Advanced Process Control

CBEg 6142

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Chapter 9

Multi-loop Control

9.1 MIMO Control Systems: Introduction

- control problems that have only one controlled variable and one manipulated variable are referred to as single-input, single-output (SISO), or *single-loop*, control problems.
- control problems that have multiple controlled variable and multiple manipulated variable are referred to as multiple-input, multiple-output (MIMO), or multi-*loop*, control problems.
- For almost all important processes, at least two variables must be controlled: product quality and throughput.

9.1 MIMO Control Systems: Introduction

Several examples of processes with two controlled variables and two manipulated variables are shown in Fig. 9.1.

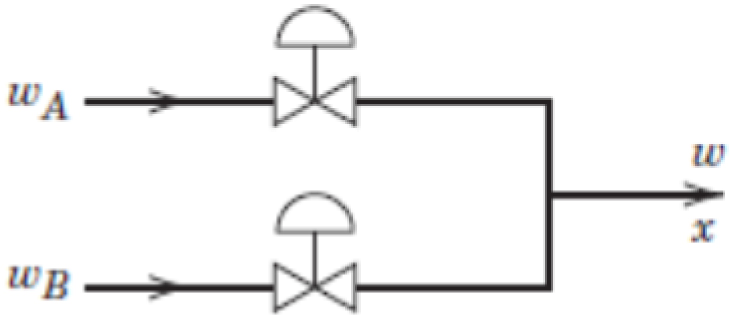


Fig 9.1 In-line blending system

Control Objectives

1. To maintain the mass flow rate, w, at set-point w_{sp}

2. To maintain the mass fration, x, at a set-point x_{sp}

Manipulated variables: w_A, w_B

- Distillation column

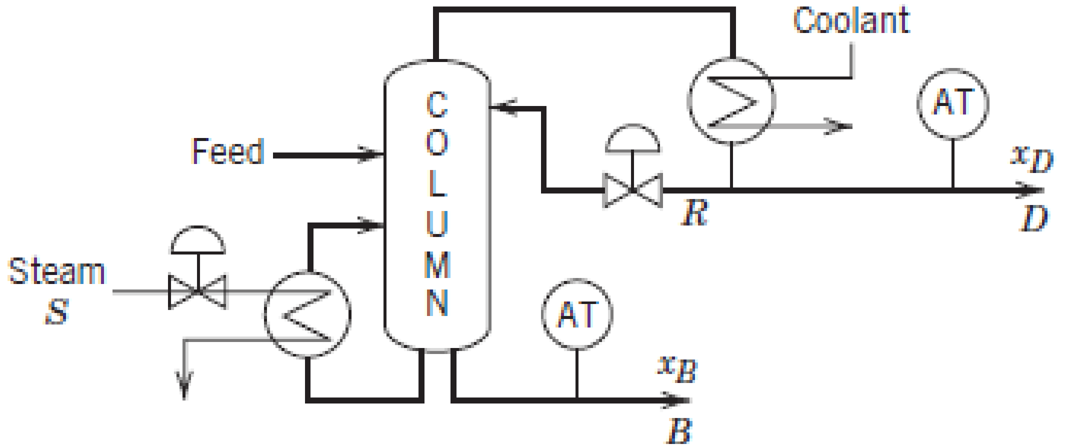


Fig 9.2 Distillation Column

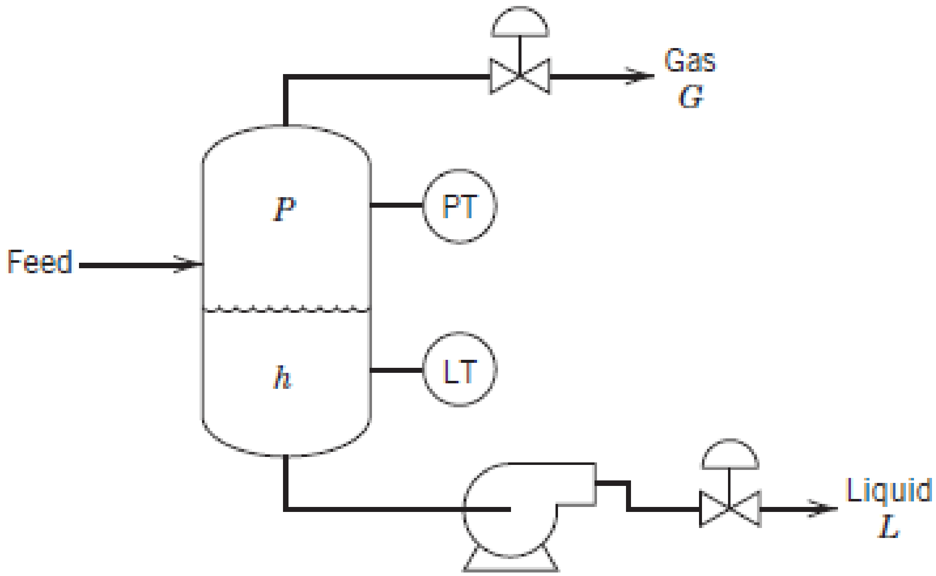
<u>Control Objectives</u>

1. To maintain the distillate composition , x_D, at set-point $x_{D,sp}$

2. To maintain the bottoms composition , x_B, at set-point $x_{B,sp}$

<u>Manipulated variables: *S*, *R*</u>

9.1 MIMO Control Systems: Introduction



Gas–liquid separator

Fig 9.3 Gas-liquid separator

<u>Control Objectives</u>

1. To maintain the pressure , P, at set-point P_{sp}

2. To maintain the liquid level , h, at set-point h_{sp}

<u>Manipulated variables: S, R</u>

- Block Diagram

SISO

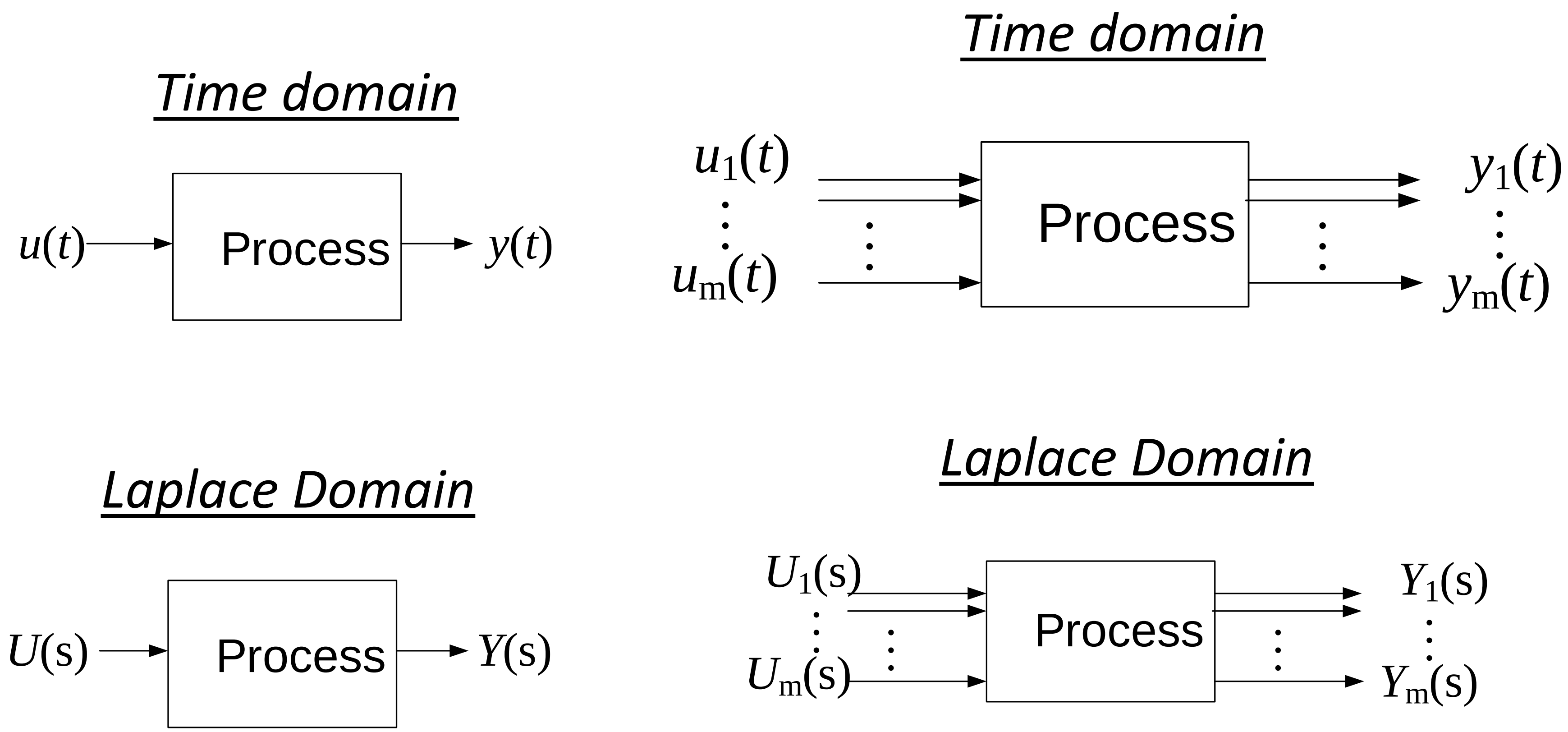


Fig 9.4 (a)Single Input Single Output (b) Multiple Input Multiple Output

9.1 MIMO Control Systems: Introduction

Two controlled variables and two manipulated variables - 4 transfer functions required

$$\frac{Y_1(s)}{U_1(s)} = G_{P11}(s), \quad \frac{Y_1(s)}{U_2(s)} = G_{P12}(s)$$
$$\frac{Y_2(s)}{U_1(s)} = G_{P21}(s), \quad \frac{Y_2(s)}{U_2(s)} = G_{P22}(s)$$

Thus, the input-output relations for the process can be written as

$$Y_1(s) = G_{P11}(s)U_1(s) + G_{P12}(s)U_2(s)$$
$$Y_2(s) = G_{P21}(s)U_1(s) + G_{P22}(s)U_2(s)$$

9.1 MIMO Control Systems: Introduction

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{p11}(s) & G_{p12}(s) \\ G_{p21}(s) & G_{p22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

Or in vector-matrix notation as

$$\underline{Y}(s) = \underline{\underline{G_P}}(s)\underline{U}(s)$$

where $\underline{Y}(s)$ and $\underline{U}(s)$ are vectors

$$\underline{Y}(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix}, \quad \underline{U}(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

Block diagram representation:Open-loop

- Block diagram representation MIMO system

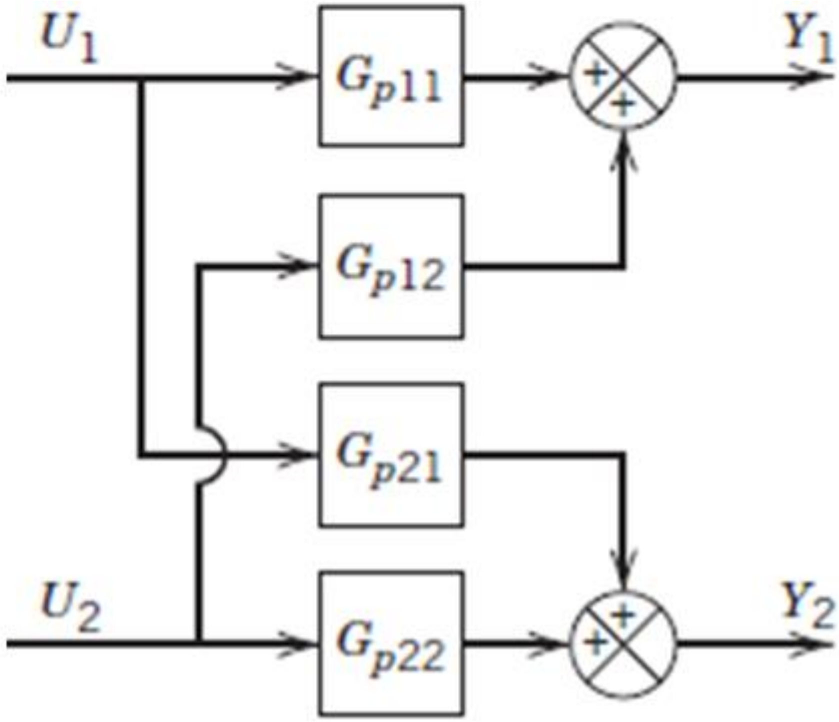


Fig 9.5 Block diagram of a 2×2 system

Block Diagram Representations: Closed-loop

- MIMO-closed loop:1-1/2-2 controller pairing

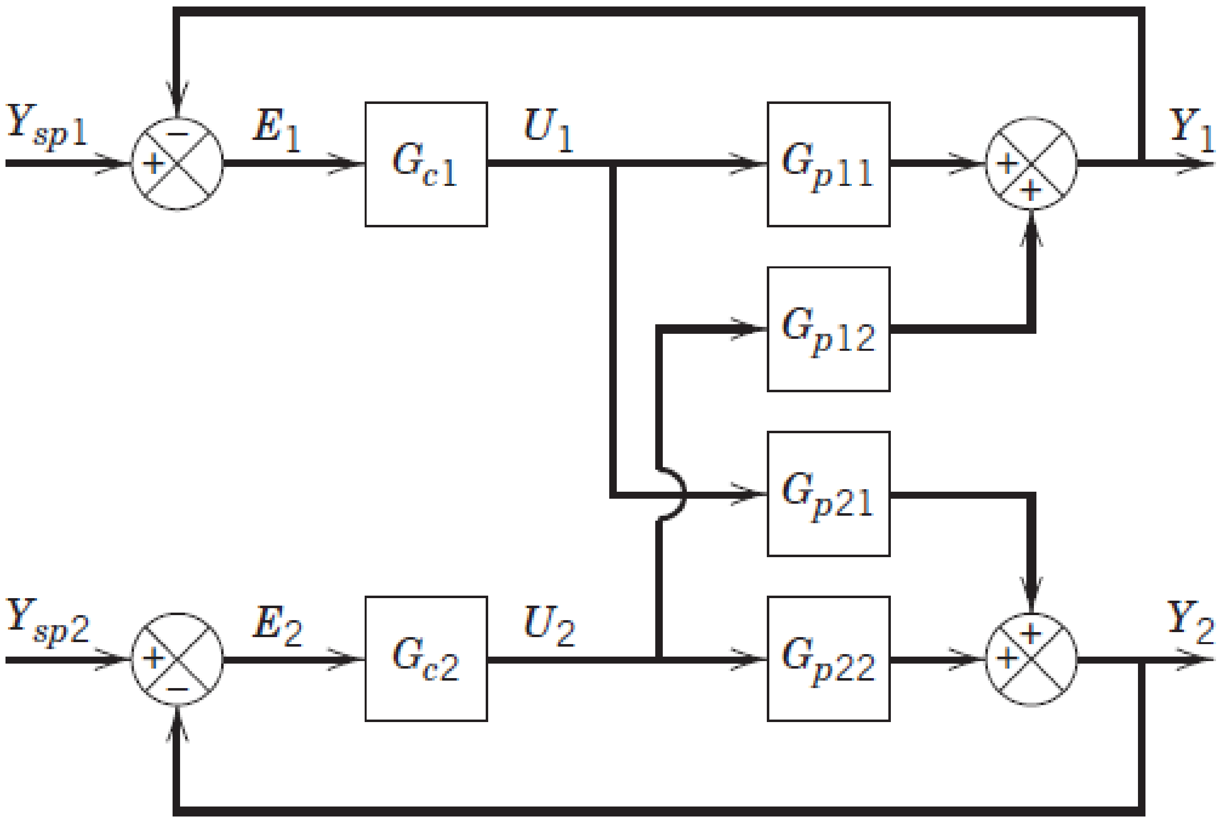


Fig 9.6 Block diagram of a 2×2 feedback control system

Block Diagram Representations

- Closed-loop :1-2/2-1 pairing

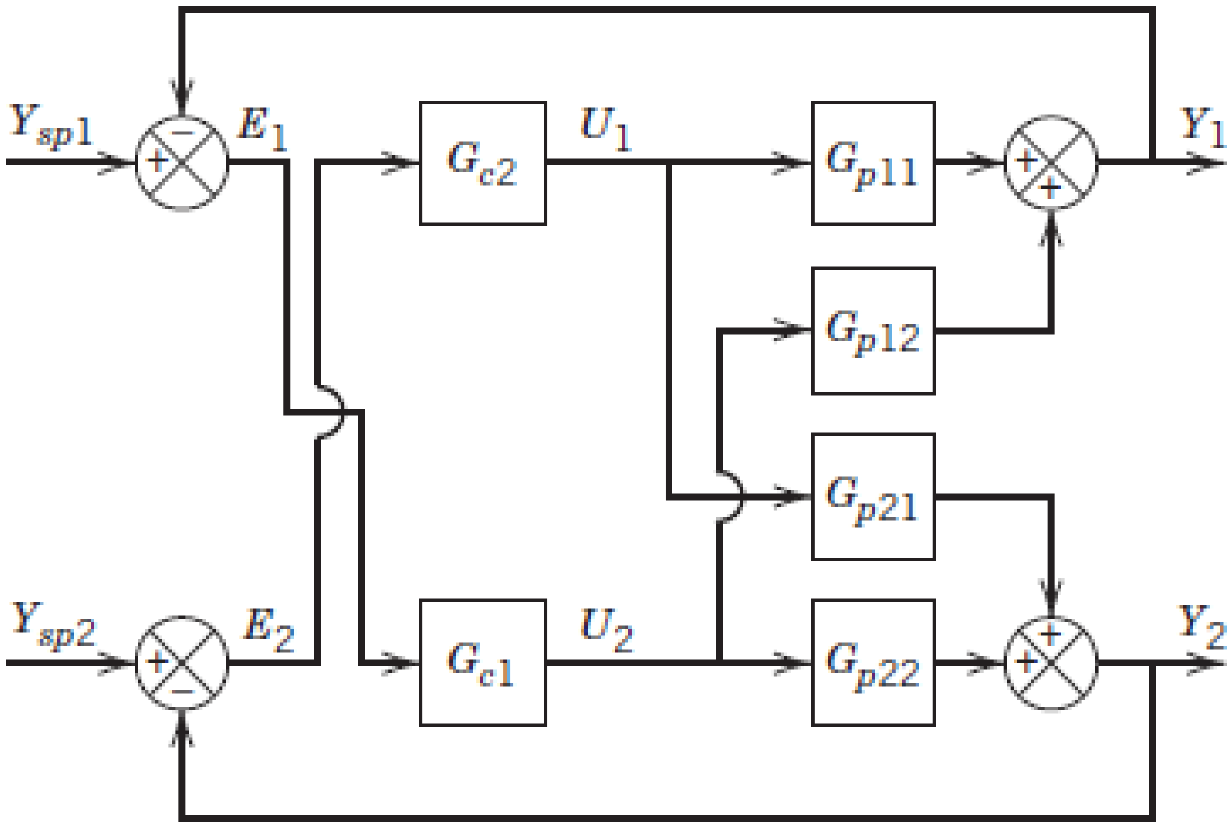


Fig 9.7 Block diagram of a 2×2 feedback control system

9.1 MIMO Control Systems: Introduction

1. The controller for loop 1 (G_{c1}) adjusts U_1 so as to force Y_1 back to the set point. However, U_1 also affects Y_2 via transfer function G_{p21}.

2. Since Y_2 has changed, the loop 2 controller (G_{c2}) adjusts U_2 to bring Y_2 back to its set point, Y_{2sp}. However, changing U_2 also affects Y_1 via transfer function G_{p12}.

9.1 MIMO Control Systems: Introduction

The control loop interactions in a 2 × 2 control problem result from the presence of a third feedback loop that contains the two controllers and two of the four process transfer functions.

Thus, for the 1-1/2-2 configuration, this hidden feedback loop contains Gc1, Gc2, Gp12, and Gp21, as shown in Fig. 9.4.

9.1 MIMO Control Systems: Introduction

The hidden feedback control loop for a 1-1/2-2 controller pairing causes two major problems:

1. It tends to destabilize the closed-loop system.

2. It makes controller tuning more difficult.

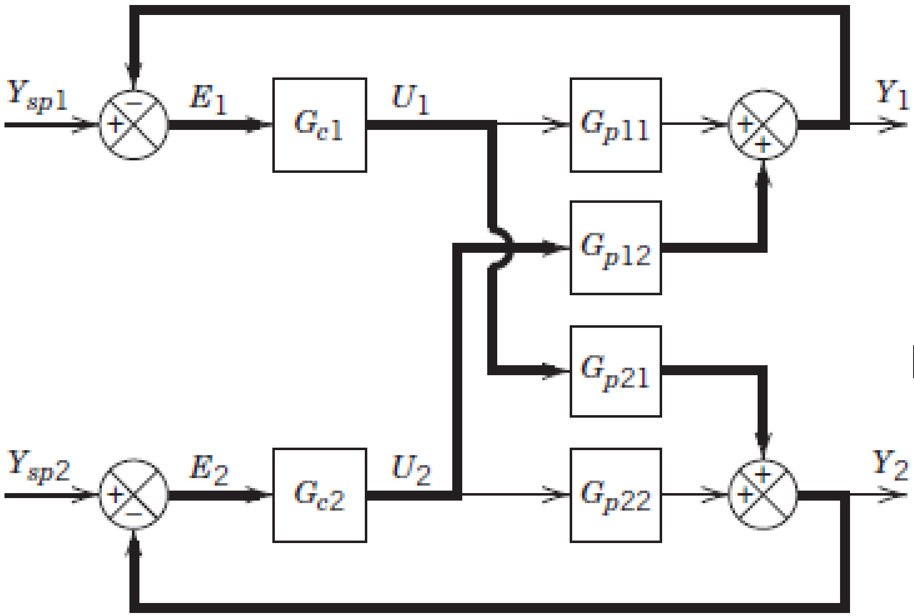


Fig 9.8 The hidden feedback loop

Example 9.1

Consider the following empirical model of a pilot-scale distillation column (Wood and Berry, 1973)

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix}$$

Suppose that a multiloop control system consisting of two PI controllers is used. Compare the closed-loop set-point changes that result if the $X_D - R/X_B - S$ pairing is selected and

9.1 MIMO Control Systems: Introduction

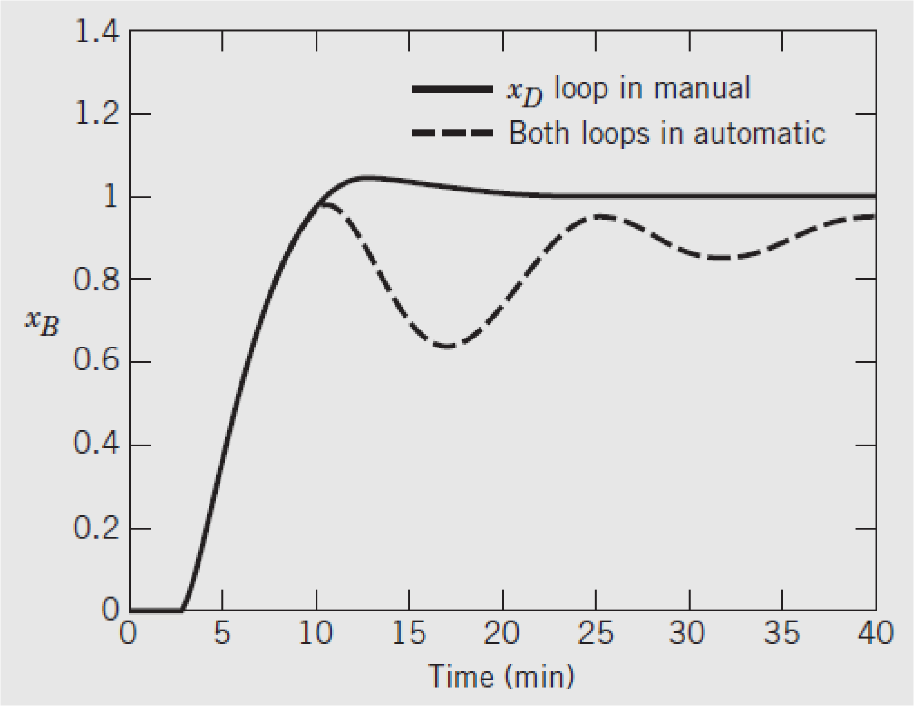
(a) A set-point change is made in each loop with the other loop in manual

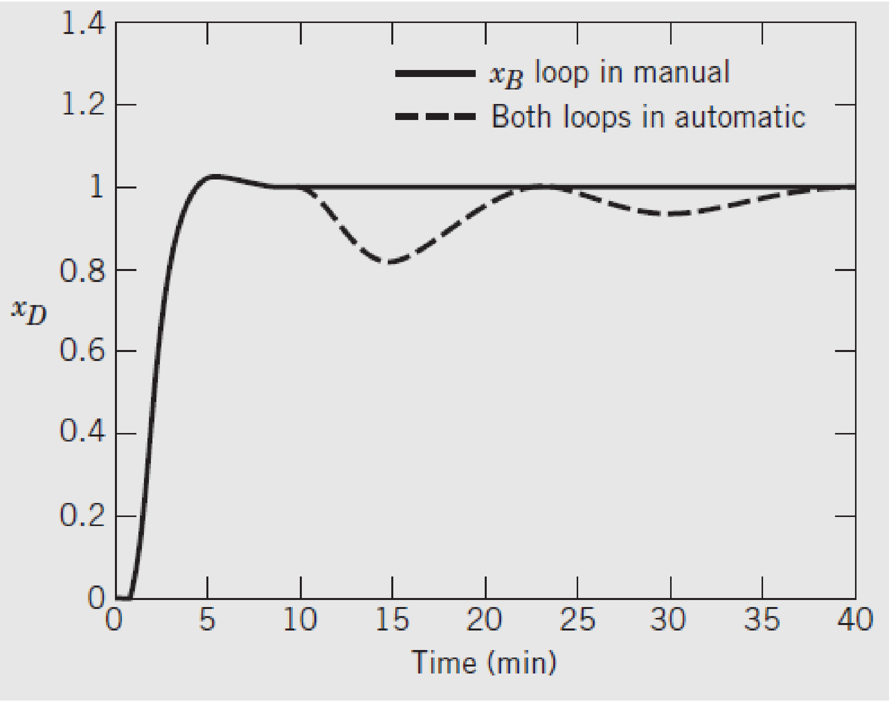
(b) The set-point changes are made with both controllers in automatic

Table 9.1 Controller Settings for Example 9.1

Controller Pairing	K_c	τ_I (min)
$x_D - R$	0.604	16.37
$x_B - S$	–0.127	14.46

9.1 MIMO Control Systems: Introduction





9.2 Loop-pairing

- How are the controlled variables and the manipulated variables should paired in a multiloop control scheme?

- An incorrect pairing can result in poor control system performance and reduced stability margins.

- If a multiloop control scheme consisting of five feedback controllers is used, there are 5! = 120 different ways of pairing the controlled and manipulated variables.

9.2 Loop-pairing

- how to determine the most effective pairing.

- There are two different approaches
  - the relative gain array method
  - singular value analysis is described

9.2.1 The Relative Gain Array

- The relative gain array (RGA)- λ is defined as

$$\lambda_{ij} \triangleq \frac{(\partial y_i/\partial u_j)_u}{(\partial y_i/\partial u_j)_y} = \frac{\text{open-loop gain}}{\text{closed-loop gain}}$$

- RGA is based on steady state gain

$$Y_1 = K_{11}U_1 + K_{12}U_2$$
$$Y_2 = K_{21}U_1 + K_{22}U_2$$

9.2.1 The Relative Gain Array

Example

Derive the RGA equations for the 2×2 shown below.

$$Y_1 = K_{11}U_1 + K_{12}U_2$$
$$Y_2 = K_{21}U_1 + K_{22}U_2$$

Solution

The open-loop gain for calculating λ_{11} is

$$\left(\frac{\partial y_1}{\partial u_1}\right)_{u_2} = K_{11}$$

9.2.1 The Relative Gain Array

- Calculating the closed-loop gain (y_2=0)

$$Y_2 = K_{21}U_1 + K_{22}U_2 = 0$$

$$U_2 = -\frac{K_{21}}{K_{22}}U_1$$

$$Y_1 = K_{11}\left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}\right)U_1$$

$$\lambda_{11} = \frac{\left(\partial y_{11} / u_{11}\right)_{u2}}{\left(\partial y_{11} / u_{11}\right)_{y2}} = \frac{K_{11}}{K_{11}\left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}\right)} = \frac{1}{\left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}\right)}$$

9.2.1 The Relative Gain Array

- Because each row and each column of $\mathbf{\Lambda}$ sums to one, the other relative gains are easily calculated, therefore

- Thus, the RGA for a 2 × 2 system can be expressed as

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$$

9.2.1 The Relative Gain Array

For higher-dimension processes, the RGA can be calculated from the expression

$$\mathbf{\Lambda} = \boldsymbol{K} \otimes \mathbf{H} \qquad (18\text{-}36)$$

where $\otimes$ denotes the Schur product (element by element multiplication):

$$\lambda_{ij} = K_{ij} H_{ij} \qquad (18\text{-}37)$$

K_{ij} is the (i, j) element of $\boldsymbol{K}$ in Eq. 18-28, and H_{ij} is the (i, j) element of $\boldsymbol{H} = (\boldsymbol{K}^{-1})^T$; that is, H_{ij} is an element of the transpose of the matrix inverse of $\boldsymbol{K}$.

9.2.1 The Relative Gain Array

λ_{11}=1 open-loop and closed-loop gains between y_1 and u_1 are identical. In this situation, opening or closing loop 2 has no effect on loop 1.

λ_{11}=0 the open-loop gain between *y*1 and *u*1 is zero, and thus *u*1 has no direct effect on *y*1. Consequently, *u*1 should be paired with *y*2 rather than *y*1 configuration should be utilized.

9.2.1 The Relative Gain Array

3. 0 < λ_{11} < 1. The closed-loop gain between *y*1 and *u*1 is larger than the open-loop gain. Within this range, the interaction between the two loops is largest when $\lambda = 0.5$.

4. λ_{11} > 1. For this situation, closing the second loop reduces the gain between *y*1 and *u*1. Thus, the control loops interact. As λ increases, the degree of interaction increases and becomes most severe as $\lambda \to \infty$. When λ is very large, it is impossible to control both outputs independently.

9.2.1 The Relative Gain Array

5. $\lambda_{11} < 0$. When λ_{11} is negative, the open-loop and closed-loop gains between *y*1 and *u*1 have different signs. Thus, opening or closing loop 2 has an adverse effect on the behavior of loop 1 such as oscillation. It follows that *y*1 should not be paired with *u*1. For $\lambda < 0$ the control loops interact by trying to “fight each other,” and the closed-loop system may become unstable. Based on these considerations, the

9.2.1 The Relative Gain Array

Recommendation: *Pair the controlled and manipulated variables so that corresponding relative gains are positive and as close to one as possible.*

9.2.1 The Relative Gain Array

Example 9.2

The relative gain array for a refinery distillation column associated with a hydrocracker discussed by Nisenfeld and Schultz (1971) is given by

$$\Lambda = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{bmatrix} 0.931 & 0.150 & 0.080 & -0.164 \\ -0.011 & -0.429 & 0.286 & 1.154 \\ -0.135 & 3.314 & -0.270 & -1.910 \\ 0.215 & -2.030 & 0.900 & 1.919 \end{bmatrix} \end{array}$$

The four controlled variables are the compositions of the top and bottom product streams ($y1$, $y2$) and the two side streams ($y3$, $y4$). The manipulated variables are the four flow rates numbered from the top of the column; for example, the top flow rate is $u1$. Find the recommended pairing using the RGA.

9.2.1 The Relative Gain Array

Solution

The recommended pairings are *y*1-*u*1, *y*2-*u*4, *y*3-*u*2, and *y*4-*u*3.

9.2.2 Singular Value Analysis

Singular value analysis (SVA) is a powerful analytical technique that can be used to solve several important control problems:

1. Selection of controlled, measured, and manipulated variables

2. Evaluation of the robustness of a proposed control strategy

3. Determination of the best multiloop control configuration

9.2.2 Singular Value Analysis

One desirable property of the gain matrix ***K*** is that the *n* linear equations in *n* unknowns be linearly independent.

In contrast, if the equations are dependent, then not all of the *n* controlled variables can be independently regulated.

This characteristic property of linear independence can be checked by several methods.

9.2.2 Singular Value Analysis

- One of these methods is the *condition number* (CN).

- Assume that ***K*** is nonsingular. Then the condition number of ***K*** is a positive number defined as the ratio of the largest and smallest nonzero singular values:

$$\mathrm{CN} = \frac{\sigma_1}{\sigma_r}$$

- If ***K*** is singular, then it is ill-conditioned, and by convention, CN=∞.

9.2.2 Singular Value Analysis

Example 9.3

Determine the preferred multiloop control strategy for a process with the following steady-state gain matrix:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.48 & 0.90 & -0.006 \\ 0.52 & 0.95 & 0.008 \\ 0.90 & -0.95 & 0.020 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

9.2.2 Singular Value Analysis

Solution

- The singular value analysis yields

$$\mathbf{W} = \begin{bmatrix} 0.5714 & 0.3766 & 0.7292 \\ 0.6035 & 0.4093 & -0.6843 \\ -0.5561 & 0.8311 & 0.0066 \end{bmatrix}$$

$$\mathbf{\Sigma} = \begin{bmatrix} 1.618 & 0 & 0 \\ 0 & 1.143 & 0 \\ 0 & 0 & 0.0097 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 0.0541 & 0.9984 & 0.0151 \\ 0.9985 & -0.0540 & -0.0068 \\ -0.0060 & 0.0154 & -0.9999 \end{bmatrix}$$

$$\text{CN} = \frac{\sigma_1}{\sigma_3} = \frac{1.618}{0.0097} = 166.5$$

9.2.2 Singular Value Analysis

Only two of the singular values ($\sigma 1$, $\sigma 2$) are of the same magnitude, but $\sigma 3$ is much smaller. The CN value suggests that only two output variables can be controlled effectively. If we eliminate one input variable and one output variable, the condition number, $\sigma 1/\sigma 2$, can be recalculated, as shown in Table 18.3.

8.2.2 Singular Value Analysis

Table 18.3 CN and λ for Different 2 × 2 Pairings, Example 18.7

Pairing Number	Controlled Variables	Manipulated Variables	CN	λ
1	y_1, y_2	u_1, u_2	184	39.0
2	y_1, y_2	u_1, u_3	72.0	0.552
3	y_1, y_2	u_2, u_3	133	0.558
4	y_1, y_3	u_2, u_1	1.51	0.640
5	y_1, y_3	u_1, u_3	69.4	0.640
6	y_1, y_3	u_2, u_3	139	1.463
7	y_2, y_3	u_2, u_1	1.45	0.634
8	y_2, y_3	u_1, u_3	338	3.25
9	y_2, y_3	u_2, u_3	67.9	0.714

Based on their having small condition numbers and acceptable values of λ, pairings 4 (*y*1-*u*2, *y*3-*u*1) and 7 (*y*2-*u*2, *y*3-*u*1) appear to be the most promising ones.

9.3 Decoulping control system

By adding additional controllers called *decouplers* to a conventional multiloop configuration, the design objective to reducing control loop interactions can be realized.

In principle, decoupling control schemes can reduce control loop interactions, and a set-point change for one controlled variable has little or no effect on the other controlled variables.

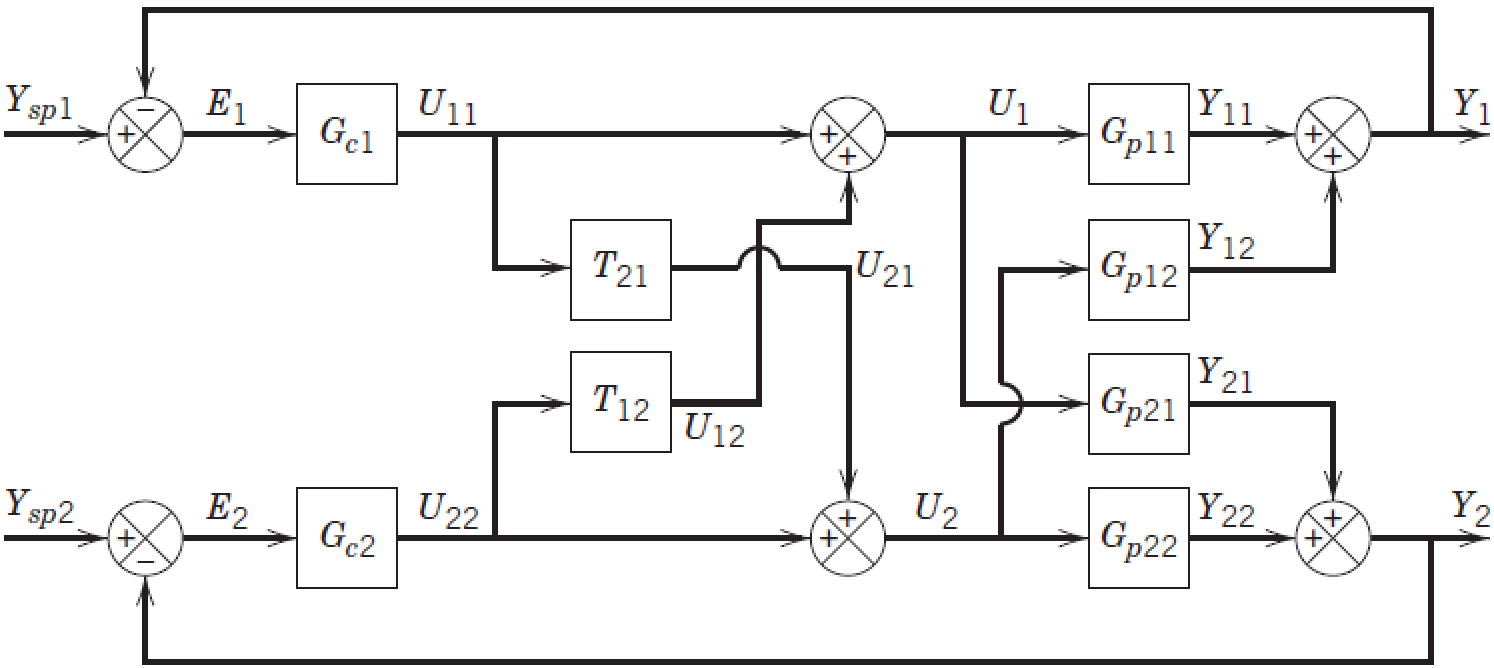
In practice, these benefits may not be fully realized due to imperfect process models.

9.3 Decoulping control system

- One type of decoupling control system for a 2 × 2 process and a 1-1/2-2 control configuration is shown in Fig. 9.9.
- Note that four controllers are used: two conventional feedback controllers, G_{c1} and G_{c2}, plus two decouplers, T_{12} and T_{21}.

9.3 Decoulping control system

- Decoulping control system (Decouplers T21, and T12)



9.3 Decoulping control system

To cancel the signal from controller G_{c1} to the second loop

$$U_{11}G_{p21} + U_{11}T_{21}G_{p22} = 0$$

Solving for the decoupler

$$T_{21} = -\frac{G_{p21}}{G_{p22}}$$

To cancel the signal from controller G_{c2} to the second loop

$$U_{11}G_{p21} + U_{11}T_{12}G_{p11} = 0$$

Solving for the decoupler

$$T_{12} = -\frac{G_{p12}}{G_{p11}}$$

9.3 Decoulping control system

- The ideal dynamic decouplers

$$T_{21} = -\frac{G_{p21}}{G_{p22}} \qquad T_{12} = -\frac{G_{p12}}{G_{p11}}$$

- *The static decoupler*

$$T_{21} = -\frac{K_{p21}}{K_{p22}} \quad \text{and} \quad T_{12} = -\frac{K_{p12}}{K_{p11}}$$

Exercise

Exercise 9.1

For a 2×2 process with the following plant transfer functions.

$$G = \begin{bmatrix} -\dfrac{7e^{-0.8s}}{4s+1} & \dfrac{2.2e^{-0.5s}}{3s+1} \\ \dfrac{5.1e^{-1.5s}}{8s+1} & \dfrac{0.4e^{-2s}}{6s+1} \end{bmatrix}$$

i. Determine the appropriate pairing
ii. For the proposed pairing check whether there is an inherent difficulty to control the process.
iii. Construct the appropriate control configuration, tune the controllers
iv. Conduct performance test for servo problem.
v. Design decouplers and conduct performance tests