Chapter Four
Computational Analysis of Laminar Convective Heat Transfer

Introduction

Now a days the study of heat transfer and energy transport has become an increasingly intense concern .in virtually every discipline concerned with physical processes involving energy production and exchanges, the need to predict, understand, and to optimize has led to detailed study of how energy is carried, distributed, and diffused in and by materials . The study of heat transfer is founded on the concepts of energy, mass, and momentum. These physical concepts have meaning because they can be related to other measurable properties, such as temperature and velocity, using the physical laws and empirical relations. When fluids are subjected to macroscopic motions due to surface or body forces, the thermal energy transferred to or from the flowing fluid can be classified as heat transfer by convection. In addition when the rate of heat transfer process is influenced by fluid motion it is also classified as convection heat transfer. When the energy causing fluid motion is produced by a thermally induced density difference in the fluid which arises because of the presence of a heated or cooled surface from buoyancy effect the process is called natural or free convection. On the other hand if the fluid process is imposed by forcing fluid passage by providing pressure difference the process is called forced convection.

1. Laminar convective heat transfer over a flat plate

1.1 Theoretical background

When a fluid at one temperature flows along a surface which is at another temperature, the behavior of the fluid cannot be described by the hydrodynamic equations alone. In addition to the hydrodynamic boundary layer, a thermal boundary layer develops. The thickness of both boundary layers is limited to the inter-surface distance. Laminar boundary layers occur in many important applications, and the techniques of boundary layer analysis has been applied to many circumstances. Solutions of the boundary layer equations are called “exact” solutions.

Consider the viscous flow over a flat plate as sketched in fig 1 below. The viscous effects are contained within a thin boundary layer adjacent to the surface; the thickness is exaggerated in fig 1 below for clarity. Immediately at the surface, the flow velocity is zero; this is the “no-slip” condition. In addition, the temperature of the fluid immediately at the surface is equal to the temperature of the surface; this is called the wall temperature T_w, as shown in fig below. Above the surface, the flow velocity increases in the Y-direction until, for all practical purposes, it equals the free stream velocity. This will occur at a height above the wall equal to δ,as shown in fig below. More precisely, δ is defined as that distance above the wall where $u=0.99u_e$; here, u_e is the velocity at the outer edge of the boundary layer. In fig 1 which illustrates the flow over a flat plate, the velocity at the edge of the boundary layer will be U_∞ ;that is $u_e = U_\infty$ for a body of general shape, u_e is the velocity obtained from an inviscid flow solution evaluated at the body surface or at the "effective body" surface.

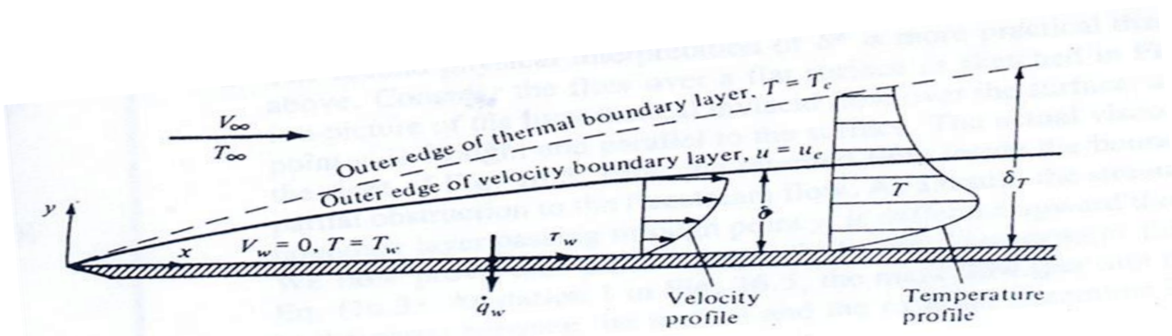


Fig 1 boundary layer properties

The quantity δ is called the velocity boundary layer thickness. At any given x station, the variation of u between y =0 and y = δ, that is u=u(y), is defined as the velocity profile within the boundary layer, as sketched in fig above. This profile is different for different x –stations. Similarly, the flow temperature will change above the wall, ranging from $T= T_w$ at y = 0 to $T=0.99T_e$ at $y = \delta_t$. Here, δ_t is defined as the thermal boundary-layer thickness. At any given x station, the variation of T between y = 0 and $y = \delta_t$, that is T=T(y), is called the temperature profile within the boundary layer. In the above case T_e is the temperature at the edge of the thermal boundary layer. For the flow over a flat plate, as sketched in fig above $T_e = T_\infty$. For a general body, T_e is obtained from inviscid flow solution evaluated at the body surface.

In general, two boundary layers can be defined: a velocity boundary layer with thickness δ and a temperature boundary layer with thickness δ_t. in general, $\delta_t \neq \delta$. The relative thicknesses depend on the Prandtl number: it can be shown that if Pr = 1, then $\delta = \delta_t$; if Pr >1, then $\delta_t < \delta$; if Pr <1, then $\delta_t > \delta$. This is clearly shown in fig below.

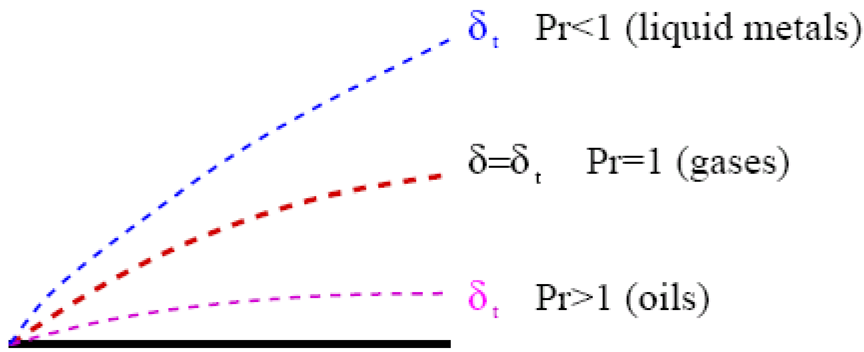


Fig 2. Variation of velocity and thermal boundary layer with prandtl number

For air at standard conditions, Pr = 0.71, hence, the thermal boundary layer is thicker than the velocity boundary layer. From fig 2 it can be seen that both boundary layer thickness increase with the distance from the leading edge, that is, $\delta = \delta(x)$ and $\delta_t = \delta_t(x)$. This can be seen clearly in fig below.

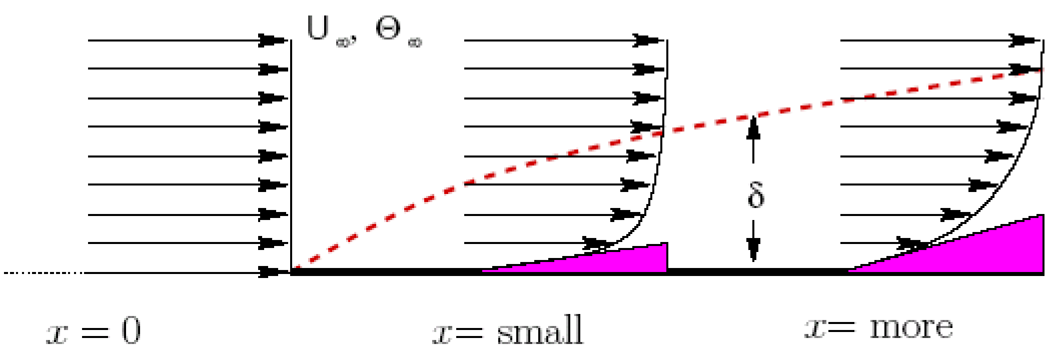


Fig 3. Variation of velocity boundary layer thickness along the plate.

The consequence of the velocity gradient at the wall is the generation shear stress at the wall,

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_w \qquad \text{---------- (1)}$$

Where $\left(\frac{\partial u}{\partial y}\right)_w$ is the velocity gradient evaluated at y = 0, that is, at the wall. Similarly, the temperature gradient at the wall generates heat transfer at the wall, which is given by

$$\dot{q}_w = -k\left(\frac{\partial T}{\partial y}\right)_w \qquad ---------- (2)$$

Where $\left(\frac{\partial T}{\partial y}\right)_w$ is the temperature gradient evaluated at y = 0; that is, at the wall. In general both τ_w and $\dot{q}_w$ are functions of distance from the leading edge; that is, $\tau_w = \tau_w(x)$ and $\dot{q}_w = \dot{q}_w(x)$.

1.2 Problem description and assumptions imposed in the analysis

The main objective of this analysis is to determine the convective heat transfer coefficient associated with the plot of velocity and temperature boundary layers using finite difference explicit forward marching scheme with fixed meshes for forced, laminar convective heat transfer over the flat plate.

For simplicity consider the following assumptions:

- Two-dimensional flow over a flat plate at zero angle of incidence
- The fluid is assumed to be Newtonian, steady and incompressible
- The flow is considered to be laminar
- The flow properties are evaluated at mean value of temperature, the so- called film temperature

2.3 Governing set of equations.

The stream approaches at a uniform velocity u_∞ . A boundary region forms in which the fluid is decelerated by viscous action. The local thickness of the boundary region is denoted by δ. The relevant Navier Stokes equations, the continuity equation and the energy equation and the boundary conditions are shown below. The dynamic viscosity of the fluid μ is a function of temperature. for gases, the temperature dependence is not great, and a constant value of μ may be used in the analysis when the temperature difference within the boundary layer is only a few hundred degrees or less. Therefore in this analysis μ is assumed constant as the working fluid is air.

The Governing Sets of Equations are:-

- Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{---------} \ (3)$$

- Momentum Equation

Momentum in x- direction

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{---------------} \ (4)$$

Momentum in y- direction

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad \text{----------------} \ (5)$$

- Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \text{---------} \ (6)$$

The above equations can be reduced further by imposing the following boundary layer assumptions.

$$\left.\begin{array}{l} \bullet\ \delta << L \quad \text{Where L is the boundary layer development length} \\ \bullet\ \dfrac{\partial^2 u}{\partial x^2} << \dfrac{\partial^2 u}{\partial y^2} = 0 \\ \bullet\ \dfrac{\partial p}{\partial x} \approx 0 \\ \bullet\ v << u \end{array}\right\} \quad (7)$$

Based on the above assumptions equations (4) and (6) reduced to:-

$$u\frac{\partial u}{\partial x}=\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{-------------} \ (8)$$

$$u\frac{\partial T}{\partial x}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{-------------} \ (9)$$

$$u\frac{\partial T}{\partial x}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \ \text{Where} \ \frac{k}{\rho c_p}=\alpha \qquad \text{---------} \ (10)$$

2.4 Boundary Conditions

The boundary conditions for the above two equations are

1) The plate surface is adiabatic, that is $\frac{\partial T}{\partial y}=0$

- At y = 0 u = 0

The above equation specifies no fluid velocity in the x direction relative to the solid surface for the fluid physically in contact with the surface at y = 0. This is the no-slip condition.

- At y = 0 T= T_w

2) The plate surface maintained at free stream temperature T_∞ and

Velocity u_∞

- At y =δ_t u = u_∞

i.e. the viscous velocity in the boundary layer approach the inviscid velocity at large (on scale of the boundary layer) distances above the surface.

- At y = δ_t T = T_∞

Numerical Analysis.

computation of velocity and temperature distribution

- First divide the flow domain in by a computational grid, as shown in figure below
- Assume all mean flow properties are constant in the control volume
- Integrate the governing equation in each cell

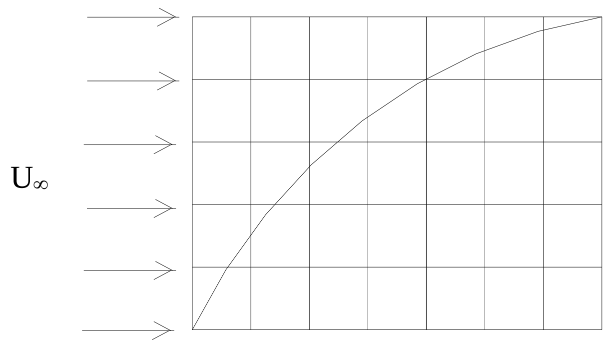


Fig 4 discretization of flow domain

Consider the following arrangement for the analysis of the computational grid

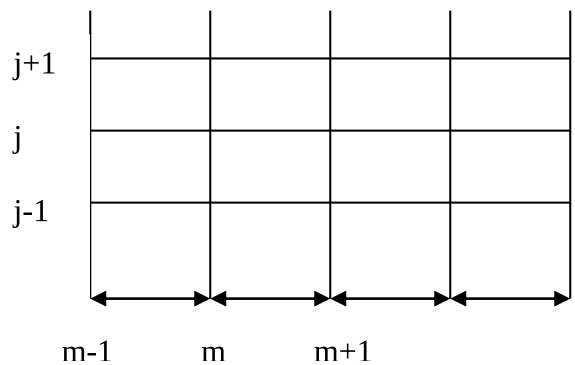


Fig 5. Discretization of flow domain

The explicit finite difference equivalent for the simplified governing equations (8) and (10) for the above element with $\Delta x = \Delta y$ are:

From equation (8)

$$u_j^{m+1} \frac{u_j^{m+1} - u_j^m}{\Delta x} = \frac{\mu}{\rho}\left(\frac{u_{j+1}^m - 2u_j^m + u_{j-1}^m}{\Delta y^2}\right) \qquad \text{-------} \; (11)$$

Solving for u_j^{m+1} gives

$$u_j^{m+1} = u_j^m\left(1 - \frac{2\mu}{\Delta y \rho u_j^m}\right) + \left(\frac{\mu}{\Delta y \rho u_j^m}\right)u_{j+1}^m + \left(\frac{\mu}{\Delta y \rho u_j^m}\right)u_{j-1}^m \quad ---- (12)$$

From equation (9)

$$u_j^m \frac{T_j^{m+1} - T_j^m}{\Delta x} = \alpha\left(\frac{T_{j+1}^m - 2T_j^m + T_{j-1}^m}{\Delta y^2}\right) + \frac{\mu}{\rho c_p}\left(\frac{u_{j+1}^m - u_{j-1}^m}{2\Delta y}\right)^2 \quad --- (13)$$

Solving for T_j^{m+1} gives

$$T_j^{m+1} = T_j^m\left(1 - \frac{2\alpha}{\Delta y u_j^m}\right) + \frac{\alpha}{\Delta y u_j^m}\left(T_{j+1}^m + T_{j-1}^m\right) + \frac{\mu}{4\rho c_p u_j^m \Delta y}\left(u_{j+1}^m - u_{j-1}^m\right)^2 \quad -- (14)$$

Computation of local and average heat transfer coefficients

- To compute the local heat transfer coefficients

From conduction heat transfer

$$q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = k\frac{\Delta T}{\Delta y} \quad --------- (15)$$

Where $\Delta T = T_m - T_w$ ($T_m = T_2$ and $T_w = T_1$

From convection heat transfer

$$q = h_x\left(T_w - T_\infty\right) \quad ------ (16)$$

Equating equations (15) and (16) and solving for h_x gives

$$h_x = k\frac{T_w - T_m}{\left(T_w - T_\infty\right)\Delta y} \quad ------ (17)$$

The average surface coefficient $\bar{h}$ from $x = 0$ to L may be obtained from $\bar{h} = \frac{\sum_{1}^{m} h_x \Delta x}{L}$ ------ (18), where L is the plate length

Matlab Code for the above case

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       MATLAB PROGRAMMING FOR CONVECTIVE LAMINAR BOUNDARY %%%%
%       LAYER ON A FLAT PLATE USING AN EXPLICIT FORWARD     %%%%
%       DIFFERENCE METHOD FOR VELOCITY AND TEMPERATURE      %%%%
%       DISTRIBUTION UNING FIXED MESH METHOD                %%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear;
cpa=4180;         %heat capacity of air
Ka=0.612;        %thermal conductivity of air
mue=5e-5;      % dynamic viscosity of air
rho=1000;          %density of air [kg/m^3]
Usurr=3;          %free stream velocity
Tsurr=315;        %surrounding temperature
Twall=500;        %wall temperature
alpha=Ka/(rho*cpa);%thermal diffusivity
dy=(8*alpha/(Usurr));% grid spacing in vertical direction
m=100;  %number of nodes along the plate
n=50;  %number of nodes in vertical direction
dx=dy;
% initializing velocity and temperature
%-------------------------------------
U=zeros(1,n);
T=zeros(1,n);
%Initial data input of velocity and temperature
%-------------------------------------
 for i=1:n
      if i==1
          Ui=0;
          Ti=Twall;
      else
          Ui=Usurr;
          Ti=Tsurr;
      end
      U(i)=Ui;
      T(i)=Ti;
  end
  %COMPUTATION OF VELOCITY DISTRIBUTION AND VELCITY BOUNDARY LAYER THICKNESS
  %-----------------------------------------------------------------------
 for j=1:m
     err=1;
     iter=0;
     dx1(1)=0;
     dx1(j+1)=j*dx;
     while err >3e-7;
         iter=iter +1;
         Uio=U;
         K=iter;
         if K==1;
             Ui=0;
         else
             Ui=Uio(K)*(1-(2*mue/(dy*rho*Uio(K))))+(mue*(Uio(K+1)+Uio(K-
1))/(dy*rho*Uio(K)));
         end
         Uii(K)=Ui;
         Velo(K)=Ui;
```

```
        delta(K)=(K-1)*dy;
        err=abs(Usurr-Ui);
    end
    deltaV(1)=0;
    deltaV(j+1)=iter*dy;
    for b=iter+1:n
        Uii(b)=Usurr;
    end

    U=Uii;
    Vel(j,:)=U;
end
Vel;
deltaV;
sum=0;
%COMPUTATION OF TEMPERATURE DISTRIBUTION AND THERMAL BOUNDARY LAYER THICKNESS
%------------------------------------------------------------------------------
for j=1:m
     err=1;
     iter=0;
      dx1(1)=0;
     dx1(j+1)=j*dx;
     dx11(j)=j*dx;
     while err > 3e-7;
         iter=iter +1;
        Tio=T;
         P=iter;
         if P==1;
             Ti=Twall;
         else
             Ti=                                                      Tio(P)*(1-
(2*alpha/(dy*Vel(j,P))))+(alpha/(dy*Vel(j,P)))*(Tio(P+1)+Tio(P-1))...
                  +mue*(Vel(P+1)+Vel(P-1))/(4*rho*cpa*Vel(j,P)*dy);
          end
         Tii(P)=Ti;
         Temp(P)=Ti;
         delta1(P)=(P-1)*dy;
         err=abs(Tsurr-Ti);
     end
   %COMPUTATION OF LOCAL AND AVERAGE  HEAT TRANSFER COEFFICIENTS
   %------------------------------------------------------------
     q(1)=0;
    q(j+1)=Ka*(Twall-Temp(2));
    hx(j)=q(j+1)/(dy*(Twall-Tsurr));
    sum=hx(j)+sum;
    deltaT(1)=0;
    deltaT(j+1)=iter*dy;
     for b=iter+1:m
         Tii(b)=Tsurr;
     end
     T=Tii;
     Te(j,:)=T;
end
h=sum*dx/(dx1(m))
    Temp;
    deltaT;
    dx1;
   %COMPUTATION OF VELOCITY BOUNDARY LAYER THCIKNESS FROM ANALYTICAL RESULT
   %------------------------------------------------------------------------
    for z=2:m
        blayer(1)=0;
      Rex(z)=(rho*Usurr*z*dx)/mue;
      blayert(z)=5*z*dx/(Rex(z))^.5;
```

```
    end
 %PLOT OF COMPUTED RESULTS
 %------------------------
 plot(Velo,delta)
xlabel('velocity [m/s]  ')
ylabel('distance from the surface of the plate in Y- direction')
title('The graphical representation of velocity  at x station ')
 grid on
 pause
 plot(Temp,delta1)
 xlabel('temperature [K] ')
ylabel('distance from the surface of the plate in Y- direction')
title('The graphical representation of temperature  at x station ')
grid on
pause
plot(10^2*dx11,hx)
xlabel('horizontal plate length [m] ')
ylabel('convective heat transfer coefficient[W/m^2K] ')
title(' variation of convective heat transfer along the plate ')
grid on
pause
plot(10^2*dx1,deltaV,'*',10^2*dx1,deltaT,'*')
xlabel('length[m]')
ylabel('boundary layer thicknes[m]')
title('Velocity and Termal Boundary-layer thicknes  profiles along the plate')
legend('velocity boundary layer thickness','temperature boundary layer thicknes')
grid on
pause
plot(10^2*dx11,blayert,'*')
xlabel('length[m]')
ylabel('boundary layer thicknes[m]')
title(' velocity Boundary-layer thicknes   profiles from analytical result along the
plate')
grid on
```

Results and discussion

1. Velocity distribution with in the boundary layer

from the following figure it can be seen that the velocity variation at a given x station is obtained using the explicit method is consistent with the velocity distribution with in the boundary layer, and it increases from zero at the wall to the free stream velocity at y = δ.The profile is also parabolic as expected for the distribution with in the boundary layer.

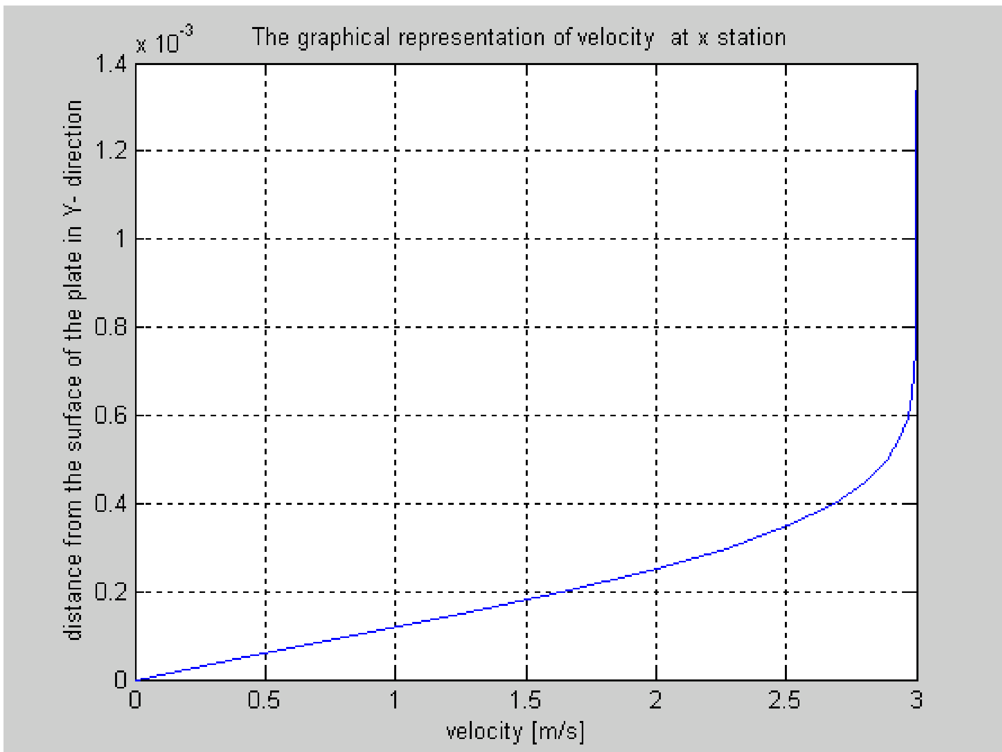


Fig 6. Velocity distribution at a given x station

2. Temperature distribution with in the boundary layer

From the following figure it can be seen that the temperature variation at a given x station is obtained using the explicit method is consistent with the temperature distribution with in the boundary layer, and it increases from the wall temperature to the free stream temperature at y = δ_t The profile is also parabolic as expected for the distribution with in the boundary layer. In this analysis the wall is assumed colder than the fluid.

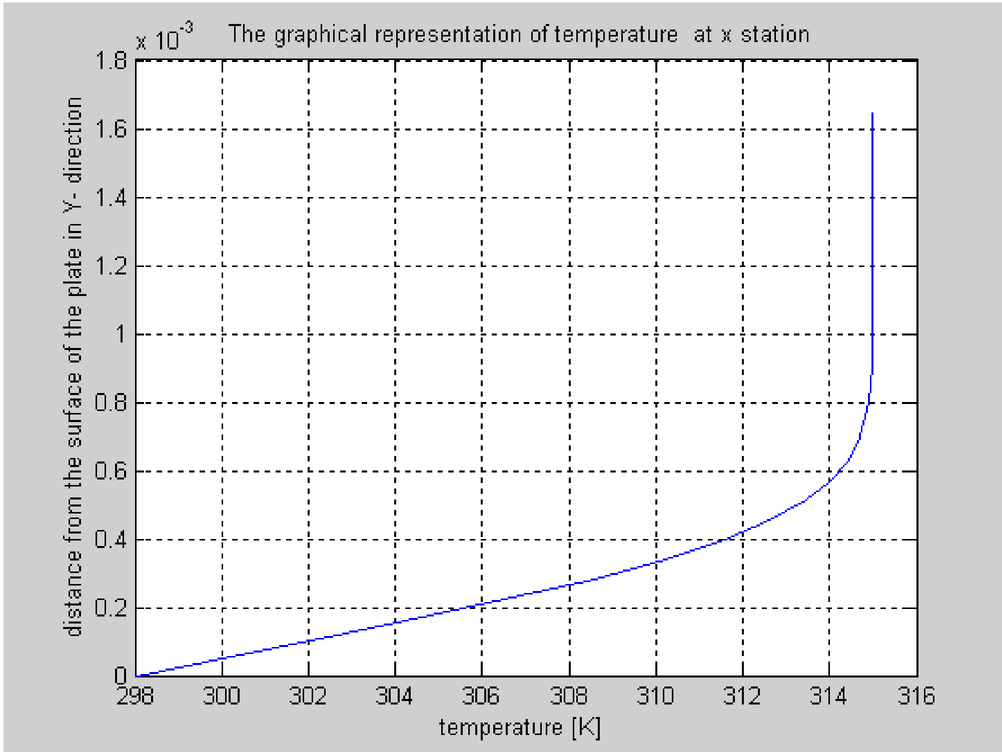


Fig 7.Temperature distribution at a given x station

3. Variation of local convective heat transfer coefficient

The local convective heat transfer coefficient varies with the distance from the leading edge of the plate. As shown in the figure below it decreases with an increase in horizontal distance along the plate. In addition the distribution is more or less hyperbolic as expected. Therefore the forward explicit method is reasonably good to compute the local heat transfer coefficient. The average convective heat transfer coefficient is also computed in the matlab program and the result agrees well with recommended average convective heat transfer coefficient for air with forced convection.

The recommended value of the average convective heat transfer coefficient for gases in forced convection is in the range of $30W/m^{2\,0}c - 300W/m^{2\,0}c$.In this analysis a value of $\bar{h} = 106.640\mathrm{W/m}^{2\,0}c$ is obtained which agrees with recommended values of $\bar{h}$.

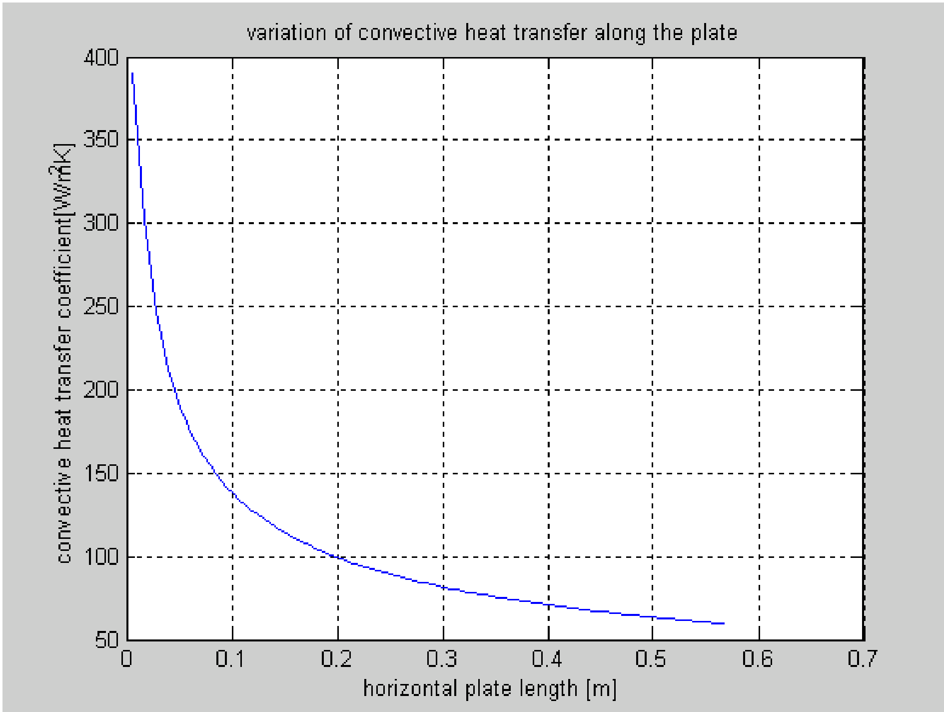


Fig 8. Variation of convective heat transfer coefficient along the plate

1. Profile of velocity and temperature boundary layer thicknesses

The velocity and temperature boundary layer thickness profiles are shown in figure below. From the figure we can see that both boundary layer thicknesses increase along the plate from the leading edge. If we compare the velocity profile with the analytical result it can be seen that the profile is the same. In addition the thermal boundary layer thickness is larger than the velocity boundary layer thickness.

This agrees with the theoretical result that for prandtl number less than one the thermal boundary layer thickness is larger than the velocity boundary layer thickness.

In this case the working fluid is air with prandtl number less than one which means that the thermal boundary layer is greater which agrees with the result obtained.Therefore the forward difference explicit method is very suitable for computing the boundary layer thickness as it does for convective heat transfer coefficient.

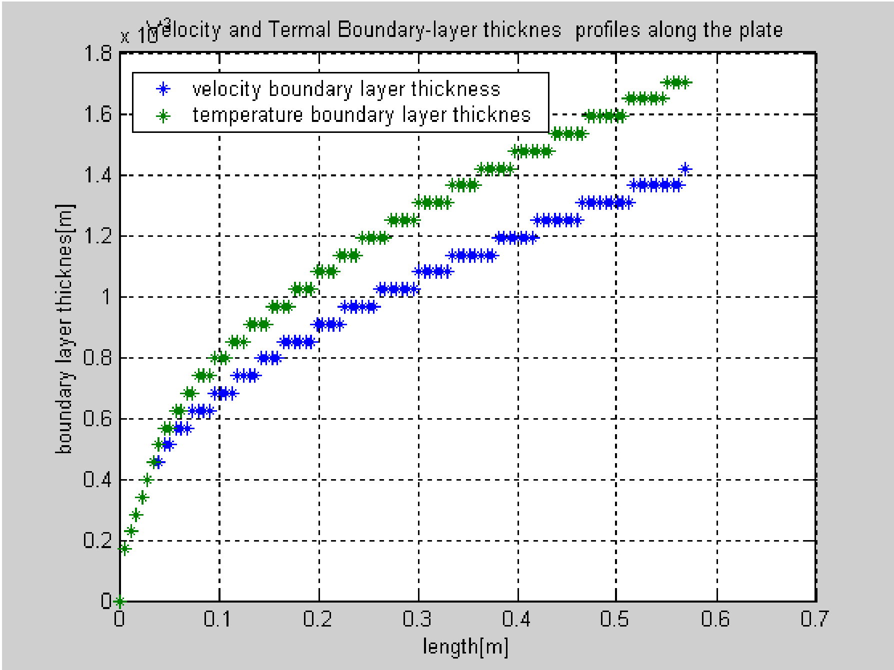


Fig 9. Velocity and thermal boundary layer thickness

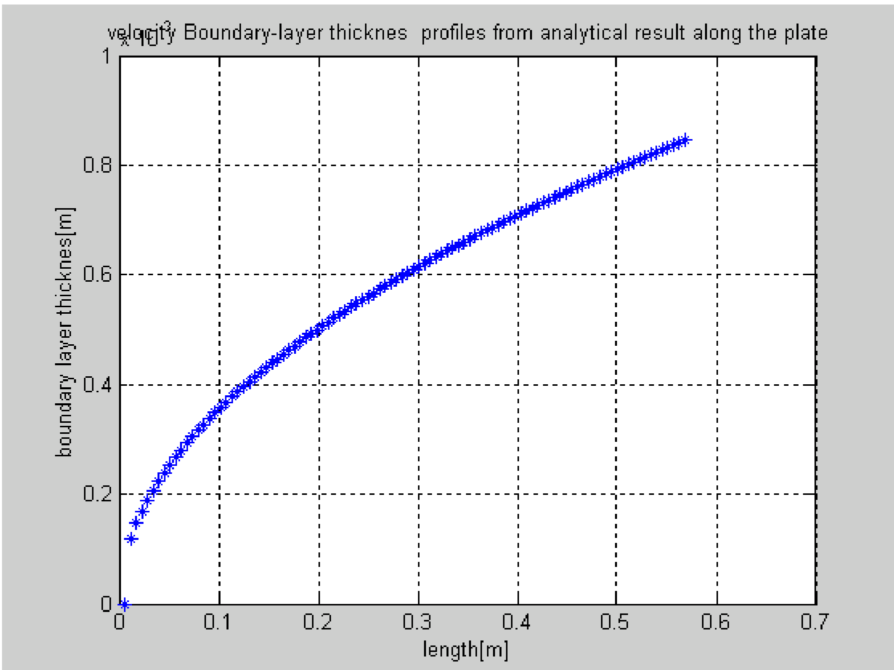


Fig 10. Velocity boundary layer thickness from analytical result

Conclusion

Generally the forward difference explicit method is well suited in the computation of boundary layer problems. More over the method gives good results for the computation of velocity and temperature distribution with in the boundary layer. Besides it is well suited for computing the distribution of the local heat transfer coefficient. From this analysis the following conclusions can be made.

- The velocity with in the boundary layer increases from zero value at the wall to the free stream velocity at the outer edge of the boundary layer
- The velocity distribution at a given x station increases in parabolic form from the surface up to the outer edge of the boundary layer.
- The temperature distribution at a given x station increase in a similar manner from the wall temperature to the free stream temperature.(In this analysis the wall is considered colder than the working fluid).
- From the result the thermal boundary layer thickness is larger than the velocity boundary layer as expected as the working fluid is air with prandtl number less than one.
- The local heat transfer coefficient decreases along the length of the plate. In addition the value of the average heat transfer coefficient calculated from this result falls with in the range of values recommended for forced convection. Hence the implementation of the forward explicit method is reasonably good at computing boundary layer problems.

Case 2: Laminar convective heat transfer between two fixed plates

- Problem description and assumptions imposed in the analysis

The main objective of this analysis is to determine the convective heat transfer coefficient associated with the plot of velocity and temperature distribution and the formation of both the velocity and temperature boundary layers using finite difference explicit method with fixed meshes of **laminar convective heat transfer between two fixed plates.**

For simplicity consider the following assumptions:

▪ Two-dimensional fluid flow between two fixed parallel plates at zero angle of incidence
▪ The fluid is assumed to be Newtonian, steady and incompressible
▪ The flow is considered to be laminar
▪ The flow properties are evaluated at mean value of temperature, the so-called film temperature

- Governing set of equations.

The stream approaches at a uniform velocity u_∞. A boundary region is formed in which the fluid is decelerated by viscous action. The local thickness of the boundary region is denoted by δ. The relevant Navier Stokes equations, the continuity equation and the energy equation and the boundary conditions are shown below. The dynamic viscosity of the fluid μ is a function of temperature. For gases, the temperature dependence is not great, and a constant value of μ may be used in the analysis when the temperature difference within the boundary layer is only a few hundred degrees or less. Therefore in this analysis μ is assumed constant as the working fluid is taken to be air.

The Governing Sets of Equations are

- 1. Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{-----------} \ (1)$$

Where: - u- is X – component of velocity

v –is Y – component of velocity

- **2. Momentum Equation**

Momentum in x- direction

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{-------------- (2)}$$

Momentum in y- direction

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad \text{---------------- (3)}$$

- **3.Relationship between shear stress and pressure gradient**

Consider the free body of any fluid having the form of an elementary parallelepiped of length dx, thickness dy and width dz as shown below.

Because of viscous effects, the velocity distribution is non-uniform, i.e., there exist a relative velocity between two adjacent layers.

These velocity gradients across the two layers setup shear stresses. Let τ represent the shear stress on the lower face AA'B'B and that on the upper face DD'C'C be given by ($\tau + (\partial\tau/\partial y)*$ dy). For a steady two – dimensional flow there will be any shear stress on the vertical faces of the element.

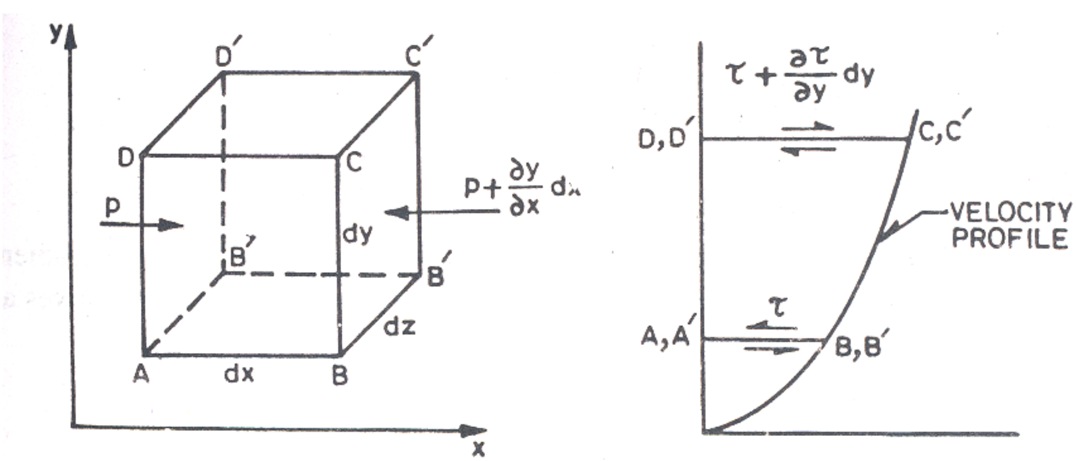


Fig1. Pressure and viscous forces on a fluid element

Shear force on the element

$= [\tau + (\partial\tau/\partial y) * dy]dx\, dz - \tau\, dxdz$

$= (\partial\tau/\partial y) * dx\, dy\, dz$

If P is the intensity of pressure on the face ADD'A', then the pressure intensity on the face BCC'B' will be $[P + (\partial P/\partial x) * dx]$. Thus

Pressure force on the element is

$= P\, dy\, dz - [P + (\partial P/\partial x) * dx]dy\, dz$

$= -(\partial P/\partial x)\, dx\, dy\, dz$

for equilibrium in steady flow, the acceleration is zero and the summation of the pressure and viscous forces in the x-direction must vanish.

$(\partial\tau/\partial y) * dx\, dy\, dz - (\partial P/\partial x)\, dx\, dy\, dz = 0$

or $\partial\tau/\partial y = -(\partial P/\partial x)$

since P =P(x) the partial derivatives can be replaced by total derivatives. Thus

$$\frac{dP}{dX} = \mu\frac{d^2U}{d^2Y} \qquad \text{--------------------(4)}$$

The flow has a zero velocity relative to an adjacent surface giving the boundary conditions:

U = 0 at y = 0; U = 0 at y = y (n) where Y = 0: Y (n)

The characteristic flow are then governed by the differential equation, $\frac{dP}{dX} = \mu\frac{d^2U}{d^2Y}$

With the knowledge that $\frac{dP}{dX}$ is independent of y, the above differential equation can be twice integrated with respect to y to give $U = \frac{1}{\mu}\frac{dP}{dX}\frac{Y^2}{2} + C_1Y + C_2$ where constants C_1 and C_2 are to be determined by introducing the boundary conditions discussed above

$C_2 = 0$ and $C_1 = -\frac{Y(n)}{2\mu}\frac{dP}{dX}$ substituting these values in the above equation

$U = \frac{1}{2\mu}(-\frac{dP}{dX})(Y(n)*Y(j) - Y^2(j))$ Then solving for $-\frac{dP}{dX}$

$-\frac{dP}{dX} = \frac{2\mu U}{(Y(n)*Y(j) - Y^2(j))}$ Multiplying both sides by (−1/ρ)

$$-\frac{1}{\rho}\frac{dP}{dX} = 2\upsilon\frac{U}{(Y(n)*Y(j) - Y^2(j))} \qquad \text{---------------- (5)}$$

- **4. Energy Equation**

For incompressible steady flow neglecting viscous dissipation and no heat generation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad \text{------------------------ (6)}$$

The above equations can be reduced further by imposing the following boundary layer assumptions.

- Momentum change in the Y-direction is neglected
- $\delta << L$ Where L is the boundary layer development length

$$\left.\begin{array}{l} \circ \quad \frac{\partial^2 u}{\partial x^2} \approx 0 \\ \circ \quad \frac{\partial p}{\partial Y} \approx 0 \\ \circ \quad \frac{\partial^2 T}{\partial x^2} \approx 0 \end{array}\right\} \qquad (7)$$

Based on the above assumptions equations are reduced to:-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{------------- (8)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{------------- (9)}$$

$$-\frac{1}{\rho}\frac{dP}{dX} = 2\upsilon\frac{U}{(Y(n)*Y(j)-Y^2(j))}$$ ------------(10)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right)$$ ------------ (11)

Where $\alpha = \frac{k}{\rho c_p}$

- **Boundary Conditions**

The boundary conditions for the above three equations are

1) **At the wall surfaces**

- At $y = 0$ $u = 0$
- At $y = 0$ $v = 0$
- At $y = y_{max}$ $u = 0$
- At $y = y_{max}$ $v = 0$
- At $y = 0$ $T = T_w$
- At $y = ymax$ $T = T_w$

The above equation specifies no fluid velocity in the both x & y direction relative to the solid surface for the fluid physically in contact with the surface at $y = 0$ and at $y=y_{max}$. This is the no-slip condition.

The fluid temperature is the same as wall temperature at $y = 0$ and at $y=y_{max}$

2) The inlet fluid is at free stream temperature T_∞ and velocity u_∞

- At $x = 0$ $U = U_\infty$
- At $x = 0$ $T = T_\infty$

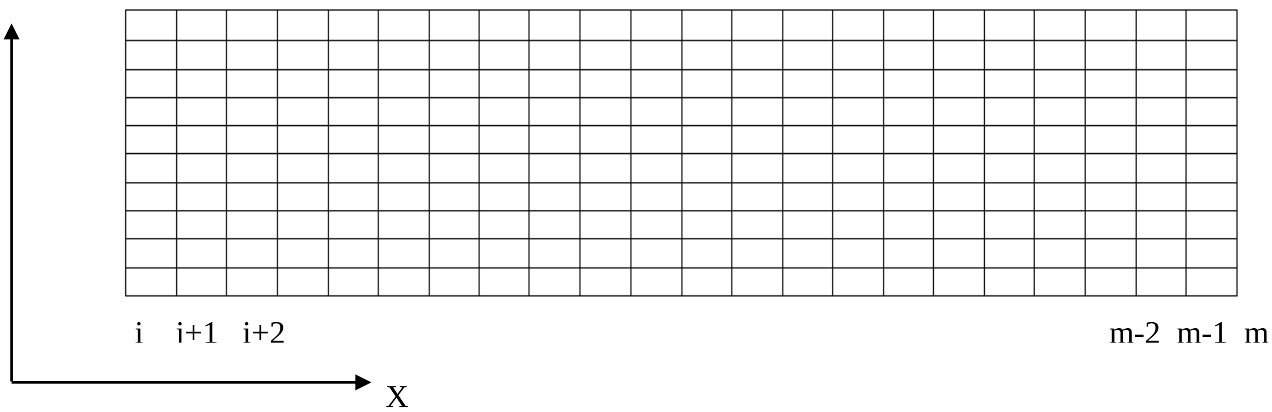
- Numerical Analysis.

a) Computation of velocity and temperature distribution

- First divide the flow domain in by a computational grid, as shown in figure below
- Assume all mean flow properties are constant in the control volume
- Integrate the governing equation in each cell

Consider the following arrangement for the analysis of the computational grid

n
n-1



The explicit finite difference discritization for the above simplified governing equations are:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} \qquad \frac{\partial u}{\partial y} = \frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y} \qquad \frac{\partial v}{\partial y} = \frac{v_{i+1,j} - v_{i+1,j-1}}{\Delta y}$$

$$\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i,j}}{\Delta x} \qquad \frac{\partial T}{\partial y} = \frac{T_{i,j+1} - T_{i,j-1}}{2\Delta y}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y} \qquad \frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y}$$

$$-\frac{1}{\rho}\frac{dP}{dX} = 2\upsilon \frac{U}{(Y(n) * Y(j) - Y^2(j))}$$

Continuity equation

$$\frac{u_{i+1,j} - u_{i,j}}{\Delta x} + \frac{v_{i+1,j} - v_{i+1,j-1}}{\Delta y} = 0$$

Solving for $v_{i+1,j}$

$$v_{i+1,j} = v_{i+1,j-1} - \left(\frac{\Delta y}{\Delta x}\right)(u_{i+1,j} - u_{i,j}) \text{ ------------------------- } (12)$$

Momentum equation

$$u_{i,j}\left(\frac{u_{i+1,j} - u_{i,j}}{\Delta x}\right) + v_{i,j}\left(\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y}\right) = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_{i,j} + \upsilon\left(\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2}\right)$$

Solving for $u_{i+1,j}$

$$u_{i+1,j} = u_{i,j} + \left(\frac{\Delta x}{u_{i,j}}\right)\left[2\upsilon\left(\frac{U_{i,j}}{Y_n * Y_{i,j} - Y_{i,j}^2}\right) + \upsilon\left(\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2}\right) - v_{i,j}\left(\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y}\right)\right] \text{-----} (13)$$

Energy equation

$$u_{i+1,j}\left(\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\right) + v_{i+1,j}\left(\frac{T_{i,j+1} - T_{i,j-1}}{2\Delta y}\right) = \alpha\left(\frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}\right)$$

Solving for $T_{i+1,j}$

$$T_{i+1,j} = T_{i,j} + \left(\frac{\Delta x}{u_{i+1,j}}\right)\left[\alpha\left(\frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}\right) - v_{i+1,j}\left(\frac{T_{i,j+1} - T_{i,j-1}}{2\Delta y}\right)\right] \text{----------------} (14)$$

b) Computation of local and average heat transfer coefficients

To compute the local heat transfer coefficients. From conduction heat transfer

$$\dot{q} = -k\left(\frac{\partial T}{\partial n}\right) = -k\frac{\Delta T}{\Delta y} \qquad \text{------------------} (15)$$

Where $\Delta T = T_n - T_w$

$$T_n = T_2 \ , \quad T_w = T_1$$

$$\dot{q} = -k\left(\frac{T_2 - T_1}{\Delta y}\right) \qquad \text{----------------} (16)$$

From convection heat transfer

$$\dot{q} = h_x(T_w - T_\infty) \qquad T_w = T_1$$

$$\dot{q} = h_x(T_1 - T_\infty) \qquad \text{-----------------} (17)$$

Equating equations (14) and (15) and solving for h_x gives

$$h_x = k\left(\frac{\left(\frac{T_1 - T_2}{\Delta y}\right)}{T_1 - T_\infty}\right) \quad \text{---------------} \ (18)$$

FDM discritization is $h_{i+1} = k\left(\frac{\left(\frac{T_{i+1,1} - T_{i+1,2}}{\Delta y}\right)}{T_{i+1,1} - T_\infty}\right)$ ---------------- (19)

The average surface coefficient $\bar{h}$ from $x = 0$ to L may be obtained from

$$\bar{h} = \frac{1}{L}\sum_{0}^{L} h_x \Delta x$$

$$\bar{h}_i = \frac{1}{L}\sum_{i=0}^{n} h_i \Delta x \quad \text{-------------} \ (20)$$

Where L is the plate length

C) Computations of velocity and thermal boundary layer

When a fluid at one temperature flows along a surface which is at another temperature, the behavior of the fluid cannot be described by the hydrodynamic equations alone. In addition to the hydrodynamic boundary layer, a thermal boundary layer develops. The thickness of both boundary layers is limited to the inter-surface distance. Laminar boundary layers occur in many important applications and the techniques of boundary layer analysis has been applied to many circumstances. Solutions of the boundary layer equations are called "exact " solutions.

In general, two boundary layers can be defined: a velocity boundary layer with thickness δ and a temperature boundary layer with thickness δ_t. in general, $\delta_t \neq \delta$. The relative thicknesses depend on the Prandtl number: it can be shown that if Pr = 1, then $\delta = \delta_t$; if Pr >1, then $\delta_t < \delta$; if Pr <1, then $\delta_t > \delta$.

δ is defined as that distance above the wall where $u=0.99u_e$; here, u_e is the velocity at the outer edge of the boundary layer.

The quantity δ is called the velocity boundary layer thickness. At any given x station, the variation of u between y =0 and y = δ,that is u=u(y), is defined as the velocity profile within the boundary layer

δ_t is defined as the thermal boundary-layer thickness. At any given x station, the variation of T between y = 0 and y = δ_t, that is T=T(y), is called the temperature profile within the boundary layer.

If $\dfrac{U_{i,j-1}}{U_{i,j}} > 0.99$ then $y_j = \delta$

If $\dfrac{T_{i,j}}{T_{i,j-1}} > 0.99$ then $y_j = \delta_t$

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Project

Obtain the results for flow between two parallel plates using case 1 as a reference. Write a mat lab code for this case (modify the code above for case 1). The problem statement, discretization and numerical method are elaborated as follows. Complete this report and submit with the results, analysis of results and conclusions included:

Expected outcomes in the results

a) Computation of velocity and temperature distribution

b) Computation of local and average heat transfer coefficients

C) Computations of velocity and thermal boundary layer