Nondeterministic Networking Methods

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Abstract: Several probabilistic methods use risk analysis for construction networking. In this study, the writers compare and evaluate five nondeterministic methods: (1) Program evaluation review technique (PERT); (2) probabilistic network evaluation technique (PNET); (3) narrow reliability bounds (NRB), (4) Monte Carlo simulation (MCS); and (5) simplified Monte Carlo simulation (SMCS). To compare the five methods, the writers solved 31 networks using each of the methods. The results obtained by each method were compared and evaluated based on the survival function and computer time. The writers found that PERT is the simplest method and yields the most optimistic results, while MCS and SMCS produce the most conservative result. The gap between the survival function obtained with PERT and that obtained with the two simulation methods (MCS and SMCS) is wider for positively skewed distributions of network activities.

Introduction

Construction operations involve many uncertain variables and require the use of risk analyses, which experts employ for construction scheduling. To perform these risk analyses, experts use networks to represent the occurrence of activities involved in the construction project. A network consists of activities and links. Each activity represents a significant and definable task in the construction project, while links are used to indicate the relationships between tasks. A sequence of activities that starts with the first activity and ends with the last one is called a path. Failure to complete the project on time occurs when one or more paths take longer to complete than expected.

A probabilistic approach incorporating the correlation among the network paths was introduced by Ang (1975); and a related unpublished study (Zaferriere 1981) performed an assessment of probabilistic methods.

The purpose of our study is to compare the use of probabilistic scheduling methods for risk analyses. The survival function and the computing speed of each nondeterministic method were used to measure its performance. In this paper, the writers present examples of different construction networks. The methods included in this research are: (1) Program evaluation review technique (PERT); (2) probabilistic network evaluation technique (PNET); (3) narrow reliability bounds (NRB); (4) Monte Carlo simulation (MCS); and (5) simplified Monte Carlo simulation (SMCS). Of these five methods, PERT and MCS are the most widely used in practice. Moreover, some state-of-the-art scheduling software use PERT or MCS to perform risk analyses, for example, super project expert and open plan. PNET, NRB, and SMCS are relatively new and have been used mostly in research.

The survival function—$S(T) = 1 - F(T)$, where $F(T)$ = the cumulative distribution function (CDF) of project duration—is used to compare the results of each method. Computer time is also measured for each of the

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five methods. In order to carry out the study, the writers developed several computer programs, and used 31 construction network cases.

The first section of this paper covers the theoretical bases of the five risk analyses techniques employed. The second section contains the results of two case studies, and the final section includes conclusions and recommendations.

THEORETICAL BACKGROUND

In this section, the writers describe the theory behind the methods employed for the study, namely, PERT, PNET, NRB, MCS, and SMCS. A more detailed description of the use of these methods for construction scheduling networks can be found in a study by Diaz (1989).

Program Evaluation Review Technique (PERT) Method

The PERT model is constructed according to three durations that the scheduler determines for each activity: optimistic, most likely, and pessimistic ["PERT" 1958].

Eqs. (1) and (2) present the expected value and standard deviation for the duration of a given activity ("PERT" 1958).

$$t_e = \frac{(a + 4M + b)}{6} \quad (1)$$

$$s = \frac{(b - a)}{d} \quad (2)$$

where t_e = expected duration, a = optimistic duration, M = most likely duration, b = pessimistic duration, s = standard deviation, and d = scaling factor.

A value of 3.2 for d is appropriate for construction purposes (Moder 1983). Therefore, 3.2 is used in this study as the value of d.

PERT uses the central limit theorem (CLT) to find the expected project duration. The CLT indicates that for independent random variables

$$E(T) = t_1 + t_2 + t_3 + \dots + t_n \quad (3)$$

and,

$$S^2 = s_1^2 + s_2^2 + s_3^2 + \dots + s_n^2 \quad (4)$$

where $E(T)$ = expected duration; t_i = expected duration of ith activity; S = standard deviation of the project; and s_i = standard deviation of ith activity.

PERT assumes that the network $F(T)$ follows a normal distribution with expected duration, $E(T)$, and standard deviation, S. Values for $F(T)$ can be found in any published standardized normal distribution table (Patel 1982). We also assume that $F(T)$ of the network is determined exclusively by the expected duration and the standard deviation of the critical path. In this study, $F(T)$ is calculated using (5), which has a maximum error of only 0.07% (Patel 1982). The scheduler has to find the probability, $S(T)$, that the network duration will be longer than a duration, T.

$$F(T) = \frac{1}{1 + \exp[-1.5957691(X)(1 + 0.044715(X^2))]} \quad (5)$$

and $S(T) = 1 - F(T)$ where $X = [E(T) - t]/S$.

$S(T)$ is calculated for as many durations, t, as the scheduler feels necessary. Using small differences between the durations drawing of the survival function (SF) curve is recommended to obtain a smooth curve.

Probabilistic Network Evaluation Technique (PNET)

PNET was introduced by Ang (1975). The algorithm used by PNET is based on the different modes of failure that a network can have. Failure, in this case, is the completion of a project in a time longer than the target duration. Each path in the network can become a mode of failure. Thus, the completion of a project can be delayed by one or more paths in the network. PNET uses the simplified solution for the combination of modes of failure explained in the following paragraphs.

First, each activity in the network must have an expected time and a standard deviation. For comparison purposes, the writers utilized the same values used in PERT analyses. The CLT is applied to determine the expected value and standard deviation for the duration of each path. Next, the paths are ranked in the order of the longest duration. If two paths have the same duration, then the one with the highest standard deviation is assigned the higher rank. Eq. (6) is used to calculate the correlation coefficient between two paths (Ang 1975)

$$R_{ij} = \frac{s_1^2 + s_2^2 + \ldots + s_k^2 + \ldots + s_n^2}{S_i \times S_j} \quad \ldots\ldots (6)$$

where R_{ij} = correlation coefficient between the paths i and j; n = number of activities common to both paths; s_k = standard deviation of the kth activity that is common to paths i and j; S_i = standard deviation of path i; and S_j = standard deviation of path j.

For every pair of paths, the correlation coefficient, R_{ij}, is compared with a correlation coefficient, Ro. Ang indicates that a value of 0.5 for Ro is appropriate for construction networks (Ang 1975). If R_{ij} is larger than the Ro value, the two paths are considered completely correlated, and R_{ij} becomes unity. If R_{ij} is smaller than the Ro value, the two paths are considered uncorrelated, and R_{ij} becomes zero. The next step is to construct a correlation matrix with ones and zeroes obtained for every pair of paths.

A network composed of 60 activities can easily have more than 100 paths. PNET considers that one path can represent a group of paths. If any two paths have a correlation coefficient value of unity, the pair will be represented by the higher-ranked path. By repeating this for every pair of correlated paths the paths that are represented by a path of higher rank are eliminated. This selection results in a smaller number of uncorrelated paths which represent the network.

The probability, P, of the network to have a duration, T, or longer is

$$P(T) = 1 - p(t_1 < T) \times p(t_2 < T) \times \ldots \times p(t_n < T) \quad \ldots\ldots (7)$$

where t_i = expected duration of the ith representative path; and n = number of representative paths (Ang 1975).

Eq. (5) provides the formula for the individual probability of each path, $p(t_i < T)$ (Patel 1982). The network SF curve is constructed by calculating P for a large number of durations T.

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Narrow Reliability Bounds Method (NRB)

The NRB method was developed for structural reliability analysis by Ditlevsen (1979), and was earlier applied for scheduling by Laferriere (1981).

Like PNET, the NRB model is based on the probability of failure of each path. Failure occurs when the network duration is longer than a predetermined target duration. A failure mode is equivalent to a network path. Each path is considered to be normally distributed with expected duration, $E(t_i)$, and standard deviation, S_i.

NRB finds two probabilities of failure for the combination of all existing paths: lower bound probability (PL) and upper bound probability (PU) (Ditlevsen 1979).

The first step in NRB is to find the expected duration and standard deviation for each activity by using procedures similar to those used in PERT. The expected duration, $E(t_i)$, and the standard deviation, S_i, of each path, i, are found by applying the CLT. Then, (6) is used to calculate the correlation coefficient, R_{ij}, between each pair of paths i, j.

The purpose of the calculation of PL and PU is to find the probability of completing the project in a duration longer than the target duration, T (or probability of failure). T must be at least as long as the expected duration of the longest path. This limit is due to the geometrical nature of the NRB solution (Ditlevsen 1979). Ditlevsen's objective is to obtain the reliability of project completion. This reliability corresponds to the reliability index. For a more detailed discussion of this approach see Ditlevsen (1979) and Diaz (1989).

Hereafter, (5) is used to calculate the individual probability of failure, $S(T)$, for every path. The next step is to rank the paths in the order of the higher probability of failure. The combined probability between the paths then can be represented by a two-dimensional figure with failure and success regions for the network (Ditlevsen 1979).

The intersection of failure regions ($F_i \cap F_j$) has upper and lower bounds. The value of the lower bound, pl, of the intersection of two modes of failure is max ($P1$, $P2$), while the upper bound, pu, is the summation of $P1$ and $P2$. Eqs. (8) and (9) are used to find the value of $P1$ and $P2$ (Ditlevsen 1979).

$$P1 = \Phi(X_k)\Phi\left[\frac{(X_l - R_{kl}X_k)}{(1 - R_{kl}^2)^{(1/2)}}\right] \qquad (8)$$

$$P2 = \Phi(X_l)\Phi\left[\frac{(X_k - R_{kl}X_l)}{(1 - R_{kl}^2)^{(1/2)}}\right] \qquad (9)$$

where k, l = any pair of paths; R_{kl} = correlation coefficient between paths k and l; $X_k = [E(t_k) - t]/S$; $\Phi(\cdot)$ = standard normal distribution function.

These upper and lower bounds, pu and pl, are used in the following calculation of PL and PU.

$$PL = P(F_1) + \sum_{i=2}^{m} \max\left[0,\ P(F_i) - \sum_{j=1}^{i-1} P(F_i \cap F_j)\right] \qquad (10)$$

$$PU = \sum_{i=1}^{m} P(F_i) - \sum_{i=2}^{m} \max_{j<i} P(F_i \cap F_j) \qquad (11)$$

where $P(F_i)$ = probability of failure of the ith ranked path; $P(F_i \cap F_j)$ = the probability of the intersection of failure modes i and j.

The probability of failure $P(T)$ of the network for the target time T is presented as follows: $PL \leq P(T) \leq PU$. The process described earlier for finding PL and PU is repeated for different target times, T, and generates pairs of values of (T, PL) and (T, PU) that form the SF curve of the network. The network is represented by two SF curves. One is constructed with PL values, and the second with PU values.

Monte Carlo Simulation (MCS) Method

MCS, a probabilistic method that includes randomness in its calculations, is recommended only for computer applications, due to the large number of calculations it requires (Diaz 1989). The model presented is a simulation procedure. Before running a simulation of the duration of a construction network the CDF of each activity is determined.

During each replication in the simulation random values in the range (0–1) are assigned to the probability of completion of the activities. Once the CDF and the probability of completion of the activities are known, their durations can be determined by solving (5) for the given random probability. Thereafter, the duration of each path is found by summing up the durations of all activities in the path. The network duration is the duration of the longest path (Diaz 1989).

The whole process is repeated as many times as necessary. A large number of replications, say 10,000, is needed to obtain very accurate results. A simulation with 1,000 replications gives satisfactory results for construction networking purposes and is affordable in cost (Moder 1983). To perform an MCS, the activities in the network are assumed to be independent. This means that the duration of any activity will not affect the duration of another.

For illustration purposes the writers use triangular approximations for the distributions of construction activities. Figs. 1(a) and 1(b) show examples of triangular distributions. The optimistic, most likely, and pessimistic durations used in MCS are determined in the same way as they are in PERT, since these values are independent of the method used (Diaz 1989).

After the desired number of replications have been run, the network durations are ranked in the order of the shortest to the longest duration. Since the MCS result is based on a number of replications, for example, 1,000 (12) gives a good approximation for the probability, P, that the project will be completed in time, T, or longer

$$P = 1 - \frac{n}{N} \quad (12)$$

where n = number of replications with project duration equal or smaller than T; and N = total number of replications.

A large number of values for P for different values of T will guarantee a smooth SF curve.

Simplified Monte Carlo Simulation (SMCS)

Simplified Monte Carlo simulation, SMCS, simplifies the scheduling network to those activities and paths that are more likely to cause delay of the construction project completion.

The SMCS method is similar to the MCS method, but elimination of path(s) and activities in the network is performed prior to the first replication. Each replication involves the calculation of the duration of each

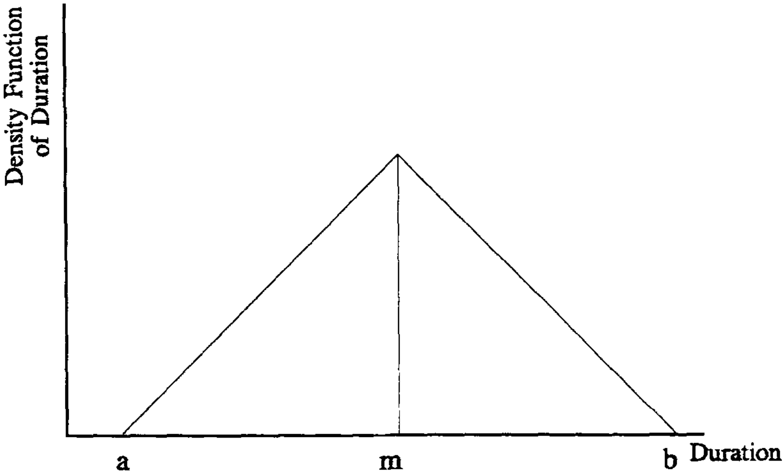

FIG. 1(*a*). Symmetric Distribution

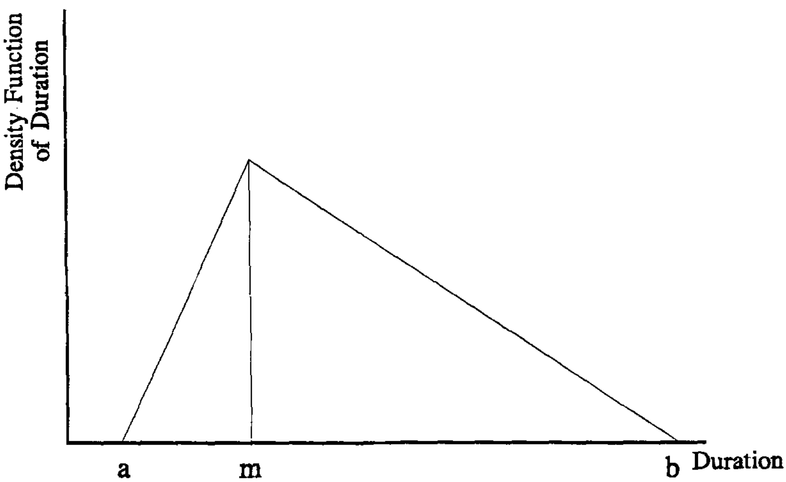

FIG. 1(*b*). Asymmetric Distribution

activity and each path in the simplified network. The project duration for a particular replication is the duration of the longest path. Although with the SMCS method the number of calculations is considerably reduced, the use of a computer is recommended.

The first step in SMCS is the calculation of the expected duration of each activity. Eq. (3) then is used to calculate the expected duration, $E(T)$, of the network. Those paths with an expected duration of less than T_{min} are not considered in further calculations. Eq. (13) gives the formula to determine T_{min} (Diaz 1989).

$$T_{min} = K \times E(T) \quad \text{.......................................} \quad (13)$$

K = a coefficient that indicates how close a path must be to the critical path if it is to cause the delay of the project, and can range from zero to one. The selection of K for a particular network is left to the scheduler's judgment. Since the effect of the value of K has not been extensively studied, the writers use a value of 2/3 for K, which results in considerable reduction of computing time and unnoticeable changes in accuracy compared to MCS (Diaz 1989).

After T_{min} is used to filter the network paths, the network activities are reduced. Those activities that are absent from any of the remaining paths are not considered in the rest of the calculations. The result of these two refinements is a simplified network, which only includes activities that are likely to cause delay of the construction project. The simulation process and the procedures used to develop the network SF curve are similar to those of the MCS method. The only difference is that, in the SMCS method, the calculations are performed using the simplified network.

Case Studies

In this study, 31 network problems extracted from various references were analyzed using each of the five methods. The cases ranged from renovation projects with 15 activities to industrial projects with the number of activities close to 100. In some of these cases, activities represent subnetworks. One of the reviewers of this paper indicated that "there is a large gap between the theory applied to networks of 15 or 20 activities and the application to networks of 300 or 400 activities." The writers agree with this statement and for this reason we included projects that had the number of activities close to 100. Since some of the activities represent subnetworks of several activities, the complete network includes more than 100 activities.

For each activity, the data set of the networks included optimistic duration, a, most likely duration, m, and pessimistic duration, b. Each network was solved for two different sets of data. The first set assumed that the activities involved had a symmetric distribution. The second set assumed positively skewed activities [see Figs. 1(a) and 1(b)]. The authors used their own judgment to determine the shape of the asymmetric distribution.

In this study, the writers considered the original expected duration as m. The a and b values were found using the following formulas:

$$B = 3.2 \times S + a \quad (14)$$

$$b - m = m - a \quad (15)$$

$$b - m = 2 \times (m - a) \quad (16)$$

Eq. (14) is derived from (2). Eqs. (15) and (16) were developed from the geometry of the distributions in Figs. 1(a) and 1(b). The symmetric set uses Eqs. (14) and (15) while the asymmetric set uses (14) and (16) (Diaz 1989).

In this section, two case studies are presented: ANG1 (Ang 1975) and OBRIEN6 (O'Brien 1984). Since the references provided only the most likely duration, m, of the activities, the writers, for the purpose of this study, used their judgment to provide values for a and b. For each case, a graphic shows the network SF curves obtained when using each of the methods studied. For the MCS and SMCS methods 1,000 replications were run, which is the number of replications recommended (Moder 1983). Common random numbers were not used to compare MCS against SMCS, since the networks used in the simulation of the same case study are different.

Case Study ANG1 and ANG1B

The case study ANG1 consists of a highway project network (see Fig. 2) (Ang 1975). Case ANG1B follows the same description as ANG1. The difference falls in the shape of the CDF of the activities. The writers chose this problem as the first case since it has been used in other studies that compared several probabilistic scheduling methods (Ang 1975; Laferriere 1981). A symmetric CDF was assumed for each activity [see Fig. 1(*a*)]. A computer program calculated the mean and standard deviation. Fig. 3 presents the results obtained by all the five methods.

The PERT results are the most liberal, and the SMCS the most conservative. The PNET curve, which falls between the PERT and the MCS curves but very close to the MCS curve, is the only curve to fall between the lower and upper bounds that were obtained with the NRB method.

For case ANG1B, an asymmetric CDF (positively skewed) for every activity was assumed. See Fig. 4 for the SF curve of this case. Also, for case ANG1B, the results are more conservative than those of ANG1. The PERT curve is the most liberal, the MCS curve the most conservative, and the PNET curve falls midway between the PERT and the MCS curves. The PNET curve is also the only curve to fall between the lower and upper bounds obtained with the NRB method. The gap between the MCS and SMCS curves, and the other curves is more pronounced in the case of ANG1B than in ANG1. In ANG1B, as well as in ANG1, the lower bound becomes closer to the PERT curve as the probability value became closer to 0.5.

Case Study OBRIEN6 and OBRIEN6B

Case study OBRIEN6 and OBRIEN6B relates to the construction of a small industrial building for a company (O'Brien 1984). Case OBRIEN6B follows the same description as OBRIEN6. The difference falls in the shape of the CDF of the activities. The network includes only major field activities. Figs. 5–7 present this case in a form similar to case ANG1 and ANG1B.

For OBRIEN6, the PERT curve is the most liberal curve and the upper bound the most conservative (see Fig. 6). The PNET curve is halfway between the MCS and PERT curves. The PERT, PNET, and lower bound curves are very similar for probabilities lower than 0.25. The MCS and SMCS curves are, once again, very similar.

For case OBRIEN6B, an asymmetric CDF (positively skewed) for every activity was assumed. As in OBRIEN6, the PERT curve is the most liberal curve (see Fig. 7). The MCS and SMCS curves are the most conservative. The PNET curve falls between the MCS and the PERT curves but closer to that of PERT. The PNET curve is also the only curve to fall between the lower and upper bounds. The MCS and SMCS curves are, again, very close to each other. The gap between the MCS and SMCS curves, and the other curves, is quite pronounced in this case.

Results

In this paper, the authors applied PERT, PNET, NRB, MCS, and SMCS to 31 network cases, and used computer programs to test the five different approaches. Specific observations for two of the case studies and general observations for the 31 cases are discussed here.

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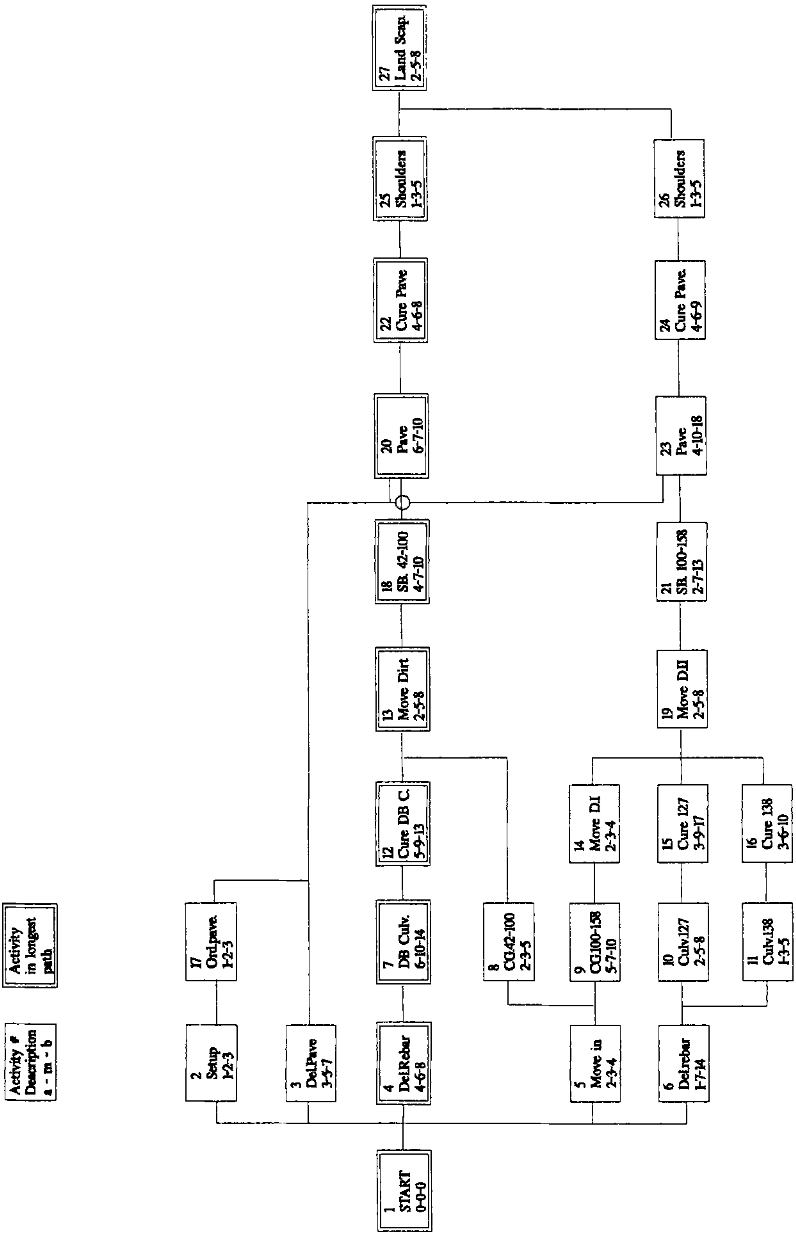

FIG. 2. Problem ANG1

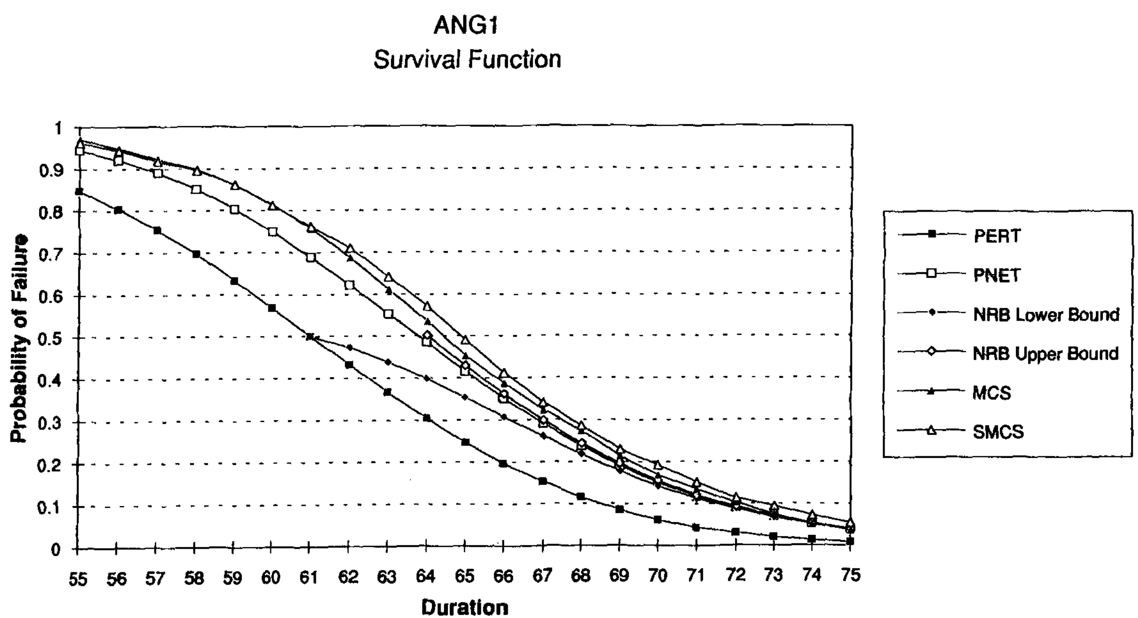

FIG. 3. Survival Function ANG1

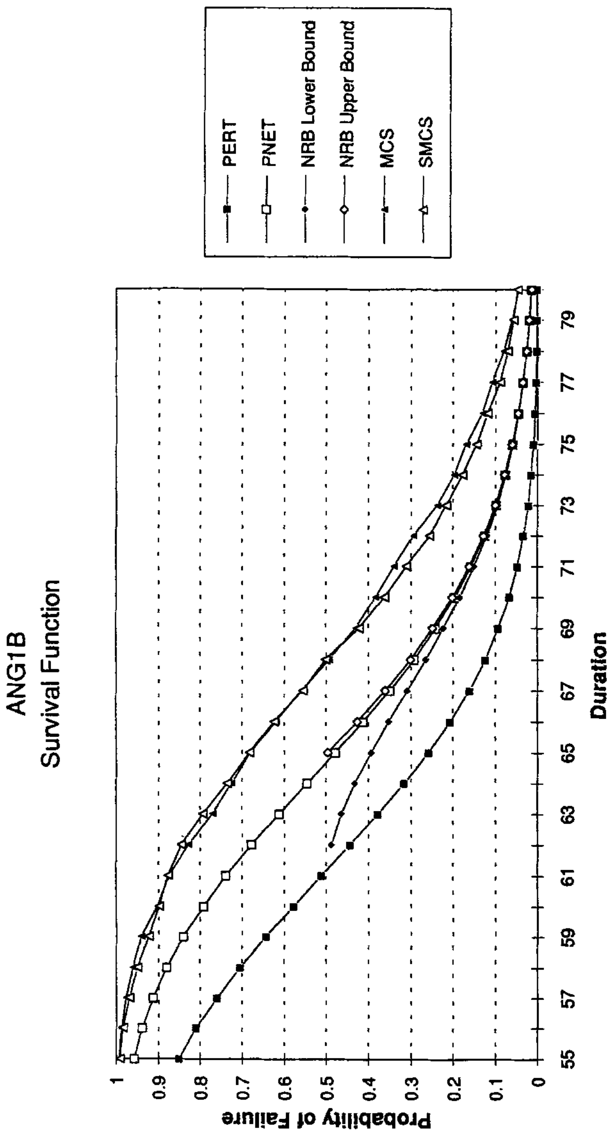

FIG. 4. Survival Function of ANG1B

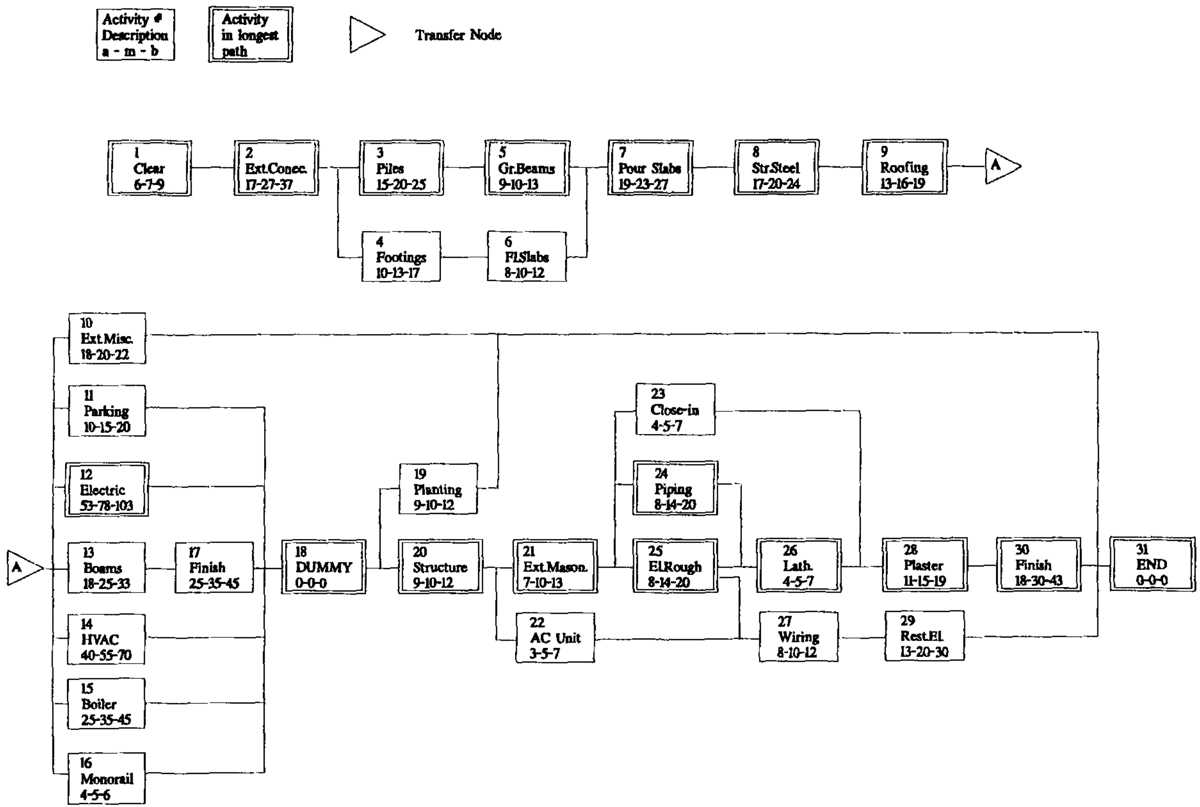

FIG. 5. Problem OBRIEN6

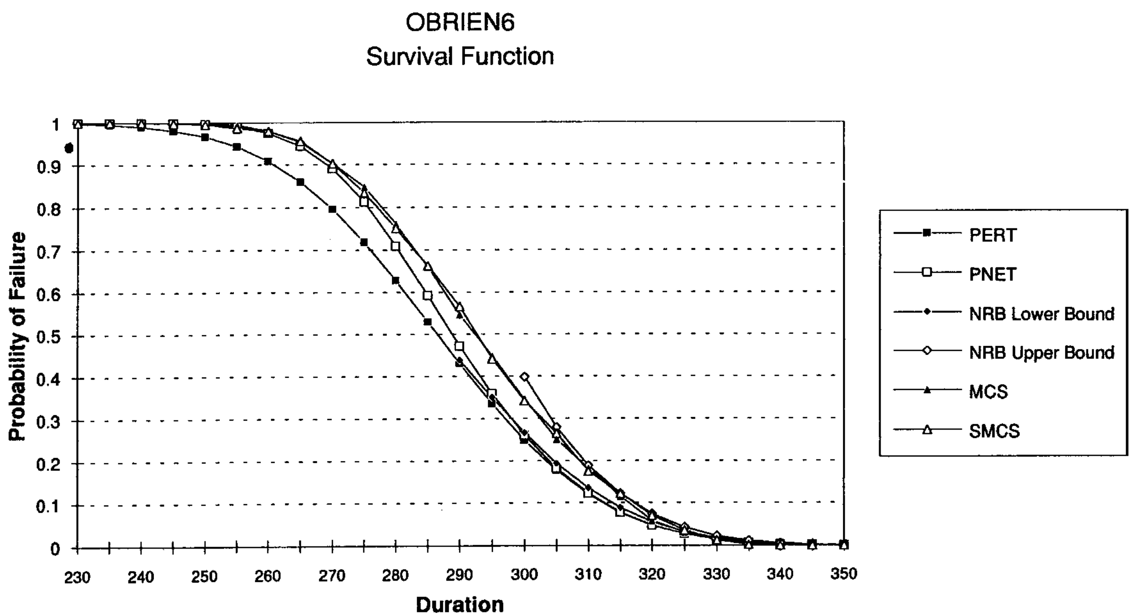

FIG. 6. Survival Function OBRIEN6

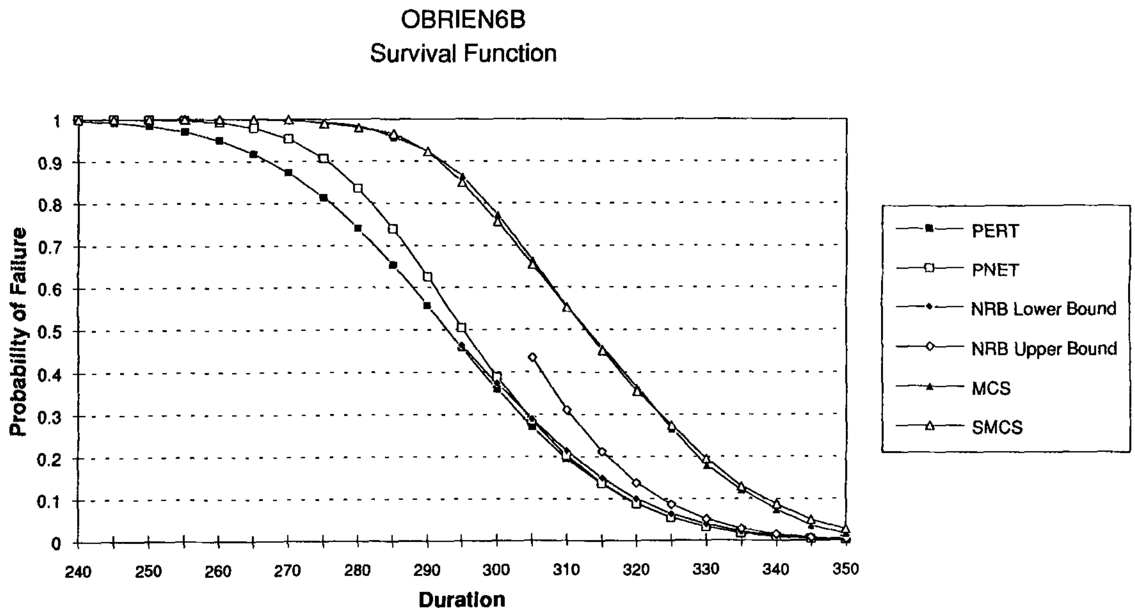

FIG. 7. Survival Function OBRIEN6B

Influence of Activities Distributions

There are consistent differences between the results obtained for a set with asymmetric (positively skewed) activity distributions and a set with symmetric activity distributions. The difference between the SF curves of the network using MCS and SMCS methods, and the SF curves using PERT, PNET, or NRB, is more noticeable in the network with asymmetric activity distributions. The results of MCS and SMCS tend to be much more conservative than the results obtained with the other three methods. This difference is due to the fact that PERT, PNET, and NRB use the expected duration, $E(t)$, and standard deviation, S, as parameters of the activities distributions; and these three methods do not account for the skewness of the activity distributions. On the other hand, the MCS and SMCS methods have the ability to simulate the skewness of the activity distributions.

Discussion of Probability of Failure

The PERT method consistently derives a liberal probability of failure, $P(T)$, for a network. This is evident in all 31 case studies. In some networks, value of $P(T)$ obtained with PERT is much lower than in the other methods. The difference between the value of $P(T)$ obtained by PERT, and that obtained by other methods, is higher when the network has several critical or near-critical paths. PERT's liberal results are due to the fact that PERT uses exclusively the expected longest path to calculate the probability of failure.

PNET yields a value of $P(T)$ higher than or equal to PERT in every case. In some cases, the SF curve obtained with PNET is similar to that obtained with MCS. In most cases, the value of $P(T)$ obtained with PNET falls between the lower and upper bounds found with the NRB method. From these observations, it can be seen that PNET gives better results than PERT because PNET considers the existence of more than one path.

The relative pessimism in the value of $P(T)$ obtained by PNET does not seem consistent. Sometimes the SF curve obtained with PNET is close to that obtained with PERT, and other times it is closer to the MCS result (see Figs. 6 and 7).

The writers found that the correlation coefficient Ro determines whether the result is liberal or conservative. In regard to choosing a Ro value, there are several considerations that a scheduler should keep in mind. A starting Ro value of 0.5 is fine for many networks; however, for some networks a higher Ro value is more convenient. If most of the paths in the network are closely correlated (share a large number of activities), Ro values between 0.60 and 0.85 give results closer to MCS results; however, if the Ro value is high, the results can become very conservative. Until more research is done about the effect of Ro, the writers suggest solving the problem using two different values of Ro, and using judgment to arrive at an appropriate answer.

Although the objective of this study did not include a parametric study of Ro, the writers noted that one of the case studies was particularly sensitive to the Ro value. For that case, the writers used two slightly different values of Ro (0.83 and 0.85). The SF curve was very liberal for 0.83 and very conservative for 0.85. This difference is due to the large number of pairs of paths that gave correlation values between 0.83 and 0.85.

The NRB method gives consistent results in the upper tail of the network SF curve. In that region of the SF curve, the lower and upper bounds are very close to each other. The lower and upper bounds obtained with NRB do not represent the complete SF curve of the network, since the NRB model can only find the probability of failure for target durations larger than the project expected duration. Although NRB is only useful for the upper tail, it seems to be very accurate for values of $P(T)$ lower than 5%. For higher values of $P(T)$, the results are at least as optimistic as those obtained with PNET or PERT. NRB has the advantage of providing the lower and upper bounds.

In most cases, MCS results in a more conservative value of $P(T)$ than that obtained with the three previous methods. This is because MCS considers every path and activity in the network as a potential cause of failure for completion of the project in a given time. MCS also has the capability of limiting the network duration to a maximum pessimistic duration. This maximum duration is the summation of the b of the critical activities. The probability for completion of a project in a time longer than the maximum duration is zero. On the other hand, the previous three methods (PERT, PNET, and NRB) use a normal distribution for the network SF curve. Any normal distribution has tails that go to the infinite. In this case, the maximum duration would be infinite; in the real world, however, no construction project lasts forever.

The SMCS method gives results similar to those obtained with the MCS method. In the networks analyzed, there is no tendency for the SMCS results to be more liberal than those obtained with the MCS method. Therefore, a simplified network can accurately represent a project network. This leads to the conclusion that the duration of some activities has little or no impact on the final duration of the network. The writers expected that this method would offer results close to those obtained with MCS, but slightly more on the liberal side. This expectation was based on the simplified network of the SMCS method. In every case, the SF curves obtained with the two methods was very similar. When determining the value of K for SMCS: the smaller the value of K, the closer T_{min} is to zero, and the closer the result of SMCS is to the result of MCS; the higher the value of K, the closer T_{min} is to the critical path, therefore, the closer the result of SMCS is to the PERT result and the higher the reduction in computer time is. A value of 2/3 for K seems adequate for most construction networks.

The writers also noted that in some cases the results obtained with SMCS were more conservative than those obtained with MCS. In theory, the results of MCS should be more conservative than the results of SMCS due to the simplification involved in SMCS. The writers concluded that the cause of conservative results of SMCS was the randomness involved in the simulation process. To obtain better results with MCS as well as with SMCS, the writers recommend using at least 10,000 replications for the simulation.

Conclusions and Recommendations

Five nondeterministic methods for construction networks were evaluated: program evaluation review technique (PERT), probabilistic network evaluation technique (PNET), narrow reliability bounds (NRB), Monte Carlo simulation (MCS), and simplified Monte Carlo simulation (SMCS). Thirty-one case studies were solved by each of the methods. This paper shows the results of two case studies.

PERT is the simplest method and consistently derives a liberal probability of failure, $P(T)$, for a network. PNET yields values of $P(T)$ greater than or equal to PERT in every case. The relative pessimism of PNET fluctuates and is sensitive to the value of Ro. NRB provides lower and upper bounds

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for the probability of the failure. Although NRB is useful only for the upper tail of the SF curve, the results are very close to the MCS results for values of $P(T)$ lower than 5%. In most cases, MCS results in more conservative values of $P(T)$ than the other methods, and SMCS provides results very similar to MCS. The difference in the results obtained by the two simulation methods, MCS and SMCS, and the other three methods, PERT, PNET, NRB, is more evident when the activities in the network have positively skewed distributions.

This study raises several questions that may be answered in future research. The writers believe that a parametric study of the influence of the value of *Ro* in the results of PNET could be of great value. The writers agree with the reviewers' comments suggesting a study of the correlation between the float time and the probability of failure in the future.

Appendix I. References

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Appendix II. Notation

The following symbols are used in this paper:

a = optimistic activity duration;
b = pessimistic activity duration;
d = scaling factor;
$E(T)$ = expected project duration;
$F(T)$ = cumulative distribution function of project duration;
K = project duration coefficient for SMCS;
M = most likely activity duration;
N = number of replications;
n = number of replications with project duration equal or smaller than T;
P = probability of project late completion;
$P1$ = intermediate value used in NRB;
$P2$ = intermediate value used in NRB;
PL = project lower-bound probability of failure;
PU = project upper-bound probability of failure;
$P(F_i)$ = probability of failure of ith ranked path;
$P(T)$ = probability of having project duration larger than T;
p = probability;

p_l = path lower bound;
pu = path upper bound;
R_{ij} = correlation coefficient between paths i and j;
Ro = correlation coefficient used in PNET;
S = project standard deviation;
S_i = standard deviation of path i;
$S(T)$ = survival function of project duration;
s = activity standard deviation;
s_i = standard deviation of the ith activity;
T = goal project duration;
T_{min} = minimum path duration;
t_e = expected activity duration;
t_i = duration of the ith activity; and
X = value from standard normal distribution.

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