CENG 6101 Project Management

Schedule Crashing and Time-Cost Tradeoff

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Crash vs. Normal Durations and Costs

- Normal duration of activity = based on resources normally available
- Normal cost = direct cost associated with normal duration (bid value)
- Crashing = to complete activity in less time than scheduled by employing additional crews, equipment, etc.

Crash vs. Normal Durations and Costs

- Crash duration = time to complete activity with extra funds or resources
- Crash cost = direct cost of completing activity by its crash duration
- Crash project duration = combination of crash and normal durations of activities
- Crash project cost = total cost associated with crash project duration

Cost-Time Relationships

- Cost slope

  = (crash cost – normal cost) /

  (normal duration – crash duration)

  = amount of funds required to reduce duration of activity by one day

Steps for Deriving Cost-Time Relationships

1. Select methods by which activity can be performed
2. Determine duration and direct cost of each
3. Plot results on graph of duration vs. direct cost
4. Connect points from right to left with straight lines sloping upwards to left

Linear Relationship

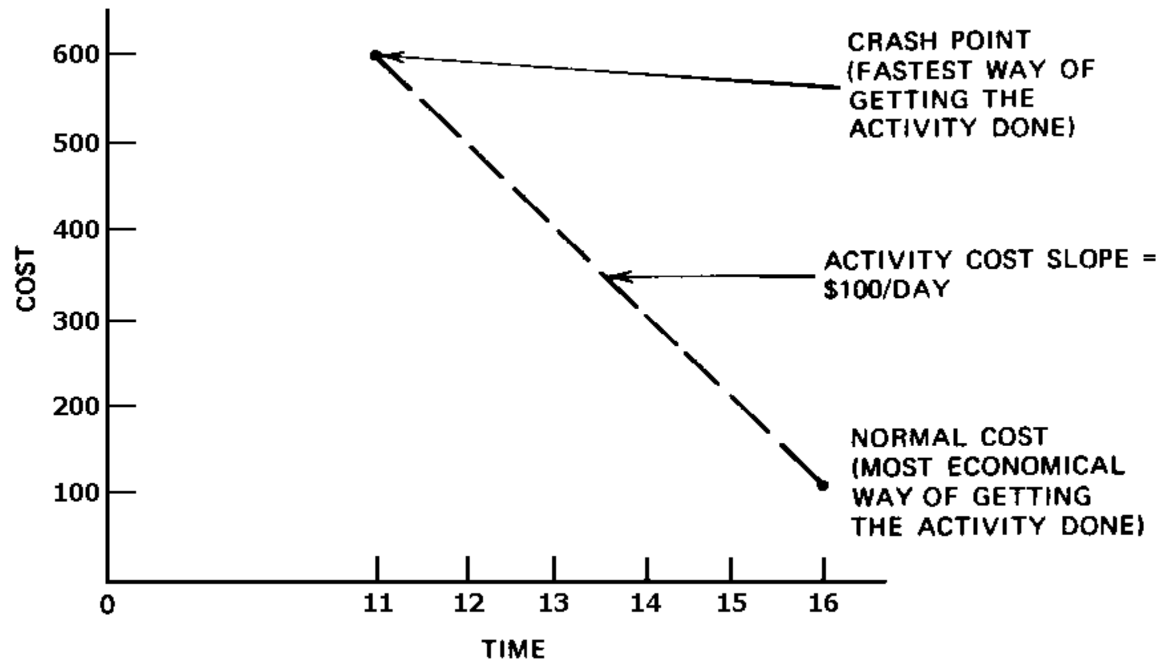


FIGURE 10.1 Linear Relationship between Time and Cost

Example: overtime work results in savings in time.

Cost slope = (600-100)/(16-11) = \$100/day

Multi-linear Relationship

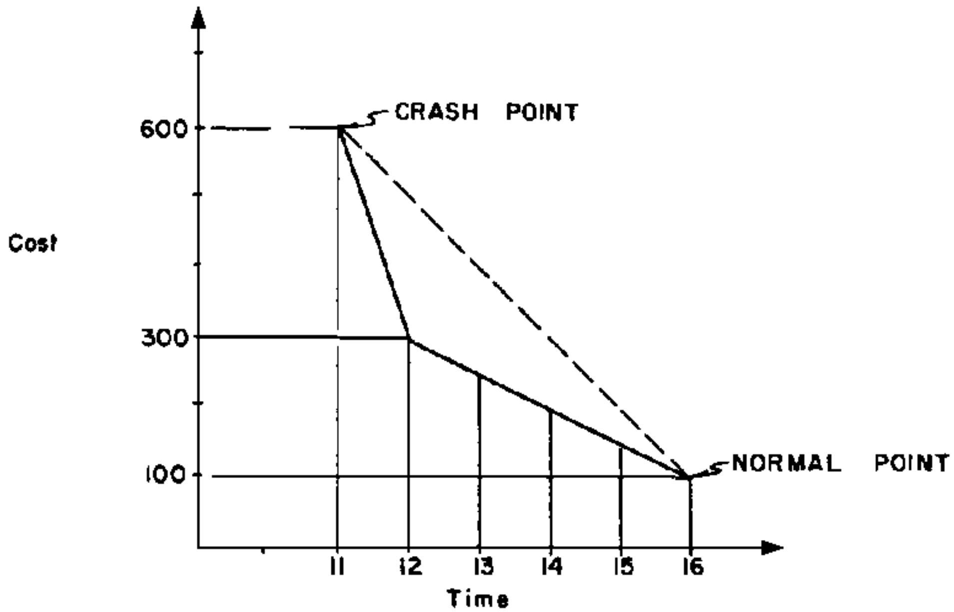


FIGURE 10.2 Multilinear Cost-Time Relationship

Two cost slopes:

Day 16-12: (300-100)/4 = $50/day

Day 12-11: (600-300)/1 = $300/day

Linear relationship for different time intervals.

16-12 days: use loaders of different capacities.

Beyond day 12: use two loaders – extra mobilization cost.

Discrete Relationship

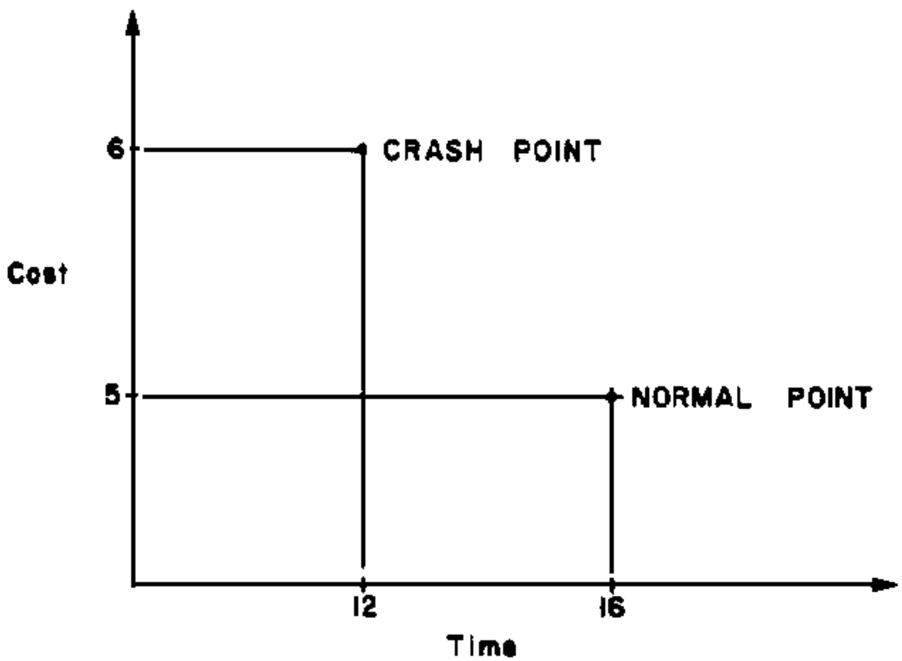
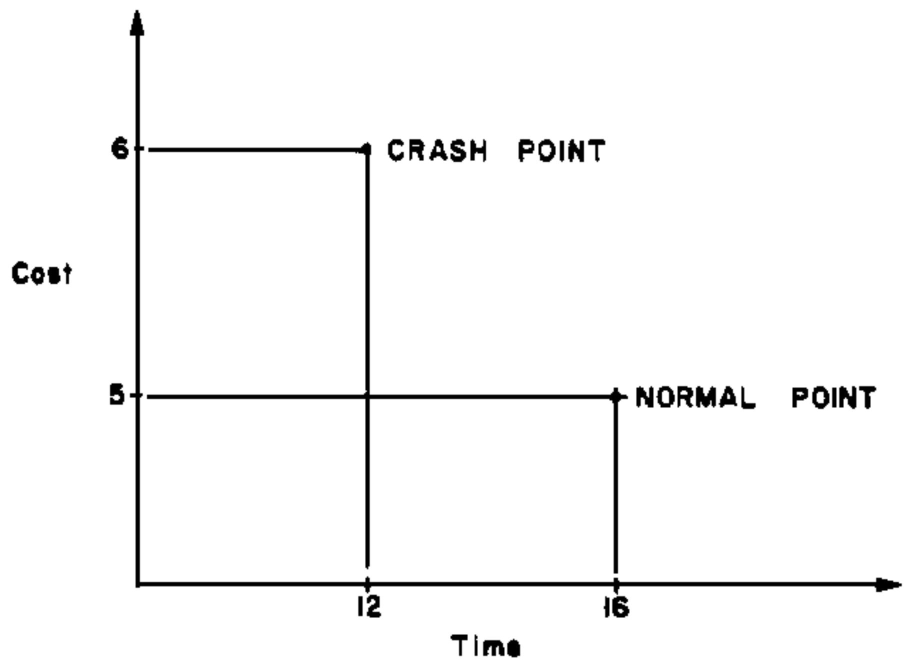


FIGURE 10.3 Discrete Cost-Time Relationship

Only 2 costs: normal cost for normal time, or crash cost for crash time; no relationship between normal and crash costs.

Tunneling project:
Drill jumbo: 16 months, $5 million
Tunnel boring machine: 12 months, $6 million

Discrete Relationship



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Curvilinear Continuous Relationship

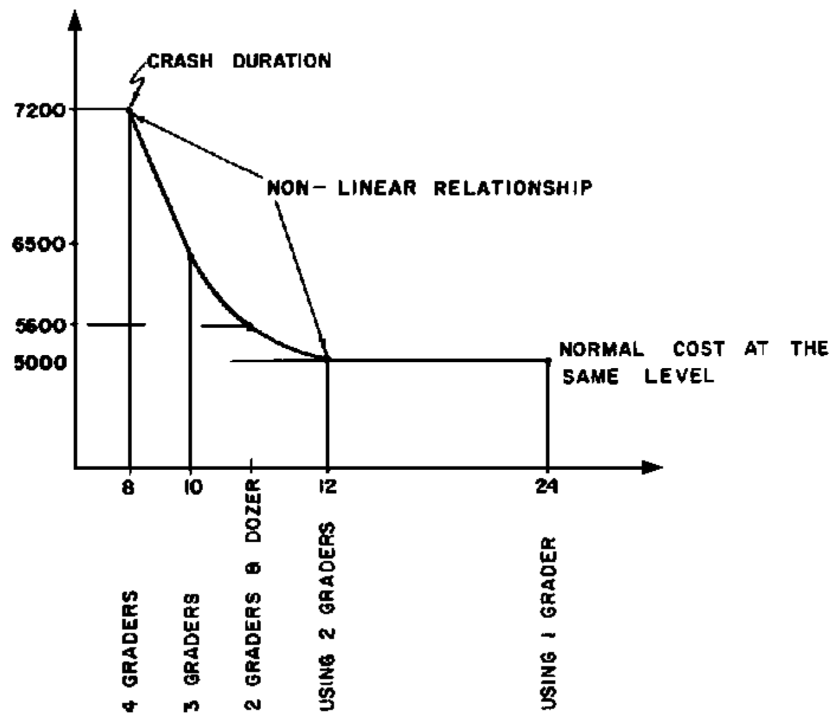

FIGURE 10.4 Curvilinear Continuous Cost-Time Relationship

Dike construction project: Use of graders and dozers of different capacities.

Continuous curve represents relationship between different points of crash costs. Curvilinear more accurate than linear but need historical data to fit curve. Linear relationship means durations and costs in between two points can be achieved.

Crash vs. Normal Durations and Costs

- Contractors’ normal and crash costs will vary and depend on: construction methods, workers’ experience, availability of equipment
- Crashing involves expediting materials, increasing labour and/or equipment
- Additional cost due to mobilization of additional equipment; overtime work; reduced productivity from shiftwork, overtime, or congested work space

Crashing Example

TABLE 10.1 Data for Example 10.1

Activity	Normal Duration	Crash Duration	Normal Cost	Crash Cost
A	8	4	$ 500.00	$ 800.00
B	4	2	300.00	400.00
C	3	2	300.00	500.00
D	6	3	800.00	1000.00
			$ 1900.00	$ 2700.00

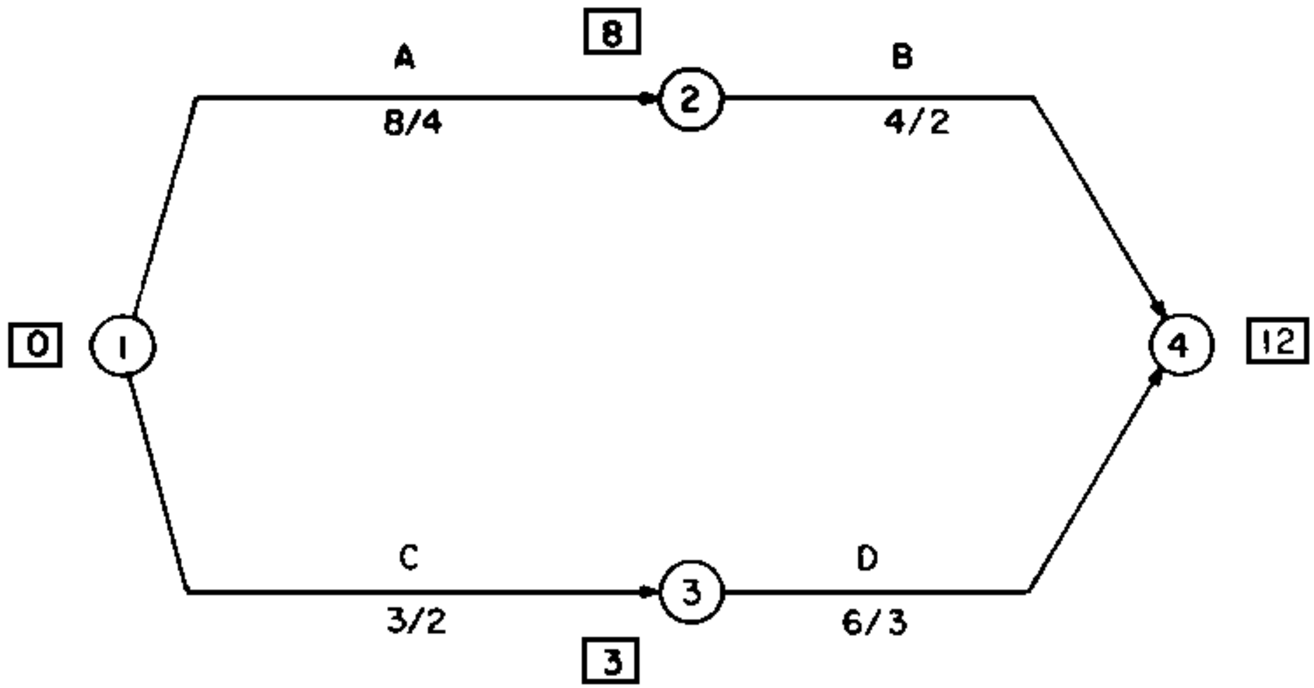

FIGURE 10.5 Example 10.1 Network

Normal duration and cost: 12 days, $1900

If crash all activities,
Crash duration and cost: 6 days, $2700

Question: Is it necessary to crash all activities to achieve crash duration?

Crashing Example

TABLE 10.2 Example 10.1 – Cost Slopes

Activity	Normal Duration	Crash Duration	Slope
A	8	4	75.00
B	4	2	50.00
C	3	2	200.00
D	6	3	66.67

Crashing Example

Need to crash activities A and B (on critical path): $A_{crash} + B_{crash} = 6$ days.

Crash activity D before C, since D has lower cost slope.

D can be crashed by 3 days; $D_{crash} + C_{normal} = 6$ days; no need to crash C.

Crash project duration = 6 days.

Crash project cost = $2500.

Crashing Example

TABLE 10.3 Example 10.1 Selective Crashing

Activity	Reduced by	Additional Cost	Total Cost
A	4	$ 300.00	$ 800.00
B	2	100.00	400.00
C	0	0	300.00
D	3	200.00	1000.00
		$ 600.00	$ 2500.00

Relaxation

- Relaxation is opposite of crashing
- May be necessary to make project more economical (especially on non-critical activities)
- Can relax non-critical activities (e.g., reduce amount of overtime) without increasing project duration
- Results in cost savings, but reduces flexibility during construction

Criticality Theorem

- Helps in selecting which activities to crash, especially when dealing with large project network
- Checks condition where crash duration of critical path is greater than normal duration of non-critical path(s), in which case do not need to consider activities on non-critical path

Criticality Theorem

- C_1 and C_2 are two paths with common end nodes
- Let C_1 be longer path (critical path)
- Question: Is it necessary to consider crashing activities on path C_2 (or just longer path C_1) to arrive at minimum crash cost solution?
- Let path whose activities must be crashed be called path of "maximum duration" for crashing purposes
- Compare C_1 to other paths to determine if it has "maximum duration" for crashing purposes

Criticality Theorem

- Path duration of C_1 is “maximum” for crashing purposes if and only if crash path duration of C_1 is greater than or equal to normal path duration of C_2, in which case do not need to consider activities on path C_2 for crashing
- Let crash duration of path C_1 be a_1, let normal duration of path C_2 be b_2
- If $a_1 >= b_2$, then C_1 is “maximum” for crashing

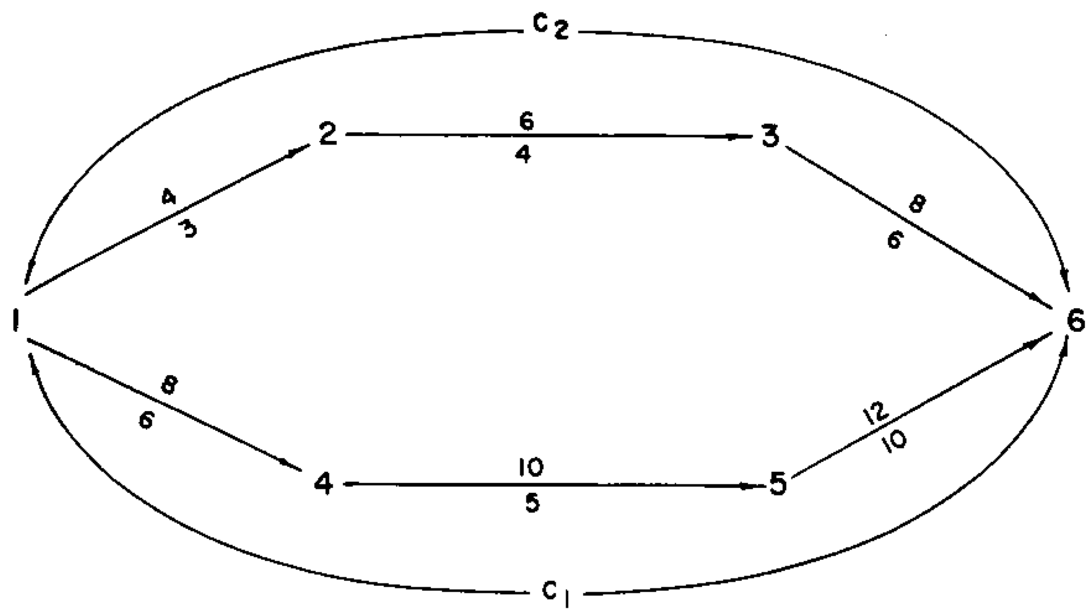

FIGURE 10.6 Crashing Example

Normal duration of C_1 = 30 days; crash duration of $C_1 = a_1$ = 21 days

Normal duration of C_2 = 18 days = b_2

Since 21 >= 18, C_1 is maximum, no need to consider C_2

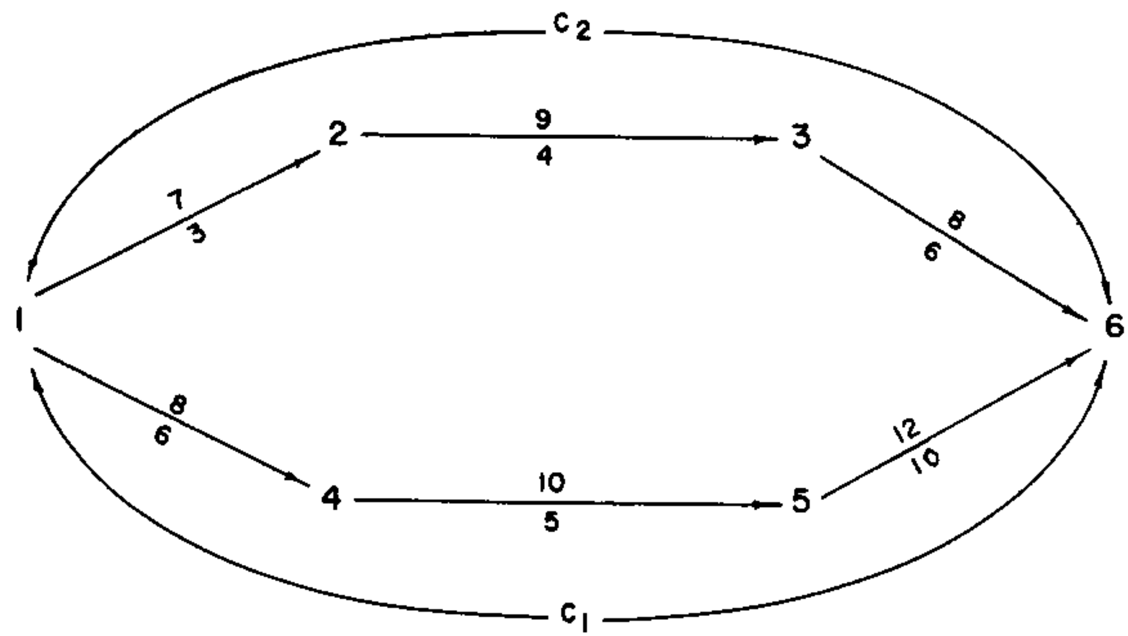

FIGURE 10.7 Another Crashing Example

Normal duration of C_1 = 30 days; crash duration of $C_1 = a_1$ = 21 days

Normal duration of C_2 = 24 days = b_2

Since 21 < 24, C_1 is NOT maximum, need to consider C_2 in combination with C_1

Procedure for Compression

1. Determine normal project duration and normal project cost.
2. Identify normal duration critical path.
3. Eliminate all noncritical activities that do not need to be crashed. This is done by successively comparing normal duration of paths parallel to the critical path to the latter path's crash duration, using the criticality theorem.

Procedure for Compression

4. Tabulate normal and crash durations and normal and crash costs for all the activities.
5. Compute and tabulate the cost slope of each activity from the following formula:

$$\text{Cost slope} = \frac{\text{crash cost} - \text{normal cost}}{\text{normal duration} - \text{crash duration}}$$

6. Proceed to determine the project time cost curve by shortening the critical activities beginning with the activity having the lowest cost slope. Each activity is shortened until (a) its crash time is reached or (b) a new critical path is formed.

Procedure for Compression

7. When a new critical path is formed, shorten the combination of activities having the lowest combined slope. Where several parallel paths exist, it is necessary to shorten each of them simultaneously if the overall project time is to be reduced.
8. At each step check to see whether float time has been introduced in any of the activities. If so, perhaps these activities can be expanded to reduce cost.
9. At each shortening cycle compute the new project cost and duration. Tabulate and plot these points on a time-cost graph.

Procedure for Compression

10. Continue until no further shortening is possible. This is the crash point.
11. Plot the indirect project costs on the same time-cost graph.
12. Add direct and indirect costs to find the total project cost at each duration.
13. Use the total project cost curve to find the optimum time (completion at lowest cost) or the cost of any other desired schedule.

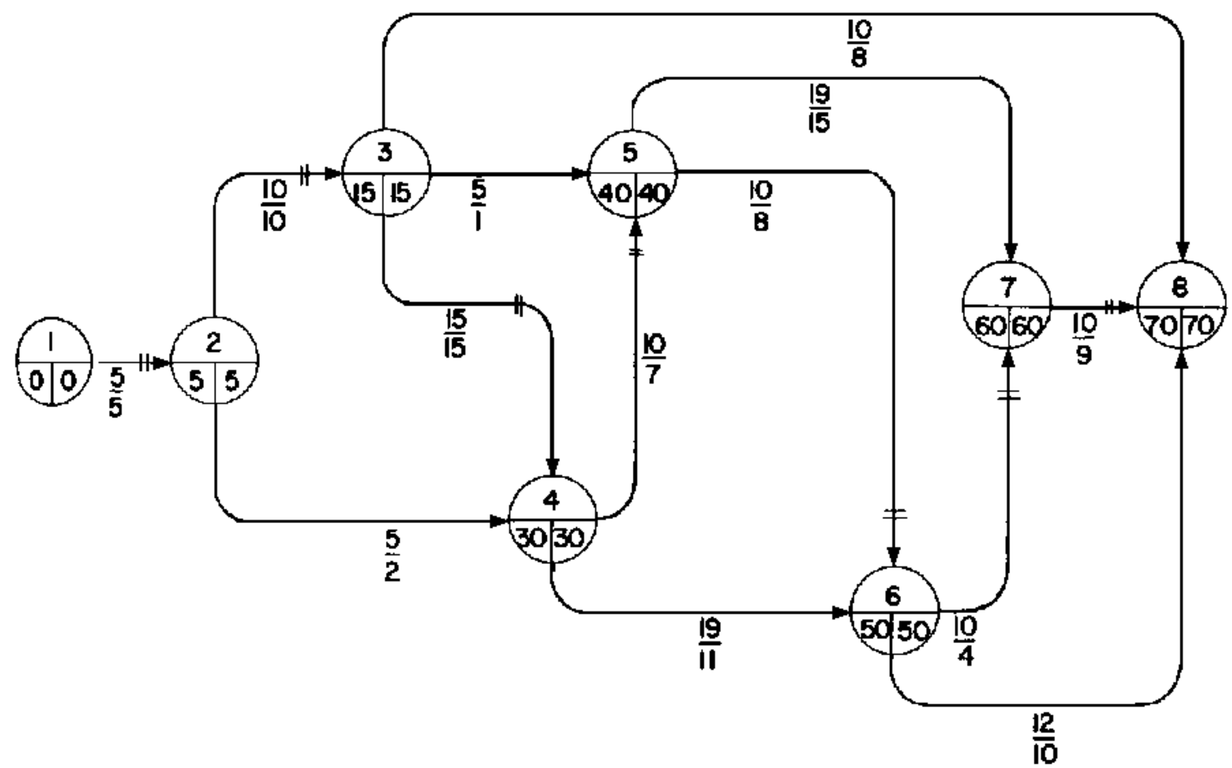

FIGURE 10.8 Network for Example 10.2

Objective: To determine minimum crash cost solution for project, associated with minimum project duration.

Normal duration = 70 days; normal cost = $6600. Critical path shown in double strokes.

Use Criticality Theorem

- Node 1 to 2: Activity 1-2 is critical, but crash duration = normal duration
- Node 2 to 4: 2-3, 3-4 (C_1), 2-4 (C_2): $a_1 = 25 > b_2 = 5$: eliminate 2-4
- Node 3 to 8: 3-4, 4-5, 5-6, 6-7, 7-8 (C_1), 3-8 (C_2): $a_1 = 43 > b_2 = 10$: eliminate 3-8
- Node 3 to 5: 3-4, 4-5 (C_1), 3-5 (C_2): $a_1 = 22 > b_2 = 5$: eliminate 3-5
- Node 4 to 5: keep 4-5 (only option)

Use Criticality Theorem

- Node 4 to 6: 4-5, 5-6 (C_1), 4-6 (C_2): $a_1 = 15 < b_2 = 19$: keep 4-6
- Node 5 to 7: 5-6, 6-7 (C_1), 5-7 (C_2): $a_1 = 12 < b_2 = 19$: keep 5-7
- Node 6 to 8: 6-7, 7-8 (C_1), 6-8 (C_2): $a_1 = 13 > b_2 = 12$: eliminate 6-8
- Node 7 to 8: keep 7-8 (only option)

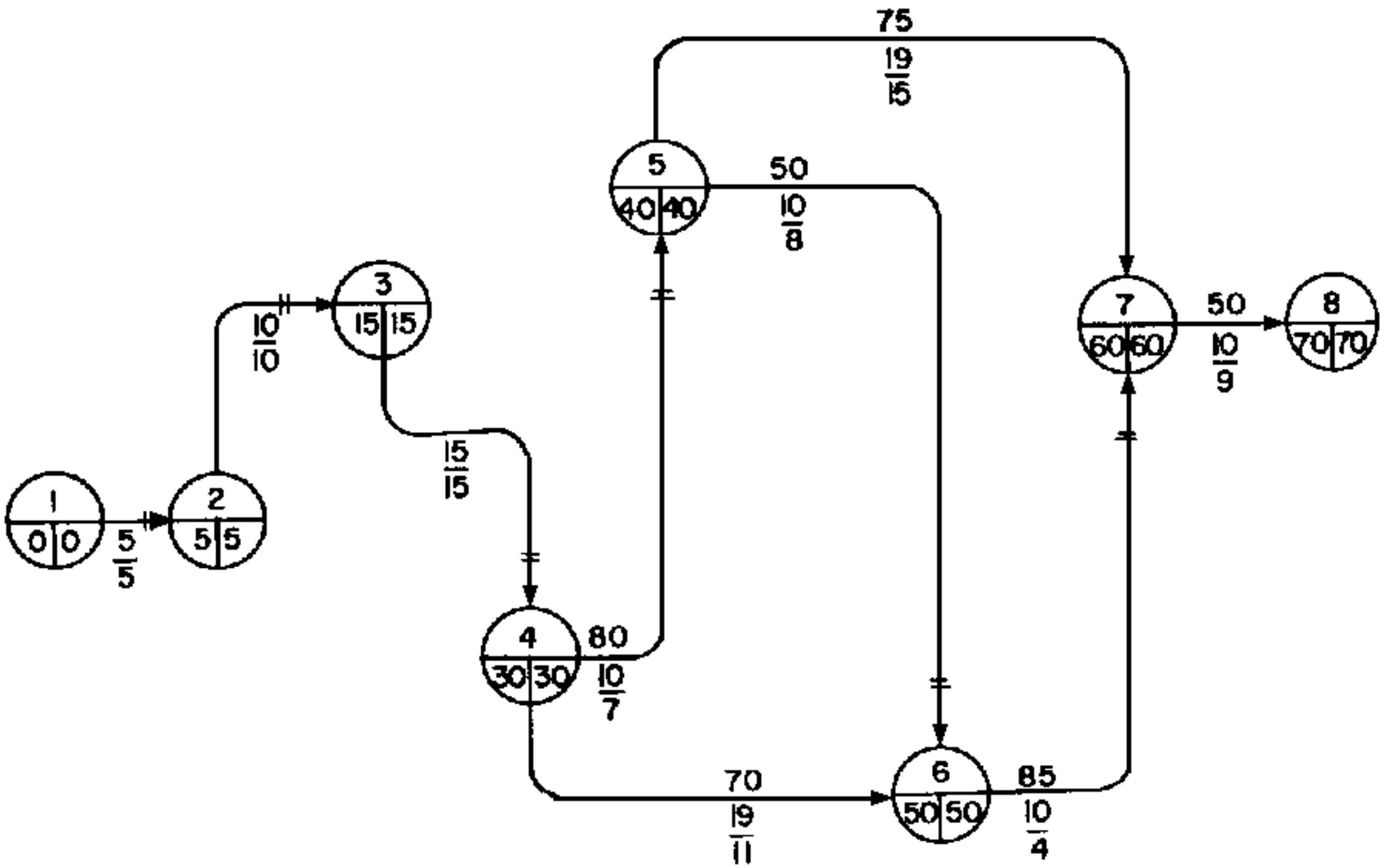

FIGURE 10.9 Skelton Network for Example 10.2

Activities eliminated:

2-4, 3-8, 3-5, 6-8 (keep all critical activities initially)

Time-Cost Tradeoff Steps

- Tabulate normal and crash durations and costs; calculate cost slopes for all activities not eliminated.
- Determine crash duration and crash cost of project by shortening critical activities.
- Begin with critical activity having lowest cost slope.
- Each activity is shortened until either:

  (1) Its crash time is reached, or

  (2) A new critical path is formed.

Time-Cost Tradeoff Example

TABLE 10.4 Example 10.2 Data

Activity	Normal Duration	Crash Duration	Slope
1-2	5	5	NA
2-3	10	10	NA
3-4	15	15	NA
4-5	10	7	80
4-6	19	11	70
5-6	10	8	50
5-7	19	15	75
6-7	10	4	85
7-8	10	9	50

Time-Cost Tradeoff Example

TABLE 10.4 Example 10.2 Data

Project Duration	Activity	Normal Duration	Crash Duration	Number of Days Reduced	Slope	Project Cost Before Crashing	Project Cost After Crashing
70						6600.00	
69	7-8	10	9	1	50		6650.00
68	5-6	10	8	1	50		6700.00
65	4-5	10	7	3	80		7150.00
	4-6	19	11	3	70		
61	5-7	19	15	4	75		7790.00
	6-7	10	4	4	85		

Time-Cost Tradeoff Example

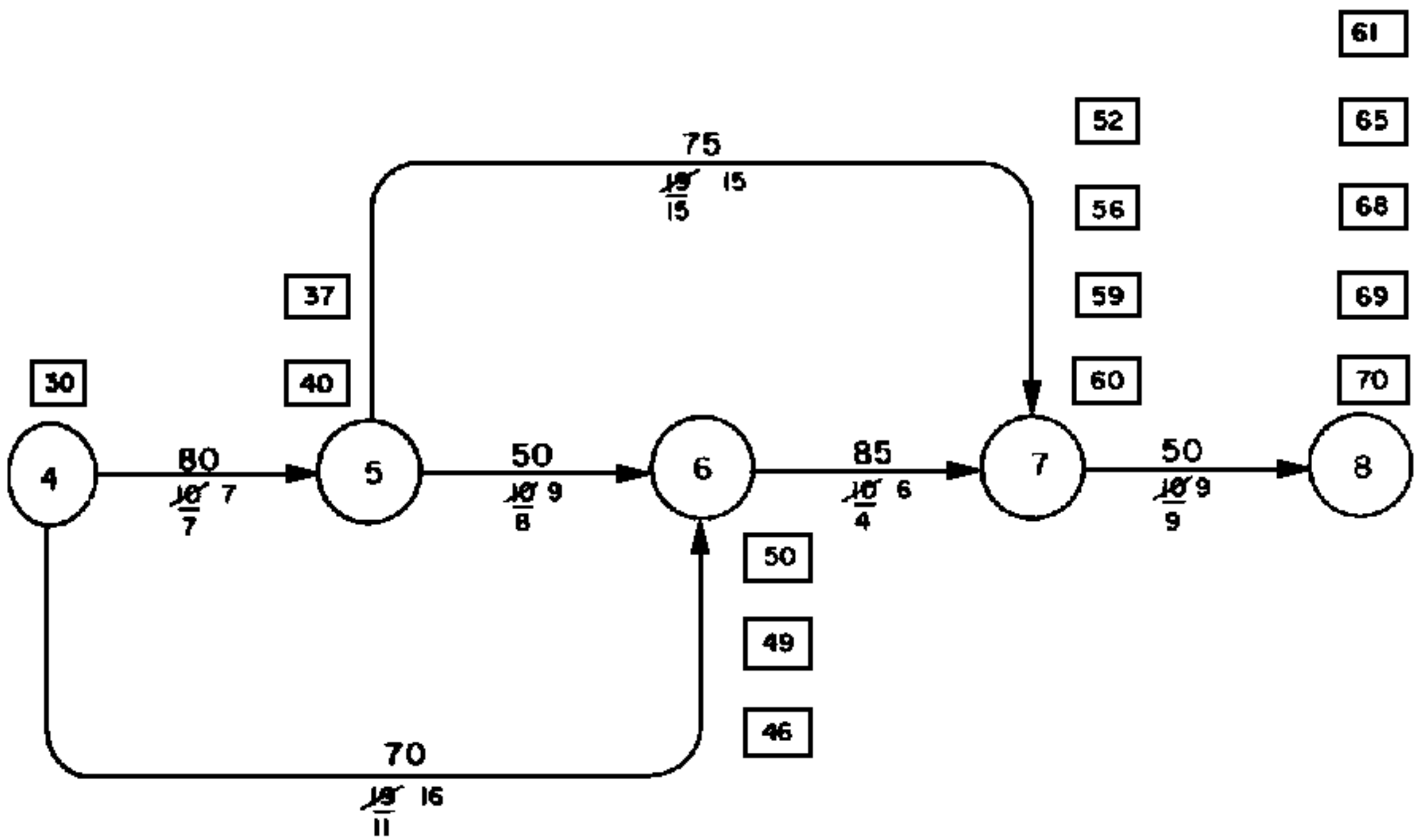

FIGURE 10.10 Example 10.2 Step by Step Crashing

Steps in Example Project

- Normal duration = 70 days; normal cost = $6600
- Crash 7-8 by one day (lowest cost slope): duration = 69, cost = $6650
- Crash 5-6 by one day; can't reduce further without reducing 4-6 and 5-7 too: duration = 68, cost = $6700
- Crashing 5-6 created 3 parallel critical paths:
  (1) 4-5 and 5-7
  (2) 4-5, 5-6, 6-7
  (3) 4-6, 6-7

Steps in Example Project

- Start with combination with lowest cost slope, crash 4-5 and 4-6 ($80 + $70 = $150/day) by 3 days each: reduces all 3 critical paths at once (4-5 crashed to limit): duration = 65, cost = $7150
- Next combination to crash is 5-7 and 6-7 for $160/day for 4 days each (5-7 crashed to limit): duration = 61 days, cost = $7790
- Last combination 4-6, 5-6, 5-7 can not be crashed further since 5-7 reached crash limit (and 4-5 did too – so that path can not be crashed)
- No activities generated float: no relaxation

Time-Cost Tradeoff Example

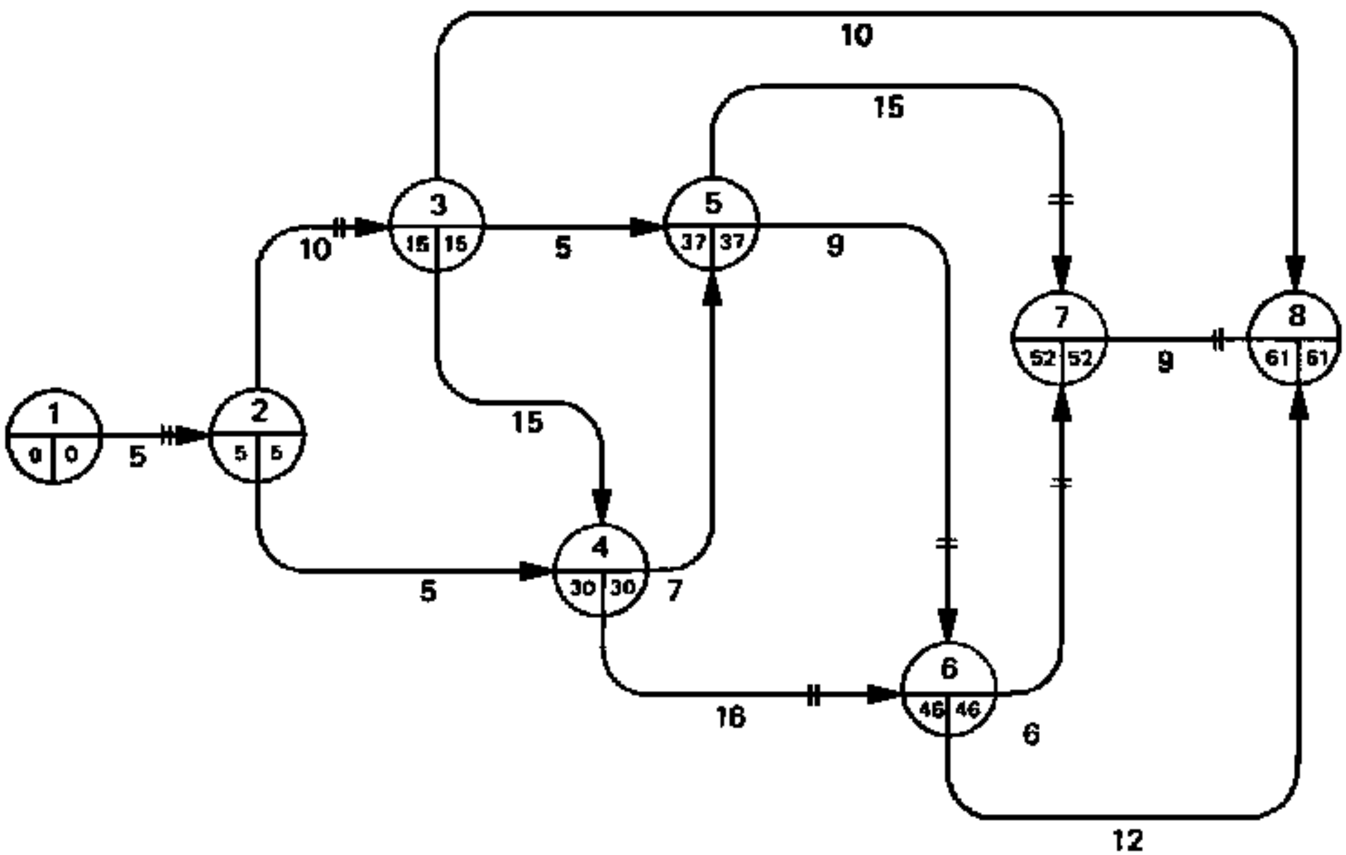

FIGURE 10.11 Example 10.2 Crashed Network

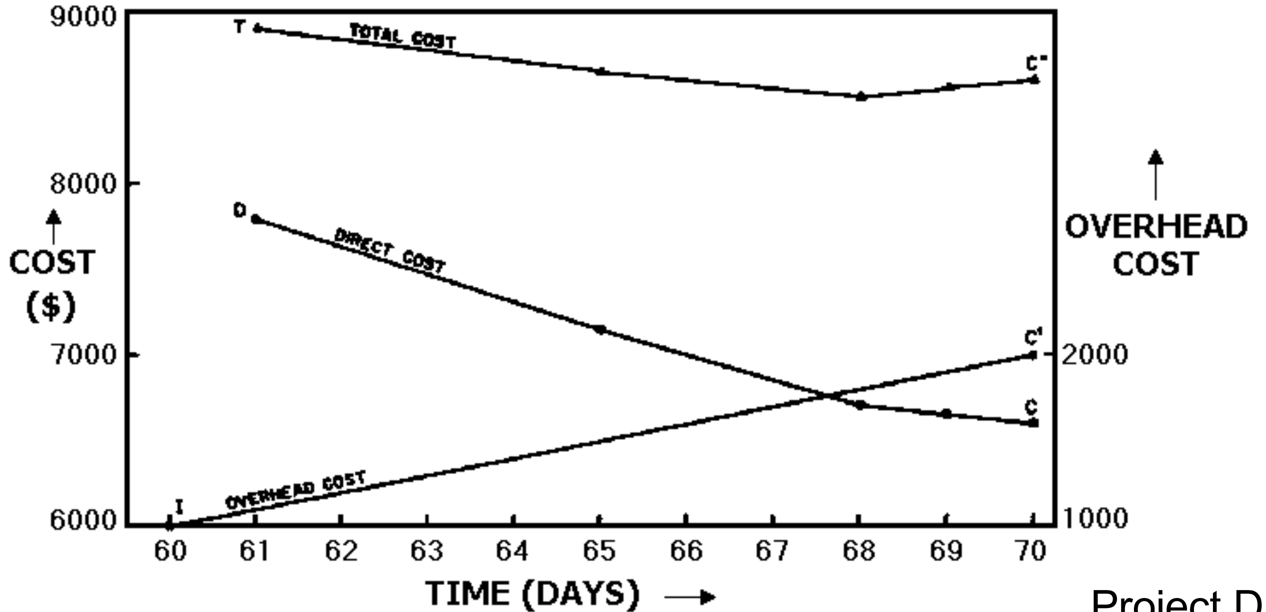

FIGURE 10.12 Time Cost Graph

Plot direct costs and indirect (overhead) costs on same graph.

Assume overhead costs:
Fixed cost of $6000 for 60 days;
$100/day for remainder of project.

Add direct and indirect costs to get total cost curve; tabulate time and cost.

Project Duration	Total Project Cost
61 days	$8890.00
65 days	$8650.00
68 days	$8500.00
69 days	$8550.00
70 days	$8600.00

Minimum project cost = $8500 (68 days).

Minimum project duration = 61 days ($8890).

Time-Cost Tradeoff

- Computer packages available for solving time-cost tradeoff (linear programming tools)
- Especially beneficial on repetitive-type projects (e.g., high rises, prefabricated modules, highways), since savings accumulated on repetitive activities and on future projects

References:

- *CIV E 601: Project Management, Lecture Notes,* Fayek, A. R. University of Alberta, 2013.
- *Project Management: Techniques in Planning and Controlling Construction Projects*, 2nd Edition, Ahuja, Dozzi, and AbouRizk, John Wiley and Sons, 1994.