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Addis Ababa Institute of Technology
School of Chemical and Bio-Engineering
CBEg 6142 Advanced Process Control

Assignment I Submission Date: ______________

1. The constant holdup homogenizer tank shown in **FIGURE Q1** is used to blend process liquids from streams 1 and 2. Both streams contain the same solution at different concentrations. The composition of the outlet stream should be maintained at 0.5 $\mathrm{mol/m^3}$. At steady state, the flow rates of stream 1 and 2 are 2.5 and 1.5 $\mathrm{m^3/min}$, respectively, and the volume of liquid in the tank is 5.6 $\mathrm{m^3}$. The composition of stream 2, c_{A2}, is constant and its value is 1.0 $\mathrm{mol/m^3}$. The difference in the densities of the inlet and outlet streams is negligible.

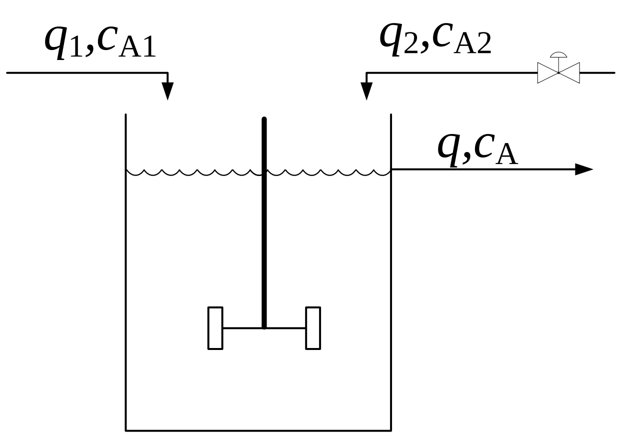

FIGURE Q1. Blending Tank

a. Develop the dynamic model of the tank in time domain.

[10 marks]

b. Develop the transfer function model and identify the transfer functions, their orders, gains and time constants.

[10 marks]

c. Determine the response, $c_A'(t)$, if a step change of 0.2 m^3/min is introduced in the inlet flow rate $q_1'(t)$.

[5 marks]

2. The transfer function two processes are given below. Determine the response of the process when a unit step change in input is introduced. Determine the time the process will take to reach 99% of its new steady state value? Sketch the responses of the processes.

a. $$G(s) = \frac{2.5e^{-1.2s}}{3s+1}$$

b. $$G(s) = \frac{2e^{-2.5s}}{(2s+1)(3s+1)}$$

[6 marks]

3. The transfer functions of three different processes are given below. Determine whether the processes represented by the transfer functions are stable or not indicating the poles causing the unstability when it is unstable. Identify whether the response is oscillatory or not.

a. $$G(s) = \frac{2(s+1)}{s(s+2)(s+3)^2}$$

b. $$G(s) = \frac{2}{s^2-4}$$

c. $$G(s) = \frac{2(s-1)}{(s+2)(s^2+8s+20)}$$

[9 marks]

4. The transfer functions of four different processes are given below. The steady state value of the output is 20. Determine, when possible, the new steady state value, for a step change of size 5 in input. If it is impossible to determine the new steady state value state the reason.

a. $$G(s) = \frac{2(s+1)}{(s+2)(s+3)^2}$$

b. $$G(s) = \frac{2(s-1)}{(s+2)(s^2+8s+20)}$$

c. $$G(s) = \frac{2(s-1)}{(s-2)(s+1)^2}$$

[5 marks]

5. The transfer function of a second order process is given below. For a unit step change in input, determine:

a. Percent overshoot

b. Decay ratio

c. Period of oscillation

d. Maximum value of $y(t)$

e. Final Value of $y(t)$

$$G(s) = \frac{24}{6s^2 + 1.2s + 3}$$

and sketch the response.

[5 marks]

Table A1: Laplace transforms for various time-domain functions

1	$f(t)$	F(s)
2	$\delta(t)$ $(unit\ impulse)$	1
3	$\begin{cases} 0 & t<0 \\ a & t\geq 0 \end{cases}$ $(step)$	$\frac{a}{s}$
4	t^n	$\frac{n!}{s^{n+1}}$
5	e^{-at} $(Exponential)$	$\frac{1}{s+a}$
6	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
7	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
8	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
9	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
10	$e^{-at}f(t)$	$F(s+a)$
11	$\frac{df}{dt}$	$sF(s)-f(0)$
12	$\frac{d^n f}{dt^n}$	$s^n F(s)-s^{n-1}f(0)-s^{n-2}f^{(1)}(0)-\ldots-sf^{(n-2)}-f^{(n-1)}(0)$
13	$\int_0^t f(t)dt$	$\frac{F(s)}{s}$
14	$\lim_{t\to\infty} f(t)$ [final value theorem]	$\lim_{s\to 0}[sF(s)]$

Table A2: Inverse Laplace transforms for various Laplace-domain functions

	F(s)	*f*(t)
1	1	$\delta(t)$ (unit impulse)
2	$\frac{1}{s}$	*S(t)* (unit step)
3	$\frac{1}{s^2}$	*t* (ramp)
4	$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$
5	$\frac{1}{s+a}$	e^{-at}
6	$\frac{1}{(s+a_1)(s+a_2)}$	$\frac{1}{a_1-a_2}\left(e^{-a_2 t}-e^{-a_1 t}\right)$
7	$\frac{s+a_3}{(s+a_1)(s+a_2)}$	$\frac{a_3-a_1}{a_2-a_1}e^{-a_1 t}+\frac{a_3-a_2}{a_1-a_2}e^{-a_2 t}$
8	$\frac{1}{(s+a)^n}$	$\frac{t^{n-1}e^{-at}}{(n-1)!}$ $(n>0)$
9	$\frac{1}{\tau s+1}$	$\frac{1}{\tau}e^{-t/\tau}$
10	$\frac{1}{s(\tau s+1)}$	$1-e^{-t/\tau}$
11	$\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$
12	$\frac{1}{(\tau s+1)^n}$	$\frac{1}{\tau^n(n-1)!}t^{n-1}e^{-t/\tau}$
13	$\frac{1}{s(\tau^2 s^2+2\zeta\tau s+1)}$	$1-e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]$