Chapter 8: Multi-carrier Transmission Techniques and OFDM

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Single-Carrier Signals over Fading Multipath Channels

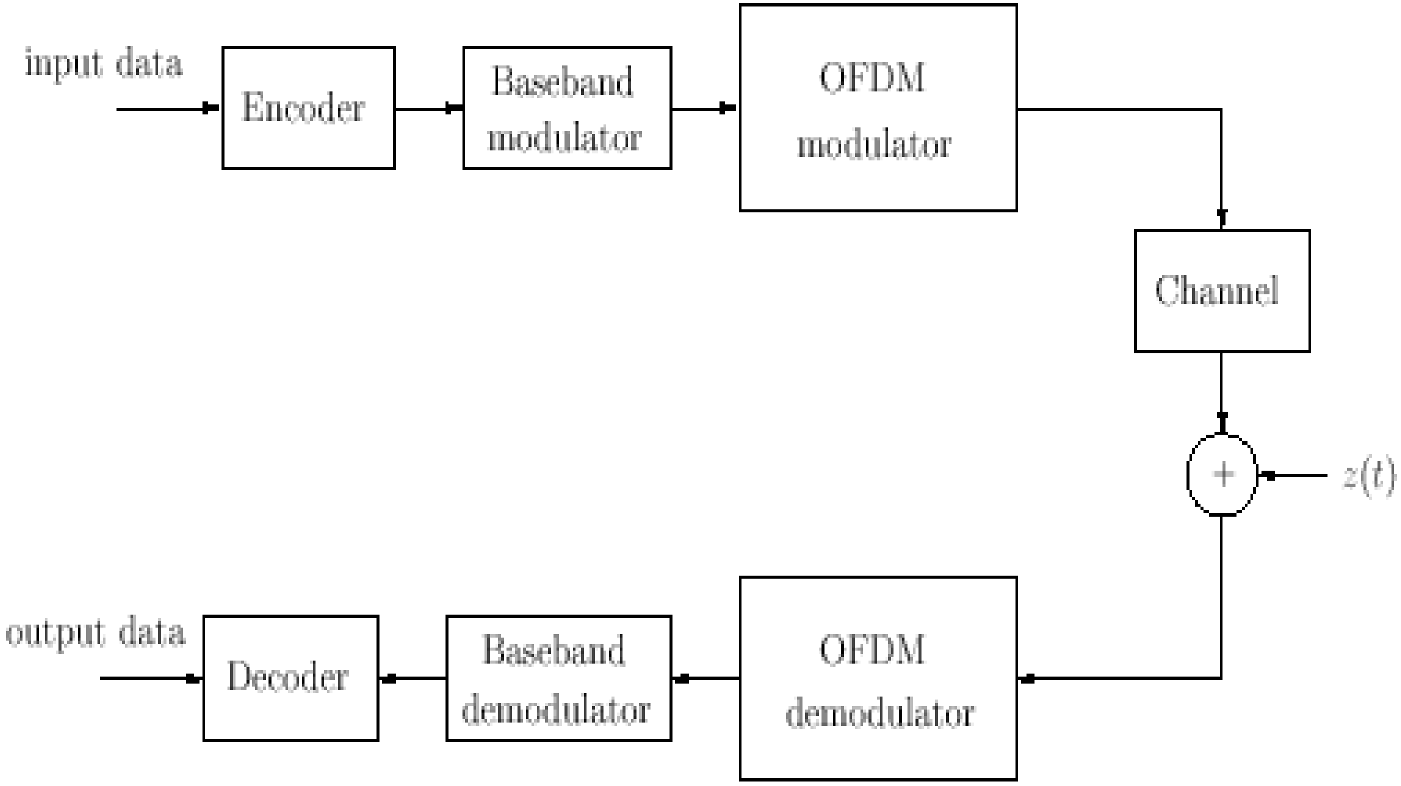
- Fading multipath channels introduce ISI (memory) into communication signals
- The length of ISI is dependent on the *transmission rate*
- Increasing the transmission data rate makes the time equalizer at the receiver *more complex*
  - Increases computational complexity
  - Increased delay due to processing
- Complexity and delay *limit the maximum data* rate that can be achieved with single carrier transmission in fading multipath channels
- OFDM is a possible solution for achieving high data rates in fading multipath environments *without* the need for *complicated time equalizers*

Orthogonal Frequency Division Multiplexing (OFDM)

- OFDM is used in many applications
  - Digital Audio Broadcasting (DAB)
  - Digital Video Broadcasting (DVB)
  - IEEE 802.11b Wireless LAN system; etc
- Application of OFDM is expected to grow in areas such as
  - Wireless multimedia communications
  - Fixed broadband Wireless Access Systems
- OFDM is a modulation applied to the data-modulated signal a second time (see figure on the next slide)
- *It transforms the high data rate stream into several low rate streams*

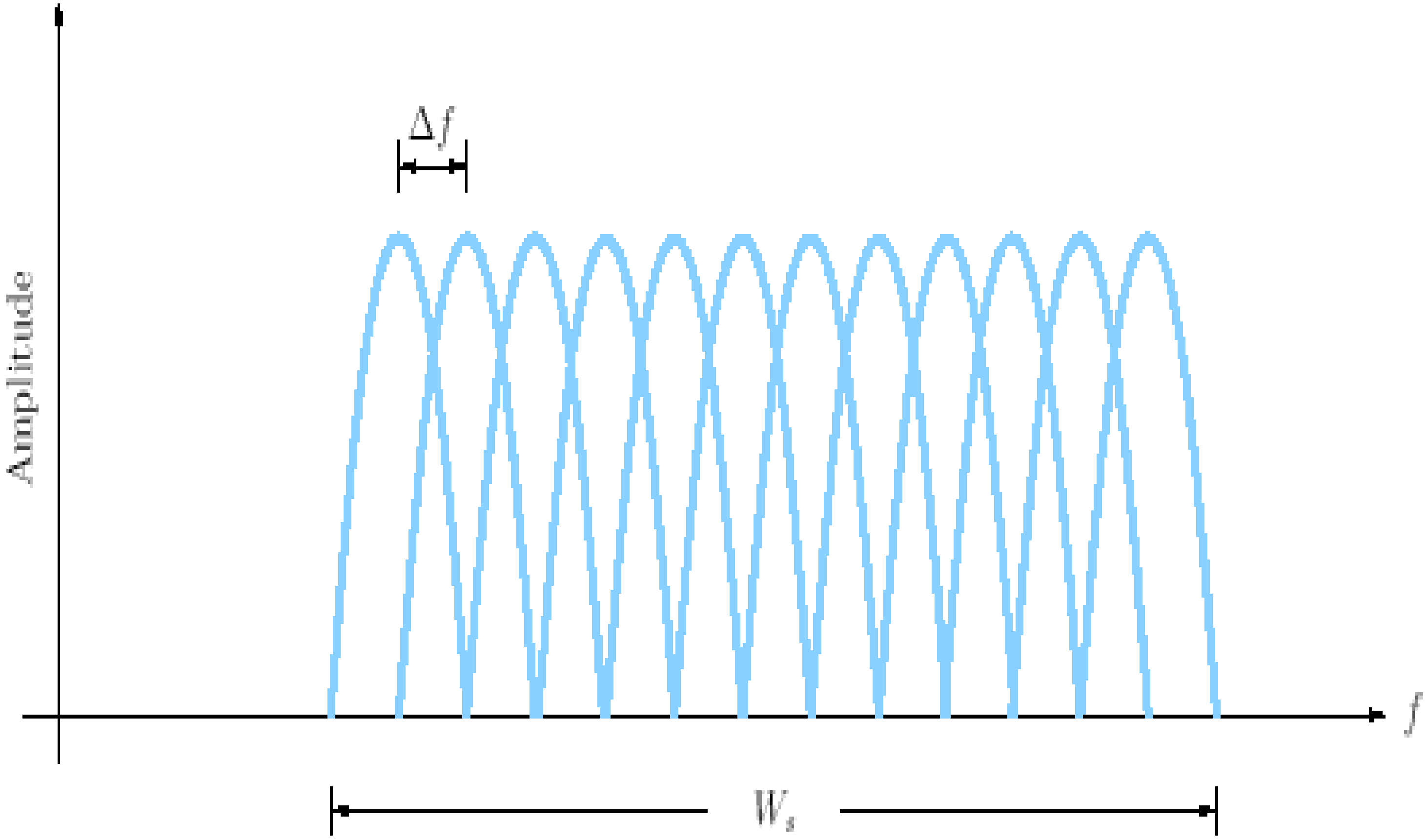
Orthogonal Frequency Division Multiplexing …

- Splits the total bandwidth into several narrowbands (equally spaced subcarriers)
- Transmits each low rate stream on a different subcarriers

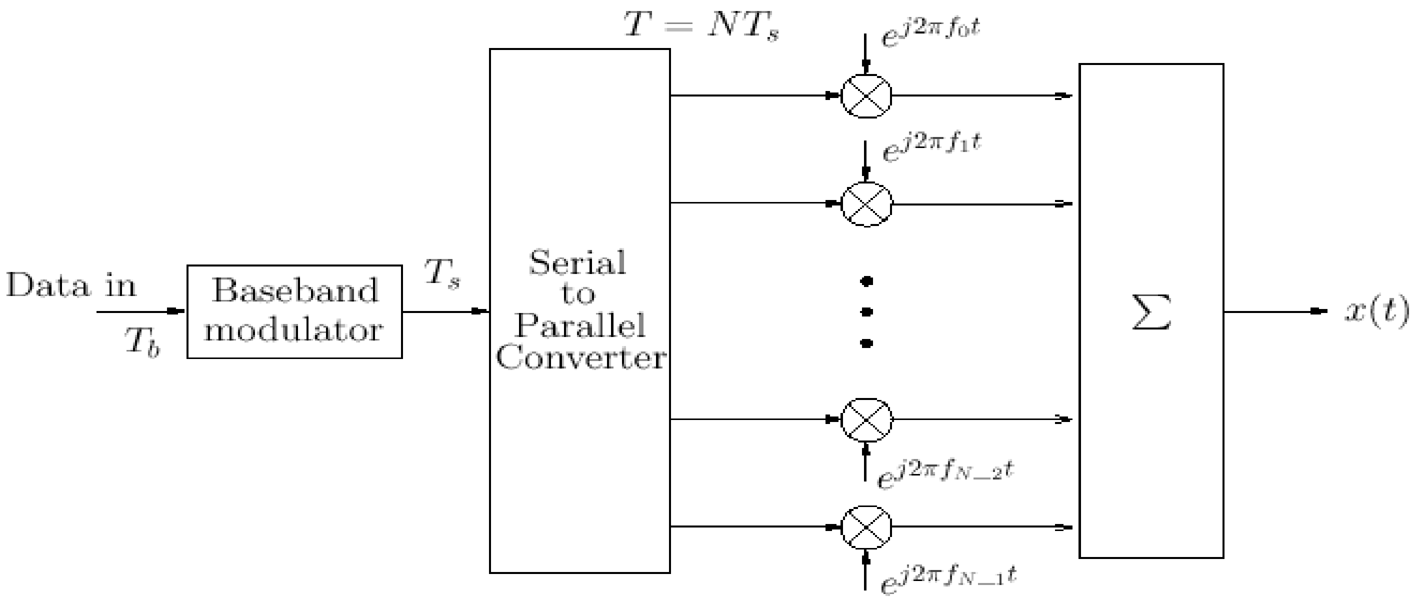


Orthogonal Carrier frequencies

- *No guard band* between the different narrowbands is needed



Orthogonal Carrier frequencies …



- The equivalent lowpass of the transmitted signal is given as

$$x(t) = \frac{1}{\sqrt{T}} \sum_{k=0}^{N-1} s_k(n) e^{j2\pi f_k t}, \quad nT \le t < (n+1)T$$

- $s_k(n)$ is PSK, QAM, or PAM baseband modulated symbol

Orthogonal Carrier frequencies …

- The transmission rate on each subcarrier is given by

$$R = \frac{1}{N}\frac{1}{T_s} = \frac{R_s}{N}$$

- An OFDM is a multi-carrier system for which each carrier is orthogonal to other carriers, i.e.,

$$\frac{1}{T}\int_0^T e^{j2\pi(f_j - f_k)t}dt = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}$$

- In this case the carriers are minimally separated with

$$\Delta f = f_{i+1} - f_i = \frac{1}{T}$$

- The equivalent lowpass of the OFDM signal becomes

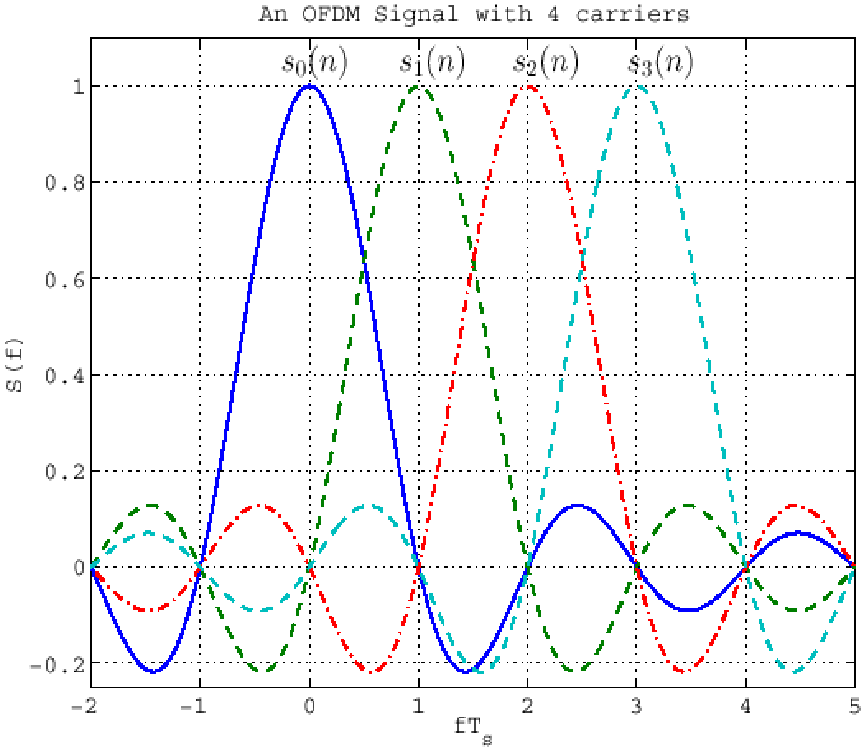
$$x(t) = \frac{1}{\sqrt{T}}\sum_{k=0}^{N-1} s_k(n)e^{j2\pi k\frac{t}{T}}, \qquad nT \leq t \leq (n+1)T$$

Orthogonal Carrier frequencies …

- Taking the Fourier transform of *x(t)* we get

$$X(f, nT) = \frac{1}{\sqrt{T}} \int_{nT}^{(n+1)T} x(t) e^{-j2\pi f t} dt = e^{-j\theta_n} \sum_{k=0}^{N-1} s_k(n) \mathrm{sinc}\,(fT - k)$$

Orthogonal Carrier frequencies …



- Notice that the symbol $s_k(n)$ is obtained by just sampling X(f, nT) at $f=k/T$

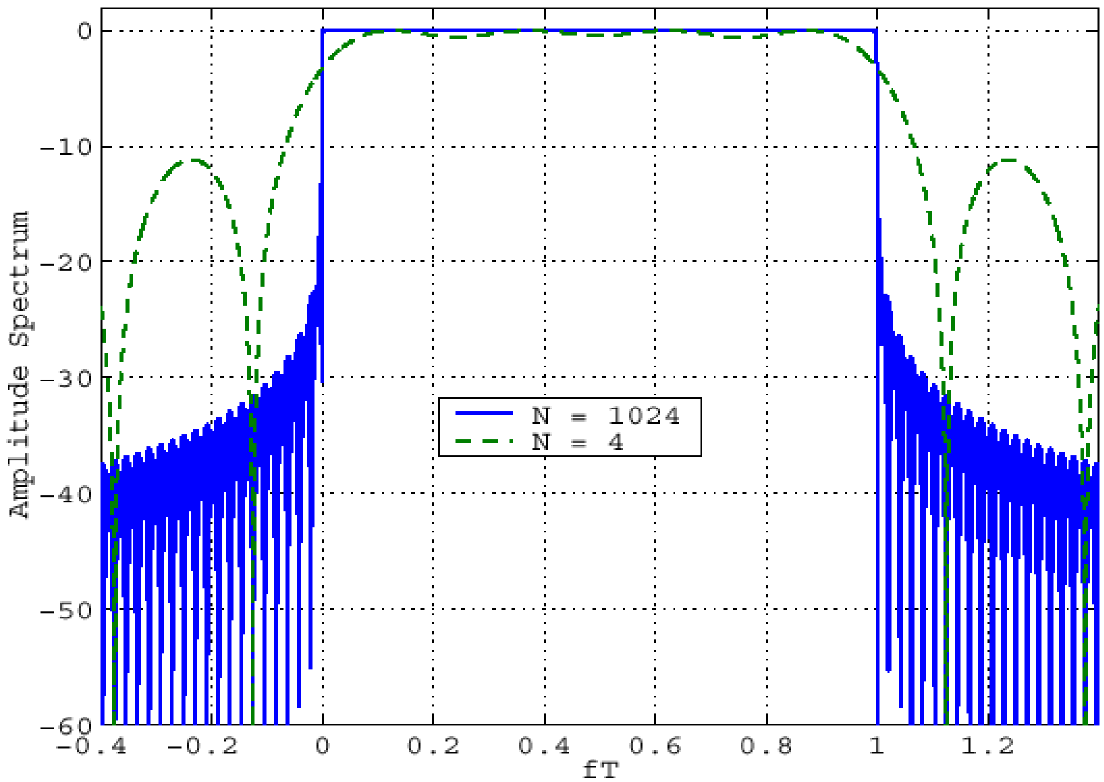
$$X\left(\frac{k}{T}, nT\right) = s_k(n), \quad k = 0, 1, \cdots, N-1$$

- Any frequency offset creates interference between all subcarrier signals

Orthogonal Carrier frequencies …

- The power spectral density of OFDM signals is

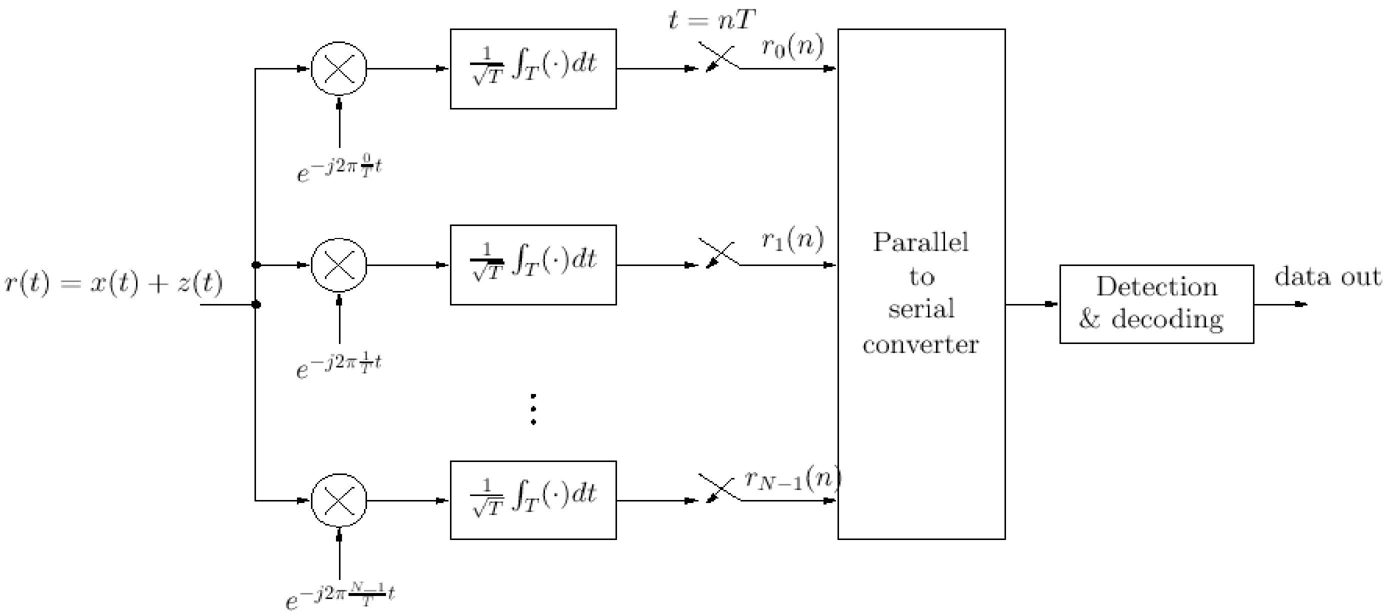
$$\boxed{X_s(f) = \sum_{n=0}^{N-1} \mathrm{sinc}^2(fT - n)} \quad \rightsquigarrow \quad W_s \approx \frac{N+1}{NT_s} = \frac{1+\beta}{T_s}$$



Detection of OFDM Signals

- Detection of OFDM signals is achieved using N correlators, each centered around a different subcarrier frequency
- The received sample at subcarrier k can be written as

$$r_k(t) = \frac{1}{\sqrt{T}} \int_{nT}^{(n+1)T} r(t) e^{-2\pi \frac{k}{T} t} dt = s_k(n) + z_k(n)$$



OFDM Signals over AWGN Channels

- The received sample at subcarrier k can be written as

$$r_m(k) = s_k(n) + z_k(n)$$

  - $s_k(n)$ is the signal corresponding to the signal point of the modulation used
  - $z_k(n)$ is complex Gaussian random variable with zero mean and variance N_0
- The performance of OFDM over AWGN channels is the same as that of single-carrier scheme that uses the same modulation
- The bandwidth efficiency in this case is 1 bit/s/Hz same as single-carrier BPSK
- The spectrum of OFDM-BPSK is much more compact than that of single-carrier BPSK

OFDM Signals over Fading Multipath Channels

- In a fading multipath environment with coherence bandwidth B_m, the equivalent lowpass of the received signal is

$$r(t) = \sum_{i=0}^{L-1} h_i(t) x\left[t - \tau_i(t)\right] + z(t)$$

- With proper selection of N, we can make the OFDM block duration much larger than the maximum delay spread of the channel, i.e.,

$$T = NT_s >> T_m = 1/B_m$$

- The output sample of subcarrier k is given by

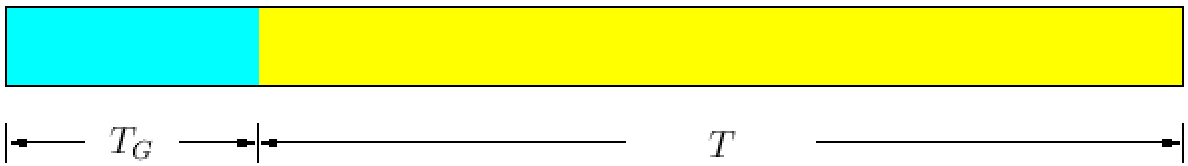
$$r_k(n) = \frac{1}{\sqrt{T}} \int_{nT}^{(n+1)T} r(t) e^{-2\pi \frac{k}{T} t} = H_k s_n(n) + \sum_{m}^{N-1} G_{m,k} s_m(n-1) - \sum_{m=0, m\neq k}^{N-1} G_{m,k} s_m(n) + z_k(n)$$

OFDM Signals over Fading Multipath Channels …

- The first term on the right side is the required signal, the second term is an ISI and the third term is the inter-carrier interference (ICI)
- Some modifications are needed in the OFDM reception scheme to eliminate the ISI and the ICI

OFDM Signals over Fading Multipath Channels …

- *Guard Interval- solution to the Problem of Multipath Fading*



- With the guard interval, the total OFDM block duration becomes

$$T = T + T_G$$

- Note that there is a power loss due the added guard interval which can be shown to be:

$$10 \log 10 \left(1 + T_G/T\right)$$

OFDM Signals over Fading Multipath Channels …

- Every OFDM block is extended by a guard interval TG, i.e.,

$$x(t) = \sum_{m=0}^{N-1} s_m(n) e^{j2\pi \frac{m}{T} t}, \quad nT - T_G \le t < (n+1)T, \quad \text{with } T_G \ge T_m$$

- At the receiver the guard interval is removed such that the received signal at the input of the receiver is

$$r(t) = \sum_{m=0}^{N-1} H(m/T; t) s_m(n) e^{j2\pi \frac{m}{T} t} + z(t), \quad nT \le t < (n+1)T$$

$H(f;t)$ **is the transfer function of the channel** $\rightsquigarrow \quad H(f;t) = \sum_{i=0}^{L-1} h_i(t) e^{-j2\pi f \tau_i(t)}$

OFDM Signals over Fading Multipath Channels …

- The output sample of subcarrier k becomes

$$r_k(n) = \frac{1}{\sqrt{T}} \int_{nT}^{(n+1)T} r(t) e^{-j2\pi \frac{k}{T} t} dt = \sqrt{\alpha_g} H(k/T; t) s_k(n) + z_k(n), \quad \alpha_g = \frac{T}{T + T_G}$$

- Which is free from ISI and ICI
- Note that OFDM with guard interval has transformed a frequency selective fading into *N* parallel flat fading channels

Discrete Representation & Implementation of OFDM

- Recall that the OFDM signal is given by

$$x(t) = \frac{1}{\sqrt{T}} \sum_{m=0}^{N-1} s_m(n) e^{j2\pi \frac{m}{T} t}, \qquad nT \le t \le (n+1)T$$

- Sampling the signal *x*(t) at time instants lT_s,

$$x_l = x(lT_s) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} s_m(n) e^{j2\pi \frac{ml}{N}}, \quad l = 0, 1, \cdots, N-1$$

such that we get a sequence of length N

$$\mathbf{x}(n) = \{x_0(n), x_1(n), \cdots, x_{N-1}(n)\} \ = \ \mathrm{IDFT}\{\mathbf{s}(n)\}$$

Discrete Representation & Implementation of OFDM

- At the receiver, the correlator output of subcarrier k can be rewritten as

$$y_k(n) = \frac{1}{\sqrt{T}} \int_0^T x(t) e^{-j2\pi \frac{k}{T} t} dt \approx \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} s_k(l) e^{-j2\pi \frac{lk}{N}}$$

- We can also view this as a sequence ***y**(n)*

$$\mathbf{y}(n) = \{y_0(n), y_1(n), \cdots, y_{N-1}(n)\} = \mathrm{DFT}\{\mathbf{x}(n)\}$$

Discrete Representation of OFDM Signals

- With guard interval, the OFDM signal is given by

$$x(t) = \frac{1}{\sqrt{T}} \sum_{m=0}^{N-1} s_m(n) e^{j2\pi \frac{m}{T} t}, \qquad nT - T_G \le t \le (n+1)T$$

- Sampling the signal *x*(t) at time instants lT_s, we get

$$x_l = x(lT_s) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} s_m(n) e^{j2\pi \frac{ml}{N}}, \quad l = N-G, \cdots, N-1, 0, 1, \cdots, N-1$$

- a sequence of length N + G

$$\tilde{\mathbf{x}}(n) = \{\underbrace{x_{N-G}(n), x_{N-G+1}(n), \cdots, x_{N-1}(n)}_{\text{Cyclic Prefix}}, x_0(n), x_1(n), \cdots, x_{N-1}(n)\}$$

Discrete Representation of OFDM Signals …

- The corresponding received sequence is denoted as follows

$$\tilde{\mathbf{v}}(n) = \{v_{-G}(n), v_{-G+1}(n), \cdots, v_{-1}(n), v_0(n), v_1(n), \cdots, v_{N-1}(n)\}$$

- The last N samples are taken as input to the DFT receiver
- The other samples are ignored (removed)

$$\mathbf{r}(n) = \text{DFT}\{v_0(n), v_1(n), \cdots, v_{N-1}(n)\}, \quad \text{with} \quad r_k(n) = \sqrt{\alpha_g} H(k/T; t) s_k(n) + z_k(n$$

Block Diagram of an OFDM communication system