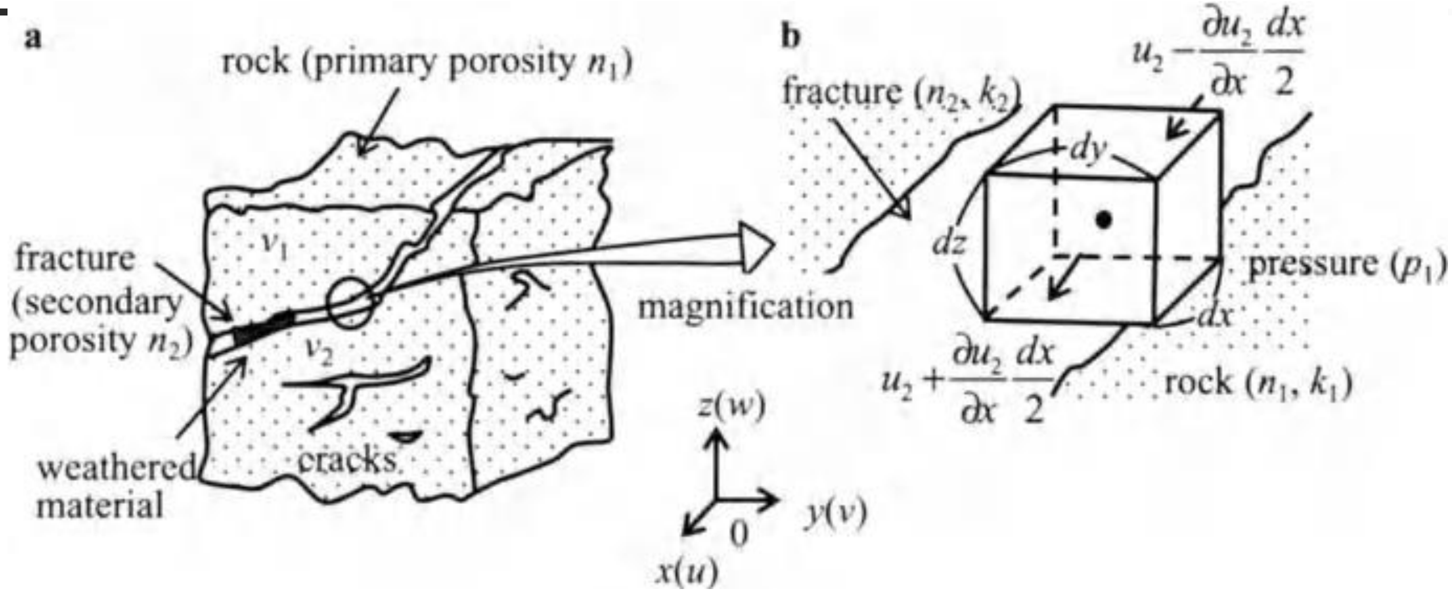
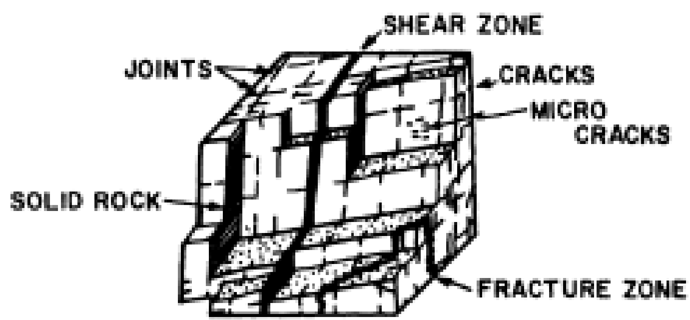
6. Approaches to groundwater flow analysis in fractured aquifers



- **A fractured medium consists of solid rock with some primary porosity cut by a system of cracks, microcracks, joints, fracture zones, and shear zones that create secondary porosity**

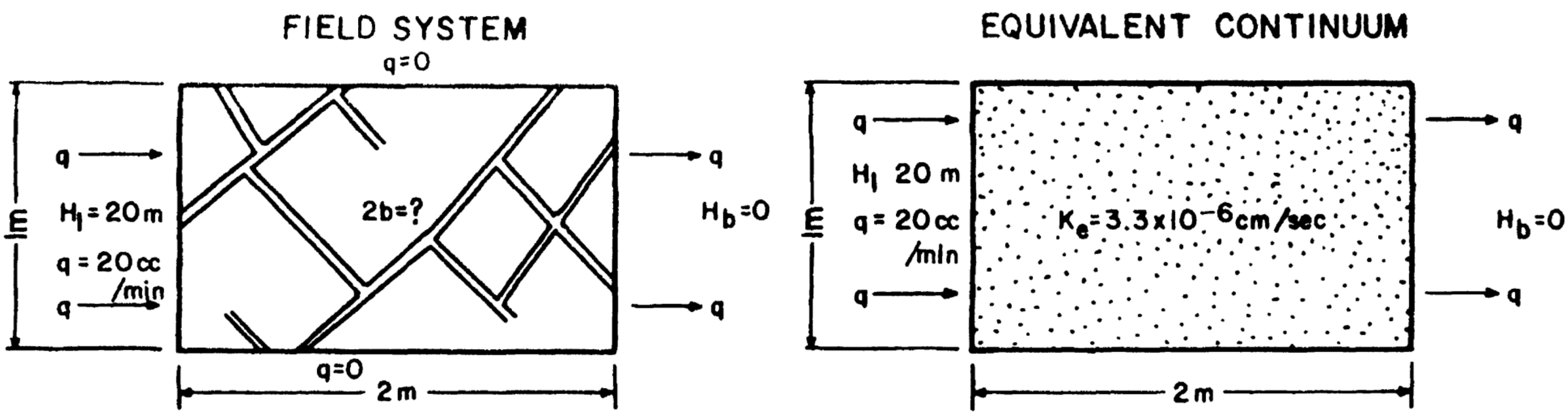


- Fractured systems are typically modeled using one or more of the following conceptual models:

1. **Equivalent porous medium (EPM)**: This model Replaces the primary and secondary hydrogeologic parameters with a continuous porous medium having so-called equivalent or effective hydraulic properties. EPM assumes that the fractured material can be treated as a continuum.
2. **Discrete fractures (DF)**: Flow through a single fracture may be idealized as occurring between two parallel plates with a uniform separation equal to the fracture aperture (2b). Typically applied to fractured media with low primary permeability such as crystalline rocks.
3. **Dual porosity (DP):** If the rock matrix containing the fracture network has significant primary permeability, a DP model may be used.

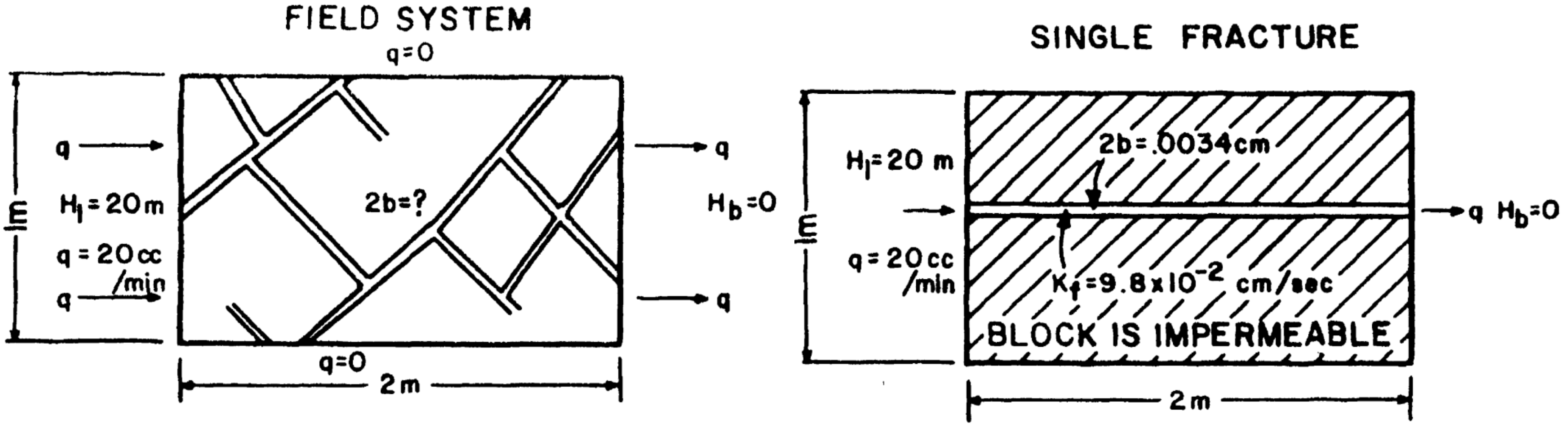
Equivalent porous Medium (EPM)

- The difficulty in applying the EPM approach arises in determining the appropriate size of the REV needed to define equivalent hydraulic properties.
- When fractures are few and far between and the unfractured block hydraulic conductivity is low, the EPM method may not be appropriate even with a large REV.
- EPM approach may adequately represent the behavior of a regional flow system, but poorly reproduces local conditions.



Discrete Fracture (DF) approach

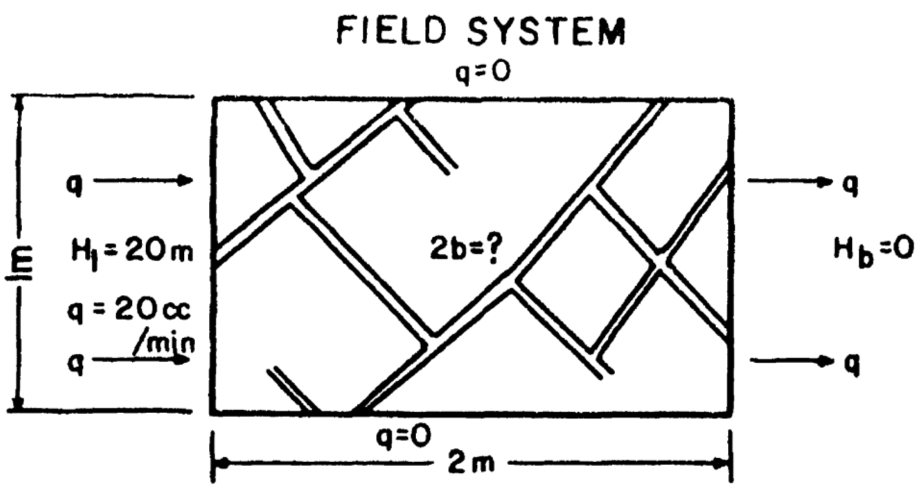
- The flow rate (Q_f): $Q_f = 2bwK_f(dh/dl)$, where w is the width, K_f $(= \rho g(2b)^2/(12\mu))$ the hydraulic conductivity of the fracture, h is hydraulic head, and *1* is the length over which the hydraulic gradient is measured, ρ is fluid density, μ is viscosity, and g is gravity. Note that Q_f is proportional to the $(2b)^3$.
- Use of a cubic model (above equation) requires a description of the fracture network, including fracture apertures and geometry. These data are extremely difficult to collect or estimate.

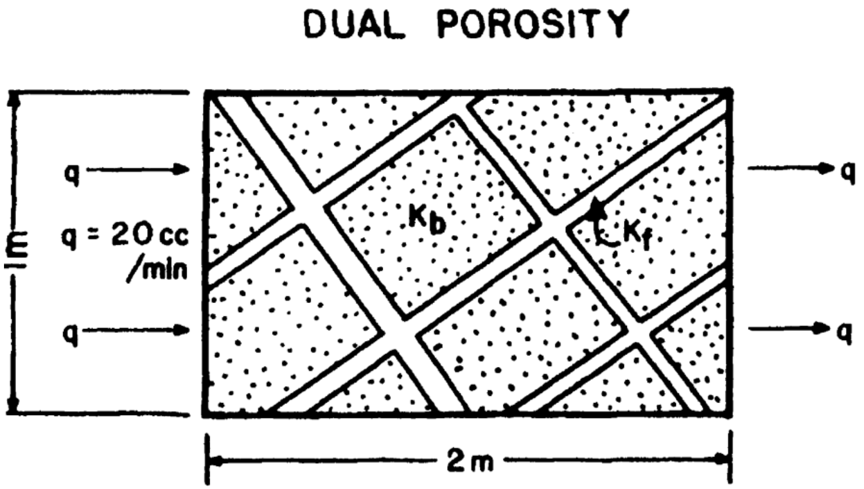


- Further complications in using a DF model arise when fracture widths are less than 10 μm and when portions of the fracture surfaces touch or are rough. Under these conditions the cubic law for flow through a fracture may not be valid.
- Furthermore, increases in stress with depth and with decreases in pore pressure (e.g., from dewatering) cause a decrease in fracture aperture. Hence, the relative orientation of the fractures and the stress field in relation to the groundwater flow field must also be considered.
- Models based on the DF approach are computationally intensive. To date, applications have been mainly to research problems.

Dual Porosity (DP)

- In this conceptual model, flow through the fractures is accompanied by exchange of water and solute to and from the surrounding porous rock matrix.
- Obviously, the fracture network as well as the properties of the porous blocks must be described prior to modeling.





Groundwater Hydraulics

Chapter 3 – Well Hydraulics

CENG 6606

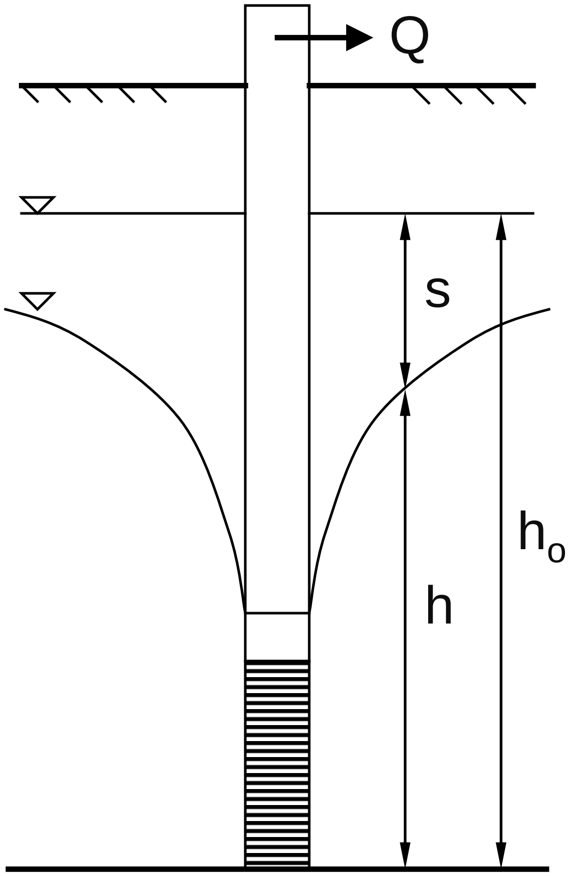
AAU

Contents

Well Hydraulics

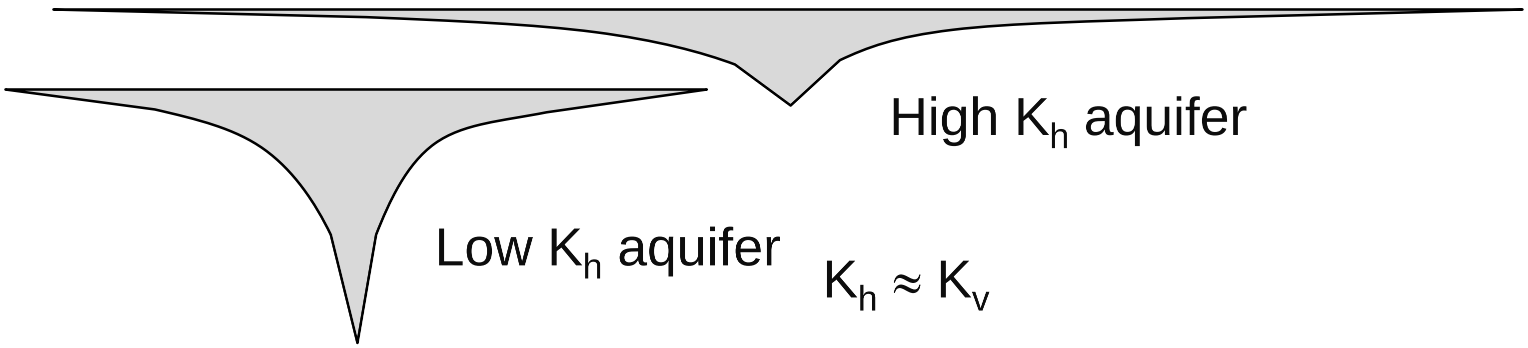
- A water well is a hydraulic structure that is designed and constructed to permit economic withdrawal of water from an aquifer
- Water well construction includes:
  - Selection of appropriate drilling methods
  - Selection of appropriate completion materials
  - Analysis and interpretation of well and aquifer performance

1. Pumping Well Terminology



- **Static Water Level** [SWL] (h_o) is the equilibrium water level before pumping commences
- **Pumping Water Level** [PWL] (h) is the water level during pumping
- **Drawdown** ($s = h_o - h$) is the difference between SWL and PWL
- **Well Yield** (Q) is the volume of water pumped per unit time
- **Specific Capacity** (Q/s) is the yield per unit drawdown

Cone of Depression

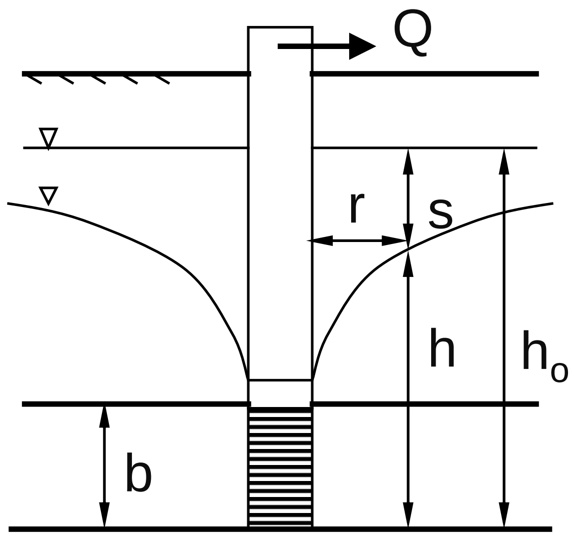


- A zone of low pressure is created centered on the pumping well
- Drawdown is maximum at the well and reduces radially
- Head gradient decreases away from the well and the pattern resembles an inverted cone called the **cone of depression**
- The cone expands over time until the inflows (from various boundaries) match the well extraction
- The shape of the equilibrium cone is controlled by hydraulic conductivity

Aquifer Characteristics

- Pump tests allow estimation of transmission and storage characteristics of aquifers
- **Transmissivity** ($T = Kb$) is the rate of flow through a vertical strip of aquifer (thickness b) of unit width under a unit hydraulic gradient
- **Storage Coefficient** ($S = S_y + S_s b$) is storage change per unit volume of aquifer per unit change in head
- **Radius of Influence** (R) for a well is the maximum horizontal extent of the cone of depression when the well is in equilibrium with inflows

2. Unsteady Radial Confined Flow



- **Assumptions**

  Isotropic, homogeneous, infinite aquifer, 2-D radial flow
- **Initial Conditions**

  $h(r,0) = h_o$ for all r
- **Boundary Conditions**

  $h(\infty,t) = h_o$ for all t

- PDE $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) = \frac{S}{T}\frac{\partial h}{\partial t}$$
- Solution is more complex than steady-state
- Change the dependent variable by letting

$$u = \frac{r^2 S}{4Tt}$$

- The ultimate solution is:

$$h_o - h = \frac{Q}{4\pi T}\int_u^\infty \left(\frac{e^{-u}}{u}\right)du$$

- where the integral is called the exponential integral written as the well function W(u)

  This is the Theis Equation

Theis PDE to ODE

- Let α = S/T (to simplify notation where α is called the inverse hydraulic diffusivity)

$$PDE: \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) = \alpha\frac{\partial h}{\partial t}$$

$$u = \frac{r^2 S}{4Tt} = \frac{\alpha r^2}{4t} \Rightarrow \frac{\partial u}{\partial r} = \frac{\alpha r}{2t} = \frac{2u}{r};\ \frac{\partial u}{\partial t} = -\frac{\alpha r^2}{4t^2} = -\frac{u}{t}$$

$$thus\ the\ PDE\ in\ terms\ of\ u:$$

$$\frac{1}{r}\frac{d}{du}\left(r\frac{\partial u}{\partial r}\frac{dh}{du}\right)\frac{\partial u}{\partial r} = \alpha\frac{\partial u}{\partial t}\frac{dh}{du}$$

Theis PDE to ODE

- Rewriting partial derivatives in terms of u

$$\frac{1}{r}\frac{d}{du}\left(r\frac{2u}{r}\frac{dh}{du}\right)\frac{2u}{r}=-\alpha\frac{u}{t}\frac{dh}{du}$$

$$\Rightarrow\frac{d}{du}\left(u\frac{dh}{du}\right)=-\alpha\frac{r^2}{4t}\frac{dh}{du}=-u\frac{dh}{du}$$

$$\Rightarrow u\frac{d}{du}\left(\frac{dh}{du}\right)+\frac{dh}{du}=-u\frac{dh}{du}\Rightarrow u\frac{d}{du}\left(\frac{dh}{du}\right)=-(u+1)\frac{dh}{du}$$

$$\Rightarrow u\frac{dh'}{du}+h'=-uh' \quad where \quad h'=\frac{dh}{du}$$

$$\Rightarrow\frac{dh'}{du}=-\left(1+\frac{1}{u}\right)h'\Rightarrow\frac{dh'}{h'}=-\left(1+\frac{1}{u}\right)du$$

Theis Integration

- The resulting ODE is: $$\frac{dh'}{h'} = -\left(1+\frac{1}{u}\right)du$$ $$\Rightarrow \ln\left(h'u\right) = c - u \Rightarrow h'u = e^c e^{-u}$$ $$\Rightarrow \lim_{u \to 0}\left(h'u\right) = \lim_{u \to 0}\left(e^c e^{-u}\right) = e^c$$
- To eliminate exp(c), use Darcy's Law: $$\lim_{r \to 0}\left(r\frac{\partial h}{\partial r}\right) = -\frac{Q}{2\pi Kb} = \lim_{u \to 0}\left(2h'u\right)$$
- Remember $$r\frac{\partial h}{\partial r} = r\frac{\partial h}{\partial u}\frac{\partial u}{\partial r} = r\frac{dh}{du}\frac{2u}{r} = 2h'u$$ $$\lim_{r \to 0}\left(r\frac{\partial h}{\partial r}\right) = -\frac{Q}{2\pi Kb} = \lim_{u \to 0}\left(2h'u\right)$$
- Simplifying: $$h' = \frac{-Q}{4\pi T} \times \frac{e^{-u}}{u}$$

Theis integration

$$h' = \frac{dh}{du} = \frac{-Q}{4\pi T} \times \frac{e^{-u}}{u} \Rightarrow h = \frac{-Q}{4\pi T} \int_u^{\infty} \left(\frac{e^{-u}}{u} \right) du + C$$

- Finally, using h(∞,t) = h_o to eliminate C:

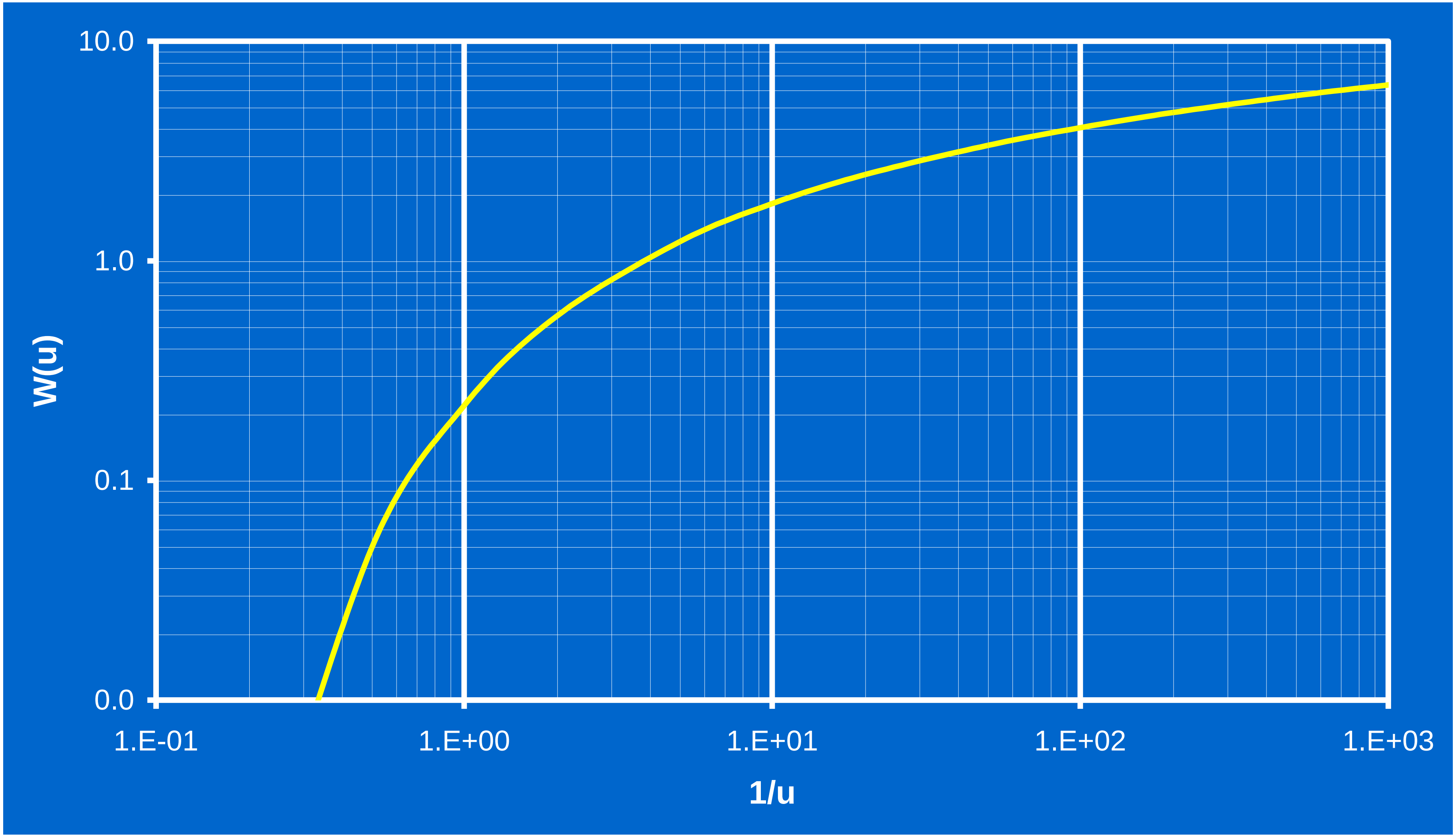
$$h_o - h = \frac{Q}{4\pi T} \int_u^{\infty} \left(\frac{e^{-u}}{u} \right) du$$

- The integral is called the exponential integral but is often written as the Theis well function

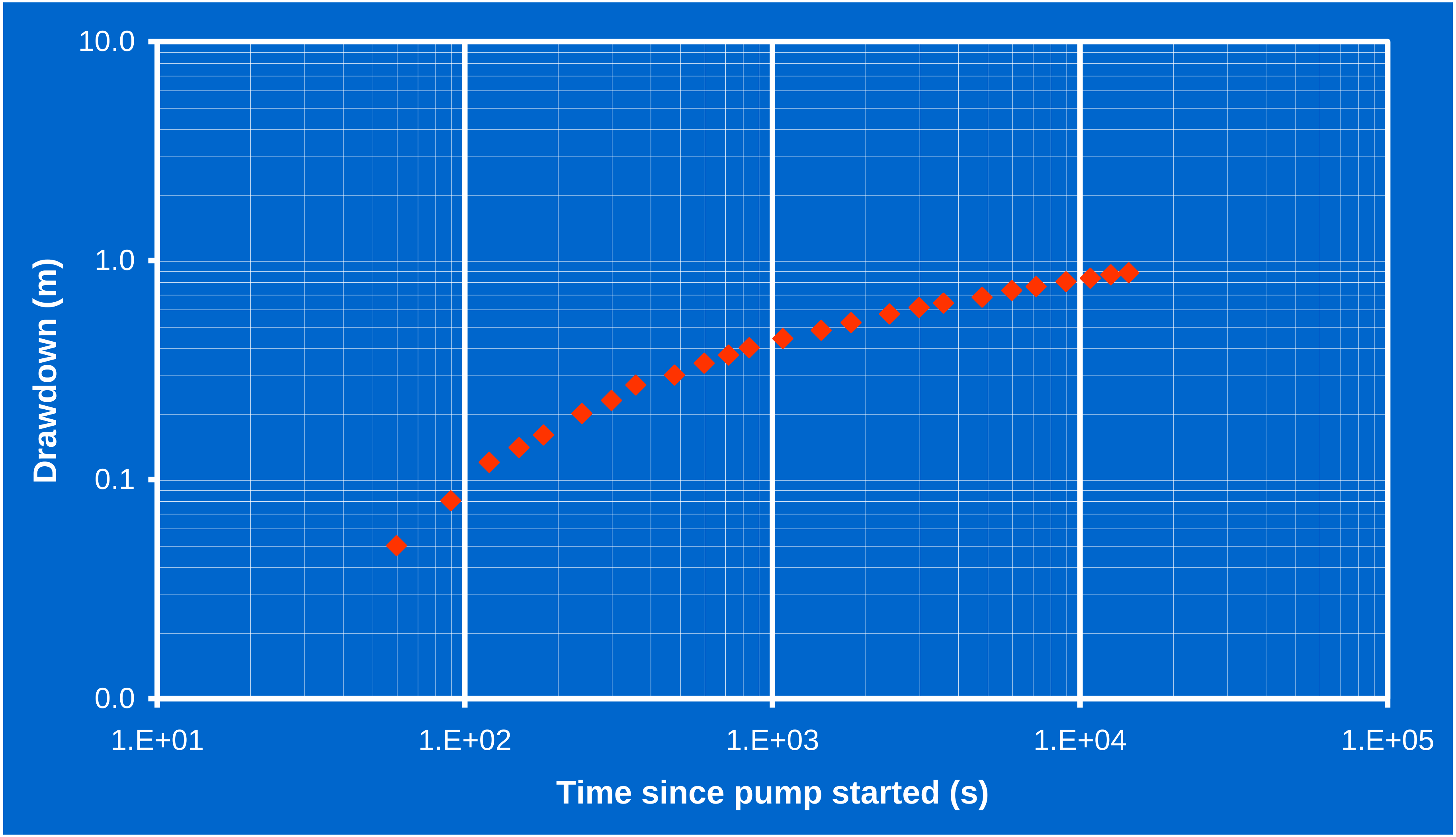
$$s = h_o - h = \frac{Q}{4\pi T} W(u)$$

- Well function is dimensionless

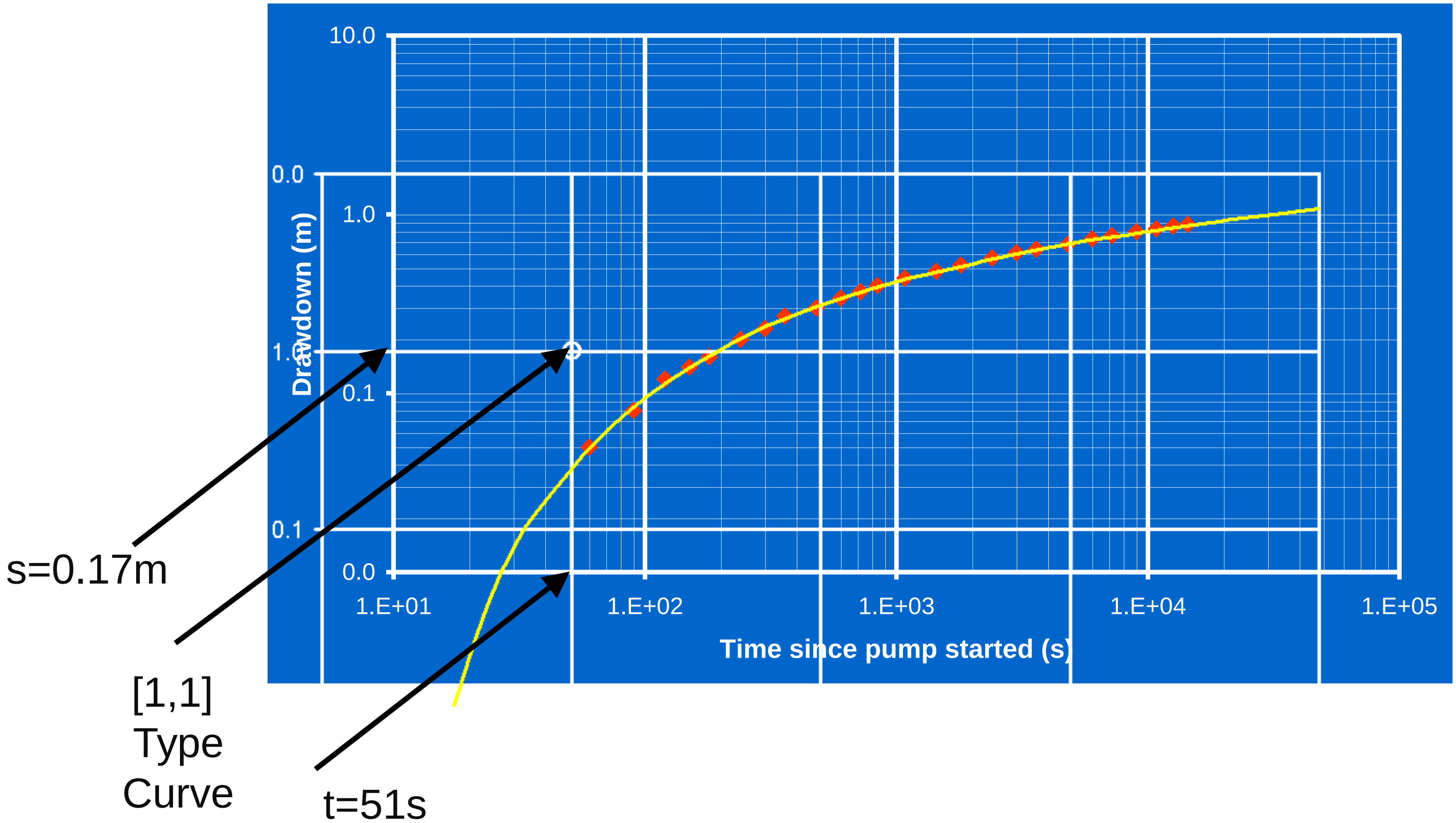
Theis Plot : 1/u vs W(u)



Theis Plot : Log(time) vs Log(drawdown)



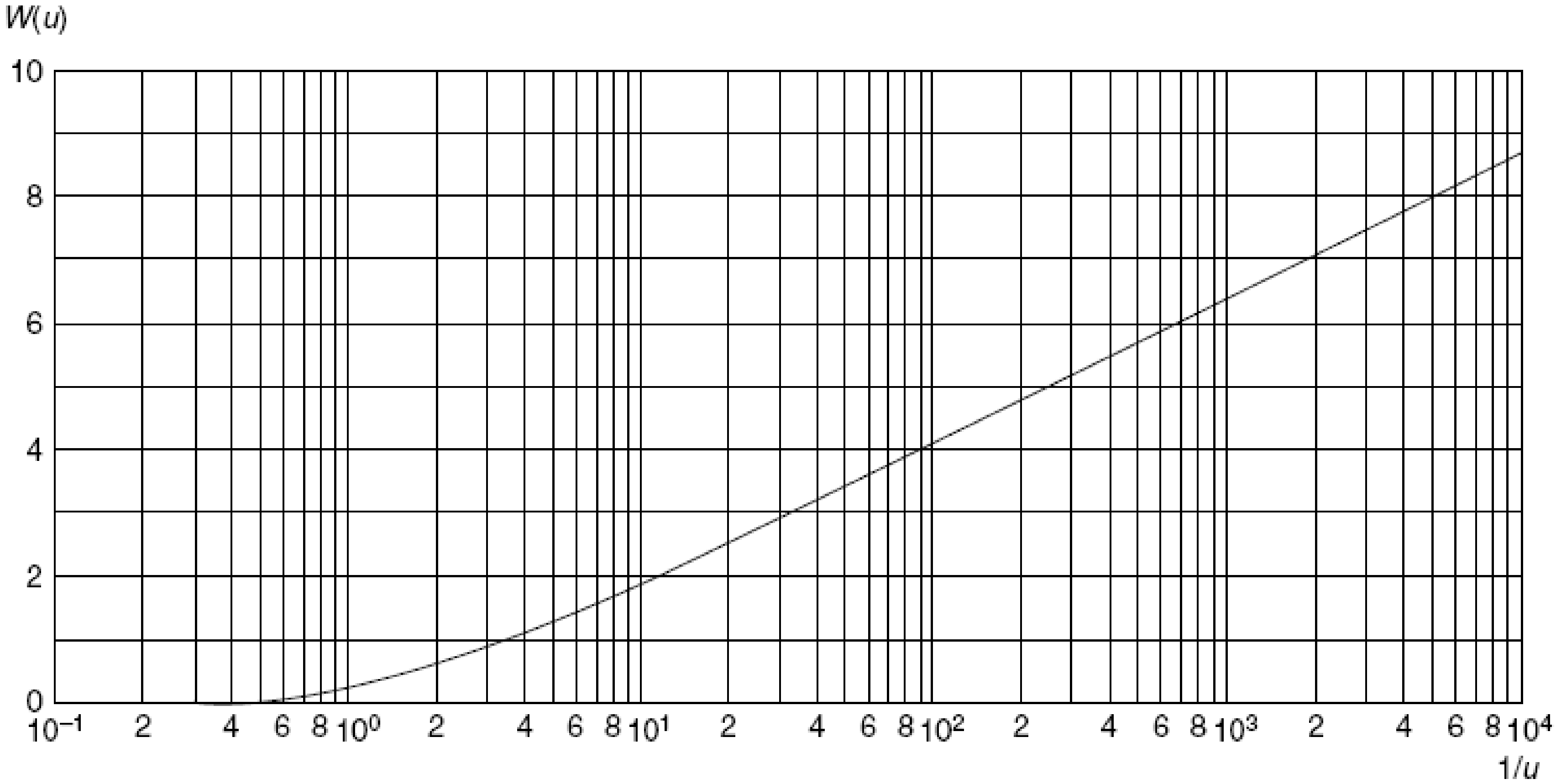
Theis Plot : Log(time) vs Log(drawdown)



Theis Analysis

1. Overlay type-curve on data-curve keeping axes parallel
2. Select a point on the type-curve (any will do but [1,1] is simplest)
3. Read off the corresponding co-ordinates on the data-curve [t_d, s_d]
4. For [1,1] on the type curve corresponding to [t_d, s_d], $T = Q/4\pi s_d$ and $S = 4Tt_d/r^2 = Qt_d/\pi r^2 s_d$
5. For the example, Q = 32 L/s or 0.032 m^3/s; r = 120 m; t_d = 51 s and s_d = 0.17 m
6. T = (0.032)/(12.56 x 0.17) = 0.015 m^2/s = 1300 m^2/d
7. S = (0.032 x 51)/(3.14 x 120 x 120 x 0.17) = 2.1 x 10^{-4}

Copper Jacob



Cooper-Jacob

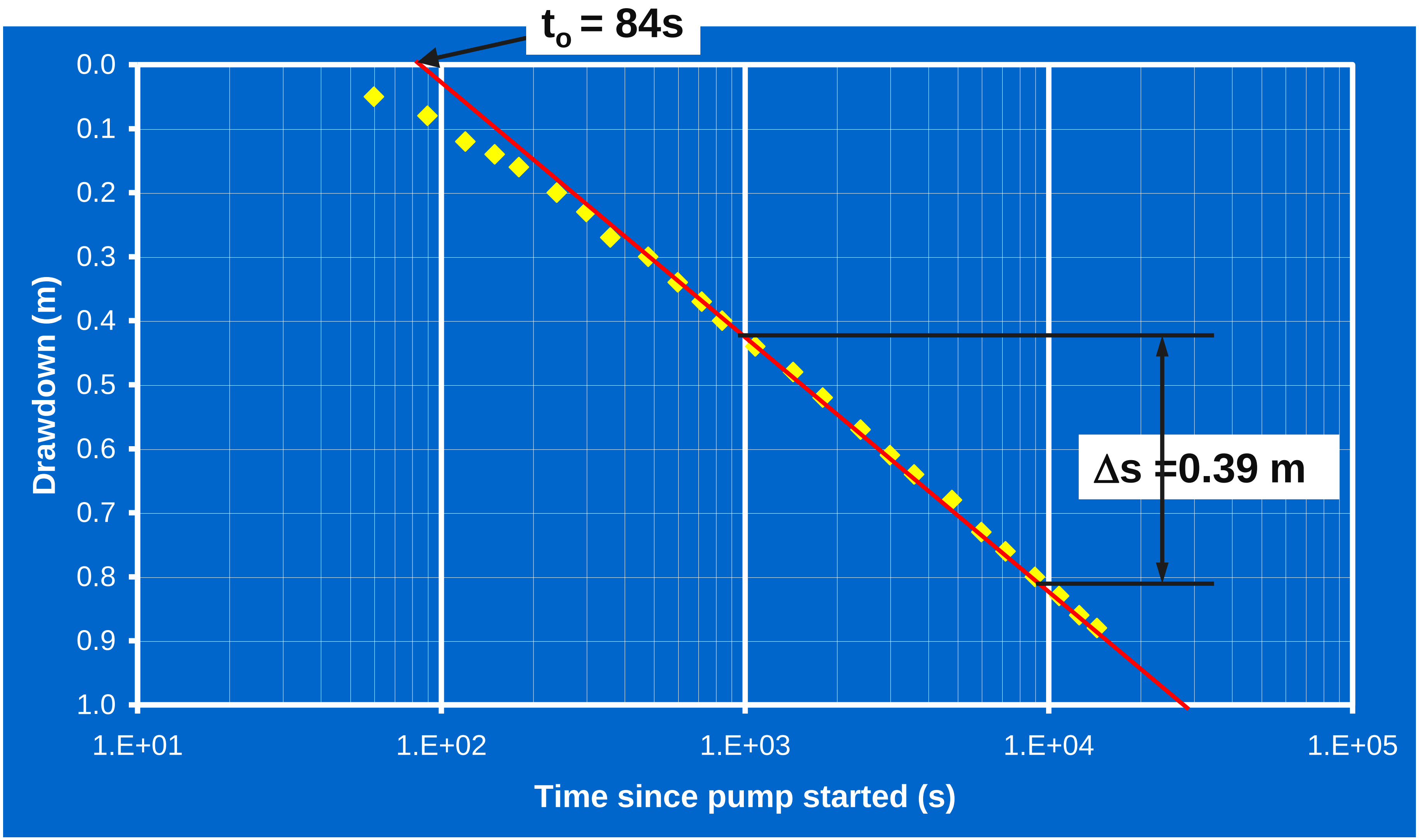
- In the above figure, the Theis well function *W(u) is plotted vs. 1/u on semi-log paper.*
- This figure shows that, for large values of 1/*u*, *the Theis well function exhibits a straight-line segment.*
- The Jacob method is based on this phenomenon. Cooper and Jacob (1946) showed that, for the straight-line segment, s can be approximated by

$$s = h_o - h = \frac{Q}{4\pi T} W(u) = \frac{Q}{4\pi T} \ln\left(\frac{2.25Tt}{r^2 S}\right) = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25Tt}{r^2 S}\right)$$

with an error less than 1%, 2%, 5%, and 10% for 1/*u larger than 30*, *20*, *10*, *and 7*, *respectively*.

- The Cooper-Jacob simplification expresses drawdown (*s*) as a linear function of *ln(t)* or *log(t)*.

Cooper-Jacob Plot : Log(t) vs s



Cooper-Jacob Analysis

- Fit straight-line to data (excluding early and late times if necessary):
- Note: at early times the Cooper-Jacob approximation may not be valid and at late times boundaries may significantly influence drawdown
- Determine intercept on the time axis for s=0
- Determine drawdown increment (Δs) for one log-cycle
- For straight-line fit,

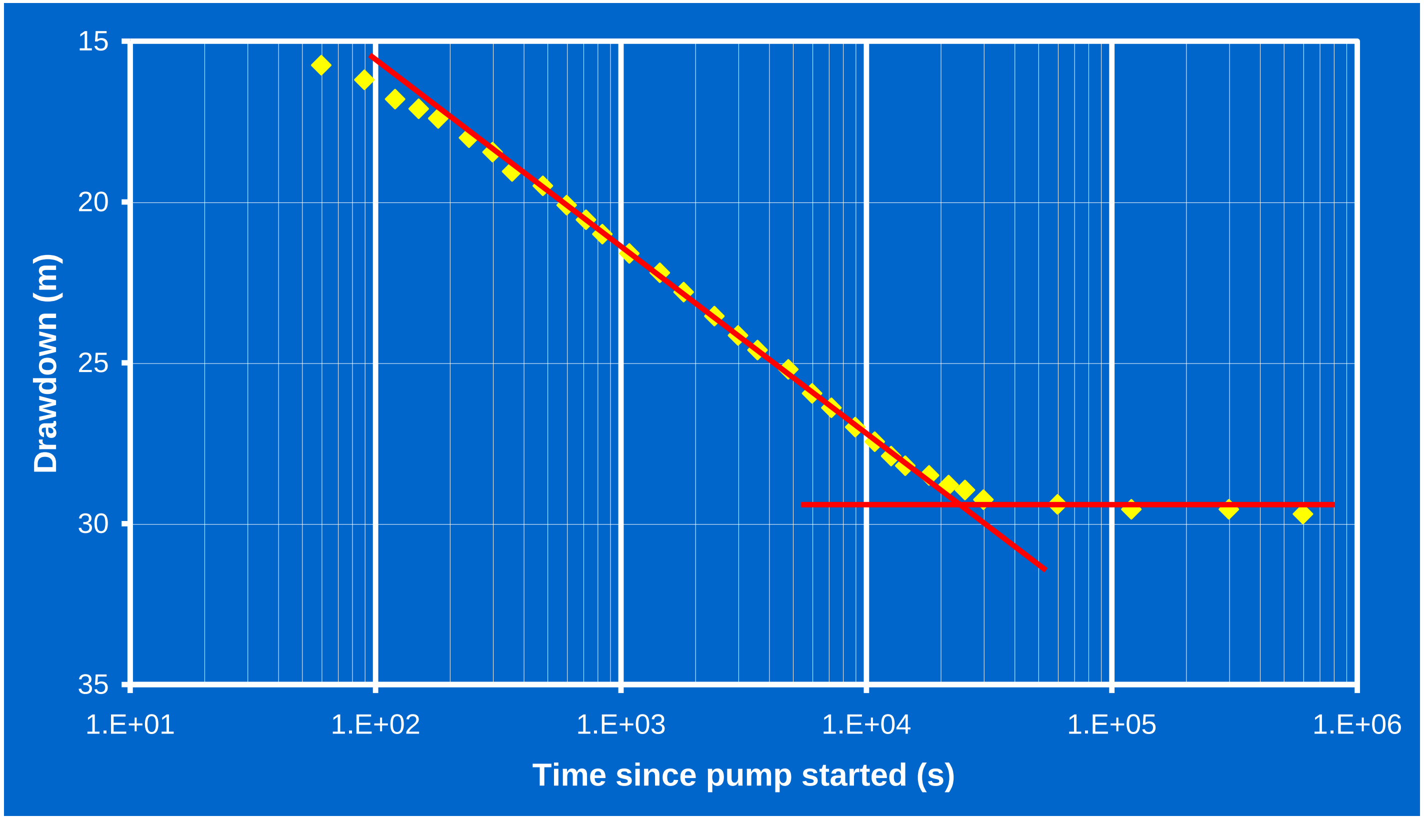
$$T = \frac{2.3Q}{4\pi\Delta s} \qquad S = \frac{2.25Tt_0}{r^2} = \frac{2.3Qt_0}{1.778\Delta s \pi r^2}$$

- For the example, Q = 32 l/s or 0.032 m^3/s; r = 120 m; t_o = 84 s and Δs = 0.39 m
- T = (2.3 x 0.032)/(12.56 x 0.39) = 0.015 m^2/s = 1300 m^2/d
- S = (2.3 x 0.032 x 84)/(1.78 x 3.14 x 120 x 120 x 0.39) = 1.9 x 10^{-4}

Theis-Cooper-Jacob Assumptions

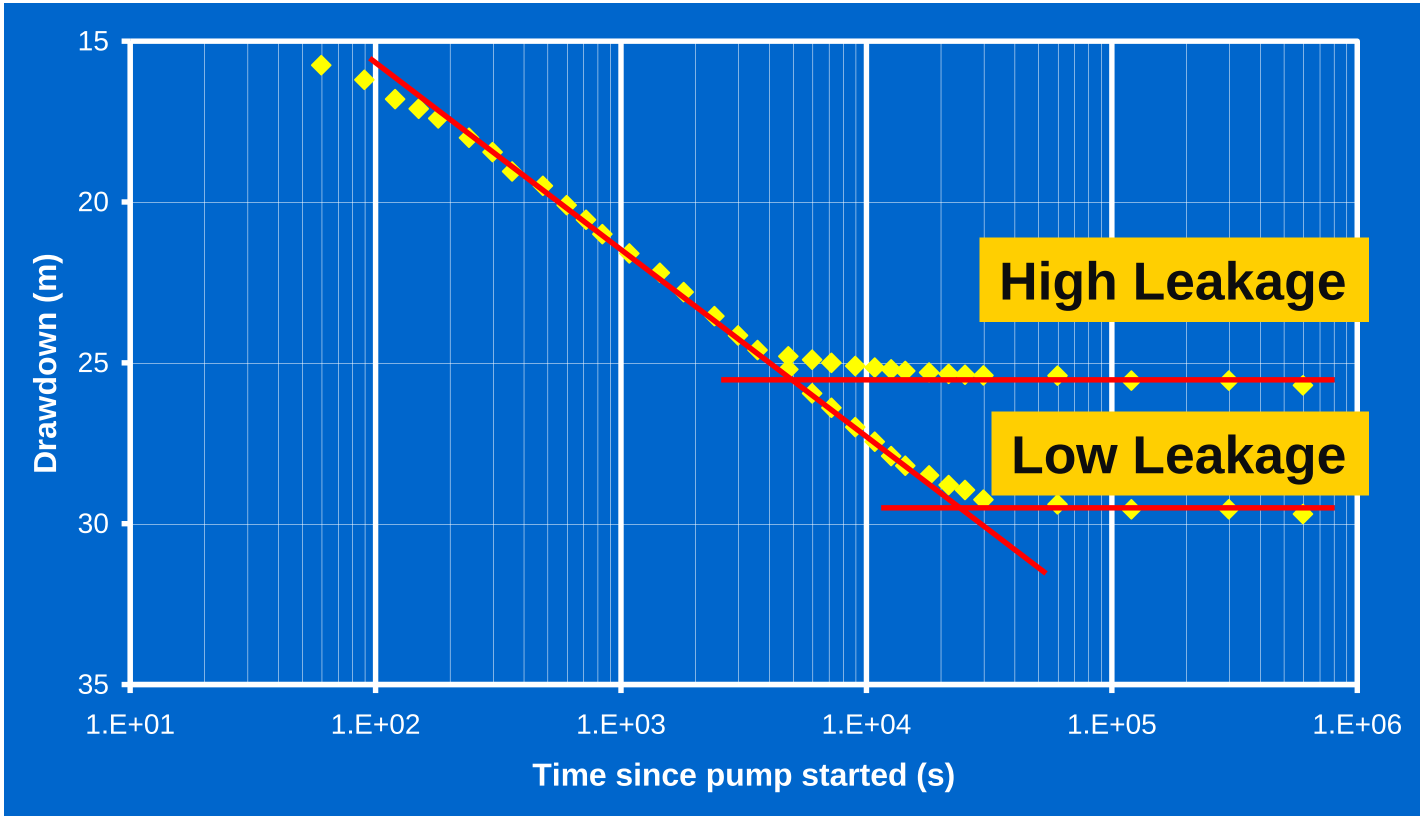
- Real aquifers rarely conform to the assumptions made for Theis-Cooper-Jacob non-equilibrium analysis
  - Isotropic, homogeneous, uniform thickness
  - Fully penetrating well
  - Laminar flow
  - Flat potentiometric surface
  - Infinite areal extent
  - No recharge
- Failure of some or all of these assumptions leads to “non-ideal” behavior and deviations from the Theis and Cooper-Jacob analytical solutions for radial unsteady flow

Recharge Effect : Recharge > Well Yield



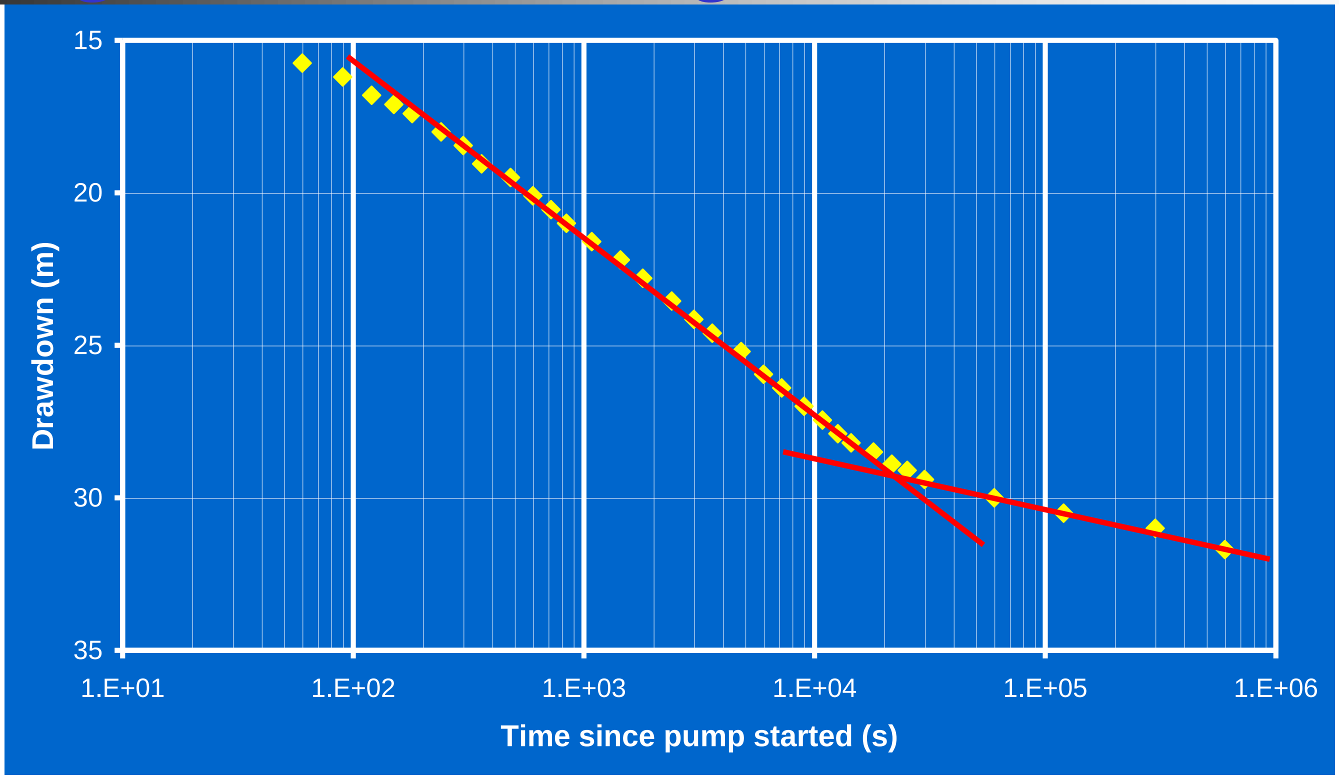
Recharge causes the slope of the log(time) vs drawdown curve to flatten as the recharge in the zone of influence of the well matches the discharge. The gradient and intercept can still be used to estimate the aquifer characteristics (T, S).

Recharge Effect : Leakage Rate



Recharge by vertical leakage from overlying (or underlying beds) can be quantified using analytical solutions developed by Jacob (1946). The analysis assumes a single uniform leaky bed.

Recharge Effect : Recharge < Well Yield



If the recharge is insufficient to match the discharge, the log(time) vs drawdown curve flattens but does not become horizontal and drawdown continues to increase at a reduced rate.

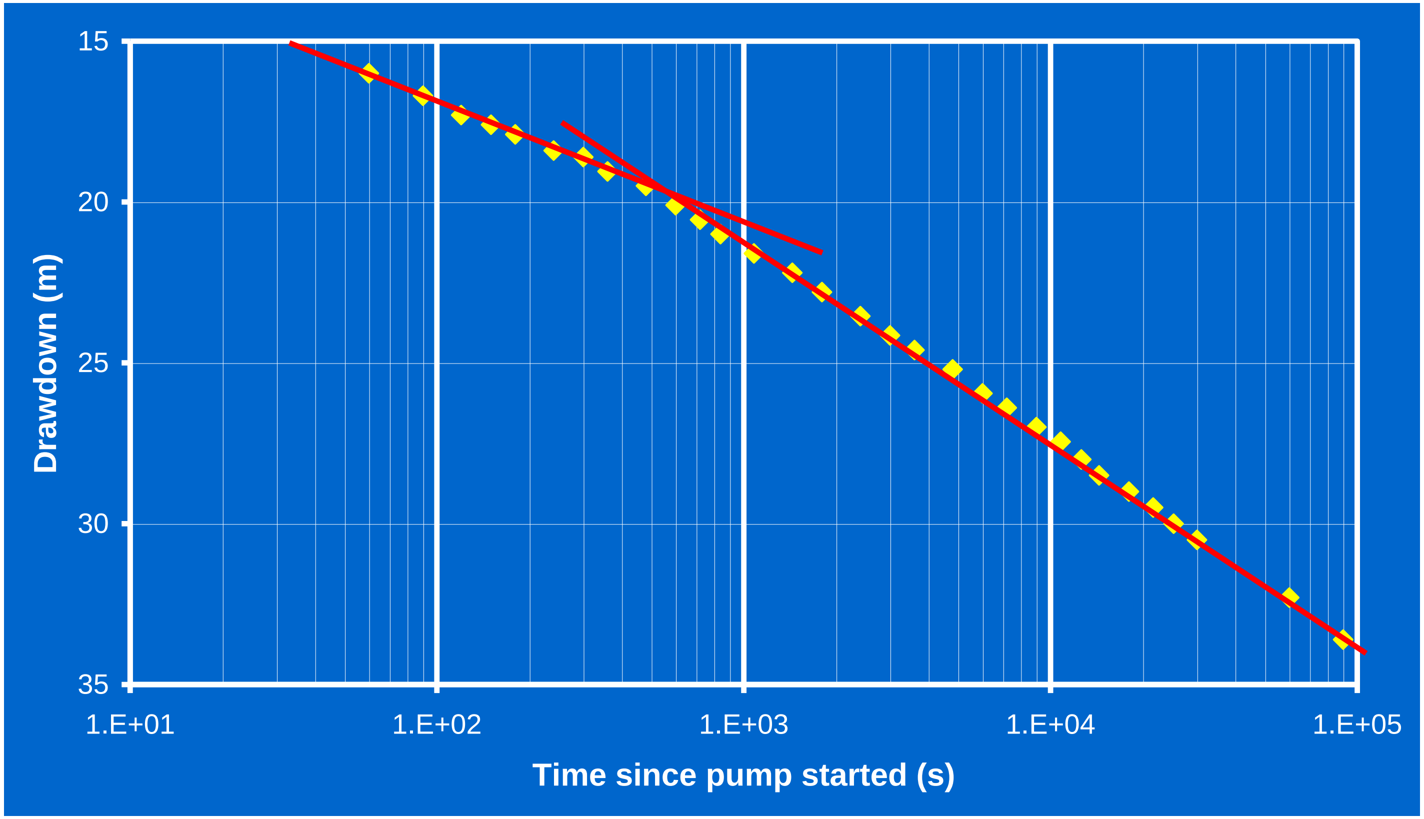
The same result will be obtained if the average T and/or S increased

T and S can be estimated from the first leg of the curve.

Sources of Recharge

- Various sources of recharge may cause deviation from the ideal Theis behavior.
- Surface water: river, stream or lake boundaries may provide a source of recharge, halting the expansion of the cone of depression.
- Vertical seepage from an overlying aquifer, through an intervening aquitard, as a result of vertical gradients created by pumping, can also provide a source of recharge.
- Where the cone of depression extends over large areas, leakage from aquitards may provide sufficient recharge.

Barrier Effect : No Flow Boundary



Steepening of the log(time) vs. drawdown curve indicates:

- An aquifer limited by a barrier boundary of some kind.
- The average transmissivity and/or storativity decreased

Aquifer characteristics (T,S) can be estimated from the first leg.

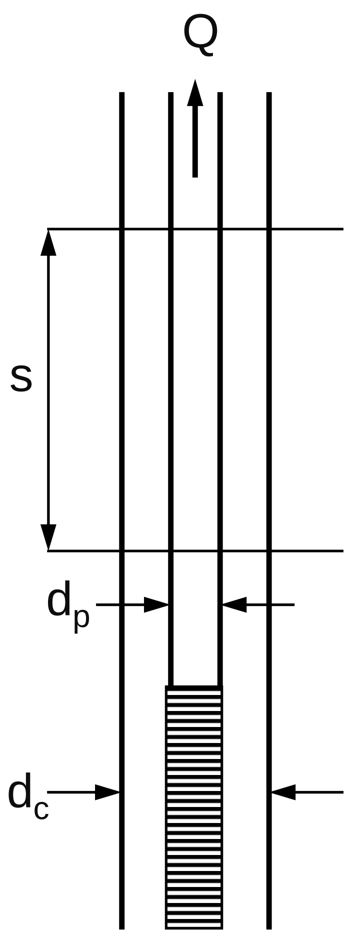
Potential Flow Barriers

- Various flow barriers may cause deviation from the ideal Theis behavior.
- Fault truncations against low permeability aquitards.
- Lenticular pinch outs and lateral facies changes associated with reduced permeability.
- Groundwater divides associated with scarp slopes.
- Spring lines with discharge captured by wells.
- Artificial barriers such as grout curtains and slurry walls.

Casing Storage

- It has been known for many decades that early time data can give erroneous results because of removal of water stored in the well casing.
- When pumping begins, this water is removed and the amount drawn from the aquifer is consequently reduced.
- The true aquifer response is masked until the casing storage is exhausted.
- Analytical solutions accounting for casing storage were developed by Papadopulos and Cooper (1967) and Ramey et al (1973)
- Unfortunately, these solutions require prior knowledge of well efficiencies and aquifer characteristics

Casing Storage



Schafer (1978) suggests that an estimate of the critical time to exhaust casing storage can be made more easily:

$$t_c = 3.75\pi(d_c^2 - d_p^2) / (Q/s) = 15\ V_a/Q$$

where: t_c is the critical time (d); d_c is the inside casing diameter (m); d_p is the outside diameter of the rising main (m); Q/s is the specific capacity of the well (m^3/d/m); V_a is the volume of water removed from the annulus between casing and rising main.

Note: It is safest to ignore data from pumped wells earlier than time t_c in wells in low-K region.

3. Distance-Drawdown

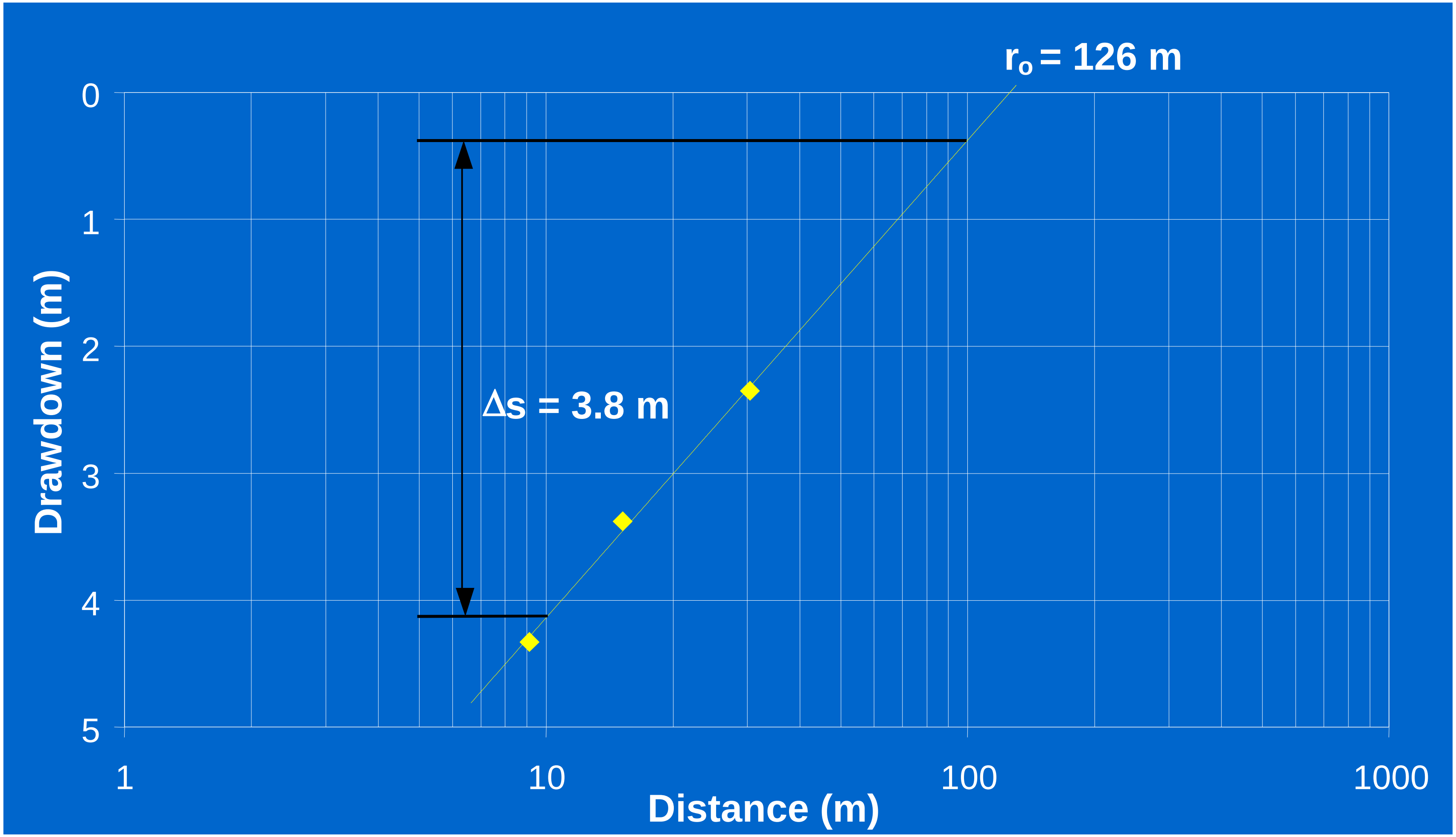
- Simultaneous drawdown data from at least three observation wells, each at different radial distances, can be used to plot a log(distance)-drawdown graph.
- The Cooper-Jacob equation, for fixed t, has the form:

$$s = \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt}{r^2 S}\right) = \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt}{S}\right) - \frac{4.6Q}{4\pi T}\log(r)$$

- So the log(distance)-drawdown curve can be used to estimate aquifer characteristics by measuring Δs for one log-cycle and the r_o intercept on the distance-axis.

$$T = \frac{4.6Q}{4\pi T(\Delta s)} \; and \; S = \frac{2.25Tt}{r_o^{\;2}}$$

Distance-Drawdown Graph



Distance drawdown Analysis

- For the example: $t = 0.35$ days and $Q = 1100\ m^3/d$

$$T = 0.366 \times 1100 / 3.8 = 106\ m^2/d$$

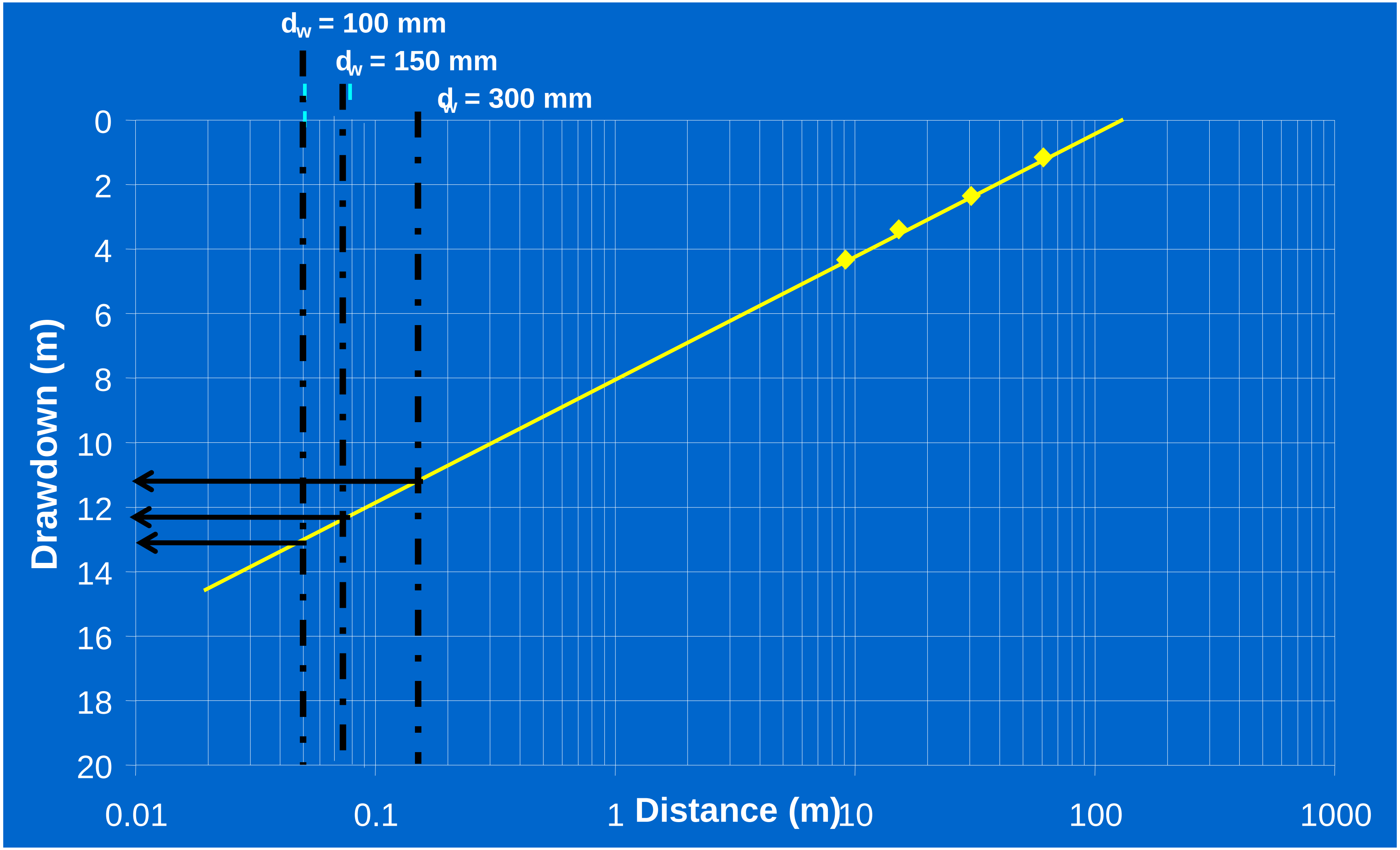
$$S = 2.25 \times 106 \times 0.35 / (126 \times 126) = 5.3 \times 10^{-3}$$

- The estimates of T and S from log(time)-drawdown and log(distance)-drawdown plots are independent of one another and so are recommended as a check for consistency in data derived from pump tests.
- Ideally 4 or 5 observation wells are needed for the distance-drawdown graph and it is recommended that T and S are computed for several different times.

Well Efficiency

- The efficiency of a pumped well can be evaluated using distance-drawdown graphs.
- The distance-drawdown graph is extended to the outer radius of the pumped well (including any filter pack) to estimate the theoretical drawdown for a 100% efficient well.
- This analysis assumes the well is fully-penetrating and the entire saturated thickness is screened.
- The theoretical drawdown (estimated) divided by the actual well drawdown (observed) is a measure of well efficiency.
- A correction is necessary for unconfined wells to allow for the reduction in saturated thickness as a result of drawdown.

Theoretical Pumped Well Drawdown



Unconfined Well Correction

- The adjusted drawdown for an unconfined well is given by:

$$s_c = s_a\left(1 - \frac{s_a}{2b}\right)$$

where b is the initial saturated thickness;
s_a is the measured drawdown; and
s_c is the corrected drawdown

- For example, if b = 20 m; s_a = 6 m; then the corrected drawdown $s_c = 0.85 s_a = 5.1$ m
- If the drawdown is not corrected, the Jacob and Theis analysis underestimates the true transmissivity under saturated conditions by a factor of s_c / s_a.

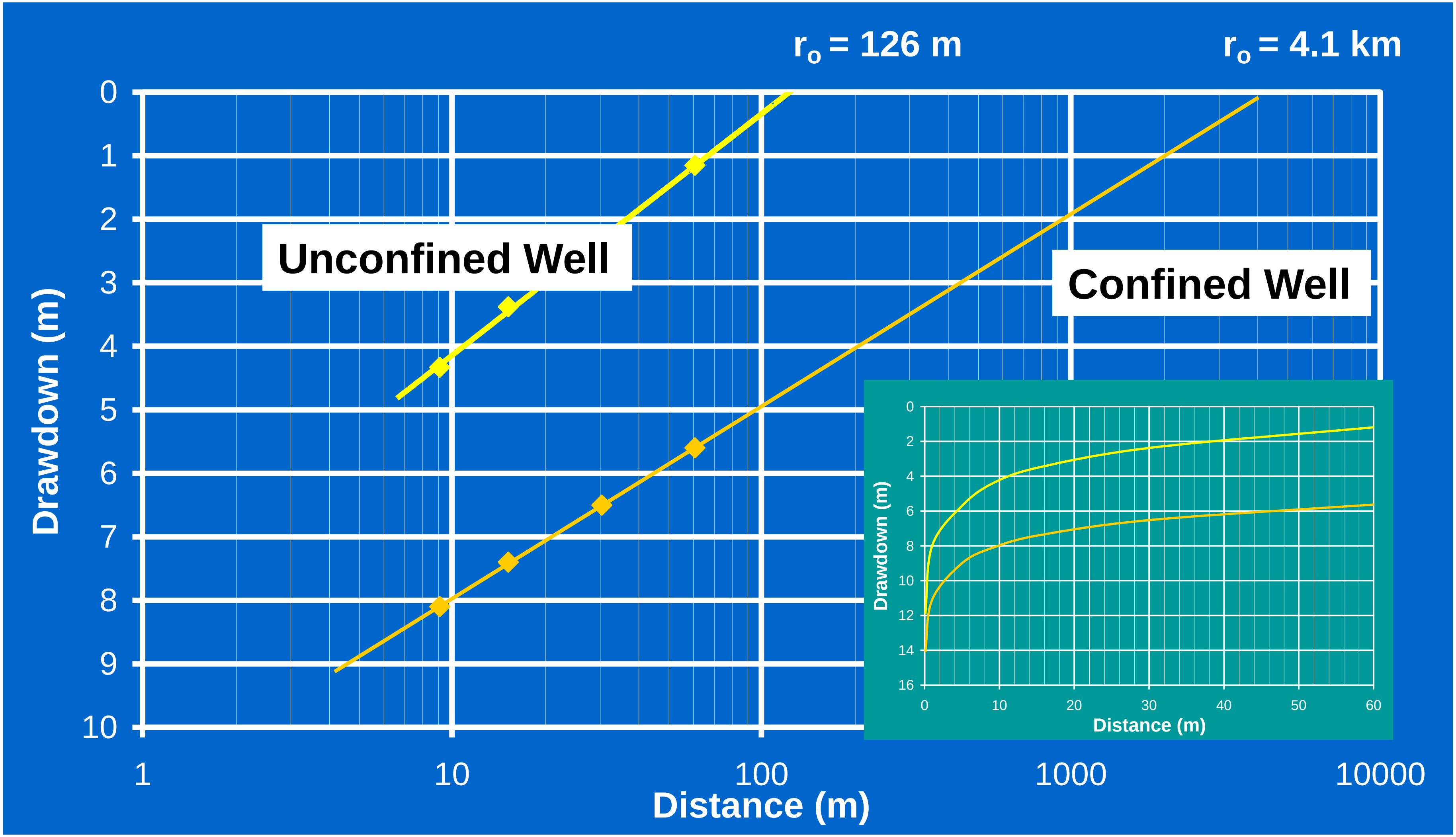
Causes of Well Inefficiency

- Factors contributing to well inefficiency (excess head loss) fall into two groups:
  - Design factors
    - Insufficient open area of screen
    - Poor distribution of open area
    - Insufficient length of screen
    - Improperly designed filter pack
  - Construction factor
    - Improper placement of screen relative to aquifer interval
    - Compaction of aquifers near by the well
    - Clogging of the aquifer by the drilling mud

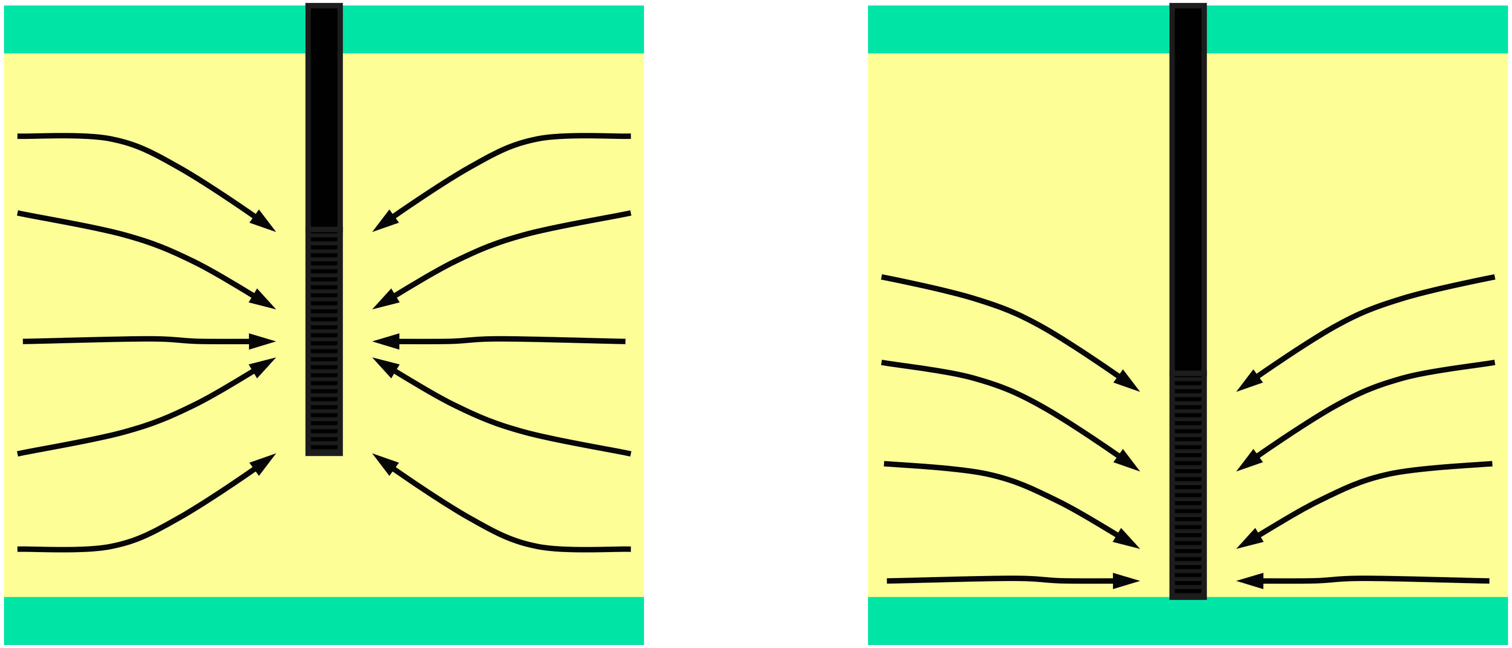
Radius of Influence

- The radius of influence of a well can be determined from a distance-drawdown plot.
- For all practical purposes, a useful comparative index is the intercept of the distance-drawdown graph on the distance axis.
- Radius of influence can be used as a guide for well spacing to avoid interference.
- Since radius of influence depends on the balance between aquifer recharge and well discharge, the radius may vary from year to year.
- For unconfined wells in productive aquifers, the radius of influence is typically a few hundred meters.
- For confined wells may have a radius of influence extending several kilometers.

Determining r_o



4. Partial Penetration



- Partial penetration effects occur when the intake of the well is less than the full thickness of the aquifer

Effects of Partial Penetration

- The flow is not strictly horizontal and radial.
- Flow-lines curve upwards and downwards as they approach the intake and flow-paths are consequently longer.
- The convergence of flow-lines and the longer flow-paths result in greater head-loss than predicted by the analytical equations.
- For a given yield (Q), the drawdown of a partially penetrating well is more than that for a fully penetrating well.
- The analysis of the partially penetrating case is difficult but Kozeny (1933) provides a practical method to estimate the change in specific capacity (Q/s).

Q/s Reduction Factors

- Kozeny (1933) gives the following approximate reduction factor to correct specific capacity (Q/s) for partial penetration effects:

$$F = \frac{L}{b}\left[1 + 7\cos\left(\frac{\pi L}{2b}\right)\sqrt{\frac{r}{2L}}\right]$$

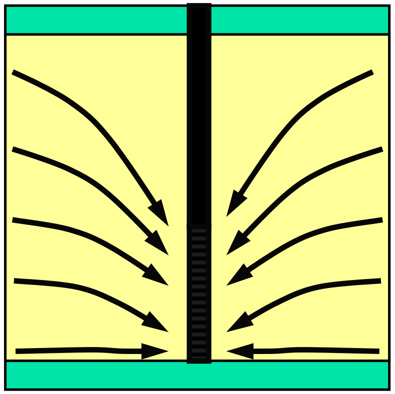
where b is the total aquifer thickness (m); r is the well radius (m); and L is screen length (m).

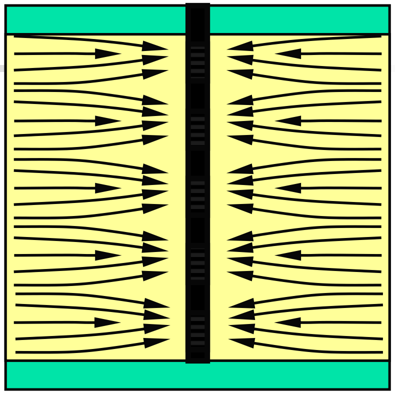
- The equation is valid for $L/b < 0.5$ and $L/r > 30$
- For a 300 mm dia. well with an aquifer thickness of 30 m and a screen length of 15 m, $L/b = 0.5$ and $2L/r = 200$ the reduction factor is:

$$F = 0.5 \times \{1 + 7 \times 0.707 \sqrt{(1/200)}\} = 0.67$$

- Other factor are provided by Muskat (1937), Hantush (1964), Huisman (1964), Neumann (1974) but they are harder to use.

Screen Design





- 300 mm dia. well with single screened interval of 15 m in aquifer of thickness 30 m.

  L/b = 0.5 and 2L/r = 200

F = 0.5 x {1 + 7 x cos (0.5π/2) √(1/200)} = 0.67

- 300 mm dia. well with 5 x 3 m solid sections alternating with 5 x 3m screened sections, in an aquifer of thickness 30 m.

  There effectively are five aquifers.

  L/b = 0.5 and 2L/r = 40

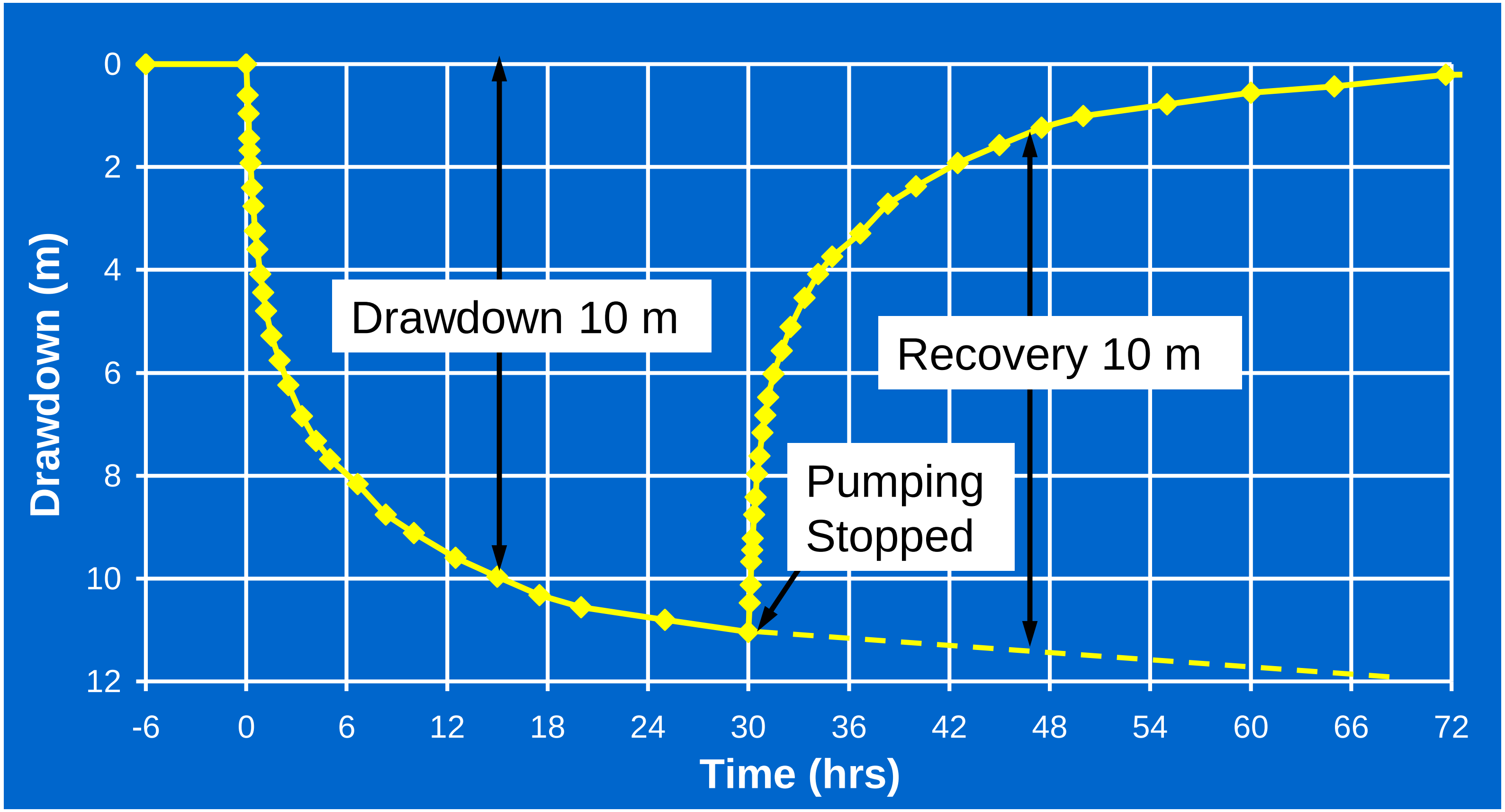
  F = 0.5 x {1 + 7 x cos(0.5π/2) √(1/40)} = 0.89

  This is clearly a much more efficient well completion.

5. Recovery Data

- When pumping is halted, water levels rise towards their pre-pumping levels.
- The rate of recovery provides a second method for calculating aquifer characteristics.
- Monitoring recovery heads is an important part of the well-testing process.
- Observation well data (from multiple wells) is preferable to that gathered from pumped wells.
- Pumped well recovery records are less useful but can be used in a more limited way to provide information on aquifer properties.

Recovery Curve



The recovery curve on a linear scale appears as an inverted image of the drawdown curve. The dotted line represent the continuation of the drawdown curve.

Superposition

- The drawdown (s) for a well pumping at a constant rate (Q) for a period (t) is given by:

$$s = h_o - h = \frac{QW(u)}{4\pi T}; \; where \; u = r^2 S / (4Tt)$$

- The effects of well recovery can be calculated by adding the effects of a pumping well to those of a recharge well using the superposition theorem.
- Applying this principle, it is assumed that, after the pump has been shut down, the well continues to be pumped at the same discharge as before, and that an imaginary recharge, equal to the discharge, is injected into the well. The recharge and discharge thus cancel each other, resulting in an idle well as is required for the recovery period.

- The drawdown (s_r) for a well recharged at a constant rate (-Q) for a period ($t' = t - t_r$) starting from time t_r (the time at which the pumping stopped) is given by:

$$s_r = -\frac{QW(u')}{4\pi T}; \; where \; u' = r^2 S \big/ (4Tt')$$

- The total (Residual) drawdown according to Theis for $t > t_r$ is:

$$s' = s + s_r = \frac{Q(W(u) - W(u'))}{4\pi T}$$

Residual Drawdown and Recovery

- The Cooper-Jacob approximation can be applied giving:

$$s' = s + s_r = \frac{Q\left(\ln\left(2.25Tt/\left(r^2S\right)\right) - \ln\left(2.25Tt'/\left(r^2S\right)\right)\right)}{4\pi T}$$

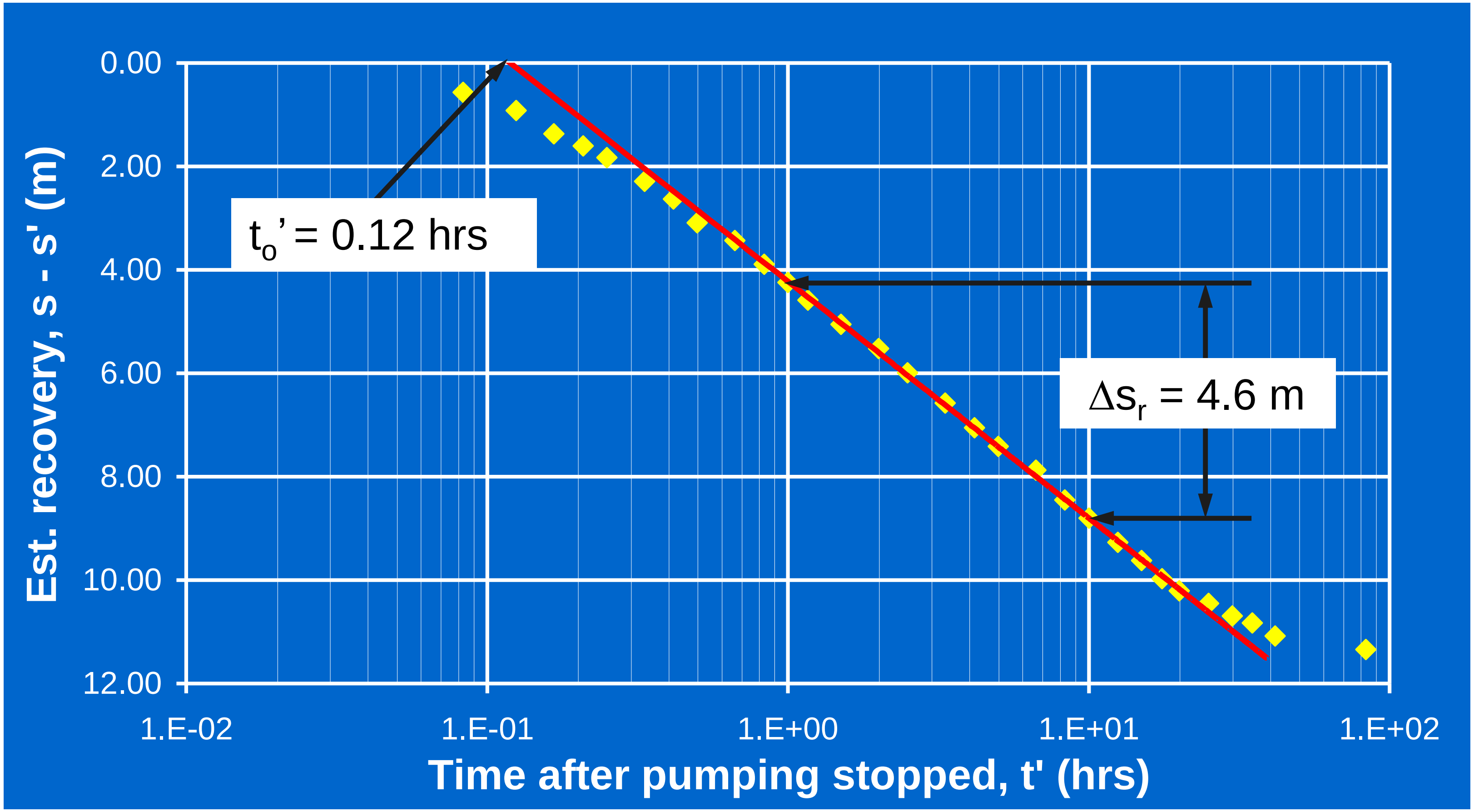
$$= \frac{Q\ln\left(t/t'\right)}{4\pi T}$$

- The equation predicting the **recovery** is:

$$s_r = \frac{-Q\left(\ln\left(2.25Tt'/\left(r^2S\right)\right)\right)}{4\pi T}$$

For $t > t_r$, the recovery s_r is the difference between the observed drawdown s' and the extrapolated pumping drawdown (s).

Time-Recovery Graph



Aquifer characteristics can be calculated from a log(time)-recovery plot but the drawdown (s) curve for the pumping phase must be extrapolated to estimate recovery (s - s')

Time-Recovery Analysis

- For a constant rate of pumping (Q), the recovery any time (t') after pumping stops:

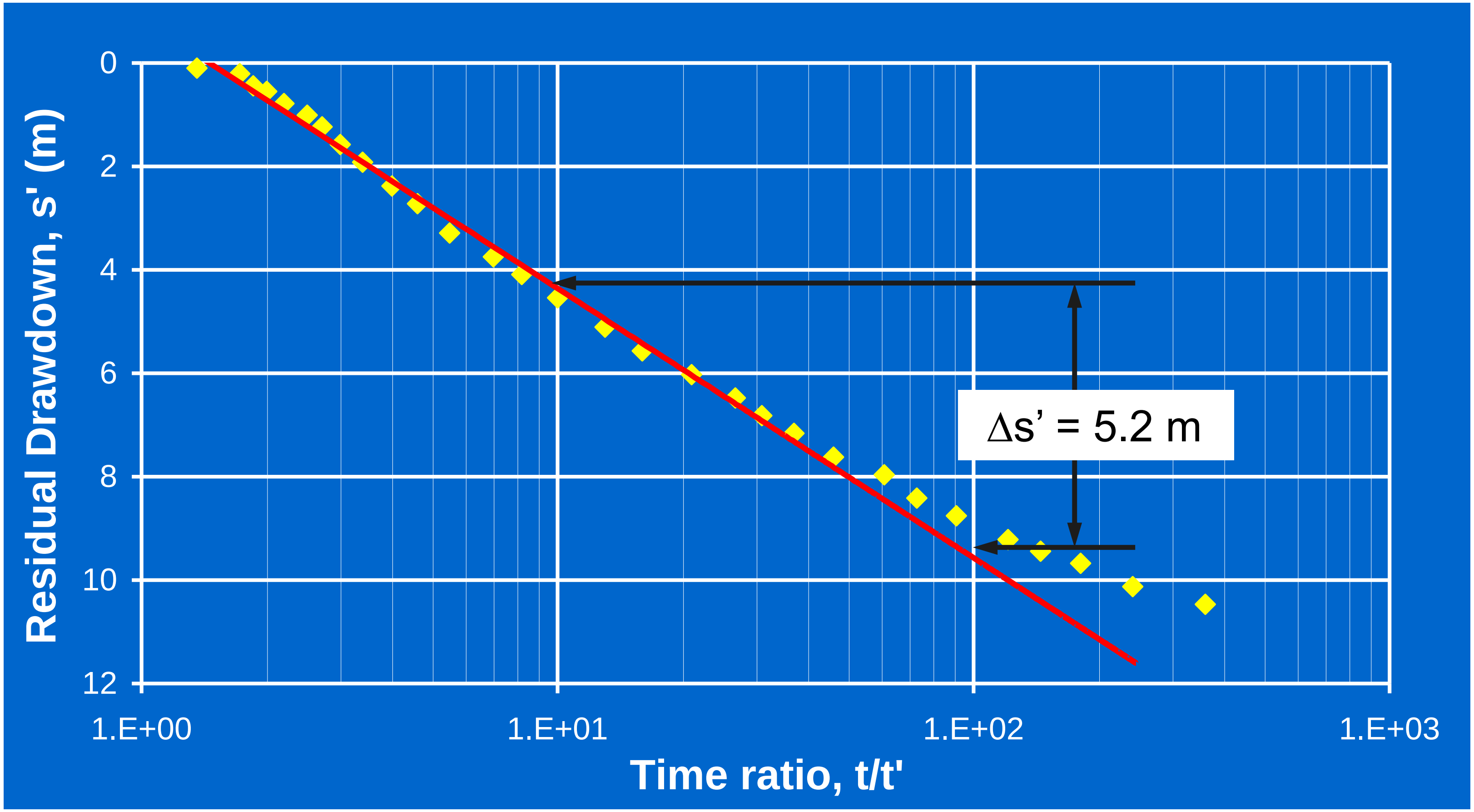
$$T = \frac{Q}{4\pi\Delta(s - s')} = \frac{-Q}{-4\pi\Delta s_r} = \frac{Q}{4\pi\Delta s_r}$$

- For the example, Δs_r = 4.6 m and Q = 1100 m^3/d so:

  $T = 1100 / (12.56 \times 4.6) = 19\ m^2/d$
- The storage coefficient can be estimated for an observation well (r = 30 m) using: $S = 4Tt_o'/r^2$
- For the example, t_o' = 0.12 and Q = 1100 m^3/d so:

  $S = 4 \times 19 \times 0.12 / (24 \times 30 \times 30) = 4.3 \times 10^{-4}$

Time-Residual Drawdown Graph



Transmissivity can be calculated from a log(time ratio)-residual drawdown (s') graph by determining the gradient. For such cases, the x-axis is log(t/t') and thus is a ratio.

Time-Residual Drawdown Analysis

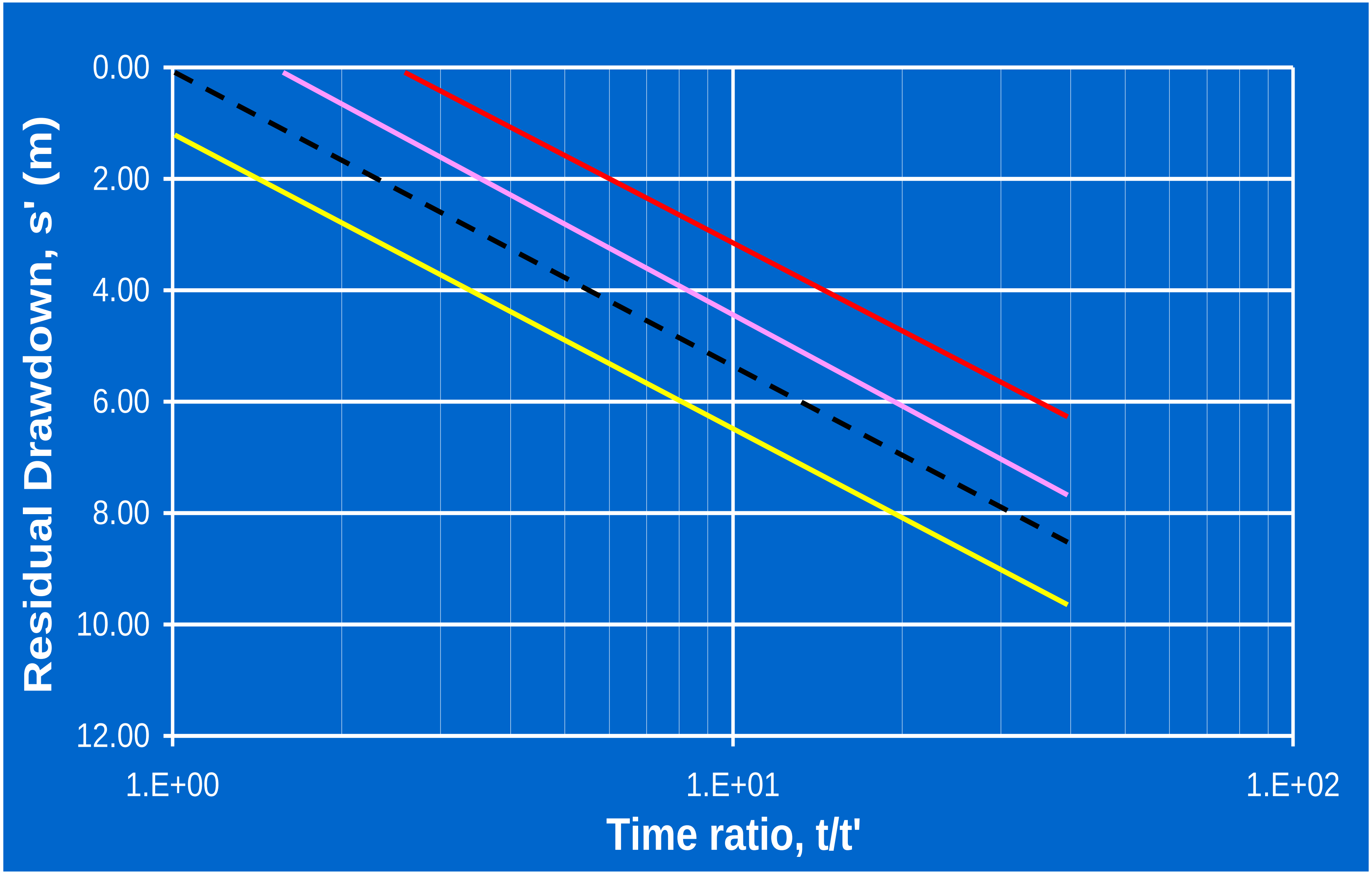
- For a constant rate of pumping (Q), the recovery any time (t') after pumping stops:

$$T = \frac{Q}{4\pi \Delta s'}$$

- For the example, $\Delta s_r = 5.2$ m and Q = 1100 m^3/d so:

  T = 1100 / (12.56 x 5.2) = 17 m^2/d
- Notice that the graph plots t/t' so the points on the LHS represent long recovery times and those on the RHS short recovery times.
- The storage coefficient cannot be estimated for the residual drawdown plot because the intercept $t / t' \rightarrow 1$ as $t' \rightarrow \infty$.
  - Remembering $t' = t - t_r$ where t_r is the elapsed pumping time before recovery starts.

Residual Drawdown for Real Aquifers



Residual Drawdown for Real Aquifers

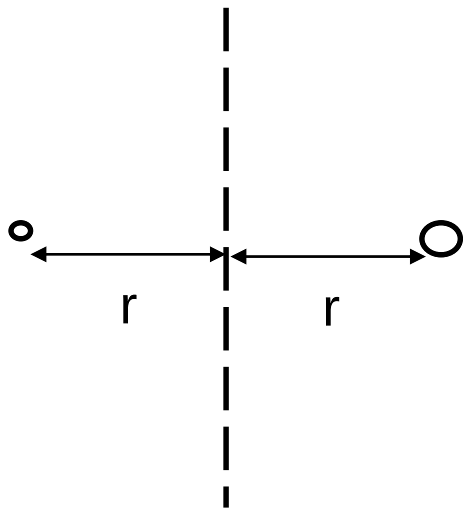
- Theoretical intercept is 1
- >> 1 indicates a recharge effect
- >1 may indicate greater S for pumping than recovery
- < 1 indicates incomplete recovery of initial head - finite aquifer volume
- << 1 indicates incomplete recovery of initial head - small aquifer volume

6. Bounded Aquifers

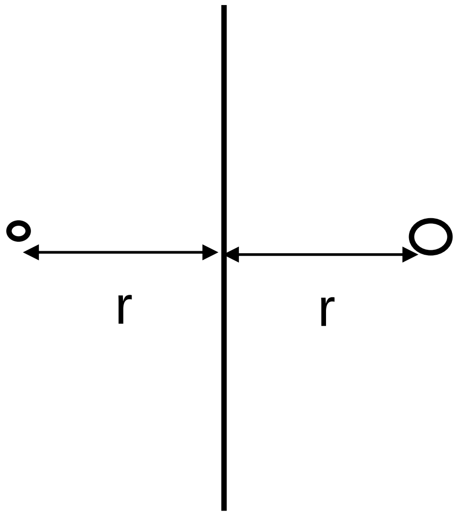
- Superposition was used to calculate well recovery by adding the effects of a pumping and recharge well starting at **different times**.
- Superposition can also be used to simulate the effects of aquifer boundaries by adding wells at **different positions**.
- For boundaries within the radius of influence, the wells that create the same effect as a boundary are called **image wells**.
- This, relatively simple application of superposition for analysis of aquifer boundaries, was described by Ferris (1959)

Image Wells

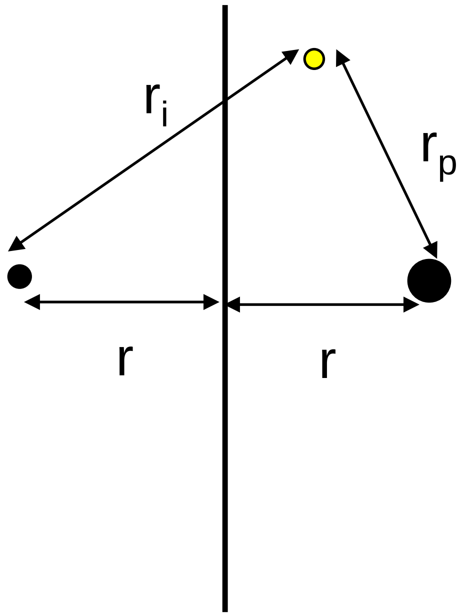
- Recharge boundaries at distance (r) are simulated by a recharge image well at an equal distance (r) across the boundary.



- Barrier boundaries at distance (r) are simulated by a pumping image well at an equal distance (r) across the boundary.



General Solution



The general solution for adding image wells to a real pumping well can be written:

$$s = s_p \pm s_i = \frac{Q}{4\pi T}\left[W(u_p) \pm W(u_i)\right]$$

$$u_p = \frac{r_p^2 S}{4Tt} \; and \; u_i = \frac{r_i^2 S}{4Tt}$$

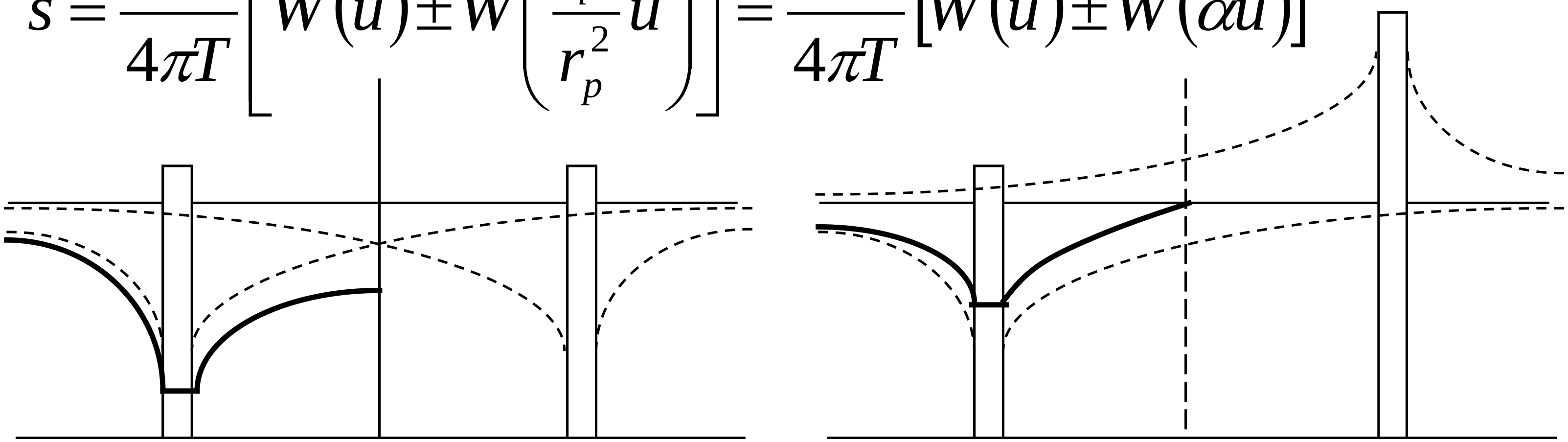
Where r_p, r_i are the distances from the pumping and image wells respectively.

- For a barrier boundary, for all points on the boundary $r_p = r_i$ the drawdown is doubled.
- For a recharge boundary, for all points on the boundary $r_p = r_i$ the drawdown is zero.

Specific Solutions

- The use of Cooper-Jacob approximation is only possible for large values of 1/u i.e. u < 0.05 for all r so the Theis well function is used:

$$s = \frac{Q}{4\pi T}\left[W(u) \pm W\left(\frac{r_i^2}{r_p^2}u\right)\right] = \frac{Q}{4\pi T}\left[W(u) \pm W(\alpha u)\right]$$



- For the barrier boundary case:

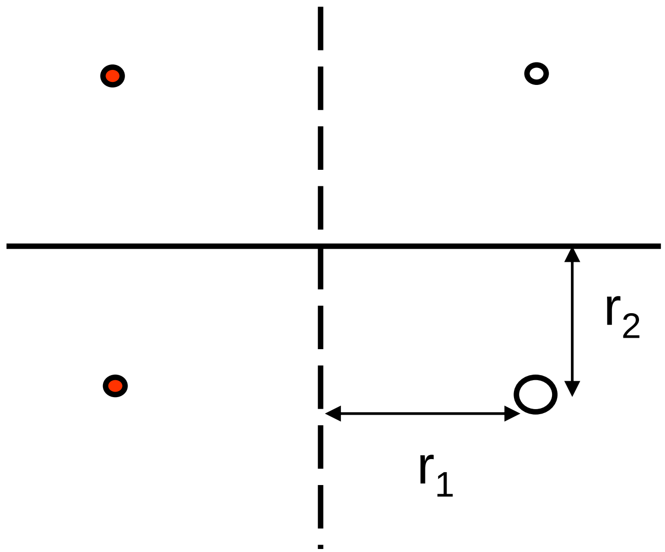
$$s = \frac{Q}{4\pi T}\left[W(u) + W(\alpha u)\right]$$

- For the recharge boundary case:

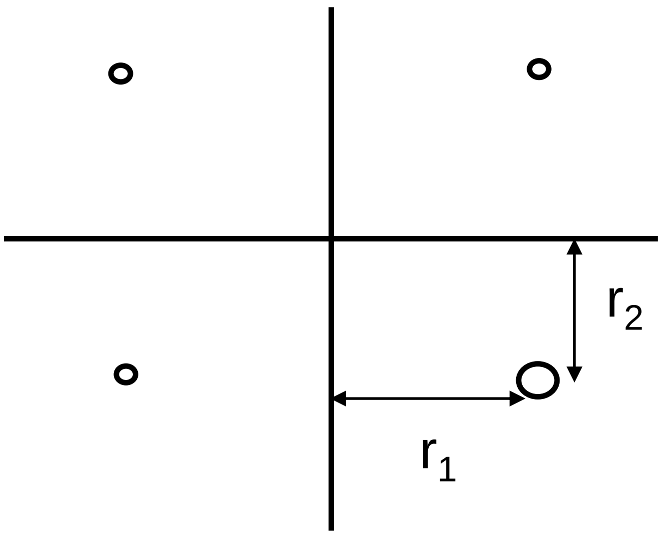
$$s = \frac{Q}{4\pi T}\left[W(u) - W(\alpha u)\right]$$

Multiple Boundaries

- A recharge boundary and a barrier boundary at right angles can be generated by two pairs of pumping and recharge wells.

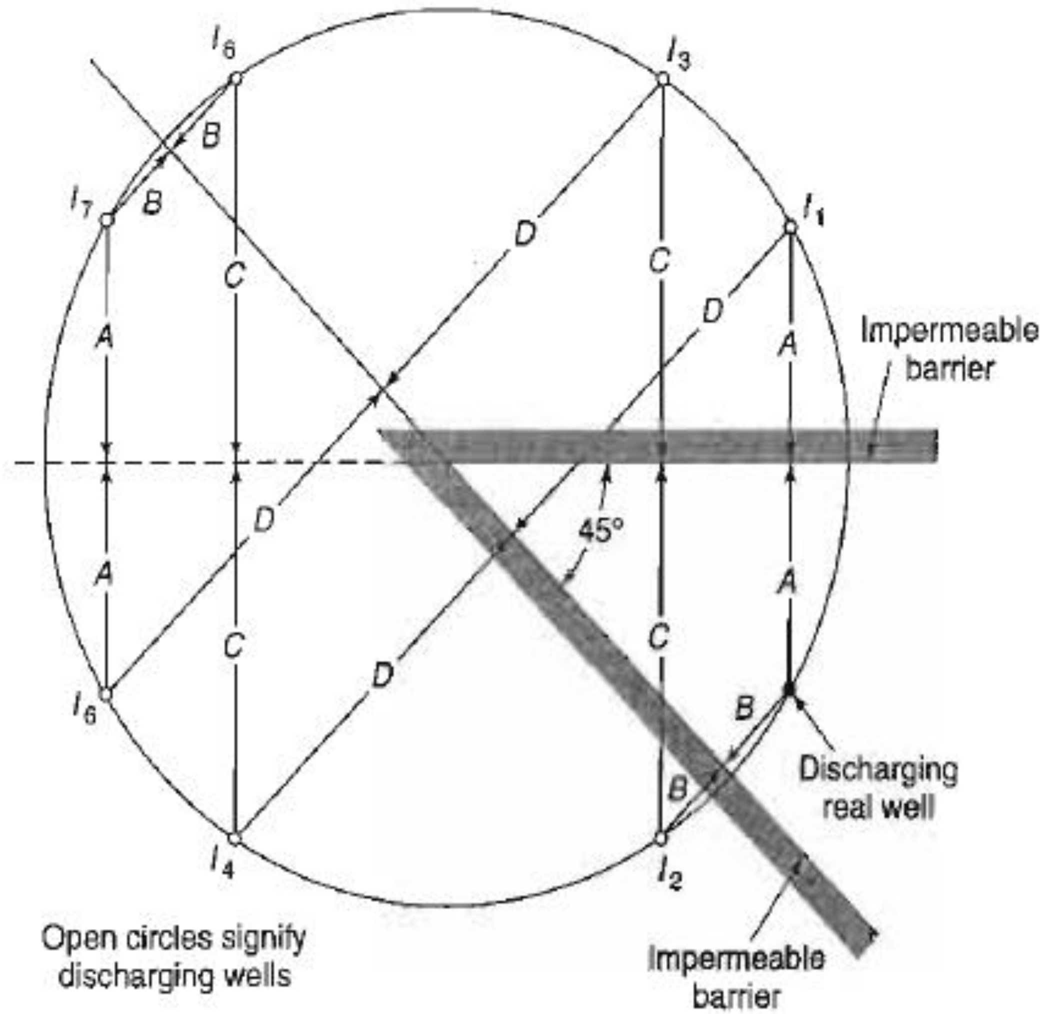


- Two barrier boundaries at right angles can be generated by superposition of an array of four pumping wells.



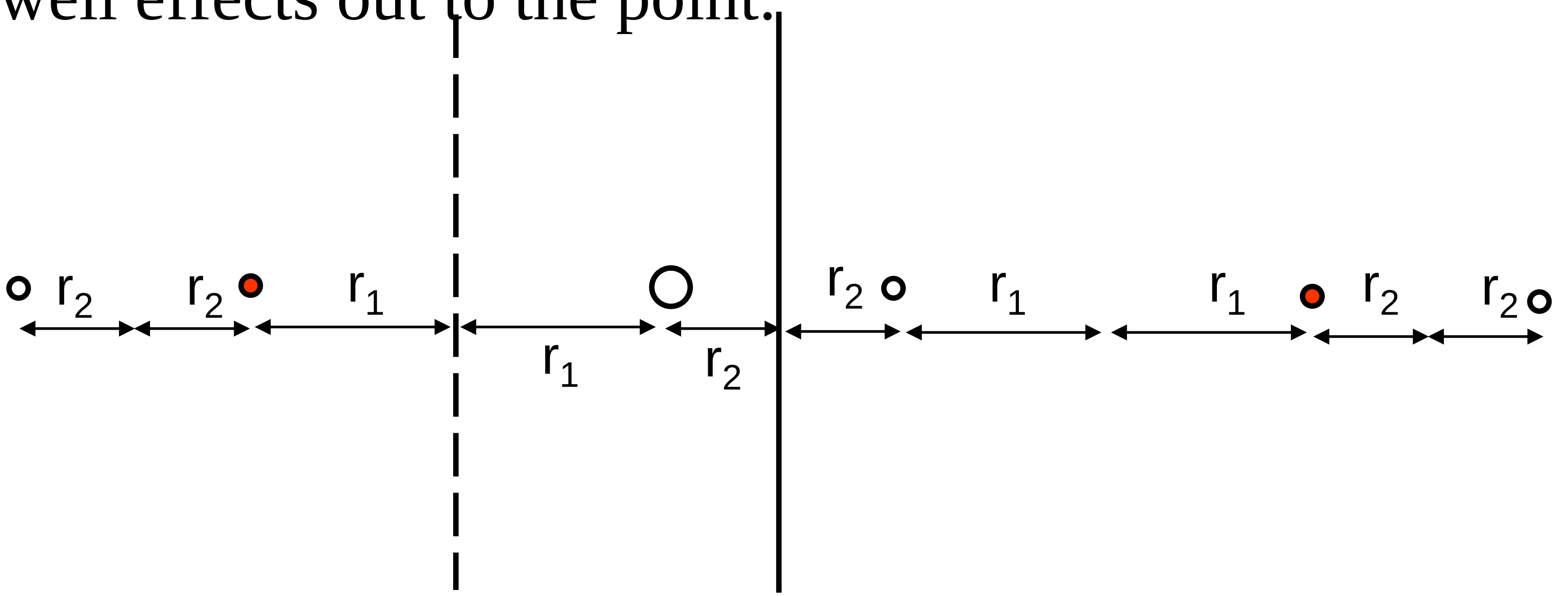
- The number of image wells, n, necessary for a wedge angle θ is given by: $n = 360/\theta - 1$.

- The image wells are usually lie on a circle centered at the apex of the wedge and radius equal to the distance between the pumping well and the apex.



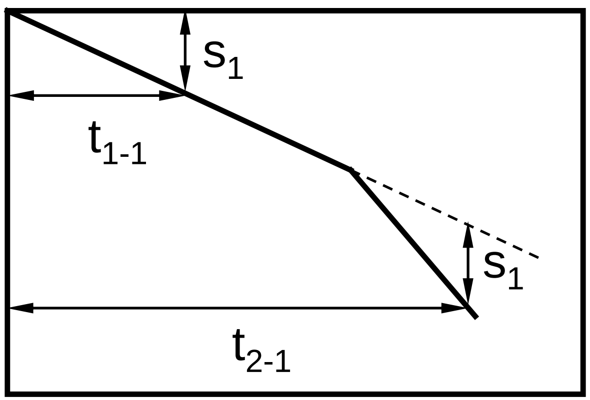
Parallel Boundaries

- A parallel recharge boundary and a barrier boundary (or any pattern with parallel boundaries) requires an infinite array of image wells.
- Each successively added secondary image well produces a residual effect at the opposite boundary.
- It is only necessary to add pairs of image wells until the next pair has negligible influence on the sum of all image well effects out to the point.

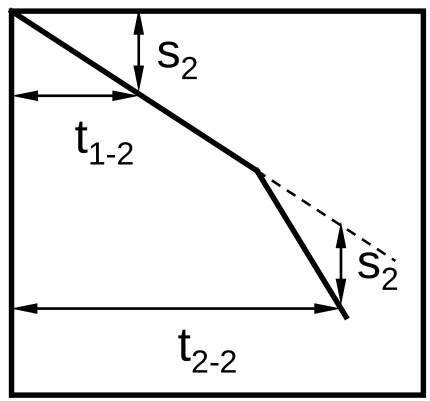


Boundary Location

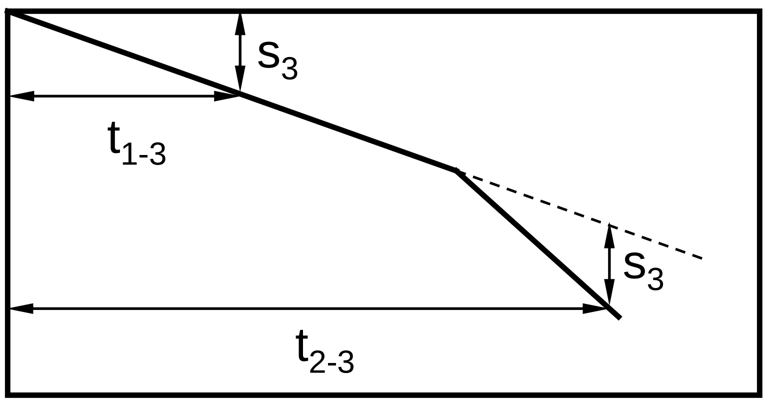
- For an observation well at distance r_1, measure off the same drawdown (s), before and after the “dog leg” on a log(time) vs. drawdown plot.



Observation well 1



Observation well 2



Observation well 3

Boundary Location

- Assuming that the "dog leg" is created by an image well at distance r_2 , if the drawdown are identical then $W(u_1) = W(u_2)$ so $u_1 = u_2$.
- Thus: $$\frac{r_1^2 S}{4Tt} = \frac{r_2^2 S}{4Tt} \Rightarrow r_2 = r_1 \sqrt{\left(\frac{t_2}{t_1}\right)}$$
  - The distance r_2 the radial distance from the observation point to the boundary.
  - Repeating for additional observation wells may help locate the boundary.