A STUDENT'S SOLUTIONS MANUAL TO ACCOMPANY

MECHANICS *of* FLUIDS,

4^{TH} EDITION, SI

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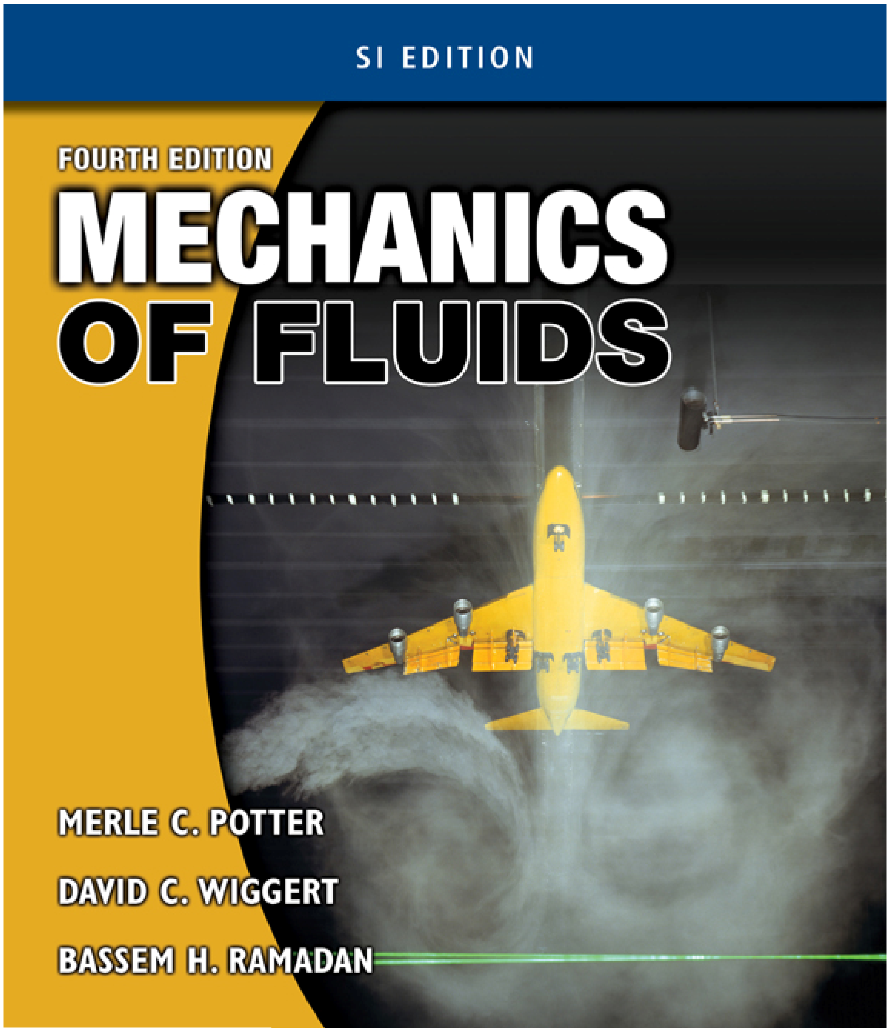

CENGAGE Learning

STUDENT'S SOLUTIONS MANUAL
TO ACCOMPANY

MECHANICS *of* FLUIDS

FOURTH EDITION, SI

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Contents

Preface for the Student

This manual provides the solutions to the problems whose answers are provided at the end of our book, ***MECHANICS OF FLUIDS,* SI**. In many cases, the solutions are not as detailed as the examples in the book; they are intended to provide the primary steps in each solution so you, the student, are able to quickly review how a problem is solved. The discussion of a subtle point, should one exist in a particular problem, is left as a task for the instructor. In general, some knowledge of a problem may be needed to fully understand all of the steps presented. This manual is not intended to be a self-paced workbook; your instructor is critically needed to provide explanations, discussions, and illustrations of the myriad of phenomena encountered in the study of fluids, but it should give you considerable help in working through a wide variety of problems.

The degree of difficulty and length of solution for each problem varies considerably. Some are relatively easy and others quite difficult. Typically, the easier problems are the first problems for a particular section.

We continue to include a number of multiple-choice problems in the earlier chapters, similar to those encountered on the Fundamentals of Engineering Exam (the old EIT Exam) and the GRE/Engineering Exam. These problems will provide a review for the Fluid Mechanics part of those exams. They are all four-part, multiple-choice problems and are located at the beginning of the appropriate chapters.

The examples and problems have been carefully solved with the hope that errors have not been introduced. Even though extreme care is taken and problems are reworked, errors creep in. We would appreciate knowing about any errors that you may find so they can be eliminated in future printings. Please send any corrections or comments to MerleCP@att.net.We have class tested most of the chapters with good response from our students, but we are sure that there are improvements to be made.

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CHAPTER 1

Basic Considerations

FE-type Exam Review Problems: Problems 1.1 to 1.13

1.1 (C)	$m = F/a$ or $\text{kg} = \text{N/m/s}^2 = \underline{\text{N}\cdot\text{s}^2/\text{m}}$
1.2 (B)	$[\mu] = [\tau/(du/dy)] = (F/L^2)/(L/T)/L = \underline{F\cdot T/L^2}$
1.3 (A)	$2.36\times10^{-8}\ \text{Pa} = 23.6\times10^{-9}\ \text{Pa} = \underline{23.6\ \text{nPa}}$
1.4 (C)	The mass is the same on earth and the moon, so we calculate the mass using the weight given on earth as: $m = W/g = 250\ \text{N}/9.81\ \text{m/s}^2 = 25.484\ \text{kg}$ Hence, the weight on the moon is: $W = mg = 25.484 \times 1.6 = \underline{40.77\ \text{N}}$
1.5 (C)	The shear stress is due to the component of the force acting tangential to the area: $F_{\text{shear}} = F\sin\theta = 4200\sin 30^\circ = 2100\ \text{N}$ $\tau = \dfrac{F_{\text{shear}}}{A} = \dfrac{2100\ \text{N}}{250\times10^{-4}\ \text{m}^2} = 84\times10^3\ \text{Pa}$ or $\underline{84\ \text{kPa}}$
1.6 (B)	– 53.6°C
1.7 (D)	Using Eqn. (1.5.3): $\rho_{\text{water}} = 1000 - \dfrac{(T-4)^2}{180} = 1000 - \dfrac{(80-4)^2}{180} = \underline{968\ \text{kg/m}^3}$
1.8 (A)	The shear stress is given by: $\tau = \mu\left\lvert\dfrac{du}{dr}\right\rvert$ We determine du/dr from the given expression for u as: $\dfrac{du}{dr} = \dfrac{d}{dr}\left[10\left(1-2500r^2\right)\right] = -50,000r$ At the wall $r = 2$ cm $= 0.02$ m. Substituting r in the above equation we get:

	$\left\|\frac{du}{dr}\right\| = 50,000r = 1000\ 1/\text{s}$ The density of water at 20°C is $10^{-3}\ \text{N}\cdot\text{s/m}^2$ Now substitute in the equation for shear stress to get $\tau = \mu\left\|\frac{du}{dr}\right\| = 10^{-3}\ \text{N}\cdot\text{s/m}^2 \times 1000\ 1/\text{s} = 1\ \text{N/m}^2 = \underline{1\ \text{Pa}}$
1.9 (D)	Using Eqn. (1.5.16), $\beta = 0$ (for clean glass tube), and $\sigma = 0.0736$ N/m for water (Table B.1 in Appendix B) we write: $h = \frac{4\sigma\cos\beta}{\rho g D} = \frac{4\times 0.0736\ \text{N/m}\times 1}{1000\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2 \times 10\times 10^{-6}\ \text{m}} = 3\ \text{m} \quad \text{or} \quad \underline{300\ \text{cm}}$ where we used N = kg×m/s^2
1.10 (C)	Density
1.11 (C)	Assume propane (C_3H_8) behaves as an ideal gas. First, determine the gas constant for propane using $R = \frac{R_u}{M} = \frac{8.314\ \text{kJ/kmol}\cdot\text{K}}{44.1\ \text{kg/kmol}} = 0.1885\ \text{kJ/kg}$ then, $m = \frac{p\cancel{V}}{RT} = \frac{800\ \text{kN/m}^2 \times 4\ \text{m}^3}{0.1885\ \text{kJ/(kg}\cdot\text{K)} \times (10+273)\ \text{K}} = 59.99\ \text{kg} \approx \underline{60\ \text{kg}}$
1.12 (B)	Consider water and ice as the system. Hence, the change in energy for the system is zero. That is, the change in energy for water should be equal to the change in energy for the ice. So, we write $\Delta E_{\text{ice}} = \Delta E_{\text{water}}$ $m_{\text{ice}} \times 320\ \text{kJ/kg} = m_{\text{water}} \times c_{\text{water}} \Delta T$ The mass of ice is calculate using $m_{ice} = \rho\cancel{V} = 5\ \text{cubes} \times \left(1000\ \text{kg/m}^3\right) \times \left(40\times 10^{-6}\ \text{m}^3/\text{cube}\right) = 0.2\ \text{kg}$ Where we assumed the density of ice to be equal to that of water, namely 1000 kg/m^3. Ice is actually slightly lighter than water, but it is not necessary for such accuracy in this problem. Similarly, the mass of water is calculated using $m_{water} = \rho\cancel{V} = \left(1000\ \text{kg/m}^3\right) \times 2\ \text{liters}\left(10^{-3}\ \text{m}^3/\text{liter}\right) = 2\ \text{kg}$ Solving for the temperature change for water we get $0.2\ \text{kg} \times 320\ \text{kJ/kg} = 2\ \text{kg} \times 4.18\ \text{kJ/kg}\cdot\text{K} \times \Delta T \quad \Rightarrow \quad \Delta T = \underline{7.66^\circ\text{C}}$

1.13 (D)	Since a dog's whistle produces sound waves at a high frequency, the speed of sound is $c = \sqrt{RT} = \sqrt{287 \text{ J/kg}\cdot\text{K} \times 323 \text{ K}} = \underline{304 \text{ m/s}}$ where we used J/kg = m^2/s^2.

Dimensions, Units, and Physical Quantities

1.16	**a)** density = $\frac{M}{L^3} = \frac{FT^2/L}{L^3} = FT^2/L^4$ **c)** power = $F \times$ velocity = $F \times L/T = FL/T$ **e)** mass flux = $\frac{M/T}{A} = \frac{FT^2/L}{L^2T} = FT/L^3$
1.18	**b)** N = [C] kg $\quad\therefore$ [C] = N/kg = (kg$\cdot$m/s^2)/kg = m/s^2
1.20	$\text{kg}\frac{\text{m}}{\text{s}^2} + c\frac{\text{m}}{\text{s}} + k\text{m} = f$. Since all terms must have the same dimensions (units) we require: [c] = kg/s, [k] = kg/s^2 = $\text{N}\cdot\text{s}^2 / \text{m}\cdot\text{s}^2 = \text{N/m}$, $[f] = \text{kg}\cdot\text{m/s}^2 = \text{N}$ Note: we could express the units on c as [c] = $\text{kg/s} = \text{N}\cdot\text{s}^2/\text{m}\cdot\text{s} = \underline{\text{N}\cdot\text{s/m}}$
1.22	**a)** 1.25×10^8 N $\qquad$ **c)** 6.7×10^8 Pa $\qquad$ **e)** 5.2×10^{-2} m^2
1.24	**a)** $20 \text{ cm/hr} = 20 \frac{\text{cm}}{\text{hr}} \times \frac{\text{m}}{100 \text{ cm}} \times \frac{\text{hr}}{3600 \text{ s}} = 5.556 \times 10^{-5} \text{m/s}$ **c)** $500 \text{ hp} = 500 \text{ hp} \times \frac{745.7 \text{ W}}{\text{hp}} = 37{,}285 \text{ W}$ **e)** $2000 \text{ kN/cm}^2 = 2000 \frac{\text{kN}}{\text{cm}^2} \times \left(\frac{100 \text{ cm}}{\text{m}}\right)^2 = 2 \times 10^{10} \text{N/m}^2$
1.26	The mass is the same on the earth and the moon, so we calculate the mass, then calculate the weight on the moon: $m = 27$ kg $\quad\therefore\ W_{\text{moon}}$ = (27 kg) $\times$ (1.63) = $\underline{44.01 \text{ N}}$

Pressure and Temperature

1.28

Use the values from Table B.3 in the Appendix:

b) At an elevation of 1000 m the atmospheric pressure is 89.85 kPa. Hence, the absolute pressure is $52.3 + 89.85 = \underline{142.2\text{ kPa}}$

d) At an elevation of 10,000 m the atmospheric pressure is 26.49 kPa, and the absolute pressure is $52.3 + 26.49 = \underline{78.8\text{ kPa}}$

1.30

$p = p_o\, \mathrm{e}^{-gz/RT} = 101\, \mathrm{e}^{-(9.81 \times 4000)/[(287) \times (15 + 273)]} = \underline{62.8\text{ kPa}}$

From Table B.3, at 4000 m: $p = 61.6$ kPa. The percent error is

$$\%\text{ error} = \frac{62.8 - 61.6}{61.6} \times 100 = \underline{1.95\ \%}$$

1.32

Using Table B.3 and linear interpolation we write:

$$T = 223.48 + \frac{10{,}600 - 10{,}000}{12{,}000 - 10{,}000}(223.6 - 216.7) = \underline{221.32\text{ K}}$$

$$\text{or } (221.32 - 273.15)\frac{5}{9} = \underline{-51.83°\text{C}}$$

1.34

The normal force due pressure is: $F_n = (120{,}000\text{ N/m}^2) \times 0.2 \times 10^{-4}\text{ m}^2 = 2.4\text{ N}$

The tangential force due to shear stress is: $F_t = 20\text{ N/m}^2 \times 0.2 \times 10^{-4}\text{ m}^2 = 0.0004\text{ N}$

The total force is $F = \sqrt{F_n^2 + F_t^2} = \underline{2.40\text{ N}}$

The angle with respect to the normal direction is $\theta = \tan^{-1}\left(\frac{0.0004}{2.4}\right) = \underline{0.0095°}$

Density and Specific Weight

1.36

Using Eq. 1.5.3 we have

$\rho = 1000 - (T - 4)^2/180 = 1000 - (70 - 4)^2/180 = \underline{976\text{ kg/m}^3}$

$\gamma = 9800 - (T - 4)^2/18 = 9800 - (70 - 4)^2/180 = \underline{9560\text{ N/m}^3}$

Using Table B.1 the density and specific weight at 70°C are

$\rho = 977.8\text{ kg/m}^3$

$\gamma = 977.8 \times 9.81 = 9592.2\text{ N/m}^3$

$$\%\text{ error for } \rho = \frac{976 - 978}{978} \times 100 = \underline{-0.20\%}$$

$$\%\text{ error for } \gamma = \frac{9560 - 9592}{9592} \times 100 = \underline{-0.33\%}$$

1.38

b) $m = \frac{\gamma \cancel{V}}{g} = \frac{12{,}400\ \text{N/m}^3 \times 500 \times 10^{-6}\text{m}^3}{9.77\ \text{m/s}^2} = \underline{0.635\ \text{kg}}$

Viscosity

1.40

Assume carbon dioxide is an ideal gas at the given conditions, then

$$\rho = \frac{p}{RT} = \frac{200\ \text{kN/m}^3}{\left(0.189\ \text{kJ/kg}\cdot\text{K}\right)\left(90 + 273\ \text{K}\right)} = 2.915\ \text{kg/m}^3$$

$$\gamma = \frac{W}{\cancel{V}} = \frac{mg}{\cancel{V}} = \rho g = 2.915\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2 = 28.6\ \text{kg/m}^2\cdot\text{s}^2 = \underline{28.6\ \text{N/m}^3}$$

From Fig. B.1 at 90°C, $\mu \cong 2\times10^{-5}\ \text{N}\cdot\text{s/m}^2$, so that the kinematic viscosity is

$$\nu = \frac{\mu}{\rho} = \frac{2\times10^{-5}\ \text{N}\cdot\text{s/m}^2}{2.915\ \text{kg/m}^3} = \underline{6.861\times10^{-6}\ \text{m}^2/\text{s}}$$

The kinematic viscosity cannot be read from Fig. B.2 since the pressure is not at 100 kPa.

1.42

The shear stress can be calculated using $\tau = \mu\left|du/dy\right|$. From the given velocity distribution, $u = 120(0.05y - y^2)$ we get $\Rightarrow \frac{du}{dy} = 120(0.05 - 2y)$

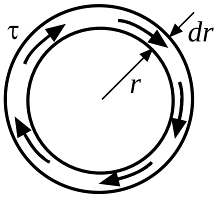
From Table B.1 at 10°C for water, $\mu = 1.308\times10^{-3}\ \text{N}\cdot\text{s/m}^2$

So, at the lower plate where y = 0, we have

$$\left|\frac{du}{dy}\right| = 120(0.05 - 0) = 6\ \text{s}^{-1} \quad\Rightarrow\quad \tau = \left(1.308\times10^{-3}\right)\times 6 = \underline{7.848\times10^{-3}\ \text{N/m}^2}$$

At the upper plate where y = 0.05 m,

$$\left|\frac{du}{dy}\right|_{y=0.05} = \left|120(0.05 - 2\times0.05)\right| = 6\ \text{s}^{-1} \quad\Rightarrow\quad \tau = \underline{7.848\times10^{-3}\ \text{N/m}^2}$$

1.44	The shear stress can be calculated using $\tau = \mu\left\|du/dr\right\|$. From the given velocity $u = 16(1 - r^2/r_o^2)$ $\Rightarrow$ we get $\frac{du}{dr} = 16(-2r/r_o^2)$. Hence, $\left\|\frac{du}{dr}\right\| = 32r/r_o^2$ At the centerline, $r = 0$, so $\frac{du}{dr} = 0$, and hence $\underline{\tau = 0}$. At $r = 0.25$ cm, $\Rightarrow \left\|\frac{du}{dr}\right\| = 32r/r_o^2 = 32\frac{0.25/100}{(0.5/100)^2} = 3200\ \text{s}^{-1}$, $\Rightarrow$ $\tau = 3200\times\mu = 3200\left(1\times10^{-3}\right) = \underline{3.2\ \text{N/m}^2}$ At the wall, $r = 0.5$ cm, $\Rightarrow \left\|\frac{du}{dr}\right\| = 32r/r_o^2 = 32\frac{0.5/100}{(0.5/100)^2} = 6400\ \text{s}^{-1}$, $\Rightarrow$ $\tau = 6400\times\mu = 6400\left(1\times10^{-3}\right) = \underline{6.4\ \text{N/m}^2}$
1.46	Use Eq.1.5.8 to calculate the torque, $T = \frac{2\pi R^3\omega L\mu}{h}$ where $h = (2.6 - 2.54)/2 = 0.03\ \text{cm} = 0.03\times10^{-2}\ \text{m}$ The angular velocity $\omega = \frac{2\pi}{60}\times 2000\ \text{rpm} = 209.4\ \text{rad/s}$ The viscosity of SAE-30 oil at 21°C is $\mu = 0.2884\ \text{Ns/m}^2$ (Figure B.2) $T = \frac{2\pi\times(1.27\times10^{-2}\ \text{m})^3\times209.4\ \text{rad/s}\times1.2\ \text{m}\times0.2884\ \text{Ns/m}^2}{(0.03\times10^{-2})\text{m}} = \underline{3.10\ \text{N}\cdot\text{m}}$ power = $\text{T}\omega = 3.1\times209.4$ = 650 W = $\underline{0.65\ \text{kW}}$
1.48	Assume a linear velocity in the fluid between the rotating disk and solid surface. The velocity of the fluid at the rotating disk is $V = r\omega$, and at the solid surface $V = 0$. So, $\frac{du}{dy} = \frac{r\omega}{h}$, where h is the spacing between the disk and solid surface, and $\omega = 2\pi\times400/60 = 41.9\ \text{rad/s}$. The torque needed to rotate the disk is *T = shear force × moment arm*  Due to the area element shown, $dT = dF\times r = \tau dA\times r$ where τ = shear stress in the fluid at the rotating disk and $dA = 2\pi r dr \ \Rightarrow\ dT = \mu\frac{du}{dy}\times2\pi r dr\times r = \mu\frac{r\omega}{h}\times2\pi r^2 dr$

	$T=\int_0^R \frac{2\pi\mu\omega}{h} r^3 dr = \frac{\pi\mu\omega}{2h}R^4$ The viscosity of water at 16°C is $\mu = 1.12\times10^{-3}\text{ Ns/m}^2$ $\Rightarrow T = \frac{\pi\mu\omega}{2h}R^4 = \frac{\pi\left(1.12\times10^{-3}\text{ Ns/m}^2\right)\left(41.9\text{ rad/s}\right)\left(\frac{0.15}{2}\right)^4}{2\times\left(2\times10^{-3}\text{ m}\right)} = \underline{1.16\times10^{-3}\text{ N}\cdot\text{m}}$
1.50	If $\tau = \mu\frac{du}{dy}$ = constant, and $\mu = Ae^{B/T} = Ae^{By/K} = Ae^{Cy}$, then $Ae^{Cy}\frac{du}{dy}$ = constant. $\therefore\ \frac{du}{dy} = De^{-Cy}$, where D is a constant. Multiply by dy and integrate to get the velocity profile $u=\int_0^y De^{-Cy}dy = -\frac{D}{C}e^{-Cy}\Big\|_0^y = E\left(e^{Cy}-1\right)$ where A, B, C, D, E, and K are constants.

Compressibility

1.54	The sound will travel across the lake at the speed of sound in water. The speed of sound in water is calculated using $c=\sqrt{\frac{B}{\rho}}$, where B is the bulk modulus of elasticity. Assuming (T = 10°C) and using Table B.1 we find $B = 211\times10^7\text{ Pa}$, and $\rho = 999.7\text{ kg/m}^3 \Rightarrow c = \sqrt{\frac{211\times10^7\text{ N/m}^2}{999.7\text{ kg/m}^3}} = 1453\text{ m/s}$ The distance across the lake is, $L = c\Delta t = 1453 \times 0.62 = \underline{901\text{ m}}$
1.56	**b)** Using Table B.2 and T = 37°C we find $B = 226\times10^7\text{ N/m}^2$, $\rho = 993\text{ kg/m}^3$ The speed of sound in water is calculated using: $c=\sqrt{\frac{B}{\rho}} \Rightarrow c = \sqrt{\left(226\times10^7\right)/993} = \underline{1508\text{ m/s}}$

Surface Tension

1.58

For a spherical droplet the pressure is given by $p = \dfrac{2\sigma}{R}$

Using Table B.1 at 15°C the surface tension $\sigma = 7.41\times10^{-2}$ N/m

$$\Rightarrow p = \frac{2\sigma}{R} = \frac{2\times 0.0741\ \text{N/m}}{5\times10^{-6}\ \text{m}} = \underline{29.6\ \text{kPa}}$$

For bubbles: $\Rightarrow p = \dfrac{4\sigma}{R} = \dfrac{2\times 0.0741\ \text{N/m}}{5\times10^{-6}\ \text{m}} = \underline{59.3\ \text{kPa}}$

1.60

For a spherical droplet the net force due to the pressure difference Δp between the inside and outside of the droplet is balanced by the surface tension force, which is expressed as:

$$\Delta p = \frac{2\sigma}{R} \Rightarrow p_{inside} - p_{outside} = \frac{2\times 0.025\ \text{N/m}}{5\times10^{-6}\ \text{m}} = 10\ \text{kPa}$$

Hence, $p_{inside} = p_{outside} + 10\ \text{kPa} = 8000\ \text{kPa} + 10\ \text{kPa} = \underline{8010\ \text{kPa}}$

In order to achieve this high pressure in the droplet, diesel fuel is usually pumped to a pressure of about 2000 bar before it is injected into the engine.

1.62

See Example 1.4:

$$h = \frac{4\sigma\cos\beta}{\rho g D} = \frac{4\times\left(0.47\ \text{N/m}\right)\cos130^\circ}{\left(13.6\times1000\ \text{kg/m}^3\right)\times9.81\ \text{m/s}^2\times\left(2\times10^{-2}\ \text{m}\right)}$$

$$= -4.53\times10^{-4}\ \text{m} = -0.453\ \text{mm}$$

Note that the minus sign indicates a capillary drop rather than a capillary rise in the tube.

1.64

Draw a free-body diagram of the floating needle as shown in the figure.

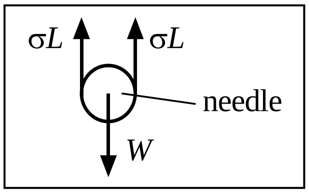
The weight of the needle and the surface tension force must balance:

$W = 2\sigma L$ or $\rho g V\!\!\!- = 2\sigma L$

The volume of the needle is $V\!\!\!- = \left(\dfrac{\pi d^2}{4} L\right)$

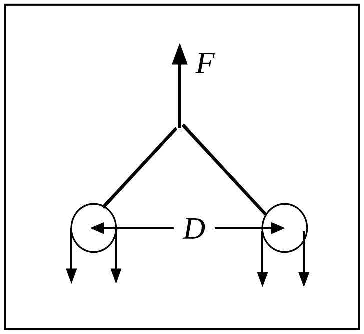
$$\Rightarrow \left(\frac{\pi d^2}{4} L\right)\rho g = 2\sigma L$$

$$\therefore d = \sqrt{\frac{8\sigma}{\pi\rho g}}$$



1.66

There is a surface tension force on the outside and on the inside of the ring. Each surface tension force = $\sigma \times \pi D$.



Neglecting the weight of the ring, the free-body diagram of the ring shows that

$$F = \underline{2\sigma\pi D}$$

Vapor Pressure

1.68

To determine the temperature, we can determine the absolute pressure and then use property tables for water.

The absolute pressure is p = –80 + 92 = 12 kPa.

From Table B.1, at 50°C water has a vapor pressure of 12.3 kPa;

so T = $\underline{50°\text{C}}$ is a maximum temperature. The water would “boil” above this temperature.

1.70

At 40°C the vapor pressure from Table B.1 is $\underline{7.38\ \text{kPa}}$. This would be the minimum pressure that could be obtained since the water would vaporize below this pressure.

1.72

The inlet pressure to a pump cannot be less than 0 kPa absolute. Assuming atmospheric pressure to be 100 kPa, we have

$$10{,}000\ \text{kPa} + 100\ \text{kPa} = 600x \qquad \therefore x = \underline{16.83\ \text{km}}.$$

Ideal Gas

1.74

Assume air is an ideal gas and calculate the density inside and outside the house using $T_{\text{in}} = 15°\text{C},\ p_{\text{in}} = 101.3\ \text{kPa},\ T_{\text{out}} = -25°\text{C},$ and $p_{\text{out}} = 85\ \text{kPa}$

$$\rho_{\text{in}} = \frac{p}{RT} = \frac{101.3\ \text{kN/m}^3}{0.287\ \text{kJ/kg}\cdot\text{K}\times(15+273\ \text{K})} = \underline{1.226\ \text{kg/m}^3}$$

$$\rho_{\text{out}} = \frac{85}{0.287\times 248} = \underline{1.19\ \text{kg/m}^3}$$

Yes. The heavier air outside enters at the bottom and the lighter air inside exits at the top. A circulation is set up and the air moves from the outside in and the inside out. This is infiltration, also known as the “chimney” effect.

1.76	The weight can be calculated using, $W=\gamma V\!\!\!\!- = \rho g V\!\!\!\!-$ Assume air in the room is at 20°C and 100 kPa and is an ideal gas: $\Rightarrow \rho = \dfrac{p}{RT} = \dfrac{100}{0.287\times 293} = 1.189\text{ kg/m}^3$ $\Rightarrow W = \gamma V\!\!\!\!- = \rho g V\!\!\!\!- = 1.189\text{ kg/m}^3 \times 9.81\text{ m/s}^2 \times (10\times 20\times 4\text{ m}^3) = \underline{9333\text{ N}}$
1.78	The pressure holding up the mass is 100 kPa. Hence, using $pA = W$, we have $100{,}000\text{ N/m}^2\times 1\text{ m}^2 = m\times 9.81\text{ m/s}^2$ This gives the mass of the air $m = 10{,}194\text{ kg}$ Since air is an ideal gas we can write $pV\!\!\!\!- = mRT$ or $V\!\!\!\!- = \dfrac{mRT}{p} = \dfrac{10{,}194\text{ kg}\times 0.287\text{ kJ/kg}\cdot\text{K}\times(15{+}273\text{ K})}{100\text{ kPa}} = 8426\text{ m}^3$ Assuming a spherical volume $\Rightarrow V\!\!\!\!- = \pi d^3/6$, which gives $d = \left(\dfrac{6\times 8426\text{ m}^3}{\pi}\right)^{1/3} = \underline{25.2\text{ m}}$

First Law

1.80

The fist law of the thermodynamics is applied to the mass:

$$Q_{1-2} - W_{1-2} = \Delta PE + \Delta KE + \Delta U$$

In this case, since there is no change in potential and internal energy, i.e., $\Delta PE = 0,$ and $\Delta U = 0,$ and there is no heat transfer to or from the system, $Q_{1-2} = 0.$ The above equation simplifies to: $-W_{1-2} = \Delta KE$

where $W_{1-2} = \int_1^2 Fdl$

a) $F = 200 \text{ N} \Rightarrow W_{1-2} = -200 \text{ N} \times 10 \text{ m} = -2000 \text{ N} \cdot \text{m}$ or J

Note that the work is negative in this case since it is done on the system.

Substituting in the first law equation we get:

$$2000 \text{ N} \cdot \text{m} = \frac{1}{2} m \left(V_2^2 - V_1^2 \right) \Rightarrow V_2 = \sqrt{(10 \text{ m/s})^2 + \frac{2 \times 2000 \text{ N} \cdot \text{m}}{15 \text{ kg}}} = \underline{19.15 \text{ m/s}}$$

b) $F = 20s \Rightarrow W_{1-2} = -\int_0^{10} 20sds = -10s^2 \Big|_0^{10} = -1000 \text{ N} \cdot \text{m}$

$$1000 \text{ N} \cdot \text{m} = \frac{1}{2} m \left(V_2^2 - V_1^2 \right) \Rightarrow V_2 = \sqrt{(10 \text{ m/s})^2 + \frac{2 \times 1000 \text{ N} \cdot \text{m}}{15 \text{ kg}}} = \underline{15.28 \text{ m/s}}$$

c) $F = 200 \cos\left(\pi s/20 \right) \Rightarrow W_{1-2} = -\int_0^{10} 200 \cos\left(\pi s/20 \right) ds$

$$\Rightarrow W_{1-2} = -200 \times \left(20/\pi \right) \sin\left(10\pi/20 \right) = -4000/\pi \text{ N} \cdot \text{m}$$

$$4000/\pi \text{ N} \cdot \text{m} = \frac{1}{2} m \left(V_2^2 - V_1^2 \right) \Rightarrow V_2 = \sqrt{(10 \text{ m/s})^2 + \frac{2 \times 4000/\pi \text{ N} \cdot \text{m}}{15 \text{ kg}}} = \underline{16.42 \text{ m/s}}$$

1.82

Applying the first law to the system (auto + water) we write:

$$Q_{1-2} - W_{1-2} = \Delta PE + \Delta KE + \Delta U$$

In this case, the kinetic energy of the automobile is converted into internal energy in the water. There is no heat transfer, $Q = 0$, no work $W = 0$, and no change in potential energy, $\Delta PE = 0$. The first law equation reduces to:

$$\frac{1}{2} m V_1^2 = \Delta U_{H_2O} = \left[mc\Delta T \right]_{H_2O}$$

where, $m_{H_2O} = \rho \cancel{V} = 1000 \text{ kg/m}^3 \times 2000 \text{ cm}^3 \times 10^{-6} \text{ m}^3/\text{cm}^3 = 2 \text{ kg}$

	For water $c = 4180 \text{ J/kg}\cdot\text{K}$. Substituting in the above equation $\frac{1}{2}\times 1500\text{ kg}\times\left(\frac{100\times 1000}{3600}\text{ m/s}\right)^2 = 2\text{ kg}\times 4180\text{ J/kg}\cdot\text{K}\times\Delta T$ $\Rightarrow \underline{\Delta T = 69.2°\text{C}}$
1.84	For a closed system the work is $W_{1-2} = \int_1^2 p\,dV$ For air (which is an ideal gas), $pV = mRT$, and $p = \frac{mRT}{V}$ $W_{1-2} = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT\int_1^2 \frac{dV}{V} = mRT\ln\frac{V_2}{V_1}$ Since T = constant, then for this process $p_1V_1 = p_2V_2$ or $\frac{V_2}{V_1} = \frac{p_1}{p_2}$ Substituting in the expression for work we get: $W_{1-2} = mRT\,\frac{p_2}{p_1}$ $W_{1-2} = 2\text{ kg}\times\left(287\text{ J/kg}\cdot\text{K}\right)\times 294\text{ K}\ln\frac{1}{2} = -116{,}980\text{ J} = \underline{-116.98\text{ kJ}}$ The 1st law states that $Q_{1-2} - W_{1-2} = m\Delta\tilde{u} = mc_v\Delta T = 0$ since $\Delta T = 0$. $\therefore Q = W = \underline{-116.98\text{ kJ}}$
1.86	For a closed system the work is $W_{1-2} = \int_1^2 p\,dV$ If p = constant, then $W_{1-2} = p\left(V_2 - V_1\right)$ Since air is an ideal gas, then $p_1V_1 = mRT_1$, and $p_2V_2 = mRT_2$ $W_{1-2} = mR\left(T_2 - T_1\right) = mR\left(2T_1 - T_1\right) = mRT_1 = 2\text{ kg}\times 0.287\text{ kJ/kg}\cdot\text{K}\times 423\text{ K} = \underline{243\text{ kJ}}$

Isentropic Flow

1.88	Since the process is adiabatic, we assume an isentropic process to estimate the maximum final pressure: $p_2 = p_1\left(\frac{T_2}{T_1}\right)^{k/k-1} = (150+100)\left(\frac{423}{293}\right)^{1.4/0.4} = 904$ kPa abs or $\underline{804 \text{ kPa gage.}}$ Note: We assumed p_{atm} = 100 kPa since it was not given. Also, a measured pressure is a gage pressure.

Speed of Sound

1.90	**b)** $c = \sqrt{kRT} = \sqrt{1.4 \times 188.9 \times 293} = \underline{266.9 \text{ m/s}}$ **d)** $c = \sqrt{kRT} = \sqrt{1.4 \times 4124 \times 293} = \underline{1301 \text{ m/s}}$ Note: We must use the units on R to be J/kg·K in the above equations.
1.92	**b)** The sound will travel at the speed of sound $c = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 293} = 343$ m/s The distance is calculated using $L = c\Delta t = 343 \times 8.32 = \underline{2854 \text{ m}}$

CHAPTER 2

Fluid Statics

FE-type Exam Review Problems: Problems 2-1 to 2-9

2.1 (C)	The pressure can be calculated using: $p = \gamma_{Hg} h$ were h is the height of mercury. $p = \gamma_{Hg} h = (13.6 \times 9810 \text{ N/m}^3) \times (28.5 \times 0.0254) = 96{,}600 \text{ Pa} = \underline{96.6 \text{ kPa}}$
2.2 (D)	Since the pressure varies in a vertical direction, then: $p = p_0 - \rho g h = 84{,}000 \text{ Pa} - 1.00 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 4000 \text{ m} = \underline{44.76 \text{ kPa}}$
2.3 (C)	$p_w = p_{atm} + \gamma_m h_m - \gamma_{water} h_w = 0 + 30{,}000 \times 0.3 - 9810 \times 0.1 = 8020 \text{ Pa} = 8.02 \text{ kPa}$ This is the gage pressure since we used $p_{atm} = 0$.
2.4 (A)	Initially, the pressure in the air is $p_{Air,1} = -\gamma H = -(13.6 \times 9810) \times 0.16 = -21{,}350 \text{ Pa}.$ After the pressure is increased we have: $p_{Air,2} = -21{,}350 + 10{,}000 = -11{,}350 = -13.6 \times 9810 H_2.$ $\therefore H_2 = 0.0851 \text{ m} = 8.51 \text{ cm}$
2.5 (B)	The moment of force P with respect to the hinge, must balance the moment of the hydrostatic force F with respect to the hinge, that is: $(2 \times \frac{5}{3}) \times P = F \times d$ $F = \gamma \bar{h} A = 9.81 \text{ kN/m}^3 \times 1 \text{ m} \times (2 \times \frac{5}{3} \times 3 \text{ m}^2)] \quad \therefore F = 98.1 \text{ kN}$ The location of F is at $y_p = \bar{y} + \frac{\bar{I}}{\bar{y} A} = 1.67 + \frac{3(3.33)^3/12}{1.67(3.33 \times 3)} = 2.22 \text{ m} \Rightarrow d = 3.33 - 2.22 = 1.11 \text{ m}$ $3.33 \times P = 98.1 \times 1.11 \quad \therefore P = 32.7 \text{ kN}$
2.6 (A)	The gate opens when the center of pressure is at the hinge: $\bar{y} = \frac{1.2 + h}{2} + 5. \quad y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = \frac{11.2 + h}{2} + \frac{b(1.2+h)^3/12}{(1.2+h)b(11.2+h)/2} = 5 + 1.2$ This can be solved by trial-and-error, or we can simply substitute one of the answers into the equation and check to see if it is correct. This yields $h = 1.08$ m.

2.7 (D)	The hydrostatic force will pass through the center, and so F_H will be balanced by the force in the hinge and the force P will be equal to F_V. $\therefore P = F_V = 9.81\times 4\times 1.2\times w + 9.81\times(\pi\times 1.2^2/4)\times w = 300. \quad \therefore w = 5.16$ m.
2.8 (A)	The weight is balanced by the buoyancy force which is given by $F_B = \gamma \cancel{V}$ where $\cancel{V}$ is the displaced volume of fluid: $900\times 9.81 = 9810\times 0.01\times 15w. \quad \therefore w = 6$ m
2.9 (A)	The pressure on the plug is due to the initial pressure plus the pressure due to the acceleration of the fluid, that is: $p_{plug} = p_{initial} + \gamma_{gasoline}\Delta Z$ where $\Delta Z = \Delta x \dfrac{a_x}{g}$ $p_{plug} = 20,000 + 6660\times(1.2\times\dfrac{5}{9.81}) = 24,070$ Pa $F_{plug} = p_{plug}A = 24,070\times\pi\times 0.02^2 = 30.25$ N

Pressure

2.12	Since $p = \gamma h$, then $h = p/\gamma$ **a)** $h = 250,000/9810 = 25.5$ m **c)** $h = 250,000/(13.6\times 9810) = 1.874$ m
2.14	This requires that $p_{water} = p_{Hg} \quad\Rightarrow\quad (\gamma h)_{water} = (\gamma h)_{Hg}$ **b)** $9810\times h = (13.6\times 9810)\times 0.75 \quad \therefore h = 10.2$ m
2.16	$\Delta p = -\gamma\Delta z \Rightarrow \Delta p = -1.27\times 9.81\ (3000) = -37,370$ Pa or -37.37 kPa
2.18	From the given information the specific gravity is $S = 1.0 + z/100$ since $S(0) = 1$ and $S(10) = 1.1$. By definition $\rho = 1000\ S$, where $\rho_{water} = 1000$ kg/m^3. Using $dp = -\gamma dz$ then, by integration we write: $\int_0^p dp = \int_0^{10} 1000(1+z/100)g\,dz = 1000g\left(z+\dfrac{z^2/2}{100}\right)$ $p = 1000\times 9.81\left(10+\dfrac{10^2}{2\times 100}\right)$ = 103,000 Pa or 103 kPa

	Note: we could have used an average S: S_{avg} = 1.05, so that ρ_{avg} = 1050 kg/m^3 and so $p=\gamma h \;\Rightarrow\; p=1050\times 9.81\times 10=103{,}005$ Pa
2.20	From Eq. (2.4.8): $p = p_{atm}[(T_0-\alpha z)/T_0]^{g/\alpha R}$ = 100 [(288 – 0.0065 × 300)/288]$^{9.81/0.0065\times 287}$ = 96.49 kPa Assuming constant density, then $p = p_{atm}-\rho g h=100-\left(\dfrac{100}{0.287\times 288}\right)\times 9.81\times 300/1000$ = 96.44 kPa % error = $\dfrac{96.44-96.49}{96.49}\times 100$ = –0.052% Since the error is small, the density variation can be ignored over heights of 300 m or less.
2.22	Eq. 1.5.11 gives $B=\rho\left.\dfrac{dp}{d\rho}\right\vert_T$ But, dp = ρgdh. Therefore $\rho g dh=\dfrac{B}{\rho}d\rho$ or $\dfrac{d\rho}{\rho^2}=\dfrac{g}{B}dh$ Integrate, using ρ_0 = 1064 kg/m^3, and B = 2.1 × 10^9 N/m^2 $\displaystyle\int_2^{\rho}\frac{d\rho}{\rho^2}=\frac{g}{B}\int_0^h dh \quad \therefore -\left(\frac{1}{\rho}-\frac{1}{1064}\right)=\left(\frac{9.81}{2.1\times 10^9\ \left(\text{N/m}^2\right)}\right)h=4.67\times 10^{-9}h$ This gives $\rho=\dfrac{1}{9.4\times 10^{-4}-4.67\times 10^{-9}h}$ Now $\displaystyle p=\int_0^h \rho g dh=\int_0^h\frac{g}{9.4\times 10^{-4}-4.67\times 10^{-9}h}dh$ $\displaystyle =\frac{g}{-4.67\times 10^{-9}}\ln(9.4\times 10^{-4}-4.67\times 10^{-9}h)$ If we assume ρ = const: $p=\rho g h=1064\times 9.81\times h=10{,}438\,h$ **b)** For h = 1500 m: p_{accurate} = 15,690 kPa and p_{estimate} = 15,657 kPa. % error $=\dfrac{15{,}657-15{,}690}{15{,}690}\times 100=$ –0.21%
2.24	Use Eq. 2.4.8: p = $101(288-0.0065z/288)^{\frac{9.81}{0.0065\times 287}}$

	a) for $z = 3000$ $\therefore p = \underline{69.9\text{ kPa}}$. **c)** for $z = 9000$ $\therefore p = \underline{30.6\text{ kPa}}$.

Manometers

2.26	Referring to Fig. 2.7b, the pressure in the pipe is: $p = \gamma h = (13.6 \times 9810) \times 0.25 = 33{,}350$ Pa or $\underline{33.35\text{ kPa}}$
2.28	Referring to Fig. 2.7a, the pressure in the pipe is $p = \rho g h$. If $p = 2400$ Pa, then $2400 = \rho g h = \rho \times 9.81\ h$ or $\rho = \dfrac{2400}{9.81\,h}$ **a)** $\rho = \dfrac{2400}{9.81 \times 0.36} = 680\text{ kg/m}^3$ $\therefore$ The fluid is $\underline{\text{gasoline}}$ **c)** $\rho = \dfrac{2400}{9.81 \times 0.245} = 999\text{ kg/m}^3$ $\therefore$ The fluid is $\underline{\text{water}}$
2.30	See Fig. 2.7b: The pressure in the pipe is given by $p_1 = -\gamma_1 h + \gamma_2 H$ $p_1 = -0.86 \times 9810 \times 0.125 + 13.6 \times 9810 \times 0.24 = 30{,}695$ Pa or $\underline{30{,}96\text{ kPa}}$
2.32	$p_{water} - p_{oil} = \gamma_{oil} \times 0.3 + \gamma_{Hg} \times H - \gamma_{water} \times 0.2$ $40{,}000 - 16{,}000 = 920 \times 9.81 \times 0.3 + 13{,}600 \times 9.81 \times H - 1000 \times 9.81 \times 0.2$ Solving for H we get: $H = 0.1743$ m or $\underline{17.43\text{ cm}}$
2.34	$p_{water} = \gamma_{Hg}(0.16) - \gamma_{water}(0.02) - \gamma_{Hg}(0.04) - \gamma_{water}(0.02)$ Using $\gamma_{water} = 9.81\text{ kN/m}^3$ and $\gamma_{Hg} = 13.6 \times 9.81\text{ kN/m}^3$ $p_{water} = \underline{15.62\text{ kPa}}$
2.36	$p_{water} - 9.81 \times 0.12 - 0.68 \times 9.81 \times 0.1 + 0.86 \times 9.81 \times 0.1 = p_{oil}$ With $p_{water} = 15$ kPa, $p_{oil} = \underline{14\text{ kPa}}$
2.38	$p_{gage} = p_{air} + \gamma_{water} \times 4$ *where,* $p_{air} = p_{atm} - \gamma_{Hg} \times H$ $\Rightarrow p_{gage} = -\gamma_{Hg} \times H + \gamma_{water} \times 4$ $p_{gage} = -13.6 \times 9.81 \times 0.16 + 9810 \times 4 = \underline{17.89\text{ kPa}}$ Note: we subtracted atmospheric pressure since we need the gage pressure.

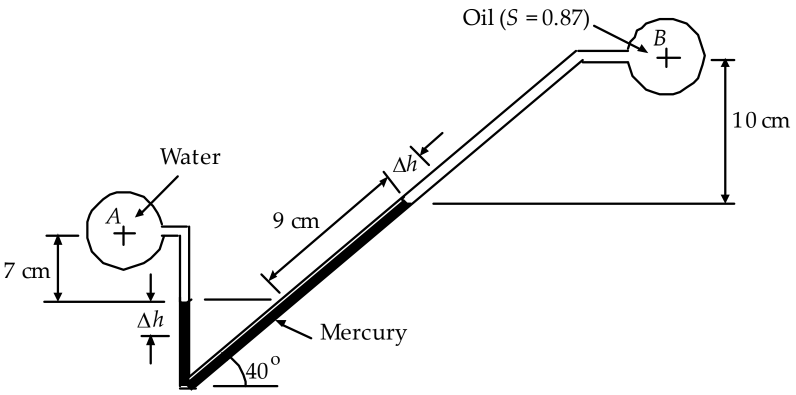
2.40

$$p + 9810 \times 0.05 + 1.59 \times 9810 \times 0.07 - 0.8 \times 9810 \times 0.1 = 13.6 \times 9810 \times 0.05$$

$$\therefore p = 5873 \text{ Pa} \qquad \text{or} \qquad \underline{5.87 \text{ kPa}}$$

2.42

The distance the mercury drops on the left equals the distance along the tube that the mercury rises on the right. This is shown in the sketch.



From the previous problem we have:

$$\left(p_B\right)_1 = p_A + \gamma_{\text{water}} \times 0.07 - \gamma_{\text{HG}} \times 0.09 \sin 40 - \gamma_{\text{oil}} \times 0.1 = 2.11 \text{ kPa} \qquad (1)$$

For the new condition:

$$\left(p_B\right)_2 = p_A + \gamma_{\text{water}} \times \left(0.07 + \Delta h\right) - \gamma_{\text{HG}} \times 0.11 \sin 40 - \gamma_{\text{oil}} \times \left(0.1 - \Delta h \sin 40\right) \qquad (2)$$

where Δh in this case is calculated from the new manometer reading as:

$$\Delta h + \Delta h / \sin 40 = 11 - 9 \text{ cm} \quad \Rightarrow \quad \Delta h = 0.783 \text{ cm}$$

Subtracting Eq.(1) from Eq.(2) yields:

$$\left(p_B\right)_2 - \left(p_B\right)_1 = \gamma_{water} \times \left(\Delta h\right) - \gamma_{HG} \times 0.02 \sin 40 - \gamma_{oil} \times \left(-\Delta h \sin 40\right)$$

Substituting the value of Δh gives:

$$\left(p_B\right)_2 = 2.11 + \left[\left(0.00783\right) - 13.6 \times 0.02 \sin 40 - 0.87 \times \left(-0.00783 \sin 40\right)\right] \times 9.81$$
$$= \underline{0.52 \text{ kPa}}$$

2.44

a) Using Eq. (2.4.16):

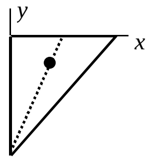
$$p_1 = \gamma_1\left(z_2 - z_1\right) + \gamma_2 h + \left(\gamma_3 - \gamma_2\right) H \quad \text{where} \quad h = z_5 - z_2 = 17 - 16 = 1 \text{ cm}$$

$$4000 = 9800(0.16 - 0.22) + 15{,}600(0.01) + (133{,}400 - 15{,}600)H$$

$$\therefore H = 0.0376 \text{ m or } \underline{3.76 \text{ cm}}$$

2.46	From No. 2.30: $p_{oil} = 14.0$ kPa From No. 2.36: $p_{oil} = p_{water} - 9.81 \times (0.12+\Delta z) - 0.68 \times 9.81 \times (0.1-2\Delta z) + 0.86 \times 9.81\times (0.1-\Delta z)$ $\therefore \Delta z = 0.0451$ m or $\underline{4.51\text{ cm}}$

Forces on Plane Areas

2.48	The hydrostatic force is calculated using: $F = \gamma \bar{h} A$ where, $\bar{h} = 10$ m, and $A = \pi R^2 = \pi\left(0.15\text{ m}\right)^2$ hence, $F = 9810 \times 10 \times \pi\left(0.15\right)^2 = \underline{6934\text{ N}}$
2.50	For saturated ground, the force on the bottom tending to lift the vault is: $F = p_c A = 9800 \times 1.5 \times (2 \times 1) = 29{,}400$ N The weight of the vault is approximately: $W = \rho g \rlap{\,-}V_{\text{walls}}$ $W = 2400 \times 9.81\left[2\left(2\times 1.5\times 0.1\right) + 2\left(2\times 1\times 0.1\right) + 20\left(0.8\times 1.3\times 0.1\right)\right] = \underline{28{,}400\text{ N}}$ The vault will tend to rise out of the ground.
2.52	**b)** Since the triangle is horizontal the force is due to the uniform pressure at a depth of 10 m. That is, $F = pA$, where $p = \gamma h = 9.81\times 10 = 98.1\text{ kN/m}^2$ The area of the triangle is $A = bh/2 = 2.828 \times 2/2 = 2.828\text{ m}^2$ $F = 98.1 \times 2.828 = \underline{277.4\text{ kN}}$
2.54	**a)** $F = \gamma \bar{h} A = 9.81\times 6\times \pi 2^2 = \underline{739.7\text{ kN}}$ $y_p = \bar{y} + \dfrac{\bar{I}}{A\bar{y}} = 6 + \dfrac{\pi \times 2^4/4}{4\pi \times 6} = 6.167\text{ m} \quad \therefore (x, y)_p = \underline{(0, -0.167)\text{ m}}$ **c)** $F = 9.81 \times (4 + 4/3) \times 6 = \underline{313.9\text{ kN}}$ $y_p = 5.333 + \dfrac{3\times 4^3/36}{5.333\times 6} = 5.50\text{ m} \quad \therefore y = -1.5$ $4/2.5 = \dfrac{1.5}{x} \quad \therefore x = 0.9375$ $\therefore (x, y)_p = \underline{(0.9375, -1.5)\text{ m}}$ 
2.56	$F = \gamma \bar{h} A = 9810 \times 6 \times 20 = 1.777 \times 10^6$ N, or 1177 kN

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 7.5 + \frac{4 \times 5^3 / 12}{7.5 \times 20} = 7.778 \text{ m}$$

$\Sigma M_{Hinge} = 0 \Rightarrow (10 - 7.778)\ 1177 = 5\ P$ $\therefore P = \underline{523 \text{ kN}}$

2.60

The vertical height of water is $h = \sqrt{1.2^2 - 0.4^2} = 1.1314$ m

The area of the gate can be split into two areas: $A = A_1 + A_2$ or

$$A = 1.2 \times 1.1314 + 0.4 \times 1.1314 = 1.8102 \text{ m}^2$$

Use 2 forces: $F_1 = \gamma \bar{h}_1 A_1 = 9810 \times 0.5657 \times (1.2 \times 1.1314) = 7534$ N

$$F_2 = \gamma \bar{h}_2 A_2 = 9810 \times \frac{1.1314}{3} \times (0.4 \times 1.1314) = 1674 \text{ N}$$

The location of F_1 is at $y_{p1} = \frac{2}{3}(1.1314) = 0.754$ m, and F_2 is at

$$y_{p2} = \bar{y} + \frac{\bar{I}_2}{A_2 \bar{y}} = \frac{1.1314}{3} + \frac{0.4 \times 1.1314^3 / 36}{0.4 \times (1.1314/2) \times (1.1314/3)} = 0.5657 \text{ m}$$

$$\Sigma M_{hinge} = 0: \ 7534 \times \frac{1.1314}{3} + 1674 \times (1.1314 - 0.5657) - 1.1314P = 0$$

$\therefore \underline{P = 3346 \text{ N}}$

2.62

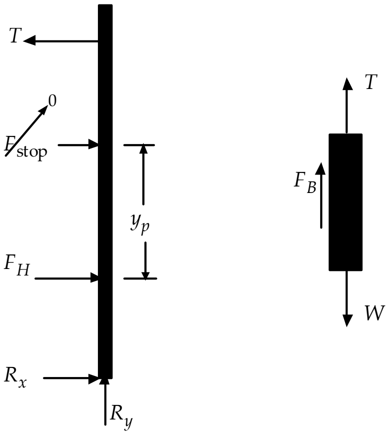
The gate is about to open when the center of pressure is at the hinge.

b) $y_p = 1.2 + H = (2.0/2 + H) + \dfrac{b \times 2^3/12}{(1+H)2b}$

$\therefore H = \underline{0.6667 \text{ m}}$

2.64

A free-body-diagram of the gate and block is sketched.



Sum forces on the block:

$$\Sigma F_y = 0 \qquad \therefore W = T + F_B$$

where F_B is the buoyancy force which is given by

$$F_B = \gamma\left[\pi R^2 (3 - H)\right]$$

Take moments about the hinge:

$$T \times 3.5 = F_H \times (3 - y_p)$$

where F_H is the hydrostatic force acting on the gate. It is, using

$$\bar{h} = 1.5 \text{ m and } A = 2\times 3 = 6 \text{ m}^2$$

$$F_H = \gamma \bar{h} A = \left(9.81 \text{ kN/m}^3\right)\left(1.5 \text{ m}\times 6 \text{ m}^2\right) = 88.29 \text{ kN}$$

From the given information

$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 1.5 + \frac{2\left(3^3\right)/12}{1.5\times 6} = 2 \text{ m}$$

$$\therefore T = \frac{88.29\times(3-2)}{3.5} = 25.23 \text{ kN}$$

$$F_B = W - T = 70 - 25.23 = 44.77 \text{ kN}. \qquad \therefore \gamma\pi R^2(3-H) = 44.77$$

$$H = 3 \text{ m} - \frac{44.77 \text{ kN}}{\left(9.81 \text{ kN/m}^3\right)\pi(1 \text{ m})^2} = \underline{1.55 \text{ m}}$$

2.66

The dam will topple if there is a net clockwise moment about “O”

The weight of the dam consists of the weight of the rectangular area + a triangular area, that is: $W = W_1 + W_2$. The force F3 acting on the bottom of the dam can be divided into two forces:

Fp1 due to the uniform pressure distribution and

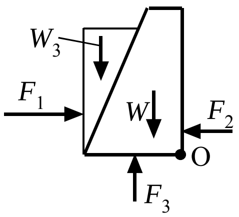
Fp2 due to the linear pressure distribution.

b)

$$W_1 = 2.4\times 9810\times 18.9\times 1.8 = 801 \text{ kN}$$

$$W_2 = 2.4\times 9810\times 18.9\times 7.2/2 = 1602 \text{ kN}$$

assume 1 m deep



$$W_3 = 9810\times(18\times 6.86/2) = 605.7 \text{ kN}$$

$$F_1 = 9810\times 9\times 18 = 1589 \text{ kN}$$

$$F_2 = 9810\times 1.5\times 3 = 44.14 \text{ kN}$$

$$F_{p1} = 9810\times 3\times 9 = 265 \text{ kN}$$

$$F_{p2} = 9810\times 15\times 9/2 = 662.2 \text{ kN}$$

ΣM_O: (1589)(6) + (265)(4.5) +

(662.2)(6) – (801)(0.9) – (44.14)(3/3)

– (1602)(4.2) – (605.7)(6.37) = 3392 kN · m > 0. $\therefore$ <u>will tip</u>

Forces on Curved Surfaces

2.68

Since all infinitesimal pressure forces pass through the center, we can place the resultant forces at the center. Since the vertical components pass through the bottom point, they produce no moment about that point. Hence, consider only horizontal forces:

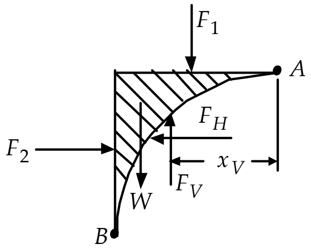
$$(F_H)_{water} = \left(\gamma \bar{h} A\right)_{water} = 9.81 \times 2 \times (4 \times 10) = 784.8 \text{ kN}$$

$$(F_H)_{oil} = \left(\gamma \bar{h} A\right)_{oil} = 0.86 \times 9.81 \times 1 \times 20 = 168.7 \text{ kN}$$

$$\Sigma M:\ 2P = 784.8 \times 2 - 168.7 \times 2. \qquad \therefore \underline{P = 616.1 \text{ kN}}$$

2.70

A free-body-diagram of the volume of water in the vicinity of the surface is shown.



Force balances in the horizontal and vertical directions give:

$$F_H = F_2$$
$$F_V = W + F_1$$

where F_H and F_V are the horizontal and vertical components of the force acting on the water by the surface *AB*. Hence

$$F_H = F_2 = \left(9.81 \text{ kN/m}^3\right)\left(8+1\right)\left(2 \times 4\right) = 706.3 \text{ kN}$$

The line of action of F_H is the same as that of F_2. Its distance from the surface is

$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 9 + \frac{4\left(2^3\right)/12}{9 \times 8} = 9.037 \text{ m}$$

To find F_V we find W and F_1:

$$W = \gamma \mathcal{V} = \left(9.81 \text{ kN/m}^3\right)\left[2 \times 2 - \frac{\pi}{4}\left(2^2\right)\right] \times 4 = 33.7 \text{ kN}$$

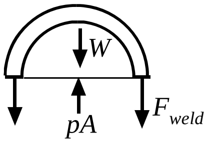
$$F_1 = 9.81 \text{ kN/m}^3\left(8 \times 2 \times 4\right) = 628 \text{ kN}$$

$$\therefore F_V = F_1 + W = 33.7 + 628 = 662 \text{ kN}$$

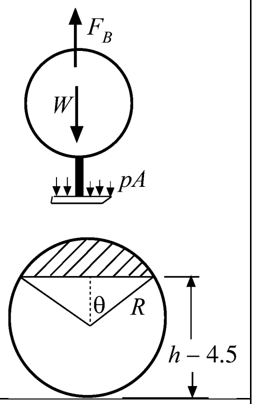
To find the line of action of F_V, we take moments at point *A*:

$$F_V \times x_V = F_1 \times d_1 + W \times d_2$$

where $d_1 = 1 \text{ m},$ and $d_2 = \dfrac{2R}{3\left(4-\pi\right)} = \dfrac{2 \times 2}{3\left(4-\pi\right)} = 1.553 \text{ m}:$

	$\therefore x_V = \frac{F_1 \times d_1 + W \times d_2}{F_V} = \frac{628\times 1 + 33.7\times 1.553}{662} = 1.028 \text{ m}$ Finally, the forces F_H and F_V that act on the surface *AB* are equal and opposite to those calculated above. So, on the surface, F_H acts to the right and F_V acts downward.
2.72	Place the resultant $\mathbf{F}_H + \mathbf{F}_V$ at the center. $\mathbf{F}_V$ passes through the hinge. The moment of $\mathbf{F}_H$ must equal the moment of P with respect to the hinge: $2 \times (9.81 \times 1 \times 10) = 2.8 \text{ P} \qquad \therefore P = \underline{70.1 \text{ kN}}$
2.74	The resultant $\mathbf{F}_H + \mathbf{F}_V$ of the unknown liquid acts through the center of the circular arc. $\mathbf{F}_V$ passes through the hinge. Thus, we use only (F_H). Assume 1 m wide: $F_H = \gamma_x \bar{h} A = \gamma_x (R/2)(R\times 1) = \gamma_x R^2/2$ The horizontal force due to the water is $F_w = \gamma_w \bar{h} A = \gamma_w (R/2)(R\times 1) = \gamma_w R^2/2$ The weight of the gate is $W = S\gamma_w \cancel{V} = 0.2\gamma_w \left(\pi R^2/4\right)\times 1$ Summing moments about the hinge: $F_w(R/3) + W(4R/3\pi) = F_H \times R$ **a)** $\left(9810\times\frac{R^2}{2}\right)\times\frac{R}{3} + \left(0.2\times 9810\frac{\pi R^2}{4}\right)\times\frac{4R}{3\pi} = \left(\gamma_x \frac{R^2}{2}\right)\times R$ $\therefore \gamma_x = \underline{4580 \text{ N/m}^3}$
2.76	The pressure in the dome is: **a)** $p = 60{,}000 - 9810 \times 3 - 0.8 \times 9810 \times 2 = 14{,}870$ Pa or 14.87 kPa The force is $F = pA_{projected} = (\pi \times 3^2) \times 14.87 = \underline{420.4 \text{ kN}}$ **b)** From a free-body diagram of the dome filled with oil:  $F_{weld} + W = pA$ Using the pressure from part (a): $F_{weld} = 14{,}870 \times \pi \times 3^2 - (0.8 \times 9810) \times \frac{1}{2}\left(\frac{4}{3}\pi \times 3^3\right) = -23{,}400$ or $\underline{-23.4 \text{ kN}}$

Buoyancy

2.78	Under static conditions the weight of the barge + load = weight of displaced water. **(a)** $20{,}000 + 250{,}000 = 9810 \times 3\,(6d + d^2/2)$. $\therefore d^2 + 12d - 18.35 = 0$ $\therefore d =$ 1.372 m
2.80	The weight of the cars will be balanced by the weight of displaced water: $15{,}000 \times 60 = 9810(7.5 \times 90 \times \Delta d\,)$ $\therefore \Delta d = 0.136$ m or 13.6 cm
2.82	$T + F_B = W$ (See Fig. 2.11 c) $T = 40{,}000 - 1.59 \times 9810 \times 2 = 8804$ N or 8.804 kN
2.86	At the limit of lifting: $F_B = W + pA$ where p is the pressure acting on the plug. **(b)** Assume $h > 4.5 + R$ and use the above equation with $R = 0.4$ m and $h = 4.92$ $F_B = \gamma_w V = \gamma_w \times 3 \times \left(\pi R^2 - A_{segment}\right) = 9810 \times 3 \times 0.2803 = 8249\text{ N}$ $W + pA = 6670\text{ N} + 9810 \times 4.92 \times \pi\left(0.1\right)^2 = 8234\text{ N}$ Hence, the plug will lift for $h > 4.92$ m 
2.88	**(a)** When the hydrometer is completely submerged in water: $W = \gamma_w V \;\Rightarrow\; (0.01 + m_{Hg})9.81 = 9810\left[\frac{\pi \times 0.015^2}{4} \times 0.15 + \frac{\pi \times 0.005^2}{4} \times 0.12\right]$ $\therefore m_{Hg} = 0.01886$ kg When the hydrometer without the stem is submerged in a fluid: $W = \gamma_x V \;\Rightarrow\; (0.01 + 0.0189)9.81 = S_x \times 9810 \times \frac{\pi\left(0.015\right)^2}{4} \times 0.15$ $\therefore S_x =$ 1.089

Stability

2.90	With ends horizontal $I_o = \pi d^4 / 64$ The displaced volume is $\mathcal{V} = W / \gamma_{water} \Rightarrow$ $\mathcal{V} = \left(\gamma_x \pi d^2 h/4\right)/9810 = 8.01\times10^{-5}\gamma_x d^3$ since $h = d$ The depth the cylinder will sink is depth = $\frac{\mathcal{V}}{A} = \frac{8.01\times10^{-5}\gamma_x d^3}{\pi d^2/4} = 10.20\times10^{-5}\gamma_x d$ The distance $\overline{CG}$ is $\overline{CG} = \frac{h}{2} - 10.2\times10^{-5}\gamma_x d/2$. Then $\overline{GM} = \frac{I_O}{\mathcal{V}} - \overline{CG} = \frac{\pi d^4/64}{8.01\times10^{-5}\gamma_x d^3} - \frac{d}{2} + 10.2\times10^{-5}\gamma_x d/2 > 0$ This gives (divide by d and multiply by γ_x): $612.8 - 0.5\,\gamma_x + 5.1\times10^{-5}\,\gamma_x^2 > 0$. Consequently $\gamma_x >$ <u>8368 N/m³</u> or $\gamma_x <$ <u>1436 N/m³</u>
2.92	As shown, $\bar{y} = \frac{16\times9+16\times4}{16+16}$ = 6.5 cm above the bottom edge. $G = \frac{4\gamma\times9.5+16\gamma\times8.5+16S_A\gamma\times4}{0.5\gamma\times8+2\gamma\times8+S_A\gamma\times16}$ = 6.5 cm $\therefore 130 + 104\,S_A = 174 + 64\,S_A$ $\therefore S_A =$ <u>1.1</u>
2.94	The centroid C is 1.5 m below the water surface $\therefore \overline{CG} = 1.5$ m Using Eq. 2.4.47: $\overline{GM} = \frac{\ell\times8^3/12}{\ell\times8\times3} - 1.5 = 1.777 - 1.5 = 0.277 > 0$ $\therefore$ <u>The barge is stable</u>

Linearly Accelerating Containers

2.96	**(a)** $p_{max} = -1000\times20\,(0-4) - 1000\,(9.81)\,(0-2) =$ <u>99,620 Pa</u> **(c)** $p_{max} = -1000\times18\,(0-3.6) - 1000\,(9.81+18)\,(0-1.8)$ = 114,860 Pa or <u>114.86 kPa</u>

2.98

Use Eq. 2.5.2:

b) $$60{,}000 = -1000\,a_x\,(-8) - 1000\,(9.81 + 10)\left(-2.5 + \sqrt{\frac{8a_x}{9.81}}\right)$$

$$60 = 8\,a_x + 49.52 - 19.81\sqrt{\frac{8a_x}{19.81}} \quad \text{or} \quad a_x - 1.31 = 1.574\sqrt{a_x}$$

$$a_x^2 - 5.1\,a_x + 1.44 = 0 \qquad \therefore a_x = 0.25,\ \underline{4.8\text{ m/s}^2}$$

2.100

a) The pressure on the end *AB* (*z* is zero at *B*) is, using Eq. 2.5.2

$$p(z) = -1000 \times 10\,(-7.626) - 1000 \times 9.81(z) = 76{,}260 - 9810\,z$$

$$\therefore F_{AB} = \int_0^{2.5} (76{,}260 - 9810z)4dz = 640{,}000\text{ N} \quad \text{or} \quad \underline{640\text{ kN}}$$

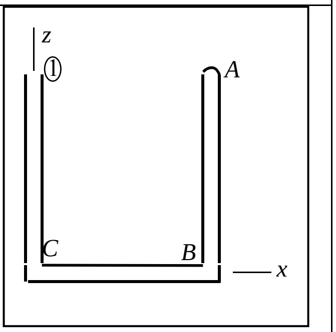
b) The pressure on the bottom *BC* is

$$p(x) = -1000 \times 10\,(x - 7.626) = 76{,}260 - 10{,}000\,x$$

$$\therefore F_{BC} = \int_0^{7.626} (76{,}260 - 10{,}000x)4dx = 1.163 \times 10^6\text{ N} \quad \text{or} \quad \underline{1163\text{ kN}}$$

2.102

Use Eq. 2.5.2 with position 1 at the open end:



b) $p_A = -1000 \times 10\,(0.9 - 0) = -\underline{9000\text{ Pa}}$

$p_B = -1000 \times 10\,(0.9) - 1000 \times 9.81(-0.6)$

$= -\underline{3114\text{ Pa}}$

$p_C = -1000 \times 9.81 \times (-0.6) = \underline{5886\text{ Pa}}$

e) $p_A = 1000 \times 18\,\left(-1.5 \times 0.625\right) = \underline{-16{,}875\text{ Pa}}$

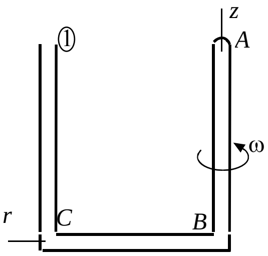
$p_B = 1000 \times 18\,\left(-1.5 \times 0.625\right) - 1000 \times 9.81\,\left(-0.625\right) = \underline{-10{,}740\text{ Pa}}$

$p_C = -1000 \times 9.81\,\left(-0.625\right) = \underline{6130\text{ Pa}}$

Rotating Containers

2.104

Use Eq. 2.6.4 with position 1 at the open end:



a) $p_A = \dfrac{1}{2} \times 1000 \times 10^2\,(0 - 0.9^2) = \underline{-40{,}500\text{ Pa}}$

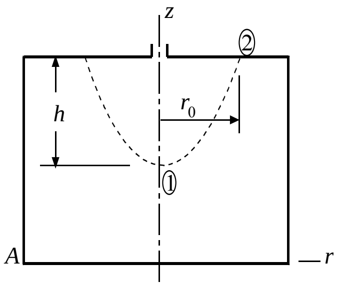
$p_B = -40{,}500 + 9810 \times 0.6 = \underline{-34{,}600\text{ Pa}}$

$p_C = 9810 \times 0.6 = \underline{5886\text{ Pa}}$

2.106

The air volume before and after is equal

$$\therefore \frac{1}{2}\pi r_0^2 h = \pi \times 0.6^2 \times 0.2 \qquad \therefore r_0^2 h = 0.144$$



(a) Using Eq. 2.6.5: $r_0^2 \times 5^2 / 2 = 9.81\, h$

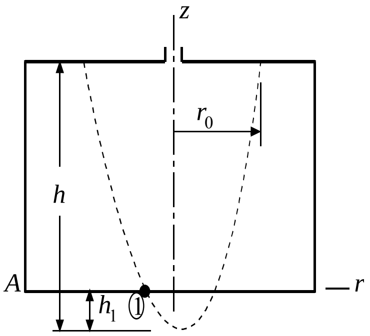
$\therefore h = 0.428$ m

$$\therefore p_A = \frac{1}{2} \times 1000 \times 5^2 \times 0.6^2 - 9810\,(-0.372)$$

= <u>8149 Pa</u>

(c) For $\omega = 10$, part of the bottom is bared

$$\pi \times 0.6^2 \times 0.2 = \frac{1}{2}\pi r_0^2 h - \frac{1}{2}\pi r_1^2 h_1$$



Using Eq. 2.6.5:

$$\frac{\omega^2 r_0^2}{2g} = h, \qquad \frac{\omega^2 r_1^2}{2g} = h_1$$

$$\therefore 0.144 = \frac{2g}{\omega^2}h^2 - \frac{2g}{\omega^2}h_1^2 \qquad \text{or}$$

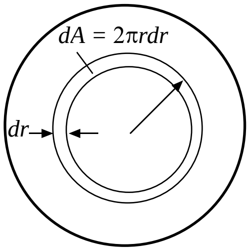
$$h^2 - h_1^2 = \frac{0.144 \times 10^2}{2 \times 9.81}$$

Also, $h - h_1 = 0.8$. $1.6\,h - 0.64 = 0.7339$. $\therefore h = 0.859$ m, $r_1 = 0.108$ m

$$\therefore p_A = \frac{1}{2} \times 1000 \times 10^2\,(0.6^2 - 0.108^2) = \underline{17{,}400\text{ Pa}}$$

2.108

$$p(r) = \frac{1}{2}\rho\omega^2 r^2 - \rho g[0 - (0.8 - h)]$$



$$p(r) = 500\omega^2 r^2 + 9810(0.8 - h) \qquad \text{if } h < 0.8$$

$$p(r) = 500\omega^2(r^2 - r_1^2) \qquad \text{if } h > 0.8$$

a) $F = \int p2\pi r dr = 2\pi \int_0^{0.6} (12{,}500r^3 + 3650r)dr = \underline{6670\text{ N}}$

(We used $h = 0.428$ m)

c) $F = \int p2\pi r dr = 2\pi \int_{-0.108}^{0.6} (50{,}000(r^3 - 0.108^2 r)dr = \underline{9520\text{ N}}$

(We used $r_1 = 0.108$ m)

CHAPTER 3

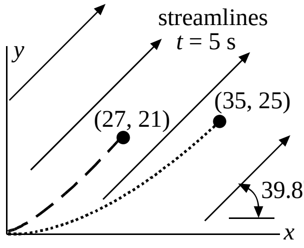
Introduction to Fluids in Motion

FE-type Exam Review Problems: Problems 3-1 to 3-9

3.1 (D)	$\hat{\mathbf{n}}\cdot\mathbf{V}=0 \qquad (n_x\hat{\mathbf{i}}+n_y\hat{\mathbf{j}})\cdot(3\hat{\mathbf{i}}-4\hat{\mathbf{j}})=0$ or $3n_x-4n_y=0$ Also $n_x^2+n_y^2=1$ since $\hat{\mathbf{n}}$ is a unit vector. A simultaneous solution yields $n_x=4/5$ and $n_y=3/5$. (Each with a negative sign would also be OK)
3.2 (C)	$\mathbf{a}=\frac{\partial \mathbf{V}}{\partial t}+u\frac{\partial \mathbf{V}}{\partial x}+v\frac{\partial \mathbf{V}}{\partial y}+w\frac{\partial \mathbf{V}}{\partial z}=2xy(2y\hat{\mathbf{i}})-y^2(2x\hat{\mathbf{i}}-2y\hat{\mathbf{j}})=-16\hat{\mathbf{i}}+8\hat{\mathbf{i}}+16\hat{\mathbf{j}}$ $\therefore \lvert a\rvert=\sqrt{(-8)^2+16^2}=17.89\text{ m/s}$
3.3 (D)	$a_x=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=u\frac{\partial u}{\partial x}=\frac{10}{(4-x)^2}\frac{\partial}{\partial x}\left[10(4-x)^{-2}\right]$ $=\frac{10}{(4-x)^2}10(-2)(-1)(4-x)^{-3}=\frac{10}{4}\times 20\times\frac{1}{8}=6.25\text{ m/s}^2$
3.4 (C)	The only velocity component is $u(x)$. We have neglected $v(x)$ since it is quite small. If $v(x)$ were not negligible, the flow would be two-dimensional.
3.5 (B)	$\frac{V^2}{2}=\frac{p}{\rho}=\frac{\gamma_{water}h}{\rho_{air}}=\frac{9810\times0.800}{1.23} \qquad \therefore V=113\text{ m/s}$
3.6 (C)	$\frac{V_1^2}{2g}+\frac{p}{\gamma}=\frac{V_2^2}{2g} \qquad \frac{V_1^2}{2g}+0.200=0.600 \qquad \therefore V=\sqrt{2\times9.81\times0.400}=2.80\text{ m/s}$
3.7 (B)	The manometer reading h implies: $\frac{V_1^2}{2}+\frac{p_1}{\rho}=\frac{V_2^2}{2}+\frac{p_2}{\rho}$ or $V_2^2=\frac{2}{1.13}(60-10.2) \qquad \therefore V_2=\underline{9.39\text{ m/s}}$ The temperature (the viscosity of the water) and the diameter of the pipe are not needed.

3.8 (A)	$\frac{\cancel{V_1^2}}{\cancel{2g}} + \frac{p_1}{\gamma} = \frac{V_2^2}{2g} + \frac{\cancel{p_2}}{\cancel{\gamma}}$ $\qquad \frac{800{,}000}{9810} = \frac{V_2^2}{2 \times 9.81}$ $\qquad \therefore V_2 = 40 \text{ m/s}$
3.9 (D)	$p_1 = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) = \frac{902}{2}\left(30^2 - 15^2\right) = \underline{304{,}400 \text{ Pa}}$

Flow Fields

3.14	**a)** $u = \frac{dx}{dt} = 2t + 2 \qquad v = \frac{dy}{dt} = 2t$ $x = t^2 + 2t + c_1 \qquad y = t^2 + c_2$ $= y + 2\sqrt{y}$ $\therefore x^2 - 2xy + y^2 = 4y \qquad \therefore \text{parabola}$ 
3.16	Lagrangian: Several college students would be hired to ride bikes around the various roads, making notes of quantities of interest. Eulerian: Several college students would be positioned at each intersection and quantities would be recorded as a function of time.
3.18	**a)** $\cos\alpha = \frac{\mathbf{V}\cdot\hat{\mathbf{i}}}{V} = \frac{1+2}{\sqrt{3^2+2^2}} = 0.832 \qquad \therefore \alpha = \underline{33.69^\circ}$ $\mathbf{V}\cdot\hat{\mathbf{n}} = 0 \qquad (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\cdot(n_x\hat{\mathbf{i}} + n_y\hat{\mathbf{j}}) = 0 \qquad \left.\begin{array}{r} 3n_x + 2n_y = 0 \\ n_x^2 + n_y^2 = 1 \end{array}\right\} \therefore \begin{array}{l} n_y = -\frac{3}{2}n_x \\ n_x^2 + \frac{9}{4}n_x^2 = 1 \end{array}$ $\therefore n_x = \frac{2}{\sqrt{13}}, \quad n_y = -\frac{3}{\sqrt{13}} \quad \text{or} \quad \underline{\hat{\mathbf{n}} = \frac{1}{\sqrt{13}}(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})}$ **c)** $\cos\alpha = \frac{\mathbf{V}\cdot\hat{\mathbf{i}}}{V} = \frac{5}{\sqrt{5^2+(-8)^2}} = 0.6202 \qquad \therefore \alpha = \underline{-51.67^\circ}$ $\mathbf{V}\cdot\hat{\mathbf{n}} = 0 \qquad (5\hat{\mathbf{i}} - 8\hat{\mathbf{j}})\cdot(n_x\hat{\mathbf{i}} + n_y\hat{\mathbf{j}}) = 0 \qquad \left.\begin{array}{r} 5n_x - 8n_y = 0 \\ n_x^2 + n_y^2 = 1 \end{array}\right\} \therefore \begin{array}{l} n_x = \frac{8}{5}n_y \\ \frac{64}{25}n_y^2 + n_y^2 = 1 \end{array}$

	$\therefore n_y = \frac{5}{\sqrt{89}},\ \ n_x = \frac{8}{\sqrt{89}}$ or $\underline{\hat{\mathbf{n}} = \frac{1}{\sqrt{89}}(8\hat{\mathbf{i}}+5\hat{\mathbf{j}})}$
3.20	**b)** $u\frac{\partial \mathbf{V}}{\partial x}+v\frac{\partial \mathbf{V}}{\partial y}+w\frac{\partial \mathbf{V}}{\partial z}+\frac{\partial \mathbf{V}}{\partial t} = 2x(2\hat{\mathbf{i}})+2y(2\hat{\mathbf{j}}) = 4x\hat{\mathbf{i}}+4y\hat{\mathbf{j}} = \underline{8\hat{\mathbf{i}}-4\hat{\mathbf{j}}}$
3.22	The vorticity $\boldsymbol{\omega} = 2\boldsymbol{\Omega}$. **a)** $\boldsymbol{\omega} = \underline{-40\hat{\mathbf{i}}}$ **c)** $\boldsymbol{\omega} = \underline{12\hat{\mathbf{i}}-4\hat{\mathbf{k}}}$
3.24	**a)** $a_r = \left(10-\frac{40}{r^2}\right)\cos\theta\left(\frac{80}{r^3}\right)\cos\theta-\left(10+\frac{40}{r^2}\right)\frac{\sin\theta}{r}\left(1-\frac{40}{r^2}\right)(-\sin\theta)$ $-\frac{1}{r}\left(10+\frac{40}{r^2}\right)^2\sin^2\theta = (10-2.5)(-1)1.25(-1)\ = \underline{9.375\text{ m/s}^2}$ $a_\theta = \left(10-\frac{40}{r^2}\right)\cos\theta\left(\frac{80}{r^3}\right)\sin\theta+\left(10+\frac{40}{r^2}\right)\frac{\sin\theta}{r}\left(10+\frac{40}{r^2}\right)\cos\theta$ $-\frac{1}{r}\left(100-\frac{1600}{r^4}\right)\sin\theta\cos\theta\ = \underline{0}$ since $\sin(180°) = 0$ $a_\phi = \underline{0}$
3.26	$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t}+u\cancel{\frac{\partial \mathbf{V}}{\partial x}}+\cancel{v}\frac{\partial \mathbf{V}}{\partial y}+\cancel{w}\frac{\partial \mathbf{V}}{\partial z} = \underline{\frac{\partial u}{\partial t}\hat{\mathbf{i}}}$ For steady flow $\partial u/\partial t = 0$ so that $\underline{\mathbf{a} = 0}$
3.28	**b)** $u = 2(1-0.5^2)(1-e^{-t/10}) = \ \underline{1.875\text{ m/s}}$ at $t = \infty$
3.30	$\frac{D\rho}{Dt} = u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}+\frac{\partial\rho}{\partial t} = 10(-1.23\times10^{-4}e^{-3000\times10^{-4}})$ $= \ \underline{-9.11\times10^{-4}\ \text{kg/m}^3\cdot\text{s}}$
3.32	$\frac{D\rho}{Dt} = u\frac{\partial\rho}{\partial x} = 4\times(0.01)\ = \underline{0.04\text{ kg/m}^3\cdot\text{s}}$

3.34	$\left.\begin{aligned} a_x &= \frac{\partial u}{\partial t} + \mathbf{V}\cdot\nabla u \\ a_y &= \frac{\partial v}{\partial t} + \mathbf{V}\cdot\nabla v \\ a_z &= \frac{\partial w}{\partial t} + \mathbf{V}\cdot\nabla w \end{aligned}\right\} \quad \therefore \mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}$
3.36	$\mathbf{\Omega} = \frac{2\pi}{24\times 60\times 60}\hat{\mathbf{k}} = 7.272\times 10^{-5}\hat{\mathbf{k}}\ \text{rad/s}$ $\mathbf{V} = 5(-0.707\hat{\mathbf{i}} - 0.707\hat{\mathbf{k}}) = -3.535\hat{\mathbf{i}} - 3.535\hat{\mathbf{k}}\ \text{m/s}$ $\mathbf{A} = 2\mathbf{\Omega}\times\mathbf{V} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})$ $= 2\times 7.272\times 10^{-5}\hat{\mathbf{k}}\times(-3.535\hat{\mathbf{i}} - 3.535\hat{\mathbf{k}}) + 7.272\times 10^{-5}\hat{\mathbf{k}}$ $\times\left[7.272\times 10^{-5}\hat{\mathbf{k}}\times(6.4\times 10^{6})(-0.707\hat{\mathbf{i}} + 0.707\hat{\mathbf{k}})\right]$ $= \underline{-52\times 10^{-5}\hat{\mathbf{j}} + 0.0224\hat{\mathbf{i}}\ \text{m/s}^2}$ Note: We have neglected the acceleration of the earth relative to the sun since it is quite small (it is $d^2\mathbf{S}/dt^2$). The component $(-51.4\times 10^{-5}\hat{\mathbf{j}})$ is the Coriolis acceleration and causes air motions to move c.w. or c.c.w. in the two hemispheres.

Classification of Fluid Flows

3.38	Steady: **a**, **c**, **e**, **f**, **h** Unsteady: **b**, **d**, **g**
3.42	**a)** inviscid **c)** inviscid **e)** viscous inside the boundary layers and separated regions. **g)** viscous.
3.46	$\text{Re} = \frac{VL}{\nu} = \frac{0.18\times 0.75}{1.4\times 10^{-5}} = 9640 \qquad \therefore \underline{\text{Turbulent}}$
3.48	**a)** $\text{Re} = \frac{VD}{\nu} = \frac{1.2\times 0.01}{1.51\times 10^{-5}} = 795 \qquad \underline{\text{Always laminar}}$
3.50	Assume the flow is parallel to the leaf. Then $3\times 10^5 = Vx_T/\nu$ $\therefore x_T = 3\times 10^5\nu/V = 3.5\times 10^5\times 1.4\times 10^{-4}/6 = 8.17\ \text{m}$ The flow is expected to be <u>laminar</u>

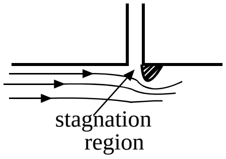
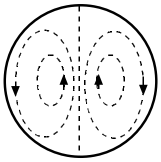
3.52	$\frac{D\rho}{Dt} = u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z} + \frac{\partial\rho}{\partial t} = 0$ For a steady, plane flow $\partial\rho/\partial t = 0$ and $w = 0$ Then $u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} = 0$

Bernoulli's Equation

3.54	$\frac{V^2}{2} = \frac{p}{\rho}$ Use ρ = 1.1 kg/m^3 **a)** $v = \sqrt{2p/\rho} = \sqrt{2 \times 2000/1.1}$ = $\underline{60 \text{ m/s}}$
3.56	$\frac{V^2}{2} + \frac{p}{\rho} = 0$ $\therefore V = \sqrt{\frac{-2p}{\rho}} = \sqrt{\frac{2 \times 2000}{1.23}}$ = $\underline{57.0 \text{ m/s}}$
3.58	$\frac{V^2}{2} + \frac{p}{\rho} = \frac{U_\infty^2}{2} + \cancel{\frac{p_\infty}{\rho}}$ **b)** Let $r = r_c$: $\underline{p_T = \frac{\rho}{2}U_\infty^2}$ **d)** Let $\theta = 90^\circ$: $\underline{p_{90} = -\frac{3}{2}\rho U_\infty^2}$
3.60	$\frac{V^2}{2} + \frac{p}{\rho} = \frac{U_\infty^2}{2} + \cancel{\frac{p_\infty}{\rho}}$ **a)** $p = \frac{\rho}{2}\left(U_\infty^2 - u^2\right) = \frac{\rho}{2}\left[10^2 - \left(10 + \frac{20\pi}{2\pi x}\right)^2\right] = 50\rho\left[1 - \left(1 + \frac{1}{x}\right)^2\right]$ $= \underline{-50\rho\left(\frac{2}{x} + \frac{1}{x^2}\right)}$ **c)** $p = \frac{\rho}{2}\left(U_\infty^2 - u^2\right) = \frac{\rho}{2}\left[30^2 - \left(30 + \frac{60\pi}{2\pi x}\right)^2\right] = 450\rho\left[1 - \left(1 + \frac{1}{x}\right)^2\right]$ $= \underline{-450\rho\left(\frac{2}{x} + \frac{1}{x^2}\right)}$

3.62	Assume the velocity in the plenum is zero. Then $\frac{V_1^2}{2}+\frac{p_1}{\rho}=\frac{V_2^2}{2}+\frac{p_2}{\rho}$ or $V_2^2=\frac{2}{1.13}(60-10.2)$ $\therefore V_2=\underline{9.39\text{ m/s}}$ We found $\rho=1.13\text{ kg / m}^3$ in Table B.2.
3.64	Bernoulli from the stream to the pitot probe: $p_T=\rho\frac{V^2}{2}+p$ Manometer: $p_T+\gamma H-\gamma_{Hg}H-\gamma h=p-\gamma h$ Then, $\rho\frac{V^2}{2}+p+\gamma H-\gamma_{Hg}H=p$ $\therefore V^2=\frac{\gamma_{Hg}-\gamma}{\rho}(2H)$ **a)** $V^2=\frac{(13.6-1)9800}{1000}(2\times 0.04)$ $\therefore V=\underline{3.14\text{ m/s}}$ **b)** $V^2=\frac{(13.6-1)9800}{1000}(2\times 0.1)$ $\therefore V=\underline{4.97\text{ m/s}}$
3.66	The pressure at 90° from Problem 3.58 is $p_{90}=-3\rho U_\infty^2/2$. The pressure at the stagnation point is $p_T=\rho U_\infty^2/2$. The manometer provides: $p_T-\gamma H=p_{90}$ $\frac{1}{2}\times 1.204U_\infty^2-9800\times 0.04=-\frac{3}{2}\times 1.204U_\infty^2.$ $\therefore U_\infty=\underline{12.76\text{ m/s}}$
3.68	Bernoulli: $\frac{V_2^2}{2g}+\cancel{\frac{p_2}{\gamma}}=\frac{V_1^2}{2g}+\frac{p_1}{\gamma}$ Manometer: $p_1+\gamma z+\gamma_{Hg}H-\gamma H-\gamma z=\frac{V_2^2}{2g}\gamma+\cancel{p_2}$ Substitute Bernoulli's into the manometer equation: $p_1+\left(\gamma_{Hg}-\gamma\right)H=\frac{V_1^2}{2g}\gamma+p_1$ **b)** Use H = 0.05 m: $\frac{V_1^2\times 9800}{2\times 9.81}=(13.6-1)9800\times 0.05$ $\therefore V_1=\underline{3.516\text{ m/s}}$ Substitute into Bernoulli: $p_1=\frac{V_2^2-V_1^2}{2g}\gamma=\frac{20^2-3.516^2}{2\times 9.81}\times 9800=\underline{193{,}600\text{ Pa}}$

3.70	Write Bernoulli's equation between points 1 and 2 along the center streamline: $p_1+\frac{\rho V_1^2}{2}+\gamma z_1=p_2+\frac{\rho V_2^2}{2}+\gamma z_2$ Since the flow is horizontal, $z_1=z_2$ and Bernoulli's equation becomes $p_1+1000\times\frac{0.5^2}{2}=p_2+1000\times\frac{1.125^2}{2}$ From fluid statics, the pressure at 1 is $p_1=\gamma h=9810\times 0.25=2452\text{ Pa}$ and at 2, using $p_2=\gamma H$, Bernoulli's equation predicts $2452+1000\times\frac{0.5^2}{2}=9810H+1000\times\frac{1.125^2}{2}$ $\quad\therefore H=0.1982\text{ m}$ or $\underline{19.82\text{ cm}}$
3.72	Assume incompressible flow ($V < 100$ m/s) with point 1 outside the wind tunnel where $p_1=0$ and $V_1=0$. Bernoulli's equation gives $0=\frac{V_2^2}{2}+\frac{p_2}{\rho}\qquad\therefore p_2=-\frac{1}{2}\rho V_2^2$ **a)** $\rho=\frac{p}{RT}=\frac{90}{0.287\times 253}=1.239\text{ kg/m}^3\quad\therefore p_2=-\frac{1}{2}\times 1.239\times 100^2=\underline{-6195\text{ Pa}}$ **c)** $\rho=\frac{p}{RT}=\frac{92}{0.287\times 293}=1.094\text{ kg/m}^3\quad\therefore p_2=-\frac{1}{2}\times 1.094\times 100^2=\underline{-5470\text{ Pa}}$
3.74	Bernoulli across nozzle: $\cancel{\frac{V_1^2}{2}}+\frac{p_1}{\rho}=\frac{V_2^2}{2}+\cancel{\frac{p_2}{\rho}}\qquad\therefore V_2=\sqrt{2p_1/\rho}$ Bernoulli to max. height: $\cancel{\frac{V_1^2}{2g}}+\frac{p_1}{\gamma}+\cancel{h_1}=\cancel{\frac{V_2^2}{2g}}+\cancel{\frac{p_2}{\gamma}}+h_2\qquad\therefore h_2=p_1/\gamma$ **a)** $V_2=\sqrt{2p_1/\rho}=\sqrt{2\times 700{,}000/1000}=\underline{37.42\text{ m/s}}$ $h_2=p_1/\gamma=700{,}000/9800=\underline{71.4\text{ m}}$ **b)** $V_2=\sqrt{2p_1/\rho}=\sqrt{2\times 1{,}400{,}000/1000}=\underline{52.92\text{ m/s}}$ $h_2=p_1/\gamma=1{,}400{,}000/9800=\underline{142.9\text{ m}}$
3.76	$\cancel{\frac{V_1^2}{2}}+\frac{p_1}{\rho}=\frac{V_2^2}{2}+\frac{p_2}{\rho}\qquad p_2=-100{,}000$ Pa, the lowest possible pressure.

	a) $\frac{600,000}{1000}=\frac{V_2^2}{2}-\frac{100,000}{1000}$ $\therefore V_2 = \underline{37.4\text{ m/s}}$ **b)** $\frac{300,000}{1000}=\frac{V_2^2}{2}-\frac{100,000}{1000}$ $\therefore V_2 = \underline{28.3\text{ m/s}}$
3.78	**b)** $p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{902}{2}\left(2^2-10^2\right)=\underline{-43,300\text{ Pa}}$ **d)** $p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{1.23}{2}\left(2^2-10^2\right)=\underline{-59.0\text{ Pa}}$
3.80	Apply Bernoulli's equation between the exit (point 2) where the radius is *R* and a point 1 in between the exit and the center of the tube at a radius *r* less than *R*: $\frac{V_1^2}{2}+\frac{p_1}{\rho}=\frac{V_2^2}{2}+\frac{p_2}{\rho}$ $\therefore p_1=\rho\frac{V_2^2-V_1^2}{2}$ Since $V_2 < V_1$, we see that p_1 is negative (a vacuum) so that the envelope would tend to rise due to the negative pressure over most of its area (except for a small area near the end of the tube).
3.82	A burr downstream of the opening will create a region that acts similar to a stagnation region thereby creating a high pressure since the velocity will be relatively low in that region. 
3.84	The higher pressure at *B* will force the fluid toward the lower pressure at *A*, especially in the wall region of slow moving fluid, thereby causing a secondary flow normal to the pipe's axis. This results in a relatively high loss for an elbow. 

CHAPTER 4

The Integral Forms of the Fundamental Laws

FE-type Exam Review Problems: Problems 4-1 to 4-15

4.1 (B)	
4.2 (D)	$\dot{m} = \rho AV = \frac{p}{RT} AV = \frac{200}{0.287 \times 293} \pi \times 0.04^2 \times 70 = 0.837 \text{ kg/s}$
4.3 (A)	Refer to the circle of Problem 4.27: $Q = AV = (\pi \times 0.4^2 \times \frac{75.7 \times 2}{360} - 0.10 \times 0.40 \times \sin 75.5^\circ) \times 3 = 0.516 \text{ m}^3\text{/s}$
4.4 (D)	$\frac{\dot{W}_P}{\gamma Q} = \cancel{\frac{V_2^2 - V_1^2}{2g}} + \frac{p_2 - p_1}{\gamma} \qquad \frac{\dot{W}_P}{\gamma \times 0.040} = \frac{1200 - 200}{\gamma}$ $\therefore \dot{W}_P = 40 \text{ kW}$ and energy required = $\frac{40}{0.85} = 47.1 \text{ kW}$
4.5 (A)	$0 = \frac{\cancel{V_2^2} - V_1^2}{2g} + \frac{p_2 - \cancel{p_1}}{\gamma} \qquad 0 = \frac{-120^2}{2 \times 9.8} + \frac{p_2}{9810}. \quad \therefore p_2 = 7{,}200{,}000 \text{ Pa}$
4.6 (C)	Manometer: $\gamma H + p_1 = \rho g \frac{V_2^2}{2g} + \cancel{p_2}$ or $9810 \times 0.02 + p_1 = \rho g \frac{V_2^2}{2g}$ Energy: $K \frac{7.96^2}{2 \times 9.81} = \frac{100{,}000}{9810} \qquad \therefore K = 3.15$ Combine the equations: $9810 \times 0.02 = 1.2 \times \frac{V_1^2}{2} \qquad \therefore V_1 = 18.1 \text{ m/s}$
4.7 (B)	$h_L = K \frac{V^2}{2g} = \frac{\Delta p}{\gamma} \qquad V = \frac{Q}{A} = \frac{0.040}{\pi \times 0.04^2} = 7.96 \text{ m/s}$ $K \frac{7.96^2}{2 \times 9.81} = \frac{100{,}000}{9810} \qquad \therefore K = 3.15$

4.8 (C)	$\frac{\dot{W}_P}{\gamma Q} = \cancel{\frac{V_2^2 - V_1^2}{2g}} + \frac{\Delta p}{\gamma}$ $\dot{W}_P = Q\Delta p = 0.040\times 400 = 16\text{ kW}$ $\frac{\dot{W}_P}{\eta} = \frac{16}{0.89} = 18.0\text{ kW}$
4.9 (D)	$36.0 + 15 = \frac{4.58^2}{2\times 9.81} + \frac{p_B}{9810} + 3.2\frac{7.16^2}{2\times 9.81}$ $\therefore p_B = \underline{416{,}000\text{ Pa}}$ In the above energy equation we used $h_L = K\frac{V^2}{2g}$ with $V = \frac{Q}{A} = \frac{0.2}{\pi\times 0.2^2} = 4.42\text{ m/s}$
4.10 (A)	$V = \frac{Q}{A} = \frac{0.1}{\pi\times 0.04^2} = 19.89\text{ m/s}$ Energy—surface to entrance: $H_P = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + K\frac{V_2^2}{2g}$ $\therefore H_P = \frac{19.89^2}{2\times 9.81} + \frac{180{,}000}{9810} + 50 + 5.6\frac{19.89^2}{2\times 9.81} = 201.4\text{ m}$ $\therefore \dot{W}_P = \gamma Q H_P / \eta_P = 9810\times 0.1\times 201.4/0.75 = \underline{263{,}000\text{ W}}$
4.11 (A)	After the pressure is found, that pressure is multiplied by the area of the window. The pressure is relatively constant over the area.
4.12 (C)	$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} = \frac{V_2^2}{2g} + \cancel{\frac{p_2}{\gamma}}.$ $p_1 = 9810\times\frac{(6.25^2 - 1)\times 12.73^2}{2\times 9.81} = 3{,}085{,}000\text{ Pa}$ $p_1 A_1 - F = \rho Q(V_2 - V_1).$ $3{,}085{,}000\times\pi\times 0.05^2 - F = 1000\times 0.1\times 12.73(6.25 - 1)$ $\therefore F = \underline{17{,}500\text{ N}}$
4.13 (D)	$-F_x = \dot{m}(V_{2x} - V_{1x}) = 1000\times 0.01\times 0.2\times 50(50\cos 60^\circ - 50) = -2500\text{ N}$
4.14 (A)	$-F_x = \dot{m}(V_{r2x} - V_{r1x}) = 1000\times\pi\times 0.02^2\times 60\times(40\cos 45^\circ - 40) = 884\text{ N}$ $\text{Power} = F_x\times V_B = 884\times 20 = 17{,}700\text{ W}$
4.15 (A)	Let the vehicle move to the right. The scoop then diverts the water to the right. Then, $F = \dot{m}(V_{2x} - V_{1x}) = 1000\times 0.05\times 2\times 60\times[60 - (-60)] = 720{,}000\text{ N}$

Basic Laws

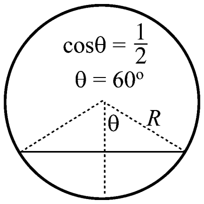
4.16	**b)** The energy transferred to or from the system must be zero: $\underline{Q - W = 0}$
4.20	**b)** The conservation of mass. **d)** The energy equation.

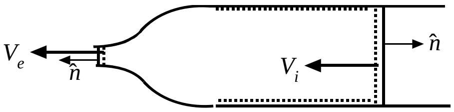
System-to-Control-Volume Transformation

4.24	$\hat{\mathbf{n}}_1 = -\frac{1}{\sqrt{2}}\hat{\mathbf{i}} - \frac{1}{\sqrt{2}}\hat{\mathbf{j}} = -0.707(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ $\quad \hat{\mathbf{n}}_2 = \underline{0.866\hat{\mathbf{i}} - 0.5\hat{\mathbf{j}}}$. $\quad \hat{\mathbf{n}}_3 = \underline{-\hat{\mathbf{j}}}$ $V_{1n} = \mathbf{V}_1 \cdot \hat{\mathbf{n}}_1 = 10\hat{\mathbf{i}} \cdot \left[-0.707(\hat{\mathbf{i}} + \hat{\mathbf{j}})\right] = \underline{-7.07\text{ m/s}}$ $V_{2n} = \mathbf{V}_2 \cdot \hat{\mathbf{n}}_2 = 10\hat{\mathbf{i}} \cdot (0.866\hat{\mathbf{i}} - 0.5\hat{\mathbf{j}}) = \underline{8.66\text{ m/s}}$ $V_{3n} = \mathbf{V}_3 \cdot \hat{\mathbf{n}}_2 = 10\hat{\mathbf{i}} \cdot (-\hat{\mathbf{j}}) = \underline{\ 0}$
4.26	$(\mathbf{B} \cdot \hat{\mathbf{n}})A = 15(0.5\hat{\mathbf{i}} + 0.866\hat{\mathbf{j}}) \cdot \hat{\mathbf{j}}(10 \times 12) = 15 \times 0.866 \times 120 = 1559\text{ cm}^3$ Volume = $15 \sin 60^\circ \times 10 \times 12 = \underline{1559\text{ cm}^3}$

Conservation of Mass

4.32	Use Eq. 4.4.2 with m_V representing the mass in the volume: $0 = \frac{dm_V}{dt} + \int_{c.s.} \rho \hat{\mathbf{n}} \cdot \mathbf{V} dA = \frac{dm_V}{dt} + \rho A_2 V_2 - \rho A_1 V_1$ $= \frac{dm_V}{dt} + \rho Q - \dot{m}$ Finally $\underline{\frac{dm_V}{dt} = \dot{m} - \rho Q}$
4.34	$A_1 V_1 = A_2 V_2$ $\quad \pi \times \frac{3^2}{10^4} \times 18 = \pi \times \frac{6^2}{10^4} V_2$ $\quad \therefore V_2 = \underline{4.5\text{ m/s}}$ $\dot{m} = \rho A V = 1000\pi \frac{3^2}{10^4} \times 18 = \underline{51\text{ kg/s}}$ $Q = AV = \pi \frac{3^2}{10^4} \times 18 = \underline{0.051\text{ m}^3\text{/s}}$

4.38	$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad \rho_1 = \frac{p_1}{RT} = \frac{500}{0.287\times393} = 4.433\ \text{kg/m}^3$ $\rho_2 = \frac{1246}{0.287\times522} = 8.317\ \text{kg/m}^3$ $4.433\ \pi\times0.05^2\times600 = 8.317\ \pi\times0.05^2\ V_2 \qquad \therefore V_2 = \underline{319.8\ \text{m/s}}$ $\dot m = \rho_1 A_1 V_1 = \underline{20.89\ \text{kg/s}} \qquad Q_1 = A_1 V_1 = \underline{4.712\ \text{m}^3/\text{s}} \qquad Q_2 = \underline{2.512\ \text{m}^3/\text{s}}$
4.40	**b)** $(2\times1.5+1.5\times1.5)\ 3 = \pi\frac{d_2^2}{4}\times\frac{2}{2} \qquad \therefore d_2 = 4.478\ \text{m}$ 
4.42	**a)** Since the area is rectangular, $V = \underline{5\ \text{m/s}}$. $\dot m = \rho AV = 1000\times0.08\times0.8\times5 = \underline{320\ \text{kg/s}} \qquad Q = \frac{\dot m}{\rho} = \underline{0.32\ \text{m}^3/\text{s}}$ **c)** $V\times0.08 = 10\times0.04+5\times0.02+5\times0.02 \qquad \therefore V = \underline{7.5\ \text{m/s}}$ $\dot m = \rho AV = 1000\times0.08\times0.8\times7.5 = \underline{480\ \text{kg/s}} \qquad \dot Q = \frac{\dot m}{\rho} = \underline{0.48\ \text{m}^3/\text{s}}$
4.44	If $dm/dt = 0$, then $\rho_1 A_1 V_1 = \rho_2 A_2 V_2 + \rho_3 A_3 V_3$. In terms of $\dot m_2$ and Q_3 this becomes, letting $\rho_1 = \rho_2 = \rho_3$ $1000\times\pi\times0.02^2\times12 = \dot m_2 + 1000\times0.01 \qquad \therefore \dot m_2 = \underline{5.08\ \text{kg/s}}$
4.46	$\dot m_{in} = \dot m_{out} + \dot m \qquad \rho\times0.2\times2\times10 = \left[\rho\int_0^{0.1}10(20y-100y^2)2dy + \rho\times0.1\times2\times10\right] + \dot m$ Note: We see that at y = 0.1 m the velocity $u(0.1)$ = 10 m/s. Thus, we integrate to y = 0.1, and between y = 0.1 and 0.2 the velocity u = 10: $4\rho = \left[\frac{4}{3}\rho + 2\rho\right] + \dot m \qquad \therefore \dot m = 0.6667\rho = \underline{0.82\ \text{kg/s}}$
4.48	$\dot m = \int\rho V dA = \int_0^{0.1}1165(1-1.42y)(y-5y^2)12\times1.5dy$ $= 20{,}970\int_0^{0.1}(y-6.42y^2+7.1y^3)\,dy = \underline{63.7\ \text{kg/s}}$ $\bar V = \frac{2}{3}u_{\max} = \frac{2}{3}\times0.6 = 0.4\ \text{m/s}$ (See Prob. 4.43b)

	$\bar{\rho}=\dfrac{1165+1000}{2}$ = 1082 kg/m^3 $\therefore \bar{\rho}\bar{V}A=1082\times0.4\times(1.5\times0.1)$ = $\underline{64.9\text{ kg/s}}$ Thus, $\bar{\rho}\bar{V}A\neq\dot{m}$ since $\rho=\rho(y)$ and $V=V(y)$ so that $\bar{\rho}\bar{V}\neq\overline{\rho V}$
4.50	$2000\times\dfrac{4}{3}\pi\times0.0015^3\dfrac{\text{m}^3\text{ of H}_2\text{O}}{\text{m}^3\text{ of air}}\times9000\times5\dfrac{\text{m}^3\text{ of air}}{\text{s}}$ = 1.5 × (1.5 h) $\therefore h$ = $\underline{0.565\text{ m}}$
4.52	$\dot{m}_{\text{in}}=\dot{m}_2+\dot{m}_3$ $\bar{V}_1$ = 20 m/s (see Prob. 4.43c) $20\times1000\pi\times0.02^2=10+1000\pi\times0.02^2\times\bar{V}_3$ $\therefore \bar{V}_3$ = $\underline{12.04\text{ m/s}}$
4.54	The control surface is close to the interface at the instant shown.  $\therefore V_i$ = interface velocity $\rho_eA_eV_e=\rho_iA_iV_i$ $1.5\times\pi\times0.15^2\times300=\dfrac{8000}{0.287\times673}\pi\times1^2\times V_i$ $\therefore V_i$ = $\underline{0.244\text{ m/s}}$
4.56	For an incompressible flow (low speed air flow) $\int_{A_1}udA=A_2V_2$ $\int_0^{0.2}20y^{1/5}\times0.8dy=\pi\times0.15^2\,V_2$ $20\times0.8\dfrac{5}{6}0.2^{6/5}=\pi\times0.15^2\,V_2$ $\therefore V_2=\underline{27.3\text{ m/s}}$
4.58	Draw a control volume around the entire set-up: $0=\dfrac{dm_{tissue}}{dt}+\rho V_2A_2-\rho V_1A_1$ $=\dot{m}_{tissue}+\rho\pi\left(\dfrac{d_2^2-d^2}{4}\right)\dot{h}_2-\rho\pi(h_1\tan\phi)^2\dot{h}_1$ or $\dot{m}_{tissue}=\underline{\rho\pi\left[\dfrac{d^2-d_2^2}{4}\dot{h}_2+h_1^2\dot{h}_1\tan^2\phi\right]}$

4.60

$$0 = \frac{dm}{dt} + \rho A_2 V_2 - \rho A_1 V_1$$

$$= \dot{m} + 1000(\pi \times 0.003^2 \times 0.02 - 10 \times 10^{-6} / 60)$$

$$\dot{m} = \underline{3.99 \times 10^{-4}\ \text{kg/s}}$$

4.62

$$0 = \frac{dm}{dt} + \rho_3 Q_3 - \rho_1 A_1 V_1 - \dot{m}_2 \quad \text{where } m = \rho A h$$

a) $0 = 1000\pi \times 0.6^2 \dot{h} + 1000 \times 0.6 / 60 - 1000\pi \times 0.02^2 \times 10 - 10$

$$\therefore \dot{h} = 0.0111 \text{ m/s} \qquad \text{or} \qquad \underline{11.1 \text{ mm/s}}$$

4.64

Choose the control volume to consist of the air volume inside the tank. The conservation of mass equation is

$$0 = \frac{d}{dt}\int_{CV} \rho d\cancel{V} + \int_{CS} \rho \mathbf{V} \cdot \mathbf{n}\, dA$$

Since the volume of the tank is constant, and for no flow into the tank, the equation is

$$0 = \cancel{V}\frac{d\rho}{dt} + \rho_e V_e A_e$$

Assuming air behaves as an ideal gas, $\rho = \dfrac{p}{RT}$. At the instant of interest, $\dfrac{d\rho}{dt} = \dfrac{1}{RT}\dfrac{dp}{dt}$. Substituting in the conservation of mass equation, we get

$$\frac{dp}{dt} = -\frac{\rho_e V_e A_e}{\cancel{V}}(RT) = -\frac{\left(1.8 \text{ kg/m}^3\right)\left(200 \text{ m/s}\right)\pi\left(0.015 \text{ m}\right)^2}{1.5 \text{ m}^3}\left[0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \times 298 \text{ K}\right]$$

$$\therefore \frac{dp}{dt} = \underline{-14.5 \text{ kPa/s}}$$

Energy Equation

4.66

$$\dot{W} = T\omega + pAV + \mu\frac{du}{dy}A_{\text{belt}}$$

$$= 20 \times 500 \times 2\pi/60 + 400 \times 0.4 \times 0.5 \times 10 + 1.81 \times 10^{-5} \times 100 \times 0.5 \times 0.8$$

$$= 1047 + 800 + 0.000724 = \underline{1847\ \text{W}}$$

4.68

80% of the power is used to increase the pressure while 20% increases the internal energy ($\dot{Q} = 0$ because of the insulation). Hence

$$\dot{m}\Delta\tilde{u} = 0.2\,\dot{W}$$

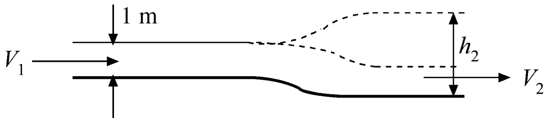
$$1000\times 0.02\times 4.18\Delta T = 0.2\times 500 \qquad \therefore \Delta T = \underline{0.836°\text{C}}$$

4.70

$$-\frac{\dot{W}_T}{\dot{m}g} = -40\times 0.89 \qquad \textbf{b)}\ \dot{W}_T = 40\times 0.89\times(90{,}000/60)\times 9.81 = \underline{523{,}900\text{ W}}$$

4.72

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1 = \frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2$$



$$\frac{4^2}{2\times 9.81}+2 = \frac{4^2}{19.62h_2^2}+h_2$$

$$2.82 = \frac{0.82}{h_2^2}+h_2$$

Continuity: $1 \times 4 = h_2\, V_2$ This can be solved by trial-and-error:

$h_2 = 2.7\text{ m}:\ \ 2.82 \underset{=}{?} 2.81 \qquad h_2 = 2.72\text{ m}:\ \ 2.82 \underset{=}{?} 2.83 \qquad \because h_2 = \underline{2.71\text{ m}}$

$h_2 = 0.6\text{ m}:\ \ 2.82 \underset{=}{?} 2.87 \qquad h_2 = 0.62:\ \ 2.82 \underset{=}{?} 2.75 \qquad \because h_2 = \underline{0.61\text{ m}}$

4.74

Manometer: Position the datum at the top of the right mercury level.

$$9810\times 0.4 + 9810z_2 + p_2 + \frac{V_2^2}{2}\times 1000 = (9810\times 13.6)\times 0.4 + 9810\times 2 + p_1$$

Divide by $\gamma = 9810$: $\quad 0.4 + z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = 13.6\times 0.4 + 2 + \frac{p_1}{\gamma} \qquad (1)$

Energy: $\quad \frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1 = \frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2 \qquad (2)$

Subtract (1) from (2): With $z_1 = 2$ m $\qquad \frac{V_1^2}{2g} = 12.6\times 0.4 \qquad \therefore V_1 = \underline{9.94\text{ m/s}}$

4.76

$Q = 7\times 10^{-3} = \pi\times(0.025)^2 V_1 \qquad \therefore V_1 = 3.75$ m/s

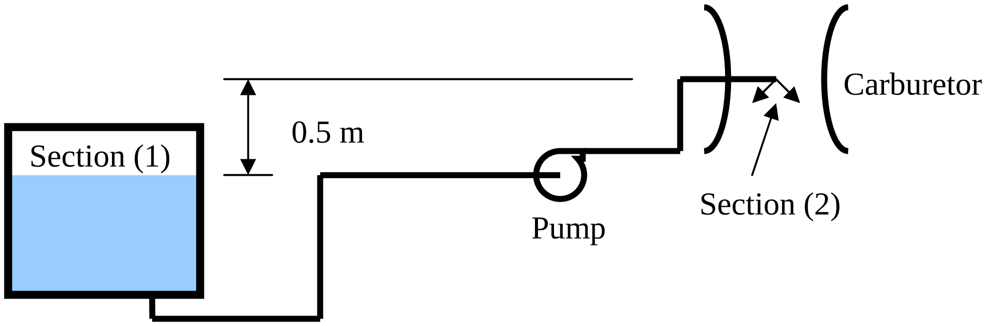
Continuity: $\quad \pi\times(0.025)^2 V_1 = \pi\times(0.0375)^2 V_2 \qquad \therefore V_2 = 1.58$ m/s

Energy: $\quad \frac{V_1^2}{2g}+\frac{p_1}{\gamma} = \frac{V_2^2}{2g}+\frac{p_2}{\gamma}+0.37\frac{V_1^2}{2g}$

	$\therefore p_2 = 414\times10^3 + 1000\left[0.63\frac{3.57^2}{19.62} - \frac{1.58^2}{19.62}\right] = 414.3$ kPa
4.78	$V_1 = Q/A_1 = \frac{0.08}{\pi\times0.03^2}$ = 28.29 m/s $\therefore V_2 = 9V_1 = 254.6$ m/s Energy: $\frac{V_1^2}{2g}+\frac{p_1}{\gamma}=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+0.2\frac{V_1^2}{2g}$ $\therefore p_1 = 9810\left[\frac{254.6^2}{2\times9.81} - 0.8\frac{28.29^2}{2\times9.81}\right] = \underline{32.1\times10^6\text{ Pa}}$
4.80	**a)** Energy: $\frac{V_0^2}{2g}+\frac{p_0}{\gamma}+z_0=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2$ $\therefore V_2 = \sqrt{2gz_0} = \sqrt{2\times9.81\times2.4} = 6.862$ m/s $Q = AV = 0.8\times1\times6.862 = \underline{5.49\text{ m}^3\text{/s}}$ For the second geometry the pressure on the surface is zero but it increases with depth. The elevation of the surface is 0.8 m: $\therefore z_0 = \frac{V_2^2}{2g}+h$ $\therefore V_2 = \sqrt{2g(z_0-h)} = \sqrt{2\times9.81\times2} = 6.264$ m/s $\therefore Q = 0.8\times6.264 = \underline{5.01\text{ m}^3\text{/s}}$ Note: z_0 is measured from the channel bottom in the 2nd geometry. $\therefore z_0 = H + h$
4.82	$\frac{V_0^2}{2g}+\frac{p_0}{\gamma}+z_0=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2$ $\frac{80{,}000}{9810}+4=\frac{V_2^2}{2\times9.81}$ $\therefore V_2 = 19.04$ m/s **b)** $Q = A_2V_2 = \pi\times0.09^2\times19.04 = \underline{0.485\text{ m}^3\text{/s}}$
4.84	Manometer: $\gamma H+\gamma z+p_1=13.6\gamma H+\gamma z+p_2$ $\therefore \frac{p_1}{\gamma}=12.6H+\frac{p_2}{\gamma}$ Energy: $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}.$ Combine energy and manometer: $12.6H=\frac{V_2^2-V_1^2}{2g}$ Continuity: $V_2=\frac{d_1^2}{d_2^2}V_1$ $\therefore V_1^2 = 12.6H\times2g\Big/\left(\frac{d_1^4}{d_2^4}-1\right)$

	$\therefore Q = V_1\pi\frac{d_1^2}{4} = \frac{\pi}{4}\left(\frac{12.6H\times 2g}{d_1^4/d_2^4-1}\right)^{1/2} d_1^2 = \underline{12.35 d_1^2 d_2^2\left(\frac{H}{d_1^4-d_2^4}\right)^{1/2}}$
4.86	**a)** Energy from surface to outlet: $\frac{V_2^2}{2g}=H \qquad \therefore V_2^2 = 2gH$ Energy from constriction to outlet: $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}$ Continuity: $V_1=4V_2$ With $p_1 = p_v$ = 2450 Pa and p_2 = 100,000 Pa $\frac{2450}{9810}+\frac{16}{2\times9.81}\times 2gH = \frac{100,000}{9810}+\frac{1}{2\times9.81}\times2gH \qquad \therefore H = \underline{0.663\text{ m}}$
4.88	Energy: surface to surface: $z_0 = z_2 + h_L \qquad \therefore 30 = 20 + 2\frac{V_2^2}{2g}$ Continuity: $V_1 = 4V_2 \quad \therefore V_1^2 = 160\,g \qquad \therefore V_2^2 = 10g$ Energy: surface to constriction: $30 = \frac{160g}{2g}+\frac{(-94,000)}{9810}+z_1$ $\therefore z_1 = -40.4$ m $\quad \therefore H = 40.4 + 20 = \underline{60.4\text{ m}}$
4.90	Velocity at exit = V_e Velocity in constriction = V_1 Velocity in pipe = V_2 Energy: surface to exit: $\frac{V_e^2}{2g}=H \qquad \therefore V_e^2=2gH$ Continuity across nozzle: $V_2=\frac{D^2}{d^2}V_e$ Also, $V_1=4V_2$ Energy: surface to constriction: $H=\frac{V_1^2}{2g}+\frac{p_v}{\gamma}$ **a)** $5=\frac{1}{2g}\left(16\times\frac{D^4}{0.2^4}\times2g\times5\right)+\frac{-97,550}{9810} \qquad \therefore \underline{D=0.131\text{ m}}$
4.92	$\dot m = pAV = 1000\times\pi\times(0.025)^2\times36 = 70.65$ kg/s $\dot W_P = 70.65\times9.81\left[\frac{9^2-36^2}{2\times9.81}+\frac{830\times1000}{9810}\right]/0.85 = 18,500\text{ W or }\underline{18.5\text{ kW}} = 24.8\text{ hp}$

4.94	$-\dot{W}_T = 2\times 1000\times 9.81\left[\frac{0-10.2^2}{2\times 9.81}+\frac{-600{,}000}{9810}\right]\times 0.87$ $\therefore \dot{W}_T = \underline{1.304\times 10^6\text{ W}}$ We used $V_2 = \frac{Q}{A_2} = \frac{2}{\pi\times 0.25^2} = 10.2$ m/s $\qquad \therefore \eta_T = \underline{0.924}$
4.96	**a)** $\dot{Q}-\dot{W}_S = \dot{m}g\left[\frac{V_2^2-V_1^2}{2g}+\frac{p_2}{\gamma_2}-\frac{p_1}{\gamma_1}+z_2-z_1+\frac{c_v}{g}(T_2-T_1)\right]$ The above is Eq. 4.5.17 with Eq. 4.5.18 and Eq. 1.7.13 $\gamma_1 = \frac{p_1 g}{RT_1} = \frac{85\times 9.81}{0.287\times 293} = 9.92\text{ N/m}^3 \qquad \gamma_2 = \frac{600\times 9.81}{0.287\ T_2} = \frac{20{,}500}{T_2}$ $\therefore -(-1{,}500{,}000) = 5\times 9.81\left[\frac{200^2}{2\times 9.81}+\frac{600{,}000T_2}{20{,}500}-\frac{85{,}000}{9.92}+\frac{716.5}{9.81}(T_2-293)\right]$ $\therefore T_2 = 572$ K or $\underline{299°\text{C}}$ Be careful of units. $p_2 = 600{,}000$ Pa $\qquad c_v = 716.5\text{ J/kg}\cdot\text{K}$
4.98	Energy: surface to exit: $-\dot{W}_T\eta_T = \dot{m}g\left[\frac{V_2^2}{2g}-20+4.5\frac{V_2^2}{2g}\right]$ $V_2 = \frac{15}{\pi\times 0.6^2} = 13.26$ m/s $\qquad \dot{m}g = Q\gamma = 15\times 9810 = 147{,}150$ N/s $-\dot{W}_T\times 0.8 = 147{,}150\left[\frac{13.26^2}{2\times 9.81}-20+4.5\frac{13.26^2}{2\times 9.81}\right] \qquad \therefore \dot{W}_T = \underline{5390\text{ kW}}$
4.100	Choose a control volume that consists of the entire system and apply conservation of energy: $H_P+\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 = H_T+\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$



We recognize that $H_T = 0$, V_1 is negligible and $h_L = 210\, V^2/2g$ where $V = Q/A$ and $V_2 = Q/A_2$. Rearranging we get:

$$H_P = \frac{p_2 - p_1}{\gamma} + \frac{V_2^2}{2g} + z_2 - z_1 + h_L$$

$$V = \frac{Q}{A} = \frac{\left(6.3\times10^{-6}\,\text{m}^3/\text{s}\right)}{\pi\left(2.5\times10^{-3}\,\text{m}\right)^2} = 0.321\text{ m/s} \quad \Rightarrow \quad h_L = 210\frac{(0.321)^2}{2\times9.81} = 1.1\text{ m}$$

$$V_2 = \frac{Q}{A_2} = \frac{6.3\times10^{-6}\,\text{m}^3/\text{s}}{\pi\left(4\times10^{-4}\,\text{m}\right)^2} = 12.53\text{ m/s}$$

Substituting the given values we get:

$$H_P = \frac{(95-100)\ \text{kN/m}^2}{6.660\ \text{kN/m}^3} + 0.5\text{ m} + \frac{(12.53\text{ m/s})^2}{2\left(9.81\text{ m/s}^2\right)} + 1.1\text{ m} = 8.85\text{ m}$$

The power input to the pump is:

$$\dot{W}_P = \gamma Q H_P/\eta_P = \left(6660\text{ N/m}^3\right)\left(6.3\times10^{-6}\text{ m}^3/\text{s}\right)\left(8.85\text{ m}\right)/0.75 = \underline{0.5\text{ W}}$$

4.102

Energy: across the nozzle: $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \qquad V_2 = \frac{5^2}{2^2}V_1 = 6.25V_1$

$$\therefore \frac{400{,}000}{9810} + \frac{V_1^2}{2\times9.81} = \frac{6.25^2V_1^2}{2\times9.81} \qquad \therefore V_1 = 4.58\text{ m/s}, \quad V_A = 7.16\text{ m/s}$$

$$V_2 = 28.6\text{ m/s}$$

Energy: surface to exit:

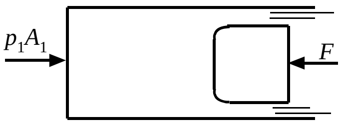
$$H_P + 15 = \frac{28.6^2}{2\times9.81} + 1.5\frac{4.58^2}{2\times9.81} + 3.2\frac{7.16^2}{2\times9.81} \qquad \therefore H_P = 36.8\text{ m}$$

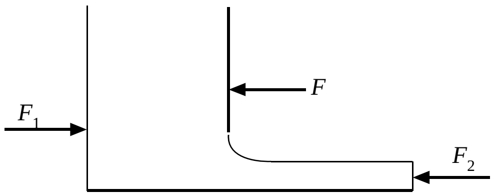
	$\therefore \dot{W}_P = \frac{\gamma Q H_P}{\eta_P} = \frac{9810\times(\pi\times 0.01^2)\times 28.6\times 36.8}{0.85} = \underline{3820\text{ W}}$ Energy: surface to "*A*": $15 = \frac{7.16^2}{2\times 9.81} + \frac{p_A}{9810} + 3.2\frac{7.16^2}{2\times 9.81} \qquad \therefore p_A = \underline{39{,}400\text{ Pa}}$ Energy: surface to "*B*": $36.0+15 = \frac{4.58^2}{2\times 9.81} + \frac{p_B}{9810} + 3.2\frac{7.16^2}{2\times 9.81} \qquad \therefore p_B = \underline{416{,}000\text{ Pa}}$
4.104	Depth on raised section = y_2. Continuity: $3\times 3 = V_2 y_2$ Energy (see Eq. 4.5.21): $\frac{3^2}{2g}+3 = \frac{V_2^2}{2g}+(0.4+y_2)$ $\therefore 3.059 = \frac{9^2}{2g\,y_2^2}+y_2 \quad$ or $\quad y_2^3 - 3.059\,y_2^2 + 4.128 = 0$ Trial-and-error: $\left.\begin{array}{ll} y_2 = 2.0: & -0.11 \overset{?}{=} 0 \\ y_2 = 1.8: & +0.05 \overset{?}{=} 0 \end{array}\right\} \quad \therefore y_2 = \underline{1.85\text{ m}}$ $\left.\begin{array}{ll} y_2 = 2.1: & -0.1 \overset{?}{=} 0 \\ y_2 = 2.3: & +0.1 \overset{?}{=} 0 \end{array}\right\} \quad \therefore y_2 = \underline{2.22\text{ m}}$ The depth that actually occurs depends on the downstream conditions. We cannot select a "correct" answer between the two.
4.106	The average velocity at section 2 is also 8 m/s. The kinetic-energy-correction factor for a parabola is 2 (see Example 4.9). The energy equation is: $\frac{V_1^2}{2g}+\frac{p_1}{\gamma} = \alpha_2\frac{\bar{V}_2^2}{2g}+\frac{p_2}{\gamma}+h_L$ $\frac{8^2}{2\times 9.81}+\frac{150{,}000}{9810} = 2\frac{8^2}{2\times 9.81}+\frac{110{,}000}{9810}+h_L \qquad \therefore h_L = \underline{0.815\text{ m}}$
4.108	**a)** $\bar{V} = \frac{1}{A}\int V dA = \frac{1}{\pi\times 0.01^2}\int_0^{0.01} 10\left(1-\frac{r^2}{0.01^2}\right)2\pi r dr = \frac{20}{0.01^2}\left[\frac{0.01^2}{2}-\frac{0.01^4}{4\times 0.01^2}\right] = 5\text{ m/s}$ $\alpha = \frac{1}{A\bar{V}^3}\int V^3 dA = \frac{1}{\pi\times 0.01^2\times 5^3}\int_0^{0.01} 10^3\left(1-\frac{r^2}{0.01^2}\right)^3 2\pi r dr$ $= \frac{2000}{0.01^2\times 5^3}\left(\frac{0.01^2}{2}-\frac{3\times 0.01^4}{4\times 0.01^2}+\frac{3\times 0.01^6}{6\times 0.01^4}-\frac{0.01^8}{8\times 0.01^6}\right) = \underline{2.00}$

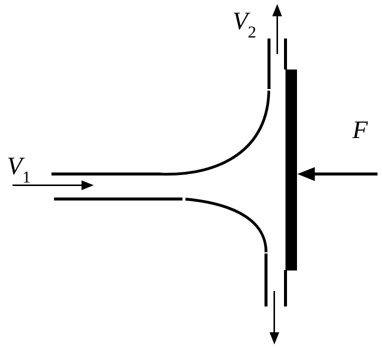
4.110	Engine power = $F_D \times V_\infty + \dot{m}\left(\frac{V_2^2 - V_1^2}{2} + \tilde{u}_2 - \tilde{u}_1\right)$ $\dot{m}_f q_f = F_D V_\infty + \dot{m}\left[\frac{V_2^2 - V_1^2}{2} + c_v (T_2 - T_1)\right]$
4.112	$0 = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 - \frac{V_1^2}{2g} - \frac{p_1}{\gamma} - z_1 + 32\frac{\nu L V}{g D^2}$ $0 = 2\frac{V^2}{2 \times 9.81} - 0.35 + 32 \times \frac{10^{-6} \times 180V}{9.81 \times 0.02^2}$ $V^2 + 14.4V - 3.434 = 0 \quad \therefore V = 0.235$ m/s and $Q = \underline{7.37 \times 10^{-5}\ \text{m}^3/\text{s}}$
4.114	Energy from surface to surface: $H_P = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 - \frac{V_1^2}{2g} - \frac{p_1}{\gamma} - z_1 + K\frac{V^2}{2g}$ **a)** $H_P = 40 + 5\frac{Q^2}{\pi \times 0.04^2 \times 2 \times 9.81} = 40 + 50.7\ Q^2$ Try $Q = 0.25$: $H_P = 43.2$ (energy) $H_P = 58$ (curve) Try $Q = 0.30$: $H_P = 44.6$ (energy) $H_P = 48$ (curve) Solution: $Q = \underline{0.32\ \text{m}^3/\text{s}}$

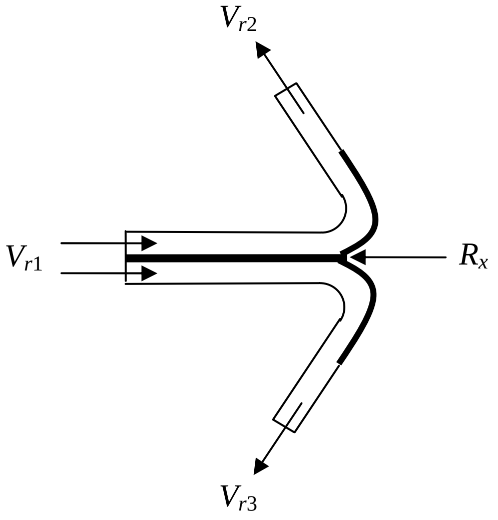
Momentum Equation

4.116	$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} = \frac{V_2^2}{2g} + \frac{p_2}{\gamma}$ $\quad V_2 = \frac{d^2}{(d/2)^2} V_1 = 4\ V_1$ **b)** $\frac{V_1^2}{2 \times 9.81} + \frac{400{,}000}{9810} = \frac{16\ V_1^2}{2 \times 9.81}$ $\quad \therefore V_1 = 7.303$ m/s $400{,}000\pi \times 0.03^2 - F = 1000\pi \times 0.03^2 \times 7.303(4 \times 7.303 - 7.303)\ \therefore F = \underline{679\ \text{N}}$

4.118	$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}$ $\quad V_0\pi\times0.01^2=V_e\times0.006\times0.15$ $\quad\therefore V_e=11.1\text{ m/s}$ $\Sigma F_x=\dot{m}(V_{2x}-V_{1x})$ **a)** $V_2=\frac{10^2}{8^2}V_1=1.562\ V_1$ $\quad\frac{V_1^2}{2\times9.81}+\frac{400{,}000}{9810}=\frac{2.441\ V_1^2}{2\times9.81}$ $\therefore V_1=23.56\text{ m/s}$ $\therefore p_1A_1-F=\dot{m}(V_2-V_1)$ $400{,}000\pi\times0.05^2-F=1000\pi\times0.05^2\times23.56(0.562\times23.56)$ $\quad\therefore F=\underline{692\text{ N}}$ **c)** $V_2=\frac{10^2}{4^2}V_1=6.25\ V_1$ $\quad\frac{V_1^2}{2g}+\frac{400{,}000}{9810}=\frac{39.06\ V_1^2}{2g}$ $\quad\therefore V_1=4.585\text{ m/s}$ $400{,}000\pi\times0.05^2-F=1000\pi\times0.05^2\times4.585(5.25\times4.585)$ $\quad\therefore F=\underline{2275\text{ N}}$
4.120	$V_2=4\ V_1$ $\quad\frac{V_1^2}{2g}+\frac{p_1}{\gamma}=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}$ $\quad\therefore\frac{15\ V_1^2}{2g}=\frac{p_1}{\gamma}$ **b)** $V_1^2=\frac{2\times9.81}{15\times9810}\times400{,}000=53.33$ $\quad\therefore V_1=7.30\text{ m/s},\ V_2=29.2\text{ m/s}$ $p_1A_1-F_x=\dot{m}(V_{2x}-V_{1x})$ $\therefore F_x=400{,}000\pi\times0.04^2+1000\pi\times0.04^2\times7.3^2=\underline{2280\text{ N}}$ $F_y=\dot{m}(V_{2y}-V_{1y})=1000\pi\times0.04^2\times7.3\times(29.2)=\underline{1071\text{ N}}$
4.122	$A_1V_1=A_2V_2$ $\quad\pi\times0.025^2\times4=\pi(0.025^2-0.02^2)V_2$ $\therefore V_2=11.11\text{ m/s}$ $\quad\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}$  $p_1=9810\left(\frac{11.11^2-4^2}{2\times9.81}\right)=53{,}700\text{ Pa}$ $p_1A_1-F=\dot{m}(V_2-V_1)$ $\therefore F=53{,}700\pi\times0.025^2-1000\pi\times0.025^2\times4(11.11-4)$ $=\underline{49.6\text{ N}}$
4.124	Continuity: $\quad 6\ V_1=0.2\ V_2$ $\quad\therefore V_2=30\ V_1$ Energy (along bottom streamline):

	$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2$  $\frac{V_2^2/900}{2\times9.81}+6=\frac{V_2^2}{2\times9.81}+0.2$ $\therefore V_2=10.67,\quad V_1=0.36$ m/s Momentum: $F_1-F_2-F=\dot{m}(V_2-V_1)$ $9810\times3(6\times4)-9810\times0.1(0.2\times4)-F=1000\times(0.2\times4)\times10.67(10.67-0.36)$ $\therefore F=\underline{618{,}000\text{ N}}$ (F acts to the right on the gate)
4.126	Continuity: $V_2y_2=V_1y_1=4V_2y_1\quad\therefore y_2=4y_1$ Use the result of Example 4.12: $y_2=\frac{1}{2}\left[-y_1+\left(y_1^2+\frac{8}{g}y_1V_1^2\right)^{1/2}\right]$ **a)** $y_2=4\times0.8=\underline{3.2\text{ m}}$ $3.2=\frac{1}{2}\left[-0.8+\left(0.8^2+\frac{8}{9.81}\times0.8\times V_1^2\right)^{1/2}\right]\quad\therefore V_1=\underline{8.86\text{ m/s}}$
4.128	Refer to Example 4.12: $\gamma\frac{y_1}{2}y_1w-\gamma\times1\times2w=\rho\times6w\left(3-\frac{6}{y_1}\right)\qquad(V_1y_1=2\times3)$ $\therefore\frac{\gamma}{2}(y_1^2-4)=18\rho\left(\frac{y_1-2}{y_1}\right)$ or $(y_1+2)y_1=\frac{36}{9.81}=3.67\quad\therefore y_1=\underline{0.77\text{ m}},\ V_1=\underline{7.8\text{ m/s}}$
4.130	$V_1A_1=2V_2A_2\qquad V_2=15\frac{\pi\times0.05^2}{2\pi\times0.025^2}=30$ m/s $\qquad\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}$ $\therefore p_1=9810\frac{30^2-15^2}{2\times9.81}=337{,}500$ Pa $\Sigma F_x=\dot{m}\left(V_{2x}-V_{1x}\right)\qquad p_1A_1-F=\dot{m}(-V_1)$ $\therefore F=p_1A_1+\dot{m}V_1$ $=337{,}500\pi\times0.05^2+1000\pi\times0.05^2\times15^2=\underline{4420\text{ N}}$

4.132	**a)** $\sum F_x = \dot{m}(V_{2x} - V_{1x}), \qquad -F = -\dot{m}V_1$ $V_1 = \frac{\dot{m}}{\rho A_1} = \frac{300}{1000\times\pi\times 0.05^2} = 38.2\text{ m/s}$ $\therefore F = 300\times 38.2 = 11,460\text{ N}$ **b)** $-F = \dot{m}_r\left(V_1 - V_B\right)\left(\cos\ \alpha - 1\right)$ $\therefore F = 300\times\frac{28.2}{38.2}(38.2-10) = \underline{6250\text{ N}}$ 
4.134	**b)** $-F = \dot{m}_r\left(V_1 - V_B\right)\left(\cos\ \alpha - 1\right) \qquad -700 = 1000\pi\times 0.04^2(V_1 - 8)^2(0.866-1)$ $V_1 = 40.24\text{ m/s}$ $\therefore \dot{m} = \rho A_1 V_1 = 1000\pi\times 0.04^2\times 40.24 = \underline{202\text{ kg/s}}$
4.136	$V_B = R\omega = 0.5\times 3.0 = 15\text{ m/s}$ $-R_x = \dot{m}(V_1 - V_B)(\cos\alpha - 1) = 1000\pi\times 0.025^2\times 40\times 25(0.5-1) \quad \therefore R_x = 982\text{ N}$ $\therefore \dot{W} = 10R_xV_B = 10\times 982\times 15 = \underline{147,300\text{ W}}$
4.138	$-F_x = \dot{m}(V_1 - V_B)(\cos 120° - 1) = 4\pi\times 0.02^2\times(400-180)^2(-0.5-1)$ $\therefore R_x = 365\text{ N}$ $V_B = 1.2\times 150 = 180\text{ m/s} \qquad \dot{W} = 15\times 365\times 180 = \underline{986,000\text{ W}}$ The *y*-component force does no work.
4.140	**b)** $\left.\begin{array}{r} 100\sin 30^\circ = V_{r1}\sin\alpha_1 \\ 100\cos 30^\circ - 40 = V_{r1}\cos\alpha_1 \end{array}\right\} \quad \therefore \alpha_1 = \underline{47^\circ}, \quad V_{r1} = V_{r2} = 68.35\text{ m/s}$ $\left.\begin{array}{r} V_2\sin 60^\circ = 68.35\sin\alpha_2 \\ V_2\cos 60^\circ = 68.35\cos\alpha_2 - 40 \end{array}\right\} \quad V_2 = 38.9\text{ m/s}, \quad \alpha_2 = \underline{29.5^\circ}$ $-R_x = \dot{m}(V_{2x} - V_{1x}) = 1000\pi\times 0.015^2\times 100(-38.9\cos 60^\circ - 100\cos 30^\circ)$ $\therefore R_x = 7500\text{ N}$ $\therefore \dot{W} = 12V_BR_x = 12\times 40\times 7500 = \underline{3.60\times 10^6\text{ W}}$
4.142	To find *F*, sum forces normal to the plate: $\Sigma F_n = \dot{m}\left[\cancel{\left(V_{out}\right)_n} - V_{1n}\right]$

	a) $\therefore F = 1000\times 0.02\times 0.4\times 40\left[-(-40\sin 60^\circ)\right] = \underline{11,080\text{ N}}$ (We have neglected friction) $\Sigma F_t = 0 = \dot m_2 V_2 + \dot m_3(-V_3) - \dot m_1 \times 40\sin 30^\circ \quad$ Bernoulli: $V_1 = V_2 = V_3$ $\left.\begin{array}{l}\therefore \quad 0 = \dot m_2 - \dot m_3 - 0.5\dot m_1 \\ \text{Continuity: } \dot m_1 = \dot m_2 + \dot m_3\end{array}\right\} \quad \begin{array}{l}\therefore \dot m_2 = 0.75\dot m_1 = 0.75\times 320 = \underline{240\text{ kg/s}} \\ \dot m_3 = \underline{80\text{ kg/s}}\end{array}$
4.144	$F = \dot m_r (V_{1r})_n = 1000\times 0.02\times 0.4(40 - V_B)^2 \sin 60^\circ \quad F_x = 8(40 - V_B^2)\sin^2 60^\circ$ $\dot W = V_B F_x = 8V_B(40 - V_B)^2 \times 0.75 = 6(1600V_B - 80V_B^2 + V_B^3)$ $\frac{d\dot W}{dV_B} = 6(1600 - 160V_B + 3V_B^2) = 0 \quad \therefore V_B = \underline{13.33\text{ m/s}}$
4.146	$F = \dot m_r (V_1 - V_B)(\cos\alpha - 1) = 90\times 0.8\times 2.5\times 13.89\times(-13.89)(-1) = 34,700\text{ N}$ $\left(V_B = \frac{50\times 1000}{3600} = 13.89\text{ m/s}\right) \quad \therefore \dot W = 34,700\times 13.89 = 482{,}000\text{ W} \quad$ or $\underline{647\text{ hp}}$
4.148	To solve this problem, choose a control volume attached to the reverse thruster vanes, as shown below. The momentum equation is applied to a free body diagram. Momentum: $-R_x = 0.5\dot m\left[(V_{r2})_x + (V_{r3})_x\right] - \dot m \times \left(V_{r1}\right)_x$ Assume the pressure in the gases equals the atmospheric pressure and that $V_{r1} = V_{r2} = V_{r3}$. Hence $(V_{r1})_x = V_{r1} = 800\text{ m/s}$, and $(V_{r2})_x = (V_{r3})_x = -V_{r1}\times\sin\alpha$ where $\alpha = 20^\circ$. Then, momentum is $-R_x = 0.5\dot m\left[-2V_{r1}\sin\alpha\right] - \dot m\times V_{r1}$ $= -\dot m V_{r1}\left(\sin\alpha + 1\right)$  Momentum: $-R_x = 0.5\dot m\left[-2V_{r1}\sin\alpha\right] - \dot m\times V_{r1}$ $= -\dot m V_{r1}\left(\sin\alpha + 1\right)$ The mass flow rate of the exhaust gases is $\dot m = \dot m_{air} + \dot m_{fuel} = \dot m_{air}\left(1 + 1/40\right) = 100\times 1.025 = 102.5\text{ kg/s}$

	Substituting the given values we calculate the reverse thrust: $R_x = (102.5\text{ kg/s})(800\text{ m/s})\left(1+\sin 20^\circ\right) = \underline{110\text{ kN}}$ Note that the thrust acting on the engine is in the opposite direction to R_x, and hence it is referred to as a reverse thrust; its purpose is to decelerate the airplane.
4.150	For this steady-state flow, we fix the boat and move the upstream air. This provides us with the steady-state flow of Fig. 4.17. This is the same as observing the flow while standing on the boat. $\dot{W} = FV_1 \quad 20{,}000 = F\times\frac{50\times 1000}{3600} \quad \therefore F = \underline{1440\text{ N}} \quad (V_1 = 13.89\text{ m/s})$ $F = \dot{m}(V_2 - V_1) \quad 1440 = 1.23\pi\times 1^2\times\frac{V_2+13.89}{2}(V_2 - 13.89) \quad \therefore V_2 = 30.6\text{ m/s}$ $\therefore Q = A_3V_3 = \pi\times 1^2\times\frac{30.6+13.89}{2} = \underline{69.9\text{ m}^3\text{/s}}$ $\eta_p = \frac{V_1}{V_3} = \frac{13.89}{22.24} = 0.625 \quad \text{or} \quad \underline{62.5\%}$
4.152	Fix the reference frame to the boat so that $V_1 = 36\times\frac{5}{18} = 10\text{ m/s}$ $V_2 = 72\times\frac{5}{18} = 20\text{ m/s}$ $\therefore F = \dot{m}(V_2 - V_1) = 1000\pi\times(0.25)^2\times\frac{10+20}{2}(20-10) = 29.44\text{ kN}$ $\dot{W} = F\times V_1 = (24\text{ kN})\times 10\text{ m/s} = 294.4\text{ kW}$ or $\underline{395\text{ hp}}$ $\dot{m} = 1000\times\pi\times(0.25)^2\times\frac{10+20}{2} = \underline{2944\text{ kg/s}}$
4.154	$0.2 = \bar{V}_1A_1 = \bar{V}_1\times .2\times 1.0 \quad \therefore \bar{V}_1 = 1\text{ m/s} \quad \therefore V_{1\,\max} = 2\text{ m/s} \quad \therefore V_1(y) = 20(0.1-y)$ flux in = $2\int_0^{0.1}\rho V^2 dy = 2\int_0^{0.1} 1000\times 20^2(0.1-y)^2 dy = 800{,}000\frac{0.1^3}{3} = 267\text{ N}$ The slope at section 1 is –20 $\quad \therefore V_2(y) = -20y + A$ Continuity: $\quad A_1\bar{V}_1 = A_2\bar{V}_2 \quad \therefore \bar{V}_2 = 2\bar{V}_1 = 2\text{ m/s}$

$$\left.\begin{aligned} V_2(0) &= A \\ V_2(0.05) &= A-1 \end{aligned}\right\} \quad \therefore \bar{V}_2 = A - 1/2$$

$$2 = A - \frac{1}{2} \quad \therefore A = 2.5 \quad \therefore V_2(y) = 2.5 - 20y$$

$$\text{flux out} = 2\int_0^{0.05} 1000(2.5-20y)^2 dy = 800{,}000\left[\frac{(y-0.125)^3}{3}\right]_0^{0.05} = \frac{800{,}000}{3}[0.00153]$$

$$= 408.3\text{ N} \quad \therefore \text{change} = 408 - 267 = \underline{141\text{ N}}$$

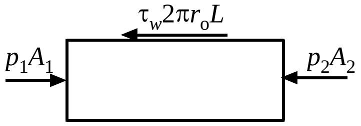
4.156

From the c.v. shown: $(p_1 - p_2)\pi r_0^2 = \tau_w 2\pi r_0 L$

$$\therefore \tau_w = \frac{\Delta p\, r_o}{2L} = \mu \left.\frac{du}{dr}\right|_w$$

$$\therefore \left.\frac{du}{dr}\right|_w = \frac{210 \times 0.019}{2 \times 9 \times 0.001}$$

$$= \underline{220\text{ m/s/m}}$$



4.158

$$\dot{m}_{top} = \rho A_1 V_1 - \rho \int V_2(y) dA$$

$$= 1.23\left[2 \times 10 \times 32 - \int_0^2 (28 + y^2) 10 dy\right] = 65.6\text{ kg/s}$$

$$-\frac{F}{2} = \int \rho V^2 dA + \dot{m}_{top} V_1 - \dot{m}_1 V_1 = 1.23\int_0^2 (28 + y^2)^2 10 dy + 65.6 \times 32 - 1.23 \times 20 \times 32^2$$

$$\therefore F = \underline{3780\text{ N}}$$

Momentum and Energy

4.160

a) Energy: $\dfrac{V_1^2}{2g} + z_1 = \dfrac{V_2^2}{2g} + z_2 + h_L$ See Problem 4.125(a)

$$\frac{8^2}{2 \times 9.81} + 0.6 = \frac{1.912^2}{2 \times 9.81} + 2.51 + h_L \qquad \therefore h_L = 1.166\text{ m}$$

$\therefore$ losses = $\gamma A_1 V_1 h_L = 9810 \times (0.6 \times 1) \times 8 \times 1.166 = \underline{54{,}900\text{ W/m}}$ of width

Moment of Momentum

4.164	$V_e = \frac{\dot{m}}{\rho A_e} = \frac{4}{1000\times 4\times \pi\times 0.004^2} = 19.89$ m/s Velocity in arm = V $\mathbf{M}_I = \int_{\text{c.v.}} \mathbf{r}\times(2\boldsymbol{\Omega}\times\mathbf{V})\rho d\cancel{V} = 4\int_0^{0.3} r\hat{\mathbf{i}}\times(-2\Omega\hat{\mathbf{k}}\times V\hat{\mathbf{i}})\rho A dr$ $= 8\rho AV\Omega\hat{\mathbf{k}}\int_0^{0.3} r dr = -0.36\rho AV\Omega\hat{\mathbf{k}}$ $\sum\mathbf{M} = \mathbf{0}$ and $\frac{d}{dt}\int_{\text{c.v.}}(\mathbf{r}\times\mathbf{V})\rho d\cancel{V} = 0$ $\int_{\text{c.s.}}(\mathbf{r}\times\mathbf{V})\mathbf{V}\cdot\hat{\mathbf{n}}\rho dA = 0.3\hat{\mathbf{i}}\times\left(0.707V_e\hat{\mathbf{j}} + 0.707V_e\hat{\mathbf{k}}\right)V_e\rho A_e$ The z-component of $\int_{c.s.}\mathbf{r}\times\mathbf{V}(\mathbf{V}\cdot\hat{\mathbf{n}})\rho dA = 0.3\times 0.707V_e^2A_e\rho$ Finally $-(M_I)_z = 0.36\rho AV\Omega = 4\times 0.3\times 0.707V_e^2A_e\rho$ Using $AV = A_eV_e$ $0.36\Omega = 4\times 0.3\times 0.707\times 19.89$ $\therefore \Omega = \underline{46.9\text{ rad/s}}$
4.166	$\dot{m} = 10 = \rho AV = 1000\pi\times 0.01^2 V_0$ $\therefore V_0 = 31.8$ m/s Continuity: $V_0\pi\times 0.01^2 = V\pi\times 0.01^2 + V_e\times 0.006(r - 0.05)$ $V_0\pi\times 0.01^2 = V_e\times 0.006\times 0.15$ $\therefore V_e = 11.1$ m/s $\therefore V = V_0 - 19.1(r - 0.05)V_e = 42.4 - 212r$ $\mathbf{M}_I = \int_0^{0.05} 2r\hat{\mathbf{i}}\times(+2\Omega\hat{\mathbf{k}}\times V_0\hat{\mathbf{i}})\rho A dr + \int_{0.05}^{0.2} 2r\hat{\mathbf{i}}\times[+2\Omega\hat{\mathbf{k}}\times(42.4 - 212r)\hat{\mathbf{i}}]\rho A dr$ $= 4\Omega V_0\rho A\hat{\mathbf{k}}\int_0^{0.05} r dr + 4\Omega\rho A\hat{\mathbf{k}}\int_{0.05}^{0.2}(42.4r - 212r^2)dr$ $= \left[\frac{42.4}{2}(0.2^2 - 0.05^2) - \frac{212}{3}(0.2^3 - 0.05^3)\right]\hat{\mathbf{k}}$ $= (0.05\Omega + 0.3\Omega)\hat{\mathbf{k}} = 0.35\Omega\hat{\mathbf{k}}$

	$\int_{0.05}^{0.2} r\hat{\mathbf{i}}\times(-V_e\hat{\mathbf{j}})V_e\rho\times 0.006dr = -11.1^2\times 1000\times 0.006\int_{0.05}^{0.2} rdr\,\hat{\mathbf{k}} = -13.86\hat{\mathbf{k}}$ $\therefore -0.35\Omega = -13.86$ $\therefore \Omega = \underline{39.6\text{ rad/s}}$
4.168	See Problem 4.165. $V_e = 19.89$ m/s $V = \frac{0.008^2}{0.02^2}\times 19.89 = 3.18$ m/s $\mathbf{M}_I = 4\int_0^{0.3} r\hat{\mathbf{i}}\times\left[(-2\Omega\hat{\mathbf{k}}\times V\hat{\mathbf{i}}) + \left(-\frac{d\Omega}{dt}\hat{\mathbf{k}}\right)\times r\hat{\mathbf{i}}\right]\rho Adr \quad A = \pi\times 0.01^2,\ A_e = \pi\times 0.004$ $= -8\rho AV\Omega\hat{\mathbf{k}}\int_0^{0.3} rdr - 4\rho A\frac{d\Omega}{dt}\hat{\mathbf{k}}\int_0^{0.3} r^2dr$ $= -360AV\Omega\hat{\mathbf{k}} - 36A\frac{d\Omega}{dt}\hat{\mathbf{k}}$ $\int_{\text{c.s.}} (\mathbf{r}\times\mathbf{V})_z(\mathbf{V}\cdot\hat{\mathbf{n}})\rho dA = 212V_e^2A_e\hat{\mathbf{k}}$ Thus, $360AV\Omega + 36A\frac{d\Omega}{dt} = 212V_e^2A_e$ or $\frac{d\Omega}{dt} + 31.8\Omega = 373$ The solution is $\Omega = Ce^{-31.8t} + 11.73$. The initial condition is $\Omega(0) = 0$ $\therefore C = -11.73.$ Finally $\Omega = \underline{11.73(1-e^{-31.8t})\text{ rad/s}}$

CHAPTER 5

The Differential Forms of the Fundamental Laws

Differential Continuity Equation

5.2	$\dot{m}_{in} - \dot{m}_{out} = \frac{\partial m_{\text{element}}}{\partial t}$. This is expressed as $\rho v_r (rd\theta dz) - \left[\rho v_r + \frac{\partial}{\partial r}(\rho v_r)dr\right](r+dr)d\theta dz + \rho v_\theta drdz - \left[\rho v_\theta + \frac{\partial}{\partial \theta}(\rho v_\theta)d\theta\right]drdz$ $+\rho v_z\left(r+\frac{dr}{2}\right)d\theta dr - \left[\rho v_z + \frac{\partial}{\partial z}(\rho v_z)dz\right]\left(r+\frac{dr}{2}\right)d\theta dr = \frac{\partial}{\partial t}\left[\rho\left(r+\frac{dr}{2}\right)d\theta drdz\right]$ Subtract terms and divide by $rd\theta drdz$: $-\frac{\rho v_r}{r} - \frac{r+dr}{r}\frac{\partial}{\partial r}(\rho v_r) - \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) - \frac{\partial}{\partial z}(\rho v_z)\frac{r+dr/2}{r} = \frac{\partial}{\partial t}\rho\frac{r+dr/2}{r}$ Since dr is an infinitesimal, $(r+dr)/r = 1$ and $(r+dr/2)/r = 1$. Hence $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) + \frac{1}{r}\rho v_r = 0$ This can be put in various forms. If ρ = const, it divides out
5.4	For a steady flow $\frac{\partial \rho}{\partial t} = 0$. Then, with $v = w = 0$, Eq. 5.2.2 yields $\frac{\partial}{\partial x}(\rho u) = 0$ or $\underline{\rho\frac{du}{dx} + u\frac{d\rho}{dx} = 0}$ Partial derivatives are not used since there is only one independent variable.
5.6	Given: $\frac{\partial}{\partial t} = 0$, $\frac{\partial \rho}{\partial z} \neq 0$. Since water can be considered to be incompressible, we demand that $D\rho/Dt = 0$. Equation (5.2.8) then provides $\underline{u\frac{\partial \rho}{\partial x} + w\frac{\partial \rho}{\partial z} = 0}$, assuming the *x*-direction to be in the direction of flow. There is no variation with *y*. Also, we demand that $\nabla \cdot \mathbf{V} = 0$, or

	$\underline{\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0}$
5.7	We can use the ideal gas law, $\rho=\frac{p}{RT}$. Then, the continuity equation (5.2.7) $\frac{D\rho}{Dt}=-\rho\nabla\cdot\mathbf{V}$ becomes, assuming *RT* to be constant $\frac{1}{RT}\frac{Dp}{Dt}=-\frac{p}{RT}\nabla\cdot\mathbf{V}$ or $\frac{1}{p}\frac{Dp}{Dt}=-\nabla\cdot\mathbf{V}$
5.8	**a)** Use cylindrical co-ordinates with $v_\theta=v_z=0$: $\frac{1}{r}\frac{\partial}{\partial r}(rv_r)=0$ Integrate: $rv_r=C$ $\therefore\underline{v_r=\frac{C}{r}}$ **b)** Use spherical coordinates with $v_\theta=v_\phi=0$: $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2v_r)=0$ Integrate: $r^2v_r=C$ $\therefore\underline{v_r=\frac{C}{r^2}}$
5.9	**(a)** Since the flow is steady and incompressible then *VA* = *constant*, where the constant is determined by using the conditions at the inlet that is, $(VA)_{inlet}=40\times1=40\ \text{m}^3/\text{s}.$ And, since the flow is inviscid, the velocity is uniform in the channel, so $u=V$. Hence, at any *x* position within the channel the velocity *u* can be calculated using $u=V=40/A.$ Since the flow area is not constant it is given by $A=2hw,$ where the vertical distance *h* is a function of *x* and can be determined as, $h=0.15x+0.5H$. Substituting, we obtain the following expression for the velocity: $u(x)=\frac{40}{2(0.15x+0.5)}=\frac{20}{0.15x+0.5}$ m/s **(b)** To determine the acceleration in the *x*-direction, we use (see Eq. 3.2.9) $a_x=u\frac{du}{dx}$ where

	$\frac{du}{dx} = -\frac{3}{(0.15x+0.5)^2}$ Hence, the expression for acceleration is $a_x = \frac{20}{0.15x+0.5}\times\frac{-3}{(0.15x+0.5)^2} = \frac{-60}{(0.15x+0.5)^3}\ \text{m/s}^2$ Note that the minus sign indicates deceleration of the fluid in the *x*-direction.
5.10	**(a)** Using the continuity equation $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$ we write $v=-\int\left(\frac{\partial u}{\partial x}\right)dy+C.$ With the result from Problem 5.9: $\frac{\partial u}{\partial x}=-\frac{3}{(0.15x+0.5)^2}=-\frac{\partial v}{\partial y}$, we integrate to find $v(x,y)=\frac{3y}{(0.15x+0.5)^2}$ and since $v=0$ at $y=0,$ then $C=0.$ **(b)** To determine the acceleration in the *y*-direction, we use (see Eq. 3.2.9) $a_y = u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}$ From part (a) we have $\frac{\partial v}{\partial x}=-\frac{0.9y}{(0.15x+0.5)^3}$ Substituting in the expression for acceleration we get $a_y=\frac{20}{0.15x+0.5}\times\frac{-0.9y}{(0.15x+0.5)^3}+\frac{3y}{(0.15x+0.5)^2}\times\frac{3}{(0.15x+0.5)^2}=\frac{-9y}{(0.15x+0.5)^4}$
5.11	$\frac{D\rho}{Dt}=-\rho\nabla\cdot\mathbf{V}=-\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=-2.3(200\times1+400\times1)=\underline{-1380\ \text{kg/m}^3\cdot\text{s}}$
5.12	In a plane flow, $u=u(x,y)$ and $v=v(x,y).$ Continuity demands that $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$ If $u=\text{const},$ then $\frac{\partial u}{\partial x}=0$ and hence $\frac{\partial v}{\partial y}=0.$ Thus, we also have $\underline{v=\text{const}}$ and $\underline{D\rho/Dt=0}.$
5.13	If $u=C_1$ and $v=C_2,$ the continuity equation provides, for an incompressible flow

	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \therefore \frac{\partial w}{\partial z}=0$ so $w=C_3$ The z-component of velocity w is also constant. We also have $\frac{D\rho}{Dt}=0=\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}$ The density may vary with x, y, z and t. It is not, necessarily, constant.
5.14	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \therefore\ A+\frac{\partial v}{\partial y}=0 \qquad \therefore\ v(x,y)=-Ay+f(x)$ But, $v(x,0)=0=f(x) \qquad \therefore\ \underline{v=-Ay}$
5.15	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \therefore\ \frac{\partial v}{\partial y}=-\frac{\partial u}{\partial x}=-\frac{(x^2+y^2)5-5x(2x)}{(x^2+y^2)^2}=-\frac{5x^2-5y^2}{(x^2+y^2)^2}$ $\therefore v(x,y)=\int\frac{5y^2-5x^2}{(x^2+y^2)^2}dy+f(x)=\frac{5y}{x^2+y^2}+f(x) \qquad f(x)=0$ $\therefore v=\underline{\frac{5y}{x^2+y^2}}$
5.16	From Table 5.1: $\frac{1}{r}\frac{\partial}{\partial r}(rv_r)=-\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}=-\frac{1}{r}\left(10+\frac{0.4}{r^2}\right)\sin\theta$ $\therefore rv_r=\int\left(10+\frac{0.4}{r^2}\right)\sin\theta dr+f(\theta)=\left(10r-\frac{0.4}{r}\right)\sin\theta+f(\theta)$ $0.2v_r(0.2,\theta)=\left(10\times 0.2-\frac{0.4}{0.2}\right)\sin\theta+f(\theta)=0 \qquad \therefore f(\theta)=0$ $\therefore v_r=\underline{\left(10-\frac{0.4}{r^2}\right)\sin\theta}$
5.17	From Table 5.1: $\frac{1}{r}\frac{\partial}{\partial r}(rv_r)=-\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}=\frac{-20}{r}\left(1+\frac{1}{r^2}\right)\cos\theta$ $\therefore rv_r=\int -20\left(1+\frac{1}{r^2}\right)\cos\theta dr+f(\theta)=-20\left(r-\frac{1}{r}\right)\cos\theta+f(\theta)$ $v_r(1,\theta)=-20(1-1)\cos\theta+f(\theta)=0 \qquad \therefore f(\theta)=0$ $\therefore v_r=\underline{-20\left(1-\frac{1}{r^2}\right)\cos\theta}$

5.18	From Table 5.1, spherical coordinates: $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) = -\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta)$ $\therefore \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) = \frac{1}{r\sin\theta}\left(10+\frac{40}{r^3}\right)2\sin\theta\cos\theta$ $\therefore r^2 v_r = \int r\left(10+\frac{40}{r^3}\right)2\cos\theta dr + f(\theta) = \left(10r^2 - \frac{80}{r}\right)\cos\theta + f(\theta)$ $4v_r(2,\theta) = \left(10\times 2^2 - \frac{80}{2}\right)\cos\theta + f(\theta) = 0 \qquad \therefore f(\theta) = 0$ $\therefore v_r = \underline{\left(10-\frac{80}{r^3}\right)\cos\theta}$
5.19	Continuity: $\frac{\partial}{\partial x}(\rho u) = 0 \qquad \therefore \rho\frac{du}{dx} + u\frac{d\rho}{dx} = 0$ $\rho = \frac{p}{RT} = \frac{124\times 10^3}{287\times 278} = 1.554\text{ kg/s} \qquad \frac{du}{dx} = \frac{159-137}{0.1} = 220\text{ m/s/m}$ $\therefore \frac{d\rho}{dx} = -\frac{\rho}{u}\frac{du}{dx} = -\frac{1.554}{147}\times 220 = \underline{-2.32\text{ kg/m}^4}$
5.20	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \frac{\partial}{\partial x}\left[-20(1-e^{-x})\right] = -20e^{-x}$ Hence, in the vicinity of the *x*-axis: $\frac{\partial v}{\partial y} = 20e^{-x}$ and $v = 20ye^{-x} + C$ But $v = 0$ if $y = 0 \qquad \therefore C = 0$ $v = 20ye^{-x} = 20(0.2)e^{-2} = \underline{0.541\text{ m/s}}$
5.21	From Table 5.1, $\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0 \qquad \frac{\partial}{\partial z}\left[-20(1-e^{-z})\right] = -20e^{-z}$ Hence, in the vicinity of the *z*-axis: $\frac{1}{r}\frac{\partial}{\partial r}(rv_r) = 20e^{-z}$ and $rv_r = \frac{r^2}{2}20e^{-z} + C$ But $v_r = 0$ if $r = 0 \qquad \therefore C = 0$ $v_r = 10re^{-z} = 10(0.2)e^{-2} = \underline{0.271\text{ m/s}}$

5.22

The velocity is zero at the stagnation point. Hence, $0 = 10 - \frac{40}{R^2}$ $\quad \therefore \underline{R = 2\text{ m}}$

The continuity equation for this plane flow is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$. Using $\frac{\partial u}{\partial x} = 80x^{-3}$, we see that $\frac{\partial v}{\partial y} = -80x^{-3}$ near the *x*-axis. Consequently, for small Δy,

$$\Delta v = -80x^{-3}\Delta y \quad \text{so that} \quad v = -80(-3)^{-3}(0.1) = \underline{0.296\text{ m/s}}$$

5.23

The velocity is zero at the stagnation point. Hence

$$0 = (40/R^2) - 10 \quad \therefore \underline{R = 2\text{ m}}$$

Use continuity from Table 5.1:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(40 - 10r^2) = -\frac{20}{r}$$

Near the negative *x*-axis continuity provides us with

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta \sin\theta\right) = \frac{20}{r}$$

Integrate, letting $\theta = 0$ from the *y*-axis: $\quad v_\theta \sin\theta = -20\cos\theta + C$

Since $v_\theta = 0$ when $\theta = 90°$, $C = 0$. Then, with $\alpha = \tan^{-1}\frac{0.1}{3} = 1.909°$

$$v_\theta = -20\frac{\cos\theta}{\sin\theta} = -20\frac{\cos 88.091}{\sin 88.091} = -20\frac{0.0333}{0.999} = \underline{0.667\text{ m/s}}$$

5.24

Continuity: $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \therefore \frac{\Delta v}{\Delta y} = -\frac{\Delta u}{\Delta x} = -\frac{13.5 - 11.3}{2\times 0.005} = -220\ \frac{\text{m/s}}{\text{m}}$

$$\therefore \Delta v = v - 0 = -220\Delta y \qquad \therefore v = -220\times 0.004 = \underline{-0.88\text{ m/s}}$$

b) $a_x = u\frac{\partial u}{\partial x} = 12.6\times(+220) = \underline{2772\text{ m/s}^2}$

Differential Momentum Equation

5.25

$\Sigma F_y = ma_y$. For the fluid particle occupying the volume of Fig. 5.3:

$$\left(\tau_{yy} + \frac{\partial\tau_{yy}}{\partial y}\frac{dy}{2}\right)dxdz + \left(\tau_{zy} + \frac{\partial\tau_{zy}}{\partial z}\frac{dz}{2}\right)dxdy + \left(\tau_{xy} + \frac{\partial\tau_{xy}}{\partial x}\frac{dx}{2}\right)dydz$$

$$-\left(\tau_{yy}-\frac{\partial\tau_{yy}}{\partial y}\frac{dy}{2}\right)dxdz-\left(\tau_{zy}-\frac{\partial\tau_{zy}}{\partial z}\frac{dz}{2}\right)dxdy-\left(\tau_{xy}-\frac{\partial\tau_{xy}}{\partial x}\frac{dx}{2}\right)dydz$$

$$+\rho g_y dx\ dy\ dz=\rho dx\ dy\ dz\frac{Dv}{Dt}$$

Dividing by $dx\ dy\ dz$, and adding and subtracting terms:

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{yy}}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}+\rho g_y=\rho\frac{Dv}{Dt}$$

5.26

Check continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\frac{(x^2+y^2)10-10x(2x)}{(x^2+y^2)^2}+\frac{(x^2+y^2)10-10y(2y)}{(x^2+y^2)^2}=0$$

Thus, it is a possible flow. For a frictionless flow, Euler's Eqs. 5.3.7, give with $g_x=g_y=0$:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}$$

$$\therefore\ \frac{\partial p}{\partial x}=-\rho\frac{10x}{x^2+y^2}\frac{10y^2-10x^2}{(x^2+y^2)^2}-\rho\frac{10y}{x^2+y^2}\frac{-20xy}{(x^2+y^2)^2}=\rho\frac{100(x^2+y^2)y}{(x^2+y^2)^3}$$

$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}$$

$$\therefore\ \frac{\partial p}{\partial y}=-\rho\frac{10x}{x^2+y^2}\frac{-20xy}{(x^2+y^2)^2}-\rho\frac{10y}{x^2+y^2}\frac{10x^2-10y^2}{(x^2+y^2)^2}=\rho\frac{100(x^2+y^2)y}{(x^2+y^2)^3}$$

$$\therefore\ \nabla p=\frac{\partial p}{\partial x}\hat{\mathbf{i}}+\frac{\partial p}{\partial y}\hat{\mathbf{j}}=\frac{100x\rho}{(x^2+y^2)^2}\hat{\mathbf{i}}+\frac{100y\rho}{(x^2+y^2)^2}\hat{\mathbf{j}}=\underline{\frac{100\rho}{(x^2+y^2)^2}\ (x\hat{\mathbf{i}}+y\hat{\mathbf{j}})}$$

5.27

Check continuity (cylindrical coord. from Table 5.1):

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}=\frac{10}{r}\left(1+\frac{1}{r^2}\right)\cos\theta+\frac{-10}{r}\left(1+\frac{1}{r^2}\right)\cos\theta=0.\quad\therefore$$ It is a possible flow. For Euler's Eqs. (let v = 0 in the momentum eqns of Table 5.1) in cylindrical coord:

$$\frac{\partial p}{\partial r}=\rho\frac{v_\theta^2}{r}-\rho v_r\frac{\partial v_r}{\partial r}-\rho\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}=\frac{100\rho}{r}\left(1+\frac{1}{r^2}\right)^2\sin^2\theta-10\rho\left(1-\frac{1}{r^2}\right)\cos^2\theta\left(\frac{20}{r^3}\right)$$

$$-\frac{10\rho}{r}\left(1+\frac{1}{r^2}\right)\sin^2\theta\left(10-\frac{10}{r^2}\right)$$

	$\frac{1}{r}\frac{\partial p}{\partial \theta} = -\rho\frac{v_r v_\theta}{r} - \rho v_r\frac{\partial v_\theta}{\partial r} - \rho\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} = \frac{100\rho}{r}\left(1-\frac{1}{r^4}\right)\sin\theta\cos\theta$ $-10\rho\left(1-\frac{1}{r^2}\right)\cos\theta\sin\theta\left(\frac{20}{r^3}\right) - \frac{100\rho}{r}\left(1+\frac{1}{r^2}\right)^2\sin\theta\cos\theta$ $\therefore \nabla p = \frac{\partial p}{\partial r}\hat{\mathbf{i}}_r + \frac{1}{r}\frac{\partial p}{\partial \theta}\hat{\mathbf{i}}_\theta = \underline{\frac{200\rho}{r^3}\left[\frac{1}{r^2}-\cos 2\theta\right]\hat{\mathbf{i}}_r - \frac{200\rho}{r^3}\sin 2\theta\hat{\mathbf{i}}_\theta}$
5.28	Follow the steps of Problem 5.27. The components of the pressure gradient are $\frac{\partial p}{\partial r} = \rho\frac{v_\theta^2 + v_\phi^2}{r} - \rho v_r\frac{\partial v_r}{\partial r} - \rho\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}$ $\frac{1}{r}\frac{\partial p}{\partial \theta} = -\rho\frac{v_r v_\theta}{r} - \rho v_r\frac{\partial v_\theta}{\partial r} - \rho\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}$
5.29	$\therefore \overline{p} = p - \left(\frac{2\mu}{3} + \lambda\right)\nabla\cdot\mathbf{V}$ $\qquad \therefore \overline{p} - p = -\left(\frac{2\mu}{3} + \lambda\right)\nabla\cdot\mathbf{V}$ $\frac{\partial\hat{\mathbf{s}}}{\partial s} \cong \frac{\Delta\hat{\mathbf{s}}}{\Delta s} = -\frac{\Delta\alpha\,\hat{\mathbf{n}}}{R\Delta\alpha} = -\frac{\hat{\mathbf{n}}}{R}$ $\frac{\partial\hat{\mathbf{s}}}{\partial t} \cong \frac{\Delta\hat{\mathbf{s}}}{\Delta t} = \frac{\hat{\mathbf{n}}\,\Delta\theta}{\Delta t} = \hat{\mathbf{n}}\frac{\partial\theta}{\partial t}$ $\therefore \frac{D\mathbf{V}}{Dt} = \underline{\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}\right)\hat{\mathbf{s}} + \left(V\frac{\partial\theta}{\partial t} - \frac{V^2}{R}\right)\hat{\mathbf{n}}}$ For steady flow, the normal acc. is $\left(-\frac{V^2}{R}\right)$, the tangential acc. is $V\frac{\partial V}{\partial s}$
5.30	For a rotating reference frame (see Eq. 3.2.15), we must add the terms due to $\mathbf{\Omega}$. Thus, Euler's equation becomes $\rho\left(\frac{D\mathbf{V}}{Dt} + 2\mathbf{\Omega}\times\mathbf{V} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}) + \frac{d\mathbf{\Omega}}{dt}\times\mathbf{r}\right) = -\nabla p - \rho\mathbf{g}$
5.31	$\tau_{xx} = -p + 2\mu\cancel{\frac{\partial u}{\partial x}} + \lambda\cancel{\nabla\cdot\mathbf{V}} = -210\text{ kPa}$ $\tau_{yy} = \tau_{zz} = -p = -210\text{ kPa}$

	$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right) = 5\times10^{-4}\left[32 - 32\times192\times0.0025\right] = 8.32\times10^{-3}$ Pa $\tau_{xz} = \tau_{yz} = 0 \qquad \frac{\tau_{xy}}{\tau_{xx}} = \frac{8.32\times10^{-3}}{210\times10^{3}} = \underline{3.96\times10^{-8}}$
5.32	$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \frac{16y}{C\,x^{9/5}} - \frac{16y^2}{C^2x^{13/5}} \qquad \therefore v(x,y) = \frac{8y^2}{Cx^{9/5}} - \frac{16y^3}{3C^2x^{13/5}} + f(x)$ $v(x,o) = 0 \qquad \therefore f(x) = 0 \qquad 8 = C\ 1000^{4/5} \qquad \therefore C = 0.0318$ $\therefore u(x,y) = 629yx^{-4/5} - 9890y^2x^{-8/5}$ $v(x,y) = \underline{252y^2x^{-9/5} - 5270y^3x^{-13/5}}$ $\tau_{xx} = -p + 2\mu\frac{\partial u}{\partial x} = -100 + 0 = \underline{-100\text{ kPa}}$ $\tau_{yy} = \tau_{zz} = -p = \underline{-100\text{ kPa}}$ $\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 2\times10^{-5}\left[629\times1000^{-4/5}\right] = \underline{5.01\times10^{-5}\text{ Pa}}$ $\tau_{xz} = \tau_{yz} = 0$
5.33	$\frac{Du}{Dt} = \cancel{\frac{\partial u}{\partial t}} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)u = (\mathbf{V}\cdot\nabla)u$ $\frac{Dv}{Dt} = \cancel{\frac{\partial v}{\partial t}} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)v = (\mathbf{V}\cdot\nabla)v$ $\frac{Dw}{Dt} = \cancel{\frac{\partial w}{\partial t}} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)w = (\mathbf{V}\cdot\nabla)w$ $\therefore \frac{D\mathbf{V}}{Dt} = \frac{Du}{Dt}\hat{\mathbf{i}} + \frac{Dv}{Dt}\hat{\mathbf{j}} + \frac{Dw}{Dt}\hat{\mathbf{k}} = \mathbf{V}\cdot\nabla(u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}) = (\mathbf{V}\cdot\nabla)\mathbf{V}$
5.34	Follow the steps that lead to Eq. 5.3.17 and add the term due to compressible effects: $\rho\frac{D\mathbf{V}}{Dt} = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{V} + \frac{\mu}{3}\frac{\partial}{\partial x}\nabla\cdot\mathbf{V}\,\hat{\mathbf{i}} + \frac{\mu}{3}\frac{\partial}{\partial y}\nabla\cdot\mathbf{V}\,\hat{\mathbf{j}} + \frac{\mu}{3}\frac{\partial}{\partial z}\nabla\cdot\mathbf{V}\,\hat{\mathbf{k}}$

	$= -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{V} + \frac{\mu}{3}\left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right)\nabla\cdot\mathbf{V}$ $\therefore \underline{\rho\frac{D\mathbf{V}}{Dt} = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{V} + \frac{\mu}{3}\nabla(\nabla\cdot\mathbf{V})}$
5.35	If $u = u(y)$, then continuity demands that $\frac{\partial v}{\partial y} = 0 \quad \therefore v = C.$ But, at y = 0 (the lower plate), v = 0 $\quad \therefore C = 0,$ and $v(x,y) = 0$ $\therefore \rho\frac{Du}{Dt} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$ $\therefore \underline{0 = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{ay^2}}$ $\rho\frac{Dv}{Dt} = \underline{0 = -\frac{\partial p}{\partial y}}$ $\rho\frac{Dw}{Dt} = 0 = -\frac{\partial p}{\partial z} + \rho(-g) \qquad \therefore \underline{\frac{\partial p}{\partial z} = -\rho g}$
5.36	The x-component Navier-Stokes equation can be written as $\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$ Based on the given conditions the following assumptions can be made: One-dimensional $(v = w = 0)$ Steady state $\left(\frac{\partial u}{\partial t} = 0\right)$ Incompressible $(\rho =$ constant$)$ Zero pressure-gradient $\left(\frac{dp}{dx} = 0\right)$ Fully-developed flow $\left(\frac{\partial u}{\partial x} = 0\right)$ A wide channel $\left(\frac{\partial u}{\partial z} = 0\right)$ The Navier-Stokes equation takes the simplified form $\mu\frac{\partial^2 u}{\partial y^2} = 0$ or $\frac{\partial^2 u}{\partial y^2} = 0.$ Integrating twice yields, $u(y) = ay + b$. To determine a and b we apply the following boundary conditions: $u = V_1$ at y = 0, and $u = -V_2$ at $y = h$. This gives $b = V_1$ and $a = -(V_1 + V_2)/h.$ The velocity distribution between the plates is

then

$$u(y) = -\left(\frac{V_1 + V_2}{h}\right)y + V_1$$

5.37

Using the *x*-component Navier-Stokes equation with *x* being vertical and the following assumptions:

One-dimensional $(v = w = 0)$ Steady state $\left(\frac{\partial u}{\partial t} = 0\right)$

Incompressible $(\rho = \text{constant})$ Fully-developed flow $\left(\frac{\partial u}{\partial x} = 0\right)$

The *x*-component Navier-Stokes equation reduces to

$$0 = -\frac{\partial p}{\partial x} - \rho g + \mu \frac{\partial^2 u}{\partial y^2}$$

Integrate the above differential equation twice (see Problem 5.36):

$$u(y) = \frac{1}{2\mu}\left(\frac{dp}{dx} + \rho g\right)y^2 + ay + b$$

Applying the no-slip boundary condition at both plates (see Problem 5.36) we get

$$u(y) = \frac{1}{2\mu}\left(\frac{dp}{dx} + \rho g\right)\left(y^2 - hy\right)$$

5.38

Assumptions: One-dimensional $(v_r = v_\theta = 0)$ Steady state $\left(\frac{\partial v_z}{\partial t} = 0\right)$

Incompressible $(\rho = \ \text{constant})$ Horizontal $(g_z = 0)$

Fully-developed flow $\left(\frac{\partial v_z}{\partial z} = 0\right)$

$$\frac{Dv_r}{Dt} = 0 = \underline{-\frac{1}{\rho}\frac{\partial p}{\partial r}} \qquad \frac{Dv_\theta}{Dt} = 0 = \underline{-\frac{1}{\rho r}\frac{\partial p}{\partial \theta}}$$

$$\rho\frac{Dv_z}{Dt} = \rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r}\frac{\partial v_z}{\partial r} + \frac{\cancel{v_\theta}}{r}\frac{\partial v_z}{\partial \theta} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right)$$

$$\underline{\therefore 0 = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right)}$$

5.39

Assumptions:

One-dimensional flow $\left(v_\theta = v_r = 0\right)$ Steady state $\left(\dfrac{\partial v_z}{\partial t} = 0\right)$

Incompressible $\left(\rho = \text{ constant}\right)$ Horizontal $(g_z = 0)$

Fully-developed flow $\left(\dfrac{\partial v_z}{\partial z} = 0\right)$

The Navier-Stokes equation in cylindrical form provides the following equation:

$$0 = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right]$$

Rearrange the above equation and integrate:

$$0 = -\frac{\partial p}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) \qquad \frac{\partial v_z}{\partial r} = \frac{1}{\mu}\frac{\partial p}{\partial z}\left(\frac{r}{2}\right) + \frac{C_1}{r}$$

Integrating again yields: $v_z(r) = \dfrac{1}{\mu}\dfrac{\partial p}{\partial z}\left(\dfrac{r^2}{4}\right) + C_1 \ln r + C_2$

C_1 and C_2 are determined using the boundary conditions: $v_z = 0$, at $r = r_o$, and $v_z = V_c$ at $r = r_i$. Hence

$$V_c = \frac{1}{\mu}\frac{\partial p}{\partial z}\left(\frac{r_i^2}{4}\right) + C_1 \ln r_i + C_2 \quad \text{and} \quad 0 = \frac{1}{\mu}\frac{\partial p}{\partial z}\left(\frac{r_o^2}{4}\right) + C_1 \ln r_o + C_2$$

Subtracting the second equation from the first yields

$$C_1 = \frac{V_c - \dfrac{1}{4\mu}\dfrac{\partial p}{\partial z}\left(r_i^2 - r_o^2\right)}{\ln\left(r_i/r_o\right)}$$

The drag force on the inner cylinder is zero when the shear stress τ_{rz} on the inner cylinder is zero, i.e., $\tau_{rz} = \mu\left[\dfrac{\partial v_r}{\partial z} + \dfrac{\partial v_z}{\partial r}\right]_{r=r_i} = 0$. Since $v_r = 0$, then $\tau_{rz} = \mu\left[\dfrac{\partial v_z}{\partial r}\right]_{r=r_i} = 0$. From the above expression for v_z we find $\left.\dfrac{\partial v_z}{\partial r}\right|_{r=r_i} = \dfrac{1}{2\mu}\dfrac{\partial p}{\partial z} r_i + \dfrac{C_1}{r_i} = 0.$ Then $C_1 = -\dfrac{1}{2\mu}\dfrac{\partial p}{\partial z} r_i^2$. Combining with the above expressions for C_1 we solve for V_c. The result is:

	$\underline{V_c = \frac{1}{4\mu}\frac{\partial p}{\partial z}\left[\left(r_i^2 - r_o^2\right) - 2r_i^2 \ln\left(r_i/r_o\right)\right]}$
5.40	Continuity: $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) = 0 \qquad \therefore r^2 v_r = C \qquad At\ r = r_1,\ v_r = 0 \qquad \therefore C = 0$ $\underline{-\frac{v_\theta^2}{r}\rho = -\frac{\partial p}{\partial r} + \mu\left(-\frac{2v_\theta}{r^2}\cot\theta\right)} \qquad \underline{0 = -\frac{\partial p}{\partial \theta} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r\sin^2\theta}\right]}$ $\underline{0 = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}}$
5.41	Assumptions: One-dimensional flow $\left(v_z = v_r = 0\right)$ Steady state $\left(\frac{\partial v_z}{\partial t} = 0\right)$ Incompressible $\left(\rho = \text{ constant}\right)$ Vertical $(g_r = g_\theta = 0)$ Developed flow $\left(\frac{\partial v_r}{\partial \theta} = 0\right)$ The simplified differential equation from Table 5.1 is $\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} = 0$ which can be re-written as $\frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) = 0.$ Integrating we get: $\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} = C_1$ The above equation can be re-written as $\frac{1}{r}\frac{\partial}{\partial r}\left(r v_\theta\right) = C_1$ Integrating again yields $r v_\theta = C_1\frac{r^2}{2} + C_2 \Rightarrow v_\theta = C_1\frac{r}{2} + \frac{C_2}{r}$ Apply the boundary conditions $v_\theta = 0$ at $r = r_i$, and $v_\theta = r_o\omega$ at $r = r_o$. We have $0 = C_1\frac{r_i}{2} + \frac{C_2}{r_i}$ and $r_o\omega = C_1\frac{r_o}{2} + \frac{C_2}{r_o}$ Solving for C_1 and C_2 yields $C_1 = -2\frac{\omega r_o^2}{r_i^2 - r_o^2}$ and $C_2 = \frac{\omega r_i^2 r_o^2}{r_i^2 - r_o^2}$

	Finally $v_\theta = \left(-\frac{\omega r_o^2}{r_i^2 - r_o^2} \right) r + \left(\frac{\omega r_i^2 r_o^2}{r_i^2 - r_o^2} \right) \frac{1}{r}$
5.42	For an incompressible flow $\nabla \cdot \mathbf{V} = 0$. Substitute Eqs. 5.3.10 into Eq. 5.3.2 and 5.3.3: $\rho \frac{Du}{Dt} = \frac{\partial}{\partial x}\left(-p + 2\mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial z} \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) + \rho g_x$ $= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial^2 u}{\partial z^2} + \mu \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \rho g_x$ $\therefore \underline{\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x}$ $\rho \frac{Dv}{Dt} = \frac{\partial}{\partial x} \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y}\left(-p + 2\mu \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) + \rho g_y$ $= -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2} + \mu \frac{\partial^2 v}{\partial z^2} + \mu \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \rho g_y$ $\therefore \underline{\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y}$ $\rho \frac{Dw}{Dt} = \frac{\partial}{\partial x} \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial z}\left(-p + 2\mu \frac{\partial w}{\partial z} \right) + \rho g_z$ $= -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial z^2} + \mu \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \rho g_z$ $\therefore \underline{\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho g_z}$
5.43	If we substitute the constitutive equations (5.3.10) into Eqs. 5.3.2 and 5.3.3., with $\mu = \mu(x, y, z)$ we arrive at $\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho g_x + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + 2 \frac{\partial \mu}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial y} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial \mu}{\partial z} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$

5.44	If plane flow is only parallel to the plate, $v = w = 0$. Continuity then demands that $\partial u/\partial x = 0$. The first equation of (5.3.14) simplifies to $\rho\left(\frac{\partial u}{\partial t} + u\frac{\cancel{\partial u}}{\cancel{\partial x}} + \cancel{v}\frac{\partial u}{\partial y} + \cancel{w}\frac{\partial u}{\partial z}\right) = -\frac{\cancel{\partial p}}{\cancel{\partial x}} + \cancel{\rho g_x} + \mu\left(\frac{\cancel{\partial^2 u}}{\cancel{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \frac{\cancel{\partial^2 u}}{\cancel{\partial z^2}}\right)$ $\underline{\rho\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2}}$ We assumed *g* to be in the *y*-direction, and since no forcing occurs other than due to the motion of the plate, we let $\partial p/\partial x = 0$.
5.45	From Eqs. 5.3.10, $-\frac{\tau_{xx}+\tau_{yy}+\tau_{zz}}{3} = p - \frac{2\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \lambda\,\nabla\cdot\mathbf{V}$ $\therefore\ \overline{p} = p - \left(\frac{2\mu}{3} + \lambda\right)\nabla\cdot\mathbf{V} \qquad \therefore\ \overline{p} - p = -\left(\frac{2\mu}{3} + \lambda\right)\nabla\cdot\mathbf{V}$

Vorticity

5.46	$(\mathbf{V}\cdot\nabla)\mathbf{V} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)(u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}})$ $\nabla\times(\mathbf{V}\cdot\nabla)\mathbf{V} = \left[\frac{\partial}{\partial y}\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) - \frac{\partial}{\partial z}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)\right]\hat{\mathbf{i}}$ $+\left[\frac{\partial}{\partial z}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial x}\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)\right]\hat{\mathbf{j}}$ $+\left[\frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) - \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\right]\hat{\mathbf{k}}$ Use the definition of vorticity: $\boldsymbol{\omega} = (\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z})\hat{\mathbf{i}} + (\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x})\hat{\mathbf{j}} + (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\hat{\mathbf{k}}$: $(\boldsymbol{\omega}\cdot\nabla)\mathbf{V} = \left[(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z})\frac{\partial}{\partial x} + (\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x})\frac{\partial}{\partial y} + (\frac{\partial v}{\partial x} - \frac{\partial x}{\partial y})\frac{\partial}{\partial z}\right](u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}})$ $(\mathbf{V}\cdot\nabla)\boldsymbol{\omega} = \left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right]\left[(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z})\hat{\mathbf{i}} + (\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x})\hat{\mathbf{j}} + (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\hat{\mathbf{k}}\right]$

	Expand the above, collect like terms, and compare coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.
5.47	Studying the vorticity components of Eq. 3.2.21, we see that $\omega_z = -\partial u/\partial y$ is the only vorticity component of interest. The third equation of Eq. 5.3.24 then simplifies to $$\frac{D\omega_z}{Dt} = \nu \nabla^2 \omega_z$$ $$= \nu \frac{\partial^2 \omega_z}{\partial y^2}$$ since changes normal to the plate are much larger than changes along the plate, i.e., $$\frac{\partial \omega_z}{\partial y} >> \frac{\partial \omega_z}{\partial x}$$
5.48	If viscous effects are negligible, as they are in a short section, Eq. 5.3.25 reduces to $$\frac{D\omega_z}{Dt} = 0$$ that is, there is no change in vorticity (along a streamline) between sections 1 and 2. Since (see Eq. 3.2.21), at section 1 $$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -10$$ we conclude that, for the lower half of the flow at section 2, $\frac{\partial u}{\partial y} = 10.$ This means the velocity profile at section 2 is a straight line with the same slope of the profile at section 1. Since we are neglecting viscosity, the flow can slip at the wall with a slip velocity u_0; hence, the velocity distribution at section 2 is $u_2(y) = u_0 + 10y.$ Continuity then allows us to calculate the profile: $$V_1 A_1 = V_2 A_2$$ $$\frac{1}{2}(10 \times 0.04)(0.04w) = (u_0 + 10 \times 0.02/2)(0.02w) \qquad \therefore u_0 = 0.3 \text{ m/s}$$ Finally $$u_2(y) = \underline{0.3 + 10y}$$
5.49	No. The first of Eqs. 5.3.24 shows that $\frac{D\omega_x}{Dt} = \omega_x \frac{\partial u}{\partial x} + \omega_y \frac{\partial u}{\partial y} + \omega_z \frac{\partial u}{\partial z}$

	neglecting viscous effects, so that ω_y, which is nonzero near the snow surface, creates ω_x through the term $\omega_y \partial u/\partial y$, since there would be a nonzero $\partial u/\partial y$ near the tree.

Differential Energy Equation

5.50	$\int_{\text{c.s.}} K\nabla T\cdot\hat{\mathbf{n}}\,dA = \int_{\text{c.v.}} \frac{\partial}{\partial t}\left(\frac{V^2}{2}+gz+\tilde{u}\right)\rho d\mathcal{V} + \int_{\text{c.s.}}\left(\frac{V^2}{2}+gz+\tilde{u}+\frac{p}{\rho}\right)\rho\mathbf{V}\cdot\hat{\mathbf{n}}dA$ $\int_{\text{c.v.}} \nabla\cdot(K\nabla T)d\mathcal{V} = \int_{\text{c.v.}} \frac{\partial}{\partial t}\left(\rho\frac{V^2}{2}+\rho gz+\rho\tilde{u}\right)d\mathcal{V} + \int_{\text{c.v.}} \nabla\cdot\rho\mathbf{V}\left(\frac{V^2}{2}+gz+\tilde{u}+\frac{p}{\rho}\right)d\mathcal{V}$ $\therefore \int_{\text{c.v.}}\left[-K\nabla^2 T+\frac{\partial}{\partial t}\left(\rho\frac{V^2}{2}+\rho gz+\rho\tilde{u}\right)+\nabla\cdot\rho\mathbf{V}\left(\frac{V^2}{2}+gz+\tilde{u}+\frac{p}{\rho}\right)\right]d\mathcal{V}=0$ $\frac{\partial}{\partial t}\rho\frac{V^2}{2}+\nabla\cdot\rho\mathbf{V}\left(\frac{V^2}{2}+gz+\frac{p}{\rho}\right)=\frac{V^2}{2}\underbrace{\left(\cancel{\frac{\partial\rho}{\partial t}+\nabla\cdot\rho\mathbf{V}}\right)}_{\text{continuity}}+\rho V\cdot\underbrace{\cancel{\left[\frac{\partial V}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}+\frac{\nabla p}{\rho}+g\nabla z\right]}}_{\text{momentum}}=0$ $\therefore -K\nabla^2 T+\frac{\partial}{\partial t}\rho\tilde{u}+\ \rho\mathbf{V}\cdot\nabla\tilde{u}=0$ or $\underline{\rho\frac{D\tilde{u}}{Dt}=K\nabla^2 T}$
5.51	Divide each side by $dx\,dy\,dz$ and observe that $\frac{\left.\frac{\partial T}{\partial x}\right\|_{x+dx}-\left.\frac{\partial T}{\partial x}\right\|_{x}}{dx}=\frac{\partial^2 T}{\partial x^2}$, $\frac{\left.\frac{\partial T}{\partial y}\right\|_{y+dy}-\left.\frac{\partial T}{\partial y}\right\|_{y}}{dy}=\frac{\partial^2 T}{\partial x^2}$, $\frac{\left.\frac{\partial T}{\partial z}\right\|_{z+dz}-\left.\frac{\partial T}{\partial z}\right\|_{z}}{dz}=\frac{\partial^2 T}{\partial z^2}$ Eq. 5.4.5 follows.
5.52	$\rho\frac{D\tilde{u}}{Dt}=\rho\frac{D(h-p/\rho)}{Dt}=\rho\frac{Dh}{Dt}-\frac{Dp}{Dt}+\frac{p}{\rho}\frac{D\rho}{Dt}=\rho\frac{Dh}{Dt}-\frac{Dp}{Dt}+\frac{p}{\rho}[-\rho\nabla\cdot\mathbf{V}]$ where we used the continuity equation: $D\rho/Dt=-\rho\nabla\cdot\mathbf{V}$. Then Eq. 5.4. 9 becomes $\rho\frac{Dh}{Dt}-\frac{Dp}{Dt}+\frac{p}{\rho}[-\rho\nabla\cdot\mathbf{V}]=K\nabla^2 T-p\nabla\cdot\mathbf{V}$ which is simplified to $\rho\frac{Dh}{Dt}=K\nabla^2 T+\frac{Dp}{Dt}$

5.53	See Eq. 5.4.9: $\tilde{u} = cT \qquad \therefore \rho c\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=K\nabla^2 T$ Neglect terms with velocity: $\underline{\rho c\frac{\partial T}{\partial t}=K\nabla^2 T}$
5.54	The dissipation function Φ involves viscous effects. For flows with extremely large velocity gradients, it becomes quite large. Then $\rho c_p \frac{DT}{Dt}=\Phi$ and $\frac{DT}{Dt}$ is large. This leads to very high temperatures on reentry vehicles.
5.55	$u=10(1-10,000r^2) \qquad \therefore \frac{\partial u}{\partial r}=-2r\times 10^5$ (r takes the place of y) From Eq. 5.4.17, $\Phi=2\mu\left[\frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2\right]=\mu\times 4r^2\times 10^{10}$ At the wall where $r=0.01$ m, $\Phi=1.8\times 10^{-5}\times 4\times 0.01^2\times 10^{10}=\underline{72\text{ N/m}^2\cdot\text{s}}$ At the centerline: $\frac{\partial u}{\partial r}=0$ so $\Phi=\underline{0}$ At a point half-way: $\Phi=1.8\times 10^{-5}\times 4\times 0.005^2\times 10^{10}=\underline{18\text{ N/m}^2\cdot\text{s}}$
5.56	**(a)** Momentum: $\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}$ Energy: $\rho c\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2$ **(b)** Momentum: $\rho\frac{\partial u}{\partial t}=\mu\frac{\partial^2 u}{\partial y^2}+\frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y}$ Energy: $\rho c\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2$

CHAPTER 6

Dimensional Analysis and Similitude

FE-type Exam Review Problems: Problems 6-1 to 6-6

6.1 (A)	The dimensions on the variables are as follows: $[\dot{W}]=[F\times V]=M\dfrac{L}{T^2}\times\dfrac{L}{T}=\dfrac{ML^2}{T^3},\quad [d]=L,\quad [\Delta p]=\dfrac{ML/T^2}{L^2}=\dfrac{M}{LT^2},\quad [V]=\dfrac{L}{T}$ First, eliminate T by dividing $\dot{W}$ by Δp. That leaves T in the denominator so divide by V leaving L^2 in the numerator. Then divide by d^2. That provides $\pi=\dfrac{\dot{W}}{\Delta pVd^2}$
6.2 (A)	$V = f(d,\ l,\ g,\ \omega,\ \mu)$. The units on the variables on the rhs are as follows: $[d]=L,\ [l]=L,\ [g]=\dfrac{L}{T^2},\ [\omega]=T^{-1},\ [\mu]=\dfrac{ML}{T}$ Because mass M occurs in only one term, it cannot enter the relationship.
6.3 (A)	$\mathrm{Re}_m=\mathrm{Re}_p\qquad \dfrac{V_mL_m}{\cancel{\nu_m}}=\dfrac{V_pL_p}{\cancel{\nu_p}}\qquad \therefore V_m=V_p\dfrac{L_p}{L_m}=12\times9=108\text{ m/s}$
6.4 (A)	$\mathrm{Re}_m=\mathrm{Re}_p\qquad \dfrac{V_mL_m}{\nu_m}=\dfrac{V_pL_p}{\nu_p}\qquad \therefore V_m=V_p\dfrac{L_p}{L_m}\dfrac{\nu_m}{\nu_p}=4\times10\dfrac{1.51\times10^{-5}}{1.31\times10^{-6}}=461\text{ m/s}$
6.5 (C)	$\mathrm{Fr}_m=\mathrm{Fr}_p\qquad \dfrac{V_m^2}{l_m\cancel{g_m}}=\dfrac{V_p^2}{l_p\cancel{g_p}}\qquad \therefore V_m=V_p\sqrt{\dfrac{l_m}{l_p}}=2\times\dfrac{1}{4}=0.5\text{ m/s}$
6.6 (A)	From Froude's number $V_m=V_p\sqrt{\dfrac{l_m}{l_p}}$. From the dimensionless force we have: $F_m^*=F_p^*\quad\text{or}\quad \dfrac{F_m}{\rho_mV_m^2l_m^2}=\dfrac{F_p}{\rho_pV_p^2l_p^2}\qquad \therefore F_p=F_m\dfrac{V_p^2}{V_m^2}\dfrac{l_p^2}{l_m^2}=10\times25\times25^2=156{,}000\text{ N}$

Introduction

6.8	**a)** $[\dot{m}] = \text{kg/s} = \text{N}\cdot\text{s}^2/\text{m}\cdot\text{s} = \text{N}\cdot\text{s/m} \quad \therefore \frac{FT}{L}$ **c)** $[\rho] = \text{kg/m}^3 = \text{N}\cdot\text{s}^2/\text{m}\cdot\text{m}^3 = \text{N}\cdot\text{s}^2/\text{m}^4 \quad \therefore \frac{FT^2}{L^4}$ **e)** $[W] = \text{N}\cdot\text{m} \quad \therefore FL$

Dimensional Analysis

6.10	$V = f(\ell,\rho,\mu) \quad [V]=\frac{L}{T},\ [\ell]=L,\ [\rho]=\frac{M}{L^3},\ [\mu]=\frac{M}{LT}$ $\therefore$ There is one $\pi -$ term: $\pi_1 = \frac{\rho V\ell}{\mu}$ $\therefore \pi_1 = f_1(\pi_2^0) = \text{Const.} \quad \therefore \underline{\rho\frac{V\ell}{\mu} = C}$ or Re = Const.
6.12	$V = f(H,g,m) \quad [V]=\frac{L}{T},\ \ [g]=\frac{L}{T^2},\ \ [m]=M,\ \ [H]=L$ $\therefore \pi_1 = \frac{gHm^0}{V^2} \quad \therefore \pi_1 = C \quad \therefore \underline{V=\sqrt{gH/C}}$
6.14	$F_D = f(d,\ell,V,\mu,\rho) \ \ [F_D]=\frac{ML}{T^2},\ [d]=L,\ [V]=\frac{L}{T},\ [\mu]=\frac{M}{LT},\ [\rho]=\frac{M}{L^3}$ $\pi_1 = F_D\ell^{a_1}V^{b_1}\rho^{c_1},\quad \pi_2 = dV^{b_2}\rho^{c_2}\ell^{a_2},\quad \pi_3 = \mu\ell^{a_3}V^{b_3}\rho^{c_3}$ $\therefore \pi_1 = \frac{F_D}{V^2\rho\ell^2},\quad \pi_2 = \frac{d}{\ell},\quad \pi_3 = \frac{\mu}{\rho V\ell}$ $\therefore \underline{\frac{F_D}{\rho\ell^2V^2} = f_1\left(\frac{d}{\ell},\frac{\mu}{\rho\ell V}\right)}$ We could write $\frac{\pi_1}{\pi_2^2} = f_2\left(\frac{1}{\pi_2},\frac{\pi_3}{\pi_2}\right)$ or $\frac{F_D}{\rho d^2V^2} = f_2\left(\frac{\ell}{d},\frac{\mu}{\rho dV}\right)$. This is equivalent to the above. Either functional form must be determined by experimentation.
6.16	$h = f(\sigma,d,\gamma,\beta,g).\ \ [h]=L,\ [\sigma]=\frac{M}{T^2},\ [d]=L,\ [\gamma]=\frac{M}{L^2T^2},\ [\beta]=1,\ [g]=\frac{L}{T^2}$ Se

	lect d,γ,g as repeating variables: $\pi_1 = hd^{a_1}\gamma^{b_1}g^{c_1},\ \ \pi_2 = \sigma d^{a_2}\gamma^{b_2}g^{c_2},\ \ \pi_3=\beta$ $\therefore \pi_1 = h/d,\ \ \pi_2 = \sigma/\gamma d^2,\ \ \pi_3 = \beta$ $\therefore \underline{h/d = f_1(\sigma/\gamma d^2,\beta)}.$ Note: gravity does not enter the answer.
6.18	$\sigma = f(M,y,I)\quad [\sigma]=\frac{M}{LT^2},\ [M]=\frac{ML^2}{T^2},\ [y]=L,\ [I]=L^4$ Given that $b=-1$, $\ \pi_1=\frac{\sigma I}{yM}=\text{Const.}\qquad \therefore \underline{\sigma = C\frac{My}{I}}$
6.20	$V=f(H,g,\rho)\qquad [V]=\frac{L}{T},\ [H]=L,\ [g]=\frac{L}{T^2},\ [\rho]=\frac{M}{L^3}$ $\therefore \pi_1 = VH^a g^b \rho^c = V\frac{\rho^0}{\sqrt{g}\sqrt{H}}=\text{Const.}\qquad \therefore V = \underline{\text{Const.}\sqrt{gH}}$ Density does not enter the expression.
6.22	$\Delta p = f(V,d,\nu,L,\varepsilon,\rho)$ $[\Delta p]=\frac{M}{LT^2},\ [V]=\frac{L}{T},\ [d]=L,\ [\nu]=\frac{L^2}{T},\ [L]=L,\ [e]=L,\ [\rho]=\frac{M}{L^3}$ Repeating variables: V,d,ρ $\pi_1=\Delta p V^{a_1}d^{b_1}\rho^{c_1},\ \pi_2=\nu\ V^{a_2}d^{b_2}\rho^{c_2},\ \ \pi_3 = L\ V^{a_3}d^{b_3}\rho^{c_3},\ \pi_4 = e\ V^{a_4}d^{b_4}\rho^{c_4}$ By inspection: $\therefore \pi_1=\frac{\Delta p}{\rho V^2},\ \pi_2=\frac{\nu}{Vd},\ \pi_3=\frac{L}{d},\ \pi_4=\frac{e}{d}$ $\pi_1 = f_1(\pi_2,\pi_3,\pi_4)\qquad \therefore \Delta p/\rho V^2 = \underline{f_1(\nu/Vd, L/d, e/d)}$
6.24	$Q = f(R,A,e,S,g).\ \ [Q]=\frac{L^3}{T},[R]=L,[A]=L^2,[e]=L,[s]=1,[g]=\frac{L}{T^2}$ There are only two basic dimensions. Choose two repeating variables, R and g. Then $\pi_1 = QR^{a_1}g^{b_1},\ \pi_2 = AR^{a_2}g^{b_2},\ \pi_3 = eR^{a_3}g^{b_3},\ \pi_4 = sR^{a_4}g^{b_4}$ $\therefore \pi_1=\frac{Q}{\sqrt{g}R^{5/2}},\ \pi_2=\frac{A}{R^2},\ \pi_3=\frac{e}{R},\ \pi_4=s$

	$\therefore \pi_1 = f_1(\pi_2,\pi_3,\pi_4) \quad \therefore \underline{Q/\sqrt{gR^5} = f_1(A/R^2, e/R, s)}$
6.26	$F_D = f(V,\mu,\rho,e,I,d)$ Repeating variables: V,ρ,d $[F_D]=\frac{ML}{T^2},\ [V]=\frac{L}{T},\ [\mu]=\frac{M}{LT},\ [\rho]=\frac{M}{L^3},\ [e]=L,\ [I]=1,\ [d]=L$ $\pi_1 = F_D V^{a_1}\rho^{b_1}d^{c_1},\ \pi_2=\mu V^{a_2}\rho^{b_2}d^{c_2},\ \pi_3 = e\ V^{a_3}\rho^{b_3}d^{c_3},\ \pi_4 = I\ V^{a_4}\rho^{b_4}d^{c_4}$ $\therefore \pi_1 = \frac{F_D}{\rho V^2 d^2},\ \pi_2=\frac{\mu}{V\rho d},\ \pi_3=\frac{e}{d},\ \pi_4 = I$ $\therefore \underline{F_D/\rho V^2 d^2 = f_1(\mu/\rho V d, e/d, I)}$
6.28	$T = f(\rho,\mu,V,D)=\rho^a\mu^bV^cD^d$ or $\frac{ML}{T^2}=\left[\frac{M}{L^3}\right]^a\left[\frac{M}{LT}\right]^b\left[\frac{L}{T}\right]^c[L]^d$ $M:\quad 1=a+b\Rightarrow a=1-b$ $T:\quad -2=-b-c\Rightarrow c=2-b$ $L:\quad 1=-3a-b+c+d\Rightarrow 1=-3(1-b)-b+(2-b)+d$ $\therefore d=2-b.\quad T=\rho^{(1-b)}\mu^bV^{(2-b)}D^{(2-b)}=\rho V^2D^2\left(\frac{\mu}{\rho VD}\right)^b$ and $\underline{\frac{T}{\rho V^2D^2}=\phi[\text{Re}]}$
6.30	$F_D = f(V,\mu,\rho,d,e,r,c)$. Repeating variables: V,ρ,d $[F_D]=\frac{ML}{T^2},\ [V]=\frac{L}{T},\ [\mu]=\frac{M}{LT},\ [\rho]=\frac{M}{L^3},\ [d]=L,\ [e]=L,\ [r]=L,\ [c]=\frac{1}{L^2}$ $\pi_1 = F_D V^{a_1}\rho^{b_1}d^{c_1},\ \pi_2=\mu V^{a_2}\rho^{b_2}d^{c_2},\ \pi_3=eV^{a_3}\rho^{b_3}d^{c_3},$ $\pi_4 = rV^{a_4}\rho^{b_4}d^{c_4},\ \pi_5=cV^{a_5}\rho^{b_5}d^{c_5}$ $\therefore \pi_1=\frac{F_D}{\rho V^2d^2},\ \pi_2=\frac{\mu}{\rho Vd},\ \pi_3=\frac{e}{d},\ \pi_4=\frac{r}{d},\ \pi_5=cd^2$ $\therefore \underline{\frac{F_D}{\rho V^2d^2}=f_1\left(\frac{\mu}{\rho Vd},\frac{e}{d},\frac{r}{d},cd^2\right)}$
6.32	$F_L = f(V,\ c,\ \rho,\ \ell_c,\ t,\ \alpha)$. Repeating variables: $V,\ \rho,\ \ell_c$ $[F_L]=\frac{ML}{T^2},\ [V]=\frac{L}{T},\ [c]=\frac{L}{T},\ [\rho]=\frac{M}{L^3},\ [\ell_c]=L,\ [t]=L,\ [\alpha]=1$ $\pi_1=F_LV^{a_1}\rho^{b_1}\ell_c{}^{c_1},\ \pi_2=cV^{a_2}\rho^{b_2}\ell_c{}^{c_2},\ \pi_3=tV^{a_3}\rho^{b_3}\ell_c{}^{c_3},\ \pi_4=\alpha V^{a_4}\rho^{b_4}\ell_c{}^{c_4}$

	$\therefore \pi_1 = \frac{F_L}{\rho V^2 {\ell_c}^2},\ \pi_2 = \frac{c}{V},\ \pi_3 = \frac{t}{\ell_c},\ \pi_4 = \alpha$ $\therefore \underline{\frac{F_L}{\rho V^2 {\ell_c}^2} = f_1\left(\frac{c}{V}, \frac{t}{\ell_c}, \alpha\right)}$
6.34	$F_D = f(V,\ \rho,\ \mu,\ d,\ L,\ \rho_c,\ \omega)$ where d is the cable diameter, L the cable length, ρ_c the cable density, and ω the vibration frequency. Repeating variables: $V,\ d,\ \rho$. The $\pi-$terms are $\pi_1 = \frac{F_D}{\rho V^2 d^2},\ \pi_2 = \frac{Vd\rho}{\mu},\ \pi_3 = \frac{d}{L},\ \pi_4 = \frac{\rho}{\rho_c},\ \pi_5 = \frac{V}{\omega d}$ We then have $\underline{\frac{F_D}{\rho V^2 d^2} = f_1\left(\frac{Vd\rho}{\mu},\ \frac{d}{L},\ \frac{\rho}{\rho_c},\ \frac{V}{\omega d}\right)}$
6.36	$T = g(f,\ \omega,\ d,\ H,\ \ell,\ N,\ h,\ \rho)$. Repeating variables: $\omega,\ d,\ \rho$ $[T] = \frac{ML^2}{T^2},\ [f] = \frac{1}{T},\ [\omega] = \frac{1}{T},\ [d] = L,\ [H] = L,\ [\ell] = L,\ [N] = 1,\ [h] = L,\ [\rho] = \frac{M}{L^3}$ $\pi_1 = \frac{T}{\rho\omega^2 d^5},\ \pi_2 = \frac{f}{\omega},\ \pi_3 = \frac{H}{d},\ \pi_4 = \frac{\ell}{d},\ \pi_5 = N,\ \pi_6 = \frac{h}{d}$ $\therefore \underline{\frac{T}{\rho\omega^2 d^5} = g_1\left(\frac{f}{\omega},\ \frac{H}{d},\ \frac{\ell}{d},\ N,\ \frac{h}{d}\right)}$
6.38	$d = f(V,\ V_j,\ D,\ \sigma,\ \rho,\ \mu,\ \rho_a)$. Repeating variables: $V_j,\ D,\ \rho$ $[d] = L,\ [V] = \frac{L}{T},\ [V_j] = \frac{L}{T},\ [D] = L,\ [\sigma] = \frac{M}{T^2},\ [\rho] = \frac{M}{L^3},\ [\mu] = \frac{M}{LT},\ [\rho_a] = \frac{M}{L^3}$ $\pi_1 = \frac{d}{D},\ \pi_2 = \frac{V}{V_j},\ \pi_3 = \frac{\sigma}{\rho V_j^2 D},\ \pi_4 = \frac{\mu}{\rho V_j D},\ \pi_5 = \frac{\rho_a}{\rho}$ $\therefore \underline{\frac{d}{D} = f_1\left(\frac{V}{V_j},\ \frac{\sigma}{\rho V_j^2 D},\ \frac{\mu}{\rho V_j D},\ \frac{\rho_a}{\rho}\right)}$
6.40	$\mu = f(D,\ H,\ \ell,\ g,\ \rho,\ V)$. D = tube dia., H = head above outlet, ℓ = tube length. Repeating variables: $D,\ V,\ \rho$ $\quad \pi_1 = \frac{\mu}{\rho VD},\ \pi_2 = \frac{H}{D},\ \pi_3 = \frac{\ell}{D},\ \pi_4 = \frac{gD}{V^2}$

	$\therefore \underline{\frac{\mu}{\rho VD} = f_1\left(\frac{H}{D}, \frac{\ell}{D}, \frac{gD}{V^2}\right)}$
6.42	$y_2 = f(V_1, y_1, \rho, g)$. Neglect viscous wall shear. $[y_2] = L,\ [V_1] = \frac{L}{T},\ [y_1] = L,\ [\rho] = \frac{M}{L^3},\ [g] = \frac{L}{T^2}$ Repeating variables: V_1, y_1, ρ $\pi_1 = \frac{y_2}{y_1},\ \pi_2 = \frac{gy_1}{V_1^2}$ (ρ does not enter the problem) $\therefore \underline{\frac{y_2}{y_1} = f\left(\frac{gy_1}{V_1^2}\right)}$

Similitude

6.44	$\frac{Q_m}{Q_p} = \frac{V_m \ell_m^2}{V_p \ell_p^2},\ \frac{\Delta p_m}{\Delta p_p} = \frac{\rho_m V_m^2}{\rho_p V_p^2},\ \frac{(F_p)_m}{(F_p)_p} = \frac{\rho_m V_m^2 \ell_m^2}{\rho_p V_p^2 \ell_p^2}$ $\frac{\tau_m}{\tau_p} = \frac{\rho_m V_m^2}{\rho_p V_p^2},\ \frac{T_m}{T_p} = \frac{\rho_m V_m^2 \ell_m^3}{\rho_p V_p^2 \ell_p^3},\ \frac{\dot{Q}_m}{\dot{Q}_p} = \frac{\rho_m V_m^3 \ell_m^2}{\rho_p V_p^3 \ell_p^2}$ ($\dot{Q}$ has same dimensions as $\dot{W}$)
6.46	**a)** $\text{Re}_m = \text{Re}_p \quad \frac{V_m d_m}{\nu_m} = \frac{V_p d_p}{\nu_p} \quad \therefore \frac{V_m}{V_p} = \frac{d_p}{d_m} = 5$ $\frac{\dot{m}_m}{\dot{m}_p} = \frac{\rho_m \ell_m^2 V_m}{\rho_p \ell_p^2 V_p} = \frac{1}{5^2} \times 5 \quad \therefore \dot{m}_m = \dot{m}_p \frac{1}{5} = 800/5 = \underline{160\text{ kg/s}}$ $\frac{\Delta p_m}{\Delta p_p} = \frac{\rho_m V_m^2}{\rho_p V_p^2} = 5^2 \quad \therefore \Delta p_m = 25\Delta p_p = 25 \times 600 = \underline{15{,}000\text{ kPa}}$
6.48	$\text{Re}_m = \text{Re}_p \quad \frac{V_m \ell_m}{\nu_m} = \frac{V_p \ell_p}{\nu_p} \quad \therefore \frac{V_m}{V_p} = \frac{\ell_p}{\ell_m}\frac{\nu_m}{\nu_p} = 10$ assuming $\frac{\nu_m}{\nu_p} = 1$ $\therefore V_m = 10V_p = 1000\text{ km/hr}$ This velocity is much too high for a model test; it is in the compressibility region. Thus, small-scale models of autos are not used. Full-scale wind tunnels are common.

6.50	Properties of the atmosphere at 8 km altitude: T = –37°C +273 = 236 K and pressure = 35.7 kPa, density = 0.526 kg/m^3, and viscosity = 1.527×10^{-5} N·s/m^2 Properties of air at standard atmosphere: T = 20°C, p = 101.325 kPa, density = 1.204 kg/m^3, dynamic viscosity = 1.82×10^{-5} N·s/m^2 Use the Reynolds number to achieve dynamic similarity, $\left(\frac{\rho VD}{\mu}\right)_m=\left(\frac{\rho VD}{\mu}\right)_p$ Then $V_m=V_p\left(\frac{\rho D}{\mu}\right)_p\left(\frac{\mu}{\rho D}\right)_m$ and $V_m=V_p\left(\frac{\rho_p\mu_m}{\rho_m\mu_p}\right)\left(\frac{D_p}{D_m}\right)=200\left(\frac{0.526}{1.204}\right)\left(\frac{1.82\times10^{-5}}{1.527\times10^{-5}}\right)\left(\frac{10}{1}\right)=1041\text{ km/hr}$ To calculate the thrust apply: $\left(\frac{T}{\rho V^2D^2}\right)_p=\left(\frac{T}{\rho V^2D^2}\right)_m$ Then $T_p=T_m\left(\frac{\rho_pV_p^2D_p^2}{\rho_mV_m^2D_m^2}\right)=10\left(\frac{0.526\times1041^2\times10^2}{1.204\times200^2\times1^2}\right)=\underline{1184\text{ N}}$
6.52	$\text{Re}_m=\text{Re}_p \quad \frac{V_md_m}{\nu_m}=\frac{V_pd_p}{\nu_p} \quad \therefore d_m=d_p\frac{V_p}{V_m}\frac{\nu_m}{\nu_p}=0.75\times1\times\frac{10^{-6}}{5\times10^{-3}}$ = 0.0015 m = 1.5 mm Find ν_{oil} using Fig. B.2. Then $\frac{\Delta p_m}{\Delta p_p}=\frac{\rho_mV_m^2}{\rho_pV_p^2}=\frac{1000}{1000\times0.9}\times1^2=\underline{1.11}$
6.54	$\text{Re}_m=\text{Re}_p \quad \therefore \frac{V_m\ell_m}{\nu_m}=\frac{V_p\ell_p}{\nu_p} \quad \text{Fr}_m=\text{Fr}_p \quad \therefore \frac{V_m^2}{\ell_mg_m}=\frac{V_p^2}{\ell_pg_p} \quad \therefore \frac{V_m}{V_p}=\sqrt{\frac{1}{30}}$ $\frac{V_m}{V_p}=\frac{\ell_p}{\ell_m}\frac{\nu_m}{\nu_p}=30\frac{\nu_m}{\nu_p}=\sqrt{\frac{1}{30}} \quad \therefore \frac{\nu_m}{\nu_p}=\frac{1}{164} \quad \therefore \nu_m=\underline{6.1\times10^{-9}\text{ m}^2\text{/s}}$ Impossible!
6.56	$\text{Fr}_m=\text{Fr}_p \quad \frac{V_m^2}{\ell_mg_m}=\frac{V_p^2}{\ell_pg_p} \quad \therefore \frac{V_m}{V_p}=\sqrt{\frac{\ell_m}{\ell_p}}$ **a)** $\frac{Q_m}{Q_p}=\frac{V_m\ell_m^2}{V_p\ell_p^2} \quad \therefore Q_m=Q_p\frac{V_m}{V_p}\frac{\ell_m^2}{\ell_p^2}=2\times\frac{1}{\sqrt{10}}\times\frac{1}{10^2}=\underline{0.00632\text{ m}^3\text{/s}}$

	b) $\frac{F_m}{F_p}=\frac{\rho_m V_m^2 \ell_m^2}{\rho_p V_p^2 \ell_p^2} \qquad \therefore F_p = F_m \frac{V_p^2}{V_m^2}\frac{\ell_p^2}{\ell_m^2} = 12\times10\times10^2 = \underline{12{,}000\text{ N}}$
6.58	Neglect viscous effects, and account for wave (gravity) effects. $\text{Fr}_m=\text{Fr}_p \quad \therefore \frac{V_m^2}{\ell_m g_m}=\frac{V_p^2}{\ell_p g_p} \quad \therefore \frac{V_m}{V_p}=\sqrt{\frac{\ell_m}{\ell_p}} \quad \frac{\omega_m}{\omega_p}=\frac{V_m/\ell_m}{V_p/\ell_p}$ $\therefore \omega_m=\omega_p\frac{V_m}{V_p}\frac{\ell_p}{\ell_m}=600\times\sqrt{\frac{1}{10}}\times10=1897\text{ rpm}$ $\frac{T_m}{T_p}=\frac{\rho_m V_m^2\ell_m^3}{\rho_p V_p^2 \ell_p^3} \qquad \therefore T_p=T_m\frac{V_p^2}{V_m^2}\frac{\ell_p^3}{\ell_m^3}=1.2\times10\times10^3=\underline{120{,}000\text{ N}\cdot\text{m}}$
6.60	Check the Reynolds number: $\text{Re}_p=V_p d_p/\nu_p=15\times2/10^{-6}=30\times10^6$ This is a high-Reynolds-number flow $\text{Re}_m=\frac{2\times2/30}{10^{-6}}=1.33\times10^5$ This may be sufficiently large for similarity. If so $\frac{\dot{W}_m}{\dot{W}_p}=\frac{\rho_m V_m^3\ell_m^2}{\rho_p V_p^3\ell_p^2}=\frac{2^3}{15^3}\times\frac{1}{30^2}=2.63\times10^{-6}$ $\therefore \dot{W}_p=(2\times2.15)/2.63\times10^{-6}=\underline{1633\text{ kW}}$
6.62	$\text{Re}_p=\frac{20\times10}{1.5\times10^{-5}}=13.3\times10^6$. This is a high-Reynolds-number flow. For the wind tunnel, let $\text{Re}_m=10^5=V_m\times0.4/1.5\times10^{-5} \qquad \therefore V_m\ge3.75\text{ m/s}$ For the water channel, let $\text{Re}_m=10^5=V_m\times0.1/1\times10^{-6} \qquad \therefore V_m\ge1.0\text{ m/s}$ <u>Either could be selected</u>. The more convenient facility would be chosen. $\frac{F_{m_1}}{F_{m_2}}=\frac{\rho_{m_1}V_{m_1}^2\ell_{m_1}^2}{\rho_{m_2}V_{m_2}^2\ell_{m_2}^2}=\frac{3.2}{F_{m_2}} \qquad \therefore F_{m_2}=3.2\frac{1000}{1.23}\frac{2.4^2}{15^2}\times\frac{0.1^2}{0.4^2}=\underline{4.16\text{ N}}$ $\frac{\dot{W}_m}{\dot{W}_p}=\frac{\rho_m V_m^3\ell_m^2}{\rho_p V_p^3\ell_p^2}=\frac{15^3\times0.4^2}{20^3\times10^2} \qquad \therefore \dot{W}_p=(15\times3.2)\frac{20^3}{15^3}\times\frac{10^2}{0.4^2}=71{,}100\text{ W or }\underline{95\text{ hp}}$

6.64	Mach No. is the significant parameter: $\mathrm{M}_m = \mathrm{M}_p$ **b)** $V_p = V_m \dfrac{c_p}{c_m} = V_m \sqrt{\dfrac{T_p}{T_m}} = 200\sqrt{\dfrac{255.7}{296}} = \underline{186 \text{ m/s}}$ $F_p = F_m \dfrac{\rho_m V_p^2 \ell_p^2}{\rho_m V_m^2 \ell_m^2} = 10 \times 0.601 \times \dfrac{186^2}{200^2} \times 20^2 = \underline{2080 \text{ N}}$
6.66	**a)** $\mathrm{Fr}_m = \mathrm{Fr}_p \quad \dfrac{V_m^2}{\ell_m g_m} = \dfrac{V_p^2}{\ell_p g_p} \quad \therefore \dfrac{V_m}{V_p} = \sqrt{\dfrac{\ell_m}{\ell_p}}$ $\dfrac{\omega_m}{\omega_p} = \dfrac{V_m}{V_p}\dfrac{\ell_p}{\ell_m} = \dfrac{1}{\sqrt{10}} \times 10 \quad \therefore \omega_m = 2000 \times \dfrac{10}{\sqrt{10}} = \underline{6320 \text{ rpm}}$ **b)** $\mathrm{Re}_m = \mathrm{Re}_p \quad \dfrac{V_m \ell_m}{\nu_m} = \dfrac{V_p \ell_p}{\nu_p} \quad \therefore \dfrac{V_m}{V_p} = \dfrac{\ell_p}{\ell_m} = 10$ $\dfrac{\omega_m}{\omega_p} = \dfrac{V_m}{V_p}\dfrac{\ell_p}{\ell_m} = 10 \times \dfrac{1}{10} = 1 \quad \therefore \omega_m = \underline{2000 \text{ rpm}}$
6.68	$\mathrm{Re}_m = \mathrm{Re}_p \quad \therefore \dfrac{V_m \ell_m}{\nu_m} = \dfrac{V_p \ell_p}{\nu_p} \quad \therefore V_m = V_p \dfrac{\ell_p}{\ell_m} = 10 \times 10 = 100 \text{ m/s}$ This is too large for a water channel. Undoubtedly this is a high-Re flow. Select a speed of <u>5 m/s</u>. For this speed $\mathrm{Re}_m = \dfrac{5 \times 0.1}{1 \times 10^{-6}} = 5 \times 10^5$ where we used $\ell_m = 0.1$ ($\ell_p = 1$ m, i.e., the dia. of the porpoise) $\omega_m = \omega_p (V_m/V_p)(\ell_p/\ell_m) = 1 \times (5/10) \times 10 = \underline{5 \text{ motions/s}}$

Normalized Differential Equations

6.70	$\rho^* = \dfrac{\rho}{\rho_0},\ t^* = tf,\ u^* = \dfrac{u}{V},\ v^* = \dfrac{v}{V},\ x^* = \dfrac{x}{\ell},\ y^* = \dfrac{y}{\ell}$. Substitute in: $f\,\rho_0 \dfrac{\partial \rho^*}{\partial t^*} + \rho_0 \dfrac{V}{\ell}\dfrac{\partial(\rho^* u^*)}{\partial x^*} + \rho_0 \dfrac{V}{\ell}\dfrac{\partial(\rho^* v^*)}{\partial y^*} = 0$ Divide by $\rho_0 V/\ell$:

	$\therefore \frac{f\ell}{V}\frac{\partial\rho^*}{\partial t^*}+\frac{\partial}{\partial x^*}(\rho^* u^*)+\frac{\partial}{\partial y^*}(\rho^* v^*)=0$ Parameter $=\underline{\frac{f\ell}{V}}$
6.72	$\mathbf{V}^*=\frac{\mathbf{V}}{U},\ t^*=\frac{tU}{\ell},\ \nabla^*=\ell\nabla,\ p^*=\frac{p}{\rho U^2},\ h^*=\frac{h}{\ell}$. Euler's equation is then $\rho\frac{U^2}{\ell}\frac{D\mathbf{V}^*}{Dt^*}=-\rho\frac{U^2}{\ell}\nabla^* p^*-\rho g\frac{\ell}{\ell}\nabla^* h^*$ Divide by $\rho U^2/\ell$: $\frac{D\mathbf{V}^*}{Dt^*}=-\nabla^* p^*-\frac{g\ell}{U^2}\nabla^* h^*$ Parameter $=\underline{\frac{g\ell}{U^2}}$
6.74	The only velocity component is *u*. Continuity then requires that $\partial u/\partial x=0$ (replace *z* with *x* and v_z with *u* in the equations written using cylindrical coordinates). The *x*-component Navier-Stokes equation is $\frac{\partial u}{\partial t}+\cancel{v_r}\frac{\partial u}{\partial r}+\frac{\cancel{v_\theta}}{r}\frac{\partial u}{\partial\theta}+u\cancel{\frac{\partial u}{\partial x}}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\cancel{g_x}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\cancel{\frac{\partial^2 u}{\partial\theta^2}}+\cancel{\frac{\partial^2 u}{\partial x^2}}\right)$ This simplifies to $\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)$ **b)** Let $u^*=u/V,\ x^*=x/d,\ t^*=t\nu/d^2,\ p^*=p/\rho V^2$ and $r^*=r/d$: $\frac{\nu V}{d^2}\frac{\partial u^*}{\partial t^*}=-\frac{\rho V^2}{\rho d}\frac{\partial p^*}{\partial x^*}+\frac{\nu V}{d^2}\left(\frac{\partial^2 u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$ The normalized equation is $\frac{\partial u^*}{\partial t^*}=-\mathrm{Re}\frac{\partial p^*}{\partial x^*}+\frac{\partial^2 u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}$ where $\mathrm{Re}=\frac{Vd}{\nu}$
6.76	$u^*=\frac{u}{U},\quad v^*=\frac{v}{U},\quad T^*=\frac{T}{T_0},\quad x^*=\frac{x}{\ell},\quad y^*=\frac{y}{\ell},\quad \nabla^{*2}=\ell^2\nabla^2$ $\rho c_p\left[\frac{UT_0}{\ell}\frac{\partial T^*}{\partial x^*}+\frac{UT_0}{\ell}\frac{\partial T^*}{\partial y^*}\right]=\frac{K}{\ell^2}T_0\nabla^{*2}T^*$ Divide by $\rho c_p UT_0/\ell$: $\frac{\partial T^*}{\partial x^*}+\frac{\partial T^*}{\partial y^*}=\frac{K}{\rho c_p U\ell}\nabla^{*2}T^*$ Parameter $=\frac{K}{\mu c_p}\frac{\mu}{\rho U\ell}=\underline{\frac{1}{\mathrm{Pr}}\frac{1}{\mathrm{Re}}}$

CHAPTER 7

Internal Flows

FE-type Exam Review Problems: Problems 7-1 to 7-12

7.1 (D)	
7.2 (A)	
7.3 (D)	The flow in a pipe may be laminar at any Reynolds number between 2000 and perhaps 40,000 depending on the character of the flow.
7.4 (D)	The friction factor *f* depends on the velocity (the Reynolds number).
7.5 (A)	
7.6 (B)	$\Delta p = f\frac{L}{D}\frac{V^2}{2g}$ $\frac{e}{D}=\frac{0.15}{20}=0.0075$ $\therefore$ Moody's diagram gives, assuming $\text{Re}>10^5$ $f=0.034$ Then $60{,}000=0.034\times\frac{15}{0.02}\frac{V^2}{2\times9.81}$ $\therefore V=6.79$ m/s and $Q=AV=\pi\times0.01^2\times6.79=0.00214\text{ m}^3\text{/s}$ Check the Reynolds number: $\text{Re}=\frac{6.79\times0.02}{10^{-6}}=1.36\times10^5$ $\therefore$ OK
7.7 (D)	$\frac{e}{D}=\frac{0.26}{80}=0.00325$ Assume $\text{Re}>3\times10^5$ Then $f=0.026$ $\frac{h_L}{L}=\sin\theta=f\frac{1}{D}\frac{V^2}{2g}$ $\sin30^\circ=0.026\frac{1}{0.08}\frac{V^2}{2\times9.81}$ $\therefore V=5.49$ m/s Check Re: $\text{Re}=\frac{VD}{\nu}=\frac{5.49\times0.08}{10^{-6}}=4.39\times10^5$ $\therefore$ OK
7.8 (B)	$\frac{e}{D}=\frac{0.26}{60}=0.0043$ $\text{Re}=\frac{4\times0.06}{8\times10^{-6}}=3\times10^4$ $\therefore f=0.033$ from Moody's diagram $\Delta p=-\gamma f\frac{L}{D}\frac{V^2}{2g}+\gamma\,\Delta h=-9810\times0.033\times\frac{20}{0.06}\frac{4^2}{2\times9.81}+9810\times20=108{,}000\text{ Pa}$
7.9 (A)	$R=\frac{A}{P}=\frac{4\times4}{4\times4}=1\text{ cm}$ $V=\frac{Q}{A}=\frac{0.02}{0.04\times0.04}=12.5\text{ m/s}$

	$\text{Re} = \frac{4RV}{\nu} = \frac{4\times 0.01\times 12.5}{10^{-6}} = 5\times 10^5 \quad \frac{e}{4R} = \frac{0.046}{4\times 10} = 0.00115 \quad \therefore f = 0.021$ $\Delta p = f\frac{L}{4R}\frac{V^2}{2g} = 0.021\times\frac{40}{4\times 0.01}\times\frac{12.5^2}{2\times 9.81} = 167 \text{ Pa}$
7.10 (B)	Viscous effects (losses) are important.
7.11 (A)	A negative pressure must not exist anywhere in a water system for a community since a leak would suck into the system possible impurities.
7.12 (C)	$Q = \frac{1}{n}AR^{2/3}S^{1/2} = \frac{1}{0.012}0.8\times 2.4\times 0.48^{2/3}\times 0.002^{1/2} = 4.39 \text{ m}^3\text{/s}$ where we have used $R = \frac{A}{P_{wetted}} = \frac{0.8\times 2.4}{0.8+0.8+2.4} = 0.48 \text{ m}$

Laminar or Turbulent Flow

7.14	$\text{Re} = \frac{Vh}{\nu} = \frac{Vh}{1\times 10^{-6}}$ **b)** $1500 = \frac{V\times 1}{1\times 10^{-6}} \quad \therefore V = \underline{0.0015 \text{ m/s}}$
7.16	$\text{Re} = \frac{Vh}{\nu} = \frac{(1/2)\times 1.4}{10^{-6}} = 700{,}000 \qquad \therefore \underline{\text{Very turbulent}}$

Entrance and Developed Flow

7.18	$\frac{L_E}{D} = 0.065\,\text{Re} \quad \text{Re} = \frac{VD}{\nu} \quad V = \frac{0.0002}{\pi\times 0.02^2} = 0.1592 \text{ m/s}$ **a)** $L_E = 0.065\times\frac{0.1592\times 0.04}{1.31\times 10^{-6}}\times 0.04 = \underline{12.6 \text{ m}}$ **c)** $L_E = 0.065\times\frac{0.1592\times 0.04}{0.661\times 10^{-6}}\times 0.04 = \underline{25.0 \text{ m}}$
7.20	$V = \frac{0.025}{\pi\times 0.03^2} = 8.84 \quad \text{Re} = \frac{8.84\times 0.06}{1.007\times 10^{-6}} = 5.3\times 10^5 \quad \therefore \text{Turbulent}$ $\therefore L_E = 120\times 0.06 = 7.2 \text{ m} \qquad \therefore \underline{\text{Developed}}$
7.22	$L_E = 0.04\,\text{Re}\times h = 0.04\times 7700\times 0.012 = \underline{3.7 \text{ m}}$

$$\left(L_E\right)_{\min} = 0.04\times 1500\times 0.012 = \underline{0.72\text{ m}}$$

7.24

$$\text{Re} = \frac{VD}{\nu} = \frac{0.2\times 0.04}{10^{-6}} = 8000$$

a) If laminar, $L_E = 0.065\times\text{Re}\times D = 0.065\times 8000\times 0.04 = \underline{20.8\text{ m}}$

$$L_i = L_E/4 = 20.8/4 = \underline{5.2\text{ m}}$$

Laminar Flow in a Pipe

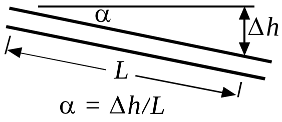
7.28

In a developed flow, *dh/dx* = slope of the pipe and *p* is a linear function of *x* so that *dp*/*dx* = const. Therefore, *d*(*p* + *γh*)/*dx* = const and it can be moved outside the integral. Then

$$\frac{2}{4\mu r_0^2}\frac{d(p+\gamma h)}{dx}\int_0^{r_0}(r^2-r_0^2)rdr = \frac{1}{2\mu r_0^2}\frac{d(p+\gamma h)}{dx}\left(\frac{r_0^4}{4}-\frac{r_0^2}{2}\times r_0^2\right) = \frac{r_0^2}{8\mu}\frac{d(p+\gamma h)}{dx}$$

7.30

$$1500 = \frac{VD}{\nu} = \frac{V\times 0.01}{6.61\times 10^{-7}} \qquad \therefore V = 0.0992\text{ m/s}$$



Eq. 7.3.14: $$\cancel{\frac{\Delta p}{L}} + \gamma\frac{\Delta h}{L} = \frac{8\mu V}{r_0^2} \qquad \therefore \frac{\Delta h}{L} = \frac{8\mu V}{r_0^2\rho g}$$

$$\therefore \alpha = \frac{8\nu V}{r_0^2 g} = \frac{8\times 6.61\times 10^{-7}\times 0.0992}{0.005^2\times 9.81} = 0.00214\text{ rad or } \underline{0.123^\circ}$$

$$Q = AV = \pi\times 0.005^2\times 0.0992 = \underline{7.79\times 10^{-6}\text{ m}^3\text{/s}}$$

7.32

Eq. 7.3.14: $$Q = \int_0^{0.01}\frac{9810(0.00015)}{2\times 10^{-3}}(0.02y - y^2)100dy$$

$$= 73{,}600\times(0.01\times 0.01^2 - \frac{0.01^3}{3}) = \underline{0.049\text{ m}^3\text{/s}}$$

For a vertical pipe $\Delta h = L.$ Thus

$$V = \frac{\rho g r_0^2}{8\mu} = \frac{g r_0^2}{8\nu} = \frac{9.81\times .01^2}{8\nu} = \frac{1.226\times 10^{-4}}{\nu}$$

b) $V = \dfrac{1.226\times 10^{-4}}{0.34/917} = 0.33\text{ m/s} \quad \therefore Q = \underline{1.04\times 10^{-4}\text{ m}^3\text{/s}} \quad \text{Re} = 17.8 \quad \therefore \underline{\text{laminar}}$

7.34	Neglect the effects of the entrance region and assume developed flow for the whole length. Also, assume $p_{\text{inlet}} = \gamma h$ (neglect $V^2/2g$ compared to 4 m). $\therefore \Delta p = \gamma h - 0 \qquad \therefore \rho g h = \frac{8\mu VL}{r_0^2} \qquad \therefore V = \frac{ghr_0^2}{8\nu L} = \frac{9.81\times 4\times 0.0025^2}{8\times 1\times 10^{-6}\times 40} = 0.766\text{ m/s}$ $\therefore Q = AV = \pi\times 0.0025^2\times 0.766 = \underline{1.5\times 10^{-5}\text{ m}^3\text{s}}$ $L_E = 0.065\times\frac{0.766\times 0.005}{1\times 10^{-6}}\times 0.005 = \underline{1.2\text{ m}}$
7.36	$\text{Re} = 2000 = \frac{VD}{\nu} = \frac{V\times 0.02}{1.5\times 10^{-5}} \qquad \therefore V = 1.5\text{ m/s}$ $\Delta p = \frac{8\mu VL}{r_0^2} = \frac{8\times 1.9\times 10^{-5}\times 1.5\times 10}{(0.01)^2} = \underline{23\text{ Pa}}$
7.38	$\text{Re} = 40{,}000 = \frac{VD}{\nu} = \frac{V\times 0.1}{1.51\times 10^{-5}} \qquad \therefore V = 6.04\text{ m/s} \quad \therefore V_{\max} = 2V = \underline{12.1\text{ m/s}}$ $\Delta p = \frac{8\mu VL}{r_0^2} = \frac{8\times 1.81\times 10^{-5}\times 6.04\times 10}{0.05^2} = \underline{3.5\text{ Pa}} \quad L_E = 0.065\times 40{,}000\times 0.1 = \underline{260\text{ m}}$
7.40	$V = -\frac{R^2}{8\mu}\frac{\Delta p + \gamma\Delta h}{L} = -\frac{R^2}{8\mu}\frac{\gamma(-L)}{L} = \frac{\gamma R^2}{8\mu} = \frac{9800\times 0.001^2}{8\times 10^{-3}} = 1.225\text{ m/s}$ $Q = AV = \pi\times 0.001^2\times 1.225 = \underline{3.85\times 10^{-6}\text{ m}^3\text{/s}}$ $\text{Re} = \frac{VD}{\nu} = \frac{122.5\times 00.02}{10^{-6}} = 2450$ It probably is not a laminar flow. Since $\text{Re} > 2000$ it would most likely be turbulent. If the pipe were smooth, disturbance and vibration free, with a well-rounded entrance it could be laminar.
7.42	**a)** Combine Eq. 7.3.12 and 7.3.15: $u = u_{\max}\left(1-\frac{r^2}{r_0^2}\right)$ $u(r) = V = 2V\left(1-\frac{r^2}{r_0^2}\right) \qquad \therefore r^2 = \frac{r_0^2}{2}$ and $\underline{r = 0.707\ r_0}$ **b)** From Eq. 7.3.17: $\tau = Cr$ where C is a constant. Then $\tau_w = Cr_0$ If $\tau = \tau_w/2$, then $\tau_w/2 = Cr$ and $r = \tau_w/2C = \underline{r_0/2}$

7.44	$\text{Re} = 20{,}000 = \frac{VD}{\nu} = \frac{V \times 0.05}{1.1 \times 10^{-6}} \qquad \therefore V = 0.44 \text{ m/s}$ $h_L = \frac{\Delta p}{\gamma} = \frac{8\mu VL}{\gamma r_0^2} = \frac{8 \times 1.1 \times 10^{-3} \times 0.44 \times 10}{9810 \times (0.025)^2} = \underline{0.0063 \text{ m}}$ $\tau_0 = \frac{r_0 \Delta p}{2L} = \frac{(0.025) \times 0.0063 \times 9810}{2 \times 10} = \underline{0.08 \text{ Pa}}$ $L_E = 0.065 \times 20{,}000 \times 0.05 = \underline{65 \text{ m}}$
7.46	See Example 7.2: $Q = -\frac{\pi}{8\mu} \frac{(-\Delta p)}{L} \left[r_2^4 - r_1^4 - \frac{(r_2^2 - r_1^2)^2}{\ln(r_2 / r_1)} \right]$ $\therefore Q = \frac{\pi \times 10}{8 \times 1.81 \times 10^{-5} \times 10} \left[0.03^4 - 0.02^4 - \frac{(0.03^2 - 0.02^2)^2}{\ln(0.03/0.02)} \right] = 7.25 \times 10^{-4} \text{ m}^3\text{/s}$ $\therefore V = \frac{Q}{A} = \frac{7.25 \times 10^{-4}}{\pi(0.03^2 - 0.02^2)} = \underline{0.462 \text{ m/s}}$ $\tau_{r_1} = \mu \frac{\partial u}{\partial r}\bigg\|_{r=r_1} = -\frac{\Delta p}{4L} \left[2r_1 - \frac{r_2^2 - r_1^2}{\ln(r_2 / r_1)} \frac{1}{r_1} \right]$ $= -\frac{10}{4 \times 10} \left[2 \times 0.02 - \frac{0.03^2 - 0.02^2}{\ln(0.03/0.02)} \times \frac{1}{0.02} \right] = \underline{0.0054 \text{ Pa}}$
7.48	From Example 7.2 $u(r) = \frac{1}{4\mu} \frac{dp}{dx} \left[r^2 - r_2^2 + \frac{r_2^2 - r_1^2}{\ln(r_1 / r_2)} \ln \frac{r}{r_2} \right]$ As $r_1 \to 0,\ \ln(r_1 / r_2) \to -\infty$ so that $\frac{r_2^2 - r_1^2}{\ln(r_1 / r_2)} \ln(r / r_2) \to 0$ Thus, $u(r) = \frac{1}{4\mu} \frac{dp}{dx} \left(r^2 - r_0^2 \right)$ where $r_2 = r_0$. See Eq. 7.3.11 As $r_1 \to r_2$, $\frac{r_2^2 - r_1^2}{\ln(r_1 / r_2)} = \frac{0}{0}$. $\therefore$Differentiate w.r.t r_1: $\frac{-2r_1}{1 / r_1} = -2r_1^2$ Also, $\ln \frac{r}{r_2} = \ln \left(1 - \frac{y}{r_2} \right) \cong -\frac{y}{r_2}$, where $y = r_2 - r$

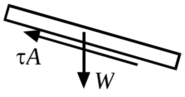
$$\therefore u(r)=\frac{1}{4\mu}\frac{dp}{dx}\left(r^2-r_2^2+\frac{2r_1^2}{r_2}y\right)=\frac{1}{4\mu}\frac{dp}{dx}\left[y^2-2r_2y+\frac{2r_1^2}{r_2}y\right]=\frac{1}{4\mu}\frac{dp}{dx}(y^2-ay)$$

Laminar Flow Between Parallel Plates

7.50

There is no pressure gradient $\therefore$ Eq. 7.4.13 gives $u=\frac{V}{a}y$

The friction balances the weight component.

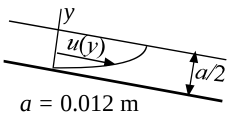


$$\tau A=W\sin\theta \qquad \tau=\mu\frac{\partial u}{\partial y}=\mu\frac{V}{a}=\mu\frac{0.2}{0.0004}$$

a) $\mu\frac{0.2}{0.0004}\times1\times1=40\sin20^\circ \qquad \therefore \mu=\underline{0.0274\ \text{N}\cdot\text{s/m}^2}$

7.52

The depth of water is $a/2$ with the maximum velocity at the surface:



$$-\frac{dh}{dx}=\sin\theta \quad \text{Hence, } u(y)=-\frac{\gamma\sin\theta}{2\mu}(y^2-ay)$$

$$Q=-\int_0^{a/2}\frac{\gamma\sin\theta}{2\mu}(y^2-ay)50dy=-\frac{50\gamma\sin\theta}{2\mu}\left(\frac{a^3}{24}-\frac{a^3}{8}\right)=\frac{25\gamma a^3}{12\mu}\sin\theta$$

$$=\frac{25}{12\times10^{-3}}\times9810\times\sin20^\circ\times0.012^3=\underline{12.1\ \text{m}^3/\text{s}}$$

$$\therefore V=\frac{12.1}{0.006\times50}=40.3\ \text{m/s}$$

$$\therefore \text{Re}=\frac{Va/2}{\nu}=\frac{40.3\times0.006}{10^{-6}}=\underline{241{,}000}$$

The assumption of laminar flow was not a good one, but we shall stay with it to answer the remaining parts.

$$u_{\max}=\frac{9810\times\sin20^\circ}{2\times10^{-3}}\left(\frac{0.012^2}{4}\right)=\underline{60.4\ \text{m/s}}$$

$$\tau_0=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}=-\frac{\gamma\sin\theta}{2}(-a)=\frac{9810\sin20^\circ}{2}\times0.012=\underline{20.1\ \text{Pa}}$$

Obviously the flow would be turbulent and the above analysis would have to be modified substantially for an actual flow.

7.54

Eq. 7.4.17: $\Delta p = \dfrac{12\mu VL}{a^2} \quad \therefore V = \dfrac{50\times 0.02^2}{12\times 1.81\times 10^{-5}\times 60} = 1.53 \text{ m/s}$

$$\therefore Q = AV = 0.02\times 0.9\times 1.53 = \underline{0.028 \text{ m}^3\text{/s}}$$

This is maximum since laminar flow is assumed. Check the Reynolds number:

$$\text{Re} = \frac{Va}{\nu} = \frac{1.53\times 0.02}{1.51\times 10^{-5}} = \underline{2030}$$

This is marginally high. Care should be taken to eliminate vibrations, disturbances, or rough walls.

7.56

Assume laminar:

$$\Delta p = \frac{12\mu VL}{a^2} \quad \therefore 4.2\times 10^6 = \frac{12\times 9.4\times 10^{-3} V\times 0.1}{(0.5\times 10^{-3})^2} \quad \therefore V = 93 \text{ m/s}$$

$$\therefore Q = AV = (0.0005\times 0.1)\times 93 = \underline{4.6\times 10^{-3} \text{ m}^3\text{/s} = 4.6 \text{ L/s}}$$

7.58

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - ay) + \frac{U}{a}y \quad \tau = \mu\frac{du}{dy} = \frac{1}{2}\frac{dp}{dx}(2y - a) + \mu\frac{U}{a}$$

a) $\tau_{y=0.006} = \dfrac{1}{2}(-20)(0.006) + 1.95\times 10^{-5}\dfrac{U}{0.006} = 0 \qquad \therefore U = \underline{18.5 \text{ m/s}}$

c) $Q = \displaystyle\int_0^{0.006}\left[\frac{1}{2\times 1.95\times 10^{-5}}(-20)(y^2 - 0.006y) + \frac{U}{0.006}y\right]dy \quad \therefore U = \underline{-6.15 \text{ m/s}}$

7.60

a) The solution for laminar flow between two parallel plates is given in Eq. (7.4.13) as

$$u(y) = \frac{1}{2\mu}\frac{d}{dx}\left(p + \gamma h\right)\left(y^2 - ay\right) + \frac{U}{a}y$$

For a horizontal flow *dh*/*dx* = 0, and hence the velocity profile is given by

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - ay\right) + \frac{U}{a}y$$

b) To determine the pressure gradient we set $u = 0$ at $y = a/2$, that is

$$0 = \frac{1}{2\mu}\frac{dp}{dx}\left(\frac{a^2}{4} - \frac{a^2}{2}\right) + \frac{U}{a}\left(\frac{a}{2}\right)$$

Simplifying and solving for the pressure gradient we get

$$\frac{dp}{dx} = 4\mu U/a^2 = 4\left(0.4 \text{ N}\cdot\text{s/m}^2\right)\left(2 \text{ m/s}\right)/\left(0.01\text{m}\right)^2 = \underline{32 \text{ kPa/m}}$$

	Note that since the pressure gradient is positive in the flow direction, it is considered to be an adverse pressure gradient.
7.62	$v_\theta = \frac{U}{a}y \qquad \therefore \tau = \mu\frac{dv_\theta}{dy} = 0.1\times\frac{0.2\times 30}{0.0008} = 750\text{ Pa}$ $T = F\times R = \tau\pi D\times L\times R = 750\pi\times 0.4\times 0.8\times 0.2 = \underline{151\text{ N}\cdot\text{m}}$
7.64	Neglect the shear on the cylinder bottom; assume a linear velocity profile: $v_\theta = \frac{U}{a}y = \frac{0.1\times 30}{0.001}y = 3000y \qquad \therefore \tau = \mu\frac{dv_\theta}{dy} = 0.42\times 3000 = 1260\text{ Pa}$ $\therefore T = F\times R = \pi DL\times\tau\times R = \pi\times 0.2\times 0.1\times 1260\times 0.1 = \underline{7.9\text{ N}\cdot\text{m}}$
7.66	Assume that all losses occur in the 8-m-long channel. The velocity through the straws and screens is so low that the associated losses will be neglected. Assume a developed flow in the channel: $V = \frac{\text{Re}\times\nu}{h} = \frac{7000\times 1.5\times 10^{-5}}{0.012} = 8.75\text{ m/s}$ $\Delta p = \frac{12\times 1.8\times 10^{-5}\times 8.75\times 8}{0.012^2} = 105\text{ Pa}$ Energy: $\dot{W}_{\text{fan}} = \frac{\Delta p}{\rho\eta}\dot{m} = \frac{\Delta p}{\rho\eta}(\rho AV) = \frac{\Delta pAV}{\eta}$ $= \frac{105\times 1.2\times 0.012\times 8.75}{0.7} = \underline{18.9\text{ W}}$

Laminar Flow Between Rotating Cylinders

7.68	Use Eq. 7.5.19: $T = \frac{4\pi\mu r_1^2 r_2^2 L\omega_1}{r_2^2 - r_1^2} = \frac{4\pi\times 0.035\times 0.02^2\times 0.03^2\times 0.4\times(3000\times 2\pi/60)}{0.03^2 - 0.02^2}$ $\therefore T = \underline{0.040\text{ N}\cdot\text{m}} \qquad \therefore \dot{W} = T\omega = 0.04\times(3000\times 2\pi/60) = \underline{12.6\text{ W}}$ $\text{Re} = \frac{\omega r_1(r_2 - r_1)\rho}{\mu} = \frac{(3000\times 2\pi/60)\times 0.02\times(0.01)\times 917}{0.035} = \underline{1650}$ $\therefore$Eq. 7.5.15 is OK

7.70

With $\omega_1 = 0,$ Eq. 7.5.15 is $v_\theta = \frac{r_2^2\omega_2}{r_2^2 - r_1^2}\left(r - \frac{r_1^2}{r}\right)$ $\quad \tau_2 = \mu r \frac{d}{dr}\left(\frac{v_\theta}{r}\right) = \frac{2\mu r_1^2\omega_2}{r_2^2 - r_1^2}$

$$\therefore T_2 = \tau_2 A_2 r_2 = \frac{2\mu r_1^2\omega_2}{r_2^2 - r_1^2} 2\pi r_2 L r_2 = \frac{4\pi\mu r_1^2 r_2^2 L\omega_2}{r_2^2 - r_1^2}$$

Turbulent Flow

7.72

Let $u = \overline{u} + u',\ v = \overline{v} + v',\ w = \overline{w} + w'.$ The continuity equation becomes

$$\frac{\partial}{\partial x}(\overline{u} + u') + \frac{\partial}{\partial y}(\overline{v} + v') + \frac{\partial}{\partial z}(\overline{w} + w') = \frac{\partial\overline{u}}{\partial x} + \frac{\partial\overline{v}}{\partial y} + \frac{\partial\overline{w}}{\partial z} + \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}$$

Now, time-average the above equation recognizing that $\overline{\frac{\partial\overline{u}}{\partial x}} = \frac{\partial\overline{u}}{\partial x}$ and $\overline{\frac{\partial u'}{\partial x}} = \frac{\partial}{\partial x}\overline{u'} = 0.$ Then, $\frac{\partial\overline{u}}{\partial x} + \frac{\partial\overline{v}}{\partial y} + \frac{\partial\overline{w}}{\partial z} = 0.$ Substitute this back into the continuity equation, so that

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

7.74

Use the fact that $\frac{\partial}{\partial y}\overline{u'v'} = \overline{\frac{\partial}{\partial y}u'v'}.$ See Eq. 7.6.2. This is equivalent to

$$\frac{\partial}{\partial y}\left(\frac{1}{T}\int_0^T u'v'dt\right) = \frac{1}{T}\int_0^T \frac{\partial}{\partial y}(u'v')dt$$

which is obviously correct. Also

$$\frac{\partial}{\partial x}u'u' + \frac{\partial}{\partial y}u'v' + \frac{\partial}{\partial z}u'w' = u'\left(\cancel{\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}}\right) + u'\frac{\partial u'}{\partial x} + v'\frac{\partial u'}{\partial y} + w'\frac{\partial u'}{\partial z}$$

Time average both sides and obtain the result.

7.76

$$\overline{u} = \Sigma u_i / 11 = \underline{16.2\ \text{m/s}} \qquad \overline{v} = \Sigma v_i / 11 = \underline{-1.6\ \text{m/s}} \qquad u' = u - \overline{u} \qquad v' = v - \overline{v}$$

t	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
u'	–0.1	9.5	–5.6	1.1	–11	–6	0.9	12.4	–9.5	3	5.4
u'^2	0.01	90.2	31.4	1.2	121	36	0.81	153.8	90.2	9	29.2
v'	3.2	–3.8	–7	5.1	5.7	–4.4	0.2	8.3	–3.6	–6.6	3.1
v'^2	10.2	14.4	49	26	32.5	19.4	0.04	68.9	13.0	43.6	9.6
$u'v'$	–0.32	–36.1	39.2	5.6	–62.7	26.4	0.2	102.9	34.2	–19.8	16.7

$$\overline{u'^2} = \underline{51.2\ \text{m}^2/\text{s}^2} \qquad \overline{v'^2} = \underline{26.1\ \text{m}^2/\text{s}^2} \qquad \overline{u'v'} = \underline{9.7\ \text{m}^2/\text{s}^2}$$

7.78

$$\eta = -\frac{\overline{u'v'}}{d\overline{u}/dy} = -\frac{-2.52}{(23-18.2)/0.03} = \underline{0.016\ \text{m}^2/\text{s}}$$

$$K_{uv} = \frac{\overline{u'v'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}} = \frac{-2.52}{\sqrt{29}\sqrt{14}} = \underline{-0.125}$$

$$\ell_m = \sqrt{\eta / \partial\overline{u} / \partial y} = \sqrt{0.016/(23-18.2)/0.03} = 0.01\ \text{m or } \underline{1\ \text{cm}}$$

7.80

b) $\text{Re} = \dfrac{0.2 \times 0.2}{10^{-6}} = 40{,}000, \quad \dfrac{e}{D} = 0.0013$

From the Moody diagram, this is in the transition zone where δ_v may be near, in magnitude, to *e*. The pipe is <u>rough</u>.

7.82

b) With $\text{Re} = 40{,}000,\ e/D = 0.0013$ the Moody diagram gives $f = 0.026.$ Then

$$\tau_0 = \frac{0.026}{8} 1000 \times 0.2^2 = 0.130\ \text{Pa} \qquad u_\tau = \sqrt{0.130/1000} = 0.01140\ \text{m/s}$$

Eq. 7.6.17:
$$u_{\max} = u_\tau \left(2.44 \ln \frac{r_o}{e} + 8.5 \right)$$
$$= 0.0114 \left(2.44 \ln \frac{0.1}{0.00026} + 8.5 \right) = \underline{0.262\ \text{m/s}}$$

7.84

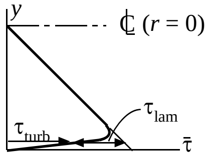
$$V = \frac{Q}{A} = \frac{70 \times 10^{-3}}{\pi \times (0.6)^2} = 6.2\ \text{m/s} \qquad \text{Re} = \frac{VD}{\nu} = \frac{6.2 \times 0.12}{1.007 \times 10^{-6}} = 7.4 \times 10^5$$

From Table 7.1, $\underline{n \cong 8.5}$. From Eq. 7.6.20

$$u_{\max} = \frac{(n+1)(2n+1)}{2n^2} \times V = \frac{9.5 \times 18}{2 \times 8.5^2} \times 6.2 = \underline{7.34\ \text{m/s}}$$

7.86

With $n = 7$, Eq. 7.6.21 gives $f = \frac{1}{n^2} = \frac{1}{49} = 0.0204$



$$\therefore \tau_0 = \frac{1}{8}\rho V^2 f = \frac{1}{2}\times 1000\times 10^2\times 0.0204 = 1020 \text{ Pa}$$

Since τ varies linearly with r and is zero at $r = 0$

$$\bar{\tau} = \tau_0 \frac{r}{r_0} = \frac{1020}{0.05} r = 20{,}400\, r$$

$$\tau_{lam} = \mu \frac{\partial \bar{u}}{\partial y} = 10^{-3}\left[u_{\max} \frac{1}{7}\frac{1}{r_0^{1/7}} y^{-6/7}\right] = \frac{10^{-3}\times 12.24}{7\times 0.05^{1/7}} y^{-6/7} = 0.00268 y^{-6/7}$$

$$\tau_{\text{turb}} = \bar{\tau} - \tau_{\text{lam}} = 20{,}400 r - 0.00268 y^{-6/7} \quad \text{where} \quad y + r = 0.05$$

$\tau_{\text{lam}}(y)$ is good away from $y = 0$ (the wall)

$$\tau_{\text{lam}}(0.00625) = 0.21, \quad \tau_{\text{lam}}(0.003125) = 0.38$$
$$\tau_{\text{lam}}(0.00156) = 0.68, \quad \tau_{\text{lam}}(0.00078) = 1.24$$

$$\frac{d\bar{p}}{dx} = -\frac{2\tau_0}{r_0} = -\frac{2\times 1020}{0.05} = \underline{-40{,}800 \text{ Pa/m}}$$

7.88

We must find u_τ :

$$V = \frac{Q}{A} = \frac{1.2}{\pi\times 0.4^2} = 19.63 \text{ m/s} \qquad \text{Re} = \frac{19.63\times 0.8}{2.2\times 10^{-4}} = 71{,}400$$

$$\frac{e}{D} = \frac{0.26}{800} = 0.000325 \quad \therefore \text{ Moody diagram} \Rightarrow f = 0.021$$

$$\tau_0 = \frac{1}{8} f\rho V^2 = \frac{1}{8}\times 0.021\times 917\times 19.63^2 = 928 \text{ Pa}$$

$$u_\tau = \sqrt{\tau_0/\rho} = \sqrt{928/917} = 1.006 \text{ m/s}$$

$$\therefore u_{\max} = 1.006\left[2.44 \ln \frac{1.006\times 0.4}{2.2\times 10^{-4}} + 5.7\right] = \underline{24.2 \text{ m/s}}$$

7.90

a) From a control volume of the 10-m section of pipe

$$\Delta p \frac{\pi D^2}{4} = \tau_0 \pi D L \qquad \therefore \tau_0 = \frac{0.12\times 5000}{4\times 10} = \underline{15 \text{ Pa}}$$

Assume $n = 7$. Then Eq. 7.6.21 gives $f = \frac{1}{n^2} = \frac{1}{49} = 0.0204$

From Eq. 7.3.19, $V^2 = \dfrac{8\tau_0}{\rho f} = \dfrac{8 \times 15}{917 \times 0.0204} = 6.41$ or $V = 2.53$ m/s

Check: $\text{Re} = \dfrac{2.53 \times 0.12}{2.2 \times 10^{-4}} = 1381.$ This suggests laminar flow. Use Eq. 7.3.14:

$$V = \frac{r_0^2 \Delta p}{8\mu L} = \frac{0.06^2 \times 5000}{8 \times (917 \times 2.2 \times 10^{-4}) \times 10} = \underline{1.12 \text{ m/s}}$$

Check: $\text{Re} = \dfrac{1.12 \times 0.12}{2.2 \times 10^{-4}} = 611$ OK

Finally, $Q = AV = \pi \times 0.06^2 \times 1.12 = \underline{0.0127 \text{ m}^3\text{/s}}$

Turbulent Flow in Pipes and Conduits

7.92

a) $\text{Re} = \dfrac{VD}{\nu} = \dfrac{(0.020/\pi \times 0.04^2) \times 0.08}{10^{-6}} = 3.18 \times 10^5$ $\dfrac{e}{D} = 0$ $\therefore f = \underline{0.0143}$

b) Eq. 7.6.26 provides *f* by trial-and-error. Try *f* = 0.0143 from Moody's diagram:

$$\frac{1}{\sqrt{f}} = 0.86 \ln\left(3.18 \times 10^5 \sqrt{0.0143}\right) - 0.8 \quad \therefore f = \underline{0.0146}$$

Another iteration may be recommended but this is quite close. The value for *f* is essentially the same using either method. The equations could be programmed on a computer.

7.94

a) $\text{Re} = \dfrac{0.025 \times 0.04}{10^{-6}} = 1000$ $\therefore$ laminar $\therefore f = \dfrac{64\nu}{VD} = \dfrac{64 \times 10^{-6}}{0.025 \times 0.04} = \underline{0.064}$

c) $\text{Re} = \dfrac{2.5 \times 0.04}{10^{-6}} = 100{,}000$ $\dfrac{e}{D} = \dfrac{0.26}{40} = 0.0065$ $\therefore f = \underline{0.034}$

7.96

$V = \dfrac{Q}{A} = \dfrac{1.65 \times 10^{-3}}{\pi (0.019)^2} = 1.45$ m/s $\text{Re} = \dfrac{VD}{\nu} = \dfrac{1.45 \times 0.038}{1.14 \times 10^{-6}} = 4.8 \times 10^4$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{180}{0.038}\frac{1.45^2}{19.62} = 508\, f$$

a) $\dfrac{e}{D} = \dfrac{0.00026}{0.038} = 0.0068$ $\therefore f = 0.035$ $\therefore h_L = 508 \times 0.035 = \underline{17.8 \text{ m}}$

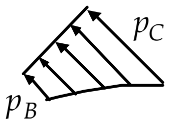
c) $\dfrac{e}{D} = \dfrac{0.000046}{0.038} = 0.0012$ $\therefore f = 0.0245$ $\therefore h_L = 508 \times 0.0245 = \underline{12.4 \text{ m}}$

7.98	$V=\frac{Q}{A}=\frac{0.08}{\pi\times0.075^2}=4.73\text{ m/s}\qquad\frac{e}{D}=\frac{0.15}{150}=0.001$ $\Delta p=\gamma f\frac{L}{D}\frac{V^2}{2g}=(917\times9.81)f\frac{10}{0.15}\times\frac{4.73^2}{2\times9.81}=6.84\times10^5 f$ **a)** $\text{Re}=\frac{4.73\times0.15}{0.0022}=322\quad\therefore\text{ laminar and }\quad\Delta p=6.84\times10^5\times\frac{64}{\text{Re}}=\underline{1.36\times10^5\text{ Pa}}$ Eq. 7.6.29 is not applicable to a laminar flow **c)** $\text{Re}=\frac{4.73\times0.15}{0.000044}=16{,}300\quad\therefore f=0.029\quad\Delta p=6.84\times10^5\times0.029=\underline{20{,}000\text{ Pa}}$ $\Delta p=9.8\times917\times1.07\frac{0.08^2\times10}{9.8\times0.15^5}\left\{\ln\left[\frac{0.15}{3.7\times50}+4.62\left(\frac{4.4\times10^{-5}\times0.15}{0.08}\right)^{0.9}\right]\right\}^{-2}$ $=20{,}700\text{ Pa}$
7.100	$V=\frac{Q}{A}=\frac{8.3\times10^{-3}}{\pi(0.03)^2}=2.94\text{ m/s}\qquad\text{Re}=\frac{2.94\times0.06}{1.31\times10^{-6}}=1.3\times10^5\qquad\frac{e}{D}=0$ $\therefore f=0.0167\quad\therefore\Delta p-\gamma\Delta h=\gamma f\frac{L}{D}\frac{V^2}{2g}\qquad(\text{Recall: }\Delta p=p_1-p_2,\ \Delta h=h_2-h_1)$ $\therefore\Delta p=9810\times0.0167\frac{90}{0.06}\times\frac{2.94^2}{19.62}+9810\times90\sin30^\circ=\underline{550\times10^3\text{ Pa.}}=\underline{550\text{ kPa}}$
7.102	$V=\frac{Q}{A}=\frac{5}{\pi\times0.4^2}=9.95\text{ m/s}\qquad\text{Re}=\frac{9.95\times0.8}{10^{-6}}=8\times10^6$ $\frac{e}{D}=\frac{1.6}{800}=0.002$ (using an average "e" value) $\quad\therefore f=0.0237$ $\therefore\Delta p=\gamma f\frac{L}{D}\frac{V^2}{2g}=9810\times0.0237\times\frac{100}{0.8}\times\frac{9.95^2}{2\times9.81}=\underline{147{,}000\text{ Pa}}$
7.104	Use Eq. 7.6.30: $h_L=\frac{\Delta p}{\gamma}=\frac{200{,}000}{9810}=20.4\text{ m}\quad\nu=10^{-6}\text{ m}^2\text{/s}$ **b)** $Q=-0.965\left[\frac{9.81\times0.04^5\times20.4}{100}\right]^{0.5}\ln\left[\frac{0.046}{3.7\times40}+\left(\frac{3.17\times10^{-12}\times100}{9.81\times0.04^3\times20.4}\right)^{0.5}\right]$ $=\underline{0.0033\text{ m}^3\text{/s}}$

7.106	Use Eq. 7.6.30: $\dot{m}=\rho Q \quad \frac{e}{3.7D}=\frac{0.046}{3.7\times120}=0.000104$ **b)** $\rho=\frac{p}{RT}=\frac{200}{0.189\times313}=3.38\ \text{kg/m}^3 \quad \therefore h_L=\frac{400}{9.81\times3.38}=12.1\ \text{m}$ $\nu=\frac{\mu}{\rho}=\frac{1.3\times10^{-5}}{3.38}=3.8\times10^{-6}\ \text{m}^2/\text{s}$ $Q=-0.965\left[\frac{9.81\times0.12^5\times12.1}{400}\right]^{0.5}\ln\left[0.000104+\left(\frac{3.17\times3.8^2\times10^{-12}\times400}{9.81\times0.12^3\times12.1}\right)^{0.5}\right]$ $=0.0205$ $\therefore \dot{m}=\underline{0.069\ \text{kg/s}}$
7.108	Use Eq. 7.6.31: $e=0 \quad D=0.66\left[e^{1.25}\left(\frac{LQ^2}{gh_L}\right)^{4.75}+\nu Q^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$ **a)** $h_L=\frac{\Delta p}{\gamma}=\frac{200{,}000}{9810}=20.4\ \text{m} \quad \nu=10^{-6}\ \text{m}^2/\text{s}$ $D=0.66\left[10^{-6}\times0.002^{9.4}\left(\frac{100}{9.81\times20.4}\right)^{5.2}\right]^{0.04}=\underline{0.032\ \text{m}}$ **c)** $h_L=\frac{\Delta p}{\gamma}=\frac{200{,}000}{7930}=25.2\ \text{m} \quad \nu=2.1\times10^{-6}\ \text{m}^2/\text{s}$ $D=0.66\left[2.1\times10^{-6}\times0.002^{9.4}\left(\frac{100}{9.81\times25.2}\right)^{5.2}\right]^{0.04}=\underline{0.031\ \text{m}}$
7.110	**a)** $V=\frac{0.4/60}{\pi D^2/4}=0.00849/D^2, \quad h_L=f\frac{L}{D}\frac{V^2}{2g}$ or $3=f\frac{1200}{D}\frac{(0.00849/D^2)^2}{2\times9.8}$ or $f=680D^5$ Guess: $f=0.02$. Then the above equation gives $D=0.124$ m Check: $V=0.551$ m/s, $\text{Re}=\frac{0.551\times0.25}{1.31\times10^{-6}}=1.05\times10^5, \quad \frac{e}{D}=\frac{0.0015}{124}=1.2\times10^{-5}$ $\therefore f=0.0175$. Use $f=0.0175$ and $\underline{D=0.121\ \text{m}}$ **b)** Use Eq. 7.6.31:

	$D = 0.66\left[(1.5\times10^{-5})^{1.25}\left(\frac{1200\times(0.4/60)^2}{9.8\times3}\right)^{4.75} + 1.31\times10^{-6}\times\left(\frac{0.4}{60}\right)^{9.4}\left(\frac{1200}{9.8\times3}\right)^{5.2}\right]^{0.04}$ $= \underline{0.127\text{ m}}$ Exiting K.E. = $\frac{V^2}{2g} = \frac{0.892^2}{2\times9.8} = 0.041\text{ m}$ ∴ negligible
7.112	Use Eq. 7.6.30: $h_L = \frac{80}{9810},\ \nu = 10^{-6}\text{ m}^2\text{/s}$, use $D = 4R = 4\frac{0.02\times0.04}{2(0.02+0.04)} = 0.027\text{ m}$ $Q = -0.965\left[\frac{9.81\times0.027^5}{2}\times\frac{80}{9810}\right]^{0.5}\ln\left[\frac{0.0015}{3.7\times27}+\left(\frac{3.17\times10^{-12}\times2\times9810}{9.81\times0.027^3\times80}\right)^{0.5}\right]$ $= \underline{0.000143\text{ m}^3\text{/s}}$
7.114	**b)** $R = \frac{A}{P_{\text{wet}}} = \frac{0.6\times1.2}{2.4} = 0.3\text{ m},\ h_L = 10{,}000\times0.0015 = 15\text{ m}$ (from energy eq.) $Q = -0.965\left[\frac{9.8\times1.2^5\times15}{10{,}000}\right]^{0.5}\ln\left[\frac{1.5}{3.7\times1200}+\left(\frac{3.17\times10^{-12}\times10{,}000}{9.8\times1.2^3\times15}\right)^{0.5}\right] = \underline{0.257\text{ m}^3\text{/s}}$

Minor Losses

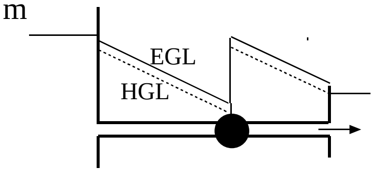
7.116	Referring to the equation $\Delta p = -\rho\frac{V^2}{R}\Delta n$ (n is in the direction of the center of curvature), we observe that Δp is negative all along the line CB from C to B in Fig. P7.115. Hence, the pressure decreases from C to B with $p_C > p_B$. Using Bernoulli's equation we see that $V_B > V_C$. Fluid moves from the high pressure region at the outside of the bend toward the low pressure region at the outside of the bend creating a secondary flow. 
7.118	**a)** $V_1 = 63.7,\quad V_2 = 15.9,\quad \rho_1 = 1.78\text{ kg/m}^3$ Energy: $0 = \frac{15.9^2 - 63.7^2}{2\times9.81} + \frac{p_2 - 50{,}000}{1.78\times9.81} + 0.4\frac{(63.7-15.9)^2}{2\times9.81}\quad \therefore p_2 = \underline{52{,}600\text{ Pa}}$
7.120	$V = \frac{Q}{A} = \frac{3.3\times10^{-3}}{\pi(0.025)^2} = 1.6\text{ m/s}$ Neglect pipe friction, i.e., $f\frac{L}{D}\frac{V^2}{2g} \cong 0$

	Energy: $0=\dfrac{1.68^2-0^2}{2\times 9.81}+0-1.8+(K+0.03)\dfrac{1.68^2}{2\times 9.81}$ $\quad\therefore \underline{K=11.48}$

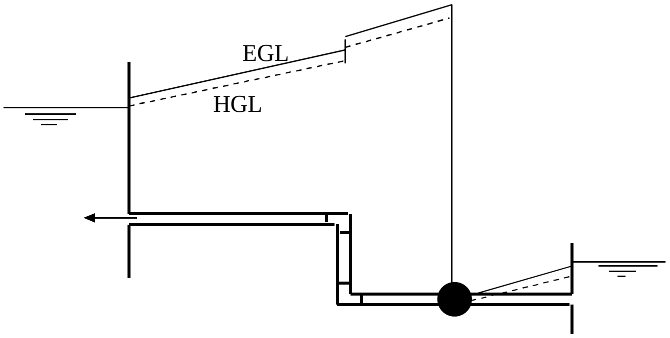
Simple Piping Systems

7.122	Assume completely turbulent regime: $\dfrac{e}{D}=\dfrac{0.046}{40}=0.0012 \quad \therefore f=0.0205, \quad \text{Re}>10^6$ Energy: $\quad 0=\dfrac{V^2}{2g}-40+\left[f\dfrac{L}{D}+2K_{\text{elbow}}+K_{\text{entrance}}\right]\dfrac{V^2}{2g}$ $40=\left[1+0.0205\times\dfrac{110}{0.04}+2\times 1.0+0.5\right]\dfrac{V^2}{2\times 9.81}$ $\quad$(Assume a screwed elbow) $\therefore V=3.62 \text{ m/s} \quad \text{Re}=\dfrac{3.62\times 0.04}{10^{-6}}=1.4\times 10^5 \quad \therefore \text{Try } f=0.022$ $\therefore V=3.50 \text{ m/s} \quad \therefore Q=AV=\pi\times 0.02^2\times 3.5=\underline{0.0044 \text{ m}^3\text{/s}}$ The EGL and HGL have sudden drops at the elbows, and a gradual slope over the pipe length.
7.124	Assume screwed elbows: $0=\dfrac{V^2}{2g}-2+\left(K_{\text{entrance}}+2K_{\text{elbows}}+f\dfrac{L}{D}\right)\dfrac{V^2}{2g}$ **b)** Assume $f=0.018: \quad 2=\left(0.8+1+2\times 0.8+0.018\dfrac{26}{0.08}\right)\dfrac{V^2}{2\times 9.81} \quad \therefore V=2.06 \text{ m/s}$ $\therefore \text{Re}=\dfrac{2.06\times 0.08}{1.14\times 10^{-6}}=1.5\times 10^5 \quad f=0.0164$ $\therefore V=2.12 \quad Q=\pi\times 0.04^2\times 2.12=\underline{0.011 \text{ m}^3\text{/s}}$
7.126	Assume constant water level, neglect the velocity head at the pipe exit and let $p_{\text{exit}}=0$. Then the energy equation says $h_L=0.8$ m: $0.8=f\dfrac{10}{0.025}\dfrac{V^2}{2\times 9.8}+2\times 1.5\dfrac{V^2}{2\times 9.8} \quad \text{or} \quad 0.8=400fV^2+0.153V^2$ Try $f=0.02$. Then $V=0.319$ m/s $\quad \text{Re}=\dfrac{0.313\times 0.025}{0.73\times 10^{-6}}=1.1\times 10^4 \quad \therefore f=0.029$ Try $f=0.029$ Then $V=0.262$ m/s $\therefore Q=AV=\pi\times 0.0125^2\times 0.262=1.28\times 10^{-4} \text{ m}^3\text{/s}$

	Finally $\Delta t = \frac{\cancel{V}}{Q} = \frac{10}{0.128} = 78.1\text{ sec}$ or $\underline{1.3\text{ min}}$
7.128	Energy: $0 = \frac{V_3^2}{2g} - \frac{p_0}{\gamma} - 1.2 + 0.8\frac{V_1^2}{2g} + f_1\frac{1.2}{0.008}\frac{V_1^2}{2g} + f_2\frac{1.2}{0.005}\frac{V_2^2}{2g}$ Continuity: $V_3 = \frac{25}{4}V_2,\ V_2 = \frac{64}{25}V_1$ Assume $f_1 = f_2 = 0.02$ Then energy says $V_1 = 2.09$ m/s Check: $\text{Re}_1 = \frac{2.09\times 0.008}{10^{-6}} = 16{,}700$ $\therefore f = 0.026$ Then $V_1 = 2.88$ m/s Finally $V_3 = 16V_1 = \underline{46.1\text{ m/s}}$
7.130	Energy: $0 = -3 + f\frac{300}{0.0094}\frac{V^2}{2\times 9.8}$ Assume turbulent: $f = 0.04$ Then $V = 0.215$ m/s Check: $\text{Re} = \frac{0.215\times 0.0094}{10^{-6}} = 2020$ $\therefore$ marginally laminar Assume laminar flow with $f = 64/\text{Re}$. Then $3 = \frac{64\times 10^{-6}}{V\times 0.0094}\times\frac{300}{0.0094}\times\frac{V^2}{2\times 9.8}$ $\therefore V = 0.271$ m/s Check: $\text{Re} = \frac{0.271\times 0.0094}{10^{-6}} = 2540$ $\therefore$ turbulent The flow is neither laminar nor turbulent but may oscillate between the two. The wall friction is too low in the laminar state so it speeds up and becomes turbulent. The wall friction is too high in the turbulent state so it slows down and becomes laminar, etc., etc.
7.132	$V = \frac{Q}{A} = \frac{0.01}{\pi\times 0.02^2} = 7.96$ m/s $\text{Re} = \frac{7.96\times 0.04}{1.14\times 10^{-6}} = 2.8\times 10^5$ $\frac{e}{D} = \frac{0.0015}{40} = 3.8\times 10^{-5}$ $\therefore f = 0.0145$ $H_P = 80 - 10 + \left(0.5 + 1.0 + 0.0145\frac{800}{0.04}\right)\frac{7.96^2}{2\times 9.81} = 1010$ m. $\therefore \dot{W}_P = \frac{\gamma Q H_P}{\eta_P} = \frac{9810\times 0.01\times 1010}{0.85} = \underline{117{,}000\text{ W}}$

	$0=\frac{7.96^2}{2\times9.81}+\frac{1670-100,000}{9810}+0-10+\left(0.5+0.0145\frac{L}{0.04}\right)\frac{7.96^2}{2\times9.81}$ $\therefore \underline{L=13.0\text{ m}}$
7.134	Energy across nozzle (neglect losses): $\frac{V_1^2}{2g}+\frac{690\times10^3}{9810}=\frac{V_2^2}{2g}=\frac{(4V_1)^2}{2g}$ $\therefore V_1=9.6\text{ m/s}$ $\text{Re}=\frac{9.6\times0.05}{1.007\times10^{-6}}=4.8\times10^5$ $\frac{e}{D}=\frac{0.000045}{0.05}=0.0009$ $\therefore f=0.02$ $H_P=\frac{(4\times9.6^2)^2}{19.62}-18+\left(0.5+0.02\frac{360}{0.05}\right)\frac{9.6^2}{19.62}=677.2\text{ m}$ $\therefore \dot{W}_P=\frac{\gamma QH_P}{\eta_P}=\frac{9810\times(\pi\times(0.025)^2\times9.6)\times677.2}{0.75}=166.97\times10^3$ W or $\underline{166.97\text{ kW}}$ $0=\frac{9.6^2}{19.62}+\frac{(2.3-100)10^3}{9810}-18+\left(0.5+0.02\frac{L}{0.05}\right)\frac{9.6^2}{19.62}$ $\therefore \underline{L=11.1\text{ m}}$
7.136	$\frac{e}{D}=0.0013$ $\therefore f=0.021$ $Q=0.0314V$ $H_P=-20+\left(0.5+1.0+0.021\frac{300}{0.2}\right)\frac{V^2}{2g}=-20+1700Q^2$ Try $\underline{Q=0.23\text{ m}^3\text{/s}}$: $(H_P)_E=70$ m $(H_P)_C=70$ m  $\therefore \dot{W}_P=\frac{9810\times0.23\times70}{0.83}=\underline{190,000\text{ W}}$ $\text{Re}=1.5\times10^6$ $\therefore$OK
7.138	$\frac{e}{D}=\frac{0.046}{160}=0.00029$ $\therefore f=0.015$ $Q=\pi\times0.08^2V=0.0201V$ $H_P=25+\left(0.5+2\times0.4+1+0.015\frac{50}{0.16}\right)\frac{Q^2}{2g\times0.0201^2}=25+882Q^2$ Try $Q=0.23\text{ m}^3\text{/s}$ $(H_P)_E=72$ m $(H_P)_{C.}=70$ m **a)** $\therefore \dot{W}_P=\frac{9810\times0.23\times72}{0.83}=\underline{195,000\text{ W}}$ **b)** $0=\frac{11.44^2}{2\times9.81}+\frac{p_2}{9810}-8+\left(0.5+0.015\frac{10}{0.16}\right)\frac{11.44^2}{2\times9.81}$ $\therefore \underline{p_{in}=-81,000\text{ Pa}}$ **c)** $72=\frac{11.44^2}{2\times9.81}+\frac{p_3}{9810}-8+\left(0.5+0.015\frac{10}{0.16}\right)\frac{11.44^2}{2\times9.81}$ $\therefore \underline{p_{out}=625,000\text{ Pa}}$

d)



7.140

$$\frac{e}{D}=\frac{1.65}{1200}=0.0014 \quad \therefore f=0.021 \quad Q=\pi\times 0.6^2\,V=1.13V$$

$$-H_T=-60+\left(0.8+1+0.021\frac{1000}{1.2}\right)\frac{Q^2}{1.13^2\times 2\times 9.81} \quad \therefore H_T=60-0.77Q^2$$

$$\dot{W}_T=\gamma Q H_T \eta_T=9810\times(60Q-0.77Q^3)\times 0.9=530{,}000Q-6800Q^3$$

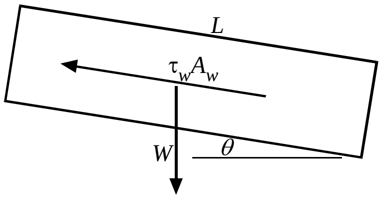
But, the characteristic curve is $\dot{W}_T=500{,}000\,Q$ (power in watts)

Hence, 500,000Q = 530,000Q – 6800Q^3 $\therefore Q$ = 2.1 m^3/s

$$\therefore \dot{W}_T=\underline{1.05\text{ MW}}$$

Open-Channel Flow

7.142



$$\tau_w A_w=W\sin\theta \qquad \sin\theta=0.001$$

or $\tau_w\times w\times 1=(w\times d\times 1)\rho_w g\sin\theta$

$$\tau_w=\rho_w dg\sin\theta=1000\times 1.8\times 9.81\times 0.001$$
$$=17.6\text{ Pa}$$

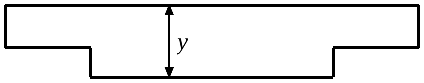
7.144

a) $Q=\frac{1.0}{n}AR^{2/3}S^{1/2}=\frac{1.0}{0.012}\times(2\times 0.6)\left(\frac{2\times 0.6}{2+2\times 0.6}\right)^{2/3}\times\ 0.001^{1/2}=\underline{1.64\text{ m}^3\text{/s}}$

b) Planed wood and finished concrete have the same "*n*" (i.e., roughness)

$\therefore$Use a value for "*e*" which is relatively small, say *e* = 0.6 mm. The solution is not too sensitive to this choice. Then, with *R* = 0.375 m

$$\frac{e}{D}=\frac{e}{4R}=\frac{0.6}{4\times 375}=0.0004 \qquad \therefore f=0.016$$

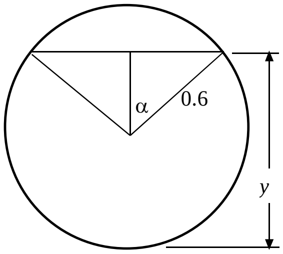
	$h_L = f\frac{L}{D}\frac{V^2}{2g} = LS \quad \therefore V^2 = \frac{2\times9.81\times0.001\times(4\times0.375)}{0.016} \quad \therefore V = 1.36\text{ m/s}$ $\therefore Q = 1.36\times2\times0.6 = \underline{1.64\text{ m}^3\text{/s}} \quad \left(\text{Check: Re} = \frac{1.36\times(4\times0.375)}{10^{-6}} = 2.0\times10^6 \quad \therefore \text{OK}\right)$
7.146	**a)** $R = \frac{1.2\times0.5+0.5\times0.5}{1.2+2\times0.5/0.707} = 0.325\text{ m} \quad \therefore Q = \frac{1.0}{0.016}\times0.85\times0.325^{2/3}\times0.001^{1/2} = \underline{0.794\text{ m}^3\text{/s}}$ $V = \frac{Q}{A} = \frac{0.794}{0.85} = \underline{0.934\text{ m/s}}$ **b)** Use an "*e*" for rough concrete, *e* = 3 mm $\therefore \frac{e}{D} = \frac{3}{4\times325} = 0.0023 \quad \therefore f = 0.025$ $h_L = LS = f\frac{L}{4R}\frac{V^2}{2g} \quad \therefore V^2 = \frac{4\times0.325\times2\times9.81\times0.001}{0.025}$ and $V = \underline{1.01\text{ m/s}}$ $\therefore Q = 1.01\times0.85 = \underline{0.86\text{ m}^3\text{/s}}$
7.148	Assume $y > 3$. $R = \frac{30+20(y-3)}{26+2(y-3)} = \frac{20y-30}{2y+20} = \frac{10y-15}{y+10}$ $100 = \frac{1.0}{0.022}(20y-30)\left(\frac{10y-15}{y+10}\right)^{2/3}\times0.001^{1/2}$ Or $6.96 = (2y-3)\left(\frac{10y-15}{y+10}\right)^{2/3}$  Try $y = 5$: $6.96 \overset{?}{=} 12.3 \quad y = 4$: $6.96 \overset{?}{=} 7.36$ $\left.\begin{array}{l} y = 3.8:\ 6.96 \overset{?}{=} 6.47 \\ y = 3.9:\ 6.96 \overset{?}{=} 6.91 \end{array}\right\} \therefore y = \underline{3.91\text{ m}}$

7.150

Try $y = 0.6$ m: $Q = \frac{1}{0.013} \times \frac{\pi}{2} \times (0.6)^2 \left(\frac{\frac{\pi}{2} \times 0.6^2}{0.6\pi} \right)^{2/3} \times 0.001^{1/2}$

$= 0.62 \text{ m}^3\text{/s} \quad \cos\alpha = \frac{y-2}{2} = \frac{y-0.6}{0.6}$

$\therefore y > 0.6$ m. $\quad R = \frac{A}{2\pi(0.6) \times \alpha / 180}$



$A = 2\pi(0.6)\frac{180 - \alpha}{180} + (y - 0.6)0.6\sin\alpha$

$Q = \frac{1}{0.013} AR^{2/3} \times 0.001^{1/2} = 0.66 \quad \therefore AR^{2/3} = 0.27$

$$\left. \begin{array}{l} \text{Try } y = 0.63\text{: } \alpha = 87.13^\circ,\ A = 0.6012 \text{ m}^2,\ R = 0.33 \text{ m. } 0.287 \overset{?}{=} 0.27 \\ \text{Try } y = 0.612\text{: } \alpha = 88.85^\circ,\ A = 0.58 \text{ m}^2,\ R = 0.312 \text{ m. } 0.267 \overset{?}{=} 0.27 \end{array} \right\} y = \underline{0.612 \text{ m}}$$

CHAPTER 8

External Flows

FE-type Exam Review Problems: Problems 8-1 to 8-8

8.1 (C)	
8.2 (C)	
8.3 (B)	$\text{Re} = \frac{VD}{\nu} = \frac{0.8\times 0.008}{1.31\times 10^{-6}} = 4880$
8.4 (B)	Assume a large Reynolds number so that C_D = 0.2. Then $F = \frac{1}{2}\rho V^2 AC_D = \frac{1}{2}\times 1.23\times\left(\frac{80\times 1000}{3600}\right)^2 \times\pi\times 5^2\times 0.2 = 4770\text{ N}$
8.5 (D)	Assume a Reynolds number of 10^5. Then C_D = 1.2 $F = \frac{1}{2}\rho V^2 AC_D \quad \therefore 60 = \frac{1}{2}\times 1.23\times 40^2\times 4\times D\times 1.2 \quad \therefore D = 0.0041\text{ m}$ $\text{Re} = \frac{VD}{\nu} = \frac{40\times 0.0041}{10^{-6}} = 1.64\times 10^5 \quad \therefore C_D = 1.2$ The assumption was OK
8.6 (C)	$\text{Re} = \frac{VD}{\nu} = \frac{4\times 0.02}{1.6\times 10^{-5}} = 5000 \quad \therefore \text{St} = 0.21 = \frac{fD}{V} = \frac{f\times 0.02}{4}$ $\therefore f = 42$ Hz (cycles/second) distance = $\frac{V}{f} = \frac{4\text{ m/s}}{42\text{ cycles/s}} = 0.095$ m/cycle
8.7 (C)	By reducing the separated flow area, the pressure in that area increases thereby reducing that part of the drag due to pressure.
8.8 (B)	From Fig. 8.12a, $C_L = 1.1 \quad C_L = \frac{F_L}{\frac{1}{2}\rho V^2 cL}$ $\therefore V^2 = \frac{2W}{\rho c L C_L} = \frac{2\times 1200\times 9.81}{1.23\times 16\times 1.1} = 1088$ and $V = 33.0$ m/s

Separated Flows

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<td>8.10</td>
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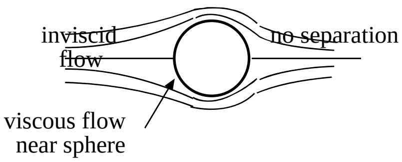
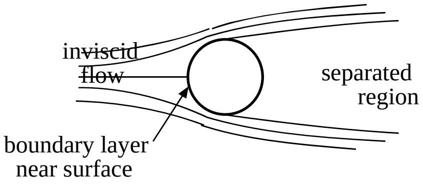
$$\mathrm{Re}=5=\frac{VD}{\nu}\qquad \therefore D=\frac{5\times1.51\times10^{-5}}{20}=\underline{3.78\times10^{-5}\text{ m}}$$





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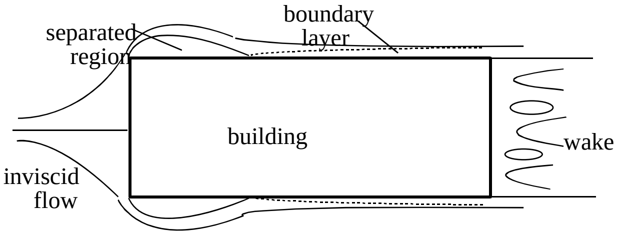

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$$\mathrm{Re}=\frac{VD}{\nu}=\frac{20\times D}{1.51\times10^{-5}}=13.25\times10^{5}D$$

b) $\mathrm{Re}=13.25\times10^{5}\times0.06=\underline{7.9\times10^{4}}$ ∴Separated flow

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$$F_{\text{total}}=F_{\text{bottom}}+F_{\text{top}}=20{,}000\times0.3\times0.3+10{,}000\times0.3\times0.3=2700\text{ N}$$

$$F_{\text{lift}}=2700\cos10^\circ=\underline{2659\text{ N}}$$

$$F_{\text{drag}}=2700\ \sin\ 10^\circ=\underline{469\text{ N}}$$

$$C_L=\frac{F_L}{\frac{1}{2}\rho V^2A}=\frac{2659}{\frac{1}{2}\times1000\times5^2\times0.3\times0.3}=\underline{2.36}$$

$$C_D=\frac{F_D}{\frac{1}{2}\rho V^2A}=\frac{469}{\frac{1}{2}\times1000\times5^2\times0.3\times0.3}=\underline{0.417}$$

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If $C_D=1.0$ for a sphere, Re = 100 (see Fig. 8.8).

$$\therefore\frac{V\times0.1}{\nu}=100,\quad V=1000\nu$$

b) $V=1000\times\dfrac{1.46\times10^{-5}}{0.015\times1.22}=0.798\text{ m/s}$

$$\therefore F_D=\frac{1}{2}\times(0.015\times1.22)\times0.798^2\pi\times0.05^2\times1.0$$

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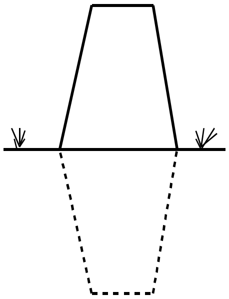
	$=\underline{4.58\times10^{-5}\text{ N}}$
8.20	The velocities associated with the two Reynolds numbers are $V_1=\dfrac{\text{Re}_1\,\nu}{D}=\dfrac{3\times10^5\times1.5\times10^{-5}}{0.0445}=101\text{ m/s}$ $V_2=\dfrac{\text{Re}_2\,\nu}{D}=\dfrac{6\times10^4\times1.5\times10^{-5}}{0.0445}=20\text{ m/s}$ The drag, between these two velocities, is reduced by a factor of 2.5, i.e., $(C_D)_{\text{high}}=0.5$ and $(C_D)_{\text{low}}=0.2.$ Thus, between 20 m/s and 100 m/s, the drag is reduced by a factor of 2.5. This would significantly lengthen the flight of the ball.
8.22	$4.2=\dfrac{1}{2}\times1000V^2\pi\times0.1^2C_D\quad\therefore V^2C_D=0.267\quad\text{Re}=\dfrac{V\times0.2}{10^{-6}}=2\times10^5V$ Try $\underline{C_D=0.5}$: $\quad\therefore V=0.73\text{ m/s}\quad\text{Re}=\underline{1.46\times10^5}\quad\therefore\text{OK}$
8.24	**a)** $\text{Re}_1=\dfrac{25\times0.05}{1.08\times10^{-5}}=1.2\times10^5\quad\text{Re}_2=1.8\times10^5\quad\text{Re}_3=2.4\times10^5$ Assume a rough cylinder (the air is highly turbulent). $\therefore(C_D)_1=0.7,\ (C_D)_2=0.8,\ (C_D)_3=0.9$ $\therefore F_D=\dfrac{1}{2}\times1.45\times25^2(0.05\times10\times0.7+0.075\times15\times0.8+0.1\times20\times0.9)=\underline{1380\text{ N}}$ $M=\dfrac{1}{2}\times1.45\times25^2(0.05\times10\times0.7\times40+0.075\times15\times0.8\times27.5+0.1\times20\times0.9\times10)$ $=\underline{25{,}700\text{ N}\cdot\text{m}}$
8.26	Since the air cannot flow around the bottom, we imagine the structure to be mirrored as shown. Then  $L/D=40/5=8\quad\therefore C_D=0.66C_{D_\infty}$ $\text{Re}_{\text{min}}=\dfrac{VD_{\text{min}}}{\nu}=\dfrac{30\times2}{1.5\times10^{-5}}=4\times10^6$ $\therefore C_D=1.0\times0.66=0.66$ $\therefore F_D=\dfrac{1}{2}\times1.22\times30^2\times\left(\dfrac{2+8}{2}\times20\right)\times0.66=\underline{36{,}000\text{ N}}$

8.28	$F_B + F_D = F_W \quad 1.22\times\frac{4}{3}\pi(0.25)^3 + \frac{1}{2}\times 1.22V^2\pi(0.25)^2 C_D = 9810S\frac{4}{3}\pi(0.25)^3$ $\text{Re} = \frac{V\times 0.5}{1.47\times10^{-5}} = 3.4\times10^4 V \qquad \therefore 1+0.153V^2 C_D = 820S$ **a)** $S = 0.005 \quad V^2 C_D = 20.26$ Assume atmospheric turbulence, i.e., rough Try $C_D = 0.4$: $V = 7.1$ m/s Re $= 2.4\times10^5$ $\therefore C_D = 0.3$ and $V = \underline{8.22\text{ m/s}}$
8.30	From Table 8.2 $C_D = 0.35 \quad F_D = \frac{1}{2}\times 1.22V^2\times 3.2\times 0.35 = 0.683V^2$ **a)** $F_D = 0.683\times\left(\frac{80\times1000}{3600}\right)^2 = 337\text{ N} \quad \therefore \dot{W} = 337\frac{80\times1000}{3600} = 7500\text{ W}$ or $\underline{10\text{ hp}}$
8.32	$\text{Re} = \frac{VD}{\nu} = \frac{(40{,}000/3600)0.6}{1.51\times10^{-5}} = 4.42\times10^5 \quad \therefore C_D = 0.35$ from Fig. 8.8. **a)** $F_D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\times 1.204\times(40{,}000/3600)^2\times 0.6\times 6\times 0.35 = \underline{93.6\text{ N}}$
8.34	With the deflector the drag coefficient is 0.76 rather than 0.96. The required power, directly related to fuel consumed, is reduced by the ratio of 0.76/0.96. The cost per year without the deflector is Cost = (200,000/1.2) × 0.25 = \$41,667 With the deflector it is Cost = 41,667 × 0.76/0.96 = \$32,986 The savings is \$41,667 – 32,986 = <u>\$8,800</u>
8.36	$F_D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\times 1.22\times(27.8\times1.6)^2\times\pi\times 0.05^2\times 1.1 = 10.43\text{ N}$ $\dot{W} = F_D\times V\times 2 = 10.43\times(27.8\times1.6)\times 2 = 226\text{ W}$ or $\underline{1.24\text{ hp}}$
8.38	The net force acting up is (use absolute pressure) $F_{\text{up}} = \frac{4}{3}\pi\times 0.4^3\times 1.21\times 9.8 - 0.5 - \frac{4}{3}\pi\times 0.4^3\frac{120}{2.077\times 293}\times 9.8 = 2.16\text{ N}$ From a force triangle (2.16 N up and F_D to the right), we see that $\tan\alpha = F_{\text{up}}/F_D$ **a)** $F_D = 2.16/\tan 80° = 0.381.$ Assume $C_D = 0.2$: $0.381 = \frac{1}{2}\times 1.21V^2\pi\times 0.4^2\times 0.2 \qquad \therefore V = 2.50\text{ m/s}$

	Check Re: $\text{Re}=\frac{2.5\times 0.8}{1.51\times 10^{-5}}=1.33\times 10^{5}$ Too low. Use C_D = 0.5: $0.381=\frac{1}{2}\times 1.21V^2\pi\times 0.4^2\times 0.5 \quad \therefore V=\underline{1.58\text{ m/s}}$ **c)** $F_D=2.16/\tan 60°=1.25.$ Assume C_D = 0.5: $1.25=\frac{1}{2}\times 1.21V^2\pi\times 0.4^2\times 0.5 \quad \therefore V=\underline{2.86\text{ m/s}}$ Check Re: $\text{Re}=\frac{2.86\times 0.8}{1.51\times 10^{-5}}=1.5\times 10^{5} \quad \therefore \text{OK}$
8.40	Power to move the sign: $F_D V=\frac{1}{2}\rho V^2 A C_D\times V$ $=\frac{1}{2}\times 1.21\times 11.11^2\times 0.72\times 1.1\times 11.11=657\text{ J/s.}$ This power comes from the engine: $657=(12{,}000\times 1000)\dot{m}\times 0.3 \quad \therefore \dot{m}=1.825\times 10^{-4}\text{ kg/s}$ Assuming the density of gas to be 900 kg/m^3 $1.825\times 10^{-4}\times 10\times 3600\times 6\times 52\times\frac{1000}{900}\times 0.30=\underline{\$683}$
8.42	$\dot{W}=40\times 746\eta=F_D\times V=\frac{1}{2}\rho V^2 A C_D\times V=\frac{1}{2}\rho A C_D V^3$ $\therefore 40\times 746\times 0.9=\frac{1}{2}\times 1.22\times 3\times 0.35V^3 \quad \therefore V=34.7\text{ m/s or }\underline{125\text{ km/hr}}$

Vortex Shedding

8.44	$40>\frac{VD}{\nu}=\frac{1.8D}{1.141\times 10^{-6}} \quad \therefore \underline{D < 2.5\times 10^{-5}\text{ m}}$ $10{,}000<\frac{VD}{\nu}=\frac{1.8D}{1.141\times 10^{-6}} \quad \therefore D>0.0063\text{ m or }\underline{0.63\text{ m}}$

8.46	$\text{St} = \frac{fD}{V} = \frac{0.002\times 2}{V}$ $\text{Re} = \frac{VD}{\nu} = \frac{V\times 2}{10^{-6}}$ Use Fig. 8.9 Try $\text{St} = 0.21$: $V = \underline{0.0191 \text{ m/s}}$ $\text{Re} = 38\times 10^{3}$ $\therefore$ OK

Streamlining

8.48	$\text{Re} = \frac{25\times 0.15}{1.5\times 10^{-5}} = 2.5\times 10^{5}$ $F_D = \frac{1}{2}\times 1.225\times 25^2\times 1.0\times 0.8\times\left(1.8\times 0.5\right) = 82.7 \text{ N}$ The coefficient 1.0 comes from Fig. 8.8 and 0.8 from Table 8.1. We have $\dot{W} = F_D\times V = 82.7\times 25 = 2067.5 \text{ W}$ or $\underline{2.77 \text{ kW}}$ $\left(C_D\right)_{\text{streamlined}} = 0.035$ $\therefore F_D = 2.9 \text{ N}$ $\dot{W} = 72.5 \text{ W}$ or $0.72 \text{ kW.} = \underline{0.1 \text{ hp}}$
8.50	$\text{Re} = \frac{VD}{\nu} = \frac{2\times 0.8}{10^{-6}} = 1.6\times 10^{6}$ $\therefore C_D = 0.45$ from Fig. 8.8. $\frac{L}{D} = \frac{4}{0.8} = 5$ $\therefore C_D = 0.62\times 0.45 = 0.28$ Because only one end is free, we double the length. $F_D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\times 1000\times 2^2\times 0.8\times 2\times 0.28 = \underline{900 \text{ N}}$ If streamlined, $C_D = 0.03\times 0.62 = 0.0186$ $\therefore F_D = \frac{1}{2}\times 1000\times 2^2\times 0.8\times 2\times 0.0186 = \underline{60 \text{ N}}$
8.52	$V = 50\times 1000/3600 = 13.9 \text{ m/s}$ $\text{Re} = \frac{13.9\times 0.3}{1.5\times 10^{-5}} = 2.8\times 10^{5}$ $\therefore C_D = 0.4$ We assumed a head diameter of 0.3 m and used the rough sphere curve. $F_D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\times 1.2\times 13.9^2(\pi\times 0.3^2/4)\times 0.4 = \underline{3.3 \text{ N}}$ $F_D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\times 1.2\times 13.9^2(\pi\times 0.3^2/4)\times 0.035 = \underline{0.29 \text{ N}}$

Cavitation

8.54

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{200{,}000}{\frac{1}{2}\times 1000\times 12^2\times 0.4\times 10} = 0.69 \qquad \therefore \underline{\alpha \cong 3^\circ}$$

$$C_D = 0.0165 = \frac{F_D}{\frac{1}{2}\times 1000\times 12^2\times 0.4\times 10} \qquad \therefore F_D = \underline{4800\text{ N}}$$

$$\sigma_{\text{crit}} = 0.75 \overset{?}{>} \frac{(9810\times 0.4 + 101{,}000) - 1670}{\frac{1}{2}\times 1000\times 12^2} = 1.43 \qquad \therefore \text{no cavitation}$$

8.56

$$p_\infty = 9810\times 5 + 101{,}000 = 150{,}000\text{ Pa} \quad p_v = 1670\text{ Pa} \quad \text{Re} = \frac{20\times 0.8}{10^{-6}} = 16\times 10^6$$

$$\sigma = \frac{150{,}000 - 1670}{\frac{1}{2}\times 1000\times 20^2} = 0.74 \qquad \therefore C_D = C_D(0)(1+\sigma) = 0.3(1+0.74) = 0.52$$

$$\therefore F_D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\times 1000\times 20^2\times\pi\times 0.4^2\times 0.52 = \underline{52{,}000\text{ N}}$$

Note: We retain 2 sig. figures since C_D is known to only 2 sig. figures.

Added Mass

8.58

$$\Sigma F = ma \qquad \textbf{a)}\ 400 - 9810\times\frac{4}{3}\pi\times 0.2^3 = \frac{400}{9.81}a \qquad \therefore a = \underline{1.75\text{ m/s}^2}$$

$$\textbf{b)}\ 400 - 9810\times\frac{4}{3}\pi\times 0.2^3 = \left(\frac{400}{9.81} + \frac{1}{2}\times 1000\times\frac{4}{3}\pi\times 0.2^3\right)a \qquad \therefore a = \underline{1.24\text{ m/s}^2}$$

Lift and Drag on Airfoils

8.60

The total aerodynamic drag consists of both lift and drag that is:

$$\mathbf{F}_{Total} = \mathbf{F}_L + \mathbf{F}_D \;\Rightarrow\; \left|\mathbf{F}_{Total}\right| = \sqrt{F_L^2 + F_D^2} = 18\text{ kN} \;\Rightarrow\; F_L^2 + F_D^2 = (18\text{ kN})^2$$

which can be combined with $F_L = 3F_D$ to yield

$$9F_D^2 + F_D^2 = 324 \;\Rightarrow\; F_D = \sqrt{324/10} = 5.69\text{ kN}$$

and hence, $F_L = 3\times 5.69 = 17.1\text{ kN}$ so

	$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 cL} = \frac{17.1\times10^3}{\frac{1}{2}\times1.2\times61.1^2\times1.3\times10} = \underline{0.587}$
8.62	$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{1000\times9.81}{\frac{1}{2}\times0.412\times80^2\times15} = 0.496 \quad \therefore \alpha = 3.2^\circ \quad C_D = 0.0065$ $\dot{W} = F_D V = \left(\frac{1}{2}\times0.412\times80^2\times15\times0.0065\right)\times80 = 10,300\text{ W}$ or $\underline{13.8\text{ hp}}$
8.64	$C_L = 1.22 = \frac{1500\times9.81+3000}{\frac{1}{2}\times1.007\times V^2\times20} \quad \therefore V = \underline{38.0\text{ m/s}}$
8.66	$C_L = 1.22 = \frac{1500\times9.81+9000}{\frac{1}{2}\times1.22\times V^2\times20} \quad \therefore V = \underline{39.9\text{ m/s}}$
8.68	**a)** $C_L = 1.72 = \frac{250,000\times9.81}{\frac{1}{2}\times1.05\times V^2\times60\times8} \quad \therefore V = 75.2\text{ m/s}$ % change = $\frac{75.2-69.8}{69.8}\times100 = \underline{7.77\%\text{ increase}}$

Vorticity, Velocity Potential, and Stream Function

8.70	$\nabla\times\left[\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} + \frac{\nabla p}{\rho} - \nu\nabla^2\mathbf{V}\right] = \mathbf{0}$ $\nabla\times\frac{\partial V}{\partial t} = \frac{\partial}{\partial t}(\nabla\times V) = \frac{\partial\omega}{\partial t} \qquad \nabla\times\frac{\nabla p}{\rho} = \frac{1}{\rho}\nabla\times\nabla p = \mathbf{0}$ $\nabla\times(\nabla^2\mathbf{V}) = \nabla^2(\nabla\times\mathbf{V}) = \nabla^2\boldsymbol{\omega}$ (we have interchanged derivatives) $\nabla\times[(\mathbf{V}\cdot\nabla)\mathbf{V}] = \nabla\times\left[\frac{1}{2}\nabla V^2 - \mathbf{V}\times(\nabla\times\mathbf{V})\right] = \frac{1}{2}\cancel{(\nabla\times\nabla V^2)} - \nabla\times(\mathbf{V}\times\boldsymbol{\omega})$ $= \mathbf{V}\,\cancel{(\nabla\cdot\boldsymbol{\omega})} - \boldsymbol{\omega}\,\cancel{(\nabla\cdot\mathbf{V})} + (\mathbf{V}\cdot\nabla)\boldsymbol{\omega} - (\boldsymbol{\omega}\cdot\nabla)\mathbf{V}$ $= (\mathbf{V}\cdot\nabla)\boldsymbol{\omega} - (\boldsymbol{\omega}\cdot\nabla)\mathbf{V}$ since $\nabla\cdot\boldsymbol{\omega} = \nabla\cdot(\nabla\times\mathbf{V}) = 0$ and $\nabla\cdot\mathbf{V} = 0$ There results: $\frac{\partial\boldsymbol{\omega}}{\partial t} + (\mathbf{V}\cdot\nabla)\boldsymbol{\omega} - (\boldsymbol{\omega}\cdot\nabla)\mathbf{V} - \nu\nabla^2\boldsymbol{\omega} = \mathbf{0}$ This is written as $\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{V} + \nu\nabla^2\boldsymbol{\omega}$

8.72

x-comp: $$\frac{\partial\omega_x}{\partial t}+u\frac{\partial\omega_x}{\partial x}+v\frac{\partial\omega_x}{\partial y}+w\frac{\partial\omega_x}{\partial z}=\omega_x\frac{\partial u}{\partial x}+\omega_y\frac{\partial u}{\partial y}+\omega_z\frac{\partial u}{\partial z}+\nu\nabla^2\omega_x$$

y-comp: $$\frac{\partial\omega_y}{\partial t}+u\frac{\partial\omega_y}{\partial x}+v\frac{\partial\omega_y}{\partial y}+w\frac{\partial\omega_y}{\partial z}=\omega_x\frac{\partial v}{\partial x}+\omega_y\frac{\partial v}{\partial y}+\omega_z\frac{\partial v}{\partial z}+\nu\nabla^2\omega_y$$

z-comp: $$\frac{\partial\omega_z}{\partial t}+u\frac{\partial\omega_z}{\partial x}+v\frac{\partial\omega_z}{\partial y}+w\frac{\partial\omega_z}{\partial z}=\omega_x\frac{\partial w}{\partial x}+\omega_y\frac{\partial w}{\partial y}+\omega_z\frac{\partial w}{\partial z}+\nu\nabla^2\omega_z$$

8.74

a) $$\nabla\times\mathbf{V}=\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\hat{\mathbf{i}}+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\hat{\mathbf{j}}+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\hat{\mathbf{k}}=\mathbf{0} \qquad \therefore \underline{\text{irrotational}}$$

$$\frac{\partial\phi}{\partial x}=10x \qquad \therefore \phi=5x^2+f(y)$$

$$\frac{\partial\phi}{\partial y}=\frac{\partial f}{\partial y}=20y \qquad \therefore f=10y^2+C \quad Let\ C=0$$

$$\therefore \underline{\phi=5x^2+10y^2}$$

c) $$\nabla\times\mathbf{V}=0\hat{\mathbf{i}}+0\hat{\mathbf{j}}+\left(\frac{-y\frac{1}{2}(x^2+y^2)^{-1/2}2x}{x^2+y^2}-\frac{-x\frac{1}{2}(x^2+y^2)^{-1/2}2y}{x^2+y^2}\right)\hat{\mathbf{k}}=\mathbf{0}$$

$$\therefore \underline{\text{irrotational}}$$

$$\frac{\partial\phi}{\partial x}=\frac{x}{\sqrt{x^2+y^2}} \qquad \therefore \phi=\sqrt{x^2+y^2}+f(y)$$

$$\frac{\partial\phi}{\partial y}=\frac{1}{2}(x^2+y^2)^{-1/2}2y+\frac{\partial f}{\partial y}=\frac{y}{\sqrt{x^2+y^2}} \qquad \therefore \frac{\partial f}{\partial y}=0 \quad \therefore f=C.\ \ \text{Let } C=0$$

$$\therefore \underline{\phi=\sqrt{x^2+y^2}}$$

8.76

$$u=\frac{\partial\psi}{\partial y}=100 \quad \therefore \psi=100y+f(x) \qquad v=-\frac{\partial\psi}{\partial x}=-\frac{df}{dx}=50$$

$$\therefore f=-50x+C$$

$$\therefore \underline{\psi(x,y)=100y-50x} \qquad \text{(We usually let } C = 0)$$

$$u=\frac{\partial\phi}{\partial x}=100 \quad \therefore \phi=100x+f(y) \qquad v=\frac{\partial\phi}{\partial y}=\frac{df}{dy}=50$$

$$\therefore f=50y+C$$

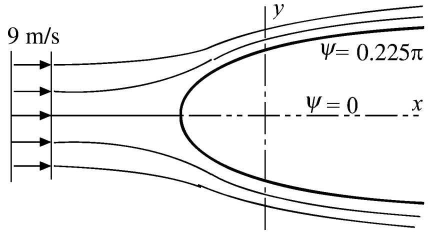
$$\therefore \underline{\phi(x,y)=100x+50y}$$

8.78	$u = \frac{\partial\psi}{\partial y} = 20\frac{2y}{x^2+y^2} = \frac{\partial\phi}{\partial x}$ $\quad\therefore \phi = -40\tan^{-1}\frac{y}{x} + f(y)$ $v = \frac{\partial\phi}{\partial y} = -\frac{40/x}{1+y^2/x^2} + \frac{\partial f}{\partial y} = -\frac{40x}{x^2+y^2} + \frac{\partial f}{\partial y} = -20\frac{2x}{x^2+y^2}$ $\quad\therefore f = C.$ Let $C = 0$ $\underline{\phi = -40\tan^{-1}\frac{y}{x}}$
8.80	**b)** Polar coord: $\phi = 10r\cos\theta + 5\ln r^2$ (See Eq. 8.5.14.) $\frac{\partial\phi}{\partial r} = 10\cos\theta + \frac{10r}{r^2} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$ $\quad\therefore \psi = 10r\sin\theta + 10\theta + f(r)$ $\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -10\sin\theta = -\frac{\partial\psi}{\partial r} = -10\sin\theta - \frac{df}{dr}$ $\quad\therefore f = \cancel{C}$ $\quad\therefore \psi = 10r\sin\theta + 10\theta$ $\therefore \underline{\psi(x, y) = 10y + 10\tan^{-1}\frac{y}{x}}$ **c)** $v = \frac{\partial\phi}{\partial y} = \frac{10y}{x^2+y^2}$ Along x-axis (y = 0) v = 0 $u = \frac{\partial\phi}{\partial x} = 10 + \frac{10x}{x^2+y^2}$ Along x-axis $u = 10 + \frac{10}{x}$ Bernoulli: $\frac{V^2}{2} + \frac{p}{\rho} + \cancel{gz} = \frac{V_\infty^2}{2} + \frac{p_\infty}{\rho} + \cancel{gz_\infty}$ (assume $z = z_\infty$) $\frac{(10+10/x)^2}{2} + \frac{p}{\rho} = \frac{10^2}{2} + \frac{100{,}000}{\rho}$ $\quad\therefore \underline{p = 100 - 50\left(\frac{2}{x} + \frac{1}{x^2}\right)\text{ kPa}}$ **e)** $a_y = \cancel{v}\partial v/\partial y + u\cancel{\partial v/\partial x} = 0$ on x-axis $a_x = u\partial u/\partial x + \cancel{v}\partial u/\partial y = (10 + 10/x)\left(-10/x^2\right)$ $\therefore a_x(-2, 0) = (10 - 5)(-10/4) = \underline{-12.5\text{ m/s}^2}$

Superposition of Simple Flows

8.82

$$\psi = 9y + \frac{0.45\pi}{2\pi}\theta = 9r\sin\theta + 0.225\theta$$



a) $v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 9\cos\theta + \frac{0.225}{r} = 0$

At $\theta = \pi,\quad \frac{0.225}{r_s} = 9 \quad \therefore r_s = 0.025 \text{ m}$

Stag. pt: $(-0.025, 0)$

c) $q = U \times H = \Delta\psi \qquad \therefore 9H = 0.225\pi \qquad \therefore H = \frac{\pi}{40}$

Thickness = $2H = \frac{\pi}{20}$ m or $\underline{0.157 \text{ m}}$

d) $v_r(1,\pi) = 9\cos\pi + \frac{0.225}{3} = -8.25 \quad \therefore \underline{u = 8.25 \text{ m/s}}$

8.84

$$\phi = \frac{2\pi}{2\pi}\ln\left[(x+1)^2 + y^2\right]^{1/2} + \frac{-2\pi}{2\pi}\ln\left[(x-1)^2 + y^2\right]^{1/2} + 2x$$

$$= \frac{1}{2}\ln\left[(x+1)^2 + y^2\right] - \frac{1}{2}\ln\left[(x-1)^2 + y^2\right] + 2x$$

$$u = \frac{\partial\phi}{\partial x} = \frac{\frac{1}{2}2(x+1)}{(x+1)^2 + y^2} - \frac{\frac{1}{2}2(x-1)}{(x-1)^2 + y^3} + 2 \qquad v = \frac{y}{(x-1)^2 + y^2} - \frac{y}{(x-1)^2 + y^2}$$

Along the x-axis ($y = 0$), $v = 0$ and $u = \frac{1}{x+1} - \frac{1}{x-1} + 2$

Set $u = 0$: $\frac{1}{x-1} - \frac{1}{x+1} = 2, \quad \text{or} \quad x^2 = 2 \quad \therefore x = \pm\sqrt{2}$

Stag. pts.: $\underline{(\sqrt{2}, 0), \quad (-\sqrt{2}, 0)}$

$$u(4,0) = \frac{1}{-4+1} - \frac{1}{-4-1} + 2 = \underline{1.867 \text{ m/s}} \qquad v(-4,0) = \underline{0}$$

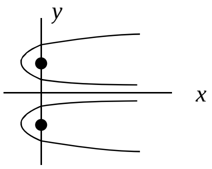
$$u(0,4) = \frac{1}{1+4^2} - \frac{-1}{1+4^2} + 2 = \underline{2.118 \text{ m/s}} \qquad v(0,4) = \frac{4}{1+4^2} - \frac{4}{1+4^2} = \underline{0}$$

8.86

$$\phi = \frac{2\pi}{2\pi}\ln\left[(y-1)^2 + x^2\right]^{1/2} + \frac{2\pi}{2\pi}\ln\left[(y+1)^2 + x^2\right]^{1/2} + U_\infty x$$

$$=\frac{1}{2}\ln\left[(y-1)^2+x^2\right]+\frac{1}{2}\ln\left[(y+1)^2+x^2\right]+U_\infty x$$

a) Stag. pts. May occur on *x*-axis, *y* = 0



$$u=\left.\frac{\partial\phi}{\partial x}\right|_{y=0}=\frac{x}{1+x^2}+\frac{x}{1+x^2}+10$$

$\therefore x^2+0.2x+1=0$ $\quad\therefore$ no stagnation points exist on the *x*-axis

(They do exist away from the *x*-axis)

Along the *y*-axis: $u(y)=10 \quad q=\int_0^h u\,dy=\frac{1}{2}(2\pi)=\pi\ \ \text{m}^2/\text{s}$

$$\therefore \pi=\int_0^h 10dy=10\ h \qquad \therefore h=\underline{0.314\ \text{m}}$$

8.88

$\psi=\frac{4\pi}{2\pi}\theta+\frac{20\pi}{2\pi}\ln r=2\theta+10\ln r$ at $(x,y)=(0,1)$ which is $(r,\theta)=(1,\pi/2)$.

b) $v_r=\frac{2}{r}$ and $v_\theta=-\frac{10}{r}$. From Table 5.1 (use the l.h.s. of momentum):

$$a_r=\frac{Dv_r}{Dt}+\frac{v_\theta^2}{r}=v_r\frac{\partial v_r}{\partial r}-\frac{v_\theta^2}{r}=\frac{2}{r}\left(-\frac{2}{r^2}\right)-\frac{100}{r^3}=-104\ \text{m/s}^2$$

$$a_\theta=\frac{Dv_\theta}{Dt}+\frac{v_r v_\theta}{r}=v_r\frac{\partial v_\theta}{\partial r}+\frac{v_r v_\theta}{r}=\frac{2}{r}\left(\frac{10}{r^2}\right)+\frac{2(-10)}{r^3}=0$$

$$\therefore \mathbf{a}(0,1)=-104\mathbf{i}_r \quad \text{or} \quad (a_x,a_y)=\underline{(0,-104)\ \text{m/s}^2}$$

c) $v_r(14.14,\pi/4)=\frac{2}{14.14}=0.1414,\quad v_\theta(14.14,\pi/4)=-\frac{10}{14.14}=-0.707\ \text{m/s}$

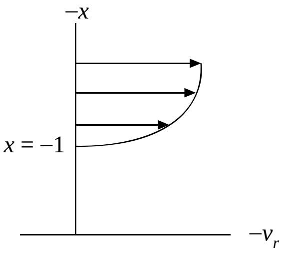
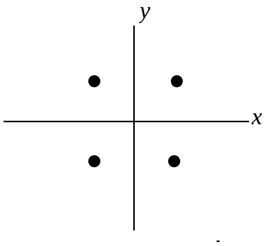
$$v_r(0.1,\pi/2)=\frac{2}{0.1}=20,\quad v_\theta(0.1,\pi/2)=\frac{-10}{-0.1}=-100\ \text{m/s}$$

Bernoulli: $\frac{20{,}000}{1.2}+\frac{0.1414^2+0.707^2}{2}=\frac{p}{1.2}+\frac{20^2+100^2}{2} \quad \therefore \underline{p=13{,}760\ \text{Pa}}$

We used $\rho_\text{air}=1.2\ \text{kg/m}^3$ at standard conditions.

8.90

b) $v_\theta=-\frac{\partial\psi}{\partial r}=-U_\infty\sin\theta-\frac{\mu\sin\theta}{r_c^2}=\left(-4-\frac{4}{1^2}\right)\sin\theta=\underline{-8\sin\theta}$

	c) $p_c = p_\infty + \rho\frac{V_\infty^2}{2} - \rho\frac{v_\theta^2}{2} = 50{,}000 + 1000\times\frac{4^2}{2} - 1000\frac{8^2\sin^2\theta}{2}$ $\therefore p_c = \underline{58-32\sin^2\theta\ \text{kPa}}$  **d)** Drag = $2\int_0^{\pi/2}(58-32\sin^2\alpha)\cos\alpha\times1\times1d\alpha - 26\times2\times1$ $= 2\left[58-32\left(\frac{1}{3}\right)\right] - 52 = \underline{42.7\text{ kN}}$ (See the figure in Problem 8.89c)
8.92	$v_\theta = -2\times20\sin\theta - \frac{\Gamma}{2\pi\times0.4}$ For one stag. pt.: $v_\theta = 0$ at $\theta = 270^\circ$: $0 = -2\times20\sin270^\circ - \frac{\Gamma}{2\pi\times0.4}$ $\therefore \Gamma = 2\times20\times2\pi\times0.4 = 100.5\text{ m}^2/\text{s}$ $\Gamma = 2\pi r_c^2\omega$ $\therefore \omega = \frac{\Gamma}{2\pi r_c^2} = \frac{100.5}{2\pi\times0.4^2} = \underline{100\text{ rad/s}}$ (See Example 8.12) Min. pressure occurs where $\lvert v_\theta\rvert$ is max, i.e., $\theta = \pi / 2$. There $v_\theta = -2\times20\times1 - \frac{100.5}{2\pi\times0.4} = 80\text{ m/s}$ $\therefore p_{\min} = p_\infty + \frac{V_\infty^2}{2}\rho - \frac{v_\theta^2}{2}\rho = 0 + \frac{20^2}{2}\times1.22 - \frac{80^2}{2}\times1.22 = \underline{-3660\text{ Pa}}$
8.94	At 9000 m, $\rho = 0.47\text{ kg/m}^3$ Lift = $\rho U_\infty\Gamma L = 0.47\times(105)\times(1400)\times18 = 1244\times10^3\text{ N} = \underline{1244\text{ kN}}$
8.96	Place four sources as shown. Then, with $q = 2\pi$ for each:  $u(x,y) = \frac{x-2}{(x-2)^2+(y-2)^2} + \frac{x+2}{(x+2)^2+(y-2)^2} + \frac{x+2}{(x+2)^2+(y+2)^2} + \frac{x-2}{(x-2)^2+(y+2)^2}$ $v(x,y) = \frac{y-2}{(x-2)^2+(y-2)^2} + \frac{y+2}{(x-2)^2+(y+2)^2} + \frac{y-2}{(x+2)^2+(y-2)^2} + \frac{y+2}{(x+2)^2+(y+2)^2}$ $\therefore$ **v**(4, 3) = $\underline{(0.729,\ 0.481)\text{ m/s}}$

Boundary Layers

8.98	$\text{Re}_{\text{crit}} = \frac{U_\infty x_T}{\nu} \quad \therefore x_T = \frac{6\times10^5 \nu}{222.2} = 2700\nu$ **b)** $\nu = \frac{\mu}{\rho} = 3.22\times10^{-5}\,\text{m}^2/\text{s} \quad \therefore x_T = 2700\times3.22\times10^{-5} = 8.7\times10^{-2}$ m or $\underline{8.7\text{ cm}}$
8.100	**a)** Use $\text{Re}_{\text{crit}} = 3\times10^5 = \frac{10x_T}{10^{-6}} \qquad \therefore x_T = 0.03$ m or $\underline{3\text{ cm}}$ **c)** Use $\text{Re}_{\text{crit}} = 3\times10^5 = \frac{10x_T}{10^{-6}} \qquad \therefore x_T = 0.03$ m or $\underline{3\text{ cm}}$ **e)** $\text{Re} = 6\times10^4 = \frac{10x_{\text{growth}}}{10^{-6}} \qquad \therefore x_{\text{growth}} = 0.006$ m or $\underline{6\text{ mm}}$
8.102	The x-coordinate is measured along the cylinder surface as shown in Fig. 8.19. The pressure distribution (see solution 8.89) on the surface is $p = p_0 - 2\rho U_\infty^2 \sin^2\alpha$ where $r_c\alpha = x$ (α is zero at the stagnation point). Then $p(x) = 20{,}000 - 2\times1000\times10^2 \sin^2(x/2)$ $= \underline{20 - 200\sin^2(x/2)\text{ kPa}}$ The velocity $U(x)$ at the edge of the b.l. is $U(x)$ on the cylinder wall: $v_\theta(r=2) = -10\sin\theta - 10\sin\theta = -20\sin(\pi-\alpha) = 20\sin\alpha$ $\therefore U(x) = \underline{20\sin(x/2)}$
8.104	The height h above the plate is $h(x) = mx + 0.4 \qquad 0.1 = m\times2 + 0.4 \qquad \therefore m = -0.15$ $\therefore h(x) = 0.4 - 0.15x$ Continuity: $6\times0.4 = U(x)h \quad \therefore U(x) = \frac{2.4}{0.4-0.15x}$ or $\underline{U(x) = \frac{16}{2.67-x}}$ Euler's Eqn: $\rho u\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} \qquad \therefore \frac{dp}{dx} = \rho\frac{16}{2.67-x}\times\frac{16}{(2.67-x)^2} = \underline{\frac{256}{(2.67-x)^3}}$

Von Kármán Integral Equation

8.106	$\tau_0 = -\delta \frac{dp}{dx} + U(x)\frac{d}{dx}\int_0^\delta \rho u dy - \frac{d}{dx}\int_0^\delta \rho u^2 dy$ $= -\delta\frac{dp}{dx} + \frac{d}{dx}\int_0^\delta \rho u U dy - \frac{dU}{dx}\left(\int_0^\delta \rho u dy\right) - \frac{d}{dx}\int_0^\delta \rho u^2 dy$ where we have used $g\frac{df}{dx} = \frac{dfg}{dx} - f\frac{dg}{dx}$ $\left(\text{Here } U = g,\ f = \int_0^\delta \rho u dy\right)$ $\therefore \tau_0 = -\delta\frac{dp}{dx} + \frac{d}{dx}\int_0^\delta \rho u(U-u)dy - \rho\frac{dU}{dx}\int_0^\delta u dy \qquad (\rho = \text{const.})$
8.108	**a)** If $dp/dx = 0$ then $\frac{dU}{dx} = 0$ and $\tau_0 = \rho\frac{d}{dx}\int_0^\delta u(U_\infty - u)dy$ $\tau_0 = \rho\frac{d}{dx}\int_0^\delta U_\infty^2 \sin\frac{\pi y}{2\delta}\left(1-\sin\frac{\pi y}{2\delta}\right)dy = \rho U_\infty^2\frac{d}{dx}\left[-\frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta} - \frac{y}{2}\right]_0^\delta = \rho U_\infty^2 \frac{d}{dx}\left(\frac{2\delta}{\pi} - \frac{\delta}{2}\right)$ Also, we have $\tau_0 = \mu\frac{\partial u}{\partial y}\Big\vert_{y=0} = \mu U_\infty \frac{\pi}{2\delta}\cos 0$ $\therefore \mu U_\infty\frac{\pi}{2\delta} = 0.137\rho U_\infty^2\frac{d\delta}{dx} \qquad \therefore \delta d\delta = 11.5\frac{\nu}{U_\infty}dx \qquad \therefore \underline{\delta = 4.79\sqrt{\frac{\nu x}{U_\infty}}}$ **b)** $\tau_0 = \mu U_\infty \frac{\pi}{2}\frac{1}{4.79}\sqrt{\frac{U_\infty}{\nu x}} = \underline{0.328\mu U_\infty\sqrt{\frac{U_\infty}{\nu x}}}$ **c)** $\frac{\partial u}{\partial x} = U_\infty\frac{\partial}{\partial x}\sin\left(\frac{\pi y}{2\times 4.79}\sqrt{\frac{U_\infty}{\nu x}}\right) = U_\infty\frac{\partial}{\partial x}\sin\left(\frac{ay}{\sqrt{x}}\right) = U_\infty\left(-\frac{ayx^{-3/2}}{2}\right)\cos\frac{ay}{\sqrt{x}} = -\frac{\partial v}{\partial y}$ $\therefore v = \int_0^\delta U_\infty \frac{0.164y}{x^{3/2}}\sqrt{\frac{U_\infty}{\nu}}\cos\left(0.328y\sqrt{\frac{U_\infty}{\nu x}}\right)dy = 0.0316\sqrt{\frac{U_\infty^3}{\nu}}\underline{\int_0^\delta y\cos\left[\left(0.189\sqrt{\frac{U_\infty}{\nu}}\right)y\right]dy}$
8.110	$\tau_0 = \frac{d}{dx}\left\{\int_0^{\delta/6}\rho 3U_\infty^2\frac{y}{\delta}\left(1-3\frac{y}{\delta}\right)dy + \int_{\delta/6}^{\delta/2}\rho U_\infty^2\left(\frac{y}{\delta}+\frac{1}{3}\right)\left(1-\frac{y}{\delta}-\frac{1}{3}\right)dy\right.$

$$+\int_{\delta/2}^{\delta}\rho U_\infty^2\left(\frac{y}{3\delta}+\frac{2}{3}\right)\left(1-\frac{y}{3\delta}-\frac{2}{3}\right)dy\Bigg\}$$

$$=\frac{d}{dx}\rho U_\infty^2(0.1358\delta)=\mu\frac{3U_\infty}{\delta}\qquad\therefore\delta\frac{d\delta}{dx}=22.08\frac{\mu}{\rho U_\infty}$$

Thus

$$\delta(x)=6.65\sqrt{\frac{\nu x}{U_\infty}},\quad\tau_0(x)=0.1358\rho U_\infty^2\left(\frac{6.65}{2}\sqrt{\frac{\nu}{U_\infty x}}\right)=\underline{0.451\rho U_\infty^2\,\mathrm{Re}_x^{-1/2}}$$

% error for $\delta=\dfrac{6.65-5}{5}\times100=\underline{33\%}$

% error for $\tau_0=\dfrac{0.451-0.332}{0.332}\times100=\underline{36\%}$

8.112

The given velocity profile is that used in Example 8.13. There we found

$$\delta=5.48\sqrt{\nu x/U_\infty}=5.48\sqrt{10^{-6}x/10}=0.00173\sqrt{x}=0.00173\sqrt{3}=0.003\text{ m}$$

Assume the streamline is outside the b.l. Continuity is then

$$10\times0.02=\int_0^{0.003}10\left(\frac{2y}{0.003}-\frac{y^2}{0.003^2}\right)dy\ +(h-0.003)10$$

$$=0.02+10h-0.03\qquad\therefore h=0.021\text{ m or }\underline{2.1\text{ cm}}$$

$$\delta_d=\frac{1}{10}\int_0^{0.003}\left(10-\frac{20y}{0.003}+\frac{10y^2}{0.003^2}\right)dy=\frac{1}{10}\big(0.03-0.03+0.01\big)=\underline{0.001\text{ m}}$$

$$h-2=2.1-2=0.1\text{ cm}\quad\text{or}\quad\underline{0.001\text{ m}}$$

The streamline moves away from the wall a distance δ_d

8.114

a) $u=U_\infty\left(\dfrac{3}{2}\dfrac{y}{\delta}-\dfrac{1}{2}\dfrac{y^3}{\delta^3}\right)\quad\delta_d=\dfrac{1}{U_\infty}\displaystyle\int_0^{\delta}U_\infty\left(1-\frac{3}{2}\frac{y}{\delta}+\frac{1}{2}\frac{y^3}{\delta^3}\right)dy=\delta-\frac{3}{4}\delta+\frac{1}{8}\delta=0.375\delta$

From Eq. 8.6.16, $\delta_d=0.375\times4.65\sqrt{\dfrac{\nu x}{U_\infty}}=\underline{1.74\sqrt{\dfrac{\nu x}{U_\infty}}}$ % error = $\underline{1.2\%}$

$$\theta=\frac{1}{U_\infty^2}\int_0^{\delta}U_\infty^2\left(\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\frac{y^3}{\delta^3}\right)\left(1-\frac{3}{2}\frac{y}{\delta}+\frac{1}{2}\frac{y^3}{\delta^3}\right)dy=0.139\delta$$

	$\therefore \theta = 0.139\times 4.65\sqrt{\frac{\nu x}{U_\infty}} = \underline{0.648\sqrt{\frac{\nu x}{U_\infty}}}$ % error = $\frac{0.648-0.644}{0.644}\times 100 = \underline{0.62\%}$
8.116	**a)** $\delta = 4.65\sqrt{\frac{\nu x}{U_\infty}} = 4.65\left(\frac{1.5\times10^{-5}\times 6}{4}\right)^{1/2} = \underline{0.0221\text{ m}}$ **b)** $\tau_0 = 0.323\rho U_\infty^2\sqrt{\frac{\nu}{xU_\infty}} = 0.323\times1.22\times4^2\left(\frac{1.5\times10^{-5}}{6\times4}\right)^{1/2} = \underline{0.00498\text{ Pa}}$ **c)** Drag = $\frac{1}{2}\rho U_\infty^2 Lw\times1.29\sqrt{\frac{\nu}{LU_\infty}} = \frac{1}{2}\times1.22\times4^2\times6\times5\times1.29\left(\frac{1.5\times10^{-5}}{6\times4}\right)^{1/2} = \underline{0.299\text{ N}}$ **d)** $\frac{\partial u}{\partial x} = U_\infty\left[-\frac{3y}{2\delta^2}+\frac{3}{2}\frac{y^3}{\delta^4}\right]\frac{d\delta}{dx}$ $= 4\left[-\frac{3\times4}{2\times4.65^2\times1.5\times10^{-5}\times3}y + \frac{3}{2}\times\frac{4^2y^3}{4.65^4\times(1.5\times10^{-5}\times3)^2}\right]\frac{d\delta}{dx}$ $\therefore \frac{\partial u}{\partial x} = 4\left[-6166y+2.53\times10^7y^3\right]\frac{4.65}{2}\sqrt{\frac{1.5\times10^{-5}}{4}}\frac{1}{\sqrt{3}} = -64.1y+2.63\times10^5y^3$ $\therefore v = \int_0^\delta -\frac{\partial u}{\partial x}dy = \frac{64.1}{2}\times0.0156^2 - \frac{2.63\times10^5}{4}\times0.0156^4 = \underline{0.00391\text{ m/s}}$ where $\delta_{x=3} = 4.65\sqrt{\frac{1.5\times10^{-5}\times3}{4}} = 0.01560\text{ m}$

Laminar and Turbulent Boundary Layers

8.118	**a)** $\delta = 0.38\times6\left(\frac{1.5\times10^{-5}}{20\times6}\right)^{0.2} = \underline{0.0949\text{ m}}$ $\tau_0 = \frac{1}{2}\times1.22\times20^2\times0.059\left(\frac{1.5\times10^{-5}}{20\times6}\right)^{0.2}$ $= \underline{0.6\text{ Pa}}$
8.120	**a)** Drag $=\frac{1}{2}\times1.225\times6^2\times(3.6\times15)\left[0.074\left(\frac{1.47\times10^{-5}}{6\times3.6}\right)^{0.2} - 1060\left(\frac{1.47\times10^{-5}}{6\times3.6}\right)\right]$ $=1191(4.32\times10^{-3}-0.72\times10^{-3}) = \underline{4.3\text{ N}}$

8.122

a) $U_\infty = \frac{60\times1000}{3600} = 16.67\text{ m/s} \quad \delta = 0.38\times100{,}000\left(\frac{1.5\times10^{-5}}{16.67\times10^{5}}\right)^{0.2} = \underline{235\text{ m}}$

$$\tau_0 = \frac{1}{2}\rho U_\infty^2 c_f = \frac{1}{2}\times1.22\times16.67^2\times\left[0.059\left(\frac{1.5\times10^{-5}}{16.67\times10^{5}}\right)^{0.2}\right] = \underline{0.0618\text{ Pa}}$$

b) $\tau_0 = \frac{1}{2}\rho U_\infty^2 c_f = \frac{1}{2}\times1.22\times16.67^2\,\frac{0.455}{\left[\ln\left(0.06\,\frac{16.67\times10^{5}}{1.5\times10^{-5}}\right)\right]^2} = \underline{0.151\text{ Pa}}$

$$\therefore u_\tau = \sqrt{\frac{0.151}{1.22}} = 0.351\text{ m/s} \quad \therefore \frac{16.67}{0.351} = 2.44\ln\frac{0.351\delta}{1.5\times10^{-5}} + 7.4 \quad \therefore \underline{\delta = 585\text{ m}}$$

Both (a) and (b) are in error; however, (b) is more accurate.

8.124

a) Use Eq. 8.6.40: $c_f = \frac{0.455}{\left[\ln\left(0.06\,\frac{90\times6}{1.47\times10^{-5}}\right)\right]^2} = \underline{0.00213}$

b) $\tau_0 = \frac{1}{2}\rho U_\infty^2 c_f = \frac{1}{2}\times1.225\times90^2\times0.00213 = \underline{10.52\text{ Pa}}$

$$u_\tau = \sqrt{\frac{10.52}{1.225}} = 2.93\text{ m/s}$$

c) $\delta_\nu = \frac{5\nu}{u_\tau} = \frac{5\times1.47\times10^{-5}}{2.93} = \underline{2.51\times10^{-5}\text{ m}}$

d) $\frac{90}{2.93} = 2.44\ln\frac{2.93\delta}{1.47\times10^{-5}} + 7.4 \qquad \therefore \underline{\delta = 0.071\text{ m}}$

8.126

Assume flat plates with $dp/dx = 0$. $C_f = \frac{0.523}{\left[\ln\left(0.06\,\frac{10\times100}{10^{-6}}\right)\right]^2} = 0.00163$

$$\therefore \text{Drag} = 2\times\frac{1}{2}\times1000\times10^2\times10\times100\times0.00163 = \underline{163{,}000\text{ N}}$$

To find $\delta_{\max}$ we need u_τ :

$$\tau_0 = \frac{1}{2}\times 1000\times 10^2 \frac{0.455}{\left[\ln\left(0.06\frac{10\times 100}{10^{-6}}\right)\right]^2} = 70.9\text{ Pa} \quad \therefore u_\tau = \sqrt{\frac{70.9}{1000}} = 0.266\text{ m/s}$$

$$\frac{10}{0.266} = 2.44\ln\frac{0.266\delta}{10^{-6}} + 7.4 \qquad \therefore \underline{\delta_{\max} = 0.89\text{ m}}$$

Laminar Boundary-Layer Equations

8.128

$$u = \frac{\partial\psi}{\partial y}, \quad \frac{\partial u}{\partial x} = \frac{\partial^2\psi}{\partial x\partial y}, \quad v = -\frac{\partial\psi}{\partial x}, \quad \frac{\partial u}{\partial y} = \frac{\partial^2\psi}{\partial y^2}, \quad \frac{\partial^2 u}{\partial y^2} = \frac{\partial^3\psi}{\partial y^3}$$

Substitute into Eq. 8.6.45 (with $dp/dx = 0$):

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = \nu\frac{\partial^3\psi}{\partial y^3}$$

8.130

$$u = \frac{\partial\psi}{\partial y} = \sqrt{U_\infty\nu x}\frac{dF}{d\eta}\frac{\partial\eta}{\partial y} = \sqrt{U_\infty\nu x}\ \ F'(\eta)\sqrt{\frac{U_\infty}{\nu x}} = \underline{U_\infty F'(\eta)}$$

We used Eq. 8.6.50 and Eqs. 8.6.48.

$$v = -\frac{\partial\psi}{\partial x} = -\frac{\partial}{\partial x}\left(\sqrt{U_\infty\nu x}\ \ F\right) = -\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}\ F - \sqrt{U_\infty\nu x}\ \ \frac{\partial F}{\partial\eta}\frac{\partial\eta}{\partial x}$$

$$= -\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}\ F - \sqrt{U_\infty\nu x}\ F'\ y\sqrt{\frac{U_\infty}{\nu}}\left(-\frac{1}{2}x^{-3/2}\right)$$

$$= -\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}\ F + \frac{y}{2}\sqrt{\frac{U_\infty}{\nu x}}\sqrt{\frac{U_\infty\nu}{x}}\ F' = \underline{\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}(\eta F' - F)}$$

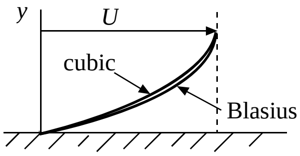
8.132

a) $\tau_0 = 0.332\times 1.22\times 5^2\sqrt{\frac{1.5\times 10^{-5}}{2\times 5}} = \underline{0.0124\text{ Pa}}$

b) $\delta = 5\sqrt{\frac{1.5\times 10^{-5}\times 2}{5}} = \underline{0.0122\text{ m}}$

c) $v_{\max} = \sqrt{\frac{\nu U_\infty}{x}}\left[\frac{1}{2}(\eta F' - F)\right]_{\max} = \sqrt{\frac{1.5\times 10^{-5}\times 5}{2}}\times 0.8605 = \underline{0.00527\text{ m/s}}$

d) $Q = \int_0^\delta u(1\times dy) = \int_0^\delta U_\infty\frac{dF}{d\eta}dy = \int_0^\delta U_\infty\frac{dF}{d\eta}d\eta\sqrt{\frac{vx}{U_\infty}}$

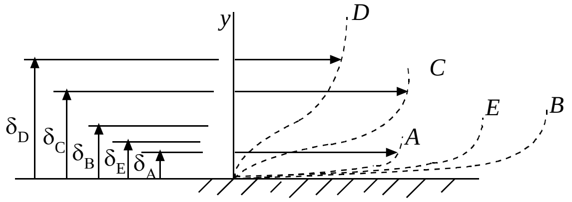
	$= U_\infty\sqrt{\frac{\nu x}{U_\infty}}[F(\delta)-F(0)] = 5\sqrt{\frac{1.5\times10^{-5}\times 2}{5}}\times 3.28 = \underline{0.04\text{ m}^3\text{/s/m}}$
8.134	At $x = 2$ m, Re = $5 \times 2/10^{-6} = 10^7$ $\therefore$ Assume turbulent from the leading edge. **a)** $\tau_0 = \frac{1}{2}\rho U_\infty^2 \frac{0.455}{\left[\ln(0.06\,\text{Re}_x)\right]^2} = \frac{1}{2}\times 1000\times 5^2 \frac{0.455}{\left[(\ln 0.06\times 10^7)\right]^2} = \underline{32.1\text{ Pa}}$ **b)** $u_\tau = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{32.1}{1000}} = 0.1792\text{ m/s}$ $\frac{5}{0.1792} = 2.44\ln\frac{0.1792\delta}{10^{-6}} + 7.4 \quad \therefore \delta = \underline{0.0248\text{ m}}$ **c)** Use the 1/7 the power-law equation: $Q = \int_0^{0.0248} 5\left(\frac{y}{0.0248}\right)^{1/7} dy = \underline{0.109\text{ m}^3\text{/s/m}}$
8.136	From Table 8.5 we interpolate for $F' = 0.5$ to be $\eta = \frac{0.5 - 0.3298}{0.6298 - 0.3298}(2-1) + 1 = 1.57$ $1.57 = \sqrt{\frac{U_\infty}{\nu x}} = y\sqrt{\frac{5}{1.5\times10^{-5}\times 2}} \quad \therefore y = 0.00385\text{ m or } \underline{3.85\text{ mm}}$ $v = \sqrt{\frac{\nu U_\infty}{x}}\left(\frac{1}{2}\right)(\eta F' - F) = \sqrt{\frac{1.5\times10^{-5}\times 5}{2}}(0.207) = \underline{0.00127\text{ m/s}}$ $\tau = \mu\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right) = F''\rho U_\infty^2\sqrt{\frac{\nu}{xU_\infty}} = 0.291(1.2)5^2\sqrt{\frac{1.5\times10^{-5}}{2\times 5}} = \underline{0.011\text{ Pa}}$
8.138	$u = U_\infty\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\right)$  For the Blasius profile: see Table 8.5. (This is only a sketch. The student is encouraged to draw the profiles to scale.)
8.140	A: $\frac{\partial p}{\partial x} < 0$. (favorable)

B: $\frac{\partial p}{\partial x} \cong 0$

C: $\frac{\partial p}{\partial x} > 0$ (unfavorable)

D: $\frac{\partial p}{\partial x} > 0$

E: $\frac{\partial p}{\partial x} < 0$



CHAPTER 9

Compressible Flow

Introduction

9.2

$c_p = c_v + R \qquad c_p = kc_v \qquad \therefore c_p = \frac{c_p}{k} + R$ or $c_p\left(1-\frac{1}{k}\right) = R$

$$\therefore \underline{c_p = \frac{Rk}{k-1}}$$

Speed of Sound

9.4

Substitute Eq. 4.5.18 into Eq. 4.5.17 and neglect potential energy change:

$$\frac{\dot{Q}-\dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} + \tilde{u}_2 - \tilde{u}_1$$

Enthalpy is defined in Thermodynamics as $h = \tilde{u} + pv = \tilde{u} + p/\rho.$ Therefore

$$\frac{\dot{Q}-\dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + h_2 - h_1$$

Assume the fluid is an ideal gas with constant specific heat so that $\Delta h = c_p \Delta T.$ Then

$$\frac{\dot{Q}-\dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + c_p(T_2 - T_1)$$

Next, let $c_p = c_v + R$ and $k = c_p/c_v$ so that $c_p / R = k/(k-1).$ Then, with the ideal gas law $p = \rho RT,$ the first law takes the form

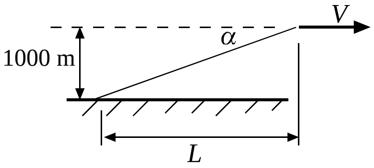
$$\frac{\dot{Q}-\dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + \frac{k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right)$$

9.6

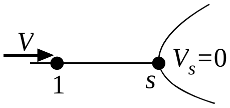
The speed of sound is given by

$$c = \sqrt{dp/d\rho}$$

For an isothermal process $TR = p/\rho = K,$ where K is a constant. This can be differentiated: $dp = Kd\rho = RTd\rho.$ Hence, the speed of sound is $c = \sqrt{RT}.$

9.8	For water Bulk modulus $=\rho\dfrac{dp}{d\rho}=2110\times10^6$ Pa Since $\rho=1000\text{ kg/m}^3$, we see that $c=\sqrt{\dfrac{dp}{d\rho}}=\sqrt{\dfrac{2110\times10^6}{1000}}=\underline{1453\text{ m/s}}$
9.10	Since c = 1450 m/s for the small wave, the time increment is $\Delta t=\dfrac{d}{c}=\dfrac{10}{1450}=\underline{0.0069\text{ s}}$
9.12	$c=\sqrt{kRT}=\sqrt{1.4\times287\times263}=325\text{ m/s}\quad\therefore d=ct=256\times1.21=\underline{393\text{ m}}$
9.14	$c=\sqrt{1.4\times287\times263}=256\text{ m/s}\quad\sin\alpha=\dfrac{1}{\text{M}}=\dfrac{c}{V}$ $\sin\alpha=0.256\quad\therefore\tan\alpha=0.2648=\dfrac{1000}{L}\quad\therefore L=3776\text{ m}$ $\Delta t=\dfrac{3776}{1000}=\underline{3.776\text{ s}}$ 
9.16	Eq. 9.2.4: $\Delta V=-\dfrac{\Delta p}{\rho c}=-\dfrac{\Delta p}{\rho\sqrt{kRT}}=-\dfrac{0.3}{1.225\sqrt{1.4\times287\times288}}=\underline{-0.35\text{ m/s}}$ Energy Eq: $\dfrac{V^2}{2}+c_pT=\dfrac{(V+\Delta V)^2}{2}+c_p(T+\Delta T)\qquad\therefore 0=V\Delta V+\cancel{\dfrac{(\Delta V)^2}{2}}+c_p\Delta T$ $\therefore\Delta T=\dfrac{-c\Delta V}{c_p}=\dfrac{-\sqrt{1.4\times287\times288}\text{ m/s}(-0.35\text{ m/s})}{1005\text{ (N-m)/(kg}\cdot\text{K)}}=0.012\text{ K}$ Use $\text{N}=\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$ (Units can be a pain!)

Isentropic Flow

9.18	**a)** $p_s = p_{\text{atm}} + 10 = 69.9 + 10 = 79.9$ kPa abs $p_1 = 69.9$ kPa abs  From 1 → s : $\frac{V_1^2}{2}+\frac{p_1}{\rho_1}=\frac{p_s}{\rho_s}$ $\quad \rho_s=\rho_1\left(\frac{p_s}{p_1}\right)^{1/k}=0.906\left(\frac{79.9}{69.9}\right)^{1/1.4}=0.997\text{ kg/m}^3$ $\therefore \frac{V_1^2}{2}+\frac{69{,}900}{0.906}=\frac{79{,}900}{0.997}$ $\quad \therefore \underline{V_1 = 77.3\text{ m/s}}$
9.20	Is $p_r < 0.5283p_0$? $\quad 0.5283\times 200 = 105.7$ kPa **a)** $p_r < 0.5283p_0$ $\quad \therefore$ choked flow $\quad \therefore \text{M}_e = 1 \quad \therefore V_e^2 = kRT_e \quad p_e = 105.7$ kPa $1000\times 298 = \frac{1.4\times 287T_e}{2}+1000T_e. \quad \therefore T_e = 248.1\text{ K}, \quad V_e = 315.8\text{ m/s}.$ $\rho_e = \frac{105.7}{0.287\times 248.1} = 1.484\text{ kg/m}^3 \quad \therefore \dot{m} = 1.484\times\pi\times 0.01^2\times 315.8 = \underline{0.1473\text{ kg/s}}$ **b)** $p_r > 0.5283p_0 \quad \therefore \text{M}_e < 1 \quad 1000\times 298 = \frac{V_e^2}{2}+\frac{1.4}{0.4}\times\frac{130{,}000}{\rho_e} \quad \frac{130}{200}=\left(\frac{\rho_e}{2.338}\right)^{1.4}$ $\rho_0 = \frac{200}{0.287\times 298} = 2.338 \quad \therefore \rho_e = 1.7187\text{ kg/m}^3 \quad \therefore V_e = 257.9\text{ m/s}$ $\therefore \dot{m} = 1.7187\times\pi\times 0.01^2\times 257.9 = \underline{0.1393\text{ kg/s}}$
9.22	**a)** $p_r < 0.5283\,p_0 \quad \therefore \text{M}_e = 1 \quad \therefore p_e = 0.5283\times 200 = 105.7\text{ kPa} \quad T_e = 0.8333\times 298 = 248.3\text{ K}$ $\rho_e = \frac{105.7}{0.287\times 248.3} = 1.483\text{ kg/m}^3 \quad V_e = \sqrt{1.4\times 287\times 248.3} = 315.9\text{ m/s}$ $\therefore \dot{m} = 1.483\times\pi\times 0.01^2\times 315.9 = \underline{0.1472\text{ kg/s}}$ **b)** $p_r > 0.5283\,p_0 \quad \therefore p_e = 130\text{ kPa}, \ \frac{p_e}{p_0} = 0.65 \quad \therefore \text{M}_e = 0.81, \quad T_e = 0.884T_0$ $\rho_e = \frac{130}{0.287\times 263.4} = 1.719\text{ kg/m}^3, \quad V_e = 0.81\sqrt{1.4\times 287\times 263.4} = 263.5\text{ m/s}$ $\therefore \dot{m} = 1.719\times\pi\times 0.01^2\times 263.5 = \underline{0.1423\text{ kg/s}}$

9.24	$p_e = 0.5283\times 400 = \underline{211.3\text{ kPa abs}}$ $\quad T_e = 0.8333\times 303 = 252.5\text{ K}$ $V_e = \sqrt{1.4\times 287\times 252.5} = 318.5\text{ m/s}$ $\quad \therefore \dot{m} = \frac{211.3}{0.287\times 252.5}\pi\times 0.05^2\times 318.5 = \underline{7.29\text{ kg/s}}$
9.26	$p_e = 0.5283\,p_0 = 101.3\text{ kPa}$ $\quad \therefore p_0 = \underline{192\text{ kPa}}$ $\quad T_e = 0.8333\times 280 = 233.3\text{ K}$ $V_e = \sqrt{1.4\times 287\times 233.3} = 306.2\text{ m/s}$ $\dot{m} = [101.3\times 10^3/(287\times 233.3)]\times\pi\times(0.3)^2\times 306.2 = \underline{1.31\text{ kg/s}}$ $p_0 = 2\times 192$ $\quad p_e = 0.5283\,p_0 = 203\text{ kPa},$ $\quad T_e = 233.3\text{ K},$ $\quad V_e = 306.2\text{ m/s}$ $\dot{m} = [203\times 10^3/(287\times 233.3)]\times\pi\times(0.03)^2\times 306.2 = \underline{2.62\text{ kg/s}}$
9.28	$5193\times 300 = \frac{1.667\times 2077\ T_e}{2} + 5193\ T_e$ $\quad \therefore T_e = 225\text{ K}$ $\quad \therefore p_e = 200\left(\frac{225}{300}\right)^{1.667/0.667}$ $= \underline{97.45\text{ kPa abs}}$ Next, $T_t = 225\text{ K},\ p_t = 97.45\text{ kPa};$ $\quad \therefore V_t = \sqrt{1.667\times 2077\times 225} = 882.6\text{ m/s}$ $\rho_t = \frac{97.45}{2.077\times 225} = 0.2085\text{ kg/m}^3$ $\quad 0.2085\times\pi\times 0\ .03^2\times 882.6 = \rho_e\pi\times 0.075^2 V_e$ $5193\times 300 = \frac{V_e^2}{2} + \frac{1.667}{0.667}\frac{p_e}{\rho_e}$ $\quad p_e = 200\left(\frac{\rho_e}{200/2.077\times 300}\right)^{1.667} = 1330\rho_e^{1.667}\text{ kPa}$ $= \frac{V_e^2}{2} + 3324\times 10^3\times 9.54 V_e^{-0.667}$ or $3.116\times 10^6 = V_e^2 + 63\ 420\times 10^3 V_e^{-0.667}$ $\quad$ Trial-and-error: $V_e = 91.8\text{ m/s}$ $\therefore \rho_e = 0.3203\text{ kg/m}^3$ and $p_e = \underline{199.4\text{ kPa abs}}$
9.30	We need to determine the Mach number at the exit. Since the M = 1 at the throat, then $A^* = A_{\text{throat}} = 9.7\text{ cm}^2$. Hence, the area ratio at the exit is $A_e/A^* = 13/9.7 = 1.34$. Using the air tables, we find two possible solutions, one for subsonic flow, and the other for supersonic flow in the diverging section of the nozzle. At the exit: *Subsonic Flow*: $\Rightarrow \text{M}_e = 0.5,\ T_e/T_0 = 0.9524,$ and $p_e/p_0 = 0.8430$ Hence, $V_e = \text{M}_e c_e = \text{M}_e\sqrt{kRT_e} = 0.5\sqrt{1.4\times 287(0.9524\times 295)} = \underline{168\text{ m/s}}$ *Supersonic Flow*: $\Rightarrow \text{M}_e = 1.76,\ T_e/T_0 = 0.6175,$ and $p_e/p_0 = 0.1850.$

	Hence, $V_e = \mathrm{M}_e c_e = \mathrm{M}_e \sqrt{kRT_e} = 1.76\sqrt{1.4\times 287\left(0.6175\times 295\right)} = \underline{476 \text{ m/s}}$
9.32	$\rho_1 = p_1 / RT_1 = (310+101.3)10^3/(287\times 289) = 4.96 \text{ kg/m}^3$ $\rho_2 = 4.96\left(\frac{351.3}{411.3}\right)^{1/1.4} = 4.43 \text{ kg/m}^3$ $V_1 \times 4.96\times (0.1)^2 = V_2 \times 4.43\times (0.05)^2 \quad \therefore V_2 = 4.48\ V_1$ $\frac{V_1^2}{2} + \frac{1.4}{0.4}\times\frac{411.3\times 10^3}{4.96} = \frac{4.48^2 V_1^2}{2} + \frac{1.4}{0.4}\times\frac{351.3\times 10^3}{4.43} \quad \therefore V_1 = 36.5 \text{ m/s}$ $\therefore \dot{m} = 4.96\pi\times (0.05)^2 \times 36.5 = \underline{1.42 \text{ kg/s}}$
9.34	$V_t^2 = kR_T \quad 1000\times 293 = \frac{1.4\times 287\,T_t}{2} + 1000\,T_t \quad \therefore T_t = 244.0 \text{ K} \quad V_t = 313.1 \text{ m/s}$ $\therefore p_t = 500\left(\frac{244}{293}\right)^{1.4/0.4} = 263.5 \text{ kPa abs} \quad \therefore \rho_t = \frac{263.5}{0.287\times 244} = 3.763 \text{ kg/m}^3$ $1000\times 293 = \frac{V_e^2}{2} + \frac{1.4}{0.4}\frac{p_e}{\rho_e} \qquad 3.763\times \pi \times 0.025^2 \times 313.1 = \rho_e \pi \times 0.075^2 V_e$ $\frac{p_e}{\rho_e^{1.4}} = \frac{263{,}500}{3.763^{1.4}}$ $\therefore 293{,}000 = \frac{V_e^2}{2} + 1.014\times 10^6 V_e^{-0.4}$. Trial-and-error: $V_e = 22.2$ m/s, 659 m/s $\therefore \rho_e = 5.897,\ 0.1987 \text{ kg/m}^3 \qquad \therefore p_e = \underline{494.2 \text{ kPa}},\ \underline{4.29 \text{ kPa abs}}$
9.36	$\mathrm{M_t} = 1 \quad \therefore p_t = 0.5283\times 830 = 438.5 \text{ kPa}, \quad T_t = 0.8333\times 289 = 241 \text{ K}$ $\therefore \rho_t = 6.34 \text{ kg/m}^3$ $\dot{m} = 14.6 = 6.34\frac{\pi d_t^2}{4}\sqrt{1.4\times 287\times 241} \quad \therefore \underline{d_t = 0.097 \text{ m}}$ $\frac{p}{p_0} = \frac{103}{830} = 0.125 \quad \therefore \mathrm{M_e} = 2.014,\ T_e = 0.552\times 289 = 160 \text{ K},\ V_e = 2.014\sqrt{1.4\times 287\times 160}$ $= \underline{253 \text{ m/s}}$ $\frac{A}{A^*} = 1.708 \quad \therefore \frac{\pi d_e^2}{4} = 1.708\frac{\pi\times 0.097^2}{4} \quad \therefore \underline{d_e = 0.127 \text{ m}}$

9.38

Using compressible flow tables for air, we determine the pressure ratio and temperature ratio for M = 2.8 to be:

$$\frac{p}{p_0} = 0.03685, \text{ and } \frac{T}{T_0} = 0.3894 \qquad \therefore\ p = 0.03685 \times p_0 = \underline{129\text{ kPa}}\text{ abs}$$

$$\text{and } T = 0.3894 \times T_0 = 125\text{ K}$$

$$\therefore\ V = \mathrm{M}c = 2.8\sqrt{kRT} = 2.8\sqrt{1.4 \times 287 \times 320} = \underline{1004\text{ m/s}}$$

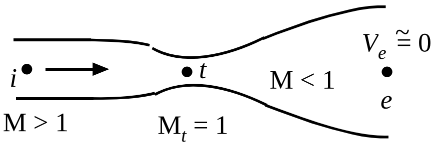
9.40

$$\text{Let } \mathrm{M}_t = 1. \text{ Neglect viscous effects. } \mathrm{M}_1 = \frac{150}{\sqrt{1.4 \times 287 \times 303}} = 0.430$$

$$\therefore\ \frac{A}{A^*} = 1.5007 \quad \therefore\ A_t = \frac{A_1}{1.5007} = \frac{\pi \times 0.05^2}{1.5007} = \frac{\pi d_t^2}{4} \quad \therefore\ d_t = 0.0816\text{ m or } \underline{8.16\text{ cm}}$$

9.42

Isentropic flow. Since $k = 1.4$ for nitrogen, the isentropic flow table may be used.



$$\text{At } \mathrm{M} = 3, \quad \frac{A}{A^*} = 4.235$$

$$V_i = 3\sqrt{1.4 \times 297 \times 373} = 1181\text{ m/s} \qquad \rho_i = \frac{100}{0.297 \times 373} = 0.9027\text{ kg/m}^3$$

$$\therefore\ A_i = \frac{\dot{m}}{\rho_i V_i} = \frac{10}{0.9027 \times 1181} = 0.00938\text{ m}^2 \quad \therefore\ A_t = \frac{0.00938}{4.235} = \underline{0.00221\text{ m}^2}$$

$$\text{At } \mathrm{M} = 3,\ T = 0.3571\ T_0,\ p = 0.02722\ p_0$$

$$\therefore T_0 = T_e = \frac{373}{0.3571} = 1044\text{ K or } \underline{772^\circ\text{C}} \qquad p_0 = \frac{100}{0.02722} = p_e = \underline{3670\text{ kPa}}\text{ abs}$$

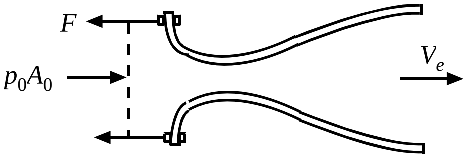
9.44

$$\text{Assume } p_e = 101\text{ kPa. Then } \rho_e = \frac{101}{0.189 \times 1273} = 0.4198\text{ kg/m}^3$$

$$F = \dot{m}V = \rho A V^2 \qquad \frac{80{,}000 \times 9.81}{6} = 0.4198\pi \times 0.25^2 V^2 \qquad \therefore\ \underline{V = 1260\text{ m/s}}$$

9.46

$$\mathrm{M_t} = 1 \quad \frac{A_e}{A^*} = 4; \quad \therefore\ \mathrm{M_e} = 2.94,\ p_e = 0.02980\ p_0$$



$$T_e = 0.3665\ T_0 = 0.3665 \times 300 = 110.0\text{ K},$$

$$p_e = 100 = 0.0298 p_0 \quad \therefore\ p_0 = 3356\text{ kPa abs}$$

$$\therefore V_e = 2.94\sqrt{1.4 \times 287 \times 109.95} = 618\text{ m/s}$$

$$\therefore F_B = \frac{-100}{0.287\times 109.95}\pi\times 0.05^2\times 618^2 + \ 3{,}356{,}000\pi\times 0.2^2 = \underline{412{,}000\text{ N}}$$

Normal Shock

9.48

a) $0.9850\times 1000 = \rho_2 V_2 \quad 80{,}000 - p_2 = 0.985\times 1000(V_2 - 1000)$

$$\frac{V_2^2 - 1000^2}{2} + \frac{1.4}{0.4}\left(\frac{p_2}{\rho_2} - 287\times 283\right) = 0 \qquad \left(\rho_1 = \frac{80}{0.287\times 283} = 0.9850\text{ kg/m}^3\right)$$

$$\frac{V_2^2}{2} - \frac{1000^2}{2} + \frac{1.4}{0.4}\times\frac{V_2}{985}(-985V_2 + 1{,}065{,}000) - 284{,}300 = 0$$

$\therefore 3V_2^2 - 3784V_2 + 784{,}300 = 0 \quad \therefore V_2 = 261\text{ m/s} \quad \rho_2 = \underline{3.774\text{ kg/m}^3}$

Substitute in and find $p_2 = \underline{808\text{ kPa abs}}$

$$\text{M}_1 = \frac{1000}{\sqrt{1.4\times 287\times 283}} = \underline{2.966} \quad T_2 = \frac{808}{0.287\times 3.774} = 746\text{ K or } \underline{473^\circ\text{C}}$$

$$\text{M}_2 = \frac{261}{\sqrt{1.4\times 287\times 746}} = \underline{0.477}$$

9.50

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\frac{T_1}{T_2} = \frac{2k\text{M}_1^2 - k + 1}{k+1}\frac{(k+1)^2\text{M}_1^2}{\left[1 + \frac{k-1}{2}\text{M}_1^2\right][4k\text{M}_1^2 - 2k + 2]} = \frac{(k+1)\text{M}_1^2}{2 + (k-1)\text{M}_1^2}$$

$\text{M}_1^2 = \dfrac{k+1}{2k}\dfrac{p_2}{p_1} + \dfrac{k-1}{2k}$ (This is Eq. 9.4.12). Substitute into above:

$$\frac{\rho_2}{\rho_1} = \frac{(k+1)\left[(k+1)\frac{p_2}{p_1} + (k-1)\right]}{4k + (k-1)\left\{(k+1)\frac{p_2}{p_1} + (k-1)\right\}} = \frac{(k+1)\left[(k+1)\frac{p_2}{p_1} + k - 1\right]}{(k+1)^2 + (k-1)(k+1)\frac{p_2}{p_1}}$$

$$= \frac{k - 1 + (k+1)p_2/p_1}{k + 1 + (k-1)p_2/p_1}$$

For a strong schock in which $\dfrac{p_2}{p_1} >> 1$, $\underline{\dfrac{\rho_2}{\rho_1} = \dfrac{k+1}{k-1}}$

9.52

If $\text{M}_2 = 0.5$, then $\text{M}_1 = 2.645 \quad \therefore V_1 = 2.645\sqrt{1.4\times 287\times 293} = \underline{908\text{ m/s}}$

$$p_2 = 8.00\times 200 = \underline{1600\text{ kPa abs}} \qquad \rho_2 = \frac{1600}{0.287\times(2.285\times 293)} = \underline{8.33\text{ kg/m}^3}$$

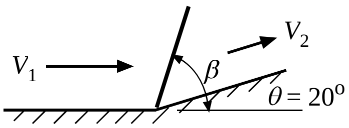
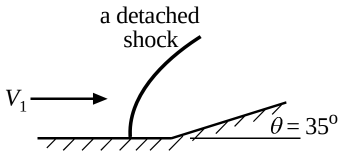
9.54	$p_1 = 0.2615\times 101 = 26.4\text{ kPa}\quad T_1 = 223.3\text{ K}\quad \text{M}_1 = 1000/\sqrt{1.4\times 287\times 223.3} = 3.34$ $\therefore \text{M}_2 = 0.4578\quad p_2 = 12.85\times 26.4 = 339\text{ kPa}\quad T_2 = 3.101\times 223.3 = 692.5\text{ K}$ For isentropic flow from 2 → 0: For M = 0.458, $p = 0.866p_0$ and $T = 0.960\ T_0\quad \therefore p_0 = 339/0.866 = \underline{391\text{ kPa abs}}\quad T_0 = 692.5/0.96 = 721\text{ K or }\underline{448^\circ\text{C}}$
9.56	$\frac{A}{A^*} = 4\quad \therefore \text{M}_\text{e} = 0.147\quad p_e = 0.985\ p_0\quad \therefore p_0 = \frac{101}{0.985} = \underline{102.5\text{ kPa abs}}$ $\text{M}_\text{t} = 1\quad p_t = 0.5283\times 102.5 = 54.15\text{ kPa}\quad T_t = 0.8333\times 298 = 248.3\text{ K}$ $\therefore \rho_t = \frac{54.15}{0.287\times 248.3} = 0.7599\text{ kg/m}^3\quad V_t = \sqrt{1.4\times 287\times 248.3} = 315.9\text{ m/s}$ $\therefore \dot{m} = 0.7599\times\pi\times 0.025^2\times 315.9 = \underline{0.471\text{ kg/s}}$. If throat area is reduced, M_t remains at 1, $\rho_t = 0.7599\text{ kg/m}^3$ and $\dot{m} = 0.7599\times\pi\times 0.02^2\times 315.9 = \underline{0.302\text{ kg/s}}$
9.58	$p_e = 100.3\text{ kPa} = p_2\quad \frac{A}{A^*} = 4\quad \therefore \text{M}_1 = 2.94,\text{ and } p_2/p_1 = 9.918$ $\therefore p_1 = \frac{100.3}{9.918} = 10.11\text{ kPa}\quad \text{At M}_1 = 2.94,\ p/p_0 = 0.0298\quad \therefore p_0 = \frac{10.11}{0.0298} = \underline{339.3\text{ kPa}}$ $\text{M}_\text{t} = 1,\ p_t = 0.5283\times 393.3 = \underline{179.2\text{ kPa}}\quad T_t = 0.8333\times 290 = 242\text{ K}$ $\therefore V_t = \sqrt{1.4\times 287\times 242} = \underline{312\text{ m/s}}$ $\text{M}_1 = 2.94,\ p_1 = \underline{10.11\text{ kPa}}\quad T_1 = 0.3665\times 290 = 106.3\text{ K}$ $\therefore V_1 = 2.94\sqrt{1.4\times 287\times 106.3} = \underline{607.6\text{ m/s}}$ $\text{M}_2 = 0.4788,\ p_e = \underline{100.3\text{ kPa}}\quad T_e = T_2 = 2.609\times 106.3 = 277.3$ $\therefore V_2 = 0.4788\sqrt{1.4\times 287\times 277.3} = \underline{160\text{ m/s}}$

Vapor Flow

9.60	$p_t = 0.546\ p_0 = 0.546\times 1200 = 655\text{ kPa}\quad T_t = 673\left(\frac{655}{1200}\right)^{0.3/1.3} = 585\text{ K}$ $\therefore \rho_t = \frac{655}{0.462\times 585} = 2.42\text{ kg/m}^3\quad V_t = \sqrt{1.3\times 462\times 585} = 593\text{ m/s}\quad (\text{M}_t = 1)$ $\dot{m} = \rho_t A_t V_t\quad \therefore 4 = 2.42\times\frac{\pi\times d_t^2}{4}\times 593\quad \therefore d_t = 0.060\text{ m or }\underline{6\text{ cm}}$

	$T_e = 673\left(\dfrac{101}{1200}\right)^{0.3/1.3} = 380.2\text{ K} \quad \therefore \rho_e = \dfrac{101}{0.462\times 380.2} = 0.575\text{ kg/m}^3$ $\dfrac{V_e^2}{2} + 1872\times 380.2 = 1872\times 673$ (Energy from $0 \rightarrow$ e) ($c_p = 1872$ J/kg·K) $\therefore V_e = 1050\text{ m/s} \quad \therefore 4 = 0.575(\pi d_e^2/4)\times 1050 \quad \therefore d_e = 0.092\text{ m}$ or $\underline{9.2\text{ cm}}$
9.62	$\mathrm{M_e} = 1,\ p_e = 0.546\times 1000 = 546\text{ kPa} \quad T_e = 643\left(\dfrac{546}{1000}\right)^{0.3/1.3} = 559\text{ K}$ $\therefore \rho_e = \dfrac{546\times 10^3}{462\times 559} = 2.11\text{ kg/m}^3 \quad V_e = \sqrt{1.3\times 462\times 559} = 579\text{ m/s}$ $3.65 = 2.11\dfrac{\pi d_e^2}{4}\times 579 \quad \therefore d_e = 0.0617\text{ m}$ or $\underline{6.17\text{ cm}}$

Oblique Shock Wave

9.64	$\mathrm{M}_1 = \dfrac{800}{\sqrt{1.4\times 287\times 303}} = 2.29$  From Fig. 9.15, $\beta = 46°,\ 79°$ **a)** $\beta = 46° \quad \therefore \mathrm{M}_{1n} = 2.29\sin 46° = 1.65$ $\therefore \mathrm{M}_{2n} = 0.654 = \mathrm{M}_2 \sin(46° - 20°) \quad \therefore \underline{\mathrm{M}_2 = 1.49}$ $p_2 = 3.01\times 40 = \underline{120.4\text{ kPa abs}} \quad T_2 = 1.423\times 303 = 431\text{ K}$ $V_2 = \sqrt{1.4\times 287\times 431}\times 1.49 = \underline{620\text{ m/s}}$ **c)** 
9.66	$\mathrm{M}_{1n} = 3.5\sin 35° = 2.01 \quad \therefore \mathrm{M}_{2n} = 0.576 \quad T_2 = 1.696\times 303 = 514\text{ K}$ $\mathrm{M}_2 = 0.576/\sin(35° - 20°) = 2.26 \quad \theta_1 = 20° = \theta_2 \quad \therefore \beta_2 = 47°$ $\mathrm{M}_{2n} = 2.26\sin 47° = 1.65 \quad \therefore \mathrm{M}_{3n} = 0.654 = \mathrm{M}_3 \sin(47° - 20°) \quad \therefore \mathrm{M}_3 = 1.44$ $T_3 = 1.423\times 514 = 731\text{ K} \quad V_3 = \mathrm{M}_3\sqrt{kRT_3} = 1.44\sqrt{1.4\times 287\times 731} = \underline{780\text{ m/s}}$

9.68

$\mathrm{M}_1 = 3,\ \theta = 10° \quad \therefore \beta_1 = 28° \quad \mathrm{M}_{1\mathrm{n}} = 3\sin 28° = 1.41 \quad \therefore \mathrm{M}_{2\mathrm{n}} = 0.736$

$\therefore p_2 = 2.153 \times 40 = 86.1 \text{ kPa abs}$

$$\mathrm{M}_2 = \frac{0.736}{\sin(28° - 10°)} = 2.38 \qquad \therefore p_3 = 6.442 \times 86.1 = \underline{555 \text{ kPa abs}}$$

$(p_3)_{\text{normal}} = 10.33 \times 40 = \underline{413 \text{ kPa abs}}$

Expansion Waves

9.70

$\theta_1 = 26.4° \quad \text{For M} = 4,\ \theta = 65.8° \quad \text{(See Fig. 9.18.)}$

$\therefore \theta = 65.8 - 26.4 = \underline{39.4°}$

$$T_2 = T_1 \frac{T_0}{T_1}\frac{T_2}{T_0} = 273 \frac{1}{0.5556} \times 0.2381 = 117 \text{ K}$$

$\therefore V_2 = 4\sqrt{1.4 \times 287 \times 117} = \underline{867 \text{ m/s}} \qquad T_2 = \underline{-156°\mathrm{C}}$

9.72

a) $\theta_1 = 39.1° \quad \theta_2 = 39.1 + 5 = 44.1° \quad \therefore \mathrm{M_u} = 2.72 \quad p_{2\mathrm{u}} = (20/0.0585) \times 0.04165$

$= \underline{14.24 \text{ kPa abs}}$

$\text{For } \theta = 5° \text{ and M} = 2.5,\ \beta = 27° \quad \mathrm{M}_{1\mathrm{n}} = 2.5\sin 27° = 1.13 \quad \therefore \mathrm{M}_{2\mathrm{n}} = 0.889$

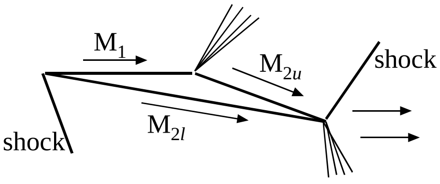
$\therefore p_{2\ell} = 1.32 \times 20 = \underline{26.4 \text{ kPa abs}} \qquad \mathrm{M}_\ell = \mathrm{M}_2 = 0.889/\sin(27° - 5°) = 2.37$

If $\theta = 5°$ with $\mathrm{M}_1 = 4$, then Fig. 9.15 $\rightarrow \beta$=18°

$\mathrm{M}_{1\mathrm{n}} = 4\sin 18° = 1.24 \quad \therefore \mathrm{M}_{2\mathrm{n}} = 0.818$

$\therefore p_{2\ell} = 1.627 \times 20 = 32.5 \text{ kPa}$

$$\therefore \mathrm{M}_{2\ell} = \frac{0.818}{\sin(18° - 5°)} = 3.64$$



$$\text{At } \mathrm{M}_1 = 4,\ \theta_1 = 65.8° \quad \text{At } 75.8°,\ \mathrm{M}_{2\mathrm{u}} = 4.88 \quad p_{2\mathrm{u}} = p_1 \frac{p_0}{p}\frac{p_2}{p_0} = 20\frac{0.002177}{0.006586}$$

$= 6.61 \text{ kPa}$

$$C_L = \frac{\text{Lift}}{\frac{1}{2}\rho V_1^2 A} = \frac{32.5A\cos 5° - 20 \times A/2 - 6.61 \times (A/2) \times \cos 10°}{\frac{1}{2} \times 1.4 \times 4^2 \times 20A} = \underline{0.0854}$$

$$C_D = \frac{\text{Drag}}{\frac{1}{2}\rho V_1^2 A} = \frac{32.5A\sin 5° - 6.61 \times (A/2) \times \sin 10°}{\frac{1}{2} \times 1.4 \times 4^2 \times 20A} = \underline{0.010}$$

CHAPTER 10

Flow in Open Channels

Introduction

10.2

$$\alpha = \cos^{-1}(1-2\times 0.3) = 1.159 \text{ rad or } 66.4°$$

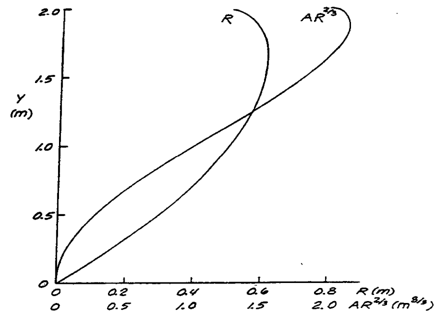
$$\frac{Q^2B}{gA^3} = \frac{Q^2 d\sin\alpha}{g\left[d^2/4\;(\alpha - \sin\alpha\cos\alpha)\right]^3}$$

$$= \frac{4^2\sin(1.159)}{9.81\times(d^5/64)\times 1.159 - \sin 1.159\cos 1.159)^3} = 1$$

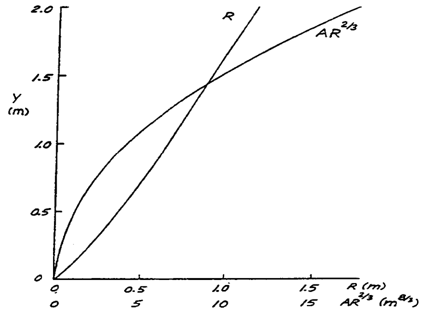
This equation reduces to: $192.1 = d^5$ $\therefore d = \underline{2.86 \text{ m}}$

Uniform Flow

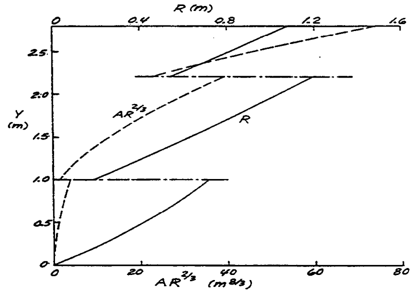
10.4



a)



b)



c)

10.6

Use Chezy-Manning equation in the form $Q = \frac{c_1}{n}\frac{A^{5/3}}{P^{2/3}}\sqrt{S_0}$. Substitute in appropriate expressions for *A* and *P*, and solve for y_0 by trial and error.

(a)

$$35 = \frac{\left[4.5y_0 + \frac{1}{2}y_0^2(2.5+3.5)\right]^{5/3}}{0.015\left[4.5 + y_0(\sqrt{1+2.5^2} + \sqrt{1+3.5^2})\right]^{2/3}}\sqrt{0.00035}$$

This reduces to $28.06 = \frac{(4.5y_0 + 3y_0^2)^{5/3}}{(4.5+6.33y_0)^{2/3}}$

Solving, $y_0 = \underline{2.15\text{ m}}$, $A = 4.5\times 2.15 + 3\times 2.15^2 = \underline{23.5\text{ m}^2}$,

$P = 4.5 + 6.33\times 2.15 = \underline{18.1\text{ m}}$

(c)

$$3.3 = \frac{\left[1.8^2/4\,(\alpha - \sin\alpha\cos\alpha)\right]^{5/3}}{0.012(1.8\alpha)^{2/3}}\sqrt{0.001}$$

This reduces to $2.63 = \frac{(\alpha - \sin\alpha\cos\alpha)^{5/3}}{\alpha^{2/3}}$

Solving, $\alpha = 1.94$ rad, $y_0 = \frac{1.8}{2}(1-\cos 1.94) = \underline{1.255\text{ m}}$

$$A = \frac{1.8^2}{4}(1.94 - \sin 1.94\cos 1.94) = \underline{1.845\text{ m}^2}$$

$P = 1.8\times 1.94 = \underline{3.5\text{ m}}$

10.8

$b = 0$, $m_1 = 8$, $m_2 = 0$

$$A = by + \frac{1}{2}y^2(m_1+m_2) = \frac{1}{2}m_1y^2 = \frac{8}{2}y^2 = 4y^2$$

$$P = b + y\left(\sqrt{1+m_1^2} + \sqrt{1+m_2^2}\right) = y\left(\sqrt{1+m_1^2}+1\right) = y\left(\sqrt{65}+1\right) = 9.062y$$

$$Q = AR^{2/3}\frac{\sqrt{S_0}}{n} = 4y^2\left(\frac{4y^2}{9.062y}\right)^{2/3}\frac{\sqrt{0.0005}}{0.015} = 3.456y^{8/3}$$

(a) $y = 0.12$ m, $Q = 3.956(0.12)^{8/3} = \underline{0.0121\text{ m}^3}$/s

(b) $Q = 0.08$ m^3/s, $y = \left(\frac{0.08}{5.456}\right)^{3/8} = \underline{0.244\text{ m}}$

Energy Concepts

10.10	$q = \sqrt{2gy^2(E-y)}$, $E =$ constant, $\frac{dq}{dy} = \frac{1}{2}\frac{4gy(E-y)-2gy^2}{\sqrt{2gy^2(E-y)}} = \frac{2gy(E-y)-gy^2}{q}$ Setting $dq/dy = 0$ and noting that $q \neq 0,$ so that the numerator is zero, one can solve for *y* at the condition $q = q_{\max}$ (note: $y = 0$ is a trivial solution): $gy[2(E-y)-y] = 0,$ $2E-3y = 0,$ $\therefore \ y = 2E/3 = y_c$
10.12	$q = V_1y_1 = 3\times 2.5 = 7.5 \text{ m}^2\text{/s}$ $E_1 = y_1 + \frac{q_2}{2gy_1^2} = 2.5 + \frac{3^2}{2\times 9.81} = 2.96 \text{ m}$ $y_c = \sqrt[3]{q^2/g} = \sqrt{7.5^2/9.81} = 1.79 \text{ m},\ E_c = \frac{3}{2}y_c = \frac{3}{2}\times 1.79 = 2.68 \text{ m}$ **(a)** $E_2 = E_1 - h = 2.96 - 0.2 = 2.76 \text{ m},\ E_2 > E_c$ $\therefore y_2$ is subcritical $\therefore E_2 = y_2 + \frac{q^2}{2gy_2^2}$, or $2.76 = y_2 + \frac{7.5^2}{2\times 9.81y_2^2} = y_2 + \frac{2.87}{y_2^2}$ Solving, $y_2 = 2.13$ m, and $y_2 + h - y_1 = 2.13 + 0.2 - 2.5 = \underline{-0.17 \text{ m}}$ **(c)** Set $E_2 = E_c = 2.68$ m and $\therefore$ maximum h is $h_{\max} = E_1 - E_2 = E_1 - E_c = 2.96 - 2.68 = 0.28 \text{ m}$ $\therefore y_c + h_{\max} - y_1 = 1.79 + 0.28 - 2.5 = \underline{-0.43 \text{ m}}$
10.14	**(a)** $q_1 = V_1y_1 = 3\times 3 = 9 \text{ m}^2\text{/s},\ Q = b_1q_1 = 3\times 9 = 27 \text{ m}^3\text{/s}$ $E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = 3 + \frac{3^2}{2\times 9.81} = 3.46 \text{ m},\ y_{c_1} = \sqrt[3]{9^2/9.81} = 2.02 \text{ m}$ $E_{c_1} = \frac{3}{2}\times 2.02 = 3.03 \text{ m}$ Without change in width at loc. 2, $E_2 = E_1 - h = 3.46 - 0.7 = 2.76 \text{ m}$ Since $E_2 < E_{c_1}$, width must change to prevent choking. Set $E_{c_2} = 2.76$ m $\therefore y_{c_2} = \frac{2}{3}\times 2.76 = 1.84 \text{ m},\ q_2 = \sqrt{gy_{c2}^3} = \sqrt{9.81\times 1.84^3} = 7.81 \text{ m}^2\text{/s}$ $\therefore b_2 = Q/q_2 = 27/7.81 = \underline{3.46 \text{ m}}$

10.16

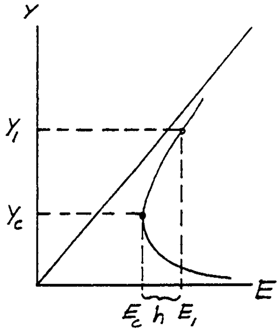
$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 2.15 + \frac{5.5^2}{2\times 9.81\times 2.15^2} = 2.484 \text{ m}$$

$$\text{Fr}_1 = \frac{q}{\sqrt{gy_1^3}} = \frac{5.5}{\sqrt{9.81\times 2.15^3}} = 0.557 \quad y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{5.5^2}{9.81}} = 1.456 \approx 1.46 \text{ m}$$

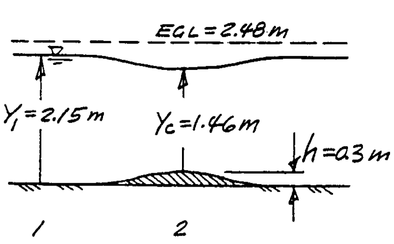
$$E_c = \frac{3}{2}y_c = \frac{3}{2}\times 1.456 = 2.183 \approx 2.18 \text{ m}$$

(a) The maximum height of the raised bottom at location 2 will be one for which the energy is a minimum:

$$E_1 = E_c + h,\ \ 2.484 = 2.183 + h,\ \ \therefore h = 2.484 - 2.183 = \underline{0.30 \text{ m}}$$



(b)



(c)

(d) Since $\text{Fr}_1 < 1$, if $h > 0.30$ m, subcritical nonuniform flow will occur upstream of the transition.

10.18

At the canal entrance, the condition of critical flow is $\text{Fr}^2 = Q^2B/gA^3 = 1,$ to be solved for the unknown width *b*.

(a) Rectangular channel: $B = b,\ A = by,\ y = y_c$

$$\therefore \frac{Q^2 b}{gb^3y^3} = \frac{Q^2}{gb^2y^3} = 1 \quad \therefore b = \frac{Q}{\sqrt{gy^3}} = \frac{18}{\sqrt{9.81\times 1}} = \underline{5.75 \text{ m}}$$

10.20

Objective is to determine the zero of the function $f(y) = Q^2B(y)/g[A(y)]^3 - 1,$ in which eqns. for *B*(*y*) and *A*(*y*) representing either a circular or trapezoidal section area can be substituted. The false position algorithm is explained in Example 10.10; this was used to determine the roots. The solutions are:

(a) $\underline{y_c = 1.00 \text{ m}},$ **(c)** $\underline{y_c = 1.81}$ m

10.22

In the lower section $(y \le 1\ \text{m})$, the area is

$$A = \int_0^y b\,d\eta = \int_0^y 3\sqrt{\eta}\,d\eta = 2y^{3/2} \quad \text{Assume } y < 1\ \text{m; then } \text{Fr}^2 = \frac{Q^2B}{gA^3} = \frac{Q^2 3\sqrt{y}}{g8y^{9/2}} = \frac{3Q^2}{8gy^4}$$

If $y_c = 1\ \text{m}$, then $\dfrac{3Q^2}{8\times 9.81\times 1^4} = 1$, or $Q = \sqrt{8\times 9.81/3} = 5.1\ \text{m}^3\text{/s}$

$\therefore$ when $Q > 5.1\ \text{m}^3\text{/s}$, critical depth will be > 1 m. Cross-sectional area including upper region (y > 1 m) is

$$A = 2(1)^{3/2} + 23(y_c - 1) = 23y_c - 21, \quad \text{and } B = 23\ \text{m}$$

(a) $\dfrac{Q^2B}{gA^3} = \dfrac{55^2\times 23}{9.81(23y_c - 21)^3} = 1, \quad (23y_c - 21)^3 = 7092,$

$$\therefore y_c = \frac{(7092)^{1/3} + 21}{23} = \underline{1.75\ \text{m}}$$

(b) $\dfrac{Q^2B}{gA^3} = \dfrac{3\times 3.5^2}{8\times 9.81 y_c^4} = 1, \quad y_c^4 = 0.468, \quad \therefore \underline{y_c = 0.83\ \text{m}}$

10.26

(a) $q = \dfrac{22.5}{15} = 1.5\ \text{m}^2\text{/s}, \quad y_c = \sqrt[3]{\dfrac{1.5^2}{9.81}} = 0.612\ \text{m}$

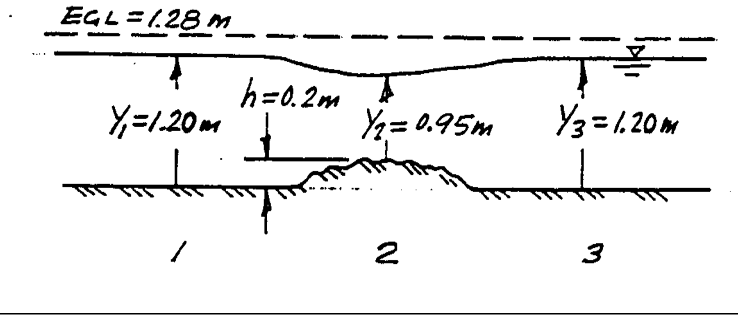
$$E_c = \frac{3}{2}\times 0.612 = 0.919\ \text{m}, \quad E_0 = 1.2 + \frac{1.5^2}{2\times 9.81\times 1.2^2} = 1.28\ \text{m} = E_3$$

$$E_2 = E_0 - h = 1.28 - 0.2 = 1.08\ \text{m} > E_3 \quad \therefore \text{no choking.}$$

Since losses are neglected throughout the transition, $y_1 = y_0$, and y_2 can be computed by writing the energy equation between locations 1 and 2:

$$1.28 = y_2 + \frac{1.5^2}{2\times 9.81\times 1.2^2} + h = y_2 + \frac{0.115}{y_2^2} + 0.2\ , \quad \therefore \underline{y_2 \approx 0.95\ \text{m}}$$

(b)



10.28

(a) $B' = 1$ m, $H' = 0.36$ m $\quad \Rightarrow B = \dfrac{B'}{0.3048} = 3.281$ and $H = \dfrac{H'}{0.3048} = 1.18$

$$Q = 0.11BH^{(1.522B)^{0.026}} = 0.11\times(3.281)(1.18)^{(1.522\times3.218)^{0.026}} = \underline{0.42\ \text{m}^3/\text{s}}$$

(b)

H'(m)	0.15	0.2	0.25	0.30	0.35	0.4	0.45
Q m^3/s	0.17	0.23	0.29	0.35	0.41	0.47	0.53

10.30

$$Q_1 = b\left(\frac{2}{3}\sqrt{\frac{2}{3}g}\right)(y_1-h)^{3/2},\quad Q_2 = b\left(\frac{2}{3}\sqrt{\frac{2}{3}g}\right)(y_2-h)^{3/2}$$

given $Q_1,\ y_1,\ Q_2,\ y_2,$ solve for b and h

$$\frac{y_1-h}{y_2-h} = \left(\frac{Q_1}{Q_2}\right)^{2/3} \quad \text{or} \quad h = \frac{y_1 - y_2(Q_1/Q_2)^{2/3}}{1-(Q_1/Q_2)^{2/3}}$$

(a) $$h = \frac{1.05-1.75(0.15/30)^{2/3}}{1-(0.15/30)^{2/3}} = \underline{1.03\ \text{m}}$$

$$\therefore b = \frac{Q_2}{\frac{2}{3}\left(\sqrt{\frac{2}{3}g}\right)(y_2-h)^{3/2}} = \frac{30}{\frac{2}{3}\left(\sqrt{\frac{2}{3}\times9.81}\right)(1.75-1.03)^{3/2}} = \underline{28.8\ \text{m}}$$

Momentum Concepts

10.32

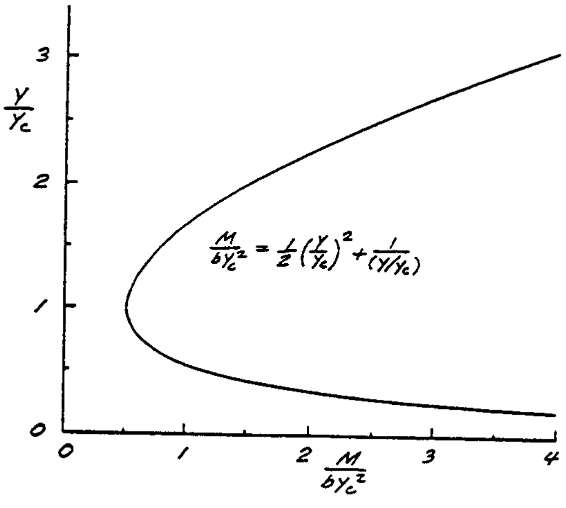
$$M = b\left(\frac{y^2}{2}+\frac{q^2}{gy}\right)$$

$$\frac{M}{by_c^2} = \frac{1}{2}\left(\frac{y}{y_c}\right)^2 + \frac{q^2}{gy_c^2 y}$$

$$= \frac{1}{2}\left(\frac{y}{y_c}\right)^2 + \frac{q^2}{gy_c^3(y/y_c)}$$

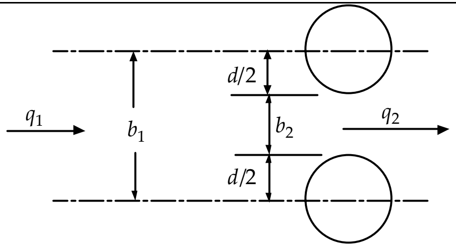
But $q^2/gy_c^3 = 1$

$$\therefore \underline{\frac{M}{by_c^2} = \frac{1}{2}\left(\frac{y}{y_c}\right)^2 + \frac{1}{(y/y_c)}}$$



10.34

(a) Conditions at location 1:



$$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = 1.8 + \frac{1.5^2}{2 \times 9.81 \times 1.8^2} = 1.835 \text{ m}$$

$$y_c = \sqrt[3]{q_1^2/g} = \sqrt[3]{1.5^2/9.81} = 0.612 \text{ m.}$$

The smallest constriction at location 2 is one that establishes critical flow, i.e., where minimum energy exists:

$\therefore E_2 = E_c$, or since $E_1 = E_2$,

$$y_{c_2} = \frac{2}{3}E_{c_2} = \frac{2}{3}E_1 = \frac{2}{3} \times 1.835 = 1.22 \text{ m}$$

$$\therefore q_2 = \sqrt{g(y_{c_2})^3} = \sqrt{9.81 \times 1.22^3} = 4.22 \text{ m}^2/\text{s}$$

$$b_2 = \frac{q_1 b_1}{q_2} = \frac{1.5 \times 6}{4.22} = 2.13 \text{ m}$$

Hence the maximum diameter is $d = b_1 - b_2 = 6 - 2.13 = \underline{3.87 \text{ m}}$

The momentum eqn. cannot be used to determine the drag since no information is given with regard to location downstream of cofferdam. But we do have available the drag relation, which will provide the required drag force:

Frontal area: $A = y_1 d = 1.8 \times 3.87 = 6.97 \text{ m}^2$

Approach velocity: $V_1 = q_1 / y_1 = 1.5/1.8 = 0.833 \text{ m/s}$

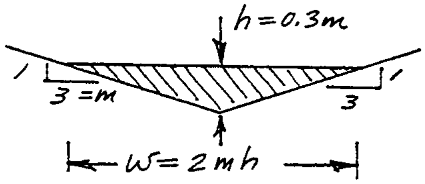
$\therefore F = C_D A \rho V_1^2 / 2 = 0.15 \times 6.97 \times 1000 \times 0.833^2 / 2 = \underline{363 \text{ N}}$, a rather insignificant drag force!

10.36

$$A_{\text{sill}} = \frac{1}{2}wh$$

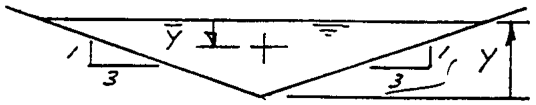
$$= \frac{1}{2} \times 2 \times 3 \times 0.3$$

$$= 0.27 \text{ m}^2$$



$$\bar{y} = \frac{1}{3}y$$

$$A = my^2 = 3y^2$$



$$M_1 = A_1\bar{y}_1 + \frac{Q^2}{gA_1} = 3\times0.5^2\times\frac{1}{3}\times0.5+\frac{Q^2}{9.81\times3\times0.5^2} = 0.125+\frac{Q^2}{7.358},$$

$$M_2 = A_2\bar{y}_2 + \frac{Q^2}{gA_2} = 3\times1.8^2\times\frac{1}{3}\times1.8+\frac{Q^2}{9.81\times3\times1.8^2} = 5.832+\frac{Q^2}{95.35},$$

$$\frac{F}{\gamma} = \frac{C_D A_{\text{sill}} Q^2}{2gA_1^2} = \frac{0.4\times0.27Q^2}{2\times9.81\times(3\times0.5^2)^2} = \frac{Q^2}{102.2}$$

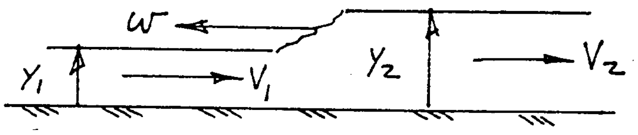
Substitute into momentum eqn: $M_1 = M_2 + F/\gamma$

$$\therefore 0.125+\frac{Q^2}{7.358} = 5.832+\frac{Q^2}{95.35}+\frac{Q^2}{102.2}$$

$$Q^2 = (5.832-0.125)\left(\frac{1}{7.58}-\frac{1}{95.35}-\frac{1}{102.2}\right)^{-1} = 49.37$$

$$\therefore \underline{Q = 7.03\ \text{m}^3/\text{s}}$$

10.38



Given condition: $V_2y_2 = 0.4V_1y_1$

Continuity eqn: $y_1(V_1+w) = y_2(V_2+w)$

Momentum eqn: $\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8\frac{(V_1+w)^2}{gy_1}}-1\right]$

Unknowns are V_2, y_2 and w. Eliminate V_2 and w, and solve for y_2.

(a) $V_2y_2 = 0.4\times1\times1.5 = 0.6$

$1.5(1+w) = y_2V_2 + y_2w = 0.6 + y_2w$, or $w = 0.9/(y_2-1.5)$

$$\therefore \frac{y_2}{1.5} = \frac{1}{2}\left[\sqrt{1+8\left(1+\frac{0.9}{y_2-1.5}\right)^2/(9.81\times1.5)}-1\right] \quad \text{or}$$

$$1-\sqrt{1+0.5437\left(1+\frac{0.9}{y_2-1.5}\right)^2}+1.333y_2 = 0$$

	$\therefore$ Solving, $y_2 = \underline{1.77\text{ m}}$ $\therefore V_2 = \frac{0.6}{y_2} = \frac{0.6}{1.77} = \underline{0.339\text{ m/s}}$ $\therefore w = \frac{0.9}{y_2 - 1.5} = \frac{0.9}{1.77 - 1.5} = \underline{3.33\text{ m/s}}$
10.40	**(a)** To compute the discharge, write the energy equation between locations 1 and 2, and solve for *q*: $q = \sqrt{\frac{2g(y_2 - y_1)}{\left(y_1^{-2} - y_2^{-2}\right)}} = \sqrt{\frac{2 \times 9.81(0.10 - 2.5)}{\left(2.5^{-2} - 0.10^{-2}\right)}} = 0.687\text{ m}^2/\text{s}$ $\therefore Q = bq = 5 \times 0.687 = \underline{3.43\text{ m}^3/\text{s}}$ **(b)** To compute depth downstream, first compute the Froude number at 2: $\text{Fr}_2 = \frac{q}{\sqrt{gy_2^3}} = \frac{0.687}{\sqrt{9.81 \times 0.10^3}} = 6.934$ $\therefore y_3 = \frac{y_2}{2}\left(\sqrt{1 + 8Fr_2^2} - 1\right) = \frac{0.10}{2}\left(\sqrt{1 + 8 \times 6.934^2} - 1\right) = \underline{0.932\text{ m}}$ **(c)** To calculate power lost, first compute the head loss in the jump: $h_j = \frac{(y_3 - y_2)^3}{4y_3y_2} = \frac{(0.932 - 0.10)^3}{4 \times 0.932 \times 0.10} = 1.544\text{ m}$ $\therefore \dot{W} = \gamma\, Qh_j = 9810 \times 3.43 \times 1.544 = \underline{5.20 \times 10^4\text{ watt, or }52.0\text{ kW}}$
10.42	Use Eqn. 10.5.16 with $F = 0$: $M_1 = \frac{y_1^2}{6}(2my_1 + 3b) + \frac{Q^2}{g(by_1 + my_1^2)}$ $= \frac{1.1^2}{6}(2 \times 3 \times 1.1 + 3 \times 5) + \frac{60^2}{9.81(5 \times 1.1 + 3 \times 1.1^2)} = 44.55\text{ m}^3$ $\therefore M_2 = M_1$ or $\frac{y_2^2}{6}(2my_2 + 3b) + \frac{Q^2}{g(by_2 + my_2^2)} = 44.55$ $\frac{y_2^2}{6}(2 \times 3y_2 + 3 \times 5) + \frac{60^2}{9.81(5y_2 + 3y_2^2)} = 44.55$

$$y_2^3 + 2.5y_2^2 + \frac{367}{5y_2 + 3y_2^2} = 44.55 \qquad \therefore \text{Solving, } y_2 = \underline{2.55 \text{ m}}$$

Energy loss across jump is h_j:

$$h_j = E_1 - E_2 = y_1 + \frac{Q^2}{2gA_1^2} - y_2 - \frac{Q^2}{2gA_2^2}$$

$$A_1 = 5\times1.1 + 3\times1.1^2 = 9.13 \text{ m}^2, \; A_2 = 5\times2.55 + 3\times2.55^2 = 32.26 \text{ m}^2,$$

$$\therefore h_j = 1.1 + \frac{60^2}{2\times9.81\times9.13^2} - 2.55 + \frac{60^2}{2\times9.81\times32.26^2} = 0.575 \text{ m}$$

$\therefore$ Power dissipated is $\dot{W}_j$:

$$\dot{W}_j = \gamma Q h_j = 9810\times60\times0.575 = \underline{3.38\times10^5 \text{W}}$$

10.44

First find *Q*, then compute y_1:

$$V_2 = Fr_2\sqrt{gy_2} = 0.4\sqrt{9.81\times1.5} = 1.534 \text{ m/s}$$

$$\therefore Q = V_2 A_2 = 1.534\times5\times1.5 = \underline{11.51 \text{ m}^3\text{/s}}$$

$$q = Q/b = 11.51/5 = 2.30 \text{ m}^2\text{/s}$$

$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{2.30^2/9.81} = 0.814 \text{ m}$$

To compute y_2, use momentum eqn, $M_1 - F/\gamma = M_2$ or

$$\left(\frac{y_1^2}{2} + \frac{q^2}{gy_1}\right) - \frac{C_D A \rho V_1^2/2}{b\gamma} = \left(\frac{y_2^2}{2} + \frac{q^2}{gy_2}\right)$$

Substitute in known data, plus the relation $V_1 = q/y_1$:

$$\frac{y_1^2}{2} + \frac{2.30^2}{9.81y_1} - \frac{0.35\times5\times0.17\times1000\times2.30^2}{5\times9810y_1^2} = \frac{1.5^2}{2} + \frac{2.30^2}{9.81\times1.5^2}$$

which reduces to $y_1^2 + \dfrac{1.078}{y_1} - \dfrac{0.0321}{y_1^2} = 2.97$

$\therefore$ Solving, $y_1 = \underline{0.346 \text{ m}}$

Nonuniform Gradually Varied Flow

10.46

(a) $q = Q/b = 0.35/1.8 = \underline{0.194\ \text{m}^2/\text{s}}$ $\qquad y_c = \sqrt[3]{q^2/g} = \sqrt[3]{0.194^2/9.81} = \underline{0.157\ \text{m}}$

Use Chezy-Manning eqn. to compute y_0:

$$\frac{Qn}{c_1\sqrt{S_0}} = \frac{0.35\times 0.012}{1\times\sqrt{0.0163}} = 0.0329 = \frac{(1.8y_0)^{5/3}}{(1.8+2y_0)^{2/3}}$$

$\therefore$ Solving, $y_0 = \underline{0.094\ \text{m}}$

$$E_c + h = \frac{3}{2}y_c + h = \frac{3}{2}\times 0.157 + 0.1 = 0.335\ \text{m}$$

$$E_0 = y_0 + \frac{q^2}{2gy_0^2} = 0.094 + \frac{0.194^2}{2\times 9.81\times 0.094^2} = 0.311\ \text{m}$$

Since $E_0 < E_c + h,$ normal conditions cannot exist at loc. 1, and choking will occur at loc. 2. Compute alternate depths at locs. 1 and 3: $E_1 = E_3 = E_c + h.$

$$y + \frac{0.194^2}{2\times 9.81y^2} = y + \frac{0.00192}{y^2} = 0.335$$

$\therefore$ Solving, $y_1 = \underline{0.316\ \text{m}}$, $y_3 = \underline{0.088\ \text{m}}$. (Note: loc. 2 is a critical control, with subcritical flow upstream and supercritical flow downstream.) A jump is located upstream of loc. 1. Find the depth conjugate to y_0:

$$\text{Fr}_0^3 = q^2/gy_o^3 = 0.194^2/(9.81\times 0.094^3) = 4.62$$

$$\therefore y_{cj} = \frac{y_0}{2}\left(\sqrt{1+8\text{Fr}_0^2} - 1\right) = \frac{0.094}{2}\left(\sqrt{1+8\times 4.62} - 1\right) = \underline{0.243\ \text{m}}$$

10.48

$q = Q/b = 33/4 = 8.25\ \text{m}^2/\text{s}$ $\qquad y_c = \sqrt[3]{q^2/g} = \sqrt[3]{8.25^2/9.81} = 1.91\ \text{m}$

Use Chezy-Manning eqn. to compute y_0:

$$\frac{Qn}{c_1\sqrt{S_0}} = \frac{33\times 0.012}{1\times\sqrt{0.00087}} = 13.43 = \frac{(4y_0)^{5/3}}{(4+2y_0)^{2/3}} \qquad \therefore \text{Solving, } y_0 = \underline{2.98\ \text{m}}$$

Since $y_0 > y_c$, mild slope conditions prevail. With $y_{\text{entrance}} < y_c$, an M_3 profile exists downstream of the entrance. For free outfall conditions, $y_{\text{exit}} \cong y_c$, and an M_2 profile exists upstream of the exit. The M_3 and M_2 profiles are separated by a hydraulic jump located approximately 260 m downstream of the entrance (determined by numerical analysis).

10.50

A spreadsheet solution is shown below. An M_3 profile is situated upstream, with a hydraulic jump at approximately 150 m followed by an M_1 profile.

Q =	20	n =	0.014	S_0 =	0.001	L =	300	g =	9.81
b =	0	m_1 =	3.5	y_u =	0.50	y_d =	2.50	c_1 =	1.00
		m_2 =	2.5						

	Depth	Residual
y_c =	1.554	0.00
y_0 =	1.809	-1.8E-09

Station	y	A	V	E	y_m	$S(y_m)$	Δx	x	y_{cj}	FM	Residual
1	0.500	0.750	26.667	36.744				0	3.769	54.49	2.77E-04
2	0.600	1.080	18.519	18.079	0.550	5.720E-01	33	33	3.324	37.97	-5.67E-05
3	0.700	1.470	13.605	10.135	0.650	2.347E-01	34	67	2.837	28.08	3.64E-06
4	0.800	1.920	10.417	6.330	0.750	1.094E-01	35	102	2.578	21.75	-5.48E-04
5	0.900	2.430	8.230	4.353	0.850	5.612E-02	36	138	2.365	17.51	-1.19E-04
6	1.000	3.000	6.667	3.265	0.950	3.101E-02	36	174	2.184	14.59	-2.52E-05
7	1.100	3.630	5.510	2.647	1.050	1.818E-02	36	210	2.028	12.56	-4.53E-06
8	1.200	4.320	4.630	2.292	1.150	1.119E-02	35	245	1.893	11.17	-1.48E-04
9	1.300	5.070	3.945	2.093	1.250	7.175E-03	32	277	1.778	10.24	-6.60E-05
10	1.400	5.880	3.401	1.990	1.350	4.760E-03	28	304	1.684	9.68	-3.90E-05
11	2.500	18.750	1.067	2.558				300			
12	2.450	18.008	1.111	2.513	2.475	1.878E-04	-56	244			
13	2.400	17.280	1.157	2.468	2.425	2.094E-04	-56	188			
14	2.350	16.568	1.207	2.424	2.375	2.340E-04	-57	131			
15	2.300	15.870	1.260	2.381	2.325	2.621E-04	-59	72			
16	2.250	15.188	1.317	2.338	2.275	2.943E-04	-60	12			
17	2.200	14.520	1.377	2.297	2.225	3.313E-04	-62	-51			

10.52

$$q = Q/b = 15/4 = 3.75\ \text{m}^2/\text{s}$$

$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{3.75^2/9.81} = \underline{1.13\ \text{m}} \qquad y_{0_1} = 0.93\ \text{m},\ y_c > y_{0_1}$$

$\therefore$ upstream reach has a <u>steep</u> slope.

$$y_{0_2} = 1.42\ \text{m},\ y_c < y_{0_2}$$

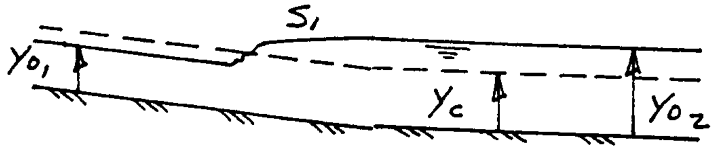
$\therefore$ downstream reach has a <u>mild</u> slope.

Compute depth conjugate to y_{0_1}:

$$\text{Fr}_{01}^2 = q^2/(gy_{01}^3) = 3.75^2/(9.81 \times 0.93^3) = 1.782$$

$$\therefore y_{cj} = \frac{y_{0_1}}{2}\left[\sqrt{1+8\text{Fr}_{01}^2} - 1\right] = \frac{0.93}{2}\left[\sqrt{1+8\times 1.782} - 1\right] = 1.35\ \text{m}$$

$\therefore$ Hydraulic jump occurs upstream, followed by an S_1 curve up to the transition.



10.54

(a) $y_c = \sqrt[3]{\dfrac{q^2}{g}} = \sqrt[3]{\dfrac{5.75^2}{9.81}} = 1.50\text{ m}.$ Since $y_{01} < y_c$ and $y_{02} > y_c$, there will be a hydraulic jump between *A* and *C*.

10.56

(a) Compute y_c using $Fr^2 = 1$:

$$\frac{Q^2B}{gA^3} = \frac{Q^2 d\sin\alpha}{g\left[(d^2/4)(\alpha - \sin\alpha\cos\alpha)\right]^3} = \frac{2.5^2 \times 2.5\sin\alpha}{9.81\left[(2.5^2/4)(\alpha - \sin\alpha\cos\alpha)\right]^3}$$

$$= 0.4175\frac{\sin\alpha}{(\alpha - \sin\alpha\cos\alpha)^3} = 1$$

$\therefore$ Solving

$$\alpha = 1.115\text{ rad and } \therefore y_c = \frac{d}{2}(1-\cos\alpha) = \frac{2.5}{2}(1-\cos 1.115) = \underline{0.70\text{ m}}$$

Compute y_0 using

$$\frac{Qn}{c_1\sqrt{S_0}} = \frac{\left[(d^2/4)(\alpha - \sin\alpha\cos\alpha)\right]^{5/3}}{(\alpha d)^{2/3}},\quad \frac{2.5\times 0.015}{1\times\sqrt{0.001}} = \frac{\left[(2.5^2/4)(\alpha - \sin\alpha\cos\alpha)\right]^{5/3}}{(\alpha\times 2.5)^{2/3}}$$

which reduces to $1.039 = \dfrac{(\alpha - \sin\alpha\cos\alpha)^{5/3}}{\alpha^{2/3}}$

$\therefore$ Solving, $\alpha = 1.361$ rad, and

$$\therefore y_0 = \frac{d}{2}(1-\cos\alpha) = \frac{2.5}{2}(1-\cos 1.361) = \underline{0.99\text{ m}}$$

Since $y_0 > y_c$, a mild slope condition exists. The water surface consists of an M_3 curve beginning at the inlet, followed by a hydraulic jump to an M_2 curve, which terminates at critical depth at the outlet. The numerically-predicted water surface and energy grade line are provided in the following table.

x (m)	y (m)	E (m)	x (m)	y (m)	E (m)
0	0.40	1.64	406	0.90	1.03
7	0.43	1.44	442	0.87	1.01
14	0.46	1.29	465	0.84	1.00
20*	0.49*	1.18*	480	0.82	0.98
20*	0.97*	1.07*	490	0.79	0.97
230	0.96	1.065	500	0.70	0.95
350	0.93	1.05	*hydraulic jump		

10.58

(a) $q = \frac{30}{5} = 6\ \text{m}^2/\text{s} \qquad y_c = \sqrt[3]{\frac{6^2}{9.81}} = 1.54\ \text{m}$

$$E_1 = 2 + \frac{6^2}{2\times 9.81\times 2^2} = 2.459\ \text{m}$$

$$E_2 = 1.8 + \frac{6^2}{2\times 9.81\times 1.8^2} = 2.366\ \text{m}$$

$$y_m = 1.9\ \text{m},\ \ A_m = 5\times 1.9 = 9.5\ \text{m}^2,\ \ P_m = 5 + 2\times 1.9 = 8.8\ \text{m},\ \ R_m = 9.5/8.8 = 1.08\ \text{m}$$

$$S(y_m) = \frac{Q^2 n^2}{A_m^2 R_m^{4/3}} = \left(\frac{30\times 0.012}{9.5}\right)^2 \frac{1}{1.08^{4/3}} = 0.0036$$

$$\therefore \Delta x = \frac{E_2 - E_1}{S_0 - S(y_m)1} = \frac{2.366 - 2.459}{-0.001 - 0.0036} = \underline{20.2\ \text{m}}$$

(b) An A_2 profile is contained within the reach (which has an adverse slope with $y_2 < y_1$)

10.60

Evaluate Q using $\Delta x = \frac{E_2 - E_1}{S_0 - S(y_m)}$

$$E_1 = y_1 + \frac{Q^2}{2gA_1^2} = 1.05 + \frac{Q^2}{2\times 9.81\times (2.5\times 1.05)^2} = 1.05 + \frac{Q^2}{135.2}$$

$$E_2 = y_2 + \frac{Q^2}{2gA_2^2} = 1.2 + \frac{Q^2}{2\times 9.81\times (2.5\times 1.2)^2} = 1.2 + \frac{Q^2}{176.6}$$

$$y_m = \frac{1}{2}(y_1 + y_2) = \frac{1}{2}(1.05 + 1.2) = 1.125\ \text{m}$$

$$S(y_m) = \left(\frac{Qn}{c_1}\right)^2 \frac{(b + 2y_m)^{4/3}}{(by_m)^{10/3}}$$

$$= \frac{Q^2 \times 0.013^2 \times (2.5+2\times1.125)^{4/3}}{1^2 \times (2.5\times1.125)^{10/3}} = 4.296\times10^{-5}Q^2$$

$$\therefore \Delta x\left[S_0 - S(y_m)\right] = E_2 - E_1$$

$$50(0.005 - 4.296\times10^{-5}Q^2) = 1.2 + \frac{Q^2}{176.6} - 1.05 - \frac{Q^2}{135.2}$$

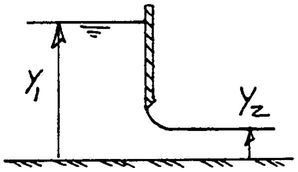
which reduces to $4.141\times10^{-4}Q^2 = 0.10$ $\therefore$ Solving, $Q = \underline{15.5\ \text{m}^3\text{/s}}$

$$y_c = \sqrt[3]{\left(\frac{Q}{b}\right)^2 \Big/ g} = \sqrt[3]{\left(\frac{15.5}{2.5}\right)^2 \Big/ 9.81} = 1.58\ \text{m}$$

$\therefore$ The profile is an $\underline{S_3 \text{ curve}}$

10.62

Write energy eqn. across the gate to compute the discharge:



$$E_1 = E_2 \quad \text{or} \quad y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$$

$$\therefore q = \sqrt{2g(y_1 - y_2)\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right) - 1}$$

$$= \sqrt{2\times9.81\times(1.85-0.35)\left(\frac{1}{0.35^2} - \frac{1}{1.85^2}\right) - 1} = \underline{1.93\ \text{m}^2\text{/s}}$$

$$\therefore Q = bq = 4\times1.93 = \underline{7.72\ \text{m}^3\text{/s}}$$

$$y_c = \sqrt[3]{q^2/g} = \sqrt{1.93^2/9.81} = \underline{0.72\ \text{m}}$$

Compute y_0 using Chezy-Manning relation:

$$\frac{Qn}{c_1\sqrt{S_0}} = \frac{7.72\times0.014}{1\times\sqrt{0.0008}} = 3.821 = \frac{(4y_0)^{5/3}}{(4+2y_0)^{2/3}}$$

$\therefore$ Solving, $y_0 = \underline{1.17\ \text{m}}$. Since $y_0 > y_c$, mild channel conditions exist.

Upstream of the gate there will be an M_1 profile, with an M_3 downstream of the gate, terminating in a hydraulic jump to normal flow conditions.

Compute the depth upstream of the jump, conjugate to y_0:

$$\text{Fr}_0^2 = q^2/gy_0^3 = 1.93^2/(9.81\times1.17^3) = 0.237$$

$$\therefore y_{cj} = \frac{y_0}{2}\left(\sqrt{1+8\text{Fr}_0^2} - 1\right) = \frac{1.17}{2}\left(\sqrt{1+8\times0.237} - 1\right) = \underline{0.41\ \text{m}}$$

Use the step method to compute water surface and energy grade line:

$$E = y + \frac{q^2}{2gy^2} = y + \frac{1.93^2}{2 \times 9.81 y^2} = y + \frac{0.190}{y^2}$$

$$S(y) = \frac{Q^2 n^2 (4+2y)^{4/3}}{(4y)^{10/3}} = \frac{7.72^2 \times 0.014^2 (4+2y)^{4/3}}{4^{10/3} y^{10/3}}$$

$$= 1.15 \times 10^{-4} (4+2y)^{4/3} y^{-10/3}$$

M_1 curve upstream of gate

y (m)	E (m)	y_m (m)	$S(y_m)$	Δx (m)	x (m)
1.85	1.906				0
		1.775	2.514×10^{-4}	–255	
1.7	1.766				–255
		1.6	3.336×10^{-4}	–390	
1.5	1.584				–645
		1.4	4.824×10^{-4}	–542	
1.3	1.412				–1187
		1.25	6.627×10^{-4}	–583	
1.2	1.332				–1770

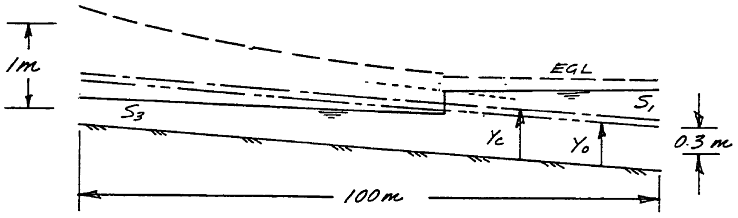
M_3 curve downstream of gate

y (m)	E (m)	y_m (m)	$S(y_m)$	Δx (m)	x (m)
0.35	1.901				0
		0.36	0.02741	5	
0.37	1.758				5
		0.38	0.02315	5	
0.39	1.639				10
		0.40	0.01973	5	
0.41	1.540				15

Hydraulic jump occurs ~5 m from the gate.

10.64

(a)



10.66

Compute normal depth using Chezy-Manning eqn:

$$\frac{Qn}{c_1\sqrt{S_0}}=\frac{15.38\times 0.017}{1\times\sqrt{0.00228}}=5.476=\frac{(3.66y_{0_2})^{5/3}}{(3.66+2y_{0_2})^{2/3}}$$

$\therefore$ Solving, $y_{0_2}=\underline{1.65\text{ m}}$

Compute critical depth:

$$q=Q/b=15.38/3.66=4.20\text{ m}^2/\text{s}\qquad \therefore y_c=\sqrt[3]{q^2/g}=\sqrt[3]{4.20^2/9.81}=\underline{1.22\text{ m}}$$

$\therefore$ Upstream of slope change the channel is steep, and downstream, the channel is mild. The hydraulic jump will terminate at normal depth.

(a) Find depth conjugate to y_{0_2}:

$$\text{Fr}_{02}^2=q^2/gy_{02}^3=4.20^2/(9.81\times 1.65^3)=0.400$$

$$\therefore y_{cj}=\frac{y_{0_2}}{2}\left(\sqrt{1+8\text{Fr}_{02}^2}-1\right)=\frac{1.65}{2}\left(\sqrt{1+8\times 0.4}-1\right)=\underline{0.87\text{ m}}$$

(b) Step method:

$$E=y+\frac{q^2}{2gy^2}=y+\frac{4.20^2}{2\times 9.81y^2}=y+\frac{0.900}{y^2}$$

$$S(y)=\frac{Q^2n^2(b+2y)^{4/3}}{(by)^{10/3}}=\frac{15.38^2\times 0.017^2(3.66+2y)^{4/3}}{(3.66)^{10/3}y^{10/3}}$$

$$=9.052\times 10^{-4}(3.66+2y)^{4/3}y^{-10/3}$$

y	E	y_m	$S(y_m)$	Δx	x
(m)	(m)	(m)		(m)	(m)
0.6	3.100				0
		0.65	0.03216	18.8	
0.7	2.537				18.8
		0.75	0.02104	17.6	
0.8	2.206				36.4
		0.835	0.01536	11.2	
0.87	2.059				$\underline{47.6}\cong L_j$

(c) An M_3 curve exists downstream of the change in channel slope, terminating in a hydraulic jump, approximately 50 m from the slope change.

10.68

This is a design analysis problem. Some extensive calculations are required.

(a) Find the depth of flow immediately upstream of the jump (call it location 1):

$$V_2 = \frac{Q}{A_2} = \frac{19}{0.75 \times 20} = 1.267 \text{ m/s}, \quad q = \frac{19}{20} = 0.95 \text{ m}^2\text{/s}.$$

$$\text{Fr}_2 = \frac{1.267}{\sqrt{9.81 \times 0.75}} = 0.467, \quad y_1 = \frac{0.75}{2}\left(\sqrt{1 + 8 \times 0.467^2} - 1\right) = 0.246 \text{ m}$$

Compute the loss across the jump, and subsequently the dissipated power:

$$h_j = E_1 - E_2 = 0.10 + \frac{0.95^2}{2 \times 9.81 \times 0.10^2} - 0.75 - \frac{0.95^2}{2 \times 9.81 \times 0.75^2} = 3.87 \text{ m}$$

$$\therefore \dot{W} = \gamma Q h_j = 9810 \times 19 \times 3.87 = 7.21 \times 10^5 \text{ watt, or } \underline{721 \text{ kW}}$$

(b) The length of the apron is the distance from the toe to downstream of the jump. First, compute the distance from y_{toe} to y_1 using a single increment of length:

$$E_{toe} = 0.10 + \frac{0.95^2}{2 \times 9.81 \times 0.10^2} = 4.700 \text{ m}, \ E_1 = 0.246 + \frac{0.95^2}{2 \times 9.81 \times 0.246^2} = 1.006 \text{ m},$$

$$y_m = \frac{1}{2}(0.10 + 0.246) = 0.173 \text{ m}, \ A_m = 20 \times 0.173 = 3.46 \text{ m}^2,$$

$$P_m = 20 + 2 \times 0.173 = 20.35 \text{ m}, \ S_m = \left(\frac{19 \times 0.014}{3.46}\right)^2 \frac{1}{(3.46/20.35)^{4/3}} = 0.06275,$$

$$\therefore x_1 - x_{toe} = \frac{E_1 - E_{toe}}{S_0 - S_m} = \frac{1.006 - 4.700}{0.0005 - 0.06275} = 59.3 \text{ m}$$

The length of the jump is six times the downstream depth, or 6 × 0.75 = 4.5 m. Hence the required length of the apron is L = 59.3 + 4.5 = 63.8 m, or approximately 65 m. Normal design would require additional length as a safety factor.

10.70

Use the varied flow function to compute the water surface profile.

$$y_0 = \left(\frac{q^2 n^2}{S_0}\right)^{3/10} = \left(\frac{1.5^2 \times 0.015^2}{0.0001}\right) = 1.627 \text{ m}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.5^2}{9.81}\right) = 0.612 \text{ m}$$

Therefore the estuary has a mild slope and an M_1 curve exists upstream of the outlet. For a wide rectangular channel (Ex. 10.16), J = 2.5, N = 3.33, M = 3, and N/J = 1.33. Use Eq. 10.7.13 to compute x:

$$x = \frac{1.627}{0.0001}\left[u - F(u,3.33) + \left(\frac{0.612}{1.627}\right)^3 \frac{2.5}{3.33} F(v,2.5)\right]$$

$$= 16,270\left[u - F(u,3.33) - 0.04F(v,2.5)\right]$$

The results of the calculations are as follows:

y (m)	u	$F(u,3.33)$	v	$F(v,2.5)$	x (m)	x – 69,800 (m)
7	4.30	0.015	6.96	0.038	69,800	0
6	3.67	0.021	5.67	0.053	59,700	–10,100
5	3.07	0.032	4.45	0.074	49,500	–20,300
4	2.46	0.052	3.31	0.117	39,500	–30,300
2	1.23	0.348	1.32	0.572	14,700	–55,100

10.72

This is a design analysis problem, and requires extensive calculations that would best be performed on a spreadsheet. Let location *A* be 400 m upstream of location *B*. Conditions at *B* are known. The energy equation between *A* and *B* is predicted using total energy $H = y + z + \alpha V^2 / 2g$:

$$H_A^* \approx H_B + h_L \approx H_B + \frac{L}{2}(S_A + S_B)$$

In the equation, H_B and S_B are known, and H_A and S_A are unknown. A trial and error solution is required. With the table of given data, location *A* is associated with x = 0, and location *B* with x = 400 m. The following parameters are computed at location *B*, where y_B = 3.0 m:

Location	A_B (m^2)	P_B (m)	R_B (m)	K_B (Eq. 10.7.8)
Side channel	60	149	0.40	11
Main channel	273	97	2.83	18,210

Now α_B, V_B, H_B, and S_B can be computed:

$$\alpha_B = \frac{(60+273)^2}{(11+18,210)^3}\left(\frac{11^3}{60^2} + \frac{18,210^3}{273^2}\right) = 1.48 \qquad \text{(Eq. 10.7.9)}$$

$$V_B = \frac{Q}{A_B} = \frac{280}{60+273} = 0.84 \text{ m/s}$$

$$S_B = \frac{280^2}{(11+18,210)^3} = 0.000236 \qquad \text{(Eq. 10.7.7)}$$

$$H_B = 3.0 + 15.1 + 1.48 \times \frac{0.84^2}{2 \times 9.81} = 18.153 \text{ m}$$

With L = 400 m, the energy equation between A and B is predicted:

$$H_A^* = 18.153 + \frac{400}{2}(S_A + 0.000236) = 18.20 + 200S_A$$

The total energy at location A is

$$H_A = y_A + z_A + \alpha_A \frac{V_A^2}{2g} = y_A + 15.0 + \alpha_A \frac{V_A^2}{2g}$$

The trial and error solution proceeds by assuming values of y_A, computing the corresponding α_A, V_A, S_A, H_A, and H_A^* until the latter two are in close agreement. For y_A = 3.2 m, the hydraulic parameters at location A are provided in this table:

Location	A_A (m^2)	P_A (m)	R_A (m)	K_A (Eq. 10.7.8)
Side channel	172	116	1.48	4470
Main channel	342	112	3.05	24,000

The corresponding values of H_A and H_A^* are

$$\alpha_A = \frac{(172+342)^2}{(4470+24,000)^3}\left(\frac{4470^3}{172^2} + \frac{24,000^3}{342^2}\right) = 1.39 \qquad \text{(Eq. 10.7.9)}$$

$$V_A = \frac{Q}{A_A} = \frac{280}{172+342} = 0.54 \text{ m/s}$$

$$S_A = \frac{280^2}{(4470+24,000)^3} = 9.67\times10^{-5} \qquad \text{(Eq. 10.7.7)}$$

$$H_A = 3.2 + 15.0 + 1.39\times\frac{0.54^2}{2\times9.81} = 18.22 \text{ m}$$

$$H_A^* = 18.200 + 200\times0.0000967 = 18.22 \text{ m}$$

Hence the depth at the upstream location is $\underline{y_A = 3.2 \text{ m}}$

CHAPTER 11

Flows in Piping Systems

Steady Flows

11.2

Substitute $V = \frac{Q}{A}$ and $H_P = \frac{\dot{W}_f}{\gamma Q}$ into the energy equation:

$$\frac{p_1}{\gamma} + \frac{Q^2}{2gA_1^2} + \frac{\dot{W}_f}{\gamma Q} = \frac{p_2}{\gamma} + \frac{Q^2}{2gA_2^2} + h_L$$

(a) Note that the kinetic energy terms can be neglected:

$$A_1 = \frac{\pi}{4} \times 0.05^2 = 0.00196 \text{ m}^2, \ A_2 = \frac{\pi}{4} \times 0.08^2 = 0.00503 \text{ m}^2,$$

$$Q = \frac{0.095}{60} = 0.00158 \text{ m}^3\text{/s}$$

$$\frac{350 \times 10^3}{9810} + \frac{0.00158^2}{2 \times 9.81 \times 0.00196^2} + \frac{\dot{W}_f}{9810 \times 0.00158} = \frac{760 \times 10^3}{9810} + \frac{0.00158^2}{2 \times 9.81 \times 0.00503^2} + 6.6$$

$$35.7 + 0.03 + 0.0645\dot{W}_f = 77.5 + 0.005 + 6.6 \qquad \therefore \underline{\dot{W}_f = 750 \text{ watt}}$$

11.4

Write energy eqn. between locs. 1 and 2:

$$H_P + z_1 = z_2 + \frac{f}{D}(L + L_e)\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}$$

Solving for D_1, $D = \left[\frac{Q^2(L + L_e)f}{2g(\pi/4)^2(H_P - (z_2 - z_1))}\right]^{1/5}$

Substitute the relations

$$f = 1.325\left[\ln\left(0.27\frac{e}{D} + 5.74\text{Re}^{-0.9}\right)\right]^{-2} \text{ and } \text{Re} = \frac{4Q}{\pi D \nu}, \quad L_e = \frac{\Sigma kD}{f},$$

and solve for *D* by successive substitution.

(a) Compute H_P from pump data using linear interpolation:

$$H_P = 136 - (136 - 127)\frac{(0.757 - 0.95)}{(0.757 - 1.01)} = 129.1 \text{ m}$$

$Q = 0.95\ \text{m}^3/\text{s}$

$$\therefore D = \left[\frac{0.95^2(500+L_e)f}{2\times 9.81(\pi/4)^2(129.1-36)}\right]^{1/5} = 0.24[(500+L_e)f]^{1/5}$$

D (m)	e/D	$\text{Re} = \frac{1.401\times 10^6}{D}$	f	$L_e = \frac{2.50}{f}$ (m)
0.3	0.003	4.67×10^6	0.026	29
0.4	0.0023	3.5×10^6	0.024	41.6
0.39	0.0023	3.6×10^6	0.024	40.6
0.39				

$\therefore \underline{D = 1.39\ \text{m}}$.

11.6

(a) Energy eqn. from A to B:

$$\left(\frac{p}{\gamma}+z\right)_A - \left(\frac{p}{\gamma}+z\right)_B = (R_1+R_2+R_3)Q^{1.85} + \left(\frac{\sum K_1}{2gA_1^2} + \frac{\sum K_2}{2gA_2^2}\right)Q^2$$

Use Hazen-Williams resistance formula for R:

$$R = \frac{10.59L}{C^{1.85}D^{4.87}}$$

$$\therefore R_1 = \frac{10.59\times 200}{100^{1.85}\times 0.2^{4.87}} = 1071$$

$$\frac{\sum K_1}{2gA_1^2} = \frac{2}{2\times 9.81\times(\pi/4)^2\times(0.2)^4} = 103.3, \quad R_2 = \frac{10.59\times 150}{120^{1.85}\times 0.25^{4.87}} = 193.4,$$

$$\frac{\sum K_2}{2gA_2^2} = \frac{3}{2\times 9.81\times(\pi/4)^2\times(0.3)^4} = 63.5, \quad R_3 = \frac{10.59\times 300}{90^{1.85}\times 0.3^{4.87}} = 271.1$$

Substitute known data into energy eqn:

$$250-107 = (1071+193+271)Q^{1.85} + (103.3+63.5)Q^2$$

$$143 = 1535Q^{1.85} + 167Q^2$$

$$F(Q) = 1535Q^{1.85} + 167Q^2 - 143, \qquad F'(Q) = 2840Q^{0.85} + 334Q$$

Iter.	$Q(\text{m}^3/\text{s})$	F	F'	$\Delta Q = -F/F'$
1	0.277^*	12.60	1046	– 0.01205
2	0.265	0.284	1007	– 0.00028
3	0.265			

$$\therefore Q=\underline{0.265\text{ m}^3\text{/s}}$$

$$^*Q\cong\left(\frac{143}{1535}\right)^{1/1.85}\quad(1^{\text{st}}\text{ iteration})$$

11.8

Write the energy equation from the liquid surface (location 1) to the valve exit (location 2):

$$\frac{p_1}{\gamma}+z_1+\frac{\dot{W}_f}{\gamma Q}=\left(\sum K+f\frac{L}{D}+1\right)\frac{Q^2}{2gA^2}+z_2$$

(a) Rearrange and substitute in known data:

$$\frac{P_1}{\gamma}+z_1-z_2+\frac{\dot{W}_f}{\gamma}\frac{1}{Q}-\left(\Sigma K+\frac{fL}{D}+1\right)\frac{Q^2}{2gA^2}=0$$

$$\frac{110\times10^3}{0.68\times9810}+24-18+\frac{1\times10^4}{0.68\times9810}\frac{1}{Q}$$

$$-\frac{(0.5+3\times0.26+2+0.015\times450/0.30+1)}{2\times9.81\times(0.7854\times0.30^2)^2}Q^2=0$$

$$22.4+\frac{1.50}{Q}-273.2Q^2=0$$

By trial and error, $\underline{Q\approx0.32\text{ m}^3\text{/s}}$

11.10

Initially work between locs. *B* and *C*:

$$L_e=\frac{D\Sigma k}{f},\ \overline{R}=\frac{8f(L+L_e)}{g\pi^2D^5}$$

$$L_{e1}=\frac{1.2\times2}{0.015}=160,\quad \overline{R}_1=\frac{8\times0.015\times260}{9.81\times\pi^2\times1.2^5}=0.1295,$$

$$L_{e2}=\frac{1\times3}{0.02}=150,\quad \overline{R}_2=\frac{8\times0.02\times1150}{9.81\times\pi^2\times1^5}=1.900,$$

$$L_{e3}=\frac{0.5\times2}{0.018}=56,\quad \overline{R}_3=\frac{8\times0.018\times1556}{9.81\times\pi^2\times0.5^5}=74.05,$$

$$L_{e4}=\frac{0.75\times4}{0.021}=143,\quad \overline{R}_4=\frac{8\times0.021\times943}{9.81\times\pi^2\times0.75^5}=6.895$$

$$\therefore W=\frac{Q^2}{\left(\sum_{i=2}^{4}\frac{1}{\sqrt{\overline{R}_i}}\right)^2}=\frac{3^2}{\left(\frac{1}{\sqrt{1.9}}+\frac{1}{\sqrt{74.05}}+\frac{1}{\sqrt{6.895}}\right)^2}=6.022\text{ m},$$

$$\therefore Q_2 = \sqrt{W/\bar{R}_2} = \sqrt{6.022/1.9} = \underline{1.780\text{ m}^3\text{/s}},$$

$$Q_3 = \sqrt{W/\bar{R}_3} = \sqrt{6.022/74.05} = \underline{0.285\text{ m}^3\text{/s}},$$

$$Q_4 = \sqrt{W/\bar{R}_4} = \sqrt{6.022/6.895} = \underline{0.935\text{ m}^3\text{/s}}.$$

Check continuity: $Q_2 + Q_3 + Q_4 = 1.78 + 0.285 + 0.935 = 3 = Q_1, \ \therefore \text{OK}$

To find $\dot{W}_P$, write energy equation from loc. *A* to *B*:

$$H_P = \bar{R}_1 Q_1^2 + \left(\frac{p}{\gamma} + z\right)_B = \bar{R}_1 Q_1^2 + W + 20$$

$$\therefore H_P = 0.1295 \times 3^2 + 6.022 + 20 = \underline{27.2\text{ m}}$$

$$\therefore \dot{W}_P = \frac{\gamma Q_1 H_P}{\eta} = \frac{9810 \times 3 \times 27.2}{0.75} = 1.07 \times 10^6\text{ W}, \quad \text{or } \dot{W}_P = \underline{1.07\text{ MW}}$$

11.12

Use SI data from Pbm 11.11 (a):

$$\sum Q = Q_1 - Q_2 - Q_3 = 0, \text{ or}$$

$$w(H) = \sqrt{\frac{30 - H}{142.1}} - \sqrt{\frac{H - 20}{232.4}} - \sqrt{\frac{H - 18}{1773}}$$

Use false position method of solution: $H_r = \dfrac{H_\ell w(H_u) - H_u w(H_\ell)}{w(H_u) - w(H_\ell)}$

Iter	H_ℓ	H_u	$w(H_\ell)$	$w(H_u)$	H_r	$w(H_r)$	Sign $w(H_\ell)w(H_r)$	ε
1	25	20	–0.02193	0.2317	24.57	–0.00562	+	—
2	24.57	20	–0.00562	0.2317	24.46	–0.00144	+	0.0045
3	24.46	20	–0.00144	0.2317	24.43	–0.00030	+	0.0012

$$\therefore Q_1 = \sqrt{(30 - 24.43)/142.1} = \underline{0.198\text{ m}^3\text{/s}}$$

$$Q_2 = \sqrt{(24.43 - 20)/232.4} = \underline{0.138\text{ m}^3\text{/s}}$$

$$Q_3 = \sqrt{(24.43 - 18)/1773} = \underline{0.060\text{ m}^3\text{/s}}$$

11.14

Continuity at junction: $Q_1 - (Q_2 + Q_3 + Q_4) = 0.$

Energy eqn. across pipe 1: $\dfrac{p_0}{\gamma} = \bar{R}_1 Q_1^2 + H$, or $Q_1 = \sqrt{(p_0/\gamma - H)/\bar{R}_1}$

in which *H* = piezometric head at junction. For each branch (i= 2, 3, 4):

$H = \bar{R}_i Q_1^2$ or $Q_i = \sqrt{H/\bar{R}_i}$

Substitute into continuity eqn: $\left[\dfrac{p_0/\gamma - H}{\bar{R}_1}\right]^{1/2} = \sum_{i=2}^{4}\left[\dfrac{H}{\bar{R}_i}\right]^{1/2} = \sqrt{H}\sum_{i=2}^{4}\dfrac{1}{\sqrt{\bar{R}_i}}$

$\therefore$ Solving for *H*,

$$H = \frac{p_0\gamma}{1 + R_1\left[\sum \dfrac{1}{\sqrt{\bar{R}_i}}\right]^2}$$

$$= \frac{300\times10^3/9810}{1+1.6\times10^4\left[\dfrac{1}{\sqrt{5.3\times10^5}}+\dfrac{1}{\sqrt{1\times10^6}}+\dfrac{1}{\sqrt{1.8\times10^6}}\right]^2}$$

$$= \frac{30.58}{1.556} = \underline{26.46\text{ m}}$$

$$\therefore Q_1 = \sqrt{(30.58-26.46)/1.6\times10^4} = \underline{0.01605\text{ m}^3\text{/s}}$$

$$Q_2 = \sqrt{26.46/5.3\times10^5} = \underline{0.00707\text{ m}^3\text{/s}}$$

$$Q_3 = \sqrt{26.46/1\times10^6} = \underline{0.00514\text{ m}^3\text{/s}}$$

$$Q_4 = \sqrt{26.46/1.8\times10^6} = \underline{0.00383\text{ m}^3\text{/s}}$$

11.16

(c) Substitute known data into energy equation, and solve for Q_1:

$$a_0 = \left\{a_1 + \bar{R}_1 + \left[\sum\frac{1}{\sqrt{\bar{R}_i}}\right]^{-2}\right\}Q_1^2$$

$$\therefore 45 = \left\{10^4 + 34{,}650 + \left[\frac{1}{\sqrt{82{,}500}}+\frac{1}{\sqrt{127{,}900}}+\frac{1}{\sqrt{115{,}500}}\right]^{-2}\right\}Q_1^2 = 56{,}410Q_1^2$$

$$\therefore Q_1 = \sqrt{45/56{,}410} = \underline{0.0282\text{ m}^3\text{/s}}$$

$$\therefore H = 45-(10^4+34{,}650)\times0.0282^2 = \underline{9.49\text{ m}}$$

$Q_2 = \sqrt{9.49/82{,}500} = \underline{0.0107\text{ m}^3\text{/s}}$ $\quad Q_3 = \sqrt{9.49/127{,}900} = \underline{0.0086\text{ m}^3\text{/s}}$

$$Q_4 = \sqrt{9.49/115{,}500} = \underline{0.0091\text{ m}^3\text{/s}}$$

11.18

(a) For each pipe compute equivalent length and resistance coefficient:

Pipe 1: $L_e = D\Sigma K/f = 0.05\times 3/0.02 = 7.5$

$$\bar{R}_1 = \frac{8f(L+L_e)}{g\pi^2 D^5} = \frac{8\times 0.02\times 37.5}{9.81\times \pi^2\times 0.05^5} = 1.983\times 10^5$$

Pipe 2: $Le = 0.075\times 5/0.025 = 15,\ \bar{R}_2 = \dfrac{8\times 0.025\times 55}{9.81\times \pi^2\times 0.075^5} = 4.788\times 10^4$

Pipe 3: $Le = 0.06\times 1/0.022 = 2.7,\quad \bar{R}_3 = \dfrac{8\times 0.022\times 62.7}{9.81\times \pi^2\times 0.06^5} = 1.466\times 10^5$

$$W = \frac{Q^2}{\left(\sum_{i=1}^{3}(\bar{R}_i)^{-1/2}\right)^2} = \frac{(0.6/60)^2}{[(1.983\times 10^5)^{-1/2} + (4.788\times 10^4)^{-1/2} + (1.466\times 10^5)^{-1/2}]^2} = 1.125\text{ m}$$

$$\therefore Q_1 = \sqrt{W/\bar{R}_1} = \sqrt{1.125/1.983\times 10^5} = \underline{0.00238\text{ m}^3\text{/s, or 143 L/min}}$$

$$Q_2 = \sqrt{W/\bar{R}_2} = \sqrt{1.125/4.788\times 10^4} = \underline{0.00485\text{ m}^3\text{/s, or 290 L/min}}$$

$$Q_3 = \sqrt{W/\bar{R}_3} = \sqrt{1.125/1.466\times 10^5} = \underline{0.00277\text{ m}^3\text{/s, or 166 L/min}}$$

11.20

Write energy eqn. from junction at *A* to suction side of pump at *B*:

$$H_{P_A} = \left(R_1 + \frac{\Sigma K}{2gA_1^2}\right)Q^2 + \frac{p_B}{\gamma} + z_B$$

where $H_{P_A} =$ pump head and $p_B =$ pressure. The only unknown in the relation is *Q*, since

$$H_{P_A} = \frac{\dot{W}_{P_A}\eta}{\gamma Q} = \frac{1\times 10^6\times 0.76}{0.81\times 9810\times Q} = \frac{95.6}{Q}$$

Evaluate the constants:

$$R_1 = \frac{8f_1L_1}{g\pi^2 D_1^5} = \frac{8\times 0.23\times 5000}{9.81\times \pi^2\times 0.75^5} = 40.0 \qquad \frac{\Sigma K}{2gA_1^2} = \frac{2}{2\times 9.81\times (\pi/4)^2\times 0.75^4} = 0.52$$

$$\frac{p_B}{\gamma} + z_B = \frac{150\times 10^3}{0.81\times 9810} + 27 = 45.9\text{ m.}$$

Substituting into the energy eqn., $\dfrac{95.6}{Q} = 40.5Q^2 + 45.9$

Solve using Newton's method: $Q = \underline{1.053\text{ m}^3\text{/s}}$ and $H_{P_A} = 95.6/1.053 = \underline{90.8\text{ m}}$

The second reach is now analyzed. Write energy eqn. from suction side of pump at *B* to the downstream reservoir:

$$\frac{p_B}{\gamma}+z_B+H_{P_B}=\left(R_2+\frac{\sum K}{2gA_2^2}\right)Q^2+50$$

Evaluate the constants:

$$R_2=\frac{8f_2L_2}{g\pi^2D_2^5}=\frac{8\times0.023\times7500}{9.81\times\pi^2\times0.75^5}=60.1 \qquad \frac{\sum K}{2gA_1^2}=\frac{10}{2\times9.81\times(\pi/4)^2\times0.75^4}=2.61$$

$$\therefore 45.9+H_{P_B}=(60.1+2.6)\times1.053^2+50 \text{ or } H_{P_B}=\underline{73.6\text{ m}}$$

$$\therefore \dot{W}_{P_B}=\frac{\gamma QH_{P_B}}{\eta}=\frac{0.81\times9810\times1.053\times73.6}{0.76}=8.10\times10^5\text{ W} \quad\text{or}$$

$$\dot{W}_{P_B}=\underline{810\text{ kW}}$$

11.22

Compute resistance coefficients:

$$R_1=\frac{10.59\text{ L}}{C^{1.85}D^{4.87}}=\frac{10.59\times200}{130^{1.85}\times0.5^{4.87}}=7.6 \qquad R_2=\frac{10.59\times600}{130^{1.85}\times0.3^{4.87}}=274.6$$

$$R_3=\frac{10.59\times1500}{130^{1.85}\times0.3^{4.87}}=686.5 \qquad R_4=\frac{10.59\times1500}{130^{1.85}\times0.4^{4.87}}=169.1$$

(a) Let $H_J=$ piezometric head at junction *J*. Assume $Q_1=2\text{ m}^3\text{/s}$ (out of *J*).

Then $H_J=30+R_1Q_1^2=30+7.6\times2^2=60.4\text{ m}$

$$Q_2=\sqrt{(250-60.4)/274.6}=0.831\text{ m}^3\text{/s (into }J)$$

$$Q_3=\sqrt{(300-60.4)/686.5}=0.591\text{ m}^3\text{/s (into }J)$$

$$Q_4=\sqrt{(200-60.4)/169.1}=0.909\text{ m}^3\text{/s (into }J)$$

$$\therefore \Delta Q=-2+0.831+0.591+0.909=+0.331$$

Assume $Q_1=2.5\text{ m}^3\text{/s}$. Then $H_J=30+7.6\times2.5^2=77.5\text{ m}$

$$Q_2=\sqrt{(250-77.5)/274.6}=0.793\text{ m}^3\text{/s}$$

$$Q_3=\sqrt{(300-77.5)/686.5}=0.569\text{ m}^3\text{/s}$$

$$Q_4=\sqrt{(200-77.5)/169.1}=0.851\text{ m}^3\text{/s}$$

$$\therefore \Delta Q=-2.5+0.793+0.569+0.851=-0.287$$

Linear interpolation to find the next estimate of Q_1 yields

$$\frac{-0.287-0.331}{2.5-2}=\frac{-0.287}{2.5-Q_1} \qquad \therefore Q_1=\underline{2.27\ \text{m}^3/\text{s}}$$

Then $H_J=30+7.6\times 2.27^2=\underline{69.2\ \text{m}}$

$$Q_2=\sqrt{(250-69.2)/274.6}=\underline{0.811\ \text{m}^3/\text{s}}$$

$$Q_3=\sqrt{(300-69.2)/686.5}=\underline{0.580\ \text{m}^3/\text{s}}$$

$$Q_4=\sqrt{(200-69.2)/169.1}=\underline{0.880\ \text{m}^3/\text{s}}$$

$$\therefore \Delta Q=-2.27+0.811+0.580+0.880=-0.001 \quad \therefore \text{OK}$$

11.24

Compute the *R*-values for each pipe:

Pipe 1: $L_e=\dfrac{1\times 0.1}{0.04}=2.5\ \text{m},\quad R_1=\dfrac{8\times 0.04\times 5.5}{9.87\times\pi^2\times 0.1^5}=1819.6$

Pipe 2: $L_e=\dfrac{2\times 0.1}{0.04}=5\ \text{m},\quad R_2=\dfrac{8\times 0.04\times 155}{9.81\times\pi^2\times 0.1^5}=5.31\times 10^4$

Pipe 3: $L_e=\dfrac{2\times 0.1}{0.04}=5\ \text{m},\quad R_3=\dfrac{8\times 0.04\times 605}{9.81\times\pi^2\times 0.1^5}=2\times 10^5$

Pipe 4: $L_e=\dfrac{4\times 0.1}{0.04}=10\ \text{m},\quad R_4=\dfrac{8\times 0.04\times 1{,}817}{9.81\times\pi^2\times 0.1^5}=77{,}750$

(a) Compute the discharge and head loss in pipe 2:

$$Q_2=5.2\times 10^{-3}\ \text{m}^3/\text{s}$$

$$\therefore (h_L)_2=R_2Q_2^2=5.31\times 10^4\times(5.2\times 10^{-3})^2=1.387\ \text{m}$$

(c) $Q_2=11.6\times 10^{-3}\ \text{m}^3/\text{s}=696\ \text{L/min},$ and from the pump curve,

H_P = 136 m

Write the energy equation from *A* to *C*:

$$H_C=H_P-(R_1+R_2)Q_2^2=136-53{,}120\times(11.6\times 10^{-3})^2=129\ \text{m}$$

The energy equation from *C* to *D* is $H_C=z_D+R_4Q_4^2$, where H_C is the hydraulic grade line at location *C*. Therefore the discharge in pipe 4 is

$$Q_4=\sqrt{\frac{H_C-z_D}{R_4}}=\sqrt{\frac{129-127}{77{,}750}}=5.07\ \text{L/s},\ \text{or}\ \underline{300\ \text{L/min}}$$

11.26

(a) From the pump curve, for H_P = 138 m, Q = 0.01 m^3/s. Write the energy equation from A to C and evaluate the hydraulic grade line

$$H_C = H_P - (h_L)_1 - (h_L)_2 = 138 - (53{,}120) \times (0.01)^2 = 132.7 \text{ m}$$

Thus the discharge in pipe 3, between location C and location B is

$$Q_3 = \sqrt{\frac{H_C - H_B}{R_3}} = \sqrt{\frac{132.7-130}{2\times 10^5}} = \underline{3.67\times 10^{-3} \text{ m}^3\text{/s or 220 L/min}}$$

11.28

$$\Delta Q_{\text{I}} = \frac{-(\pm W_1 \pm W_2) - 10}{G_1 + G_2}, \quad \Delta Q_{\text{II}} = \frac{-(\pm W_2 \pm W_3) - 15}{G_2 + G_3}$$

$$\pm W_i = \bar{R} Q_i |Q_i|, \quad G_i = 2\bar{R}|Q_i|$$

Loop	**Pipe**	***Q***	***W***	***G***	***Q***	***W***	***G***	***Q***
I	1	–3.5	–24.50	14.00	–2.46	–12.10	9.84	–2.30
	2	0	0	0	+0.36	+0.26	1.44	+0.43
			Σ 24.50	14.00		Σ –11.84	11.28	

$$\Delta Q_{\text{I}} = \frac{+24.50 - 10}{14.00} = +1.04 \qquad \Delta Q_{\text{I}} = \frac{+11.84 - 10}{11.28} = +0.16$$

II	2	0	0	0	– 0.36	– 0.26	1.44	– 0.43
	3	– 3.5	– 24.5	14.00	– 2.82	– 15.90	11.28	+ 2.73
			Σ –24.50	14.00		Σ –16.16	12.72	

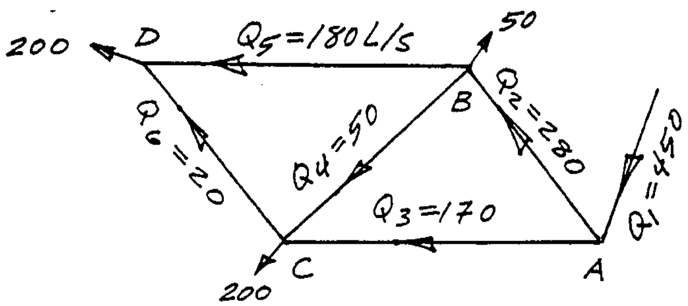
$$\Delta Q_{\text{II}} = \frac{+24.50 - 15}{14.00} = +0.68 \qquad \Delta Q_{\text{II}} = \frac{+16.16 - 15}{12.72} = +0.09$$

$$H_{\text{Junction}} = 50 - \bar{R}_1 Q_1^2 = 50 - 2\times 2.3^2 = \underline{39.4}$$

$$\therefore \underline{Q_1 = 2.30} \text{ (into } J\text{)}, \quad \underline{Q_2 = 0.43} \text{ (into } J\text{)}, \quad \underline{Q_3 = 2.73} \text{ (out of } J\text{)}$$

11.30

First, continuity is satisfied as shown in the figure:



(a) The hydraulic grade lines at the nodes (i.e., the nodal piezometric heads can now be computed:

$$H_A = 100 - \bar{R}_1 Q_1^2 = 100 - 20 \times 0.45^2 = \underline{95.95 \text{ m}}$$

$$H_B = H_A - \bar{R}_2 Q_2^2 = 95.95 - 51 \times 0.28^2 = \underline{91.95 \text{ m}}$$

$$H_C = H_A - \bar{R}_3 Q_3^2 = 95.95 - 280 \times 0.17^2 = \underline{87.86 \text{ m}}$$

$$H_D = H_B - \bar{R}_5 Q_5^2 = 91.95 - 310 \times 0.18^2 = \underline{81.91 \text{ m}}$$

(b) $p_A = \gamma(H_A - z_A) = 9810(95.95 - 10) = \underline{8.43 \times 10^5 \text{ Pa}}$

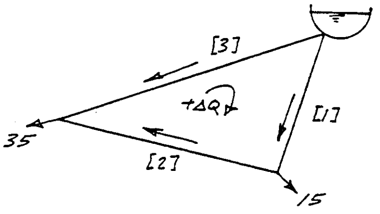
$$p_B = \gamma(H_B - z_B) = 9810(91.95 - 20) = \underline{7.06 \times 10^5 \text{ Pa}}$$

$$p_C = \gamma(H_C - z_C) = 9810(87.86 - 5) = \underline{8.13 \times 10^5 \text{ Pa}}$$

$$p_D = \gamma(H_D - z_D) = 9810(81.91 - 0) = \underline{8.04 \times 10^5 \text{ Pa}}$$

11.32

Assume flow directions as shown.



Then $\Delta Q = \dfrac{-(\bar{R}_1 Q_1^2 + \bar{R}_2 Q_2^2 - \bar{R}_3 Q_3^2)}{2(\bar{R}_1 Q_1 + \bar{R}_2 Q_2 + \bar{R}_3 Q_3)}$

Initial flow assumption:
$Q_1 = 25,\ Q_2 = 10,\ Q_3 = 25$

1st iteration: $\Delta Q = \dfrac{-(3 \times 25^2 + 5 \times 10^2 - 2 \times 25^2)}{2(3 \times 25 + 5 \times 10 + 2 \times 25)} = -3.21$

$\therefore Q_1 = 25 - 3.21 = 21.79,\ Q_2 = 10 - 3.21 = 6.79,\ Q_3 = 25 + 3.21 = 28.21$

2^{nd} iteration: $\Delta Q = \dfrac{-(3 \times 21.79^2 + 5 \times 6.79^2 - 2 \times 28.21^2)}{2(3 \times 21.79 + 5 \times 6.79 + 2 \times 28.21)} = -0.20$

$\therefore Q_1 = 21.79 - 0.20 = 21.59, \quad Q_2 = 6.79 - 0.20 = 6.59,$

$Q_3 = 28.21 + 0.20 = 28.41$

3rd iteration: $\Delta Q = \dfrac{-(3\times 21.59^2 + 5\times 6.59^2 - 2\times 28.41^2)}{2(3\times 21.59 + 5\times 6.59 + 2\times 28.41)} = -0.0041$

$\therefore \underline{Q_1 \cong 21.6}, \quad \underline{Q_2 \cong 6.6}, \quad \underline{Q_3 \cong 28.4}.$ Units are L/s

11.34

A Mathcad solution is provided below. The solution converges after 3 iterations.

Given: ORIGIN := 1

$$L := \begin{pmatrix} 200 \\ 300 \\ 120 \end{pmatrix}\cdot m \qquad D := \begin{pmatrix} 1500 \\ 1000 \\ 1200 \end{pmatrix}\cdot mm \qquad K := \begin{pmatrix} 2 \\ 0 \\ 10 \end{pmatrix} \qquad \gamma := 9810\cdot\frac{newton}{m^3}$$

$$\Delta Z := 50\cdot m \qquad e := 1\cdot mm \qquad W_P := 1920\cdot kW \qquad \eta := 0.82$$

Resistance coefficients: $i := 1..3$

$$f_i := 1.325\cdot\left(\ln\left(0.27\cdot\frac{e}{D_i}\right)\right)^{-2} \qquad f = \begin{pmatrix} 0.018 \\ 0.02 \\ 0.019 \end{pmatrix}$$

$$R_i := \frac{\left[8\cdot f_i\cdot\left(L_i + \frac{D_i\cdot K_i}{f_i}\right)\right]}{g\cdot\pi^2\cdot(D_i)^5} \qquad R = \begin{pmatrix} 0.071 \\ 0.487 \\ 0.473 \end{pmatrix}\frac{s^2}{m^5}$$

Initial flow estimate (note that the discharge is common to all three lines): $Q_1 := 2\cdot\frac{m^3}{s}$

Hardy Cross iteration:

$$\Delta Q(Q) := \frac{-(R_1 + R_2 + R_3)\cdot Q\cdot|Q| + \frac{W_P\cdot\eta}{\gamma\cdot Q} - \Delta Z}{2\cdot(R_1 + R_2 + R_3)\cdot Q + \frac{W_P\cdot\eta}{\gamma\cdot Q^2}}$$

$$N := 4 \qquad j := 1..N \qquad Q_{j+1} := Q_j + \Delta Q(Q_j)$$

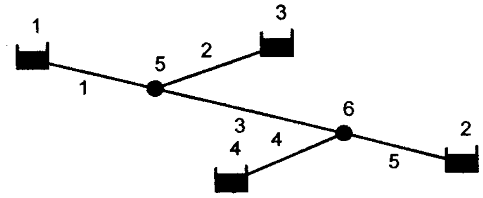
Solution: $Q = \begin{pmatrix} 2 \\ 2.59 \\ 2.762 \\ 2.771 \\ 2.771 \end{pmatrix}\frac{m^3}{s}$

Residual: $\Delta Q(Q_j) =$ $\frac{m^3}{s}$

0.59
0.172
$8.456\cdot10^{-3}$
$1.762\cdot10^{-5}$

11.36

The solution was obtained using EPANET, Version 2.0.



```
Link - Node Table:
----------------------------------------------------------------
Link           Start          End                 Length  Diameter
ID             Node           Node                   m        mm
----------------------------------------------------------------
1              1              5                     200       100
2              5              3                     150        50
3              5              6                     500       100
4              6              4                      35        50
5              6              2                     120       100

Node Results:
----------------------------------------------------------------
Node                  Demand      Head  Pressure   Quality
ID                       LPS         m         m
----------------------------------------------------------------
5                       0.00    121.72    121.72    0.00
6                       0.00    116.23    116.23    0.00
1                      -9.28    125.00      0.00    0.00 Reservoir
2                       7.15    115.00      0.00    0.00 Reservoir
3                       1.58    118.00      0.00    0.00 Reservoir
4                       0.55    116.00      0.00    0.00 Reservoir

Link Results:
----------------------------------------------------------------
Link                    Flow  Velocity  Headloss    Status
ID                       LPS       m/s      m/km
----------------------------------------------------------------
1                       9.28      1.18     16.40      Open
2                       1.58      0.81     24.80      Open
3                       7.69      0.98     10.98      Open
4                       0.55      0.28      6.55      Open
5                       7.15      0.91     10.24      Open
```

11.38

(c) $$\Delta Q=\frac{-(R_1+R_2+R_{3)})|Q|Q+(a_0+a_1Q+a_2Q^2+a_3Q^3)+z_A-z_B}{2(R_1+R_2+R_{3)})|Q|-(a_1+2a_2Q+3a_3Q^2)}$$

(d) $R_1=\frac{8f}{g\pi^2}\frac{L_1}{D_1^5}=0.00516,\quad R_2=\frac{8f}{g\pi^2}\frac{L_2}{N^2D_2^5}=2.538,\quad R_3=\frac{8f}{g\pi^2}\frac{L_3}{D_3^5}=0.0103$

Using the equation in part (c), Q = 2.061 m^3/s after 5 iterations, beginning with an initial estimate Q = 3 m^3/s.

Iteration	Q	ΔQ
1	3	– 0.611
2	2.389	– 0.277
3	2.113	– 0.050
4	2.062	– 0.0014
5	2.061	-5.1×10^{-13}

Unsteady Flow

11.44

$$V_{ss} = \sqrt{\frac{2g(H_1 - H_3)}{fL/D + K}} = \sqrt{\frac{2\times 9.81\times 3}{0.025\times 2500/0.1+0.15}} = 0.307 \text{ m/s}$$

$$V = 0.99V_{ss} = 0.99\times 0.307 = 0.304 \text{ m/s}$$

$$t = \frac{V_{ss}L}{2g(H_1 - H_3)}\ln\left[\frac{(V_{ss}+V)(V_{ss}-V_0)}{(V_{ss}-V)(V_{ss}+V_0)}\right]$$

$$= \frac{0.307\times 2500}{2\times 9.81\times 3}\ln\left[\frac{(0.307+0.304)(0.307-0)}{(0.307-0.304)(0.307+0)}\right] = \underline{69.0 \text{ s}}$$

11.46

First compute the initial velocity V_0, with H_1-H_3 = 8 m, and K_0 = 275:

$$V_0 = \sqrt{\frac{2\times 9.81\times 8}{\frac{0.015\times 800}{0.05}+275}} = 0.552 \text{ m/s}$$

The final steady-state velocity is, with K_{ss} = 5:

$$V_{SS} = \sqrt{\frac{2\times 9.81\times 8}{\frac{0.015\times 800}{0.05}+5}} = 0.800 \text{ m/s}$$

The steady-state discharge is $Q = 0.7854\times 0.05^2\times 0.800 = \underline{0.00157 \text{ m}^3/\text{s}}$, and the time to reach 95% of that value is

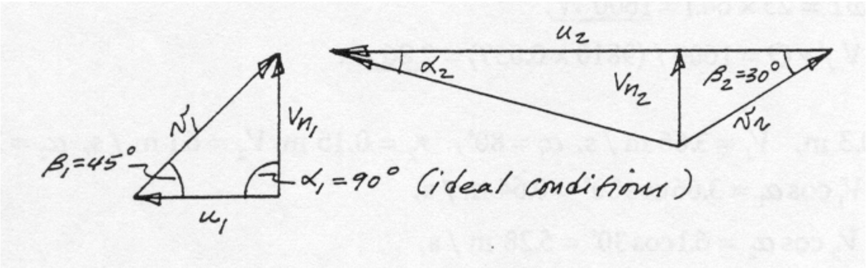
$$V = 0.95V_{ss} = 0.95\times 0.800 = 0.760 \text{ m/s}$$

$$\therefore t = \frac{0.800\times 800}{2\times 9.81\times 8}\ln\left[\frac{(0.800+0.760)(0.800-0.552)}{(0.800-0.760)(0.800+0.552)}\right] = \underline{8.03 \text{ s}}$$

CHAPTER 12

Turbomachinery

Elementary Theory

12.2	 $\omega = 800\times\pi/30 = 83.8$ rad/s $u_1 = \omega r_1 = 83.8\times 0.04 = 3.35$ m/s $u_2 = \omega r_2 = 83.8\times 0.125 = 10.48$ m/s $Q = 2\pi r_1 b_1 V_{n_1}$, but $V_{n_1} = u_1$ since $\beta_1 = 45^\circ$ $\therefore Q = 2\pi\times 0.04\times 0.05\times 3.35 = \underline{0.0421\ \text{m}^3/\text{s}}$ $V_{n_2} = \dfrac{Q}{2\pi r_2 b_2} = \dfrac{0.0421}{2\pi\times 0.125\times 0.025} = 2.14$ m/s $V_{t_2} = u_2 - \dfrac{V_{n_2}}{\tan\beta_2} = 10.48 - \dfrac{2.14}{\tan 30^\circ} = 6.77$ m/s $V_{t_1} = 0 \quad (\alpha_1 = 90^\circ$ under ideal conditions) $\therefore T = \rho Q\left(r_2 V_{t_2} - r_1 V_{t_1}\right) = 1000\times 0.0421(0.125\times 6.77 - 0) = \underline{35.6\ \text{N}\cdot\text{m}}$ $\dot{W}_P = \omega T = 83.8\times 35.6 = \underline{2980\ \text{W}}$ $H_t = \omega T/\gamma Q = 2980/(9810\times 0.0421) = \underline{7.22\ \text{m}}$
12.4	$Q = 7.6\times 10^{-3}\ \text{m}^3/\text{s}$ $\omega = 2000\times\dfrac{\pi}{30} = 209$ rad/s

	$V_{n_2} = \frac{Q}{2\pi r_2 b_2} = \frac{7.6\times10^{-3}}{2\pi\times0.0625\times0.01} = 1.94 \text{ m/s}$ $u_2 = \omega r_2 = 209\times0.0625 = 13.06 \text{ m/s}$ $\therefore H_t = \frac{u_2}{g}(u_2 - V_{n_2}\cot\beta_2) = \frac{13.06}{9.81}(13.06 - 1.94\cot 60°) = \underline{15.9 \text{ m}}$ $\dot{W} = \gamma Q H_t = 9810\times0.8\times7.6\times10^{-3}\times15.9 = \underline{948 \text{ W}}$ $\dot{W} = 948/746 = \underline{1.21 \text{ hp}}$
12.6	Compute loss in suction pipe: $h_L = \left(f\frac{L}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$ $= \left(0.015\times\frac{11}{0.1} + 2\times0.19 + 0.8\right)\frac{0.05^2}{2\times9.81\times\left(\frac{\pi}{4}\right)^2\times0.1^4} = 5.85 \text{ m}$ Water at 20°C: $\gamma = 9792 \text{ N/m}^3$, $p_v = 2340$ Pa. Substitute known data into NPSH relation, solving for Δz: $\therefore \Delta z = \frac{p_{\text{atm}} - p_v}{\gamma} - h_L - \text{NPSH} = \frac{101\times10^3 - 2340}{9792} - 5.85 - 3 = \underline{1.23 \text{ m}}$

Dimensional Analysis and Similitude

12.8	$Q_1 = \frac{\omega_1}{\omega_2}Q_2 = \frac{N_1}{N_2}Q_2 = \frac{970}{1200}\times0.8 = 0.65 \text{ m}^3/\text{s}$ $\therefore$ from Fig. 12.9, $H_1 \cong 11.5$ m and $\dot{W}_1 \cong 91$ kW. $\therefore H_2 = \left(\frac{\omega_2}{\omega_1}\right)^2 H_1 = \left(\frac{1200}{970}\right)^2\times11.5 = \underline{17.6 \text{ m}}$ $\therefore \dot{\text{W}}_2 = \left(\frac{\omega_2}{\omega_1}\right)^3 \dot{\text{W}}_1 = \left(\frac{1200}{970}\right)^3\times91 = \underline{172 \text{ kW}}$
12.10	$C_Q = \frac{Q}{\omega D^3} = \frac{Q/3600}{304\times0.205^3} = 1.061\times10^{-4}Q \qquad (Q \text{ in m}^3/\text{h})$

$$C_H = \frac{gH}{\omega^2 D^2} = \frac{9.81H}{304^2 \times 0.205^2} = 2.526\times 10^{-3} H \quad (H \text{ in m})$$

Tabulate C_Q and C_H using selected values of Q and H from Fig. 12.6:

Q (m^3/h)	$C_Q\times10^{-3}$	H (m)	$C_H\times10^{-1}$
0	0	54	1.36
50	0.53	53	1.34
100	1.06	52	1.31
150	1.59	50	1.26
200	2.12	47	1.19
250	2.65	41	1.04
300	3.18	33	0.83

The dimensionless curve shown in Fig. 12.12 is for the 240-mm impeller. Since the impellers are not the same (240 mm versus 205 mm), dynamic similitude does not exist, and thus the curves are not the same.

12.12

Compute the specific speed: $\Omega_P = \dfrac{\omega\sqrt{Q}}{(gH_P)^{3/4}} = \dfrac{1800\times\frac{\pi}{30}\times\sqrt{0.15}}{(9.81\times 22)^{3/4}} = \underline{1.30}$,

hence use a <u>mixed flow pump</u>. As an alternate, since Ω_P is close to unity, a radial flow pump could be employed.

12.14

Fig. 12.13: At $\eta = 0.75$ (best eff.), $C_Q \cong 0.048$,

$$C_H \cong 0.018,\ C_{\dot W} \cong 0.0011,\ C_{\text{NPSH}} \cong 0.023.$$

$$\omega = 750\times\frac{\pi}{30} = 78.5 \text{ rad/s}$$

(a) $$D = \left(\frac{1240\times10^{-3}}{0.049\times 78.5}\right)^{1/3} = \underline{0.685 \text{ m}}$$

$$H = \frac{0.018\times78.5^2\times0.685^2}{32.2} = \underline{5.3 \text{ m}}$$

$$H_{\text{NPSH}} = \frac{0.023\times78.5^2\times0.685^2}{9.81} = \underline{6.78 \text{ m}}$$

$$\dot W = 0.0011\times1000\times78.5^3\times0.685^5 = 80,250 \text{ W} = 80.25 \text{ kW}$$

or $$\dot W = 82.25/0.746 = \underline{107.6 \text{ hp}}$$

12.16

$$\omega = 600\times\frac{\pi}{30} = 62.8 \text{ rad / s},\quad Q = 22.7 / 60 = 0.378 \text{ m}^3 \text{ / s},$$

$$\Omega_P = \frac{\omega\sqrt{Q}}{(gH_P)^{3/4}} = \frac{62.8\sqrt{0.378}}{(9.81\times 19.5)^{3/4}} = \underline{0.751} \qquad \therefore \text{Use a } \underline{\text{radial flow pump}}.$$

12.18

$$\frac{H_2}{H_1} = \left(\frac{\omega_2}{\omega_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 = 2 \qquad \frac{Q_2}{Q_1} = \left(\frac{\omega_2}{\omega_1}\right)\left(\frac{D_2}{D_1}\right)^3 = 2$$

$$\therefore \left(\frac{\omega_2}{\omega_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{\omega_2}{\omega_1}\right)\left(\frac{D_2}{D_1}\right)^3 \quad \text{or} \quad \frac{\omega_2}{\omega_1} = \frac{D_2}{D_1}$$

$$\therefore \left(\frac{\omega_2}{\omega_1}\right)^4 = 2, \quad \omega_2 = \sqrt[4]{2}\ \omega_1 = \underline{1.19\ \omega_1} \quad \text{and} \quad D_2 = \sqrt[4]{2}\ D_1 = \underline{1.19\ D_1}$$

12.20

Assume a pump speed N = 2000 rpm or $\omega = 2000 \times \frac{\pi}{30} = 209$ rad/s

$$H_P = \dot{W}_f/(\gamma Q) = 200\times 10^3/(8830\times 0.66) = 34.3 \text{ m}$$

$$\therefore \Omega_P = \frac{\omega\sqrt{Q}}{(gH_P)^{3/4}} = \frac{209\sqrt{0.66}}{(9.81\times 34.3)^{3/4}} = 2.16$$

The specific speed suggests a mixed-flow pump. However, if N = 1000 rpm, a radial-flow pump may be appropriate. Consider both possibilities.

<u>Mixed flow:</u> from Fig. 12.14, at best η:

$$C_{\dot{W}} \cong 0.0117, \quad C_Q \cong 0.148, \quad C_H \cong 0.067$$

Use

$$C_Q = \frac{Q}{\omega D^3} \quad \text{and} \quad C_H = \frac{gH_P}{\omega^2 D^2}$$

Combining and solving for D and ω

$$D = \sqrt{\frac{Q/C_Q}{\sqrt{gH_P/C_H}}} \quad \text{and} \quad \omega = \frac{Q}{C_Q D^3}$$

$$\therefore D = \sqrt{\frac{0.66/0.148}{\sqrt{9.81\times 34.3/0.067}}} = \underline{0.251 \text{ m}}$$

$$\omega = \frac{0.66}{0.148\times 0.251^3} = \underline{282 \text{ rad/s}} \quad \text{or} \quad N = 282\times 30/\pi = \underline{2674 \text{ rpm}}$$

$$\rho = \frac{\gamma}{g} = \frac{8830}{9.81} = 900 \text{ kg/m}^3$$

$$\therefore \dot{W}_P = C_{\dot{W}} \rho \omega^3 D^5 = 0.0117 \times 900 \times 282^3 \times 0.251^5$$

$$= \underline{2.35 \times 10^5 \text{ W}} \text{ or } \underline{235 \text{ kW}}$$

Radial flow: from Fig. 12.12, at best η:

$C_{\dot{W}} \cong 0.027, \quad C_Q \cong 0.0165, \quad C_H \cong 0.125$

$$\therefore D = \sqrt{\frac{0.66/0.0165}{\sqrt{9.81 \times 34.3/0.125}}} = \underline{0.878 \text{ m}}$$

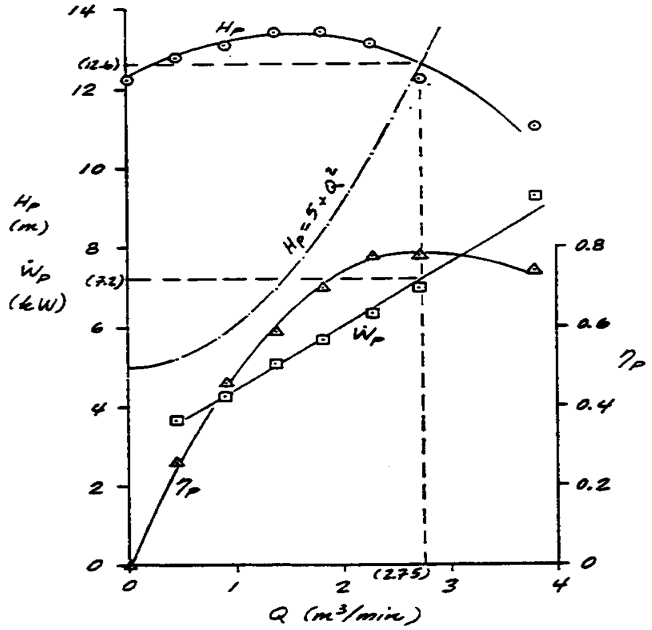
$$\omega = \frac{0.66}{0.0165 \times 0.878^3} = \underline{59.1 \text{ rad/s}} \quad \text{or } N = \frac{59.1 \times 30}{\pi} = \underline{564 \text{ rpm}}$$

$$\dot{W}_P = 0.0027 \times 900 \times 59.1^3 \times 0.878^5 = \underline{2.62 \times 10^5 \text{ W}} \quad \text{or} \quad \underline{262 \text{ kW}}$$

Hence, a mixed-flow pump is preferred.

Use of Turbopumps

12.22

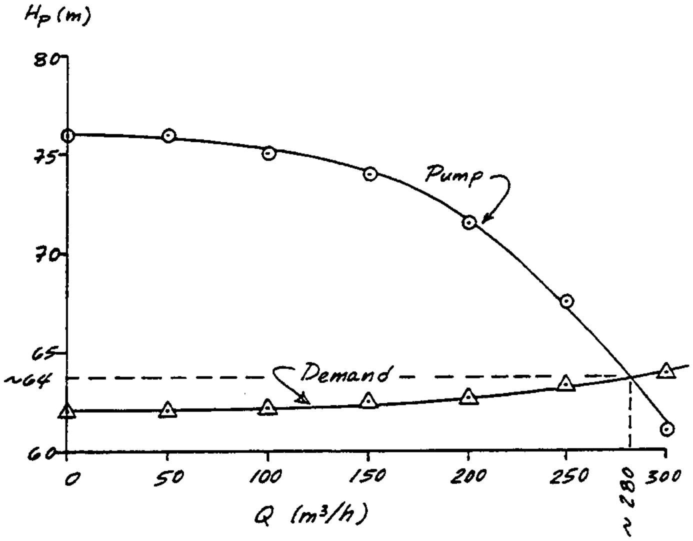


The intersection of the system demand curve with the head-discharge curve yields $\underline{Q = 2.75 \text{ m}^3/\text{min}}$, $\underline{H_P = 12.6 \text{ m}}$, $\underline{\dot{W}_P = 7.2 \text{ kW}}$.

12.24

$N_1 = 1350$ rpm, $Q_1 = 2.75$ m^3 / min, $H_1 = 12.6$ m,

$\dot{W}_1 = 7.2$ kW, $N_2 = 1200$ rpm, $D_2 = D_1$

	$\therefore Q_2 = Q_1\left(\frac{N_2}{N_1}\right)\left(\frac{D_2}{D_1}\right)^3 = 2.75\left(\frac{1200}{1350}\right) = \underline{2.44\text{ m}^3\text{/min}}$ $H_2 = H_1\left(\frac{N_2}{N_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 = 12.6\left(\frac{1200}{1350}\right)^2 = \underline{9.96\text{ m}}$ $\dot{W}_2 = \dot{W}_1\left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 = 7.2\left(\frac{1200}{1350}\right)^3 = \underline{5.06\text{ kW}}$ Efficiency will remain approximately the same.
12.26	Compute system demand: $H_P = \left(f\frac{L}{D} + \Sigma K\right)\frac{V^2}{2g} = \left(0.01\times\frac{14}{0.3} + 4\times 0.1\right)\frac{3^2}{2\times 9.81} = 0.40\text{ m}$ $Q = VA = 3\times\frac{\pi}{4}\times 0.3^2 = \underline{0.212\text{ m}^2}$ $\omega = 300\times\pi/30 = 31.4\text{ rad/s}$ $\therefore \Omega_P = \frac{\omega\sqrt{Q}}{(gH_P)^{3/4}} = \frac{31.4\sqrt{0.212}}{(9.81\times 0.4)^{3/4}} = 5.19$ $\therefore$ $\underline{\text{Axial pump}}$ is appropriate.
12.28	(a)  The intersection of the pump curve with the demand curve yields $H_P \cong \underline{64\text{ m}}$ and $Q \cong \underline{280\text{ m}^3/\text{h}}$. $\therefore$ From Fig. 12.6, $\dot{W}_P \cong \underline{64\text{ kW}}$ and NPSH $\cong$ $\underline{8.3\text{ m}}$

12.30

Use energy eqn. to establish system demand:

$$f \cong 1.325\left[\ln 0.27\frac{e}{D}\right]^{-2} = 1.325\left[\ln 0.27\times\frac{0.000255}{0.45}\right]^{-2} = 0.017$$

$$H_P = \Delta z + f\frac{L}{D}\frac{Q^2}{2gA^2} = 192 + \frac{0.017\times 5000}{2\times 9.81\times(\pi/4)^2\times(0.45)^5}Q^2 = 192 + 381Q^2$$

From Fig. 12.6, at best η_P, $Q \cong \frac{240\text{ m}^3}{3600\text{ s}} = \underline{0.067\text{ m}^3\text{/s}}$, and $H_P \cong 65\text{ m}$

Assume three pumps in series, so that $H_P = 3\times 65 = 195\text{ m}$. Then the demand head is

$$H_P = 192 + 381\times(0.067)^2 = \underline{193.7\text{ m}}$$

Hence <u>three pumps in</u> <u>series</u> are appropriate. The required power is

$$\dot{W}_P = \gamma Q H_P/\eta_P = 9810\times 0.86\times 0.067\times 195/0.75 = 146{,}965\text{ W}$$

or $\dot{W}_P = 146{,}965/746 = \underline{197\text{ hp}}$

12.32

(a) For water at 80°C, $p_v = 46.4\times 10^3$ Pa, and $\rho = 9553\text{ kg/m}^3$. Write the energy equation from the inlet (section *i*) to the location of cavitation in the pump:

$$\frac{p_i}{\gamma} + \frac{V_i^2}{2g} = \frac{p_v}{\gamma} + NPSH$$

$$\therefore NPSH = \frac{p_i - p_v}{\gamma} + \frac{V_i^2}{2g} = \frac{(83-46.4)\times 10^3}{9533} + \frac{6^2}{2\times 9.81} = \underline{5.67\text{ m}}$$

(b) $NPSH_1 = 5.67$ m, $N_1 = 2400$ rpm, $N_2 = 1000$ rpm, $D_2/D_1 = 4$

$$\therefore NPSH_2 = NPSH_1\left(\frac{N_2}{N_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 = 5.67\left(\frac{1000}{2400}\right)^2(4)^2 = \underline{15.8\text{ m}}$$

12.34

Given:

$$L = 200\text{ m}, D = 0.05\text{ m}, \Delta z = z_2 - z_1 = -3\text{ m}, V = 3\text{ m/s}, \nu = 6\times 10^{-7}\text{ m}^2\text{/s},$$
$$\rho = 1593\text{ kg/m}^3, p_v = 86.2\times 10^3\text{ Pa}, p_a = 101\times 10^3\text{ Pa}$$

Compute the pump head:

$$\text{Re} = \frac{3 \times 0.05}{6 \times 10^{-7}} = 2.5 \times 10^5$$

$$f = 1.325\left[\ln\left(5.74\text{Re}^{-0.9}\right)\right]^{-2} = 1.325\left[\ln 5.74 \times (2.5 \times 10^5)^{-0.9}\right]^{-2} = 0.015$$

$$H_P = \Delta z + \left(1 + \frac{fL}{D}\right)\frac{V^2}{2g} = -3 + \left(1 + \frac{0.015 \times 200}{0.05}\right)\frac{3^2}{2 \times 9.81} = 25.0 \text{ m}$$

(a) Choose a radial-flow pump. Use Fig. P12.35 to select the size and speed:

$$C_H = 0.124, \;\; C_Q = 0.0165, \;\; \eta = 0.75$$

$$Q = 3 \times \frac{\pi}{4} \times 0.05^2 = 0.00589 \text{ m}^2$$

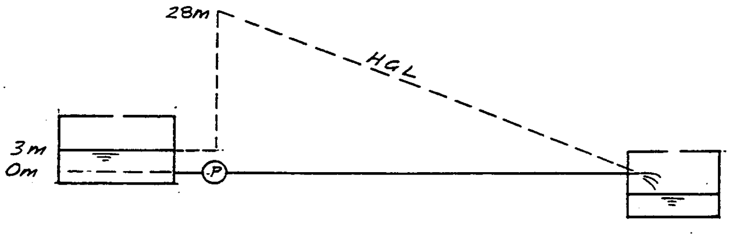
$$\therefore D = \sqrt[4]{\frac{C_H Q^2}{C_Q^2 g H_P}} = \sqrt[4]{\frac{0.124 \times 0.00589^2}{0.0165^2 \times 9.81 \times 25.0}} = \underline{0.090 \text{ m}}$$

$$\therefore \omega = \frac{Q}{C_Q D^3} = \frac{0.00589}{0.0165 \times 0.090^3} = 490 \text{ rad/s, or } \; N = 490 \times \frac{30}{\pi} = \underline{4680 \text{ rpm}}$$

(b) Available net positive suction head:

$$NPSH = \frac{p_a - p_v}{\rho g} - \Delta z = \frac{101 \times 10^3 - 86.2 \times 10^3}{1593 \times 9.81} + 3 = \underline{3.95 \text{ m}}$$

(c)



Turbines

12.36	$\omega = 120\times\pi/30 = 12.6$ rad/s $\alpha_1 = \cot^{-1}(2\pi r_1^2 b_1 \omega/Q + \cot\beta_1)$ $= \cot^{-1}(2\pi\times 4.5^2\times 0.85\times 12.6/150 + \cot 75^\circ) = \underline{6.1^\circ}$ $V_{t_1} = u_1 + V_{n_1}\cot\beta_1 = \omega r_1 + \dfrac{Q}{2\pi r_1 b_1}\cot\beta_1$ $= 12.6\times 4.5 + \dfrac{150}{2\pi\times 4.5\times 0.85}\cot 75^\circ = 58.37\ \text{m/s}$ $V_{t_2} = u_2 + V_{n_2}\cot\beta_2 = \omega r_2 + \dfrac{Q}{2\pi r_2 b_2}\cot\beta_2$ $= 12.6\times 2.5 + \dfrac{150}{2\pi\times 2.5\times 0.85}\cot 100^\circ = 29.52\ \text{m/s}$ $\therefore T = \rho Q(r_1 V_{t_1} - r_2 V_{t_2})$ $= 1000\times 150(4.5\times 58.37 - 2.5\times 29.52) = \underline{2.83\times 10^7\ \text{N}\cdot\text{m}}$ $\dot{W}_T = \omega T = 12.6\times 2.83\times 10^7 = \underline{3.57\times 10^8\ \text{W}}$ or $\underline{357\ \text{MW}}$ Under ideal conditions $\eta_T = 1$, and $\dot{W}_T = \dot{W}_f$, hence $H_T = \dot{W}_T/\gamma Q = 3.57\times 10^8/(9810\times 150) = \underline{243\ \text{m}}$
12.38	$N_2 = 240$ rpm, $\dot{W}_2 = 2200$ kW, $D_2 = 0.9$ m, $\dot{W}_1 = 9$kW, $H_1 = 7.5$ m From the similarity rules $\dfrac{\dot{W}_2}{\dot{W}_1} = \left(\dfrac{N_2}{N_1}\right)^3\left(\dfrac{D_2}{D_1}\right)^5$ and $\dfrac{H_2}{H_1} = \left(\dfrac{N_2}{N_1}\right)^2\left(\dfrac{D_2}{D_1}\right)^2$ Substitute second eqn. into the first to eliminate (N_2/N_1), and solve for D_1: $\therefore D_1 = D_2\left(\dfrac{\dot{W}_1}{\dot{W}_2}\right)^{1/2}\left(\dfrac{H_2}{H_1}\right)^{3/4} = 0.9\left(\dfrac{9}{2200}\right)^{1/2}\left(\dfrac{45}{7.5}\right)^{3/4} = \underline{0.221\ \text{m}}$ $N_2 = N_1\left(\dfrac{H_1}{H_2}\right)^{1/2}\left(\dfrac{D_2}{D_1}\right) = 240\left(\dfrac{7.5}{45}\right)^{1/2}\left(\dfrac{0.9}{0.221}\right) = \underline{399\ \text{rpm}}$

12.40

Write energy eqn. from upper reservoir (loc. 1) to surge tank (loc. 2) and solve for *Q*:

$$Q=\left[2gA^2\frac{D}{f\,L}(z_1-z_2)\right]^{1/2}$$

$$=\left[\frac{2\times 9.81\times(\pi/4)^2\times 0.85^5}{0.025\times 2000}(650-648.5)\right]^{1/2}=\underline{0.401\text{ m}^3/\text{s}}$$

Apply energy eqn. from loc. 2 to lower reservoir (loc. 3) and determine H_T:

$$H_T=z_2-z_3-\left(f\frac{L}{D}+K_v\right)\frac{Q^2}{2gA^2}$$

$$=648.5-595-\left(\frac{0.02\times 100}{0.85}+1\right)\frac{0.401^2}{2\times 9.81\times\left(\frac{\pi}{4}\right)^2\times 0.85^5}=\underline{53.4\text{ m}}$$

$$\therefore \dot{W}_T=\gamma Q H_T\eta_T=9810\times 0.401\times 53.4\times 0.9=\underline{1.89\times 10^5\text{ W}}\text{ or }\underline{189\text{ kW}}$$

∴ From Fig. 12.32, use a <u>Francis turbine</u>.

A representative value of the specific speed is 2 (Fig. 12.20):

$$\therefore \omega=\frac{\Omega_T(gH_T)^{5/4}}{(\dot{W}_T/\rho)^{1/2}}=\frac{2(9.81\times 53.3)^{5/4}}{(2.67\times 10^5/1000)^{1/2}}=\underline{306\text{ rad/s}}$$

or $N=306\times 30/\pi=\underline{2920\text{ rpm}}$

12.42

$\omega=200\times\pi/30=20.94\text{ rad/s}$ and from Fig. P12.42 at best efficiency

(η_T = 0.8), $\phi\approx 0.42$, $c_v = 0.94$

$$V_1=c_v\sqrt{2gH_T}=0.94\sqrt{2\times 9.81\times 120}=45.6\text{ m/s}$$

$$\therefore Q=\frac{\dot{W}_T}{\gamma H_T\eta_T}=\frac{4.5\times 10^6}{9810\times 120\times 0.8}=\underline{4.78\text{ m}^3/\text{s}}$$

This is the discharge from all of the jets.

Determine the wheel radius *r* : $r=\dfrac{\phi\sqrt{2gH_T}}{\omega}=\dfrac{0.42\sqrt{2\times 9.81\times 120}}{20.94}=0.973\text{ m}$

Hence, the diameter of the wheel is $2r=2\times 0.973=\underline{1.95\text{ m}}$

Compute diameter of one jet: $D_j=2r/8=1.95/8=\underline{0.244\text{ m}}$, or $\underline{244\text{ mm}}$

Let N_j = no. of jets. Then each jet has a discharge of Q/N_j and an area

$$\frac{Q/N_j}{V_1}=\frac{\pi}{4}D_j^2 \qquad \therefore \text{Solving for } N_j:$$

$$N_j=\frac{Q}{V_1}\frac{1}{\left(\frac{\pi}{4}D_j^2\right)}=\frac{4.78}{45.6\times\left(\frac{\pi}{4}\right)\times 0.244^2}=\underline{2.24}$$

$\therefore$ Use <u>three</u> jets $\qquad \Omega_T=\frac{\omega(\dot{W}_T/\rho)^{1/2}}{(gH_T)^{5/4}}=\frac{20.94(4.5\times10^6/1000)^{1/2}}{(9.81\times120)^{5/4}}=\underline{0.204}$

12.44

$Q=2082/6=347\ \text{m}^3/\text{sec}$ (one unit)

$$H_T=\frac{\dot{W}_T}{\gamma Q\eta_T}=\frac{427,300\times746}{9810\times347\times0.85}=110.1\ \text{m}$$

Write energy eqn. from upper reservoir (loc. 1) to lake (loc. 2):

$$z_1=\left(f\frac{L}{D}+\Sigma K\right)\frac{Q^2}{2gA^2}+H_T+z_2$$

$$\therefore 309=\left(\frac{0.01\times390}{D}+0.5\right)\frac{347^2}{2\times9.81\times\left(\frac{\pi}{4}\right)^2D^4}+110.1+196.5$$

which reduces to $\quad 2.4-\frac{38{,}840}{D^5}-\frac{4980}{D^4}=0$

$\therefore$ Solving, $\underline{D\cong 8\ \text{m}}$

$\therefore$ From Fig. 12.32, a <u>Francis or pump/turbine</u> unit is indicated.

12.46

(a) Let H be the total head and Q the discharge delivered to the turbine; then

$$H_T=0.95H=0.95\times305=289.8\ \text{m}$$

and $\quad Q=\frac{\dot{W}_T}{\gamma H_T\eta_T}=\frac{10.4\times10^6}{9810\times289.8\times0.85}=4.30\ \text{m}^3/\text{s}.$

Write energy eqn. from reservoir to turbine outlet:

$$H=H_T+\left(f\frac{L}{D}+\Sigma K\right)\frac{Q^2}{2gA^2}$$

$$\therefore 305=289.8+\left(\frac{0.02\times3000}{D}+2\right)\frac{4.3^2}{2\times9.81\times(\pi/4)^2D^4}$$

	which reduces to $15.2-\frac{91.67}{D^5}-\frac{3.06}{D^4}=0$ $\therefore$ Solving $\underline{D = 1.45\text{ m}}$ **(b)** Compute jet velocity: $V_1=c_v\sqrt{2gH_T}=0.98\sqrt{2\times9.81\times289.8}=73.9\text{ m/s}$ The flow through one nozzle is $Q/4$ and the jet area is $\pi D_j^2/4$. Hence $\frac{\pi D_j^2}{4}=\frac{Q/4}{V_1}=\frac{4.3/4}{73.9}=0.0146\text{ m}^2$ $\therefore D_j=\sqrt{\frac{4\times0.0146}{\pi}}=\underline{0.136\text{ m}}$
12.48	$\dot{W}_T=1000\left[\frac{4.15\times(9.81\times3.7)^{5/4}}{50\times\pi/30}\right]^2=4.99\times10^6$ W (one unit). Total power developed is 9.21×10^6 W. Hence, required number of units is $9.21/4.99=1.8$ $\therefore$ Use two turbines.
12.50	**(a)** $H_T=z_1-z_2-\frac{fL}{D}\frac{Q^2}{2gA^2}$ $=915-892-\frac{0.015\times350\times0.25^2}{0.3\times2\times9.81\times(0.7854\times0.3^2)^2}=11.8\text{ m}$ $\dot{W}_T=\gamma QH_T\eta_T=9810\times0.25\times11.8\times0.85=2.46\times10^4$ W or $\underline{24.6\text{ kW}}$ **(b)** Compute the specific speed of the turbine: $\omega=N\frac{\pi}{30}=1200\times\frac{\pi}{30}=126\text{ rad/s}$ $\therefore\Omega_T=\frac{\omega\sqrt{\dot{W}_T/\rho}}{(gH_T)^{5/4}}=\frac{126\sqrt{2.46\times10^4/1000}}{(9.81\times11.8)^{5/4}}=1.65$ Hence, from Fig. 12.20, a Francis turbine is appropriate. **(c)** From Fig. 12.24, the turbine with Ω_T = 1.063 is chosen: C_H = 0.23, C_Q = 0.13, and η_T = 0.91.

$$D = \sqrt[4]{\frac{C_H Q^2}{C_Q^2 g H_T}} = \sqrt[4]{\frac{0.23\times 0.25^2}{0.13^2\times 9.81\times 11.8}} = 0.293 \text{ m or approximately } \underline{0.30 \text{ m}}$$

$$\omega = \frac{Q}{C_Q D^3} = \frac{0.25}{0.13\times 0.30^3} = 71.2 \text{ rad/s or } N = 71.2\times\frac{30}{\pi} = \underline{680 \text{ rpm}}$$

$$\dot{W}_T = \gamma Q H_T \eta_T = 9810\times 0.25\times 11.8\times 0.91 = 2.63\times 10^4 \text{ W, or } \underline{26.3 \text{ kW}}$$

Calculate a new specific speed based on the final design data:

$$\Omega_T = \frac{71.2\times\sqrt{2.63\times 10^4/1000}}{(9.81\times 11.8)^{5/4}} = \underline{0.96}$$

An acceptable value according to Fig. 12.20.

CHAPTER 13

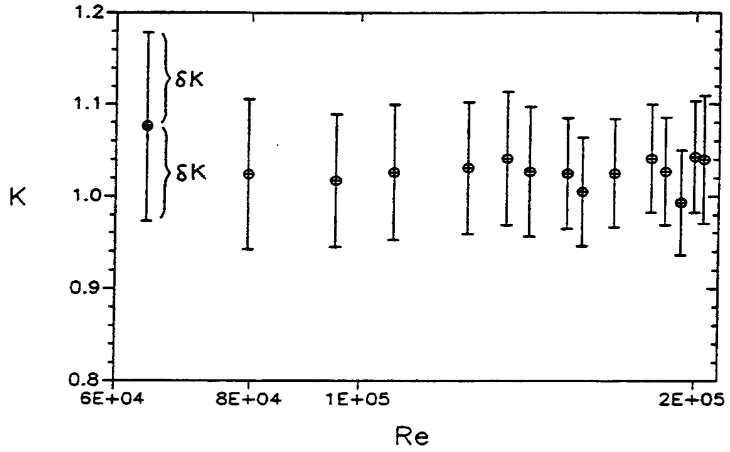
Measurements in Fluid Mechanics

Introduction

13.2	**a)** $\frac{V^2}{2}\times\rho = p = \frac{\gamma_{Hg}h}{100} \quad \therefore \frac{V^2}{2}\times 1.22 = \left(9810\times 13.6\right)\frac{h}{100}$ $\therefore V = 46.8\sqrt{h}$ and $\underline{C = 46.8}$ **b)** $\frac{V^2}{2}\times(0.8217\times 1.22) = (9810\times 13.6)\frac{h}{100} \quad \therefore V = 51.6\sqrt{h}$ and $\underline{C = 51.6}$
13.4	$Q = \frac{\cancel{V}}{\Delta t} = \frac{1.9\times 10^{-3}}{10\times 60} = \underline{3.2\times 10^{-6}\ \text{m}^3/\text{s}}$ $\dot{m} = \rho Q = 997\times 3.2\times 10^{-6} = 3.19\times 10^{-3}\ \text{kg/s}$ $V = \frac{Q}{A} = \frac{3.2\times 10^{-6}}{(\pi\times(0.0025)^2/4)} = \underline{0.65\ \text{m/s}} \quad \therefore \text{Re} = \frac{0.65\times 0.0025}{9\times 10^{-7}} = \underline{1805}$ The flow is <u>laminar</u>.
13.6	$Q = \left[\pi\times 1^2\times 10 + \pi(2^2 - 1^2)\times 9.9 + \pi(3^2 - 2^2)\times 9.5 + \pi(4^2 - 3^2)\times 8.65 + \right.$ $\left. \pi(4.5^2 - 4^2)\times 6.65 + \pi(5^2 - 4.5^2)\times 2.6\right]\times 10^{-4} = \underline{0.0592\ \text{m}^3/\text{s}}$ $V = \frac{Q}{A} = \frac{0.0592}{\pi\times 0.05^2} = \underline{7.54\ \text{m/s}}$
13.8	See the sketch above: $\frac{p_1 - p_2}{\gamma_{Hg} - \gamma} = H. \quad \therefore p_1 - p_2 = 9810(13.6 - 1)\times 0.12 = 14{,}830\ \text{Pa.}$ $\therefore h_1 - h_2 = \frac{p_1 - p_2}{9810} = \frac{14{,}830}{9810} = 1.512\ \text{m} \qquad \frac{D_0}{D} = \frac{15}{24} = 0.625$

	a) Assume: $\text{Re}=10^5 \quad \therefore K=0.68 \quad \therefore Q=0.68\pi\times0.075^2\sqrt{2\times9.81\times1.512}=0.0654\text{ m}^3/\text{s}$ Check: $V=\frac{Q}{A}=1.45\text{ m/s} \quad \text{Re}=\frac{1.45\times0.24}{10^{-6}}=3.5\times10^5$ $\therefore K=0.67$ and $\underline{Q=0.064\text{ m}^3/\text{s}}$
13.10	$Q=KA\sqrt{\frac{R\Delta p}{D\rho}}$ where $K=1-\frac{6.5}{\sqrt{\text{Re}}} \quad \text{Re}=\frac{VD}{\nu}=\frac{Q4}{\pi D\nu}$ **a)** Assume $\text{Re}=10^5$ Then $K=0.98 \quad Q=0.98\pi\times0.05^2\sqrt{\frac{0.2\times80{,}000}{0.1\times1000}}=\underline{0.0974\text{ m}^3/\text{s}}$ Check $V=\frac{Q}{A}=\frac{0.0974}{\pi\times0.05^2}=12.4\text{ m/s} \quad \therefore \text{Re}=\frac{12.4\times0.1}{10^{-6}}=1.2\times10^6 \quad \therefore K=0.99$ $\therefore$OK
13.12	$A_0=\frac{\pi}{4}\times\left(\frac{33.3}{1000}\right)^2=8.709\times10^{-4}\text{m}^2 \qquad \beta=33.3/54=0.617$ $K=\frac{Q}{A_0\sqrt{2g(S-1)\Delta h}}$ (Δh in meters of mercury) $=\frac{Q}{8.709\times10^{-4}\sqrt{2\times9.81\times(13.6-1)\Delta h}}=73.03\frac{Q}{\sqrt{\Delta h}}=K(Q,\Delta h)$ $\delta K=\sqrt{[K(Q,\Delta h)-K(Q+\delta Q,\Delta h)]^2+[K(Q,\Delta h)-K(Q,\Delta h+\delta(\Delta h))]^2}$

No.	$K(Q,\Delta h)$	$K(Q+\delta Q,\Delta h)$	$K(Q,\Delta h+\delta(\Delta h))$	δK
1	1.076	1.147	1.002	0.1026
2	1.024	1.093	0.9805	0.0816
3	1.017	1.082	0.9862	0.0719
4	1.026	1.095	1.001	0.0734
5	1.031	1.100	1.012	0.0716
6	1.041	1.112	1.025	0.0728
7	1.027	1.096	1.012	0.0706
8	1.025	1.084	1.013	0.0602
9	1.005	1.063	0.9938	0.0591
10	1.025	1.083	1.015	0.0589
11	1.041	1.099	1.032	0.0587
12	1.027	1.085	1.018	0.0587
13	1.043	1.103	1.036	0.0604
14	1.040	1.109	1.033	0.0694
15	0.9935	1.050	0.9864	0.0569



Compare the above with Fig. 13.10, curve labeled “Venturi meters and nozzles” β = 0.6.

13.14

i	$(S-1)\Delta h$ (m water)	$x_i^{(1)}$	$y_i^{(2)}$	$x_i y_i$	x_i^2
1	0.164	-1.8079	-6.3890	11.5507	3.2685
2	0.277	-1.2837	-6.1754	7.9274	1.6479
3	0.403	-0.9088	-5.9955	5.4487	0.8259
4	0.504	-0.6852	-5.8746	4.0253	0.4695
5	0.680	-0.3857	-5.7199	2.2062	0.1488
6	0.781	-0.2472	-5.6408	1.3944	0.06111
7	0.882	-0.1256	-5.5940	0.7026	0.01578
8	1.033	0.03247	-5.5165	-0.1791	0.001054
9	1.147	0.1371	-5.4846	-0.7519	0.01880
10	1.260	0.2311	-5.4171	-1.2519	0.05341
11	1.424	0.3535	-5.3412	-1.8881	0.1250
12	1.550	0.4383	-5.3124	-2.3284	0.1921
13	1.688	0.5235	-5.2533	-2.7501	0.2741
14	1.764	0.5676	-5.2344	-2.9710	0.3222
15	1.764	0.5676	-5.2805	-2.9972	0.3222
	Σ	-2.5929	-84.2292	+18.1376	+7.7464

(1) $x_i = \ln[(S-1)\Delta h]$ $S = 13.6$ (Δh in meters mercury) (2) $y_i = \ln Q$ (Q in m^3/s)

$$\therefore m = \frac{18.1376 - (-2.5929)(-84.2292)\ /\ 15}{7.7464 - (2.5929)^2\ /\ 15} = \underline{0.4902},$$

$$b = \frac{-84.2292 - 0.4902(-2.5929)}{15} = -5.5305 \qquad \therefore C = \exp(-5.5305) = \underline{0.00396}.$$

From Problem 13.12, $A_0 = 8.709 \times 10^{-4}\,\mathrm{m}^2$ and $K_{\text{avg}} = 1.029$. Hence in Eq. 13.3.8

$K_{\text{avg}} A_0 \sqrt{2g} = 1.029 \times (8.709 \times 10^{-4})\sqrt{2 \times 9.81} = 0.00397,$ and the exponent is $\underline{0.500}$.