Chapter Three
Rainfall-Runoff Analysis

- **Linear System Theory and Rainfall-Runoff Analysis**
  - Unit hydrograph theory
    - From Stream flow Data
    - Synthetically
    - “Fitted” Distributions
  - Instantaneous unit hydrograph (IUH)
  - IUH analysis methods:
    - S-Hydrograph
    - Conceptual model
    - Fitting Harmonic analysis Fourier transforms
    - Theoretically from Laplace transforms
- **River and Reservoir Flood Routing**
  - Flood Routing
  - Reservoir flood routing methods:
  - Linear Muskingum method:
  - Multiple reach Muskingum method
  - Nonlinear Muskingum method:

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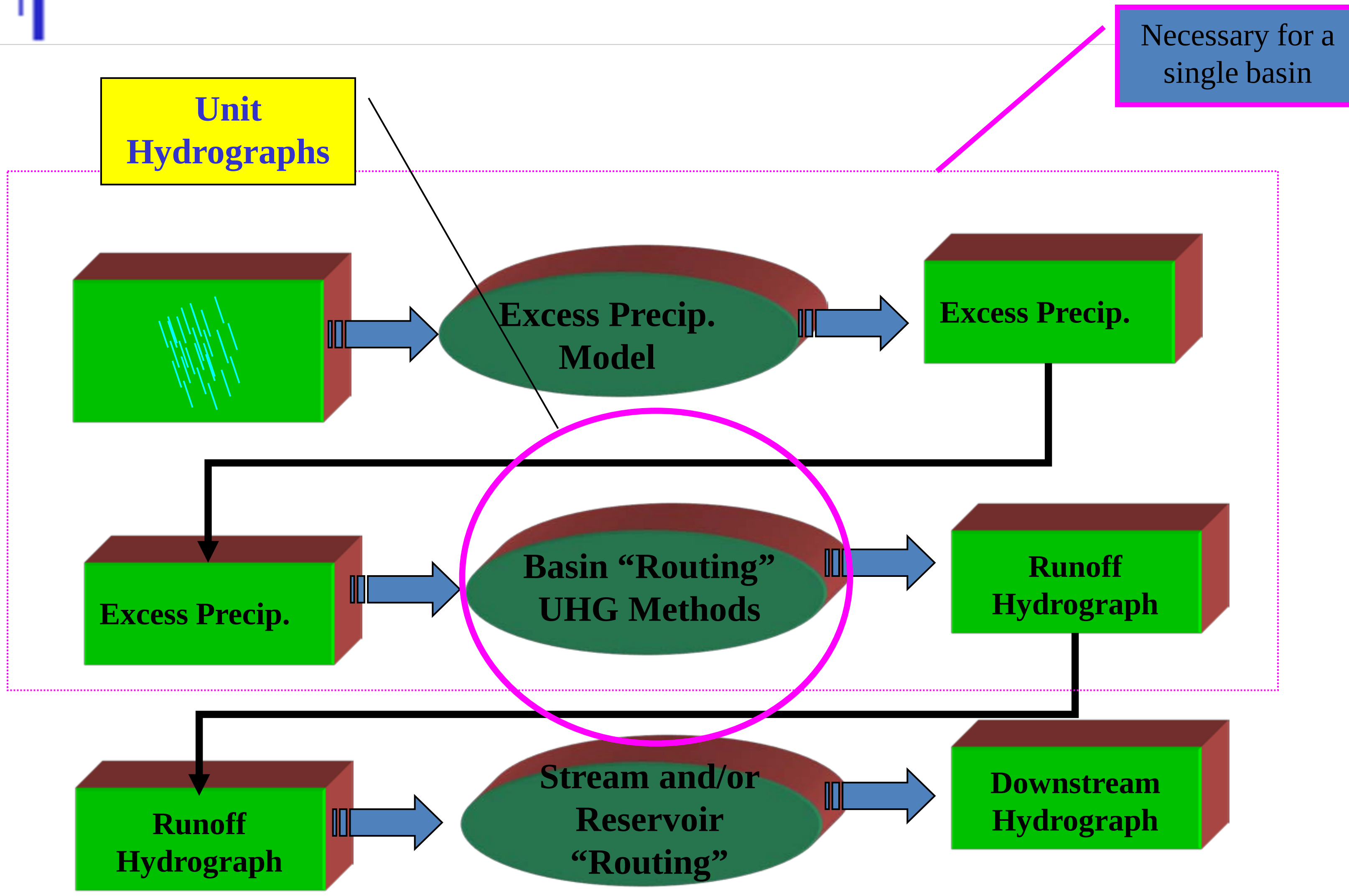
Unit Hydrograph Theory

- A mathematical concept
- Linear in nature
- Definition

  The hydrograph that results from 1-unit of excess precipitation (or runoff) spread uniformly in space and time over a watershed for a given duration.

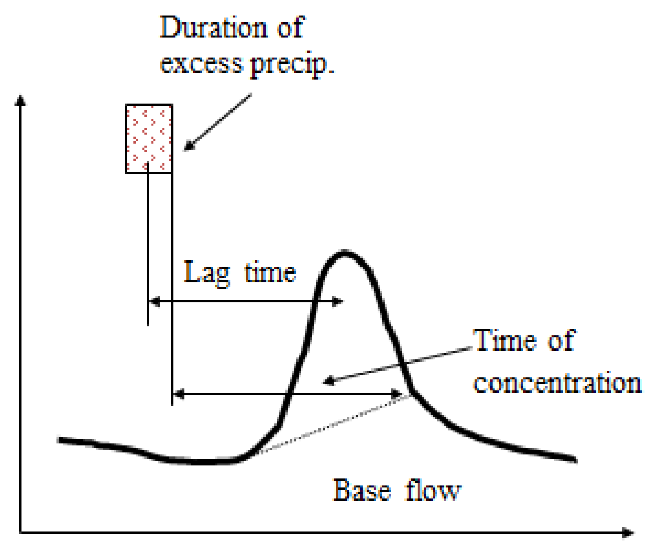
- The key points :
  - ✓1-unit of EXCESS precipitation
  - ✓Spread uniformly over space - evenly over the watershed
  - ✓Uniformly in time - the excess rate is constant over the time interval
  - ✓There is a given duration

The Basic Process



Unit Hydrograph "Lingo"

- Duration
- Lag Time
- Time of Concentration
- Rising Limb
- Recession Limb (falling limb)
- Peak Flow
- Time to Peak (rise time)
- Recession Curve
- Separation
- Base flow



Methods of Developing UHG's

- From Stream flow Data
- Synthetically
  - Snyder
  - SCS
  - Time-Area (Clark, 1945)
- "Fitted" Distributions

Derived Unit Hydrograph

Rules of Thumb :

↘… the storm should be fairly uniform in nature and the excess precipitation should be equally as uniform throughout the basin. This may require the initial conditions throughout the basin to be spatially similar.

↘… Second, the storm should be relatively constant in time, meaning that there should be no breaks or periods of no precipitation.

↘… Finally, the storm should produce at least an inch of excess precipitation (the area under the hydrograph after correcting for base-flow).

Deriving a UHG from a Storm

- Separate the base flow from direct runoff,
- Determine the volume of direct runoff, and
- Divide the ordinate of the direct runoff hydrograph by observed runoff depth.

$$r_d = \frac{\Delta t \sum_{i=1}^{n} q_i}{A}$$

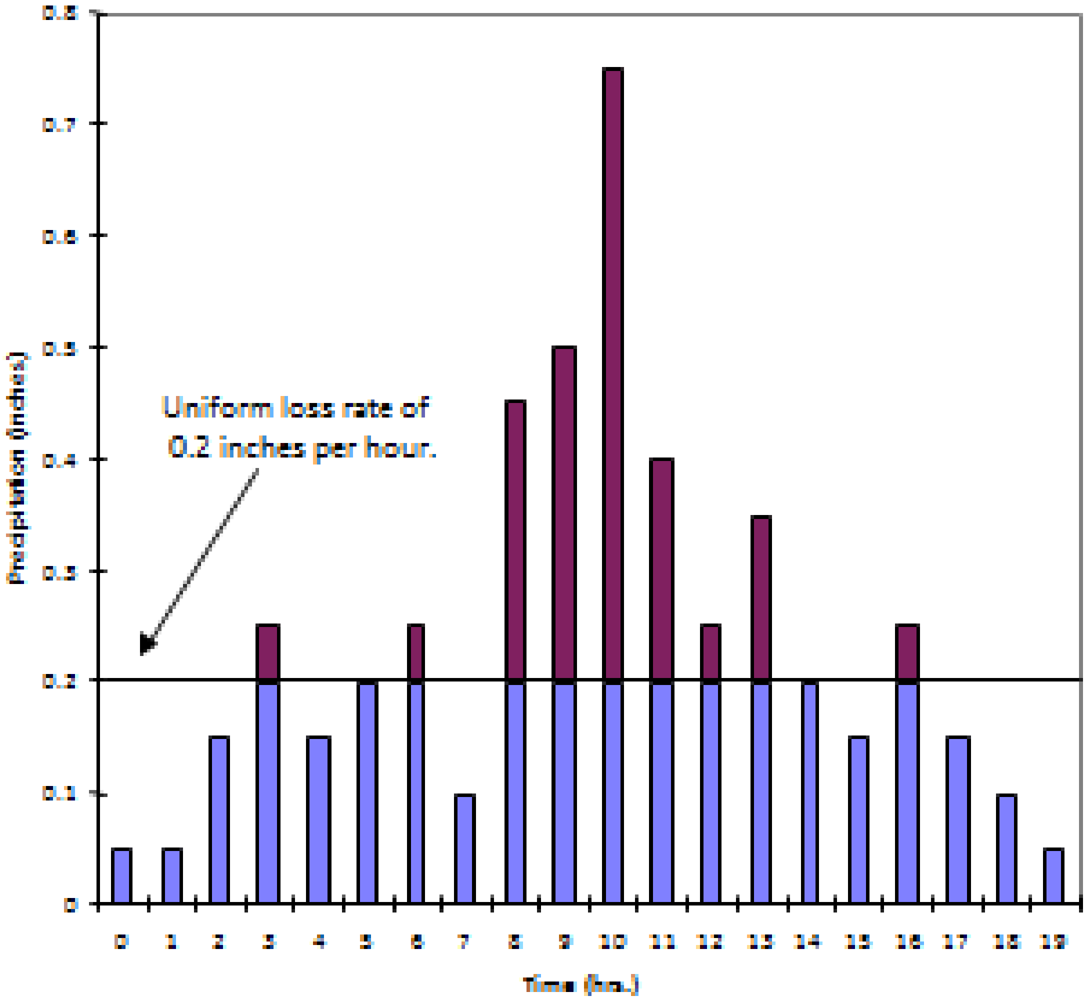
Determine Duration of UHG

- The duration of the derived unit hydrograph is found by examining the precipitation for the event and determining that precipitation which is in excess.

- This is generally accomplished by plotting the precipitation in hyetograph form and drawing a horizontal line such that the precipitation above this line is equal to the depth of excess precipitation as previously determined.

- This horizontal line is generally referred to as the ϕ-index and is based on the assumption of a constant or uniform infiltration rate.

Estimating Excess Precip.



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Example

Given the following hydrograph of a given watershed having drainage area of 104 km^2 derive the unit hydrograph for the watershed

Date	Hour	Stream flow	Base flow
16-Feb	06:00	11	8
	08:00	170	8
	10:00	260	6
	12:00	266	6
	14:00	226	8
	16:00	188	9
	18:00	157	11
	20:00	130	12
	22:00	108	14
	24:00	91	16
17-Feb	02:00	76	17
	04:00	64	19
	06:00	54	21
	08:00	46	22
	10:00	38	24
	12:00	32	26
	14:00	27	27

Synthetic UHG's

- Snyder
- SCS
- Time-area

Snyder

- Since peak flow and time of peak flow are two of the most important parameters characterizing a unit hydrograph, the Snyder method employs factors defining these parameters, which are then used in the synthesis of the unit graph (Snyder, 1938).

- The parameters are C_p, (peak flow factor), and C_t, (lag factor).

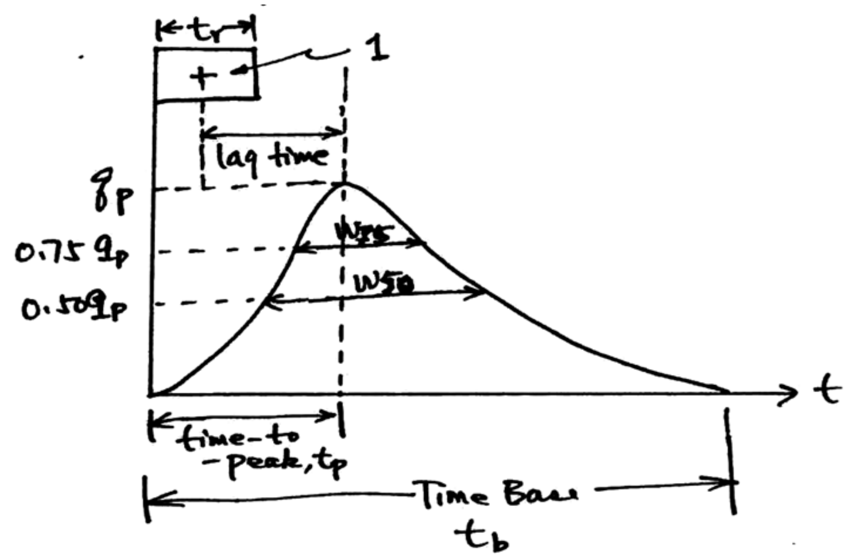
- The basic assumption in this method is that basins which have **similar physiographic characteristics** are located in the same area will have **similar values of C_t and C_p**.

- Therefore, the method is prefer to develop UH for un-gaged basins, which are near or similar to gaged basins for which these coefficients can be determined.

Snyder's Method

- Study area: US highlands. 10 ~ 10,000 mi^2 or (30 ~30,000 km^2)

- Properties characterizes the response of watershed under various rainfall inputs:
  - (a) Lag time (t_L);
  - (b) Duration of rainfall excess (tr);
  - (c) Time base of UH (t_b);
  - (d) Time to peak (t_p);
  - (e) Peak discharge of UH (q_p);
  - (f) Hydrograph time width at 50% and 75% (W50, W75) of peak flow



Snyder's Method

<u>Lag time</u> (t_p): time from the center of rainfall – excess to the UH peak

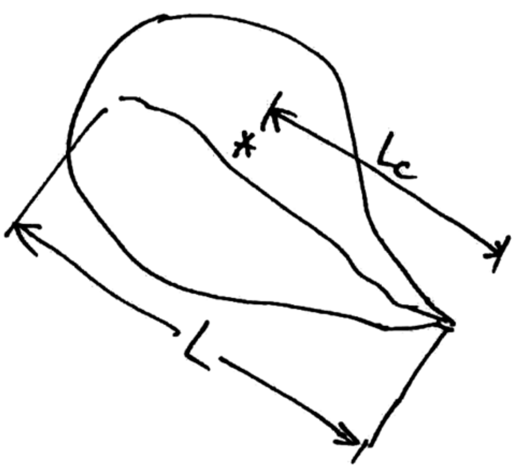
$$t_p = C_t \, (L \, L_c)^{0.3}$$

Where t_p = Time of lag [hrs];

C_t = Coefficient which is a function of watershed slope and shape, (~ 1.35 – 1.60 , for steeper slope, C_t is smaller);

L = length of the main channel [km] or [mi];

L_c = length along the main channel to the point nearest to the watershed centroid



Snyder's Method

UH Duration (t_r):

$$t_r = t_p / 5.5$$

where t_r and t_p are in [hrs].

If the duration of UH is other than t_r, then the lag time needs to be adjusted as

$$t_{np} = t_p + 0.25\,(t_R - t_r)$$

where t_{np} = adjusted lag time; t_R = desired UH duration.

UH Peak Discharge (q_p):

$$q_p = \frac{2.78 A C_p}{t_p}$$

where C_p = coefficient accounting for flood wave and storage condition, 0.56 ~ 0.69 as miller; 0.23 - 0.67 clark;

q_p =specific discharge, [m^3/s]

Snyder's Method

Time Base (T_b): $t_b = (72 + 3t_p)$ or $T_b = 5(t_{np} + 0.5\, t_r)$

UH Widths:

$$W_{50} = \frac{5.87}{q_{pru}^{1.08}} \qquad W_{75} = \frac{3.354}{q_{pru}^{1.08}}$$

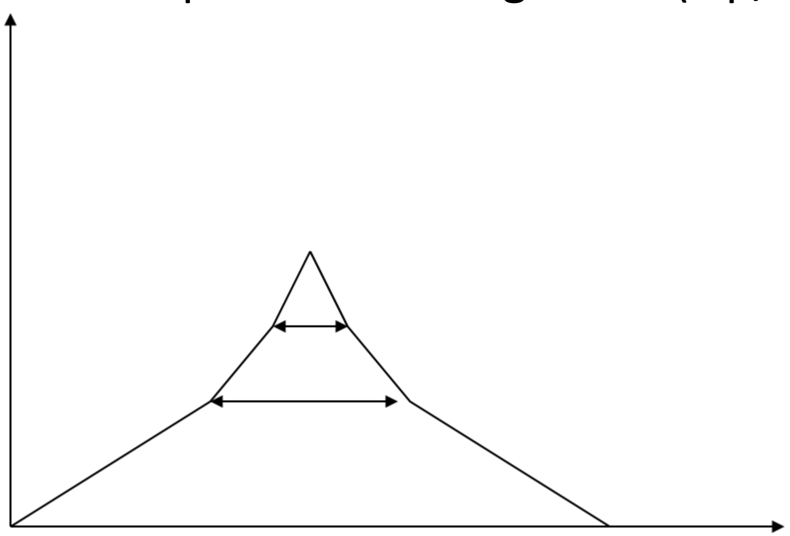
where

W_{50}, are in hours; Usually, 2/3 of the width after the peak of UH

W_{75} are in hours; Usually, 1/3 of the width is before UH peak

q_{pru} = the peak discharge of the UH per unit drainage area (Qp/A)

The final shape of the Snyder unit hydrograph is controlled by the equations for width at 50% and 75% of the peak of the UHG



Example

Two catchments A and B are considered meteorologically similar . Their catchment characteristics are given below

Catchment A	Catchment B
L= 30km	L= 45km
L_{ca} = 15km	L_{ca}= 25km
A= 250km^2	A= 400km^2

For catchment A, a 2hr unit hydrograph was developed and was found to have a peak discharge of 50m3/sec. The time to peak from the beginning of the rainfall excess in this unit hydrograph was 9.0hr. Using the Snyder's method develop a unit hydrograph for catchment B.

SCS Dimensionless UH

Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
0	.000	.000
.1	.030	.001
.2	.100	.006
.3	.190	.012
.4	.310	.035
.5	.470	.065
.6	.660	.107
.7	.820	.163
.8	.930	.228
.9	.990	.300
1.0	1.000	.375
1.1	.990	.450
1.2	.930	.522
1.3	.860	.589
1.4	.780	.650
1.5	.680	.700
1.6	.560	.751
1.7	.460	.790
1.8	.390	.822
1.9	.330	.849
2.0	.280	.871
2.2	.207	.908
2.4	.147	.934
2.6	.107	.953
2.8	.077	.967
3.0	.055	.977
3.2	.040	.984
3.4	.029	.989

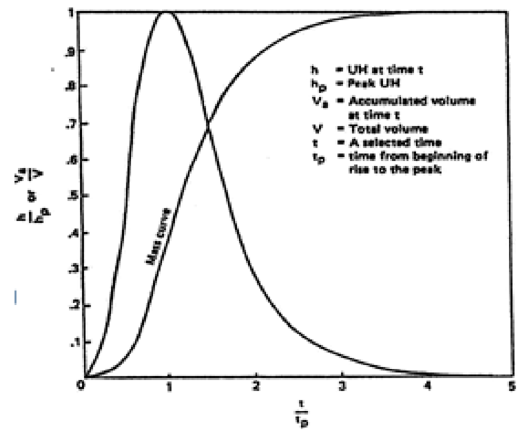


Figure 16.4 Dimensionless unit hydrograph and mass curve (after Soil Conservation Service, 1971).

Triangular Representation

$$T_b = 2.67 \text{ x } T_p$$

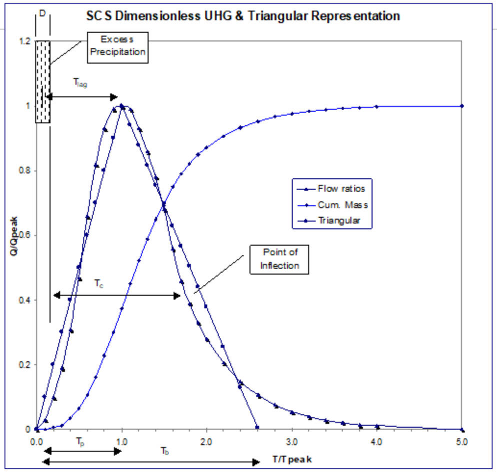
$$T_r = T_b - T_p = 1.67 \text{ x } T_p$$

$$Q = \frac{q_p T_p}{2} + \frac{q_p T_r}{2} = \frac{q_p}{2}(T_p + T_r)$$

$$q_p = \frac{2Q}{T_p + T_r}$$

$$q_p = \frac{654.33 \text{ x } 2 \text{ x } A \text{ x } Q}{T_p + T_r}$$

$$q_p = \frac{484\, A\, Q}{T_p}$$



The 645.33 is the conversion used for delivering 1-inch of runoff (the area under the unit hydrograph) from 1-square mile in 1-hour (3600 seconds).

Triangular Representation

Comes from the initial assumption that 3/8 of the volume under the UHG is under the rising limb and the remaining 5/8 is under the recession limb.

General Description	Peaking Factor	Limb Ratio (Recession to Rising)
Urban areas; steep slopes	575	1.25
Typical SCS	484	1.67
Mixed urban/rural	400	2.25
Rural, rolling hills	300	3.33
Rural, slight slopes	200	5.5
Rural, very flat	100	12.0

Duration and times

$$T_p = \frac{D}{2} + L$$

$$T_c + D = 1.7\,T_p$$

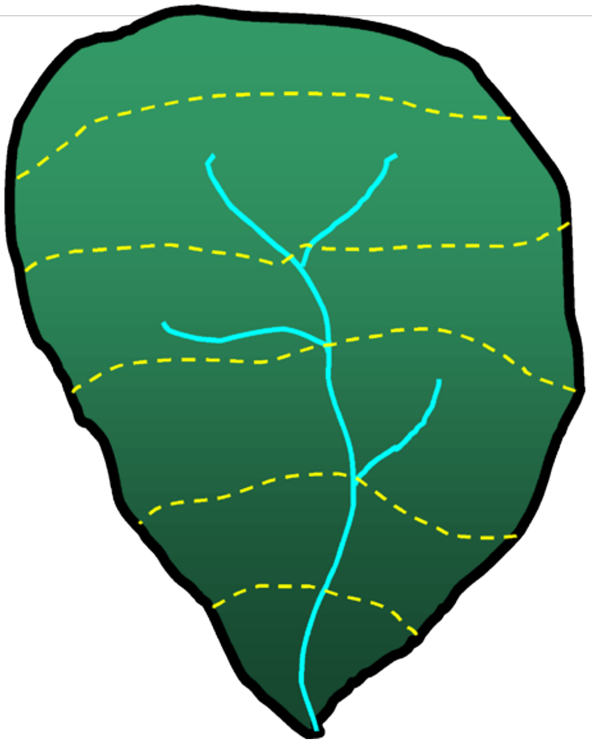
$$L = 0.6 * T_c$$

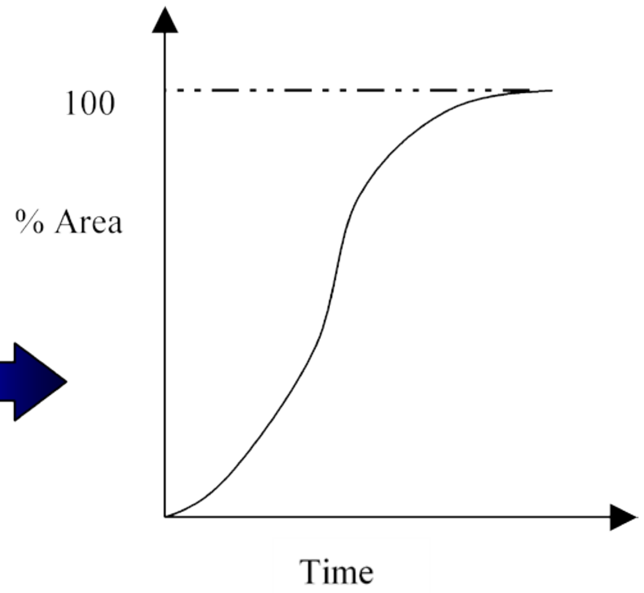
$$\frac{D}{2} + 0.6\,T_c = T_p$$

For estimation purposes :

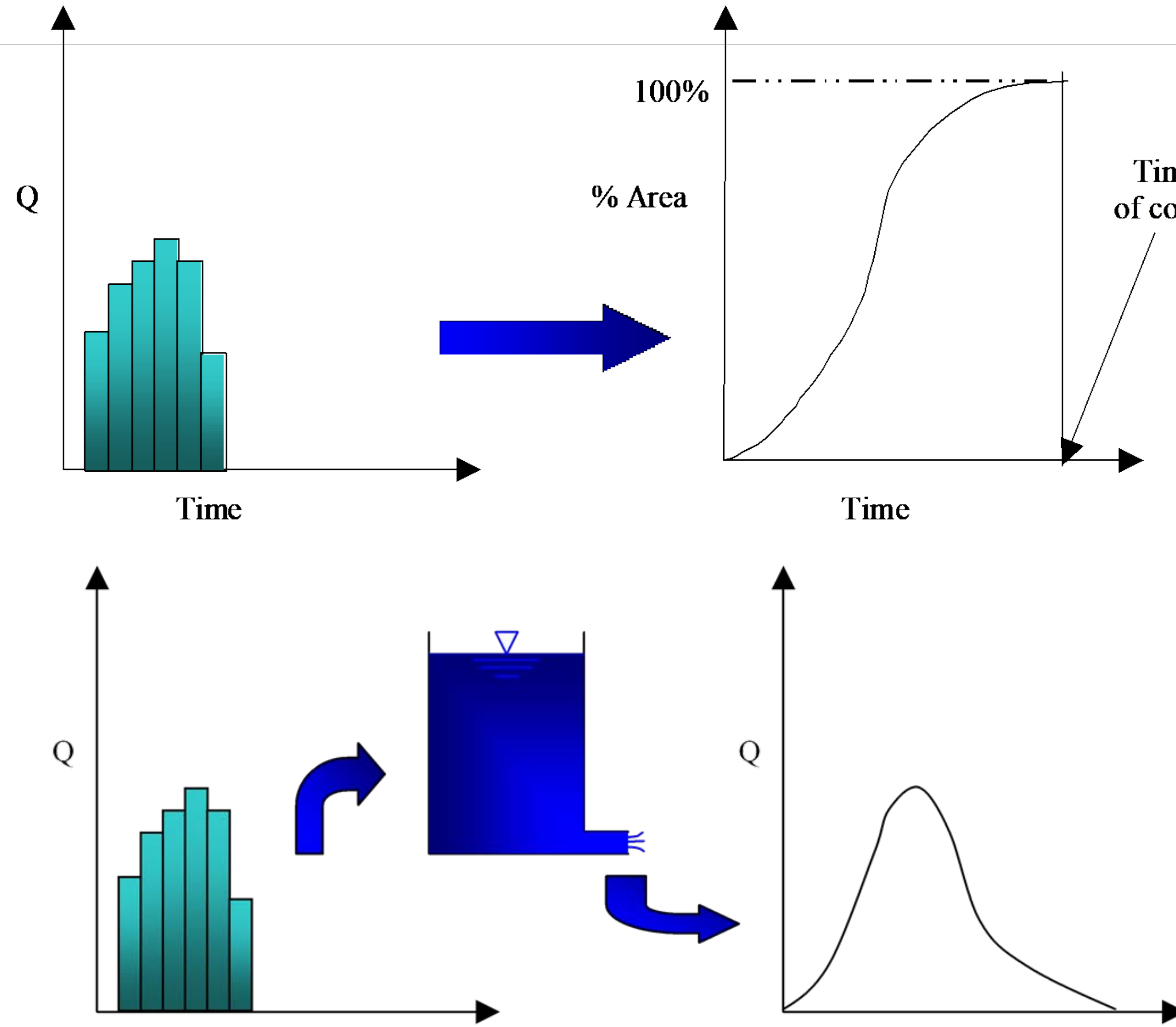
$$D = 0.133\,T_c$$

Time-Area





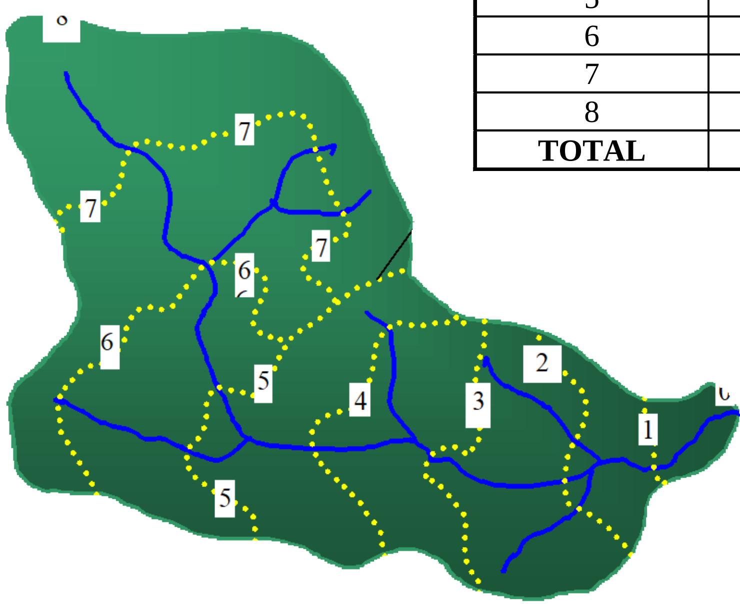
Time-Area



Hypothetical Example

- A 190 mi^2 watershed is divided into 8 isochrones of travel time.
- The linear reservoir routing coefficient, R, estimated as 5.5 hours.
- A time interval of 2.0 hours will be used for the computations.

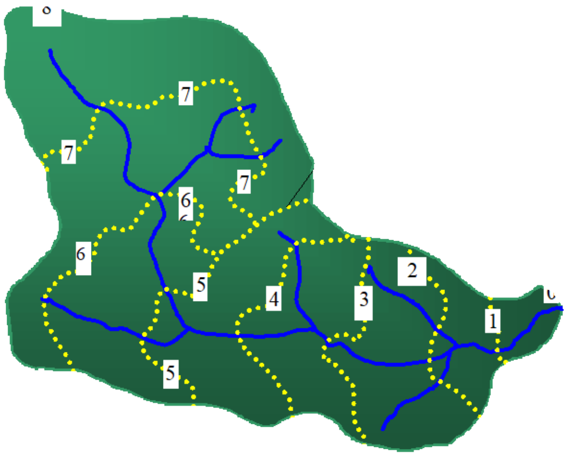
Map Area #	Bounding Isochrones	Area (mi^2)	Cumulative Area (mi^2)	Cumulative Time (hrs)
1	0-1	5	5	1.0
2	1-2	9	14	2.0
3	2-3	23	37	3.0
4	3-4	19	58	4.0
5	4-5	27	85	5.0
6	5-6	26	111	6.0
7	6-7	39	150	7.0
8	7-8	40	190	8.0
TOTAL		**190**	**190**	**8.0**

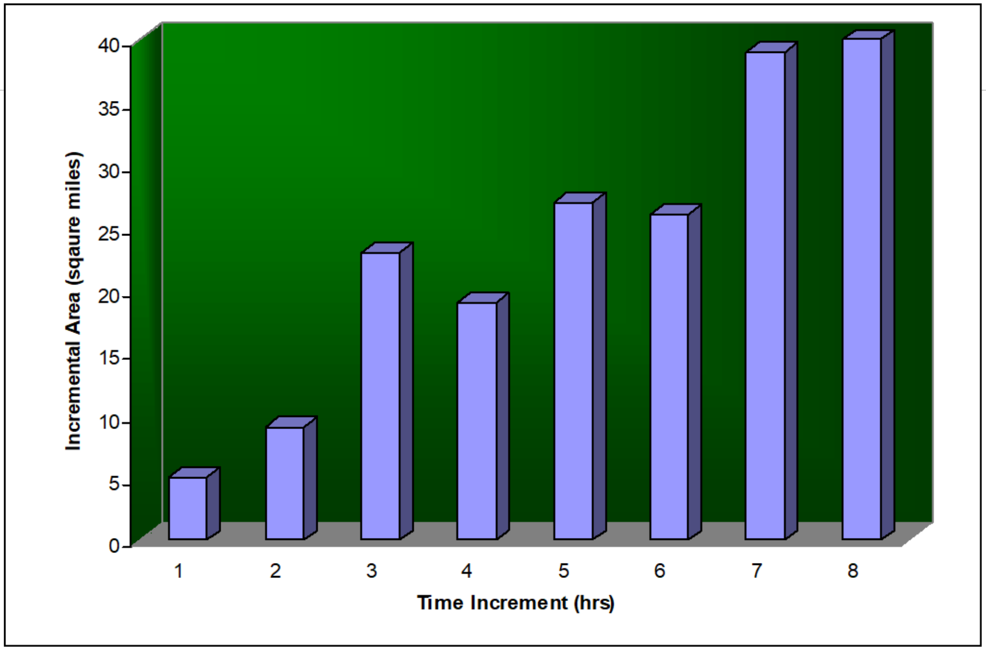


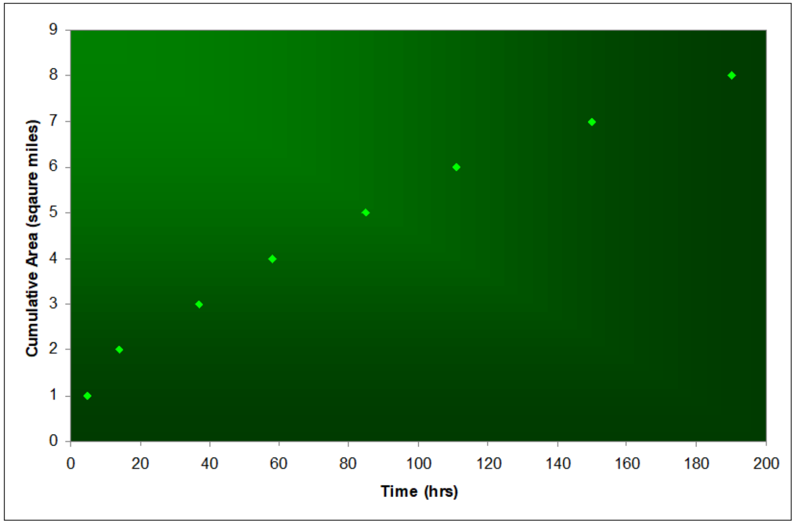
Rule of Thumb

R - The linear reservoir routing coefficient can be estimated as approximately 0.75 times the time of concentration.

Incremental Area







$$TA_i = 1.414 T_i^{1.5} \quad \text{for } (0 \le \text{Ti} \le 0.5)$$

$$1 - TA_i = 1.414(1 - T_i)^{1.5} \quad \text{for } (0.5 \le \text{Ti} \le 1.0)$$

Synthetic time-area curve - The U.S. Army Corps of Engineers (HEC 1990)

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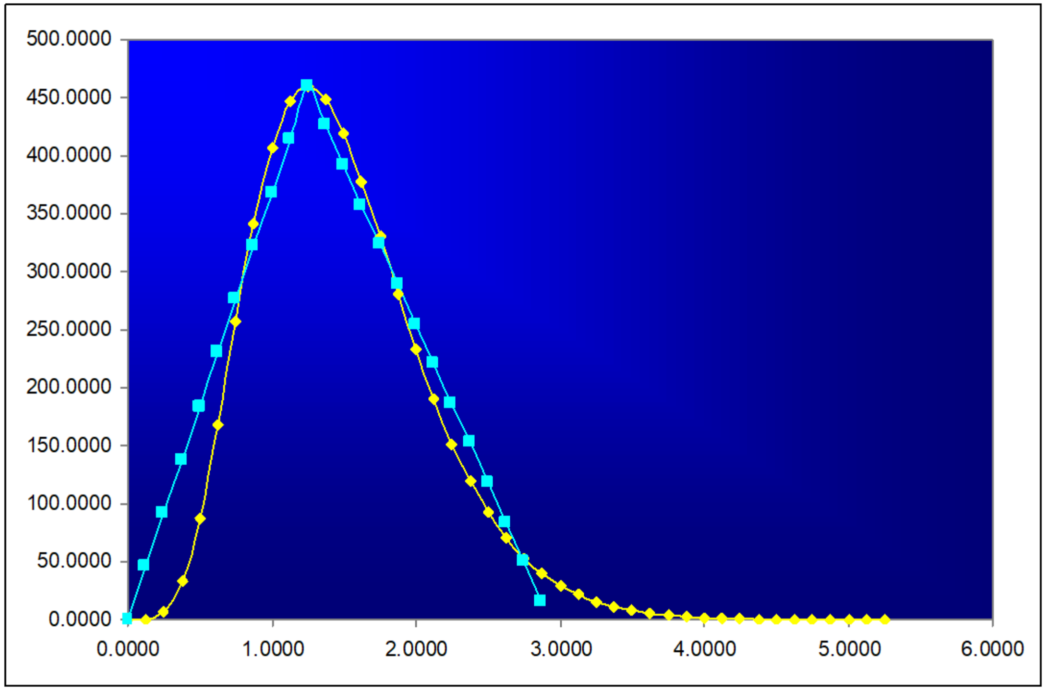
General guiding Rule in synthetic UH

- The UH is better calculated from strong and isolated storm events… Not many of them happened in the last 120 days
- **Verification**:
  - Find The watershed with the perfect storms and storm information
  - Create a synthetic Hydrograph from the UH and test it for other stream gages in the watershed
  - Compare the peak flows from the UH and the rationale method (C i A)

Fitting with Probability distribution

Fitting a Gamma Distribution

$$f(t;a,b)=\frac{t^{a}e^{-t/b}}{b^{a+1}\Gamma(a+1)}$$



Project in UH

- Take one river gauging station from the country gauging stations dataset.
- Develop UH for the gauging station based on gauged data
- Develop the catchment characteristics
- Develop UH synthetic UH for that catchment
- Compare the peak of UH and give your opinion on the results