CENG 6101 Project Management

Scheduling of Non-Repetitive Construction Projects: PERT

Abraham Assefa Tsehayae, PhD

November, 2017

TO DO

1. What is PERT?
2. Why use PERT over CPM?
3. PERT vs. CPM
4. Procedure for a project network
5. PERT network and calculations
6. Case study

WHAT IS PERT?

- PERT stands for Program Evaluation and Review Technique, a methodology developed by the U.S. Navy in the 1958 to manage the POLARIS submarine missile program
- A project management tool used for management of non-repetitive projects, where the time and cost estimates tend to be quite uncertain; this technique uses probabilistic time estimates
- PERT is used to schedule, organize, and coordinate tasks within a project

WHY USE PERT OVER CPM?

- PERT enables us to answer two main questions:
  1. What project duration will provide a probability p of meeting it?
  2. What is the probability of finishing a project in x days/months/years?
- When you have a high degree of uncertainty surrounding your project, stochastic schedules are preferred to deterministic schedules (like CPM)
- PERT environment is estimated on the basis of optimistic, most likely, and pessimistic durations for each activity; these durations can be arrived at in various ways (heuristic, data, etc.)

WHY USE PERT OVER CPM?

- Advantages of PERT
  - Accounts for uncertainty
  - More realistic
- Limitations of PERT
  - Time and labour intensive
  - Mostly used on large, complex projects
- Both use Network Diagrams

  - CPM: deterministic

  - PERT: probabilistic
- CPM: one estimate, PERT: several estimates

WHY USE PERT OVER CPM?

- Both useful at many stages of project management
- Both are mathematically simple
- Both give critical path and float time
- Both provide project documentation
- Both are useful in monitoring costs

WHY USE PERT OVER CPM?

Example: Placing concrete

Placing concrete might be delayed due to:

- Rain
- Low labour productivity
- Equipment breakdown

PERT allows scheduler to account for such uncertainties in activity durations

PERT PROCEDURE FOR A PROJECT NETWORK

1. Estimate the optimistic, most likely, and pessimistic durations for each activity
2. Do a forward and backward pass, and calculate the mean/expected project duration and variance of project completion time
3. Using the Central Limit Theorem, assume that the project completion time has a normal distribution function
4. Given a duration x, calculate Z (standard normal variate) and use a normal distribution table to assess the probability of completing the project in x units of time or less, OR
5. Given a probability p, read Z from a normal distribution table and calculate the duration x that will give this probability of completion

PERT NETWORK AND CALCULATIONS

Step 1: Identify activity durations:

- Optimistic Estimate (O) – duration expected to occur under best circumstances
- Most Likely Estimate (ML) – duration expected to occur under normal circumstances
- Pessimistic Estimate (P) – duration expected to occur under worst circumstances

PERT NETWORK AND CALCULATIONS

Step 2: Calculate mean/expected duration (μ) for each activity (assuming Beta distribution for activity durations):

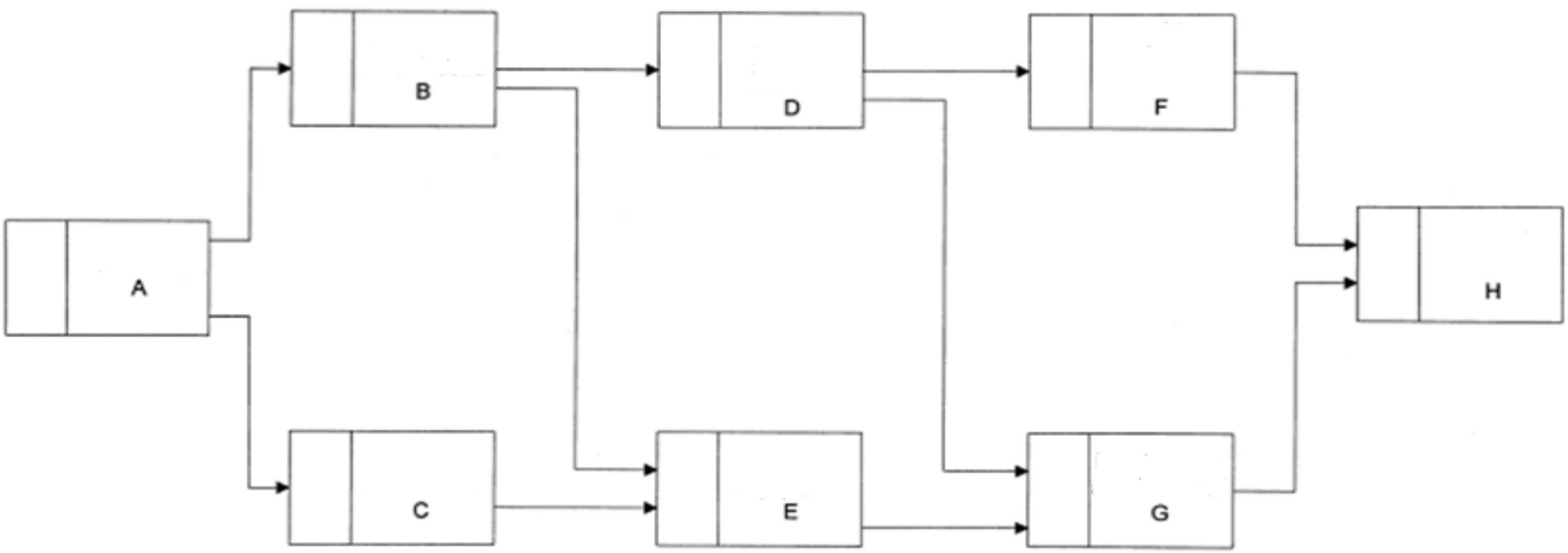
$$\mu = \frac{(1 * O + 4 * ML + 1 * P)}{6}$$

Step 3: Calculate variance (σ^2) and standard deviation (σ) for each activity:

$$\sigma^2 = \left(\frac{(P-O)}{6}\right)^2$$
$$\sigma = \left(\frac{(P-O)}{6}\right)$$

The higher the σ^2 and σ, the greater the amount of uncertainty in the activity duration

Step 4: Plot network diagram and perform calculations



PERT NETWORK AND CALCULATIONS

Step 5: Network calculations

Traditional CPM early and late date calculations, but treat activity durations as mean activity duration, so:

$$ES + \mu = EF$$

$$LF - \mu = LS$$

Step 5: Network calculations

- Forward Pass (Based on CPM)

-For activity A

$EF_A = ES_A + \mu_A = \mu_A$

-For activity C

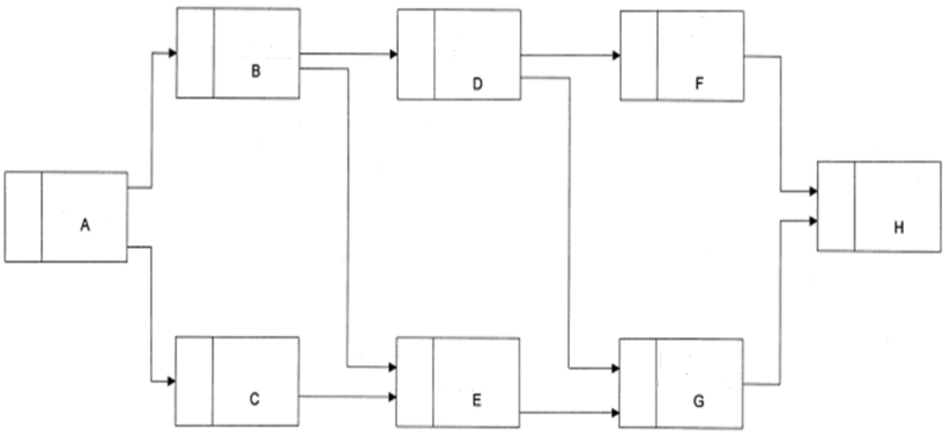
$ES_C = EF_A$

$EF_C = ES_C + \mu_C$

-For activity E

$ES_E = Max(EF_B, EF_c)$

$EF_E = ES_E + \mu_E$



Step 5: Network calculations

- Backward Pass (Based on CPM)

-For activity A

$LS_A = LF_A - \mu_A$

-For activity C

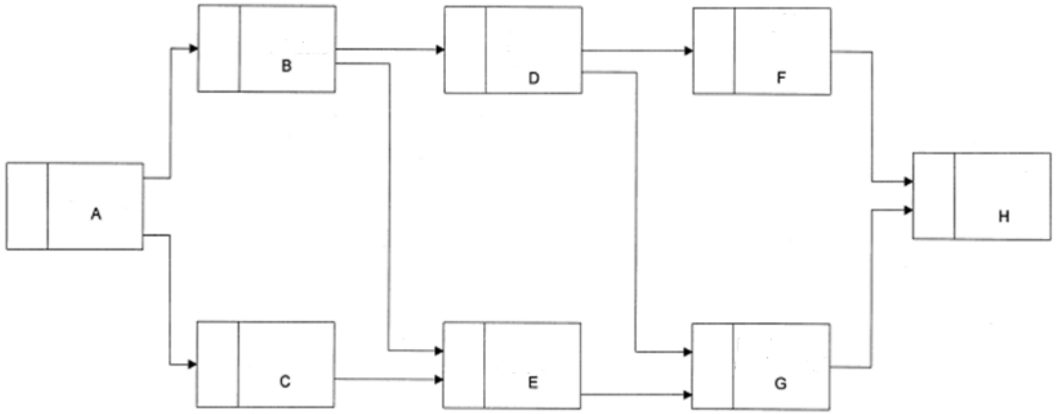
$LF_C = LS_E$

$LS_C = LF_C - \mu_C$

-For activity B

$LF_B = \text{Min}\ (LS_D, LS_E)$

$LS_B = LF_B - \mu_B$



PERT NETWORK AND CALCULATIONS

Step 5: Network calculations

Project duration ($\bar{T}$) $=\sum_i^n \mu_i$, where μ_i is mean duration of the *n* activities on the longest path

$V^2 = \sum_i^n \sigma^2{}_i$, where $\sigma^2{}_i$ is variance of the n activities on the longest path

Step 6: Determining probabilities

- Assume that there are n activities along the critical path and that the activity durations are independent

- Assume that each activity duration has a given distribution D (e.g., a beta distribution) with a finite mean and variance

Step 6: Determining probabilities

- If the number of activities on critical path is large, then the distribution of the mean of the activity durations approaches a normal distribution, according to Central Limit Theorem
- So, if n is large enough, distribution of the project completion time T can be approximated with normal distribution with mean $\overline{\mathrm{T}}$ and variance V^2 as follows:

Step 6: Determining probabilities

- $\overline{T} = \sum_{ij=1}^{n} \mu_{ij} = \mu_{1j} + \mu_{2j} + \ldots + \mu_{nj}$

such that j belongs to longest path leading to terminal node and i refers to the n activities on the path

- $V^2 = \sum_{ij=1}^{n} \sigma^2{}_{ij} = \sigma^2{}_{1j} + \sigma^2{}_{2j} + \cdots + \sigma^2{}_{nj}$

such that j belongs to longest path leading to terminal node and i refers to the n activities on the path

Note: Longest path is not necessarily the path with highest uncertainty; could have higher uncertainty along non-critical paths

Step 6: Determining probabilities

- Since $\overline{\mathrm{T}}$ and V^2 are assumed to follow a normal distribution, statistical analysis can be performed to make probabilistic statements
- What is the probability of completing the project in x time units or less?
- What project duration would give a probability of $y\%$ of completing the project in time?

Step 6: Determining probabilities

- Normal distribution with mean μ and variance σ^2 has a probability density function (PDF) and cumulative distribution function (CDF) as shown below:

PDF

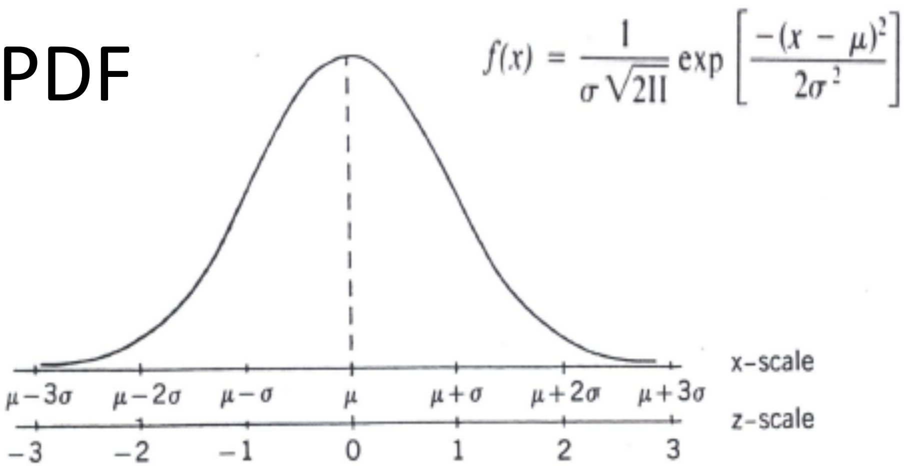


FIGURE 15.4 Normal Distribution

CDF

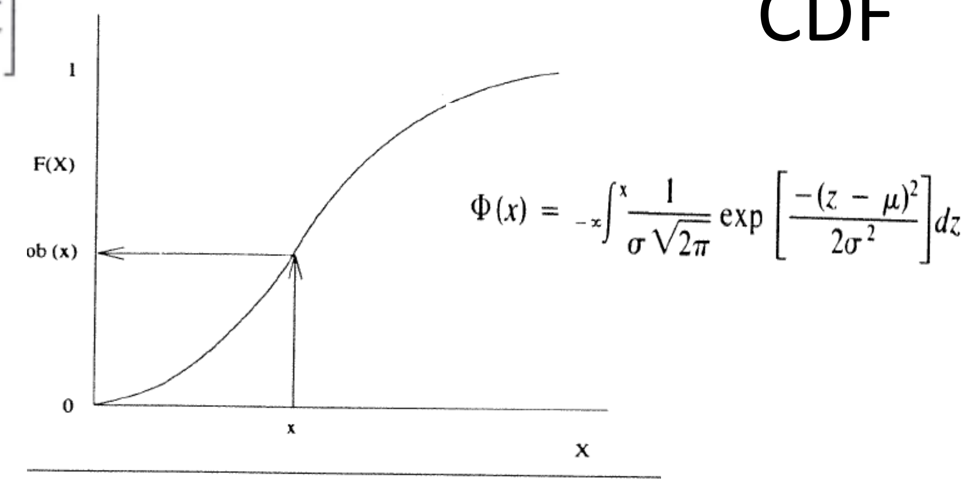


FIGURE 15.5 Sample Normal Distribution CDF

Step 6: Determining probabilities

- Using CDF, for any given value of x we can determine probability of having a number $\leq x$ by reading $F(x)$ from graph, or using the equation shown below, or using a normal distribution table (next slide)

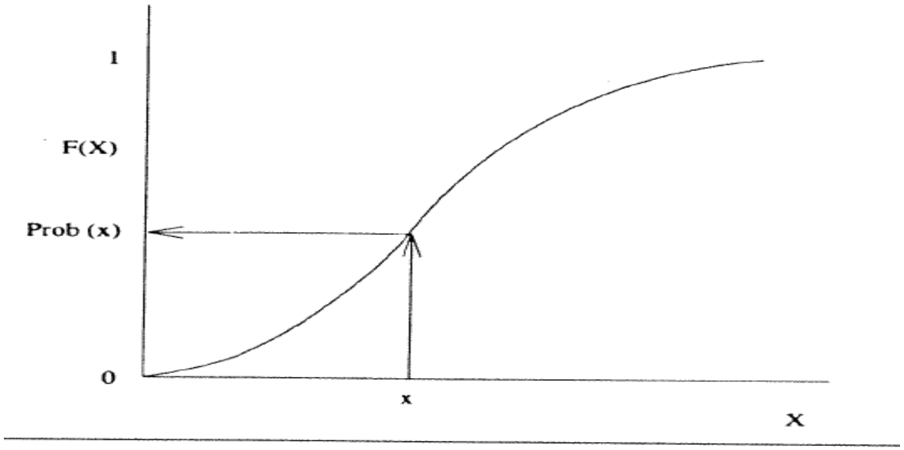


FIGURE 15.5 Sample Normal Distribution CDF

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(z-\mu)^2}{2\sigma^2}\right] dz$$

Step 6: Determining probabilities
From normal distribution table for $+Z$:

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189

Step 6: Determining probabilities
From normal distribution table for $-Z$:

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Step 6: Determining probabilities

If x is normally distributed with mean μ and variance σ^2 , then random variable Z :

$$Z = \frac{(x - \mu)}{\sqrt{\sigma^2}}$$

is normally distributed with mean $\mu = 0$ and variance $\sigma^2 = 1$ and Z is referred to as standard normal variate; therefore to determine probability of completing project in x time units or less, read probability corresponding to value of Z from normal distribution table

Step 6: Determining probabilities

Sometimes, probability tables are provided for positive values of Z only:

- Use symmetric properties of normal distribution, so that:

$$\varphi(-z) = 1 - \varphi(z)$$

where φ is cumulative probability density function of normal distribution

- So, read value corresponding to $+Z$ in table and calculate probability of $-Z$ by subtracting value read from table from 1.0

Step 6: Determining probabilities

For a project network:

Since $\overline{\mathrm{T}}$ = mean and V^2= variance for the project completion time (assumed to have a normal distribution)

-Calculate $\overline{\mathrm{T}}$ and V^2

- Given values of x calculate Z and use normal distribution table to assess probability of completing project in x days or less, OR
- Given a probability, read Z from table and calculate duration x which will give this probability of completion.

Case Study

Activity	Dependency	Predecessor Relationship Type	O	ML	P
A			10	16	20
B	A	FS	7	10	20
C	A	FS	5	7	8
D	B	FS	15	18	21
E	B,C	FS,FS	25	30	32
F	D	FS	6	9	12
G	D,E	FS,FS	21	25	28
H	F,G	FS,FS	6	8	9

Step 1: Determine activity durations

Activity	O	ML	P
A	10	16	20
B	7	10	20
C	5	7	8
D	15	18	21
E	25	30	32
F	6	9	12
G	21	25	28
H	6	8	9

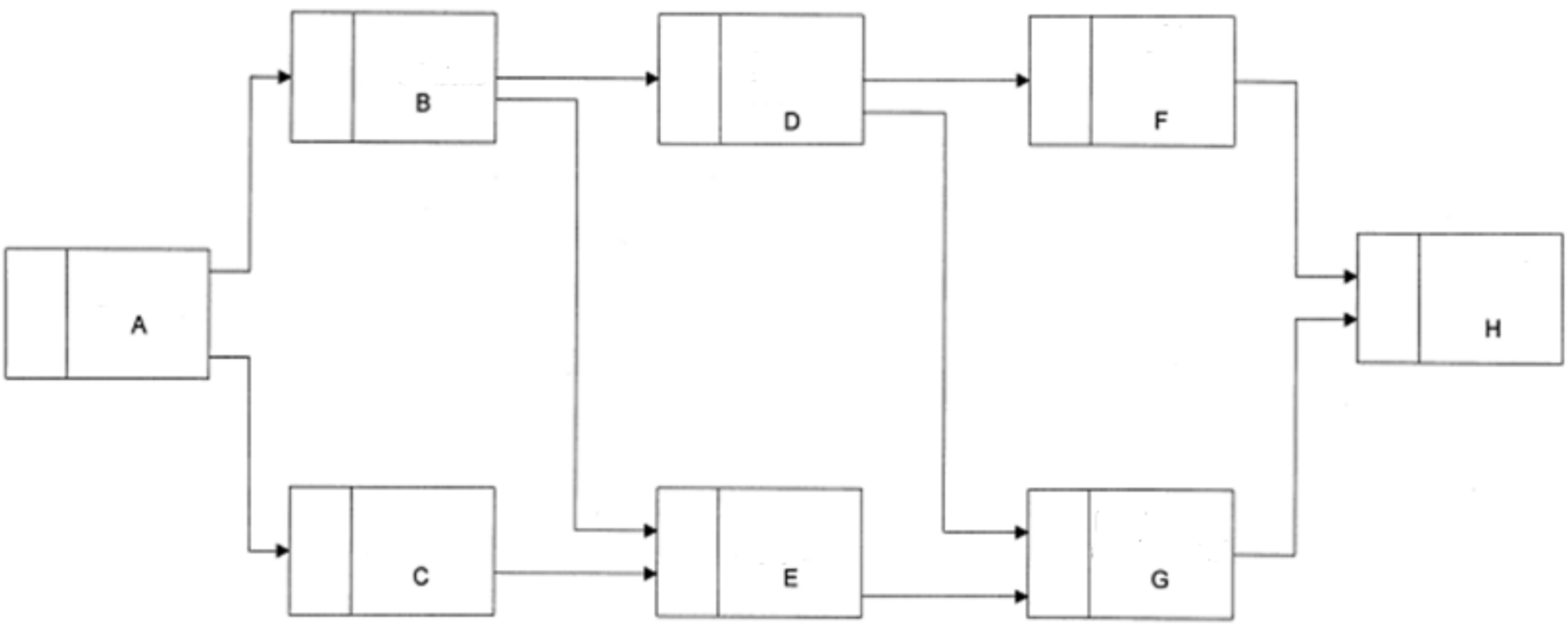
Step 2: Calculate mean/expected duration for each activity

Activity	Dependency	Predecessor Relationship Type	O	ML	P	μ
A			10	16	20	**15.67**
B	A	FS	7	10	20	**11.17**
C	A	FS	5	7	8	**6.83**
D	B	FS	15	18	21	**18.00**
E	B,C	FS,FS	25	30	32	**29.50**
F	D	FS	6	9	12	**9.00**
G	D,E	FS,FS	21	25	28	**24.83**
H	F,G	FS,FS	6	8	9	**7.83**

Step 3: Calculate variance and standard deviation for each activity

Activity	Dependency	Predecessor Relationship Type	O	ML	P	μ	σ^2	σ
A			10	16	20	15.67	**2.78**	1.67
B	A	FS	7	10	20	11.17	**4.69**	2.17
C	A	FS	5	7	8	6.83	**0.25**	0.50
D	B	FS	15	18	21	18.00	**1.00**	1.00
E	B,C	FS,FS	25	30	32	29.50	**1.36**	1.17
F	D	FS	6	9	12	9.00	**1.00**	1.00
G	D,E	FS,FS	21	25	28	24.83	**1.36**	1.17
H	F,G	FS,FS	6	8	9	7.83	**0.25**	0.50

Step 4: Plot network diagram



Step 5: Perform network calculations

Activity	μ	ES	EF	LS	LF	σ^2
A	**15.67**	**0**	**15.67**	**0**	**15.67**	**2.78**
B	**11.17**	**15.67**	**26.84**	**15.67**	**26.84**	**4.69**
C	6.83	15.67	22.5	20.01	26.84	
D	18.00	26.84	44.84	38.3	56.34	
E	**29.50**	**26.87**	**56.34**	**26.87**	**56.34**	**1.36**
F	9.00	44.84	53.84	72.17	81.17	
G	**24.83**	**56.34**	**81.17**	**56.34**	**81.17**	**1.36**
H	**7.83**	**81.17**	**89**	**81.17**	**89**	**0.25**
					V^2= Variance of Critical path $\Sigma\sigma^2$	**10.44**

Critical Path is: A – B – E – G - H

$$\overline{T} = 15.67 + 11.17 + 29.50 + 24.83 + 7.83 = 89.00$$

$$V^2 = 2.78 + 4.69 + 1.36 + 1.36 + 0.25 = 10.44$$

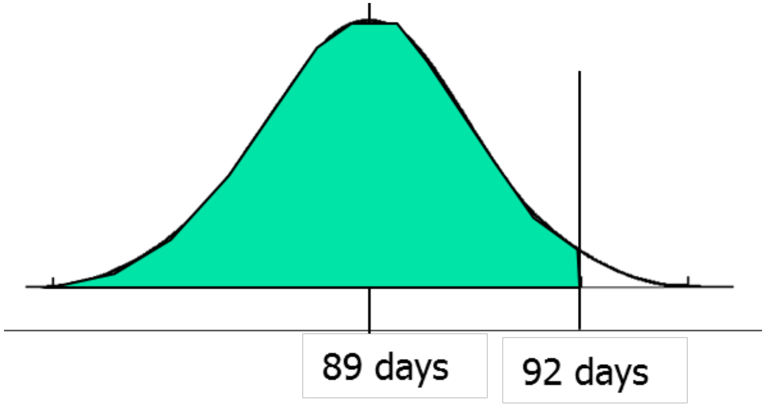
- Step 6: Determining probabilities
- Probability of project completion in 92 days or less:

$\therefore\ x = 92$

- $Z = \dfrac{(x - \overline{T})}{\sqrt{V^2}} = \dfrac{(92 - 89)}{\sqrt{10.44}} = 0.9285 \approx 0.93$
- From normal distribution Table 1:

$\rightarrow\ p = 0.82381$

$\therefore$ 82.4% chance of completing project in 92 days or less



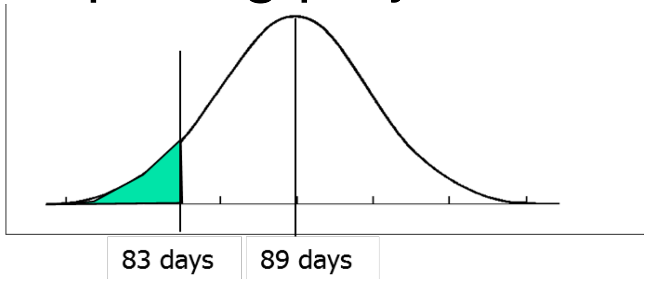
Step 6: Determining probabilities

Probability of project completion in 83 days or less:

$\therefore\ x = 83$

- $Z = \frac{(x - \bar{T})}{\sqrt{V^2}} = \frac{(83 - 89)}{\sqrt{10.44}} = -1.8569 \approx -1.86$
- From $-Z$ normal distribution table: $\rightarrow p = 0.03144$
- Alternatively,
- From $+Z$ normal distribution table:
- For $Z = 1.86, p' = 0.96856 \rightarrow p = 1 - 0.96856 \approx 0.03144$

$\therefore$ 3.1% chance of completing project in 83 days or less



PERT NETWORK AND CALCULATIONS

Step 6: Determining probabilities

Probability of project completion in:

- Minimum project duration such that probability of completing it on time is 90%:

∴ Probability = 0.90

- From normal distribution table for p = 0.90, Z = 1.28

$$Z = \frac{(x - \overline{T})}{\sqrt{V^2}} = \frac{(x - 89)}{\sqrt{10.44}} = 1.28 \rightarrow (x - 89) = 1.28 * \sqrt{10.44} = 4.14$$

$$x = 89 + 4.14 = 93.14 \text{ Days}$$

Step 6: Determining probabilities

Probability of project completion in:

- If a contractor uses a 10% contingency, what probability of completion does this duration correspond to?

With 10% contingency

- $x = 1.1 * \overline{\mathrm{T}} = 1.1 * \ 89 = 97.90 \text{ Days}$
- $\mathrm{Z} = \frac{(\mathrm{x} - \overline{\mathrm{T}})}{\sqrt{\mathrm{V}^2}} = \frac{(97.90 - 89)}{\sqrt{10.44}} = 2.75 \ \rightarrow \mathrm{p} = 0.99702$

∴ 99.7% probability of completing on time (97.9 days)

References:

- *CIV E 601: Project Management, Lecture Notes,* Fayek, A. R. University of Alberta, 2013.