Advanced Process Control

CBEg 6142

School of Chemical and Bio-Engineering
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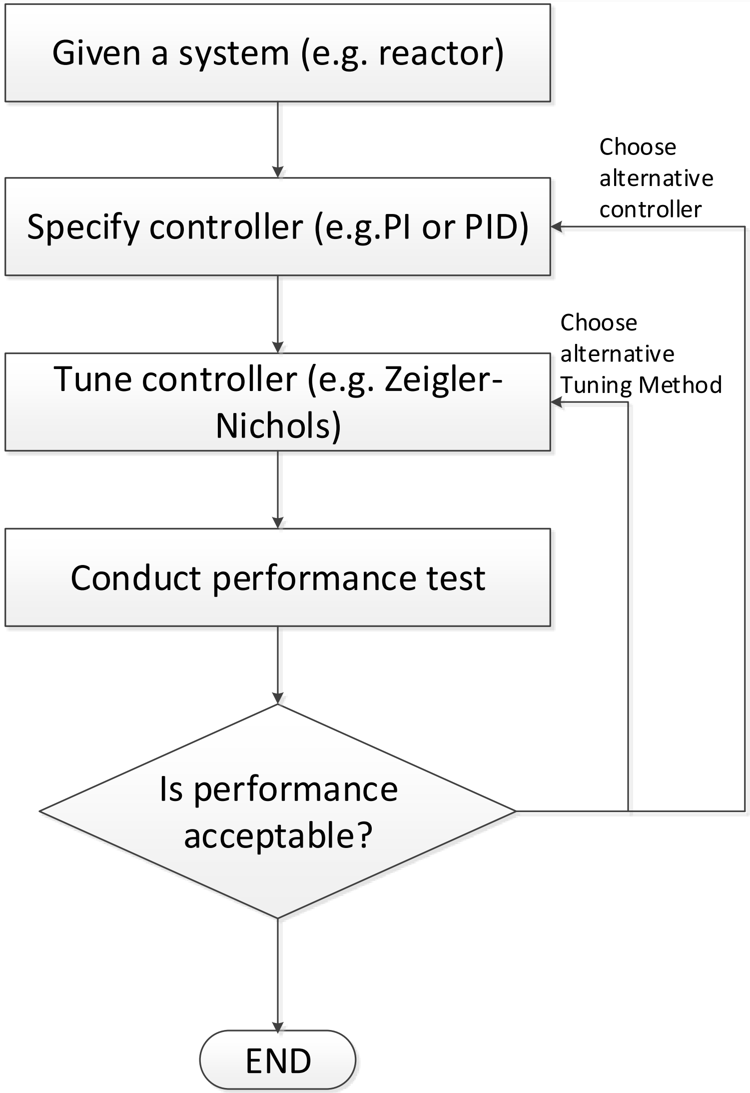
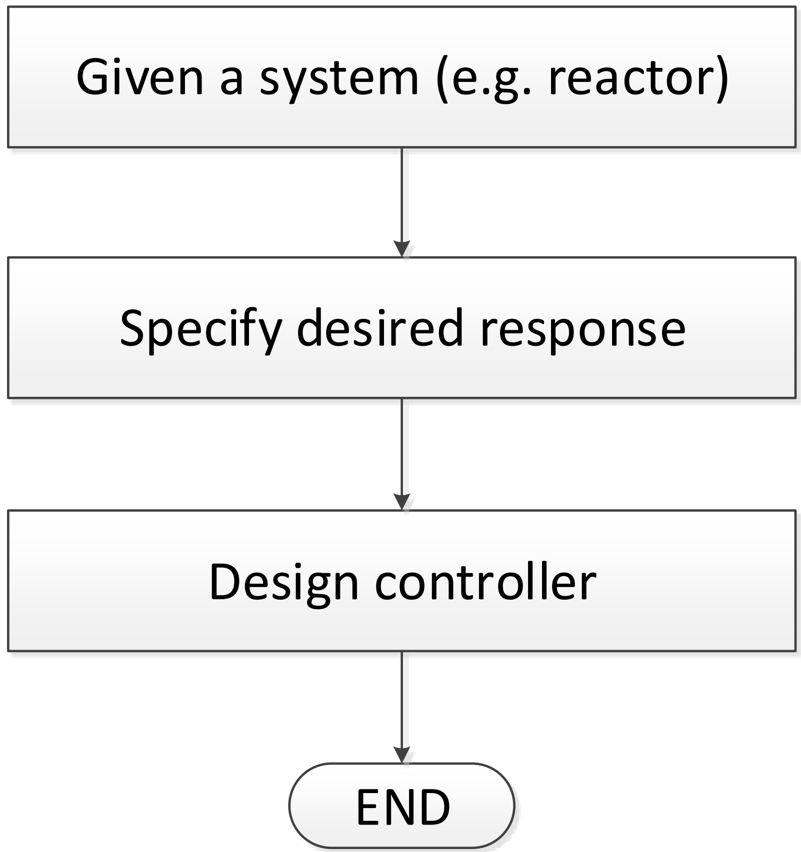
Chapter 6

Direct Synthesis Controller

Controller Design Classical Approach

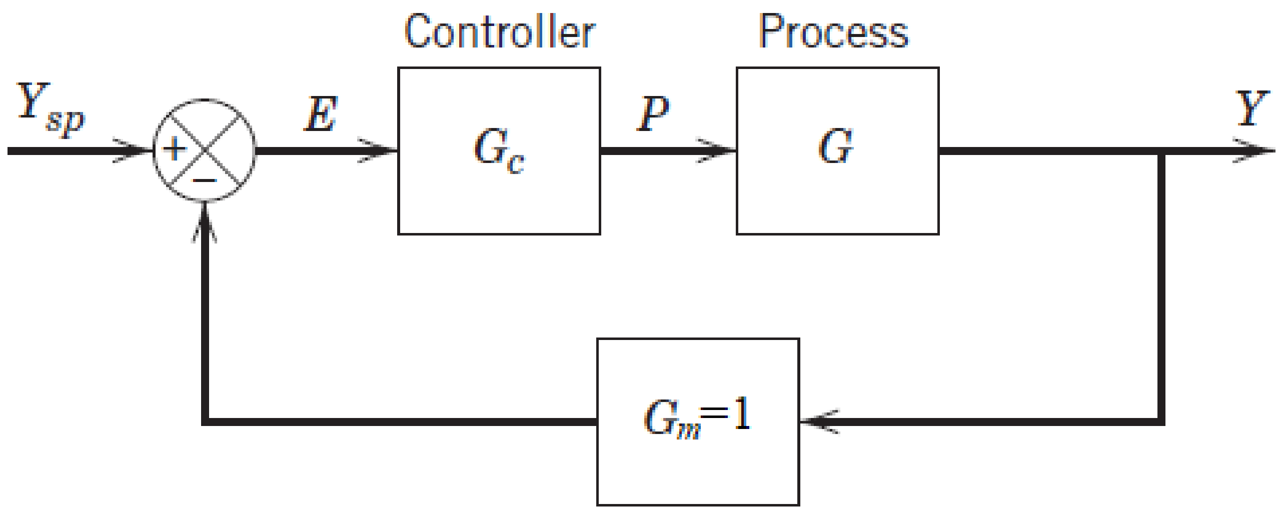


Direct Synthesis Controller



Direct Synthesis Controller

The feedback control loop for servo problem



The closed-loop equation for servo problem is given by

$$Y = \frac{G_c G}{1 + G_c G} Y_{sp} \qquad (6.1)$$

Direct Synthesis Controller

Assuming a desired response for y(s) to be:

$$Y = Q_d Y_{sp} \qquad (6.2)$$

Using (6.2) in (6.1)

$$Q_d = \frac{G_c G}{1 + G_c G} \qquad (6.3)$$

Rearranging (6.3) we get the design equation

$$G_c = \frac{1}{G}\frac{Q_d}{1 - Q_d} \qquad (6.4)$$

Direct Synthesis Controller

Example 6.1

The transfer function of a process is given below. The desired closed loop response should have a time constant of 5min and no offset is tolerated. Design a controller using the direct synthesis approach.

$$G = \frac{0.25}{8s + 1}$$

$$Q_d = \frac{1}{5s + 1}$$

No offset

Time constant

Direct Synthesis Controller

Solution

$$G_c = \frac{8s+1}{0.25}\frac{1/(5s+1)}{1-1/(5s+1)}$$

$$= \frac{8s+1}{0.25}\frac{1}{(5s+1)-1}$$

$$= 4\frac{8s+1}{5s}$$

$$= 6.4\left(1+\frac{1}{8s}\right)$$

This a PI controller with K_c=6.4 and τ_I= 8 min

Example 6.2

The transfer function of a process is given below. The desired closed loop response should have a transfer function given by Q_d. Design a controller using the direct synthesis approach. Use 1st order Taylor series expansion for the time delay.

$$G = \frac{0.2e^{-1.2s}}{3.6s + 1}$$

$$Q_d = \frac{e^{-1.2s}}{s + 1}$$

Direct Synthesis Controller

Solution

$$G_c = \frac{(3.6s+1)}{0.2}\frac{e^{-1.2s}/(s+1)}{1-e^{-1.2s}/(s+1)}$$

$$= \frac{(3.2s+1)}{0.2}\frac{1}{s+1-e^{-1.2s}}$$

$$= \frac{(3.2s+1)}{0.2}\frac{1}{s+1-(1-1.2s)}$$

$$G_c = \frac{(3.2s+1)}{0.44s} = \frac{3.2}{0.44}\left(1+\frac{1}{3.2s}\right) = 7.27\left(1+\frac{1}{3.2s}\right)$$

This a PI controller with K_c= 7.27 and τ_I= 3.2 min

Direct Synthesis Controller

Example 6.3

For process that are estimated by first order plus time delay (FOPTD) and second order plus time delay (SOPTD) models given below. Design controllers using the direct synthesis approach. Use 1st order Taylor series expansion for the time delay.

$$G = \frac{Ke^{-\theta s}}{\tau s + 1}$$

$$G = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Direct Synthesis Controller

Solution

$$Q_d = \frac{1e^{-\theta s}}{\tau_c s + 1}$$

$$G = \frac{Ke^{-\theta s}}{\tau s + 1}$$

$$G_c = \frac{\tau s + 1}{Ke^{-\theta s}} \frac{\dfrac{1e^{-\theta s}}{\tau_c s + 1}}{1 - \dfrac{1e^{-\theta s}}{\tau_c s + 1}} = \frac{\tau s + 1}{K} \frac{1}{\tau_c s + 1 - e^{-\theta s}}$$

Using first order Taylor's expansion $e^{-\theta s} = 1 - \theta s$

$$G_c = \frac{\tau s + 1}{K(\tau_c + \theta)s} = \frac{\tau}{K(\tau_c + \theta)}\left(1 + \frac{1}{\tau s}\right)$$

This is a PI controller with $K_c = \frac{\tau}{K(\tau_c + \theta)}$ and $\tau_I = \tau$

Direct Synthesis Controller

Solution

$$Q_d = \frac{1e^{-\theta s}}{\tau_c s + 1}$$

$$G = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Inserting and simplifying

$$G_c = \frac{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1}{K(\tau_c+\theta)s} = \frac{\tau_1 + \tau_2}{K(\tau_c+\theta)} + \frac{1}{K(\tau_c+\theta)s} + \frac{\tau_1\tau_2}{K(\tau_c+\theta)}s$$

$$G_c = \frac{\tau_1 + \tau_2}{K(\tau_c+\theta)}\left(1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1\tau_2}{\tau_1 + \tau_2}s\right)$$

This is a PID controller with $K_c = \frac{\tau_1+\tau_2}{K(\tau_c+\theta)}$ $\quad \tau_I = \tau_1 + \tau_2$ *and* $\tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2}$

Exercise 6.1

The transfer function of three processes are given below. The desired closed loop response should have no-offset . Fore each of the three processes design a controller using the direct synthesis approach. Determine, the time desired time constant by yourself. Use Skogestad's half rule whenever necessary.

i. $G = \frac{3.5(-0.5s+1)\, e^{-0.8s}}{(4.2s+1)(3.2s+1)}$

ii. $G = \frac{0.5e^{-1.4s}}{(4.2s+1)(1.7s+1)(0.8s+1)}$

iii. $G = \frac{0.5(-0.8s+1)e^{-1.4s}}{(4.2s+1)(1.7s+1)(0.8s+1)}$