Chapter 3

Inventory Management

Introduction to Inventory Management

- Inventory is the stock of any item or resource held to meet future demand and can include: raw materials, finished products, component parts , and work-in-process.
- **Inventory management** is the planning and controlling of inventories in order to meet the competitive priorities of the organization.
  - Effective inventory management is essential for realizing the full potential of any value chain.
- Inventory management requires information about expected demands, amounts on hand and amounts on order for every item stocked at all locations.
  - The appropriate timing and size of the reorder quantities must also be determined.

Introduction..

- **Types of Inventory:**
  - **Cycle Inventory:** The portion of total inventory that varies directly with lot size (*Q*).

    Average cycle inventory = ?

    - **Lot Sizing**: The determination of how frequently and in what quantity to order inventory.
  - **Safety Stock Inventory:** Surplus inventory that a company holds to protect against uncertainties in demand, lead time and supply changes.
  - **Anticipation Inventory:** is used to absorb uneven rates of demand or supply, which businesses often face.

Introduction.….

- **Pipeline Inventory:** Inventory moving from point to point in the materials flow system.

$$\text{Pipeline inventory} = D_L = dL$$

D_L *is the average demand for the item per period (d) times the number of periods in the item's lead time (L).*

- Function of Inventory:

1. To "decouple" or separate various parts of the production process, i.e. to maintain independence of operations.

2. To meet unexpected demand & to provide high levels of customer service.

Introduction…..

3. To smooth production requirements by meeting seasonal or cyclical variations in demand.
4. To protect against stock-outs.
5. To provide a safeguard for variation in raw material delivery time.
6. To provide a stock of goods that will provide a “selection” for customers.
7. To take advantage of economic purchase-order size.

Introduction…..

- Cost of Inventory:
  - Holding (or carrying) costs
    - Costs for storage, handling, insurance, etc
  - Setup (or production change) costs
    - Costs to prepare a machine or process for manufacturing an order, eg. arranging specific equipment setups, etc
  - Ordering costs (costs of replenishing inventory)
    - Costs of placing an order and receiving goods
  - Shortage costs
    - Costs incurred when demand exceeds supply.

Application of Inventory Management

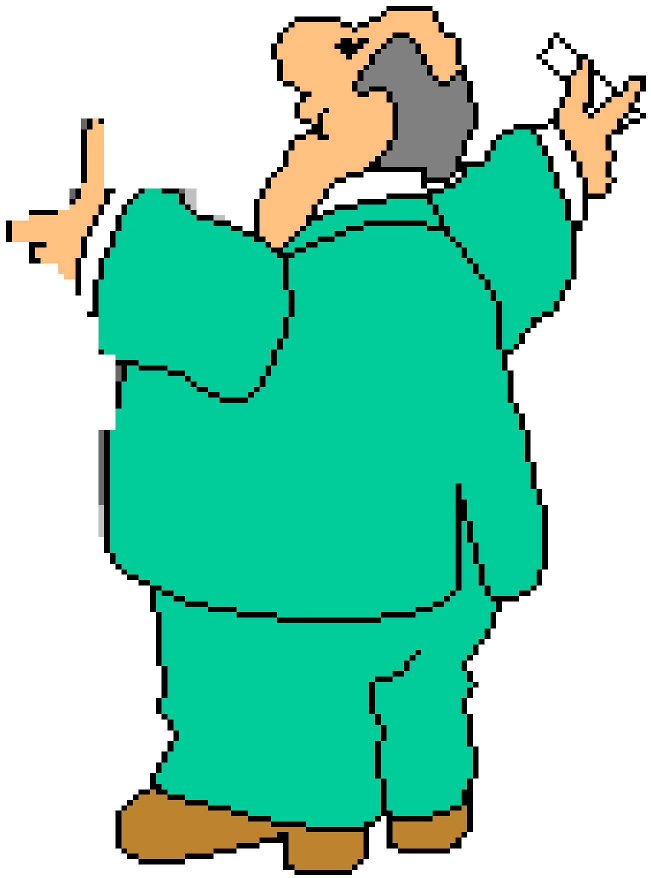
proper management of inventory is important for;

- ➢Manufacturing industry
- ➢Service industry
- ➢Shops
- ➢Pharmaceutical shops
- ➢Etc.

Inventory Models

- **Inventory Decision Questions**

How Much?
When?



Types of Inventory Models

- Deterministic Inventory Model:
  - When demand and lead time for an item are constant.
- Probabilistic Inventory Model:
  - When demand and lead time for an item are not constant.

Independent and Dependent Demand

- **Independent demand -** the demand for item is independent of the demand for any other item in inventory.
- **Dependent demand -** the demand for item is dependent upon the demand for some other item in the inventory.

Inventory Models for Independent Demand

Need to determine when and how much to order

- Basic economic order quantity
- Production order quantity
- Quantity discount model

- ***Economic Order Quantity Model (EOQ)***
  - ✓Fixed Order Quantity Models.
- ***Economic Order Quantity Model Assumptions***
  - ✓Demand for the product is known with certainty, is constant and uniform throughout the period.
  - ✓Lead time (time from ordering to receipt) is known and constant.
  - ✓Price per unit of product is constant (no quantity discounts).
  - ✓Inventory holding cost is based on average inventory.

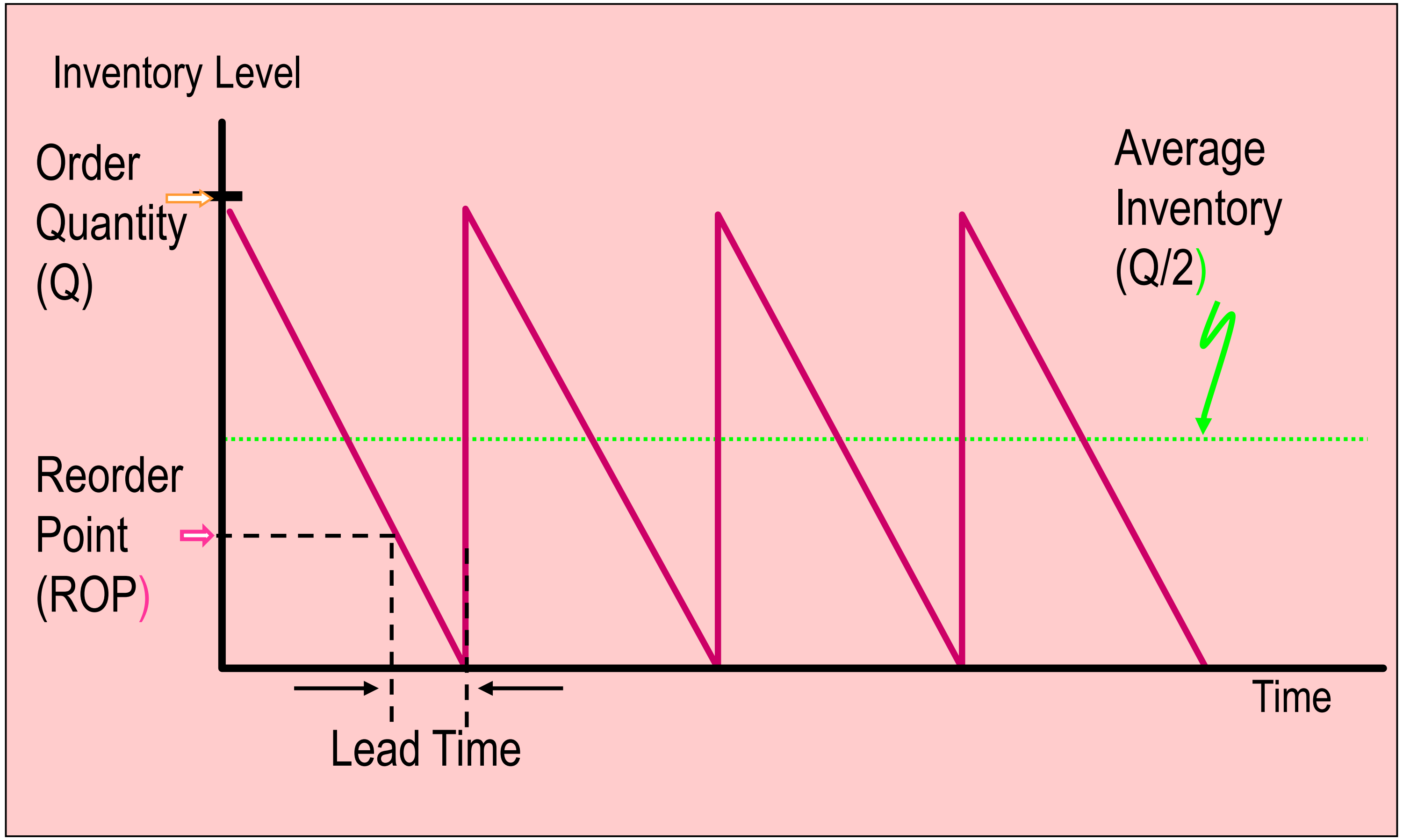
EOQ cont…

- Ordering or setup costs are constant.
- All demands for the product will be satisfied (no back orders are allowed).
- No stockouts (shortages) are allowed.
- The order quantity is received all at once. (Instantaneous receipt of material in a single lot).

The goal is to calculate the order quantitiy that minimizes total cost

EOQ cont…

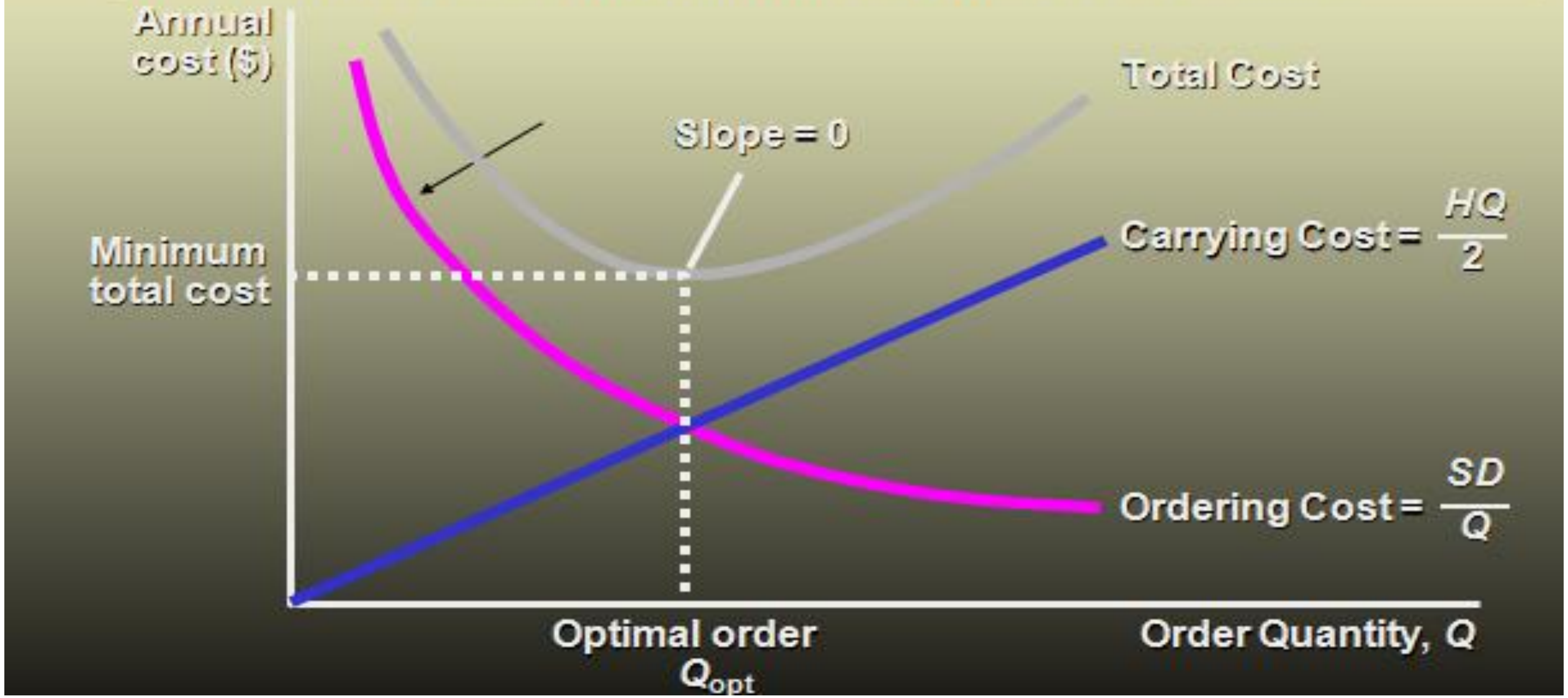
- ***EOQ Model***



EOQ cont…

- *EOQ Cost Model: How Much to Order?*

By adding the holding and ordering costs together, we determine the total cost curve, which in turn is used to find the optimal order quantity that minimizes total costs



EOQ cont…

Basic Fixed-Order Quantity (EOQ) Model Formula

Total Annual Cost = Annual Purchase Cost + Annual Ordering Cost + Annual Holding Cost

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

TC=Total annual cost
D =Annual demand
C =Cost per unit
Q =Order quantity
S =Cost of placing an order or setup cost
R =Reorder point
L =Lead time
H=Annual holding and storage cost per unit of inventory

EOQ cont…

$$TC = \frac{SD}{Q} + \frac{HQ}{2}$$

$$\frac{\partial TC}{\partial Q} = \frac{SD}{Q^2} + \frac{H}{2}$$

$$0 = \frac{SD}{Q^2} + \frac{H}{2}$$

$$EOQ = Q\ opt. = \sqrt{\frac{2SD}{H}}$$

EOQ…

How much to order?:

$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(\text{Annual Demand})(\text{Order or Setup Cost})}{\text{Annual Holding Cost}}}$$

When to order?

We also need a reorder point to tell us when to place an order

$$\text{Reorder point, } R = \bar{d}\,L$$

$\bar{d}$ = average daily demand (constant)

L = Lead time (constant)

EOQ cont…

$$\text{Optimal Order Quantity} = Q^* = \sqrt{\frac{2 \times D \times S}{H}}$$

$$\text{Expected Number of Orders} = N = \frac{D}{Q^*}$$

$$\text{Expected Time Between Orders} = T = \frac{\text{Working Days / Year}}{N}$$

$$d = \frac{D}{\text{Working Days / Year}}$$

$$ROP = d \times L$$

EOQ cont…

Given the information below, what are the EOQ and reorder point?

Annual Demand = 1,000 units

Days per year considered in average daily demand = 365

Cost to place an order = $10

Holding cost per unit per year = $2.50

Lead time = 7 days

Cost per unit = $15

EOQ cont…

$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1{,}000)(10)}{2.50}} = 89.443 \text{ units or } 90 \text{ units}$$

$$\bar{d} = \frac{1{,}000 \text{ units / year}}{365 \text{ days / year}} = 2.74 \text{ units / day}$$

$$\text{Reorder point, } R = \bar{d}\,L = 2.74 \text{units / day (7days)} = 19.18 \text{ or } 20 \text{ units}$$

In summary, you place an optimal order of 90 units. In the course of using the units to meet demand, when you only have 20 units left, place the next order of 90 units.

EOQ cont…

Orders per year $= D/Q_{opt}$

$= 1000/90$

$= 11$ orders/year

Order cycle time $= 365/(D/Qopt)$

$= 365/11$

$= 33.1$days

$$TC_{min} = \frac{SD}{Q} + \frac{HQ}{2}$$

$$TC_{min} = \frac{(10)(1,000)}{90} + \frac{(2,5)(90)}{2}$$

$TC_{min} = \$\ 111 + \$111 = 22\ \$$

EOQ…

- Determine the economic order quantity and the reorder point given the following…

  - Annual Demand = 10,000 units
  - Days per year considered in average daily demand = 365
  - Cost to place an order = $10
  - Holding cost per unit per year = 10% of cost per unit
  - Lead time = 10 days
  - Cost per unit = $15

EOQ…

$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(10{,}000)(10)}{1.50}} = 365.148 \text{ units, or } 366 \text{ units}$$

$$\bar{d} = \frac{10{,}000 \text{ units / year}}{365 \text{ days / year}} = 27.397 \text{ units / day}$$

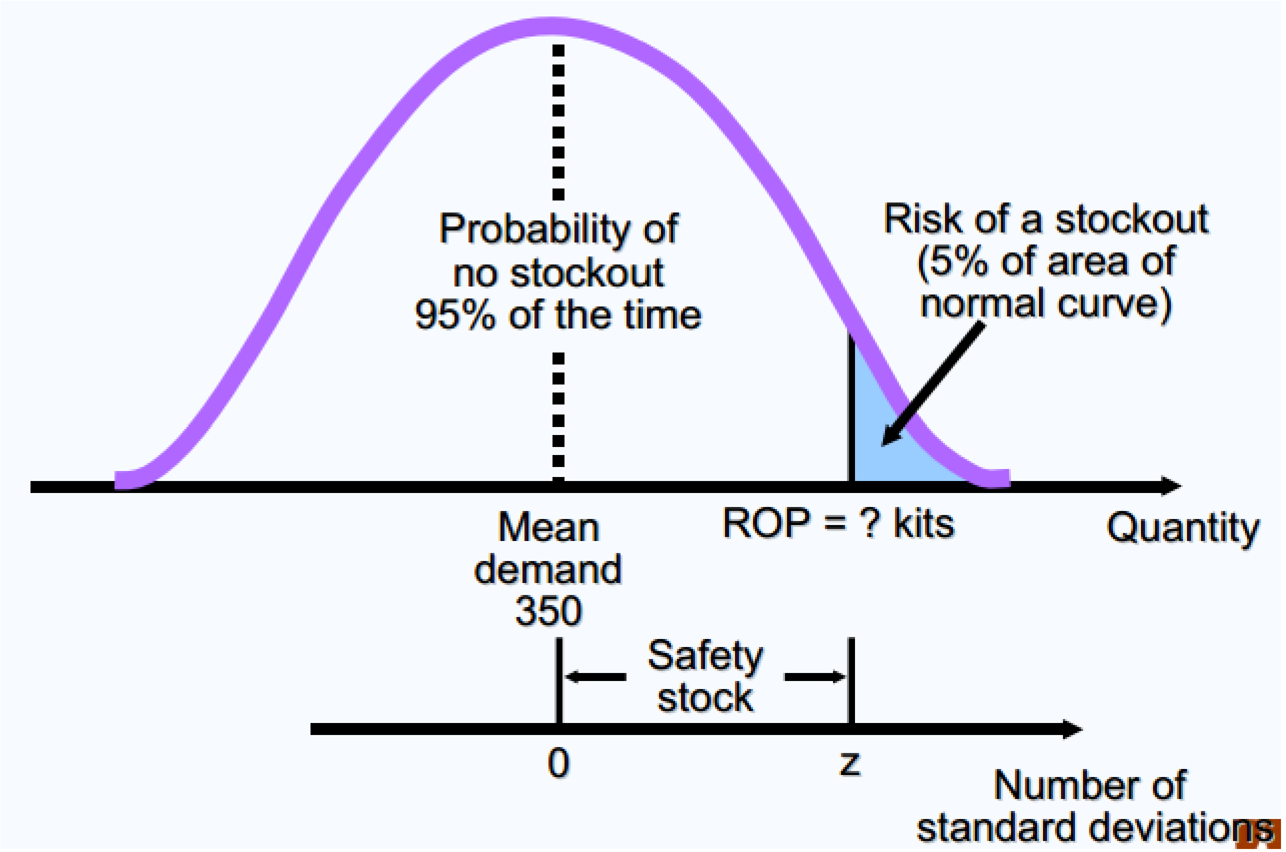
$$R = \bar{d}\,L = 27.397 \text{ units / day } (10 \text{ days}) = 273.97 \text{ or } 274 \text{ units}$$

Place an order for 366 units. When in the course of using the inventory you are left with only 274 units, place the next order of 366 units.

Probabilistic Models and Safety Stock

- Used when demand is not constant or certain
- Use safety stock to achieve a desired service level and avoid stock outs
- ROP = d x L + ss
- **Annual stock out costs** = the sum of the units short x the probability x the stock out cost/unit
- x the number of orders per year

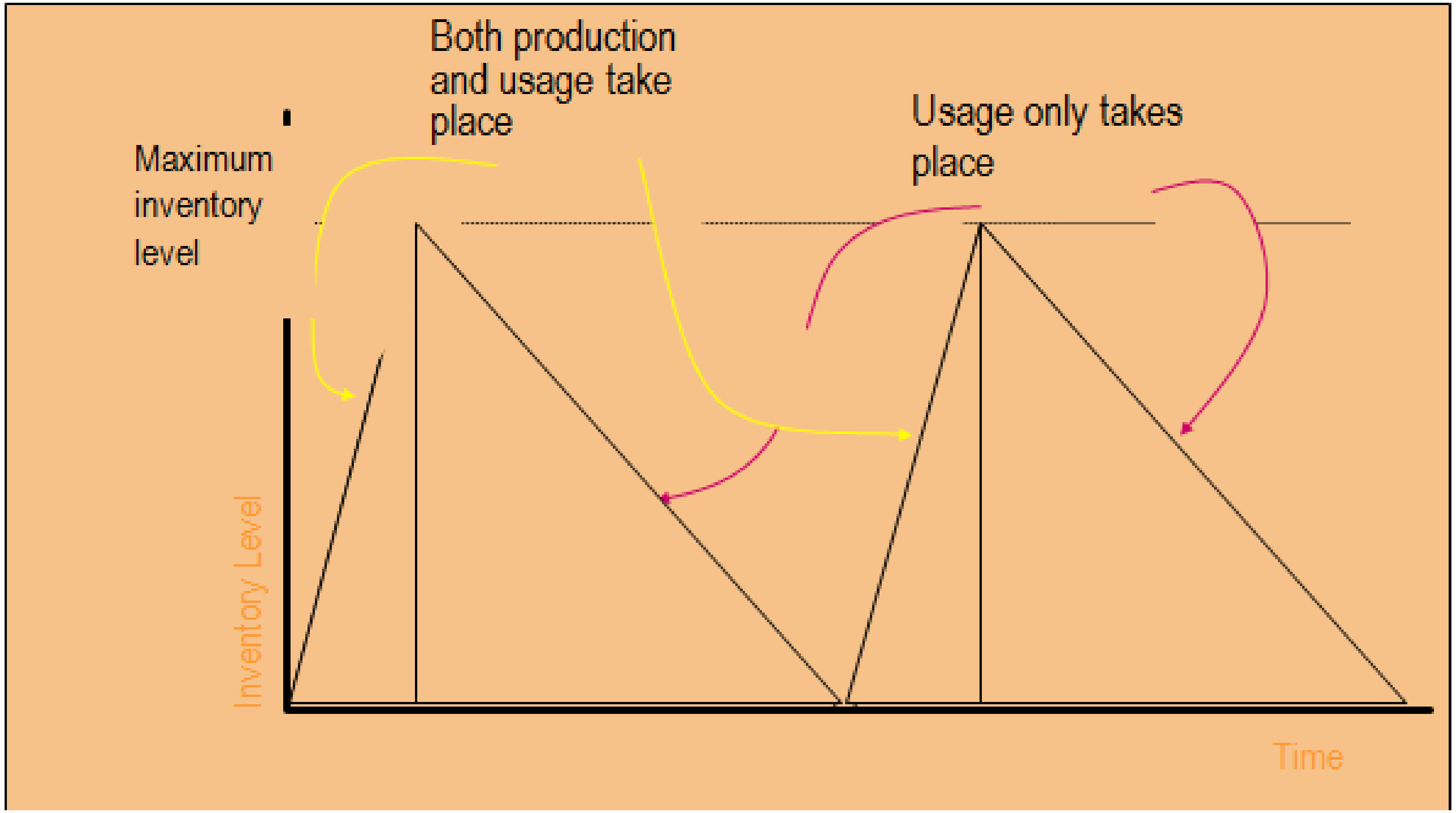
Probabilistic Demand



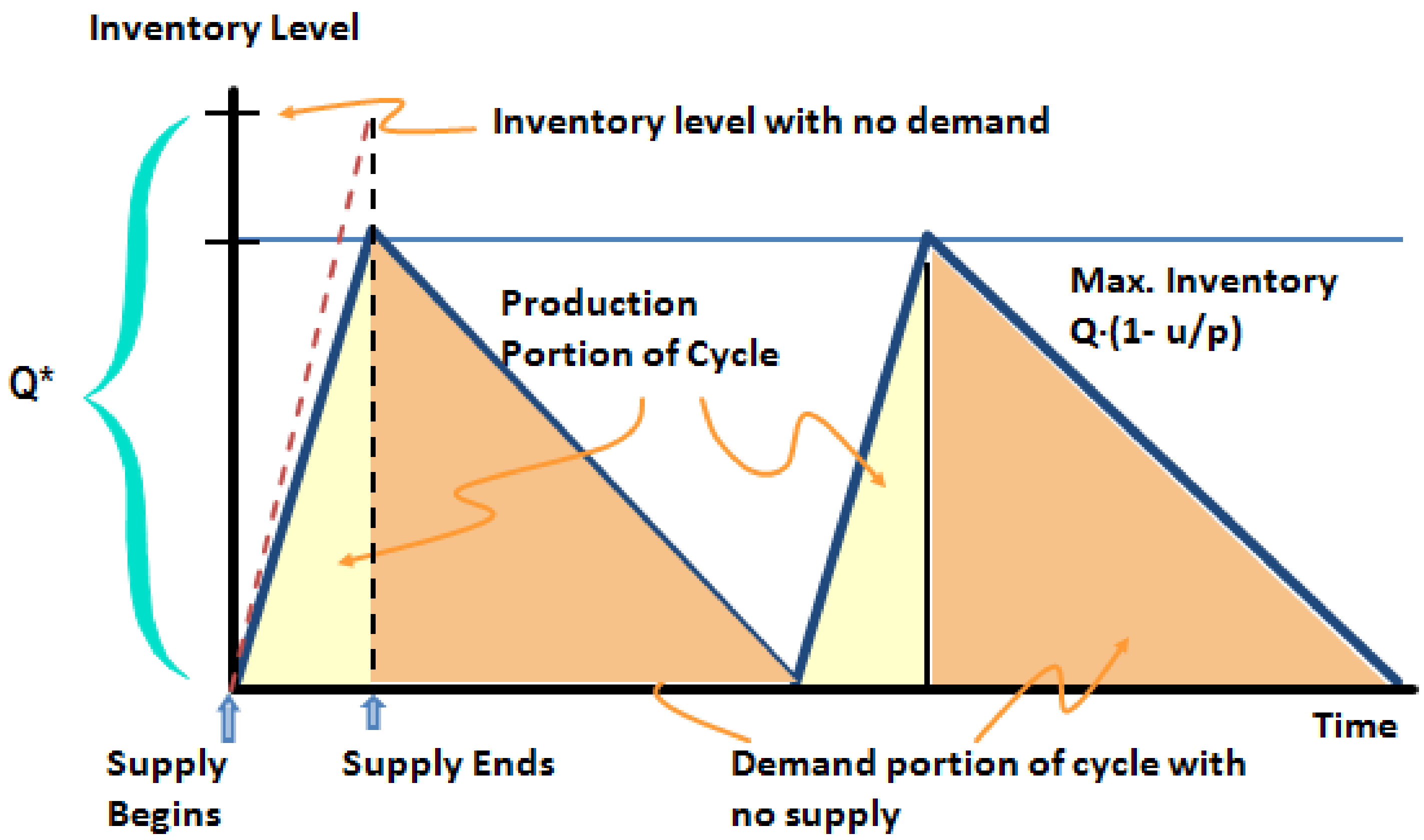
Production Order Quantity (Economic Lot Size) Model

- Production is done in batches or lots.
- Capacity to produce a part exceeds that part's usage or demand rate.
- Allows partial receipt of material.
  - ➢ Other EOQ assumptions apply
- Suited for production environment.
  - ➢ Material produced, used immediately
  - ➢ Provides production lot size
- Lower holding cost than EOQ model.
- Answers how much to order and when to order.

Production order quantity model cont…



Production order quantity model cont…



Production order quantity model cont…

POQ Model Equations

Production Order Quantity $= Q^*_p = \sqrt{\dfrac{2*D*S}{H*\left(1 - \dfrac{u}{p}\right)}}$

Maximum inventory level $= Q^*\left(1 - \dfrac{u}{p}\right)$

Setup Cost $= \dfrac{D}{Q} * S$

Holding Cost $= 1/2 * H * Q \left(1 - \dfrac{u}{p}\right)$

D = Demand per year

S = Setup cost

H = Holding cost

d = Demand per day

p = Production per day

$$Q_o = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}}$$

Production order quantity model cont…

- Example:

H = \$0.75 per yard S = \$150 D = 10,000 yards

u = 10,000/311 = 32.2 yards per day p = 150 yards per day

$$POQ_{opt} = \sqrt{\frac{2SD}{H\left(1 - \frac{u}{p}\right)}} = \sqrt{\frac{2(150)(10{,}000)}{0.75\left(1 - \frac{32.2}{150}\right)}} = 2{,}256.8 \text{ yards}$$

$$TC = \frac{SD}{Q} + \frac{HQ}{2}\left(1 - \frac{u}{p}\right) = \$1{,}329$$

$$\text{Production run} = \frac{Q}{p} = \frac{2{,}256.8}{150} = 15.05 \text{ days per order}$$

Production order quantity model cont…

- Example cont..

H = \$0.75 per yard S = \$150 D = 10,000 yards

u = 10,000/311 = 32.2 yards per day p = 150 yards per day

$$\text{Number of production runs} = \frac{D}{Q} = \frac{10{,}000}{2{,}256.8} = 4.43 \text{ runs/year}$$

$$\text{Maximum inventory level} = Q\left(1 - \frac{u}{p}\right) = 2{,}256.8\left(1 - \frac{32.2}{150}\right)$$

$$= 1{,}772 \text{ yards}$$

$$\text{Production run} = \frac{Q}{p} = \frac{2{,}256.8}{150} = 15.05 \text{ days per order}$$

Quantity Discount Model

- Answers how much to order & when to order.
- Allows quantity discounts.
  - Price per unit decreases as order quantity increases.
  - Other EOQ assumptions apply.
- Trade-off is between lower price & increased holding cost.

Total cost with purchasing cost

$$TC = \frac{SD}{Q} + \frac{iCQ}{2} + PD$$

Where P: Unit Price

Quantity Discount model cont…

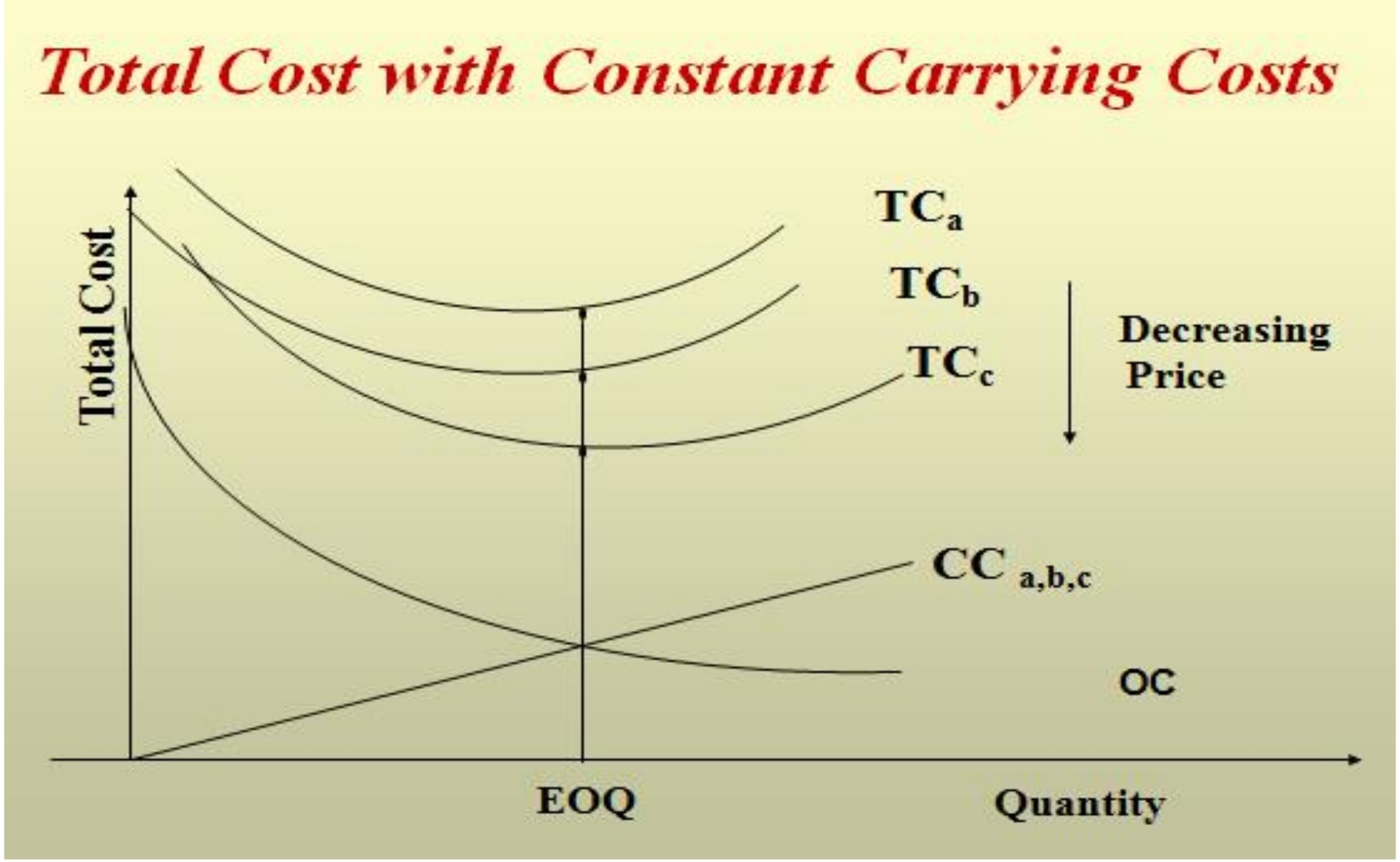
- Based on the same assumptions as the EOQ model, the price-break model has a similar Q_{opt} formula:

$$Q_{OPT} = \sqrt{\frac{2DS}{iC}} = \sqrt{\frac{2(\text{Annual Demand})(\text{Order or Setup Cost})}{\text{Annual Holding Cost}}}$$

i = percentage of unit cost attributed to carrying inventory
C = cost per unit

Since "C" changes for each price-break, the formula above will have to be used with each price-break cost value

Quantity Discount model cont…



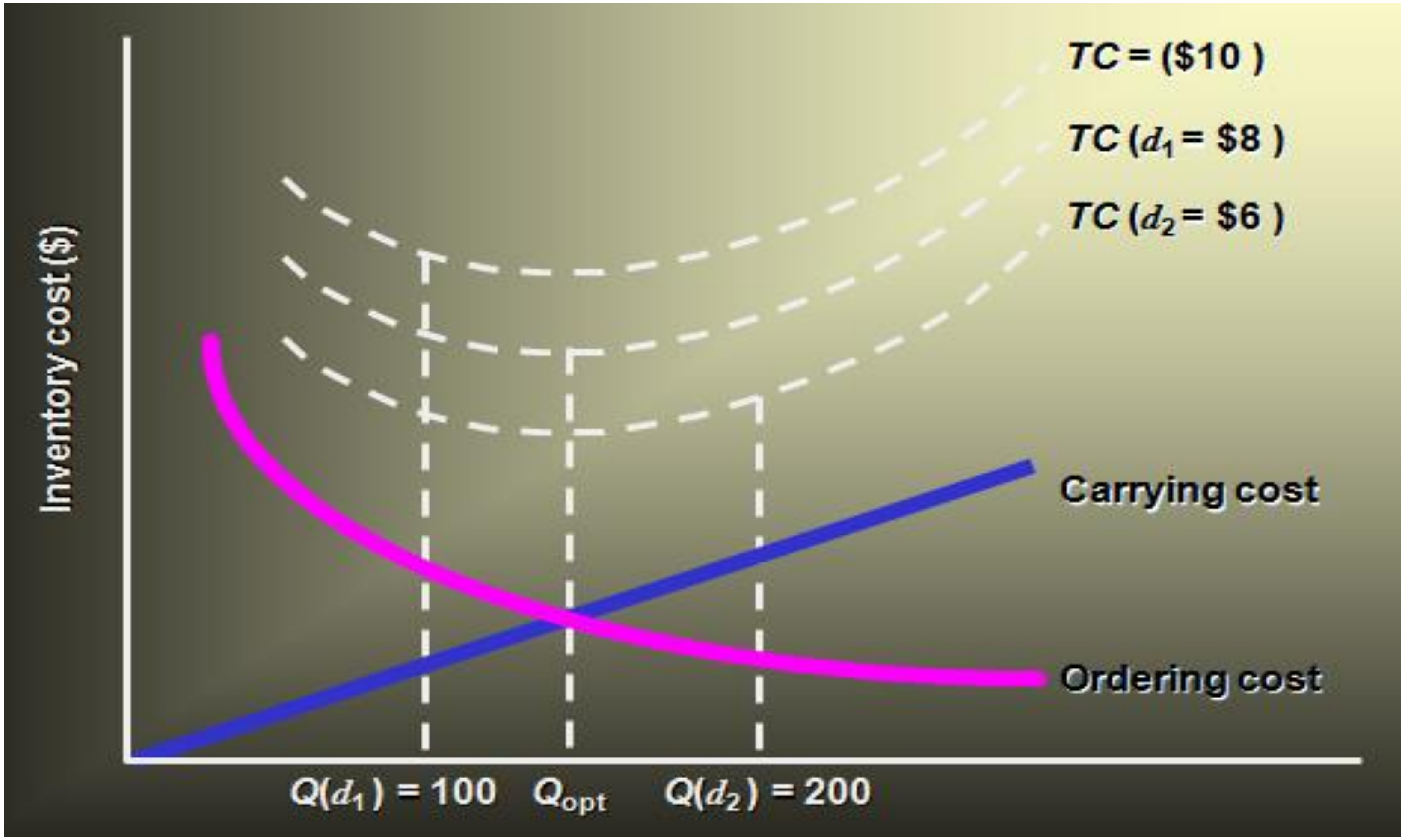
Quantity Discount model cont…

- Examples:

ORDER SIZE	PRICE
0 - 99	\$10
100 - 199	8 (d_1)
200+	6 (d_2)

- For this problem holding cost is given as a constant value, not as a percentage of price, so the optimal order quantity is the same for each of the price ranges. (see the figure)

Quantity Discount model cont…



Quantity Discount model cont…

- Example:

QUANTITY	PRICE
1 - 49	$1,400
50 - 89	1,100
90+	900

$S =$ $2,500
$H =$ $190 per computer
$D =$ 200

$$Q_{opt} = \sqrt{\frac{2SD}{H}} = \sqrt{\frac{2(2500)(200)}{190}} = 72.5 \text{ PCs}$$

For $Q = 72.5$

$$TC = \frac{SD}{Q_{opt}} + \frac{H\,Q_{opt}}{2} + PD = \$233{,}784$$

For $Q = 90$

$$TC = \frac{SD}{Q} + \frac{H\,Q}{2} + PD = \$194{,}105$$

Quantity Discount model cont…

- Example:
- A company has a chance to reduce their inventory ordering costs by placing larger quantity orders using the price-break order quantity schedule below. What should their optimal order quantity be if this company purchases this single inventory item with an e-mail ordering cost of $4, a carrying cost rate of 2% of the inventory cost of the item, and an annual demand of 10,000 units?

Order Quantity(units)	Price/unit($)
0 to 2,499	$1.20
2,500 to 3,999	1.00
4,000 or more	.98

Quantity Discount model cont…

- solution:

Annual Demand (D)= 10,000 units

Cost to place an order (S)= $4

Carrying cost % of total cost (i)= 2%

Cost per unit (C) = $1.20, $1.00, $0.98

Next, determine if the computed Q_{opt} values are feasible or not

Interval from 4000 & more, the Q_{opt} value is not feasible

$$Q_{OPT} = \sqrt{\frac{2DS}{iC}} = \sqrt{\frac{2(10,000)(4)}{0.02(0.98)}} = 2,020 \text{ units}$$

Interval from 2500-3999, the Q_{opt} value is not feasible

$$Q_{OPT} = \sqrt{\frac{2DS}{iC}} = \sqrt{\frac{2(10,000)(4)}{0.02(1.00)}} = 2,000 \text{ units}$$

Interval from 0 to 2499, the Q_{opt} value is feasible

$$Q_{OPT} = \sqrt{\frac{2DS}{iC}} = \sqrt{\frac{2(10,000)(4)}{0.02(1.20)}} = 1,826 \text{ units}$$

Quantity Discount model cont…

Next, we plug the true Q_{opt} values into the total cost annual cost function to determine the total cost under each price-break

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}iC$$

TC(0-2499)=(10000*1.20)+(10000/1826)*4+(1826/2)(0.02*1.20)
= $12,043.82
TC(2500-3999)= $10,041
TC(4000&more)=$9,949.20

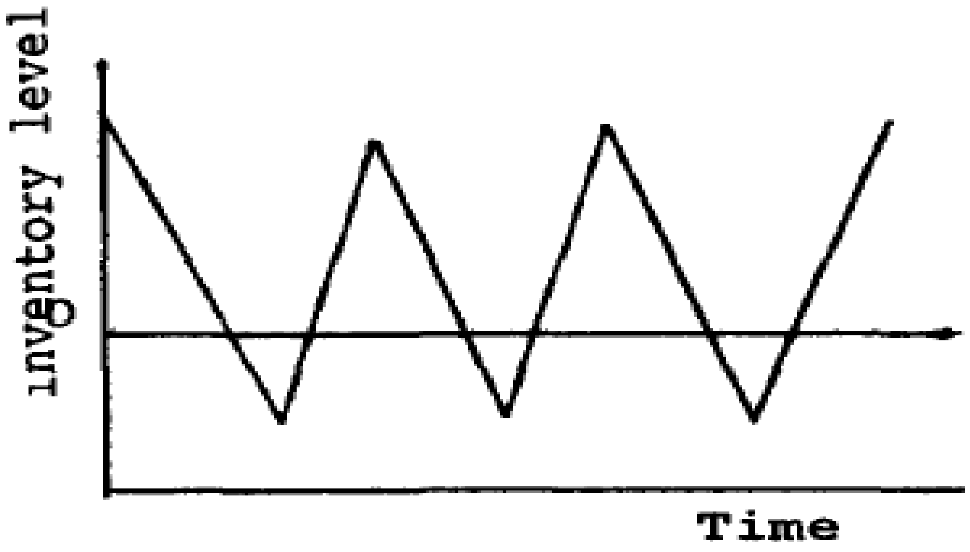
Finally, we select the least costly Q_{opt}, which in this problem occurs in the 4000 & more interval. In summary, our optimal order quantity is 4000 units

Quantity Discount model cont…

- Example 2.
- The maintenance department of a large hospital uses about 816 cases of liquid cleanser annually. Ordering costs are $12, carrying costs are $4 per case per year, and the new price schedule indicates that orders of less than 50 cases will cost $20 per case, 50 to 79 cases will cost $18 per case, 80 to 99 cases will cost $17 per case, and larger orders will cost $16 per case. Determine the optimal order quantity and the total cost.

Finite Input Rate Backlogging Allowed

- The basic deterministic single item model with static demand.

Fi  d

Finite input…

- **Optimal order quantity(Q*) for this model is calculated by :**

$$Q^* = \sqrt{\frac{2AD}{ic(1-\frac{D}{P})} - \frac{D^2}{ic(ic+r)}} * \sqrt{\frac{ic+r}{r}}$$

Where,

A= Cost of placing an order

D= Demand rate in units per year

i = Annual inventory carrying cost rate

c = unit variable cost

r = shortage cost per unit per year

P= production rate in units per year

Infinite Input Rate Backlogging Allowed

- **The basic deterministic single item model with static demand.**

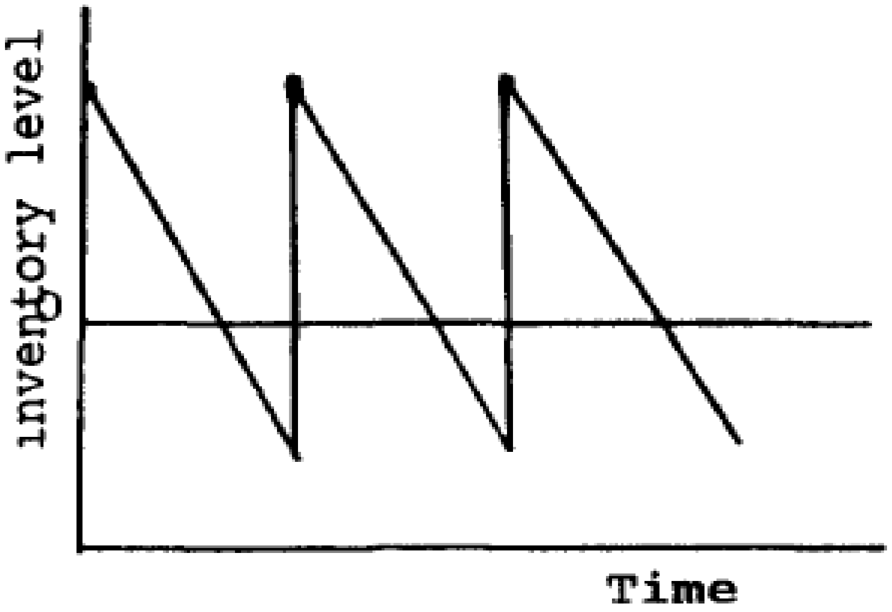


Fig. Infinite input rate backlogging allowed

Infinite Input Rate…

- Optimal order quantity(Q*) for this model is calculated by:

$$Q^* = \sqrt{\frac{2AD}{rc} - \frac{\pi D^2}{ic(ic+r)}} \;*\; \sqrt{\frac{ic+r}{r}}$$

- Where,

π = shortage cost per unit, independent of period of short

- Reading Assignment:

❑ Finite and infinite input rates with no backlogs

2. Stochastic Inventory Model

- Demand and lead time are not constant.
- Answer how much & when to order.
- It assumes that the demand over a period of time is normally distributed with a mean and a standard deviation.
- It considers only the probability of running out of stock.

2.1 Single Period Inventory Models

- Single period refers to the situation where the inventory will only be demanded in one time duration, and cannot be transferred to the next time duration.
  - Newspaper selling is such an example. The newspaper ordered for today will not be sold tomorrow. Fashion selling is another example. Spring-summer designs will not sell during the autumn-winter season.

Single Period…

- Optimal order quantity for a single-period inventory model :
  - The increment analysis addresses the how-much-to-order question by comparing the cost or loss of ordering one additional unit with the cost or loss of not ordering one additional unit.
  - C_o= cost per unit of *overestimating demand*
    - This cost represents the loss of ordering one additional unit which will not sell.
  - C_u = cost per unit of *underestimating demand*
    - This cost represents the loss of not ordering one additional unit which could have been sold.

Single Period…

- Suppose that the probability of the demand of the inventory items being more than a certain level y is $P(D>y)$, and that the probability of the demand of the inventory items being less than or equal to this level y is $P(D \leq y)$. Then, the *expected loss (EL) will be either of the following:*
- For over estimation: $EL(y+1) = C_o \, P(D \leq y)$
- For underestimation: $EL(y) = C_u \, P(D>y)$

Single Period… cont…

- From the study of Probability, it is known that

$$P(D>y^*) = 1 - P(D \leq y^*)$$

$$P(D \leq y^*) = C_u/(C_u + C_o)$$

- The above expression provides the general condition for the optimal order quantity y^* in the single-period inventory model. The determination of y^* depends on the probability distribution.

Single Period…

- Ex: (uniform probability distribution)
- Emma's Shoe Shop is to order some new design men's shoes for the next spring-summer season. The shoes cost £40 per pair and retail £60 per pair. If there are still shoes not sold by the end of July, they will be put on clearance sale in August at the price of £30 per pair. It is expected that all the remaining shoes can be sold during the sale.
- For the size 10D shoes, it is found that the demand can be described by the uniform probability distribution, shown in Figure below . The demand range is between 350 and 650 pairs, with average, or expected, demand of 500 pairs of shoes.

Single Period…

- Determine the order quantity.

Solution:

- It is essential to work out the overestimating cost C_o and the underestimating cost C_u.
- The cost per pair of overestimating demand is equal to the purchase cost minus the sale price per pair; that is

$$C_o = £40 - £30 = £10$$

- The cost per pair of underestimating demand is the difference between the regular selling price and the and the purchase cost; that is

Single Period…

$C_u = £60 - £40 = £20$

Then,

$P(D \leq y^*) = C_u / (C_u + C_o) = 20/(20+10) = 2/3$

- We can find the optimal order quantity y^* by referring to the assumed probability distribution shown in Figure below and finding the value of y that will provide $P(D \leq y^*) = 2/3$. To do this, we note that in the uniform distribution the probability is evenly distributed over the entire range of 350 - 650.

Single Period…

- Thus, we can satisfy the expression for y^* by moving two-thirds of the way from 350 to 650. That gives

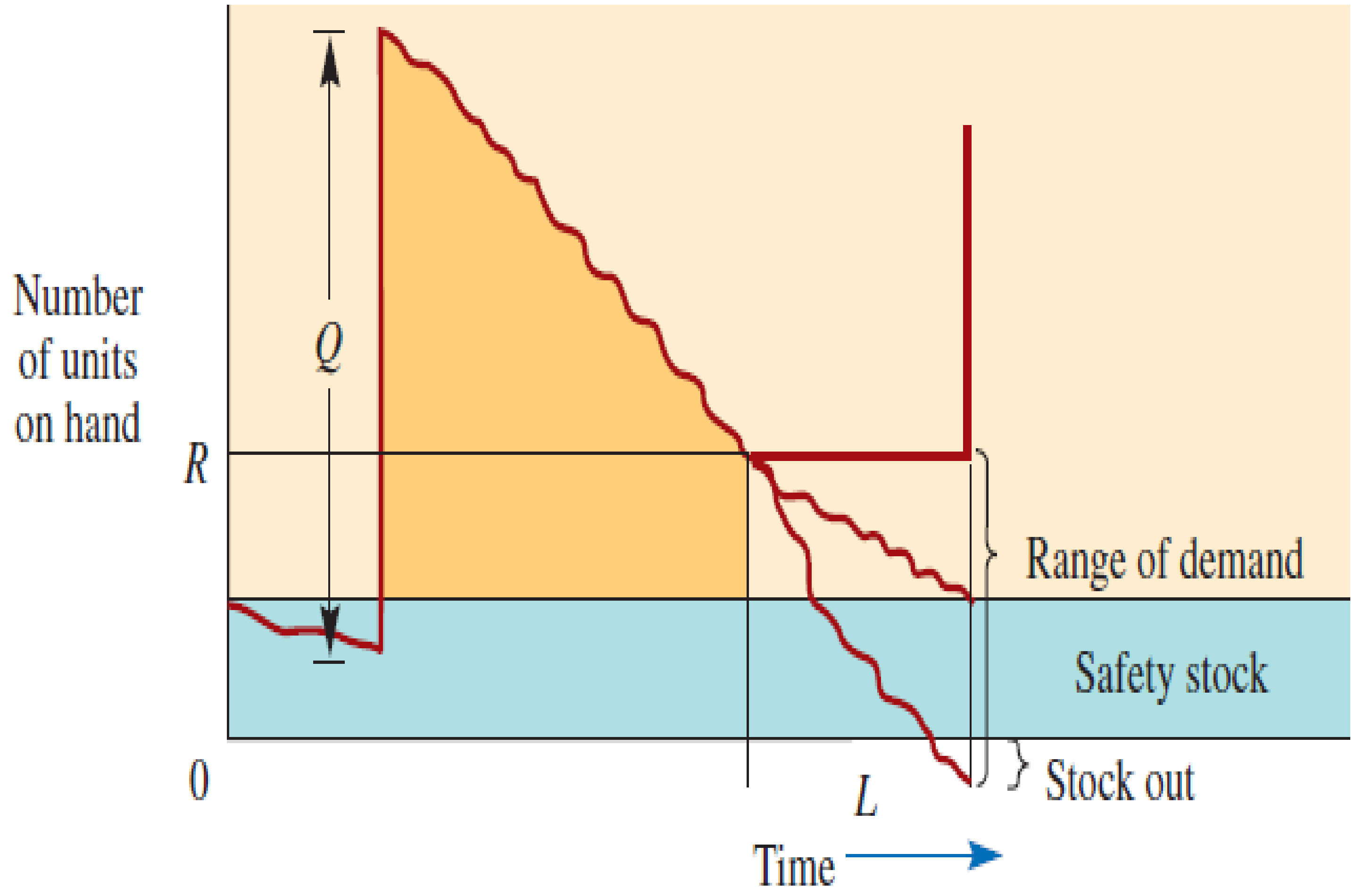
$$350 + 2/3\ (650-350) = 550 \text{ pairs}$$

namely, the optimal order quantity of size 10D shoes is 550 pairs.

2.2 Fixed Order Quantity Model

- It attempts to determine the specific point, *R* , *at which an order* will be placed and the size of that order, *Q* .
- In the majority of cases, though, demand is not constant but varies from day to day. Safety stock must therefore be maintained to provide some level of protection against stock outs.
- Safety stock can be defined as the amount of inventory carried in addition to the expected demand.

Fixed Order Quantity Model cont…



Fixed Order Quantity Model cont…

- The quantity to be ordered, *Q , is calculated in the usual way considering the demand, shortage* cost, ordering cost, and holding cost.
- A fixed–order quantity model can be used to compute *Q , such as the simple Q opt model.*
- The reorder point is then set to cover the expected demand during the lead time plus a safety stock determined by the desired service level.

Fixed Order Quantity Model cont…

- The reorder point is

$$R = \bar{d}L + Z\sigma_L$$

where,

R = *Reorder point in units*

$\bar{d}$ = *Average daily demand*

L = *Lead time in days (time between placing an order and receiving the items)*

z = *Number of standard deviations for a specified service probability*

σ_L = Standard deviation of usage during lead time

Fixed Order Quantity Model cont…

- The term $Z\sigma_L$ is the amount of safety stock. Note that if safety stock is positive, the effect is to place a reorder sooner. That is, *R without safety stock is simply the average demand* during the lead time.
- If lead time usage was expected to be 20, for example, and safety stock was computed to be 5 units, then the order would be placed sooner, when 25 units remained. The greater the safety stock, the sooner the order is placed.

Fixed Order Quantity Model cont…

- For the daily demand situation, *d can be a forecast demand using any of the models*. For example, if a 30-day period was used to calculate d , *then* a simple average would be

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} = \frac{\sum_{i=1}^{30} d_i}{30}$$

where n is the number of days.

The standard deviation of the daily demand is

$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{30} (d_i - \bar{d})^2}{30}}$$

Fixed Order Quantity Model cont…

$$\sigma_L = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_L^2}$$

- Consider an economic order quantity case where annual demand *D = 1,000 units, economic order* quantity *Q = 200 units, the desired probability of not stocking out P = .95, the standard deviation* of demand during lead time *L = 25 units, and lead time L = 15 days. Determine the reorder point.*
- Assume that demand is over a 250-workday year.

Fixed Order Quantity Model cont…

SOLUTION

In our example, $\bar{d} = \frac{1000}{250} = 4$, and lead time is 15 days. We use the equation

$$R = \bar{d}L + z\sigma_L$$
$$= 4(15) + z(25)$$

In this case, z is 1.64.

Completing the solution for R, we have

$$R = 4(15) + 1.64(25) = 60 + 41 = 101 \text{ units}$$

This says that when the stock on hand gets down to 101 units, order 200 more.

2.3 Fixed – Time Period Models

- In a fixed–time period system, inventory is counted only at particular times, such as every
  week or every month.
- Counting inventory and placing orders periodically are desirable in situations such as when vendors make routine visits to customers and take orders for their complete line of products, or when buyers want to combine orders to save transportation costs.

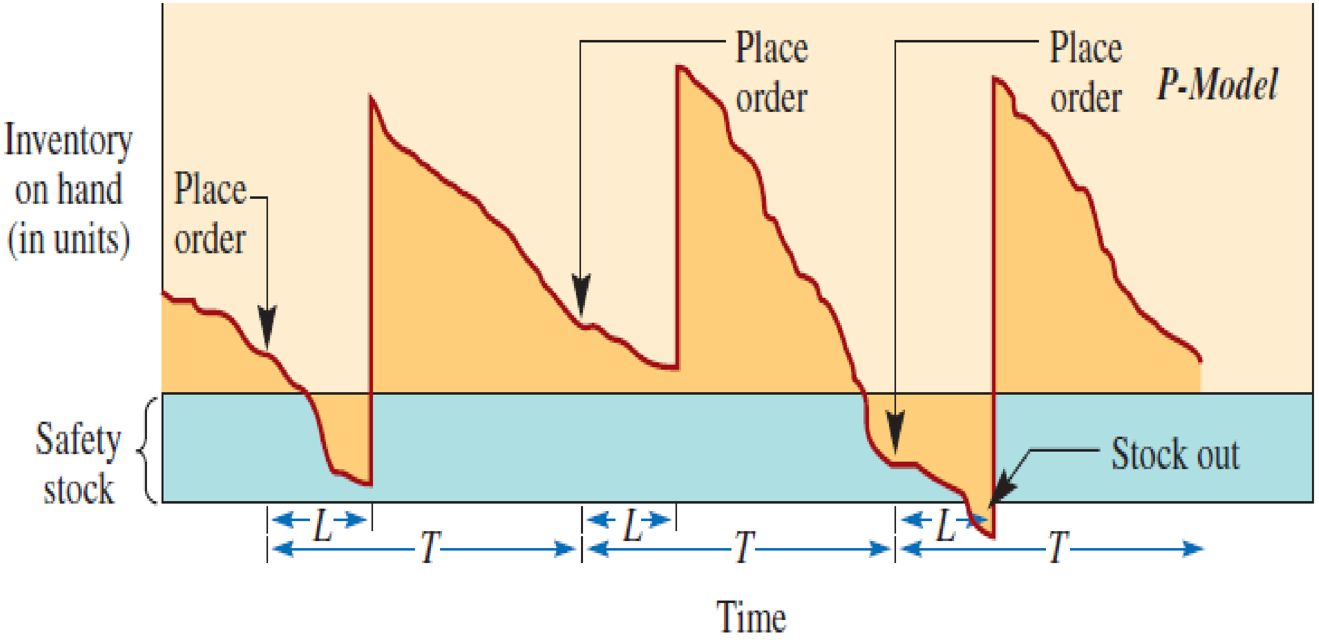
Fixed – Time Period Models cont...

- Fixed–time period models generate order quantities that vary from period to period, depending on the usage rates. These generally require a higher level of safety stock than a fixed–order quantity system.
- The fixed–order quantity system assumes continual tracking of inventory on hand, with an order immediately placed when the reorder point is reached.
- In contrast, the standard fixed–time period models assume that inventory is counted only at the time specified for review.

Fixed – Time Period Models cont...

- In a fixed–time period system, reorders are placed at the time of review (*T*), *and the safety* stock that must be reordered is

$$\text{Safety stock} = z\sigma_{T+L}$$



Fixed – Time Period Models cont...

- The above figure shows a fixed–time period system with a review cycle of *T and a constant* lead time of *L. In this case, demand is randomly distributed about a mean d . The quantity* to order, *q , is*

$$\underset{\text{quantity}}{\text{Order}} = \begin{matrix}\text{Average demand}\\\text{over the}\\\text{vulnerable period}\end{matrix} + \begin{matrix}\text{Safety}\\\text{stock}\end{matrix} - \begin{matrix}\text{Inventory currently}\\\text{on hand (plus on}\\\text{order, if any)}\end{matrix}$$

$$q = \bar{d}(T+L) + z\sigma_{T+L} - I$$

Fixed – Time Period Models cont...

where

q = Quantity to be ordered

T = The number of days between reviews

L = Lead time in days (time between placing an order and receiving it)

$\bar{d}$ = Forecast average daily demand

z = Number of standard deviations for a specified service probability

σ_{T+L} = Standard deviation of demand over the review and lead time

I = Current inventory level (includes items on order)

Note: The demand, lead time, review period, and so forth can be any time units such as days, weeks, or years so long as they are consistent throughout the equation.

In this model, demand ($\bar{d}$) can be forecast and revised each review period if desired, or the yearly average may be used if appropriate. We assume that demand is normally distributed.

Fixed – Time Period Models cont...

- Daily demand for a product is 10 units with a standard deviation of 3 units. The review period is 30 days, and lead time is 14 days. Management has set a policy of satisfying 98 percent of demand from items in stock. At the beginning of this review period, there are 150 units in inventory. How many units should be ordered?

SOLUTION

The quantity to order is

$$q = \bar{d}(T + L) + z\sigma_{T+L} - I$$
$$= 10(30 + 14) + z\sigma_{T+L} - 150$$

Because each day is independent and σ_d is constant,

$$\sigma_{T+L} = \sqrt{(T + L)\sigma_d^2} = \sqrt{(30 + 14)(3)^2} = 19.90$$

The z value for $P = 0.98$ is 2.05.

The quantity to order, then, is

$$q = \bar{d}(T + L) + z\sigma_{T+L} - I = 10(30 + 14) + 2.05(19.90) - 150 = 331 \text{ units}$$

To ensure a 98 percent probability of not stocking out, order 331 units at this review period.

The Newsboy Model

- It is a single period inventory model.
- This model is applied to solve problems related with the daily news paper.

- At the start of each day, a newsboy must decide on the number of papers to purchase. *Daily sales cannot be predicted exactly*, and are represented by the random variable, D.
- The newsboy must carefully consider these costs:

$$c_o = \text{unit cost of overage}$$

$$c_u = \text{unit cost of underage}$$

- It can be shown that the optimal number of papers to purchase is the fractal of the demand distribution given by

$$P = c_u / (c_u + c_o).$$

Newsboy cont..

- The classic illustration of this problem involves a newsboy who must purchase a quantity of newspapers for the day's sale. The purchase cost of the papers is \$0.10 and they are sold to customers for a price of \$0.25. Papers unsold at the end of the day are returned to the publisher for \$0.02. The boy does not like to disappoint his customers (who might turn elsewhere for supply), so he estimates a "good will" cost of \$0.15 for each customer who is not be satisfied if the supply of papers runs out. The boy has kept a record of sales and shortages, and estimates that the mean demand during the day is 250 and the standard deviation is 50. A Normal distribution is assumed. How many papers should he purchase?

This is a single-period problem because today's newspapers will be obsolete tomorrow.

Newsboy cont..

Standard Normal Probabilities

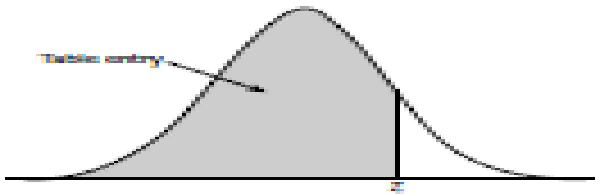


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Newsboy cont..

- $P = 0.7895$
- Z score = $0.8022 \sim 0.805$

$Q^* = \mu + Z\sigma$

$= 250 + 0.805*50$

$= 290.2$

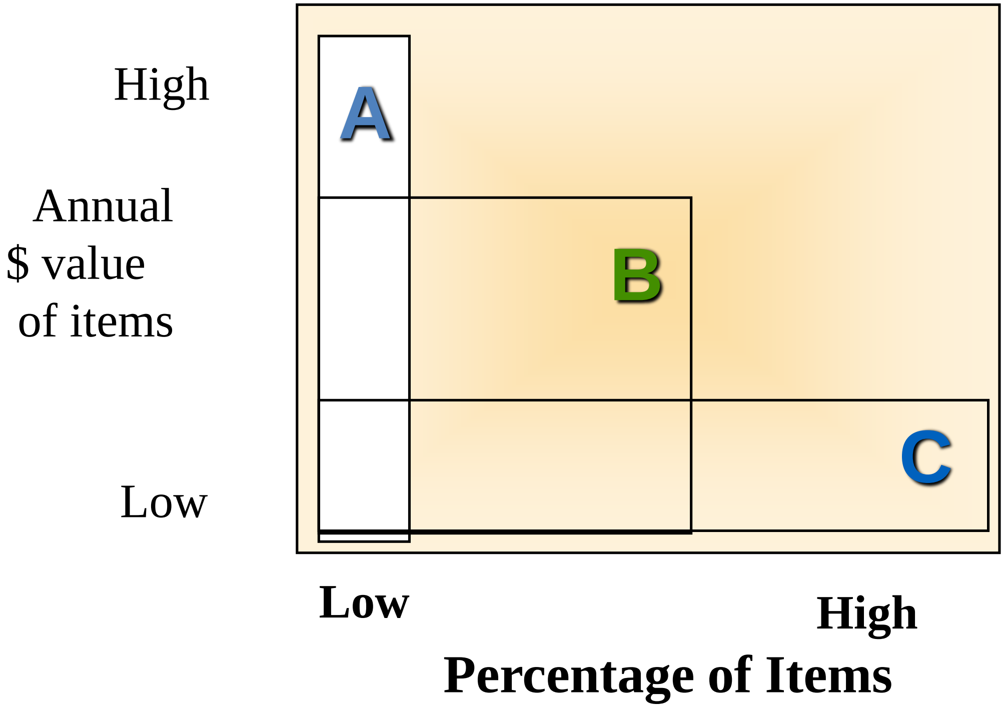
Rounding up, we suggest that the newsboy should purchase 291 papers for the day. The risk of a shortage during the day is

ABC Analysis

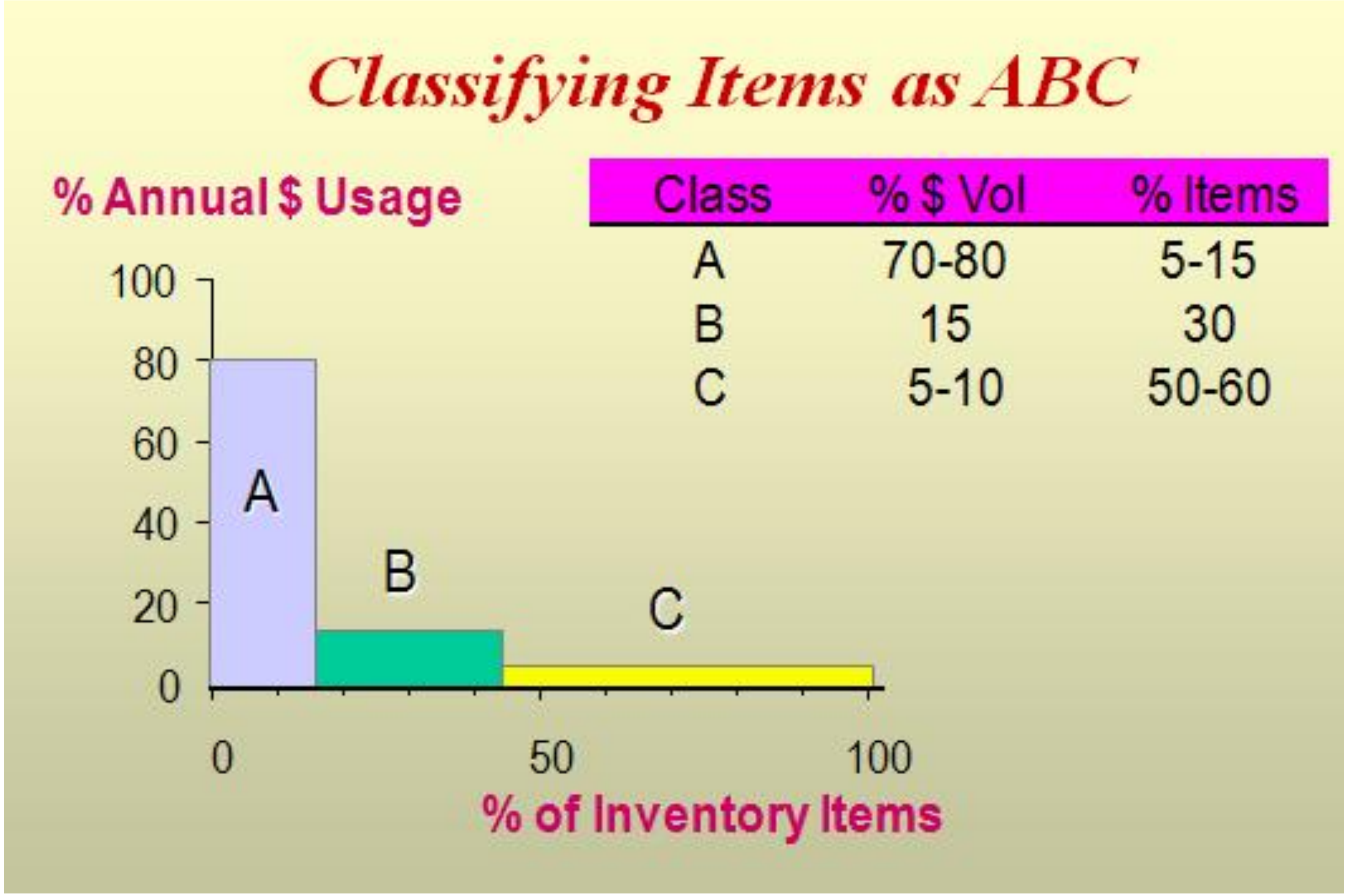
- Demand volume and value of items vary
- Items kept in inventory are not of equal importance in terms of:
  - dollars invested
  - profit potential
  - sales or usage volume
  - stock-out penalties

ABC …

- Classifying inventory according to some measure of importance and allocating control efforts accordingly.
  - **A** - very important
  - **B** - mod. important
  - **C** - least important



ABC …



Classifying Items as ABC
% Annual $ Usage
Class
% $ Vol
% Items
A
70-80
5-15
B
15
30
C
5-10
50-60
100
80
60
40
20
0
A
B
C
0
50
100
% of Inventory Items

ABC …

ABC Classification

PART	UNIT COST	ANNUAL USAGE
1	$ 60	90
2	350	40
3	30	130
4	80	60
5	30	100
6	20	180
7	10	170
8	320	50
9	510	60
10	20	120

ABC …

ABC Classification

PART	TOTAL VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY	% CUMMULATIVE
9	$30,600	35.9	6.0	6.0
8	16,000	18.7	5.0	11.0
2	14,000	16.4	4.0	15.0
1	5,400	6.3	9.0	24.0
4	4,800	5.6	6.0	30.0
3	3,900	4.6	10.0	40.0
6	3,600	4.2	18.0	58.0
5	3,000	3.5	13.0	71.0
10	2,400	2.8	12.0	83.0
7	1,700	2.0	17.0	100.0
	$85,400			

ABC …

ABC Classification

PART	TOTAL VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY		% CUMMULATIVE
9	$30,600	35.9	6.0		6.0
8	16,000	18.7	5.0	A	11.0
2	14,000	16.4	4.0		15.0
1	5,400	6.3	9.0		24.0
4	4,800	5.6	6.0	B	30.0
3	3,900	4.6	10.0		40.0
6	3,600	4.2	18.0		58.0
5	3,000	3.5	13.0		71.0
10	2,400	2.8	12.0	C	83.0
7	1,700	2.0	17.0		100.0
	$85,400				

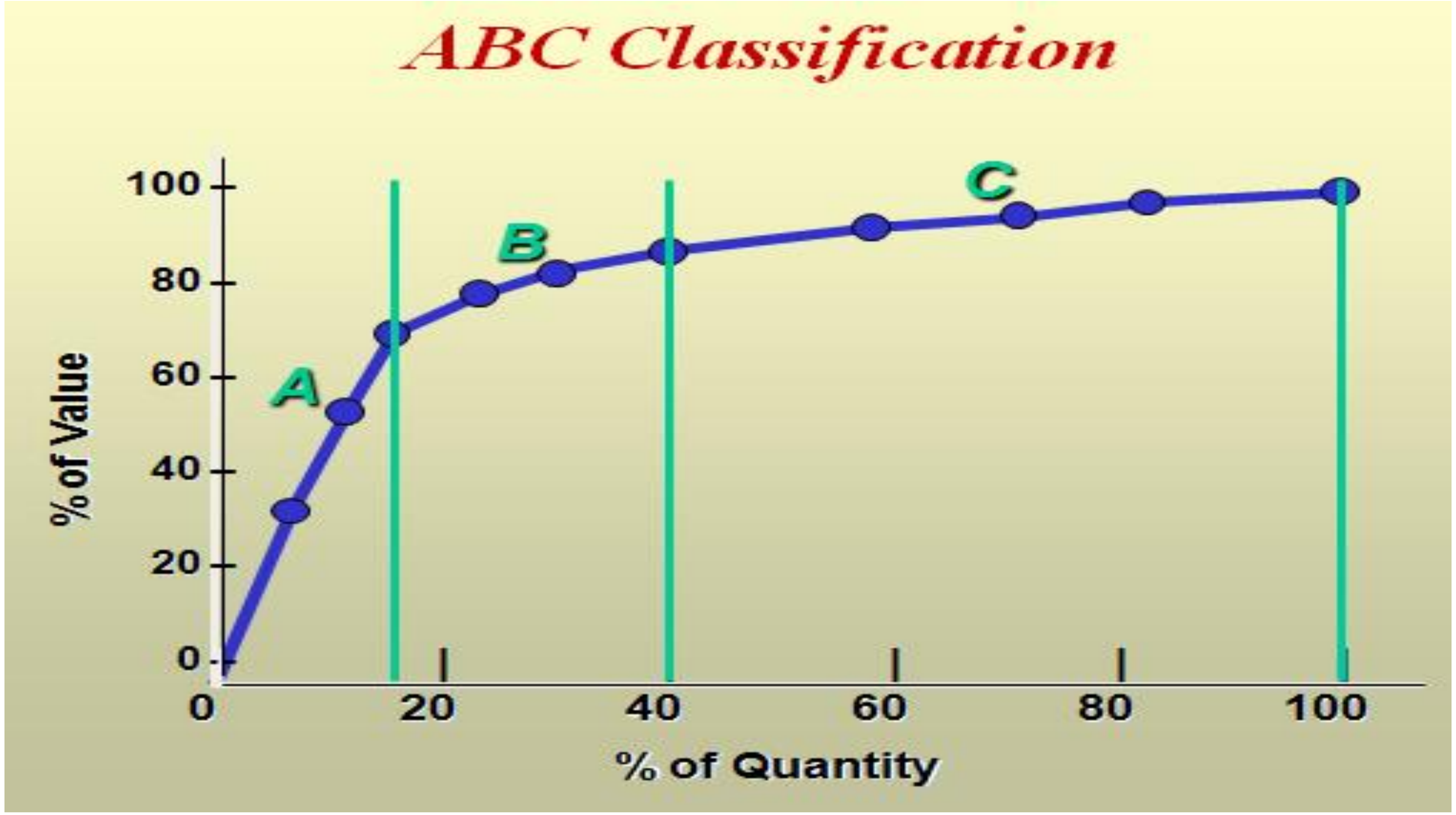
ABC …

ABC Classification

PART	TOTAL VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY	% CUMMULATIVE
9	$30,600	35.9	6.0	6.0
7	1,700	2.0	17.0	100.0
	$85,400			

CLASS	ITEMS	% OF TOTAL VALUE	% OF TOTAL QUANTITY
A	9, 8, 2	71.0	15.0
B	1, 4, 3	16.5	25.0
C	6, 5, 10, 7	12.5	60.0

ABC …



ABC Classification
100
80
60
40
20
0
% of Value
A
B
C
0
20
40
60
80
100
% of Quantity


Any question?