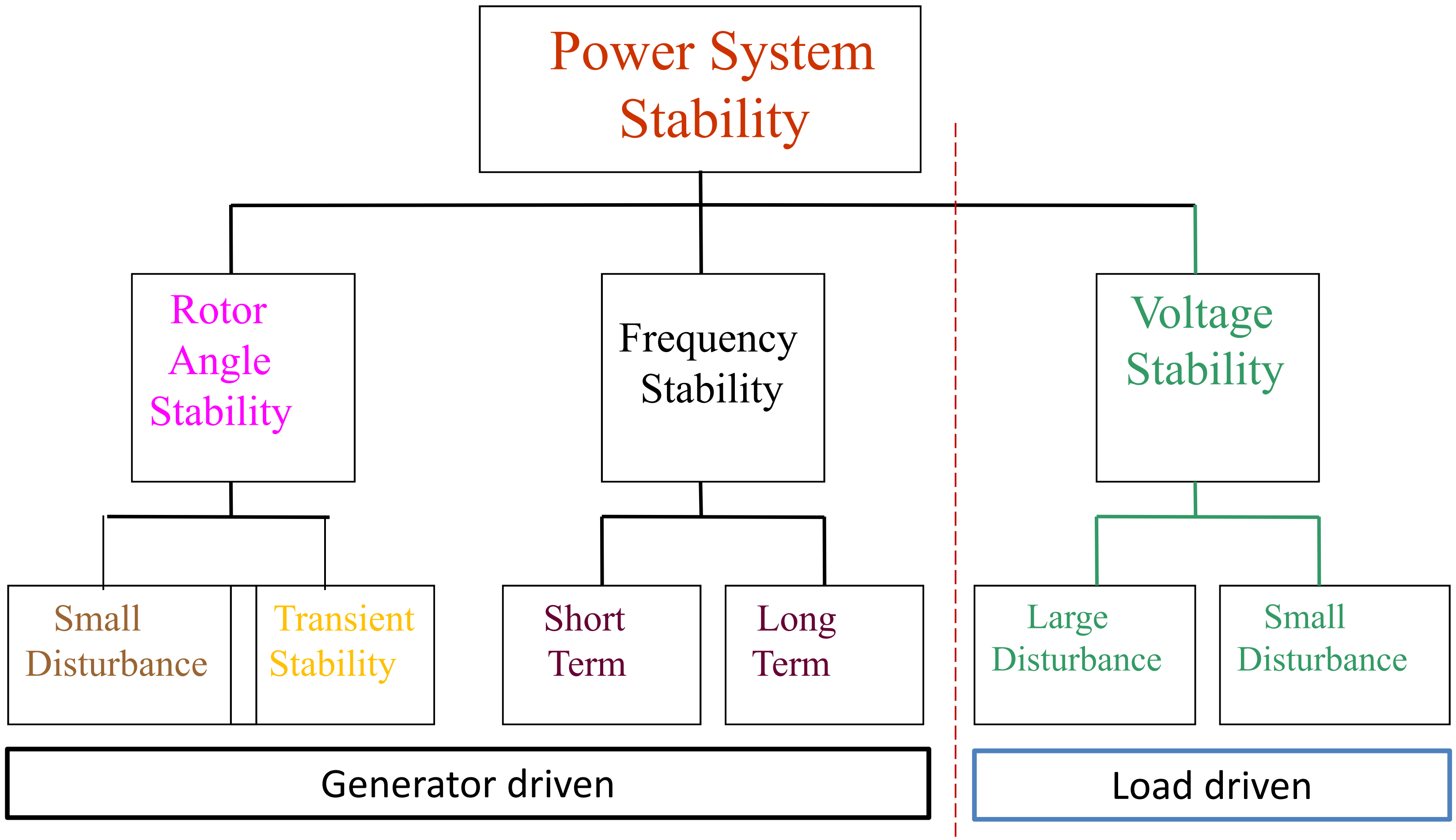
VOLTAGE STABILITY

IEEE/CIGRE Classification of Power System Stability[1]



1. P. Kundur, J. Paserba, V. Ajjarapu , Andersson, G.; Bose, A.; Canizares, C.; Hatziargyriou, N.; Hill, D.; Stankovic, A.; Taylor, C.; Van Cutsem, T.; Vittal, V "**Definitions and Classification of Power System Stability** " IEEE/CIGRE Joint Task Force on Stability Terms and Definitions, IEEE transactions on Power Systems, Volume 19, Issue 3, pp. 1387-1401 August 2004

OUTLINE

1. What is „Voltage stability"?
2. Classification of voltage stability
   - 2.1 Short-term voltage stability
   - 2.2 Long-term voltage stability
3. Static voltage stability assessment
   - 3.1 PV curve
   - 3.2 QV curve

1. What is voltage stability?

What is "Voltage Stability"?

- Voltage Stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance.
  – Instability may result in the form of a progressive fall or rise of voltages of some buses

What is "Voltage Stability" (cont'd)?

Definition of voltage stability related terms according to IEEE :

- Voltage Stability is the ability of a system to maintain voltage so that when load admittance is increased, load power will increase, and so that both power and voltage are controllable.
- Voltage Collapse is the process by which voltage instability leads to loss of voltage in a significant part of the system.
- Voltage Security is the ability of a system, not only to operate stably, but also to remain stable (as far as the maintenance of system voltage is concerned) following any reasonably credible contingency or adverse system change.
  - A system enters a state of voltage instability when a disturbance, increase in load, or system changes causes voltage to drop quickly or drift downward, and operators and automatic system controls fail to halt the decay.
  - The voltage decay may take just a few seconds (short term voltage stability problem) or ten to twenty minutes (long term voltage stability problem).
    - ✓ If the decay continues unabated, steady-state angular instability or voltage collapse will occur.

Voltage security vs. Voltage instability

- Voltage security is the ability of the system to maintain adequate and controllable voltage levels at all system load buses.
  - The main concern is to make sure that voltages remain inside a specified range and do not affect the operation of customer devices or after a credible contingency

- Voltage security problem:
  1. Low voltage: voltage level is below a pre-defined range.
     - low voltage does not necessarily imply voltage instability
  2. Voltage instability: an uncontrolled voltage decline.
     - voltage value in the normal range does not necessarily imply continuous voltage security

Voltage instability cont…

- Possible outcomes of voltage instability:
  - Loss of load in an area
  - Tripping of lines and other elements leading to cascading outages
    - ✓ Loss of synchronism of some generators may result from the outages or from operating condition that violate excitation current limit

Voltage instability cont..

- Equipment that exacerbate voltage instability:
  - Induction motors
  - Distribution voltage regulators
  - Tap-changing transformers
  - Thermostatic loads

  Problem: The last three devices - during normal operation - attempt to restore the power consumed by the loads

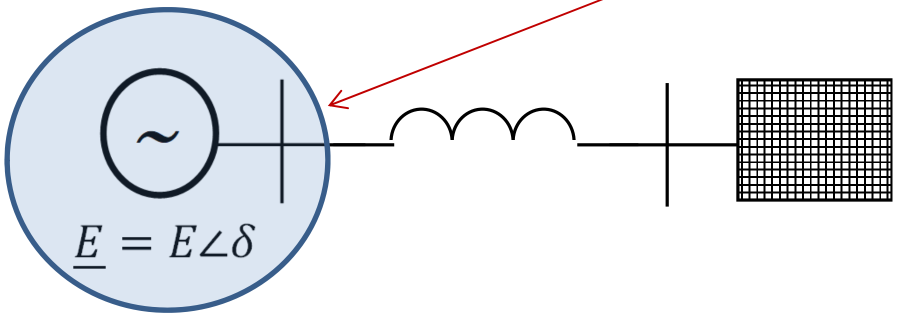
  (during a contingency situation: this characteristic is not desirable)

  - A run down situation causing voltage instability occurs when the load dynamics attempt to restore power consumption beyond the capability of the transmission network and the connected generation

Voltage stability vs. transient stability

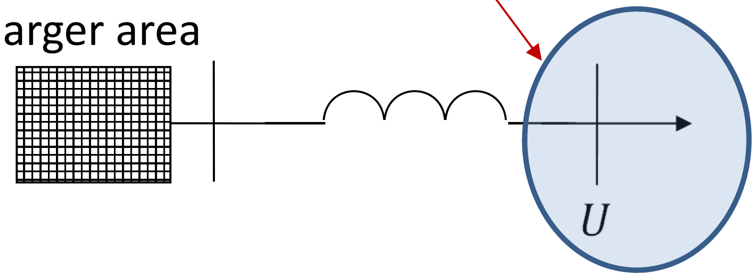
Transient stability: example: remote generator connected to a large system

- pure rotor angle stability concern
- transient stability is generator stability



Voltage stability: example: radial feeder supplying power from large system to a remote load

- pure voltage stability concern
- voltage stability is load stability
- typically involves a load area in a power system, but can cascade to blackout in larger area



Angle and voltage stability phenomena can interact with one another:

- e.g., rotor angle swings may cause voltage swings

2. Classification of voltage stability

Voltage stability classification

Normal operation:

- acceptable voltage levels at all system buses; voltage is controllable in the operating point; and it would survive a contingency or change in the system

Short-term (transient) voltage stability:

- is characterized by a large disturbance and a rapid response of the power system and its components, e.g. induction motors
- time frame is one to several seconds which is also a period in which automatic control devices at generators react.

Long-term voltage stability or collapse:

- is characterized by a large disturbance and subsequent process of load restoration or load change
- time frame: 0.5 - 30 min

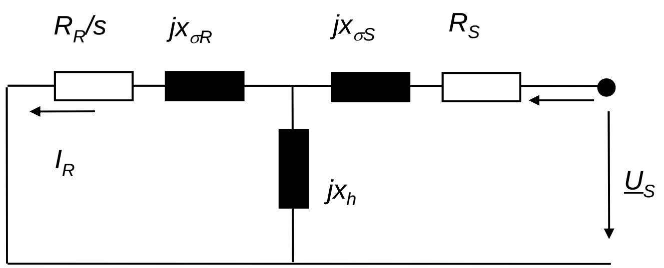
2.1 Short-term voltage stability

Short-term voltage stability

Short-term voltage stability

- A common scenario is a large disturbance such as a multi-phase fault near a load center that decelerates motors
- Following fault clearing with possible transmission outages, motors draw very high current while simultaneously attempting to reaccelerate
  - may stall if the power system is weak
  - massive loss of load and possibly area instability and voltage collapse may follow.

$$s = \frac{\omega_S - \omega_R}{\omega_S}$$

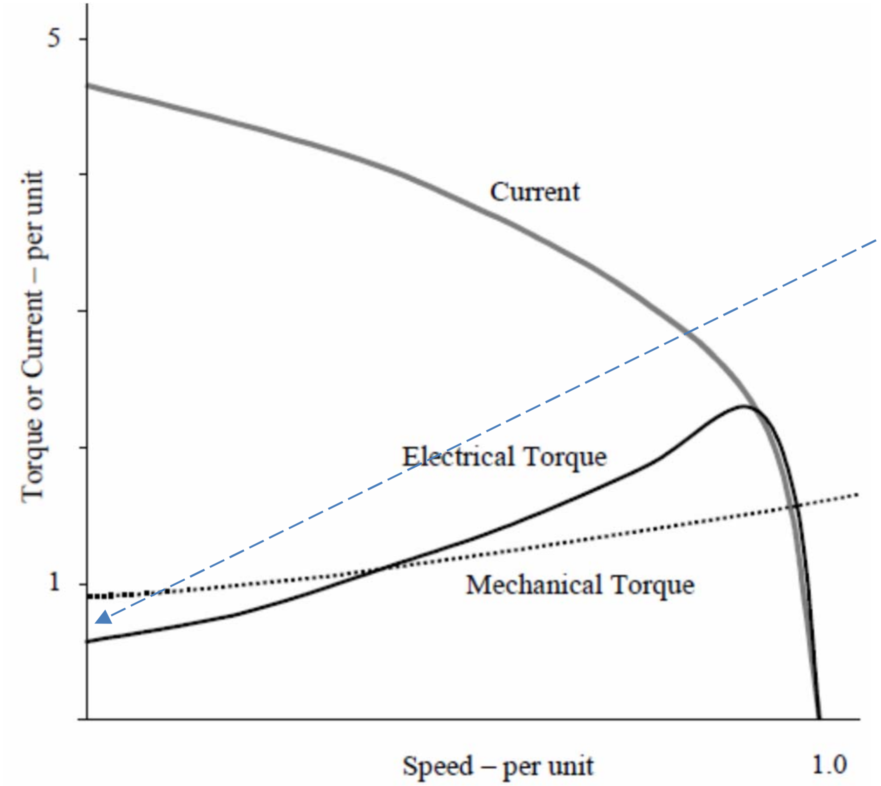


Short-term voltage stability

Short-term voltage stability

- Short term voltage stability is associated with induction motors, especially residential air conditioners and heat pumps:
  - Short circuits slow low-inertia air-conditioner compressor motors, requiring high current similar to the starting current
  - Motors may stall, preventing fast voltage recovery after short-circuit clearing
  - Compressor motors are tripped only after overheating, 3 – 15 s after stalling
  - Cascading motor stalling within few seconds may follow

Short term voltage stability - illustration



Generic torque – speed and current – speed characteristics of an air conditioner compressor motor.

Basic characteristics

– Electrical torque ~ U^2

– Low power factor at low speed

– Mechanical torque ~ ambient temperature.

Electrical torque > mechanical torque:

- motors will reaccelerate to near normal speed

If not:

- Motors will rundown and stall - drawing high currents at low power factor
- ✓ Stalling of motors near ends of feeders may cause cascaded stalling of other motors
- ✓ Motor tripping may occur:
  - by motor control or protection in response to low voltage,
  - after a few seconds by power system overcurrent relays,
  - or after many seconds by motor thermal protection.

Motor loads - cause of short term voltage instability

- The problem: motor loads draw very high current when starting or when slowed because of disturbances.
  - Heavily loaded, constant torque type mechanical loads are the main culprits
  - The potential for voltage stability problems is increased since both shunt capacitor bank reactive power, and induction motor electrical torque decrease with the *square* of the voltage.
- Because of mechanical and electrical torque characteristics, and low inertia, residential air conditioner compressor motors on phases affected by short circuits will slow considerably and draw high current.
  - *All* may stall if the network is weak

Motor loads - cause of short term voltage instability

- Unless disconnected by power system overcurrent relays, most residential air conditioners trip only by thermal protection many seconds after stalling.
  - Typical air conditioners will automatically restart in about 30 s.
- Commercial and industrial motors generally will trip during severe voltage reduction. Some commercial motors have a protection module that trips at around 70% voltage with about 0.1 s delay.
  - This tripping may be very advantageous in avoiding voltage collapse

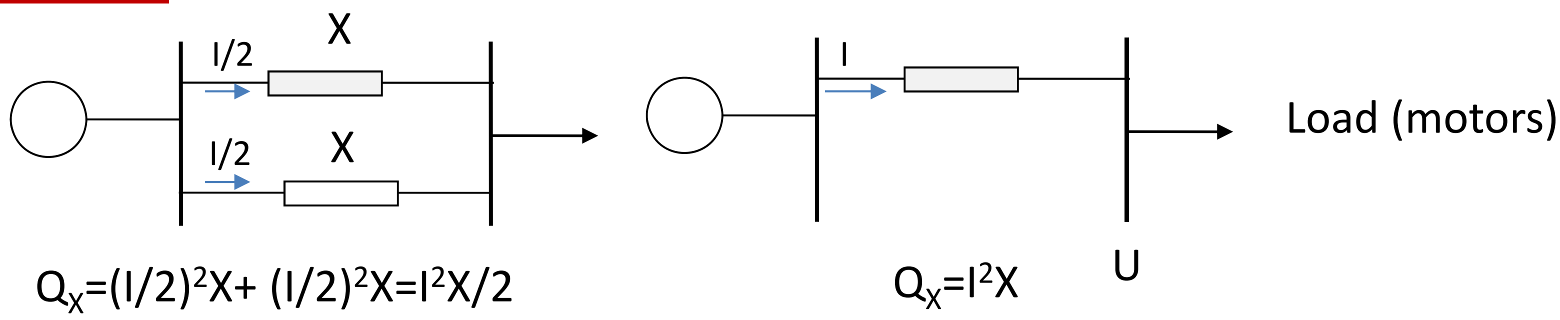
2.2 Long-term voltage stability

Long term voltage stability

- The ability of the network to transfer power from point of production to point of consumption results in:
  - Voltage drop mainly due to reactive power transfer
  - Reactive power loss (I^2X) mainly due to active power transfer
- This capability is determined by:
  - the ability of the system to control voltages following large disturbances such as system faults, loss of generator or circuit contingency
  - the load characteristics and the interaction of both control and protection systems
- A criterion for voltage stability is that, following a given disturbance and system control actions, voltages at all buses attain acceptable steady state values

Voltage instability mechanism basics

Simple example:

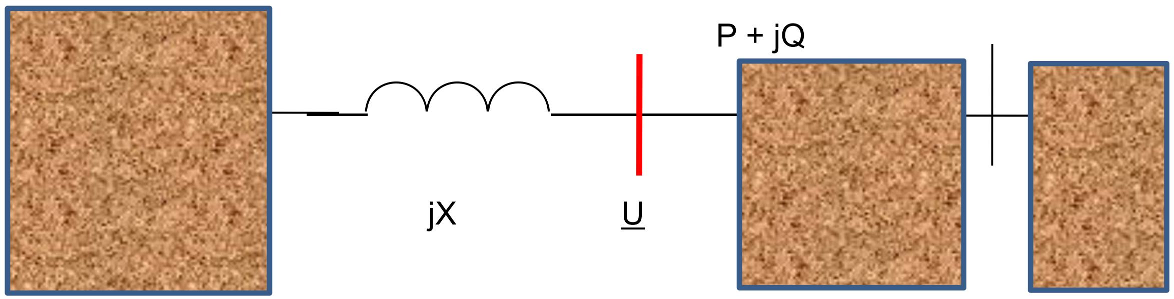


- Assume one of the transmission lines trips due to fault/heavy loading → Voltage (U) will sag due to the increased voltage drop ($\Delta U = I X$, instead of $I X/2$)
  - Induction motors will slow down and absorb more reactive power, and perhaps stall (for a large voltage drop)
  - Voltage control devices (e.g. tap changing transformer) try to restore the normal voltage
  - This stresses and overloads the system even further, possibly resulting in voltage instability and collapse
- Voltage stability is load stability, related to load demand versus load supply capability

Voltage stability cont..

– Long Term:

- Involves slow acting equipment:
  - Tap changing transformers
  - Thermostatically controlled loads
  - Generator excitation current limiters
- Instability is due to the loss of long-term equilibrium

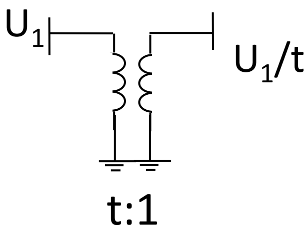


Voltage instability ↔ collapse of U

Tap-changing transformer

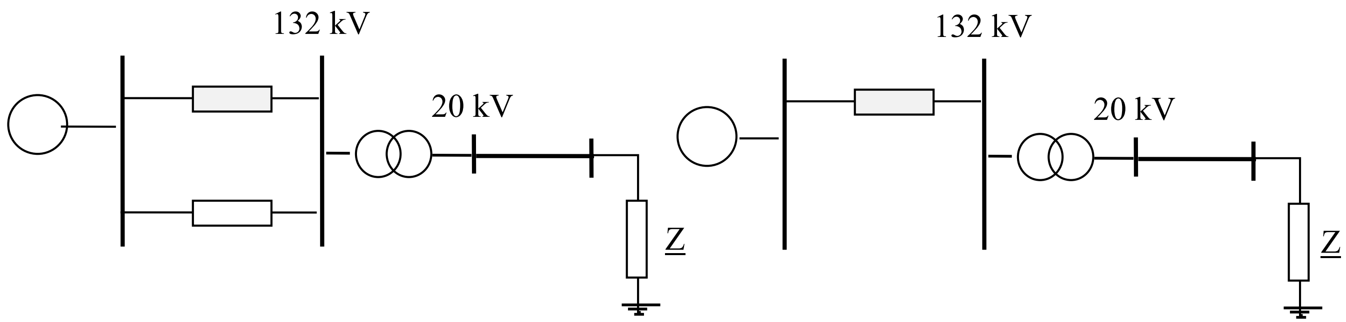
Transformer tap changers

- Load tap changers (LTC, alternatively OLTC, ULTC, TCUL) are transformers that connect the transmission or sub-transmission systems to the distribution systems.
- They are typically equipped with regulation capability that allows them to control the voltage on the low voltage side so that voltage deviation on the high voltage side is not seen on the low voltage side.



- t may range from 0.85 - 1.15 pu
- For incoming low primary voltage, the LTC will decrease t in order to try to raise the outgoing (secondary) voltage, i.e. U_1/t.
- If performed in 60 steps, a single step may introduce ca. 0.005 pu voltage change
- a change of one step can require up to 5 s
- a dead-band in the equivalent of 2-3 taps used to prevent excessive tap activity

Transformer tap-changer



- Effect of transformer tap changer action during voltage sag
  - Tap changing transformers try to restore the normal voltage at the load by increasing the voltage at transformer secondary terminal

As seen from the HV side:

<u>Example</u>:

Previous:

$$\underline{Z}' = \underline{Z}\left(\frac{132}{20}\right)^2 = 43.56\ \underline{Z}$$

After tap-changer action

$$\underline{Z}' = \underline{Z}\left(\frac{132}{22}\right)^2 = 36\ \underline{Z}$$

Assuming the TCUL changes the voltage ratio from 132/20 to 132/22 kV to compensate for the increased voltage drop on the transmission line.

  - This stresses and overloads the power system even further, resulting in voltage instability and collapse

Role of transformer tap-changer cont..

- Typical disturbance leading to voltage instability involves loss of generator or loss of a major transmission line
  - The disturbance causes high reactive power losses and voltage sags in load areas
- The tap changer senses low voltages and acts to restore the voltage, thereby restoring load power levels
  - This causes further sags of transmission voltages
- Generator farther away must then provide reactive power; may lead to generator over-excitation limit
  - The result may be partial or complete voltage collapse

Tap changers

- LTC in regulating mode (limit not reached)
  - voltage decline on the high voltage side does not result in voltage decline at the load
  - in steady-state, constant Z leads to constant power
  - a simple load model for voltage instability analysis, for systems using LTC, is constant power
- There are two qualifications to using such a simple (constant power) model, however:
  1. "Fast" voltage dips are seen at the low side (since LTC action typically requires minutes), and if the dip is low enough, induction motors may trip, resulting in an immediate decrease in load power.
  2. Once the LTC hits its limit (minimum t), then the voltage on the low voltage side begins to decline, and it becomes necessary to model the load - voltage sensitivity

Load model

Thermostatic load recovery

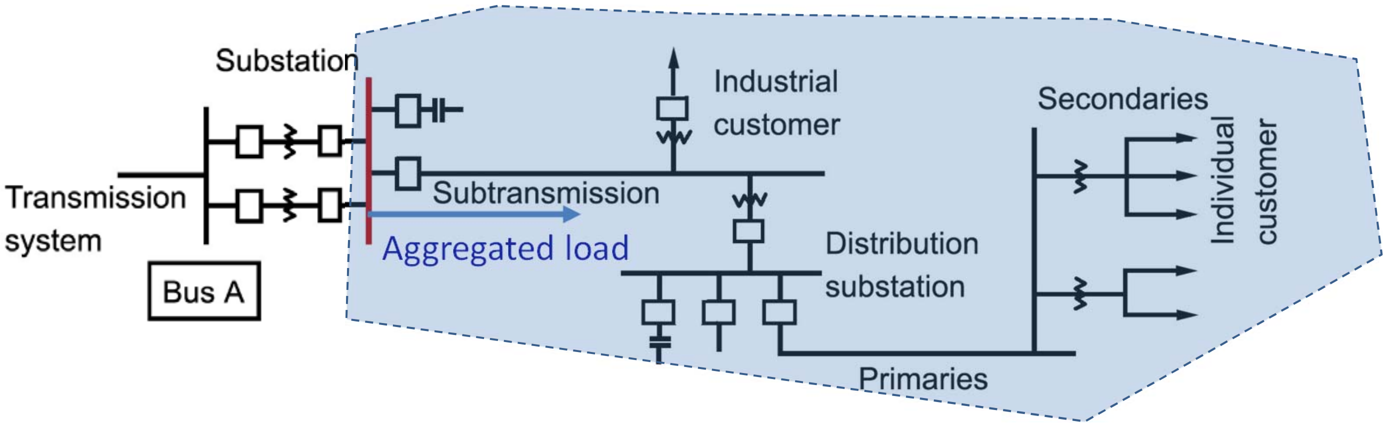
- Heating load is the most common type of thermostatic load
- Other thermostatic loads include space heaters/coolers, water heaters, and refrigerators
- When voltage drops, thermostatic loads initially decrease in power consumption. When the voltage remains low for a few minutes, the load regulation devices (thermostats) will start the loads or will maintain them for longer periods so that more of them are on at the same time.
  - This characteristic tends to exacerbate voltage stability problems during a contingency situation

Load Modelling

- A typical load bus to be represented in stability studies is composed of a large number of devices:
  - fluorescent and incandescent lamps, refrigerators, heaters, compressors, furnaces, and so on
- The composition of the load changes, depending on many factors, including:
  - time
  - weather conditions
  - state of the economy
- The exact composition at any particular time is difficult to estimate. Even if the load composition were known, it would be impractical to represent each individual component.
- For the above reasons, load representation is based on considerable amount of simplification.

Basic Load Modelling Concepts

- The aggregated load is usually represented at a transmission substation
- includes, in addition to the connected load devices, the effects of step-down transformers, sub-transmission and distribution feeders, voltage regulators, and var compensation, etc



- Load models are traditionally classified into:
  - static load models
  - dynamic load models

Static Load Models

- Express the load characteristics as algebraic functions of bus voltage magnitude and frequency.
- Traditionally, voltage dependency has been represented by the exponential model:

$$P = P_0\left(\frac{U}{U_0}\right)^a \qquad Q = Q_0\left(\frac{U}{U_0}\right)^b$$

P_0, Q_0, and U_0 are the values of the respective variables at the initial operating condition.

- For composite loads,
  - exponent "a" ranges between 0.5 and 1.8
  - exponent "b" ranges between 1.5 and 6
- The exponent "b" is a nonlinear function of voltage. This is caused by magnetic saturation of distribution transformers and motors.

Static load models

$$P = P_0 \left(\frac{U}{U_0}\right)^a \qquad Q = Q_0 \left(\frac{U}{U_0}\right)^b$$

Typical values for a and b (example):

	a	b
Incandescent lamps	1.54	-
Room air conditioner	0.50	2.5
Furnace fan	0.08	1.6
Battery charger	2.59	4.06
Electronic compact florescent	1.0	0.40
Conventional florescent	2.07	3.21

ZIP load model

- An alternative static model widely used is the *polynomial model*:

$$P = P_0\left(p_1\left(\frac{U}{U_0}\right)^2 + p_2\left(\frac{U}{U_0}\right) + p_3\right) \qquad Q = Q_0\left(q_1\left(\frac{U}{U_0}\right)^2 + q_2\left(\frac{U}{U_0}\right) + q_3\right)$$

- This model is commonly referred to as the "**ZIP**" model, as it is composed of constant impedance (Z), constant current (I), and constant power (P) components.
- The frequency dependency of load characteristics is usually represented by multiplying the exponential or polynomial model by a factor:

For example,

$$P = P_0\left(p_1\left(\frac{U}{U_0}\right)^2 + p_2\left(\frac{U}{U_0}\right) + p_3\right)(1 + K_{pf}\Delta f) \qquad Q = Q_0\left(q_1\left(\frac{U}{U_0}\right)^2 + q_2\left(\frac{U}{U_0}\right) + q_3\right)(1 + K_{qf}\Delta f)$$

where Δf is the frequency deviation (f-f_0). Typically, K_{pf} ranges from 0 to 3.0, and K_{qf} ranges from -2.0 to 0.

Response of most loads is fast and steady state reached quickly, at least for modest changes in V and f.

- use of static model justified in such cases

Effect of lad charactrisitc on voltage stability

- Risk of voltage instability is reduced when the demand drops together with the voltage
    - Smaller I smaller Q = I^2X

- With regard to voltage instability:
    - constant impedance load (p_1) is favorable since power decreases with square of voltage
    - constant current load (p_2) is OK since power decreases linearly with voltage.
    - Constant power load (p_3) is unfavorable since power does not change with the voltage
- Exponential or polynomial load models cannot adequately represent:
    - Thermostatic load recovery
    - Induction motor stalling/tripping
    - Load tap changers

Dynamic Load Models

- In many cases, it is necessary to account for the dynamics of loads. For example, in studies of
  - inter-area oscillations and voltage stability
  - systems with large concentrations of motors
- Typically, motors consume 60% to 70% of total energy supplied by a power system
  - dynamics attributable to motors are usually the most significant aspects
- Other dynamic aspects of load components include:
  - Extinction of discharge (mercury vapor, sodium vapor, fluorescent) lamps when voltage drops below 0.7 to 0.8 pu and restart after 1 or 2 seconds delay when voltage recovers.
  - Operation of protective relays. For example, starter contactors of industrial motors drop open when voltage drops below 0.55 to 0.75 pu.
  - Thermostatic controlled loads such as space heaters/coolers, water heaters and refrigerators - operate longer during low voltages and hence, total number of devices increase in a few minutes.
  - Response of ULTCs on distribution transformers and voltage regulators

Modelling of Induction Motors

- The general procedure is similar to that of a synchronous machine
  - first write basic equations in terms of phase (a,b,c) variables
  - then, transform equations into 'dq' reference frame
- In developing the model of an induction motor it is worth noting the following regarding its features which differ from those of the synchronous machine:
  - rotor has a symmetrical structure; hence, d and q axis equivalent circuits are identical
  - rotor speed is not fixed; this has an impact on the selection of dq reference frame
  - there is no excitation source applied to the rotor; consequently the rotor circuit dynamics are determined by slip rather than by excitation control.
  - currents induced in shorted rotor windings produce a field with the same number of poles as in the stator; therefore, rotor windings may be represented by equivalent 3-phase winding

System of equations

- The 'dq' transformation:
  - the preferred reference frame is one with axes rotating at synchronous speed, rather than at rotor speed
- The machine equations in dq reference frame:

- Stator flux linkages:

$$\Psi_{ds} = L_{ss} i_{ds} + L_m i_{dr}$$

$$\Psi_{qs} = L_{ss} i_{qs} + L_m i_{qr}$$

- Rotor flux linkages:

$$\Psi_{dr} = L_{rr} i_{dr} + L_m i_{ds}$$

$$\Psi_{qr} = L_{rr} i_{qr} + L_m i_{qs}$$

- Stator voltages:

$$V_{ds} = R_s i_{ds} - \omega_s \Psi_{qs} + p\Psi_{ds}$$

$$V_{qs} = R_s i_{qs} + \omega_s \Psi_{ds} + p\Psi_{qs}$$

- Rotor voltages:

$$V_{dr} = R_r i_{dr} - (p\theta_r)\Psi_{qr} + p\Psi_{dr}$$

$$V_{qr} = R_r i_{qr} + (p\theta_r)\Psi_{dr} + p\Psi_{qr}$$

The term $p\theta_r$ is the slip angular velocity and represents the relative angular velocity between the rotor and the reference *dq* axes

Synchronous Motor Model

- A synchronous motor is modelled in the same manner as a synchronous generator
  - the only difference is that, instead of the prime mover providing mechanical torque input to the generator, the motor drives a mechanical load

- As in the case of an induction motor, a commonly used expression for the load torque is

$$T_m = T_0 \omega_r^m$$

Rotor acceleration equation is $$\frac{d\omega_r}{dt} = \frac{1}{2H}\left(T_e - T_m\right)$$

Acquisition of Load Model Parameters

- Two basic approaches:
  - measurement-based approach
  - component based approach
- Measurement-based approach
  - load characteristics measured at representative substations and feeders at selected times
  - parameters of loads throughout the system extrapolated from the above
- Component-based approach
  - involves building up the load model from information on its constituent parts
  - load supplied at a bulk power delivery point categorized into *load classes* such as residential, commercial, and industrial
  - each load class represented in terms of its components such as lighting, heating, refrigeration
  - individual devices represented by their known characteristics

Load Class Static Characteristics

Sample characteristics of different load classes.

Load Class	Power Factor	∂P/∂V	∂Q/∂V	∂P/∂f	∂Q/∂f
Residential					
- summer	**0.9**	**1.2**	**2.9**	**0.8**	**-2.2**
- winter	**0.99**	**1.5**	**3.2**	**1.0**	**-1.5**
Commercial					
- summer	**0.85**	**0.99**	**3.5**	**1.2**	**-1.6**
- winter	**0.9**	**1.3**	**3.1**	**1.5**	**-1.1**
Industrial	**0.85**	**0.18**	**6.0**	**2.6**	**1.6**
Power plant auxiliaries	**0.8**	**0.1**	**1.6**	**2.9**	**1.8**

Dynamic Characteristics

The following are sample data for induction motor equivalents representing three different types of load.

(i) The composite dynamic characteristics of a feeder supplying predominantly a commercial load:

$R_s = 0.001$ $X_s = 0.23$ $X_r = 0.23$

$X_m = 5.77$ $R_r = 0.012$ $H = 0.663$ $m = 5.0$

(ii) A large industrial motor:

$R_s = 0.012$ $X_s = 0.07$ $X_r = 0.165$

$X_m = 3.6$ $R_r = 0.01$ $H = 1.6$ $m = 2.0$

(iii) A small industrial motor:

$R_x = 0.025$ $X_s = 0.10$ $X_r = 0.17$

$X_m = 3.1$ $R_r = 0.02$ $H = 0.9$ $m = 2.0$

Component Static Characteristics

The following table summarizes typical voltage and frequency dependent characteristics of a number of load components.

Component	Power Factor	∂P/∂V	∂Q/∂V	∂P/∂f	∂Q/∂f
Air conditioner					
- 3-phase central	0.90	0.088	2.5	0.98	-1.3
- 1-phase central	0.96	0.202	2.3	0.90	-2.7
- window type	0.82	0.468	2.5	0.56	-2.8
Water heaters, Range top, oven, Deep fryer	1.0	2.0	0	0	0
Dishwasher	0.99	1.8	3.6	0	-1.4
Clothes washer	0.65	0.08	1.6	3.0	1.8
Clothes dryer	0.99	2.0	3.2	0	-2.5
Refrigerator	0.8	0.77	2.5	0.53	-1.5
Television	0.8	2.0	5.1	0	-4.5
Incandescent lights	1.0	1.55	0	0	0
Fluorescent lights	0.9	0.96	7.4	1.0	-2.8
Industrial motors	0.88	0.07	0.5	2.5	1.2
Fan motors	0.87	0.08	1.6	2.9	1.7
Agricultural pumps	0.85	1.4	1.4	5.0	4.0
Arc furnace	0.70	2.3	1.6	-1.0	-1.0
Transformer (unloaded)	0.64	3.4	11.5	0	-11.8

Synchronous machine capability curve

Operational limits of synchronous generator

- The loading limits of a synchronous generator are determined by its size and construction, i. e, by:
  - Cooling of the rotor and stator windings,
  - Iron saturation limit,
  - The capacity of the excitation system, etc.
- These technical limits must be observed during operation at all times.
- The operational chart of the synchronous machine shows these boundaries

Operational chart of a synchronous machine

1. **Limit: Maximum armature current**

- The maximum apparent power *S* is the limit with regard to stator heating
  - This limit is given by the equation:

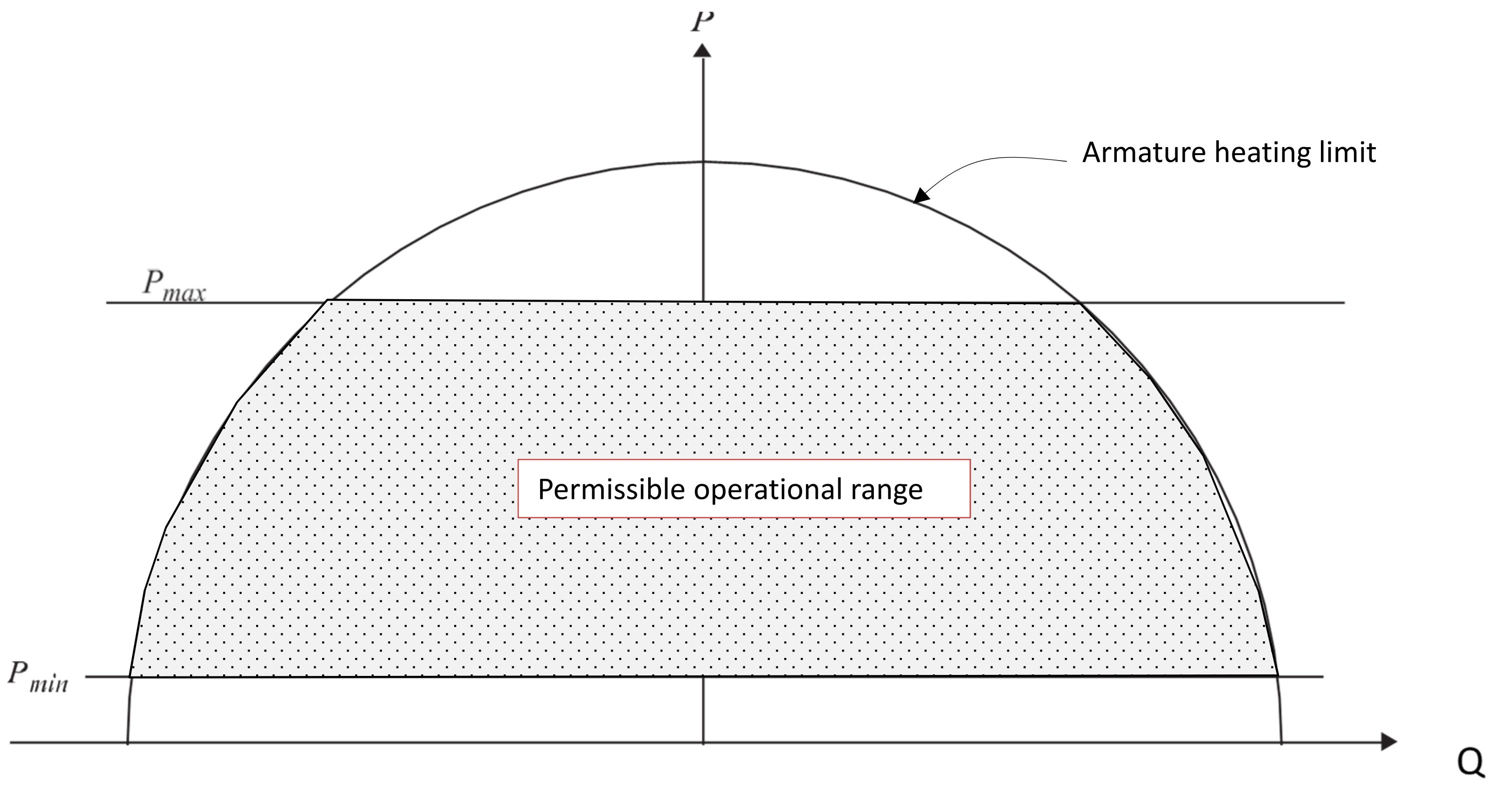
$$S^2 = P^2 + Q^2 = konst.$$

Minimum/maximum active power limit

2. Limit: minimum and maximum active power loading

- For thermal power plants, these limits (Pmin, Pmax) are determined by the minimum and maximum steam flow through the turbine
- For hydropower generators: Pmin = 0, and Pmax determined by the maximum flow rate of the water.
  - The minimum and maximum deliverable active power, Pmin and Pmax, are shown as horizontal lines in the operational chart

Active power / armature heating limits



Under excitation limit

3. Limit: Steady state under-excitation limit

$$\underline{I}_a = \frac{\underline{U}_p - U}{jX_d} = \frac{U_p\angle\delta - U}{jX_d} = \frac{U_p\angle\delta - \pi/2 + jU}{X_d}$$

Up: excitation voltage
U: terminal voltage

$$\underline{S} = 3\cdot U\cdot \underline{I}_a^* = 3\cdot U\cdot \frac{U_p\angle-\delta+(\pi/2) - jU}{X_d} = \frac{3\cdot U\cdot U_p\cdot\sin\delta}{X_d} + j\frac{3\cdot U\cdot U_p\cdot\cos\delta - U^2}{X_d}$$

$$P = \frac{3\cdot U\cdot U_p\cdot\sin\delta}{X_d} = \omega_m\cdot M \qquad Q = \frac{3\cdot U\cdot U_p\cdot\cos\delta - U^2}{X_d}$$

- The theoretical stability limit is δ = 90°.
  - To allow a saftey margin, however, a maximum power angle is limited to δ_{max} = 70° (for example).

Stability limit

$$\sin\delta = \frac{P \cdot X_d}{3 \cdot U \cdot U_p}$$

$$\cos\delta = \frac{Q \cdot X_d + U^2}{3 \cdot U \cdot U_p}$$

$$\tan\delta = \frac{P \cdot X_d}{Q \cdot X_d + U^2} \rightarrow P = Q \cdot \tan\delta + \frac{U^2}{X_d} \cdot \tan\delta$$

The stability limit in a Q – P plane (assuming δ_{max} = 70°) is:

- a straight line with the slope: tan 70° = 2.75
- X-intercept: $P = 0 \rightarrow Q = -\frac{U^2}{X_d}$

Minimum/maximum excitation

4. Limit: Maximum allowable rotor heating ($U_{P,max}$)

- Most generators are equipped with AVR
- By changing the excitation current, the excitation voltage and thus also the terminal voltage of the generator can be controlled.
- The voltage regulation specifies a minimum excitation voltage $U_{p,min}$.
- The maximum excitation voltage $U_{p,max}$ is predetermined by the maximum permissible heating of the rotor winding.
  - These two limits can be represented in the operating diagram by two circles.

$$\sin\delta = \frac{P \cdot X_d}{3 \cdot U \cdot U_p} \qquad \cos\delta = \frac{Q \cdot X_d + U^2}{3 \cdot U \cdot U_p}$$

$$\left(\frac{3 \cdot U \cdot U_p}{X_d}\right)^2 = P^2 + \left(Q + \frac{U^2}{X_d}\right)^2$$

Maximum/maximum excitation

Locus:

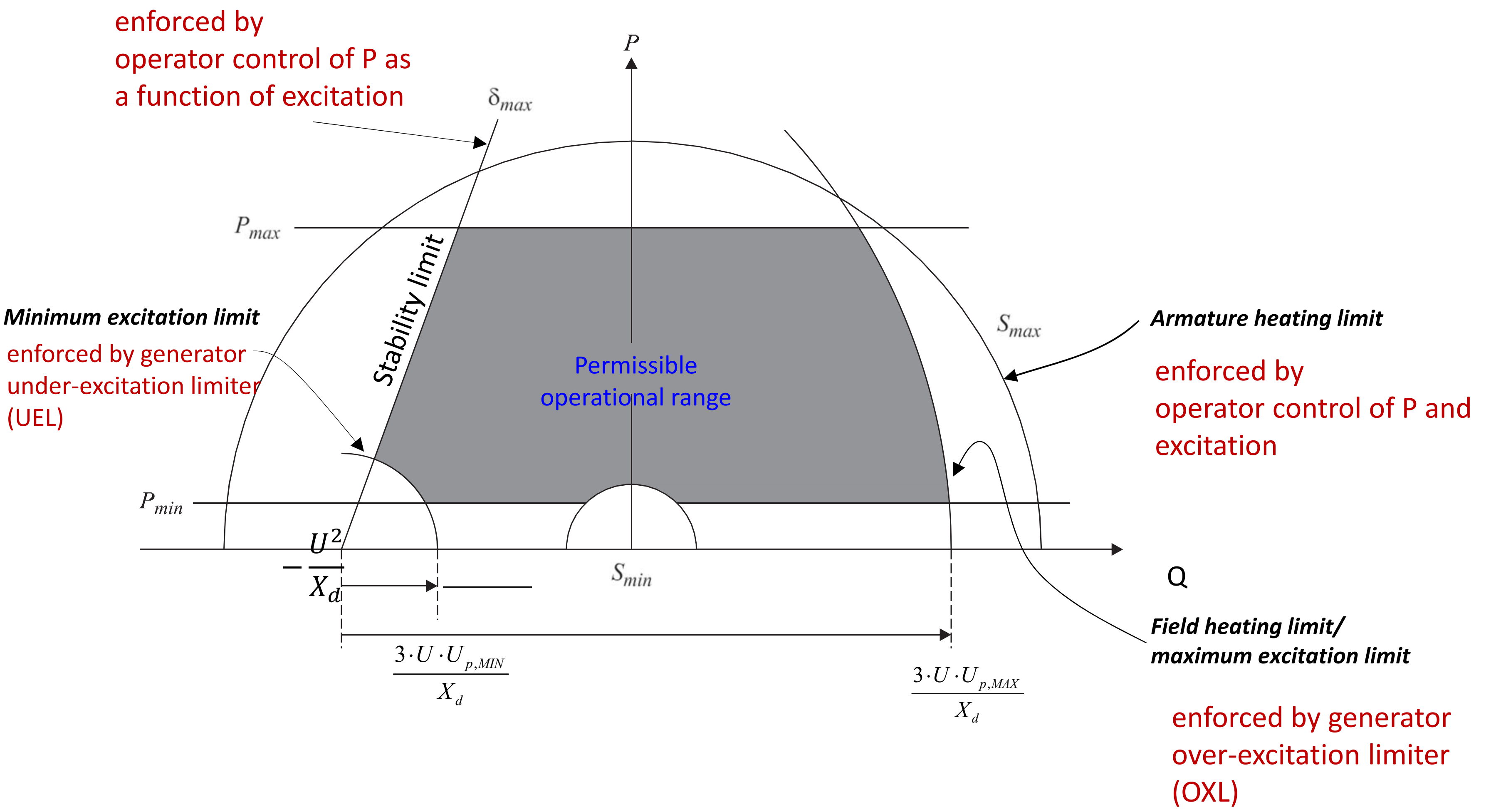
$$\left(\frac{3\cdot U\cdot U_p}{X_d}\right)^2 = P^2 + \left(Q + \frac{U^2}{X_d}\right)^2$$

Equation of a circle with origin: $\left(-\frac{U^2}{X_d}, 0\right)$ and radius: $\frac{3\cdot U\cdot U_p}{X_d}$

Minimum excitation $\frac{3\cdot U\cdot U_{p,MIN}}{X_d}$

Maximum excitation: $\frac{3\cdot U\cdot U_{p,MAX}}{X_d}$

Operational diagram



Effect of generator reactive power limit

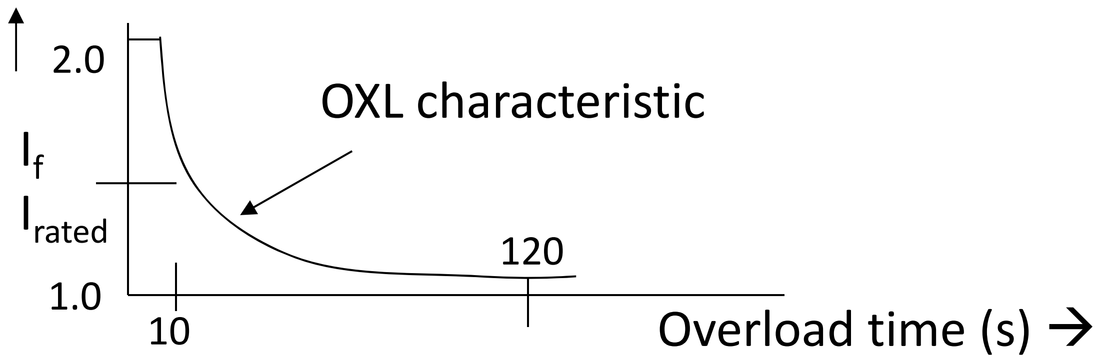
- Voltage instability is typically preceded by generators hitting their upper reactive limit (OXL)
  - Accurate modeling of Q_{max} is very important for the analysis of voltage instability
- Most power flow programs represent generator Q_{max} as fixed.
  - However, this is an approximation. In reality, Q_{max} is not fixed. The generator capability diagram shows quite clearly that Q_{max} is a function of P and becomes more restrictive as P increases.
  - A first-order approximation(instead of using fixed Q_{max}) is to model Q_{max} as a function of P.

Effect of generator reactive power limit

- Q_{max} is limited by the Over-eXcitation Limiter (OXL). The field circuit has a rated steady-state field current $I_{f\text{-}max}$, set by field circuit heating limitations. Since heating is proportional to:

$$\int_{\text{overload time}} I_f^{\,2}\,dt$$

- Small overloads can be tolerated for longer times. Therefore, most modern OXLs are set with a time-inverse characteristic:



- As soon as the OXL acts to limit I_f, then no further increase in reactive power is possible.
- When drawing PV or QV curves, the action of a generator hitting Q_{max}, will manifest itself as a sharp discontinuity in the curve.

Basic countermeasures

- Basic strategy:
  - Apply shunt capacitor banks, mainly in distribution and load area transmission substations to minimize reactive power transmission, allowing automatically controlled reactive power reserve at generators
  - Design and operate transmission network for high, flat voltage profile to minimize I^2X losses
- Switched shunt capacitor banks:
  - Local or wide-area control
- Series capacitor banks
- Static var compensators or STATCOMs for short-term voltage stability:
- Load shedding:
  - Local undervoltage or wide-area load shedding

3. Static voltage stability assessment methods

Voltage security assessment

- Voltage security quantified by margins or indices
  - Candidate margins or indices?
  - Voltage monitoring sufficient?
  - Additionally: reactive power reserves need to be monitored
- Generator Time-Overload Capability
- Field circuit overload usually occurs first, but armature current overload also important:
  - Field current (generator excitation) overload caused by outages and load restoration is key mechanism of long-term voltage instability
  - Field current closely related to reactive power output. But armature current overload also usually reduced by field current reduction (reactive power rather than active power typically reduced).
- Generator automatic voltage regulator includes limiter of field current time-overload:
  - Time frame is tens of seconds
  - Limiter set inside time-overload capability required by standards

Analysis Methods

Two different approaches are used to analyse voltage stability problem in power systems

- Static analysis
- Dynamic analysis

Static analysis methods

- The static approach
  - Static methods involve the static model of power system components. These methods are important when the power system is in operation and planning stages, in order to prepare an adequate plan for meeting the reactive power requirements during different types of contingencies arising during its operation.

- The static approach involves the computation of only algebraic equations
  - effectively analyzes the viability of a specific operating point of the power system
  - provides important information such as sensitivity or stability margin
  - much faster than dynamic approaches
  - enables the analysis of power systems over a wide range of system conditions

Dynamic methods

- The dynamic approach
  - The dynamic methods use time domain simulations to reveal the voltage collapse mechanism such as why and how the voltage collapse occurs. Dynamic methods analyze the effect of dynamic loads, on load tap changes (OLTC), generator over excitation limiters on voltage collapse
- Dynamic analysis provides the most accurate indication of the time responses of the system.
- Therefore, dynamic analysis is necessary for fast voltage collapse situations, such as loss of generation and system faults, especially concerning the complex sequence of events that lead to the instability.
- However, dynamic simulations fail to provide information such as the sensitivity or margin of stability.
- Dynamic simulations require the dynamic models of the components and parameters
- Simulation may be time- consuming

Voltage stability assessment

- Dynamic simulation essential for short-term voltage stability (similar to angle stability):
  - Dynamic models for motor load, generators, SVCs
- Dynamic simulation also useful for long-term voltage stability:
  - Coordination of controls
  - Greater accuracy and insight

Static voltage stability assessment

Static simulation using power flow program for approximate analysis and screening:

P-V Curves:

- Post-disturbance loadability
- Assessment of whether an operating point is secure or not

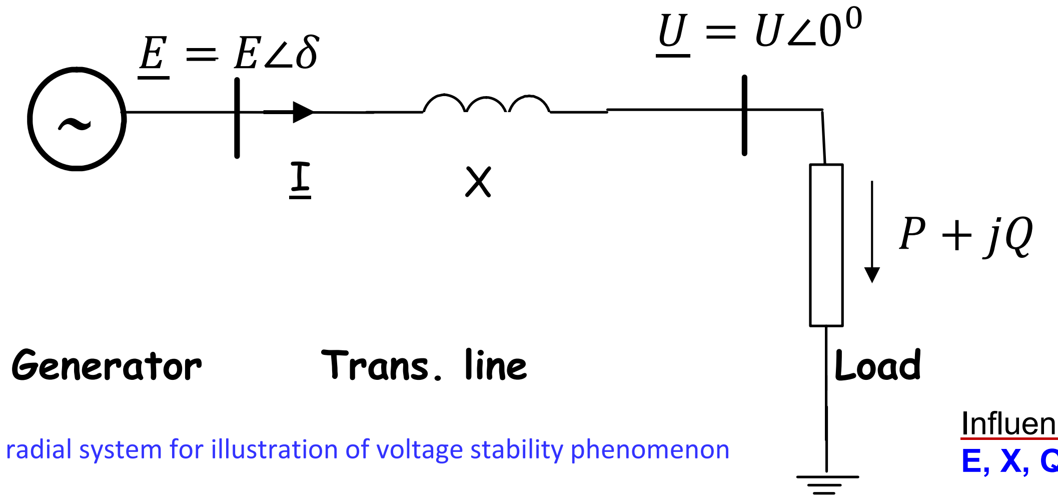
Q-V / V-Q Curves:

- Tests bus strength and helps determine possible reactive power compensation requirement

PV Curve

Derivation of the PV curve

- The PV curve (also called the "nose curve") vividly demonstrates the nature of the voltage stability problem



A simple radial system for illustration of voltage stability phenomenon

$U = f\ (P)$

Influencing factors:
E, X, Q

$$\underline{I} = \frac{\underline{E} - U}{jX}$$

$$\underline{S} = P + j\,Q = U\,\underline{I}^* = U\left(\frac{\underline{E} - U}{jX}\right)^*$$

$$P = \frac{E\,U}{X}\sin\delta$$

$$Q = \frac{E\,U}{X}\cos\delta - \frac{U^2}{X}$$

PV curve cont…

$$P = \frac{E\,U}{X}\sin\delta$$

$$Q = \frac{E\,U}{X}\cos\delta - \frac{U^2}{X}$$

$$\sin\delta = \frac{PX}{EU}$$

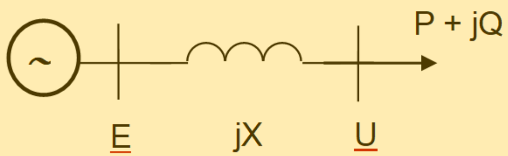
$$\cos\delta = \frac{QX + U^2}{X}$$

$$\cos^2\delta + \sin^2\delta = 1: \quad \left(\frac{P\,X}{E\,U}\right)^2 + \left(\frac{Q\,X + U^2}{E\,U}\right)^2 = 1$$

$$\frac{U^4}{E^4} + \frac{U^2}{E^4}\cdot\left(2\cdot Q\cdot X - E^2\right) + \frac{X^2}{E^4}\cdot\left(P^2 + Q^2\right) = 0$$

$$\left(\frac{U^2}{X}\right)^2 + \frac{U^2}{X}\left(2\,Q - \frac{E^2}{X}\right) + P^2 + Q^2 = 0$$

Voltage at the load (U) as a function of E, X, P and Q



PV curve cont ...

$$\frac{U^4}{E^4}+\frac{U^2}{E^4}\cdot\left(2\cdot Q\cdot X-E^2\right)+\frac{X^2}{E^4}\cdot\left(P^2+Q^2\right)=0$$

Let $v=\frac{U}{E},\ p=\frac{P\cdot X}{E^2},\ q=\frac{Q\cdot X}{E^2}\Rightarrow v^4+v^2\cdot\left(2\cdot q-1\right)+p^2+q^2=0$

Let φ be the power factor angle of the load, such that q = p tan φ;

The above equation can then be re-written as:

$$v^4+v^2\cdot\left(2\cdot p\cdot tan\,\varphi-1\right)+p^2+p^2\cdot sec^2\,\varphi=0 \qquad with \quad (sec\,\varphi=1/cos\,\varphi)$$

The equation can be converted into a quadratic equation with $x = v^2$, whose solution is:

$$x=v^2=\frac{-(2\ p\tan\varphi\ -1)\pm\sqrt{(2\ p\tan\varphi\ -1)^2-4\,p^2\,(1+\ tan^2\varphi)}}{2}$$

+ : 2 solutions for v, 1 pos. and the other neg.

- : 2 solutions for v, 1 pos. and the other neg.

Solution

$$x = v^2 = \frac{-(2\ p\tan\varphi\ \ -1) \pm \sqrt{(2\ p\tan\varphi\ \ -1)^2\ -4\,p^2\,(1+\ tan^2\varphi)}}{2}$$

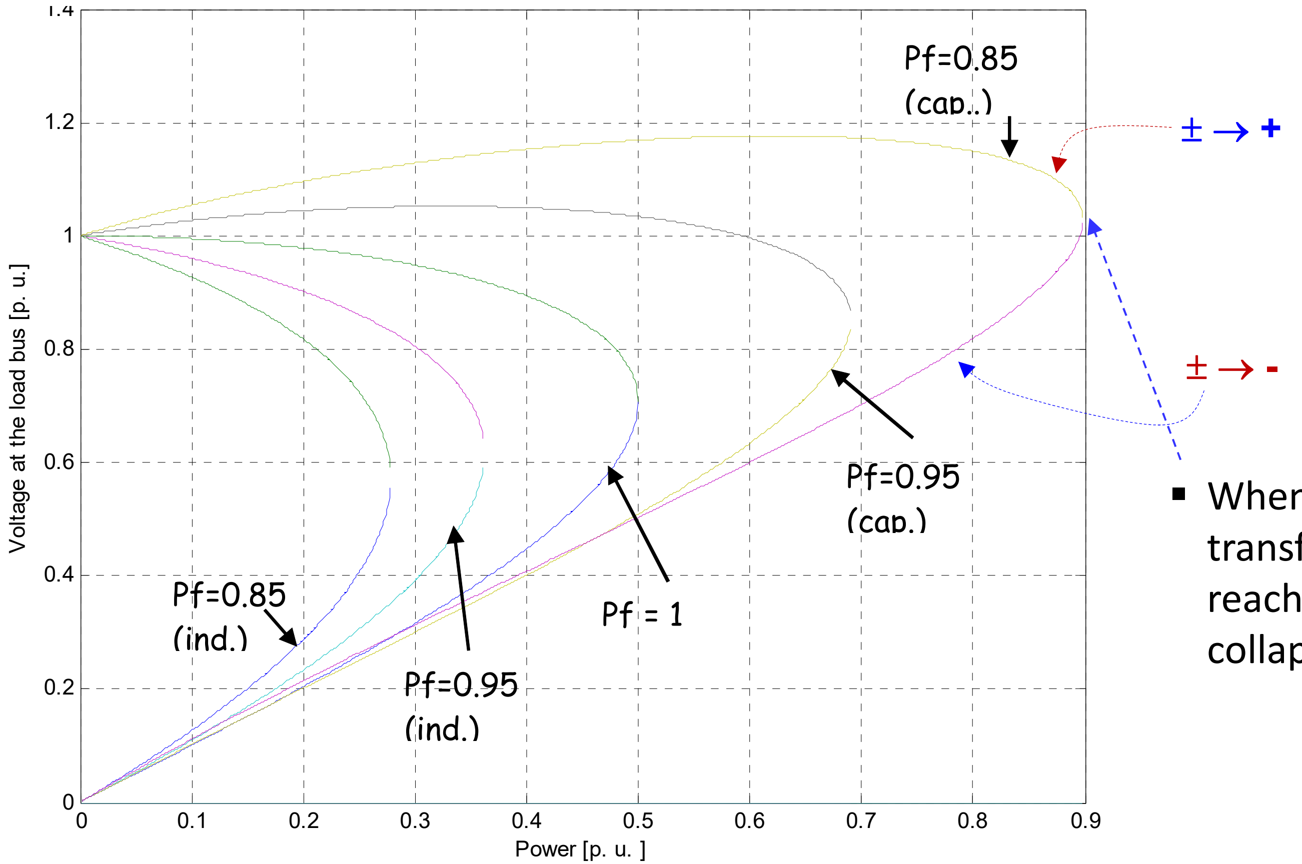
v (the voltage at the load in per unit with E as the base value) has four solutions, i.e.

- "±" replaced with "+", $v_{1,2} = \pm\sqrt{the\ above\ expression}$
- "±" replaced with "-", $v_{3,4} = \pm\sqrt{the\ above\ expression}$

In each case, only one of the two solutions is physically meaningful (positive voltage value).

Solution – the PV Curve

- “±” replaced with “+”, and the (positive) square root of this expression (p as a variable) is the solution for the upper part of the curve (STABLE OPERATING REGION).
- “±” replaced with “-”, and the (positive) square root of this expression (p as a variable) is the solution for the lower part of the curve (UNSTABLE OPERATING REGION).



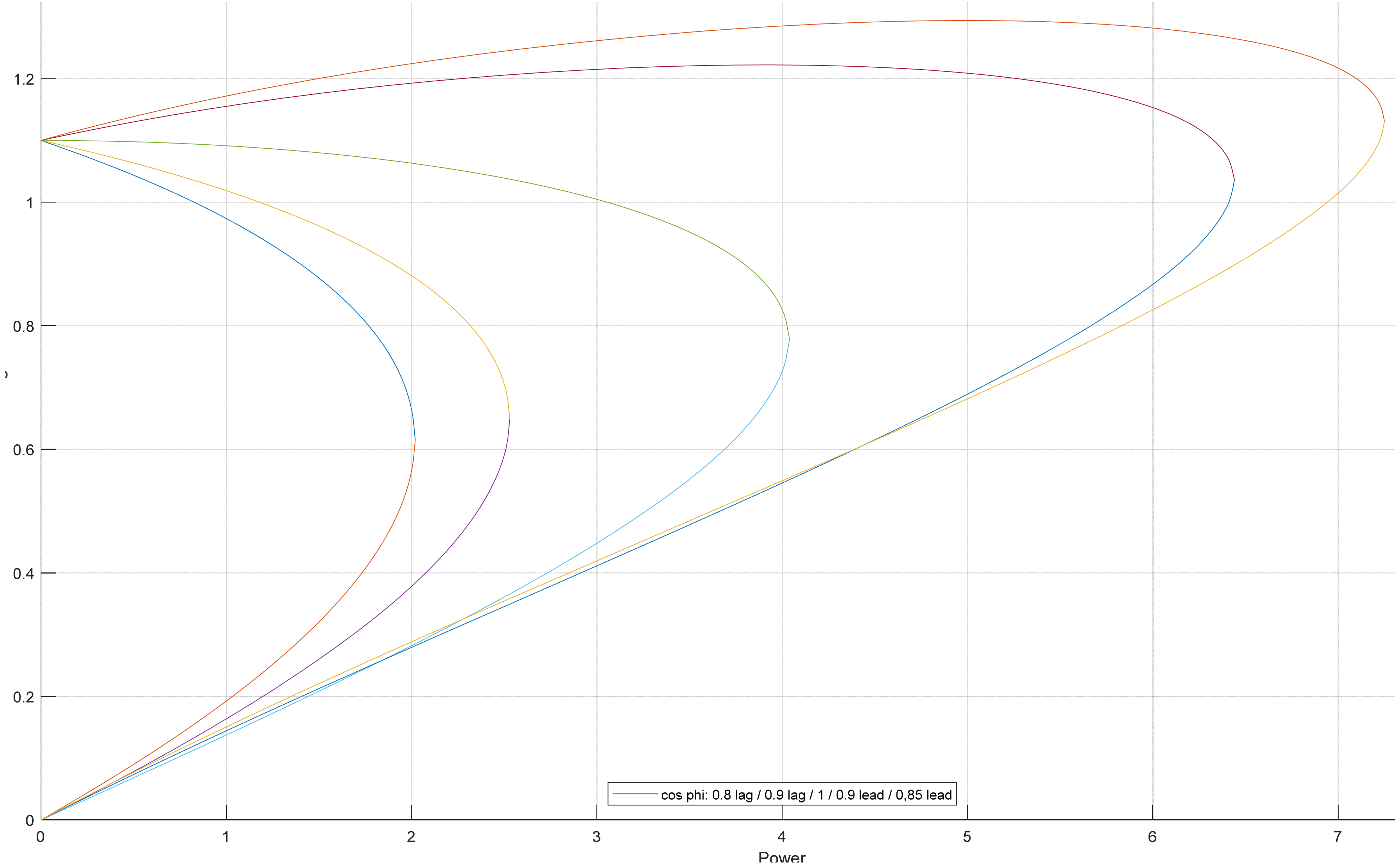
- When the power transfer limit is reached the voltage collapses

Voltage versus power curve for various load power factors. (These families of curves are called PV (Nose) Curves

MATLAB code for drawing the PV curve

```
R=0;% line resistance
X=0.15; % line reactance
E=1.1; % sending-end voltage in p. u.
P2=[0:0.01:8];% range of loads in p. u.
cosphi=[0.8 0.9 1 -0.9 -0.85];% negative --> leading
tanphi=tan(acos(cosphi));
for j=1:5
   k=0; A=zeros; B=zeros;term1=zeros;term2=zeros;x=zeros;y1=zeros;y2=zeros;
   for i=1:800
      A(i)=P2(i)*(R+X*tanphi(j));
      B(i)=P2(i)*(X-R*tanphi(j));
      term1(i)=(2*A(i)-E^2)/2;
      term2(i)=term1(i)*term1(i)-A(i)*A(i)-B(i)*B(i);
      if term2(i)>=-0.002
         k=k+1;
         y1(k)=sqrt(-term1(i)+sqrt(term2(i)));% upper part of the curve
         y2(k)=sqrt(-term1(i)-sqrt(term2(i))); % lower part of the curve
         x(k)=P2(i);
      end
   end
   plot(x,y1,x,y2)
   hold on
end
grid on
xlabel('Power')
ylabel('Voltage')
legend('cos phi: 0.8 lag / 0.9 lag / 1 / 0.9 lead / 0,85 lead')
legend('Location','south')
```

The MATLAB code should result in the following figure



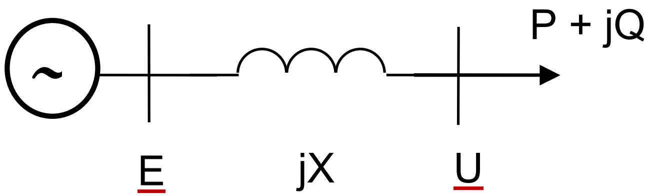
Remarks on PV curve

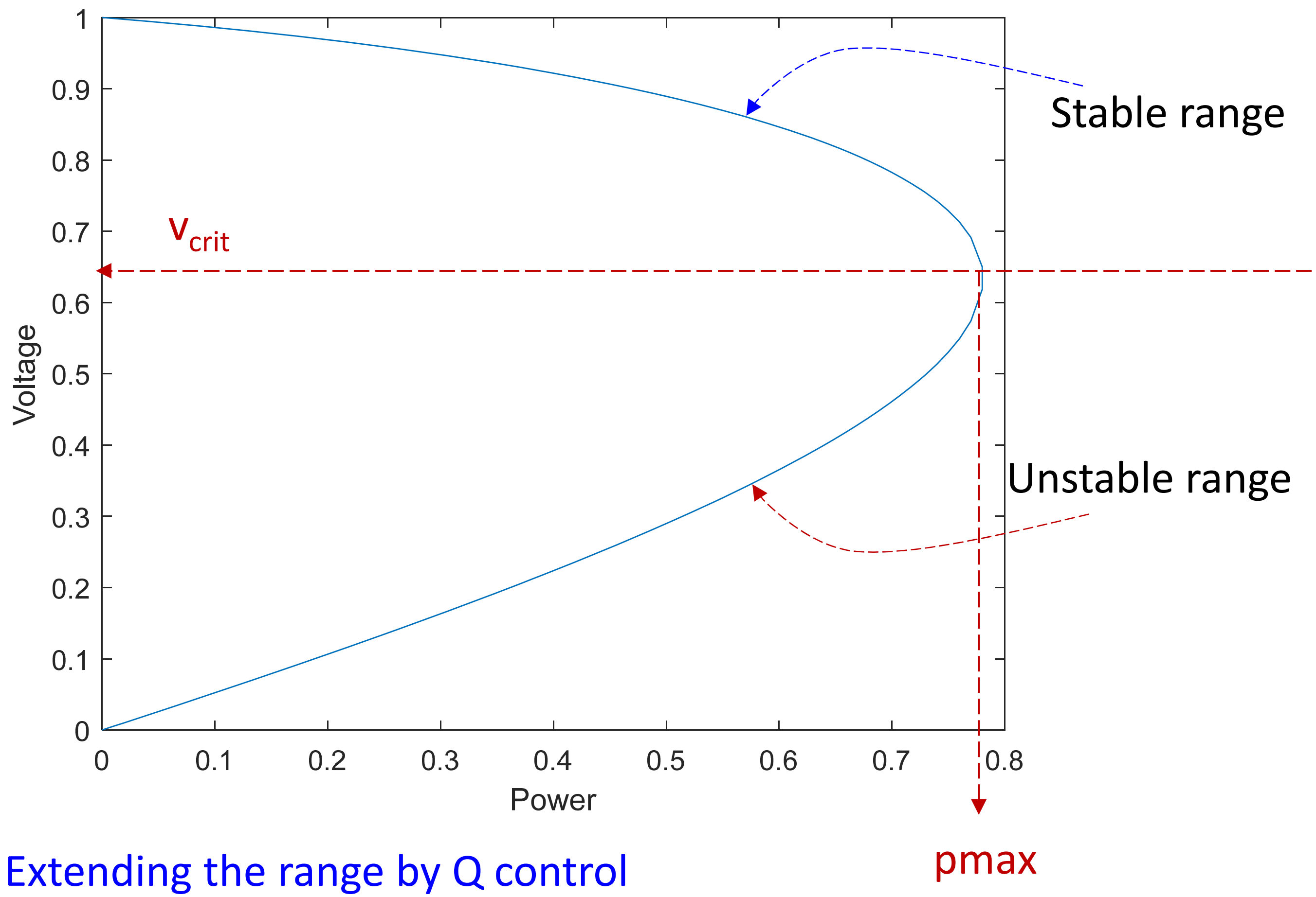
- Each curve (for a given E and power factor) has a maximum power that can be delivered to the load. This value is typically called the maximum system load or the system loadability.
  - If the load is increased beyond the loadability limit, the voltages will decline uncontrollably.

Remarks on PV curve

- In the lagging or unity power factor condition, it is clear that the voltage decreases as the load power increases until the loadability limit.
  - In this case, the voltage instability phenomena is detectable, i.e., the operator will be aware that voltages are declining before the loadability is exceeded.
- In the leading power factor case, one observes that the voltage is flat, or perhaps even increasing a little, until just before the loadability limit.
  - Thus, in the leading condition, voltage instability is not easily detectable.
  - The leading condition occurs when the load is highly compensated.

Maximum power, critical voltage





- Extending the range by Q control
- Extending the range by E control

Increasing maximum loadability through var compensation

$$\left(\frac{U^2}{X}\right)^2 + \frac{U^2}{X}\left(2\,Q - \frac{E^2}{X}\right) + P^2 + Q^2 = 0 \qquad (1)$$

$$\frac{\Delta U}{\Delta Q} \approx \frac{\partial U}{\partial Q}$$

Assuming P and E : const., and differentiating (1) w. r. t. Q:

$$2\,Q + \frac{2\,U^2}{X} + 4\,\frac{Q\,U}{X}\,\frac{\partial U}{\partial Q} + 4\,\frac{Q\,U^3}{X^2}\,\frac{\partial U}{\partial Q} = 2\,\frac{U}{X}\,\frac{\partial U}{\partial Q}\,\frac{E^2}{X}$$

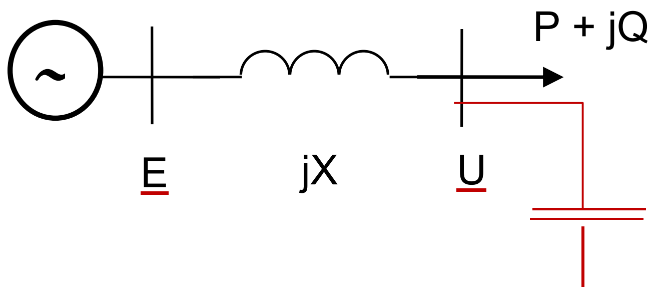
- Re-arranging:

$$\frac{\partial U}{\partial Q} = \frac{Q + \dfrac{U^2}{X}}{\dfrac{U\,E^2}{X^2} - \dfrac{2\,U}{X}\left(Q + \dfrac{U^2}{X}\right)}$$

Increasing maximum loadability through var compensation

$$\left(\frac{U^2}{X}\right)^2 + \frac{U^2}{X}\left(2\,Q - \frac{E^2}{X}\right) + P^2 + Q^2 = 0 \rightarrow \qquad (1)$$

$$\frac{U^2}{X} = \left(\frac{E^2}{2X} - Q\right) \pm \sqrt{\left(\frac{E^2}{2X} - Q\right)^2 - (P^2 + Q^2)}$$



- As observed from the bus, injecting reactive power into the bus is the same as reducing Q supplied by the transmission line, i.e. becomes ($Q - Q_C$).

Increasing maximum loadability – var compensation

$$\frac{\partial U}{\partial Q} = \frac{Q + \frac{U^2}{X}}{\frac{U\,E^2}{X^2} - \frac{2\,U}{X}\left(Q + \frac{U^2}{X}\right)}$$

With:

$$\frac{U^2}{X} = \left(\frac{E^2}{2X} - Q\right) \pm \sqrt{\left(\frac{E^2}{2X} - Q\right)^2 - (P^2 + Q^2)}$$

$$\frac{\partial U}{\partial Q} = \frac{\Delta U}{\Delta Q} = \frac{\frac{E^2}{2X} \pm \sqrt{\left(\frac{E^2}{2X} - Q\right)^2 - (P^2 + Q^2)}}{\mp \frac{U}{X}\sqrt{\left(\frac{E^2}{2X} - Q\right)^2 - (P^2 + Q^2)}}$$

"+" in the upper part of the nose curve

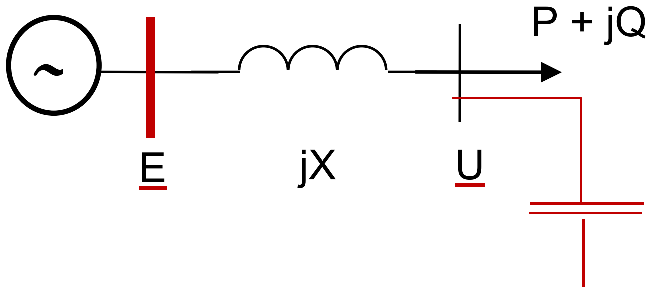
"-" since it is multiplied by "-" during simplification

$$\frac{\Delta U}{\Delta Q} < 0 \; in\ the\ upper\ half\ and\ \frac{\Delta U}{\Delta Q} > 0 \; in\ the\ lower\ half$$

Thus:

- We can raise U by reducing Q for a given P as long as the operating point remains in the upper region of the curve.
- In the lower (unstable) part of the nose curve this is not possible.

Increasing maximum loadability - raising E



$$\left(\frac{U^2}{X}\right)^2 + \frac{U^2}{X}\left(2\,Q - \frac{E^2}{X}\right) + P^2 + Q^2 = 0$$

Differentiating w. r. t. E:

$$\frac{4\,Q\,U}{X}\frac{\partial U}{\partial E} + \frac{4\,U^3}{X^2}\frac{\partial U}{\partial E} = \frac{2\,U}{X}\frac{E^2}{X}\frac{\partial U}{\partial E} + \frac{2\,U^2\,E}{X^2}$$

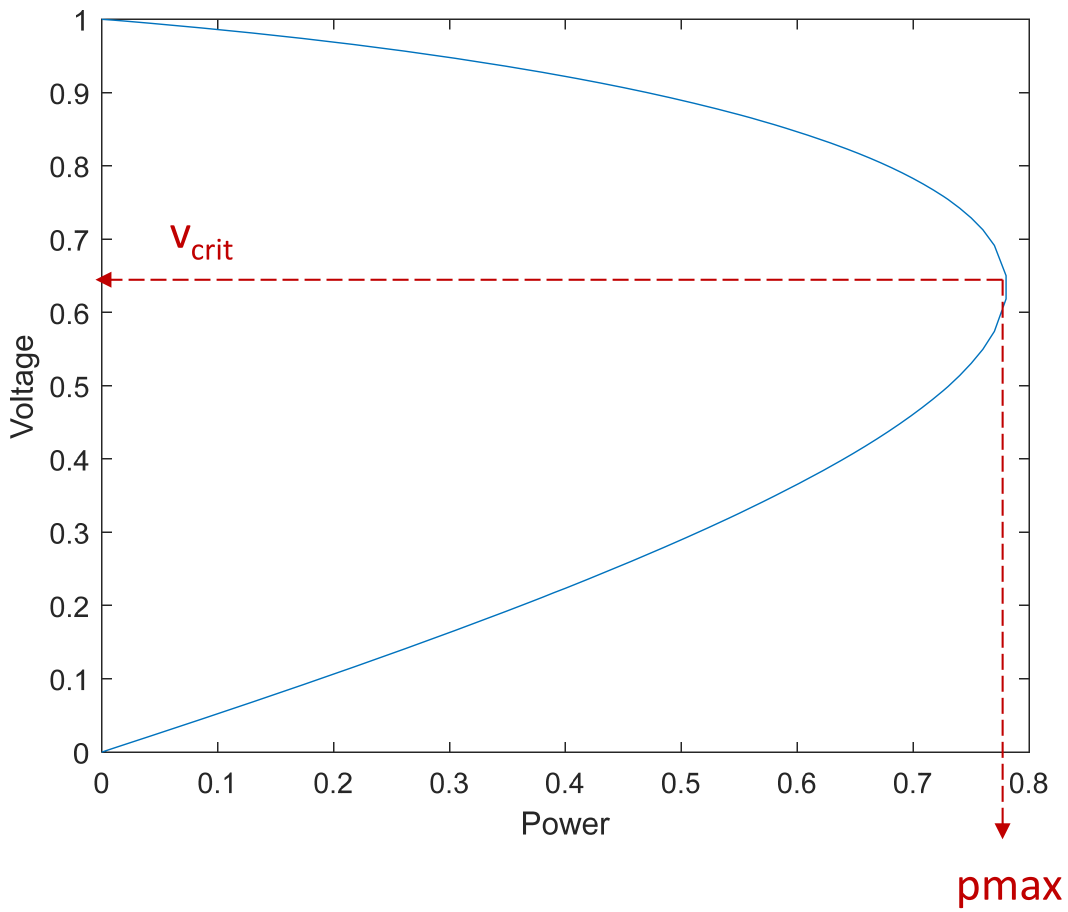
With $\frac{U^2}{X} = \left(\frac{E^2}{2X} - Q\right) \pm \sqrt{\left(\frac{E^2}{2X} - Q\right)^2 - (P^2 + Q^2)}$

$$\frac{\partial U}{\partial E} = \frac{\frac{E}{2}\left(\left(\frac{E^2}{2X} - Q\right) \pm \sqrt{\left(\frac{E^2}{2X} - Q\right)^2 - (P^2 + Q^2)}\right)}{\pm 2\sqrt{\left(\frac{E^2}{2X} - Q\right)^2 - (P^2 + Q^2)}} > 0$$

"+" in the upper part of the nose curve

- In the upper part of the nose curve, the voltage at the load end can be raised by increasing E

Maximum power, critical voltage



Maximum power p_{max}

E : sending-end voltage (given)

X : line reactance(given)

R : neglected

$$x = v^2 = \frac{-(2\ p\tan\varphi\ -1)\ \pm\ \sqrt{(2\ p\tan\varphi\ -1)^2\ -4\,p^2\,(1+\ tan^2\varphi)}}{2}$$

The maximum active power (p_{max}) that can be delivered to the load at the receiving-end of the transmission line without loss of voltage stability:

- "±" replaced with "+" → the (positive) square root is the upper part
- "±" replaced with "-" → the (positive) square root is the lower part
- at maximum power point the two are equal;
  - this is the case, when the term inside the square root is zero

$$(2\,p\tan\varphi - 1)^2 - 4\,p^2(1+\ tan^2\varphi) = 0\ \rightarrow p^2 +\ p\tan\varphi - \frac{1}{4} = 0$$

$$p_{1,2} = -\frac{\tan\varphi}{2} \pm \sqrt{\left(\frac{\tan\varphi}{2}\right)^2 + \frac{1}{4}} \rightarrow$$

$$p =\ p_{max} = \frac{1\ -\sin\varphi}{2\cos\varphi} = \frac{\cos\varphi}{2\,(1+\sin\varphi)}$$

Critical voltage v_{crit}

$$x = v^2 = \frac{-(2\ p\tan\varphi\ -1) \pm \sqrt{(2\ p\tan\varphi\ -1)^2 - 4\,p^2\,(1+\ tan^2\varphi)}}{2}$$

Example:

$$p = p_{max} = \frac{\cos\varphi}{2\,(1+\sin\varphi)} \rightarrow$$

Cos φ	p_{max}
0.85 (ind.)	0.278
1	0.5
0.85 (cap.)	0.898

Critical voltage v_{crit}

S= P+jQ

$$x = v^2 = \frac{-(2\ p \tan\varphi\ - 1) \pm \sqrt{(2\ p\tan\varphi\ - 1)^2\ - 4\,p^2\,(1 + \ tan^2\varphi)}}{2}$$

Example:

$$v_{crit} = \sqrt{\frac{-(2\ p\tan\varphi\ - 1)}{2}} = \sqrt{\frac{-\dfrac{\sin\varphi}{(1+\sin\varphi)} + 1}{2}}$$

$$p = p_{max} = \frac{\cos\varphi}{2\,(1+\sin\varphi)}$$

$$v_{crit} = \frac{1}{\sqrt{2}\,\sqrt{1+\sin\varphi}}$$

380 kV line,
200 km long,
X_b = 0.25 Ω/km

Sending-end voltage (E)
380 kV

$$\frac{E^2}{X} = \frac{380\ kV^2}{50} = 2888\ MW$$

Cos φ	Pmax (p.u.)	P_{max} MW	V_{crit}(L-L) kV	V_{crit} (p.u.)
0.85 ind	0.278	802.86	184.68	48.6%
1	0.5	1444	268.66	70.7%
0.85 cap.	0.898	2593.4	390.64	102.8%

QV Curve

Q-V / V-Q Curves:

– Tests bus strength and helps determine possible reactive power compensation requirement

Derivation of the Q-V Curve

$$P = \frac{E\,U}{X}\sin\delta c \longrightarrow \sin\delta = \frac{PX}{EU}$$

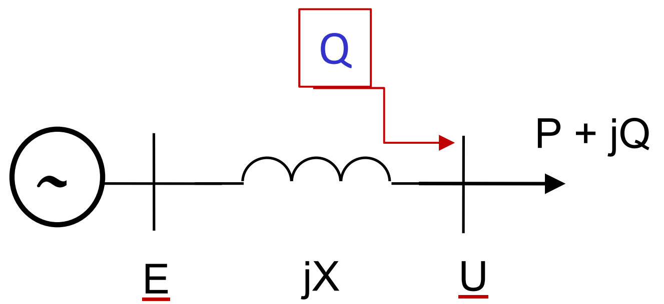
$$\cos^2\delta + \sin^2\delta = 1 \longrightarrow \cos\delta = \sqrt{1-\sin^2\delta} = \sqrt{1-\left(\frac{PX}{EU}\right)^2}$$

$$Q = \frac{E\,U}{X}\cos\delta - \frac{U^2}{X}$$

$$= \frac{E\,U}{X}\sqrt{1-\left(\frac{PX}{EU}\right)^2} - \frac{U^2}{X}$$

$$Q = \sqrt{\left(\frac{E\,U}{X}\right)^2 - P^2} - \frac{U^2}{X}$$

The basic relationship for drawing the QV curve



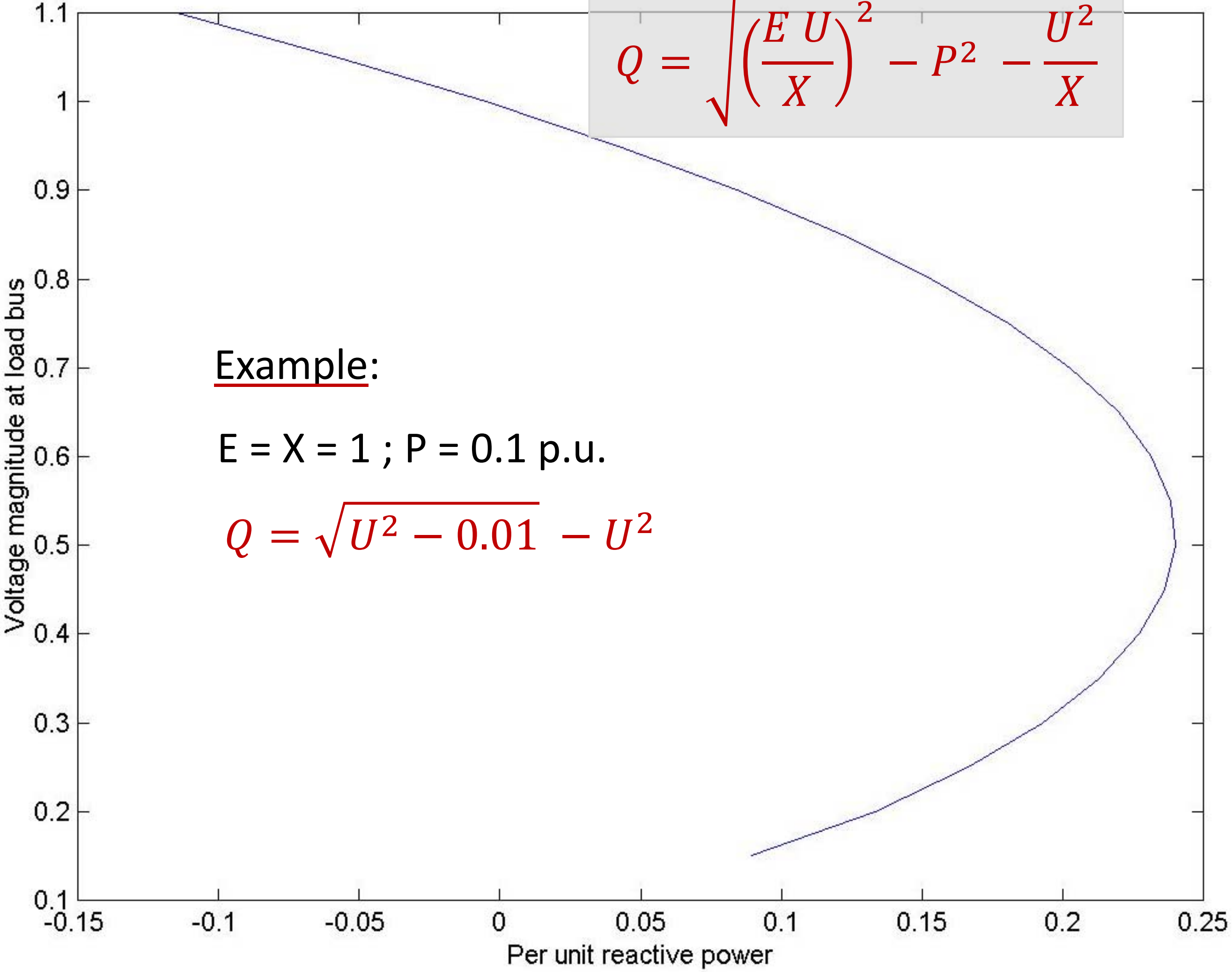
Q: the reactive power that can be supplied to the load by the source via the line

QV Curve

- Power flow programs cannot solve the power flow algorithm below the “nose” of a P-V curve

- Q-V curve can be calculated using standard power flow programs

$$Q = \sqrt{\left(\frac{E\ U}{X}\right)^2 - P^2} - \frac{U^2}{X}$$



U

Q

Example:

E = X = 1 ; P = 0.1 p.u.

$$Q = \sqrt{U^2 - 0.01} - U^2$$

Drawing QV curve using power flow program

- To draw the QV curve using a power flow software:
  1. define the target bus as a voltage controlled bus with a sufficient range of reactive limit
  2. Specify the desired value for |U|
  3. Solve the power flow
  4. Read the Q delivered by the source at the load
  5. Repeat 2 - 4 for a range of voltage values

VQ curve – (QV curve with axes exchanged)

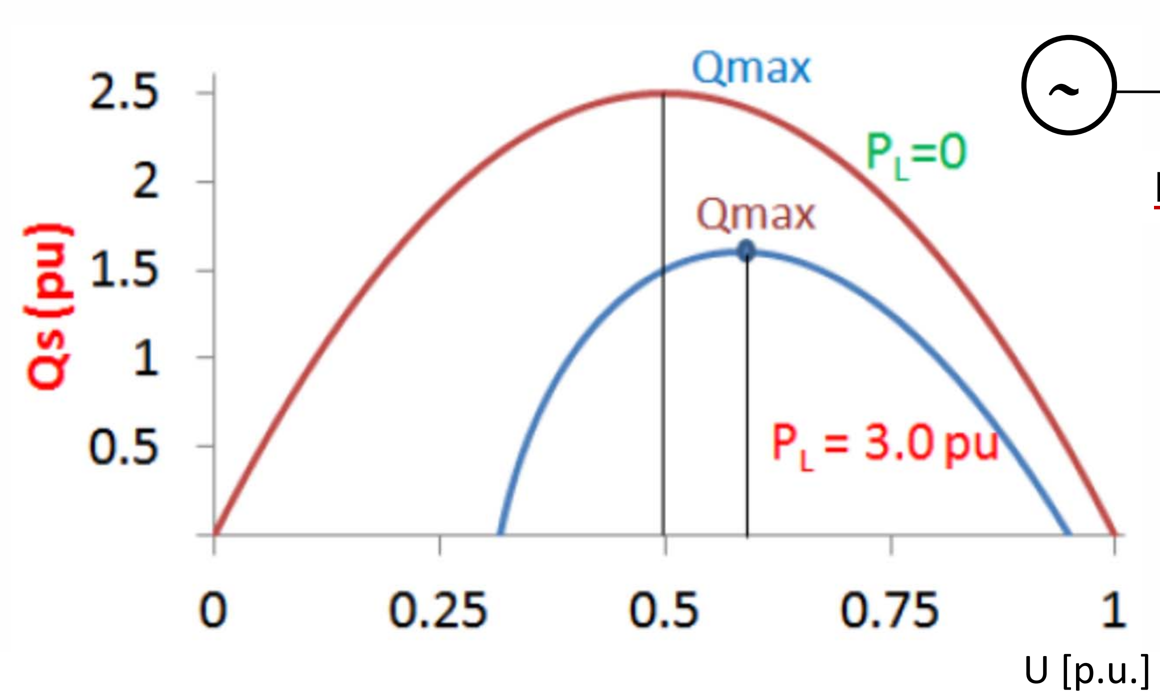
$$P_L(U) = P_S(U) = \frac{E\,U}{X}\sin\delta$$

P_L : active power absobed by the load
P_S : active power delivered by the source

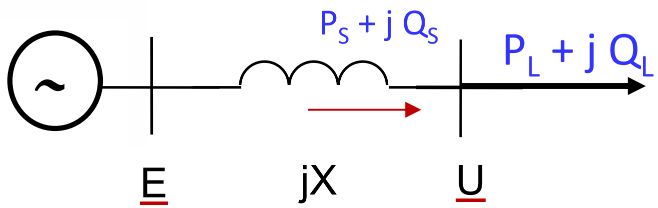
Q_S : reactive power delivered to the load by the source

$$Q_S(U) = \frac{E\,U}{X}\cos\delta - \frac{U^2}{X}$$

$$Q_S(U) = \sqrt{\left(\frac{E\,U}{X}\right)^2 - \{P_L(U)\}^2} - \frac{U^2}{X}$$



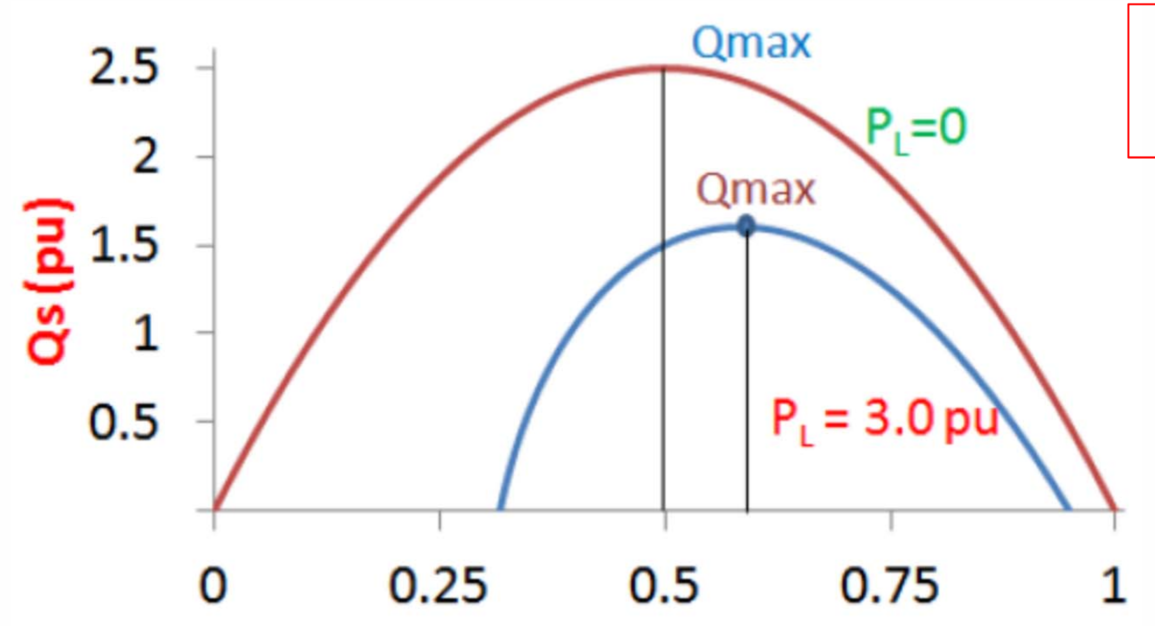
$Q_S - U$ Characteristics for $P_L = 0$ and $P_L > 0$



P_L shifts the parabola downwards and to the right

E = 1; X = 0.1 (example)

Maximum deliverable reactive power by the source



$$Q_S(U) = \sqrt{\left(\frac{E\,U}{X}\right)^2 - \{P_L(U)\}^2} - \frac{U^2}{X}$$

$$Q_S = \frac{E\,U - U^2}{X}$$

For $P_L = 0$:
$Q_S = 0$ at: $U = E$ and $U = 0$

The voltage (U_{max}) correponding to maximum value of Q_S (Q_{max}):

$P_L = 0$

$$\frac{dQ_S}{dU} = E - 2U = 0 \rightarrow U_{max} = E/2$$

$$Q_{max} = \frac{E^2}{4X}$$

$P_L > 0$

$$U_{max} = \sqrt{\left(\frac{E}{2}\right)^2 + \left(\frac{P_L(U)\,X}{E}\right)^2}$$

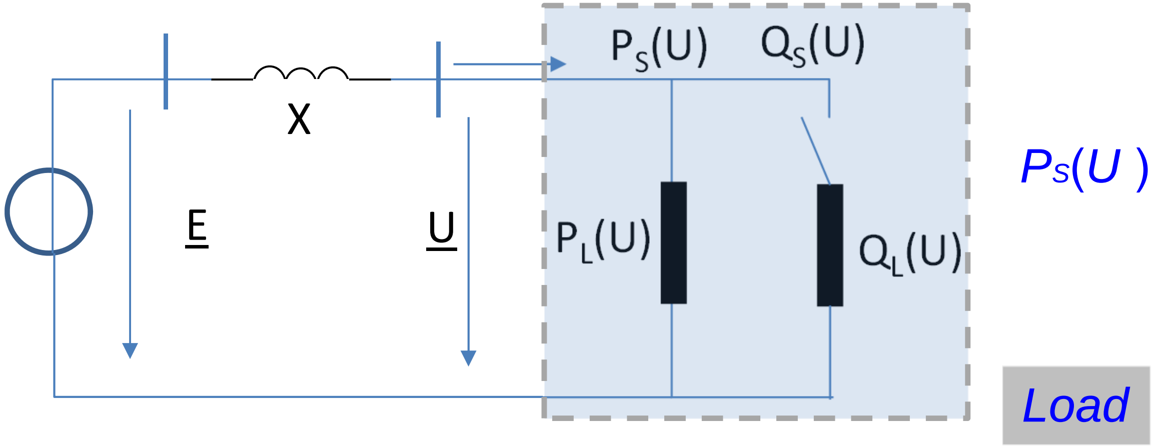
$$U_{max} > \frac{E}{2}$$

Voltage stability assessment

The d(ΔQ)/dV criterion

- This voltage stability criterion is based on the assessment of the capability of the system to supply reactive power for a given amount of active power demand.

$P_S(U)$ and $Q_S(U)$: active and reactive power, respectively, supplied by the source to the load

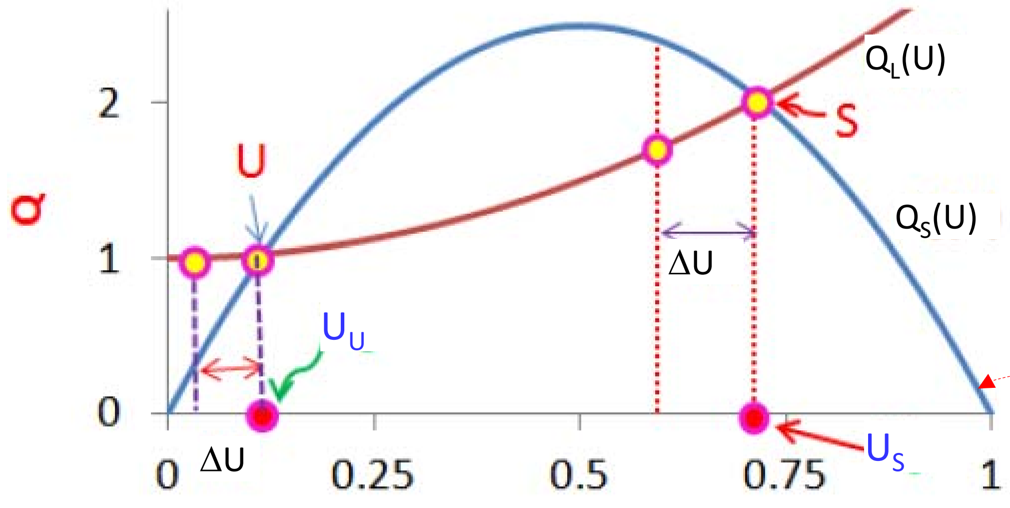


$P_S(U) = P_L(U)$

Normal operation (*i.e. operation with no compensation*): $Q_S(U) = Q_L(U)$

$P_L(U)$ and $Q_L(U)$: active and reactive power demands, respectively

Stable/unstable operating point



- at equilibrium, the supply must equal the demand i.e., $Q_L(U) = Q_S(U)$
- the two possible equilibrium points are U_S and U_U.

$$Q_S(U) = \sqrt{\left(\frac{E\,U}{X}\right)^2 - \{P_L(U)\}^2} - \frac{U^2}{X}$$

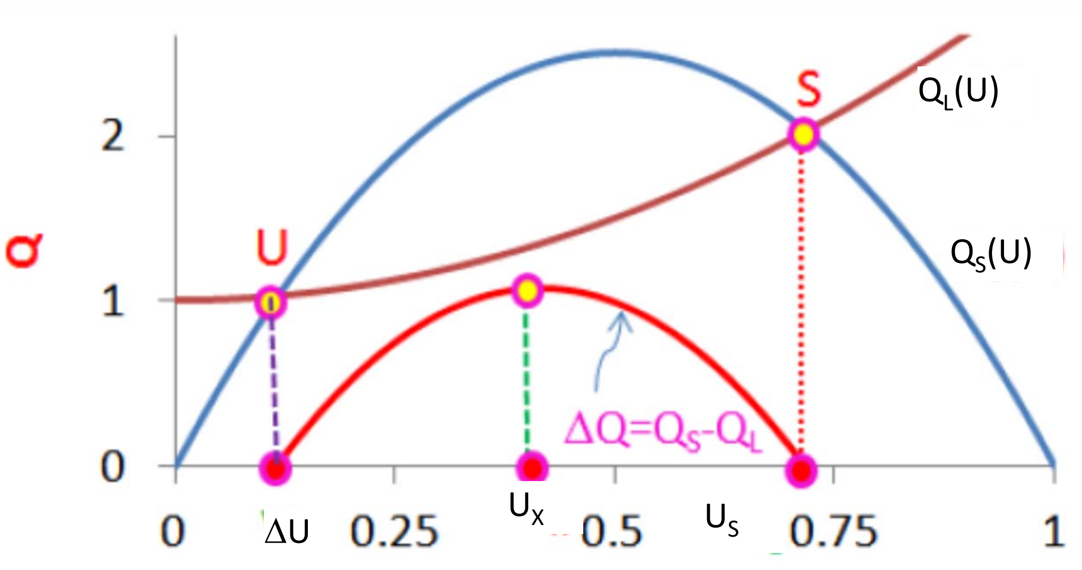
For a given E, X, P_L

Assume a small reduction in voltage ΔU:
At point 'S', this reduction will result in:
$Q_S(U) > Q_L(U)$

- excess reactive power (supplied by the source) will raise U →this will force the voltage to return to point S.
  - ✓ Stable operating point
- at point 'U' the situation is the reverse

The system is stable at point 'S', unstable at point 'U'

Analytical formulation of the *dQ/dV* criterion



$\frac{d\,\Delta Q}{dU}$ negative at the stable point 'S' and positive at the unstable point 'U'.

Conclusion:

For stability (in addition to equilibrium):

$$\frac{d\,\Delta Q}{dU} = \frac{d\,(Q_S - Q_L)}{dU} < 0 \quad \Rightarrow \quad \boxed{\frac{d\,(Q_S)}{dU} < \frac{d\,(Q_L)}{dU}}$$

Analytical formulation of the *dQ/dV* criterion cont…

The active and reactive power supplied to the load are functions of two variables U, δ

$$P_S(U) = P_L(U) = f_P(U,\delta) \qquad Q_S(U) = f_Q(U,\delta)$$

Neglecting losses; assuming load bus, i.e. no reactive power generation

Hence, the incremental values of ΔP_S and ΔQ_S can be expressed as :

$$\Delta P_L = \Delta P_S = \frac{\partial P_S}{\partial U}\Delta U + \frac{\partial P_S}{\partial \delta}\Delta\delta \qquad \Delta Q_S = \frac{\partial Q_S}{\partial U}\Delta U + \frac{\partial Q_S}{\partial \delta}\Delta\delta$$

$$\Delta\delta = \left(\frac{\partial P_S}{\partial \delta}\right)^{-1}\left(\Delta P_L - \frac{\partial P_S}{\partial U}\Delta U\right)$$

Substituting $\Delta\delta$ in the expression of ΔQ_S:

$$\frac{\Delta Q_S}{\Delta U} = \frac{\partial Q_S}{\partial U} + \frac{\partial Q_S}{\partial \delta}\left(\frac{\partial P_S}{\partial \delta}\right)^{-1}\left(\frac{\Delta P_L}{\Delta U} - \frac{\partial P_S}{\partial U}\right)$$

For small ΔU values:

$$\frac{dQ_S}{dU} \cong \frac{\partial Q_S}{\partial U} + \frac{\partial Q_S}{\partial \delta}\left(\frac{\partial P_S}{\partial \delta}\right)^{-1}\left(\frac{\Delta P_L}{\Delta U} - \frac{\partial P_S}{\partial U}\right)$$

Evaluation of the derivatives

$$P_S(U,\delta) = \frac{E\,U}{X}\sin\delta \qquad Q_S(U,\delta) = \frac{E\,U}{X}\cos\delta - \frac{U^2}{X}$$

$$\frac{\partial P_S}{\partial \delta} = \frac{E\,U}{X}\cos\delta \qquad \frac{\partial P_S}{\partial U} = \frac{E}{X}\sin\delta \qquad \frac{\partial Q_S}{\partial \delta} = -\frac{E\,U}{X}\sin\delta \qquad \frac{\partial Q_S}{\partial U} = \frac{E}{X}\cos\delta - 2\frac{U}{X}$$

$$\frac{dQ_S}{dU} \cong \frac{\partial Q_S}{\partial U} + \frac{\partial Q_S}{\partial \delta}\left(\frac{\partial P_S}{\partial \delta}\right)^{-1}\left(\frac{\Delta P_L}{\Delta U} - \frac{\partial P_S}{\partial U}\right)$$

$$\frac{dQ_S}{dU} \cong \frac{E}{X}\cos\delta - 2\frac{U}{X} - \frac{E\,U}{X}\sin\delta\,\frac{X}{EU\cos\delta}\left(\frac{dP_L}{dU} - \frac{E}{X}\sin\delta\right)$$

$$\cong \frac{E}{X}\cos\delta - 2\frac{U}{X} - \tan\delta\left(\frac{dP_L}{dU} - \frac{E}{X}\sin\delta\right)$$

$$\cong -2\frac{U}{X} + \frac{E}{X}(\cos\delta + \tan\delta\sin\delta) - \tan\delta\,\frac{dP_L}{dU}$$

$$\boxed{\frac{dQ_S}{dU} \cong \frac{E}{X\cos\delta} - \left(\frac{dP_L}{dU}\tan\delta + 2\frac{U}{X}\right)}$$

Formulation of the stability criterion

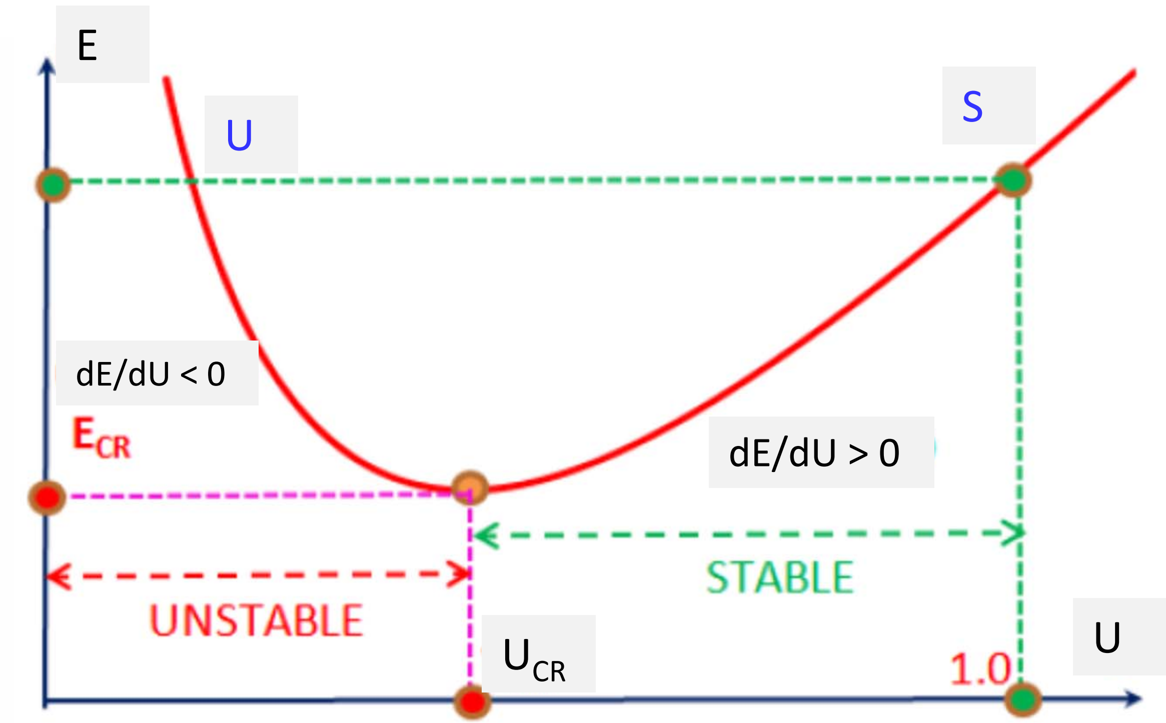
$$\frac{d\,(Q_S)}{dU} < \frac{d\,(Q_L)}{dU} \rightarrow$$

$$\frac{dQ_L}{dU} > \left(\frac{E}{X\cos\delta} - \left(\frac{dP_L}{dU}\tan\delta + 2\frac{U}{X}\right)\right)$$

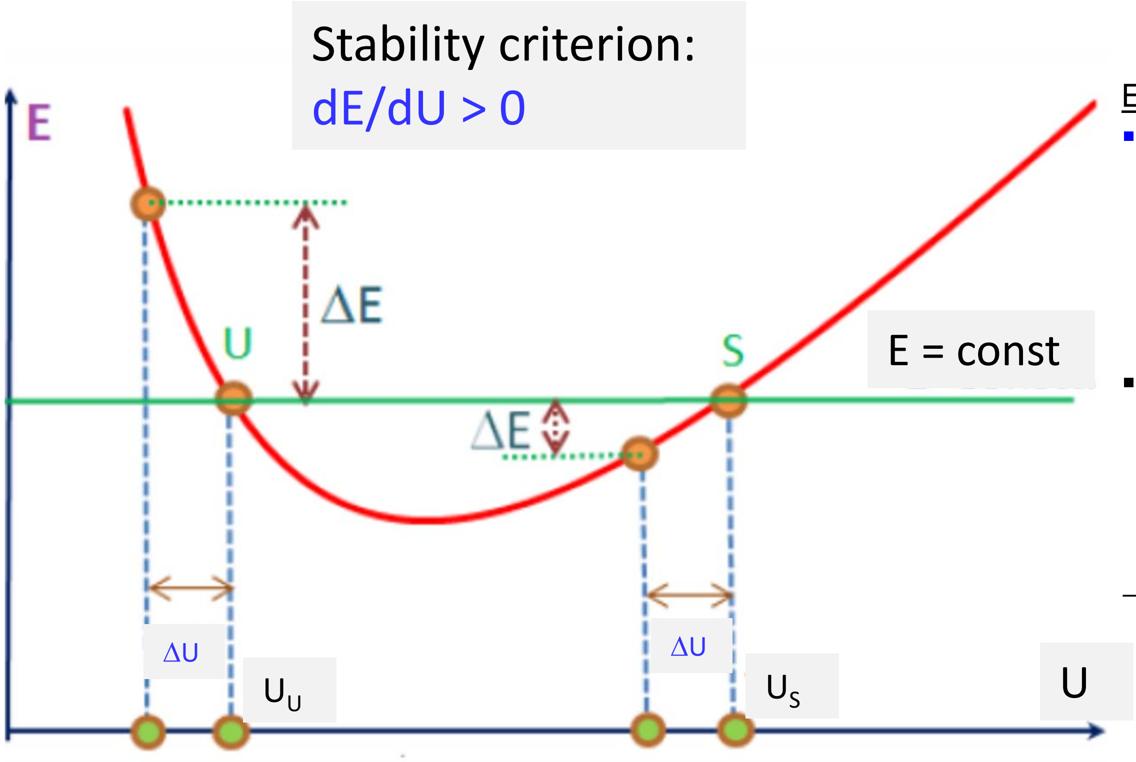
- The derivative of load active and reactive power w.r.t voltage ($\frac{dP_L}{dU}, \frac{dQ_L}{dU}$) are calculated from the load characteristics expressed in terms of U.
- For a load reactive power characteristic $Q_L(U)$, different $Q_S(U)$ (supplied reactive power) can be plotted for different values of source voltage E.

dE/dV criterion

$$\left(\frac{P\,X}{E\,U}\right)^2 + \left(\frac{Q\,X + U^2}{E\,U}\right)^2 = 1 \;\rightarrow\; E = \sqrt{\left(\frac{U^2 + Q\,X}{U}\right)^2 + \left(\frac{P\,X}{U}\right)^2}$$



dE/dV criterion



Equilibrium point 'S':

- Assume reduction in the load voltage U by ΔU. *This* will cause a reduction in the required value of E(U). As the available E is constant and greater than the required value, it will force the voltage back to its initial value U.
- Similarly, when the voltage increases by ΔU, the required emf E(U) to maintain the enhanced voltage (U +ΔU) is larger than the available E. Hence, the voltage is forced to return to its initial value U by the constant E.

→ stable operating point

Equilibrium point 'U':

- Assume the voltage is reduced by ΔU; to sustain this voltage, a higher value of emf E(U) is required. But E is constant and less than the required value of emf E(U), this will result in further reduction in voltage. The voltage moves away further from the equilibrium point.
- Similarly, if the voltage is increased by ΔU , then, the required emf E(U) is smaller than the available E. The larger available E will cause the voltage to increase further away from the initial equilibrium point 'U'.

→ unstable operating point