Chapter 5: Carrier & Symbol Synchronization

Addis Ababa Institute of Technology
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Graduate Program
School of Electrical and Computer Engineering

Overview

- Signal parameter estimation
  - Likelihood function
  - Carrier recovery & symbol synchronization
- Carrier phase estimation
- Symbol timing estimation

Signal Parameter Estimation

- Propagation *delay* from the transmitter is generally unknown at the receiver
- How to synchronously *sample* the output of the demodulator?
- *Symbol timing* must be *derived* or *extracted* from the received signal
- Moreover, *frequency offset* must be estimated at the receiver for *phase-coherent* detection, which results from
  - Propagation delay
  - Frequency drift at the local oscillator
- ***What are methods for carrier and symbol synchronization?***

Signal Parameter Estimation …

- Assume the channel delays the transmitted signal and also adds noise to it
- Thus the received signal will be

$$r(t) = s(t-\tau) + n(t) \quad \text{where} \quad s(t) = Re\left(s_l(t)e^{j2\pi f_c t}\right)$$

  - Where τ is propagation delay and $s_l(t)$ is the equivalent low pass signal
- We can also express $r(t)$ as

$$r(t) = Re\left[\left(s_l(t-\tau)\right)e^{j\phi} + z(t)\right]e^{j2\pi f_c t}$$

- Where $\phi = -2\pi f_c \tau$ is the phase shift due to delay τ

Signal Parameter Estimation …

- Note that ϕ is a function of f_c and τ
  - I.e., we need to estimate both f_c and τ to know ϕ
- The carrier signal generated at the receiver may in general *not be* in *synchronous* with the transmitter
  - Over time the two oscillators may be drifting slowly in opposite directions
- Furthermore, the precision with which one may synchronize in time depends on signal interval T
- Estimation error in τ must be a small fraction of T
  - Usually 1% of T
- But this level of precision may not be adequate in the estimation of ϕ since f_c is generally *large* and ***small estimation error results in significant phase error***

Signal Parameter Estimation …

- We have to estimate both ϕ and τ to demodulate and detect the signal
- Express the received signal as

$$r(t) = s(t; \phi, \tau) + n(t)$$

- And denote the parameter vector $\{\phi,\tau\}$ by ψ such that $s(t; \phi, \tau) = s(t; \psi)$

- Two criteria widely used in signal parameter estimation
  1. ***Maximum Likelihood (ML)*** criterion: ψ is treated as deterministic but unknown
  2. ***Maximum a posteriori probability (MAP)*** criterion: ψ is modeled as random & characterized by a priori probability density function $p(\psi)$

Signal Parameter Estimation …

- Orthonormal expansion of $r(t)$: Using N orthogonal functions $\{f_n(t)\}$ we may represent $r(t)$ by vector of coefficients $\mathbf{r} \equiv [r_1, r_2, r_3, \ldots.. r_N]$
  - In ML, the estimate of ψ is the value that maximizes $p(\mathbf{r}|\psi)$
  - In MAP the value of ψ that maximizes the a posteriori probability density function is sought

$$p(\psi/\mathbf{r}) = \frac{p(\mathbf{r}|\psi)p(\psi)}{p(\mathbf{r})}$$

- In the absence of any prior knowledge of the properties of ψ, we can assume $p(\psi)$ is ***uniform*** over a range of values of the parameter
- In such a case, the value of ψ that maximizes $p(\mathbf{r}|\psi)$ also maximizes $p(\psi|\mathbf{r})$, ***i.e., MAP and ML are identical***

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Likelihood Function

- In what follows, we view the parameters ϕ and τ *unknown* but *deterministic*
  - Hence, adopt the ML criterion in estimating them
- Also the observation interval $T_0 \geq T$, also called ***one-shot observation***, is used as a basis for continuously updating the estimate (tracking)
- Since the additive noise n(t) is WG with zero mean

$$p(\mathbf{r}|\psi) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^N \exp\left(-\sum_{n=1}^{N} \frac{[r_n - s_n(\psi)]^2}{2\sigma^2}\right)$$

- Where

$$r_n = \int_{T_0} r(t) f_n(t) dt$$

and

$$s_n(\psi) = \int_{T_0} s(t;\psi) f_n(t) dt$$

Likelihood Function …

$$\lim_{N\to\infty} \frac{1}{2\sigma^2}\sum_{n=1}^{N}\left[r_n - s_n(t;\psi)\right]^2 = \frac{1}{N_0}\int_{T_0}\left[r(t)-s(t;\psi)\right]^2 dt$$

$(\ Show\ this?)$

- The maximization of $p(\mathbf{r}|\psi)$ with respect to the signal parameter ψ is equivalent to the maximization of the ***likelihood function***

$$\Lambda(\psi) = \exp\left\{-\frac{1}{N_0}\int_{T_0}\left[r(t) - s(t;\psi)\right]^2\right\} dt$$

Overview

- Signal parameter estimation
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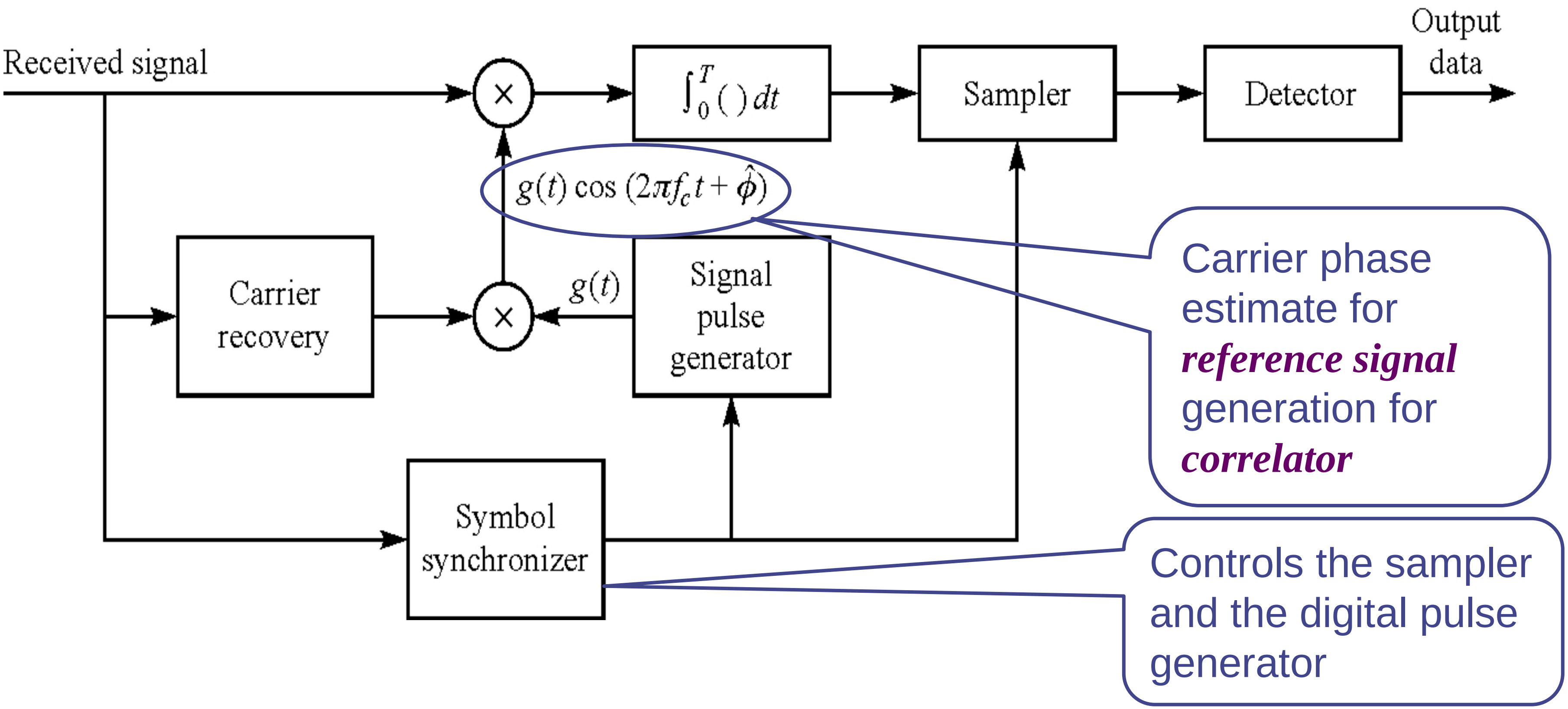
Carrier Recovery & Symbol Synchronization

- Consider the binary PSK (or binary PAM) signal demodulator and detector block diagram shown below

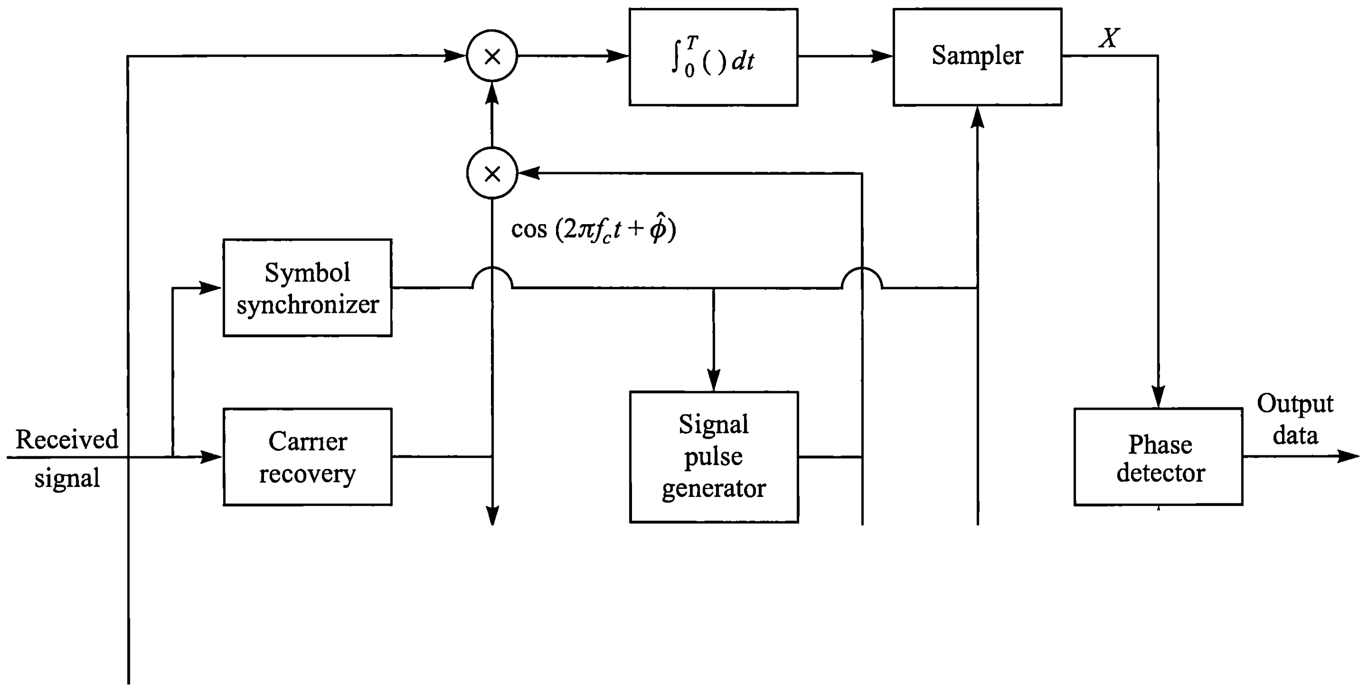


Carrier and Symbol Synchronization …

- Carrier phase estimate is used in generating the phase reference signal $g(t)\cos(2\pi f_c t + \hat{\phi})$ for the correlator
- Symbol synchronizer controls the sampler and the output of the signal pulse generator
- If *g(t)* is rectangular the signal generator can be omitted

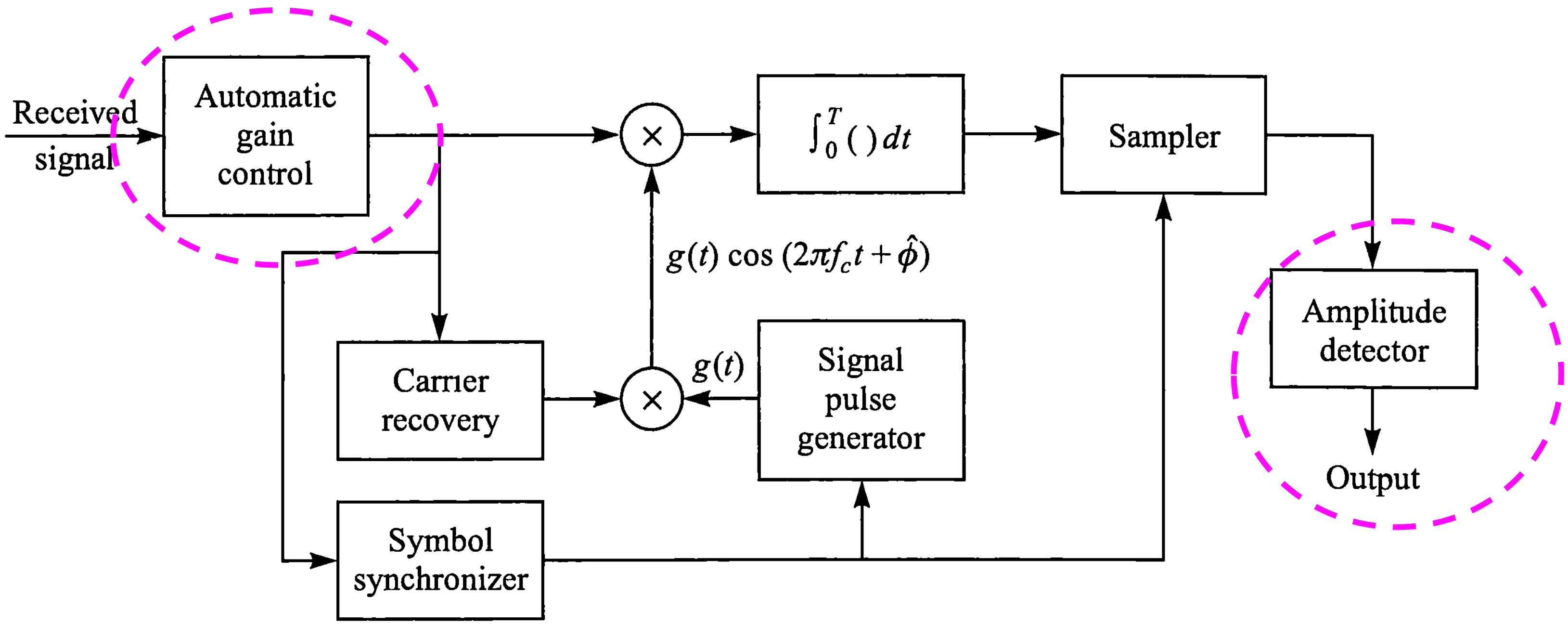
- The block diagram of an M-ary PSK demodulator is shown in the next slide
- Two correlators (or matched filters) are used to correlate the received signal with the two quadrature carrier signals
- Phase detector is used (compares the received signal phases with the possible transmitted signal phases)

Carrier and Symbol Synchronization …



Carrier and Symbol Synchronization …

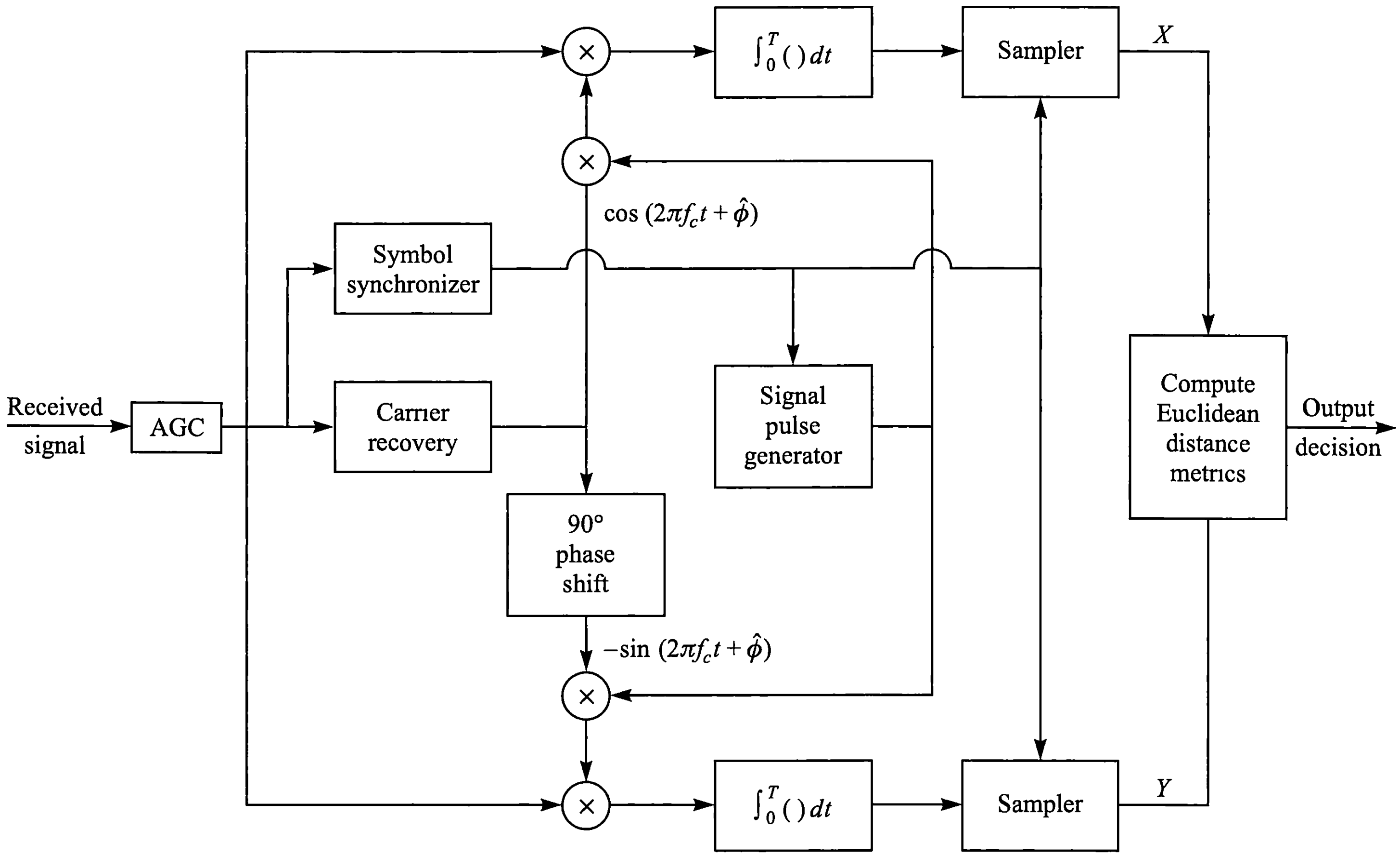
- The same arrangement can be used for M-ary PAM by introducing an automatic gain control at the front end and making the detector an "***amplitude detector***"



Block diagram of an M-ary PAM receiver

Carrier and Symbol Synchronization …

- The block diagram of a QAM demodulator is shown below



Block diagram of a QAM receiver

Overview

- Signal parameter estimation
- Carrier phase estimation
  - ML carrier phase estimation
  - Phase-locked loop
  - Effect of Additive noise on phase estimate
  - Decision-directed loops
- Symbol timing estimation

Carrier Phase Estimation

- Two methods for carrier phase estimation are:

1. Use of ***pilot signal*** that allows the receiver to extract the carrier frequency and phase of the received signal
    - Pilot signal is unmodulated carrier component that is tracked by a ***Phase Locked Loop*** (PLL) which is designed to be narrowband

2. Derive the carrier phase ***estimate directly*** from the modulated signal
    - Total transmitter power is used to transmit the information bearing signal only
    - This is ***widely used*** in practice and in our analysis we assume the signal is transmitted via suppressed carrier

Carrier Phase Estimation …

- As an *illustration* of the effect of phase error, consider the demodulation of DSB/SC AM signal

$$s(t) = A(t)\cos(2\pi f_c t + \phi)$$

- Demodulate the signal using a carrier reference signal

$$c(t) = \cos(2\pi f_c t + \hat{\phi}) \quad such\ that$$

$$c(t)s(t) = \frac{1}{2}A(t)\cos(\phi - \hat{\phi}) + \frac{1}{2}A(t)\cos(4\pi f_c t + \phi + \hat{\phi})$$

- The double frequency term is removed by the *low pass filter* (integrator) such that the output is

$$y(t) = \frac{1}{2}A(t)\cos(\phi - \hat{\phi})$$

Carrier Phase Estimation …

- Note that the effect of the error $(\phi - \hat{\phi})$ is to reduce the amplitude by the factor $cos(\phi - \hat{\phi})$ and power by the square of this factor
  - Note 10º error $\rightarrow$ 0.13dB and 30º $\rightarrow$ 1.25 dB
- The effect of phase error is much more severe in QAM and multiphase PSK which are usually represented by

$$s(t) = A(t)cos(2\pi f_c t + \phi) - B(t)sin(2\pi f_c t + \phi)$$

- This is demodulated using two quadrature carriers

$$c_c(t) = cos(2\pi f_c t + \hat{\phi})$$

$$c_s(t) = -sin(2\pi f_c t + \hat{\phi})$$

Carrier Phase Estimation …

- Multiplying s(t) by $c_c(t)$ followed by low-pass filtering yields the phase component

$$y_I(t) = \frac{1}{2}A(t)\cos(\phi - \hat{\phi}) + \frac{1}{2}B(t)\sin(\phi - \hat{\phi})$$

- And multiplying s(t) by $c_s(t)$ and low pass filtering yields the quadrature component

$$y_Q(t) = \frac{1}{2}B(t)\cos(\phi - \hat{\phi}) + \frac{1}{2}A(t)\sin(\phi - \hat{\phi})$$

- Results:
  - Power reduction by a factor of $\cos^2(\phi - \hat{\phi})$
  - ***Cross-talk interference*** from the in-phase and quadrature components causing a higher degradation in performance

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Maximum Likelihood Carrier Phase Estimation

- Assume the delay τ is constant
- The likelihood function will be a function of ϕ and not of ψ

$$\Lambda(\phi) = \exp\left[-\frac{1}{N_0}\int_{T_0}\left(r(t) - s(t,\phi)\right)^2\right]dt$$

$$= C\exp\left(-\frac{1}{N_0}\int_{T_0} r^2(t)\,dt + \frac{2}{N_0}\int_{T_0} r(t)\,s(t,\phi)\,dt - \frac{1}{N_0}\int_{T_0} s^2(t,\phi)\right)$$

- 1st term is independent of ϕ and 3rd term is a *constant* and equal to the energy over the observation time T_0
- Hence,

$$\Lambda(\phi) = C\exp\left(\frac{2}{N_0}\int_{T_0} r(t)\,s(t,\phi)\,dt\right)$$

Maximum Likelihood Carrier Phase Estimation …

- C is a constant independent of ϕ
- Equivalently, we can seek the value of ϕ that maximizes *log $\Lambda(\phi)$* such that

$$\ln \Lambda(\phi) = \Lambda_L(\phi) = \frac{2}{N_0}\int_{T_0} r(t)s(t,\phi)\,dt + \ln C$$

- The ML estimate $\hat{\phi}_{ML}$ is the value of ϕ that maximizes $\Lambda_L(\phi)$

$$\Lambda_L(\phi) = \frac{2}{N_0}\int_{T_0} r(t)s(t,\phi)\,dt + \ln C \approx \frac{2}{N_0}\int_{T_0} r(t)s(t,\phi)\,dt$$

Maximum Likelihood Carrier Phase Estimation …

- Example: Consider the transmission of unmodulated signal $A\cos 2\pi f_c t$. The received signal is $r(t) = A\cos(2\pi f_c t + \phi) + n(t)$
- Then, the log likelihood function will be

$$\Lambda_L(\phi) = \frac{2A}{N_0}\int_{T_0} r(t)\cos(2\pi f_c t + \phi)\,dt$$

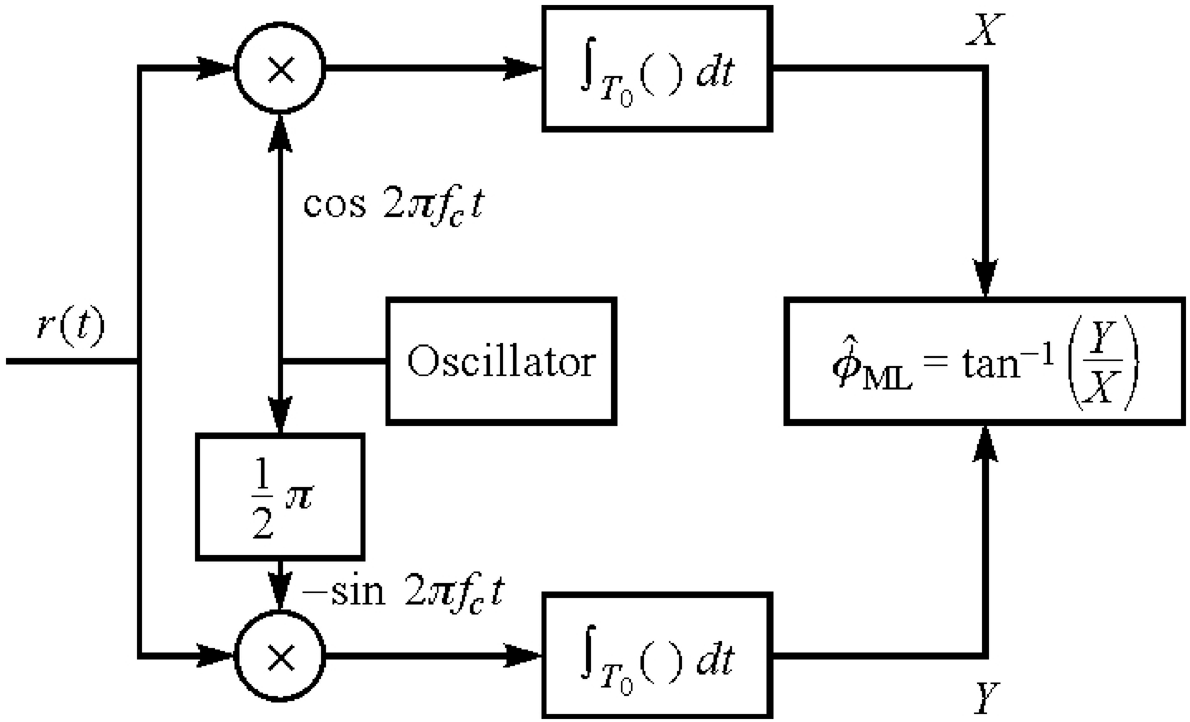
- Differentiating $\Lambda_L(\phi)$ and equating to zero we can find the value of ϕ that maximizes the likelihood function

$$\frac{d\Lambda_L(\phi)}{d\phi} = \int_{T_0} r(t)\sin(2\pi f_c t + \hat{\phi}_{ML})\,dt = 0; \quad yields$$

$$\hat{\phi}_{ML} = -\tan^{-1}\left(\frac{\int_{T_0} r(t)\sin 2\pi f_c t\,dt}{\int_{T_0} r(t)\cos 2\pi f_c t\,dt}\right)$$

Carrier and Symbol Synchronization …

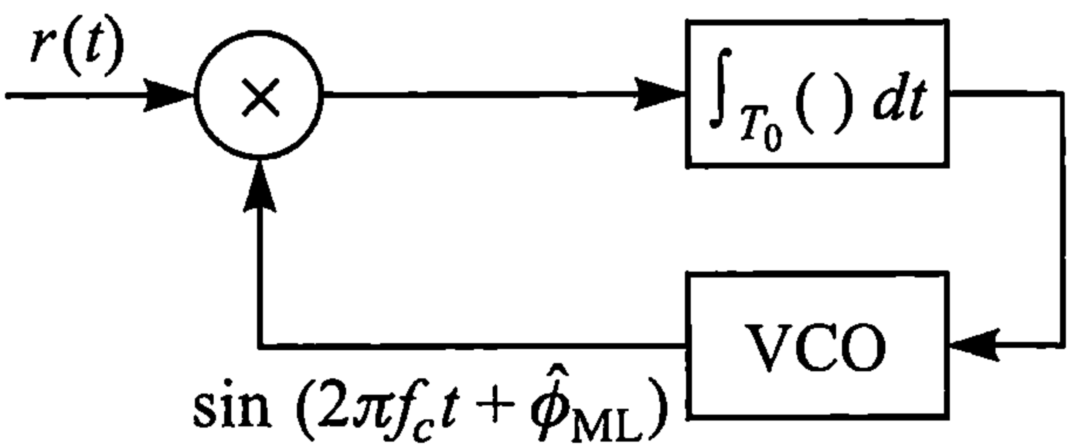
- Observe $$\hat{\phi}_{ML} = -\tan^{-1}\left(\int_{T_0} r(t)\sin 2\pi f_c t\, dt \Big/ \int_{T_0} r(t)\cos 2\pi f_c t\, dt \right)$$



A (one-shot) ML estimate of the phase of an unmodulated carrier

Maximum Likelihood Carrier Phase Estimation …

- Note that: $\int_{T_0} r(t)\sin(2\pi f_c t + \hat{\phi}_{ML})\,dt = 0$ implies the use of a loop to extract the estimate as illustrated below
- The loop filter is an integrator whose bandwidth is proportional to the reciprocal of the integration interval T_o



A PLL for obtaining the ML estimate of the phase of an unmodulated carrier

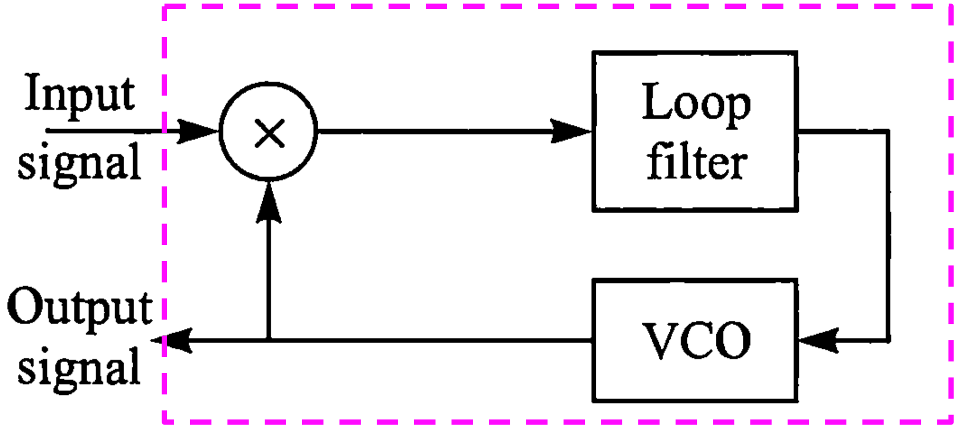
Overview

- Signal parameter estimation
- Carrier phase estimation
  - Maximum-likelihood carrier phase estimation
  - Phase-locked loop
  - Effect of additive noise on phase estimate
  - Decision-directed loops
- Symbol timing estimation

Phase-locked Loop

- *Phase-locked loop* (PLL) consists of a multiplier, a loop filter, and a voltage-controlled oscillator (VCO)
- Assume that the input to the PLL is a $\cos(2\pi f_c t+\phi)$ and the output of the VCO $\sin(2\pi f_c t+\hat{\phi})$
- Then

$$e(t) = \cos(2\pi f_c t+\phi)\sin(2\pi f_c t+\hat{\phi})$$
$$= \frac{1}{2}\sin(\hat{\phi}-\phi)+\frac{1}{2}\sin(4\pi f_c t+\phi+\hat{\phi})$$



Phase-locked Loop …

- The *loop filter* is a low-pass filter with transfer function

$$G(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s}$$

- τ_1 and τ_2 are design parameters ($\tau_1 >> \tau_2$) that control the bandwidth of the loop
- Output of the loop filter gives control voltage $\nu(t)$ for VCO
- The VCO is basically a sinusoidal signal generator with an instantaneous phase given by

$$2\pi f_c t + \hat{\phi}(t) = 2\pi f_c t + k\int_{-\infty}^{t} v(\tau)\, d\tau$$

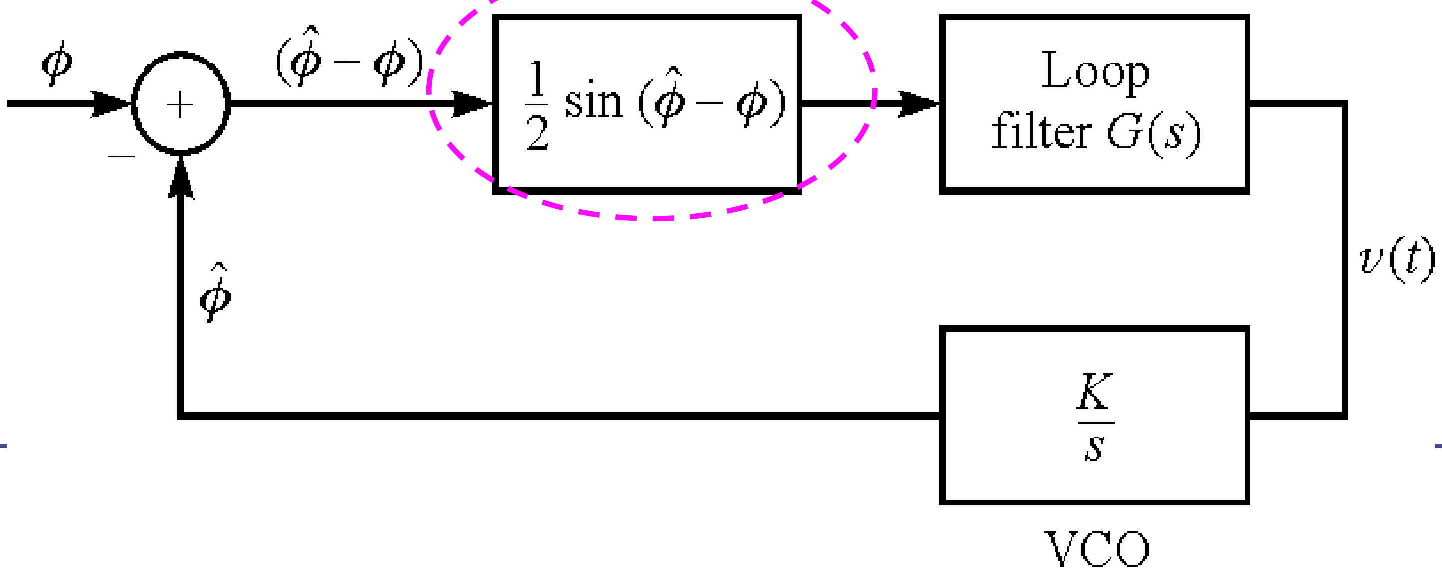
- where K is a gain constant in rad/V

Phase-locked Loop …

- Neglecting the double-frequency term, the PLL may be implemented as shown below
  - It is a *non-linear* system unless $\sin(\hat{\phi}-\phi) \approx \hat{\phi}-\phi$
- The linearized PLL is characterized by the closed-loop transfer function (see pages 342-343 of the text)

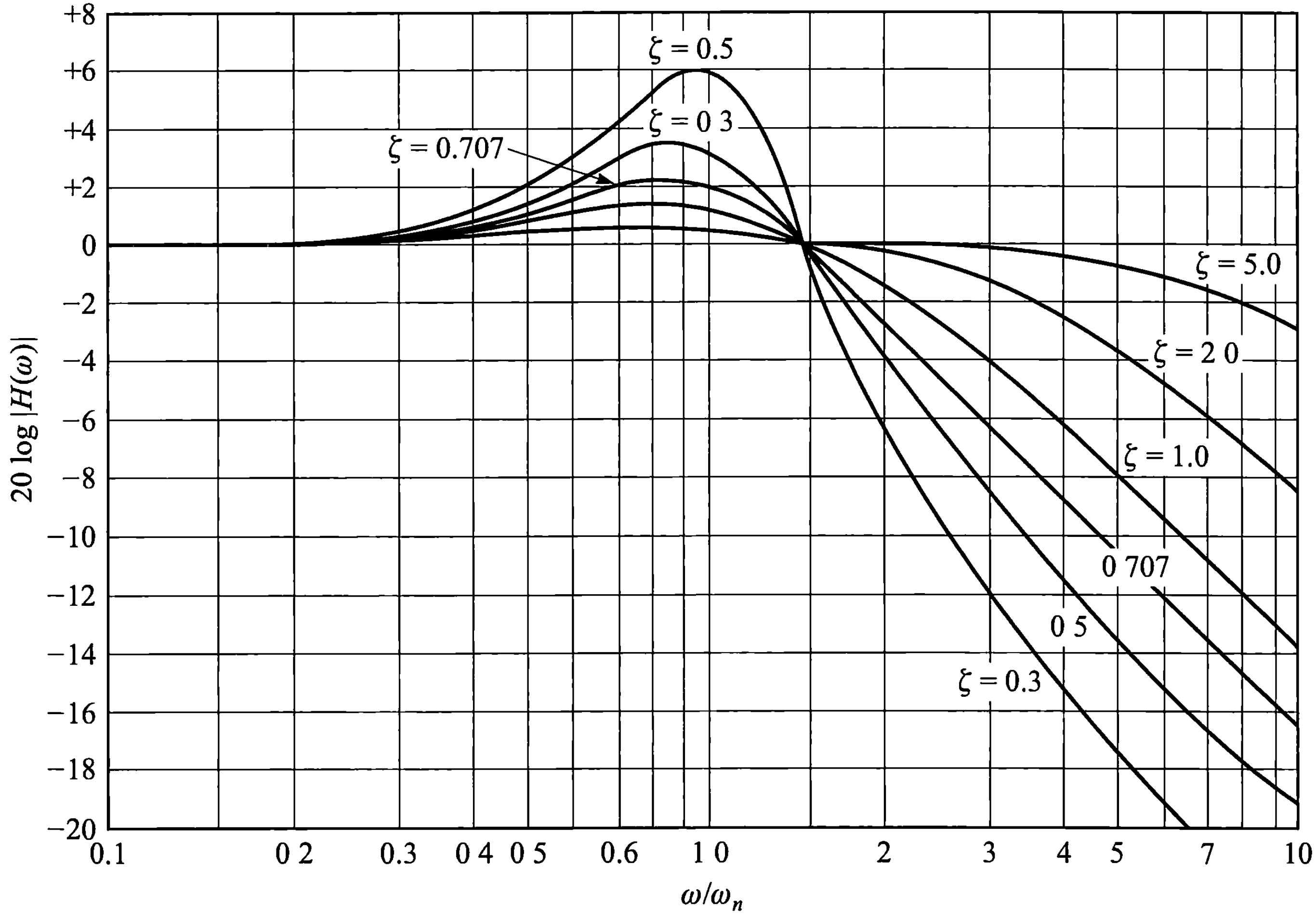
$$H(s)=\frac{1+\tau_2 s}{1+(\tau_2+1/K)s+(\tau_1/K)s^2}$$

  - Where K is the gain parameter



Phase-locked Loop …

- Frequency response of the closed-loop transfer function



Overview

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Effect of Additive Noise on Phase Estimate

- Assume narrowband noise at the input of the PLL and that the PLL is tracking a sinusoidal signal of the form

$$s(t) = A_c cos(2\pi f_c t + \phi(t))$$

- That is corrupted by additive narrowband noise

$$n(t) = x(t) cos 2\pi f_c t - y(t) sin 2\pi f_c t$$

- Where *x(t)* and *y(t)* are assumed to be *statistically independent, stationary and Gaussian* with power spectral density $N_0/2$ W/Hz
- Using trigonometric identities n(t) can be expressed as

$$n(t) = n_c(t) cos(2\pi f_c t + \phi(t)) - n_s(t) sin(2\pi f_c t + \phi(t))$$

Effect of Additive Noise on Phase Estimate

- Where

$$n_c(t) = x(t)\cos\phi(t) + y(t)\sin\phi(t)$$
$$n_s(t) = -x(t)\sin\phi(t) + y(t)\cos\phi(t)$$

- Note that

$$n_c(t) + jn_s(t) = \left(x(t) + jY(t)\right)e^{-j\phi(t)}$$

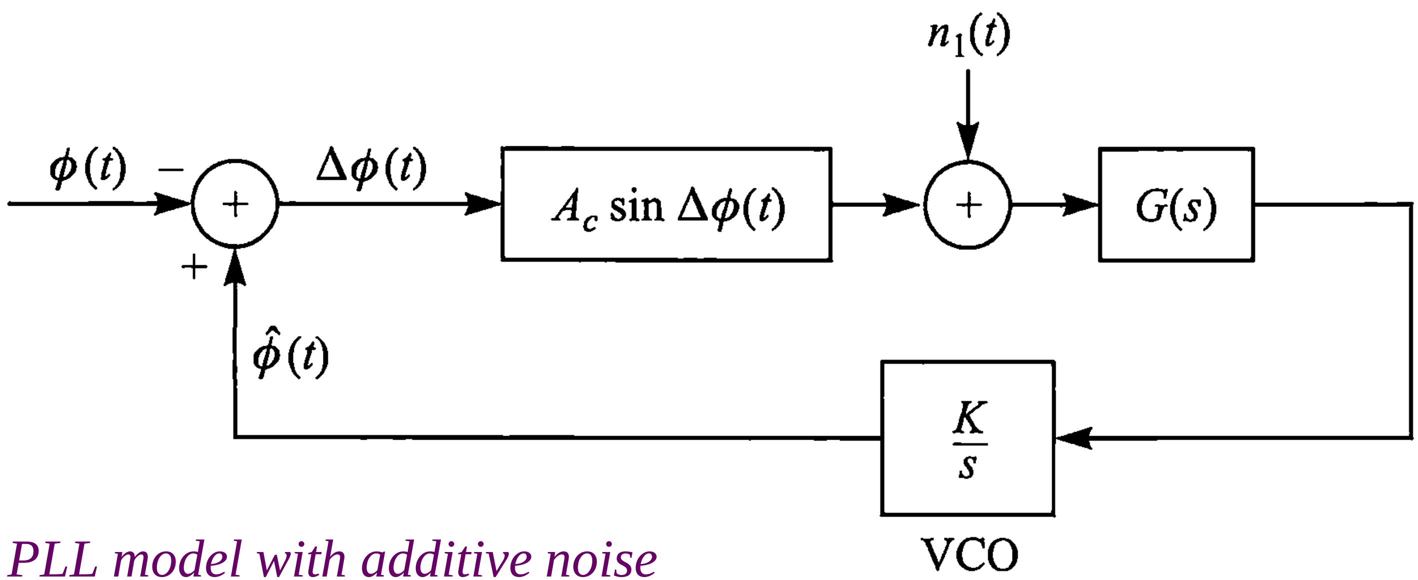
- Such that $n_c(t)$ and $n_s(t)$ have the same statistical properties as $x(t)$ and $y(t)$

Effect of Additive Noise on Phase Estimate …

- If *s(t)+n(t)* is multiplied by the output of VCO and the double frequency terms are ignored, the input to the loop filter is a noise corrupted signal

$$\begin{aligned} e(t) &= A_c \sin\Delta\phi + n_c(t)\sin\Delta\phi - n_s(t)\cos\Delta\phi \\ &= A_c \sin\Delta\phi + n_1(t) \end{aligned}$$

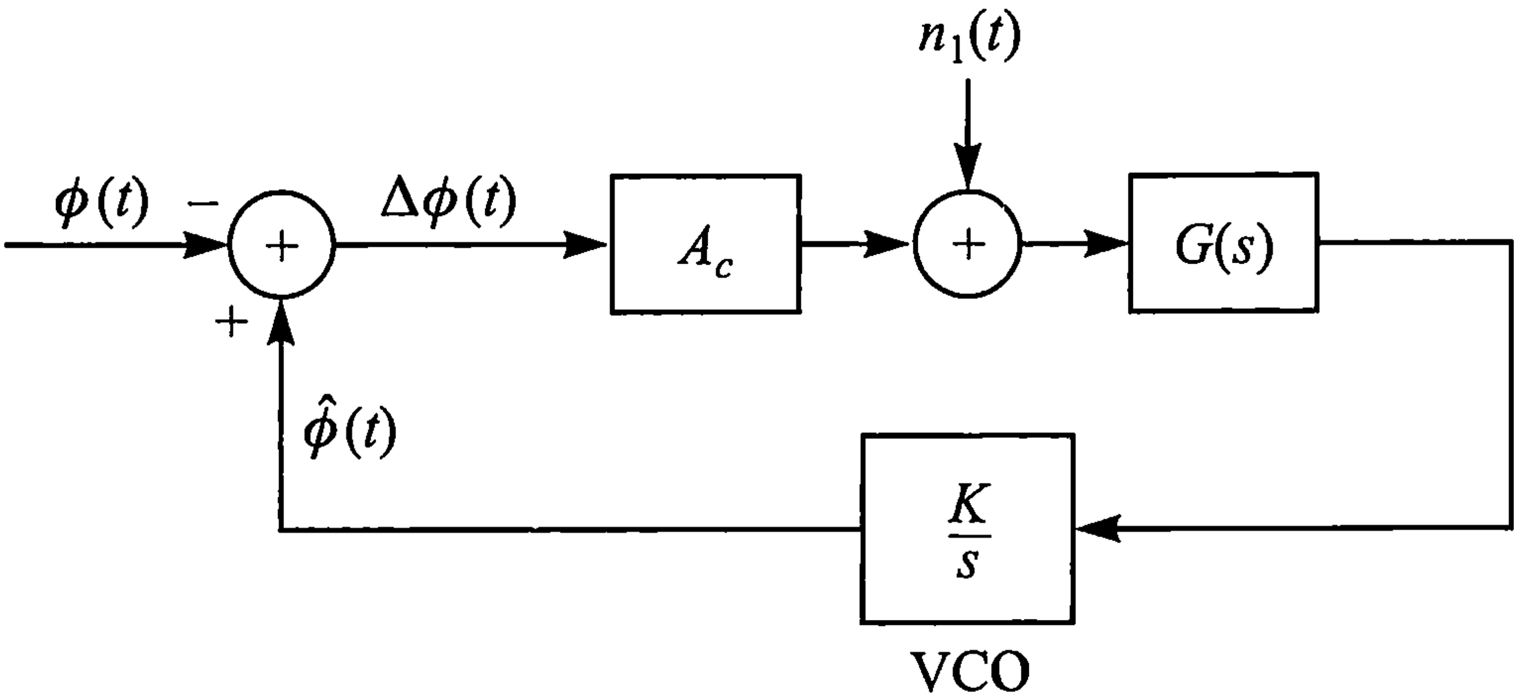
- Where $\Delta\phi = \phi - \hat{\phi}$ is the *phase error*



Equivalent PLL model with additive noise

Effect of Additive Noise on Phase Estimate …

- If the power of the incoming signal $P_c = \frac{1}{2} A_c^2$ is larger than the noise power, the PLL may be linearized by making $\sin \Delta\phi(t) \approx \Delta\phi(t)$



Linearized PLL model with additive noise

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- Signal parameter estimation
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Decision-directed Loops …

- How to maximize $\Lambda(\phi)$ or $\Lambda_L(\phi)$ when the signal s(t; ϕ) carries the information sequence $\{I_n\}$?
- Carrier recovery when the signal is modulated uses *decision-directed* loops

- In such cases, one can use one of two approaches
  1. Assume $\{I_n\}$ is known or
  2. Treat $\{I_n\}$ as a random sequence and average it over its statistics

- In *decision-directed* parameter estimation, we assume the information sequence $\{I_n\}$ over the observation interval has been *estimated*
  - In the absence of demodulation error $\tilde{\mathbf{I}}_n = \mathbf{I}_n$

Decision-directed Loops …

- In this case s(t; ϕ) is completely known except for the carrier phase
- Consider *decision-directed* phase estimate for *linear modulation* technique for which the received equivalent low pass may be expressed as

$$r_l(t) = e^{-j\phi} \sum I_n \, g(t - nT) + z(t) = s_l(t) e^{-j\phi} + z(t)$$

- Where $s_l(t)$ is a known signal if the sequence $\{I_n\}$ is assumed known

Decision-directed Loops …

- The *likelihood* and *log-likelihood* functions for the equivalent low pass signal are

$$\Lambda(\phi) = C\exp\left[\operatorname{Re}\left(\frac{1}{N_0}\int_{T_0} r_l(t)s_l^*(t)e^{j\phi}dt\right)\right]$$

$$\Lambda_L(\phi) = \operatorname{Re}\left[\left(\frac{1}{N_0}\int_{T_0} r_l(t)s_l^*(t)dt\right)e^{j\phi}\right]$$

- Substituting for $s_l(t)$ and observation interval of $T_0=KT$

$$\Lambda_L(\phi) = \operatorname{Re}\left(e^{j\phi}\frac{1}{N_0}\sum_{n=0}^{K-1} I_n^* \int_{nT}^{(n+1)T} r_l(t)g^*(t-nT)dt\right)$$

$$= \operatorname{Re}\left(e^{j\phi}\frac{1}{N_0}\sum_{n=0}^{K-1} I_n^* y_n\right)$$

Decision-directed Loops …

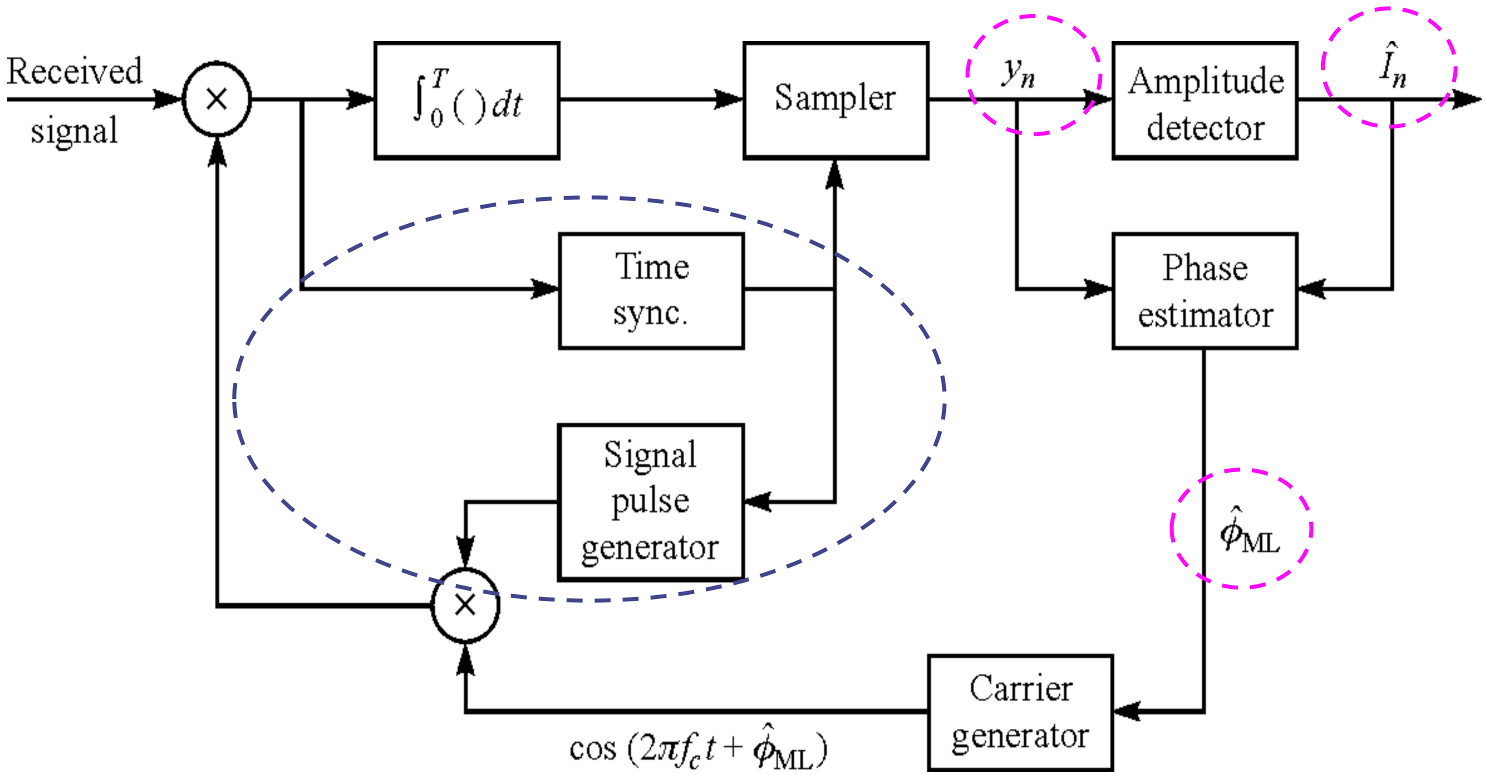
- Where $y_n = \int_{nT}^{(n+1)T} r(t)\, g^*(t-nT)\, dt$ is the output of a matched filter in the *n*th interval
- Then, $\Lambda_L(\phi) = Re\left(\frac{1}{N_0}\sum_{n=0}^{K-1} I_n^* y_n\right)\cos\phi - Im\left(\frac{1}{N_0}\sum_{n=0}^{K-1} I_n^* y_n\right)\sin\phi$
- Differentiating $\Lambda_L(\phi)$ with respect to ϕ and equating to zero, we obtain the phase estimate as

$$\hat{\phi}_{ML} = -\tan^{-1}\left[Im\left(\sum_{n=0}^{K-1} I_n^* y_n\right) \Big/ Re\left(\sum_{n=0}^{K-1} I_n^* y_n\right)\right]$$

- This is called *decision-directed* (or *decision feedback*) carrier phase estimate

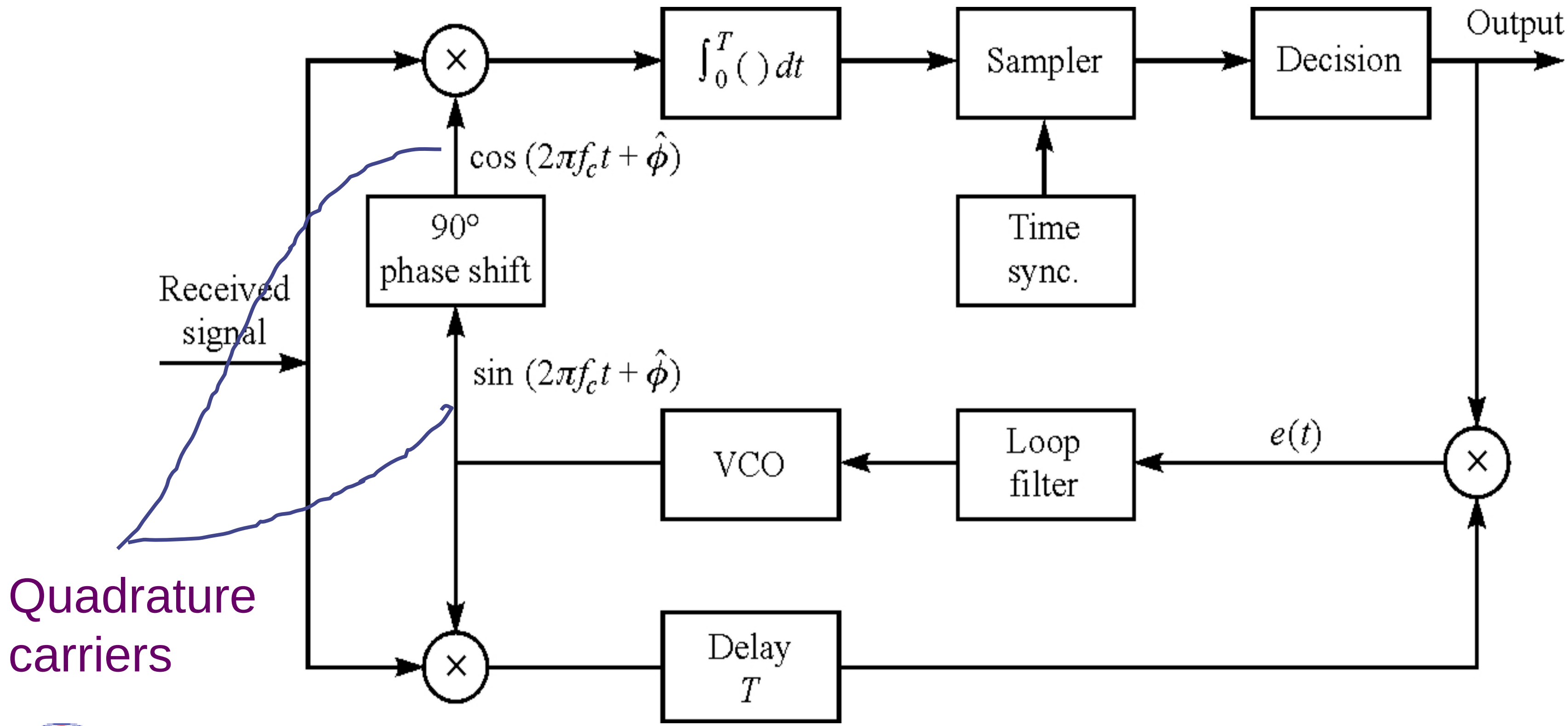
Decision-directed Loops …

- Here $E\{\hat{\phi}_{ML}\} = \phi$ and the estimate is unbiased
- Block diagram of DSB *PAM* signal receiver with *decision-directed* carrier phase estimation



Decision-directed Loops …

- For a DSB PAM signal of $A(t)\cos(2\pi f_c t+\phi)$, where $A(t)=A_m g(t)$ and $g(t)$ is assumed rectangular pulse of duration T
- Carrier recovery with a decision-feedback PLL is shown below



Decision-directed Loops …

- Output of the first multiplier and input to the integrator is given by

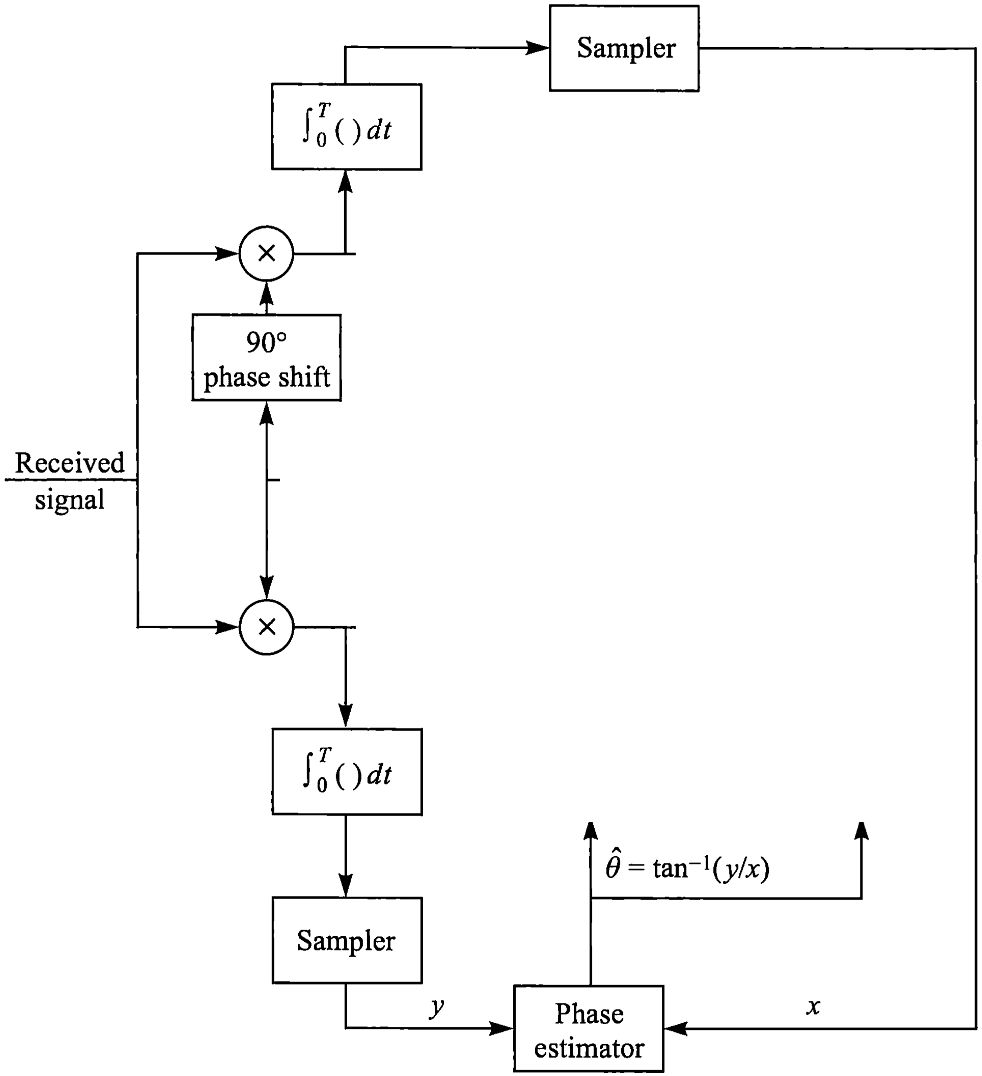
$$r(t)\cos(2\pi f_c t+\hat{\phi}) = \rho_s(t) = \frac{1}{2}\left[A(t)+n_c(t)\right]\cos\Delta\phi$$
$$-\frac{1}{2}n_s(t)\sin\Delta\phi + double \;\; frequency \;\; terms$$

- ρ_s(t) is used to recover information carried by $A(t)=A_m g(t)$
- Detector makes decision on received symbols every T sec.
  - In the absence of error it reconstructs *A(t)* free of any noise
- Reconstructed signal multiplied by 2nd quadrature carrier
- Then:

$$e(t) = \frac{1}{2}A(t)\left\{\left[A(t)+n_c(t)\right]\sin\Delta\phi - n_s(t)\cos\Delta\phi\right\} + Double \;\; frequency \;\; terms$$
$$= \frac{1}{2}A^2(t)\sin\Delta\phi + \frac{1}{2}A(t)\left[n_c(t)\sin\Delta\phi - n_s(t)\cos\Delta\phi\right] + \ldots$$

Decision-directed Loops …

- The desired component $A^2(t)\sin\Delta\phi$, which contains the phase error, drives the loop filter
- Carrier recovery system for M-ary PSK using decision feedback PLL (DFPLL) is shown in the next figure



Decision-directed Loops …

- For the case of M-ary PSK using DFPLL, the received signal is demodulated to yield the phase estimate

$$\hat{\theta}_m = \frac{2\pi}{M}(m-1) \quad m = 1,2,\ldots\ldots M$$

- Which, in the absence of decision error, is phase of the transmitted signal θ_m
- The two outputs of quadrature multipliers are delayed by symbol duration T & multiplied by $cos\theta_m$ and $sin\theta_m$ to yield

1.
$$r(t)cos(2\pi f_c t+\hat{\phi})sin\theta_m = \frac{1}{2}\left[Acos\theta_m + n_c(t)\right]sin\theta_m cos(\phi-\hat{\phi})$$
$$-\frac{1}{2}\left[Asin\theta_m + n_s(t)\right]sin\theta_m sin(\phi-\hat{\phi}) + double\ freq.\ terms$$

Decision-directed Loops …

2. $$r(t)\sin(2\pi f_c t+\hat{\phi})\cos\theta_m = \frac{1}{2}\left[A\cos\theta_m + n_c(t)\right]\cos\theta_m \sin(\phi-\hat{\phi})$$
$$-\frac{1}{2}\left[A\sin\theta_m + n_s(t)\right]\cos\theta_m \cos(\phi-\hat{\phi}) + double\ \ freq.\ terms$$

- The error signal is the sum of these two which reduces to

$$e(t) = -\frac{1}{2}A\sin(\phi-\hat{\phi}) + \frac{1}{2}n_c(t)\sin(\phi-\hat{\phi}-\theta_m)$$
$$+\frac{1}{2}n_s(t)\cos(\phi-\hat{\phi}-\theta_m) \quad + Double\ \ frequency\ \ terms$$

- The error signal drives the loop filter that provides the control signal to the VCO
- Note that the two quadrature noise components are additive

Decision-directed Loops …

- There are no product of the noise terms and thus no *additional power loss* associated with decision-feedback PLL
- The ML estimate of ϕ given by the earlier equation is also appropriate for QAM

Overview

- Signal parameter estimation
- Carrier phase estimation
- Symbol timing estimation
  - ML timing estimation

Symbol Timing Estimation

- Modulator output must be sampled periodically at the symbol rate, i.e., at precise sampling time instants $t_m = mT+\tau$
  - Where T is the symbol interval and τ is the nominal delay
- Periodic sampling requires clock signal at the receiver
- Extraction of clock signal called *symbol synchronization or timing recovery*
  - Is critical for a synchronous digital communication system
- The receiver must know
  - The frequency 1/ T and
  - Where to take the samples within each symbol interval
- The choice of sampling instant within the symbol interval of duration T is called the *timing phase*

Symbol Timing Estimation …

- Symbol synchronization can be accomplished in one of the following ways

1. Tx and Rx clocks are synchronized to a *master clock* which provides precise timing signal (VLF < 30 KHz)
2. Tx transmits the clock frequency (1/T) or (n/T) along with the information symbol
   - Rx employs narrowband filter to extract the clock signal for sampling (*simple but power inefficient*)
3. Clock signal is extracted from the received data symbol (*our focus is on this)*
   - Will cover a *decision-directed* method next

Symbol Timing Estimation - ML Timing Estimation

- Consider the problem of estimating the time delay τ for a baseband PAM waveform

$$r(t;\tau) = s(t;\tau) + n(t)$$

- where

$$s(t;\tau) = \sum I_n g(t - nT - \tau)$$

- Assume the information symbols from the output of the de-modulator are *known transmitted* sequences
- Then the log-likelihood function has the form

$$\Lambda_L(\tau) = C_L \int_{T_0} r(t) s(t;\tau)\, dt$$

$$= C_L \sum_n I_n \int_{T_0} r(t) g(t - nT - \tau)\, dt = C_L \sum_n I_n y_n(\tau)$$

Symbol Timing Estimation - ML Timing Estimation …

- Where $y_n(t)$ is defined as

$$y_n(\tau) = \int_{T_0} r(t)\, g(t - nT - \tau)\, dt$$

- A necessary and sufficient condition for $\hat{\tau}$ to be the ML estimate of τ is that

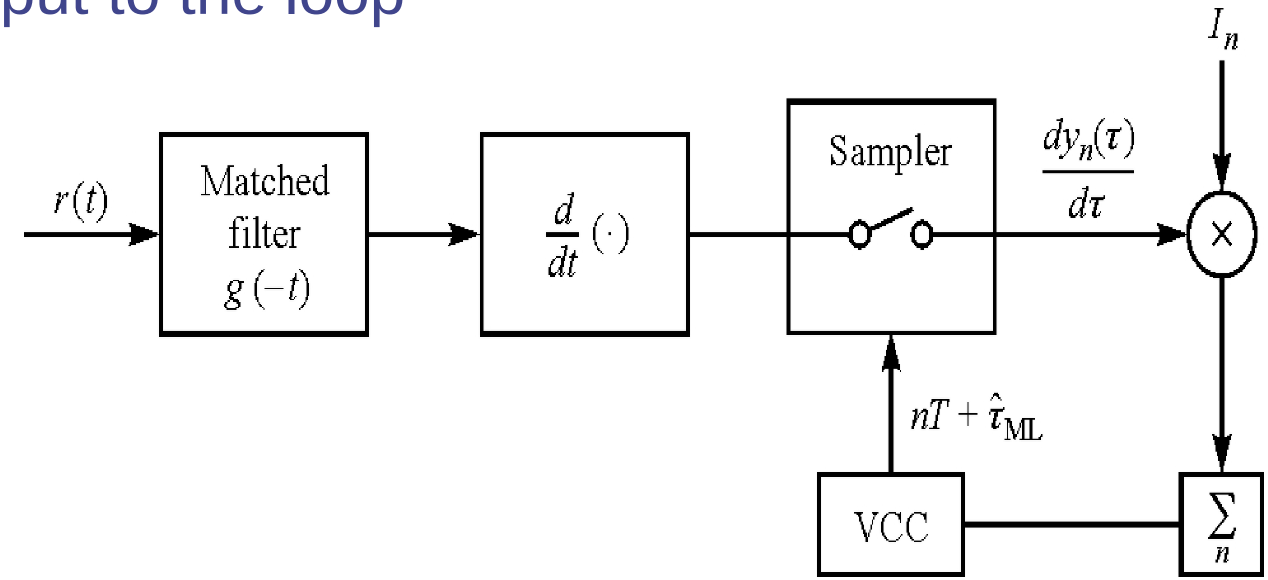
$$\frac{d\Lambda_L(\tau)}{d\tau} = \sum_n I_n \frac{d}{d\tau} \int_{T_0} r(t)\, g(t - nT - \tau)\, dt$$

$$= \sum_n I_n \frac{d}{d\tau} \left[y_n(\tau) \right] = 0$$

- The above result suggests the implementation of the tracking loop shown in next slide

Symbol Timing Estimation - ML Timing Estimation …

- The summation in the loop serves as the loop filter whose bandwidth is controlled by the length of the sliding window in the summation
- Output of the loop filter drives the *voltage controlled clock* (VCC) or VCO which in turn controls the sampling times for the input to the loop



Decision-directed ML estimation of timing for baseband PAM

Symbol Timing Estimation - ML Timing Estimation …

- Note that since the information sequence $\{I_n\}$ is assumed known and used in the estimation of τ, the estimate is *decision-directed*
- This method can be extended to signal formats such as QAM and PSK by using the equivalent low pass form of these signals