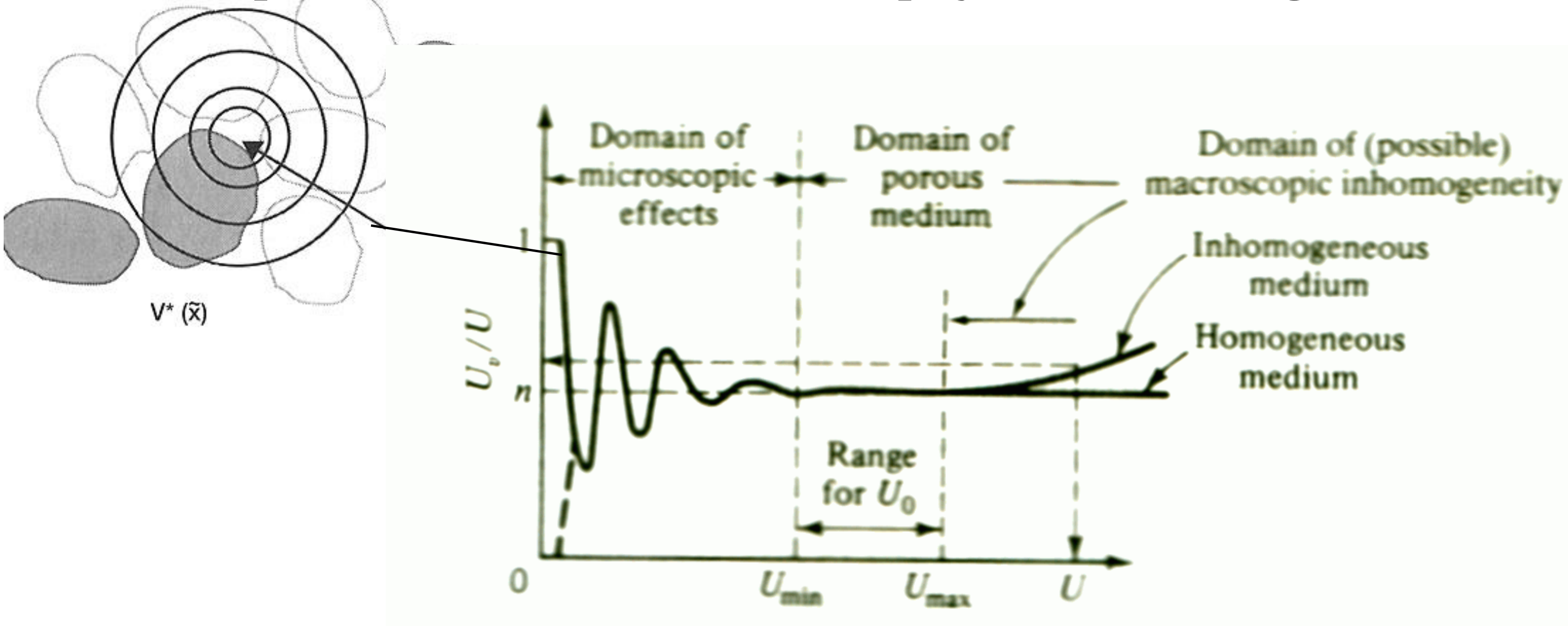
Examples of fractured media

# Porous Vs Fractured media

- **Heterogeneity and associated length scales:** A porous system is inherently heterogeneous. Mathematically, we can represent heterogeneity as; $Z(\mathbf{x}_1) \neq Z(\mathbf{x}_2)$ in which Z is a general medium property, e.g., hydraulic conductivity
- **Four length scales of heterogeneity**
  - Microscopic : at the level of pores or grains of the medium
  - Macroscopic: at the level of core plugs
  - Megascopic: at the level of the entire reservoirs which may have large fractures and faults.
  - Gigascopic: heterogeneities at this scale may contain many megascopically heterogeneous reservoirs.
- Note that not all the four heterogeneities are important to all porous media.

# Porous Vs Fractured media

- Any property of a medium is an average taken over a suitably selected volume of the medium, which is generally called representative elementary volume (REV)
- Concept of REV: Dimension and physical meaning




(After Bear and Verruijt, 1987)

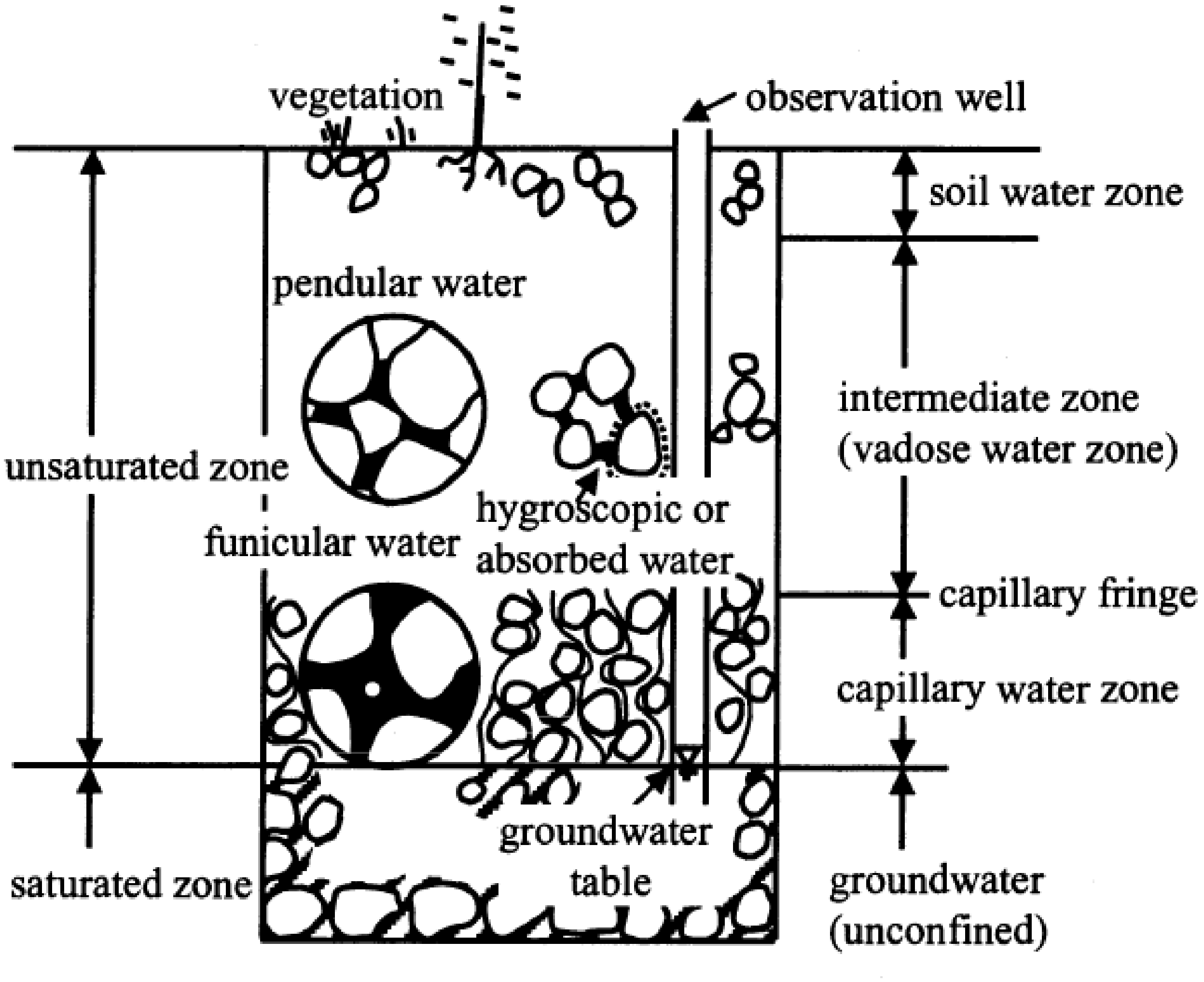
Fig. 1.5. Definition of porosity and Representative Elementary Volume.

# Porous Vs Fractured media

- Heterogeneities are spatially correlated at all scales
  - Fractal theories can be used to tell the dependence of property values in various regions of the medium on the length scale of observation
- Long-range correlation of medium properties leads to the spatial variation of interconnectivity of various regions of the medium
  - Percolation theories can be used to tell how the interconnectivity of various regions of a given system affects its overall properties
  - Spatial variation of interconnectivity is more significant in fractured media than in porous media
- Porous media is the focus of this course

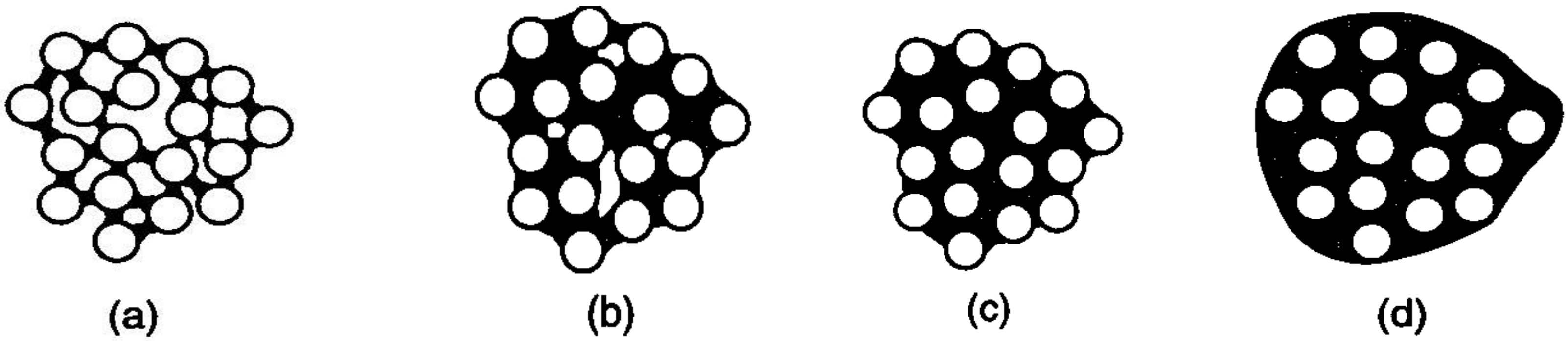
# 8. Driving forces of groundwater flow

- Moisture distribution



# Types of water Bonding

a) Pendular-looks like bridge, but particles not immersed in liquid
b) Funicular-thicker bridges but not completely filled
c) Capillary-particles at edge of cluster not completely wetted by liquid
d) Droplet-all particles completely wet



# 8. Driving forces …

- **Figure** shows qualitative relationships between water content (pendular, funicular and saturated cases) and driving forces (surface tension, gravity and pressure head).