CHAPTER SIX

Steady state laminar flow film condensation on vertical plane

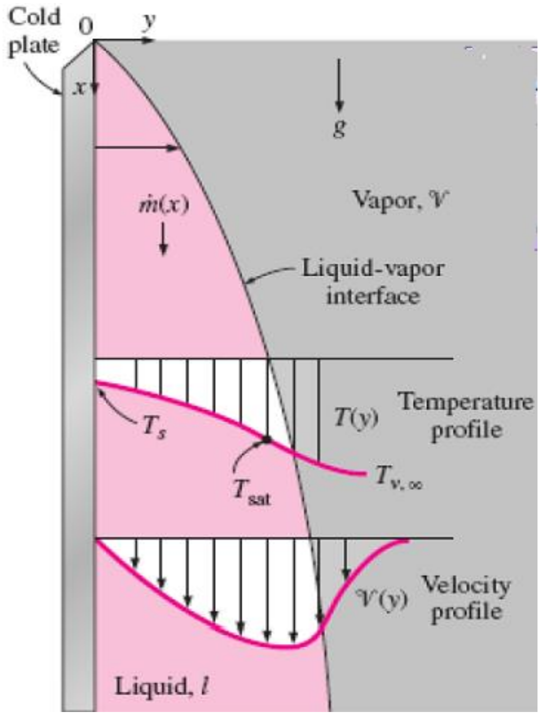
Condensation: happen when vapor comes in contact with a surface which is at lower temperature than the saturation temperature of the vapor.

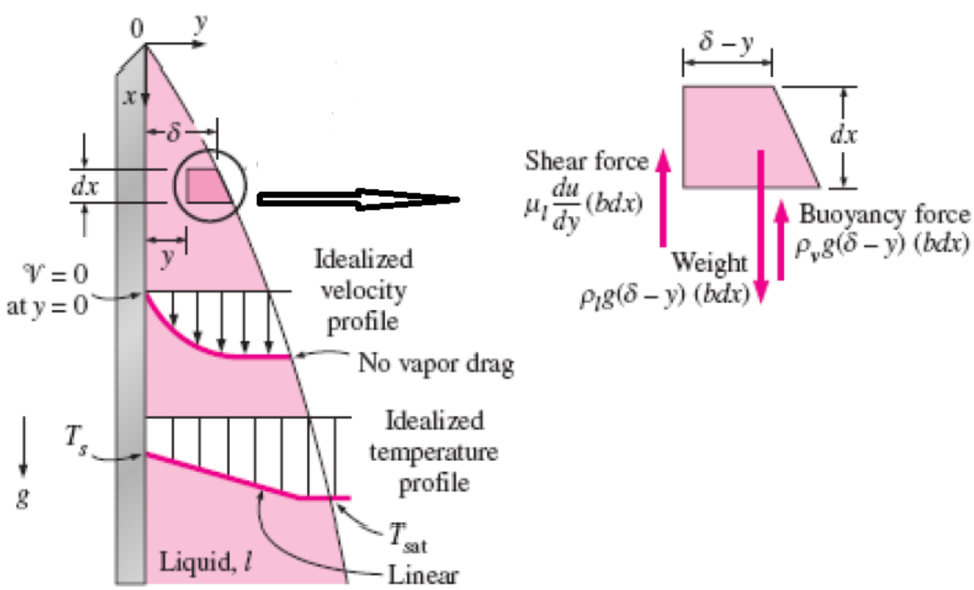
Two types of condensation:

1. Film condensation
2. Drop wise condensation

- ✓ If the condensate tends to wet the surface and then forms a liquid film the process is called **film condensation.**
- ✓ If the condensate doesn't wet the surface but instead collects in growing droplets on the cooled surface the process is called **Drop wise condensation.**
- ✓ Most type of condensation in heat transfer mechanism is **film condensation**
- ✓ Heat transfer mode in the liquid film is assumed as conduction because the flow is laminar.
- ✓ Flow parameter that best describe the film-wise condensation process along the flow direction are:
    - i. Flow velocity at each layer(Node)
    - ii. Temperature distribution along the film thickness T(y)
    - iii. The film thickness and
    - iv. The condensation mass growth

Analytical approach to solve film condensation parameters on vertical plate


Cold plate
0
y
x
g
$\dot{m}(x)$
Vapor, $\mathcal{V}$
Liquid-vapor interface
$T(y)$
Temperature profile
T_s
T_{sat}
$T_{v,\infty}$
$\mathcal{V}(y)$
Velocity profile
Liquid, l


0
y
x
δ
dx
y
$\mathcal{V} = 0$ at y = 0
Idealized velocity profile
No vapor drag
Idealized temperature profile
T_s
g
T_{sat}
Linear
Liquid, l
$\delta - y$
dx
Shear force $\mu_l \frac{du}{dy}(bdx)$
Buoyancy force $\rho_v g(\delta - y)(bdx)$
Weight $\rho_l g(\delta - y)(bdx)$

The Newton's second law of motion (summation of force) for the volume element in vertical x- direction can be written as:

$\sum F_x = ma_x = 0$ [Acceleration a_x assumed to be zero] [1]

$\sum F_{downward} \downarrow = \sum F_{upward} \uparrow$

Weight = shear force +Bouncy force

$$\rho_l g(\delta - y)(bdx) = \mu \frac{\partial u}{\partial y}(bdx) + \rho_v g(\delta - y)(bdx) \qquad [2]$$

1. Velocity profile

Rearranging equation [2] and solving for $\frac{\partial u}{\partial y}$ we get:

$$\mu \frac{\partial u}{\partial y} = \rho_l g(\delta - y) - \rho_v g(\delta - y)$$

$$\frac{\partial u}{\partial y} = \frac{(\rho_l - \rho_v) g(\delta - y)}{\mu}$$

Integrating from y=0 where u=0 (due to no slip condition) to y=y where u = u(y)

$$\int_0^{u(y)} \partial u = \int_0^y \frac{(\rho_l - \rho_v) g(\delta - y)}{\mu} dy$$

$$u(y) = \int_0^y [\frac{(\rho_l - \rho_v) g\delta}{\mu} - \frac{(\rho_l - \rho_v) gy}{\mu}] dy$$

$$\boldsymbol{u(y) = \frac{(\rho_l - \rho_v) g\left(\delta y - \frac{y^2}{2}\right)}{\mu}} \qquad \textbf{[3]}$$

2. Mass flow rate of condensate

The mass flow rate of the condensate at a location x where the boundary layer thickness is δ is determined from:

$$\dot{m}(x) = \int_A^{.} \rho_l u(y) dA = \int_{y=0}^{\delta} \rho_l u(y)(bdy)$$

$$\dot{m}(x) = \int_0^{\delta} \frac{\rho_l(\rho_l - \rho_v) g}{\mu} (\left(\delta y - \frac{y^2}{2}\right) bdy$$

$$\dot{m}(x) = \frac{\rho_l(\rho_l - \rho_v)gb}{\mu}\left(\frac{\delta y^2}{2} - \frac{y^3}{6}\right)\begin{matrix}\delta\\0\end{matrix}$$

$$\boldsymbol{\dot{m}(x)} = \frac{\boldsymbol{\rho_l(\rho_l - \rho_v)gb\delta^3}}{\boldsymbol{3\mu}} \qquad [4]$$

3. Film thickness

Derivate equation [4] with respect to x we get:

$$\frac{d\boldsymbol{\dot{m}(x)}}{dx} = \frac{d}{dx}\left(\frac{\boldsymbol{\rho_l(\rho_l - \rho_v)gb\delta^3}}{\boldsymbol{3\mu}}\right)$$

$$\frac{d\boldsymbol{\dot{m}}}{dx} = \frac{\boldsymbol{\rho_l(\rho_l - \rho_v)gb\delta^2}}{\boldsymbol{\mu}}\frac{\boldsymbol{d\delta}}{\boldsymbol{dx}} \qquad [5]$$

The rate of heat transfer from the vapor to plate through the liquid film is simply equal to the heat release as the vapor is condensed and expressed as:

$$d\dot{Q} = h_{fg}d\dot{m} = k_l(bdx)\frac{T_{sat} - T_s}{\delta}$$

Rearranging the equation we get:

$$\frac{d\boldsymbol{\dot{m}}}{dx} = \frac{k_l b(T_{sat} - T_s)}{h_{fg}\delta} \qquad [6]$$

Equating equation [5] and [6] and rearranging:

$$\frac{k_l b(T_{sat} - T_s)}{h_{fg}\delta} = \frac{\rho_l(\rho_l - \rho_v)gb\delta^2}{\mu}\frac{d\delta}{dx}$$

$$\delta^3 d\delta = \frac{k_l b\mu(T_{sat} - T_s)}{h_{fg}\rho_l(\rho_l - \rho_v)gb}dx$$

Integrating from x =0 where $\delta = 0$ to $x = x$ $where$ $\delta = \delta(x)$:

$$\int_0^{\delta(x)}\delta^3 d\delta = \int_0^x \frac{k_l b\mu(T_{sat} - T_s)}{h_{fg}\rho_l(\rho_l - \rho_v)gb}dx$$

$$\delta(\boldsymbol{x}) = \left(\frac{\boldsymbol{4k_l b\mu(T_{sat} - T_s)}}{\boldsymbol{h_{fg}\rho_l(\rho_l - \rho_v)gb}}\boldsymbol{x}\right)^{\boldsymbol{1/4}} \qquad [7]$$

4. Temperature distribution along the film thickness

Assuming linear temperature distribution across the condensate film

$$T = ay + b$$

And from boundary conditions:

$T = T_{sat}\ at\ y = \delta$

$T = T_{wall}\ or\ T_s\quad at\ y = 0$

Applying boundary condition and solving for $\boldsymbol{a}$ *and* $\boldsymbol{b}$ we get:

$$\boldsymbol{T = \frac{T_{sat} - T_w}{\delta} y + T_{wall}} \qquad \textbf{[8]}$$

Energy Equation

Energy equation for 2D steady laminar condensation is

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_x\frac{\partial^2 T}{\partial x^2} + k_y\frac{\partial^2 T}{\partial y^2} \quad \text{.........} (17)$$

Heat conduction is the flow direction along the plate is nearly zero. Thus, we can assume that the first term after equality is zero. Hence the equation becomes

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_y\frac{\partial^2 T}{\partial y^2}$$

We can rewrite this equation in the following manner

$$v = \frac{\partial y}{\partial t} = \frac{\partial x}{\partial t}\frac{\partial y}{\partial x} = u\frac{\partial y}{\partial x}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + u\frac{\partial y}{\partial x}\frac{\partial T}{\partial y}\right) = k_y\frac{\partial^2 T}{\partial y^2}$$

$$\rho c_p u\left(\frac{\partial T}{\partial x} + \frac{\partial y}{\partial x}\frac{\partial T}{\partial y}\right) = k_y\frac{\partial^2 T}{\partial y^2}$$

And rearranging

$$\frac{\partial T}{\partial x} = \frac{k}{\rho c_p u}\frac{\partial^2 T}{\partial y^2} - \frac{\partial y}{\partial x}\frac{\partial T}{\partial y} \quad \text{.....................................} (18)$$

Numerical analysis

Discretizing energy, and momentum equation using explicit finite difference scheme as shown below

a. Mass conservation

Referring back to the analytical forms, we obtain that

$$\frac{d\dot{m}}{dx} = \frac{(\rho_l - \rho_v)}{v} g\delta^2 \frac{d\delta}{dx} \quad \text{.............................} \quad d\dot{m} = \frac{-k\frac{\partial T}{\partial y}dx}{h_{fg}}$$

$$\frac{\Delta\dot{m}}{\Delta x} = \delta_n + \frac{\Delta\dot{m}v}{((\rho_l - \rho_v))g\delta_n^2} \quad \text{...} (19)$$

As density of vapor is much less than liquid condensate $(\rho_v < \rho_l)$

$$\delta_{n+1} = \delta_n + \frac{\Delta\dot{m}v}{\rho_l g\delta_n^2}$$

$$y_{i+1} = y_i + \frac{\delta_{n+1}}{m-1} \quad \text{..} (20)$$

Where m *is number of nodes in the y- direction*

n *is the number of steps in the x-direction*

From heat equation, the condensate mass in terms of interface temperature difference on the liquid and vapor side is

$$\frac{d\dot{m}}{dx} = \frac{k(T_v - T_w)}{dy.h_{fg}}dx$$

$$\Delta\dot{m} = \frac{k(T_m - T_{m-1})}{\Delta y.h_{fg}}\Delta x$$

$$\dot{m}_{n+1} = \dot{m} + \frac{k(T_m - T_{m-1})}{\Delta y.h_{fg}}\Delta x$$

Momentum equation

From the force balance on the fluid element it was found that velocity is parabolic in the form of an equation

$$u^{n+1} = \left(\frac{\rho}{\mu}\right)g\left(\delta_{n+1}.y_i - \frac{y_i^2}{2}\right)$$

C. Energy equation

From the energy equation $\frac{\partial T}{\partial x} = \frac{k}{\rho c_p u}\frac{\partial^2 T}{\partial y^2} - \frac{\partial y}{\partial x}\frac{\partial T}{\partial y}$ .... (18)

$$\frac{T_i^{n+1} - T_i^n}{\Delta x} = \frac{k}{\rho c_p u^n}\left(\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta y)^2}\right) - \left(\frac{y_i^{n+1} - y_i^n}{\Delta x}\right)\left(\frac{T_{i+1}^n - T_{i-1}^n}{2\Delta y}\right)$$

Let $\alpha = \frac{k}{\rho c_p}$

$$T_i^{n=1} = \left(\frac{\Delta x\alpha}{u_i^n(\Delta y)^2} + \frac{y_i^{n+1} - y_i^n}{2\Delta y}\right)T_{i-1}^n + \left(1 - \frac{2\Delta x\alpha}{u_i^n(\Delta y)^2}\right)T_i^n + \left(\frac{\Delta x\alpha}{u_i^n(\Delta y)^2} + \frac{y_i^{n+1} - y_i^n}{2\Delta y}\right)T_{i+1}^n \text{ ....(19)}$$

Empirical relations for the following properties

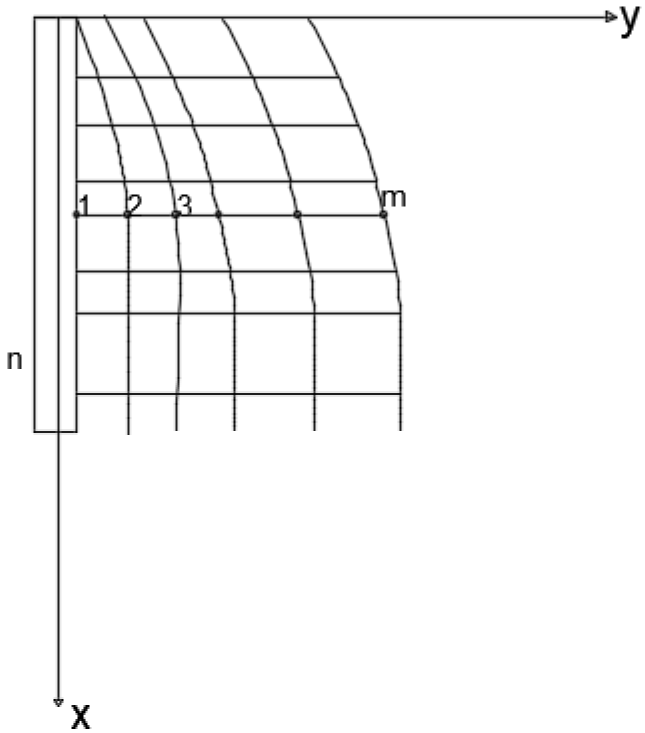
$$\rho_y = 3.6*10^{-6}(T_i - T_w)^3 - 0.0037(T_i - T_w)^2 - 0.08(T_i - T_w) + 1000$$

$$k_y = -1.1*10^{-10}(T_i - T_w)^4 + 7.8*10^{-8}(T_i - T_w)^3 - 2.15*10^{-5}(T_i - T_w)^2 + 0.0029(T_i - T_w) + 0.55$$

$$v_y = 5.4*10^{-15}(T_i - T_w)^4 + 2.7*10^{-12}(T_i - T_w)^3 + 5.1*10^{-10}(T_i - T_w)^2 - 4.4*10^{-8}(T_i - T_w) + 1.7*10^{-6}$$

Solution algorithm

As the system could not start from zero certain values are also assumed for starting. Besides the boundary condition



$T = T_{sat}$ at $y = \delta$

$T = T_w$ at $y = 0$ and $u = 0$ at $y = 0$

1. Assume small $\delta, \Delta\dot{m}, u$ discretize δ in y direction in to m-1 divisions
2. Approximate the temperature at each node using equation 19

$$T_i^{n=1} = \left(\frac{\Delta x\alpha}{u_i^n(\Delta y)^2} + \frac{y_i^{n+1} - y_i^n}{2\Delta y}\right)T_{i-1}^n + \left(1 - \frac{2\Delta x\alpha}{u_i^n(\Delta y)^2}\right)T_i^n + \left(\frac{\Delta x\alpha}{u_i^n(\Delta y)^2} + \frac{y_i^{n+1} - y_i^n}{2\Delta y}\right)T_{i+1}^n \text{ for } i = 2 \text{ to } m-1$$

3. Solve the condensate mass

$$\dot{m}_{n+1} = \dot{m} + \frac{k(T_m - T_{m-1})}{\Delta y.h_{fg}}\Delta x$$

4. Solve the boundary thickness

$$\delta_{n+1} = \delta_n + \frac{\Delta \dot{m} v}{\rho_l g \delta_n^2}$$

5. Solve the velocity at each node

$$u^{n+1} = \left(\frac{\rho}{\mu}\right)g\left(\delta_{n+1}.y_i - \frac{y_i^2}{2}\right)$$

6. Update the coordinates using

$$y_{i+1} = y_i + \frac{\delta_{n+1}}{m-1}, \text{ for } i = 1 \text{ to } m-1$$