Chapter 7

RADIATION
HEAT TRANSFER

Yilma Tadesse(PhD)

School of Mechanical and Industrial Engineering
AAiT
Addis Ababa University

Introduction

- *Radiative heat transfer* or *thermal radiation* is the science of transferring energy in the form of electromagnetic waves.
- Unlike heat conduction, electromagnetic waves do not require a medium for their propagation. Therefore, because of their ability to travel across vacuum, thermal radiation becomes the dominant mode of heat transfer in low pressure (vacuum) and outer-space applications.

- Another distinguishing characteristic between conduction (and convection, if aided by flow) and thermal radiation is their temperature dependence. While conductive and convective fluxes are more or less linearly dependent on temperature differences, radiative heat fluxes tend to be proportional to differences in the fourth power of temperature (or even higher).

- For this reason, radiation tends to become the dominant mode of heat transfer in high-temperature applications, such as combustion (fires, furnaces, rocket nozzles), nuclear reactions (solar emission, nuclear weapons), and others.

- All materials continuously emit and absorb electromagnetic waves, or photons, by changing their internal energy on a molecular level. Strength of emission and absorption of radiative energy depend on the temperature of the material, as well as on the wavelength λ, frequency ν, or wave number η, that characterizes the electromagnetic waves,

$$\lambda = \frac{c}{\nu} = \frac{1}{\eta}$$

Thermal radiation is that electromagnetic radiation emitted by a body as a result of its temperature. Unlike conduction and convection, it requires no matter for the transfer. All electromagnetic radiations are propagated at the speed of light, given as the product of wavelength and frequency.

$$c = \lambda\nu$$

1Å (angstrom) = 10^{-8} cm.

A portion of the electromagnetic spectrum is shown in **figchp11\fig11.1.pptx**

Thermal radiation lies in the range about 0.1 to 100μm. The visible light is between 0.35 to 0.75 μm.

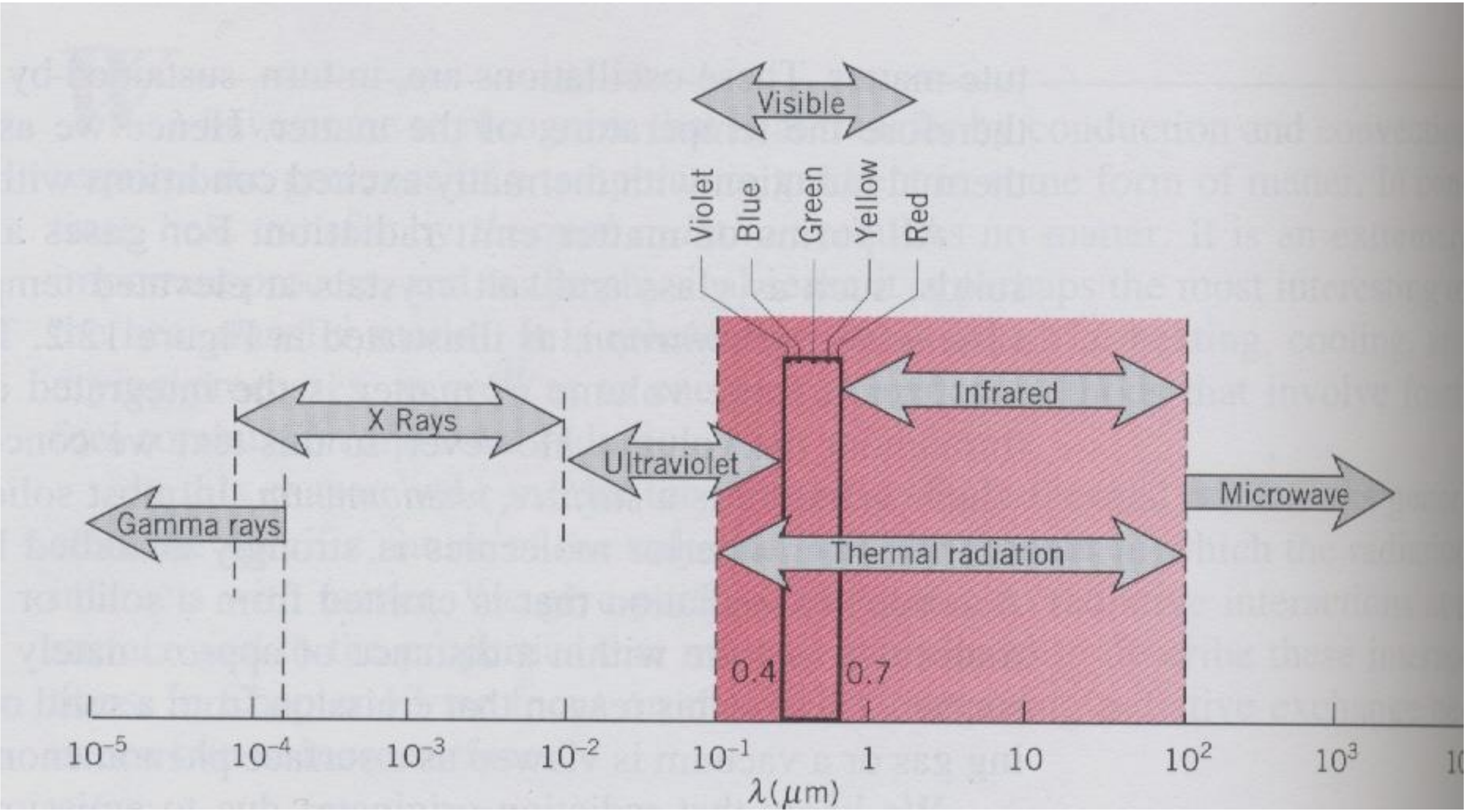


Fig.11.1 Spectrum of electromagnetic radiation

The modern theory views thermal radiation as the propagation of a collection of particles called photons or quanta with quantum of energy given by

$E = h\nu$ h=6.625x10^{-34} J.s (Planck's constant)

Using $E = mc^2 = h\nu$ one can find the momentum of a photon as

Momentum $= mc = h\nu/c$

Quantum – statistical thermodynamics gives the energy density of radiation per unit volume and per unit wave length as

$$u_\lambda = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \qquad k = Boltzmann\ constant$$

$$= 1.38066x10^{-23}\ J/molecule.K$$

When the above is integrated over all wavelengths it gives

$$E_b = \sigma T^4$$

The above is called the Stefan-Boltzmann law, E_b is the energy(W) radiated per unit time and per unit area by the ideal radiator, and σ is the Stefan-Boltzmann constant given by

$\sigma = 5.669 \times 10^{-8}\ W/m^2.K^4$

Radiation Properties

When radiant energy is incident on a surface (called irradiation), part of the radiation is reflected, part is absorbed, and part is transmitted as shown in **figchp11\fig11.2.pptx** .

For irradiation given by G

$G = \alpha G + \rho G + \tau G$ or $\alpha + \rho + \tau = 1$

α = Absorptivity ρ = Reflectivity τ = Transmissivity

For solid bodies that do not transmit

$\alpha + \rho = 1$

Two types of reflections:

Specular- incidence and reflection angles are

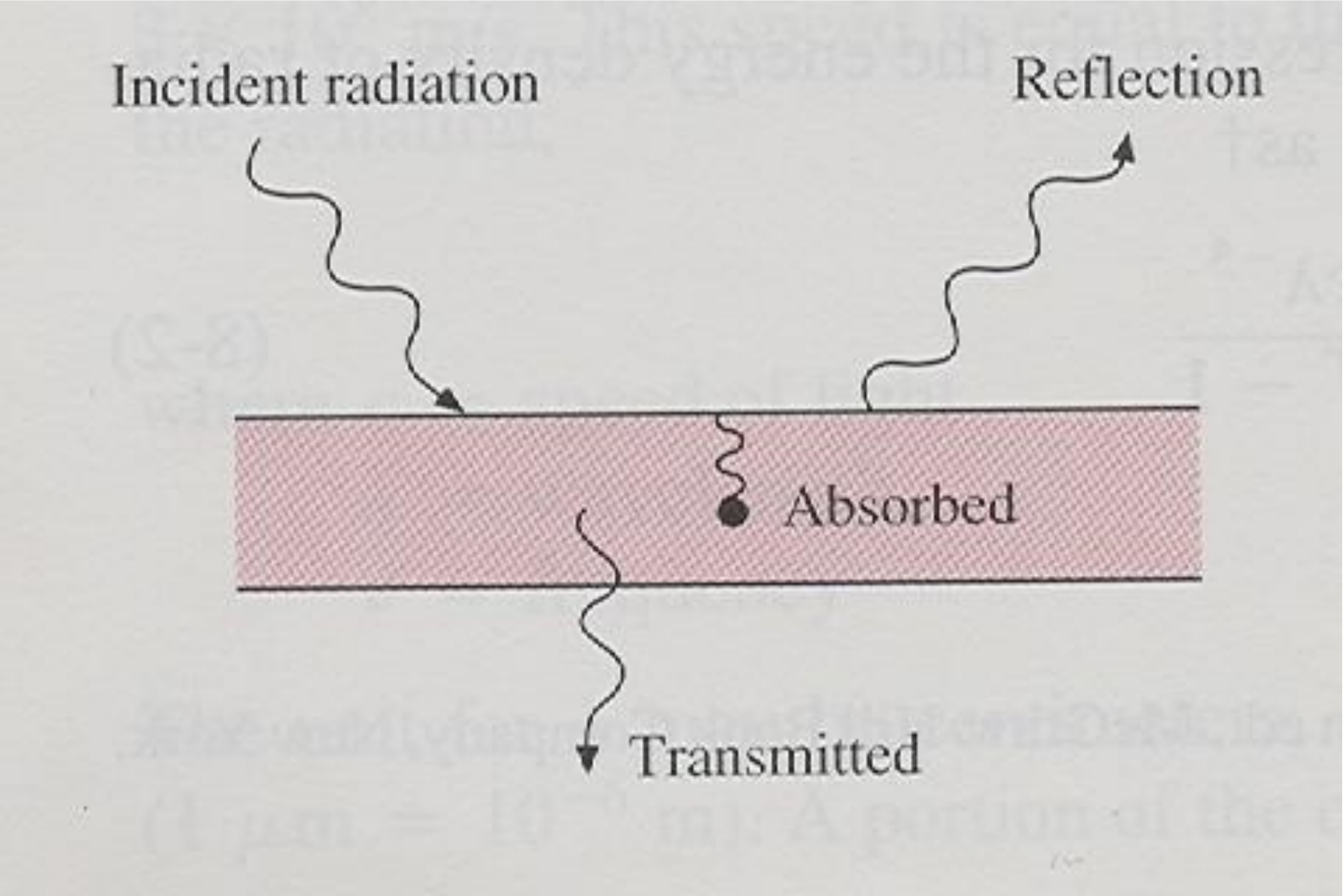


Fig.11.2 Sketch showing effects of radiation

equal.

Diffuse – incident beam is distributed uniformly in all directions after reflection **figchp11\fig11.3.pptx**

The emissive power of a body E is defined as the energy emitted by the body per unit area per unit time. Shown in **figchp11\fig11.4.pptx** the black enclosure will absorb all the incident radiation falling upon it. It will also emit radiation according to the T^4 law. Let the radiant flux arriving at some area in the enclosure be q_i W/m^2. If a body is placed inside the enclosure and allowed to come to equilibrium, the energy absorbed and emitted by the body are equal.

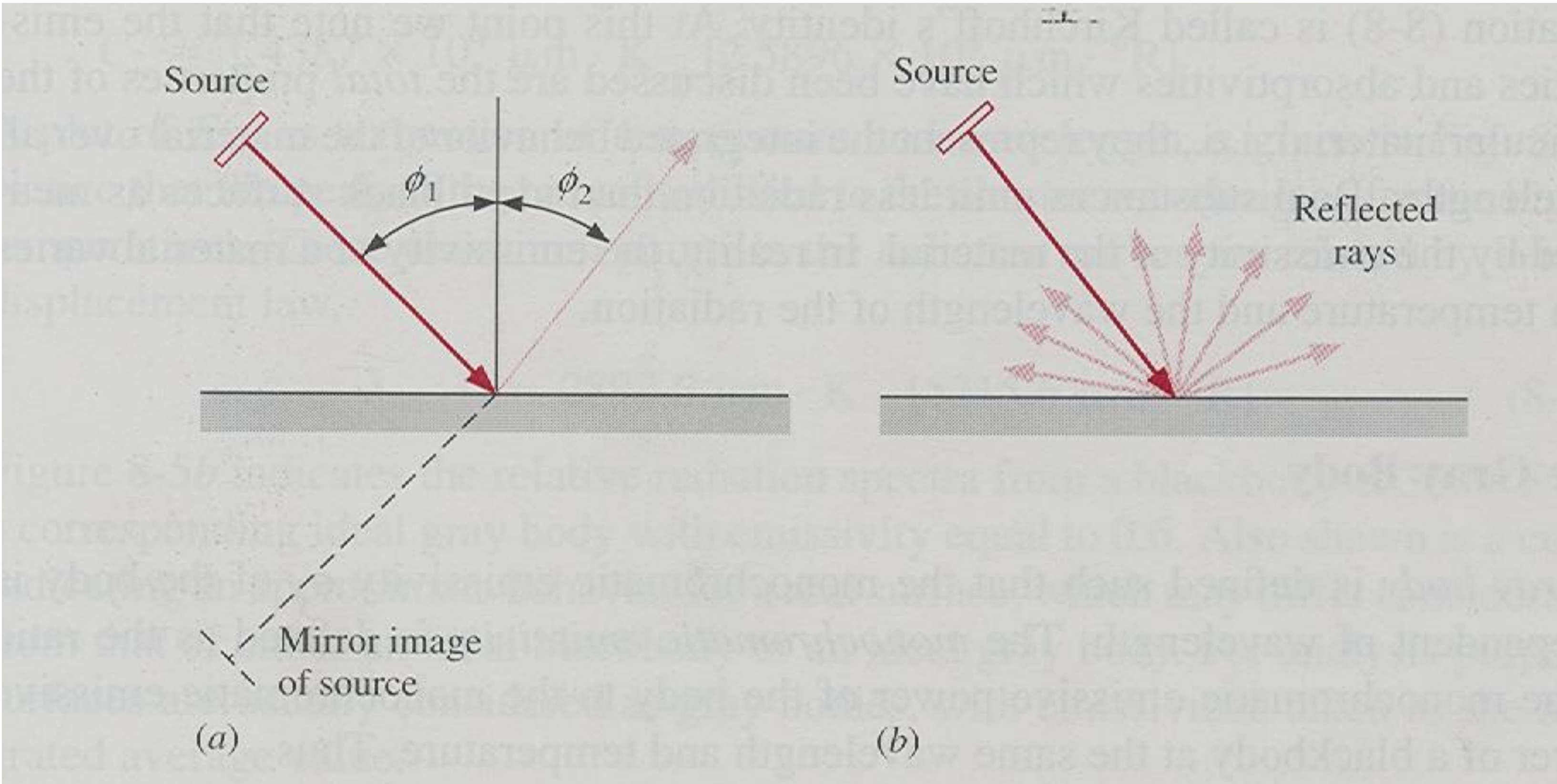


Fig.11.3 (a) Specular ($\phi_1 = \phi_2$) and (b) diffuse reflection

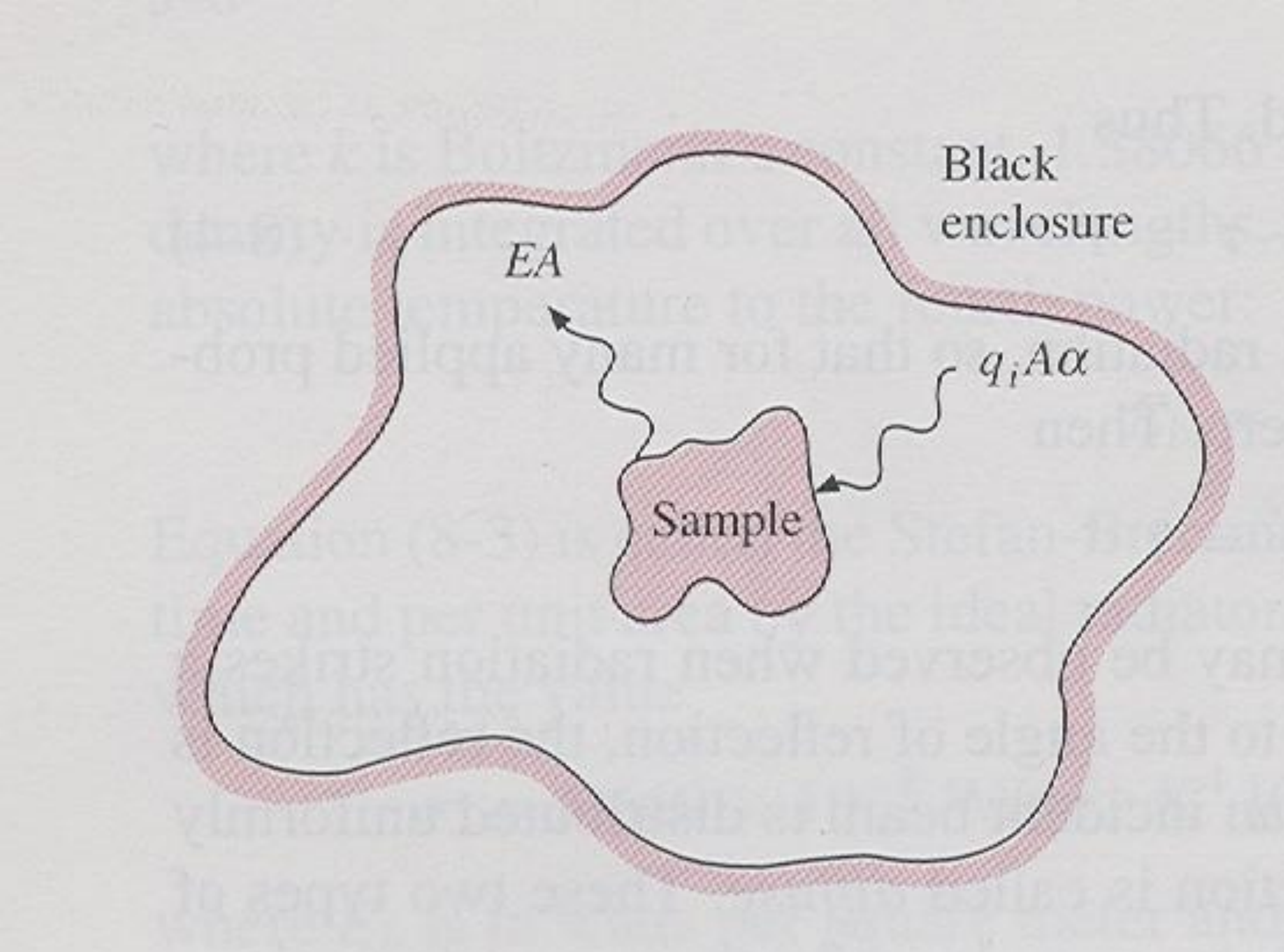


Fig.11.4 Model used to derive Kirchoff's law

At equilibrium

$$EA = q_i A\alpha$$

If the body had been a black body, then

$$E_b A = q_i A(1)$$

The above will give the ratio of the emissive power of a body to the emissive power of a blackbody at the same temperature as the absorptivity. This ratio is also defined as the emissivity ε of the body, given as

$$\varepsilon = \frac{E}{E_b} = \alpha$$

The equality of α and ε is called Kirchoff's identity.

The Gray Body

A gray body has its monochromatic emissivity ε_λ independent of the wavelength. Monochromatic emissivity is defined as the ratio of the monochromatic emissive power of the body to the monochromatic emissive power of a black body at the same wavelength and temperature.

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$

The total emissivity of the body and that of a blackbody can be determined as

$$E = \int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda \qquad and \qquad E_b = \int_0^\infty E_{b\lambda} d\lambda = \sigma T^4$$

From the above

$$\varepsilon = \frac{E}{E_b} = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda}{\sigma T^4}$$

If the gray body condition is imposed, ε_λ = constant, the above equation reduces to

$$\varepsilon = \varepsilon_\lambda$$

It has to be noted that the emissivities of various substances vary widely with wavelength, temperature, and surface condition.

For a blackbody, according to Planck, $E_{b\lambda}$ (spectral emissive power) is given by

$$E_{b\lambda} = \frac{u_\lambda c}{4} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}$$

λ=wavelength, μm

T=temperature, K

$C_1 = 3.743 \times 10^8$ W.μm^4/m^2

$C_2 = 1.4387 \times 10^4$ μm . K

- This emissive power is plotted in **figchp11\fig11.5.pptx** . Close observation of the curves shows a shift of the peak points to the shorter wavelengths for higher temperatures. This shift is defined by Wien's displacement law given by

$$\lambda_{max} T = 2897.6 \ \mu m.K$$

- The sun at 5800 K is considered as a black body. The maximum emission is in the visible range and this appears as white. For a black body at 1000K, peak emission occurs at 2.90 μm (not visible), with some of the emitted radiation appearing visible as red light.

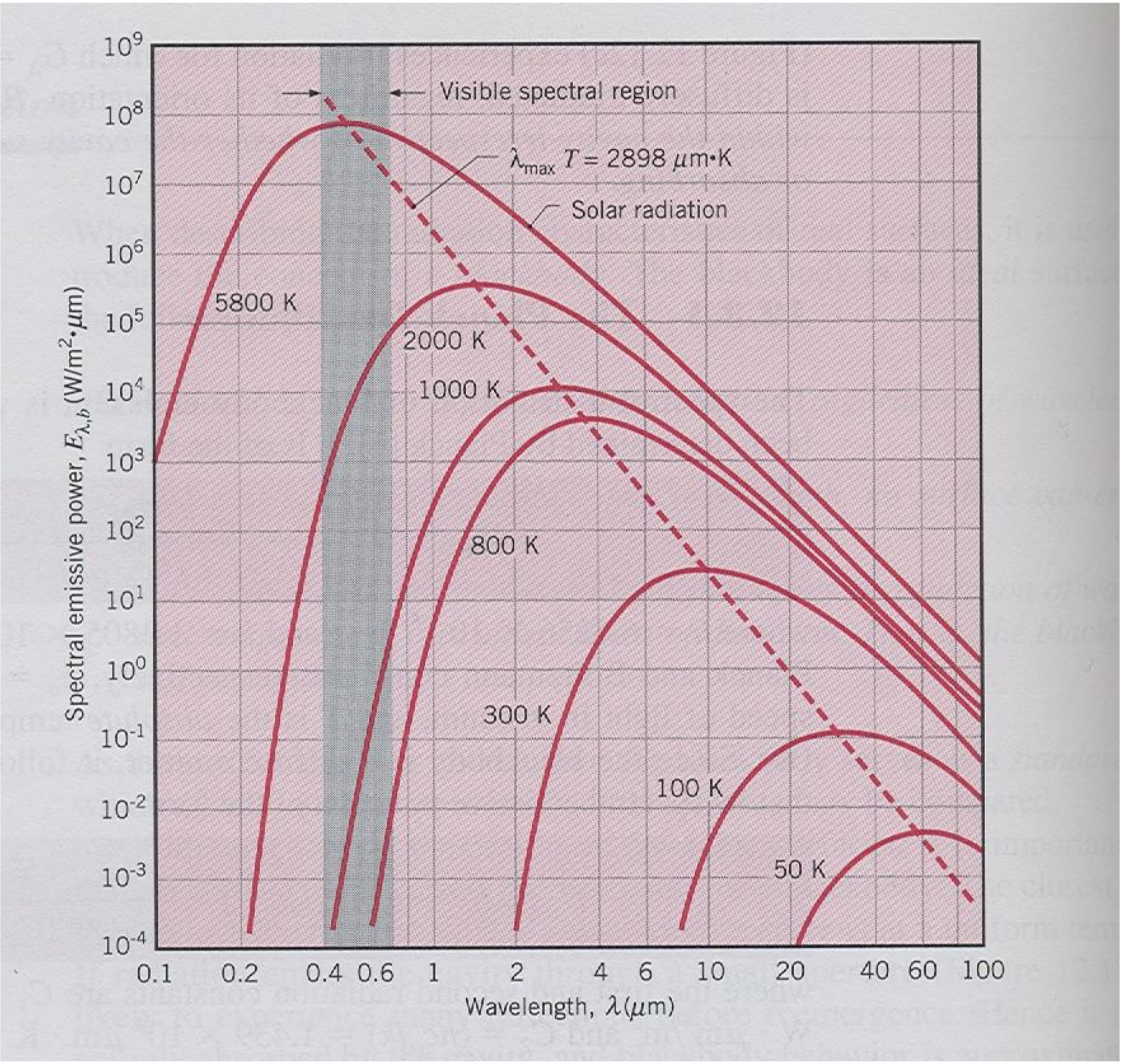


Fig.11.5 Spectral blackbody emissive power

figchp11\fig11.6.pptx shows the spectral energy density of a black body at 1922 K, a corresponding gray body with ε = 0.6 and approximate behavior of a real surface.

Band Emissions

Frequently it will be of interest to get the amount of energy radiated from a black body in a certain specified wavelength range, **figchp11\fig11.7.pptx**. This is expressed as a fraction given by

$$\frac{E_{b_{0-\lambda}}}{E_{b_{0-\infty}}} = \frac{\int_0^{\lambda} E_{b\lambda} d\lambda}{\int_0^{\infty} E_{b\lambda} d\lambda}$$

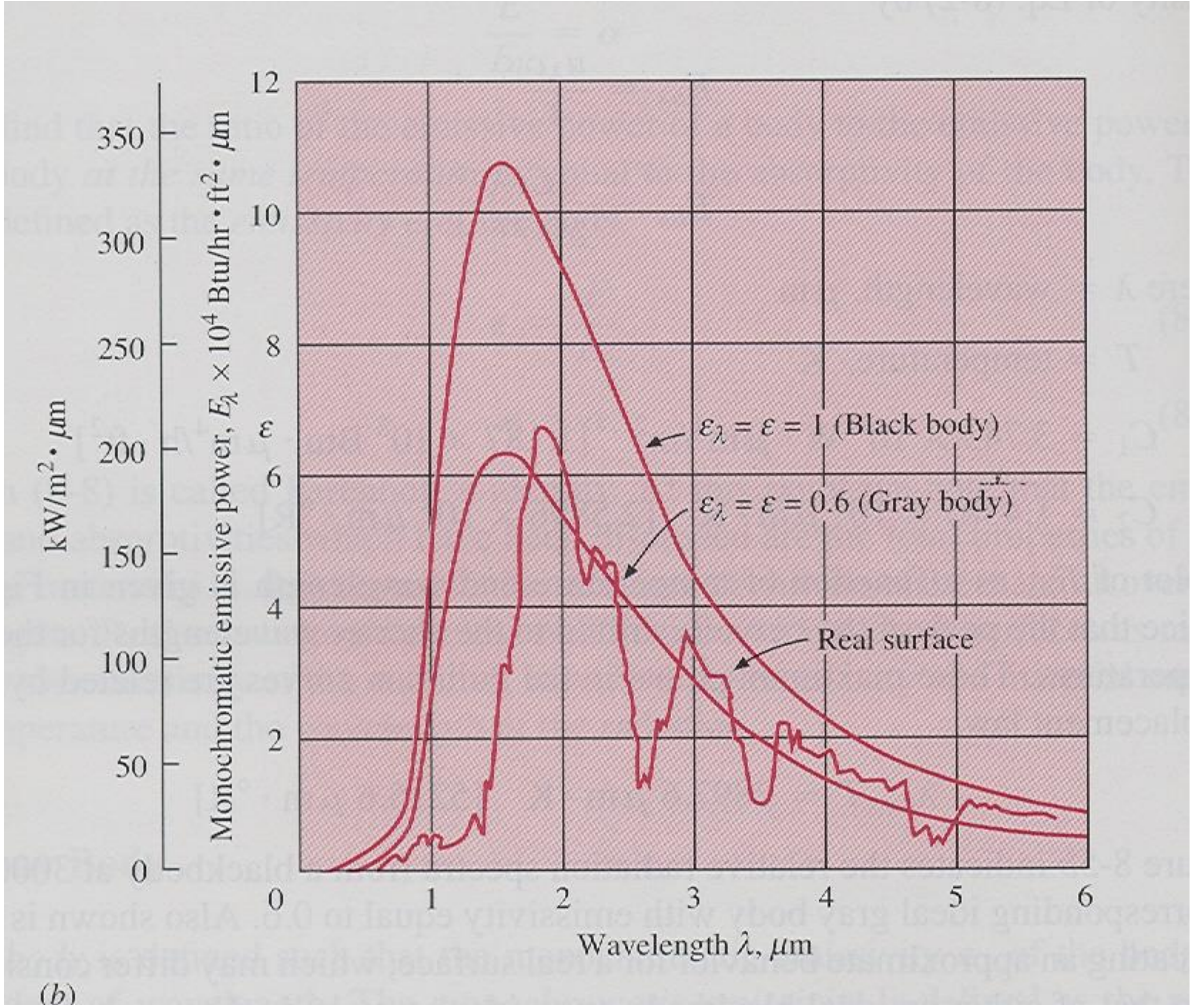


Fig.11.6 Comparison of emissive power of ideal blackbodies, and gray bodies with that of a real surface

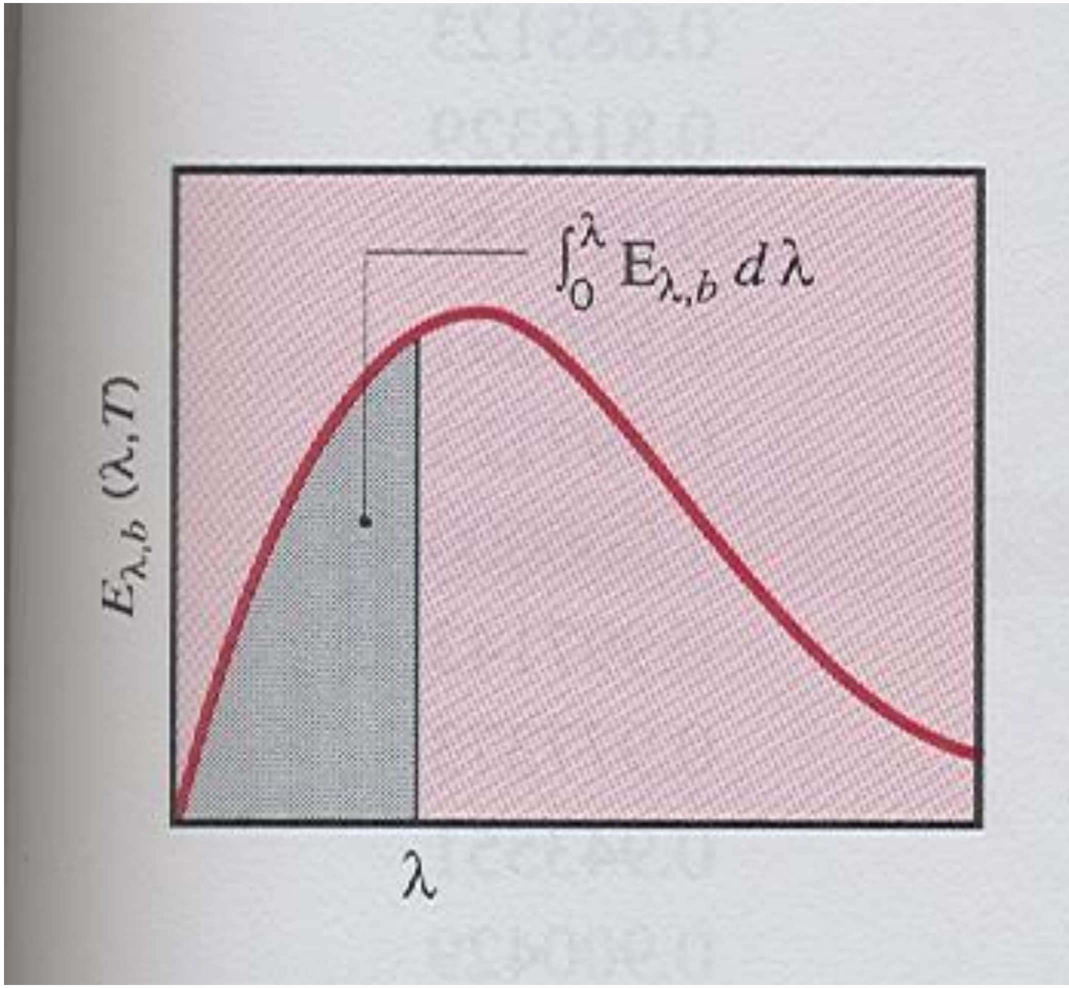


Fig.11.7 Blackbody radiation emission in the spectral band 0 to λ

Rearranging the spectral emission equation as

$$\frac{E_{b\lambda}}{T^5} = \frac{C_1}{(\lambda T)^5 (e^{C_2/\lambda T} - 1)} = f(\lambda T)$$

The results of the above have been tabulated (Table 1) and graphically in **figchp11\fig11.8.pptx** .

For radiant energy emitted between wavelengths λ_1 and λ_2

$$E_{b_{\lambda 1-\lambda 2}} = E_{b_{0-\infty}} \left(\frac{E_{b_{0-\lambda 2}}}{E_{b_{0-\infty}}} - \frac{E_{b_{0-\lambda 1}}}{E_{b_{0-\infty}}} \right) \qquad E_{b_{0-\infty}} = \sigma T^4$$

From practical observations, ordinary glass is transparent to solar radiation while not transmitting

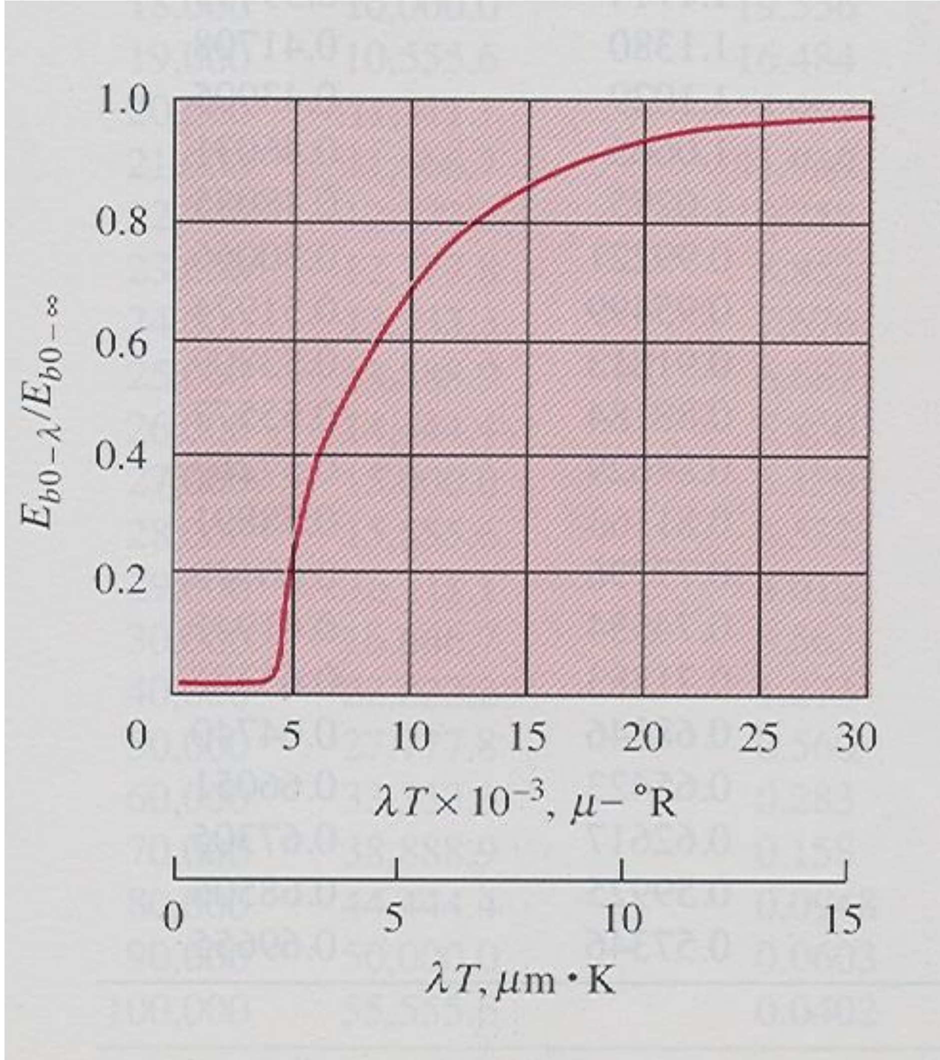


Fig. 11.8 Fraction of blackbody radiation in wavelength interval

earthly radiations. This is what is called the greenhouse effect.

Solar radiation approximates that of a black body at 5800K. Ordinary window glass transmits radiation up to about 2.5 μm. This gives $\lambda T = 2.5 \times 5800 = 14500$ μm.K. Referring to the table, about 97 % of the radiation emitted is transmitted through the glass. Glass is transparent for solar radiation. Whereas earthly radiations at about 300 K $\lambda T = 2.5 \times 300 = 750$ μm.K. The table shows only a minute fraction (less than 0.001 percent) of this radiation is transmitted. Glass is opaque for earthly radiations. There comes the greenhouse effect!

Example 11.1

A glass plate 30 cm square is used to view radiation from a furnace. The transmissivity of the glass is 0.5 from 0.2 to 3.5 μm. The emissivity may be assumed to be 0.3 up to 3.5 μm and 0.9 above that. The transmissivity of the glass is zero, except in the range from 0.2 to 3.5 μm. Assuming that the furnace is a blackbody at 2000^oC, calculate the energy absorbed in the glass and the energy transmitted.

Solution

$$T = 2000^oC = 2273\ K$$

$\lambda_1 T = (0.2)(2273) = 454.6\ \mu m.K$

$\lambda_2 T = (3.5)(2273) = 7955.5\ \mu m.K$

$A = (0.3)^2 = 0.09\ m^2$

From table

$$\frac{E_{b_{0-\lambda 1}}}{\sigma T^4} = 0 \qquad \frac{E_{b_{0-\lambda 2}}}{\sigma T^4} = 0.85443$$

$\sigma T^4 = (5.669 \times 10^{-8})(2273)^4 = 1.5133 \times 10^6\ W/m^2$

Total incident radiation is

$0.2\ \mu m < \lambda < 3.5\ \mu m$

$= (1513.3)(0.85443 - 0)0.09 = 116.4\ kW$

Total radiation transmitted = (0.5) (116.4) = 58.2 kW

$$Radiation\,absorbed$$

$$=(0.3)(116.4)=34.92\,kW \quad for\ 0<\lambda<3.5\,\mu m$$

$$=0.9(1-0.85443)(1513.3)(0.09)=17.84\,kW$$

$$for\ 3.5<\lambda<\infty\,\mu m$$

Total radiation absorbed = 34.92 +17.84 = 52.76 kW

Radiation Shape Factor

Given two black surfaces which see each other, as shown in **figchp11\fig11.9.pptx** , a general expression for energy exchange between such surfaces at different temperatures will be required. This will require the concept of radiation shape factors or view factors. These are defined as follows.

$F_{1\text{-}2}$ = fraction of energy leaving surface 1 which reaches surface 2

$F_{2\text{-}1}$ = fraction of energy leaving surface 2 which reaches surface 1

$F_{m\text{-}n}$ = fraction of energy leaving surface m which reaches surface n

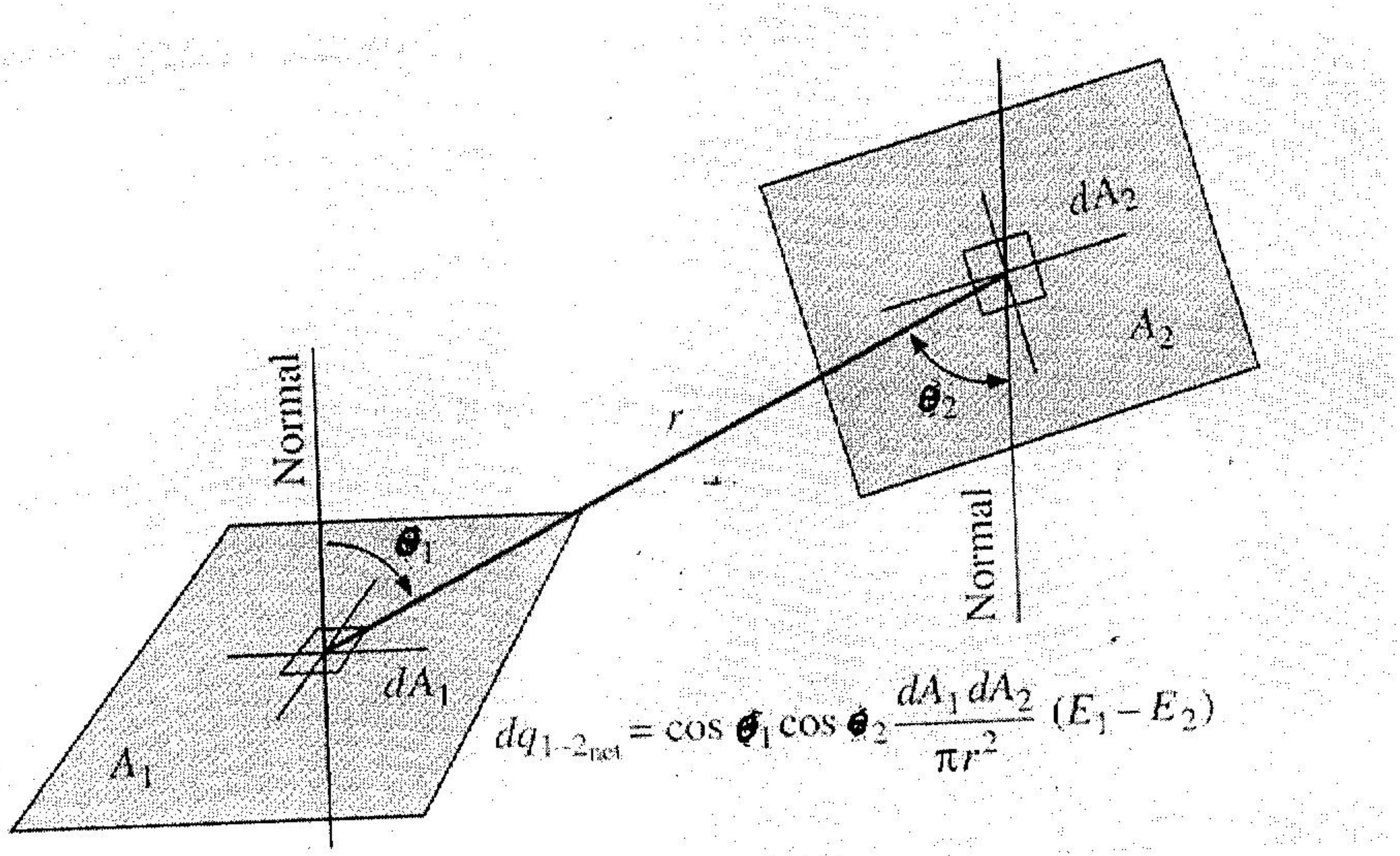


Fig. 11.9 Area elements used in deriving shape factor

The energy leaving surface 1 and arriving at surface 2 is $E_{b1}A_1F_{12}$

and the energy leaving surface 2 and arriving at surface 1 is $E_{b2}A_2F_{21}$

All radiations falling on black surfaces will be completely absorbed.

The net energy exchange is given by

$$Q_{1\text{-}2} = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$$

For $T_1 = T_2$, $Q_{1\text{-}2} = 0$

This will give $A_1F_{12} = A_2F_{21}$

This reciprocity relation will hold true for all situations.

The net heat exchange will therefore be

$$Q_{1\text{-}2} = A_1F_{12}(E_{b1} – E_{b2}) = A_2F_{21}(E_{b1} – E_{b2})$$

The general reciprocity relation for any two surfaces i and j will be

$$A_iF_{ij} = A_jF_{ji}$$

The direction of emission from dA_1 is given with reference to the zenith and azimuthal angles as shown in **figchp11\fig11.10.pptx** . This radiation passes through a differential area dA_n which is normal to the path of the radiation. This area subtends a solid angle $d\omega$ when viewed from a point on dA_1. The similarity of the angle subtended by an arc and

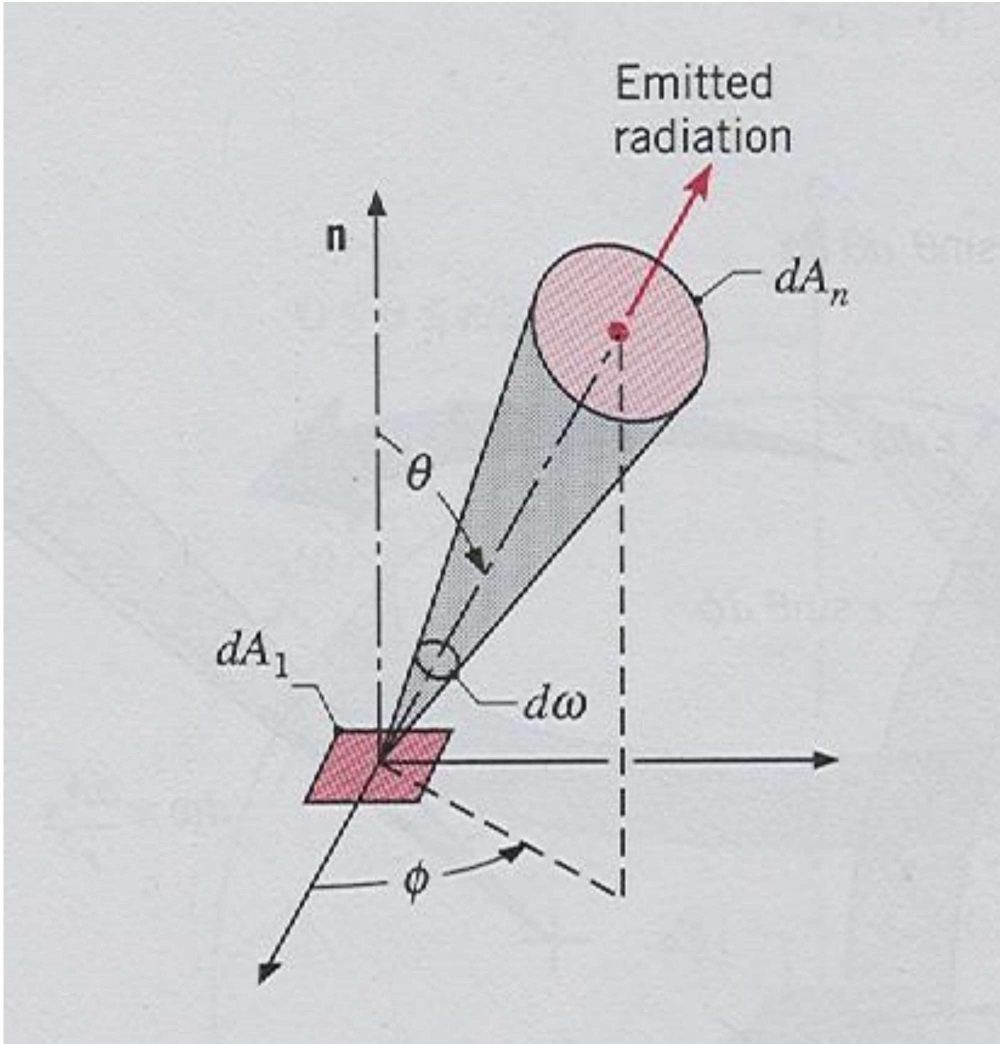


Fig.11.10 Emission of radiation from a differential area dA_1 into a solid angle $d\omega$ subtended by dA_n at a point dA_1

the solid angle subtended by an area is shown in [figchp11\fig11.11.pptx](figchp11\fig11.11.pptx) . The plane angle $d\alpha$ has a unit of radians while that of $d\omega$ is the steradian (sr).

To determine a general relation for shape factors, consider the angles θ_1 and θ_2, the angles with reference to the normals of the surfaces. The projection of dA_1 on the line between centers is

$$dA_1 \cos \theta_1$$

The radiation intensity is that emitted per unit area and per unit of solid angle in a certain specified direction. This is given by I_b considering a black surface.

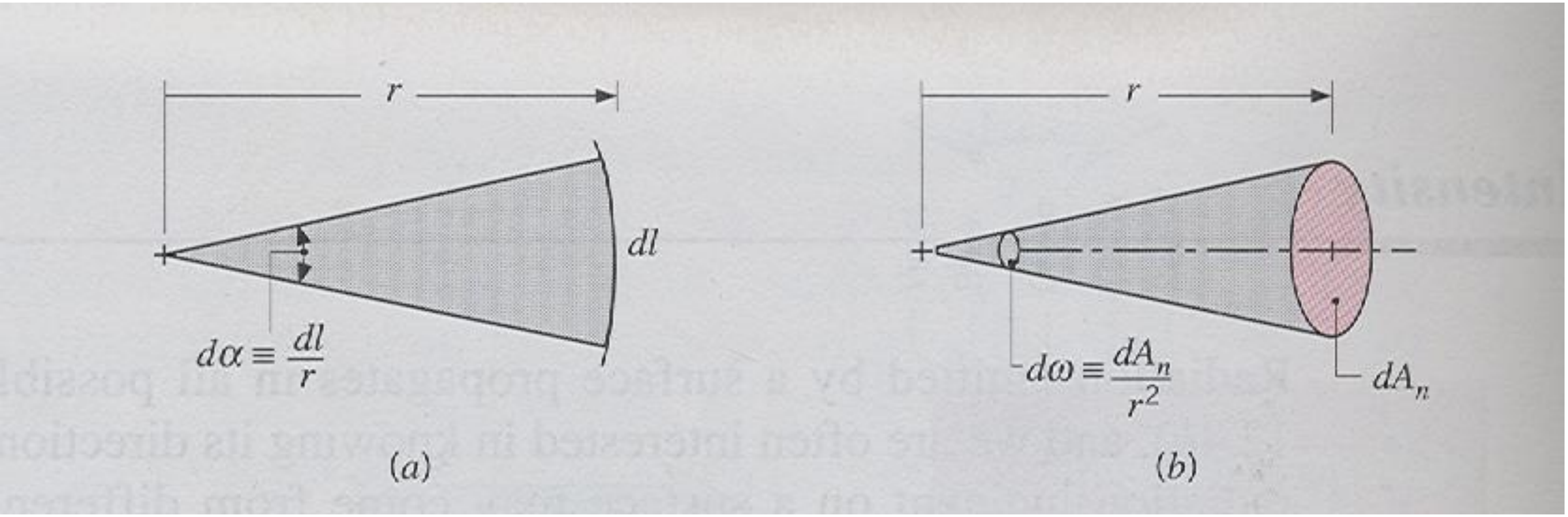


Fig.11.11 Definition of (a) plane and (b) solid angles

The differential solid angle can easily be determined as shown in **figchp11\fig11.12.pptx** . This is given by

$$\frac{dA_n}{r^2} = \sin\theta d\theta d\phi = d\omega$$

Thus the energy leaving dA_1 in the direction of θ_1 is

$I_b \, dA_1 \cos \theta_1$

The radiation arriving at some areal element dA_n at a distance r from A_1 would be

$I_b \, dA_1 \cos \theta_1 \, (d\omega)$

The intensity from the differential area can be determined in terms of the emissive power by

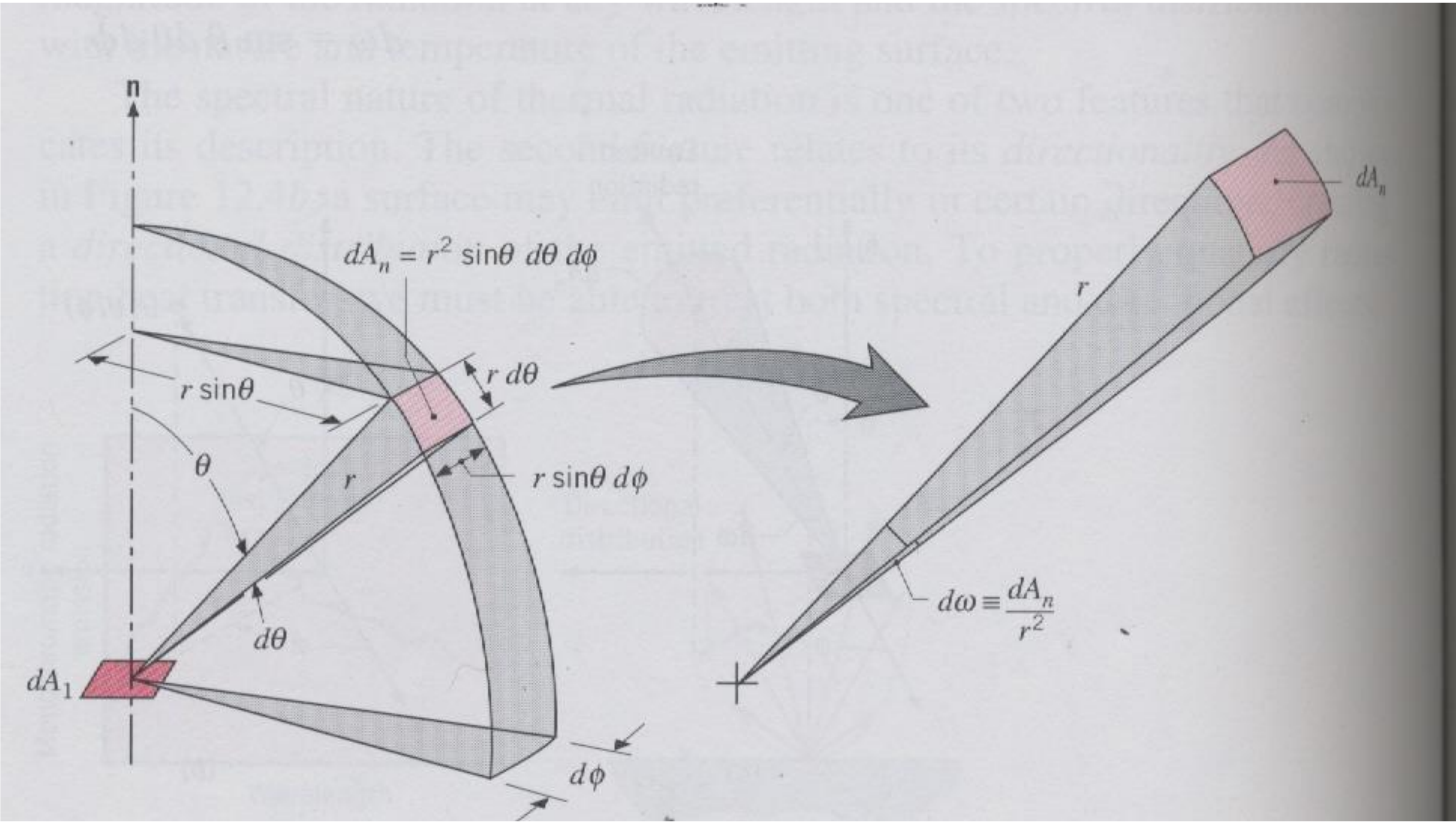


Fig.11.12 The solid angle subtended by dA_n at a point on dA_1 in the spherical coordinate system

integrating over a hemisphere enclosing the elemental area dA_1 as shown in **figchp11\fig11.13.pptx** .

$$E_b dA_1 = I_b dA_1 \int_0^{2\pi} \int_0^{\pi/2} \sin\theta \cos\theta d\theta d\phi$$

$$= \pi I_b dA_1$$

$$E_b = \pi I_b$$

With respect to the line, r, connecting the two differential areas dA_1 and dA_2, the area dA_n is given by

$dA_n = \cos \theta_2 \, dA_2$

This will give the energy leaving dA_1 and arriving

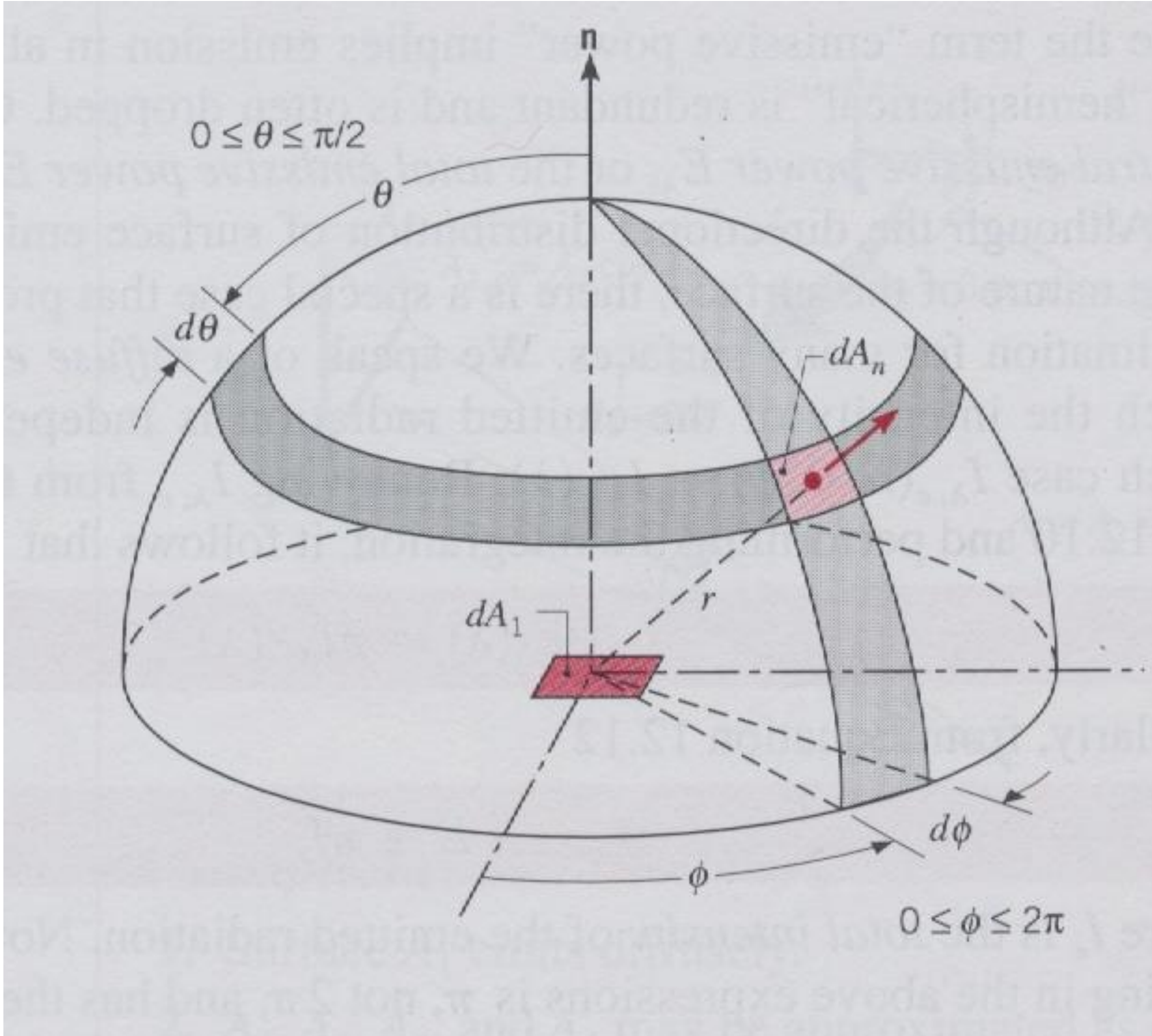


Fig.11.13 Emission from a differential element of area dA_1 into a hypothetical hemisphere centered at a point on dA_1

at dA_2 as $$dq_{1-2} = I_b dA_1 \cos\theta_1 d\omega = E_{b1} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{\pi r^2}$$

$$q_{1-2} = E_{b1} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{\pi r^2} = E_{b1} A_1 F_{12}$$

And the energy leaving dA_2 and arriving at dA_1 will be

$$dq_{2-1} = E_{b2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{\pi r^2}$$

$$q_{2-1} = E_{b2} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{\pi r^2} = E_{b2} A_2 F_{21}$$

As the integrals are exactly the same, the above equations give the reciprocity relation

$$A_i F_{ij} = A_j F_{ji}$$

The view factor for an enclosure with N surfaces with temperatures T_1, T_2, …., T_N is given by

$$\sum_{j=1}^{N} F_{ij} = 1$$

The term F_{ii} is non zero if it sees itself.

For radiation exchange in an enclosure of N surfaces, a total of N^2 view factors is needed as arranged in the matrix form

$$\begin{bmatrix} F_{11} & F_{12} & \ldots\ldots F_{1N} \\ F_{21} & F_{22} & \ldots\ldots F_{2N} \\ . & . & . \\ . & . & . \\ F_{N1} & F_{N2} & \ldots\ldots F_{NN} \end{bmatrix}$$

Out of this N^2 view factors, which require N^2 equations, there are N equations formed by the summation rule and N(N-1)/2 equations formed by the reciprocity relations. This will then require only $(N^2-N(N-1)/2)=N(N-1)/2$ view factors to be determined. For a three surface enclosure we need to determine three view factors only to completely determine the view factors.

As an example consider a two surface enclosure involving two spheres as shown in **figchp11\fig11.15.pptx** . For this we will need to determine four view factors (F_{11}, F_{12}, F_{21}, F_{22}).

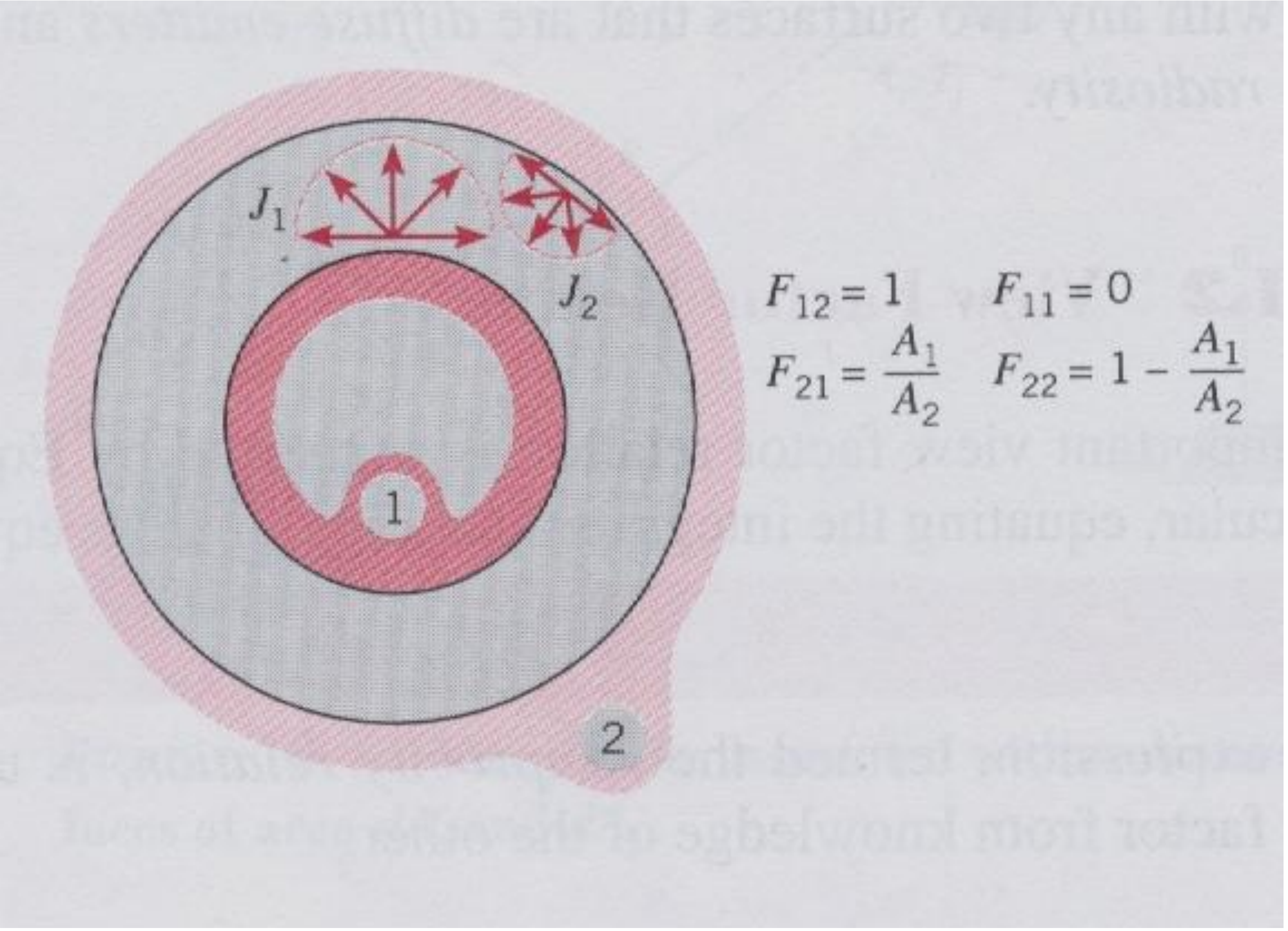


Fig.11.15 View factors for the enclosure formed by two spheres

Only N(N-1)/2 view factors need to be determined to completely get the values of the view factors. One view factor is to be determined directly. By inspection $F_{11} = 0$. For the rest use the equations formed by summation given by

$$F_{11} + F_{12} = 1 \qquad F_{12} = 1$$

$$F_{21} + F_{22} = 1$$

And the reciprocity relation

$$A_1F_{12} = A_2F_{21}$$

(three equations and three unknowns)

$$F_{21} = A_2 / A_1 \qquad F_{22} = 1 - F_{21} = 1 - A_2 / A_1$$

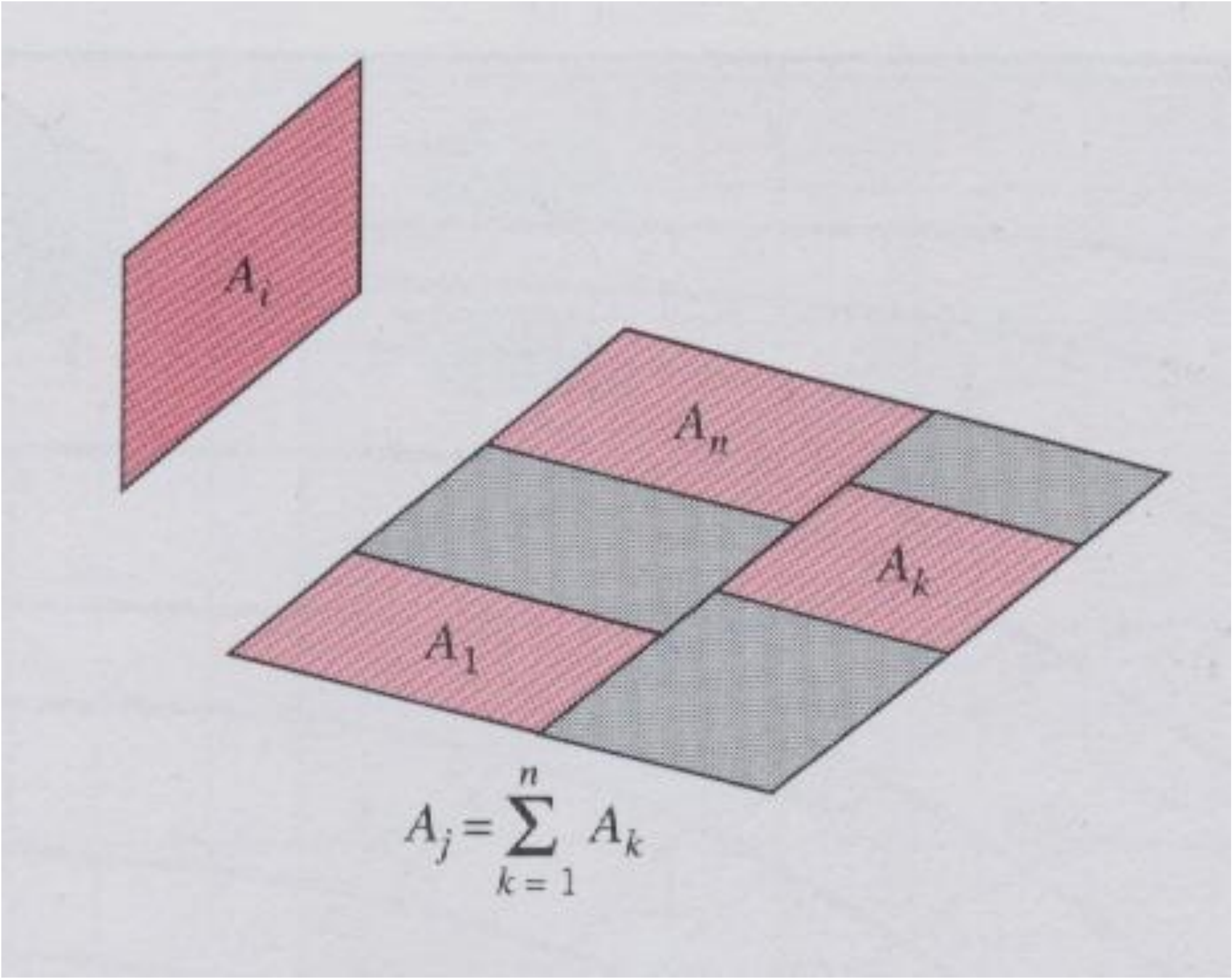


Fig.11.14 Areas used to illustrate view factor relations

For other complicated geometries, the double integral equations have been solved and the results given in tables and graphs.(tables 2&3, and graphs 1, 2, and 3)

For view factors to a subdivided surface shown in **figchp11\fig11.14.pptx**, consider the radiation from surface i to surface j, which is divided into n components, the view factor is given as a summation

$$F_{i(j)} = \sum_{k=1}^{n} F_{ik} \qquad [(j) equivalent to (1,2,..., k,...n)]$$

The view factor when radiation originates from a subdivided surface can be determined as follows:

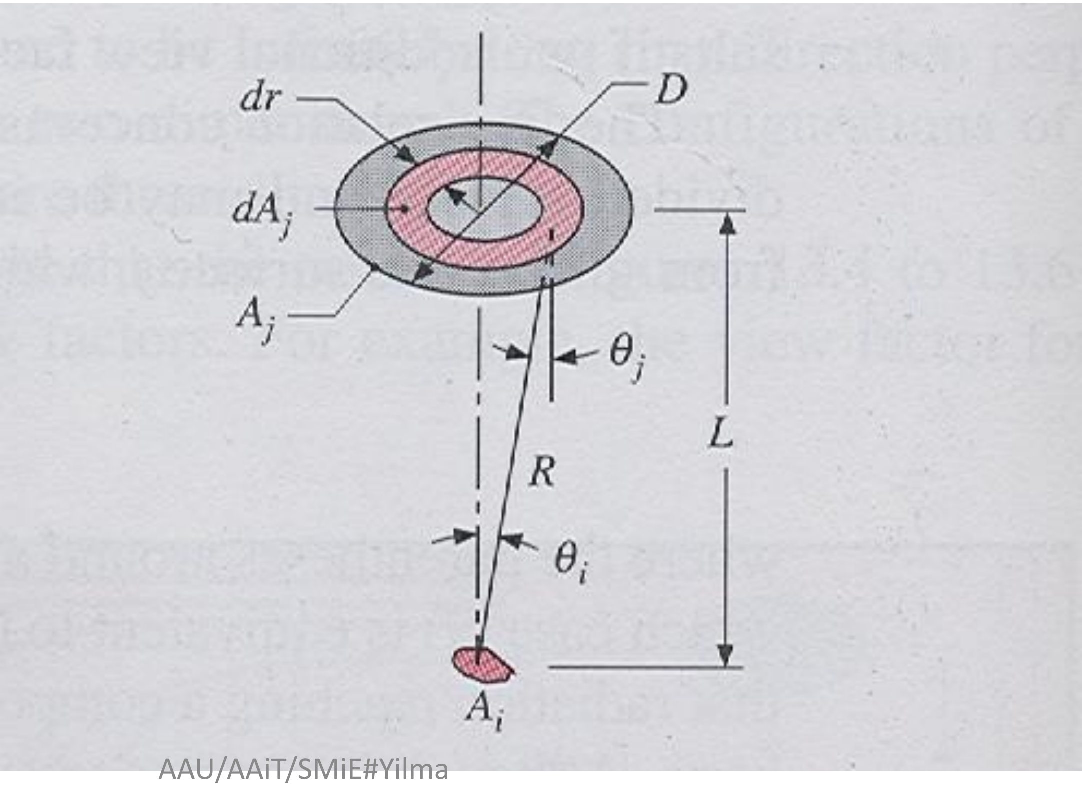
Multiplying the above equation by A_i and applying the reciprocity relation gives

$$A_i F_{i(j)} = A_{(j)} F_{(j)i} = \sum_{k=1}^{n} A_k F_{ki}$$

$$F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k}$$

Example 11.2

Consider a diffuse circular disk of diameter D and area A_j and a plane diffuse surface of area $A_i << A_j$. The surfaces are parallel, and A_i is located at a distance L from the centre of A_j. Obtain an expression for the view factor F_{ij}.



Solution

We will use
$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \cos\theta_i \cos\theta_j \frac{dA_i dA_j}{\pi R^2}$$

θ_i, θ_j, and R are approximately independent of position on A_i, the above reduces to
$$F_{ij} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j = \int_{A_j} \frac{\cos^2\theta}{\pi R^2} dA_j \qquad (\theta_i = \theta_j)$$

Using $R^2 = r^2 + L^2$, $\cos\theta = (L/R)$ and $dA_j = 2\pi r dr$, the integration will give

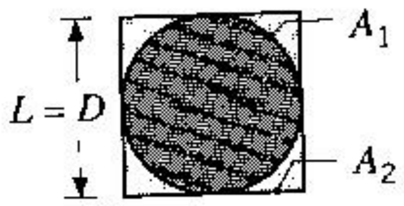
$$F_{ij} = 2L^2 \int_0^{D/2} \frac{r dr}{(r^2 + L^2)^2} = \frac{D^2}{D^2 + 4L^2}$$

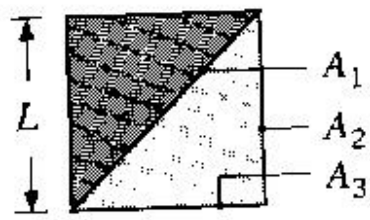
Example 11.3

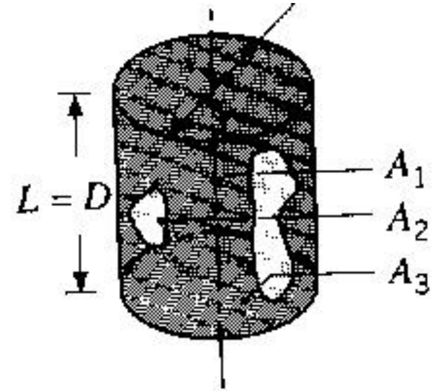
Determine all the view factors for the following geometries.

1. Sphere of diameter D inside a cubical box of length L=D.
2. Diagonal partition within a long square duct.
3. End and side of a circular tube of equal length and diameter.

Solution







1. Sphere within a cube:

$F_{12} = 1 \qquad F_{21} = (A_1/A_2)F_{12} = (\pi D^2/(6L^2) \times 1 = \pi/6$

From summation relation

$F_{11} + F_{12} = 1 \quad \rightarrow \quad F_{11} = 0$

$F_{21} + F_{22} = 1 \quad \rightarrow \quad F_{22} = (1 - \pi/6)$

2. Partition within a square duct

By inspection $F_{11} = F_{22} = F_{33} = 0$

Summation equations

$F_{12} + F_{13} = 1$ (symmetry $F_{12} = F_{13} = 0.5$)

$F_{21} + F_{23} = 1$

$F_{31} + F_{32} = 1$

$A_2 = A_3 = L \qquad A_1 = (\sqrt{2})L$

Reciprocity

$A_1F_{13} = A_3F_{31} \qquad F_{31} = (A_1/A_3)F_{13} = (\sqrt{2})F_{13} = 0.71$

$A_1F_{12} = A_2F_{21} \qquad F_{21} = (A_1/A_2)F_{12} = (\sqrt{2})F_{12} = 0.71$

$A_2F_{23} = A_3F_{32} \qquad F_{32} = (A_2/A_3)F_{23} = F_{23}$

$F_{23} = 1 - F_{21} = 1 - 0.71 = 0.29$

$F_{32} = F_{23} = 0.29$

3. Circular tube:

Using Graph 2 with $r_3/L = 0.5$ and $L/r_1 = 2$ will give $F_{31} \approx 0.17$

$F_{11} = 0$ $F_{33} = 0$

$A_1 = A_3 = (\pi D^2/4)$ $A_2 = \pi D^2$

Summation equations

$F_{12} + F_{13} = 1$

$F_{21} + F_{22} + F_{23} = 1$ (symmetry $F_{21} = F_{23}$)

$F_{31} + F_{32} = 1$ ($F_{32} = 1 - F_{31} = 0.83$)

Reciprocity

$A_2F_{23} = A_3F_{32}$ $F_{23} = (A_3/A_2)F_{32} = (1/4)F_{32} = 0.208$ $F_{21} = 0.208$

$A_1F_{13} = A_3F_{31}$ $F_{13} = (A_3/A_1)F_{31} = F_{31} = 0.17$

$F_{22} = 1 - (F_{21} + F_{23}) = 0.58$

$A_1F_{12} = A_2F_{21}$ $F_{12} = (A_2/A_1)F_{21} = (4)F_{21} = 0.83$

Radiation Exchange Between Surfaces

When radiation falls on an opaque surface there will be a possibility of absorption and reflection. In an enclosure there will be multiple reflections with partial absorptions.

Blackbody Radiation Exchange

The simplest radiation exchange will be between black surfaces where there will be no possibility of reflection.

The following terms will need to be defined.

G = irradiation

= total radiation incident upon a surface per unit time per unit area

J = radiosity

= total radiation which leaves a surface per unit time per unit area

For a black surface radiosity is the same as the emission.

For the analysis of radiative heat transfer between black surfaces, we will use **figchp11\fig11.16.pptx**.

Define $q_{i \rightarrow j}$ as the rate at which radiation leaves surface i and is intercepted by surface j. This can be expressed as

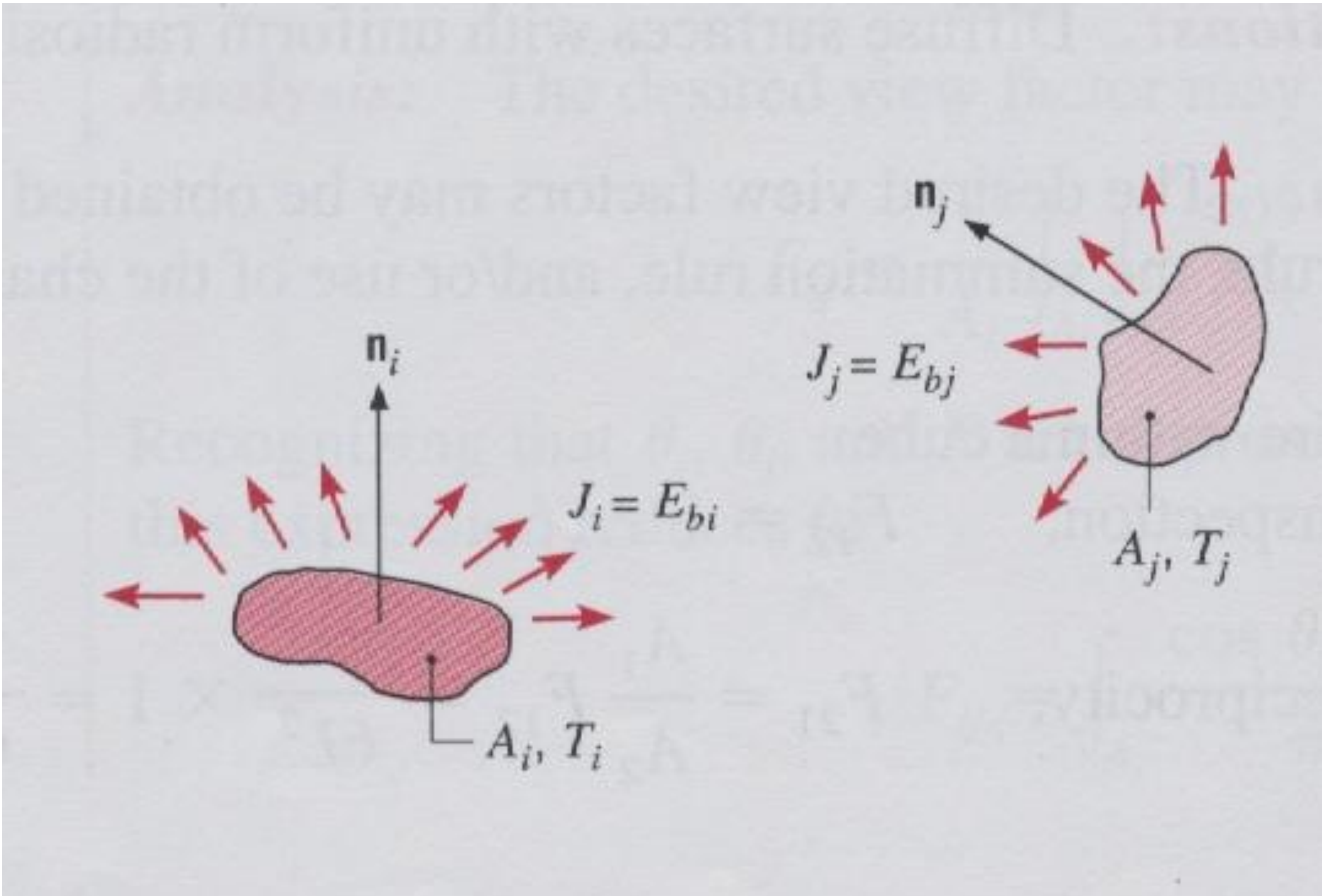


Fig.11.16 Radiation transfer between two surfaces that may be approximated as black bodies

$$q_{i\to j} = (A_i J_i) F_{ij} = A_i F_{ij} E_{bi}$$

Similarly

$$q_{j\to i} = A_j F_{ji} E_{bj}$$

Net radiative exchange will be

$$q_{ij} = q_{i\to j} - q_{j\to i}$$

Substitution gives

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj} = A_i F_{ij} (J_i - J_j) = A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

This will allow the construction of a thermal network that satisfies

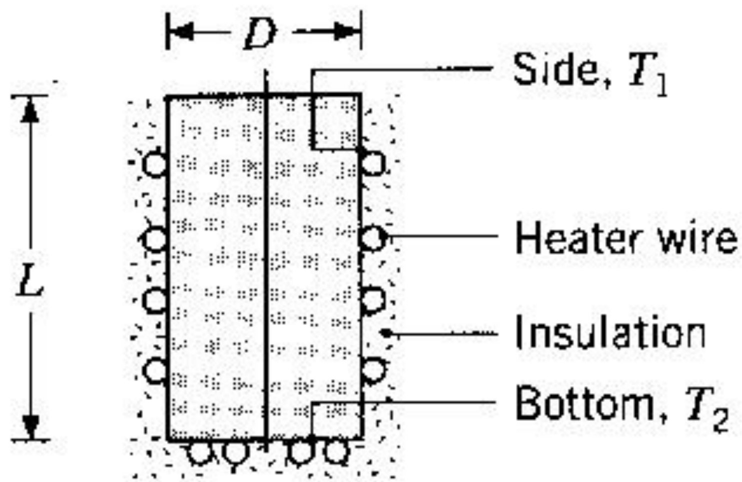
$$\frac{J_i - J_j}{1/A_i F_{ij}} = q_{ij} \qquad R = \frac{1}{A_i F_{ij}}$$

For surface i being in an enclosure and interacting with N surfaces at different temperatures, the above equation can extended to

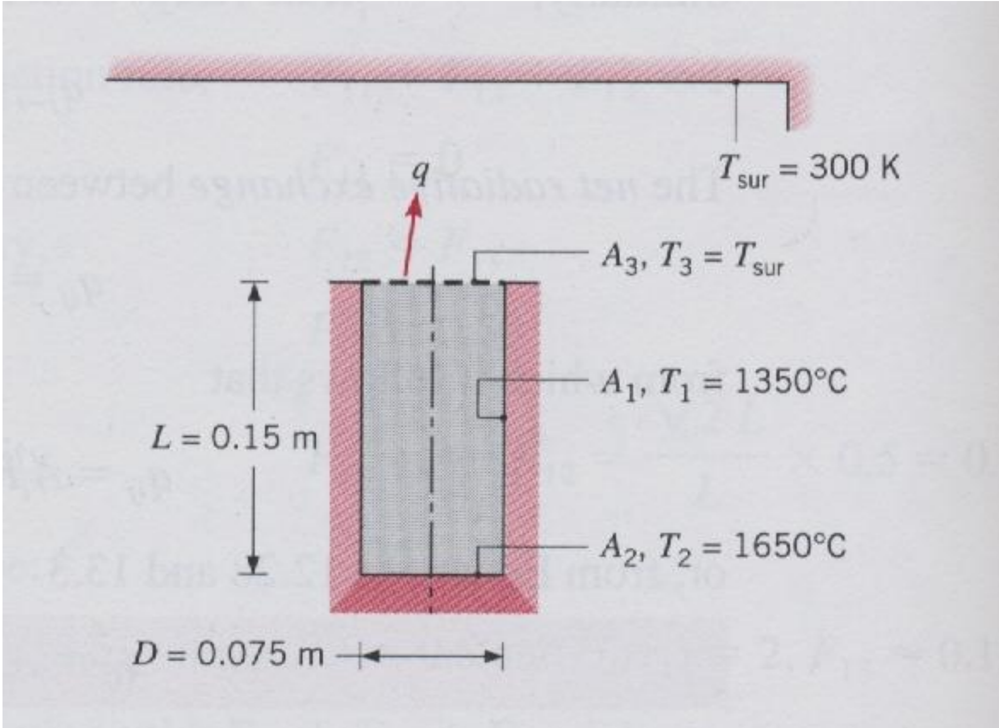
$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

Example 11.4

A furnace cavity, which is in the form of a cylinder of 75 mm diameter and 150 mm length, is open at one end to large surroundings that are at 27°C. The sides and bottom, which may be approximated as black bodies, are heated electrically, well insulated, and maintained at temperatures of 1350 and 1650ºC, respectively. How much power is required to maintain the furnace conditions.



Solution



Since the surrounding is large it may be treated as a black body. Here the heat transfer by convection will be assumed to be negligible compared to the radiative heat transfer.

With $T_3 = T_{sur}$, the heat loss can be expressed as

$q = q_{13} + q_{23}$

Using appropriate equations for radiation between black surfaces

$$q = A_1 F_{13} \sigma (T_1^4 - T_3^4) + A_2 F_{23} \sigma (T_2^4 - T_3^4)$$

For the two opposing surfaces (top and bottom), using $(r_j/L) = (0.0375/0.15) = 0.25$ and

$(L/r_i) = (0.15/0.375) = =4$

$F_{23} = 0.06$ (From view factor graphs)

Use summation rule

$F_{21} + F_{23} = 1$ $F_{21} = 1\text{-}0.006 = 0.94$

Use reciprocity relation

$A_1F_{12} = A_2F_{21}$ to get

$$F_{12} = \frac{A_2}{A_1}F_{21} = \frac{\pi(0.075)^2/4}{\pi(0.075)(0.15)}x0.94 = 0.118$$

From symmetry $F_{13} = F_{12}$

Substitution in q gives

$$q = (\pi x0.75x0.15)(0.118x5.67x10^{-8}$$

$$[(1623)^4 - (300)^4] + \left(\frac{\pi}{4}\right)(0.075)^2 x0.06$$

$$x5.67x10^{-8}[(1923)^4 - (300)^4]$$

$q = 1639 + 205 = 1844$ W

Radiative exchange between nonblackbodies

Here for an opaque body, the radiosity will also involve the reflected part from the irradiation as shown in **figchp11\fig11.17.pptx** . More complication is when the reflection is back and forth between the heat transfer surfaces several times.

The radiosity is given by

$J = \varepsilon E_b + \rho G$

Using $\rho = 1 - \alpha = 1 - \varepsilon$

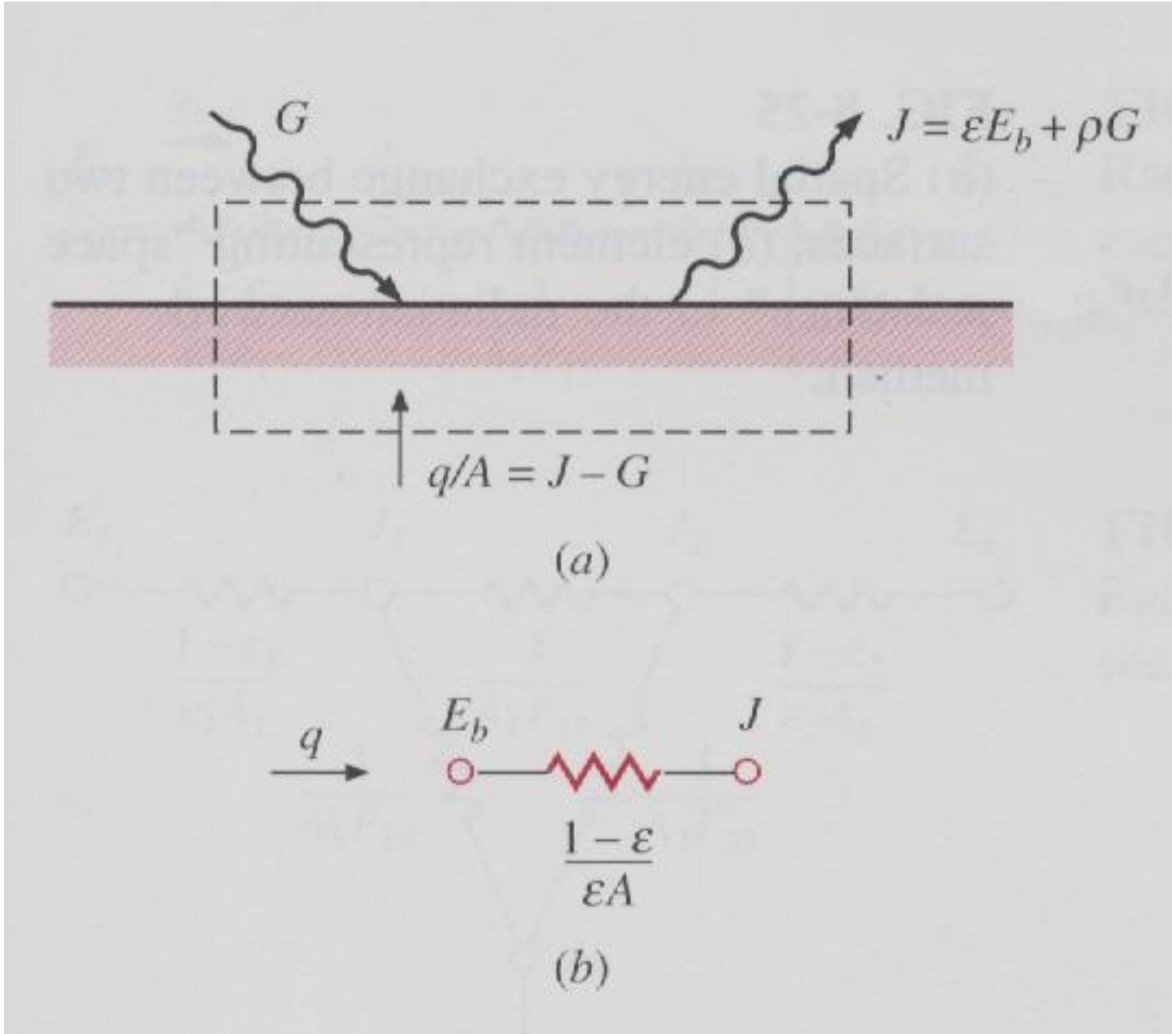


Fig.11.17 (a) Surface energy balance for opaque materials; (b) element representing "surface resistance" in the radiation network method

the radiosity expression becomes

$J = \varepsilon E_b + (1 - \varepsilon)G \quad G = (J - \varepsilon E_b)/(1 - \varepsilon)$

The difference between the radiosity and the irradiation gives net energy leaving the surface as

$(q/A) = J - G = J - (J - \varepsilon E_b)/(1 - \varepsilon)$

After substitution of G and simplification gives

$$q = \frac{\varepsilon A}{1-\varepsilon}(E_b - J) \quad or \quad q = \frac{E_b - J}{(1-\varepsilon)/\varepsilon A}$$

The above allows the construction of a network with the surface resistance as indicated.

If we consider the radiant energy exchange between two surfaces, A_1 and A_2, the net heat transfer from surface 1 to surface 2 can easily be determined as

$$q_{1\text{-}2} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

Using the reciprocity relation $A_1 F_{12} = A_2 F_{21}$

$$q_{1\text{-}2} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21}$$

For network construction the above can be written as

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$

where the resistance is indicated as space resistance.

The radiation exchange between two surfaces which exchange heat with each other and nothing else can be represented as a network given by **figchp11\fig11.18.pptx** . From this network the net heat transfer from surface 1 to surface 2 can easily be determined as

$$q_{net} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

For other two surface enclosures, Table 4 gives the necessary information.

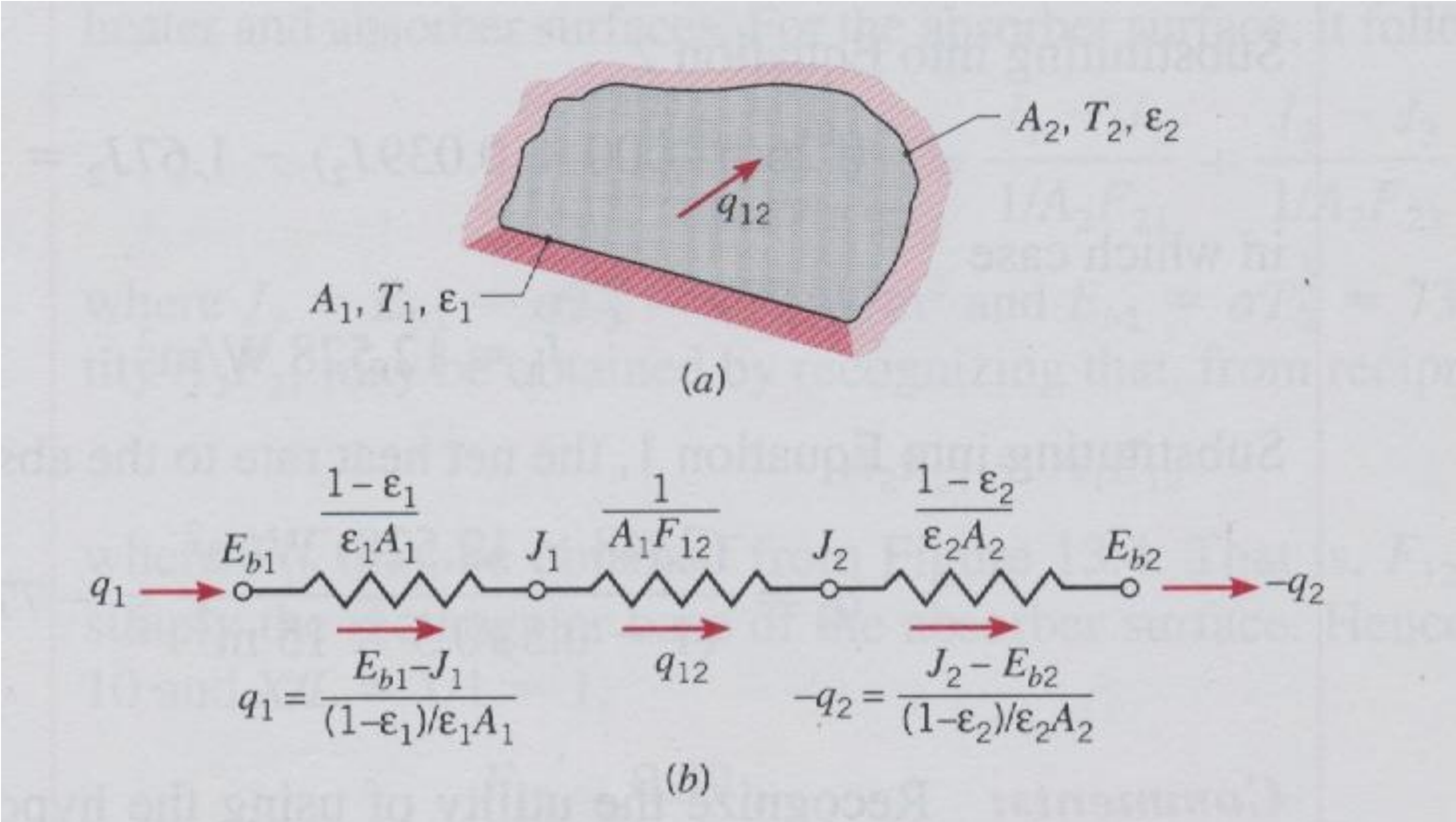


Fig.11.18 The two surface enclosure with network representation

For a three body problem, the network is given in **figchp11\fig11.19.pptx** .

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1F_{12}} \qquad q_{1-3} = \frac{J_1 - J_3}{1/A_1F_{13}}$$

Kirchoff's current law can be used to determine the radiosities. Sum of heat transfers to a node is zero.

This can be extended for a radiative interaction of a surface with other surfaces that form an enclosure as

$$q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}}$$

For any number N of surfaces forming the enclosure there will be N equations with J_N unknowns.

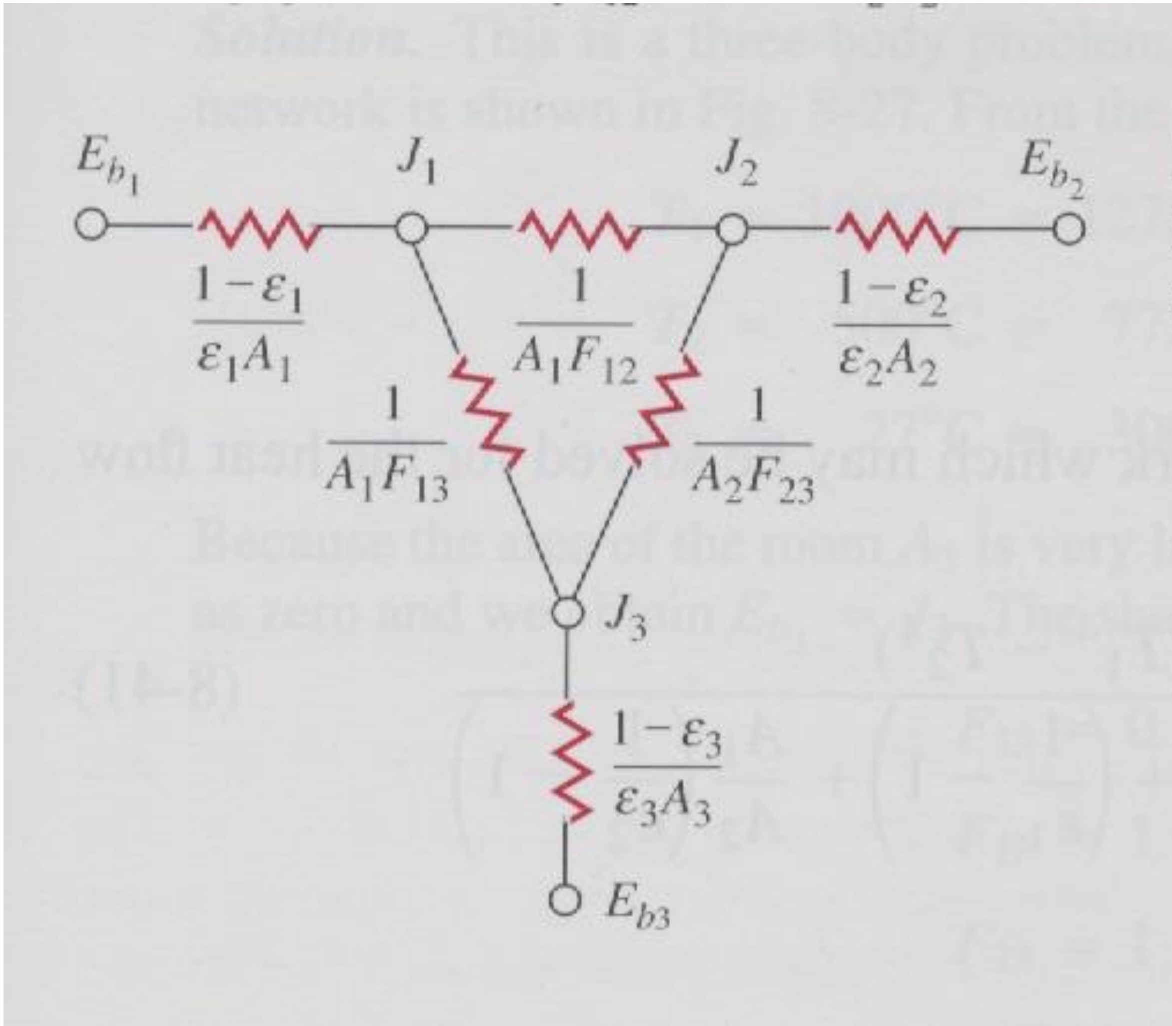


Fig.11.19 Radiation network for three surfaces which see each other and nothing else

Radiation Shields

Radiation shields use low emissivity materials (high reflectivity) placed between radiating surfaces as shown in **figchp11\fig11.20.pptx** (a).

If such a surface is placed additional surface and space resistances will be created, thus reducing the heat transfer. The network is shown in (b). The heat transfer rate can easily be determined from the series resistance network as

$$q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1}} + \frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2}}}$$

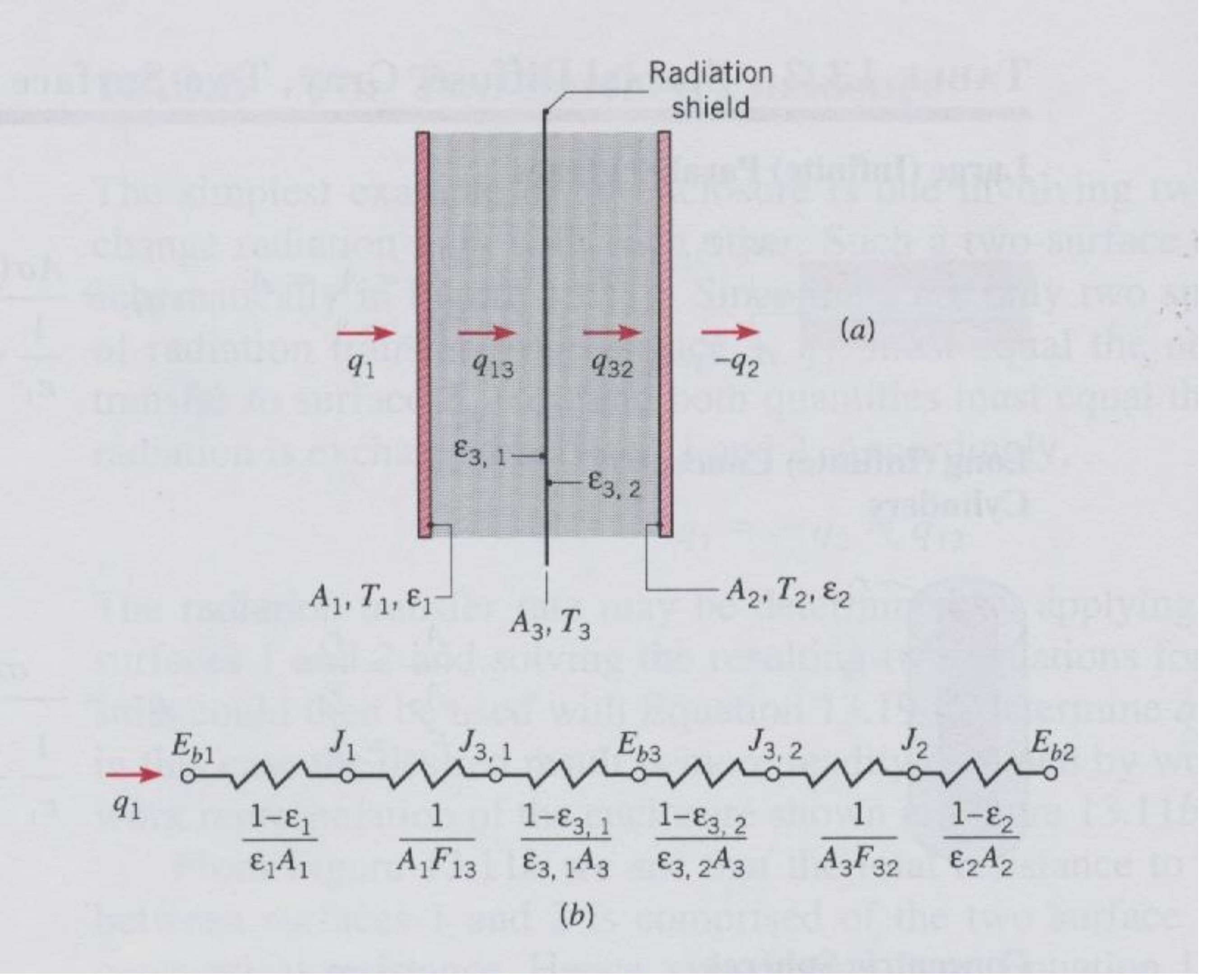


Fig.11.20 Radiation exchange between large parallel planes with a radiation shield and its network representation

Insulated surfaces and Surfaces with large areas.

For a perfectly insulated surface or that reradiates all the energy incident upon it, the heat flow from such a surface is zero. This makes the potential difference across the surface resistance to be zero, resulting in $J=E_b$. The insulated surface does not have zero resistance.

Large surface area ($A \rightarrow \infty$) has a surface resistance approaching zero. This behaves as a black body as it tends to absorb all the radiant energy falling on it. For this the surface resistance is zero ($\varepsilon=1$) and this gives $J = E_b$. Thus the two cases – insulated surface and surface with a large area – both have $J = E_b$.

If two flat or convex surfaces are connected by or enclosed in a reradiating surface as shown in the combustion furnace (**figchp11\fig11.21.pptx** for the schematic **figchp11\fig11.22.pptx**), as no net heat is exchanged with this body, $J_R = E_{bR}$.

$F_{1R} = 1 - F_{12}$ $\quad$ $F_{11} = F_{22} = 0$

$F_{2R} = 1 - F_{21}$

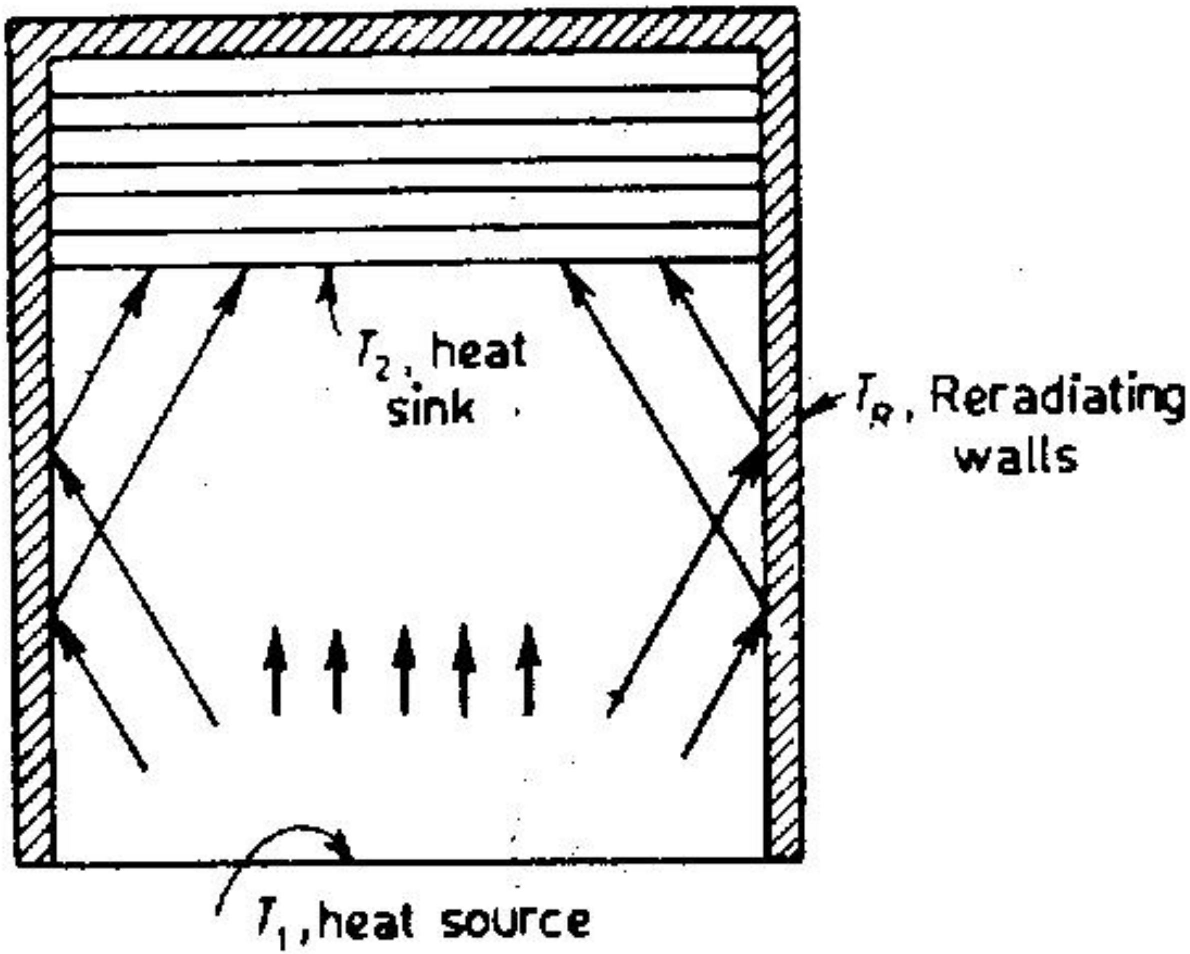


Fig.13.21 Enclosure with reradiating surface

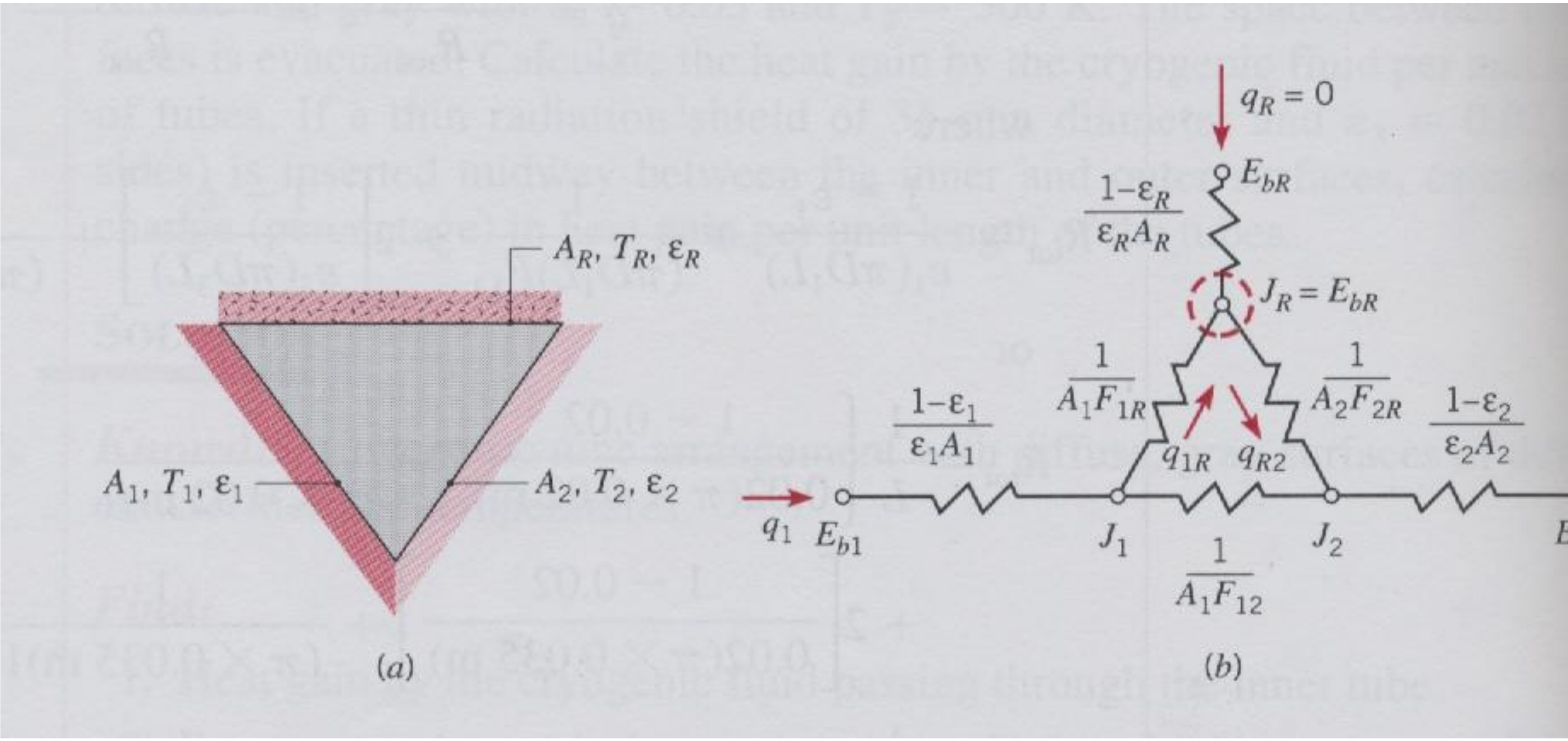


Fig.11.22 A three surface enclosure with one surface reradiating and the network representation

The network is a simple series parallel arrangement which can be shown to give

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12} + [(1/A_1F_{1R}) + (1/A_2F_{2R})]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

After determining J_1 and J_2, then J_R can be determined from

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} - \frac{J_R - J_2}{(1/A_2F_{2R})} = 0$$

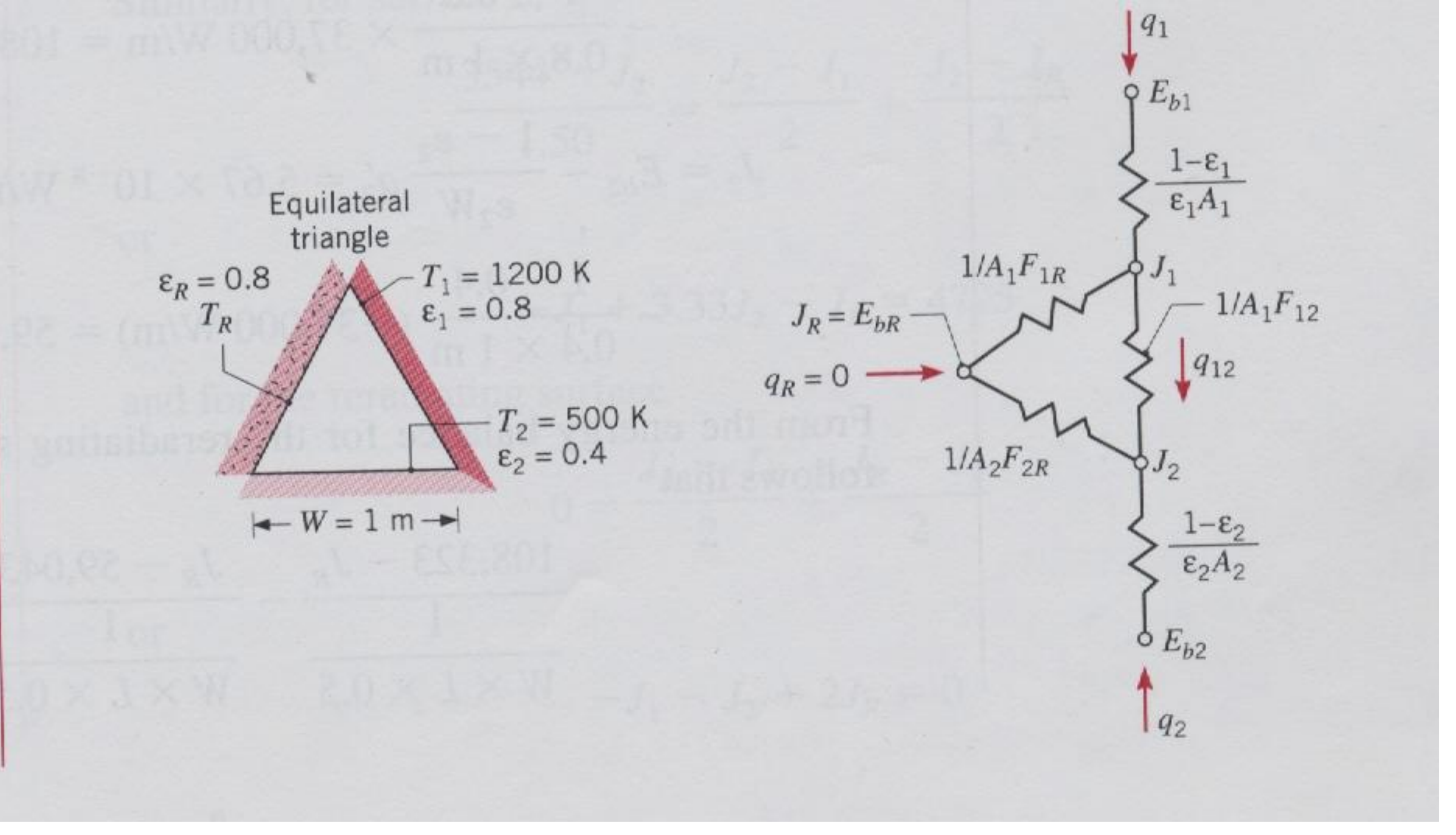
Since $J_R = \sigma T_R^4$ the temperature of the reradating surface can be determined

Example 11.5

A paint baking oven consists of a long, triangular duct in which a heated surface is maintained at 1200 K and another surface is insulated. Painted panels, which are maintained at 500 K, occupy the third surface. The triangle is of width W = 1 m on a side, and the heated insulated surfaces have an emissivity of 0.8. The emissivity of the panels is 0.4. During the steady-state operation, at what rate must energy be supplied to the heated side per unit length of the duct to maintain its temperature at 1200 K? What is the temperature of the insulation surface?

Solution

The system will be modeled as a three surface enclosure as shown in the figure below



1. The heat transfer rate to be supplied is determined from

$$q_1 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + [(1/A_1 F_{1R}) + (1/A_2 F_{2R})]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Symmetry: $F_{12} = F_{1R} = F_{2R}$

$A_1 = A_2 = WL$ L is length of duct

$$q_1^{'} = \frac{q_1}{L} = \frac{5.67x10^{-8}(1200^4 - 500^4)}{\frac{1-0.8}{0.8x1} + \frac{1}{1x0.5 + [2+2]^{-1}} + \frac{1-0.4}{0.4x1}}$$

$$or \qquad q_1^{'} = 37kW/m = -q_2^{'}$$

2. For the temperature of the insulated surface use will be made of the equality of J_R and E_{bR}. To get J_R use

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} - \frac{J_R - J_2}{(1/A_2F_{2R})} = 0$$

$$J_1 = E_{b1} - \frac{1-\varepsilon_1}{\varepsilon_1 W} q_1' = 5.67x10^{-8}(1200)^4 - \frac{1-0.8}{0.8x1}x(37000)$$

$$= 108323 W/m^2$$

$$J_2 = E_{b2} - \frac{1-\varepsilon_2}{\varepsilon_2 W} q_2' = 5.67x10^{-8}(500)^4 - \frac{1-0.4}{0.4x1}x(-37000)$$

$$= 59043 W/m^2$$

Substitution gives

$$\frac{108323 - J_R}{\dfrac{1}{W\,x\,L\,x\,0.5}} - \frac{J_R - 59043}{\dfrac{1}{W\,x\,L\,x\,0.5}} = 0$$

This gives

$$J_R = 83683\,W/m^2 = E_{bR} = \sigma T_R^4$$

$$T_R = \left(\frac{83683}{5.67x10^{-8}}\right)^{\frac{1}{4}} = 1102K$$