CENG 6108 Construction Economics

Time Value of Money and Cash Flow Analysis

Abraham Assefa Tsehayae, PhD

April, 2017

TO DO

1. Introduction
2. Interest and Interest Rates
3. Effective and Nominal Interest Rates
4. Continuous Compounding
5. Cash Flow Diagrams
6. Equivalence
7. Cash Flow Analysis

Time Value of Money

- Importance of Time
  - Money can have different values at different times.
  - $10,000 now is worth more than $10,000 a year from now even if there is no inflation, as it can earn money during the interval by:
    - Doing business with it (Productive Use of Money) or
    - Investing it in a bank and earn interest on it.
  - Thus, interest rate is the more identifiable and accepted measure of the earning power of money.
  - Interest is, thus, usually accepted as the time value of money and indication of its earning power
  - However, the buying power of money is related to inflation, a concept closely related to interest rate.

Time Value of Money

- If we assume
  - that money can always be invested in the bank (or some other reliable source) now to gain a return with interest later
  - that as rational actors, we never make an investment which, we know, offers less money than we could get in the bank

- Then
  - Money in the ***present*** can be thought as of “*equal worth*” to a larger amount of money in the future
  - money in the ***future*** can be thought of as having an equal worth to a lesser “*present value*” of money

Interest Rate

- The rate of interest i is the percentage of the money we pay for its use over a time period.
  - The interest rate is referred to by different names such as rent, cost of money, and value of money.
  - In investment terminology, it is also called the minimum acceptable rate of return or MARR
- To compare the value of money at different points in time, we need to use an acceptable interest rate.
- The interest rate will depend on the:
  - The position in time that the money is needed,
  - The length of time it is required. For short periods, it is assumed that the economy is stable and the risk is predictable.
  - Availability of money in the financial market.

Interest

- Money has a Time Value
  - Money earns money over time

- Example
  - Deposit Br 1,000,000 today in a saving account at interest rate of 10%

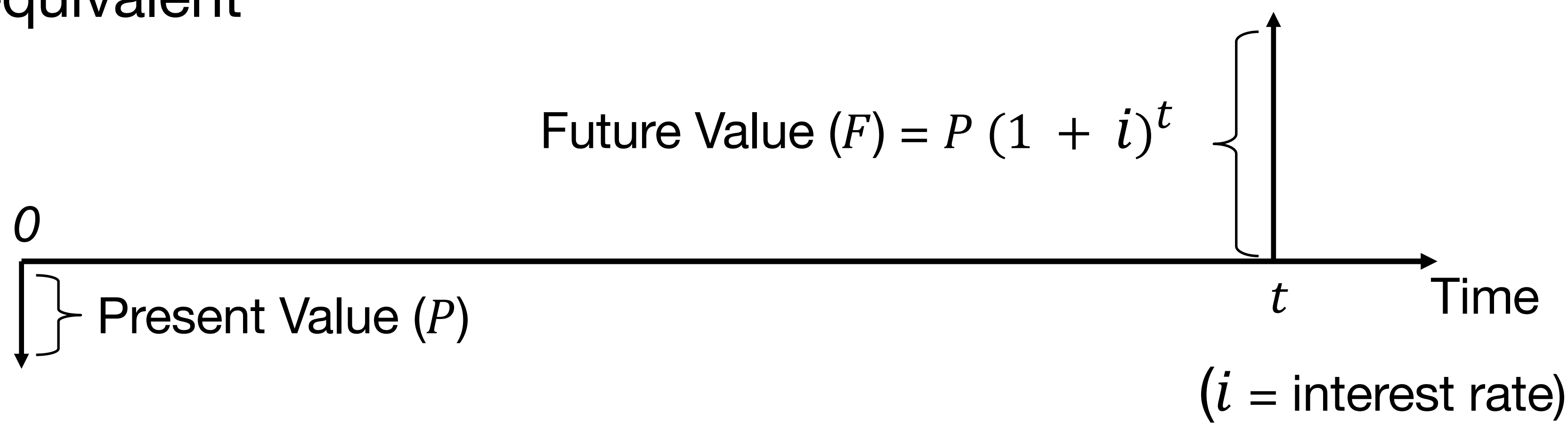
Original Amount	+	Interest Amount	=	Total Amount
$ 1,000,000	$\xrightarrow{10\%}$	Earns $ 100,000	$\xrightarrow[\text{1Year}]{\text{After}}$	$ 1,100,000

Function of Interest Rate & Time

$ 1,000,000 at present (P) = $ 1,100,000 at future time (F)

Present and Future Values

- Because we can flexibly switch from one such value to another without cost, we can view these values as equivalent



- Given a reliable source offering annual return i (i.e., interest), we can shift without additional costs between cash *P* at time *0* and $P(1+i)^t$ at time t

Major Elements & Symbols

i = Interests rate per interest period

N = Number of interest periods

P = Equivalent value of amount of money at time zero (Present Worth)

F = Equivalent value of amount of money at time N (Future Worth)

A = An End-Of-Period payment or receipt in a uniform series (Annuity)

Methods of Calculating Interest (i)

- Simple Interest
  - Considers the interest to be earned on only the Principal amount during each interest period (the interest earned during each period does not earn additional interest in the remaining periods)

For one period $I = i\,P$

e.g.) $P = \$100, i = 10\%$

$$I = 0.1 * 100 = \$10$$

Methods of Calculating Interest (i)

- Simple Interest
  - Considers the interest to be earned on only the Principal amount during each interest period (the interest earned during each period does not earn additional interest in the remaining periods)

For one period $$I = i\,P$$

For N periods $$I = (i\,P)\,N$$

e.g.) $P = \$100, i = 10\%$

$$I = 0.1 * 100 = \$10$$

If $N = 5$,

$$I = 0.1 * 100 * 5 = \$50$$

Methods of Calculating Interest (i)

- Simple Interest
  - Considers the interest to be earned on only the Principal amount during each interest period (the interest earned during each period does not earn additional interest in the remaining periods)

e.g.) $P = \$100, i = 10\%$

For one period $$I = i\,P$$

$$I = 0.1 * 100 = \$10$$

For N periods $$I = (i\,P)\,N$$

If $N = 5$,
$$I = 0.1 * 100 * 5 = \$50$$

$\vdots$

$i\,N$ is a linear function of time

F @ end of N Periods

$$F = P + I = P(1 + i\,N)$$

$$F = 100 + 50 = \$150$$

Methods of Calculating Interest (i)

- Compound Interest
  - The interest in each period is based on the total amount owed at the end of the previous period *(the original principal plus the accumulated interest)*

\$100

If you deposit (\$ $\boldsymbol{P}$) at interest rate i

10%

What you would have at the end of period one

\$10

$$I_1 = i\,P$$

$$F_1 = P + i\,P = P(1 + i)$$

\$110

Methods of Calculating Interest (i)

- Compound Interest
  - The interest in each period is based on the total amount owed at the end of the previous period *(the original principal plus the accumulated interest)*

$100

If you deposit (\$ P) at interest rate i

10%

What you would have at the end of period one

$10

$$I_1 = i\,P$$

$$F_1 = P + i\,P = P(1 + i)$$

$110

$$I_2 = i\,F_1$$

$$\$110 * 0.1$$

$$F_2 = F_1(1 + i)$$

$$\$110 * (1 + 0.1)$$

Methods of Calculating Interest (i)

- Compound Interest
  - The interest in each period is based on the total amount owed at the end of the previous period *(the original principal plus the accumulated interest)*

$100

If you deposit (\$ P) at interest rate i

10%

What you would have at the end of period one

$$I_1 = i\,P$$

$10

$$F_1 = P + i\,P = P(1 + i)$$

$110

$110*(1+0.1)

$110*0.1 $I_2 = i\,F_1$

$$F_2 = F_1(1 + i)$$

$121*0.1 $I_3 = i\,F_2$

$$F_3 = F_2(1 + i)$$

…

…

$121*(1+0.1)

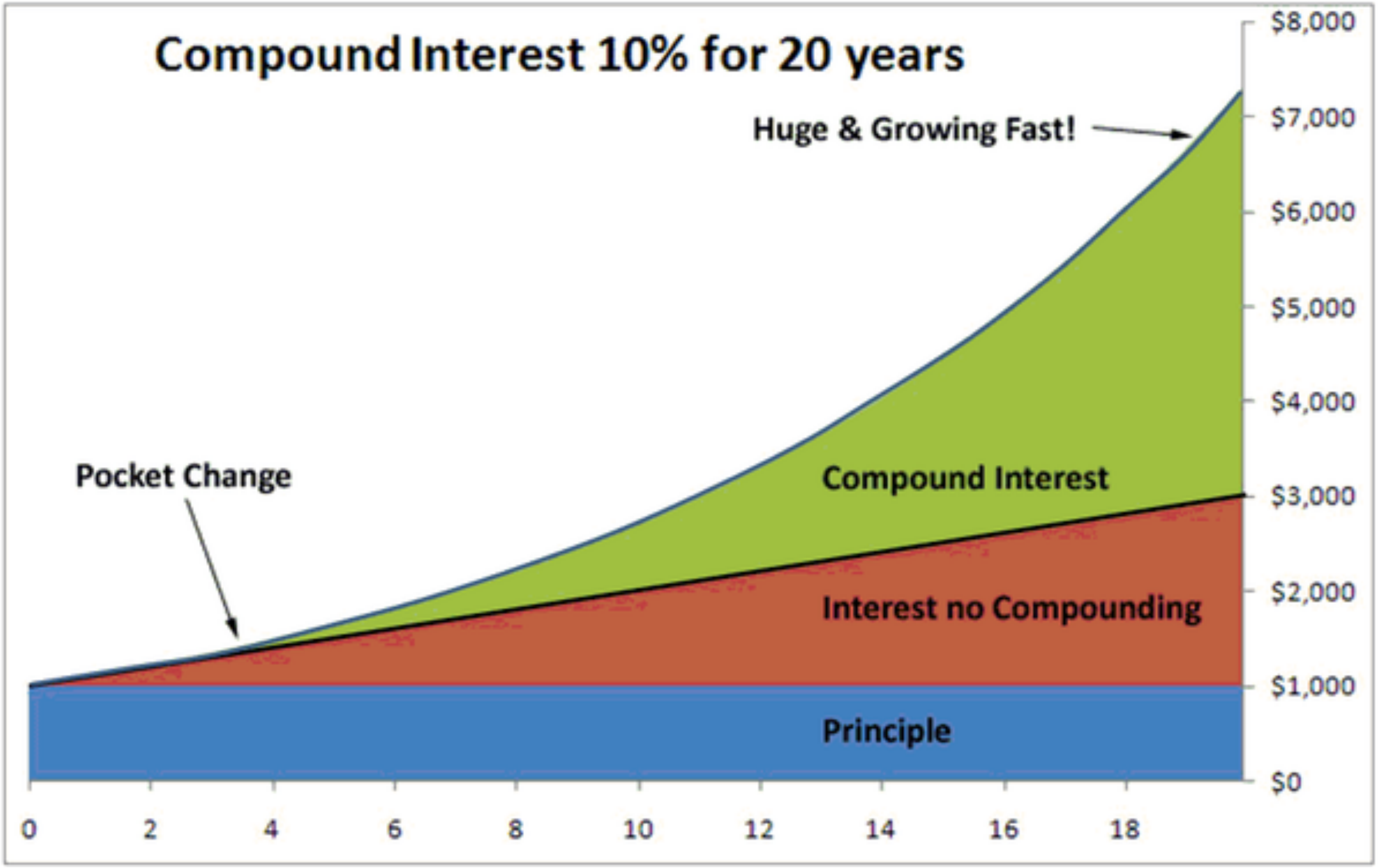
Methods of Calculating Interest (i)

- Compound Interest

End of Period	(A) Amount Owed	(B) Interest for Next Period	(C) = (A) + (B) Amount Owed for Next Period	
0	P	Pi	P+Pi	= P(1+i)
1	P(1+i)	P(1+i) i	P(1+i) + P(1+i)i	= P(1+i)^2
2	P(1+i)^2	P(1+i)^2 i	P(1+i)^2 + P(1+i)^2 i	= P(1+i)^3
3	P(1+i)^3	P(1+i)^3 i	P(1+i)^3 + P(1+i)^3 i	= P(1+i)^4
.				
.				
n-1	P(1+i)$^{n-1}$	P(1+i)$^{n-1}$ i	P(1+i)$^{n-1}$ + P(1+i)$^{n-1}$ I	= P(1+i)^n
n	P(1+i)^n			

- Compound interest (I_c): $I_c = P(1+i)^n - P$

Methods of Calculating Interest (i)



Source: https://goo.gl/images/S7cvRi

Interest Rates

- Interest rates are stated for some period, like a year, while the computation of interest is based on shorter compounding sub-periods such as months.
  - 12% per year interest compounded yearly? Or 1% per month interest compounded monthly?

- Nominal Interest Rate:
  - Is the conventional method of stating the annual interest rate.
  - It is calculated by multiplying the interest rate per compounding period by the number of compounding periods per year.
  - Suppose the nominal interest rate is r and the time period is divided into m equal sub-periods, then the interest rate for each sub-period is: $i_s = r/m$
  - Example: Nominal interest rate of 18% per year, compounded monthly, implies: $i_{s,monthly} = \frac{r}{m} = \frac{18\%}{12} = 1.5\%$

Interest Rates

- Effective Interest Rate:
  - Is the actual but not usually stated interest rate.
  - It is determined by converting a given interest rate with an arbitrary compounding period (normally less than a year) to an equivalent interest rate with a one-year compounding period.
  - The compound interest every sub-period will be:

$$F = P(1 + i_s)^m$$

  - The effective interest rate, i_e, that yields the same future amount F at the end of the full period from the present amount P:

$$P(1 + i_s)^m = P(1 + i_e)$$

$$(1 + i_s)^m = (1 + i_e)$$

$$i_e = (1 + i_s)^m - 1$$

  - To convert from a nominal rate r to an annual effective rate:

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Interest Rates

- Example A
- A local bank announces that a deposit over $1,000 will receive a monthly interest of 0.5%. If you leave $10,000 in this account, how much would you have at the end of one year?
- $F = P(1+i)^n = 10{,}000 * (1+0.005)^{12} = 10{,}000 * (1.062) = 10{,}620$
- This means that over a one-year period, $620 has been added to our money, which is the same as a 6.2% annual interest rate.
- We can see that this is not 12 times the monthly interest rate of 0.5% which is 6%. The difference between the 6.2% and 6% rates is the result of compounding monthly rather than annually.

Interest Rates

- Example B
- The annual interest rate is 6%, and the interest is compounded quarterly. What is the quarterly nominal interest rate? What is the effective annual interest rate if compounded quarterly and monthly?
- Nominal Interest Rate:
- $i_{s,quaterly} = \frac{r}{m} = \frac{6\%}{4} = 1.5\%$ $\quad\quad i_{s,monthly} = \frac{r}{m} = \frac{6\%}{12} = 0.5\%$
- Effective Interest Rate, quarterly:
- $i_e = \left(1 + i_{s,Q}\right)^m - 1 = (1 + 0.015)^4 - 1 = 0.0613 = 6.13\%$
- Effective Interest Rate, monthly:
- $i_e = \left(1 + i_{s,M}\right)^m - 1 = (1 + 0.005)^{12} - 1 = 0.0617 = 6.17\%$

Interest Rates

- Example C
- A Credit Card company charges a nominal 24% interest on overdue accounts, compounded daily? What is the effective interest rate?
- Assuming 365 days per year:
- $i_{s,daily} = \frac{r}{m} = \frac{0.24}{365} = 0.0006575$
- The effective interest rate (per year):
- $i_e = \left(1 + i_{s,D}\right)^m - 1 = (1 + 0.0006575\ )^{365} - 1 = 0.271 = 27.1\%$

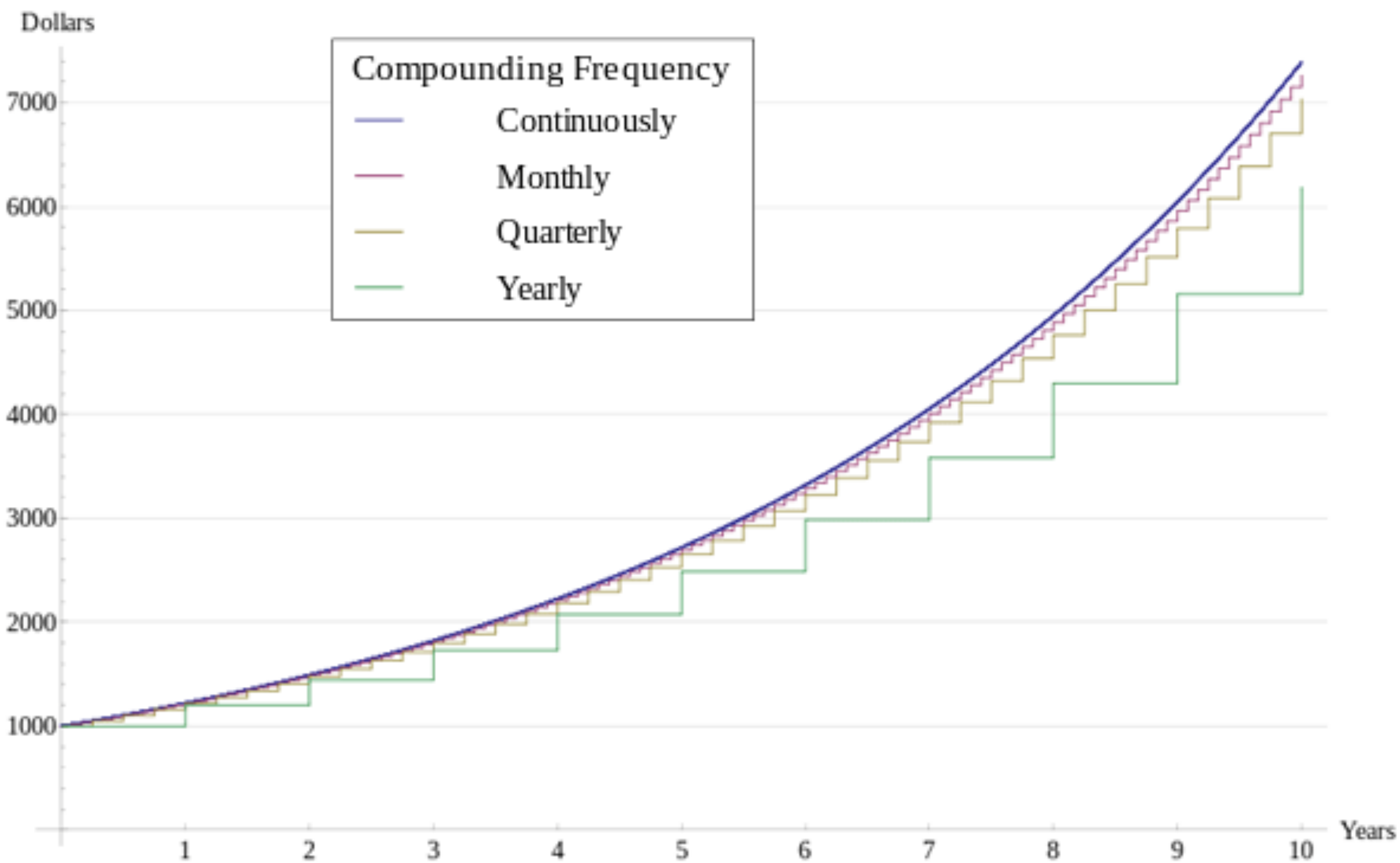
Interest Rates

- Continuous Compounding:
  - Compounding can be done yearly, quarterly, monthly, daily, or even on smaller time sub-periods.
  - If the period is made infinitesimally small, the interest will be compounded continuously.
  - Continuously compounding equation:

$$i_e = \lim_{m\to\infty}\left(1+\frac{r}{m}\right)^m - 1 \quad \rightarrow \quad \lim_{m\to\infty}\left(1+\frac{r}{m}\right)^m = e^r \quad \rightarrow \quad i_e = e^r - 1$$

- Example D
- An investment in a new stock is expected to return a nominal interest rate of 40%, compounded continuously. What is the effective interest rate earned by this stock?
- $i_e = e^r - 1 = e^{0.4} - 1 = 1.492 - 1 = 0.492 \; or \; 49.2\%$

Methods of Calculating Interest (i)



Source: https://goo.gl/images/YyMxde

Cash Flow Diagram

- The graphic presentation of the costs and benefits over the time.
- This is the time profile of all the costs and benefits. It is a presentation of what costs have to be incurred and what benefits are received at all points in time.
- The following conventions are used:
  - The horizontal axis represents time
  - The vertical axis represents costs and benefits
  - Costs are shown by downward arrows
  - Benefits are shown by upward arrows
  - All the benefits and/or costs incurred during a period are assumed to have been incurred at the end of that period. Since the period is normally a year, this is called the ''end of the year" rule.

Cash Flow Diagram

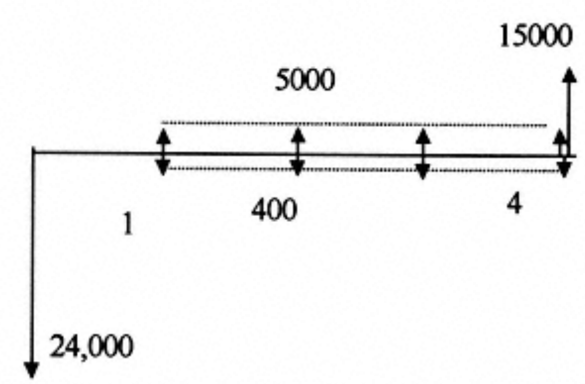
- Example C
- A car leasing company buys a car from a wholesaler for \$24,000 and leases it to a customer for four years at \$5,000 per year. Since the maintenance is not included in the lease, the leasing company has to spend \$400 per year in servicing the car. At the end of the four years, the leasing company takes back the car and sells it to a secondhand car dealer for \$15,000.

Cash Flow Diagram

- Example C
- Step 1:
  - Draw the horizontal axis to represent 1,2,3, and 4 years.
- Step 2:
  - At time zero, i.e., the beginning of year 1, the leasing company spends $24,000. Hence, at time zero, on the horizontal axis, a downward arrow represents this number.
- Step 3:
  - At the end of year 1, the company receives $5,000 from his customer. This is represented by an upward arrow at the end of year 1. The customer also spends $400 for maintaining the car; this is represented by a downward arrow.
  - The situations at years 2 and 3 are exactly the same as year 1 and are the presentations on the cash flow diagram exactly as for the first year.

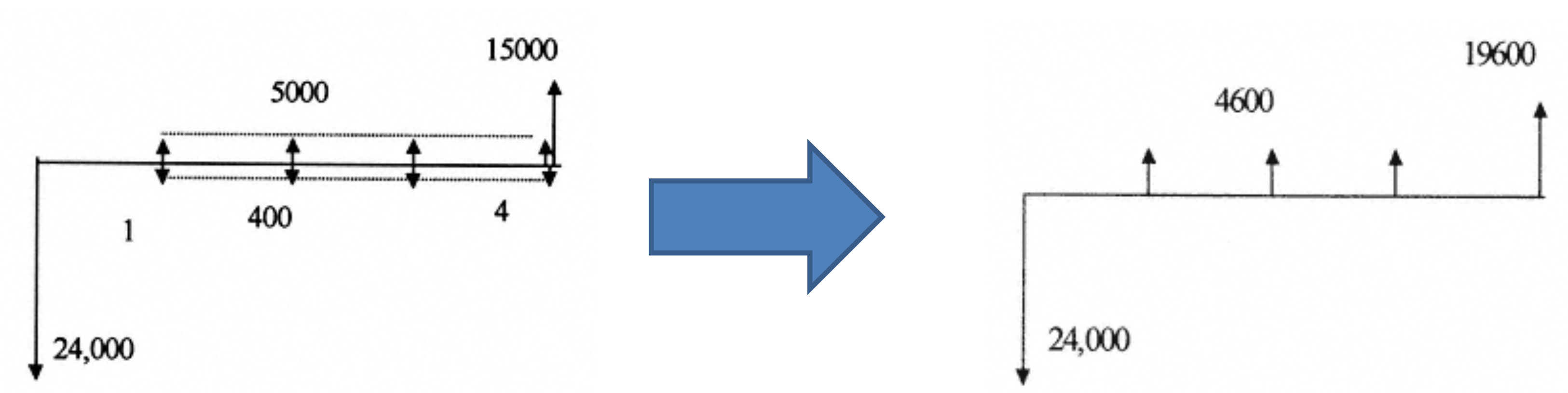
Cash Flow Diagram

- Step 4:
  - At the end of the fourth year, in addition to the income and the expenditure as in the previous years, the leasing company receives $15,000 by selling the car. This additional income is represented by an upward arrow.
  - The project ends at this time, so we have nothing else to insert in the cash flow diagram. We have represented all the costs and benefits in the cash flow.
  - At this point, it is a good idea to go back through the life of the project and make sure that nothing as expressed in the description of the project is left out.
  - Cash flow diagram

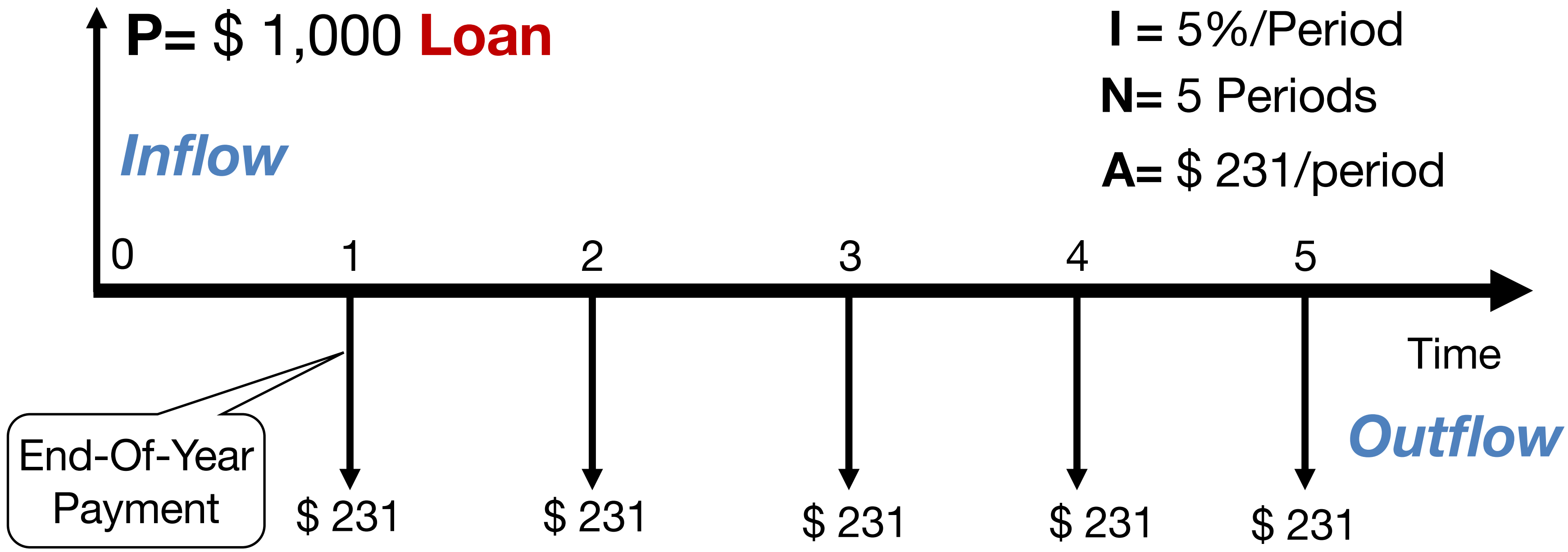


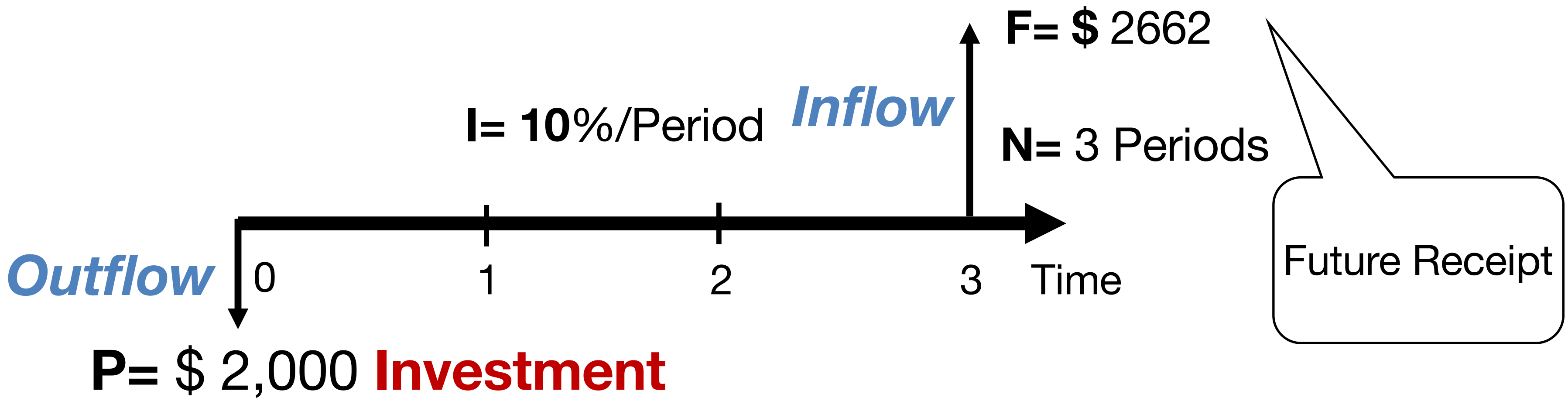
Cash Flow Diagram

- Netted cash flow diagram
  - After the cost and benefits of each year are deducted from each other:



Cash Flow Diagram: Example





Nonmonetary Costs and Benefits

- In order to perform financial analysis, all costs and benefits should be presented in monetary values.
- However, certain benefits are not easily convertible to monetary values (e.g., Beautifulness of façade of an office, Scenic views along a roadway, etc.)
  - The value of anything can be estimated by measuring the cost of not having it.
- Sunk costs: are any costs related to a project incurred before the zero time of the analysis.
- They do not enter in the analysis, however, we try to recover them through the project.
  - We need to make sure that we are not sending good money after bad money.

Equivalence

- Engineering decisions involve costs and benefits that occur at different time.
- Making such decision requires that the costs and benefits at different time to be compared, and this requires us to say certain values at different times are equivalent.
- Equivalence is a condition that exists when the value of a cost at one time is equivalent to the related benefit received at a different time.
- Three concepts of equivalence exist:
  - Mathematical equivalence
  - Decisional equivalence
  - Market equivalence

Equivalence

Mathematical equivalence

- Implies mathematical equivalence based on mathematical relationship.

Decisional equivalence

- Implies the decision maker is indifferent between two alternatives.

Market equivalence

- Implies that there is a market for money that permits cash flows in the future to be exchanged for cash flows in the present, and vice versa.

Equivalence calculations are usually made to compare alternatives:

- They need to have a common time basis.
- Equivalence is dependent on interest rate.
- Equivalence is maintained regardless of anything.

Cash Flow Analysis

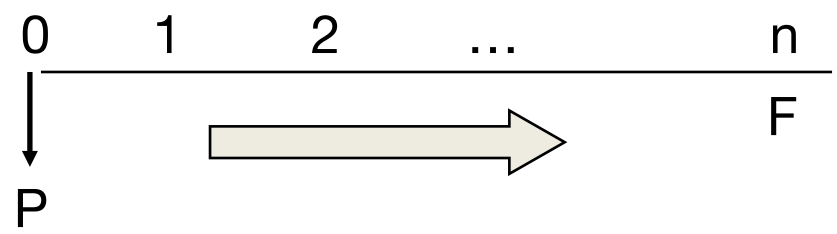
- Interest rate is used to determine whether different patterns of cash flows are equivalent, based on functions that define the mathematical equivalence among the cash flow patterns.
  - $F: Compound\ Interest\ Factors$
- Timing of Cash Flows
  - The actual timing of cash flows can be very complicated and irregular (e.g. Fuel station)
  - Because of the complexity and irregularity of cash flows, simple models are used:
    - Discrete models (DC): Assumes that all cash flows and compounding of cash flows occur at the ends of conventionally defined periods like months or years.
    - Continuous models (CC): Assumes that cash flows and their compounding occur continuously over time.
  - However, both model types are an approximation.

Compounding Interest Factors: DC

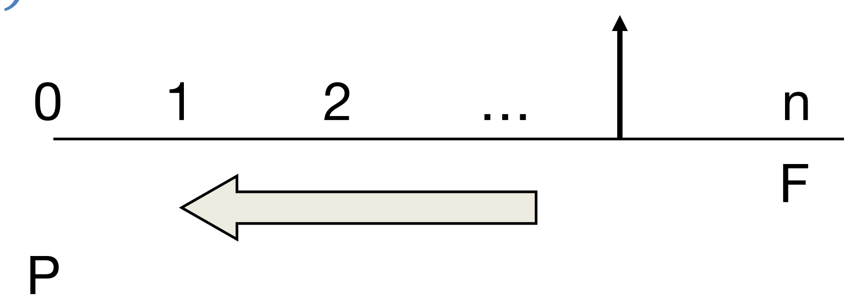
- Compound interest factors (CIF) permit cash flow analysis to be done conveniently using Tables or Spreadsheets
- Four discrete cash flow patterns are examined:
  - Single disbursement or receipt,
  - Annuity: set of equal disbursements or receipts over a sequence of periods,
  - Arithmetic gradient series: set of disbursements or receipts that change by a constant amount from one period to the next in a sequence of periods,
  - Geometric gradient series: set of disbursements or receipts that change by a constant proportion from one period to the next in a sequence of periods
- Principle of discrete compounding:
  - Compounding periods are of equal length
  - Each disbursement and receipt occurs at the end of a period
  - Annuities and gradients coincide with the ends of sequential periods

Single Payment CIF

- Single Payment Compound Amount Factor $F = P \times CIF$
- Compound Amount Factor: $(F/P, i, n)$
  - $F = P\,(1 + i\,)^n = P(F/P, i, N)$
  - $(F/P, i, N) = (1 + i\,)^n$

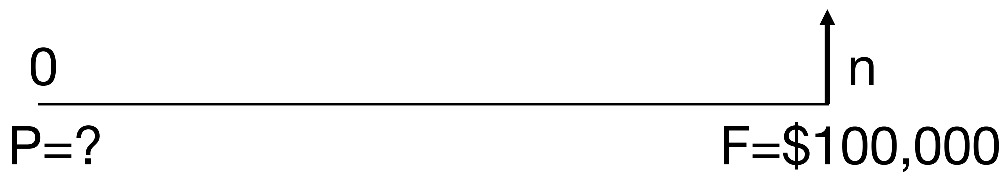


- Present Worth Factor: $(P/F, i, n)$
  - $P = F/\,(1 + i\,)^n = F(P/F, i, N)$
  - $(P/F, i, N) = \dfrac{1}{(1 + i\,)^n}$



Single Sum of Cash Flow

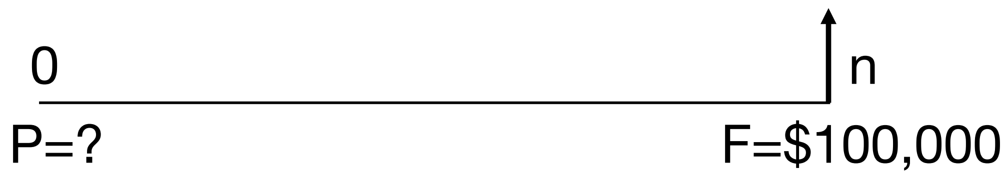
- Example
  - If you wish to have $100,000 at the end of five years in an account that pays 12 percent annually, how much would you need to deposit now?



Single Sum of Cash Flow

- Example

  - If you wish to have \$100,000 at the end of five years in an account that pays 12 percent annually, how much would you need to deposit now?



$$P = F(1+i)^{-n} = F\ (P/F, 12\%, 5)$$

$$= F * 0.5674 \text{ (From Table)}$$

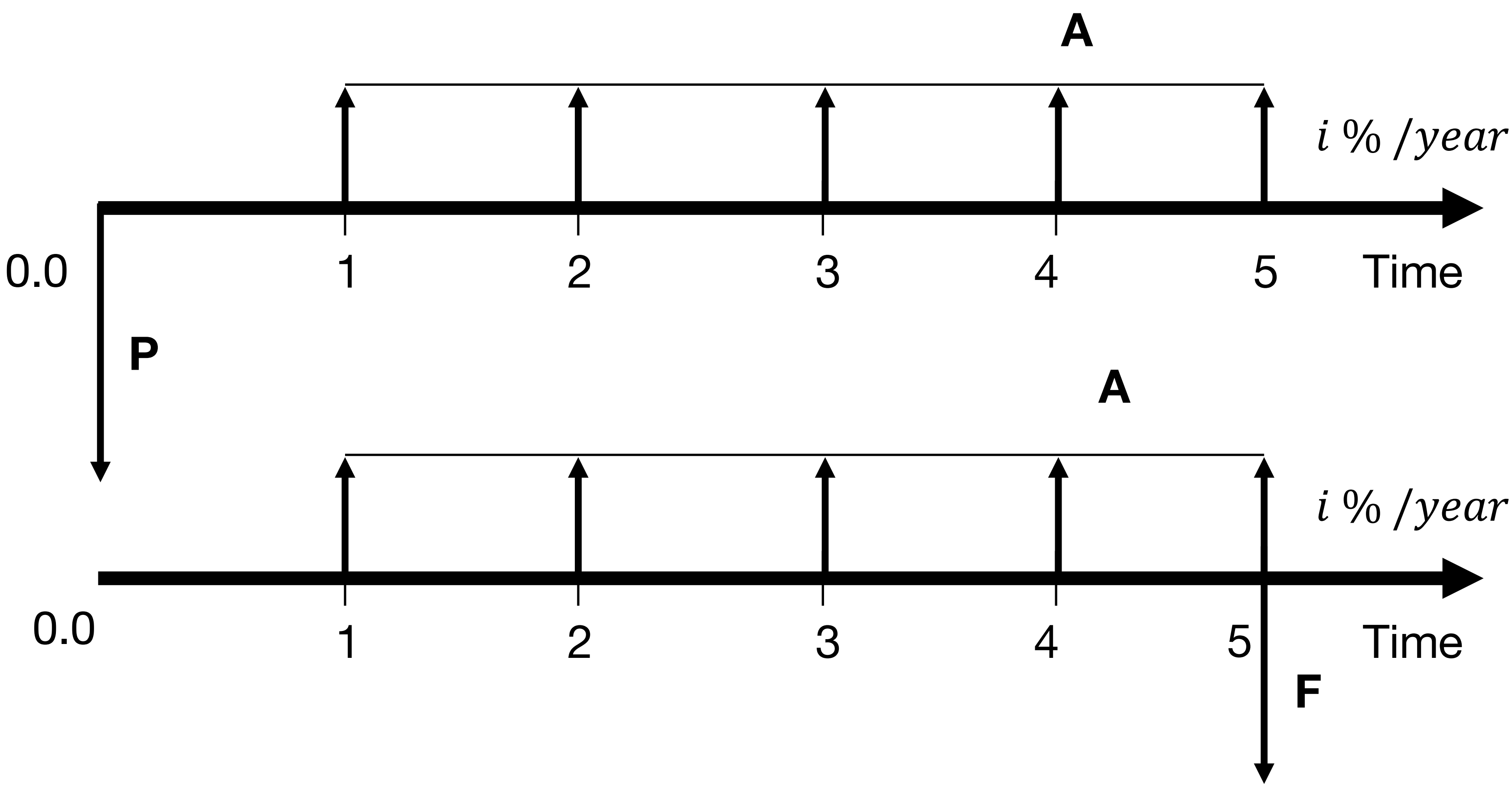
[Compounding Interest Factors.pdf]

$$P = 100{,}000 \times (1+0.12)^{-5}$$

$$= 100{,}000 \times 0.5674 = \$56{,}740$$

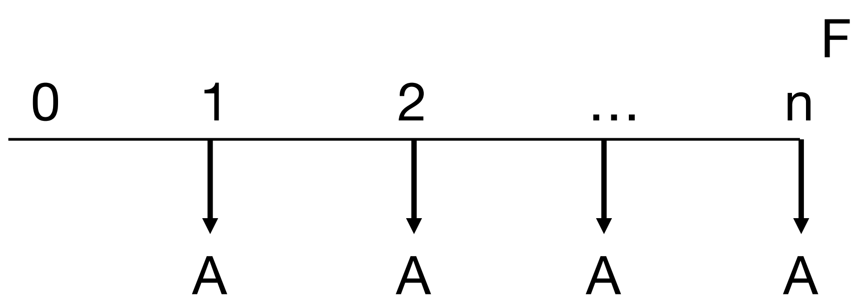
Series of Cash Flows

- Uniform series of cash flows
  - A series of equal cash flows at regular intervals



Uniform Series of Cash Flow

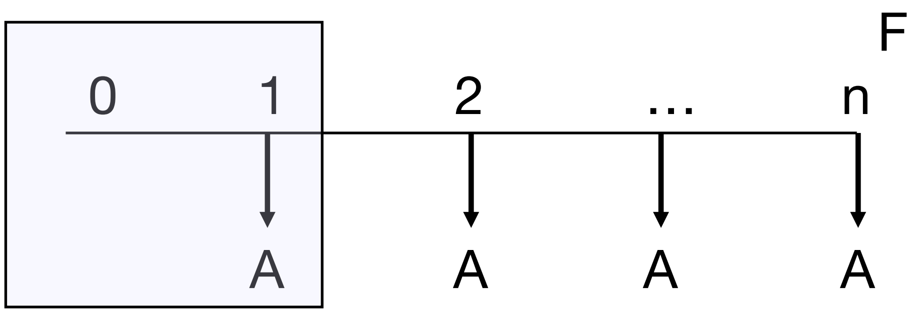
- Uniform Series Compound Amount Factor
  - Factor that will make your annuity value a future value in series payment
  - Uniform Series Compound Amount Factor: $(F/A, i, n)$
  - $(F/A, i, n) = \frac{(1+i)^n - 1}{i}$



Annuity occurs at the end of the interest period

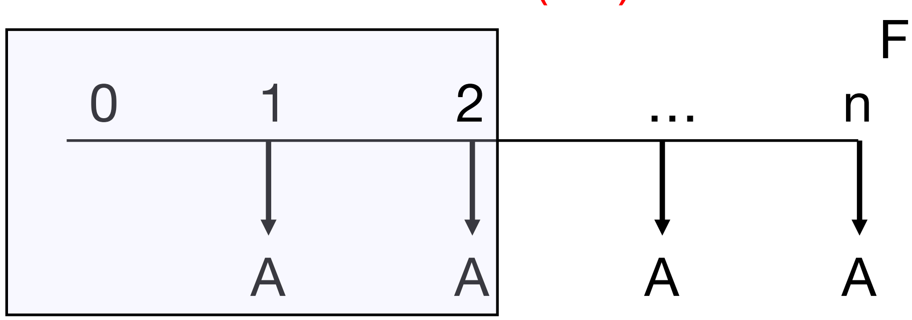
Uniform Series of Cash Flow

$F = A$

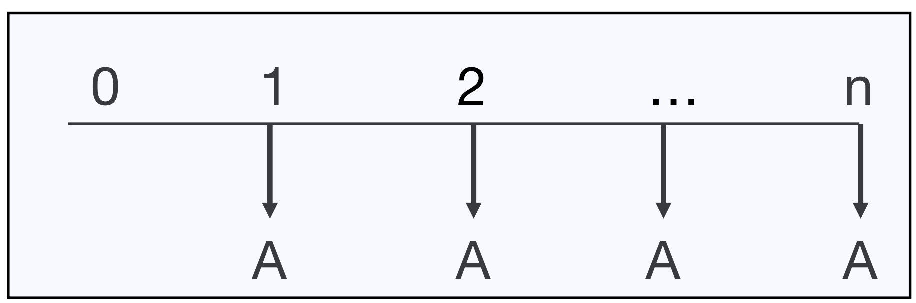


- Uniform Series Compound Amount Factor

$F = A + A(1+i)$

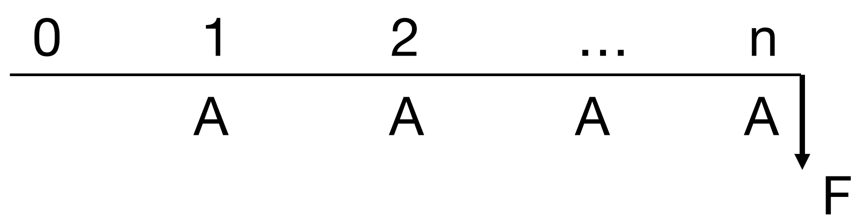


$F = A + A(1+i) + \dots + A(1+i)^{n-1}$



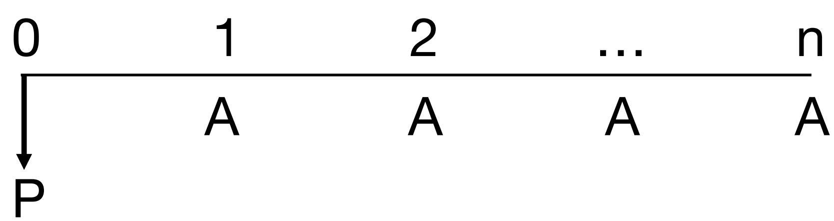
Uniform Series of Cash Flow

- Uniform Series Sinking Fund Factor
  - Factor that will make your future value annuity value in series payment
  - Sinking fund factor: $(A/F, i, n)$
  - $(A/F, i, n) = \frac{i}{(1+i)^n - 1}$



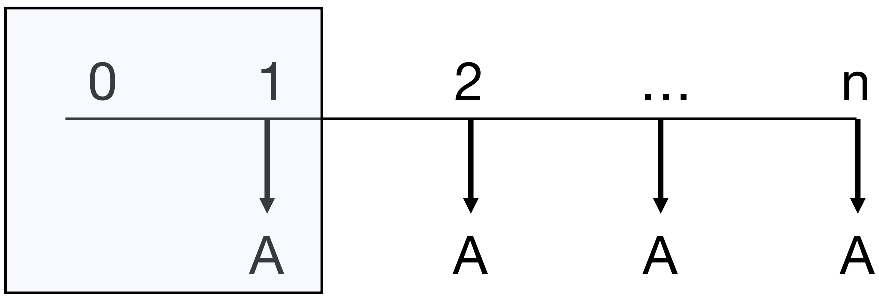
Uniform Series of Cash Flow

- Uniform Series Present Value Factor
  - Factor that will make your annuity value present value in series payment
  - Uniform Series Compound Amount Factor: $(P/A, i, n)$
  - $(P/A, i, n) = \frac{(1+i)^n - 1}{i(1+i)^n}$



Uniform Series of Cash Flow

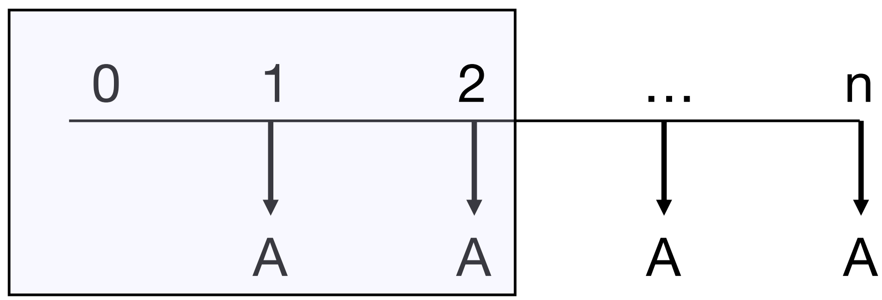
$P = A/\,(1 + i\,)$



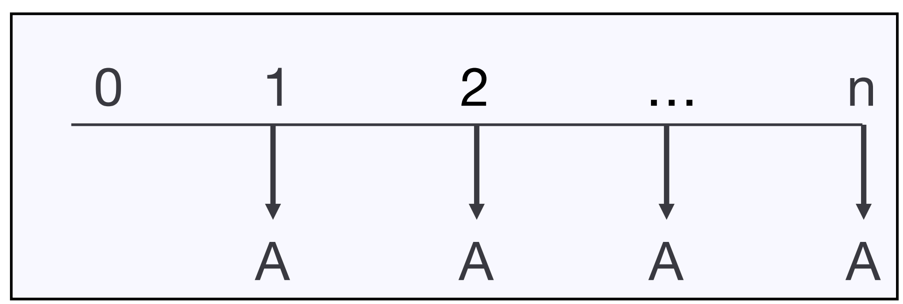
- Uniform Series

  Present Value Factor

$P = A/(1 + i\,) + A/(1 + i\,)^2$



$P = A/(1 + i\,) + A/(1 + i\,)^2 + \ldots + A/(1 + i\,)^n$



Examples

A. A private bank advertise a “10%” loan. You need to borrow $1,000 and the deal you are offered is the following:

- You pay $1,100 in 11 equal $100 amounts, starting one month from today
- In addition, there is a $25 administration fee for loan, payable immediately, and a procession fee of $10 per payment
- There is also a $20 non-optional closing fee to be included in the last payment

Recognizing fees as a form of interest payment, what is the actual effective interest rate?

Examples

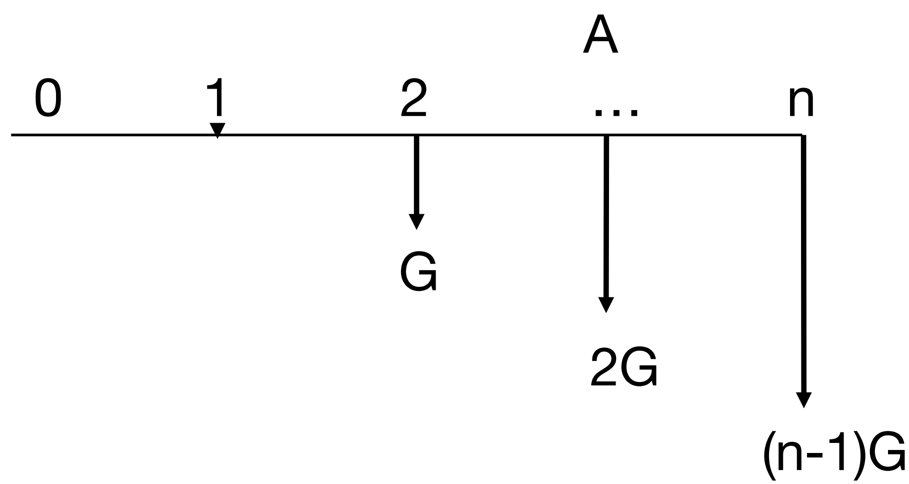
B. A senior accountant wants to retire as soon as she has enough money invested in a special bank account (paying 14% interest, compounded annually) to provide here with an annual income of $25,000. She is able to save $10,000 per year, and the account now holds $5,000. If she just turned 20, and expects to die in 50 years, how old will she be when she retires? There should be no money left when she turns 70.

Example

C. A building is offered for sale in Bahir Dar with a 15 year mortgage rate at 40% compounded annually, and 20% down payment. Annual payments are to be made. The initial cost of the building is 5 million Birr. What yearly payment is required?

Arithmetic Gradient Series of Cash Flow

- Arithmetic Gradient Series Compound Amount Factor
  - A series of disbursements or receipts that starts at zero at end of first period and then increases at a constant amount from period to period.
  - Arithmetic Gradient to Annuity Conversion Factor: $(A/G, i, n)$
  - $(A/G, i, n) = \frac{1}{i} - \frac{n}{(1+i)^n - 1}$



The sum of annuity plus an arithmetic gradient pattern can be used for analysis.

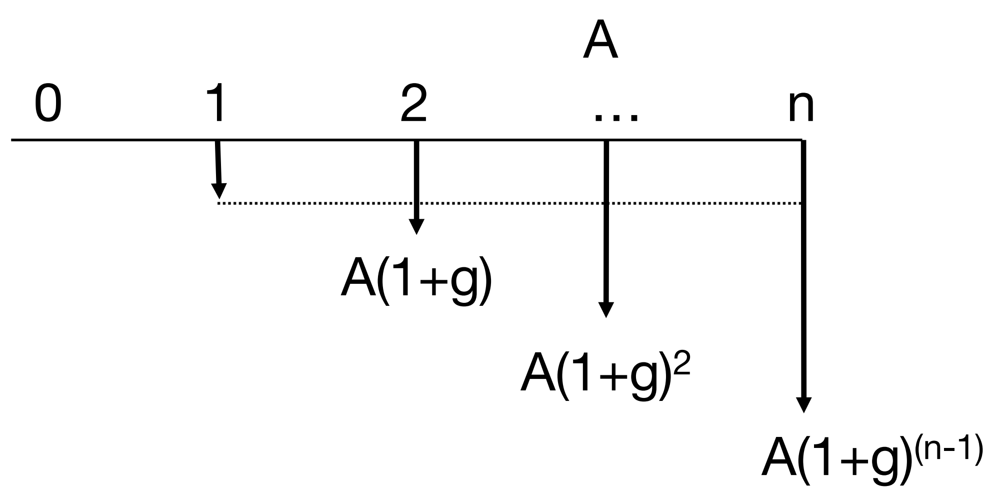
Examples

D. The benefits of a revised production schedule for a seasonal manufacturer will not be realized until the peak summer months. Net savings will be $1100, $1200, $1300, $1400, and $1500 at the end of months 5, 6, 7, 8, and 9, respectively. It is now the beginning of the month 1. Assume 365 days per year, 30 days per month. What is the present worth if the savings if nominal interest is:

- (a) 12% per year, compounded monthly?
- (b) 12% per year, compounded daily?

Geometric Gradient Series of Cash Flow

- Geometric Gradient Series Compound Amount Factor
  - A series of disbursements or receipts that increase or decrease by a constant percentage each period.
    - Inflation or deflation, Productivity Improvement, Market Size
  - The base value of the series is A and the ”growth” rate of the series (either increase or decrease) is G



Geometric Gradient Series of Cash Flow

- Geometric Gradient Series Compound Amount Factor
  - Geometric Gradient to Present Worth Conversion Factor: $(P/A, g, i, n)$
  - $P = \frac{A}{1+i} + \frac{A\,(1+g)}{(1+i)^2} + \cdots + \frac{A\,(1+g)^{n-1}}{(1+i)^n}$
  - Let $i^o = \frac{1+i}{1+g} - 1 \rightarrow \frac{1}{1+i^o} = \frac{1+g}{1+i}$
  - $(P/A, g, i, N) = \frac{(P/A, g, i^o, N)}{1+g} = \left(\frac{(1+i^o)^{n-1}}{i^o(1+i^o)^n}\right)\frac{1}{1+g}$

Cases	Use
$i > g > 0: \rightarrow i^o > 0$	Tables or Functions
$g > i > 0: \rightarrow i^o < 0$	Formula
$g = i > 0: \rightarrow i^o = 0$	$P = \mathrm{N}(\frac{A}{1+g})$
$g < 0$: Growth is negative	Tables or Functions

Examples

E. It is January 1 of this year. You are starting your new job tomorrow, having just finished your engineering degree at the end of the last term. Your take-home pay for this year will be \$36,000. It will be paid to you in equal amounts at the end of each month, starting at the of January. There is a cost of living clause in your contract that says that each subsequent January you will get an increase of 3% in your yearly salary. In addition to your salary, your family sends you a \$2,000 birthday present at the end of each June.

If you have decided to save 10% of your monthly salary and 50% of your birthday gift for your retirement and interest is 1% per month, how much will you save at the end of 5 years.

Present Worth Computations ($n \to \infty$)

- For long life projects, cash flows can be treated as continued indefinitely.
  - Capitalized Value or Present Worth of an infinitely long uniform series cash flow: $(P/A, i, n)$

  - $P = \lim_{n\to\infty} A(P/A, i, n)$

$$= A \lim_{n\to\infty} \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= A \lim_{n\to\infty} \left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$

$$= \frac{A}{i}$$

Examples

F. If a town is considering building a by-pass for truck traffic around the downtown commercial area. The by-pass will provide merchants and shoppers with benefits with estimated value of $500,000 per year. Maintenance costs will be $125,000 per year. If the by-pass is properly maintained, it will provide benefits for a very long time. If the interest rate is 10%, what is the present worth of benefits minus maintenance costs?

Continuous Compounding and Continuous Cash Flow

- Two forms of continuous modeling:
  - Discrete cash flows with continuous compounding
  - Continuous cash flows with continuous compounding
- Discrete cash flows with continuous compounding: The formulas are obtained by substituting the effective continuous compounding for the effective rate with discrete compounding: $i_e = e^r - 1 \rightarrow (1 + i_e)^n = e^{rn}$
- Compound interest factors are obtained by substituting $e^r - 1$ for i
- $(P/A, i, n) = \frac{(1+i)^n - 1}{i(1+i)^n} \rightarrow (P/A, i, n) = \frac{e^{rn} - 1}{(e^r - 1)e^{rn}}$

Continuous Compounding and Continuous Cash Flow

- Continuous cash flows with continuous compounding
  - Derived using integral calculus

  - Continuous uniform series compound amount factors: denoted by $(P/\bar{A}, r, T)$
  - $(P/\bar{A}, r, T) = \frac{e^{rT}-1}{re^{rT}}$

  - Continuous sinking funding factor:
  - $(F/\bar{A}, r, T) = \frac{e^{rT}-1}{r}$

Examples

G. Saving from a new widget grinder are estimated to be \$10,000 per year. The grinder will last 20 years and will have no scrap values at the end of that time. Assume that the savings are generated as a continuous flow. The effective interest rate is 15% compounded continuously. What is the present worth of the savings?

References:

- Fraser, N.M., Jewkes, E., Bernhardt, I., Tajima, M. (2006). *Engineering Economics in Canada*. 3rd edition, Prentice Hall.
- *CIV E 608: Construction Equipment, Lecture Notes, Sang,* University of Alberta, 2015.