Surveying for Engineers

J. Uren and W.F. Price

Second Edition

SURVEYING FOR ENGINEERS

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Surveying for Engineers

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Second Edition

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Contents

Preface to the Second Edition

In the seven years that have passed since the publication of the first edition of *Surveying for Engineers* there have been several developments in engineering surveying, particularly in Electromagnetic Distance Measurement (EDM), microcomputers, data processing and in the acceptance of the need for a greater degree of control when setting out engineering works.

Consequently, when compiling this second edition, we have attempted to keep in step with current attitudes and, although the style of the book remains the same, several significant changes have been made.

The optical levels and levelling chapters have been combined as have the three distance measurement chapters. Much of the optical distance measurement material has been removed since such equipment and methods have been largely superseded by EDM techniques.

New chapters have been introduced dealing with triangulation, trilateration, intersection and resection since engineers are coming into contact with these techniques to a much greater extent than was the case in the past.

Within existing chapters, many amendments and changes have been incorporated. The setting-out chapter has been extended since this topic has gained in importance in recent years. The new Department of Transport design standards for highways are discussed in the curve chapters, trigonometrical heighting is introduced into the theodolites chapter, computerised plotting methods are discussed in the detail surveying chapter and more worked examples have been included throughout the book.

An introductory chapter has been added to provide a background to the subject and a guide to the professional and government bodies which are involved in surveying.

Although the book has been written with civil engineering students in mind, it is hoped that it will also be found useful by practising engineers as well as by any other students who undertake engineering surveying as a subsidiary subject.

The text covers engineering surveying up to the end of virtually all first-year and most second-year degree, diploma and BTEC courses in civil engineering, engineering geology, geography, surveying and other related disciplines at universities, polytechnics and colleges of technology. Other courses for which it is thought useful are O and A level surveying and the professional examinations of the various civil engineering related institutions.

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1

Introduction

Surveying, to the majority of engineers, is the process of measuring lengths, height differences and angles on site either for the preparation of large-scale plans or in order that engineering works can be located in their correct positions on the ground. The correct term for this is *engineering surveying* and it falls under the general title of land surveying.

Land surveying itself is only one of several different types of surveying which come under the auspices of the surveyors' professional body, The Royal Institution of Chartered Surveyors (RICS).

1.1 The Royal Institution of Chartered Surveyors

The RICS has seven divisions as shown in figure 1.1. The scope of these divisions is described by the RICS as follows.

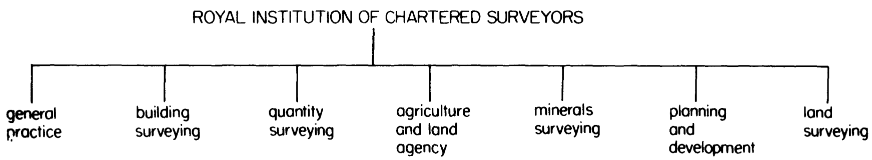


Figure 1.1 *Divisions of the RICS*

1.1.1 General Practice

The basic expertise of the General Practice Surveyor is valuation, that is, the assessment of what an interest in property is worth at a particular time. This may be in connection with purchase, sale, letting, investment, mortgage, rating, insurance, compensation or taxation.

A General Practice Surveyor may also be involved with Estate Agency (the negotiation for sale or purchase, leasing or auction of all types of property) and Estate Management (the management and maintenance of residential, commercial and industrial property, acting on behalf of both landlord and tenant). Some specialise in housing or in the valuation and auction of furniture and works of art and plant and machinery.

1.1.2 Building Surveying

The Building Surveyor offers a specialist service on all matters relating to construction; for example, the restoration of old buildings and the construction of new, building maintenance, the administration and control of contracts, building law and regulations. Building Surveyors carry out structural surveys and provide services to the general public, commerce and industry and public authorities.

1.1.3 Quantity Surveying

The Quantity Surveyor is an expert in the financial, contractual and communication aspects of the construction industry and is the impartial link between the client, the architect, the engineer and the builder. The Quantity Surveyor controls the cost of constructing a building, a road or a complex engineering installation, for example, from the design stage to the final completion of the contract. The QS is responsible for the contractual negotiation, for monitoring the progress of construction and agreeing the final account.

1.1.4 Land Agency and Agriculture

Land Agents and Agricultural Surveyors traditionally work in rural surroundings. The majority are employed in private practice where their work includes the valuation, sale and management of rural property and the sale by auction of live and dead stock; they are also concerned with forestry, farm management and rural planning.

1.1.5 Minerals Surveying

Minerals Surveyors plan the development and future of mineral workings. They advise on planning applications and appeals, mining law and working rights, mining subsidence and damage, the environmental effects of mines and the rehabilitation of derelict land. A Minerals Surveyor also manages and values mineral estates, and surveys mineral workings on the surface or in deep underground mines.

1.1.6 Planning and Development

Planning and Development Surveyors specialise in all aspects of urban and rural planning, working as part of a team and offering advice on economics and amenities, conservation and urban renewal schemes. Development Surveyors work closely with the planners to implement their plans within a given timescale and budget.

1.1.7 Land Surveying

The Land Surveyor is trained to measure land and its physical features accurately, and to record these features in the form of a map or a plan. It is work such as this which makes possible the publications of the Ordnance Survey of Great Britain (see section 1.7). Land Surveyors also undertake measurement and positioning for construction works.

1.2 Land Surveying

Land surveying can be broken down into several subsections as shown in figure 1.2. However, it must be stressed that there is considerable overlap between these sections, particularly as regards the basic methods and instruments used. The scope of each section is described by the RICS as follows.

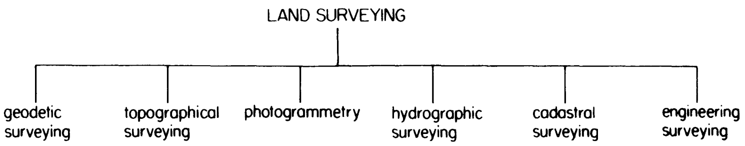


Figure 1.2 *Components of land surveying*

1.2.1 Geodetic Surveying

Geodetic surveys cover such a large area that the curved shape of the Earth has to be taken into account. Such surveys involve advanced mathematical theory and require precise measurements to provide a framework of accurately located points. These points can be used to map an entire continent, to measure the true size and shape of the Earth or to carry out associated scientific studies such as the determination of the Earth's magnetic field, or to detect continental drift.

1.2.2 Topographical Surveying

Topographical surveys establish the position and shape of natural and man-made features over a given area, usually for the purpose of producing a map of an area.

Such surveys are usually classified according to the scale of the final map. Small-scale surveys cover large areas such as an entire continent, country or county, and may range in scale from 1:1 000 000 to 1:50 000 like the familiar Ordnance Survey maps of Great Britain. Medium-scale maps range in scale from about 1:10 000 to 1:1000 and may cover the area of a small town, for example. Large-scale maps show details and present information which cannot be obtained from a map purchased in a shop and are therefore usually commissioned for a specific purpose. These maps range in scale from 1:500 up to 1:50 or larger and are often provided to meet the needs of architects, civil engineers, or government departments.

1.2.3 Photogrammetry

Surveys at most scales may be undertaken by photogrammetry, using photographs taken with special cameras from an aircraft or on the ground. Viewed in pairs, the photographs produce three-dimensional images of the surveyed features, from which maps or numerical data can be produced, usually with the aid of sophisticated and expensive stereo-plotting machines and computers.

1.2.4 Hydrographic Surveying

Hydrographic surveys at many different scales are carried out afloat, on rivers, canals, lakes, seas and oceans.

Hydrographic surveyors fix the position of their survey vessel, to determine the depth of water and to investigate the nature of the sea bed. Their traditional role for centuries has been to map the coastlines and sea bed in order to produce navigational charts for mariners. More recently, much of their work has been for offshore oil exploration and production.

Hydrographic surveys are also used in the design, construction and maintenance of harbours, inland water routes, river and sea defences, in control of pollution and in scientific studies of the ocean.

1.2.5 Cadastral Surveying

Cadastral surveys are those which establish and record the boundaries and ownership of land and property. In the United Kingdom, cadastral surveys are carried out by Her Majesty's Land Registry, a government department, and are based on the topographical detail appearing on Ordnance Survey maps. Cadastral work is mainly limited to overseas countries where National Land Registry Systems are under development.

1.2.6 Engineering Surveying

Engineering surveying can be considered to be that part of land surveying which applies to the construction industry in some way. Its requirements and purposes are discussed in the following section.

1.3 Engineering Surveying

The term engineering surveying is a general expression for any survey work carried out in connection with the construction of particular engineering features, for example, roads, railways, pipelines, dams, power stations, airports and so on. Its main purposes are listed below.

(1) To produce up-to-date plans of the areas in which engineering projects are to be built. The scales of the plans are usually considerably larger than those produced in the other forms of land surveying; 1:500 is a commonly used engineering survey scale although larger scales such as 1:200, 1:100 and 1:50 are also used. Cross sections and longitudinal sections are often drawn with exaggerated vertical scales (see chapter 12). These plans form the basis for the design of the construction, and hence the reliability of the design depends to a great extent on the precision and thoroughness with which the original survey is carried out.

(2) To determine the necessary areas and volumes of land and materials that may be required during the construction.

(3) To ensure that the construction is built in its correct relative and absolute position on the ground.

(4) To record the final as-built position of the construction including any amendments.

(5) To provide permanent control points from which particularly important projects can be surveyed, for example, monitoring the faces of dams to check for any movement.

In order that these aims can be achieved, equipment and techniques of sufficient precision should be used both before and during construction. However, it is not always necessary to use the highest possible precision; some projects may only require angles and distances to be measured to 01′ and 0.1 m, whereas others may require precisions of 01″ and 0.001 m. It is important that the engineer realises this and chooses equipment and techniques accordingly. To help with this choice, the precisions of the various equipment and techniques are emphasised throughout the book.

1.4 Principles of Engineering Surveying

The principles of engineering surveying are straightforward and follow a logical step-by-step sequence as follows.

(1) Carry out a reconnaissance, that is, look round the area and choose suitable positions for the location of control points. Adopt the universal surveying adage of *working from the whole to the part*, that is, choose a small number of primary (first-order) control points which form a well-defined network of figures covering the whole area and break these down into smaller networks of figures, as necessary, covering particular parts inside the main area by establishing secondary (second-order) and, if necessary, tertiary (third-order) control points.

(2) Construct the points. This can range from establishing concrete pillars to simply driving wooden pegs into the ground.

(3) Take field measurements of all the necessary angles, heights and distances. Take extra measurements for checking purposes, these are known as redundant observations. All the information should be recorded carefully in field books or on booking forms.

(4) Calculate the positions of the control points, checking the calculations wherever possible. Usually, both the elevation and the plan position (coordinates) of each point are calculated.

(5) If a plan is to be produced, additional field measurements are taken to locate the existing features in the area. This is known as detail surveying. The drawing is produced and passed on to the designer who uses it as a basis for the design of the particular engineering project.

(6) If the control points are to be used for setting out work, calculations are undertaken to obtain the relative angles and distances required to establish the exact position of the engineering feature from the control points and the setting out is then undertaken.

(7) Additional drawings may be produced from the field measurements if cross sections, longitudinal sections or other plan information is required. Further calculations may be carried out to obtain area, volume or other information.

1.5 Scale

All engineering plans and drawings are produced at particular scales, for example, 1:500, 1:100 and so on. The scale value indicates the ratio of horizontal and/or vertical plan distances to horizontal and/or vertical ground distances that was used when the drawing was produced, for example, a horizontal plan having a scale of 1:50 indicates that for a line AB

$$\frac{\text{horizontal plan length AB}}{\text{horizontal ground length AB}} = \frac{1}{50}$$

and, if line AB as measured on the plan = 18.2 mm, then

$$\text{horizontal ground length AB} = 18.2 \times 50 = 910 \text{ mm}$$

The term 'large-scale' indicates a small ratio, for example, 1:10, 1:20, whereas the term 'small-scale' indicates a large ratio, for example, 1:50 000.

On engineering drawings, scales are usually chosen to be as large as possible to enable features to be drawn as they actually appear on the ground. If too small a scale is chosen then it may not be physically possible to draw true representations of features and in such cases conventional symbols are used; this is a technique commonly adopted by the Ordnance Survey.

It must be stressed that the scale value of any engineering drawing or plan must always be indicated on the drawing itself. Without this it is incomplete and it is impossible to scale dimensions from the plan with complete confidence.

1.6 Units

Wherever possible throughout the text, Systѐme International (SI) units are used although other widely accepted units are introduced as necessary. Those units which are most commonly used in engineering surveying are as follows.

(1) *Units of length*

millimetre (mm), metre (m), kilometre (km)

$$\begin{aligned} 1\ \text{mm} &= 10^{-3}\ \text{m} = 10^{-6}\ \text{km} \\ 10^{3}\ \text{mm} &= 1\ \text{m} = 10^{-3}\ \text{km} \\ 10^{6}\ \text{mm} &= 10^{3}\ \text{m} = 1\ \text{km} \end{aligned}$$

(2) *Unit of area*

square metre (m^2)

(3) *Unit of volume*

cubic metre (m^3)

(4) *Units of angle*

The SI unit of angle is the radian (rad). However, most surveying instruments measure in degrees (°), minutes (′) and seconds (″) and some European countries use the gon (^{g}), formerly the grad, as a unit of angle. The relationship between these systems is as follows

$$1\ \text{circumference} = 2\pi\ \text{rad} = 360^\circ = 400^g$$

and, taking π to be 3.141 592 654 gives

$$\begin{aligned} 90^\circ &= 1.570\,796\,327\ \text{rad} = 100^g \\ 1^\circ &= 0.017\,453\,293\ \text{rad} = 1.111\,111\,111^g \\ 1' &= 0.000\,290\,888\ \text{rad} = 0.018\,518\,519^g \\ 1'' &= 0.000\,004\,848\ \text{rad} = 0.000\,308\,642^g \end{aligned}$$

$$\begin{aligned} 1\ \text{rad} &= 57.295\,779\,513^\circ = 63.661\,977\,236^g \\ 0.01\ \text{rad} &= 34.377\,467\,708' = 0.636\,619\,772^g \\ 0.0001\ \text{rad} &= 20.626\,480\,625'' = 0.006\,366\,198^g \end{aligned}$$

$$\begin{aligned} 100^g &= 90^\circ = 1.570\,796\,327\ \text{rad} \\ 1^g &= 0.9^\circ = 0.015\,707\,963\ \text{rad} \\ 0.1^g &= 5.4' = 0.001\,570\,796\ \text{rad} \\ 0.01^g &= 32.4'' = 0.000\,157\,080\ \text{rad} \end{aligned}$$

A useful approximate relationship which can be used to convert small angles from seconds of arc to their equivalent radian values is

$$\theta\ \text{rad} = \frac{\theta''}{206\,265} = \theta''\sin 1'' = \frac{\theta''}{\text{cosec}\ 1''}$$

1.7 The Ordnance Survey

The Ordnance Survey (OS) is the principal surveying and mapping organisation in Great Britain. Its work includes geodetic surveys and associated scientific work, topographical surveys and the production of maps of Great Britain at various scales.

Its range of map production is extremely wide and maps are available from the small-scale Routeplanner map, which is revised every year and contains the whole of Great Britain on one sheet at a scale of 1:625 000, to the large-scale 1:1250 maps, which each represent a ground area of 500 m x 500 m.

As far as engineering surveying is concerned, the OS maps of particular interest are those at the scales of 1:1250, 1:2500, 1:10 000 and occasionally 1:50 000.

(1) *1:1250*

These are the largest scale maps published by the OS. They cover urban areas of not less than 10 km^2 containing a population of 20 000 or more. The National Grid (see section 5.11) is shown at 100 m intervals and height information is depicted by means of spot heights and bench marks (see section 2.1.5).

(2) *1:2500*

These cover most of the country except mountain and moorland areas and large urban areas. The National Grid is shown at 100 m intervals and height information is depicted by means of spot heights and bench marks. These are the smallest scale OS maps which show bench marks.

(3) *1:10 000*

These maps cover the whole of the country. They are the largest scale of Ordnance Survey mapping to cover mountain and moorland areas and to show contours. The National Grid is shown at 1000 m intervals.

(4) *1:50 000*

This scale is covered by the Landranger series of OS maps. Each map covers an area of 40 km x 40 km and the National Grid is shown at 1000 m intervals. Altogether, 204 maps cover England, Scotland and Wales.

(5) *Digital Maps*

An increasing number of large-scale OS maps are now being produced by automated digital methods.

A digital map is defined as the representation of conventional map detail in a form suitable for manipulation by computer. Once the map data has been digitised, it can be used to plot a map at any scale, combine the data from several maps and select the particular features required. In this way, maps at 1:10 000 scale are being produced from the data originally digitised for 1:1250 and 1:2500 scale map production. The digital 1:10 000 scale maps are printed in three colours (water in blue, contours in brown and other features in black) instead of two colours as on conventionally printed maps at this scale.

Digital map data is available from the OS on magnetic tape in a variety of formats for use with most types of mainframe computer. A program is available for plotting the data.

1.7.1 Other OS Services

In addition to their wide range of maps, the Ordnance Survey provides many other services, some of which are of particular interest to engineering surveyors. These include

(1) *SUSI, Supply of Unpublished Survey Information*

This service provides the most up to date large-scale mapping information available.

Before a new edition of the 1:1250 or 1:2500 series maps is drawn and printed, locally based OS surveyors note and survey any ground changes on to their working drawings known as Master Survey Drawings (MSDs). Potential customers may call at local OS offices and examine the MSD of the area they require in order to determine whether it contains the information they need. Some offices are equipped to provide film or paper copies of the MSD on demand.

(2) *SIM, Survey Information of Microfilm*

This service provides copies of published 1:1250 and 1:2500 scale maps and of surveyors MSDs containing fixed amounts of subsequent survey change.

When a new map edition is published, a microfilm copy is made and distributed to certain OS agents equipped with special microfilm viewer and printout equipment. Customers can view the map they require and obtain a paper printout at original map scale on demand.

Microfilm copies of the surveyors MSD are produced when the drawing shows a fixed amount of survey change, and paper printouts at scale from this updated microfilm are also available from OS Microfilm Agents. This updating process continues until the amount of recorded change warrants the publication of a new map edition when the cycle will begin again.

(3) *Survey Control and Levelling Information*

To produce its maps, the OS has a framework of survey control points (triangulation stations, see section 6.3.2) and height marks (bench marks, see section 2.1.5) throughout Great Britain. These points are used frequently in engineering work and survey information is available as follows

(a) *Triangulation Stations*. Station descriptions are available which include the name, nature of station, National Grid coordinates, elevation and many other details.

(b) *Minor Control Points*. Detailed descriptions of these points which are established in urban areas to control the large-scale surveys are available.

(c) *Levelling*. Precise heighting information in Great Britain is available from the OS in the form of bench mark lists. Each list provides information on bench marks in a one kilometre square area and carries the same reference number of the corresponding National Grid 1:2500 scale map.

TABLE 1.1
Pocket Calculator Functions for Engineering Surveying

Function or Facility	Notes
Display	Should be at least 8 digit, preferably 10.
Arithmetic	Basic functions required.
Trigonometrical	sin, $\sin^{-1}$, cos, $\cos^{-1}$, tan, $\tan^{-1}$ essential.
Degrees, rad, gon (grad)	Facility for using trigonometrical functions in degree, rad and gon (grad) modes useful.
Decimal degrees	Conversion between deg, min, sec and decimal degrees needed.
Polar/Rectangular	Conversion between polar (bearing and distance) and rectangular form (ΔE and ΔN) simplifies co-ordinate calculations greatly.
Programmable	Useful (but not essential) for most calculations provided that program storage is available for repeat calculations. Some models use plug-in modules to extend programming capability.
Printer connection	Hard-copy facility avoids transposition errors.
General purpose	$1/x$, x^3, $\sqrt{x}$, y^x occur frequently in engineering surveying.
Logarithms	$\log x$, 10^x, $\ell n\ x$, e^x sometimes used.
Floating Point	Essential when dealing with large or small numbers.
Rechargeable Batteries	Preferable with AC current use.
Storage registers (memory)	Useful in complicated problems.
Pre-programmed constants	π required.
Statistical functions	$\bar{x}$ and s not essential but sometimes convenient.

1.8 Aims and Limitations of this Book

Engineers use land surveying simply as one of the means by which they can undertake their work and there is a definite limit to the surveying knowledge required by them, beyond which the surveying becomes of interest rather than importance. Specialist land surveyors are usually called in to deal with any unusual problems.

The main aim of this book, therefore, is to provide a thorough grounding in the basic land surveying techniques required in engineering. Its originality is not so much in the topics it contains, but more in the emphasis placed on each topic and the depth to which each is covered.

Although it is likely that engineers will come into contact with hydrographic surveying and photogrammetry at some stage in their careers, these are specialist subjects which require considerably more space to cover them to sufficient depth than that available here. Consequently, they are not included and space is instead given to a thorough discussion of the equipment and techniques used in general site work and plan production. Further information on hydrographic surveying and photogrammetry can be found in the references listed in section 1.9.

The text is limited mainly to *plane surveying*, that is, the effect of the curvature of the Earth is ignored. This is a valid limitation since the effect of the curvature of the Earth is negligible for areas up to 200 km^2 and it is unlikely that many engineering projects or construction sites will exceed this. The only time that curvature is considered in the text is in the section dealing with trigonometrical heighting where long sight lengths may be used.

Modern equipment is discussed at all times except where the more traditional equipment is ideal for illustrating a particular technique, and the use of electronic calculators and computers is discussed wherever applicable. Since the pocket calculator has now become an essential aid to engineering surveyors and is used extensively in their work, anyone thinking of purchasing a calculator is recommended to study table 1.1 before doing so, in order to ensure that the final choice has those features which will be most useful for surveying calculations.

A note of caution must be introduced at this point. Although the methods involved in engineering surveying can be studied in textbooks, such is the practical nature of the subject that no amount of reading will turn a student into a competent engineering surveyor. Only by undertaking some practical surveying, under site conditions, and learning how to combine the techniques and equipment as discussed in this text will the student eventually become proficient and produce satisfactory results.

1.9 Further Reading

W. A. Seymour, *A History of the Ordnance Survey* (Dawson, 1980).

J. B. Harley, *Ordnance Survey Maps: A Descriptive Manual* (Ordnance Survey, Southampton, 1975).

A. E. Ingham, *Hydrography for the Surveyor and Engineer, 2nd Edition* (Crosby Lockwood Staples, London, 1984).

P. R. Wolf, *Elements of Photogrammetry, 2nd Edition* (McGraw-Hill, Tokyo, 1983).

2

Levelling

Levelling is the name given to the process of measuring the difference in elevation between two or more points. In engineering surveying, levelling has many applications and is used at all stages in construction projects from the initial site survey through to the final setting out. Specialised equipment is required to undertake levelling: an *optical level* with its *tripod* and a *levelling staff.*

2.1 Levelling Terminology

2.1.1 Level Line

When levelling, the heights of points on the Earth's surface are determined and these must all be based on the same reference height for consistency. Such a reference height is a *level line* or *level surface* and is defined as a surface on which all points are normal to the direction of gravity as defined by a suspended plumb bob. Since the surface of the Earth is curved, level surfaces are also curved, as shown in figure 2.1.

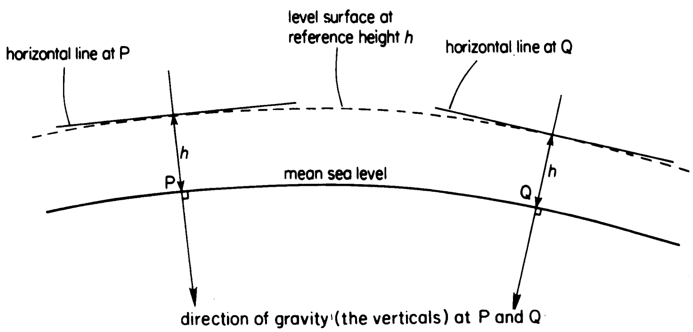


Figure 2.1 *Level and horizontal lines*

2.1.2 Horizontal Line

A *horizontal line* is one which is normal to the direction of gravity at a particular point, as shown in figure 2.1, and is, therefore, tangential to the level surface at each point chosen.

The difference between a horizontal line and a level line is called *curvature*, a factor discussed further in section 2.8.4.

2.1.3 Datum

In levelling operations, a level line is chosen to which the elevation of all points is related and is known as a *datum* or *datum surface*. This can be any surface but the most commonly used datum is *mean sea level* and, for Great Britain, this is the mean sea level as measured at Newlyn in Cornwall. Since the Ordnance Survey (OS) of Great Britain use this datum, it is called the *Ordnance Datum* and any heights referred to Ordnance Datum are said to be *Above Ordnance Datum* (AOD). All heights marked on OS maps and plans will be AOD.

2.1.4 Reduced Level

The height of a point relative to the chosen datum is said to be its *reduced level* (RL).

2.1.5 Bench Marks

These are permanent reference marks or points, the reduced levels of which have been accurately determined by levelling.

Ordnance bench marks (OBMs) are those which have been established by the Ordnance Survey throughout Great Britain and are based on the Ordnance Datum. The most common type are permanently marked on buildings and walls by a cut in vertical brickwork or masonry, an arrow or crowsfoot mark indicating the bench

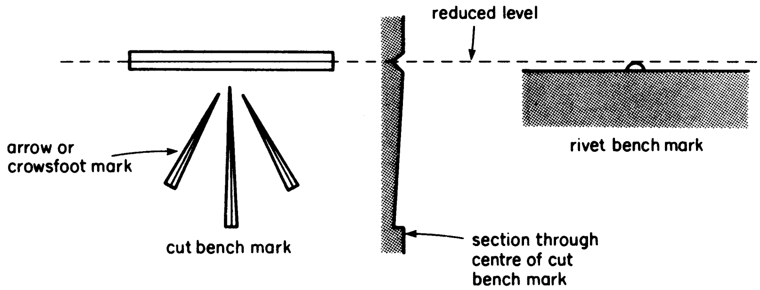


Figure 2.2 *Bench marks*

mark. On horizontal surfaces, OBMs consist of a rivet or bolt, the position of the RL being shown in figure 2.2 for both types.

Temporary or *transferred bench marks* (TBMs) are marks set up on stable points near construction sites to which all levelling operations on that particular site will be referred. These are often used when there is no OBM close to the site. The height of a TBM may be assumed at some convenient value, usually 100.00 m, or may be accurately established by levelling from the nearest OBM. Various suggestions for the construction of TBMs are given in chapter 14.

2.2 Optical Levels

This instrument, usually referred to as *a level* is used to establish a horizontal line of sight at each point where it is set up. A *horizontal line* (see also section 2.1.2) is one which is normal to the *vertical*. The direction of the vertical is that which a freely suspended plumb line takes up, that is, the direction of gravity (see figure 2.3).

The level consists of a telescope and a spirit level which help to establish a horizontal line. The telescope provides an accurate line of sight and enables the level to be used over distances suitable for surveying purposes. The spirit level, fixed to the telescope, enables the optical axis of the telescope to be set in a horizontal position.

2.2.1 The Surveying Telescope

Since the type of telescope used in levels is also used in theodolites (see chapter 3), the method of construction is considered in detail.

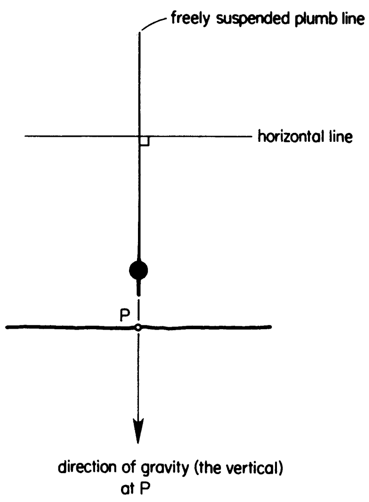


Figure 2.3 *The vertical*

The surveying telescope is *internally focusing* as shown in figure 2.4. Incorporated in the design of the telescope are special cross lines which, when the telescope is adjusted correctly, are seen clearly in the field of view. These lines provide a reference against which measurements can be taken. This part of the telescope is called

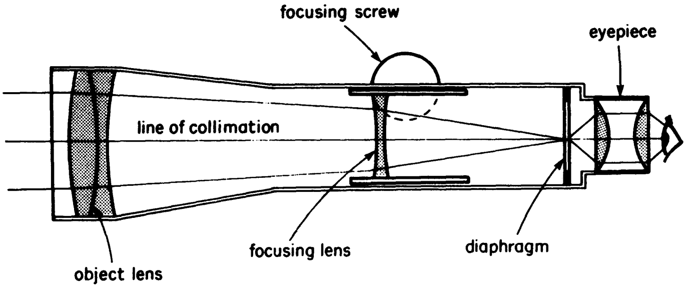


Figure 2.4 *Internal focusing telescope shown correctly adjusted*

the *diaphragm* and consists of a circle of plane glass upon which a series of lines is etched, the more common patterns being shown in figure 2.5. Conventionally, the vertical and horizontal lines are called the *cross hairs*.

The object lens, focusing lens, diaphragm and eyepiece are all mounted on the same optical axis and the imaginary line passing through the centre of the cross hairs and the optical centre of the object lens is called the *line of collimation* or the *line of sight*. When using the level, all readings are taken using this line. The diaphragm is held in the telescope by means of four adjusting screws so that the position of the line of collimation within the telescope can be moved (see section 2.2.9).

The action of the telescope is as follows. Light rays from a distant point pass through the object lens and are brought to focus in the plane of the diaphragm by axial movement of the concave lens. This is achieved by mounting the concave lens on a tube within the telescope, this tube being connected, via a rack and pinion, to a focusing screw attached to the side of the telescope. The eyepiece, a combination of lenses, has a fixed focal point that lies outside the lens combination and, by moving the eyepiece, this point can be made to coincide with the plane of the diaphragm. Since the image of the object has already been focused at the diaphragm, an observer will see in the field of view of the telescope the distant point focused against the cross hairs marked on the diaphragm. Furthermore, the optical arrangement is such that the object viewed through the eyepiece is magnified.

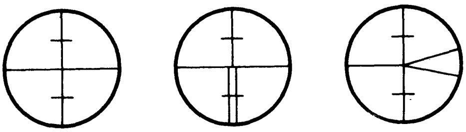


Figure 2.5 *Diaphragm patterns*

2.2.2 Parallax

It must be realised that for the surveying telescope to operate correctly the image of a distant point or object must fall exactly in the plane of the diaphragm and the eyepiece must be adjusted so that its focal point is also in the plane of the diaphragm. Failure to achieve either of these settings results in a condition called *parallax* and this is a major cause of error in both levelling and theodolite work. Parallax can be detected by moving the eye to different parts of the eyepiece when viewing a distant object; if different parts of the object appear against the cross hairs then the telescope has not been properly focused and parallax is present, as seen in figure 2.6.

It is impossible to take accurate readings under these circumstances since the line of sight alters for different positions of the eye. Parallax *must* be removed *before* any readings are taken when using *any* optical instrument with an adjustable eyepiece.

To remove parallax, the eyepiece is first adjusted while viewing a light background, for example, the sky or a booking sheet, until the cross hairs appear in sharp focus. The distant point at which readings are required is now sighted and brought into focus and is viewed while moving the eye. If the object and cross hairs do not move relative to each other then parallax has been eliminated; if there is apparent movement then the procedure should be repeated.

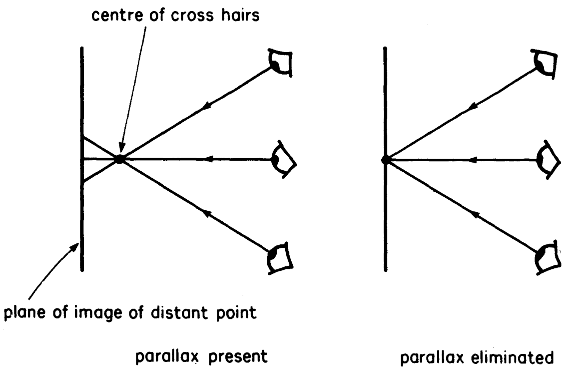


Figure 2.6 *Parallax*

2.2.3 The Spirit Level

All surveying instrument spirit levels (including those of the theodolite) are constructed in a manner similar to that shown in figure 2.7.

The bubble tube is a barrel-shaped glass tube (or vial) partially filled with alcohol or ether, chosen because of their very low freezing points. Marked on the glass vial is a series of graduations and the imaginary tangent to the surface of the vial at the

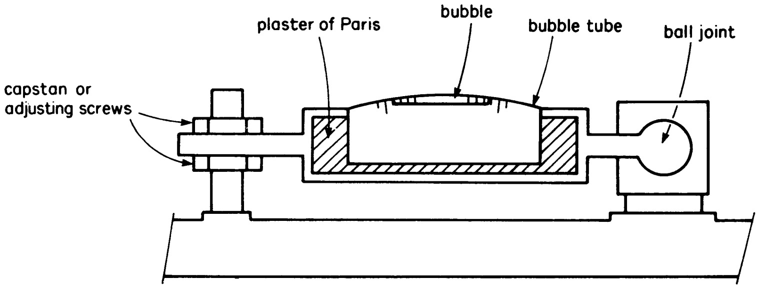


Figure 2.7 *Section of a spirit level*

centre of these graduations is known as the *principal tangent* of the spirit level. The remaining space in the tube is an air bubble which, under the action of gravity, always comes to rest so that the ends of the bubble are equally displaced about the vertical.

If the bubble takes up a position in the tube with its ends an equal number of graduations (or divisions) either side of the centre of the vial (see figure 2.8*a*), the principal tangent is horizontal and is normal to the vertical.

If the bubble does not lie in the centre of the vial, the principal tangent is not horizontal (see figure 2.8*b*).

This property of the spirit level is used in levels as a means of setting the line of collimation horizontal (see section 2.2.4).

A feature of many levels is a *coincidence bubble reader* or split bubble in which an arrangement of prisms is used, as in figure 2.9, to observe both ends of the bubble simultaneously. Instead of adjusting the bubble to be between graduations, a horizontal setting of the tube is achieved by bringing the ends of the bubble into coincidence. In most instruments a magnified image of the bubble ends is seen, enabling a very accurate setting of the bubble to be achieved.

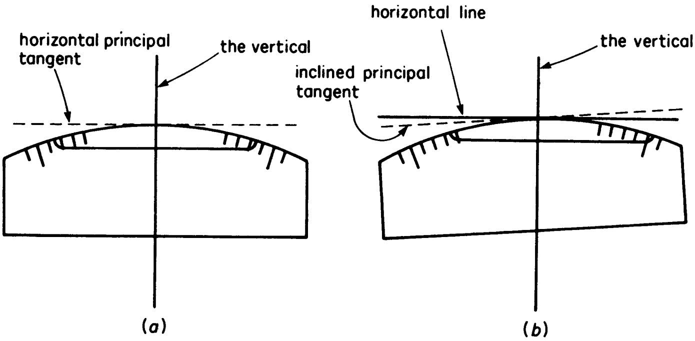


Figure 2.8 *Principal tangent*

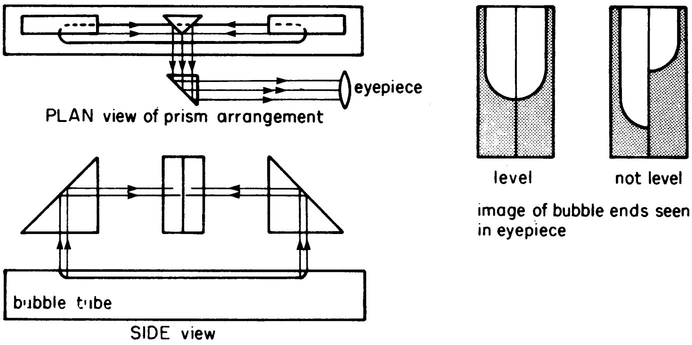


Figure 2.9 *Coincidence bubble reader*

2.2.4 Principle of the Level

By attaching a surveying telescope to a spirit level such that the principal tangent is parallel to the line of collimation, a horizontal line of collimation may be set. This is achieved by adjusting the inclination of the telescope until the bubble lies in the centre of its graduations; the line of collimation now coincides with the horizontal plane through the instrument, as shown in figure 2.10. The method of obtaining this coincidence varies slightly from instrument to instrument.

2.2.5 The Tilting Level

Figure 2.11 shows photographs of a tilting level. On this instrument the telescope is not rigidly attached to the tribrach and can be tilted a small amount in the vertical plane about a pivot placed below the telescope. Hence the name *tilting level*. The amount of tilt is controlled by the tilting screw placed near the telescope eyepiece.

The levelling head comprises the trivet stage, some form of levelling device and tribrach. The telescope and main spirit level can be moved independently of the tribrach over a limited range and a second level is fitted to the instrument and is mounted on the tribrach. This is always a small circular level known as a *pond level* which contains a pond bubble that can be levelled independently of the main spirit level fixed to the telescope.

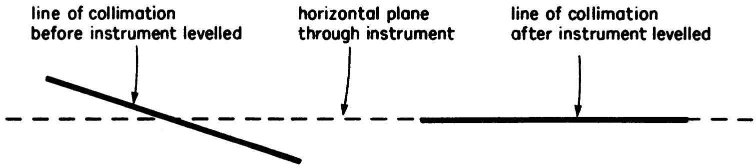
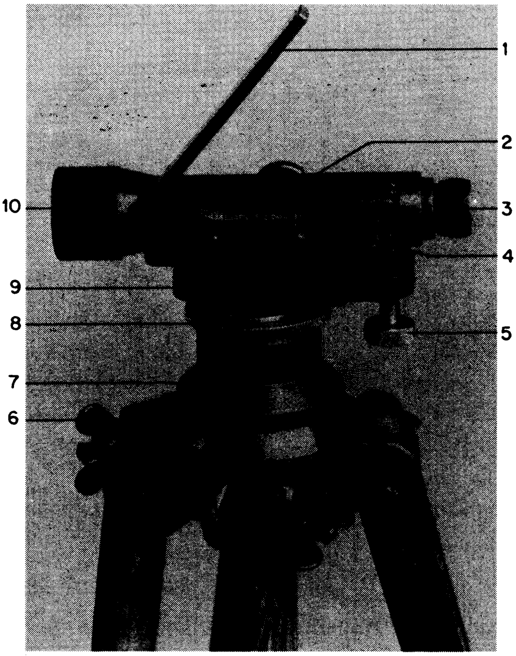
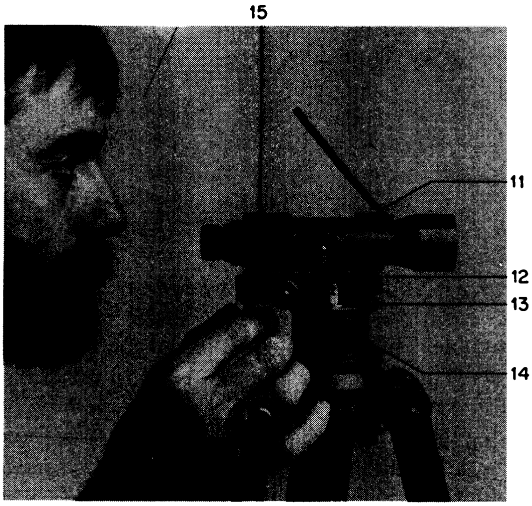


Figure 2.10 *Principle of the level*



(a)



(b)

Figure 2.11 *Tilting level: (a) general features; (b) main spirit level being set using tilting screw. 1. Reflecting mirror (for main spirit level); 2. main spirit level; 3. eyepiece; 4. capstan screws; 5. tilting screw; 6. wing nut; 7. spherical levelling head; 8. horizontal circle; 9. tribrach; 10. object lens; 11. focusing screw; 12. pivot; 13. clamp, 14. tangent screw; 15. pond level*

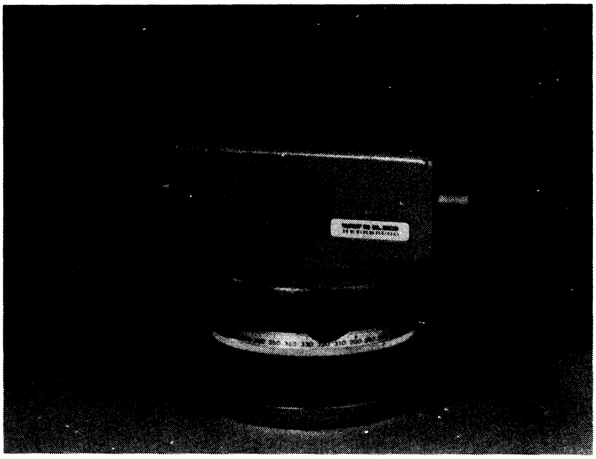


Figure 2.12 *Tilting level with conventional footscrew arrangement (courtesy Wild Heerbrugg (UK) Ltd)*

As with many surveying instruments, various designs are possible. Figure 2.11 shows a tilting level in which the conventional three footscrew arrangement (see figure 2.12) is replaced by a spherical joint which enables the level to be quickly, but only approximately, levelled.

Many tilting levels incorporate coincidence bubble readers for greater accuracy in setting the main bubble (see figure 2.9).

2.2.6 Use of the Tilting Level

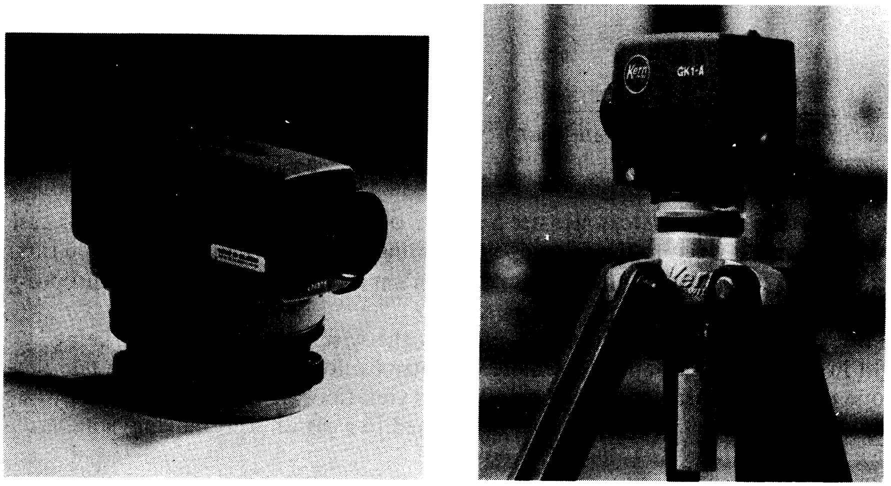
After setting up the tripod with its top levelled by eye, the level is attached to it. Using the footscrews or spherical joint, the tribrach is set approximately level by centralising the pond bubble. This ensures that the instrument is almost level.

Parallax is now removed and the telescope rotated until it is pointing in the direction in which the first reading is required. The telescope is lightly clamped in position and the pointing finely adjusted using the slow motion screw. At the position of reading so set, the tilting screw is turned until the main bubble in the spirit level attached to the telescope is brought to the centre of its run or coincidence is obtained. This ensures that the optical axis of the telescope or, more precisely, the line of collimation is exactly horizontal in the direction in which the reading is to be taken. When the telescope is rotated to other directions, the main bubble will change its position for each setting of the telescope since the standing axis is not exactly vertical. Therefore, *the main bubble must be relevelled before every reading is taken*.

2.2.7 The Automatic Level

In the automatic level, examples of which are shown in figure 2.13, a spirit bubble is no longer used to set a horizontal line of collimation. Instead, the line of collimation is directed through a system of *compensators* which ensure that the line of sight viewed through the telescope is horizontal even if the optical axis of the telescope tube itself is not horizontal.

Whatever type of automatic level is used it must be levelled within approximately 15′ of the vertical to allow the compensators to work. This is achieved by either a three footscrew arrangement or a spherical joint used in conjunction with a pond bubble mounted on the tribrach.



(a) (b)

Figure 2.13 *Automatic levels (note absence of main spirit level); (a) courtesy of Wild Heerbrugg (UK) Ltd; (b) courtesy of Kern and Co. Ltd*

The principle of the compensator is shown in figure 2.14, which has been exaggerated for clarity. The level is set up in a similar manner to the tilting level so that the pond bubble on the tribrach is centralised. This sets the standing axis of the instrument to within a few minutes of the vertical at the point of setting up. The small angle δ (δ is less than 15′) between the standing axis and the vertical is also the angle of tilt of the telescope. The function of the compensator, C, is to deviate all horizontal rays of light entering the telescope tube at the same height as P (about which the telescope is pivoted by angle δ) through the centre of the cross hairs, D.

Free suspension compensators operate using a system of mirrors or prisms which, being freely suspended within the telescope tube, take up a position according to the angle of tilt of the telescope.

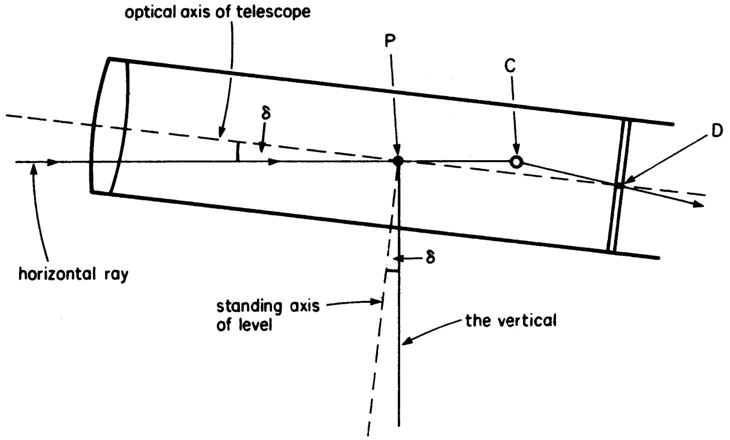


Figure 2.14 ***Principle of automatic level***

Mechanical compensators also use a system of prisms or mirrors, but, in this case, one (or more) is fixed to the telescope tube using a spring. When the telescope tilts, the spring is deformed by the weight of the prism attached to it and this is proportional to the amount of tilt of the telescope.

Both systems use some form of *damping*, otherwise the compensators, being light in weight, would tend to oscillate violently for long periods when the telescope is moved or affected by wind and other vibrations. Air damping is usually employed in which the compensator is attached to a piston, this moving in a closed cylinder.

Every commercially available automatic level uses a different compensating system and the description of all these is beyond the scope of this book. However, one example of a free suspension compensator is given since these are more common than mechanical compensators.

Figures 2.15*a* and *b* show a compensator made up of three prisms. Two of these are suspended and one is attached to the telescope housing. When the telescope is horizontal, the line of sight is as shown in figure 2.15*a*. If the standing axis does not coincide with the vertical, the telescope tilts through angle δ from the horizontal at P (see figure 2.15*b*). The suspended prisms, however, also tilt through angle δ so as to deflect a horizontal ray at P through the cross hair intersection, D.

2.2.8 Use of the Automatic Level

When an automatic level has been roughly levelled, the compensator automatically moves to a position to establish a horizontal line of sight. Therefore, no further levelling is required after the initial levelling.

As with all types of level, parallax must be removed before any readings are taken.

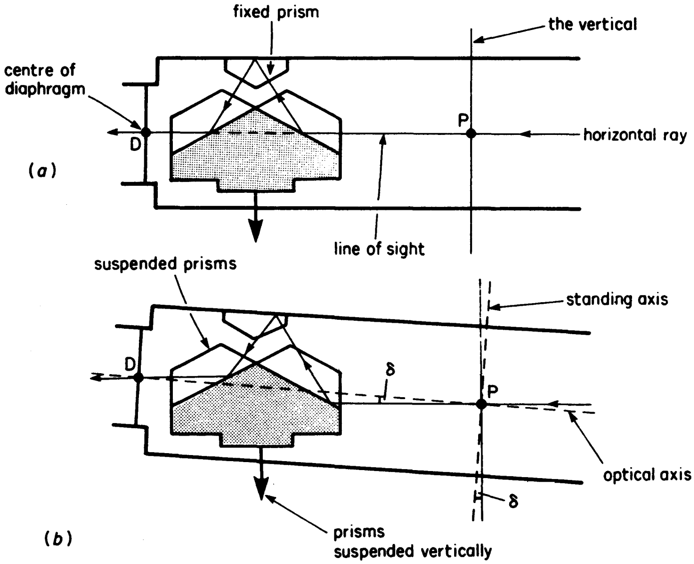


Figure 2.15 *Free suspension compensator*

In addition to the levelling procedure and parallax removal, a test should be made to see if the compensator is functioning before readings commence. One of the levelling footscrews should be moved slightly off level and, if the reading to a levelling staff (section 2.3) remains constant, the compensator is working. If the reading changes, it may be necessary to gently tap the telescope to free the compensator. On some automatic levels this procedure is not necessary since a button is attached to the level which is pressed when the staff has been sighted. If the compensator is working, the horizontal hair is seen to move and then return immediately to the horizontal line of sight.

The particular advantage of the automatic level is the greater speed with which accurate levelling may be carried out compared with using conventional levels. This is attributable to the fact that there is no main bubble to centralise. In addition, the compensating system eliminates errors caused by either forgetting to set the bubble or setting it inaccurately.

A disadvantage with automatic levels is that either a strong wind blowing on the instrument or machinery operating nearby will cause the compensator to oscillate, resulting in vibrating images. To overcome this the mean of several readings should be taken.

2.2.9 Permanent Adjustment of the Level

In the preceding sections, the operations necessary to set up various types of level have been described, the objective being in all cases to level the instruments such that the line of collimation, as viewed through the eyepiece, is horizontal. Adjustments of this nature are called *temporary adjustments* since these are carried out for every instrument position and, in some cases, for every pointing of the telescope.

So far, for every level discussed, the assumption has been made that once the temporary adjustments have been completed, the observed line of collimation is exactly horizontal. This, however, will only occur in a perfectly adjusted level, a case seldom met in practice. Hence, some checking method is required to ensure that the level is correctly adjusted. This is known as a *permanent adjustment* and should be undertaken at regular intervals during the working life of the equipment, for example, once a week, depending on its usage.

Tilting level adjustment

The only permanent adjustment check necessary for a tilting level is to ensure that the line of collimation is parallel to the principal tangent so that when the main bubble is centred the line of collimation is horizontal.

If the line of collimation is not set exactly horizontal then a *collimation error* is present in the level.

The usual method of testing and adjusting a level is to carry out a two-peg test. Alternatively, reciprocal levelling can be used (see section 2.10.2).

The *two-peg test* is carried out as follows, with reference to figures 2.16*a* and *b*.

(1) On fairly level ground, hammer in two pegs A and B a maximum of 60 m apart. Let this distance be L metres.

(2) Set up the level exactly midway between the pegs at point C and level carefully.

(3) Place a levelling staff (see section 2.3) at each peg in turn and obtain readings S_1 and S_2, as in figure 2.16*a*.

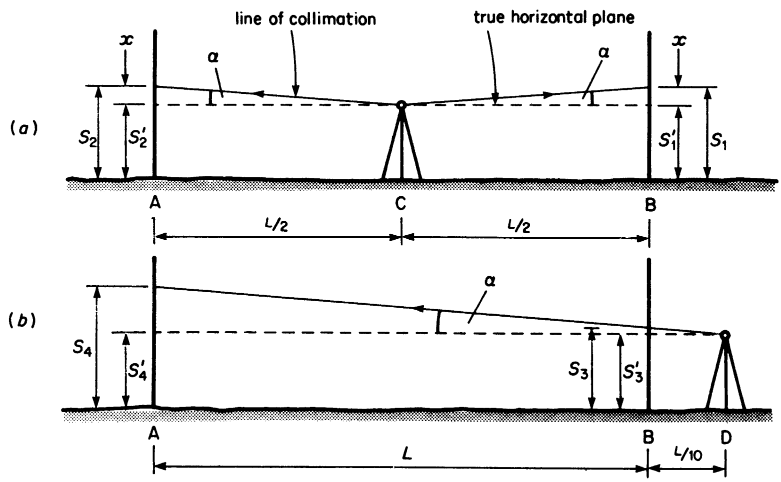


Figure 2.16 *Two-peg test*

Since AC = CB the error, x, in the readings S_1 and S_2, will be the same. This error is due to the collimation error, the effect of which is to incline the line of collimation by angle α. This gives

$$S_1 - S_2 = (S_1' + x) - (S_2' + x) = S_1' - S_2'$$

= *true difference in height* between A and B.

In figure 2.16 the assumption has been made that the line of collimation lies above the true horizontal plane. Even if this is not the case it does not affect the calculation procedure since the sign of the collimation error is obtained in the calculation as shown in the example at the end of this section.

(4) Move the level so that it is, preferably, $L/10$ m from peg B at D (see figure 2.16*b*) and take readings S_3 at B and S_4 at A. Compute the *apparent difference in height* between A and B from $(S_3 - S_4)$.

If the instrument is in adjustment $(S_1 - S_2) = (S_3 - S_4)$.

If there is any difference between the apparent and true values, this has occurred in a distance of L metres and hence

$$\text{Collimation error } (e) = (S_1 - S_2) - (S_3 - S_4) \text{ m per } L \text{ metres.}$$

If the error is found to be less than ±3 mm per 60 m the level is not adjusted. Instead, any readings taken must be observed over equal or short lengths so that the collimation error cancels out or is negligible.

(5) To adjust the instrument at point D, the correct reading that should be obtained at A, S_4', is computed from

$$S_4' = S_4 - \frac{66}{60}\,e$$

A check on this reading is obtained by computing S_3' and by comparing $(S_3' - S_4')$ with the true difference in height (see the worked example in this section).

(6) With the level still at D and observing the staff reading S_4 at A, the tilting screw is adjusted until a reading of S_4' is obtained. However, this causes the main bubble to move from the centre of its run, so it is brought back to the centre by adjusting the bubble capstan screws (see figure 2.7) with a pair of small spanners.

(7) The test should be repeated to ensure that the adjustment has been successful.

Automatic level adjustment

As with a tilting level, the standing axis of an automatic level is set only approximately vertical, the compensators automatically correcting for any slight variation from the vertical. Consequently, the two-peg test or reciprocal levelling (see section 2.10.2) must again be carried out to ensure that once the pond bubble is central the line of collimation observed is horizontal.

Having deduced the correct reading S_4' as before (see figure 2.16*b*), the adjustment can be made by one of two methods.

For most instruments the cross hairs are moved using the diaphragm adjusting screws until the appropriate reading is obtained.

In some levels, however, it is necessary that the compensator itself is adjusted. Since this is a delicate operation, the level should be returned to the manufacturer for adjustment under laboratory conditions.

Some automatic levels have, in addition to a movable diaphragm, a special adjusting screw for the compensator. When adjusting such an instrument, the compensator screw should never be touched as its setting is precisely carried out by the manufacturer.

Worked example

Question

The readings obtained from a two-peg test carried out on an automatic level with a single level staff set up alternately at two pegs A and B placed 50 m apart were as follows:

(1) With the level midway between A and B

staff reading at A = 1.283 m
staff reading at B = 0.860 m

(2) With the level positioned 5 m fiom peg B on the line AB produced

staff reading at A = 1.612 m
staff reading at B = 1.219 m

Calculate

(1) The collimation error of the level per 50 m of sight
(2) The reading that should have been observed on the staff at A from the level in position 5 m from B.

Solution

(1) Referring to figures 2.16*a* and *b*

$S_1 = 0.860$ m $\quad S_2 = 1.283$ m $\quad S_3 = 1.219$ m $\quad S_4 = 1.612$ m
collimation error, $e = (0.860 - 1.283) - (1.219 - 1.612)$

$$= \mathbf{-0.030\ m\ per\ 50\ m}$$

(2) For the instrument in position 5 m from peg B, the reading that should have been obtained on the staff when held at A is

$$S_4' = 1.612 - \frac{55}{50}(-0.030) = \mathbf{1.645\ m}$$

This is checked by computing $(S_3' - S_4')$ and by comparing with $(S_1 - S_2)$ as follows

$$S_3' = 1.219 - \frac{5}{50}(-0.030) = 1.222 \text{ m}$$

Hence

$$(S_3' - S_4') = 1.222 - 1.645 = -0.423 = (S_1 - S_2)$$

(checks)

When solving problems of this nature it is important that the lettering sequence given in figure 2.16 for S_1 to S_4 is adhered to. If it is not, incorrect answers will be obtained.

2.3 The Levelling Staff

The *levelling staff* is used for measuring distances vertically above or below points on which it is held relative to a line of collimation as defined by the level. Many types of staff are in current use and these can have lengths of 3, 4, 4.25 or 5 m. The staff can be rigid, telescopic, hinged, folding or socketed in three sections for ease of carrying and is usually made of metal, although wooden staves are still available. The staff markings can take various forms but the 'E'-type staff face conforming to British Standard (BS) 4484, as shown in figure 2.17, is the most common. The staff can be read directly to 0.01 m, with estimation to 0.001 m.

Since the staff is used to measure a vertical distance it must be held vertically and some staves are fitted with periscope-type handles and a pond bubble to assist in this operation. If no permanent bubble is fitted, a detachable pond bubble may

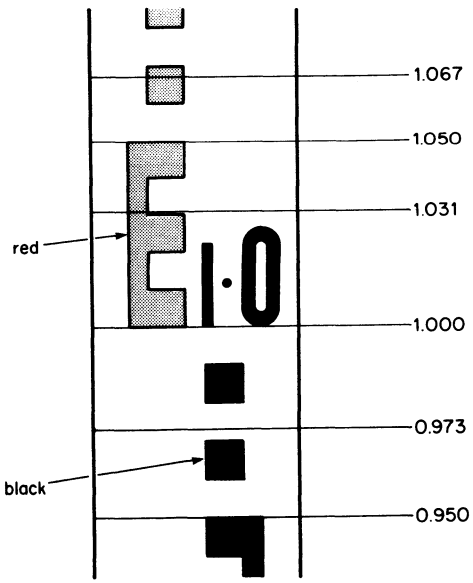


Figure 2.17 *Levelling staff with reading examples*

be used. This device is mounted on a metal angle bracket and is held against the staff when levelling. If no bubble is available, the staff should be slowly swung back and forth through the vertical and the lowest reading noted. This will be the reading when the staff is vertical.

2.4 Principles of Levelling

In a correctly levelled instrument, the line of collimation coincides with a horizontal line which lies in a horizontal plane. If the height of this plane is known, the heights of ground points can be found.

In figure 2.18, a level has been set up at point I_1 and readings R_1 and R_2 recorded with the staff placed vertically in turn at ground points A and B. If the reduced level of A (RL_A) is known then, by adding staff reading R_1 to RL_A, the reduced level of the line of collimation at instrument position I_1 is obtained. This is known as the *height of the plane of collimation* (HPC) or the *collimation level.* Thus

$$\text{collimation level at } I_1 = RL_A + R_1$$

From figure 2.18, it can be seen that to obtain the reduced level of point B (RL_B), staff reading R_2 must be subtracted from the collimation level, hence

$$\begin{aligned} RL_B &= \text{collimation level} - R_2 = (RL_A + R_1) - R_2 \\ &= RL_A + (R_1 - R_2) \end{aligned}$$

Since the direction of levelling is from A to B, the reading on A, R_1, is known as a *back sight* (BS) and that on B, R_2, a *fore sight* (FS).

From the above expression for RL_B and considering figure 2.18, the height difference between A and B is given by, in both magnitude and sign, $(R_1 - R_2)$. Furthermore, since R_1 is greater than R_2 and hence $(R_1 - R_2)$ is positive, the base of the staff must have risen from A to B and the expression $(R_1 - R_2)$ is known as a *rise*.

Referring to figure 2.18, assume the level is now moved to a new position I_2 in order that the reduced level of C may be found. Reading R_3 is first taken with the staff still at point B but with its face turned towards I_2. This will be the back sight at position I_2, the fore sight R_4 being taken with the staff at C. At point B, both a

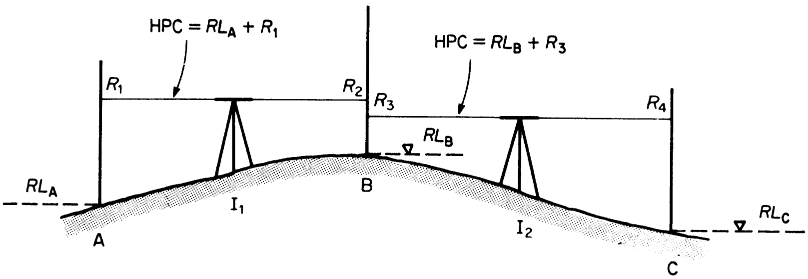


Figure 2.18 ***Principles of levelling***

FS and a BS have been recorded consecutively, each from a different instrument position. A point such as B is called a *change point* (CP).

From the staff readings taken at I_2, the reduced level of C (RL_C) is calculated from

$$RL_C = RL_B + (R_3 - R_4)$$

The height difference between B and C is given both in magnitude and sign by $(R_3 - R_4)$. In this case, since R_3 is smaller than R_4, $(R_3 - R_4)$ is negative. The base of the staff must, therefore, have fallen from B to C and the expression $(R_3 - R_4)$ is known as a *fall*.

In practice, a BS is the first reading taken after the instrument has been set up and is always to a point of known or calculated reduced level. Conversely, a FS is the last reading taken before the instrument is moved. Any readings taken between the BS and FS from the same instrument position are known as *intermediate sights* (IS).

2.5 Field Procedure

A more complicated levelling sequence is shown in cross section in figure 2.19*a*, in which an engineer has levelled from an OBM to a TBM to find the reduced levels of points A to E. Figure 2.19*b* shows the levelling in plan view. The field procedure is as follows.

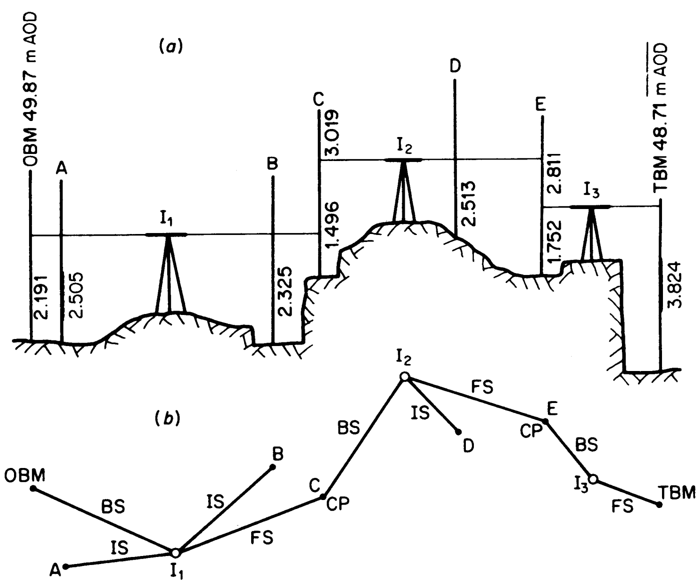


Figure 2.19 *Levelling sequence*

(1) The level is set up at some convenient position I_1 and a BS of 2.191 m taken to the OBM, the foot of the staff being held on the OBM and the staff held vertically.

(2) The staff is moved to points A and B in turn and readings taken. These are intermediate sights of 2.505 m and 2.325 m respectively.

(3) A change point must be used in order to reach D owing to the nature of the ground. Therefore, a change point is chosen at C and the staff is moved to C and a reading of 1.496 m taken. This is a FS.

(4) While the staff remains at C, the instrument is moved to another position, I_2. A reading is taken from the new position to the staff at C. This is a BS of 3.019 m.

(5) The staff is moved to D and E in turn and readings taken of 2.513 m (IS) and 2.811 m (FS) respectively, E being another CP.

(6) Finally, the level is moved to I_3, a BS of 1.752 m taken to E and a FS of 3.824 m taken to the TBM.

(7) The final staff position is at a point of known RL. This is most important as all levelling fieldwork must start and finish at points of known reduced level, otherwise it is not possible to detect misclosures in the levelling (see section 2.6).

2.6 Booking and Reduced Level Calculations

The booking and reduction of the readings discussed in section 2.5 can be done by one of two methods.

2.6.1 The Rise and Fall Method

The readings are shown booked by the rise and fall method in table 2.1. These are normally recorded in a level book containing all the relevant columns. Each line of

TABLE 2.1
Rise and Fall Method
(all values in metres)

BS	IS	FS	RISE	FALL	INITIAL RL	ADJ	ADJ RL	REMARKS
2.191					49.87	-	49.87	OBM 49.87 AOD
	2.505			0.314	49.556	+0.003	49.56	A
	2.325		0.180		49.736	+0.003	49.74	B
3.019		1.496	0.829		50.565	+0.003	50.57	C (CP)
	2.513		0.506		51.071	+0.006	51.08	D
1.752		2.811		0.298	50.773	+0.006	50.78	E (CP)
		3.824		2.072	48.701	+0.009	48.71	TBM 48.71 AOD
6.962		8.131	1.515	2.684	49.87			
		6.962		1.515	48.701			
		+1.169		+1.169	+ 1.169			

the level book corresponds to a staff position and this is confirmed by the entries in the 'remarks' column. The calculation proceeds in the following manner, in which the reduced level of a point is related to that of a previous point.

(1) From the OBM to A there is a fall (see figure 2.19). A BS of 2.191 m has been recorded at the OBM and an IS of 2.505 m at A. The resulting height difference is given by (2.191 – 2.505) = –0.314 m. The negative sign indicates the fall and is entered against point A. This fall is subtracted from the RL of the OBM to obtain the initial reduced level of A as 49.556 m.

(2) The procedure is repeated and the height difference from A to B is given by (2.505 – 2.325) = +0.180 m. The positive sign indicates a rise and this is entered opposite B. The RL of B is (RL_A + 0.180) = 49.736 m.

TABLE 2.2
Height of Collimation Method
(all values in metres)

BS	IS	FS	HPC	INITIAL RL	ADJ	ADJ RL	REMARKS
2.191			52.061	49.87	-	49.87	OBM 49.87 AOD
	2.505			49.556	+0.003	49.56	A
	2.325			49.736	+0.003	49.74	B
3.019		1.496	53.584	50.565	+0.003	50.57	C (CP)
	2.513			51.071	+0.006	51.08	D
1.752		2.811	52.525	50.773	+0.006	50.78	E (CP)
		3.824		48.701	+0.009	48.71	TBM 48.71 AOD
6.962		8.131		49.87			
		6.962		48.701			
		+1.169		+ 1.169			

(3) This calculation is repeated until the initial reduced level of the TBM is calculated, at which point a comparison can be made with the known value (see (6) below).

(4) When calculating the rises or falls the figures in the FS or IS columns must be subtracted from the figures in the line immediately above, either in the same column or one column to the left. At a CP, the FS is subtracted from the IS or BS in the line above and the BS on the same line as the FS is then used to continue the calculation with the next IS or FS in the line below.

(5) When the INITIAL RL column of the table has been completed, a check on the arithmetic involved is possible and must always be applied. This check is

$$\Sigma(FS) - \Sigma(BS) = \Sigma(FALLS) - \Sigma(RISES) = \text{FIRST RL} - \text{LAST RL}$$

It is normal to enter these summations at the foot of each relevant column in the levelling table (see table 2.1). Obviously, agreement must be obtained for all three parts of the check and it is stressed that this only provides a check on the INITIAL RL calculations and does not provide an indication of the accuracy of the readings.

(6) In table 2.1, the difference between the calculated and known values of the RL of the TBM is –0.009 m. This is known as the *misclosure* and gives an indication of the accuracy of the levelling. If the misclosure is outside the *allowable misclosure* (see section 2.7) then the levelling must be repeated. If the misclosure is within the allowable value then it is distributed throughout the reduced levels. The usual method of correction is to apply an equal, but cumulative, amount of the misclosure to each instrument position, the sign of the adjustment being opposite to that of the misclosure. Table 2.1 shows a misclosure of –0.009 m, hence a total adjustment of +0.009 m must be distributed. Since there are three instrument positions, +0.003 m is added to the reduced levels found from each instrument position. The distribution is shown in the ADJ (adjustment) column of table 2.1, in which the following cumulative adjustments have been applied. Levels A, B and C, +0.003 m; levels D and E, +(0.003 + 0.003) = +0.006 m; the TBM, +(0.003 + 0.003 + 0.003) = +0.009 m. No adjustment is applied to the initial bench mark since this level cannot be altered.

(7) The adjustments are applied to the INITIAL RL values to give the ADJ (adjusted) RL values as shown in table 2.1. These adjusted RL values are used in any subsequent calculations and are quoted only to the same number of decimal places of metres as the reduced levels of the OBM and TBM used.

2.6.2 The Height of Collimation Method

The level book for the reduction of the levelling of figure 2.19 is shown in the height of collimation form in table 2.2. This method of reducing levels is based on the HPC being calculated for each instrument position and proceeds as follows.

(1) If the BS reading taken to the OBM is added to the RL of this bench mark, then the HPC for the instrument position I_1 will be obtained. This will be 49.87 + 2.191 = 52.061 m and is entered in the appropriate column.

(2) To obtain the INITIAL reduced levels of A, B and C the staff readings to those points are now subtracted from the HPC. The relevant calculations are

$$\text{RL of A} = 52.061 - 2.505 = 49.556 \text{ m}$$

$$\text{RL of B} = 52.061 - 2.325 = 49.736 \text{ m}$$

$$\text{RL of C} = 52.061 - 1.496 = 50.565 \text{ m}$$

(3) At point C, a change point, the instrument is moved to position I_2 and a new HPC is established. This collimation level is obtained by adding the BS at C to the RL found for C from I_1. For position I_2, the HPC is 50.565 + 3.019 = 53.584 m. The staff readings to D and E are now subtracted from this to obtain their reduced levels.

(4) The procedure continues until the initial reduced level of the TBM is calculated and the misclosure found as before. With the INITIAL RL column in the table completed, only a two-sided check can be applied:

$$\Sigma(\text{FS}) - \Sigma(\text{BS}) = \text{FIRST RL} - \text{LAST RL}$$

After applying the check, any misclosure is distributed as for the rise and fall method, the ADJ RL values again being quoted to the same number of decimal places of metres as the reduced levels of the OBM and TBM used.

2.6.3 Summary of the Two Methods

The rise and fall method, although it involves more arithmetic, is preferred since it checks all the reduced level calculations whereas the collimation method does not check the calculations of the intermediate reduced levels.

However, the collimation method is quicker where a lot of intermediate sights have been taken since fewer calculations are required and it is a good method to use when setting out levels where, usually, many readings are taken from each instrument position.

2.7 Accuracy in Levelling

For normal engineering work and site surveys the *allowable misclosure* for any levelling sequence is given by

$$\textit{allowable misclosure} = \pm 5\sqrt{n}\ \text{mm}$$

where n is the number of instrument positions. For example, the allowable misclosure for tables 2.1 and 2.2 is $\pm 5\sqrt{3} = \pm 9$ mm.

When the actual and allowable misclosures are compared and it is found that the actual value is greater than the allowable value, the levelling should be repeated. If, however, the actual value is less than the allowable value, the misclosure should be distributed equally between the instrument positions as described in section 2.6.1.

2.8 Errors in Levelling

Many sources of error exist in levelling and those most commonly met in practice are discussed.

2.8.1 Errors in the Equipment

Collimation error

This can be a serious source of error in levelling if sight lengths from one instrument position are not equal, since the collimation error is proportional to the difference in sight lengths. Hence, in all types of levelling, sights should be kept equal, particularly back sights and fore sights. Also, before using any level it is advisable to carry out a two-peg test to ensure that the collimation error is as small as possible (see section 2.2.9).

Parallax

This effect, described in section 2.2.2, must be eliminated before any readings are taken.

Defects of the staff

It is possible that staff graduations may be incorrect and new or repaired staves should be checked against a steel tape. Particular attention should be paid to the base of the staff to see if it has become badly worn. If this is the case then the staff has a *zero error*. This does not affect height differences if the same staff is used for all the levelling but introduces errors if two staves are being used for the same series of levels. When using a three-section staff, it is important to ensure that the staff is properly extended by examining the graduations on either side of each joint. If these joints become loose, the staff should be returned for repair.

Tripod defects

The stability of tripods should be checked before any fieldwork commences by testing to see if the tripod head is secure, that the metal shoes at the base of each leg are not loose and that, once extended, the legs can be tightened sufficiently. When fitted, the wing nuts must be tightened before readings are taken.

2.8.2 Field Errors

Staff not vertical

Since the staff is used to measure a vertical difference between the ground and the line of collimation, failure to hold the staff vertical will result in incorrect readings. As stated in section 2.3, the staff is held vertical with the aid of periscope-type handles and a pond bubble, or it is rocked. At frequent intervals the pond bubble should be checked against a plumb line and adjusted if necessary.

Unstable ground

When the instrument is set up on soft ground and bituminous surfaces on hot days, an effect often overlooked is that the tripod legs may sink into the ground or rise slightly while readings are being taken. This alters the height of collimation and it is therefore advisable to choose firm ground on which to set up the level and tripod, and to ensure that the tripod shoes are pushed well into the ground.

Similar effects can occur with the staff and for this reason it is particularly important that change points should be at stable positions such as manhole covers, kerbstones, concrete surfaces, and so on. This ensures that the base of the staff remains at the same level during all observations to its position. If a stable point cannot be found for a change point, a *change plate* or *footplate* should be used on soft ground. Alternatively, a large stone firmly pushed into the ground can be used.

For both the level and staff, the effect of soft ground is greatly reduced if readings are taken in quick succession.

Handling the instrument and tripod

As well as vertical displacement, the HPC may be altered for any set-up if the tripod is held or leant against. When levelling, avoid contact with the tripod and only use the level by light contact through the fingertips. If at any stage the tripod is disturbed, it will be necessary to relevel the instrument and to repeat all the readings taken from that instrument position.

Instrument not level

For automatic levels this source of error is unusual but, for a tilting level in which the tilting screw has to be adjusted for each reading, this is a common mistake. The best procedure here is to ensure that the main bubble is centralised before and after a reading is taken.

When using an automatic level, the compensator may stick, causing the observed line of collimation to vary from the horizontal. Methods of overcoming this are given in section 2.2.8.

2.8.3 Reading and Booking Errors

Extra care must be taken when reading the staff since an inverted image is obtained with tilting levels. Very often this results in faulty readings being recorded by inexperienced observers, although the difficulty of reading an inverted staff image usually diminishes with practice.

Another source of reading error is sighting the staff over too long a distance, when it becomes impossible to take accurate readings. It is, therefore, recommended that sighting distances should be limited to 60 m but, where absolutely unavoidable, this may be increased to a maximum of 100 m.

Many mistakes are made during the booking of staff readings, and the general rule is that staff sightings must be carefully entered into the levelling table immediately after reading.

2.8.4 The Effects of Curvature and Refraction on Levelling

Consider a level set up at point A as in figure 2.20 and that a staff is held at B in the vertical position. If the level is adjusted correctly the line of collimation A′B′ will be horizontal and will follow the horizontal line to intersect the staff at B′. The line of collimation at the instrument position A′ will, however, be tangential to the level line through the instrument (see figure 2.20). If the Earth is considered to be a sphere, this level line is equidistant from the centre of the Earth at all points (see section 2.1.1).

Since the level line intersects the staff at B″, the staff reading at B will be too great by B′B″ (= c). This is caused by the curvature of the Earth, and to account for the incorrect staff reading a *curvature correction*, c, can be computed as follows.

In triangle A′B′O let L be the length of sight A′B′, R the radius of the earth at mean sea level, taken to be 6375 km (= OA = OB), h the height of the line of colli-

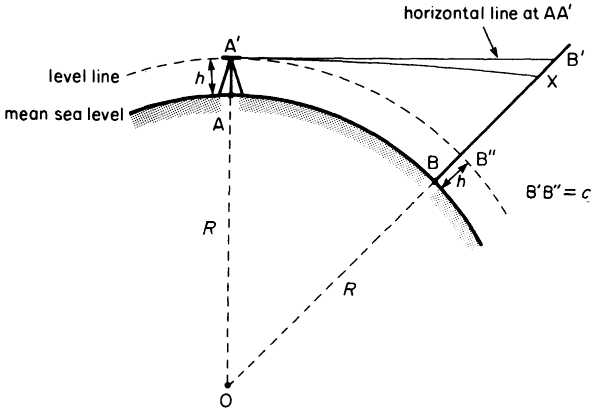


Figure 2.20 *Curvature and refraction*

mation at A above mean sea level (= AA′ = BB″ = height of the level line A′B″) and D the length of level line of sight A′B″.

By Pythagoras

$$(R + h + c)^2 = (R + h)^2 + L^2$$

or

$$c(2R + 2h + c) = L^2$$

so that

$$c = L^2/(2R + 2h + c)$$

Since c and h are small compared with R they can be ignored in the expression, hence $c = L^2/2R$.

However, L is difficult to ascertain, but since c is very small compared with L it is assumed that $L = D$.

If c is in metres and D in kilometres, $c = 0.0785\ D^2$

The effect of *refraction* is to bend the line of sight towards the Earth to follow line A′X, as shown in figure 2.20. Refraction is a variable effect depending on atmospheric conditions but for ordinary work the line A′X can be considered to be circular with a radius seven times that of R. In magnitude, therefore, refraction has a value of 1/7 that of curvature but is of opposite sign. The combined curvature and refraction correction is thus $(6/7) \times 0.0785D^2 = 0.0673D^2$. The combined correction for a length of sight of 120 m amounts to –0.001 m and the effect of both is thus negligible when undertaking levelling if sightings are less than 120 m, as should always be the case.

However, curvature and refraction effects cannot always be ignored when calculating heights using theodolite methods and this is discussed in section 3.4.

2.8.5 Weather Conditions

Windy conditions cause the level to vibrate and give rise to difficulties in holding the staff steady. Readings cannot be recorded accurately under these circumstances unless the instrument is sheltered and the minimum number of sections of the staff used.

In hot weather, the effects of refraction are serious and produce a shimmering effect near ground level. This makes it impossible to read accurately the bottom metre of the staff which, consequently, should not be used.

2.9 Summary of the Levelling Fieldwork

When levelling, the following practice should be adhered to if many of the sources of error are to be avoided.

(1) Levelling should always start and finish at points of known reduced level so that misclosures can be detected. When only one bench mark is available, levelling lines must be run in loops starting and finishing at the bench mark.

(2) Where possible, all sight lengths should be below 60 m.

(3) The staff must be held vertically by suitable use of a pond bubble or by rocking the staff and noting the minimum reading.

(4) BS and FS lengths should be kept equal for each instrument position. For engineering applications, many IS readings may be taken from each set-up. Under these circumstances it is important that the level has no more than a small collimation error.

(5) Readings should be booked immediately after they are observed and important readings, particularly at change points, should be checked.

(6) The rise and fall method of reduction should be used when heighting reference or control points.

2.10 Additional Levelling Methods

2.10.1 Inverted Staff

Occasionally, it may be necessary to determine the reduced levels of points such as the soffit of a bridge, underpass or canopy. Generally, these points will be above the line of collimation. To obtain the reduced levels of such points, the staff is held upside down in an inverted position with its base on the elevated points. When booking an inverted staff reading it is entered in the levelling table with a *minus sign*, the calculation proceeding in the normal way, taking this sign into account.

An example of a levelling line including inverted staff readings is shown in figure 2.21, table 2.3 showing the reduction of these readings.

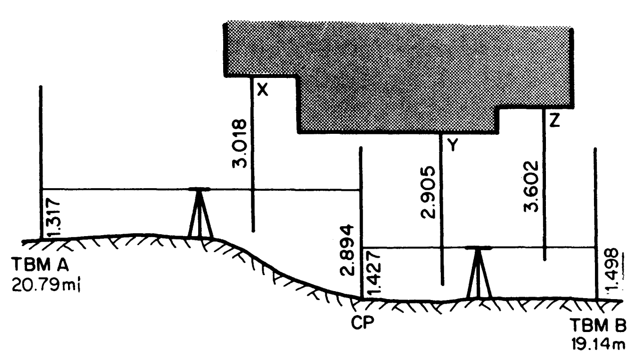


Figure 2.21 *Inverted staff levelling*

TABLE 2.3
Inverted Staff Readings
(all values in metres)

BS	IS	FS	RISE	FALL	INITIAL RL	ADJ	ADJ RL	REMARKS
1.317					20.79	-	20.79	TBM A 20.79
	-3.018		4.335		25.125	-0.001	25.12	X
1.427		2.894		5.912	19.213	-0.001	19.21	CP
	-2.905		4.332		23.545	-0.002	23.54	Y
	-3.602		0.697		24.242	-0.002	24.24	Z
		1.498		5.100	19.142	-0.002	19.14	TBM B 19.14
2.744		4.392	9.364	11.012	20.79			
		2.744		9.364	19.142			
		+1.648		+ 1.648	+ 1.648			

Each inverted reading is denoted by a minus sign and the rise or fall computed accordingly. For example, the rise from TBM A to point X is 1.317 – (–3.018) = 4.335 m. Similarly, the fall from point Z to TBM B is –3.602 – 1.498 = –5.100 m.

An inverted staff position must *not* be used as a change point since there is often difficulty in keeping the staff vertical and in keeping its base in the same position for more than one reading.

2.10.2 Reciprocal Levelling

True differences in height are obtained by ensuring that BS and FS lengths are equal when levelling. This eliminates the effect of any collimation error that may be present in the optical level used.

There are certain cases, however, when it may not be possible to take readings with equal sight lengths as, for instance, when a line of levels has to be taken over a wide gap such as a river. In these cases, the technique of *reciprocal levelling* can be adopted. Reciprocal levelling also provides an alternative to the two-peg test as a means of determining the amount of collimation error present in an optical level.

Figure 2.22 shows two points A and B on opposite sides of a wide river. The line of collimation has been assumed to be elevated above the horizontal plane. This may not be the case but does not affect the calculations. To obtain the true difference in level between A and B

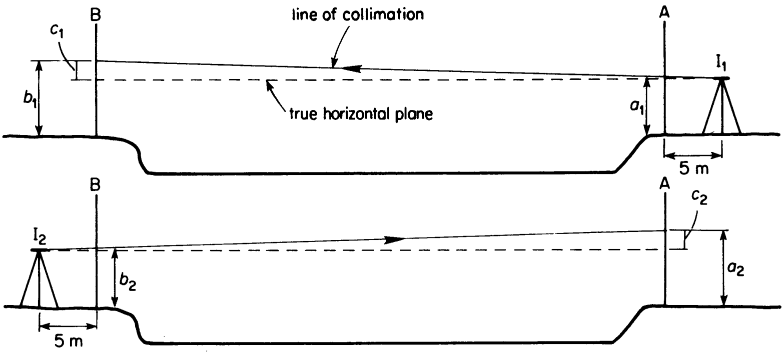


Figure 2.22 ***Reciprocal levelling***

(1) A level is placed at I_1, about 5 m from A, and a staff is held vertically at A and B. Staff readings are taken at A (a_1) and B (b_1).

(2) The level is next taken to position I_2 where readings a_2 and b_2 are recorded.

Let the true difference in level between A and B be ΔH. For instrument position I_1

$$\Delta H_{AB} = a_1 - (b_1 - c_1) \tag{2.1}$$

where c_1 is the effect of the collimation error between A and B. For instrument position I_2

$$\Delta H_{AB} = (a_2 - c_2) - b_2 \tag{2.2}$$

c_2 being the effect of the collimation error between B and A.

Adding equations (2.1) and (2.2) gives

$$\Delta H_{AB} = \tfrac{1}{2}[(a_1 - b_1 + c_1) + (a_2 - c_2 - b_2)]$$

Since the observations are taken over the same sighting distances with the same level, the effects of the collimation error will be the same for both cases. Hence $c_1 = c_2$ and

$$\Delta H_{AB} = \tfrac{1}{2}[(a_1 - b_1) + (a_2 - b_2)] \tag{2.3}$$

This result indicates that the true difference in level is the mean of the two observed differences in level recorded at I_1 and I_2.

Subtracting equations (2.1) and (2.2) gives

$$0 = a_1 - (b_1 - c_1) - (a_2 - c_2) + b_2$$

but $c_1 = c_2 = c$, hence the collimation error (e) in the level is given by

$$e = \tfrac{1}{2}[(a_2 - b_2) - (a_1 - b_1)] \text{ per } L \text{ metres} \tag{2.4}$$

where L is the horizontal distance between A and B (see also section 2.2.9).

When reciprocal levelling with one level, the two sets of observations must follow each other as soon as possible. Where this is not possible, two levels have to be used simultaneously. It must be realised that the levels should have the same collimation error or the true height difference will not be obtained.

Question

Reciprocal levelling across a river estuary using a single level and staff gave the following results between points A and B. The horizontal distance AB was measured by EDM as 55.33 m.

Instrument position	Staff position	Staff reading (m)
X	A	1.564
X	B	2.382
Y	A	2.247
Y	B	3.101

(1) Determine the RL of B if that of A is 5.79 m AOD.
(2) Calculate the collimation error in the level per 60 m of sight.

Solution

(1) *The RL of B*
With reference to figure 2.22 and equation (2.3)

$$\Delta H_{AB} = \tfrac{1}{2}[(1.564 - 2.382) + (2.247 - 3.101)]$$
$$= \tfrac{1}{2}[(-0.818) + (-0.854)] = -0.836 \text{ m}$$

Hence, RL of B = 5.79 – 0.836 = **4.95 m AOD** (rounded to 2 decimal places)
(2) *The collimation error per 60 m of sight*
From equation (2.4)

$$e = \tfrac{1}{2}[(2.247 - 3.101) - (1.564 - 2.382)] \text{ per } 55.33 \text{ m}$$
$$= -0.036 \text{ m}/55.33 \text{ m}$$

therefore

$$\textbf{collimation error/60 m} = \frac{-0.036 \times 60}{55.33} = \mathbf{-0.039\ m}$$

This is too large a value and should be removed (see section 2.2.9).

2.11 Applications of Levelling

Levelling has many uses in civil engineering construction. Levels are need principally in setting out, sectioning and contouring. Contouring is described in detail in section 2.12.

2.11.1 Setting Out

The applications of levelling in setting out are fully described in chapter 14.

2.11.2 Sectioning

This aspect of levelling is usually undertaken for construction work such as roadworks, railways and pipelines. Two types of section are often necessary and these are called *longitudinal* and *cross sections*.

On many road schemes, the longitudinal section and cross sections can be generated by a computer interrogating a Digital Terrain Model (DTM). This is discussed further in section 10.16.

Longitudinal sections

In engineering surveying, a *longitudinal section* (or *profile*) is taken along the complete length of the proposed centre line of the construction showing the existing ground level. Levelling is used to measure heights at points on the centre line so that the profile can be plotted.

Generally, this type of section provides data for determining the most economic *formation level*, this being the level to which existing ground is formed by construction methods. The optimum position for the formation level is usually found by drawing the longitudinal section with the *mass haul diagram* (see chapter 13).

The fieldwork in longitudinal sectioning normally involves two operations.

Firstly, the centre line of the section must be set out on the ground and marked with pegs. For most works, this is done by theodolite and some form of distance measurement so that pegs are placed at regular intervals (frequently 20 m) along the centre line. Further details of the techniques involved in this stage are given in chapters 9, 10 and 14. Secondly, as soon as the centre line has been established levelling can commence.

The levelling techniques adopted should all conform with the general rules already put forward and these will dictate where the level is to be set up, what bench marks are used and when change points are necessary. For longitudinal sections, it is usually sufficiently accurate to record readings to the nearest 0.01 m. Levels are taken at the following points, the object being to survey the ground profile as accurately as possible.

(1) At the top and ground level of each centre-line peg, noting the through chainage of the peg.

(2) At points on the centre line at which the ground slope changes.

(3) Where features cross the centre line, such as fences, hedges, roads, pavements, ditches and so on. At points where, for example, roads or pavements cross the centre line, levels should be taken at the top and bottom of kerbs. At ditches

and streams, the levels at the top and bottom of any banks as well as bed levels are required.

(4) Where necessary, inverted staff readings to underpasses and bridge soffits would be taken.

In order to be able to plot levels obtained in addition to those taken at the centre-line pegs, the position of each extra point on the centre line must be known. These distances are recorded by measurement with a tape, the tape being positioned horizontally between appropriate centre-line pegs.

The method of booking longitudinal sections should always be by the height of collimation method since many intermediate sights will be taken. Distances denoting chainage should be recorded for each level and most commercially available level books have a special column for this purpose. Careful booking is required to ensure that each level is entered in the level book with the correct chainage. Good use should be made of the 'remarks' column in this type of levelling so that each point can be clearly identified when plotting.

When all the fieldwork has been completed and the level book checked, the results can be plotted. The longitudinal section for a small valley is shown in figure 2.23*b* and its associated level book in figure 2.23*a*. A longitudinal section is shown in chapter 12, figure 12.12.

Cross sections

A longitudinal section provides information only along the centre line of a proposed project. For works such as sewers or pipelines, which usually are only of a narrow extent in the form of a trench cut along the surveyed centre line, a longitudinal section provides sufficient data for the construction to be planned and carried out. However, in the construction of other projects such as roads and railways, existing ground level information at right angles to the centre line is required. This is provided by taking *cross sections*. These are sections taken at right angles to the centre line such that information is obtained over the full width of the proposed construction.

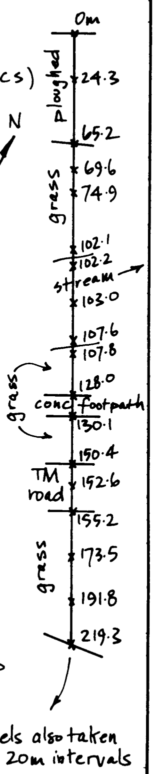
For the best possible accuracy in sectioning a cross section should be taken at every point levelled on the longitudinal section. Since this would involve a considerable amount of fieldwork, this rule is generally not observed and cross sections are, instead, taken at regular intervals along the centre line usually where pegs have been established. A right angle is set out at each cross section either by eye for short lengths or by theodolite for long distances or where greater accuracy is needed. A ranging rod is placed on either side of the centre line to mark each cross section.

The longitudinal section and the cross sections are usually levelled in the same operation. Starting at an OBM, levels are taken at each centre-line peg and at intervals along each cross section. These intervals may be regular, for example, 10 m, 20 m, 30 m on either side of the centre-line peg or, where the ground is undulating, levels should be taken at all changes of slope such that a good representation of existing ground level is obtained over the full width of the construction. The process is continued taking both longitudinal and cross section levels in the one operation and the levelling is finally closed on another known point.

(*a*)

BS	IS	FS	HPC	RL	DIST	REMARKS
	LEVELS TAKEN FOR PIPELINE CROSSING OF BRAMBER VALLEY					
	JU/WPP			19 November 1984		Level N°27 Staff LS14
0·55			35·58	35·03		OBM 35·03m AOD
	0·48			35·10	0	fence
	0·42			35·16	20	
	0·50			35·08	24·3	change of slope (CS)
	1·13			34·45	40	
	1·57			34·01	60	
	1·65			33·93	65·2	fence
	1·82			33·76	69·6	CS
	2·49			33·09	74·9	CS
	2·50			33·08	80	
	2·60			32·98	100	
	2·61			32·97	102·1	bank
	3·47			32·11	102·2	bed
	2·78			32·80	103·0	water level
	3·40			32·18	107·6	bed
	2·58			33·00	107·8	bank
	2·56			33·02	120	
2·20		2·40	35·38	33·18	128·0	edge of footpath
	2·21			33·17	130·1	edge of footpath
	2·03			33·35	140	
	2·00			33·38	150·4	edge of road
	1·92			33·46	152·6	℄
	1·98			33·40	155·2	edge of road
	1·87			33·51	160	
	1·68			33·70	173·5	CS
	1·40			33·98	180	
	0·81			34·57	191·8	CS
	0·66			34·72	200	
	0·48			34·90	219·3	fence
		0·22		35·16		OBM 35·16m AOD
Σ2·75		Σ2·62		35·03		
		2·75		35·16		
		−0·13		−0·13		

stream



(*b*)

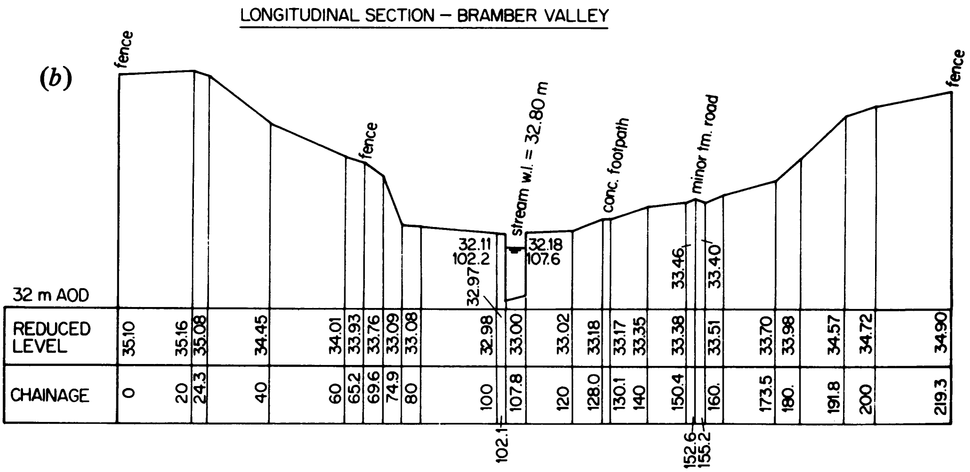


Figure 2.23 *Longitudinal section: (a) level book; (b) format for drawing section*

Such a line of levels can be very long and can involve many staff readings and it is possible for errors to occur at stages in the procedure. The result is that if a large misclosure is found, all the levelling will have to be repeated, often a soul destroying task. Therefore, to provide regular checks on the levelling it is good practice to include points of known height such as traverse stations or TBMs at regular intervals in the line of levels and then, if a large discrepancy is found, it can be isolated into a short stretch of the work.

Examples of plotted cross sections are shown in figure 2.24 and figure 12.13,

BS	IS	FS	HPC	RL	DIST	REMARKS
	BADGER LANE - EXISTING LEVELS					CHAINAGE 420m
	JU/WPP			26 November 1984		Level No 16 Staff LS 8
1·32			150·47	149·15		TBM 149·15m ROADNAIL on ℄ AT CH 420
					LEFT	
	1·43			149·04	2·3	channel
	1·26			149·21	2·3	kerb
	1·29			149·18	3·4	footpath edge
	1·13			149·34	4	ground levels
	1·20			149·27	5	ground levels
	1·42			149·05	6	ground levels
	1·68			148·79	7	ground levels
	2·11			148·36	8	ground levels
					RIGHT	
	1·45			149·02	2·1	channel
	1·28			149·19	2·1	kerb
	1·32			149·15	3·2	footpath edge
	1·15			149·32	4	ground levels
	1·10			149·37	5	ground levels
	1·01			149·46	6	ground levels
	0·63			149·84	7	ground levels
	0·46			150·01	8	ground levels
		1·32		149·15		TBM 149·15m on ℄ at CH420

Note: LEFT and RIGHT viewed when facing direction of increasing chainage

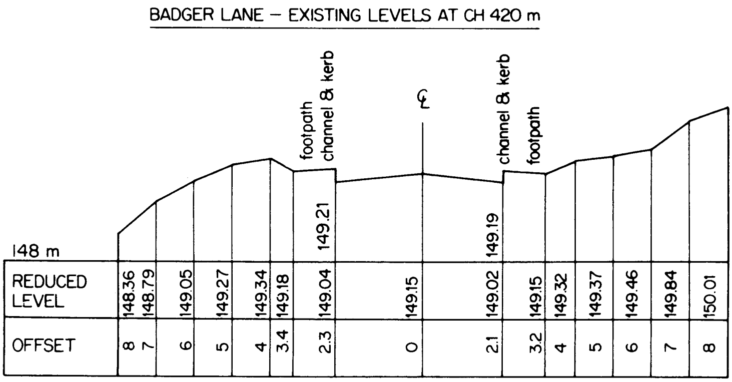


Figure 2.24 *Cross section drawing and associated level book*

and the applications of the results in earthwork calculations are considered in chapters 12 and 13.

2.12 Contouring

A *contour* is defined as a line joining points of the same height above or below a datum. These are shown so that the relief or topography of an area can be interpreted, a factor greatly used in civil engineering design and construction.

The difference in height between successive contours is known as the *contour* or *vertical interval* and this interval dictates the accuracy to which the ground is represented. The value chosen for any application depends on

(1) the intended use of the plan
(2) the scale of the plan
(3) the costs involved
(4) the nature of the terrain.

Generally, a small vertical interval of up to 1 m is required for engineering projects, for large-scale plans and for surveys on fairly even sites. In hilly or broken terrain and at small scales, a wider vertical interval is used. Very often, a compromise has to be reached on the value chosen since a smaller interval requires more fieldwork time, thus increasing the cost of the survey.

Contours can be obtained either directly or indirectly using mathematical or graphical interpolation techniques. Once plotted, in addition to indicating the relief of an area, contours can be used to provide sectional information.

2.12.1 Direct Contouring

In this method the positions of contours are located on the ground by levelling.

A level is set up in the area so that as much ground as possible can be covered by staff observations from the instrument position. A back sight is taken to a bench mark or other point of known reduced level and the height of collimation calculated. For example

$$\text{RL of bench mark} = 51.87 \text{ m AOD}$$

$$\text{BS} = 1.78 \text{ m}$$

$$\text{HPC} = 53.65 \text{ m}$$

To locate each contour the required staff readings are

$$\text{At 50 m contour} = 53.65 - 50.00 = 3.65 \text{ m}$$

$$\text{51 m contour} = 53.65 - 51.00 = 2.65 \text{ m}$$

$$\text{52 m contour} = 53.65 - 52.00 = 1.65 \text{ m}$$

$$\text{53 m contour} = 53.65 - 53.00 = 0.65 \text{ m}$$

Considering the 52 m contour, the surveyor directs the person holding the staff to move until a staff reading of 1.65 m is obtained. At this point a signal is given by the surveyor so that the staff position can be marked with a peg or chain arrow. With the staff in other positions the procedure is repeated until the complete 52 m contour is clearly marked on the ground. Only one contour is set out at any one time.

When other contours are located, care must be taken to ensure that the pegs or chain arrows of different contours are coded so that one set cannot be mistaken for another. As soon as all the contours have been marked on the ground the plan positions of all the pegs or chain arrows have to be established. This can be done by some convenient detail surveying method as given in chapter 8.

Since two operations are involved the method takes longer than others. The advantage of the technique is that it is accurate.

2.12.2 Indirect Contouring

This involves the heighting of points that do not, in general, coincide with the contour positions. Instead, the points levelled are used as a framework on which contours are later interpolated on a drawing.

Two of the more common methods of indirect contouring involve taking levels either on a regular grid pattern or at carefully selected points.

Grid levelling

The area to be contoured is divided into a series of lines forming squares and ground levels are taken at the intersection of the grid lines.

The sides of the squares can vary from 5 to 30 m, the actual figure depending on the accuracy required and on the nature of the ground surface. The more irregular the ground surface the greater the concentration of grid points. Methods of setting out the grid are numerous and one such method is considered here.

Four lines of ranging rods are set out by taping, as shown in figure 2.25, such that each ranging rod marks a grid point. Stepping of the tape will be necessary to establish a horizontal grid. To obtain the ground level at each grid point the person holding the staff lines the staff in with the two ranging rods in each direction that intersect at the point being levelled, and a reading is taken. The procedure is repeated at all grid points. Where a ranging rod marks a grid point the staff is placed against the rod and the reading taken.

When taking each reading, a suitable reference system should be adopted, for example, B8 as shown in figure 2.25, and rigorously maintained during the location of each point and the booking of each reading.

Following the fieldwork, the levels are reduced, the grid is plotted and the contours interpolated either graphically or mathematically, taking into account the general shape of the land as observed during the fieldwork.

This method of contouring is ideally suited to gently sloping areas but the setting out of the grid on a large area can take a considerable time. Furthermore, if visibility is restricted across the site, difficulties can occur when locating grid points.

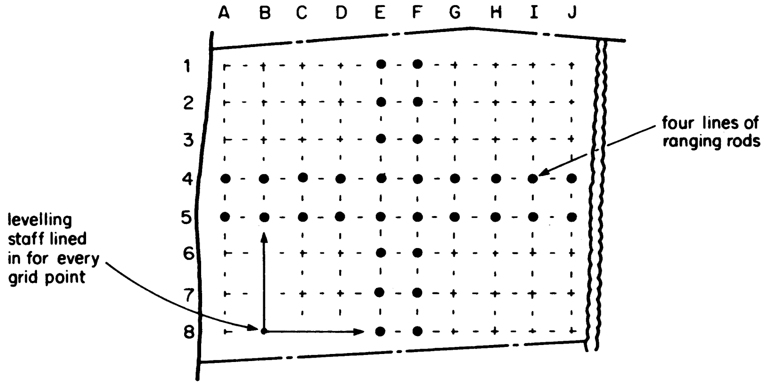


Figure 2.25 *Grid levelling*

Contours from selected points

For large areas or areas containing a lot of detail, contours can be drawn from levels taken at points of detail or at prominent points on open ground such as obvious changes of slope. These points will have been plotted on the plan by one of the methods discussed in chapter 8 and hence the position of each level, or *spot height* as it is called, is known. These spot heights will form a random pattern but the contours are drawn by interpolation as in grid levelling.

This technique is obviously well suited to detail surveying and is the usual method of contouring such surveys. As in all methods, a sufficient number of levels must be recorded so that the ground surface can be accurately represented on the site plan.

2.12.3 Interpolating Contours

In the direct method of contouring, spot heights are located at exact contour values, plotted on a plan and individual contours are drawn by joining spot heights of equal value with a smooth curve.

In the indirect methods, the plotted spot heights will not be at exact contour values and it is necessary to locate points between them on the plan which do have exact contour values. This is known as *interpolation* and it can be carried out either mathematically or graphically.

The assumption is made when undertaking interpolation that the surface of the ground slopes uniformly between the spot heights. Hence, careful positioning of spot heights in the field is essential if accurate contours are to be produced.

Mathematical Interpolation

This can be a laborious process when there are a large number of spot heights.

The height difference between each spot height is calculated and used with the horizontal distance between them to calculate the position on the line joining the spot heights at which the required contour is located.

With reference to figure 2.26, in which the positions of the 36 m and 37 m contours are to be located between two spot heights A and B of reduced level 37.2 m and 35.8 m AOD respectively, by simple proportion

$$\frac{0.2}{x} = \frac{1.2}{y} = \frac{1.4}{28.7}$$

from which

$$x = 4.1 \text{ m}$$

$$y = 24.6 \text{ m}$$

Horizontal distances x and y are scaled along line BA on the plan to fix the positions of the 36 m and 37 m contours respectively.

When all the exact contour positions have been plotted, they are joined by smooth curves as in the direct method.

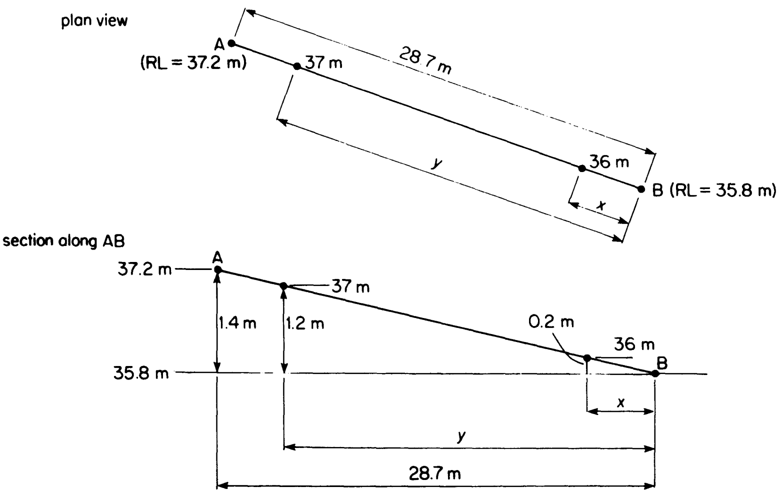


Figure 2.26 *Mathematical interpolation of contours*

Graphical Interpolation

This is a much quicker method where there are large numbers of spot heights. The procedure is as follows.

(1) A piece of tracing paper is prepared with a series of equally spaced horizontal lines as shown in figure 2.27*a*. Every tenth line is drawn heavier than the others.

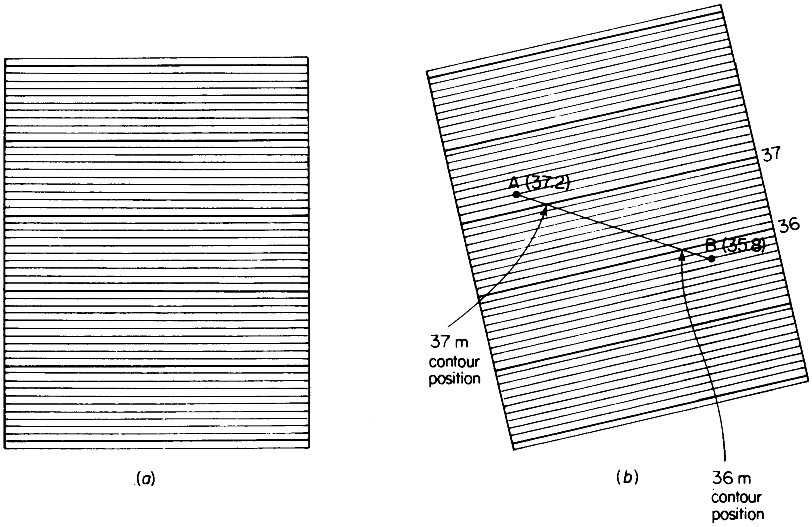


Figure 2.27 *Graphical interpolation of contours*

(2) The tracing paper is then laid between pairs of spot heights and rotated until the horizontal lines corresponding to the known spot height values pass through the points as shown in figure 2.27*b*.

(3) The heavy lines indicate the positions of the contour lines where they pass over the line joining the spot heights and these positions are pricked through on to the drawing paper using a sharp point.

(4) The reduced level of each contour is written lightly next to its position. When all the exact contour positions have been located they are joined by smooth curves.

2.12.4 Obtaining Sections from Contours

It is possible to use contours to obtain sectional information for use in the initial planning of such projects as roads, pipelines, earthworks and reservoirs.

Figure 2.28 shows part of a contoured plan of an area. The line XX is the proposed route for a straight section of a road centre line and relevant cross sections are shown at chainages 525 m to 625 m. Using the contours, the approximate shape of the longitudinal and cross sections can be obtained by scaling height and distance information from the plan at points where the section lines cut contours as shown in figure 2.29.

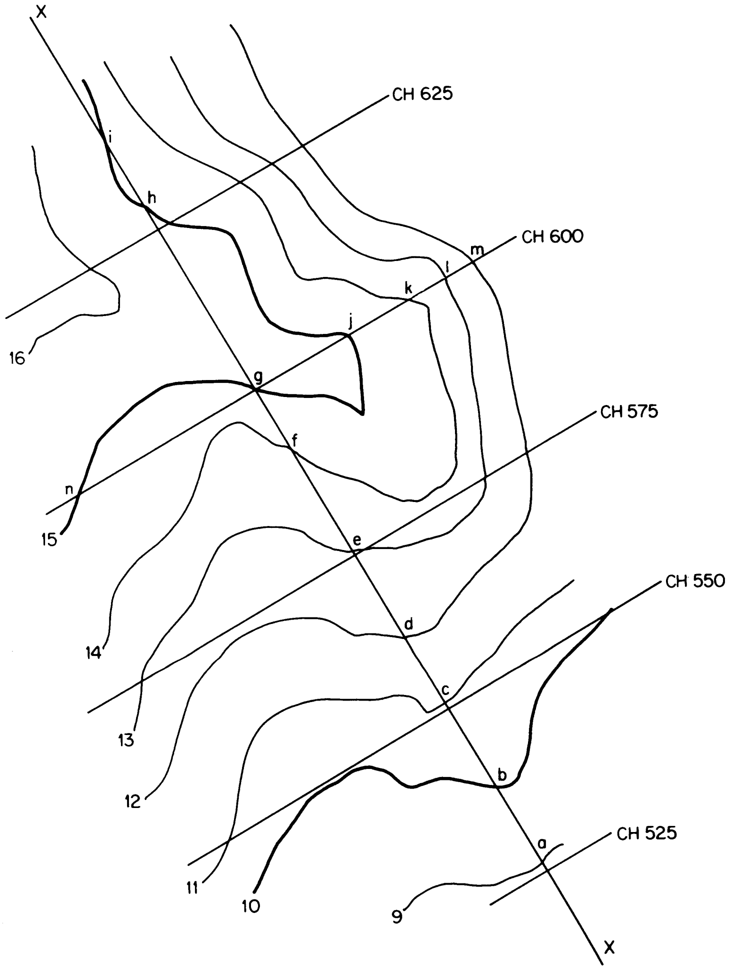


Figure 2.28 *Contoured plan*

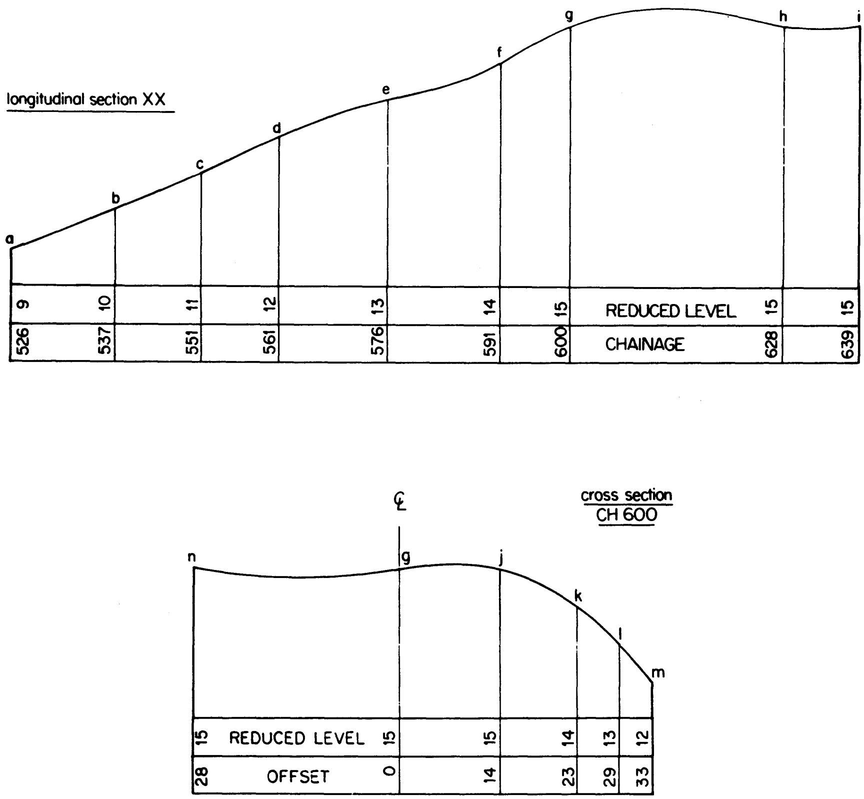


Figure 2.29 *Longitudinal and cross sections from contours*

2.13 Further Reading

M. A. R. Cooper, *Modern Theodolites and Levels, 2nd Edition* (Granada, London, 1982).

3

Theodolites and their Use

Theodolites are precision instruments used for measuring angles in the horizontal and vertical planes.

3.1 Principles of Angle Measurement

Figure 3.1 shows two points S and T and a theodolite set up on a tripod over a ground point R. The RL of S is considerably greater than that of R which, in turn, is considerably greater than that of T.

The theodolite is mounted at point L, a vertical distance *h* above R for ease of observation.

The *horizontal angle* at L between S and T is angle MLN, where M and N are the vertical projections of S and T on to the horizontal plane through L.

The *vertical angles* to S and T from L are angle SLM (an *angle of elevation*) and angle TLN (an *angle of depression*).

In order to measure horizontal and vertical angles, the theodolite must be *centred* over point R using a plumbing device and must be *levelled* to bring the angle read-

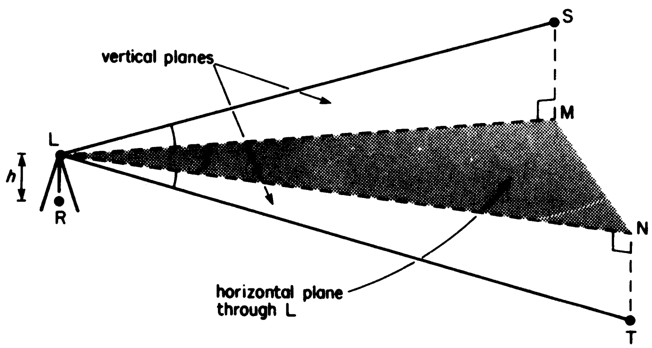


Figure 3.1 ***Horizontal and vertical angles***

ing systems of the theodolite into the appropriate planes. Although centring and levelling ensure that horizontal angles measured at point L are the same as those that would have been measured if the theodolite had been set on the ground at point R, the vertical angles from L are not the same as those from R and hence the value of h, the *height of the instrument*, must be taken into account when height differences are being calculated.

3.2 Constructional Features of Theodolites

All types of theodolite are similar in construction and the general features are shown in figure 3.2. The various parts of the theodolite and their functions will now be described.

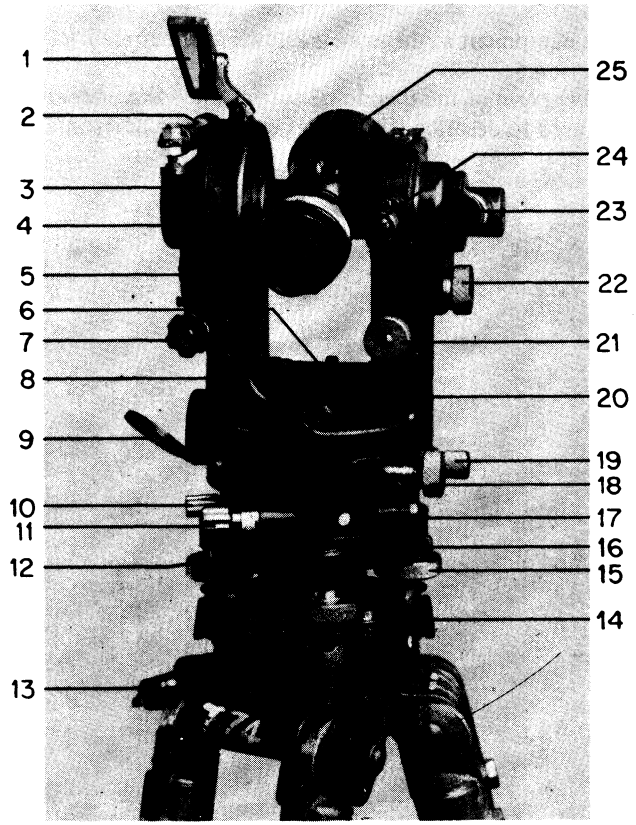


Figure 3.2 *Theodolite (on face left): 1. reflecting mirror for altitude level; 2. altitude level; 3. vertical circle; 4. main telescope focus; 5. eyepiece; 6. plate level; 7. altitude level setting screw; 8. standard; 9. reflecting mirror for reading system; 10. and 11. lower plate clamp and tangent screw; 12. horizontal circle; 13. wing nut; 14. trivet; 15. levelling footscrew; 16. fine centring clamp; 17. tribrach; 18. and 19. upper plate tangent screw and clamp; 20. standard; 21. telescope tangent screw; 22. micrometer screw; 23. circle reading telescope; 24. telescope clamp; 25. telescope objective*

3.2.1 Basic Components of a Theodolite

(1) The *trivet stage* forms the base of the instrument. In order to be able to centre the theodolite, some tripods have a *clamping screw* for fixing the trivet to the tripod. This enables the trivet to take up a variable position on the tripod head.

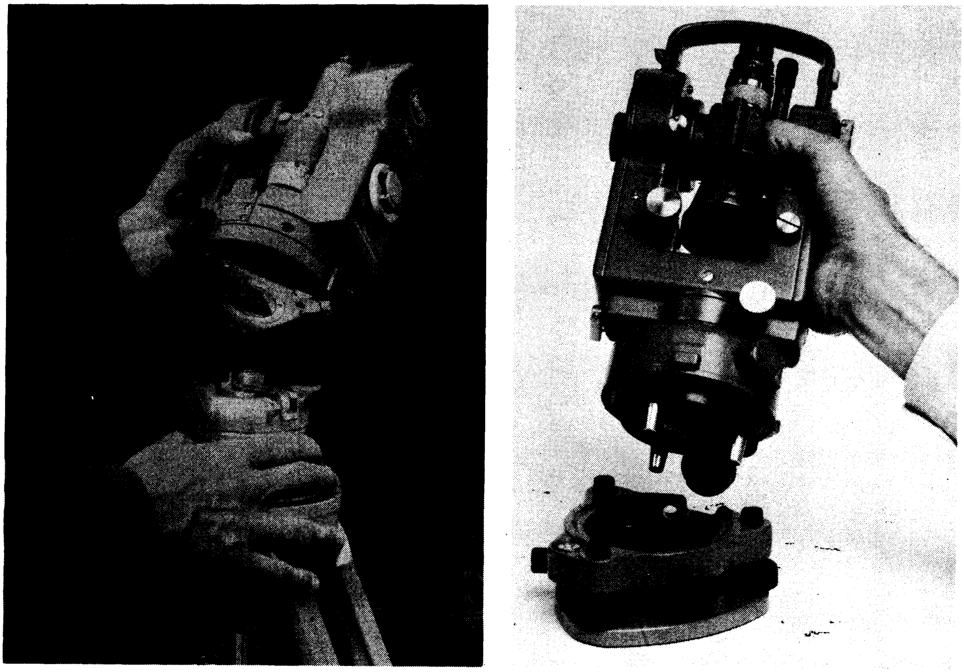
The trivet also carries the feet of three *threaded levelling footscrews*.

(2) The *tribrach* supports the remainder of the instrument and is supported in turn by the levelling footscrews. The tribrach can, therefore, be levelled independently of the trivet stage.

Many instruments have the facility for detaching the upper part of the theodolite from the tribrach. A special target or other piece of equipment can then be centred in exactly the same position occupied by the theodolite, as shown in figure 3.3. This ensures that angular and linear measurements are carried out between the same positions, thereby reducing errors, particularly *centring errors* (see section 3.3.4).

The use of the equipment in this way is known as the *three-tripod system* and is described in section 5.7.

(3) The *lower plate* of the theodolite carries the *horizontal circle*. The term *glass arc* is often used to describe theodolites because the horizontal and vertical



(a) (b)

Figure 3.3 *Forced centring: (a) Kern system (courtesy Kern and Co. Ltd); (b) Wild system* circles on which the angle graduations are photographically etched are made of glass. Many types of glass arc theodolite are available, varying in reading precision from 1′ to 0.1″ although 20″ and 1″ reading theodolites are most commonly used in engineering surveying.

(4) An *upper plate* or *alidade* is recessed into and can be free to rotate within the lower plate. The upper plate carries the horizontal circle reading system. The various circle reading systems are described in section 3.2.2.

The *plate level* is also fixed to the upper plate which is identical to the spirit level of an optical level as shown in figure 2.7 and is mounted on the upper plate.

(5) On earlier models, the upper and lower plates each have a separate clamp and *slow motion* or *tangent screw* and, to distinguish these, the upper plate screws are milled and the lower plate screws are serrated.

For this type of theodolite, if the lower plate is clamped and the upper plate free, rotation in azimuth gives different readings on the horizontal scale. If the lower plate is free and the upper plate clamped, rotation in azimuth retains the horizontal scale reading, that is, the horizontal circle rotates.

Most modern theodolites do not have a lower plate clamp and tangent screw. There is a facility for altering the position of the horizontal circle within the instrument and this is achieved using one control only, called the *horizontal circle setting screw*.

(6) The upper plate also supports two frames called the *standards*. Supported in bearings carried on the standards is the *trunnion* or *transit axis* of the theodolite.

(7) Attached to the trunnion axis are the *main telescope*, the *circle reading telescope*, the *micrometer screw* and the *vertical circle*. The micrometer screw is used when horizontal and vertical circle readings are being taken (see section 3.2.2).

The *focusing screw* of the telescope is fitted concentrically with the barrel of the telescope and the diaphragm (and also the circles) can be illuminated for night or tunnel work.

When the main telescope is rotated in altitude about the trunnion axis from one direction to face in the opposite direction it has been *transitted*. The side of the main telescope, viewed from the eyepiece, containing the vertical circle is called the *face*.

The construction of the main telescope is similar to those used in optical levels as described in section 2.2 and it can be clamped in the vertical plane, a *tangent screw* being provided for fine vertical movement. Fine horizontal movement is achieved using the upper plate tangent screw (and lower plate tangent screw, if fitted).

(8) The vertical circles of theodolites are not all graduated in the same way and it is necessary to reduce the readings to obtain the required vertical angles (see section 3.2.2). Some of the graduation systems in use are shown in figure 3.4.

(9) Attached to the vertical circle is the *altitude level*. In order that vertical angles can be recorded with respect to the horizontal plane through the trunnion axis, the altitude bubble is centred prior to reading the vertical circle by turning the *altitude level setting screw*. Alternatively, the altitude level may be of the coincidence type as described for optical levels in section 2.2. Movement of the altitude level setting screw does not alter the direction of pointing of the main telescope.

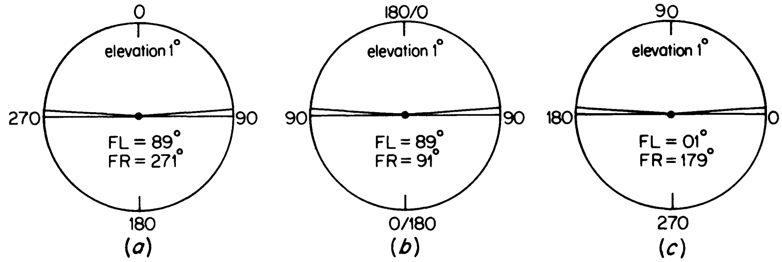


Figure 3.4 *Vertical circle graduations*

On some theodolites, the altitude bubble may be replaced by an *automatic vertical index* which requires no manual setting. Once such a theodolite has been levelled using the plate level it is ready to read vertical angles.

(10) The arrangement of the axes of the theodolite is shown in figure 3.5. When the instrument is *levelled*, the vertical axis is made to coincide with the vertical at the point where the instrument is set up. This is achieved by using the levelling footscrews and plate level as described in section 3.3.1.

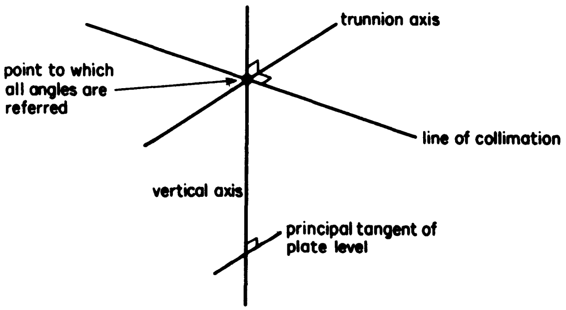


Figure 3.5 *Theodolite axes*

(11) *Centring* the theodolite involves setting the vertical axis directly above a particular point. A hook is provided so that a plumb line can be suspended underneath the tribrach or centring clamp in order to roughly centre the instrument within 5 mm. Fine centring is done using the *optical plummet*. This consists of a small eyepiece, either built into the tribrach or the alidade, the line of sight of which is deviated by 90° so that a point corresponding to the vertical axis can be viewed on the ground. The two types of optical plummet are shown in figure 3.6.

Some instrument tripods can be fitted with a *centring rod* as a further method of improving centring accuracy. The rod either forms part of the tripod or is detachable.

As shown in figure 3.7, the top of the rod is attached to an adaptor plate which, when the rod is moved, slides on the tripod head. A pond level fixed to the rod enables it to be set vertically. When the rod is placed in a vertical position with its

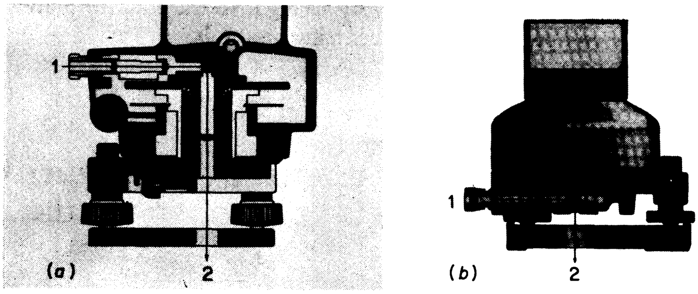


Figure 3.6 *Sections through lower halves of theodolites showing: (a) optical plummet mounted on alidade; (b) optical plummet mounted on tribrach. 1. Eyepiece; 2. line of sight along vertical axis (courtsey of Wild Heerbrugg (UK) Ltd)*

(a) (b)

Figure 3.7 *Kern centring rod. (a) Section through centring rod: 1. adaptor plate for theodolite; 2. tripod head with spherical surface; 3. trivet; 4. clamp; 5. pond level; 6. centring rod; 7. tripod leg. (b) Centring rod in use (courtesy Kern and Co. Ltd)* base centred over a station mark, the rod and hence the adaptor plate is clamped and the theodolite is centred automatically by fixing it to the adaptor plate.

Table 3.1 shows a comparison of three centring methods.

TABLE 3.1

Method	Advantages	Disadvantages	Accuracy of centring over point
Suspended Plumb bob.	Cheap	Difficult to use in windy conditions.	1 - 2 mm
Optical Plummet.	Not affected by weather.	Must be in good adjustment. Takes longer to use.	1 mm
Centring Rod.	Quicker than optical plummet. Useful in hilly terrain.	Extra piece of equipment to carry.	1 mm

3.2.2 Circle Reading Methods

Since the standards are hollow on modern theodolites and the circles are made of transparent glass, it is possible to direct light into the instrument and through the circles using prisms, and to magnify and read the images using a circle reading telescope. There are three types of scale reading systems in common use.

Optical scale reading system

In this reading system, a fixed plate of transparent glass, upon which are etched two scales from 0′ to 60′, is mounted in the optical path of the light directed through the horizontal and vertical circles, as shown in figure 3.8 for the Wild T16.

When viewed through the circle eyepiece these two scales are seen superimposed on portions of the horizontal and vertical circles and are highly magnified. Readings are obtained directly from the fixed scales, as shown for the Wild T16 instrument in figure 3.9. The length of each scale corresponds exactly to the distance between the images of the circle graduations and there is no possibility of ambiguity.

This system is often referred to as the *direct reading* system since no micrometer adjustment is required (see next sections) to obtain readings.

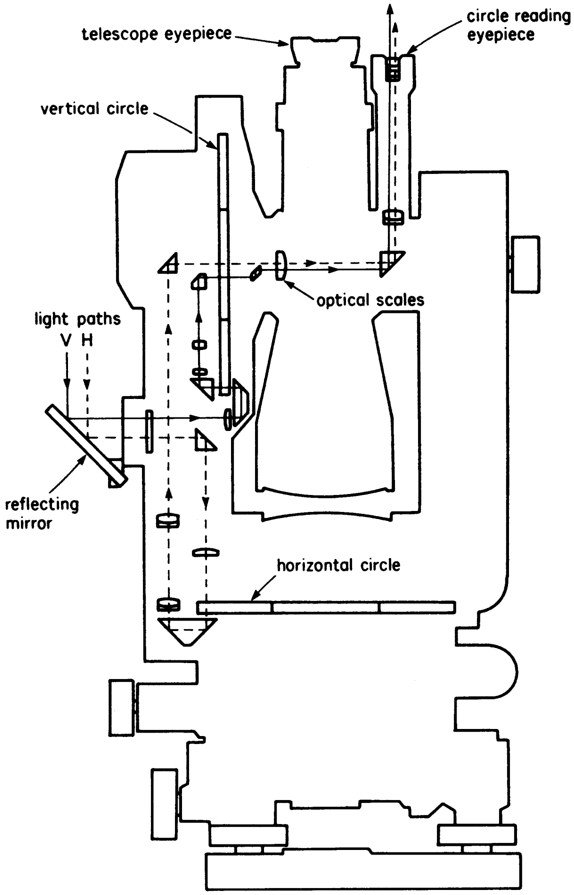


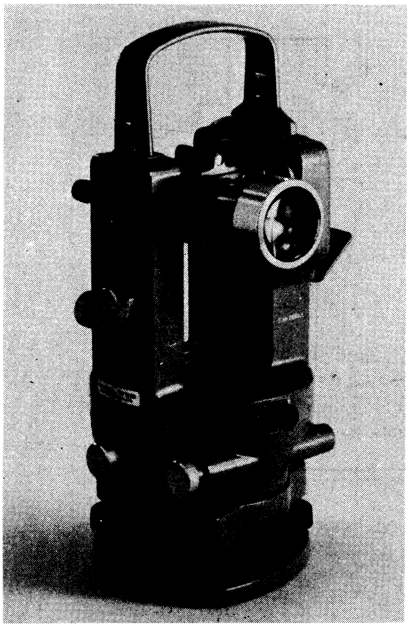
Figure 3.8 *Wild T16 reading system (courtesy Wild Heerbrugg (UK) Ltd)*

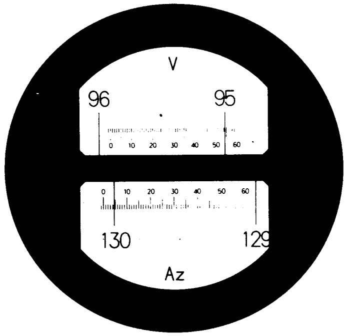
Only one side of the circle is seen by this method and any circle eccentricity is not eliminated but these errors are likely to be less than the reading accuracy which is direct to 1′ with estimation to 0.1′.

Single-reading optical micrometer reading system

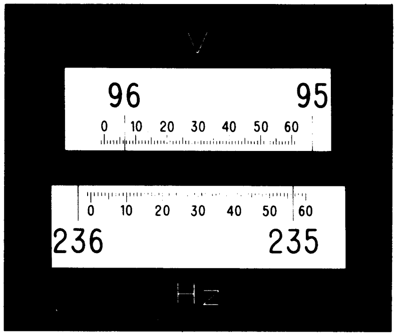
This reading system does not have a fixed scale mounted in the optical path. Instead, an optical micrometer is built into the instrument on the standard containing the reading telescope. The micrometer arrangement and optical paths for such a theodolite are shown in figure 3.10 for the Wild T1.

The important part of the optical micrometer is the parallel-sided glass block. This can be rotated by turning the micrometer screw to which the block is geared.





vertical circle reading 95° 54.4'
horizontal circle reading 130° 04.6'



vertical circle reading 96° 06.5'
horizontal circle reading 235° 56.4'

Figure 3.9 *Wild T16 and reading examples (courtesy Wild Heerbrugg (UK) Ltd)*

If light from the circles enters the block at a right angle it will pass through undeviated, as shown in figure 3.11*a*. If, however, the block is rotated through an angle θ, the light will be deviated by an amount, d, parallel to the incident ray (see figure 3.11*b*). It can be shown that the amount of this shift, d, is directly proportional to the angle of rotation of the block, θ. This principle is used to obtain angle readings using index marks built into the optical path which are seen superimposed on the circle images. Suppose the horizontal scale is set as in figure 3.12; the reading will be $62^\circ + x$. By turning the micrometer screw and hence the parallel-sided glass block, the 62° graduation can be displaced laterally until it appears to coincide with the index marks as in figure 3.12. The horizontal scale has, therefore, effectively been moved an amount x proportional to the rotation of the glass block. This angular rotation is recorded on a micrometer scale attached to the glass block, the relevant portion of which is seen in the circle eyepiece.

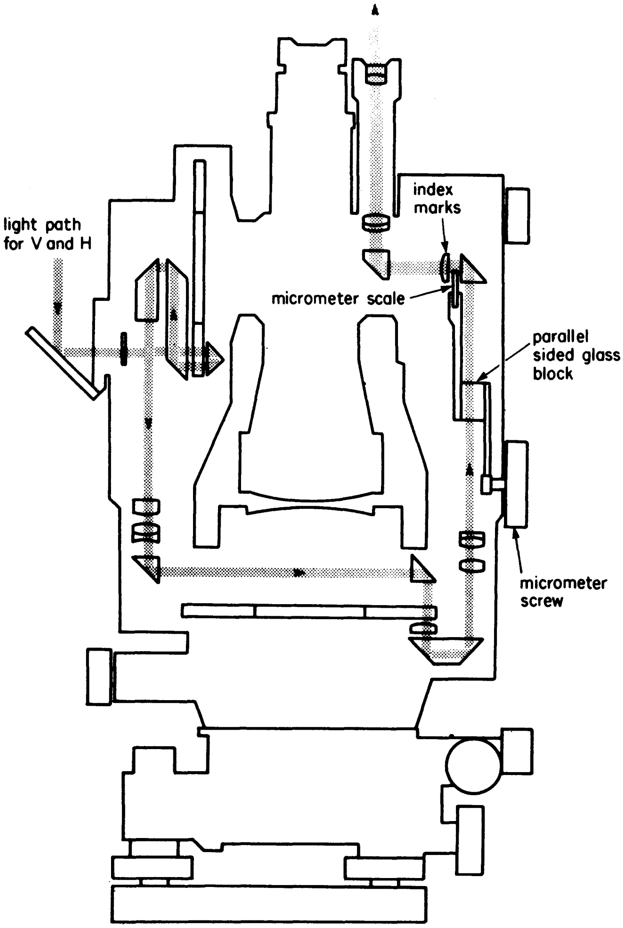


Figure 3.10 *Wild T1 reading system (courtesy Wild Heerbrugg (UK) Ltd)*

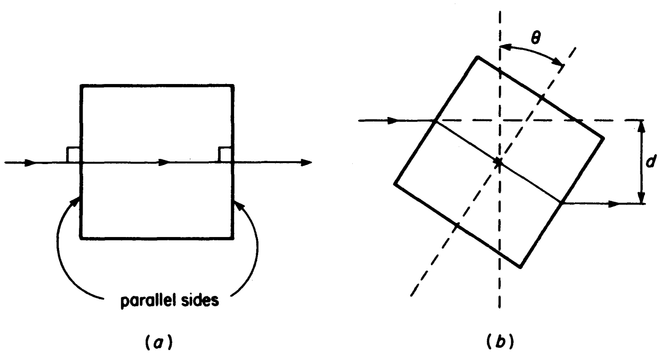


Figure 3.11 *Parallel-sided glass block*

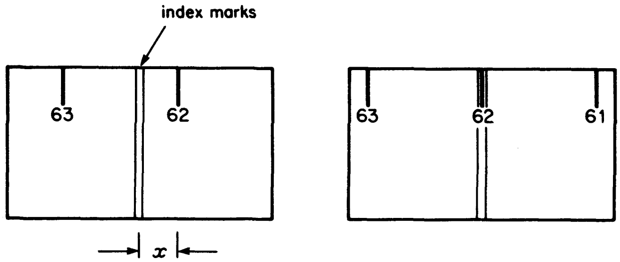


Figure 3.12

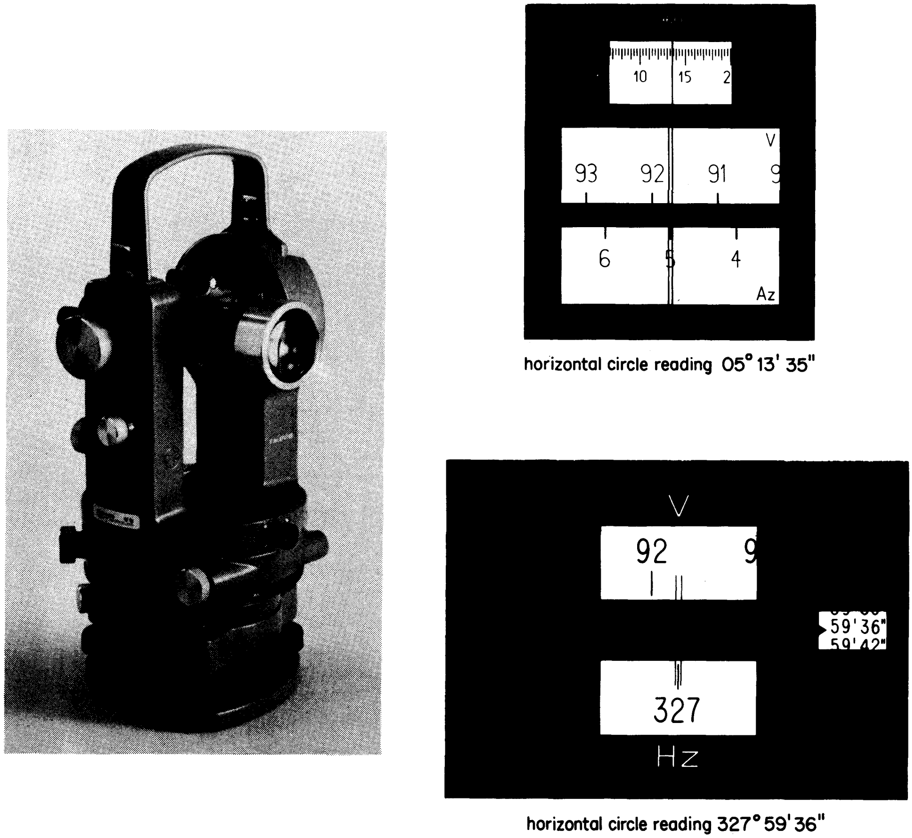


Figure 3.13 *Wild T1 and reading examples (courtesy Wild Heerbrugg (UK) Ltd)*

The circle readings are made up of two parts, as shown in figure 3.13. As with the optical scale system, only one side of the circle is read, hence the term *single-reading* optical micrometer. The reading accuracy of the Wild T1 is direct to 20″ with estimation to 5″, or direct to 6″ with estimation to 3″.

Double-reading optical micrometer reading system

When reading horizontal (or vertical) angles on a theodolite, if the opposite sides of the circle are read simultaneously and meaned, the effects of any *circle eccentricity errors* are eliminated. This is demonstrated in figure 3.14, which shows the horizontal circle in plan view (the same theory can be applied to the vertical circle). If the

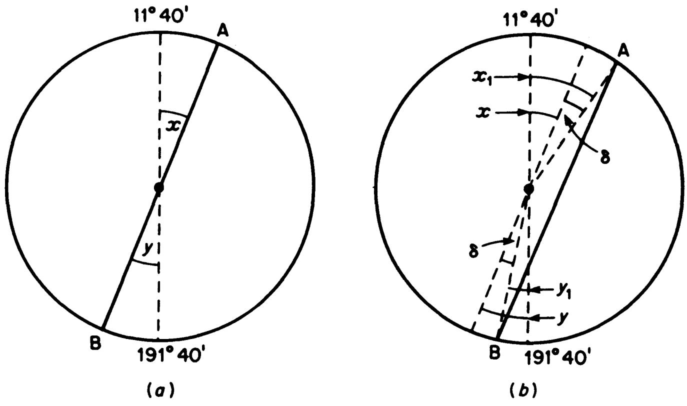


Figure 3.14 *Circle eccentricity errors*

line joining two diametrically opposed points A and B on the upper plate corresponds with the centre of the horizontal circle on the lower plate (figure 3.14*a*) the readings at A (11° 40′ + x) and B (191° 40′ + y) would be recorded with a mean of 11° 40′ + $\frac{1}{2}(x + y)$. The B degrees are not taken into account. In this case, $x = y$ and, in theory, only one of the readings, A or B, need be taken. In most cases, however, there is a small displacement between the centre of the horizontal circle and the line joining the two points A and B (see figure 3.14*b*). The readings here would be A (11° 40′ + x_1) and B (191° 40′ + y_1), where $x_1 = x + \delta$ and $y_1 = y - \delta$, δ being the eccentricity error. If only one side of the circle is read, the error δ will be included, but if the mean is taken, this gives

$$11° \, 40' + \tfrac{1}{2}(x_1 + y_1) = 11° \, 40' + \tfrac{1}{2}((x + \delta) + (y - \delta))$$
$$= 11° \, 40' + \tfrac{1}{2}(x + y)$$

which is the same value obtained when assuming that $\delta = 0$.

If only one side of the circle is read, circle eccentricity errors will be eliminated by reading on both faces *provided the value of* δ *remains constant*. Reading only one side of the circle is an acceptable practice when using 20″ theodolites but, when using theodolites reading to 1″ or less, the effects of a variable circle eccentricity must be accounted for and this is achieved with the double-reading optical micrometer system. However, instead of noting two separate readings at opposite ends of the circles and calculating the mean, an arrangement is used whereby only one reading is necessary for each setting. With reference to figure 3.14*b*, imagine that the readings at the opposite ends of the circle were to be deviated through optical paths so that they were viewed simultaneously in the circle eyepiece, as shown in figure 3.15.

If the 11° 40′ and 191° 40′ graduations are made to appear to coincide using parallel-sided glass blocks, they will be optically deviated by amounts x_1 and y_1 respectively. The mean reading of 11° 40′ + $\frac{1}{2}(x_1 + y_1)$ will be free from circle eccentricity errors provided a suitable optical micrometer system is used to record the mean of the lateral displacement of the two circle images.

The optical arrangement of the double reading Wild T2 is shown in figure 3.16. The optical micrometer basically consists of two parallel-sided glass plates which rotate equally in opposite directions. The images of each side of the circle are brought into coincidence by rotating a single micrometer screw geared to these plates, the amount of rotation being recorded, via a cam, on the moving micrometer scale. The optics are designed so that the micrometer scale reading is the mean of the two circle displacements, free of eccentricity error. The method of reading the Wild T2 is illustrated in figure 3.17.

Double-reading theodolites can use slightly differing optical micrometer systems. In some there is only one parallel plate and this displaces an image of one side of the circle so that it coincides with the other side which is stationary. The parallel plates are replaced by wedges in some designs.

Very often, these instruments do not show the horizontal and vertical scales together in the same field of view. A change-over switch is provided to switch from one scale to the other.

3.2.3 Electronic Theodolites

A theodolite that produces a digital output of direction or angle is known as an *electronic theodolite*, examples of which are shown in figure 3.18.

When using such an instrument, the operator does not have to look into a circle reading telescope or set a micrometer screw to give a reading; instead the circle readings are displayed automatically using light emitting diodes (LEDs) or a liquid crystal display (LCD) similar to those found on hand-held calculators.

To obtain a digital readout, the glass circles of electronic theodolites are coded by photographically and chemically etching on to them a pattern similar to that shown in figure 3.19. Within the alidade, incident light from internal sources is directed through each coded circle and the light pattern emerging through a circle is detected by an array of photodiodes. These are fitted opposite the light sources on the other side of each circle.

When measuring horizontal angles, the alidade is rotated with respect to the horizontal circle which causes the amount of incident light passing through the horizontal circle to vary in proportion to the angle through which the alidade has been rotated. This varying light intensity is changed into an electrical signal by the

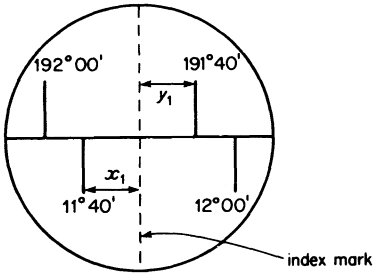


Figure 3.15

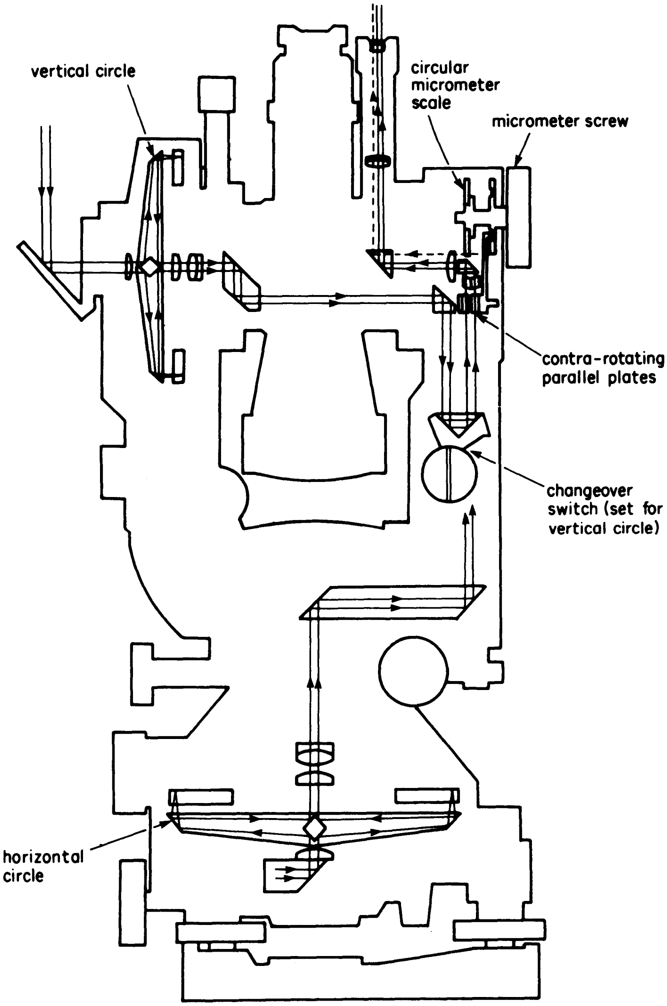
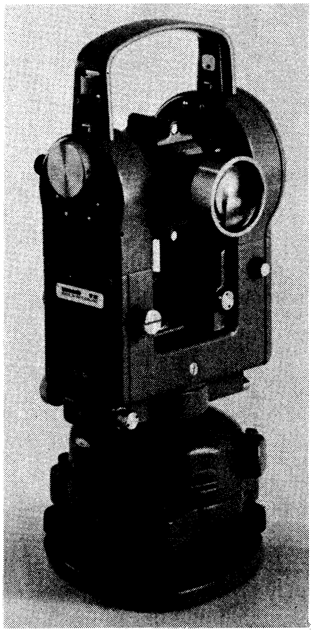
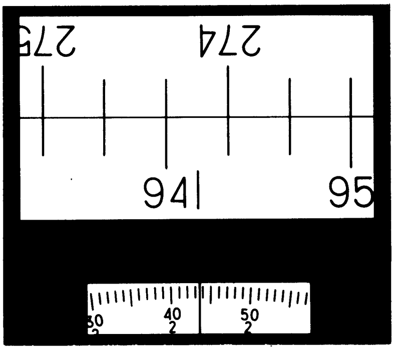
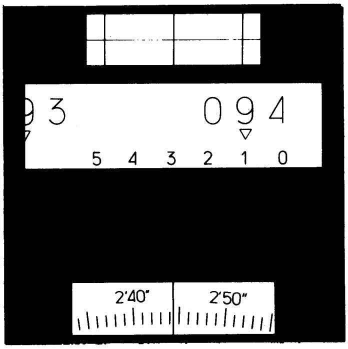


Figure 3.16 *Wild T2 reading system (courtesy Wild Heerbrugg (UK) Ltd)*





horizontal or vertical circle reading 94° 12'44"



horizontal or vertical circle reading 94° 12'44"

Figure 3.17 *Wild T2 and reading examples (courtesy Wild Heerbrugg (UK) Ltd)*

photodiodes and this in turn is passed to a microprocessor which converts the signal into an angular output.

A similar system is used for vertical angles.

Both horizontal and vertical outputs can be either displayed and recorded manually or transferred directly into a data storage unit (see section 4.13.2) for future computations.

All types of electronic theodolite can have an EDM unit fitted to enable angles and distances to be measured simultaneously. If the same microprocessor controls both the circle reading and EDM functions, then the instrument is referred to as an *electronic tacheometer*. These instruments are described in section 4.13.2.

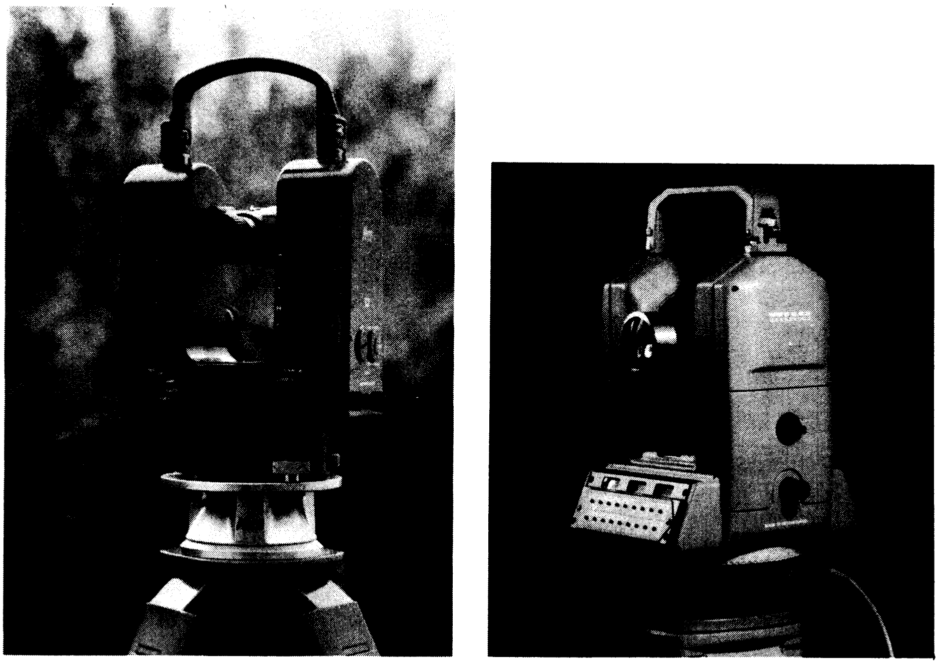


Figure 3.18 *Electronic theodolites (courtesy of Kern and Co. Ltd and Wild Heerbrugg (UK) Ltd)*

3.3 Field Procedures

3.3.1 Setting up the Theodolite

The following notes assume that the theodolite is to be erected over a ground mark which is a peg driven into the ground. A nail driven into the top of the peg defines the exact position for centring. The mark is referred to as station W.

(1) Leaving the instrument in its case, the tripod is first set up over station W. The legs are placed an equal distance from the peg and their height adjusted to suit the observer. The tripod head should be made as level as possible by eye.

(2) The plumb bob supplied with the instrument is suspended from the tripod head so that it is immediately above the station mark. The line and distance between the plumb bob and the peg are noted and each leg, in turn, is moved parallel to this line for the appropriate distance. The tripod head is relevelled and the procedure repeated until the plumb bob is within a few millimetres of the nail.

The tripod legs are now firmly pushed into the ground and all the wing nuts tightened.

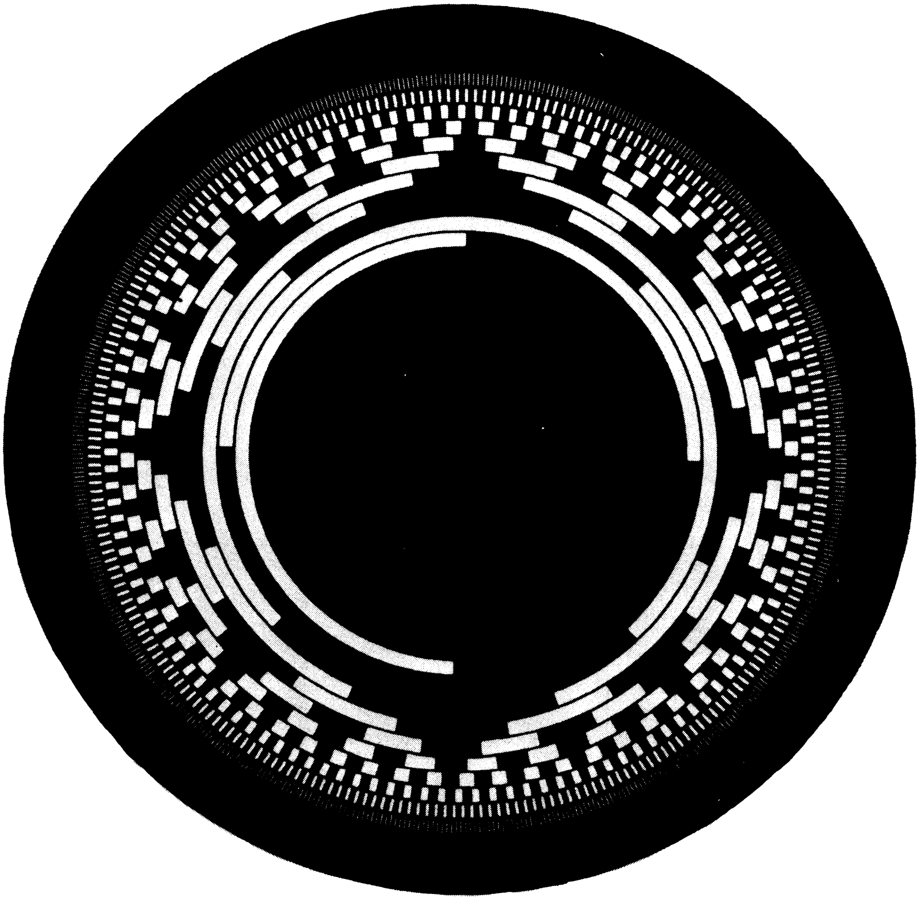


Figure 3.19 *Binary coded glass circle*

(3) The theodolite is carefully taken out of its case, its exact position being noted to assist in replacement, and is securely attached to the tripod head. Whenever carrying a theodolite, always hold it by the standards and *not* the telescope.

(4) If *centring* is to be carried out using an optical plummet, it is essential that the line of collimation of the plummet is vertical, that is, the vertical axis of the theodolite must be truly vertical before the optical plummet can be used. Therefore, the theodolite is *finely levelled* before centring takes place.

The *fine levelling* procedure is as follows.

(a) The alidade is rotated until the plate level is parallel to two footscrews as in figure 3.20*a*. These footscrews are turned until the plate level bubble is brought to the centre of its run. The levelling footscrews should be turned in opposite directions simultaneously, remembering that the bubble will move in a direction corresponding to the movement of the left thumb.

(b) The alidade is turned through 90° clockwise (see figure 3.20*b*) and the bubble centred again using the third footscrew only.

(c) The above operations are repeated until the bubble is central in positions (a) and (b).

(d) The alidade is now turned until it is in a position 180° clockwise from (a) as in figure 3.20*c*. The position of the bubble is noted.

(e) The alidade is turned through a further 90° clockwise as in figure 3.20*d* and the position of the bubble again noted.

(f) If the bubble is still in the centre of its run for both conditions (d) and (e) the theodolite is level and no further adjustment is needed. If the bubble is not central it should be off centre by the same amount in both conditions (d) and (e). This may be, for example, two divisions to the left.

(g) To remove the error, the alidade is returned to its initial position (figure 3.20*a*) and, using the two footscrews parallel to the plate level, the bubble is placed in such a position that *half* the error is taken out; for example, in the case quoted, so that it is one division to the left.

(h) The alidade is then turned through 90° clockwise as in figure 3.20*b* and half the error again taken out such that, for the example quoted, it is again one division to the left.

(i) Conditions (g) and (h) are repeated until half the error is taken out for both positions.

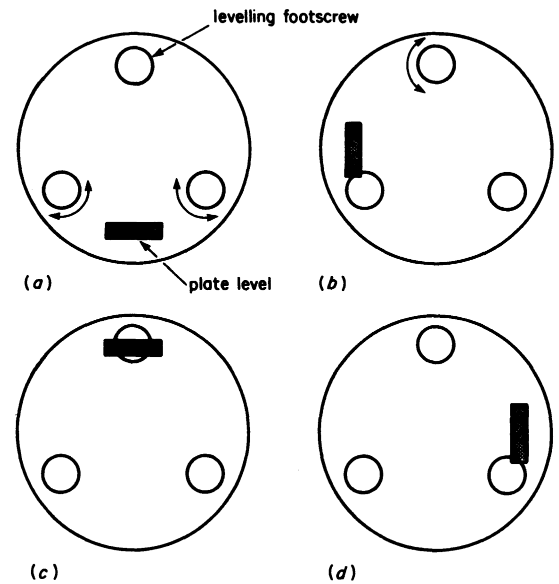


Figure 3.20 *Fine levelling*

(j) The alidade is now slowly rotated through 360° and the plate level bubble should remain in the same position.

The theodolite has now been finely levelled and the vertical axis of the instrument is truly vertical.

From the foregoing discussion, it can be seen that the plate bubble is not necessarily in the centre of its run when the theodolite is level. However, as long as the bubble is always set up at the position found by this procedure the theodolite can be used perfectly satisfactorily until a *permanent adjustment* of the plate level can be carried out (see section 3.5).

(5) The centring clamp directly underneath the trivet is now released and this allows the theodolite to move a small amount on the tripod head. The ground mark is viewed through the optical plummet and the alidade positioned such that the plummet cross hairs intersect the ground mark. This is known as *fine centring*.

For some types of theodolite, centring is carried out by releasing a clamp directly underneath the tribrach, thus enabling the theodolite to be moved on the tribrach.

Whatever centring method is used, this may upset the fine levelling and, after each centring movement, it may be necessary to relevel the instrument and to re-centre.

(6) If the theodolite centring is carried out using a plumb bob, the centring clamp is released and the instrument positioned so that the plumb bob is suspended directly over the station mark. This is done *before* the theodolite is levelled.

(7) When the theodolite has been levelled and centred, parallax is eliminated by accurately focusing the cross hairs against a light background and focusing the instrument on a distant target (see section 2.2.2).

At this stage the theodolite is ready for reading angles and this procedure is described in section 3.3.2.

Having completed all the angular observations, the theodolite is carefully removed from the tripod head and returned to its case. The tripod is folded up *after* releasing the wing nuts. Before removing the theodolite from the tripod head, the three footscrews should be set central in their runs.

If other stations are to be occupied, the theodolite must *never* be left on the tripod when moving between stations since this can distort the axes and, if the operator trips and falls, the instrument may be severely damaged.

3.3.2 Reading, Booking and Calculating Angles

It is assumed that the theodolite has been set up over a point W as described in section 3.3.1 and that the horizontal and vertical angles to three distant points X, Y and Z are to be measured. Targets must be set up at these points and suitable types of target are discussed in section 5.5.2.

Horizontal angles

The observation procedure starts with the selection of one station as the *reference object* (RO). This point will be the most reliable and preferably the most distant of all the stations to be sighted. All the horizontal angles are referred to this point as shown in table 3.2, in which the horizontal angles XWY and XWZ are required.

TABLE 3.2
Angle Booking

STATION W

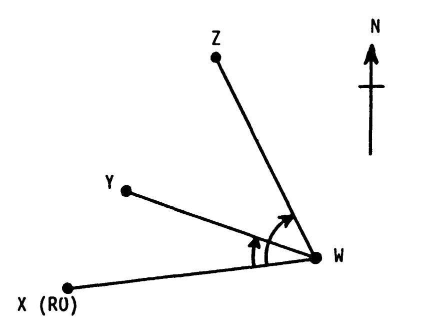
HORIZONTAL CIRCLE

POINT	FACE LEFT			FACE RIGHT			MEAN			REDUCED DIRECTION			FINAL ANGLES
X (RO)	00	03	50	180	04	30	00	04	10	00	00	00	
Y	17	22	10	197	23	10	17	22	40	17	18	30	
Z	83	58	50	264	00	00	83	59	25	83	55	15	
X (RO)	90	12	30	270	13	30	90	13	00	00	00	00	
Y	107	31	10	287	32	10	107	31	40	17	18	40	
Z	174	07	30	354	08	40	174	08	05	83	55	05	
													Clockwise
													$X\hat{W}Y$ =
													17° 18' 35"
													Clockwise
													$X\hat{W}Z$ =
													83° 55' 10"

VERTICAL CIRCLE

POINT	TARGET HEIGHT	FACE LEFT			FACE RIGHT			REDUCED FACE LEFT			REDUCED FACE RIGHT			FINAL ANGLE		
X	1.16 m	88	10	30	271	51	20	01	49	30	01	51	20	+ 01	50	25
Y	1.52 m	89	34	50	270	27	30	00	25	10	00	27	30	+ 00	26	20
Z	1.47 m	92	44	10	267	13	40	02	44	10	02	46	20	- 02	45	15

DIAGRAM



REMARKS

SURVEY TITLE	A3 Road Improvement
DATE	11 July 1985
THEODOLITE HEIGHT	1.48 m
THEODOLITE NUMBER	816421
OBSERVER	J.U.
BOOKER	W.F.P.

The procedure is as follows.

(1) Set the horizontal circle reading to approximately 00° 10′ using the horizontal circle setting screw and sight the RO on *face left* (FL) obtaining exact coincidence between the vertical hair and the target set up at the RO. Read the horizontal circle.

(2) Swinging the telescope to the *right*, sight Y and Z in turn and record the horizontal circle readings at both sightings.

(3) Transit the telescope so that the theodolite is now on *face right* (FR), sight Z and record the horizontal circle reading.

(4) Swing *left* to Y and X recording the horizontal circle readings.

(5) At this stage, *one round* of angles has been completed. The theodolite is changed to face left and *the zero* changed by setting the horizontal circle to read approximately 90° when sighting X, the RO. The setting of the minutes should also be different from that of the first round.

(6) Repeat steps (2) to (4) inclusive to complete a second round of angles.

At least two rounds of angles should be taken at each station in order to detect errors when the angles are computed since each round is independently observed. Both rounds must be computed and compared *before* the instrument and tripod are moved. It is important to use the same point on the vertical hair when sighting. Do not use the intersection of the cross hairs since setting coincidence here is time consuming and unnecessary.

Vertical angles

These should be read after the horizontal angles to avoid confusion in the booking of the results. Vertical angles can be observed in any order of the stations. General points in the procedure are given below for the booking shown in table 3.2.

(1) It is usual to take all face left readings first. The horizontal hair is used for setting coincidence in this case and it is again not necessary to use the intersection of the cross hairs but it is important that the same point on the horizontal hair is used on both faces.

(2) The altitude bubble (if fitted) must be brought to the middle of its run before every reading is taken. When reading the vertical circle it is necessary for the recorded angles to be reduced. This is shown in table 3.2.

(3) Readings should again be taken on both faces but in this case only *one* round of angles need be taken.

Booking and calculating angles

Table 3.2 shows the horizontal and vertical angle booking and calculation for this example.

The mean horizontal circle readings are obtained by averaging the FL and FR readings. To simplify these calculations, the degrees of the FL readings are carried through and only the minutes and seconds values are meaned. These mean horizontal circle readings are then reduced to the RO in the Reduced Direction column to give the horizontal angles. The final horizontal angles are obtained by meaning the values obtained from each round.

From the readings obtained, it can be seen that the vertical circle is graduated as shown in figure 3.4a and therefore it is necessary to reduce the FL and FR readings to ascertain whether the angles are either elevation (+) or depression (–). The final vertical angles are obtained by meaning these reduced FL and FR readings.

In addition, the following procedures should be adopted.

(1) For both types of angle, the stations are booked in clockwise order. This should be the order of observation.

(2) If a single figure occurs in any reading, for example, a 2 or a 4, this should be recorded as 02 or 04. If a mistake is made the number should always be re-written, for example, if a 4 is written and should be 5, this should be recorded as ~~4~~ 5, not 5.

(3) Never copy out observations from one field sheet to another.

(4) The booker, as readings are entered, should be checking for consistency in *horizontal collimation* on horizontal angles and *vertical collimation* on vertical angles. These effects are described in section 3.5 and, referring to table 3.2, the checks are as follows.

For horizontal angles, the difference (FL – FR) is computed for each sighting considering minutes and seconds only. This gives the following results for the first round.

Station	(FL – FR)
X	–00′ 40″
Y	–01′ 00″
Z	–01′ 10″

This shows the readings to be satisfactory since (FL – FR), for a 20″ theodolite, should agree to within 1′ for each point observed, considering the magnitude and the sign of the difference. If, for example, the difference for station Z was –11′ 10″ then an *operator error* of 10′ is immediately apparent. In such a case, the readings for station Z would be checked.

A similar process is applied to the vertical circle readings to check for consistency in vertical collimation.

For a 1″ theodolite, (FL – FR) should agree within a few seconds, depending on the length of sight and the type of target used.

3.3.3 Importance of Observing Procedure

The method of reading angles may be thought to be somewhat lengthy and repetitious but it is necessary to eliminate certain instrumental errors discussed in section 3.5.

(1) By taking the mean of FL and FR readings for horizontal and vertical angles, the effects of *horizontal collimation, vertical collimation* and *trunnion axis dislevelment* are all eliminated.

(2) Observing on both faces also removes any errors associated with an *inclined diaphragm* provided the same positions are used on each cross hair for observing.

(3) In addition to circle eccentricity (see section 3.2.2), the horizontal circle axis may not coincide with the vertical axis. Furthermore, the graduations may be irregular. These effects are very small and are reduced by changing the zero between rounds. Two rounds of angles would not be sufficient to reduce these errors significantly: the reason for observing two rounds is to provide a check on observations.

(4) The effect of an *inclined vertical axis* (plate level not set correctly) is not eliminated by observing on both faces but any error arising from this is negligible if the theodolite is carefully levelled. Since this error is proportional to the tangent of the vertical angle of the sighting, care should be taken when recording angles to points at significantly different elevations.

3.3.4 The Effect of Miscentring the Theodolite

Suppose a horizontal angle ABC (θ) is to be measured but, owing to miscentring, the theodolite is set up over B′ instead of B as in figure 3.21. As a result, horizontal angle AB′C is measured.

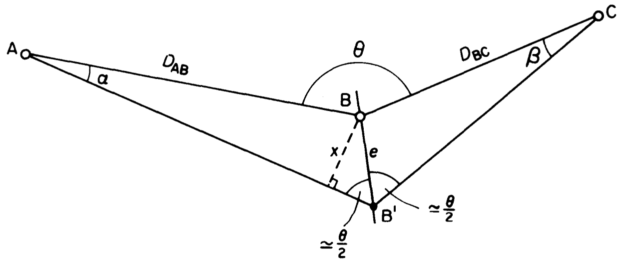


Figure 3.21 *Miscentring*

The miscentring distance, e, is equal to distance BB′ and the maximum error in θ will occur when distance e bisects the observed angle AB′C as shown in figure 3.21.

The total error in angle ABC will be $(\alpha + \beta)$.

With reference to figure 3.21, since α is very small it can be assumed that

$$x = D_{AB}\,\alpha \;(\alpha \text{ in radians})$$

But

$$\sin(\theta/2) = (x/e)$$

Hence

$$x = e \sin(\theta/2)$$

Therefore

$$\alpha = (e/D_{AB}) \sin(\theta/2) \;(\alpha \text{ in radians})$$

since α (in radians) = α'' sin 1″ for small angles.

Then

$$\alpha'' = \frac{e}{D_{AB}} \sin(\theta/2) \operatorname{cosec} 1'' \; (\operatorname{cosec} 1'' = 206\,265) \qquad (3.1a)$$

Similarly

$$\beta'' = \frac{e}{D_{BC}} \sin(\theta/2) \operatorname{cosec} 1'' \qquad (3.1b)$$

The significance of this is that for relatively small values of D, α and β will be large. Therefore, care must be taken in centring when sighting over short distances. The worked example in section 3.7.1 illustrates this point.

3.4 Height Measurement by Theodolite (Trigonometrical Heighting)

If the reduced levels of several points some distance apart in hilly terrain are required then levelling can be a very tedious task. However, if an accuracy in the order of ±100 mm is acceptable and the points are visible from other points of known elevation, an alternative and much quicker method is to use a theodolite.

The technique of using a theodolite to obtain heights is known as *trigonometrical heighting* and involves the measurement of the vertical angle between a known point and the point of unknown height. Since the slope distance between the points is required in the calculation, trigonometrical heighting is best undertaken with the aid of an EDM system fitted to the theodolite.

The EDM reflector is set up over the point being sighted and the vertical angle to it is measured with a theodolite reading to 1″ or better. Several measurements of the vertical angle are taken and the mean value used.

Because the slope length of the line of sight between the points may be in the order of kilometres, it is necessary to take into consideration the effects of the curvature of the Earth and the refraction of light by the atmosphere.

In *single ended trigonometrical heighting* (see section 3.4.1), the observations are taken from one end of the line only and curvature and refraction must be allowed for in the calculations.

In *reciprocal trigonometrical heighting* (see section 3.4.2), observations are taken from each end of the line but not at the same time. Curvature and refraction must again be taken into account.

In *simultaneous reciprocal trigonometrical heighting* (see section 3.4.3), the observations are taken from each end of the line at exactly the same time in order that the curvature and refraction effects will cancel each other out in the calculations. The simultaneous method also provides a means of measuring the coefficient of atmospheric refraction.

If care is taken, the accuracy of heights obtained by each method over distances of several kilometres can be as follows

single ended method	±200 mm
reciprocal method	±100 mm
simultaneous method	± 50 mm

The accuracy of heights obtained by single ended and reciprocal methods depends to a great extent on the value of the coefficient of atmospheric refraction used in the calculations.

3.4.1 Single Ended Observations

Figure 3.22 shows a single ended observation carried out between points A and B. In this case, the theodolite is reading an angle of elevation, θ, between the horizontal at T and the direction of the telescope pointing along TS.

If the height difference between A and B was calculated using θ, an incorrect result would be obtained because of curvature and refraction effects. Between A and B the curvature of the Earth is represented by vertical distance FG which is the difference between the level and horizontal lines through T over distance AB. Refraction causes the theodolite line of sight to be deviated along TP although vertical angles are measured along TS.

From figure 3.22, the height of B (H_B) relative to the height of A (H_A) is given by

$$H_B = H_A + i + L \sin [(+\theta) + (\gamma - \alpha)] - b$$

where

i = height of theodolite trunnion axis above point A
b = height of EDM reflector above point B
L = slope distance between A and B obtained from the EDM readout
θ = vertical angle obtained from the theodolite T
γ = curvature angle between A and B
α = refraction angle between A and B.

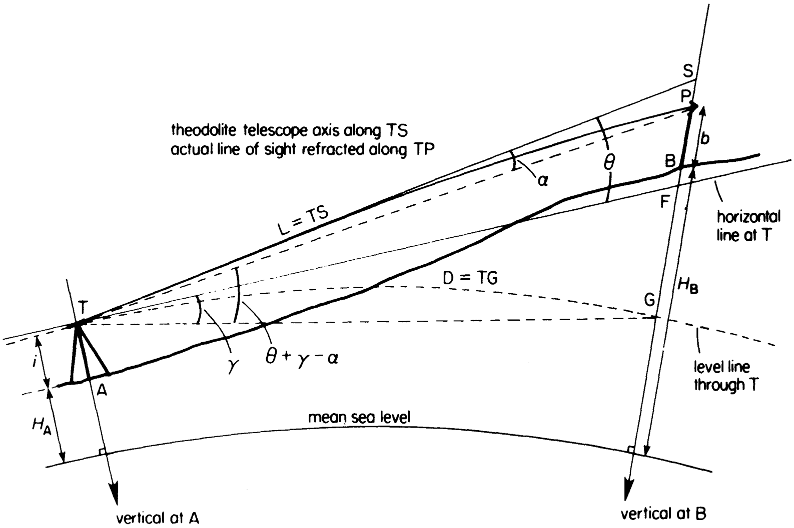


Figure 3.22 *Single ended observation*

The angle $\delta\theta = (\gamma - \alpha)$ can be considered as a correction to the observed vertical angle to account for curvature and refraction. The correction $\delta\theta$ is obtained as follows with reference to figure 3.22.

$$\gamma \simeq \frac{\mathrm{FG}}{D} \text{ rad} \quad \text{and} \quad \alpha \simeq \frac{\mathrm{SP}}{D} \text{ rad}$$

where $D = L \cos \theta$ = horizontal distance between A and B. Therefore

$$\delta\theta = \frac{1}{D}(\mathrm{FG} - \mathrm{SP}) \text{ radians} \tag{3.2}$$

From section 2.8.4

$$\mathrm{FG} = D^2/2R$$

where R = average radius of the Earth between A and B, and from figure 3.22

$$\mathrm{SP} = \alpha D$$

The coefficient of atmospheric refraction, k, is given by

$$k = (\alpha/\beta)$$

where β = the angle subtended by AB at the centre of the Earth. Therefore

$$\mathrm{SP} = k\beta D = \frac{k\,D^2}{R} \quad (\text{since } \beta \simeq D/R)$$

Substituting for FG and SP in equation (3.2) gives

$$\delta\theta = \frac{1}{D}\left[\frac{D^2}{2R} - \frac{kD^2}{R}\right] \text{ radians}$$

From which

$$\delta\theta = \frac{D(1-2k)}{2R(\sin 1'')} \text{ seconds} \tag{3.3}$$

This leads to the following general equation for single ended trigonometrical heighting, which can be applied to all cases:

$$H_{\mathrm{B}} = H_{\mathrm{A}} + i - b + L \sin[(\pm\theta) + \delta\theta] \tag{3.4}$$

where $+\theta$ is used for an angle of elevation and $-\theta$ is used for an angle of depression.

When using equation (3.3) in Great Britain, the value of R is often taken as 6375 km and the value of k is usually assumed as 0.07. However, the value of k is open to some doubt because of atmospheric uncertainties and, as a result, it is recommended that for any particular survey the simultaneous method should be used wherever possible.

If any single ended observations are necessary, the value of k which should be applied can be calculated from simultaneous readings taken at approximately the same time.

The worked example in section 3.7.2 shows how single ended observations are used to calculate the heights of points.

3.4.2 Reciprocal Observations

Although each direction is not necessarily observed on the same day, the accuracy obtained from this method will be improved if the same time of day is used for each observation since the k values should be comparable.

The two observations are each computed as for the single ended method, the final height difference being obtained by meaning the two individual height differences.

3.4.3 Simultaneous Reciprocal Observations

In this method two theodolites are required in order that observations can be taken from each end at exactly the same time to eliminate the effect of refraction. Since the sighting distances in each direction are also exactly the same, the effect of curvature is also eliminated.

The worked example in section 3.7.2 shows how the simultaneous reciprocal method can be used both to calculate heights of points and to calculate a value of k for use in single ended observations.

3.5 Adjustments of a Theodolite

There are two types of adjustment necessary, these being the *station* or *temporary adjustments* and the *permanent adjustments*.

The station adjustments are carried out each time the theodolite is set up and have been described in section 3.3.1. These adjustments are fine centring, fine levelling and removing parallax.

Figure 3.5 shows the arrangement of the axes of the theodolite when it is in *perfect adjustment*. This configuration is rarely achieved in practice and the purpose of the permanent adjustments is to set the instrument so that the axes take up positions as close as possible to those shown in figure 3.5. The permanent adjustments should be carried out when first using an unknown instrument and periodically thereafter since the setting of the axes tends to alter with continual use of the theodolite.

3.5.1 Plate Level Adjustment

The aim of this test is to set the vertical axis truly vertical when the plate level bubble is central. In other words, the plate level principal tangent is to be set perpendicular to the vertical axis.

(1) Level the theodolite as described in section 3.3.1 until the plate level bubble is in the same position for a complete 360° rotation of the alidade. The bubble is not necessarily in the middle of its run.

(2) In this position the vertical axis is truly vertical and only the bubble is out relative to its main divisions.

(3) Bring the bubble back to the centre of its run using the bubble capstan screws (see figure 2.7).

3.5.2 Horizontal Collimation Adjustment

The aim of this test is to set the line of sight (line of collimation) perpendicular to the trunnion axis. This error can be detected as soon as face left and face right readings have been taken. For example, the theodolite used in table 3.2 has a horizontal collimation error since FL and FR readings do not differ by *exactly* 180°. This error is caused by a displaced vertical hair and can be reduced using the following adjustment.

(1) Set up and level the theodolite on reasonably flat ground such that there is a clear view of approximately 100 m on either side. A marking arrow is placed at point A, approximately 100 m from the instrument, and the vertical hair aligned to it on *face left* (see figure 3.23*a*).

(2) The telescope is transitted and a second arrow placed at point B, again approximately 100 m from the instrument. The theodolite is now on *face right* (see figure 3.23*b*).

(3) Keeping *face right*, the telescope is rotated in azimuth and exact coincidence obtained at A (figure 3.23*c*).

(4) The telescope is again transitted so that it is now *face left*. If there is no collimation error, B will be intersected. Usually, however, B is not intersected and a third arrow is placed on the line of sight next to B, at C (figure 3.23*d*). The distance BC represents four times the collimation error and, if it is small, it is usually ignored (see section 3.3.3).

(5) If BC is greater than 10 mm, the error should be removed and to help in this a fourth arrow is placed at D such that CD = DF.

(6) The vertical hair is moved using the diaphragm adjusting screws until point D is intersected.

(7) To check the adjustment, transit the telescope, reintersect A and retransit. The vertical hair should exactly intersect F.

3.5.3 Diaphragm Orientation

In carrying out the horizontal collimation adjustment, the diaphragm is moved. This may upset the setting of the vertical hair in a plane perpendicular to the trunnion axis so that it no longer sweeps out a vertical plane when the trunnion axis is horizontal (see section 3.5.5).

Assuming that a horizontal collimation adjustment has just been completed, the following procedure should be adopted.

(1) Relevel the instrument carefully and sight A on either face.

(2) Move the telescope up and down while observing A. If the vertical hair stays on point A then it is set correctly.

(3) If adjustment is necessary, the diaphragm is moved until the vertical hair remains on point A while moving the telescope in altitude.

Tests 3.5.2 and 3.5.3 are interdependent and both tests are undertaken consecutively until a satisfactory result is obtained for each.

The diaphragm is constructed by the instrument manufacturer so that the horizontal and vertical hairs are perpendicular. Setting the vertical hair vertical therefore sets the horizontal hair in a horizontal plane.

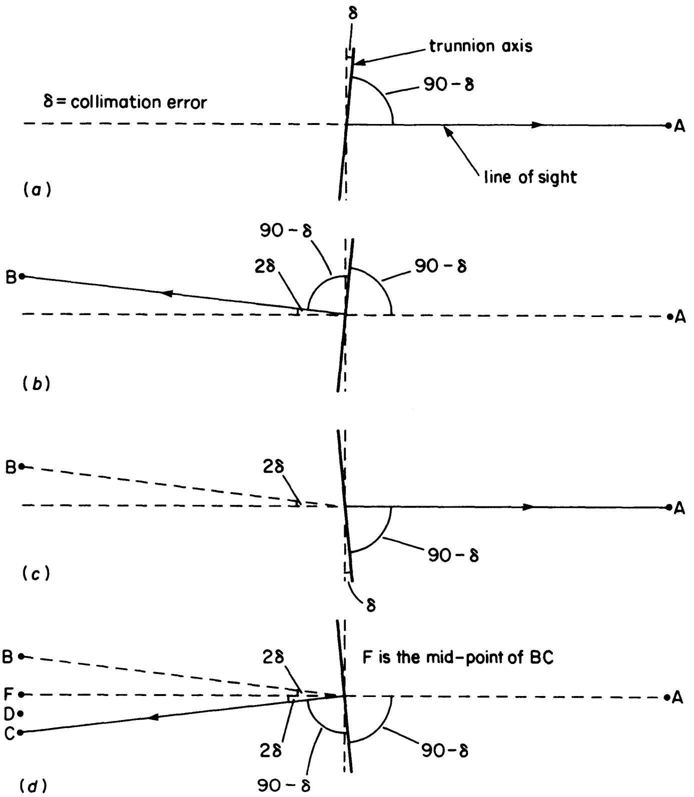


Figure 3.23 *Plan view of horizontal collimation test*

3.5.4 Adjustment of the Vertical Indices (Index Error or Vertical Collimation)

The aim of this test is to set the vertical circle to some multiple of 90° when the line of sight is horizontal and the altitude bubble is central (if fitted).

This error is shown by the difference between FL and FR vertical angles. The adjustment is as follows.

(1) Direct the telescope on to a vertically held levelling staff positioned about 50 m away from the instrument, and centralise the altitude bubble.

(2) Using the vertical circle tangent screw, set the vertical circle to read exactly a multiple of 90° and note the staff reading.

(3) Transit the telescope and repeat (1) and (2).

(4) If the indices are in adjustment, the staff readings will be the same. If a difference of more than 10 mm occurs, the error has to be removed, otherwise it is ignored (see section 3.3.3).

(5) The horizontal hair is set at the mean of the two readings using the telescope tangent screw. This causes the vertical angle reading to move from the hori-

zontal. The angle is reset to the horizontal using the altitude bubble setting screw. This causes the altitude bubble to move off centre. The bubble is recentred using the bubble capstan screws.

For a theodolite with an automatic vertical index, the manufacturer's handbook should be consulted for the correct adjustment procedure.

3.5.5 Trunnion Axis Dislevelment

The purpose of this test is to set the trunnion axis perpendicular to the vertical axis. The trunnion axis will then be horizontal when the instrument is levelled. If the trunnion axis is not horizontal the telescope will not define a vertical plane and this will give rise to incorrect vertical and horizontal angles.

In glass arc theodolites, it is rare for this not to be the case owing to their excellent construction and, consequently, most do not provide for this adjustment. However, satisfactory results will be obtained by meaning FL and FR readings.

3.5.6 Adjustment of the Optical Plummet (if fitted)

The line of collimation of an optical plummet must coincide with the vertical axis of the theodolite when it is levelled. Two tests are possible, depending on the type of instrument used.

If the optical plummet is on the alidade and can be rotated about the vertical axis (figure 3.6a)

Secure a piece of paper on the ground below the instrument and make a mark where the optical plummet intersects it. Rotate the alidade through 180° in azimuth and make a second mark. If the marks coincide, the plummet is in adjustment. If not, the correct position of the plummet axis is given by a point midway between the two marks.

Consult the instrument handbook and adjust either the diaphragm (cross hairs) or objective lens on the optical plummet.

If the optical plummet is on the tribrach and cannot be rotated without disturbing the levelling (figure 3.6b)

Set the theodolite on its side on a bench with its base facing a wall and mark the point on the wall intersected by the optical plummet. Rotate the tribrach through 180° and again mark the wall. If both marks coincide, the plummet is in adjustment. If not, the plummet diaphragm should be adjusted to intersect a point midway between the two marks.

3.6 Further Reading

M. A. R. Cooper, *Modern Theodolites and Levels, 2nd Edition* (Granada, London, 1982).

3.7 Worked Examples

3.7.1 Miscentring a Theodolite

Question

From a traverse station Y, the horizontal angle between two stations X and Z was measured with a 1″ theodolite as 123°18′42″.

The theodolite at Y was miscentred by 9 mm and the horizontal distances YX and YZ were measured as 69.41 m and 47.32 m respectively.

Calculate the maximum angular error in angle XYZ owing to the theodolite being miscentred.

Solution

For the maximum angular error, equation (3.1) gives

$$\alpha = \frac{0.009}{69.41} \sin\left[\frac{123^\circ}{2}\right] 206\,265 = 23.5''$$

$$\beta = \frac{0.009}{47.32} \sin\left[\frac{123^\circ}{2}\right] 206\,265 = 34.5''$$

Therefore

maximum angular error = $(\alpha + \beta)$ = 58″

3.7.2 Simultaneous Reciprocal Trigonometrical Heighting

Question

Simultaneous reciprocal trigonometrical heighting observations were taken from station A to station B and from station B to station A as follows

At station A		At station B	
Instrument height	= 1.49 m	Instrument height	= 1.53 m
Target height	= 1.50 m	Target height	= 1.75 m
Vertical angle to B	= −01°17′26″	Vertical angle to A	= +01°17′03″

Immediately after these simultaneous observations, the following single ended observation was taken from station A to a station C:

Target height at C = 1.96 m
Vertical angle to C = −02°24′53″

The height of station A was 117.43 m AOD and the slope distances AB and AC were measured using EDM equipment as 1863.12 m and 1543.28 m, respectively. The radius of the Earth is 6375 km. Calculate

(1) the height of station B

(2) the value of the coefficient of atmospheric refraction which prevailed during the observations

(3) the height of station C.

Solution

(1) The height of station B
From the observation at station A, equation (3.4) gives

$$H_B = H_A + i - b + L \sin [(-\theta) + \delta\theta] \tag{3.5}$$

But $\delta\theta$ can be ignored when simultaneous observations are taken, therefore

$$H_B = 117.43 + 1.49 - 1.75 + 1863.12 \sin(-01°17'26'')$$

From which

$$H_B = 75.208 = 75.21 \text{ m}$$

From the observation at station B, equation (3.4) gives

$$H_A = H_B + i - b + L \sin [(+\theta) + \delta\theta] \tag{3.6}$$

Which, again ignoring $\delta\theta$, gives

$$117.43 = H_B + 1.53 - 1.50 + 1863.12 \sin(+01°17'03'')$$

From which

$$H_B = 75.646 = 75.65 \text{ m}$$

Therefore

$$\textbf{Height of B} = \frac{75.21 + 75.65}{2} = \textbf{75.43 m AOD}$$

(2) The value of k
From the two height differences calculated above, the true height difference is obtained from

$$H_A - H_B \text{ from the observation at A} = 42.222 \text{ m}$$
$$H_A - H_B \text{ from the observation at B} = 41.784 \text{ m}$$

Hence

$$\text{True height difference} = \frac{42.222 + 41.784}{2} = 42.003 \text{ m}$$

This is substituted into equations (3.5) and (3.6) in turn to calculate first $\delta\theta$ and then k.

Substitution into equation (3.5) gives

$$H_B - H_A = -42.003 = 1.49 - 1.75 + 1863.12 \sin [(-01°17'26'') + \delta\theta]$$

From which

$$\delta\theta = 24.27''$$

From equation (3.3)

$$\delta\theta = \frac{D(1 - 2k)}{2R (\sin 1'')} \text{ seconds}$$

where

$$D = L \cos\theta = 1863.12 \cos(01°17'26'') = 1862.65 \text{ m}$$

Hence

$$24.27 = \frac{1.86265}{2(6375)}(1 - 2k)206\,265$$

From which

$$k = 0.0973$$

Substituting into equation (3.6) and solving for $\delta\theta$ and then for k gives

$$\delta\theta = 24.20'' \quad \text{and} \quad k = 0.0984$$

Hence

$$\textbf{mean value of } \boldsymbol{k} = \frac{0.0973 + 0.0984}{2} = \mathbf{0.098}$$

(3) The height of station C

Since the observation to C was taken immediately after the simultaneous reciprocal observations, $k = 0.098$ can be used, therefore

$$D_{AC} = 1543.28 \cos(02°24'53'') = 1541.91 \text{ m}$$

And

$$\delta\theta = \frac{1.541\,91}{2(6375)}(1 - 2(0.098))206\,265 = 20.06''$$

Substituting into equation (3.4) gives

$$H_C = 117.43 + 1.49 - 1.96 + 1543.28 \sin[(-02°24'53'') + 20.06'']$$

From which

$$H_C = 52.09 = \mathbf{52.1 \text{ m AOD}}$$

4

Distance Measurement

The measurement of distance is one of the fundamental operations in engineering surveying and is carried out by taping, tacheometric or Electromagnetic Distance Measurement (EDM) techniques. Whichever of these is used, the usual requirement in engineering surveying is for *horizontal distances*. Various methods of obtaining horizontal distances using these techniques are discussed in this chapter.

4.1 Steel Tapes

The most commonly used steel tapes now available are in 20 m, 30 m or 100 m lengths, either encased in plastic or leather boxes with a recessed winding handle, or mounted on an open winding frame with a folding handle. The former usually incorporate a small loop or grip on the end of the tape, this marking the zero point, whereas on open wound tapes the zero is marked on the band itself. Examples of steel tapes and their graduations are shown in figure 4.1.

Various systems are used for graduating the tape and it is, therefore, essential to ascertain at which point on a tape the zero point is marked and to inspect the tape markings before fieldwork commences. All steel tapes are manufactured so that they measure their nominal length at a specific temperature and under a certain pull. These standard conditions, very often 20°C and 50 N, are printed somewhere on the first metre of the tape. The effects of variations from the standard conditions are discussed in sections 4.2.4 and 4.2.5.

4.2 Steel Taping: Fieldwork and Corrections

Distance measurement using steel tapes involves determining the straight-line distance between two points.

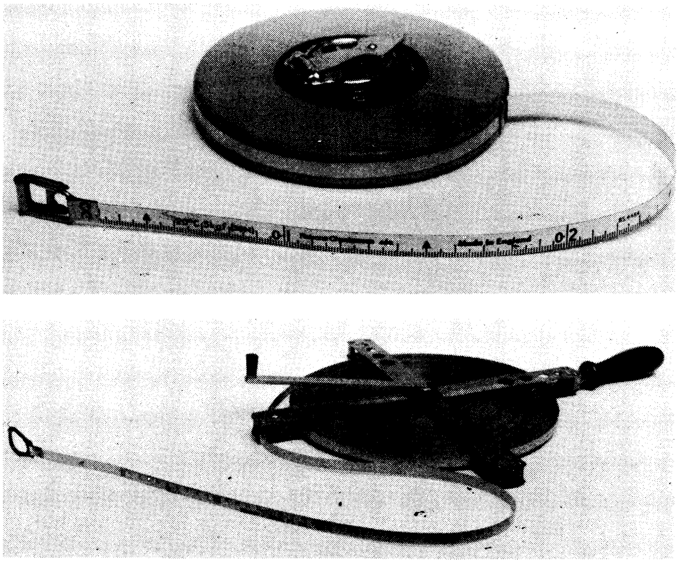


Figure 4.1 *Steel tapes (note the different zero points)*

4.2.1 Ranging

When the length to be measured is less than that of the steel tape, measurements are carried out by unwinding and laying the tape along the straight line between the points. The zero of the tape (or some convenient graduation) is held against one point, the tape is straightened, pulled taut and the distance read directly on the tape at the other point.

When the length of the line between two points exceeds that of the tape, some form of alignment is necessary to ensure that the tape is positioned along the straight line required. This is known as *ranging* and is achieved using *ranging rods* and *marking arrows* (see figure 4.2).

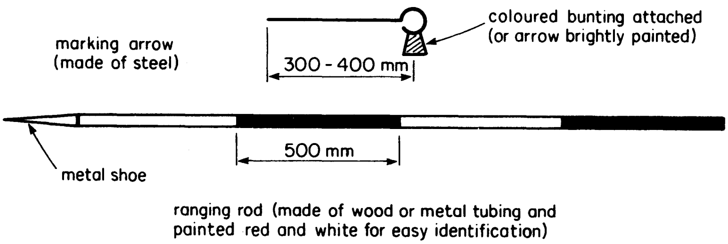


Figure 4.2 *Ranging rod and marking arrow*

For measuring long lines two people are required, identified as the *leader* and the *follower*, the procedure being as follows for a line AB. This method of measurement is known as *ranging by eye*.

(1) Ranging rods are erected as vertical as possible at the points A and B and, for a measure in the direction of A to B, the zero point of the tape is set against A by the follower.

(2) The leader, carrying a third ranging rod, unwinds the tape and walks towards point B, stopping just short of a tape length, at which point the ranging rod is held vertically.

(3) The follower removes the ranging rod at A and, stepping a few paces behind point A, lines up the ranging rod held by the leader with point A and with the rod at B. This lining-in should be done by the follower sighting as low as possible on the poles.

(4) The tape is now straightened and laid against the rod at B by the leader, pulled taut and the tape length marked by placing an arrow on line.

(5) For the next tape length the leader and the follower move ahead simultaneously with the tape unwound, the procedure being repeated but with the follower now at the first marking arrow. Before leaving point A, the follower replaces the ranging rod at A as this will be sighted on the return measurement from B to A, which should always be taken as a check for gross errors.

(6) As measurement proceeds the follower picks up each arrow and, on completion, the number of arrows held by the follower indicates the number of whole tape lengths measured. This number of tape lengths plus the section at the end less than a tape length gives the total length of the line.

The accuracy of ranging may be improved by using a *theodolite* (see chapter 3). The theodolite is set over one point and sighted on to the other, thereby establishing the line. The taping procedure is slightly altered, each intermediate setting of a ranging rod being achieved using the theodolite. This method, however, requires three people, one for aligning and two for measuring and takes slightly longer in the field.

4.2.2 Slope Measurements and Slope Corrections

The method of ranging described in section 4.2.1 can be carried out for any line, either sloping or level. Since all surveying calculations, plans and setting-out designs are based or drawn in the horizontal plane, any sloping length measured must be reduced to the horizontal before being used for calculations or plotting. This can be achieved by calculating a *slope correction* for the measured length or by measuring the horizontal equivalent of the slope directly in the field.

Consider figure 4.3*a* which shows a sloping line AB. To record the horizontal distance D between A and B, the *method of stepping* may be employed in which a series of horizontal measurements is taken. To measure D_1 the tape zero is held at A and the tape then held horizontally and on line towards B against a previously lined-in ranging rod. The horizontality of the tape should, if possible, be checked by a third person viewing it from one side some distance away.

At some convenient tape graduation (preferably a whole metre mark) the horizontal distance is transferred to ground level using a *plumb line* (a string line with a weight attached) or a *drop arrow* (a marking arrow to which a weight is attached).

The tape is now moved forward to measure D_2 in a similar manner. It is recommended that the maximum length of an unsupported tape should be 10 m and that this should be considerably shorter on steep slopes since the maximum height through which a distance is transferred should be 1.5 m.

As an alternative to stepping, the slope angle, θ, can be determined and the horizontal distance D calculated from the measured slope distance L as shown in figure 4.3*b*. Alternatively, a correction can be computed from

$$C = L(1 - \cos\theta)$$

hence

$$D = L - C$$

This correction is always negative.

The slope angle can be measured using an *Abney level*, a hand-held device shown in figure 4.4. To use the Abney level, an observer first distinctly marks his eye height (h in figure 4.3*b*) on a ranging rod which is then placed at point B. Standing at point A and looking down the sighting tube, the cross-wire is seen and is set against the mark on the ranging rod at B. The observer's line of sight will be A′B′, which is parallel to AB (see figure 4.3*b*). To record θ the milled wheel is turned until the image of the bubble appears centrally against the cross-wire when viewed through the sighting tube. A fine adjustment is provided by the slow motion screw.

A simple vernier, attached to the milled wheel, is then read with the aid of a small reading glass against the scale attached to the sighting tube. This gives a measure of θ to within 10 minutes of arc.

Where better accuracy is required, a theodolite can be used to measure θ. The theodolite is set up at A and the slope angle measured along A′B′ (see figure 4.3*b*). In this case, h will be the height of the theodolite trunnion axis above ground level. Details of the use of the theodolite are given in chapter 3.

Comparing the two methods of obtaining horizontal distance, stepping is more useful when the ground between points is very irregular, whereas the Abney level or theodolite are suitable only for measurements taken on uniform slopes.

A third method is available if the height difference between the two points is known and the slope between them is uniform. In figure 4.3*c*, if Δh is the height difference between A and B, then by Pythagoras

$$D = (L^2 - \Delta h^2)^{\frac{1}{2}} = L(1 - \Delta h^2/L^2)^{\frac{1}{2}}$$

Using the binomial theorem this expands as

$$D = L\,[1 - (\Delta h^2/2L^2) - (\Delta h^4/8L^4) - \ldots] \tag{4.1}$$

All terms in equation (4.1) with a higher power than the square can be omitted, giving

$$D = L - \Delta h^2/2L$$

Hence, a *slope correction* of $-(\Delta h^2/2L)$ is obtained and it is always negative.

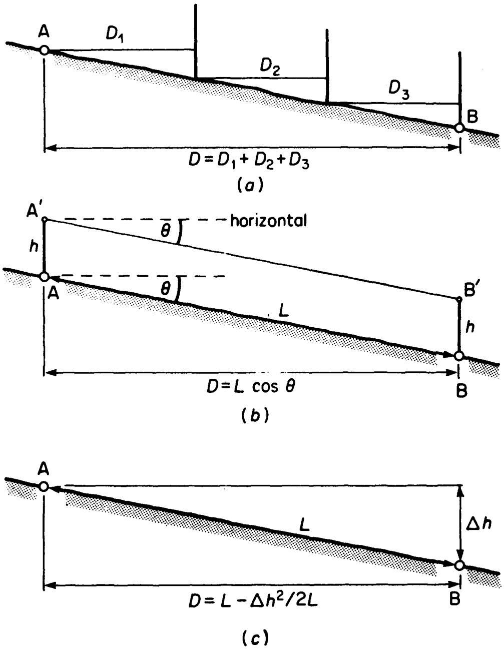


Figure 4.3 *Slope measurements*

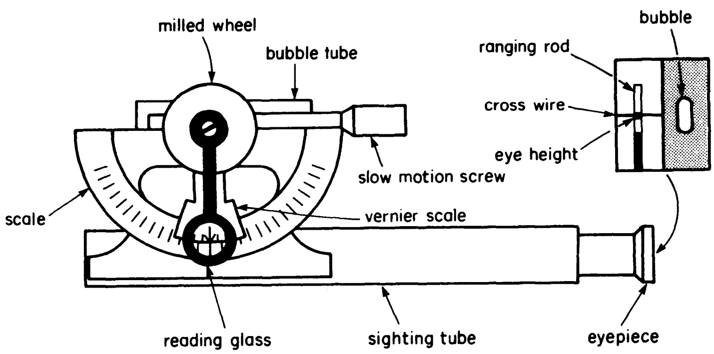


Figure 4.4 *Abney level*

4.2.3 Standardisation

Under given conditions a tape has a certain nominal length. However, with a lot of use, tapes tend to alter in length and a 30 m tape may be reading, say, 30.011 m or

29.967 m over a full length. As this effect can produce serious errors in length measurement, *standardisation* of steel tapes should be carried out frequently against a reference tape. This should be done on a smooth, flat surface such as a surfaced road or footpath. The reference tape should not be used for any fieldwork and should be checked by the manufacturer as often as possible. From standardisation measurements a correction is computed as follows.

If L is the recorded length of a line, l the nominal tape length (say 30 m) and l' the standardisation length (say 30.011 m), then

$$\text{Corrected length} = L(l'/l)$$

Alternatively, a correction can be computed from

$$C = L((l' - l)/l)$$

The sign of the correction depends on the values of l' and l.

4.2.4 Tensioning

The steel used for tapes, in common with many metals, is elastic and the tape length varies with applied tension. This effect tends to be overlooked by an inexperienced engineer and, consequently, errors can arise in measured lines.

Every steel tape is manufactured and calibrated with a standard tension applied, a typical figure being 50 N. Therefore, instead of merely pulling the tape taut, an improvement in accuracy is obtained if the tape is pulled at its standard tension. This is achieved using *spring balances* specially made for use in ground taping together with a device called a *roller grip*. When measuring, one end of the tape is held firm near the zero mark, the spring balance and roller grip are hooked to the other end of the tape and the spring balance handle is pulled until its sliding index indicates that the correct tension is applied, as shown in figure 4.5. This tension is then maintained while measurements are taken.

When setting out, this method of tensioning can be difficult and a *constant tension handle* can be used to minimise errors. The use of the constant tension handle is shown in figure 4.6 and since the correct tension is always applied to the tape it is particularly suitable for use by unskilled operatives.

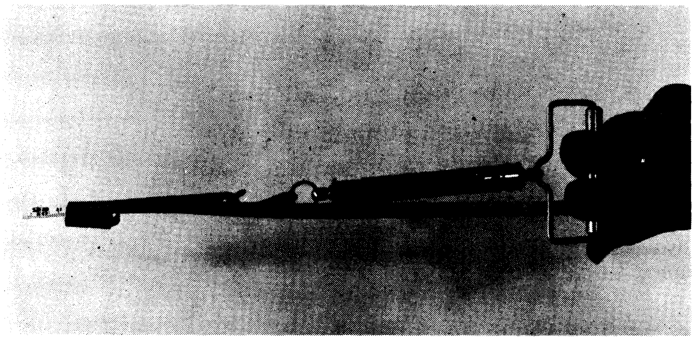


Figure 4.5 *Tensioning equipment (Crown copyright)*

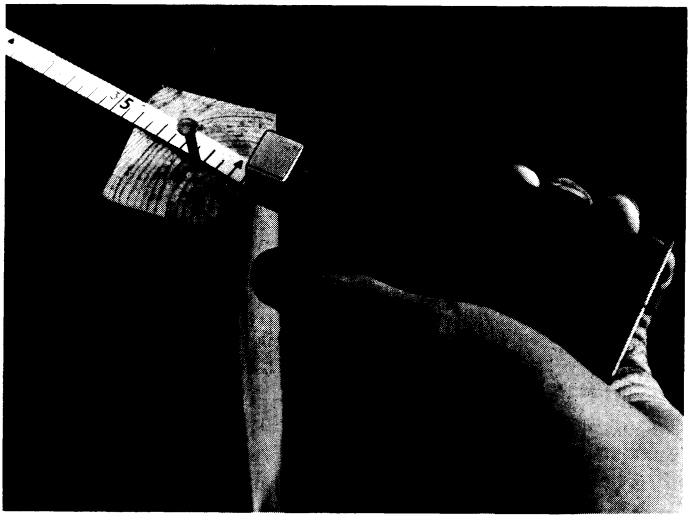


Figure 4.6 *Constant tension handle (Crown copyright)*

Should a tape be subjected to a pull other than the standardising value, it can be shown that a correction to an observed length is given by

$$C = L(T_F - T_S)/AE$$

where T_F is the tension applied to the tape in the field (N), T_S is the standardisation tension (N), A the cross-sectional area of the tape (mm^2), E the modulus of elasticity for the tape material (for steel tapes, typically 200 000 N mm^{-2}) and L the recorded length of line (m). The sign of the correction depends on the magnitudes of T_F and T_S.

4.2.5 Temperature Variations

In addition to the points covered in 4.2.3 and 4.2.4, steel tapes contract and expand with temperature variations and are, therefore, calibrated at a standard temperature, usually 20°C.

In order to improve accuracy, the temperature of the tape has to be recorded since it will seldom be used at 20°C, and special surveying thermometers are used for this purpose. When using the tape along the ground, measurement of the air temperature can give a different reading from that obtained close to the ground, so it is normal to place the thermometer alongside the tape at ground level. For this reason, the thermometers are usually metal-cased for protection. When in use they should be left in position until a steady reading is obtained since the metal casing can take some time to reach a constant temperature. It is also necessary to have the tape in position for some time before readings are taken to allow it also to reach the ambient temperature. It is bad practice to measure a distance in the field in winter with a tape that has just been removed from a heated office.

The temperature correction is applied as follows. If α is the coefficient of expansion of the tape metal (for example, $\alpha = 0.000\,011\,2$ per °C for steel), t_S the temperature of standardisation (usually 20°C), t_F the mean field temperature (°C) and L the observed length (m), the correction to L is given by $C = \alpha L(t_F - t_S)$ and its sign is given by the magnitudes of t_F and t_S.

4.2.6 Sag (Catenary)

When the ground between two points is very irregular, surface taping can prove to be a difficult process and it may be necessary to suspend the tape above the ground between the points in order to measure the distance between them. This can be done by holding the tape in tension between tripods or wooden stakes, the stakes being driven in approximately 1 m above ground level. For long lines, these tripods or stakes must be aligned by theodolite before taping commences. When measuring distances less than a tape length on site between elevated points on structures, the tape may be suspended for ease of measurement.

Whatever the case, the tape will sag under its own weight in the shape of a catenary curve as shown in figure 4.7.

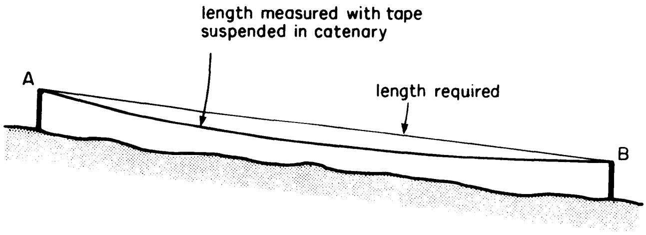


Figure 4.7 *Measurement in catenary*

Since the distance required is the chord AB, a sag correction must be applied to the catenary length measured. This correction is given by

$$C = -\frac{w^2 L^3 \cos^2 \theta}{24 T_F{}^2}$$

where θ is the angle of slope between tape supports, w the weight of the tape per metre length (N/m), T_F the tension applied to the tape (N) and L the length of the supported tape (m).

4.2.7 Combined Formula

The corrections discussed in the preceding sections are usually calculated separately and then used in the following equation

$$D = L - \text{slope} \pm \text{standardisation} \pm \text{tension} \pm \text{temperature} \pm \text{sag} \tag{4.2}$$

where

D = horizontal length of the line
L = length recorded on the steel tape.

Equation (4.2) can be used both when measuring and when setting out horizontal distances, as shown in the worked examples in section 4.5.

4.2.8 Booking of Taped Lines

An example booking sheet for a taped line is shown in figure 4.8. From the example, note the following

(1) Each whole tape length (for example, 30 m) is recorded as measurement proceeds as this guards against a gross error of 30 m.
(2) The slope angles are written alongside the linear measures and the total length is recorded at changes of slope.
(3) The corrections can be computed on the field sheet and the true horizontal length of the line calculated.
(4) The example shows the line measured in one direction only. It should be standard practice to measure a line twice, the second measure being in the reverse direction (Y to X in the example shown in figure 4.8). If the two measures agree (for example, to 1 in 10 000 for engineering surveys) then a mean is computed for the line; if not, a further measure is necessary.

4.3 Steel Taping: Errors and Accuracy

The principal sources of error are discussed below and an assessment of the effect each has on accuracy is given.

(1) An incorrect tape length is a serious source of error and standardisation is essential, suitable corrections being applied to all measures.
(2) Incorrect slope measurements can result in errors in the calculation of horizontal distances. To achieve an accuracy better than 1 in 5000, an Abney level should not be used to measure slopes in excess of 4°. Care must also be exercised when stepping to ensure that the tape does not sag excessively, that it is held horizontally and that the horizontal distance is transferred vertically.
(3) When the correct tension is not applied to a tape, incorrect lengths are obtained. This is the most neglected aspect of taping. It is recommended that for all linear measurements, especially in setting out where accuracy better than 1 in 5000 is required, tapes should be tensioned correctly.
(4) Ignoring temperature variations also gives rise to errors and, as with tensioning, it is stressed that for accuracy to be better than 1 in 5000, the field temperature should be recorded and a temperature correction applied.
(5) When taping, a straight-line distance is required and if the tape is poorly aligned or not straightened properly this will not be the case. If a tape is 0.6 m off line in the centre of two 30 m tape lengths the resulting accuracy is 1 in 5000.

CORRECTIONS

(i) STANDARDISATION

Nominal length = 30m

Standardised length

= 30.012m

$$C = 101.234 \times \frac{30.012-30}{30}$$

$= \underline{+\ 0.040m}$

(ii) TEMPERATURE

C = 0.0000112 x 101.234

x (10-20)

$= \underline{-0.011m}$

(iii) SLOPE CORRECTIONS

1. $30.00(1-\cos 1^{\circ}20')$

$= \underline{-0.008m}$

2. $45.50(1-\cos 1^{\circ}50')$

$= \underline{-0.023m}$

3. $25.73(1-\cos 2^{\circ}30')$

$= \underline{-0.024m}$

Total slope correction

$= \underline{-0.055m}$

XY = 101.234 +0.040

-0.011 -0.055

XY = 101.208m

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NOTES

(a) Mean temperature

= $10^{\circ}C$

(b) Tape positioned on the ground and pulled at standard tension

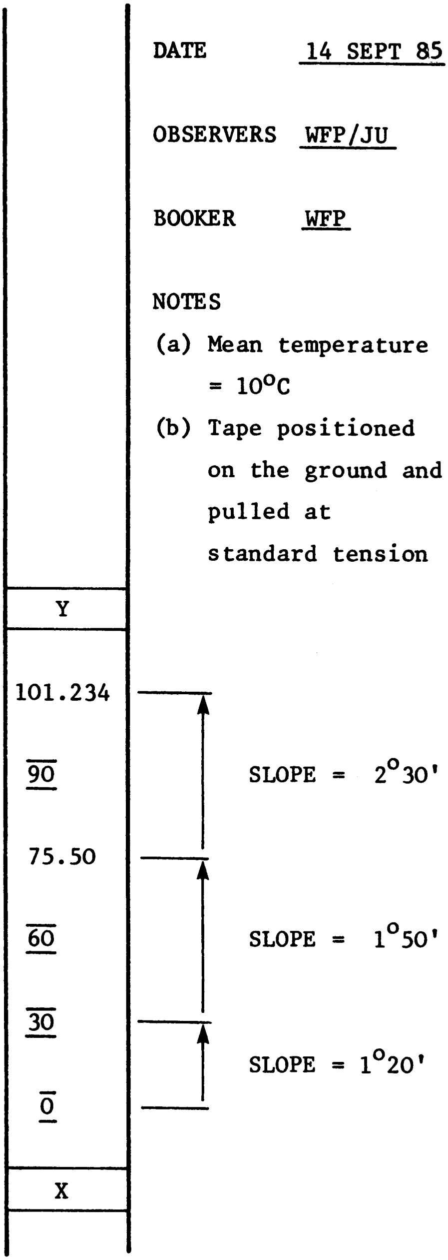


Figure 4.8 *Example booking of a taped line*

(6) The effects of sag must be considered for more accurate work.

(7) Mistakes in reading the tape, in booking and in recording the number of tape lengths are gross errors and the chance of detecting any such error is greatly increased if the line is measured twice, once in each direction.

Considering the above factors, the general rules for steel taping can be summarised as follows.

(1) For a maximum accuracy of 1 in 5000, measurements can be taken over most ground surfaces if only standardisation and slope corrections are applied. Slope angles should be measured using an Abney level or stepping can be employed.

(2) If the tape is tensioned correctly and temperature variations are taken into account, the accuracy is increased to approximately 1 in 10 000. On specially prepared surfaces or over spans less than a tape length, accuracies of 1 in 20 000 can be achieved. This assumes that sufficient care is taken when standardising the tape and in reducing slope measurements to the horizontal.

(3) To further increase accuracy (in excess of 1 in 20 000), sag corrections should be applied and on long lines ranging by theodolite is recommended.

4.4 Steel Taping: Applications

The steel tape has applications in nearly every aspect of surveying since it is a cheap form of distance measurement and yet retains a suitable accuracy.

Some of the uses of steel taping in engineering surveying are shown in table 4.1, together with an indication of the accuracy normally required for each type of project. Further details of the types of work listed in table 4.1 are given in subsequent chapters.

4.5 Steel Taping: Worked Examples

4.5.1 Measuring a Horizontal Distance with a Steel Tape

Question

A steel tape of nominal length 30 m was used to measure a line AB by suspending it between supports. The following measurements were recorded.

Line	Length measured	Slope angle	Mean temperature	Tension applied
AB	29.872 m	$3°40'$	5°C	120 N

The standardised length of the tape was known to be 30.014 m at 20°C and 50 N tension.

If the tape weighs 0.17 N/m and has a cross-sectional area of 2 mm^2, calculate the horizontal length of AB.

Young's modulus (E) for the tape material is 200 kN/mm^2 and the coefficient of thermal expansion (α) is 0.000 011 2 per °C.

TABLE 4.1

Type of work	Accuracy required
Location of spoil heaps and soft detail.	1 in 500 - 1 in 5000
Setting out sewer pipelines. Location of hard detail.	1 in 5000 - 1 in 10 000
Measuring traverse legs. Setting out road centrelines, grids, baselines, offset pegs. General site setting out, setting out buildings, establishing secondary control.	1 in 10 000 - 1 in 20 000
Setting out primary control.	1 in 20 000 upwards

Note: Refer to section 4.3 for guidance on how to achieve the various accuracies listed.

Solution

A series of corrections is computed as follows

$$\text{slope correction} = -L(1-\cos\theta) = -29.872(1-\cos 3°40')$$
$$= \mathbf{-0.0611\ m}$$

$$\text{standardisation correction} = L((l'-l)/l)$$
$$= 29.872((30.014-30.000)/30.000)$$
$$= \mathbf{+\ 0.0139\ m}$$

$$\text{tension correction} = L(T_F - T_S)/AE = 29.872(120-50)/(2 \times 200\,000)$$
$$= \mathbf{+\ 0.0052\ m}$$

$$\text{temperature correction} = \alpha(t_F - t_S)L$$
$$= 0.000\,011\,2 \times (5-20) \times 29.872$$
$$= \mathbf{-\ 0.0050\ m}$$

$$\text{sag (catenary) correction} = -\frac{w^2 L^3 \cos^2 \theta}{24\, T_F^{\,2}}$$

$$= -\frac{(0.17)^2 \times (29.872)^3 \times \cos^2 3°40'}{24 \times (120)^2}$$

$$= -\mathbf{0.0022\ m}$$

The horizontal length of AB is given by substituting the corrections into equation (4.2) as follows

horizontal length AB

$$= 29.872 - 0.0611 + 0.0139 + 0.0052 - 0.0050 - 0.0022$$

$= 29.8228 =$ **29.823 m** (rounded to the nearest mm)

4.5.2 Setting Out a Slope Distance with a Steel Tape

Question

On a construction site, a point R is to be set out from a point S using a steel tape of nominal length 50 m. The horizontal length of SR is designed as 35.000 m and it lies on a constant slope of 03°27′.

During the setting out the steel tape is laid on the ground and pulled at a tension of 70 N, the mean temperature being 12°C.

The standardised length of the tape at 50 N tension and 20°C is 50.027 m. The coefficient of thermal expansion of the tape material is 0.000 011 2 per °C, Young's modulus is 200 kN/mm^2 and the cross-sectional area of the tape is 2.4 mm^2.

Calculate the length that should be set out on the tape along the direction SR to establish the exact position of point R.

Solution

Equation (4.2) is again used but in this case D is known and L must be calculated.

The slope, standardisation, tension and temperature corrections must all be calculated. The sag correction does not apply since the tape is laid along the ground.

Although L is not known, for the purposes of calculating the corrections it is sufficiently accurate to use D instead of L in the individual formulae. Therefore

$$\text{slope correction} = -D(1 - \cos\theta)$$

$$= -35.000(1 - \cos 03°27') = -\mathbf{0.0634\ m}$$

$$\text{standardisation correction} = D((l' - l)/l)$$

$$= 35.000((50.027 - 50.000)/50.000)$$

$$= +\mathbf{0.0189\ m}$$

$$\text{tension correction} = D(T_F - T_S)/AE$$

$$= 35.000(70 - 50)/(2.4 \times 200\,000)$$

$$= +\mathbf{0.0015\ m}$$

$$\text{temperature correction} = \alpha(t_F - t_S)D$$
$$= 0.000\,011\,2(12 - 20)35.000$$
$$= -\mathbf{0.0031\ m}$$

The slope length SR is obtained from equation (4.2) as follows

$$D_{SR} = L_{SR} - \text{slope} \pm \text{standardisation} \pm \text{tension} \pm \text{temperature}$$

From which

$$35.000 = L_{SR} - 0.0634 + 0.0189 + 0.0015 - 0.0031$$

Therefore

$$L_{SR} = 35.0461 = \mathbf{35.046\ m}\ \text{(rounded to the nearest mm)}$$

4.6 Other Types of Tape

In addition to steel tapes, the following tapes are sometimes used in engineering surveys.

Synthetic tapes (*fibreglass, plastic* or *woven*) are available in a variety of lengths. When compared to steel tapes, synthetic tapes are lighter, more flexible and less likely to break but they tend to stretch much more when pulled. As a result, synthetic tapes are used mainly in detail surveying (see chapter 8), sectioning (see section 2.11.2) and in similar work where precisions in the order of 1 in 1000 are acceptable in linear measurements.

Invar tapes are made from an alloy of nickel and steel and have a coefficient of thermal expansion approximately one-tenth or less that of steel. Consequently, these tapes are almost independent of temperature changes and are ideal for use where very precise measurements are required. However, since invar tapes are expensive and must be handled with great care to avoid bends and kinks, they are not used for ordinary work.

4.7 Chaining

The land chain, the simplest form of linear measuring device, is, in metric form, 20 m, 30 m, 50 m or 100 m in length. The 20 m and 30 m chains are usually constructed from stiff wire and consist of links each 200 mm in length as shown in figure 4.9.

The links are connected by three small rings which give the chain flexibility. The brass handles at each end form part of the measurement. Every metre along the chain is marked by a tag and every fifth metre numbered.

The 50 m and 100 m chains are made of plastic. They are marked at 0.1 m and 0.05 m intervals and should be standardised regularly since the plastic can stretch.

Comparing the steel tape and chain it is obvious that the chain is very robust in construction and can be handled fairly roughly. Repairs are easily carried out in the field and the chain can be cleaned by washing and drying. In contrast, the steel tape is easily broken and has to be handled with reasonable care, maintenance being more

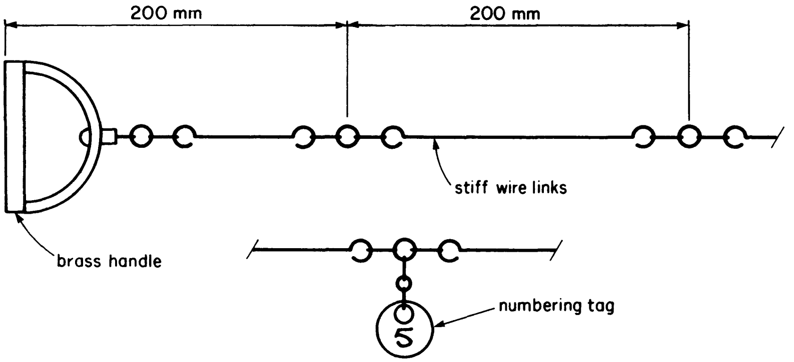


Figure 4.9 *Section of 20 m and 30 m chains*

troublesome. The steel tape, however, is much more accurate than the chain and this is the important difference as regards distance measurement.

The fieldwork involved in the measurement of a line using a chain is identical to that using a steel tape but the accuracy expected when measuring with a chain and applying slope and standardisation corrections is only about 1 in 1000. Any errors due to temperature variations and incorrect tensioning are, therefore, negigible and are not considered in chaining. Similarly, the effects of poor alignment and sag are not so critical. When measuring slope angles, an Abney level should be used or the line stepped. Booking is carried out as described for steel tapes in section 4.2.8.

The only useful application of the chain is in the production of site plans, further details of which are given in chapter 8.

4.8 Optical Distance Measurement

Two disadvantages with taping are, firstly, that the measuring process takes place on the ground (if the terrain is undulating this can be very difficult) and, secondly, when a lot of linear measurements are required, taping can be laborious and time consuming.

Optical distance measurement (ODM) techniques overcome the first problem in that they are undertaken above ground level, and overcome the second problem since they can usually be carried out in a shorter time than that required for surface taping.

The ODM technique which has the greatest application in engineering surveying is *stadia tacheometry* and this is discussed in section 4.9. The more specialised technique of *subtense tacheometry* is discussed in section 4.10. Other ODM techniques involving special tacheometers and attachments are not discussed since these have been superseded by electromagnetic distance measuring methods (see sections 4.11 to 4.15).

4.9 Stadia Tacheometry

This uses a theodolite or level and a levelling staff. It involves the use of the two short lines marked on the diaphragm of the majority of theodolite and level telescopes. These lines are called the *stadia hairs* or *stadia lines* and are marked as shown in figure 4.10. The distance between the stadia hairs is fixed and is called the *stadia interval*.

If observations are made to a levelling staff, the stadia hairs, when viewed through the instrument telescope, will appear to cover a certain length (s) of the staff, the value of s depending on the horizontal distance (D) between the instrument and staff (see figure 4.10).

4.9.1 Basic Principle

Figure 4.11 shows the optical system for measurement of a horizontal distance D between the vertical axis of an *externally focusing telescope* and a graduated staff. Although now obsolete, this type of telescope is simpler in design than modern telescopes and for this reason is used to demonstrate tacheometric principles (modern telescopes are considered in section 4.9.2).

The externally focusing telescope consists of two concentric tubes. The eyepiece and diaphragm are mounted at the end of one tube and focusing is achieved by movement of the object lens which is fixed at the end of the other tube (see figure 4.11).

In figure 4.11, the axis of the telescope is horizontal, f is the focal length of the object lens, ab the stadia interval (i) and AB the staff intercept (s).
Now $D = l + f + d$ and $l/s = f/i$
Therefore

$$D = (f/i)s + (f + d)$$

For a particular instrument, the ratio (f/i) is a constant known as the *multiplying constant* (K). The distance $(f + d)$ is known as the *additive constant* (C). This term

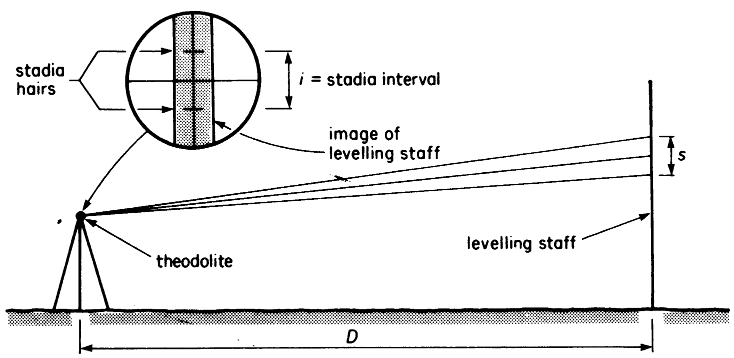


Figure 4.10 *Stadia tacheometry*

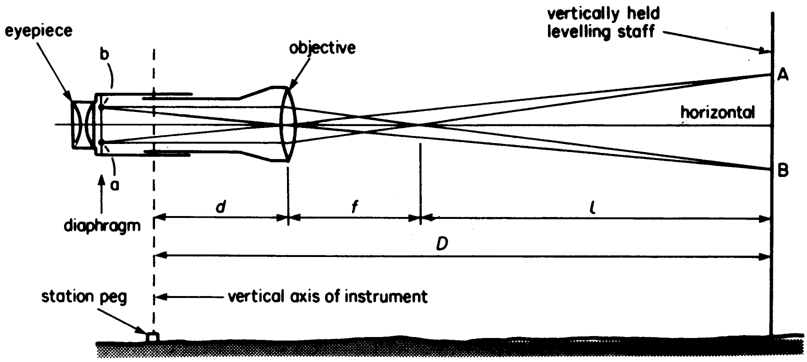


Figure 4.11 *Stadia principle*

will vary slightly when focusing is achieved owing to the movement of the object lens, that is, the value of d varies while focusing. However, for practical purposes this variation is small and the term $(f + d)$ can be considered constant. Hence, the stadia equation can be stated as $D = Ks + C$.

For ease of calculation of D, most theodolites and levels are designed such that $K(= f/i) = 100$. Further, it is obviously inconvenient to have an additive constant and it would be useful if $C = 0$. To achieve this, some externally focusing telescopes have an extra lens, known as an *anallactic lens*, placed in the telescope tube at a fixed length from the object lens. This lens has the effect of making $C = 0$ and hence the stadia formula becomes $D = 100s$. Such a telescope is known as an *anallactic telescope.*

4.9.2 The Internally Focusing Telescope

All modern telescopes use a concave lens, placed *inside* the telescope tube, to enable internal focusing to be carried out (see section 2.2.1).

The derivation of the stadia formula for internally focusing telescopes is very complicated but it can be shown that the result $D = Ks + C$ is still valid. Strictly, in this case, K is a variable as well as C but, in practice, the variation in K will be very small and can be ignored. Furthermore, by suitable optical design, modern telescopes can be assumed to be anallactic and the value of C is taken to be zero.

These assumptions break down if the horizontal distance is less than approximately 10 m to 20 m.

4.9.3 Theory with Inclined Line of Sight

Although a stadia survey could be carried out with the telescope horizontal, work would be tedious in hilly terrain and so the basic formula must be modified to cover the general case when the telescope is inclined.

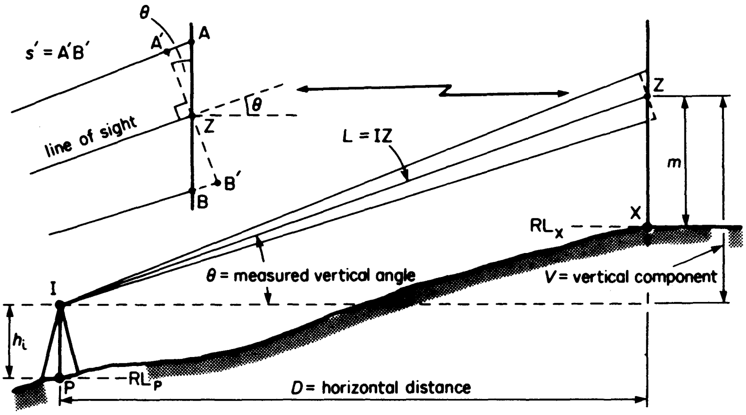


Figure 4.12 *Stadia with inclined line of sight*

Figure 4.12 shows a *vertically* held levelling staff observed with an internally focusing telescope of which the line of sight is inclined to the horizontal.

Since A′B′ is the staff intercept that would be recorded on a staff at Z held perpendicular to the line of sight, $L = Ks' + C$. If the line of sight IZ makes an angle θ with the horizontal, A′B′ makes an angle of θ with the vertical. Therefore

$$L = Ks \cos\theta + C$$

But $\quad D = L \cos\theta$

hence $\quad \boldsymbol{D = Ks \cos^2\theta + C\cos\theta}$

From the one pointing of the telescope, the vertical component of $D(V)$ is also obtained as follows

$$V = L\sin\theta = Ks\cos\theta\sin\theta + C\sin\theta$$

hence $\quad \boldsymbol{V = \tfrac{1}{2}(Ks\sin 2\theta) + C\sin\theta}$

The vertical component is used in the calculation of reduced levels. Figure 4.12 shows the observation of a staff held at X by a theodolite at P with the line of sight above the horizontal. The heights of these points are RL_X and RL_P above datum. If h_i (= IP) is the height of the instrument at P above the station mark at P and m (= ZX) is the centre hair reading of the staff at X, then $RL_X = RL_P + h_i + V - m$.

In general it can be stated that

$$\boldsymbol{RL_X = RL_P + h_i \pm V - m}$$

where V is positive for an angle of elevation and negative for an angle of depression.

This demonstrates one of the advantages of stadia tacheometry, namely, for one pointing of the instrument with the staff at an unknown point, both the horizontal distance to and reduced level of that point can be found.

4.9.4 Accuracy and Sources of Error in Vertical Staff Stadia Tacheometry

The accuracy of basic stadia tacheometry depends on two categories of error, instrumental errors and field errors.

Instrumental errors

These include

(1) An incorrectly assumed value for K, the multiplying constant, that is, an error in the construction of the diaphragm.

(2) Errors arising out of the assumption that modern telescopes are anallactic and that the stadia formula $D = Ks + C$ is applicable when, strictly, both K and C are variable.

The possible errors due to (1) and (2) above limit the overall accuracy of distance measurement by stadia tacheometry to 1 in 1000.

Field errors

These can occur from the following sources

(1) When observing the staff, incorrect readings may be recorded which result in an error in the staff intercept, s. Assuming $K = 100$, an error of ± 1 mm in the value of s results in an error of ± 100 mm in D.

Since the staff reading accuracy decreases as D increases, the maximum length of a tacheometric sight should be 100 m.

(2) Nonverticality of the staff can be a serious source of error. This and poor accuracy of staff readings form the worst two sources of error. The error in distance due to the nonverticality of the staff is proportional to both the angle of elevation of the sighting and the length of the sighting. Hence, a large error can be caused by steep sightings, long sightings or a combination of both. It is advisable not to exceed $\theta = \pm 10°$ for all stadia tacheometry.

(3) A further source of error is in reading the vertical circle of the theodolite. If the line of sight is limited to $\pm 10°$, errors arising from this source will be small provided no misreading or noncentring of the altitude bubble takes place. Usually, it is sufficiently accurate to measure the vertical angle to $\pm 1'$ and, although it is possible to improve this reading accuracy, it is seldom worth doing so owing to the magnitude of all the other errors previously discussed.

Considering all the sources of error, the overall accuracy expected for distance measurement is 1 in 500 and the best possible accuracy is only 1 in 1000.

The vertical component V, is subject to the same sources of error described above for distances, and the accuracy expected is approximately $\pm$ 50 mm.

4.9.5 Applications of Stadia Tacheometry

Vertical staff tacheometry is ideally suited for detail surveying by radiation techniques. This is discussed fully in chapter 8.

Since the best possible accuracy obtainable is only 1 in 1000, the method is best restricted to the production of contoured site plans and should not be used to measure distances where precisions better than this are required.

4.10 Subtense Tacheometry

Subtense procedures involve a method in which no graduated staff is used. Instead, a bar of fixed length, called a *subtense bar*, is positioned at one end of the line and a theodolite at the other. The angle subtended by the bar is measured using a 1″ theodolite and, knowing the length of the bar, the distance can be calculated since it is proportional to the *subtense angle*.

4.10.1 Subtense Principle

Figure 4.13 illustrates the subtense method. The subtense bar is positioned horizontally along AB and is of length b. It is positioned at right angles to the line being measured, C being the mid-point of the bar. Points D, E and F lie in the horizontal plane through X (the theodolite position) vertically below A, B and C.

The theodolite will record the horizontal angle DXE (ϕ) between the vertical planes AXD and BXE. The horizontal distance XF(D) is given by

$$D = (b/2) \cot (\phi/2)$$

Since the ends of the subtense bar could be positioned horizontally anywhere in the vertical lines through D and E, the horizontal angle ϕ is always recorded by the theodolite. Therefore, the *horizontal distance* (D) is obtained directly without the need for measuring the slope angle.

4.10.2 The Subtense Bar

A typical subtense bar, mounted on a tripod, is shown in figure 4.14. It is usually arranged to fit into a standard tribrach and can therefore be part of a three-tripod system. The bar can be set horizontal using the tribrach footscrews in conjunction with a levelling bubble attached to the bar. A sighting device is fixed to the bar which enables it to be set at right angles to the line being measured.

On the bar are two targets which are set a precise distance apart, usually 2 m. Since subtense bars are usually metallic, changes in temperature will affect the length of bar between targets and this can give rise to serious errors in the measured distance. Consequently, all modern subtense bars are made of invar, a metal with an extremely low coefficient of expansion. To further check any expansion of the invar, various compensating systems have been developed and these ensure that the length between targets remains virtually unchanged for a large variation in temperature. The invar strip and compensating system are usually housed in a sealed tube for protection, the targets being spring or tension mounted at the ends of the tube. The accuracy obtainable by most manufacturers for the stability in the nominal length of the bar is ±0.05 mm for a temperature change of ±30°C. This indicates that a bar length of 2 m is known with a proportional error of 1 in 40 000.

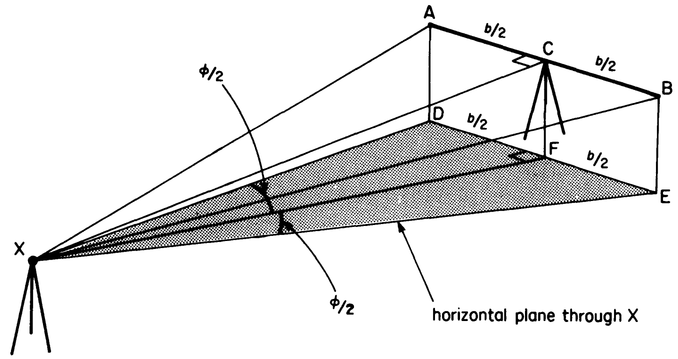


Figure 4.13 *Subtense principle*

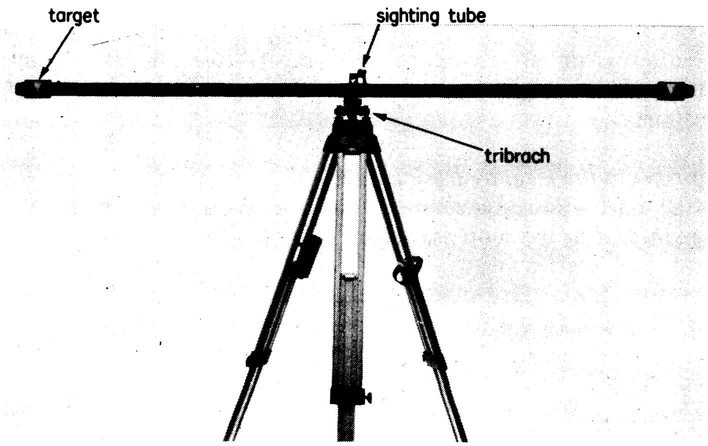


Figure 4.14 *Subtense bar (courtesy Wild Heerbrugg (UK) Ltd)*

4.10.3 Subtense Angle

The magnitude of a subtense angle is usually of the order of 1° to 2° and must be measured with an accuracy of ± 1″ to obtain the required accuracy in distance measurement (see section 4.10.4). A 1″ theodolite is, therefore, usually employed and it is necessary to measure the subtense angle a number of times to achieve ± 1″ accuracy. Normally, ten repeated readings are recorded, the mean result being used in the calculation of distance. It is good practice to use different parts of the horizontal circle when taking these measurements. Since both targets are at the same elevation there will be no error due to trunnion axis dislevelment and all measures can be taken on a single face.

4.10.4 Accuracy and Sources of Error in Subtense Tacheometry

A distance measured by subtense methods is subject to the following sources of error.

(1) Incorrect length of bar.

Since most bars are constructed with a proportional accuracy of 1 in 40 000 (see section 4.10.2), errors from this source are negligible.

(2) Incorrect setting of the bar at right angles to the line being measured.

To maintain an accuracy of 1 in 20 000, the bar must not be misaligned by more than $\pm 34'$. When properly adjusted, the sighting telescope attached to the bar can achieve an accuracy of a few minutes, so that this source of error can be disregarded.

(3) Nonhorizontality and poor centring of the bar and the theodolite over the station marks.

Errors occur if the bar is not levelled or set over the station mark correctly. As most subtense bars are mounted in tribrachs, errors from this category may be neglected provided the optical plummet and levelling bubble on the bar are in good adjustment.

(4) Errors in the measured subtense angle.

This is the most serious source of error in subtense work and great care must be taken when measuring the subtense angle.

If the subtense angle is measured with a $1''$ theodolite and the mean of ten repeated readings is used (see section 4.10.3), the error in the subtense angle should not exceed $\pm 1''$ and, if a 2 m subtense bar is used, the following proportional accuracies can be achieved.

D (m)	40	50	60	70	80	90	100	150	200
Proportional error (1 in n)	10 000	8330	6670	5830	5000	4500	4170	2730	2060

For distances less than 40 m, the proportional accuracy is not increased since the subtense bar targets appear large when viewed through the theodolite; accurate angular measurements therefore become difficult.

4.10.5 Applications of Subtense Tacheometry

Subtense methods are suitable only for measuring traverse lines. Subtense fieldwork is slow but can give good accuracy. One important factor to consider is that the line of sight on subtense measurements can be very steep with no loss of accuracy in the horizontal distance. This may be useful in quarries, in connecting rooftop traverses to ground level and in setting out where steep slopes are involved on such projects as bridge works, tunnelling, tall buildings and so on.

4.11 Electromagnetic Distance Measurement

The rapid development of Electromagnetic Distance Measurement (EDM) equipment in recent years has enabled the surveyor and engineer to measure distances, particularly over long ranges, much more easily and to a higher precision than is possible using taping or optical methods. As a result of these technical advances, many changes have taken place in surveying techniques. For example, the application of traversing and combined networks in control surveys covering large areas is now possible with the same or better precision than triangulation; detail surveying using theodolite-mounted EDM devices gives rise to more efficient methods of producing maps and plans and many modern setting-out techniques would be impossible without EDM equipment.

To use an EDM system, the instrument is set over one end of the line to be measured and some form of reflector is set over the other end such that the line of sight between the instrument and the reflector is unobstructed. An electromagnetic wave is transmitted from the instrument towards the reflector where part of it is returned to the instrument. By comparing the transmitted and received waves, the instrument is able to compute and display the required distance.

Since there are at present in excess of fifty different EDM systems available, any detailed operating instructions for any particular instrument have been excluded. Such information is available in the handbooks supplied by manufacturers for their respective instruments.

4.12 Electromagnetic Waves

When a length is measured with EDM equipment, no visible linear device is used to determine the length as, for instance, when a tape is aligned in successive lengths along the line being measured. The question often asked is what, then, are electromagnetic waves?

For the simplest treatment they can be considered to be the means by which electrical energy is conveyed through a medium, particularly the atmosphere. If an electric current is fed to an aerial this creates an electrical disturbance in and around the aerial. The disturbance is not confined to the aerial but spreads out into space by varying the electric and magnetic fields in the medium surrounding the aerial. Therefore, energy is propagated outwards and, since the energy is transmitted by varying electric and magnetic fields, the energy is said to be propagated by electromagnetic waves.

The electromagnetic waves so created require no material medium to support them and can be propagated in a vacuum or in the atmosphere. The type of electromagnetic wave generated depends on many factors but, principally, on the nature of the electrical signal used to generate the waves.

4.12.1 Properties of Electromagnetic Waves

Although electromagnetic waves are extremely complex in nature, they can be represented in their simplest form as periodic sinusoidal waves and therefore have predictable properties. Associated with periodic waves are certain characteristics by which all electromagnetic radiation is defined.

Figure 4.15 shows a sinusoidal waveform which has the following properties.

(1) The wave completes *a cycle* in moving from such identical points as A to E or D to H on the wave and the number of times in one second the wave completes a cycle is termed *the frequency* of the wave. The frequency is represented by f hertz, 1 hertz (Hz) being 1 cycle per second.

(2) *The wavelength* of a wave is the distance which separates two identical points on the wave or is that length traversed in one cycle by the wave and is denoted by λ metres.

(3) *The period* is the time taken by the wave to travel through one cycle or one wavelength and is represented by T seconds.

(4) *The velocity* of the wave is the remaining property.

Whereas frequency, wavelength and period can all vary according to the electrical disturbance producing the wave, the velocity (v) of an electromagnetic wave depends on the medium through which it is travelling. The velocity of an electromagnetic wave in a vacuum is termed *the speed of light* and is given the symbol c. The value of c is known at the present time as 299 792 458 m/s.

All of the above properties of electromagnetic waves are related as follows

$$f = (c/\lambda) = (1/T)$$

For simple calculations, c is assumed to be 3×10^8 m/s.

A further term associated with periodic waves is the *phase* of the wave. As far as EDM is concerned, this is a convenient method of identifying fractions of a wavelength or cycle. The symbol normally used is ϕ , often quoted in degrees, and one cycle or wavelength has a phase ranging from 0° to 360°. The points shown in figure 4.15 have the following phase values.

	A	B	C	D	E	F	G	H
ϕ degrees	360 or 0	90	180	270	360 or 0	90	180	270

As can be seen, a quoted phase value can apply to the same point on *any* cycle or wavelength. This has importance when measuring lengths using electromagnetic waves (see section 4.12.2).

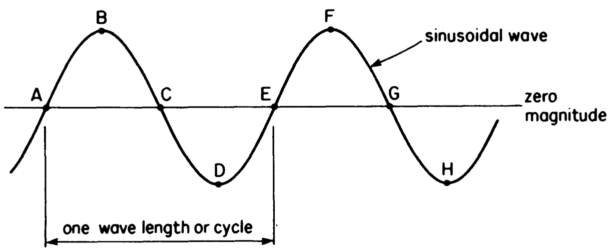


Figure 4.15 *Electromagnetic wave (sine wave)*

4.12.2 Phase Comparison

In an EDM system, distance is determined by measuring the *phase difference* between transmitted and reflected signals. This phase difference is usually expressed as a fraction of a cycle which can be converted into distance when the frequency and velocity of the wave are known.

The methods involved in measuring by phase comparison are outlined as follows.

In figure 4.16*a*, an EDM instrument has been set up at A and a reflector at B so that distance AB = D can be measured.

Figure 4.16*b* shows the same EDM configuration as in figure 4.16*a*, but only the details of the electromagnetic wave path have been shown. The wave is transmitted from A towards B, is instantly reflected at B and received back at A. For clarity, the same sequence is shown in figure 4.16*c* but the return wave has been opened out. Points A and A′ are the same since the transmitter and receiver would be side by side in the same unit at A.

From figure 4.16*c* it is apparent that the distance covered by the wave in travelling from A to A′ is given by

$$2D = n\lambda + \Delta\lambda$$

where D is the distance between A and B, λ the wavelength of the measuring unit, n the whole number of wavelengths travelled by the wave and $\Delta\lambda$ the fraction of a wavelength travelled by the wave.

Thus, the distance D is made up of two separate elements and is determined by two processes.

(1) The *phase comparison* or $\Delta\lambda$ measurement is achieved using electrical phase detectors.

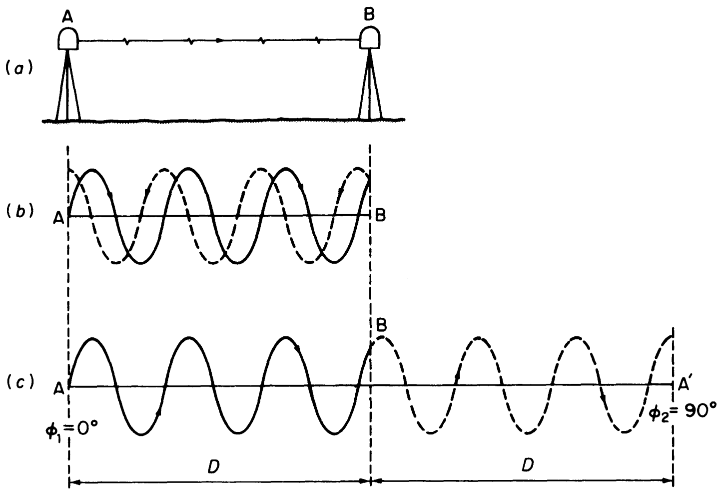


Figure 4.16 *Principle of phase comparison*

Consider a phase detector, built into the unit at A, which senses or measures the phase of the electromagnetic wave as it is transmitted from A. Let this be ϕ_1 degrees. Assume the same detector also measures the phase of the wave as it returns at A′ (ϕ_2°). These two can be compared to give a measure of $\Delta\lambda$ using the relationship

$$\Delta\lambda = \frac{\text{phase difference in degrees}}{360} \times \lambda = \frac{(\phi_2 - \phi_1)^\circ}{360} \times \lambda$$

The phase value ϕ_2 can apply to any incoming wavelength at A′ and the phase comparison can only provide a means of determining by how much the wave travels in excess of a whole number of wavelengths.

(2) Some method of determining $n\lambda$, the other element comprising the unknown distance, is required. This is often referred to as *resolving the ambiguity* of the phase comparison and can be carried out by one of three methods.

(a) The measuring wavelength can be increased manually in multiples of 10 so that a coarse measurement of D is made, enabling n to be deduced.

(b) D can be found by measuring the line using three (or more) different, but closely related, wavelengths, to form simultaneous equations of the form $2D = n\lambda + \Delta\lambda$. These can be solved, making certain assumptions, to give a value for D.

(c) Most modern instruments use electromechanical or electronic devices to solve this problem automatically, the machine displaying the required distance D.

4.12.3 Analogy with Taping

Referring to the example of figure 4.16, assume the measuring wavelength is 30 m. From the diagram $n = 6$, $\phi_1 = 0°$ and $\phi_2 = 90°$.

The double distance is given by

$$2D = n\lambda + \Delta\lambda = n\lambda + \frac{(\phi_2 - \phi_1)}{360} \times \lambda = (6 \times 30) + \frac{(90 - 0)}{360} \times 30$$

Hence

$$D = 93.75 \text{ m}$$

Imagine the distance between A and B was to be measured with a tape x metres in length. Following section 4.2.1, this would involve aligning the tape in successive lengths along the line AB (giving mx where m is the number of whole tape lengths) and noting the fraction of a tape length remaining (Δx) to complete the measurement. Hence $D = mx + \Delta x$.

If $x = 30$ m, measurement of AB would be recorded as

$$D = 3 \times 30 + 3.75 = 93.75 \text{ m}$$

Measurement of a length using electromagnetic waves is, therefore, directly analogous to taping, indeed it can be said that in EDM the electromagnetic wave has replaced the tape as the measuring medium.

4.12.4 Modulation

When designing an EDM instrument, a manufacturer must decide what frequency (or wavelength) to use in the phase comparison or measuring process.

Modern phase comparison techniques are capable of resolving to 1/10 000 of the wavelength used in the measuring process and, assuming ±10 mm to be the worst accuracy requirement for surveying equipment, the longest measuring wavelength should be 100 m. This accuracy requirement sets a lower frequency limit of 3 MHz to the measuring process and to increase accuracy it might be thought that the obvious answer would be to use an extremely high frequency of propagation. Although this would be the ideal solution, it cannot be done at the moment owing to limitations in electronic technology. At the present time, it is difficult to use phase comparison techniques at frequencies greater than 500 MHz (λ = 0.6 m).

This presents a problem because electromagnetic waves in the frequency range 3-500 MHz are easily absorbed when transmitted through the atmosphere. This difficulty can be overcome by transmitting high power electromagnetic waves, but this is impractical in the case of portable surveying equipment.

In order to be able to transmit the measuring waves through the atmosphere, the process of *modulation* is used. Using this process, the measuring wave (the one used in the phase comparison) is electronically superimposed on a carrier wave of much higher frequency, this higher frequency being chosen to correspond to an *atmospheric window* where much less attenuation of the wave occurs. In the EDM measurements the carrier wave is thus transmitted and is, in fact, acting as a medium for carrying the distance information.

Two methods of modulating the carrier wave are used in EDM.

Amplitude modulation

With amplitude modulation, the carrier wave has a constant frequency and the modulating wave (the measuring wave) information is conveyed by the amplitude of the carrier, as shown in figure 4.17.

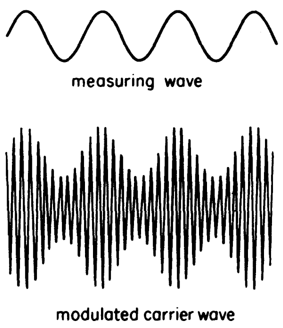


Figure 4.17 *Amplitude modulation*

Frequency modulation

When frequency modulated, a carrier wave has a constant amplitude but its frequency varies in proportion to the amplitude of the modulating wave, as seen in figure 4.18.

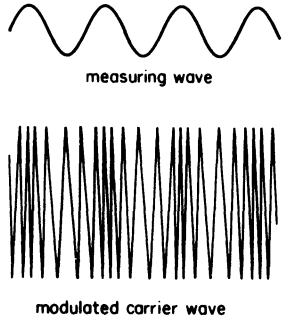


Figure 4.18 *Frequency modulation*

4.13 Instrument Characteristics

EDM instruments are conveniently classified according to the type of carrier wave employed. This section briefly describes the characteristics predominant for each type of instrument.

4.13.1 Microwave Instruments

The carrier frequency used by this group of instruments is typically 10 GHz (1 GHz = 10^9 Hz). This order of frequency is capable of being transmitted over large distances of up to 100 km in clear conditions and a characteristic of this EDM instrument is long range.

As phase comparison techniques are used to measure the distance and a signal has to be returned to the point of comparison, so some form of reflector must be used at the remote end of the line. A *passive reflector* used at these frequencies would return only a very weak signal for comparison and, hence, electronic reflection of the signal has to be used. This is achieved by placing at the remote terminal of the line another instrument, which in some cases is identical to the measuring or *master instrument*. This *remote instrument* receives the transmitted signal, amplifies it and retransmits it back to the master in exactly the phase at which it was received. Phase comparison is thus possible and, since the signal is amplified as well as reflected, a greater working range is obtained.

Microwave EDM instruments thus require *two* instruments and *two* operators in order to measure a length.

A *speech facility* between the master and the remote is provided on all microwave instruments to help the operators proceed through the measuring sequence.

At microwave frequencies, the signals are radiated from small aerials, called *dipoles*, mounted on the front of each instrument. These radiators produce a directional signal with a beam width varying from 2° to 20°, depending on the instrument, so alignment of the master and remote units is not critical.

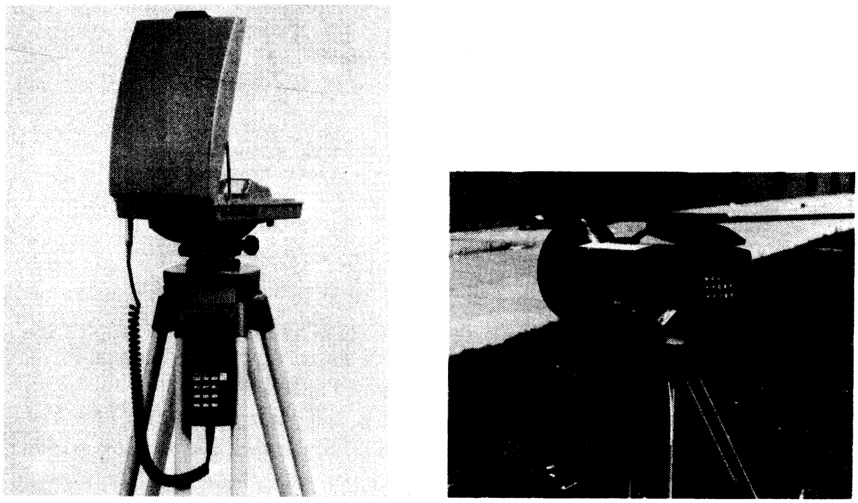
Frequency modulation is used in all microwave instruments and in most units the method of varying the measuring wavelength in multiples of 10 is used to obtain an unambiguous measurement of distance.

Typical microwave instruments are shown in figure 4.19. The maximum range of such instruments is 25-50 km, the accuracy being of the order of ± 10 mm ± 3 mm/km. The fixed component of the accuracy is due to instrumental errors and the variable component is due to atmospheric errors.

In civil engineering, microwave instruments are used mainly in the establishment of control for very large projects and in road and pipeline traverses.

4.13.2 Infrared Instruments

The near infrared radiation band, of wavelength about 0.9 μm, is used for carrier waves by instruments in this group. The reason for the predominance of infrared instruments in EDM is due to the carrier wave source which, in every case, is a gallium arsenide (GaAs) infrared emitting diode. These diodes can be very easily directly amplitude modulated at the high frequencies required for EDM and thus



(a) (b)

Figure 4.19 *Microwave EDM instruments: (a) Tellumat CMW20; (b) Tellumat CMW6 (courtesy Tellumat Ltd)* provide a simple and inexpensive method of producing a modulated carrier wave.

Since infrared is a form of high frequency electromagnetic radiation, the transmitted power in infrared carrier waves falls off rapidly with distance and the range of such instruments is not as great as that of microwave units. To overcome signal loss, the infrared carrier is transmitted as a highly collimated beam using a lens/mirror system and the beam divergence is usually less than 15 minutes of arc. This requires that the transmitted wave should be carefully aligned as it has to be reflected at the remote end of the line.

Since the infrared wavelength is close to the visible light spectrum, infrared carrier waves can be treated as beams of light and a plane mirror could be used to reflect them, but this would require very accurate alignment. Instead, a special form of reflector known as a *corner cube prism* (or *retroreflector*) is always used. These are constructed from the corners of cubes of glass which have been cut away in a plane making an angle of 45° with the faces of the cube as shown in figure 4.20.

Infrared or visible light, directed into the cut face, is reflected by the inner surfaces of the prism which are highly silvered. Such a reflector will return a beam along a path exactly parallel to the incident path over a range of angles of incidence of about 20° to the normal of the front face of the prism. As a result, the alignment of the prism is not critical and it is quickly set in the field.

Hence, infrared instruments require one measuring unit and work in conjunction with passive corner cube reflectors.

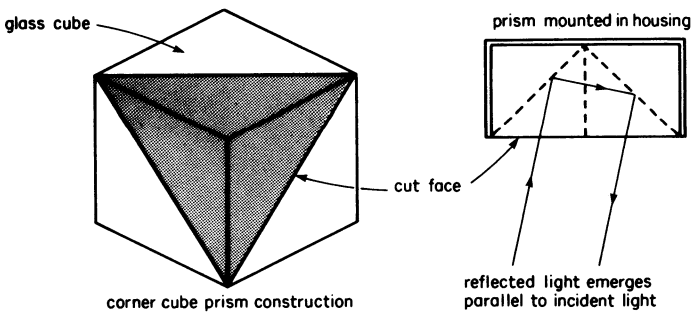
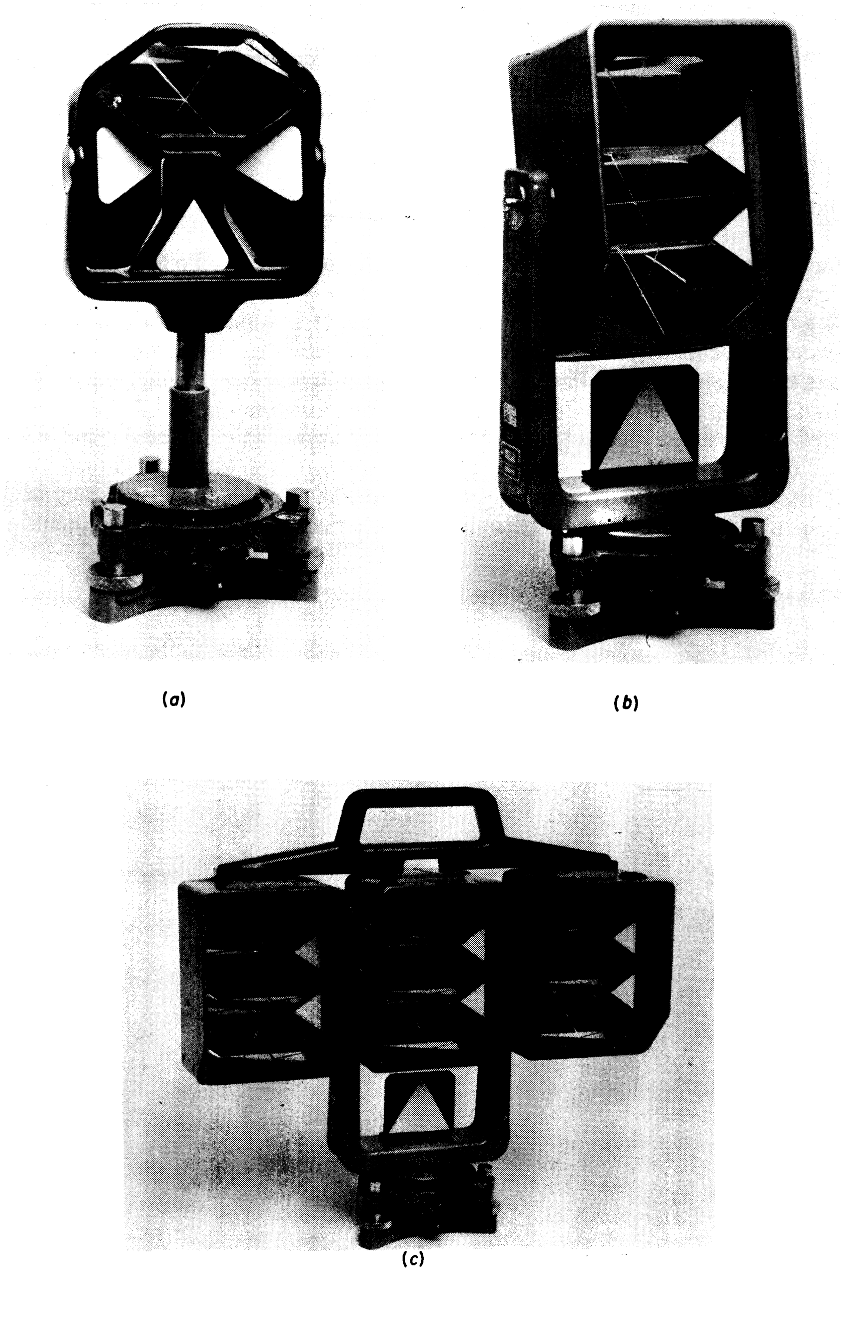


Figure 4.20 *Corner cube prism (retroreflector)*

To obtain an unambiguous measurement of distance, many ingenious systems have been developed for infrared instruments and all use electromechanical or electronic devices, the total distance being displayed automatically.

The main disadvantage with infrared systems is that their power output is low and, consequently, the range of such instruments is limited to 1 km or less with a single prism. However, range can be extended to 2 or 3 km in most systems using reflectors consisting of 3 or 9 prism arrays as shown in figure 4.21. For most engineering applications in developed countries, these ranges are adequate and, owing to the fact that infrared instruments can be made very light and compact, they are used extensively in engineering and land surveying.



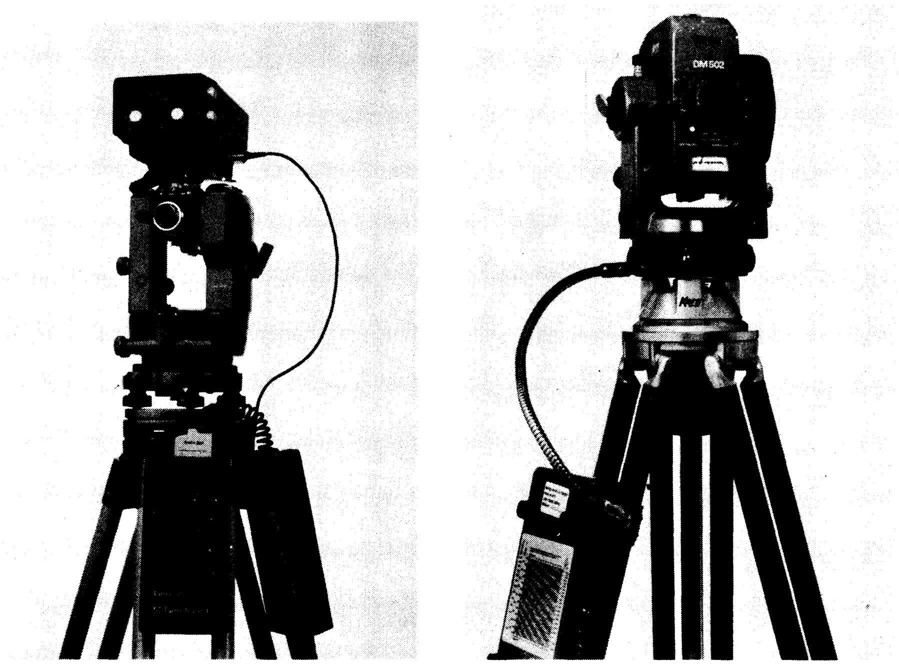
(a) (b) (c)

Figure 4.21 *Prism arrays: (a) single reflector with offset target; (b) triple reflector; (c) nine prism reflector*

The accuracy obtainable from infrared instruments is usually about ±10 mm, irrespective of distance in most cases.

Without doubt, the most useful facility with infrared instruments is that they can be combined with a theodolite in some way since the infrared units are light and compact. This facility enables angles and directions to be measured simultaneously and two types of system can be identified: combined theodolite and EDM systems and electronic tacheometers.

In the *combined theodolite and EDM systems*, a specially designed lightweight EDM unit is attached to a standard theodolite as shown in figure 4.22, the EDM unit and theodolite being operated independently in the field. To aid setting out work when using *pole-mounted reflectors* (see figure 4.23), some EDM units include a *continuous readout facility* or tracking mode in which the distance measurement is repeated automatically approximately once per second. To enable the assistant holding the reflector to take readings directly, a remote receiver or communication system can be used. A *remote receiver* (see figure 4.24) is attached to a reflector using a special lock and the receiver decodes the infrared signal transmitted from the EDM unit to display slope or horizontal distances on a readout built into the rear of the receiver. A *communication system* consists of an EDM instrument fitted with a small microphone inside the control panel and a small



(a) (b)

Figure 4.22 *Combined theodolite and EDM systems: (a) Geodimeter 220 (courtesy Geotronics (UK) Ltd); (b) Kern DM502 (courtesy Kern and Co. Ltd)*

Figure 4.23 *Pole-mounted reflector*

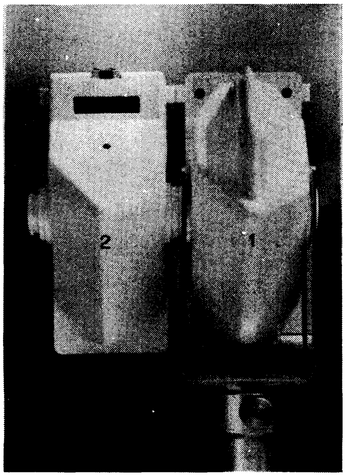


Figure 4.24 *Remote receiver: 1. rear of prism housing; 2. remote receiver displaying distance of 87.120 m (courtesy Kern and Co. Ltd)*

receiver and loudspeaker which are attached to the pole-mounted reflector (see figure 4.25). One-way voice communication is then possible from instrument to reflector using the infrared as a carrier wave. Combined theodolite and EDM systems represent great improvements over taping, and for distances in excess of one steel tape length such systems are gradually replacing other setting-out methods.

The *electronic tacheometer* is a combination of an electronic theodolite (see section 3.2.3) and some form of EDM device as shown in figure 4.26. All electronic tacheometers are purpose-built angle and distance measuring systems which sometimes cannot be separated (see figure 4.27). These instruments are controlled by a keyboard, this in turn being connected to a microprocessor that is either built into or attached to the unit. The microprocessor controls the angle and distance measuring systems and can also act as a calculator for slope reductions, height calculations, rectangular coordinate calculations and so on.

A feature of such systems is a portable *data storage unit* or *recording unit* (see figure 4.28) which is capable of storing digital angle and distance information in solid state form, the data being fed into the unit via a microprocessor or by hand. These storage units have sufficient memory for a working day and the contents of the unit can be transferred on to magnetic tape or transmitted by telephone to an office for processing by computer. Alternatively, the storage unit can have its own microprocessor or it can be connected to a portable microcomputer to enable survey computations to be performed in the field.

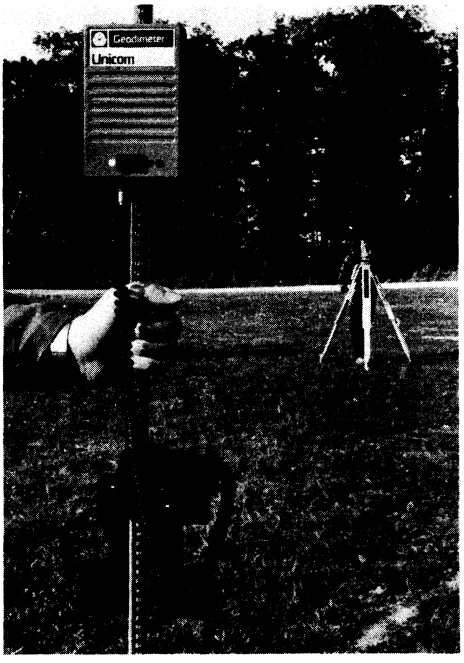


Figure 4.25 *Communication system (courtesy Geotronics (UK) Ltd)*

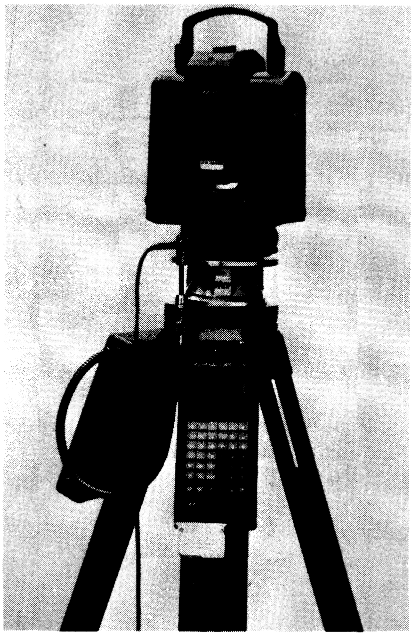
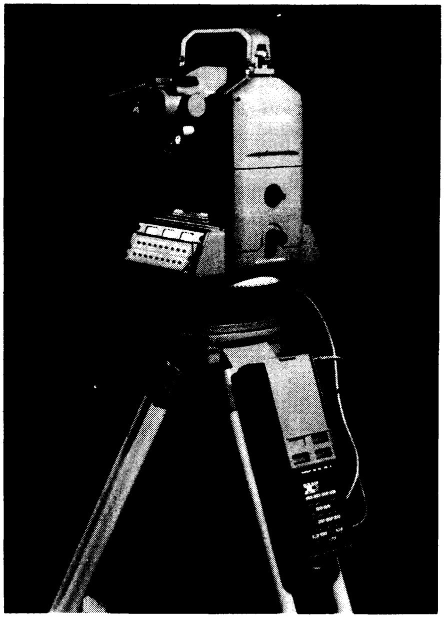



Figure 4.26 *Electronic tacheometers – combined EDM and electronic theodolite (courtesy Kern and Co. Ltd and Wild Heerbrugg (UK) Ltd)*

4.13.3 Laser Instruments

The helium-neon laser ($\lambda = 0.6328\ \mu m$, $f = 4.74 \times 10^{14}$ Hz) has been used as a carrier wave in a number of EDM instruments and the most successful application has been in long range distance measurers of relatively good accuracy. Long ranges are possible due to the capability of the laser to produce a high power in a beam of very low angular divergence, and an improved accuracy is possible since the laser is a form of coherent and stable radiation that can be frequency modulated. Typical maximum ranges for laser instruments are between 30 and 60 km with accuracies of ± 5 mm ± 1 ppm.

When long ranges are measured by EDM, the atmospheric uncertainties impose a limit on the precision attainable but, by measuring distances with *two-colour* instruments, accuracies of ±0.1 to ±0.2 ppm over ranges in excess of 20 km have been recorded. This improvement in accuracy is achieved by measuring each distance simultaneously using two lasers of differing wavelength, which enables atmospheric effects to be accounted for.

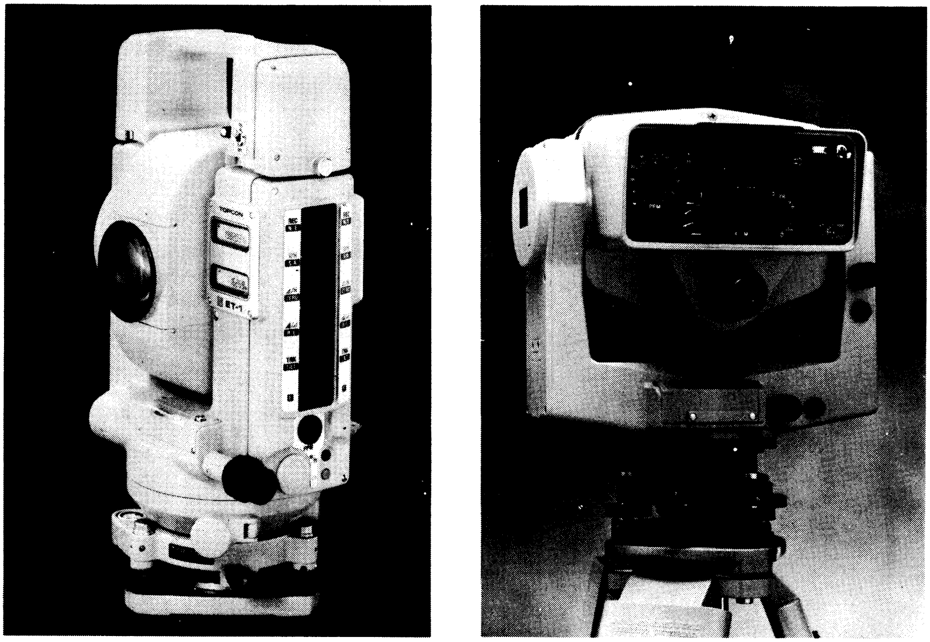


Figure 4.27 *Electronic tacheometers – purpose built (courtesy Hall and Watts Ltd and Geotronics (UK) Ltd)*

4.14 EDM Corrections

When measurements are taken using an EDM instrument, atmospheric and instrumental effects may give rise to errors in the distances displayed and corrections are required to account for these. In addition, it is usual to apply a series of geometric corrections to the slope distances measured in order that horizontal distances may be obtained.

4.14.1 Atmospheric Effects

All electromagnetic waves, when moving through a vacuum, travel with the same velocity (c) but when travelling in the atmosphere their velocity (v) is reduced from the free space value owing to the retarding action of the atmosphere.

Consequently, the velocity of the carrier and measuring (or modulating) waves will vary for all measurements.

Since v is a variable depending on atmospheric conditions, the modulating wavelength will also vary since it is given by $\lambda = v/f$. The significance of this is that the measuring unit λ is not constant and the distance recorded by the instrument will be in error.

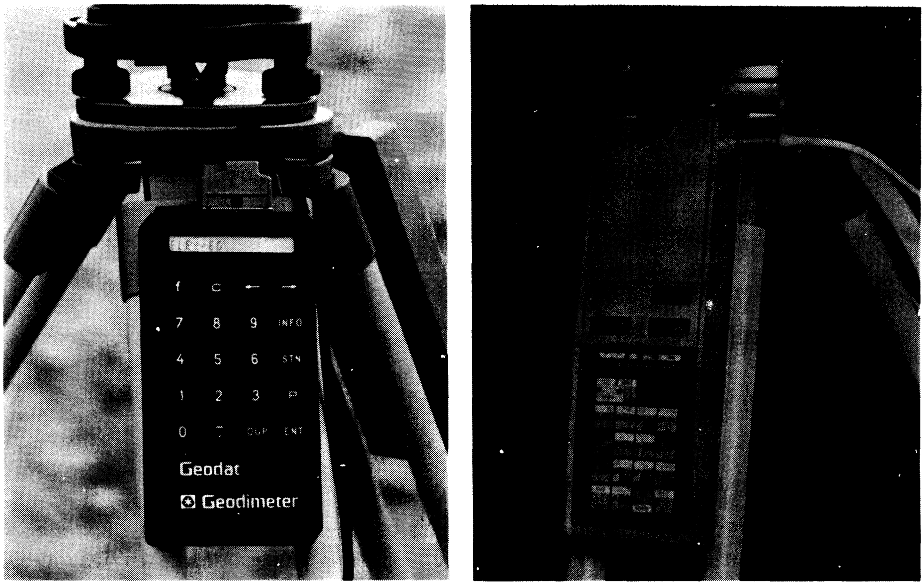


Figure 4.28 *Data storage or recording units (courtesy Geotronics (UK) Ltd and Wild Heerbrugg (UK) Ltd)*

To correct for this, many of the short range infrared systems use an *atmospheric correction switch* which is set according to the atmospheric pressure and temperature prevailing at the time of measurement, these being measured on site. Charts and tables provided by the manufacturer enable the temperature and pressure to be converted into an appropriate switch setting. In effect, changing the setting on the switch changes the frequency of the measuring wave. This compensates for the change in velocity and keeps the wavelength of the measuring unit constant.

Another method of removing atmospheric effects in EDM measurements is to enter corrections directly into the EDM unit using a dial mounted on the instrument for this purpose. As with the atmospheric correction switch, the atmospheric conditions must be measured and the correction, usually in ppm (= parts per million or the correction to the distance in mm/km), is deduced from charts supplied with the instrument.

Whatever method is used to correct for atmospheric effects, it is evident that this requires meteorological conditions to be determined at some stage in the measurement of an EDM line. Great care should be taken when recording this data as the main factor that limits the accuracy of any EDM measurement is the uncertainty in the meteorological conditions. It is worth remembering that temperatures estimated to ±5°C will produce an error of about 10 ppm in the distance as will atmospheric pressures estimated to ±25 mm Hg. Also, since the atmospheric correction is proportional to the distance being measured, extra care should be taken in the recording of meteorological conditions when measuring long lines.

4.14.2 Instrumental Errors

All EDM measurements are subject to the following instrumental errors.

Scale error (or frequency drift)

This is caused by variations in the modulation frequency, f, of the EDM instrument and the error is therefore proportional to the distance measured. Consequently, the effect is much more noticeable on long lines and can usually be ignored for the short range instruments.

Zero error (or index error)

This occurs if there are differences in the mechanical, electrical and optical centres of the EDM instrument and reflectors. Analogous to miscentring a theodolite (see section 3.3.4), the error is not dependent on range and care must be taken to eliminate it.

Cyclic error (or instrument nonlinearity)

This error is caused by unwanted interference between electrical signals generated in the EDM unit and can be investigated by measuring a series of known distances spread over the measuring wavelength of the instrument. If a calibration curve of (observed – known) distances is plotted against distance and a periodic wave is obtained, the EDM instrument has a cyclic error. The effect of this can usually be ignored for ordinary engineering surveys but may have significance on longer lines or where a high precision is required.

4.14.3 Further Corrections

When the corrected slope distance (L) has been obtained from an EDM measurement, further corrections must be applied to it. If lines of less than 10 km are considered, the following corrections are necessary.

Slope correction

This is the same as for taping as discussed in section 4.2.2.

$$\text{Slope correction} = \frac{\Delta H^2}{2L} \quad \text{or} \quad L(1 - \cos\theta)$$

where

horizontal distance = D = (L – slope correction)
ΔH is the height difference between the instrument and the reflector and
θ is the vertical angle along the line of measurement.

Height correction

When a survey is to be based on the National Grid coordinate system (see section 5.11), the line measured must be reduced to its equivalent length at mean sea level

(MSL) (see section 2.1.3). The height or MSL correction is given by (see also section 6.3.3)

$$\text{Height correction} = -\frac{Dh_m}{R}$$

where h_m is the mean height of the instrument and reflector above MSL and R the radius of the Earth (6375 km).

The correction is *negative* unless a line below MSL is measured.

Scale factor (F)

The grid distance must be used for National Grid calculations (see section 5.11)

$$\text{grid distance} = \text{horizontal MSL distance} \times F$$

4.15 Applications of EDM to Civil Engineering and Surveying

Generally, the use of EDM in engineering surveying operations results in a saving in time and, in most cases, an improvement in the accuracy of distance measurement when compared with taping (particularly in excess of one steel tape length) and with optical methods.

When using EDM, rapid and accurate surveying of detail is possible owing to the long ranges attainable and fewer control stations are required (in comparison with stadia surveys discussed in section 8.5). Consequently, EDM has replaced tacheometric methods and chain surveying for the production of site plans. In addition, both the combined theodolite/EDM and electronic tacheometer systems are extremely well adapted to forming digital terrain models (DTMs, see section 8.8) and if data storage units are used these can be interfaced directly with the computer forming the model.

As angles and distances can be measured simultaneously with the latest infrared short range equipment, many setting out operations are now simplified. Some instruments have a continuous readout facility and this enables distances of many hundreds of metres to be set out in one sighting, often over ground that would be unsuitable for taping. The ability to take measurements across congested sites is also a great advantage.

As a consequence of the continuing development of EDM instruments, some setting-out techniques are changing. In roadworks, EDM can be used to coordinate the major control points for the initial survey of the route and then can use these stations to establish the road centre line by polar methods (see section 14.8.2 and section 14.17) rather than tangential angles methods (see sections 9.9.1 and 10.8.2).

Another instance of changing techniques is that buildings can be set out from two or more instrument stations by polar coordinates rather than by using a theodolite to establish right angles (where appropriate) at each corner.

These methods have been helped by the advent of advanced pocket calculators and computers and many local authorities now issue contractors with a computer printout for setting out engineering works in the form of bearing and distance tables (see table 10.2).

A further use of EDM in civil engineering is in tunnelling, where it is used in the surveying necessary for the establishment of headings at ground level and for the measurement of the depths of shafts. EDM has also been used successfully for positioning piles and other inshore marine structures.

Although EDM equipment has tremendous potential in civil engineering, particularly for the measurement of slope and horizontal distances, its main drawback is its cost. The theodolite-mounted infrared devices such as those of figure 4.22 represent the least expensive systems currently available and their use is now common in civil engineering. Electronic tacheometers (figures 4.26 and 4.27), although much more sophisticated, are not used extensively by civil engineering contractors since their high cost is justified only by organisations who can maintain a large volume of survey work. Prices are changing rapidly and it would serve no purpose to quote current rates here, but suffice it to say that EDM equipment is very expensive and, although it can be hired at a daily, weekly or longer rate, these hire charges are also very high. This cost must be taken into account when planning a project and weighed against the cost and accuracy of the alternative methods, that is, ODM and taping.

4.16 Further Reading

C. D. Burnside, *Electromagnetic Distance Measurement, 2nd Edition* (Granada, London, 1982).

S. H. Laurila, *Electronic Surveying in Practice* (Wiley, 1983).

5

Traversing

One of the principles of engineering surveying, as discussed in section 1.4, is that horizontal and vertical control must be established for surveying detail and for setting out engineering projects. A traverse is one means of providing a network of horizontal control in which position is determined by a combination of angle and distance measurement between successive lines joining control stations.

5.1 Types of Traverse

5.1.1 Closed Traverses

Two cases have to be distinguished with this type of traverse. In figure 5.1, a traverse has been run from station X (of known position) to stations 1, 2, 3 and another known point Y. Traverse X123Y is, therefore, closed at Y. This type of

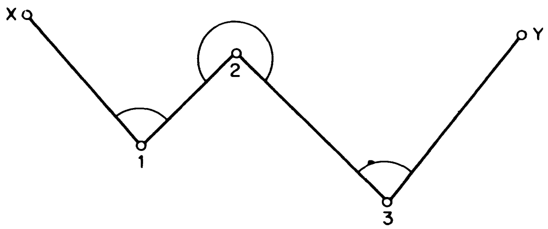


Figure 5.1 *Link traverse*

traverse is called a *link, connecting or closed-route traverse*.

In figure 5.2, a traverse starts at station X and returns to the same point X via stations 1, 2 and 3. Station X can be of known position or can have an assumed position. In this case the traverse is called a *polygon, loop or closed-ring traverse* since it closes back on itself.

In both types of closed traverse there is an external check on the observations since the traverses start and finish on known or assumed points.

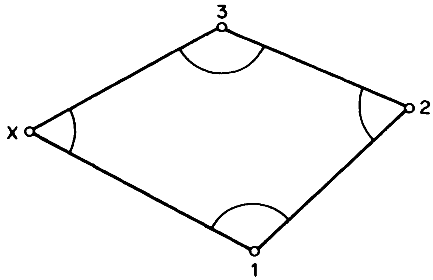


Figure 5.2 *Polygon traverse*

5.1.2 Open Traverses

These commence at a known point and finish at an unknown point and, therefore, are not closed. They are used only in exceptional circumstances since there is no external check on the measurements.

5.2 Traverse Specifications and Accuracy

The accuracy of a traverse is governed largely by the type of equipment used and the observing and measuring techniques employed. These are dictated by the purpose of the survey.

Many types of traverse are possible but three broad groups can be defined and are given in table 5.1.

The most common type of traverse for general engineering work and site surveys would be of typical accuracy 1 in 10 000. The chapter notes are concerned mainly with an expected accuracy range of about 1 in 5000 to 1 in 20 000.

An important factor when selecting traversing equipment is that the various instruments should produce roughly the same order of precision, that is, it is pointless using a 1″ theodolite to measure traverse angles if the lengths are being measured with a chain. Table 5.1 gives a general indication of the grouping of suitable equipment.

5.3 Bearings and Coordinates

5.3.1 Whole-circle Bearings

To establish the direction of a line between two points on the ground, its bearing has to be determined.

The whole-circle bearing (WCB) of a line is measured in a clockwise direction in the range 0° to 360° from a specified reference or north direction. Examples of whole-circle bearings are given in figure 5.3.

TABLE 5.1
General Traverse Specifications

TYPE	TYPICAL ACCURACY	PURPOSE	ANGULAR MEASUREMENT	DISTANCE MEASUREMENT
Geodetic	1 in 50 000 or better	(1) Major Control for mapping large areas (2) Provision of very accurate reference points for engineering surveys	0.1" theodolite	EDM
General	1 in 5000 to 1 in 50 000	(1) General engineering surveys, that is, setting out and site surveys (2) Secondary control for mapping large areas	1" or 20" theodolite	EDM, steel tapes, subtense methods
Low Accuracy	1 in 500 to 1 in 5000	(1) Small scale detail surveys (2) Rough large scale detail surveys (3) Preliminary or reconnaissance surveys	20" or 1' theodolite	Synthetic tapes, chains, stadia tacheometry

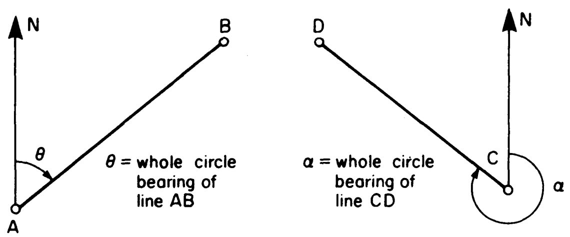


Figure 5.3 *Whole-circle bearings*

5.3.2 North Directions

The specified reference or north direction on which bearings are based may be true north, magnetic north, some entirely arbitrary direction assigned as north, or grid north.

True north

The accurate determination of this direction is undertaken only for special surveys. True north is not normally used in traversing for engineering surveys. However, an approximate value can be scaled from Ordnance Survey (OS) maps.

Magnetic north

This is determined by a freely suspended magnetic needle and can be measured with a prismatic compass.

A *prismatic compass* is a hand-held device which consists, basically, of a magnetic needle freely supported at its centre and usually immersed in oil to dampen oscillations. The needle will always indicate magnetic north although it can be unreliable in areas of strong local magnetic attraction and also when held near metal objects. For this reason, a prismatic compass must never be held against a metal ranging rod when taking readings.

The compass is graduated from $0°$ to $360°$ in half-degree intervals and is so designed that the magnetic bearing is obtained directly. The precision of the reading system is, at best, only $\pm 15'$ owing to difficulties in holding the compass steady and the rather crude sighting system.

Magnetic north is used only in reconnaissance surveys or to give a general indication of north when an arbitrary north is chosen for the survey.

Arbitrary north

Arbitrary north is most commonly used to define bearings in engineering traverses.

Any convenient direction is usually chosen to represent north even though it is not, in general, a true or magnetic north direction.

If a link traverse is being run between sets of known stations then the north direction may be determined from the values previously assigned to these known points.

Grid north

This north direction is based on the National Grid, which is discussed in section 5.11.

5.3.3 Rectangular Coordinates

The coordinate system adopted for most survey purposes is a plane, rectangular system using two axes at right angles to one another as in Cartesian geometry. One is termed the north (N) axis and the other the east (E) axis. The scale along both axes is *always* the same.

With reference to figure 5.4, any particular point P has an easting (E) and a northing (N) coordinate, always quoted in the order easting, northing unless otherwise stated.

The position of each traverse station in a scheme in relation to all the others is specified in terms of these E and N coordinates.

Bearings are related to the north axis of the coordinate system.

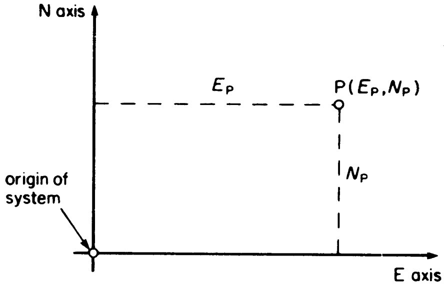


Figure 5.4 *Rectangular coordinate system*

For *all* types of survey and engineering works, the origin is taken at the extreme south and west of the area so that all coordinates are *positive*. If, at some stage in a survey, negative coordinates arise, the origin should be moved such that all coordinates will again be positive.

5.4 Traversing Fieldwork: Reconnaissance

5.4.1 General

This is one of the most important aspects of any surveying operation and must always be undertaken *before* any angles or lengths are measured.

(1) The main aim of the reconnaissance is to locate suitable positions for stations and hence a poorly executed reconnaissance can result in difficulties at later stages in the survey, leading to wasted time and inaccurate work.

(2) An overall picture of the area is obtained by walking all over the site (more than once is recommended), keeping in mind the requirements of the survey and balancing this against the accuracy and hence method of survey to be used. If an existing map or plan of the area is available, this is a useful aid at this stage.

(3) Where possible, *work from the whole to the part* as described in section 1.4, but an attempt should be made to keep the number of stations to a minimum.

(4) The lengths of traverse legs should be kept as long as possible to minimise the effect of any centring errors (see sections 3.3.4 and 5.5.1).

(5) If the traverse is being run for a detail survey then the method which is to be used for this subsequent operation must be considered.

For most sites a polygon traverse is usually sited around the perimeter of the area at points of maximum visibility. It should be possible to observe cross checks or lines across the area to enable other points inside the area to be fixed and also to assist in the location of angular errors.

Traverses for roadworks and pipelines generally require a link traverse since these sites tend to be long and narrow. The shape of the road or pipeline dictates the shape of the traverse.

(6) If the linear measurements are to be carried out using a tape or chain the ground conditions between stations should be suitable for this purpose. Try to avoid steep slopes or badly broken ground along the traverse lines. It is also better if there are as few changes of slope as possible. Roads and paths that have been surfaced are usually good for ground measurements.

(7) Stations should be located such that they are clearly intervisible, preferably at ground level, that is, with a theodolite set up at one point, it should be possible to see the ground marks at adjacent stations and as many others as possible. This eases the angular measurement process and enhances its accuracy.

(8) Stations should be placed in firm, level ground so that the theodolite and tripod are supported adequately when observing angles at the stations.

Very often stations are used for a site survey and at a later stage for setting out. Since some time may elapse between the site survey and the start of the construction the choice of firm ground in order to prevent the stations moving in any way becomes even more important. It is sometimes necessary to install semi-permanent stations (see section 5.4.2).

(9) Owing to the effects of lateral refraction and shimmer, traverse lines of sight should be well above ground level (greater than 1 m) for most of their length to avoid any possible angular errors due to rays passing close to ground level (*grazing rays*). These effects are serious in hot weather.

(10) When the stations have been sited, a sketch of the traverse should be prepared *approximately to scale*. The stations are given reference letters or numbers. This greatly assists in the planning and checking of fieldwork.

5.4.2 Station Marking

When a reconnaissance is completed, the stations have to be marked for the duration, or longer, of the survey.

Station markers must be permanent, not easily disturbed and they should be clearly visible. The construction and type of station depends on the requirements of the survey.

For general purpose traverses, wooden pegs are used which are hammered into the ground until the top of the peg is almost flush with ground level. If it is not possible to drive the whole length of the peg into hard ground the excess above the ground should be sawn off. This is necessary since a long length of peg left above the ground is liable to be knocked. A nail should be tapped into the top of the peg to define the exact position of the station.

Figure 5.5 shows such a station. Several months use is possible with this type of marker.

Stations in roadways can be marked with 75 mm pipe nails driven flush with the surface. The nail surround should be painted for easy identification. These marks are fairly permanent, but it is usually prudent to enquire if the road is to be resurfaced in the near future.

A more permanent station would normally require marks set in concrete; common station designs are shown in figure 14.2. These have to be placed with the permission of land owners as subsurface concrete blocks placed in a field could do considerable damage to farm machinery.

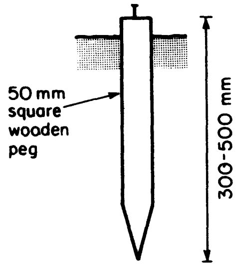


Figure 5.5 *Station peg*

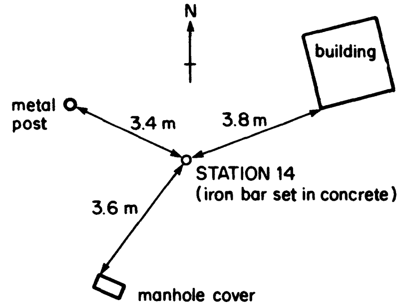


Figure 5.6 *Witnessing sketch*

A *reference* or *witnessing sketch* of the features surrounding each station should be prepared, especially if the stations are to be left for any time before being used, or if they will be required again at a much later stage. Measurements are taken from the station to nearby permanent features to enable it to be relocated. A typical sketch is shown in figure 5.6.

5.5 Traversing Fieldwork: Angular Measurement

Once the traverse stations have been placed in the ground the next stage in the field procedure is to use a theodolite to measure the included angles between the lines.

This requires two basic operations: setting the theodolite over each station mark, and observing the directions to the required stations.

In most cases it will be necessary to provide a signal at the observed stations since the station marks may not be directly visible. The theodolite and signals have to be erected perpendicularly above the station marks, otherwise centring errors will result.

5.5.1 Centring Errors

The measurement of traverse angles requires that the theodolite and signals be located in succession at each station. If this operation is not carried out accurately, centring errors are introduced, the effect of which depends on the length of the traverse leg, as discussed in section 3.3.4.

If a target displacement of 10 mm occurs on a 300 m traverse leg, the resulting angular error is $7''$. The same displacement on a 30 m leg will produce an angular error of $70''$. If this occurred during a traverse, the error would be carried through the rest of the traverse, and all subsequent bearings would be incorrect.

Hence, the effect of relatively small centring errors can be serious on short traverse legs.

If the theodolite is also displaced a further source of error arises.

The conclusion is that with both theodolites and signals care in centring is vital, especially when traverse legs are short.

5.5.2 Station Signals

Signals should be perfectly straight objects set up vertically and centred exactly over the station mark.

If a signal is not vertical then a centring error will be introduced, even though the base of the signal may be centred accurately over the mark. This is demonstrated by figure 5.7.

From figure 5.7, the lower the point of observation on the target, the smaller will be the centring error. For this reason, the lowest visible point on any signal should always be observed when measuring angles. However, this does not apply to special traverse targets (see section 5.7).

A subject not often considered is the width or diameter of a signal. It is a waste of time trying to observe a direction to a ranging rod when the line of sight is only 30 m, since accurate bisection is difficult to achieve. The width of a target should be proportional to the length of sight. Suggestions for some simple types of target are as follows.

(1) The station mark should be observed directly if possible. This can often be the case over short lines if the mark is a nail in the top of a wooden peg.

(2) If the mark cannot be seen directly, a pencil held on the mark can be used for convenience.

(3) A marking arrow can be held with its point on the mark or inserted in the top of the peg on the line of sight directly behind the nail.

(4) A tripod can be set up such that a plumb bob can be suspended from it directly over the mark. The plumb line can then be observed. Care must be exercised to ensure that the plumb bob does not rest on the mark as the string will then no longer be vertical, as shown in figure 5.8.

(5) For longer lines, ranging poles can be used. These must be carefully centred over the mark and must be held vertically by hand or in a ranging rod stand. The lowest part of the rod must be observed.

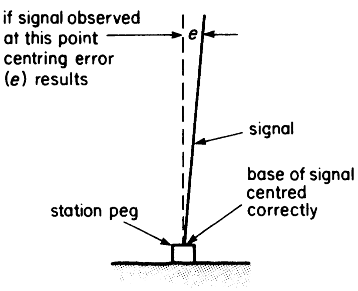


Figure 5.7 *Signal not vertical*

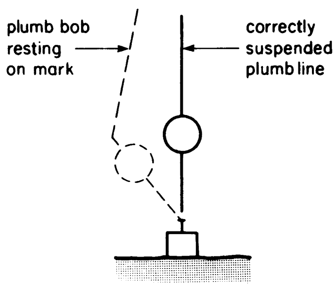


Figure 5.8 *Plumb line not vertical*

5.5.3 Abstraction of Angles

The general case at any station in a traverse is that the angle to be measured will be between some signal at a *back station* and some signal at a *forward station*, as shown in figure 5.9.

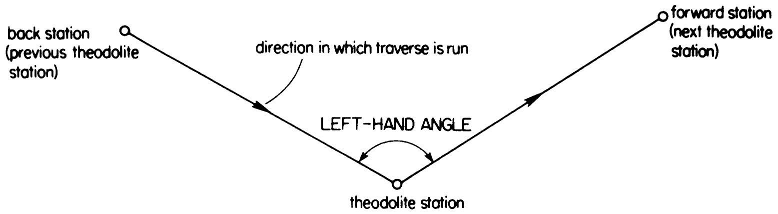


Figure 5.9 ***Left-hand angle***

When readings have been taken to both stations, the angle abstracted may be either

(1) *left-hand angle* = mean forward circle reading – mean back circle reading

or (2) *right-hand angle* = mean back circle reading – mean forward circle reading.

For computations, *either* can be chosen, but, for a particular traverse, it is important that the *same* angle should be abstracted at every station.

It is conventional, however, that the left-hand angle is chosen, the reason being that in computations the left-hand angle is added to the bearing to the back station to give the bearing to the forward station. If the right-hand angle is abstracted, this would have to be subtracted, an operation that is more liable to error (see section 5.8.3).

For polygon traverses, the *internal* angles of the polygon will be the left-hand angles if fieldwork proceeds in an *anticlockwise* direction around the traverse.

5.5.4 Field Procedure and Booking

The method given in chapter 3 for the reading and booking of angles should be adhered to whenever possible.

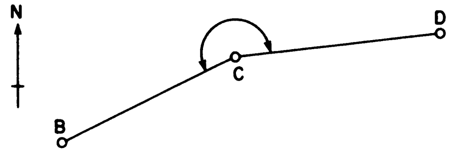
In the case where no standard booking forms are available, the angles can be entered in a field book, as in figure 5.10, in which *two complete rounds* of angles have been observed and the *zero changed* between rounds. The reasons for this are discussed in section 3.3.2.

AT STATION C

STATION	FL	FR	MEAN	ANGLE
① B	00° 07' 20"	180° 07' 10"	00° 07' 20"	
D	192° 23' 40"	12° 23' 20"	192° 23' 30"	192° 16' 10"
② B	87° 32' 40"	267° 32' 20"	87° 32' 30"	
D	279° 49' 20"	99° 49' 00"	279° 49' 10"	192° 16' 40"

MEAN ANGLE = 192° 16' 30"

DIAGRAM



OBSERVER WFP

BOOKER JU   DATE 5th September 1985

Figure 5.10 *Booking traverse angles*

5.5.5 Errors in Angular Measurements

The various sources of error that may arise when measuring traverse angles are summarised as follows.

(1) Inaccurate centring of the theodolite or signal.
(2) Nonverticality of the signal.
(3) Inaccurate bisection of the signal.
(4) Parallax not eliminated.
(5) Lateral refraction, wind and atmospheric effects.
(6) Theodolite not level and not in adjustment (see sections 3.3.3 and 3.5).
(7) Incorrect use of the theodolite.
(8) Mistakes in reading and booking.

5.6 Traversing Fieldwork: Distance Measurement

For the purposes of traversing, distance measurement of the traverse legs is normally undertaken using steel taping or EDM, both of which are discussed in chapter 4.

5.7 The Three-tripod System

Very often, short traverse legs are unavoidable, for example, in surveys in mines and tunnels and on congested sites. Some manufacturers provide special equipment for use on short traverse lines and it is known as the three-tripod system. This type of equipment is now used extensively in ordinary traverse work.

As described in section 3.2.1, the main feature of the system is that the body of the theodolite can be lifted from the tribrach and replaced by a special target. Thus, with the use of three or more tripods, the theodolites and targets can occupy the same positions and centring errors are greatly reduced. Distance measuring equipment can also be placed in the tribrachs and linear measurements are therefore made between exactly the same points as angular measurements.

The system operates as follows, with reference to figure 5.11.

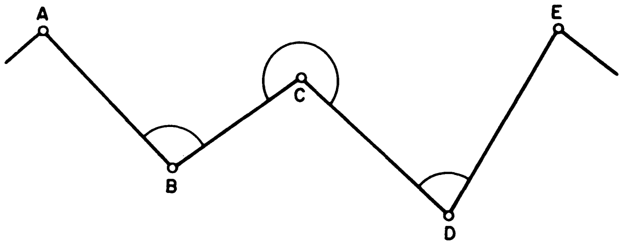


Figure 5.11 ***Three-tripod traversing***

When angle ABC is measured

(1) At A a tripod is set up and a tribrach attached to the tripod head. A special target is placed into the tribrach and clamped in position. The target or tribrach will have a tube or pond bubble attached so that the target can be set vertical by levelling using the tribrach footscrews. In order to be able to centre the target, the tribrach usually has an optical plummet.

(2) At B the theodolite is set up in the normal manner.

(3) At C a tripod and target is set up as at A.

When angle BCD is measured

(1) At A the tripod and target are moved to D, where the target is again centred and set vertical.

(2) At B the theodolite is unclamped, removed from its tribrach and interchanged with the target at C. Hence, at B and C, the tripods and tribrachs remain undisturbed and there is no need for recentring.

When angle CDE is measured

(1) At B the tripod and target are moved to E.

(2) The theodolite and target at C and D are interchanged, the tribrachs (and centring) remaining undisturbed.

The process is repeated for the whole traverse. If *four* tripods (or more) are used this speeds up the fieldwork considerably as tripods can be moved and positioned while angles are being measured.

Three tripod systems are more expensive than basic systems and more equipment has to be moved around the site by the engineer and his assistants. However, the advantages of the dramatic reduction of centring errors and a saving in time far outweigh these disadvantages.

5.8 Traversing Calculations

5.8.1 Abstract of Fieldwork

When all the traverse fieldwork has been completed, a single sheet or record containing the mean angles observed and mean horizontal (corrected) lengths measured should be prepared. It is preferable to show all the data on a sketch of the traverse as this helps in the following calculations and can minimise the chance of a mistake.

Such an abstraction of field data is shown in figure 5.12, the angles and lengths being entered on to a traverse diagram. The example shown in figure 5.12 will be referred to through section 5.8.

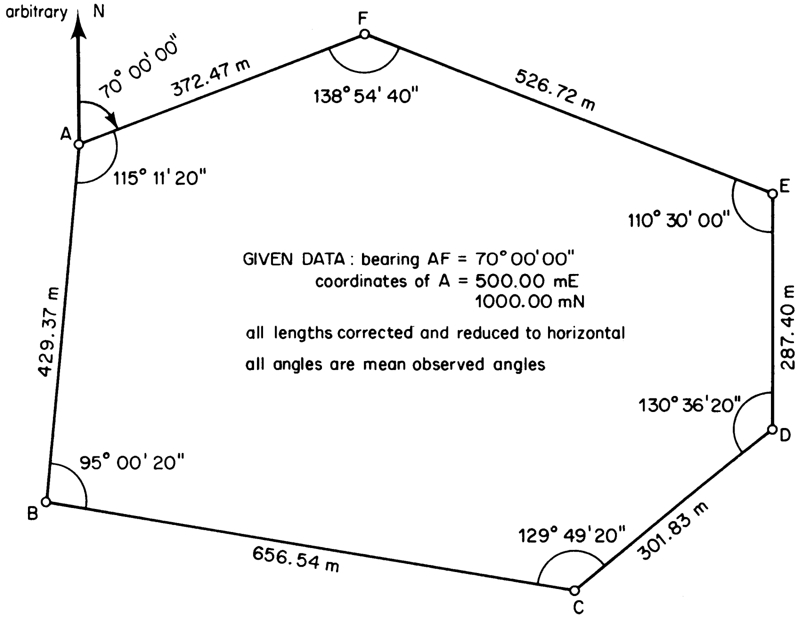


Figure 5.12 *Traverse diagram*

5.8.2 Angular Misclosure

Determination of misclosure

For a closed traverse, before any coordinate calculations can commence, the whole circle bearings of all the lines have to be calculated, the initial stage in the process being to check that the observed angles sum to the required value.

The observed angles of a *polygon traverse* can be either the *internal* or *external* angles; whichever is abstracted depends on the direction in which the traverse is run (see section 5.8.3).

The angular misclosures are found by comparing the sum of the observed angles with one of the following theoretical values.

(1) Sum of *internal* angles $= (2n - 4) \times 90^\circ$

or (2) Sum of *external* angles $= (2n + 4) \times 90^\circ$

where n is the number of angles or sides of the polygon.

For the traverse, the observed angles are summed and a check is made according to one of the above formulae.

When the bearings in a *link traverse* are calculated, an *initial back bearing* (see section 5.8.3) can usually be determined from known points at the start of the traverse and, to check the observed angles, a *final forward bearing* (see section 5.8.3) is computed from known points at the end of the traverse. The method for obtaining a bearing from coordinates is given in section 5.10. The angular misclosure in a link traverse is found by using the following theoretical relationship

sum of *left-hand angles* = (final forward bearing − initial back bearing) + $(n - 1) \times 180^\circ$

where n is the number of left-hand angles measured.

For both types of traverse, care must be taken to ensure that the correct angles have been abstracted and summed, that is, the internal *or* external angles in a polygon traverse and the left-hand angles in a link traverse. When the angles have been summed and checked, a very large misclosure probably means that an incorrect angle has been included, or one of the angles has been excluded.

Allowable misclosure

Owing to the effects of occasional miscentring, slight misreading and small bisection errors, a small misclosure will result when the summation check is made.

The allowable misclosure (E) is $E'' = \pm KS(N)^{\frac{1}{2}}$ where N is the number of traverse stations, S the smallest reading interval on the theodolite in seconds, for example, 60″, 20″, 1″, and K the multiplication factor of 1 to 3, depending on weather conditions, number of rounds taken, and so on. The allowable misclosure for the traverse shown in figure 5.12 varies from approximately 50″ to 150″ (assuming a 20″ theodolite was used).

Adjustment

When the actual misclosure is known and is compared to its allowable value, two cases may arise.

(1) *If the misclosure is acceptable* (less than the allowable) it is divided *equally* between the left-hand angles. An equal distribution is the only acceptable method since each angle is measured in the same way and there is an equal chance of the misclosure having occurred in any of the angles.

No attempt should be made to distribute the misclosure in proportion to the size of an angle.

(2) *If the misclosure is not acceptable* (greater than the allowable) the angles should be remeasured if no gross error can be located in the angle bookings or summation.

It may be possible to isolate a gross error in a small section of the traverse if check lines have been observed across it.

Example of angular misclosure and adjustment

The determination of the misclosure and adjustment of the angles of the polygon traverse given in figure 5.12 is shown in table 5.2.

Example 5.12.2 at the end of this chapter shows how the angles in a link traverse are adjusted.

5.8.3 Calculation of Whole-circle Bearings

Types and determination of bearings

Consider figure 5.13, which shows two legs of a traverse. The decision has been made to calculate the traverse in the direction . . . X to Y to Z . . . This defines the bearings as follows.

Bearings XY and YZ are *forward bearings* since they are in the same direction in which calculations are proceeding.

Bearings YX and ZY are *back bearings* since they are opposite to the direction in which the traverse calculation is proceeding.

Directions ZY and YZ differ by ± 180°, as do those of YX and XY. Therefore, the forward bearing of a line differs from the back bearing by ± 180°. Since whole-circle bearings must lie in the range 0° to 360°, some multiple of 360° is either subtracted from or added to a forward or back bearing outside this range to bring the resulting bearing into the range 0° to 360°. For example

$$\text{a bearing of } 520^\circ = \text{a bearing of } (520 - 360)^\circ = 160^\circ$$

$$\text{a bearing of } -200^\circ = \text{a bearing of } (-200 + 360)^\circ = 160^\circ$$

Bearings are calculated relative to the selected north line, usually starting from a given or assumed bearing for one line.

If bearing YX was known, this could be drawn as shown in figure 5.13 relative to the selected north line.

For the direction of computation shown in figure 5.13, the *left-hand angle* γ_Y has been abstracted and the known bearing YX is a *back bearing*.

If γ_Y is added to the back bearing YX it can be seen from figure 5.13 that the resulting angle will be the forward bearing YZ. Thus

$$\text{forward bearing YZ} = \text{back bearing YX} + \gamma_Y$$

Therefore, in general, for any particular traverse station

$$\textit{forward bearing} = \textit{back bearing} + \textit{left-hand angle}$$

For polygon traverses when working in an *anticlockwise* direction around the

TABLE 5.2

STATION	LEFT HAND ANGLE	ADJUSTMENT	ADJUSTED LEFT HAND ANGLE
A	115° 11' 20"	- 20"	115° 11' 00"
B	95° 00' 20"	- 20"	95° 00' 00"
C	129° 49' 20"	- 20"	129° 49' 00"
D	130° 36' 20"	- 20"	130° 36' 00"
E	110° 30' 00"	- 20"	110° 29' 40"
F	138° 54' 40"	- 20"	138° 54' 20"
Sums	720° 02' 00"	- 02' 00"	720° 00' 00"

Required Sum = ((2x6) - 4) x 90
= 720° 00' 00"

Misclosure = + 02' 00"

Adjustment per angle
= - (02' 00")/6
= - 20"

traverse, the left-hand angles will be the *internal* angles of the traverse and when working in a *clockwise* direction, the left-hand angles will be the *external* angles.

Either clockwise or anticlockwise can be run since abstracting the left-hand angles will always give the correct angles for the bearings computations.

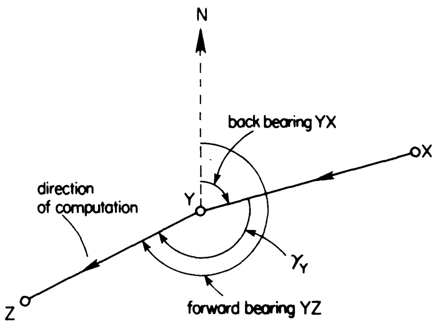


Figure 5.13 ***Whole-circle bearing calculation***

Example of bearing calculation

Some of the bearings of the lines of the traverse shown in figure 5.12 will now be computed using adjusted left-hand angles. Figures 5.14 and 5.15 show sections of this traverse.

At station A in figure 5.14

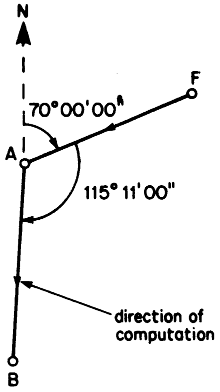


Figure 5.14

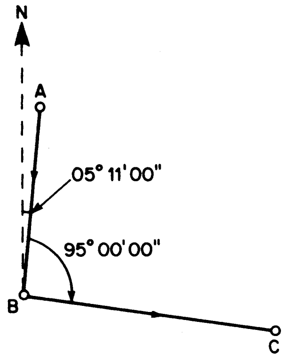


Figure 5.15

forward bearing AB = back bearing AF + left-hand angle at A
= 70° 00′ 00″ (given) + 115° 11′ 00″
= 185° 11′ 00″

At station B in figure 5.15

forward bearing BC = back bearing BA + left-hand angle at B

But back bearing BA = forward bearing AB ± 180°
= 185° 11′ 00″ ± 180°
= 365° 11′ 00″ *or* 05° 11′ 00″
= 05° 11′ 00″ (to keep bearing in range 0 to 360°)

Hence *forward bearing* BC = 05° 11′ 00″ + 95° 00′ 00″
= 100° 11′ 00″

The bearings of all the lines can be computed in a similar manner; the complete calculation is given in table 5.3.

Every bearing calculation finishes by recalculating the initial (given) bearing. This final computed bearing must be in agreement with the initial bearing and, if any difference occurs, an *arithmetic mistake* has been made, and the bearing calculation must be checked before proceeding to the next stage in the calculation.

5.8.4 Computation of Coordinate Differences

The next stage in the traverse computation is the determination of the coordinate differences of the traverse lines.

The information available at this point will be the bearings and horizontal lengths of all the lines.

TABLE 5.3

LINE / STATION / LINE	BACK BEARING / ADJUSTED LEFT HAND ANGLE / FORWARD BEARING			WHOLE CIRCLE BEARING θ			HORIZONTAL DISTANCE D	COORDINATE DIFFERENCES: CALCULATED ΔE	CALCULATED ΔN	ADJUSTMENTS δE	ADJUSTMENTS δN	ADJUSTED ΔE	ADJUSTED ΔN	COORDINATES E	COORDINATES N	STATION
AF	70	00	00											500.00	1000.00	A
A	115	11	00													
AB	185	11	00	185	11	00	429.37	-38.79	-427.61	-0.04	+0.02	-38.83	-427.59	461.17	572.41	B
BA	05	11	00													
B	95	00	00													
BC	100	11	00	100	11	00	656.54	+646.20	-116.08	-0.05	+0.03	+646.15	-116.05	1107.32	456.36	C
CB	280	11	00													
C	129	49	00													
CD	50	00	00	50	00	00	301.83	+231.22	+194.01	-0.03	+0.01	+231.19	+194.02	1338.51	650.38	D
DC	230	00	00													
D	130	36	00													
DE	00	36	00	00	36	00	287.40	+3.01	+287.38	-0.02	+0.01	+2.99	+287.39	1341.50	937.77	E
ED	180	36	00													
E	110	29	40													
EF	291	05	40	291	05	40	526.72	-491.42	+189.57	-0.04	+0.03	-491.46	+189.60	850.04	1127.37	F
FE	111	05	40													
F	138	54	20													
FA	250	00	00	250	00	00	372.47	-350.01	-127.39	-0.03	+0.02	-350.04	-127.37	500.00	1000.00	A
							Σ 2574	Σ+0.21	Σ-0.12	Σ-0.21	Σ+0.12	Σ 0.00	Σ 0.00			

ACTUAL SUM OF LEFT HAND ANGLES = 720° 02' 00"

REQUIRED SUM OF LEFT HAND ANGLES = $(2 \times 6 - 4) \times 90^\circ = 720^\circ$

MISCLOSURE = + 02' 00"

ADJUSTMENT TO EACH OBSERVED ANGLE = - 20"

$e_E = +0.21$

ADJUSTMENT TO $\Delta E/\Delta N$ BY BOWDITCH

$e_N = -0.12$

$e = \left((+0.21)^2 + (-0.12)^2\right)^{\frac{1}{2}} = 0.24$

FRACTIONAL LINEAR MISCLOSURE = 1 in 10 700

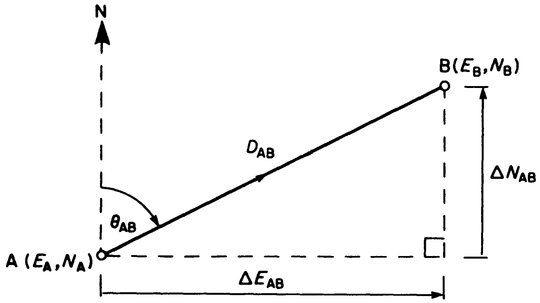


Figure 5.16 *Coordinate differences*

Coordinate differences

The coordinate differences are computed as shown in figure 5.16. Coordinates of station A are E_A, N_A which are known. Coordinates of station B are E_B, N_B which are to be calculated.

θ_{AB} = whole circle bearing of line AB

D_{AB} = horizontal length of line AB

ΔE_{AB} = *eastings difference* in moving from station A to station B

ΔN_{AB} = *northings difference* in moving from station A to station B

With reference to figure 5.16

$$E_B = E_A + \Delta E_{AB} = E_A + D_{AB} \sin \theta_{AB} \quad (5.1)$$

$$N_B = N_A + \Delta N_{AB} = N_A + D_{AB} \cos \theta_{AB} \quad (5.2)$$

For the traverse, each line is considered separately and the coordinate differences ΔE and ΔN computed.

If a pocket calculator is used, values of ΔE and ΔN can be obtained directly from equations (5.1) and (5.2) for any value of θ. Alternatively, the *polar/rectangular key* found on most calculators can be used. Since the method by which this is achieved depends on the make of calculator, the handbook supplied with the calculator should be consulted.

Examples of coordinate difference calculation

The traverse data of figure 5.12 are again used in the following examples. The bearings are the whole-circle bearings given in table 5.3.

Consider line AB in both figure 5.12 and figure 5.17.

From equation (5.1)

$$\begin{aligned}\Delta E_{AB} &= D_{AB} \sin \theta_{AB} \\ &= 429.37 \sin 185^\circ\ 11'\ 00'' = 429.37\ (-0.09034) \\ &= -38.79 \text{ m}\end{aligned}$$

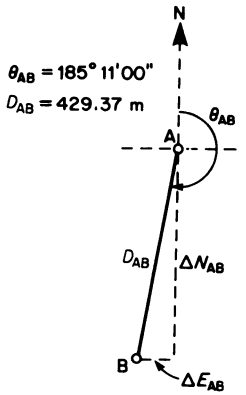


Figure 5.17

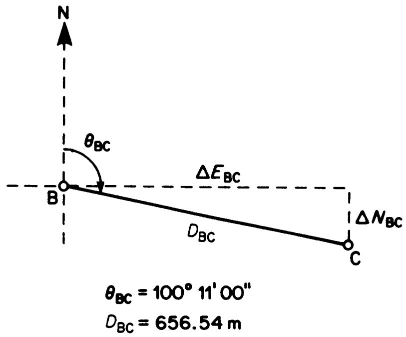


Figure 5.18

Similarly, from equation (5.2)

$$\begin{aligned}\Delta N_{AB} &= D_{AB} \cos \theta_{AB} \\ &= 429.37 \cos 185° \; 11' \; 00'' = 429.37\,(-0.99591) \\ &= -427.61 \text{ m}\end{aligned}$$

For line BC shown in figures 5.12 and 5.18, the coordinate differences are given by

$$\begin{aligned}\Delta E_{BC} &= D_{BC} \sin \theta_{BC} \\ &= 656.54 \sin 100° \; 11' \; 00'' = 656.54\,(+0.98425) \\ &= +646.20 \text{ m} \\ \Delta N_{BC} &= D_{BC} \cos \theta_{BC} \\ &= 656.54 \cos 100° \; 11' \; 00'' = 656.54\,(-0.17680) \\ &= -116.08 \text{ m}\end{aligned}$$

As with the bearing calculations, the coordinate difference results are always presented in tabular form since errors are easier to detect.

For the traverse ABCDEFA (figure 5.12) all the calculations for coordinate differences are given in table 5.3.

5.8.5 Misclosure

When the ΔE and ΔN values have been computed for the whole traverse as in table 5.3, checks can be applied to the computation.

For *polygon* traverses these are

$$\Sigma\Delta E = 0 \text{ and } \Sigma\Delta N = 0$$

since the traverse starts and finishes at the same point.

For link traverses (figure 5.1) these are

$$\Sigma\Delta E = E_Y - E_X \quad \text{and} \quad \Sigma\Delta N = N_Y - N_X$$

where station X is the starting point and station Y the final point of the traverse. Since stations X and Y are of known position, the values of $E_Y - E_X$ and $N_Y - N_X$ can be calculated.

In both cases, owing to field errors in measuring the angles and lengths, there will normally be a misclosure on returning to the starting point on a polygon traverse or on arrival at the final known station in a link traverse.

This *linear* misclosure is computed and any adjustment is allocated appropriately.

Therefore, before the station coordinates are calculated, the ΔE and ΔN values found for the traverse are summed and the misclosures, e_E and e_N, are found by comparing the summations with those expected.

These misclosures form a measure of the linear misclosure of the traverse and can be used to determine the accuracy of the survey. Consider figure 5.19 which shows the starting point A of a polygon traverse.

Owing to field errors, the traverse ends at A′ instead of A. The *linear misclosure*, e, is given by

$$e = (e_E^2 + e_N^2)^{\frac{1}{2}}$$

To obtain a measure of the accuracy of the traverse, this misclosure is compared with the total length of the traverse legs, ΣD, to give the *fractional linear misclosure*, where

fractional linear misclosure = 1 in ($\Sigma D/e$) (for example, 1 in 10 000)

This fractional misclosure is *always* computed for a traverse and is compared with the value required for the type of survey being undertaken. For appropriate values of the fractional linear misclosure see table 5.1.

If, on comparison, the fractional linear misclosure is better than the required value, the traverse fieldwork is satisfactory and the misclosures, e_E and e_N are distributed throughout the traverse.

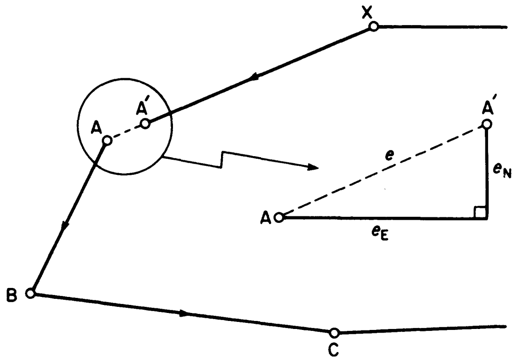


Figure 5.19 ***Traverse misclosure***

If, on comparison, the fractional linear misclosure is worse than that required, there is most likely an error in the measured lengths of one or more of the legs. The calculations should, however, be thoroughly checked before remeasuring any lengths.

An example determination of the fractional linear misclosure can be obtained from table 5.3, remembering that the traverse is a polygon. From the table

$$(1)\ \Sigma\Delta E = -38.79 + 646.20 + 231.22 + 3.01 - 491.42 - 350.01$$
$$= +\ 0.21 \text{ m}$$

Hence

$$e_E = +\ 0.21 \text{ m since } \Sigma\Delta E \text{ should be zero}$$

$$(2)\ \Sigma\Delta N = -427.61 - 116.08 + 194.01 + 278.38 + 189.57 - 127.39$$
$$= -\ 0.12 \text{ m}$$

Hence

$$e_N = -\ 0.12 \text{ m since } \Sigma\Delta N \text{ should also be zero}$$

Therefore, *linear misclosure* = $e = [(+0.21)^2 + (-0.12)^2]^{\frac{1}{2}} = 0.24$ m. From figure 5.12 and table 5.3 $\Sigma D = 2574$ m.

Therefore, *fractional linear misclosure* = 1 in (2574/0.24) $\approx$ 1 in 10 700.

This procedure is shown at the bottom of table 5.3.

5.8.6 Distribution of the Misclosure

Many methods of adjusting the linear misclosure of a traverse are possible but, for everyday engineering traverses of accuracy up to 1 in 20 000, one of two methods is normally used.

Bowditch method

The values of the adjustment found by this method are directly proportional to the length of the individual traverse lines.

Adjustment to ΔE (or ΔN) for one particular traverse leg

$$= \delta E \text{ (or } \delta N) = -e_E \text{ (or } -e_N) \times \frac{\text{length of traverse leg concerned}}{\text{total length of the traverse}}$$

Transit method

In this method, adjustments are proportional to the values of ΔE and ΔN for the various lines.

Adjustment to ΔE (or ΔN) for one particular traverse leg

$$= \delta E \text{ (or } \delta N) = -e_E \text{ (or } -e_N) \times \frac{\Delta E \text{ (or } \Delta N) \text{ of the traverse leg concerned}}{\text{absolute } \Sigma\Delta E \text{ (or } \Sigma\Delta N) \text{ for the traverse}}$$

For both methods, the negative signs are necessary since if e_E (or e_N) is positive, the *adjustments* will be negative, and if e_E (or e_N) is negative the *adjustments* will be positive.

For the *Bowditch* method, the adjustment of the values of ΔE and ΔN given in table 5.3 is as follows.

The misclosures have already been determined as $e_E = +0.21$ m and $e_N = -0.12$ m, and the total length of the traverse is 2574 m.

For line AB

$$\delta E_{AB} = -0.21 \times (429/2574) = -0.04 \text{ m}$$

$$\delta N_{AB} = +0.12 \times (429/2574) = +0.02 \text{ m}$$

For line BC

$$\delta E_{BC} = -0.21 \times (657/2574) = -0.05 \text{ m}$$

$$\delta N_{BC} = +0.12 \times (657/2574) = +0.03 \text{ m}$$

This process is repeated for the whole traverse. These adjustments, applied to the ΔE and ΔN values, would normally be tabulated as shown in table 5.3.

Applying the *transit* method to the same example gives

$$\text{absolute } \Sigma\Delta E = 1761 \text{ m and absolute } \Sigma\Delta N = 1342 \text{ m}$$

Hence *for line AB*

$$\delta E_{AB} = -0.21 \times (39/1761) = -0.00 \text{ m}$$

$$\delta N_{AB} = +0.12 \times (428/1342) = +0.04 \text{ m}$$

and *for line BC*

$$\delta E_{BC} = -0.21 \times (646/1761) = -0.08 \text{ m}$$

$$\delta N_{BC} = +0.12 \times (116/1342) = +0.01 \text{ m}$$

Again, the computation is repeated for each line of the traverse.

Both the transit method and Bowditch's method will alter the original bearings by a very small amount. It is *not* necessary to recalculate these bearings unless the traverse is to be used for subsequent control work such as setting out.

Checks on both methods of adjustment should be undertaken as follows. If the adjustment has been carried out successfully

$$\Sigma\delta E \textit{ should} = -e_E$$

$$\Sigma\delta N \textit{ should} = -e_N$$

These checks must be carried out *before* calculating the adjusted ΔE and ΔN values.

5.8.7 Calculation of the Final Coordinates

For *polygon* traverses, in order to compute the coordinates of the stations, the coordinates of the starting point have to be known. These starting coordinates may either be assumed for an area to give positive coordinates for the whole survey or, occasionally, may be given if a previously coordinated station is used to start the traverse.

For *link* traverses, the coordinates of the starting and finishing points will be known from a previous survey and the coordinates will be determined relative to these known values.

The coordinates of each point are obtained by adding or subtracting the adjusted ΔE and ΔN values as necessary, working around the traverse.

When all the coordinates have been calculated, there is a final check to be applied.

For a *polygon* traverse, the final and initial coordinates should be equal as these represent the same station.

For a *link* traverse, the final coordinates should equal those of the second known point.

If this check does not hold, there is an *arithmetical mistake* and the calculations should be investigated until it is found.

At this stage for the polygon traverse which has been referred to throughout this discussion (that shown in figure 5.12), the adjusted ΔE and ΔN values have now been determined and, since the coordinates of the starting point, station A, have been given as 500.00 mE and 1000.00 mN, the coordinates of the other traverse stations can be obtained from these initial coordinates and the adjusted ΔE and ΔN values found by the Bowditch method. For example

$$(1)\ E_B = E_A \pm \Delta E_{AB} = 500.00 - 38.83 = 461.17 \text{ m}$$

$$N_B = N_A \pm \Delta N_{AB} = 1000.00 - 427.59 = 572.41 \text{ m}$$

$$(2)\ E_C = E_B \pm \Delta E_{BC} = 461.17 + 646.15 = 1107.32 \text{ m}$$

$$N_C = N_B \pm \Delta N_{BC} = 572.41 - 116.05 = 456.36 \text{ m}$$

This process is repeated until station A is recoordinated as a check. The complete calculation is shown in table 5.3.

5.8.8 The Traverse Table

For each particular step in the traverse computation every calculation should be tabulated.

There are many variations of the layout that can be adopted but the format given in table 5.3 is recommended.

Table 5.3 shows the calculation for the polygon traverse ABCDEFA of figure 5.12. This table should be thoroughly studied, referring to the relevant preceding sections of this chapter to enable a complete understanding of how the table is compiled to be gained.

5.8.9 Precision of Computation

When using calculators and computers, care must be taken to use only the appropriate amount of the eight (sometimes more) significant figures presented by the display.

It must be realised that *any quantity calculated cannot be quoted to a higher precision than that of the data supplied or that of the field observations.*

Some examples are given to demonstrate this principle.

(1) In section 5.8.4, all the coordinate differences were calculated to the nearest 0.01 m.

Using a calculator, it would have been possible to compute, for example, ΔE_{AB} as −38.790524 m but without significance since the traverse legs were only measured to the nearest 0.01 m. Hence, the figure is rounded to −38.79 m since the coordinate differences can also, at best, be quoted only to the nearest 0.01 m.

(2) With reference to the example traverse of figure 5.12, the coordinates of station A were given as 500.00 mE and 1000.00 mN.

These are written in this manner to indicate that the position of station A is known to the nearest 0.01 m. Thus, all coordinates derived from this station can, at best, be quoted only to the nearest 0.01 m.

If the coordinates of station A were recorded as 500.0 mE and 1000.0 mN, this implies that the position of A is known only to the nearest 0.1 m. If this were the case, all subsequent coordinates can, at best, be quoted only to the nearest 0.1 m even though the coordinate differences can, perhaps, be quoted to 0.01 m.

When the coordinates are quoted as 500.000 mE and 1000.000 mN, it is now possible to quote coordinates of other stations to 0.001 m so long as the fieldwork techniques used warrant this.

However, since the ΔE and ΔN values in the example traverse are known only to the nearest 0.01 m, all the coordinates derived from those of station A can be determined only to 0.01 m, even if the coordinates of station A are quoted to 0.001 m.

This reaffirms the earlier statement that it is the least precise component in the calculations which determines the precision of the final result.

The above notes refer not only to traversing but also to any calculations in engineering.

5.9 Plotting Traverse Stations

When the final coordinates have been computed for a traverse, the stations have to be plotted if a site plan is being prepared.

A common mistake is to plot the stations using the bearings or left-hand angles and the lengths between stations. This is known as plotting by angle (or bearing) and distance. Such a method is NEVER used in traversing and the preferred and most accurate method of plotting traverse stations is to use the computed coordinates.

The methods given in the following sections for plotting traverse station coordinates apply also to triangulation and other stations (see chapters 6 and 7) when these have to be used in plan production.

5.9.1 Orientating the Survey and Plot

Before commencing any plot (that is, before constructing the grid), the extent of the survey should be taken into account such that the plotted survey will fall centrally on to the sheet.

In the case where the north direction is stipulated, the north–south and east–west extents of the area should be determined and the stations plotted for the best fit on the sheet.

If an arbitrary north is to be used, the best method of ensuring a good fit is to assign a bearing of 90° or 270° to the longest side. This line is then positioned parallel to the longest side of the sheet so that the survey will fit the paper properly.

Sometimes it may be necessary to set the arbitrary north to a particular direction in order to ensure that the survey will fit a particular sheet size. This will often be the case with long, narrow site surveys where, to save paper and for convenience, the plot of the survey is to go on to a single or a minimum number of sheets or the minimum length of a roll. The boundary of the survey should be roughly sketched and positioned until a suitable fit is obtained. Again, the longest or most convenient line should be assigned a suitable arbitrary bearing.

Where the north point is arbitrary, it has to be established before the coordinate calculation takes place. In order to estimate the extent of the survey it should be sketched, roughly to scale, using the left-hand angles and the lengths between stations.

5.9.2 Coordinate Grid

The first stage in plotting coordinates is to establish a coordinate grid.

Coordinated lines are drawn at specific intervals, for example, 10 m, 50 m, 100 m in both the east and north directions to form a pattern of squares. The stations are then plotted in relation to the grid.

When drawing the grid, T-squares and set squares should *not* be used since they are not accurate enough. Instead, the grid is constructed in the following manner.

(1) From each corner of the plotting sheet two diagonals are drawn, as shown in figure 5.20*a*.

(2) From the intersection of these diagonals an equal distance is scaled off along each diagonal using a *beam compass*. This scaled distance must be large, see figure 5.20*b*.

(3) The four marked points on the diagonals are joined using a *steel straight edge* to form a rectangle. This rectangle will be perfectly true and is used as the basis for the coordinate grid (see figure 5.20*c*).

On all site plans and maps it is conventional to have the north point (true, magnetic or arbitrary) on the drawing such that the north direction is from the bottom to the top of the sheet and roughly parallel to the sides of the sheet. This will be achieved if the grid framework is constructed as described.

(4) By scaling equal distances along the top and bottom lines of the rectangle and joining the points, the vertical (E) grid lines will be formed. The horizontal (N) grid lines are formed in a similar manner using the other sides of the rectangle (see figure 5.20*d* and *e*).

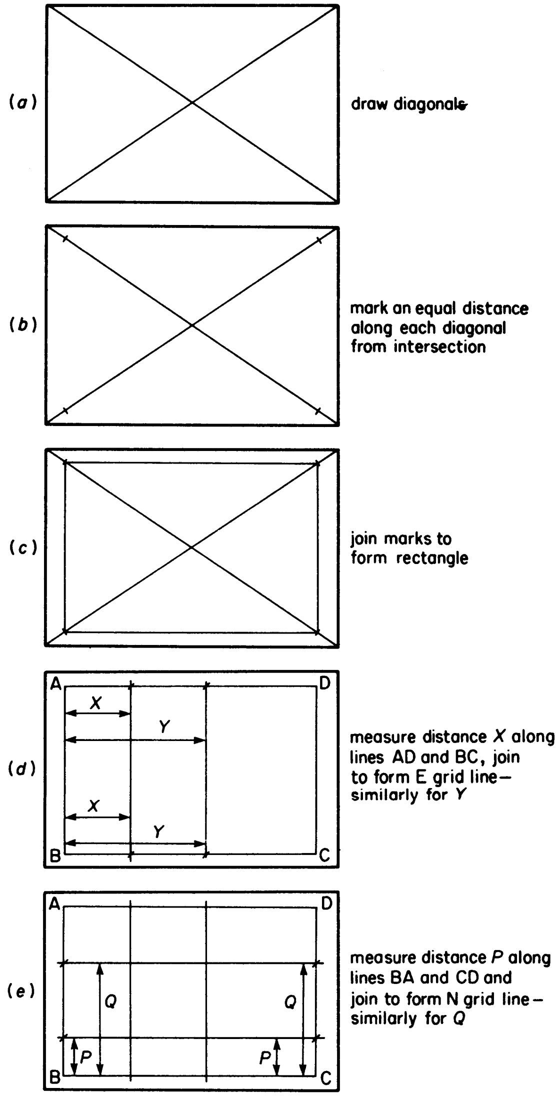


Figure 5.20 *Establishing coordinate grid*

All lines must be drawn with the aid of a steel straight edge and all measurements must be taken from lines AB and BC and not from one grid line to the next. This avoids accumulating errors.

(5) The grid lines should now be numbered accordingly. The size of a grid square should not be greater than 100 mm by 100 mm. It is not necessary to plot the origin of the survey if it lies outside the area concerned.

5.9.3 Station Plotting

When plotting by coordinates, the procedure is as follows.

Let the station to be plotted have coordinates 283.62 mE, 427.45 mN and let it be plotted on a 100 m grid previously prepared as described in section 5.9.2.

(1) The grid intersection 200 mE, 400 mN is located on the prepared grid.

(2) Along the 400 mN line, 83.62 m is scaled off from the 200 mE intersection towards the 300 mE intersection and point a is located (see figure 5.21). Similarly, point b is located along the 500 mN line. Points a and b are joined with a pencil line.

(3) Along the 200 mE line, 27.45 m is scaled from the 400 mN intersection towards the 500 mN intersection to locate point c. Point d is found by scaling 27.45 m along the 300 mE line. Points c and d are joined.

(4) The intersection of lines ab and cd gives the position of the station.

(5) To check the plotted position, dimensions X and Y are measured from the plot and compared with their expected values. In this case, X *should* equal $100.00 - 83.62 = 16.38$ m and Y *should* equal $100.00 - 27.45 = 72.55$ m.

(6) When all the stations have been plotted, the lengths between the plotted stations are measured and compared with their accepted values.

(7) The traverse lines are added by carefully joining the plotted stations. This is to aid in the location of detail (see section 8.5).

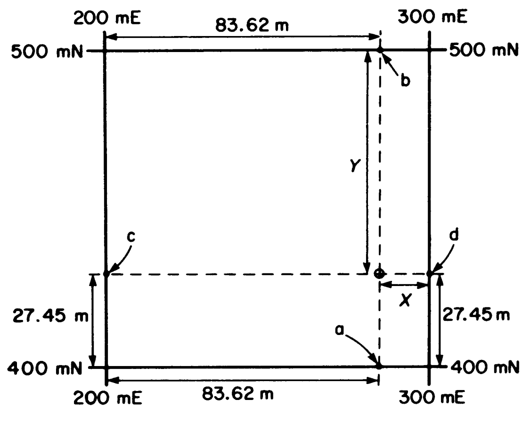


Figure 5.21 *Station plotting*

5.10 Whole-circle Bearing and Distance Calculation from Coordinates

In section 5.8.4 it was shown that, knowing the whole-circle bearing and length of a line, the coordinates of one end of the line could be computed if the coordinates of the other end were known.

For the reverse case, where the coordinates of two points are known, it is possible to compute the whole-circle bearing and the horizontal distance of the line between the two points.

This type of calculation is commonly used in engineering surveying when setting out works by polar coordinates. This is discussed in section 14.8.2.

In link traversing, it is often necessary to calculate the whole-circle bearings of the start and finish lines from the coordinates of the existing, known stations.

The WCBs and horizontal distances can be calculated by one of two methods; either by considering the *quadrant* in which the line falls or by using the *rectangular/polar conversion key* found on most calculators. If this type of calculation is undertaken using a computer which does not have a rectangular/polar facility, any programs written must be based on the quadrants method.

5.10.1 By Quadrants

Referring to figure 5.22, suppose the coordinates of stations A and B are known to be (E_A, N_A) and (E_B, N_B) and that the whole-circle bearing of line AB (θ_{AB}) and the horizontal length of AB (D_{AB}) are to be calculated. The procedure is as follows.

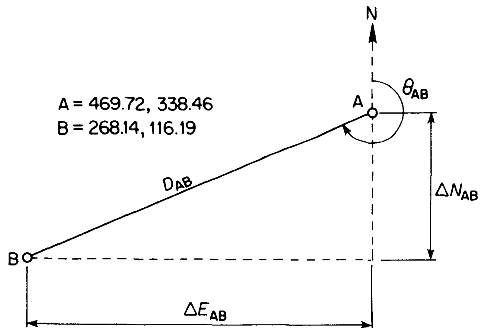


Figure 5.22

(1) A sketch showing the relative positions of the two stations should *always* be drawn in order to determine in which *quadrant* the line falls. This is most important as the greatest source of error in this type of calculation is wrong identification of quadrant. For whole circle bearings the quadrants are shown in figure 5.23.

(2) θ_{AB} is given by (in figure 5.22)

$$\theta_{AB} = \tan^{-1} (\Delta E_{AB}/\Delta N_{AB}) + 180° = \tan^{-1} [(E_B - E_A)/(N_B - N_A)] + 180°$$

$$= \tan^{-1} \left[\frac{268.14 - 469.72}{116.19 - 338.46}\right] + 180° = \tan^{-1} \left[\frac{-201.58}{-222.27}\right] + 180°$$

$= \tan^{-1} (0.906\,915) + 180°$

$= 42° \; 12' \; 19'' + 180°$

Hence

$$\theta_{AB} = 222° \; 12' \; 19''$$

It must be realised that, *in general*, the final value of θ_{AB} will depend on the quadrant of the line and a set of rules, based on the quadrant in which the line falls,

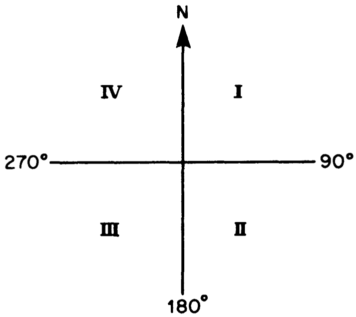


Figure 5.23 *Quadrant*

can be proposed to determine the whole circle bearing. These rules are shown in table 5.4.

(3) Having now found θ_{AB}, D_{AB} is given by

$$D_{AB} = (\Delta E_{AB}/\sin \theta_{AB}) = (\Delta N_{AB}/\cos \theta_{AB})$$

For figure 5.22

$$D_{AB} = \frac{-201.58}{\sin 222° \; 12' \; 19''} = \frac{-201.58}{-0.671\,789} = 300.06 \text{ m}$$

$$= \frac{-222.27}{\cos 222° \; 12' \; 19''} = \frac{-222.27}{-0.740\,743} = 300.06 \text{ m} \quad \text{(check)}$$

When evaluating D, *both* of the above should be calculated as a check against gross error. In the case where *small* differences occur between the two results, the correct answer is given by the trigonometric function which is the slower changing,

TABLE 5.4

QUADRANT	I	II/III	IV
FORMULA	$\theta = \tan^{-1}(\Delta E/\Delta N)$	$\theta = \tan^{-1}(\Delta E/\Delta N) + 180^{0}$	$\theta = \tan^{-1}(\Delta E/\Delta N) + 360^{0}$
Note:	($\Delta E/\Delta N$) must be calculated allowing for their signs. For a line XY, $\Delta E_{xy} = E_y - E_x$ and $\Delta N_{xy} = N_y - N_x$		

for example, if $\theta = 5°$, D found from $(\Delta N/\cos\theta)$ gives the more accurate answer since the cosine function is changing less rapidly than the sine function at this angle value.

Alternatively, D may be given by $D = (\Delta E^2 + \Delta N^2)^{\frac{1}{2}}$. If an electronic calculator is being used then this method will give a satisfactory answer but provides no check on the result. For this reason, the method involving the trigonometrical functions is preferred.

Examples of this type of calculation are given in section 5.11 and section 5.12.2.

5.10.2 By Rectangular/Polar Conversion

If a calculator is available which is fitted with a rectangular/polar key, values of D and θ can be obtained directly. When using this function, the coordinate values must be entered into the calculator in the correct sequence otherwise the wrong bearing will be obtained. In all cases, if θ is displayed as having a negative value, 360° must be added to give the correct whole-circle bearing. This is due to the fact that calculators display θ either between 0° and +180° or between 0° and −180°.

5.11 The National Grid

All Ordnance Survey (OS) maps and plans in Great Britain are based on a rectangular coordinate system known as the *National Grid*. Whole-circle bearings on this co-ordinate system are related to the north axis of the National Grid, a direction known as *grid north*. The network of stations which form the basis of this grid are known as *triangulation stations* (see section 6.3.2) and these are specially constructed pillars or permanent marks on prominent features such as churches, tall buildings, water towers and so on. Triangulation stations, which are distributed throughout the country, have been very accurately coordinated on the National Grid by the OS and are, therefore, very useful in many engineering projects, particularly road schemes, when used as reference or control points for setting out works (see section 14.6) or as known points in link traverses.

The National Grid is derived from a *map projection*. This is a means of representing the curved surface of the Earth on a plane so that maps and grids can be drawn. In forming the National Grid, the relative positions of points on the grid are altered slightly from their ground positions as a result of using a map projection to account for the curvature of the Earth. Therefore, distances and bearings calculated from National Grid coordinates will not, in some cases, agree with their equivalent measured in the field.

To convert measured distances to grid distances the *scale factor* (F) must be used as follows

$$\text{grid distance} = \text{measured distance} \times F$$

The scale factor varies across the country, as shown in table 5.5.

In practice, bearings derived from National Grid coordinates are assumed to agree with those obtained from measurements, provided the length of any individual line is less than 10 km.

TABLE 5.5

National Grid Easting (km)		Scale Factor (F)
400	400	0.999 60
410	390	60
420	380	61
430	370	61
440	360	62
450	350	63
460	340	65
470	330	66
480	320	68
490	310	70
500	300	72
510	290	75
520	280	78
530	270	81
540	260	84
550	250	88
560	240	92
570	230	0.999 96
580	220	1.000 00
590	210	04
600	200	09
610	190	14
620	180	20
630	170	25
640	160	31
650	150	1.000 37

5.11.1 Examples

Use of local scale factor in traversing

Suppose the traverse shown in figure 5.12 was to be based on the National Grid. In this case the coordinates of station A would be given as National Grid values, the given bearing AF would have to be related to grid north and all the measured distances would have to be converted to grid distances in order to calculate the National Grid coordinates of stations B to F.

If the National Grid easting of station A was approximately 451 km E, the scale factor for the traverse would be 0.999 63 (see table 5.5). Hence

$$\text{grid distance AB} = 429.37 \times 0.999\,63 = 429.21 \text{ m}$$

$$\text{grid distance BC} = 656.64 \times 0.999\,63 = 656.30 \text{ m}$$

and so on for the other traverse legs.

These lengths would be used in place of those in table 5.3.

Use of local scale factor in setting out

In a road scheme, let the National Grid coordinates of a point on the road centre line be 612 910.74 mE, 157 062.28 mN. This point is to be set out by polar coordinates (see section 14.8.2) from a nearby traverse station with National Grid coordinates 613 112.33 mE, 157 238.91 mN.

The setting out distance is calculated as follows.

$$\Delta E = 613\,112.33 - 612\,910.74 = 201.59 \text{ m}$$

$$\Delta N = 157\,238.91 - 157\,062.28 = 176.63 \text{ m}$$

$$\text{grid distance} = (\Delta E^2 + \Delta N^2)^{\frac{1}{2}} = 268.02 \text{ m}$$

$$F = 1.000\,16 \text{ (by interpolation from table 5.5)}$$

Therefore

$$\text{horizontal setting out distance} = (268.02/1.000\,16) = 267.98 \text{ m}$$

5.12 Worked Examples

5.12.1 Polygon Traverse

Question

The traverse diagram of figure 5.24 is a field abstract for a polygon traverse ABCDEA.

Calculate the adjusted coordinates of stations B, C, D and E, adjusting any misclosure by the Bowditch method.

The coordinates of station A are 500.00 mE, 500.00 mN and the line AB has an assumed whole circle bearing of 90°00′00″.

Solution

The complete solution is given in the traverse table shown in table 5.6.

(1) Since the *external* angles are given, these will be the left-hand angles if the solution follows the *clockwise* direction. For this traverse, no attempt should be made to compute in an anticlockwise direction as this would involve subtraction of angles and errors may result.

(2) The bearing calculation always starts with the assumed or given bearing. In the example, bearing AB is given as 90° and is, for the clockwise direction, a forward bearing and is entered as such in the traverse table.

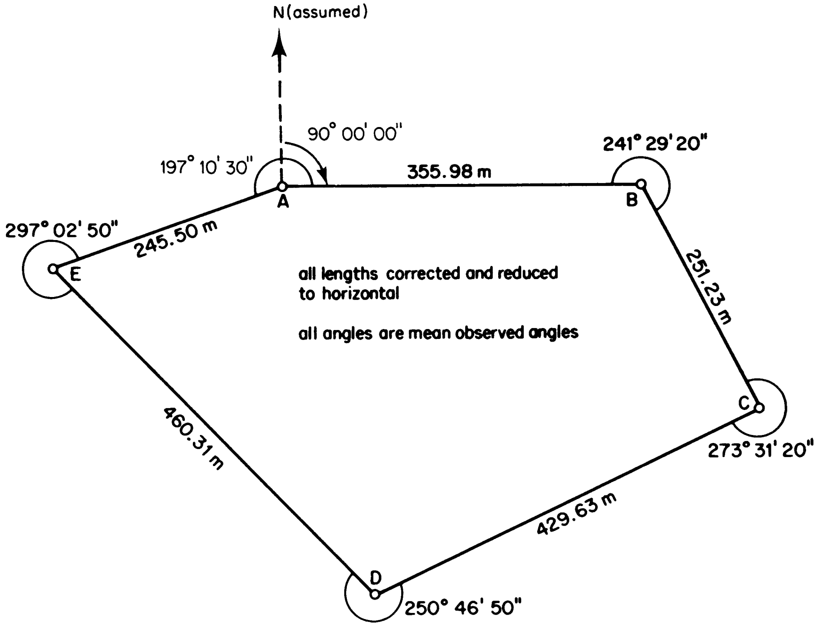


Figure 5.24

(3) The sum of the left-hand angles gives a misclosure of +00′50″ and since there are five angles, each has an adjustment of −10″.

(4) The fractional linear misclosure is rounded off to 1 in 14 500. It is not necessary to quote this to better than three significant figures. 1 in 14 500 would be acceptable for most engineering work.

(5) Adjustment of the ΔE and ΔN values by the Bowditch method gives the adjustments as shown. For example calculations, consider line CD.

$$\begin{aligned}\delta E_{CD} &= -\,e_E \times (\text{length CD}/\Sigma D)\\ &= -0.07 \times (430/1743) = -0.02 \text{ m}\\ \delta N_{CD} &= -\,e_N \times (\text{length CD}/\Sigma D)\\ &= -0.10 \times (430/1743) = -0.03 \text{ m}\end{aligned}$$

Note that lengths of each line and ΣD need only be used to three significant figures for required adjustments of two significant figures and that the total of the individual adjustments for the ΔE and ΔN values *must equal* $-e_E$ and $-e_N$ respectively.

(6) The coordinate computation starts and ends with the station of known position A. The final check is to ensure that the derived coordinates of A agree with the start coordinates of A.

TABLE 5.6

LINE / STATION / LINE	BACK BEARING / ADJUSTED LEFT HAND ANGLE / FORWARD BEARING			WHOLE CIRCLE BEARING θ			HORIZONTAL DISTANCE D	COORDINATE DIFFERENCES: CALCULATED ΔE	CALCULATED ΔN	ADJUSTMENTS δE	ADJUSTMENTS δN	ADJUSTED ΔE	ADJUSTED ΔN	COORDINATES E	COORDINATES N	STATION
														500.00	500.00	A
AB	90	00	00	90	00	00	355.98	+355.98	0.00	-0.01	-0.02	+355.97	-0.02	855.97	499.98	B
BA	270	00	00													
B	241	29	10													
BC	151	29	10	151	29	10	251.23	+119.93	-220.76	-0.01	-0.01	+119.92	-220.77	975.89	279.21	C
CB	331	29	10													
C	273	31	10													
CD	245	00	20	245	00	20	429.63	-389.39	-181.53	-0.02	-0.03	-389.41	-181.56	586.48	97.65	D
DC	65	00	20													
D	250	46	40													
DE	315	47	00	315	47	00	460.31	-321.01	+329.91	-0.02	-0:03	-321.03	+329.88	265.45	427.53	E
ED	135	47	00													
E	297	02	40													
EA	72	49	40	72	49	40	245.50	+234.56	+72.48	-0.01	-0.01	+234.55	+72.47	500.00	500.00	A
AE	252	49	40													
A	197	10	20													
AB	90	00	00													
							Σ 1743	Σ+0.07	Σ0.10	Σ-0.07	Σ-0.10	Σ 0.00	Σ 0.00			

ACTUAL SUM OF LEFT HAND ANGLES = 1260° 00' 50"

REQUIRED SUM OF LEFT HAND ANGLES = (2 X 5 + 4) X 90° = 1260°

MISCLOSURE = + 00'50"

ADJUSTMENT TO EACH OBSERVED ANGLE = -10"

e_E = +0.07

e_E = +0.10

$e = ((+0.07)^2 + (+0.10)^2)^{\frac{1}{2}} = 0.12$

FRACTIONAL LINEAR MISCLOSURE = 1 in 14 500

ADJUSTMENT TO $\Delta E/\Delta N$ BY BOWDITCH

5.12.2 Link Traverse

Question

A link traverse was run between stations A and X as shown in the traverse diagram of figure 5.25.

The coordinates of the controlling stations at the ends of the traverse are as follows

	E (m)	N (m)
A	1769.15	2094.72
B	1057.28	2492.39
X	2334.71	1747.32
Y	2995.85	1616.18

Calculate the coordinates of stations 1, 2, 3 and 4, adjusting any misclosure by the Transit method.

Solution

The complete solution is given in the traverse table shown in table 5.7.

(1) The solution follows the direction A to X as this will give the left-hand angles, as shown in figure 5.25.

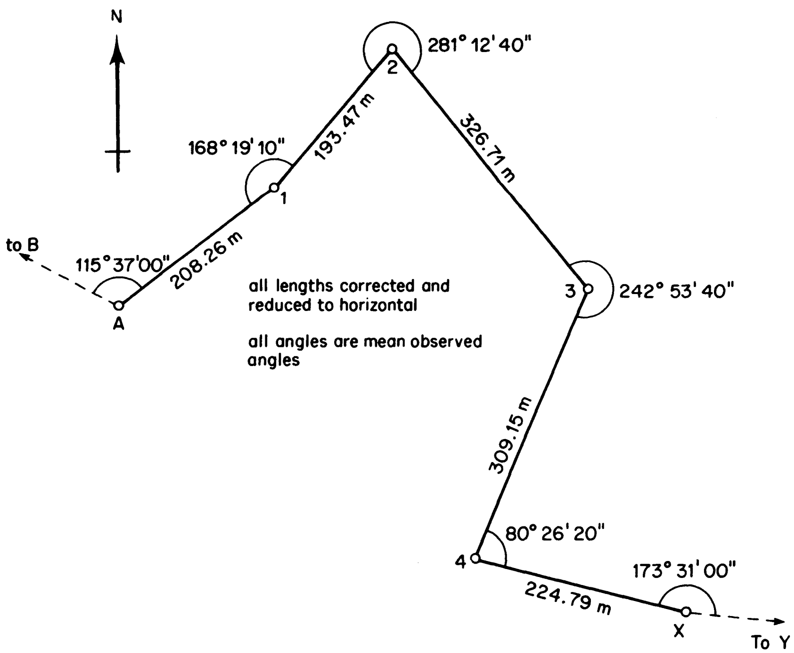


Figure 5.25

(2) When link traversing, the starting and closing bearings may either be given directly or implied by the coordinates of the stations used to start and end the traverse. In this case, coordinates are given and it is necessary to compute the initial and final bearings.

(a) *Initial back bearing AB*

Figure 5.25 is a sketch of the traverse, approximately to scale, and, therefore, shows that the bearing AB is in the *fourth* quadrant. Hence, the whole-circle bearing, θ_{AB}, is given by (see section 5.10.1)

$$\begin{aligned}\theta_{AB} &= \tan^{-1}(\Delta E_{AB}/\Delta N_{AB}) + 360^\circ \\ &= \tan^{-1}[(1057.28 - 1769.15)/(2492.39 - 2094.72)] + 360^\circ \\ &= \tan^{-1}(-711.87/397.67) + 360^\circ \\ &= \tan^{-1}(1.790\,10) + 360^\circ\end{aligned}$$

Hence

$$\theta_{AB} = -60^\circ 48'40'' + 360^\circ$$

Therefore

$$\theta_{AB} = 299^\circ 11'20''$$

Alternatively, a rectangular/polar conversion can be used as described in section 5.10.2.

(b) *Final forward bearing XY*

From figure 5.25, the bearing XY lies in the *second* quadrant hence

$$\begin{aligned}\theta_{XY} &= \tan^{-1}(\Delta E_{XY}/\Delta N_{XY}) + 180^\circ \\ &= \tan^{-1}[(2995.85 - 2334.71)/(1616.18 - 1747.32)] + 180^\circ \\ &= \tan^{-1}(661.14/-131.14) + 180^\circ \\ &= \tan^{-1}(-5.041\,48) + 180^\circ\end{aligned}$$

Hence

$$\theta_{XY} = -78^\circ 46'50'' + 180^\circ$$

Therefore

$$\theta_{XY} = 101^\circ 13'10''$$

Again, a rectangular/polar conversion can also be used.

(3) The angular misclosure is found as follows (see also section 5.8.2)

$$\text{Sum of left-hand angles} = 1061^\circ 59'50''$$

$$\begin{aligned}&(\text{final forward bearing} - \text{initial back bearing}) + (n - 1) \times 180^\circ \\ &\quad = (101^\circ 13'10'' - 299^\circ 11'20'') + (5 \times 180^\circ) \\ &\quad = (461^\circ 13'10'' - 299^\circ 11'20'') + 900^\circ \\ &\quad = 1062^\circ 01'50''\end{aligned}$$

The misclosure is, therefore, $-02'00''$ and each left-hand angle is adjusted by adding $20''$ to it.

TABLE 5.7

LINE / STATION / LINE	BACK BEARING / ADJUSTED LEFT HAND ANGLE / FORWARD BEARING			WHOLE CIRCLE BEARING θ			HORIZONTAL DISTANCE D	COORDINATE DIFFERENCES: CALCULATED ΔE	CALCULATED ΔN	ADJUSTMENTS δE	ADJUSTMENTS δN	ADJUSTED ΔE	ADJUSTED ΔN	COORDINATES E	COORDINATES N	STATION
AB	299	11	20													
A	115	37	20											1769.15	2094.72	A
A1	54	48	40	54	48	40	208.26	+170.20	+120.01	+0.02	+0.01	+170.22	+120.02	1939.37	2214.74	1
1A	234	48	40													
1	168	19	30													
12	43	08	10	43	08	10	193.47	+132.28	+141.18	+0.02	+0.01	+132.30	+141.19	2071.67	2355.93	2
21	223	08	10													
2	281	13	00													
23	144	21	10	144	21	10	326.71	+190.40	-265.49	+0.02	+0.02	+190.42	-265.47	2262.09	2090.46	3
32	324	21	10													
3	242	54	00													
34	207	15	10	207	15	10	309.15	-141.57	-274.83	+0.02	+0.02	-141.55	-274.81	2120.54	1815.65	4
43	27	15	10													
4	80	26	40													
4X	107	41	50	107	41	50	224.79	+214.15	-68.33	+0.02	+0.00	+214.17	-68.33	2334.71	1747.32	X
X4	287	41	50													
X	173	31	20													
XY	101	13	10	101	13	10										
							Σ 1262	Σ+565.46	Σ-347.46	Σ+0.10	Σ+0.06	Σ+565.56	Σ-347.40			

ACTUAL SUM OF LEFT HAND ANGLES = 1061°59'50"

REQUIRED SUM OF LEFT HAND ANGLES = 1062°01'50"

MISCLOSURE = -02'00"

ADJUSTMENT TO EACH OBSERVED ANGLE = +20"

$\Sigma\Delta E = +565.46$ $\qquad$ $\Sigma\Delta N = -347.46$ $\qquad$ abs $\Sigma\Delta E = 849$

$E_X - E_A = +565.56$ $\qquad$ $N_X - N_A = -347.40$ $\qquad$ abs $\Sigma\Delta N = 870$

$e_E = -0.10$ $\qquad$ $e_N = -0.06$ $\qquad$ ADJUSTMENT TO $\Delta E/\Delta N$ BY TRANSIT

$e = ((-0.10)^2 + (-0.06)^2)^{\frac{1}{2}} = 0.12$

FRACTIONAL LINEAR MISCLOSURE = 1 in 10 500

(4) To evaluate the misclosures e_E and e_N for the link traverse the following formulae are used (see also section 5.8.5)

$$\Sigma\Delta E - (E_X - E_A) = e_E$$
$$\Sigma\Delta N - (N_X - N_A) = e_N$$

These are evaluated as shown in table 5.7.

(5) Adjustments to the ΔE and ΔN values are by the Transit method. Example derivations are given for the line joining stations 1 and 2 as follows.

$$\delta E_{12} = -e_E \times (\Delta E/\text{abs}\ \Sigma\Delta E) = +0.10 \times (132/849)$$
$$= +0.02\ \text{m}$$
$$\delta N_{12} = -e_N \times (\Delta N/\text{abs}\ \Sigma\Delta N) = +0.06 \times (141/870)$$
$$= +0.01\ \text{m}$$

The terms abs $\Sigma\Delta E$ and abs $\Sigma\Delta N$ are the summations of the ΔE and ΔN values regardless of sign.

The ΔE, ΔN, abs $\Sigma\Delta E$ and abs $\Sigma\Delta N$ values are required only to three significant figures.

(6) The check on the final coordinates is satisfactory since the derived coordinates for station X agree with those given.

6

Triangulation and Trilateration

In the previous chapter, the method of establishing horizontal control by traversing is described. However, horizontal control can also be established by such methods as triangulation, trilateration, intersection and resection. Triangulation and trilateration are described in this chapter and intersection and resection in chapter 7.

A *triangulation network* consists of a series of single or overlapping triangles as shown in figure 6.1, the points (or vertices) of each triangle forming control stations. Position is determined by measuring all the angles in the network and by measuring the length of one or more baselines such as AB or HJ in figure 6.1. Starting at a baseline, application of the Sine Rule in each triangle throughout the network enables the lengths of all triangle sides to be calculated. These lengths, when combined with the measured angles, enable the coordinates of the stations to be computed.

A *trilateration network* also takes the form of a series of single or overlapping triangles but in this case position is determined by measuring all the distances in the network instead of all the angles. To enable station coordinates to be calculated,

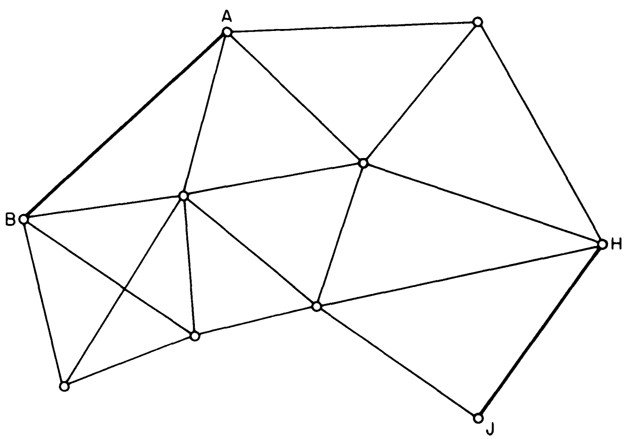


Figure 6.1 *Triangulation network*

the measured distances are combined with angle values derived from the side lengths of each triangle.

Until the advent of EDM, the measurement of distances in a trilateration scheme with sufficient accuracy was a very difficult and time consuming process and because of this trilateration techniques were seldom used for establishing horizontal control. Traversing techniques were also limited since it was not possible to maintain a uniformly high accuracy when traversing over long distances. As a result, triangulation was used extensively in the past to provide control for surveys covering very large areas. For example, the triangulation network throughout Great Britain that provides control for mapping was first established by the Ordnance Survey (see section 1.7) between 1783 and 1853, and was subsequently resurveyed from 1935 to 1962.

Nowadays, however, because of the high precision and accuracy of modern EDM equipment (see section 4.13), traversing, triangulation and trilateration can all be used as methods of establishing horizontal control.

Triangulation and trilateration are often used together in *combined networks* in which a combination of angles and distances is measured. When properly observed and adjusted, combined networks are the most accurate form of horizontal control.

For engineering surveys, separate triangulation or trilateration networks are often used on sites where control is required to be spread over large areas. Such projects may include major roads and bridges, dams, pipeline crossings, irrigation schemes and so on. Although combined networks have the same applications, they are usually used to provide reference points for monitoring and for other precise engineering work.

Once a control system has been established on site, the stations in the network can be used either for control extension or directly in detail surveying, setting out and other everyday engineering surveying activities.

6.1 Triangulation Specifications

As in traversing, triangulation surveys can be carried out using a number of different field techniques, each giving a different standard of precision in the values obtained for the coordinates of the stations. The specifications given in table 6.1 are a general guide to triangulation classification and associated precisions, the precision of a survey being linked to the field methods and type of equipment used.

For ordinary engineering work and site surveys, a precision in the range 1 in 10 000 to 1 in 20 000 is normally required for horizontal control and the notes in this chapter are directed towards such second-order surveys only.

6.2 Triangulation Figures

Although triangulation schemes could be made up entirely from single triangles as in figure 6.2*a*, it is often better to use a more complicated network involving such figures as braced quadrilaterals (figure 6.2*b*) and centre point polygons (figures 6.2*c* and 6.2*d*). Compared to a network consisting of simple triangles, these figures usually require more fieldwork and the subsequent computations are often more complicated. However, the advantage of incorporating such figures into a network

TABLE 6.1
General Triangulation Specifications

	First-Order Primary	Second-Order Secondary	Third-Order Tertiary
Typical precision (or misclosure in Length of baselines)	1 in 100 000	1 in 20 000	1 in 5000
Applications	(1) National control networks (2) Reference points for engineering projects (3) Scientific and geodetic studies	(1) Secondary framework supporting national network (2) Engineering projects (3) Topographical surveys	(1) Small scale topographic mapping and detail surveys (2) Engineering projects
Triangle misclosure average less than maximum less than	 1" 3"	 3" 5"	 5" 10"
Number of rounds of angles normally required	16	4	2
Theodolite required	0.1" or 0.2"	1"	1"

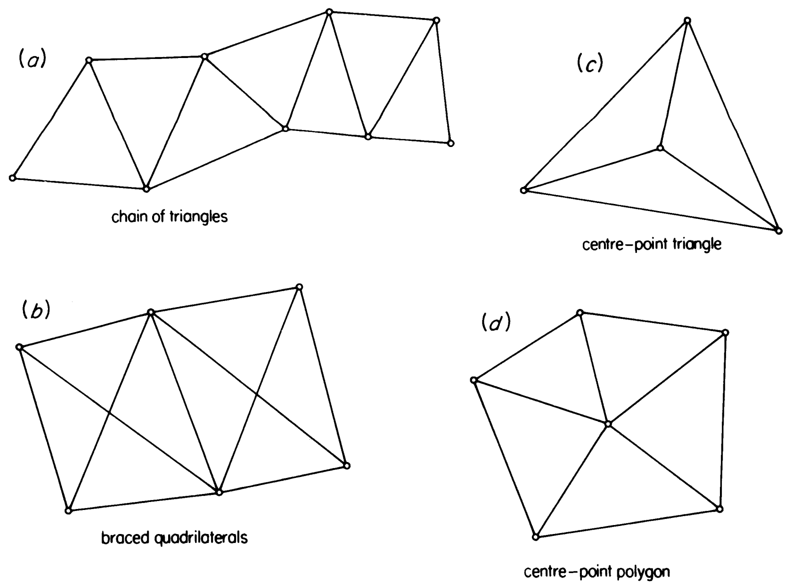


Figure 6.2 *Triangulation figures*

is that more checks can be applied to the observed angles. For instance, only one check is possible with individual triangles (Σ angles = 180°) whereas four checks are possible with a braced quadrilateral (see section 6.4.1).

The inclusion of figures more elaborate than simple triangles in a control scheme *strengthens the network* by increasing the number of redundant measurements taken, this in turn increasing the number of checks possible on the fieldwork.

In triangulation computations (see section 6.4), lengths are obtained for use in coordinate calculations by applying the Sine Rule to the various triangles making up the network. This should be borne in mind when establishing the positions of control points since a further means of strengthening a network is to avoid the use of small angles since the sine function is less reliable for small angles than for those approaching 90°. As a guide, wherever possible, angles of less than 25° should not be included in triangulation figures and, if this is the case, the network is said to be *well-conditioned*.

The final choice of figure in any triangulation scheme is usually determined by the topography of the site, the precision required and the need for a strong network.

6.3 Triangulation Fieldwork

The methods that can be used to establish and observe a triangulation network vary considerably between primary and tertiary schemes and it is again emphasised that the following sections are concerned solely with the application of second-order triangulation to civil engineering and construction sites (see also section 6.1).

6.3.1 Reconnaissance

The reconnaissance for a triangulation scheme is the most important part of the survey and is carried out to determine the positions of the control stations in the network. Since this is linked to the size and shape of the figures to be used in the scheme and to the number of angles to be observed, the reconnaissance will determine the amount of fieldwork that will have to be undertaken.

To start the reconnaissance, information relevant to the survey area should be gathered, especially that relating to any previous surveys. Such information may include existing maps, aerial photographs and any site surveys already prepared for the construction project.

From this information, a network diagram should be prepared, approximately to scale, showing proposed locations for the stations.

Following this, it is essential that the survey area is visited, at which time the final positions for the stations are chosen.

Many of the guidelines given in section 5.4 for reconnaissance when traversing are also applicable here, but for second-order triangulation particular attention must be paid to the following.

(1) When establishing the stations, it is essential that the strongest network is obtained. In general, it is advisable not to include angles less than 25° in the scheme and braced quadrilaterals and centre point polygons should be included wherever

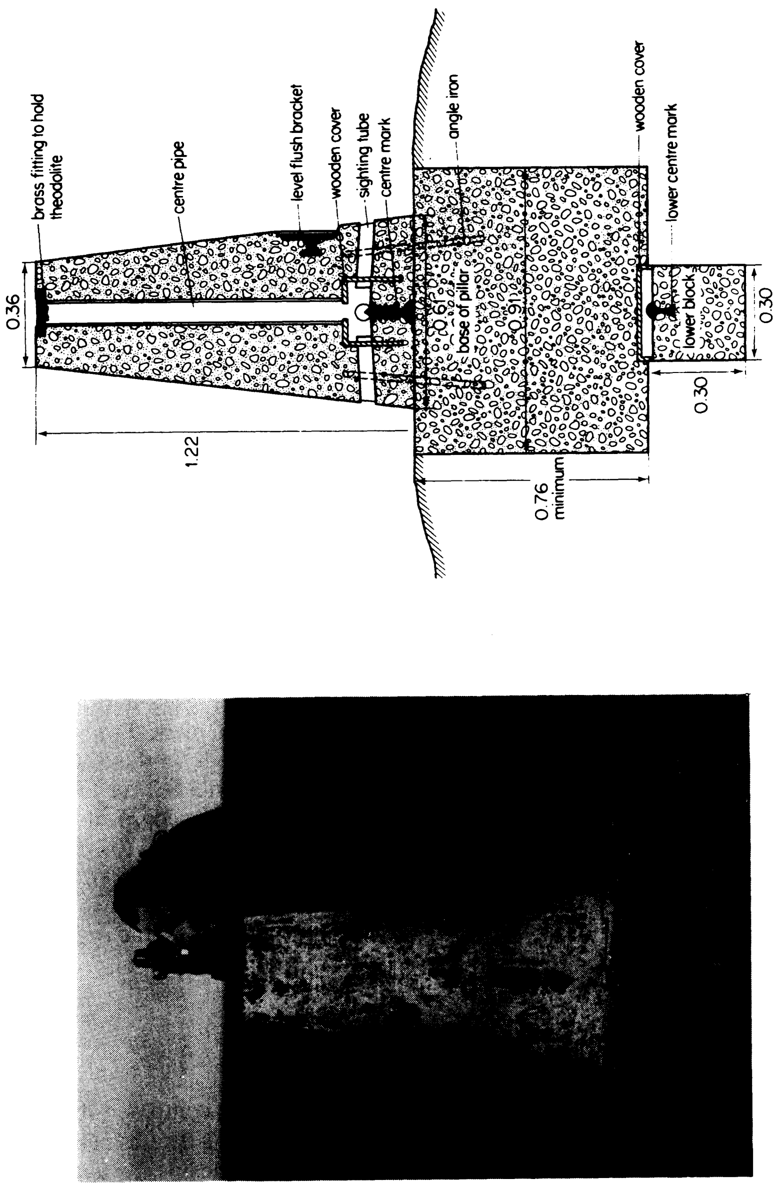


Figure 6.3 OS triangulation pillar

possible. A reliable diagram is required for determining the strength of the network and, to construct this, it may be necessary to take approximate measurements of some angles and distances in the field to supplement the network diagram already prepared.

(2) Since lines of sight between all adjacent stations must be uninterrupted, wherever possible, stations should be placed on high ground overlooking the construction site.

(3) As with all control surveys, the layout of stations in relation to the survey work for which they are intended must be carefully planned.

(4) The stations should be located in positions which provide at least one and preferably two long lines for use as baselines. The longer the baselines, the stronger the figure.

6.3.2 Station Marks and Signals (Targets)

Upon completion of the reconnaissance, the exact positions for the survey stations are marked in some way.

The triangulation stations set up by the OS are permanently marked by using an elaborate arrangement in which a metal plate is set into a concrete pillar, both of these being centred over an underground marker as in figure 6.3. Although this type of station construction could be used in engineering surveys, the cost is high and a less expensive pillar is shown in figure 14.2. Also shown in figure 14.2 is a suitable design for a mark set into a buried concrete block. For surveys of a temporary nature (a few months only) wooden pegs can be used for station markers and these are discussed in section 5.4.2.

For each station in the network, reference sketches should be prepared (see figure 5.6).

To enable angles to be observed in a triangulation scheme, each station must have some form of signal erected vertically above the station mark. For engineering sites, the type of signal used is usually temporary and depends on the distance over which the target is sighted. Those signals suggested in section 5.5.2 for traversing apply equally to small triangulation networks and their use is recommended.

For both theodolites and targets on small triangulation schemes, forced centring equipment, where theodolite and target are compatible with the same tribrach, should be used if available. This equipment is shown in figure 3.3. If theodolites equipped with centring rods are to be used to observe angles, the rod itself can be used as a built in target (see figure 3.7).

6.3.3 Baseline Measurement

In triangulation work, the lengths of all the triangle sides are computed throughout the network to enable the coordinates of all the stations to be determined. Since these computed lengths are derived from one or more baselines, it is essential that every effort is made to measure the baselines as precisely as possible since the computed triangle side lengths cannot be more precise than the measured baseline lengths (see also section 5.8.9).

EDM equipment, described in sections 4.11 to 4.15, is usually used to measure baselines and long lines should be chosen for measurement to improve the strength of the network. When using the EDM equipment, the meteorological conditions at the time of measurement must be monitored carefully and suitable corrections made (see section 4.14.1). Any systematic instrumental errors present in the equipment must be allowed for by careful calibration of the EDM unit. The instrument manufacturer may offer a calibration service but this can be carried out on site using methods given in the references to chapter 4.

Care must also be taken to reduce the line measurement from the slope to the horizontal. This can be achieved either by observing vertical angles or by determining the reduced levels of each end of the baseline, the necessary corrections for this being given in section 4.14.3.

For surveys that are to be based on the National Grid (see section 5.11), the scale factor is applied to the measured distance and, if the distance has been measured at an appreciable elevation, it must be reduced to its equivalent length at mean sea level (MSL), since MSL is the datum height for the National Grid. The correction to MSL is derived as follows.

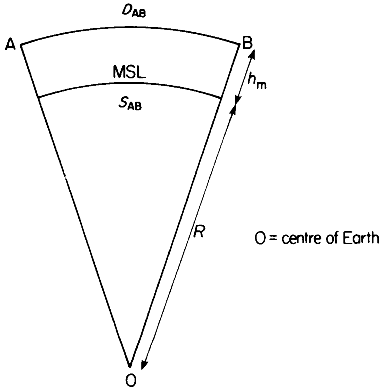


Figure 6.4 *Reduction to mean sea level*

In figure 6.4, the horizontal distance between A and B, D_{AB}, has been measured at a mean height h_m above sea level. By proportion, the equivalent value of D_{AB} at sea level, S_{AB}, is obtained from

$$\frac{S_{AB}}{R} = \frac{D_{AB}}{R + h_m}$$

or

$$S_{AB} = (D_{AB}) \frac{R}{R + h_m}$$

where R is the average radius of the Earth, which is 6375 km for Great Britain.

Alternatively, a correction to D_{AB} can be computed from

$$C = D_{AB} \frac{h_m}{R}$$

where $S_{AB} = D_{AB} - C$ (the correction is always negative for lines *above* MSL).

The following example shows the application of the MSL correction.

Question

The horizontal length of a baseline FG is measured by EDM as 879.842 m (corrected for meteorological and systematic effects). If the heights of stations F and G are $h_F = 415.17$ m AOD and $h_G = 427.20$ m AOD, calculate the length of FG at MSL.

Solution

The mean sea level distance S_{FG} is given by

$$S_{FG} = (D_{FG}) \frac{R}{R + h_m} = \frac{(879.842)\,6\,375\,000}{(6\,375\,000 + 421)} = \mathbf{879.784\ m}$$

Since

$$h_m = \frac{415.17 + 427.20}{2} = 421.19 \text{ m}$$

Alternatively, a MSL correction can be computed using

$$C = D_{FG} \frac{h_m}{R} = \frac{879.842(421.19)}{6\,375\,000} = 0.058 \text{ m}$$

which gives

$$S_{FG} = D_{FG} - C = 879.842 - 0.058 = \mathbf{879.784\ m}$$

The proportional effect of the MSL correction in this case is approximately 1 in 15 000.

If suitable terrain can be found, an alternative to using EDM is to measure baselines using steel or invar tapes. The procedures for this are given in chapter 4 but it should be noted that, compared to EDM, it is more difficult to maintain accuracy when taping lines greater than a few hundred metres in length unless catenary taping can be utilised. This, however, is a very time consuming and labour intensive method of taping which is not normally used in second-order surveys.

For some surveys, the baseline length is given in the form of coordinates for two (or more) stations already fixed by a previous survey. If such a fixed baseline has been specified for triangulation work, it is advisable to check the length between and the condition of the station marks forming the baseline before beginning the survey.

6.3.4 Angle Measurement

The instrument required for measurement of the angles in triangulation networks is a 0.1″/0.2″ or 1″ double reading optical micrometer theodolite as described in section 3.2.2 or an electronic theodolite as described in section 3.2.3. The theodolite is set up and the angles are observed and booked in rounds using the methods given in section 3.3.

Since the reliability of any triangulation network depends greatly on the angles, it is essential that a good accuracy is obtained when measuring these, and extra care must be taken to centre the instrument and targets to reduce to a minimum any centring errors (see section 3.3.4). It is also necessary to protect the instrument from the heating effects of the sun and from vibration. To overcome these difficulties, an umbrella can be used to shield the theodolite and tripod from direct sunlight, and heavy weights such as sandbags can be attached to the tripod to reduce the effects of wind.

The effects of the sun causing differential heating and wind causing vibration are greatly reduced if the instrument is supported on a pillar (see figure 6.3) and for all first-order surveys their use is essential, and they should also be used, if possible, for second-order work.

When observing triangulation angles, all the field procedures used should aim to minimise any instrumental and gross errors; section 3.3.2 gives details of the correct procedures.

Since many rounds of each angle may be required, readings should be spread evenly over the range of the horizontal circle and micrometer run in order to reduce any errors in the theodolite reading system. For each round, the zero should be changed by an interval I, where

$$I = \frac{180^\circ}{n} + \frac{r}{n}$$

where

n = number of rounds to be observed
r = run of micrometer.

If, for example, a 1″ reading theodolite having a micrometer run of 10′ were to be used on four rounds, the zeros should be 00°00′00″, 45°02′30″, 90°05′00″ and 135°07′30″.

6.3.5 Orientation

As in traversing, the North axis of the rectangular grid on which the triangulation scheme is based must be orientated to a specified direction. In engineering work, one of a number of north directions may be selected as described in section 5.3.2.

Generally, it is usual to set the scheme to align with one of the following.

(1) The National Grid, by using a baseline defined by two existing OS pillars. The coordinates of the points can be used to calculate the bearing of the baseline.

(2) Any other grid, by using existing points defined by another survey.

(3) Any other north direction to suit site conditions such as a structural or site grid (see section 14.6.2).

6.4 Triangulation Computations

The methods used to calculate the rectangular coordinates in triangulation networks are very similar to those used in traversing (see section 5.8) and follow a similar sequence as shown in the following sections.

6.4.1 Adjustment of Angles

The observed angles in triangulation schemes can be adjusted using either rigorous or semi-rigorous methods.

A *rigorous method* is one which allows all the angles in a network to be adjusted simultaneously. Since these techniques are very complicated they are usually applied to first-order surveys and are beyond the scope of this book.

Semi-rigorous methods, unlike rigorous adjustments, adopt a step-by-step approach in which each figure throughout a network is adjusted in turn. As a result of this, it is possible to obtain slightly different values for the adjusted angles, depending on which route of computation is chosen through the network.

The only semi-rigorous adjustment required for a network consisting of simple triangles is that the sum of the angles in each triangle must equal 180° and each angle is adjusted by one-third of the misclosure in the triangle in question. Since only one check is possible on the observed angles in a triangle, triangulation networks consisting of triangles alone are not strong unless frequent check baselines are introduced into the scheme.

When networks consist of more complicated figures, such as braced quadrilaterals and centre point polygons, additional checks on the angular measurements are made possible since extra geometric conditions are present in each figure. Table 6.2 shows the number of conditions that exist in the commonly used triangulation figures.

The semi-rigorous adjustment of the more complex triangulation figures is usually carried out by the *method of equal shifts*. This is now shown for a braced quadrilateral and an equal shifts adjustment of a centre point triangle is given in section 6.4.3.

TABLE 6.2
Adjustment Conditions in Triangulation and Trilateration Figures

Figure	Number of Conditions	
	Triangulation	Trilateration*
Simple triangle	1	0
Braced quadrilateral	4	1
Centre point triangle	5	1
Centre point quadrilateral	6	1
Centre point pentagon	7	1

* see section 6.6

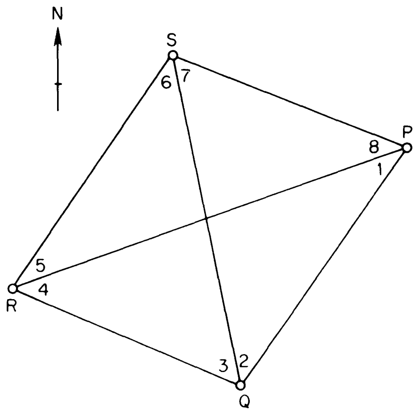
Question

The field abstract of figure 6.5 shows the observed angles for a braced quadrilateral PQRS.

Calculate the adjusted values of all the angles in this quadrilateral using the method of equal shifts.

Solution

The complete angular adjustment for the quadrilateral is given in table 6.3.



Angle	Observed value
1	30° 20' 50"
2	54 10 45
3	55 44 38
4	39 43 39
5	41 53 49
6	42 37 47
7	54 54 56
8	40 33 30

Station	Coordinates mE	Coordinates mN
P	1885.82	1632.47
Q	1401.00	1045.76

Figure 6.5 ***Abstract for braced quadrilateral***

Four geometric conditions must be satisfied when adjusting the observed angles of a braced quadrilateral: three *angle conditions* and a *side condition*. The angle conditions are, referring to the numbering system of figure 6.5

(*a*) Σangles 1 to 8 = 360°
(*b*) Angles 1 + 2 = Angles 5 + 6
(*c*) Angles 3 + 4 = Angles 7 + 8.

In this example

$$\Sigma \text{angles 1 to 8} = 359°59'54''$$

and 0.75″ is added to each angle to adjust them to 360° as shown in columns 1 and 2 of table 6.3.

Angle conditions (*b*) and (*c*) are known as *adjustments to opposites* and for figure 6.5

$$\begin{aligned} \text{Angles } 1 + 2 &= 84°31'35'' \\ \text{Angles } 5 + 6 &= \underline{84°31'36''} \\ \text{Difference } \Delta &= \underline{\qquad 01''} \end{aligned}$$

TABLE 6.3

Equal Shifts Adjustment of Braced Quadrilateral

Angle	Observed value	Adjustment to 360°	Adjustment to opposites	First adjusted angles	Side adjustment	Final adjusted angles	Final angles
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	30°20'50"	+0.75"	+0.25"	30°20'51.00"	+2.13"	30°20'53.13"	30°20'53"
2	54°10'45"	+0.75"	+0.25"	54°10'46.00"	-2.13"	54°10'43.87"	54°10'44"
3	55°44'38"	+0.75"	+2.25"	55°44'41.00"	+2.13"	55°44'43.13"	55°44'43"
4	39°43'39"	+0.75"	+2.25"	39°43'42.00"	-2.13"	39°43'39.87"	39°43'40"
5	41°53'49"	+0.75"	-0.25"	41°53'49.50"	+2.13"	41°53'51.63"	41°53'52"
6	42°37'47"	+0.75"	-0.25"	42°37'47.50"	-2.13"	42°37'45.37"	42°37'45"
7	54°54'56"	+0.75"	-2.25"	54°54'54.50"	+2.13"	54°54'56.63"	54°54'57"
8	40°33'30"	+0.75"	-2.25"	40°33'28.50"	-2.13"	40°33'26.37"	40°33'26"
	359°59'54"			360°00'00.00"		360°00'00.00"	360°00'00"

Angles 1 + 2 = 84°31'35"
Angles 5 + 6 = 84°31'36"
Δ = 1"
$\frac{\Delta}{4}$ = 0.25"

Angles 3 + 4 = 95°28'17"
Angles 7 + 8 = 95°28'26"
Δ = 9"
$\frac{\Delta}{4}$ = 2.25"

a = 0.228 202 099
b = 4.206 102 699
c = 0.228 221 840
d = 4.179 873 602
$|v''|$ = 2.13"

Final a = 0.228 212 011
c = 0.228 211 989 (check)

In order to satisfy the condition (angles 1 + 2 = angles 5 + 6), each angle must be changed by an amount $\Delta/4 = 0.25''$. Since $(5 + 6) > (1 + 2)$ in this case, 0.25″ is subtracted from 5 and 6 and added to 1 and 2. These adjustments, including those for 3 + 4 and 7 + 8 are shown in column 3 and in the lower part of table 6.3.

Application of the adjustment to 360° and the adjustments to opposites gives the first adjusted angles of column 4.

The fourth geometric condition to be satisfied in a braced quadrilateral is a side condition of the form

$$|v''| = \frac{a - c}{(ab + cd)\sin 1''}$$

where

$|v''|$ = *magnitude* of a side adjustment to be applied to each first adjusted angle
a = sin 1 × sin 3 × sin 5 × sin 7
b = cot 1 + cot 3 + cot 5 + cot 7
c = sin 2 × sin 4 × sin 6 × sin 8
d = cot 2 + cot 4 + cot 6 + cot 8

a, b, c and *d* are all computed using the first adjusted angles.

For PQRS of figure 6.5

$a = 0.228\,202\,099$ $\quad c = 0.228\,221\,840$
$b = 4.206\,102\,699$ $\quad d = 4.179\,873\,602$

which gives

$$|v''| = 2.1''$$

Since $c > a$ for this quadrilateral, the side adjustment of 2.1″ is subtracted from the angles used to compute c, that is, angles 2, 4, 6 and 8, and added to the angles used to compute a, that is, angles 1, 3, 5 and 7. The application of the side adjustment is shown in columns 5 and 6 of table 6.3.

In the case where $a > c$ for a quadrilateral, the side adjustment is added to angles 2, 4, 6 and 8 and subtracted from angles 1, 3, 5 and 7.

It is important to note that the application of the side adjustment given here (and the adjustments to opposites) refers only to the numbering sequence adopted in figure 6.5.

The side adjustment is checked by computing further a and c values using the final adjusted angles of column 6. In *all* adjustments, a should equal c or very nearly so to give $|v''| = 0$ which indicates a properly satisfied side condition. Since the final a and c values in table 6.3 agree, when rounded, to the seventh decimal place, the side adjustment has been applied correctly. If the final a and c values in any side adjustment do not agree to at least the sixth decimal place, the adjustment has not been applied correctly and should be checked. A common mistake is to allocate the incorrect + or – sign to the adjustment such that it is added when it should have been subtracted and vice versa.

Although not always necessary, column 7 shows the final adjusted angles of column 6 rounded to the same precision as the original observations.

6.4.2 Coordinate Calculations

The procedures involved in the calculation of coordinates are best illustrated using the braced quadrilateral PQRS of figure 6.5.

The baseline length D_{PQ} and bearing θ_{PQ} for PQRS are given by

$$D_{PQ} = [(E_Q - E_P)^2 + (N_Q - N_P)^2]^{\frac{1}{2}} = 761.104 \text{ m}$$

$$\theta_{PQ} = \tan^{-1}\left[\frac{E_Q - E_P}{N_Q - N_P}\right] + 180° \qquad = 219°34'05''$$

By application of the Sine Rule to triangle PQR, the following can be written

$$\frac{\sin 1}{D_{QR}} = \frac{\sin(2+3)}{D_{PR}} = \frac{\sin 4}{D_{PQ}}$$

or

$$D_{PR} = \frac{\sin(2+3)}{\sin 4} D_{PQ}$$

and

$$D_{QR} = \frac{\sin 1}{\sin 4} D_{PQ}$$

Since all the angles have been observed in the quadrilateral, these equations give the unknown side lengths D_{PR} and D_{QR}. A similar set of calculations in triangle PQS will give the unknown side lengths in that triangle.

The other triangles QRS and PSR in the quadrilateral should also be solved since these triangles provide a check on the side length RS and show the consistency in calculating the side lengths QS and PR.

Table 6.4 shows the complete calculation of the side lengths in PQRS using the final adjusted angles calculated in section 6.4.1.

The bearings (θ) of PQRS can be evaluated as follows.
For triangle PQR

$$\begin{array}{rl} \theta_{PQ} = & 219°34'05'' \\ +\,1 = & \underline{\ \ 30°20'53''} \\ \theta_{PR} = & 249°54'58'' \end{array} \qquad\qquad \begin{array}{rl} \theta_{QP} = & 39°34'05'' \\ -(2+3) = & \underline{109°55'27''} \\ \theta_{QR} = & 289°38'38'' \end{array}$$

$$\theta_{RQ} - \theta_{RP} = 109°38'38'' - 69°54'58'' = 39°43'40'' = \text{angle } 4$$

(check)

TABLE 6.4
Side Length Calculation for a Braced Quadrilateral

Triangle	Side	Formula	Station	Angle	Horizontal Length (m)
PQR	PQ	Baseline			761.104
			P	1 = 30°20'53"	
			Q	2 + 3 = 109°55'27"	
			R	4 = 39°43'40"	
				180°00'00"	
	PR	D_{PQ} sin (2 + 3)/sin 4			1119.546
	QR	D_{PQ} sin 1/sin 4			601.666
PQS	PQ	Baseline			761.104
			P	1 + 8 = 70°54'19"	
			Q	2 = 54°10'44"	
			S	7 = 54°54'57"	
				180°00'00"	
	PS	D_{PQ} sin 2/sin 7			754.165
	QS	D_{PQ} sin (1 + 8)/sin 7			878.919
QRS	QR	Baseline			601.666
			Q	3 = 55°44'43"	
			R	4 + 5 = 81°37'32"	
			S	6 = 42°37'45"	
				180°00'00"	
	QS	D_{QR} sin (4 + 5)/sin 6			878.922 (check)
	RS	D_{QR} sin 3/sin 6			734.296
PSR	PS	Baseline			754.165
			P	8 = 40°33'26"	
			S	6 + 7 = 97°32'42"	
			R	5 = 41°53'52"	
				180°00'00"	
	PR	D_{PS} sin (6 + 7)/sin 5			1119.544 (check)
	SR	D_{PS} sin 8/sin 5			734.293 (check)

For triangle PQS

$$
\begin{array}{rlrl}
\theta_{PQ} & = 219^\circ 34'05'' & \theta_{QP} & = 39^\circ 34'05'' \\
+(1+8) & = 70^\circ 54'19'' & -2 & = 54^\circ 10'44'' \\
\hline
\theta_{PS} & = 290^\circ 28'24'' & \theta_{QS} & = 345^\circ 23'21''
\end{array}
$$

$$\theta_{SQ} - \theta_{SP} = 165^\circ 23'21'' - 110^\circ 28'24'' = 54^\circ 54'57'' = \text{angle } 7$$

(check)

When all the lengths and bearings of the quadrilateral sides have been computed, the coordinates of R and S are evaluated by computing traverses to include these unknown stations. The method is identical to that for a link traverse (see section 5.12.2) and the coordinates of R and S are derived in traverses QRP and QSP as shown in table 6.5.

TABLE 6.5
Coordinate Calculations in a Braced Quadrilateral

Triangle	Side	Bearing	Horizontal Length (m)	ΔE(m)	ΔN(m)	Co-ordinates mE	mN	Station
PQR						1401.00	1045.76	Q
	QR	289°38'38"	601.666	-566.649	202.264			
						834.35	1248.02	R
	RP	69°54'58"	1119.545	1051.466	384.447			
						1885.82	1632.47 (check)	P
PQS						1401.00	1045.76	Q
	QS	345°23'21"	878.921	-221.710	850.498			
						1179.29	1896.26	S
	SP	110°28'24"	754.165	706.528	-263.785			
						1885.82	1632.47 (check)	P

6.4.3 Worked Example: Adjustment and Computation of a Centre Point Triangle

Question

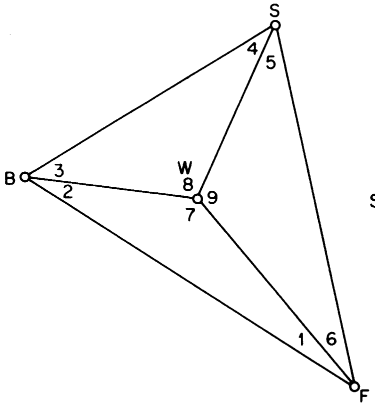
The field abstract for a triangulation scheme established for a small construction site is shown in figure 6.6. Using this data, calculate the coordinates of stations S and W.

Solution

The procedure for solving the centre point triangle is as follows.

(1) The observed angles are first adjusted using the equal shifts method as shown in table 6.6. The geometric conditions to be satisfied in a centre point triangle are

(*a*) the angles in any triangle must sum to 180° (refer to columns 1 and 2 in table 6.6)



Angle	Observed value
1	26° 10' 48"
2	27 37 16
3	35 46 10
4	32 57 52
5	28 23 12
6	29 04 37
7	126 11 59
8	111 15 52
9	122 32 02

Station	Coordinates mE	mN
F	719.37	250.00
B	250.00	447.15

Figure 6.6 ***Abstract for centre-point triangle***

(*b*) the angles at the centre station must sum to 360° without altering any previous adjustment (see columns 3 and 4)

(*c*) the side condition $|v''| = \dfrac{a - c}{(ab + cd)\sin 1''}$

where

$$a = \sin 1 \times \sin 3 \times \sin 5 \qquad b = \cot 1 + \cot 3 + \cot 5$$
$$c = \sin 2 \times \sin 4 \times \sin 6 \qquad d = \cot 2 + \cot 4 + \cot 6$$

The application of the side condition is shown in columns 5, 6 and 7 of table 6.6.

(2) The coordinates of the baseline FB are used to give $\theta_{FB} = 292°47'02''$ and $D_{FB} = 509.09(4)$ m. Since the coordinates of F and B are given for the network, they must NOT be altered.

(3) The lengths of the sides of all the triangles are calculated as shown in table 6.7.

(4) The bearings of the sides of all the triangles are calculated in the sequences given below.

For triangle FBW

$$\begin{array}{lcl} \theta_{FB} = 292°47'02'' & \quad & \theta_{BF} = 112°47'02'' \\ +1 \;\; = \;\; 26°10'45'' & & -2 \;\; = \;\; 27°37'16'' \\ \hline \theta_{FW} = 318°57'47'' & & \theta_{BW} = \;\; 85°09'46'' \\ \hline \end{array}$$

$$\theta_{WB} - \theta_{WF} = 126°11'59'' = \text{angle } 7 \qquad \text{(check)}$$

TABLE 6.6

Equal Shifts Adjustment of a Centre-point Triangle

ngle	Observed Value	Adjustment to 180°	Adjusted Centre Angles	Adjustment to 360°	First Adjusted Angles	Side Adjustment	Final Adjusted Angles	Final Angles
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	26°10'48"	-1.0"		-0.5"	26°10'46.5"	-1.3"	26°10'45.2"	26°10'45"
2	27°37'16"	-1.0"		-0.5"	27°37'14.5"	+1.3"	27°37'15.8"	27°37'16"
7	126°11'59"	-1.0"	126°11'58"	+1.0"	126°11'59		126°11'59"	126°11'59"
	180°00'03"				180°00'00.0"			
3	35°46'10"	+2.0"		-0.5"	35°46'11.5"	-1.3"	35°46'10.2"	35°46'10"
4	32°57'52"	+2.0"		-0.5"	32°57'53.5"	+1.3"	32°57'54.8"	32°57'55"
8	111°15'52"	+2.0"	111°15'54"	+1.0"	111°15'55"		111°15'55"	111°15'55"
	179°59'54"				180°00'00.0"			
5	28°23'12"	+3.0"		-0.5"	28°23'14.5"	-1.3"	28°23'13.2"	28°23'13"
6	29°04'37"	+3.0"		-0.5"	29°04'39.5"	+1.3"	29°04'40.8"	29°04'41"
9	122°32'02"	+3.0"	122°32'05"	+1.0"	122°32'06"		122°32'06"	122°32'06"
	179°59'51"		359°59'57"		180°00'00.0"			

a = sin 1 sin 3 sin 5 = 0.122 607 224
b = cot 1 + cot 3 + cot 5 = 5.272 607 166
c = sin 2 sin 4 sin 6 = 0.122 599 318
d = cot 2 + cot 4 + cot 6 = 5.251 373 055

$$|v''| = \frac{a - c}{(ab + cd)\sin 1''} = 1.3'' \quad a > c$$

Final a = 0.122 603 150
c = 0.122 603 375 (check)

TABLE 6.7
Calculation of Side Lengths in a Centre-point Triangle

Triangle	Side	Formula	Station	Angle	Horizontal Length (m)
FBW	FB	Baseline			509.094
			F	1 = 26°10'45"	
			B	2 = 27°37'16"	
			W	7 = 126°11'59"	
				180°00'00"	
	FW	D_{FB} sin 2/sin 7			292.489
	WB	D_{FB} sin 1/sin 7			278.330
WBS	WB	Baseline			278.330
			W	8 = 111°15'55"	
			B	3 = 35°46'10"	
			S	4 = 32°57'55"	
				180°00'00"	
	BS	D_{WB} sin 8/sin 4			476.685
	WS	D_{WB} sin 3/sin 4			298.992
WFS	WF	Baseline			292.489
			W	9 = 122°32'06"	
			F	6 = 29°04'41"	
			S	5 = 28°23'13"	
				180°00'00"	
	FS	D_{WF} sin 9/sin 5			518.667
	WS	D_{WF} sin 6/sin 5			298.996 (check)

For triangle WBS

$$\begin{array}{llll} \theta_{WB} & = 265°09'46'' & \theta_{BW} & = \ \ 85°09'46'' \\ +8 & = 111°15'55'' & -3 & = \ \ 35°46'10'' \\ \hline \theta_{WS} & = \ \ 16°25'41'' & \theta_{BS} & = \ \ 49°23'36'' \end{array}$$

$$\theta_{SB} - \theta_{SW} = 32°57'55'' = \text{angle } 4 \qquad \text{(check)}$$

For triangle FWS

$$\begin{array}{llll} \theta_{FW} & = 318°57'47'' & \theta_{WF} & = 138°57'47'' \\ +6 & = \ \ 29°04'41'' & -9 & = 122°32'06'' \\ \hline \theta_{FS} & = 348°02'28'' & \theta_{WS} & = \ \ 16°25'41'' \end{array} \qquad \text{(check)}$$

$$\theta_{SW} - \theta_{SF} = 28°23'13'' = \text{angle } 5 \qquad \text{(check)}$$

(5) The coordinates of W and S are evaluated in link traverses BWF and BSF as shown in table 6.8.

TABLE 6.8
Coordinate Calculations in a Centre-point Triangle

Triangle	Side	Bearing	Horizontal Length (m)	ΔE(m)	ΔN(m)	Co-ordinates mE	mN	Station
BWF						250.00	447.15	B
	BW	85°09'46"	278.330	277.339	23.470			
						527.34	470.62	W
	WF	138°57'47"	292.489	192.032	-220.620			
						719.37	250.00	F
BSF						250.00	447.15	B
	BS	49°23'36"	476.685	361.897	310.256			
						611.90	757.41	S
	SF	168°02'28"	518.667	107.473	-507.410			
						719.37	250.00	F

6.5 Reduction to Centre (Eccentric Stations)

It is often useful in triangulation schemes to incorporate prominent points such as church spires into the network since these form very good natural targets which are visible from considerable distances. Unfortunately, it is very difficult and sometimes impossible to centre a theodolite over such a station, which makes direct angle measurement equally difficult or impossible. This problem can be overcome if the angles at the spire are deduced from triangles that include the spire as one of the stations, but this is not good practice since it is not then possible to apply checks (Σ angles = 180°) in these triangles.

A field procedure that can be adopted to obtain angles at the spire or other mark, while at the same time maintaining a checking facility, is to set the theodolite at a point as near as possible to the spire. From this *eccentric station*, angles are observed to all points in the network as if the inaccessible point (spire) itself was being occupied.

Since the angles observed at the eccentric station will be slightly different to those which would have been observed at the inaccessible or main station, corrections are applied to the measured angles to give those applicable to the main station. This correction procedure is known as *reduction to centre*.

In figure 6.7, stations C, Y and M are triangulation stations, station C being inaccessible. At station E, the eccentric station, the directions to C, Y and M are observed and the distance EC measured. This distance should be as small as possible and measured as accurately as possible. The observed directions to Y and M are converted to their equivalents at station C by applying corrections c_1 and c_2 which are obtained from

$$c'' = \pm \frac{d \sin \alpha}{D \sin 1''}$$

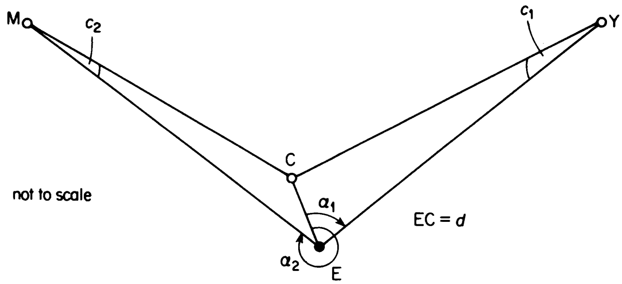


Figure 6.7 *Reduction to centre*

where

c'' = correction (in seconds) to a direction
d = eccentric distance EC
α = clockwise angle measured at E from the inaccessible station to a control station
D = distance from E (or C) to the same station corresponding to α.

The sign of the correction is given by $\sin\alpha$ and if the ratio $d/D < 1/40$ the formula cannot be used, but the ratio is not likely to be less than this in practice. The distances D are found by making provisional solutions of the triangles formed by the main station and the observed stations.

For figure 6.7

$$c_1'' = \frac{d \sin \alpha_1}{D_{\mathrm{EY}} \sin 1''} \quad \text{and} \quad c_2'' = \frac{d \sin \alpha_2}{D_{\mathrm{EM}} \sin 1''}$$

So that

direction CY = direction EY + c_1
direction CM = direction EM − c_2 (since $\alpha_2 > 180°$)

or

angle $\mathrm{M\hat{C}Y}$ in main triangulation scheme
= direction CY − direction CM
= direction EY + c_1 − (direction EM − c_2)
= angle $\mathrm{M\hat{E}Y}$ + c_1 + c_2

The following worked example shows the application of the reduction to centre technique.

Question

At inaccessible station L in a triangulation network an eccentric station E was set up nearby in order to obtain angle $\mathrm{P\hat{L}M}$. The following measurements were taken

$$D_{LE} = 3.321 \text{ m} \qquad D_{PM} = 1011.07 \text{ m}$$
$$\widehat{PEM} = 78°22'45'' \qquad \widehat{MEL} = 71°36'28''$$
$$\widehat{LPM} = 63°38'12'' \qquad \widehat{LMP} = 38°01'54''$$

Calculate the value of angle $\widehat{PLM}$.

Solution

The station geometry for this example is shown in figure 6.8. Converting the observed angles taken at E into clockwise directions gives, with direction EL as RO

$$\text{direction EP} = \alpha_1 = 210°00'47''$$
$$\text{direction EM} = \alpha_2 = 288°23'32''$$

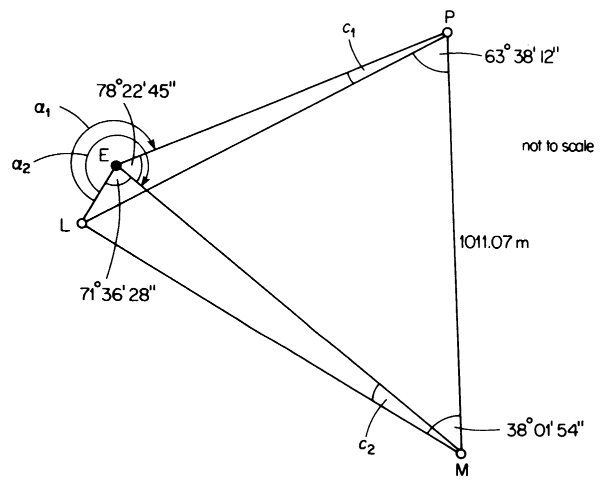


Figure 6.8

To obtain directions LP and LM and hence angle $\widehat{PLM}$, corrections c_1 and c_2 are required where, with EL = d

$$c_1'' = \frac{d \sin \alpha_1}{D_{LP} \sin 1''} \qquad c_2'' = \frac{d \sin \alpha_2}{D_{LM} \sin 1''}$$

A provisional solution of triangle PLM gives

$$\widehat{PLM} = 180° - (\widehat{LPM} + \widehat{PML}) = 78°19'54''$$
$$D_{LP} = D_{PM} (\sin \widehat{PML} / \sin \widehat{PLM}) = 636.06 \text{ m}$$
$$D_{LM} = D_{PM} (\sin \widehat{LPM} / \sin \widehat{PLM}) = 925.03 \text{ m}$$

Hence

$$c_1'' = \frac{3.321 \sin 210°00'47''}{636.06 \sin 1''} = -539'' = -08'59''$$

$$c_2'' = \frac{3.321 \sin 288°23'32''}{925.03 \sin 1''} = -703'' = -11'43''$$

The corrected directions from L are

$$\begin{aligned}\text{direction LP} &= \text{direction EP} + c_1'' = 210°00'47'' - 08'59''\\ &= 209°51'48''\\ \text{direction LM} &= \text{direction EM} + c_2'' = 288°23'32'' - 11'43''\\ &= 288°11'49''\end{aligned}$$

Therefore

$$\text{P}\hat{\text{L}}\text{M} = \text{direction LM} - \text{direction LP} = 78°20'01''$$

6.6 Trilateration

A trilateration network, like triangulation, may consist of a series of single triangles, braced quadrilaterals and centre point polygons or combinations of these but, unlike triangulation, the observed quantities are the distances not the angles. The precise measurement of distances on a large scale has been made possible by the development of EDM.

Many of the field techniques employed in triangulation (see section 6.3) are applicable to trilateration and the reconnaissance to establish the shape of a trilateration network follows the same general principles as that for triangulation.

The distances should also be measured in accordance with the guidelines given for triangulation as should any angles observed to establish bearings in the network.

The methods used in trilateration computations are very similar to those used in triangulation and proceed as follows.

(1) Using the measured or calculated baseline length and all the measured lengths, the angles in the network are calculated using the Cosine Rule. This is carried out by breaking down all the figures into their constituent triangles and proceeding as follows.

In triangle ABC of figure 6.9, the angles are given by

$$\cos \alpha = \frac{D_{AC}^2 + D_{AB}^2 - D_{BC}^2}{2D_{AC}D_{AB}}$$

$$\cos \beta = \frac{D_{AB}^2 + D_{BC}^2 - D_{AC}^2}{2D_{AB}D_{BC}}$$

$$\cos \gamma = \frac{D_{AC}^2 + D_{BC}^2 - D_{AB}^2}{2D_{AC}D_{AB}}$$

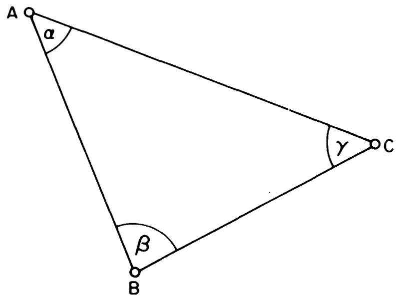


Figure 6.9 *Trilateration*

(2) Each triangle is checked using $\alpha + \beta + \gamma = 180°$.

(3) The angles derived in (1) are adjusted using the method of equal shifts.

(4) Starting at the baseline, all the lengths of the sides in the network are computed using the adjusted angles. In addition, the bearings of all the triangle sides throughout the network are computed, again using the adjusted angles.

(5) The computed bearings and distances obtained in (4) are used to calculate coordinates throughout the network.

A disadvantage of trilateration is that, compared to triangulation, the number of geometric conditions for adjustment is less than that for equivalent figures. Taking, for example, a simple triangle such as BFD in figure 6.10, three observations are made in triangulation (angles α, β and γ) to fix two unknowns, E_F and N_F. This gives rise to one redundant measurement and one adjustment condition $\alpha + \beta + \gamma = 180°$. If the same triangle were part of a trilateration network, two observations would be made (lengths b and d) to fix two unknowns and, consequently, no redundancy and no adjustment condition exist.

The use of single triangles in control extension by trilateration is, therefore, not recommended.

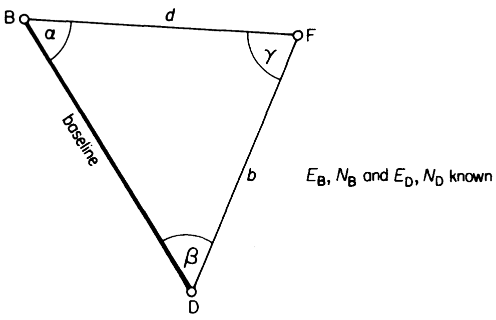


Figure 6.10

The number of conditions for adjustment in various triangulation and trilateration figures is compared in table 6.2. From this, it is evident that triangulation should always be used in preference to trilateration on small control schemes since more checking of fieldwork is possible.

6.7 Combined Networks

The most effective use that can be made of trilateration is to combine it with a triangulation scheme to form a *combined network*. This greatly improves the strength of both types of control surveys.

The ideal combined network is one which has all its angles and distances measured but a triangulation or trilateration network can be enhanced by adding just a few distances or angles to those already measured.

Without doubt, properly planned and observed combined networks can be the best possible type of horizontal control but the adjustment of such schemes must be carried out rigorously to allow for the many geometric conditions possible. Since the rigorous procedures required are usually applied to first-order surveys, details are not given in this book but further reading on the topic is suggested at the end of this chapter.

6.8 Further Reading

E. M. Mikhail and G. Gracie, *Analysis and Adjustment of Survey Measurements* (Van Nostrand Reinhold, New York, 1981).

P. Richardus, *Project Surveying, 2nd Edition* (A. A. Balkema, Rotterdam and Boston, 1984).

7

Intersection and Resection

Two techniques commonly employed in extending horizontal control surveys and in setting out are intersection and resection.

Intersection is a method of locating a point without actually occupying it. In figure 7.1, points A and B are stations in a control network already surveyed and, in order to coordinate unknown point C which lies at the intersection of the lines from A and B, angles α and β are observed.

Resection is a method of locating a point by taking angle observations from it to at least three known stations in a network. In figure 7.2, point W can be fixed by observing angles α and β subtended at resection point W by control stations D, C and L.

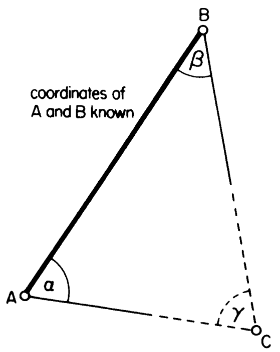


Figure 7.1 *Intersection*

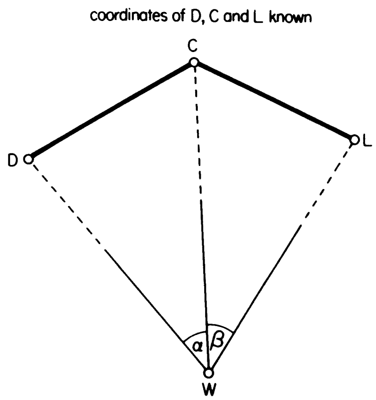


Figure 7.2 *Resection*

7.1 Intersection

Intersections can be computed in three ways.

7.1.1 Intersection by Solution of Triangle

In triangle ABC of figure 7.1, the length and bearing of baseline AB are given by

$$D_{AB} = [(E_B - E_A)^2 + (N_B - N_A)^2]^{\frac{1}{2}}$$

$$\theta_{AB} = \tan^{-1}\left[\frac{E_B - E_A}{N_B - N_A}\right]$$

The Sine Rule gives

$$D_{BC} = \frac{\sin\alpha}{\sin\gamma} D_{AB} \qquad D_{AC} = \frac{\sin\beta}{\sin\gamma} D_{AB}$$

where

$$\gamma = 180° - (\alpha + \beta)$$

The bearings in the triangle are given by

$$\theta_{AC} = \theta_{AB} + \alpha \qquad \theta_{BC} = \theta_{BA} - \beta$$

These bearings and distances are used to compute the coordinates of A along line AC as

$$E_C = E_A + D_{AC} \sin\theta_{AC} \qquad N_C = N_A + D_{AC} \cos\theta_{AC}$$

The computations are checked along line BC using

$$E_C = E_B + D_{BC} \sin\theta_{BC} \qquad N_C = N_B + D_{BC} \cos\theta_{BC}$$

7.1.2 Intersection using Angles

Adopting the clockwise lettering sequence used in figure 7.1, the coordinates of C can be obtained directly from

$$E_C = \frac{(N_B - N_A) + E_A \cot\beta + E_B \cot\alpha}{\cot\alpha + \cot\beta}$$

$$N_C = \frac{(E_A - E_B) + N_A \cot\beta + N_B \cot\alpha}{\cot\alpha + \cot\beta}$$

A disadvantage of this method compared to solving the triangle is that no check is possible on the calculations.

7.1.3 Intersection using Bearings

If the bearings of lines AC (θ_{AC}) and BC (θ_{BC}) in figure 7.1 are known, the co-ordinates of C are given by

$$E_C = \frac{E_A \cot\theta_{AC} - E_B \cot\theta_{BC} - N_A + N_B}{\cot\theta_{AC} - \cot\theta_{BC}}$$

$$N_C = \frac{N_A \tan\theta_{AC} - N_B \tan\theta_{BC} - E_A + E_B}{\tan\theta_{AC} - \tan\theta_{BC}}$$

As with intersection using angles, no check on the computations is possible.

7.1.4 Intersection from Two Baselines

When solving intersections using the formulae given in the previous sections, two quantities are observed in each case (α and β or θ_{AC} and θ_{BC}) to define two unknowns E_C and N_C. Consequently, no redundancy exists in the fixation and it is not possible to check the observations. It is, of course, possible to check the computations when solving the triangle but this method does not enable the angles α and β to be checked.

One method of detecting gross errors in the observations is to observe additional angles from a second baseline. This is shown in figure 7.3 where the angles δ and ϕ have been added to those already observed in figure 7.1.

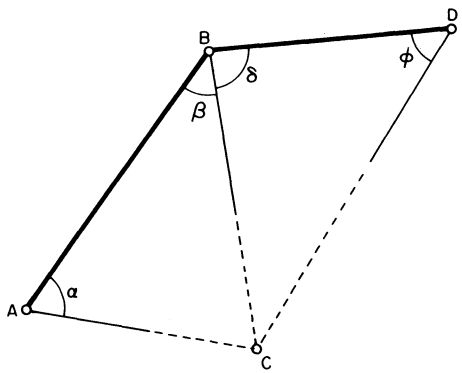


Figure 7.3 *Intersection from two baselines*

The coordinates of point C in figure 7.3 are found by solving the intersections formed by the triangles ABC and BDC, the two sets of coordinates obtained being compared. If the differences between the two intersections are small, it is assumed that the observations contain no gross errors and the average coordinates from the two sets are taken as the final values.

7.1.5 Worked Example: Intersection

Question

The coordinates of stations S, A and L are E_S = 1309.12 mE, N_S = 1170.50 mN, E_A = 1525.43 mE, N_A = 958.87 mN, E_L = 1231.08 mE and N_L = 565.81 mN. Calculate the coordinates of point B which has been located by intersection from stations S, A and L by observing the following angles: $B\hat{S}A = 85°38'49''$, $S\hat{A}B = 55°50'53''$, $B\hat{A}L = 41°41'48''$ and $A\hat{L}B = 68°09'32''$.

Solution

Referring to figure 7.4 and clockwise triangle SAB, the coordinates of B are given by the angles method (see section7.1.2) as

$$E_B = \frac{(N_A - N_S) + E_S \cot S\hat{A}B + E_A \cot B\hat{S}A}{\cot B\hat{S}A + \cot S\hat{A}B}$$

$$= \frac{(958.87 - 1170.50) + 1309.12 \cot 55°50'53'' + 1525.43 \cot 85°38'49''}{\cot 85°38'49'' + \cot 55°50'53''}$$

$$= 1050.45 \text{ m}$$

$$N_B = \frac{(E_S - E_A) + N_S \cot S\hat{A}B + N_A \cot B\hat{S}A}{\cot B\hat{S}A + \cot S\hat{A}B}$$

$$= \frac{(1309.12 - 1525.43\ ) + 1170.50 \cot 55°50'53'' + 958.87 \cot 85°38'49''}{\cot 85°38'49'' + \cot 55°50'53''}$$

$$= 862.45 \text{ m}$$

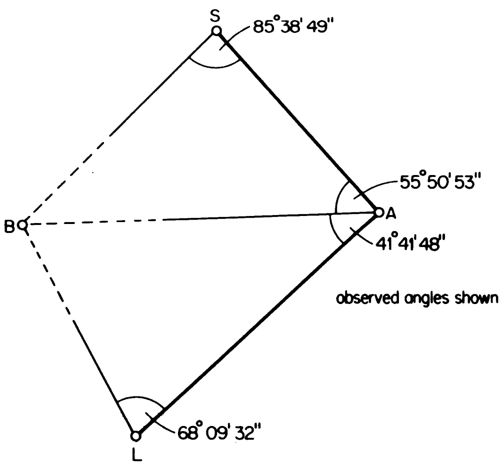


Figure 7.4

To check the fieldwork and computations, the intersection of B in triangle BAL must also be computed as follows

$$E_B = \frac{(N_L - N_A) + E_A \cot A\hat{L}B + E_L \cot B\hat{A}L}{\cot B\hat{A}L + \cot A\hat{L}B} = 1050.50 \text{ m}$$

$$N_B = \frac{(E_A - E_L) + N_A \cot A\hat{L}B + N_L \cot B\hat{A}L}{\cot B\hat{A}L + \cot A\hat{L}B} = 862.46 \text{ m}$$

Since the two results for E_B and N_B agree within 0.05 m, no gross error has occurred in the observations and the final coordinates are the mean values from the two sets, that is

$$\boldsymbol{E_B = 1050.48 \text{ m} \quad N_B = 862.46 \text{ m}}$$

7.2 Resection

A resection is carried out in the field by observing the angles subtended at the unknown point by at least three known stations and in the *three-point resections* shown in figure 7.5, P is located in each case by measurement of angles α and β.

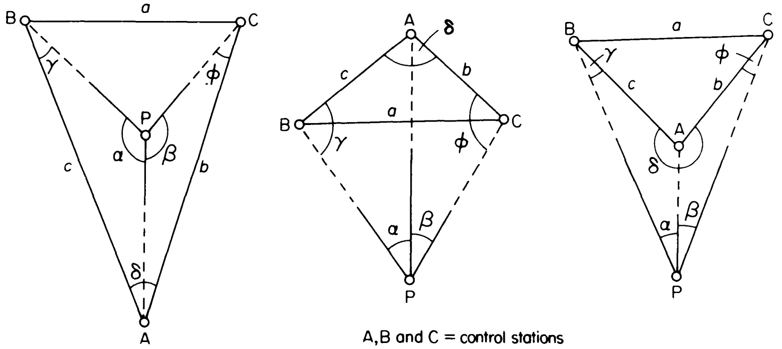


Figure 7.5 *Possible configurations for a three-point resection*

A three-point resection can be solved in a number of ways. However, no matter which method is used, it must be noted that if points A, B, C and P in figure 7.5 all lie on the circumference of the same circle then the resection is indeterminate. This condition is present when $\delta + \alpha + \beta = 180°$.

One method of solving a three-point resection is as follows. In each quadrilateral ABPC of figure 7.5

$$\alpha + \beta + \gamma + \phi + \delta = 360°$$

or

$$\gamma = [360° - (\alpha + \beta + \delta)] - \phi = R - \phi$$

where R can be deduced.

In triangles ABP and APC

$$D_{AP} = \frac{\sin \gamma}{\sin \alpha} c = \frac{\sin \phi}{\sin \beta} b$$

From which

$$K = \frac{\sin \gamma}{\sin \phi} = \frac{b \sin \alpha}{c \sin \beta}$$

which can be evaluated.

Substituting $\gamma = R - \phi$ gives a further expression for K

$$K = \frac{\sin(R - \phi)}{\sin \phi} = \frac{\sin R \cos \phi - \cos R \sin \phi}{\sin \phi} = \sin R \cot \phi - \cos R$$

Therefore

$$\cot \phi = \frac{K + \cos R}{\sin R}$$

This expression enables ϕ and all the angles in ABPC to be found, which in turn enables the coordinates of P to be calculated by solving triangles ABP and APC (see section 7.1.1). Both triangles are solved in order to provide a check on the calculations since the coordinates found for P in each triangle should be identical.

Although the calculations can be checked in three-point resections, the fieldwork cannot be checked since a unique position is obtained for the resected point by observing only three directions and deriving two angles.

To introduce some redundant data into a resection requires further directions to be observed and for most engineering surveys it is normal to observe four directions (giving three angles), the extra angle being used to check the fieldwork. The method of applying this check in a *four-point resection* is as follows.

(1) Choose three directions out of the four observed and compute a three-point resection. The observed angles (or combinations of these) that give the two resection angles nearest to 90° should be used in this calculation.

(2) Using the coordinates of P found in (1), *calculate* the value of one of the angles not used in the three-point resection.

(3) Compare the angle calculated in (2) with its observed value. If the two are in close agreement, it is assumed that no gross error has occurred in the observations and the resection coordinates obtained in (1) are accepted for further work.

7.2.1 Worked Example: Four-point Resection

Question

Using the resection data given in table 7.1, calculate the coordinates of point A.

Solution

The layout of the four control stations and point A are shown in figure 7.6. Using

TABLE 7.1

Station	m_E	m_N	Angle	Observed value
M	845.11	1952.50	$M\hat{A}P$	30°40'11"
P	1312.59	2205.90	$P\hat{A}T$	26°47'52"
T	1621.29	1835.07	$T\hat{A}W$	56°47'08"
W	1729.04	1158.60		

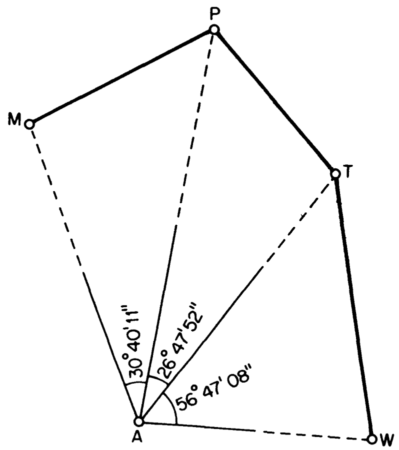


Figure 7.6

this, observed directions AM, AT and AW are selected for the coordinate calculation since the geometry of these directions gives resection angles at A closest to the optimum of 90°. The remaining direction AP will be used to check the fieldwork.

For the resection formed at A by M, T and W (see figure 7.7), angle γ is given by

$$\gamma = \cot^{-1}\left[\frac{K + \cos R}{\sin R}\right]$$

where

$$K = \frac{D_{\text{MT}} \sin \beta}{D_{\text{WT}} \sin \alpha} = \frac{785.01 \sin 56°47'08''}{685.00 \sin 57°28'03''} = 1.137\,219$$

and

$$R = 360° - (\alpha + \beta + \delta) = 138°05'37''$$

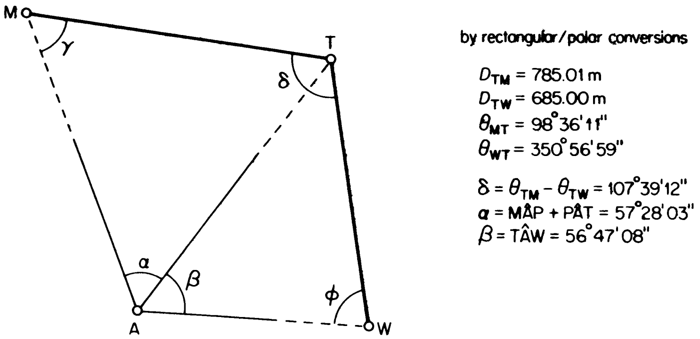


Figure 7.7

which gives

$$\gamma = \cot^{-1}\left[\frac{1.137\,219 + \cos 138°05'37''}{\sin 138°05'37''}\right] = \cot^{-1}(0.588\,372)$$

$$= 59°31'43''$$

The coordinates of A are found by solving triangle AMT as follows

$$\mathrm{M\hat{T}A} = 180° - (\gamma + \alpha) = 180° - (59°31'43'' + 57°28'03'') = 63°00'14''$$

$$D_{\mathrm{MA}} = \frac{\sin \mathrm{M\hat{T}A}}{\sin \alpha} D_{\mathrm{MT}} = \frac{\sin 63°00'14''}{\sin 57°28'03''}(785.01) = 829.66 \text{ m}$$

$$\theta_{\mathrm{MA}} = \theta_{\mathrm{MT}} + \gamma = 98°36'11'' + 59°31'43'' = 158°07'54''$$

$$E_{\mathrm{A}} = E_{\mathrm{M}} + D_{\mathrm{MA}} \sin \theta_{\mathrm{MA}} = 1154.14 \text{ m}$$

$$N_{\mathrm{A}} = N_{\mathrm{M}} + D_{\mathrm{MA}} \cos \theta_{\mathrm{MA}} = 1182.54 \text{ m}$$

A check on the *computations only* is provided by solving triangle ATW in which

$$\phi = R - \gamma = 78°33'54'' \quad \mathrm{A\hat{T}W} = 180° - (\phi + \beta) = 44°38'58''$$

$$D_{\mathrm{WA}} = \frac{\sin \mathrm{A\hat{T}W}}{\sin \beta} D_{\mathrm{WT}} = 575.40 \text{ m} \quad \theta_{\mathrm{WA}} = \theta_{\mathrm{WT}} - \phi = 272°23'05''$$

$$E_{\mathrm{A}} = E_{\mathrm{W}} + D_{\mathrm{WA}} \sin \theta_{\mathrm{WA}} = 1154.14 \text{ m}$$

$$N_{\mathrm{A}} = N_{\mathrm{W}} + D_{\mathrm{WA}} \cos \theta_{\mathrm{WA}} = 1182.54 \text{ m}$$

The *observations* for the resection are checked, in this example, by comparing the observed and calculated values for angle $\mathrm{M\hat{A}P}$.

The coordinates found above for A give, by calculation

$$\theta_{\mathrm{AP}} = 08°48'05'' \qquad \theta_{\mathrm{AM}} = 338°07'53''$$

From which

$$\mathrm{M\hat{A}P} = \theta_{\mathrm{AP}} - \theta_{\mathrm{AM}} = 30°40'12''$$

Since $\mathrm{M\hat{A}P}$ (observed) = 30°40′11″, a difference of only 1″ exists between the two values and therefore no gross errors have occurred. Hence, the coordinates of A are

$$\boldsymbol{E_{\mathrm{A}} = 1154.14\ \mathrm{m}, \quad N_{\mathrm{A}} = 1182.54\ \mathrm{m}}$$

8

Detail Surveying and Plotting

In this chapter, methods are described for the preparation of plans at the common engineering scales of between 1:50 and 1:1000. The drawings so produced are used as records of existing areas or as a basis for the design and setting out of engineering works.

8.1 Control Networks

All detail surveys are based on a framework of control stations which must be established on the ground before detail can be located. These stations can be fixed by traversing (see chapter 5), by triangulation or trilateration (see chapter 6) or occasionally by using intersection and resection (see chapter 7).

Upon completion of the control survey, the stations are plotted using their co-ordinates (see section 5.9). All plotting of control stations should be done to an accuracy of 0.2 mm and, when plotted, the lengths between the stations should be scaled from the plan and checked against their ground values. Following this, all the survey lines from which detail is to be located are constructed on the survey plan.

The methods by which detail can be added to the basic control network are discussed in section 8.5.

8.2 Drawing Paper and Film

The type of paper or film used for survey drawings is usually some form of plastic film or cartridge paper.

Plastic film is most commonly used as this has an extremely good dimensional stability, this being the most important factor when selecting a material on which to plot a survey. Pencil or ink can be used on plastic quite easily and both of these can be erased without leaving unsightly marks. In addition, plastic is waterproof and is a transparent material which enables direct contact copies of the finished plan to be taken.

Cartridge paper is frequently used for pencil drawings and temporary ink work. Since many different grades of this paper are made, care must be taken to select a grade that has a good dimensional stability and a texture that is capable of taking repeated erasures without becoming fibrous or tearing. The ability of the paper to take ink without absorbing it to such an extent that line thicknesses are affected is also important.

Other types of drawing medium are NOT recommended for survey drawings as these do not, in general, have a suitable dimensional stability.

When plotting the survey, it is usual to prepare a master drawing from field notes and a tracing is made of this on to plastic film for the purposes of reproduction (see section 8.6).

8.3 Detail

The term 'detail' is a general one that implies features both above and below ground level and at ground level.

Buildings, roads, walls and other constructed features are called *hard detail*, whereas natural features including rivers and vegetation are known as *soft detail*. Other definitions include *overhead detail* (for example, power and telephone lines) and *underground detail* (for example, water pipes and sewer runs).

Many types of symbols are used for representing detail and a standard format has yet to be universally agreed. Those symbols and abbreviations shown in figure 8.1 are fairly comprehensive and their use is recommended. However, it will be noted from figure 8.1 that more abbreviations rather than symbols are given for detail. This is due to the fact that, at the large scales used for engineering surveys, the actual shapes of many features can be plotted to scale and, therefore, do not need to be represented by a symbol.

When detail surveying, the amount and type of detail that is located (or *picked up*) for any particular survey varies enormously with the scale (see section 1.5) and the intended use of the plan.

8.4 Specifications for Detail Surveys

The accuracy required in detail surveying should always be considered before the survey is started. This is governed by two factors: the scale of the finished plan, and the accuracy with which it can be plotted.

For *plan positions*, it is usual to assume a plotting accuracy for detail of 0.5 mm and for various scales this will correspond to certain distances on the ground. These lengths are an indication of the accuracy required at the scales in question. However, even if a plan is to be plotted at 1:500, part of it may at a later date be enlarged to 1:50 so it is always better to take measurements in the field to a greater accuracy than that required for the initial plan. A good compromise is obtained by recording all distances, where possible, to the nearest 0.01 m.

For *contours*, the vertical interval depends on the scale of the plan and suitable vertical intervals are listed in table 8.1 for general purpose engineering surveys. Usually, the accuracy of a contour is guaranteed to one-half of the vertical interval.

All *spot levels* taken on soft surfaces (for example, grass) should be recorded to the nearest 0.05 m and those taken on hard surfaces to 0.01 m. In areas where there is insufficient detail at which spot levels can be taken, a grid of spot levels should be

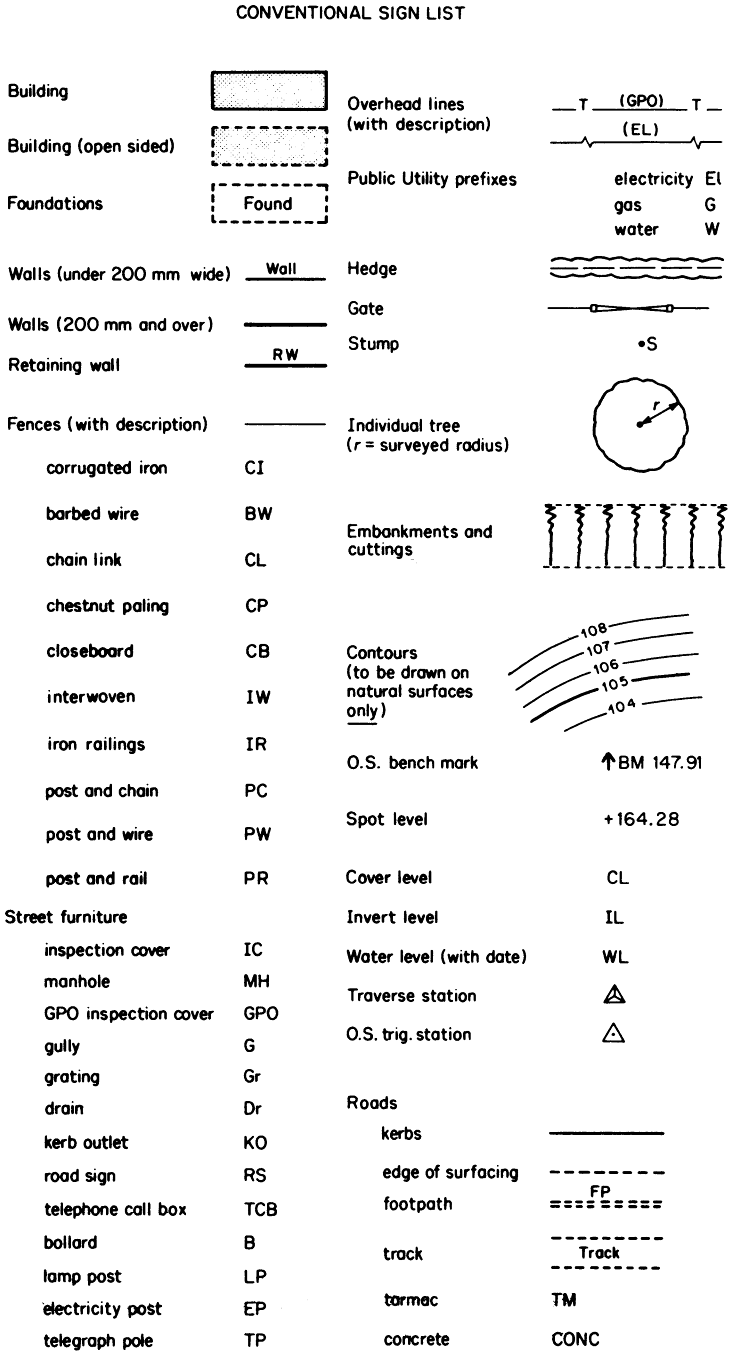

CONVENTIONAL SIGN LIST
Building
Building (open sided)
Foundations
Found
Walls (under 200 mm wide)
Wall
Walls (200 mm and over)
Retaining wall
RW
Fences (with description)
corrugated iron CI
barbed wire BW
chain link CL
chestnut paling CP
closeboard CB
interwoven IW
iron railings IR
post and chain PC
post and wire PW
post and rail PR
Street furniture
inspection cover IC
manhole MH
GPO inspection cover GPO
gully G
grating Gr
drain Dr
kerb outlet KO
road sign RS
telephone call box TCB
bollard B
lamp post LP
electricity post EP
telegraph pole TP
Overhead lines
(with description)
T (GPO) T
(EL)
Public Utility prefixes
electricity El
gas G
water W
Hedge
Gate
Stump
•S
Individual tree
(r = surveyed radius)
r
Embankments and
cuttings
Contours
(to be drawn on
natural surfaces
only)
108
107
106
105
104
O.S. bench mark
BM 147.91
Spot level
+164.28
Cover level CL
Invert level IL
Water level (with date) WL
Traverse station
O.S. trig. station
Roads
kerbs
edge of surfacing
footpath
FP
track
Track
tarmac TM
concrete CONC

Figure 8.1

surveyed in the field and plotted on the plan, the size of the grid depending on the scale, as shown in table 8.1.

TABLE 8.1

SCALE	1:50	1:100	1:200	1:500	1:1000
Contour vertical interval	0.05m	0.1m	0.25m	0.5m	1m
Spot level grid size	2m	5m	10m	20m	40m

8.5 Locating Detail

Detail can be located from the control point framework by one of two methods, either by using offsets and ties or by using radiation methods.

Offsets and ties can only locate detail in the plan position. If height information is required, spot levels must be obtained at a later date by levelling at points of detail that have already been located.

Radiation methods usually enable both plan and height information to be obtained.

8.5.1 Offsets and Ties

Figure 8.2*a* shows the method of *offsets* in which lengths x and y are recorded in the field in order to locate a point of detail P. The offsets are taken at right angles to the lines running between control points. A variation on the offset method is shown in figure 8.2*b* where *ties* from two (or more) points are used to locate P.

In practice, a tape or a chain is usually laid along the control line to measure the x distances shown in figure 8.2, and the offsets and ties are usually measured using a synthetic tape.

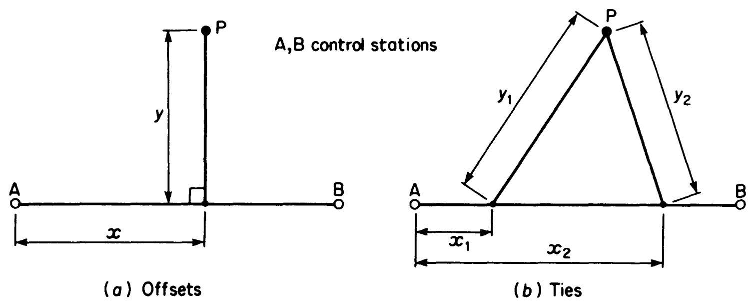


Figure 8.2

Since an offset is a line measured at right angles to a survey line to a particular feature, it is necessary to establish a right angle. This is achieved by holding the zero of the tape on the point of detail and swinging the tape over the chain. The minimum reading obtained occurs at the perpendicular and the position of the tape at this point indicates the distance along the tape or chain for the offset measurement.

For best results, an offset should never be longer than 10 m owing to possible errors in the tape length and the uncertainty of establishing an accurate right angle. Where detail has to be located beyond 10 m from the survey line, ties should be used. Usually two ties are taken but, occasionally, if the distances involved are long, three should be measured. The maximum length of a tie should not be greater than one tape length.

As well as measuring offsets and ties, the dimensions of certain features may be recorded, for example, the widths of paths or roads with parallel sides, the spread and girth of trees, the lengths around buildings and the radius of circular features. Sometimes it is acceptable to survey rectangular buildings by fixing the two corners of the longest side by ties or offsets and by recording the remaining dimensions and plotting accordingly.

Detail surveying using offsets and ties can only locate detail in the plan position. Since all survey plans must include height information this has to be added at some stage in the survey by taking spot heights at points of detail that have already been located (see section 2.12.2).

8.5.2 Booking Offsets and Ties

When booking offsets and ties, the fieldbook should always be neat and consistent and, as a general standard, all fieldwork must be capable of being plotted by someone who was not involved in the field survey. An emphasis should, therefore, be placed on clear, legible writing and large diagrams.

Long survey lines should be continued over as many pages as necessary and each new line should be started on a new page. It is important that all necessary information is recorded and explanatory notes should be given where appropriate since nothing should be left to memory. This applies especially to unusual features which do not have a conventional symbol.

Figure 8.3 shows a typical booking. This may be drawn either in a field book or on loose-leaf sheets. Conventionally, a double line is drawn through the centre of the page and this represents the survey line. Entries start at the bottom of the page and, standing at station A, facing station B, detail that is on the right-hand side of the line is booked on the right-hand side of the page and vice versa. Only continuous lengths from station A are recorded in between the double lines, the total length between A and B being written sideways with a line drawn on each side. No attempt is made to draw the sketch to scale and complicated features are exaggerated. The running dimensions around the buildings are also shown in figure 8.3 and are distinguished from other measurements by being inserted in brackets.

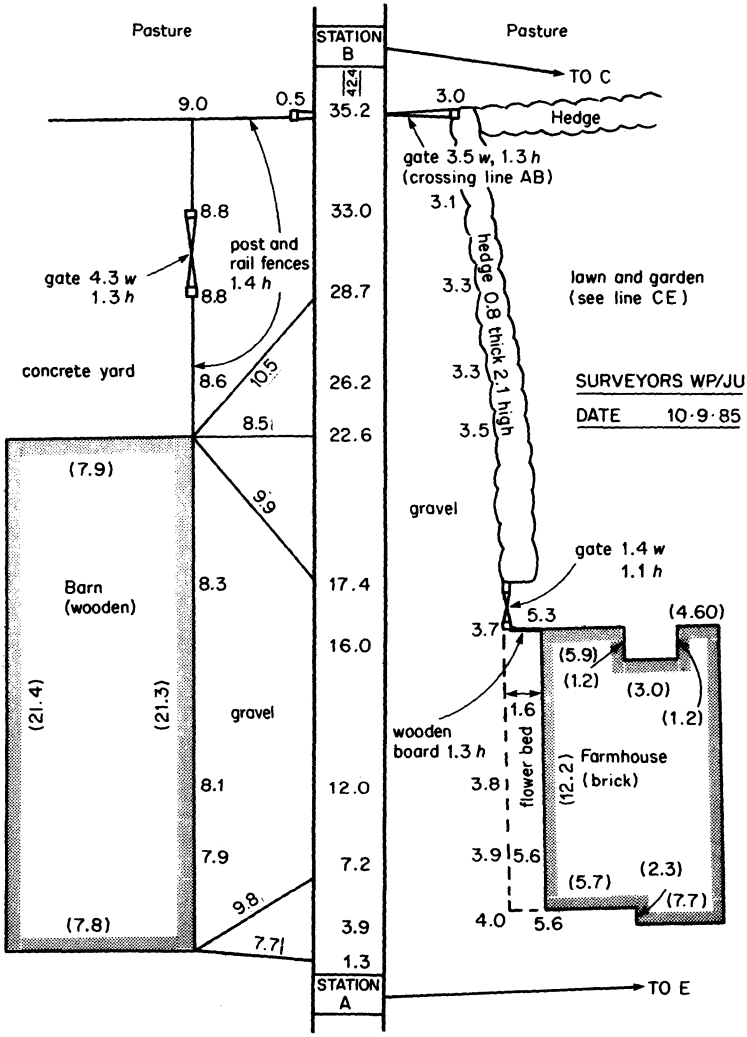


Figure 8.3 *Example booking for offsets and ties*

8.5.3 Plotting Offsets and Ties

When the basic control network has been plotted (see section 8.1), the detail for each control line can be added by marking off the distances along each line corresponding to the points on the tape or chain at which offsets and ties were taken. From these points, the relevant offset and tie lengths can be scaled to fix the points of detail with the aid of a set-square and a pair of compasses.

If spot levels have been taken by levelling at some of the points of detail, these can be added and contours interpolated from them (see section 2.12.3).

All construction marks are erased after the detail and contours have been added to the plan.

8.5.4 Radiation by Stadia Tacheometry

A full description of the theory of stadia tacheometry is given in section 4.9 and its application to detail surveying is considered here.

Stadia tacheometry is used to locate points of detail by the radiation method, the basis of which is shown in figure 8.4, where r and θ are measured in the field to locate P.

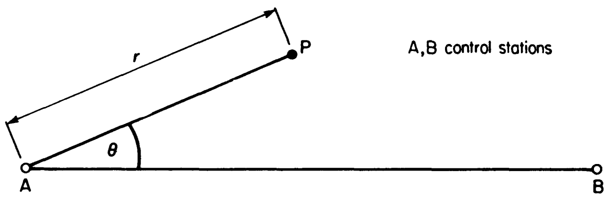


Figure 8.4

The component r is measured by tacheometry and θ by reading the horizontal circle of the theodolite or level used. An important difference between stadia tacheometry and the method of offsets and ties is that the height of each point is obtained in addition to its plan position. Tacheometry can, therefore, be used effectively in contouring, particularly in open areas where there are no points of clearly defined detail. It can, also, be used for a complete detail survey but, assuming a plotting accuracy of 0.5 mm, stadia tacheometry can be used only at scales less than 1:200 or for soft detail.

8.5.5 Fieldwork for Stadia Tacheometry

A network of control stations is again used as a base for the survey and, during the reconnaissance, it must be remembered that the length of a single tacheometric observation is limited to 100 m. This implies that, for each station, the maximum coverage on unrestricted sites should be a radius of 100 m.

Several methods of observation are possible and the following description is given as a general purpose approach. It is assumed that the staff is held vertically, that a theodolite is being used and that the reduced levels of the control stations are known.

(1) Set the instrument up over a station mark and centre and level it in the usual way. For a detail survey it is standard practice to measure horizontal and vertical angles on one face only and hence the theodolite should be in good adjustment.

(2) Measure and record the height of the trunnion axis above the station mark.

(3) Select a suitable station as RO, sight this point and record the horizontal circle reading. It may be necessary to erect a target at the RO if it is not well defined. All the detail in the radiation pattern will now be fixed in relation to this chosen direction. Some engineers prefer to set the horizontal circle to zero along the direction to the RO, although this is not essential.

(4) With the staff in position at a point of detail, rotate the telescope until the staff is aligned along the vertical hair in the field of view. Turn the vertical slow motion screw until the lowest reading stadia hair is set to a convenient mark on the staff such as 1 m, 2 m or the nearest 0.1 m. Read and record the three hairs.

A check can be applied to the stadia readings since the centre or middle reading should be the mean of the other two within ± 2 mm.

(5) Signal the person holding the staff to move to the next staff point.

(6) To save time, while the staff is moving, the vertical circle is read, first levelling the altitude bubble. This is followed by a reading of the horizontal circle. These readings need only be taken with an accuracy of ± 1′.

(7) The procedure is repeated until all the observations have been completed. As far as is practicable, each of the staff points should be selected in a clockwise order to keep the amount of walking done by the person holding the staff to a minimum.

(8) The final sighting should be to the RO to check that the setting of the horizontal circle has not been altered during observations. If it has, all the readings are unreliable and should be remeasured. Hence, it is advisable that, during a long series of tacheometric readings, a sighting on to the RO should be taken after, say, every 10 points of detail.

8.5.6 Booking Stadia Tacheometry Observations

A systematic approach to booking is essential.

Various systems of booking can be used and a sample field sheet, suitable for most types of work, is shown in table 8.2. All the information in columns (1) to (4) is recorded in the field, the remainder being computed in the office at a later stage. The vertical circle readings entered in column (2) must be those as read directly on the instrument, reduction being carried out in column (5) where necessary. Particular note should be paid to the accuracy of the computation. The horizontal distance in column (7) is recorded to 0.1 m and the reduced levels in column (10) to 0.01 m.

The booking form should also incorporate a sketch identifying all the staff points (see figure 8.5). In addition, this diagram should indicate miscellaneous information such as types of vegetation, widths of tracks and roads, heights and types of fences and so on.

8.5.7 Plotting Stadia Tacheometry

The network of control stations is first plotted as described in section 8.1. To plot the detail and spot levels, a protractor and a scale rule are required. With reference

TABLE 8.2
Example Tacheometric Booking

SURVEY FLAG HOUSING DEVELOPMENT	REDUCED LEVEL STATION D	47.15	OBSERVER JU
INSTRUMENT STATION D	HEIGHT OF INSTRUMENT	1.73	BOOKER WFP
NOTES ______	RL OF INSTRUMENT AXIS	48.88	DATE 1 AUG 85

STAFF POINT	VERTICAL CIRCLE	HORIZONTAL CIRCLE	STAFF READINGS CENTRE m	STAFF READINGS UPPER	STAFF READINGS LOWER	Θ	s	D	V	± V-m	REDUCED LEVEL	REMARKS
E (RO)		00-00										Station E
1	355-40	08-05	0.932	1.064	0.800	-04-20	0.264	26.2	-1.99	-2.92	45.96	Edge of gravel track
2	357-20	51-20	0.359	0.618	0.100	-02-40	0.518	51.7	-2.41	-2.77	46.11	Post and barbed wire fence
3	358-07	124-44	0.709	0.818	0.600	-01-53	0.218	21.8	-0.72	-1.43	47.45	Fence meets track
4	02-14	143-15	1.675	1.750	1.600	+02-14	0.150	15.0	+0.58	-1.10	47.78	Hedge next to track
5	00-31	297-52	1.974	2.048	1.900	+00-31	0.148	14.8	+0.13	-1.84	47.04	Grass meets track
6	00-26	286-10	1.402	1.604	1.200	+00-26	0.404	40.4	+0.31	-1.09	47.79	Hedge next to track
7	00-27	293-34	2.098	2.295	1.900	+00-27	0.395	39.5	+0.31	-1.79	47.09	Opposite 6, edge of grass
8	358-14	305-38	1.420	1.639	1.200	-01-46	0.439	43.9	-1.35	-2.77	46.11	S Corner of store
9	358-12	314-26	1.786	1.971	1.600	-01-48	0.371	37.1	-1.16	-2.95	45.93	SE Corner of store
10	358-10	330-53	1.678	1.956	1.400	-01-50	0.556	55.5	-1.78	-3.46	45.42	NE Corner of store
E (RO)		00-00										Station E
(1)	(2)	(3)	(4)			(5)	(6)	(7)	(8)	(9)	(10)	

to figure 8.5 and table 8.2, the procedure is as follows to plot the detail located from station D.

(1) Attach the protractor to the survey plan using masking tape such that its centre is at station D and it is orientated to give the same reading to the RO, station E, as was obtained in the field on the horizontal circle of the theodolite; in this case, 00°00′.

(2) Plot the positions of the horizontal circle readings taken to the detail points around the edge of the protractor. Identify each by its staff position, that is, D1, D2, D3 and so on.

(3) Remove the protractor and very faintly join point D to the plotted horizontal circle positions. Extend these lines.

(4) Using the calculated horizontal distance values from table 8.2, measure from point D along each direction, allowing for the scale of the plan, to fix the plan positions of the points of detail.

(5) Write the appropriate reduced level value taken from table 8.2 next to each point of detail.

(6) Using the field sketches, the detail is now filled in between these points and the contours drawn by interpolation (see section 2.12.3). All construction marks are erased after the detail and contours have been added.

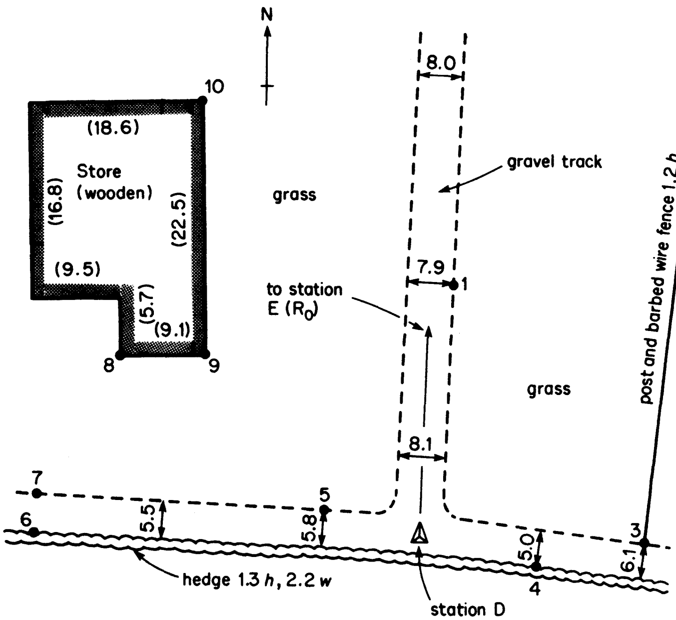


Figure 8.5 *Example sketch for a tacheomatic survey*

8.5.8 Radiation using a Theodolite and Tape

This technique is very similar to stadia tacheometry, except that no staff is read and no vertical angles recorded. Instead, the distance to each point is measured directly using a steel or synthetic tape with the tape being held horizontally in each case. The disadvantages of this are that no levels are obtained and the range is limited to one tape length unless ranging is introduced. The best application of this method is in dense detail where stadia tacheometry would become tedious owing to the amount of office work involved, and especially over distances less than 30 m.

8.5.9 Radiation using Electromagnetic Distance-measuring Equipment

This technique uses combined theodolite and EDM systems or electronic tacheometers (see section 4.13.2). Using the latest equipment, slope distances can be obtained directly with accuracies of ± 10 mm at ranges of up to 500 m. Many instruments have a simple calculator built into the distance measuring unit that will compute the horizontal distance and the height difference to a point of detail when the vertical angle, measured in the normal way, is fed into the calculator. Heights are, therefore, obtained simultaneously with horizontal distance and have a similar accuracy.

The latest developments in electromagnetic distance measuring equipment are in computerised tacheometry whereby the field observations are fed directly into a recording unit (see section 4.13.2). These units are then interfaced with a computer for automatic plotting of the plan.

The main disadvantage of this and electromagnetic techniques in general is the cost of such systems.

8.6 The Completed Survey Plan

Figure 8.6 shows parts of a survey plan based on a traverse network. Included in the title block are details of the location of the survey, the key showing the symbols and abbreviations used, the scale, the date of the survey and the names of the surveyors involved in the production of the plan. On the finished plan, a north sign is added (not shown in figure 8.6) and the coordinate grid information is transferred to the border.

Contour lines have been shown on *natural surfaces only* and have NOT been drawn through embankments and cuttings which have their own symbol as indicated in the key.

The traverse lines have been removed but the positions of the traverse stations are shown since they may be required for future use.

In practice, the original survey plan is usually prepared by hand by the surveyor who undertook the fieldwork. This plan would not normally have a title block or border. However, when completed, this master drawing prepared by the surveyor is passed to a drawing office where all the detail is traced in ink on to plastic film and the title block and border are added.

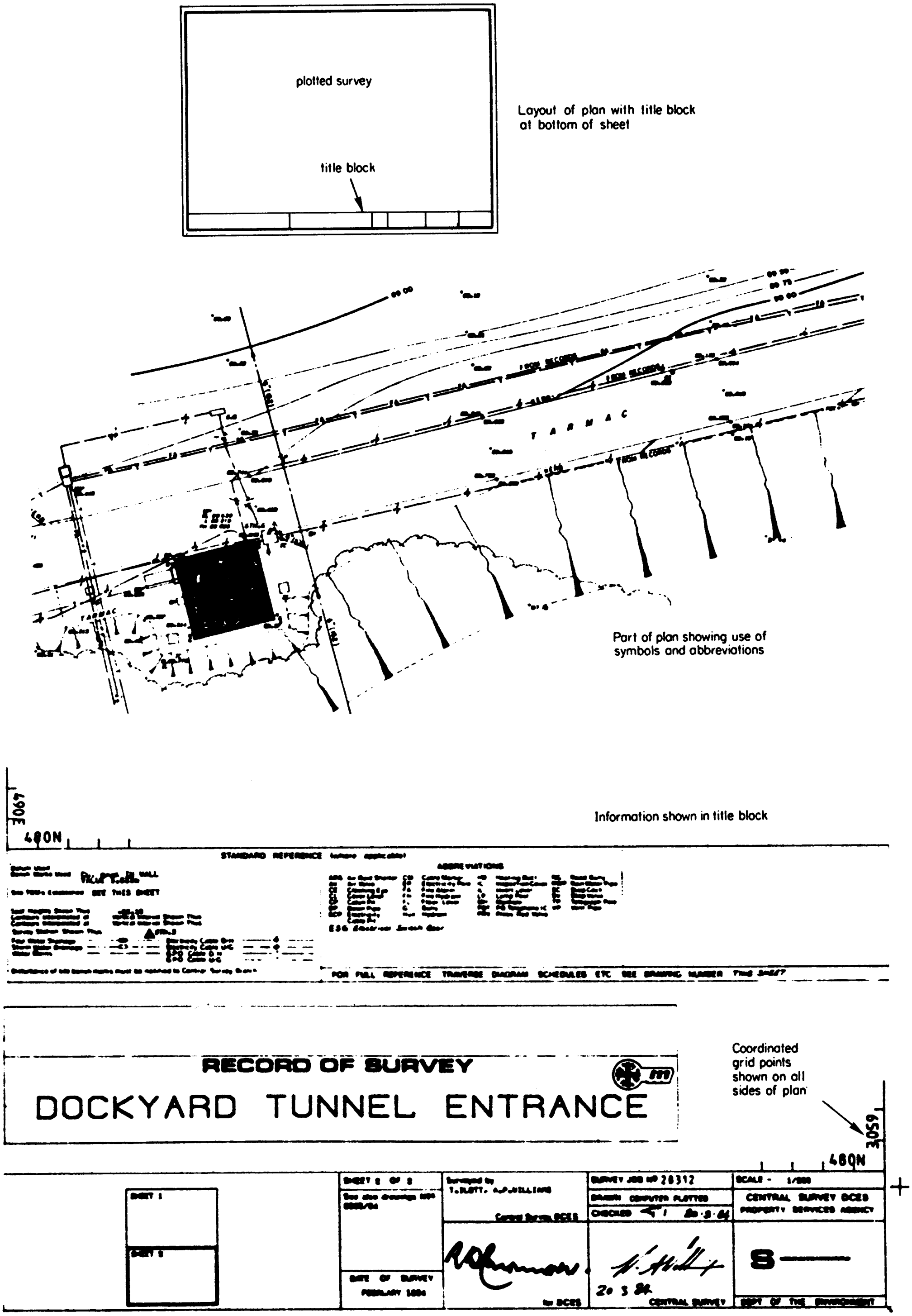


Figure 8.6 *Components of survey plan (based on a plan produced by the Central Survey Branch, PSA, DoE)*

Photocopies of the traced drawing are usually taken for use on site. Consequently, extreme care must be taken with the tracing since any errors in it will be transferred to all the copies taken.

8.7 Computer Aided Plotting

In the previous sections of this chapter, methods have been given for plotting large-scale detail surveys by hand. The process of hand drawing survey plans is extremely time consuming and very often the individual surveyor or engineer does not have the ability to achieve a high standard of presentation in inked work. This problem is usually overcome by passing the initial drawing to a drawing office for tracing and annotating, but this adds to the expense and increases the time taken for the survey.

In recent years, a number of advances have been made in computer technology such that many powerful desk top computers are now available with sophisticated peripherals. Using these, many survey organisations and large civil engineering contractors have developed their own in-house systems for the plotting of survey plans by computer. In addition, manufactured systems are also available for automated plotting of survey plans. The elements of an automated survey plotting system are shown in block form in figure 8.7. The various stages involved are described in the following sections.

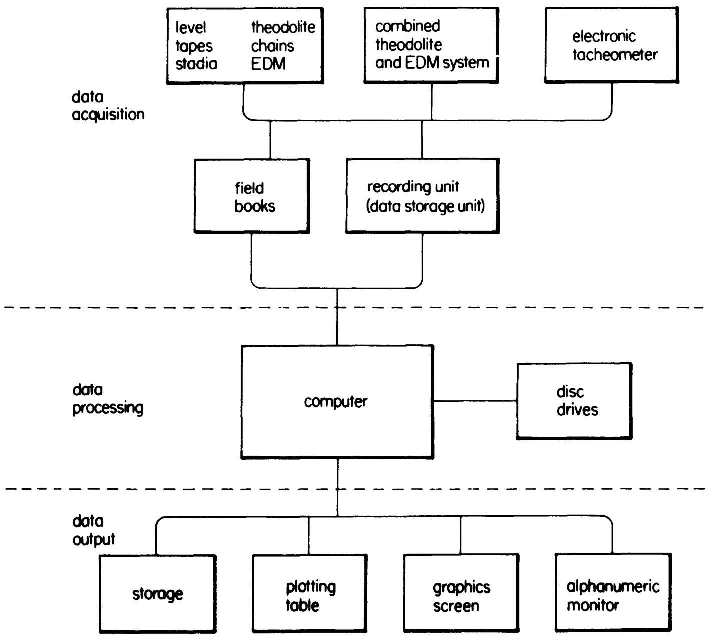


Figure 8.7 *Computer aided plotting system*

8.7.1 Data Acquisition

Field data can be fed into the computer automatically by using a data storage unit (see figure 4.28) or it can be entered manually from traditional field sheets. For each point surveyed in the field, a code is used to define the type of detail being observed, the annotation (if any) to be shown on the plan at the point of detail and any other information such as running dimensions or names of features. In effect, this code replaces the field sketch but it is recommended that sketches be drawn as aids to the subsequent plotting.

The electronic tacheometer (see section 4.13.2) is capable of recording angles and distances with their appropriate codes directly into a data storage unit. The contents of the data storage unit can be transferred on to cassette tape, the tape being mailed to the office for computer processing or the contents can be transferred directly from the data storage unit into the computer. Suitable interfaces are required for this movement of data.

Combined theodolite and EDM systems can also be used in conjunction with a data storage unit but in many cases the data acquisition is semi-automatic in that the readings from the theodolite and EDM unit, with their appropriate codes, have to be entered into the data storage unit by hand. Data transfer to the computer is then the same as for the electronic tacheometer system.

Using the traditional approach, all field observations can be entered on to field sheets and, with the aid of accompanying sketches, the data is coded and transferred into the computer using the keyboard.

8.7.2 Data Processing

Once entered into the computer, all data are stored in a field observation file. With the aid of specially devised software, the computer operator checks RO readings and then the three-dimensional coordinates (E, N and RL) are automatically computed for each point of detail surveyed in the field. This information, together with the code for each point, is stored in a coordinate file. At this stage, it is necessary to begin editing the coordinate file in order to ensure that information shown on the final plot is in the correct position, is annotated correctly and fits the descriptions given by the field surveyor. This editing process is carried out using software often known as an *interactive graphics* routine. The interactive graphics enable any small area of the coordinate file to be selected for display on the graphics screen. Information viewed can then be changed, moved or erased on the graphics screen as desired by using a light pen (see figure 8.8) or an electronic cursor. As points are changed, new coordinates are computed and the point code altered accordingly. This information is displayed, via the coordinate file, on the alphanumeric monitor. Various layers of information or combination of layers can be presented to the screen for editing. These layers are made up by the coordinate file using the point codes and the layers can contain such data as characters, linework, symbols, buildings, spot levels, roads, water features, underground services, overhead lines and so on.

The coordinate file is also used to produce computer generated contours. Since contours should not be drawn across certain features such as embankments, cuttings,

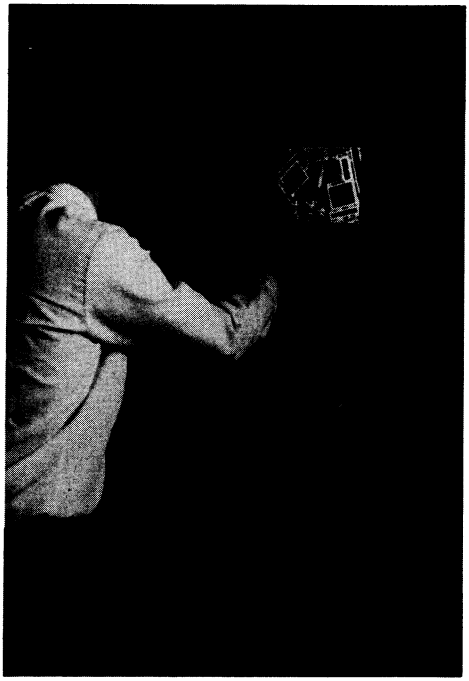


Figure 8.8 *Interactive graphics using light pen (courtesy of the Central Survey Branch, PSA, DoE)*

ditches, buildings and so on, the computer is instructed not to draw contours through these features. This is achieved by labelling, in the coordinate file, the edges of the embankments and so on with suitable codes. The contour information is usually contained in a separate information layer.

8.7.3 Data Output

After all editing has been carried out, the plan is drawn by a plotting table or a flat-bed plotter in ink on plastic film. Sections of computer plots are shown in figures 8.9 and 8.10. The contours shown across the tarmac surface in figure 8.10 have been drawn deliberately as part of a runway resurfacing survey. As an alternative to a full plot, various layers can be drawn for specialised uses, for example, a plan highlighting underground services can be drawn if required.

In addition to a graphical output, the contents of the coordinate file can be stored on tape or disc for future use. In some systems, the coordinate file is used to create DTMs (Digital Terrain Models) which are discussed in the following section.

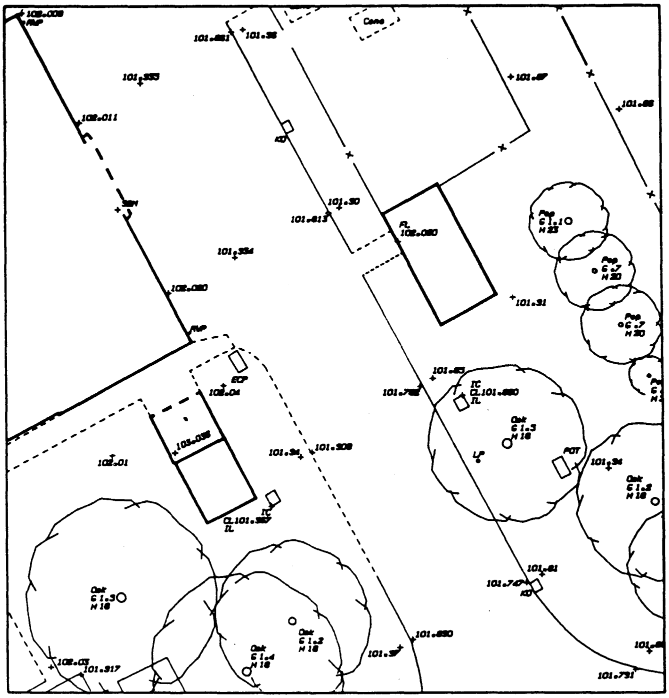


Figure 8.9 *Part of a survey plotted by computer (produced by the Central Survey Branch, PSA, DoE)*

8.8 Digital Terrain Models (DTMs)

A DTM is a means of representing the shape of natural surfaces in digital form suitable for storage in a computer. To form a DTM, a detail survey is carried out in the area for which the DTM is required. Since the shape of natural surfaces varies in a random way, the network of points surveyed to represent the shape of the ground will usually form a random pattern consisting of horizontal coordinates with associated heights.

In many cases, photogrammetric methods involving aerial surveys are used to provide surface information for DTMs. These methods are well suited to obtaining three-dimensional information over large areas where ground techniques would become laborious.

A DTM is usually formed from the field data using one of the following techniques.

A *square grid DTM* is one in which data points are obtained at the nodes of a square grid. This model is formed by the computer interpolating the height of each grid node from the field data provided.

A *triangular grid DTM* is one in which data points are interpolated at the corners of linked triangles which are positioned to give the best representation of the ground surface.

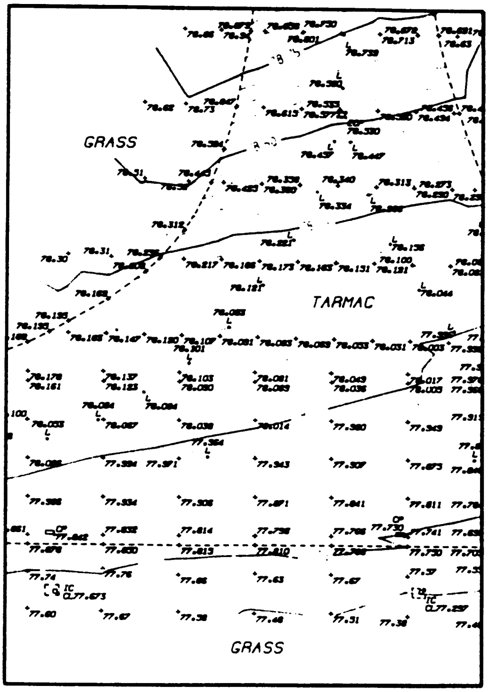


Figure 8.10 *Computer generated contours (produced by the Central Survey Branch, PSA, DoE)*

A *string DTM* is one which uses the field data directly to form the model. In these models, ground information is recorded as a series of two or three-dimensional strings. For example, existing roads, buildings, hedges and so on can form strings and these can be surveyed in sequence in the field. Three-dimensional coordinates can be calculated for each point surveyed and the line joining each discrete point represents the string stored in a computer. In such models, contours are a form of two-dimensional string and point strings can also be defined for detail such as individual trees, pylons and so on.

The methods given in section 8.7.1 for data acquisition in computer aided plotting also apply to data acquisition for a DTM but the computer functions and data output are different in many respects. With appropriate computer programs, a DTM can be used to produce a survey plan and this can be drawn if a suitable plotter is available. However, DTMs have many more applications than plan production and, when combined with a suitable computer system, DTMs have been applied very successfully to road alignment design and to the estimation of earthwork quantities. These applications are discussed in section 10.16.

8.9 Further Reading

Department of Transport, *Technical Memorandum H5/78, Model Contract for Topographical Survey Contracts* (Department of Transport, 1978).

9

Circular Curves

In the design of roads and railways, straight sections of road or track are connected by curves of constant or varying radius as shown in figure 9.1. The purpose of the curves is to deflect the road through the angle between the two straights, θ. For this reason, θ is known as *the deflection angle*.

The curves shown in figure 9.1 are *horizontal curves* since all measurements in their design and construction are considered in the horizontal plane. The two main types of horizontal curve are

(1) *circular curves*, which are curves of constant radius as shown in figure 9.1*a*
(2) *transition curves*, which are curves of varying radius as shown in figure 9.1*b*.

This chapter covers the design and setting out of circular curves and chapter 10 covers transition curves.

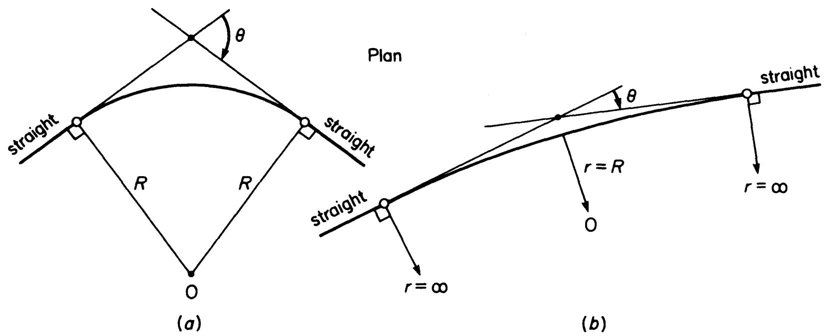


Figure 9.1 *Horizontal curves: (a) circular curve; (b) transition curve*

9.1 Types of Circular Curve

A *simple circular curve* consists of one arc of constant radius, as shown in figure 9.2. A *compound circular curve* consists of two or more circular curves of different

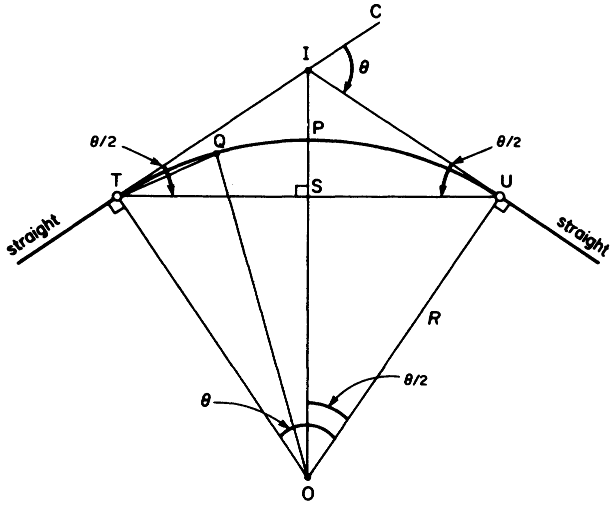


Figure 9.2 *Circular curve geometry*

radii. The centres of the curves lie on the same side of the common tangent, as shown in figure 9.17 in section 9.11.

A *reverse circular curve* consists of two consecutive circular curves, which may or may not have the same radii, the centres of which lie on opposite sides of the common tangent, as shown in figure 9.18 in section 9.12.

9.2 Terminology of Circular Curves

Figure 9.2 illustrates some of the terminology of horizontal curves and it is important that these terms are fully understood before proceeding with the derivations of the formulae used.

In figure 9.2
Q is any point on the circular curve TPU
S is the mid-point of the long chord TSU
P is the mid-point of the circular curve TPU
intersection point = I
tangent points = T and U
deflection angle = θ = external angle at I = angle CIU
radius of curvature = R
centre of curvature = O
intersection angle = $(180° - \theta)$ = internal angle at I = angle TIU
long chord = TU
tangential angle = for example, angle ITQ = angle from the tangent length at T (or U) to any point on the curve
mid-ordinate = PS
radius angle = angle TOU = deflection angle CIU

external distance = PI
tangent length = IT = IU

9.2.1 Important Relationships in Circular Curves

In figure 9.2, triangle ITU is isosceles, therefore angle ITU = angle IUT = $\theta/2$. Hence, referring to figure 9.3:

The tangential angle, α, at T to any point, X, on the curve TU is equal to half the angle subtended at the centre of curvature, O, by the chord from T to that point.

Similarly, in figure 9.4:

The tangential angle, β, at any point, X, on the curve to any forward point, Y, on the curve is equal to half the angle subtended at the centre by the chord between the two points.

Another useful relationship is illustrated in figure 9.5, which is a combination of figures 9.3 and 9.4.

From figure 9.3, angle TOX = 2 α, hence angle ITX = α.

From figure 9.4, angle XOY = 2 β, hence angle AXY = β.

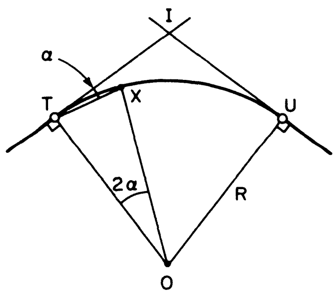


Figure 9.3

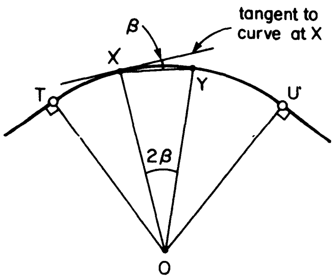


Figure 9.4

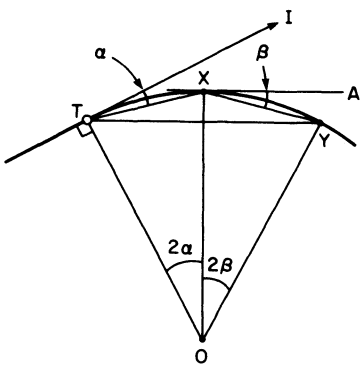


Figure 9.5

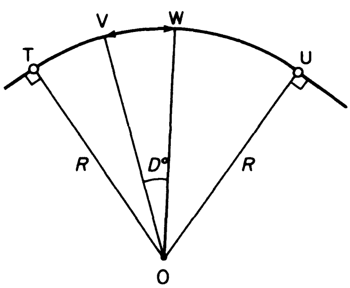


Figure 9.6

Therefore, in figure 9.5, angle TOY = $2(\alpha + \beta)$ and it follows that angle ITY = $(\alpha + \beta)$. In words, this can be stated as

The tangential angle to any point on the curve is equal to the sum of the tangential angles from each chord up to that point.

The relationships illustrated in figures 9.3, 9.4 and 9.5 are used when setting out the curves by the method of tangential angles (see section 9.9.1).

9.2.2 Useful Lengths

From the geometry of figure 9.2, the following can be derived

tangent length (IT and IU)	$= R \tan \theta/2$
external distance (PI)	$= R (\sec (\theta/2) - 1)$
mid-ordinate (PS)	$= R (1 - \cos (\theta/2))$
long chord (TU)	$= 2R \sin \theta/2$

9.3 Radius and Degree Curves

Circular curves can be referred to in one of two ways.

(1) In terms of their radius, for example, a 750 m curve. This is known as a *radius curve*.

(2) In terms of the angle subtended at the centre of the curve by a 100 m arc, for example, a 2° curve. This is known as a *degree curve*, and is shown in figure 9.6.

In figure 9.6, arc VW = 100 m and subtends an angle of $D°$ at the centre of curvature O. The curve TU is, therefore, a $D°$ degree curve.

The relationship between the two types of curve is given by the formula $DR = (18\,000/\pi)$, in which D is in degrees and R in metres, for example, a curve of radius 1500 m is equivalent to

$$D° = \frac{18\,000}{(1500\,\pi)} = \frac{12}{\pi} = 3.820°$$

that is, a 1500 m radius curve = a 3.820° degree curve.

9.4 Length of Circular Curves (L_c)

(1) For a radius curve, $L_c = (R\,\theta)$ m, where R is in metres and θ is in radians.

(2) For a degree curve, $L_c = (100\,\theta/D)$ m, where θ and D are in the same units, that is, degrees or radians.

9.5 Through Chainage

Through chainage or *chainage* is simply a distance and is usually in metres. It is a measure of the length from the starting point of the scheme to the particular point

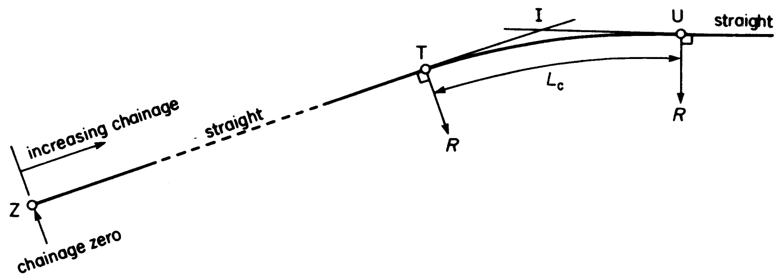


Figure 9.7 *Chainage along a circular curve*

in question and is used in road, railway, pipeline and tunnel construction as a means of referencing any point on the centre line.

Figure 9.7 shows a circular curve, of length L_c and radius R running between two tangent points T and U, which occurs in the centre line of a new road. As shown in figure 9.7, chainage increases along the centre line and is measured from the point (Z) at which the new construction begins. Z is known as *the position of zero chainage.*

Chainage continues to increase from Z along the centre line until a curve tangent point such as T is reached. At T, the chainage can continue to increase in two directions, either along the curve (that is, from T towards U) or along the straight (that is, from T along the line TI produced). Hence it is possible to calculate the chainage of the intersection point I.

In the design stage, when only the positions of the straights will be known, chainage is considered along the straights. However, once the design has been completed and the lengths of all the curves are known, the centre line becomes the important feature and chainage values must be calculated from the position of zero chainage along the centre line only. This is done in order that pegs can be placed at regular intervals along the centre line to enable earthwork quantities to be calculated (see chapters 12 and 13). Hence if the chainage of the intersection point, I, is known and the curve is then designed, the chainages of tangent points T and U,which both lie on the centre line, can be found as follows with reference to figure 9.7

$$\text{through chainage of T} = \text{through chainage of I} - \text{IT}$$
$$\text{through chainage of U} = \text{through chainage of T} + L_c$$

A common mistake in the calculation of through chainage is to assume that (TI + IU) = L_c. This is not correct. Similarly, the chainage of U does not equal the chainage of I + IU. To avoid such errors the following rule must be obeyed:

When calculating through chainage from a point which does not lie on the centre line (for example, point I in figure 9.7) it is necessary to first calculate the chainage of a point which lies further back on the centre line (that is, in the direction of zero chainage) before proceeding in a forward direction on the centre line.

9.6 Design of Circular Curves

In circular curve design there are three main variables: the deflection angle θ, the radius of curvature R and the design speed v.

All new roads are designed for a particular speed and the chosen value depends on the type and position of the proposed road (see section 10.3). The Department of Transport (DTp) stipulate design speeds for particular classes of road. This leaves θ and R to be determined.

When designing new roads, there is usually a specific area (often referred to as a *band of interest*) within which the proposed road must fall to avoid certain areas of land and unnecessary demolition. When improving roads, this band of interest is usually very clearly defined and is often limited to the immediate area next to the road being improved. Hence, in both cases, there will be a limited range of values for both θ and R in order that the finished road will fall within this band of interest.

If at all possible, θ must be measured accurately in the field before the design begins. If this is not immediately possible, an approximate value of θ can be measured using a protractor from the two straights drawn on the plan of the area. This value is then used for an initial design which is later amended once θ has been accurately determined. The alteration will, however, be slight and the approximate value of θ is ideal for ensuring that the design will fit adequately into the area.

R is chosen with reference to design values again stipulated by the DTp. These values are discussed in much greater detail in chapter 10 but basically they limit the value of the minimum radius which can be used at a particular speed for a wholly circular curve. If a radius value below the minimum is used it is necessary to incorporate transition curves into the design.

An initial radius value, greater than the minimum without transitions is chosen, the tangent lengths are calculated using R and θ and they are fitted on the plan. If there are no problems of fit this initial design value can be used, otherwise a new radius value would be chosen and a new fit obtained. Eventually, a suitable R value would be selected. The design is completed by calculating the *superelevation* required for the curve. This is fully discussed in section 10.2.

This trial and error method is suitable if any value of R above the minimum without transitions can be used and literally thousands of designs are possible and will all be perfectly acceptable. However, if a curve has to have particular tangent lengths, the following procedure can be adopted

(1) exact tangent length = R tan $\theta/2$
(2) only R is unknown, hence it can be calculated
(3) R should be checked against the DTp values (see chapter 10) to ensure that it is greater than the minimum without transitions. If it is not, transitions must be incorporated.

Having obtained θ and chosen a value for R by one of the methods just described, the rest of the design procedure can be followed through and the curve set out on site.

The following sections, 9.7 to 9.10 inclusive, deal with the design method and the setting out procedures in much greater detail.

9.7 Location of the Intersection and Tangent Points in the Field

It is not sufficiently accurate to scale the position of the tangent points from a plan and they must be accurately set out on site. The procedure is as follows with reference to figure 9.8.

(1) Locate the two tangent lines AC and BD and define them by means of a suitable target (see section 5.5.2)

(2) Set a theodolite up on one of the lines (say AC) and sight towards the intersection of the two tangents at I.

(3) Drive in two pegs x and y on the line AC such that BD will intersect the line xy. The exact position of the tangent line should be marked by nails in the top of the pegs (see figure 5.5).

(4) Join pegs x and y by means of a string line.

(5) Set up the theodolite on BD pointing towards I and fix the position of I by driving in a peg where the line of sight from BD intersects the string line.

(6) Set up the theodolite over I and measure angle AIB, hence angle θ.

(7) Calculate tangent lengths IT and IU using $R \tan \theta/2$.

(8) Measure back from I to T and U, drive in pegs and mark the exact points by nails in the tops of the pegs.

(9) Check the setting out by measuring angle ITU, which should equal $\theta/2$.

The use of two theodolites simplifies the procedure by eliminating steps (3) and (4).

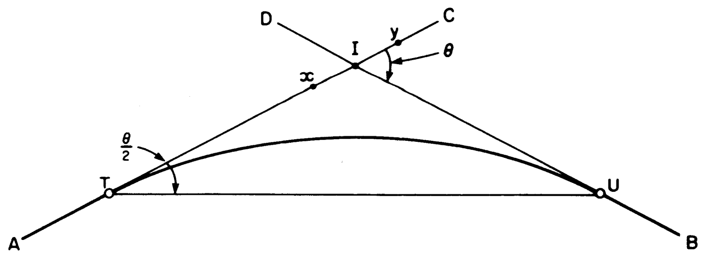


Figure 9.8

9.8 Location of the Tangent Points when the Intersection Point is Inaccessible

Occasionally, it is impossible to use the method described in section 9.7 owing to the intersection point falling on a very steep hillside, in marshy ground or in a lake or river and so on. In such cases, the following procedure should be adopted to determine θ and locate the tangent points T and U. Consider figure 9.9.

(1) Choose points A and B somewhere on the tangents such that it is possible to sight from A to B and B to A and also to measure AB.

(2) Measure AB.

(3) Measure angles α and β, deduce γ and hence θ.

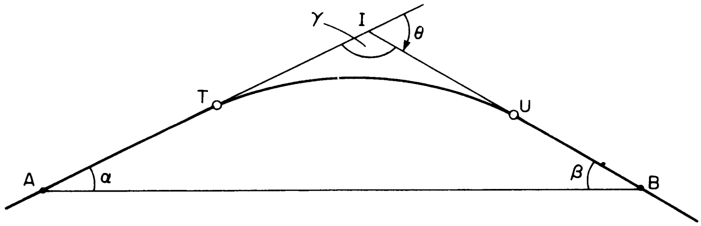


Figure 9.9

(4) Use the Sine Rule to calculate IA and IB.

(5) Calculate IT and IU from R tan $\theta/2$.

(6) AT = IA – IT and BU = IB – IU, hence set out T and U. If A and B are chosen to be on the other side of T and U, AT and BU will have negative values.

(7) If possible, sight from T to U to check. Measure angle ITU which should equal $\theta/2$.

9.9 Setting Out Circular Curves

Several methods are considered, of which the tangential angles methods using either one or two theodolites and the methods involving coordinates are the most accurate.

9.9.1 Tangential Angles Method

The method requires one theodolite and a tape. The formula used for the tangential angles is derived as follows and uses the relationships developed in section 9.2.1. Consider figure 9.10, in which tangential angles α_1 and α_2 are required. The assumption is made that arc TK = chord TK if chord $\leqslant R/20$.

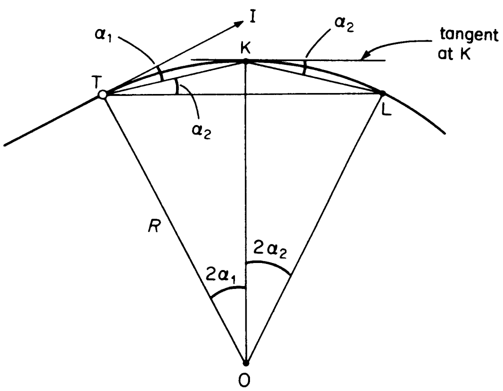


Figure 9.10

Therefore

$$\text{chord TK} = R2\alpha_1 \ (\alpha_1 \text{ in radians})$$

Hence

$$\alpha_1 = (\text{TK}/2R) \times (180/\pi) \text{ degrees}$$

Similarly

$$\alpha_2 = (\text{KL}/2R) \times (180/\pi) \text{ degrees}$$

Note that the chord for α_2 is KL not TL.
In general

$$\alpha = (180/2\pi) \times (\text{chord length/radius}) \text{ degrees}$$

$$= 1718.9 \times (\text{chord length/radius}) \text{ minutes}$$

Calculation procedure

(1) Determine the total length of the curve.

(2) Select a suitable chord length $\leqslant (R/20)$, for example, 10 m, 20 m. This will leave a sub-chord at the end and it is usually necessary to have an initial sub-chord in order to maintain equal chord lengths.

This is very important since pegs are usually placed on the centre line of the curve at exact multiples of through chainage to help in subsequent earthwork calculations, for example, pegs would be required at chainages 0 m, 20 m, 40 m, 60 m and so on if a 20 m chord has been selected. The chord must be $\leqslant (R/20)$ in order that the assumptions made in the derivation of the formula still apply.

(3) A series of tangential angles is obtained from the formula previously derived, for example, α_1, $(\alpha_1 + \alpha_2)$, $(\alpha_1 + \alpha_2 + \alpha_3)$ and so on corresponding to chords TK, KL, LM and so on.

In practice, $\alpha_2 = \alpha_3 = \alpha_4$ and so on, since all the chords except the first and the last will be equal. Therefore, usually only three tangential angles need to be calculated.

(4) All the *cumulative* angles are measured from the tangent point with reference to the tangent line IT but the chord lengths swung are *individual*, not cumulative.

(5) The results are normally tabulated before setting out the curve on site.

Setting-out procedure

(1) The tangent points are fixed and the theodolite is set up at one of them, preferably the one from which the curve swings to the right. This ensures that the tangential angles set on the horizontal circle will increase from 0°.

(2) The intersection point is sighted such that the horizontal circle is reading zero.

(3) The tangential angle for the first chord is set on the horizontal circle.

(4) The first chord is then set out by lining in the tape with the theodolite and marking off the length of the chord from the tangent point.

The chord lengths used in the calculations are considered in the horizontal plane, therefore the chord lengths set out must be either stepped or slope lengths must be calculated and used.

(5) The horizontal scale of the theodolite is set to the value of the first two tangential angles, that is ($\alpha_1 + \alpha_2$), and the tape again lined in.

With one end of the tape on the point fixed for the first chord, the length of the second chord is marked off. In practice, points are normally located by a peg in the top of which a nail is driven to within 5 to 10 mm of the top to mark the exact position. The end of the tape can be secured over the nail while the next point is located.

(6) This procedure is repeated until point U is set out. As a check, the tangential angle ITU should equal $\theta/2$.

An example based on this method is given in section 9.15.1.

9.9.2 Setting Out using Two Theodolites

This method is based on the tangential angles method and is used when the ground between the tangent points is of such a character that taping proves difficult, for example, very steep slopes, undulating ground, ploughed fields or if the curve is partly over marshy ground. The method is as follows and is shown in figure 9.11.

Two theodolites are used, one being set at each tangent point. One disadvantage of the method is that two of everything are required, for example, two engineers, two instruments and, preferably, two assistants to locate the pegs.

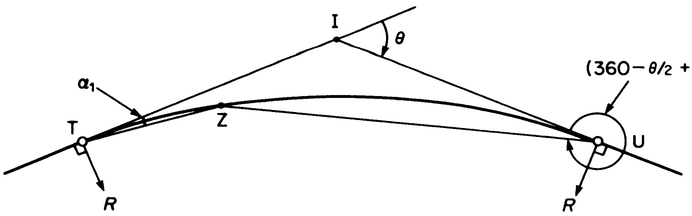


Figure 9.11

The method adopted is one of intersecting points on the curve with the theodolites.

In figure 9.11, to fix point Z

α_1 is set out from T relative to IT and

$(360^\circ - (\theta/2) + \alpha_1)$ is set out from U relative to UI.

The two lines of sight intersect at Z where an assistant drives in a peg. Good liaison between the groups is essential and, for large curves, two-way radios are a very useful aid.

9.9.3 Offsets from the Tangent Lengths

This method requires two tapes or a chain and a tape. It is suitable for short curves and it may be used to set out additional points between those previously established by the tangential angles method. This is often necessary to give a better definition of the centre line. Consider figure 9.12.

Required The offset AB, from a point A on the tangent, to the curve.

In triangle OBC

$$OB^2 = OC^2 + BC^2$$

Therefore

$$R^2 = (R - X)^2 + Y^2$$

From here there are two routes

either

$$(1)\, R - X = \sqrt{(R^2 - Y^2)} \text{ hence } X = R - \sqrt{(R^2 - Y^2)}$$

or

$$(2)\, R^2 = R^2 - 2RX + X^2 + Y^2$$

Dividing through by $2R$ gives

$$X = (Y^2/2R) + (X^2/2R)$$

but $(X^2/2R)$ will be very small since R is very large compared with X, therefore it can be neglected. Therefore

$$X = (Y^2/2R) \qquad (9.1)$$

Equation (9.1) is accurate only for large radii curves and will give errors for small radii curves where the effect of neglecting the second term cannot be justified.

Once the tangent points are fixed, the lines of the tangents can be defined using a theodolite or ranging rods and the offsets (X) set off at right angles at distances (Y) from T and then from U. Half the curve is set out from each tangent point.

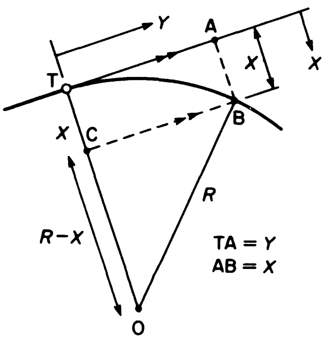


Figure 9.12

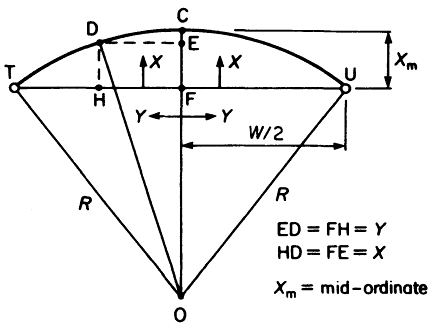


Figure 9.13

9.9.4 Offsets from the Long Chord

This method also uses two tapes or a chain and tape. It is suitable for curves of small radius such as boundary walls and kerb lines at road intersections. Also, it is a very useful method when the tangent lengths are inaccessible and method 9.9.3 cannot be used.

Consider figure 9.13.

Required The offset HD from the long chord TU at a distance Y from F. In this method all offsets are established from the mid-point F of the long chord TU. Let the length of chord TU = W. In triangle TFO

$$\mathrm{OT}^2 = \mathrm{OF}^2 + \mathrm{TF}^2$$

Therefore

$$R^2 = (R - X_\mathrm{m})^2 + (W/2)^2$$

Hence

$$(R - X_\mathrm{m}) = \sqrt{(R^2 - (W/2)^2)}$$

Therefore

$$X_\mathrm{m} = R - \sqrt{(R^2 - (W/2)^2)} \tag{9.2}$$

In triangle ODE

$$\mathrm{OD}^2 = \mathrm{OE}^2 + \mathrm{DE}^2$$

Therefore

$$R^2 = (\mathrm{OF} + X)^2 + Y^2$$

Hence

$$(\mathrm{OF} + X) = \sqrt{(R^2 - Y^2)} \tag{9.3}$$

But

$$\mathrm{OF} = (R - X_\mathrm{m})$$

Therefore from equation (9.2)

$$\mathrm{OF} = \sqrt{(R^2 - (W/2)^2)}$$

Therefore from equation (9.3)

$$X = \sqrt{(R^2 - Y^2)} - \sqrt{(R^2 - (W/2)^2)}$$

Once the tangent points are fixed, the long chord can be defined and point F established. The offsets are then calculated at regular intervals from point F, firstly along FT and secondly along FU.

Again, it is very useful to tabulate the offsets from FT and FU before beginning the setting out.

When setting out, the distance Y to a particular point is measured from F towards T and U and the corresponding offset X set out at right angles at that point.

9.9.5 Setting-out Methods involving Coordinates

These methods are nowadays used in preference to the more traditional techniques.

In such methods, which are suitable for all horizontal curves, the National Grid or local coordinates of points on the curve are calculated and these points are then fixed by either

(1) *intersection* from two of the control points in the main survey network surrounding the proposed scheme (see figure 9.14); or

(2) *bearing and distance* (*polar rays*) from control points in the main survey network (see figure 9.15). To fix point A, α is turned off from direction PQ and distance PA measured and to fix point B, β is turned off and distance PB measured.

For a complete curve, consider figure 9.16. Points A, B, C, D, E and F are points to be set out at regular intervals of through chainage on the curve from control points P and Q.

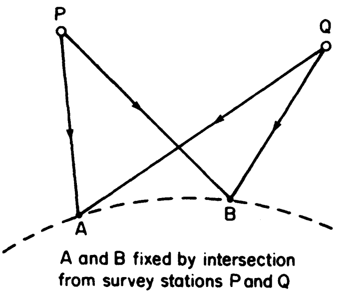


Figure 9.14

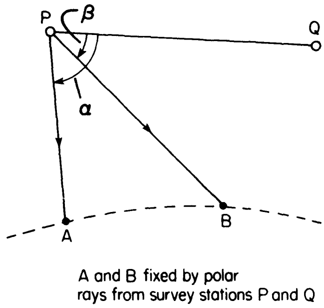


Figure 9.15

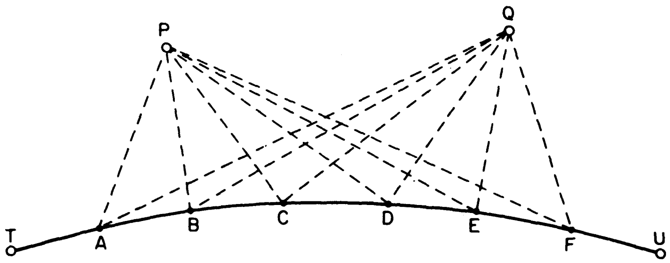


Figure 9.16

Procedure

(1) Locate T and U as discussed in sections 9.7 and 9.8.

(2) Obtain coordinates of T and U either by taking intersection observations from P and Q or by including T and U in a traverse with P and Q.

(3) Calculate chord lengths TA, AB, BC and so on and respective tangential angles as normal.

(4) Calculate bearings TA, AB, BC and so on.

(5) Calculate the coordinates of points A to U from T, treating TABCDEFU as a closed route traverse.

(6) Derive bearings PA and QA, PB and QB, PC and QC, and so on from their respective coordinates.

(7) Calculate the lengths PA and QA, PB and QB, PC and QC, and so on from their respective coordinates.

(8) Set out the curve by either

(i) *intersection* from P and Q using bearings PA and QA, PB and QB, and so on; or

(ii) *polar rays* from P or Q, using bearings PA or QA, PB or QB, and so on, and lengths PA or QA, PB or QB, and so on.

Section 9.15.2 shows an example involving the setting out of a circular curve by intersection from nearby traverse stations.

9.9.6 Summary of Setting-out Methods

Several methods have been discussed and the one finally chosen depends, to some extent, on the project involved.

Generally, tangential angles methods and methods involving coordinates are used owing to their greater accuracy, and the latter are now widely used since calculators and computers have greatly reduced the laborious time that was once associated with the calculation of coordinates. Indeed, computers and desk calculators are now used for all but the simplest curves.

The advantages and disadvantages of coordinate based methods are discussed in section 10.12.

Offset methods are used for less important curves, for example, housing estates, minor roads, kerb lines, boundary walls and so on, and the long chord method is particularly useful when the tangent length is inaccessible for some reason and the tangent length method cannot be used.

9.10 Obstructions to Setting Out

If care has been taken in route location and in the choice of a suitable radius there should be no obstruction to setting out other than the need to clear the ground surface. However, should obstructions arise, one of the coordinate methods of setting out described in section 9.9.5 can be used to set out the sections of the curve on either side of the obstruction to allow work to proceed. Once the obstruction has been removed, the same method can be employed to establish the missing section of the centre line.

9.11 Compound Circular Curves

These consist of two or more consecutive circular curves of different radii without any intervening straight section.

The object of such curves is to avoid certain points, the crossing of which would involve great expense and which cannot be avoided by a simple circular curve.

Today they are uncommon since there is a change in the *radial force* (see section 10.1) at the junction of the curves which go to make up the compound curve. The effect of this, if the change is marked, can be to give a definite jerk to passengers, particularly in trains.

To overcome this problem, either very large radii should be used to minimise the forces involved or transition curves should be used instead of the compound curve.

A typical two-curve compound curve is shown in figure 9.17.

In figure 9.17, AB = common tangent through T_c and $(\alpha + \beta) = \theta$.

The design of such a curve is best done by treating the two sections separately and choosing suitable values for α, β, R_1 and R_2 and proceeding as for two simple circular curves, that is, T_1T_c and T_cT_2.

In compound circular curves, the tangent lengths IT_1 and IT_2 are not equal.

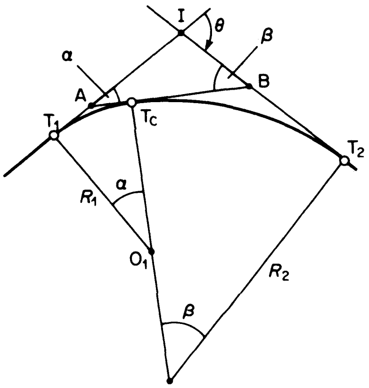


Figure 9.17 *Compound curve*

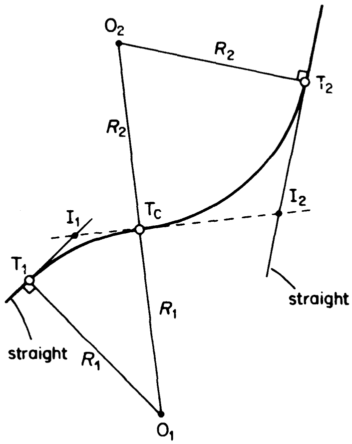


Figure 9.18 *Reverse curve*

9.12 Reverse Circular Curves

These curves consist of two consecutive curves of the same or different radii without any intervening straight section and with their centres of curvature falling on opposite sides of the common tangent. They are much more common than compound circular curves and, like such compound curves, they can be used to avoid obstacles. Often, however, they are used to connect two straights which are very nearly parallel and which would otherwise require a very long simple circular curve.

A typical reverse circular curve is shown in figure 9.18. In order to connect the two straights T_1I_1 and T_2I_2 it is necessary to introduce a third straight I_1I_2. A trial and error method using several different straights is employed until a suitable point, T_c, is chosen.

Once the point T_c has been decided, the reverse curve can be considered as two separate simple curves with no intermediate straight section, that is, T_1T_c and T_cT_2.

With reference to figure 9.18, $T_1I_1 = I_1T_c$ and $T_cI_2 = I_2T_2$ but I_1T_c does not necessarily equal T_cI_2.

9.13 Summary of Circular Curves

Although circular curves are straightforward in nature, much of their terminology also applies to transition curves and it is vital, therefore, that a good understanding of circular curves is attained before proceeding to study transitions.

Nowadays, although still used, circular curves tend to be used in conjunction with transition curves rather than in isolation and there is an increasing tendency to omit the circular curve section of such composite curves and design wholly transitional curves. This is discussed much more fully in chapter 10.

9.14 Further Reading

Department of Transport, Roads and Local Transport Directorate, *Departmental Standard TD 9/81, Road Layout and Geometry: Highway Link Design* (Department of Transport, 1981).

Department of Transport, Highways and Traffic Directorate, *Departmental Advice Note TA 43/84: Highway Link Design* (Department of Transport, 1984).

9.15 Worked Examples

9.15.1 Setting out by the Tangential Angles Method

Question

It is required to connect two straights whose deflection angle is 13° 16′00″ by a circular curve of radius 600 m.

Make the necessary calculations for setting out the curve by the tangential angles method if the through chainage of the intersection point is 2745.72 m.

Use a chord length of 25 m and sub-chords at the beginning and end of the curve to ensure that the pegs are placed at exact 25 m multiples of through chainage.

Solution

Consider figure 9.19

$$\textit{Tangent length} = R \tan \theta/2 = 600 \tan 6°38'00'' = 69.78 \text{ m}$$

Therefore

$$\textit{through chainage of}\ \mathrm{T} = 2745.72 - 69.78 = 2675.94 \text{ m}$$

To round this figure to 2700 m (the next multiple of 25 m) an initial sub-chord is required. Hence

$$\text{length of initial sub-chord} = 2700 - 2675.94 = 24.06 \text{ m}$$

$$\textit{length of circular curve} = R\,\theta = (600 \times 13.2667 \times \pi)/180 = 138.93 \text{ m}$$

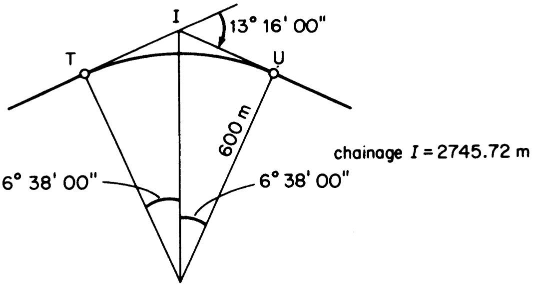


Figure 9.19

Therefore

$$\textit{through chainage of } U = 2675.94 + 138.93 = 2814.87 \text{ m}$$

Hence a final sub-chord is also required since 25 m chords can only be used up to chainage 2800 m. Therefore

$$\text{length of final sub-chord} = 2814.87 - 2800 = 14.87 \text{ m}$$

Hence three chords are necessary

initial sub-chord of	24.06 m
general chord of	25.00 m
final sub-chord of	14.87 m

The tangential angles for these chords are obtained from the formula $\alpha = 1718.9 \times$ (chord length/radius) min as follows

$$\begin{aligned} &\text{for initial sub-chord} = 1718.9 \times (24.06/600) = 68.93' = 01°08'56'' \\ &\text{for general chord} \quad = 1718.9 \times (25.00/600) = 71.62' = 01°11'37'' \\ &\text{for final sub-chord} \quad = 1718.9 \times (14.87/600) = 42.60' = 00°42'36'' \end{aligned}$$

Applying these to the whole curve, the tabulated results are shown in table 9.1. The points on the centre line are designated C_1, C_2, C_3, C_4 and C_5 for use in example 9.15.2.

As a check, the final cumulative tangential angle shown in table 9.1 should equal $\theta/2$ within a few seconds. Also the sum of the chords should equal the total length of the circular arc.

Note that since α is proportional to the chord length, any chords of equal length will have the same tangential angle and this is simply added to the cumulative total.

9.15.2 Setting out from Coordinates by Intersection

Question

The circular curve designed in example 9.15.1 is to be set out by intersection methods from two nearby traverse stations A and B. The position of the tangent point, T, is set out on the ground and its coordinates are obtained by taking obser-

TABLE 9.1

Point	Chainage (m)	Chord Length (m)	Individual Tangential Angle	Cumulative Tangential Angle
T	2675.94	0	00° 00' 00"	00° 00' 00"
C_1	2700.00	24.06	01° 08' 56" (α_1)	01° 08' 56"
C_2	2725.00	25.00	01° 11' 37" (α_2)	02° 20' 33"
C_3	2750.00	25.00	01° 11' 37" (α_3)	03° 32' 10"
C_4	2775.00	25.00	01° 11' 37" (α_4)	04° 43' 47"
C_5	2800.00	25.00	01° 11' 37" (α_5)	05° 55' 24"
U	2814.87	14.87	00° 42' 36" (α_6)	06° 38' 00"
		Σ 138.93 (checks)		

vations to it from A and B. Observations taken from T to the intersection point, I, enable the whole-circle bearing of TI to be calculated as 63°27′14″.

The coordinates of A, B and T are as follows

A 829.17 mE, 724.43 mN
B 915.73 mE, 691.77 mN
C 798.32 mE, 666.29 mN

Using the relevant data from example 9.15.1, calculate

(1) the coordinates of all the points on the centre line of the curve which lie at exact 25 m multiples of through chainage

(2) the bearing AB and the bearings from A required to establish the directions to all these points

(3) the bearing BA and the bearings from B required to establish the directions to all these points.

Solution

Figure 9.20 shows all the points to be set out together with traverse stations A and B.

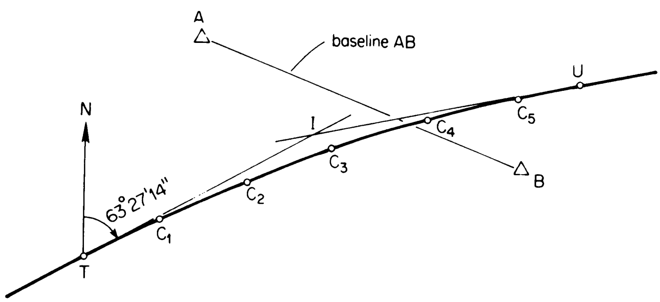


Figure 9.20

(1) Coordinates of all the points on the centre line

Coordinates of C_1

With reference to figure 9.21 and table 9.1

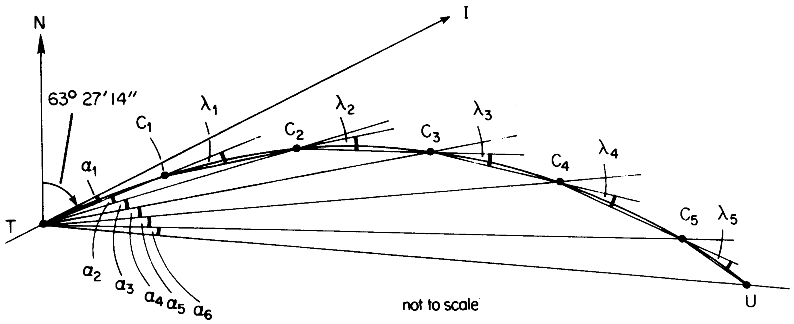


Figure 9.21

$$\text{bearing TC}_1 = \text{bearing TI} + \alpha_1$$
$$= 63°27'14'' + 01°08'56'' = 64°36'10''$$

$$\text{horizontal length TC}_1 = 24.06 \text{ m}$$

Therefore

$$\Delta E_{TC_1} = 24.06 \sin 64°36'10'' = +21.735 \text{ m}$$
$$\Delta N_{TC_1} = 24.06 \cos 64°36'10'' = +10.319 \text{ m}$$

Hence

$$E_{C_1} = E_T + (\Delta E_{TC_1})$$
$$= 798.32 + 21.735 = \mathbf{820.055 \text{ m}}$$
$$N_{C_1} = N_T + (\Delta N_{TC_1})$$
$$= 666.29 + 10.319 = \mathbf{676.609 \text{ m}}$$

These are retained with three decimal places for calculation purposes but are finally rounded to two decimal places.

Coordinates of C_2

With reference to figure 9.22 and table 9.1

$$\lambda_1 + (90° - \alpha_1) + (90° - \alpha_2) = 180°$$

Hence

$$\lambda_1 = \alpha_1 + \alpha_2$$
$$= 01°08'56'' + 00°11'37'' = 02°20'33''$$

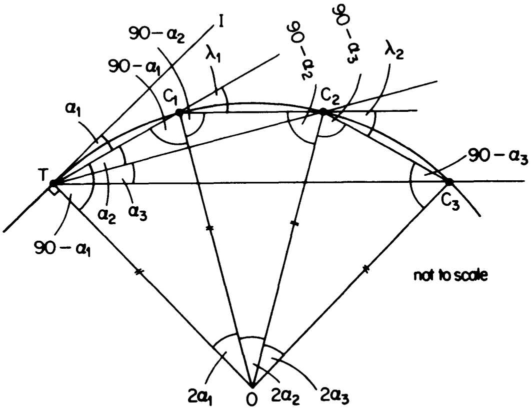


Figure 9.22

Therefore

$$\text{bearing } C_1C_2 = \text{bearing } TC_1 + \lambda_1$$
$$= 64°36'10'' + 02°20'33'' = 66°56'43''$$

From table 9.1, horizontal length C_1C_2 = 25.00 m, therefore

$$\Delta E_{C_1C_2} = 25.00 \sin 66°56'43'' = +23.003 \text{ m}$$
$$\Delta N_{C_1C_2} = 25.00 \cos 66°56'43'' = +\ \ 9.790 \text{ m}$$

Hence

$$E_{C_2} = E_{C_1} + (\Delta E_{C_1C_2})$$
$$= 820.055 + 23.003 = \mathbf{843.058\ m}$$
$$N_{C_2} = N_{C_1} + (\Delta N_{C_1C_2})$$
$$= 676.609 + 9.790 = \mathbf{686.399\ m}$$

Coordinates of C_3

With reference to figures 9.21, 9.22 and table 9.1

$$\lambda_2 = \alpha_2 + \alpha_3$$
$$= 01°11'37'' + 01°11'37'' = 02°23'14''$$
$$\text{bearing } C_2C_3 = \text{bearing } C_1C_2 + \lambda_2$$
$$= 66°56'43'' + 02°23'14'' = 69°19'57''$$

From table 9.1, horizontal length C_2C_3 = 25.00 m, therefore

$$\Delta E_{C_2C_3} = 25.00 \sin 69°19'57'' = +23.391 \text{ m}$$
$$\Delta N_{C_2C_3} = 25.00 \cos 69°19'57'' = +\ \ 8.824 \text{ m}$$

Therefore

$$E_{C_3} = E_{C_2} + (\Delta E_{C_2C_3})$$
$$= 843.058 + 23.391 = \mathbf{866.449\ m}$$

$$N_{C_3} = N_{C_2} + (\Delta N_{C_2C_3})$$
$$= 686.399 + 8.824 = \mathbf{695.223\ m}$$

Coordinates of C_4 and C_5

These are calculated by repeating the procedure used to calculate the coordinates of C_3 from those of C_2. The values obtained are

C_4 = 890.187 mE, 703.065 mN

C_5 = 914.231 mE, 709.911 mN

Coordinates of U

These are calculated twice to provide a check.

Firstly, they are calculated from point C_5 by repeating the procedure used to calculate the coordinates of C_3 from those of C_2. The values obtained are

U = 928.660 mE, 713.505 mN

Secondly, they are calculated by working along the straights from T to I to U as follows

bearing TI = 63°27′14″

horizontal length TI = 69.78 m (see example 9.15.1)

Hence

$$\Delta E_{TI} = 69.78 \sin 63°27'14'' = +\ 62.423 \text{ m}$$

$$\Delta N_{TI} = 69.78 \cos 63°27'14'' = +\ 31.186 \text{ m}$$

Therefore

$$E_I = E_T + (\Delta E_{TI})$$
$$= 798.32 + 62.423 = 860.743 \text{ m}$$

$$N_I = N_T + (\Delta N_{TI})$$
$$= 666.29 + 31.186 = 697.476 \text{ m}$$

From example 9.15.1, θ = 13°16′00″, hence

$$\text{bearing IU} = \text{bearing TI} + \theta$$
$$= 63°27'14'' + 13°16'00'' = 76°43'14''$$

horizontal length IU = 69.78 m

Therefore

$$\Delta E_{IU} = 69.78 \sin 76°43'14'' = +\ 67.914 \text{ m}$$

$$\Delta N_{IU} = 69.78 \cos 76°43'14'' = +\ 16.029 \text{ m}$$

From which

$$E_U = E_I + (\Delta E_{IU})$$
$$= 860.743 + 67.914 = \mathbf{928.657\ m}$$

$$N_U = N_I + (\Delta N_{IU})$$
$$= 697.476 + 16.029 = \mathbf{713.505\ m}$$

These check, within a few millimetres, the values obtained for the coordinates of U calculated around the curve.

All the coordinates are listed in table 9.2 and have been rounded to two decimal places.

(2) Bearing AB and the bearings to the points from A

These are calculated from the coordinates of the points using either the quadrants method or by using rectangular/polar conversions as discussed in section 5.10. The bearings are listed in table 9.2.

(3) Bearing BA and the bearings to the points from B

Again, one of the methods discussed in section 5.10 is used. The bearings are listed in table 9.2.

TABLE 9.2

Point	Chainage (m)	Coordinates mE	mN	Bearing from A °	'	"	Bearing from B °	'	"
T	2675.94	798.32	666.29	207	56	59	257	45	23
C_1	2700.00	820.05(5)	676.61	190	47	34	260	59	44
C_2	2725.00	843.06	686.40	159	56	24	265	46	26
C_3	2750.00	866.45	695.22	128	04	39	274	00	33
C_4	2775.00	890.19	703.06(5)	109	17	51	293	51	17
C_5	2800.00	914.23	709.91	99	41	09	355	16	36
U	2814.87	928.66	713.50(5)	96	15	55	30	44	59
Bearing AB = 110° 40' 19"				Bearing BA = 290° 40' 19"					

10

Transition Curves

A transition curve differs from a circular curve in that its radius is constantly changing. As may be expected, such curves involve more complex formulae than curves of constant radius and their design can be complicated. Circular curves are unquestionably more easy to design than transition curves—they are easily set out on site—and so the questions naturally arise, why are transition curves necessary, and why is it not possible to use circular curves to join all intersecting straights?

10.1 Radial Force and Design Speed

The reason for the two types of curve is due to the *radial force* acting on the vehicle as it travels round the curve.

A vehicle travelling with a constant speed v along a curve of radius r is subjected to a radial force P such that $P = (mv^2/r)$, where m is the mass of the vehicle.

This force is, in effect, trying to push the vehicle back on to a straight course.

On a straight road, r = infinity, therefore $P = 0$
On a circular curve of radius R, $r = R$, therefore $P = (mv^2/R)$

Roads and railways are designed for particular speeds and hence v, the *design speed*, is constant for any given road; v is, in fact, the 85 percentile speed, that is, the speed not normally exceeded by 85 per cent of the vehicles using the road.

Similarly, the mass of the vehicle can be assumed constant, therefore $P \propto 1/r$, that is, the smaller the radius, the greater the force.

Therefore, any vehicle leaving a straight section of road and entering a circular curve section of radius R will experience the full force (mv^2/R) instantaneously.

If R is small, the practical effect of this is for the vehicle to skid sideways, away from the centre of curvature, as the full radial force is applied.

To counteract this, the Department of Transport (DTp) lay down minimum radii for wholly circular curves. These are discussed in section 10.3. If it is necessary to go below the minimum stipulated radius at a particular speed, transition curves must be incorporated into the design.

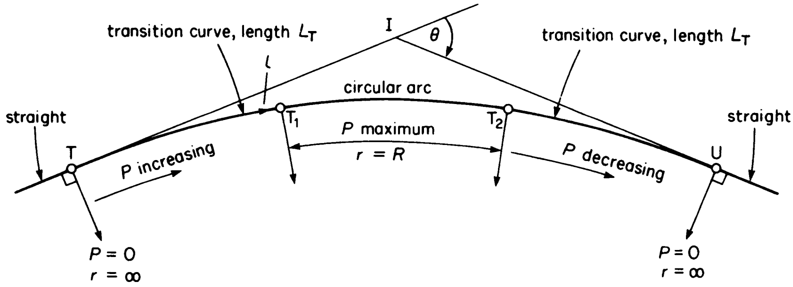


Figure 10.1 *Composite curve*

Transition curves are curves in which the radius changes from infinity to a particular value. The effect of this is to gradually increase the radial force P from zero to its highest value and thereby reduce its effect.

Usually the road curve consists of two transitions and a circular curve (see figure 10.1).

For a vehicle travelling from T to U, the force P gradually increases from zero to its maximum on the circular curve and then decreases to zero again. This greatly reduces the tendency to skid and reduces the discomfort experienced by passengers in the vehicles. This is one of the purposes of transition curves; by introducing the radial force gradually and uniformly they minimise passenger discomfort. However, to achieve this they must have a certain property. Consider figure 10.1.

For a constant speed v, the force P acting on the vehicle is (mv^2/r). Since any given curve is designed for a particular speed and the mass of a vehicle can be assumed constant, it follows that $P \propto 1/r$.

However, if the force is to be introduced uniformly along the curve, it also follows that P must be proportional to l, where l is the length along the curve from the entry tangent point to the point in question.

Combination of these two requirements gives $l \propto 1/r$ or $rl = K$, where K is a constant. If L_T is the total length of each transition and R the radius of the circular curve, then $RL_T = K$.

Hence, if the transition curve is to introduce the radial force in a gradual and uniform manner it must have the property that the product of the radius of curvature at any point on the curve and the length of the curve up to that point is a constant value. This is the definition of a spiral and because of this, transition curves are also known as *transition spirals*. The types of curves used are discussed further in section 10.6.

A further purpose of transition curves is to gradually introduce *superelevation* and this is discussed in section 10.2.

10.2 Superelevation

Although transition curves can be used to introduce the radial force gradually in an attempt to minimise its effect, this effect can also be greatly reduced and even

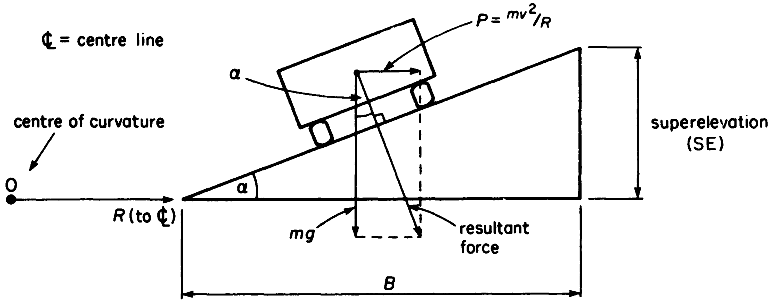


Figure 10.2 *Superelevation*

eliminated by raising one side of the roadway or one side of the track relative to the other. This procedure is shown in figure 10.2 and the difference in height between the road channels is known as the *superelevation*. By applying such superelevation, the resultant force (see figure 10.2) can be made to act perpendicularly to the road surface, thereby forcing the vehicle down on to the road surface rather than throwing it off.

The maximum superelevation (SE) occurs when r is a minimum. With reference to figure 10.2.

$$\tan \alpha = (mv^2/R)/mg$$

Therefore

$$\tan \alpha = (v^2/gR)$$

But

$$\text{SE} = B \tan \alpha$$

Therefore

$$\textit{maximum}\ \text{SE} = (Bv^2/gR)$$

This value is constant on the circular curve and is gradually introduced on the entry transition curve and gradually reduced on the exit transition curve. If a wholly circular curve has been designed, between one-half and two-thirds of the superelevation should be introduced on the approach straight and the remainder at the beginning of the curve. The superelevation should be run out into the straight at the end of the curve in a similar manner.

For high design speeds, wide carriageways and small radii, the maximum superelevation will be very large and if actually constructed will be alarming to drivers approaching the curve. Also, any vehicle travelling below the design speed will tend to slip down the road surface and the driver will have to understeer to compensate. Should the maximum SE be constructed then any vehicle travelling at the design speed will travel round the curve without the driver needing to adjust the steering wheel.

Therefore, because of these aesthetic effects, the DTp stipulate maximum and minimum values for superelevation.

10.2.1 Maximum and Minimum Allowable Values of Superelevation

The DTp lay down the following rules for *maximum* superelevation.

(1) It should normally balance out only 45 per cent of the radial force P.

(2) It should not normally be steeper than 7 per cent (approximately 1 in 14.5) and, wherever possible, should be kept within the desirable value of 5 per cent (1 in 20).

(3) On sharp curves in urban areas superelevation shall be limited to 5 per cent.

Therefore, although the *maximum theoretical* SE = (Bv^2/gR), in practice, the *maximum allowable* SE = 0.45 (Bv^2/gR). Also, once calculated, if this maximum allowable value gives a cross slope greater than 7 per cent then only 7 per cent should be used, for example, even if the design requires a superelevation of 10 per cent, only 7 per cent should be used.

The other 55 per cent of the radial force and any extra superelevation not accounted for in the final design are assumed to be taken by the friction between the road surface and the tyres of the vehicle. Hence the reason for vehicles skidding in wet or greasy conditions.

Expressing v in kph, R in metres and substituting for g gives the *maximum allowable superelevation* as

$$\text{SE} = \frac{B \times v^2}{282.8 \times R} \text{ metres}$$

or, expressing the maximum allowable superelevation as a percentage, s, such that $s = 100(\text{SE})/B$ gives

$$s \text{ per cent} = \frac{v^2}{2.828 \times R} = \frac{v^2}{2\sqrt{2} \times R}$$

These expressions for maximum allowable superelevation hold for values of R down to the absolute minimum values only (see section 10.3).

The *minimum allowable* SE, to allow for drainage, is 1 in 40, that is, 2.5 per cent.

Further details on superelevation can be found in the references given in section 10.17.

10.3 Current Department of Transport Design Standards

The DTp stipulates allowable radii for particular design speeds. These are shown in table 10.1 which has been reproduced from the DTp publication *Departmental Standard TD 9/81, Road Layout and Geometry: Highway Link Design*. This standard replaces previous DTp publications including *Layout of Roads in Rural Areas* and *Roads in Urban Areas*.

An advice note, TA 43/84, referenced in section 10.17, provides a useful guide to TD 9/81.

TABLE 10.1
Current Department of Transport Highway Design Standards (published here by permission of the Controller of Her Majesty's Stationery Office)

DESIGN SPEED kph.	120	100	85	70	60	50	v^2/R
A. STOPPING SIGHT DISTANCE m.							
A1 Desirable Minimum	295	215	160	120	90	70	
A2 Absolute Minimum	215	160	120	90	70	50	
B. HORIZONTAL CURVATURE m.							
B1 Minimum R * without elimination of Adverse Camber and Transitions	2880	2040	1440	1020	720	510	5
B2 Minimum R * with Superelevation of 2.5%	2040	1440	1020	720	510	360	7.07
B3 " " " " " 3.5%	1440	1020	720	510	360	255	10
B4 Desirable Minimum R " " 5%	1020	720	510	360	255	180	14.14
B5 Absolute Minimum R " " 7%	720	510	360	255	180	127	20
B6 Limiting Radius " " 7% at sites of special difficulty (Category B Design Speeds only)	510	360	255	180	127	90	28.28
C. VERTICAL CURVATURE							
C1 FOSD Overtaking Crest K Value	*	400	285	200	142	100	
C2 Desirable Minimum * Crest K Value	182	100	55	30	17	10	
C3 Absolute Minimum " " "	100	55	30	17	10	6.5	
C4 Absolute Minimum Sag K Value	37	26	20	20	13	9	
D. OVERTAKING SIGHT DISTANCE							
D1 Full Overtaking Sight Distance FOSD m.	*	580	490	410	345	290	

* Not recommended for use in the design of single carriageways

10.3.1 Use of the Design Standards

It is strongly recommended that the DTp publication from which table 10.1 has been taken, together with its advice note, be studied in great detail before the commencement of any highway link design (see section 10.17).

The following example is included merely as a guide to the use of the standard in horizontal curve design work. Many factors, which are discussed in great detail in the standard itself, may influence the final choice of design.

Examples of the use of table 10.1 in vertical curve design are discussed in chapter 11.

Question

Two intersecting straights on a section of a highway designed for a speed of 85 kph are to be joined using a horizontal curve. With reference to the current DTp design standards, summarise the various choices of radii that are available.

Solution

From table 10.1, if a wholly circular curve is to be used, the minimum value of R must be 1440 m (row B1).

If transition curves are to be included in the design then the following radii are permissible for various superelevation values (rows B2 to B5):

for superelevation = 2.5 per cent, R must be $\geqslant$ 1020 m
for superelevation $\leqslant$ 3.5 per cent, R must be $\geqslant$ 720 m
for superelevation $\leqslant$ 5 per cent, R must be $\geqslant$ 510 m (Desirable Minimum)
for superelevation $\leqslant$ 7 per cent, R must be $\geqslant$ 360 m (Absolute Minimum)

In certain special cases, R can be lowered to 255 m (row B6) provided that 7 per cent superelevation is used. However, wherever possible, radii values greater than the desirable minimum ones should be used.

10.4 Use of Transition Curves

Transition curves can be used to join intersecting straights in one of two ways.

10.4.1 Composite Curves

Figure 10.1 shows such a design. Transition curves of equal length are used on either side of a circular curve of radius R.

Although this type of design has widespread use, it has the disadvantage that the radius and hence the radial force is constant on the circular section and, if this force is large, the length of the circular section represents a danger length over which the maximum force applies. The values given for limiting radii in table 10.1 do greatly reduce this occurrence but the use of transitions as described in section 10.4.2 is often preferred.

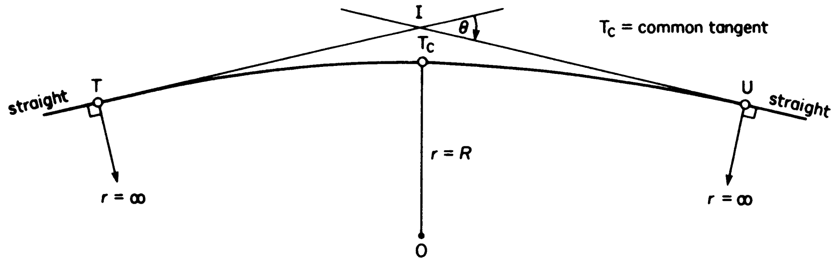


Figure 10.3 *Wholly transitional curve*

10.4.2 Wholly Transitional Curves

Figure 10.3 shows such a design.

This curve has a constantly changing radius and hence a constantly changing force, therefore there is only a short length over which the force is high and hence safety is increased. It is not always possible, however, to fit this type of curve between the two intersecting straights.

The road shown in figure 10.3 consists of transitions of equal length and these are discussed further in section 10.14.

10.5 Length of Transition Curve to be Used (L_T)

Whatever length is used, it must be checked to ensure that passenger discomfort is minimised. This depends on a parameter known as the *rate of change of radial acceleration* (c).

In practice, the value of c is kept below a certain maximum value and the length of curve is calculated from it.

10.5.1 Rate of Change of Radial Acceleration

Consider figure 10.4.

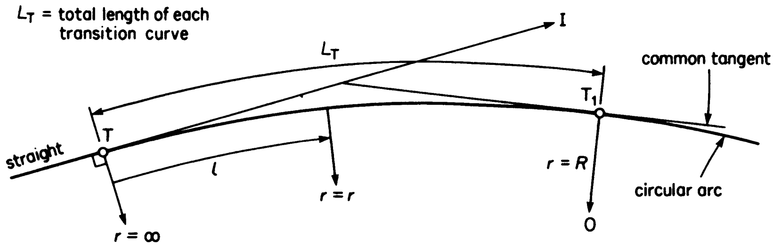


Figure 10.4

The radial force at any point on the curve is given by $P = (mv^2/r)$, but force = mass × acceleration, hence the radial acceleration at any point on the curve is given by (v^2/r).

Since v is constant for any given curve, the radial acceleration is inversely proportional to the radius. Therefore, the rate at which the radial acceleration changes is inversely proportional to the rate at which the radius changes. The faster the change in radius, the greater the rate of change of radial acceleration and hence the faster the introduction of the radial force, resulting in a greater passenger discomfort

The transition curve must, therefore, be long enough to ensure that the radius can be changed at a slow enough rate in order that the radial force can change at a rate which is acceptable to passengers.

The rate of change of radial acceleration, therefore, should be treated as a safety or comfort factor the value of which has an upper limit beyond which discomfort is too great. The DTp recommended maximum value of c is 0.3 m/s^3 although this can be increased to 0.6 m/s^3 in difficult cases.

A summary of the design method together with the final choice of a c value is given in section 10.13.

10.5.2 Length from Rate of Change of Radial Acceleration

In figure 10.4

the radial acceleration at $T_1 = (v^2/R)$ and
the radial acceleration at T = zero

Therefore, the change in radial acceleration from T to $T_1 = (v^2/R)$, but the time taken to travel along the transition curve $= L_T/v$. Hence, the rate of change of radial acceleration $= c = (v^2/R)/(L_T/v)$. Therefore

$$c = (v^3/L_T R)$$

Hence

$$L_T = (v^3/cR) \qquad \text{where } v \text{ is in m/s}$$

If v is in kph

$$L_T = (v^3/3.6^3 cR) \text{ metres}$$

and this is the formula used in the design of transition curves.

10.6 Type of Transition Curve to be Used

Although an expression for the length of the transition curve is now known, it is still not possible to set out the curve on site. This requires the equation of the transition.

In the following sections, two different transitions are considered, the *clothoid* and the *cubic parabola*.

In section 10.1 it was shown that for a transition curve the expression $rl = K$ must apply, that is, the radius of curvature must decrease in proportion to the length. The clothoid is such a curve and, because of this it is usually referred to as the ideal transition curve or the ideal transition spiral. It is discussed in section 10.7.

Another transition curve in common use is the cubic parabola and, although it does not have the property that rl is always constant, it can be used over a certain range and the design calculations involved are much simpler than those for the clothoid. The cubic parabola is discussed in section 10.8.

It is recommended that the clothoid section be studied first since much of the cubic parabola theory is based on it.

10.7 The Clothoid

Figure 10.5 shows two points M and N close together on the transition curve. ϕ is the deviation angle between the tangent at M and the straight IT; δ is the tangential angle to M from T with reference to IT; x is the offset to M from the straight IT at a distance y from T; l is the length from T to any point, M, on the curve.

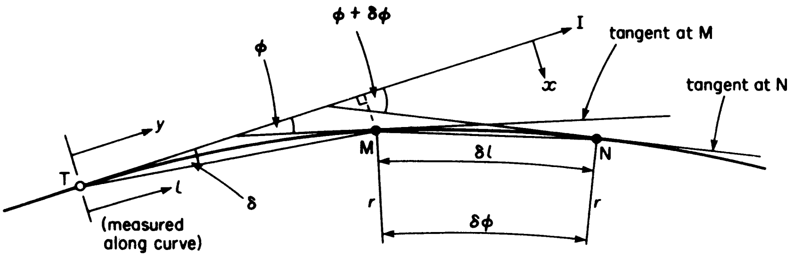


Figure 10.5

The distance MN on the curve is considered short enough to assume that the radius of curvature at both M and N is the same. Therefore

$$\delta l = r\delta\phi$$

but it has been shown that $rl = K$ is required, hence

$$\delta\phi = \frac{l}{K}\ \delta l$$

Integration gives $\phi = l^2/2K$ + constant, but when $l = 0$, $\phi = 0$, hence the constant = 0. Therefore

$$\phi = l^2/2K$$

but $K = rl = RL_{\mathrm{T}}$ hence

$$\phi = l^2/2RL_{\mathrm{T}} \ (\phi \text{ being in radians})$$

This is the basic equation of the clothoid. If its conditions are satisfied and speed is constant, radial force will be introduced uniformly.

The maximum value of ϕ will occur at the common tangent between the transition and the circular curve, that is, when $l = L_T$, hence

$$\phi_{max} = L_T/2R \text{ (in radians)}$$

10.7.1 Setting Out the Clothoid by Offsets from the Tangent Length

Figure 10.6 shows an enlarged section of figure 10.5. Since M and N are close, it can be assumed that curve length MN is equal to chord length MN and expressions for δx and δy can be derived as follows.

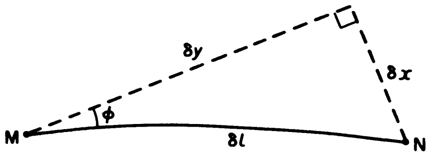


Figure 10.6

$$\delta x = \delta l \sin\phi = (\phi - \phi^3/3! + \phi^5/5! - \ldots)\delta l$$
$$= [(l^2/2K) - (l^2/2K)^3/3! + (l^2/2K)^5/5! - \ldots]\,\delta l$$

Integration gives $x = (l^3/6K) - (l^7/336K^3) + (l^{11}/42\,240K^5) - \ldots$

$$\delta y = \delta l \cos\phi = (1 - \phi^2/2! + \phi^4/4! - \ldots)\delta l$$
$$= [1 - (l^2/2K)^2/2! + (l^2/2K)^4/4! - \ldots]\,\delta l$$

Integration gives $y = [l - (l^5/40K^2) + (l^9/3456K^4) - \ldots]$

There are no constants of integration since $x = y = 0$ when $l = 0$.
These formulae can be used to set out the clothoid as follows. In figure 10.7

(1) choose l and calculate x and y;
(2) set out x at right angles to the tangent length a distance y from T towards I.

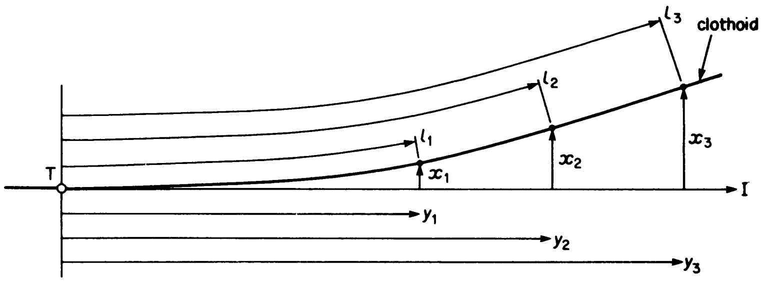


Figure 10.7

However, calculation of x and y is difficult since they are both in the form of infinite series. Special tables have been produced, however, listing values for these series up to and including the second term and the widespread use of electronic calculators has also eased the calculation process.

10.7.2 Setting Out the Clothoid by Tangential Angles

With reference to figure 10.5, $\tan \delta = x/y$, hence, by calculating x and y for a particular length l along the curve, δ can be calculated. Infinite series are again involved.

The calculation procedure and the method of setting out are identical to those for the cubic parabola and are dealt with in section 10.8.2.

10.8 The Cubic Parabola

The cubic parabola formulae are derived by making certain assumptions in the derivation of the clothoid formulae.

10.8.1 Setting Out the Cubic Parabola by Offsets from the Tangent Length

In section 10.7.1, formulae involving infinite series were developed for setting out the clothoid by means of offsets from the tangent. The assumption is now made that the second and subsequent terms in these formulae can be neglected. Hence $x = (l^3/6K)$ and $y = l$. Substituting for l gives $x = (y^3/6K)$. But $K = rl = RL_T$ hence

$$x = (y^3/6RL_T)$$

This is the basic equation of the cubic parabola and it can be used to set out the curve by offsets from the tangent lengths in a similar manner to that shown for the clothoid in figure 10.7. In this case, however, since it is assumed that the length is the same whether measured along the curve or along the tangent, the offset, x, is calculated for different values of y and set out as shown in figure 10.7.

10.8.2 Setting Out the Cubic Parabola by Tangential Angles

With reference to figure 10.5, $\tan \delta = x/y$. But $x = y^3/6RL_T$ for the cubic parabola, hence

$$\tan \delta = (y^3/6RL_T)/y = (y^2/6RL_T)$$

Here another assumption is made in that only small angles are considered. Therefore $\tan \delta = \delta$ radians, hence

$$\delta = y^2/6RL_T \text{ radians}$$

A useful relationship can be developed between δ, the tangential angle, and ϕ, the deviation angle, as follows. $\phi = l^2/2RL_T$ is the basic equation of the clothoid, but for the cubic parabola $y = l$. Therefore

$$\phi = y^2/2RL_T \text{ for the cubic parabola}$$

However

$$\delta = y^2/6RL_T \text{ for the cubic parabola}$$

Hence it follows that for the cubic parabola

$$\delta = \phi/3 \text{ and } \delta_{max} = \phi_{max}/3$$

This relationship is shown in figure 10.8.

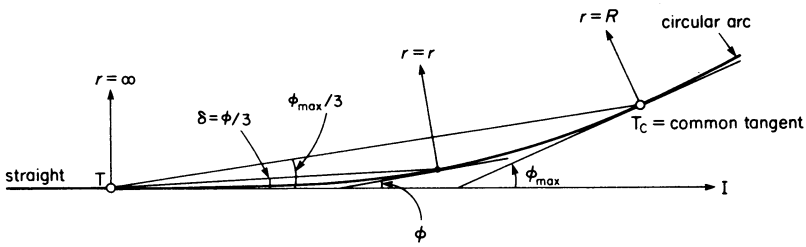


Figure 10.8

In order that the tangential angles can be set out by theodolite an expression in terms of degrees or minutes is necessary, therefore

$$\delta = (y^2/6RL_T)(180/\pi)\,60 \text{ minutes}$$

Hence

$$\delta = (1800/\pi RL_T)\,l^2 \text{ minutes}$$

since $y = l$.
The actual setting-out procedure is as follows.

Setting out the first peg

(1) l_1 is chosen as a chord length such that it is $\leqslant R/20$, where R is the minimum radius of curvature.

(2) δ_1 is calculated from l_1.

(3) A theodolite is set at T, aligned to I with a reading of zero and δ_1 is turned off.

(4) A chord of length l_1 is swung from T and lined in at point A as shown in figure 10.9.

Setting out the second and subsequent pegs

(1) δ_2 is calculated from l_2.

(2) δ_2 is set on the horizontal circle of the theodolite.

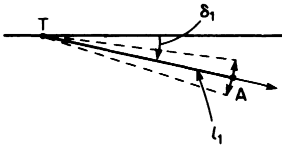


Figure 10.9

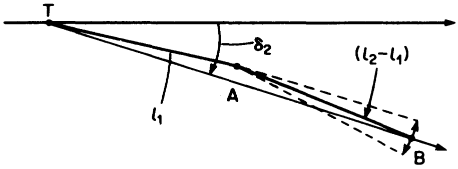


Figure 10.10

(3) A chord of length $(l_2 - l_1)$ is swung from A and lined in at point B using the theodolite as shown in figure 10.10.

(4) The system is repeated for all subsequent setting out points. Often, as with circular curves, a sub-chord is necessary at the beginning of the curve to maintain pegs at exact multiples of through chainage and hence a final sub-chord is often required to set out the common tangent between the transition and circular curves.

10.8.3 Validity of the Assumptions made in the Derivation of the Cubic Parabola Setting-out Formulae

Three assumptions are made during the derivation of the formulae.

(1) The second and subsequent terms in the expansion of sin ϕ and cos ϕ are neglected as being too small. This will depend on the value of ϕ.

(2) Tan δ is assumed to equal δ radians. Since $\delta = \phi/3$ this will also depend on the value of ϕ.

(3) y is assumed to equal l, that is, the length along the tangent is assumed to equal the length along the curve. Again, the value of ϕ will be critical since the greater the deviation, the less likely is this assumption to be true.

Hence, all the assumptions are valid and the cubic parabola can be used as a transition curve only if ϕ is below some acceptable value.

If the deviation angle remains below approximately 12°, there is no difference between the clothoid and the cubic parabola. However, beyond 12° the assumptions made in the derivation of the cubic parabola formulae begin to break down and, to maintain accuracy, further terms must be included, thereby losing the advantage offered by the simple equations. In fact, as shown in figure 10.11, once the deviation angle reaches 24°06′ γ no longer equals ϕ, even if the formulae are expressed as infinite series, and the cubic parabola becomes useless as a transition curve because its radius of curvature begins to increase with its length, that is, rl is no longer constant. Hence, in theory, the cubic parabola can be used as a transition curve only if ϕ_{max} is less than approximately 24° but, in practice, it tends to be restricted to curves where ϕ_{max} is less approximately 12° and δ_{max} is less than approximately 4° (since $\phi_{max} = \delta_{max}/3$) in order that the simple formulae can be used.

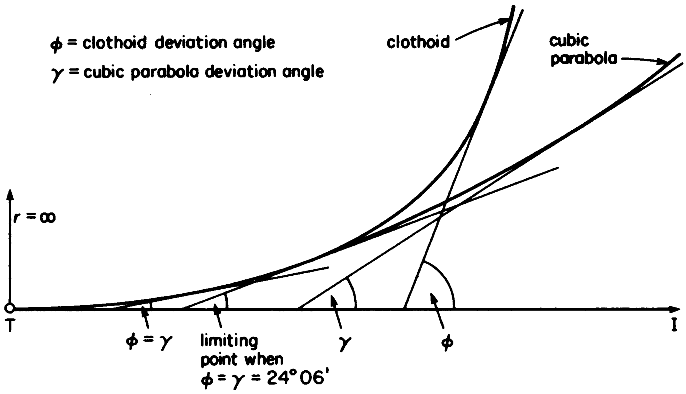


Figure 10.11

10.9 Choice of Transition Curve

In practice both the clothoid and the cubic parabola are used. The angles involved are usually well below the limiting values for the cubic parabola and hence the final choice is usually one of convenience or habit.

The remainder of the chapter is devoted to the cubic parabola.

It would appear that there is enough known about the cubic parabola to enable it to be set out on the ground. This is not true as one parameter remains to be calculated and this is known as the *shift*. It is necessary to calculate the shift in order that a value can be obtained for the tangent lengths.

10.10 The Shift of a Cubic Parabola

Figure 10.12 shows a typical composite curve arrangement. The dotted arc between V and W represents a circular curve of radius $(R + S)$ which has been replaced by a circular curve T_1T_2 of radius R plus two transition curves, entry TT_1 and exit T_2U. By doing this the original curve VW has been shifted inwards a distance S, where S = VG = WK. This distance S is known as the shift.

The tangent points and the lengths of the original curve and the new curve are not the same and the lengths of the circular arcs are not the same.

Figure 10.13 shows an enlargement of the left-hand side of figure 10.12. In quadrilateral VJT_1O

$$\text{angle OVJ} = \text{angle JT}_1\text{O} = 90^\circ$$

Hence

$$\text{angle IJT}_1 = \text{angle T}_1\text{OV} = \phi_{max}$$

$$\textit{shift} = S = VG = (\text{VH} - \text{GH}) = (\text{MT}_1 - (\text{GO} - \text{HO}))$$

But, from the cubic parabola equation

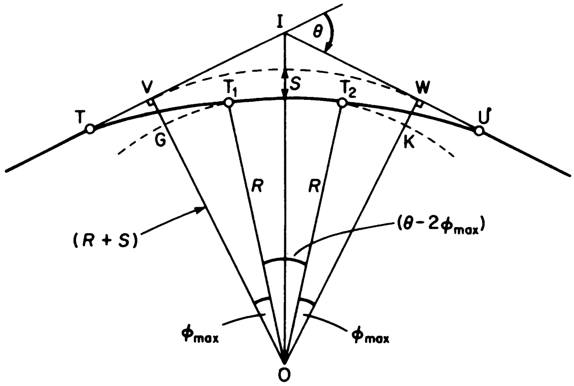


Figure 10.12

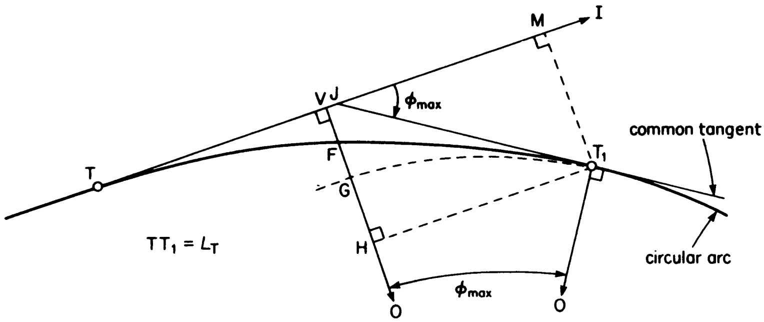


Figure 10.13

$$x = (y^3/6RL_T)$$

When $y = L_T$, $x = MT_1$, therefore

$$MT_1 = L_T{}^3/6RL_T$$

Hence

$$S = L_T{}^3/6RL_T - (R - R\cos\phi_{max})$$
$$= L_T{}^2/6R - R\,[1 - (1 - \phi^2_{max}/2! + \phi^4_{max}/4! - \ldots)]$$

This expression for S involves an infinite series but, again assuming small deviation angles, terms greater than ϕ^2_{max} can be neglected as being too small. Hence

$$S = L_T{}^2/6R - R\,\phi^2_{max}/2!$$

Therefore

$$S = L_T{}^2/6R - (R/2)\,(L_T/2R)^2$$

Hence

$$S = L_T{}^2/24R$$

and this is the formula for the shift of a cubic parabola transition curve.

10.10.1 Characteristics of the Shift and the Cubic Parabola

In figure 10.13, F is the point at which the shift meets the transition curve.

Since the angles involved are small, it is assumed that $FT_1 = GT_1$ and since GT_1 forms part of the circular curve and is equal to $R\phi_{max}$ it follows that $FT_1 = R\phi_{max}$. But $\phi_{max} = L_T/2R$, hence $FT_1 = R(L_T/2R) = L_T/2$. Hence FT must also equal $L_T/2$. Using the formula $x = y^3/6RL_T$ and the assumption that $y = l$, when $y = L_T/2$ and $x =$ VF then

$$VF = (L_T/2)^3/6RL_T = L_T{}^2/48R$$

But

$$VG = S = L_T{}^2/24R$$

Therefore

$$VF = \tfrac{1}{2} \times \text{shift} = FG$$

This gives the property that the shift at VG is bisected by the transition curve and the transition curve is bisected by the shift. Figure 10.14 shows the effect of this on the geometry of a composite curve. Hence the total length of the composite curve (L_{total}) is given by either

$$L_{total} = TF + FF' + F'U = L_T/2 + R\theta + L_T/2$$

or

$$L_{total} = TT_1 + T_1T_2 + T_2U = L_T + R(\theta - 2\phi_{max}) + L_T$$

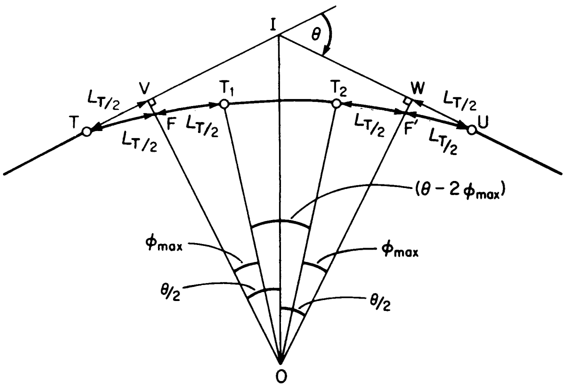


Figure 10.14

10.11 Setting Out a Composite Curve by Traditional Methods

This method assumes that the intersection point, I, can be located. The calculations involved are shown in the example in section 10.18.1 and the steps involved in the setting-out procedure are summarised below.

10.11.1 Location of the Tangent Points

With reference to figure 10.14

(1) Calculate the shift from $S = L_T{}^2/24R$.
(2) Calculate IV = $(R + S) \tan \theta/2$.
(3) VT = $L_T/2$, hence IT = $(R + S) \tan \theta/2 + L_T/2$ = IU.
(4) Measure back from I to locate T and forward from I along the other straight to locate U.

10.11.2 Setting Out the Entry Transition Curve from T to T_1

This can be done using either offsets from the tangent length or by tangential angles. The tangential angles method is preferred since a theodolite will be required to set out the circular arc. Point T_1 will be the last one established on this curve.

10.11.3 Setting Out the Central Circular Arc from T_1 to T_2

In order that this can be set out it is necessary to establish the line of the common tangent at T_1.

The final tangential angle from T to T_1 will be $\delta_{max} = \phi_{max}/3$. This is shown in figure 10.15. The procedure is as follows.

(1) Move the theodolite to T_1, align back to T with the horizontal circle reading $(180° - 2\,\delta_{max})$.
(2) Rotate the telescope in azimuth until a reading of 00°00′00″ is obtained. This is the common tangent along T_1N.
(3) Set out the circular arc from T_1 to T_2 using tangential angles calculated from the circular curve formula

$$\alpha = 1718.9 \times (\text{chord length/radius}) \text{ minutes}$$

(4) Finally, point T_2, the second common tangent point is established.

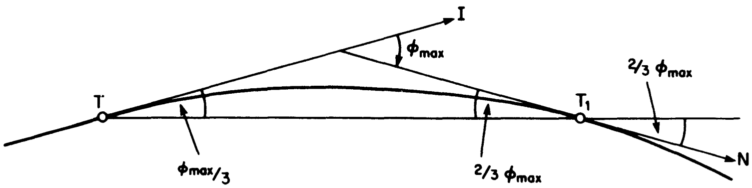


Figure 10.15

10.11.4 Setting Out the Exit Transition Curve from U to T_2

Normally this curve is set out from U to T_2. The theodolite is moved to U, aligned on I with reading 360°00′00″ and the curve is set out by the tangential angles method, the angles being subtracted from 360°00′00″. This again establishes the position of T_2 and the two positions of T_2 provide a check on the setting out.

10.12 Setting Out by Coordinates

Section 10.11 described the traditional method of establishing a curve from its tangent points. Other methods that are used are those involving coordinates. Such methods use intersection techniques and polar coordinates (bearing and distance) to establish the centre line and are very useful since they enable the centre line to be re-established as and when necessary, since during construction the centre line may have to be located several times. Such methods were discussed in section 9.9.5. They are equally applicable to transition curves and they are usually calculated with the aid of a computer and the results presented on a printout.

Many forms of computer program and printout can be used and only one example can be given here.

Table 10.2 shows a typical format for a printout and gives all the information required to set out the curve shown in figure 10.16. The curve is to be set out by

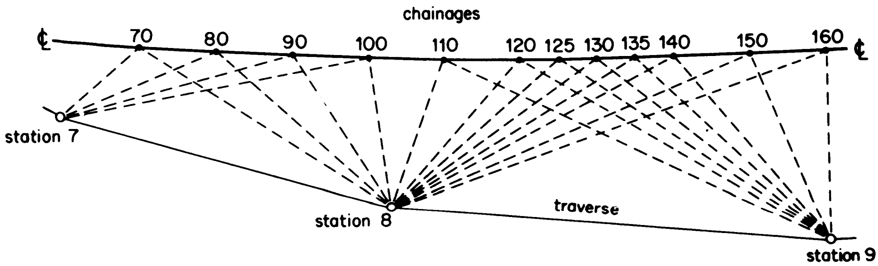


Figure 10.16

polar coordinates from nearby traverse stations, each centre-line point being established from one station and checked from another. The calculations undertaken to produce table 10.2 are summarised as follows.

(1) The coordinates of the traverse stations are found from the original site traverse.

(2) The horizontal alignment is designed and the intersection and tangent points located on the ground. They are incorporated into the original site traverse and their coordinates calculated.

(3) Suitable chord lengths are chosen to ensure that the centre line is pegged at exact multiples of through chainage and the tangential angles are calculated for both the transition curves and the central circular arc.

TABLE 10.2
Example Computer Printout Format

JOB REFERENCE J01777

PORTSMOUTH RAIL BRIDGE MAIN ALIGNMENT CH 70 TO CH 160

CENTRELINE CHAINAGE TO BE SET OUT	STATION REFERENCE NUMBER	WHOLE CIRCLE BEARING DEG.	MIN.	SEC.	HORIZONTAL DISTANCE FROM STATION TO CENTRELINE (M)	STATION REFERENCE NUMBER	WHOLE CIRCLE BEARING DEG.	MIN.	SEC.	HORIZONTAL DISTANCE FROM STATION TO CENTRELINE (M)
70	7	24	10	57	13.695	8	276	02	31	38.734
80	7	41	38	08	22.183	8	285	31	19	30.504
90	7	49	00	38	31.585	8	301	13	45	23.726
100	7	52	53	24	41.281	8	325	47	54	19.939

WHOLE CIRCLE BEARING FROM STATION 7 TO STATION 8 = 79 DEG. 12 MIN. 09 SEC.

WHOLE CIRCLE BEARING FROM STATION 8 TO STATION 7 = 259 DEG. 12 MIN. 09 SEC.

CENTRELINE CHAINAGE TO BE SET OUT	STATION REFERENCE NUMBER	WHOLE CIRCLE BEARING DEG.	MIN.	SEC.	HORIZONTAL DISTANCE FROM STATION TO CENTRELINE (M)	STATION REFERENCE NUMBER	WHOLE CIRCLE BEARING DEG.	MIN.	SEC.	HORIZONTAL DISTANCE FROM STATION TO CENTRELINE (M)
110	8	354	04	27	20.844	9	270	04	27	54.792
120	8	15	22	10	25.954	9	275	26	51	45.969
125	8	22	43	13	29.480	9	278	59	41	41.768
130	8	28	24	28	33.392	9	283	19	50	37.772
135	8	32	51	00	37.571	9	288	40	14	34.052
140	8	36	22	09	41.937	9	295	15	48	30.709
150	8	41	30	43	51.034	9	313	00	40	25.732
160	8	45	01	39	60.436	9	335	51	19	24.166

WHOLE CIRCLE BEARING FROM STATION 8 TO STATION 9 = 68 DEG. 34 MIN. 09 SEC.

WHOLE CIRCLE BEARING FROM STATION 9 TO STATION 8 = 248 DEG. 34 MIN. 09 SEC.

(4) The coordinates of the points to be established on the centre line are calculated using the chord lengths, tangential angles and the coordinates of the intersection and tangent points.

(5) The nearest two traverse stations which are visible from and which will give a good intersection to each proposed centre-line point are found and the polar coordinates calculated from each traverse station to the centre-line point.

(6) The computer repeats this procedure for all the points on the centre line and a printout is obtained with a format similar to that shown in table 10.2.

An example showing the calculations involved when a composite curve is to be set out from coordinates is given in section 10.18.2.

10.12.1 Advantages of using Coordinates

(1) If the printout is available, the work can be set out by anyone who is capable of using a theodolite. A knowledge of curve design is not necessary.

(2) When setting out from nearby traverse stations, the construction work can proceed unhindered since there is no need to set the theodolite at the tangent points.

(3) Any disturbed pegs can quickly be relocated from the traverse stations. When the tangent points are used, however, they themselves can often be lost during construction and have to be relocated.

(4) During the construction it will be necessary to relocate the centre line several times as the various stages in the operation are reached. This is easily done from nearby traverse stations using coordinates.

(5) Each point on the curve is fixed independently of any other point on the curve and this removes the chance of errors accumulating from one point to the next as may occur when setting out by tangential angles.

(6) Key sections of the curve can be set out in isolation, for example, a bridge centre line, in order that work can progress in more than one area of the site.

(7) Obstacles can be by-passed.

10.12.2 Disadvantages of using Coordinates

(1) There is very little check on the final setting out. Large errors will be noticed when the curve does not take the required shape but small errors could pass unnoticed.

(2) Often long distances are involved in the bearing and distance method and, if only tapes are available, accurate measurement can be difficult to achieve. This problem can be overcome by using either intersection methods or electromagnetic distance measuring techniques (see chapter 4).

10.13 A Design Method for a Composite Curve

Figure 10.17 shows the composite curve that is to be designed. The design is based on the fact that the composite curve must deflect the road through angle θ.

The circular curve takes $(\theta - 2\phi_{max})$ and each transition takes ϕ_{max}.

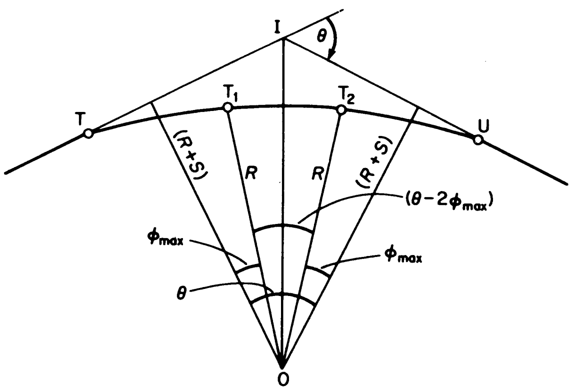


Figure 10.17

Given: design speed, v, and the road type.
Problem: to calculate a suitable curve to fit between the straights TI and IU.
Solution: Before detailing the design method, it must be noted that there are many solutions to this problem, all of them perfectly acceptable. Hence, the following method can only be a guide to design from which a suitable rather than a unique solution can be found. The procedure is based on the DTp design standards and is as follows.

(1) The deflection angle, θ, must, if possible, be accurately measured on site. This is discussed in section 9.6.

(2) Use a value of R greater than the desirable minimum radius for the design speed and road type in question and let $c = 0.3$ m/s^3, that is, start off with the recommended limiting values for both R and c so that they can be amended later if necessary.

(3) Calculate the length of each transition from $L_T = v^3/3.6^3\, cR$.

(4) Calculate the shift, S, from $S = L_T{}^2/24R$.

(5) Calculate the tangent lengths IT and IU from $(R + S)\tan\theta/2 + L_T/2$.

(6) The working drawings should show the two straights superimposed on the existing area. The calculated lengths IT and IU should now be fitted on the plan to see if they are acceptable. Owing to the *band of interest* discussed in section 9.6, it may be necessary to alter the lengths of IT and IU in order to obtain a suitable fit. This can be done by altering R and/or c.

Ideally, R should be greater than the desirable minimum value and c must be kept below 0.6 m/s^3. However, if necessary R can be reduced to the limiting value with 7 per cent superelevation to reduce the effect of the large radial force that may result.

The process is an iterative one and ends when the tangent lengths are of an acceptable length to fit the given situation.

(7) Once a suitable radius has been found, calculate ϕ_{max} from $L_T/2R$ radians.

(8) Calculate $(\theta - 2\phi_{max})$, hence the length of the circular arc from $R(\theta - 2\phi_{max})$.

(9) Calculate the superelevation (see section 10.2).
(10) Set out the curve on site using one of the methods discussed earlier.

The examples given in sections 10.18.1 and 10.18.2 show the calculations involved when setting out a composite curve.

10.13.1 Important Consideration in Design

Often a horizontal alignment is designed in conjunction with a vertical alignment and it is necessary that they should be of the same length.

Therefore, as a precaution, the total length of the composite curve should be calculated using one of the formulae given in section 10.10.1, and checked to ensure that it is greater than the required length of vertical curve. If it is not, then the length of horizontal curve should be increased to that of the vertical curve.

This need to equate the horizontal and vertical alignments is discussed in section 11.9.1.

10.14 Wholly Transitional Curves

These are curves which consist only of transitions. They can be considered as a composite curve which has a central circular arc of zero length. Figure 10.18 shows such a curve.

Wholly transitional curves have the advantage that there is only one point at which the radial force is a maximum and, therefore, the safety is increased. Unfortunately, it is not always possible to fit a wholly transitional curve into a given situation.

This section deals with the design of wholly transitional curves with equal tangent lengths only. Although it is possible to design and construct wholly transitional curves with unequal tangent lengths by using a different rate of change of radial acceleration for each half of the curve, they are rarely used and space does not permit a discussion on their method of design.

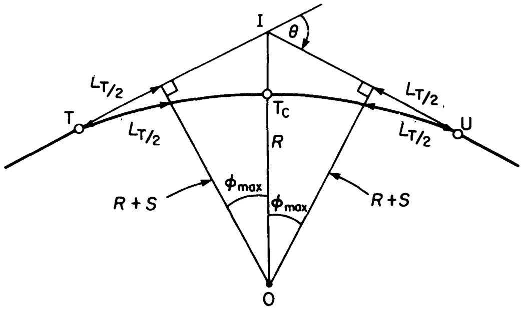


Figure 10.18 *Wholly transitional curve*

Wholly transitional curves with equal tangents have a very interesting property. With reference to figure 10.18, since the circular arc is missing, it follows that

$$\theta = 2\phi_{\text{max}} \quad \text{but} \quad \phi_{\text{max}} = L_{\text{T}}/2R$$

Hence

$$\theta = 2L_{\text{T}}/2R = L_{\text{T}}/R$$

Therefore, for a wholly transitional curve

$$\theta = L_{\text{T}}/R \tag{10.1}$$

In addition, all the other transition curve equations still apply and consequently the equation for length must still apply, that is

$$L_{\text{T}} = v^3/3.6^3\ cR \tag{10.2}$$

From equations (10.1) and (10.2)

$$R\theta = v^3/3.6^3\ cR$$

Therefore

$$R = (v^3/3.6^3\ c\theta)^{\frac{1}{2}} \text{ metres}$$

This leads to the property of wholly transitional curves that for any given two straights there is only one symmetrical wholly transitional curve that will fit between them for a given design speed if the rate of change of radial acceleration is maintained at a particular value, that is, since v and θ are usually fixed, R has a unique value if c is maintained at, say, 0.3 m/s^3.

This is, in fact, the method of designing such curves and it is summarised as follows

(1) Choose a value for c, usually near to 0.3 m/s^3.

(2) Substitute this into the equation $R = (v^3/3.6^3\ c\theta)^{\frac{1}{2}}$ and hence calculate the minimum radius of curvature.

(3) The radius value must be checked against the DTp values. R must, if possible, be greater than the desirable minimum value and must always be greater than the limiting value.

If R checks, L_{T} can be calculated using either equation (10.1) or equation (10.2).

If R does not check then the value of c must be reduced and the calculation repeated.

The example given in section 10.18.3 shows the way in which the radius value is checked.

(4) Having calculated L_{T} it is necessary to ensure that the curve will fit within the band of interest (see section 9.6). The assumption is usually made with wholly transitional curves that the length along the tangent is equal to the length of the transition curve. This is shown in figure 10.19.

However, for accurate work, it is best to use the formula previously derived for IT and IU, namely

$$\text{IT} = \text{IU} = (R + S) \tan \theta/2 + L_{\text{T}}/2$$

Hence, the tangent length is checked for fit on the working drawings. If it does not fit, it is necessary to return to the start of the calculations and change some of the

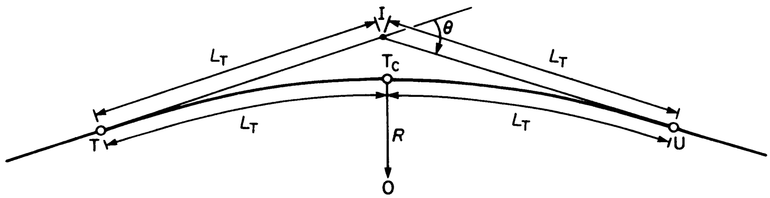


Figure 10.19

variables, either v, θ or c. It is not always possible to fit a wholly transitional curve between straights within the limits stipulated by the DTp.

(5) The superelevation is calculated and the curve is set out by either tangential angles, offsets or coordinates.

10.15 Summary of Horizontal Curve Design

In sections 9.6, 10.13 and 10.14, methods for designing wholly circular, composite and wholly transitional curves were discussed. Usually, these three techniques are combined into one general design and considered as possible solutions to the same problem, the aim being to design the best curve to fit a particular set of conditions.

Often, only the design speed and class of road are known and the problem becomes one of choosing the ideal combination of θ, R and c to fit into the band of interest concerned while maintaining current design standards.

If a vertical curve is designed in conjunction with the horizontal curve, the problem is further complicated by the need to make the two curves compatible.

Hence, when undertaken manually, the design can be tedious and time consuming. Fortunately, the iterative processes involved are ideal for solution by computer and such methods are now in widespread use. The basic steps of the design are written into the computer program and the curve parameters, v, θ, c and R together with chainage values, reduced levels and any external constraints are fed into the computer which runs the program and calculates suitable values for the radius of curvature, deflection angle, rate of change of radial acceleration, superelevation values, tangential angles, chord lengths and so on. These results are presented in list form on a printout from the computer.

In addition, if the program is suitably modified and coordinates of nearby traverse stations are fed in, as discussed in section 10.12, polar coordinates for setting out the curve can be obtained similar to those shown in table 10.2. The examples given in section 10.18 show the steps involved.

10.16 Computer Aided Road Design

In highway alignment design, many factors such as design standards, topography, environment and the visual impact of the road have to be considered. This creates a demand for a number of alternative routes to be studied for any given road scheme and, for each route, the ability to produce a visual representation or model of the proposed road is highly desirable as a means of checking design work and for presentation at public hearings.

The preparation of different alignments by hand methods involves much work and the drawing of perspective views for each design manually is an almost impossible task. However, by using a computer system in road design, these problems can be overcome to such an extent that many trial designs can be studied and presented with relative ease.

At present, various computer systems are available for highway design and, in Great Britain, the two most widely used systems are BIPS (British Integrated Program System for highway design) which is operated by the Department of Transport, and MOSS (MOdelling SyStems) which is operated by MOSS Systems Ltd. Although these and other computer systems use widely differing program suites to produce a road design, the general concepts are similar and the block diagram of figure 10.20 shows the various stages involved. These are described briefly as follows.

Initially, a digital terrain model (DTM) is produced of the area covered by the corridor or band of interest. The DTM is formed using air or ground survey methods as described in section 8.8 and is essentially a map of the area stored digitally in a computer. In addition to surface information, the results of any site investigations can also be stored in the DTM. Such data may include ground-water conditions, geotechnical characteristics of the area and any other properties which may affect the design.

After the DTM has been completed, many trial alignments can be studied by the computer. For horizontal alignments, two methods are used by the computer: the conventional method in which straight sections of road are joined by circular and/or transition curves (see chapter 9 and previous sections of this chapter) or a new technique based on curves known as *cubic splines*. A cubic spline is a curve of continually changing radius, the equation of which takes the form of a cubic polynomial. Cubic splines are specified to fit between given location points, for example, straights, end points of a scheme, points the curve must pass through to avoid obstacles and so on.

For vertical alignment design (see chapter 11), three methods are used by the computer: the traditional method based on intersecting gradients and parabolic curves, an extension of this traditional method in which parabolic curves are fitted

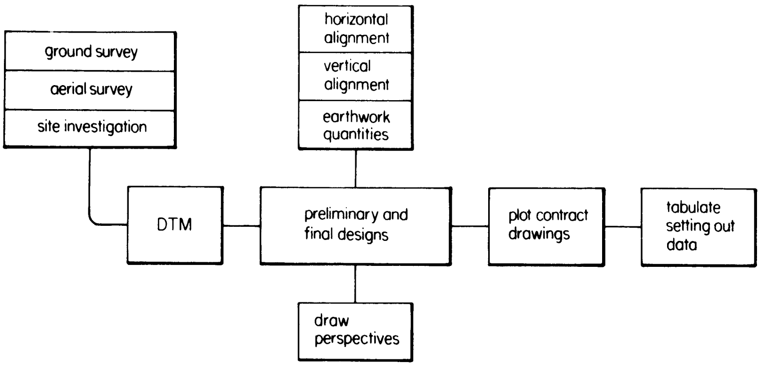


Figure 10.20 *Stages involved in computer aided road design*

to various fixed elements along the horizontal alignment such as sections of gradient, bridges, tie-ins to existing road junctions and so on, and the cubic spline method mentioned above.

Each combined horizontal and vertical alignment, as designed by the computer, is passed through the DTM and the computer produces a longitudinal section, as many cross sections as desired and an estimate of the earthwork quantities involved. This considerably shortens the time required to carry out these procedures by manual methods, details of which can be found in chapters 12 and 13. In addition, for any alignment, the computer system can also produce perspective drawings showing views along the proposed road. Such drawings can be used for visually checking the design and for preparing material for reports, exhibitions and public enquiries. The flexibility of the BIPS and MOSS systems enables any amount of design data to be combined with DTMs and it is possible to carry out a much more thorough preliminary design than that which could ever be undertaken by conventional methods.

As soon as the optimum alignment has been chosen, further data is entered into the DTM to enable a set of contract drawings to be produced by the computer interfaced with a suitable plotter. If all the relevant information for the optimum road alignment is computerised, these drawings will consist of a series of plans showing all aspects of the road construction including longitudinal and cross sections along the main alignment and also at interchanges, junctions, sliproads and so on. Based on these, schedules of earthwork quantities can be produced by the computer along any section of road and setting-out tables can be computed giving angles and distances relative to existing survey stations.

The greatest benefits of using a computer system in road design are the ability to investigate different alignments and a reduction in the overall time taken for the design and production of contract drawings. If the design should change at any time, these changes can be entered reasonably quickly into the system and modified drawings produced.

Since the development of the necessary software for a computer aided design facility requires personnel with extensive computing experience and a computer with a large storage capacity, they can be set up only by government departments such as the Department of Transport or by specialist firms such as MOSS Systems Ltd. In the case of the Department of Transport, the expense involved in developing BIPS is justified by the large volume of design work undertaken by the department each year. For any smaller organisation requiring a design facility, a software package can be purchased (for example, from MOSS) and programmed into a computer. This can involve a considerable capital outlay which limits the use of computer aided design systems at present. In the near future, however, it is expected that the development of microcomputers will reduce the costs involved.

Further information on computer aided road design systems can be found in references given in the following section.

10.17 Further Reading

Department of Transport, Roads and Local Transport Directorate, *Departmental Standard TD 9/81, Road Layout and Geometry: Highway Link Design*

(Department of Transport, 1981).
Department of Transport, Highways and Traffic Directorate, *Departmental Advice Note TA 43/84: Highway Link Design* (Department of Transport, 1984).
Department of Transport, *BIPS 3 Introductory Guide* (Department of Transport).
MOSS Systems Ltd, *MOSS Surface Modelling by Computer* (1984).

10.18 Worked Examples

10.18.1 Setting Out a Composite Curve by the Tangential Angles Method

Question

The deflection angle between two straights is measured as 14°28′26″. The straights are to be joined by a composite horizontal curve consisting of a central circular arc and two transition curves of equal length.

The design speed of the road is 85 kph and the radius of the circular curve is 600 m.

If the through chainage of the intersection point is 461.34 m, draw up the setting-out table for the three curves at exact 20 m multiples of through chainage using the tangential angles method. The rate of change of radial acceleration should be taken as 0.3 m/s^3.

Solution

Consider figure 10.21.

Design of entry transition, from T to T_1

$$L_T = v^3/3.6^3\ cR = (85^3/3.6^3 \times 0.3 \times 600) = 73.13 \text{ m}$$

$$S = L_T{}^2/24R = (73.13^2/24 \times 600) = 0.37 \text{ m}$$

$$\text{IT} = (R + S)\tan\theta/2 + L_T/2 = 76.24 + 36.56 = 112.80 \text{ m}$$

Therefore

through chainage of T $= 461.34 - 112.80 = 348.54$ m
through chainage of $\text{T}_1 = 348.54 + 73.13 = 421.67$ m

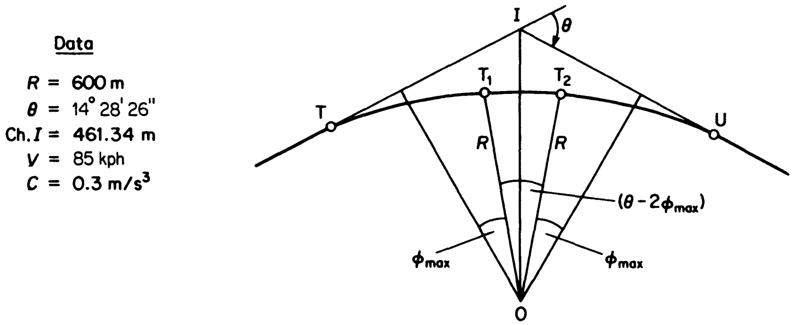


Figure 10.21

TABLE 10.3

Through Chainage (m)	Chord Length (m)	ℓ (m)	δ (minutes)	Cumulative Clockwise Tangential Angle from T relative to TI °	′	″
348.54 (T)	0	0	0	00	00	00
360.00 (C_1)	11.46	11.46	1.715	00	01	43 (δ_1)
380.00 (C_2)	20.00	31.46	12.924	00	12	55 (δ_2)
400.00 (C_3)	20.00	51.46	34.579	00	34	35 (δ_3)
420.00 (C_4)	20.00	71.46	66.681	01	06	41 (δ_4)
421.67 (T_1)	1.67	73.13	69.834	01	09	50 (δmax)
	Σ73.13 (checks)					

Therefore, to keep to exact 20 m through chainage values, the chord lengths for the entry transition curve are as follows

initial sub-chord length = 11.46 m

general chord length = 20.00 m

final sub-chord length = 1.67 m

Using the formula for tangential angles, $\delta = (1800\, l^2)/(\pi R L_T)$, table 10.3 is obtained. As a further check on table 10.3, $\phi_{max}/3$ should be calculated and compared with δ_{max}

$$\phi_{max} = (L_T/2R) \text{ rad} = 209.50 \text{ min} = 03°29'30''$$

Hence

$$\phi_{max}/3 = 01°09'50'' \quad \text{(checks)}$$

Design of the central circular arc, from T_1 to T_2

The circular arc takes $(\theta - 2\phi_{max})$; $\phi_{max} = 03°29'30''$, hence $2\phi_{max} = 06°59'00''$ Therefore

$$(\theta - 2\phi_{max}) = 07°29'26'' = 0.130\,73 \text{ rad}$$

Therefore

$$\textit{length of circular arc} = L_c = R(\theta - 2\phi_{max})$$
$$= 600\,(0.130\,73) = 78.44$$

Hence

$$\textit{through chainage of } T_2 = 421.67 + 78.44 = 500.11 \text{ m}$$

Therefore, using 20 m chords and keeping to exact 20 m multiples of through chainage, the chord lengths for the circular arc are as follows

initial sub-chord length = 18.33 m

general chord length = 20.00 m

final sub-chord length = 0.11 m

TABLE 10.4

Through Chainage (m)	Chord Length (m)	Tangential Angle for each chord °	'	"	Cumulative Clockwise Tangential Angle from T relative to the common tangent °	'	"
421.67 (T_1)	0	00	00	00	00	00	00
440.00 (C_5)	18.33	00	52	31 (α_1)	00	52	31
460.00 (C_6)	20.00	00	57	18 (α_2)	01	49	49
480.00 (C_7)	20.00	00	57	18 (α_3)	02	47	07
500.00 (C_8)	20.00	00	57	18 (α_4)	03	44	25
500.11 (T_2)	0.11	00	00	19 (α_5)	03	44	44
	Σ78.44 (checks)						

Using the formula for circular curve tangential angles, $\alpha = 1718.9$ (chord length/radius) min, table 10.4 is obtained. As a check on table 10.4, the final cumulative tangential angle should equal $(\theta - 2\phi_{\max})/2$ within a few seconds

$$(\theta - 2\phi_{\max})/2 = 03°44'43'' \quad \text{(checks)}$$

Design of the exit transition curve, from U to T_2 (that is, in the opposite direction)

Since the curve is symmetrical, the length of the exit transition again equals 73.13 m. Therefore

through chainage of U = 500.11 + 73.13 = 573.24 m

To keep to exact 20 m multiples of through chainage, the chord lengths for the exit transition curve, working from U to T_2, are as follows

initial sub-chord length from U = 13.24 m

general chord length = 20.00 m

final sub-chord length to T_2 = 19.89 m

Again, using $\delta = (1800\, l^2)/(\pi R L_T)$ min, the setting-out table shown in table 10.5 is obtained.

TABLE 10.5

Through Chainage (m)	Chord Length (m)	l (m)	δ °	'	"	Cumulative Clockwise Tangential Angle from U relative to UI °	'	"
573.24 (U)	0	0	00	00	00	360	00	00
560.00 (C_{11})	13.24	13.24	00	02	17 (δ_{11})	359	57	43
540.00 (C_{10})	20.00	33.24	00	14	26 (δ_{10})	359	45	34
520.00 (C_9)	20.00	53.24	00	37	01 (δ_9)	359	22	59
500.11 (T_2)	19.89	73.13	01	09	50 (δ_{max})	358	50	10
	Σ73.13 (checks)							

The check that $\delta_{max} = \phi_{max}/3$ must again be applied

$$\delta_{max} = (360° - 358°50'10'') = 01°09'50''$$

$$\phi_{max}/3 = 01°09'50'' \quad \text{(checks)}$$

The tangential angles for the exit transition curve are subtracted from 360° since it is set out from U to T_2. The two positions of T_2 provide a check on the setting out.

10.18.2 Setting Out a Composite Curve by Coordinate Methods

Question

The composite curve calculated in the worked example in section 10.18.1 is to be set out by bearing and distance methods from two horizontal control points G and H. The intersection point, I, and the entry tangent point, T, have been set out on site and the coordinates of these, together with those of points G and H are listed in table 10.6. Using the data calculated in the worked example in section 10.18.1, calculate

TABLE 10.6

Point	mE	mN
G	727.61	893.83
H	940.57	886.28
I	789.14	863.72
T	704.95	788.64

(1) the coordinates of all the pegs that are to be placed along the centre line,
(2) the bearing GH that must be set on the theodolite at G and the bearings and horizontal lengths from G that are necessary to set out all the pegs on the centre line using a combined theodolite and EDM system.

Solution

Figure 10.22 shows all the points to be set out. Their chainage values and the required tangential angles and chords are listed in tables 10.3, 10.4 and 10.5.

(1) Coordinates of all the points on the centre line

Coordinates of C_1
From figure 10.23 and table 10.3

$$\text{bearing } TC_1 = \text{bearing } TI + \delta_1$$

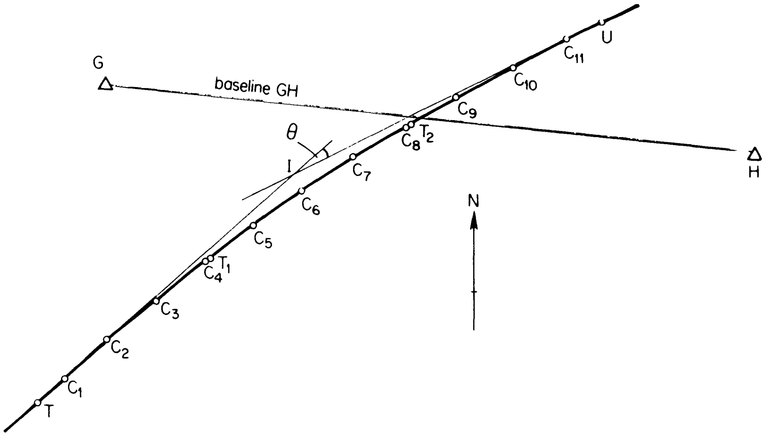


Figure 10.22

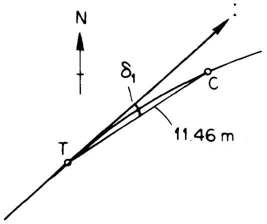


Figure 10.23

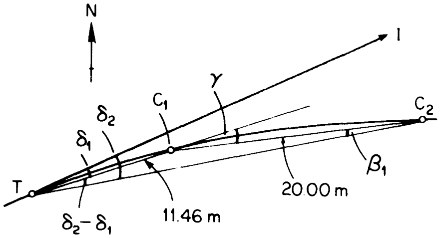


Figure 10.24

But

$$\Delta E_{\mathrm{TI}} = E_{\mathrm{I}} - E_{\mathrm{T}} = 789.14 - 704.95 = 84.19 \text{ m}$$

$$\Delta N_{\mathrm{TI}} = N_{\mathrm{I}} - N_{\mathrm{T}} = 863.72 - 788.64 = 75.08 \text{ m}$$

and, from a rectangular/polar conversion

$$\text{bearing TI} = 48°16'25''$$

Hence

$$\text{bearing TC}_1 = 48°16'25'' + 00°01'43'' = 48°18'08''$$

Therefore, since the horizontal length of TC_1 = 11.46 m

$$\Delta E_{\mathrm{TC}_1} = 11.46 \sin 48°18'08'' = +\ 8.557 \text{ m}$$

$$\Delta N_{\mathrm{TC}_1} = 11.46 \cos 48°18'08'' = +\ 7.623 \text{ m}$$

Therefore, *the coordinates of C_1* are

$$E_{\mathrm{C}_1} = E_{\mathrm{T}} + (\Delta E_{\mathrm{TC}_1}) = 704.95 + 8.557 = \mathbf{713.507\ m}$$

$$N_{C_1} = N_T + (\Delta N_{TC_1}) = 788.64 + 7.623 = \mathbf{796.263\ m}$$

These are retained with three decimal places for calculation purposes but are finally rounded to two decimal places.

Coordinates of C_2
With reference to figure 10.24, application of the Sine Rule in triangle TC_1C_2 gives

$$\frac{TC_1}{\sin \beta_1} = \frac{C_1C_2}{\sin (\delta_2 - \delta_1)}$$

Substituting values from table 10.3 gives

$$\sin \beta_1 = \frac{11.46}{20.00} (\sin 00°11'12'')$$

Hence

$$\beta_1 = 00°06'25''$$

Therefore

$$\gamma = \beta_1 + (\delta_2 - \delta_1) = 00°17'37''$$

and

$$\text{bearing } C_1C_2 = \text{bearing } TC_1 + \gamma$$
$$= 48°18'08'' + 00°17'37'' = 48°35'45''$$

Therefore, the *coordinates of C_2* are obtained as follows

$$\Delta E_{C_1C_2} = 20.00 \sin 48°35'45'' = +\ 15.001\ \text{m}$$

$$\Delta N_{C_1C_2} = 20.00 \cos 48°35'45'' = +\ 13.227\ \text{m}$$

$$E_{C_2} = E_{C_1} + 15.001 = \mathbf{728.508\ m}$$

$$N_{C_2} = N_{C_1} + 13.227 = \mathbf{809.490\ m}$$

Coordinates of C_3
With reference to figure 10.25, the chord length TC_2 can be taken to equal the

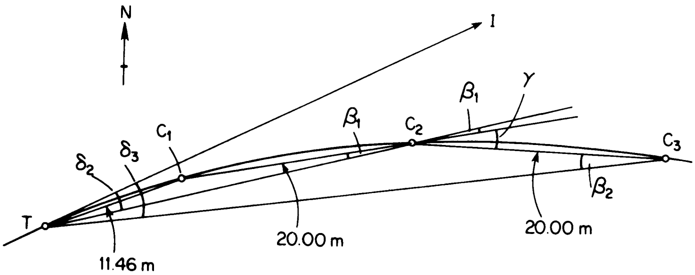


Figure 10.25

curve length TC_2, that is

$$TC_2 = TC_1 + C_1C_2 = 31.46 \text{ m}$$

In triangle TC_2C_3

$$\frac{TC_2}{\sin \beta_2} = \frac{C_2C_3}{\sin (\delta_3 - \delta_2)}$$

$$\sin \beta_2 = \frac{31.46}{20.00} (\sin 00°21'40'')$$

$$\beta_2 = 00°34'05''$$

And, since $(\gamma + \beta_1) = (\delta_3 - \delta_2) + \beta_2$

$$\gamma = 00°21'40'' + 00°34'05'' - 00°06'25''$$
$$= 00°49'20''$$

and

$$\text{bearing } C_2C_3 = \text{bearing } C_1C_2 + \gamma$$
$$= 48°35'45'' + 00°49'20''$$
$$= 49°25'05''$$

Therefore, the *coordinates of* C_3 are obtained as follows

$$\Delta E_{C_2C_3} = 20.00 \sin 49°25'05'' = +\ 15.190 \text{ m}$$

$$\Delta N_{C_2C_3} = 20.00 \cos 49°25'05'' = +\ 13.011 \text{ m}$$

$$E_{C_3} = E_{C_2} + 15.190 = \mathbf{743.698\ m}$$

$$N_{C_3} = N_{C_2} + 13.011 = \mathbf{822.501\ m}$$

Coordinates of C_4 and T_1

The coordinates of C_4 and T_1 are calculated from those of C_3 and C_4 respectively by repeating the procedure used to calculate the coordinates of C_3 from those of C_2. The values obtained are as follows

$$C_4 = \mathbf{759.188\ mE,\ 835.152\ mN}$$

$$T_1 = \mathbf{760.498\ mE,\ 836.187\ mN}$$

Coordinates of C_5

Point C_5 lies on the central circular arc as shown in figure 10.26. From this figure and the data in table 10.4

$$\text{bearing } T_1Z = \text{bearing TI} + \phi_{max}$$
$$= 48°16'25'' + 03°29'30'' = 51°45'55''$$

and

$$\text{bearing } T_1C_5 = \text{bearing } T_1Z + \alpha_1$$
$$= 51°45'55'' + 00°52'31'' = 52°38'26''$$

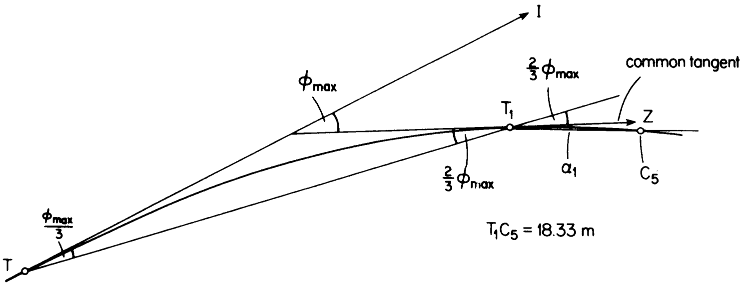


Figure 10.26

Hence the *coordinates of C_5* are obtained as follows

$$\Delta E_{T_1C_5} = 18.33 \sin 52°38'26'' = +\ 14.569 \text{ m}$$

$$\Delta N_{T_1C_5} = 18.33 \cos 52°38'26'' = +\ 11.123 \text{ m}$$

$$E_{C_5} = E_{T_1} + 14.569 = \mathbf{775.067\ m}$$

$$N_{C_5} = N_{T_1} + 11.123 = \mathbf{847.310\ m}$$

Coordinates of C_6
With reference to figure 10.27 and table 10.4

$$\lambda = (\alpha_1 + \alpha_2) = 01°49'49''$$

$$\text{bearing } C_5C_6 = \text{bearing } T_1C_5 + \lambda$$

$$= 52°38'26'' + 01°49'49'' = 54°28'15''$$

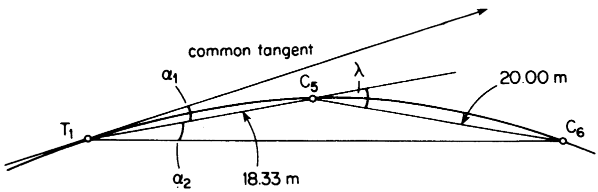


Figure 10.27

And the *coordinates of C_6* are obtained as follows

$$\Delta E_{C_5C_6} = 20.00 \sin 54°28'15'' = +\ 16.276 \text{ m}$$

$$\Delta N_{C_5C_6} = 20.00 \cos 54°28'15'' = +\ 11.622 \text{ m}$$

$$E_{C_6} = E_{C_5} + 16.276 = \mathbf{791.343\ m}$$

$$N_{C_6} = N_{C_5} + 11.622 = \mathbf{858.932\ m}$$

Coordinates of C_7, C_8 and T_2
These coordinates are calculated using procedures similar to those used in the worked example in section 9.15.2 in the Circular Curves chapter. The values obtained are as follows

$$\mathbf{C_7 = 807.998\ mE, 870.005\ mN}$$
$$\mathbf{C_8 = 825.013\ mE, 880.517\ mN}$$
$$\mathbf{T_2 = 825.108\ mE, 880.573\ mN}$$

Coordinates of U, C_{11}, C_{10}, C_9 and T_2
Using the deflection angle, bearing IU is calculated from

$$\text{bearing IU} = \text{bearing TI} + \theta$$
$$= 48°16'25'' + 14°28'26'' = 62°44'51''$$

Using the tangent length IU, the *coordinates of U* are calculated from those of I as follows

$$\Delta E_{\text{IU}} = 112.80 \sin 62°44'51'' = +\ 100.279\ \text{m}$$
$$\Delta N_{\text{IU}} = 112.80 \cos 62°44'51'' = +\ \ 51.653\ \text{m}$$
$$E_{\text{U}} = E_{\text{I}} + 100.279 = \mathbf{889.419\ m}$$
$$N_{\text{U}} = N_{\text{I}} + 51.653 = \mathbf{915.373\ m}$$

Starting from U and working back to T_2, the *coordinates of points C_{11}, C_{10}, C_9 and T_2* are calculated by repeating the procedures used to calculate the coordinates of points C_1, C_2, C_3, C_4 and T_1. The data required for this is given in table 10.5. The coordinate values obtained are as follows

$$\mathbf{C_{11} = 877.653\ mE, 909.302\ mN}$$
$$\mathbf{C_{10} = 859.933\ mE, 900.028\ mN}$$
$$\mathbf{C_9 = 842.356\ mE, 890.486\ mN}$$
$$\mathbf{T_2 = 825.109\ mE, 880.579\ mN}$$

The coordinates of T_2 are calculated twice and this provides a check on the calculations. In this example, the two sets of coordinates for T_2 differ by 0.001 m in the eastings and by 0.006 m in the northings which is perfectly acceptable.

The coordinates of all the points on the curve are listed in table 10.7 and have been rounded to two decimal places.

(2) Setting out data from point G

The bearings and lengths are calculated from the coordinates listed in table 10.7 using one of the methods discussed in section 5.10. The required bearings and lengths are also listed in table 10.7.

TABLE 10.7

Point	Through Chainage (m)	Coordinates mE	Coordinates mN	Bearing from G °	'	"	Horizontal Length from G (m)
T	348.54	704.95	788.64	192	09	25	107.60
C_1	360.00	713.51	796.26	188	13	23	98.58
C_2	380.00	728.51	809.49	179	23	19	84.34
C_3	400.00	743.70	822.50	167	17	18	73.12
C_4	420.00	759.19	835.15	151	42	43	66.64
T_1	421.67	760.50	836.19	150	17	26	66.36
C_5	440.00	775.07	847.31	134	25	37	66.46
C_6	460.00	791.34	858.93	118	42	22	72.66
C_7	480.00	808.00	870.01	106	30	17	83.84
C_8	500.00	825.01	880.52	97	46	53	98.31
T_2	500.11	825.11	880.58	97	44	20	98.40
C_9	520.00	842.36	890.49	91	40	02	114.80
C_{10}	540.00	859.93	900.03	87	19	02	132.47
C_{11}	560.00	877.65	909.30	84	06	48	150.84
U	573.24	889.42	915.37	82	25	03	163.24

Bearing GH = 92° 01' 50"

10.18.3 A Wholly Transitional Curve

Question

Part of a proposed rural road consists of two straights which intersect at an angle of 168°16′. These are to be joined using a wholly transitional horizontal curve having equal tangent lengths. The design speed of the road is to be 100 kph and the rate of change of radial acceleration 0.20 m/s³.

Calculate the minimum radius of the curve and comment on its suitability.

Solution

Consider figure 10.28. From section 10.14

$$R = (v^3/3.6^3\ c\theta)^{\frac{1}{2}}$$

But

$$\theta = 11°44' = 0.20479 \text{ rad}$$

Therefore

$$R = (100^3/3.6^3 \times 0.20 \times 0.20479)^{\frac{1}{2}} = \mathbf{723.40\ m}$$

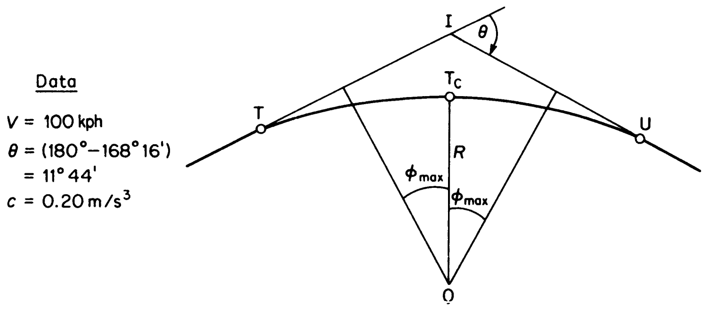


Figure 10.28

From table 10.1, the desirable minimum radius for this road is 720 m, the absolute minimum radius is 510 m and the limiting radius at sites of special difficulty is 360 m, hence this design is acceptable. If the radius had been less than the desirable minimum it would have been advisable to alter one of the variables, either v, θ or c, to increase the radius above the desirable minimum value.

11

Vertical Curves

In the same way as horizontal curves are used to connect intersecting straights in the horizontal plane, vertical curves are used to connect intersecting straights in the vertical plane. These straights are usually referred to as *gradients*.

As with horizontal curves, vertical curves are designed for particular speed values and the design speed is constant for each particular vertical curve.

11.1 Gradients

These are usually expressed as percentages, for example, 1 in 50 = 2 per cent, 1 in 25 = 4 per cent. The Department of Transport (DTp) recommend *desirable* and *absolute maximum gradient values* for all new highways and these are shown in table 11.1. Wherever possible, the desirable maximum values should not be exceeded. Any gradient steeper than 4 per cent on motorways and 8 per cent on all other highways is considered to be substandard.

For drainage purposes the channels should have a minimum gradient of 0.5 per cent. This is achieved on level sections of road by steepening the channels between gullies while the road itself remains level.

Further details concerning gradients can be found in the DTp publications referenced in section 11.15.

TABLE 11.1
(published here by permission of the Controller of Her Majesty's Stationery Office)

Type of Road	Desirable Maximum Gradient	Absolute Maximum Gradient
Motorways	3%	4%
Dual Carriageways	4%	8%
Single Carriageways	6%	8%

In the design calculations, which are discussed later in the chapter, the *algebraic difference* between the gradients is used. This necessitates the introduction of the sign convention that gradients rising in the direction of increasing chainage are considered to be positive and those falling are considered to be negative.

This leads to the *six* different types of vertical curve. These are shown in figure 11.1 together with the value of the algebraic difference (A). Note that A can be either positive or negative and is calculated in the direction of increasing chainage.

Throughout the remainder of this chapter, reference will be made to the terms *crest curve* and *sag curve* and, in order to avoid confusion, these terms are defined as follows. A *crest curve*, which can also be referred to as a summit or hogging curve, is one for which the algebraic difference of the gradients is positive, and a *sag curve*, which can also be referred to as a valley or sagging curve, is one for which the algebraic difference of the gradients is negative.

Hence, in figure 11.1, (*a*), (*b*) and (*f*) are crest curves and (*c*), (*d*) and (*e*) are sag curves.

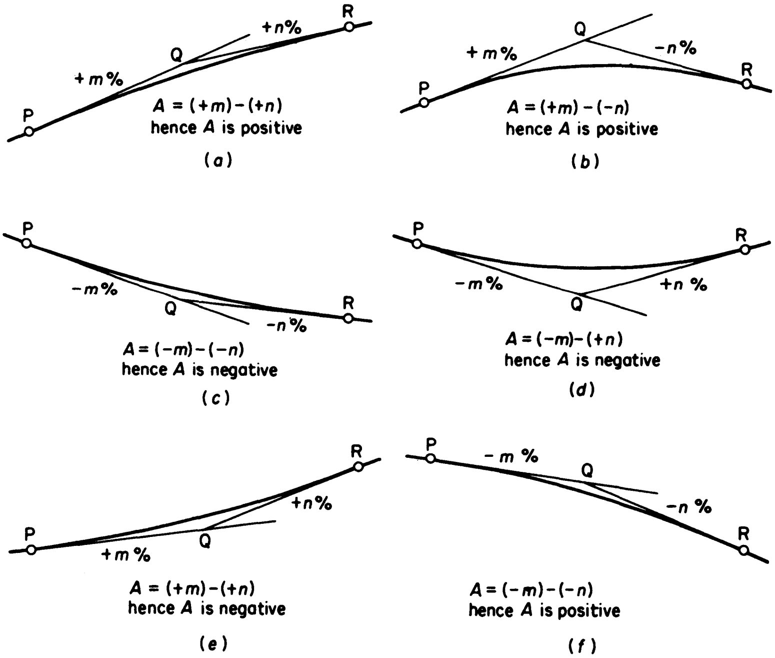


Figure 11.1 *Types of vertical curve*

11.2 Purposes of Vertical Curves

There are two main requirements in the design and construction of vertical curves.

11.2.1 Adequate Visibility

In order that vehicles travelling at the design speed can stop or overtake safely it is essential that oncoming vehicles or any obstructions in the road can be seen clearly and in good time.

This requirement is achieved by the use of *sight distances* and *K-values* which are discussed in sections 11.6 and 11.7.

11.2.2 Passenger Comfort and Safety

As the vehicle travels along the curve a radial force, similar to that which occurs in horizontal curves, acts on the vehicle in the vertical plane. This has the effect of trying to force the vehicle away from the centre of curvature of the vertical curve. In crest design this could cause the vehicle to leave the road surface, as in the case of hump-back bridges, while in sag design the underside of the vehicle could come into contact with the surface, particularly where the gradients are steep and opposed. This results in both discomfort and danger to passengers travelling and must, therefore, be minimised. This is achieved firstly by restricting the gradients (see table 11.1), which has the effect of reducing the force, and secondly by choosing a suitable type and length of curve such that this reduced force is introduced as gradually and uniformly as possible. The *K-values* discussed in section 11.7 also ensure that sufficient comfort is provided.

11.3 Type of Curve Used

In practice, owing to the restrictions placed on the gradients, vertical curves can be categorised as *flat*; the definition of a flat curve is that if its length is L_{V} and its radius R then $L_{\mathrm{V}}/R < 1/10$. This definition does assume that the vertical curve forms part of a circle of radius R but, again owing to the restricted gradients, there is no appreciable difference between a circular arc, an ellipse or a parabola and the definition can be applied to all three types of curve by approximating the value of R.

The final choice of curve is governed by the requirement for passenger safety and comfort discussed in section 11.2.2. In practice a *parabolic curve* is used to achieve a uniform rate of change of gradient and therefore a uniform introduction of the vertical radial force. This uniformity of rate of change of gradient is shown as follows

$$x = cy^2,\ \mathrm{d}x/\mathrm{d}y = 2cy,\ \mathrm{d}^2x/\mathrm{d}y^2 = 2c = \text{constant}$$

11.4 Assumptions made in Vertical Curve Calculations

The choice of a parabola simplifies the calculations and further simplifications are possible if certain assumptions are made. Consider figure 11.2, which is greatly exaggerated for clarity and shows a parabolic vertical curve having equal tangent lengths joining two intersecting gradients PQ and QR.

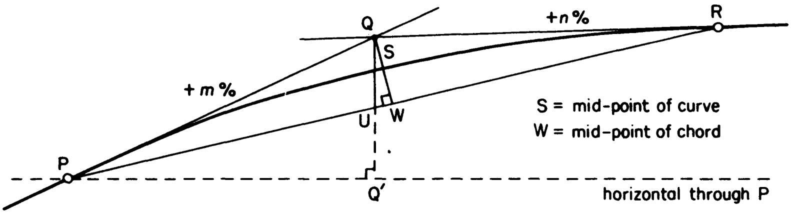


Figure 11.2

The assumptions are as follows

(1) Chord PWR = arc PSR = PQ + QR.

(2) Length along tangents = horizontal length, that is PQ = PQ′.

Assumptions (1) and (2) are very important since they are saying that the length is the same whether measured along the tangents, the chord, the horizontal or the curve itself.

(3) QU = QW, that is, there is no difference in lengths measured either in the vertical plane or perpendicular to the entry tangent length.

In general, vertical curves are designed such that the two tangent lengths are equal, that is, PQ = QR, but it is possible to design vertical curves with unequal tangents and these are discussed in section 11.13.

These assumptions are valid if the DTp recommendations for gradients as listed in table 11.1 are adhered to.

11.5 Equation of the Vertical Curve

Since the curve is to be parabolic, the equation of the curve will be of the form $x = cy^2$, y being measured along the tangent length and x being set off at right angles to it. In fact, from the assumptions, x can also be set off in a vertical direction without introducing any appreciable error.

The basic equation is usually modified to a general equation containing some of the parameters involved in the vertical curve design. This general equation will be developed for the equal tangent length crest curve shown in figure 11.3, but the same equation can be derived for sags and applies to all six possible combinations of gradient.

Consider figure 11.3. Let QS = e and let the total length of the curve = L_V. Using the assumptions

$$\text{level of Q above P} = (m/100)(L_V/2) = (mL_V/200)$$

$$\text{level of R below Q} = (n/100)(L_V/2) = (nL_V/200)$$

Hence

$$\text{level of R above P} = (mL_V/200) - (nL_V/200) = (m-n)L_V/200$$

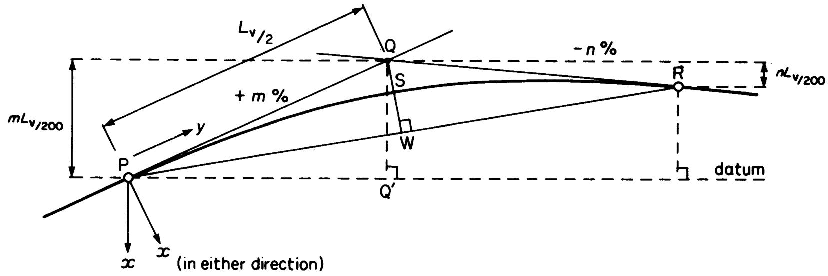


Figure 11.3

But, from the assumptions, PW = WR, therefore,

$$\text{level of W above P} = (m - n)L_V/400$$

But, from the properties of the parabola

$$QS = QW/2 = SW$$

Therefore

$$QS = \tfrac{1}{2}(mL_V/200 - (m - n)L_V/400) = (m + n)L_V/800$$

But $(m + n)$ = algebraic difference of the gradients = A, therefore

$$QS = e = L_V A/800$$

The equation of the parabola is $x = cy^2$, therefore at point Q, when $y = L_V/2$, $x = e$, hence

$$e = c(L_V/2)^2$$

Therefore

$$c = e/(L_V/2)^2$$

Therefore, substituting in the equation of the parabola gives

$$x = ey^2/(L_V/2)^2$$

But, from above

$$e = L_V A/800$$

Therefore

$$x = Ay^2/200L_V$$

11.6 Sight Distances

The length of curve to be used in any given situation depends on the *sight distance*. This is simply the distance of visibility from one side of the curve to the other.

There are two categories of sight distance

(1) *Stopping Sight Distance (SSD)* which is the theoretical forward sight distance required by a driver in order to stop safely and comfortably when faced with an unexpected hazard on the carriageway, and

(2) *Full Overtaking Sight Distance (FOSD)* which is the length of visibility required by drivers of vehicles to enable them to overtake vehicles ahead of them in safety and comfort.

Since it requires a greater distance to overtake than to stop, the *FOSD* values are greater than the *SSD* values.

When designing vertical curves, it is essential to know whether safe overtaking is to be included in the design. If it is then the *FOSD* must be incorporated, if it is not then the *SSD* must be incorporated.

It is usually necessary to consider whether to design for overtaking only at crest curves on single carriageways since overtaking should not be a problem on dual carriageways and visibility is usually more than adequate for overtaking at sag curves on single carriageways.

The DTp specify sight distances for both stopping and overtaking at various design speeds and these are shown in sections A and D respectively of table 10.1. They were obtained as follows.

The SSD ensures that there is an envelope of clear visibility such that, at one extreme, drivers of low vehicles are provided with sufficient visibility to see low objects, while, at the other extreme, drivers of high vehicles are provided with visibility to see a significant portion of other vehicles. This envelope of visibility is shown in figure 11.4 in which 1.05 m represents the drivers' eye height for low vehicles and 2.00 m that for high vehicles; a lower object height of 0.26 m is used to include the rear tail lights of other vehicles and an upper object height of 2.00 m ensures that a sufficient portion of a vehicle ahead can be seen to identify it as such.

The *FOSD* ensures that there is an envelope of clear visibility between the 1.05 m and 2.00 m drivers' eye heights above the centre of the carriageway as shown in figure 11.5.

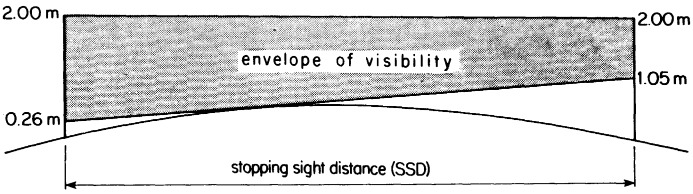


Figure 11.4 *Measurement of SSD*

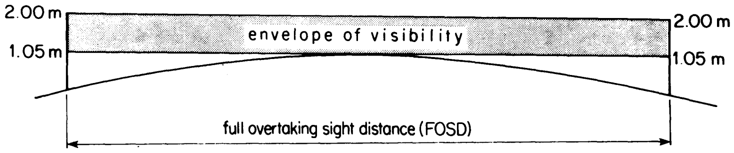


Figure 11.5 *Measurement of FOSD*

11.7 *K*-values

In the past it was necessary to use the appropriate sight distance for the road type and design speed in question to calculate the minimum length of the vertical curve required. Nowadays, however, constants, known as *K-values*, have been introduced by the DTp and greatly simplify the calculations.

The minimum length of vertical curve (*min* L_V) for any given road is obtained from the formula

$$min\ L_V = K\,A \text{ metres} \tag{11.1}$$

where K is the constant obtained from the DTp standards for the particular road type and design speed in question and A is the algebraic difference of the gradients, the absolute value (always positive) being used.

Section C of table 10.1 shows the current DTp K-values for various design speeds. The K-values ensure that the minimum length of vertical curve obtained from equation (11.1) contains adequate visibility and provides sufficient comfort.

There are three categories of K-values for crests and one category of K-values for sags. The units of K are metres and their values have been derived from the sight distances discussed in section 11.6.

11.7.1 Crest K-values

If a full overtaking facility is to be included in the design of single carriageways then the *FOSD crest K-values* given in row C1 of table 10.1 should be used in equation (11.1).

If overtaking is not considered in the design then, if possible, to ensure more than adequate visibility, K-values in excess of the *desirable minimum crest K-values* given in row C2 of table 10.1 should be used. If, owing to site constraints, this cannot be done, then it is permissible to use K-values as low as the *absolute minimum crest K-values* given in row C3 of table 10.1. These still ensure adequate visibility.

11.7.2 Sag K-values

Only one set of K-values is given for sags since overtaking visibility is usually unrestricted on this type of vertical curve. Row C4 of table 10.1 lists the *absolute minimum sag K-values* which will ensure adequate visibility and comfort.

Examples of the use of K-values are given in the following section.

11.8 Use of *K*-values

Example (1)

Dual carriageway, design speed 85 kph, Crest.

From table 10.1

FOSD crest K-value	= 285 m
desirable minimum crest K-value	= 55 m
absolute minimum crest K-value	= 30 m

Since a dual carriageway is being designed, overtaking is not critical. Therefore, from equation (11.1)

if possible, use $L_V \geqslant 55A$ metres
otherwise use $L_V \geqslant 30A$ metres

Example (2)

Single carriageway, design speed 60 kph, Crest.
From table 10.1

FOSD crest K-value	= 142 m
desirable minimum crest K-value	= 17 m
absolute minimum crest K-value	= 10 m

Since a single carriageway is being designed, a decision has to be made as to whether or not full overtaking is to be allowed for in the design.
If full overtaking is to be included, equation (11.1) gives

$$min\ L_V = 142A \text{ metres}$$

If full overtaking is not to be included, it would appear that, from equation (11.1)

$$min\ L_V = 17A \text{ metres}$$

However, the current DTp standards state that for crests on single carriageways, unless *FOSD* crest K-values can be used, it is sufficient to use only the absolute minimum crest K-values since the use of the desirable minimum crest K-values may result in sections of road having dubious visibility for overtaking.
In summary, this means that on single carriageway crests, overtaking should be either easily achieved or not possible at all. Hence, in this example

if possible, use $L_V \geqslant 142A$ metres
otherwise use $L_V = 10A$ metres

Further details on restrictions involved in the design of single carriageways can be found in the current DTp standards referenced in section 11.15.

Example (3)

Single carriageway, design speed 100 kph, Sag.
From table 10.1

absolute minimum sag K-value = 26 m

Therefore, from equation (11.1), use $L_V \geqslant 26A$ metres.

11.9 Length of Vertical Curve to be Used

Often the value for the minimum length of curve obtained from the K-values is not used, a greater length being chosen. This may be done for several reasons, for example, it may be necessary to fit the curve into particular site conditions. However, there is another factor which must be considered before deciding on the final length of a vertical curve. This is the necessity to try to fit the vertical alignment of the road to the horizontal alignment.

11.9.1 Phasing of Vertical and Horizontal Aligments

Usually, when designing new roads or improving existing alignments, the procedure is as follows.

(1) Design or redesign the horizontal alignment.

(2) Take reduced levels at regular intervals along the proposed centre line and plot a longitudinal section (see section 2.11.2).

(3) Superimpose chosen gradients on the longitudinal section, altering their percentage gradient and position as necessary to try to balance out any *cut* and *fill* (see section 12.2 and chapter 13) and also to try to get the vertical curve tangent points to coincide with those of the horizontal curve.

It is this third stage which often gives the final length of the vertical curve. The tangent points of the vertical curve must, wherever possible, coincide exactly with the tangent points of the horizontal curve, where applicable. This is to avoid the creation of optical illusions. If a vertical curve is started during a horizontal curve then to a driver travelling along the curve the road appears disjointed owing to the vertical directional change of the vertical alignment being inflicted on the horizontal curve at a point where the horizontal radial force and superelevation may be severe. This can lead to driver error and must be avoided wherever possible.

In most cases, the horizontal curve will be greater in length than the minimum required for the vertical curve and it will be necessary to increase the vertical curve length to that of the horizontal curve. Should the minimum vertical curve length be greater than the length of the horizontal curve then the opposite will apply.

When phasing vertical and horizontal alignments, the curves should run between the start and finish tangent points and not between any two tangent points. This is shown in figure 11.6. To introduce the two alignments at different tangent points would again create optical illusions.

11.10 Setting Out the Vertical Curve

Once the length and gradients have been decided, it is necessary to plot the curve on the longitudinal section as a check on the design and then set it out on the ground.

In order that these can be done, it is necessary to calculate the reduced levels (RL) of points along the proposed centre line.

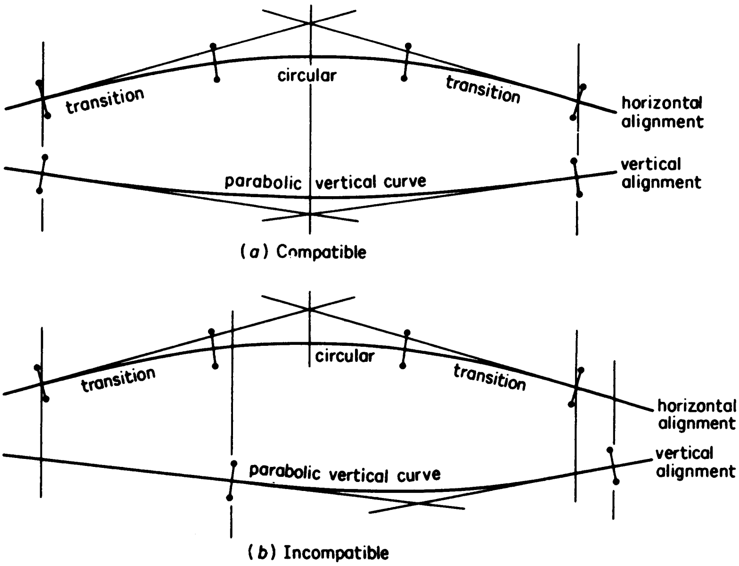


Figure 11.6 *Phasing of horizontal and vertical alignments*

With reference to figure 11.7, if P is datum level, the level of any point Z on the curve with respect to P is given by ΔH, where

$$\Delta H = [(m)y/100 - (A)y^2/200L_V]$$

This is a general expression and ΔH can be either positive or negative, depending on the signs of m and A.

All ΔH values are related to the RL of P and should be added to or subtracted from this to obtain the reduced levels of points on the curve which lie a general distance y along the curve from P.

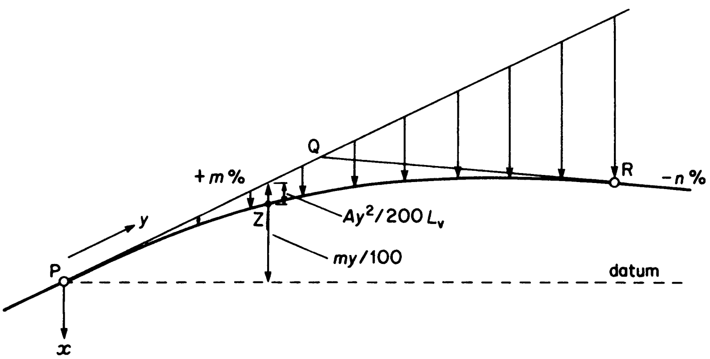


Figure 11.7

Hence, reduced levels of points on the curve can be calculated and plotted on the longitudinal section (see section 12.2). If the design is acceptable, the reduced levels of the points are set out on site using sight rails as described in chapter 14.

11.11 Highest Point of a Crest, Lowest Point of a Sag

In order that drainage gullies can be positioned effectively, it is necessary to know the through chainage and reduced level of the highest or lowest point of the vertical curve. The highest point of a crest occurs when ΔH is a maximum and the lowest point of a sag occurs when ΔH is a minimum.

For a maximum or minimum value of ΔH, $\mathrm{d}(\Delta H)/\mathrm{d}y = 0$, therefore

$$\frac{\mathrm{d}}{\mathrm{d}y}(\Delta H) = \frac{m}{100} - \frac{Ay}{100L_v} = 0$$

Hence, $m/100 = Ay/100L_v$, therefore $y = L_v m/A$ for a maximum or minimum value ΔH. This gives the point along the curve at which the maximum or minimum level occurs. To find the reduced level at this point it is necessary to substitute this expression for y back into the equation for ΔH. Therefore

$$\Delta H_{max/min} = (m/100)\,(L_v m/A) - (A/200L_v)\,(L_v{}^2 m^2/A^2)$$

Hence

$$\Delta H_{max/min} = L_v m^2/200A$$

above or below point P.

11.12 Summary of Vertical Curve Design

Problem

To design a vertical curve to fit between two gradients for a particular design speed.

Solution

(1) Calculate A.

(2) From the current DTp design standards obtain the appropriate K-value for the design speed and road type.

(3) Use the K-value to calculate the minimum required length of vertical curve.

At this stage it may be necessary to phase the vertical and horizontal alignment as described in section 11.9.1 and an alteration in gradients may be necessary. An attempt should also be made on the longitudinal section to balance out cut and fill.

(4) The reduced levels of the entry and exit tangent points on the vertical curve should be measured in the field once the final length has been chosen and these positions are known.

(5) The formula for ΔH is used together with the reduced level of the entry tangent point to calculate the reduced levels of points on the curve itself. As a

check on the calculations, the reduced level of the exit tangent point should be calculated using the formula and it should equal that found in (4).

(6) The curve is plotted on the longitudinal section by plotting the reduced levels calculated in (5) and, if acceptable, is set out on site using sight rails set some convenient height above the formation level.

11.13 Vertical Curves with Unequal Tangent Lengths

The foregoing discussion has been limited to vertical curves having equal tangent lengths. These are easy to design and can be fitted to the majority of cases but, occasionally, either to meet particular site conditions or to avoid large amounts of cut and/or fill, it becomes necessary to design a curve having unequal tangent lengths.

With reference to figure 11.8, the easiest method of designing such a curve is to introduce a third gradient BCD which splits the total curve PR into two consecutive

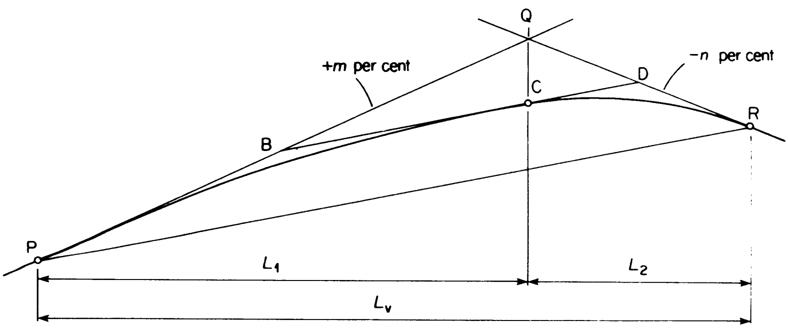


Figure 11.8

equal tangent length curves PC and CR. The common tangent line BCD is parallel to the chord PR and C is the common tangent point between the two curves.

The first curve PC is equal in length to the entry tangent length PQ and the second curve CR is equal in length to the exit tangent length QR. B is the mid-point of PQ and D is the mid-point of QR.

From figure 11.8

$$PC = L_1, \; CR = L_2$$

$$L_v = L_1 + L_2$$

$$PB = BC = \frac{PQ}{2} = \frac{L_1}{2}$$

$$CD = DR = \frac{QR}{2} = \frac{L_2}{2}$$

When calculating RLs at regular chainage intervals along the curves, each curve is treated as a separate equal tangent length vertical curve. The worked example in section 11.16.2 shows how this is done.

11.14 Computer Aided Road Design

Nowadays, vertical alignments are often designed in conjunction with horizontal alignments using complete highway design computer systems such as BIPS and MOSS. These are discussed in detail in section 10.16.

11.15 Further Reading

Department of Transport, Roads and Local Transport Directorate, *Departmental Standard TD/81, Road Layout and Geometry: Highway Link Design* (Department of Transport, 1981).

Department of Transport, Highways and Traffic Directorate, *Departmental Advice Note TA 43/84: Highway Link Design* (Department of Transport, 1984).

11.16 Worked Examples

11.16.1 Vertical Curve having Equal Tangent Lengths

Question

The reduced level at the intersection of a rising gradient of 1.5 per cent and a falling gradient of 1.0 per cent on a proposed road is 93.60 m AOD. Given that the K-value for this particular road is 55, the through chainage of the intersection point is 671.34 m and the vertical curve is to have equal tangent lengths, calculate

(1) the through chainages of the tangent points of the vertical curve if the minimum required length is to be used

(2) the reduced levels of the tangent points and the reduced levels at exact 20 m multiples of through chainage along the curve

(3) the position and level of the highest point on the curve.

Solution

Figure 11.9 shows the curve in question. Minimum $L_V = KA = 55 \times 2.5 = 137.5$ m. Therefore

$$\textit{through chainage of } P = 671.34 - (137.5/2) = \mathbf{602.59\ m}$$

$$\textit{through chainage of } R = 671.34 + (137.5/2) = \mathbf{740.09\ m}$$

From the diagram it is obvious that P and R are both lower than Q, therefore, ignoring the signs of m and n

$$\textit{reduced level of } P = 93.60 - (mL_V/200) = 93.60 - 1.03$$

$$= \mathbf{92.57\ m}$$

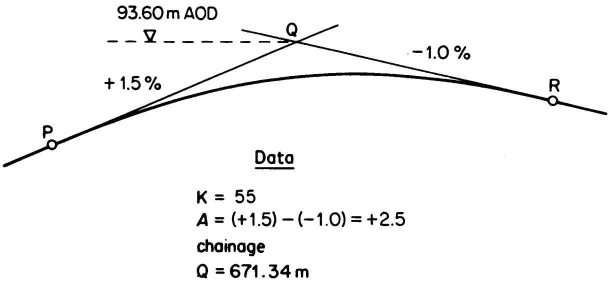


Figure 11.9

$$\textit{reduced level of } R = 93.60 - (nL_V/200) = 93.60 - 0.69$$
$$= \mathbf{92.91\ m}$$

To keep to exact multiples of 20 m of through chainage there will need to be an initial short value of y of 620.00 − 602.59 = 17.41 m. y will increase in steps of 20 m and the final y will be equal to the length of the curve, that is 137.5 m.

The reduced levels of points on the curve are given by

$$RL = 92.57 + [(m)y/100 - (A)y^2/200L_V]$$

working from P towards R.

The results are tabulated and shown in table 11.2.

As a check on the calculations, the reduced level of R should equal that calculated earlier as is the case in this example.

The *highest point* on the curve occurs when

$$y = L_V m/A = (137.5 \times 1.5)/2.5 = \mathbf{82.50\ m}$$

TABLE 11.2
(all quantities are in metres)

Chainage	y	$(m)y/100$	$(A)y^2/200L_V$	ΔH	RL
602.59 (P)	0	0	0	0	92.57
620.00	17.41	+0.261	+0.028	+0.233	92.80
640.00	37.41	+0.561	+0.127	+0.434	93.00
660.00	57.41	+0.861	+0.300	+0.561	93.13
680.00	77.41	+1.161	+0.545	+0.616	93.19
700.00	97.41	+1.461	+0.863	+0.598	93.17
720.00	117.41	+1.761	+1.253	+0.508	93.08
740.00	137.41	+2.061	+1.717	+0.344	92.91
740.09 (R)	137.50	+2.063	+1.719	+0.344	92.91

The *highest level* on the curve = $92.57 + L_V m^2/200A$

$$= 92.57 + (137.5 \times 1.5^2)/(200 \times 2.5) = \mathbf{93.19\ m}$$

These can be confirmed by inspection of table 11.2.
In this example both m and A are positive and the positive sign has been retained in the calculations. When either m or A is negative, the negative sign should also be retained and taken into account in the equation for ΔH in order that ΔH can have the correct sign.

The reduced levels shown in table 11.2 are rounded to the nearest 10 mm since the initial data was only quoted to this precision.

11.16.2 Vertical Curve having Unequal Tangent Lengths

Question

A parabolic vertical curve is to connect a −2.50 per cent gradient to a +3.50 per cent gradient on a highway designed for a speed of 100 kph. The K-value for the highway is 26 and the minimum required length is to be used.

The reduced level and through chainage of the intersection point of the gradients are 59.34 m AOD and 617.49 m respectively and, in order to meet particular site conditions, the through chainage of the entry tangent point is to be 553.17 m. Calculate

(1) the reduced levels of the tangent points,
(2) the reduced levels at exact 20 m multiples of through chainage along the curve.

Solution

$$A = (-2.50) - (+3.50) = -6.00$$

Hence

$$min\ L_V = 26 \times 6.00 = 156.00\ \text{m}$$

Figure 11.10 shows the required curve and, from this, the tangent lengths are

$$L_1 = \text{PQ} = 617.49 - 553.17 = 64.32\ \text{m}$$

$$L_2 = \text{QR} = 156.00 - 64.32 = 91.68\ \text{m}$$

Since these are unequal, a third gradient BCD is introduced as discussed in section 11.13.

(1) Reduced levels of P and R

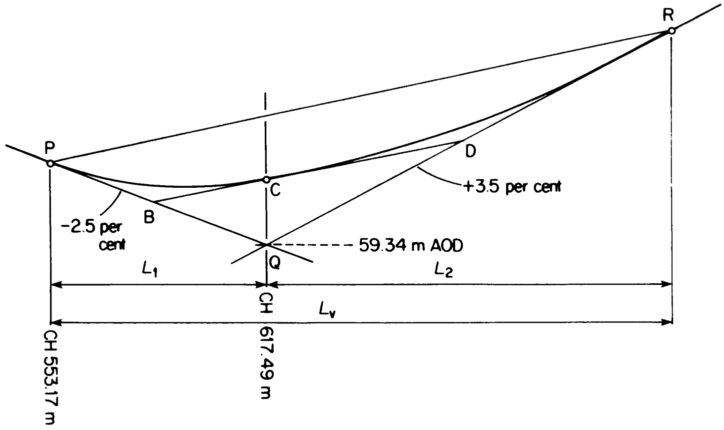


Figure 11.10

From figure 11.10, it can be seen that

$$\mathbf{RL_P} = \mathrm{RL_Q} + \frac{(2.50)\mathrm{PQ}}{100} = 59.34 + \frac{(2.50)64.32}{100} = \mathbf{60.95\ m\ AOD}$$

$$\mathbf{RL_R} = \mathrm{RL_Q} + \frac{(3.50)\mathrm{QR}}{100} = 59.34 + \frac{(3.50)91.68}{100} = \mathbf{62.55\ m\ AOD}$$

(2) Reduced levels at exact 20 m multiples of through chainage along the curve

through chainage of P = 553.17 m

through chainage of R = through chainage of P + L_V

= 553.17 + 156.00 = 709.17 m

Also, from figure 11.10, it can be seen that

$$\text{gradient of BCD} = \text{gradient of PR}$$

$$= \frac{(-2.50)L_1 + (+3.50)L_2}{L_V} \text{ per cent}$$

$$= \frac{(-2.50)64.32 + (+3.50)91.68}{156.00} \text{ per cent}$$

$$= +1.03\%$$

For the vertical curve PC, the reduced levels are calculated from P to C with reference to $\mathrm{RL_P}$ using

$$\mathrm{RL} = \mathrm{RL_P} + [(m)y/100 - (A)y^2/200L_1]$$

TABLE 11.3
(all quantities are in metres)

Chainage	y	$(m)y/100$	$(A)y^2/200L_1$	ΔH	RL
553.17 (P)	0	0	0	0	60.95
560.00	6.83	-0.17	-0.01	-0.16	60.79
580.00	26.83	-0.67	-0.20	-0.47	60.48
600.00	46.83	-1.17	-0.60	-0.57	60.38
617.49 (C)	64.32	-1.61	-1.14	-0.47	60.48

where $m = -2.50$ per cent, $L_1 = 64.32$ m, $A = (-2.50) - (+1.03) = -3.53$ per cent, through chainage of C = through chainage of Q = 617.49 m.
The RLs calculated along curve PC are shown in table 11.3.

For the vertical curve CR, the reduced levels are calculated from C to R with reference to RL_C using

$$RL = RL_C + [(m)y/100 - (A)y^2/200L_2]$$

where $m = +1.03$ per cent, $L_2 = 91.68$ m, $A = (+1.03) - (+3.50) = -2.47$.

The RLs calculated along curve CR are shown in table 11.4. Note that the value obtained for the RL of point R in table 11.4 agrees with the value obtained in the solution to the first part of the question.

TABLE 11.4
(all quantities are in metres)

Chainage	y	$(m)y/100$	$(A)y^2/200L_2$	ΔH	RL
617.49 (C)	0	0	0	0	60.48
620.00	2.51	+0.03	0.00	+0.03	60.51
640.00	22.51	+0.23	-0.07	+0.30	60.78
660.00	42.51	+0.44	-0.24	+0.68	61.16
680.00	62.51	+0.64	-0.53	+1.17	61.65
700.00	82.51	+0.85	-0.92	+1.77	62.25
709.17 (R)	91.68	+0.94	-1.13	+2.07	62.55

12

Calculation of Areas and Volumes

With the high costs of areas of land and volumes of material it is vital that the engineer makes as accurate a measurement as possible of any such quantities involved in a particular project.

This chapter deals with some of the more important and most often used techniques in the calculation of areas and volumes and for convenience is divided into three sections, that is, calculation of plan areas, calculation of cross-sectional areas and the calculation of volumes.

12.1 Calculation of Plan Areas

Such areas fall into one of three categories; they are either straight sided, irregular sided or a combination of both.

12.1.1 Areas enclosed by Straight Lines

Into this category fall areas enclosed by traverse, triangulation, trilateration or detail survey lines. The results obtained for such areas will be exact since correct geometric equations and theorems can be applied.

Areas from triangles

The straight-sided figure can be divided into well-conditioned triangles, the areas of which can be calculated using one of the following formulae.

(1) Area = $\sqrt{[S(S-a)(S-b)(S-c)]}$ where a, b and c are the lengths of the sides of the triangle and $S = \frac{1}{2}(a+b+c)$.

(2) Area = $\frac{1}{2}$ (base of triangle $\times$ height of triangle).

(3) Area = $\frac{1}{2}a\,b \sin C$ where C is the angle contained between side lengths a and b.

The area of any straight-sided figure can be calculated by splitting it into triangles and summing the individual areas.

Areas from coordinates

In traverse, triangulation and trilateration calculations, the coordinates of the junctions of the sides of a straight-sided figure are calculated and it is possible to use them to calculate the area enclosed by the control network lines. This is achieved using the *cross coordinate method.*

Consider figure 12.1, which shows a three-sided clockwise control network ABC. The required area = ABC.

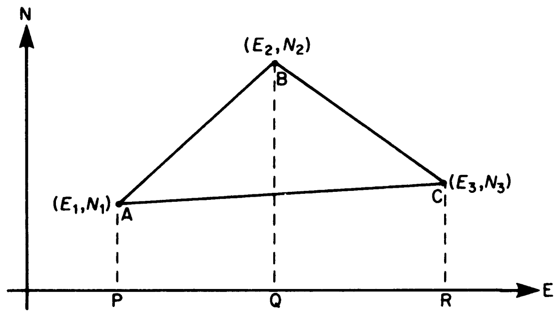


Figure 12.1

$$\text{area of ABC} = \text{area of ABQP} + \text{area of BCRQ} - \text{area of ACRP} \quad (12.1)$$

These figures are trapezia for which the area is obtained from

$$\text{area of trapezium} = (\text{mean height} \times \text{width})$$

Therefore

$$\text{area of ABQP} = \tfrac{1}{2}(N_1 + N_2)(E_2 - E_1)$$

Hence equation (12.1) becomes

$$\text{area ABC} = \tfrac{1}{2}(N_1 + N_2)(E_2 - E_1) + \tfrac{1}{2}(N_2 + N_3)(E_3 - E_2) - \tfrac{1}{2}(N_1 + N_3)(E_3 - E_1)$$

Therefore

$$2 \times \text{area ABC} = N_1E_2 - N_1E_1 + N_2E_2 - N_2E_1 + N_2E_3 - N_2E_2 + N_3E_3 - N_3E_2 - N_1E_3 + N_1E_1 - N_3E_3 + N_3E_1$$

Rearranging, this gives

$$2 \times \text{area ABC} = (N_1E_2 + N_2E_3 + N_3E_1) - (E_1N_2 + E_2N_3 + E_3N_1)$$

The similarity between the two brackets should be noted.

Although the example given is only for a three-sided figure, the formula can be applied to a figure containing N sides and the general formula for such a case is given by

$$2 \times \text{area} = (N_1E_2 + N_2E_3 + N_3E_4 + \ldots + N_{N-1}E_N + N_NE_1) - (E_1N_2 + E_2N_3 + E_3N_4 + \ldots + E_{N-1}N_N + E_NN_1)$$

If the figure is numbered in the opposite direction, the signs of the two brackets are reversed.

The cross coordinate method can be used to subdivide straight-sided areas as shown in the worked example in section 12.4.1 and can also be used to calculate the area of irregular cross sections as discussed in section 12.2.5.

12.1.2 Areas Enclosed by Irregular Lines

For such cases only approximate results can be achieved. However, methods are adopted which will give the best approximations.

Give and take lines

In this method an irregular-sided figure is divided into triangles or trapezia, the irregular boundaries being replaced by straight lines such that any small areas excluded from the survey by the lines are balanced by other small areas outside the survey but included as shown in figure 12.2.

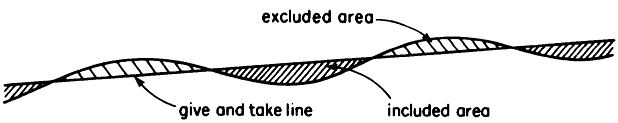


Figure 12.2

The positions of these lines can be estimated by eye on a survey plan. The area is then calculated using one of the straight-sided methods.

Graphical method

This method involves the use of a transparent overlay of squared paper which is laid over the drawing or plan. The number of squares and parts of squares which are enclosed by the area is counted and, knowing the plan scale, the area represented by each square is known and hence the total area can be computed. This can be a very accurate method if a small grid is used.

Mathematical methods

The following two methods make a mathematical attempt to calculate the area of an irregular-sided figure.

Trapezoidal rule Figure 12.3 shows a control network contained inside an area having irregular sides. The shaded area is that remaining to be calculated after using one of the straight-sided methods to calculate the area enclosed by the control network lines.

Figure 12.4 shows an enlargement of a section of figure 12.3. The offsets O_1, $O_2, O_3 \ldots O_8$ are either measured directly in the field or scaled from a plan.

The trapezoidal rule assumes that if the interval between the offsets is small, the boundary can be approximated to a straight line between the offsets. Hence, figure 12.4 is assumed to be made up of a series of trapezia as shown in figure 12.5. Therefore, in figure 12.5

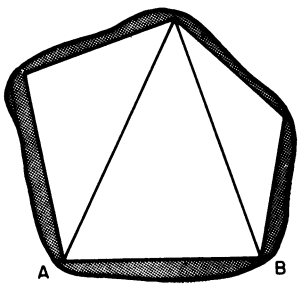


Figure 12.3

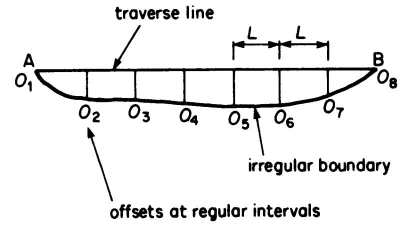


Figure 12.4

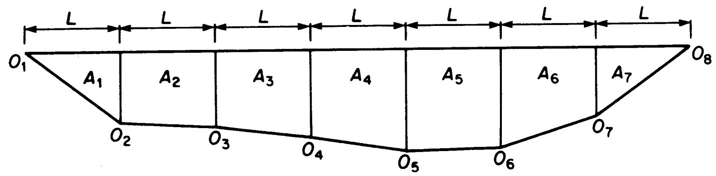


Figure 12.5

$$A_1 = \frac{(O_1 + O_2)}{2} L; \quad A_2 = \frac{(O_2 + O_3)}{2} L, \text{etc.}$$

Hence, for N offsets, the total area (A) is given by

$$A = \frac{(O_1 + O_2)}{2} L + \frac{(O_2 + O_3)}{2} L + \ldots + \frac{(O_{N-1} + O_N)}{2} L$$

Which leads to the general trapezoidal rule shown below

$$A = \frac{L}{2} (O_1 + O_N + 2(O_2 + O_3 + O_4 + \ldots + O_{N-1}))$$

The trapezoidal rule applies to any number of offsets.

Consider the following example.

Question

The following offsets, 8 m apart, were measured at right angles from a traverse line to an irregular boundary.

0 m 2.3 m 5.5 m 7.9 m 8.6 m 6.9 m 7.3 m 6.2 m 3.1 m 0 m

Calculate the area between the traverse line and the irregular boundary.

Solution

$$\textbf{Area} = \frac{8.0}{2}\,(0 + 0 + 2(2.3 + 5.5 + 7.9 + 8.6 + 6.9 + 7.3 + 6.2 + 3.1))$$

$$= 4 \times 2(47.8) = \mathbf{382.4\ m^2}$$

Simpson's rule This method assumes that instead of being made up of a series of straight lines, the boundary consists of a series of parabolic arcs. A more accurate result is obtained since a better approximation of the true shape of the irregular boundary is achieved. Figure 12.6 shows this applied to figure 12.5.

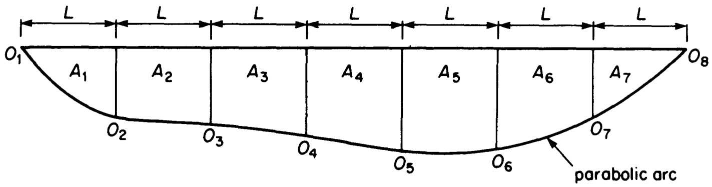


Figure 12.6

Simpson's rule considers offsets in sets of three and it can be shown that the area between offset 1 and 3 is given by

$$A_1 + A_2 = \frac{L}{3}\,(O_1 + 4O_2 + O_3)$$

Similarly

$$A_3 + A_4 = \frac{L}{3}\,(O_3 + 4O_4 + O_5)$$

Hence, in general

$$\text{Total area} = \frac{L}{3}\,(O_1 + O_N + 4\,\Sigma\text{ even offsets} + 2\,\Sigma\text{ remaining odd offsets})$$

However, N MUST be an ODD number for Simpson's rule to apply.

When faced with an even number of offsets, as in figure 12.6, when using Simpson's rule, the final offset must be omitted (for example, O_8), the rest of the area calculated and the last small area calculated as a trapezium (that is, using the trapezoidal rule). Consider the example given during the derivation of the trapezoidal rule solved using Simpson's rule.

Solution

There are an even number of offsets, 10, hence calculate the area between 1 and 9 by Simpson's rule and the area between 9 and 10 by the trapezoidal rule.

$$\text{Area}_{1-9} = \frac{8.0}{3}\,[0 + 3.1 + 4(2.3 + 7.9 + 6.9 + 6.2) + 2(5.5 + 8.6 + 7.3)]$$

$$= \frac{8.0}{3}\,[3.1 + 4(23.3) + 2(21.4)$$

$$= \frac{8.0 \times 139.1}{3} = 370.9\ \text{m}^2$$

$$\text{Area}_{9-10} = \frac{8.0}{2}\,(3.1 + 0) = 12.4\ \text{m}^2$$

Therefore

total area = 370.9 + 12.4 = **383.3 m²**

Note the difference between this result and that obtained using the trapezoidal rule. Simpson's rule will give the more accurate result when the boundary is genuinely irregular and the trapezoidal rule will give the more accurate result when the boundary is almost a series of straight lines. In general for irregular-sided figures, Simpson's rule should be used.

12.1.3 The Planimeter

The planimeter is a mechanical device for determining the area of any irregular-sided plane figure. A high degree of accuracy can be achieved. Figure 12.7 shows the main features of a planimeter. The integrating unit can also be in digital form as shown in figure 12.8.

The area is obtained from the measuring unit consisting of an integrating disc which revolves and alters the reading as the tracing point is moved round the perimeter of the figure. It is possible to read to 1/1000 of a revolution of the disc. The reading is directly related to the length of the tracing arm.

On a *fixed tracing arm instrument* the readings are obtained directly in mm^2 and then have to be converted according to the plan scale to obtain the ground area.

On a *movable arm instrument* the tracing arm length can be set to particular values depending on the plan scale such that the readings obtained give the ground area directly.

The planimeter can be used as follows.

With the pole block outside the figure

This is the most common method of use and is shown in figure 12.9.

(1) Point A is marked and the scale read.

(2) The tracing point is moved clockwise round the perimeter of the figure back to point A and the scale is again read.

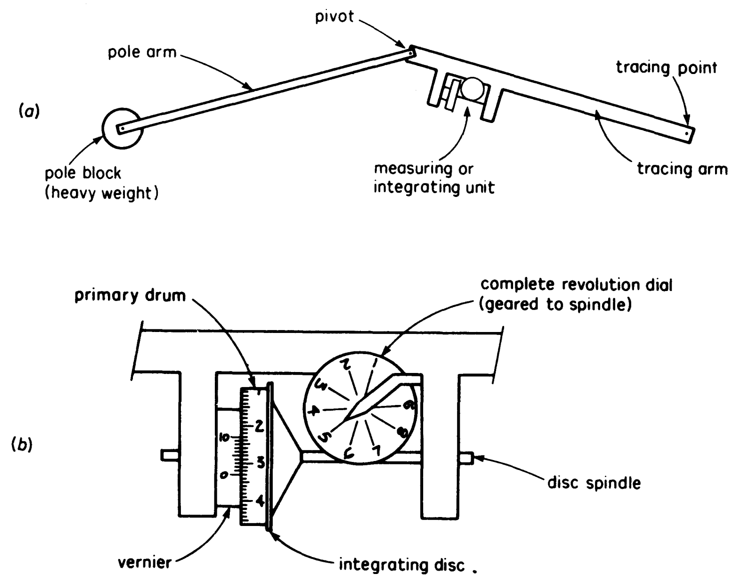


Figure 12.7 *Planimeter: (a) main features; (b) integrating unit*

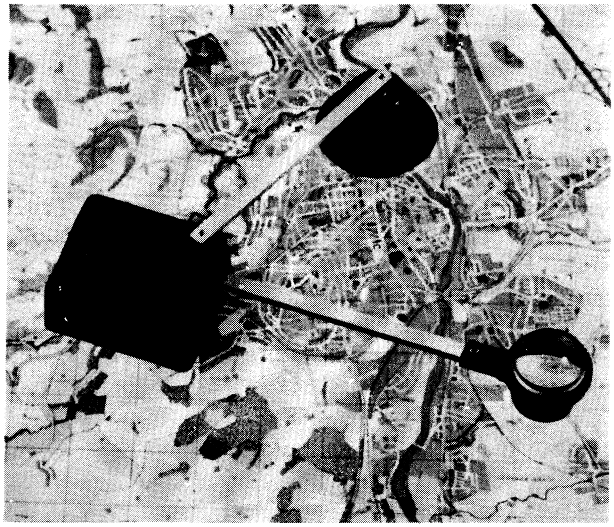


Figure 12.8 *Digital planimeter*

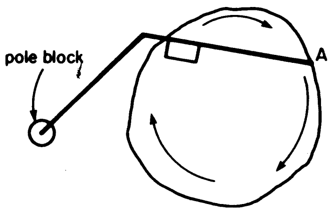


Figure 12.9

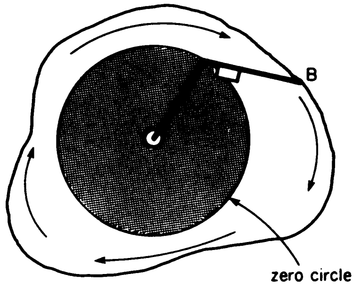


Figure 12.10

(3) The difference between the two readings multiplied by any necessary factor gives the ground area.

With the pole block inside the figure

This is essentially the same as for the pole block outside, but a constant must be added, as shown in figure 12.10. The shaded area is known as the *zero circle* and is that area which is not registered on the scale owing to the disc being dragged at right angles to its direction of rotation during the measurement. This constant is given with the planimeter and will also have to be converted as necessary.

Whichever method is used, the planimeter should always be checked over a known area and if a discrepancy is found a further correction factor should be computed and applied to all the planimeter readings. *Testing bars* are usually provided with the planimeter for this purpose.

12.2 Calculation of Cross-sectional Areas

In the construction of a road, railway, large diameter underground pipeline or similar, having set out the proposed centre line on the ground, levels are taken at regular intervals both along it and at right angles to it to obtain the longitudinal and cross sections. This is shown in figure 12.11, the fieldwork being described in detail in section 2.11.2.

When plotting the *longitudinal section*, the vertical alignment is designed and the formation levels along the centre line are calculated. A typical longitudinal section showing the formation level is shown in figure 12.12.

Each *cross section* (CS) is drawn and the area between the existing and proposed levels is calculated. Figure 12.13 shows typical cross sections.

Both the longitudinal section and the cross sections are usually drawn with their horizontal and vertical scales at different values, that is

Scales for longitudinal section

horizontal—as road layout drawings, for example, 1 in 500

vertical—exaggerated, for example, 1 in 100

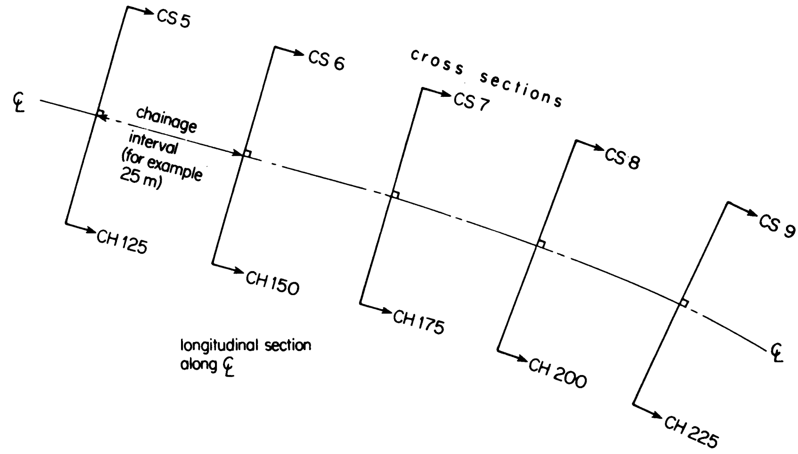


Figure 12.11

Scales for cross sections

horizontal—exaggerated, for example, in 1 in 200

vertical—exaggerated, for example, 1 in 50

The reason for exaggerating the vertical scales of both sections and the horizontal scale of the cross sections is to give a clear picture of the exact shape of the sections.

If the cross sections have different horizontal and vertical scales it is still possible to calculate their areas either by the graphical method or by using a planimeter as normal and applying a conversion factor. Consider the following example.

Question

A cross-sectional area was measured using a fixed arm planimeter which gave readings directly in mm^2. The initial planimeter reading was set to zero and the final reading was 7362. If the horizontal scale of the cross section was 1 in 200 and the vertical scale 1 in 100, calculate the true area represented by the cross section.

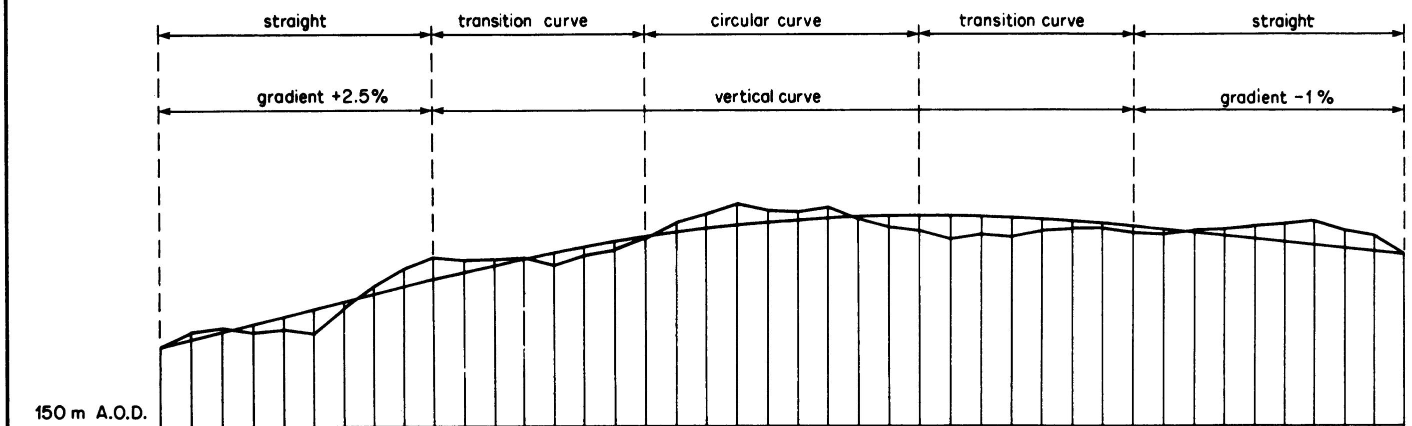
Solution

Difference between planimeter readings = 7362 mm^2. But 1 mm^2 does, in fact, represent an area (200 mm x 100 mm) since the horizontal and vertical scales are 1 in 200 and 1 in 100 respectively. Hence

$$7362 \text{ mm}^2 = (7362 \times 200 \times 100) \text{ mm}^2 = \mathbf{147.24\ m^2}$$

Once the areas of all the cross sections have been obtained they are used to calculate the volumes of material to be either excavated (*cut*) or imported (*fill*) between consecutive cross sections. Such volume calculations are considered in section 12.3.

Although, in the example given above, a planimeter was used to measure the cross-sectional area, this is not the only method available; any of the methods discussed in section 12.1 can be employed or, alternatively, one of the following methods can be used. Five types of cross section are considered.



CHAINAGE	EXISTING	PROPOSED
0	155.17	155.17
	156.12	155.67
	156.55	156.17
	156.20	156.67
	156.41	157.17
100	156.13	157.67
	157.72	158.17
	159.20	158.67
	160.35	159.17
	161.16	159.67
200	160.92	160.15
	160.95	160.61
	161.08	161.03
	160.59	161.43
	161.21	161.79
300	161.54	162.12
	162.37	162.42
	163.36	162.70
	163.93	162.94
	164.58	163.15
400	164.15	163.33
	164.07	163.48
	164.33	163.60
	163.60	163.69
	163.09	163.75
500	162.84	163.77
	162.34	163.77
	162.61	163.74
	162.43	163.68
	162.37	163.58
600	162.99	163.46
	163.02	163.30
	162.59	163.12
	162.59	162.92
	162.91	162.72
700	162.98	162.52
	163.17	162.32
	163.30	162.12
	163.52	161.92
	162.82	161.72
800	162.48	161.52
820	161.32	161.32

DRAWING DETAILS (including title, names of local authority/contractor, name and location of scheme, horizontal and vertical scales, date, etc.)

Figure 12.12 *Example of a longitudinal section*

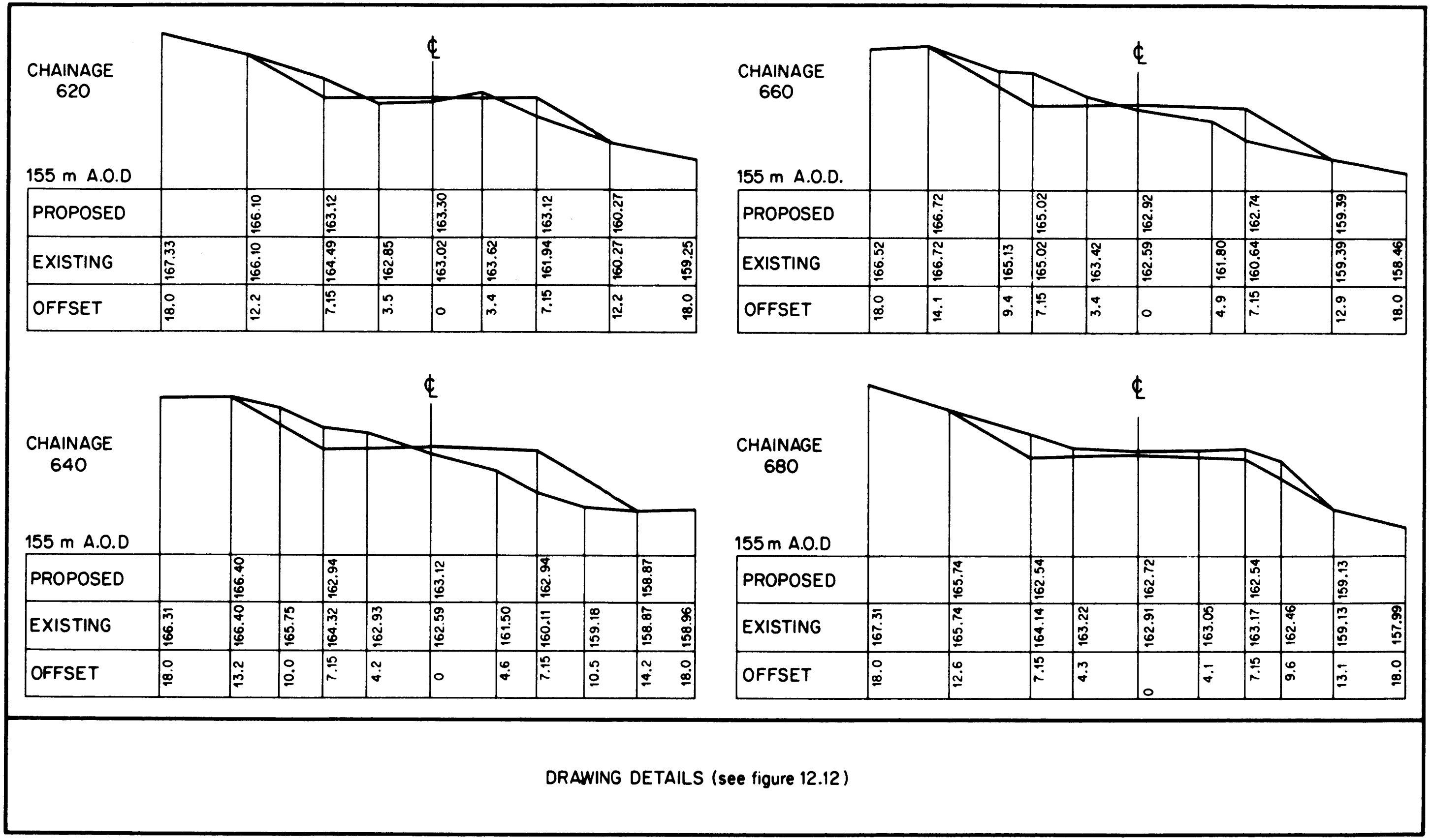


Figure 12.13 *Example cross sections*

12.2.1 Existing Ground Level Horizontal

Figure 12.14 shows a sectional drawing of a cutting formed in an area where existing ground level is constant.

From figure 12.14

$$\textit{area of cross section} = h(2b + nh)$$

$$\textit{plan width} = 2W = 2(b + nh)$$

For an embankment, the diagram is inverted and the same formulae apply.

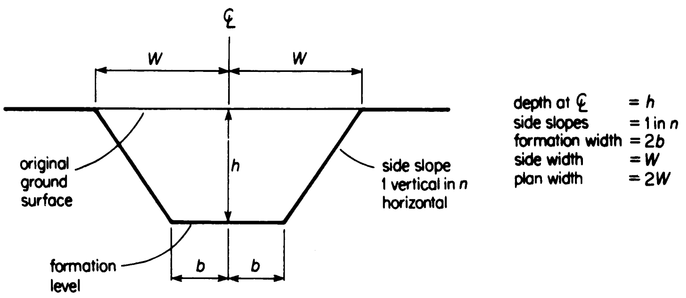


Figure 12.14 *Level section*

12.2.2 The Two-level Section

A cutting with a constant transverse slope is shown in figure 12.15, where W_G = greater side width; W_L = lesser side width; h = depth of cut on centre line from

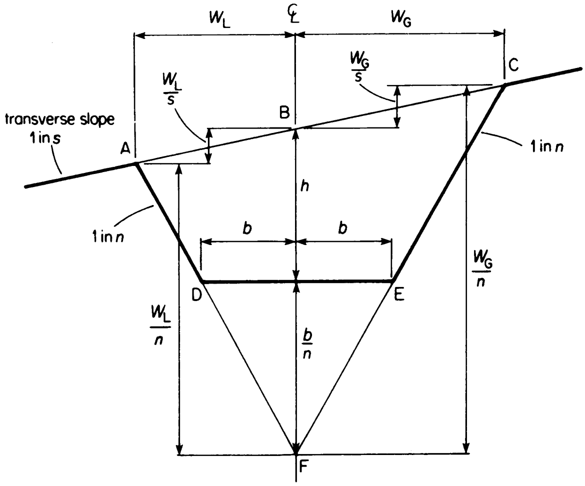


Figure 12.15 *Two-level section*

existing to proposed level; 1 in n = side slope; 1 in s = ground or transverse slope.

Considering vertical distances at the centre line gives

$$\frac{W_L}{n} = \frac{b}{n} + h - \frac{W_L}{s}$$

and

$$\frac{W_G}{n} = \frac{b}{n} + h + \frac{W_G}{s}$$

Multiplying by sn gives

$$W_L s = bs + hsn - W_L n$$

and

$$W_G s = bs + hsn + W_G n$$

Hence

$$W_L = \frac{s(b + nh)}{s + n}$$

and

$$W_G = \frac{s(b + nh)}{s - n}$$

The *plan width* is given by $(W_L + W_G)$. The *cross-sectional area* (A) of the cutting is given by

$$A = \text{area ABF} + \text{area BCF} - \text{area DEF, hence}$$

$$A = \frac{1}{2}\left[h + \frac{b}{n}\right](W_L + W_G) - \frac{b^2}{n}$$

Again, for embankments, figure 12.15 is inverted and the same formulae apply.

12.2.3 The Three-level Section

A cutting with a transverse slope which changes gradient at the centre line is shown in figure 12.16, where W_1 and W_2 are the side widths, h = depth of cut on the centre line from the existing to the proposed levels, 1 in n = side slope, 1 in s_1 and 1 in s_2 = transverse slopes.

Cross sections of this type are best considered as consisting of two separate half sections on either side of the centre line. There are eight possible types of half section as shown in figure 12.17.

Using techniques similar to those used to derive the formulae for the two-level section in figure 12.15, it is possible to derive the following formulae for three-level sections.

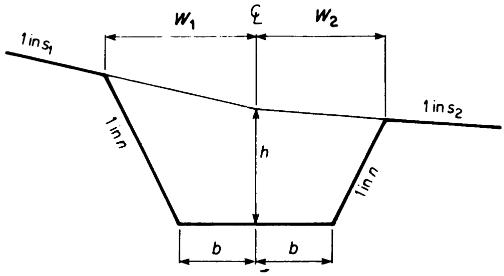


Figure 12.16 *Three-level section*

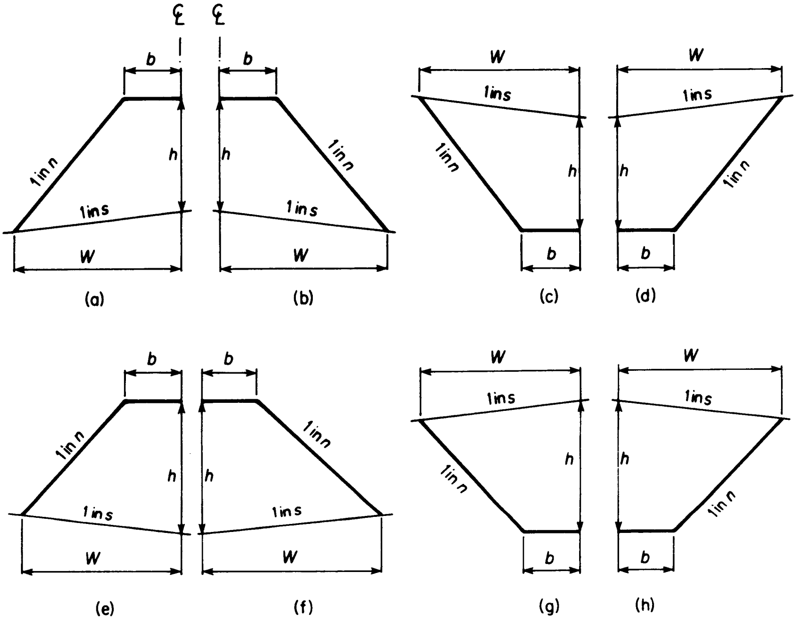


Figure 12.17 *Half sections*

For the half sections shown in figure 12.17*a, b, c* and *d*

$$W = \frac{s(b + nh)}{(s - n)}$$

For the half sections shown in figure 12.17*e, f, g* and *h*

$$W = \frac{s(b + nh)}{(s + n)}$$

The cross-sectional area (A) of any combination of any two of the eight types of half section is given by

$$A = \frac{1}{2}\left(h + \frac{b}{n}\right) \text{(sum of side widths)} - \frac{b^2}{n}$$

12.2.4 Cross Sections involving both Cut and Fill

Figure 12.18 shows the four types of section that can occur in practice where the depth of cut or fill on the centre line is not great enough to give either a full cutting or a full embankment but instead gives a cross section consisting partly of cut and partly of fill. Such a section can occur when a road is being built around the side of a hill and is used for economic reasons since the cut section can be used to provide the fill section and very little earth-moving distance is involved.

With reference to figure 12.18, h = depth of cut or fill on the centre line from the existing to the proposed levels, W_1 and W_2 = the side widths, A_1 and A_2 = the areas of cut or fill, 1 in s = transverse slope, 1 in n and 1 in m = side slopes.

Two different side slopes are shown since often a different side slope is used for cut compared to that used for fill.

Formulae for W_1, W_2, A_1 and A_2 can be derived as follows by again considering vertical distances along the centre line. Consider figure 12.19 which shows a more detailed version of the cross section shown in figure 12.18*a*.

$$\frac{W_1}{n} = \frac{b}{n} + h_1$$

and

$$\frac{W_2}{m} = \frac{b}{m} + h_2$$

But

$$h_1 = \frac{W_1}{s} - h$$

and

$$h_2 = \frac{W_2}{s} + h$$

Substituting for h_1 and h_2 and multiplying by sn and sm respectively gives

$$W_1 s = bs + W_1 n - hsn$$

and

$$W_2 s = bs + W_2 m + hsm$$

Hence

$$W_1 = s\,\frac{b - nh}{s - n}$$

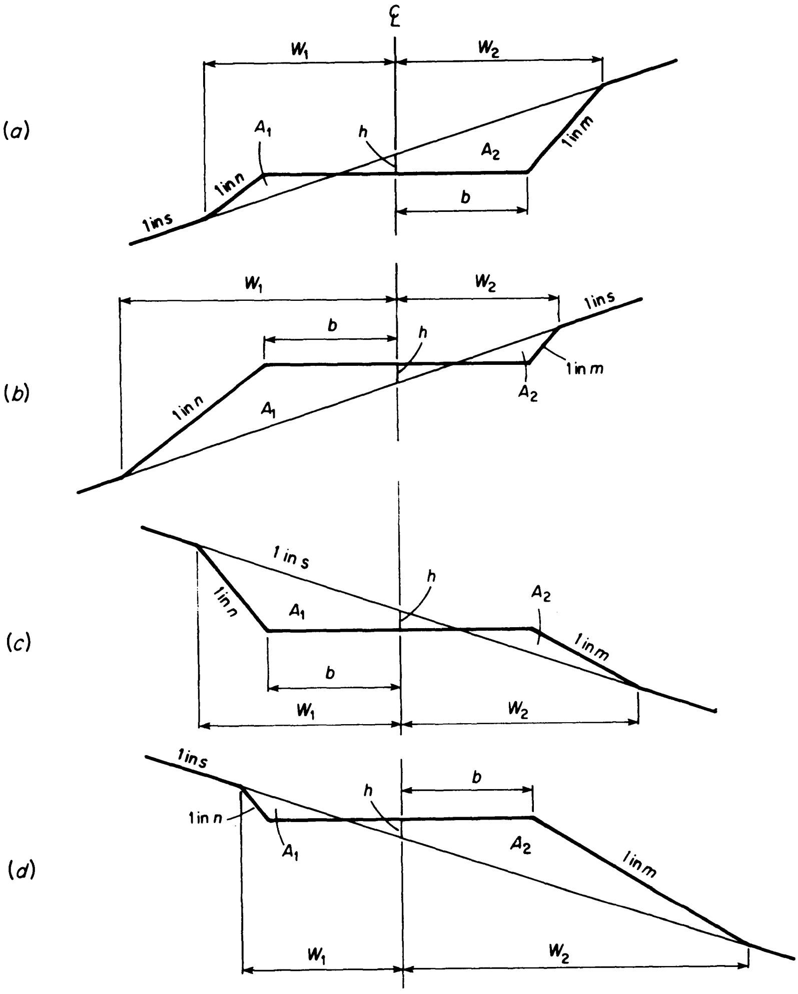


Figure 12.18 *Sections involving cut and fill*

and

$$W_2 = s\,\frac{b + mh}{s - m}$$

The *plan width* is given by $(W_1 + W_2)$.

The *cross-sectional areas* are obtained as follows

$$A_1 = \tfrac{1}{2}(b - sh)h_1$$

and

$$A_2 = \tfrac{1}{2}(b + sh)h_2$$

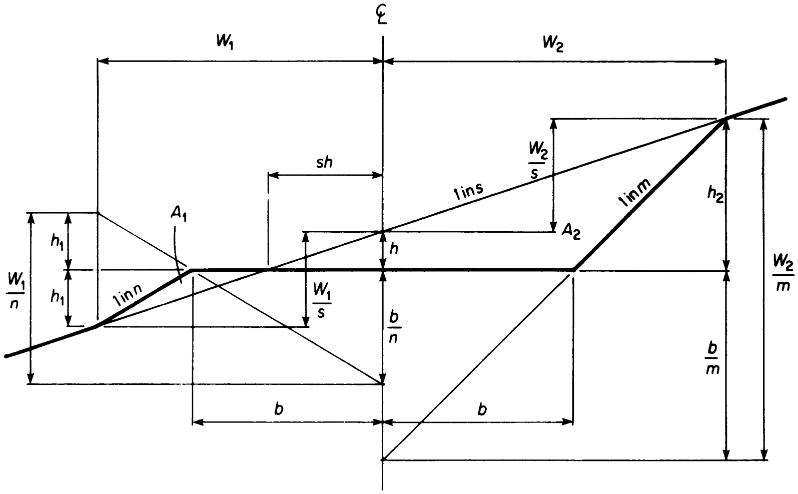


Figure 12.19

but

$$h_1 = \frac{W_1}{s} - h$$

$$= \frac{b - nh}{s - n} - h$$

$$= \frac{b - sh}{s - n}$$

and

$$h_2 = \frac{W_2}{s} + h$$

$$= \frac{b + mh}{s - m} + h$$

$$= \frac{b + sh}{s - m}$$

Hence

$$A_1 = \frac{(b - sh)^2}{2(s - n)}$$

and

$$A_2 = \frac{(b + sh)^2}{2(s - m)}$$

The formulae derived above for W_1, W_2, A_1 and A_2 apply to cross sections similar to those shown in figure 12.18*a* and *d*.

For cross sections similar to those shown in figure 12.18*b* and *c*, the formulae are amended slightly as follows

$$W_1 = s\,\frac{b + nh}{s - n} \qquad W_2 = s\,\frac{b - mh}{s - m}$$

$$A_1 = \frac{(b + sh)^2}{2(s - n)} \qquad A_2 = \frac{(b - sh)^2}{2(s - m)}$$

With any cross section partly in cut and partly in fill, it is essential that a drawing be produced so that the correct formulae can be used.

The worked example in section 12.4.2 illustrates the application of these formulae.

12.2.5 Irregular Sections

Figure 12.20 shows a cutting, the ground surface of which has been surveyed using the levelling methods discussed in section 2.11.2. For each point surveyed, the RL and offset distance (x) will be known.

Although the area of such a section could be found using a planimeter or by a mathematical method the *cross coordinate method* (see section 12.1.1) can also be used. In order to apply this method, a coordinate system which has its origin at the intersection of the formation level and the centre line is used. Offset distances (x values) to the right of the centre line are taken as positive and those to the left of the centre line are taken as negative; heights (y values) above the formation level are considered to be positive and those below the formation level are considered to be negative. For figure 12.20, the points defining the section will have the coordinates given in table 12.1. These coordinates are used to obtain the area of this section by substituting them into the following cross coordinate formula

$$\begin{aligned} 2 \times \text{Area} = {} & (N_1E_2 + N_2E_3 + N_3E_4 + \ldots + N_{11}E_1) \\ & - (E_1N_2 + E_2N_3 + E_3N_4 + \ldots + E_{11}N_1) \end{aligned}$$

since a clockwise order is given in figure 12.20 for the points.

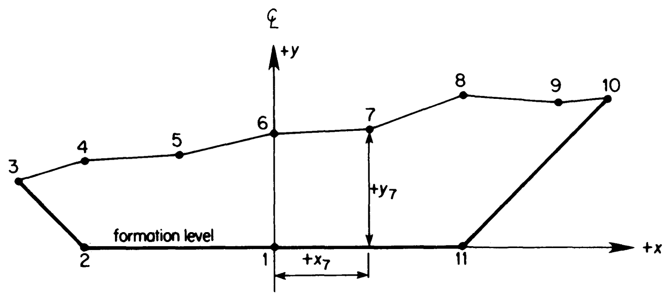


Figure 12.20 *Irregular section*

TABLE 12.1

Point n=	1	2	3	4	5	6	7	8	9	10	11
E_n	0	$-b$	$-x_3$	$-x_4$	$-x_5$	0	x_7	x_8	x_9	x_{10}	b
N_n	0	0	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	0

A similar process can be applied to embankments and also to those sections involving both cut and fill. For the cut and fill sections, separate calculations are required.

In this type of area calculation, careful attention must be paid to the algebraic signs involved.

12.2.6 Using the Cross-sectional Areas and Side Widths

The cross-sectional areas are used to calculate volumes and this is discussed in section 12.3. However, before they can be used to obtain volumes it is necessary to modify their calculated values by making allowance for the depth of the road construction. The cross-sectional area of the road construction should be calculated or scaled from a plan of the construction and added to cross-sectional areas in cut and subtracted from cross-sectional areas in fill. Cross sections partly in cut and partly in fill should be inspected on the cross-sectional drawings and their areas modified accordingly.

The side widths, either separately or together in the form of the plan width, are used to mark the extent of the embankments and cuttings on the working drawings.

This is shown in figure 12.21. They help firstly in the calculation of the area of

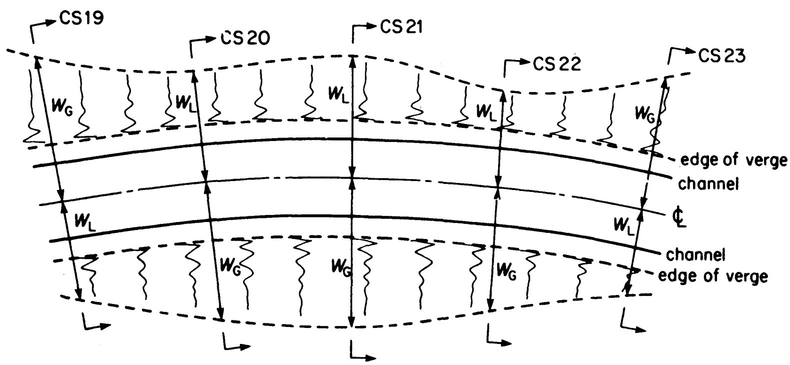


Figure 12.21

land which must be obtained for the construction and secondly in the calculation of the area of the site to be cleared and stripped of topsoil before construction can begin.

12.3 Calculation of Volumes

Volumes of earthworks can be calculated in several ways, three of which are considered here: volumes from cross sections, volumes from spot heights and volumes from contours.

12.3.1 Volumes from Cross Sections

The cross sections calculated in section 12.2 are used to calculate the volume contained between them. Two methods are considered, both comparable to the trapezoidal rule and Simpson's rule for areas.

End areas method

This is comparable to the trapezoidal rule for areas. If two cross-sectional areas A_1 and A_2 are a horizontal distance d_1 apart, the volume contained between them (V_1) is given by

$$V_1 = d_1 \frac{(A_1 + A_2)}{2}$$

This leads to the general formula for a series of N cross sections

$$\begin{aligned} V_{\text{total}} &= V_1 + V_2 + V_3 + \ldots + V_{N-1} \\ &= d_1 \frac{(A_1 + A_2)}{2} + d_2 \frac{(A_2 + A_3)}{2} + d_3 \frac{(A_3 + A_4)}{2} + \ldots \\ &+ d_{N-1} \frac{(A_{N-1} + A_N)}{2} \end{aligned}$$

and, if $d_1 = d_2 = d_3 = d_{N-1} = d$

$$\text{total volume} = \frac{d}{2} (A_1 + A_N + 2(A_2 + A_3 + \ldots A_{N-1}))$$

The end areas method will give accurate results if the cross-sectional areas are of the same order of magnitude.

Prismoidal formula

This is comparable to Simpson's rule for areas and is more accurate than the end areas method.

The volume contained between a series of cross sections a constant distance

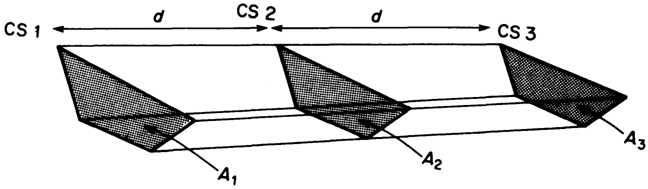


Figure 12.22

apart can be approximated to the volume of *a prismoid* which is a solid figure with plane parallel ends and plane sides. This is shown in figure 12.22.

It can be shown that for a series of three cross sections the volume, V_{1-3}, contained between them is given by

$$V_{1-3} = \frac{d}{3}(A_1 + 4A_2 + A_3)$$

This is the *prismoidal formula* and is used for earthwork calculations of cuttings and embankments and gives a true volume if either

(1) the transverse slopes at right angles to the centre line are straight and the longitudinal profile on the centre line is parabolic, or

(2) the transverse slopes are parabolic and the longitudinal profile is a straight line.

Hence, unless the ground profile is regular both transversely and longitudinally it is likely that errors will be introduced in assuming that the figure is prismoidal over its entire length. These errors, however, are small and the volume obtained is a good approximation to the true volume.

If figure 12.22 is extended to include cross section 4 (A_4) and cross section 5 (A_5), the volume from CS3 to CS5 (V_{3-5}) is given by

$$V_{3-5} = \frac{d}{3}(A_3 + 4A_4 + A_5)$$

Therefore, the total volume from CS1 to CS5 (V) is

$$V = \frac{d}{3}(A_1 + 4A_2 + 2A_3 + 4A_4 + A_5)$$

This leads to a general formula for N cross sections, where N MUST be ODD, as follows

$$V = \frac{d}{3}(A_1 + A_N + 4\ \Sigma \text{ even areas} + 2\ \Sigma \text{ remaining odd areas})$$

This is often referred to as Simpson's rule for volumes and should be used wherever possible. The worked example given in section 12.4.2 illustrates the application of the prismoidal formula.

Effect of curvature on volume

The foregoing discussion has assumed that the cross sections are taken on a straight road or similar. However, where a horizontal curve occurs the cross sections will no longer be parallel to each other and errors will result in volumes calculated from either the end areas method or the prismoidal formula.

To overcome this, *Pappus' theorem* must be used and this states that a volume swept out by a plane constant area revolving about a fixed axis is given by the product of the cross-sectional area and the distance moved by the centre of gravity of the section.

Hence the volumes of cuttings which occur on circular curves can be calculated with a better degree of accuracy. Figure 12.23 shows an unsymmetrical cross section in which the centroid is situated at a horizontal distance c from the centre line, where c is referred to as the *eccentricity*. The centroid may be on either side of the centre line according to the transverse slope.

Figure 12.24 shows this cross section occurring on a circular curve of radius R.

$$\text{Length of path of centroid} = (R + c)\,\theta \text{ rad}$$

From Pappus' theorem, the volume swept out is $V = A(R + c)\,\theta$ rad. But

$$\theta \text{ rad} = L/R, \text{ hence } V = LA(R + c)/R.$$

Therefore

$$V = L\left(A + \frac{Ac}{R}\right) = LA\left(1 + \frac{c}{R}\right) = LA'$$

In the expression for V, LA is the volume of a prismoid of length L and the term Ac/R can be regarded as the correction to be made to the cross-sectional area before calculating the volume as that of a normal prismoid. The corrected area can be expressed as

$$A' = A\left(1 \pm \frac{c}{R}\right)$$

The ± sign is necessary since the centroid can lie on either side of the centre line. The negative sign is adopted if the centroid lies on the same side of the centre line as the centre of curvature and the positive sign if on the other side.

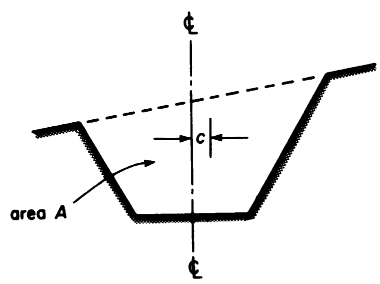


Figure 12.23

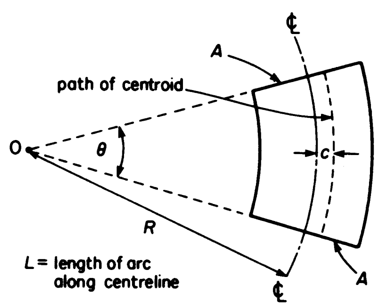


Figure 12.24

In practice, the shape of the cross section will not be constant so that neither A nor c will be constant. However, the ratio c/R will usually be small and it is generally sufficiently accurate to calculate the correction for each cross section and to use either the end areas method or the prismoidal formula to determine the volume.

Example of volume calculation from cross sections

Figure 12.25 shows a longitudinal section for a series of cross sections taken at 20 m intervals along the proposed centre line of a road, together with the resulting cross sections. The procedure for calculating the volume contained between CS1 and CS6 is as follows.

(1) Calculate cross-sectional areas CS1 to CS6 using one of the methods discussed in sections 12.1 and 12.2.

(2) Calculate the volume using either the end areas method or the prismoidal formula. If the cross sections are all of the same type, one of the formulae, preferably the prismoidal, can be used. However, if the cross sections are changing as in figure 12.25 it is best to work from one cross section to the next as follows.

CS1 to CS

$$\text{Volume of cut} = \frac{20}{2}\,(110.6 + 64.3) = \mathbf{1749\ m^3}$$

$$\text{Volume of fill} = \text{zero}$$

CS2 to CS3

$$\text{Volume of cut} = \frac{20}{2}\,(64.3 + 36.2) = \mathbf{1005\ m^3}$$

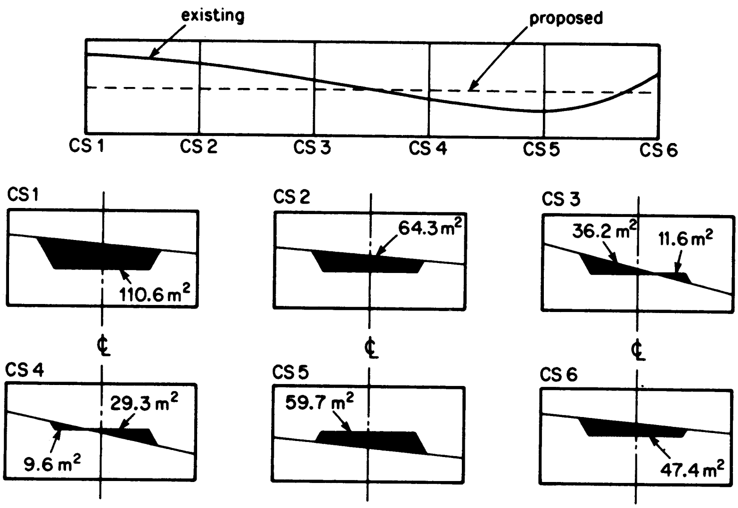


Figure 12.25

The volume of fill presents a problem. Between CS2 and CS3 there is a point at which the fill begins. A good estimate of the position of this point must be made to enable accurate volume figures to be obtained. This is best done by assuming that the rate of increase of fill between CS2 and CS3 is the same as that between CS3 and CS4.

Between CS3 and CS4, the area of fill increases from 11.6 m^2 to 29.3 m^2 in a distance of 20 m. If this is extrapolated back as shown in figure 12.26, the point at which the fill begins between CS2 and CS3 can be found.

In figure 12.26

$$\frac{d_1}{11.6} = \frac{20}{(29.3 - 11.6)}$$

hence

$$d_1 = 13.1 \text{ m}$$

Therefore

$$\text{volume of fill} = \frac{13.1}{2}\ (0 + 11.6) = \mathbf{76\ m^3}$$

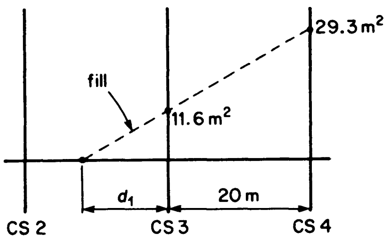


Figure 12.26

This extrapolation method will work only if the area of fill at CS4 is greater than twice the area of fill at CS3 otherwise a meaningless result will be obtained, that is, $d_1 > d$. The only solution to such an occurrence is to inspect the cross-sectional drawings and the longitudinal section and to make a reasoned estimate of the position at which the fill begins.

The extrapolation method is suitable for both cut and fill, whether increasing or decreasing.

CS3 to CS4

$$\text{volume of cut} = \frac{20}{2}\ (36.2 + 9.6) = \mathbf{458\ m^3}$$

$$\text{volume of fill} = \frac{20}{2}\ (11.6 + 29.3) = \mathbf{409\ m^3}$$

CS3 to CS5 The volume of cut must be calculated using the extrapolation method. From CS3 to CS4 the area of cut decreases from 36.2 m^2 to 9.6 m^2 in a distance of

20 m. Hence, the distance from CS4 towards CS5 at which the cut decreases (d_2) is given by

$$d_2 = \frac{20 \times 9.6}{36.2 - 9.6} = 7.2 \text{ m}$$

Therefore

$$\text{volume of cut} = \frac{7.2}{2}(9.6 + 0) = \mathbf{35\ m^3}$$

$$\text{volume of fill} = \frac{20}{2}(29.3 + 59.7) = \mathbf{890\ m^3}$$

CS5 to CS6 In this case the cross section changes from all fill to all cut. Again, for accuracy, it is necessary to estimate the position where the fill section ends and the cut section begins. The relevant part of the longitudinal section is shown in figure 12.27.

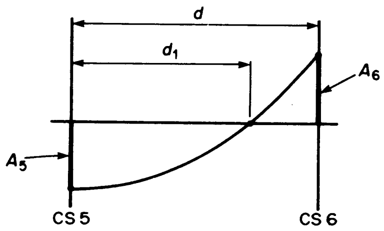


Figure 12.27

A linear relationship involving the cross-sectional areas can be used

$$\frac{d_1}{A_5} = \frac{d}{A_5 + A_6}$$

hence

$$d_1 = \frac{d A_5}{A_5 + A_6}$$

The volume of fill is obtained from $[d_1/2]\ (A_5 + 0)$ and the volume of cut is obtained from $[(d - d_1)/2]\ (0 + A_6)$. In this case

$$d_1 = \frac{20 \times 59.7}{59.7 + 47.4} = 11.1 \text{ m}$$

Therefore

$$\text{volume of fill} = \frac{11.1}{2}\ (59.7 + 0) = \mathbf{331\ m^3}$$

$$\text{volume of cut} = \frac{(20 - 11.1)}{2}\ (0 + 47.4) = \mathbf{211\ m^3}$$

This linear method applies equally when the cross section changes from all cut to all fill.

12.3.2 Volumes from Spot Heights

This method is used to obtain the volume of large deep excavations such as basements, underground tanks and so on where the formation level can be sloping, horizontal or terraced.

A square, rectangular or triangular grid is established on the ground and spot levels are taken at each grid intersection as described in section 2.12.2. The smaller the grid the greater will be the accuracy of the volume calculated but the amount of fieldwork increases so a compromise is usually reached.

The formation level at each grid point must be known and hence the depth of cut from the existing to the proposed level at each grid intersection can be calculated.

Figure 12.28 shows a 10 m square grid with the depths of cut marked at each grid intersection. Consider the volume contained in grid square $h_1h_2h_6h_5$; this is shown in figure 12.29.

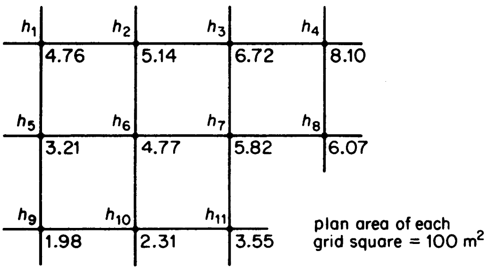


Figure 12.28

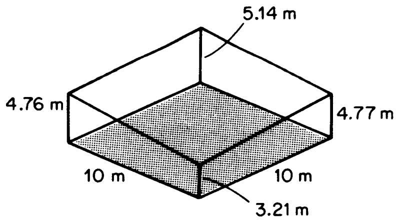


Figure 12.29

It is assumed that the surface slope is constant between grid intersections, hence the volume is given by

$$\text{volume} = \text{mean height} \times \text{plan area}$$
$$= \tfrac{1}{4}(4.76 + 5.14 + 4.77 + 3.21) \times 100 = \mathbf{447\ m^3}$$

A similar method can be applied to each individual grid square and this leads to a general formula for square or rectangular grids

$$\text{total volume} = \frac{A}{4}(\Sigma \text{ single depths} + 2\,\Sigma \text{ double depths} + 3\,\Sigma \text{ triple depths} + 4\,\Sigma \text{ quadruple depths}) + \delta V$$

where A = plan area of each grid square; single depths = depths such as h_1 and h_4 which are used once; double depths = depths such as h_2 and h_3 which are used twice; triple depths = depths such as h_7 which are used three times; quadruple depths = depths such as h_6 which are used four times; δV = the total volume outside the grid which is calculated separately.

Hence, in the example shown in figure 12.28

$$\text{volume contained within the grid area} = \frac{100}{4}[4.76 + 8.10 + 6.07 + 1.98 + 3.55 + 2(5.14 + 6.72 + 3.21 + 2.31) + 3(5.82) + 4(4.77)]$$

$$= 25(24.46 + 34.76 + 17.46 + 19.08) = \mathbf{2394\ m^3}$$

The result is only an approximation since it has been assumed that the surface slope is constant between spot heights.

If a triangular grid is used, the general formula must be modified as follows

(1) $A'/3$ must replace $A/4$ where A' = plan area of each triangle and
(2) depths appearing in five and six triangles must be included.

12.3.3 Volumes from Contours

This method is particularly suitable for calculating very large volumes such as those of reservoirs, earth dams, spoil heaps and so on.

The system adopted is to calculate the plan area enclosed by each contour and then treat this as a cross-sectional area. The contour interval provides the distance between cross sections and either the prismoidal or end areas method is used to calculate the volume. If the prismoidal method is used, the number of contours must be odd.

The plan area contained by each contour can be calculated using a planimeter or one of the other methods discussed in section 12.1. The graphical method is particularly suitable in this case.

The accuracy of the result depends to a large extent on the contour interval but normally great accuracy is not required, for example in reservoir capacity calculations, volumes to the nearest 1000 m^3 are more than adequate. Consider the following example.

Figure 12.30 shows a plan of a proposed reservoir and dam wall. The vertical interval is 5 m and the water level of the reservoir is to be 148 m. The capacity of the reservoir is required.

The volume of water that can be stored between the contours can be found by reference to figure 12.31, which shows a cross section through the reservoir and the plan areas enclosed by each contour and the dam wall.

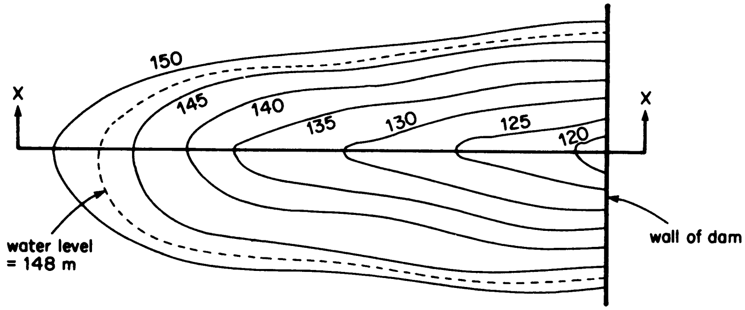


Figure 12.30

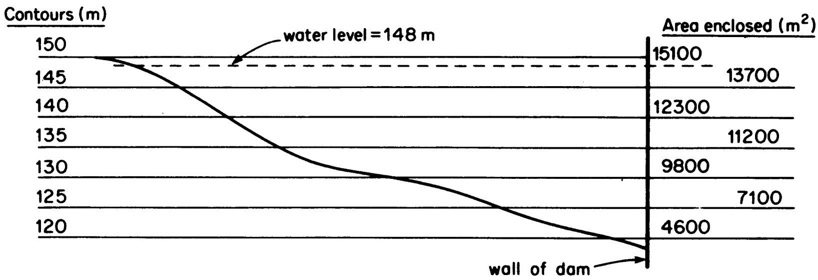


Figure 12.31

total volume = volume between 148 m and 145 m contours +
volume between 145 m and 120 m contours +
small volume below 120 m contour.

Volume between 148 m and 145 m contours is found by the end areas method to be

$$= \left(\frac{15\,100 + 13\,700}{2}\right) \times 3 = \mathbf{43\,200\ m^3}$$

Volume between 145 m and 120 m contours can also be found by the end areas method to be

$$= \frac{5}{2}\,(13\,700 + 4600 + 2(12\,300 + 11\,200 + 9800 + 7100))$$

$$= \mathbf{247\,750\ m^3}$$

The small volume below the 120 m contour can be found by decreasing the contour interval to say, 1 m and using the end areas method or the prismoidal formula. Alternatively, if it is very small, it may be neglected. Let this volume $= \delta V$. Therefore

$$\text{total volume} = 43\,200 + 247\,750 + \delta V \qquad (12.2)$$
$$= \mathbf{(290\,950 + \delta V)\ m^3}$$

(this would usually be rounded to the nearest 1000 m^3). The second term in equation (12.2) was obtained by the end areas method applied between contours 145 m and 120 m. Alternatively, the prismoidal formula could have been used between the 145 m and 125 m contours (to keep the number of contours ODD) and the end areas method between the 125 m and 120 m contours. If this is done, the volume between the 145 m and 120 m contours is calculated to be **248 583 m^3**.

12.4 Worked Examples

12.4.1 Division of an Area using the Cross Coordinate Method

Question

The polygon traverse PQRSTP shown in figure 12.32 is to be divided into two equal areas by a straight line that must pass through point R and which meets line TP at Z. The coordinates of the points are given in table 12.2. Calculate the coordinates of point Z.

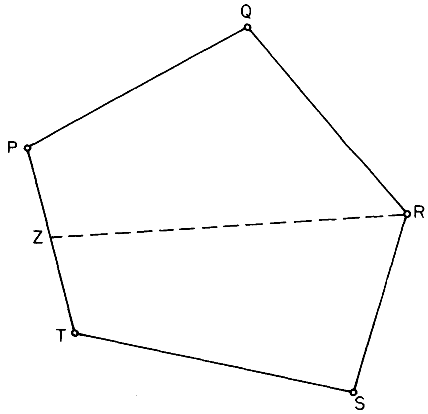


Figure 12.32

Solution

The traverse is lettered and specified in a clockwise direction. Hence, using the clockwise version of the cross coordinate method gives

$$\text{area PQRSTP} = \tfrac{1}{2}(1\,452\,532 - 1\,314\,662) = 68\,935\ m^2$$

Therefore

$$\text{area PQRZP} = \text{area ZRSTZ} = (68\,935/2) = 34\,467.5\ m^2$$

TABLE 12.2

Point	mE	mN
P	613.26	418.11
Q	806.71	523.16
R	942.17	366.84
S	901.89	203.18
T	652.08	259.26

Let point Z have coordinates (E_Z, N_Z).

Applying the clockwise version of the cross coordinate method to area PQRZP gives

$$68\,935 = 213\,432.58 - 51.27E_Z - 328.91N_Z$$

from which

$$E_Z = 2818.365 - 6.41525N_Z \tag{12.3}$$

A similar application to area ZRSTZ gives

$$E_Z = 2.69651N_Z - 286.765 \tag{12.4}$$

Solving equations (12.3) and (12.4) gives

$$E_Z = 632.16 \text{ m}, N_Z = 340.78 \text{ m}$$

As a check, since Z lies on the line PT

$$\frac{E_T - E_P}{N_T - N_P} \text{ should equal } \frac{E_Z - E_P}{N_Z - N_P}$$

Substituting the coordinates of P, T and Z gives

$$-0.2444 = -0.2444$$

which checks the coordinates of Z as calculated above.

12.4.2 Cross Sectional Area and Volume Calculations

Question

The centre line of a proposed road of formation width 12.00 m is to fall at a slope of 1 in 100 from chainage 50 m to chainage 150 m.

The existing ground levels on the centre line at chainages 50 m, 100 m and 150 m are 71.62 m, 72.34 m and 69.31 m respectively and the ground slopes at 1 in 3 at right angles to the proposed centre line.

If the centre line formation level at chainage 50 m is 71.22 m and side slopes are to be 1 in 1 in cut and 1 in 2 in fill, calculate the volumes of cut and fill between chainages 50 m and 150 m.

Solution

Figure 12.33 shows the longitudinal section from chainage 50 m to chainage 150 m. Hence

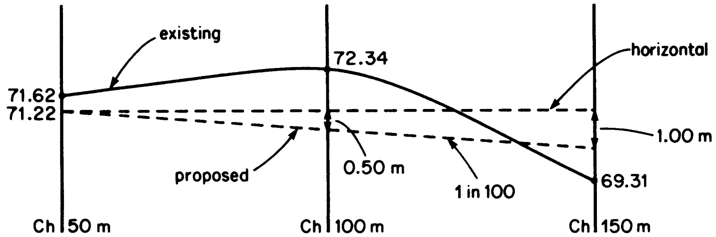


Figure 12.33

the centre-line formation level at chainage 100 m = 71.22 − 0.50

= 70.72 m

the centre-line formation level at chainage 150 m = 71.22 − 1.00

= 70.22 m

Figure 12.34 shows the three cross sections.

Since all the cross sections are part in cut and part in fill the formulae derived in section 12.2.4 apply

Cross Section 50 m $s = 3, b = 6$ m, $n = 2, m = 1$

$h = 71.62 - 71.22 = +0.40$ m, that is, cut at the centre line

This cross section is similar to that shown in figure 12.18*a*, hence

$$\textit{area of cut} = A_2 = \frac{(b + sh)^2}{2(s - m)} = \frac{(6 + 3 \times 0.40)^2}{2(3 - 1)} = \mathbf{12.96\ m^2}$$

$$\textit{area of fill} = A_1 = \frac{(b - sh)^2}{2(s - n)} = \frac{(6 - 3 \times 0.40)^2}{2(3 - 2)} = \mathbf{11.52\ m^2}$$

Cross Section 100 m $s = 3, b = 6$ m, $n = 2, m = 1$

$h = 72.34 - 70.72 = +1.62$ m, that is, cut at the centre line

This cross section is similar to cross section 50 m, hence again

$$\textit{area of cut} = A_2 = \frac{(6 + 3 \times 1.62)^2}{2(3 - 1)} = \mathbf{29.48\ m^2}$$

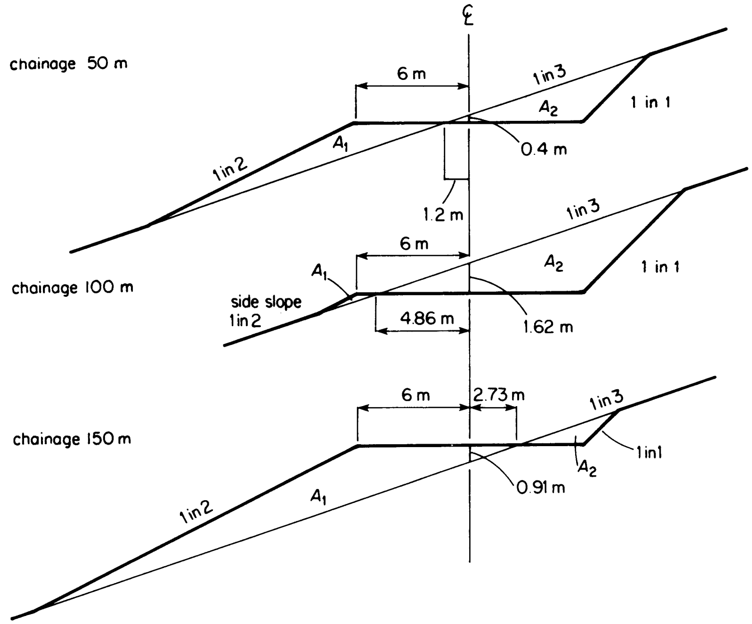


Figure 12.34

$$\textit{area of fill} = A_1 = \frac{(6 - 3 \times 1.62)^2}{2(3-2)} = \mathbf{0.65\ m^2}$$

Cross Section 150 m $\quad = 3, b = 6$ m, $n = 2, m = 1$

$h = 69.31 - 70.22 = -0.91$ m, that is, fill at the centre line

This cross section is similar to that shown in figure 12.18*b*, hence

$$\textit{area of cut} = A_2 = \frac{(b - sh)^2}{2(s-m)} = \frac{(6 - 3 \times 0.91)^2}{2(3-1)} = \mathbf{2.67\ m^2}$$

$$\textit{area of fill} = A_1 = \frac{(b + sh)^2}{2(s-n)} = \frac{(6 + 3 \times 0.91)^2}{2(3-2)} = \mathbf{38.11\ m^2}$$

The prismoidal formula can be used to calculate the volumes since the number of cross sections is odd, hence

$$\textit{volume of cut} = \frac{50}{3}\,[12.96 + 2.67 + 4(29.48)] = \mathbf{2225.8\ m^3}$$

$$\textit{volume of fill} = \frac{50}{3}\,[11.52 + 38.11 + 4(0.65)] = \mathbf{870.5\ m^3}$$

These figures would normally be rounded to at least the nearest cubic metre.

13

Mass Haul Diagrams

During the construction of long engineering projects such as roads, railways, pipelines and canals there may be a considerable quantity of earth required to be brought on to the site to form embankments and to be removed from the site during the formation of cuttings.

The earth brought to form embankments may come from another section of the site such as a tip formed from excavated material (known as a *spoil heap*) or may be imported on to the site from a nearby quarry. Any earth brought on to the site is said to have been *borrowed*.

The earth excavated to form cuttings may be deposited in tips at regular intervals along the project to form spoil heaps for later use in embankment formation or may be *wasted* either by spreading the earth at right angles to the centre line to form verges or by carting it away from the site area and depositing it in suitable local areas.

This movement of earth throughout the site can be very expensive and, since the majority of the cost of such projects is usually given over to the earth-moving, it is essential that considerable care is taken when planning the way in which material is handled during the construction. The *mass haul diagram* is a graph of volume against chainage which greatly helps in planning such earth-moving.

The x axis represents the chainage along the project from the position of zero chainage.

The y axis represents the aggregate volume of material up to any chainage from the position of zero chainage.

When constructing the mass haul diagram, volumes of cut are considered positive and volumes of fill are considered negative. The vertical and horizontal axes of the mass haul diagram are usually drawn at different scales to exaggerate the diagram and thereby facilitate its use.

The mass haul diagram considers only earth moved in a direction longitudinal to the direction of the centre line of the project and does not take into account any volume of material moved at right angles to the centre line.

13.1 Formation Level and the Mass Haul Diagram

Since the mass haul diagram is simply a graph of aggregate volume against chainage it will be noted that if the volume is continually decreasing with chainage, the project is all embankment and all the material will have to be imported on to the site since there will be no fill material available for use. Such an occurrence will involve a great deal of earth-moving and is obviously not an ideal solution.

If a better attempt had been made in the selection of a suitable formation level such that some areas of cut were balanced out by some areas of fill, a more economical solution would result. Because of this vital connection between the formation level and the mass haul diagram the two are usually drawn together as shown in figure 13.1 and section 13.2 describes its method of construction.

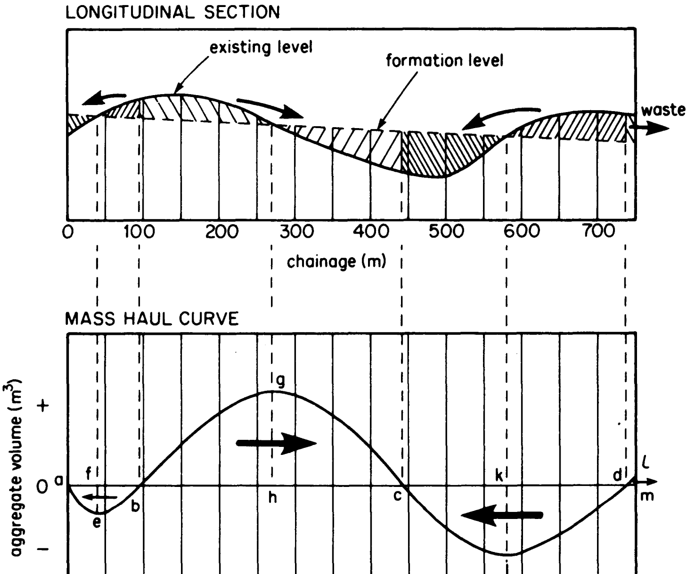


Figure 13.1

13.2 Drawing the Diagram

Figure 13.1 was drawn as follows.

(1) The cross-sectional areas are calculated at regular intervals along the project, in this case every 50 m.

(2) The volumes between consecutive areas and the aggregate volume along the site are calculated, cut being positive and fill negative.

(3) Before plotting, a table is drawn up as shown in table 13.1. One of the columns in table 13.1 shows bulking and shrinkage factors. These are necessary

TABLE 13.1
Mass Haul Diagram Calculations

Chainage (metres)	Individual volume (m^3) Cut(+)	Fill(-)	Bulking/ Shrinkage Factors	Corrected Individual Volumes (m^3) Cut(+)	Fill(-)	Aggregate Volume (m^3) Cut(+) Fill(-)
0	-	-	-	-	-	0
50	40	800	1.1	44	800	- 756
100	730	-	1.1	803	-	+ 47
150	910	-	1.1	1001	-	+1048
200	760	-	1.1	836	-	+1884
250	450	-	1.1	495	-	+2379
300	80	110	1.1	88	110	+2357
350	-	520	-	-	520	+1837
400	-	900	-	-	900	+ 937
450	-	1120	-	-	1120	- 183
500	-	970	-	-	970	-1153
550	-	620	-	-	620	-1773
600	200	200	0.8	160	200	-1813
650	590	-	0.8	472	-	-1341
700	850	-	0.8	680	-	- 661
750	1120	-	0.8	896	-	+ 235

owing to the fact that material usually occupies a different volume when it is used in a man-made construction to that which it occupied in natural conditions. Very few soils can be compacted back to their original volume.

If 100 m^3 of rock are excavated and then used for filling, they may occupy 110 m^3 even after careful compaction and the rock is said to have undergone *bulking* and has a *bulking factor* of 1.1.

If 100 m^3 of clay are excavated and then used for filling they may occupy only 80 m^3 after compaction and the clay is said to have undergone *shrinkage* and has a *shrinkage factor* of 0.8.

Owing to the variable nature of the same material when found in different parts of the country, it is impossible to standardise bulking and shrinkage factors for different soil and rock types. Therefore, a list of such factors has deliberately not been included since it would indicate a uniformity that, in practice, does not exist. Instead, it is recommended that local knowledge of the materials in question should be considered together with tests on soil and rock samples from the area so that reliable bulking and shrinkage factors (which apply only to that particular site) can be determined.

As far as the mass haul diagram is concerned, it is the volumes of fill that are critical, for example, if the hole in the ground is 1000 m^3, the required volume is that amount of cut which will fill the hole. There are two methods which can be used to allow for such bulking and shrinkage. Either the calculated volumes of fill can be amended by dividing them by the factors applying to the type of material available for fill or the calculated volumes of cut can be amended by multiplying them by the factors applying to the type of material in the cut. In table 13.1 the latter has been done.

(4) The longitudinal section along the proposed centre line is plotted, the proposed formation level being included.

(5) The axes of the mass haul diagram are drawn underneath the longitudinal profile such that chainage zero on the profile coincides with chainage zero on the diagram.

(6) The aggregate volume up to chainage 50 is plotted at $x = 50$ m. The aggregate volume up to chainage 100 is plotted at $x = 100$ m and so on for the rest of the diagram.

(7) The points are joined by curves or straight lines to obtain the finished mass haul diagram.

13.3 Terminology

(1) *Haul distance* is the distance from the point of excavation to the point where the material is to be tipped.

(2) *Average haul distance* is the distance from the centre of gravity of the excavation to the centre of gravity of the tip.

(3) *Free haul distance* is that distance, usually specified in the contract, over which a charge is levied only for the volume of earth excavated and not its movement. This is discussed further in section 13.5.

(4) *Free haul volume* is that volume of material which is moved through the free haul distance.

(5) *Overhaul distance* is that distance, in *excess* of the free haul distance, over which it may be necessary to transport material. See section 13.5.

(6) *Overhaul volume* is that volume of material which is moved in excess of the free haul distance.

(7) *Haul*. This is the term used when calculating the costs involved in the earth-moving and is equal to the sum of the products of each volume of material and the distance through which it is moved. It is equal to the total volume of the excavation multiplied by the average haul distance and on the mass haul diagram is equal to the area contained between the curve and *balancing line* (see section 13.4).

(8) *Freehaul* is that part of the haul which is contained within the free haul distance.

(9) *Overhaul* is that part of the haul which remains after the freehaul has been removed. It is equal to the product of the overhaul volume and the overhaul distance

(10) *Waste* is that volume of material which must be exported from a section of the site owing to a surplus or unsuitability.

(11) *Borrow* is that volume of material which must be imported into a section of the site owing to a deficiency of suitable material.

13.4 Properties of the Mass Haul Curve

Consider figure 13.1.

(1) When the curve rises the project is in cut since the aggregate volume is increasing, for example section ebg. When the curve falls the project is in fill since the aggregate volume is decreasing, for example section gcj. Hence, the end of a

section in cut is shown by a maximum point on the curve, for example point g, and the end of a section in fill is shown by a minimum point on the curve, for example point j.

The vertical distance between a maximum point and the next forward minimum represents the volume of an embankment, for example (gh + kj), and the vertical distance between a minimum point and the next forward maximum represents the volume of a cutting, for example (ef + gh).

(2) Any horizontal line which cuts the mass haul curve at two or more points balances cut and fill between those points and because of this is known as a *balancing line*.

In figure 13.1, the x axis is a balancing line and the volumes between chainages a and b, b and c, and c and d are balanced out, that is, as long as the material is suitable, all the cut material between a and d can be used to provide the exact amount of fill required between a and d.

The x axis, however, does not always provide the best balancing line and this is discussed further in section 13.6.

When a balancing line has been drawn on the curve, any area lying above the balancing line signifies that the material must be moved to the right and any area lying below the balancing line signifies that the material must be moved to the left. In figure 13.1, the arrows on the longitudinal section and the mass haul diagram indicate these directions of haul.

The length of balancing line between intersection points is the maximum haul distance in that section, for example the maximum haul distance in section bc is (chainage c – chainage b).

(3) The area of the mass haul diagram contained between the curve and the balancing line is equal to the *haul* in that section, for example afbea, bgchb and ckdjc.

If the horizontal scale is 1 mm = R m and the vertical scale is 1 mm = S m^3, then an area of T mm^2 represents a haul of TRS m^3 m. This area could be measured using one of the methods discussed in chapter 12. Note that the units of haul are m^3 m (one cubic metre moved through one metre).

Instead of calculating centres of gravity of excavations and tips, which can be a difficult task, the *average haul distance* in each section can be easily found by dividing the *haul* in that section by the *volume* in that section, for example

$$\text{the } \textit{average haul distance} \text{ between b and c} = \frac{\text{area bgchb}}{\text{gh}} \frac{\text{m}^3\ \text{m}}{\text{m}^3}$$

(4) If a surplus volume remains, this is *waste* and must be removed from the site, for example l m; if a deficiency of earth is found at the end of the project this is *borrow* and must be imported on to the site. It is possible for waste and borrow to occur at any point along the site and this is discussed in section 13.6.

13.5 Economics of Mass Haul Diagrams

When costing the earth-moving, there are four basic costs which are usually included in the contract for the project.

(1) *Cost of freehaul*

Any earth moved over distances not greater than the free haul distance is costed only on the excavation of its volume, that is £A per m^3.

(2) *Cost of overhaul*

Any earth moved over distances greater than the free haul distance is charged both for its volume and for the distance in excess of the free haul distance over which it is moved. This charge can be specified either for units of haul, that is, £B per m^3 m, or for units of volume, that is £C per m^3.

(3) *Cost of waste*

Any surplus or unsuitable material which must be removed from the site and deposited in a tip is usually charged on units of volume, that is, £D per m^3. This charge can vary from one section of the site to another depending on the nearness of tips.

(4) *Cost of borrow*

Any extra material which must be brought on to the site to make up a deficiency is also usually charged on units of volume, that is, £E per m^3. This charge can also vary from one section of the site to another depending on the nearness of borrow pits or spoil heaps.

The following example illustrates how the costs of freehaul and overhaul can be calculated. The example given in section 13.6.1 illustrates how the costs of borrowing and wasting can affect the final decision as to how the earth should be moved around the site.

13.5.1 Example of Costing using the Mass Haul Diagram

Question

In a project for which a section of the mass haul diagram is shown in figure 13.2, the free haul distance is specified as 100 m. Calculate the cost of earth-moving in

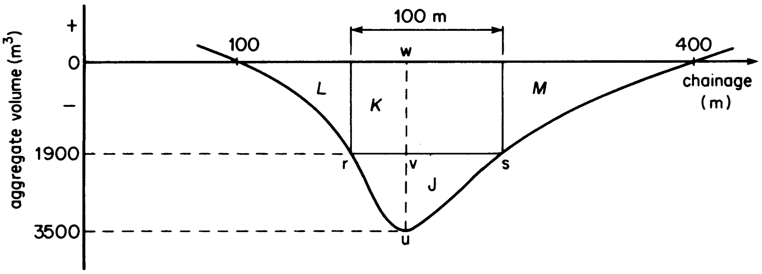


Figure 13.2

the section between chainages 100 m and 400 m if the charge for moving material within the free haul distance is £*A* per m^3 and that for moving any overhaul is £*B* per m^3 m.

The x axis should be taken as the balancing line and the areas between the curve and the balancing line in figure 13.2 were measured with a planimeter and, on conversion, found to be as follows

$$\text{area of } (J + K + L + M) = 396\,000 \text{ m}^3 \text{ m}$$

$$\text{area } J = 181\,300$$

Solution

This type of problem can be solved in one of two ways.

Solution 1 – Using planimeter areas only

Between chainages 100 m and 400 m, the x axis balances cut and fill and the total volume to be moved in that section is given in figure 13.2 as uw = 3500 m³.

The free haul distance of 100 m is fitted to figure 13.2 so that it touches the curve at two points r and s This means that the volume uv is the free haul volume and is, therefore, only charged for volume.

$$\text{uv} = (3500 - 1900)\text{m}^3 = 1600 \text{ m}^3$$

Therefore, area J can be removed since it is costed as **£(1600*A*).**

This leaves volume vw, which is equal to 1900 m³, to be considered. This volume is the overhaul volume and has to be moved over a distance greater than the free haul distance. This distance through which it is moved has two components, the free haul distance and the overhaul distance, and this leads to two costs.

(1) The overhaul volume moved through the free haul distance is costed on its volume only. This is area K in figure 13.2. The cost = **£(1900*A*).**

(2) The overhaul volume moved through the overhaul distance is the overhaul and is shown in figure 13.2 as areas L and M. The cost is that involved in moving area M to area L and is obtained as follows.

$$\begin{aligned}\text{area contained in } L \text{ and } M &= (J + K + L + M) - (J + K)\\ &= 396\,000 - (181\,300 + (1900 \times 100))\\ &= 24\,700 \text{ m}^3 \text{ m}\end{aligned}$$

Hence

$$\text{cost of this overhaul} = \mathbf{£(24\,700}\boldsymbol{B}\mathbf{)}$$

Therefore

$$\begin{aligned}\textit{total cost} &= \text{free haul volume cost + overhaul volume costs}\\ &= £(1600A) + (£(1900A) + £(24\,700B))\\ &= \mathbf{£(3500}\boldsymbol{A}\mathbf{) + £(24\,700}\boldsymbol{B}\mathbf{)}\end{aligned}$$

Solution 2 – Using average haul distance and overhaul distance

Average haul distance between chainages 100 m and 400 m

$$= \frac{\text{haul between chainages 100 m and 400 m}}{\text{total volume between chainages 100 m and 400 m}}$$

$$= \frac{396\,000}{3500} = 113 \text{ m}$$

but the free haul distance = 100 m, hence

overhaul distance = 113 − 100 = 13 m

therefore

overhaul = overhaul volume × overhaul distance

$= 1900 \times 13 = 24\,700$ m^3 m

As for solution 1, the cost of moving material over the free haul distance

= (free haul volume + overhaul volume) × £*A*

= £3500*A* (areas *J* and *K*)

cost of overhaul = £24 700*B* (moving area *M* to area *L*)

therefore

total cost = £3500*A* + £24 700*B*

13.6 Choice of Balancing Line

In the example given in section 13.5.1, the *x* axis was used as the balancing line. This is not always ideal. Figure 13.3 shows three possible balancing lines for the same mass haul diagram. In figure 13.3*a* the *x* axis has been used and this results in waste near chainage 230 m.

In figure 13.3*b* a balancing line is shown which gives wastage near chainage 0 m. This may be better and cheaper if local conditions provide a suitable wasting point near chainage zero.

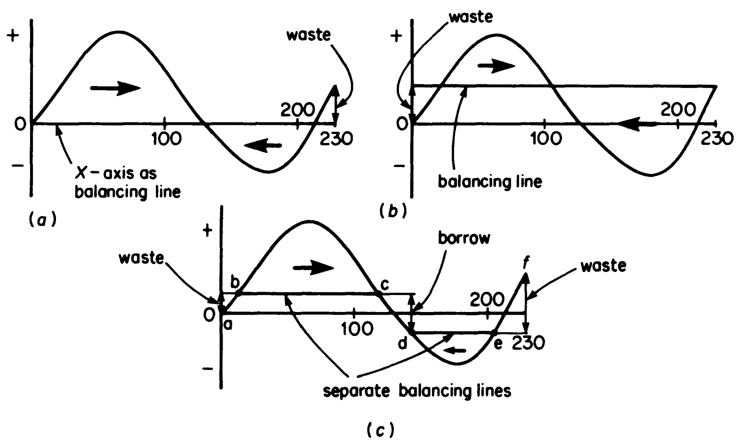


Figure 13.3

In figure 13.3*c* two different balancing lines have been used, bc and de. This results in waste near chainage 0 m where the curve is rising from a to b, borrow near chainage 125 m where the curve is falling from c to d and waste near chainage 210 m where the curve is rising from e to f. The two waste sections may be used to satisfy the central borrow requirement if economically viable.

Which choice is best depends on local conditions, for example, proximity of borrow pits, quarries and suitable tipping sites. However, the following factors should be considered before a final choice is made.

(1) The use of more than one balancing line results in waste and borrow at intermediate points along the project which will involve extra excavation and transportation of material.

(2) Short, unconnected, balancing lines are often more economical than one long continuous balancing line, especially where the balancing lines are shorter than the free haul distance since no overhaul costs will be involved.

(3) The direction of haul can be important. It is better to haul downhill to save power and, if long uphill hauls are involved, it may be better to waste at the lower points and borrow at the higher points.

(4) The main criterion should be one of economy. The free haul limit should be exceeded as little as possible in order that the amount of overhaul can be minimised.

(5) The haul is given by the area contained between the mass haul curve and the balancing line. Since the haul consists of freehaul and overhaul, if the haul area on the diagram can be minimised, the majority of it will be freehaul and hence overhaul will also be minimised. Therefore, the most economical solution from the haul aspect is to minimise the area between the curve and the balancing line. However, as shown in figure 13.3*c*, this can result in large amounts of waste and borrow at intermediate points along the project. The true economics can only be found by considering all the probable costs, that is, those of hauling, wasting and borrowing.

(6) Where long haul distances are involved, it may be more economical to waste material from the excavation at some point within the free haul limit at one end of the site and to borrow material from a location within the free haul limit at the other end of the site rather than cart the material a great distance from one end of the site to the other.

This possibility will become economical when the cost of excavating and hauling one cubic metre to fill from one end of the site to the other equals the cost of excavating and hauling one cubic metre to waste at one end of the site plus the cost of excavating and hauling one cubic metre to fill from a borrow pit at the other end of the site.

In practice, several different sets of balancing lines are tried and each costed separately with reference to the costs of wasting, borrowing and hauling. The most economical solution is usually adopted. The following example illustrates how this can be done.

13.6.1 Example of the Use of Balancing Lines in Costing

Question

In a project for which a section of the mass haul diagram is shown in figure 13.4, the free haul distance is specified as 200 m. The earth-moving charges are as follows

cost of free haul volume = £A per m^3

cost of overhaul volume = £B per m^3

cost of borrowing = £E per m^3

Calculate the costs of each of the following alternatives

(1) borrowing at chainage 1000 m only
(2) borrowing at chainage 0 m only
(3) borrowing at chainage 300 m only.

Solution

The 200 m free haul distance is added to figure 13.4 as shown, that is, rs = tu = 200 m. The volumes corresponding to the horizontal lines rs and tu are interpolated from the curve to be +433 m^3 and −2007 m^3 respectively.

(1) Borrowing at chainage 1000 m only
In this case, acg is used as a balancing line and borrow is required at g (chainage 1000 m) to close the loop cefgc.

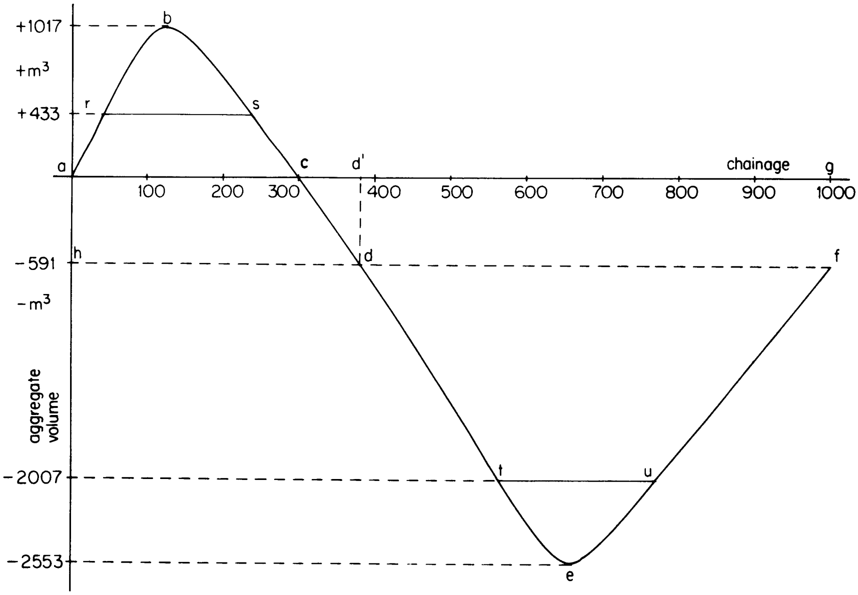


Figure 13.4

free haul volume in section ac = 1017 − 433 = 584 m^3

free haul volume in section cg = 2553 − 2007 = 546 m^3

hence

total free haul volume = 584 + 546 = *1130 m^3*

overhaul volume in section ac = 433 m^3

overhaul volume in section cg = 2007 m^3

hence

total overhaul volume = 2440 m^3

borrow at g = 591 m^3

therefore

cost of borrowing at chainage 1000 m only

= £1130*A* + £2440*B* + £591*E*

(2) Borrowing at chainage 0 m only
In this case, hdf is used as a balancing line and borrow is required at h (chainage 0 m) to close the loop habdh.
The *total free haul volume* again equals *1130 m^3*

overhaul volume in section hd = 433 + 591 = 1024 m^3

overhaul volume in section df = 2007 − 591 = 1416 m^3

hence

total overhaul volume = 2440 m^3

borrow at h = 591 m^3

therefore

cost of borrowing at chainage 0 m only

= £1130*A* + £2440*B* + £591*E*

This is the same as the cost of borrowing at chainage 1000 m provided that the cost of borrow is the same at chainages 0 m and 1000 m.

(3) Borrowing at chainage 300 m only
In this case, two separate balancing lines ac and df are used and borrow is required at c (chainage 300 m) to fill the gap between c and d.
As before, *total free haul volume = 1130 m^3*, however

overhaul volume in section ac = 433 m^3

overhaul volume in section df = 2007 − 591 = 1416 m^3

hence

total overhaul volume = 1849 m^3

borrow at c = 591 m^3

therefore

cost of borrowing at chainage 300 m only

= £1130*A* + £1849*B* + £591*E*

This is the cheapest alternative assuming that the costs of borrow at chainages 0 m, 300 m and 1000 m are all equal. Considerably less overhaul is required when borrowing at chainage 300 m only.

13.7 Uses of Mass Haul Diagrams

Mass haul diagrams can be used in several ways.

13.7.1 In Design

In section 13.1, the close link between the mass haul diagram and the formation level was discussed. If several formation levels are tried and a mass haul diagram constructed for each, that formation which gives the most economical result and maintains any stipulated standards, for example, gradient restrictions in vertical curve design, can be used.

Nowadays, mass haul diagrams tend to be produced using computer graphics and this greatly reduces the time required to obtain several different possible mass haul diagrams for comparison purposes. Both the BIPS and MOSS computer road design systems discussed in section 10.16 have subroutines for determination of earthwork quantities.

13.7.2 In Financing

Once the formation level has been designed, the mass haul diagram can be used to indicate the most economical method of moving the earth around the project and a good estimate of the overall cost of the earth-moving can be calculated.

13.7.3 In Construction

The required volumes of material are known before construction begins, enabling suitable plant and machinery to be chosen, sites for spoil heaps and borrow pits to be located and directions of haul to be established.

13.7.4 In Planning Ahead

The mass haul diagram can be used to indicate the effect that other engineering works, for example tunnels and bridges, within the overall project will have on the earth-moving. Such constructions upset the pattern of the mass haul diagram by restricting the directions of haul but, since the volumes and hence the quantities of any waste and borrow will be known, suitable areas for spoil heaps and borrow pits can be located in advance of construction, enabling work to proceed smoothly.

14

Setting Out

A definition often used for setting out is that it is the reverse of surveying. What is meant by this is that whereas surveying is the process of producing a plan or map of a particular area, setting out begins with the plan and ends with some particular engineering project correctly positioned in the area. This definition can be misleading since it implies that setting out and surveying are opposites. This is not true.. Most of the techniques and equipment used in surveying are also used in setting out and it is important to realise that setting out is simply one application of surveying.

A better definition of setting out is provided by the International Organisation for Standardization (ISO) in their publication ISO/DP 7078 Building Construction which states that

'Setting Out is the establishment of the marks and lines to define the position and level of the elements for the construction work so that works may proceed with reference to them. This process may be contrasted with the purpose of Surveying which is to determine by measurement the positions of existing features.'

Attitudes towards setting out vary enormously from site to site, but, frequently, insufficient importance is attached to the process and it tends to be rushed to, supposedly, save time. Unfortunately, such haphazard procedures can lead to errors which in turn require costly corrections.

The British Standard Code of Practice for Accuracy in Building, BS 5606, specifies particular techniques and equipment to be used in setting out as do several international bodies, notably the International Council for Building Research, Studies and Documentation (CIB), the International Federation of Surveyors (FIG) and the International Organisation for Standardization (ISO). Further information on these bodies and their publications are given in sections 14.18 and 14.19.

However, although progress is being made in the production of national and international standards, the main problems of the lack of education in and the poor knowledge of suitable setting out procedures remain. Good knowledge is vital since, despite the lack of importance often attached to the process, setting out is probably the most important stage in any civil engineering construction. Errors in setting out cause delays which leave machinery and plant idle, resulting in additional costs.

This chapter deals with some of the techniques and equipment used in setting out and, since horizontal and vertical curves are discussed in other chapters, concentrates on setting out procedures used for other engineering schemes.

14.1 Personnel Involved in Setting Out and Construction

The *Client* or *Promoter* is the person, company or government department who requires the particular scheme to be undertaken and finances the project. Usually, the Promoter has no engineering knowledge and therefore commissions an *Engineer* (possibly a firm of Consulting Engineers or the City Engineer of a local authority) to provide the professional expertise. A formal contract is sometimes established between these two parties.

It is the responsibility of the *Engineer* to investigate various solutions, to undertake the site investigation and to prepare the necessary calculations, drawings, specifications and quantities for the proposed scheme. The calculations and drawings give the form and construction of the work; the specifications describe all materials and workmanship and the quantities are used as a method of costing the work and, subsequently, as a means of calculating payment for the work executed.

Upon completion of these documents, the scheme is put out to contractors to submit tenders. From the tenders received, a *Contractor* is nominated to carry out the work and a contract is, subsequently, made between the Contractor and the Promoter.

Hence, three parties are now involved, the Promoter, the Engineer and the Contractor.

Although the Engineer is not legally a party to the contract between the Promoter and the Contractor, the Engineer acts as the agent of the Promoter and ensures that work is carried out in accordance with the drawings, calculations, specifications and quantities. The Promoter is rarely, if ever, seen on site. The Engineer is represented on site by the *Resident Engineer* (RE). The Contractor is represented on site by the *Agent*.

The responsibilities of the Resident Engineer are usually described in the document known as the conditions of contract. The RE is in the employ of the Engineer who has overall responsibility for the contract. Many responsibilities are vested in the Resident Engineer by the Engineer. The RE is helped on site by a staff which can include assistant resident engineers and clerks of works.

The Agent, being in the employ of the Contractor, is responsible for the actual construction of the works. The Agent is a combination of engineer, manager and administrator who supervises assistant agents and site foremen who are involved in the day to day construction of the work.

Many large organisations employ a Contracts Manager who mainly supervises financial dealings on several contracts and is a link between head office and site.

As regards setting out, the Resident Engineer and the Agent usually work in close cooperation and they have to meet frequently to discuss the work. The Agent undertakes the setting out and it is checked by the Resident Engineer. Good communication is essential since, although the Resident Engineer checks the work, the setting out is the responsibility of the Contractor and the cost of correcting any errors in the setting out has to be paid for by the Contractor, providing the Resident Engineer has supplied reliable information.

Setting-out records, to monitor the progress, accuracy and any changes from the original design should be kept by both engineering parties as the scheme proceeds. These can be used to settle claims but more usually provide the basis for amending the working drawings and help in costing the various stages of the project.

Further detailed information on the topics discussed in this section can be found in references given in section 14.19.

14.2 Aims of Setting Out

There are two main aims when undertaking setting-out operations.

(1) The new structure must be correct in all three dimensions both relatively and absolutely, that is, it must be of the correct size, in the correct plan position and at the correct level.

(2) Once setting out begins it must proceed quickly and with little or no delay in order that costs can be minimised.

There are many techniques used in practice to achieve these aims but all are based on three general principles.

(1) Horizontal control points must be established within or near the design area from which the design points can be set out in their correct plan positions. This is *horizontal control* and is discussed in section 14.6.

(2) Reference marks of known height relative to an agreed datum are required within or near the design area from which the design points can be set out at their correct level. This is *vertical control* and is discussed in section 14.7.

(3) Accurate methods must be adopted to establish design points from this horizontal and vertical control. This involves *positioning techniques* and is discussed in section 14.8.

In addition, the chances of achieving the aims and minimising errors will be greatly increased if the setting out operations are planned well in advance, taking into account the considerations discussed in the following section.

14.3 Important Considerations

14.3.1 Recording and Filing Information

As work proceeds, the quantity of level and offset books, booking forms and other setting out documents will quickly grow. Therefore, some form of filing or storage system is necessary to give easy access to information.

14.3.2 Care of Instruments

All instruments to be used on the site should be inspected and checked before work commences and again at regular intervals during the work, that is, once per week when used daily and at least once every month. In the case of levels and theodolites, the permanent adjustments should be carried out and regular checks maintained. All other equipment such as chains, tapes and ranging rods should be kept clean and oiled where necessary.

All equipment must be stored carefully in a dry place. Tripod-mounted equipment should not be left unattended over a reference mark since such marks may be near the traffic routes on site and the legs are easily knocked, resulting in damage to essential and expensive equipment.

Further information on the care of instruments can be found in the national and international standards referenced in section 14.19.

14.3.3 Maintaining Accuracy

Once the control framework of plan and level points has been established, all design points must be set out from this and not from any design points already established. This avoids cumulative errors from one design point to the next.

14.3.4 Regular Site Inspection

The engineer should inspect the site daily for signs of moved or missing pegs. A peg may be disturbed and replaced without the engineer being informed. Points of known level should be checked at regular intervals, preferably at least once a week, and points of known plan position should be checked from similar marks nearby. All the control points must be permanently and clearly marked and protected on site.

14.3.5 Error Detection

Once the setting out has begun it should be independently checked wherever possible. This ensures that any errors are detected and can be corrected at an early stage.

There is nothing to be gained from hiding errors as this does not removed them; they will only reappear at a later stage when dealing with them will be that much more difficult and expensive.

14.3.6 Communication on Site

Lack of communication is one of the main causes of errors on construction sites. The engineer must understand exactly what has to be done *before* going ahead and doing it.

14.4 Stages in Setting Out

Setting out can be divided into two broad stages but the division is not easily defined and a certain amount of overlap is inevitable.

14.4.1 First Stage Setting Out

The first stage in any construction is to set it out on the ground in its correct position. This involves preliminary operations followed by the establishment of horizontal and vertical control points on or near the site. This control is used to establish corner points of buildings, road centre lines or any other major design points on the scheme. The boundary of the scheme is also established to enable site clearance to begin. The first stage ends once excavation to foundation level and construction to ground floor level or similar have been completed. The relevant sections of this chapter are 14.5 to 14.10 inclusive.

14.4.2 Second Stage Setting Out

This continues on from the first stage, beginning at the ground floor slab or similar. Horizontal and vertical control points are transferred from the area round the structure on to the structure itself and used to establish the various elements required in the floor by floor construction. The relevant sections of this chapter are 14.11 to 14.15 inclusive.

14.5 Preliminaries to Setting Out

The most important of these are listed below and they must be undertaken with a great deal of care and not rushed. A mistake or lack of thought at this stage in the construction can be extremely costly at a later date.

14.5.1 Checking the Design

The design information must be checked before any setting out is done. Occasionally it may not be possible to set out part of the design owing to obstacles on site and this must be referred back to the designer.

14.5.2 Reconnaissance

The engineer should become familiar with the area and check for any irregularities or faults in the ground surface which may cause problems. An on-site inspection is vital; simply looking at the site plan in the office is not sufficient. Any discrepancies between the area and the plan should be noted.

During this reconnaissance, suitable positions for reference marks can be temporarily marked with ranging rods or wooden pegs.

14.5.3 Survey Stations

With a little foresight, survey stations used to produce the survey plan can be left in position to form a basis for the horizontal control of the site. These must, whenever possible, be made permanent and protected to avoid their disturbance. A suitable method of construction and protection is shown in figure 14.6 and figure 14.7.

14.5.4 Agreed Bench Marks

To provide a basis for vertical control, all levels on site will normally be reduced to a nearby *ordnance bench mark* (OBM). The actual OBM used will be agreed in writing between the Engineer and the Contractor.

This bench mark is known as the *master bench mark* (MBM) and is used firstly to establish points of known level near to the proposed structure, these are *transferred bench marks* (TBMs), and secondly if there are other OBMs nearby their heights should be checked with reference to the MBM and their amended heights used. This ensures that only one MBM is used. The accuracy of this levelling should be within ± 0.010 m.

It is also possible to use any nearby horizontal control stations as temporary bench marks, providing they have been permanently marked.

14.5.5 Plans

Before any form of construction can begin, a preliminary survey is required. This may be undertaken by the Engineer (see section 14.1) or a team of surveyors and the result will be a contoured plan of the area at a suitable scale (usually 1 : 500 or larger) showing all the existing detail. It is usually prepared from a traverse or from a triangulation scheme and the traverse stations are often left in position to provide control points. This first plan is known as the *site* or *survey plan*.

The Engineer takes this site plan and uses it as a basis for the design of the project. The proposed scheme is drawn on the site plan and this becomes the *layout* or *working drawings*. All relevant dimensions are shown and a set of documents relating to the project and the drawings is included. These form part of the scheme when it is put out to tender. The Contractor who is awarded the job will be given these drawings.

The Contractor uses these layout drawings to decide on the location of the horizontal and vertical control points in the area from which the project is to be set out and on the positions of site offices, stores, access points and spoil heaps. All this information together with all angles and lengths necessary to relocate the control points should they become disturbed is recorded on a copy of the original site plan and forms what is known as the *setting-out plan*.

As work proceeds, it may be necessary to make slight amendments to the original design to overcome unforeseen problems. These will be agreed between the Resident Engineer and the Agent. Any such alterations are recorded on a copy of the working drawings to form what is known as the *as built drawing* or *record drawing*.

14.6 Methods of Horizontal Control

This is the establishment of reference marks of known plan position, that is, known coordinates, from which design points on the project may be set out by one of the methods discussed in section 14.8.2.

The process used in establishing horizontal control is one of *working from the whole to the part*. This entails the use of a major control network enclosing the

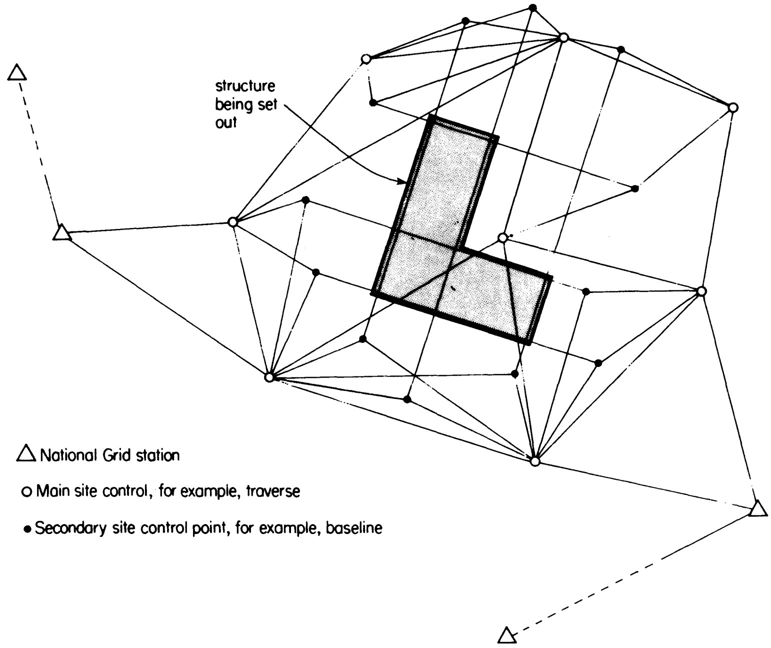


Figure 14.1 *Site control*

whole area, which in Great Britain would be several points on the National Grid. From these National Grid stations, main site control stations are established from which the design is set out. Figure 14.1 shows a scheme involving National Grid stations, main site control stations and secondary site control in the form of baselines.

If National Grid stations are to be used, any distances calculated from their co-ordinates which are used to establish further control points will have to be corrected using the scale factor, as shown in section 5.11.1.

The main site control may be the original control stations used in the production of the site plan prior to design work. If this is the case and they are to be used in the setting-out operation, they must be resurveyed before setting out commences since they may alter position owing to settlement or vandalism in the time period between the end of the original survey and the start of the setting-out operations.

The main control points should be located as near as possible to the site in open positions for ease of working, but well away (up to 100 m if necessary since this is easily accommodated by modern EDM equipment) from the construction areas and traffic routes on site to avoid them being disturbed. Since design points are to be established from them, they must be clearly visible and as many proposed design points as possible should be capable of being set out from each control point.

The construction and protection of control points is very important. Wooden pegs are often used for nonpermanent stations but they are not recommended owing to their vulnerability. Should they be the only means available, figure 5.5 shows suitable dimensions.

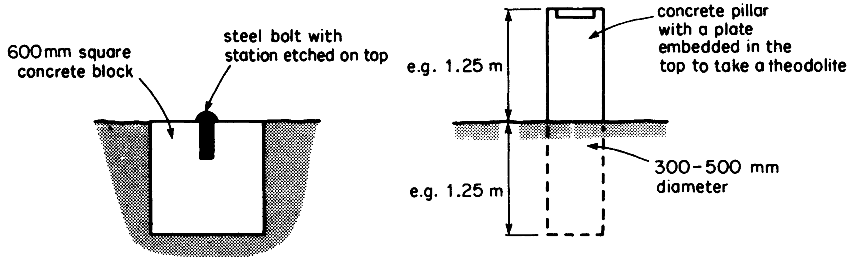


Figure 14.2

For longer life the wooden peg can be surrounded in concrete but, preferably, permanent stations similar to those illustrated in figure 14.2 should be built.

All points must be protected and painted so that they can be easily found. Figure 14.7 shows a method for protecting a transferred bench mark and this is also recommended for horizontal control points.

Once established and coordinated, the main site control points are used to set out design points of the proposed structure. They are, generally, used in one of the following ways.

14.6.1 Baselines

Main site control points, such as traverse stations, can be used to establish baselines from which setting out can be undertaken. Examples are shown in figures 14.1 and 14.3.

Subsidiary lines can be set off from the baseline to establish design corner points.

The baseline may be specified by the designer and included in the contract between the Promoter and the Contractor.

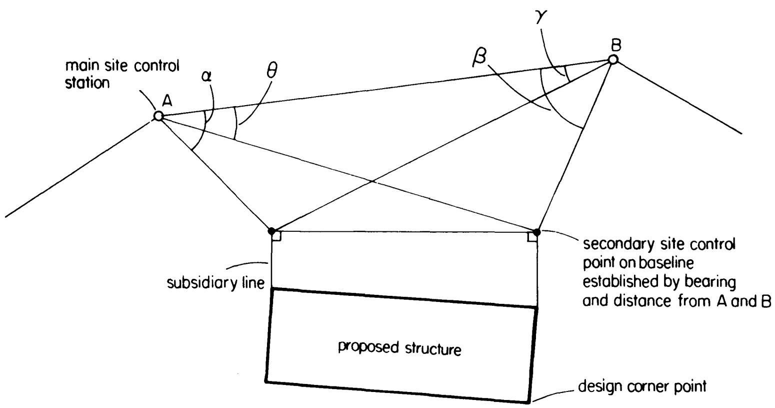


Figure 14.3 ***Baseline***

Baselines can take many forms: they can run between existing buildings; mark the boundary of existing development; be the direction of a proposed pipeline or the centre line of a new road.

The accuracy is increased if two baselines at right angles to each other are used on site. Design points can be established by offsetting from both lines or a grid system can be set up to provide additional control points in the area enclosed by the baselines.

The use of baselines to form grids leads to the use of reference grids on site.

14.6.2 Reference Grids

A control grid enables points to be set up over a large area. Several different grids can be used in setting out.

(1) *The survey grid* is drawn on the survey plan from the original traverse or triangulation scheme. The grid points have known eastings and northings related either to some arbitrary origin or to the National Grid. Control points on this grid are represented by the original control stations.

(2) *The site grid* is used by the designer. It is usually related in some way to the survey grid and should, if possible, actually be the survey grid, the advantage of this being that if the original control stations have been permanently marked then the designed points will be on the same coordinate system and setting out is greatly simplified. If no original control stations remain, the designer usually specifies the positions of several points in the site grid which are then set out on site prior to any construction. These form the site grid on the ground.

Since all design positions will be in terms of the site grid coordinates, the setting out is easily achieved as shown in figure 14.4.

The grid itself may be marked with wooden pegs set in concrete, the interval between points being small enough to enable every design point to be set out from at least two and preferably three grid points but large enough to ensure that movement on site is not restricted.

(3) *The structural grid* is established around a particular building or structure which contains much detail, such as columns, which cannot be set out with sufficient accuracy from the site grid. An example of its use is in the location of column centres (section 14.13).

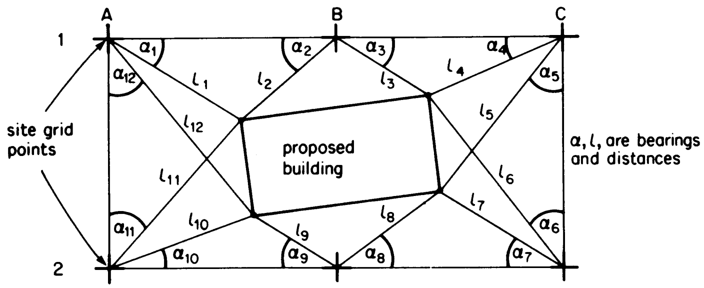


Figure 14.4 *Site grid*

The structural grid is usually established from the site grid points and uses the same coordinate system.

(4) *The secondary grid* is established inside the structure from the structural grid when it is no longer possible to use the structural grid to establish internal features of the building owing to vision becoming obscured.

Note: Errors can be introduced in the setting out each time one grid system is established from another hence, wherever possible, only one grid system should be used to set out the design points.

14.6.3 Offset Pegs

Whether used in the form of a baseline or a grid, the horizontal reference marks are used to establish points on the proposed structure. For example, in figure 14.4, the corners of a building have been established by polar coordinates from a site grid.

However, as soon as excavations for the foundations begin, the corner pegs will be lost. To avoid having to re-establish these from reference points, extra pegs are located on the lines of the sides of the building but offset back from the true corner positions. Figure 14.5 shows these *offset pegs* in use.

The offset distance should be great enough to avoid the offset pegs being disturbed during excavation.

These pegs enable the corners to be re-established at a later date and are often used with *profile boards* in the construction of buildings; this is further discussed in section 14.7.5. Offset pegs can be used in all forms of engineering construction to aid in the relocation of points after excavation.

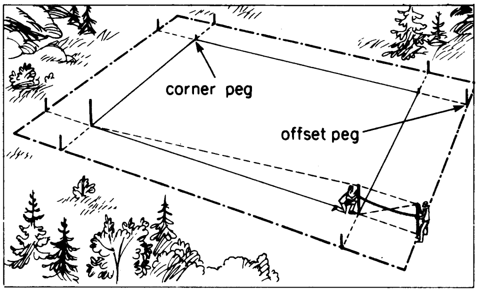


Figure 14.5 *Offset pegs*

14.7 Methods of Vertical Control

This is the establishment of reference marks of known height relative to some specified datum. The MBM discussed in section 14.5.4 is usually the datum used.

These points of known height are used to define a reference plane in space, parallel to and usually offset from a selected plane of the proposed construction. This plane may be horizontal, for example, a floor level in the case of a building, or inclined, for example, an embankment slope in earthwork construction.

Where coordinated grid points are set out for horizontal control, the points are often levelled to provide vertical control. All such levelling should be done from the MBM or a nearby TBM and not from any other point of known elevation.

14.7.1 Transferred or Temporary Bench Marks (TBMs)

The positions of TBMs should be fixed during the initial site reconnaissance so that their construction can be completed in good time and they can be allowed to settle before levelling them in. For this reason, permanent, existing features should be used wherever possible. 20 mm diameter steel bolts 100 mm long driven into existing door steps, ledges, footpaths, low walls and so on are ideal and any TBMs constructed on the sides of walls should take the form of etched lines about 75 mm long with the height and the familiar crowsfoot sign marked nearby.

Where TBMs are constructed on site, a design similar to that shown in figure 14.6 is recommended.

Each TBM is referenced by a number or letter on the site plan and the setting-out plan and should be protected since re-establishment can be time consuming. A suitable method of protection is shown in figure 14.7.

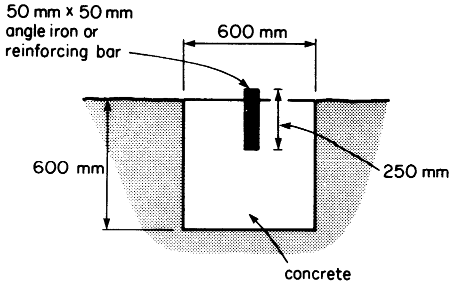


Figure 14.6

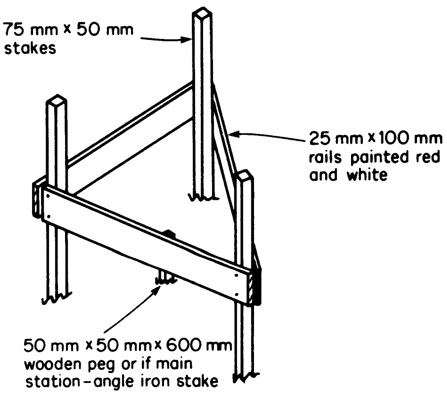


Figure 14.7

All TBMs are relative to the agreed MBM or some other agreed datum (see section 14.5.4). It is vital that the correct datum is used since the design levels are usually based on this.

There should never be more than 100 m between TBMs on site and the accuracy of levelling should be within the following limits

site TBM relative to the MBM	± 0.010 m
spot levels on soft surfaces relative to a TBM	± 0.010 m
spot levels on hard surfaces relative to a TBM	± 0.005 m

Because TBMs are vulnerable, they must be checked by relevelling at regular intervals and, as soon as the project has reached a suitable stage, TBMs should be established on permanent points on the new construction.

14.7.2 Sight Rails

These consist of a horizontal timber cross piece nailed to a single upright or a pair of uprights driven into the ground. Figure 14.8 shows several different types of sight rail.

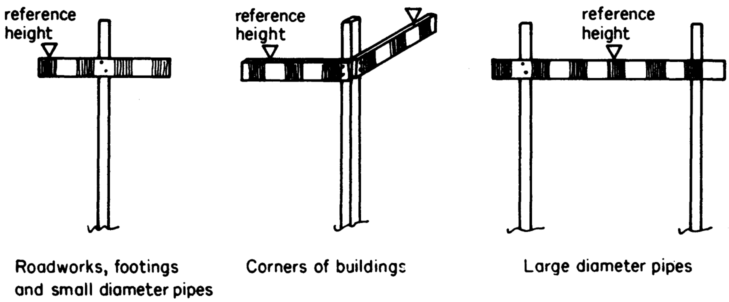


Figure 14.8 *Sight rails*

The upper edge of the cross piece is set to a convenient height above the required plane of the structure, usually to the nearest half metre, and should be at a height above ground to ensure convenient alignment by eye with the upper edge. The level of the top edge of the cross piece is usually written on the sight rail together with the length of traveller required. *Travellers* are discussed in section 14.7.3. *Double sight rails* are discussed in section 14.9.4.

Sight rails are usually offset 2 or 3 metres at right angles to construction lines to avoid them being damaged as excavation proceeds. This is shown in figure 14.9.

14.7.3 Travellers and Boning Rods

A *traveller* is similar in appearance to a sight rail on a single support and is portable. The length from upper edge to base should be a convenient dimension to the nearest half metre.

Travellers are used in conjunction with sight rails. The sight rails are set some convenient value above the required plane and the travellers are constructed so that their length is equal to this value. As excavation proceeds, the traveller is sighted in between the sight rails and used to monitor the cutting or filling. Excavation or compaction stops when the tops of the sight rails and the traveller are all in line.

Figure 14.9 shows a traveller and sight rails in use in the excavation of a trench and figure 14.10 shows the ways in which travellers and sight rails can be used to monitor cutting and filling in earthwork construction.

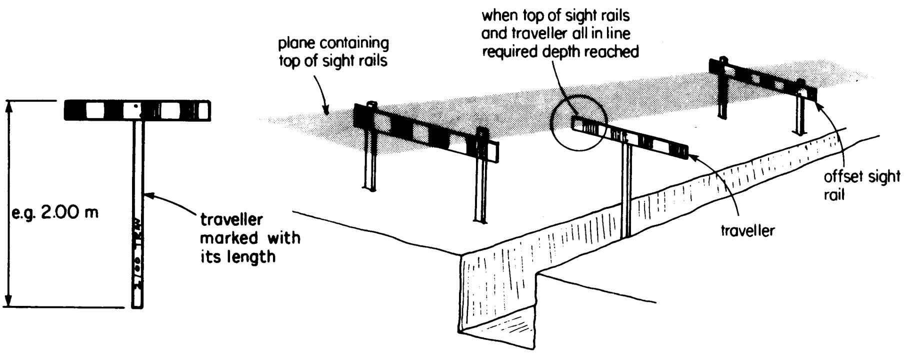


Figure 14.9

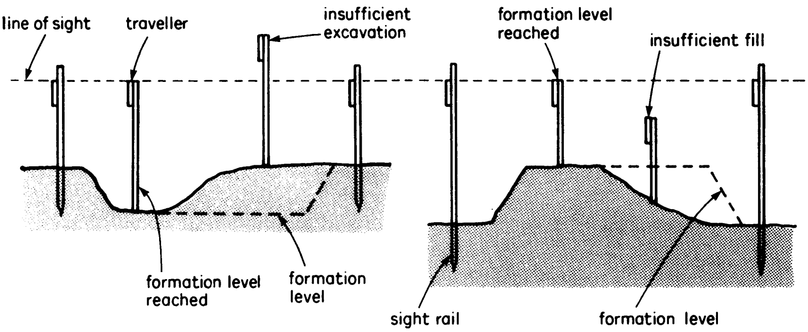


Figure 14.10

Boning rods are discussed in section 14.9.6.

There are several different types of traveller. *Free-standing travellers* are frequently used in the control of superelevation on roads, a suitable foot being added to the normal traveller as shown in figure 14.11. *Pipelaying travellers* are discussed in section 14.9.6.

14.7.4 Slope Rails or Batter Boards

For controlling side slopes in embankments and cuttings, sloping rails are used. These are known as *slope rails* or *batter boards*.

For an *embankment*, the slope rails usually define a plane parallel to but offset some perpendicular or vertical distance from the proposed embankment slope and the slope rails are usually offset a distance x to prevent them being covered during filling. Travellers are used to control the slope as shown in figure 14.12.

For a *cutting*, the wooden stakes supporting the slope rail are usually offset some horizontal distance from the edge of the proposed cutting to prevent them being disturbed during excavation. This is shown in figure 14.13.

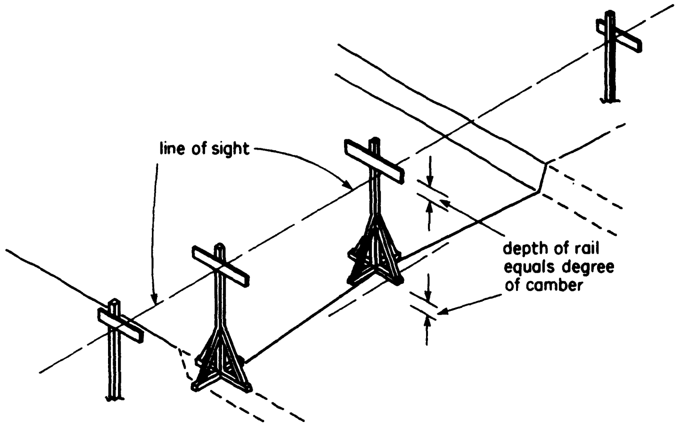


Figure 14.11 *Free standing traveller*

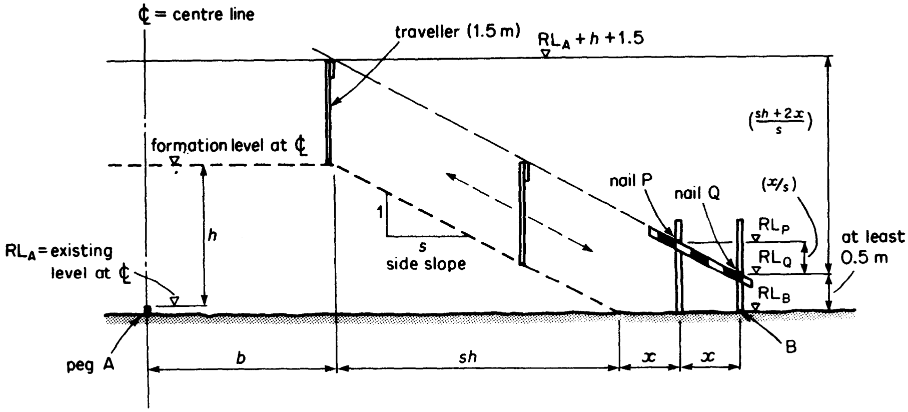


Figure 14.12 *Slope rail for an embankment*

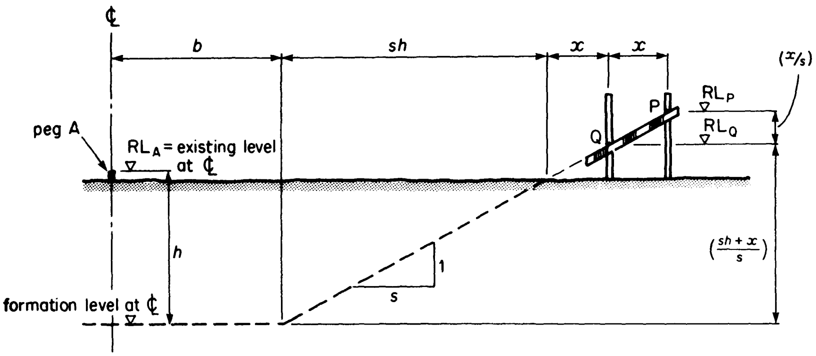


Figure 14.13 *Slope rail for a cutting*

Any relevant information is usually marked on the slope rails, for example, chainage of centre line, distance from wooden stakes to centre line, length of traveller and side slope.

The calculations associated with slope rails are those necessary to obtain the reduced levels at which two nails P and Q should be placed on the wooden stakes holding the slope rail in order that it can be correctly positioned.

For an *embankment*, assuming a 1.5 m traveller is to be used as shown in figure 14.12, the reduced levels of P and Q are obtained as follows. Existing ground is assumed to be level

$$RL_Q = (RL_A + h + 1.5) - (sh + 2x)/s$$

$$RL_P = RL_Q + (x/s)$$

RL_A, h and s will be known and x should be chosen (usually 1 m).

Once calculated, the reduced level of nail Q (RL_Q) and the reduced level of the ground below Q (RL_B) should be compared to ensure that $(RL_Q - RL_B) > 0.5$ m, otherwise a longer traveller will have to be used to enable sighting in to be undertaken from a comfortable height.

For a *cutting*, a traveller is not required, as shown in figure 14.13, and the reduced levels of P and Q are obtained as follows. Existing ground is assumed to be level.

$$RL_Q = (RL_A - h) + (sh + x)/s$$

$$RL_P = RL_Q + (x/s)$$

As for an embankment, RL_A, h and s will be known and x should be chosen (usually 1 m).

14.7.5 Profile Boards

These are very similar to sight rails but are used to define corners or sides of buildings.

In section 14.6.3 it was shown that offset pegs are used to enable building corners to be relocated after foundation excavation.

Normally a *profile board* is erected near each offset peg and used in exactly the same way as a sight rail, a traveller being used between profile boards to monitor excavation.

Figure 14.14 shows profile boards and offset pegs at the four corners of a proposed building.

The arrangement shown in figure 14.14 is quite an elaborate one and a simpler, more often used type of corner arrangement is shown in figure 14.15. Nails or saw-cuts are placed in the tops of the profile boards to define the width of the foundations and the line of the outside face of the wall. String or piano wire is stretched between opposite profile boards to guide the width of cut while a traveller is used to control the depth of cut.

A variation on corner profiles is to use a *continuous profile* all round the building set to a particular level above the required structural plane. Figure 14.16 shows such a profile with a gap left for access into the building area.

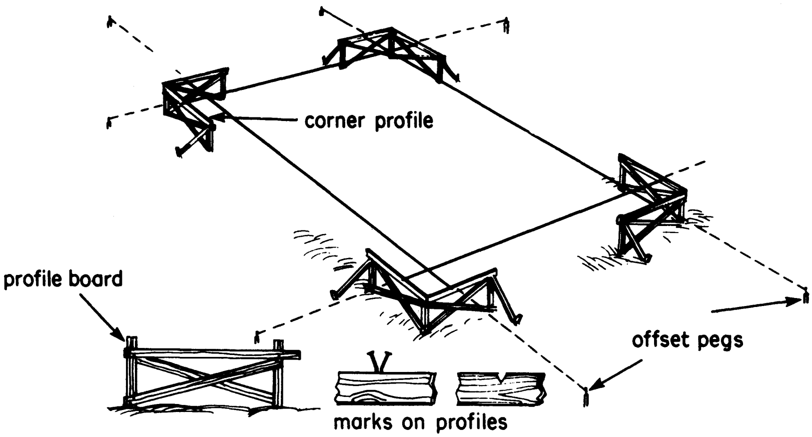


Figure 14.14 *Profile boards*

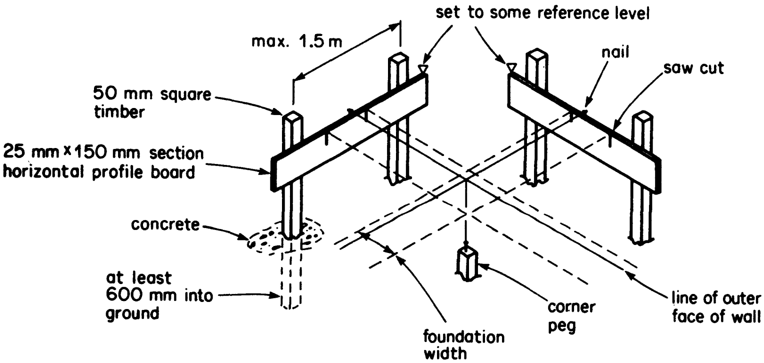


Figure 14.15 *Profile boards*

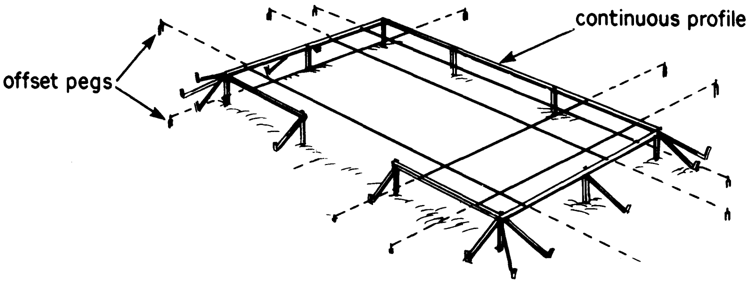


Figure 14.16 *Continuous profile*

The advantage of a continuous profile is that the lines of the internal walls can be marked on the profile and strung across to guide construction.

Another type of profile is a *transverse profile* and this is used together with a traveller to monitor the excavation of deep trenches as shown in figure 14.17.

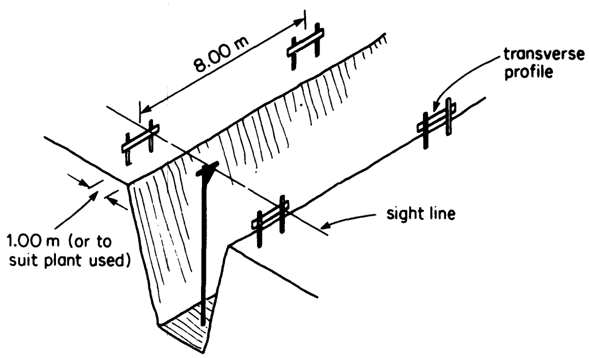


Figure 14.17 *Transverse profile*

14.8 Positioning Techniques

Several methods of locating design points are available.

14.8.1 From Existing Detail

On small sites or for single buildings, the location of the new structure may have to be fixed by running a line between corners of existing buildings and offsetting from this. The offset dimensions have to be scaled from the plan but this can be inaccurate and it is not recommended.

14.8.2 From Coordinates

These are undoubtedly the best methods. Design points will be coordinated in terms of the site grid or referenced to a baseline and they can be established by one of the following techniques.

(1) By calculation of the *bearing and distance* from at least three horizontal control points to each design point (this is known as setting out by polar coordinates) as shown in figure 14.18*a*.

The angle α in figure 14.18*a* can be set out by one of two methods. In one method, α is the angle to be set out after being calculated from α = WCB(ST) – WCB(SA). The length l and the WCBs of ST and SA are calculated from the coordinates of S, T and A as described in section 5.10. In the alternative method, the horizontal

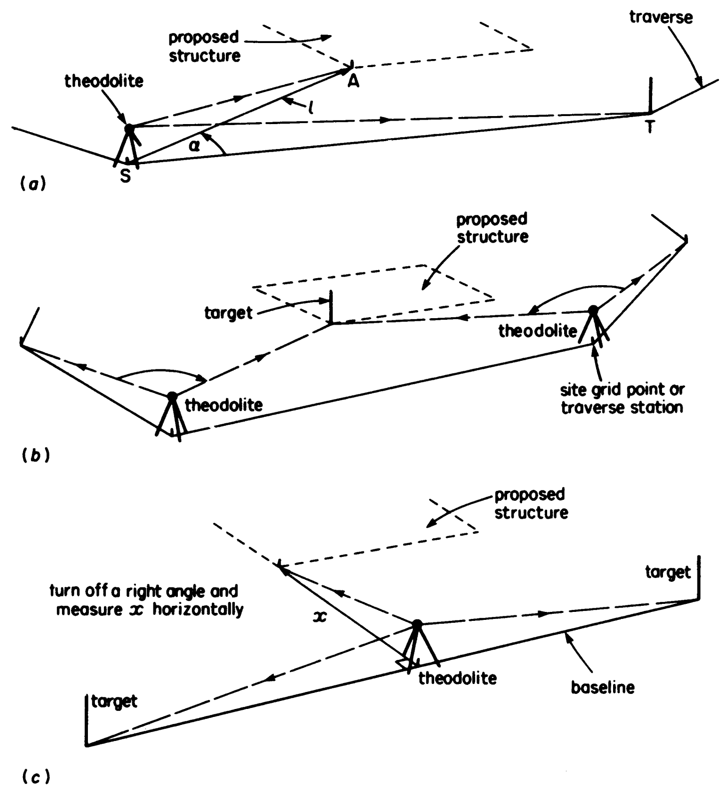


Figure 14.18 *Positioning techniques using coordinates*

circle of the theodolite is set to read the WCB of ST and the telescope aligned on point T with the instrument at station S. The telescope is then rotated towards point A until the WCB of SA is read on the horizontal circle. In both methods, l is the horizontal length to be set out from S to A and its value is calculated using methods described in section 5.10.

(2) By *intersection*, with two theodolites, from two of the control points using bearings only and checking from a third. Intersection is shown in figure 14.18*b*.

(3) By *offsetting* from one or more baselines as shown in figure 14.18*c*, the offsets being calculated from the coordinates of the ends of the baselines and the design point coordinates. If only one baseline is used, extra care should be taken since there is very little check on the set out points.

Whichever method is used, the following points must be taken into consideration.

(1) All angles must be set off using a correctly adjusted theodolite otherwise both faces should be used and the mean position taken.

(2) Since the design dimensions will be in the horizontal plane, any distance set out with a steel tape should be stepped to a plumb line or computation of the

slope distance will be necessary. The slope can be measured either using an Abney level between two ranging rods or, for greater accuracy, a theodolite should be used, again taking readings on both faces. Further corrections may be necessary if high precision is required (see section 4.2). If an EDM unit and pole-mounted reflector are used to set out distances, the slope distances displayed should be reduced to the horizontal.

(3) It is recommended that, wherever possible, each design point be set out from at least three control points. This increases the accuracy since the effect of one of the control points being out of position is reduced.

(4) To locate each design point, a large cross section wooden peg should be driven into the ground at the point and the exact design position marked on top of the peg with a fine tipped pen. A nail is then hammered into the peg at this point.

(5) Right angles should be set out by theodolite and the angle turned on both faces using opposite sides of the horizontal circle to remove eccentricity and graduation errors, for example. on face right use 0° to 90° and on face left use 180° to 270°. The mean of two pointings is the correct angle.

Applications of coordinate methods of setting out are discussed in section 14.17.

14.8.3 From Free Station Points

This technique is shown in figure 14.19 and is a combination of resection (see section 7.2) and setting out from coordinates (see section 14.8.2). It is particularly applicable to large sites where the coordinates of prominent features and targets on nearby buildings or parts of the construction are known. The procedure is as follows

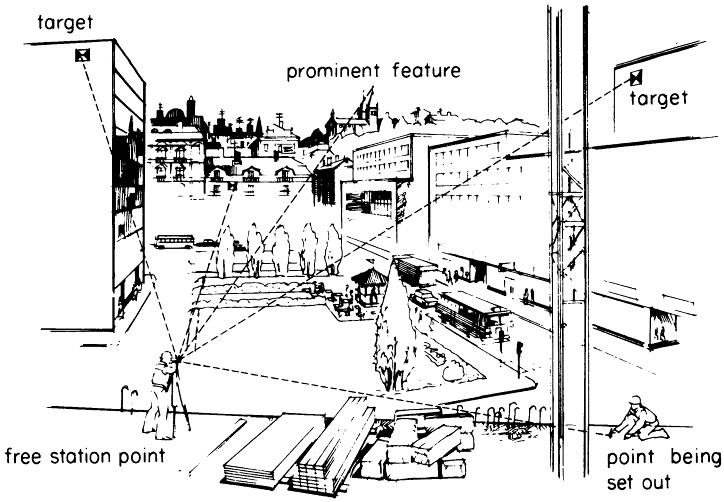


Figure 14.19 *Free station point*

(1) The theodolite is set up at some suitable place in the vicinity of the points which are to be set out. This gives rise to the term *free station* since the choice of theodolite position is arbitrary.

(2) A resection is carried out to fix the position of the free station. Preferably, observations should be taken to four known main site control points rather than the minimum of three (see section 7.2).

(3) The coordinates of the free station are calculated.

(4) One of the methods described in section 14.8.2 is used to set out the required points using the theodolite at the free station; usually the method of polar coordinates is chosen.

If free station points are to be used widely on a particular site, it is essential that there is a sufficient number of well-established control points around the site to enable enough obstruction free sightings to be achieved while construction proceeds. For good results with this method, theodolites reading to 1″ or better should be used.

14.9 Setting Out a Pipeline

The foregoing principles are now considered in relation to the setting out of a gravity sewer pipeline. The whole operation falls within the category of first stage setting out.

14.9.1 General Considerations

Sewers normally follow the natural fall in the land and are laid at gradients which will induce a self-cleansing velocity. Such gradients vary according to the material and diameter of the pipe. Figure 14.20 shows a sight rail offset at right angles to a pipeline laid in granular bedding in a trench.

Depth of cover is, normally, kept to a minimum but the sewer pipe must have a concrete surround at least 150 mm in thickness where cover is less than 1 m or

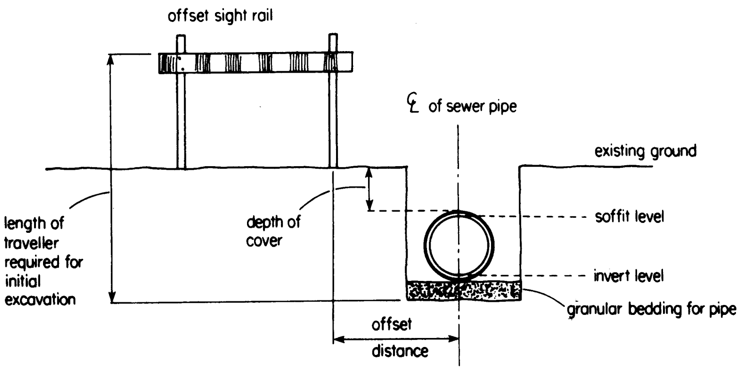


Figure 14.20

greater than 7 m. This is to avoid cracking of pipes owing to surface or earth pressures.

14.9.2 Horizontal Control

The working drawings will show the directions of the sewer pipes and the positions of manholes.

The line of the sewer is normally pegged at 20 to 30 m intervals using coordinate methods of positioning from reference points or in relation to existing detail. Alternatively, the direction of the line can be set out by theodolite and pegs sighted in.

Manholes are set out at least every 100 m and also at pipe branches and changes of gradient, the actual positions being controlled as discussed in section 14.9.5.

14.9.3 Vertical Control

This involves the erection of sight rails some convenient height above the invert level of the pipe (see figure 14.20).

The method of excavation should be known in advance such that the sight rails will not be covered by the excavated material (the *spoil*).

A suitable scheme for both horizontal and vertical control is shown in figure 14.21.

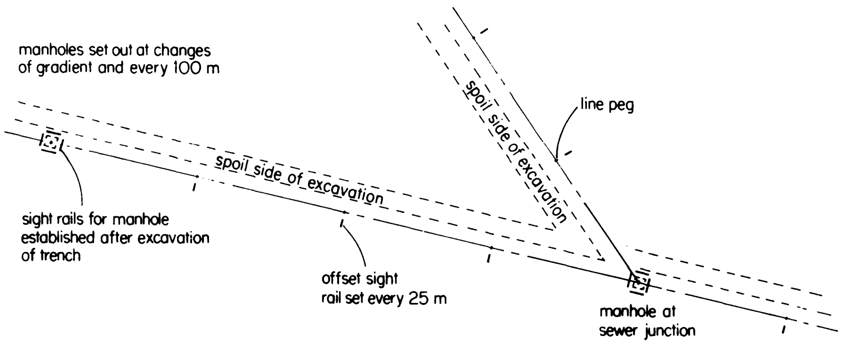


Figure 14.21 *Layout of horizontal and vertical control for a pipeline*

14.9.4 Erection and Use of Sight Rails

The sight rail upright or uprights are hammered firmly into the ground, usually offset from the line rather than straddling it. Using a nearby TBM and levelling equipment, the reduced levels of the tops of the uprights are determined. Knowing the proposed depth of excavation, a suitable traveller is chosen and the difference between the level of the top of each upright and the level at which the top edge of the cross piece is to be set is calculated (see section 14.20.1). Figure 14.22 shows examples of sight rails fixed in position. The excavation is monitored by lining in the traveller as shown in figure 14.23.

Where the natural slope of the ground is not approximately parallel to the proposed pipe gradient, *double sight rails* can be used as shown in figure 14.24.

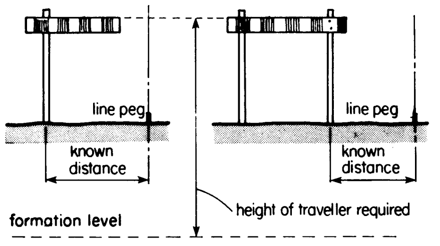


Figure 14.22

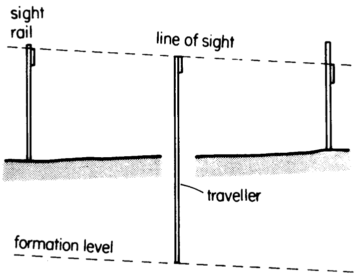


Figure 14.23

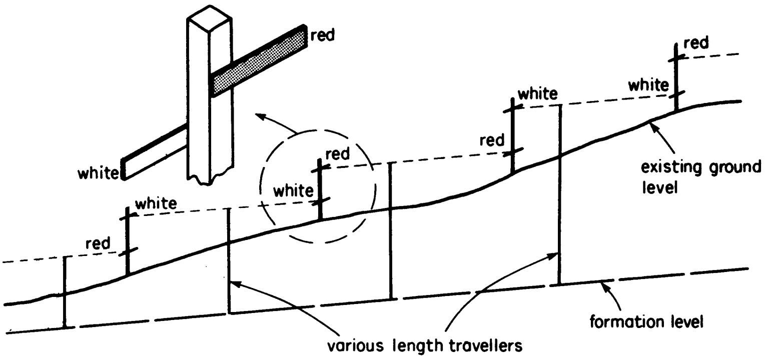


Figure 14.24 *Double sight rails*

14.9.5 Manholes

Control for manholes is usually established after the trench has been excavated and can be done by using sight rails as shown in figure 14.25 or by using an offset peg as shown in figure 14.26.

14.9.6 Pipelaying

On completion of the excavation, the sight rail control is transferred to pegs in the bottom of the trench as shown in figure 14.27. The top of each peg is set at the invert level of the pipe.

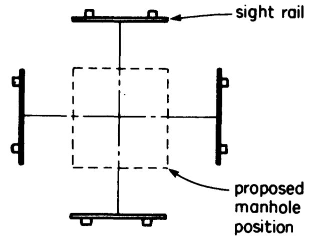


Figure 14.25

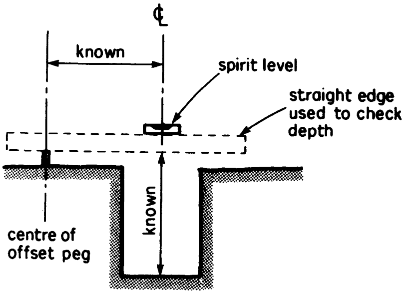


Figure 14.26

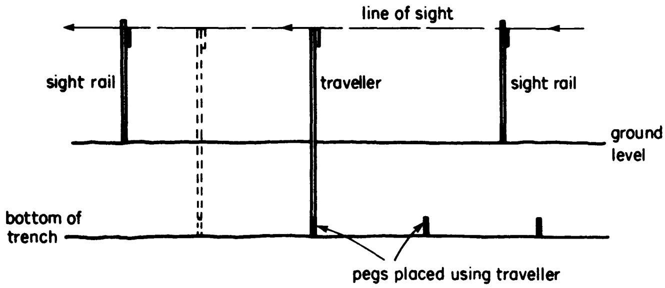


Figure 14.27 *Setting pegs in trench with traveller*

Pipes are usually laid in some form of bedding and a *pipelaying traveller* is useful for this purpose. Figure 14.28 shows such a traveller and its method of use.

Pipes are laid from the lower end with sockets facing uphill. They can be bedded in using a straight edge inside each pipe until the projecting edge just touches the next forward peg or the pipelaying traveller can be used. Alternatively, three travellers can be used together as shown in figure 14.29. When used like this the travellers are known as *boning rods*.

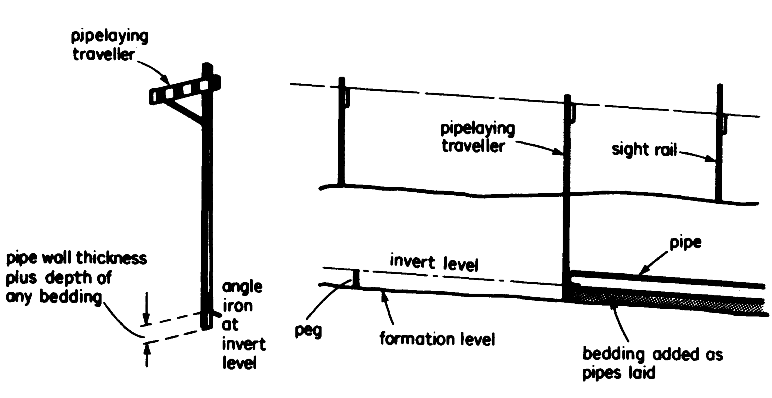


Figure 14.28 *Pipelaying traveller*

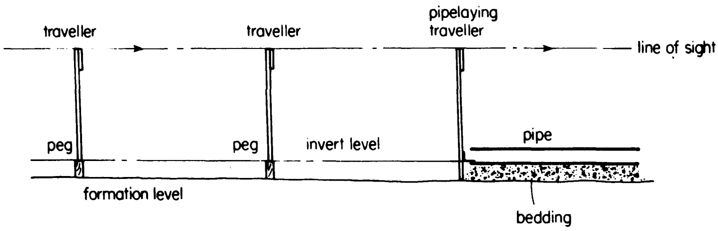


Figure 14.29 *Boning rods*

14.10 Setting Out a Building to Ground-floor Level

This also comes into the category of first stage setting out. It is summarised below.

It is vital to remember when setting out that, since dimensions, whether scaled or designed, are almost always horizontal, slope must be allowed for in surface taping on sloping ground. The slope correction is additive when setting out.

(1) Two corners of the building will be set out from the baseline, site grid or traverse stations using one of the methods shown in figure 14.18.

(2) From these two corners, the sides will be set out using a theodolite to turn off right angles as shown in figure 14.30.

The exact positions of each corner will be marked in the top of wooden pegs by nails.

Offset pegs must be established at the same time as the corner pegs (see figure 14.5).

(3) The diagonals are checked as shown in figure 14.31 and the nails re-positioned on the tops of the pegs as necessary.

(4) Profile boards are erected at each corner or a continuous profile is used (see figures 14.14, 14.15 and 14.16) and excavation begins. The next step is to construct the foundations; these can take several forms but for the purposes of the remainder of the chapter it will be assumed that concrete foundations have been used and a concrete ground floor slab laid. This would have required formwork to contain the wet concrete and this could have been set out by aligning the shuttering with string lines strung between the profiles.

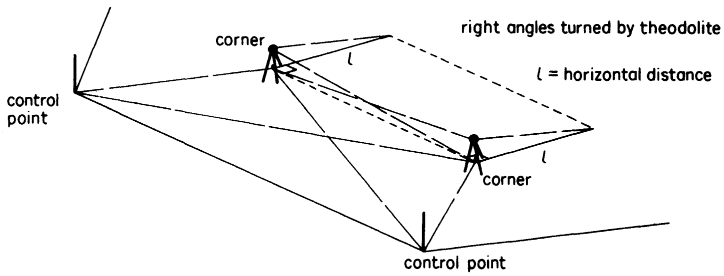


Figure 14.30 *Setting out building sides by right angles*

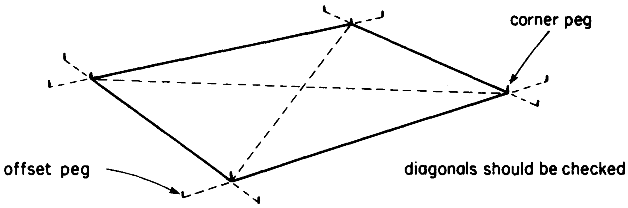


Figure 14.31 *Checking diagonals*

14.11 Transfer of Control to Ground-floor Slab

This is done for horizontal control by setting a theodolite and target over opposite pairs of offset pegs as shown in figure 14.32 and for vertical control as shown in figure 14.33.

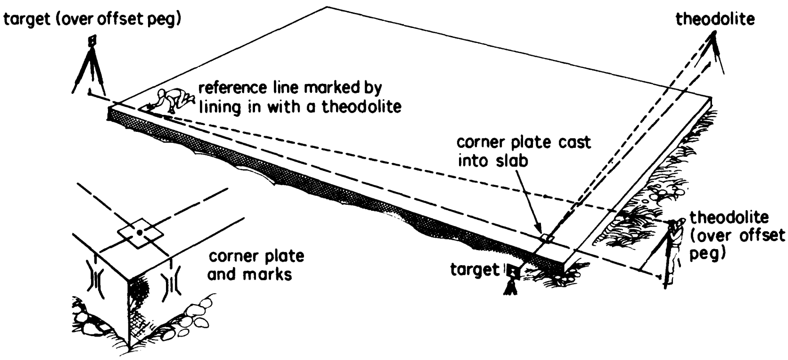


Figure 14.32 *Transfer of horizontal control to ground floor slab*

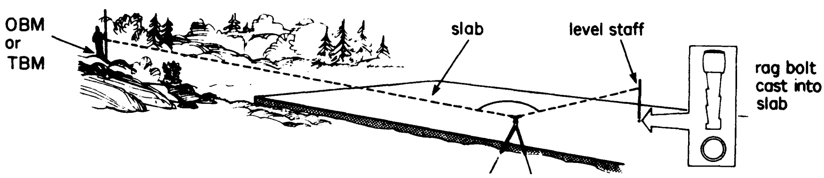


Figure 14.33 *Transfer of vertical control to ground floor slab*

14.12 Setting Out Formwork

The points required for formwork can be set-out with reference to the control plates by marking the lines between these plates as shown in figure 14.34.

On method of marking these lines on the slab is by means of chalked string held taut and fixed at each corner position. The string is pulled vertically away

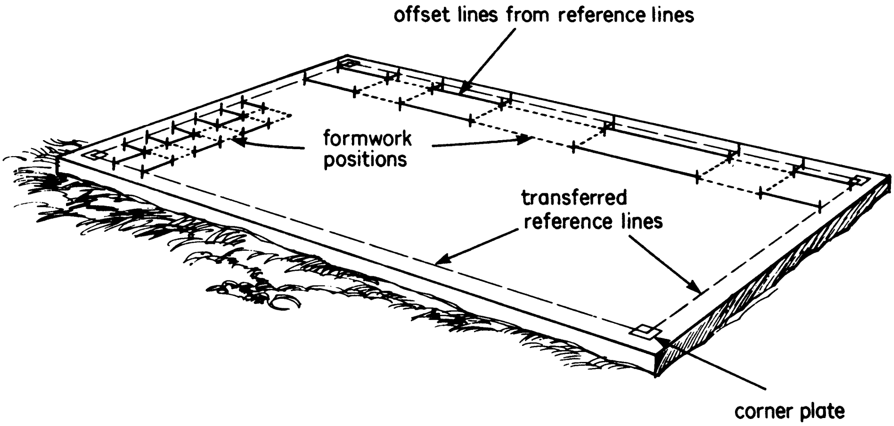


Figure 14.34 *Setting out formwork*

from the slab and released. It hits the surface of the slab, marking it with the chalk.

These slab markings are used as guidelines for positioning the formwork and should be extended to check the positioning as shown in figure 14.35.

14.13 Setting Out Column Positions

Where columns are used, they can be set out with the aid of a structural grid as discussed in section 14.6.2. Column centres should be positioned to within ± 2 to 5 mm of their design position. The structural grid enables this to be achieved.

Figure 14.36 shows the structural grid of wooden pegs set out to coincide with the lines of columns. The pegs can either be level with the ground floor slab or profile boards can be used.

Lines are strung across the slab between the pegs or profiles to define the column centres. If the pegs are at slab level the column positions are marked directly. If profiles are used, a theodolite can be used to transfer the lines to the slab surface. The intersections of the lines define the column centres.

Once the centres are marked, the bolt positions for steel columns can be accurately established with a template, equal in size to the column base, placed exactly at the marked point. For reinforced concrete columns, the centres are established in exactly the same way but usually prior to the slab being laid so that the reinforcing starter bars can be placed in position.

14.14 Controlling Verticality in Multi-storey Structures

The setting-out lines marked on the ground floor slab (see figure 14.32) must be transferred to each higher floor as construction proceeds. If they can be transferred accurately, the verticality will be maintained.

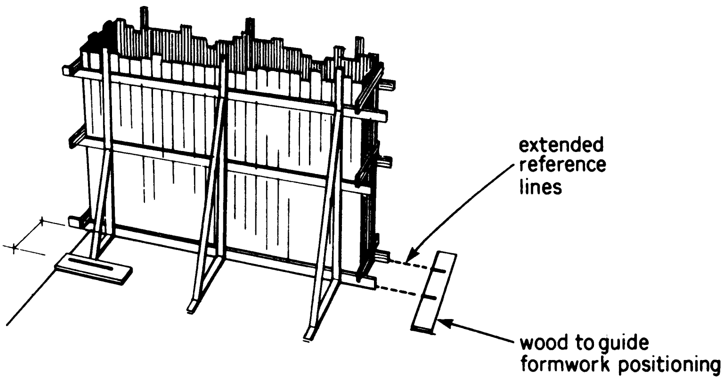


Figure 14.35 *Setting out formwork*

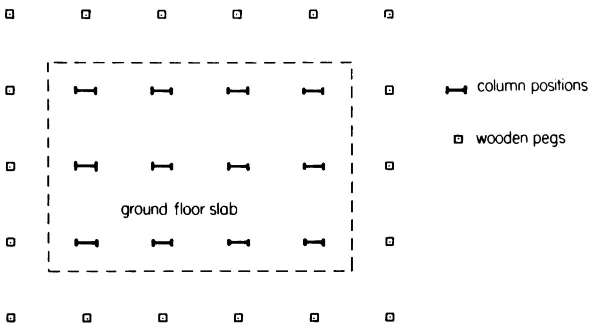


Figure 14.36 *Setting out column positions*

The maintenance of verticality is the most important setting out feature of multi-storey structures. Four methods are discussed.

14.14.1 Plumb Bob Methods

Plumb bobs, usually weighing 3 kg, suspended on piano wire or nylon are used. Two plumb bobs are needed in order to provide a reference line from which the upper floors may be controlled. To dampen oscillations, the bob is often suspended in a transparent drum of oil.

The bob is suspended from an upper floor and moved until it hangs over the reference mark on the ground floor slab. Holes and openings must be provided in the floors to allow the plumb bob to hang through and a centring frame is necessary to cover the opening to enable the exact point to be fixed.

Wind currents in the structure can be a problem and this method can be time consuming if great accuracy is required. It is, however, a very useful method when constructing lift shafts.

14.14.2 Theodolite Methods

These methods assume that the theodolite is in perfect adjustment so that its line of sight will describe a vertical plane when rotated about its trunnion axis. Two methods are considered.

Using a theodolite only

The theodolite is set up on extensions of each reference line marked on the ground floor slab in turn and the telescope is sighted on to the particular line being transferred. The telescope is elevated to the required floor and the point at which the line of sight meets the floor is marked. This is repeated at all four corners and eight points in all are transferred as shown in figure 14.37.

Once the eight marks are transferred, they are joined and the distances between them measured as checks.

If the theodolite is not in perfect adjustment, the points must be transferred using both faces and the mean position used. In addition, because of the large angles of elevation involved, the theodolite must be carefully levelled.

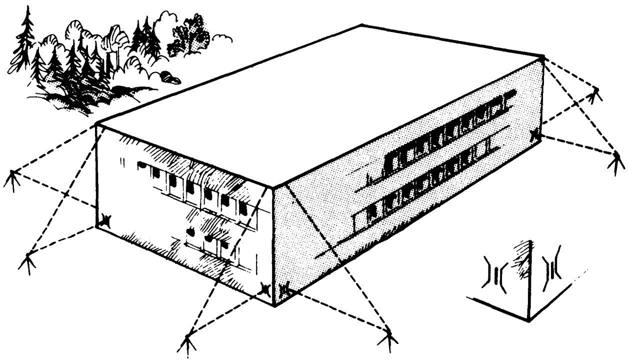


Figure 14.37 *Transfer of control in multi-storey structures by theodolite*

Using a theodolite and targets

In figure 14.38, A and B are the offset pegs.

(1) The theodolite is set over reference mark A, carefully levelled and aligned on the reference line marked on the side of the slab (see figure 14.32).

(2) The line of sight is transferred to the higher floor and a target accurately positioned.

(3) A three-tripod traverse system is used and the target replaces the theodolite and vice versa.

(4) The theodolite, now at C, is sighted on to the target at A, transitted and used to line in a second target at D. Both faces must be used and the mean position adopted for D.

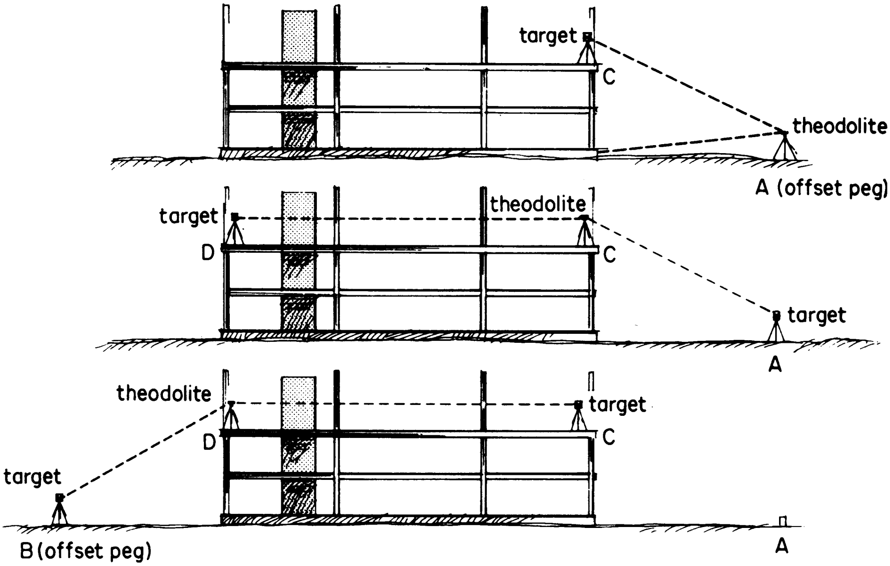


Figure 14.38 *Transfer of control in multi-storey structures by theodolite and target*

(5) A three-tripod traverse system is again used and the theodolite checks the line by sighting down to the reference mark at B, again using both faces.

(6) It may be necessary to repeat the process if a slight discrepancy is found.

(7) The procedure is repeated along the other sides of the building.

14.14.3 Optical Plumbing Methods

The optical plummet of a theodolite can be used or a diagonal eyepiece can be fitted to a theodolite but the best method is to use an optical plumbing device specially manufactured for the purpose. Holes and openings must be provided in the floors and a centring frame may be used to establish the exact position.

There are several variations of optical plummet. They consist of either one prism which can be rotated to plumb up or down, or two prisms which enable simultaneous up and down plumbing to be carried out.

Figures 14.39*a* and *b* show an optical plummet being used to plumb upwards to a special centring device. This device is moved until the line of sight passes through its centre.

Figure 14.39*c* shows downward plumbing. In this case the movable head of the optical plummet is adjusted until the reference mark is bisected by the cross hairs. The centring device is then set in place beneath the tripod and moved until the line of sight passes through its centre.

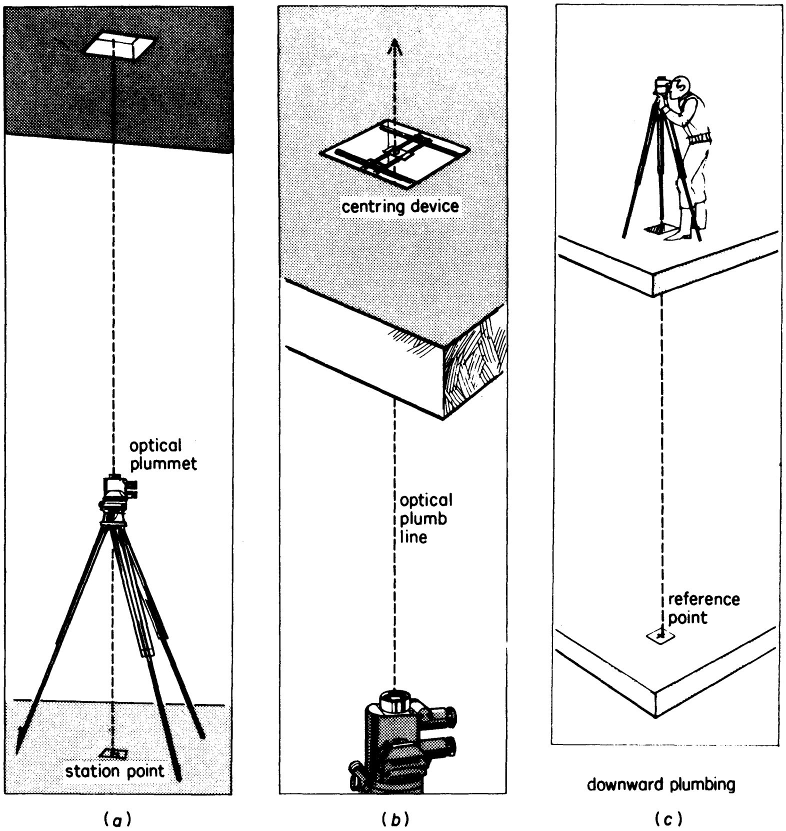


Figure 14.39 *Optical plumbing*

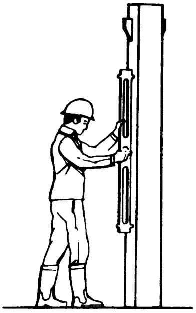
14.14.4 Column Verticality

Columns one storey high are best checked by means of a long spirit level held up against them as shown in figure 14.40.

Multi-storey columns are best checked with a theodolite as shown in figure 14.41*a* and *b*. Either the edges or the centre lines of each column are plumbed with the vertical hairs of two theodolites by elevating and depressing the telescopes. The theodolites used must be free from error and carefully levelled; as a further precaution, both faces should be used to check the verticality.

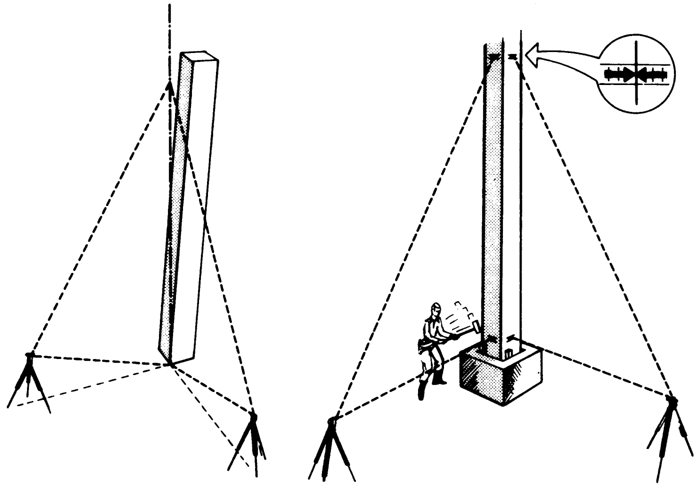
14.15 Transferring Height from Floor to Floor

Height can be transferred by means of a weighted steel tape measuring each time from a datum in the base of the structure as shown in figure 14.42. The base datum levels should be set in the bottom of lift wells, service ducts and so on, such that an unrestricted taping line to roof level is provided.



plumbing a column by using a spirit level

Figure 14.40



plumbing a multi-storey column by using a theodolite

(*a*)

plumbing the centre line of the column

(*b*)

Figure 14.41

Each floor is then provided with TBMs in key positions from which normal levelling methods can be used to transfer levels on each floor.

Alternatively, if there are cast-*in-situ* stairs present, a level and staff can be used to level up and down the stairs as shown in figure 14.43. Note that both up and down levelling must be done as a check.

14.16 Setting Out Using Laser Instruments

Although a detailed description of laser techniques and equipment is beyond the scope of this book, laser instruments are being used increasingly in setting-out operations and a few of the more common applications are discussed here.

The laser generates a light beam of high intensity and of low angular divergence, hence it can be projected over long distances without spreading significantly. These characteristics are utilised in specially designed laser equipment and it is possible to carry out many alignment and levelling operations by laser.

The type of laser used in surveying equipment is usually a helium-neon gas laser which produces a bright red beam which can be seen clearly when intercepted by a target. The lasers used in construction and surveying are low power with outputs in the range 1 mW to 5 mW and these represent no hazard when the beam or its reflection strikes the skin or the clothes of anyone in the vicinity. However, an output of 1 mW to 5 mW presents a serious hazard to the eyes and on no account should anyone look directly into a laser beam and optical instruments should never be used to find the position of the beam. Whenever lasers are being used, warning

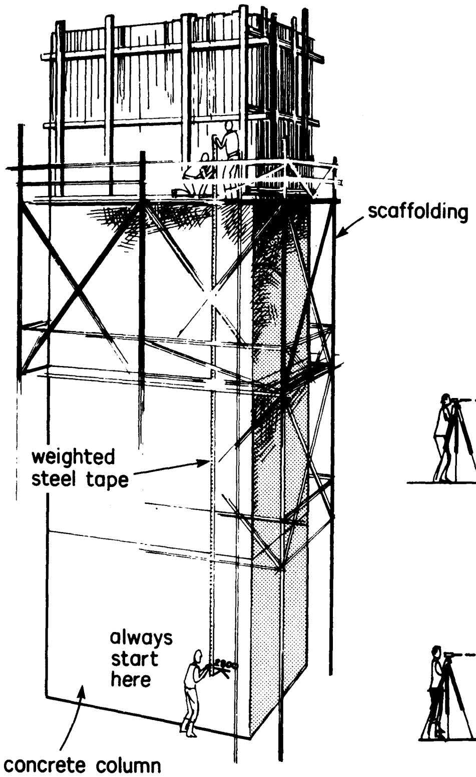


Figure 14.42 *Transfer of height from floor to floor using steel tape*

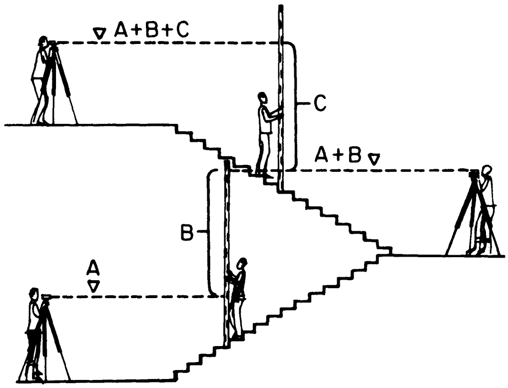


Figure 14.43 *Transfer of height from floor to floor using levelling*

signs must be placed in prominent positions. A detailed discussion of laser safety can be found in *A Guide to the Safe Use of Lasers in Surveying and Construction*, published by the RICS.

Although the use of laser equipment can have many advantages, such equipment is generally expensive and its purchase is justified only by those contractors with a high volume of setting-out work in which specialised laser equipment can be exploited fully.

Two types of laser instrument are used in setting out and these are classified as alignment lasers and rotating head lasers.

14.16.1 Alignment Lasers

This type of laser produces a single beam output which, when used for alignment purposes, has the important advantage of producing a constantly present reference line which can be used without interrupting construction.

The *laser theodolite* is either a purpose-built instrument or a standard theodolite converted into a laser theodolite by means of a laser eyepiece attachment (see figure 14.44). The laser beam projected by these theodolites coincides exactly with the line of collimation and is focused using the telescope focusing screw to appear as a dot in the centre of the cross hairs (see figure 14.45). A special filter in the eyepiece prevents any hazardous level of radiation reaching the observer's eye. This beam can be intercepted with the aid of suitable targets over daylight ranges of 200-300 m and night ranges of 400-600 m. This type of instrument can be used in place of a conventional theodolite in almost any alignment or intersection technique and, once set up, the theodolite can be left unattended. However, since the instrument could be accidentally knocked or vibration of nearby machinery could deflect the beam it is essential that regular checks are taken to ensure that the beam is in its intended position.

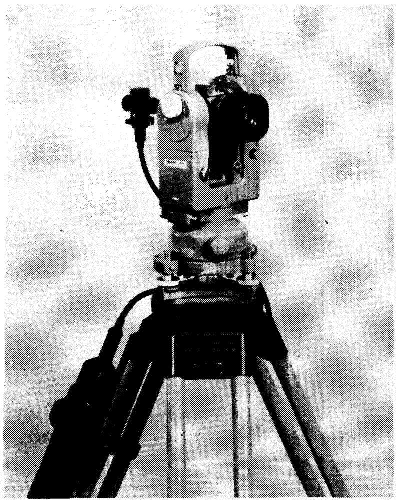


Figure 14.44 *Laser theodolite (courtesy Wild Heerbrugg (UK) Ltd)*

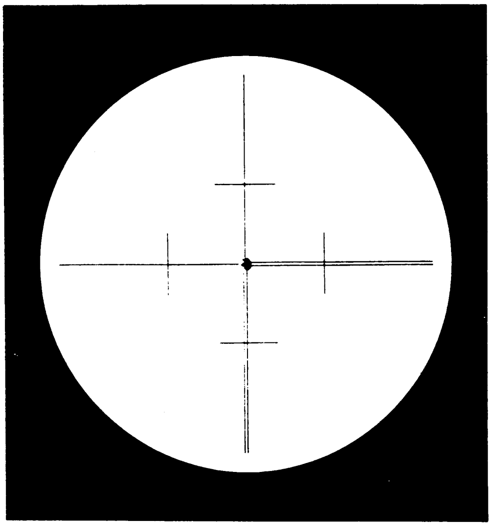


Figure 14.45

For *controlling verticality*, a laser eyepiece can be attached to an optical plummet. This is set up on the ground floor slab directly over the reference mark and the beam is projected vertically either up the outside of the building or through special openings in the floors. The beam is intercepted as it passes the floor to be referenced by the use of plastic targets fitted in the openings or attached to the edge of the slab. The point at which the beam meets the target is marked to provide the reference. The essential requirement of the system is to ensure that the beam is truly vertical.

When *setting out pipelines*, the use of a laser system eliminates the need for sight rails and pipelaying travellers. A specially made pipelaying laser is shown in figure 14.46 and this can be set up on a stable base positioned either within the pipe itself,

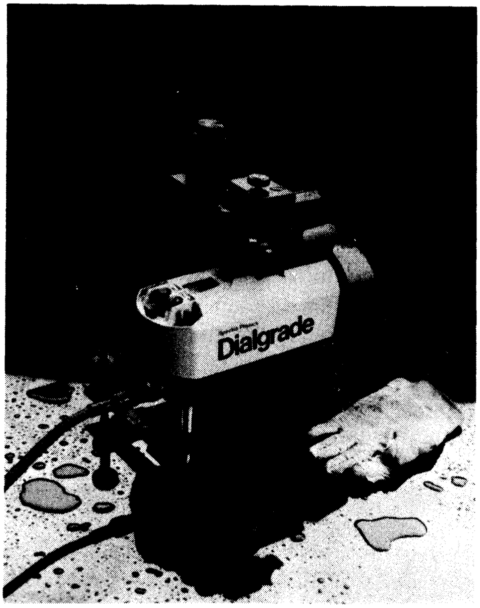


Figure 14.46 *Pipelaying laser (courtesy Spectra-Physics Ltd)*

in a manhole or supported on a tripod. Figure 14.47 shows typical arrangements. When placed in a manhope or pipe, the unit will self-level provided it has been rough levelled to within ± 5°. The laser is correctly aligned along the direction in which the pipe is to be laid and the gradient of the pipe is set on the grade indicator of the laser. During pipelaying, a plastic target is placed in the open end of the pipe which is then moved horizontally and/or vertically until the laser beam hits the centre of the target as shown in figure 14.48. The pipe is then carefully bedded in that position. The target is then removed and the procedure repeated. If the laser is unintentionally moved off grade, the beam blinks on and off to provide a warning of this until the unit has relevelled itself.

The laser has been used very successfully in *tunnel alignment* because the beam is capable of showing the intended line of the tunnel on a continuous basis. If tunnelling machines are used, the process of laser guidance can be made automatic using photoelectric cells to detect the beam, the cells in turn controlling the movement of the machine. Since tunnel alignment by laser is a complicated setting-out process, the development of such systems is usually undertaken by specialist surveyors and engineers.

14.16.2 Rotating Head Lasers

These instruments, examples of which are shown in figure 14.49, are also known as laser levels. They are usually mounted on a tripod and transmit a laser beam which

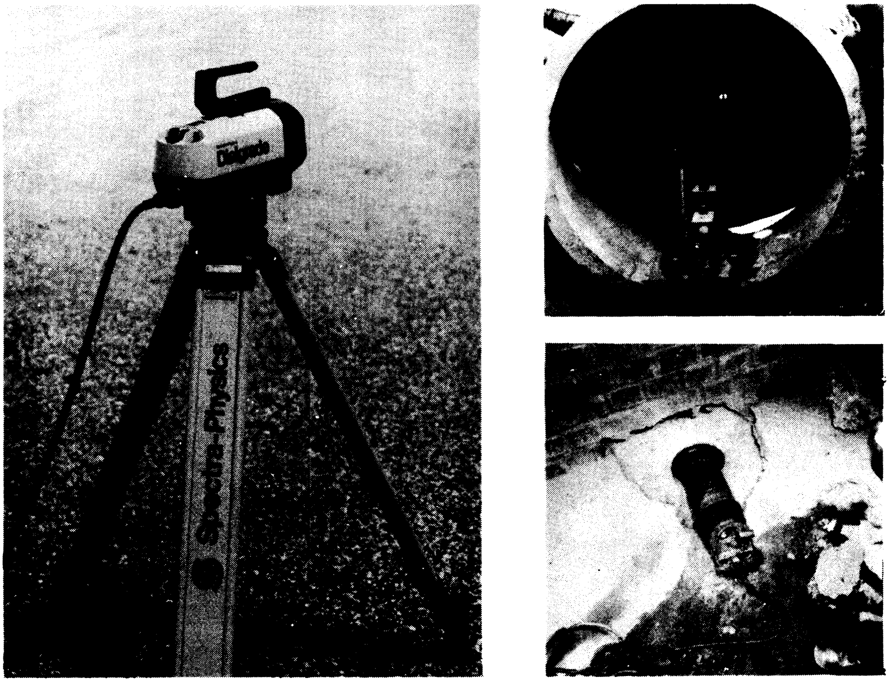


Figure 14.47 *Pipelaying laser set-ups (courtesy Spectra-Physics Ltd)*

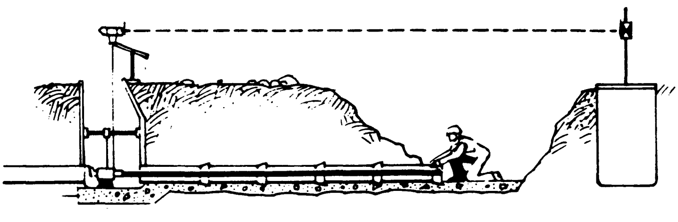


Figure 14.48 *Pipelaying with the aid of a laser*

can be made to rotate at varying speeds to form a horizontal reference plane. The reduced level of the reference plane can be obtained and this replaces the plane of collimation of an optical level in setting out and levelling.

Many laser levels are self-levelling and have a built-in checking facility which operates by shutting off the laser beam if it is knocked off level.

Rotating lasers are used in conjunction with a sliding battery-powered sensor which is attached to a specially made levelling staff. The sensor will search for the laser reference plane and then lock on to it, enabling the person holding the staff to take a reading. This allows conventional levelling to be done and, if the laser level

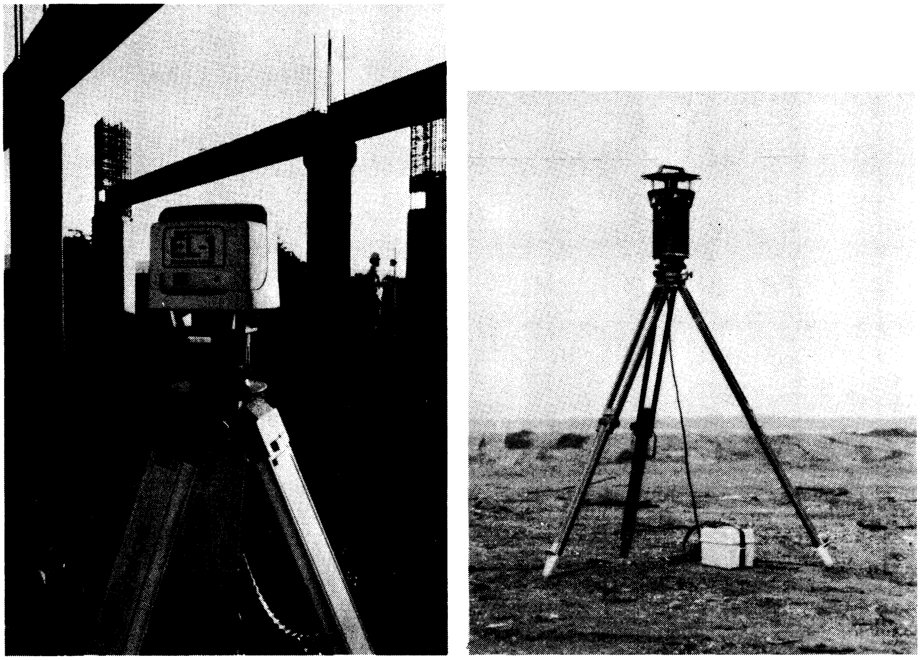


Figure 14.49 *Rotating head lasers (courtesy Spectra-Physics Ltd)*

can be set up near the centre of a construction site, many operatives can use the reference plane simultaneously. This can be achieved on reasonably open and level sites.

When used for setting out foundations, floor levels and so on, the sensor can be fixed at some desired reading and the staff used as a form of traveller, the laser reference plane replacing sight rails as shown in figure 14.50.

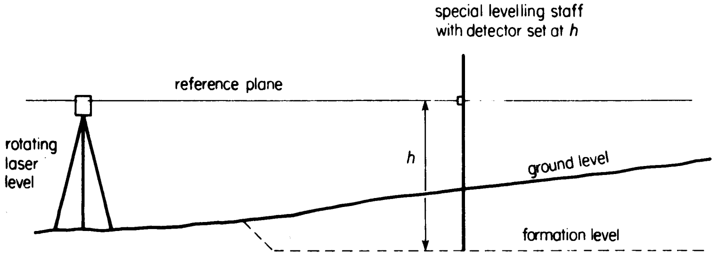
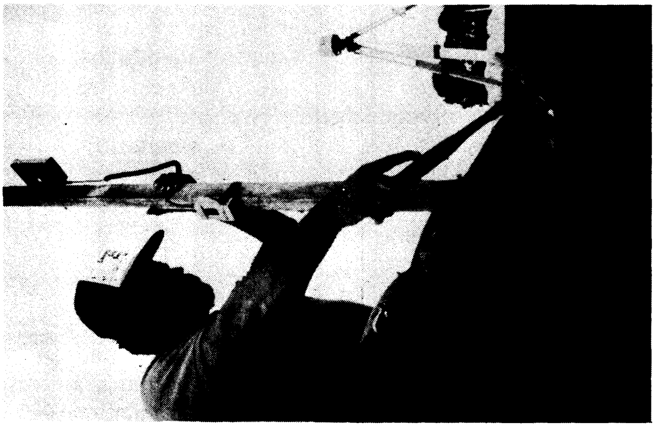
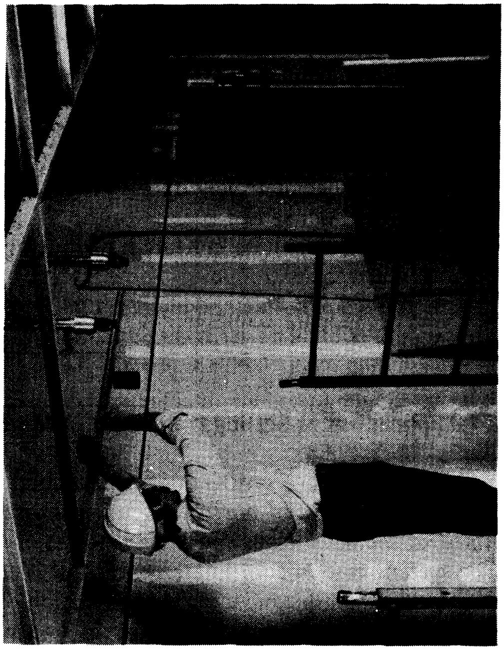


Figure 14.50 ***Setting out with rotating head laser level***



(a)

(b)



(c)

Figure 14.51 ***Setting out (a) formwork, (b) floor levels and (c) internal fittings***

Figure 14.51 shows further applications of a laser level. In these applications, the reduced level of the horizontal reference plane is determined by setting the base of the special levelling staff on a point of known reduced level. The maximum recommended distance from the laser to the staff is approximately 80–100 m.

Laser levels have also been used for controlling earth-moving and grading as shown in figure 14.52. For this, the sensor unit is fixed outside an earth-mover or grader cab and a special display unit is mounted inside the cab in full view of the driver. When the laser sensor is close to the reference plane, the lights on the display unit are energised and these indicate how much material is required to be removed or filled to reach formation level. Some lasers of this type also have the facility of being able to set up an inclined plane for use in grading, and systems are available for the remote control of earth digging and asphalt laying equipment.

14.17 Applications of Setting Out from Coordinates

Coordinate methods of setting out are described in section 14.8.2 and their application in the setting out of horizontal curves is discussed in section 10.12, where an appraisal of the advantages and disadvantages of such methods can be found.

The system used in coordinate setting out is that of establishing design points by either bearings and distances or by intersection using bearings only from nearby control points. The bearings and distances are calculated from the coordinates of the control and design points by the method described in section 5.10.

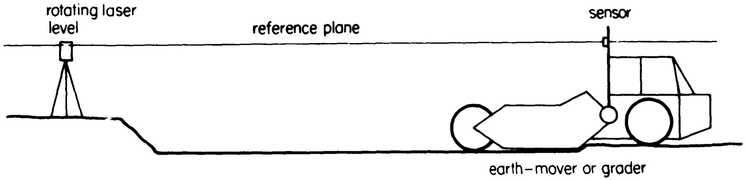
If the bearing and distance method is used, a theodolite and some form of distance measuring system are required whereas if the intersection method is used, two theodolites are necessary. These two techniques are described in section 14.8.2.

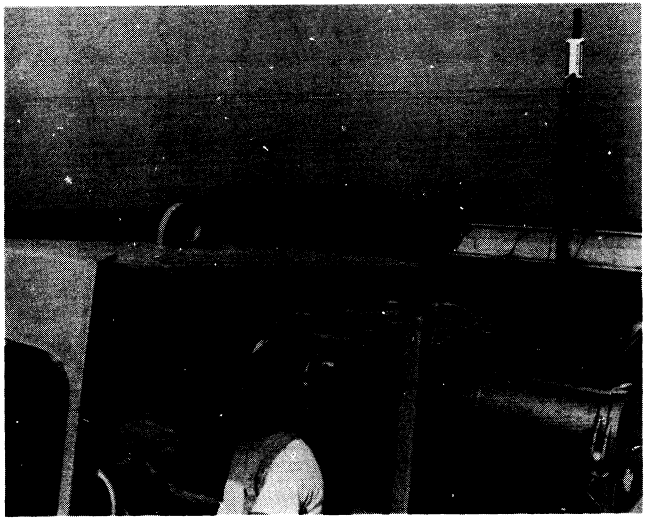
The development of EDM equipment in which the reflecting unit can be mounted on a movable pole, enables distances to be set out very accurately and quickly regardless of terrain and this is being used increasingly in bearing and distance methods. However, EDM does have its limitations in setting out. It is ideal for use with a theodolite in bearing and distance methods for establishing site grids and other control points but such methods should not be used where alignment is critical, for example, setting out column centres, since the alignment obtained would not be satisfactory owing to slight angular and/or distance errors. In any alignment situation, it is best either to use one theodolite to establish the line and measure distances along it using a steel tape or EDM or, preferably, to use two theodolites positioned at right angles such that the intersection of their lines of sight establishes the point which can be located by lining in a suitable target.

In general, on site EDM would not be used if a steel tape could be used satisfactorily and usually all lengths less than approximately 50 m would be set out and checked using steel tapes. EDM would be used in cases where distances in excess of approximately 50 m were involved and over very uneven ground where steel taping would be difficult.

In all setting out, if the design is based on National Grid coordinates, the scale factor must be taken into account as described in section 5.11.

The great advantage of coordinate methods is that they can be used to set out virtually any civil engineering construction providing the points to be located and the control points are on the same rectangular coordinate system. The calculations





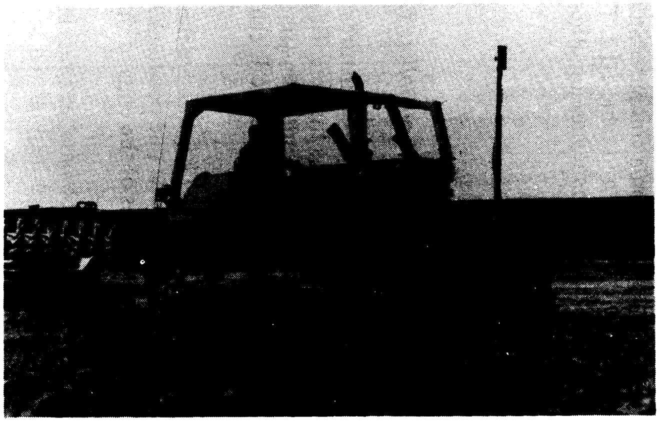


Figure 14.52 *Earthmoving and grading by laser (courtesy Spectra-Physics Ltd)*

can be undertaken on a computer and the results presented on a printout similar in format to that shown in table 10.2. The following two examples demonstrate the versatility of coordinate based methods.

14.17.1 Setting Out and Controlling Piling Work

The equipment used in piling disturbs the ground, takes up a lot of space and obstructs sightings across the area. Hence, it is not possible to establish all the pile positions before setting out begins since they are very likely to be disturbed during construction. Coordinate methods can be used to overcome this difficulty as follows.

(1) Before piling begins a baseline is decided upon and the lengths and angles necessary to set out the pile positions from each end of the baseline are calculated from the coordinates of the ends of the baseline and the design coordinates of the pile positions.

The position of the baseline must be carefully chosen so that taping and sighting from each end will not be hindered by the piling rig. Figure 14.53 shows a suitable scheme.

(2) Each bearing is set out by theodolite from one end of the baseline and checked from the other. The distances are measured using a steel tape or, if possible, EDM equipment with the reflecting unit mounted on a movable pole.

(3) The initial two or three positions are set out and the piling rig follows the path shown in figure 14.53.

(4) The engineer goes on ahead and establishes the other pile positions as work proceeds.

(5) A variation is to use two baselines on opposite sides of the area and establish the pile positions from four positions instead of two.

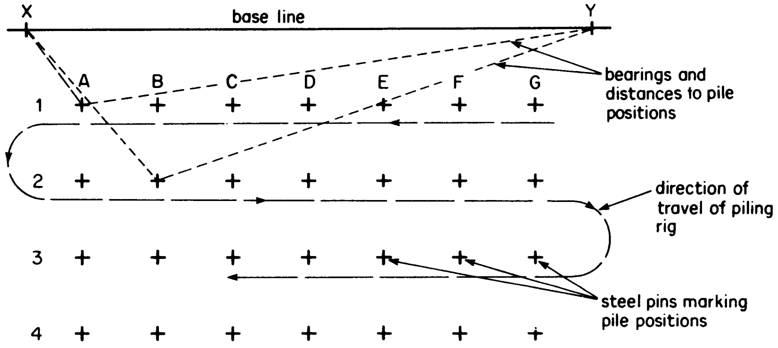


Figure 14.53

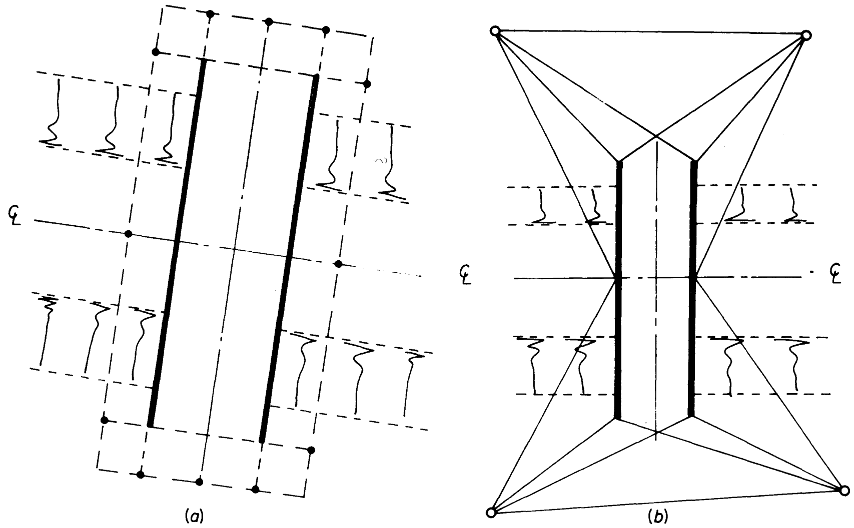


Figure 14.54 *Setting out bridges: (a) using structural grid; (b) by bearing and distance*

14.17.2 Setting Out Bridges

Figure 14.54 shows the plan view of a bridge to carry one road over another.

Procedure

(1) The centre lines of the two roads are set out by one of the methods discussed in chapters 9 and 10.

(2) The bridge is set out in advance of the road construction. Secondary site control points are established either in the form of a structural grid, which itself is set up from main site control stations by bearings and distances (see figure 14.54*a*), or, if this is not possible, in positions from which the bridge abutments can be set out by bearings and distances. These positions may be at traverse stations or site grid points (see figure 14.54*b*).

Whichever method is used, all the points must be permanently marked and protected to avoid their disturbance during construction, and positioned well away from the traffic routes on site.

(3) TBMs are set up. These can be separate levelled points or a control point can be levelled and used as a TBM.

(4) If the method shown in figure 14.54*a* is used, the distances from the secondary site control points to abutment design points are calculated and set out by steel tape or EDM equipment, the directions being established by theodolite.

If the method shown in figure 14.54*b* is used, the bearings and distances from the secondary site control points are calculated from their respective coordinates such that each design point can be established from at least two and, preferably, three control points.

(5) The design points are set out and their positions checked.

(6) Offset pegs are established to allow excavation and foundation work to proceed and to enable the abutments to be relocated as and when required.

(7) Once the foundations are established, the formwork, steel or precast units can be positioned with reference to the offset pegs.

For multi-span bridges, a structural grid can again be established from the site grid or traverse stations as shown in figure 14.55 and the centres of the abutments and piers set out from this. Since points A to P may be used many times during the construction, they should be positioned well away from site traffic and site operations and permanently marked and protected.

Each pier can be established by setting out from its centre position using offset pegs and profiles to mark the excavation area as shown in figure 14.56.

The required levels of the tops of the piers and the subsequent deck will be established from TBMs set up nearby either by conventional levelling techniques or by using a weighted steel tape as shown in figure 14.42.

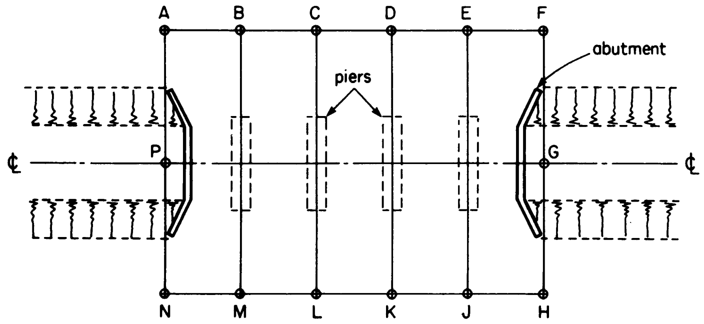


Figure 14.55

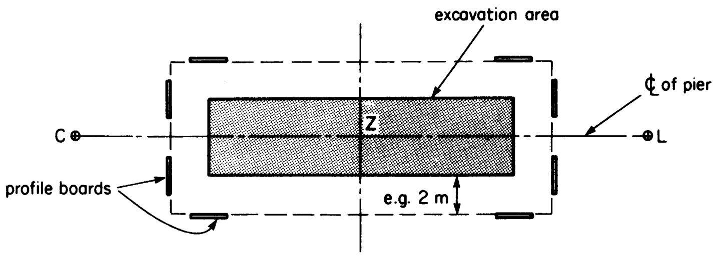


Figure 14.56

14.18 Accuracy of Setting Out

The accuracy to which setting out operations should be carried out has often been neglected in the past. However, national and international standards are now available which give recommendations regarding this question of accuracy. *BS 5606*,

TABLE 14.1 *The Accuracy in Use of Measuring Instruments*

Measurement	Instrument	Expected Precision	Comment
Linear	30 m carbon steel tape for general use	±5 mm up to and including 5 m ±10 mm for > 5 m and ⩽ 25 m ±15 mm for over 25 m	With sag eliminated and slope correction applied.
	30 m carbon steel tape for use in precise work	±3 mm up to and including 10 m ±6 mm for > 10 m and ⩽ 30 m	At correct tension and with slope, sag and temperature corrections applied.
Angular	30 m carbon steel tape on uneven terrain	±5′ (±25 mm in 15 m)	For right angles only.
	30 m carbon steel tape on flat surfaces	±2′ (±10 mm in 15 m)	With sag and slope corrections applied.
	Vernier theodolite* reading directly to 20″ (centred by plumbline)	±40″ (±10 mm in 50 m)	Mean of two sights, one on each face with readings in opposite quadrants of the horizontal circle.
	Glass arc theodolite* (with optical plummet or centring rod) reading directly to 20″	±20″ (± 5 mm in 50 m)	Scale readings estimated to the nearest 5 s. Mean of two sights, one on each face with readings in opposite quadrants of the horizontal circle.
	Glass arc theodolite (with optical plummet or centring rod) reading directly to 1″	±5″ (±2 mm in 80 m)	Mean of two sights, one on each face with readings in opposite quadrants of the horizontal circle.
Verticality	Spirit level	±10 mm in 3 m	For an instrument ⩾ 750 mm in length.
	Plumb-bob (3 kg) freely suspended	±5 mm in 5 m	To be used in still conditions only.
	Plumb-bob (3 kg) immersed in oil to restrict movement	±5 mm in 10 m	To be used in still conditions only.
	Theodolite (with optical plummet or centring rod) and diagonal eye-piece	±5 mm in 30 m	Mean of at least four projected points, each one established at a 90° interval around the horizontal circle.
	Optical plumbing device	±5 mm in 100 m	Automatic plumbing device incorporating a pendulous prism instead of a levelling bubble. Four readings to be taken in each quadrant of the horizontal circle and the mean value of readings in opposite quadrants accepted.
Levels	Spirit level	±5 mm in 5 m distance	For an instrument ⩾ 750 mm in length.
	Optical level	±3 mm per single sight of up to 60 m† ±10 mm per km.	Where possible sight lengths to be equal.

The distances given are the suggested maximum practical ranges for each instrument.
*If a single sight only is made when using a correctly adjusted theodolite to establish an angle the likely errors will be increased by a factor of 3. Therefore, a single sight should not be taken.
†Value based on measured data.

British Standard Code of Practice on Accuracy in Building, published in 1978, specifies permissible errors for various setting out activities, although it applies only to buildings and not to all civil engineering works. *ISO 4463, Measurement Methods for Building*, published by the International Organisation for Standardization (ISO), recommends accuracy requirements for all normal types of building construction but does not deal with specialist operations such as those required for precision machinery.

Table 14.1 shows the expected precisions of various pieces of equipment when used in engineering surveying by reasonably proficient operators. The equipment is assumed to be in correct adjustment. The data in the table are taken from the British Standard Code BS 5606. The values shown in the table should be used both by the designer when specifying the precision expected in the design in order that what is designed can actually be set out, and by the engineer undertaking the setting out in order that equipment can be chosen which will maintain the design standards and specifications.

The international standard for setting out, ISO 4463, specifies the precisions within which the adjusted coordinates of control points should be established. Other precisions are specified, including those for levelling and controlling verticality.

A further international standard, *ISO/DP 8322 Procedure for determining the accuracy in use of measuring instruments*, is also recommended for study prior to undertaking setting-out operations.

Section 14.19 lists these and other publications which will be found very useful by anyone concerned with the correct methods of setting out on construction sites.

14.19 Further Reading

A. C. Twort, *Civil Engineering Supervision and Management* (Arnold, London, 1972).

Bulletin M83: 16, *Measuring Practice on the Building Site*, CIB Report No. 69, (National Swedish Institute for Building Research, 1983).

A Manual of Setting-out Procedures (CIRIA, 1973).

BS 5606, *British Standard Code of Practice for Accuracy in Building* (British Standards Institution, London, 1978).

ISO 4463: *Measurement methods for building – Setting out and measurement – Permissible measuring deviations* (International Organisation for Standardization, Geneva).

ISO/DP 8322 *Procedure for determining the accuracy in use of measuring instruments* (International Organisation for Standardization, Geneva).

BRE Digest 234, *Accuracy in Setting Out* (Building Research Establishment, 1980).

14.20 Worked Examples

14.20.1 Pipeline Example

Question

An existing sewer at P is to be continued to Q and R on a falling gradient of 1 in 150 for plan distances of 27.12 m and 54.11 m consecutively, where the positions of P, Q and R are defined by wooden uprights.

Given the following level observations, calculate the difference in level between the top of each upright and the position at which the top edge of each sight rail must be set at P, Q and R if a 2.5 m traveller is to be used.

Level reading to staff on TBM on wall (RL 89.52 m AOD)	0.39 m
Level reading to staff on top of upright at P	0.16 m
Level reading to staff on top of upright at Q	0.35 m
Level reading to staff on top of upright at R	1.17 m
Level reading to staff on invert of existing sewer at P	2.84 m

All readings were taken from the same instrument position.

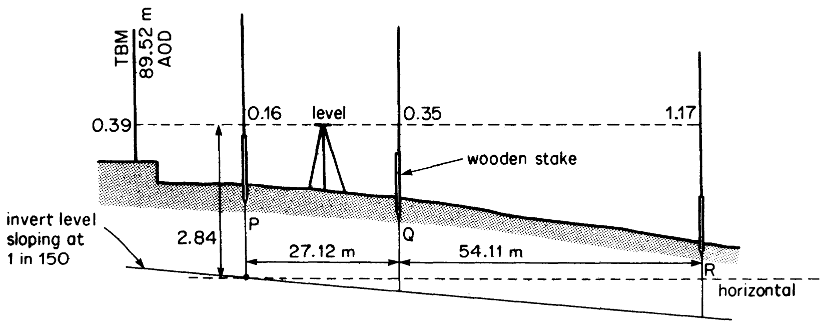


Figure 14.57

Solution

Consider figure 14.57.

Height of collimation of instrument	= 89.52 + 0.39 = 89.91 m
Invert level of existing sewer at P	= 89.91 – 2.84 = 87.07 m
Hence, sight rail top edge level at P	= 87.07 + 2.50 = 89.57 m
Level of top of upright at P	= 89.91 – 0.16 = 89.75 m
Hence, upright level – sight rail level	= 89.75 – 89.57 = +0.18 m

That is, the top edge of the sight rail must be fixed **0.18 m** *below* the top of the upright at P.

Fall of sewer from P to Q	$= -27.12 \times (1/150) = -0.18$ m
Hence, invert level at Q	$= 87.07 - 0.18 = 86.89$ m
Hence, sight rail top edge level at Q	$= 86.89 + 2.50 = 89.39$ m
But, level of top of upright at Q	$= 89.91 - 0.35 = 89.56$ m

Hence, upright level – sight rail level = 89.56 – 89.39 = +0.17 m. That is, the top edge of the sight rail must be fixed **0.17 m** *below* the top of the upright at Q.

Fall of sewer from P to R	$= -(27.12 + 54.11)/150 = -0.54$ m
Hence, invert level at R	$= 87.07 - 0.54 = 86.53$ m
Hence, sight rail top edge level at R	$= 86.53 + 2.50 = 89.03$ m
But, level of top of upright at R	$= 89.91 - 1.17 = 88.74$ m
Hence, upright level – sight rail level	$= 88.74 - 89.03 = -0.29$ m

That is, the top edge of the sight rail must be fixed **0.29 m** *above* the top of the upright at R.

This is achieved by nailing the sight rail to an extension piece to form a short traveller and then nailing this to the upright such that it adds 0.29 m to its height.

14.20.2 Coordinate Example

Question

A rectangular building having plan sides of 75.36 m and 23.24 m is to be set out with its major axis aligned precisely east–west on a coordinate system. Coordinates of the SE corner have been fixed as (348.92, 591.76) and this corner is to be fixed by theodolite intersections from two stations P and Q whose respective coordinates are (296.51, 540.32) and (371.30, 522.22). All dimensions are in metres.

Existing ground levels at the corners of the proposed structure were determined as follows

SE (156.82 m AOD), SW (149.73 m AOD), NE (151.45 m AOD),

NW (146.53 m AOD)

Calculate

(1) The respective clockwise angles (to the nearest 20″) to be set off at P relative to PQ and at Q relative to QP in order to intersect the position of the SE corner.

(2) Surface setting-out measurements around the four sides of the building together with the two diagonals, assuming even gradients along all lines.

Solution

Consider figure 14.58.

(1) *Calculation of α and β*

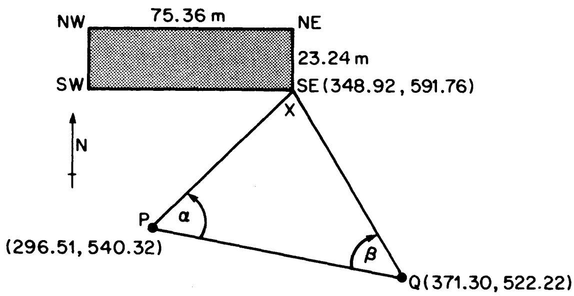


Figure 14.58

Let the SE corner of the building be X.

easting of X	348.92	northing of X	591.76
easting of P	296.51	northing of P	540.32
ΔE_{PX}	+52.41	ΔN_{PX}	+51.44

Therefore from a rectangular/polar conversion

bearing PX = 45°32′07″

easting of X	348.92	northing of X	591.76
easting of Q	371.30	northing of Q	522.22
ΔE_{QX}	−22.38	ΔN_{QX}	+69.54

Therefore from a rectangular/polar conversion

bearing QX = 342°09′37″

easting of Q	371.30	northing of Q	522.22
easting of P	296.51	northing of P	540.32
ΔE_{PQ}	+74.79	ΔN_{PQ}	−18.10

Therefore from a rectangular/polar conversion

bearing PQ = 103°36′17″

Therefore

angle α = bearing PQ − bearing PX = 58°04 10″

Hence

clockwise angle to be set off at P relative to PQ = 360° − 58°04′10″

= **301°56′00″**

and

angle β = bearing QX − bearing QP = 58°33′20″

Hence

clockwise angle to be set off at Q relative to QP = **58°33′20″**

Both answers have been rounded to the nearest 20″.

(2) *Calculation of surface measurements*

Slope correction = $+(\Delta h^2/2L)$ (see section 4.2.2) where Δh is the height difference and L the slope distance (but horizontal distance may be used without significant error).

From SE to SW corners, Δh = 156.82 – 149.73 = 7.09; Δh^2 = 50.27

From NE to NW corners, Δh = 151.45 – 146.53 = 4.92; Δh^2 = 24.21

From SE to NE corners, Δh = 156.82 – 151.45 = 5.37; Δh^2 = 28.84

From SW to NW corners, Δh = 149.73 – 146.53 = 3.20; Δh^2 = 10.24

Slope distances are as follows

SE to SW corners = 75.36 + (50.27/(2 x 75.36)) = 75.36 + 0.33
= **75.69 m**

NE to NW corners = 75.36 + (24.21/(2 x 75.36)) = 75.36 + 0.16
= **75.52 m**

SE to NE corners = 23.24 + (28.84/(2 x 23.24)) = 23.24 + 0.62
= **23.86 m**

SW to NW corners = 23.24 + (10.24/(2 x 23.24)) = 23.24 + 0.22
= **23.46 m**

For the diagonals

Horizontal diagonals = $(75.36^2 + 23.24^2)^{\frac{1}{2}}$ = 78.86

From SE to NW corners, Δh = 156.82 – 146.53 = 10.29; Δh^2 = 105.88

From SW to NE corners, Δh = 151.45 – 149.73 = 1.72; Δh^2 = 2.96

Diagonal slope distances are as follows

SE to NW corners = 78.86 + (105.88/(2 x 78.86)) = 78.86 + 0.67
= **79.53 m**

SW to NE corners = 78.86 + (2.96/(2 x 78.86)) = 78.86 + 0.02
= **78.88 m**

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