MECHANICS *of* MATERIALS

6TH EDITION

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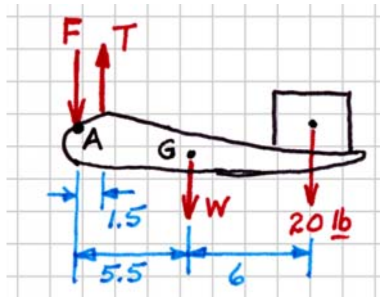
INSTRUCTOR SOLUTIONS MANUAL

Chapter 1

1-1*

From a free-body diagram of the forearm, the equilibrium equations give



$\uparrow \Sigma F_y = 0:$ $T - F - W - 20 = 0$

$\circlearrowleft \Sigma M_F = 0:$ $1.5T - 5.5W - 11.5(20) = 0$

$T = 3.667W + 153.33$ lb .... **Ans.**

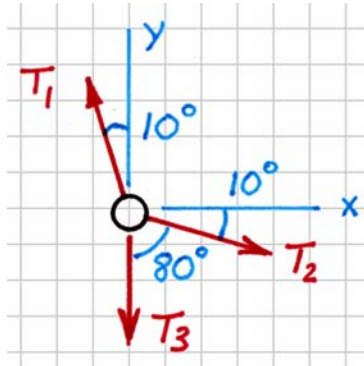
$F = 2.667W + 133.33$ lb .... **Ans.**

1-2*

From a free-body diagram of the ring, the equations of equilibrium

$\rightarrow \Sigma F_x = 0: \qquad T_2 \cos 10° - T_1 \sin 10° = 0$

$\uparrow \Sigma F_y = 0: \qquad T_1 \cos 10° - T_2 \sin 10° - 175(9.81) = 0$



are solved to get

$T_1 = 5.67128 T_2$

$T_1 = 1799$ N .... **Ans.**

$T_2 = 317$ N .... **Ans.**

$T_3 = 175(9.81) = 1717$ N .... **Ans.**

1-3

The equations of equilibrium

$\rightarrow \Sigma F_x = 0:$ $N_A \sin 30° - N_B \sin 30° = 0$

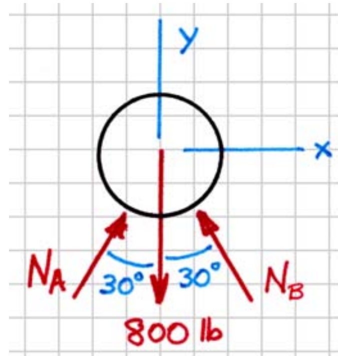
$\uparrow \Sigma F_y = 0:$ $N_A \cos 30° + N_B \cos 30° - 800 = 0$

are solved to get

$N_A = N_B$

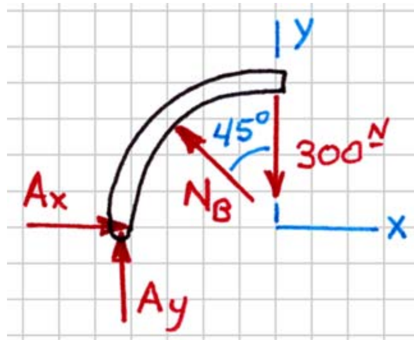
$\mathbf{N}_A = 462$ lb ∡ 60° .... **Ans.**

$\mathbf{N}_B = 462$ lb ∡ 60° .... **Ans.**



1-4*

The equations of equilibrium



$\rightarrow \Sigma F_x = 0: \qquad A_x - N_B \sin 45^\circ = 0$

$\uparrow \Sigma F_y = 0: \qquad A_y + N_B \cos 45^\circ - 300 = 0$

$\circlearrowleft \Sigma M_A = 0: \qquad 1.5\left(N_B \cos 45^\circ\right) - 1.5(300) = 0$

are solved to get

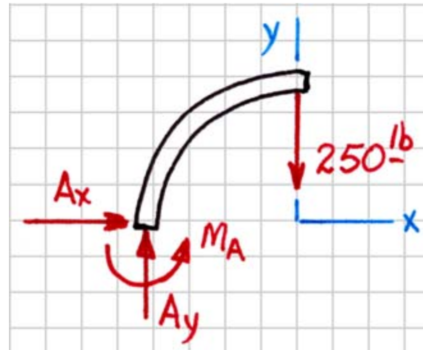
$N_B = 424.264 \text{ N} \cong 424 \text{ N}$ ⦣ 45............................ **Ans.**

$A_x = 300 \text{ N} \qquad A_y = 0 \text{ N}$

$\mathbf{A} = 300 \text{ N} \rightarrow$ .... **Ans.**

1-5

The equations of equilibrium



$\rightarrow \Sigma F_x = 0:$ $A_x = 0$

$\uparrow \Sigma F_y = 0:$ $A_y - 250 = 0$

$\circlearrowleft \Sigma M_A = 0:$ $M_A - 3(250) = 0$

are solved to get

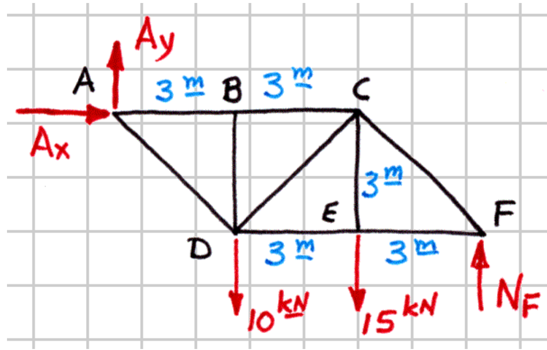
$A_x = 0 \text{ lb}$ $A_y = 250 \text{ lb}$

$\mathbf{A} = 250 \text{ lb} \uparrow$ ....**Ans.**

$M_A = 750 \text{ lb} \cdot \text{ft} \circlearrowleft$ ....**Ans.**

1-6

From an overall free-body diagram, the equations of equilibrium



$$\rightarrow \Sigma F_x = 0: \qquad A_x = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y - 10 - 15 + N_F = 0$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 9N_F - 3(10) - 6(15) = 0$$

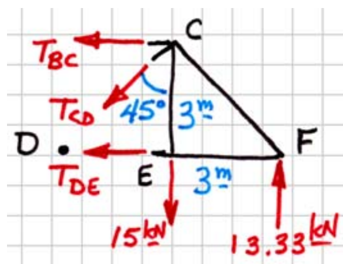
are solved to get

$$A_x = 0 \text{ kN}$$

$$A_y = 11.6667 \text{ kN} \uparrow$$

$$N_F = 13.3333 \text{ kN} \uparrow$$

Then, from a free-body diagram of the right hand section of the truss, the equations of equilibrium



$$\circlearrowleft \Sigma M_C = 0: \qquad 3(13.3333) - 3T_{DE} = 0$$

$$\circlearrowleft \Sigma M_D = 0: \qquad 3T_{BC} - 3(15) + 6(13.3333) = 0$$

$$\uparrow \Sigma F_y = 0: \qquad 13.3333 - 15 - T_{CD} \cos 45^\circ = 0$$

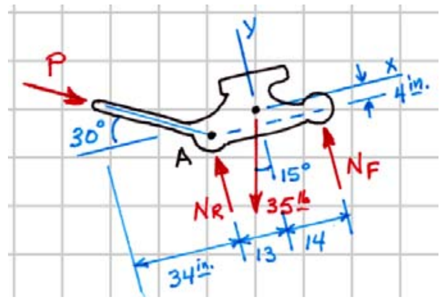
are solved to get

$T_{DE} = 13.33$ kN (T) .... **Ans.**

$T_{BC} = -11.67$ kN $= 11.67$ kN (C) .... **Ans.**

$T_{CD} = -2.36$ kN $= 2.36$ kN (C) .... **Ans.**

1-7*



$\circlearrowleft \Sigma M_A = 0: \qquad 27N_F - 13(33\cos 15°) + 4(35\sin 15°) = 0$

$N_F = 14.93561 \text{ lb} \cong 14.94 \text{ lb}$ ∠ 75° .... **Ans.**

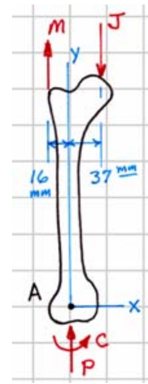
Z $\Sigma F_x = 0: \qquad P\cos 30° - 35\sin 15° = 0$

$P = 10.46005 \text{ lb} \cong 10.46 \text{ lb}$ ∠ 15° .... **Ans.**

^ $\Sigma F_y = 0: \qquad N_R + N_F - P\sin 30° - 35\cos 15° = 0$

$N_R = 24.1 \text{ lb}$ ∠ 75° .... **Ans.**

1-8



$\uparrow \Sigma F_y = 0: \qquad P + 4060 - 5210 = 0$

$P = 1150 \text{ N}$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0: \qquad C - 16(4060) - 37(5210) = 0$

$C = 257,730 \text{ N} \cdot \text{mm} \cong 258 \text{ N} \cdot \text{m}$ .... **Ans.**

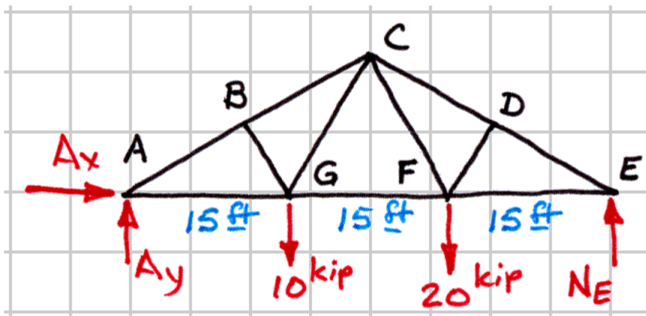
1-9*

From an overall free-body diagram, the equations of equilibrium

$\rightarrow \Sigma F_x = 0: \qquad A_x = 0$

$\uparrow \Sigma F_y = 0: \qquad A_y - 10 - 20 + N_E = 0$

$\circlearrowleft \Sigma M_A = 0: \qquad 45N_E - 15(10) - 30(20) = 0$



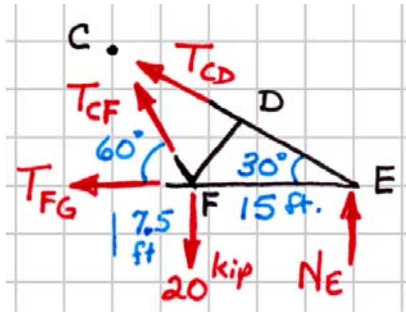
are solved to get

$A_x = 0 \text{ kip}$

$A_y = 13.3333 \text{ kip} \uparrow$

$N_E = 16.6667 \text{ kip} \uparrow$

Then, from a free-body diagram of the right hand section of the truss, the equations of equilibrium



$\circlearrowleft \Sigma M_F = 0: \qquad 15N_E + 15(T_{DE} \sin 30°) = 0$

$\circlearrowleft \Sigma M_E = 0: \qquad 15(20) - 15(T_{CF} \sin 60°) = 0$

$\circlearrowleft \Sigma M_C = 0: \qquad 22.5N_E - 7.5(20) - (15 \cos 30°)T_{FG} = 0$

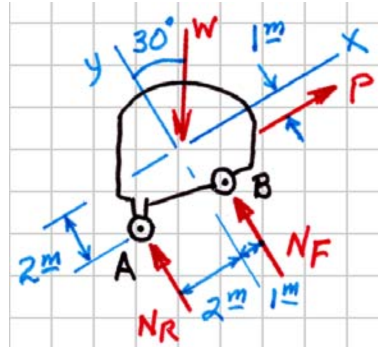
are solved to get

$T_{CD} = -33.3333 \text{ kip} \cong 33.3 \text{ kip (C)}$ .... **Ans.**

$T_{CF} = +23.094 \text{ kip} \cong 23.1 \text{ kip (T)}$ .... **Ans.**

$T_{FG} = +17.32 \text{ kip} = 17.32 \text{ kip (T)}$ .... **Ans.**

1-10*



$$W = 2000(9.81) = 19{,}620 \text{ N}$$

Z $\Sigma F_x = 0:$ $\quad P - W \sin 30° = 0$

$\circlearrowleft \Sigma M_A = 0:$ $\quad 2(W \sin 30°) - 2(W \cos 30°) + 3N_F - 1P = 0$

^ $\Sigma F_y = 0:$ $\quad N_R + N_F - W \cos 30° = 0$

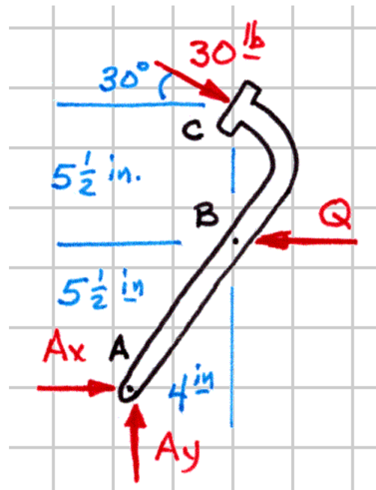
$P = 9810 \text{ N} = 9.81 \text{ kN} \measuredangle\ 30°$ .... **Ans.**

$N_R = 8933.806 \text{ N} \cong 8.93 \text{ kN} \measuredangle\ 60°$ .... **Ans.**

$N_F = 8057.612 \text{ N} \cong 8.06 \text{ kN} \measuredangle\ 60°$ .... **Ans.**

1-11

From a free-body diagram of the brake pedal, the equilibrium equations are solved to get the forces



$$\circlearrowleft \Sigma M_A = 0: \qquad 5.5Q - (30\cos 30°)(11) - (30\sin 30°)(4) = 0$$

$$Q = 62.871 \text{ lb}$$

$$\rightarrow \Sigma F_x = 0: \qquad A_x - Q + 30\cos 30° = 0$$

$$A_x = 36.890 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \qquad A_y - 30\sin 30° = 0$$

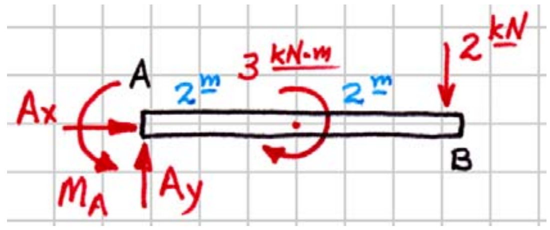
$$A_y = 15.00 \text{ lb}$$

$\mathbf{A} = 39.8$ lb ∡ 22.13° .... **Ans.**

$\mathbf{Q} = 62.9$ lb ← .... **Ans.**

1-12*

From a free-body diagram of the beam, the equilibrium equations are solved to get the forces and moment



$\rightarrow \Sigma F_x = 0: \qquad A_x = 0$

$\uparrow \Sigma F_y = 0: \qquad A_y - 2 = 0$

$A_y = 2 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0: \qquad M_A - 2(4) - 3 = 0$

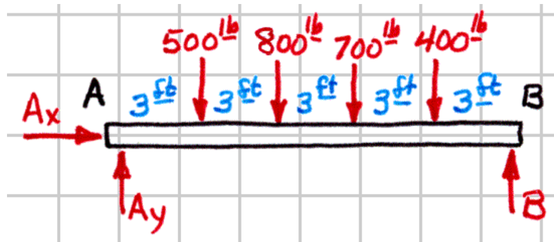
$M_A = 11 \text{ kN} \cdot \text{m}$

$\mathbf{A} = 2 \text{ kN} \uparrow$ .... **Ans.**

$\mathbf{M}_A = 11 \text{ kN} \cdot \text{m} \circlearrowleft$ .... **Ans.**

1-13

From a free-body diagram of the beam, the equilibrium equations are solved to get the forces



$\rightarrow \Sigma F_x = 0:\qquad A_x = 0$

$\circlearrowleft \Sigma M_A = 0:\qquad 15B - 3(500) - 6(800) - 9(700) - 12(400) = 0$

$B = 1160 \text{ lb}$

$\uparrow \Sigma F_y = 0:\qquad A_y + B - 500 - 800 - 700 - 400 = 0$

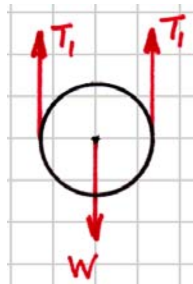
$A_y = 1240 \text{ lb}$

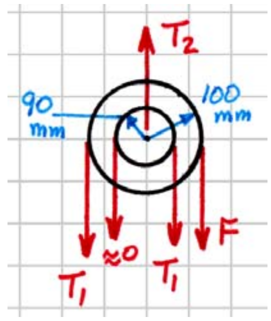
$\mathbf{A} = 1240 \text{ lb} \uparrow$ .... **Ans.**

$\mathbf{B} = 1160 \text{ lb} \uparrow$ .... **Ans.**

1-14

$$W = 450(9.81) = 4414.50 \text{ N}$$





From a free-body diagram of the lower pulley, vertical equilibrium equation gives the tension

$$\uparrow \Sigma F_y = 0: \qquad 2T_1 - 4414.50 = 0$$

$$T_1 = 2207.25 \text{ N}$$

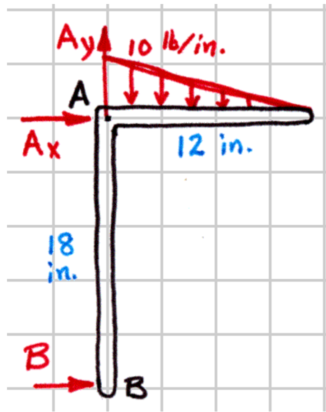
Then, from a free-body diagram of the upper pulley, moment equilibrium equation gives the force F

$$\circlearrowleft \Sigma M_{axle} = 0: \qquad 100T_1 - 90T_1 - 100F = 0$$

$F = 221$ N .... **Ans.**

1-15*

From a free-body diagram of the bracket, the equilibrium equations



$$\circlearrowleft \Sigma M_A = 0: \qquad 18B - \left[(12)(10)/2\right](12/3) = 0$$

$$\rightarrow \Sigma F_x = 0: \qquad A_x + B = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y - \left[(12)(10)/2\right] = 0$$

Are solved to get the forces

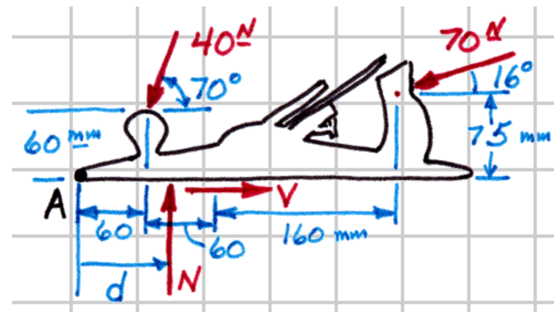
$$B = 13.333 \text{ lb}$$

$$A_x = -13.333 \text{ lb} \qquad A_y = 60.0 \text{ lb}$$

$\mathbf{A} = 61.464 \text{ lb} \cong 61.5 \text{ lb} \measuredangle\ 77.47°$ .... **Ans.**

$\mathbf{B} = 13.33 \text{ lb} \rightarrow$ .... **Ans.**

1-16



(a) From a free-body diagram of the plane, the force equilibrium equations are solved to get the forces

$$\rightarrow \Sigma F_x = 0: \qquad V - 40\cos 70° - 70\cos 16° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad N - 40\sin 70° - 70\sin 16° = 0$$

$V = 81.0$ N .... **Ans.**

$N = 56.882 \text{ N} \cong 56.9$ N .... **Ans.**

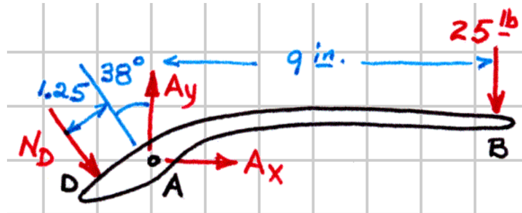
(b) Then the moment equilibrium equation gives the required location of the normal force

$$\circlearrowleft \Sigma M_A = 0: \qquad (70\cos 16°)(75) - (70\sin 16°)(280) + Nd$$
$$+ (40\cos 70°)(60) - (40\sin 70°)(60) = 0$$

$d = 31.5$ mm

31.5 mm (from the left end of plane) .... **Ans.**

1-17*



$\circlearrowleft \Sigma M_A = 0: \qquad 1.25 N_D - 9(25) = 0$

$N_D = 180.0$ lb .... **Ans.**

$\rightarrow \Sigma F_x = 0: \qquad A_x + N_D \sin 38° = 0$

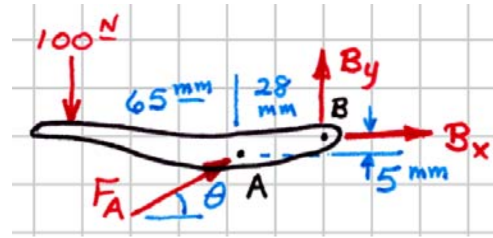
$\uparrow \Sigma F_y = 0: \qquad A_y - 25 - N_D \cos 38° = 0$

$A_x = -110.8$ lb $\qquad A_y = 166.8$ lb

A = 200.3 lb ∠ 56.4° .... **Ans.**

1-18*

From a free-body diagram of the upper handle, moment equilibrium gives



$$\rightarrow \Sigma F_x = 0: \qquad F_A \cos\theta + B_x = 0$$

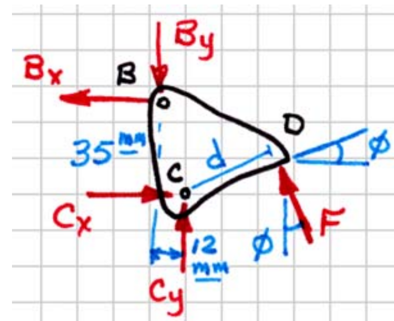
$$\uparrow \Sigma F_y = 0: \qquad F_A \sin\theta - 100 + B_y = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad 93(100) - 28(F_A \sin\theta) + 5(F_A \cos\theta) = 0$$

$$\theta = \tan^{-1}\frac{30}{50} = 30.964°$$

$$F_A = 919.116 \text{ N}$$

$$B_x = -788.136 \text{ N} \qquad B_y = -372.881 \text{ N}$$



Then a free body diagram of the upper jaw gives

$$\circlearrowleft \Sigma M_C = 0: \qquad Fd + 12B_y + 35B_x = 0$$

$$d = \sqrt{35^2 + 15^2} = 38.0789 \text{ mm}$$

Therefore

$$F = 842 \text{ N} \; ⦣ \; 66.8° \text{ on the jaw}$$

$\mathbf{F} = 842$ N ⦥ 66.8° on the block.. **Ans.**

1-19

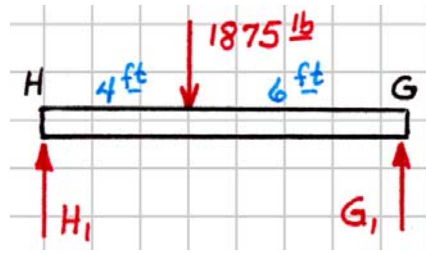
Use $W = 3750\text{ lb}$ (the weight carried by one truss). Then by symmetry (or from equilibrium of a free-body diagram of the entire truss) each support carries half of the total weight

$$E_y = A_y = W/2 = 1875\text{ lb}$$

Also by symmetry (or equilibrium of a free-body diagram of the truck), the truck's weight is divided equally between its front and rear wheels.

$$N_F = N_R = W/2 = 1875\text{ lb}$$

Then equilibrium of the floor panel between pins *G* and *H* gives



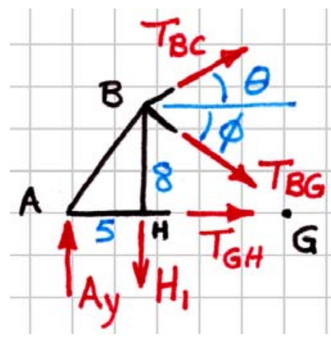
$$\circlearrowleft \Sigma M_H = 0: \qquad 10G_1 - 4(1875) = 0$$

$$G_1 = 750\text{ lb}$$

$$\circlearrowleft \Sigma M_G = 0: \qquad 6(1875) - 10H_1 = 0$$

$$H_1 = 1125\text{ lb}$$

Next, from a free-body diagram of a section of the left side of the truss



$$\theta = \tan^{-1}\frac{5}{10} = 26.565° \qquad \phi = \tan^{-1}\frac{8}{10} = 38.660°$$

$$\circlearrowleft \Sigma M_B = 0: \qquad 8T_{GH} - 5(1875) = 0$$

$$T_{GH} = 1171.875\text{ lb}$$

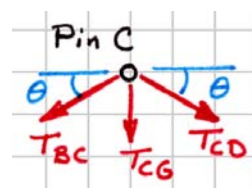
$$\circlearrowleft \Sigma M_G = 0: \quad -15(1875) - 8(T_{BC}\cos\theta) - 10(T_{BC}\sin\theta) + 10(1125) = 0$$

$T_{BC} = -1451.295\text{ lb} \cong 1451\text{ lb (C)}$ .... **Ans.**

$$\uparrow \Sigma F_y = 0: \qquad 1875 + T_{BC}\sin\theta - T_{BG}\sin\phi - 1125 = 0$$

$T_{BG} = +161.6\text{ lb} = 161.6\text{ lb (T)}$ .... **Ans.**

Finally, from a free-body diagram of pin *C*



$$\rightarrow \Sigma F_x = 0: \qquad T_{CD}\cos\theta - T_{BC}\cos\theta = 0$$

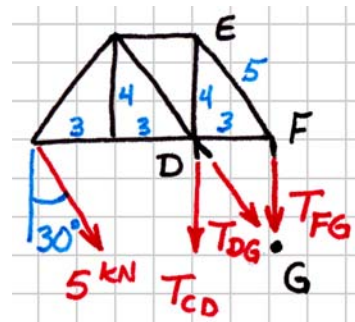
$$\uparrow \Sigma F_y = 0: \qquad -T_{BC}\sin\theta - T_{CD}\sin\theta - T_{CG} = 0$$

$$T_{CD} = -1451.295\text{ lb}$$

$T_{CG} = +1298\text{ lb} = 1298\text{ lb (T)}$ .... **Ans.**

1-20*

Cut a section through *CD*, *DG*, and *FG*, and draw a free-body diagram of the upper-portion of the truss. The equilibrium equations give



$$\circlearrowleft \Sigma M_G = 0: \qquad (5\cos 30°)(9) - (5\sin 30°)(4) + T_{CD}(3) = 0$$

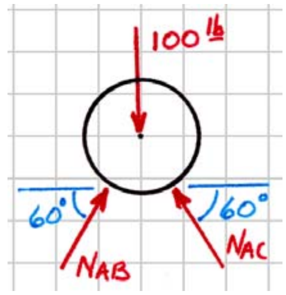
$$\circlearrowleft \Sigma M_D = 0: \qquad (5\cos 30°)(6) - T_{FG}(3) = 0$$

$T_{CD} = -9.66 \text{ kN} = 9.66 \text{ kN (C)}$ .... **Ans.**

$T_{FG} = 8.66 \text{ kN (T)}$ .... **Ans.**

1-21

A free-body diagram of cylinder *A* gives

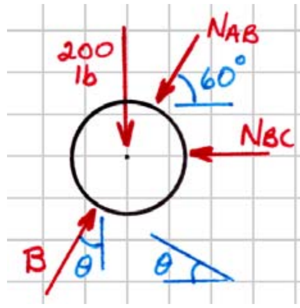


$\rightarrow \Sigma F_x = 0: \qquad N_{AB} \cos 60° - N_{AC} \cos 60° = 0$

$\uparrow \Sigma F_y = 0: \qquad N_{AB} \sin 60° + N_{AC} \sin 60° - 100 = 0$

$$N_{AB} = N_{AC} = 57.73503 \text{ lb}$$

Then, from a free-body diagram of cylinder *B*



$\rightarrow \Sigma F_x = 0: \qquad B \sin\theta - N_{AB} \cos 60° - N_{BC} = 0$

$\uparrow \Sigma F_y = 0: \qquad B \cos\theta - N_{AB} \sin 60° - 200 = 0$

The minimum force occurs when $N_{BC} = 0$, therefore

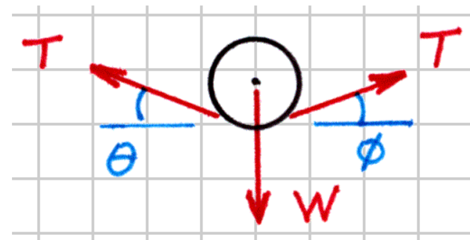
$$\frac{B\sin\theta}{B\cos\theta} = \tan\theta = \frac{N_{AB}\cos 60°}{N_{AB}\sin 60° + 200}$$

$\theta = 6.59°$ .... **Ans.**

1-22

$$W = 250(9.81) = 2452.50 \text{ N}$$

From a free-body diagram of the pulley

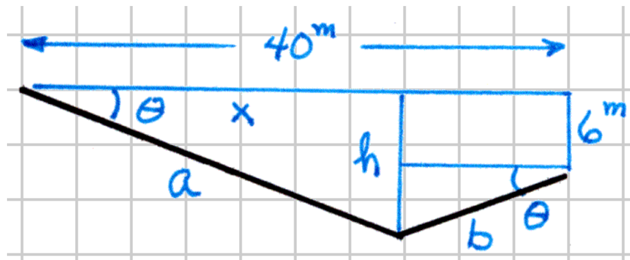


$$\rightarrow \Sigma F_x = 0: \qquad T\cos\phi - T\cos\theta = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T\sin\phi + T\sin\theta - 2452.50 = 0$$

$$\phi = \theta$$

From the geometry of the cable



$$a + b = 42 \text{ m}$$

$$(a+b)\cos\theta = 40 \text{ m}$$

$$\theta = \cos^{-1}\frac{40}{42} = 17.7528°$$

Also from the geometry of the cable

$$h = a\sin\theta = 6 + b\sin\theta$$

Therefore

$$(a-b) = 6/\sin\theta = 19.67789 \text{ m}$$

which together with $a + b = 42$ m gives

$$a = 30.83895 \text{ m} \qquad b = 11.16105 \text{ m}$$

$x = a\cos\theta = 29.4$ m .... **Ans.**

$T = 4020$ N .... **Ans.**

1-23*

Max pull occurs when either the rear wheels begin to slip $\left(B = 0.8B_y\right)$ or when the front wheels start to lift off the ground $\left(A_y = 0\right)$. The force which makes the front wheels lift off of the ground is

$$\circlearrowleft \Sigma M_B = 0: \qquad 8(15,000) - 5(P\cos 30°) - 10(P\sin 30°) = 0$$

$P = 12,862 \text{ lb} \cong 12.86 \text{ kip}$ .... **Ans.**

Checking the amount of friction required and the amount of friction available for this pulling force

$$\rightarrow \Sigma F_x = 0: \qquad P\sin 30° - B_x = 0$$

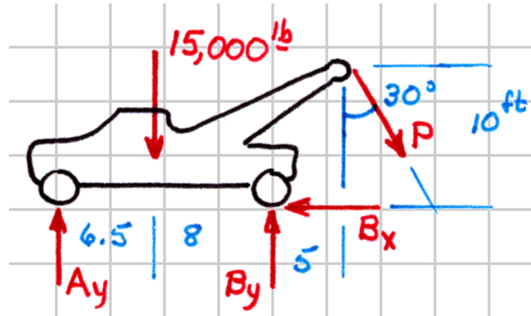
$$\uparrow \Sigma F_y = 0: \qquad A_y + B_y - 15,000 - P\cos 30° = 0$$

$$A_y = 0$$

$$B_x = 6431 \text{ lb (friction required)}$$

$$B_y = 26,138 \text{ lb}$$

$$0.8(26138) = 20,911 \text{ lb (friction available)}$$



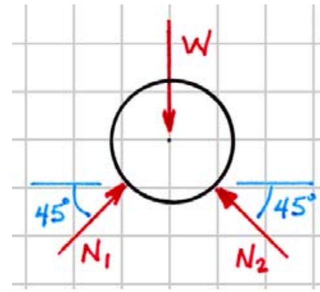
Since the friction required is much less than the friction available, we made the correct guess.

1-24

Divide the weight by 2 since there are two frames

$$W = 200(9.81)/2 = 981 \text{ N}$$

Then from a free-body diagram of the drum

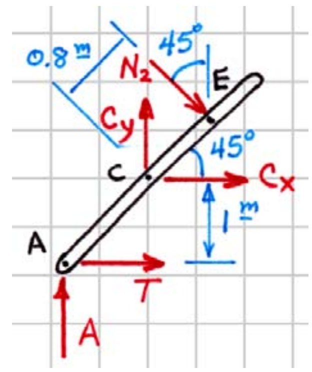


$\rightarrow \Sigma F_x = 0: \qquad N_1 \cos 45° - N_2 \cos 45° = 0$

$\uparrow \Sigma F_y = 0: \qquad 0.4 N_1 \sin 45° + N_2 \sin 45° - 981 = 0$

$N_1 = N_2 = 693.672 \text{ N} \cong 694 \text{ N}$ .... **Ans.**

Finally from a free-body diagram of one leg



$\circlearrowleft \Sigma M_C = 0: \qquad 1T - 1A - 0.8 N_2 = 0$

$\rightarrow \Sigma F_x = 0: \qquad T + C_x + N_2 \sin 45° = 0$

$\uparrow \Sigma F_y = 0: \qquad A + C_y - N_2 \cos 45° = 0$

where by symmetry (or from overall equilibrium)

$A = 981/2 = 490.5 \text{ N}$ .... **Ans.**

and then

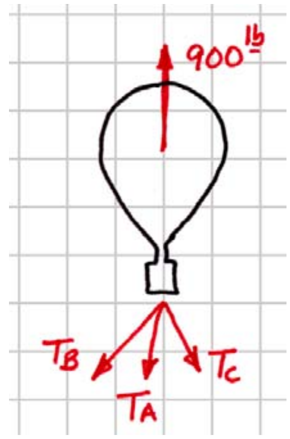
$T = 1045.4376 \text{ N} \cong 1045 \text{ N}$ .... **Ans.**

$C_x = -1535.94 \text{ N} \cong 1536 \text{ N} \leftarrow$ .... **Ans.**

$C_y = 0 \text{ N}$ .... **Ans.**

1-25*

The components of the three tension forces are



$$\mathbf{T}_A = T_A \frac{20\mathbf{i} + 30\mathbf{j} - 50\mathbf{k}}{\sqrt{20^2 + 30^2 + 50^2}}$$

$$= 0.32444 T_A \mathbf{i} + 0.48666 T_A \mathbf{j} - 0.81111 T_A \mathbf{k}$$

$$\mathbf{T}_B = T_B \frac{16\mathbf{i} - 25\mathbf{j} - 50\mathbf{k}}{\sqrt{16^2 + 25^2 + 50^2}}$$

$$= 0.27517 T_B \mathbf{i} - 0.42995 T_B \mathbf{j} - 0.85990 T_B \mathbf{k}$$

$$\mathbf{T}_C = T_C \frac{-25\mathbf{i} - 15\mathbf{j} - 50\mathbf{k}}{\sqrt{25^2 + 15^2 + 50^2}}$$

$$= -0.43193 T_C \mathbf{i} - 0.25916 T_C \mathbf{j} - 0.86387 T_C \mathbf{k}$$

Then the *x*-, *y*-, and *z*-components of the force equilibrium equation give

x: $\quad 0.32444 T_A + 0.27517 T_B - 0.43193 T_C = 0$

y: $\quad 0.48666 T_A - 0.42995 T_B - 0.25916 T_C = 0$

z: $\quad -0.81111 T_A - 0.85990 T_B - 0.86387 T_C + 900 = 0$

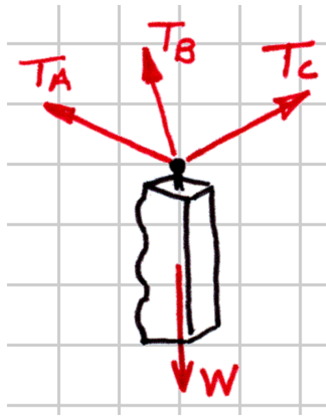
$T_A = 418.214 \text{ lb} \cong 418 \text{ lb}$ .... **Ans.**

$T_B = 205.219 \text{ lb} \cong 205 \text{ lb}$ .... **Ans.**

$T_C = 444.876 \text{ lb} \cong 445 \text{ lb}$ .... **Ans.**

1-26*

The components of the forces are



$$\mathbf{W} = -100(9.81)\mathbf{k} = -981\mathbf{k}\ \text{N}$$

$$\mathbf{T}_A = T_A \frac{4\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}}{\sqrt{4^2 + 8^2 + 5^2}}$$
$$= 0.39036 T_A \mathbf{i} - 0.78072 T_A \mathbf{j} + 0.48795 T_A \mathbf{k}$$

$$\mathbf{T}_B = T_B \frac{-6\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}}{\sqrt{6^2 + 8^2 + 5^2}}$$
$$= -0.53666 T_B \mathbf{i} - 0.71554 T_B \mathbf{j} + 0.44721 T_B \mathbf{k}$$

$$\mathbf{T}_C = T_C \frac{8\mathbf{j} + 5\mathbf{k}}{\sqrt{8^2 + 5^2}} = 0.84800 T_C \mathbf{j} + 0.53000 T_C \mathbf{k}$$

Then the *x*-, *y*-, and *z*-components of the force equilibrium equation give

$x:$ $0.39036 T_A - 0.53666 T_B = 0$

$y:$ $-0.78072 T_A - 0.71554 T_B + 0.84800 T_C = 0$

$z:$ $0.48795 T_A + 0.44721 T_B + 0.53000 T_C - 981 = 0$

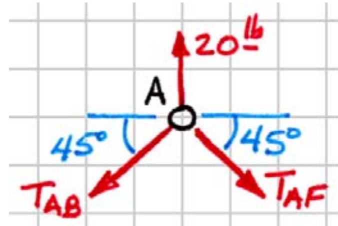
$T_A = 603.139\ \text{N} \cong 603\ \text{N}$ .... **Ans.**

$T_B = 438.716\ \text{N} \cong 439\ \text{N}$ .... **Ans.**

$T_C = 925.473\ \text{N} \cong 925\ \text{N}$ .... **Ans.**

1-27

From a free-body diagram of pin *A*

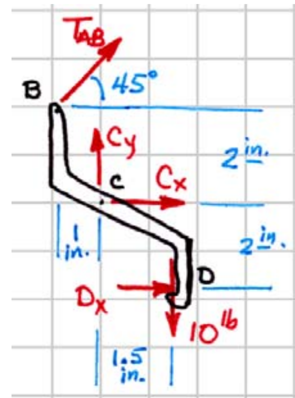


$\rightarrow \Sigma F_x = 0:$ $T_{AF}\cos 45° - T_{AB}\cos 45° = 0$

$\uparrow \Sigma F_y = 0:$ $20 - T_{AB}\sin 45° - T_{AF}\sin 45° = 0$

$T_{AB} = T_{AF} = 14.14214 \text{ lb} \cong 14.14 \text{ lb}$ .... **Ans.**

Finally from a free-body diagram of *BCD*



$\rightarrow \Sigma F_x = 0:$ $T_{AB}\cos 45° + C_x + D_x = 0$

$\uparrow \Sigma F_y = 0:$ $T_{AB}\sin 45° + C_y - D_y = 0$

$\circlearrowleft \Sigma M_C = 0:$ $2D_x - 1.5(10) - 2(T_{AB}\cos 45°) - 1(T_{AB}\sin 45°) = 0$

where by symmetry (or from overall equilibrium)

$$D_y = 20/2 = 10 \text{ lb} \downarrow$$

and then

$$C_x = -32.500 \text{ lb} \cong 32.5 \text{ lb} \leftarrow \qquad C_y = 0 \text{ lb}$$

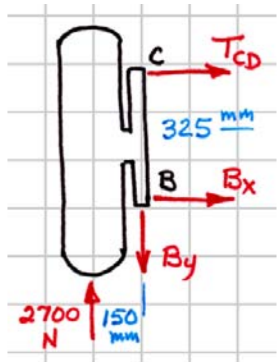
$$D_x = 22.500 \text{ lb} \cong 22.5 \text{ lb} \rightarrow$$

C $= 32.5$ lb $\leftarrow$ .... **Ans.**

D $= 24.6$ lb ↘ $24.0°$ .... **Ans.**

1-28

From a free body diagram of the wheel and arm *BC*



$$\rightarrow \Sigma F_x = 0: \qquad T_{CD} + B_x = 0$$

$$\uparrow \Sigma F_y = 0: \qquad 2700 - B_y = 0$$

$$\circlearrowleft \Sigma M_B = 0: \qquad -150(2700) - 325 T_{CD} = 0$$

$$B_x = 1246 \text{ N} \qquad B_y = 2700 \text{ N}$$

B = 2970 N ∠ 65.2° (on *AB*) .... **Ans.**

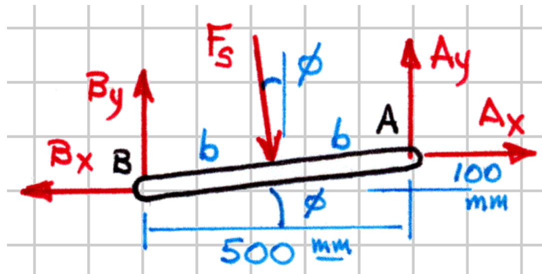
$T_{CD} = -1246.154 \text{ N} \cong 1246 \text{ N (C)}$ .... **Ans.**

Then from a free-body diagram of the arm *AB* (and assuming that the spring pushes perpendicularly against the arm)

$$\circlearrowleft \Sigma M_A = 0: \qquad -100(1246.154) - 500(2700) + bF_S = 0$$

$$\rightarrow \Sigma F_x = 0: \qquad A_x - 1246.154 + F_S \sin\phi = 0$$

$$\uparrow \Sigma F_y = 0: \qquad 2700 - F_S \cos\phi + A_y = 0$$



$$\phi = \tan^{-1}\frac{50}{250} = 11.310°$$

$$b = \sqrt{50^2 + 250^2} = 254.951 \text{ mm}$$

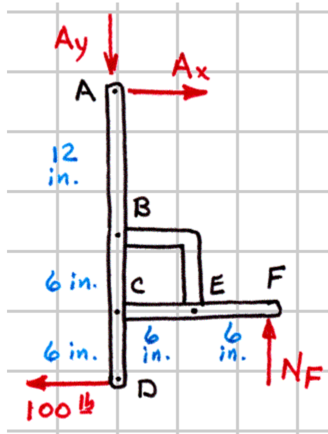
$F_S = 5783.92 \text{ N} \cong 5780 \text{ N (C)}$ .... **Ans.**

$$A_x = 111.827 \text{ N} \qquad A_y = 2971.60 \text{ N}$$

A = 2970 N ∠ 87.8° .... **Ans.**

1-29*

First equilibrium of an overall free-body diagram gives



$\rightarrow \Sigma F_x = 0: \qquad A_x - 100 = 0$

$\uparrow \Sigma F_y = 0: \qquad 200 - A_y = 0$

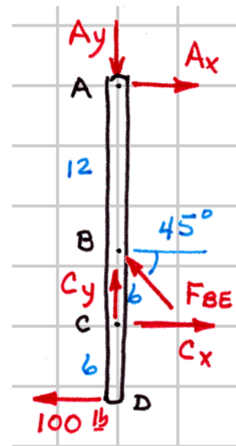
$\circlearrowleft \Sigma M_A = 0: \qquad 12N_F - 24(100) = 0$

$A_x = 100 \text{ lb} \qquad A_y = 200 \text{ lb}$

$N_F = 200 \text{ lb}$

A = 224 lb $\measuredangle$ 63.4° ....**Ans.**

Then from a free-body diagram of the bar *ABCD*



$\rightarrow \Sigma F_x = 0: \qquad A_x + C_x - F_{BE}\cos 45° - 100 = 0$

$\uparrow \Sigma F_y = 0: \qquad C_y + F_E \sin 45° - A_y = 0$

$\circlearrowleft \Sigma M_C = 0: \qquad -18A_x + 6(F_{BE}\cos 45°) - 6(100) = 0$

$C_x = 400 \text{ lb} \qquad C_y = -200 \text{ lb}$

$F_{BE} = 565.685 \text{ lb} \cong 566 \text{ lb}$ $\measuredangle$ 45° .... **Ans.**

C = 447 lb $\measuredangle$ 26.6° .... **Ans.**

1-30 The total weight of water carried by each truss is

$$W_T = 1000(9.81)(0.2)(3.6)(2) = 14{,}126.40 \text{ N}$$

By symmetry (or overall equilibrium) the support reactions at A and F are

$$A_y = F = W_T/2 = 7063.20 \text{ N}$$

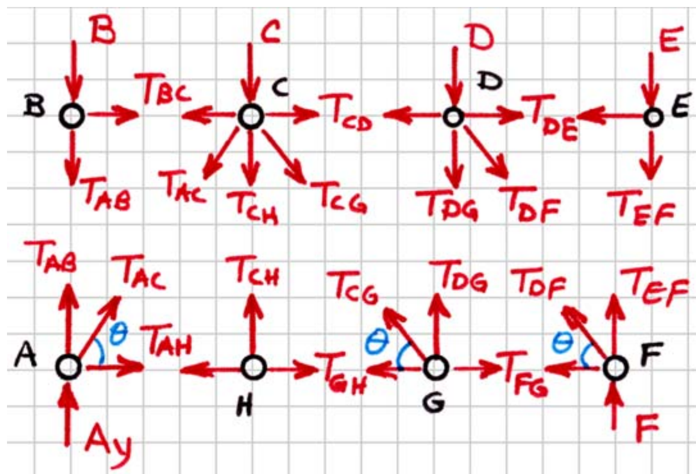
The weight of water carried by each roof panel is

$$W_P = 1000(9.81)(0.2)(1.2)(2) = 4708.80 \text{ N}$$

and by symmetry (or equilibrium of the panel) half of this load is carried by the pins at each end of the panel

$$B = E = W_P/2 = 2354.40 \text{ N} \qquad C = D = 2(W_P/2) = 4708.80 \text{ N}$$

$$\sin\theta = 0.8000 \qquad \cos\theta = 0.6000$$



Then, equilibrium of Pin B gives

$$\rightarrow \Sigma F_x = 0: \qquad T_{BC} = 0$$

$$\uparrow \Sigma F_y = 0: \qquad -T_{AB} - 2354.40 = 0$$

$$T_{AB} = -2354.40 \text{ N}$$

Pin E:

$$\rightarrow \Sigma F_x = 0: \qquad -T_{DE} = 0$$

$$\uparrow \Sigma F_y = 0: \qquad -T_{EF} - 2354.40 = 0$$

$$T_{EF} = -2354.40 \text{ N}$$

Pin A:

$$\rightarrow \Sigma F_x = 0: \qquad T_{AH} + T_{AC}\cos\theta = 0 \qquad T_{AH} = 3531.60 \text{ N}$$

$$\uparrow \Sigma F_y = 0: \qquad 7063.20 + (-2354.40) + T_{AC}\sin\theta = 0 \qquad T_{AC} = -5886.00 \text{ N}$$

Pin H:

$$\rightarrow \Sigma F_x = 0: \qquad T_{GH} - (3531.60) = 0 \qquad T_{GH} = 3531.60 \text{ N}$$

$$\uparrow \Sigma F_y = 0: \qquad T_{CH} = 0$$

Pin C:

$$\rightarrow \Sigma F_x = 0: \qquad T_{CD} + T_{CG}\cos\theta - (-5886)\cos\theta - (0) = 0 \qquad T_{CD} = -3531.60 \text{ N}$$

$$\uparrow \Sigma F_y = 0: \qquad -4708.80 - (-5886)\sin\theta - (0) - T_{CG}\sin\theta = 0 \qquad T_{CG} = 0 \text{ N}$$

Pin G:

$$\rightarrow \Sigma F_x = 0: \qquad T_{FG} - (3531.60) - (0)\cos\theta = 0 \qquad T_{FG} = 3531.60 \text{ N}$$

$$\uparrow \Sigma F_y = 0: \qquad (0)\sin\theta + T_{DG} = 0 \qquad T_{DG} = 0 \text{ N}$$

Pin D:

$$\uparrow \Sigma F_y = 0: \qquad -4708.80 - (0) - T_{DF}\sin\theta = 0 \qquad T_{DF} = -5886.00 \text{ N}$$

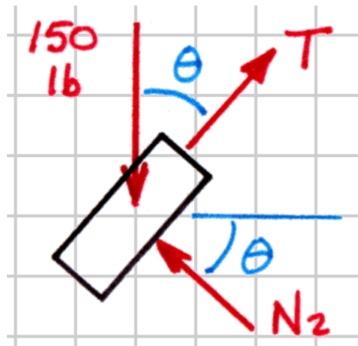
1-30 (cont.)

The member forces are

AB:	2350 N (C)	*DE*:	0 N	**Ans.**
AC:	5890 N (C)	*DF*:	5890 N (C)	**Ans.**
AH:	3530 N (T)	*DG*:	0 N	**Ans.**
BC:	0 N	*EF*:	2350 N (C)	**Ans.**
CD:	3530 N (C)	*FG*:	3530 N (T)	**Ans.**
CG:	0 N	*GH*:	3530 N (T)	**Ans.**
CH:	0 N			**Ans.**

1-31

The equations of equilibrium for the two blocks are



$$\rightarrow \Sigma F_x = 0: \qquad T\sin\theta - N_2\cos\theta = 0$$

$$-T\cos\theta + N_1\sin\theta = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T\cos\theta + N_2\sin\theta - 150 = 0$$

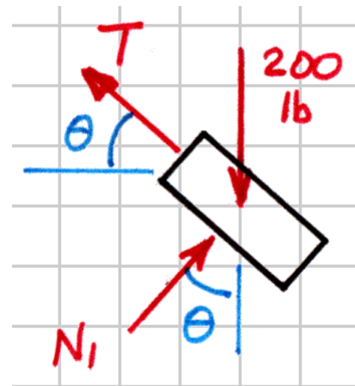
$$T\sin\theta + N_1\cos\theta - 200 = 0$$

Adding the second and third equation together gives

$$N_1\sin\theta + N_2\sin\theta = 150$$

while subtracting the first equation from the last equation gives

$$N_1\cos\theta + N_2\cos\theta = 200$$



Dividing these two equations gives

$$\frac{(N_1+N_2)\sin\theta}{(N_1+N_2)\cos\theta} = \tan\theta = \frac{150}{200}$$

$$\theta = 36.87°$$

$$N_1 + N_2 = 250 \text{ lb}$$

Now the first two equations can be rewritten

$$T\sin^2\theta - N_2\sin\theta\cos\theta = 0$$

$$-T\cos^2\theta + N_1\sin\theta\cos\theta = 0$$

and subtracting the second equation from the first gives

$$T\left(\sin^2\theta + \cos^2\theta\right) = (N_1+N_2)\sin\theta\cos\theta$$

$$T(1) = (250)(0.6000)(0.8000)$$

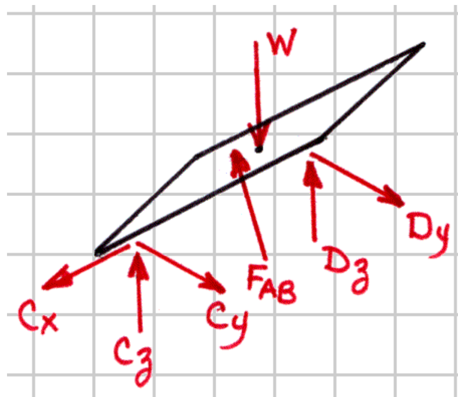
$$T = 120 \text{ lb}$$

(a) $N_1 = 160$ lb .... $N_2 = 90$ lb .... **Ans.**

(b) $T = 120$ lb .... **Ans.**

(c) $\theta = 36.87°$ .... **Ans.**

1-32*



ABC is an equilateral triangle with a height of $h = 950 \sin 60° = 822.724$ mm, and

$$\mathbf{W} = -60(9.81)\mathbf{k} = -588.60\mathbf{k} \text{ N}$$

$$\mathbf{F}_{AB} = F_{AB}\frac{-475\mathbf{j} + 822.724\mathbf{k}}{950}$$
$$= -0.50000 F_{AB}\mathbf{j} + 0.86603 F_{AB}\mathbf{k}$$

Moment equilibrium about *C*

$$\Sigma\mathbf{M}_C = \mathbf{0}: \quad (-0.95\mathbf{i}) \times (B_y\mathbf{j} + B_z\mathbf{k})$$
$$(0.95\mathbf{j}) \times (-0.50000 F_{AB}\mathbf{j} + 0.86603 F_{AB}\mathbf{k})$$
$$+(-0.475\mathbf{i} + 0.23750\mathbf{j} + 0.411362\mathbf{k}) \times (-588.6\mathbf{k}) = \mathbf{0}$$

has components

x: $\quad 0.82272 F_{AB} - 139.7925 = 0$

y: $\quad 0.95 B_z - 279.585 = 0$

z: $\quad 0 - 0.95 B_y = 0$

$F_{AB} = 169.915 \text{ N} \cong 169.9 \text{ N (C)}$ .... **Ans.**

$B_y = 0 \text{ N}$ .... **Ans.**

$B_z = 294.300 \text{ N} \cong 294 \text{ N}$ .... **Ans.**

Then the *x*-, *y*-, and *z*-components of the force equilibrium equation give

x: $\quad C_x = 0$

y: $\quad C_y + (0) - 0.5(169.915) = 0$

z: $\quad 294.300 + C_z - 588.6 + 0.86603(169.915) = 0$

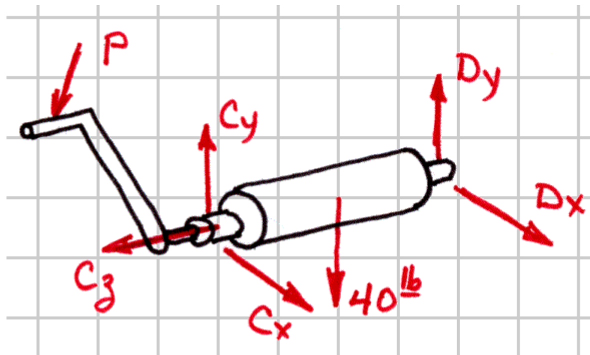
$C_x = 0 \text{ N}$ .... **Ans.**

$C_y = 84.958 \text{ N} \cong 85.0 \text{ N}$ .... **Ans.**

$C_z = 147.15 \text{ N} \cong 147.2 \text{ N}$ .... **Ans.**

1-33

$\mathbf{P} = -P\mathbf{j}$



Moment equilibrium about C

$\Sigma \mathbf{M}_C = \mathbf{0}:$

$$(-50\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j})$$
$$+(-20\mathbf{k} + 2.5\mathbf{i}) \times (-40\mathbf{j})$$
$$+(11\mathbf{k} - 17.32051\mathbf{i} + 10\mathbf{j}) \times (-P\mathbf{j}) = \mathbf{0}$$

has components

$x: \quad +50D_y - 800 + 11P = 0$

$y: \quad -50D_x = 0$

$z: \quad -100 + 17.32051P = 0$

$P = 5.77350 \text{ lb} \cong 5.77 \text{ lb}$ .... **Ans.**

$D_x = 0 \text{ lb}$ .... **Ans.**

$D_y = 14.7298 \text{ lb} \cong 14.73 \text{ lb}$ .... **Ans.**

Then the *x*-, *y*-, and *z*-components of the force equilibrium equation give

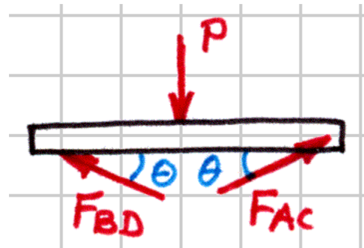
$x: \quad C_x + 0 = 0$ $\qquad C_x = 0 \text{ lb}$ .... **Ans.**

$y: \quad C_y + 14.7298 - 40 - 5.77350 = 0$ $\qquad C_y = 31.044 \text{ lb} \cong 31.0 \text{ lb}$ .... **Ans.**

$z: \quad C_z = 0$ $\qquad C_z = 0 \text{ lb}$ .... **Ans.**

1-34*

From a free-body diagram of the platform the equilibrium equations give



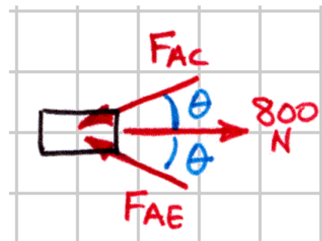
$$\rightarrow \Sigma F_x = 0: \qquad F_{AC}\cos\theta - F_{BD}\cos\theta = 0$$

$$\uparrow \Sigma F_y = 0: \qquad F_{AC}\sin\theta + F_{BD}\sin\theta - P = 0$$

$$F_{BD} = F_{AC}$$

$$P = 2F_{AC}\sin\theta$$

Then from a free-body diagram of the screw-block *A*, the equilibrium equations give



$$\rightarrow \Sigma F_x = 0: \quad 800 - F_{AC}\cos\theta - F_{AE}\cos\theta = 0$$

$$\uparrow \Sigma F_y = 0: \qquad F_{AE}\sin\theta - F_{AC}\sin\theta = 0$$

$$F_{AE} = F_{AC} = \frac{400}{\cos\theta}$$

Therefore

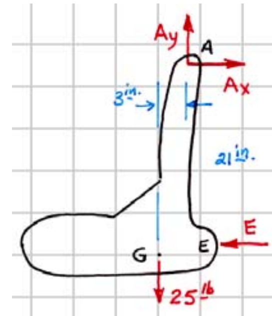
$\theta = 15°$ $\quad F_{AC} = 414.110\text{ N}$ $\quad P = 214\text{ N}$ .... **Ans.**

$\theta = 30°$ $\quad F_{AC} = 461.880\text{ N}$ $\quad P = 462\text{ N}$ .... **Ans.**

$\theta = 45°$ $\quad F_{AC} = 565.685\text{ N}$ $\quad P = 800\text{ N}$ .... **Ans.**

1-35*

From an overall free-body diagram, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad A_x - E = 0$

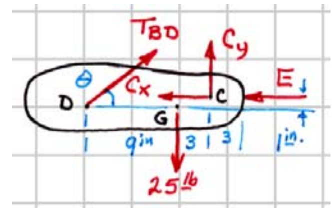
$\uparrow \Sigma F_y = 0: \qquad A_y - 25 = 0$

$\circlearrowleft \Sigma M_A = 0: \qquad 3(25) - 21E = 0$

$$A_x = E = 3.57143 \text{ lb}$$

$$A_y = 25 \text{ lb}$$

Assume that the weight of the seat back is very small compared to the weight of the seat. Then the center of gravity of the seat and the center of gravity of the entire chair are the same point. A free-body diagram of the seat gives



$\rightarrow \Sigma F_x = 0: \qquad T_{BD} \cos\theta - C_x = 0$

$\uparrow \Sigma F_y = 0: \qquad C_y + T_{BD} \sin\theta - 25 = 0$

$\circlearrowleft \Sigma M_C = 0: \qquad 3(25) - 12(T_{BD} \sin\theta) + 1(T_{BD} \cos\theta) = 0$

in which $\theta = \tan^{-1}\dfrac{10}{12} = 39.806°$

$T_{BD} = 10.8476 \text{ lb} \qquad C_x = 4.76188 \text{ lb} \qquad C_y = 18.0555 \text{ lb}$

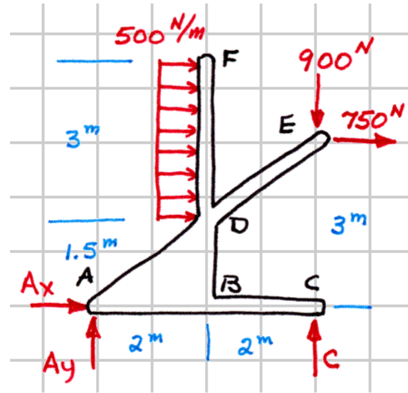
A = 25.3 lb 81.87° .... **Ans.**

$\mathbf{T}_{BD}$ = 10.85 lb 39.81° .... **Ans.**

C = 18.67 lb 75.23° .... **Ans.**

1-36

From an overall free-body diagram, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad A_x + (500 \times 3) + 750 = 0$$

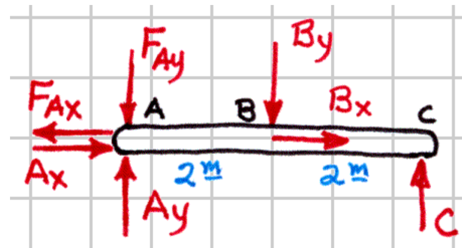
$$\uparrow \Sigma F_y = 0: \qquad A_y - 900 + C = 0$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 4C - (3 \times 500)(3) - 4(900) - 3(750) = 0$$

$$C = 2587.50 \text{ N}$$

$$A_x = -2250 \text{ N} \qquad A_y = -1687.50 \text{ N}$$

Next, draw a free-body diagram of the bar *ABC*. Note that the forces A_x and A_y are the forces exerted on bar *ABC* by the support (the same forces that are shown on the overall free-body diagram) and that the forces F_{Ax} and F_{Ay} are the forces exerted on bar *ABC* by the bar *ADE*. The equations of equilibrium give



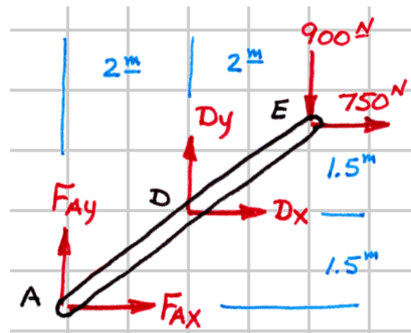
$$\rightarrow \Sigma F_x = 0: \qquad A_x + B_x - F_{Ax} = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y - F_{Ay} - B_y + C = 0$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 4(2587.50) - 2B_y = 0$$

$$B_y = 5175.00 \text{ N} \qquad F_{Ay} = -4275.00 \text{ N}$$

Finally, from a free-body diagram of the bar *ADE*



$$\rightarrow \Sigma F_x = 0: \qquad F_{Ax} + D_x + 750 = 0$$

$$\uparrow \Sigma F_y = 0: \qquad F_{Ay} + D_y - 900 = 0$$

$$\circlearrowleft \Sigma M_D = 0: \qquad 1.5F_{Ax} - 2(-4275) - 2(900) - 1.5(750) = 0$$

$$F_{Ax} = -3750 \text{ N}$$

$$D_x = 3000 \text{ N} \qquad D_y = 5175 \text{ N}$$

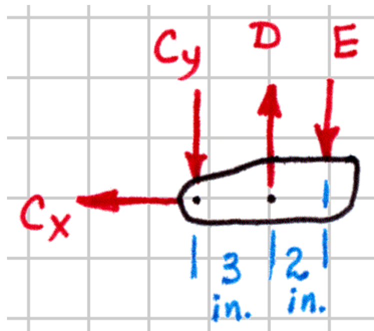
$\mathbf{A} = 5690$ N 48.7° .... **Ans.**

$\mathbf{D} = 5980$ N 59.9° .... **Ans.**

$\mathbf{E} = 1170$ N 50.2° .... **Ans.**

1-37

First draw a free-body diagram of the lower jaw, and write the equations of equilibrium



$\rightarrow \Sigma F_x = 0: \quad -C_x = 0$

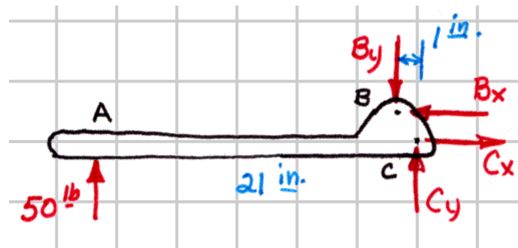
$\uparrow \Sigma F_y = 0: \quad D - C_y - E = 0$

$\circlearrowleft \Sigma M_D = 0: \quad 3C_y - 2E = 0$

$C_x = 0$

$E = 1.5C_y$

Next, from a free-body diagram of the lower handle, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \quad 0 - B_x = 0$

$\uparrow \Sigma F_y = 0: \quad 50 + C_y - B_y = 0$

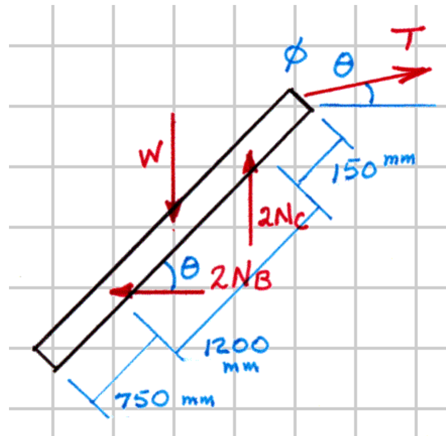
$\circlearrowleft \Sigma M_B = 0: \quad 1C_y - 20(50) = 0$

$C_y = 1000 \text{ lb}$

$B_x = 0 \text{ lb}$ $\quad B_y = 1050 \text{ lb}$

$E = 1500 \text{ lb}$ .... **Ans.**

1-38*



$$W = 100(9.81) = 981 \text{ N}$$

$$\cos\theta = \frac{2500 - 1875}{1200} \qquad \theta = 58.612°$$

$$\tan\phi = \frac{300 - 150\sin\theta}{1875 - 150\cos\theta} \qquad \phi = 5.466°$$

There are two rollers (one on each side of the door) at *B* and *D*, hence the $2N_B$ and $2N_C$ on the free-body diagram

$\rightarrow \Sigma F_x = 0: \qquad T\cos\phi - 2N_B = 0$

$\uparrow \Sigma F_y = 0: \qquad T\sin\phi + 2N_C - 981 = 0$

$\circlearrowleft \Sigma M_D = 0: \quad (1050\cos\theta)(981) - (150\cos\theta)(2N_C) - (1350\sin\theta)(2N_B) = 0$

Substituting the first two equations into the third gives

$$(1050\cos\theta)(981) - (150\cos\theta)(981 - T\sin\phi) - (1350\sin\theta)(T\cos\phi) = 0$$

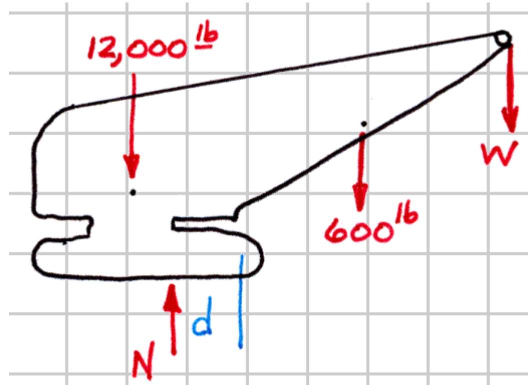
$T = 403.455 \text{ N} \cong 403 \text{ N}$ .... **Ans.**

$N_B = 200.812 \text{ N} \cong 201 \text{ N}$ .... **Ans.**

$N_C = 471.283 \text{ N} \cong 471 \text{ N}$ .... **Ans.**

1-39*

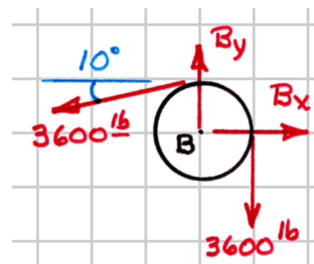
(a) First draw an overall free-body diagram. The force of the ground on the track of the crane is equivalent to a single concentrated force N acting at some location on the treads. As the load increases, the distance d gets smaller and smaller. The maximum load that the crane can lift corresponds to $d = 0$. Then, summing moments about the point where the normal force acts gives



$$9(12,000) - (12\cos 30° - 1)(600) - (24\cos 30° - 1 + 1)(W) = 0$$

$W = 4930$ lb .... **Ans.**

(b) Next, from a free-body diagram of the pulley at the end of the boom the equations of equilibrium give



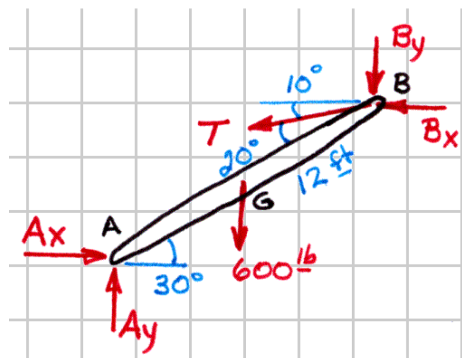
$\rightarrow \Sigma F_x = 0$: $\quad B_x - 3600\cos 10° = 0$

$\uparrow \Sigma F_y = 0$: $\quad B_y - 3600 - 3600\sin 10° = 0$

$B_x = 3545.3079$ lb

$B_y = 4225.1334$ lb

Finally, from a free-body diagram of the boom the equations of equilibrium give



$\rightarrow \Sigma F_x = 0$: $\quad A_x - (3545.3079) - T\cos 10° = 0$

$\uparrow \Sigma F_y = 0$: $\quad A_y - 600 - T\sin 10° - (4225.1334) = 0$

$\circlearrowleft \Sigma M_A = 0$:

$$24(T\sin 20°) + (24\sin 30°)(3545.3079) - (12\cos 30°)(600) - (24\cos 30°)(4225.1334) = 0$$

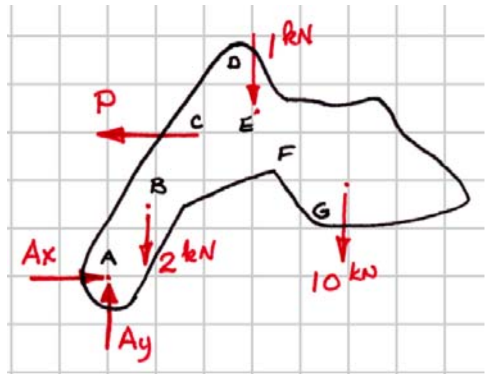
$T = 6275.1466 \text{ lb} \cong 6280$ lb .... **Ans.**

(c) $A_x = 9725.1209$ lb $\qquad A_y = 5914.8012$ lb

$\mathbf{A} = 11,380$ lb ∡ 31.3° .... **Ans.**

1-40

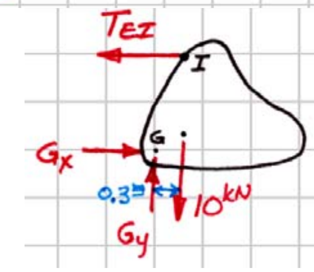
From an overall free-body diagram, the equations of equilibrium give



$$\circlearrowleft \Sigma M_A = 0: \quad (1.8\cos 30°)P - (0.9\sin 30°)(2)$$
$$-(2.7\sin 30° + 0.6\sin 30°)(1)$$
$$-(2.7\sin 30° + 1.8\sin 30° + 0.3)(10) = 0$$

$P = 17.99$ kN .... **Ans.**

Next, from a free-body diagram of the bucket, the equations of equilibrium give

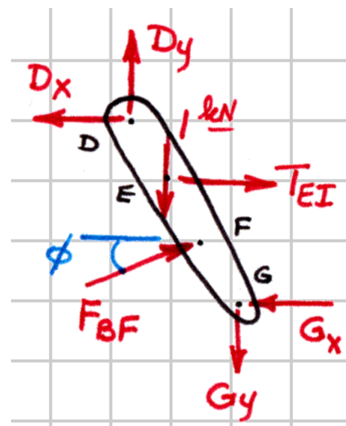


$$\rightarrow \Sigma F_x = 0: \qquad G_x - T_{EI} = 0$$
$$\uparrow \Sigma F_y = 0: \qquad G_y - 10 = 0$$
$$\circlearrowleft \Sigma M_G = 0: \quad (1.2\cos 30°)T_{EI} - 0.3(10) = 0$$

$T_{EI} = 2.88675 \text{ kN} \cong 2.89 \text{ kN (T)}$ .... **Ans.**

$G_x = 2.88675$ kN $\qquad G_y = 10.00$ kN

Finally, from a free-body diagram of the arm *DEFG*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad (2.88675) - D_x - (2.88675) + F_{BF}\cos\phi = 0$$
$$\uparrow \Sigma F_y = 0: \qquad D_y - (10) - 1 + F_{BF}\sin\phi = 0$$
$$\circlearrowleft \Sigma M_D = 0: \quad (0.6\cos 30°)(2.88675) - (0.6\sin 30°)(1)$$
$$+(1.2\cos 30°)(F_{BF}\cos\phi) + (1.2\sin 30°)(F_{BF}\sin\phi)$$
$$-(1.8\cos 30°)(2.88675) - (1.8\sin 30°)(10) = 0$$

in which

$$\tan\phi = \frac{1.8\cos 30° - 1.2\cos 30°}{1.8\sin 30° + 1.2\sin 30°} \qquad \phi = 19.107°$$

$F_{BF} = 10.43807 \text{ kN} \cong 10.44 \text{ kN (C)}$ .... **Ans.**

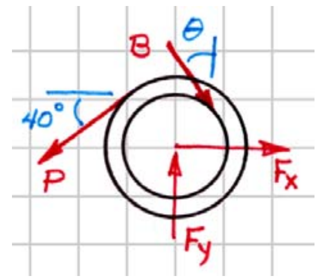
$D_x = 9.86303$ kN $\qquad D_y = 7.58627$ kN

D $= 12.44$ kN ∠ 37.6° .... **Ans.**

G $= 10.41$ kN ∠ 73.9° .... **Ans.**

1-41

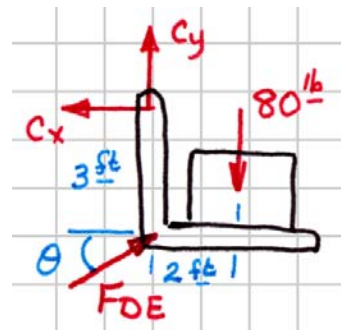
First draw a free-body diagram of the wheel. It is stated that the pin *B* is at or near the surface of the wheel. Then, the equations of equilibrium give



$$\circlearrowleft \Sigma M_{axle} = 0: \qquad 2P - 2B = 0$$

$$P = B$$

Next, from a free-body diagram of the platform, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad F_{DE}\cos\theta - C_x = 0$$

$$\uparrow \Sigma F_y = 0: \qquad C_y + F_{DE}\sin\theta - 80 = 0$$

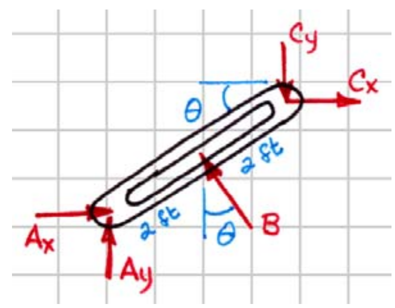
$$\circlearrowleft \Sigma M_C = 0: \qquad 3\left(F_{DE}\cos\theta\right) - 2(80) = 0$$

$$\theta = \sin^{-1}\frac{2}{4} = 30°$$

$$F_{DE} = 61.5840 \text{ lb}$$

$$C_x = 53.3333 \text{ lb} \qquad C_y = 49.2080 \text{ lb}$$

Finally, from a free-body diagram of the arm *ABC*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad A_x + (53.3333) - B\sin\theta = 0 = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y + B\cos\theta - (49.2080) = 0$$

$$\circlearrowleft \Sigma M_A = 0: \quad 2B - (4\cos\theta)(49.2080) - (4\sin\theta)(53.3333) = 0$$

$$B = 138.5641 \text{ lb}$$

$$A_x = 15.9487 \text{ lb} \qquad A_y = 70.7920 \text{ lb}$$

A = 72.6 lb ∠ 77.3° .... **Ans.**

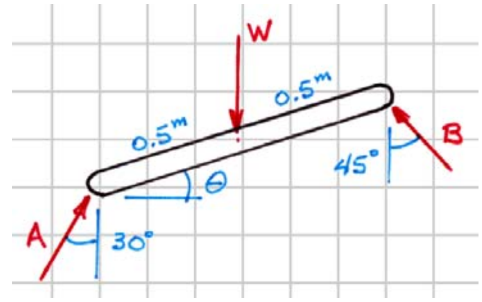
B = 138.6 lb ∠ 60° .... **Ans.**

C = 72.6 lb ∠ 42.7° .... **Ans.**

1-42*

The weight of the bar *AB* is

$$W = 25(9.81) = 245.25 \text{ N}$$



From a free-body diagram of the bar *AB*, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad A\sin 30° - B\sin 45° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A\cos 30° + B\cos 45° - W = 0$$

$$\circlearrowleft \Sigma M_B = 0: \qquad (0.5\cos\theta)W - (1\cos\theta)(A\cos 30°) + (1\sin\theta)(A\sin 30°) = 0$$

Since $\sin 45° = \cos 45°$ adding the first two equations together gives

$$A(\sin 30° + \cos 30°) = W$$

Substituting this result into the third equation gives

$$A(\cos 30° \cos\theta - \sin 30° \sin\theta) = 0.5\cos\theta\left[A(\sin 30° + \cos 30°)\right]$$

Dividing by $A\cos\theta$ gives

$$2(\cos 30° - \sin 30° \tan\theta) = (\sin 30° + \cos 30°)$$

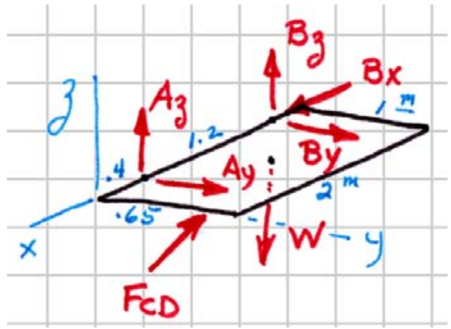
$$\tan\theta = \frac{2\cos 30° - (\sin 30° + \cos 30°)}{2\sin 30°}$$

$\theta = 20.10°$ .... **Ans.**

(Actually, this result is independent of both the length and weight of the bar.)

1-43

$$W = 25(9.81) = 245.25 \text{ N}$$



$$\mathbf{F}_{CD} = F_{CD}\frac{0.65\mathbf{j} + 0.95\mathbf{k}}{\sqrt{0.65^2 + 0.95^2}}$$

$$= 0.56468F_{CD}\mathbf{j} + 0.82531F_{CD}\mathbf{k}$$

Moment equilibrium about B

$\Sigma\mathbf{M}_B = \mathbf{0}$:

$$(1.2\mathbf{i})\times(A_y\mathbf{j} + A_z\mathbf{k}) + (0.6\mathbf{i} + 0.5\mathbf{j})\times(-245.25\mathbf{k})$$

$$+(1.6\mathbf{i} + 0.65\mathbf{j})\times(0.56468F_{CD}\mathbf{j} + 0.82531F_{CD}\mathbf{k}) = \mathbf{0}$$

has components

x: $-122.625 + 0.53645F_{CD} = 0$

y: $-1.2A_z + 147.150 - 1.32050F_{CD} = 0$

z: $1.2A_y + 0.90349F_{CD} = 0$

$F_{CD} = 228.586 \text{ N} \cong 229 \text{ N}$ .... **Ans.**

$A_y = -172.104 \text{ N} \cong -172.1 \text{ N}$ .... **Ans.**

$A_z = -129.915 \text{ N} \cong 129.9 \text{ N}$ .... **Ans.**

Then the x-, y-, and z-components of the force equilibrium equation give

x: $B_x = 0$

y: $(-172.104) + B_y + 0.56468(228.586) = 0$

z: $(-129.915) + B_z + 0.82531(228.586) - 245.25 = 0$

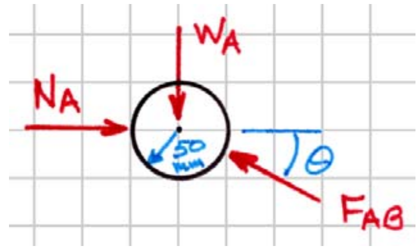
$B_x = 0 \text{ N}$ .... **Ans.**

$B_y = 43.026 \text{ N} \cong 43.0 \text{ N}$ .... **Ans.**

$B_z = 185.511 \text{ N} \cong 185.5 \text{ N}$ .... **Ans.**

1-44

From a free-body diagram of pipe *A*, the equations of equilibrium are

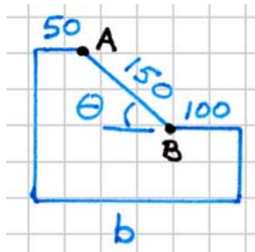


$\rightarrow \Sigma F_x = 0: \qquad N_A - F_{AB}\sin\theta = 0$

$\uparrow \Sigma F_y = 0: \qquad F_{AB}\cos\theta - 5(9.81) = 0$

where

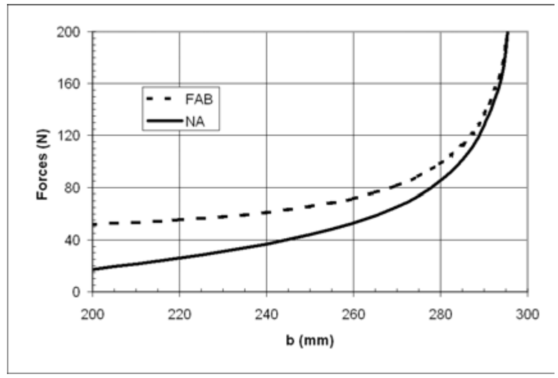
$$\theta = \sin^{-1}\frac{b-150}{150}$$



Therefore

$$F_{AB} = \frac{49.05}{\cos\theta}\ \text{N}$$

$$N_A = F_{AB}\sin\theta\ \text{N}$$



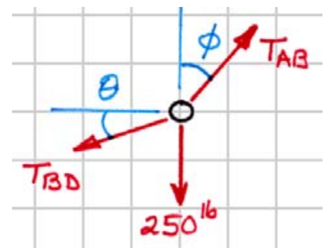
(a) $N_A < 50\ \text{N}$ for $b < 260\ \text{mm}$ ....**Ans.**

(b) $F_{AB} < 100\ \text{N}$ for $b < 280\ \text{mm}$ ....**Ans.**

(c) $F_{AB} < 200\ \text{N}$ for $b < 295\ \text{mm}$ ....**Ans.**

1-45

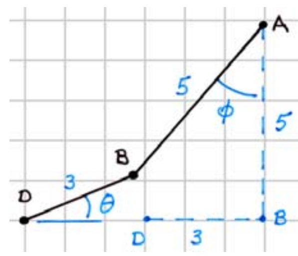
From a free-body diagram of the ring, the equations of equilibrium are



$\rightarrow \Sigma F_x = 0: \qquad T_{AB} \sin\phi - T_{BD} \cos\theta = 0$

$\uparrow \Sigma F_y = 0: \qquad T_{AB} \cos\phi - T_{BD} \sin\theta - 250 = 0$

where

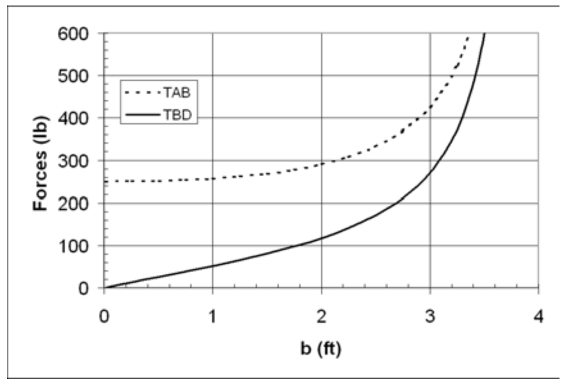


$$\phi = \sin^{-1}\frac{b}{5}$$

$$\theta = \sin^{-1}\frac{5(1-\cos\phi)}{3}$$

Therefore

$$T_{AB} = \frac{T_{BD}\cos\theta}{\sin\phi}$$



$$\mathbf{T}_{BD} = \frac{250\sin\phi}{\cos\theta\cos\phi - \sin\theta\sin\phi} \text{ lb}$$

$$T_{AB} = \frac{250\cos\theta}{\cos\theta\cos\phi - \sin\theta\sin\phi} \text{ lb}$$

(a) b_{max} occurs when T_{BD} goes negative (after it goes to infinity);

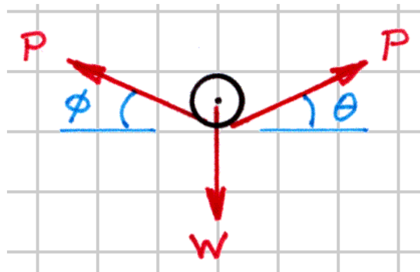
$b_{max} \cong 3.90$ ft .... **Ans.**

(At this point, the rope will straight from *D* to *B* to *A*.)

(c) To pull further to the side, the worker needs a longer rope to pull on or he needs to attach his rope lower - closer to the crate.......................... **Ans.**

1-46

$$W = 50(9.81) = 490.50 \text{ N}$$



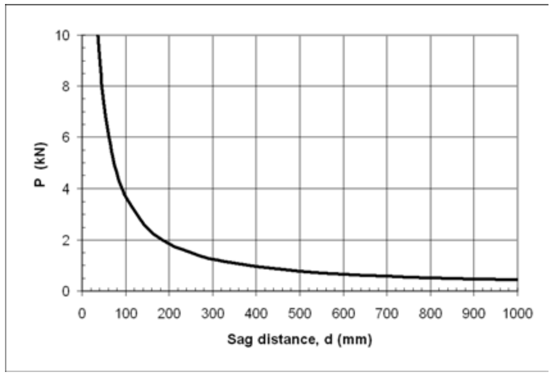
The cable is continuous, therefore the tension in the cable is continuous (equal to the force P); and from a free-body diagram of the pulley, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad P\cos\theta - P\cos\phi = 0$$

$$\uparrow \Sigma F_y = 0: \qquad P\sin\theta + P\sin\phi - W = 0$$

$$\phi = \theta = \tan^{-1}\frac{d}{1.5}$$

$$P = \frac{W}{2\sin\theta}$$



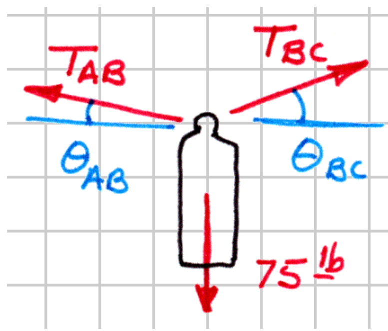
(a) $P < 2W$ $\quad d > 387$ mm .... **Ans.**

(b) $P < 4W$ $\quad d > 189$ mm .... **Ans.**

(c) $P < 8W$ $\quad d > 94$ mm .... **Ans.**

1-47

$$\theta_{AB} = \tan^{-1}\frac{d}{20} \qquad \theta_{BC} = \tan^{-1}\frac{d}{10}$$



From a free-body diagram of the stop light, the equations of equilibrium give

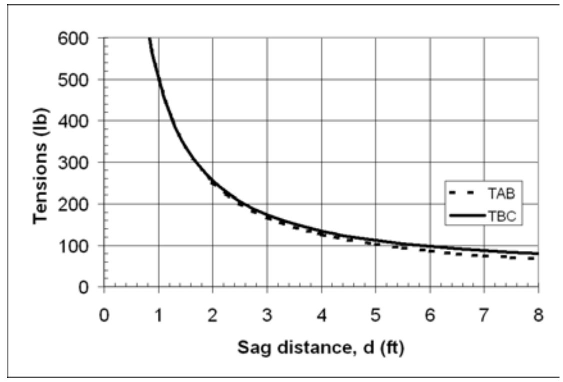
$$\rightarrow \Sigma F_x = 0: \qquad T_{BC}\cos\theta_{BC} - T_{AB}\cos\theta_{AB} = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{AB}\sin\theta_{AB} + T_{BC}\sin\theta_{BC} - 75 = 0$$

Solving yields

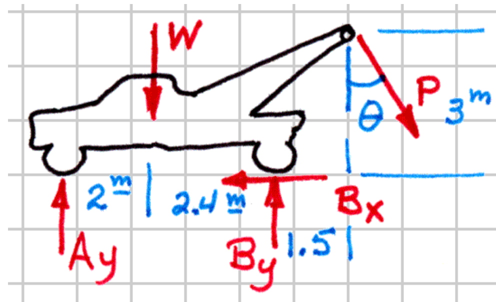
$$T_{AB} = \frac{75\cos\theta_{BC}}{\sin\theta_{BC}\cos\theta_{AB} + \cos\theta_{BC}\sin\theta_{AB}}$$

$$T_{BC} = \frac{75\cos\theta_{AB}}{\sin\theta_{BC}\cos\theta_{AB} + \cos\theta_{BC}\sin\theta_{AB}}$$



1-48

$$W = 6800(9.81) = 66,708 \text{ N}$$



From a free-body diagram of the truck, the equilibrium equations are

$$\rightarrow \Sigma F_x = 0: \qquad P\sin\theta - B_x = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y + B_y - 66,708 - P\cos\theta = 0$$

$$\circlearrowleft \Sigma M_B = 0: \qquad (66,708)(2.4) - 4.4A_y - (P\cos\theta)(1.5) - (P\sin\theta)(3) = 0$$

The fourth equation needed to solve for the four unknowns is either $A_y = 0$ (the front wheels are on the verge of lifting off the ground) or $B_x = 0.8B_y$ (the rear wheels are on the verge of slipping). Guessing that the front wheels are on the verge of lifting off the ground gives the solution

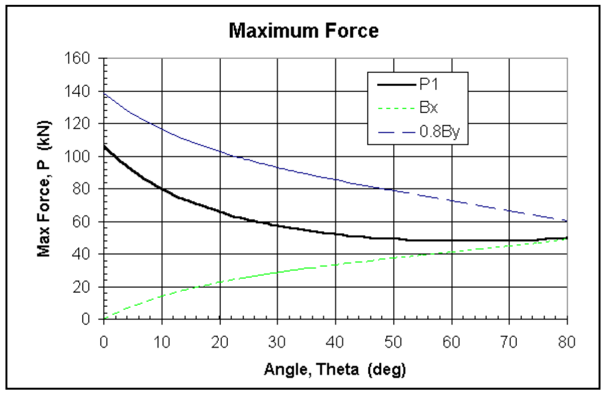
$$A_y = 0 \text{ N}$$

$$P = \frac{66,708(2.4)}{1.5\cos\theta + 3\sin\theta} \text{ N}$$

$$B_x = P\sin\theta \text{ N}$$

$$B_y = 66,708 + P\cos\theta \text{ N}$$

The forces B_x and $0.8B_y$ are plotted on the same graph as the force P. Since B_x is always less than $0.8B_y$, the guess that the front wheels are on the verge of lifting off the ground was the correct guess, and the solution is valid for all values of θ.



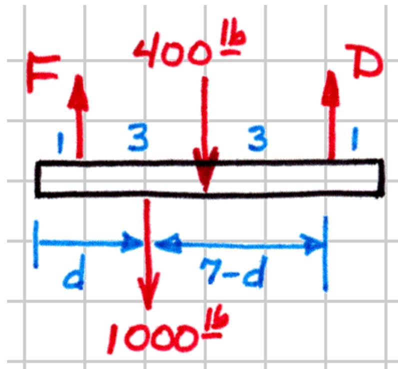
$E = 1500$ lb .... **Ans.**

$\mathbf{T}_{BD} = 10.85$ lb ↘ 39.81° .... **Ans.**

$\mathbf{C} = 18.67$ lb ↙ 75.23° .... **Ans.**

1-49

From a free-body diagram of the I-beam, the equations of equilibrium give



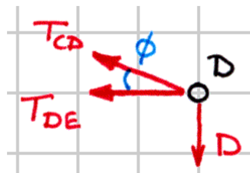
$$\circlearrowleft \Sigma M_D = 0: \qquad 400(3) + 1000(7-d) + 6F = 0$$

$$\circlearrowleft \Sigma M_F = 0: \qquad 6D - 1000(d-1) - 400(3) = 0$$

$$D = \frac{1000(d-1)+1200}{6}$$

$$F = \frac{1000(7-d)+1200}{6}$$

Next, from a free-body diagram of the joint *D*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad -T_{CD}\cos\phi - T_{DE} = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{CD}\sin\phi - D = 0$$

$$T_{CD} = \frac{D}{\sin\phi} \qquad\qquad T_{DE} = \frac{-D}{\tan\phi}$$

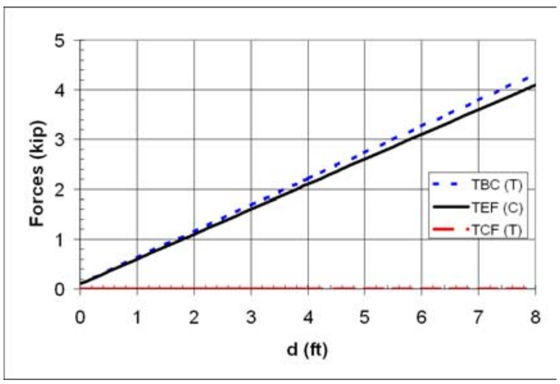
in which

$$\phi = \tan^{-1}\frac{1}{3} = 18.435°$$

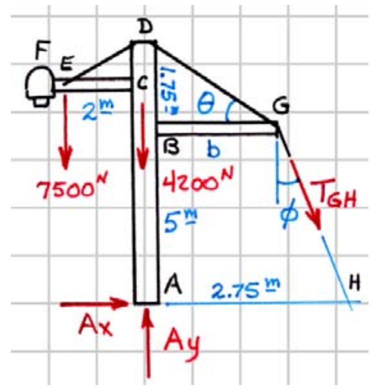
By inspection, members *CE* and *CF* are both zero-force members. Therefore the tension force in member *BC* will be the same as the tension force in member *CD* and the compression force in member *EF* will be the same as the compression force in member *DE*,

$$T_{BC} = \frac{D}{\sin\phi} \qquad\qquad T_{CF} = 0$$

$$T_{EF} = \frac{-D}{\tan\phi}$$



1-50



From an overall free-body diagram of the light pole,

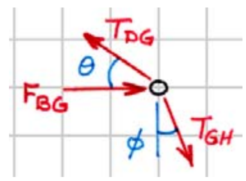
$$\theta = \tan^{-1}\frac{1.75}{b} \qquad \phi = \tan^{-1}\frac{2.75-b}{5}$$

the moment equation of equilibrium gives

$$\circlearrowleft \Sigma M_A = 0: \qquad 2(7500) - 2.75(T_{GH}\cos\theta) = 0$$

$$T_{GH} = \frac{2(7500)}{2.75\cos\theta}$$

It will be assumed that the cable *DG* supports the end of the arm *BG* and that the connection of the horizontal arm *BG* to the vertical pole *ABCD* exerts negligible moment on the arm. (If the connection could provide a sufficient moment, then the cable between *D* and *G* would not be necessary.)



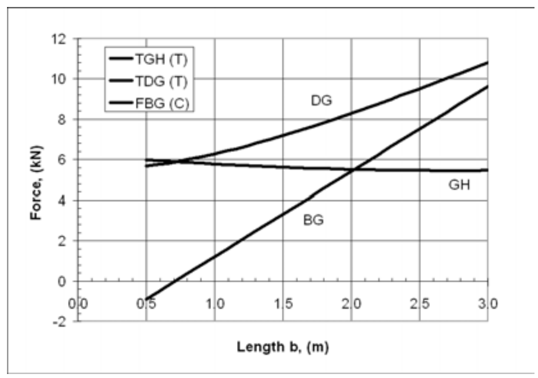
Then, from a free-body diagram of the pin *G*, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad F_{BG} + T_{GH}\sin\phi - T_{DG}\cos\theta = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{DG}\sin\theta - T_{GH}\cos\phi = 0$$

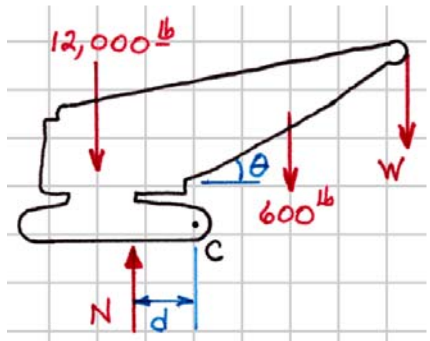
$$T_{DG} = \frac{T_{GH}\cos\phi}{\sin\theta}$$

$$F_{BG} = T_{GH}\,\frac{\cos\phi\cos\theta - \sin\phi\sin\theta}{\sin\theta}$$



1-51

From an overall free-body diagram of the crane, the equations of equilibrium give



$$\uparrow \Sigma F_y = 0: \qquad N - 12{,}000 - 600 - W = 0$$

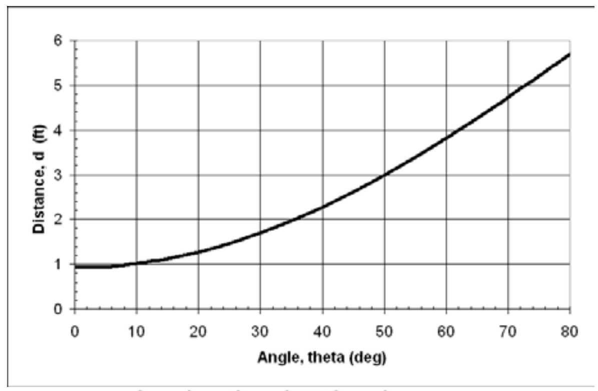
$$\circlearrowleft \Sigma M_C = 0: \qquad (9)(12{,}000) - (12\cos\theta - 1)(600) - (24\cos\theta - 1 + 1)W - Nd = 0$$

$$N = (12{,}600 + W)\ \text{lb}$$

$$d = \frac{108{,}600 - (7200 + 24W)\cos\theta}{12{,}600 + W}\ \text{ft}$$

(a) For $W = 3600$ lb

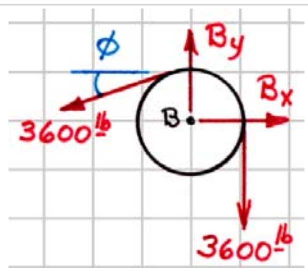
$$d = \frac{108{,}600 - 93{,}600\cos\theta}{16{,}200}\ \text{ft}$$



(b) From a free-body diagram of the pulley at *B*,

$$\tan\phi = \frac{24\sin\theta - 6}{24\cos\theta + 9}$$

and the equations of equilibrium give

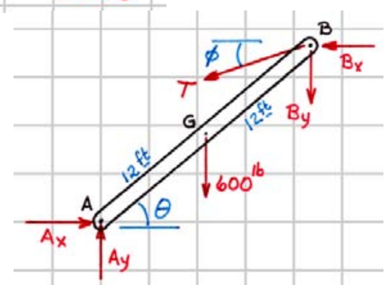


$$\rightarrow \Sigma F_x = 0: \qquad B_x - 3600\cos\phi = 0$$

$$\uparrow \Sigma F_y = 0: \qquad B_y - 3600 - 3600\sin\phi = 0$$

$$B_x = 3600\cos\phi \qquad B_y = 3600(1 + \sin\phi)$$

From a free-body diagram of the boom, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad A_x - B_x - T\cos\phi = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y - B_y - T\sin\phi - 600 = 0$$

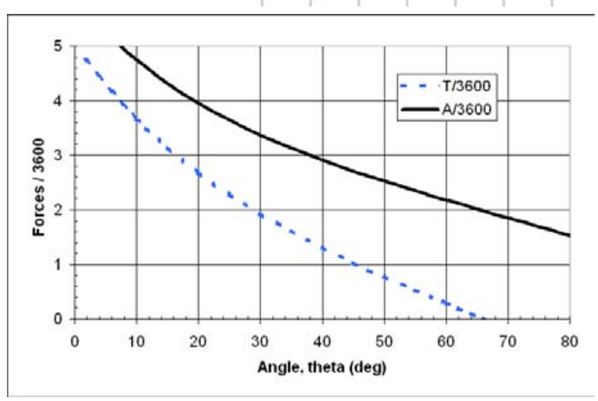
$$\circlearrowleft \Sigma M_A = 0: \qquad [24\sin(\theta - \phi)]T - (12\cos\theta)(600) + (24\sin\theta)B_x - (24\cos\theta)B_y = 0$$

$$T = \frac{(7200 + 24B_y)\cos\theta - 24B_x\sin\theta}{24\sin(\theta - \phi)}$$

$$A_x = B_x + T\cos\phi$$

$$A_y = B_y + T\sin\phi + 600$$

$$A = \sqrt{A_x^2 + A_y^2}$$



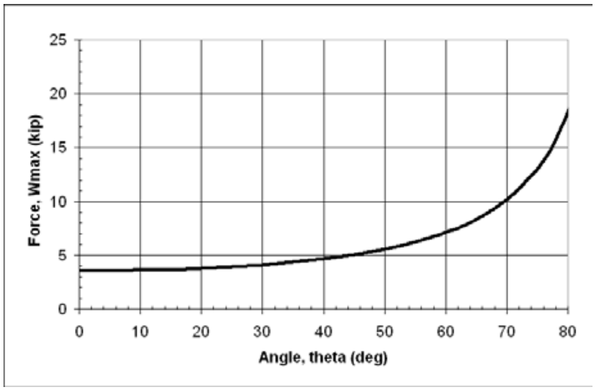
Note that the tension force becomes negative for an angle of about $66°$. Since negative forces in the cable are not possible, the boom would topple over onto the top of the cab of the crane if the operator tried to lift higher than $66°$.

1-51 (cont.)

(c) For $d = 1$ ft

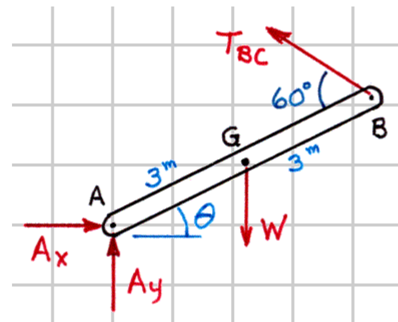
$$d = \frac{108,600 - (7200 + 24W)\cos\theta}{12,600 + W} = 1 \text{ ft}$$

$$W_{\max} = \frac{96,000 - 7200\cos\theta}{1 + 24\cos\theta} \text{ lb}$$



1-52

$$W = 250(9.81) = 2452.50\text{ N}$$

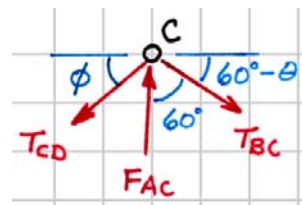


(a) From a free-body diagram of the post *AB*, moment equilibrium gives

$$\circlearrowleft \Sigma M_B = 0: \qquad 6(T_{BC}\sin 60°) - (3\cos\theta)W = 0$$

$$T_{BC} = \frac{(2452.50)(3\cos\theta)}{6\sin 60°}$$

Since the pin at *A* is frictionless and the weight of the brace *AC* is neglected, the brace *AC* is a two-force member and from a free-body diagram of the pin *C*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad F_{AC}\cos(60°+\theta) + T_{BC}\cos(60°-\theta) - T_{CD}\cos\phi = 0$$

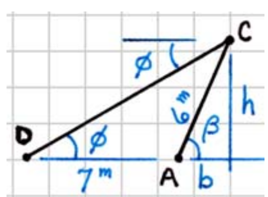
$$\uparrow \Sigma F_y = 0: \qquad F_{AC}\sin(60°+\theta) - T_{BC}\sin(60°-\theta) - T_{CD}\sin\phi = 0$$

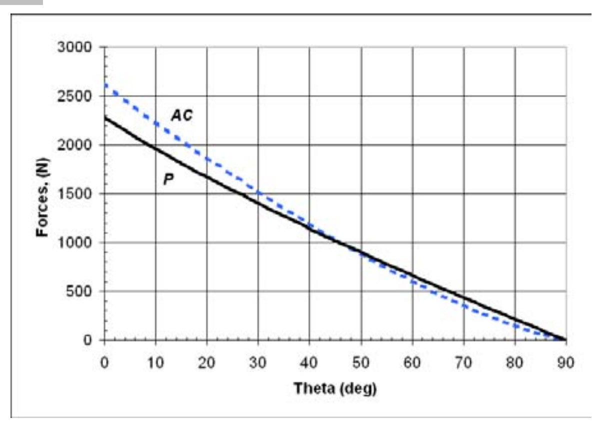
$$F_{AC} = \frac{\cos(60°-\theta)\sin\phi + \sin(60°-\theta)\cos\phi}{\sin(60°+\theta)\cos\phi - \cos(60°+\theta)\sin\phi}T_{BC}$$

$$T_{CD} = \frac{F_{AC}\cos(60°+\theta) + T_{BC}\cos(60°-\theta)}{\cos\phi} = P$$

in which

$$\tan\phi = \frac{6\sin\beta}{7+6\cos\beta} = \frac{6\sin(60°+\theta)}{7+6\cos(60°+\theta)}$$

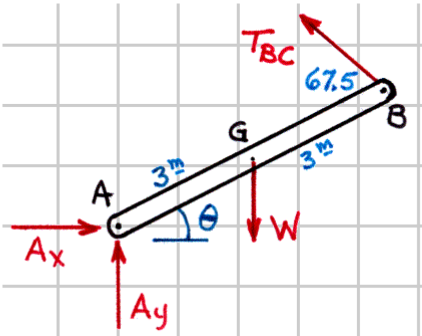




For this arrangement, the force P necessary to start raising the post (2300 N) is almost as large as the weight of the post (2450 N).

1-52 (cont.)

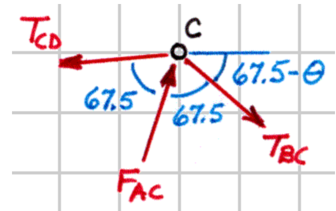
(b) From a free-body diagram of the post *AB*, moment equilibrium now gives



$$\circlearrowleft \Sigma M_B = 0: \quad 6\left(T_{BC}\sin 67.5°\right) - \left(3\cos\theta\right)W = 0$$

$$T_{BC} = \frac{(2452.50)(3\cos\theta)}{6\sin 67.5°}$$

Again, since the pin at *A* is frictionless and the weight of the brace *AC* is neglected, the brace *AC* is a two-force member and from a free-body diagram of the pin *C*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0:$$

$$F_{AC}\cos(45° + \theta) + T_{BC}\cos(67.5° - \theta) - T_{CD}\cos(22.5° - \theta) = 0$$

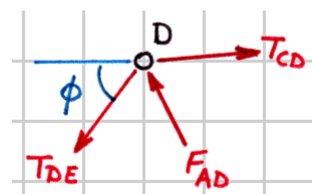
$$\uparrow \Sigma F_y = 0:$$

$$F_{AC}\sin(45° + \theta) - T_{BC}\sin(67.5° - \theta) + T_{CD}\sin(22.5° - \theta) = 0$$

$$F_{AC} = \frac{\sin(67.5° - \theta)\cos(22.5° - \theta) - \cos(67.5° - \theta)\sin(22.5° - \theta)}{\sin(45° + \theta)\cos(22.5° - \theta) + \cos(45° + \theta)\sin(22.5° - \theta)} T_{BC}$$

$$T_{CD} = \frac{F_{AC}\cos(45° + \theta) + T_{BC}\cos(67.5° - \theta)}{\cos(22.5° - \theta)}$$

Finally, since the weight of the brace *AD* is also neglected, the brace *AD* is also a two-force member and from a free-body diagram of the pin *D*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \quad -F_{AD}\cos(90° - \theta) + T_{CD}\cos(22.5° - \theta) - T_{DE}\cos\phi = 0$$

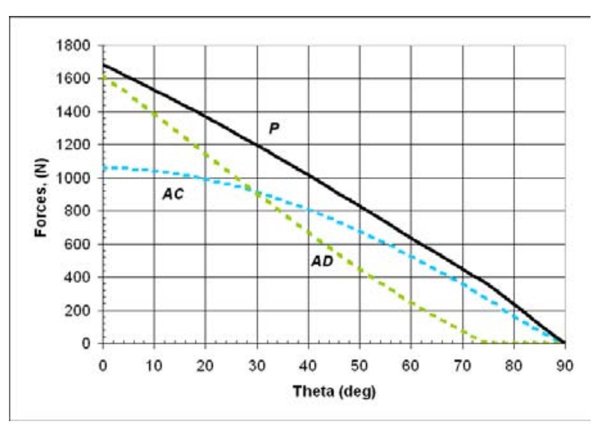
$$\uparrow \Sigma F_y = 0: \quad F_{AD}\sin(90° - \theta) - T_{CD}\sin(22.5° - \theta) - T_{DE}\sin\phi = 0$$

$$F_{AD} = \frac{\cos(22.5° - \theta)\sin\phi + \sin(22.5° - \theta)\cos\phi}{\cos(90° - \theta)\sin\phi + \sin(90° - \theta)\cos\phi} T_{CD}$$

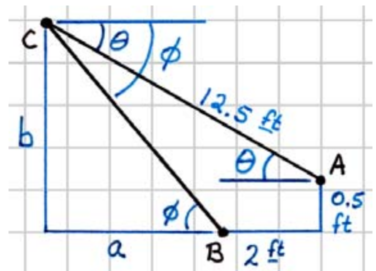
$$T_{DE} = \frac{T_{CD}\cos(22.5° - \theta) - F_{AD}\cos(90° - \theta)}{\cos\phi} = P$$

Note that the force in the brace *AD* goes to zero at about $\theta = 75°$. For $75° \le \theta \le 90°$, the solution becomes similar to that of part a (with the angle between the post and the brace *AC* $45°$ rather than $60°$).

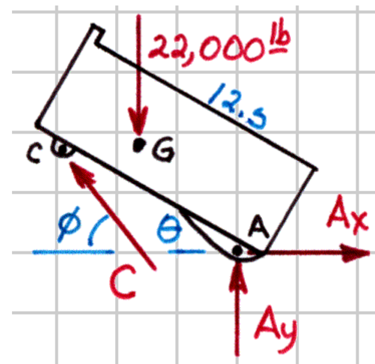
For this arrangement, the force P necessary to start raising the post (1700 N) is about 25% less than the force required using a single brace (part a).



1-53



$$a = 12.5\cos\theta - 2$$
$$b = 12.5\sin\theta + 0.5$$
$$\phi = \tan^{-1}\frac{b}{a}$$



From a free-body diagram of the truck box, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad A_x - C\cos\phi = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y + C\sin\phi - 22{,}000 = 0$$

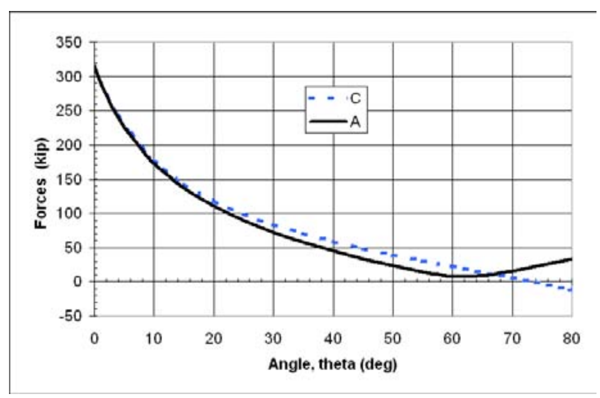
$$\circlearrowleft \Sigma M_A = 0: \qquad 8.5(22{,}000\cos\theta) - 2.5(22{,}000\sin\theta) - 12.5\left[C\sin(\phi-\theta)\right] = 0$$

$$C = \frac{187{,}000\cos\theta - 55{,}000\sin\theta}{12.5\sin(\phi-\theta)}$$

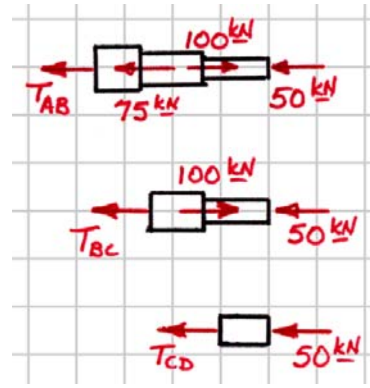
$$A_x = C\cos\phi$$

$$A_y = 22{,}000 - C\sin\phi$$

$$A = \sqrt{A_x^2 + A_y^2}$$



1-54*



$\rightarrow \Sigma F_x = 0: \qquad -T_{AB} - 75 + 100 - 50 = 0$

$T_{AB} = -25 \text{ kN} = 25 \text{ kN (C)}$ .... **Ans.**

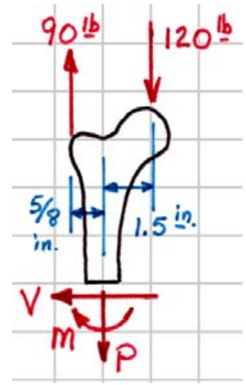
$\rightarrow \Sigma F_x = 0: \qquad -T_{BC} + 100 - 50 = 0$

$T_{BC} = +50 \text{ kN} = 50 \text{ kN (T)}$ .... **Ans.**

$\rightarrow \Sigma F_x = 0: \qquad -T_{CD} - 50 = 0$

$T_{CD} = -50 \text{ kN} = 50 \text{ kN (C)}$ .... **Ans.**

1-55*



$\rightarrow \Sigma F_x = 0: \quad -V = 0$

$\uparrow \Sigma F_y = 0: \quad 90 - 120 - P = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \quad -M - 90(5/8) - 120(1.5) = 0$

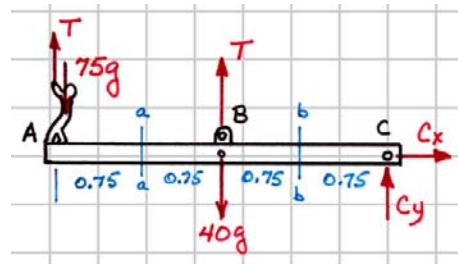
$V = 0$ lb .... **Ans.**

$P = -30$ lb $= 30$ lb (C) .... **Ans.**

$M = -236$ lb · in. .... **Ans.**

1-56

Next, from a free-body diagram of the man standing on the beam, the equations of equilibrium give



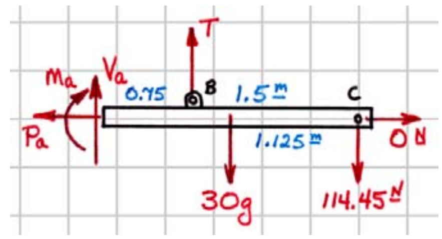
$\rightarrow \Sigma F_x = 0: \qquad C_x = 0$

$\uparrow \Sigma F_y = 0: \qquad T - 75(9.81) + T - 40(9.81) + C_y = 0$

$\circlearrowleft \Sigma M_C = 0: \qquad 3\left[75(9.81)\right] + 1.5\left[40(9.81)\right] - 3T - 1.5T = 0$

$T = 621.300 \text{ N} \qquad C_y = -114.450 \text{ N}$

Next, draw a free-body diagram of the portion of the beam between section *a–a* and the right end of the beam. Note that since one-fourth of the beam has been "cut away," only three-fourths of the total beam weight is included on the free-body diagram. The equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad -P_a + 0 = 0$

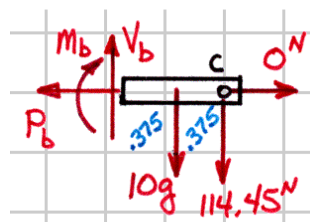
$\uparrow \Sigma F_y = 0: \qquad V_a + (621.3) - 30(9.81) - 114.450 = 0$

$\circlearrowleft \Sigma M_a = 0: \qquad -M_a + 0.75(621.3) - 1.125\left[30(9.81)\right] - 2.25(114.450) = 0$

$P_a = 0 \text{ N}$ .... $V_a = -213 \text{ N}$ .... **Ans.**

$M_a = -122.6 \text{ N} \cdot \text{m}$ .... **Ans.**

Finally, draw a free-body diagram of the portion of the beam between section *b–b* and the right end of the beam. This time three-fourths of the beam has been "cut away" and only one-fourth of the total beam weight is included on the free-body diagram. The equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad -P_b + 0 = 0$

$\uparrow \Sigma F_y = 0: \qquad V_b - 10(9.81) - 114.450 = 0$

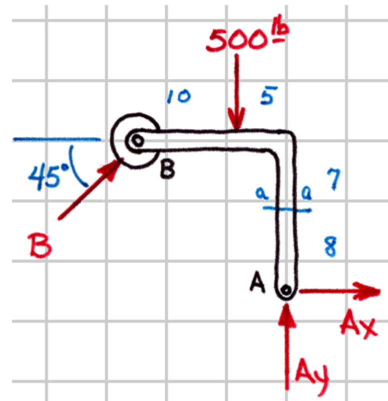
$\circlearrowleft \Sigma M_b = 0: \qquad -M_b - 0.375\left[10(9.81)\right] - 0.75(114.450) = 0$

$P_b = 0 \text{ N}$ .... $V_b = 213 \text{ N}$ .... **Ans.**

$M_b = -122.6 \text{ N} \cdot \text{m}$ .... **Ans.**

1-57*

From an overall free-body diagram of the bracket, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad A_x + B\cos 45° = 0$$

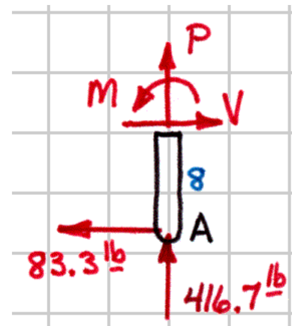
$$\uparrow \Sigma F_y = 0: \qquad A_y + B\sin 45° - 500 = 0$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 5(500) - 15(B\cos 45°) - 15(B\sin 45°) = 0$$

$$B = 117.85113 \text{ lb}$$

$$A_x = -83.3333 \text{ lb} \qquad A_y = 416.6667 \text{ lb}$$

Next, draw a free-body diagram of the portion of the bracket between section *a*–*a* and pin *A*. The equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad V - 83.3333 = 0$$

$$\uparrow \Sigma F_y = 0: \qquad P + 416.6667 = 0$$

$$\circlearrowleft \Sigma M_a = 0: \qquad M - 8(83.3333) = 0$$

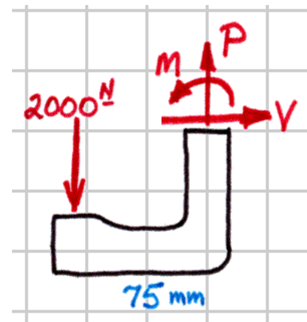
$P = -417 \text{ lb} = 417 \text{ lb (C)}$ .... **Ans.**

$V = 83.3 \text{ lb}$ .... **Ans.**

$M = 667 \text{ lb} \cdot \text{in.}$ .... **Ans.**

1-58

From a free-body diagram of the lower half of the clamp, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \quad V = 0$

$\uparrow \Sigma F_y = 0: \quad P - 2000 = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \quad M + 0.075(2000) = 0$

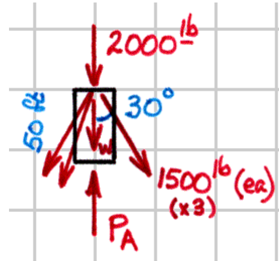
$P = 2000$ N (T) .... **Ans.**

$V = 0$ N .... **Ans.**

$M = -150.0$ N·m .... **Ans.**

1-59*

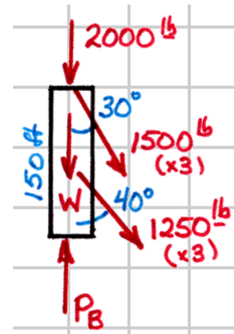
Draw free-body diagrams of the sections of the tower between the points of interest and the top. For each section, the vertical component of the equation of equilibrium gives



$$\uparrow \Sigma F_y = 0: \qquad P_A - 2000 - 3(1500\cos 30°) - W = 0$$

$$W = 40(50) = 2000 \text{ lb}$$

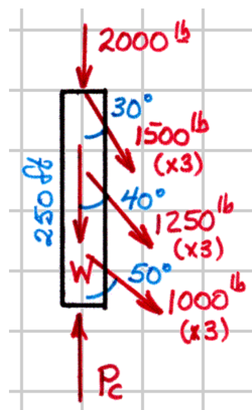
$$P_A = 7900 \text{ lb}$$ .... **Ans.**



$$\uparrow \Sigma F_y = 0: \quad P_B - 2000 - 3(1500\cos 30°) - 3(1250\cos 40°) - W = 0$$

$$W = 40(150) = 6000 \text{ lb}$$

$$P_B = 14,770 \text{ lb}$$ .... **Ans.**

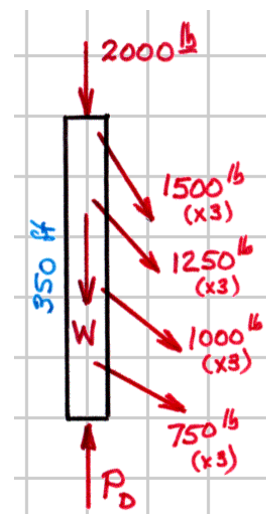


$$\uparrow \Sigma F_y = 0: \quad P_C - 2000 - 3(1500\cos 30°) - W$$

$$-3(1250\cos 40°) - 3(1000\cos 50°) = 0$$

$$W = 40(250) = 10,000 \text{ lb}$$

$$P_C = 20,700 \text{ lb}$$ .... **Ans.**



$$\uparrow \Sigma F_y = 0: \quad P_D - 2000 - 3(1500\cos 30°) - W$$

$$-3(1250\cos 40°) - 3(1000\cos 50°)$$

$$-3(750\cos 60°) = 0$$

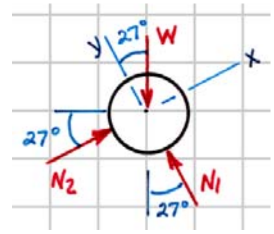
$$W = 40(350) = 14,000 \text{ lb}$$

$$P_D = 25,800 \text{ lb}$$ .... **Ans.**

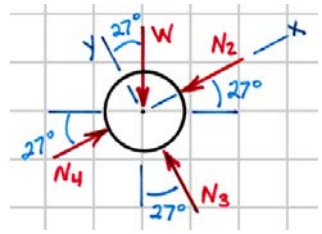
1-60*

The weights of the two cylinders are the same

$$W = 50(9.81) = 490.5 \text{ N}$$



First draw a free-body diagram of the upper cylinder, and write the equations of equilibrium



Z $\Sigma F_x = 0: \qquad N_2 - W \sin 27° = 0$

^ $\Sigma F_y = 0: \qquad N_1 - W \cos 27° = 0$

$$N_1 = 437.0387 \text{ N}$$

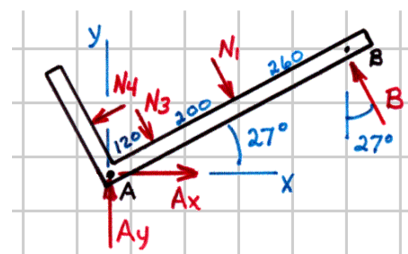
$$N_2 = 222.6823 \text{ N}$$

Next, from a free-body diagram of the lower cylinder, the equations of equilibrium give

Z $\Sigma F_x = 0: \qquad N_4 - N_2 - W \sin 27° = 0$

^ $\Sigma F_y = 0: \qquad N_3 - W \cos 27° = 0$

$$N_3 = 437.0387 \text{ N} \qquad N_4 = 445.3647 \text{ N}$$



Next, from a free-body diagram of the rack, the equations of equilibrium give

$\rightarrow \Sigma F_x = 0: \qquad A_x - N_4 \cos 27° + (N_1 + N_3) \sin 27° - B \sin 27° = 0$

$\uparrow \Sigma F_y = 0: \qquad A_y - N_4 \sin 27° - (N_1 + N_3) \cos 27° + B \cos 27° = 0$

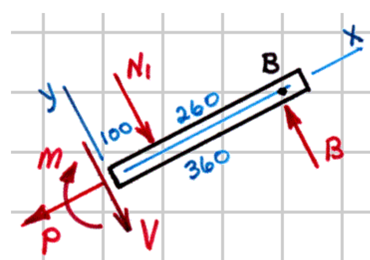
$\circlearrowleft \Sigma M_A = 0: \qquad 120(445.3647) - 320(437.0387)$

$$-120(437.0387) + 580B = 0$$

$$B = 239.4022 \text{ N}$$

$$A_x = 902.3320 \text{ N} \qquad A_y = 767.6911 \text{ N}$$

Finally, from a free-body diagram of the upper portion of the rack (between section *a* and the roller support *B*), the equations of equilibrium give



Z $\Sigma F_x = 0: \qquad -P = 0$

^ $\Sigma F_y = 0: \qquad B - V - N_1 = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \qquad 0.360B - M - 0.100N_1 = 0$

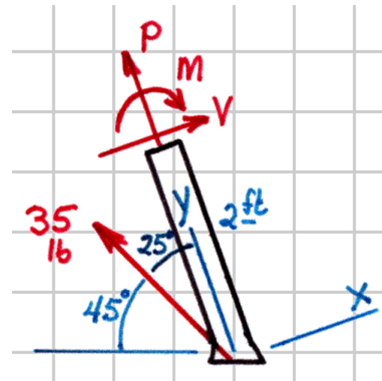
$P = 0 \text{ N}$ .... **Ans.**

$V = -197.6 \text{ N}$ .... **Ans.**

$M = 42.5 \text{ N} \cdot \text{m}$ .... **Ans.**

1-61

From a free-body diagram of the lower portion of the crutch (between section *a* and the bottom *B*), the equations of equilibrium give



Z $\Sigma F_x = 0: \qquad V - 35\sin 25° = 0$

^ $\Sigma F_y = 0: \qquad P + 35\cos 25° = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \qquad -M - 2(35\sin 25°) = 0$

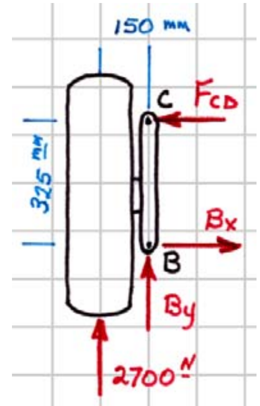
$P = -31.7 \text{ lb} = 31.7 \text{ lb (C)}$ .... **Ans.**

$V = 14.79 \text{ lb}$ .... **Ans.**

$M = 29.6 \text{ lb}\cdot\text{ft}$ .... **Ans.**

1-62

From a free-body diagram of the wheel, the equations of equilibrium give



$$\circlearrowleft \Sigma M_B = 0: \qquad 325F_{CD} - 150(2700) = 0$$

$$F_{CD} = 1246 \text{ N}$$

Since *CD* is a two-force member, the axial force on every cross section is the same

$P = 1246$ N (C) .... **Ans.**

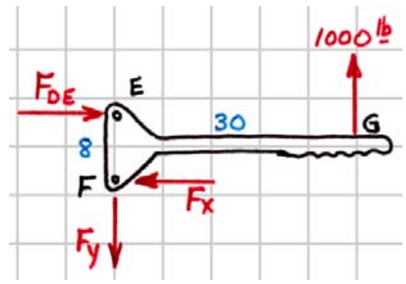
and the shear force and the bending moment are both zero

$V = 0$ N .... **Ans.**

$M = 0 \text{ N} \cdot \text{m}$ .... **Ans.**

1-63

From a free-body diagram of the handle, the equations of equilibrium give



$\circlearrowleft \Sigma M_F = 0: \qquad 30(1000) - 8F_{DE} = 0$

$F_{DE} = 3750 \text{ lb}$

Since *DE* is a two-force member, the axial force on every cross section is the same

$P = 3750 \text{ lb (C)}$ .... **Ans.**

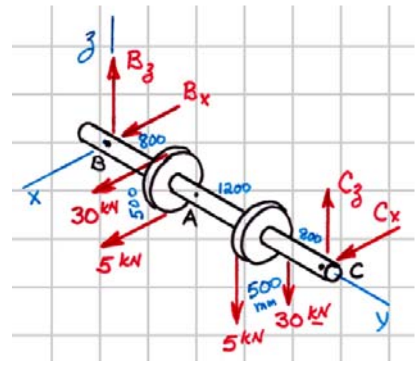
and the shear force and the bending moment are both zero

$V = 0 \text{ lb}$ .... **Ans.**

$M = 0 \text{ lb} \cdot \text{ft}$ .... **Ans.**

1-64*

First draw an overall free-body diagram of the shaft and sum moments about the bearing *B*



$\Sigma \mathbf{M}_C = \mathbf{0}:$

$$(-800\mathbf{j}+250\mathbf{i})\times(-5\mathbf{k})+(-800\mathbf{j}-250\mathbf{i})\times(-30\mathbf{k})$$
$$+(-2000\mathbf{j}+250\mathbf{k})\times(30\mathbf{i})+(-2000\mathbf{j}-250\mathbf{k})\times(5\mathbf{i})$$
$$+(-2800\mathbf{j})\times(B_x\mathbf{i}+B_z\mathbf{k})=\mathbf{0}$$

The *x*-, *y*-, and *z*-components of this equation give

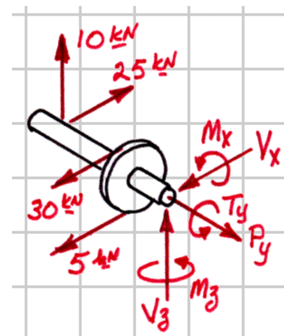
$x: \quad 4000+24,000-2800B_z=0$

$y: \quad 1250-7500+7500-1250=0$

$z: \quad 60,000+10,000+2800B_x=0$

$B_x=-25.00 \text{ kN} \qquad B_z=10.00 \text{ kN}$

Next draw a free-body diagram of the portion of the shaft between the bearing *B* and the section at *A* and write the equations of equilibrium



$\Sigma F_x=0: \quad V_x+30+5-25=0$

$\Sigma F_y=0: \quad P_y=0$

$\Sigma F_z=0: \quad V_z+10=0$

$V_x=-10 \text{ kN}$ .... **Ans.**

$P_y=0 \text{ kN}$ .... **Ans.**

$V_z=-10 \text{ kN}$ .... **Ans.**

$\Sigma \mathbf{M}_{cut}=\mathbf{0}:$

$$(M_x\mathbf{i}+T_y\mathbf{j}+M_z\mathbf{k})+(-0.4\mathbf{j}+0.25\mathbf{k})\times(30\mathbf{i})$$
$$+(-0.4\mathbf{j}-0.25\mathbf{k})\times(5\mathbf{i})+(-1.2\mathbf{j})\times(-25\mathbf{i}+10\mathbf{k})=\mathbf{0}$$

The *x*-, *y*-, and *z*-components of this equation give

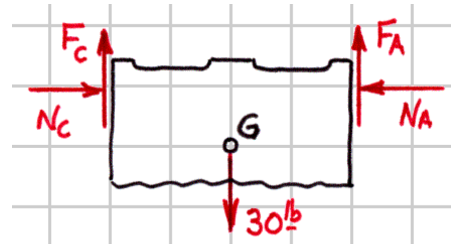
$x: \quad M_x-12=0$ $\qquad M_x=12 \text{ kN}\cdot\text{m}$ .... **Ans.**

$y: \quad T_y+7.5-1.25=0$ $\qquad T_y=-6.25 \text{ kN}\cdot\text{m}$ .... **Ans.**

$z: \quad M_z-30+12+2=0$ $\qquad M_z=16 \text{ kN}\cdot\text{m}$ .... **Ans.**

1-65*

First draw a free-body diagram of the blocks, and write the equations of equilibrium



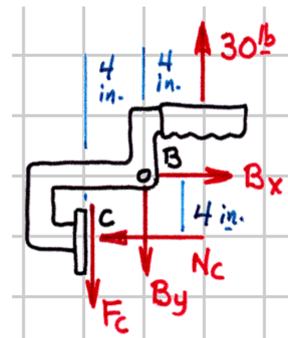
$\rightarrow \Sigma F_x = 0: \quad N_C - N_A = 0$

$\uparrow \Sigma F_y = 0: \quad F_C + F_A - 30 = 0$

$\circlearrowleft \Sigma M_A = 0: \quad 8(30) - 16(F_C) = 0$

$N_C = N_A \qquad F_C = F_A = 15 \text{ lb}$

Next, from a free-body diagram of the left handle, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \quad B_x - N_C = 0$

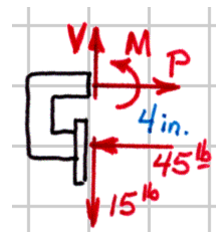
$\uparrow \Sigma F_y = 0: \quad 30 - B_y - (15) = 0$

$\circlearrowleft \Sigma M_B = 0: \quad 4(30) + 4(15) - 4N_C = 0$

$N_C = 45 \text{ lb}$

$B_x = 45 \text{ lb} \qquad B_y = 15 \text{ lb}$

Next, from a free-body diagram of the lower section of the left handle, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \quad P - 45 = 0$

$\uparrow \Sigma F_y = 0: \quad V - 15 = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \quad M - 4(45) = 0$

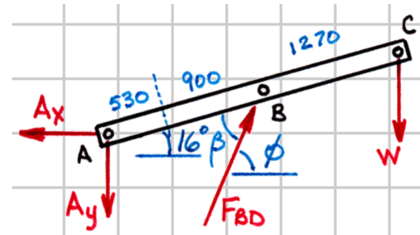
$P = 45 \text{ lb}$ .... **Ans.**

$V = 15 \text{ lb}$ .... **Ans.**

$M = 180 \text{ lb} \cdot \text{in.}$ .... **Ans.**

1-66

$$W = 360(9.81) = 3531.60 \text{ N}$$

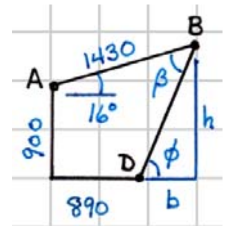


From a free-body diagram of the bar *ABC*, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad -A_x + F_{BD}\cos\phi = 0$$

$$\uparrow \Sigma F_y = 0: \qquad -A_y + F_{BD}\sin\phi - W = 0$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 1430(F_{BD}\sin\beta) - (2700\cos 16°)W = 0$$



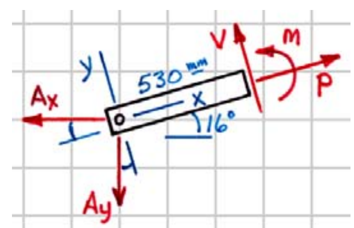
$$\tan\phi = \frac{h}{b} = \frac{900 + 1430\sin 16°}{1430\cos 16° - 890}$$

$$\phi = 69.471° \qquad \beta = \phi - 16° = 53.471°$$

$$F_{BD} = 7976.730 \text{ N}$$

$$A_x = 2797.2911 \text{ N} \qquad A_y = 3938.5663 \text{ N}$$

Then, from a free-body diagram of the left section of the bar, the equations of equilibrium give



$$Z \;\; \Sigma F_x = 0: \qquad P - A_x\cos 16° - A_y\sin 16° = 0$$

$$\wedge \;\; \Sigma F_y = 0: \qquad V + A_x\sin 16° - A_y\cos 16° = 0$$

$$\circlearrowleft \Sigma M_{cut} = 0: \quad M - (0.530\sin 16°)A_x + (0.530\cos 16°)A_y = 0$$

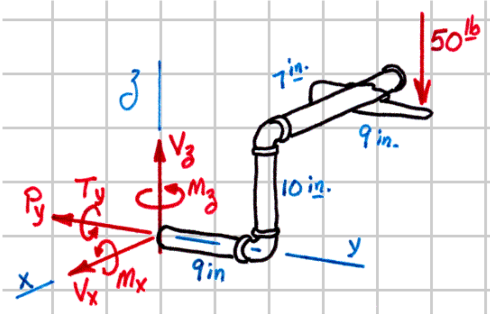
$P = 3770$ N .... **Ans.**

$V = 3010$ N .... **Ans.**

$M = -1598$ N·m .... **Ans.**

1-67

From a free-body diagram of the pipe the moment equilibrium equation



$\Sigma \mathbf{M}_{cut} = \mathbf{0}:$

$$\left(M_x\mathbf{i} + T_y\mathbf{j} + M_z\mathbf{k}\right) + \left(-7\mathbf{i} + 18\mathbf{j} + 10\mathbf{k}\right) \times \left(-50\mathbf{k}\right) = \mathbf{0}$$

has *x*-, *y*-, and *z*-components

$x: \quad M_x - 900 = 0 \qquad M_x = 900 \text{ lb} \cdot \text{in.}$ .... **Ans.**

$y: \quad T_y - 350 = 0 \qquad T_y = 350 \text{ lb} \cdot \text{in.}$ .... **Ans.**

$z: \quad M_z = 0 \qquad M_z = 0 \text{ lb} \cdot \text{in.}$ .... **Ans.**

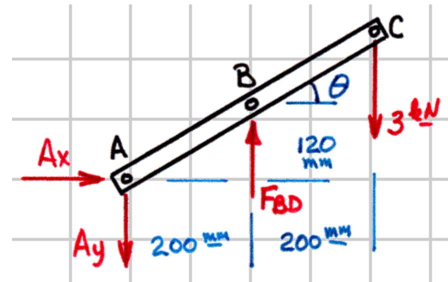
and the force equilibrium equation has components

$\Sigma F_x = 0: \quad V_x = 0 \qquad V_x = 0 \text{ lb}$ .... **Ans.**

$\Sigma F_y = 0: \quad P_y = 0 \qquad P_y = 0 \text{ lb}$ .... **Ans.**

$\Sigma F_z = 0: \quad V_z - 50 = 0 \qquad V_z = 50 \text{ lb}$ .... **Ans.**

1-68*



From a free-body diagram of the bar *ABC*, the equations of equilibrium give

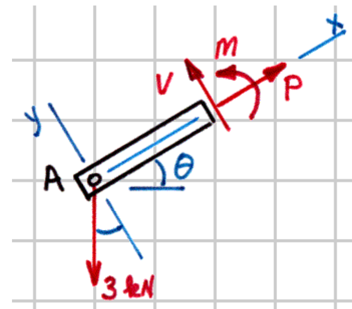
$$\rightarrow \Sigma F_x = 0: \qquad A_x = 0$$

$$\uparrow \Sigma F_y = 0: \qquad -A_y + F_{BD} - 3 = 0$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 200F_{BD} - 400(3) = 0$$

$$F_{BD} = 6 \text{ N}$$

$$A_x = 0 \text{ N} \qquad A_y = 3 \text{ N}$$



Then, from a free-body diagram of the left section of the bar, the equations of equilibrium give

$$\text{Z} \;\; \Sigma F_x = 0: \qquad P - (3)\sin\theta = 0$$

$$\wedge \;\; \Sigma F_y = 0: \qquad V - (3)\cos\theta = 0$$

$$\circlearrowleft \Sigma M_{cut} = 0: \qquad (0.1)(3) - M = 0$$

in which

$$\theta = \tan^{-1}\frac{120}{200} = 30.964°$$

Then

$P = 1.543$ kN .... **Ans.**

$V = 2.57$ kN .... **Ans.**

$M = 0.3$ kN·m .... **Ans.**

1-69

From a free-body diagram of a part of the hook, the equations of equilibrium give

Z $\Sigma F_x = 0:$ $V - 10\cos\theta = 0$

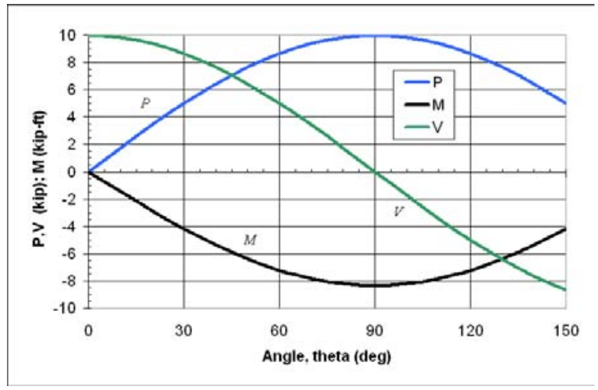
^ $\Sigma F_y = 0:$ $P - 10\sin\theta = 0$

↺ $\Sigma M_{cut} = 0:$ $-M - 10(10\sin\theta) = 0$

$P = 10\sin\theta$ kip .... **Ans.**

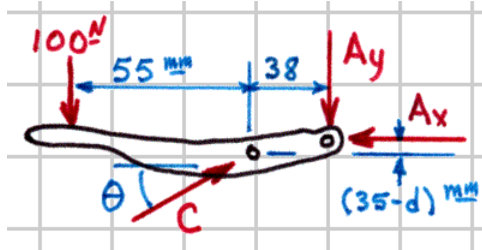
$V = 10\cos\theta$ kip .... **Ans.**

$M = -100\sin\theta$ kip·in. .... **Ans.**



1-70

From a free-body diagram of the top handle



$$\circlearrowleft \Sigma M_A = 0: \qquad 93(100) - 38(C\sin\theta) + (35-d)(C\cos\theta) = 0$$

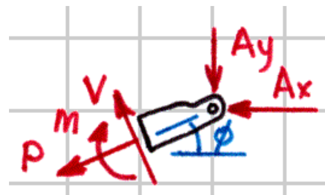
$$C = \frac{9300}{38\sin\theta - (35-d)\cos\theta}$$

Note that $\tan\theta = d/40$; therefore $d = 40\tan\theta$ and

$$C = \frac{9300}{38\sin\theta - 35\cos\theta + 40\sin\theta} = \frac{9300}{78\sin\theta - 35\cos\theta}$$

$$\rightarrow \Sigma F_x = 0: \qquad C\cos\theta - A_x = 0 \qquad A_x = C\cos\theta$$

$$\uparrow \Sigma F_y = 0: \qquad C\sin\theta - A_y - 100 = 0 \qquad A_y = C\sin\theta - 1000$$



Next, from a free-body diagram of the part of the top handle to the right of section *a*–*a* the equations of equilibrium give

$$[\quad \Sigma F_x = 0: \qquad P + A_x\cos\phi + A_y\sin\phi = 0$$

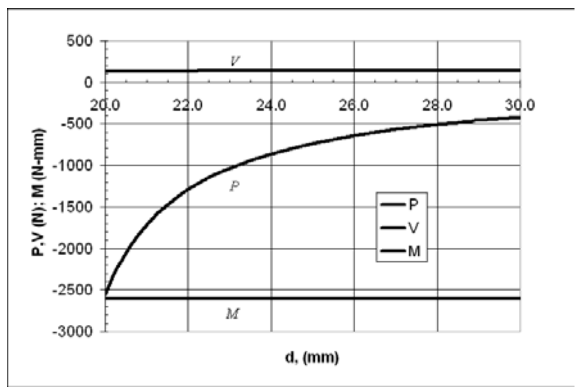
$$\wedge \quad \Sigma F_y = 0: \qquad V + A_x\sin\phi - A_y\cos\phi = 0$$

$$\circlearrowleft \Sigma M_{cut} = 0: \qquad A_x(18\tan\phi) - M - 18A_y = 0$$

$P = (-A_x\cos\phi - A_y\sin\phi)$ N .... **Ans.**

$V = (A_y\cos\phi - A_x\sin\phi)$ N .... **Ans.**

$M = [A_x(18\tan\phi) - 18A_y]$ N·mm .... **Ans.**



1-71

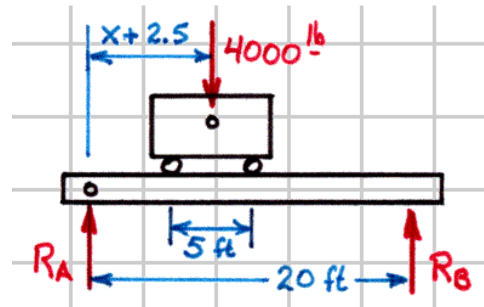
(a) From a free-body diagram of the complete beam, the equations of equilibrium give

$$\circlearrowleft \Sigma M_A = 0: \qquad 20R_B - (x + 2.5)(4000) = 0$$

$$\uparrow \Sigma F_y = 0: \qquad R_A + R_B - 4000 = 0$$

$$R_B = 200(x + 2.5) = (200x + 500)\ \text{lb}$$

$$R_A = (3500 - 200x)\ \text{lb}$$

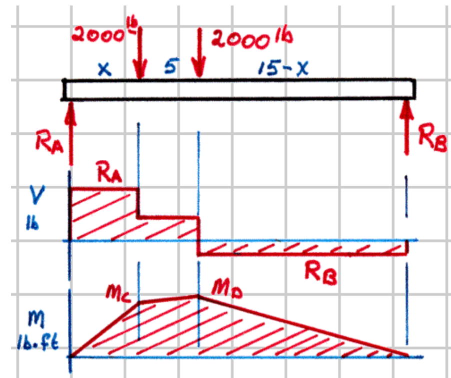


Load, shear force, and bending moment diagrams are as shown. The moment under the left wheel is

$$M_C = xR_A = (3500x - 200x^2)\ \text{lb} \cdot \text{ft}$$

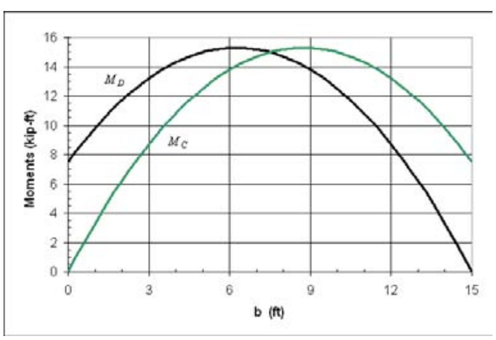
and the moment under the right wheel is

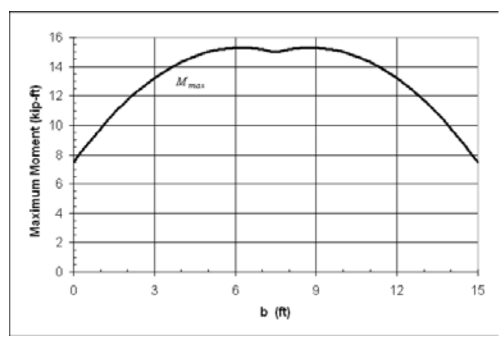
$$M_D = (15 - x)R_B = (15 - x)(200x + 500)\ \text{lb} \cdot \text{ft}$$



These moments are graphed below. Notice that the maximum moment occurs under the right wheel when $0 \le x \le 7.5$ m (and the right wheel is closer to the center of the beam) and under the right wheel when $7.5 \le x \le 15$ m (and the left wheel is closer to the center of the beam).

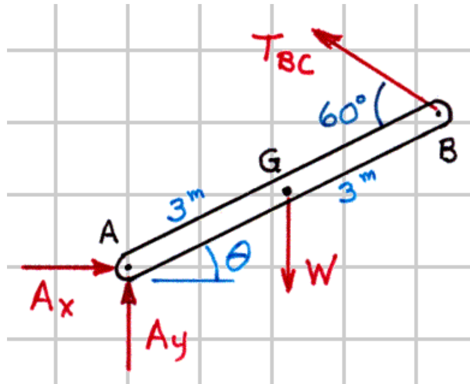
(b) The graph of maximum moment is also shown below.





1-72

$$W = 250(9.81) = 2452.50 \text{ N}$$



(a) From a free-body diagram of the post *AB*, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad A_x - T_{BC}\cos(60° - \theta) = 0$$

$$\uparrow \Sigma F_y = 0: \qquad A_y + T_{BC}\sin(60° - \theta) - W = 0$$

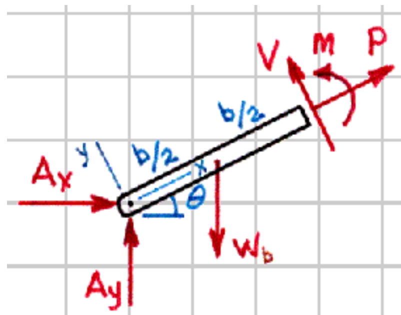
$$\circlearrowleft \Sigma M_B = 0: \qquad 6(T_{BC}\sin 60°) - (3\cos\theta)W = 0$$

$$T_{BC} = \left[\frac{(2452.50)(3\cos\theta)}{6\sin 60°}\right] \text{ N}$$

$$A_x = \left[T_{BC}\cos(60° - \theta)\right] \text{ N}$$

$$\left[A_y = 2452.50 - T_{BC}\sin(60° - \theta)\right] \text{ N}$$

Next, draw a free-body diagram of the lower portion of *ABC*. The weight of this portion is proportional to its length



$$W_b = \frac{Wb}{L} = \frac{2452.50b}{6} = (408.75b) \text{ N}$$

Then the equations of equilibrium give

$$Z \ \Sigma F_x = 0: \qquad P + A_x\cos\theta + A_y\sin\theta - W_b\sin\theta = 0$$

$$\wedge \ \Sigma F_y = 0: \qquad V - A_x\sin\theta + A_y\cos\theta - W_b\cos\theta = 0$$

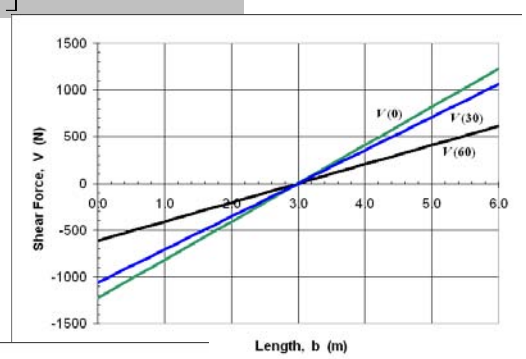
$$\circlearrowleft \Sigma M_{cut} = 0: \qquad M + W_b\left[(b/2)\cos\theta\right]$$

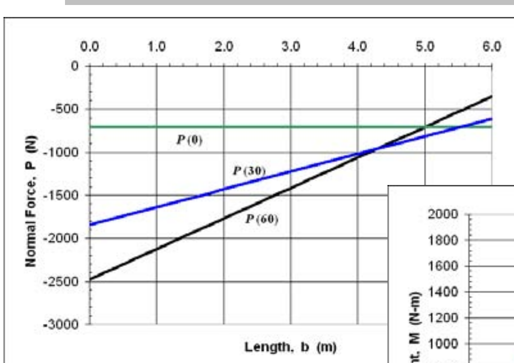
$$+ A_x(b\sin\theta) - A_y(b\cos\theta) = 0$$

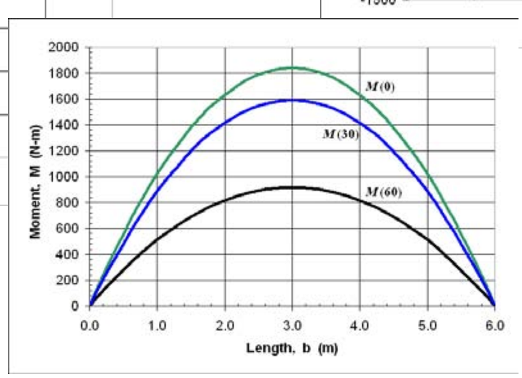
$$M = \left[A_y(b\cos\theta) - A_x(b\sin\theta) - 204.375b^2\cos\theta\right] \text{ N}\cdot\text{m}$$ ....**Ans.**

$$P = \left[(408.75b - A_y)\sin\theta - A_x\cos\theta\right] \text{ N}$$ .....**Ans.**

$$V = \left[(408.75b - A_y)\cos\theta + A_x\sin\theta\right] \text{ N}$$ .....**Ans.**

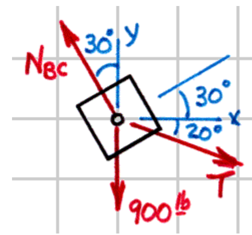






1-73*

From a free-body diagram of the collar, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0:$ $\quad T\cos 20° - N_{BC}\sin 30° = 0$

$\uparrow \Sigma F_y = 0:$ $\quad N_{BC}\cos 30° - T\sin 20° - 900 = 0$

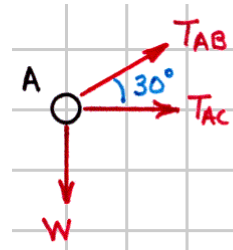
$T = 700$ lb ↘ 20° ....**Ans.**

$N_{BC} = 1316$ lb ↖ 60° ....**Ans.**

1-74*

$$W = 500(9.81) = 4905 \text{ N}$$

First draw a free-body diagram of joint *A*, and write the equations of equilibrium



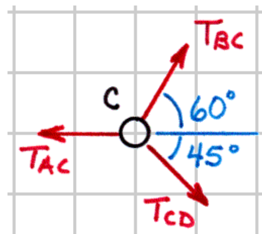
$$\rightarrow \Sigma F_x = 0: \qquad T_{AC} + T_{AB} \cos 30° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{AB} \sin 30° - W = 0$$

$T_{AB} = 9810 \text{ N} = 9.81 \text{ kN (T)}$ .... **Ans.**

$T_{BC} = -8495.709 \text{ N} \cong 8.50 \text{ kN (C)}$ .... **Ans.**

Next, from a free-body diagram of joint *C*,
the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad T_{AC} + T_{BC} \cos 60° + T_{CD} \cos 45° = 0$$

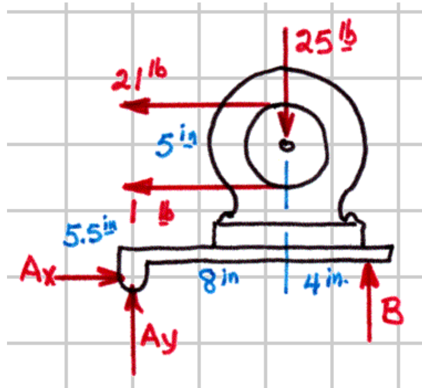
$$\uparrow \Sigma F_y = 0: \qquad T_{BC} \sin 60° - T_{CD} \sin 45° = 0$$

$T_{BC} = -6219.291 \text{ N} = 6.22 \text{ kN (C)}$ .... **Ans.**

$T_{CD} = -7617.044 \text{ N} \cong 7.62 \text{ kN (C)}$ .... **Ans.**

1-75

From a free-body diagram of the motor and support, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad A_x - 21 - 1 = 0$

$\uparrow \Sigma F_y = 0: \qquad A_y + B - 25 = 0$

$\circlearrowleft \Sigma M_D = 0: \qquad 12B - 8(25) + 10.5(21) + 5.5(1) = 0$

$B = -2.16667 \text{ lb} \cong 2.17 \text{ lb} \downarrow$ .... **Ans.**

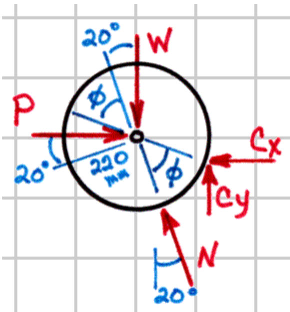
$A_x = 22 \text{ lb} \qquad A_y = 27.16667 \text{ lb}$

$\mathbf{A} = 35.0 \text{ lb} \measuredangle\ 51.0°$ .... **Ans.**

1-76*

$$W = 135(9.81) = 1324.50 \text{ N}$$

$$\phi = \cos^{-1}\frac{220-75}{220} = 48.769°$$



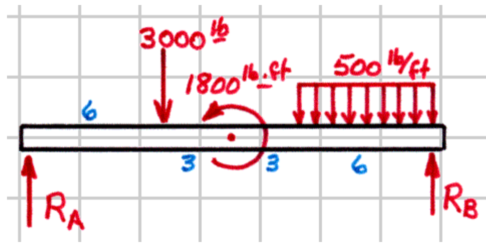
First draw a free-body diagram of the cylinder. When the cylinder just starts to rotate about the step, the normal force N becomes zero. Then moment equilibrium gives

$$\circlearrowleft \Sigma M_C = 0: \qquad 220\left[W\sin\left(20°+\phi\right)\right] - 220\left[P\cos\left(20°+\phi\right)\right] = 0$$

$P = 3410$ N ....**Ans.**

1-77

(a) From a free-body diagram of the entire beam, the equations of equilibrium give



$$\uparrow \Sigma F_y = 0: \qquad R_A + R_B - 3000 - 500(6) = 0$$

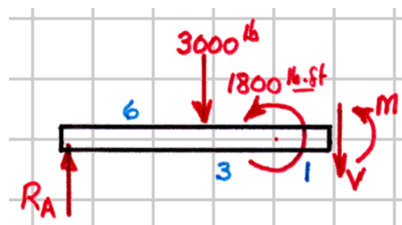
$$\circlearrowleft \Sigma M_B = 0:$$

$$[500(6)](3) + 3000(12) + 1800 - 18R_A = 0$$

$R_A = 2600$ lb .... **Ans.**

$R_B = 3400$ lb .... **Ans.**

(b) Next, from a free-body diagram of the left end of the beam, the equations of equilibrium give



$$\uparrow \Sigma F_y = 0: \qquad (2600) - 3000 - V = 0$$

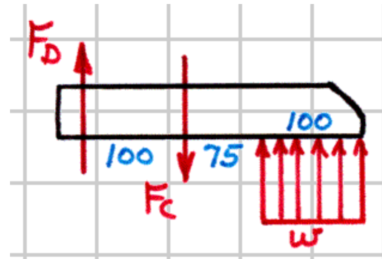
$$\circlearrowleft \Sigma M_{cut} = 0: \qquad M + 1800 + 3000(4) - (2600)(10) = 0$$

$V = -400$ lb .... **Ans.**

$M = 12,200 \text{ lb} \cdot \text{ft}$ .... **Ans.**

1-78

First draw a free-body diagram of the upper jaw. The resultant of the distributed load is a 300 N force acting through the centroid of the distributed load. The equations of equilibrium give



$\circlearrowleft \Sigma M_D = 0$: $\quad 225(300) - 100F_C = 0$

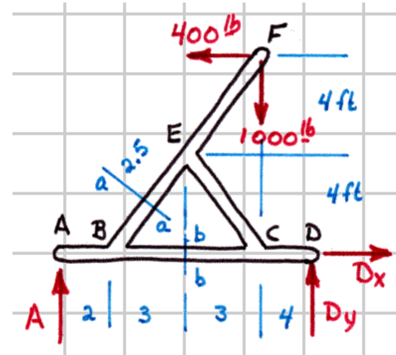
$\circlearrowleft \Sigma M_C = 0$: $\quad 125(300) - 100F_D = 0$

$F_C = 675 \text{ N} \downarrow$ .... **Ans.**

$F_D = 375 \text{ N} \uparrow$ .... **Ans.**

1-79*

(a) First, from a free-body diagram of the entire frame, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad D_x - 400 = 0$$

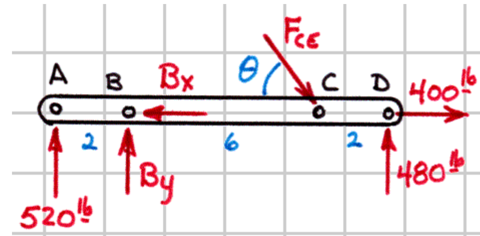
$$\uparrow \Sigma F_y = 0: \qquad A + D_y - 1000 = 0$$

$$\circlearrowleft \Sigma M_D = 0: \qquad 8(400) + 2(1000) - 10A = 0$$

$$A = 520 \text{ lb}$$

$$D_x = 400 \text{ lb} \qquad D_y = 480 \text{ lb}$$

Next, from a free-body diagram of bar *ABCD*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad (400) - B_x + F_{CE}\cos\theta = 0$$

$$\uparrow \Sigma F_y = 0: \qquad (520) + B_y - F_{CE}\sin\theta + 480 = 0$$

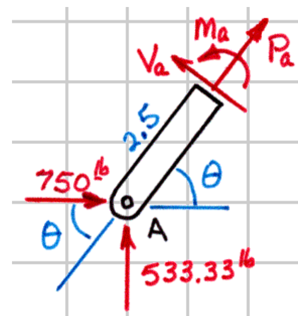
$$\circlearrowleft \Sigma M_B = 0: \qquad 8(480) - 2(520) - 6(F_{CE}\sin\theta) = 0$$

$$\sin\theta = 4/5 \qquad \cos\theta = 3/5$$

$$F_{CE} = 583.333 \text{ lb}$$

$$B_x = 750 \text{ lb} \qquad B_y = -533.333 \text{ lb}$$

Next, from a free-body diagram of the lower end of bar *BEF*, the equations of equilibrium give



Z $\Sigma F_x = 0: \qquad P_a + 750\cos\theta + 533.333\sin\theta = 0$

^ $\Sigma F_y = 0: \qquad V_a - 750\sin\theta + 533.333\cos\theta = 0$

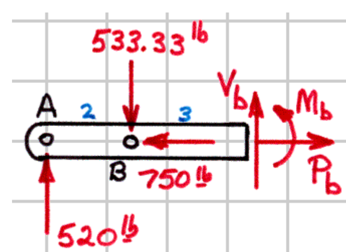
$$\circlearrowleft \Sigma M_B = 0: \qquad M_a + 2.5(750\sin\theta) - 2.5(533.333\cos\theta) = 0$$

$P_a = -876.667 \text{ lb} \cong 877 \text{ lb (C)}$ ....**Ans.**

$V_a = 280 \text{ lb}$ ....**Ans.**

$M_a = -700 \text{ lb}\cdot\text{ft}$ ....**Ans.**

(b) Finally, from a free-body diagram of the left end of bar *ABCD*, the equations of equilibrium give



Z $\Sigma F_x = 0: \qquad P_b - 750 = 0$

^ $\Sigma F_y = 0: \qquad V_b + 520 - 533.333 = 0$

$$\circlearrowleft \Sigma M_B = 0: \qquad M_b + 3(533.333) - 5(520) = 0$$

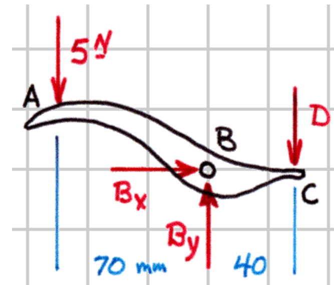
$P_b = 750 \text{ lb (T)}$ ....**Ans.**

$V_b = 13.33 \text{ lb}$ ....**Ans.**

$M_b = 1000 \text{ lb}\cdot\text{ft}$ ....**Ans.**

1-80

Next, from a free-body diagram of the upper handle, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0:$ $B_x = 0$

$\uparrow \Sigma F_y = 0:$ $B_y - 5 - D = 0$

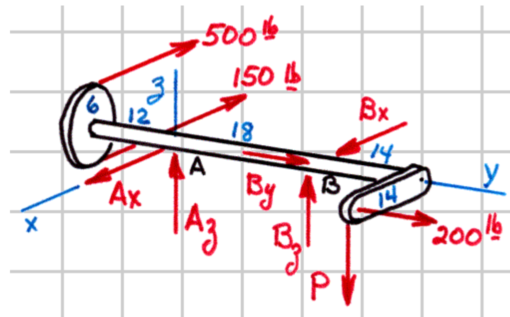
$\circlearrowleft \Sigma M_B = 0:$ $70(5) - 40D = 0$

$\mathbf{D} = 8.75 \text{ N} \downarrow$ .... **Ans.**

$B_x = 0 \text{ N}$ $B_y = 13.75 \text{ N}$

$\mathbf{B} = 13.75 \text{ N} \downarrow$ (on the pin) .... **Ans.**

1-81*



Draw a free-body diagram of the shaft.
The moment equation of equilibrium

$$\Sigma \mathbf{M}_A = \mathbf{0}:$$

$$(18\mathbf{j})\times(B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k})+(14\mathbf{i}+32\mathbf{j})\times(200\mathbf{j}-P\mathbf{k})$$
$$+(-12\mathbf{j}+6\mathbf{k})\times(-500\mathbf{i})+(-12\mathbf{j}-6\mathbf{k})\times(-150\mathbf{i})=\mathbf{0}$$

has components

x: $18B_z - 32P = 0$ — $B_z = 266.667 \text{ lb} \cong 267 \text{ lb}$ .... **Ans.**

y: $-14P - 3000 + 900 = 0$ — $P = 150 \text{ lb}$ .... **Ans.**

z: $-18B_x + 2800 - 6000 - 1800 = 0$ — $B_x = -277.778 \text{ lb} \cong 278 \text{ lb}$ .... **Ans.**

Then the *x*-, *y*-, and *z*-components of the force equilibrium equation give

x: $A_x + (-277.778) - 500 - 150 = 0$ — $A_x = 928 \text{ lb}$ .... **Ans.**

y: $200 + B_y = 0$ — $B_y = -200 \text{ lb}$ .... **Ans.**

z: $A_z + (266.667) - (150) = 0$ — $A_z = -116.7 \text{ lb}$ .... **Ans.**

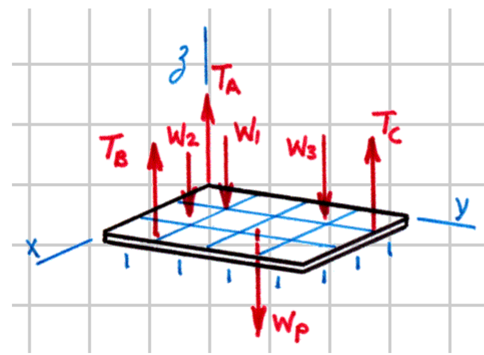
1-82*

$$W_1 = 300(9.81) = 2943 \text{ N}$$

$$W_2 = 100(9.81) = 981 \text{ N}$$

$$W_3 = 200(9.81) = 1962 \text{ N}$$

$$W_P = 500(9.81) = 4905 \text{ N}$$



Draw a free-body diagram of the platform.
The moment equation of equilibrium

$$\Sigma \mathbf{M}_O = \mathbf{0}: \quad (\mathbf{i}+\mathbf{j})\times(-2943\mathbf{k}) + (2\mathbf{i}+\mathbf{j})\times(-981\mathbf{k})$$
$$+(3\mathbf{i}+\mathbf{j})\times(T_B\mathbf{k}) + (\mathbf{i}+3\mathbf{j})\times(-1962\mathbf{k})$$
$$+(\mathbf{i}+4\mathbf{j})\times(T_C\mathbf{k}) + (1.5\mathbf{i}+2\mathbf{j})\times(-4905\mathbf{k}) = \mathbf{0}$$

has components

$$x: \quad -2943 - 981 + T_B - 5886 + 4T_C - 9810 = 0$$

$$y: \quad 2943 + 1962 - 3T_B + 1962 - T_C + 7357.5 = 0$$

$T_B = 3388.909 \text{ N} \cong 3390 \text{ N}$ .... **Ans.**

$T_C = 4057.773 \text{ N} \cong 4060 \text{ N}$ .... **Ans.**

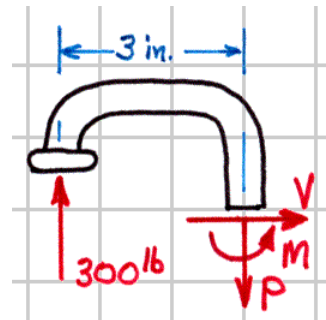
and the *z*-component of the force equilibrium equation gives

$$z: \quad T_A + (3388.909) + (4057.773) - (2943) - (981) - (1962) - (4905) = 0$$

$T_A = 3344.318 \text{ N} \cong 3340 \text{ N}$ .... **Ans.**

1-83

From a free-body diagram of the upper half of the clamp, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad V = 0$$

$$\uparrow \Sigma F_y = 0: \qquad 300 - P = 0$$

$$\circlearrowleft \Sigma M_{cut} = 0: \qquad M - 3(300) = 0$$

$P = 300$ lb (T) .... **Ans.**

$V = 0$ lb .... **Ans.**

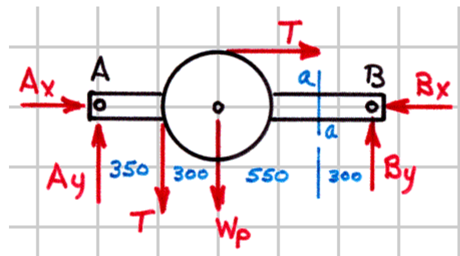
$M = 900$ lb · ft .... **Ans.**

1-84

$$T = W = 100(9.81) = 981 \text{ N}$$

$$W_P = 50(9.81) = 490.50 \text{ N}$$

First draw a free-body diagram of the arm *AB* and the pulley. The moment equation of equilibrium gives

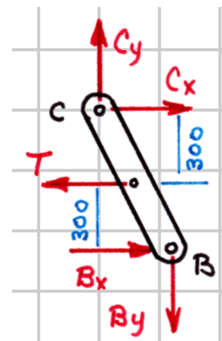


$\circlearrowleft \Sigma M_A = 0:$

$$1500B_y - 300(981) - 350(981) - 650(490.5) = 0$$

$$B_y = 637.650 \text{ N}$$

Next, from a free-body diagram of the bar *BC*, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \quad C_x - (981) + B_x = 0$

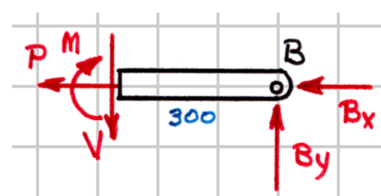
$\uparrow \Sigma F_y = 0: \quad C_y - (637.5) = 0$

$\circlearrowleft \Sigma M_C = 0: \quad 600B_x - 300(981) - 300(637.650) = 0$

$$B_x = 809.325 \text{ N}$$

$$C_x = 171.675 \text{ N} \qquad C_y = 637.650 \text{ N}$$

Finally, from a free-body diagram of the right end of bar *AB*, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \quad -P - (809.325) = 0$

$\uparrow \Sigma F_y = 0: \quad (637.650) - V = 0$

$\circlearrowleft \Sigma M_B = 0: 300(637.650) - M = 0$

$P = -809 \text{ N} = 809 \text{ N (C)}$ .... **Ans.**

$V = 638 \text{ N}$ .... **Ans.**

$M = 191.3 \text{ N} \cdot \text{m}$ .... **Ans.**

2-1*

$$\sigma = \frac{N}{A} = \frac{10}{A} \le 30 \text{ ksi}$$

$$A = \frac{\pi}{4}\left(1^2 - d_i^2\right) \ge \frac{1}{3} \text{ in}^2$$

$$d_i \le 0.75867 \text{ in.}$$

$$t = \frac{d_o - d_i}{2} \ge 0.1207 \text{ in.}$$ .... **Ans.**

2-2*

$$\sigma_A = \frac{N_A}{A_A} = \frac{40(10^3)}{(0.025)(0.015)} = 106.6(10^6)\ \text{N/m}^2 = 106.6\ \text{MPa (T)}$$ .... **Ans.**

$$\sigma_B = \frac{N_B}{A_B} = \frac{50(10^3)}{(0.025)(0.015)} = 133.3(10^6)\ \text{N/m}^2 = 133.3\ \text{MPa (T)}$$ .... **Ans.**

$$\sigma_C = \frac{N_C}{A_C} = \frac{20(10^3)}{(0.025)(0.015)} = 53.3(10^6)\ \text{N/m}^2 = 53.3\ \text{MPa (T)}$$ .... **Ans.**

2-3

$\uparrow \Sigma F_y = 0:$ $N_{AB} - 90 - 2(75) + 2(60) - 2(125) = 0$ $N_{AB} = 370 \text{ kip (C)}$

$\uparrow \Sigma F_y = 0:$ $N_{BC} - 90 - 2(75) + 2(60) = 0$ $N_{BC} = 120 \text{ kip (C)}$

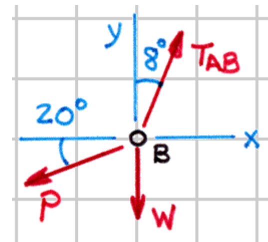
$\uparrow \Sigma F_y = 0:$ $N_{CD} - 90 - 2(75) = 0$ $N_{CD} = 240 \text{ kip (C)}$

$\sigma_{AB} = \dfrac{N_{AB}}{A_{AB}} = \dfrac{370}{16} = 23.1 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{BC} = \dfrac{N_{BC}}{A_{BC}} = \dfrac{120}{4} = 30.0 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{CD} = \dfrac{N_{CD}}{A_{CD}} = \dfrac{240}{12} = 20.0 \text{ ksi (C)}$ .... **Ans.**

2-4*



$$W = 130(9.81) = 1275.30 \text{ N}$$

From a free-body diagram of joint *B*, the equations of equilibrium give

$\rightarrow \Sigma F_x = 0:$ $\quad T_{AB} \sin 8° - P \cos 20° = 0$

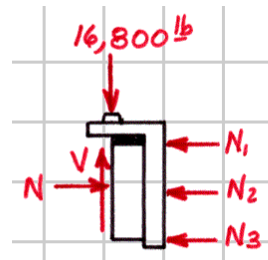
$\uparrow \Sigma F_y = 0:$ $\quad T_{AB} \cos 8° - P \sin 20° - (1275.3) = 0$

$$T_{AB} = 1357.261 \text{ N}$$

$$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{1357.261}{\pi(0.015)^2/4} = 7.68(10^6) \text{ N/m}^2 = 7.68 \text{ MPa (T)}$$ .... **Ans.**

2-5

From a free-body diagram of a portion of the wood block, the equations of equilibrium give



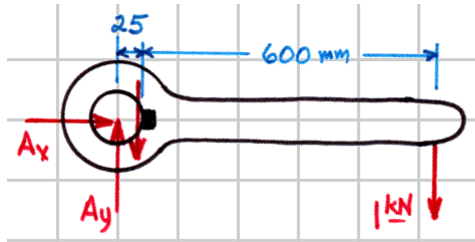
$$\uparrow \Sigma F_y = 0: \qquad V - 16,800 = 0$$

$$V = 16,800 \text{ lb}$$

$$\tau = \frac{V}{A} = \frac{16800}{(8 \times 2)} = 1050 \text{ psi}$$ .... **Ans.**

2-6

From a free-body diagram of the lever, moment equilibrium about the center of the shaft gives



$$\circlearrowleft \Sigma M = 0: \qquad 25V - 625(1) = 0$$

$$V = 25 \text{ kN}$$

$$\tau = \frac{V}{A} = \frac{25(10^3)}{0.020a} \le 125(10^6) \text{ N/m}^2$$

$a \ge 0.0100 \text{ m} = 10.00 \text{ mm}$ ....**Ans.**

2-7*

$$\tau = \frac{V}{A} = \frac{8000}{\pi(2)L_1} = \frac{8000}{\pi(1.5)L_2} \le 500 \text{ psi}$$

$L_1 \ge 2.55$ in. .... **Ans.**

$L_2 \ge 3.40$ in. .... **Ans.**

2-8

$$\tau = \frac{V}{A} = \frac{P}{\pi(0.100)(0.025)} \le 75(10^6) \text{ N/m}^2 \qquad P \le 589(10^3) \text{ N}$$

$$\sigma = \frac{N}{A} = \frac{P}{\pi(0.150^2 - 0.100^2)/4} \le 100(10^6) \text{ N/m}^2 \qquad P \le 982(10^3) \text{ N}$$

$P_{max} = 589 \text{ kN}$ .... **Ans.**

2-9*

$$\tau = \frac{V}{A} = \frac{100(2000)}{\pi d\,(0.5)} = 40\left(10^3\right) \text{ psi}$$

$d = 3.18$ in. .... **Ans.**

2-10*

Assuming that the pulleys rotate freely, the tension will be the same throughout the length of the cord, and

$$T = W = 45(9.81) = 441.45 \text{ N}$$

$$\sigma = \frac{T}{A} = \frac{441.45}{\pi(0.010)^2/4} = 5.62(10^6) \text{ N/m}^2 = 5.62 \text{ MPa (T)}$$ .... **Ans.**

2-11

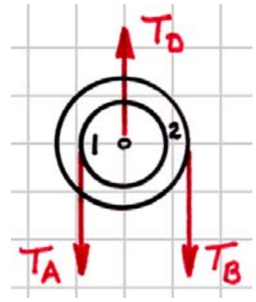
Assume that the girl's arms are vertical. Each arm will carry half of the girl's weight and

(a) $\sigma = \frac{N}{A} = \frac{125/2}{\pi(1)^2/4} = 79.6 \text{ psi (T)}$ .... **Ans.**

(b) $\sigma = \frac{N}{A} = \frac{125/2}{\pi(1^2 - 0.6^2)/4} = 124.3 \text{ psi (T)}$ .... **Ans.**

2-12

$$W = 225(9.81) = 2207.25 \text{ N}$$

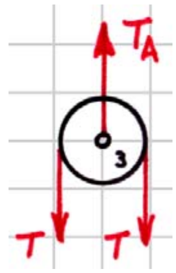


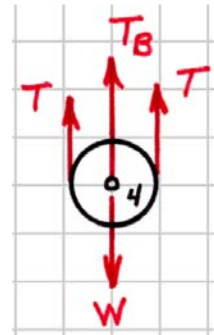
From a free-body diagram of pulley 4, the equations of equilibrium give

$$\uparrow \Sigma F_y = 0: \qquad 2T + T_B - (2207.25) = 0$$

Next, from a free-body diagram of the double pulley (1 and 2), the equations of equilibrium give

$$\circlearrowleft \Sigma M_{axle} = 0: \qquad 100T_A - 300T_B = 0$$





Finally, from a free-body diagram of the pulley 3, the equations of equilibrium give

$$\uparrow \Sigma F_y = 0: \qquad T_A - 2T = 0$$

Combining these three equations gives

$$T_A = 1655.4375 \text{ N}$$

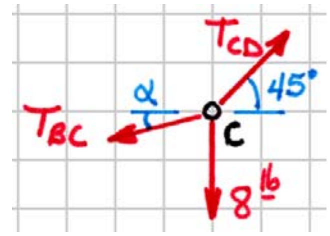
$$T_B = 551.8125 \text{ N}$$

$$T = 827.7188 \text{ N}$$

$$\sigma_C = \frac{T}{A} = \frac{827.7188}{\pi(0.015)^2/4} = 4.68(10^6) \text{ N/m}^2 = 4.68 \text{ MPa (T)}$$ .... **Ans.**

2-13*

From free-body diagrams of the rings *B* and *C*, the equations of equilibrium give

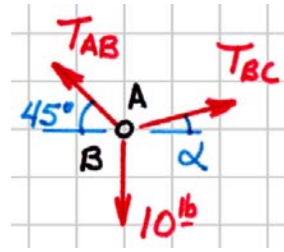


$$\rightarrow \Sigma F_x = 0: \qquad -T_{AB}\cos 45° + T_{BC}\cos\alpha = 0$$

$$-T_{BC}\cos\alpha + T_{CD}\cos 45° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{AB}\sin 45° + T_{BC}\sin\alpha - 10 = 0$$

$$-T_{BC}\sin\alpha + T_{CD}\sin 45° - 8 = 0$$



Adding the first pair of equations together gives

$$T_{AB} = T_{CD}$$

Then, adding the second pair of equations together gives

$$T_{AB} = T_{CD} = 12.72792 \text{ lb}$$

Now, the third equation can be written

$$T_{BC}\sin\alpha = 10 - T_{AB}\sin 45°$$

and the first equation can be written

$$T_{BC}\cos\alpha = T_{AB}\cos 45°$$

Dividing this pair of equations gives

$$\frac{T_{BC}\sin\alpha}{T_{BC}\cos\alpha} = \tan\alpha = \frac{10 - (12.72792)\sin 45°}{(12.72792)\cos 45°} \qquad \text{or} \qquad \alpha = 6.340°$$

$$T_{BC} = 9.05539 \text{ lb}$$

$$\sigma_{\max} = \frac{T_{AB}}{A} = \frac{12.72792}{\pi d^2/4} \le 18(10^3) \text{ psi}$$

$d \ge 0.0300$ in. .... **Ans.**

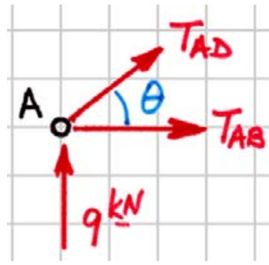
2-14*

$$\theta = \tan^{-1}\frac{1.5}{2} = 36.870° \qquad \sin\theta = 3/5 \qquad \cos\theta = 4/5$$

From symmetry (or overall equilibrium), each support carries half of the total load

$$R_A = R_C = 18/2 = 9\text{ kN}$$

Then, from a free-body diagram of the joint *A*, the equations of equilibrium give



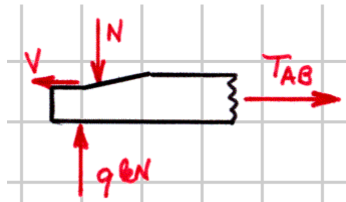
$$\rightarrow \Sigma F_x = 0: \qquad T_{AD}\cos\theta + T_{AB} = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{AD}\sin\theta + 9 = 0$$

$$T_{AD} = -15.00\text{ kN}$$

$$T_{AB} = 12.00\text{ kN}$$

Finally, from a free-body diagram of the bottom chord, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad (12.00) - V = 0$$

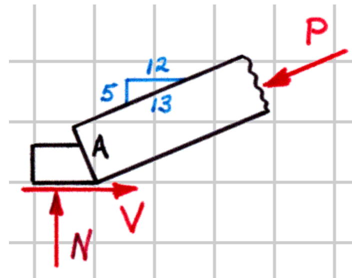
$$V = 12\text{ kN}$$

$$\tau = \frac{V}{A} = \frac{12(10^3)}{0.100a} \le 2.25(10^6)\ \text{N/m}^2$$

$a \ge 0.0533\text{ m} = 53.3\text{ mm}$ .... **Ans.**

2-15

$$V = \tau A = 225(5\times 4) = 4500 \text{ lb}$$



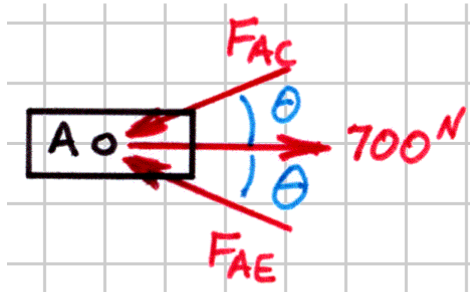
From a free-body diagram of member *AB* and the end of the bottom chord, the equations of equilibrium give

$\rightarrow \Sigma F_x = 0:$ $\quad V - (12/13)P = 0$

$P = 4870 \text{ lb}$ .... **Ans.**

2-16*

From a free-body diagram of the block A, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \quad 700 - F_{AC}\cos\theta - F_{AE}\cos\theta = 0$$

$$\uparrow \Sigma F_y = 0: \quad F_{AE}\sin\theta - F_{AC}\sin\theta = 0$$

$$F_{AC} = F_{AE} = \frac{700}{2\cos\theta} \text{ N}$$

$$\tau = \frac{V}{A} = \frac{F_{AC}}{\pi(0.010)^2/4} \text{ N/m}^2$$

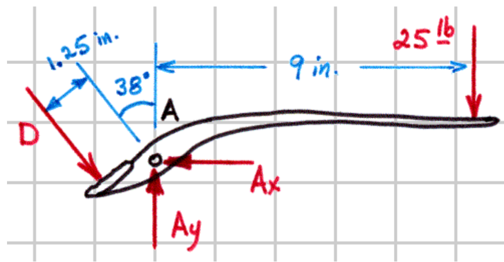
(a) $\theta = 15°$ $F_{AC} = 362.347$ N $\tau = 4.61$ MPa .... **Ans.**

(b) $\theta = 30°$ $F_{AC} = 404.145$ N $\tau = 5.15$ MPa .... **Ans.**

(c) $\theta = 45°$ $F_{AC} = 494.975$ N $\tau = 6.30$ MPa .... **Ans.**

2-17

From a free-body diagram of the upper handle, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad D\sin 38° - A_x = 0$

$\uparrow \Sigma F_y = 0: \qquad A_y - D\cos 38° - 25 = 0$

$\circlearrowleft \Sigma M_A = 0: \qquad 1.25D - 9(25) = 0$

$D = 180 \text{ lb}$

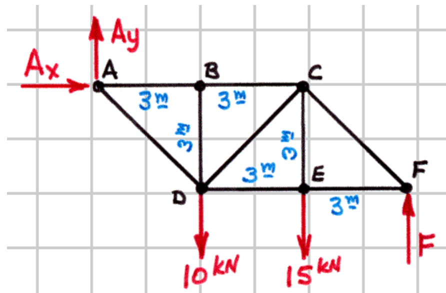
$A_x = 110.8191 \text{ lb} \qquad A_y = 166.8419 \text{ lb}$

$A = \sqrt{A_x^2 + A_y^2} = 200.2925 \text{ lb}$

$\tau = \dfrac{V}{A} = \dfrac{200.2925}{\pi(0.25)^2/4} = 4080 \text{ psi}$ .... **Ans.**

2-18

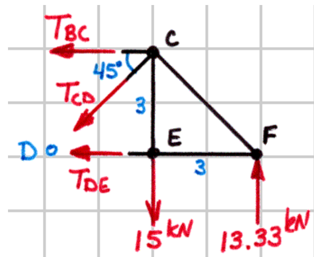
From an overall free-body diagram of the truss, the equations of equilibrium give



$$\circlearrowleft \Sigma M_A = 0: \quad 9F - 3(10) - 6(15) = 0$$

$$F = 13.3333 \text{ kN}$$

Then, from a free-body diagram of the right half of the truss, the equations of equilibrium give



$$\circlearrowleft \Sigma M_C = 0: \quad 3(13.3333) - 3T_{DE} = 0$$

$$T_{DE} = 13.3333 \text{ kN}$$

(a) $$\sigma_{DE} = \frac{13.3333(10^3)}{750(10^{-6})} = 17.78(10^6) \text{ N/m}^2 = 17.78 \text{ MPa}$$ **Ans.**

$$\circlearrowleft \Sigma M_D = 0: \quad 3T_{BC} - 3(15) + 6(13.3333) = 0$$

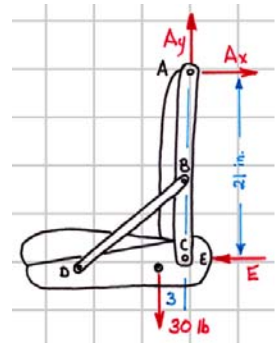
$$T_{BC} = -11.6667 \text{ kN}$$

$$\sigma_{BC} = \frac{11.6667(10^3)}{A_{BC}} \le 30(10^6) \text{ N/m}^2$$

(b) $$A_{BC} \ge 388.89(10^{-6}) \text{ m}^2 = 389 \text{ mm}^2$$ .... **Ans.**

2-19*

From an overall free-body diagram of the seat, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad A_x - E = 0$

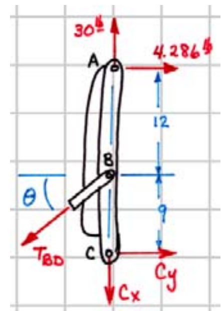
$\uparrow \Sigma F_y = 0: \qquad A_y - 30 = 0$

$\circlearrowleft \Sigma M_A = 0: \qquad 3(30) - 21E = 0$

$E = 4.28571 \text{ lb}$

$A_x = 4.28571 \text{ lb} \qquad A_y = 30 \text{ lb}$

Next, from a free-body diagram of the seat back, the equations of equilibrium give



$\circlearrowleft \Sigma M_C = 0: \qquad 9(T_{BD} \cos\theta) - 21(4.28571) = 0$

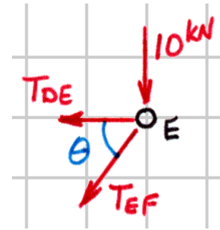
$\theta = \tan^{-1} \dfrac{10}{12} = 39.806°$

$T_{BD} = 13.01707 \text{ lb}$

$\tau = \dfrac{V_B}{A} = \dfrac{13.01707}{\pi (3/8)^2 / 4} = 117.9 \text{ psi}$ ....**Ans.**

2-20

From a free-body diagram of the pin *E*, the equations of equilibrium give



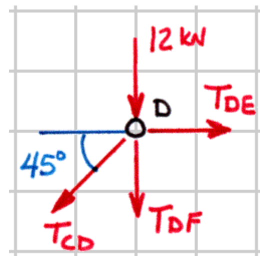
$\rightarrow \Sigma F_x = 0: \qquad -T_{DE} - T_{EF} \cos\theta = 0$

$\uparrow \Sigma F_y = 0: \qquad -10 - T_{EF} \sin\theta = 0$

$$\theta = \tan^{-1}\frac{4}{3} = 53.130°$$

$$T_{DE} = 7.500 \text{ kN} \qquad T_{EF} = -12.500 \text{ kN}$$

Next, from a free-body diagram of the pin *D*, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad (7.5) - T_{CD} \cos 45° = 0$

$\uparrow \Sigma F_y = 0: \qquad -12 - T_{CD} \sin 45° - T_{DF} = 0$

$$T_{CD} = 10.60660 \text{ kN} \qquad T_{DF} = -19.500 \text{ kN}$$

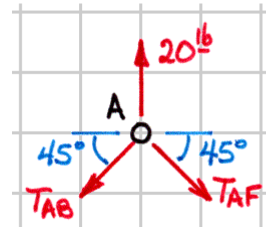
(a) $\sigma_{CD} = \dfrac{T}{A} = \dfrac{10.6066(10^3)}{624(10^{-6})} = 17.00(10^6) \text{ N/m}^2 = 17.00 \text{ MPa (T)}$ .... **Ans.**

$$\sigma_{DF} = \frac{T}{A} = \frac{19.5(10^3)}{A} \le 25(10^6) \text{ N/m}^2$$

(b) $A_{DF} \ge 780(10^{-6}) \text{ m}^2 = 780 \text{ mm}^2$ .... **Ans.**

2-21

From a free-body diagram of the pin *A*, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \qquad T_{AF} \cos 45° - T_{AB} \cos 45° = 0$

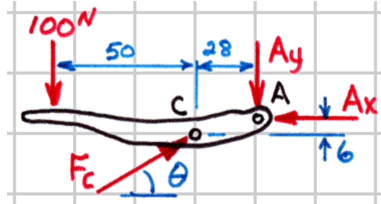
$\uparrow \Sigma F_y = 0: \qquad 20 - T_{AF} \sin 45° - T_{AB} \sin 45° = 0$

$$T_{AB} = T_{AF} = 14.14214 \text{ lb}$$

$$\tau_B = \frac{V_B}{A} = \frac{14.14214}{\pi (1/8)^2 / 4} = 1152 \text{ psi}$$ ....**Ans.**

2-22*

From a free-body diagram of the upper handle, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \quad F_C \cos\theta - A_x = 0$$

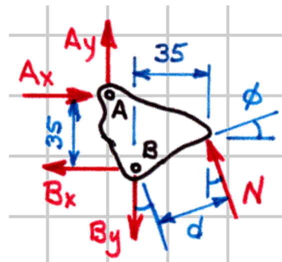
$$\uparrow \Sigma F_y = 0: \quad F_C \sin\theta - 100 - A_y = 0$$

$$\circlearrowleft \Sigma M_A = 0: \quad 93(100) - 28(F_C \sin\theta) + 5(F_C \cos\theta) = 0$$

$$F_C = 919.1059 \text{ N} \qquad \theta = \tan^{-1}\frac{30}{50} = 30.964°$$

$$A_x = 788.1248 \text{ N} \qquad A_y = 372.8794 \text{ N} \qquad A = \sqrt{A_x^2 + A_y^2} = 871.8829 \text{ N}$$

$$\tau_A = \frac{V_A}{A} = \frac{871.8829}{2\pi(0.004)^2/4} = 34.7(10^6) \text{ N/m}^2 = 34.7 \text{ MPa} \quad \textbf{Ans.}$$



Then, from a free-body diagram of the upper jaw, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \quad (788.1248) - B_x - N\sin\phi = 0$$

$$\uparrow \Sigma F_y = 0: \quad (372.8794) - B_y + N\cos\phi = 0$$

$$\circlearrowleft \Sigma M_A = 0: \quad Nd - 35(788.1248) - 12(372.8794) = 0$$

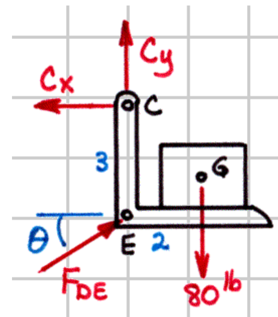
$$N = 841.9084 \text{ N} \qquad d = \sqrt{15^2 + 35^2} = 38.0789 \text{ mm} \qquad \phi = \tan^{-1}\frac{15}{35} = 23.199°$$

$$B_x = 456.4753 \text{ N} \qquad B_y = 1146.7130 \text{ N} \qquad B = \sqrt{B_x^2 + B_y^2} = 1234.229 \text{ N}$$

$$\tau_B = \frac{V_B}{A} = \frac{1234.229}{2\pi(0.005)^2/4} = 31.4(10^6) \text{ N/m}^2 = 31.4 \text{ MPa} \quad \textbf{Ans.}$$

2-23*

From a free-body diagram of the platform, the equations of equilibrium give



$$\circlearrowleft \Sigma M_C = 0: \qquad 3\left(F_{DE}\cos 30°\right) - 2(80) = 0$$

$$F_{DE} = 61.58403 \text{ lb}$$

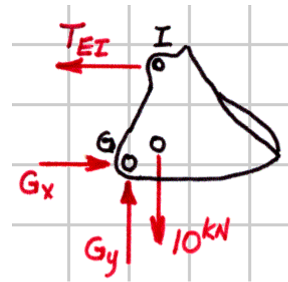
(a) Member *DE* is a two-force member and the stress on every cross section is the same

$$\sigma_{DE} = \frac{F_{DE}}{A} = \frac{61.58403}{1.25} = 49.3 \text{ psi}$$ ....**Ans.**

(b) $$\tau_E = \frac{V_E}{A} = \frac{61.58403}{2\left[\pi(0.25)^2/4\right]} = 627 \text{ psi}$$ ....**Ans.**

2-24

From a free-body diagram of the bucket, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad G_x - T_{EI} = 0$$

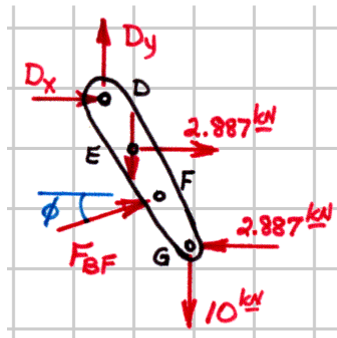
$$\uparrow \Sigma F_y = 0: \qquad G_y - 10 = 0$$

$$\circlearrowleft \Sigma M_G = 0: \quad (1.2\cos 30°)T_{EI} - 0.3(10) = 0$$

$$T_{EI} = 2.88675 \text{ kN}$$

$$G_x = 2.88675 \text{ kN} \qquad G_y = 10 \text{ kN}$$

Then, from a free-body diagram of the arm *DEFG*, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \quad D_x + (2.88675) - (2.88675) + F_{BF}\cos\phi = 0$$

$$\uparrow \Sigma F_y = 0: \qquad D_y - (1) + F_{BF}\sin\phi - (10) = 0$$

$$\circlearrowleft \Sigma M_D = 0:$$

$$(0.6\cos 30°)(2.88675) - (0.6\sin 30°)(1)$$
$$+(1.2\cos 30°)(F_{BF}\cos\phi) + (1.2\sin 30°)(F_{BF}\sin\phi)$$
$$-(1.8\cos 30°)(2.88675) - (1.8\sin 30°)(10) = 0$$

$$\phi = \tan^{-1}\frac{1.8\cos 30° - 1.2\cos 30°}{1.8\sin 30° + 1.2\sin 30°} = 19.107°$$

$$F_{BF} = 10.43809 \text{ kN}$$

$$D_x = -9.86304 \text{ kN} \qquad D_y = 7.58327 \text{ kN}$$

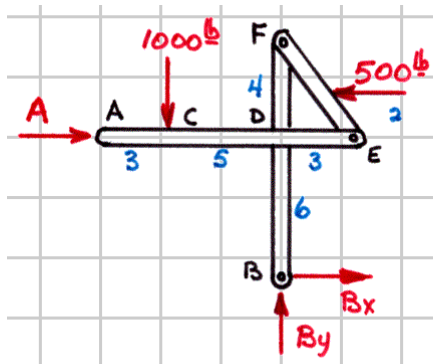
$$D = \sqrt{D_x^2 + D_y^2} = 12.44128 \text{ kN}$$

$$\tau_D = \frac{V_D}{A} = \frac{12.44128(10^3)}{2[\pi d^2/4]} \le 120(10^6) \text{ N/m}^2$$

$d \ge 0.00812$ m $= 8.12$ mm .... **Ans.**

2-25*

First, from an overall free-body diagram of the frame, the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \quad B_x - 500 + 1500 = 0$$

$$\uparrow \Sigma F_y = 0: \qquad B_y - 1000 = 0$$

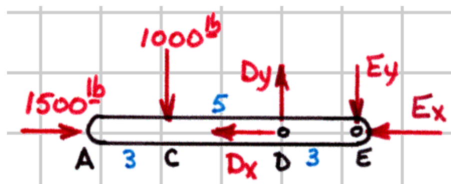
$$\circlearrowleft \Sigma M_B = 0: \qquad 8(500) + 5(1000) - 6A = 0$$

$$A = 1500 \text{ lb}$$

$$B_x = -1000 \text{ lb}$$

$$B_y = 1000 \text{ lb}$$

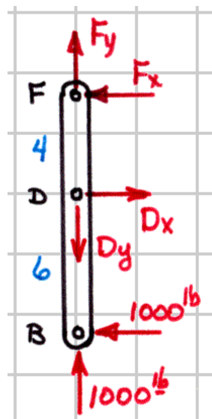
Next, from a free-body diagram of the horizontal member *ACDE*, the equations of equilibrium give



$$\circlearrowleft \Sigma M_E = 0: \qquad 8(1000) - 3D_y = 0$$

$$D_y = 2666.667 \text{ lb}$$

Finally, from a free-body diagram of the vertical member *BDF*, the equations of equilibrium give



$$\circlearrowleft \Sigma M_F = 0: \qquad 4D_x - 10(1000) = 0$$

$$D_x = 2500 \text{ lb}$$

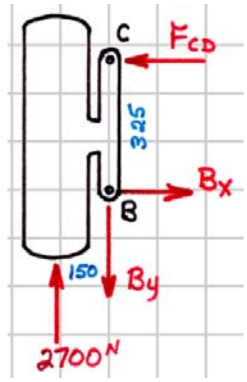
$$D = \sqrt{D_x^2 + D_y^2} = 3655.285 \text{ lb}$$

$$\tau_D = \frac{V_D}{A} = \frac{3655.285}{\pi d^2/4} \le 7500 \text{ psi}$$

$d \ge 0.788$ in. .... **Ans.**

2-26

From a free-body diagram of the wheel, the equations of equilibrium give



$\rightarrow \Sigma F_x = 0: \quad B_x - F_{CD} = 0$

$\uparrow \Sigma F_y = 0: \quad 2700 - B_y = 0$

$\circlearrowleft \Sigma M_B = 0: \quad 325 F_{CD} - 150(2700) = 0$

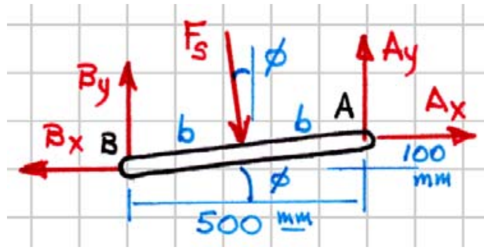
$$F_{CD} = 1246.15385 \text{ N}$$

$$B_x = 1246.15385 \text{ N}$$

$$B_y = 2700 \text{ N}$$

$$B = \sqrt{B_x^2 + B_y^2} = 2973.701 \text{ N}$$

Then from a free-body diagram of the arm *AB* (and assuming that the spring pushes perpendicularly against the arm)



$\circlearrowleft \Sigma M_A = 0: \quad -100(1246.154) - 500(2700) + bF_S = 0$

$\rightarrow \Sigma F_x = 0: \quad A_x - (1246.154) + F_S \sin\phi = 0$

$\uparrow \Sigma F_y = 0: \quad 2700 - F_S \cos\phi + A_y = 0$

$$b = \sqrt{50^2 + 250^2} = 254.951 \text{ mm}$$

$$\phi = \tan^{-1}\frac{50}{250} = 11.310°$$

$$F_S = 5783.917 \text{ N}$$

$$A_x = 111.8278 \text{ N} \qquad A_y = 2971.5958 \text{ N}$$

$$A = \sqrt{A_x^2 + A_y^2} = 2973.699 \text{ N}$$

$$\tau_A = \frac{V_A}{A} = \frac{2973.699}{2\left[\pi d_A^2/4\right]} \le 125\left(10^6\right) \text{ N/m}^2$$

$d_A \ge 0.00389 \text{ m} = 3.89 \text{ mm}$ .... **Ans.**

$$\tau_B = \frac{V_B}{A} = \frac{2973.701}{2\left[\pi d_B^2/4\right]} \le 125\left(10^6\right) \text{ N/m}^2$$

$d_B \ge 0.00389 \text{ m} = 3.89 \text{ mm}$ .... **Ans.**

The forces on pins *C* and *D* are equal (both equal to the force in the member *CD*) and their diameters will be the same

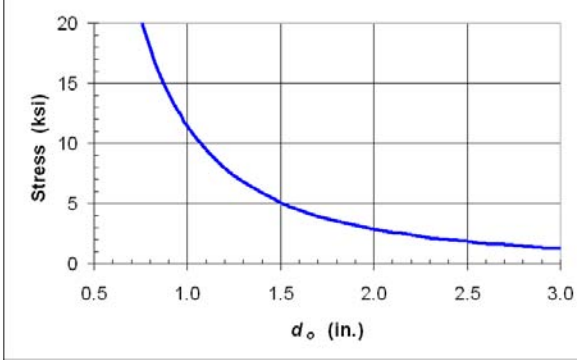
$$\tau_C = \frac{T_{CD}}{A} = \frac{1246.154}{2\left[\pi d_C^2/4\right]} \le 125\left(10^6\right) \text{ N/m}^2$$

$d_C = d_D \ge 0.00252 \text{ m} = 2.52 \text{ mm}$ .... **Ans.**

2-27

$$d_i = d_o - 2(0.1d_o) = 0.8d_o$$

$$\sigma = \frac{P}{A} = \frac{9000}{\pi\left(d_o^2 - d_i^2\right)/4} = \frac{11,459.156}{d_o^2} \text{ psi}$$



For standard steel pipe,

$$\sigma = \frac{P}{A} = \frac{9000}{A} \le 12,000 \text{ psi}$$

$$A = 0.75 \text{ in.}^2$$

$d = 1.5$ in. .... **Ans.**

2-28

For a uniformly distributed load

$$\sigma = \frac{P}{A} = \frac{P}{\pi d^2/4} \qquad d^2 = \frac{4P}{\pi\sigma} = \frac{4(150)(10^3)}{\pi(3.25)(10^6)} = 0.05876$$

$d = 0.242 \text{ m} = 242 \text{ mm}$ .... **Ans.**

For the flexible bearing plate:

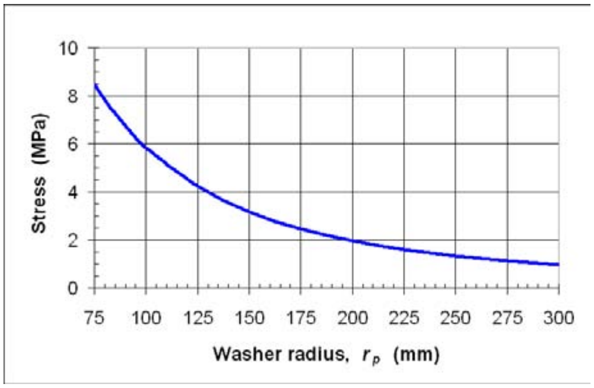
$$\sigma = \sigma_{\max} \qquad 0 \le r \le r_c \qquad r_c = 75 \text{ mm}$$

$$\sigma = \frac{\sigma_{\max}(r_p - 0.2r_c - 0.8r)}{r_p - r_c} \qquad r_c \le r \le r_p$$

$$P = \int_A \sigma\, dA = \int_0^{r_c} \sigma_{\max}(2\pi r dr) + \int_{r_c}^{r_p} \frac{\sigma_{\max}(r_p - 0.2r_c - 0.8r)}{r_p - r_c}(2\pi r dr)$$

$$= \frac{\pi}{3}\sigma_{\max}\left(0.8r_c^2 + 0.8r_p r_c + 1.4r_p^2\right) = 150 \text{ kN}$$

$$\sigma_{\max} = \frac{450(10^3)}{\pi\left(0.8r_c^2 + 0.8r_p r_c + 1.4r_p^2\right)} \quad \text{N/m}^2$$



For $\sigma_{\max} = 3.25$ MPa

$d = 2r_p \cong 296$ mm .... **Ans.**

For $d_p = 150$ mm, $(r_p = 75 \text{ mm})$

$\sigma_{\max} = 8.49$ MPa

For $d_p = 400$ mm, $(r_p = 200 \text{ mm})$

$\sigma_{\max} = 1.98$ MPa $\qquad D = \dfrac{8.49 - 1.98}{8.49}(100) = 76.7\%$ .... **Ans.**

For $d_p = 600$ mm, $(r_p = 300 \text{ mm})$

$\sigma_{\max} = 0.94$ MPa $\qquad D = \dfrac{8.49 - 0.94}{8.49}(100) = 89.0\%$ .... **Ans.**

2-29

For a uniformly distributed load

$$\sigma = \frac{P}{A} = \frac{P}{\pi\left(d_c^2 - 2^2\right)/4} \qquad d_c^2 = \frac{4P}{\pi\sigma} + 4 = \frac{4(50)}{\pi(10)} + 4 = 10.366$$

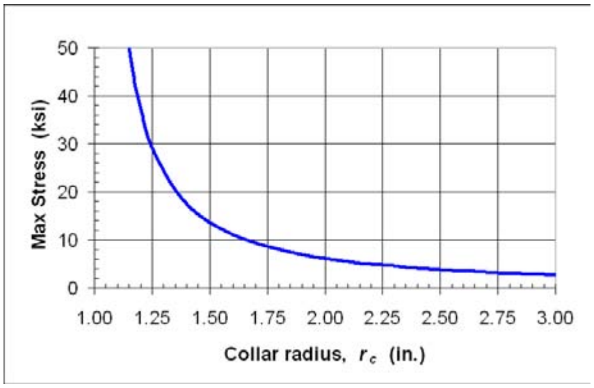
$d_c = 3.22$ in. .... **Ans.**

For the flexible collar: $\sigma = \dfrac{\sigma_{\max}(5-r)}{4}$

$$P = \int_A \sigma\, dA = \int_1^{r_c} \frac{\sigma_{\max}(5-r)}{4}(2\pi r dr)$$

$$= \frac{\pi}{4}\sigma_{\max}\left[5\left(r_c^2 - 1\right) - \frac{2}{3}\left(r_c^3 - 1\right)\right] = 50 \text{ kip}$$

$$\sigma_{\max} = \frac{200}{\pi\left[5\left(r_c^2 - 1\right) - \frac{2}{3}\left(r_c^3 - 1\right)\right]} \text{ ksi}$$



For $\sigma_{\max} = 10$ ksi

$d_c = 2r_c \cong 3.32$ in. .... **Ans.**

For $d_c = 2.4$ in., $(r_c = 1.2$ in.$)$

$\sigma_{\max} = 37.13$ ksi

For $d_c = 3.2$ in., $(r_c = 1.6$ in.$)$

$\sigma_{\max} = 11.10$ ksi $\qquad D = \dfrac{37.13 - 11.10}{37.13}(100) = 70.1\%$ .... **Ans.**

For $d_c = 4.0$ in., $(r_c = 2.0$ in.$)$

$\sigma_{\max} = 6.16$ ksi $\qquad D = \dfrac{37.13 - 6.16}{37.13}(100) = 83.4\%$ .... **Ans.**

For $d_c = 5.0$ in., $(r_c = 2.5$ in.$)$

$\sigma_{\max} = 3.86$ ksi $\qquad D = \dfrac{37.13 - 3.86}{37.13}(100) = 89.6\%$ .... **Ans.**

2-30

For a uniformly distributed load

$$\sigma = \frac{P}{A} = \frac{P}{\pi\left(d_w^2 - 0.040^2\right)/4} \qquad d_w^2 = \frac{4P}{\pi\sigma} + 0.040^2 = \frac{4(80)(10^3)}{\pi(2.8)(10^6)} + 0.0016$$

$$d_w = 0.1949 \text{ m} = 194.9 \text{ mm}$$ .... **Ans.**

For the flexible washer:

$$\sigma = \sigma_{max} \qquad 0.020 \text{ mm} \le r \le 0.030 \text{ mm}$$

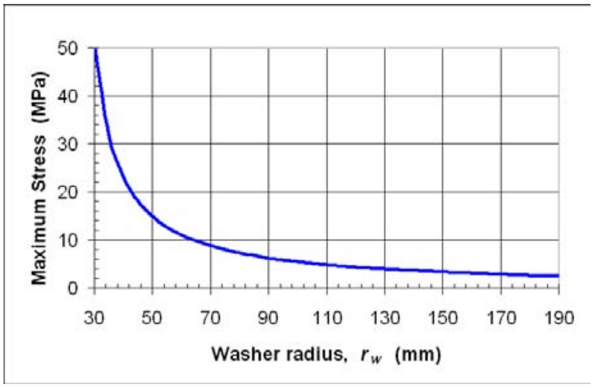
$$\sigma = \frac{0.030\sigma_{max}}{r} \qquad 0.030 \text{ mm} \le r \le r_w$$

$$P = \int_A \sigma\, dA = \int_{0.020}^{0.030} \sigma_{max}\left(2\pi r dr\right) + \int_{0.030}^{r_w} \frac{0.030\sigma_{max}}{r}\left(2\pi r dr\right)$$

$$= \pi\sigma_{max}\left(0.030^2 - 0.020^2\right) + 0.060\sigma_{max}\pi\left(r_w - 0.030\right)$$

$$= \pi\sigma_{max}\left[0.0005 + 0.060\left(r_w - 0.030\right)\right] = 80 \text{ kN}$$

$$\sigma_{max} = \frac{80(10^3)}{\pi\left[0.0005 + 0.060\left(r_w - 0.030\right)\right]} \text{ N/m}^2$$



For $\sigma_{max} = 2.8$ MPa

$$d_w = 2r_w \cong 346 \text{ mm}$$ .... **Ans.**

For no washer

$$\sigma_{max} = \frac{80(10^3)}{\pi\left(0.030^2 - 0.020^2\right)}$$

$$= 50.93(10^6) \text{ N/m}^2 = 50.93 \text{ MPa}$$

For $d_w = 200$ mm, $(r_w = 100 \text{ mm})$

$$\sigma_{max} = 5.42 \text{ MPa} \qquad D = \frac{50.93 - 5.42}{50.93}(100) = 89.4\%$$ .... **Ans.**

For $d_w = 300$ mm, $(r_w = 150 \text{ mm})$

$$\sigma_{max} = 3.31 \text{ MPa} \qquad D = \frac{50.93 - 3.31}{50.93}(100) = 93.5\%$$ .... **Ans.**

2-31*

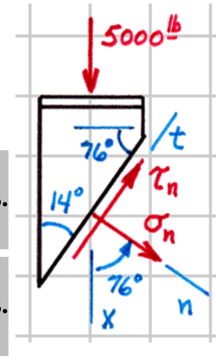
$P = -5000$ lb

$A = 4 \times 4 = 16$ in.^{2}

$\theta = 90° - 14° = 76°$

$$\sigma_n = \frac{(-5000)}{2(16)}\left[1 + \cos 2(76°)\right] = -18.29 \text{ psi} = 18.29 \text{ psi (C)}$$ ....**Ans.**

$$\tau_n = \frac{-(-5000)}{2(16)}\sin 2(76°) = +73.4 \text{ psi}$$ ....**Ans.**

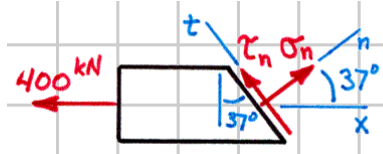


2-32*

$P = 400\text{ kN}$

$A = 75 \times 45\text{ mm}^2$

$\theta = 37°$



$$\sigma_n = \frac{(400)(10^3)}{2(0.075 \times 0.045)}\left[1 + \cos 2(37°)\right]$$

$\sigma_n = +75.6(10^6)\text{ N/m}^2 = 75.6\text{ MPa (T)}$ .... **Ans.**

$$\tau_n = \frac{-(400 \times 10^3)}{2(0.075 \times 0.045)}\sin 2(37°)$$

$\tau_n = -57.0(10^6)\text{ N/m}^2 = -57.0\text{ MPa}$ .... **Ans.**

2-33

$$\sigma_{\max} = \frac{P}{A} = \frac{270}{2 \times 6} = 22.5 \text{ ksi (T)}$$ .... **Ans.**

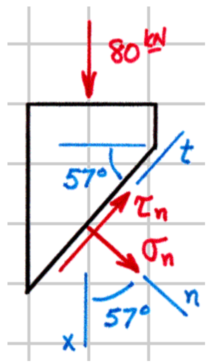
$$\tau_{\max} = \frac{P}{2A} = \frac{270}{2(2 \times 6)} = 11.25 \text{ ksi}$$ .... **Ans.**

2-34

$P = -80 \text{ kN}$

$A = \pi(75)^2/4 = 4417.865 \text{ mm}^2$

$\theta = 57°$



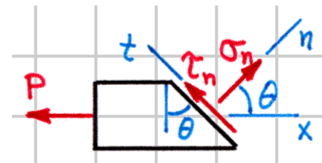
$$\sigma_n = \frac{(-80)(10^3)}{2(4417.865)(10^{-6})}\left[1+\cos 2(57°)\right]$$

$\sigma_n = -5.37(10^6) \text{ N/m}^2 = 5.37 \text{ MPa (C)}$ .... **Ans.**

$$\tau_n = \frac{-(-80\times 10^3)}{2(4417.865)(10^{-6})}\sin 2(57°)$$

$\tau_n = +8.27(10^6) \text{ N/m}^2 = +8.27 \text{ MPa}$ .... **Ans.**

2-35*



$$\sigma_n = \frac{P}{2(4\times1)}[1+\cos 2\theta] = 12 \text{ ksi}$$

$$\tau_n = \frac{-P}{2(4\times1)}\sin 2\theta = -9 \text{ ksi}$$

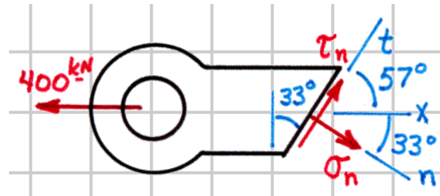
$$\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{9}{12}$$

Solve by trial and error to get

$\theta = 36.870°$ .... **Ans.**

$P = 75.0 \text{ kip}$ .... **Ans.**

2-36*



$$\sigma_n = \frac{(400)(10^3)}{2(0.1t)}\left[1+\cos 2(-33°)\right] \le 70(10^6)\ \text{N/m}^2$$

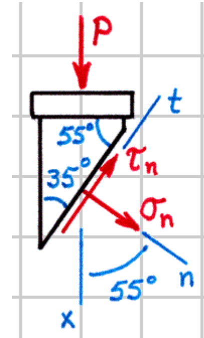
$$t \ge 0.0402\ \text{m} = 40.2\ \text{mm}$$

$$\tau_n = \frac{-(400\times 10^3)}{2(0.1t)}\sin 2(-33°) \le 45(10^6)\ \text{N/m}^2$$

$$t \ge 0.0406\ \text{m} = 40.6\ \text{mm}$$

$t_{\min} = 40.6$ mm .... **Ans.**

2-37



$$\tau_a = \frac{-(-P)}{2(4\times 8)}\sin 2(55°) = 2 \text{ ksi}$$

(a) $P = 136.21475 \text{ kip} \cong 136.2 \text{ kip (C)}$ ....**Ans.**

$$\sigma_a = \frac{(-136.21475)}{2(4\times 8)}\left[1+\cos 2(55°)\right]$$

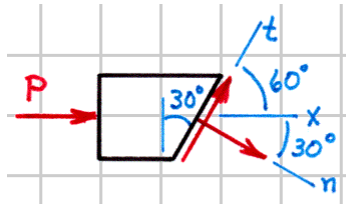
(b) $\sigma_a = -1.400 \text{ ksi} = 1.400 \text{ ksi (C)}$ ....**Ans.**

(c) $\sigma_{\max} = \frac{P}{A} = \frac{136.21475}{4\times 8} = 4.26 \text{ ksi}$ .... **Ans.**

$\tau_{\max} = \frac{P}{2A} = \frac{136.21475}{2(4\times 8)} = 2.13 \text{ ksi}$ .... **Ans.**

2-38

$$\sigma_n = \frac{P}{2(0.200)(0.120)}\left[1+\cos 2(-30°)\right] \le 13.60(10^6) \text{ N/m}^2$$



$$P \le 435.2(10^3) \text{ N}$$

$$\tau_n = \frac{-P}{2(0.200)(0.120)}\sin 2(-30°) \le 5.25(10^6) \text{ N/m}^2$$

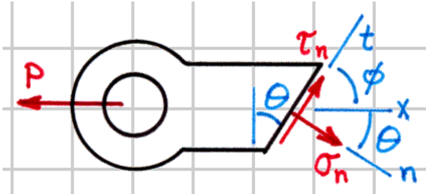
$$P \le 291.0(10^3) \text{ N}$$

$$\tau_{\max} = \frac{P}{2A} = \frac{P}{2(0.200)(0.120)} \le 8.75(10^6) \text{ N/m}^2$$

$$P \le 420(10^3) \text{ N}$$

$P_{\max} = 291$ kN .... **Ans.**

2-39*



$$\sigma_n = \frac{P}{2(4\times 1)}\left[1+\cos 2(-\theta)\right] \le 12 \text{ ksi}$$

$$\tau_n = \frac{-P}{2(4\times 1)}\sin 2(-\theta) \le 9 \text{ ksi}$$

Optimum angle occurs when $\sigma_n = 12$ ksi and $\tau_n = 9$ ksi

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{9}{12}$$

Solve by trial and error to get $\theta = 36.870°$

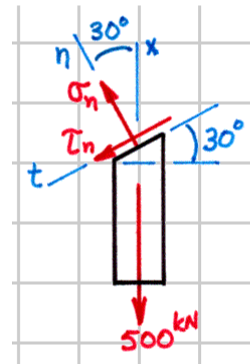
(a) $\phi = 90° - \theta = 53.13°$ .... **Ans.**

(b) $P = 75.0$ kip .... **Ans.**

2-40*

$T_{AB} = 500 \text{ kN (T)}$ $T_{BC} = 300 \text{ kN (T)}$ $T_{top} = 250 \text{ kN (T)}$

(a) On plane *a–a*



$$\sigma_n = \frac{(500)(10^3)}{2(0.200)(0.100)}\left[1+\cos 2(30°)\right]$$

$$\sigma_n = 18.75(10^6) \text{ N/m}^2 = 18.75 \text{ MPa (T)}$$ ....**Ans.**

$$\tau_n = \frac{-(500)(10^3)}{2(0.200)(0.100)}\sin 2(30°)$$

$$\tau_n = -10.83(10^6) \text{ N/m}^2 = -10.83 \text{ MPa}$$ ....**Ans.**

(b) The maximum stresses in the bar occur in the section with the maximum load

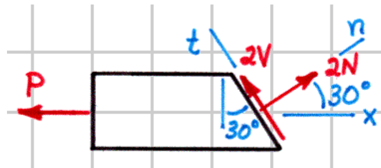
$$P_{\max} = T_{AB} = 500 \text{ kN (T)}$$

$$\sigma_{\max} = \frac{P}{A} = \frac{500(10^3)}{(0.200)(0.100)} = 25.0(10^6) \text{ N/m}^2 = 25.0 \text{ MPa (T)}$$ .... **Ans.**

$$\tau_{\max} = \frac{P}{2A} = \frac{500(10^3)}{2(0.200)(0.100)} = 12.50(10^6) \text{ N/m}^2 = 12.50 \text{ MPa}$$ .... **Ans.**

2-41

There are two bolts and they each carry a normal force of N and a shear force of V. Equilibrium of the eyebar gives



Z $\Sigma F_n = 0:$ $\qquad 2N - P\cos 30° = 0$

^ $\Sigma F_t = 0:$ $\qquad 2V + P\sin 30° = 0$

$$N = \frac{P\cos 30°}{2} \le 12\left[\frac{\pi(0.5)^2}{4}\right] \text{ kip}$$

$$P \le 5.441 \text{ kip}$$

$$V = \frac{P\sin 30°}{2} \le 8\left[\frac{\pi(0.5)^2}{4}\right] \text{ kip}$$

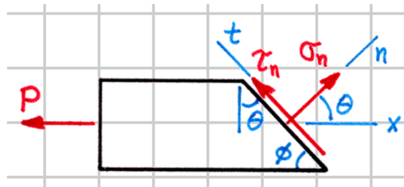
$$P \le 6.283 \text{ kip}$$

$P_{max} = 5.44$ kip .... **Ans.**

2-42

$$\sigma_n = \frac{P}{2(0.050)(0.100)}\left[1+\cos 2(-\theta)\right] \le 5\left(10^6\right)\ \text{N/m}^2$$

$$\tau_n = \frac{-P}{2(0.050)(0.100)}\sin 2(-\theta) \le 3\left(10^6\right)\ \text{N/m}^2$$



Optimum angle occurs when $\sigma_n = 5\left(10^6\right)\ \text{N/m}^2$ and $\tau_n = 3\left(10^6\right)\ \text{N/m}^2$

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{30}{50}$$

Solve by trial and error to get $\theta = 30.964°$

(a) $\phi = 90° - \theta = 59.036°$ .... **Ans.**

(b) $P = 34.0\left(10^3\right)\ \text{N} = 34.0\ \text{kN}$ .... **Ans.**

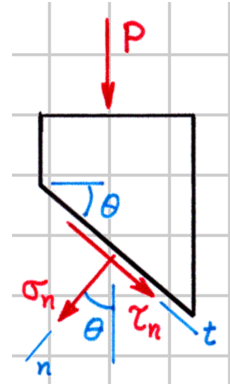
2-43

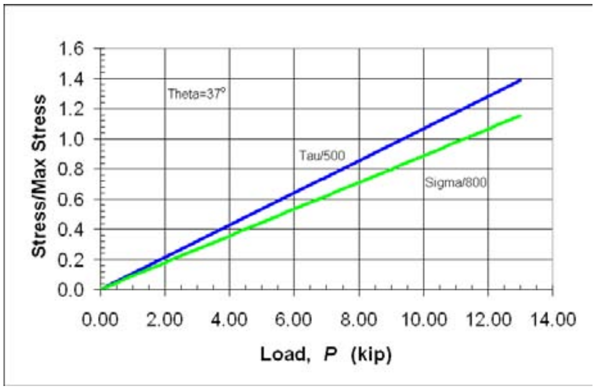
$$\sigma_n = \frac{P}{2(3\times 3)}\left[1+\cos 2(-\theta)\right] \text{ psi}$$

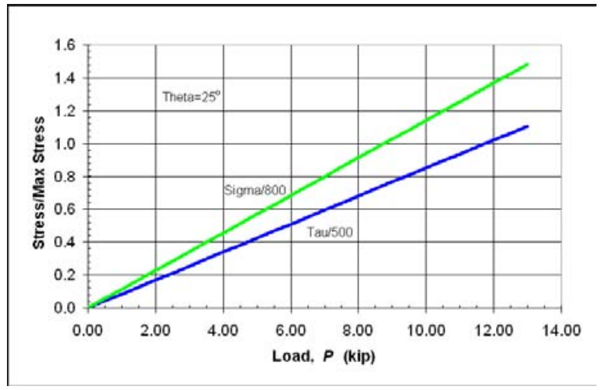
$$\tau_n = \frac{-P}{2(3\times 3)}\sin 2(-\theta) \text{ psi}$$

$$\frac{\sigma_n}{800} = \frac{P}{14,400}[1+\cos 2\theta]$$

$$\frac{\tau_n}{500} = \frac{P}{9000}\sin 2\theta$$







When $\theta = 37°$ the shear stress reaches its maximum value $(\tau_n/500 = 1)$ first at which point

$P_{max} \cong 9.36$ kip .... **Ans.**

When $\theta = 25°$ the normal stress reaches its maximum value $(\sigma_n/800 = 1)$ first at which point

$P_{max} \cong 8.77$ kip .... **Ans.**

For simultaneous control

$$\frac{\sigma_n}{800} = \frac{P}{14,400}[1+\cos 2\theta] = \frac{P}{9000}\sin 2\theta = \frac{\tau_n}{500} = 1$$

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta} = \tan\theta = \frac{9000}{14,400}$$

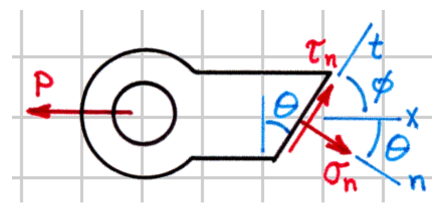
$\theta = 32.01°$ .... **Ans.**

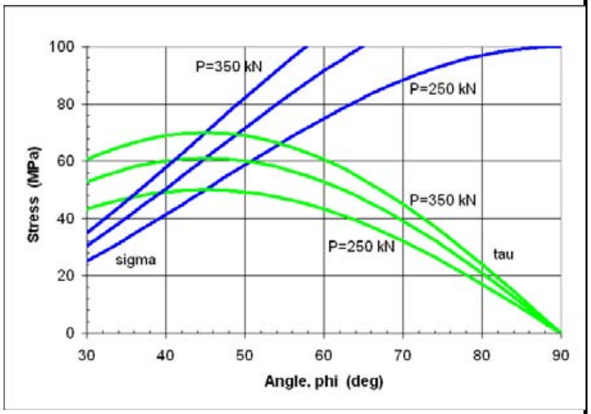
2-44

$$\sigma_n = \frac{250(10^3)}{2(0.100)(0.025)}\left[1+\cos 2(-\theta)\right] \text{ N/m}^2$$
$$= 50(1+\cos 2\theta) \text{ MPa}$$

$$\tau_n = \frac{-250(10^3)}{2(0.100)(0.025)}\sin 2(-\theta) \text{ N/m}^2$$
$$= 50\sin 2\theta \text{ MPa}$$

$$\theta = 90° - \phi$$





For $\sigma_n \le 80$ MPa and $\tau_n \le 60$ MPa

$P = 250$ kN	$30° \le \phi \le 63°$
$P = 305$ kN	$30° \le \phi \le 40°$
	$50° \le \phi \le 54°$
$P = 350$ kN	$\phi < 30°$

2-45*

The given values are

$$\sigma_x = 20 \text{ ksi} \qquad \sigma_y = -10 \text{ ksi} \qquad \tau_{xy} = 0 \text{ ksi} \qquad \theta_{ab} = -26°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$= (20)\cos^2(-26°) + (-10)\sin^2(-26°) + 2(0)\sin(-26°)\cos(-26°)$$

$\sigma_{ab} = +14.23 \text{ ksi} = 14.23 \text{ ksi (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(20) - (-10)]\sin(-26°)\cos(-26°) + (0)[\cos^2(-26°) - \sin^2(-26°)]$$

$\tau_{ab} = +11.82 \text{ ksi}$ .... **Ans.**

2-46*

The given values are

$$\sigma_x = 95 \text{ MPa} \qquad \sigma_y = 125 \text{ MPa} \qquad \tau_{xy} = 0 \text{ MPa} \qquad \theta_{ab} = 110°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (95)\cos^2(110°) + (125)\sin^2(110°) + 2(0)\sin(110°)\cos(110°)$$

$\sigma_{ab} = +121.5 \text{ MPa} = 121.5 \text{ MPa (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(95)-(125)]\sin(110°)\cos(110°) + (0)[\cos^2(110°) - \sin^2(110°)]$$

$\tau_{ab} = -9.64 \text{ MPa}$ .... **Ans.**

2-47

The given values are

$$\sigma_x = 0 \text{ ksi} \qquad \sigma_y = 0 \text{ ksi} \qquad \tau_{xy} = 15 \text{ ksi} \qquad \theta_{ab} = 30°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (0)\cos^2(30°) + (0)\sin^2(30°) + 2(15)\sin(30°)\cos(30°)$$

$\sigma_{ab} = +12.99 \text{ ksi} = 12.99 \text{ ksi (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(0)-(0)]\sin(30°)\cos(30°) + (15)[\cos^2(30°) - \sin^2(30°)]$$

$\tau_{ab} = +7.50 \text{ ksi}$ .... **Ans.**

2-48*

The given values are

$$\sigma_x = -65 \text{ MPa} \qquad \sigma_y = -125 \text{ MPa} \qquad \tau_{xy} = 75 \text{ MPa} \qquad \theta_{ab} = 145°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (-65)\cos^2(145°) + (-125)\sin^2(145°) + 2(75)\sin(145°)\cos(145°)$$

$\sigma_{ab} = -155.2 \text{ MPa} = 155.2 \text{ MPa (C)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(-65) - (-125)]\sin(145°)\cos(145°) + (75)[\cos^2(145°) - \sin^2(145°)]$$

$\tau_{ab} = +53.8 \text{ MPa}$ .... **Ans.**

2-49

The given values are

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = 6 \text{ ksi} \qquad \tau_{xy} = 15 \text{ ksi} \qquad \theta_{ab} = 155°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$= (18)\cos^2(155°) + (6)\sin^2(155°) + 2(15)\sin(155°)\cos(155°)$$

$\sigma_{ab} = +4.37 \text{ ksi} = 4.37 \text{ ksi (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(18)-(6)]\sin(155°)\cos(155°) + (15)[\cos^2(155°) - \sin^2(155°)]$$

$\tau_{ab} = +14.24 \text{ ksi}$ .... **Ans.**

2-50

The given values are

$$\sigma_x = -170 \text{ MPa} \qquad \sigma_y = 0 \text{ MPa} \qquad \tau_{xy} = -70 \text{ MPa} \qquad \theta_{ab} = 145°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (-170)\cos^2(145°) + (0)\sin^2(145°) + 2(-70)\sin(145°)\cos(145°)$$

$\sigma_{ab} = -48.3 \text{ MPa} = 48.3 \text{ MPa (C)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(-170)-(0)]\sin(145°)\cos(145°) + (-70)[\cos^2(145°) - \sin^2(145°)]$$

$\tau_{ab} = -103.8 \text{ MPa}$ .... **Ans.**

2-51*

The given values are

$$\sigma_x = 0 \text{ psi} \qquad \sigma_y = \frac{-5000}{(4)(4)} = -312.5 \text{ psi} \qquad \tau_{xy} = 0 \text{ psi} \qquad \theta = 166°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$= (0)\cos^2(166°) + (-312.5)\sin^2(166°) + 2(0)\sin(166°)\cos(166°)$$

$\sigma_n = -18.29 \text{ psi} = 18.29 \text{ psi (C)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -\left[(0) - (-312.5)\right]\sin(166°)\cos(166°) + (0)\left[\cos^2(166°) - \sin^2(166°)\right]$$

$\tau_n = +73.4 \text{ psi}$ .... **Ans.**

2-52*

The given values are

$$\sigma_x = \frac{400\left(10^3\right)}{(0.100)(0.040)} = 100\left(10^6\right) \text{ N/m}^2 = 100 \text{ MPa}$$

$$\sigma_y = 0 \text{ MPa} \qquad \tau_{xy} = 0 \text{ MPa} \qquad \theta = -33°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (100)\cos^2(-33°) + (0)\sin^2(-33°) + 2(0)\sin(-33°)\cos(-33°)$$

$\sigma = +70.3$ MPa $= 70.3$ MPa (T) .... **Ans.**

$$\tau_{nt} = -\left(\sigma_x - \sigma_y\right)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

$$= -\left[(100) - (0)\right]\sin(-33°)\cos(-33°) + (0)\left[\cos^2(-33°) - \sin^2(-33°)\right]$$

$\tau_{ab} = +45.7$ MPa .... **Ans.**

2-53

The given values are

$$\sigma_x = 0 \text{ psi} \qquad \sigma_y = \frac{-P}{9} \text{ psi} \qquad \tau_{xy} = 0 \text{ ksi} \qquad \theta_{aa} = \tan^{-1}\frac{4}{3} = 53.130°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (0) + (P/9)\sin^2(53.13°) + (0) \le 800 \text{ psi}$$

$$P \le 11,250 \text{ lb}$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= (P/9)\sin(53.13°)\cos(53.13°) + (0) \le 500 \text{ psi}$$

$$P \le 9375 \text{ lb}$$

$P \le 9.37$ kip .... **Ans.**

2-54
The given values are

$$\sigma_x = \tau_{xy} = 0 \text{ MPa} \qquad \sigma_y = \frac{-P}{(0.100)(0.200)} = -50P \text{ N/m}^2 \qquad \theta_{ab} = -35°$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(0)-(-50P)]\sin(-35°)\cos(-35°) + (0) = 15(10^6) \text{ N/m}^2$$

(a) $P = 638.51(10^3) \text{ N} \cong 639 \text{ kN (C)}$ .... **Ans.**

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (0) + (-50)(638.51)(10^3)\sin^2(-35°) + (0)$$

(b) $\sigma_{ab} = -10.52(10^6) \text{ N/m}^2 = 10.52 \text{ MPa (C)}$ .... **Ans.**

2-55*

The given values are

$$\sigma_x = \tau_{xy} = 0 \text{ ksi} \qquad \sigma_y = -32/b^2 \text{ ksi} \qquad \theta = -20°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (0) + \left(-32/b^2\right)\sin^2(-20°) + (0) \le 3.5 \text{ ksi}$$

$$b \ge 1.034 \text{ in.}$$

$$\tau_{nt} = -\left(\sigma_x - \sigma_y\right)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

$$= -\left[(0) - \left(-32/b^2\right)\right]\sin(-20°)\cos(-20°) + (0) \le 0.8 \text{ ksi}$$

$b \ge 3.59$ in. .... **Ans.**

2-56

The given values are

$$\sigma_x = \frac{-P}{(0.2)(0.12)} = -41.667P \text{ N/m}^2 \qquad \sigma_y = \tau_{xy} = 0 \text{ MPa} \qquad \theta = -30°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (-41.667P)\cos^2(-30°) + (0) + (0) \le 13.60(10^6) \text{ N/m}^2$$

$$P \le 435.2(10^3) \text{ N}$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(-41.667P) - (0)]\sin(-30°)\cos(-30°) + (0) \le 5.25(10^6) \text{ N/m}^2$$

$$P \le 291(10^3) \text{ N}$$

$P_{max} = 291$ kN .... **Ans.**

2-57*

The given values are

$$\sigma_x = 0 \text{ ksi} \qquad \theta = -\tan^{-1}\frac{4}{3} = -53.130° \qquad \sin\theta = -0.8000 \qquad \cos\theta = 0.6000$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (0) + \sigma_y(-0.8)^2 + 2\tau_{xy}(-0.8)(0.6) = 4800 \text{ psi}$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -\left[(0) - \sigma_y\right](-0.8)(0.6) + \tau_{xy}\left[(0.6)^2 - (-0.8)^2\right] = 1500 \text{ psi}$$

$$0.64\sigma_y - 0.96\tau_{xy} = 4800 \text{ psi}$$

$$-0.48\sigma_y - 0.28\tau_{xy} = 1500 \text{ psi}$$

(a) $\tau_{xy} = -5100 \text{ psi}$ .... **Ans.**

(b) $\sigma_y = -150 \text{ psi} = 150 \text{ psi (C)}$ .... **Ans.**

2-58*

The given values are

$$\sigma_y = 0 \text{ MPa} \qquad \tau_{xy} = 25 \text{ MPa} \qquad \theta_{ab} = 90° + \tan^{-1}\frac{5}{12} = 112.620°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= \sigma_x \cos^2\theta_{ab} + (0) + 2(25)\sin\theta_{ab}\cos\theta_{ab} = 15 \text{ MPa}$$

(a) $\sigma_x = 221.398 \text{ MPa} \cong 221 \text{ MPa (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(221.398) - (0)]\sin\theta_{ab}\cos\theta_{ab} + (25)(\cos^2\theta_{ab} - \sin^2\theta_{ab})$$

(b) $\tau_{ab} = +61.0 \text{ MPa}$ .... **Ans.**

2-59

(a) The given values are

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = 13 \text{ ksi} \qquad \tau_{xy} = 6 \text{ ksi} \qquad \theta_{ab} = -19°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (18)\cos^2(-19°) + (13)\sin^2(-19°) + 2(6)\sin(-19°)\cos(-19°)$$

$\sigma_{ab} = +13.78 \text{ ksi} = 13.78 \text{ ksi (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(18)-(13)]\sin(-19°)\cos(-19°) + (6)[\cos^2(-19°) - \sin^2(-19°)]$$

$\tau_{ab} = +6.27 \text{ ksi}$ .... **Ans.**

(b) The given values are

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = 13 \text{ ksi} \qquad \tau_{xy} = 6 \text{ ksi} \qquad \theta_n = 26° \qquad \theta_t = 116°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (18)\cos^2(26°) + (13)\sin^2(26°) + 2(6)\sin(26°)\cos(26°)$$

$\sigma_n = +21.77 \text{ ksi} = 21.77 \text{ ksi (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(18)-(13)]\sin(26°)\cos(26°) + (6)[\cos^2(26°) - \sin^2(26°)]$$

$\tau_{nt} = +1.724 \text{ ksi}$ .... **Ans.**

$$\sigma_t = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (18)\cos^2(116°) + (13)\sin^2(116°) + 2(6)\sin(116°)\cos(116°)$$

$\sigma_t = +9.23 \text{ ksi} = 9.23 \text{ ksi (T)}$ .... **Ans.**

2-60*

(a) Using *x*-*y*-coordinates rotated to align with the *n*-*t*-coordinates

$$\sigma_n = \sigma_x = 200 \text{ MPa} \qquad \sigma_t = \sigma_y = 50 \text{ MPa} \qquad \tau_{nt} = \tau_{xy} = 0 \text{ MPa} \qquad \theta_{aa} = 135°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$= (200)\cos^2(135°) + (50)\sin^2(135°) + (0)$$

$\sigma_{aa} = +125.0 \text{ MPa} = 125.0 \text{ MPa (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(200) - (50)]\sin(135°)\cos(135°) + (0)$$

$\tau_{aa} = +75.0 \text{ MPa}$ .... **Ans.**

(b) Using the original coordinate system with the *x*- and *y*-axes horizontal and vertical

$$\theta_n = 18° \qquad \theta_t = 108°$$

$$\sigma_n = 200 \text{ MPa} \qquad 200 = \sigma_x \cos^2(18°) + \sigma_y \sin^2(18°) + 2\tau_{xy} \sin(18°)\cos(18°)$$

$$\sigma_t = 50 \text{ MPa} \qquad 50 = \sigma_x \cos^2(108°) + \sigma_y \sin^2(108°) + 2\tau_{xy} \sin(108°)\cos(108°)$$

$$\tau_{nt} = 0 \text{ MPa} \qquad 0 = -(\sigma_x - \sigma_y)\sin(18°)\cos(18°) + \tau_{xy}\left[\cos^2(18°) - \sin^2(18°)\right]$$

$\sigma_x = 185.7 \text{ MPa (T)}$ .... $\sigma_y = 64.3 \text{ MPa (T)}$ .... $\tau_{xy} = 44.1 \text{ MPa}$ .... **Ans.**

2-61

The given values are

$$\sigma_x = 8 \text{ ksi} \qquad \sigma_y = 0 \text{ ksi} \qquad \theta_{aa} = 90° + \tan^{-1}\frac{3}{4} = 126.870°$$

$$\sigma_n = 8 \text{ ksi} \qquad 8 = (8)\cos^2 126.870° + (0) + 2\tau_{xy} \sin 126.870° \cos 126.870°$$

(a) $\tau_{xy} = \tau_h = \tau_v = -5.3333 \text{ ksi} \cong -5.33 \text{ ksi}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(8)-(0)]\sin\theta_{aa}\cos\theta_{aa} + (-5.3333)[\cos^2\theta_{aa} - \sin^2\theta_{aa}]$$

(b) $\tau_a = +5.33 \text{ ksi}$ .... **Ans.**

2-62
The given values are

$$\sigma_y = 0 \text{ MPa} \qquad \tau_{xy} = 25 \text{ MPa} \qquad \theta_{ab} = 90° + \tan^{-1}\frac{5}{12} = 112.620°$$

$$\sigma_n = 15 \text{ MPa} \qquad 15 = \sigma_x \cos^2(112.620°) + (0) + 2(25)\sin(112.620°)\cos(112.620°)$$

(a) $\sigma_x = +221.40 \text{ MPa} \cong 221 \text{ MPa (T)}$ .... **Ans.**

(b)

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(-221.40) - (0)]\sin\theta_{ab}\cos\theta_{ab} + (25)[\cos^2\theta_{ab} - \sin^2\theta_{ab}]$$

$\tau_{ab} = +61.0 \text{ MPa}$ .... **Ans.**

2-63*

The given values are

$$\sigma_y = 2\sigma_x \qquad \tau_{xy} = 0 \text{ ksi} \qquad \theta = 35°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= \sigma_x \cos^2(35°) + (2\sigma_x)\sin^2(35°) + (0) \le 10 \text{ ksi}$$

$$\sigma_x \le 7.52451 \text{ ksi}$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[\sigma_x - (2\sigma_x)]\sin(35°)\cos(35°) + (0) \le 7 \text{ ksi}$$

$$\sigma_x \le 14.8985 \text{ ksi}$$

$(\sigma_x)_{max} = 7.52$ ksi .... **Ans.**

2-64

The given values are $\theta_{bb} = 126.870°$ $\theta_{cc} = 36.870°$

$\sigma_{bb} = 125 \text{ MPa}$ $125 = \sigma_x \cos^2(126.87°) + \sigma_y \sin^2(126.87°) + (0)$

$\sigma_{cc} = -225 \text{ MPa}$ $-225 = \sigma_x \cos^2(36.87°) + \sigma_y \sin^2(36.87°) + (0)$

$\tau_{bc} = 0 \text{ MPa}$ $0 = -(\sigma_x - \sigma_y)\sin(126.87°)\cos(126.87°) + (0)$

$$0.3600\sigma_x + 0.6400\sigma_y - 0.9600\tau_{xy} = 125$$

$$0.6400\sigma_x + 0.3600\sigma_y + 0.9600\tau_{xy} = -225$$

$$0.4800\sigma_x - 0.4800\sigma_y - 0.2800\tau_{xy} = 0$$

(a) $\sigma_x = -99.0 \text{ MPa} = 99.0 \text{ MPa (C)}$ .... **Ans.**

$\tau_{xy} = -168.0 \text{ MPa}$ .... **Ans.**

(b) $\sigma_y = -1.000 \text{ MPa} = 1.000 \text{ MPa (C)}$ .... **Ans.**

$\tau_{xy} = -168.0 \text{ MPa}$ .... **Ans.**

2-65

The given values are $\sigma_x = \frac{P}{4}$ ksi $\sigma_y = 0$ ksi $\tau_{xy} = 0$ ksi

$\sigma_{ab} = 12$ ksi $12 = \sigma_x \cos^2\theta + (0) + (0)$

$\tau_{ab} = -9$ ksi $-9 = -[\sigma_x - (0)]\sin\theta\cos\theta + (0)$

$\frac{\sigma_x \sin\theta\cos\theta}{\sigma_x \cos^2\theta} = \tan\theta = \frac{9}{12}$ $\theta = 36.870°$ .... **Ans.**

$\sigma_x = 18.750 \text{ ksi} = P/4$ $P = 75.0$ kip .... **Ans.**

2-66

(a) $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$

$\sigma_n = \left[(60)\cos^2\theta + (0) + 2(-40)\sin\theta\cos\theta\right]$ MPa .... **Ans.**

$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$

$\tau_{nt} = \left\{-\left[(60)-(0)\right]\sin\theta\cos\theta + (-40)(\cos^2\theta - \sin^2\theta)\right\}$ MPa .... **Ans.**

(b) At $\theta = 153.435°$

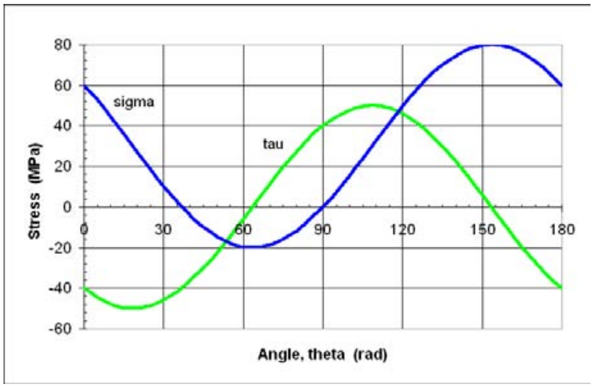
$\sigma_{max} = 80$ MPa .... **Ans.**

$\tau = 0$ MPa .... **Ans.**

At $\theta = 63.435°$

$\sigma_{min} = -20$ MPa .... **Ans.**

$\tau = 0$ MPa .... **Ans.**



(c) At $\theta = 108.435°$

$\tau_{max} = 50$ MPa .... **Ans.**

$\sigma = 30$ MPa .... **Ans.**

At $\theta = 18.435°$

$\tau_{min} = -50$ MPa .... **Ans.**

$\sigma = 30$ MPa .... **Ans.**

2-67

(a) $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$

$\sigma_n = \left[(18)\cos^2\theta + (13)\sin^2\theta + 2(6)\sin\theta\cos\theta\right]$ ksi .... **Ans.**

$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$

$= \left\{-\left[(18)-(13)\right]\sin\theta\cos\theta + (6)\left(\cos^2\theta - \sin^2\theta\right)\right\}$ ksi .... **Ans.**

(b) At $\theta = 33.69°$

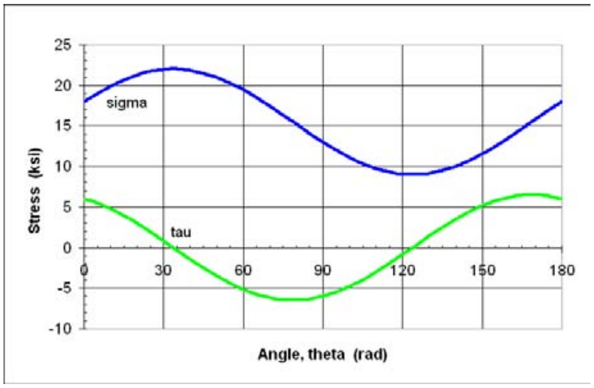
$\sigma_{max} = 22$ ksi .... **Ans.**

$\tau = 0$ MPa .... **Ans.**

At $\theta = 123.69°$

$\sigma_{min} = 9$ ksi .... **Ans.**

$\tau = 0$ MPa .... **Ans.**



(c) At $\theta = 168.69°$

$\tau_{max} = 6.5$ ksi .... **Ans.**

$\sigma = 15.5$ ksi .... **Ans.**

At $\theta = 78.69°$

$\tau_{max} = -6.5$ ksi .... **Ans.**

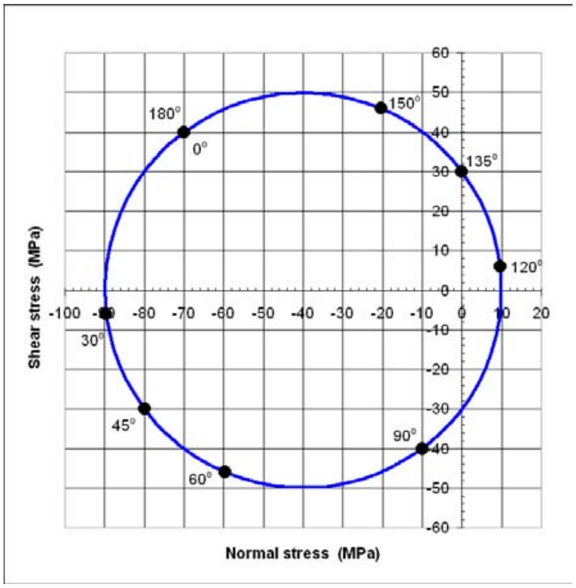
$\sigma = 15.5$ ksi .... **Ans.**

2-68

Note that the θ on the figure is for the surface rather than for the normal to the surface. Therefore, need to use $\phi = 90° - \theta$ in the transformation equations.

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$
$$+2\tau_{xy}\sin\theta\cos\theta$$
$$= (-10)\cos^2\phi + (-70)\sin^2\phi$$
$$+2(40)\sin\phi\cos\phi$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta$$
$$+\tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$
$$= -\left[(-10)-(-70)\right]\sin\phi\cos\phi$$
$$+40\left(\cos^2\phi - \sin^2\phi\right)$$



2-69
The given values are

$$\sigma_x = -15 \text{ ksi} \qquad \sigma_y = 10 \text{ ksi} \qquad \tau_{xy} = 8 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(8)}{(-15)-(10)} = -16.310°, \quad 73.690°$$

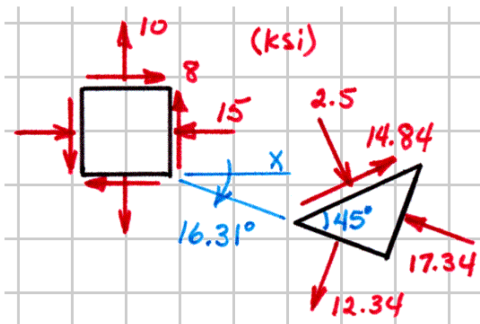
When $\theta_p = -16.310°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= (-15)\cos^2\theta_p + (10)\sin^2\theta_p + 2(8)\sin\theta_p\cos\theta_p$$
$$= -17.341 \text{ ksi} = \sigma_{p2}$$

$$\sigma_{p1} = \sigma_x + \sigma_y - \sigma_{p2} = 12.341 \text{ ksi}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 14.841 \text{ ksi}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = -2.500 \text{ ksi}$$



(a) $\sigma_{p1} = 12.34$ ksi (T) ∡ 73.69° .... **Ans.**

$\sigma_{p2} = 17.34$ ksi (C) ∡ 16.31° .... **Ans.**

$\tau_{max} = \tau_p = 14.841$ ksi .... **Ans.**

2-70

The given values are

$$\sigma_x = 50 \text{ MPa} \qquad \sigma_y = 20 \text{ MPa} \qquad \tau_{xy} = 40 \text{ MPa}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(40)}{(50)-(20)} = 34.722°, \quad -55.278°$$

When $\theta_p = 34.722°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

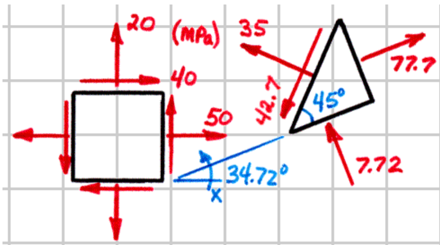
$$= (50)\cos^2\theta_p + (20)\sin^2\theta_p + 2(40)\sin\theta_p\cos\theta_p$$

$$= 77.720 \text{ MPa} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = -7.720 \text{ MPa}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 42.720 \text{ MPa}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = 35.00 \text{ MPa}$$



(a) $\sigma_{p1} = 77.7$ MPa (T) ∡ 34.72° .... **Ans.**

$\sigma_{p2} = 7.72$ MPa (C) ∢ 55.28° .... **Ans.**

$\tau_{max} = \tau_p = 42.7$ MPa .... **Ans.**

2-71
The given values are

$$\sigma_x = -15 \text{ ksi} \qquad \sigma_y = 10 \text{ ksi} \qquad \tau_{xy} = 8 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(8)}{(-15)-(10)} = -16.310°, \quad 73.690°$$

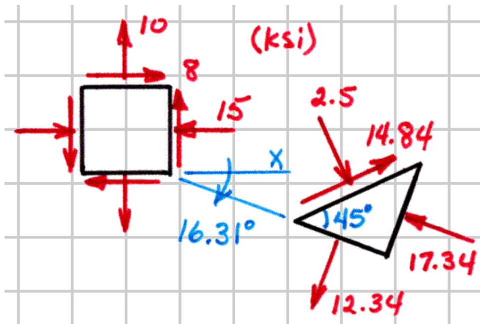
When $\theta_p = -16.310°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= (-15)\cos^2\theta_p + (10)\sin^2\theta_p + 2(8)\sin\theta_p\cos\theta_p$$
$$= -17.341 \text{ ksi} = \sigma_{p2}$$

$$\sigma_{p1} = \sigma_x + \sigma_y - \sigma_{p2} = 12.341 \text{ ksi}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 14.841 \text{ ksi}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = -2.500 \text{ ksi}$$



(a) $\sigma_{p1} = 12.34$ ksi (T) ∡ 73.69° .... **Ans.**

$\sigma_{p2} = 17.34$ ksi (C) ∡ 16.31° .... **Ans.**

$\tau_{max} = \tau_p = 14.841$ ksi .... **Ans.**

2-72

The given values are

$$\sigma_x = 200 \text{ MPa} \qquad \sigma_y = 0 \text{ MPa} \qquad \tau_{xy} = 25 \text{ MPa}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(25)}{(200)-(0)} = 7.018°, \quad -82.982°$$

When $\theta_p = 7.018°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

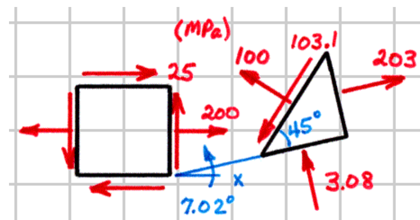
$$= (200)\cos^2\theta_p + (0) + 2(25)\sin\theta_p\cos\theta_p$$

$$= 203.078 \text{ MPa} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = -3.078 \text{ MPa}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 103.078 \text{ MPa}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = 100.00 \text{ MPa}$$



(a) $\sigma_{p1} = 203 \text{ MPa (T)}$ 7.02° .... **Ans.**

$\sigma_{p2} = 3.08 \text{ MPa (C)}$ 82.98° .... **Ans.**

$\tau_{max} = \tau_p = 103.1 \text{ MPa}$ .... **Ans.**

2-73

The given values are

$$\sigma_x = 12 \text{ ksi} \qquad \sigma_y = -4 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-6)}{(12)-(-4)} = -18.435°, \quad 71.565°$$

When $\theta_p = -18.435°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

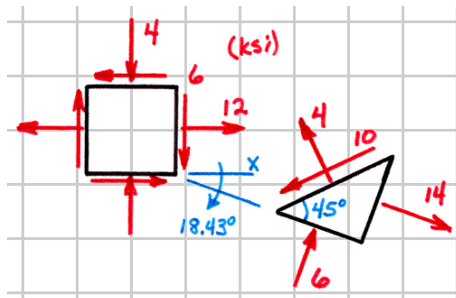
$$= (12)\cos^2\theta_p + (-4)\sin^2\theta_p + 2(-6)\sin\theta_p\cos\theta_p$$

$$= 14.000 \text{ ksi} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = -6.000 \text{ ksi}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 10.000 \text{ ksi}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = 4.000 \text{ ksi}$$



(a) $\sigma_{p1} = 14.00$ ksi (T) $\measuredangle$ 18.43° .... **Ans.**

$\sigma_{p2} = 6.00$ ksi (C) $\measuredangle$ 71.57° .... **Ans.**

$\tau_{max} = \tau_p = 10.00$ ksi .... **Ans.**

2-74
The given values are

$$\sigma_x = 75 \text{ MPa} \qquad \sigma_y = -25 \text{ MPa} \qquad \tau_{xy} = -35 \text{ MPa}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-35)}{(75)-(-25)} = -17.496°, \quad 72.504°$$

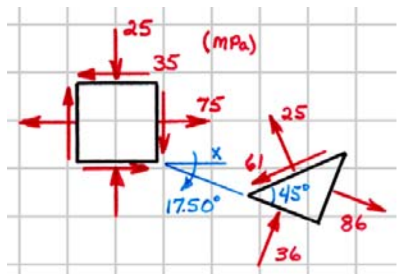
When $\theta_p = -17.496°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= (75)\cos^2\theta_p + (-25)\sin^2\theta_p + 2(-35)\sin\theta_p\cos\theta_p$$
$$= 86.033 \text{ MPa} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = -36.033 \text{ MPa}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 61.033 \text{ MPa}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = 25.00 \text{ MPa}$$



(a) $\sigma_{p1} = 86.0$ MPa (T) ↘ 17.50° .... **Ans.**

$\sigma_{p2} = 36.0$ MPa (C) ∡ 72.50° .... **Ans.**

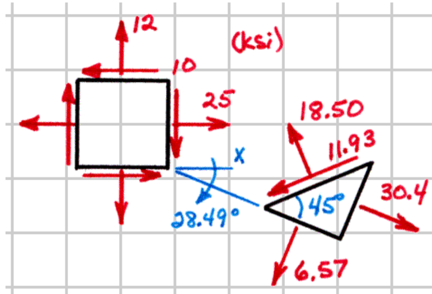
$\tau_{max} = \tau_p = 61.0$ MPa .... **Ans.**

2-75

The given values are

$$\sigma_x = 25 \text{ ksi} \qquad \sigma_y = 12 \text{ ksi} \qquad \tau_{xy} = -10 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-10)}{(25)-(12)} = -28.488°, \quad 61.512°$$



When $\theta_p = -28.488°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

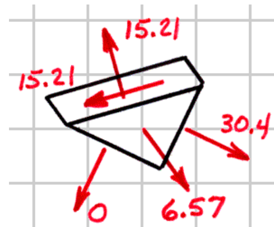
$$= (25)\cos^2\theta_p + (12)\sin^2\theta_p + 2(-10)\sin\theta_p\cos\theta_p$$

$$= 30.427 \text{ ksi} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = 6.573 \text{ ksi}$$

$$\tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 11.927 \text{ ksi} \qquad \sigma_n = 18.500 \text{ ksi}$$

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = 15.213 \text{ ksi} \qquad \sigma_n = 15.213 \text{ ksi}$$



(a) $\sigma_{p1} = 30.43$ ksi (T) ⦪ 28.49° ....**Ans.**

$\sigma_{p2} = 6.57$ ksi (T) ⦩ 61.51° ....**Ans.**

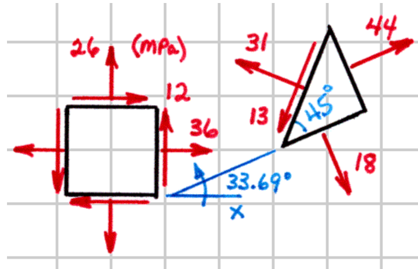
$\tau_p = 11.93$ ksi .... $\tau_{max} = 15.21$ ksi .... **Ans.**

2-76

The given values are

$$\sigma_x = 36 \text{ MPa} \qquad \sigma_y = 26 \text{ MPa} \qquad \tau_{xy} = 12 \text{ MPa}$$

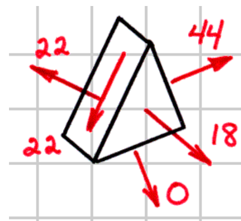
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(12)}{(36)-(26)} = 33.690°, \quad -56.310°$$



When $\theta_p = 33.690°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= (36)\cos^2\theta_p + (26)\sin^2\theta_p + 2(12)\sin\theta_p\cos\theta_p$$
$$= 44.000 \text{ MPa} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = 18.000 \text{ MPa}$$



$$\tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 13.00 \text{ MPa} \qquad \sigma_n = 31.00 \text{ MPa}$$

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = 22.00 \text{ MPa} \qquad \sigma_n = 22.00 \text{ MPa}$$

(a) $\sigma_{p1} = 44.0$ MPa (T) ∡ 33.69° ....**Ans.**

$\sigma_{p2} = 18.00$ MPa (T) ∡ 56.31° ....**Ans.**

$\tau_p = 13.00$ MPa .... $\tau_{max} = 22.0$ MPa ....**Ans.**

2-77

The given values are

$$\sigma_x = -2 \text{ ksi} \qquad \sigma_y = -14 \text{ ksi} \qquad \tau_{xy} = -8 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-8)}{(-2)-(-14)} = -26.565°, \quad 63.435°$$

When $\theta_p = -26.565°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

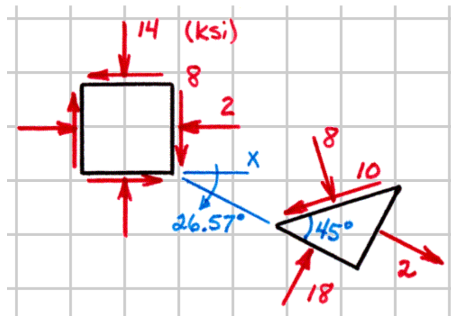
$$= (-2)\cos^2\theta_p + (-14)\sin^2\theta_p + 2(-8)\sin\theta_p\cos\theta_p$$

$$= 2.000 \text{ ksi} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = -18.00 \text{ ksi}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 10.00 \text{ ksi}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = -8.00 \text{ ksi}$$



(a) $\sigma_{p1} = 2.00$ ksi (T) ⦣ 26.57° .... **Ans.**

$\sigma_{p2} = 18.00$ ksi (C) ⦣ 63.43° .... **Ans.**

$\tau_{max} = \tau_p = 10.00$ ksi .... **Ans.**

2-78

The given values are

$$\sigma_x = -170 \text{ MPa} \qquad \sigma_y = 0 \text{ MPa} \qquad \tau_{xy} = -70 \text{ MPa}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-70)}{(-170)-(0)} = 19.736°, \quad -70.264°$$

When $\theta_p = 19.736°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

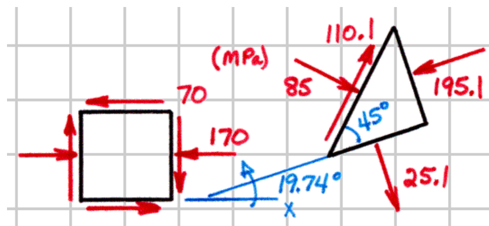
$$= (-170)\cos^2\theta_p + (0) + 2(-70)\sin\theta_p\cos\theta_p$$

$$= -195.114 \text{ MPa} = \sigma_{p2}$$

$$\sigma_{p1} = \sigma_x + \sigma_y - \sigma_{p2} = 25.114 \text{ MPa}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 110.114 \text{ MPa}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = -85.00 \text{ MPa}$$



$\sigma_{p1} = 25.1$ MPa (T) ⦩ 70.26° .... **Ans.**

$\sigma_{p2} = 195.1$ MPa (C) ∡ 19.74° .... **Ans.**

$\tau_{max} = \tau_p = 110.1$ MPa .... **Ans.**

2-79

The given values are

$$\sigma_x = \sigma_y = 0 \text{ ksi} \qquad \tau_{xy} = 15 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(15)}{(0)-(0)} = 45.00°, \quad -45.00°$$

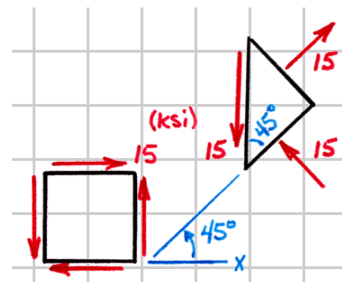
When $\theta_p = 45.00°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= (0) + (0) + 2(15)\sin\theta_p \cos\theta_p$$
$$= 15.00 \text{ ksi} = \sigma_{p1}$$

$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = -15.00 \text{ ksi}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 15.00 \text{ ksi}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = 0 \text{ ksi}$$



$\sigma_{p1} = 15.00$ ksi (T) ∡ 45.00° .... **Ans.**

$\sigma_{p2} = 15.00$ ksi (C) ∡ 45.00° .... **Ans.**

$\tau_{max} = \tau_p = 15.00$ ksi .... **Ans.**

2-80

(a) The given values are

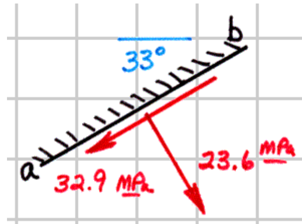
$$\sigma_x = -27\text{ MPa} \qquad \sigma_y = 45\text{ MPa} \qquad \tau_{xy} = 0\text{ MPa} \qquad \theta_{ab} = -57°$$

$$\sigma_n = (-27)\cos^2(-57°) + (45)\sin^2(-57°) + (0)$$

$\sigma_{ab} = 23.6\text{ MPa(T)}$ .... **Ans.**

$$\tau_{nt} = -[(-27) - (45)]\sin(-57°)\cos(-57°) + (0)$$

$\tau_{ab} = -32.9\text{ MPa}$ .... **Ans.**



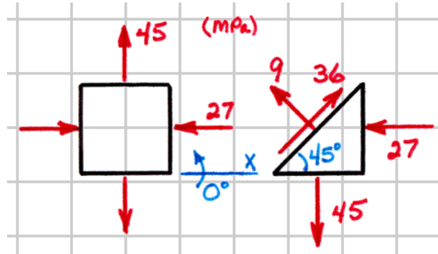
(b) Since there are no shear stresses on the horizontal and vertical surfaces, they are principal surfaces and the stresses on them are principal stresses.

$$\sigma_{p1} = \sigma_y = 45\text{ MPa}$$

$$\sigma_{p2} = \sigma_x = -27\text{ MPa}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 36.00\text{ MPa}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = 9.00\text{ MPa}$$



$\sigma_{p1} = 45.0\text{ MPa (T)} \uparrow$ .... **Ans.**

$\sigma_{p2} = 27.0\text{ MPa (C)} \rightarrow$ .... **Ans.**

$\tau_{max} = \tau_p = 36.0\text{ MPa}$ .... **Ans.**

2-81

The given values are

$$\sigma_x = (\text{T}) \qquad \sigma_y = (\text{C}) \qquad \tau_{xy} = 8 \text{ ksi} \qquad \sigma_{p1} = 12 \text{ ksi} \qquad \sigma_{p2} = -20 \text{ ksi}$$

We know that $\sigma_x + \sigma_y = \sigma_{p1} + \sigma_{p2} = -8 \text{ ksi}$

and that $$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-8}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (8)^2} = 12 \text{ ksi}$$

which gives $\sigma_x - \sigma_y = 27.71281 \text{ ksi}$

Therefore

$$\sigma_x = 9.85641 \text{ ksi} \qquad \sigma_y = -17.85641 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(8)}{27.71281} = 15.00°, \quad -75.00°$$

When $\theta_p = 15.00°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (9.85641)\cos^2\theta_p + (-17.85641)\sin^2\theta_p + 2(8)\sin\theta_p\cos\theta_p$$

$$= 12 \text{ ksi} = \sigma_{p1}$$

$\sigma_x = 9.86 \text{ ksi (T)}$ .... **Ans.**

$\sigma_y = 17.86 \text{ ksi (C)}$ .... **Ans.**

$\theta_{p1} = 15.00°$ .... **Ans.**

2-83

The given values are

$$\sigma_y = -\sigma_C \qquad \sigma_x = -4\sigma_C = 4\sigma_y \qquad \tau_{xy} = 0 \text{ ksi} \qquad \theta = -30°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$= (-4\sigma_C)\cos^2(-30°) + (-\sigma_C)\sin^2(-30°) + (0) \le -300 \text{ psi}$$

$$\sigma_C \le 92.3 \text{ psi}$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(-4\sigma_C) - (-\sigma_C)]\sin(-30°)\cos(-30°) + (0) \le 125 \text{ psi}$$

$$\sigma_C \le 96.2 \text{ psi}$$

$(\sigma_C)_{max} = 92.3$ psi .... **Ans.**

2-84

(a) The given values are

$$\sigma_x = -10 \text{ MPa} \qquad \sigma_y = -70 \text{ MPa} \qquad \tau_{xy} = 40 \text{ MPa} \qquad \theta_{ab} = -28°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

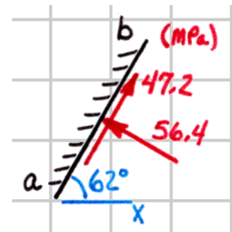
$$= (-10)\cos^2(-28°) + (-70)\sin^2(-28°) + 2(40)\sin(-28°)\cos(-28°)$$

$\sigma_{ab} = -56.4 \text{ MPa} = 56.4 \text{ MPa (C)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(-10)-(-70)]\sin(-28°)\cos(-28°) + (40)[\cos^2(-28°) - \sin^2(-28°)]$$

$\tau_{ab} = 47.2 \text{ MPa}$ .... **Ans.**



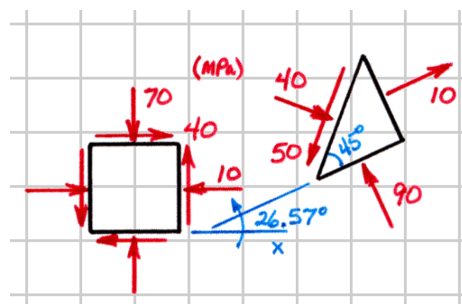
(b) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(40)}{(-10)-(-70)} = 26.565°, \quad -63.435°$$

When $\theta_p = 26.565°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (-10)\cos^2\theta_p + (-70)\sin^2\theta_p + 2(40)\sin\theta_p\cos\theta_p$$

$$= 10.00 \text{ MPa} = \sigma_{p1}$$



$$\sigma_{p2} = \sigma_x + \sigma_y - \sigma_{p1} = -90.00 \text{ MPa}$$

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 50.00 \text{ MPa}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = -40.00 \text{ MPa}$$

$\sigma_{p1} = 10.00 \text{ MPa (T)}$ ∡ 26.565° .... **Ans.**

$\sigma_{p2} = 90.0 \text{ MPa (C)}$ ∡ 63.565° .... **Ans.**

$\tau_{max} = \tau_p = 50.0 \text{ MPa}$ .... **Ans.**

2-85

The given values for use in drawing Mohr's circle are

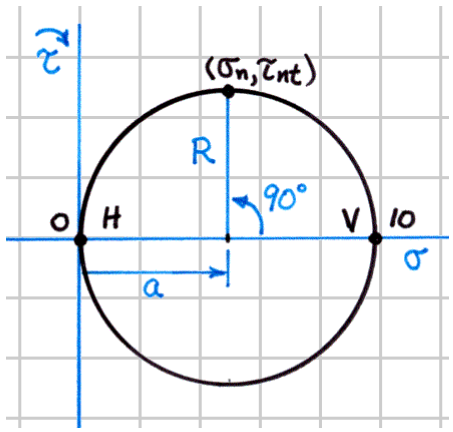
$$\sigma_x = \sigma_{p1} = 10 \text{ ksi}$$

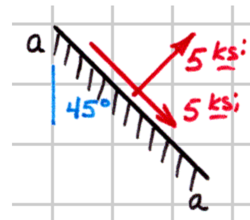
$$\sigma_y = \sigma_{p2} = 0 \text{ ksi}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ ksi}$$

$$a = \frac{10+0}{2} = 5 \text{ ksi}$$

$$R = \frac{10-0}{2} = 5 \text{ ksi}$$





$\sigma_{aa} = 5 + 5\cos 90° = 5 \text{ ksi (T)}$ .... **Ans.**

$\tau_{aa} = 5\sin 90° = 5 \text{ ksi (CW)} = -5 \text{ ksi}$ .... **Ans.**

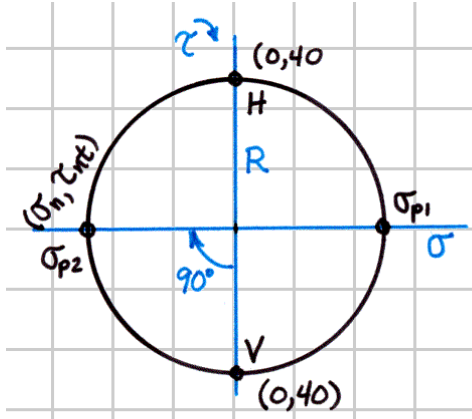
2-86

The given values for use in drawing Mohr's circle are

$$\sigma_x = \sigma_y = 0 \text{ MPa}$$

$$\tau_{xy} = 40 \text{ MPa}$$

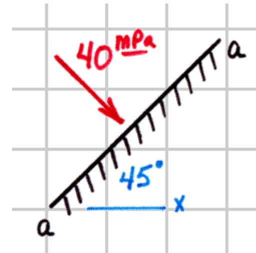
$$\sigma_z = \sigma_{p3} = 0 \text{ MPa}$$



$$R = 40 \text{ MPa}$$

$\sigma_{aa} = \sigma_{p2} = -40 \text{ MPa} = 40 \text{ MPa (C)}$ .... **Ans.**

$\tau_{aa} = 0 \text{ MPa}$ .... **Ans.**



2-87

The given values for use in drawing Mohr's circle are

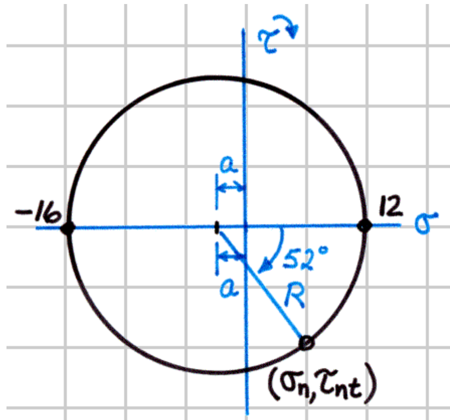
$$\sigma_x = \sigma_{p1} = 12 \text{ ksi}$$

$$\sigma_y = \sigma_{p2} = -16 \text{ ksi}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ ksi}$$

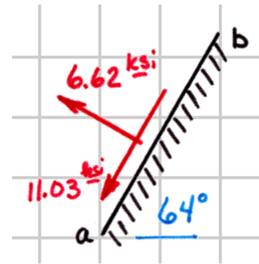
$$a = \frac{12 + (-16)}{2} = -2 \text{ ksi}$$

$$R = \frac{12 - (-16)}{2} = 14 \text{ ksi}$$



$\sigma_{ab} = -2 + 14\cos 52° = 6.62 \text{ ksi (T)}$ .... **Ans.**

$\tau_{ab} = 14\sin 52° = 11.03 \text{ ksi (CCW)} = +11.03 \text{ ksi}$ .... **Ans.**



2-88

The given values for use in drawing Mohr's circle are

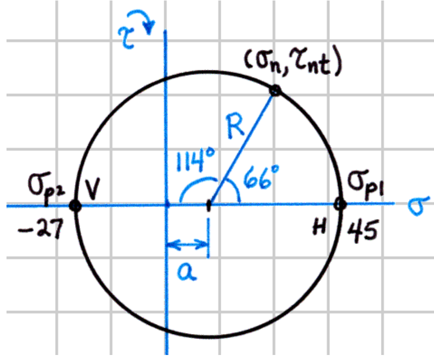
$$\sigma_x = \sigma_{p2} = -27 \text{ MPa}$$

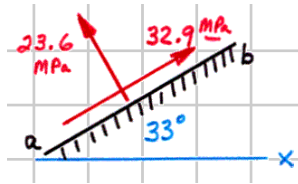
$$\sigma_y = \sigma_{p1} = 45 \text{ MPa}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ MPa}$$

$$a = \frac{(-27)+45}{2} = 9 \text{ MPa}$$

$$R = \frac{45-(-27)}{2} = 36 \text{ MPa}$$





$\sigma_{ab} = 9 + 36\cos 66° = 23.6$ MPa (T) .... **Ans.**

$\tau_{ab} = 36\sin 66° = 32.9$ MPa (CW) $= -32.9$ MPa .... **Ans.**

2-89

The given values for use in drawing Mohr's circle are

$\sigma_x = 25$ ksi

$\sigma_y = 7$ ksi

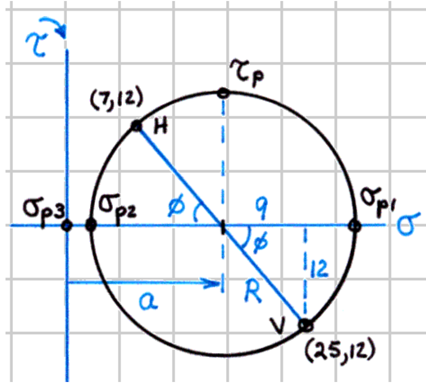
$\tau_{xy} = 12$ ksi

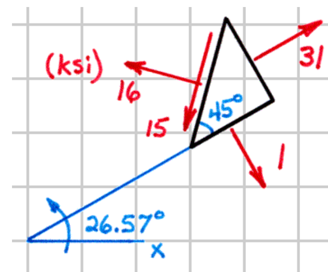
$\sigma_z = \sigma_{p3} = 0$ ksi

$$a = \frac{25+7}{2} = 16.00 \text{ ksi}$$

$$R = \sqrt{9^2 + 12^2} = 15.00 \text{ ksi}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{12}{9} = 26.57° \text{ (CCW)}$$





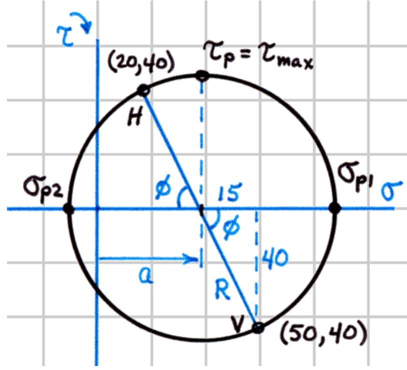
$\sigma_{p1} = 16.00 + 15.00 = 31.0$ ksi (T) ∡ 26.57° .... **Ans.**

$\sigma_{p2} = 16.00 - 15.00 = 1.0$ ksi (T) ∡ 63.43° .... **Ans.**

$\tau_p = R = 15$ ksi .... **Ans.**

$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (31 - 0)/2 = 15.5$ ksi (out of plane) .... **Ans.**

2-90
The given values for use in drawing Mohr's circle are



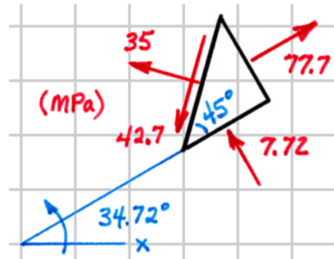
$$\sigma_x = 50 \text{ MPa}$$

$$\sigma_y = 20 \text{ MPa}$$

$$\tau_{xy} = 40 \text{ MPa}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ MPa}$$

$$a = \frac{50+20}{2} = 35.00 \text{ MPa}$$



$$R = \sqrt{15^2 + 40^2} = 42.72 \text{ MPa}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{40}{15} = 34.72° \text{ (CCW)}$$

$\tau_{max} = \tau_p = R = 42.7$ MPa ....**Ans.**

$\sigma_{p1} = 35.00 + 42.7 = 77.7$ MPa (T) ∡ 34.72° ....**Ans.**

$\sigma_{p2} = 35.00 - 42.72 = -7.72$ MPa $= 7.72$ MPa (C) ∡ 55.28° .... **Ans.**

2-91

The given values for use in drawing Mohr's circle are

$$\sigma_x = -15 \text{ ksi}$$

$$\sigma_y = 10 \text{ ksi}$$

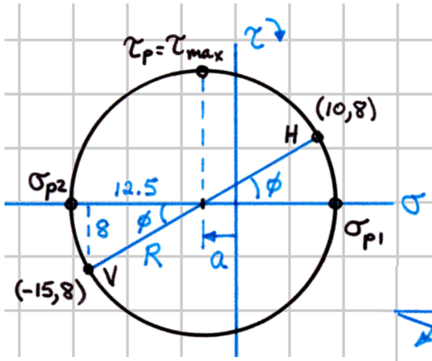
$$\tau_{xy} = 8 \text{ ksi}$$

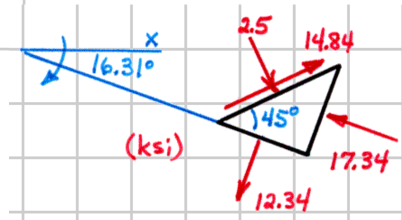
$$\sigma_z = \sigma_{p3} = 0 \text{ ksi}$$

$$a = \frac{(-15)+10}{2} = -2.50 \text{ ksi}$$

$$R = \sqrt{12.5^2 + 8^2} = 14.841 \text{ ksi}$$

$$\theta_{p2} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{8}{12.5} = 16.31° \text{ (CW)}$$





$\sigma_{p1} = -2.50 + 14.84 = 12.34$ ksi (T) ∡ 73.69° .... **Ans.**

$\sigma_{p2} = -2.50 - 14.84 = -17.34$ ksi $= 17.34$ ksi (C) ∡ 16.31° .... **Ans.**

$\tau_{max} = \tau_p = R = 14.84$ ksi .... **Ans.**

2-92

The given values for use in drawing Mohr's circle are

$\sigma_x = 50 \text{ MPa}$ $\sigma_y = 0 \text{ MPa}$

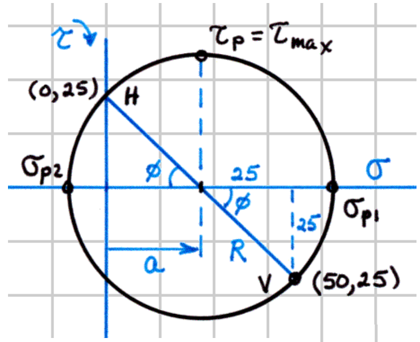
$\tau_{xy} = 25 \text{ MPa}$

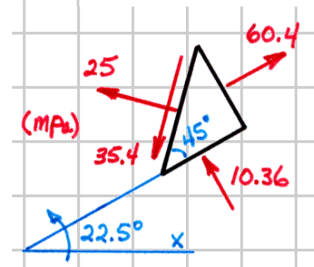
$\sigma_z = \sigma_{p3} = 0 \text{ MPa}$

$a = (50+0)/2 = 25.00 \text{ MPa}$

$R = \sqrt{25^2 + 25^2} = 35.355 \text{ MPa}$

$\theta_{p1} = \dfrac{\phi}{2} = \dfrac{1}{2}\tan^{-1}\dfrac{25}{25} = 22.50° \text{ (CCW)}$





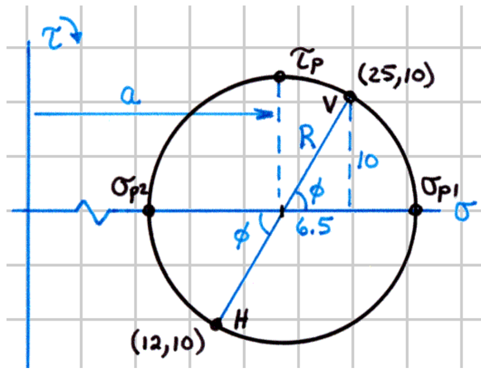
$\tau_{\max} = \tau_p = R = 35.4 \text{ MPa}$ ....**Ans.**

$\sigma_{p1} = 25.0 + 35.4 = 60.4 \text{ MPa (T)}$ ∡ 22.50° ....**Ans.**

$\sigma_{p2} = 25.00 - 35.36 = -10.36 \text{ MPa} = 10.36 \text{ MPa (C)}$ ⦣ 67.50° .... **Ans.**

2-93

The given values for use in drawing Mohr's circle are



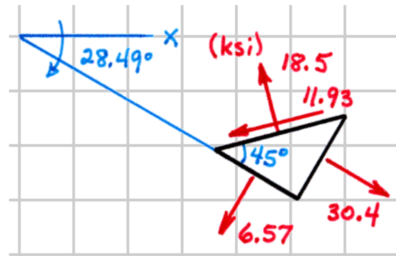
$$\sigma_x = 25 \text{ ksi}$$

$$\sigma_y = 12 \text{ ksi}$$

$$\tau_{xy} = -10 \text{ ksi}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ ksi}$$

$$a = \frac{25+12}{2} = 18.50 \text{ ksi}$$



$$R = \sqrt{6.5^2 + 10^2} = 11.927 \text{ ksi}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{10}{6.5} = 28.488° \text{ (CW)}$$

$\sigma_{p1} = 18.5 + 11.9 = 30.4$ ksi (T) ∡ 28.49° .... **Ans.**

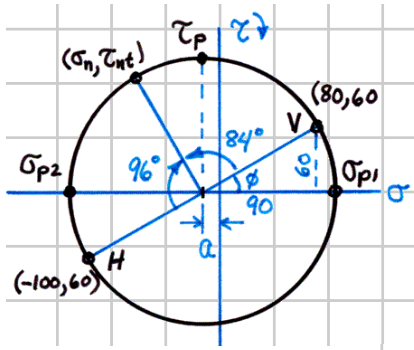
$\sigma_{p2} = 18.50 - 11.93 = 6.57$ ksi (T) ∡ 61.51° .... **Ans.**

$\tau_p = R = 11.93$ ksi .... **Ans.**

$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (30.427 - 0)/2 \cong 15.21$ ksi (out of plane) .... **Ans.**

2-94

The given values for use in drawing Mohr's circle are



$$\sigma_x = 80 \text{ MPa}$$

$$\sigma_y = -100 \text{ MPa}$$

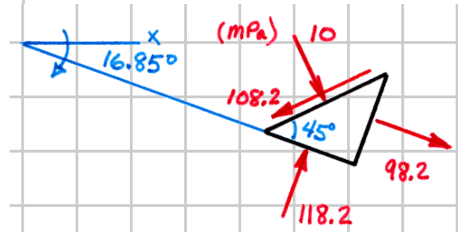
$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ MPa}$$

$$a = \frac{80-100}{2} = -10.00 \text{ MPa}$$

$$R = \sqrt{90^2 + 60^2} = 108.17 \text{ MPa}$$

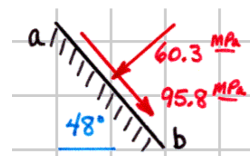
$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{60}{90} = 16.845° \text{ (CW)}$$



(a) $\sigma_{p1} = -10.0 + 108.2 = 98.2 \text{ MPa (T)}$ 16.85° .... **Ans.**

$\sigma_{p2} = -10.0 - 108.2 = -118.2 \text{ MPa} = 118.2 \text{ MPa (C)}$ 73.15° .... **Ans.**

$\tau_{max} = \tau_p = R = 108.2 \text{ MPa}$ .... **Ans.**

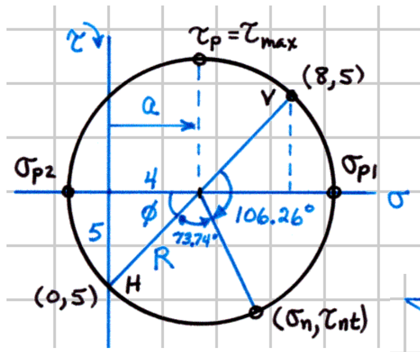


(b) $\sigma_{ab} = -10 - 108.17\cos 62.310° = -60.3 \text{ MPa} = 60.3 \text{ MPa (C)}$ ....**Ans.**

$\tau_{ab} = 108.17\sin 62.310° = 95.8 \text{ MPa (CW)} = -95.8 \text{ MPa}$ ....**Ans.**

2-95

The given values for use in drawing Mohr's circle are



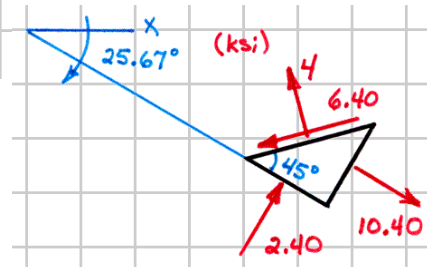
$$\sigma_x = 8 \text{ ksi}$$

$$\sigma_y = 0 \text{ ksi}$$

$$\tau_{xy} = -5 \text{ ksi}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ ksi}$$

$$a = \frac{8+0}{2} = 4.00 \text{ ksi}$$

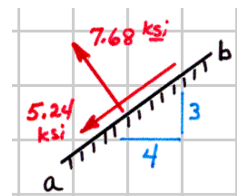


$$R = \sqrt{4^2 + 5^2} = 6.403 \text{ ksi}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{5}{4} = 25.670° \text{ (CW)}$$

(a) $\sigma_{p1} = 4.00 + 6.40 = 10.40$ ksi (T) ↘ 25.67° .... **Ans.**

$\sigma_{p2} = 4.00 - 6.40 = -2.40$ ksi $= 2.40$ ksi (C) ∡ 64.33° .... **Ans.**



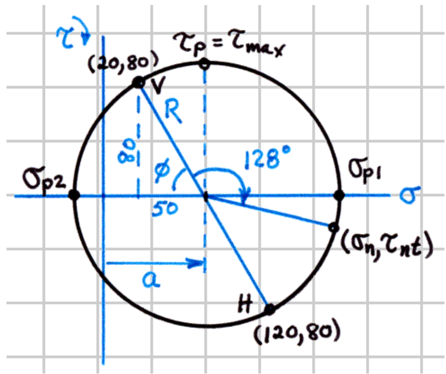
$\tau_{max} = \tau_p = R = 6.40$ ksi .... **Ans.**

(b) $\sigma_{ab} = 4.00 + 6.403\cos 54.920° = 7.68$ ksi (T) .... **Ans.**

$\tau_{ab} = 6.403\sin 54.920° = 5.24$ ksi (CCW) $= +5.24$ ksi .... **Ans.**

2-96

The given values for use in drawing Mohr's circle are



$$\sigma_x = 20 \text{ MPa}$$

$$\sigma_y = 120 \text{ MPa}$$

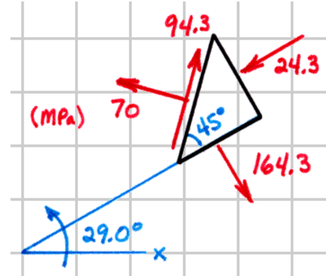
$$\tau_{xy} = -80 \text{ MPa}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ MPa}$$

$$a = \frac{20+120}{2} = 70.00 \text{ MPa}$$

$$R = \sqrt{50^2 + 80^2} = 94.340 \text{ MPa}$$

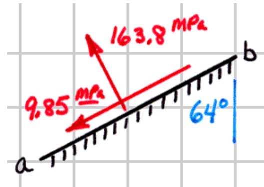
$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{80}{50} = 28.997° \text{ (CCW)}$$



(a) $\sigma_{p1} = 70.00 + 94.3 = 164.3$ MPa (T) ∡ 61.00° .... **Ans.**

$\sigma_{p2} = 70.00 - 94.3 = -24.3$ MPa $= 24.3$ MPa (C) ∡ 29.00° .... **Ans.**

$\tau_{max} = \tau_p = R = 94.3$ MPa .... **Ans.**

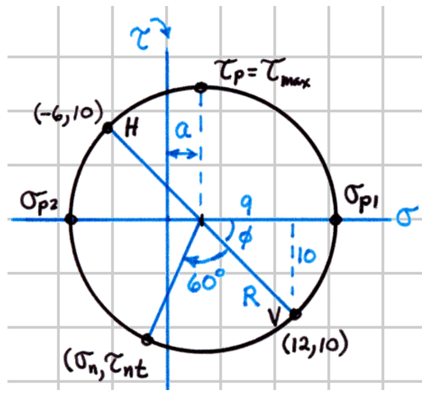


(b) $\sigma_{ab} = 70 + 94.340\cos 5.995° = 163.82$ MPa (T) .... **Ans.**

$\tau_{ab} = 94.340\sin 5.995° = 9.85$ MPa (CCW) $= 9.85$ MPa .... **Ans.**

2-97

The given values for use in drawing Mohr's circle are



$$\sigma_x = 12 \text{ ksi}$$

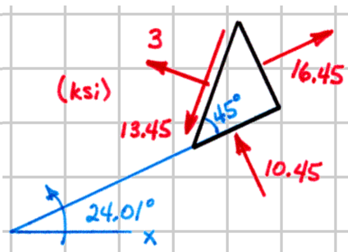
$$\sigma_y = -6 \text{ ksi}$$

$$\tau_{xy} = 10 \text{ ksi}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ ksi}$$

$$a = \frac{12+(-6)}{2} = 3.00 \text{ ksi}$$

$$R = \sqrt{9^2 + 10^2} = 13.454 \text{ ksi}$$



$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{10}{9} = 24.006° \text{ (CCW)}$$

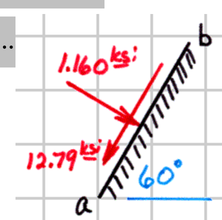
(a) $\sigma_{p1} = 3.00 + 13.45 = 16.45 \text{ ksi (T)} \measuredangle\ 24.01°$ .... **Ans.**

$\sigma_{p2} = 3.00 - 13.45 = -10.45 \text{ ksi} = 10.45 \text{ ksi (C)}\ 65.99°$ .... **Ans.**

$\tau_{max} = \tau_p = R = 13.45 \text{ ksi}$ ....

(b) $\sigma_{ab} = 3.00 - 13.454\cos 71.987° = -1.160 \text{ ksi} = 1.160 \text{ ksi (C)}$ ...... **Ans.**

$\tau_{ab} = 13.454\sin 71.987° = 12.79 \text{ ksi (CCW)} = 12.79 \text{ ksi}$ .... **Ans.**



2-98*

The given stress values are:

$$\sigma_x = 40 \text{ MPa} \qquad \sigma_y = -20 \text{ MPa} \qquad \sigma_z = 20 \text{ MPa} \qquad \theta_x = 40° \qquad \theta_y = 75°$$

$$\tau_{xy} = 40 \text{ MPa} \qquad \tau_{yz} = 0 \text{ MPa} \qquad \tau_{zx} = 30 \text{ MPa} \qquad \theta_z = 54°$$

Then

$$S_x = \sigma_x \cos\theta_x + \tau_{yx} \cos\theta_y + \tau_{zx} \cos\theta_z$$
$$= 40\cos 40° + 40\cos 75° + 30\cos 54° = 58.628 \text{ MPa}$$
$$S_y = \tau_{xy} \cos\theta_x + \sigma_y \cos\theta_y + \tau_{zy} \cos\theta_z$$
$$= 40\cos 40° - 20\cos 75° + 0 = 25.465 \text{ MPa}$$
$$S_z = \tau_{xz} \cos\theta_x + \tau_{yz} \cos\theta_y + \sigma_z \cos\theta_z$$
$$= 30\cos 40° + 0 + 20\cos 54° = 34.737 \text{ MPa}$$

$$S = \sqrt{S_x^2 + S_y^2 + S_z^2} = \sqrt{(58.628)^2 + (25.465)^2 + (34.737)^2} = 72.749 \text{ MPa}$$

$$\sigma_n = S_x \cos\theta_x + S_y \cos\theta_y + S_z \cos\theta_z$$
$$= 58.628\cos 40° + 25.465\cos 75° + 34.737\cos 54°$$

$\sigma_n = 71.920 \text{ MPa} \cong 71.9 \text{ MPa (T)}$ .... **Ans.**

$\tau_n = \sqrt{S^2 - \sigma_n^2} = \sqrt{(72.749)^2 - (71.920)^2} = 10.95 \text{ MPa}$ .... **Ans.**

2-99*

The given stress values are:

$$\sigma_x = 14 \text{ ksi} \qquad \sigma_y = 12 \text{ ksi} \qquad \sigma_z = 10 \text{ ksi} \qquad \theta_x = 40° \qquad \theta_y = 60°$$

$$\tau_{xy} = 4 \text{ ksi} \qquad \tau_{yz} = -4 \text{ ksi} \qquad \tau_{zx} = 0 \text{ ksi} \qquad \theta_z = 66.2°$$

Then

$$S_x = \sigma_x \cos\theta_x + \tau_{yx} \cos\theta_y + \tau_{zx} \cos\theta_z$$
$$= 14\cos 40° + 4\cos 60° + 0 = 12.7246 \text{ ksi}$$
$$S_y = \tau_{xy} \cos\theta_x + \sigma_y \cos\theta_y + \tau_{zy} \cos\theta_z$$
$$= 4\cos 40° + 12\cos 60° - 4\cos 66.2° = 7.4500 \text{ ksi}$$
$$S_z = \tau_{xz} \cos\theta_x + \tau_{yz} \cos\theta_y + \sigma_z \cos\theta_z$$
$$= 0 - 4\cos 60° + 10\cos 66.2° = 2.0355 \text{ ksi}$$

$$S = \sqrt{S_x^2 + S_y^2 + S_z^2} = \sqrt{(12.7246)^2 + (7.4500)^2 + (2.0355)^2} = 14.8849 \text{ ksi}$$

$$\sigma_n = S_x \cos\theta_x + S_y \cos\theta_y + S_z \cos\theta_z$$
$$= 12.7246\cos 40° + 7.4500\cos 60° + 2.0355\cos 66.2°$$

$\sigma_n = 14.2939 \text{ ksi} \cong 14.29 \text{ ksi (T)}$ .... **Ans.**

$\tau_n = \sqrt{S^2 - \sigma_n^2} = \sqrt{(14.8849)^2 - (14.2939)^2} = 4.15 \text{ ksi}$ .... **Ans.**

2-100

The given stress values are:

$$\sigma_x = 60 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \sigma_z = 60 \text{ MPa} \qquad \theta_x = 60° \qquad \theta_y = 70°$$

$$\tau_{xy} = 120 \text{ MPa} \qquad \tau_{yz} = 75 \text{ MPa} \qquad \tau_{zx} = 90 \text{ MPa} \qquad \theta_z = 37.3°$$

Then

$$S_x = \sigma_x \cos\theta_x + \tau_{yx} \cos\theta_y + \tau_{zx} \cos\theta_z$$
$$= 60\cos 60° + 120\cos 70° + 90\cos 37.3° = 142.6347 \text{ MPa}$$

$$S_y = \tau_{xy} \cos\theta_x + \sigma_y \cos\theta_y + \tau_{zy} \cos\theta_z$$
$$= 120\cos 60° + 90\cos 70° + 75\cos 37.3° = 150.4421 \text{ MPa}$$

$$S_z = \tau_{xz} \cos\theta_x + \tau_{yz} \cos\theta_y + \sigma_z \cos\theta_z$$
$$= 90\cos 60° + 75\cos 70° + 60\cos 37.3° = 118.3797 \text{ MPa}$$

$$S = \sqrt{S_x^2 + S_y^2 + S_z^2} = \sqrt{(142.6347)^2 + (150.4421)^2 + (118.3797)^2} = 238.7283 \text{ MPa}$$

$$\sigma_n = S_x \cos\theta_x + S_y \cos\theta_y + S_z \cos\theta_z$$
$$= 142.6347\cos 60° + 150.4421\cos 70° + 118.3797\cos 37.3°$$

$\sigma_n = 216.9390 \text{ MPa} \cong 217 \text{ MPa (T)}$ .... **Ans.**

$\tau_n = \sqrt{S^2 - \sigma_n^2} = \sqrt{(238.7283)^2 - (216.9390)^2} = 99.6 \text{ MPa}$ .... **Ans.**

2-101*

The given stress values are:

$$\sigma_x = \sigma_y = \sigma_z = 0 \text{ ksi} \qquad \theta_x = \theta_y = \theta_z$$

$$\tau_{xy} = 6 \text{ ksi} \qquad \tau_{yz} = 10 \text{ ksi} \qquad \tau_{zx} = 8 \text{ ksi}$$

But $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

Therefore $\cos\theta_x = \cos\theta_y = \cos\theta_z = 1/\sqrt{3}$

$$S_x = \sigma_x \cos\theta_x + \tau_{yx}\cos\theta_y + \tau_{zx}\cos\theta_z = (0+6+8)/\sqrt{3} = 8.0829 \text{ ksi}$$

$$S_y = \tau_{xy}\cos\theta_x + \sigma_y\cos\theta_y + \tau_{zy}\cos\theta_z = (6+0+10)/\sqrt{3} = 9.2376 \text{ ksi}$$

$$S_z = \tau_{xz}\cos\theta_x + \tau_{yz}\cos\theta_y + \sigma_z\cos\theta_z = (8+10+0)/\sqrt{3} = 10.3923 \text{ ksi}$$

$$S = \sqrt{S_x^2 + S_y^2 + S_z^2} = \sqrt{(8.0839)^2 + (9.2376)^2 + (10.3923)^2} = 16.0831 \text{ ksi}$$

$$\sigma_n = S_x\cos\theta_x + S_y\cos\theta_y + S_z\cos\theta_z = (8.0829 + 9.2376 + 10.3923)/\sqrt{3}$$

$\sigma_n = 16.00 \text{ ksi} = 16.00 \text{ ksi (T)}$ .... **Ans.**

$\tau_n = \sqrt{S^2 - \sigma_n^2} = \sqrt{(16.0831)^2 - (16.00)^2} = 1.633 \text{ ksi}$ .... **Ans.**

2-102
The given stress values are:

$$\sigma_x = 72 \text{ MPa} \qquad \sigma_y = -32 \text{ MPa} \qquad \sigma_z = 0 \text{ MPa} \qquad \theta_x = \theta_y = \theta_z$$

$$\tau_{xy} = 21 \text{ MPa} \qquad \tau_{yz} = 0 \text{ MPa} \qquad \tau_{zx} = 21 \text{ MPa}$$

But $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

Therefore $\cos\theta_x = \cos\theta_y = \cos\theta_z = 1/\sqrt{3}$

$$S_x = \sigma_x \cos\theta_x + \tau_{yx}\cos\theta_y + \tau_{zx}\cos\theta_z = (72+21+21)/\sqrt{3} = 65.8179 \text{ MPa}$$

$$S_y = \tau_{xy}\cos\theta_x + \sigma_y\cos\theta_y + \tau_{zy}\cos\theta_z = (21-32+0)/\sqrt{3} = -6.3509 \text{ MPa}$$

$$S_z = \tau_{xz}\cos\theta_x + \tau_{yz}\cos\theta_y + \sigma_z\cos\theta_z = (21+0+0)/\sqrt{3} = 12.1244 \text{ MPa}$$

$$S = \sqrt{S_x^2 + S_y^2 + S_z^2} = \sqrt{(65.8179)^2 + (6.3509)^2 + (12.1244)^2} = 67.2260 \text{ MPa}$$

$$\sigma_n = S_x\cos\theta_x + S_y\cos\theta_y + S_z\cos\theta_z = (65.8179 - 6.3509 + 12.1244)/\sqrt{3}$$

$\sigma_n = 41.3333 \text{ MPa} \cong 41.3 \text{ MPa (T)}$ .... **Ans.**

$\tau_n = \sqrt{S^2 - \sigma_n^2} = \sqrt{(67.2260)^2 - (41.3333)^2} = 53.0 \text{ MPa}$ .... **Ans.**

2-103*

The given stress values are:

$$\sigma_x = 12 \text{ ksi} \qquad \sigma_y = -10 \text{ ksi} \qquad \sigma_z = 8 \text{ ksi}$$

$$\tau_{xy} = 8 \text{ ksi} \qquad \tau_{yz} = -10 \text{ ksi} \qquad \tau_{zx} = 12 \text{ ksi}$$

$$\sigma_x + \sigma_y + \sigma_z = 10 \text{ ksi}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -412 \text{ ksi}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = -3152 \text{ ksi}^3$$

$$\sigma_p^3 - (10)\sigma_p^2 + (-412)\sigma_p - (-3152) = 0$$

$\sigma_{p1} = \sigma_{\max} = 22.1706 \text{ ksi} \cong 22.2 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = 7.3013 \text{ ksi} \cong 7.30 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = -19.4719 \text{ ksi} \cong 19.47 \text{ ksi (C)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{22.1706 - (-19.4719)}{2} = 20.8 \text{ ksi}$ .... **Ans.**

2-104*

The given stress values are:

$$\sigma_x = 40 \text{ MPa} \qquad \sigma_y = -20 \text{ MPa} \qquad \sigma_z = 20 \text{ MPa}$$

$$\tau_{xy} = 40 \text{ MPa} \qquad \tau_{yz} = 0 \text{ MPa} \qquad \tau_{zx} = 30 \text{ MPa}$$

$$\sigma_x + \sigma_y + \sigma_z = 40 \text{ MPa}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -2900 \text{ MPa}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = -30{,}000 \text{ MPa}^3$$

$$\sigma_p^3 - (40)\sigma_p^2 + (-2900)\sigma_p - (-30{,}000) = 0$$

$\sigma_{p1} = \sigma_{\max} = 73.7908 \text{ MPa} \cong 73.8 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = 9.4107 \text{ MPa} \cong 9.41 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = -43.2014 \text{ MPa} \cong 43.2 \text{ MPa (C)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{73.7908 - (-43.2014)}{2} = 58.5 \text{ MPa}$ .... **Ans.**

2-105

The given stress values are:

$$\sigma_x = 14 \text{ ksi} \qquad \sigma_y = 12 \text{ ksi} \qquad \sigma_z = 10 \text{ ksi}$$

$$\tau_{xy} = 4 \text{ ksi} \qquad \tau_{yz} = -4 \text{ ksi} \qquad \tau_{zx} = 0 \text{ ksi}$$

$$\sigma_x + \sigma_y + \sigma_z = 36 \text{ ksi}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = 396 \text{ ksi}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = 1296 \text{ ksi}^3$$

$$\sigma_p^3 - (36)\sigma_p^2 + (396)\sigma_p - (1296) = 0$$

$\sigma_{p1} = \sigma_{\max} = 18.00 \text{ ksi} \cong 18.00 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = 12.00 \text{ ksi} \cong 12.00 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = 6.00 \text{ ksi} \cong 6.00 \text{ ksi (T)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{18-(6)}{2} = 6.00 \text{ ksi}$ .... **Ans.**

2-106*

The given stress values are:

$$\sigma_x = 60 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \sigma_z = 60 \text{ MPa}$$

$$\tau_{xy} = 120 \text{ MPa} \qquad \tau_{yz} = 75 \text{ MPa} \qquad \tau_{zx} = 90 \text{ MPa}$$

$$\sigma_x + \sigma_y + \sigma_z = 210 \text{ MPa}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -13,725 \text{ MPa}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = 13,500 \text{ MPa}^3$$

$$\sigma_p^3 - (210)\sigma_p^2 + (-13,725)\sigma_p - (13,500) = 0$$

$\sigma_{p1} = \sigma_{\max} = 262.485 \text{ MPa} \cong 262 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = -1.000 \text{ MPa} \cong 1.000 \text{ MPa (C)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = -51.485 \text{ MPa} \cong 51.5 \text{ MPa (C)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{262.485 - (-51.485)}{2} = 157.0 \text{ MPa}$ .... **Ans.**

2-107

The given stress values are:

$$\sigma_x = \sigma_y = \sigma_z = 0 \text{ ksi} \qquad \tau_{xy} = 6 \text{ ksi} \qquad \tau_{yz} = 10 \text{ ksi} \qquad \tau_{zx} = 8 \text{ ksi}$$

$$\sigma_x + \sigma_y + \sigma_z = 0 \text{ ksi}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -200 \text{ ksi}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = 960 \text{ ksi}^3$$

$$\sigma_p^3 - (0)\sigma_p^2 + (-200)\sigma_p - (960) = 0$$

$\sigma_{p1} = \sigma_{\max} = 16.1116 \text{ ksi} \cong 16.11 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = -5.7511 \text{ ksi} \cong 5.75 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = -10.3605 \text{ ksi} \cong 10.36 \text{ ksi (C)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{16.1116 - (-10.3605)}{2} = 13.24 \text{ ksi}$ .... **Ans.**

2-108

The given stress values are:

$$\sigma_x = 72 \text{ MPa} \qquad \sigma_y = -32 \text{ MPa} \qquad \sigma_z = 0 \text{ MPa}$$

$$\tau_{xy} = 21 \text{ MPa} \qquad \tau_{yz} = 0 \text{ MPa} \qquad \tau_{zx} = 21 \text{ MPa}$$

$$\sigma_x + \sigma_y + \sigma_z = 40 \text{ MPa}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -3186 \text{ MPa}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = 14{,}112 \text{ MPa}^3$$

$$\sigma_p^3 - (40)\sigma_p^2 + (-3186)\sigma_p - (14{,}112) = 0$$

$\sigma_{p1} = \sigma_{\max} = 81.3151 \text{ MPa} \cong 81.3 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = -4.7457 \text{ MPa} \cong 4.75 \text{ MPa (C)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = -36.5695 \text{ MPa} \cong 36.6 \text{ MPa (C)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{81.3151 - (-36.5695)}{2} = 58.9 \text{ MPa}$ .... **Ans.**

2-109*

The given stress values are:

$$\sigma_x = -18 \text{ ksi} \qquad \sigma_y = -15 \text{ ksi} \qquad \sigma_z = -12 \text{ ksi}$$

$$\tau_{xy} = -15 \text{ ksi} \qquad \tau_{yz} = 12 \text{ ksi} \qquad \tau_{zx} = -9 \text{ ksi}$$

(a)

$$\sigma_x + \sigma_y + \sigma_z = -45 \text{ ksi}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = 216 \text{ ksi}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = 6507 \text{ ksi}^3$$

$$\sigma_p^3 - (-45)\sigma_p^2 + (216)\sigma_p - (6507) = 0$$

$\sigma_{p1} = \sigma_{\max} = 9.1477 \text{ ksi} \cong 9.15 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = -22.4191 \text{ ksi} \cong 22.4 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = -31.7286 \text{ ksi} \cong 31.7 \text{ ksi (C)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{9.1477 - (-31.7286)}{2} = 20.4 \text{ ksi}$ .... **Ans.**

(b) For $\sigma_{p3} = -31.7286 \text{ ksi}$

$$\left[(-31.7286) - (-18)\right]l - (-15)m - (-9)n = 0 \qquad (1)$$

$$\left[(-31.7286) - (-15)\right]m - (-15)l - (12)n = 0 \qquad (2)$$

$$\left[(-31.7286) - (-12)\right]n - (-9)l - (12)m = 0 \qquad (3)$$

From Eqs. (1) and (2) $\quad m = 1.01021l$

From Eqs. (2) and (3) $\quad n = -0.15827l$

$$l^2 + m^2 + n^2 = l^2 + (1.01021l)^2 + (-0.15827l)^2 = 1$$

$l = 0.69919$ $\quad \theta_{x3} = 45.64°$ .... **Ans.**

$m = 0.70632$ $\quad \theta_{y3} = 45.06°$ .... **Ans.**

$n = -0.11066$ $\quad \theta_{z3} = 96.35°$ .... **Ans.**

2-110*

The given stress values are:

$$\sigma_x = 75 \text{ MPa} \qquad \sigma_y = 35 \text{ MPa} \qquad \sigma_z = 55 \text{ MPa}$$

$$\tau_{xy} = 45 \text{ MPa} \qquad \tau_{yz} = 28 \text{ MPa} \qquad \tau_{zx} = 36 \text{ MPa}$$

(a) $$\sigma_x + \sigma_y + \sigma_z = 165 \text{ MPa}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = 4570 \text{ MPa}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = 19{,}560 \text{ MPa}^3$$

$$\sigma_p^3 - (165)\sigma_p^2 + (4570)\sigma_p - (19{,}560) = 0$$

$\sigma_{p1} = \sigma_{\max} = 131.3380 \text{ MPa} \cong 131.3 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = 28.4218 \text{ MPa} \cong 28.4 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = 5.2399 \text{ MPa} \cong 5.24 \text{ MPa (T)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{131.33880 - (5.2399)}{2} = 63.0 \text{ MPa}$ .... **Ans.**

(b) For $\sigma_{p1} = 131.3380 \text{ MPa}$

$$\left[(131.3380) - (75)\right]l - (45)m - (36)n = 0 \qquad (1)$$

$$\left[(131.3380) - (35)\right]m - (45)l - (28)n = 0 \qquad (2)$$

$$\left[(131.3380) - (55)\right]n - (36)l - (28)m = 0 \qquad (3)$$

From Eqs. (1) and (2) $m = 0.67626l$

From Eqs. (2) and (3) $n = 0.71963l$

$$l^2 + m^2 + n^2 = l^2 + (0.67626l)^2 + (0.71963l)^2 = 1$$

$l = 0.71153$ $\theta_{x1} = 44.64°$ .... **Ans.**

$m = 0.48118$ $\theta_{y1} = 61.24°$ .... **Ans.**

$n = 0.51204$ $\theta_{z1} = 59.20°$ .... **Ans.**

2-111

The given stress values are:

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = 12 \text{ ksi} \qquad \sigma_z = 6 \text{ ksi}$$

$$\tau_{xy} = 12 \text{ ksi} \qquad \tau_{yz} = -6 \text{ ksi} \qquad \tau_{zx} = 9 \text{ ksi}$$

(a) $\sigma_x + \sigma_y + \sigma_z = 36 \text{ ksi}$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = 135 \text{ ksi}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = -2484 \text{ ksi}^3$$

$$\sigma_p^3 - (36)\sigma_p^2 + (135)\sigma_p - (-2484) = 0$$

$\sigma_{p1} = \sigma_{max} = 28.0170 \text{ ksi} \cong 28.0 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{int} = 14.2186 \text{ ksi} \cong 14.22 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{min} = -6.2355 \text{ ksi} \cong 6.24 \text{ ksi (C)}$ .... **Ans.**

$\tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{28.0170 - (-6.2355)}{2} = 17.13 \text{ ksi}$ .... **Ans.**

(b) For $\sigma_{p1} = 28.0170 \text{ ksi}$

$$[(28.0170) - (18)]l - (12)m - (9)n = 0 \qquad (1)$$

$$[(28.0170) - (12)]m - (12)l - (-6)n = 0 \qquad (2)$$

$$[(28.0170) - (6)]n - (9)l - (-6)m = 0 \qquad (3)$$

From Eqs. (1) and (2) $m = 0.66384l$

From Eqs. (2) and (3) $n = 0.22787l$

$$l^2 + m^2 + n^2 = l^2 + (0.66384l)^2 + (0.22787l)^2 = 1$$

$l = 0.81852$ $\theta_{x1} = 35.06°$ .... **Ans.**

$m = 0.54336$ $\theta_{y1} = 57.09°$ .... **Ans.**

$n = 0.18652$ $\theta_{z1} = 79.25°$ .... **Ans.**

2-112

The given stress values are:

$$\sigma_x = 100 \text{ MPa} \qquad \sigma_y = -100 \text{ MPa} \qquad \sigma_z = 80 \text{ MPa}$$

$$\tau_{xy} = 50 \text{ MPa} \qquad \tau_{yz} = -70 \text{ MPa} \qquad \tau_{zx} = -64 \text{ MPa}$$

(a) $\sigma_x + \sigma_y + \sigma_z = 80 \text{ MPa}$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -21,496 \text{ MPa}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = -632,400 \text{ MPa}^3$$

$$\sigma_p^3 - (80)\sigma_p^2 + (-21,496)\sigma_p - (-632,400) = 0$$

$\sigma_{p1} = \sigma_{max} = 179.9330 \text{ MPa} \cong 179.9 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{int} = 27.5659 \text{ MPa} \cong 27.6 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{min} = -127.4990 \text{ MPa} \cong 127.5 \text{ MPa (C)}$ .... **Ans.**

$\tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{179.9330 - (-127.4990)}{2} = 153.7160 \text{ MPa}$ .... **Ans.**

(b) For $\sigma_{p3} = -127.4990 \text{ MPa}$

$$\left[(-127.4990) - (100)\right]l - (50)m - (-64)n = 0 \qquad (1)$$

$$\left[(-127.4990) - (-100)\right]m - (50)l - (-70)n = 0 \qquad (2)$$

$$\left[(-127.4990) - (80)\right]n - (-64)l - (-70)m = 0 \qquad (3)$$

From Eqs. (1) and (2) $\quad m = -7.31291l$

From Eqs. (2) and (3) $\quad n = -2.15885l$

$$l^2 + m^2 + n^2 = l^2 + (7.31591l)^2 + (2.15885l)^2 = 1$$

$l = 0.13004$ $\qquad \theta_{x3} = 82.53°$ .... **Ans.**

$m = -0.95094$ $\qquad \theta_{y3} = 161.98°$ .... **Ans.**

$n = -0.28073$ $\qquad \theta_{z3} = 106.30°$ .... **Ans.**

2-113*

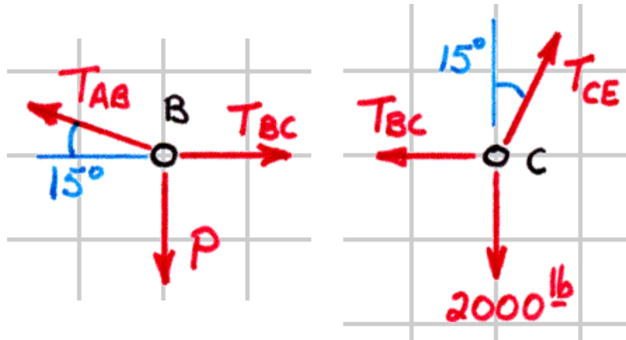
From a free-body diagram of the ring *C*, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad T_{CE} \sin 15° - T_{BC} = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{CE} \cos 15° - 2000 = 0$$

$$T_{CE} = 2070.552 \text{ lb}$$

$$T_{BC} = 535.898 \text{ lb}$$



Then, from a free-body diagram of the ring *B* the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad T_{BC} - T_{AB} \cos 15° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{AB} \sin 15° - P = 0$$

$$T_{AB} = 554.803 \text{ lb}$$

$$P = 143.594 \text{ lb}$$

$$\sigma_{AB} = \frac{T_{AB}}{A} = \frac{554.803}{\pi(0.25)^2/4} = 11{,}302 \text{ psi} \cong 11.30 \text{ ksi (T)}$$ .... **Ans.**

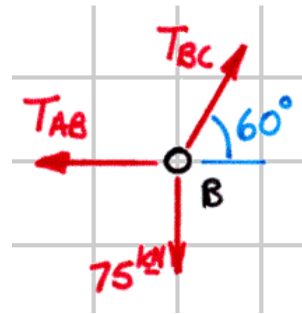
$$\sigma_{CE} = \frac{T_{CE}}{A} = \frac{2070.552}{\pi(0.25)^2/4} = 42{,}181 \text{ psi} \cong 42.2 \text{ ksi (T)}$$ .... **Ans.**

2-114*

From a free-body diagram of the pin *B*, the equations of equilibrium give

$$\rightarrow \Sigma F_x = 0: \qquad -T_{AB} + T_{BC}\cos 60° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{BC}\sin 60° - 75 = 0$$



$$T_{AB} = 43.30127 \text{ kN} \qquad T_{BC} = 86.60254 \text{ kN}$$

$$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{43.30127\left(10^3\right)}{A_{AB}} \le 75\left(10^6\right) \text{ N/m}^2$$

$$\sigma_{BC} = \frac{T_{BC}}{A_{BC}} = \frac{\left(86.6025410^3\right)}{A_{BC}} \le 75\left(10^6\right) \text{ N/m}^2$$

$A_{AB} \ge 577\left(10^{-6}\right) \text{ m}^2 = 577 \text{ mm}^2$ .... **Ans.**

$A_{BC} \ge 1155\left(10^{-6}\right) \text{ m}^2 = 1155 \text{ mm}^2$ .... **Ans.**

$$\tau_A = \frac{V}{A} = \frac{43{,}301.27}{\pi d_A^2/4} \le 100\left(10^6\right) \text{ N/m}^2$$

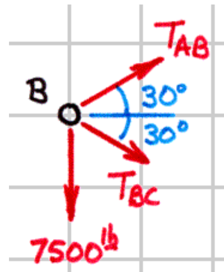
$$\tau_C = \frac{V}{A} = \frac{86{,}602.54}{\pi d_C^2/4} \le 100\left(10^6\right) \text{ N/m}^2$$

$d_A \ge 23.5\left(10^{-3}\right) \text{ m} = 23.5 \text{ mm}$ .... **Ans.**

$d_C \ge 33.2\left(10^{-3}\right) \text{ m} = 33.2 \text{ mm}$ .... **Ans.**

2-115

From a free-body diagram of the pin *B*,
the equations of equilibrium give

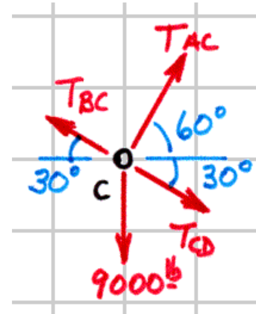


$$\rightarrow \Sigma F_x = 0: \qquad T_{AB}\cos 30° + T_{BC}\cos 30° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{AB}\sin 30° - T_{BC}\sin 30° - 7500 = 0$$

$$T_{AB} = 7500 \text{ lb} \qquad T_{BC} = -7500 \text{ lb}$$

Then, from a free-body diagram of the pin *B*
the equations of equilibrium give



$$\rightarrow \Sigma F_x = 0: \qquad T_{AC}\cos 60° + T_{CD}\cos 30° - T_{BC}\cos 30° = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{AC}\sin 60° - T_{CD}\sin 30° + T_{BC}\sin 30° - 9000 = 0$$

$$T_{AC} = 7794.229 \text{ lb}$$

$$P = -12{,}000 \text{ lb}$$

$$\sigma_{AC} = \frac{T_{AC}}{A} = \frac{7794.229}{1.477} = 5277 \text{ psi} \cong 5.28 \text{ ksi (T)}$$ .... **Ans.**

$$\sigma_{CD} = \frac{T_{CD}}{A} = \frac{12{,}000}{A_{CD}} \leq 3500 \text{ psi}$$

$$A_{CD} \geq 3.43 \text{ in.}^2$$ .... **Ans.**

2-116*

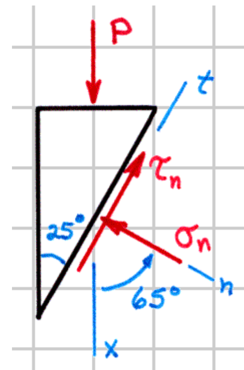
$$\sigma_n = \frac{P}{2(0.150)(0.180)}\left[1+\cos 2(65°)\right] \le 12(10^6)\ \text{N/m}^2$$

$$P \le 1.814(10^6)\ \text{N}$$

$$\tau_n = \frac{P}{2(0.150)(0.180)}\sin 2(65°) \le 1.40(10^6)\ \text{N/m}^2$$

$$P \le 98.7(10^3)\ \text{N}$$

$P_{max} = 98.7$ kN ....**Ans.**



2-117

The given values are

$$\sigma_x = 12 \text{ ksi} \qquad \sigma_y = 28 \text{ ksi} \qquad \tau_{xy} = 7 \text{ ksi} \qquad \theta_{ab} = 23°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

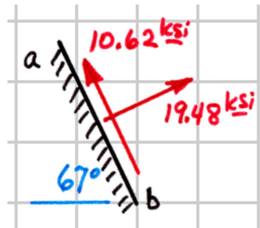
$$= (12)\cos^2(23°) + (28)\sin^2(23°) + 2(7)\sin(23°)\cos(23°)$$

$\sigma_{ab} = +19.48 \text{ ksi} = 19.48 \text{ ksi (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -[(12)-(28)]\sin(23°)\cos(23°) + (7)[\cos^2(23°) - \sin^2(23°)]$$

$\tau_{ab} = +10.62 \text{ ksi}$ .... **Ans.**



2-118

The given values are

$$\sigma_x = -35 \text{ MPa} \qquad \sigma_y = 45 \text{ MPa} \qquad \tau_{xy} = -18 \text{ MPa}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-18)}{(-35)-(45)} = 12.114°, \quad -77.886°$$

When $\theta_p = 12.114°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (-35)\cos^2\theta_p + (45)\sin^2\theta_p + 2(-18)\sin\theta_p\cos\theta_p$$

$$= -38.863 \text{ MPa} = \sigma_{p2}$$

$$\sigma_{p1} = \sigma_x + \sigma_y - \sigma_{p2} = 48.863 \text{ MPa}$$

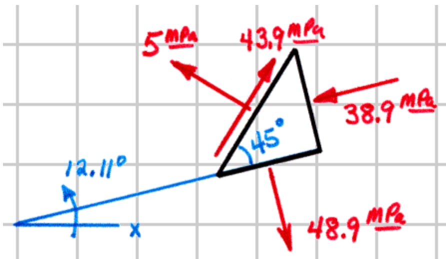
$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 43.863 \text{ MPa}$$

$$\sigma_{n45} = (\sigma_{p1} + \sigma_{p2})/2 = 5 \text{ MPa}$$

$\sigma_{p1} = 48.9$ MPa (T) ∡ 77.89° .... **Ans.**

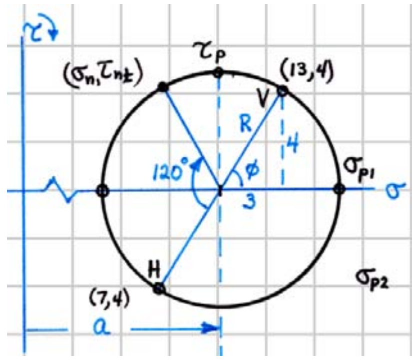
$\sigma_{p2} = 38.9$ MPa (C) ∡ 12.11° .... **Ans.**

$\tau_{max} = \tau_p = 43.9$ MPa .... **Ans.**



2-119*

The given values for use in drawing Mohr's circle are



$$\sigma_x = 13 \text{ ksi}$$

$$\sigma_y = 7 \text{ ksi}$$

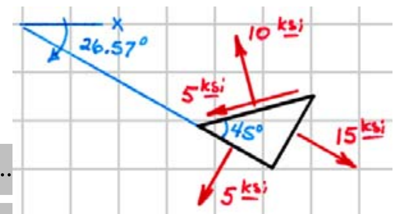
$$\tau_{xy} = -4 \text{ ksi}$$

$$\sigma_z = \sigma_{p3} = 0 \text{ ksi}$$

$$a = \frac{13+7}{2} = 10.00 \text{ ksi}$$

$$R = \sqrt{3^2 + 4^2} = 5.00 \text{ ksi}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{4}{3} = 26.57° \text{ (CW)}$$

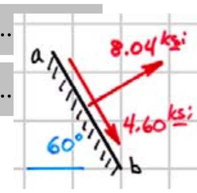


(a) $\sigma_{p1} = 10.00 + 5.00 = 15.00 \text{ ksi (T)} \quad 26.57°$ ....

$\sigma_{p2} = 10.00 - 5.00 = 5.0 \text{ ksi (T)} \quad 63.43°$ .... **Ans.**

$\tau_p = R = 5 \text{ ksi}$ ....

$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (15-0)/2 = 7.50 \text{ ksi (out of plane)}$ ....



(b) $\sigma_{ab} = 10 - 5.00\cos 66.870° = 8.04 \text{ ksi} = 8.04 \text{ ksi (T)}$ .... **Ans.**

$\tau_{ab} = 5.00\sin 66.870° = 4.60 \text{ ksi (CW)} = -4.60 \text{ ksi}$ .... **Ans.**

2-120

The given stress values are:

$$\sigma_x = 53 \text{ MPa} \quad \sigma_y = -28 \text{ MPa} \quad \sigma_z = 36 \text{ MPa} \quad \theta_x = 40° \quad \theta_y = 75°$$

$$\tau_{xy} = 24 \text{ MPa} \quad \tau_{yz} = -18 \text{ MPa} \quad \tau_{zx} = 46 \text{ MPa} \quad \theta_z = 54°$$

(a)

$$S_x = \sigma_x \cos\theta_x + \tau_{yx}\cos\theta_y + \tau_{zx}\cos\theta_z$$

$$= 53\cos 40° + 24\cos 75° + 46\cos 54° = 73.8501 \text{ MPa}$$

$$S_y = \tau_{xy}\cos\theta_x + \sigma_y\cos\theta_y + \tau_{zy}\cos\theta_z$$

$$= 24\cos 40° - 28\cos 75° - 18\cos 54° = 0.5578 \text{ MPa}$$

$$S_z = \tau_{xz}\cos\theta_x + \tau_{yz}\cos\theta_y + \sigma_z\cos\theta_z$$

$$= 46\cos 40° - 18\cos 75° + 36\cos 54° = 51.7385 \text{ MPa}$$

$$S = \sqrt{S_x^2 + S_y^2 + S_z^2} = \sqrt{(73.8501)^2 + (0.5578)^2 + (51.7395)^2} = 90.1728 \text{ MPa}$$

$$\sigma_n = S_x\cos\theta_x + S_y\cos\theta_y + S_z\cos\theta_z$$

$$= 73.8501\cos 40° + 0.5578\cos 75° + 51.7395\cos 54°$$

$\sigma_n = 87.1285 \text{ MPa} \cong 87.1 \text{ MPa (T)}$ .... **Ans.**

$\tau_n = \sqrt{S^2 - \sigma_n^2} = \sqrt{(90.1728)^2 - (87.1285)^2} = 23.2 \text{ MPa}$ .... **Ans.**

2-120 (cont.)

(b) $\sigma_x + \sigma_y + \sigma_z = 61 \text{ MPa}$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -3600 \text{ MPa}^2$$

$$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} = -71{,}828 \text{ MPa}^3$$

$$\sigma_p^3 - (61)\sigma_p^2 + (-3600)\sigma_p - (-71{,}828) = 0$$

$\sigma_{p1} = \sigma_{\max} = 91.7133 \text{ MPa} \cong 91.7 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{\text{int}} = 16.5662 \text{ MPa} \cong 16.57 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p3} = \sigma_{\min} = -47.2785 \text{ MPa} \cong 47.3 \text{ MPa (C)}$ .... **Ans.**

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{91.7133 - (-47.2785)}{2} = 69.5 \text{ MPa}$ .... **Ans.**

(c) For $\sigma_{p1} = 91.7133 \text{ MPa}$

$$[(91.7133) - (53)]l - (24)m - (46)n = 0 \qquad (1)$$

$$[(91.7133) - (-28)]m - (24)l - (-18)n = 0 \qquad (2)$$

$$[(91.7133) - (36)]n - (46)l - (-18)m = 0 \qquad (3)$$

From Eqs. (1) and (2) $m = 0.08023l$

From Eqs. (2) and (3) $n = 0.79973l$

$$l^2 + m^2 + n^2 = l^2 + (0.08023l)^2 + (0.79973l)^2 = 1$$

$l = 0.77944$ $\theta_{x1} = 38.79°$ .... **Ans.**

$m = 0.06253$ $\theta_{y1} = 86.41°$ .... **Ans.**

$n = 0.62334$ $\theta_{z1} = 51.44°$ .... **Ans.**

2-121 Equation 2-22 is

$$\sigma_p^3 - \left(\sigma_x + \sigma_y + \sigma_z\right)\sigma_p^2 + \left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2\right)\sigma_p$$
$$-\left(\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx}\right) = 0$$

For plane stress $\sigma_z = \tau_{zx} = \tau_{yz} = 0$

$$\sigma_p^3 - \left(\sigma_x + \sigma_y\right)\sigma_p^2 + \left(\sigma_x\sigma_y - \tau_{xy}^2\right)\sigma_p = 0$$
$$\sigma_p\left[\sigma_p^2 - \left(\sigma_x + \sigma_y\right)\sigma_p + \left(\sigma_x\sigma_y - \tau_{xy}^2\right)\right] = 0$$
$$\sigma_p = \frac{\left(\sigma_x + \sigma_y\right) \pm \sqrt{\left(\sigma_x + \sigma_y\right)^2 - 4\left(\sigma_x\sigma_y - \tau_{xy}^2\right)}}{2} = \frac{\left(\sigma_x + \sigma_y\right)}{2} \pm \sqrt{\frac{\left(\sigma_x + \sigma_y\right)^2 - 4\left(\sigma_x\sigma_y - \tau_{xy}^2\right)}{4}}$$
$$= \frac{\left(\sigma_x + \sigma_y\right)}{2} \pm \sqrt{\frac{\sigma_x^2 - 2\sigma_x\sigma_y + \sigma_y^2}{4} + \tau_{xy}^2} = \frac{\left(\sigma_x + \sigma_y\right)}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

which is Eq. 2-15.

3-1*

$$\varepsilon = \frac{\Delta L}{L} = \frac{0.625}{(25)(12)} = 0.00208 \text{ in./in.} = 2080\ \mu\text{in./in.}$$ .... **Ans.**

3-2*

$$\varepsilon = \frac{\Delta L}{L} \qquad 1200\left(10^{-6}\right) = \frac{\Delta L}{400}$$

$\Delta L = 0.480\text{ mm}$ .... **Ans.**

3-3

$$\varepsilon_{avg} = \frac{\Delta L}{L} = \frac{1.5 + 0.450(2)}{8} = 0.300 \text{ in./in.}$$ .... **Ans.**

3-4

(a) $$\varepsilon_{avg} = \frac{\Delta L}{L} = \frac{5+5.5+6.5+9+19.5+7+6+5}{200}$$

$\varepsilon_{avg} = 0.317$ m/m .... **Ans.**

(b) $$\varepsilon_{avg} = \frac{9+19.5}{50}$$

$\left(\varepsilon_{avg}\right)_{\max} = 0.570$ m/m .... **Ans.**

3-5*

$$\gamma = \frac{\Delta y}{L_x} = \frac{0.1}{0.5} = 0.0200 \text{ in./in.} = 0.0200 \text{ rad}$$ .... **Ans.**

3-6*

$$\gamma_{xy} = \tan^{-1}\frac{0.380}{500} + \tan^{-1}\frac{0.200}{250} = 0.08938° = 0.001560 \text{ rad}$$

$\gamma_{xy} = 1560\ \mu\text{rad}$ .... **Ans.**

3-7

$$\gamma_{xy} = 90° - 89.92° = 0.0800° = 0.001396 \text{ rad} = 1396\ \mu\text{rad}$$ .... **Ans.**

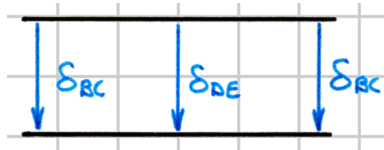
3-8

(a) $L_{AB} = \sqrt{2^2 + 199^2} = 199.01005 \text{ mm}$

$\varepsilon_{avg} = \dfrac{L_{AB} - 200}{200} = \dfrac{-0.98995}{200} = -0.00495 \text{ m/m} = -4950\ \mu\text{m/m}$ .... **Ans.**

(b) $\gamma = \dfrac{\Delta x}{L_y} = \dfrac{2}{200} = 0.0100 \text{ rad}$ .... **Ans.**

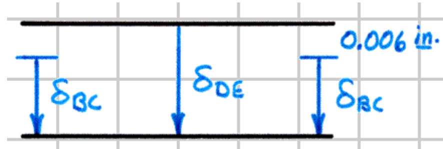
3-9*



(a) $\delta_{BC} = \delta_{DE} = \varepsilon_{DE} L_{DE} = 800\left(10^{-6}\right)\left(70\right) = 0.056000 \text{ in.}$

$$\varepsilon_{BC} = \frac{\delta_{BC}}{L_{BC}} = \frac{0.056000}{40} = 0.001400 \text{ in./in.}$$

$\varepsilon_{BC} = 1400\ \mu\text{in./in.}$ .... **Ans.**

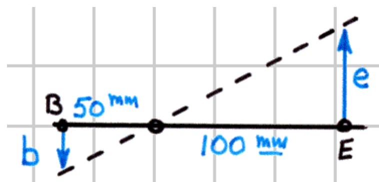


(b) $\delta_{BC} = \delta_{DE} - 0.006 = 0.050000 \text{ in.}$

$$\varepsilon_{BC} = \frac{\delta_{BC}}{L_{BC}} = \frac{0.050000}{40} = 0.001250 \text{ in./in.}$$

$\varepsilon_{BC} = 1250\ \mu\text{in./in.}$ .... **Ans.**

3-10*



$$\delta_D = \varepsilon_D L_D = 0.0075(150) = 1.1250 \text{ mm}$$

$$\frac{e}{100} = \frac{b}{50} \qquad e = 2b$$

(a) $b = \delta_D + 0.09 \text{ mm}$ $\qquad e = \delta_{CE}$

$$\delta_{CE} = 2(\delta_D + 0.09) = 2(1.1250 + 0.09) = 2.4300 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\delta_{CE}}{L_{CE}} = \frac{2.4300}{300} = 0.00810 \text{ m/m} = 8100\ \mu\text{m/m}$$ .... **Ans.**

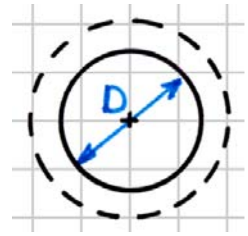
(b) $b = \delta_D + 0.09 \text{ mm}$ $\qquad e = \delta_{CE} + 0.10 \text{ mm}$

$$(\delta_{CE} + 0.10) = 2(\delta_D + 0.09) = 2(1.12500 + 0.09) = 2.4300 \text{ mm}$$

$$\delta_{CE} = 2.3300 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\delta_{CE}}{L_{CE}} = \frac{2.3300}{300} = 0.00777 \text{ m/m} = 7770\ \mu\text{m/m}$$ .... **Ans.**

3-11



(a) $$\varepsilon_{avg} = \frac{\Delta d}{d} = \frac{2.15 - 2.00}{2.00} = 0.0750 \text{ in./in.}$$ ....**Ans.**

(b) $$\varepsilon_{circ} = \frac{\Delta circ}{circ} = \frac{circ' - circ}{circ} = \frac{2\pi d' - 2\pi d}{2\pi d} = \frac{d' - d}{d} = \frac{\Delta d}{d}$$

$$\varepsilon_{circ} = \frac{\Delta d}{d} = \varepsilon_d = 0.0750 \text{ in./in.}$$ ....**Ans.**

3-12*

$$\varepsilon = kx^2 \qquad 1250\left(10^{-6}\right) = k\left(1500\right)^2$$

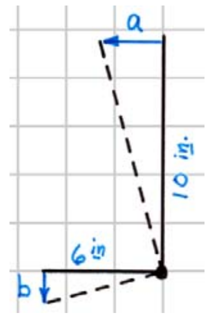
$$k = 5.55556\left(10^{-10}\right)\ /\text{mm}^2 \qquad \varepsilon = 5.55556\left(10^{-10}\right)x^2\ \text{m/m}$$

(a) $$\Delta L = \int_0^{3000} kx^2 dx = \left.\frac{kx^3}{3}\right|_0^{3000} = 9\left(10^9\right)k = 5.00\ \text{mm}$$ .... **Ans.**

(b) $$\varepsilon_{avg} = \frac{\Delta L}{L} = \frac{5.00}{3000} = 0.001667\ \text{m/m} = 1667\ \mu\text{m/m}$$ .... **Ans.**

(c) $$\varepsilon_{\max} = \varepsilon_{x=3000} = k\left(3000\right)^2 = 0.00500\ \text{m/m} = 5000\ \mu\text{m/m}$$ .... **Ans.**

3-13



$$\delta_B = \varepsilon_B L_B = 0.0014(15) = 0.02100 \text{ in.}$$

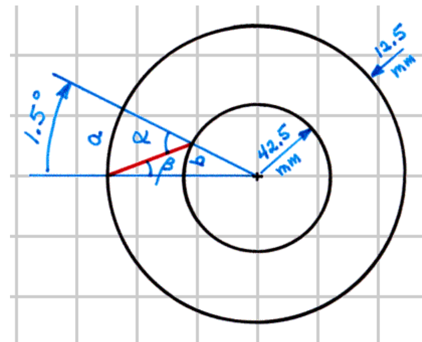
$$\frac{a}{10} = \frac{b}{6} \qquad a = \frac{5b}{3}$$

$$a = \delta_{AD} \qquad b = \delta_B + 0.009 \text{ in.}$$

$$\delta_{AD} = \frac{5(\delta_B + 0.009)}{3} = \frac{5(0.02100 + 0.009)}{3} = 0.05000 \text{ in.}$$

$$\varepsilon_{AD} = \frac{\delta_{AD}}{L_{AD}} = \frac{0.05000}{50} = 0.001000 \text{ in./in.} = 1000\ \mu\text{in./in.}$$ .... **Ans.**

3-14



$$1.5° = 0.02618 \text{ rad}$$

$$a = 55(0.02618) = 1.43990 \text{ mm}$$

$$b = 42.5(0.02618) = 1.11265 \text{ mm}$$

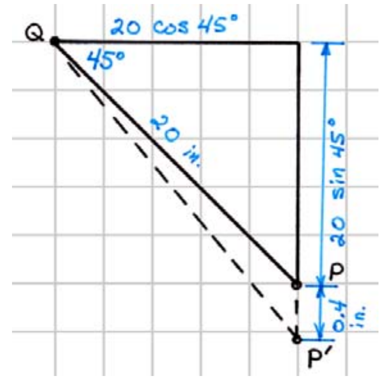
$$\alpha = \tan^{-1}\frac{a}{12.5} = \tan^{-1}\frac{1.43990}{12.5} = 6.5710° = 0.1147 \text{ rad}$$

$$\beta = \tan^{-1}\frac{b}{12.5} = \tan^{-1}\frac{1.11265}{12.5} = 5.0866° = 0.0888 \text{ rad}$$

(a) $\gamma_{r\theta} = \alpha = 0.1147 \text{ rad}$ .... **Ans.**

(b) $\gamma_{r\theta} = \beta = 0.0888 \text{ rad}$ .... **Ans.**

3-15*



$$\alpha = \angle P'QR = \tan^{-1}\frac{20\sin 45° + 0.4}{20\cos 45°}$$

$$= 45.7989°$$

$$\theta' = \angle QP'R = 180° - 2\alpha$$

$$= 180° - 2(45.7989°) = 88.4021°$$

$$\gamma = 90° - \theta' = 90° - (88.4021°) = 1.5979°$$

$\gamma = 0.0279$ rad .... **Ans.**

3-16*

The given values are

$$\varepsilon_x = -2000\ \mu\text{m/m} \qquad \varepsilon_y = -1500\ \mu\text{m/m}$$

$$\gamma_{xy} = 1250\ \mu\text{rad} \qquad \theta_n = -35°$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

$$= (-2000)\cos^2(-35°) + (-1500)\sin^2(-35°) + (1250)\sin(-35°)\cos(-35°)$$

$\varepsilon_n = -2420\ \mu\text{m/m}$ .... **Ans.**

3-17*

The given values are

$$\varepsilon_x = 880\ \mu\text{in./in.} \qquad \varepsilon_y = 960\ \mu\text{in./in.} \qquad \gamma_{xy} = -750\ \mu\text{rad}$$

(a) $\theta_{AC} = \tan^{-1}(2/4) = 26.565°$

$$\varepsilon_{AC} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (880)\cos^2(26.565°) + (960)\sin^2(26.565°) + (-750)\sin(26.565°)\cos(26.565°)$$

$\varepsilon_{AC} = 596\ \mu\text{in./in.}$ .... **Ans.**

(b) $\theta_{BD} = -\tan^{-1}(2/4) = -26.565°$

$$\varepsilon_{BD} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (880)\cos^2(-26.565°) + (960)\sin^2(-26.565°) + (-750)\sin(-26.565°)\cos(-26.565°)$$

$\varepsilon_{BD} = 1196\ \mu\text{in./in.}$ .... **Ans.**

3-18

The given values are

$$\varepsilon_x = 1750\ \mu\text{m/m} \qquad \varepsilon_y = -2200\ \mu\text{m/m} \qquad \gamma_{xy} = -800\ \mu\text{rad}$$

(a) $\theta_n = -45°$ $\qquad \theta_t = +45°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (1750)\cos^2(-45°) + (-2200)\sin^2(-45°) + (-800)\sin(-45°)\cos(-45°)$$

$\varepsilon_n = 175\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_t = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (1750)\cos^2(45°) + (-2200)\sin^2(45°) + (-800)\sin(45°)\cos(45°)$$

$\varepsilon_t = -625\ \mu\text{m/m}$ .... **Ans.**

(b)

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -2\left[(1750) - (-2200)\right]\sin(-45°)\cos(-45°) + (-800)\left[\cos^2(-45°) - \sin^2(-45°)\right]$$

$\gamma_{nt} = 3950\ \mu\text{rad}$ .... **Ans.**

3-19*

(a) $\varepsilon_x = \dfrac{\Delta L}{L} = \dfrac{0.06}{30} = 0.00200 \text{ in./in.} = 2000\ \mu\text{in./in.}$ .... **Ans.**

$\varepsilon_y = \dfrac{\Delta L}{L} = \dfrac{-0.03}{30} = -0.00100 \text{ in./in.} = 1000\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{xy} = 0\ \mu\text{rad}$ .... **Ans.**

(b) $\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$

$= (2000)\cos^2(30°) + (-1000)\sin^2(30°) + (0)$

$\varepsilon_n = 1250\ \mu\text{in./in.}$ .... **Ans.**

3-20*

$$L'_x = \sqrt{(100.5)^2 + (0.1)^2} = 100.0500500 \text{ mm}$$

$$L'_y = \sqrt{(0.02)^2 + (49.99)^2} = 49.9900040 \text{ mm}$$

$$\theta'_{xy} = 90° - \left[\tan^{-1}\frac{0.02}{50}\right] + \left[\tan^{-1}\frac{0.1}{100}\right] = 90.03438°$$

$$\varepsilon_x = \frac{\Delta L_x}{L_x} = \frac{0.0500500}{100} = 501\left(10^{-6}\right) \text{ m/m} = 500\ \mu\text{m/m}$$ .... **Ans.**

$$\varepsilon_y = \frac{\Delta L_y}{L_y} = \frac{49.9900040 - 50}{50} = -199.9\left(10^{-6}\right) \text{ m/m} = -199.9\ \mu\text{m/m}$$ .... **Ans.**

$$\gamma_{xy} = 90° - \theta'_{xy} = -0.03438° = -600\left(10^{-6}\right) \text{ rad} = -600\ \mu\text{rad}$$ .... **Ans.**

3-21

The given values are

$$\varepsilon_x = -800\ \mu\text{in./in.} \qquad \varepsilon_y = 640\ \mu\text{in./in.} \qquad \gamma_{xy} = -960\ \mu\text{rad}$$

$$\theta_n = 42° \qquad \theta_t = 132°$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (-800)\cos^2(42°) + (640)\sin^2(42°) + (-960)\sin(42°)\cos(42°)$$

$\varepsilon_n = -633\ \mu\text{in./in.}$ .... **Ans.**

$$\varepsilon_t = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (-800)\cos^2(132°) + (640)\sin^2(132°) + (-960)\sin(132°)\cos(132°)$$

$\varepsilon_t = 473\ \mu\text{in./in.}$ .... **Ans.**

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -2\left[(-800) - (640)\right]\sin(42°)\cos(42°) + (-960)\left[\cos^2(42°) - \sin^2(42°)\right]$$

$\gamma_{nt} = 1332\ \mu\text{rad}$ .... **Ans.**

3-22

The given values are

$$\varepsilon_x = 720\ \mu\text{m/m} \qquad \varepsilon_y = -480\ \mu\text{m/m} \qquad \gamma_{xy} = 360\ \mu\text{rad}$$

$$\theta_n = -30° \qquad \theta_t = 60°$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (720)\cos^2(-30°) + (-480)\sin^2(-30°) + (360)\sin(-30°)\cos(-30°)$$

$\varepsilon_n = 264\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_t = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (720)\cos^2(60°) + (-480)\sin^2(60°) + (360)\sin(60°)\cos(60°)$$

$\varepsilon_t = -24.1\ \mu\text{m/m}$ .... **Ans.**

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -2[(720) - (-480)]\sin(-30°)\cos(-30°) + (360)[\cos^2(-30°) - \sin^2(-30°)]$$

$\gamma_{nt} = 1219\ \mu\text{rad}$ .... **Ans.**

3-23

Equation (b) is

$$\left(1+\varepsilon_n\right)^2 dn^2 = \left(1+\varepsilon_x\right)^2 dn^2 \cos^2\theta + \left(1+\varepsilon_y\right)^2 dn^2 \sin^2\theta$$

$$+2dn^2 \sin\theta\cos\theta\left(1+\varepsilon_x\right)\left(1+\varepsilon_y\right)\sin\gamma_{xy}$$

$$1+2\varepsilon_n+\varepsilon_n^2 = \left(1+2\varepsilon_x+\varepsilon_x^2\right)\cos^2\theta + \left(1+2\varepsilon_y+\varepsilon_y^2\right)\sin^2\theta$$

$$+2\sin\theta\cos\theta\left(\sin\gamma_{xy}+\varepsilon_x\sin\gamma_{xy}+\varepsilon_y\sin\gamma_{xy}+\varepsilon_x\varepsilon_y\sin\gamma_{xy}\right)$$

But $\sin^2\theta+\cos^2\theta=1$, therefore

$$2\varepsilon_n+\varepsilon_n^2 = \left(2\varepsilon_x+\varepsilon_x^2\right)\cos^2\theta + \left(2\varepsilon_y+\varepsilon_y^2\right)\sin^2\theta$$

$$+2\sin\theta\cos\theta\left(\sin\gamma_{xy}+\varepsilon_x\sin\gamma_{xy}+\varepsilon_y\sin\gamma_{xy}+\varepsilon_x\varepsilon_y\sin\gamma_{xy}\right)$$

and then the small strain approximation $\varepsilon_n^2 = \varepsilon_n$, $\varepsilon_x^2 = \varepsilon_x$, and $\varepsilon_y^2 = \varepsilon_y$. Also, $\sin\gamma_{xy} \cong \gamma_{xy}$, therefore $\varepsilon_x\gamma_{xy}$, $\varepsilon_y\gamma_{xy}$, and $\varepsilon_x\varepsilon_y\gamma_{xy}$ are all $= \gamma_{xy}$, and Eq. (b) can be written

$$2\varepsilon_n = 2\varepsilon_x\cos^2\theta + 2\varepsilon_y\sin^2\theta + 2\gamma_{xy}\sin\theta\cos\theta$$

which upon dividing through by 2 is Eq. 3-7*a*.

3-24*

The given values are

$$\varepsilon_x = \varepsilon_{AD} = -600\ \mu\text{m/m}$$

$$\varepsilon_{AB} = -1200\ \mu\text{m/m} \qquad \theta_{AB} = \tan^{-1}\frac{240}{200} = 50.194°$$

$$\varepsilon_{BD} = 750\ \mu\text{m/m} \qquad \theta_{BD} = -\theta_{AB} = -50.194°$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_{AB} = (-600)\cos^2\theta_{AB} + \varepsilon_y \sin^2\theta_{AB} + \gamma_{xy}\sin\theta_{AB}\cos\theta_{AB} = -1200$$

$$\varepsilon_{BD} = (-600)\cos^2\theta_{BD} + \varepsilon_y \sin^2\theta_{BD} + \gamma_{xy}\sin\theta_{BD}\cos\theta_{BD} = 750$$

$$0.59016\varepsilon_y + 0.49180\gamma_{xy} = -954.098$$

$$0.59016\varepsilon_y - 0.49180\gamma_{xy} = 995.902$$

$\varepsilon_y = 35.4\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{nt} = -1983\ \mu\text{rad}$ .... **Ans.**

3-25*

The given values are

$$\varepsilon_x = 1250\ \mu\text{in./in.} \qquad \varepsilon_n = 1575\ \mu\text{in./in.} \qquad \varepsilon_t = 1350\ \mu\text{in./in.}$$

$$\theta_n = 45° \qquad \theta_t = 135°$$

$$\varepsilon_n = (1250)\cos^2(45°) + \varepsilon_y \sin^2(45°) + \gamma_{xy}\sin(45°)\cos(45°) = 1575$$

$$\varepsilon_t = (1250)\cos^2(135°) + \varepsilon_y \sin^2(135°) + \gamma_{xy}\sin(135°)\cos(135°) = 1350$$

$$0.5\varepsilon_y + 0.5\gamma_{xy} = 950$$

$$0.5\varepsilon_y - 0.5\gamma_{xy} = 725$$

$$\varepsilon_y = 1675\ \mu\text{in./in.} \qquad \gamma_{xy} = 225\ \mu\text{rad}$$

(a) $\gamma_{nt} = -2\left[(1250)-(1675)\right]\sin(45°)\cos(45°) + (225)\left[\cos^2(45°) - \sin^2(45°)\right]$

$\gamma_{nt} = 425\ \mu\text{rad}$ .... **Ans.**

(b) $\varepsilon_y = 1675\ \mu\text{in./in.}$ .... **Ans.**

3-26

The given values are

$$\varepsilon_x = 1950\ \mu\text{m/m} \qquad \varepsilon_y = -1625\ \mu\text{m/m} \qquad \varepsilon_n = -1275\ \mu\text{m/m}$$

$$\sin\theta_n = 3/5 \qquad \cos\theta_n = 4/5$$

(a) $$\varepsilon_n = (1950)\cos^2\theta_n + (-1625)\sin^2\theta_n + \gamma_{xy}\sin\theta_n\cos\theta_n = -1275$$

$\gamma_{xy} = -4037.500\ \mu\text{rad} \cong -4040\ \mu\text{rad}$ .... **Ans.**

(b) $$\varepsilon_{QR} = (1950)\cos^2(-\theta_n) + (-1625)\sin^2(-\theta_n) + (-4037.5)\sin(-\theta_n)\cos(-\theta_n)$$

$\varepsilon_{QR} = 2600\ \mu\text{m/m}$ .... **Ans.**

3-27*

The given values are

$$\varepsilon_x = 600\ \mu\text{in./in.} \qquad \varepsilon_y = -200\ \mu\text{in./in.} \qquad \gamma_{xy} = -480\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-480)}{(600)-(-200)} = -15.482°, \quad 74.518°$$

When $\theta_p = -15.482°$

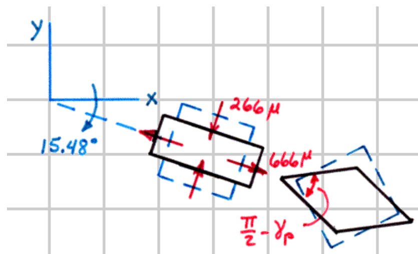
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (600)\cos^2\theta_p + (-200)\sin^2\theta_p + (-480)\sin\theta_p\cos\theta_p$$

$$= 666.476\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -266.476\ \mu\text{in./in.}$$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 932.952\ \mu\text{rad}$$



$\varepsilon_{p1} = +666\ \mu\text{in./in.}$ 15.48° .... **Ans.**

$\varepsilon_{p2} = -266\ \mu\text{in./in.}$ 74.52° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 933\ \mu\text{rad}$ .... **Ans.**

3-28*

The given values are

$$\varepsilon_x = 960\ \mu\text{m/m} \qquad \varepsilon_y = -320\ \mu\text{m/m} \qquad \gamma_{xy} = 500\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(500)}{(960)-(-320)} = 10.668°, \quad -79.332°$$

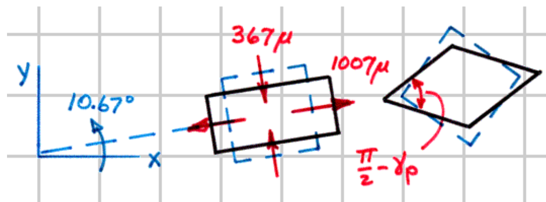
When $\theta_p = 10.668°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (960)\cos^2\theta_p + (-320)\sin^2\theta_p + (500)\sin\theta_p\cos\theta_p = 1007.095\ \mu\text{m/m} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -367.095\ \mu\text{m/m}$$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1374.190\ \mu\text{rad}$$



$\varepsilon_{p1} = +1007\ \mu\text{m/m}$ ∡ 10.67° .... **Ans.**

$\varepsilon_{p2} = -367\ \mu\text{m/m}$ ∡ 79.33° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 1374\ \mu\text{rad}$ .... **Ans.**

3-29

The given values are

$$\varepsilon_x = 900\ \mu\text{in./in.} \qquad \varepsilon_y = -300\ \mu\text{in./in.} \qquad \gamma_{xy} = 480\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(480)}{(900)-(-300)} = 10.901°, \quad -79.099°$$

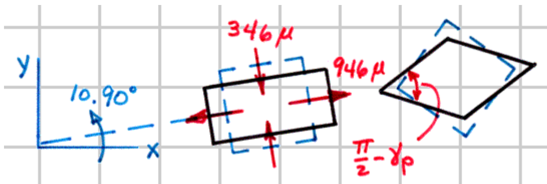
When $\theta_p = 10.901°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (900)\cos^2\theta_p + (-300)\sin^2\theta_p + (480)\sin\theta_p\cos\theta_p = 946.220\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -346.220\ \mu\text{in./in.}$$

$$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1292.440\ \mu\text{rad}$$



$\varepsilon_{p1} = +946\ \mu\text{in./in.}$ ∡ 10.90° .... **Ans.**

$\varepsilon_{p2} = -346\ \mu\text{in./in.}$ ∡ 79.10° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{\max} = \gamma_p = 1292\ \mu\text{rad}$ .... **Ans.**

3-30

The given values are

$$\varepsilon_x = -900\ \mu\text{m/m} \qquad \varepsilon_y = 600\ \mu\text{m/m} \qquad \gamma_{xy} = -420\ \mu\text{rad}$$

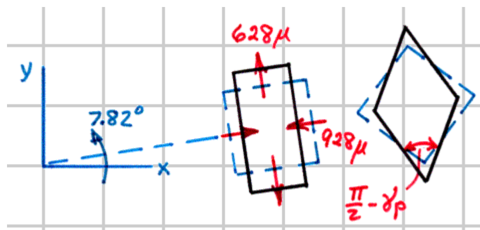
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-420)}{(-900)-(600)} = 7.821°, \quad -82.179°$$

When $\theta_p = 7.821°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$
$$= (-900)\cos^2\theta_p + (600)\sin^2\theta_p + (-420)\sin\theta_p\cos\theta_p$$
$$= -928.845\ \mu\text{m/m} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 628.845\ \mu\text{m/m}$$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1557.690\ \mu\text{rad}$$



$\varepsilon_{p1} = +629\ \mu\text{m/m}$ ⦩ 82.18° .... **Ans.**

$\varepsilon_{p2} = -929\ \mu\text{m/m}$ ∡ 7.82° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 1558\ \mu\text{rad}$ .... **Ans.**

3-31*

The given values are

$$\varepsilon_x = 750 \ \mu\text{in./in.} \qquad \varepsilon_y = -1000 \ \mu\text{in./in.} \qquad \gamma_{xy} = 360 \ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(360)}{(750)-(-1000)} = 5.812°, \quad -84.188°$$

When $\theta_p = 5.812°$

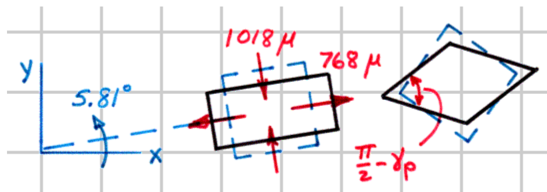
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (750)\cos^2\theta_p + (-1000)\sin^2\theta_p + (360)\sin\theta_p\cos\theta_p$$

$$= 768.322 \ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -1018.322 \ \mu\text{in./in.}$$

$$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1786.64 \ \mu\text{rad}$$



$\varepsilon_{p1} = +768 \ \mu\text{in./in.}$ ∡ $5.81°$ .... **Ans.**

$\varepsilon_{p2} = -1018 \ \mu\text{in./in.}$ ∡ $84.19°$ .... **Ans.**

$\varepsilon_{p3} = 0 \ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{\max} = \gamma_p = 1787 \ \mu\text{rad}$ .... **Ans.**

3-32

The given values are

$$\varepsilon_x = -750\ \mu\text{m/m} \qquad \varepsilon_y = 410\ \mu\text{m/m} \qquad \gamma_{xy} = -250\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-250)}{(-750)-(410)} = 6.081°, \quad -83.919°$$

When $\theta_p = 6.081°$

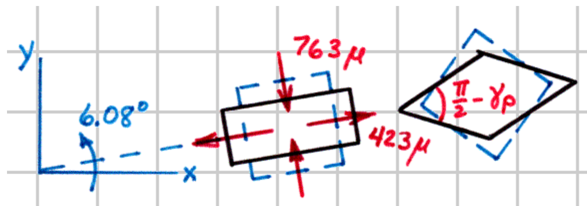
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (-750)\cos^2\theta_p + (410)\sin^2\theta_p + (-250)\sin\theta_p\cos\theta_p$$

$$= -763.317\ \mu\text{m/m} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 423.317\ \mu\text{m/m}$$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1186.634\ \mu\text{rad}$$



$\varepsilon_{p1} = +423\ \mu$m/m ∠ 83.92° .... **Ans.**

$\varepsilon_{p2} = -763\ \mu$m/m ∠ 6.08° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu$m/m .... **Ans.**

$\gamma_{max} = \gamma_p = 1187\ \mu$rad .... **Ans.**

3-33*

The given values are

$$\varepsilon_x = 720\ \mu\text{in./in.} \qquad \varepsilon_y = 520\ \mu\text{in./in.} \qquad \gamma_{xy} = 480\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(480)}{(720)-(520)} = 33.690°, \quad -56.310°$$

When $\theta_p = 33.690°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

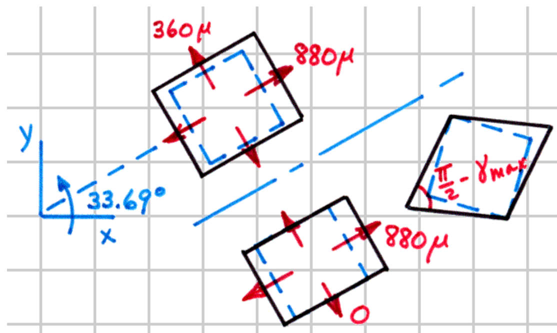
$$= (720)\cos^2\theta_p + (520)\sin^2\theta_p + (480)\sin\theta_p\cos\theta_p$$

$$= 880\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = 360\ \mu\text{in./in.}$$

$$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 520\ \mu\text{rad}$$

$$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 880 - 0 = 880\ \mu\text{rad}$$



$\varepsilon_{p1} = +880\ \mu\text{in./in.}$ ∡ 33.69° .... **Ans.**

$\varepsilon_{p2} = +360\ \mu\text{in./in.}$ ∡ 56.31° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_p = 520\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = 880\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-34*

The given values are

$$\varepsilon_x = -540\ \mu\text{m/m} \qquad \varepsilon_y = -980\ \mu\text{m/m} \qquad \gamma_{xy} = 560\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(560)}{(-540)-(-980)} = 25.921°, \quad -64.079°$$

When $\theta_p = 25.921°$

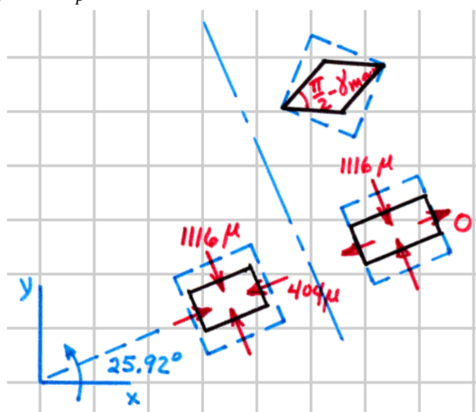
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (-540)\cos^2\theta_p + (-980)\sin^2\theta_p + (560)\sin\theta_p\cos\theta_p$$

$$= -403.910\ \mu\text{m/m} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -1116.090\ \mu\text{m/m}$$

$$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 712.180\ \mu\text{rad}$$



$\varepsilon_{p1} = -404\ \mu\text{m/m}$ ∡ 25.92° .... **Ans.**

$\varepsilon_{p2} = -1116\ \mu\text{m/m}$ ∡ 64.08° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_p = 712\ \mu\text{rad}$ .... **Ans.**

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 0 - (-1116) = 1116\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-35

The given values are

$$\varepsilon_x = 864\ \mu\text{in./in.} \qquad \varepsilon_y = 432\ \mu\text{in./in.} \qquad \gamma_{xy} = 288\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(288)}{(864)-(432)} = 16.845°, \quad -73.155°$$

When $\theta_p = 16.845°$

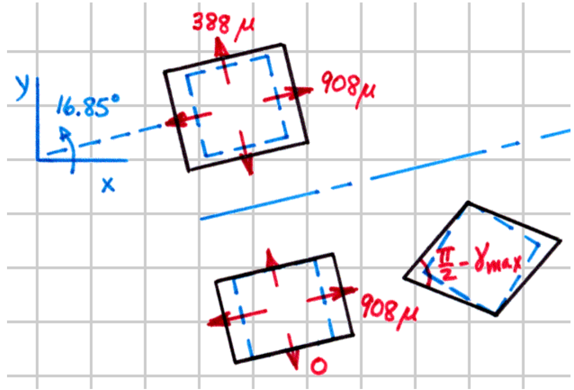
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (864)\cos^2\theta_p + (432)\sin^2\theta_p + (288)\sin\theta_p\cos\theta_p$$

$$= 907.600\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = 388.400\ \mu\text{in./in.}$$

$$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 519.200\ \mu\text{rad}$$



$\varepsilon_{p1} = +908\ \mu\text{in./in.}$ ∡ 16.85° .... **Ans.**

$\varepsilon_{p2} = +388\ \mu\text{in./in.}$ ∡ 73.15° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_p = 519\ \mu\text{rad}$ .... **Ans.**

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 908 - 0 = 908\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-36*

The given values are

$$\varepsilon_x = 900\ \mu\text{m/m} \qquad \varepsilon_y = 650\ \mu\text{m/m} \qquad \gamma_{xy} = 300\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(300)}{(900)-(650)} = 25.097°, \quad -64.903°$$

When $\theta_p = 25.097°$

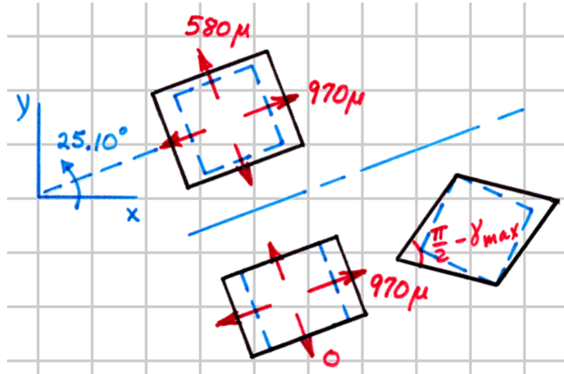
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (900)\cos^2\theta_p + (650)\sin^2\theta_p + (300)\sin\theta_p\cos\theta_p$$

$$= 970.256\ \mu\text{m/m} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = 579.744\ \mu\text{m/m}$$

$$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 390.512\ \mu\text{rad}$$



$\varepsilon_{p1} = +970\ \mu\text{m/m}$ ∡ 25.10° .... **Ans.**

$\varepsilon_{p2} = +580\ \mu\text{m/m}$ ∡ 64.90° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_p = 391\ \mu\text{rad}$ .... **Ans.**

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 970 - 0 = 970\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-37

The given values are

$$\varepsilon_x = -325\ \mu\text{in./in.} \qquad \varepsilon_y = -625\ \mu\text{in./in.} \qquad \gamma_{xy} = 680\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(680)}{(-325)-(-625)} = 33.097°, \quad -56.903°$$

When $\theta_p = 33.097°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (-325)\cos^2\theta_p + (-625)\sin^2\theta_p + (680)\sin\theta_p\cos\theta_p$$

$$= -103.382\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -846.618\ \mu\text{in./in.}$$

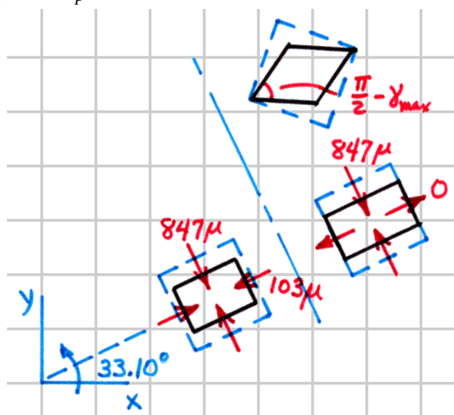
$$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 743.236\ \mu\text{rad}$$

$\varepsilon_{p1} = -103.4\ \mu\text{in./in.}$ ∡ $33.10°$ .... **Ans.**

$\varepsilon_{p2} = -847\ \mu\text{in./in.}$ ∡ $56.90°$ .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_p = 743\ \mu\text{rad}$ .... **Ans.**



$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 0 - (-847) = 847\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-38

The given values are

$$\varepsilon_x = -900\ \mu\text{m/m} \qquad \varepsilon_y = -650\ \mu\text{m/m} \qquad \gamma_{xy} = -600\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-600)}{(-900)-(-650)} = 33.690°, \quad -56.310°$$

When $\theta_p = 33.690°$

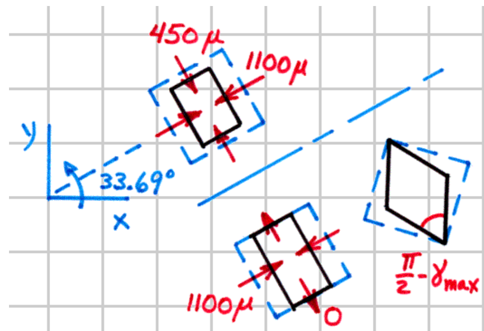
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (-900)\cos^2\theta_p + (-650)\sin^2\theta_p + (-600)\sin\theta_p\cos\theta_p$$

$$= -1100\ \mu\text{m/m} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = -450\ \mu\text{m/m}$$

$$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 650\ \mu\text{rad}$$



$\varepsilon_{p1} = -450\ \mu\text{m/m}$ ∡ 56.31° .... **Ans.**

$\varepsilon_{p2} = -1100\ \mu\text{m/m}$ ∡ 33.69° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_p = 650\ \mu\text{rad}$ .... **Ans.**

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 0 - (-1100) = 1100\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-39*

The given values are

$$\varepsilon_x = 480\ \mu\text{in./in.} \qquad \varepsilon_y = -1200\ \mu\text{in./in.} \qquad \varepsilon_{p2} = -1400\ \mu\text{in./in.}$$

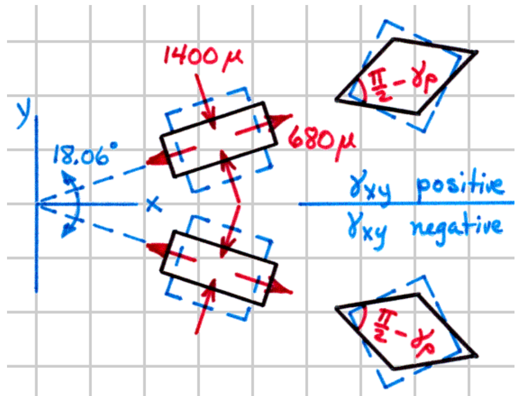
$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = +680\ \mu\text{in./in.}$$

$$680 = \frac{480 + (-1200)}{2} + \sqrt{\left[\frac{480 - (-1200)}{2}\right]^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = \pm 1226.377\ \mu\text{rad} \cong \pm 1226\ \mu\text{rad}$ .... **Ans.**

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(\pm 1226.377)}{(480) - (-1200)} = \pm 18.064^\circ, \quad \text{m}71.936^\circ$$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 2080\ \mu\text{rad}$$



Using $\theta_p = +18.064^\circ$

$\varepsilon_{p1} = +680\ \mu\text{in./in.}$ ∡ 18.06° .... **Ans.**

$\varepsilon_{p2} = -1400\ \mu\text{in./in.}$ ⦩ 71.94° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 2080\ \mu\text{rad}$ .... **Ans.**

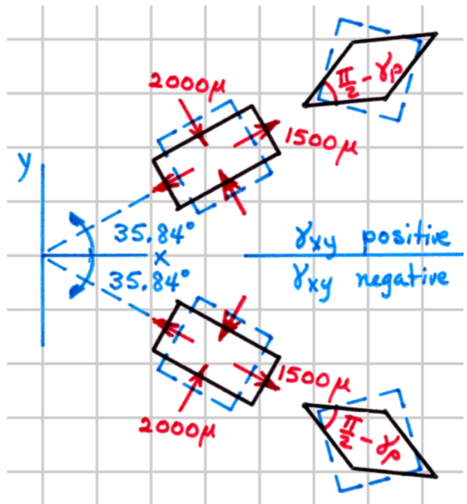
3-40*
The given values are

$$\varepsilon_x = 300\ \mu\text{m/m} \qquad \varepsilon_y = -800\ \mu\text{m/m} \qquad \varepsilon_{p1} = 1500\ \mu\text{m/m}$$

$\varepsilon_{p2} = -2000\ \mu\text{m/m}$ .... **Ans.**

$$1500 = \frac{300+(-800)}{2} + \sqrt{\left[\frac{300-(-800)}{2}\right]^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = \pm 3322.650\ \mu\text{rad} \cong \pm 3320\ \mu\text{rad}$ .... **Ans.**



$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(\pm 3322.650)}{(300)-(-800)}$$

$$= \pm 35.841°, \quad \text{m}54.159°$$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 3500\ \mu\text{rad}$$

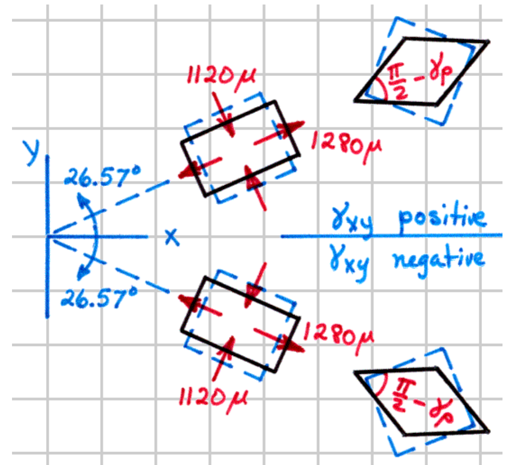
$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 3500\ \mu\text{rad}$ .... **Ans.**

3-41

The given values are

$$\varepsilon_x = 800\ \mu\text{in./in.} \qquad \varepsilon_{p1} = 1280\ \mu\text{in./in.} \qquad \gamma_{\max} = 2400\ \mu\text{rad}$$



Assuming that $\gamma_{\max} = \gamma_p$ gives

$$\gamma_{\max} = \gamma_p = (1280) - \varepsilon_{p2} = 2400$$

$$\varepsilon_{p2} = -1120\ \mu\text{in./in.}$$

Then $\varepsilon_y = \varepsilon_{p1} + \varepsilon_{p2} - \varepsilon_x = (1280) + (-1120) - (800)$

$\varepsilon_y = -640\ \mu\text{in./in.}$ .... **Ans.**

$$1280 = \frac{800 + (-640)}{2} + \sqrt{\left[\frac{800 - (-640)}{2}\right]^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = \pm 1920.00\ \mu\text{rad} \cong \pm 1920\ \mu\text{rad}$ .... **Ans.**

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(\pm 1920)}{(800) - (-640)} = \pm 26.565°, \quad \text{m}63.435°$$

Using $\theta_p = +26.565°$

$\varepsilon_{p2} = -1120\ \mu\text{in./in.}$ ∡ 71.94° .... **Ans.**

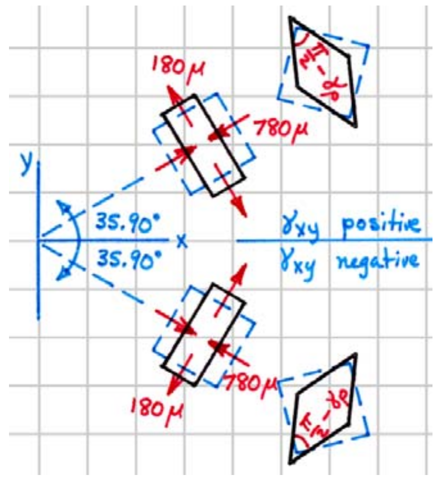
$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

3-42

The given values are

$$\varepsilon_x = -450\ \mu\text{m/m} \qquad \varepsilon_{p2} = -780\ \mu\text{m/m} \qquad \gamma_{max} = 960\ \mu\text{rad}$$

Assuming that $\gamma_{max} = \gamma_p$ gives



$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - (-780) = 960$$

$$\varepsilon_{p1} = 180.00\ \mu\text{m/m}$$

Then $\varepsilon_y = \varepsilon_{p1} + \varepsilon_{p2} - \varepsilon_x = (180) + (-780) - (-450)$

$\varepsilon_y = -150.0\ \mu\text{m/m}$ .... **Ans.**

$$180 = \frac{(-450)+(-150)}{2} + \sqrt{\left[\frac{(-450)-(-150)}{2}\right]^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = \pm 911.921\ \mu\text{rad} \cong \pm 912\ \mu\text{rad}$ .... **Ans.**

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(\pm 911.921)}{(-450)-(-150)} = \mp 35.895°, \quad \pm 54.105°$$

Using $\theta_p = -35.895°$

$\varepsilon_{p1} = +180.0\ \mu\text{m/m}\ \measuredangle\ 54.10°$ .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

3-43*

The given values are

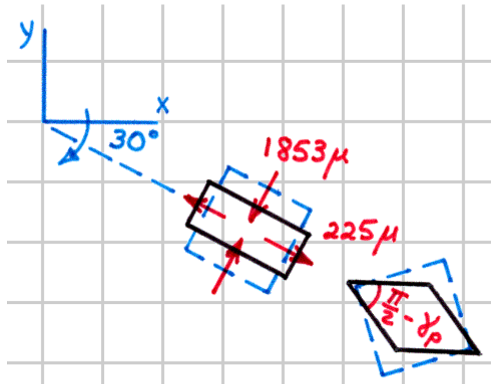
$$\gamma_{xy} = -1800\ \mu\text{rad} \qquad \varepsilon_{p1} = 225\ \mu\text{in./in.} \qquad \theta_p = -30°$$

$$\tan 2(-30°) = \frac{(-1800)}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_x - \varepsilon_y = 1039.230\ \mu\text{in./in.}$$

$$225 = \frac{\varepsilon_x + \varepsilon_y}{2} + \sqrt{\left[\frac{(1039.230)}{2}\right]^2 + \left(\frac{-1800}{2}\right)^2}$$

$$\varepsilon_x + \varepsilon_y = -1628.461\ \mu\text{in./in.}$$



$\varepsilon_x = -294.615\ \mu\text{in./in.} \cong -295\ \mu\text{in./in.}$ .... **Ans.**

$\varepsilon_y = -1333.845\ \mu\text{in./in.} \cong -1334\ \mu\text{in./in.}$ .... **Ans.**

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -1853.461\ \mu\text{in./in.} \cong -1853\ \mu\text{in./in.}$$

$\varepsilon_{p2} = -1853\ \mu\text{in./in.}\ \measuredangle\ 60°$ .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 2078\ \mu\text{rad}$ .... **Ans.**

3-44

The given values are

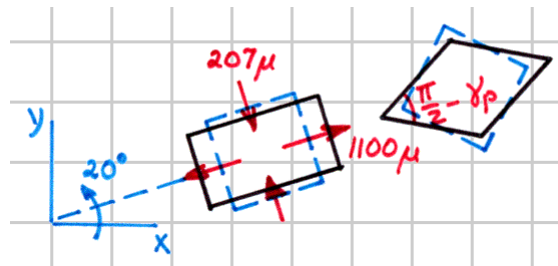
$$\gamma_{xy} = 840\ \mu\text{rad} \qquad \varepsilon_{p1} = 1100\ \mu\text{m/m} \qquad \theta_p = 20°$$

$$\tan 2(20°) = \frac{(840)}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_x - \varepsilon_y = 1001.073\ \mu\text{m/m}$$

$$1100 = \frac{\varepsilon_x + \varepsilon_y}{2} + \sqrt{\left[\frac{(1001.073)}{2}\right]^2 + \left(\frac{840}{2}\right)^2}$$

$$\varepsilon_x + \varepsilon_y = 893.192\ \mu\text{m/m}$$



$\varepsilon_x = +947.133\ \mu\text{m/m} \cong 947\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_y = -53.940\ \mu\text{m/m} \cong -53.9\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -206.808\ \mu\text{m/m} \cong -207\ \mu\text{m/m}$$

$\varepsilon_{p2} = -207\ \mu\text{m/m}\ \measuredangle\ 70°$ .... **Ans.**

$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1306.808\ \mu\text{rad} \cong 1307\ \mu\text{rad}$ .... **Ans.**

3-45

The given values are

$$\varepsilon_y = -750\ \mu\text{in./in.} \qquad \gamma_{xy} = -750\ \mu\text{rad} \qquad \varepsilon_{p2} = -1500\ \mu\text{in./in.}$$

$$-1500 = \frac{\varepsilon_x + (-750)}{2} + \sqrt{\left[\frac{\varepsilon_x - (-750)}{2}\right]^2 + \left(\frac{-750}{2}\right)^2}$$

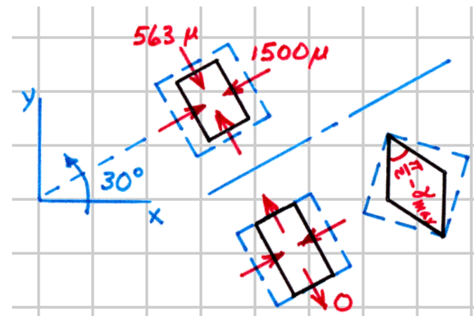
$\varepsilon_x = -1312.500\ \mu\text{in./in.} \cong -1313\ \mu\text{in./in.}$ .... **Ans.**

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-750)}{(-1312.5)-(-750)} = 26.565°, \quad -63.435°$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = -562.500\ \mu\text{in./in.}$$

$$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 937.500\ \mu\text{rad}$$

$$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = (0) - (-1500) = 1500\ \mu\text{rad}$$



$\varepsilon_{p1} = -563\ \mu\text{in./in.}$ ∠ 63.43° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_p = 937\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = 1500\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-46*

The given values are

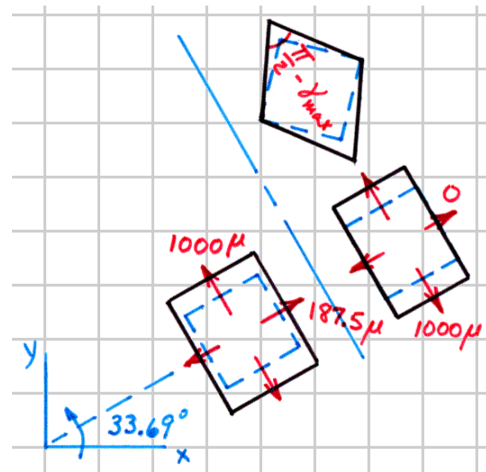
$$\varepsilon_y = 750\ \mu\text{m/m} \qquad \gamma_{xy} = -750\ \mu\text{rad} \qquad \varepsilon_{p1} = 1000\ \mu\text{m/m}$$

$$1000 = \frac{\varepsilon_x + (750)}{2} + \sqrt{\left[\frac{\varepsilon_x - (750)}{2}\right]^2 + \left(\frac{-750}{2}\right)^2}$$

$\varepsilon_x = 437.50\ \mu\text{m/m} \cong +437\ \mu\text{m/m}$ .... **Ans.**

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-750)}{(437.5)-(750)}$$

$\theta_p = 33.690°, \quad -56.310°$ .... **Ans.**



$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = 187.50\ \mu\text{m/m}$$

$\varepsilon_{p2} \cong +187.5\ \mu\text{m/m}$ .... **Ans.**

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 813\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 1000 - 0 = 1000\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

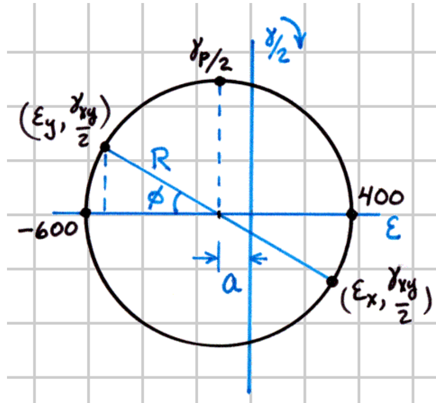
3-47*

The given values for use in drawing Mohr's circle are

$$\varepsilon_{p1} = 400\ \mu\text{in./in.}$$

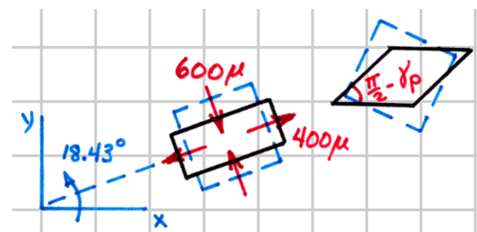
$$\varepsilon_{p2} = -600\ \mu\text{in./in.}$$

$$\theta_p = \frac{\phi}{2} = 18.43°$$



$$a = \frac{400 + (-600)}{2} = -100\ \mu\text{in./in.}$$

$$R = \frac{400 - (-600)}{2} = 500\ \mu\text{in./in.}$$



$\varepsilon_x = -100 + 500\cos 36.86° = +300\ \mu\text{in./in.}$ .... **Ans.**

$\varepsilon_y = -100 - 500\cos 36.86° = -500\ \mu\text{in./in.}$ .... **Ans.**

$$\gamma_{xy} = 2(500\sin 36.86°) = 600\ \mu\text{rad (CCW)}$$

$\gamma_{xy} = +600\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 2R = 1000\ \mu\text{rad}$ .... **Ans.**

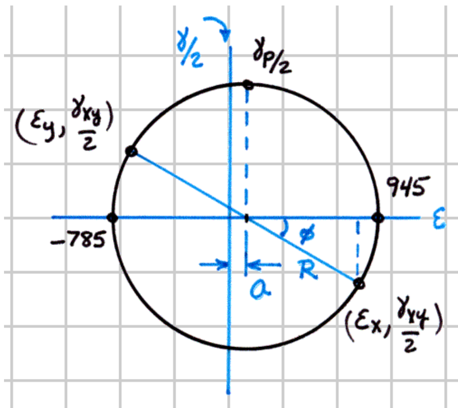
3-48*

The given values for use in drawing Mohr's circle are

$$\varepsilon_{p1} = 945\ \mu\text{m/m}$$

$$\varepsilon_{p2} = -785\ \mu\text{m/m}$$

$$\theta_p = \frac{\phi}{2} = 16.85°$$

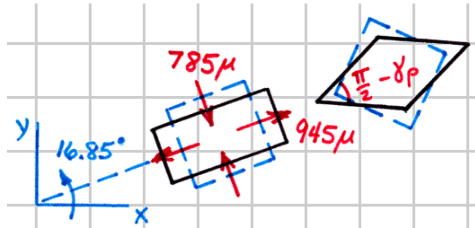


$$a = \frac{945 + (-785)}{2} = 80\ \mu\text{m/m}$$

$$R = \frac{945 - (-785)}{2} = 865\ \mu\text{m/m}$$

$\varepsilon_x = 80 + 865\cos 33.70° = +800\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_y = 80 - 865\cos 33.70° = -640\ \mu\text{m/m}$ .... **Ans.**



$$\gamma_{xy} = 2(865\sin 33.70°) = 960\ \mu\text{rad (CCW)}$$

$\gamma_{xy} = +960\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 2R = 1730\ \mu\text{rad}$ .... **Ans.**

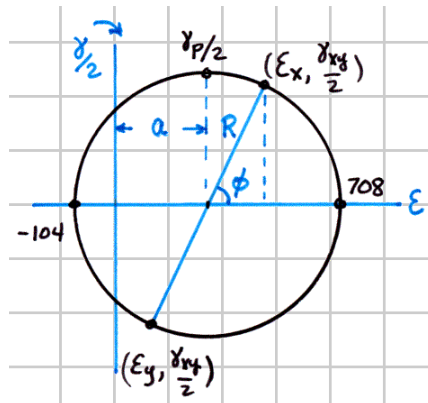
3-49

The given values for use in drawing Mohr's circle are

$$\varepsilon_{p1} = 708\ \mu\text{in./in.}$$

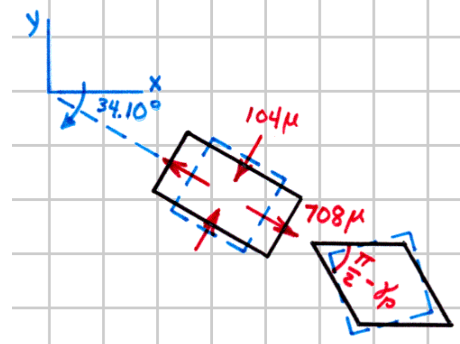
$$\varepsilon_{p2} = -104\ \mu\text{in./in.}$$

$$\theta_p = \frac{\phi}{2} = -34.10°$$



$$a = \frac{708 + (-104)}{2} = 302\ \mu\text{in./in.}$$

$$R = \frac{708 - (-104)}{2} = 406\ \mu\text{in./in.}$$



$\varepsilon_x = 302 + 406\cos 68.20° = +453\ \mu\text{in./in.}$ .... **Ans.**

$\varepsilon_y = 302 - 406\cos 68.20° = +151.2\ \mu\text{in./in.}$ .... **Ans.**

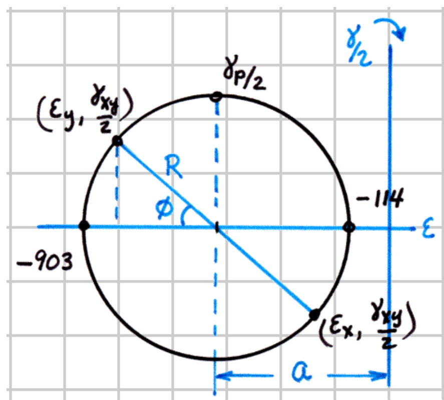
$$\gamma_{xy} = 2(406\sin 68.20°) = 754\ \mu\text{rad (CW)}$$

$\gamma_{xy} = -754\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 2R = 812\ \mu\text{rad}$ .... **Ans.**

3-50

The given values for use in drawing Mohr's circle are



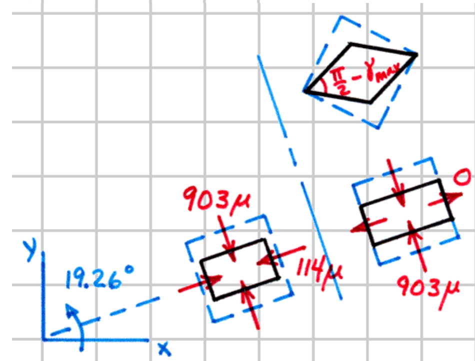
$$\varepsilon_{p1} = -114\ \mu\text{m/m}$$

$$\varepsilon_{p2} = -903\ \mu\text{m/m}$$

$$\theta_p = \frac{\phi}{2} = 19.26°$$

$$a = \frac{(-114) + (-903)}{2} = -508.5\ \mu\text{m/m}$$

$$R = \frac{(-114) - (-903)}{2} = 394.5\ \mu\text{m/m}$$



$\varepsilon_x = -508.5 + 394.5\cos 38.52° = -199.8\ \mu\text{m/m}$ **Ans.**

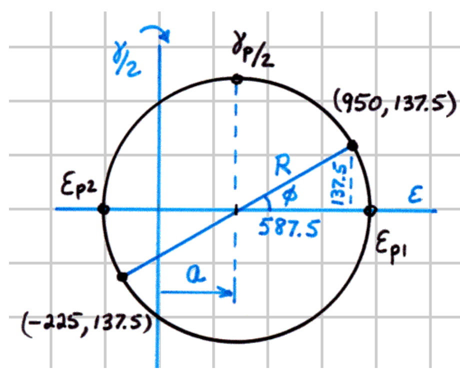
$\varepsilon_y = -508.5 - 394.5\cos 38.52° = -817\ \mu\text{m/m}$ .... **Ans.**

$$\gamma_{xy} = 2(394.5\sin 38.52°) = 491\ \mu\text{rad (CCW)}$$

$\gamma_{xy} = +491\mu\text{rad}$ .... **Ans.**

$\gamma_p = 2R = 789\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = (\varepsilon_{max} - \varepsilon_{min}) = 0 - (-903) = 903\ \mu\text{rad (out-of-plane)}$ .... **Ans.**



3-51*

The given values for use in drawing Mohr's circle are

$$\varepsilon_x = 950\ \mu\text{in./in.}$$

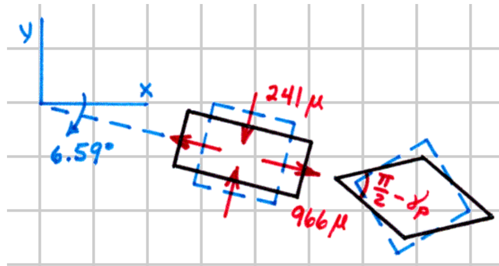
$$\varepsilon_y = -225\ \mu\text{in./in.}$$

$$\gamma_{xy} = -275\ \mu\text{rad}$$

$$a = \frac{950 + (-225)}{2} = 362.5\ \mu\text{in./in.}$$

$$R = \sqrt{(587.5)^2 + (137.5)^2} = 603.38\ \mu\text{in./in.}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{137.5}{587.5} = 6.59°\ \text{(CW)}$$



$$\varepsilon_{p1} = 362.5 + 603.38 = +965.88\ \mu\text{in./in.}$$

$\varepsilon_{p1} \cong +966\ \mu\text{in./in.}$ ↘ 6.59° .... **Ans.**

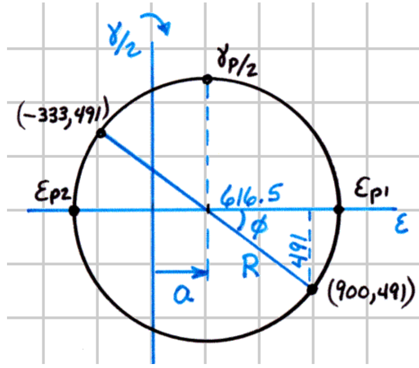
$$\varepsilon_{p2} = 362.5 - 603.38 = -240.88\ \mu\text{in./in.}$$

$\varepsilon_{p2} \cong -241\ \mu\text{in./in.}$ ∡ 83.41° .... **Ans.**

$\gamma_{\max} = \gamma_p = 2R = 1207\ \mu\text{rad}$ .... **Ans.**

3-52*

The given values for use in drawing Mohr's circle are



$$\varepsilon_x = 900\ \mu\text{m/m}$$

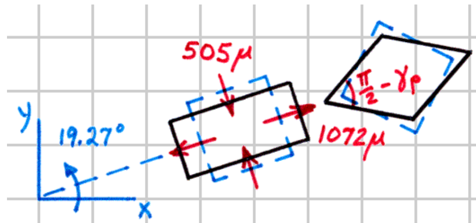
$$\varepsilon_y = -333\ \mu\text{m/m}$$

$$\gamma_{xy} = 982\ \mu\text{rad}$$

$$a = \frac{900+(-333)}{2} = 283.5\ \mu\text{m/m}$$

$$R = \sqrt{(616.5)^2 + (491)^2} = 788.133\ \mu\text{m/m}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{491}{616.5} = 19.267^\circ\ \text{(CCW)}$$



$$\varepsilon_{p1} = 283.5 + 788.133 = +1071.633\ \mu\text{m/m}$$

$\varepsilon_{p1} \cong +1072\ \mu\text{m/m}\ \measuredangle\ 19.27^\circ$ .... **Ans.**

$$\varepsilon_{p2} = 283.5 - 788.133 = -504.633\ \mu\text{m/m}$$

$\varepsilon_{p2} \cong -505\ \mu\text{m/m}\ \measuredangle\ 70.73^\circ$ .... **Ans.**

$\gamma_{max} = \gamma_p = 2R = 1576\ \mu\text{rad}$ .... **Ans.**

3-53

The given values for use in drawing Mohr's circle are

$$\varepsilon_x = 750\ \mu\text{in./in.}$$

$$\varepsilon_y = 390\ \mu\text{in./in.}$$

$$\gamma_{xy} = -900\ \mu\text{rad}$$

$$a = \frac{750+390}{2} = 570\ \mu\text{in./in.}$$

$$R = \sqrt{(180)^2 + (450)^2} = 484.66\ \mu\text{in./in.}$$

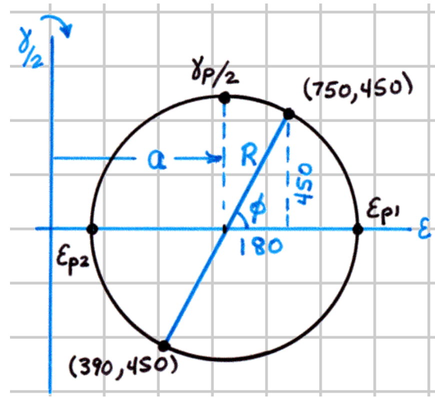
$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{450}{180} = 34.100°\ \text{(CW)}$$

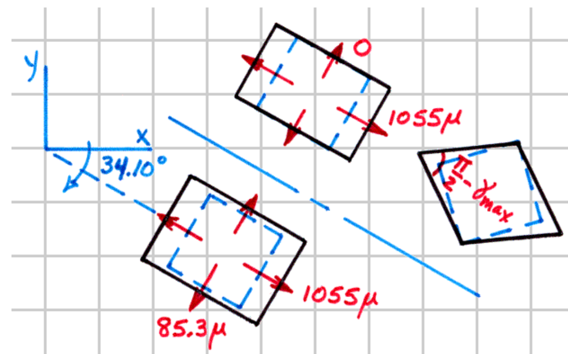
$$\varepsilon_{p1} = 570 + 484.66 = +1054.66\ \mu\text{in./in.}$$

$\varepsilon_{p1} \cong +1055\ \mu\text{in./in.}$ 34.10° ....**Ans.**

$$\varepsilon_{p2} = 570 - 484.66 = +85.34\ \mu\text{in./in.}$$

$\varepsilon_{p2} \cong +85.3\ \mu\text{in./in.}$ 55.90° ....**Ans.**





$\gamma_p = 2R = 969\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 1055 - 0 = 1055\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-54

The given values for use in drawing Mohr's circle are

$$\varepsilon_x = 600\ \mu\text{m/m}$$

$$\varepsilon_y = 480\ \mu\text{m/m}$$

$$\gamma_{xy} = 480\ \mu\text{rad}$$

$$a = \frac{600+480}{2} = 540\ \mu\text{m/m}$$

$$R = \sqrt{(60)^2 + (240)^2} = 247.386\ \mu\text{m/m}$$

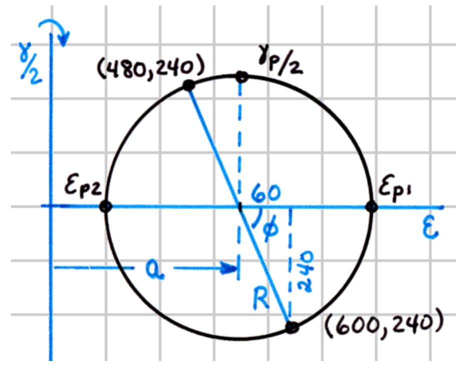
$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{240}{60} = 37.982°\ \text{(CCW)}$$

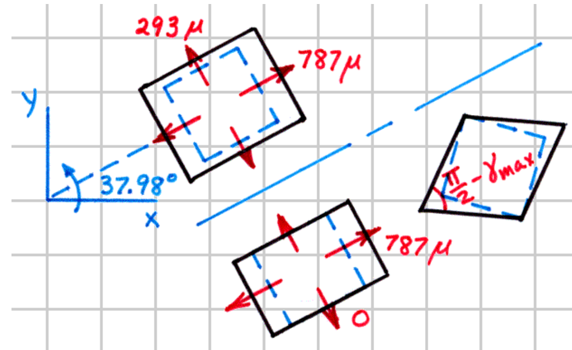
$$\varepsilon_{p1} = 540 + 247.386 = +787.386\ \mu\text{m/m}$$

$\varepsilon_{p1} \cong +787\ \mu\text{m/m}\ \measuredangle\ 37.98°$ ....**Ans.**

$$\varepsilon_{p2} = 540 - 247.386 = +292.614\ \mu\text{m/m}$$

$\varepsilon_{p2} \cong +293\ \mu\text{m/m}\ \rotatebox[origin=c]{180}{$\measuredangle$}\ 52.02°$ ....**Ans.**



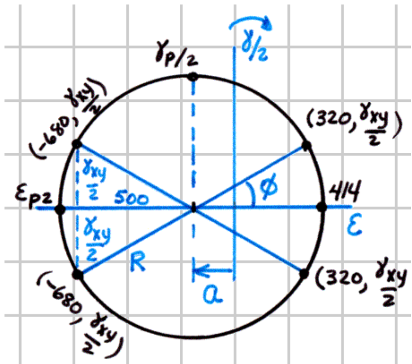


$\gamma_p = 2R = 495\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 787 - 0 = 787\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-55*

The given values for use in drawing Mohr's circle are



$$\varepsilon_x = -680\ \mu\text{in./in.}$$

$$\varepsilon_y = 320\ \mu\text{in./in.}$$

$$\varepsilon_{p1} = 414\ \mu\text{in./in.}$$

$$a = \frac{(-680)+(320)}{2} = -180\ \mu\text{in./in.}$$

$$R = 414 + 180 = 594\ \mu\text{in./in.}$$

$$\theta_p = \frac{\phi}{2} = \frac{1}{2}\cos^{-1}\frac{500}{594} = 16.34°$$

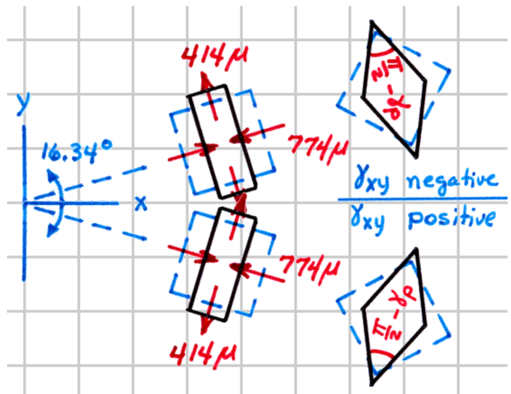
$$\gamma_{xy} = 2(594)\sin 32.68°$$

$\gamma_{xy} = \pm 641\ \mu\text{rad}$ .... **Ans.**

$$\varepsilon_{p2} = (-180) - 594$$

$\varepsilon_{p2} = -774\ \mu\text{in./in.}$ .... **Ans.**

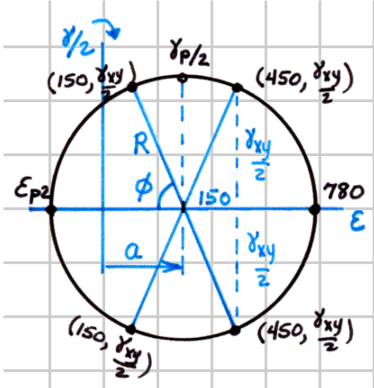
$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**



$\gamma_{max} = \gamma_p = 2R = 1188\ \mu\text{rad}$ .... **Ans.**

3-56*

The given values for use in drawing Mohr's circle are



$$\varepsilon_x = 450\ \mu\text{m/m}$$

$$\varepsilon_y = 150\ \mu\text{m/m}$$

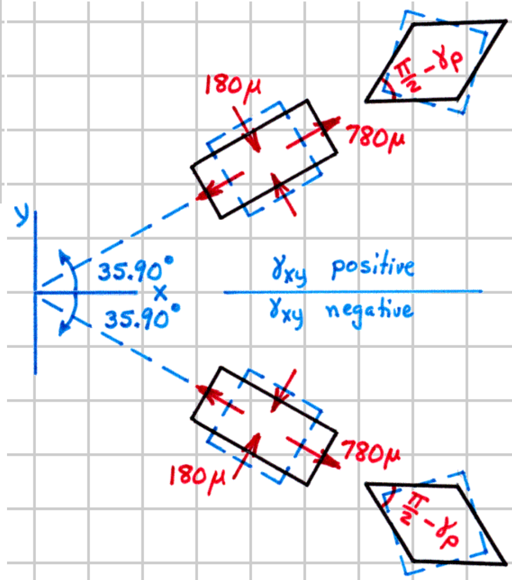
$$\varepsilon_{p1} = 780\ \mu\text{m/m}$$

$$a = \frac{(450)+(150)}{2} = 300\ \mu\text{m/m}$$

$$R = 780 - 300 = 480\ \mu\text{m/m}$$

$$\theta_p = \frac{\phi}{2} = \frac{1}{2}\cos^{-1}\frac{150}{480} = 35.895°$$

$$\gamma_{xy} = 2(480)\sin 71.790°$$



$\gamma_{xy} = \pm 912\ \mu\text{rad}$ .... **Ans.**

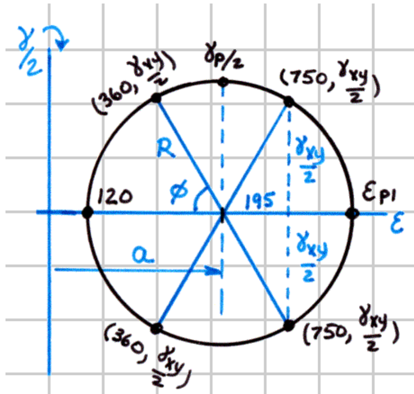
$\varepsilon_{p2} = (300) - (480) = -180\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{max} = \gamma_p = 2R = 960\ \mu\text{rad}$ .... **Ans.**

3-57

The given values for use in drawing Mohr's circle are



$$\varepsilon_x = 360\ \mu\text{in./in.}$$

$$\varepsilon_y = 750\ \mu\text{in./in.}$$

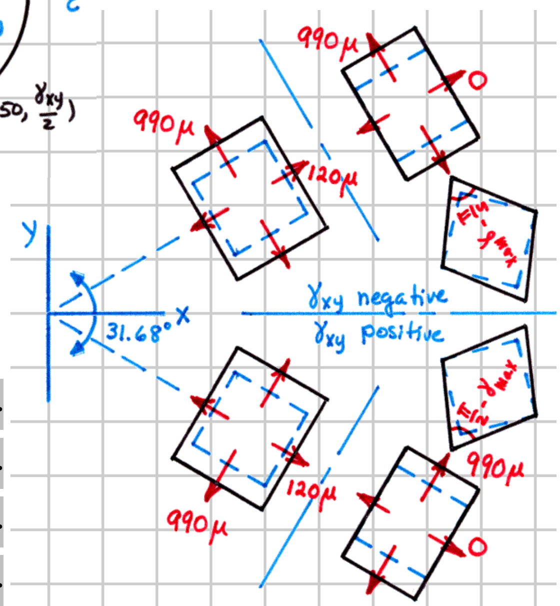
$$\varepsilon_{p2} = 120\ \mu\text{in./in.}$$

$$a = \frac{(360)+(750)}{2} = 555\ \mu\text{in./in.}$$

$$R = 555 - 120 = 435\ \mu\text{in./in.}$$

$$\theta_p = \frac{\phi}{2} = \frac{1}{2}\cos^{-1}\frac{195}{435} = 31.683°$$

$$\gamma_{xy} = 2(435)\sin 63.367°$$



$\gamma_{xy} = \pm 778\ \mu\text{rad}$ .... **Ans.**

$\varepsilon_{p1} = 555 + 435 = +990\ \mu\text{in./in.}$ .... **Ans.**

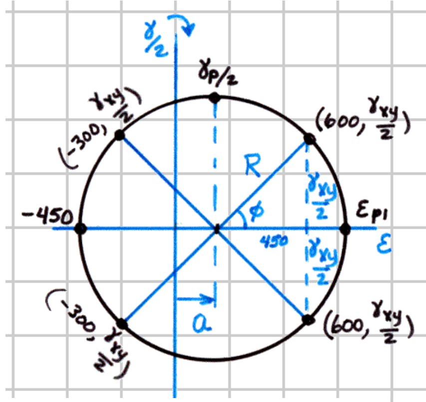
$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_p = 2R = 870\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 990 - 0 = 990\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-58

The given values for use in drawing Mohr's circle are



$$\varepsilon_x = -300\ \mu\text{m/m}$$

$$\varepsilon_y = 600\ \mu\text{m/m}$$

$$\varepsilon_{p2} = -450\ \mu\text{m/m}$$

$$a = \frac{(-300)+(600)}{2} = 150\ \mu\text{m/m}$$

$$R = 150 + 450 = 600\ \mu\text{m/m}$$

$$\theta_p = \frac{\phi}{2} = \frac{1}{2}\cos^{-1}\frac{450}{600} = 20.705°$$

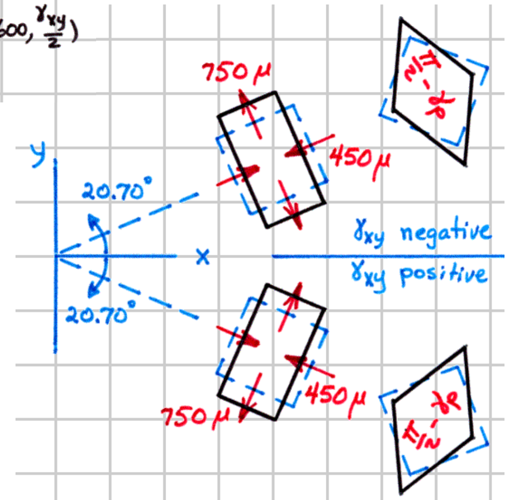
$$\gamma_{xy} = 2(600)\sin 41.410°$$

$\gamma_{xy} = \pm 794\ \mu\text{rad}$ ....**Ans.**

$\varepsilon_{p1} = (150) + (600) = +750\ \mu\text{m/m}$ ....**Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ ....**Ans.**

$\gamma_{max} = \gamma_p = 2R = 1200\ \mu\text{rad}$ ....**Ans.**



3-59*

(a) The given values are $\varepsilon_a = \varepsilon_x = 750\ \mu\text{in./in.}$ $\qquad \varepsilon_b = \varepsilon_{45°} = -125\ \mu\text{in./in.}$

$\varepsilon_c = \varepsilon_y = -250\ \mu\text{in./in.}$ $\qquad \nu = 0.30$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (750)\cos^2(45°) + (-250)\sin^2(45°) + \gamma_{xy}\sin(45°)\cos(45°) = 125$$

Therefore: $\varepsilon_x = +750\ \mu\text{in./in.}$ .... $\varepsilon_y = -250\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{xy} = -750\ \mu\text{rad}$ .... **Ans.**

(b) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-750)}{(750)-(-250)} = -18.435°, \quad 71.565°$$

When $\theta_p = 71.565°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$
$$= (750)\cos^2\theta_p + (-250)\sin^2\theta_p + (-750)\sin\theta_p\cos\theta_p$$
$$= -375\ \mu\text{in./in.} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 875\ \mu\text{in./in.}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.30}{1-0.30}\big[(750)+(-250)\big] = -214\ \mu\text{in./in.}$$

$\varepsilon_{p1} = +875\ \mu\text{in./in.}$ 18.43° .... **Ans.**

$\varepsilon_{p2} = -375\ \mu\text{in./in.}$ 71.57° .... **Ans.**

$\varepsilon_{p3} = -214\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1250\ \mu\text{rad}$ .... **Ans.**

3-60*

(a) The given values are $\varepsilon_a = \varepsilon_x = -555\ \mu\text{m/m}$ $\varepsilon_b = \varepsilon_{120°} = 925\ \mu\text{m/m}$

$\varepsilon_c = \varepsilon_{240°} = 740\ \mu\text{m/m}$ $\nu = 0.30$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (-555)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 925$$

$$\varepsilon_c = (-555)\cos^2(240°) + \varepsilon_y \sin^2(240°) + \gamma_{xy}\sin(240°)\cos(240°) = 740$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 1063.750$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = 878.750$$

Therefore: $\varepsilon_x = -555\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_y = 1295.00\ \mu\text{m/m} = +1295\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{xy} = -213.620\ \mu\text{rad} \cong -214\ \mu\text{rad}$ .... **Ans.**

(b) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-213.620)}{(-555)-(1295)} = 3.293°,\quad -86.707°$$

When $\theta_p = 3.293°$

$$\varepsilon_n = (-555)\cos^2\theta_p + (1295)\sin^2\theta_p + (-213.620)\sin\theta_p\cos\theta_p$$
$$= -561.146\ \mu\text{m/m} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 1301.146\ \mu\text{m/m}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.30}{1-0.30}\left[(-555)+(1295)\right] = -317\ \mu\text{m/m}$$

$\varepsilon_{p1} = +1301\ \mu\text{m/m}$ ∡ $86.71°$ .... **Ans.**

$\varepsilon_{p2} = -561\ \mu\text{m/m}$ ∡ $3.29°$ .... **Ans.**

$\varepsilon_{p3} = -317\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1862\ \mu\text{rad}$ .... **Ans.**

3-61

(a) The given values are $\varepsilon_a = \varepsilon_x = 800\ \mu\text{in./in.}$ $\varepsilon_c = \varepsilon_y = 600\ \mu\text{in./in.}$

$$\varepsilon_b = \varepsilon_n = 950\ \mu\text{in./in.} \qquad \theta_b = \tan^{-1}\frac{3}{4} = 36.870° \qquad \nu = 0.33$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\varepsilon_b = (800)\cos^2\theta_b + (600)\sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b = 950$$

Therefore: $\varepsilon_x = +800\ \mu\text{in./in.}$ .... $\varepsilon_y = 600\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{xy} = 462.500\ \mu\text{rad} \cong 463\ \mu\text{rad}$ .... **Ans.**

(b)
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(462.5)}{(800)-(600)} = 33.307°, \quad -56.693°$$

When $\theta_p = 33.307°$

$$\varepsilon_n = (800)\cos^2\theta_p + (600)\sin^2\theta_p + (462.5)\sin\theta_p\cos\theta_p$$
$$= 951.946\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = 448.054\ \mu\text{in./in.}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.33}{1-0.33}\left[(800)+(600)\right] = -689.55\ \mu\text{in./in.}$$

$\varepsilon_{p1} = +952\ \mu\text{in./in.}\ \angle\ 33.31°$ .... **Ans.**

$\varepsilon_{p2} = +448\ \mu\text{in./in.}\ \angle\ 56.69°$ .... **Ans.**

$\varepsilon_{p3} = -690\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 504\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{p1} - \varepsilon_{p3} = 1641\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

3-62*

(a) The given values are $\varepsilon_a = \varepsilon_x = 780\ \mu\text{m/m}$ $\varepsilon_b = \varepsilon_{120°} = 345\ \mu\text{m/m}$

$\varepsilon_c = \varepsilon_{60°} = -332\ \mu\text{m/m}$ $\nu = 0.33$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\varepsilon_b = (780)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy} \sin(120°)\cos(120°) = 345$$

$$\varepsilon_c = (780)\cos^2(60°) + \varepsilon_y \sin^2(60°) + \gamma_{xy} \sin(60°)\cos(60°) = -332$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 150$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = -527$$

Therefore: $\varepsilon_x = +780\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_y = -251.333\ \mu\text{m/m} \cong -251\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{xy} = -781.732\ \mu\text{rad} \cong -782\ \mu\text{rad}$ .... **Ans.**

(b) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-781.732)}{(780)-(-251.333)} = -18.581°, \quad 71.419°$$

When $\theta_p = -18.581°$

$$\varepsilon_n = (780)\cos^2\theta_p + (-251.333)\sin^2\theta_p + (-781.732)\sin\theta_p\cos\theta_p$$

$$= 911.395\ \mu\text{m/m} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -382.728\ \mu\text{m/m}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.33}{1-0.33}\left[(780)+(-251.333)\right] = -260.388\ \mu\text{m/m}$$

$\varepsilon_{p1} = +911\ \mu\text{m/m}$ ⦩ $18.58°$ .... **Ans.**

$\varepsilon_{p2} = -383\ \mu\text{m/m}$ ∡ $71.42°$ .... **Ans.**

$\varepsilon_{p3} = -260\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1294\ \mu\text{rad}$ .... **Ans.**

3-63*

The given values are $\varepsilon_a = \varepsilon_x = 36\ \mu\text{in./in.}$ $\quad \varepsilon_b = \varepsilon_{45°} = 310\ \mu\text{in./in.}$

$\varepsilon_c = \varepsilon_y = 150\ \mu\text{in./in.}$ $\quad \nu = 0.30$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (36)\cos^2(45°) + (150)\sin^2(45°) + \gamma_{xy}\sin(45°)\cos(45°) = 310$$

$$\gamma_{xy} = 434.00\ \mu\text{rad}$$

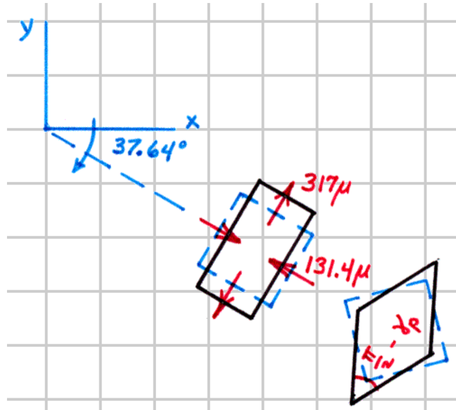
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(434)}{(36)-(150)} = -37.641°, \quad 52.359°$$

When $\theta_p = -37.641°$

$$\varepsilon_n = (36)\cos^2\theta_p + (150)\sin^2\theta_p + (434)\sin\theta_p\cos\theta_p$$
$$= -131.361\ \mu\text{in./in.} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 317.361\ \mu\text{in./in.}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.30}{1-0.30}\left[(36)+(150)\right]$$
$$= -79.714\ \mu\text{in./in.}$$



$\varepsilon_{p1} = +317\ \mu\text{in./in.}$ ∡ 52.36° .... **Ans.**

$\varepsilon_{p2} = -131.4\ \mu\text{in./in.}$ ∡ 37.64° .... **Ans.**

$\varepsilon_{p3} = -79.7\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 449\ \mu\text{rad}$ .... **Ans.**

3-64

The given values are $\varepsilon_a = \varepsilon_x = 525\ \mu\text{m/m}$ $\varepsilon_b = \varepsilon_{45°} = 450\ \mu\text{m/m}$

$\varepsilon_c = \varepsilon_{135°} = 1425\ \mu\text{m/m}$ $\nu = 0.30$

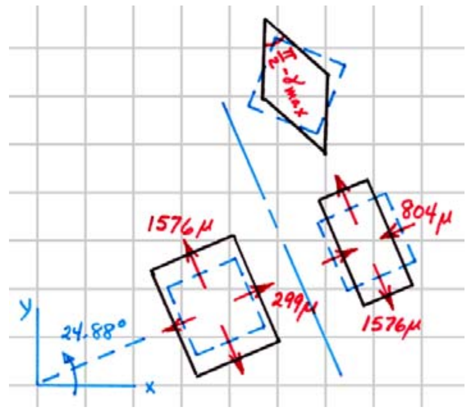
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (525)\cos^2(45°) + \varepsilon_y \sin^2(45°) + \gamma_{xy}\sin(45°)\cos(45°) = 450$$

$$\varepsilon_c = (525)\cos^2(135°) + \varepsilon_y \sin^2(135°) + \gamma_{xy}\sin(135°)\cos(135°) = 1425$$

$$0.5000\varepsilon_y + 0.5000\gamma_{xy} = 187.5$$

$$0.5000\varepsilon_y - 0.5000\gamma_{xy} = 1162.5$$



$$\varepsilon_y = 1350\ \mu\text{m/m} \qquad \gamma_{xy} = -975\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-975)}{(525)-(1350)}$$

$$= 24.882°, \quad -65.118°$$

When $\theta_p = 24.882°$

$$\varepsilon_n = (525)\cos^2\theta_p + (1350)\sin^2\theta_p + (-975)\sin\theta_p\cos\theta_p$$

$$= 298.898\ \mu\text{m/m} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 1576.102\ \mu\text{m/m}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.30}{1-0.30}\left[(525)+(1350)\right] = -803.571\ \mu\text{m/m}$$

$\varepsilon_{p1} = +1576\ \mu\text{m/m}$ ⦫ $65.12°$ .... **Ans.**

$\varepsilon_{p2} = +299\ \mu\text{m/m}$ ∡ $24.88°$ .... **Ans.**

$\varepsilon_{p3} = -804\ \mu\text{m/m}$ .... **Ans.**

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1277\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{p1} - \varepsilon_{p3} = 2380\ \mu\text{rad}$ .... **Ans.**

3-65*

(a) The given values are $\varepsilon_a = \varepsilon_x = 875\ \mu\text{in./in.}$ $\varepsilon_c = \varepsilon_y = 350\ \mu\text{in./in.}$

$\varepsilon_b = \varepsilon_n = 700\ \mu\text{in./in.}$ $\theta_b = \tan^{-1}\dfrac{4}{3} = 53.130°$ $\nu = 0.30$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (875)\cos^2\theta_b + (350)\sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b = 700$$

$$\gamma_{xy} = 335.417\ \mu\text{rad}$$

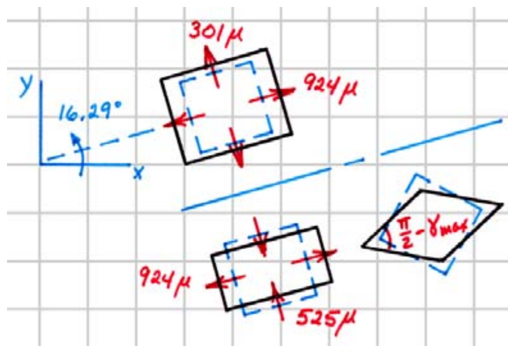
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(335.417)}{(875)-(350)} = 16.287°,\quad -73.713°$$

When $\theta_p = 16.287°$

$$\varepsilon_n = (875)\cos^2\theta_p + (350)\sin^2\theta_p + (335.417)\sin\theta_p\cos\theta_p$$
$$= 924.00\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = 301.00\ \mu\text{in./in.}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$
$$= \frac{-0.30}{1-0.30}\left[(875)+(350)\right]$$
$$= -525.00\ \mu\text{in./in.}$$



$\varepsilon_{p1} = +924\ \mu\text{in./in.}$ ∡ $16.29°$ .... **Ans.**

$\varepsilon_{p2} = +301\ \mu\text{in./in.}$ ∢ $73.71°$ .... **Ans.**

$\varepsilon_{p3} = -525\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 623\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{p1} - \varepsilon_{p3} = 1449\ \mu\text{rad}$ (out-of-plane) .... **Ans.**

(b) $\varepsilon_n = (875)\cos^2(120°) + (350)\sin^2(120°) + (335.417)\sin(120°)\cos(120°)$

$\varepsilon_n = +336\ \mu\text{in./in.}$ .... **Ans.**

3-66

(a) The given values are $\varepsilon_a = \varepsilon_x = 875\ \mu\text{m/m}$ $\quad \varepsilon_b = \varepsilon_{120°} = 700\ \mu\text{m/m}$

$\varepsilon_c = \varepsilon_{60°} = -650\ \mu\text{m/m}$ $\quad \nu = 0.33$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (875)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 700$$

$$\varepsilon_c = (875)\cos^2(60°) + \varepsilon_y \sin^2(60°) + \gamma_{xy}\sin(60°)\cos(60°) = -650$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 481.25$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = -868.75$$

$$\varepsilon_y = -258.333\ \mu\text{m/m}$$

$$\gamma_{xy} = -1558.846\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-1558.846)}{(875)-(-258.333)}$$

$$= -26.991°, \quad 63.009°$$

When $\theta_p = -26.991°$

$$\varepsilon_n = (875)\cos^2\theta_p + (-258.333)\sin^2\theta_p + (-1558.846)\sin\theta_p\cos\theta_p$$

$$= 1271.978\ \mu\text{m/m} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -655.312\ \mu\text{m/m}$$

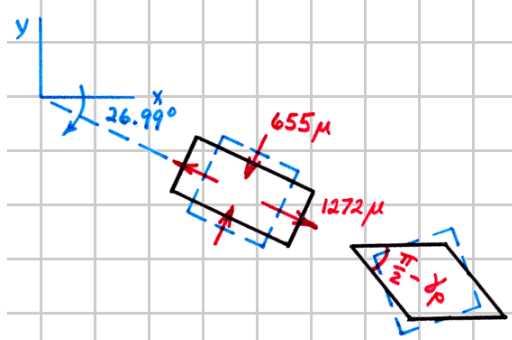
$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.33}{1-0.33}\left[(875) + (-258.333)\right] = -303.732\ \mu\text{m/m}$$

$\varepsilon_{p1} = +1272\ \mu\text{m/m}$ ↘ $26.99°$ .... **Ans.**

$\varepsilon_{p2} = -655\ \mu\text{m/m}$ ↗ $63.01°$ .... **Ans.**

$\varepsilon_{p3} = -304\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1927\ \mu\text{rad}$ .... **Ans.**



(b) $$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta + \sin^2\theta)$$

$$= -2\left[(875) - (258.333)\right]\sin(40°)\cos(40°)$$

$$+ (-1558.846)\left[\cos^2(40°) + \sin^2(40°)\right]$$

$\gamma_{nt} = -1387\ \mu\text{rad}$ .... **Ans.**

3-67*

(a) The given values are $\varepsilon_a = \varepsilon_x = 800\ \mu\text{in./in.}$ $\varepsilon_b = \varepsilon_y = 950\ \mu\text{in./in.}$

$\varepsilon_c = \varepsilon_{120°} = 600\ \mu\text{in./in.}$ $\nu = 0.33$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_c = (800)\cos^2(120°) + (950)\sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 600$$

$$\gamma_{xy} = 721.688\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(721.688)}{(800)-(950)} = -39.129°, \quad 50.871°$$

When $\theta_p = -39.129°$

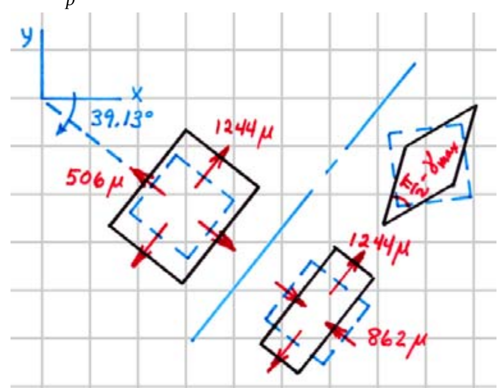
$$\varepsilon_n = (800)\cos^2\theta_p + (950)\sin^2\theta_p + (721.688)\sin\theta_p\cos\theta_p$$

$$= 506.444\ \mu\text{in./in.} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 1243.556\ \mu\text{in./in.}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.33}{1-0.33}\left[(800)+(950)\right]$$

$$= -861.94\ \mu\text{in./in.}$$



$\varepsilon_{p1} = +1244\ \mu\text{in./in.}$ ∡ 50.87° ....**Ans.**

$\varepsilon_{p2} = +506\ \mu\text{in./in.}$ ∡ 39.13° ....**Ans.**

$\varepsilon_{p3} = -862\ \mu\text{in./in.}$ ....**Ans.**

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 737\ \mu\text{rad}$ ....**Ans.**

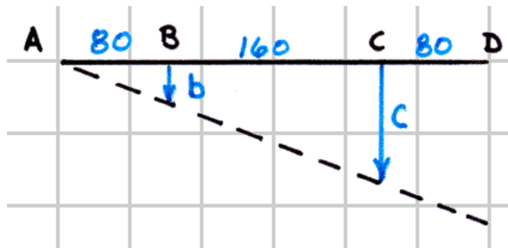
$\gamma_{max} = \varepsilon_{p1} - \varepsilon_{p3} = 2110\ \mu\text{rad}$ ....**Ans.**

(b) $\varepsilon_n = (800)\cos^2(200°) + (950)\sin^2(200°) + (721.688)\sin(200°)\cos(200°)$

$\varepsilon_n = +1049\ \mu\text{in./in.}$ ....**Ans.**

3-68*

$$\delta_{BF} = \varepsilon_{BF} L_{BF} = 400\left(10^{-6}\right)\left(1000\right) = 0.400 \text{ mm}$$



$$\frac{b}{80} = \frac{c}{240}$$

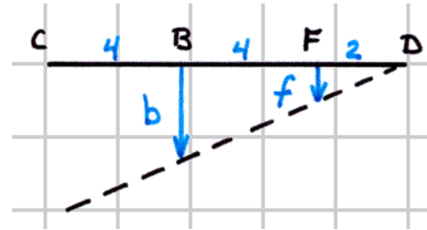
$$c = 3b = 3\delta_{BF} = 1.200 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\delta_{CE}}{L_{CE}} = \frac{c}{600} = 2000\left(10^{-6}\right) \text{ m/m}$$

$\varepsilon_{CE} = 2000\ \mu\text{m/m}$ .... **Ans.**

3-69*

$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0015(15) = 0.02250 \text{ in.}$$



$$\frac{b}{6} = \frac{f}{2} \qquad f = \frac{b}{3} = \frac{\delta_{AB}}{3} = 0.00750 \text{ in.}$$

(a) $f = \delta_{EF} = 0.00750$ in.

$$\varepsilon_{EF} = \frac{\delta_{EF}}{L_{EF}} = \frac{0.00750}{8} = 937\left(10^{-6}\right) \text{ in./in.}$$

$\varepsilon_{EF} = 937\ \mu$in./in. .... **Ans.**

(b) $f = 0.005 + \delta_{EF} = 0.00750$ in. $\qquad \delta_{EF} = 0.00250$ in.

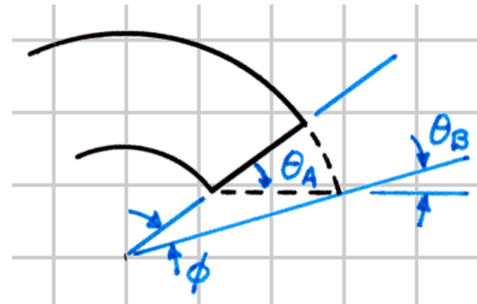
$$\varepsilon_{EF} = \frac{\delta_{EF}}{L_{EF}} = \frac{0.00250}{8} = 313\left(10^{-6}\right) \text{ in./in.}$$

$\varepsilon_{EF} = 313\ \mu$in./in. .... **Ans.**

3-70

$$\gamma_A = \theta_A = \frac{R_2\phi}{R_2 - R_1}$$ .... **Ans.**

$$\gamma_B = \theta_B = \frac{R_1\phi}{R_2 - R_1}$$ .... **Ans.**



3-71*

The given values are

$$\varepsilon_x = 3200\ \mu\text{in./in.} \qquad \varepsilon_y = 1500\ \mu\text{in./in.} \qquad \gamma_{xy} = 1000\ \mu\text{rad} \qquad \theta_n = 45°$$

(a) $$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\varepsilon_n = (3200)\cos^2(45°) + (1500)\sin^2(45°) + (1000)\sin(45°)\cos(45°)$$

$\varepsilon_n = 2850\ \mu\text{in./in.}$ .... **Ans.**

(b) $$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\gamma_{nt} = -2[(3200) - (1500)]\sin(45°)\cos(45°) + (1000)[\cos^2(45°) - \sin^2(45°)]$$

$\gamma_{nt} = -1700\ \mu\text{rad}$ .... **Ans.**

3-72

The given values are

$$\varepsilon_x = 1500\ \mu\text{m/m} \qquad \varepsilon_y = -1250\ \mu\text{m/m} \qquad \gamma_{xy} = 1000\ \mu\text{rad}$$

$$\theta_{BD} = -\tan^{-1}\frac{150}{200} = -36.870°$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_{BD} = (1500)\cos^2\theta_{BD} + (-1250)\sin^2\theta_{BD} + (1000)\sin\theta_{BD}\cos\theta_{BD}$$

$\varepsilon_{BD} = 30.0\ \mu\text{m/m}$ .... **Ans.**

3-73

The given values are

$$\varepsilon_x = 1000\ \mu\text{in./in.} \qquad \varepsilon_y = -800\ \mu\text{in./in.} \qquad \gamma_{xy} = -800\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-800)}{(1000)-(-800)} = -11.981°, \quad 78.019°$$

When $\theta_p = -11.981°$

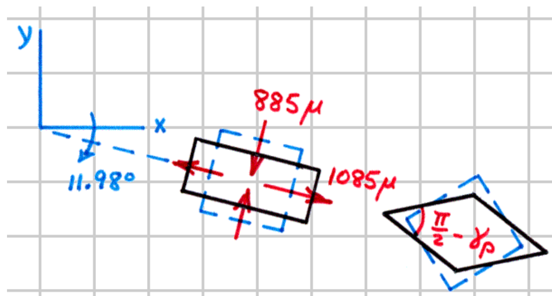
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (1000)\cos^2\theta_p + (-800)\sin^2\theta_p + (-800)\sin\theta_p\cos\theta_p$$

$$= 1084.886\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -884.886\ \mu\text{in./in.}$$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1969.772\ \mu\text{rad}$$



$\varepsilon_{p1} = +1085\ \mu\text{in./in.}$ ↘ $11.98°$ .... **Ans.**

$\varepsilon_{p2} = -885\ \mu\text{in./in.}$ ↗ $78.02°$ .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{max} = 1970\ \mu\text{rad}$ .... **Ans.**

3-74*

The given values are

$$\varepsilon_x = -600\ \mu\text{m/m} \qquad \varepsilon_y = 1200\ \mu\text{m/m} \qquad \gamma_{xy} = 2000\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(2000)}{(-600)-(1200)} = -24.006°, \quad 65.996°$$

When $\theta_p = -24.006°$

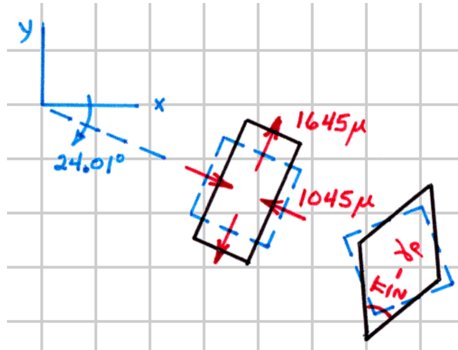
$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (-600)\cos^2\theta_p + (1200)\sin^2\theta_p + (2000)\sin\theta_p\cos\theta_p$$

$$= -1045.362\ \mu\text{m/m} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 1645.362\ \mu\text{m/m}$$

$$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 2690.724\ \mu\text{rad}$$



$\varepsilon_{p1} = +1645\ \mu\text{m/m}$ ∡ 65.99° .... **Ans.**

$\varepsilon_{p2} = -1045\ \mu\text{m/m}$ ∡ 24.01° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{\max} = \gamma_p = 2690\ \mu\text{rad}$ .... **Ans.**

3-75

The given values are $\varepsilon_a = \varepsilon_x = 600\ \mu\text{in./in.}$ $\varepsilon_b = \varepsilon_{45°} = 500\ \mu\text{in./in.}$

$\varepsilon_c = \varepsilon_y = -200\ \mu\text{in./in.}$ $\nu = 0.30$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (600)\cos^2(45°) + (-200)\sin^2(45°) + \gamma_{xy}\sin(45°)\cos(45°) = 500$$

$$\gamma_{xy} = 600.00\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(600)}{(600)-(-200)} = 18.435°, \quad -71.565°$$

When $\theta_p = 18.435°$

$$\varepsilon_n = (600)\cos^2\theta_p + (-200)\sin^2\theta_p + (600)\sin\theta_p\cos\theta_p$$
$$= 700.00\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -300.00\ \mu\text{in./in.}$$

$\varepsilon_{p1} = +700\ \mu\text{in./in.}$ ∡ 18.43° .... **Ans.**

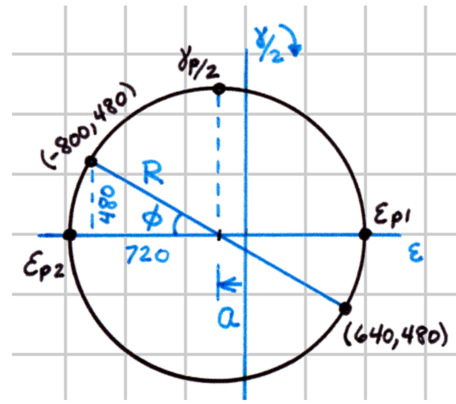
$\varepsilon_{p2} = -300\ \mu\text{in./in.}$ ∡ 71.57° .... **Ans.**

$\varepsilon_{p3} = \varepsilon_z = \dfrac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \dfrac{-0.30}{1-0.30}\left[(600)+(-200)\right] = -171.4\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1000\ \mu\text{rad}$ .... **Ans.**

3-76*

The given values for use in drawing Mohr's circle are



$$\varepsilon_x = -800\ \mu\text{m/m}$$

$$\varepsilon_y = 640\ \mu\text{m/m}$$

$$\gamma_{xy} = -960\ \mu\text{rad}$$

$$a = \frac{(-800)+(640)}{2} = -80\ \mu\text{m/m}$$

$$R = \sqrt{(720)^2 + (480)^2} = 865.332\ \mu\text{m/m}$$

$$\theta_{p1} = \frac{\phi}{2} = \frac{1}{2}\tan^{-1}\frac{480}{720} = 16.845°\ \text{(CCW)}$$

$\varepsilon_{p1} = (-80)+(865) = +785\ \mu\text{m/m}$ 73.15° .... **Ans.**

$\varepsilon_{p2} = (-80)-(865) = -945\ \mu\text{m/m}$ 16.85° .... **Ans.**

$\varepsilon_{p3} = 0\ \mu\text{m/m}$ 73.15° .... **Ans.**

$\gamma_{max} = \gamma_p = 2R = 1731\ \mu\text{rad}$ .... **Ans.**

3-77

The given values are $\varepsilon_a = \varepsilon_x = 800\ \mu\text{in./in.}$ $\quad \varepsilon_b = \varepsilon_{120°} = 960\ \mu\text{in./in.}$

$\varepsilon_c = \varepsilon_{240°} = 800\ \mu\text{in./in.}$ $\quad \nu = 0.33$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (800)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 960$$

$$\varepsilon_c = (800)\cos^2(240°) + \varepsilon_y \sin^2(240°) + \gamma_{xy}\sin(240°)\cos(240°) = 800$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 760$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = 600$$

$$\varepsilon_y = 906.667\ \mu\text{in./in.}$$

$$\gamma_{xy} = -184.752\ \mu\text{rad}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-184.752)}{(800)-(906.667)} = 30.00°, \quad -60.00°$$

When $\theta_p = 30.00°$

$$\varepsilon_n = (800)\cos^2\theta_p + (906.667)\sin^2\theta_p + (-184.752)\sin\theta_p\cos\theta_p$$
$$= 746.667\ \mu\text{in./in.} = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 960.00\ \mu\text{in./in.}$$

3-77 (cont.)

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.33}{1-0.33}\left[(800)+(906.667)\right] = -840.597\ \mu\text{m/m}$$

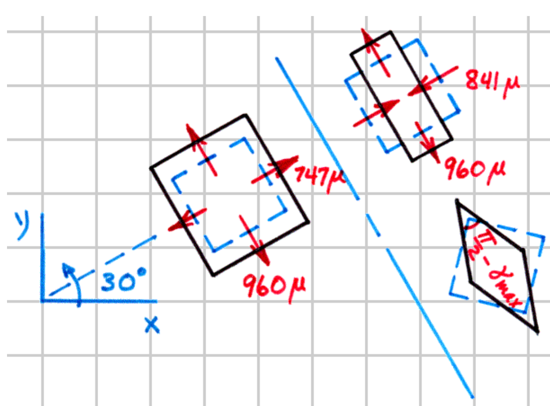
$\varepsilon_{p1} = +960\ \mu\text{m/m}$ 60.00° .... **Ans.**

$\varepsilon_{p2} = +747\ \mu\text{m/m}$ 30.00° .... **Ans.**

$\varepsilon_{p3} = -841\ \mu\text{m/m}$ .... **Ans.**

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 213\ \mu\text{rad}$ .... **Ans.**

$\gamma_{max} = \varepsilon_{p1} - \varepsilon_{p3} = 1801\ \mu\text{rad}$ .... **Ans.**



4-1*

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.5)^2}{4} = 1.76715 \text{ in.}^2 \qquad \sigma = \frac{F}{A} = \frac{53}{1.76715} = 29.9919 \text{ ksi}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{0.48}{(20\times 12)} = 0.002000 \text{ in./in.} \qquad \varepsilon_t = \frac{\Delta d}{d} = \frac{-0.001}{1.5} = 666.667(10^{-6}) \text{ in./in.}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{29.9919}{0.002000} = 15,000 \text{ ksi}$$ .... **Ans.**

$$\nu = \frac{-\varepsilon_t}{\varepsilon_1} = \frac{-666.667(10^{-6})}{0.002000} = 0.333$$ .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{15,000}{2(1+0.333)} = 5630 \text{ ksi}$$ .... **Ans.**

4-2*

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.015)^2}{4} = 176.715(10^{-6}) \text{ mm}^2$$

$$\sigma = \frac{F}{A} = \frac{62.6(10^3)}{176.715(10^{-6})} = 354.244(10^6) \text{ N/m}^2 = 354.244 \text{ MPa}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{0.90}{200} = 0.004500 \text{ m/m} \qquad \varepsilon_t = \frac{\Delta d}{d} = \frac{-0.022}{15} = -1.46667(10^{-3}) \text{ m/m}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{354.244(10^6)}{0.004500} = 78.721(10^9) \text{ N/m}^2 \cong 78.7 \text{ GPa}$$ .... **Ans.**

$$\nu = \frac{-\varepsilon_t}{\varepsilon_1} = \frac{-(-1.46667)(10^{-3})}{0.004500} = 0.326$$ .... **Ans.**

$$\sigma = \sigma_{PL} = 354 \text{ MPa}$$ .... **Ans.**

4-3

(a) $\sigma = \frac{F}{A} = \frac{10}{(2)(0.25)} = 20.0 \text{ ksi}$ .... **Ans.**

(b) $E = \frac{\sigma}{\varepsilon} = \frac{20}{0.08/(5\times 12)} = 15,000 \text{ ksi}$ .... **Ans.**

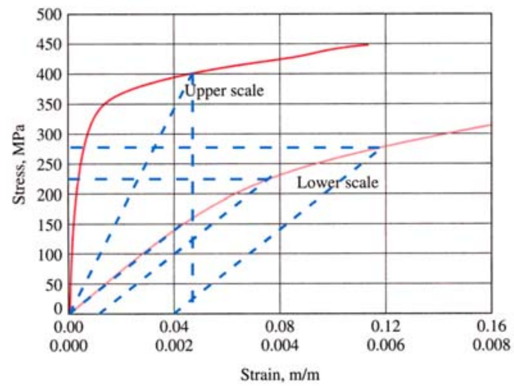
(c) $\nu = -\varepsilon_t/\varepsilon_1$

$0.25 = \frac{-(\delta_{0.25}/0.25)}{0.08/(5\times 12)}$ $\delta_{0.25} = -0.0000833 \text{ in.}$ .... **Ans.**

$0.25 = \frac{-(\delta_2/2)}{0.08/(5\times 12)}$ $\delta_2 = -0.000667 \text{ in.}$ .... **Ans.**

4-4*

$$A_o = \pi(5.64)^2/4 = 24.98 \text{ mm}^2$$
$$= 24.98(10^{-6}) \text{ m}^2$$
$$\Delta D = \nu\varepsilon_{frac} D = 0.30(0.115)(5.64)$$
$$= 0.19458 \text{ mm}$$
$$A_f = \pi(5.64 - 0.19458)^2/4$$
$$= 23.29 \text{ mm}^2 = 23.29(10^{-6}) \text{ m}^2$$



From the $\sigma - \varepsilon$ diagram:

(a) $E = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(139-0)(10^6)}{0.002-0} = 69.5(10^9) \text{ N/m}^2 = 69.5 \text{ GPa}$ .... **Ans.**

(b) $\sigma_{PL} \cong 139 \text{ MPa}$ .... **Ans.**

(c) $\sigma_{ult} \cong 450 \text{ MPa}$ .... **Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 220 \text{ MPa}$ .... **Ans.**

(e) $\sigma_{ys}(0.20\%) \cong 278 \text{ MPa}$ .... **Ans.**

(f) $\sigma_t = \sigma_{ult} \cong 450 \text{ MPa}$ .... **Ans.**

(g) $\sigma_{ft} = \dfrac{P_f}{A_f} = \dfrac{\sigma_f A_o}{A_f} \cong \dfrac{450(24.98)}{23.29} = 483 \text{ MPa}$ .... **Ans.**

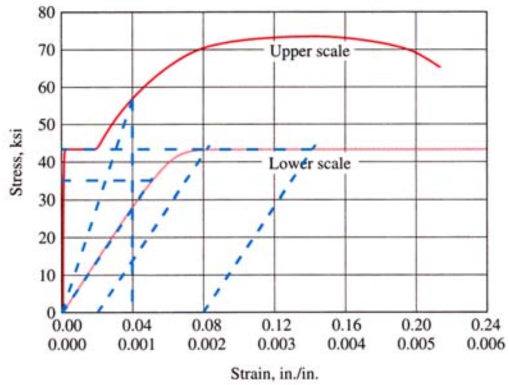
(h) $E_t = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(410-393)(10^6)}{0.06-0.04} = 850(10^6) \text{ N/m}^2 = 850 \text{ MPa}$ .... **Ans.**

(i) $E_s = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(410-0)(10^6)}{0.06} = 6.83(10^9) \text{ N/m}^2 = 6.83 \text{ GPa}$ .... **Ans.**

4-5

$$A_o = \pi(0.25)^2/4 = 0.04909 \text{ in.}^2$$

$$A_f = \pi(0.212)^2/4 = 0.03530 \text{ in.}^2$$



From the $\sigma - \varepsilon$ diagram:

(a) $E = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{34.5 - 0}{0.00125 - 0} = 27,600 \text{ ksi}$ ....**Ans.**

(b) $\sigma_{PL} \cong 36 \text{ ksi}$ ....**Ans.**

(c) $\sigma_{ult} \cong 73 \text{ ksi}$ ....**Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 43 \text{ ksi}$ .... **Ans.**

(e) $\sigma_{ys}(0.20\%) \cong 43 \text{ ksi}$ .... **Ans.**

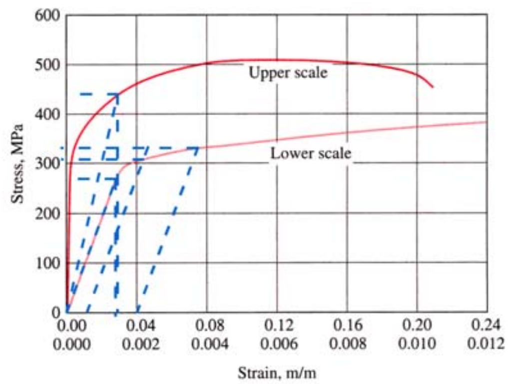
(f) $\sigma_t \cong 65 \text{ ksi}$ .... **Ans.**

(g) $\sigma_{ft} = \dfrac{P_f}{A_f} = \dfrac{\sigma_f A_o}{A_f} \cong \dfrac{65(0.04909)}{0.03530} = 90 \text{ ksi}$ .... **Ans.**

(h) $E_t = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{64 - 50}{0.06 - 0.03} = 467 \text{ ksi}$ .... **Ans.**

(i) $E_s = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{56 - 0}{0.04} = 1400 \text{ ksi}$ .... **Ans.**

4-6



$$A_o = \pi(5.64)^2/4 = 24.98 \text{ mm}^2 = 24.98(10^{-6}) \text{ m}^2$$

$$A_f = \pi(4.75)^2/4 = 17.72 \text{ mm}^2 = 17.72(10^{-6}) \text{ m}^2$$

From the $\sigma - \varepsilon$ diagram:

(a) $E = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(225-0)(10^6)}{0.0012-0}$

$E \cong 187(10^9) \text{ N/m}^2 = 187 \text{ GPa}$ .... **Ans.**

(b) $\sigma_{PL} \cong 270 \text{ MPa}$ .... **Ans.**

(c) $\sigma_{ult} \cong 510 \text{ MPa}$ .... **Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 305 \text{ MPa}$ .... **Ans.**

(e) $\sigma_{ys}(0.20\%) \cong 328 \text{ MPa}$ .... **Ans.**

(f) $\sigma_t \cong 450 \text{ MPa}$ .... **Ans.**

(g) $\sigma_{ft} = \dfrac{P_f}{A_f} = \dfrac{\sigma_f A_o}{A_f} \cong \dfrac{450(24.98)}{17.72} = 634 \text{ MPa}$ .... **Ans.**

(h) $E_t = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(460-410)(10^6)}{0.04-0.02} = 2.50(10^9) \text{ N/m}^2 = 2.50 \text{ GPa}$ .... **Ans.**

(i) $E_s = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(440-0)(10^6)}{0.03} = 14.67(10^9) \text{ N/m}^2 = 14.67 \text{ GPa}$ .... **Ans.**

4-7*

$$A_o = \pi(0.505)^2/4 = 0.200 \text{ in.}^2 \qquad A_f = \pi(0.425)^2/4 = 0.142 \text{ in.}^2$$

First calculate stresses and strains from the given data and draw the $\sigma - \varepsilon$ diagram (next page). Then, from the $\sigma - \varepsilon$ diagram:

(a) $E = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{32-0}{0.0012-0} = 26,700 \text{ ksi}$ .... **Ans.**

(b) $\sigma_{PL} \cong 38 \text{ ksi}$ .... **Ans.**

(c) $\sigma_{ult} \cong 73 \text{ ksi}$ .... **Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 43 \text{ ksi}$ .... **Ans.**

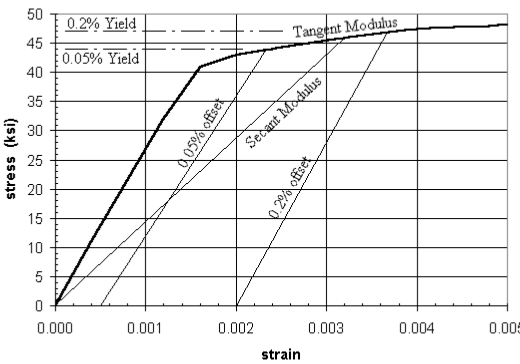
(e) $\sigma_{ys}(0.20\%) \cong 47 \text{ ksi}$ .... **Ans.**

(f) $\sigma_t \cong 65 \text{ ksi}$ .... **Ans.**

(g) $\sigma_{ft} = \dfrac{P_f}{A_f} = \dfrac{\sigma_f A_o}{A_f} \cong \dfrac{13}{0.142} = 91 \text{ ksi}$ .... **Ans.**

(h) $E_t = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{7.75}{0.0032} = 2400 \text{ ksi}$ .... **Ans.**

(i) $E_s = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{46-0}{0.0032} = 14,400 \text{ ksi}$ .... **Ans.**

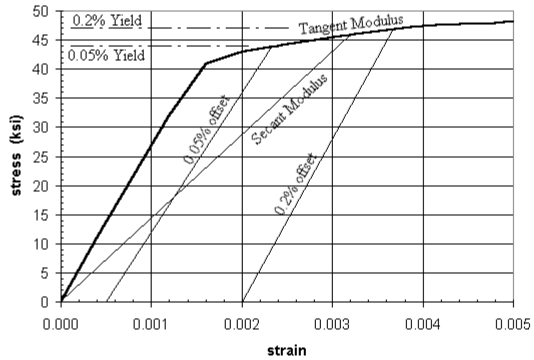


4-8

$$A_o = \pi(11.28)^2/4 = 99.93 \text{ mm}^2$$

$$A_f = \pi(9.50)^2/4 = 70.88 \text{ mm}^2$$

First calculate stresses and strains from the given data and draw the $\sigma - \varepsilon$ diagram (next page). Then, from the $\sigma - \varepsilon$ diagram:



(a) $$E = \frac{\Delta\sigma}{\Delta\varepsilon} \cong \frac{(222-0)(10^6)}{0.0012-0}$$

$E \cong 185(10^9) \text{ N/m}^2 = 185 \text{ GPa}$ .... **Ans.**

(b) $\sigma_{PL} \cong 270 \text{ MPa}$ .... **Ans.**

(c) $\sigma_{ult} \cong 510 \text{ MPa}$ .... **Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 305 \text{ MPa}$ .... **Ans.**

(e) $\sigma_{ys}(0.20\%) \cong 328 \text{ MPa}$ .... **Ans.**

(f) $\sigma_t \cong 450 \text{ MPa}$ .... **Ans.**

(g) $\sigma_{ft} = \dfrac{P_f}{A_f} = \dfrac{\sigma_f A_o}{A_f} \cong \dfrac{450(99.93)}{70.88} = 634 \text{ MPa}$ .... **Ans.**

(h) $E_t = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{80(10^6)}{0.0048} = 16.7(10^9) \text{ N/m}^2 = 16.7 \text{ GPa}$ .... **Ans.**

(i) $E_s = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{315(10^6)}{0.0029} = 109(10^9) \text{ N/m}^2 = 109 \text{ GPa}$ .... **Ans.**

4-9*

The given values are

$$\varepsilon_x = 900\ \mu\text{in./in.} \qquad \varepsilon_y = -300\ \mu\text{in./in.} \qquad \gamma_{xy} = -400\ \mu\text{rad}$$

$$E = 10,000\ \text{ksi} \qquad \nu = 0.30$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{10,000}{1-(0.30)^2}\left[(900) + 0.30(-300)\right]\left(10^{-6}\right)$$

$\sigma_x = +8.90\ \text{ksi} = 8.90\ \text{ksi (T)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{10,000}{1-(0.30)^2}\left[(-300) + 0.30(900)\right]\left(10^{-6}\right)$$

$\sigma_y = -0.330\ \text{ksi} = 0.330\ \text{ksi (C)}$ .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{10,000}{2(1+0.30)} = 3846.15\ \text{ksi}$$

$\tau_{xy} = G\gamma_{xy} = 3846.15\left(-400\times10^{-6}\right) = -1.538\ \text{ksi}$ .... **Ans.**

4-10*

The given values are

$$\varepsilon_x = 1175\ \mu\text{m/m} \qquad \varepsilon_y = -1250\ \mu\text{m/m} \qquad \gamma_{xy} = 850\ \mu\text{rad}$$

$$E = 190\text{ GPa} \qquad \nu = 0.25$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{\left(190\times10^3\right)}{1-\left(0.25\right)^2}\left[\left(1175\right)+0.25\left(-1250\right)\right]\left(10^{-6}\right)$$

$\sigma_x = +174.8\text{ MPa} = 174.8\text{ MPa (T)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{\left(190\times10^3\right)}{1-\left(0.25\right)^2}\left[\left(-1250\right)+0.25\left(1175\right)\right]\left(10^{-6}\right)$$

$\sigma_y = -193.8\text{ MPa} = 193.8\text{ MPa (C)}$ .... **Ans.**

$$G = \frac{E}{2\left(1+\nu\right)} = \frac{190}{2\left(1+0.25\right)} = 76.00\text{ GPa}$$

$\tau_{xy} = G\gamma_{xy} = \left(76.0\times10^3\right)\left(850\times10^{-6}\right) = +64.6\text{ MPa}$ .... **Ans.**

4-11

The given values are

$$\varepsilon_x = 500\ \mu\text{in./in.} \qquad \varepsilon_y = 250\ \mu\text{in./in.} \qquad \gamma_{xy} = 150\ \mu\text{rad}$$

$$E = 15{,}000\text{ ksi} \qquad \nu = 0.34$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{15{,}000}{1-(0.34)^2}\left[(500)+0.34(250)\right]\left(10^{-6}\right)$$

$\sigma_x = +9.92\text{ ksi} = 9.92\text{ ksi (T)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{15{,}000}{1-(0.34)^2}\left[(250)+0.34(500)\right]\left(10^{-6}\right)$$

$\sigma_y = +7.12\text{ ksi} = 7.12\text{ ksi (T)}$ .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{15{,}000}{2(1+0.34)} = 5597.01\text{ ksi}$$

$\tau_{xy} = G\gamma_{xy} = 5597.01\left(150\times10^{-6}\right) = +0.840\text{ ksi}$ .... **Ans.**

4-12
The given values are

$$\varepsilon_x = 1000\ \mu\text{m/m} \qquad \varepsilon_y = 400\ \mu\text{m/m} \qquad \gamma_{xy} = 800\ \mu\text{rad}$$

$$E = 210\text{ GPa} \qquad \nu = 0.25$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{\left(210\times10^3\right)}{1-\left(0.25\right)^2}\left[\left(1000\right)+0.25\left(400\right)\right]\left(10^{-6}\right)$$

$\sigma_x = +246\text{ MPa} = 246\text{ MPa (T)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{\left(210\times10^3\right)}{1-\left(0.25\right)^2}\left[\left(400\right)+0.25\left(1000\right)\right]\left(10^{-6}\right)$$

$\sigma_y = +145.6\text{ MPa} = 145.6\text{ MPa (T)}$ .... **Ans.**

$$G = \frac{E}{2\left(1+\nu\right)} = \frac{210}{2\left(1+0.25\right)} = 84.00\text{ GPa}$$

$\tau_{xy} = G\gamma_{xy} = \left(84.0\times10^3\right)\left(800\times10^{-6}\right) = +67.2\text{ MPa}$ .... **Ans.**

4-13*

The given values are

$$\sigma_x = 15,000 \text{ psi} \qquad \sigma_y = 5000 \text{ psi} \qquad \sigma_z = 7500 \text{ psi} \qquad E = 30,000 \text{ ksi}$$

$$\tau_{xy} = 5500 \text{ psi} \qquad \tau_{yz} = 4750 \text{ psi} \qquad \tau_{zx} = 3200 \text{ psi} \qquad \nu = 0.30$$

$$G = \frac{E}{2(1+\nu)} = \frac{30,000}{2(1+0.30)} = 11,538.46 \text{ ksi}$$

$$\varepsilon_x = \frac{\sigma_x - \nu(\sigma_y + \sigma_z)}{E} = \frac{15,000 - 0.30(5000 + 7500)}{(30\times 10^6)}$$

$\varepsilon_x = +375(10^{-6})$ in./in. $= +375\ \mu$in./in. .... **Ans.**

$$\varepsilon_y = \frac{\sigma_y - \nu(\sigma_x + \sigma_z)}{E} = \frac{5000 - 0.30(15,000 + 7500)}{(30\times 10^6)}$$

$\varepsilon_y = -58.0(10^{-6})$ in./in. $= -58.0\ \mu$in./in. .... **Ans.**

$$\varepsilon_z = \frac{\sigma_z - \nu(\sigma_x + \sigma_y)}{E} = \frac{7500 - 0.30(15,000 + 5000)}{(30\times 10^6)}$$

$\varepsilon_z = +50.0(10^{-6})$ in./in. $= +50.0\ \mu$in./in. .... **Ans.**

$\gamma_{xy} = \dfrac{\tau_{xy}}{G} = \dfrac{5500}{(11.53846\times 10^6)} = +477(10^{-6})$ rad $= +477\ \mu$rad .... **Ans.**

$\gamma_{yz} = \dfrac{\tau_{yz}}{G} = \dfrac{4750}{(11.53846\times 10^6)} = +412(10^{-6})$ rad $= +412\ \mu$rad .... **Ans.**

$\gamma_{zx} = \dfrac{\tau_{zx}}{G} = \dfrac{3200}{(11.53846\times 10^6)} = +277(10^{-6})$ rad $= +277\ \mu$rad .... **Ans.**

4-14*

The given values are

$$\sigma_x = 120 \text{ MPa} \qquad \sigma_y = -85 \text{ MPa} \qquad \sigma_z = 45 \text{ MPa} \qquad E = 73 \text{ GPa}$$

$$\tau_{xy} = 35 \text{ MPa} \qquad \tau_{yz} = 48 \text{ MPa} \qquad \tau_{zx} = 76 \text{ MPa} \qquad \nu = 0.33$$

$$G = \frac{E}{2(1+\nu)} = \frac{73}{2(1+0.33)} = 27.444 \text{ GPa}$$

$$\varepsilon_x = \frac{\sigma_x - \nu(\sigma_y + \sigma_z)}{E} = \frac{120 - 0.33(-85+45)}{(73\times10^9)}$$

$$\varepsilon_x = +1825(10^{-6}) \text{ m/m} = +1825\ \mu\text{m/m}$$ .... **Ans.**

$$\varepsilon_y = \frac{\sigma_y - \nu(\sigma_x + \sigma_z)}{E} = \frac{(-85) - 0.33(120+45)}{(73\times10^9)}$$

$$\varepsilon_y = -1910(10^{-6}) \text{ m/m} = -1910\ \mu\text{m/m}$$ .... **Ans.**

$$\varepsilon_z = \frac{\sigma_z - \nu(\sigma_x + \sigma_y)}{E} = \frac{45 - 0.33(120-85)}{(73\times10^9)}$$

$$\varepsilon_z = +458(10^{-6}) \text{ m/m} = +458\ \mu\text{m/m}$$ .... **Ans.**

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{35\times10^6}{(27.444\times10^9)} = 1275(10^{-6}) \text{ rad} = 1275\ \mu\text{rad}$$ .... **Ans.**

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = \frac{48\times10^6}{(27.444\times10^9)} = 1749(10^{-6}) \text{ rad} = 1749\ \mu\text{rad}$$ .... **Ans.**

$$\gamma_{zx} = \frac{\tau_{zx}}{G} = \frac{76}{(11.53846\times10^6)} = 2770(10^{-6}) \text{ rad} = 2770\ \mu\text{rad}$$ .... **Ans.**

4-15

$$\gamma_{xy} = \frac{\Delta_y}{L_x} = \frac{0.001}{0.5} = 0.00200 \text{ in./in.}$$

$$\tau_{xy} = G\gamma_{xy} = (3000)(0.00200) = 6 \text{ psi}$$

$P = 2\tau_{xy} A = 2(6)(2 \times 4) = 96$ lb .... **Ans.**

4-16*

The given values are $E = 73\text{ GPa}$ $\nu = 0.33$

$$\varepsilon_a = \varepsilon_x = 875\ \mu\text{m/m} \qquad \varepsilon_b = \varepsilon_{120°} = 700\ \mu\text{m/m} \qquad \varepsilon_c = \varepsilon_{60°} = -650\ \mu\text{m/m}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (875)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 700$$

$$\varepsilon_c = (875)\cos^2(60°) + \varepsilon_y \sin^2(60°) + \gamma_{xy}\sin(60°)\cos(60°) = -650$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 481.25$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = -868.75$$

$$\varepsilon_y = -258.33\ \mu\text{m/m} \qquad \gamma_{xy} = -1558.85\ \mu\text{rad}$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{(73\times10^3)}{1-(0.33)^2}\left[(875) + 0.33(-258.33)\right](10^{-6})$$

$\sigma_x = +64.7\text{ MPa} = 64.7\text{ MPa (T)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{(73\times10^3)}{1-(0.33)^2}\left[(-258.33) + 0.33(875)\right](10^{-6})$$

$\sigma_y = +2.49\text{ MPa} = 2.49\text{ MPa (T)}$ .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{73}{2(1+0.33)} = 27.444\text{ GPa}$$

$\tau_{xy} = G\gamma_{xy} = (27.444\times10^3)(-1558.85\times10^{-6}) = -42.8\text{ MPa}$ .... **Ans.**

4-17

The given values are $E = 30,000 \text{ ksi}$ $\nu = 0.30$

$$\varepsilon_a = \varepsilon_x = 650\ \mu\text{in./in.} \qquad \varepsilon_b = \varepsilon_{45^\circ} = 475\ \mu\text{in./in.} \qquad \varepsilon_c = \varepsilon_y = -250\ \mu\text{in./in.}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (650)\cos^2(45^\circ) + (-250)\sin^2(45^\circ) + \gamma_{xy}\sin(45^\circ)\cos(45^\circ) = 475$$

$$\gamma_{xy} = +550\ \mu\text{rad}$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{30,000}{1-(0.30)^2}\left[(650) + 0.30(-250)\right](10^{-6})$$

$\sigma_x = +18.96 \text{ ksi} = 18.96 \text{ ksi (T)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{30,000}{1-(0.30)^2}\left[(-250) + 0.30(650)\right](10^{-6})$$

$\sigma_y = -1.813 \text{ ksi} = 1.813 \text{ ksi (C)}$ .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{30,000}{2(1+0.30)} = 11,538.46 \text{ ksi}$$

$\tau_{xy} = G\gamma_{xy} = (11,538.46)(550\times10^{-6}) = +6.35 \text{ ksi}$ .... **Ans.**

4-18

The given values are $E = 200 \text{ GPa}$ $\nu = 0.30$

$$\varepsilon_a = \varepsilon_x = 540\ \mu\text{m/m} \qquad \varepsilon_b = \varepsilon_{45°} = 930\ \mu\text{m/m} \qquad \varepsilon_c = \varepsilon_y = 20\ \mu\text{m/m}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\varepsilon_b = (540)\cos^2(45°) + (20)\sin^2(45°) + \gamma_{xy}\sin(45°)\cos(45°) = 930$$

$$\gamma_{xy} = +1300\ \mu\text{rad}$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{\left(200\times10^3\right)}{1-(0.30)^2}\left[(540) + 0.30(20)\right]\left(10^{-6}\right)$$

$\sigma_x = +120.0 \text{ MPa} = 120.0 \text{ MPa (T)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{\left(200\times10^3\right)}{1-(0.30)^2}\left[(20) + 0.30(540)\right]\left(10^{-6}\right)$$

$\sigma_y = +40.0 \text{ MPa} = 40.0 \text{ MPa (T)}$ .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{200}{2(1+0.30)} = 76.923 \text{ GPa}$$

$\tau_{xy} = G\gamma_{xy} = \left(76.923\times10^3\right)\left(1300\times10^{-6}\right) = +100.0 \text{ MPa}$ .... **Ans.**

4-19*

The given values are $E = 30,000$ ksi $\quad \nu = 0.30$

$\sigma_x = 8$ ksi $\quad \sigma_y = 0$ ksi $\quad \tau_{xy} = -5$ ksi

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E} = \frac{(8) - 0.30(0)}{30,000} = 267(10^{-6}) = 267\ \mu\text{in./in.}$$ .... **Ans.**

$$\varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E} = \frac{(0) - 0.30(8)}{30,000} = -80(10^{-6}) = -80\ \mu\text{in./in.}$$ .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{30,000}{2(1+0.30)} = 11,538.46\ \text{ksi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-5}{11,538.46} = -433(10^{-6}) = -433\ \mu\text{rad}$$ .... **Ans.**

4-20*

The given values are $E = 200\text{ GPa}$ $\nu = 0.30$

$$\varepsilon_a = \varepsilon_x = -555\ \mu\text{m/m} \qquad \varepsilon_b = \varepsilon_{120°} = 925\ \mu\text{m/m} \qquad \varepsilon_c = \varepsilon_{-120°} = 740\ \mu\text{m/m}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (-555)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 925$$

$$\varepsilon_c = (-555)\cos^2(-120°) + \varepsilon_y \sin^2(-120°) + \gamma_{xy}\sin(-120°)\cos(-120°) = 740$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 1063.75$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = 878.75$$

$$\varepsilon_y = 1295.0\ \mu\text{m/m} \qquad \gamma_{xy} = -213.620\ \mu\text{rad}$$

(a) $$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{\left(200\times10^3\right)}{1-(0.30)^2}\left[(-555)+0.30(1295)\right]\left(10^{-6}\right)$$

$\sigma_x = -36.593\text{ MPa} \cong 36.6\text{ MPa (C)}$ .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{\left(200\times10^3\right)}{1-(0.30)^2}\left[(1295)+0.30(-555)\right]\left(10^{-6}\right)$$

$\sigma_y = +248.022\text{ MPa} \cong 248\text{ MPa (T)}$ .... **Ans.**

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} = \frac{200\times10^3}{2(1+0.30)}\left(-213.620\times10^{-6}\right)$$

$\tau_{xy} = -16.4323\text{ MPa} \cong -16.43\text{ MPa}$ .... **Ans.**

(b) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-16.4323)}{(-36.593)-(248.022)} = 3.293°, \quad -86.707°$$

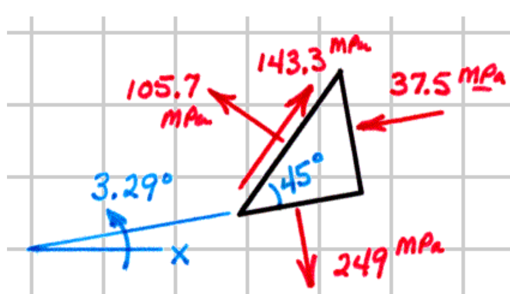
When $\theta_p = 3.293°$

$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= (-36.593)\cos^2\theta_p + (248.022)\sin^2\theta_p + 2(-16.4323)\sin\theta_p\cos\theta_p$$
$$= -37.539\text{ MPa} = \sigma_{p2}$$

$$\sigma_{p1} = \sigma_x + \sigma_y - \sigma_{p2} = 248.968\text{ MPa}$$

$$\tau_{max} = \tau_p = \left(\sigma_{p1} - \sigma_{p2}\right)/2 = 143.3\text{ MPa}$$

$$\sigma_{n45} = \left(\sigma_{p1} + \sigma_{p2}\right)/2 = 105.7\text{ MPa}$$



$\sigma_{p1} = 249\text{ MPa (T)}$ ⦩ $86.71°$ .... **Ans.**

$\sigma_{p2} = 37.5\text{ MPa (C)}$ ⦨ $3.29°$ .... **Ans.**

$\tau_{max} = \tau_p = 143.3\text{ MPa}$ .... **Ans.**

4-21*

The given values are $E = 30,000$ ksi $\nu = 0.30$

$$\varepsilon_a = \varepsilon_x = 1000\ \mu\text{in./in.} \qquad \varepsilon_b = \varepsilon_{60°} = 2000\ \mu\text{in./in.} \qquad \varepsilon_c = \varepsilon_{120°} = 1200\ \mu\text{in./in.}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (1000)\cos^2(60°) + \varepsilon_y \sin^2(60°) + \gamma_{xy}\sin(60°)\cos(60°) = 2000$$

$$\varepsilon_c = (1000)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 1200$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = 1750$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 950$$

$$\varepsilon_y = 1800\ \mu\text{in./in.} \qquad \gamma_{xy} = 923.760\ \mu\text{rad}$$

$$\varepsilon_p = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{(1000)+(1800)}{2} \pm \sqrt{\left(\frac{(1000)-(1800)}{2}\right)^2 + \left(\frac{923.760}{2}\right)^2}$$

$$= 1400 \pm 611.01\ \mu\text{in./in.}$$

$$\varepsilon_{p1} = 1400 + 611.01\ \mu\text{in./in.} = 2011.01\ \mu\text{in./in.} \quad (< 2200\ \mu\text{in./in. - okay})$$

$$\varepsilon_{p2} = 1400 - 611.01\ \mu\text{in./in.} = 788.99\ \mu\text{in./in.}$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu(\varepsilon_x + \varepsilon_y)}{1-\nu} = \frac{-0.30\left[(1000)+(1800)\right]}{1-0.30} = -1200\ \mu\text{in./in.}$$

$$\gamma_{max} = (2011) - (-1200)\ \mu\text{rad} = 3211\ \mu\text{rad} \quad (> 2500\ \mu\text{rad - design fails})$$

$$\sigma_{p1} = \frac{E}{1-\nu^2}(\varepsilon_{p1} + \nu\varepsilon_{p2}) = \frac{(30,000)}{1-(0.30)^2}\left[(2011.01) + 0.30(788.99)\right](10^{-6})$$

$$= 74.1\ \text{ksi} \quad (> 74\ \text{ksi - design fails})$$

$$\tau_{max} = G\gamma_{max} = \frac{E}{2(1+\nu)}\gamma_{max} = \frac{(30,000)}{2(1+0.30)}(3211\times 10^{-6})$$

$$= 37.05\ \text{ksi} \quad (< 40\ \text{ksi - okay})$$

Design fails since both γ_{max} and σ_{p1} are above the design limits. .... **Ans.**

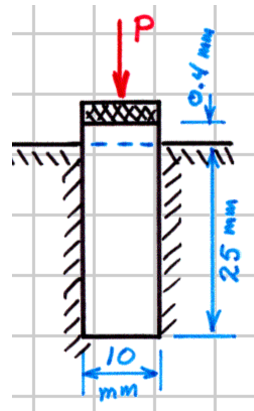
4-22

The given values are

$$\delta_x = 0 \text{ mm} \qquad \delta_y = 0 \text{ mm} \qquad \delta_z = -0.4 \text{ mm} \qquad \sigma_z = -P/A$$

Then

$$\varepsilon_x = 0 = \frac{\sigma_x - 0.4(\sigma_y + \sigma_z)}{1400} \qquad \varepsilon_y = 0 = \frac{\sigma_y - 0.4(\sigma_x + \sigma_z)}{1400}$$



$$\varepsilon_z = \frac{-0.4}{25.4} = \frac{\sigma_z - 0.4(\sigma_x + \sigma_y)}{1400}$$

$$\sigma_x - 0.4\sigma_y - 0.4\sigma_z = 0 \text{ MPa}$$

$$-0.4\sigma_x + \sigma_y - 0.4\sigma_z = 0 \text{ MPa}$$

$$-0.4\sigma_x - 0.4\sigma_y + \sigma_z = -22.04724 \text{ MPa}$$

$$\sigma_x = \sigma_y = -31.496 \text{ MPa} \qquad \sigma_z = -47.244 \text{ MPa}$$

$$P = -\sigma_z A = -(22.04724 \times 10^6)(0.010 \times 0.010) = 4.72(10^3) \text{ N}$$

$P = 4.72$ kN .... **Ans.**

4-23

The given values are $E = 10,600$ ksi $\nu = 0.33$

$\varepsilon_a = \varepsilon_x = 875\ \mu\text{in./in.}$ $\varepsilon_b = \varepsilon_{135°} = 700\ \mu\text{in./in.}$ $\varepsilon_c = \varepsilon_{-135°} = -350\ \mu\text{in./in.}$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (875)\cos^2(135°) + \varepsilon_y \sin^2(135°) + \gamma_{xy}\sin(135°)\cos(135°) = 700$$

$$\varepsilon_c = (875)\cos^2(-135°) + \varepsilon_y \sin^2(-135°) + \gamma_{xy}\sin(-135°)\cos(-135°) = -350$$

$$0.5000\varepsilon_y - 0.5000\gamma_{xy} = 262.5$$

$$0.5000\varepsilon_y + 0.5000\gamma_{xy} = -787.5$$

$$\varepsilon_y = -525.00\ \mu\text{in./in.}$$

$$\gamma_{xy} = -1050.00\ \mu\text{rad}$$

(a) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-1050)}{(875)-(-525)} = -18.435°,\quad 71.565°$$

When $\theta_p = -18.435°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (875)\cos^2\theta_p + (-525)\sin^2\theta_p + (-1050)\sin\theta_p\cos\theta_p$$

$$= 1050.00\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -700.00\ \mu\text{in./in.}$$

$\varepsilon_{p1} = 1050\ \mu\text{in./in.}$ ⦩ $18.43°$ .... **Ans.**

$\varepsilon_{p2} = -700\ \mu\text{in./in.}$ ⦨ $71.57°$ .... **Ans.**

$\varepsilon_{p3} = \dfrac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \dfrac{-0.33}{1-0.33}\left[(875)+(-525)\right] = -172.4\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1750\ \mu\text{rad}$ .... **Ans.**

(b) $$\sigma_{p1} = \frac{E}{1-\nu^2}(\varepsilon_{p1} + \nu\varepsilon_{p2}) = \frac{(10,600)}{1-(0.33)^2}\left[(1050)+0.33(-700)\right](10^{-6})$$

$\sigma_{p1} = +9.7423\text{ ksi} \cong 9.74\text{ ksi (T)}$ .... **Ans.**

$$\sigma_{p2} = \frac{E}{1-\nu^2}(\varepsilon_{p2} + \nu\varepsilon_{p1}) = \frac{(10,600)}{1-(0.33)^2}\left[(-700)+0.33(1050)\right](10^{-6})$$

$\sigma_{p2} = -4.2050\text{ ksi} \cong 4.21\text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = 0\text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 6.97\text{ ksi}$ .... **Ans.**

4-24

The given values are $E = 200$ GPa $\nu = 0.30$

$\varepsilon_{ai} = 500\ \mu\text{m/m}$ $\varepsilon_{hi} = 750\ \mu\text{m/m}$ $\sigma_{ri} = -p = -100$ MPa

$\varepsilon_{ao} = 500\ \mu\text{m/m}$ $\varepsilon_{ho} = 100\ \mu\text{m/m}$ $\sigma_{ro} = 0$ MPa

On the inside:

$$\sigma_{ri} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{ri} + \nu(\varepsilon_{ai} + \varepsilon_{hi})\right]$$

$$-100\left(10^6\right) = \frac{200\left(10^9\right)}{(1+0.30)(1-0.60)}\left[(1-0.30)\varepsilon_{ri} + 0.30(500+750)\right]\left(10^{-6}\right)$$

$$\varepsilon_{ri} = -907.1\ \mu\text{m/m}$$

$$\sigma_{ai} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{ai} + \nu(\varepsilon_{ri} + \varepsilon_{hi})\right]$$

$$= \frac{200\left(10^9\right)}{(1+0.30)(1-0.60)}\left[(1-0.30)(500) + 0.30(-907.1+750)\right]\left(10^{-6}\right)$$

$\sigma_{ai} = 116.5\left(10^6\right)\ \text{N/m}^2 = 116.5$ MPa (T) .... **Ans.**

$$\sigma_{hi} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{hi} + \nu(\varepsilon_{ri} + \varepsilon_{ai})\right]$$

$$= \frac{200\left(10^9\right)}{(1+0.30)(1-0.60)}\left[(1-0.30)(750) + 0.30(-907.1+500)\right]\left(10^{-6}\right)$$

$\sigma_{hi} = 155.0\left(10^6\right)\ \text{N/m}^2 = 155.0$ MPa (T) .... **Ans.**

On the outside:

$$\sigma_{ao} = \frac{E}{1-\nu^2}\left(\varepsilon_{ao} + \nu\varepsilon_{ho}\right) = \frac{200\left(10^9\right)}{1-(0.30)^2}\left[(500) + 0.30(100)\right]\left(10^{-6}\right)$$

$\sigma_{ao} = 116.5\left(10^6\right)\ \text{N/m}^2 = 116.5$ MPa (T) .... **Ans.**

$$\sigma_{ho} = \frac{E}{1-\nu^2}\left(\varepsilon_{ho} + \nu\varepsilon_{ao}\right) = \frac{200\left(10^9\right)}{1-(0.30)^2}\left[(100) + 0.30(500)\right]\left(10^{-6}\right)$$

$\sigma_{ho} = 54.9\left(10^6\right)\ \text{N/m}^2 = 54.9$ MPa (T) .... **Ans.**

4-25*

$$300 - \tau_i(\pi 3.25)(8) = 0 \qquad \tau_i = 3.67281 \text{ psi} = 2000\gamma_i$$

$$\gamma_i = 1836.403\ \mu\text{rad}$$

$$300 - \tau_o(\pi 4.25)(8) = 0 \qquad \tau_o = 2.80862 \text{ psi} = 2000\gamma_o$$

$$\gamma_o = 1404.308\ \mu\text{rad}$$

$$\gamma_{avg} = \frac{1836.403 + 1404.308}{2} = 1620.356\ \mu\text{rad} \cong \frac{\Delta x}{0.5}$$

$$\Delta x \cong 0.5\left(1620.356 \times 10^{-6}\right) = 8.10\left(10^{-4}\right) \text{ in.}$$ .... **Ans.**

4-26*

(a) $\delta_L = \alpha \Delta T L = (12.1\times 10^{-6})(70)(2500) = 2.12 \text{ mm}$ .... **Ans.**

$\delta_{do} = \alpha \Delta T d_o = (12.1\times 10^{-6})(70)(105) = 0.0889 \text{ mm}$ .... **Ans.**

$\delta_{di} = \alpha \Delta T d_i = (12.1\times 10^{-6})(70)(70) = 0.0593 \text{ mm}$ .... **Ans.**

(b) $\delta_L = \alpha \Delta T L = (12.1\times 10^{-6})(-85)(2500) = -2.57 \text{ mm}$ .... **Ans.**

$\delta_{do} = \alpha \Delta T d_o = (12.1\times 10^{-6})(-85)(105) = -0.1080 \text{ mm}$ .... **Ans.**

$\delta_{di} = \alpha \Delta T d_i = (12.1\times 10^{-6})(-85)(70) = -0.0720 \text{ mm}$ .... **Ans.**

4-27*

$$\delta = \varepsilon L = \frac{PL}{AE} + \alpha \Delta T L$$

$$0.05 = \frac{3000(4\times 12)}{(1\times 2)(30\times 10^6)} + (6.6\times 10^{-6})(\Delta T)(4\times 12)$$

$\Delta T = 150.3\ °\text{F}$ .... **Ans.**

4-28

$$\delta = \varepsilon L = \frac{PL}{AE} + \alpha \Delta T L$$

$$= 0 + \left(22.5 \times 10^{-6}\right)\left(-80\right)\left(40 \times 10^{3}\right)$$

$\delta = -72.0$ mm .... **Ans.**

4-29

$$\delta = \varepsilon L = \frac{PL}{AE} + \alpha \Delta T L$$

$\delta_L = 0 + \left(6.5\times10^{-6}\right)\left(250\right)\left(225\times12\right) = 4.39$ in. .... **Ans.**

$\delta_d = 0 + \left(6.5\times10^{-6}\right)\left(250\right)\left(12\times12\right) = 0.234$ in. .... **Ans.**

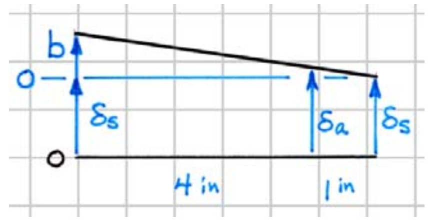
4-30*

$$99.8 + \delta_{br} = 100 + \delta_{st} \qquad \delta = \varepsilon L = \frac{PL}{AE} + \alpha \Delta T L$$

$$99.8 + \left[0 + \left(16.9 \times 10^{-6}\right)\left(\Delta T\right)\left(99.8\right)\right] = 100 + \left[0 + \left(11.9 \times 10^{-6}\right)\left(\Delta T\right)\left(100\right)\right]$$

$\Delta T = 403\ ^\circ\text{C}$ .... **Ans.**

4-31*



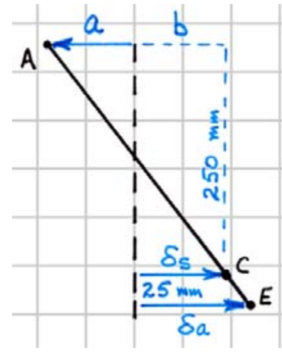
$$\delta = \varepsilon L = \frac{PL}{AE} + \alpha \Delta T L$$

$$\delta_a = 0 + \left(12.5\times10^{-6}\right)(80)(20) = 0.020000 \text{ in.}$$

$$\delta_s = 0 + \left(6.6\times10^{-6}\right)(80)(20) = 0.0105600 \text{ in.}$$

$b = 5\left(\delta_a - \delta_s\right) = 5\left(0.02 - 0.01056\right) = 0.0472$ in. ↑ .... **Ans.**

4-32



$$\delta = \varepsilon L = \frac{PL}{AE} + \alpha \Delta T L$$

$$\delta_a = 0 + \left(22.5\times10^{-6}\right)(75)(300) = 0.50625 \text{ mm}$$

$$\delta_s = 0 + \left(11.9\times10^{-6}\right)(75)(300) = 0.26775 \text{ mm} = b$$

$$a + b = \frac{250}{25}(\delta_a - \delta_s) = \frac{250}{25}(0.50625 - 0.26775)$$

$a = 2.12 \text{ mm} \leftarrow$ ....**Ans.**

4-33

$$\varepsilon_d = \varepsilon_\sigma + \varepsilon_T = \frac{\sigma_d - \nu\sigma_a}{E} + \alpha\Delta T$$

$$= \frac{0 - 0.33(4)}{(10{,}000)\left[\pi(0.25)^2/4\right]} + \left(12.5\times10^{-6}\right)(60) = \frac{\delta_d}{0.25}$$

$\delta_d = +0.000860$ in. .... **Ans.**

4-34*

$$P = W = 2500(9.81) = 24,525 \text{ N}$$

(a) $\sigma = \dfrac{P}{A} = \dfrac{24,525}{\pi(0.025)^2/4} = 49.962(10^6) \text{ N/m}^2 \cong 50.0 \text{ MPa}$ .... **Ans.**

(b) $\varepsilon = \dfrac{\sigma}{E} + \alpha\Delta T = \dfrac{49.962(10^6)}{73(10^9)} + (22.5\times10^{-6})(-50)$

$\varepsilon = -441(10^{-6}) = -441\ \mu\text{m/m}$ .... **Ans.**

(c) $\varepsilon_d = \varepsilon_\sigma + \varepsilon_T = \dfrac{\sigma_d - \nu\sigma_a}{E} + \alpha\Delta T$

$$\varepsilon_d = \frac{0 - 0.33(49.962\times10^6)}{73\times10^9} + (22.5\times10^{-6})(-50) = \frac{\delta_d}{25}$$

$\delta_d = -0.0338 \text{ mm}$ .... **Ans.**

4-35

$$\varepsilon_d = \frac{P}{AE} + \alpha\Delta T = \frac{(25-10)}{(1/2)(1/32)(30\times10^6)} + (6.5\times10^{-6})(100-72) = 214(10^{-6})$$

$$\Delta L = \varepsilon L = (214\times10^{-6})(100\times12) = +0.257 \text{ in.}$$

correction = –0.257 in. .... **Ans.**

4-36

$$T_{AB} = T_{BC} = P = 25 \text{ kN}$$

(a) $$\delta_{AB} = \frac{PL}{AE} + \alpha \Delta T L = \frac{(25{,}000)(200)}{\left[\pi (0.050)^2 / 4\right](200 \times 10^9)} + (12 \times 10^{-6})(20)(200)$$

$\delta_{AB} = 0.0607$ mm .... **Ans.**

(b) $$\delta_{BC} = \frac{PL}{AE} + \alpha \Delta T L = \frac{(25{,}000)(150)}{\left[\pi (0.025)^2 / 4\right](70 \times 10^9)} + (22.5 \times 10^{-6})(20)(150)$$

$\delta_{BC} = 0.1766$ mm .... **Ans.**

(c) $\Delta_C = \delta_{AB} + \delta_{BC} = 0.237 \text{ mm} \rightarrow$ .... **Ans.**

4-37*

From Table 4-1 for the T300/5208 material

$$E_1 = 26,300 \text{ ksi} \qquad E_2 = 1494 \text{ ksi} \qquad G_{12} = 1040 \text{ ksi} \qquad \nu_{12} = 0.28$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{1494}{26,300}(0.28) = 0.01591$$

The given data are

$$\varepsilon_1 = 2000\ \mu\text{in./in.} \qquad \varepsilon_2 = 4000\ \mu\text{in./in.} \qquad \gamma_{12} = 1500\ \mu\text{rad}$$

$$\sigma_1 = \frac{E_1}{1-\nu_{12}\nu_{21}}(\varepsilon_1 + \nu_{21}\varepsilon_2) = \frac{26,300}{1-(0.28)(0.01591)}\left[(2000) + 0.01591(4000)\right](10^{-6})$$

$\sigma_1 = +54.5 \text{ ksi} = 54.5 \text{ ksi (T)}$ .... **Ans.**

$$\sigma_2 = \frac{E_2}{1-\nu_{12}\nu_{21}}(\varepsilon_2 + \nu_{12}\varepsilon_1) = \frac{1494}{1-(0.28)(0.01591)}\left[(4000) + 0.28(2000)\right](10^{-6})$$

$\sigma_2 = +6.84 \text{ ksi} = 6.84 \text{ ksi (T)}$ .... **Ans.**

$\tau_{12} = G_{12}\gamma_{12} = (1040)(1500\times10^{-6}) = +1.560 \text{ ksi}$ .... **Ans.**

4-38*

From Table 4-1 for the Scotchply 1002 Glass/Epoxy material

$$E_1 = 36.8 \text{ GPa} \qquad E_2 = 8.27 \text{ GPa} \qquad G_{12} = 4.14 \text{ GPa} \qquad \nu_{12} = 0.26$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{8.27}{36.8}(0.26) = 0.05843$$

The given data are

$$\sigma_1 = 30 \text{ MPa} \qquad \sigma_2 = -2 \text{ MPa} \qquad \tau_{12} = 0.3 \text{ MPa}$$

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\frac{\sigma_2}{E_2} = \frac{30}{36.8(10^3)} - (0.05843)\frac{-2}{8.27(10^3)} = 829(10^{-6}) \text{ m/m}$$

$\varepsilon_1 = 829\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_2 = \frac{\sigma_2}{E_2} - \nu_{12}\frac{\sigma_1}{E_1} = \frac{-2}{8.27(10^3)} - (0.26)\frac{30}{36.8(10^3)} = -454(10^{-6}) \text{ m/m}$$

$\varepsilon_2 = -454\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{12} = \dfrac{\tau_{12}}{G_{12}} = \dfrac{0.3}{4.14(10^3)} = +72.5(10^{-6}) \text{ rad} = +72.5\ \mu\text{rad}$ .... **Ans.**

4-39

From Table 4-1 for the T300/5208 material

$$E_1 = 26{,}300 \text{ ksi} \qquad E_2 = 1494 \text{ ksi} \qquad G_{12} = 1040 \text{ ksi} \qquad \nu_{12} = 0.28$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{1494}{26{,}300}(0.28) = 0.01591$$

The given data are

$$\sigma_1 = 40 \text{ ksi} \qquad \sigma_2 = -10 \text{ ksi} \qquad \tau_{12} = 2 \text{ ksi}$$

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\frac{\sigma_2}{E_2} = \frac{40}{26{,}300} - (0.01591)\frac{(-10)}{1494} = 1627(10^{-6}) \text{ in./in.}$$

$\varepsilon_1 = 1627\ \mu\text{in./in.}$ .... **Ans.**

$$\varepsilon_2 = \frac{\sigma_2}{E_2} - \nu_{12}\frac{\sigma_1}{E_1} = \frac{(-10)}{1494} - (0.28)\frac{40}{26{,}300} = -7120(10^{-6}) \text{ in./in.}$$

$\varepsilon_2 = -7120\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{12} = \dfrac{\tau_{12}}{G_{12}} = \dfrac{2}{1040} = +1923(10^{-6}) \text{ rad} = +1923\ \mu\text{rad}$ .... **Ans.**

4-40

From Table 4-1 for the Boron/epoxy material

$$E_1 = 200 \text{ GPa} \qquad E_2 = 20 \text{ GPa} \qquad G_{12} = 6 \text{ GPa} \qquad \nu_{12} = 0.23$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{20}{200}(0.23) = 0.02300$$

The given data are

$$\varepsilon_1 = 1000\ \mu\text{m/m} \qquad \varepsilon_2 = 500\ \mu\text{m/m} \qquad \gamma_{12} = 300\ \mu\text{rad}$$

$$\sigma_1 = \frac{E_1}{1-\nu_{12}\nu_{21}}(\varepsilon_1 + \nu_{21}\varepsilon_2) = \frac{200(10^3)}{1-(0.23)(0.023)}\left[(1000) + 0.023(500)\right](10^{-6})$$

$\sigma_1 = +203 \text{ MPa} = 203 \text{ MPa (T)}$ .... **Ans.**

$$\sigma_2 = \frac{E_2}{1-\nu_{12}\nu_{21}}(\varepsilon_2 + \nu_{12}\varepsilon_1) = \frac{20(10^3)}{1-(0.23)(0.023)}\left[(500) + 0.23(1000)\right](10^{-6})$$

$\sigma_2 = +14.68 \text{ MPa} = 14.68 \text{ MPa (T)}$ .... **Ans.**

$\tau_{12} = G_{12}\gamma_{12} = (6\times 10^3)(300\times 10^{-6}) = +1.800 \text{ MPa}$ .... **Ans.**

4-41*

From Table 4-1 for the T300/5208 material

$$E_1 = 26,300 \text{ ksi} \qquad E_2 = 1494 \text{ ksi} \qquad G_{12} = 1040 \text{ ksi} \qquad \nu_{12} = 0.28$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{1494}{26,300}(0.28) = 0.01591$$

The given data are

$$\sigma_1 = 5 \text{ ksi} \qquad \sigma_2 = 5 \text{ ksi} \qquad \tau_{12} = 1 \text{ ksi}$$

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\frac{\sigma_2}{E_2} = \frac{5}{26,300} - (0.01591)\frac{5}{1494} = 0.00013687 \text{ in./in.}$$

$$\varepsilon_2 = \frac{\sigma_2}{E_2} - \nu_{12}\frac{\sigma_1}{E_1} = \frac{5}{1494} - (0.28)\frac{5}{26,300} = 0.003293 \text{ in./in.}$$

$\delta_x = \varepsilon_2 L = (0.003293)(10) = 0.0329$ in. .... **Ans.**

$\delta_y = \varepsilon_1 L = (0.00013687)(10) = 0.001369$ in. .... **Ans.**

4-42

From Table 4-1 for the Scotchply 1002 Glass/Epoxy material

$$E_1 = 36.8 \text{ GPa} \qquad E_2 = 8.27 \text{ GPa} \qquad G_{12} = 4.14 \text{ GPa} \qquad \nu_{12} = 0.26$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{8.27}{36.8}(0.26) = 0.05843$$

The given data are

$$\sigma_1 = 5 \text{ MPa} \qquad \sigma_2 = -2 \text{ MPa} \qquad \tau_{12} = 0 \text{ MPa}$$

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\frac{\sigma_2}{E_2} = \frac{5}{36.8(10^3)} - (0.05843)\frac{(-2)}{8.27(10^3)} = 150.00(10^{-6}) \text{ m/m}$$

$$\delta_x = \varepsilon_1 L = (0.00015000)(125) = 0.01875 \text{ mm}$$ .... **Ans.**

$$\varepsilon_2 = \frac{\sigma_2}{E_2} - \nu_{12}\frac{\sigma_1}{E_1} = \frac{(-2)}{8.27(10^3)} - (0.26)\frac{5}{36.8(10^3)} = -277.16(10^{-6}) \text{ m/m}$$

$$\delta_y = \varepsilon_2 L = (-0.00027716)(100) = -0.0277 \text{ mm}$$ .... **Ans.**

4-43

Since $\sigma_y = 0$, $\varepsilon_x = \sigma_x / E_x$ or $\sigma_x = E_x \varepsilon_x$

The total load carried by the composite is carried partially by the fibers and partially by the matrix. Thus,

$$P = \sigma_x A = \sigma_f A_f + \sigma_m A_m = E_f \varepsilon_f A_f + E_m \varepsilon_m A_m$$

where $\varepsilon_f = \varepsilon_m = \varepsilon_x$ since the fibers and the matrix are bonded together. Therefore,

$$\sigma_x = \frac{E_f A_f + E_m A_m}{A} \varepsilon_x = E_x \varepsilon_x$$

from which

$$E_x = \frac{E_f A_f + E_m A_m}{A} = \frac{E_f A_f}{A} + \frac{E_m A_m}{A} = E_f V_f + E_m V_m$$ .... **Ans.**

4-44

The total load carried by the composite is carried partially by the fibers and partially by the matrix. Thus,

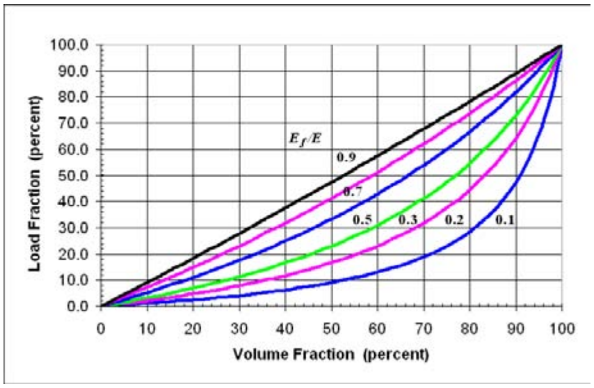
$$P = P_f + P_m = \sigma_f A_f + \sigma_m A_m$$

Therefore,

$$\frac{P_f}{P} = \frac{\sigma_f A_f}{\sigma_f A_f + \sigma_m A_m} = \frac{E_f \varepsilon_f A_f}{E_f \varepsilon_f A_f + E_m \varepsilon_m A_m}$$

where $\varepsilon_f = \varepsilon_m = \varepsilon_x$ since the fibers and the matrix are bonded together. Therefore,

$$\frac{P_f}{P} = \frac{E_f A_f}{E_f A_f + E_m A_m} = \frac{E_f A_f}{E_f A_f + E_m \left(A - A_f\right)}$$



Dividing both the numerator and the denominator by the total area A gives

$$\frac{P_f}{P} = \frac{E_f \left(A_f / A\right)}{E_f \left(A_f / A\right) + E_m \left(A_m / A\right)} = \frac{E_f V_f}{E_f V_f + E_m \left(1 - V_f\right)}$$ .... **Ans.**

4-45*

(a) $E_{75} = \dfrac{50-0}{0.002-0} = 25,000 \text{ ksi}$ .... **Ans.**

$E_{1600} = \dfrac{26-0}{0.002-0} = 13,000 \text{ ksi}$ .... **Ans.**

(b) $\sigma_y\left(75°\right) \cong 57 \text{ ksi}$ .... **Ans.**

$\sigma_y\left(1600°\right) \cong 21 \text{ ksi}$ .... **Ans.**

4-46*

The given values are

$$\sigma_x = 120 \text{ MPa} \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = 60 \text{ MPa}$$

$$E = 70 \text{ GPa} \qquad \nu = 0.33$$

$$\varepsilon_x = \frac{\sigma_x - \nu\left(\sigma_y + \sigma_z\right)}{E} = \frac{(120) - 0.33\left[(-80) + 0\right]}{70\left(10^3\right)} = +0.00209143 = +2091.43\ \mu\text{m/m}$$

$$\varepsilon_y = \frac{\sigma_y - \nu\left(\sigma_x + \sigma_z\right)}{E} = \frac{(-80) - 0.33\left[(120) + 0\right]}{70\left(10^3\right)} = -0.00170857 = -1708.57\ \mu\text{m/m}$$

$$G = \frac{E}{2(1+\nu)} = \frac{70}{2(1+0.33)} = 26.3158 \text{ GPa}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{60}{26.3158\left(10^3\right)} = +0.002280 \text{ rad} = +2280\ \mu\text{rad}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (2091.43)\cos^2(-30°) + (-1708.57)\sin^2(-30°) + (2280)\sin(-30°)\cos(-30°)$$

$\varepsilon_n = 154.2\ \mu\text{m/m}$ .... **Ans.**

4-47

The given values are

$$\sigma_x = 12 \text{ ksi} \qquad \sigma_y = -4 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$$

$$E = 30{,}000 \text{ ksi} \qquad \nu = 0.30$$

$$\varepsilon_x = \frac{\sigma_x - \nu\left(\sigma_y + \sigma_z\right)}{E} = \frac{\left(12\right) - 0.30\left[\left(-4\right) + 0\right]}{30{,}000}$$

$\varepsilon_x = \varepsilon_a = +0.000440 \text{ in./in.} = +440\ \mu\text{in./in.}$ .... **Ans.**

$$\varepsilon_y = \frac{\sigma_y - \nu\left(\sigma_x + \sigma_z\right)}{E} = \frac{\left(-4\right) - 0.30\left[\left(12\right) + 0\right]}{30{,}000}$$

$\varepsilon_y = \varepsilon_b = -0.00025333 \text{ in./in.} \cong -253\ \mu\text{in./in.}$ .... **Ans.**

$$G = \frac{E}{2\left(1+\nu\right)} = \frac{30{,}000}{2\left(1+0.30\right)} = 11{,}538.46 \text{ ksi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-6}{11{,}538.46} = -0.000520 \text{ rad} = -520\ \mu\text{rad}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= \left(440\right)\cos^2\left(120°\right) + \left(-253.33\right)\sin^2\left(120°\right) + \left(-520\right)\sin\left(120°\right)\cos\left(120°\right)$$

$\varepsilon_n = \varepsilon_c = 145.2\ \mu\text{in./in.}$ .... **Ans.**

4-48*

The given values are

$$\sigma_x = -100 \text{ MPa} \qquad \sigma_z = 0 \text{ MPa} \qquad E = 210 \text{ GPa} \qquad G = 80 \text{ GPa}$$

$$E = 2(1+\nu)G \qquad 210 = 2(1+\nu)(80) \qquad \nu = 0.31250$$

$$\varepsilon_y = \frac{\sigma_y - \nu(\sigma_x + \sigma_z)}{E} = \frac{\sigma_y - 0.31250\left[(-100)+0\right]}{210(10^3)} = 0$$

$$\sigma_y = -31.250 \text{ MPa}$$

$$\varepsilon_z = \frac{\sigma_z - \nu(\sigma_x + \sigma_y)}{E} = \frac{0 - 0.31250\left[(-100)+(-31.250)\right]}{210(10^3)} = 0.0001953 \text{ m/m}$$

$\delta = \varepsilon_z L = (0.0001953)(10) = 0.001953 \text{ mm}$ .... **Ans.**

4-49

$$E = 29,000 \text{ ksi} \qquad G = 11,000 \text{ ksi} \qquad E = 2(1+\nu)G$$

$$29,000 = 2(1+\nu)(11,000) \qquad \nu = 0.31818$$

$$\delta_{xa} = \varepsilon_{xa} L_{xa} = \varepsilon_{xa}(2) = \frac{6 - 0.31818\sigma_y}{29,000}(2)$$

$$\delta_{xb} = \varepsilon_{xb} L_{xb} = \varepsilon_{xb}(3) = \frac{5}{29,000}(3)$$

But $\delta_{xa} = \delta_{xb}$, therefore

$$12 - 0.63636\sigma_y = 15$$

$\sigma_y = -4.71$ ksi .... **Ans.**

4-50

(a) $E_1 = E_2 = \dfrac{400\left(10^6\right) - 0}{0.002 - 0} = 200\left(10^9\right)\ \text{N/m}^2 = 200\ \text{GPa}$ .... **Ans.**

(b) $\sigma_{y1} = 350\ \text{MPa}$ .... **Ans.**

$\sigma_{y2} = 1000\ \text{MPa}$ .... **Ans.**

4-51*

Assume series of rails all initially separated by 0.125 in.
When heated, rails expand from center in both directions.

(a) $\delta = 0.125 = \alpha \, \Delta T \, L = \left(6.6\times10^{-6}\right)\left(\Delta T\right)\left(55\times12\right)$

$\Delta T = 28.7 \ ^\circ\text{F}$

Rails touch when $T = 60 + 28.7 = 88.7 \ ^\circ\text{F}$ .... **Ans.**

(b) $\delta = \left(6.6\times10^{-6}\right)\left(-50\right)\left(55\times12\right) = -0.21780 \text{ in.}$

$gap = 0.125 + 0.2178 = 0.3428 \text{ in.} \cong 0.343 \text{ in.}$ .... **Ans.**

4-52

The given values are

$$\sigma_x = 72 \text{ MPa} \qquad \sigma_y = 36 \text{ MPa} \qquad \tau_{xy} = -24 \text{ MPa}$$

$$E = 100 \text{ GPa} \qquad \nu = 0.28$$

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E} = \frac{(72) - 0.28(36)}{100(10^3)} = +0.00061920 = +619.20\ \mu\text{m/m}$$

$\varepsilon_x = \varepsilon_a = +619\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E} = \frac{(36) - 0.28(72)}{100(10^3)} = 0.00015840 = 158.40\ \mu\text{m/m}$$

$$G = \frac{E}{2(1+\nu)} = \frac{100}{2(1+0.28)} = 39.0625 \text{ GPa}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-24}{39.0625(10^3)} = -0.00061440 \text{ rad} = -614.40\ \mu\text{rad}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (619.20)\cos^2(45°) + (158.40)\sin^2(45°) + (-614.40)\sin(45°)\cos(45°)$$

$\varepsilon_n = \varepsilon_b = +81.6\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$= (619.20)\cos^2(135°) + (158.40)\sin^2(135°) + (-614.40)\sin(135°)\cos(135°)$$

$\varepsilon_n = \varepsilon_c = +696\ \mu\text{m/m}$ .... **Ans.**

4-53

The given values are

$$\sigma_x = 8.5 \text{ ksi} \qquad \sigma_y = 4.5 \text{ ksi} \qquad \tau_{xy} = 6 \text{ ksi}$$

$$E = 30,000 \text{ ksi} \qquad \nu = 0.30$$

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E} = \frac{(8.5) - 0.30(4.5)}{30,000} = +0.00023833 = +238.33 \ \mu\text{in./in.}$$

$$\varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E} = \frac{(4.5) - 0.30(8.5)}{30,000} = 0.00006500 \text{ in./in.} \cong +65.00 \ \mu\text{in./in.}$$

$$G = \frac{E}{2(1+\nu)} = \frac{30,000}{2(1+0.30)} = 11,538.46 \text{ ksi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{6}{11,538.46} = 0.00052000 \text{ rad} = 520.00 \ \mu\text{rad}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (238.33)\cos^2(20°) + (65.00)\sin^2(20°) + (520.00)\sin(20°)\cos(20°)$$

$\varepsilon_n = 385 \ \mu\text{in./in.}$ .... **Ans.**

4-54*

The given values are $E = 73$ GPa $\nu = 0.33$

$\varepsilon_x = 825\ \mu\text{m/m}$ $\varepsilon_y = 950\ \mu\text{m/m}$ $\gamma_{xy} = 680\ \mu\text{rad}$

(a) $$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{\left(73\times10^3\right)}{1-\left(0.33\right)^2}\left[\left(825\right)+0.33\left(950\right)\right]\left(10^{-6}\right)$$

$\sigma_x = +93.2673$ MPa $\cong +93.3$ MPa (T) .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{\left(73\times10^3\right)}{1-\left(0.33\right)^2}\left[\left(950\right)+0.33\left(825\right)\right]\left(10^{-6}\right)$$

$\sigma_y = +100.1282$ MPa $\cong 100.1$ MPa (T) .... **Ans.**

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2\left(1+\nu\right)}\gamma_{xy} = \frac{73\times10^3}{2\left(1+0.33\right)}\left(680\times10^{-6}\right)$$

$\tau_{xy} = +18.6617$ MPa $\cong +18.66$ MPa .... **Ans.**

(b) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x-\sigma_y} = \frac{1}{2}\tan^{-1}\frac{2\left(18.6617\right)}{\left(93.2673\right)-\left(100.1282\right)} = -39.792°, \quad 50.208°$$

When $\theta_p = -39.792$

$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= \left(93.2673\right)\cos^2\theta_p + \left(100.1282\right)\sin^2\theta_p + 2\left(18.6617\right)\sin\theta_p\cos\theta_p$$

$$= +77.7234 \text{ MPa} = \sigma_{p2}$$

$\sigma_{p1} = \sigma_x + \sigma_y - \sigma_{p2} = 115.6721$ MPa

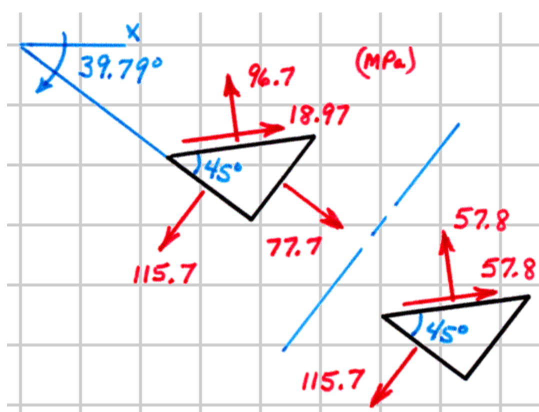
$\sigma_{p1} = 115.7$ MPa (T) ∠ 50.21° .... **Ans.**

$\sigma_{p2} = 77.7$ MPa (T) ∠ 39.79° .... **Ans.**

$\sigma_{p3} = 0$ MPa .... **Ans.**

$\tau_p = \left(\sigma_{p1}-\sigma_{p2}\right)/2 = 18.97$ MPa .... **Ans.**

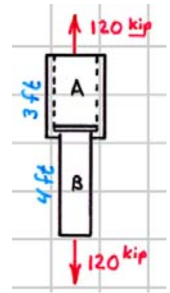
$\sigma_{n45} = \left(\sigma_{p1}+\sigma_{p2}\right)/2 = 96.7$ MPa



$\tau_{max} = \left(\sigma_{p1}-\sigma_{p3}\right)/2 = 57.8$ MPa (out-of-plane) .... **Ans.**

$\sigma_{n45} = \left(\sigma_{p1}+\sigma_{p3}\right)/2 = 57.8$ MPa

5-1*



$$A_A = \pi\left(6^2 - 4.5^2\right)/4 = 12.37002 \text{ in.}^2 \qquad P_A = P_B = 120 \text{ kip}$$

$$A_B = \pi 4^2/4 = 12.56637 \text{ in.}^2$$

$$\delta_{total} = \sum \frac{PL}{AE} = \frac{(120)(3\times 12)}{(12.37002)(30,000)} + \frac{(120)(4\times 12)}{(12.56637)(10,600)}$$

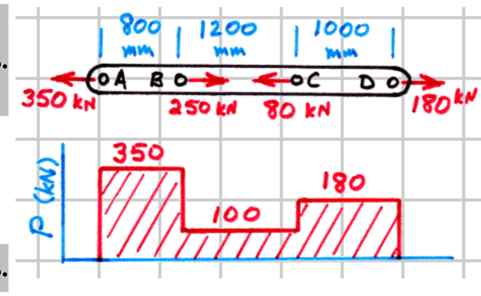
$\delta = 0.0549$ in. .... **Ans.**

5-2*

$$A = (0.100)(0.025) = 0.002500 \text{ m}^2$$

(a) $$\delta_{AB} = \frac{PL}{AE} = \frac{(350\times10^3)(800)}{(0.002500)(200\times10^9)} = 0.560 \text{ mm}$$ .... **Ans.**

(b) $$\delta_{BC} = \frac{(100\times10^3)(1200)}{(0.002500)(200\times10^9)} = 0.240 \text{ mm}$$ .... **Ans.**



(c) $$\delta_{CD} = \frac{(180\times10^3)(1000)}{(0.002500)(200\times10^9)} = 0.360 \text{ mm}$$

$$\delta_{total} = 0.560 + 0.240 + 0.360 = 1.160 \text{ mm}$$ .... **Ans.**

5-3

(a) $A_{pipe} = \pi\left(6^2 - 4.8^2\right)/4 = 10.17876 \text{ in.}^2$

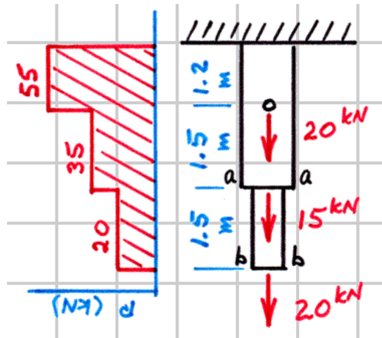
$\sigma_{avg} = \frac{P}{A} = \frac{30}{10.17876} = 2.95 \text{ ksi (C)}$ .... **Ans.**

(b) $\delta = \frac{PL}{AE} = \frac{(-30)(24)}{(10.17876)(29,000)} = -0.0024392 \text{ in.} \cong 0.00244 \text{ in. (shorten)}$ .... **Ans.**

(c) $\varepsilon_{avg} = \frac{\delta}{L} = \frac{-0.0024392}{24} = -101.6\left(10^{-6}\right) = -101.6\ \mu\text{in./in.}$ .... **Ans.**

5-4*

$A_T = \pi(25)^2/4 = 490.9 \text{ mm}^2$ $A_B = \pi(15)^2/4 = 176.71 \text{ mm}^2$



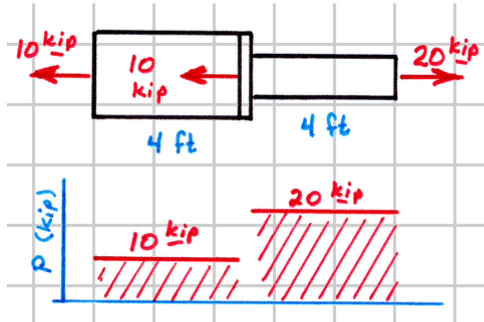
(a) $\delta_{a-a} = \sum \frac{PL}{AE} = \frac{(55\times10^3)(1200)+(35\times10^3)(1500)}{(490.9\times10^{-6})(73\times10^9)}$

$\delta_{a-a} = 3.30693 \text{ mm} \cong 3.31 \text{ mm}$ .... **Ans.**

(b) $\delta_{b-b} = \sum \frac{PL}{AE} = 3.30693 + \frac{(20\times10^3)(1500)}{(176.71\times10^{-6})(73\times10^9)}$

$\delta_{b-b} = 0.563 \text{ mm}$ .... **Ans.**

5-5



(a) $$\delta = \sum \frac{PL}{AE} = \frac{(10)(4\times 12)}{\left[\pi (1.5)^2/4\right](30{,}000)} + \frac{(20)(4\times 12)}{\left[\pi (1)^2/4\right](30{,}000)}$$

$\delta = 0.0498$ in. .... **Ans.**

(b) $$W = \gamma V = \gamma \left[\frac{\pi (1.5)^2}{4}(48) + \frac{\pi (1)^2}{4}(48)\right] = \gamma \left[\frac{\pi d^2}{4}(96)\right] \qquad d = 1.27475 \text{ in.}$$

$$\delta = \sum \frac{PL}{AE} = \frac{(10)(4\times 12) + (20)(4\times 12)}{\left[\pi (1.27475)^2/4\right](30{,}000)} = 0.0376 \text{ in.}$$ .... **Ans.**

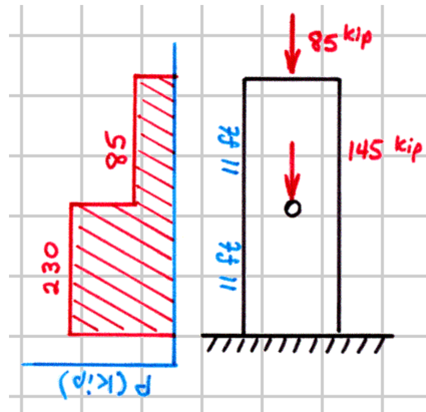
5-6

$$\sigma = \frac{N}{A} = \frac{P}{(0.025\times 0.075)} \le 100(10^6)\ \text{N/m}^2 \qquad P \le 187.5(10^3)\ \text{N}$$

$$\delta = \frac{PL}{AE} = \frac{P(2000)}{(0.025\times 0.075)(70\times 10^9)} \le 4\ \text{mm} \qquad P \le 262.5(10^3)\ \text{N}$$

$P_{max} = 187.5$ kN .... **Ans.**

5-7*



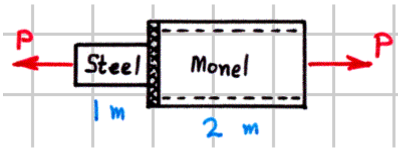
(a) $$\delta = \frac{PL}{AE} = \frac{(230)(11\times 12)}{(9)(29,000)} + \frac{(20)(4\times 12)}{\left[\pi (1)^2/4\right](30,000)}$$

$\delta = 0.11632 \text{ in.} \cong 0.1163 \text{ in.}$ .... **Ans.**

(b) $$\delta = \sum \frac{PL}{AE} = 0.11632 + \frac{(85)(11\times 12)}{(9)(29,000)}$$

$\delta = 0.1593 \text{ in.}$ .... **Ans.**

5-8

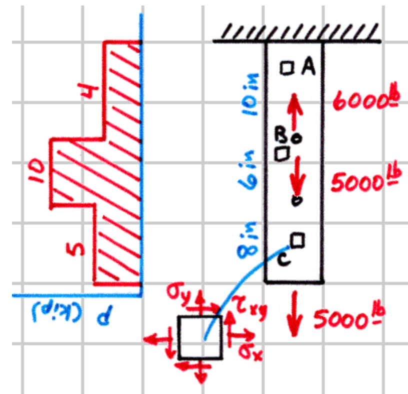


$$\delta = \sum \frac{PL}{AE} = \frac{P(1000)}{\left[\pi(0.030)^2/4\right](200\times10^9)} + \frac{P(2000)}{\left[\pi(0.060^2 - 0.040^2)/4\right](180\times10^9)} = 3\text{ mm}$$

$P = 212(10^3)$ N $= 212$ kN .... **Ans.**

5-9*

(a) $\delta = \sum \frac{PL}{AE} = \frac{(5)(8)+(10)(6)+(4)(10)}{(1\times 2)(10,000)}$



$\delta = 0.00700$ in. .... **Ans.**

(b) $\sigma_{xA} = \tau_{xyA} = 0$ .... **Ans.**

$\sigma_{yA} = \frac{P}{A} = \frac{4}{(1\times 2)} = 2.00$ ksi (T) .... **Ans.**

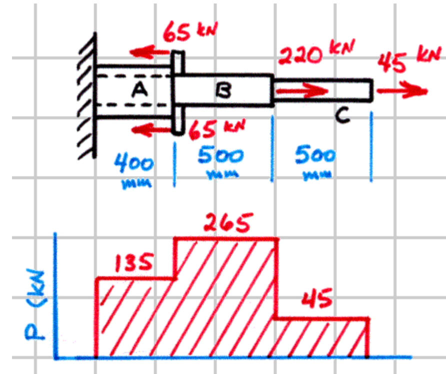
$\sigma_{xB} = \tau_{xyB} = 0$ .... **Ans.**

$\sigma_{yB} = \frac{P}{A} = \frac{10}{(1\times 2)} = 5.00$ ksi (T) .... **Ans.**

$\sigma_{xC} = \tau_{xyC} = 0$ .... **Ans.**

$\sigma_{yC} = \frac{P}{A} = \frac{5}{(1\times 2)} = 2.50$ ksi (T) .... **Ans.**

5-10*



$$A_A = \pi\left(60^2 - 50^2\right)/4 = 863.938 \text{ mm}^2$$

$$A_B = \pi(50)^2/4 = 1963.495 \text{ mm}^2$$

$$A_C = \pi(25)^2/4 = 490.874 \text{ mm}^2$$

(a) $$\delta_A = \frac{PL}{AE} = \frac{\left(135\times10^3\right)(400)}{\left(863.938\times10^{-6}\right)\left(200\times10^9\right)}$$

$\delta_A = 0.31252 \text{ mm} \cong 0.313 \text{ mm}$ .... **Ans.**

(b) $$\delta_{total} = \sum\frac{PL}{AE} = 0.31252 + \frac{\left(265\times10^3\right)(500)}{\left(1963.435\times10^{-6}\right)\left(73\times10^9\right)} + \frac{\left(45\times10^3\right)(500)}{\left(490.874\times10^{-6}\right)\left(73\times10^9\right)}$$

$\delta_{total} = 1.865 \text{ mm}$ .... **Ans.**

(c) $\varepsilon_A = \dfrac{\delta}{L} = \dfrac{0.31252}{400} = 781\left(10^{-6}\right) = 781\ \mu\text{m/m}$ .... **Ans.**

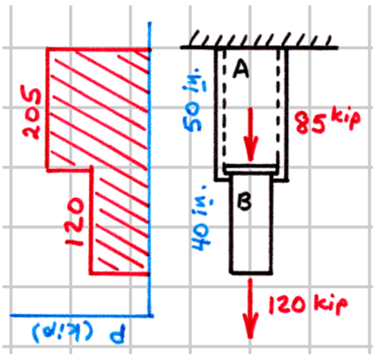
(d) $$\sigma_A = \frac{N}{A} = \frac{135\left(10^3\right)}{863.938} = 156.3\left(10^6\right)\text{ N/m}^2 = 156.3 \text{ MPa}$$

$$\sigma_B = \frac{N}{A} = \frac{265\left(10^3\right)}{1963.495} = 135.0\left(10^6\right)\text{ N/m}^2 = 135.0 \text{ MPa}$$

$$\sigma_C = \frac{N}{A} = \frac{45\left(10^3\right)}{490.874} = 91.7\left(10^6\right)\text{ N/m}^2 = 91.7 \text{ MPa}$$

$\sigma_{\max} = 156.3 \text{ MPa}$ .... **Ans.**

5-11



$$A_A = \pi\left(6^2 - 4.5^2\right)/4 = 12.3700 \text{ in.}^2$$

$$A_B = \pi(4)^2/4 = 12.5664 \text{ in.}^2$$

(a) $$\delta_A = \frac{PL}{AE} = \frac{(205)(50)}{(12.3700)(29{,}000)}$$

$\delta_A = 0.02857 \text{ in.} \cong 0.0286 \text{ in.}$ .... **Ans.**

(b) $$\delta_{total} = \sum \frac{PL}{AE} = 0.02857 + \frac{(120)(40)}{(12.5664)(10{,}600)}$$

$\delta_{total} = 0.0646 \text{ in.}$ .... **Ans.**

(c) $\sigma_{\max B} = \dfrac{N}{A} = \dfrac{120}{12.5664} = 9.5493 \text{ ksi} \cong 9.55 \text{ ksi}$ .... **Ans.**

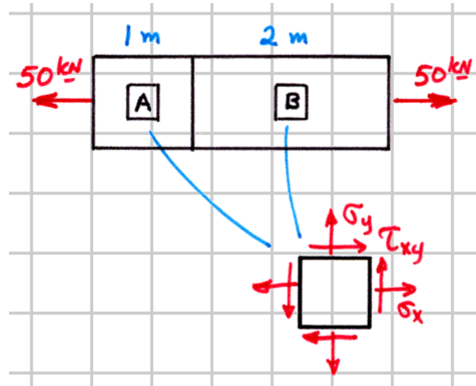
$\tau_{\max B} = \dfrac{\sigma_{\max B}}{2} = 4.7747 \text{ ksi} \cong 4.77 \text{ ksi}$ .... **Ans.**

(d) $$\varepsilon_{longB} = \frac{\sigma}{E} = \frac{9.5493}{10{,}600} = 900.8774\left(10^{-6}\right)$$

$$\varepsilon_{latB} = -\nu\varepsilon_{longB} = -(0.33)\left(900.8774\times10^{-6}\right) = -297.2895\left(10^{-6}\right)$$

$\delta_{dB} = \varepsilon_{latB} d = \left(-297.2895\times10^{-6}\right)(4) = -1.189\left(10^{-3}\right) \text{ in.}$ .... **Ans.**

5-12*



$$A_A = (25)(25) = 625 \text{ mm}^2$$

$$A_B = \pi(25)^2/4 = 490.874 \text{ mm}^2$$

(a) $$\delta = \sum \frac{PL}{AE} = \frac{(50\times10^3)(1000)}{(625\times10^{-6})(73\times10^9)} + \frac{(50\times10^3)(2000)}{(490.874\times10^{-6})(73\times10^9)}$$

$\delta = 3.89$ mm .... **Ans.**

(b) $\sigma_{xA} = \dfrac{N}{A} = \dfrac{50(10^3)}{625(10^{-6})} = 80(10^6)\ \text{N/m}^2 = 80$ MPa .... **Ans.**

$\sigma_{yA} = \tau_{xyA} = 0$ MPa .... **Ans.**

$\sigma_{xB} = \dfrac{N}{A} = \dfrac{50(10^3)}{490.874(10^{-6})} = 101.859(10^6)\ \text{N/m}^2 \cong 101.9$ MPa .... **Ans.**

$\sigma_{yB} = \tau_{xyB} = 0$ MPa .... **Ans.**

(c) $\varepsilon_{xB} = \dfrac{\sigma}{E} = \dfrac{101.859(10^6)}{73(10^9)} = 1395.331(10^{-6}) \cong 1395\ \mu\text{m/m}$ .... **Ans.**

(d) $$\varepsilon_{yB} = -\nu\varepsilon_{xB} = -(0.33)(1395.331\times10^{-6}) = -460.459(10^{-6})$$

$\delta_d = \varepsilon_{yB} d = (-460.459\times10^{-6})(25) = -0.01151$ mm .... **Ans.**

5-13*

$$A = \pi (0.75)^2 / 4 = 0.44179 \text{ in.}^2$$

(a) $\delta_A = \dfrac{PL}{AE} = \dfrac{(5)(3 \times 12)}{(0.44179)(29,000)} = 0.01405 \text{ in.}$ .... **Ans.**

(b) $\sigma_x = \dfrac{N}{A} = \dfrac{5}{0.44179} = 11.3177 \text{ ksi} \cong 11.32 \text{ ksi}$ .... **Ans.**

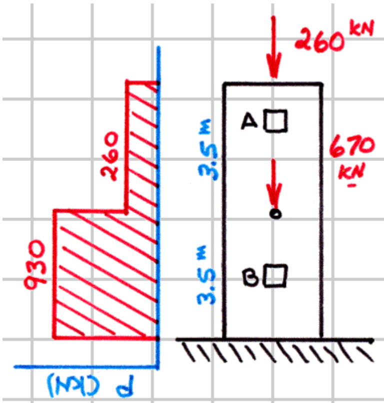
$\sigma_y = \tau_{xy} = 0 \text{ ksi}$ .... **Ans.**

(c) $\varepsilon_x = \dfrac{\sigma}{E} = \dfrac{11.31769}{29,000} = 390.265(10^{-6}) \cong 390\ \mu\text{in./in.}$ .... **Ans.**

$\varepsilon_y = -\nu\varepsilon_x = -(0.30)(390.265 \times 10^{-6}) = -117.1(10^{-6}) \cong -117.1\ \mu\text{in./in.}$ .... **Ans.**

$\gamma_{xy} = \dfrac{\tau_{xy}}{G} = 0\ \mu\text{rad}$ .... **Ans.**

5-14



(a) $\delta = \sum \dfrac{PL}{AE} = \dfrac{(260\times10^3)(3500)+(930\times10^3)(3500)}{(9485\times10^{-6})(200\times10^9)}$

$\delta = 2.20$ mm .... **Ans.**

(b) $\sigma_{xA} = \tau_{xyA} = 0$ MPa .... **Ans.**

$\sigma_{yA} = \dfrac{N}{A} = \dfrac{260(10^3)}{9485(10^{-6})} = 27.4117(10^6)\ \text{N/m}^2$

$\sigma_{yA} \cong 27.4$ MPa (C) .... **Ans.**

$\sigma_{xB} = \tau_{xyB} = 0$ MPa .... **Ans.**

$\sigma_{yB} = \dfrac{N}{A} = \dfrac{930(10^3)}{9485(10^{-6})} = 98.0496(10^6)\ \text{N/m}^2 \cong 98.0$ MPa (C) .... **Ans.**

(c) $E = 2(1+\nu)G \qquad 200 = 2(1+\nu)(76) \qquad \nu = 0.31579$

$\varepsilon_{yA} = \dfrac{\sigma}{E} = \dfrac{-27.4117(10^6)}{200(10^9)} = -137.059(10^{-6}) \cong -137.1\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_{xA} = -\nu\varepsilon_{yA} = -(0.31579)(-137.059) = +43.3\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{xyA} = 0\ \mu\text{m/m}$ .... **Ans.**

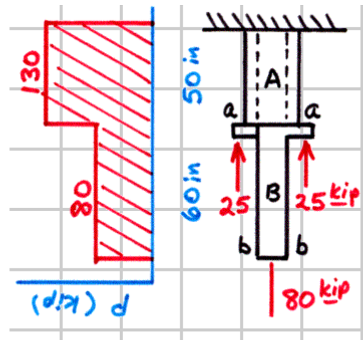
$\varepsilon_{yB} = \dfrac{\sigma}{E} = \dfrac{-98.0496(10^6)}{200(10^9)} = -490.2478(10^{-6}) \cong -490\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_{xB} = -\nu\varepsilon_{yB} = -(0.31579)(-490.2478) = +154.8\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{xyB} = 0\ \mu\text{m/m}$ .... **Ans.**

5-15

$A_A = \pi\left(4^2 - 2^2\right)/4 = 9.42478 \text{ in.}^2$ $\qquad A_B = \pi\left(2\right)^2/4 = 3.14159 \text{ in.}^2$



(a) $\delta_{a-a} = \dfrac{PL}{AE} = \dfrac{(130)(50)}{(9.42478)(15,000)}$

$\delta_{a-a} = 0.045978 \text{ in.} \cong 0.0460 \text{ in.} \uparrow$ .... **Ans.**

(b) $\delta_{b-b} = 0.045978 + \dfrac{(80)(60)}{(3.14159)(30,000)} = 0.0969 \text{ in.} \uparrow$ ..... **Ans.**

(c) $\sigma_{yA} = \dfrac{N}{A} = \dfrac{130}{9.42478} = 13.7934 \text{ ksi} \cong 13.79 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{yB} = \dfrac{80}{3.14159} = 25.4648 \text{ ksi} \cong 25.5 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{xB} = \tau_{xyB} = 0 \text{ ksi}$ .... $\sigma_{xA} = \tau_{xyA} = 0 \text{ ksi}$ .... **Ans.**

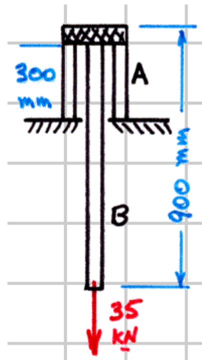
(d) $E = 2(1+\nu)G \qquad 15,000 = 2(1+\nu)(5600) \qquad \nu = 0.33929$

$\delta_d = \varepsilon_x d = \left(-\nu\varepsilon_y\right) d = \dfrac{-\nu\sigma_y}{E} d$

$\delta_{do} = \dfrac{-(0.33929)(-13.7934)}{30,000}(4) = 0.000624 \text{ in.}$ .... **Ans.**

$\delta_{di} = \dfrac{-(0.33929)(-13.7934)}{30,000}(2) = 0.000312 \text{ in.}$ .... **Ans.**

5-16*



$$\delta = \sum \frac{PL}{AE} = \frac{\left(35\times10^3\right)(900)}{\left[\pi(0.025)^2/4\right]\left(200\times10^9\right)} + \frac{\left(35\times10^3\right)(300)}{A\left(73\times10^9\right)} \le 0.40 \text{ mm}$$

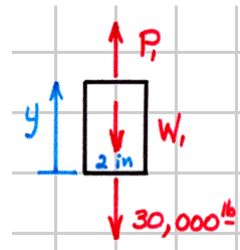
$$A \ge 1.81740\left(10^{-3}\right)\text{ m}^2 = 1817.40 \text{ mm}^2$$

$$\frac{\pi\left(75^2 - d_i^2\right)}{4} \ge 1817.40 \text{ mm}^2 \qquad d_i \le 57.5414 \text{ mm}$$

$t_{min} = (75 - 57.5414)/2 = 8.73 \text{ mm}$ .... **Ans.**

5-17*

For the uniform section:



$$W_1 = \gamma V = 0.284(2\times 5)y = 0.284y$$

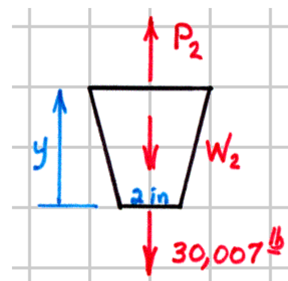
$$P_1 = 30{,}000 + 0.284y$$

$$\delta_U = \int_0^{25} \frac{(30{,}000 + 0.284y)\,dy}{(2\times 5)(29\times 10^6)}$$

$$= (1.03448\times 10^{-3})(25) + (9.79310\times 10^{-9})\left(\frac{25^2}{2}\right)$$

$$= 0.02587 \text{ in.}$$

For the tapered section:



$$W_2 = \gamma V = 0.284(y + 0.0125y^2)$$

$$P_2 = 30{,}007 + 0.284(y + 0.0125y^2)$$

$$A_y = b_y t = \left(2 + \frac{y}{20}\right)(0.5) = 1 + 0.025y$$

$$d\delta = \frac{P_y dy}{A_y E} = \frac{30{,}007 + 0.284y + 0.00355y^2}{(1+0.025y)(29\times 10^6)}dy$$

$$\delta_T = \int_0^{60} \frac{30{,}007 + 0.284y + 0.00355y^2}{(1+0.025y)(29\times 10^6)}dy$$

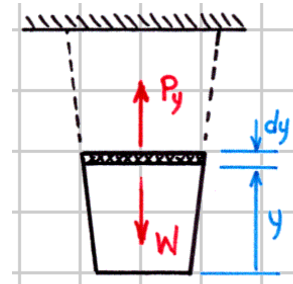
$$= 0.04139\ln(1+0.025y)\Big|_0^{60} + \frac{0.284(1600)}{(29\times 10^6)}\Big[(1+0.025y) - \ln(1+0.025y)\Big]_0^{60}$$

$$+ \frac{0.00355(64{,}000)}{(29\times 10^6)}\left[\frac{(1+0.025y)^2}{2} - 2(1+0.025y) + \ln(1+0.025y)\right]_0^{60}$$

$$= 0.03793 \text{ in.}$$

$\delta_{total} = \delta_U + \delta_T = 0.0638$ in. .... **Ans.**

5-18



$$P_y = \frac{\gamma}{3}\left[\frac{\pi r^2}{L^2}(L+y)^3 - \pi r^2 L\right]$$

$$A_y = \pi\left[\frac{r}{L}(L+y)\right]^2 = \frac{\pi r^2}{L^2}(L+y)^2$$

$$d\delta = \frac{P_y dy}{A_y E} = \frac{\gamma}{3E}\left[(L+y) - \frac{L^3}{(L+y)^2}\right]dy$$

$$\delta = \frac{\gamma}{3E}\int_0^L \left[(L+y) - \frac{L^3}{(L+y)^2}\right]dy = \frac{\gamma}{3E}\left[\frac{(L+y)^2}{2} + \frac{L^3}{(L+y)}\right]_0^L = \frac{\gamma L^2}{3E}$$ .... **Ans.**

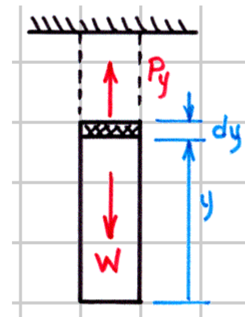
5-19*

$P_y = W = \gamma A y$ $\qquad \sigma = P/A = \gamma y = K\varepsilon^{1/2}$

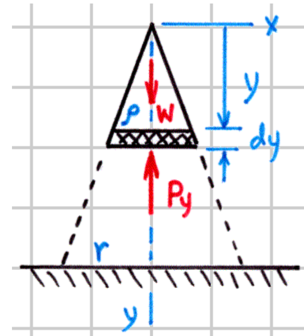
$\varepsilon = (\gamma y / K)^2$

$$d\delta = \varepsilon dy = \left(\frac{\gamma y}{K}\right)^2 dy$$

$$\delta = \int_0^L \left(\frac{\gamma y}{K}\right)^2 dy = \frac{\gamma^2}{K^2}\left[\frac{y^3}{3}\right]_0^L = \frac{\gamma^2 L^3}{3K^2}$$ ....**Ans.**



5-21

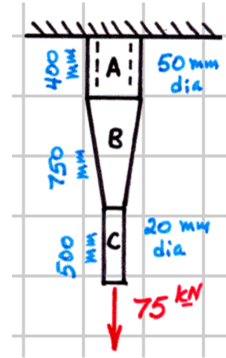


$$P_y = W = \gamma V = \gamma \frac{\pi \rho^2 y}{3} \qquad A_y = \pi \rho^2$$

$$d\delta = \frac{P_y dy}{A_y E} = \frac{\gamma \pi \rho^2 y/3}{\pi \rho^2 E} dy = \frac{\gamma y}{3E} dy$$

$$\delta = \int_0^L \left(\frac{\gamma y}{3E} \right) dy = \frac{\gamma y^2}{6E} \bigg|_0^L = \frac{\gamma L^2}{6E}$$ ....**Ans.**

5-22*



$$A_A = \pi\left(50^2 - 30^2\right)/4 = 1256.6\text{ mm}^2$$

$$A_C = \pi\left(20^2\right)/4 = 314.16\text{ mm}^2$$

$$\delta_A = \frac{PL}{AE} = \frac{\left(75\times10^3\right)(400)}{\left(1256.6\times10^{-6}\right)\left(70\times10^9\right)} = 0.3411\text{ mm}$$

$$\delta_C = \frac{PL}{AE} = \frac{\left(75\times10^3\right)(500)}{\left(314.16\times10^{-6}\right)\left(70\times10^9\right)} = 1.7052\text{ mm}$$

For the tapered section: $\rho = 0.01(1+2y)$

$$A = \pi\rho^2 = \pi(0.01)^2(1+2y)^2$$

$$d\delta = \frac{Pdy}{A_yE} = \frac{\left(75\times10^3\right)dy}{\pi(0.01)^2(1+2y)^2\left(70\times10^9\right)} = \frac{3.4105\left(10^{-3}\right)dy}{(1+2y)^2}$$

$$\delta = 3.4105\left(10^{-3}\right)\int_0^{0.75}\left(\frac{dy}{(1+2y)^2}\right) = 3.4105\left(10^{-3}\right)\frac{-1}{2(1+2y)}\Bigg|_0^{0.75}$$

$$= 1.0231\left(10^{-3}\right)\text{ m} = 1.0231\text{ mm}$$

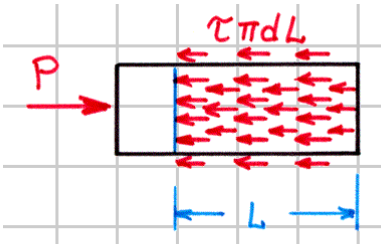
$$\delta_{total} = \delta_A + \delta_B + \delta_C = 0.3411 + 1.0231 + 1.7052$$

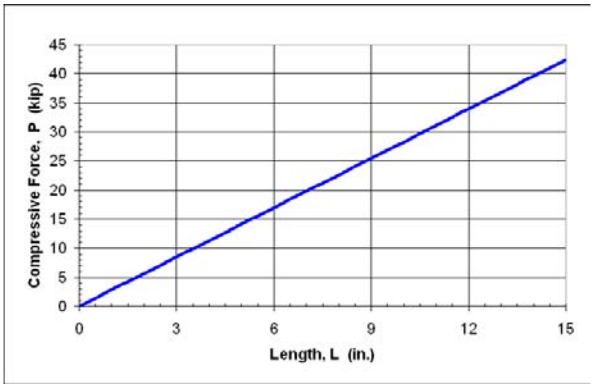
δ_{total} 3.0694 mm ≅ 3.07 mm .... **Ans.**

5-23

(a) $P = \tau \pi d L = \pi(300)(3)L = 900\pi L$ lb

$0 \text{ in.} \le L \le 15 \text{ in.}$

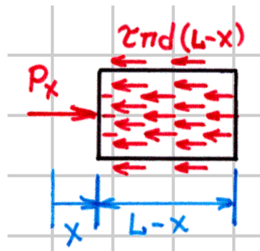




(b) $P_x = \tau \pi d(L - x) = 900\pi(15 - x)$ lb

$$\sigma_x = \frac{P_x}{A} = \frac{900\pi(15-x)}{\pi(3)^2/4} = 400(15-x) \text{ psi}$$

$0 \text{ in.} \le x \le 15 \text{ in.}$

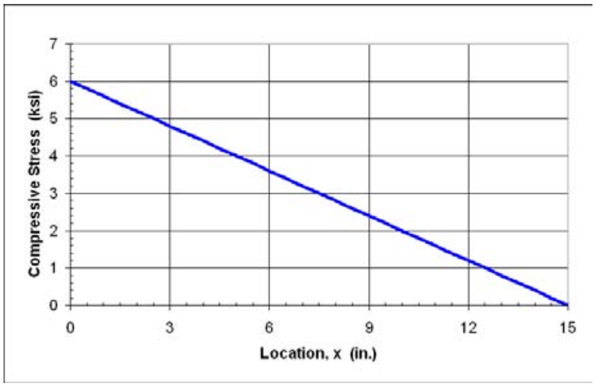


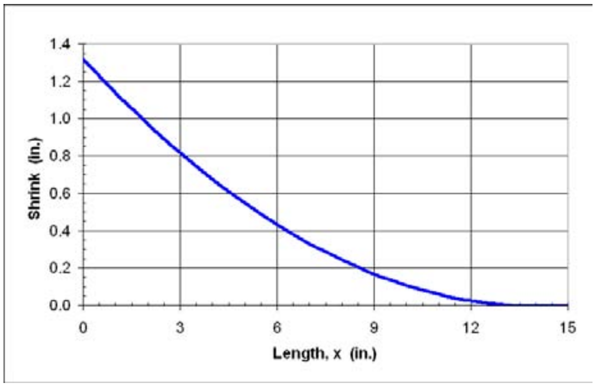
(c) $$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y - \nu\sigma_z}{E} = \frac{(400)(15-x) - 2(0.30)(600)}{30(10^6)}$$

(Both σ_x and σ_y are compressive, therefore ε_x will be a shrink.)

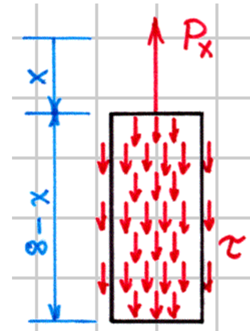
$$\delta = \frac{1}{E}\int_x^{15} \varepsilon_x \, ds = \frac{1}{E}\int_x^{15} (5640 - 400s)\, ds = \frac{5640(15-x) - 200(225-x^2)}{30(10^6)} \text{ in.}$$

$0 \text{ in.} \le x \le 15 \text{ in.}$





5-24



(a) $$P_x = \int_x^8 \tau\pi(0.300)\,ds$$

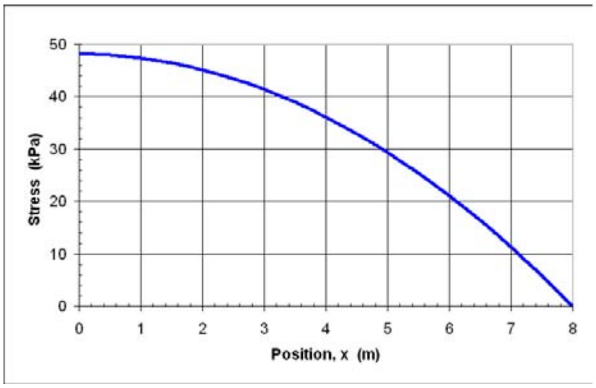
$$\sigma_x = \frac{P_x}{A} = \frac{1}{\pi(0.300)^2/4}\int_x^8 \pi(400s)(1-\sin 28°)(\tan 28°)(0.300)\,ds$$

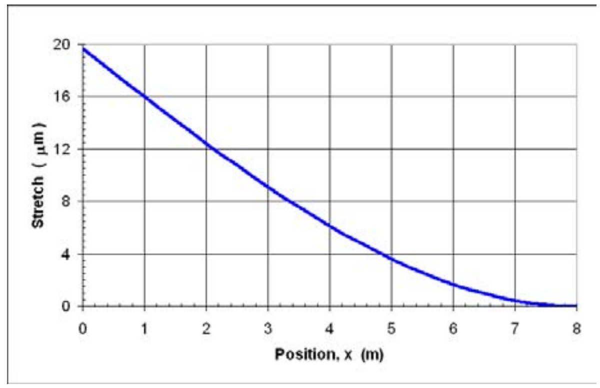
$$\sigma_x = 1504.46\left(\frac{8^2}{2}-\frac{x^2}{2}\right) = 752.23\left(64-x^2\right)\ \text{N/m}^2$$

$$0\ \text{m} \le x \le 8\ \text{m}$$

(b) $$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y - \nu\sigma_z}{E} = \frac{(752.23)\left(64-x^2\right)-2(0.30)(400)(1-\sin 28°)}{13\left(10^9\right)}$$

$$\varepsilon_x = 3.6935\left(10^{-6}\right)-5.7864\left(10^{-8}\right)x^2$$





$$\delta = \int_x^8 \varepsilon_x\,ds = \int_x^8 \left[3.6935\left(10^{-6}\right)-5.7864\left(10^{-8}\right)s^2\right]ds$$

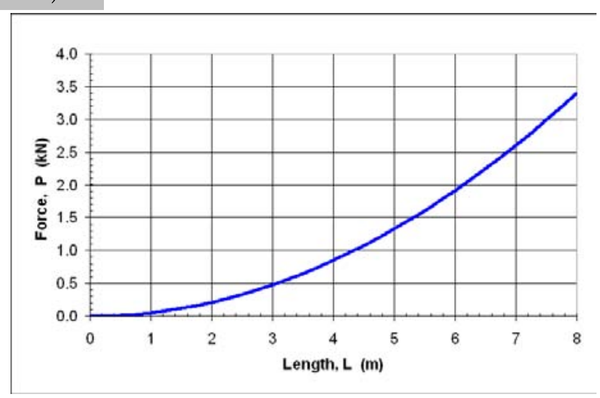
$$\delta = 3.6935\left(10^{-6}\right)(8-x)-1.9288\left(10^{-8}\right)\left(8^3-x^3\right)\ \text{m} \qquad 0\ \text{m} \le x \le 8\ \text{m}$$

(c) $$P = \sigma_{x=0}\,\pi(0.300)^2/4$$

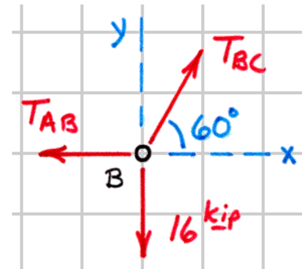
$$= 752.23L^2\,\pi(0.300)^2/4$$

$$P = 53.1721L^2\ \text{N}$$

$$0\ \text{m} \le L \le 8\ \text{m}$$



5-25*



$\rightarrow \Sigma F_x = 0$: $\quad T_{BC}\cos 60° - T_{AB} = 0$

$\uparrow \Sigma F_y = 0$: $\quad T_{BC}\sin 60° - 16 = 0$

$T_{AB} = 9.2376$ kip (T)

$T_{BC} = 18.4752$ kip (T)

(a) $\quad \delta_{AB} = \dfrac{PL}{AE} = \dfrac{(9.2376)(80)}{(0.6)(10,600)} = 0.11620$ in.

$\delta_{AB} \cong 0.1162$ in. (stretch) .... **Ans.**

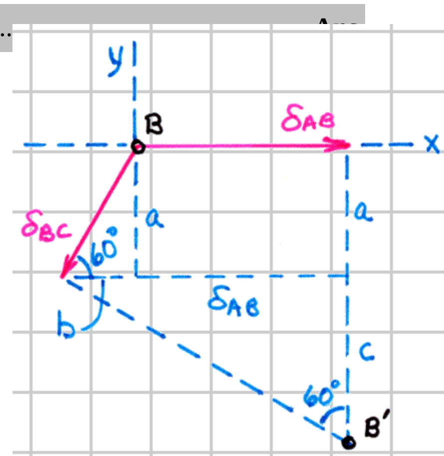
$\delta_{BC} = \dfrac{(18.4752)(160)}{(1.25)(29,000)} = 0.08155$ in.

$\delta_{BC} \cong 0.0815$ in. (stretch) .... **Ans.**

(b) $\quad \sin 60° = a/\delta_{BC} \qquad a = 0.07062$ in.

$\cos 60° = b/\delta_{BC} \qquad b = 0.04077$ in.

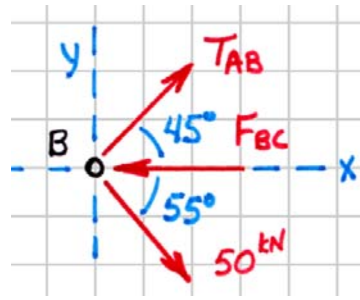
$\tan 60° = \dfrac{b + \delta_{AB}}{c} \qquad c = 0.09063$ in.



$u_B = \delta_{AB} = 0.1162$ in. $\rightarrow$ .... **Ans.**

$v_B = a + c = 0.1613$ in. $\downarrow$ .... **Ans.**

5-26



$\rightarrow \Sigma F_x = 0: \qquad T_{AB}\cos 45° - F_{BC} + 50\cos 55° = 0$

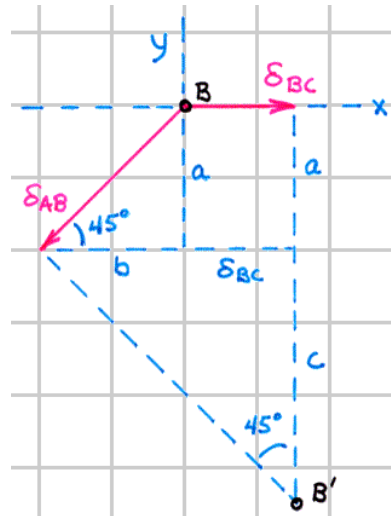
$\uparrow \Sigma F_y = 0: \qquad T_{AB}\sin 45° - 50\sin 55° = 0$

$T_{AB} = 57.9228 \text{ kN (T)} \qquad F_{BC} = 69.6364 \text{ kN (C)}$

(a) $L_{AB} = \sqrt{1^2 + 1^2} = 1.41421 \text{ m}$

$$\delta_{AB} = \frac{PL}{AE} = \frac{(57.9228\times 10^3)(1414.21)}{(450\times 10^{-6})(96\times 10^9)} = 1.89619 \text{ mm}$$

$\delta_{AB} \cong 1.896 \text{ mm (stretch)}$ .... **Ans.**



$$\delta_{BC} = \frac{(69.6364\times 10^3)(1000)}{(1450\times 10^{-6})(180\times 10^9)} = 0.26681 \text{ mm}$$

$\delta_{BC} \cong 0.267 \text{ mm (shrink)}$ .... **Ans.**

(b) $\sin 45° = a/\delta_{AB} \qquad a = 1.34081 \text{ mm} = b$

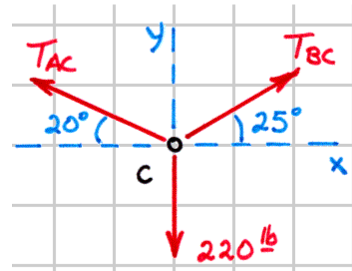
$\cos 45° = b/\delta_{AB} \qquad b = 1.34081 \text{ mm}$

$\tan 45° = \dfrac{b + \delta_{BC}}{c} \qquad c = 1.60761 \text{ mm}$

$u_B = \delta_{BC} = 0.267 \text{ mm} \rightarrow$ .... **Ans.**

$v_B = a + c = 2.95 \text{ mm} \downarrow$ .... **Ans.**

5-27



$\rightarrow \Sigma F_x = 0: \qquad T_{BC}\cos 25° - T_{AC}\cos 20° = 0$

$\uparrow \Sigma F_y = 0: \qquad T_{BC}\sin 25° + T_{AC}\sin 20° - 220 = 0$

$T_{AC} = 281.9768 \text{ lb (T)} \qquad T_{BC} = 292.3648 \text{ lb (T)}$

(a) $\sigma_{AC} = \dfrac{P}{A} = \dfrac{(281.9768)}{(0.015)} = 18,798 \text{ psi} \cong 18.80 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{BC} = \dfrac{(292.3648)}{(0.015)} = 19,491 \text{ psi} \cong 19.49 \text{ ksi (T)}$ .... **Ans.**

(b) $L_{AC} = 20/\cos 20° = 21.28356 \text{ ft} = 255.4027 \text{ in.}$

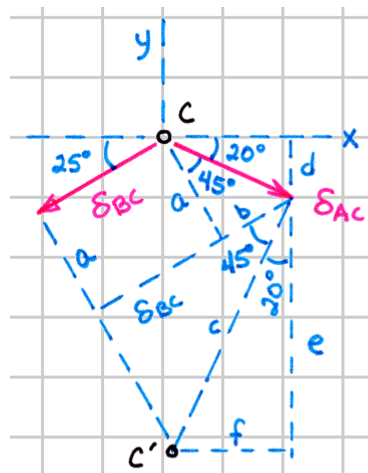
$L_{BC} = 20/\cos 25° = 22.06956 \text{ ft} = 264.8107 \text{ in.}$

$\delta_{AC} = \dfrac{PL}{AE} = \dfrac{(281.9768)(255.4027)}{(0.015)(29,000\times 10^3)} = 0.16556 \text{ in.}$

$\delta_{AC} \cong 0.1656 \text{ in. (stretch)}$ .... **Ans.**

$\delta_{BC} = \dfrac{(292.3648)(264.8107)}{(0.015)(29,000\times 10^3)} = 0.17798 \text{ in.}$

$\delta_{BC} \cong 0.1780 \text{ in. (stretch)}$ .... **Ans.**



(c) $a = \delta_{AC}\cos 45° = 0.11707 \text{ in.}$

$b = \delta_{AC}\sin 45° = 0.11707 \text{ in.}$

$d = \delta_{AC}\sin 20° = 0.05662 \text{ in.}$

$\cos 45° = \dfrac{b + \delta_{BC}}{c} \qquad c = 0.41726 \text{ in.}$

$e = c\cos 20° = 0.39210 \text{ in.}$

$v_C = d + e = 0.449 \text{ in.} \downarrow$ .... **Ans.**

5-28*

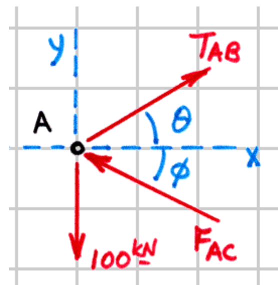
$$\theta = \tan^{-1}(2.5/6) = 22.6199° \qquad L_{AB} = \sqrt{2.5^2 + 6^2} = 6.5000 \text{ m}$$

$$\phi = \tan^{-1}(4.5/6) = 36.8699° \qquad L_{AC} = \sqrt{4.5^2 + 6^2} = 7.5000 \text{ m}$$

$\rightarrow \Sigma F_x = 0:$ $\quad T_{AB}\cos\theta - F_{AC}\cos\phi = 0$

$\uparrow \Sigma F_y = 0:$ $\quad T_{AB}\sin\theta + F_{AC}\sin\phi - 100 = 0$

$T_{AB} = 92.8571$ kN (T) $\qquad F_{AC} = 107.1432$ kN (C)



(a) $$\sigma_{AB} = \frac{P}{A} = \frac{(92.8571\times10^3)}{(620\times10^{-6})} = 149.8(10^6) \text{ N/m}^2$$

$\sigma_{AB} = 149.8$ MPa (T) .... **Ans.**

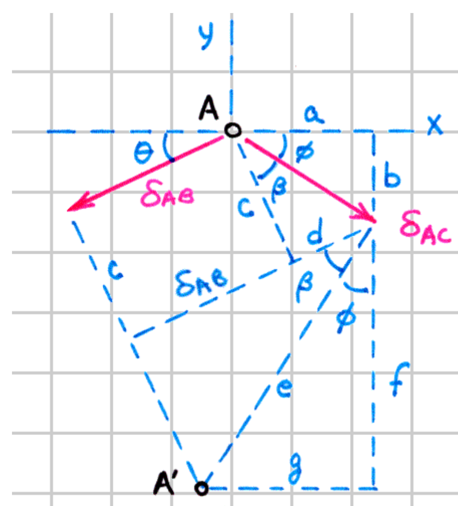
$$\sigma_{AC} = \frac{(107.1432\times10^3)}{(1000\times10^{-6})} = 107.1(10^6) \text{ N/m}^2 = 107.1 \text{ MPa (C)}$$ .... **Ans.**

(b) $$\delta_{AB} = \frac{PL}{AE} = \frac{(92.8571\times10^3)(6500)}{(620\times10^{-6})(200\times10^9)} = 4.86751 \text{ mm}$$

$\delta_{AB} \cong 4.87$ mm (stretch) .... **Ans.**

$$\delta_{AC} = \frac{(107.1432\times10^3)(7500)}{(1000\times10^{-6})(200\times10^9)} = 4.01787 \text{ mm}$$

$\delta_{AC} \cong 4.02$ mm (shrink) .... **Ans.**



(c) $$\beta = 180° - \theta - \phi - 90° = 30.5102°$$

$$a = \delta_{AC}\cos\phi = 3.21430 \text{ mm}$$

$$b = \delta_{AC}\sin\phi = 2.41072 \text{ mm}$$

$$d = \delta_{AC}\sin\beta = 2.03984 \text{ mm}$$

$$e = \frac{\delta_{AB} + d}{\cos\beta} = 8.01746 \text{ mm}$$

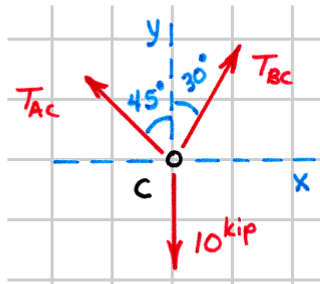
$$f = e\cos\phi = 6.41397 \text{ mm}$$

$$g = e\sin\phi = 4.81048 \text{ mm}$$

$u_A = a - g = -1.596 \text{ mm} = 1.596 \text{ mm} \leftarrow$ .... **Ans.**

$v_A = b + f = 8.82 \text{ mm} \downarrow$ .... **Ans.**

5-29



$\rightarrow \Sigma F_x = 0: \qquad T_{BC} \sin 30° - T_{AC} \sin 45° = 0$

$\uparrow \Sigma F_y = 0: \qquad T_{BC} \cos 30° + T_{AC} \cos 45° - 10 = 0$

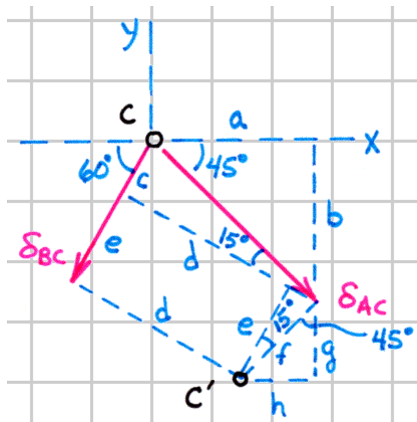
$T_{AC} = 5.17638 \text{ kip (T)}$

$T_{BC} = 7.32051 \text{ kip (T)}$

(a) $\delta_{AC} = \dfrac{PL}{AE} = \dfrac{(5.17638)(10\times 12)}{(0.326)(10,600)} = 0.17976 \text{ in.}$

$\delta_{AC} \cong 0.1798 \text{ in. (stretch)}$ .... **Ans.**

$\delta_{BC} = \dfrac{(7.32051)(15\times 12)}{(0.508)(29,000)} = 0.08944 \text{ in.}$



$\delta_{BC} \cong 0.0894 \text{ in. (stretch)}$ .... **Ans.**

(b) $a = \delta_{AC} \cos 45° = 0.12711 \text{ in.}$

$b = \delta_{AC} \sin 45° = 0.12711 \text{ in.}$

$c = \delta_{AC} \sin 15° = 0.04653 \text{ in.}$

$d = \delta_{AC} \cos 15° = 0.17363 \text{ in.}$

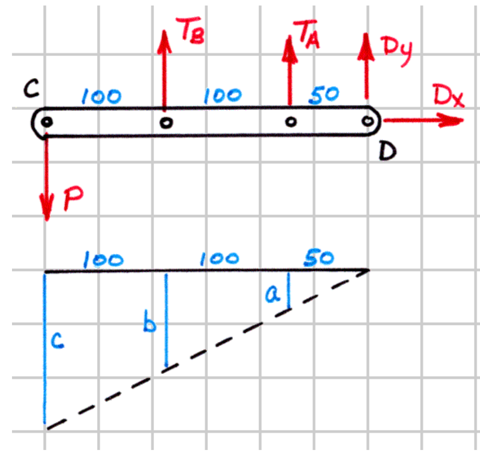
$e = \delta_{BC} - c = 0.04291 \text{ in.}$

$f = e/\cos 15° = 0.04443 \text{ in.}$

$g = f \cos 45° = 0.03142 \text{ in.}$ $\qquad u_C = a - h = 0.0957 \text{ in.} \rightarrow$ .... **Ans.**

$h = f \sin 45° = 0.03142 \text{ in.}$ $\qquad v_C = b + g = 0.1585 \text{ in.} \downarrow$ .... **Ans.**

5-30



(a) $\varepsilon_B = \dfrac{\delta}{L} = \dfrac{\delta_B}{375} = 1500\left(10^{-6}\right)$

$\delta_B = b = 0.56250$ mm

$v_C = c = (250/150)b = 0.937$ mm $\downarrow$ .... **Ans.**

(b) $\delta_A = a = (50/150)b = 0.18750$ mm

$$\delta_A = \frac{T_A(200)}{\left(1250\times10^{-6}\right)\left(200\times10^{9}\right)} = 0.18750 \text{ mm}$$

$T_A = 234{,}375$ N

$$\delta_B = \frac{T_B(375)}{\left(940\times10^{-6}\right)\left(100\times10^{9}\right)} = 0.56250 \text{ mm}$$

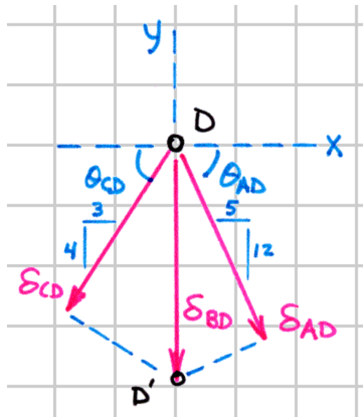
$T_B = 141{,}000$ N

$\circlearrowleft \Sigma M_D = 0: \qquad 250P - 150(141.000) - 50(234.375) = 0$

$P = 131.5$ kN .... **Ans.**

5-31*

$\delta_{CD} = \varepsilon_{CD} L_{CD} = (680\times10^{-6})(72) = 0.048960 \text{ in.} \cong 0.0490 \text{ in.}$ .... **Ans.**



$$\delta_{CD} = \frac{PL}{AE} = \frac{T_{CD}(6\times12)}{(2.5)(29,000)} = 0.048960 \text{ in.}$$

$$T_{CD} = 49.300 \text{ kip (T)}$$

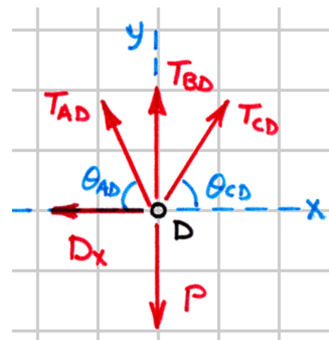
$\sin\theta_{CD} = 4/5$ $\quad\cos\theta_{CD} = 3/5$

$\sin\theta_{AD} = 12/13$ $\quad\cos\theta_{AD} = 5/13$

$\delta_{CD} = \delta_{BD}\sin\theta_{BD}$ $\quad\delta_{AD} = \delta_{BD}\sin\theta_{AD}$

$\delta_{BD} = 0.061200 \text{ in.} \cong 0.0612 \text{ in.}$ .... **Ans.**

$\delta_{AD} = 0.056492 \text{ in.} \cong 0.0565 \text{ in.}$ .... **Ans.**



$$\delta_{BD} = \frac{T_{BD}(6\times12)}{(2.5)(6500)} = 0.061200 \text{ in.}$$

$$T_{BD} = 13.8125 \text{ kip (T)}$$

$$\delta_{AD} = \frac{T_{AD}(6\times12)}{(2.5)(26,000)} = 0.056492 \text{ in.}$$

$$T_{AD} = 51.0000 \text{ kip (T)}$$

$\uparrow\Sigma F_y = 0: \quad T_{CD}\sin\theta_{CD} + T_{AD}\sin\theta_{AD} + T_{BD} - P = 0$

$P = 100.3 \text{ kip}$ .... **Ans.**

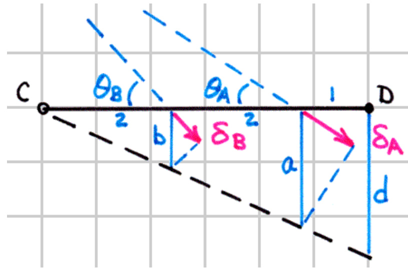
5-32

$$\delta_A = \varepsilon_A L_A = \left(625\times10^{-6}\right)\left(5000\right) = 3.12500 \text{ mm}$$

$$\delta_A = a\sin\theta_A = \left(3/5\right)a$$

$$a = 5.20833 \text{ mm}$$

$$d = \left(5/4\right)a = 6.51 \text{ mm } \downarrow$$ .... **Ans.**

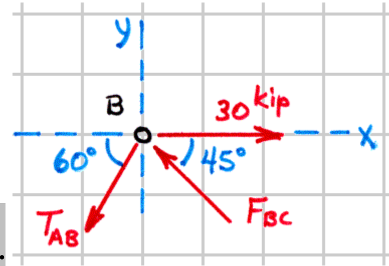


5-33

$\rightarrow \Sigma F_x = 0: \qquad 30 - F_{BC}\cos 45° - T_{AB}\cos 60° = 0$

$\uparrow \Sigma F_y = 0: \qquad F_{BC}\sin 45° - T_{AB}\sin 60° = 0$

$T_{AB} = 21.9615 \text{ kip (T)} \qquad F_{BC} = 26.8973 \text{ kip (C)}$



(a) $\sigma_{\max AB} = \dfrac{P}{A} = \dfrac{(21.9615)}{(1.25)} = 17.57 \text{ ksi (T)}$ .... **Ans.**

$\tau_{\max AB} = \sigma_{\max AB}/2 = 8.78 \text{ ksi}$ .... **Ans.**

(b) $\sigma_{\max BC} = \dfrac{(26.8973)}{(2.50)} = 10.76 \text{ ksi (C)}$ .... **Ans.**

$\tau_{\max BC} = \sigma_{\max BC}/2 = 5.38 \text{ ksi}$ .... **Ans.**

(c) $L_{AB} = 30/\sin 60° = 34.6410 \text{ in.} \qquad L_{BC} = 30/\sin 45° = 42.4264 \text{ in.}$

$$\delta_{AB} = \frac{PL}{AE} = \frac{(21.9615)(34.6410)}{(1.25)(10,600)} = 0.0574166 \text{ in.}$$

$\delta_{AB} \cong 0.0574 \text{ in. (stretch)}$ .... **Ans.**

$$\delta_{BC} = \frac{(26.8973)(42.4264)}{(2.5)(29,000)} = 0.0157401 \text{ in.}$$

$\delta_{BC} \cong 0.01574 \text{ in. (shrink)}$ .... **Ans.**

(d) $a = \delta_{AB}\cos 30° = 0.0497242 \text{ in.}$

$b = \delta_{AB}\sin 30° = 0.0287083 \text{ in.}$

$c = \delta_{AB}\sin 15° = 0.0148605 \text{ in.}$

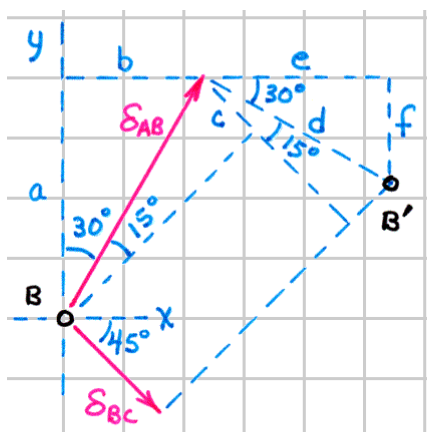
$$d = \frac{c + \delta_{BC}}{\cos 15°} = 0.0316800 \text{ in.}$$

$e = d\cos 30° = 0.0274657 \text{ in.}$

$f = d\sin 30° = 0.0158400 \text{ in.}$

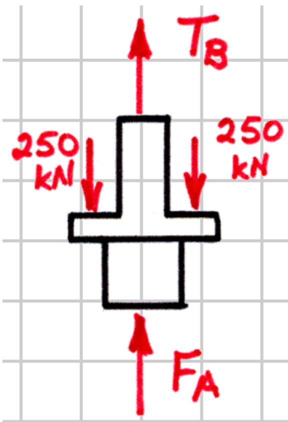
$u_B = b + e = 0.0561 \text{ in.} \rightarrow$ .... **Ans.**

$v_B = a - f = 0.0339 \text{ in.} \uparrow$ .... **Ans.**



5-34*

$$A_A = \frac{\pi\left(0.100^2 - 0.050^2\right)}{4} = 5.89049\left(10^{-3}\right)\text{ m}^2 \qquad A_B = \frac{\pi\left(0.050\right)^2}{4} = 1.96350\left(10^{-3}\right)\text{ m}^2$$



$$\uparrow \Sigma F_y = 0: \qquad F_A + T_B - 500 = 0$$

$$F_A + T_B = 500\text{ kN} \qquad \text{(a)}$$

$$\delta_A = \delta_B$$

$$\frac{F_A(1500)}{\left(5.89049\times10^{-3}\right)\left(100\times10^9\right)} = \frac{T_B(2000)}{\left(1.96350\times10^{-3}\right)\left(200\times10^9\right)}$$

$$F_A = 2.000 T_B \qquad \text{(b)}$$

$$F_A = 333.333\text{ kN (C)} \qquad T_B = 166.6667\text{ kN (T)}$$

(a) $\sigma_A = \frac{P}{A} = \frac{\left(333.333\times10^3\right)}{\left(5.89049\times10^{-3}\right)} = 56.6\left(10^6\right)\text{ N/m}^2 = 56.6\text{ MPa (C)}$ .... **Ans.**

$\sigma_{BC} = \frac{\left(166.6667\times10^3\right)}{\left(1.96350\times10^{-3}\right)} = 84.9\left(10^6\right)\text{ N/m}^2 = 84.9\text{ MPa (T)}$ .... **Ans.**

(b) $v_C = \delta_B = \frac{\left(166.6667\times10^3\right)(2000)}{\left(1.96350\times10^{-3}\right)\left(200\times10^9\right)} = 0.849\text{ mm} \downarrow$ .... **Ans.**

5-35*

$$\uparrow \Sigma F_y = 0: \quad 2F_S + F_W - 700 = 0$$

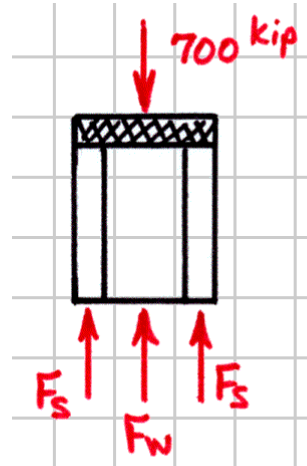
$$2F_S + F_W = 700 \text{ kip} \qquad \text{(a)}$$

$$\delta_S = \delta_W$$

$$\frac{F_S(20)}{(2\times 7.5)(29,000)} = \frac{F_W(20)}{(7.5\times 7.5)(1800)}$$

$$F_S = 4.29630F_W \qquad \text{(b)}$$

$$F_W = 72.9730 \text{ kip (C)} \qquad F_S = 313.5138 \text{ kip (C)}$$

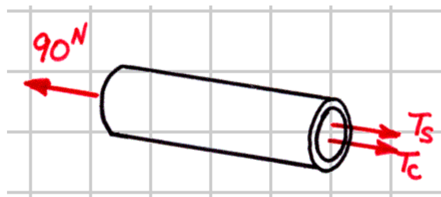


(a) $\sigma_W = \dfrac{P}{A} = \dfrac{(72.9730)}{(7.5\times 7.5)} = 1.297 \text{ ksi (C)}$ .... **Ans.**

$\sigma_S = \dfrac{(313.5138)}{(2\times 7.5)} = 20.9 \text{ ksi (C)}$ .... **Ans.**

(b) $\delta_S = \dfrac{(313.5138)(20)}{(2\times 7.5)(29,000)} = 0.01441 \text{ in.}$ .... **Ans.**

5-36



$$T_S + T_C = 90 \text{ N} \qquad \text{(a)}$$

$$\delta_S = \delta_C$$

$$\frac{T_S L}{\left[\pi\left(2^2 - 1.5^2\right)/4\right]\left(14\times10^9\right)} = \frac{T_C L}{\left[\pi(1.5)^2/4\right]\left(7\times10^9\right)}$$

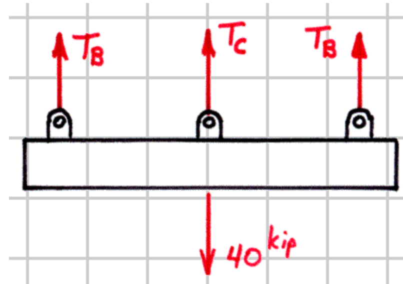
$$T_S = 1.55556 T_C \qquad \text{(b)}$$

$T_C = 35.2$ N (T) .... **Ans.**

$T_S = 54.8$ N (T) .... **Ans.**

5-37*

$$2T_B + T_C = 40 \text{ kip} \qquad \text{(a)}$$



$$\delta_B = \delta_C$$

$$\frac{T_B(36)}{(2)(10{,}600)} = \frac{T_C(36)}{(2)(28{,}000)}$$

$$T_C = 1.32075 T_B \qquad \text{(b)}$$

$$T_B = 12.0455 \text{ kip (T)}$$

$$T_C = 15.9090 \text{ kip (T)}$$

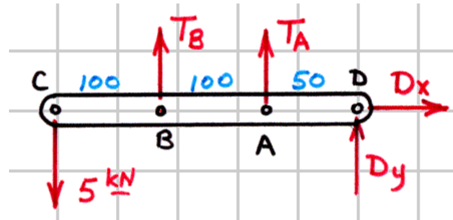
$$\sigma_B = \frac{P}{A} = \frac{(12.0455)}{(2)} = 6.02 \text{ ksi (T)}$$ .... **Ans.**

$$\sigma_C = \frac{(15.9090)}{(2)} = 7.95 \text{ ksi (T)}$$ .... **Ans.**

5-38

$$\circlearrowleft \Sigma M_D = 0: \qquad 250(5) - 50T_A - 150T_B = 0$$

$$T_A + 3T_B = 25 \text{ kN} \qquad \text{(a)}$$

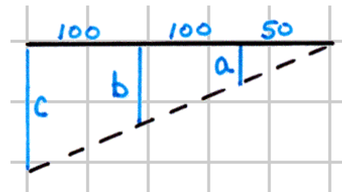


$$b = 3a$$

$$\delta_B = 3\delta_A$$

$$\frac{T_B(400)}{(80\times10^{-6})(200\times10^{9})} = 3\frac{T_A(200)}{(80\times10^{-6})(200\times10^{9})}$$

$$T_B = 1.5T_A \qquad \text{(b)}$$

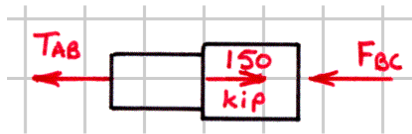


(a) $T_A = 4.5455 \text{ kN} \cong 4.55 \text{ kN (T)}$ .... **Ans.**

$T_B = 6.82 \text{ kN (T)}$ .... **Ans.**

(b) $v_C = 5a = 5\delta_A = 5\left[\dfrac{(4.5455\times10^{3})(200)}{(80\times10^{-6})(200\times10^{9})}\right] = 1.136 \text{ mm} \downarrow$ .... **Ans.**

5-39



$$T_{AB} + F_{BC} = 150 \text{ kip} \qquad \text{(a)}$$

$$\delta_{AB} = \delta_{BC}$$

$$\frac{T_{AB}(24)}{\left[\pi(2.25)^2/4\right](10{,}600)} = \frac{F_{BC}(24)}{\left[\pi(3.2)^2/4\right](10{,}600)}$$

$$F_{BC} = 2.02272 T_{AB} \qquad \text{(b)}$$

$T_{AB} = 49.6242$ kip (T) $\qquad F_{BC} = 100.3758$ kip (C)

(a) $u_B = \delta_{AB} = \dfrac{(49.6242)(24)}{\left[\pi(2.25)^2/4\right](10{,}600)} = 0.02826 \text{ in.} \cong 0.0283 \text{ in.} \rightarrow$ .... **Ans.**

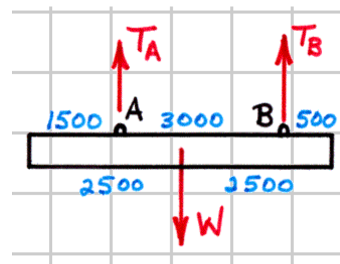
(b) $\varepsilon_{AB} = \dfrac{\delta_{AB}}{L_{AB}} = \dfrac{0.02826}{24} = 1177.425\left(10^{-6}\right) \cong 1177\ \mu\text{in./in.}$ .... **Ans.**

(c) $\varepsilon_d = -\nu\varepsilon_{AB} = -(0.33)\left(1177.425\times10^{-6}\right) = -388.550\left(10^{-6}\right)$ in./in.

$\delta_d = \varepsilon_d d = \left(-388.550\times10^{-6}\right)(2.25) = -0.874\left(10^{-3}\right)$ in. .... **Ans.**

5-40*

$$W = 4500(9.81) = 44{,}145 \text{ N} = 44.145 \text{ kN}$$



$\uparrow \Sigma F_y = 0:$ $\quad T_A + T_B - 44{,}145 = 0$

$\circlearrowleft \Sigma M_A = 0:$ $\quad 3000 T_B - 1000(44.145) = 0$

$T_A = 29.430 \text{ kN}$ $\quad T_B = 14.715 \text{ kN}$

$$\delta_A = \delta_B$$

$$\frac{(29.430\times 10^3)(1200)}{\left[\pi d_A^2/4\right](70\times 10^9)} = \frac{(14.715\times 10^3)(1800)}{\left[\pi d_B^2/4\right](200\times 10^9)}$$

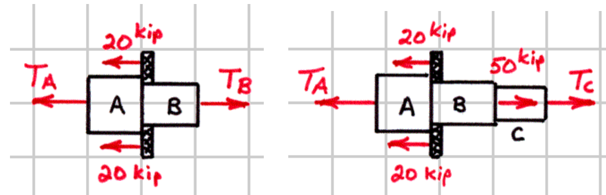
$d_A/d_B = 1.952$ .... **Ans.**

5-41*

$$T_B = T_A + 40 \text{ kip} \qquad \text{(a)}$$

$$T_C = T_A - 10 \text{ kip} \qquad \text{(b)}$$

$$\delta_A + \delta_B + \delta_C = 0 \qquad \text{(c)}$$



$$\frac{T_A(20)}{\left[\pi\left(2.5^2 - 2^2\right)/4\right](30{,}000)} + \frac{(T_A + 40)(24)}{\left[\pi(2)^2/4\right](10{,}000)} + \frac{(T_A - 10)(24)}{\left[\pi(1)^2/4\right](10{,}000)} = 0$$

$T_A = 0$ kip $\qquad T_B = 40$ kip (T) $\qquad T_C = -10$ kip $= -10$ kip (C)

(a) $\sigma_A = \dfrac{P}{A} = 0$ ksi .... **Ans.**

$\sigma_B = \dfrac{(40)}{\pi(2)^2/4} = 12.73$ ksi (T) .... **Ans.**

$\sigma_C = \dfrac{(-10)}{\pi(1)^2/4} = -12.73$ ksi $= 12.73$ ksi (C) .... **Ans.**

(b) $u_{a-a} = \delta_C = \dfrac{(-10)(24)}{\left[\pi(1)^2/4\right](10{,}000)} = -0.0306$ in. $= 0.0306$ in. $\rightarrow$ .... **Ans.**

5-42*

$$F_s + F_a = 30 \text{ kN} \qquad \text{(a)}$$

$$\delta_s = \delta_a$$

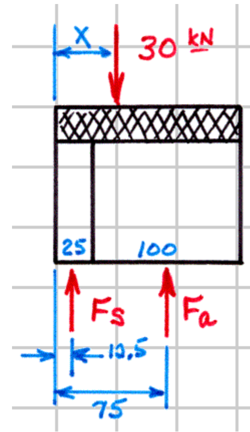
$$\frac{F_s L}{(0.025 \times 0.100)(200 \times 10^9)} = \frac{F_a L}{(0.100 \times 0.100)(73 \times 10^9)}$$

$$F_a = 1.4600 F_s \qquad \text{(b)}$$

$$F_s = 12.1951 \text{ kN (C)}$$

$$F_a = 17.8049 \text{ kN (C)}$$

$$x = \frac{12.1951(12.5) + 17.8049(75)}{30} = 49.6 \text{ mm}$$ .... **Ans.**



5-43

$$T_A + F_B = 95 \text{ kip} \qquad \text{(a)}$$

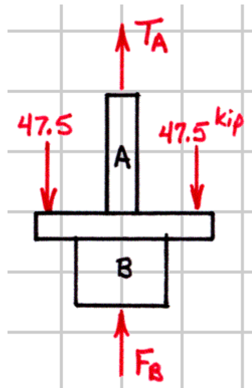
$$\delta_A = \delta_B + 0.015 \text{ in.}$$

$$\frac{T_A(72)}{(1.25)(30{,}000)} = \frac{F_B(24)}{(3.75)(15{,}000)} + 0.015$$

$$T_A = 0.22222 F_B + 7.8125 \text{ kip} \qquad \text{(b)}$$

$$T_A = 23.6648 \text{ kip (T)}$$

$$F_B = 71.3352 \text{ kip (C)}$$

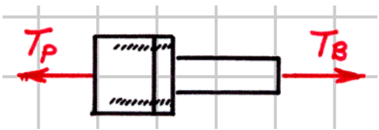


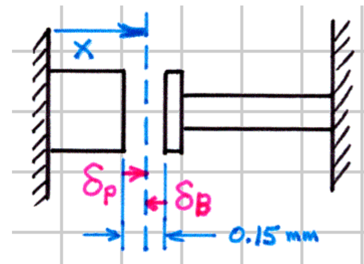
(a) $\sigma_A = \frac{N}{A} = \frac{23.6648}{1.25} = 18.93 \text{ ksi (T)}$ .... **Ans.**

$\sigma_B = \frac{N}{A} = \frac{71.3352}{3.75} = 19.02 \text{ ksi (C)}$ .... **Ans.**

(b) $v_C = \delta_A = \frac{(23.6648)(72)}{(1.25)(30{,}000)} = 0.0454 \text{ in. } \downarrow$ .... **Ans.**

5-44*





$$T_P = T_B \quad \text{(a)}$$

$$\delta_P + \delta_B = 0.15 \text{ mm} \quad \text{(b)}$$

$$\frac{T_P(200)}{\left[\pi(0.15)^2/4\right](2.1\times10^9)} + \frac{T_B(400)}{\left[\pi(0.045)^2/4\right](100\times10^9)} = 0.15$$

$$T_P = T_B = 18{,}976.74 \text{ N}$$

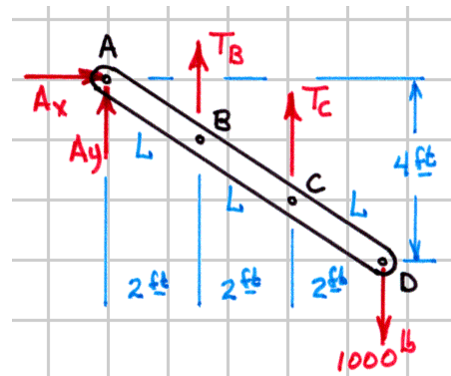
(a) $$\sigma_P = \frac{N}{A} = \frac{18{,}976.74}{\pi(0.15)^2/4} = 1.074(10^6) \text{ N/m}^2$$

$\sigma_P = 1.074$ MPa (T) .... **Ans.**

$\sigma_B = \dfrac{N}{A} = \dfrac{18{,}976.74}{\pi(0.045)^2/4} = 11.93(10^6) \text{ N/m}^2 = 11.93$ MPa (T) .... **Ans.**

(b) $x = 200 + \delta_P = 200 + \dfrac{(18{,}976.74)(200)}{\left[\pi(0.15)^2/4\right](2.1\times10^9)} = 200.1023$ mm .... **Ans.**

5-45*



$$\circlearrowleft \Sigma M_A = 0: \qquad 2T_B + 4T_C - 6(1000) = 0$$

$$T_B + 2T_C = 3000 = 0 \qquad \text{(a)}$$

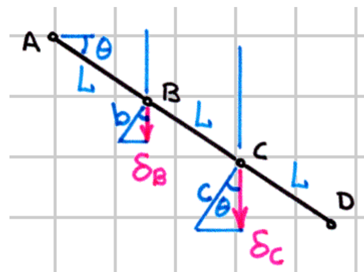
$$\delta_B = b\cos\theta = (L\Delta\theta)\cos\theta \qquad L_B = (1/3)(48) = 16 \text{ in.}$$

$$\delta_C = c\cos\theta = (2L\Delta\theta)\cos\theta \qquad L_C = (2/3)(48) = 32 \text{ in.}$$

$$\delta_C = 2\delta_B \qquad \text{(b)}$$

$$\frac{T_C(32)}{(0.3)(29,000)} = 2\frac{T_B(16)}{(0.3)(29,000)}$$

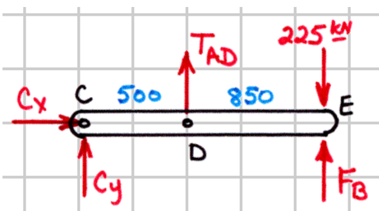
$$T_B = T_C = 1.000 \text{ kip (T)}$$



$$\delta_B = \frac{(1.000)\left[1(48)/3\right]}{(0.3)(29,000)} = 0.001839 \text{ in.}$$ .... **Ans.**

$$\delta_C = 2\delta_B = 0.00368 \text{ in.}$$ .... **Ans.**

5-46

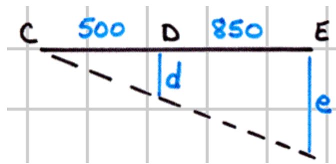


$\rightarrow \Sigma F_x = 0:$ $\quad C_x = 0$

$\uparrow \Sigma F_y = 0:$ $\quad C_y + T_{AD} + F_B - 225 = 0$

$\circlearrowleft \Sigma M_C = 0:$ $\quad 500T_{AD} + 1350F_B - 1350(225) = 0$

$$10T_{AD} + 27F_B = 6075 \text{ kN} \qquad \text{(a)}$$



$$\delta_B = (1350/500)\,\delta_{AD}$$

$$\frac{F_B(375)}{(4500\times10^{-6})(12\times10^{9})} = \left(\frac{1350}{500}\right)\left[\frac{T_{AD}(450)}{(300\times10^{-6})(200\times10^{9})}\right]$$

$$F_B = 2.91600T_{AD} \qquad \text{(b)}$$

$T_{AD} = 68.4646$ kN $\qquad F_B = 199.6427$ kN

$C_x = 0$ kN $\qquad C_y = -43.1073$ kN $\qquad C = \sqrt{C_x^2 + C_y^2} = 43.1073$ kN

(a) $\sigma_{AD} = \dfrac{N}{A} = \dfrac{68.4646(10^3)}{300(10^{-6})} = 228(10^6)$ N/m^2

$\sigma_{AD} = 228$ MPa (T) .... **Ans.**

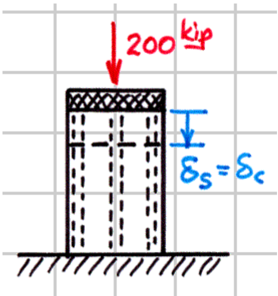
$\sigma_B = \dfrac{199.6427(10^3)}{4500(10^{-6})} = 44.4(10^6)$ N/m$^2 = 44.4$ MPa (C) .... **Ans.**

(b) $\tau_C = \dfrac{V}{A} = \dfrac{43.1073(10^3)}{2\left[\pi(0.020)^2/4\right]} = 68.6(10^6)$ N/m$^2 = 68.6$ MPa .... **Ans.**

(c) $v_D = \delta_{AD} = \dfrac{(68.4646\times10^3)(450)}{(300\times10^{-6})(200\times10^9)} = 0.513$ mm $\downarrow$ .... **Ans.**

5-47

$$5F_S + F_C = 200 \text{ kip} \quad \text{(a)}$$



$$\delta_S = \delta_C$$

$$\frac{F_S L}{\left[\pi(1)^2/4\right](29{,}000)} = \frac{F_C L}{A_C(4500)} \quad \text{(b)}$$

$$A_C = b^2 - 5\left[\pi(1)^2/4\right] \quad \text{(c)}$$

If $F_S = F_{S\max} = \sigma_S A = (18)\left[\pi(1)^2/4\right] = 14.1372 \text{ kip}$, then

$$F_C = 129.3142 \text{ kip} \qquad A_C = 46.2977 \text{ in.}^2 \qquad b = 7.09 \text{ in.}$$

and $\sigma_C = \dfrac{F}{A} = \dfrac{129.3142}{46.2977} = 2.79 \text{ ksi} > \sigma_{C\max} = 1.4 \text{ ksi}$ (not correct guess)

If $\sigma_C = F_C/A_C = \sigma_{C\max} = 1.4 \text{ ksi}$, then

$$F_S = 7.0860 \text{ kip} \qquad F_C = 164.5698 \text{ kip} \qquad A_C = 117.5499 \text{ in.}^2 \qquad b = 11.02 \text{ in.}$$

and $\sigma_S = \dfrac{F}{A} = \dfrac{7.0860}{\pi(1)^2/4} = 9.02 \text{ ksi} < \sigma_{S\max} = 18 \text{ ksi}$ (correct guess)

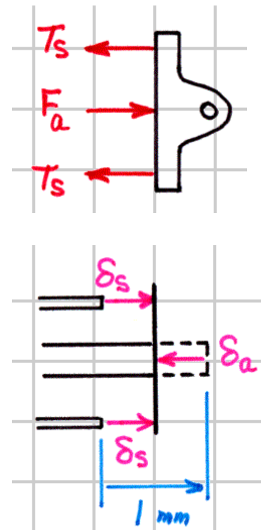
Therefore $b = 11.02 \text{ in.}$ .... **Ans.**

5-48*

$$2T_s = F_a \qquad \text{(a)}$$

$$\delta_s + \delta_a = 1 \text{ mm} \qquad \text{(b)}$$

$$\frac{T_s(330)}{(120\times10^{-6})(190\times10^{9})} + \frac{F_a(251)}{(625\times10^{-6})(73\times10^{9})} = 1 \text{ mm}$$



$$T_s = 39.2520(10^3) \text{ N} = 39.2520 \text{ kN}$$

$$F_a = 78.5040(10^3) \text{ N} = 78.5040 \text{ kN}$$

(a) $$\sigma_s = \frac{N}{A} = \frac{39.2520(10^3)}{120(10^{-6})} = 327(10^6) \text{ N/m}^2$$

$\sigma_s = 327$ MPa (T) .... **Ans.**

$$\sigma_a = \frac{78.5040(10^3)}{625(10^{-6})} = 125.6(10^6) \text{ N/m}^2 = 125.6 \text{ MPa (C)}$$ .... **Ans.**

(b) $$\delta_a = \frac{(78.5040\times10^3)(251)}{(625\times10^{-6})(73\times10^9)} = 0.432 \text{ mm (shrink)}$$ .... **Ans.**

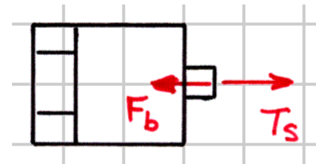
5-49*

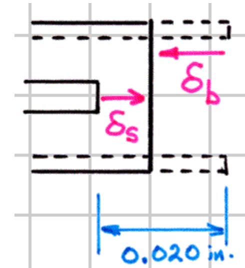
$$T_s = F_b \qquad \text{(a)}$$

$$\delta_b + \delta_s = 0.020 \text{ in.}$$

$$\frac{F_b(6)}{(0.375)(15{,}000)} + \frac{T_s(6)}{\left[\pi(0.5)^2/4\right](30{,}000)} = 0.02 \text{ in.}$$

$$T_s = F_b = 9.59114 \text{ kip} \qquad \text{(b)}$$



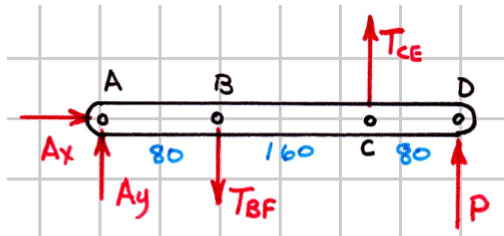


$$\sigma_s = \frac{N}{A} = \frac{9.59114}{\pi(0.5)^2/4} = 48.8 \text{ ksi (T)}$$ ....**Ans.**

$$\sigma_b = \frac{9.59114}{0.375} = 25.6 \text{ ksi (C)}$$ ....**Ans.**

5-50

Initially, bar CE is not connected to bar $ABCD$ $(T_{CE}=0)$ and the load $P=50$ kN is applied to the end of bar $ABCD$. Then



$$\circlearrowleft \Sigma M_A = 0: \qquad 320(50) - 80T_{BF} = 0$$

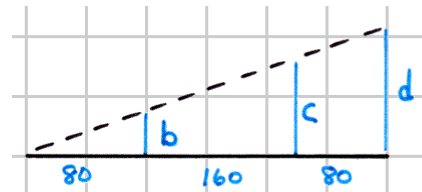
$$T_{BF} = 200 \text{ kN}$$

$$\delta_{BF} = \frac{(200\times 10^3)(1000)}{(1200\times 10^{-6})(210\times 10^9)} = 0.79365 \text{ mm}$$

At this point, bar CE is connected to bar $ABCD$ $(T_{CE}\neq 0)$ and the applied load P is removed $(P=0)$. Then

$$\circlearrowleft \Sigma M_A = 0: \qquad 240T_{CE} - 80T_{BF} = 0$$

$$T_{BF} = 3T_{CE} \qquad \text{(a)}$$



$$c = 3b = 3\delta_{BF}$$

$$c = 3(0.79365) - \delta_{CE} = 2.38095 - \delta_{CE}$$

$$3\delta_{BF} = 2.38095 - \delta_{CE} \qquad \text{(b)}$$

$$3\left[\frac{T_{BF}(1000)}{(1200\times 10^{-6})(210\times 10^9)}\right] = 2.38095 - \frac{T_{CE}(600)}{(900\times 10^{-6})(73\times 10^9)}$$

$$T_{BF} = 159.2727(10^3) \text{ N} = 159.2727 \text{ kN}$$

$$T_{CE} = 53.0909(10^3) \text{ N} = 53.0909 \text{ kN}$$

(a) $$\sigma_{CE} = \frac{N}{A} = \frac{53.0909(10^3)}{900(10^{-6})} = 59.0(10^6) \text{ N/m}^2$$

$$\sigma_{CE} = \frac{N}{A} = \frac{53.0909(10^3)}{900(10^{-6})} = 59.0(10^6) \text{ N/m}^2 = 59.0 \text{ MPa (T)}$$ .... **Ans.**

(b) $$\delta_{BF} = \frac{(159.2727\times 10^3)(1000)}{(1200\times 10^{-6})(210\times 10^9)} = 0.63203 \text{ mm}$$

$$v_D = d = 4b = 4\delta_{BF} = 2.53 \text{ mm} \uparrow$$ .... **Ans.**

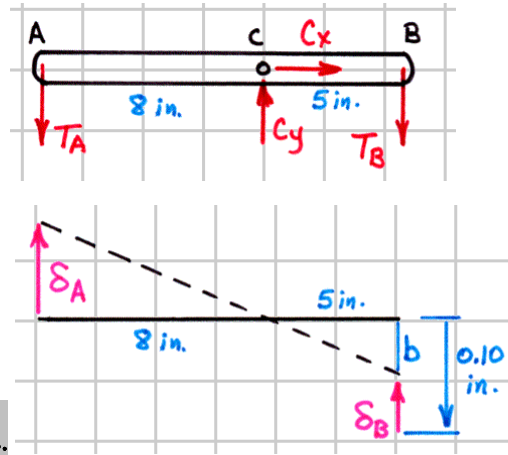
5-51

$\circlearrowleft \Sigma M_C = 0: \qquad 8T_A - 5T_B = 0$

$$T_B = 1.6T_A \qquad \text{(a)}$$

$$\delta_A = (8/5)b = (8/5)(0.1 - \delta_B)$$

$$\delta_A = 1.6(0.1 - \delta_B) \text{ in.} \qquad \text{(b)}$$



$$\frac{T_A(60)}{0.5(29,000)} = 1.6\left[0.1 - \frac{T_B(60)}{(1.2)(15,000)}\right]$$

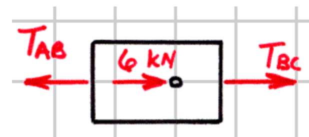
$$T_A = 12.6270 \text{ kip} \qquad T_B = 20.2032 \text{ kip}$$

(a) $\sigma_A = \dfrac{N}{A} = \dfrac{12.6270}{0.5} = 25.3 \text{ ksi (T)}$ ....**Ans.**

(b) $v_A = \delta_A = \dfrac{(12.6270)(60)}{(0.5)(29,000)} = 0.0523 \text{ in.} \uparrow$ .... **Ans.**

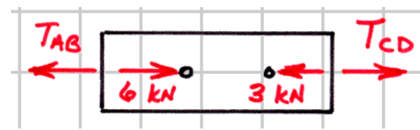
5-52*

$$A = \pi(0.030)^2/4 = 706.858(10^{-6})\ \text{m}^2$$



$$T_{BC} = (T_{AB} - 6)\ \text{kN} \qquad \text{(a)}$$

$$T_{CD} = (T_{AB} - 3)\ \text{kN} \qquad \text{(b)}$$



$$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0 \qquad \text{(c)}$$

$$\frac{T_{AB}(1000) + T_{BC}(1000) + T_{CD}(1000)}{(706.858\times10^{-6})(73\times10^9)} = 0$$

$$T_{AB} + (T_{AB} - 6) + (T_{AB} - 3) = 0$$

$$T_{AB} = 3\ \text{kN} \qquad T_{BC} = -3\ \text{kN} \qquad T_{CD} = 0\ \text{kN}$$

(a) $\sigma_{AB} = \dfrac{N}{A} = \dfrac{3000}{706.858(10^{-6})} = 4.24(10^6)\ \text{N/m}^2 = 4.24\ \text{MPa (T)}$ .... **Ans.**

$\sigma_{BC} = 4.24\ \text{MPa (C)}$ .... $\sigma_{CD} = 0\ \text{MPa}$ .... **Ans.**

(b) $\tau_{\max} = \sigma_{\max}/2 = 2.12\ \text{MPa}$ .... **Ans.**

(c) $\delta_{BC} = \dfrac{(3000)(1000)}{(706.858\times10^{-6})(73\times10^9)} = 0.058139\ \text{mm} \cong 0.0581\ \text{mm (shrink)}$ .... **Ans.**

(d) $E = 2(1+\nu)G \qquad 73 = 2(1+\nu)(28) \qquad \nu = 0.30357$

$$\varepsilon_{BC} = \frac{\delta}{L} = \frac{-0.058139}{1000} = -58.139(10^{-6})\ \text{m/m}$$

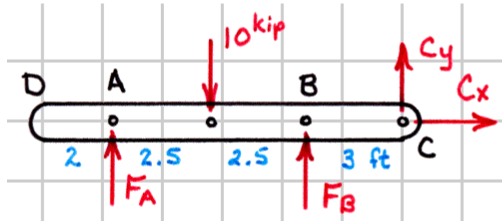
$$\varepsilon_{dBC} = -\nu\varepsilon_{BC} = -(0.30357)(-58.139\times10^{-6}) = +17.649(10^{-6})\ \text{m/m}$$

$\delta_d = \varepsilon_d d = (+17.649\times10^{-6})(30) = +0.529(10^{-3})\ \text{mm (expands)}$ .... **Ans.**

(e) $\delta_{AB} = -\delta_{BC} = +0.0581\ \text{mm (stretch)}$

$\varepsilon_{AB} = \dfrac{\delta}{L} = \dfrac{+0.058139}{1000} = +58.139(10^{-6})\ \text{m/m} \cong +58.1\ \mu\text{m/m}$ .... **Ans.**

5-53*

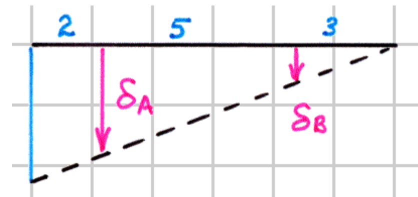


$\rightarrow \Sigma F_x = 0: \qquad C_x = 0$

$\uparrow \Sigma F_y = 0: \qquad F_A + F_B - 10 + C_y = 0$

$\circlearrowleft \Sigma M_C = 0: \qquad 5.5(10) - 8F_A - 3F_B = 0$

$$8F_A + 3F_B = 55 \text{ kip} \qquad \text{(a)}$$



$$\delta_A = (8/3)\delta_B$$

$$\frac{F_A(48)}{(2.25)(10,600)} = \frac{8}{3}\left[\frac{F_B(36)}{(1.75)(28,000)}\right]$$

$$F_B = 1.02725 F_A \qquad \text{(b)}$$

$$F_A = 4.96311 \text{ kip} \qquad F_B = 5.09835 \text{ kip} \qquad C_y = -0.06146 \text{ kip}$$

(a) $\sigma_A = \dfrac{N}{A} = \dfrac{4.96311}{2.25} = 2.2058 \text{ ksi} \cong 2.21 \text{ ksi (C)}$ .... **Ans.**

$\sigma_B = \dfrac{5.09835}{1.75} = 2.9133 \text{ ksi} \cong 2.91 \text{ ksi (C)}$ .... **Ans.**

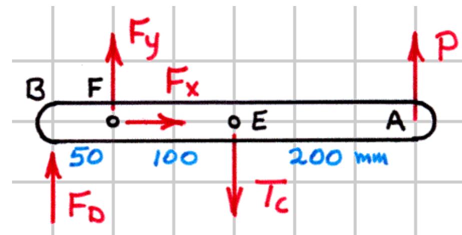
(b) $\tau_{\max A} = \sigma_A/2 = 1.103 \text{ ksi}$ .... $\tau_{\max B} = \sigma_B/2 = 1.457 \text{ ksi}$ .... **Ans.**

(c) $C = \sqrt{C_x^2 + C_y^2} = 61.46 \text{ lb}$

$\tau_C = \dfrac{V}{A} = \dfrac{61.46}{2\left[\pi(0.5)^2/4\right]} = 156.5 \text{ psi}$ .... **Ans.**

(d) $v_D = \dfrac{10}{8}a = \dfrac{10\delta_A}{8} = \dfrac{10(4.96311)(48)}{8(2.25)(10,600)} = 0.01249 \text{ in.} \downarrow$ .... **Ans.**

5-54



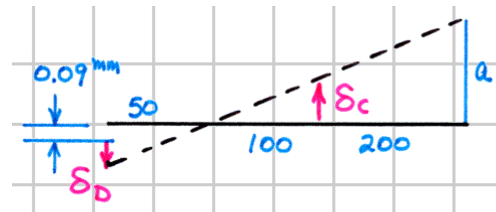
$\circlearrowleft \Sigma M_F = 0: \quad 300P - 50F_D - 100T_C = 0$

$$F_D + 2T_C = 6P \qquad \text{(a)}$$

$$\delta_C = 2(\delta_D + 0.09) \text{ mm}$$

$$\frac{T_C(300)}{(600\times10^{-6})(200\times10^9)} = 2\left[\frac{F_D(150)}{(2500\times10^{-6})(100\times10^9)} + 0.09\right]$$

$$T_C = 0.48000F_D + 72,000 \text{ N} \qquad \text{(b)}$$



Guess that

$$T_C = T_{C\max} = \sigma A$$

$$= (215\times10^6)(600\times10^{-6}) = 129,000 \text{ N}$$

Then $F_D = 118,750$ N $\qquad P = 62,792$ N

and $\sigma_D = \dfrac{N}{A} = \dfrac{118,750}{2500(10^{-6})} = 47.5(10^6) \text{ N/m}^2 = 47.5 \text{ MPa}$

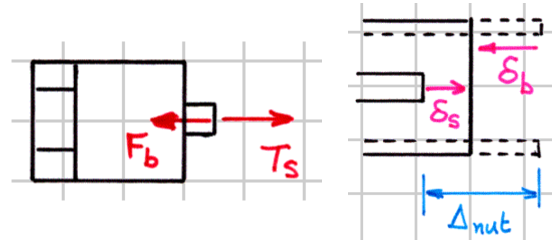
Since $\sigma_D = 47.5 \text{ MPa} < \sigma_{max} = 95 \text{ MPa}$, the guess was correct and

$P_{max} = 62.8$ kN .... **Ans.**

5-55

$$T_s = F_b \qquad \text{(a)}$$

$$\delta_s + \delta_b = \Delta_{nut} = \frac{0.125\theta}{360} \qquad \text{(b)}$$



$$\frac{T_s(14)}{(0.785)(30,000)} + \frac{F_b(12)}{(1.767)(15,000)} = \frac{0.125\theta}{360}$$

$$T_s = F_b = 0.33156\theta \text{ kip}$$

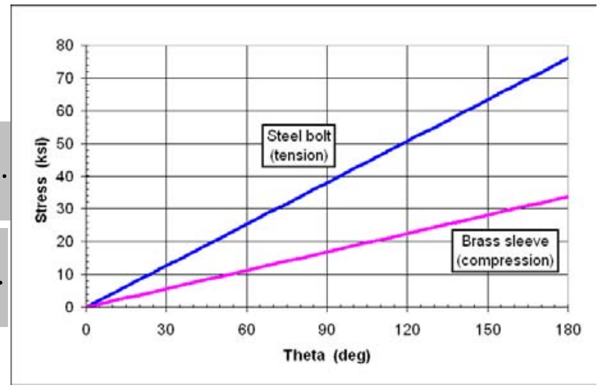
(a) $$\sigma_s = \frac{N}{A} = \frac{0.33156\theta}{0.785} = 0.42237\theta \text{ ksi (T)}$$

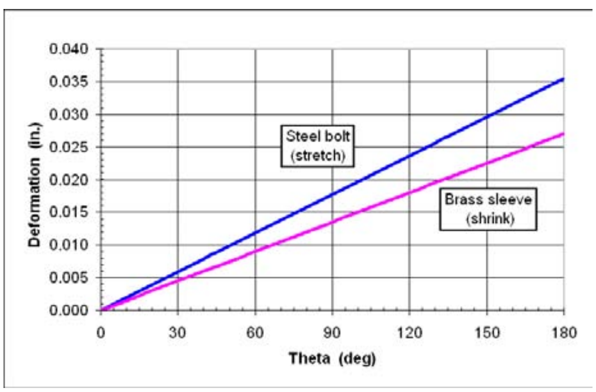
$$\sigma_b = \frac{0.33156\theta}{1.767} = 0.18764\theta \text{ ksi (C)}$$

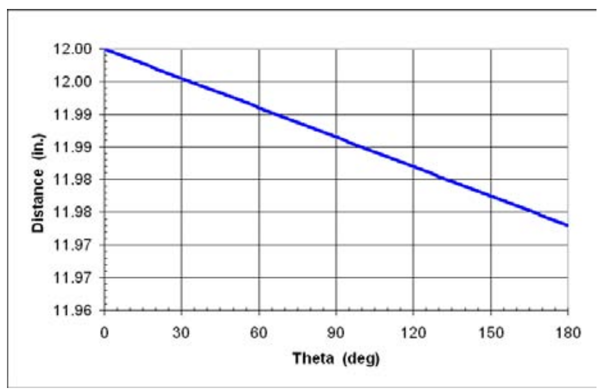
(b) $$\delta_s = \frac{(0.33156\theta)(14)}{(0.785)(30,000)} = 0.19711\theta\left(10^{-3}\right) \text{ in.}$$

$$\delta_b = \frac{(0.33156\theta)(12)}{(1.767)(15,000)} = 0.15011\theta\left(10^{-3}\right) \text{ in.}$$

(c) $$L = 12 - \delta_b = 12 - 0.15011\theta\left(10^{-3}\right) \text{ in.}$$





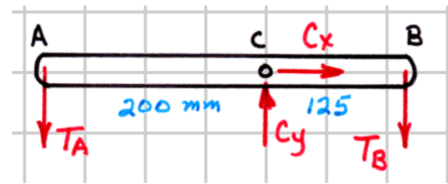


5-56

$$\circlearrowleft \Sigma M_C = 0: \quad 200T_A - 125T_B = 0$$

$$T_B = 1.6T_A$$

(a)



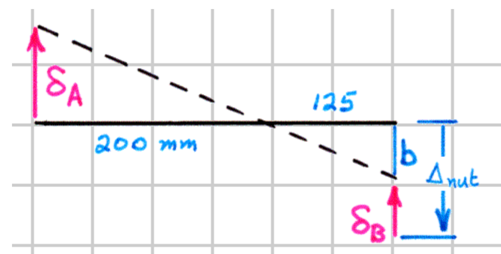
$$\delta_A = (200/125)b = (200/125)(\Delta_{nut} - \delta_B)$$

$$\delta_A = 1.6(\Delta_{nut} - \delta_B) \text{ mm}$$

$$\frac{T_A(1500)}{(350\times10^{-6})(200\times10^{9})} = 1.6\left[\frac{2.5\theta}{360} - \frac{T_B(1500)}{(750\times10^{-6})(100\times10^{9})}\right]$$

$$T_A + 1.49333T_B = 518.5185\theta \text{ N}$$

(b)

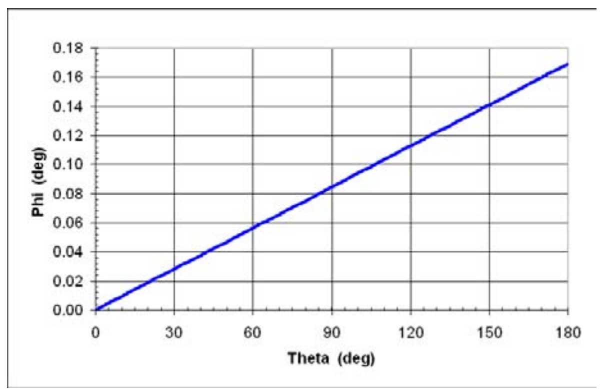


$$T_A = 152.98854\theta \text{ N}$$

$$T_B = 244.7766\theta \text{ N}$$

(a) $$\sigma_A = \frac{N}{A} = \frac{152.98854\theta}{350\times10^{-6}} = 0.43710\theta(10^{6}) \text{ N/m}^2$$

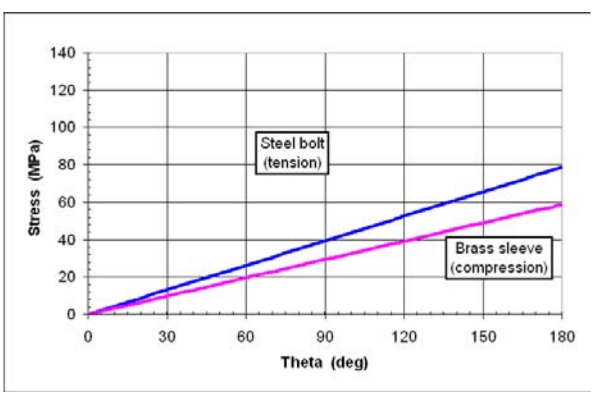
$$\sigma_A = \frac{N}{A} = \frac{244.7766\theta}{750\times10^{-6}} = 0.0.32637\theta(10^{6}) \text{ N/m}^2$$

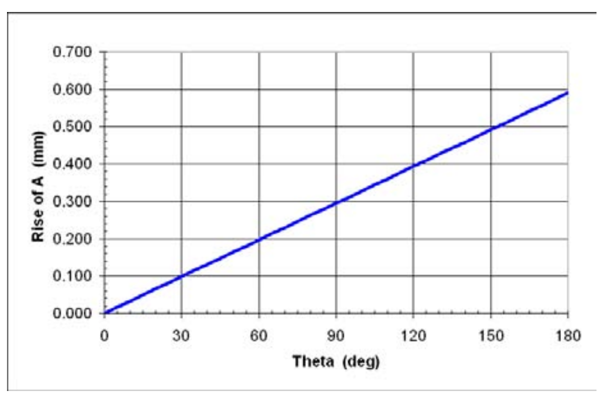


(b) $$v_A = \delta_A = \frac{(152.98854\theta)(1500)}{(350\times10^{-6})(200\times10^{9})}$$

$$v_A = 3.27826\theta(10^{-3}) \text{ mm}$$

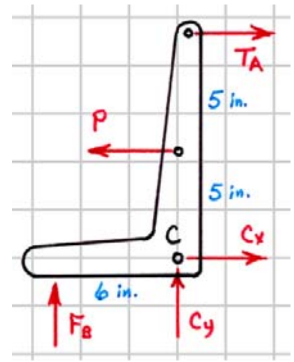
(c) $$\tan\phi = \frac{\delta_A}{200} = 16.69130\theta(10^{-6})$$





5-57

$\circlearrowleft \Sigma M_C = 0: \qquad 5P - 10T_A - 6F_B = 0$



$$10T_A + 6F_B = 5P \qquad \text{(a)}$$

$$\delta_A = (10/6)(\delta_B + 0.009) = (10\delta_B/6) + 0.015 \text{ in.}$$

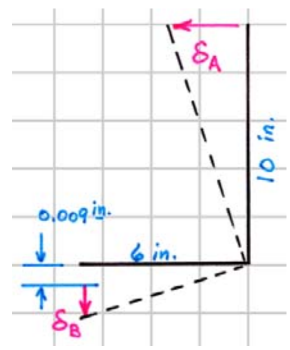
If $\delta_A \le 0.015$ in., then

$$\delta_B = 0 \qquad F_B = 0 \qquad \sigma_B = 0$$

$$T_A = P/2 \qquad \sigma_A = T_A/2 = P/4$$

and $$\delta_A = \frac{(P/2)(50)}{(2)(10{,}000)} = 0.00125P \text{ in.}$$

If $\delta_A \ge 0.015$ in., then



$$10T_A + 6F_B = 5P \qquad \text{(a)}$$

$$\delta_A = (10\delta_B/6) + 0.015 \text{ in.} \qquad \text{(b)}$$

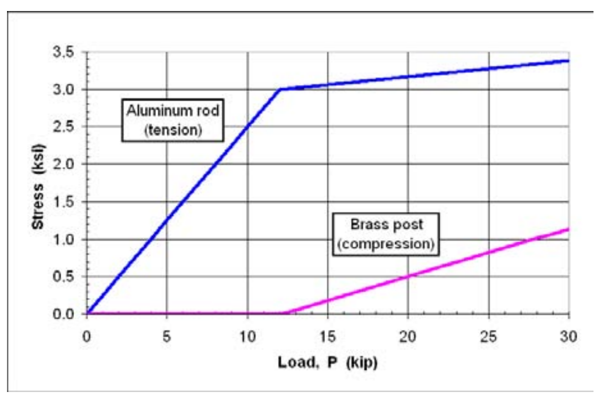
$$\frac{T_A(50)}{(2)(10{,}000)} = \frac{10}{6}\left[\frac{F_B(15)}{(12)(15{,}000)}\right] + 0.015$$

$$T_A = (0.042373P + 5.4915) \text{ kip}$$

$$F_B = (0.76271P - 9.15254) \text{ kip}$$

(a) $$\sigma_A = \frac{0.042373P + 5.4915}{2} = (0.02119P + 2.74575) \text{ ksi (T)}$$

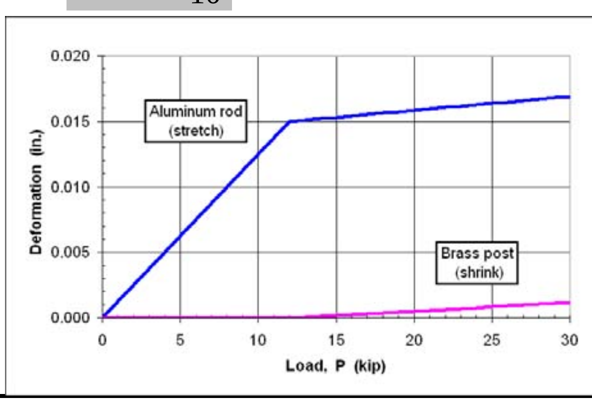
$$\sigma_B = \frac{0.76271P - 9.15254}{12} = (0.06356P - 0.76271) \text{ ksi (C)}$$

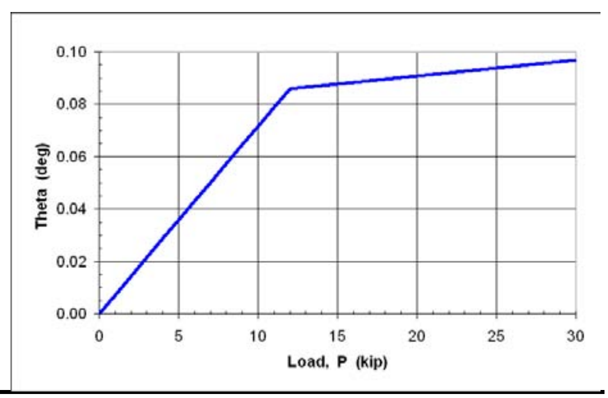


(b) $$\delta_A = \frac{(0.042373P + 5.4915)(50)}{(2)(10{,}000)} \text{ in.}$$

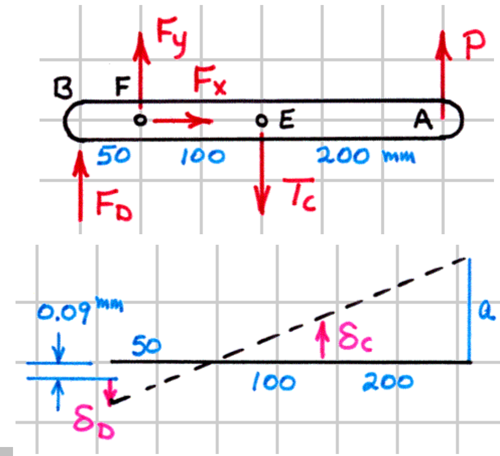
$$\delta_B = \frac{(0.76271P - 9.15254)(50)}{(12)(15{,}000)} \text{ in.}$$

(c) $$\theta = \tan^{-1}\frac{\delta_A}{10}$$





5-58



$$\circlearrowleft \Sigma M_F = 0: \quad 300P - 50F_D - 100T_C = 0$$

$$F_D + 2T_C = 6P \qquad \text{(a)}$$

$$\delta_C = 2(\delta_D + 0.09) \text{ mm}$$

If $\delta_C \leq 0.18$ mm , then

$$\delta_D = 0 \qquad F_D = 0 \qquad \sigma_D = 0$$

$$T_C = 3P$$

$$\sigma_C = \frac{3P}{600(10^{-6})} = 0.00500P(10^6) \text{ N/m}^2$$

and $$\delta_C = \frac{(3P)(300)}{(600\times10^{-6})(73\times10^9)} = 20.5480P(10^{-6}) \text{ mm}$$

If $\delta_C \leq 0.18$ mm , then

$$F_D + 2T_C = 6P \qquad \text{(a)}$$

$$\delta_C = 2(\delta_D + 0.09) \text{ mm}$$

$$\frac{T_C(300)}{(600\times10^{-6})(73\times10^9)} = 2\left[\frac{F_D(150)}{(2500\times10^{-6})(100\times10^9)} + 0.09\right] \text{ mm}$$

$$T_C = (0.17520F_D + 26,280) \text{ N} \qquad \text{(b)}$$

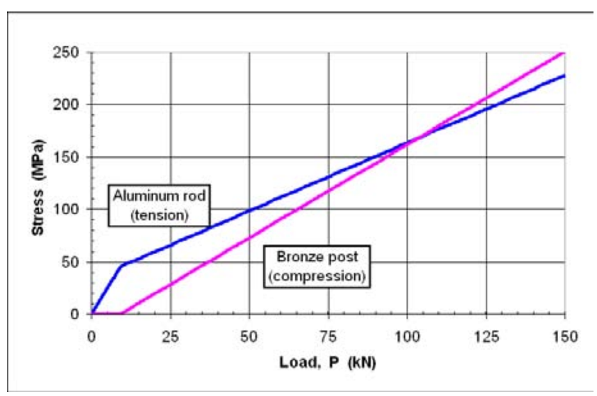
$$T_C = (0.77844P + 19,831) \text{ N}$$

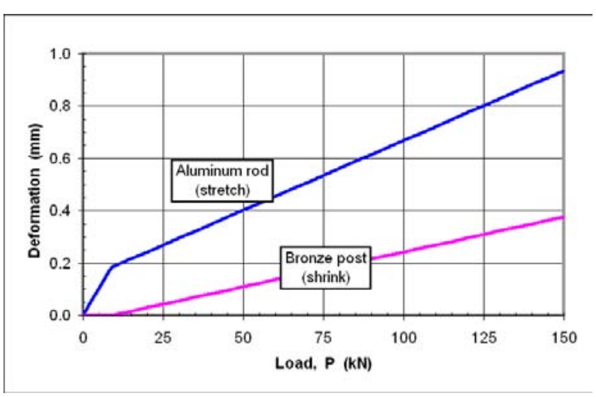
$$F_D = (4.44313P - 39,662) \text{ N}$$

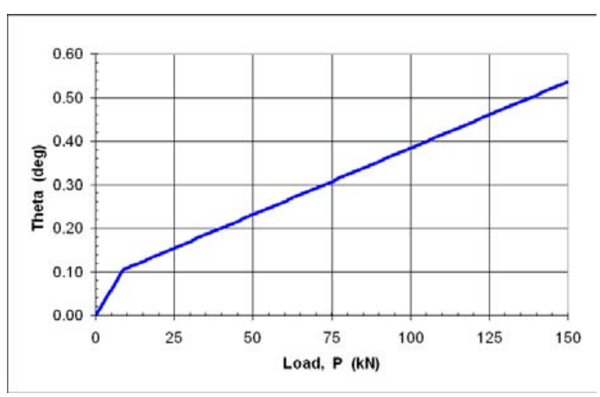
(b) $$\delta_C = \frac{(0.77844P + 19,831)(300)}{(600\times10^{-6})(73\times10^9)} \text{ mm}$$

$$\delta_D = \frac{(4.44313P - 39,662)(150)}{(2500\times10^{-6})(100\times10^9)} \text{ mm}$$

(c) $$\theta = \tan^{-1}\frac{\delta_C}{100}$$







5-59*

(a) $$\delta = 0 = \left(\frac{\sigma}{E} + \alpha\Delta T\right)L = \left[\frac{\sigma}{10,600} + \left(12.5\times10^{-6}\right)\left(-100\right)\right]\left(80\right)$$

$\sigma = 13.25$ ksi .... **Ans.**

(b) $\delta = 0$ (rigid supports) .... **Ans.**

(c) $\sigma = 0$ (nothing to exert a force) .... **Ans.**

(d) $$\delta = \left(\frac{\sigma}{E} + \alpha\Delta T\right)L = \left[0 + \left(12.5\times10^{-6}\right)\left(-100\right)\right]\left(80\right)$$

$\delta = -0.1000$ in. .... **Ans.**

5-60*

(a) $$\delta = \left(\frac{\sigma}{E} + \alpha \Delta T\right) L = \left[\frac{\sigma}{70\left(10^9\right)} + \left(22.5\times10^{-6}\right)(-55)\right](6000) = -1 \text{ mm}$$

$\sigma = 74.9583\left(10^6\right) \text{ N/m}^2 \cong 75.0 \text{ MPa}$ .... **Ans.**

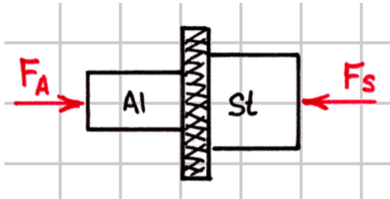
(b) $$\delta_d = \left(\frac{-\nu\sigma}{E} + \alpha \Delta T\right) d = \left[\frac{-(0.346)\left(74.9583\times10^6\right)}{70\left(10^9\right)} + \left(22.5\times10^{-6}\right)(-55)\right](50)$$

$\delta_d = -0.0804 \text{ mm}$ .... **Ans.**

5-61

$F_A = F_S$ $\quad A_A = \pi 3^2/4 = 7.06858 \text{ in.}^2$

$\delta_{total} = \delta_A + \delta_S = 0$ $\quad A_S = \pi 4^2/4 = 12.5664 \text{ in.}^2$



$$\delta = \left(\frac{F}{AE} + \alpha \Delta T\right) L$$

$$\left[\frac{(-F_A)}{(7.06858)(10,600)} + (12.5\times10^{-6})(80)\right](30) + \left[\frac{(-F_S)}{(12.5664)(30,000)} + (6.6\times10^{-6})(80)\right](20) = 0$$

$F_A = F_S = 89.4493$ kip (both C)

(a) $\sigma_A = \dfrac{F}{A} = \dfrac{89.4493}{7.06858} = 12.65$ ksi (C) .... **Ans.**

$\sigma_S = \dfrac{89.4493}{12.5664} = 7.12$ ksi (C) .... **Ans.**

(b) $\delta_d = \left(\dfrac{-\nu\sigma}{E} + \alpha \Delta T\right) d = \left[\dfrac{-(0.30)(-89.4493)}{(12.5664)(30,000)} + (6.6\times10^{-6})(80)\right](4)$

$\delta_d = +0.00240$ in. .... **Ans.**

(c) $u_B = \delta_A = \left[\dfrac{(-89.4493)}{(7.06858)(10,600)} + (12.5\times10^{-6})(80)\right](30)$

$u_B = -0.00581 \text{ in.} = 0.00581 \text{ in.} \leftarrow$ .... **Ans.**

5-62

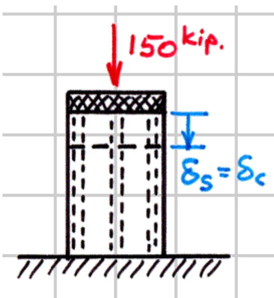
$$\delta_{summer} = \left(\frac{\sigma}{E} + \alpha \Delta T\right) L = \left[\frac{15\left(10^6\right)}{200\left(10^9\right)} + \left(11.9\times10^{-6}\right)(0)\right] L$$

$$\delta_{winter} = \left[\frac{\sigma}{200\left(10^9\right)} + \left(11.9\times10^{-6}\right)(-40)\right] L = \delta_{summer}$$

$$\left[\frac{\sigma}{200\left(10^9\right)} + \left(11.9\times10^{-6}\right)(-40)\right] L = \left[\frac{15\left(10^6\right)}{200\left(10^9\right)} + \left(11.9\times10^{-6}\right)(0)\right] L$$

$\sigma = 110.2\left(10^6\right)\text{ N/m}^2 = 110.2\text{ MPa}$ .... **Ans.**

5-63*



$$A_S = \pi(0.75)^2/4 = 0.44179 \text{ in.}^2$$

$$A_C = (10)^2 - 9(0.44179) = 96.0239 \text{ in.}^2$$

$$9F_S + F_C = 150 \text{ kip} \qquad \text{(a)}$$

$$\delta_S = \delta_C$$

$$\frac{(-F_S)(24)}{(0.44179)(30,000)} + (6.6\times10^{-6})(100)(24)$$

$$= \frac{(-F_C)(24)}{(96.0239)(4500)} + (6.0\times10^{-6})(100)(24)$$

$$32.60305F_S - F_C = 25.92646 \text{ kip} \qquad \text{(b)}$$

$$F_C = 111.9428 \text{ kip} \qquad F_S = 4.22869 \text{ kip}$$

(a) $\sigma_C = \dfrac{111.9418}{96.0239} = 1.166 \text{ ksi (C)}$ .... **Ans.**

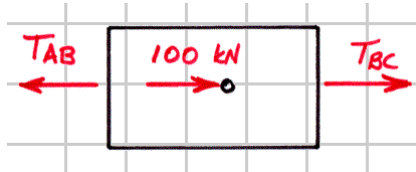
$\sigma_S = \dfrac{4.22869}{0.44179} = 9.57 \text{ ksi (C)}$ .... **Ans.**

(c) $\delta = \delta_S = \dfrac{(-4.22869)(24)}{(0.44179)(30,000)} + (6.6\times10^{-6})(100)(24)$

$\delta = +0.00818 \text{ in.}$ .... **Ans.**

5-64*

$$A_{AB} = A_{BC} = \pi(50)^2/4 = 1963.495 \text{ mm}^2$$



$$T_{AB} = T_{BC} + 100 \text{ kN} \qquad \text{(a)}$$

$$\delta_{AB} + \delta_{BC} = 0$$

$$\frac{(T_{AB} \times 10^3)(200)}{(1963.495 \times 10^{-6})(200 \times 10^9)} + (11.9 \times 10^{-6})(-20)(200)$$

$$+ \frac{(T_{BC} \times 10^3)(200)}{(1963.495 \times 10^{-6})(200 \times 10^9)} + (11.9 \times 10^{-6})(-20)(200) = 0$$

$$T_{AB} + T_{BC} = 186.9248 \text{ kN} \qquad \text{(b)}$$

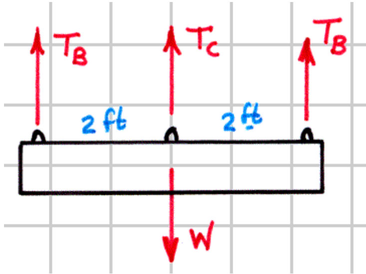
$$T_{AB} = 143.4624 \text{ kN} \qquad T_{BC} = 43.4624 \text{ kN}$$

$$\sigma_{AB} = \frac{143.4624(10^3)}{1963.495(10^{-6})} = 73.1(10^6) \text{ N/m}^2 = 73.1 \text{ MPa}$$ .... **Ans.**

$$\sigma_{BC} = \frac{43.4624(10^3)}{1963.495(10^{-6})} = 22.1(10^6) \text{ N/m}^2 = 22.1 \text{ MPa}$$ .... **Ans.**

5-65

$$T_C + 2T_B = W$$



If $W = 0$, then

$$T_C = -2T_B \qquad \text{(a)}$$

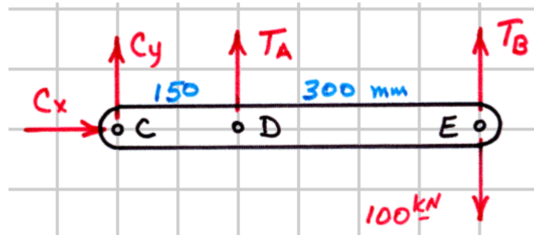
But since wires cannot sustain compressive forces, neither of the tensions can be negative and the only solution of Eq. (a) is

$$T_C = T_B = 0$$

regardless of any temperature change. And if the tensions are all zero, the stresses are also all zero

$\sigma_B = \sigma_C = 0$ .... **Ans.**

5-66*

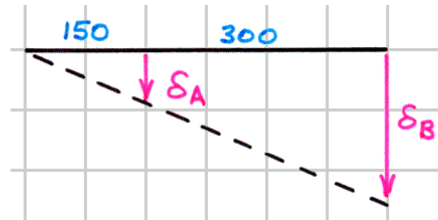


$\rightarrow \Sigma F_x = 0: \qquad C_x = 0$

$\uparrow \Sigma F_y = 0: \qquad C_y + T_A + T_B - 100 = 0$

$\circlearrowleft \Sigma M_C = 0: \qquad 150T_A + 450T_B - 450(100) = 0$

$$T_A + 3T_B = 300 \text{ kN} \qquad \text{(a)}$$



$$\delta_B = (450/150)\delta_A$$

$$\left[\frac{T_B(500)}{(300\times10^{-6})(70\times10^{9})} + (22.5\times10^{-6})(-25)(500)\right]$$

$$= \left(\frac{450}{150}\right)\left[\frac{T_A(250)}{(1200\times10^{-6})(210\times10^{9})} + (11.9\times10^{-6})(-25)(250)\right]$$

$$T_A - 8T_B = -19{,}530 \text{ N} = -19.530 \text{ kN} \qquad \text{(b)}$$

$T_A = 212.8555$ kN $\qquad T_B = 29.0482$ kN

$C_x = 0$ kN $\qquad C_y = -141.904$ kN $\qquad C = \sqrt{C_x^2 + C_y^2} = 141.904$ kN

(a) $\sigma_A = \dfrac{212.8555(10^3)}{1200(10^{-6})} = 177.4(10^6)$ N/m^2 $= 177.4$ MPa (T) .... **Ans.**

$\sigma_B = \dfrac{29.0482(10^3)}{300(10^{-6})} = 96.8(10^6)$ N/m^2 $= 96.8$ MPa (T) .... **Ans.**

(b) $\tau_{\max A} = \sigma_{\max A}/2 = 88.7$ MPa .... **Ans.**

$\tau_{\max B} = \sigma_{\max B}/2 = 48.4$ MPa .... **Ans.**

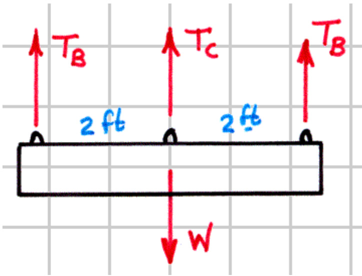
(c) $\tau_C = \dfrac{V}{A} = \dfrac{141.904(10^3)}{2\left[\pi(0.020)^2/4\right]} = 226(10^6)$ N/m^2 $= 226$ MPa .... **Ans.**

(d) $v_E = \delta_B = \dfrac{(29.0482)(500)}{(300\times10^{-6})(70\times10^{9})} + (22.5\times10^{-6})(-25)(500)$

$v_E = 0.410$ mm $\downarrow$ .... **Ans.**

5-67

$$T_C + 2T_B = W = 5 \text{ kip}$$



Assume that the wires are adjusted such that the weight is evenly supported by the three wires prior to the temperature change. Then

$$T_C = T_B = 5/3 = 1.66667 \text{ kip}$$

$$\delta_B = \frac{(1.66667)(3\times 12)}{(0.25)(29,000)} = 0.0082759 \text{ in.}$$

$$\delta_C = \frac{(1.66667)(5\times 12)}{(0.5)(10,600)} = 0.0188679 \text{ in.}$$

After the temperature change of $+50\ °\text{F}$

$$T_C + 2T_B = 5 \text{ kip} \qquad \text{(a)}$$

and assuming that the wires stay taught (in tension) the additional stretch of the wires must be equal

$$\delta_B - 0.0082759 = \delta_C - 0.0188679$$

$$\left[\frac{T_B(36)}{(0.25)(29,000)} + (6.6\times 10^{-6})(50)(36)\right] - 0.0082759$$

$$= \left[\frac{T_C(60)}{(0.5)(10,600)} + (12.5\times 10^{-6})(50)(60)\right] - 0.0188679$$

$$T_B - 2.27987T_C = 3.02647 \text{ kip} \qquad \text{(b)}$$

$$T_B = 2.59469 \text{ kip} \qquad T_C = -0.18939 \text{ kip}$$

But since wires cannot sustain compressive forces, neither of the tensions can be negative. Therefore, the wire C must have become slack, all of the weight is being carried by the wires B, and

$$T_B = 5/2 = 2.500 \text{ kip} \qquad T_C = 0 \text{ kip}$$

$\sigma_B = \dfrac{2.500}{0.25} = 10.00 \text{ ksi}$ .... **Ans.**

$\sigma_C = 0 \text{ ksi}$ .... **Ans.**

5-68

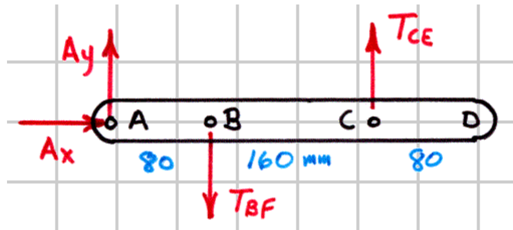
$\rightarrow \Sigma F_x = 0:$ $A_x = 0$

$\uparrow \Sigma F_y = 0:$ $A_y + T_{CE} - T_{BF} = 0$

$\circlearrowleft \Sigma M_A = 0:$ $240T_{CE} - 80T_{BF} = 0$

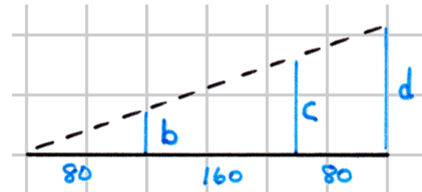
$T_{BF} = 3T_{CE}$

(a)



$c = 3b$ $b = \delta_{BF}$ $c = -\delta_{CE}$

$(-\delta_{CE}) = 3\delta_{BF}$



$$-\left[\frac{T_{CE}(600)}{(900\times10^{-6})(73\times10^{9})} + (22.5\times10^{-6})(-60)(600)\right]$$

$$= 3\left[\frac{T_{BF}(1000)}{(1200\times10^{-6})(210\times10^{9})} + (11.9\times10^{-6})(-60)(1000)\right]$$

$T_{CE} + 1.30357T_{BF} = 323,244.00 \text{ N}$

(b)

$T_{BF} = 197,472.87 \text{ N}$ $T_{CE} = 65,824.29 \text{ N}$ $A_y = 131,648.58 \text{ N}$

(a) $\sigma_{BF} = \dfrac{197,472.87}{1200(10^{-6})} = 164.6(10^6) \text{ N/m}^2 = 164.6 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{CE} = \dfrac{65,824.29}{900(10^{-6})} = 73.1(10^6) \text{ N/m}^2 = 73.1 \text{ MPa (T)}$ .... **Ans.**

(b) $A = \sqrt{A_x^2 + A_y^2} = 131,648.58 \text{ N}$

$\tau_A = \dfrac{131,641.58}{\pi(0.030)^2/4} = 186.2(10^6) \text{ N/m}^2 = 186.2 \text{ MPa}$ .... **Ans.**

$\tau_B = \dfrac{197,472.87}{2\left[\pi(0.030)^2/4\right]} = 139.7(10^6) \text{ N/m}^2 = 139.7 \text{ MPa}$ .... **Ans.**

$\tau_C = \dfrac{65,824.29}{\pi(0.030)^2/4} = 93.1(10^6) \text{ N/m}^2 = 93.1 \text{ MPa}$ .... **Ans.**

(c) $\delta_{BF} = \dfrac{(197,472.87)(1000)}{(1200\times10^{-6})(210\times10^{9})} + (11.9\times10^{-6})(-60)(1000) = 0.069623 \text{ mm}$

$v_D = d = 4b = 4\delta_{BF} = 0.278 \text{ mm} \uparrow$ .... **Ans.**

5-69*

Initially,

$$\delta_{init} = \frac{PL}{AE} + \alpha \Delta T\, L = \frac{(200)(120)}{(0.15)(10.6\times 10^{6})} + (12.5\times 10^{-6})(0)(120)$$

$$= 0.01509434 \text{ in.}$$

First determine the temperature rise required to close the gap so the weight rests on the floor

$$\delta = \frac{(200)(120)}{(0.15)(10.6\times 10^{6})} + (12.5\times 10^{-6})(\Delta T)(120)$$

$$= \delta_{init} + 0.08 = 0.02309434 \text{ in.} \qquad \Delta_T = 53.33\ ^\circ\text{F}$$

(a) Not touching the floor. Therefore, $T = W = 200$ lb and

$$\sigma = \frac{200}{0.15} = 1333 \text{ psi (T)}$$ .... **Ans.**

(b) Now the weight is partially resting on the floor so $T < W$ and

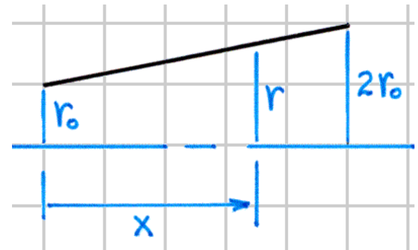
$$\delta = \frac{\sigma(120)}{(10.6\times 10^{6})} + (12.5\times 10^{-6})(60)(120) = 0.02309434 \text{ in.}$$

$$\sigma = -5910 \text{ psi}$$

But the wire cannot support a compression. Therefore, at this temperature the wire has become slack, and the weight rests totally on the floor.

$$\sigma = 0 \text{ psi}$$ .... **Ans.**

5-70*



$$\frac{d\delta}{dx} = \frac{\sigma}{E} + \alpha \Delta T = \frac{P}{AE} + \alpha \Delta T$$

$$A = \pi r^2 = \pi \left[(1+x) r_o \right]^2$$

$$= \pi (1+x)^2 r_o^2 = (1+x)^2 A_o$$

$$\delta = \int d\delta = \int_0^1 \left(\frac{P}{AE} + \alpha \Delta T \right) dx = 0$$

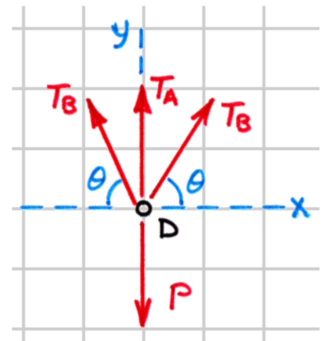
$$\frac{P}{E} \int_0^1 \left(\frac{dx}{(1+x)^2 A_o} \right) + \alpha \Delta T (1) = \frac{P}{EA_o} \left[\frac{-1}{(1+x)} \right]_0^1 + \alpha \Delta T = \frac{P}{2EA_o} + \alpha \Delta T = 0$$

$$\frac{P}{A_o} = -2E\alpha \, \Delta T$$

$$\sigma = \frac{P}{A} = \frac{P}{(1+x)^2 A_o} = \frac{-2E\alpha \, \Delta T}{(1+x)^2} = \frac{-2\left(74\times 10^9\right)\left(12.5\times 10^{-6}\right)(50)}{(1+x)^2}$$

$\sigma = \dfrac{-92.5\left(10^6\right)}{(1+x)^2}$ N/m^2 $= \dfrac{92.5}{(1+x)^2}$ MPa (C) .... **Ans.**

5-71*



$$\sin\theta = 4/5 \qquad P = 200 \text{ kip}$$

$$T_A + 2(4/5)T_B = 200 \text{ kip} \qquad \text{(a)}$$

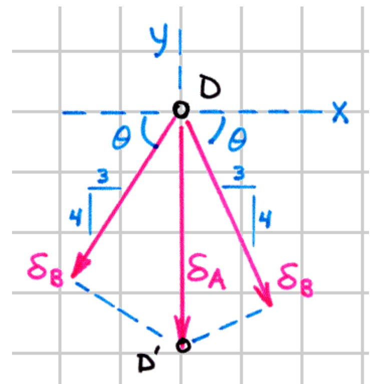
$$\delta_B = (4/5)\delta_A$$

$$\frac{T_B(5\times 12)}{(2.5)(30{,}000)} + (6.6\times 10^{-6})(30)(5\times 12)$$

$$= \left(\frac{4}{5}\right)\left[\frac{T_A(4\times 12)}{(3)(15{,}000)} + (9.4\times 10^{-6})(-50)(4\times 12)\right]$$

$$T_B = 1.06667T_A - 37.41000 \text{ kip} \qquad \text{(b)}$$

$$T_A = 96.0055 \text{ kip (T)} \qquad T_B = 64.9966 \text{ kip (T)}$$



(a) $\sigma_A = \dfrac{96.0055}{3} = 32.0$ ksi ....**Ans.**

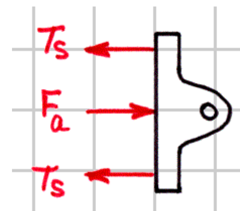
$\sigma_B = \dfrac{64.9966}{2.5} = 26.0$ ksi ....**Ans.**

(b) $v_C = \delta_A = \dfrac{(96.0055)(4\times 12)}{(3)(15{,}000)} + (9.4\times 10^{-6})(-50)(4\times 12)$

$v_C = 0.0799$ in. ....**Ans.**

5-72

$$2T_s = F_a \qquad \text{(a)}$$

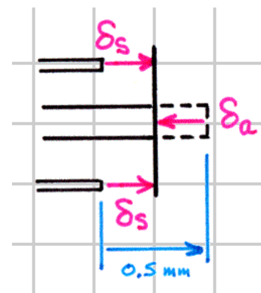


$$\delta_s + \delta_a = 0.5 \text{ mm}$$

$$\left[\frac{T_s(330)}{(115\times10^{-6})(190\times10^{9})} + (17.3\times10^{-6})(100)(330)\right]$$

$$+\left[\frac{F_a(250.5)}{(625\times10^{-6})(73\times10^{9})} - (22.5\times10^{-6})(100)(250.5)\right] = 0.5 \text{ mm}$$

$$F_a + 2.75079T_s = 89{,}742.83 \text{ N} \qquad \text{(b)}$$



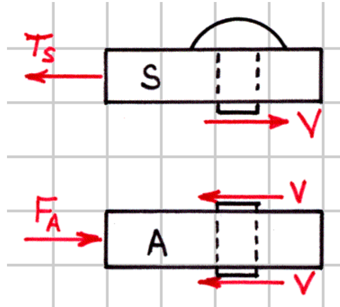
$T_s = 18{,}890.08$ N $\qquad F_a = 37{,}780.16$ N

$$\sigma_s = \frac{18{,}890.08}{115(10^{-6})} = 164.3(10^6) \text{ N/m}^2$$

$\sigma_s = 164.3$ MPa (T) .... **Ans.**

$\sigma_a = \dfrac{37{,}780.16}{625(10^{-6})} = 60.4(10^6)$ N/m^2 $= 60.4$ MPa (C) .... **Ans.**

5-73



$$T_S = V \qquad F_A = 2V = 2T_S \qquad \text{(a,b)}$$

$$\delta_S = \delta_A$$

$$\frac{T_S(7)}{(0.5\times 1)(29{,}000)} + (6.6\times 10^{-6})(-40)(7)$$

$$= \left[\frac{(-F_A)(7)}{(0.5\times 1)(10{,}600)} + (12.5\times 10^{-6})(-40)(7)\right]$$

$$T_S + 2.73585F_A = -3.42200 \text{ kip} \qquad \text{(c)}$$

$$T_S = V = -0.52876 \text{ kip} \qquad F_A = 2V = -1.05752 \text{ kip}$$

(The negative means that the steel is actually in compression and the aluminum is actually in tension.)

$$\tau = \frac{0.52876}{\pi(0.5)^2/4} = 2.69 \text{ ksi}$$ .... **Ans.**

5-74

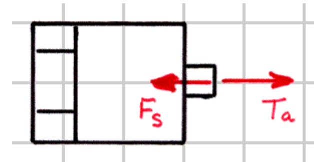
$$\frac{d\delta}{dx} = \frac{\sigma}{E} + \alpha\Delta T = \frac{\sigma}{E} + \alpha\left(\frac{100x^2}{L^2}\right) \qquad \delta = 0$$

$$\delta = \int_0^L \left[\frac{\sigma}{E} + \frac{100\alpha x^2}{L^2}\right] dx = \frac{\sigma L}{E} + \frac{100\alpha L^3}{3L^2} = 0$$

$$\sigma = \frac{100\alpha E}{3} = \frac{100\left(22.5\times10^{-6}\right)\left(70\times10^{9}\right)}{3}$$

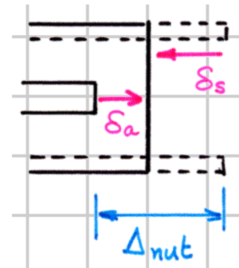
$\sigma = 52.5\left(10^6\right) \text{ N/m}^2 = 52.5 \text{ MPa}$ .... **Ans.**

5-75



$$T_a = F_s \qquad \text{(a)}$$

$$\delta_a + \delta_s = \Delta_{nut} \qquad \text{(b)}$$



$$\left[\frac{T_a(14)}{(1.4)(10,000)} + \left(12.5\times10^{-6}\right)(\Delta T)(14)\right] + \left[\frac{F_s(12)}{(0.400)(30,000)} - \left(6.6\times10^{-6}\right)(\Delta T)(12)\right] = \Delta_{nut}$$

Initially,

$$T_s = F_b = 3.500 \text{ kip} \qquad \Delta T = 0$$

$$\Delta_{nut} = 0.00700 \text{ in.}$$

As the temperature rises,

$$\left[\frac{T_a(14)}{(1.4)(10,000)} + \left(12.5\times10^{-6}\right)(\Delta T)(14)\right] + \left[\frac{F_s(12)}{(0.400)(30,000)} - \left(6.6\times10^{-6}\right)(\Delta T)(12)\right] = 0.00700 \text{ in.}$$

$$T_a = F_s = \left(3.5 - 0.0479000\Delta T\right) \text{ kip (bolt - tension; sleeve - compression)}$$

Note that at a temperature of about $73\ °\text{F}$ the force in the bolt and in the sleeve both go to zero. Beyond this point, the two pieces separate and no longer exert forces on each other – the forces and stresses both become zero

(a) $$\sigma_a = \frac{N}{A} = \frac{\left(3.5 - 0.0479000\Delta T\right)}{1.4} = \left(2.5 - 0.0342143\Delta T\right) \text{ ksi (T)} \qquad 0\ °\text{F} \le \Delta T \le 73\ °\text{F}$$

$$\sigma_s = \frac{\left(3.5 - 0.0479000\Delta T\right)}{0.4} = \left(8.75 - 0.119750\Delta T\right) \text{ ksi (C)} \qquad 0\ °\text{F} \le \Delta T \le 73\ °\text{F}$$

5-75 (cont.)

$\sigma_a = \sigma_s = 0 \text{ ksi}$ $\quad 73\ °\text{F} \le \Delta T \le 100\ °\text{F}$

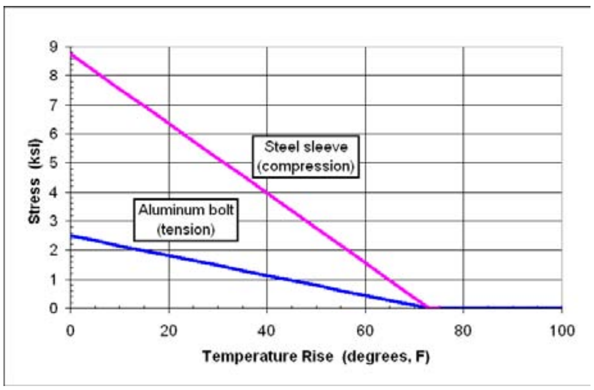
(b) $$\delta_a = \frac{(3.5 - 0.0479000\Delta T)(14)}{(1.4)(10{,}000)} + (12.5\times10^{-6})(\Delta T)(14)$$

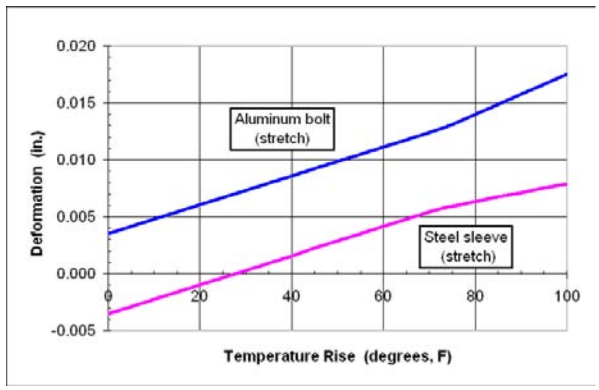
$$\delta_a = (3.500 + 0.12710\Delta T)(10^{-3}) \text{ in. (stretch)} \qquad 0\ °\text{F} \le \Delta T \le 73\ °\text{F}$$

$$\delta_s = \frac{(3.5 - 0.0479000\Delta T)(12)}{(0.400)(30{,}000)} - (6.6\times10^{-6})(\Delta T)(12)$$
$$= (3.500 - 0.12710\Delta T)(10^{-3}) \text{ in. (shrink)} \qquad 0\ °\text{F} \le \Delta T \le 73\ °\text{F}$$

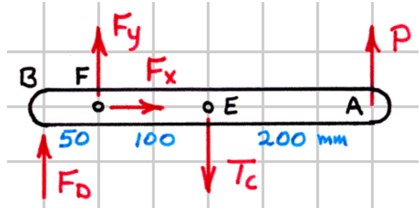
$$\delta_a = (12.5\times10^{-6})(\Delta T)(14) = 0.17500\Delta T\,(10^{-3}) \text{ in. (stretch)} \qquad 73\ °\text{F} \le \Delta T \le 100\ °\text{F}$$

$$\delta_s = -(6.6\times10^{-6})(\Delta T)(12) = -0.0.07920\Delta T\,(10^{-3}) \text{ in. (shrink)} \qquad 73\ °\text{F} \le \Delta T \le 100\ °\text{F}$$





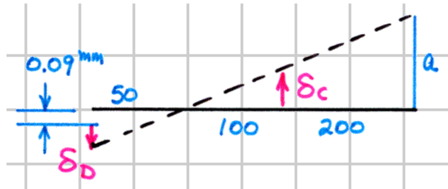
5-76



$$P = 35 \text{ kN}$$

$$\circlearrowleft \Sigma M_F = 0: \quad 300(35) - 50F_D - 100T_C = 0$$

$$F_D + 2T_C = 210 \text{ kN} \qquad \text{(a)}$$



$$\delta_C = 2(\delta_D + 0.09) \text{ mm}$$

$$\left[\frac{T_C(300)}{(600\times10^{-6})(200\times10^{9})} + (11.9\times10^{-6})(\Delta T)(300)\right]$$

$$= 2\left[\frac{F_D(150)}{(2500\times10^{-6})(73\times10^{9})} - (22.5\times10^{-6})(\Delta T)(150) + 0.09\right]$$

$$T_C = (0.65753F_D - 4.12800\Delta T + 72.000) \text{ kN} \qquad \text{(b)}$$

$$T_C = (90.7455 - 1.78311\Delta T) \text{ kN} \qquad F_D = (28.5090 + 3.56621\Delta T) \text{ kN}$$

(a) $$\sigma_C = \frac{(90.7455 - 1.78311\Delta T)(10^3)}{600(10^{-6})} = (151.2425 - 2.97185\Delta T) \text{ MPa (T)}$$

5-76 (cont.)

$$\sigma_D = \frac{(28.5090 + 3.56621\Delta T)(10^3)}{2500(10^{-6})} = (11.4036 + 1.42648\Delta T) \text{ MPa (C)}$$

(b)

$$\delta_C = \frac{(90.7455 - 1.78311\Delta T)(10^3)(300)}{(600\times10^{-6})(200\times10^9)} + (11.9\times10^{-6})(\Delta T)(300)$$

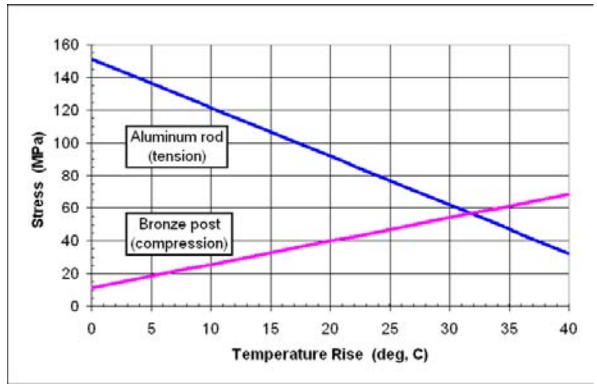
$$\delta_C = (0.22686 - 0.00088778\Delta T) \text{ mm}$$

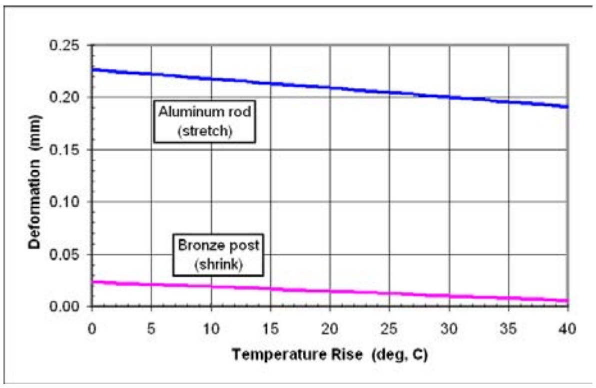
$$\delta_D = \frac{(28.5090 + 3.56621\Delta T)(10^3)(150)}{(2500\times10^{-6})(73\times10^9)} - (22.5\times10^{-6})(\Delta T)(150)$$

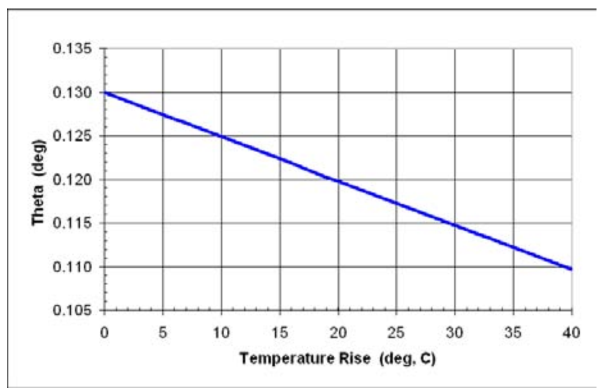
$$\delta_D = (0.02343 + 0.00044387\Delta T) \text{ mm}$$

(c)

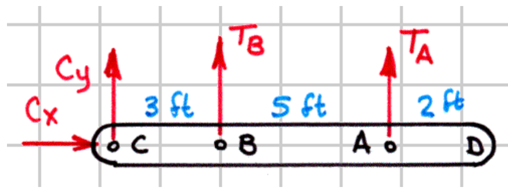
$$\theta = \tan^{-1}\frac{\delta_C}{100}$$







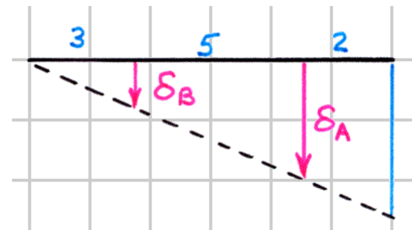
5-77



$$\circlearrowleft \Sigma M_C = 0: \qquad 8T_A + 3T_B = 0$$

$$8\sigma_A(1.75) + 3\sigma_B(2.25) = 0$$

$$\sigma_B = -2.07407\sigma_A \qquad \text{(a)}$$



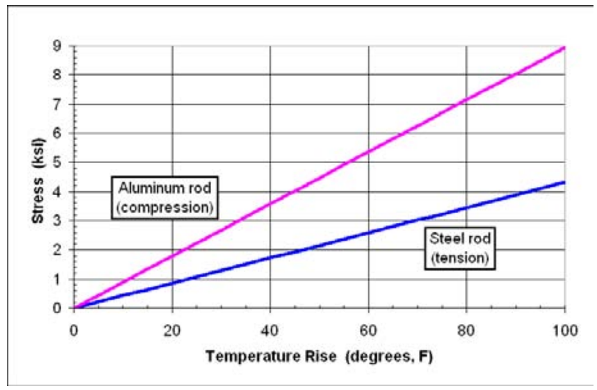
$$\delta_A = (8/3)\delta_B$$

$$\left[\frac{\sigma_A(48)}{(28,000)} + (6.6\times10^{-6})(\Delta T)(48)\right] = \frac{8}{3}\left[\frac{\sigma_B(36)}{(10,600)} + (12.5\times10^{-6})(\Delta T)(36)\right]$$

$$\sigma_A - 5.2830\sigma_B = 0.51520\Delta T \text{ ksi} \qquad \text{(b)}$$

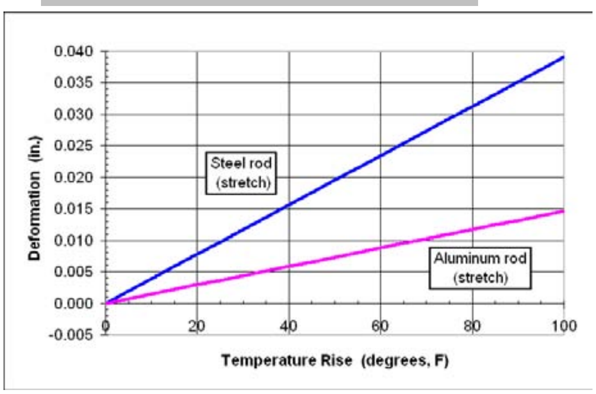
(a) $\sigma_A = 0.043087\Delta T$ ksi (T)

$\sigma_B = -0.089365\Delta T = 0.089365\Delta T$ ksi (C)

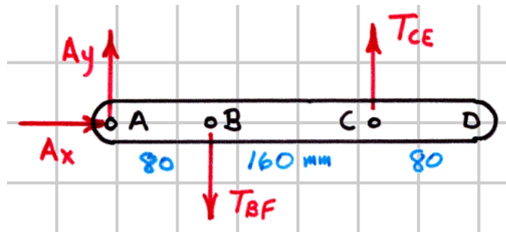


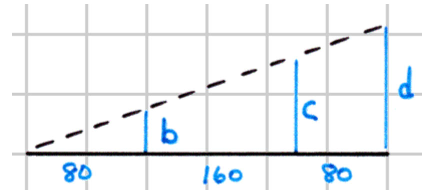
(b) $\delta_A = 390.7(10^{-6})\Delta T$ in. (stretch)

$\delta_B = 146.51(10^{-6})\Delta T$ in. (stretch)



5-78





$$\circlearrowleft \Sigma M_A = 0: \qquad 240T_{CE} - 80T_{BF} = 0$$

$$T_{BF} = 3T_{CE}$$

$$\sigma_{BF}\left(1200\times10^{-6}\right) = 3\sigma_{CE}\left(900\times10^{-6}\right)$$

$$\sigma_{BF} = 2.2500\sigma_{CE} \qquad \text{(a)}$$

$$c = 3b \qquad \text{(b)}$$

$$b = \delta_{BF} = \frac{\sigma_{BF}(1000)}{\left(210\times10^{9}\right)} = 4.76190\left(10^{-9}\right)\sigma_{BF} \text{ mm}$$

(The temperature of *BF* never changes.)

$$c = -\left[\frac{\sigma_{CE}(600)}{\left(73\times10^{9}\right)} + \left(22.5\times10^{-6}\right)(\Delta T)(600)\right]$$

$$= \left[-8.21918\left(10^{-9}\right)\sigma_{CE} - 13.500\left(10^{-3}\right)(\Delta T)\right] \text{ mm}$$

When $\Delta T = 0$ (after *CE* has been heated and the pin inserted),

$$b = c = \sigma_{CE} = \sigma_{BF} = 0$$

$$\delta_{CE} = \left(22.5\times10^{-6}\right)(80)(600) = 1.0800 \text{ mm}$$

As *CE* cools down

$$c = \left[-8.21918\left(10^{-9}\right)\sigma_{CE} - 13.500\left(10^{-3}\right)(\Delta T)\right] = 3b = 3\left[4.76190\left(10^{-9}\right)\sigma_{BF}\right]$$

$$14.28570\sigma_{BF} + 8.21918\sigma_{CE} = -13.500\left(10^{6}\right)(\Delta T) \qquad \text{(b)}$$

5-78 (cont.)

(a) $\sigma_{BF} = -0.75256\left(10^6\right)\Delta T \text{ N/m}^2$ .... **Ans.**

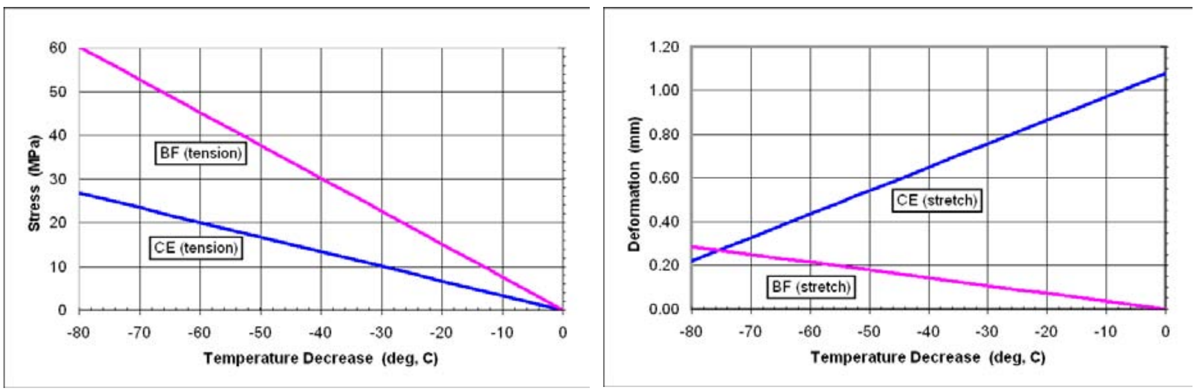
$\sigma_{CE} = -0.33447\left(10^6\right)\Delta T \text{ N/m}^2$ .... **Ans.**

(Note that since ΔT is negative the stresses will be positive or tension stresses.)

(b) $$\delta_{BF} = \frac{\left[-0.75256\left(10^6\right)\Delta T\right](1000)}{\left(210\times10^9\right)} = -3.58364\left(10^{-3}\right)\Delta T \text{ mm}$$ .... **Ans.**

$$c = -8.21918\left(10^{-9}\right)\left[-0.33447\left(10^6\right)\Delta T\right] - 13.500\left(10^{-3}\right)\Delta T$$
$$= -10.75093\left(10^{-3}\right)\Delta T$$

$\delta_{CE} = 1.080 - c = \left[1.080 + 10.75093\left(10^{-3}\right)\Delta T\right] \text{ mm}$ .... **Ans.**



5-79*

For the fillet: $D/d = 3/2 = 1.50$ $r/d = 0.4/2 = 0.20$

From Fig. 5-20(*c*): $K_t \cong 1.73$

$$\sigma = K_t \frac{P}{A_t} = 1.73 \frac{P}{(2)(0.25)} \le 66 \text{ ksi} \qquad P \le 19.08 \text{ kip}$$

For the hole: $d/w = 0.5/3 = 0.1667$

From Fig. 5-20(*b*): $K_t \cong 2.48$

$$\sigma = K_t \frac{P}{A_t} = 2.48 \frac{P}{(3-0.5)(0.25)} \le 66 \text{ ksi} \qquad P \le 16.63 \text{ kip}$$

$P_{max} = 16.63$ kip .... **Ans.**

5-80

For the small hole: $d/w = 16/160 = 0.10$

From Fig. 5-20(*b*): $K_t \cong 2.6$

$$\sigma = K_t \frac{P}{A_t} = 2.6 \frac{P}{(0.160 - 0.016)(0.010)} \le 760(10^6) \text{ N/m}^2 \qquad P \le 421(10^3) \text{ N}$$

For the large hole: $d/w = 64/160 = 0.40$

From Fig. 5-20(*b*): $K_t \cong 2.2$

$$\sigma = K_t \frac{P}{A_t} = 2.2 \frac{P}{(0.160 - 0.064)(0.010)} \le 760(10^6) \text{ N/m}^2 \qquad P \le 332(10^3) \text{ N}$$

$P_{max} = 332$ kN .... **Ans.**

5-81

(a) With no hole: $\sigma_A = \dfrac{500}{(4)(1/8)} = 1000 \text{ psi}$ .... **Ans.**

With a hole: $d/w = (1/64)/4 = 0.0039$

From Fig. 5-20(*b*): $K_t \cong 3.00$

$\sigma_A = 3.00 \dfrac{500}{(4 - 1/64)(1/8)} = 3012 \text{ psi}$ .... **Ans.**

(b) At *A*: $\theta = 0°$ $\sigma_r = 0$ $\tau_{r\theta} = 0$

Eq. 5-6: $\sigma_\theta = \sigma(1 + 2\cos 2\theta) = 1000[1 + 2\cos 2(0°)] = 3000 \text{ psi}$ .... **Ans.**

(c) With a 1-in. hole: $d/w = 1/4 = 0.25$ $K_t \cong 2.35$

$\sigma_A = 2.35 \dfrac{500}{(4 - 1)(1/8)} = 3133 \text{ psi}$ .... **Ans.**

$\sigma_\theta = \sigma(1 + 2\cos 2\theta) = 1000[1 + 2\cos 2(0°)] = 3000 \text{ psi}$ .... **Ans.**

(d) With a 2-in. hole: $d/w = 2/4 = 0.50$ $K_t \cong 2.12$

$\sigma_A = 2.12 \dfrac{500}{(4 - 2)(1/8)} = 4240 \text{ psi}$ .... **Ans.**

$\sigma_\theta = \sigma(1 + 2\cos 2\theta) = 1000[1 + 2\cos 2(0°)] = 3000 \text{ psi}$ .... **Ans.**

(Since Eq. 5-6 assumes a plate of infinite width, the size of the hole has no effect in the equation.)

5-82*

For the fillet: $D/d = 80/40 = 2.0$

$r/d = r/40$

$$\sigma = K_t \frac{P}{A_t} = K_t \frac{100\left(10^3\right)}{(0.040)(0.020)} \leq 205\left(10^6\right) \text{ N/m}^2$$

$$K_t \leq 1.64$$

From Fig. 5-20(*c*): $r/d = r/40 \geq 0.3$

$r_{\min} \cong 12.00$ mm .... **Ans.**

5-83*

For the grooves: $\frac{r}{b} = \frac{0.5}{B-1}$ $\frac{d}{r} = 1$

$$\sigma = K_t \frac{P}{A_t} = K_t \frac{10}{(B-1)(0.5)} \leq 20 \text{ ksi} \qquad \text{(a)}$$

Solve by systematic trial and error.

First try $K_t \cong 2$:

Eq. (a) gives $B-1=2$

then $r/b = 0.5/2 = 0.25$

Fig. 5-20(*a*): $K_t \cong 1.92$

Next try $K_t \cong 1.92$:

Eq. (a) gives $B-1=1.92$

then $r/b = 0.5/1.92 = 0.26$

Fig. 5-20(*a*): $K_t \cong 1.92$

Therefore $B_{min} = 1.92 + 1 = 2.92$ in. .... **Ans.**

5-84

At *A*: $\sigma_x = 0 \qquad \tau_{xy} = 0$

$$d/w = 80/200 = 0.40$$

From Fig. 5-20(*b*): $K_t \cong 2.2$

$$\sigma_y = K_t \frac{P}{A_t} = 2.2 \frac{180(10^3)}{(0.200 - 0.080)(0.025)} = 132.0(10^6) \text{ N/m}^2$$

$$\nu = \frac{E}{2G} - 1 = \frac{73}{2(28)} - 1 = 0.30357$$

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E} = \frac{0 - (0.30357)(132.0 \times 10^6)}{73(10^9)}$$

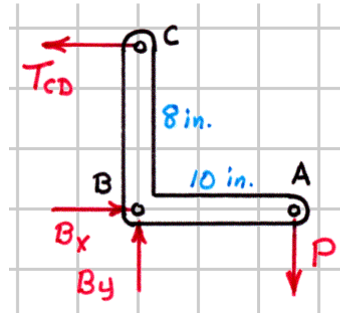
$\varepsilon_x = -549(10^{-6}) = -549\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E} = \frac{132.0(10^6) - 0}{73(10^9)}$$

$\varepsilon_y = 1808(10^{-6}) = +1808\ \mu\text{m/m}$ .... **Ans.**

$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0\ \mu\text{rad}$ .... **Ans.**

5-85*



$\circlearrowleft \Sigma M_B = 0: \qquad 8T_{CD} - 10P = 0$

(a) $T_{CD} = 1.25P = 1.25(28) = 35$ kip

$\sigma_{CD} = \dfrac{(35)}{1.00} = 35.0$ ksi .... **Ans.**

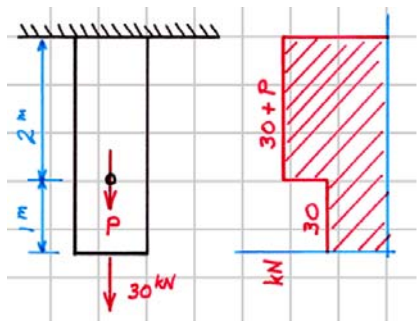
(b) $T_{CD} = 1.25P = 1.25(35) = 43.75$ kip

$\sigma_{CD} = \dfrac{(43.75)}{1.00} = 43.75 \text{ ksi} \cong 43.7$ ksi .... **Ans.**

(c) $\varepsilon_{CD} = 0.004 + \dfrac{43.75 - 42}{1400} = 0.0052500$ in./in.

$\delta_{CD} = \varepsilon_{CD} L_{CD} = (0.0052500)(8) = 0.0420$ in. .... **Ans.**

5-86*



$$A = \frac{\pi}{4}(20)^2 = 314.1593 \text{ mm}^2$$

(a) $$\sigma_{top} = \frac{50(10^3)}{314.1593(10^{-6})} = 159.2(10^6) \text{ N/m}^2$$

$\sigma_{top} = 159.2$ MPa .... **Ans.**

$$\sigma_{bottom} = \frac{30(10^3)}{314.1593(10^{-6})} = 95.5(10^6) \text{ N/m}^2$$

$\sigma_{bottom} = 95.5$ MPa .... **Ans.**

(b) $\sigma_{top} = \frac{95(10^3)}{314.1593(10^{-6})} = 302(10^6) \text{ N/m}^2 = 302$ MPa .... **Ans.**

$\sigma_{bottom} = \frac{30(10^3)}{314.1593(10^{-6})} = 95.5(10^6) \text{ N/m}^2 = 95.5$ MPa .... **Ans.**

5-87

(a) $$\sigma_{AB} = \frac{10}{\pi(1.25)^2/4} = 8.14873 \text{ ksi} \quad (< 42 \text{ ksi})$$

$$\sigma_{BC} = \frac{10}{\pi(0.75)^2/4} = 22.63537 \text{ ksi} \quad (< 42 \text{ ksi})$$

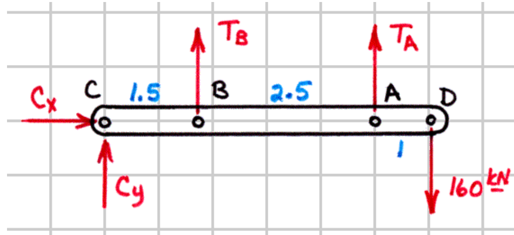
$$\delta = \left[\frac{8.14873}{10,500}(10)\right] + \left[\frac{22.63537}{10,500}(10)\right] = 0.0293 \text{ in.}$$ .... **Ans.**

(b) $$\sigma_{AB} = \frac{20}{\pi(1.25)^2/4} = 16.29747 \text{ ksi} \quad (< 42 \text{ ksi})$$

$$\sigma_{BC} = \frac{20}{\pi(0.75)^2/4} = 45.27074 \text{ ksi} \quad (> 42 \text{ ksi})$$

$$\delta = \left[\frac{1629747}{10,500}(10)\right] + \left[\frac{42}{10,500} + \frac{45.27074 - 42}{1400}\right](10) = 0.0789 \text{ in.}$$ .... **Ans.**

5-88*



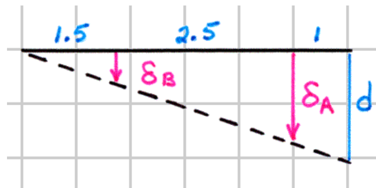
$\rightarrow \Sigma F_x = 0: \qquad C_x = 0$

$\uparrow \Sigma F_y = 0: \qquad C_y + T_A + T_B - 160 = 0$

$\circlearrowleft \Sigma M_C = 0: \qquad 1.5T_B + 4T_A - 5(160) = 0$

$$1.5T_B + 4T_A = 800 \text{ kN} \qquad \text{(a)}$$

$$\delta_A = (4/1.5)\delta_B \qquad \text{(b)}$$



Try elastic solution …

$$\frac{T_A(2000)}{(500\times10^{-6})(73\times10^{9})} = \left(\frac{4}{1.5}\right)\left[\frac{T_B(1500)}{(750\times10^{-6})(210\times10^{9})}\right]$$

$$T_B = 2.15753T_A$$

$$T_A = 110.55372 \text{ kN} \qquad T_B = 238.52343 \text{ kN}$$

$$\sigma_A = \frac{110.55362(10^3)}{500(10^{-6})} = 221(10^6) \text{ N/m}^2 \quad (< 330 \text{ MPa - elastic})$$

$$\sigma_B = \frac{238.52343(10^3)}{750(10^{-6})} = 318(10^6) \text{ N/m}^2 \quad (> 275 \text{ MPa - plastic})$$

(a) Therefore, $\sigma_B = 275$ MPa (T) .... **Ans.**

$$T_B = (275\times10^6)(750\times10^{-6}) = 206,250 \text{ N} \qquad T_A = 122,656 \text{ N}$$

$\sigma_A = \dfrac{122,656}{500(10^{-6})} = 245(10^6) \text{ N/m}^2 = 245 \text{ MPa (T)}$ (still elastic) .... **Ans.**

(b) $C_x = 0$ kN $\qquad C_y = -168.906$ kN $\qquad C = \sqrt{C_x^2 + C_y^2} = 168,906$ kN

$\tau_C = \dfrac{168,906}{2\left[\pi(0.030)^2/4\right]} = 119.5(10^6) \text{ N/m}^2 = 119.5 \text{ MPa}$ .... **Ans.**

(c) $\tau_{\max A} = \sigma_{\max A}/2 = 122.5$ MPa .... **Ans.**

$\tau_{\max B} = \sigma_{\max B}/2 = 137.5$ MPa .... **Ans.**

(d) $v_D = \dfrac{5\delta_A}{4} = \dfrac{5}{4}\left[\dfrac{(122,656)(2000)}{(500\times10^{-6})(73\times10^{9})}\right] = 8.40 \text{ mm} \downarrow$ .... **Ans.**

5-89*

$$2T_S + T_A = P \qquad 2(1.6\sigma_S) + (3.2\sigma_A) = P \qquad \text{(a)}$$

$$\delta_S = \delta_A$$

$$\frac{\sigma_S(8)}{(29{,}000)} = \frac{\sigma_A(10)}{(10{,}600)} \qquad \sigma_S = 3.41981\sigma_A \qquad \text{(b)}$$

$$\sigma_S = 100 \text{ ksi} \qquad \sigma_A = 29.2414 \text{ ksi}$$

$$T_S = 100(1.6) = 160 \text{ kip (T)} \qquad T_A = 29.2414(3.2) = 93.572 \text{ kip (T)}$$

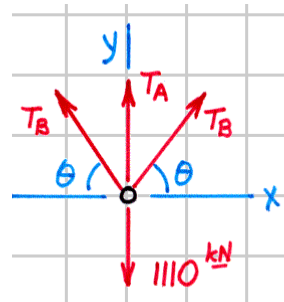
$P_{max} = 93.572 + 2(160) = 414$ kip .... **Ans.**

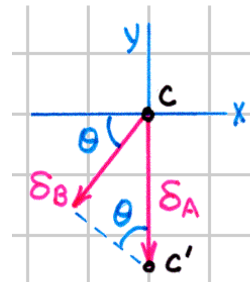
5-90

$$\sin\theta = 4/5$$

$$T_A + 2(4/5)T_B = 1110 \text{ kN} \qquad \text{(a)}$$

$$\delta_B = (4/5)\delta_A \qquad \text{(b)}$$





Try elastic solution …

$$\frac{T_B(1500)}{(1500\times10^{-6})(200\times10^{9})} = \left(\frac{4}{5}\right)\left[\frac{T_A(1200)}{(1500\times10^{-6})(72\times10^{9})}\right]$$

$$T_B = 1.77778T_A$$

$$T_A = 288.7283 \text{ kN} \qquad T_B = 513.2948 \text{ kN}$$

$$\sigma_A = \frac{288.7283(10^3)}{1500(10^{-6})} = 192.5(10^6) \text{ N/m}^2 \quad (< 380 \text{ MPa - elastic})$$

$$\sigma_B = \frac{513.2948(10^3)}{1500(10^{-6})} = 342(10^6) \text{ N/m}^2 \quad (> 250 \text{ MPa - plastic})$$

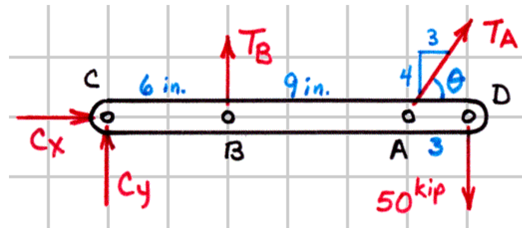
(a) Therefore, $\sigma_B = 250$ MPa (T) .... **Ans.**

$$T_B = (250\times10^6)(1500\times10^{-6}) = 375{,}000 \text{ N} \qquad T_A = 510{,}000 \text{ N}$$

$$\sigma_A = \frac{510{,}000}{1500(10^{-6})} = 340(10^6) \text{ N/m}^2 = 340 \text{ MPa (T) (still elastic)}$$ .... **Ans.**

(b) $$v_C = \delta_A = \frac{(510{,}000)(1200)}{(1500\times10^{-6})(72\times10^9)} = 5.67 \text{ mm} \downarrow$$ .... **Ans.**

5-91*

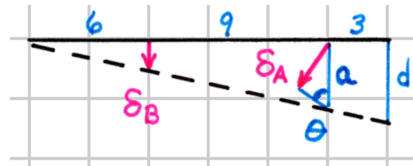


$$\rightarrow \Sigma F_x = 0: \qquad C_x + (3/5)T_A = 0$$

$$\uparrow \Sigma F_y = 0: \qquad C_y + T_B + (4/5)T_A - 50 = 0$$

$$\circlearrowleft \Sigma M_C = 0: \qquad 6T_B + 15\left[(4/5)T_A\right] - 18(50) = 0$$

$$T_B + 2T_A = 150 \text{ kip} \qquad \text{(a)}$$



$$\delta_A = (4/5)a = (4/5)\left[(15/6)\delta_B\right]$$

$$\delta_A = 2\delta_B \qquad \text{(b)}$$

Try elastic solution …

$$\frac{T_A(40)}{(1.5)(10{,}500)} = 2\left[\frac{T_B(20)}{(1.5)(29{,}000)}\right]$$

$$T_B = 2.76190 T_A$$

$$T_A = 31.50 \text{ kip} \qquad T_B = 87.00 \text{ kip}$$

$$\sigma_A = \frac{31.5}{1.5} = 21.0 \text{ ksi} \quad (< 55 \text{ ksi - elastic})$$

$$\sigma_B = \frac{87.00}{1.5} = 58.0 \text{ ksi} \quad (> 36 \text{ ksi - plastic})$$

(a) Therefore, $\sigma_B = 36$ ksi (T) .... **Ans.**

$$T_B = (36)(1.5) = 54.00 \text{ kip} \qquad T_A = (150 - 54)/2 = 48.00 \text{ kip}$$

$\sigma_A = \dfrac{48.00}{1.5} = 32.0$ ksi (T) (still elastic) .... **Ans.**

(b) $C_x = -28.80$ kip $\qquad C_y = -42.40$ kip $\qquad C = \sqrt{C_x^2 + C_y^2} = 51.2562$ kip

$\tau_C = \dfrac{51.2562}{2\left[\pi(1)^2/4\right]} = 32.6$ ksi .... **Ans.**

5-92

$$E_A = \frac{350\left(10^6\right)}{0.005} = 70.0\left(10^9\right)\ \text{N/m}^2 \qquad E_S = \frac{1400\left(10^6\right)}{0.007} = 200.0\left(10^9\right)\ \text{N/m}^2$$

$$T_A + T_S = 530\ \text{kN} \qquad \text{(a)}$$

$$\delta_A = \delta_S \qquad \text{(b)}$$

Try elastic solution …

$$\frac{T_A(750)}{\left(315\times10^{-6}\right)\left(70\times10^9\right)} = \frac{T_S(750)}{\left(315\times10^{-6}\right)\left(200\times10^9\right)} \qquad T_S = 2.85714 T_A$$

$$T_A = 137.407\ \text{kN} \qquad T_S = 392.593\ \text{kN}$$

$$\sigma_A = \frac{137,407}{315\left(10^{-6}\right)} = 436\left(10^6\right)\ \text{N/m}^2 \quad (> 350\ \text{MPa - plastic})$$

$$\sigma_S = \frac{392,593}{315\left(10^{-6}\right)} = 1246\left(10^6\right)\ \text{N/m}^2 \quad (< 1400\ \text{MPa - elastic})$$

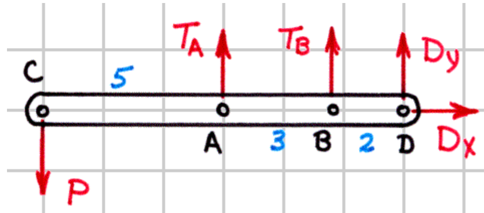
Therefore, $\sigma_A = 350\ \text{MPa (T)}$ $\quad T_A = \left(350\times10^6\right)\left(315\times10^{-6}\right) = 110,250\ \text{N}$

$$T_S = 530,000 - 110,250 = 419,750\ \text{N}$$

$$\sigma_S = \frac{419,750}{315\left(10^{-6}\right)} = 1333\left(10^6\right)\ \text{N/m}^2 = 1333\ \text{MPa (T)} \quad \text{(still elastic)}$$

$$v = \delta_S = \frac{(419,750)(750)}{\left(315\times10^{-6}\right)\left(200\times10^9\right)} = 5.00\ \text{mm} \downarrow$$ .... **Ans.**

5-93

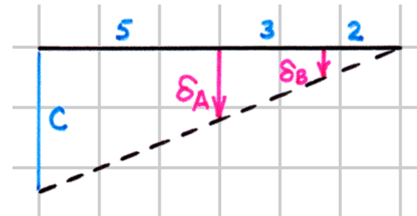


$\circlearrowleft \Sigma M_D = 0: \qquad 10P - 5T_A - 2T_B = 0$

$$5T_A + 2T_B = 10P \qquad \text{(a)}$$

$$\delta_A = (5/2)\delta_B = 2.5\delta_B \qquad \text{(b)}$$

$$c = (10/5)\delta_A = 2\delta_A \qquad \text{(c)}$$



(a) $P = 40$ kip

Try elastic solution …

$$\frac{T_A(10)}{(2)(10{,}000)} = 2.5\left[\frac{T_B(10)}{(2.5)(30{,}000)}\right]$$

$$T_B = 1.5T_A \qquad 5T_A + 2T_B = 10(40) = 400 \text{ kip}$$

$$T_A = 50.0 \text{ kip} \qquad T_B = 75.0 \text{ kip}$$

$\sigma_A = \dfrac{50.0}{2} = 25.0 \text{ ksi} \quad (< 34 \text{ ksi - elastic})$ .... **Ans.**

$\sigma_B = \dfrac{75.0}{2.5} = 30.0 \text{ ksi} \quad (< 36 \text{ ksi - elastic})$ .... **Ans.**

(b) $P = 60$ kip

Try elastic solution …

$$T_B = 1.5T_A \qquad 5T_A + 2T_B = 10(60) = 600 \text{ kip}$$

$$T_A = 75.0 \text{ kip} \qquad T_B = 112.5 \text{ kip}$$

$$\sigma_A = \frac{75}{2} = 37.5 \text{ ksi} \quad (> 34 \text{ ksi - yields})$$

$$\sigma_B = \frac{112.5}{2.5} = 45.0 \text{ ksi} \quad (> 36 \text{ ksi - plastic})$$

Therefore, $\sigma_B = 36$ ksi (T) .... **Ans.**

$$T_B = (36)(2.5) = 90.0 \text{ kip} \qquad T_A = 84.0 \text{ kip}$$

$\sigma_A = \dfrac{84.00}{2} = 42.0$ ksi (T) .... **Ans.**

5-93 (cont.)

(c) $P = 65$ kip $\qquad 5T_A + 2T_B = 10(65) = 650$ kip

$\sigma_B = 36$ ksi (T) $\qquad T_B = (36)(2.5) = 90.0$ kip

$T_A = 94.0$ kip

$\sigma_A = \dfrac{94.0}{2} = 47.0$ ksi $\qquad \varepsilon_A \cong 0.006$ in./in.

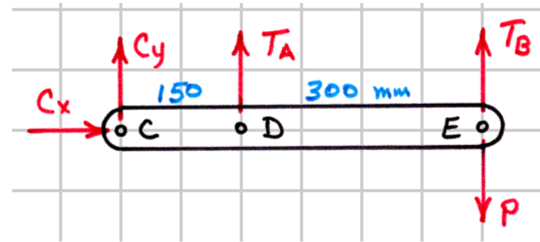
$v_C = 2\delta_A = 2\varepsilon_A L_A = 2(0.006)(10) = 0.1200$ in. $\downarrow$ .... **Ans.**

(d) $P = 40$ kip $\qquad \varepsilon_A = \dfrac{25}{10,000} = 0.00250$ in./in. .... **Ans.**

(e) $P = 65$ kip $\qquad \varepsilon_A \cong 0.006$ in./in. .... **Ans.**

$\varepsilon_B = \dfrac{\delta_B}{L_B} = \dfrac{\delta_A/2.5}{10} = \dfrac{\varepsilon_A}{2.5} = 0.00240$ in./in. .... **Ans.**

5-94



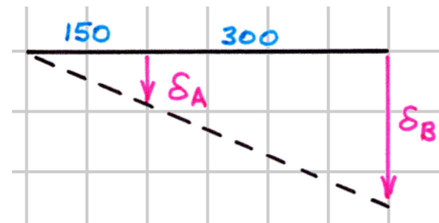
$$\circlearrowleft \Sigma M_C = 0: \qquad 150T_A + 450T_B - 450P = 0$$

$$T_A + 3T_B = 3P$$

$$\sigma_A\left(500\times10^{-6}\right) + 3\sigma_B\left(750\times10^{-6}\right) = 3P$$

$$\sigma_A + 4.5\sigma_B = 6000P \qquad \text{(a)}$$

$$\delta_B = 3\delta_A \qquad \text{(b)}$$



If $\varepsilon_B \le 0.0013095$ m/m, then

$$\frac{\sigma_B(500)}{\left(210\times10^9\right)} = 3\left[\frac{\sigma_A(250)}{\left(73\times10^9\right)}\right]$$

$$\sigma_B = 4.3151\sigma_A \qquad \text{(b)}$$

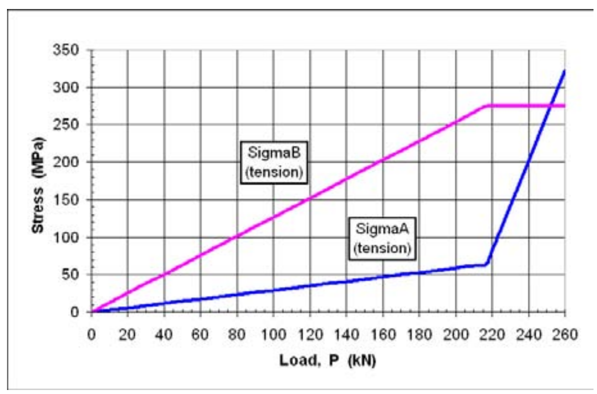
From Eqs. (a) and (b):

$$\sigma_A = 293.9P \text{ N/m}^2\text{(T)}$$

$$\sigma_B = 1268.0P \text{ N/m}^2\text{(T)}$$

$$\delta_A = \frac{\sigma_A(250)}{\left(73\times10^9\right)} \text{ mm}$$

$$\delta_B = \frac{\sigma_B(500)}{\left(210\times10^9\right)} \text{ mm}$$



If $\varepsilon_B \ge 0.0013095$ m/m, then

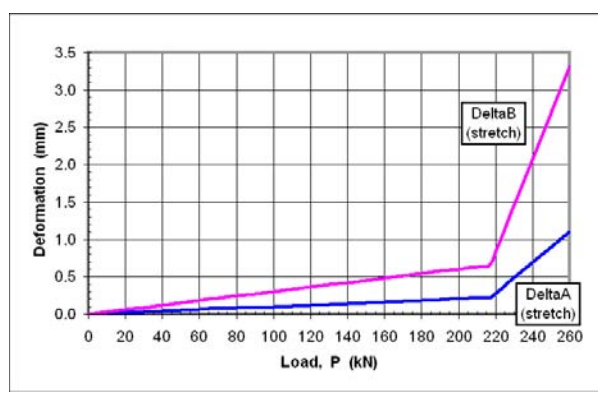
$$\sigma_B = 275\left(10^6\right) \text{ N/m}^2\text{(T)}$$

and from Eq. (a):

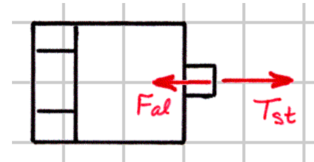
$$\sigma_A = 6000P - 1237.5\left(10^6\right) \text{ N/m}^2\text{(T)}$$

$$\delta_A = \frac{\sigma_A(250)}{\left(73\times10^9\right)} \text{ mm}$$

$$\delta_B = 3\delta_A$$



5-95

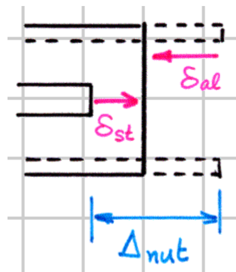


$$A_{st} = \pi(0.75)^2/4 = 0.44179 \text{ in}^2$$

$$F_{al} = T_{st}$$

$$\sigma_{al}(0.40) = \sigma_{st}(0.44179)$$

$$\sigma_{al} = 1.10447\sigma_{st} \qquad \text{(a)}$$



$$\delta_{st} + \delta_{al} = \Delta_{nut} = \frac{0.125\theta}{360} \qquad \text{(b)}$$

$$\delta_{st} = \frac{\sigma_{st}(12)}{(30{,}000)} = 0.400(10^{-3})\sigma_{st}$$

If $\varepsilon_{al} \le 0.004$ then

$$\delta_{al} = \frac{\sigma_{al}(10)}{(10{,}500)} = 0.95238(10^{-3})\sigma_{al}$$

$$0.400(10^{-3})\sigma_{st} + 0.95238(10^{-3})\sigma_{al} = \frac{0.125\theta}{360} \qquad \text{(b)}$$

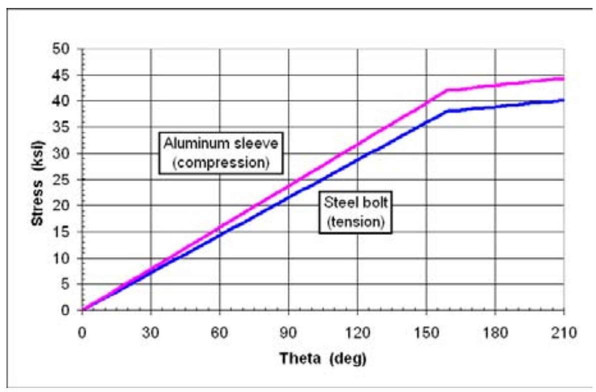
From Eqs. (a) and (b)

(a) $\sigma_{st} = 0.239150\theta$ ksi (T)

$\sigma_{al} = 1.10447\sigma_{st} = 0.26414\theta$ ksi (C)

(b) $\delta_{st} = 0.400(10^{-3})\sigma_{st} = 0.09566(10^{-3})\theta$ in.

$\delta_{al} = 0.95238(10^{-3})\sigma_{al} = 0.25156(10^{-3})\theta$ in.



If $\varepsilon_{al} \ge 0.004$ then

$$\delta_{al} = \left[0.004 + \frac{(\sigma_{al} - 42)}{(1400)}\right](10) = 7.14286(10^{-3})\sigma_{al} - 0.2600$$

$$0.400(10^{-3})\sigma_{st} + \left[7.14286(10^{-3})\sigma_{al} - 0.2600\right] = \frac{0.125\theta}{360} \qquad \text{(b)}$$

5-95 (cont.)

Now from Eqs. (a) and (b)

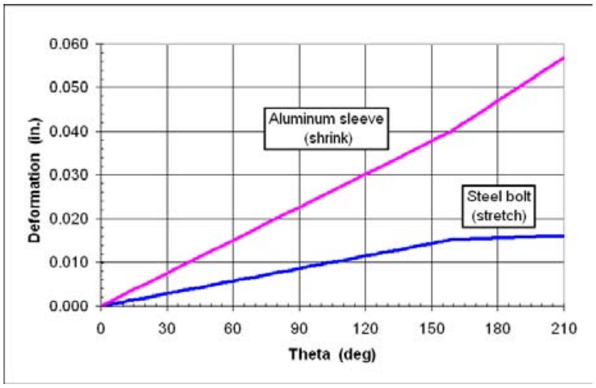
(a) $\sigma_{st} = (31.3666 + 0.041889\theta)$ ksi (T)

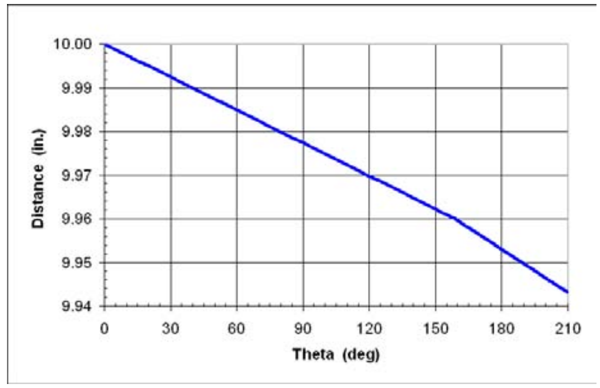
$\sigma_{al} = 1.10447\sigma_{st} = (34.6434 + 0.0462265\theta)$ ksi (C)

(b) $\delta_{st} = 0.400(10^{-3})\sigma_{st} = (12.5466 + 0.01676\theta)(10^{-3})$ in.

$\delta_{al} = 7.14286(10^{-3})\sigma_{al} - 0.2600 = (-12.5470 + 0.33046\theta)(10^{-3})$ in.

(c) $L = 10 - \delta_{al}$





5-96*

$$\sigma = \frac{pr}{2t} = \frac{(100)(148)}{2(2)} = 3700 \text{ kPa} = 3.70 \text{ MPa}$$ .... **Ans.**

5-97*

$$\sigma_h = \frac{pr}{t} = \frac{(800)(5)}{t} \leq 10{,}000 \text{ psi}$$

$t \geq 0.400$ in. .... **Ans.**

5-98

$$\sigma_a = \frac{pr}{2t} = \frac{p(1625-22)}{2(22)} \le 45.0 \text{ MPa} \qquad p \le 1.235 \text{ MPa}$$

$$\sigma_h = \frac{pr}{t} = \frac{p(1625-22)}{(22)} \le 100.0 \text{ MPa} \qquad p \le 1.372 \text{ MPa}$$

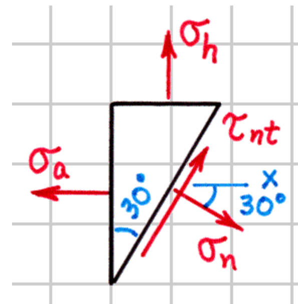
$p_{max} = 1.235$ MPa .... **Ans.**

5-99

$$\sigma = \frac{pr}{2t} = \frac{(100)\left[(17.5\times 12) - 7/8\right]}{2(7/8)} = 11{,}950 \text{ psi}$$

$\sigma = 11.95$ ksi .... **Ans.**

5-100*



$$\sigma_x = \sigma_a = \frac{pr}{2t} = \frac{(950)(500-50)}{2(50)} = 4275 \text{ kPa} = 4.275 \text{ MPa}$$

$$\sigma_y = \sigma_h = \frac{pr}{t} = \frac{(950)(500-50)}{(50)} = 8550 \text{ kPa} = 8.550 \text{ MPa}$$

$$\tau_{xy} = 0 \text{ MPa} \qquad \theta = -30°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= 4.275\cos^2(-30°) + 8.55\sin^2(-30°) + 0$$

$\sigma_n = 5.34$ MPa (T) .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -(4.275 - 8.55)\sin(-30°)\cos(-30°) + 0$$

$\tau_{nt} = -1.851$ MPa .... **Ans.**

5-101*

(a) $\sigma_a = \dfrac{pr}{2t} = \dfrac{(200)(3\times12)}{2(0.5)} = 7200 \text{ psi} = 7.20 \text{ ksi}$ .... **Ans.**

$\sigma_h = \dfrac{pr}{t} = \dfrac{(200)(3\times12)}{(0.5)} = 14,400 \text{ psi} = 14.40 \text{ ksi}$ .... **Ans.**

(b) $E = 2(1+\nu)G$ $\nu = \dfrac{29,000}{2(11,000)} - 1 = 0.31818$

$\varepsilon_a = \dfrac{\sigma_a - \nu\sigma_h}{E} = \dfrac{7.2 - 0.31818(14.4)}{29,000} = 90.3(10^{-6}) = 90.3\ \mu\text{in./in.}$ .... **Ans.**

$\varepsilon_h = \dfrac{\sigma_h - \nu\sigma_a}{E} = \dfrac{14.4 - 0.31818(7.2)}{29,000} = 418(10^{-6}) = 418\ \mu\text{in./in.}$ .... **Ans.**

5-102

$$p = \gamma h = (850 \times 9.81)(6) = 50{,}031.00 \text{ N/m}^2$$

$$\sigma_h = \frac{pr}{t} = \frac{(50{,}031.00)(10)}{t} \le 80(10^6) \text{ N/m}^2$$

$t \ge 0.00625 \text{ m} = 6.25 \text{ mm}$ .... **Ans.**

5-103

(a) $p = \gamma h = (62.4)(50) = 3120 \text{ psf} = 21.6667 \text{ psi}$

$\sigma_a = 0 \text{ ksi}$ (not including the weight of the tank) .... **Ans.**

$$\sigma_h = \frac{pr}{t} = \frac{(21.6667)(6\times12-0.5)}{(0.5)} = 3098 \text{ psi} \cong 3.10 \text{ ksi}$$ .... **Ans.**

(b) $p = \gamma h = (62.4)(25) = 1560 \text{ psf} = 10.8333 \text{ psi}$

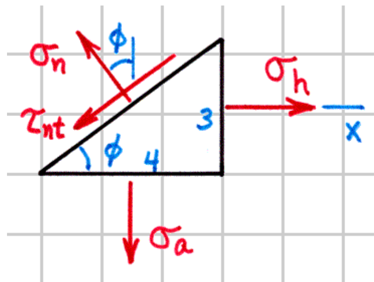
$\sigma_a = 0 \text{ ksi}$ (not including the weight of the tank) .... **Ans.**

$$\sigma_h = \frac{pr}{t} = \frac{(10.8333)(6\times12-0.5)}{(0.5)} = 1549 \text{ psi} = 1.549 \text{ ksi}$$ .... **Ans.**

5-104*

(a) $$\sigma_x = \sigma_h = \frac{pr}{t} = \frac{(2800\times 10^3)(600)}{(20)}$$

$$= 84.00(10^6)\ \text{N/m}^2 = 84.00\ \text{MPa}$$



$$\sigma_y = \sigma_a = \frac{pr}{2t} = \frac{(2800\times 10^3)(600)}{2(20)}$$

$$= 42.00(10^6)\ \text{N/m}^2 = 42.00\ \text{MPa}$$

$$\tau_{xy} = 0\ \text{MPa} \qquad \phi = \tan^{-1}\frac{3}{4} = 36.870° \qquad \theta = 90° + \phi = 126.870°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= 84\cos^2(126.870°) + 42\sin^2(126.870°) + 0$$

$\sigma_n = 57.1$ MPa (T) .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -(84-42)\sin(126.870°)\cos(126.870°) + 0$$

$\tau_{nt} = +20.2$ MPa .... **Ans.**

(b) $$E = 2(1+\nu)G \qquad \nu = \frac{200}{2(76)} - 1 = 0.31579$$

$\varepsilon_a = \dfrac{\sigma_a - \nu\sigma_h}{E} = \dfrac{[42 - 0.31579(84)](10^6)}{200(10^9)} = 77.4(10^{-6}) = 77.4\ \mu\text{m/m}$ .... **Ans.**

$\varepsilon_h = \dfrac{\sigma_h - \nu\sigma_a}{E} = \dfrac{[84 - 0.31579(42)](10^6)}{200(10^9)} = 354(10^{-6}) = 354\ \mu\text{m/m}$ .... **Ans.**

(c) $$\sigma_{\max} = \sigma_h = 84.00\ \text{MPa} \qquad \sigma_{\min} = \sigma_z = 0\ \text{MPa}$$

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{84.00 - 0}{2} = 42.0$ MPa .... **Ans.**

(d) $$\sigma_{\max} = \sigma_h = 84.00\ \text{MPa} \qquad \sigma_{\min} = \sigma_z = -2.800\ \text{MPa}$$

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{84.00 - (-2.800)}{2} = 43.4$ MPa .... **Ans.**

5-105*

$$E = 2(1+\nu)G \qquad \nu = \frac{30,000}{2(11,600)} - 1 = 0.29310$$

(a) $$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) = \frac{30,000}{1-0.29310^2}\left[619 + 0.29310(330)\right](10^{-6})$$

$\sigma_1 = 23.490 \text{ ksi} \cong 23.5 \text{ ksi}$ .... **Ans.**

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1) = \frac{30,000}{1-0.29310^2}\left[330 + 0.29310(619)\right](10^{-6})$$

$\sigma_2 = 16.785 \text{ ksi} \cong 16.79 \text{ ksi}$ .... **Ans.**

(b) $$\sigma_x = \sigma_a = \frac{pr}{2t} = \frac{p(9.875)}{2(0.125)} = 39.5p \text{ ksi} \qquad \tau_{xy} = 0 \text{ ksi}$$

$$\sigma_y = \sigma_h = \frac{pr}{t} = \frac{p(9.875)}{(0.125)} = 79.0p \text{ ksi}$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\sigma_2 = (39.5p)\cos^2(30°) + (79.0p)\sin^2(30°) + 0 = 16.785 \text{ ksi}$$

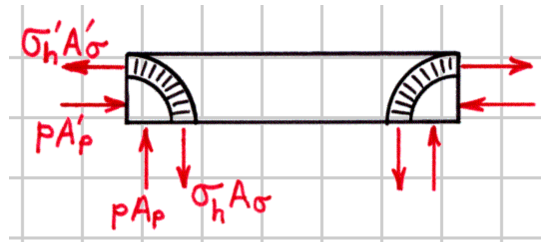
$p = 0.342 \text{ ksi} = 342 \text{ psi}$ .... **Ans.**

5-106

At point *A*:

$$\uparrow \Sigma F_y = 0: \qquad pA_p - \sigma_h A_\sigma = 0$$

$$p\left[\pi R^2 - \pi(R-r)^2\right] - \sigma_h\left[2\pi(R-r)t\right] = 0$$



$$\sigma_h = \frac{pr(2R-r)}{2t(R-r)} \quad \text{.......... \textbf{Ans.}}$$

$$\frac{\sigma_h}{r_r} + \frac{\sigma_a}{r_a} = \frac{p}{t} \qquad r_h = r \qquad r_a = -(R-r)$$

$$\sigma_a = \frac{p}{t}(r_a) - \frac{\sigma_h}{r_h}(r_a) = \frac{p}{t}\left[-(R-r)\right] - \frac{p(2R-r)}{2t(R-r)}\left[-(R-r)\right] = \frac{pr}{2t} \quad \text{.......... \textbf{Ans.}}$$

At point *B*:

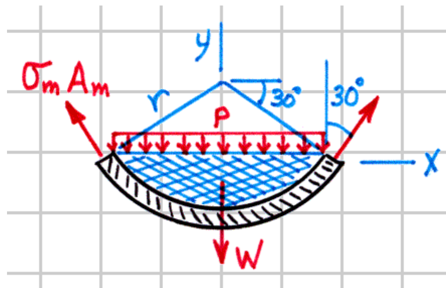
$$\uparrow \Sigma F_y = 0: \qquad p\left[\pi(R+r)^2 - \pi R^2\right] - \sigma_h\left[2\pi(R+r)t\right] = 0$$

$$\sigma_h = \frac{pr(2R+r)}{2t(R+r)} \quad \text{.......... \textbf{Ans.}}$$

$$\frac{\sigma_h}{r_r} + \frac{\sigma_a}{r_a} = \frac{p}{t} \qquad r_h = r \qquad r_a = R+r$$

$$\sigma_a = \frac{p}{t}(r_a) - \frac{\sigma_h}{r_h}(r_a) = \frac{p}{t}(R+r) - \frac{p(2R+r)}{2t(R+r)}(R+r) = \frac{pr}{2t} \quad \text{.......... \textbf{Ans.}}$$

5-107



$$p = \gamma y \qquad \theta = \sin^{-1}\frac{r/2}{r} = 30°$$

$$W = \int \gamma\, dV = \int_{r/2}^{r} \gamma\pi\left(r^2 - y^2\right) dy$$

$$= \gamma\pi\left[r^2 y - \frac{y^3}{3}\right]_{r/2}^{r} = \frac{5\gamma\pi r^3}{24}$$

$$x = \sqrt{r^2 - y^2} = \sqrt{r^2 - (r/2)^2} = \frac{r\sqrt{3}}{2}$$

$\uparrow \Sigma F_y = 0:$ $\qquad \sigma_m A_m \cos 30° - W - pA_p = 0$

$$\sigma_m\left(2\pi x t\right)\cos 30° - W - p\left(\pi x^2\right) = 0$$

$$\sigma_m\left(2\pi t\right)\left(\frac{r\sqrt{3}}{2}\right)\cos 30° - \left(\frac{5\gamma\pi r^3}{24}\right) - \pi\left(\frac{\gamma r}{2}\right)\left(\frac{r\sqrt{3}}{2}\right)^2 = 0$$

$\sigma_m = \dfrac{7\gamma r^2}{18t}$ .... **Ans.**

$$\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t} \qquad r_m = r_t = r$$

$$\frac{7\gamma r}{18t} + \frac{\sigma_t}{r} = \frac{\gamma r}{2t}$$

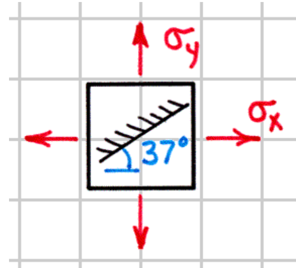
$\sigma_t = \dfrac{\gamma r^2}{9t}$ .... **Ans.**

5-108

$$\sigma_x = \sigma_h = \frac{pr}{t} = \frac{p(600)}{20} = 30p \text{ N/m}^2$$

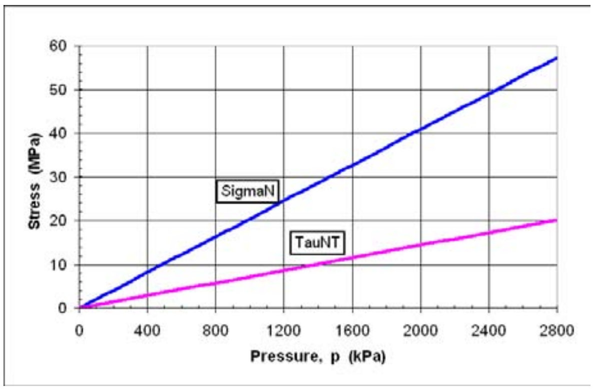
$$\sigma_y = \sigma_a = \frac{pr}{2t} = \frac{p(600)}{2(20)} = 15p \text{ N/m}^2$$

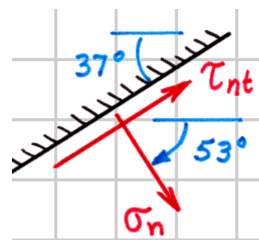
$$\tau_{xy} = 0 \text{ N/m}^2$$



$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = 30p\cos^2(-53°) + 15p\sin^2(-53°)$$

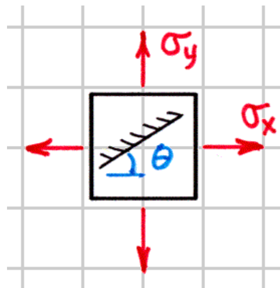
$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = -(30p - 15p)\sin(-53°)\cos(-53°)$$





5-109

$$\sigma_x = \sigma_h = \frac{pr}{t} = \frac{(200)(24)}{(0.75)} = 6400 \text{ psi}$$



$$\sigma_y = \sigma_a = \frac{pr}{2t} = \frac{(200)(24)}{2(0.75)} = 3200 \text{ psi}$$

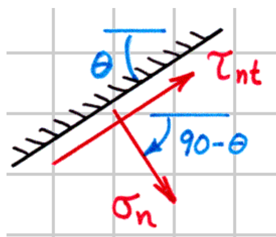
$$\tau_{xy} = 0 \text{ psi}$$

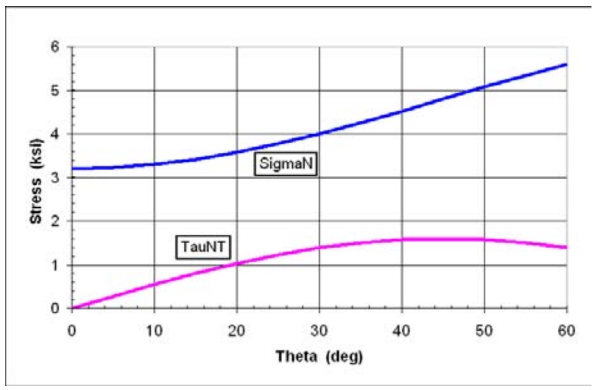
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\sigma_n = (6400)\cos^2(\theta - 90°) + 3200\sin^2(\theta - 90°)$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\tau_{nt} = -(6400 - 3200)\sin(\theta - 90°)\cos(\theta - 90°)$$





5-110*

$$\sigma_x = \frac{pr}{2t} - \frac{P}{A} = \frac{(2\times10^6)(500)}{2(20)} - \frac{(40\times10^3)}{\pi(1.040^2 - 1^2)/4}$$

$\sigma_x = 24.4(10^6)\ \text{N/m}^2 = 24.4\ \text{MPa}$ .... **Ans.**

$$\sigma_y = \frac{pr}{t} = \frac{(2\times10^6)(500)}{(20)}$$

$\sigma_y = 50.0(10^6)\ \text{N/m}^2 = 50.0\ \text{MPa}$ .... **Ans.**

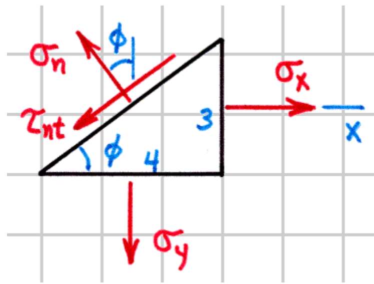
$\tau_{xy} = 0\ \text{MPa}$ .... **Ans.**

5-111

$$\sigma_x = \frac{pr}{2t} + \frac{P}{A} = \frac{(300)(1.5\times 12)}{2(0.375)} + \frac{(P)}{\pi\left(18.75^2 - 18^2\right)/4} \le 18\left(10^3\right) \text{ psi}$$

$P \le 234\left(10^3\right)$ lb $= 234$ kip .... **Ans.**

5-112



$$\sigma_x = \sigma_h = \frac{pr}{t} = \frac{(2800\times10^3)(600)}{(20)}$$

$$= 84.00(10^6)\ \text{N/m}^2 = 84.00\ \text{MPa}$$

$$\sigma_y = \frac{pr}{2t} - \frac{P}{A} = \frac{(2800\times10^3)(600)}{2(20)} - \frac{130(10^3)}{\pi(1.28^2 - 1.2^2)/4}$$

$$= 41.1657(10^6)\ \text{N/m}^2 = 41.1657\ \text{MPa}$$

$\tau_{xy} = 0\ \text{MPa}$ $\quad\phi = \tan^{-1}\frac{3}{4} = 36.870°$ $\quad\theta = 90° + \phi = 126.870°$

(a) $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

$\sigma_n = 84\cos^2(126.870°) + 41.1657\sin^2(126.870°) = 56.6\ \text{MPa (T)}$ .... **Ans.**

(b) $\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$

$\tau_{nt} = -(84 - 41.1657)\sin(126.870°)\cos(126.870°) = +20.6\ \text{MPa}$ .... **Ans.**

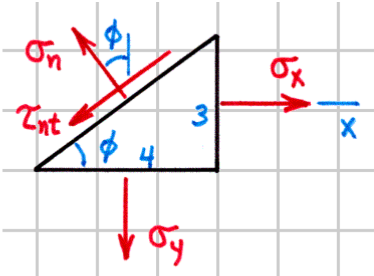
(c) $\sigma_{max} = \sigma_h = 84.00\ \text{MPa}$ $\quad\sigma_{min} = \sigma_z = 0\ \text{MPa}$

$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{84.00 - 0}{2} = 42.0\ \text{MPa}$ .... **Ans.**

(d) $\sigma_{max} = \sigma_h = 84.00\ \text{MPa}$ $\quad\sigma_{min} = \sigma_z = -2.800\ \text{MPa}$

$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{84.00 - (-2.800)}{2} = 43.4\ \text{MPa}$ .... **Ans.**

5-113*



$$\sigma_x = \frac{pr}{2t} + \frac{P}{A} = \frac{p(20)}{2(0.4)} - \frac{(10,000)}{\pi\left(20.8^2 - 20^2\right)/4}$$

$$= 25p + 390.08564 \text{ psi}$$

$$\sigma_y = \frac{pr}{t} = \frac{p(20)}{(0.4)} = 50p$$

$$\tau_{xy} = 0 \text{ MPa} \qquad \phi = \tan^{-1}\frac{3}{4} = 36.870° \qquad \theta = 90° + \phi = 126.870°$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= (25p + 390.08564)\cos^2(126.870°) + (50p)\sin^2(126.870°) + 0$$

$$= 41p + 140.43150 \le 11,000$$

$p = 265$ psi .... **Ans.**

5-114*

$$\sigma_{axial} = \frac{pr}{2t} + \frac{P}{A} = \frac{(2500)(500)}{2(20)} + \frac{50(10^3)}{\pi(1.04^2 - 1^2)/4}$$

$$= 811.421(10^3)\ \text{N/m}^2 = 811.421\ \text{kPa}$$

$$\sigma_{hoop} = \frac{pr}{t} = \frac{(2500)(500)}{(20)} = 62.500(10^3)\ \text{N/m}^2 = 62.500\ \text{kPa}$$

$$E = 2(1+\nu)G \qquad \nu = \frac{200}{2(76)} - 1 = 0.31579$$

$$\varepsilon_{axial} = \frac{\sigma_{axial} - \nu\sigma_{hoop}}{E} = \frac{811{,}421 - 0.31579(62{,}500)}{200(10^9)}$$

$\varepsilon_{axial} = 3.96(10^{-6}) = 3.96\ \mu\text{m/m}$ .... **Ans.**

$$\varepsilon_{hoop} = \frac{\sigma_{hoop} - \nu\sigma_{axial}}{E} = \frac{62{,}500 - 0.31579(811{,}421)}{200(10^9)}$$

$\varepsilon_{hoop} = -0.969(10^{-6}) = -0.969\ \mu\text{m/m}$ .... **Ans.**

5-115

$$\sigma_x = \frac{pr}{2t} + \frac{P}{A} = \frac{(100)(2\times 12)}{2(0.25)} + \frac{(5000)}{\pi\left(48.5^2 - 48^2\right)/4} = 4931.94 \text{ psi}$$

$$\sigma_y = \frac{pr}{t} = \frac{(100)(2\times 12)}{(0.25)} = 9600.00 \text{ psi}$$

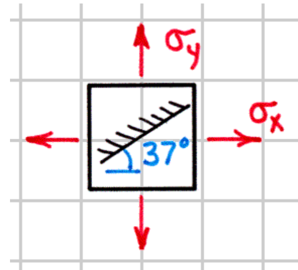
Outside: $\sigma_{\max} = 9600$ psi $\sigma_{\min} = 0$ psi

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{9600 - 0}{2} = 4800 \text{ psi}$$ .... **Ans.**

$\sigma_{\max} = 9600$ psi $\sigma_{\min} = -p = -100$ psi

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{9600 - (-100)}{2} = 4850 \text{ psi}$$ .... **Ans.**

5-116



$$\sigma_x = \sigma_h = \frac{pr}{t} = \frac{p(600)}{(20)} = 30p \text{ N/m}^2$$

$$\sigma_y = \frac{pr}{2t} - \frac{P}{A} = \frac{p(600)}{2(20)} - \frac{130(10^3)}{\pi(1.28^2 - 1.2^2)/4}$$

$$= 15p - 834,280 \text{ N/m}^2$$

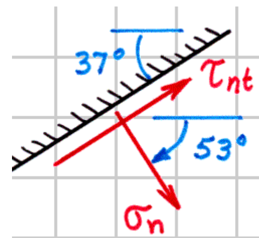
$$\tau_{xy} = 0 \text{ N/m}^2$$

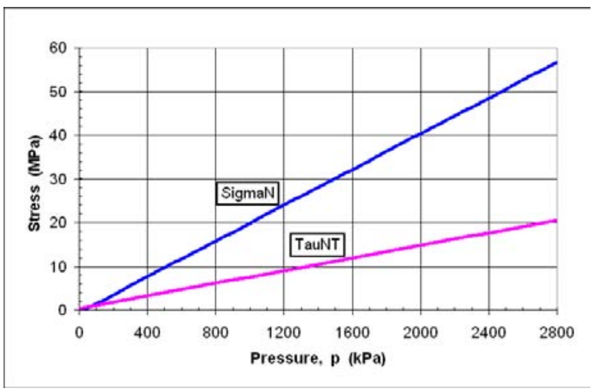
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\sigma_n = (30p)\cos^2(-53°) + (15p - 834,280)\sin^2(-53°) \text{ N/m}^2$$

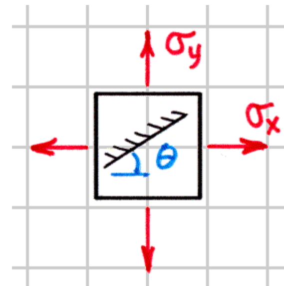
$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\tau_{nt} = -\left[(30p) - (15p - 834,280)\right]\sin(-53°)\cos(-53°) \text{ N/m}^2$$





5-117



$$\sigma_x = \sigma_h = \frac{pr}{t} = \frac{(200)(24)}{(0.75)} = 6400 \text{ psi}$$

$$\sigma_y = \frac{pr}{2t} - \frac{P}{A} = \frac{(200)(24)}{2(0.75)} - \frac{30,000}{\pi(4\times 12)(0.75)} = 2934.74 \text{ psi}$$

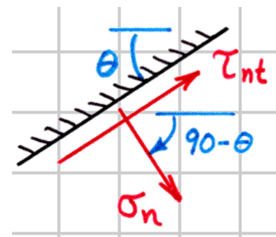
$$\tau_{xy} = 0 \text{ psi}$$

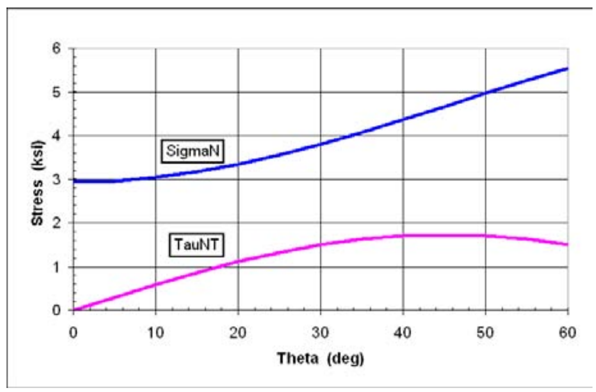
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\sigma_n = (6400)\cos^2(\theta - 90°) + 2934.74\sin^2(\theta - 90°)$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\tau_{nt} = -(6400 - 2934.74)\sin(\theta - 90°)\cos(\theta - 90°)$$





5-118*

(a) $$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{125^2(75)}{200^2 - 125^2}\left(1 + \frac{200^2}{125^2}\right) = 171.2 \text{ MPa}$$ .... **Ans.**

(b) $$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{b^2}\right) = \frac{125^2(75)}{200^2 - 125^2}\left(1 + \frac{200^2}{200^2}\right) = 96.2 \text{ MPa}$$ .... **Ans.**

(c) $$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_t - (-p)}{2} = \frac{171.2 - (-75)}{2} = 123.1 \text{ MPa}$$ .... **Ans.**

5-119

$$t = b - a = 0.1a \qquad b = 1.1a$$

From thick-walled cylinder equations:

$$\sigma_t = \frac{a^2 p}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{a^2 p}{(1.1a)^2 - a^2}\left(1 + \frac{(1.1a)^2}{a^2}\right) = 10.524p$$

From thin-walled cylinder equations:

$$\sigma_h = \sigma_t = \frac{pr}{2t} = \frac{pa}{0.1a} = 10p$$

$$Error = \frac{10.524p - 10p}{10.524p}(100) = 4.979 \cong 5\%$$ .... **Ans.**

5-120

$$\sigma_{\max} = \sigma_t \quad \text{at} \quad \rho = a$$

$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{100^2 p_i}{150^2 - 100^2}\left(1 + \frac{150^2}{100^2}\right) \le 430 \text{ MPa}$$

$p_i \le 165.4$ MPa .... **Ans.**

5-121*

$$\sigma_{\max} = \sigma_t \quad \text{at} \quad \rho = a$$

$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{2^2 p_i}{4^2 - 2^2}\left(1 + \frac{4^2}{2^2}\right) = 1.6667\, p_i$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_t - (-p)}{2} = \frac{1.6667\, p_i - (-p_i)}{2} \le 24 \text{ ksi}$$

$p_i \le 18.00$ ksi .... **Ans.**

5-122

(a) $$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{100^2(75)}{150^2 - 100^2}\left(1 + \frac{150^2}{100^2}\right) = 195.0 \text{ MPa}$$ .... **Ans.**

(b) At $\rho = 125$ mm

$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{\rho^2}\right) = \frac{100^2(75)}{150^2 - 100^2}\left(1 + \frac{150^2}{125^2}\right) = 146.4 \text{ MPa}$$

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{\rho^2}\right) = \frac{100^2(75)}{150^2 - 100^2}\left(1 - \frac{150^2}{125^2}\right) = -26.4 \text{ MPa}$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_t - \sigma_r}{2} = \frac{146.4 - (-26.4)}{2} = 86.4 \text{ MPa}$$ .... **Ans.**

5-123*

(a) $$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{4^2(25)}{7^2 - 4^2}\left(1 + \frac{7^2}{4^2}\right) = 49.242 \text{ ksi}$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_t - (-p)}{2} = \frac{49.242 - (-25)}{2} = 37.1 \text{ ksi}$$ .... **Ans.**

(b) At $\rho = 5.5$ in.

$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{\rho^2}\right) = \frac{4^2(25)}{7^2 - 4^2}\left(1 + \frac{7^2}{5.5^2}\right) = 31.8 \text{ ksi}$$ .... **Ans.**

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{\rho^2}\right) = \frac{4^2(25)}{7^2 - 4^2}\left(1 - \frac{7^2}{5.5^2}\right) = -7.51 \text{ ksi}$$ .... **Ans.**

5-124*

$$\sigma_t = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{a^2 b^2 \left(p_i - p_o\right)}{\left(b^2 - a^2\right)\rho^2}$$

$$= \frac{25^2(85) - 125^2(30)}{125^2 - 25^2} + \frac{(25)^2(125)^2(85-30)}{\left(125^2 - 25^2\right)\rho^2} = \left(-27.708 + \frac{35,807}{\rho^2}\right) \text{ MPa}$$

$$F = \int_{50}^{75}\left[(-27.708) + \frac{35,807}{\rho^2}\right] d\rho = \left[(-27.708)\rho - \frac{35,807}{\rho}\right]_{50}^{75}$$

$F = -454\left(10^3\right)$ N/m $= -454$ kN/m .... **Ans.**

5-125

(a) $$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{8^2(20)}{b^2 - 8^2}\left(1 + \frac{b^2}{8^2}\right) \le 36 \text{ ksi}$$

$b \ge 14.967 \text{ in.} \cong 14.97 \text{ in.}$ .... **Ans.**

(b) $$E = 2(1+\nu)G \qquad \nu = \frac{30,000}{2(11,600)} - 1 = 0.29310$$

$$\delta_a = \frac{a^2 p_i}{(b^2 - a^2)Ea}\left[(1-\nu)a^2 + (1+\nu)b^2\right]$$

$$= \frac{8^2(20)}{(14.967^2 - 8^2)(30,000)(8)}\left[(1-0.29310)(8)^2 + (1+0.29310)(14.967)^2\right]$$

$$\delta_a = 0.01162 \text{ in.}$$

$\Delta D_i = 2\delta_a = 2(0.01162) = 0.0223 \text{ in.}$ .... **Ans.**

5-126*

From Table B-18 $\sigma_y = 250 \text{ MPa}$

$$\sigma = \frac{P}{A} = \frac{100\left(10^3\right)}{A} \le \frac{250\left(10^6\right)}{1.6}$$

$$A \ge 640\left(10^{-6}\right) \text{ m}^2 = 640 \text{ mm}^2$$

From Table B-14 $d_{\text{min}} = 51 \text{ mm}$ .... **Ans.**

`

5-127*

From Table B-17 $\sigma_y = 36 \text{ ksi}$

$$\sigma = \frac{P}{A} = \frac{80}{A} \leq \frac{36}{3}$$

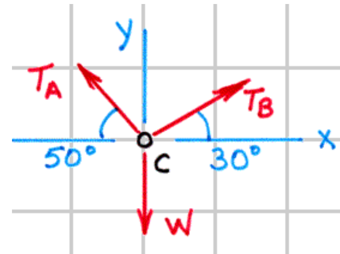
$$A \geq 6.667 \text{ in.}^2$$

From Table B2, sections with $A \geq 6.667 \text{ in.}^2$ include $\text{W}6\times 25,\quad \text{W}8\times 24,\quad \text{W}10\times 30,\quad \text{W}12\times 30$

The lightest section is $\text{W}8\times 24$ .... **Ans.**

5-128

$$W = 2000(9.81) = 19,620 \text{ N}$$



$\rightarrow \Sigma F_x = 0:$ $\quad T_B \cos 30° - T_A \cos 50° = 0$

$\uparrow \Sigma F_y = 0:$ $\quad T_B \sin 30° + T_A \sin 50° - 19,620 = 0$

$T_A = 17,253.54 \text{ N (T)}$ $\qquad T_B = 12,806.05 \text{ N (T)}$

From Table B-18 $\qquad \sigma_y = 250 \text{ MPa}$

$$\sigma_A = \frac{P}{A} = \frac{17,253.54}{A} \leq \frac{250(10^6)}{1.75} \qquad A \geq 120.771(10^{-6}) \text{ m}^2$$

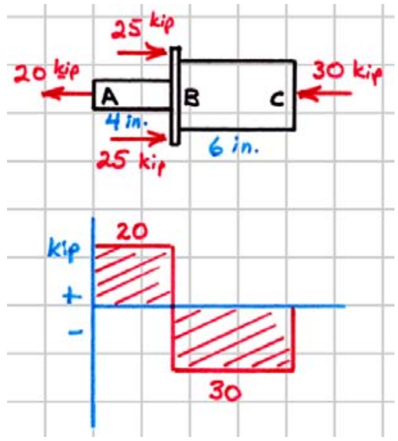
$$\sigma_B = \frac{12,806.05}{A} \leq \frac{250(10^6)}{1.75} \qquad A \geq 89.642(10^{-6}) \text{ m}^2$$

$$A = \pi d^2/4 \geq 120.771 \text{ mm}^2 \qquad d \geq 12.40 \text{ mm}$$

$d_{min} = 13 \text{ mm}$ .... **Ans.**

5-129*

From Table B-17 $\sigma_y = 50\text{ ksi}$



$$\sigma_{AB} = \frac{P}{A} = \frac{20}{\pi d_{AB}^2/4} \le \frac{50}{1.5}$$

$$d_{AB} \ge 0.874\text{ in.} = d_{\min AB}$$ **Ans.**

$$\sigma_{BC} = \frac{P}{A} = \frac{30}{\pi d_{BC}^2/4} \le \frac{50}{1.5}$$

$$d_{BC} \ge 1.070\text{ in.} = d_{\min BC}$$ **Ans.**

5-130*

From Table B-18 concrete $\sigma_f = 34 \text{ MPa}$ $E = 31 \text{ GPa}$

steel $\sigma_y = 250 \text{ MPa}$ $E = 200 \text{ GPa}$

$$A_S = b_S^2 - b_C^2 \qquad A_C = b_C^2 \qquad A_C = 10A_S$$

$$F_C + F_S = 1000 \text{ kN} \qquad \delta_C = \delta_S$$

$$\frac{F_C L}{A_C\left(31\times10^9\right)} = \frac{F_S L}{A_S\left(200\times10^9\right)}$$

$$F_S = 392{,}156.86 \text{ N} \qquad F_C = 607{,}843.14 \text{ N}$$

Try $\sigma_C = \dfrac{P}{A} = \dfrac{607{,}843.14}{A_C} \le \dfrac{34\left(10^6\right)}{1.4}$ $\qquad A_C \ge 25.0288\left(10^{-3}\right) \text{ m}^2$

Then $A_S = 2.50288\left(10^{-3}\right) \text{ m}^2$

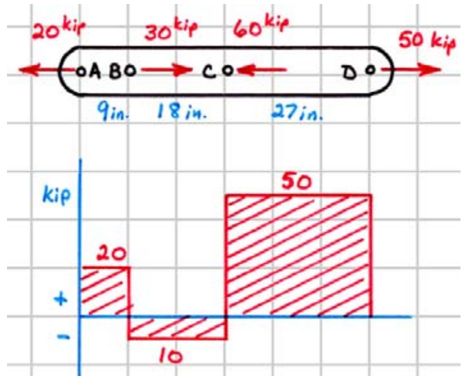
$$\sigma_S = \frac{392{,}156.86}{2.50288\left(10^{-3}\right)} = 156.7\left(10^6\right) \text{ N/m}^2 \le 250 \text{ MPa} \quad \text{(correct guess)}$$

$b_C = \sqrt{25.0288\left(10^{-3}\right)} = 0.1582 \text{ m} = 158.2 \text{ mm}$ .... **Ans.**

$$A_S = b_S^2 - b_C^2 = 2.50288\left(10^{-3}\right)$$

$b_S = \sqrt{25.0288\left(10^{-3}\right) + 2.50288\left(10^{-3}\right)} = 0.1659 \text{ m} = 165.9 \text{ mm}$ .... **Ans.**

5-131



From Table B-17 $\sigma_y = 53\text{ ksi}$

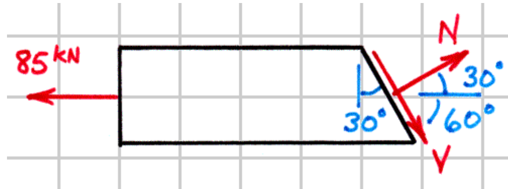
$$\sigma_{AB} = \frac{P}{A} = \frac{20}{1w} \leq \frac{53}{1.75} \qquad w \geq 0.66038\text{ in.}$$

$$\sigma_{BC} = \frac{10}{1w} \leq \frac{53}{1.75} \qquad w \geq 0.33019\text{ in.}$$

$$\sigma_{CD} = \frac{50}{1w} \leq \frac{53}{1.75} \qquad w \geq 1.65094\text{ in.}$$

$w_{\min} = 1.651\text{ in.}$ **Ans.**

5-132



$\rightarrow \Sigma F_x = 0: \quad N\cos 30° + V\cos 60° - 85 = 0$

$\uparrow \Sigma F_y = 0: \quad N\sin 30° - V\sin 60° = 0$

$N = 73.6122 \text{ kN} \qquad V = 42.500 \text{ kN}$

$$\sigma = \frac{N}{A} = \frac{73{,}612.2}{2\left(\pi d^2/4\right)} \le \frac{1035\left(10^6\right)}{1.5}$$

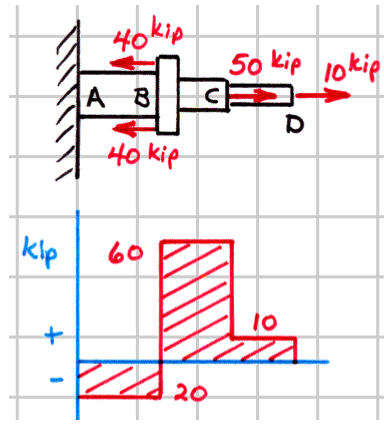
(There are two bolts)

$$d \ge 8.24\left(10^{-3}\right) \text{ m}$$

$$\tau = \frac{V}{A} = \frac{42{,}500}{2\left(\pi d^2/4\right)} \le \frac{620\left(10^6\right)}{1.5} \qquad d \ge 8.09\left(10^{-3}\right) \text{ m}$$

$d_{\min} = 8.24$ mm .... **Ans.**

5-133*



$$\sigma_{AB} = \frac{P}{A} = \frac{20}{\pi(2.50)^2/4} = 4.07 \text{ ksi (C)}$$ .... **Ans.**

$$\sigma_{BC} = \frac{60}{\pi(1.50)^2/4} = 34.0 \text{ ksi (T)}$$ .... **Ans.**

$$\sigma_{CD} = \frac{10}{\pi(1)^2/4} = 12.73 \text{ ksi (T)}$$ .... **Ans.**

$$\delta_{total} = \sum\frac{PL}{AE} = \frac{(-20)(15)}{\left[\pi(2.50)^2/4\right](30,000)} + \frac{(60)(15)}{\left[\pi(1.50)^2/4\right](30,000)} + \frac{(10)(15)}{\left[\pi(1)^2/4\right](30,000)}$$

$\delta = +0.0213$ in. .... **Ans.**

5-134*

$$\sigma_B = \frac{P}{A} = \frac{P}{\pi(0.050)^2/4} \le 200(10^6) \qquad P \le 392.7(10^3)\text{ N}$$

$$\sigma_S = \frac{P}{A} = \frac{P}{\pi(0.032)^2/4} \le 500(10^6) \qquad P \le 402.1(10^3)\text{ N}$$

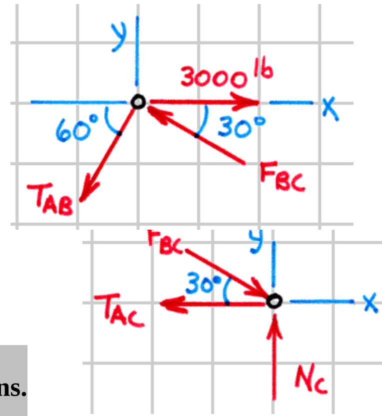
$$\delta_{total} = \delta_B + \delta_S$$

$$\frac{P(1500)}{\left[\pi(0.050)^2/4\right](100\times 10^9)} + \frac{P(1000)}{\left[\pi(0.032)^2/4\right](190\times 10^9)} \le 5.60\text{ mm}$$

$$P \le 394.8(10^3)\text{ N}$$

$P_{\max} = 393$ kN .... **Ans.**

5-135



$\rightarrow \Sigma F_x = 0:$ $\quad F_{BC}\cos 30° + T_{AB}\cos 60° - 3000 = 0$

$\uparrow \Sigma F_y = 0:$ $\quad F_{BC}\sin 30° - T_{AB}\sin 60° = 0$

$T_{AB} = 1500 \text{ lb}$

$F_{BC} = 2598.076 \text{ lb}$

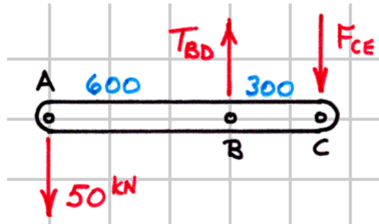
$\rightarrow \Sigma F_x = 0:$ $\quad F_{BC}\cos 30° - T_{AC} = 0$

$T_{AC} = 2250 \text{ lb}$

$\sigma_{AC} = \dfrac{N}{A} = \dfrac{2.250}{\pi(0.5)^2/4} = 11.46 \text{ ksi}$ ....**Ans.**

$\delta_{AC} = \dfrac{PL}{AE} = \dfrac{(2.250)(30)}{\left[\pi(0.5)^2/4\right](30{,}000)} = +0.01146 \text{ in.}$ .... **Ans.**

5-136*



$\uparrow \Sigma F_y = 0:$ $T_{BD} - F_{CE} - 50 = 0$

$\circlearrowleft \Sigma M_C = 0:$ $900(50) - 300T_{BD} = 0$

$T_{BD} = 150 \text{ kN (T)}$

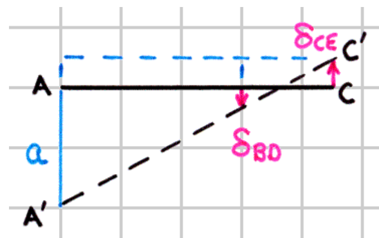
$F_{CE} = 100 \text{ kN (C)}$

$\sigma_{BD} = \dfrac{N}{A} = \dfrac{150(10^3)}{1250(10^{-6})} = 120.0(10^6) \text{ N/m}^2 = 120.0 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{CE} = \dfrac{100(10^3)}{750(10^{-6})} = 133.3(10^6) \text{ N/m}^2 = 133.3 \text{ MPa (C)}$ .... **Ans.**

$$\delta_{BD} = \frac{PL}{AE} = \frac{(150\times10^3)(600)}{[1250\times10^{-6}](73\times10^9)} = 0.98630 \text{ mm}$$

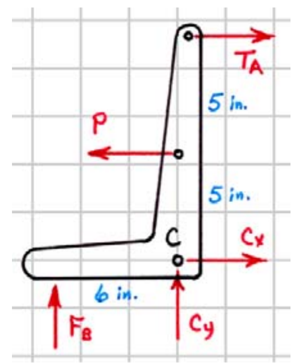
$$\delta_{CE} = \frac{PL}{AE} = \frac{(100\times10^3)(400)}{[750\times10^{-6}](200\times10^9)} = 0.26667 \text{ mm}$$



$$\frac{\delta_{CE} + \delta_{BD}}{300} = \frac{\delta_{CE} + a}{900}$$

$v_A = a = 3.49 \text{ mm} \downarrow$ .... **Ans.**

5-137



$\circlearrowleft \Sigma M_C = 0: \qquad 5P - 10T_A - 6F_B = 0$

$$10T_A + 6F_B = 5P \qquad \text{(a)}$$

$$\delta_A = (10/6)(\delta_B + 0.009) = (10\delta_B/6) + 0.015 \text{ in.}$$

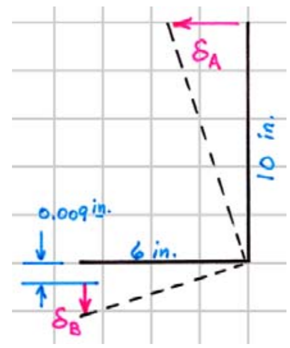
$$\frac{T_A(50)}{(1.24)(30{,}000)} = \frac{10}{6}\left[\frac{F_B(15)}{(4)(15{,}000)}\right] + 0.015 \qquad \text{(b)}$$

If $\sigma_A = \sigma_{\max} = 30 \text{ ksi}$, then

$$T_A = 30(1.24) = 37.20 \text{ kip}$$

$$F_B = 84.00 \text{ kip} \qquad P = 175.2 \text{ kip}$$

$$\sigma_B = \frac{84.00}{4} = 21 \text{ ksi} > 20 \text{ ksi} \quad \text{(wrong guess)}$$



If $\sigma_B = \sigma_{\max} = 20 \text{ ksi}$, then

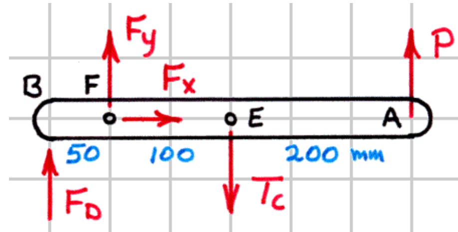
$$F_B = 20(4) = 80.00 \text{ kip}$$

$$T_A = 35.9600 \text{ kip} \qquad P = 167.9 \text{ kip}$$

$$\sigma_A = \frac{35.9600}{1.24} = 29.00 \text{ ksi} < 230\text{ksi} \quad \text{(correct guess)}$$

$$P_{\max} = 167.9 \text{ kip}$$

5-138



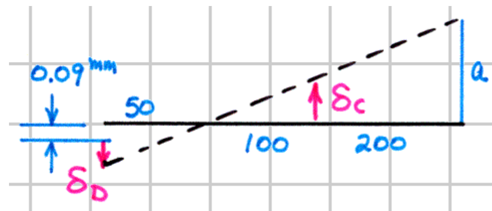
$\circlearrowleft \Sigma M_F = 0: \quad 300P - 50F_D - 100T_C = 0$

$$F_D + 2T_C = 6P \qquad \text{(a)}$$

$$\delta_C = 2(\delta_D + 0.09) \text{ mm}$$

$$\frac{T_C(300)}{(625\times10^{-6})(73\times10^9)} = 2\left[\frac{F_D(150)}{(2500\times10^{-6})(12\times10^9)} + 0.09\right]$$

$$T_C = (1.52083F_D + 2737.50) \text{ N} \qquad \text{(b)}$$



Guess that

$$T_C = T_{C\max} = \sigma A$$

$$= (100\times10^6)(625\times10^{-6}) = 62,500 \text{ N}$$

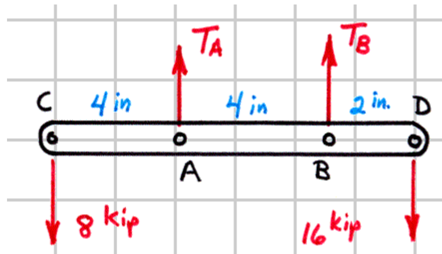
Then $F_D = 39,295.98$ N $\qquad P = 27,383$ N

and $\sigma_D = \dfrac{N}{A} = \dfrac{39,295.98}{2500(10^{-6})} = 15.72(10^6) \text{ N/m}^2 = 15.72 \text{ MPa}$

Since $\sigma_D = 15.72 \text{ MPa} < \sigma_{\max} = 30 \text{ MPa}$, the guess was correct and

$P_{\max} = 27.4$ kN .... **Ans.**

5-139*



$\uparrow \Sigma F_y = 0: \qquad T_A + T_B - 8 - 16 = 0$

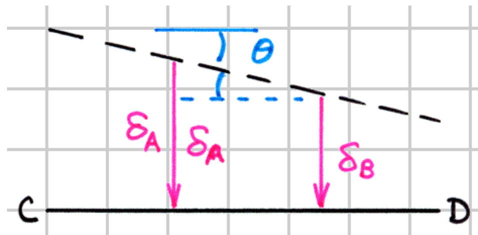
$\circlearrowleft \Sigma M_B = 0: \qquad 8(8) - 4T_A - 2(16) = 0$

$$T_A = 8.00 \text{ kip (T)}$$

$$T_B = 16.00 \text{ kip (T)}$$

$$\delta_A = \delta_B + 4\tan\theta = \delta_B + 4(5/10{,}000) = (\delta_B + 0.00200) \text{ in.}$$

$$\frac{\sigma_A(10)}{(10{,}600)} = \frac{\sigma_B(16)}{(15{,}000)} + 0.00200 \qquad \sigma_A = 1.13067\sigma_B + 2.12000$$



Guess that

$$\sigma_B = \sigma_{B\max} = 15 \text{ ksi}$$

Then $\quad \sigma_A = 19.0802 \text{ ksi}$

Since $\sigma_A < 20 \text{ ksi} = \sigma_{A\max}$, the guess was correct and

(a) $\quad \sigma_A = \dfrac{N}{A} = \dfrac{8}{A} \le 19.0802 \text{ ksi}$

$A_A \ge 0.419 \text{ in.}^2 = A_{A\min}$ .... **Ans.**

$\sigma_B = \dfrac{N}{A} = \dfrac{16}{A} \le 15 \text{ ksi} \qquad A_B \ge 1.067 \text{ in.}^2 = A_{B\min}$ .... **Ans.**

(b) $\quad \delta_A = \dfrac{\sigma L}{E} = \dfrac{(19.0802)(10)}{(10{,}600)} = 0.01800 \text{ in.}$ .... **Ans.**

$\delta_B = \dfrac{(15)(16)}{(15{,}000)} = 0.01600 \text{ in.}$ .... **Ans.**

5-140

$$T_B = T_S \qquad \text{(a)}$$

$$\delta_B + \delta_S = 0$$

$$\frac{T_B(800)}{\left[\pi(0.090)^2/4\right](100\times10^9)} + (17.6\times10^{-6})(-70)(800)$$

$$+\frac{T_S(480)}{\left[\pi(0.050)^2/4\right](200\times10^9)} + (11.9\times10^{-6})(-70)(480) = 0$$

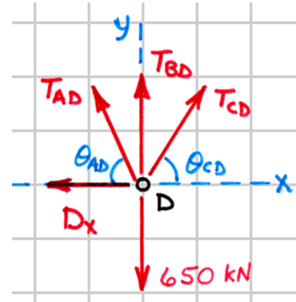
$$T_B + 0.97200 T_S = 1.101724(10^6) \text{ N} \qquad \text{(b)}$$

$$T_B = T_S = 558,683 \text{ kN (both T)}$$

$\sigma_B = \dfrac{N}{A} = \dfrac{558,683}{\pi(0.090)^2/4} = 87.8(10^6) \text{ N/m}^2 = 87.8 \text{ MPa (T)}$ .... **Ans.**

$\sigma_S = \dfrac{558,683}{\pi(0.050)^2/4} = 285(10^6) \text{ N/m}^2 = 285 \text{ MPa (T)}$ .... **Ans.**

5-141



$$\sin\theta_{CD} = 4/5 \qquad \cos\theta_{CD} = 3/5$$

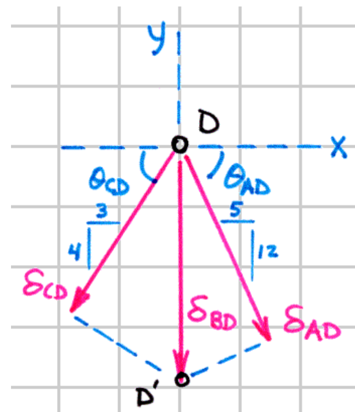
$$\sin\theta_{AD} = 12/13 \qquad \cos\theta_{AD} = 5/13$$

$$A_{AD} = A_{BD} = A_{CD} = (50\times 25) = 1250 \text{ mm}^2$$

$$(12/13)T_{AD} + T_{BD} + (4/5)T_{CD} = 650 \text{ kN}$$

$$\left[(12/13)\sigma_{AD} + \sigma_{BD} + (4/5)\sigma_{CD}\right](1250\times 10^{-6}) = 650 \text{ kN}$$

$$(12/13)\sigma_{AD} + \sigma_{BD} + (4/5)\sigma_{CD} = 520(10^6) \text{ N/m}^2 = 520 \text{ MPa} \qquad \text{(a)}$$



$$\delta_{AD} = \delta_{BD}\sin\theta_{AD} = (12/13)\delta_{BD} \qquad \text{(b)}$$

$$\delta_{CD} = \delta_{BD}\sin\theta_{CD} = (4/5)\delta_{BD} \qquad \text{(c)}$$

Assume all bars elastic:

$$\frac{\sigma_{AD}L}{(180)} = \left(\frac{12}{13}\right)\left[\frac{\sigma_{BD}L}{(40)}\right] \qquad \sigma_{AD} = 4.15385\sigma_{BD} \qquad \text{(b)}$$

$$\frac{\sigma_{CD}L}{A(200)} = \left(\frac{4}{5}\right)\left[\frac{\sigma_{BD}L}{A(40)}\right] \qquad \sigma_{CD} = 4.00000\sigma_{BD} \qquad \text{(c)}$$

$$\sigma_{AD} = 268.8468 \text{ MPa} < 400 \text{ MPa (elastic)}$$

$$\sigma_{BD} = 64.7223 \text{ MPa} < 100 \text{ MPa (elastic)}$$

$$\sigma_{CD} = 258.8893 \text{ MPa} > 240 \text{ MPa (plastic)}$$

Therefore

(a) $\sigma_{CD} = \sigma_y = 240 \text{ MPa (T)}$ .... **Ans.**

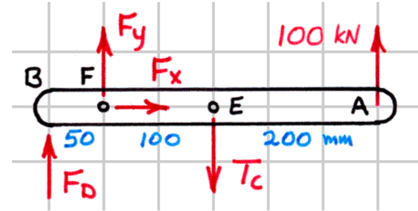
and from Eqs. (a) and (b)

$\sigma_{BD} = 67.8482 \text{ MPa (T)} \cong 67.8 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{AD} = 282 \text{ MPa (T)}$ .... **Ans.**

(b) $v_D = \delta_{BD} = \dfrac{\sigma L}{E} = \dfrac{(67.8482\times 10^6)(4000)}{(40\times 10^9)} = 6.78 \text{ mm} \downarrow$ .... **Ans.**

5-142*



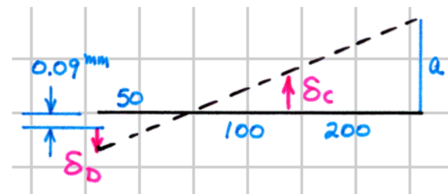
$$\circlearrowleft \Sigma M_F = 0: \quad 300(100) - 50F_D - 100T_C = 0$$

$$F_D + 2T_C = 600 \text{ kN}$$

$$\sigma_D(2000\times10^{-6}) + 2\sigma_C(600\times10^{-6}) = 600 \text{ kN}$$

$$5\sigma_D + 3\sigma_C = 1500(10^6) \text{ N/m}^2 = 1500 \text{ MPa} \qquad \text{(a)}$$

$$\delta_C = 2(\delta_D + 0.09) \text{ mm} \qquad \text{(b)}$$



Assume both bars elastic. Then

$$\frac{\sigma_C(300)}{(200\times10^9)} = 2\left[\frac{\sigma_D(150)}{(100\times10^9)} + 0.09\right]$$

$$\sigma_C = 2\sigma_D + 120(10^6) \text{ N/m}^2 = 2\sigma_D + 120 \text{ MPa} \qquad \text{(b)}$$

$$\sigma_C = 327.273 \text{ MPa} > 240 \text{ MPa (plastic)}$$

$$\sigma_D = 103.636 \text{ MPa} < 410 \text{ MPa (elastic)}$$

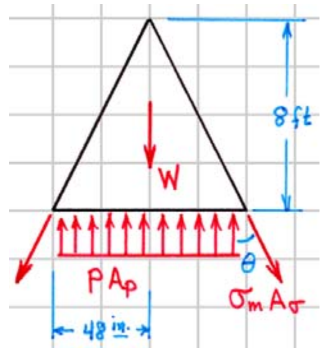
Therefore

(a) $\sigma_C = \sigma_y = 240.0$ MPa .... **Ans.**

and from Eq. (a) $\sigma_D = 156.0$ MPa .... **Ans.**

(b) $v_A = a = 6(\delta_D + 0.09) = 6\left[\frac{(156.0\times10^6)(150)}{(100\times10^9)} + 0.09\right] = 1.944 \text{ mm} \uparrow$ .... **Ans.**

5-143*



$$\theta = \tan^{-1}\frac{4}{8} = 26.565°$$

$$p = \gamma y = (62.4)(8) = 499.20 \text{ psf } = 3.46667 \text{ psi}$$

$$W = \gamma V = \frac{\gamma \pi r^2 h}{3} = \frac{(62.4)\pi(4)^2(8)}{3} = 8364.176 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad pA_p - W - \sigma_m A_\sigma \cos 26.565° = 0$$

$$(499.20)\pi(4)^2 - 8364.176 - \sigma_m 2\pi(48)(1/8)\cos 26.565° = 0$$

$\sigma_m = 496.109 \text{ psi} \cong 496 \text{ psi (T)}$ .... **Ans.**

$$\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t} \qquad r_m = \infty \qquad r_t = r/\cos\theta = 53.6656 \text{ in.}$$

$\sigma_t = \dfrac{pr_t}{t} = \dfrac{(3.46667)(53.6656)}{1/8} = 1488 \text{ psi (T)}$ .... **Ans.**

5-144

$$\sigma_a = \frac{pr}{2t} = \sigma_x \qquad \sigma_h = \frac{pr}{t} = \sigma_y \qquad \tau_{xy} = 0$$

$$\varepsilon_a = \frac{\sigma_a}{E} - \frac{\nu\sigma_h}{E} = \frac{pr}{2tE} - \frac{\nu pr}{tE}$$

(a) $$p = \frac{\varepsilon_a 2tE}{r(1-2\nu)} = \frac{(300\times10^{-6})(20)(200\times10^{9})}{(1000-10)(1-0.6)} = 3.030303(10^{6})\ \text{N/m}^2$$

$p = 3.030303\ \text{MPa} \cong 3.03\ \text{MPa}$ .... **Ans.**

(b) $$\sigma_a = \frac{pr}{2t} = \frac{(3.030303)(1000-10)}{2(10)} = 150.0\ \text{MPa}$$ .... **Ans.**

$$\sigma_h = \frac{pr}{t} = \frac{(3.030303)(1000-10)}{(10)} = 300.0\ \text{MPa}$$ .... **Ans.**

(c) $\sigma_z = -p = -3.030303\ \text{MPa}$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{(300)-(-3.030303)}{2} = 151.5\ \text{MPa}$$ .... **Ans.**

(d) $$\varepsilon_h = \frac{\sigma_h - \nu\sigma_a}{E} = \frac{(300\times10^{6}) - 0.3(150\times10^{6})}{200(10^{9})} = 1275(10^{-6})$$

$\varepsilon_h = 1275\ \mu\text{m/m}$ .... **Ans.**

5-145

$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{1.5^2\, p_i}{3.5^2 - 1.5^2}\left(1 + \frac{3.5^2}{1.5^2}\right) = 50 \text{ ksi}$$

$p_i = 34.5$ ksi .... **Ans.**

5-146*

(a) $$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \frac{100^2(125)}{225^2 - 100^2}\left(1 + \frac{225^2}{100^2}\right) = 186.5 \text{ MPa}$$ .... **Ans.**

(b) $$\delta_a = \frac{a^2 p_i}{(b^2 - a^2)Ea}\left[(1-\nu)a^2 + (1+\nu)b^2\right]$$

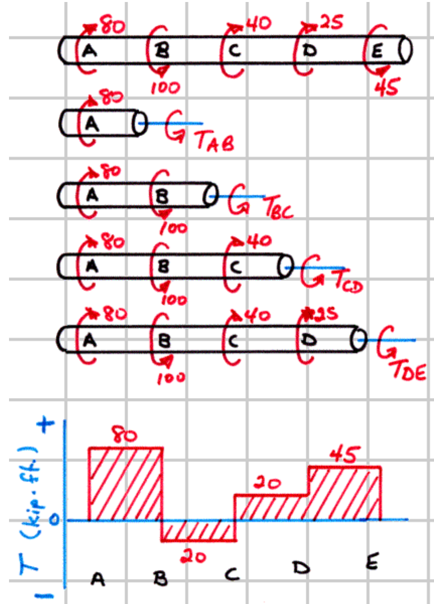
$$\delta_a = \frac{100^2(125)}{(225^2 - 100^2)(210{,}000)(100)}\left[(1-0.30)(100)^2 + (1+0.30)(225)^2\right]$$

$$= 0.106685 \text{ mm}$$

$$\Delta D_i = 2\delta_a = 2(0.106685) = 0.213 \text{ mm}$$ .... **Ans.**

6-1*

(a) Free-body diagrams for parts of the shaft to the left of sections in the intervals *AB*, *BC*, *CD*, and *DE* of the shaft are shown. From the free-body diagrams:



$\circlearrowleft \Sigma M = 0: \quad T_{AB} - 80 = 0$

$T_{AB} = +80 \text{ kip} \cdot \text{ft}$ .... **Ans.**

$\circlearrowleft \Sigma M = 0: \quad T_{BC} - 80 + 100 = 0$

$T_{BC} = -20 \text{ kip} \cdot \text{ft}$ .... **Ans.**

$\circlearrowleft \Sigma M = 0: \quad T_{CD} - 80 + 100 - 40 = 0$

$T_{CD} = +20 \text{ kip} \cdot \text{ft}$ .... **Ans.**

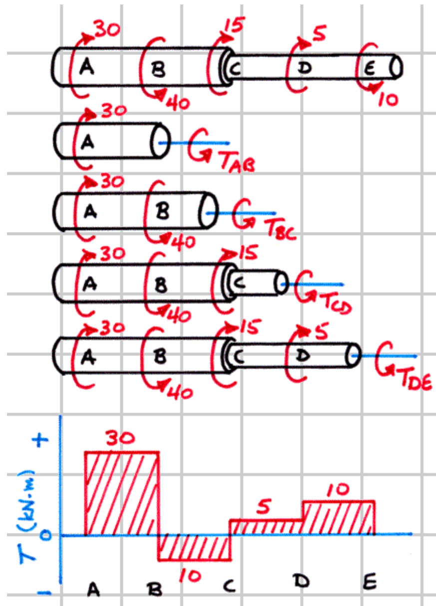
$\circlearrowleft \Sigma M = 0: \quad T_{DE} - 80 + 100 - 40 - 25 = 0$

$T_{DE} = +45 \text{ kip} \cdot \text{ft}$ .... **Ans.**

(b) A torque diagram for the shaft is shown below the free-body diagrams.

6-2*

Free-body diagrams for parts of the shaft to the left of sections in the intervals *AB*, *BC*, *CD*, and *DE* of the shaft are shown. From the free-body diagrams:



$\circlearrowleft \Sigma M = 0: \quad T_{AB} - 30 = 0$

(b) $T_{AB} = +30 \text{ kN} \cdot \text{m} = T_{\max}$ .... **Ans.**

$\circlearrowleft \Sigma M = 0: \quad T_{BC} - 30 + 40 = 0$

$T_{BC} = -10 \text{ kN} \cdot \text{m}$ .... **Ans.**

$\circlearrowleft \Sigma M = 0: \quad T_{CD} - 30 + 40 - 15 = 0$

$T_{CD} = +5 \text{ kN} \cdot \text{m}$ .... **Ans.**

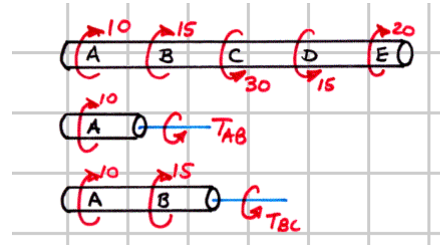
$\circlearrowleft \Sigma M = 0: \quad T_{DE} - 30 + 40 - 15 - 5 = 0$

$T_{DE} = +10 \text{ kN} \cdot \text{m}$ .... **Ans.**

(a) A torque diagram for the shaft is shown below the free-body diagrams.

6-3

(a) Free-body diagrams for parts of the shaft to the left of sections in the intervals *AB*, *BC*, *CD*, and *DE* of the shaft are shown. From the free-body diagrams:

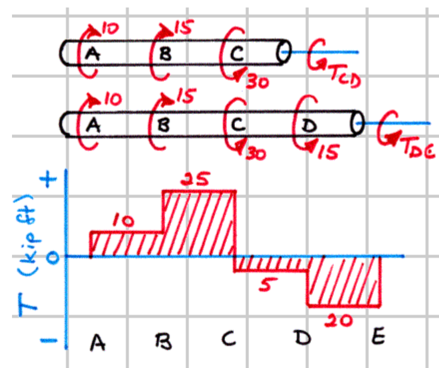


$$\circlearrowleft \Sigma M = 0: \qquad T_{AB} - 10 = 0$$

$$T_{AB} = +10 \text{ kip}\cdot\text{ft}$$ ....**Ans.**

$$\circlearrowleft \Sigma M = 0: \qquad T_{BC} - 10 - 15 = 0$$

$$T_{BC} = +25 \text{ kip}\cdot\text{ft} = T_{\max}$$ ....**Ans.**



$$\circlearrowleft \Sigma M = 0: \qquad T_{CD} - 10 - 15 + 30 = 0$$

$$T_{CD} = -5 \text{ kip}\cdot\text{ft}$$ ....**Ans.**

$$\circlearrowleft \Sigma M = 0: \qquad T_{DE} - 10 - 15 + 30 + 15 = 0$$

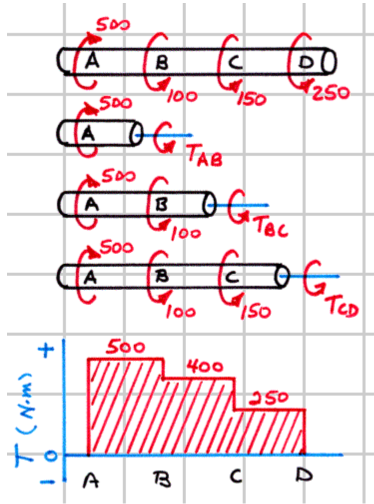
$$T_{DE} = -20 \text{ kip}\cdot\text{ft}$$ ....**Ans.**

A torque diagram for the shaft is shown below the free-body diagrams.

(b) $T_{\max} = T_{BC} = +25 \text{ kip}\cdot\text{ft}$ ....**Ans.**

6-4*

(a) Free-body diagrams for parts of the shaft to the left of sections in the intervals *AB*, *BC*, and *CD* of the shaft are shown. From the free-body diagrams:



$\circlearrowleft \Sigma M = 0: \quad T_{AB} - 500 = 0$

$T_{AB} = +500 \text{ N} \cdot \text{m}$ .... **Ans.**

$\circlearrowleft \Sigma M = 0: \quad T_{BC} - 500 + 100 = 0$

$T_{BC} = +400 \text{ N} \cdot \text{m}$ .... **Ans.**

$\circlearrowleft \Sigma M = 0: \quad T_{CD} - 500 + 100 + 150 = 0$

$T_{CD} = +250 \text{ N} \cdot \text{m}$ .... **Ans.**

(b) A torque diagram for the shaft is shown below the free-body diagrams.

6-5

$$J = \pi d^4 / 32 = \pi (2)^4 / 32 = 1.57080 \text{ in.}^4$$

(a) $\tau = \dfrac{Tc}{J} = \dfrac{(18)(1)}{(1.5708)} = 11.46 \text{ ksi}$ .... **Ans.**

(b) $\theta = \dfrac{TL}{JG} = \dfrac{(18)(6 \times 12)}{(1.5708)(12{,}000)} = 0.0688 \text{ rad}$ .... **Ans.**

6-6

$$J = \pi d^4/32 = \pi\left(120^4 - 80^4\right)/32 = 16.33628\left(10^6\right)\ \text{mm}^4$$

(a) $$\tau_o = \frac{Tc}{J} = \frac{(28,000)(0.060)}{\left(16.33628\times10^{-6}\right)} = 102.8\left(10^6\right)\ \text{N/m}^2 = 102.8\ \text{MPa}$$ .... **Ans.**

(b) $$\tau_i = \frac{(28,000)(0.040)}{\left(16.33628\times10^{-6}\right)} = 68.6\left(10^6\right)\ \text{N/m}^2 = 68.6\ \text{MPa}$$ .... **Ans.**

(c) $$\theta = \frac{TL}{JG} = \frac{(28,000)(2)}{\left(16.33628\times10^{-6}\right)\left(80\times10^9\right)} = 0.0429\ \text{rad}$$ .... **Ans.**

(d) $$A = \pi\left(120^2 - 80^2\right)/4 = \pi r^2 \qquad r = 44.72136\ \text{mm}$$

$$J = \pi r^4/2 = \pi(44.72136)^4/2 = 6.28319\left(10^6\right)\ \text{mm}^4$$

$$\theta = \frac{TL}{JG} = \frac{(28,000)(2)}{\left(6.28319\times10^{-6}\right)\left(80\times10^9\right)} = 0.1114\ \text{rad}$$ .... **Ans.**

6-7*

$$\tau = \frac{Tc}{J} = \frac{(2.2\times 12)(d/2)}{(\pi d^4/32)} \le 14.5 \text{ ksi} \qquad d \ge 2.10 \text{ in.}$$

$$\theta = \frac{TL}{JG} = \frac{(2.2\times 12)(6.5\times 12)}{(\pi d^4/32)(4000)} \le \frac{5\pi}{180} \text{ rad} \qquad d \ge 2.78 \text{ in.}$$

$d_{\min} = 2.78$ in. .... **Ans.**

6-8

$$J_{AB} = \pi d^4/32 = \pi(80)^4/32 = 4.02124\left(10^6\right)\ \text{mm}^4$$

$$J_{AC} = \pi d^4/32 = \pi(65)^4/32 = 1.75248\left(10^6\right)\ \text{mm}^4$$

(a) $$\tau_{AB} = \frac{Tc}{J} = \frac{(6000)(0.040)}{\left(4.02124\times10^{-6}\right)} = 59.7\left(10^6\right)\ \text{N/m}^2 = 59.7\ \text{MPa}$$ .... **Ans.**

$$\tau_{AC} = \frac{Tc}{J} = \frac{(4000)(0.0325)}{\left(1.75248\times10^{-6}\right)} = 74.2\left(10^6\right)\ \text{N/m}^2 = 74.2\ \text{MPa}$$ .... **Ans.**

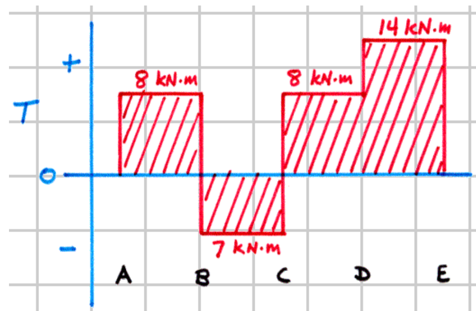
(b) $$\theta_{B/A} = \left[\frac{TL}{JG}\right]_{AB} = \frac{(6000)(2.25)}{\left(4.02124\times10^{-6}\right)\left(80\times10^9\right)} = 0.04197\ \text{rad} \cong 0.0420\ \text{rad}$$ .... **Ans.**

(c) $$\theta_{C/A} = \left[\frac{TL}{JG}\right]_{AC} = \frac{(4000)(1.60)}{\left(1.75248\times10^{-6}\right)\left(80\times10^9\right)} = 0.04565\ \text{rad}$$

$$\theta_{C/B} = \theta_{C/A} + \theta_{B/A} = 0.04565 - 0.04197 = 0.00368\ \text{rad}$$ .... **Ans.**

6-10*

$$J = \pi d^4/32 = \pi(100)^4/32 = 9.81748(10^6)\ \text{mm}^4$$



(a) $T_{max} = 14\ \text{kN}\cdot\text{m}$

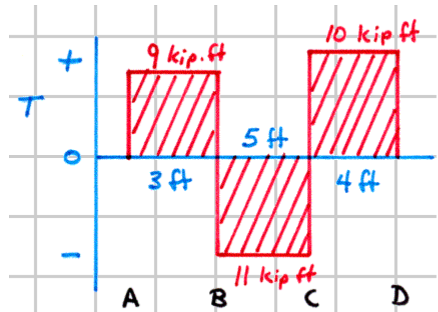
$$\tau_{max} = \frac{Tc}{J} = \frac{(14{,}000)(0.050)}{(9.81748\times10^{-6})} = 71.3(10^6)\ \text{N/m}^2$$

$\tau_{max} = 71.3\ \text{MPa}$ ....**Ans.**

(b) $\theta_{D/B} = \dfrac{TL}{JG} = \dfrac{(-7000)(1.50)+(+8000)(1.50)}{(9.81748\times10^{-6})(80\times10^9)} = +0.001910\ \text{rad}$ .... **Ans.**

(c) $\theta_{E/A} = \dfrac{(8000-7000+8000+14{,}000)(1.50)}{(9.81748\times10^{-6})(80\times10^9)} = +0.0439\ \text{rad}$ .... **Ans.**

6-11



$$J = \pi d^4/32 = \pi\left(4^4 - 2^4\right)/32 = 23.56194 \text{ in.}^4$$

(a) $T_{max} = T_{BC} = 11 \text{ kip}\cdot\text{ft}$

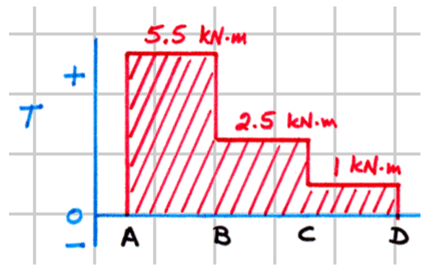
$$\tau_{max} = \frac{Tc}{J} = \frac{(11\times 12)(2)}{(23.56194)} = 11.20 \text{ ksi}$$ .... **Ans.**

(b) $$\theta_{D/B} = \frac{TL}{JG} = \frac{(-11\times 12)(5\times 12) + (+10\times 12)(4\times 12)}{(23.56194)(12,000)}$$

$\theta_{D/B} = -0.00764 \text{ rad}$ .... **Ans.**

(c) $$\theta_{D/A} = \frac{(+9\times 12)(3\times 12)}{(23.56194)(12,000)} + (-0.00764) = 0.00611 \text{ rad}$$ .... **Ans.**

6-12*



(a) $T_{max} = T_{AB} = 5.5 \text{ kN} \cdot \text{m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{(5500)(d/2)}{(\pi d^4/32)} \leq 100(10^6) \text{ N/m}^2$$

$d \geq 0.0654 \text{ m} = 65.4 \text{ mm}$ .... **Ans.**

(b) $$\theta_{D/A} = \frac{(5500)(2)+(2500)(2)+(1000)(2)}{\left[\pi(0.075)^4/32\right](80\times 10^9)}$$

$\theta_{D/A} = 0.0724 \text{ rad}$ .... **Ans.**

6-13

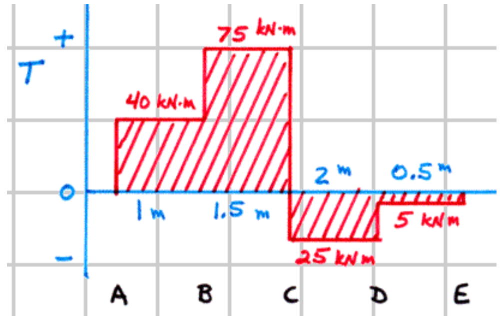
$$\tau = \frac{Tc}{J} = \frac{T(0.875)}{\left[\pi(1.75)^4/32\right]} \le 8 \text{ ksi} \qquad T \le 8.42 \text{ kip}\cdot\text{in.}$$

$$\theta = \frac{TL}{JG} = \frac{T(3\times12)}{\left[\pi(2.5)^4/32\right](4000)} + \frac{T(4\times12)}{\left[\pi(1.75)^4/32\right](4000)} \le 0.04 \text{ rad}$$

$$T \le 2.60 \text{ kip}\cdot\text{in.}$$

$T_{max} = 2.60$ kip·in. .... **Ans.**

6-14



$$J_{AB} = J_{BC} = \pi(160)^4/32 = 64.33982(10^6) \text{ mm}^4$$

$$J_{CD} = \pi(100)^4/32 = 9.81749(10^6) \text{ mm}^4$$

$$J_{DE} = \pi(50)^4/32 = 0.61359(10^6) \text{ mm}^4$$

(a) $$\tau_3 = \frac{Tc}{J} = \frac{(25{,}000)(0.050)}{(9.81749\times10^{-6})} = 127.3(10^6) \text{ N/m}^2$$

$\tau_3 = 127.3$ MPa .... **Ans.**

(b) $$\theta_2 = \frac{TL}{JG} = \frac{(40{,}000)(1)+(75{,}000)(1)}{(64.33982\times10^{-6})(80\times10^9)} = 0.0223 \text{ rad}$$ .... **Ans.**

(c) $$\theta_{E/A} = \frac{(40{,}000)(1)+(75{,}000)(1.5)}{(64.33982\times10^{-6})(80\times10^9)} + \frac{(-25{,}000)(2)}{(9.81748\times10^{-6})(80\times10^9)}$$

$$+\frac{(-5000)(0.5)}{(0.61359\times10^{-6})(80\times10^9)} = -0.0850 \text{ rad}$$

$\theta_{E/A} = -0.0850$ rad .... **Ans.**

6-15*

$T_{AB} = T$ $T_{CD} = 2T$

(a) $\tau = \dfrac{Tc}{J} = \dfrac{T(0.75)}{\pi(1.5)^4/32} \le 15 \text{ ksi}$ $T \le 9.94020 \text{ kip}\cdot\text{in.}$

$\tau = \dfrac{Tc}{J} = \dfrac{(2T)(1)}{\pi(2)^4/32} \le 15 \text{ ksi}$ $T \le 11.78 \text{ kip}\cdot\text{in.}$

$T_{max} = 9.94 \text{ kip}\cdot\text{in.}$ .... **Ans.**

$$\theta_{CD} = \frac{TL}{JG} = \frac{(2\times 9.94020)(3\times 12)}{\left[\pi(2)^4/32\right](12{,}000)} = 0.03797 \text{ rad}$$

$\theta_A = 2(0.03797) + \dfrac{(9.94020)(4\times 12)}{\left[\pi(1.5)^4/32\right](12{,}000)} = 0.1559 \text{ rad}$ .... **Ans.**

6-16

(a) $\tau = Tc/J = (2000)(0.040)/J \le 50(10^6)\ \text{N/m}^2$

$J \ge 1.600(10^{-6})\ \text{m}^4 = 1.600(10^6)\ \text{mm}^4$

$J = \dfrac{\pi(80^4 - d_i^4)}{32} \ge 1.600(10^6)\ \text{mm}^4$ $d_i \le 70.5$ mm .... **Ans.**

(b) $\theta = \dfrac{TL}{JG} = \dfrac{(2000)(3.5)}{\left[\pi(0.050)^4/32\right](80\times10^9)} + \dfrac{(2000)(2.5)}{J(28\times10^9)} \le 0.25$ rad

$J \ge 1.66272(10^{-6})\ \text{m}^4 = 1.66272(10^6)\ \text{mm}^4$

$J = \dfrac{\pi(80^4 - d_i^4)}{32} \ge 1.66272(10^6)\ \text{mm}^4$ $d_i \le 70.0$ mm .... **Ans.**

6-17*

(a) $$\tau = \frac{Tc}{J} = \frac{(30\times 12)(d/2)}{(\pi d^4/32)} \le 12,000 \text{ psi}$$

$d_{AB} \ge 0.535$ in. .... **Ans.**

(b) $$T_{CD} = (2/5)(30) = 12 \text{ lb}\cdot\text{ft}$$

$$\tau = \frac{Tc}{J} = \frac{(12\times 12)(d/2)}{(\pi d^4/32)} \le 12,000 \text{ psi}$$

$d_{CD} \ge 0.394$ in. .... **Ans.**

(c) $$\theta = \frac{(12\times 12)L}{\left[\pi(0.394)^4/32\right](3.8\times 10^6)} \le 0.5 \text{ rad}$$

$L \le 31.2$ in. .... **Ans.**

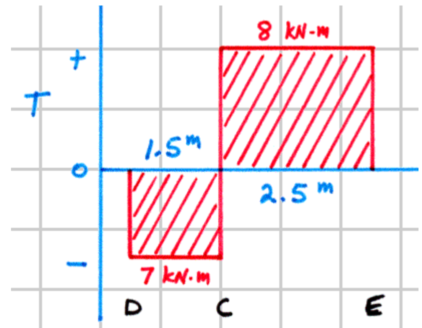
6-18*

(a) $\tau_{max} = \frac{Tc}{J} = \frac{(45,000)(0.075)}{\pi(0.075)^4/2} = 67.9(10^6) \text{ N/m}^2 = 67.9 \text{ MPa}$ .... **Ans.**

(b) $T_C = (150/450)(45) = 15 \text{ kN}\cdot\text{m}$ $T_D = 15 - 8 = 7 \text{ kN}\cdot\text{m}$

$\tau_{max} = \tau_{CE} = \frac{(8000)(0.040)}{\pi(0.040)^4/2} = 79.6(10^6) \text{ N/m}^2$

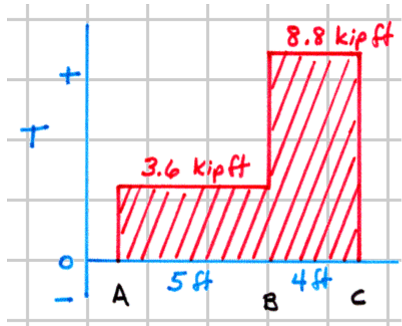
$\tau_{max} = 79.6 \text{ MPa}$ .... **Ans.**



(c) $\theta = \frac{TL}{JG} = \frac{(8000)(2.5)+(-7000)(1.5)}{\left[\pi(0.040)^4/2\right](80\times10^9)}$

$\theta = 0.0295 \text{ rad}$ .... **Ans.**

6-19



(a) $$\tau_{AB} = \frac{Tc}{J} = \frac{(3.6\times12)(D_1/2)}{\pi D_1^4/32} = 18 \text{ ksi}$$

$D_1 = 2.30$ in. .... **Ans.**

$$\tau_{BC} = \frac{(8.8\times12)(D_2/2)}{\pi D_2^4/32} = 18 \text{ ksi}$$

$D_2 = 3.10$ in. .... **Ans.**

(b) $$\theta = \frac{TL}{JG} = \frac{(3.6\times12)(5\times12)+(8.8\times12)(4\times12)}{(\pi D^4/32)(11{,}600)} = 0.15 \text{ rad}$$

$D = 2.59$ in. .... **Ans.**

6-20*

With the left end of the shaft at $x = 0$
and the right end of the shaft at $x = L$

$$\rho = r + mx$$

$$J = \pi\rho^4 / 2 = \pi\left(r + mx\right)^4 / 2$$

$$d\theta = \frac{T\,dx}{JG} = \frac{T\,dx}{\left[\pi\left(r+mx\right)^4/2\right]G} = \frac{2T\,dx}{\pi G\left(r+mx\right)^4}$$

$$\theta = \int d\theta = \int_0^L \frac{2T\,dx}{\pi G\left(r+mx\right)^4} = \frac{2T}{\pi G}\int_0^L \frac{dx}{\left(r+mx\right)^4} = \frac{2T}{\pi G}\left[\frac{-1}{3m\left(r+mx\right)^3}\right]_0^L$$

$$\theta = \frac{2T}{3\pi G m r^3}\left[\frac{\left(r+mL\right)^3 - r^3}{\left(r+mL\right)^3}\right]$$ .... **Ans.**

6-21

With the left end of the shaft at $x = 0$
and the right end of the shaft at $x = L$

$$T = qx \qquad d\theta = \frac{T\,dx}{JG} = \frac{(qx)\,dx}{(\pi c^4/2)G} = \frac{2qx\,dx}{\pi G c^4}$$

$$\theta = \int d\theta = \int_0^L \frac{2qx\,dx}{\pi G c^4} = \frac{2q}{\pi G c^4}\int_0^L x\,dx = \frac{qL^2}{\pi G c^4}$$ .... **Ans.**

6-22

With the left end of the shaft at $x = L$
and the right end of the shaft at $x = 2L$

$$\rho = rx/L$$

$$J = \frac{\pi}{2}\left(\rho^4 - R^4\right) = \frac{\pi}{2L^4}\left(r^4x^4 - R^4L^4\right)$$

$$d\theta = \frac{T\,dx}{JG} = \frac{2TL^4}{\pi G}\left(\frac{dx}{r^4x^4 - R^4L^4}\right)$$

$$\theta = \int d\theta = \frac{2TL^4}{\pi G}\int_L^{2L}\left(\frac{dx}{r^4x^4 - R^4L^4}\right)$$

$$\theta = \frac{TL}{2\pi GR^3 r}\left[\ln\left(\frac{2r-R}{r-R}\right)\left(\frac{r+R}{2r+R}\right) - 2\tan^{-1}\left(\frac{2r}{R}\right) + 2\tan^{-1}\left(\frac{r}{R}\right)\right]$$ .... **Ans.**

6-23*

With the left end of the shaft at $x = L$
and the right end of the shaft at $x = 2L$

$$\rho = rx/L$$

$$J = \rho^2 A = \rho^2 \left(2\pi\rho t\right) = 2\pi\rho^3 t = 2\pi\left(\frac{rx}{L}\right)^3 t = \frac{2\pi r^3 t x^3}{L^3}$$

$$d\theta = \frac{T\,dx}{JG} = \frac{TL^3}{2\pi r^3 tG}\left(\frac{dx}{x^3}\right)$$

$$\theta = \int d\theta = \frac{TL^3}{2\pi r^3 tG}\int_L^{2L}\frac{dx}{x^3} = \frac{3TL}{16Gt\pi r^3}$$ .... **Ans.**

6-24

With the left end of the shaft at $x = 0$
and the right end of the shaft at $x = L$

$$T = \frac{qx^2}{2L} \qquad d\theta = \frac{T\,dx}{JG} = \frac{\left(qx^2/2L\right)dx}{\left(\pi c^4/2\right)G} = \frac{qx^2\,dx}{\pi GLc^4}$$

$$\theta = \int d\theta = \frac{q}{\pi GLc^4}\int_0^L x^2\,dx = \frac{qL^2}{3\pi Gc^4}$$ .... **Ans.**

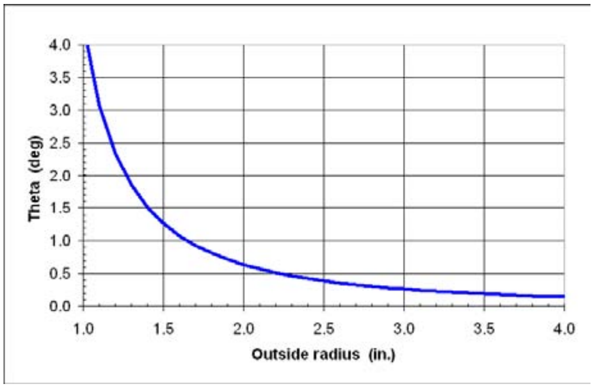
6-25

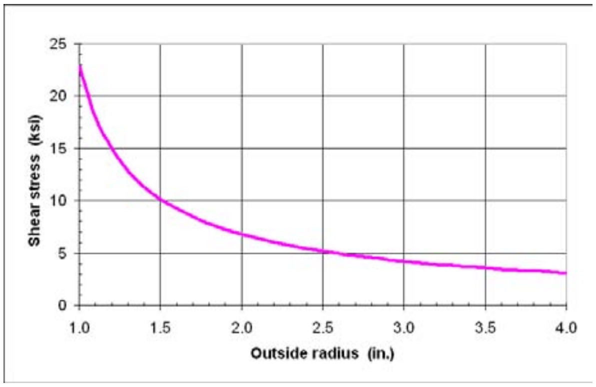
$$A = \pi\left(r_o^2 - r_i^2\right) = 3 \text{ in.}^2 \qquad r_i^2 = r_o^2 - (3/\pi)$$

$$J = \pi\left(r_o^4 - r_i^4\right)/2 = \pi r_o^4/2 - \pi\left[r_o^2 - (3/\pi)\right]^2/2 = 3r_o^2 - (4.5/\pi)$$

(a) $$\theta = \frac{TL}{JG} = \frac{(3\times 12)(3\times 12)}{\left[3r_o^2 - (4.5/\pi)\right](11{,}000)} = \left(\frac{0.246758}{2\pi r_o^2 - 3}\right) \text{ rad}$$

(b) $$\tau_c = \frac{Tc}{J} = \frac{(3\times 12)(r_o)}{\left[3r_o^2 - (4.5/\pi)\right]} = \left(\frac{75.3982 r_o}{2\pi r_o - 3}\right) \text{ ksi}$$





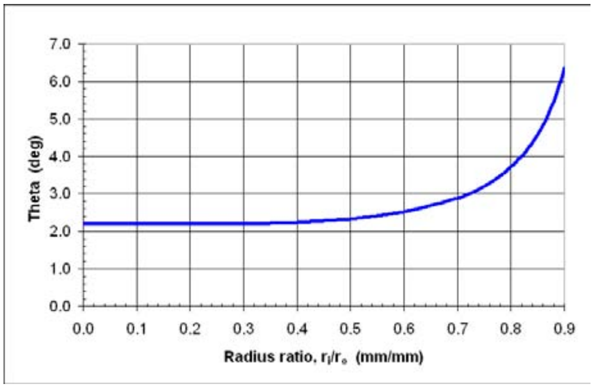
6-26

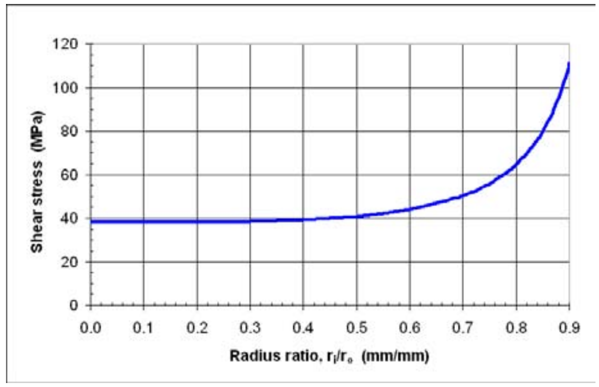
$$J = \pi\left(r_o^4 - r_i^4\right)/2 = \pi r_o^4\left[1-\left(r_i/r_o\right)^4\right]/2$$

$$= \pi\left(50\right)^4\left[1-\left(r_i/r_o\right)^4\right]/2 = \left(9.81748\times10^6\right)\left[1-\left(r_i/r_o\right)^4\right]\text{ mm}^4$$

$$\theta = \frac{TL}{JG} = \frac{(7500)(2)}{\left(9.81748\times10^{-6}\right)\left[1-\left(r_i/r_o\right)^4\right]\left(40\times10^9\right)} = \frac{0.0381972}{\left[1-\left(r_i/r_o\right)^4\right]}\text{ rad}$$

$$\tau_c = \frac{Tc}{J} = \frac{(7500)(0.050)}{\left(9.81748\times10^{-6}\right)\left[1-\left(r_i/r_o\right)^4\right]} = \frac{38.1972\times10^6}{\left[1-\left(r_i/r_o\right)^4\right]}\text{ N/m}^2$$





6-27

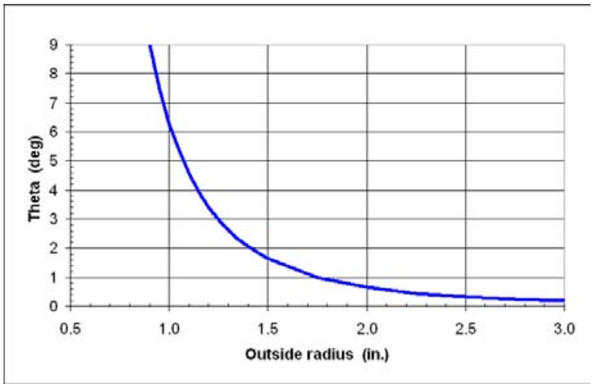
$$r_i = (r_o - 0.25) \text{ in.}$$

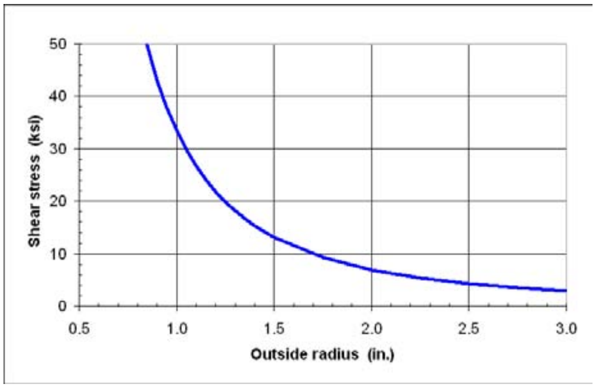
$$J = \pi\left(r_o^4 - r_i^4\right)/2 = \left[\pi r_o^4/2 - \pi(r_o - 0.25)^4/2\right] \text{ in.}^4$$

(a) $$\theta = \frac{TL}{JG} = \frac{(3\times 12)(3\times 12)}{\left[\pi r_o^4/2 - \pi(r_o - 0.25)^4/2\right](11{,}000)} = \left[\frac{0.0750054}{r_o^4 - (r_o - 0.25)^4}\right] \text{ rad}$$

(b) $$\tau_c = \frac{Tc}{J} = \frac{(3\times 12)(r_o)}{\left[\pi r_o^4/2 - \pi(r_o - 0.25)^4/2\right]} = \left[\frac{22.91831 r_o}{r_o^4 - (r_o - 0.25)^4}\right] \text{ ksi}$$

(c) As the outside radius increases, the shear stress decreases. Between $r_o = 0.5$ in. and $r_o = 1.5$ in. the decrease in shear stress is very dramatic. Between $r_o = 1.5$ in. and $r_o = 2.0$ in. the decrease in shear stress is much less dramatic. And beyond $r_o = 2.0$ in. the decrease in shear stress is very slight. Therefore, a reasonable maximum value for the outside radius would be around $r_o = 1.5$ in.





6-28

$$\theta_{D/A} = \frac{TL_{AB}}{J_{AB}G} + \frac{T(1.5 - L_{AB})}{J_{BC}G} + \frac{T(0.75)}{J_{CD}G}$$

$$T_{AB} = T_{BC} = T_{CD} = T = 2500 \text{ N} \cdot \text{m}$$

$$G_{AB} = G_{BC} = G_{CD} = G = 28(10^9) \text{ N/m}^2$$

$$J_{BC} = \pi d^4/32 = \pi(100)^4/32 = 9.81748(10^6) \text{ mm}^4$$

$$J_{CD} = \pi(75)^4/32 = 3.10631(10^6) \text{ mm}^4$$

(a) For $d = 75$ mm

$$J_{AB} = \pi(100^4 - 75^4)/32$$

$$= 6.71117(10^6) \text{ mm}^4$$

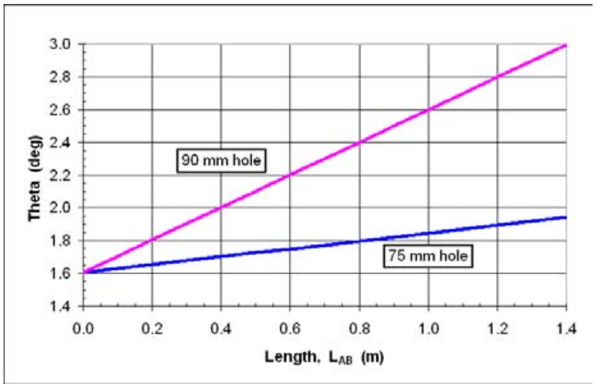
$$\theta_{D/A} = (4.20948 L_{AB} + 28.0135)(10^{-3}) \text{ rad}$$

(b) For $d = 90$ mm

$$J_{AB} = \pi(100^4 - 90^4)/32$$

$$= 3.37623(10^6) \text{ mm}^4$$

$$\theta_{D/A} = (17.3508 L_{AB} + 28.0135)(10^{-3}) \text{ rad}$$



6-29*

(a) $\sigma_{max} = \tau_c = \dfrac{Tc}{J} = \dfrac{(15)(1.5)}{\pi(1.5)^4/2} = 2.83\text{ ksi (T)}$ .... **Ans.**

(b) $\sigma_{max} = \tau_c = \dfrac{Tc}{J} = \dfrac{(15)(1)}{\pi(1)^4/2} = 9.55\text{ ksi (C)}$ .... **Ans.**

(c) $\theta = \dfrac{TL}{JG} = \dfrac{(-15)(3\times 12)}{\left[\pi(1.5)^4/2\right](12,000)} + \dfrac{(-15)(4\times 12)}{\left[\pi(1)^4/2\right](12,000)} = -0.0439\text{ rad}$ .... **Ans.**

6-30*

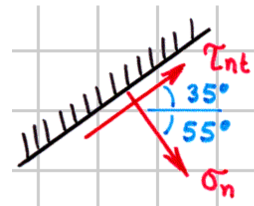
$$\sigma_{max} = \tau_{max} = Tc/J \le 75 \text{ MPa} \le 80 \text{ MPa}$$

$$\frac{T_{max}(0.045)}{\pi(0.090^4 - 0.050^4)/32} = 75(10^6) \text{ N/m}^2$$

$T_{max} = 9710 \text{ N} \cdot \text{m} = 9.71 \text{ kN} \cdot \text{m}$ .... **Ans.**

6-31

$\sigma_x = \sigma_y = 0$ ksi $\qquad \theta = -55°$



$$\tau_{xy} = \frac{Tc}{J} = \frac{-(1.00)(0.75)}{\pi\left(1.5^4 - 1.35^4\right)/32} = -4.38797 \text{ ksi}$$

(a) $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

$\sigma_n = 0 + 0 + 2(-4.38797)\sin(-55°)\cos(-55°) = 4.12$ ksi (T) .... **Ans.**

$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$

$\tau_{nt} = 0 + (-4.38797)\left[\cos^2(-55°) - \sin^2(-55°)\right] = +1.501$ ksi .... **Ans.**

(b) $\sigma_{\max T} = \tau_{xy} = 4.39$ ksi (T) .... **Ans.**

$\sigma_{\max C} = \tau_{xy} = 4.39$ ksi (C) .... **Ans.**

6-32

$$J_{60} = \pi\left(120^4 - 60^4\right)\Big/32 = 19.08517\left(10^6\right)\ \text{mm}^4$$

$$J_{100} = \pi\left(120^4 - 100^4\right)\Big/32 = 10.54004\left(10^6\right)\ \text{mm}^4$$

(a) $$\sigma_{\max} = \tau = \frac{Tc}{J} = \frac{(7500)(0.060)}{19.08517\left(10^{-6}\right)} = 23.6\left(10^6\right)\ \text{N/m}^2 = 23.6\ \text{MPa}$$ .... **Ans.**

(b) $$\sigma_{\max} = \frac{(7500)(0.060)}{10.54004\left(10^{-6}\right)} = 42.7\left(10^6\right)\ \text{N/m}^2 = 42.7\ \text{MPa}$$ .... **Ans.**

(c) $$\theta_{60} = \frac{TL}{JG} = \frac{(-7500)(2)}{\left(19.08517\times10^{-6}\right)\left(80\times10^9\right)} = -0.00982\ \text{rad}$$ .... **Ans.**

$$\theta_{100} = \frac{TL}{JG} = \frac{(-7500)(2)}{\left(10.54004\times10^{-6}\right)\left(80\times10^9\right)} = -0.01779\ \text{rad}$$ .... **Ans.**

6-33*

(a) $T_{BC} = (4/12)(1500) = 500 \text{ lb}\cdot\text{ft}$ .... **Ans.**

(b) $T_{CD} = 500 - 250 = 250 \text{ lb}\cdot\text{ft}$ .... **Ans.**

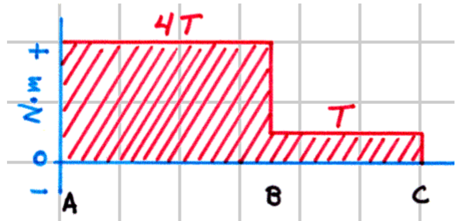
(c) $$\tau_{\max}(motor) = \frac{Tc}{J} = \frac{(1500\times 12)(1)}{\pi(1)^4/2} = 11,459 \text{ psi} \cong 11.46 \text{ ksi}$$

$\sigma_{\max}(motor) = \tau_{\max}(motor) = 11.46 \text{ ksi (T\&C)}$ .... **Ans.**

$$\tau_{\max}(power) = \frac{(500\times 12)(0.625)}{\pi(1.25)^4/32} = 15,646 \text{ psi} \cong 15.65 \text{ ksi}$$

$\sigma_{\max}(power) = \tau_{\max}(power) = 15.65 \text{ ksi (T\&C)}$ .... **Ans.**

6-34*



$$\tau_{AB} = \frac{Tc}{J} = \frac{(4T)(0.080)}{\pi(0.080)^4/2} \leq 150(10^6) \text{ N/m}^2$$

$$T \leq 30.2(10^3) \text{ N}\cdot\text{m}$$

$$\sigma_{AB} = \tau_{AB} = 150(10^6) \text{ N/m}^2 \leq 260(10^6) \text{ N/m}^2$$

$$\sigma_{BC} = \tau_{BC} = \frac{T(0.050)}{\pi(0.050)^4/2} \leq 75(10^6) \text{ N/m}^2 \leq 125(10^6) \text{ N/m}^2$$

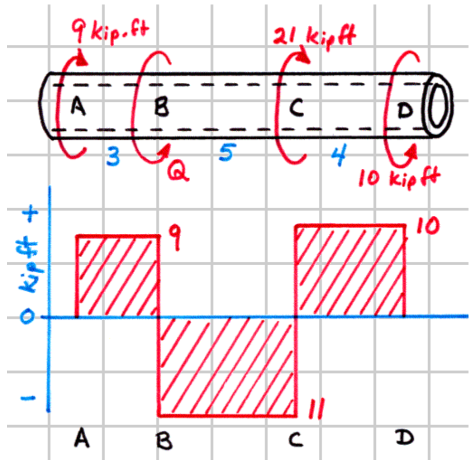
$$T \leq 14.73(10^3) \text{ N}\cdot\text{m}$$

$$\theta = \frac{TL}{JG} = \frac{(4T)(2)}{\left[\pi(0.080)^4/2\right](45\times10^9)} + \frac{(T)(1.5)}{\left[\pi(0.050)^4/2\right](76\times10^9)} \leq \frac{2.5\pi}{180} \text{ rad}$$

$$T \leq 9140 \text{ N}\cdot\text{m}$$

$T_{\max} = 9.14 \text{ kN}\cdot\text{m}$ .... **Ans.**

6-35



$\circlearrowleft \Sigma M = 0:$ $\quad -9 + Q - 21 + 10 = 0$

$$Q = 20 \text{ kip}\cdot\text{ft}$$

$$T_{max} = T_{BC} = 11 \text{ kip}\cdot\text{ft}$$

(a) $$\tau_{BC} = \frac{Tc}{J} = \frac{(11\times 12)(2)}{\pi(2^4 - 1^4)/2} = 11.20 \text{ ksi}$$

$\sigma_{max} = \tau_{BC} = 11.20$ ksi (C) .... **Ans.**

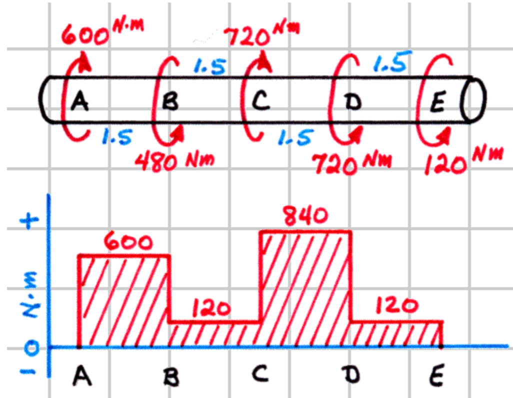
(b) $$\tau_{BC} = \frac{Tc}{J} = \frac{(11\times 12)(1)}{\pi(2^4 - 1^4)/2} = 5.60 \text{ ksi}$$

$\sigma_{max} = \tau_{BC} = 5.60$ ksi (C) .... **Ans.**

(c) $\theta = \dfrac{TL}{JG} = \dfrac{(9\times 3 - 11\times 5 + 10\times 4)(12\times 12)}{\left[\pi(2^4 - 1^4)/2\right](12{,}000)} = +0.00611$ rad .... **Ans.**

6-36

$$J = \pi d^4/32 = \pi(20)^4/2 = 0.251327\left(10^6\right)\ \text{mm}^4$$



(a) $$\tau_{AB} = \frac{Tc}{J} = \frac{(600)(0.020)}{\left(0.251327\times10^{-6}\right)} = 47.7\left(10^6\right)\ \text{N/m}^2$$

$\tau_{AB} = 47.7$ MPa ....**Ans.**

$$\tau_{BC} = \frac{(120)(0.020)}{\left(0.251327\times10^{-6}\right)} = 9.55\left(10^6\right)\ \text{N/m}^2$$

$\tau_{BC} = 9.55$ MPa .... **Ans.**

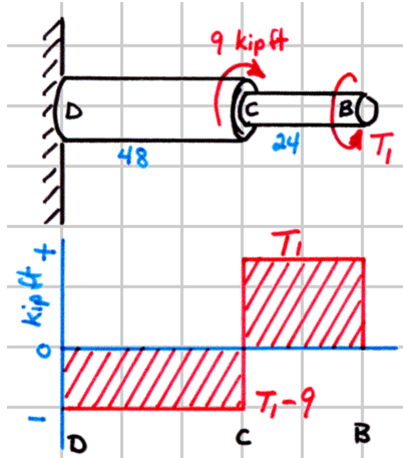
$$\tau_{CD} = \frac{(840)(0.020)}{\left(0.251327\times10^{-6}\right)} = 66.8\left(10^6\right)\ \text{N/m}^2 = 66.8\ \text{MPa}$$ .... **Ans.**

$$\tau_{DE} = \frac{(120)(0.020)}{\left(0.251327\times10^{-6}\right)} = 9.55\left(10^6\right)\ \text{N/m}^2 = 9.55\ \text{MPa}$$ .... **Ans.**

(b) $\sigma_{\max T} = \sigma_{\max C} = \tau_{\max T} = 66.8$ MPa (T&C) .... **Ans.**

(c) $$\theta = \frac{TL}{JG} = \frac{(600+120+840+120)(1.5)}{\left(0.251327\times10^{-6}\right)\left(76\times10^9\right)} = 0.1319\ \text{rad}$$ .... **Ans.**

6-37



(a) $$\theta = \frac{TL}{JG} = \frac{0.172}{6} \text{ rad}$$

$$\frac{(T_1 \times 12)(24)}{\left[\pi(1)^4/2\right](12{,}000)} + \frac{\left[(T_1 - 9)\times 12\right](48)}{\left[\pi(2)^4/2\right](12{,}000)} = \frac{0.172}{6} \text{ rad}$$

$T_1 = 2.66776 \text{ kip}\cdot\text{ft} \cong 32.0 \text{ kip}\cdot\text{in.}$ ....**Ans.**

(b) $$\tau_{BC} = \frac{Tc}{J} = \frac{(2.66776 \times 12)(1)}{\pi(1)^4/2} = 20.4 \text{ ksi}$$

$\sigma_{BC} = \tau_{BC} = 20.4 \text{ ksi (T)}$ ....**Ans.**

(c) $\sigma_{CD} = \tau_{CD} = \dfrac{\left[(2.66776 - 9)\times 12\right](2)}{\pi(2)^4/2} = 6.05 \text{ ksi (C)}$ .... **Ans.**

6-38*

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(180)T}{60} = 240\left(10^3\right)\ \text{N}\cdot\text{m/s}$$

$$T = 12.73240\left(10^3\right)\ \text{N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(12,732.40)(d/2)}{\pi d^4/32} = 80\left(10^6\right)\ \text{N/m}^2$$

$d \geq 0.0932\ \text{m} = 93.2\ \text{mm}$ .... **Ans.**

6-39

$$\theta = \frac{TL}{JG} = \frac{T(20\times 12)}{\left[\pi(4)^4/32\right](12,000)} = 0.06 \text{ rad}$$

$$T = 75.39822 \text{ kip}\cdot\text{in.} = 6.28319 \text{ kip}\cdot\text{ft}$$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(270)(6283.19)}{(60)(550)} = 323 \text{ hp}$$ .... **Ans.**

6-40*

$$\tau = \frac{Tc}{J} = \frac{T(0.050)}{\pi(0.050^4 - 0.030^4)/2} = 80(10^6)\ \text{N/m}^2 \qquad T = 13,672.211\ \text{N}\cdot\text{m}$$

(a) $$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(200)(13,672.211)}{60} = 286(10^3)\ \text{N}\cdot\text{m/s}$$

$Power = 286\ \text{kW}$ .... **Ans.**

(b) $$\theta = \frac{TL}{JG} = \frac{(13,672.211)(3)}{\left[\pi(0.050^4 - 0.030^4)/2\right](80\times 10^9)} = 0.0600\ \text{rad}$$ .... **Ans.**

6-41*

$$Power = T\omega = 2\pi NT/60 = 2\pi(60)T/60 = (20,000 \times 550) \text{ lb} \cdot \text{ft/s}$$

$$T = 1.750704(10^6) \text{ lb} \cdot \text{ft} = 21.00845(10^6) \text{ lb} \cdot \text{in.}$$

(a) $$\tau = \frac{Tc}{J} = \frac{(21.00845 \times 10^3)(15)}{\pi(15)^4/2} = 3.96 \text{ ksi}$$ .... **Ans.**

(b) $$\theta = \frac{TL}{JG} = \frac{(21.00845 \times 10^3)(20 \times 12)}{\left[\pi(15)^4/2\right](12,000)} = 0.00528 \text{ rad}$$ .... **Ans.**

6-42

$$Power = T\omega = 2\pi(400)T/60 = 200(10^3) \text{ N}\cdot\text{m/s} \qquad T = 4774.648 \text{ N}\cdot\text{m}$$

(a) $$\tau = \frac{Tc}{J} = \frac{(4774.648)(d/2)}{\pi d^4/32} = 70(10^6) \text{ N/m}^2 \qquad d = 0.0703 \text{ m}$$

$$\theta = \frac{TL}{JG} = \frac{(4774.648)(1.5)}{[\pi d^4/32](80\times 10^9)} = 0.045 \text{ rad} \qquad d = 0.0671 \text{ m}$$

$d_{min} = 70.3$ mm .... **Ans.**

(b) $$\tau = \frac{T(0.075/2)}{\pi(0.075)^4/32} = 50(10^6) \text{ N/m}^2 \qquad T = 4141.748 \text{ N}\cdot\text{m}$$

$$Power = \frac{2\pi N(4141.748)}{60} = 200(10^3) \text{ N}\cdot\text{m/s}$$

$N = 461$ rpm .... **Ans.**

6-43

$$Power = T\omega = 2\pi NT/60 = 2\pi(3800)T/60 = (162\times 550)\ \text{lb}\cdot\text{ft/s}$$

$$T = 223.90588\ \text{lb}\cdot\text{ft} = 2686.8705\ \text{lb}\cdot\text{in.}$$

(a) $$\tau = \frac{(2686.8705)(d/2)}{\pi d^4/32} = 5000\ \text{psi}$$

$d = 1.39878\ \text{in.} \cong 1.399\ \text{in.}$ .... **Ans.**

(b) $$\tau = \frac{Tc}{J} = \frac{(2686.8705)(1.5)}{J} = 5000\ \text{psi}$$ $J = 0.80606\ \text{in.}^4$

$$J = \frac{\pi\left(3^4 - d_i^4\right)}{32} = 0.80606\ \text{in.}^4$$

$d_i = 2.92090\ \text{in.} \cong 2.92\ \text{in.}$ .... **Ans.**

(c) $$\frac{W_h - W_s}{W_s}(100) = \frac{\left[\pi\left(3^2 - 2.92090^2\right)/4\right] - \left[\pi(1.39878)^2/4\right]}{\left[\pi(1.39878)^2/4\right]}(100) = -76.1$$

$\%reduction = 76.1\%$ .... **Ans.**

6-44

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(1800)T}{60} = 1200(10^3)\ \text{N}\cdot\text{m/s}$$

$$T = 6366.1977\ \text{N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(6366.1977)(d/2)}{\pi\left[d^4 - (0.75d)^4\right]/32} = 100(10^6)\ \text{N/m}^2 \qquad d = 0.0780\ \text{m}$$

$$\theta = \frac{TL}{JG} = \frac{(6366.1977)(3)}{\left\{\pi\left[d^4 - (0.75d)^4\right]/32\right\}(80\times 10^9)} = 0.20\ \text{rad} \qquad d = 0.0844\ \text{m}$$

$d_{min} = 84.4$ mm .... **Ans.**

6-45*

Motor shaft: $Power = T\omega = \dfrac{2\pi NT}{60} = \dfrac{2\pi(1800)T}{60} = (350\times 550)\ \text{lb}\cdot\text{ft/s}$

$$T = 1021.24422\ \text{lb}\cdot\text{ft} = 12,254.93\ \text{lb}\cdot\text{in.}$$

$$\tau = \frac{Tc}{J} = \frac{(12.25493)(d/2)}{\pi d^4/32} = 15\ \text{ksi} \qquad d = 1.61\ \text{in.}$$

$$\theta = \frac{TL}{JG} = \frac{(12.25493)(10\times 12)}{(\pi d^4/32)(12,000)} = 0.10\ \text{rad} \qquad d = 1.880\ \text{in.}$$

$d_{\min}(motor) = 1.880$ in. .... **Ans.**

Power shaft: $Power = T\omega = \dfrac{2\pi NT}{60} = \dfrac{2\pi(200)T}{60} = (350\times 550)\ \text{lb}\cdot\text{ft/s}$

$$T = 9191.19796\ \text{lb}\cdot\text{ft} = 110,294.4\ \text{lb}\cdot\text{in.}$$

$$\tau = \frac{Tc}{J} = \frac{(110.2994)(d/2)}{\pi d^4/32} = 15\ \text{ksi} \qquad d = 3.35\ \text{in.}$$

$$\theta = \frac{TL}{JG} = \frac{(110.2994)(10\times 12)}{(\pi d^4/32)(12,000)} = 0.10\ \text{rad} \qquad d = 3.26\ \text{in.}$$

$d_{\min}(power) = 3.35$ in. .... **Ans.**

6-46*

$$Power = T\omega = 2\pi NT/60$$

$$2\pi(250)T_{AB}/60 = 200(10^3)\ \text{N}\cdot\text{m/s} \qquad T_{AB} = 7639.4373\ \text{N}\cdot\text{m}$$

$$2\pi(250)T_{BC}/60 = 75(10^3)\ \text{N}\cdot\text{m/s} \qquad T_{BC} = 2864.7890\ \text{N}\cdot\text{m}$$

(a) $$\tau = \frac{Tc}{J} = \frac{(7639.4373)(d/2)}{\pi d^4/32} = 75(10^6)\ \text{N/m}^2 \qquad d = 0.0804\ \text{m}$$

$d_1 = 80.4$ mm .... **Ans.**

(b) $$\tau = \frac{Tc}{J} = \frac{(2864.7890)(d/2)}{\pi d^4/32} = 75(10^6)\ \text{N/m}^2 \qquad d = 0.0579\ \text{m}$$

$d_2 = 57.9$ mm .... **Ans.**

(c) $$\theta = \frac{TL}{JG} = \frac{(7639.4373)(1)+(2864.7890)(2)}{\left[\pi(0.075)^4/32\right](80\times 10^9)} = 0.0538\ \text{rad}$$ .... **Ans.**

6-47

(a) Motor shaft: $Power = T\omega = \dfrac{2\pi NT}{60} = \dfrac{2\pi(360)T_1}{60} = (100\times 550)\ \text{lb}\cdot\text{ft/s}$

$$T_1 = 1458.92031\ \text{lb}\cdot\text{ft} = 17{,}507.0\ \text{lb}\cdot\text{in.}$$

$$\tau = \frac{Tc}{J} = \frac{(17.5070)(d_1/2)}{\pi d_1^4/32} = 12\ \text{ksi}$$

$d_1 = 1.951$ in. .... **Ans.**

(b) Power shaft: $N_{power} = (96/16)N_{motor} = 6(360) = 2160$ rpm

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(2160)T}{60} = (100\times 550)\ \text{lb}\cdot\text{ft/s}$$

$$T_2 = 243.15339\ \text{lb}\cdot\text{ft} = 2917.84\ \text{lb}\cdot\text{in.}\ \left(= T_1/6\right)$$

$$\tau = \frac{(2.91784)(d_2/2)}{\pi d_2^4/32} = 12\ \text{ksi}$$

$d_2 = 1.074$ in. .... **Ans.**

6-48

(a) Shaft D: $N_D = (48/24)\,N_E = 2(400) = 800 \text{ rpm}$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(800)T_D}{60} = 40(10^3)\ \text{N}\cdot\text{m/s} \qquad T_D = 477.4648\ \text{N}\cdot\text{m}$$

$$\tau = \frac{(477.4648)(d_D/2)}{\pi d_D^4/32} = 70(10^6)\ \text{N/m}^2 \qquad d_D = 0.0326\ \text{m} = 32.6\ \text{mm}$$ .... **Ans.**

(b) Shaft E: $$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(400)T_D}{60} = 180(10^3)\ \text{N}\cdot\text{m/s}$$

$$T_E = 4297.1835\ \text{N}\cdot\text{m}$$

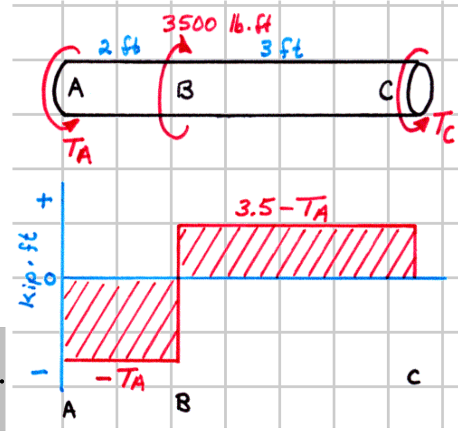
$$\tau = \frac{(4297.1835)(d_E/2)}{\pi d_E^4/32} = 70(10^6)\ \text{N/m}^2 \qquad d_E = 0.0679\ \text{m} = 67.9\ \text{mm}$$ .... **Ans.**

(c) Shaft F: $N_F = (48/24)\,N_E = 2(400) = 800 \text{ rpm}$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(800)T_E}{60} = 80(10^3)\ \text{N}\cdot\text{m/s} \qquad T_E = 954.9297\ \text{N}\cdot\text{m}$$

$$\tau = \frac{(954.9297)(d_E/2)}{\pi d_E^4/32} = 70(10^6)\ \text{N/m}^2 \qquad d_E = 0.0411\ \text{m} = 41.1\ \text{mm}$$ .... **Ans.**

6-49*



Equilibrium: Torque diagram

Deformations: $\theta_{AB} + \theta_{BC} = 0$ $\theta = TL/JG$

$$\frac{(-T_A)(2)}{JG} + \frac{[(3.5\times12) - T_A](3)}{JG} = 0$$

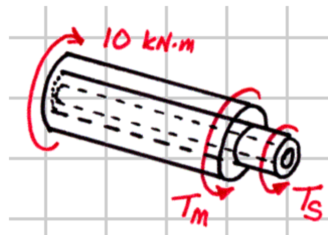
$$T_A = 25.200 \text{ kip}\cdot\text{in.} = -T_{AB}$$

$$T_{BC} = (3.5\times12) - T_A = 16.800 \text{ kip}\cdot\text{in.}$$

(a) $\tau_{max} = \dfrac{T_{AB}c}{J} = \dfrac{(25.200)(1)}{\pi(2)^4/32} = 16.04 \text{ ksi}$ .... **Ans.**

(b) $\theta = \theta_{AB} = \dfrac{TL}{JG} = \dfrac{(-25.200)(2\times12)}{[\pi(2)^4/32](12{,}000)} = -0.0321 \text{ rad}$ .... **Ans.**

6-50*



$$J_S = \pi d^4/32 = \pi\left(125^4 - 100^4\right)/32 = 14.15097\left(10^6\right)\ \text{mm}^4$$

$$J_M = \pi\left(175^4 - 125^4\right)/32 = 68.10875\left(10^6\right)\ \text{mm}^4$$

Equilibrium: $T_S + T_M = 10\ \text{kN}\cdot\text{m}$ (a)

Deformations: $\theta_S = \theta_M$ $\theta = TL/JG$

$$\frac{T_S L}{\left(14.15097\times10^{-6}\right)\left(80\times10^9\right)} = \frac{T_M L}{\left(68.10875\times10^{-6}\right)\left(65\times10^9\right)}$$

$$T_M = 3.91057 T_S \quad \text{(b)}$$

$T_S = 2.03642\ \text{kN}\cdot\text{m}$ $T_M = 7.96358\ \text{kN}\cdot\text{m}$

(a) $\tau_S = \dfrac{Tc}{J} = \dfrac{(2036.42)(0.0625)}{\left(14.15097\times10^{-6}\right)} = 8.99\left(10^6\right)\ \text{N/m}^2 = 8.99\ \text{MPa}$ .... **Ans.**

$\tau_M = \dfrac{(7963.58)(0.0875)}{\left(68.10875\times10^{-6}\right)} = 10.23\left(10^6\right)\ \text{N/m}^2 = 10.23\ \text{MPa}$ .... **Ans.**

(b) $\theta = \theta_S = \dfrac{TL}{JG} = \dfrac{(2036.42)(2)}{\left(14.15097\times10^{-6}\right)\left(80\times10^9\right)} = 0.00360\ \text{rad}$ .... **Ans.**

6-51

(a) $\tau = \dfrac{Tc}{J} = \dfrac{T(1.5)}{\pi(3)^4/32} \le 15 \text{ ksi}$

$T \le 79.5 \text{ kip}\cdot\text{in.}$ .... **Ans.**

(b) $\theta_S = \theta_A$ $\qquad \theta = TL/JG$

$$\frac{T_S L}{\left[\pi(3)^4/32\right](11,600)} = \frac{T_A L}{\left[\pi\left(3.5^4 - 3^4\right)/32\right](4000)} \qquad T_S = 3.40127 T_A$$

Guess that $\tau_S = \dfrac{Tc}{J} = \dfrac{T_S(1.5)}{\pi(3)^4/32} = \tau_{max} = 15 \text{ ksi}$

Then $T_S = 79.52156 \text{ kip}\cdot\text{in.} \qquad T_A = 23.37998 \text{ kip}\cdot\text{in.}$

and $\tau_A = \dfrac{(23.37998)(1.75)}{\pi\left(3.5^4 - 3^4\right)/32} = 6.03 \text{ ksi} \le 12 \text{ ksi} = \tau_{max}$ (correct guess)

Therefore $T_{max} = T_A + T_S = 102.9 \text{ kip}\cdot\text{in.}$ .... **Ans.**

6-52*

$$J_B = \pi d^4/32 = \pi\left(80^4 - 60^4\right)/32 = 2.74889\left(10^6\right)\ \text{mm}^4$$

$$J_A = \pi\left(60\right)^4/32 = 1.272345\left(10^6\right)\ \text{mm}^4$$

$$\frac{T_B L}{\left(2.74889\times10^{-6}\right)\left(45\times10^9\right)} = \frac{T_A L}{\left(1.272345\times10^{-6}\right)\left(28\times10^9\right)}$$

$$T_B = 3.47222 T_A$$

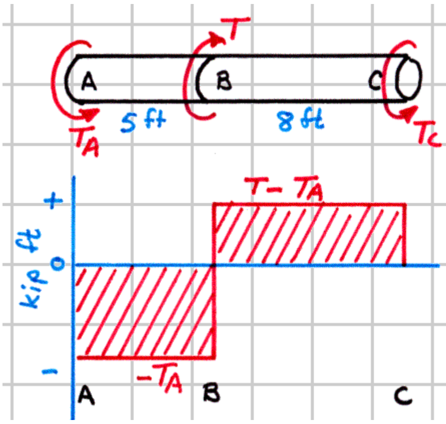
(a) $$\tau_B = \frac{Tc}{J} = \frac{T_B\left(0.040\right)}{\left(2.74889\times10^{-6}\right)} = 150\left(10^6\right)\ \text{N/m}^2$$

$T_B = 10{,}308.35\ \text{N}\cdot\text{m}$ $\qquad$ $T_A = 2968.807\ \text{N}\cdot\text{m}$

$T_{max} = T_B + T_A = 13.28\ \text{kN}\cdot\text{m}$ .... **Ans.**

(b) $$\tau_A = \frac{Tc}{J} = \frac{\left(2968.807\right)\left(0.030\right)}{\left(1.272345\times10^{-6}\right)} = 70.0\left(10^6\right)\ \text{N/m}^2 = 70.0\ \text{MPa}$$ .... **Ans.**

6-53



Equilibrium: Torque diagram

Deformations: $\theta_{AB} + \theta_{BC} = 0 \qquad \theta = TL/JG$

$$\frac{(-T_A)(5\times 12)}{J(11{,}600)} + \frac{(T - T_A)(8\times 12)}{J(6500)} = 0$$

$$T = 1.35022 T_A$$

Guess that $\tau_B = \dfrac{Tc}{J} = \dfrac{(T - T_A)(1.5)}{\pi(3)^4/32} = \tau_{max} = 6 \text{ ksi}$

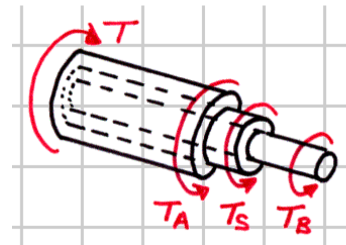
Then $T - T_A = 31.80863 \text{ kip}\cdot\text{in.}$

$T_A = 90.82586 \text{ kip}\cdot\text{in.} = -T_S$

and $\tau_S = \dfrac{(90.82586)(1.5)}{\pi(3)^4/32} = 17.13 \text{ ksi} \le 18 \text{ ksi} = \tau_{max}$ (correct guess)

Therefore $T_{max} = 1.35022(90.82586) = 122.6 \text{ kip}\cdot\text{in.}$ .... **Ans.**

6-54*



$$J_B = \pi d^4/32 = \pi(40)^4/32 = 0.251327(10^6) \text{ mm}^4$$

$$J_S = \pi(80^4 - 40^4)/32 = 3.76991(10^6) \text{ mm}^4$$

$$J_A = \pi(100^4 - 80^4)/32 = 5.79624(10^6) \text{ mm}^4$$

Equilibrium: $T = T_A + T_S + T_B$ (a)

Deformations: $\theta_A = \theta_S = \theta_B$ $\theta = TL/JG$

$$\frac{T_A L}{(5.79624\times10^{-6})(28\times10^9)} = \frac{T_S L}{(3.76991\times10^{-6})(80\times10^9)} = \frac{T_B L}{(0.251327\times10^{-6})(39\times10^9)}$$

$T_A = 16.5577 T_B$ (b)

$T_S = 30.7693 T_B$ (c)

(a) $16.5577T_B + 30.7693T_B + T_B = 15 \text{ kN}\cdot\text{m}$

$T_B = 0.31039 \text{ kN}\cdot\text{m}$ $T_A = 5.13928 \text{ kN}\cdot\text{m}$ $T_S = 9.55034 \text{ kN}\cdot\text{m}$

$\tau_B = \frac{Tc}{J} = \frac{(310.39)(0.020)}{(0.251327\times10^{-6})} = 24.7(10^6) \text{ N/m}^2 = 24.7 \text{ MPa}$ .... **Ans.**

$\tau_A = \frac{(5139.28)(0.050)}{(5.79624\times10^{-6})} = 44.3(10^6) \text{ N/m}^2 = 44.3 \text{ MPa}$ .... **Ans.**

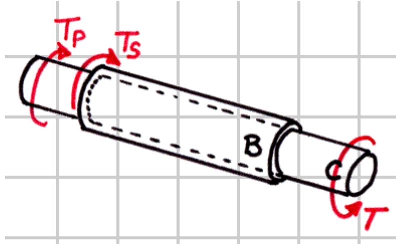
$\tau_S = \frac{(9550.34)(0.040)}{(3.76991\times10^{-6})} = 101.3(10^6) \text{ N/m}^2 = 101.3 \text{ MPa}$ .... **Ans.**

(b) $16.5577T_B + 30.7693T_B + T_B = 10 \text{ kN}\cdot\text{m}$

$T_B = 0.206924 \text{ kN}\cdot\text{m}$ $T_A = 3.42618 \text{ kN}\cdot\text{m}$ $T_S = 6.36690 \text{ kN}\cdot\text{m}$

$\theta = \theta_B = \frac{TL}{JG} = \frac{(206.924)(3)}{(0.251327\times10^{-6})(39\times10^9)} = 0.0633 \text{ rad}$ .... **Ans.**

6-55*



Equilibrium: $T_S + T_P = T = 1000 \text{ lb}\cdot\text{in.}$ (a)

Deformations: $\theta_{AB,P} = \theta_{AB,S}$ $\theta = TL/JG$

(a) $$\frac{T_P(12)}{\left[\pi(2)^4/32\right](150)} = \frac{T_S(12)}{\left[\pi\left(2.25^4 - 2^4\right)/32\right](12{,}000)}$$

$$T_S = 48.1445 T_P \quad \text{(b)}$$

$T_P = 20.3482 \text{ lb}\cdot\text{in.}$ $T_S = 979.6518 \text{ lb}\cdot\text{in.}$

$$\theta = \frac{TL}{JG} = \frac{(20.3482)(12) + (1000)(4)}{\left[\pi(2)^4/32\right]\left(150\times 10^3\right)} = 0.01801 \text{ rad}$$ .... **Ans.**

(b) $$\theta = \frac{(1000)(4)}{\left[\pi(2)^4/32\right]\left(150\times 10^3\right)} = 0.01698 \text{ rad}$$ .... **Ans.**

(c) $$\%error = \frac{0.01801 - 0.01698}{0.01801}(100) = 5.75\%$$ .... **Ans.**

6-56

$$J_S = \pi\left(100^4 - 50^4\right)/32 = 9.20388\left(10^6\right)\ \text{mm}^4$$

$$J_M = \pi\left(125^4 - 102^4\right)/32 = 13.34170\left(10^6\right)\ \text{mm}^4$$

$$\theta_S = \theta_M \qquad \theta = TL/JG$$

$$\frac{T_S L}{\left(9.20388\times10^{-6}\right)\left(80\times10^9\right)} = \frac{T_M L}{\left(13.34170\times10^{-6}\right)\left(65\times10^9\right)} \qquad T_S = 0.84906 T_M$$

Guess that $\tau_S = \dfrac{Tc}{J} = \dfrac{T_S\left(0.05\right)}{\left(9.20388\times10^{-6}\right)} = \tau_{\max} = 70\left(10^6\right)\ \text{N/m}^2$

Then $T_S = 12{,}885.4\ \text{N}\cdot\text{m} \qquad T_M = 15{,}176.2\ \text{N}\cdot\text{m}$

and $\tau_M = \dfrac{\left(15{,}176.2\right)\left(0.0625\right)}{\left(13.34170\times10^{-6}\right)} = 71.1\left(10^6\right)\ \text{N/m}^2$

$\le 85\ \text{MPa} = \tau_{\max}$ (correct guess)

(a) $T_{\max} = T_S + T_M = 12.885 + 15.176 = 28.061\ \text{kN}\cdot\text{m} \cong 28.1\ \text{kN}\cdot\text{m}$ .... **Ans.**

(b) $\theta = \theta_S = \dfrac{TL}{JG} = \dfrac{\left(12{,}885.4\right)\left(2.5\right)}{\left(9.20388\times10^{-6}\right)\left(80\times10^9\right)} = 0.0437\ \text{rad}$ .... **Ans.**

6-57*

Equilibrium: $T = T_S + T_A = 1.5T_A$ $T_S = 0.5T_A$

Deformations: $\theta_A = \theta_S$ $\theta = TL/JG$

$$\frac{T_A L}{\left[\pi(2)^4/32\right](4000)} = \frac{(0.5T_A)L}{J_S(12,000)} \qquad J_S = 0.26180 \text{ in.}^4$$

$$J_S = \frac{\pi\left(d^4 - 2^4\right)}{32} = 0.26180 \text{ in.}^4 \qquad d = 2.07858 \text{ in.} = 2(1+t)$$

$t = 0.0393$ in. .... **Ans.**

6-58

$T_B = T_S$

$\theta_B = \theta_S \qquad \theta = TL/JG$

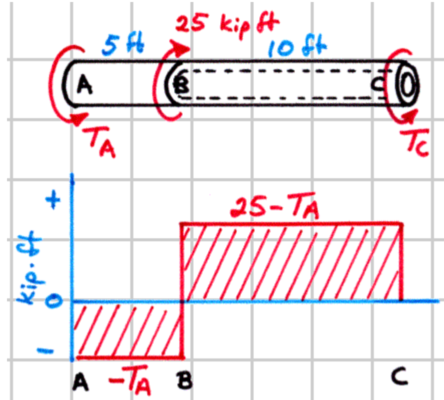
$$\frac{T_B L}{J_B\left(45\times10^9\right)} = \frac{T_S L}{J_S\left(80\times10^9\right)} \qquad J_B = 1.77778 J_S$$

$$J_B = \frac{\pi\left(100^4 - d^4\right)}{32} = \left(1.77778\right)\frac{\pi d^4}{32} = 1.77778 J_S$$

$d = 77.5$ mm .... **Ans.**

6-59

Equilibrium: Torque diagram



Deformations: $\theta_{AB} + \theta_{BC} = 0 \qquad \theta = TL/JG$

$$\frac{(-T_A)(5\times 12)}{\left[\pi (4)^4/32\right]G} + \frac{\left[(25\times 12) - T_A\right](10\times 12)}{\left[\pi \left(4^4 - 2^4\right)/32\right]G} = 0$$

$$T_A = 204.255 \text{ kip}\cdot\text{in.} = -T_{AB}$$

$$T_{BC} = (25\times 12) - T_A = 95.7447 \text{ kip}\cdot\text{in.}$$

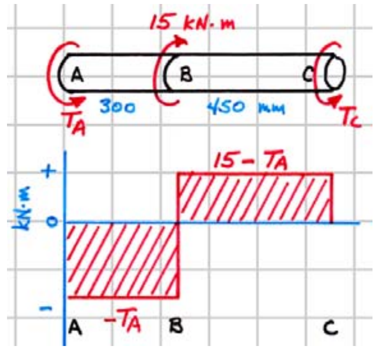
(a) $$\tau_{AB} = \frac{Tc}{J} = \frac{(204.255)(2)}{\pi (4)^4/32} = 16.25 \text{ ksi}$$

$$\tau_{BC} = \frac{(95.7447)(2)}{\pi \left(4^4 - 2^4\right)/32} = 8.13 \text{ ksi}$$

$\tau_{\max} = \tau_{AB} = 16.25$ ksi .... **Ans.**

(b) $$\theta = \theta_{AB} = \frac{TL}{JG} = \frac{(-204.255)(5\times 12)}{\left[\pi (4)^4/32\right](12{,}000)} = -0.0406 \text{ rad}$$ .... **Ans.**

6-60*



$$J_B = \pi d^4/32 = \pi(75)^4/32 = 3.10631(10^6)\ \text{mm}^4$$

$$J_M = \pi(60)^4/32 = 1.272345(10^6)\ \text{mm}^4$$

Equilibrium: Torque diagram

Deformations: $\theta_{AB} + \theta_{BC} = 0 \qquad \theta = TL/JG$

$$\frac{(-T_A)(0.300)}{(3.10631\times10^{-6})(39\times10^9)} + \frac{(15{,}000 - T_A)[0.450]}{(1.272345\times10^{-6})(65\times10^9)} = 0$$

$$T_A = 10{,}308.49\ \text{N}\cdot\text{m} = -T_{AB} \qquad T_{BC} = 15{,}000 - T_A = 4691.51\ \text{N}\cdot\text{m}$$

(a) $\tau_{AB} = \dfrac{Tc}{J} = \dfrac{(10{,}308.49)(0.0375)}{(3.10631\times10^{-6})} = 124.4(10^6)\ \text{N/m}^2 = 124.4\ \text{MPa}$ .... **Ans.**

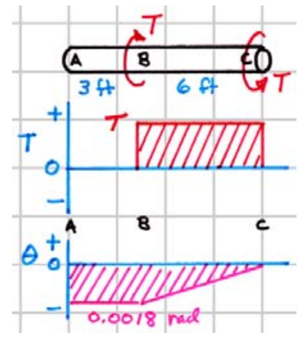
$\tau_{BC} = \dfrac{(4691.51)(0.030)}{(1.272345\times10^{-6})} = 110.6(10^6)\ \text{N/m}^2 = 110.6\ \text{MPa}$ .... **Ans.**

(b) $\theta = \theta_{AB} = \dfrac{TL}{JG} = \dfrac{(-10{,}308.49)(0.300)}{(3.10631\times10^{-6})(39\times10^9)} = -0.0255\ \text{rad}$ .... **Ans.**

(c) $\sigma_{\max T} = \sigma_{\max C} = \tau_{AB} = 124.4\ \text{MPa}$ .... **Ans.**

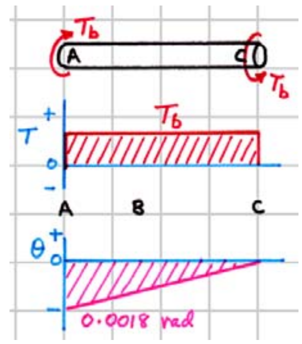
6-61

$$J = \pi d^4/32 = \pi(6)^4/32 = 127.2345 \text{ in}^4$$



(a) $$\theta = \frac{TL}{GJ} = \frac{T(6\times 12)}{(127.2345)(12{,}000)} = 0.0018 \text{ rad}$$

$T = 38.17035 \text{ kip}\cdot\text{in.} \cong 3.18 \text{ kip}\cdot\text{ft}$ ....**Ans.**



(b) $$\theta = \frac{TL}{GJ} = \frac{T_b(9\times 12)}{(127.2345)(12{,}000)} = 0.0018 \text{ rad}$$

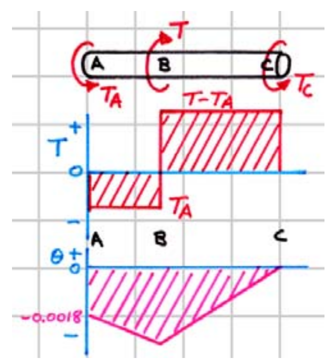
$T_b = 25.44690 \text{ kip}\cdot\text{in.}$

$$\tau = \frac{Tc}{J} = \frac{(25.44690)(3)}{(127.2345)} = 0.600 \text{ ksi}$$

$\tau = 600 \text{ psi}$ .... **Ans.**

(c) Equilibrium: Torque diagram

Deformations: $\theta_A + \theta_{AB} + \theta_{BC} = 0$ $\theta = TL/JG$



$$-0.0018 + \frac{(-T_A)(3\times 12)}{(127.2345)(12{,}000)} + \frac{(T - T_A)(6\times 12)}{(127.2345)(12{,}000)} = 0$$

Guess that $$\tau_{AB} = \frac{Tc}{J} = \frac{(T_A)(3)}{(127.2345)} = \tau_{\max} = 10 \text{ ksi}$$

Then $T_A = 424.1150 \text{ kip}\cdot\text{in.} = -T_{AB}$

$T - T_A = 250.2279 \text{ kip}\cdot\text{in.} = T_{BC}$

and $$\tau_{BC} = \frac{(250.2279)(3)}{(127.2345)} = 5.90 \text{ ksi} \le 10 \text{ ksi} = \tau_{\max} \quad \text{(correct guess)}$$

Therefore $T_{\max} = 250.2279 + 424.1150 = 674.3429 \text{ kip}\cdot\text{in.}$

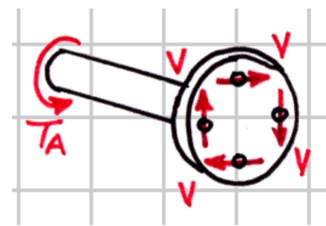
$T_{\max} = 56.2 \text{ kip}\cdot\text{ft}$ .... **Ans.**

6-62*

$\circlearrowleft \Sigma M = 0:\qquad T_A - 4(0.075V) = 0$

$T_A = 0.300V = 0.300(\tau A)$

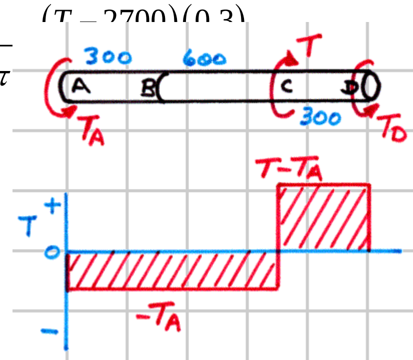
$= 0.300(60\times10^6)(150\times10^{-6}) = 2700\text{ N}\cdot\text{m}$



Equilibrium: Torque diagram

Deformations: $\theta_{AB} + \theta_{BC} + \theta_{CD} = 0 \qquad \theta = TL/JG$

$$\frac{(-2700)(0.3)}{\left[\pi(0.075)^4/32\right](28\times10^9)} + \frac{(-2700)(0.6)}{\left[\pi(0.075)^4/32\right](80\times10^9)} + \frac{(T-2700)(0.3)}{\left[\pi \right.}$$



(a) $T = 15{,}814.3\text{ N}\cdot\text{m} \cong 15.81\text{ kN}\cdot\text{m}$ .... **Ans.**

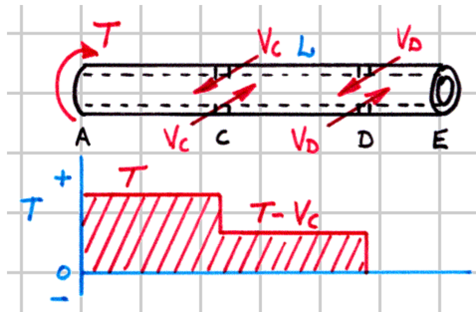
(b) $T_{CD} = 15{,}814.3 - 2700 = 13{,}114.3\text{ N}\cdot\text{m}$

$T_{\max}(steel) = T_{CD} = 13{,}114.3\text{ N}\cdot\text{m}$

$$\tau_{\max} = \tau_{CD} = \frac{Tc}{J} = \frac{(13{,}114.3)(0.0375)}{\left[\pi(0.075)^4/32\right]}$$

$\tau_{\max} = 158.3(10^6)\text{ N/m}^2 = 158.3\text{ MPa}$ .... **Ans.**

6-63*

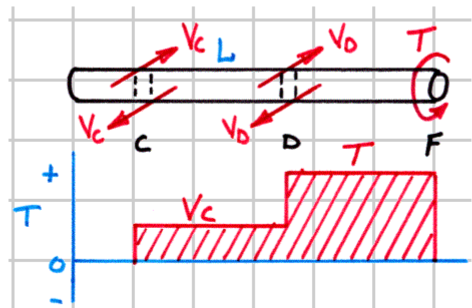


The torque of the shear force V_C is $T_C = V_C(1) = V_C$ and the torque of the shear force V_D is $T_D = V_D(1) = V_D$.

Equilibrium: Torque diagrams

Deformations: $\theta_{CD,h} = \theta_{CD,s}$ $\theta = TL/JG$

$$\frac{(1000 - V_C)L}{\left[\pi(2^4 - 1^4)/32\right](12\times10^6)} = \frac{(V_C)L}{\left[\pi(1)^4/32\right](12\times10^6)}$$



Therefore $V_C = 62.500$ lb

and $V_D = 1000 - V_C = 1000 - 62.5 = 937.500$ lb

$V_C = \tau A = (25{,}000)(\pi d_C^2/4) = 62.500$ lb

$d_C = 0.0564$ in. .... **Ans.**

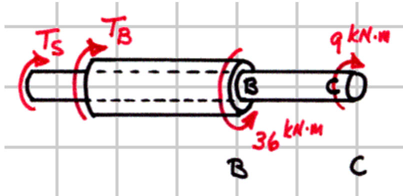
$V_D = \tau A = (25{,}000)(\pi d_D^2/4) = 937.500$ lb

$d_D = 0.219$ in. .... **Ans.**

6-64

$$J_S = \pi d^4/32 = \pi(80)^4/32 = 4.02124(10^6)\ \text{mm}^4$$

$$J_B = \pi(120^4 - 80^4)/32 = 16.33628(10^6)\ \text{mm}^4$$



Equilibrium: $T_S + T_B = 27\ \text{kN}\cdot\text{m}$ (a)

Deformations: $\theta_{AB,S} = \theta_{AB,B}$ $\theta = TL/JG$

$$\frac{T_S(1.5)}{(4.02124\times10^{-6})(86\times10^9)} = \frac{T_B(1.5)}{(16.33628\times10^{-6})(39\times10^9)}$$

$T_B = 1.84230 T_S$ (b)

$T_B = 17.50065\ \text{kN}\cdot\text{m}$ $T_S = 9.49935\ \text{kN}\cdot\text{m}$

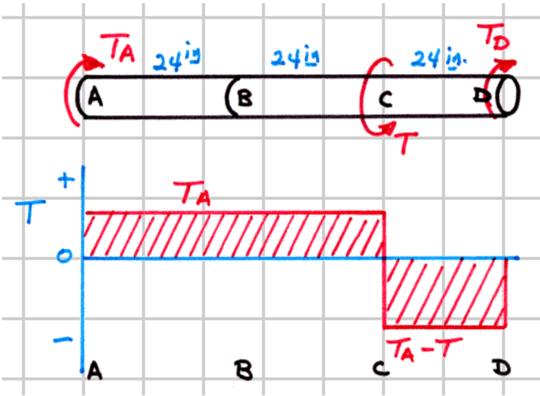
(a) $\tau_S = \dfrac{Tc}{J} = \dfrac{(9,499.35)(0.040)}{(4.02124\times10^{-6})} = 94.5(10^6)\ \text{N/m}^2 = 94.5\ \text{MPa}$ .... **Ans.**

(b) $\tau_B = \dfrac{(17,500.65)(0.060)}{(16.33628\times10^{-6})} = 64.3(10^6)\ \text{N/m}^2 = 64.3\ \text{MPa}$ .... **Ans.**

(c) $\sigma_{\max} = \tau_{\max} = 64.3\ \text{MPa (C)}$ .... **Ans.**

(d) $\theta = \dfrac{TL}{JG} = \dfrac{(9,499.35)(1.5) + (-9000)(1)}{(4.02124\times10^{-6})(86\times10^9)} = 0.01518\ \text{rad}$ .... **Ans.**

6-65



Equilibrium: Torque diagram

Deformations: $\theta_{AB} + \theta_{BC} + \theta_{CD} = 0 \qquad \theta = TL/JG$

$$\frac{T_A(24)}{J(6000)} + \frac{T_A(24)}{J(12{,}000)} + \frac{(T_A - T)(24)}{J(12{,}000)} = 0$$

Guess that $\tau_{CD} = \dfrac{Tc}{J} = \dfrac{(T_A - T)(2)}{\pi(4)^4/32} = \tau_{\max} = -12 \text{ ksi}$

Then $T_A - T = -150.79645 \text{ kip}\cdot\text{in.} = T_{CD}$

$T_A = 50.26548 \text{ kip}\cdot\text{in.} = T_{AB} = T_{BC}$

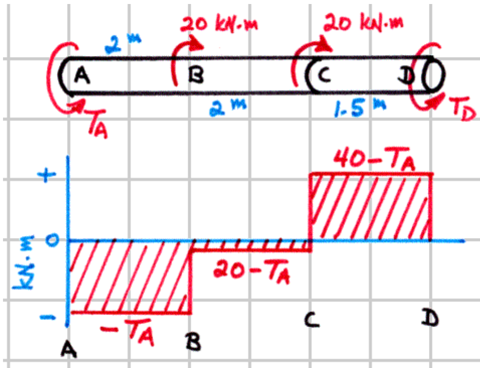
$$\tau_{AB} = \frac{(50.26548)(2)}{\pi(4)^4/32} = 4.00 \text{ ksi} \le 5 \text{ ksi} = \tau_{\max} \quad \text{(okay)}$$

and $\tau_{BC} = \dfrac{(50.26548)(2)}{\pi(4)^4/32} = 4.00 \text{ ksi} \le 12 \text{ ksi} = \tau_{\max} \quad \text{(okay)}$

Therefore $T_{\max} = 50.26548 + 150.79645 = 201.062 \text{ kip}\cdot\text{in.} \cong 201 \text{ kip}\cdot\text{in.}$ .... **Ans.**

6-66*

$$J = \pi d^4/32 = \pi(100)^4/32 = 9.81748(10^6) \text{ mm}^4$$



Equilibrium: Torque diagram

Deformations: $\theta_{AB} + \theta_{BC} + \theta_{CD} = 0 \qquad \theta = TL/JG$

$$\frac{(-T_A)(2)}{J(80\times10^9)} + \frac{(20-T_A)(2)}{J(80\times10^9)} + \frac{(40-T_A)(1.5)}{J(40\times10^9)} = 0$$

$$T_A = 22.85714 \text{ kN}\cdot\text{m} = -T_{AB}$$

$$T_{BC} = 20 - T_A = -2.85715 \text{ kN}\cdot\text{m}$$

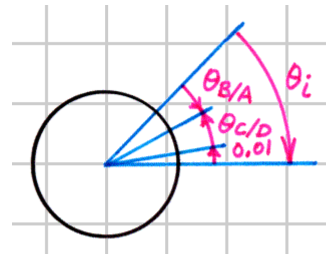
$$T_{CD} = 40 - T_A = 17.14286 \text{ kN}\cdot\text{m}$$

(a) $\tau_B = \dfrac{Tc}{J} = \dfrac{(17{,}142.86)(0.050)}{(9.81748\times10^{-6})} = 87.3(10^6) \text{ N/m}^2 = 87.3 \text{ MPa}$ .... **Ans.**

(b) $\tau_S = \dfrac{(22{,}857.14)(0.050)}{(9.81748\times10^{-6})} = 116.4(10^6) \text{ N/m}^2 = 116.4 \text{ MPa}$ .... **Ans.**

(c) $\theta = \dfrac{TL}{JG} = \dfrac{(-22{,}857.14)(2)}{(9.81748\times10^{-6})(80\times10^9)} = -0.0582 \text{ rad}$ .... **Ans.**

6-67



$$J_{AB} = \pi d^4/32 = \pi\left(6^4 - 4^4\right)/32 = 102.1018 \text{ in}^4$$

$$J_{BC} = \pi(4)^4/32 = 25.1327 \text{ in}^4$$

Initially $\quad \theta_i = \dfrac{TL}{GJ} = \dfrac{(40\times 12)(6\times 12)}{(102.1018)(12{,}000)} = 0.02821 \text{ rad}$

After the torque is removed $\quad T_{AB} = T_{CD}$, and

Deformations: $\quad \theta_{B/A} + \theta_{C/D} + \theta_{slip} = \theta_i \qquad \theta = TL/JG$

$$\frac{T_{AB}(6\times 12)}{(102.1018)(12{,}000)} + \frac{T_{BC}(4\times 12)}{(25.1327)(4000)} + 0.010 = 0.02821 \text{ rad}$$

$$T_{AB} = T_{CD} = 33.9540 \text{ kip}\cdot\text{in.}$$

(a) $\tau_A = \dfrac{Tc}{J} = \dfrac{(33.9540)(2)}{(25.1327)} = 2.70 \text{ ksi}$ .... **Ans.**

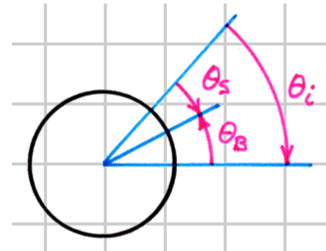
(b) $\tau_S = \dfrac{(33.9540)(3)}{(102.1018)} = 0.998 \text{ ksi}$ .... **Ans.**

(c) $\theta = \dfrac{TL}{GJ} = \dfrac{(-33.9540)(6\times 12)}{(102.1018)(12{,}000)} = -0.001995 \text{ rad}$ .... **Ans.**

6-68

$$J_S = \pi d^4/32 = \pi(80)^4/32 = 4.02124(10^6)\ \text{mm}^4$$

$$J_B = \pi(160^4 - 140^4)/32 = 26.62500(10^6)\ \text{mm}^4$$



Initially $\theta_i = \dfrac{TL}{GJ} = \dfrac{(10{,}000)(0.800)}{(4.02124\times10^{-6})(80\times10^9)} = 0.02487\ \text{rad}$

After the torque is removed $T_S = T_B$, and

Deformations: $\theta_S + \theta_B = \theta_i$ $\theta = TL/JG$

$$\frac{T_S(0.800)}{(4.02124\times10^{-6})(80\times10^9)} + \frac{T_B(0.800)}{(26.62500\times10^{-6})(40\times10^9)} = 0.02487\ \text{rad}$$

$$T_S = T_B = 7680.1087\ \text{N}\cdot\text{m}$$

(a) $\tau_B = \dfrac{Tc}{J} = \dfrac{(7680.1087)(0.080)}{(26.62500\times10^{-6})} = 23.1(10^6)\ \text{N/m}^2 = 23.1\ \text{MPa}$ .... **Ans.**

(b) $\tau_S = \dfrac{(7680.1087)(0.040)}{(4.02124\times10^{-6})} = 76.4(10^6)\ \text{N/m}^2 = 76.4\ \text{MPa}$ .... **Ans.**

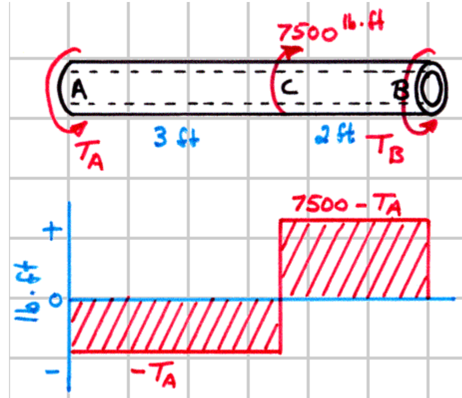
(c) $\theta = \dfrac{TL}{GJ} = \dfrac{(-7680.1087)(0.800)}{(4.02124\times10^{-6})(80\times10^9)} = -0.01910\ \text{rad}$ .... **Ans.**

6-69

$$J = \pi d^4/32 = \pi\left(r_o^4 - r_i^4\right)/2 = \pi r_o^4\left[1-\left(r_i/r_o\right)^4\right]/2 = 8\pi\left[1-\left(r_i/r_o\right)^4\right] \text{ in.}^4$$

<u>Equilibrium:</u> Torque diagram

<u>Deformations:</u> $\theta_{AC} + \theta_{CB} = 0$ $\theta = TL/JG$



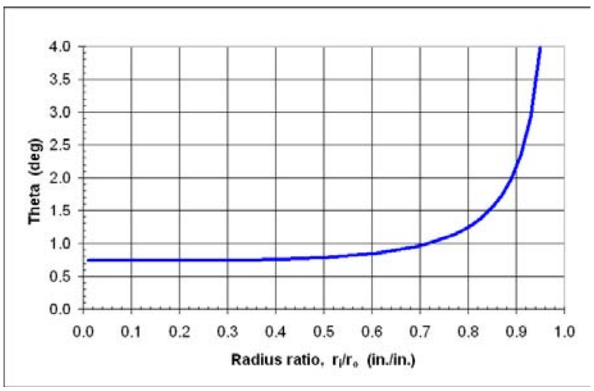
$$\frac{(-T_A)(3\times 12)}{J(4000)} + \frac{(7.5-T_A)(2\times 12)}{J(4000)} = 0$$

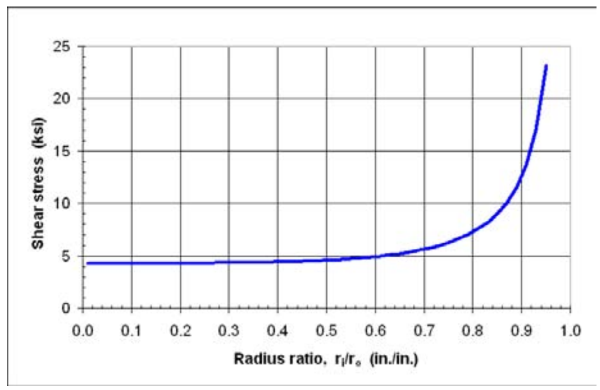
$$T_A = 3.000 \text{ kip}\cdot\text{ft} = T_{AC}$$

$$T_{CB} = 7.5 - T_A = 4.500 \text{ kip}\cdot\text{ft}$$

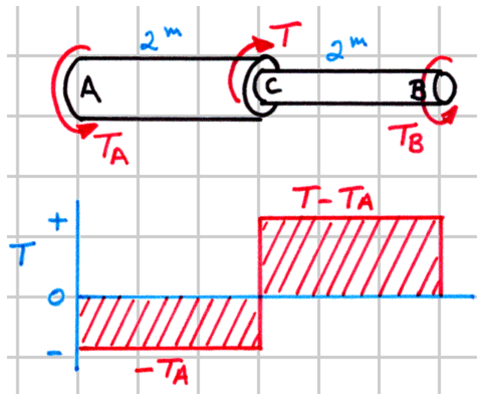
(a) $$\theta = \frac{TL}{JG} = \frac{(-3\times 12)(3\times 12)}{8\pi\left[1-(r_i/r_o)^4\right](4000)} \text{ rad}$$

(b) $$\tau = \frac{Tc}{J} = \frac{(4.5\times 12)(2)}{8\pi\left[1-(r_i/r_o)^4\right]} \text{ ksi}$$





6-70



Equilibrium: Torque diagram

Deformations: $\theta_{AC} + \theta_{CB} = 0$ $\theta = TL/JG$

$$\frac{(-T_A)(2)}{J_s\left(80\times10^9\right)} + \frac{(T-T_A)(2)}{J_b\left(40\times10^9\right)} = 0$$

$$T_A\left[1+\left(J_b/2J_s\right)\right] = T$$

$$J_s = \pi d_s^4/32 \qquad J_b = \pi d_b^4/32$$

$$d_b + d_s = 200 \text{ mm}$$

In terms of the diameter ratio $r = (d_b/d_s)$

$$d_s = 200/(1+r) \qquad d_b = rd_s = 200r/(1+r)$$

Method of solution:

Guess $\tau_s = \dfrac{T_s(d_s/2)}{J_s} = \tau_s(\max)$

Compute $T_A = -T_s$, $T - T_A = T_b$, T, τ_b, and θ_{AC}

Check $\tau_b \le \tau_b(\max)$ and $\theta_{AC} \le \theta_{\max}$

Guess $\tau_b = \dfrac{T_b(d_b/2)}{J_b} = \tau_b(\max)$

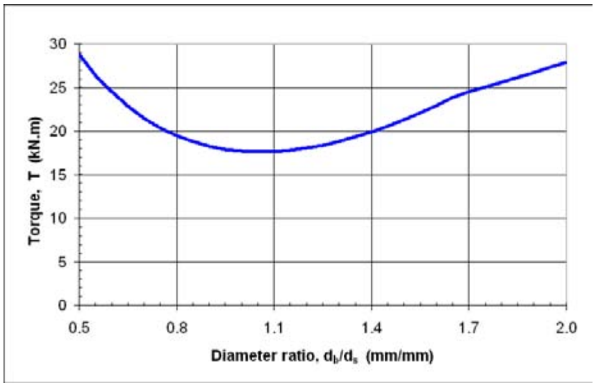
Compute $T - T_A = T_b$, $T_s = -T_A$, T, τ_s, and θ_{AC}

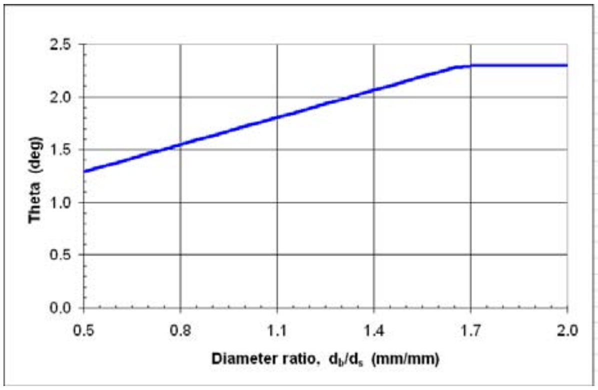
Check $\tau_s \le \tau_s(\max)$ and $\theta_{AC} \le \theta_{\max}$

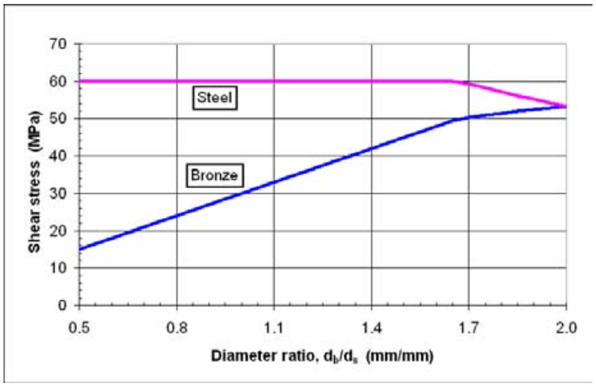
Guess $\theta_{AC} = \theta_{\max}$

Compute $T_A = -T_s$, $T - T_A = T_b$, T, τ_b, and τ_s

Check $\tau_b \le \tau_b(\max)$ and $\tau_s \le \tau_s(\max)$







6-71

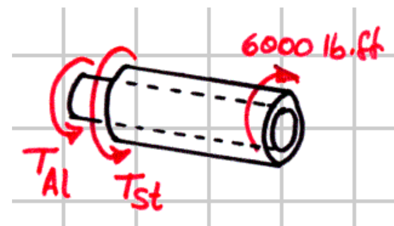
$$J_S = \frac{\pi d^4}{32} = \frac{\pi\left(4^4 - d_a^4\right)}{32} \text{ in.}^4 \qquad J_{Al} = \frac{\pi d_a^4}{32} \text{ in.}^4$$

Equilibrium: $T_{St} + T_{Al} = 6.00 \text{ kip}\cdot\text{ft} = 72.00 \text{ kip}\cdot\text{in.}$ (a)

Deformations: $\theta_{ST} = \theta_{Al}$ $\qquad \theta = TL/JG$

$$\frac{T_{St}L}{J_{St}(12{,}000)} = \frac{T_{Al}L}{J_{Al}(4000)} \qquad T_{St} = \frac{3J_{St}T_{Al}}{J_{Al}} \quad \text{(b)}$$

$$T_{Al} = \frac{72J_{Al}}{J_{Al} + 3J_{St}} = \frac{36d_a^4}{384 - d_a^4} \text{ kip}\cdot\text{in.}$$

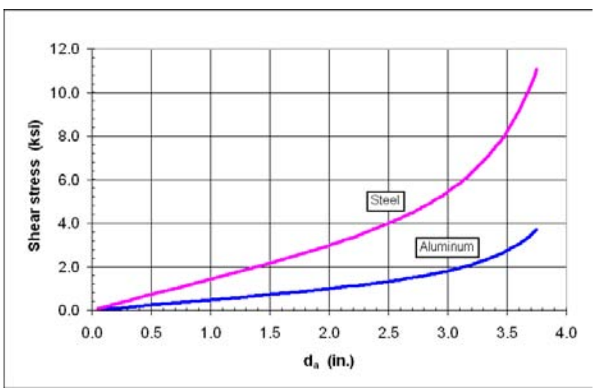


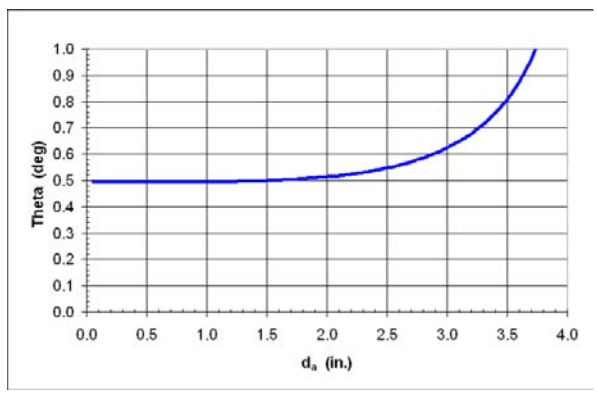
$$T_{St} = \frac{216J_{St}}{J_{Al} + 3J_{St}} = \frac{108\left(256 - d_a^4\right)}{384 - d_a^4} \text{ kip}\cdot\text{in.}$$

$$\tau_{Al} = \frac{T_{Al}(d_a/2)}{J_{Al}} = \frac{183.3465d_a}{384 - d_a^4} \text{ ksi}$$

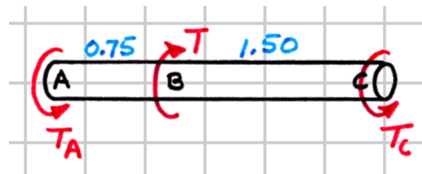
$$\tau_{St} = \frac{T_{St}(d_a/2)}{J_{St}} = \frac{72J_{Al}}{J_{Al} + 3J_{St}} = \frac{550.0395d_a}{384 - d_a^4} \text{ ksi}$$

$$\theta = \frac{T_{Al}36}{J_{Al}(4000)} = \frac{3.30024}{384 - d_a^4} \text{ rad}$$





6-72



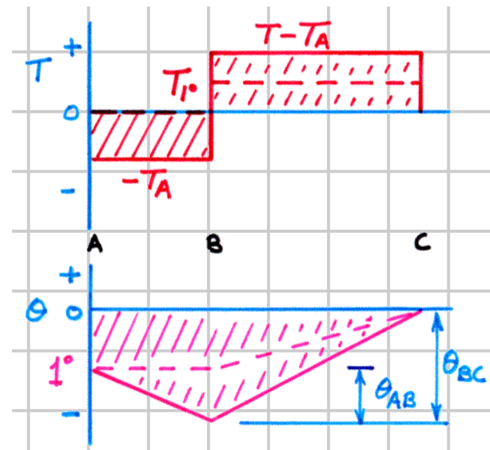
(a)

$$J = \frac{\pi d^4}{32} = \frac{\pi (120)^4}{32} = 20.3575(10^6)\text{ mm}^4$$

$$A_b = \frac{\pi d^2}{4} = \frac{\pi (18)^2}{4} = 254.4690\text{ mm}^2$$

Equilibrium: $T_A + T_C = T$

Deformations: $\theta_{total} = \theta_{slip} + \theta_{AB} + \theta_{BC} = 0$ $\qquad \theta = TL/JG$



$$\theta_{AB} = \frac{(-T_A)(0.75)}{(20.3575\times 10^{-6})(80\times 10^9)} = -0.460518(10^{-6})T_A\text{ rad}$$

$$\theta_{BC} = \frac{(T - T_A)(1.50)}{(20.3575\times 10^{-6})(80\times 10^9)} = 0.921036(10^{-6})(T - T_A)\text{ rad}$$

If $\theta_{BC} \le 1° = 0.0174533\text{ rad}$, then $\qquad T_A = \theta_{AB} = 0$

$$T_{BC} = T - T_A = T = 1.08573(10^6)\theta_{BC}\text{ N}$$

If $\theta_{BC} = 1° = 0.0174533\text{ rad}$, then $\qquad T_A = \theta_{AB} = 0$

$$T_{BC} = T = 18{,}949.63\text{ N}$$

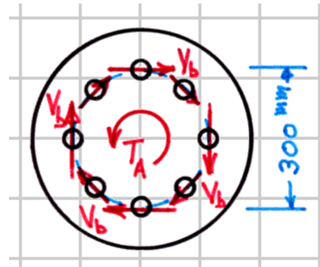
If $\theta_{BC} \ge 1° = 0.0174533\text{ rad}$, then

$$(-0.0174533) + \left[-0.460518(10^{-6})T_A\right] + \left[0.921036(10^{-6})(T - T_A)\right] = 0$$

6-72 (cont.)

$$T_A = (0.666667T - 12,633.09)\ \text{N} = -T_{AB}$$

$$T_{BC} = T - T_A = (0.333333T + 12,633.09)\ \text{N}$$



(a) $$\tau_{AB} = \frac{Tc}{J} = \frac{(T_A)(0.060)}{(20.3575\times10^{-6})} = (2947.32T_A)\ \text{N/m}^2$$

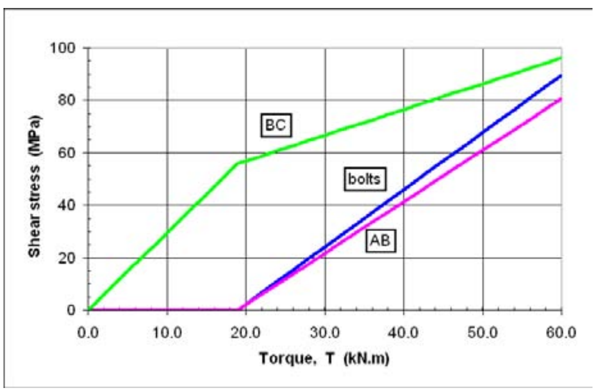
$$\tau_{BC} = \frac{Tc}{J} = \frac{(T - T_A)(0.060)}{(20.3575\times10^{-6})} = 2947.32(T - T_A)\ \text{N/m}^2$$

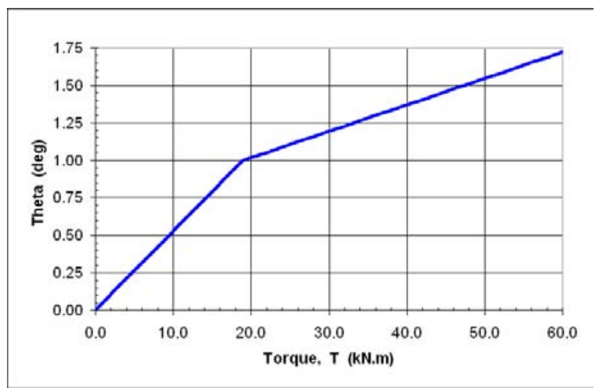
$$T_A = 8[0.150V_b] = 8[0.150\tau_b](254.4690\times10^{-6}) = 305.3628(10^{-6})\tau_b$$

(b) $$\tau_b = \frac{(T_A)}{(305.3628\times10^{-6})} = \frac{(0.666667T - 12,633.09)}{(305.3628\times10^{-6})}$$

$$\tau_b = \left[2183.197T - 41.37076(10^6)\right]\ \text{N/m}^2$$

(c) $$\theta_B = \theta_{BC} = 0.921036(10^{-6})(T - T_A)\ \text{rad}$$





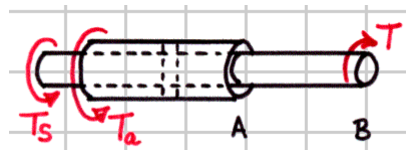
6-73

Assume that the steel shaft extends all the way through the aluminum shell and attaches to the wall at *D*.

$$J_a = \frac{\pi d^4}{32} = \frac{\pi\left(3^4 - 2^4\right)}{32} = 6.38136 \text{ in.}^4 \qquad J_a = \frac{\pi\left(2^4\right)}{32} = 1.57080 \text{ in.}^4$$

Equilibrium: $T_s + T_a = T$ (a)

Deformations: $\theta_{CD,s} = \theta_{CD,a} \qquad \theta = TL/JG$



(b)

$$\frac{T_s(12)}{(1.57080)(12,000)} = \frac{T_a(12)}{(6.38136)(4000)}$$

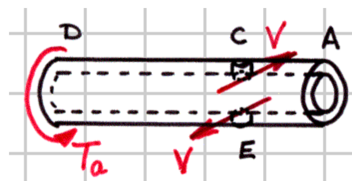
$$T_a = 1.35416 T_s$$

$$T_a = 0.575221T \qquad T_{s,CD} = 0.424779T \qquad T_{s,CB} = T$$

(a) $$\tau_a = \frac{Td}{J} = \frac{(0.575221T)(1.5)}{(6.38136)} = 0.135211T \text{ ksi}$$

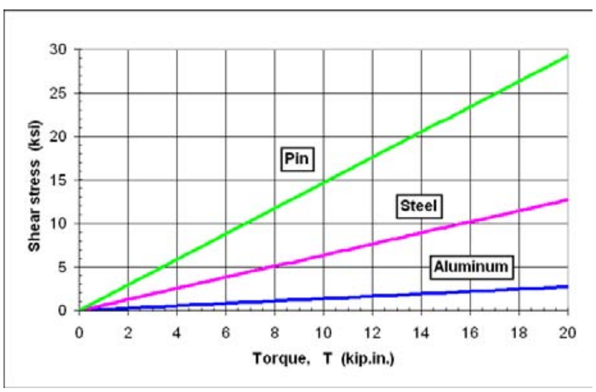
$$\tau_s = \frac{(T)(1)}{(1.57080)} = 0.636618T \text{ ksi}$$

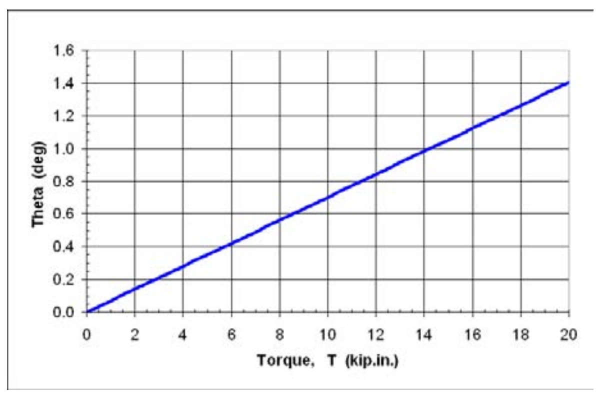
(b) $$T_a = 2V = 2\left(\tau_b A_b\right)$$



$$\tau_b = \frac{V}{2A_p} = \frac{0.575221T}{2\left[\pi(0.5)^2/4\right]} = 1.46479T \text{ ksi}$$

(c) $$\theta = \frac{(0.424779T)(12)}{(1.57080)(12,000)} + \frac{(T)(18)}{(1.57080)(12,000)} = 0.00122535T \text{ rad}$$





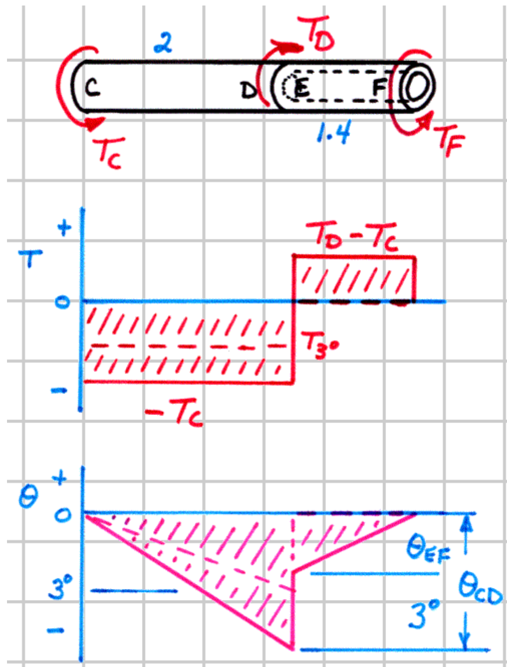
6-74

$$J_s = \frac{\pi d^4}{32} = \frac{\pi(120)^4}{32} = 20.35752\left(10^6\right)\ \text{mm}^4$$

$$\theta = TL/JG$$

$$J_a = \frac{\pi\left(120^4 - 60^4\right)}{32} = 19.08518\left(10^6\right)\ \text{mm}^4$$

$$J_b = \frac{\pi(60)^4}{32} = 1.272345\left(10^6\right)\ \text{mm}^4$$



Equilibrium: $T_s = -T_C$ (a)

$T_D - T_C = T_a + T_b$ (b)

Deformations: $\theta_{total} = \theta_{CD} + \theta_{slip} + \theta_{EF} = 0$ (c)

$\theta_{EF,a} = \theta_{EF,b}$ (d)

$$\theta_{CD} = \frac{(-T_C)(2)}{\left(20.35752\times10^{-6}\right)\left(80\times10^9\right)}$$

$$= -1.22805\left(10^{-6}\right)(T_C)\ \text{rad}$$

$$\theta_{EF,a} = \frac{(T_a)(1.40)}{\left(19.08518\times10^{-6}\right)\left(28\times10^9\right)}$$

$$= 2.61983\left(10^{-6}\right)(T_a)\ \text{rad}$$

6-74 (cont.)

$$\theta_{EF,b} = \frac{(T_b)(1.40)}{(1.272345\times10^{-6})(45\times10^{9})} = 24.45179(10^{-6})(T_b) \text{ rad}$$

If $\theta_{CD} \le 3° = 0.0523599 \text{ rad}$, then $T_C = T_D$ $T_a = T_b = \theta_{EF} = 0$

If $\theta_{CD} \ge 3° = 0.0523599 \text{ rad}$, then

$$2.61983(10^{-6})(T_a) = 24.45179(10^{-6})(T_b) \qquad \text{(d)}$$

$$T_a = 9.33335 T_b$$

$$-1.22805(10^{-6})T_C + \frac{3\pi}{180} + 2.61983(10^{-6})(T_a) = 0 \qquad \text{(c)}$$

$$T_C = [42,636.60 + 2.13333 T_a] \text{ N} = [42,636.60 + 19.91107 T_b] \text{ N}$$

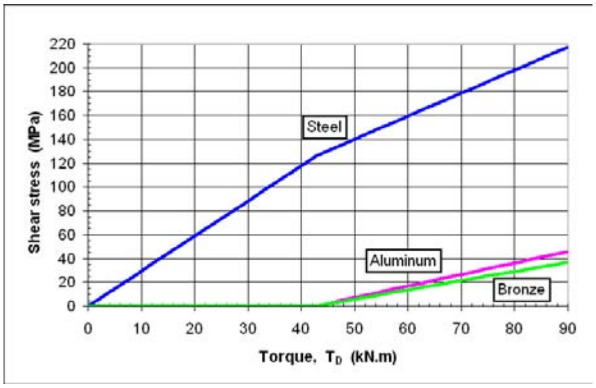
$$T_D = (42,636.60 + 19.91107 T_b) + (9.33335 T_b) + T_b \qquad \text{(b)}$$

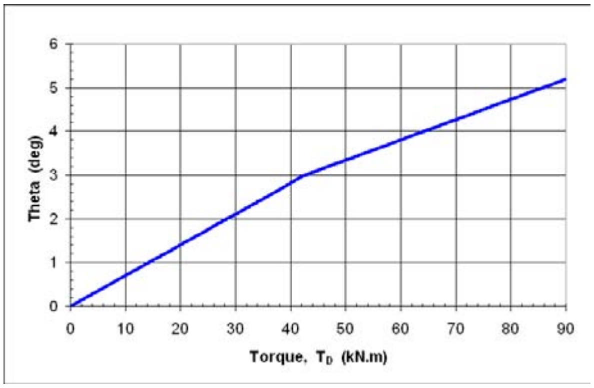
$$T_b = (0.0330639 T_D - 1409.734) \text{ N} \qquad T_a = (0.308597 T_D - 13,157.54) \text{ N}$$

$$T_s = -(0.658338 T_D + 14,567.29) \text{ N}$$

$$\tau_a = \frac{(T_a)(0.060)}{(19.08518\times10^{-6})} \qquad \tau_b = \frac{(T_b)(0.030)}{(1.272345\times10^{-6})} \qquad \tau_s = \frac{(T_s)(0.030)}{(20.35752\times10^{-6})}$$

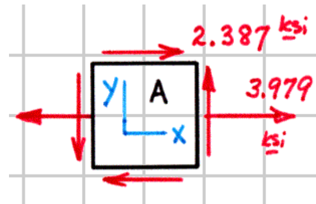
$$\theta_D = \theta_{CD} = \frac{(T_s)(2)}{(20.35752\times10^{-6})(80\times10^{9})}$$





6-75*

$$\sigma_x = \frac{N}{A} = \frac{50}{\pi(4)^2/4} = 3.97887 \text{ ksi} \qquad \sigma_y = 0 \text{ ksi}$$



$$\tau_{xy} = \frac{Tc}{J} = \frac{(30)(2)}{\pi(4)^4/32} = 2.38732 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(2.38732)}{(3.97887)-(0)} = 25.10°, \quad -64.90°$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{p1,p2} = \frac{3.97887}{2} \pm \sqrt{\left(\frac{3.97887}{2}\right)^2 + (2.38732)^2} = 1.98944 \pm 3.10760 \text{ ksi}$$

$\sigma_{p1} = 1.98944 + 3.10760 = 5.10 \text{ ksi (T)} \measuredangle\ 25.10°$ .... **Ans.**

$\sigma_{p2} = 1.98944 - 3.10760 = -1.118 \text{ ksi} = 1.118 \text{ ksi (C)}\ 64.90°$ .... **Ans.**

$\sigma_{p3} = \sigma_z = 0 \text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = 3.10760 \text{ ksi} \cong 3.11 \text{ ksi}$ .... **Ans.**

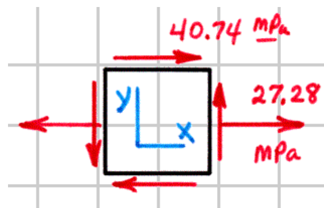
6-76*

$$\sigma_x = \frac{N}{A} = \frac{1500\left(10^3\right)}{\pi\left(0.400^2 - 0.300^2\right)/4} = 27.2837\left(10^6\right)\text{ N/m}^2 \cong 27.3\text{ MPa}$$ .... **Ans.**

$$\tau_{xy} = \frac{Tc}{J} = \frac{\left(350\times10^3\right)\left(0.200\right)}{\pi\left(0.400^4 - 0.300^4\right)/32} = 40.7437\left(10^6\right)\text{ N/m}^2 \cong 40.7\text{ MPa}$$ .... **Ans.**

$\sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$ MPa .... **Ans.**

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2\left(40.7437\right)}{\left(27.2837\right)-\left(0\right)} = 35.74°, \quad -54.26°$$



$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{p1,p2} = \frac{27.2837}{2} \pm \sqrt{\left(\frac{27.2837}{2}\right)^2 + \left(40.7437\right)^2} = 13.6419 \pm 42.9668\text{ MPa}$$

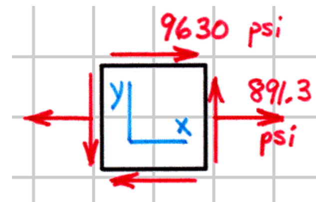
$\sigma_{p1} = 13.6419 + 42.9668 = 56.6$ MPa (T) ∡ 35.74° .... **Ans.**

$\sigma_{p2} = 13.6419 - 42.9668 = -29.3$ MPa $= 29.3$ MPa (C) ∡ 54.26° .... **Ans.**

$\sigma_{p3} = \sigma_z = 0$ Mpa .... **Ans.**

$\tau_{max} = \tau_p = 42.9668$ MPa $\cong 43.0$ MPa .... **Ans.**

6-77



$$\sigma_x = \frac{N}{A} = \frac{2800}{\pi(2)^2/4} = 891.268 \text{ psi} \qquad \sigma_y = 0 \text{ psi}$$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(1500)T}{60} = (360 \times 550) \text{ lb}\cdot\text{ft/s}$$

$$T = 1260.50715 \text{ lb}\cdot\text{ft} = 15{,}126.09 \text{ lb}\cdot\text{in.}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(15{,}126.09)(1)}{\pi(2)^4/32} = 9629.565 \text{ psi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(9629.565)}{(891.268)-(0)} = 43.68°, \quad -46.32°$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{p1,p2} = \frac{0.891268}{2} \pm \sqrt{\left(\frac{0.891268}{2}\right)^2 + (9.629565)^2} = 0.44563 \pm 9.63987 \text{ ksi}$$

$\sigma_{p1} = 0.44563 + 9.63987 = 10.08$ ksi (T) ∡ 43.68° .... **Ans.**

$\sigma_{p2} = 0.44563 - 9.63987 = -9.19$ ksi $= 9.19$ ksi (C) ∡ 46.32° .... **Ans.**

$\sigma_{p3} = \sigma_z = 0$ ksi .... **Ans.**

$\tau_{max} = \tau_p = 9.63987$ ksi $\cong 9.64$ ksi .... **Ans.**

6-78*

$$\sigma_x = \frac{N}{A} = \frac{125\left(10^3\right)}{\pi\left(0.060\right)^2/4} = 44.20971\left(10^6\right)\text{ N/m}^2 \qquad \sigma_y = 0\text{ N/m}^2$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{T\left(0.030\right)}{\pi\left(0.060\right)^4/32} = 23{,}578.51T\text{ N/m}^2$$

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{p1} = \frac{44.20971}{2} + \sqrt{\left(\frac{44.20971}{2}\right)^2 + \left(23.57851T \times 10^{-3}\right)^2} \le 100\text{ MPa}$$

$T \le 3167.83\text{ N}\cdot\text{m}$ $\qquad$ $T_{\max} = 3.17\text{ kN}\cdot\text{m}$ .... **Ans.**

6-79

$$\tau_{xy} = \frac{Tc}{J} = \frac{(100)(2)}{\pi(4)^4/32} = 7.95775 \text{ ksi} \qquad \sigma_y = 0 \text{ ksi}$$

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (7.95775)^2} \le 18 \text{ ksi}$$

$$\sigma_x \le 14.48190 \text{ ksi}$$

$$P_{\max} = \sigma_x A = (14.48190)\left[\frac{\pi(4)^2}{4}\right] = 182.0 \text{ kip}$$ .... **Ans.**

6-80*

$$\tau_{xy} = \frac{Tc}{J} = \frac{(35\times10^3)(0.075)}{\pi(0.150)^4/32} = 52.81586 \text{ N/m}^2 \qquad \sigma_y = 0 \text{ N/m}^2$$

$$\tau_p = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (52.81586)^2} \le 60 \text{ MPa}$$

$$|\sigma_x| \le 56.93802 \text{ MPa}$$

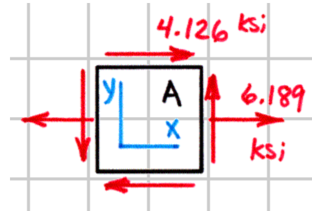
$$\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (52.81586)^2} \ge -96 \text{ MPa}$$

$$\sigma_x \ge -66.94255 \text{ MPa}$$

$$P_{\max} = \sigma_x A = (56.93802\times10^6)\left[\frac{\pi(0.150)^2}{4}\right] = 1.006(10^6) \text{ N}$$

$P_{\max} = 1006$ kN (C) .... **Ans.**

6-81*



$$\sigma_x = \frac{N}{A} = \frac{175}{\pi(6)^2/4} = 6.18936 \text{ ksi} \qquad \sigma_y = 0 \text{ ksi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(175)(3)}{\pi(6)^4/32} = 4.12624 \text{ ksi}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(4.12624)}{(6.18936)-(0)} = 26.57°, \quad -63.43°$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{p1,p2} = \frac{6.18936}{2} \pm \sqrt{\left(\frac{6.18936}{2}\right)^2 + (4.12624)^2} = 3.09468 \pm 5.15780 \text{ ksi}$$

$\sigma_{p1} = 3.09468 + 5.15780 = 8.25$ ksi (T) ∡ 26.57° .... **Ans.**

$\sigma_{p2} = 3.09468 - 5.15780 = -2.06$ ksi $= 2.06$ ksi (C) ∡ 63.43° .... **Ans.**

$\sigma_{p3} = \sigma_z = 0$ ksi .... **Ans.**

$\tau_{max} = \tau_p = 5.15780$ ksi $\cong 5.16$ ksi .... **Ans.**

6-82

$$A_{AB} = \frac{\pi d^2}{4} = \frac{\pi(100)^2}{4} = 7853.98 \text{ mm}^2 \qquad A_{BC} = \frac{\pi(160^2 - 100^2)}{4} = 12,252.21 \text{ mm}^2$$

$$J_{AB} = \frac{\pi(100)^4}{32} = 9.81748(10^6) \text{ mm}^4 \qquad J_{BC} = \frac{\pi(160^4 - 100^4)}{32} = 54.5223(10^6) \text{ mm}^4$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{7853.98(10^{-6})} = 127.324P$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{12,252.21(10^{-6})} = 81.6180P$$

$$\tau_{AB} = \frac{Tc}{J} = \frac{(10,000)(0.050)}{(9.81748 \times 10^{-6})} = 50.9296(10^6) \text{ N/m}^2 = 50.9296 \text{ MPa}$$

$$\tau_{BC} = \frac{Tc}{J} = \frac{(30,000)(0.080)}{(54.5223 \times 10^{-6})} = 44.0187(10^6) \text{ N/m}^2 = 44.0187 \text{ MPa}$$

Since both σ_{AB} and τ_{AB} are larger than σ_{BC} and τ_{BC}, the maximum stresses will occur in section *AB*,

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (50.9296)^2} \le 140 \text{ MPa}$$

$$\sigma_x \le 121.473 \text{ MPa}$$

$$\tau_p = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (50.9296)^2} \le 80 \text{ MPa}$$

$$\sigma_x \le 123.389 \text{ MPa}$$

$$P_{max} = \sigma_x A = (121.473 \times 10^6)(7853.98 \times 10^{-6}) = 954(10^3) \text{ N}$$

$P_{max} = 954$ kN (C) .... **Ans.**

6-83*

$$A_4 = \frac{\pi d^2}{4} = \frac{\pi(4)^2}{4} = 12.5664 \text{ in.}^2 \qquad A_6 = \frac{\pi(6)^2}{4} = 28.2743 \text{ in.}^2$$

$$J_4 = \frac{\pi d^4}{32} = \frac{\pi(4)^4}{32} = 25.1327 \text{ in.}^4 \qquad J_6 = \frac{\pi(6)^4}{32} = 127.2345 \text{ in.}^4$$

$$\sigma_4 = \frac{P}{A_4} = \frac{125}{12.5664} = 9.94716 \text{ ksi} \qquad \tau_4 = \frac{Tc}{J} = \frac{T(2)}{(25.1327)} = 0.0795776T$$

$$\sigma_6 = \frac{P}{A_6} = \frac{125}{28.2743} = 4.42098 \text{ ksi} \qquad \tau_6 = \frac{Tc}{J} = \frac{(3T)(3)}{(127.2345)} = 0.0707355T$$

Since both σ_4 and τ_4 are larger than σ_6 and τ_6,
the maximum stresses will occur in the 4-in. section

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{9.94716}{2} + \sqrt{\left(\frac{9.94716}{2}\right)^2 + \tau_{xy}^2} \le 15 \text{ ksi}$$

$$\tau_{xy} \le 8.70589 \text{ ksi}$$

$$\tau_p = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{9.94716}{2}\right)^2 + \tau_{xy}^2} \le 10 \text{ ksi}$$

$$\tau_{xy} \le 8.67545 \text{ ksi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(T)(2)}{25.1327} = 8.67545 \text{ ksi}$$

$T_{max} = 109.0 \text{ kip} \cdot \text{in.}$ .... **Ans.**

6-84

$$\sigma_x = \frac{N}{A} = \frac{(-200)(10^3)}{\pi\left[d^2 - (0.5d)^2\right]/4} = \frac{-339{,}530.545}{d^2} \text{ N/m}^2 \qquad \sigma_y = 0 \text{ N/m}^2$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(7500)(0.5d)}{\pi\left[d^4 - (0.5d)^4\right]/32} = \frac{40{,}743.665}{d^3} \text{ N/m}^2$$

$$\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-169{,}765.273}{d^2} - \sqrt{\left(\frac{-169{,}765.273}{d^2}\right)^2 + \left(\frac{40{,}743.665}{d^3}\right)^2} \le -100(10^6) \text{ N/m}^2$$

Simplifying yields

$$10(10^6)d^6 - 33.9531(10^3)d^4 - 1.66005 = 0$$

from which $d = 0.0830 \text{ m} = 83.0 \text{ mm}$ .... **Ans.**

6-85

$$\sigma_x = \frac{N}{A} = \frac{20}{\pi d^2/4} = \frac{25.46479}{\text{d}^2} \text{ ksi} \qquad \sigma_y = 0 \text{ ksi}$$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(1800)T}{60} = (240 \times 550) \text{ lb}\cdot\text{ft/s}$$

$$T = 700.28175 \text{ lb}\cdot\text{ft} = 8403.3810 \text{ lb}\cdot\text{in.}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(8.403381)(0.5d)}{\pi d^4/32} = \frac{42.79807}{d^3} \text{ ksi}$$

$$\sigma_{p1} = \frac{12.73240}{d^2} + \sqrt{\left(\frac{12.73240}{d^2}\right)^2 + \left(\frac{42.79807}{d^3}\right)^2} \le 15 \text{ ksi}$$

Simplifying yields

$$225d^6 - 381.9720d^4 - 1831.6746 = 0$$

from which

$d = 1.662$ in. .... **Ans.**

6-86*

$$\sigma_{45} = \frac{E}{1-\nu^2}\left[\varepsilon_a + \nu\varepsilon_b\right] = \frac{210\left(10^3\right)}{1-\left(0.30\right)^2}\left[1084+\left(0.30\right)\left(-754\right)\right]\left(10^{-6}\right) = 197.9539 \text{ MPa}$$

$$\sigma_{-45} = \frac{E}{1-\nu^2}\left[\varepsilon_b + \nu\varepsilon_a\right] = \frac{210\left(10^3\right)}{1-\left(0.30\right)^2}\left[\left(-754\right)+\left(0.30\right)\left(1084\right)\right]\left(10^{-6}\right) = -98.9539 \text{ MPa}$$

$$\sigma_{45} = \sigma_x \cos^2\left(45°\right) + \sigma_y \sin^2\left(45°\right) + 2\tau_{xy}\sin\left(45°\right)\cos\left(45°\right)$$

$$\sigma_{-45} = \sigma_x \cos^2\left(-45°\right) + \sigma_y \sin^2\left(-45°\right) + 2\tau_{xy}\sin\left(-45°\right)\cos\left(-45°\right)$$

For a shaft subjected to an axial load P and a torque T: $\sigma_y = 0 \text{ N/m}^2$

Therefore $0.50\sigma_x + \tau_{xy} = 197.9539 \text{ MPa}$

$0.50\sigma_x - \tau_{xy} = -98.9539 \text{ MPa}$

Solving yields $\sigma_x = 99.0000 \text{ MPa}$ $\tau_{xy} = 148.4539 \text{ MPa}$

$$P = \sigma_x A = \left(99\times10^6\right)\frac{\pi\left(0.025\right)^2}{4} = 48.6\left(10^3\right) \text{ N} = 48.6 \text{ kN}$$ .... **Ans.**

$$T = \frac{\tau_{xy} J}{c} = \frac{\left(148.4539\times10^6\right)\pi\left(0.025\right)^4}{32\left(0.0125\right)} = 455\left(10^3\right) \text{ N}\cdot\text{m} = 455 \text{ kN}\cdot\text{m}$$ .... **Ans.**

6-87*

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

$$= (36)\cos^2(45°) + (150)\sin^2(45°) + \gamma_{xy} \sin(45°)\cos(45°) = 310$$

$$\gamma_{xy} = 434.00 \ \mu\text{rad}$$

$$\sigma_a = \sigma_x = \frac{E}{1-\nu^2}\left[\varepsilon_a + \nu\varepsilon_b\right] = \frac{30,000}{1-(0.30)^2}\left[36 + (0.30)(150)\right](10^{-6}) = 2.67033 \text{ ksi}$$

$$\sigma_b = \sigma_y = \frac{E}{1-\nu^2}\left[\varepsilon_b + \nu\varepsilon_a\right] = \frac{30,000}{1-(0.30)^2}\left[(150) + (0.30)(36)\right](10^{-6}) = 5.30110 \text{ ksi}$$

$$p = \frac{2\sigma_a t}{r} = \frac{2(2.67033)(0.375)}{10} = 200 \text{ psi}$$ .... **Ans.**

$$\tau_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)} = \frac{(30,000)(434.00\times10^{-6})}{2(1+0.30)} = 5.00769 \text{ ksi}$$

$$J = \pi d^4/32 = \pi\left(20.75^4 - 20^4\right)/32 = 2492.075 \text{ in}^4$$

$$T = \frac{\tau_{xy} J}{c} = \frac{(5007.69)(2492.075)}{(10.375)}$$

$$T = 1.20285(10^6) \text{ lb}\cdot\text{in.} = 100.2 \text{ kip}\cdot\text{ft}$$ .... **Ans.**

6-88

$$\sigma_{45} = \frac{E}{1-\nu^2}\left[\varepsilon_a + \nu\varepsilon_b\right] = \frac{200\left(10^3\right)}{1-\left(0.30\right)^2}\left[1414+\left(0.30\right)\left(-212\right)\right]\left(10^{-6}\right) = 296.7912 \text{ MPa}$$

$$\sigma_{-45} = \frac{E}{1-\nu^2}\left[\varepsilon_b + \nu\varepsilon_a\right] = \frac{200\left(10^3\right)}{1-\left(0.30\right)^2}\left[\left(-212\right)+\left(0.30\right)\left(1414\right)\right]\left(10^{-6}\right) = 46.6374 \text{ MPa}$$

$$\sigma_{45} = \sigma_x \cos^2\left(45°\right) + \sigma_y \sin^2\left(45°\right) + 2\tau_{xy}\sin\left(45°\right)\cos\left(45°\right)$$

$$\sigma_{-45} = \sigma_x \cos^2\left(-45°\right) + \sigma_y \sin^2\left(-45°\right) + 2\tau_{xy}\sin\left(-45°\right)\cos\left(-45°\right)$$

For a shaft subjected to an axial load P and a torque T: $\sigma_y = 0 \text{ N/m}^2$

Therefore $0.50\sigma_x + \tau_{xy} = 296.7912 \text{ MPa}$

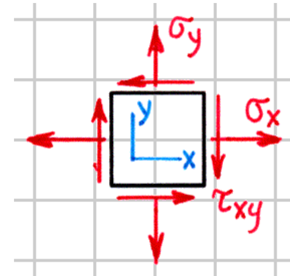
$0.50\sigma_x - \tau_{xy} = 46.6374 \text{ MPa}$

Solving yields $\sigma_x = 343.4286 \text{ MPa}$ $\tau_{xy} = 125.0769 \text{ MPa}$

$$P = \sigma_x A = \left(343.4286\times10^6\right)\frac{\pi\left(0.050\right)^2}{4} = 674\left(10^3\right) \text{ N} = 674 \text{ kN}$$ .... **Ans.**

$$T = \frac{\tau_{xy} J}{c} = \frac{\left(125.0769\times10^6\right)\pi\left(0.050\right)^4}{32\left(0.025\right)} = 3.07\left(10^3\right) \text{ N}\cdot\text{m} = 3.07 \text{ kN}\cdot\text{m}$$ .... **Ans.**

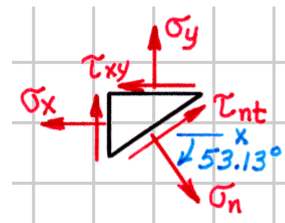
6-89



$$\sigma_x = \frac{pr}{t} = \frac{(360)(24)}{(0.6)} = 14,400 \text{ psi} = 14.400 \text{ ksi}$$

$$\sigma_y = \frac{pr}{2t} - \frac{P}{A} = \frac{14.400}{2} - \frac{28.000}{\pi\left(24.6^2 - 24^2\right)/4} = 5.97741 \text{ ksi}$$

$$\tau_{xy} = \frac{-Tc}{J} = \frac{-(550\times 12)(24.6)}{\pi\left(24.6^4 - 24^4\right)/32} = -48.01566 \text{ ksi}$$



(a) $\theta = -\tan^{-1}(4/3) = -53.130°$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$= (14.4)\cos^2(-53.130°) + (5.97741)\sin^2(-53.130°) + 2(-48.01566)\sin(-53.130°)\cos(-53.130°)$$

$\sigma_n = +55.1 \text{ ksi} = 55.1 \text{ ksi (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$
$$= -[(14.4)-(5.97741)]\sin(-53.130°)\cos(-53.130°) + (-48.01566)\left[\cos^2(-53.130°) - \sin^2(-53.130°)\right]$$

$\tau_{nt} = 17.49 \text{ ksi}$ .... **Ans.**

(b) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-48.01566)}{(14.4)-(5.97741)} = -42.494°, \quad +47.506°$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{(14.4)+(5.97741)}{2} \pm \sqrt{\left[\frac{(14.4)-(5.97741)}{2}\right]^2 + (-48.01566)^2}$$

$$\sigma_{p1} = (10.18871) + (48.19999) = 58.3887 \text{ ksi}$$

$\sigma_{p1} \cong 58.4 \text{ ksi (T)}$ ∠ 42.494° .... **Ans.**

$$\sigma_{p2} = (10.18871) - (48.19999) = -38.0113 \text{ ksi}$$

$\sigma_{p2} \cong 38.0 \text{ ksi (C)}$ ∠ 47.506° .... **Ans.**

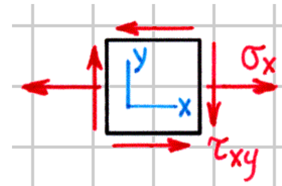
$\sigma_{p3} = -p = -(0.360) \text{ ksi} = 0.360 \text{ ksi (C)}$ .... **Ans.**

$\tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{(58.3887)-(-38.0113)}{2} = 48.2 \text{ ksi}$ .... **Ans.**

6-90

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi d_s^2}{4} = \frac{\pi (0.120)^2}{4} \qquad d_i = \sqrt{d_o^2 - (0.120)^2}$$

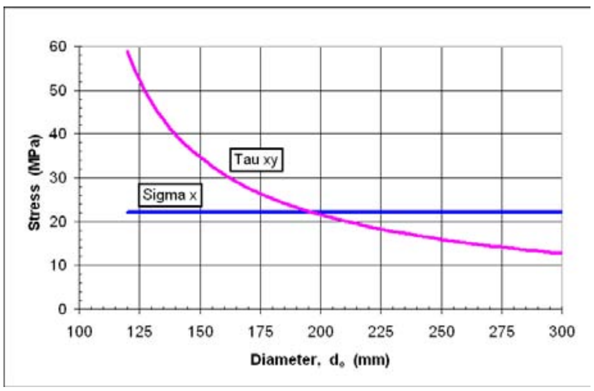
$$J = \pi d^4 / 32 = \pi\left(d_o^4 - d_i^4\right)/32$$

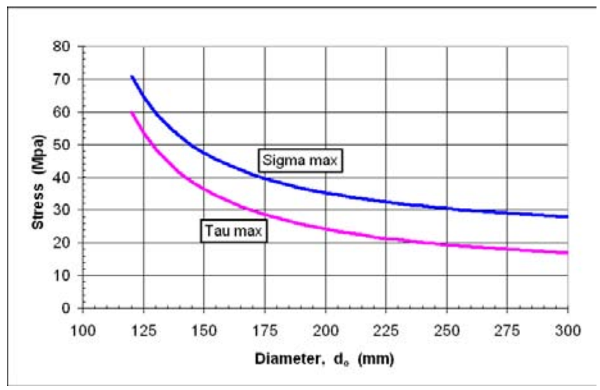


$$\sigma_x = \frac{P}{A} = \frac{250\left(10^3\right)}{\pi (0.120)^2/4} = 22.10485\left(10^6\right) \text{ N/m}^2$$

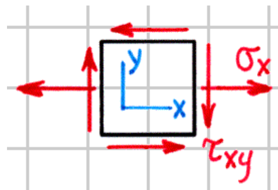
$$\tau_{xy} = \frac{-Tc}{J} = \frac{-(20{,}000)\left(d_o/2\right)}{\pi\left(d_o^4 - d_i^4\right)/32} = \frac{-101.85916\left(10^3\right) d_o}{\left(d_o^4 - d_i^4\right)} \text{ N/m}^2$$

$$\sigma_{p1} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} \qquad \tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$





6-91



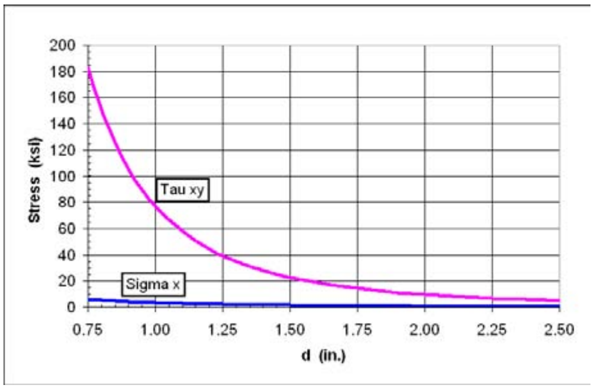
$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(1500)T}{60} = (360\times 550)\ \text{lb}\cdot\text{ft/s}$$

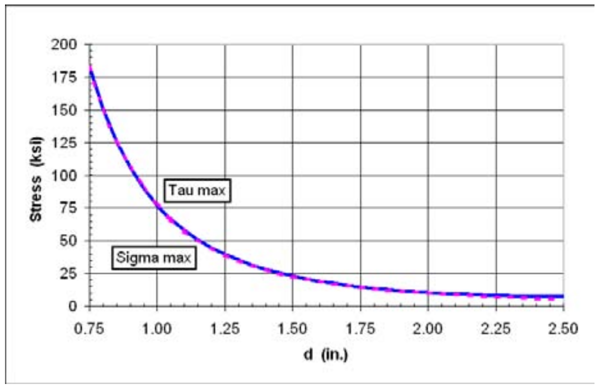
$$T = 1260.50715\ \text{lb}\cdot\text{ft} = 15{,}126.09\ \text{lb}\cdot\text{in.}$$

$$\sigma_x = \frac{P}{A} = \frac{2800}{\pi d^2/4} = \frac{3565.071}{d^2}\ \text{psi}$$

$$\tau_{xy} = \frac{-Tc}{J} = \frac{-(15{,}126.09)(d/2)}{\pi d^4/32} = \frac{-77{,}036.54}{d^3}\ \text{psi}$$

$$\sigma_{p1} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} \qquad \tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$





6-92*

$$r/d = 12/100 = 0.12$$

$$D/d = 135/100 = 1.35$$

Therefore, from Fig. 6-25*b* $K_t \cong 1.6$

$$J = \pi d^4/32 = \pi(100)4/32 = 9.81748(10^6)\ \text{mm}^4$$

$$\tau_{\max} = K_t \frac{Tc}{J} = (1.6)\frac{(10{,}000)(0.050)}{(9.81748\times10^{-6})}$$

$$\tau_{\max} = 81.5(10^6)\ \text{N/m}^2 = 81.5\ \text{MPa}$$ .... **Ans.**

6-93*

$$r/d = 0.15/3 = 0.05$$

$$D/d = 4/3 = 1.3333$$

Therefore, from Fig. 6-25*b* $K_t \cong 2.0$

$$J = \pi d^4/32 = \pi (3)^4/32 = 7.95216 \text{ in.}^4$$

$$\tau_{\max} = K_t \frac{Tc}{J} = (2)\frac{(4\times 12)(1.5)}{(7.95216)} = 18.11 \text{ ksi}$$ .... **Ans.**

6-94

$$\tau = K_t \frac{Tc}{J} = K_t \frac{(614)(0.025)}{\left[\pi(0.050)^4/32\right]} = 40(10^6) \text{ N/m}^2$$

$K_t = 1.60$ $\qquad r/d = 4.5/50 = 0.09$

Therefore, from Fig. 6-25*b* $\quad D/d \cong 1.2$

$D = 1.2d = 1.2(50) = 60$ mm .... **Ans.**

6-95

$$r/d = 0.125/6 = 0.021 \qquad D/d = 8/6 = 1.3333$$

Therefore, from Fig. 6-25*b* $K_t \cong 2.6$

$$\tau_{\max} = K_t \frac{Tc}{J} = (2.6)\frac{T(3)}{\pi(6)^4/32} = 12 \text{ ksi}$$

$T = 195.7 \text{ kip}\cdot\text{in.}$ .... **Ans.**

6-96*

$$h/r = 5/5 = 1 \qquad\qquad r/d = 5/100 = 0.05$$

Therefore, from Fig. 6-25*a* $K_t \cong 1.85$

$$\tau_{\max} = K_t \frac{Tc}{J} = (1.85)\frac{T(0.050)}{\pi(0.100)^4/32} = 60(10^6) \text{ N/m}^2$$

$T = 6.37(10^3) \text{ N}\cdot\text{m} = 6.37 \text{ kN}\cdot\text{m}$ .... **Ans.**

6-97

$$K_t = \tau_{\max}/\tau_{nom} = 12/8 = 1.50 \qquad D/d = 5/4 = 1.25$$

Therefore, from Fig. 6-25*b* $\quad r/d \cong 0.125$

$r = 0.125d = 0.125(4) = 0.50$ in. .... **Ans.**

6-98*

$$\tau_{max} = K_t \frac{Tc}{J} = K_t \frac{(3270)(d/2)}{\pi d^4/32} = 60(10^6) \text{ N/m}^2$$

$$d^3 = 277.56622(10^{-6})K_t \qquad \text{(a)}$$

Guess $K_t \cong 2.0$ Then Eq. (*a*) $d = 0.08219$ m

$r/d = 5/82.19 = 0.061$ $D/d = 100/82.19 = 1.22$

and from Fig. 6-25*b* $K_t \cong 1.8$

2nd guess $K_t \cong 1.8$ Then Eq. (*a*) $d = 0.07935$ m

$r/d = 5/79.35 = 0.063$ $D/d = 100/79.35 = 1.26$

and from Fig. 6-25*b* $K_t \cong 1.8$

Therefore, the 2nd guess was correct, and $d = 79$ mm .... **Ans.**

6-99

$$h/r = 0.5/0.25 = 2 \qquad r/d = 0.25/1 = 0.25$$

Therefore, from Fig. 6-25*a* $K_t \cong 1.65$

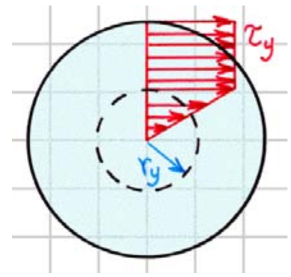
$$\tau_{\max} = K_t \frac{Tc}{J} = (1.65)\frac{(500)(0.5)}{\pi(1)^4/32} = 4201.69 \text{ psi} = \frac{\sigma_y}{3}$$

$\sigma_y = 12,605.1 \text{ ksi} \cong 12.61 \text{ ksi}$ .... **Ans.**

6-100*

(a) $$\tau = \frac{Tc}{J} = \frac{T(0.050)}{\pi(0.100)^4/32} = 120(10^6)\ \text{N/m}^2$$

$$T = 23.5619(10^3)\ \text{N}\cdot\text{m} \cong 23.6\ \text{kN}\cdot\text{m}$$ ....**Ans.**



(b) Plastic zone $0 \le \rho \le c$ $\tau_\rho = \tau_y = 120\ \text{MPa}$

$$T = \int \rho(\tau\, dA) = \int_0^{0.050} \rho(120\times 10^6)(2\pi\rho)\, d\rho$$

$$= 240\pi(10^6)\left[\frac{0.050^3}{3}\right] = 31.4159(10^3)\ \text{N}\cdot\text{m}$$

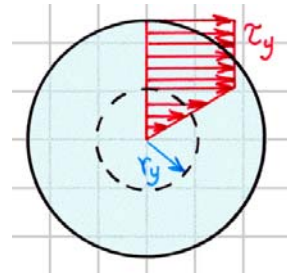
$$= 31.4159\ \text{kN}\cdot\text{m}$$

$$\%Inc = \frac{31.4159 - 23.5619}{23.5619}(100) = 33.3\%$$ .... **Ans.**

6-101*

Elastic zone $0 \le \rho \le r_y$ $\tau_\rho = \dfrac{\tau_y \rho}{r_y} = \dfrac{24\rho}{r_y}$ ksi

Plastic zone $r_y \le \rho \le c$ $\tau_\rho = \tau_y = 24$ ksi



(a) $r_y = 1.5$ in.

$$T = \int \rho(\tau\, dA) = \int_0^{1.5} \rho \frac{24\rho}{1.5}(2\pi\rho)\, d\rho + \int_{1.5}^{2} \rho(24)(2\pi\rho)\, d\rho$$

$$= \frac{48\pi}{1.5}\left[\frac{1.5^4}{4}\right] + (48\pi)\left[\frac{2^3 - 1.5^3}{3}\right]$$

$T = 360$ kip·in. .... **Ans.**

(b) $r_y = 1.0$ in.

$$T = \int \rho(\tau\, dA) = \int_0^{1} \rho \frac{24\rho}{1}(2\pi\rho)\, d\rho + \int_{1}^{2} \rho(24)(2\pi\rho)\, d\rho$$

$$= \frac{48\pi}{1}\left[\frac{1^4}{4}\right] + (48\pi)\left[\frac{2^3 - 1^3}{3}\right]$$

$T = 390$ kip·in. .... **Ans.**

6-102

Elastic zone $0 \le \rho \le r_y$ $\tau_\rho = \dfrac{\tau_y \rho}{r_y} = \dfrac{140\left(10^6\right)\rho}{r_y}\ \text{N/m}^2$

Plastic zone $r_y \le \rho \le c$ $\tau_\rho = \tau_y = 140\left(10^6\right)\ \text{N/m}^2$

(a) $r_y = 40\text{ mm}$

$$T = \int \rho\left(\tau\, dA\right) = \int_{0.025}^{0.040} \rho \frac{140\left(10^6\right)\rho}{0.040}\left(2\pi\rho\right) d\rho + \int_{0.040}^{0.050} \rho\left[140\left(10^6\right)\right]\left(2\pi\rho\right) d\rho$$

$$= \frac{280\left(10^6\right)\pi}{0.040}\left[\frac{0.040^4 - 0.025^4}{4}\right] + 280\left(10^6\right)\pi\left[\frac{0.050^3 - 0.040^3}{3}\right]$$

$T = 29.8\left(10^3\right)\ \text{N}\cdot\text{m} = 29.8\ \text{kN}\cdot\text{m}$ .... **Ans.**

(b) $r_y = 25\text{ mm}$

$$T = \int \rho\left(\tau\, dA\right) = \int_{0.025}^{0.050} \rho\left[140\left(10^6\right)\right]\left(2\pi\rho\right) d\rho$$

$$= 280\left(10^6\right)\pi\left[\frac{0.050^3 - 0.025^3}{3}\right]$$

$T = 32.1\left(10^3\right)\ \text{N}\cdot\text{m} = 32.1\ \text{kN}\cdot\text{m}$ .... **Ans.**

6-103*

(a) $\gamma_c = \frac{c}{r}\gamma = \frac{c}{r}\left(\frac{\tau_y}{G}\right) = \frac{1.50}{0.75}\left(\frac{24}{12,000}\right) = 0.004 = 4000\ \mu\text{rad}$ .... **Ans.**

(b) Elastic zone $0 \le \rho \le 0.75$ in. $\tau_\rho = \tau_y \rho / r_y = 24\rho/0.75 = 32.00\rho$ ksi

Plastic zone $0.75\text{ in.} \le \rho \le 1.5$ in. $\tau_\rho = \tau_y = 24$ ksi

$$T = \int \rho(\tau\, dA) = \int_0^{0.75} \rho(32\rho)(2\pi\rho)\, d\rho + \int_{0.75}^{1.50} \rho(24)(2\pi\rho)\, d\rho$$

$$= (64\pi)\left[\frac{0.75^4}{4}\right] + (48\pi)\left[\frac{1.50^3 - 0.75^3}{3}\right]$$

$T = 164.3$ kip·in. .... **Ans.**

6-104

(a) $\theta = 8° = 0.13963 \text{ rad}$ $\gamma = r\theta/L$

$$\gamma_{\max} = \frac{c\theta}{L} = \frac{0.050(0.13963)}{3} = 0.002327 \cong 2330\ \mu\text{rad}$$ .... **Ans.**

(b) $\gamma_y = \dfrac{\tau_y}{G} = \dfrac{140}{80{,}000} = 0.00175 \text{ rad}$

$$r_y = \frac{\gamma_r L}{\theta} = \frac{0.00175(3)}{0.13963} = 0.03760 \text{ m}$$

Elastic zone $0.025 \text{ m} \le \rho \le 0.03760 \text{ m}$ $\tau_\rho = \dfrac{\tau_y \rho}{r_y} = \dfrac{140\rho}{0.03760} = 3723.40\rho \text{ MPa}$

Plastic zone $0.03760 \text{ m} \le \rho \le 0.050 \text{ m}$ $\tau_\rho = \tau_y = 140 \text{ MPa}$

$$T = \int \rho(\tau\, dA) = \int_{0.025}^{0.0376} \rho\left[3723.40(10^6)\rho\right](2\pi\rho)\,d\rho + \int_{0.0376}^{0.050} \rho\left[140(10^6)\right](2\pi\rho)\,d\rho$$

$$= 5848.71(10^6)\left[0.0376^4 - 0.025^4\right] + 293.215(10^6)\left[0.050^3 - 0.0376^3\right]$$

$$T = 30.5(10^3) \text{ N}\cdot\text{m} = 30.5 \text{ kN}\cdot\text{m}$$ .... **Ans.**

(c) Elastic zone since $25 \text{ mm} < 37.6 \text{ mm}$

$$\tau_\rho = \frac{\rho\tau_y}{r_y} = \frac{25(140)}{37.6} = 93.1 \text{ MPa}$$ .... **Ans.**

6-105

(a) $\theta = 5° = 0.0872665 \text{ rad}$ $\gamma = r\theta/L$

$$\gamma_{\max} = \frac{c\theta}{L} = \frac{2(0.0872665)}{(5\times 12)} = 0.0029089 \cong 2910\ \mu\text{rad}$$ .... **Ans.**

(b) $\gamma_y = \frac{\tau_y}{G} = \frac{18}{12,000} = 0.00150 \text{ rad}$ $r_y = \frac{\gamma_r L}{\theta} = \frac{0.001500(5\times 12)}{0.0872665} = 1.03132 \text{ in.}$

Elastic zone $0 \text{ in.} \le \rho \le 1.03132 \text{ in.}$ $\tau_\rho = \frac{\tau_y \rho}{r_y} = \frac{18\rho}{1.03132} = 17.45330\rho \text{ ksi}$

Plastic zone $1.03132 \text{ in.} \le \rho \le 2.0 \text{ in.}$ $\tau_\rho = \tau_y = 18 \text{ ksi}$

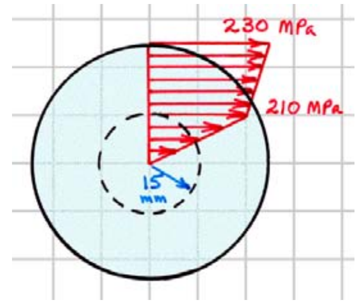
$$T = \int \rho(\tau\, dA) = \int_0^{1.03132} \rho(17.45330\rho)(2\pi\rho)\, d\rho + \int_{1.03132}^{2.0} \rho(18)(2\pi\rho)\, d\rho$$
$$= (27.41558)\left[1.03132^4\right] + (37.69911)\left[2.0^3 - 1.03132^3\right]$$

$T = 291 \text{ kip}\cdot\text{in.}$ .... **Ans.**

(c) Elastic zone since $0.50 \text{ in.} < 1.03132 \text{ in.}$

$$\tau_\rho = \frac{\rho\tau_y}{r_i} = \frac{(0.50)(18)}{1.03132} = 8.73 \text{ ksi}$$ .... **Ans.**

6-106*



(a) $$r_y = \frac{\gamma_y r_o}{\gamma_o} = \frac{7.5(25)}{12.5} = 15 \text{ mm}$$

$$\gamma_\rho = \frac{\rho \gamma_o}{r_o} = \frac{0.0125\rho}{0.025} = 0.500\rho \text{ rad}$$

$$G_1 = \frac{210(10^6)}{0.0075} = 28.00(10^9) \text{ N/m}^2$$

$$0 \le \gamma \le 0.0075 \text{ rad}$$

$$G_2 = \frac{(230-210)(10^6)}{0.0125-0.0075} = 4.00(10^9) \text{ N/m}^2 \qquad 0.0075 \text{ rad} \le \gamma \le 0.0125 \text{ rad}$$

$$\tau_{\rho 1} = G_1 \gamma_\rho = 28(10^9)(0.5\rho) = \left[14.00(10^9)\rho\right] \text{ N/m}^2 \qquad 0 \le \rho \le 15 \text{ mm}$$

$$\tau_{\rho 2} = 180(10^6) + G_2 \gamma_\rho = \left[180(10^6) + 2.00(10^9)\rho\right] \text{ N/m}^2 \qquad 15 \text{ mm} \le \rho \le 25 \text{ mm}$$

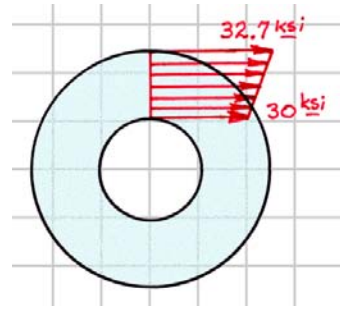
$$T = \int \rho(\tau\, dA) = \int_0^{0.015} \rho\left[14(10^9)\rho\right](2\pi\rho)\, d\rho + \int_{0.015}^{0.025} \rho\left[180(10^6) + 2(10^9)\rho\right](2\pi\rho)\, d\rho$$

$$= 21{,}991.15(10^6)\left[0.015^4\right] + 376.9911(10^6)\left[0.025^3 - 0.015^3\right] + 3141.5927(10^6)\left[0.025^4 - 0.015^4\right]$$

$$T = 6.80(10^3) \text{ N}\cdot\text{m} = 6.80 \text{ kN}\cdot\text{m}$$ .... **Ans.**

(b) $$\theta = \frac{\gamma_y L}{r_y} = \frac{0.0075(1)}{0.015} = 0.500 \text{ rad} \cong 28.6^\circ$$ .... **Ans.**

6-107*



(a) $$\gamma_o = \frac{r_o \gamma_i}{r_i} = \frac{(2.00)(0.0075)}{(1.25)} = 0.0120 \text{ rad} < 0.0125 \text{ rad}$$

$$\gamma_\rho = \frac{\rho \gamma_i}{r_i} = \frac{0.0075\rho}{1.25} = 0.00600\rho \text{ rad}$$

$$G_2 = \frac{33-30}{0.0125-0.0075} = 600 \text{ ksi}$$

$$0.0075 \text{ rad} \le \gamma \le 0.0125 \text{ rad}$$

$$\tau_\rho = 25.5 + G_2\gamma_\rho = [25.5 + 3.600\rho] \text{ ksi} \qquad 1.25 \text{ in.} \le \rho \le 2.0 \text{ in.}$$

$$T = \int \rho(\tau\, dA) = +\int_{1.25}^{2.0} \rho[25.5 + 3.6\rho](2\pi\rho)\, d\rho$$

$$= 53.40708\left[2.0^3 - 1.25^3\right] + 5.65487\left[2.0^4 - 1.25^4\right]$$

$T = 400 \text{ kip} \cdot \text{in.}$ .... **Ans.**

(b) $\theta = \frac{\gamma_y L}{r_y} = \frac{0.0075(3\times 12)}{1.25} = 0.216 \text{ rad} \cong 12.4°$ .... **Ans.**

6-108

$$\gamma = \frac{\theta\rho}{L} = \frac{0.300\rho}{2} = 0.150\rho \text{ rad}$$

$$r_y = \frac{\gamma_y \rho}{\gamma_\rho} = \frac{(0.0035)\rho}{(0.150\rho)} = 0.023333 \text{ m} = 23.333 \text{ mm}$$

$$\tau = 2910\gamma^{0.74} = 2910(0.150\rho)^{0.74} = \left[714.826\rho^{0.74}\right] \text{ MPa} \qquad 0 \le \rho \le 23.333 \text{ mm}$$

$$\tau = 533\gamma^{0.44} = 533(0.150\rho)^{0.44} = \left[231.317\rho^{0.44}\right] \text{ MPa} \qquad 23.333 \text{ mm} \le \rho \le 40 \text{ mm}$$

$$T = \int \rho(\tau\, dA) = \int_0^{0.02333} \rho\left[714.826\left(10^6\right)\rho^{0.74}\right](2\pi\rho)\, d\rho$$

$$+ \int_{0.02333}^{0.040} \rho\left[231.317\rho^{0.44}\left(10^6\right)\right](2\pi\rho)\, d\rho$$

$$= 1200.905\left(10^6\right)\left[0.02333^{3.74}\right] + 422.502\left(10^6\right)\left[0.040^{3.44} - 0.02333^{3.44}\right]$$

$T = 6.48\left(10^3\right) \text{ N}\cdot\text{m} = 6.48 \text{ kN}\cdot\text{m}$ .... **Ans.**

6-109

(a) $$T = \int \rho(\tau\, dA) = \int_0^R \rho\left[k\gamma^{1/2}\right](2\pi\rho)\, d\rho \qquad \gamma = \rho\theta/L$$

$$T = \int_0^R \rho k \left[\frac{\rho\theta}{L}\right]^{1/2}(2\pi\rho)\, d\rho = 2\pi k\sqrt{\theta/L}\int_0^R \rho^{5/2} d\rho = \frac{4\pi k}{7} R^{7/2}\sqrt{\theta/L}$$

Solving for θ yields $$\theta = \frac{49LT^2}{16\pi^2 k^2 R^7}$$ .... **Ans.**

(b) $$\gamma_{\max} = \frac{R\theta}{L} = \frac{49T^2}{16\pi^2 k^2 R^6}$$

$$\tau_{\max} = k\gamma_{\max}^{1/2} = k\left[\frac{49T^2}{16\pi^2 k^2 R^6}\right]^{1/2} = \frac{7T}{4\pi R^3}$$ .... **Ans.**

6-110*

$$\tau_b(\max) = 60 \text{ MPa} \le 84 \text{ MPa} \qquad \text{(all elastic)}$$

$$J_b = \pi d^4/32 = \pi(80)^4/32 = 4.02124(10^6) \text{ mm}^4$$

$$T_b = \frac{\tau_{\max} J}{c} = \frac{(60\times10^6)(4.02124\times10^{-6})}{(0.040)} = 6.03186(10^3) \text{ N}\cdot\text{m} = 6.03186 \text{ kN}\cdot\text{m}$$

$$\theta_b = \theta_{B/C} = \left(\frac{\tau L}{cG}\right)_b = \frac{(60\times10^6)(2.5)}{(0.040)(45\times10^9)} = 0.083333 \text{ rad} = \theta_s$$

$$\gamma_s(\max) = \frac{c\theta}{L} = \frac{(0.040)(0.083333)}{1.5} = 0.002222 \text{ rad} > 0.0015 \text{ rad}$$

Therefore, part of the steel shaft will be in plastic deformation,
and the yield surface location is located using

$$r_y = \frac{\gamma_y L}{\theta} = \frac{(0.0015)(1.5)}{0.083333} = 0.02700 \text{ m} = 27.00 \text{ mm}$$

Elastic zone $\quad 0 \text{ m} \le \rho \le 0.027 \text{ m} \qquad \tau_\rho = \dfrac{\tau_y \rho}{r_y} = \dfrac{120\rho}{0.0270} = 4444.44\rho \text{ MPa}$

Plastic zone $\quad 0.027 \text{ m} \le \rho \le 0.040 \text{ m} \qquad \tau_\rho = \tau_y = 120 \text{ MPa}$

$$T_s = \int \rho(\tau\, dA) = \int_0^{0.027} \rho\left[4444.44\rho(10^6)\right](2\pi\rho)\, d\rho + \int_{0.027}^{0.040} \rho\left[120(10^6)\right](2\pi\rho)\, d\rho$$
$$= 6981.32(10^6)\left[0.0270^4\right] + 251.327(10^6)\left[0.040^3 - 0.0270^3\right]$$

$$T = 14.8482(10^3) \text{ N}\cdot\text{m} = 14.8482 \text{ kN}\cdot\text{m}$$

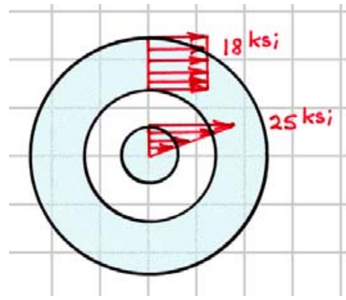
$$T = T_b + T_s = 6.03186 + 14.8482 = 20.8801 \text{ kN}\cdot\text{m} \cong 20.9 \text{ kN}\cdot\text{m}$$ .... **Ans.**

6-111*

$$\gamma_b(\max) = \frac{c\theta}{L} = \frac{(1)(0.30)}{60} = 0.00500 \text{ rad}$$

$$\tau_b(\max) = G\gamma_b(\max) = 5000(0.005) = 25.0 \text{ ksi} \le 30 \text{ ksi} \qquad \text{(all elastic)}$$

$$J_b = \pi d^4/32 = \pi(2)^4/32 = 1.57080 \text{ in}^4$$



$$T_b = \frac{\tau_{\max} J}{c} = \frac{(25.0)(1.57080)}{(1)} = 39.2700 \text{ kip}\cdot\text{in.}$$

$$\gamma_s(\min) = \frac{r_i\theta}{L} = \frac{(2)(0.30)}{60} = 0.01000 \text{ rad}$$

Therefore, all of the steel shaft will be in plastic deformation, and

$$\tau_s = \tau_y = 18 \text{ ksi}$$

$$T_s = \int \rho(\tau\, dA) = \int_2^3 \rho[18](2\pi\rho)\, d\rho = 37.69911\left[3^3 - 2^3\right] = 716.283 \text{ kip}\cdot\text{in.}$$

$$T = T_b + T_s = 39.270 + 716.283 = 755.55 \text{ kip}\cdot\text{in.} \cong 756 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

6-112

(a) $J_{AB} = \pi d^4/32 = \pi(160)^4/32 = 64.3398(10^6)\text{ mm}^4$

$\circlearrowleft \Sigma M = 0: \quad 200 - 125 - T_{AB} = 0 \qquad T_{AB} = +75\text{ kN}\cdot\text{m}$

$$\tau_{AB} = \frac{Tc}{J} = \frac{(75{,}000)(0.080)}{(64.3398\times10^{-6})} = 93.2549(10^6)\text{ N/m}^2 < 120\text{ MPa} \quad \text{(elastic)}$$

$\tau_{AB} \cong 93.3\text{ MPa}$ .... **Ans.**

$T_{BC} = 125\text{ kN}\cdot\text{m} > T_{AB} = 75\text{ kN}\cdot\text{m}$ (assume steel is fully plastic)

$$T_s = \int \rho(\tau\, dA) = \int_{0.050}^{0.080} \rho\left[120(10^6)\right](2\pi\rho)\,d\rho = 251.3274(10^6)\left[0.080^3 - 0.050^3\right]$$

$$= 97.2637(10^3)\text{ N}\cdot\text{m} = 97.2637\text{ kN}\cdot\text{m}$$

$$T_b = T_{BC} - T_s = 125 - 97.2637 = 27.7363\text{ kN}\cdot\text{m}$$

$$J_b = \pi d^4/32 = \pi(100)^4/32 = 9.81748(10^6)\text{ mm}^4$$

$$\tau_b = \frac{Tc}{J} = \frac{(27{,}736.3)(0.050)}{(9.81748\times10^{-6})} = 141.2598(10^6)\text{ N/m}^2 < 240\text{ MPa} \quad \text{(elastic)}$$

$\tau_b \cong 141.3\text{ MPa}$ .... **Ans.**

At $r = 50$ mm $\qquad \gamma_s = \gamma_b = \dfrac{\tau_b}{G_b} = \dfrac{141.2598(10^6)}{40(10^9)} = 0.0035315\text{ rad}$

For the steel: $\qquad \gamma_y = \dfrac{\tau_y}{G_b} = \dfrac{120(10^6)}{80(10^9)} = 0.001500\text{ rad} < 0.0035315\text{ rad}$

Therefore, the steel is fully plastic in *AB* as assumed and

$T_s \cong 97.3\text{ kN}\cdot\text{m} \qquad \tau_s = \tau_y = 120\text{ MPa}$ .... **Ans.**

(b) Since the steel in *AB* is elastic and the bronze in *BC* is elastic:

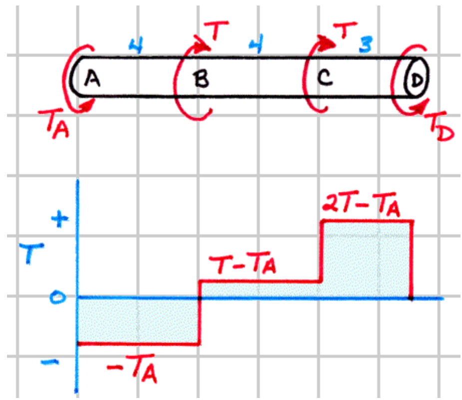
$$\theta_{D/A} = \theta_{B/A} + \theta_{C/B} + \theta_{D/C} = \left(\frac{T_{AB}L_{AB}}{J_{AB}G_s}\right) + \left(\frac{T_bL_{BC}}{J_bG_b}\right) + 0$$

$$= \frac{(75{,}000)(2)}{(64.3398\times10^{-6})(80\times10^9)} + \frac{(27{,}736.3)(1.5)}{(9.81748\times10^{-6})(40\times10^9)}$$

$\theta_{D/A} = -0.0769\text{ rad}$ .... **Ans.**

6-113

(a) For section *AB* (steel)



$$\gamma_y = \frac{\tau_y}{G_s} = \frac{18}{12{,}000} = 0.001500 \text{ rad}$$

$$\gamma_{AB} = \frac{r_{AB}\theta_B}{L_{AB}} = \frac{(2)(0.072)}{(4\times 12)}$$

$$= 0.003 \text{ rad} > 0.001500 \text{ rad} \quad \text{(steel yields)}$$

$$r_y = \frac{\gamma_y L_{AB}}{\theta_B} = \frac{(0.0015)(4\times 12)}{(0.072)} = 1.00 \text{ in.}$$

Elastic zone $0 \text{ in.} \le \rho \le 1.00 \text{ in.}$ $\qquad \tau_\rho = \dfrac{\tau_y \rho}{r_y} = \dfrac{18\rho}{1.00} = 18\rho \text{ ksi}$

Plastic zone $1.00 \text{ in.} \le \rho \le 2.0 \text{ in.}$ $\qquad \tau_\rho = \tau_y = 18 \text{ ksi}$

$$T_A = -T_{AB} = \int \rho(\tau\, dA) = \int_0^{1.0} \rho(18\rho)(2\pi\rho)\,d\rho + \int_{1.0}^{2.0} \rho(18)(2\pi\rho)\,d\rho$$

$$= (28.2743)\left[1.0^4\right] + (37.69911)\left[2.0^3 - 1.0^3\right] = 292.168 \text{ kip}\cdot\text{in.}$$

In section *BD* assume elastic action since $\tau_{yb} = 35 \text{ ksi} ? \ \tau_{ys} = 18 \text{ ksi}$

$\circlearrowleft \Sigma M = 0: \qquad T_A + T_D - 2T = 0 \qquad\qquad J = \pi d^4/32 = \pi(4)^4/32 = 25.1327 \text{ in.}^4$

$$T_D = 2T - T_A = 2T - 292.168 \text{ kip}\cdot\text{in.}$$

$$\theta_{D/A} = 0 = \theta_{D/C} + \theta_{C/B} + \theta_{B/A} = \left[\frac{(2T - T_A)L_{CD}}{JG_s}\right] + \left[\frac{(T - T_A)L_{BC}}{JG_s}\right] - 0.072 \text{ rad}$$

$$\frac{(2T - 292.168)(3\times 12)}{(25.1327)(6000)} + \frac{(T - 292.168)(4\times 12)}{(25.1327)(12{,}000)} - 0.072 = 0$$

$T = 295.702 \text{ kip}\cdot\text{in.} \cong 296 \text{ kip}\cdot\text{in.}$ .... **Ans.**

(b) $T_{BC} = T - T_A = 295.702 - 292.168 = 3.534 \text{ kip}\cdot\text{in.}$

$$\tau_s = \frac{Tc}{J} = \frac{(3.534)(2)}{(25.1327)} = 0.2812 \text{ ksi} < \tau_{ys} = 18 \text{ ksi} \quad \text{(elastic)}$$

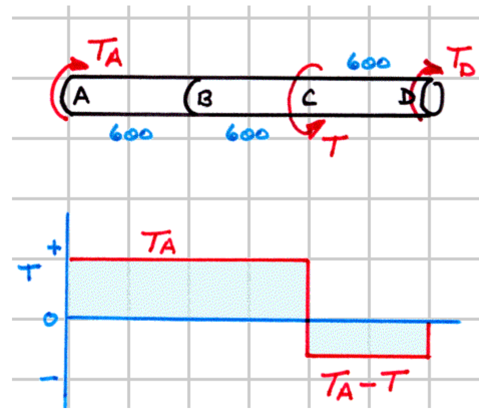
$$T_{CD} = 2T - T_A = 2(295.702) - 292.168 = 299.236 \text{ kip}\cdot\text{in.}$$

$$\tau_b = \frac{Tc}{J} = \frac{(299.236)(2)}{(25.1327)} = 23.812 \text{ ksi} < \tau_b = 35 \text{ ksi} \quad \text{(elastic)}$$

Therefore $\tau_{\max}(steel) = \tau_y = 18 \text{ ksi}$ (in section *AB*) .... **Ans.**

$\tau_{\max}(bronze) = 23.8 \text{ ksi}$ (in section *CD*) .... **Ans.**

6-114



$$\tau_{AB} = \tau_b = 50 \text{ MPa} < \tau_{yb} = 60 \text{ MPa} \text{ (elastic)}$$

$$\tau_{BC} = \tau_s = 50 \text{ MPa} < \tau_{ys} = 120 \text{ MPa} \text{ (elastic)}$$

$$J = \pi d^4/32 = \pi(100)^4/32 = 9.81748(10^6) \text{ mm}^4$$

$$T_A = T_{AB} = \frac{\tau_b J}{c} = \frac{(50\times10^6)(9.81748\times10^{-6})}{(0.050)}$$

$$= 9817.48 \text{ N}\cdot\text{m} = 9.81748 \text{ kN}\cdot\text{m}$$

$$\theta_{D/A} = 0 = \theta_{D/C} + \theta_{C/B} + \theta_{B/A}$$

$$= \theta_{D/C} + \left[\frac{(T_A)L_{BC}}{JG_s}\right] + \left[\frac{(T_A)L_{AB}}{JG_b}\right]$$

$$\theta_{D/C} + \frac{(9{,}817.48)(0.600)}{(9.81748\times10^{-6})(80\times10^9)} + \frac{(9{,}817.48)(0.600)}{(9.81748\times10^{-6})(40\times10^9)} = 0$$

$$\theta_{D/C} = -0.02250 \text{ rad}$$

For segment *CD*:

$$\gamma_{ys} = \frac{\tau_{ys}}{G_s} = \frac{120(10^6)}{80(10^9)} = 0.001500 \text{ rad}$$

$$\gamma_{CD} = \frac{r\theta_{CD}}{L_{CD}} = \frac{(50)(0.0225)}{(600)} = 0.001875 \text{ rad} > 0.001500 \text{ rad} \quad \text{(part plastic)}$$

$$r_y = \frac{\gamma_y L_{CD}}{\theta_C} = \frac{(0.0015)(0.600)}{(0.0225)} = 0.0400 \text{ m} = 40.0 \text{ mm}$$

Elastic zone $0 \text{ in.} \le \rho \le 40 \text{ mm}$ $\qquad \tau_\rho = \dfrac{\tau_y \rho}{r_y} = \dfrac{120\rho}{0.040} = 3000\rho \text{ MPa}$

Plastic zone $40 \text{ mm} \le \rho \le 50 \text{ mm}$ $\qquad \tau_\rho = \tau_y = 120 \text{ MPa}$

$$T_D = -T_{CD} = \int \rho(\tau\, dA) = \int_0^{0.040} \rho\left[3000\rho(10^6)\right](2\pi\rho)\,d\rho + \int_{0.040}^{0.050} \rho\left[120(10^6)\right](2\pi\rho)\,d\rho$$

$$= 4712.3889(10^6)\left[0.040^4\right] + 251.3274(10^6)\left[0.050^3 - 0.040^3\right]$$

$$= 27.3947(10^3) \text{ N}\cdot\text{m} = 27.3947 \text{ kN}\cdot\text{m}$$

$T = T_A + T_D = 9.81748 + 27.3947 = 37.2 \text{ kN}\cdot\text{m}$ .... **Ans.**

6-115

$$J = \pi d^4/32 = \pi(4)^4/32 = 25.13274 \text{ in.}^4 \qquad \tau_y = T_y c/J$$

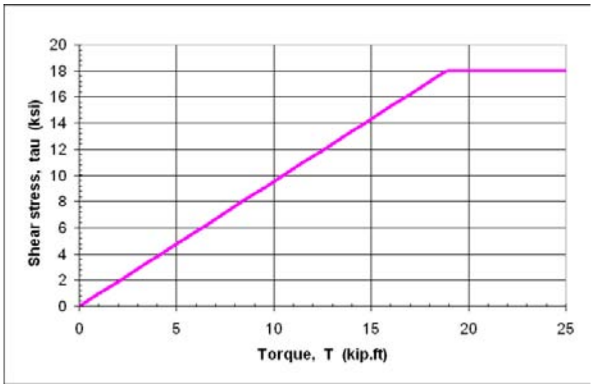
$$T_y = \frac{\tau_{\max} J}{c} = \frac{(18)(25.13274)}{(2)} = 226.1947 \text{ kip}\cdot\text{in.} = 18.8496 \text{ kip}\cdot\text{ft}$$

For $0 \le T \le 18.85 \text{ kip}\cdot\text{ft}$

$$\tau = \frac{Tc}{J} = \frac{(T\times 12)(2)}{(25.13274)} = 0.95493T \text{ ksi}$$

$$\theta = \frac{TL}{JG} = \frac{(T\times 12)(10\times 12)}{(25.13274)(12{,}000)}$$

$$= 4.77465(10^{-3})T \text{ rad}$$



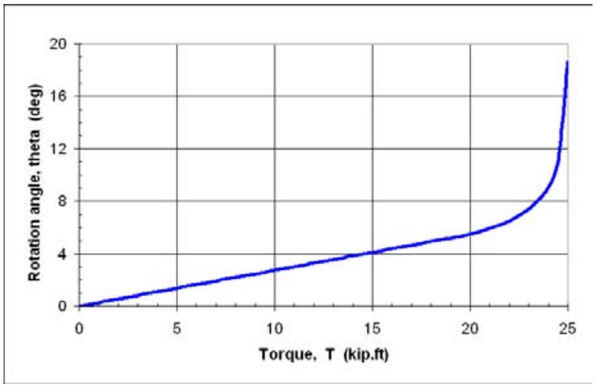
For $18.85 \text{ kip}\cdot\text{ft} \le T \le 25 \text{ kip}\cdot\text{ft}$

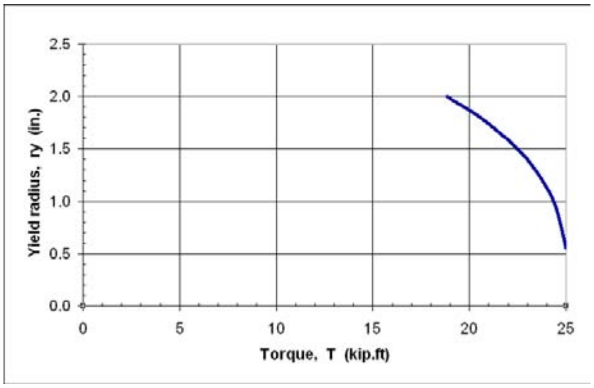
$$(T\times 12) = \int \rho(\tau\, dA) = \int_0^r \rho\left(\frac{18\rho}{r}\right)(2\pi\rho)\, d\rho + \int_r^{2.0} \rho(18)(2\pi\rho)\, d\rho$$

$$= \frac{9\pi}{r}\left[r^4\right] + 12\pi\left[2.0^3 - r^3\right] = 96\pi - 3\pi r^3$$

$$r_y = \sqrt[3]{32 - (4T/\pi)} \qquad \tau_{\max} = \tau_y = 18 \text{ ksi}$$

$$\theta = \frac{\gamma_y L}{r_y} = \frac{\tau_y L}{G r_y} = \frac{(18)(10\times 12)}{(12{,}000) r_y} = \frac{0.1800}{r_y}$$





6-116

$$J = \pi d^4/32 = \pi(120)^4/32 = 20.35752(10^6) \text{ mm}^4 \qquad \tau_y = T_y c/J$$

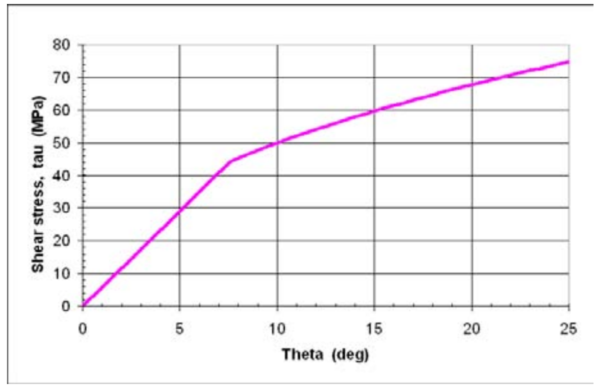
$$T_y = \frac{\tau_{\max} J}{c} = \frac{(44.3\times10^6)(20.35752\times10^{-6})}{(0.060)} = 15{,}030.636 \text{ N}\cdot\text{m}$$

For $0 \le T \le 15.03 \text{ kN}\cdot\text{m}$

$$\tau = \frac{Tc}{J} = \frac{T(0.060)}{20.35752(10^{-6})} = 2947.314T \text{ N/m}^2$$

$$\theta = \frac{TL}{JG} = \frac{T(3)}{(20.35752\times10^{-6})(16.66\times10^9)}$$

$$= 8.84548(10^{-6})T \text{ rad}$$



When $T = 15{,}030.636 \text{ N}\cdot\text{m}$ $\qquad \theta = 0.13295 \text{ rad} \cong 7.618°$

For $7.618° \le \theta$

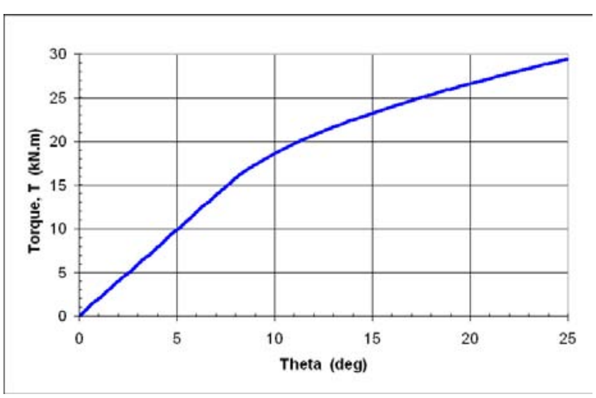
$$\theta = \frac{\gamma_y L}{r_y} = \frac{\tau_y L}{G r_y} = \frac{(44.3\times10^6)(3)}{(16.66\times10^9) r_y} = \frac{7.97719(10^{-3})}{r_y}$$

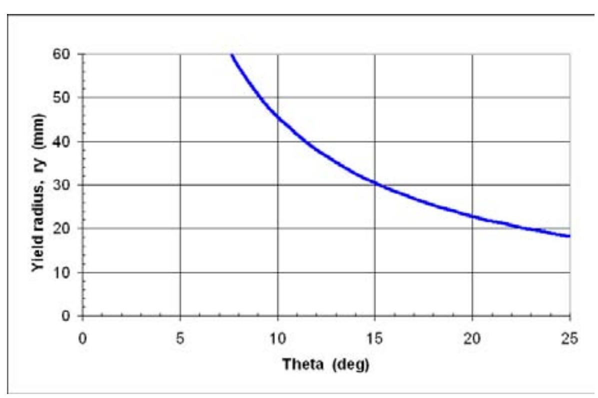
$$r_y = \frac{7.97719(10^{-3})}{\theta} \text{ m} = \frac{7.97719}{\theta} \text{ mm}$$

$$\tau_{\max} = 602\gamma^{0.44} = 602\left(\frac{\rho\theta}{L}\right)^{0.44} = 602\left(\frac{0.060\theta}{3}\right)^{0.44} \text{ MPa}$$

$$T = \int \rho(\tau\, dA) = \int_0^r \rho\left(\frac{44.3\times10^6 \rho}{r}\right)(2\pi\rho)\, d\rho + \int_r^{0.060} \rho(602\times10^6)\left(\frac{\rho\theta}{3}\right)^{0.44}(2\pi\rho)\, d\rho$$

$$= 22.15(10^6)\pi r_y^3 + 350(10^6)\pi\left(\frac{\theta}{3}\right)^{0.44}\left[0.060^{3.44} - r_y^{3.44}\right] \text{ N}\cdot\text{m}$$





6-117

$$J_a = \pi(1)^4/32 = 0.09817477 \text{ in}^4 \qquad J_s = \pi(0.5)^4/32 = 0.00613592 \text{ in}^4$$

$$T_{ya} = \frac{\tau_{\max} J_a}{c_a} = \frac{(30,000)(0.09817477)}{(0.5)} = 5890.49 \text{ lb}\cdot\text{in.} = 490.874 \text{ lb}\cdot\text{ft}$$

$$T_{ys} = \frac{\tau_{\max} J_s}{c_s} = \frac{(18,000)(0.00613592)}{(0.25)} = 441.786 \text{ lb}\cdot\text{in.} = 36.8155 \text{ lb}\cdot\text{ft}$$

For $0 \le T \le T_y$ $\qquad \theta = TL/JG$

$$\theta_a = \frac{(T_a \times 12)(10)}{(0.09817477)(30,000/0.0075)} = 0.3055775(10^{-3})T_a \text{ rad} \qquad (1)$$

$$\theta_s = \frac{(T_s \times 12)(30)}{(0.00613592)(18,000/0.0015)} = 4.88924(10^{-3})T_s \text{ rad} \qquad (2)$$

$$\theta_a = \theta_s \qquad T = T_a + T_s$$

$$T_a = 16.000 T_s \qquad T = 17.000 T_s$$

If $T_a = T_{ya} = 5890.49 \text{ lb}\cdot\text{in.}$, then $\qquad T_s = 368.1556 \text{ lb}\cdot\text{in.}$

$$T_{\max} = T_a + T_s = 6258.6456 \text{ lb}\cdot\text{in.} = 521.5538 \text{ lb}\cdot\text{ft}$$

If $T_s \ge T_{ys}$ (where r is the yield boundary)

$$(T_s \times 12) = \int_0^r \rho\left(\frac{18,000\rho}{r}\right)(2\pi\rho)\,d\rho + \int_r^{0.25} \rho(18,000)(2\pi\rho)\,d\rho$$

$$= \frac{9000\pi}{r}\left[r^4\right] + 12,000\pi\left[0.25^3 - r^3\right] = 187.50\pi - 3000\pi r^3 \qquad (3)$$

$$r = \sqrt[3]{62.500 - (0.004T_s/\pi)} \qquad \tau_{\max} = \tau_y = 18 \text{ ksi}$$

$$\theta_s = \frac{\tau_{\max} L}{Gr} = \frac{(18)(30)}{(18/0.0015)r} = \frac{0.04500}{r} \text{ rad} \qquad (4)$$

6-117 (cont.)

If $T_a \geq T_{ya}$ (where r is the yield boundary)

$$\tau_a = 30\rho/r \qquad \tau \leq 30 \text{ ksi} \qquad \rho \leq r$$

$$\tau_a = 30 + \frac{3(\gamma - 0.0075)}{0.0125 - 0.0075} = 25.5 + \frac{4.5\rho}{r} \qquad \tau \geq 30 \text{ ksi} \qquad \rho \geq r$$

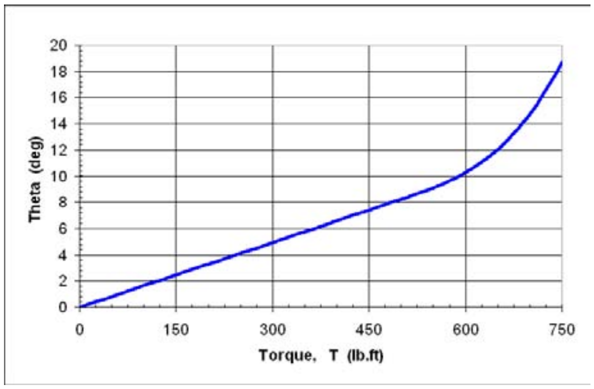
$$(T_a \times 12) = \int_0^r \rho\left(\frac{30{,}000\rho}{r}\right)(2\pi\rho)\,d\rho + \int_r^{0.50} \rho\left(25{,}500 + \frac{4500\rho}{r}\right)(2\pi\rho)\,d\rho$$

$$= \frac{15{,}000\pi}{r}\left[r^4\right] + 17{,}000\pi\left[0.50^3 - r^3\right] + \frac{2250\pi}{r}\left[0.50^4 - r^4\right] \qquad (5)$$

$$= 2125.0\pi + \frac{140.625\pi}{r} - 4250\pi r^3$$

$$\theta_a = \frac{\tau_{\max} L}{Gr} = \frac{(30)(10)}{(30/0.0075)r} = \frac{0.07500}{r} \text{ rad} \qquad (6)$$

Computer approach:



1. Increment $\theta = \theta_a = \theta_s$
2. Compute T_s and T_a using Eqs. (1) and (2)
   If $T_s \geq 36.82 \text{ lb}\cdot\text{ft}$,
   use Eqs. (3) and (4).
   If $T_a \geq 490.9 \text{ lb}\cdot\text{ft}$,
   use Eqs. (5) and (6).
3. Compute $T = T_a + T_s$
4. Plot θ versus T
5. Repeat until $T = 750 \text{ lb}\cdot\text{ft}$

6-118

$$J_a = \pi(20)^4/32 = 15.70796(10^3)\ \text{mm}^4$$

$$J_s = \pi(25)^4/32 = 38.34952(10^3)\ \text{mm}^4$$

$$T_{ya} = \frac{\tau_{\max} J_a}{c_a} = \frac{(210\times10^6)(15.70796\times10^{-9})}{(0.010)} = 329.867\ \text{N}\cdot\text{m}$$

$$T_{ys} = \frac{\tau_{\max} J_s}{c_s} = \frac{(120\times10^6)(38.34952\times10^{-9})}{(0.0125)} = 368.155\ \text{N}\cdot\text{m}$$

For $0 \le T \le T_y$ $\qquad \theta = TL/JG$

$$\theta_a = \frac{(T)(0.250)}{(15.70796\times10^{-9})(210\times10^6/0.0075)} = 568.411(10^{-6})T\ \text{rad}$$

$$\theta_s = \frac{(T)(0.300)}{(38.34952\times10^{-9})(120\times10^6/0.0015)} = 97.7848(10^{-6})T\ \text{rad}$$

If $T \ge T_{ys}$ (where r is the yield boundary)

$$T = \int_0^r \rho\left[\frac{120(10^6)\rho}{r}\right](2\pi\rho)\,d\rho + \int_r^c \rho(120\times10^6)(2\pi\rho)\,d\rho$$

$$= \frac{60(10^6)\pi}{r}\left[r^4\right] + 80(10^6)\pi\left[c^3 - r^3\right] = 20(10^6)\pi\left(4c^3 - r^3\right)$$

$$r = \sqrt[3]{4c^3 - \frac{T}{20\pi(10^6)}} \qquad \text{where} \qquad c = 0.0125\ \text{m} \qquad (1)$$

$$\theta_s = \frac{\tau_{\max} L}{Gr} = \frac{(120\times10^6)(0.300)}{(120\times10^6/0.0015)r} = \frac{450(10^{-6})}{r}\ \text{rad} \qquad (2)$$

6-118 (cont.)

If $T \geq T_{ya}$ (where r is the yield boundary)

$$\tau \leq 210 \text{ MPa} \qquad (\rho \leq r) \qquad \tau_a = 210\rho/r$$

$$\tau \geq 210 \text{ MPa} \qquad (\rho \geq r) \qquad \tau_a = 210 + 4000(\gamma - 0.0075) = 180 + 4000\gamma$$

$$= 180 + 4000\left(\frac{0.0075\rho}{r}\right) = 180 + \frac{30\rho}{r}$$

$$T = \int_0^r \rho\left(\frac{210\rho}{r}\right)(10^6)(2\pi\rho)\,d\rho + \int_r^c \rho\left(180 + \frac{30\rho}{r}\right)(10^6)(2\pi\rho)\,d\rho$$

$$= \frac{105(10^6)\pi}{r}\left[r^4\right] + 120(10^6)\pi\left[c^3 - r^3\right] + \frac{15(10^6)\pi}{r}\left[c^4 - r^4\right]$$

$$= 2\pi(10^6)\left[60c^3 + \frac{7.5c^4}{r} - 15r^3\right] \qquad \text{where } c = 0.010 \text{ m}$$

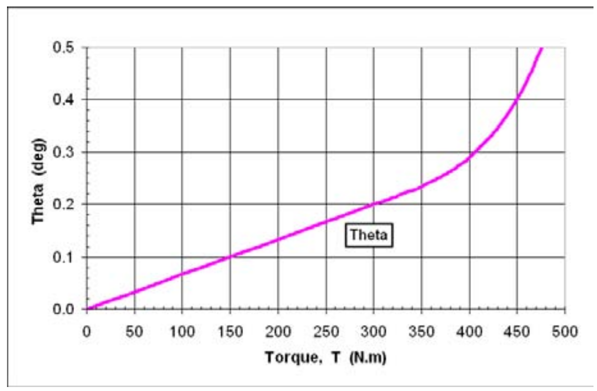
For a given torque T, solve for r. For example, using the Newton-Raphson iteration method, let

$$f(r) = \frac{Tr}{2\pi(10^6)} + 15r^4 - 7.5c^4 - 60c^3 r = 0$$

$$f'(r) = \frac{T}{2\pi(10^6)} + 60r^3 - 60c^3$$

Then $\quad r^{(n+1)} = r^{(n)} - (f/f')$

Guess that $r^{(0)} = c$ and iterate until the value for r no longer changes.



Using the r from the Newton-Raphson solution, calculate

$$\theta_a = \frac{\tau_{\max} L}{Gr} = \frac{(210\times10^6)(0.250)}{(210\times10^6/0.0075)r} = \frac{1.875(10^{-3})}{r} \text{ rad} \qquad (3)$$

Use Eqs. (1) and (2) to determine θ_s and Eq. (3) to determine θ_a. Then,

$$\theta = \theta_a + \theta_s$$

6-119*

For the square bar: $b/a = 1/1 = 1$

The value of α from Fig. 6-30 is: $\alpha \cong 0.21$

$T_{max} = \tau_{max}\alpha a^2 b = (12)(0.21)(1)^2(1) = 2.52 \text{ kip} \cdot \text{in.}$ .... **Ans.**

6-120*

(a) For the rectangular bar: $b/a = 75/40 = 1.875$

The value of α from Fig. 6-30 is: $\alpha \cong 0.245$

$$T_{max} = \tau_{max} \alpha a^2 b = \left(25\times10^6\right)(0.245)(0.040)^2(0.075) = 735 \text{ N}\cdot\text{m}$$ .... **Ans.**

For the square bar: $b/a = a/a = 1/1 = 1$

The value of α from Fig. 6-30 is: $\alpha \cong 0.21$

and $a = \sqrt{(40)(75)} = 54.77 \text{ mm}$

$$T_{max} = \tau_{max} \alpha a^2 b = \left(25\times10^6\right)(0.21)(0.05477)^2(0.05477) = 863 \text{ N}\cdot\text{m}$$ .... **Ans.**

(b) For the rectangular bar: $\beta \cong 0.225$

$$\theta = \frac{TL}{\beta a^3 bG} = \frac{(735)(0.400)}{(0.225)(0.040)^3(0.075)\left(28\times10^9\right)} = 0.00972 \text{ rad}$$ .... **Ans.**

For the square bar: $\beta \cong 0.15$

$$\theta = \frac{TL}{\beta a^3 bG} = \frac{(863)(0.400)}{(0.15)(0.05477)^3(0.05477)\left(28\times10^9\right)} = 0.00913 \text{ rad}$$ **Ans.**

6-121

(a) For the square bar: $b/a = 1.5/1.5 = 1$

The value of α from Fig. 6-30 is: $\alpha \cong 0.21$

$$T_{max} = \tau_{max}\alpha a^2 b = (12)(0.21)(1.5)^2(1.5) = 8.505 \text{ kip}\cdot\text{in.} \cong 8.51 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

For the circular bar: $J = \pi d^4/32 = \pi(1.5)^4/32 = 0.4970 \text{ in.}^4$

$$T_{max} = \frac{\tau_{max} J}{c} = \frac{(12)(0.4970)}{0.75} = 7.952 \text{ kip}\cdot\text{in.} \cong 7.95 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

(b) For the square bar: $\beta \cong 0.15$

$$\theta = \frac{TL}{\beta a^3 bG} = \frac{(8.505)(3\times 12)}{(0.15)(1.5)^3(1.5)(4000)} = 0.1008 \text{ rad}$$ .... **Ans.**

For the circular bar:

$$\theta = \frac{TL}{JG} = \frac{(7.952)(3\times 12)}{(0.4970)(4000)} = 0.1440 \text{ rad}$$ .... **Ans.**

6-122*

$$\tau = \frac{T}{2At} \qquad A = \pi r^2/4 = \pi(100)^2/4 = 7854.0 \text{ mm}^2$$

$$t = \frac{T}{2A\tau} = \frac{(2000)}{2(7854.0\times10^{-6})(40\times10^{6})} = 0.00318 \text{ m} = 3.18 \text{ mm}$$ .... **Ans.**

6-123

$$\tau = \frac{T}{2At} \qquad A = (6)(8) + \pi(3)^2 = 76.274 \text{ in}^2$$

$$t = \frac{T}{2A\tau} = \frac{(125)}{2(76.274)(8)} = 0.1229 \text{ in.}$$ .... **Ans.**

6-124

(a) For the circle: $d = 500/\pi = 159.155 \text{ mm}$

$$\tau = \frac{T}{2At} \qquad A = \pi r^2/4 = \pi(159.155)^2/4 = 19,894.38 \text{ mm}^2$$

$$T_{max} = 2At\tau_{max} = 2(19.89438\times10^{-3})(0.003)(75\times10^6)$$

$$T_{max} = 8.95(10^3) \text{ N}\cdot\text{m} = 8.95 \text{ kN}\cdot\text{m}$$ .... **Ans.**

(b) For the equilateral triangle: $a = 500/3 = 166.667 \text{ mm}$

$$\tau = \frac{T}{2At} \qquad A = \frac{(166.667)^2\cos 30°}{2} = 12,028.13 \text{ mm}^2$$

$$T_{max} = 2At\tau_{max} = 2(12.02813\times10^{-3})(0.003)(75\times10^6)$$

$$T_{max} = 5.41(10^3) \text{ N}\cdot\text{m} = 5.41 \text{ kN}\cdot\text{m}$$ .... **Ans.**

(c) For the square: $a = 500/4 = 125 \text{ mm}$

$$\tau = \frac{T}{2At} \qquad A = (125)^2 = 15,625 \text{ mm}^2$$

$$T_{max} = 2At\tau_{max} = 2(15.625\times10^{-3})(0.003)(75\times10^6)$$

$$T_{max} = 7.03(10^3) \text{ N}\cdot\text{m} = 7.03 \text{ kN}\cdot\text{m}$$ .... **Ans.**

(d) For the rectangle: $a = 100 \text{ mm}$ $b = 150 \text{ mm}$

$$\tau = \frac{T}{2At} \qquad A = (100)(150) = 15,000 \text{ mm}^2$$

$$T_{max} = 2At\tau_{max} = 2(15\times10^{-3})(0.003)(75\times10^6)$$

$$T_{max} = 6.75(10^3) \text{ N}\cdot\text{m} = 6.75 \text{ kN}\cdot\text{m}$$ .... **Ans.**

6-125*

For the rectangle: $a = 2$ in. $b = 3$ in. $b/a = 3/2 = 1.5$

(a) The value of α from Fig. 6-30 is: $\alpha \cong 0.23$

$$T_{\max} = T_{AB} = T_2 - T_1 = 30 - 10 = 20 \text{ kip} \cdot \text{in.}$$

$$\tau_{\max} = \frac{T_{\max}}{\alpha a^2 b} = \frac{20}{0.23(2)^2(3)} = 7.25 \text{ ksi}$$ ...... **Ans.**

(b) The value of β from Fig. 6-30 is: $\beta \cong 0.20$

$$\theta_{C/A} = \theta_{C/B} + \theta_{B/A} = \left(\frac{TL}{\beta a^3 bG}\right)_{C/B} + \left(\frac{TL}{\beta a^3 bG}\right)_{B/A}$$

$$= \frac{(-10)(30)}{(0.20)(2)^3(3)(4000)} + \frac{(20)(30)}{(0.20)(2)^3(3)(4000)}$$

$\theta = +0.01563$ rad ...... **Ans.**

6-126*

$$\tau = \frac{T}{2At} \qquad A = (65)(95) = 6175 \text{ mm}^2$$

$$T_{AB} = T_2 - T_1 = 2T - 2T = 0 \text{ kN}\cdot\text{m}$$

$$T_{max} = T_{BC} = T_1 = 2T = 2At_{min}\tau_{max}$$

$$= 2\left(6175\times 10^{-6}\right)(0.005)\left(80\times 10^{6}\right) = 4.940\left(10^{3}\right) \text{ N}\cdot\text{m}$$

$T = 4.94/2 = 2.47 \text{ kN}\cdot\text{m}$ .... **Ans.**

6-127

(a) For the square bar: $b/a = a/a = 1$

The value of α from Fig. 6-30 is: $\alpha \cong 0.21$

$$T_{\max} = \tau_{\max}\alpha a^2 b = \tau_{\max}(0.21)(a)^2(a) = 0.21a^3\tau_{\max}$$

For the circular bar: $A = a^2 = \pi d^2/4$ $d = 1.12838a$

$$T_{\max} = \frac{\tau_{\max} J}{c} = \frac{\tau_{\max}\left[\pi(1.12838a)^4/32\right]}{1.12838a/2} = 0.28209a^3\tau_{\max}$$

$$\frac{T_{circle}}{T_{square}} = \frac{0.28209a^3\tau_{\max}}{0.21a^3\tau_{\max}} = 1.34329 \cong 1.343$$ .... **Ans.**

(b) For the square bar: $\beta \cong 0.15$

$$\theta = \frac{TL}{\beta a^3 bG} = \frac{TL}{(.15)a^3 aG} = \frac{6.667TL}{a^4 G}$$

For the circular bar:

$$\theta = \frac{TL}{JG} = \frac{1.34329TL}{\left[\pi(1.12838a)^4/32\right]G} = \frac{8.44009TL}{a^4 G}$$

$$\frac{\theta_{circle}}{\theta_{square}} = \frac{8.44009TL/a^4G}{6.667TL/a^4G} = 1.266$$ .... **Ans.**

6-128

$T_{AB} = T$ $\qquad T_{BC} = 2T$ $\qquad T_{CD} = -T$

For the square bar: $\quad b/a = 50/50 = 1$

From Fig. 6-30: $\quad \alpha \cong 0.21 \qquad \beta \cong 0.15$

For the stress specification: $\quad \tau_{max} \le 80 \text{ MPa}$

$$T_{max} = T_{BC} = 2T \le \tau_{max}\alpha a^2 b = \left(80\times10^6\right)(0.21)(0.050)^2(0.050) = 2100 \text{ N}\cdot\text{m}$$

$$T \le 1050 \text{ N}\cdot\text{m}$$

For the deformation specification: $\quad \theta \le 0.035 \text{ rad}$

$$\theta_{D/A} = \theta_{B/A} + \theta_{C/B} + \theta_{D/C}$$

$$\theta_{D/A} = \frac{\left(T_{AB} + T_{BC} + T_{CD}\right)L}{\beta a^3 bG} = \frac{\left(T + 2T - T\right)(0.400)}{(.15)(0.050)^3(0.050)\left(28\times10^9\right)} \le 0.035 \text{ rad}$$

$$T \le 1148 \text{ N}\cdot\text{m}$$

$T_{max} = 1050 \text{ N}\cdot\text{m}$ .... **Ans.**

6-129*

$$\tau = \frac{T}{2At} \qquad A = (12.5)(40) = 500 \text{ in}^2$$

$$T = 2At_{\text{min}}\tau_{\text{max}} = 2(500)(0.04)(8) = 320 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

6-130

$$A = \pi(500)^2 \frac{210}{360} + \pi(320)^2 \frac{150}{360} + 2\left[\frac{500+320}{2}\right](695\cos 15°)$$

$$= 1.142671(10^6)\text{ mm}^2 = 1.142671\text{ m}^2$$

$$t = \frac{T}{2A\tau} = \frac{200(10^3)}{2(1.142671)(50\times 10^6)} = 0.001750\text{ m} = 1.750\text{ mm}$$ .... **Ans.**

6-131

Equilibrium: $T_A - 8 - 0 + T_D = 0$ $T_A + T_D = 8\text{ kip}\cdot\text{in.}$

$T_{AB} = -T_A$ $T_{BCD} = (8 - T_A)\text{ kip}\cdot\text{in.}$

For the square bar: $b/a = 1.5/1.5 = 1$

From Fig. 6-30: $\alpha \cong 0.21$ $\beta \cong 0.15$

Deformation: $\theta_{D/A} = \theta_{B/A} + \theta_{D/B} = 0$

$$\theta_{D/A} = \frac{(-T_A)(1.5)}{\beta a^3 bG} + \frac{(8 - T_A)(3.0)}{\beta a^3 bG} = 0\text{ rad}$$

$4.5T_A = 24\text{ kip}\cdot\text{in.}$

$T_A = 5.3333\text{ kip}\cdot\text{in.} \cong 5.33\text{ kip}\cdot\text{in.}$ .... **Ans.**

$T_D = 2.6667\text{ kip}\cdot\text{in.} \cong 2.67\text{ kip}\cdot\text{in.}$ .... **Ans.**

6-132*

$$\sigma_{max} = \sigma_y / FS = 250/1.25 = 200 \text{ MPa} \qquad \tau_{max} = \sigma_{max}/2 = 100 \text{ MPa}$$

For 60 rpm:

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(60)T}{60} = 150(10^3) \text{ N}\cdot\text{m/s}$$

$$T = 23{,}873.24 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(23{,}873.24)(d/2)}{\pi d^4/32} = 100(10^6) \text{ N/m}^2 \qquad d = 0.1067 \text{ m}$$

Use shaft with $d = 110$ mm .... **Ans.**

For 6000 rpm:

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(6000)T}{60} = 150(10^3) \text{ N}\cdot\text{m/s}$$

$$T = 238.7324 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(238.7324)(d/2)}{\pi d^4/32} = 100(10^6) \text{ N/m}^2 \qquad d = 0.0230 \text{ m}$$

Use shaft with $d = 25$ mm .... **Ans.**

Use a speed of 6000 rpm if weight is critical. .... **Ans.**

6-133*

$$\sigma_{max} = \sigma_y / FS = 62/1.5 = 41.3333 \text{ ksi} \qquad \tau_{max} = \sigma_{max}/2 = 20.6667 \text{ ksi}$$

$$\tau = \frac{Tc}{J} = \frac{(1200\times 12)(c)}{J} \le 20,666.7 \text{ psi}$$

$$J/c \ge 0.69677 \text{ in}^3$$

$$\frac{J}{c} = \frac{I_x + I_y}{c} = \frac{2I}{c} = 2S \ge 0.69677 \text{ in}^3$$

$$S \ge 0.34839 \text{ in}^3$$

From Table A-13 Use a 2-in. diameter pipe. .... **Ans.**

6-134

$\sigma_{max} = \sigma_y / FS = 250/1.5 = 166.6667 \text{ MPa}$ $\tau_{max} = \sigma_{max}/2 = 83.3333 \text{ MPa}$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(60)T}{60} = 150(10^3) \text{ N}\cdot\text{m/s}$$

$$T = 23{,}873.24 \text{ N}\cdot\text{m}$$

(a) $$\tau = \frac{Tc}{J} = \frac{(23{,}873.24)c}{J} \le 83.3333(10^6) \text{ N/m}^2$$

$$J/c \ge 286.4789(10^{-6}) \text{ m}^3 = 286.4789(10^3) \text{ mm}^3$$

$$\frac{J}{c} = \frac{I_x + I_y}{c} = \frac{2I}{c} = 2S \ge 286.4789(10^3) \text{ mm}^3 \qquad S \ge 143.239(10^3) \text{ mm}^3$$

Use 203 mm diameter .... **Ans.**

(b) $$\tau = \frac{Tc}{J} = \frac{(23{,}873.24)(d/2)}{\pi d^4/32} = 83.3333(10^6) \text{ N/m}^2 \qquad d = 0.1134 \text{ m}$$

Use shaft with $d = 120$ mm .... **Ans.**

(c) $$W(pipe) = 42.46 \text{ kg/m} \qquad W(solid) = (7870)\frac{\pi(0.120)^2}{4} = 89.01 \text{ kg/m}$$

$W(solid)/W(pipe) = 89.01/42.46 = 2.10$ .... **Ans.**

6-135

$$\sigma_{max} = \sigma_y / FS = 36/2 = 18 \text{ ksi} \qquad \tau_{max} = \sigma_{max}/2 = 9 \text{ ksi}$$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(200)T}{60} = (100 \times 550) \text{ lb}\cdot\text{ft/s}$$

$$T = 2626.057 \text{ lb}\cdot\text{ft} = 31{,}512.38 \text{ lb}\cdot\text{in.}$$

(a) $$\tau = \frac{Tc}{J} = \frac{(31.51238)(c)}{J} \le 9 \text{ ksi} \qquad J/c \ge 3.50141 \text{ in}^3$$

$$\frac{J}{c} = \frac{I_x + I_y}{c} = \frac{2I}{c} = 2S \ge 3.50141 \text{ in}^3 \qquad S \ge 1.75070 \text{ in}^3$$

Use a 3-in. diameter pipe.. **Ans.**

(b) $$\tau = \frac{Tc}{J} = \frac{(31.51238)(d/2)}{\pi d^4/32} \le 9 \text{ ksi} \qquad d \ge 2.6126 \text{ in.}$$

Use shaft with $d = 2\tfrac{5}{8}$ in. .... **Ans.**

(c) $$W(pipe) = 7.58 \text{ lb/ft}$$

$$W(solid) = (0.284)\frac{\pi(2.625)^2}{4} = 1.54 \text{ lb/in.} = 18.44 \text{ lb/ft}$$

Use hollow pipe if weight is critical... **Ans.**

6-136*

$\sigma_{max} = \sigma_y / FS = 360/3 = 120 \text{ MPa}$ $\tau_{max} = \sigma_{max}/2 = 60 \text{ MPa}$

$$T_{max} = T_{AB} = 1000 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(1000)(d/2)}{\pi d^4/32} = 60(10^6) \text{ N/m}^2$$

$d \geq 0.04395 \text{ m} = 43.95 \text{ mm}$

Use shaft with $d = 50$ mm .... **Ans.**

6-137

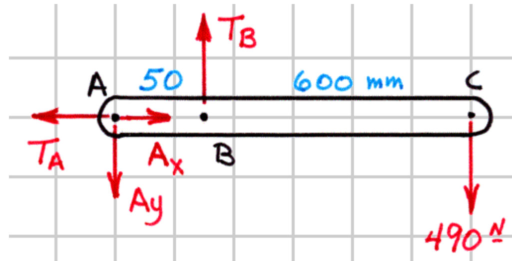
$\sigma_{\max} = \sigma_y / FS = 36/2.25 = 16$ ksi $\qquad \tau_{\max} = \sigma_{\max}/2 = 8$ ksi

$T_{\max} = T_{AB} = 30$ kip·in.

$$\tau = \frac{Tc}{J} = \frac{(30)(d/2)}{\pi d^4/32} \le 8 \text{ ksi} \qquad d \ge 2.673 \text{ in.}$$

Use shaft with $d = 2\tfrac{3}{4}$ in. .... **Ans.**

6-138*



$$\sigma_{max} = \sigma_y / FS = 360/3 = 120 \text{ MPa}$$

$$\tau_{max} = \sigma_{max}/2 = 60 \text{ MPa}$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 50T_B - 650(490) = 0$$

$$T_B = 6370 \text{ N}$$

If the drum is tending to rotate clockwise, then

$$6370 = T_A e^{0.2(3\pi/2)} \qquad T_A = 2482.14 \text{ N}$$

and the torque on the axle of the drum is

$$T = 6370(0.25) - 2482.14(0.25) = 971.965 \text{ N}\cdot\text{m}$$

If the drum is tending to rotate counter-clockwise, then

$$T_A = 6370 e^{0.2(3\pi/2)} \qquad T_A = 16,347.54 \text{ N}$$

and the torque on the axle of the drum is

$$T = 16,347.54(0.25) - 6370(0.25) = 2494.38 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2494.38)(d/2)}{\pi d^4/32} \le 60(10^6) \text{ N/m}^2$$

$$d \ge 0.05960 \text{ m} = 59.60 \text{ mm}$$

Use shaft with $d = 60$ mm .... **Ans.**

6-139

$$T_{max} = T_{AB} = 380 \text{ lb}\cdot\text{ft} = 4560 \text{ lb}\cdot\text{in.}$$

(a) For the shaft: $\sigma_{max} = \sigma_y / FS = 53/2 = 26.5 \text{ ksi}$ $\tau_{max} = \sigma_{max}/2 = 13.25 \text{ ksi}$

$$\tau = \frac{Tc}{J} = \frac{(4.560)(d/2)}{\pi d^4/32} \le 13.25 \text{ ksi} \qquad d \ge 1.206 \text{ in.}$$

Use shaft with $d = 1\frac{1}{4}$ in. .... **Ans.**

(b) For the bolts: $\sigma_{max} = \sigma_y / FS = 36/1.5 = 24 \text{ ksi}$ $\tau_{max} = \sigma_{max}/2 = 12 \text{ ksi}$

$$T_{AB} = 4V\left(\frac{d_1}{2}\right) = 4(\tau A)\left(\frac{d_1}{2}\right) = 2\tau\left(\frac{\pi d_b^2}{4}\right)d_1 = \frac{\tau\pi d_b^2 d_1}{2}$$

$$4.560 = \frac{(12)\pi d_b^2 (3.5)}{2} \qquad d_b = 0.2629 \text{ in.}$$

Use bolts with $d = \frac{5}{16}$ in. .... **Ans.**

6-140

For a given value of power, the values of torque are the same in the shaft and in the collar.

$$\tau_s = \frac{Tc}{J} = \frac{T\left(d_s/2\right)}{\pi d_s^4/32} = \frac{16Td_s}{\pi d_s^4} \qquad \tau_c = \frac{Tc}{J} = \frac{T\left(d_c/2\right)}{\pi\left(d_c^4 - d_s^4\right)/32} = \frac{16Td_c}{\pi\left(d_c^4 - d_s^4\right)}$$

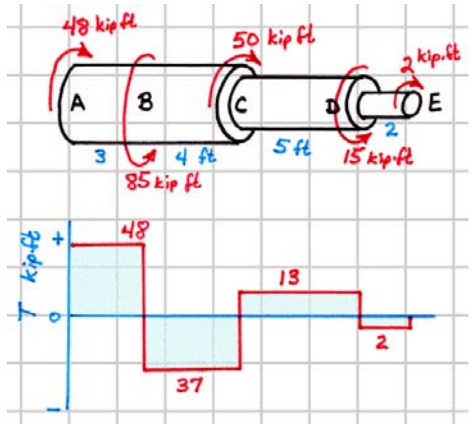
For the same shear stress in the shaft and in the collar

$$\frac{16Td_s}{\pi d_s^4} = \frac{16Td_c}{\pi\left(d_c^4 - d_s^4\right)} \qquad \frac{d_c^4 - d_s^4}{d_s^4} = \frac{d_c}{d_s}$$

$$\left(\frac{d_c}{d_s}\right)^4 - \left(\frac{d_c}{d_s}\right) - 1 = 0$$

$d_c/d_s = 1.221$ (independent of the material) .... **Ans.**

6-141*



(a) $$\tau_{AB} = \frac{Tc}{J} = \frac{(48\times 12)(3)}{\pi (6)^4/32} = 13.58 \text{ ksi}$$

$$\tau_{CD} = \frac{Tc}{J} = \frac{(13\times 12)(2)}{\pi (4)^4/32} = 12.41 \text{ ksi}$$

$$\tau_{DE} = \frac{Tc}{J} = \frac{(2\times 12)(1)}{\pi (2)^4/32} = 15.28 \text{ ksi}$$

$\tau_{\max} = \tau_{DE} = 15.28$ ksi .... **Ans.**

(b) $$\theta_{E/A} = \theta_{B/A} + \theta_{C/B} + \theta_{D/C} + \theta_{E/D}$$

$$\theta_{E/A} = \frac{TL}{JG} = \frac{(48\times 12)(3\times 12)}{\left[\pi (6)^4/32\right](12{,}000)} + \frac{(-37\times 12)(4\times 12)}{\left[\pi (6)^4/32\right](12{,}000)} + \frac{(13\times 12)(5\times 12)}{\left[\pi (4)^4/32\right](12{,}000)} + \frac{(-2\times 12)(2\times 12)}{\left[\pi (2)^4/32\right](12{,}000)}$$

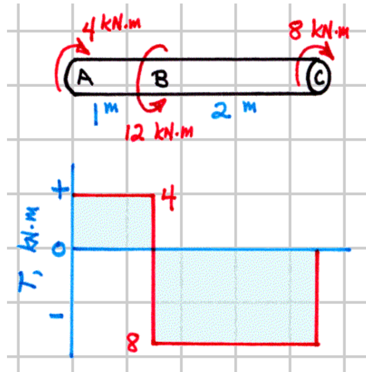
$\theta_{E/A} = 0.1002\left(10^{-3}\right)$ rad .... **Ans.**

(c) $$\theta_{C/A} = \theta_{B/A} + \theta_{C/B}$$

$$\theta_{C/A} = \frac{(48\times 12)(3\times 12)}{\left[\pi (6)^4/32\right](12{,}000)} + \frac{(-37\times 12)(4\times 12)}{\left[\pi (6)^4/32\right](12{,}000)}$$

$\theta_{C/A} = -0.377\left(10^{-3}\right)$ rad .... **Ans.**

6-142*



(a) $\tau_{AB} = \dfrac{Tc}{J} = \dfrac{(4000)(d_1/2)}{\pi d_1^4/32} \le 80(10^6)\ \text{N/m}^2$

$d_1 \ge 0.0634\ \text{m} = 63.4\ \text{mm}$ .... **Ans.**

$\tau_{BC} = \dfrac{Tc}{J} = \dfrac{(8000)(d_2/2)}{\pi d_2^4/32} \le 80(10^6)\ \text{N/m}^2$

$d_2 \ge 0.0799\ \text{m} = 79.9\ \text{mm}$ .... **Ans.**

(b) $\theta = TL/JG$

$$\theta_{C/A} = \frac{(4000)(1) + (-8000)(2)}{\left[\pi(0.075)^4/32\right](80\times 10^9)} = -0.0483\ \text{rad}$$ .... **Ans.**

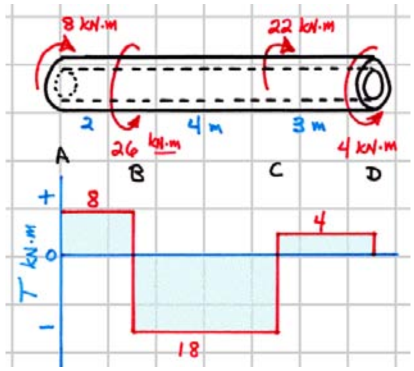
6-143

(a) $$\tau = \frac{Tc}{J} = \frac{T(0.625)}{\pi\left(1.25^4 - 1.12^4\right)/32} \le 8 \text{ ksi}$$

$T \le 1.091 \text{ kip} \cdot \text{in.}$ .... **Ans.**

(b) $$\theta_{C/A} = \frac{(1)(3\times 12)}{\left[\pi\left(1.25^4 - 1.12^4\right)/32\right](3800)} = 0.1112 \text{ rad}$$ .... **Ans.**

6-144*



(a) $$\tau = \frac{Tc}{J} = \frac{(18{,}000)(d/2)}{\pi\left[d^4 - (0.5d)^4\right]/32} \le 100\left(10^6\right)\ \text{N/m}^2$$

$d_{min} = 0.0993\ \text{m} = 99.3\ \text{mm}$ .... **Ans.**

(b) $$\theta = \frac{TL}{JG} = \frac{(8000)(2) + (-18{,}000)(4) + (4000)(3)}{\left\{\pi\left[0.120^4 - 0.060^4\right]/32\right\}\left(80\times 10^9\right)}$$

$\theta = -0.0288\ \text{rad}$ .... **Ans.**

6-145

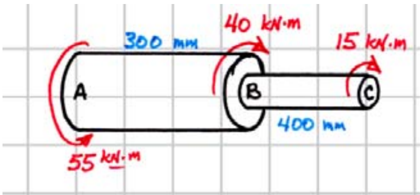
$$\theta = \frac{TL}{JG} = \frac{T(3\times 12)}{\left[\pi(2.5)^4/32\right](4000)} \le 0.052 \text{ rad} \qquad T \le 22.1575 \text{ kip}\cdot\text{in.} = 1.84646 \text{ kip}\cdot\text{ft}$$

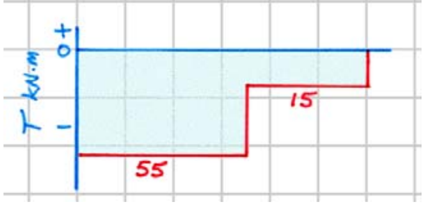
$$\tau = \frac{Tc}{J} = \frac{T(1.25)}{\pi(2.5)^4/32} \le 10 \text{ ksi} \qquad T \le 30.6796 \text{ kip}\cdot\text{in.} = 2.55663 \text{ kip}\cdot\text{ft}$$

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(500)(1846.46)}{60} = (96,680.3) \text{ lb}\cdot\text{ft/s}$$

$Power = 96,680.3/550 = 175.8$ hp .... **Ans.**

6-146





(a) $$\tau_{AB} = \frac{Tc}{J} = \frac{(55,000)(0.075)}{\pi(0.150)^4/32} \leq 83.0(10^6)\ \text{N/m}^2$$

$$\sigma_{AB} = \tau_{AB} = 83.0\ \text{MPa (T)}$$ .... **Ans.**

(b) $$\tau_{BC} = \frac{(15,000)(0.050)}{\pi(0.100)^4/32} \leq 76.4(10^6)\ \text{N/m}^2$$

$$\sigma_{BC} = \tau_{BC} = 76.4\ \text{MPa (C)}$$ .... **Ans.**

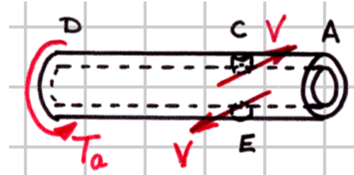
(c) $$\theta = TL/JG$$

$$\theta_C = \frac{(-55,000)(0.300)}{\left[\pi(0.150)^4/32\right](80\times10^9)} + \frac{(-15,000)(0.400)}{\left[\pi(0.100)^4/32\right](80\times10^9)}$$

$$\theta_C = -0.01179\ \text{rad}$$ .... **Ans.**

6-147*

(a) First look at equilibrium of the aluminum shell and the ends of the pins:

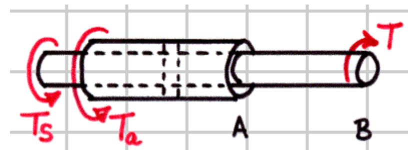


$$T_a = V(2) = (5)\left[\frac{\pi(0.5)^2}{4}\right](2) = 1.963495 \text{ kip}\cdot\text{in.}$$

$$J_a = \frac{\pi d^4}{32} = \frac{\pi(3^4 - 2^4)}{32} = 6.38136 \text{ in.}^4 \qquad J_s = \frac{\pi(2^4)}{32} = 1.57080 \text{ in.}^4$$

Assume that the steel shaft extends all the way through the aluminum shell and attaches to the wall at *D*. Then, the portion of the shafts between the wall and the pin must rotate the same amount

$$\theta_{CD,s} = \theta_{CD,a} \qquad \theta = TL/JG$$



$$\frac{T_s(12)}{(1.57080)(12{,}000)} = \frac{T_a(12)}{(6.38136)(4000)}$$

$$T_s = 0.73846 T_a = 0.73846(1.963495)$$

$$= 1.44997 \text{ kip}\cdot\text{in.}$$

$T = T_a + T_s = 1.963495 + 0.73846 = 3.41346 \text{ kip}\cdot\text{in.} \cong 3.41 \text{ kip}\cdot\text{in.}$ .... **Ans.**

(b) $\tau_a = \frac{Td}{J} = \frac{(1.963495)(1.5)}{(6.38136)} = 0.462 \text{ ksi} = 462 \text{ psi}$ .... **Ans.**

(c) $\theta = \frac{(-1.44997)(12)}{(1.57080)(12{,}000)} + \frac{(-3.41346)(18)}{(1.57080)(12{,}000)} = -0.00418 \text{ rad}$ .... **Ans.**

6-148

$$J_{AB,s} = \pi d^4/32 = \pi(160)^4/32 = 64.33982(10^6)\ \text{mm}^4$$

$$J_{BC,s} = \pi(160^4 - 100^4)/32 = 54.52234(10^6)\ \text{mm}^4$$

$$J_{BC,b} = \pi(100)^4/32 = 9.81748(10^6)\ \text{mm}^4$$

Equilibrium: $T_s + T_b = 75\ \text{kN}\cdot\text{m}$ (a)

Deformations: $\theta_{BC,s} = \theta_{BC,b}$ $\theta = TL/JG$

$$\frac{T_s(1.5)}{(54.52234\times10^{-6})(80\times10^9)} = \frac{T_b(1.5)}{(9.81748\times10^{-6})(40\times10^9)}$$

$$T_s = 11.10720 T_b \qquad \text{(b)}$$

$T_b = 6.19466\ \text{kN}\cdot\text{m}$ $T_s = 68.80534\ \text{kN}\cdot\text{m}$

In *AB*: $$\tau_s = \frac{Tc}{J} = \frac{(85,000)(0.08)}{(64.33982\times10^{-6})} = 105.6888(10^6)\ \text{N/m}^2 = 105.6888\ \text{MPa}$$

In *BC*: $$\tau_s = \frac{(68,805.34)(0.08)}{(54.52234\times10^{-6})} = 101.0(10^6)\ \text{N/m}^2 = 101.0\ \text{MPa}$$

$$\tau_b = \frac{(6194.66)(0.05)}{(9.81748\times10^{-6})} = 31.5(10^6)\ \text{N/m}^2 = 31.5\ \text{MPa}$$

(a) $\tau_{\max,s} = 105.7\ \text{MPa}$ .... $\tau_{\max,b} = 31.5\ \text{MPa}$ .... **Ans.**

(b) $\theta_D = \theta_{B/A} + \theta_{C/B} + \theta_{D/C}$ $\theta_D = TL/JG$

$$\theta_D = \frac{(85,000)(2)}{(64.33982\times10^{-6})(80\times10^9)} + \frac{(-68,805.34)(1.5)}{(54.52234\times10^{-6})(80\times10^9)} + 0$$

$\theta_D = 0.00937\ \text{rad}$ .... **Ans.**

6-149

$$\tau = \frac{T}{2At} \qquad A = (6\times 2) + \pi(1)^2/2 = 13.57080 \text{ in}^2$$

$$t = \frac{T}{2A\tau} = \frac{(1850\times 12)}{2(13.57080)(8000)} = 0.1022 \text{ in.}$$ .... **Ans.**

6-150*

$$\tau = \frac{T}{2At} = \frac{(12,000)}{2\left(53,000 \times 10^{-6}\right)(0.0013)}$$

$\tau = 87.1\left(10^{6}\right)\ \text{N/m}^2 = 87.1\ \text{MPa}$ .... **Ans.**

6-151*

$$Power = T\omega = \frac{2\pi NT}{60} = \frac{2\pi(300)T}{60} = (200\times 550)\ \text{lb}\cdot\text{ft/s}$$

$$T = 3501.4087\ \text{lb}\cdot\text{ft} = 42{,}016.9\ \text{lb}\cdot\text{in.}$$

$$\tau = \frac{Tc}{J} = \frac{(42.0169)(d/2)}{\pi d^4/32} \le 15.9\ \text{ksi} \qquad d \ge 2.3787\ \text{in.}$$

$$\theta = \frac{TL}{JG} = \frac{(42.0169)(3\times 12)}{(\pi d^4/32)(11{,}600)} \le 1.5° = 0.026180\ \text{rad} \qquad d \ge 2.6689\ \text{in.}$$

use $d = 2\tfrac{3}{4}$ in. .... **Ans.**

6-152

$$J = \pi d^4 / 32 = \pi (100)^4 / 32 = 9.81748(10^6) \text{ mm}^4$$

$$\theta_o = \frac{TL}{JG} = \frac{(15,000)(1.2)}{(9.81748\times10^{-6})(40\times10^9)} = 0.04584 \text{ rad}$$

After the torque at *B* is removed $\theta_{total} = \theta_{ABC} + \theta_{slip} + \theta_{CD} = 0$

$$\frac{(-T)(2.2)}{(9.81748\times10^{-6})(40\times10^9)} + 0.04584 + \frac{(-T)(1.6)}{(9.81748\times10^{-6})(80\times10^9)} = 0$$

$T = 2149.808 \text{ N}\cdot\text{m}$ .... **Ans.**

$\tau = \dfrac{Tc}{J} = \dfrac{(2149.808)(0.030)}{(9.81748\times10^{-6})} = 50.7(10^6) \text{ N/m}^2 = 50.7 \text{ MPa}$ .... **Ans.**

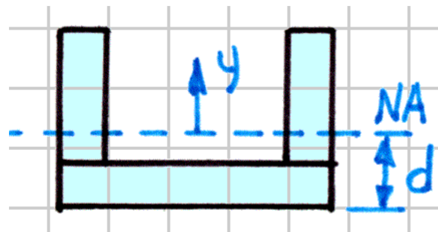
7-1*

$$I = \frac{bh^3}{12} = \frac{(4)(6)^3}{12} = 72.00 \text{ in.}^4$$

$$M_r = \frac{\sigma I}{c} = \frac{(1000)(72.00)}{3} = 24,000 \text{ lb} \cdot \text{in.} = 24.0 \text{ kip} \cdot \text{in.}$$ .... **Ans.**

7-2*

$$d = \frac{M_x}{A} = \frac{(100)\left[(200)(50)\right] + (25)\left[(200)(50)\right] + (100)\left[(200)(50)\right]}{3\left[(200)(50)\right]} = 75 \text{ mm}$$



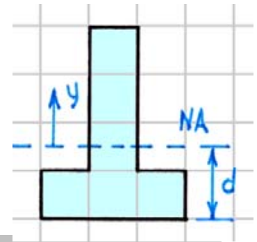
$$I = \frac{(100)(125)^3}{3} + \frac{(300)(75)^3}{3} - \frac{(200)(25)^3}{3}$$

$$= 106.25\left(10^6\right) \text{ mm}^4$$

$$\sigma = \frac{-M_r y}{I} = \frac{-(-10{,}000)(0.125)}{\left(106.25\times 10^{-6}\right)}$$

$\sigma = +11.76\left(10^6\right) \text{ N/m}^2 = 11.76 \text{ MPa (T)}$ .... **Ans.**

7-3

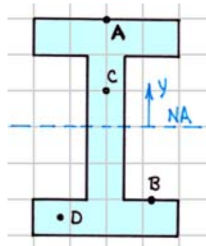


$$d = \frac{M_x}{A} = \frac{(1)\left[(6)(2)\right] + (5)\left[(6)(2)\right]}{2\left[(6)(2)\right]} = 3 \text{ in.}$$

$$I = \frac{(2)(5)^3}{3} + \frac{(6)(3)^3}{3} - \frac{(4)(1)^3}{3} = 136.00 \text{ in.}^4$$

$$\sigma = \frac{-M_r y}{I} = \frac{-(4000 \times 12)(5)}{(136.00)} = -667 \text{ psi} = 667 \text{ psi (C)}$$ .... **Ans.**

7-4*



$$I = \frac{(200)(300)^3}{12} - \frac{(150)(200)^3}{12} = 350(10^6) \text{ mm}^4$$

$$M_r = \frac{-\sigma_A I}{y_A} = \frac{-(-7.5\times10^6)(350\times10^{-6})}{0.150} = 17{,}500 \text{ N}\cdot\text{m}$$

(a) $\sigma_B = \dfrac{-M_r y}{I} = \dfrac{-(17{,}500)(-0.100)}{(350\times10^{-6})} = 5.00(10^6) \text{ N/m}^2 = 5.00 \text{ MPa (T)}$ .... **Ans.**

(b) $\sigma_C = \dfrac{-(17{,}500)(0.050)}{(350\times10^{-6})} = -2.50(10^6) \text{ N/m}^2 = 2.50 \text{ MPa (C)}$ .... **Ans.**

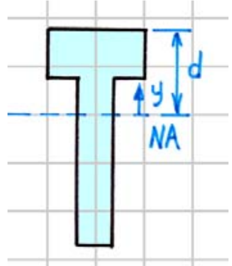
(c) $\sigma_D = \dfrac{-(17{,}500)(-0.125)}{(350\times10^{-6})} = 6.25(10^6) \text{ N/m}^2 = 6.25 \text{ MPa (T)}$ .... **Ans.**

7-5

$$I = \frac{(6)(10)^3}{12} - \frac{(4)(6)^3}{12} = 428 \text{ in.}^4$$

$$M_r = \frac{\sigma I}{c} = \frac{(1200)(428)}{5} = 102{,}700 \text{ lb} \cdot \text{in.} = 102.7 \text{ kip} \cdot \text{in.}$$ .... **Ans.**

7-6



$$d = \frac{M_x}{A} = \frac{(25)\left[(100)(50)\right] + (150)\left[(200)(37.5)\right]}{(100)(50) + (200)(37.5)} = 100 \text{ mm}$$

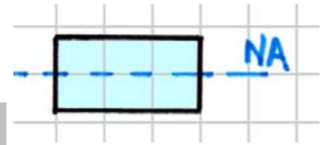
$$I = \frac{(100)(100)^3}{3} - \frac{(62.5)(50)^3}{3} + \frac{(37.5)(150)^3}{3}$$

$$= 72.92\left(10^6\right) \text{ mm}^4$$

$$M_r = \frac{\sigma I}{c} = \frac{\left(200\times10^6\right)\left(72.92\times10^{-6}\right)}{0.150} = 97.2\left(10^3\right) \text{ N}\cdot\text{m} = 97.2 \text{ kN}\cdot\text{m}$$ .... **Ans.**

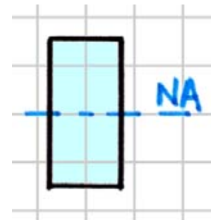
7-7*

(a) $$I = \frac{bh^3}{12} = \frac{(4)(2)^3}{12} = 2.667\text{ in.}^4$$



$$M_r = \frac{\sigma I}{c} = \frac{(8)(2.667)}{1} = 21.3\text{ kip}\cdot\text{in.}$$ ....**Ans.**

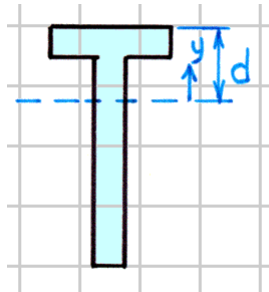
(b) $$I = \frac{bh^3}{12} = \frac{(2)(4)^3}{12} = 10.667\text{ in.}^4$$



$$M_r = \frac{\sigma I}{c} = \frac{(8)(10.667)}{2} = 85.3\text{ kip}\cdot\text{in.}$$ ....**Ans.**

7-8

$$d = \frac{M_x}{A} = \frac{(12.5)\left[(100)(25)\right] + (125)\left[(200)(25)\right]}{(100)(25) + (200)(25)} = 87.5 \text{ mm}$$



$$I = \frac{(100)(87.5)^3}{3} - \frac{(50)(62.5)^3}{3} + \frac{(25)(137.5)^3}{3}$$

$$= 39.93\left(10^6\right) \text{ mm}^4$$

(a) $$\sigma_{bot} = \frac{-M_r y}{I} = \frac{-(-10{,}000)(-0.1375)}{\left(39.93 \times 10^{-6}\right)} = -34.4\left(10^6\right) \text{ N/m}^2$$

$\sigma_{bot} = 34.4$ MPa (C) .... **Ans.**

(b) $$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(-10{,}000)(0.0875)}{\left(39.93 \times 10^{-6}\right)} = +21.9\left(10^6\right) \text{ N/m}^2$$

$\sigma_{top} = 21.9$ MPa (T) .... **Ans.**

7-9*

(a) From Table B-3 for an S 24×80 section $d = 2c = 24.00 \text{ in.}$ $I = 2100 \text{ in.}^4$

$$M_r = \frac{\sigma I}{c} = \frac{(18)(2100)}{12.00} = 3150 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

(b) $$I = 2100 + 2\left[\frac{(8)(0.75)^3}{12} + (8\times 0.75)(12.375)^2\right] = 3938 \text{ in.}^4$$

$$M_r = \frac{\sigma I}{c} = \frac{(18)(3938)}{12.75} = 5560 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

7-10*

From Table B-8 for an $\text{L}102\times102\times12.7\text{-mm angle}$

$$S = 32.3\left(10^3\right)\ \text{mm}^3$$

$$M_r = \sigma S = \left(120\times10^6\right)\left(2\times32.3\times10^{-6}\right) = 7752\ \text{N}\cdot\text{m} \cong 7.75\ \text{kN}\cdot\text{m}$$ .... **Ans.**

7-11

$$I = \frac{(16)(28)^3}{12} - \frac{(15)(24)^3}{12} = 11,989 \text{ in.}^4$$

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(1000\times 12)(14)}{(11,989)} = -14.01 \text{ ksi} = 14.01 \text{ ksi (C)}$$

$$\sigma_{bottom} = \frac{-M_r y}{I} = \frac{-(1000\times 12)(-14)}{(11,989)} = +14.01 \text{ ksi} = 14.01 \text{ ksi (T)}$$

$\sigma_{\max} = 14.01$ ksi (T, on bottom; C, on top)) .... **Ans.**

7-12*

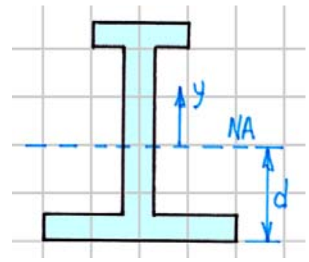
$$d = \frac{M_x}{A} = \frac{(237.5)\left[(100)(25)\right]+(125)\left[(25)(200)\right]+(12.5)\left[(200)(25)\right]}{(100)(25)+(25)(200)+(200)(25)}$$

$$= 102.5 \text{ mm}$$

$$I = \frac{(100)(147.5)^3}{3} - \frac{(75)(122.5)^3}{3} + \frac{(200)(102.5)^3}{3} - \frac{(175)(77.5)^3}{3}$$

$$= 105.65\left(10^6\right) \text{ mm}^4$$

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(-3000)(0.1475)}{\left(105.65\times10^{-6}\right)} = +4.19\left(10^6\right) \text{ N/m}^2$$

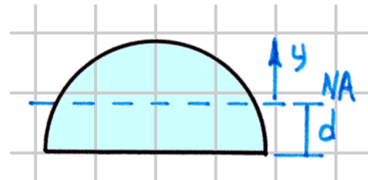


$\sigma_{top} = 4.19$ MPa (T) .... **Ans.**

$$\sigma_{bottom} = \frac{-(-3000)(-0.1025)}{\left(105.65\times10^{-6}\right)} = -2.91\left(10^6\right) \text{ N/m}^2$$

$\sigma_{bottom} = 2.91$ MPa (C) .... **Ans.**

7-13



$$d = \frac{4r}{3\pi} = \frac{4(3)}{3\pi} = 1.2732 \text{ in.}$$

$$I = \frac{\pi r^4}{8} - \frac{8r^4}{9\pi} = \frac{\pi(3)^4}{8} - \frac{8(3)^4}{9\pi} = 8.890 \text{ in.}^4$$

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(-20)(1.7268)}{(8.890)} = +3.88 \text{ ksi} = 3.88 \text{ ksi (T)}$$ .... **Ans.**

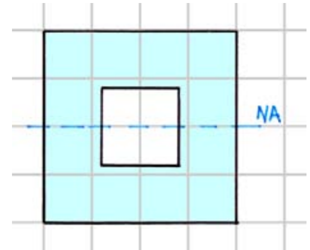
$$\sigma_{bottom} = \frac{-(-20)(-1.2732)}{(8.890)} = -2.86 \text{ ksi} = 2.86 \text{ ksi (C)}$$ .... **Ans.**

7-14

$$I = \frac{(50)(50)^3}{12} - \frac{(20)(20)^3}{12} = 507.5\left(10^3\right)\ \text{mm}^4$$

$$M_r = \frac{\sigma I}{c} = \frac{\left(110\times10^6\right)\left(507.5\times10^{-9}\right)}{0.025} = 2233\ \text{N}\cdot\text{m}$$

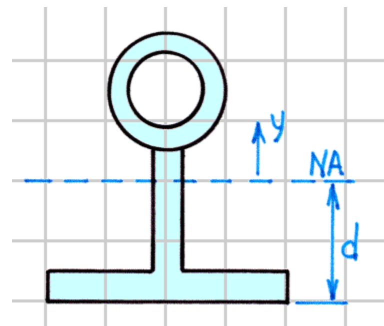
$M_r \cong 2.23\ \text{kN}\cdot\text{m}$ .... **Ans.**



7-15*

$$d = \frac{M_x}{A} = \frac{(0.5)\left[(8)(1)\right]+(3)\left[(4)(1)\right]+(7)\left[\pi\left(4^2-3^2\right)/4\right]}{(8)(1)+(4)(1)+\pi\left(4^2-3^2\right)/4} = 3.113 \text{ in.}$$

$$I = \frac{(8)(3.113)^3}{3} - \frac{(7)(2.113)^3}{3} + \frac{(1)(1.887)^3}{3} + \frac{\pi\left(4^4-3^4\right)}{64} + \frac{\pi\left(4^2-3^2\right)}{4}(3.887)^2 = 152.33 \text{ in.}^4$$



(a) $$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(30\times 12)(5.887)}{(152.33)} = +13.91 \text{ ksi}$$

$\sigma_{top} = 13.91$ ksi (T) .... **Ans.**

(b) $\sigma_{bottom} = \dfrac{-(30\times 12)(-3.113)}{(152.33)} = -7.36 \text{ ksi} = 7.36 \text{ ksi (C)}$ .... **Ans.**

7-16*

(a) $\sigma = E\varepsilon = \left(210\times10^9\right)\left(1200\times10^{-6}\right) = 252.00\left(10^6\right)\text{ N/m}^2 = 252\text{ MPa}$ .... **Ans.**

(b) $$I = \frac{(50)(50)^3}{12} = 0.5208\left(10^6\right)\text{ mm}^4$$

$$M_r = \frac{-\sigma I}{y} = \frac{-\left(252.00\times10^6\right)\left(0.5208\times10^{-6}\right)}{0.025} = -5250\text{ N}\cdot\text{m}$$

$M_r = -5.25\text{ kN}\cdot\text{m}$ .... **Ans.**

7-17

$$\rho = R + (h/2) = 12 + (h/2) \text{ in.}$$

$$\sigma_x = E\varepsilon_x = E\frac{c}{\rho} = \frac{E(h/2)}{12 + (h/2)}$$

$$h = \frac{24\sigma_x}{E - \sigma_x} = \frac{24(36)}{29{,}000 - 36} = 0.0298 \text{ in.}$$ .... **Ans.**

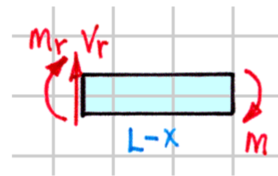
7-18*

$$\rho = R + (h/2) = R + (25/2) = (R + 12.5)\text{ mm}$$

$$\sigma_x = E\varepsilon_x = E\frac{c}{\rho} = \frac{E(12.5)}{R + 12.5}$$

$$R = \frac{E(12.5)}{\sigma_x} - 12.5 = \frac{(73,000)(12.5)}{100} - 12.5 = 9113\text{ mm} \cong 9.11\text{ m}$$ .... **Ans.**

7-19



$$\circlearrowleft \Sigma M_{cut} = 0: \qquad -M_r - M = 0$$

$$M_r = -M = -15 \text{ kip} \cdot \text{in.}$$

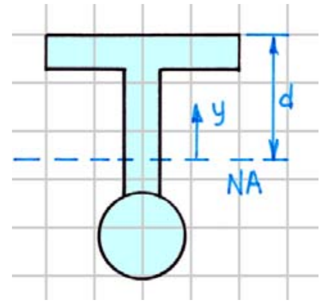
(The internal resisting moment is the same over the entire length of the beam.)

$$I = \frac{(2)(2)^3}{12} = 1.3333 \text{ in.}^4$$

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(-15)(1)}{(1.3333)} = +11.25 \text{ ksi} = 11.25 \text{ ksi (T)}$$ .... **Ans.**

7-20

$$d = \frac{M_x}{A} = \frac{(12.5)\left[(250)(25)\right] + (100)\left[(25)(150)\right] + (225)\left[\pi(100)^2/4\right]}{\left[(250)(25)\right] + \left[(25)(150)\right] + \left[\pi(100)^2/4\right]} = 124.36 \text{ mm}$$



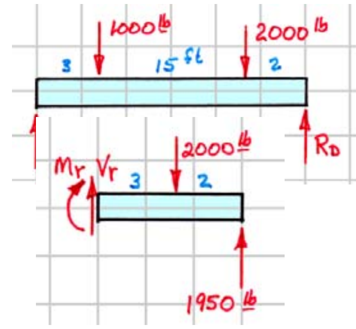
$$I = \frac{(250)(124.36)^3}{3} - \frac{(225)(99.36)^3}{3} + \frac{(25)(50.64)^3}{3}$$

$$+ \frac{\pi(100)^4}{64} + \frac{\pi(100)^2}{4}(100.64)^2 = 172.24\left(10^6\right) \text{ mm}^4$$

$$\sigma_{bottom} = \frac{-M_r y}{I} = \frac{-\left(100\times10^3\right)(-0.15064)}{\left(172.24\times10^{-6}\right)} = +87.5\left(10^6\right) \text{ N/m}^2$$

$\sigma_{bottom} = 87.5$ MPa (T) .... **Ans.**

7-21*



$$\circlearrowleft \Sigma M_C = 0: \qquad 20R_D - 3(1000) - 18(2000) = 0$$

$$R_D = 1950 \text{ lb} \uparrow$$

$$\circlearrowleft \Sigma M_{cut} = 0: \qquad -M_r + 5(1950) - 3(2000) = 0$$

$$M_r = 3750 \text{ lb}\cdot\text{ft} = 3.750 \text{ kip}\cdot\text{ft}$$

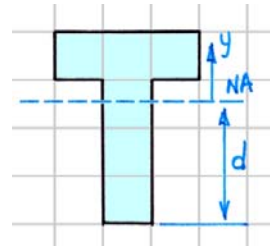
$$I = \frac{\pi\left(2^4 - 1.5^4\right)}{4} = 8.590 \text{ in.}^4$$

$$\sigma_A = \frac{-M_r y}{I} = \frac{-(3.750\times 12)(2)}{(8.590)} = -10.48 \text{ ksi} = 10.48 \text{ ksi (C)}$$ .... **Ans.**

$$\sigma_B = \frac{-(3.750\times 12)(-1.5)}{(8.590)} = +7.86 \text{ ksi} = 7.86 \text{ ksi (T)}$$ .... **Ans.**

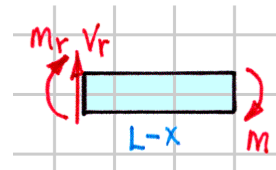
7-22

$$d = \frac{M_x}{A} = \frac{(140)\left[(120)(40)\right] + (60)\left[(40)(120)\right]}{\left[(120)(40)\right] + \left[(40)(120)\right]} = 100 \text{ mm}$$



$$I = \frac{(120)(60)^3}{3} - \frac{(80)(20)^3}{3} + \frac{(40)(100)^3}{3}$$

$$= 21.76\left(10^6\right) \text{ mm}^4$$



$\circlearrowleft \Sigma M_{cut} = 0: \qquad -M_r - M = 0$

$$M_r = -M$$

(The internal resisting moment is the same over the entire length of the beam.)

At the top of the beam $(\sigma = 90 \text{ MPa T})$

$$M = -M_r = \frac{\sigma I}{y} = \frac{\left(90 \times 10^6\right)\left(21.76 \times 10^{-6}\right)}{(0.060)} = +32.6 \text{ kN} \cdot \text{m}$$

At the bottom of the beam $(\sigma = 140 \text{ MPa C})$

$$M = -M_r = \frac{\sigma I}{y} = \frac{\left(-140 \times 10^6\right)\left(21.76 \times 10^{-6}\right)}{(-0.100)} = +30.5 \text{ kN} \cdot \text{m}$$

$M_{\max} = 30.5 \text{ kN} \cdot \text{m} \circlearrowright$ .... **Ans.**

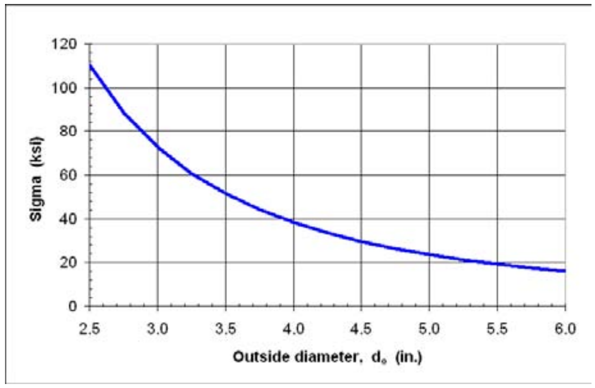
7-23

$$d_i = d_o - 0.5$$

(a) $$\sigma_{\max} = \frac{M_r c}{I} = \frac{(100)(d_o/2)}{\pi\left[d_o^4 - (d_o - 0.5)^4\right]/64}$$

$$\sigma_{\max} = \frac{32(100)d_o}{\pi\left[d_o^4 - (d_o - 0.5)^4\right]} \text{ ksi}$$

(b) $d_{\min} \cong 3.9$ in. ....**Ans.**



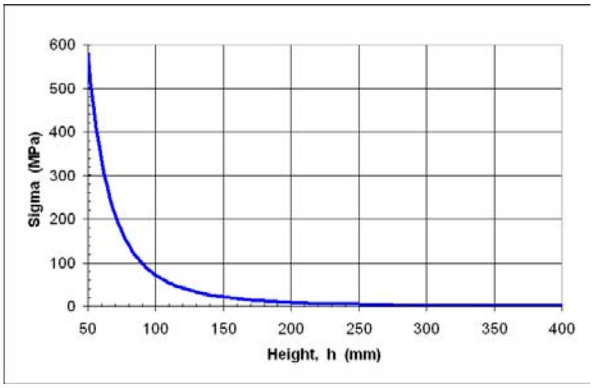
7-24

$$I = \frac{bh^3}{12} = \frac{b(2b)^3}{12} = \frac{8b^4}{12}$$

(a) $$\sigma_{\max} = \frac{M_r c}{I} = \frac{(6000)(h/2)}{(8b^4/12)} = \frac{12(6000)(b)}{8b^4}$$

$$\sigma_{\max} = \frac{12(6000)(b)}{8b^4} = \frac{(9000)}{b^3} \text{ N/m}^2$$

(b) $h_{\min} \cong 150$ mm .... **Ans.**



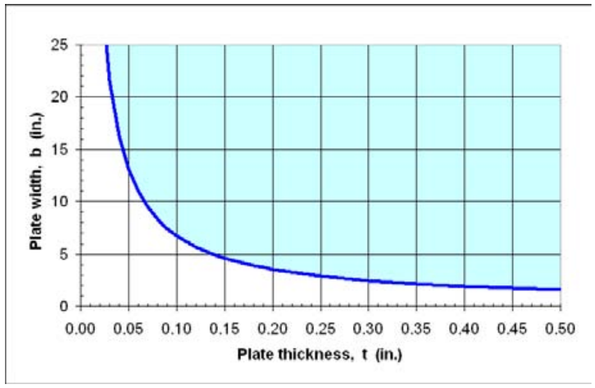
7-25

From Table B-3 for an $S4\times 9.5$ section $d = 2c = 4.00$ in. $A = 2.79$ in.^{2}

$I = 6.79$ in.^{4} $S = 3.39$ in.^{3}

$$M_1 = \sigma S = (20)(3.39) = 67.8 \text{ kip}\cdot\text{in.}$$

$$M_R = 1.75M = (1.75)(67.8) = 118.65 \text{ kip}\cdot\text{in.}$$



$$I_R = 6.79 + 2\left[\frac{bt^3}{12} + (bt)(2+0.5t)^2\right]$$

$$\sigma = \frac{M_R c_R}{I_R} = \frac{(118.65)(2+t)}{I_R} \le 20 \text{ ksi}$$

$$I_R \ge \frac{(118.65)(2+t)}{20} = 5.9325(2+t) \text{ in.}^4$$

$$6.79 + 2\left[\frac{bt^3}{12} + (bt)(2+0.5t)^2\right] \ge 5.9325(2+t)$$

$$b \ge \frac{30.45 + 35.595t}{t^3 + 12t(2+0.5t)^2}$$

7-26

$$A_a = (200)(30) = 6000 \text{ mm}^2 \qquad A_b = 2(150)t + (200)w = 6000 \text{ mm}^2$$

$$w = (30 - 1.5t) \text{ mm}$$

$$I_a = \frac{(30)(200)^3}{12} = 20.00(10^6) \text{ mm}^4$$

$$M_a = \frac{\sigma I}{c} = \frac{(150\times10^6)(20\times10^{-6})}{0.100} = 30,000 \text{ N}\cdot\text{m}$$

$$I_b = \frac{w(200)^3}{12} + 2\left[\frac{(150)t^3}{12} + (150t)(100+0.5t)^2\right]$$

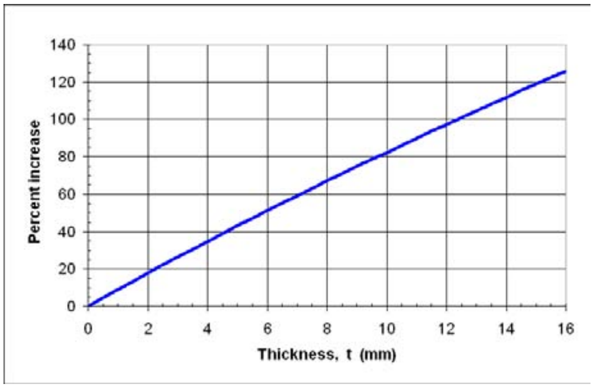
$$= 100t^3 + 30,000 + 2(10^6)t + 20(10^6) \text{ mm}^4$$

$$M_b = \frac{\sigma I}{c} = \frac{(150\times10^6)I_b}{0.100+t} \text{ N}\cdot\text{m}$$

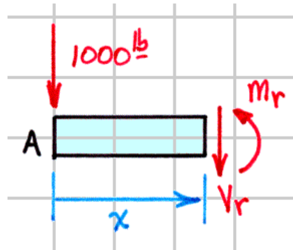
$$\%Inc = \frac{M_b - M_a}{M_a}(100)$$

$$= \left(\frac{M_b}{30,000} - 1\right)(100)$$

$$\%Inc = \left(\frac{5000 I_b}{0.100+t} - 1\right)(100)$$



7-27*



$\uparrow \Sigma F_y = 0:$ $\qquad -1000 - V_r = 0$

$V_r = -1000$ lb ....**Ans.**

$\circlearrowleft \Sigma M_{cut} = 0:$ $\qquad M_r + 1000x = 0$

$M_r = (-1000x)$ lb · ft ....**Ans.**

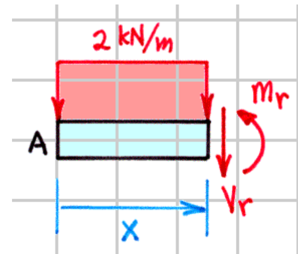
7-28*

$\uparrow \Sigma F_y = 0:$ $-(2x) - V_r = 0$

$V_r = (-2x)$ kN .... **Ans.**

$\circlearrowleft \Sigma M_{cut} = 0:$ $M_r + (2x)(x/2) = 0$

$M_r = (-x^2)$ kN·m .... **Ans.**



7-29

$\circlearrowleft \Sigma M_B = 0: \quad (200 \times 10)(5) - (500)(2) - 10R_A = 0 \qquad R_A = 900 \text{ lb}$

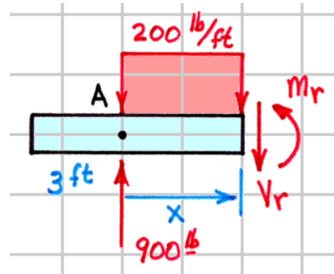
$0 \le x \le 10 \text{ ft}$

$\uparrow \Sigma F_y = 0: \quad 900 - (200x) - V_r = 0$

$V_r = (900 - 200x) \text{ lb}$ .... **Ans.**

$\circlearrowleft \Sigma M_{cut} = 0: \quad M_r + (200x)(x/2) - 900x = 0$

$M_r = (900x - 100x^2) \text{ lb} \cdot \text{ft}$ .... **Ans.**



7-30*

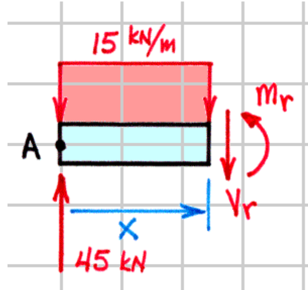
$\circlearrowleft \Sigma M_B = 0: \quad (15 \times 4)(6) - 8R_A = 0 \quad R_A = 45 \text{ kN}$

$\uparrow \Sigma F_y = 0: \quad 45 - (15x) - V_r = 0$

$V_r = (45 - 15x) \text{ kN}$ ....**Ans.**

$\circlearrowleft \Sigma M_{cut} = 0: \quad M_r - 45x + (15x)(x/2) = 0$

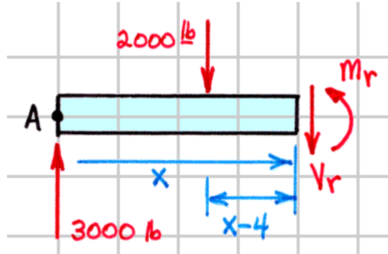
$M_r = (45x - 7.5x^2) \text{ kN} \cdot \text{m}$ ....**Ans.**



7-31

$\circlearrowleft \Sigma M_B = 0: \qquad (5000)(4) + (2000)(8) - 12R_A = 0 \qquad R_A = 3000 \text{ lb}$

$4 \text{ ft} \le x \le 8 \text{ ft}$



$\uparrow \Sigma F_y = 0: \qquad 3000 - 2000 - V_r = 0$

$V_r = (1000) \text{ lb}$ .... **Ans.**

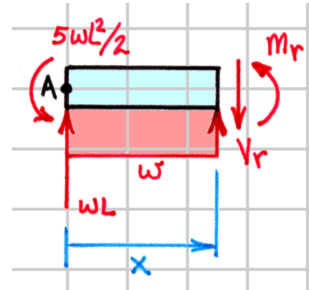
$\circlearrowleft \Sigma M_{cut} = 0: \qquad M_r - 3000x + (2000)(x-4) = 0$

$M_r = (1000x + 8000) \text{ lb} \cdot \text{ft}$ .... **Ans.**

7-32

$\uparrow \Sigma F_y = 0:$ $\quad R_A + (wL) - (2wL) = 0$ $\quad R_A = wL \uparrow$

$\circlearrowleft \Sigma M_A = 0:$ $\quad M_A + (wL)(L/2) - (2wL)(3L/2) = 0$ $\quad M_A = 5wL^2/2 \circlearrowleft$



$\uparrow \Sigma F_y = 0:$ $\quad (wL) + (wx) - V_r = 0$

$V_r = wL + wx = w(L + x)$ ....**Ans.**

$\circlearrowleft \Sigma M_{cut} = 0:$ $\quad M_r + (5wL^2/2) - (wL)x - (wx)(x/2) = 0$

$M_r = \dfrac{wx^2}{2} + wLx - \dfrac{5wL^2}{2} = \dfrac{w}{2}(x^2 + 2Lx - 5L^2)$ ....**Ans.**

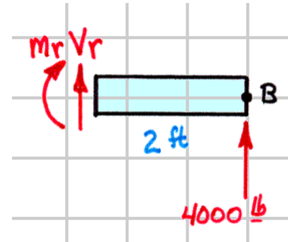
7-33*

$\circlearrowleft \Sigma M_A = 0: \quad 12R_B - (2000)(4) - (5000)(8) = 0 \qquad R_B = 4000 \text{ lb}$

$\circlearrowleft \Sigma M_{cut} = 0: \quad (4000)(2) - M_r = 0$

$$M_r = (8000) \text{ lb}\cdot\text{ft}$$

$$I = \frac{bh^3}{12} = \frac{(3)(8)^3}{12} = 128.00 \text{ in.}^4$$

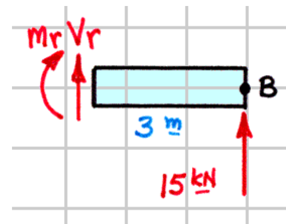


On the bottom of the beam $(y = -4 \text{ in.})$

$$\sigma = \frac{-M_r y}{I} = \frac{-(8\times 12)(-4)}{(128.00)} = +3.00 \text{ ksi} = 3.00 \text{ ksi (T)}$$ .... **Ans.**

7-34

From Table B-4 for an $S254\times52$ section $d = 2c = 254.00$ mm $I = 61.2(10^6)$ mm^4



$\circlearrowleft \Sigma M_A = 0:$ $8R_B - (15\times 4)(2) = 0$ $R_B = 15$ kN

$\circlearrowleft \Sigma M_{cut} = 0:$ $-M_r + (15)(3) = 0$ $M_r = 45$ kN·m

On the bottom of the beam $(y = -127.0 \text{ mm})$

$$\sigma = \frac{-M_r y}{I} = \frac{-(45,000)(-0.127)}{(61.2\times10^{-6})} = 93.4(10^6) \text{ N/m}^2$$

$\sigma = 93.4$ MPa (T) .... **Ans.**

On the top of the beam $(y = +127.0 \text{ mm})$

$$\sigma = \frac{-(45,000)(+0.127)}{(61.2\times10^{-6})} = -93.4(10^6) \text{ N/m}^2 = 93.4 \text{ MPa (C)}$$ .... **Ans.**

7-35*

$\circlearrowleft \Sigma M_B = 0: \quad (2000)(13) + (1000 \times 6)(5) - 10R_A = 0 \qquad R_A = 5600 \text{ lb}$

$\circlearrowleft \Sigma M_A = 0: \quad (2000)(3) - (1000 \times 6)(5) + 10R_B = 0 \qquad R_B = 2400 \text{ lb}$

(a) $V_r = (-2000) \text{ lb}$ .... **Ans.**

$M_r = -2000(x+3) = [-2000x - 6000] \text{ lb} \cdot \text{ft}$ .... **Ans.**

(b) $V_r = -2000 + 5600 = (3600) \text{ lb}$ .... **Ans.**

$M_r = -2000(x+3) + 5600x = [3600x - 6000] \text{ lb} \cdot \text{ft}$ .... **Ans.**

(c) $V_r = -2000 + 5600 - 1000(x-2) = (-1000x + 5600) \text{ lb}$ .... **Ans.**

$M_r = -2000(x+3) + 5600x - 1000(x-2)(x-2)/2$

$M_r = [-500x^2 + 5600x - 8000] \text{ lb} \cdot \text{ft}$ .... **Ans.**

(d) $V_r = -2000 + 5600 - 1000(6) = (-2400) \text{ lb}$ .... **Ans.**

$M_r = -2000(x+3) + 5600x - 1000(6)(x-5)$

$M_r = [-2400x + 24,000] \text{ lb} \cdot \text{ft}$ .... **Ans.**

7-36*

$\circlearrowleft \Sigma M_B = 0: \quad (12\times 6)(9)+(24)(4)-10R_A = 0 \quad R_A = 74.4 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0: \quad -(12\times 6)(1)-(24)(6)+10R_B = 0 \quad R_B = 21.6 \text{ kN}$

(a) $V_r = -12(x+2) = (-12x-24) \text{ kN}$ .... **Ans.**

$M_r = -[(12)(x+2)](x+2)/2 = [-6x^2 - 24x - 24] \text{ kN}\cdot\text{m}$ .... **Ans.**

(b) $V_r = -12(x+2)+74.4 = (-12x+50.4) \text{ kN}$ .... **Ans.**

$M_r = -[(12)(x+2)](x+2)/2 + 74.4x = [-6x^2 + 50.4x - 24] \text{ kN}\cdot\text{m}$ .... **Ans.**

(c) $V_r = -12(6)+74.4 = (2.4) \text{ kN}$ .... **Ans.**

$M_r = -[12(6)](x-1)+74.4x = [2.4x+72] \text{ kN}\cdot\text{m}$ .... **Ans.**

(d) $V_r = -12(6)+74.4-24 = (-21.6) \text{ kN}$ .... **Ans.**

$M_r = -[12(6)](x-1)+74.4x-24(x-6) = [-21.6x+216] \text{ kN}\cdot\text{m}$ .... **Ans.**

7-37

$\circlearrowleft \Sigma M_B = 0: \quad \left[(1/2)(1500)(12)\right](4) - 12R_A = 0 \quad R_A = 3000 \text{ lb}$

$\circlearrowleft \Sigma M_A = 0: \quad 12R_B - \left[(1/2)(1500)(12)\right](8) = 0 \quad R_B = 6000 \text{ lb}$

(a) $V_r = 3000 - \left(\frac{1}{2}\right)\left(\frac{1500x}{12}\right)(x) = \left(-62.5x^2 + 3000\right) \text{ lb}$ .... **Ans.**

$M_r = 3000x - \left[\left(\frac{1}{2}\right)\left(\frac{1500x}{12}\right)(x)\right]\left(\frac{x}{3}\right) = \left[-20.83x^3 - 3000x\right] \text{ lb}\cdot\text{ft}$ .... **Ans.**

(b) $\frac{dV_r}{dx} = -125x = 0$ Solving yields: $x = 0$

Therefore, the maximum shear force occurs either at the beginning or end of the region:

$V_{x=0} = 3000 \text{ lb} \qquad V_{x=12} = -6000 \text{ lb}$

$V_{\max} = V_{x=12} = (-6000) \text{ lb}$ .... **Ans.**

$\frac{dM_r}{dx} = -62.5x^2 + 3000 = 0$ Solving yields: $x = 6.928 \text{ ft}$

$M_{x=0} = 0 \text{ lb}\cdot\text{ft} \qquad M_{x=6.928} = 13{,}858 \text{ lb}\cdot\text{ft} \qquad M_{x=12} = 0 \text{ lb}\cdot\text{ft}$

$M_{\max} = M_{x=6.928} = 13{,}858 \text{ lb}\cdot\text{ft} \cong 13.86 \text{ kip}\cdot\text{ft}$ .... **Ans.**

7-38*

$\circlearrowleft \Sigma M_B = 0: \quad (9)(4) + (15\times 2)(1.5) - 3R_A = 0 \quad R_A = 27.0 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0: \quad 3R_B + (9)(1) - (15\times 2)(1.5) = 0 \quad R_B = 12.0 \text{ kN}$

(a) $V_r = -9 + 27 - 15(x - 0.5) = (-15x + 25.5) \text{ kN}$ .... **Ans.**

$M_r = -9(x+1) + 27x - [(15)(x-0.5)](x-0.5)/2$

$M_r = [-7.5x^2 + 25.5x - 10.88] \text{ kN}\cdot\text{m}$ .... **Ans.**

(b) From Table B-4 for an S178×30 section $d = 2c = 177.8 \text{ mm}$

$I = 17.6(10^6) \text{ mm}^4 \qquad S = 198(10^3) \text{ mm}^3$

$M_{1.5} = -9(2.5) + 27(1.5) - [(15)(1)](0.5) = +10.5 \text{ kN}\cdot\text{m}$

$y = -c + 15 = -88.9 + 15 = -73.9 \text{ mm}$

$$\sigma = \frac{-M_r y}{I} = \frac{-(10{,}500)(-0.0739)}{(17.6\times 10^{-6})} = +44.1(10^6) \text{ N/m}^2$$

$\sigma = 44.1 \text{ MPa (T)}$ .... **Ans.**

(c) $$\sigma = \frac{M_r}{S} = \frac{(10{,}500)}{(198\times 10^{-6})} = 53.0(10^6) \text{ N/m}^2$$

$\sigma_{max} = 53.0 \text{ MPa (C, top; T, bottom)}$ .... **Ans.**

7-39

$\circlearrowleft \Sigma M_B = 0: \quad [(500)(6)](7) + [(800)(5)](2.5) - 10R_A = 0 \qquad R_A = 1100 \text{ lb}$

$\circlearrowleft \Sigma M_A = 0: \quad 10R_B - [(500)(6)](3) - [(800)(5)](12.5) = 0 \qquad R_B = 5900 \text{ lb}$

(a) $V_r = 1100 - (500)(6) = (-1900) \text{ lb}$ .... **Ans.**

$M_r = 1100x - [(500)(6)](x-3) = [-1900x + 9000] \text{ lb}\cdot\text{ft}$ .... **Ans.**

(b) From Table B-3 for an S8×23 section $\qquad d = 2c = 8.00 \text{ in.}$

$I = 64.9 \text{ in.}^4 \qquad S = 16.2(10^3) \text{ in.}^3$

$M_3 = 1100(3) - [(500)(3)](1.5) = 1050 \text{ lb}\cdot\text{ft} \qquad y = c - 1 = 4 - 1 = +3 \text{ in.}$

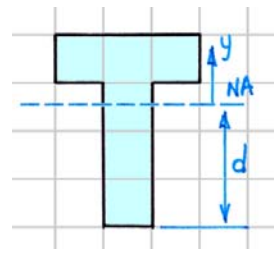
$\sigma = \dfrac{-M_r y}{I} = \dfrac{-(1050\times 12)(3)}{(64.9)} = -582 \text{ psi} = 582 \text{ psi (C)}$ .... **Ans.**

(c) $\sigma_{\max} = \dfrac{M_r}{S} = \dfrac{(1050\times 12)}{(16.2)} = 778 \text{ psi (C, top; T bottom)}$ .... **Ans.**

7-40

$\circlearrowleft \Sigma M_B = 0: \qquad (3\times 2)(3) - 4R_A = 0 \qquad R_A = 4.50 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0: \qquad 4R_B - (3\times 2)(1) = 0 \qquad R_B = 1.50 \text{ kN}$



$$d = \frac{M_x}{A} = \frac{(140)\left[(120)(40)\right] + (60)\left[(40)(120)\right]}{\left[(120)(40)\right] + \left[(40)(120)\right]} = 100 \text{ mm}$$

$$I = \frac{(120)(60)^3}{3} - \frac{(80)(20)^3}{3} + \frac{(40)(100)^3}{3} = 21.76\left(10^6\right) \text{ mm}^4$$

$$M_3 = R_B(1) = (1.50)(1) = 1.500 \text{ kN}\cdot\text{m}$$

At the top of the section $(y = +60 \text{ mm})$

$$\sigma = \frac{-M_r y}{I} = \frac{-(1500)(0.060)}{\left(21.76\times 10^{-6}\right)} = -4.14\left(10^6\right) \text{ N/m}^2$$

$\sigma = 4.14$ MPa (C) .... **Ans.**

At the bottom of the section $(y = -100 \text{ mm})$

$\sigma = \dfrac{-(1500)(-0.100)}{\left(21.76\times 10^{-6}\right)} = +6.89\left(10^6\right) \text{ N/m}^2 = 6.89$ MPa (T) .... **Ans.**

7-41*

$$R_A = R_B = \frac{1}{2}\int_0^{10} 1000 \sin\left(\frac{\pi s}{10}\right) ds = \left[\frac{-5000}{\pi}\cos\left(\frac{\pi s}{10}\right)\right]_0^{10} = \left(\frac{10,000}{\pi}\right) \text{ lb}$$

(a) $$V_r = R_A - \int_0^x w\, ds = \left(\frac{10,000}{\pi}\right) - \int_0^x 1000 \sin\left(\frac{\pi s}{10}\right) ds$$

$$V_r = \left(\frac{10,000}{\pi}\right) + \left[\frac{10,000}{\pi}\cos\left(\frac{\pi s}{10}\right)\right]_0^x = \left[\frac{10,000}{\pi}\cos\left(\frac{\pi x}{10}\right)\right] \text{ lb}$$ .... **Ans.**

$$M_r = R_A x - \int_0^x w(x-s)\, ds = \left(\frac{10,000x}{\pi}\right) - \int_0^x 1000(x-s)\sin\left(\frac{\pi s}{10}\right) ds$$

$$= \left(\frac{10,000x}{\pi}\right) + \left[\frac{10,000x}{\pi}\cos\left(\frac{\pi s}{10}\right)\right]_0^x + \left[\frac{100,000}{\pi^2}\sin\left(\frac{\pi s}{10}\right)\right]_0^x - \left[\frac{10,000s}{\pi}\cos\left(\frac{\pi s}{10}\right)\right]_0^x$$

$$M_r = \left[\frac{100,000}{\pi^2}\sin\left(\frac{\pi x}{10}\right)\right] \text{ lb}\cdot\text{ft} = \left[10.13\sin\left(\frac{\pi x}{10}\right)\right] \text{ kip}\cdot\text{ft}$$ .... **Ans.**

(b) $$V_{max} = V_{x=0} = V_{x=10} = \frac{10,000}{\pi} = 3183 \text{ lb} \cong 3.18 \text{ kip}$$ .... **Ans.**

$$M_{max} = M_{x=5} = \frac{100,000}{\pi^2} = 10,132 \text{ lb}\cdot\text{ft} \cong 10.13 \text{ kip}\cdot\text{ft}$$ .... **Ans.**

7-42*

$$\circlearrowleft \Sigma M_B = 0: \qquad \int_0^4 w(4-s)\,ds - 4R_A = 0$$

$$R_A = \frac{1}{4}\int_0^4 (4-s)\left[25\cos\left(\frac{\pi s}{8}\right)\right]dx$$

$$= \left[\frac{200}{\pi}\sin\left(\frac{\pi s}{8}\right)\right]_0^4 - \left[\frac{400}{\pi^2}\cos\left(\frac{\pi s}{8}\right)\right]_0^4 - \left[\frac{50s}{\pi}\sin\left(\frac{\pi s}{8}\right)\right]_0^4 = \left(\frac{400}{\pi^2}\right) \text{ kN}$$

(a) $$V_r = R_A - \int_0^x w\,ds = \left(\frac{400}{\pi^2}\right) - \int_0^x 25\cos\left(\frac{\pi s}{8}\right)ds = \left(\frac{400}{\pi^2}\right) - \left[\frac{200}{\pi}\sin\left(\frac{\pi s}{8}\right)\right]_0^x$$

$V_r = \left[\left(\frac{400}{\pi^2}\right) - \frac{200}{\pi}\sin\left(\frac{\pi x}{8}\right)\right]$ kN .... **Ans.**

$$M_r = R_A x - \int_0^x w(x-s)\,ds = \left(\frac{400x}{\pi^2}\right) - \int_0^x (x-s)\left[25\cos\left(\frac{\pi s}{8}\right)\right]ds$$

$$= \left(\frac{400x}{\pi^2}\right) - \left[\frac{200}{\pi}\sin\left(\frac{\pi s}{8}\right)\right]_0^x + \left[\frac{1600}{\pi^2}\cos\left(\frac{\pi s}{8}\right)\right]_0^x + \left[\frac{200s}{\pi}\sin\left(\frac{\pi s}{8}\right)\right]_0^x$$

$M_r = \left[\frac{400(x-4)}{\pi^2} + \frac{1600}{\pi^2}\cos\left(\frac{\pi x}{8}\right)\right]$ kN·m .... **Ans.**

(b) $V_{max} = V_{x=0} = \frac{400}{\pi^2} = 40.5$ kN .... **Ans.**

$$\frac{dM_r}{dx} = \frac{400}{\pi^2} - \frac{200}{\pi}\sin\left(\frac{\pi x}{8}\right) = 0 \qquad\qquad x = 1.7573 \text{ m}$$

$M_{max} = M_{x=1.7573} = 34.1$ kN·m .... **Ans.**

7-43

$$\circlearrowleft \Sigma M_B = 0: \qquad \int_0^{10} w(10-s)\,ds - 10R_A = 0$$

$$R_A = \frac{1}{10}\int_0^{10}(10-s)(10s^2)\,ds = \left[\frac{10s^3}{3} - \frac{s^4}{4}\right]_0^{10} = (833.3) \text{ lb}$$

(a) $V_r = R_A - \int_0^x w\,ds = 833.3 - \int_0^x 10s^2\,ds = (833 - 3.33x^3) \text{ lb}$ .... **Ans.**

$$M_r = R_A x - \int_0^x w(x-s)\,ds = 833.3x - \int_0^x 10s^2(x-s)\,ds$$

$M_r = (833x - 0.8333x^4) \text{ lb}\cdot\text{ft}$ .... **Ans.**

(b) $V_{max} = V_{x=10} = 833.3 - 3.333(10)^3 = -2500 \text{ lb}$ .... **Ans.**

$$\frac{dM_r}{dx} = 833.3 - 3.333x^3 = 0 \qquad x = 6.300 \text{ ft}$$

$M_{max} = M_{x=6.300} = 3.94 \text{ kip}\cdot\text{ft}$ .... **Ans.**

7-44

$\circlearrowleft \Sigma M_B = 0: \qquad \int_0^8 w(8-s)\,ds - 8R_A = 0$

$$R_A = \frac{1}{8}\int_0^8 (64-s^2)(8-s)\,ds = \frac{1}{8}\int_0^8 (s^3 - 8s^2 - 64s + 512)\,ds = 213.3 \text{ kN}$$

(a) $V_r = R_A - \int_0^x w\,ds = 213.3 - \int_0^x (64-s^2)\,ds = 213.3 - 64x + \dfrac{x^3}{3}$

$V_r = [0.333x^3 - 64.0x + 213]$ kN .... **Ans.**

$$M_r = R_A x - \int_0^x w(x-s)\,ds = 213.3x - \int_0^x (64-s^2)(x-s)\,ds$$

$$= 213.3x - \frac{x^4}{4} + \frac{x^4}{3} + 32x^2 - 64x^2$$

$M_r = [0.0833x^4 - 32.0x^2 + 213x]$ kN·m .... **Ans.**

(b) $\dfrac{dV_r}{dx} = x^2 - 64 = 0 \qquad\qquad x = 8$ m

Therefore, the maximum shear force occurs
either at the beginning or end of the region:

$V_{x=0} = +213.3$ kN $\qquad\qquad V_{x=8} = -128.0$ kN

$V_{max} = V_{x=0} = 213$ kN .... **Ans.**

$\dfrac{dM_r}{dx} = 0.3333x^3 - 64.0x + 213.3 = 0 \qquad\qquad x = 3.570$ m

$M_{max} = M_{x=3.570} = 367$ kN·m .... **Ans.**

7-45

$\circlearrowleft \Sigma M_B = 0: \quad \left[(1000)(10)\right](5) + 5P - 10R_A = 0 \qquad R_A = R_B = (5000 + 0.5P)\text{ lb}$

$$M_P = (5000 + 0.5P)(5) - \left[(1000)(5)\right](2.5) = (12{,}500 + 2.5P)\text{ lb}\cdot\text{ft}$$

From Table B-5 for a $C10\times 15.3$ section

$$S = 13.5\text{ in.}^3$$

$$M_P = \sigma_{\max} S = (16)(2\times 13.5) = 432\text{ kip}\cdot\text{in.} = 36\text{ kip}\cdot\text{ft}$$

$$P = \frac{M_P - 12.5}{2.5} = \frac{36 - 12.5}{2.5} = 9.40\text{ kip}$$ .... **Ans.**

7-46

$\circlearrowleft \Sigma M_A = 0: \quad 5R_D - 5b = 0 \quad R_D = (b) \text{ kN}$

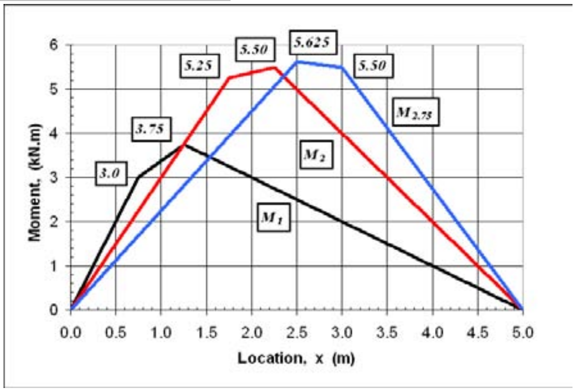
$\circlearrowleft \Sigma M_D = 0: \quad 5(5-b) - 5R_A = 0 \quad R_A = (5-b) \text{ kN}$

(a) $0 \le x \le (b-0.25) \text{ m} \quad M_r = \left[(5-b)x\right] \text{ kN}\cdot\text{m}$

$(b-0.25) \text{ m} \le x \le (b+0.25) \text{ m} \quad M_r = \left[(5-b)x - 2.5(x-b+0.25)\right] \text{ kN}\cdot\text{m}$

$(b+0.25) \text{ m} \le x \le 5 \text{ m} \quad M_r = \left[(5-x)b\right] \text{ kN}\cdot\text{m}$

(Note that for any position of the crane, the maximum bending moment occurs under the wheel closest to the center of the beam.)



(b) $M_B = M_{x=(b-0.25)} = (5-b)(b-0.25)$

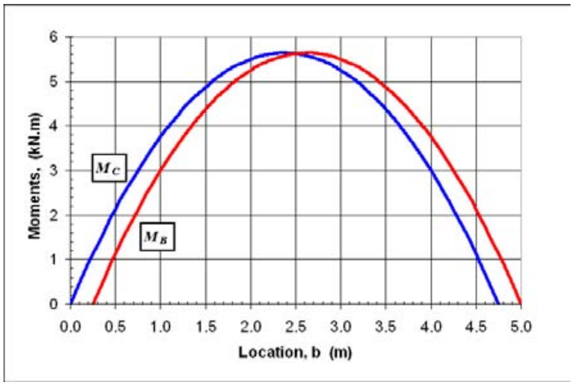
$M_B = \left[-b^2 + 5.25b - 1.25\right] \text{ kN}\cdot\text{m}$

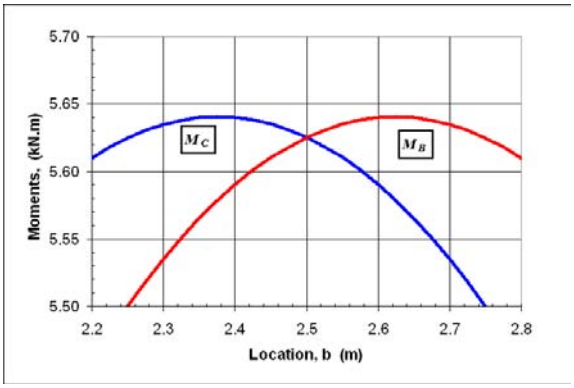
$M_C = M_{x=(b+0.25)} = \left[(4.75-b)b\right] \text{ kN}\cdot\text{m}$

(c) $\dfrac{dM_B}{dx} = -2b + 5.25 = 0$

$b = 2.625 \text{ m} \quad M_{B\max} = 5.64 \text{ kN}\cdot\text{m}$

$\dfrac{dM_C}{dx} = 4.75 - 2b = 0 \quad b = 2.375 \text{ m} \quad M_{C\max} = 5.64 \text{ kN}\cdot\text{m}$





7-47

$R_B = R_C = wL/2 = (1200)(15)/2 = 9000$ lb

(a) $0 \le x \le d$ ft $\quad M_r = -(1200)(x)(x/2) = \left[-600x^2\right]$ lb·ft

$d \le x \le (15-d)$ ft $\quad M_r = -(1200)(x)(x/2) + 900(x-d)$

$= \left[-600x^2 + 9000x - 9000d\right]$ lb·ft

$(15-d)$ ft $\le x \le 15$ ft $\quad M_r = -(1200)(15-x)(15-x)/2$

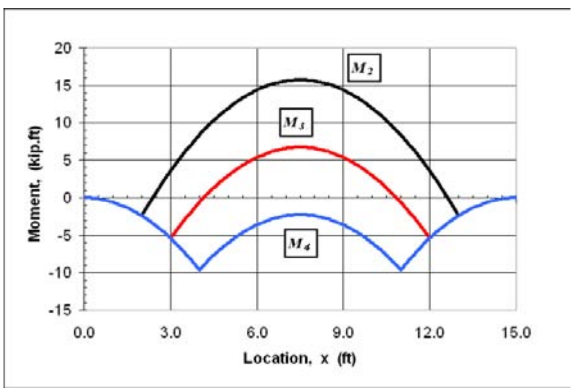
$= \left[-600x^2 + 18,000x - 135,000\right]$ lb·ft

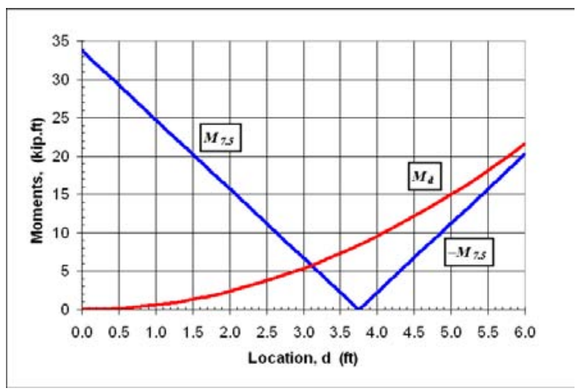
(b) $AB: \quad M_{\max AB} = M_{x=d} = \left[-600d^2\right]$ lb·ft

$BC: \quad \dfrac{dM}{dx} = -1200x + 9000 = 0 \qquad x = 7.5$ ft

$M_{\max BC} = M_{x=7.5} = [33,750 - 9000d]$ lb·ft

(c) $-M_{\max AB} = M_{\max BC} \qquad 600d^2 = 33,750 - 9000d \qquad d = 3.107$ ft





7-48

$\circlearrowleft \Sigma M_B = 0: (5-d)R_C - (15\times 6)(2) = 0$ $\qquad R_C = \left(\dfrac{180}{5-d}\right)$ kN

$\circlearrowleft \Sigma M_C = 0: (15\times 6)(3-d) - (5-d)R_B = 0$ $\qquad R_A = \left[\dfrac{90(3-d)}{5-d}\right]$ kN

(a) $0 \le x \le 1$ m $\qquad M_r = -(15x)(x/2) = (-7.5x^2)$ kN·m

$1\text{ m} \le x \le (6-d)$ m $\qquad M_r = -(15x)(x/2) + \dfrac{90(3-d)}{5-d}(x-1)$

$$M_r = \left[-7.5x^2 + \frac{90(3-d)}{5-d}(x-1)\right] \text{ kN}\cdot\text{m}$$

$(6-d)\text{ m} \le x \le 6$ m $\qquad M_r = -15(6-x)(6-x)/2$

$$M_r = \left[-7.5x^2 + 90x - 270\right] \text{ kN}\cdot\text{m}$$

(b) $M_{AB\max} = -7.5(1)^2 = -7.5$ kN·m

$1\text{ m} \le x \le (6-d)$ m $\qquad \dfrac{dM_r}{dx} = -15x + \dfrac{90(3-d)}{5-d} = 0 \qquad x = \dfrac{6(3-d)}{5-d}$ m

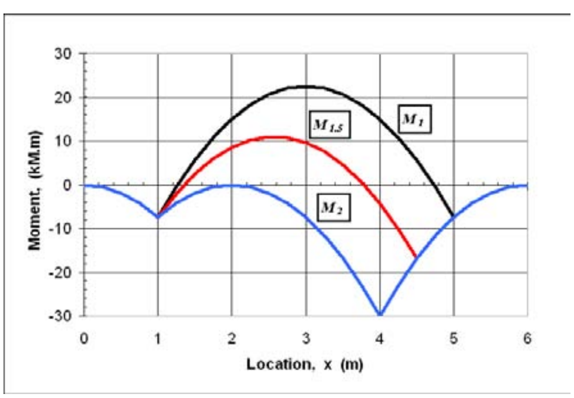
Note that when $d \ge 2.6$ m, this gives a location for the maximum bending moment which is outside the range for which the bending moment equation is valid $(x \le 1.0\text{ m})$. Furthermore, when $d = 1.369$ m, the bending moment at the right support is greater than the internal maximum. Therefore, for $d \ge 1.369$ m the maximum bending moment in the region *BC* is the same as the maximum bending moment in the region *CD*.

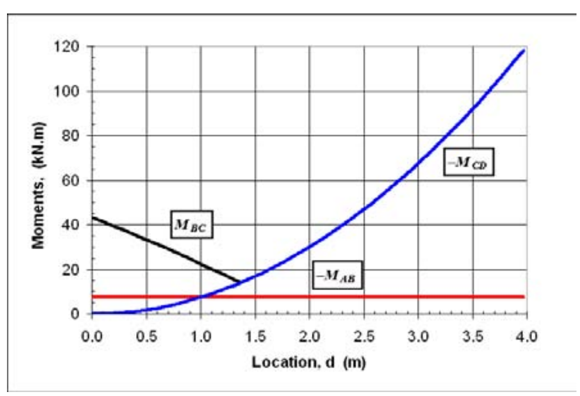
$M_{BC\max} = \left[\dfrac{180d^2 - 900d + 1080}{(5-d)^2}\right]$ kN·m $\qquad d \le 1.369$ m

$M_{BC\max} = M_{CD\max} = \left[-7.5d^2\right]$ kN·m $\qquad d \ge 1.369$ m

$M_{CD\max} = \left[-7.5d^2\right]$ kN·m

(c) $M_{BC\max} = -M_{CD\max} \qquad \dfrac{180d^2 - 900d + 1080}{(5-d)^2} = 7.50d^2 \qquad d = 1.369$ m





7-49*

$\circlearrowleft \Sigma M_D = 0:$

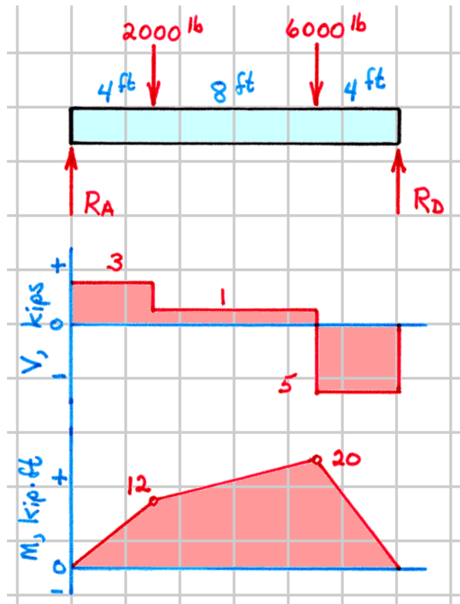
$$(2000)(12)+(6000)(4)-16R_A = 0$$

$$R_A = 3000 \text{ lb} \uparrow$$

$\circlearrowleft \Sigma M_A = 0:$

$$16R_D - (2000)(4) - (6000)(12) = 0$$

$$R_D = 5000 \text{ lb} \uparrow$$



7-50*

$\circlearrowleft \Sigma M_A = 0:$

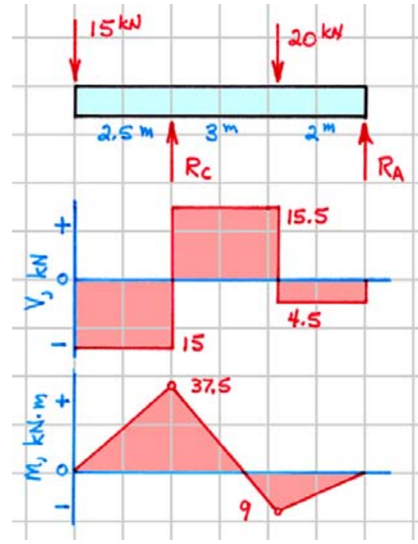
$$(15)(7.5) + (20)(2) - 5R_C = 0$$

$$R_C = 30.5 \text{ kN} \uparrow$$

$\circlearrowleft \Sigma M_C = 0:$

$$5R_A + (15)(2.5) - (20)(3) = 0$$

$$R_A = 4.50 \text{ kN} \uparrow$$



7-51

$\circlearrowleft \Sigma M_C = 0:$

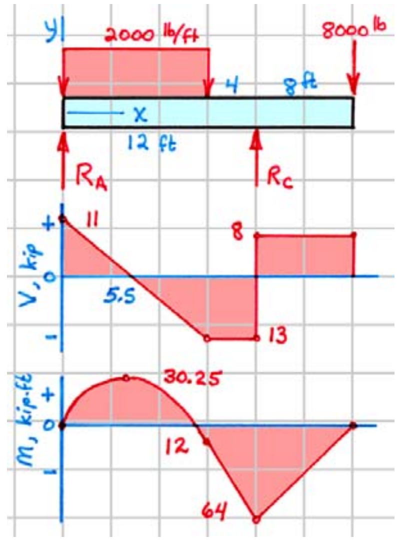
$$(2000\times 12)(10)-(8000)(8)-16R_A=0$$

$$R_A = 11{,}000 \text{ lb}\uparrow$$

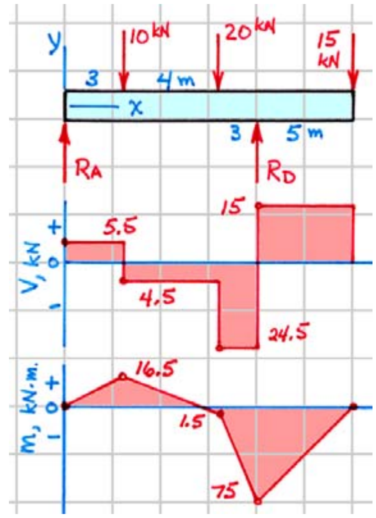
$\circlearrowleft \Sigma M_A = 0:$

$$16R_C-(2000\times 12)(6)-(8000)(24)=0$$

$$R_C = 21{,}000 \text{ lb}\uparrow$$



7-52*



$\circlearrowleft \Sigma M_D = 0:$

$$(10)(7)+(20)(3)-(15)(5)-10R_A = 0$$

$$R_A = 5.50 \text{ kN} \uparrow$$

$\circlearrowleft \Sigma M_A = 0:$

$$10R_D-(10)(3)-(20)(7)-(15)(15) = 0$$

$$R_D = 39.5 \text{ kN} \uparrow$$

7-53

$\circlearrowleft \Sigma M_D = 0:$

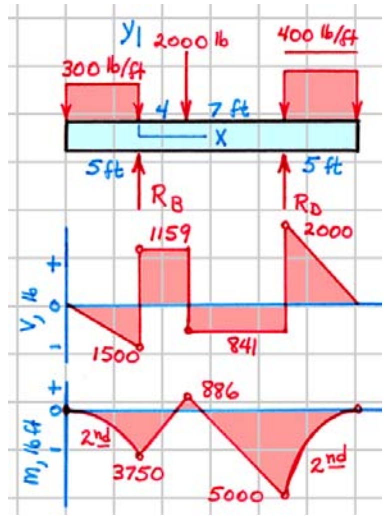
$$(300\times 5)(13.5)-(400\times 5)(2.5)+(2000)(7)-11R_B = 0$$

$$R_B = 2659 \text{ lb}\uparrow$$

$\circlearrowleft \Sigma M_B = 0:$

$$11R_D + (300\times 5)(2.5)-(400\times 5)(13.5)-(2000)(4) = 0$$

$$R_D = 2841 \text{ lb}\uparrow$$



7-54

$\circlearrowleft \Sigma M_D = 0:$

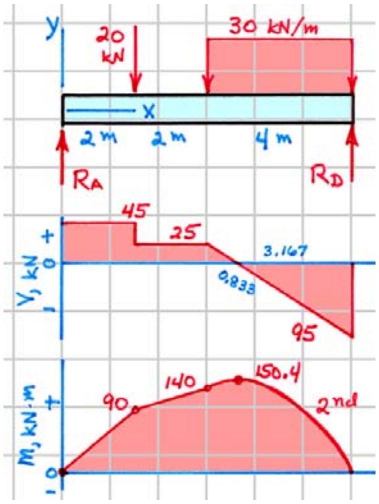
$$(30\times 4)(2)+(20)(6)-8R_A=0$$

$$R_A = 45 \text{ kN}\uparrow$$

$\circlearrowleft \Sigma M_A = 0:$

$$8R_D-(20)(2)-(30\times 4)(6)=0$$

$$R_D = 95 \text{ kN}\uparrow$$



7-55*

$\circlearrowleft \Sigma M_D = 0:$

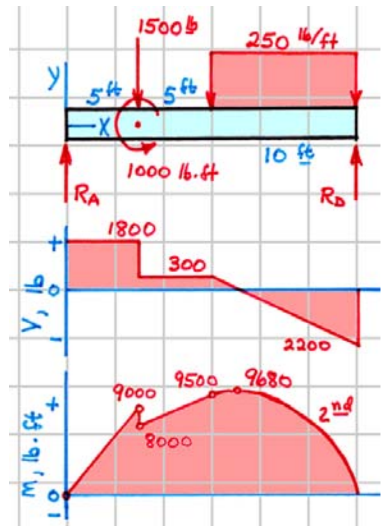
$$(250\times10)(5)+(1500)(15)+(1000)-20R_A = 0$$

$$R_A = 1800 \text{ lb} \uparrow$$

$\circlearrowleft \Sigma M_A = 0:$

$$20R_D + (1000)-(1500)(5)-(250\times10)(15) = 0$$

$$R_D = 2200 \text{ lb} \uparrow$$



7-56

$\circlearrowleft \Sigma M_D = 0:$

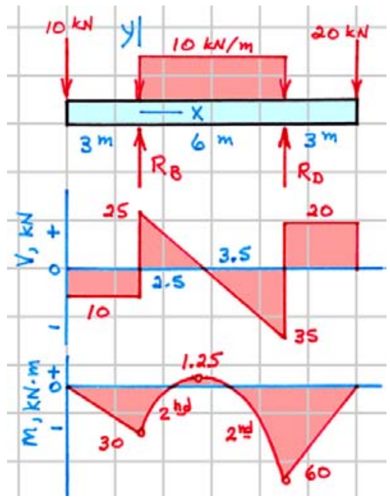
$$(10)(9)+(10\times 6)(3)-(20)(3)-6R_B = 0$$

$$R_B = 35 \text{ kN} \uparrow$$

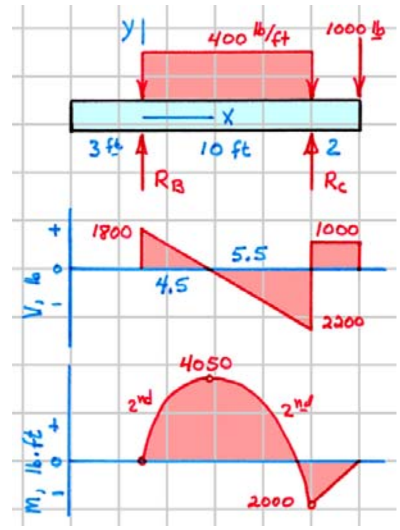
$\circlearrowleft \Sigma M_B = 0:$

$$6R_D + (10)(3)-(10\times 6)(3)-(20)(9) = 0$$

$$R_D = 55 \text{ kN} \uparrow$$



7-57*

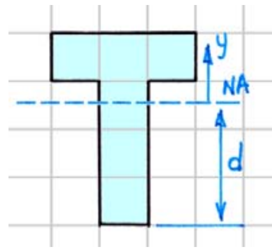


$\circlearrowleft \Sigma M_C = 0$: $(400\times10)(5)-(1000)(2)-10R_B = 0$

$$R_B = 1800 \text{ lb}\uparrow$$

$\circlearrowleft \Sigma M_A = 0$: $10R_C-(400\times10)(5)-(1000)(12) = 0$

$$R_C = 3200 \text{ lb}\uparrow$$



$$d = \frac{(7)\big[(6)(2)\big]+(3)\big[(2)(6)\big]}{\big[(6)(2)\big]+\big[(2)(6)\big]}$$

$$= 5.00 \text{ in.}$$

$$I = \frac{(6)(3)^3}{3}-\frac{(4)(1)^3}{3}+\frac{(2)(5)^3}{3}$$

$$= 136.0 \text{ in.}^4$$

From the moment diagram: $M_{\max} = +4050 \text{ lb}\cdot\text{ft}\ ,\quad -2000 \text{ lb}\cdot\text{ft}$

At the section where $M = +4050 \text{ lb}\cdot\text{ft}$

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(4050\times12)(3)}{(136.00)} = -1072 \text{ psi} = 1072 \text{ psi (C)}$$

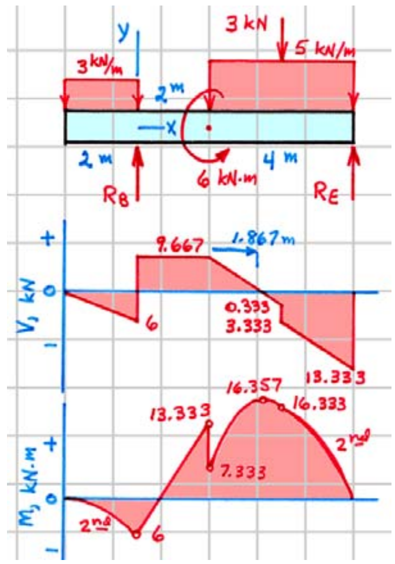
$$\sigma_{bottom} = \frac{-(4050\times12)(-5)}{(136.00)} = +1787 \text{ psi} = 1787 \text{ psi (T)}$$

At the section where $M = -2000 \text{ lb}\cdot\text{ft}$

$$\sigma_{top} = \frac{-(-2000\times12)(3)}{(136.00)} = +529 \text{ psi} = 529 \text{ psi (T)} = \sigma_{\max T}$$ .... **Ans.**

$$\sigma_{bottom} = \frac{-(-2000\times12)(-5)}{(136.00)} = -882 \text{ psi} = 882 \text{ psi (C)} = \sigma_{\max C}$$ .... **Ans.**

7-58*



$\circlearrowleft \Sigma M_E = 0:$

$$(3\times2)(7)+(6)+(5\times4)(2)+(3)(2)-6R_B = 0$$

$$R_B = 15.667 \text{ kN} \uparrow$$

$\circlearrowleft \Sigma M_B = 0:$

$$6R_E + (3\times2)(1)+(6)-(5\times4)(4)-(3)(4) = 0$$

$$R_E = 13.333 \text{ kN} \uparrow$$

From the moment diagram: $M_{\max} = +16.357 \text{ kN}\cdot\text{m}$, $-6 \text{ kN}\cdot\text{m}$

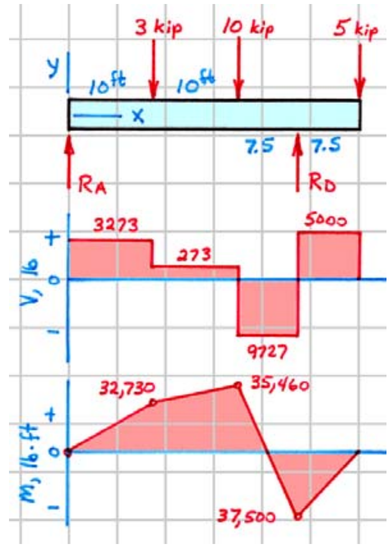
From Table B-2 for a W102×19 section : $S = 89.5(10^3) \text{ mm}^3$

$$\sigma = \frac{M_r}{S} = \frac{(16.357)}{(89.5\times10^{-6})} = +182.8(10^6) \text{ N/m}^2$$

$\sigma = 182.8$ MPa (T, bottom; C, top) .... **Ans.**

Both stresses would be less at the section where $M = -6 \text{ kN}\cdot\text{m}$.

7-59



$\circlearrowleft \Sigma M_D = 0:$

$$(3)(17.5)+(10)(7.5)-(5)(7.5)-27.5R_A = 0$$

$$R_A = 3.2727 \text{ kip} \uparrow$$

$\circlearrowleft \Sigma M_A = 0:$

$$27.5R_D-(3)(10)-(10)(20)-(5)(35) = 0$$

$$R_D = 14.7272 \text{ kip} \uparrow$$

From the moment diagram: $M_{\max} = +35,460 \text{ lb}\cdot\text{ft}$, $-37,500 \text{ lb}\cdot\text{ft}$

From Table B-3 for an $\text{S}18\times 70$ section :

$$d = 2c = 18.00 \text{ in.} \qquad I = 926 \text{ in.}^4$$

$$I = I_C + I_P = 926 + 2\left[\frac{(10)(1)^3}{12}+(10\times 1)(9.5)^2\right] = 2733 \text{ in.}^4$$

At the top of the section $y = 9+1 = 10 \text{ in.}$:

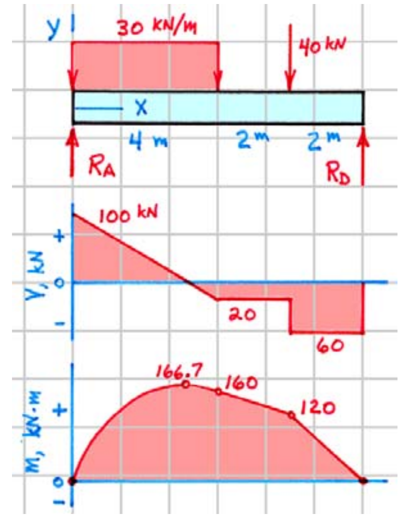
$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(-37.5\times 12)(10)}{(2733)} = +1.647 \text{ ksi} = 1.647 \text{ ksi (T)}$$ .... **Ans.**

At the bottom of the section $y = -10 \text{ in.}$:

$$\sigma_{bottom} = \frac{-(-37.5\times 12)(-10)}{(2733)} = -1.647 \text{ ksi} = 1.647 \text{ ksi (C)}$$ .... **Ans.**

Both stresses would be less at the section where $M = +35,460 \text{ lb}\cdot\text{ft}$.

7-60



$$\circlearrowleft \Sigma M_D = 0: \qquad (30\times 4)(6)+(40)(2)-8R_A = 0$$

$$R_A = 100 \text{ kN}\uparrow$$

$$\circlearrowleft \Sigma M_A = 0: \qquad 8R_D-(30\times 4)(2)-(40)(6) = 0$$

$$R_D = 60 \text{ kN}\uparrow$$

From the moment diagram: $M_{\max} = +166.67 \text{ kN}\cdot\text{m}$

From Table B-6 for a $\text{C}\,254\times 45$ channel: $d = 2c = 254.0$ mm

$$I_{X-X} = 42.9\left(10^6\right) \text{ mm}^4$$

$$I = I_C + I_P$$

$$= 2\left(42.9\times 10^6\right)+2\left[\frac{(250)(25)^3}{12}+(250\times 25)(139.5)^2\right]$$

$$= 329.4\left(10^6\right) \text{ mm}^4$$

At the top of the section $y = c+25 = 127+25 = 152$ mm:

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-\left(166.67\times 10^3\right)(0.152)}{\left(329.4\times 10^{-6}\right)} = -76.9\left(10^6\right) \text{ N/m}^2$$

$\sigma_{top} = 76.9$ MPa (C) .... **Ans.**

At the bottom of the section $y = -152$ mm:

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-\left(166.67\times 10^3\right)(-0.152)}{\left(329.4\times 10^{-6}\right)} = +76.9\left(10^6\right) \text{ N/m}^2$$

$\sigma_{bottom} = 76.9$ MPa (T) .... **Ans.**

7-61

$$\circlearrowleft \Sigma M_B = 0: \qquad (1000\times 12)(6) + (6000)(6) - 12R_A = 0$$

$$R_A = R_B = 9000 \text{ lb} \uparrow$$

From the moment diagram: $M_{\max} = +36 \text{ kip}\cdot\text{ft}$

From Table B-11 for a $WT8\times 25$ section :

$$d = 2c = 8.130 \text{ in.} \qquad y_{top} = y_C = 1.89 \text{ in.}$$

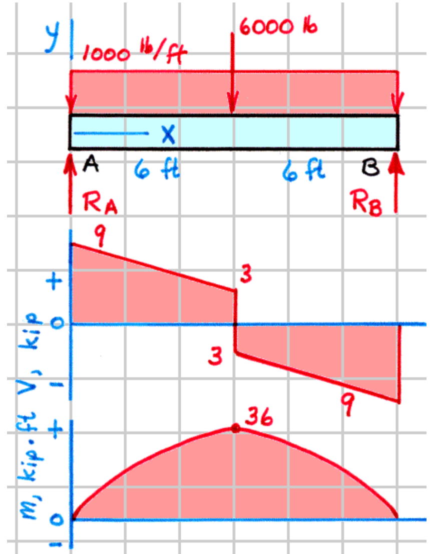
$$I = 42.3 \text{ in.}^4 \qquad y_{bottom} = -(d - y_C) = -6.24 \text{ in.}$$

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(36.0\times 12)(1.89)}{(42.3)} = -19.30 \text{ ksi}$$

$\sigma_{top} = 19.30$ ksi (C) .... **Ans.**

$$\sigma_{bottom} = \frac{-M_r y}{I} = \frac{-(36.0\times 12)(-6.24)}{(42.3)} = +63.7 \text{ ksi}$$

$\sigma_{bottom} = 63.7$ ksi (T) .... **Ans.**



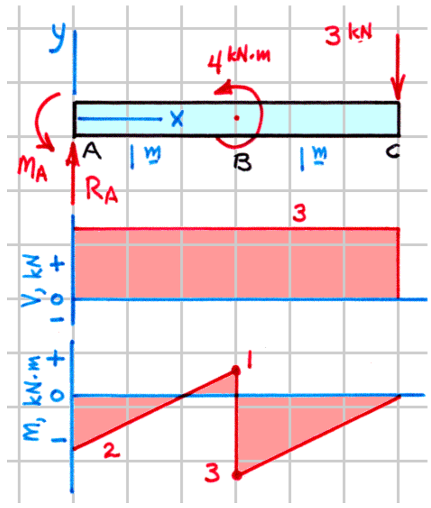
7-62*

$\uparrow \Sigma F_y = 0: \qquad R_A - 3 = 0$

$R_A = 3 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_A = 0: \qquad M_A + (4) - (3)(2) = 0$

$M_A = 2 \text{ kN}\cdot\text{m} \circlearrowleft$



From the moment diagram: $M_{max} = +1 \text{ kN}\cdot\text{m}, \quad -3 \text{ kN}\cdot\text{m}$

From Table B-6 for a $\text{C}\,254 \times 30$ channel:

$w_f = 69.6 \text{ mm} \qquad y_{top} = x_C = 15.4 \text{ mm}$

$I = 1.17(10^6) \text{ mm}^4 \qquad y_{bottom} = -(w_f - x_C) = -54.2 \text{ mm}$

At the section where $M = -3.00 \text{ kN}\cdot\text{m}$:

$$\sigma_{top} = \frac{-(-3000)(0.0154)}{(1.17\times10^{-6})} = +39.5(10^6) \text{ N/m}^2 = 39.5 \text{ MPa (T)}$$

$$\sigma_{bottom} = \frac{-(-3000)(-0.0542)}{(1.17\times10^{-6})} = -139.0(10^6) \text{ N/m}^2 = 139.0 \text{ MPa (C)}$$

At the section where $M = +1.00 \text{ kN}\cdot\text{m}$:

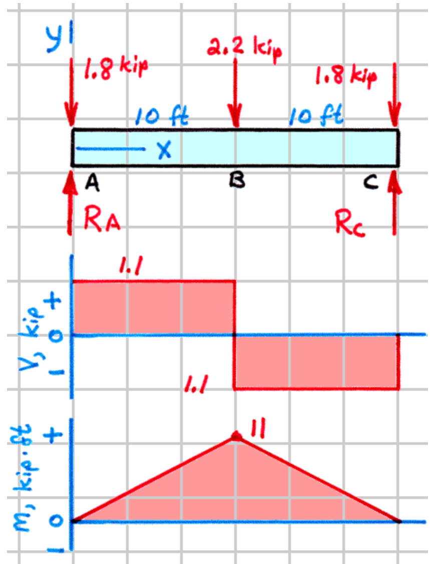
$$\sigma_{top} = \frac{-(1000)(0.0154)}{(1.17\times10^{-6})} = -13.16(10^6) \text{ N/m}^2 = 13.16 \text{ MPa (C)}$$

$$\sigma_{bottom} = \frac{-(1000)(-0.0542)}{(1.17\times10^{-6})} = +46.3(10^6) \text{ N/m}^2 = 46.3 \text{ MPa (T)}$$

Therefore, $\sigma_{\max T} = 46.3 \text{ MPa}$ .... **Ans.**

$\sigma_{\max C} = 139.0 \text{ MPa}$ .... **Ans.**

7-63



$\circlearrowleft \Sigma M_C = 0: \qquad (1.8)(20) + (2.2)(10) - 20 R_A = 0$

$R_A = R_B = 2.90 \text{ kip} \uparrow$

From the moment diagram: $M_{max} = +11.00 \text{ kip} \cdot \text{ft}$

From Table B-15 for an 8×8-in. timber:

$S = 70.3 \text{ in.}^3$

At the top and bottom of the timber:

$$\sigma_{max} = \frac{M_r}{S} = \frac{(11{,}000 \times 12)}{(70.3)} = 1878 \text{ psi}$$

$\sigma_{max} = 1878$ psi (T, bottom; C, top) .... **Ans.**

7-64*

$\circlearrowleft \Sigma M_D = 0:$

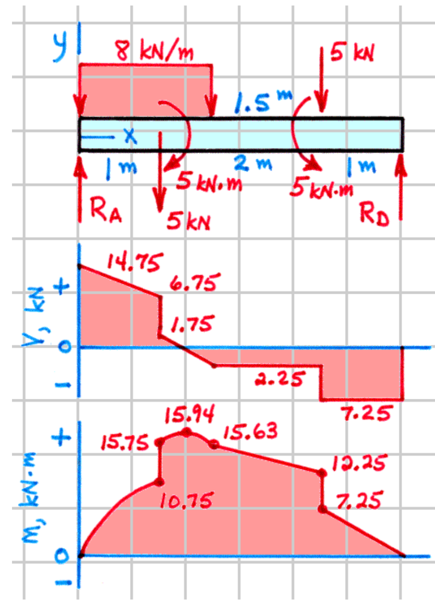
$$(5)(1)+(5)+(5)(3)-(5)+(8\times 1.5)(3.25)-4R_A = 0$$

$$R_A = 14.75 \text{ kN} \uparrow$$

$\circlearrowleft \Sigma M_A = 0:$

$$4R_D-(8\times 1.5)(0.75)-(5)(1)-(5)+(5)-(5)(3) = 0$$

$$R_D = 7.25 \text{ kN} \uparrow$$



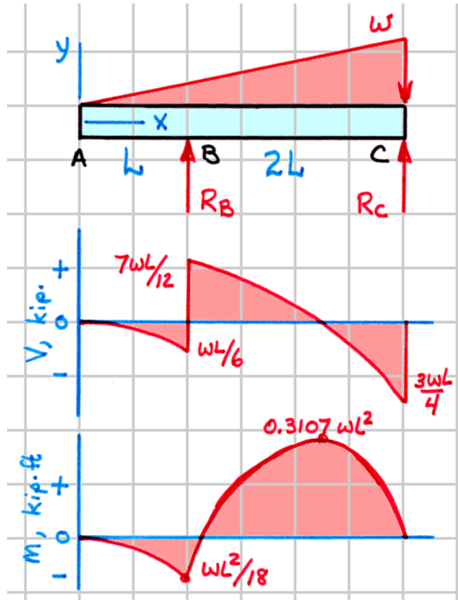
7-65*

$\circlearrowleft \Sigma M_C = 0: \qquad \left[(w)(3L)/2\right](L) - 2LR_B = 0$

$R_B = (3wL/4) \uparrow$

$\circlearrowleft \Sigma M_B = 0: \qquad (2L)R_C - \left[(w)(3L)/2\right](L) = 0$

$R_C = (3wL/4) \uparrow$



The maximum moment occurs where the shear force goes to zero,

$$V_r = \frac{3wL}{4} - \frac{(wx/3L)(x)}{2} = 0$$

$$x = 3L/\sqrt{2} \cong 2.121L$$

$$M_{2.121L} = \left(\frac{3wL}{4}\right)(1.121L) - \frac{(w/3L)(2.121L)^3}{6}$$

$$= 0.3107wL^2$$

From the moment diagram: $\quad M_{max} = 0.3107wL^2$

From Table B-3 for an S15×50 section: $\quad S = 64.8 \text{ in.}^3$

Therefore

$$\sigma = \frac{M}{S} = \frac{0.3107w(5\times 12)^2}{64.8} \le 15 \text{ ksi}$$

$$w \le \frac{(15)(64.8)}{0.3107(5\times 12)^2} = 0.869 \text{ kip/in.} = 10.43 \text{ kip/ft}$$ .... **Ans.**

7-66

For the complete structure:

$$\circlearrowleft \Sigma M_D = 0: \qquad 3R_C - (3)(1.5) - (1.5\times 3)(1.5) = 0$$

$$R_C = 3.75 \text{ kN} = 3.75 \text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \qquad R_D - (3) - (1.5\times 3) + (3.75) = 0$$

$$R_D = 3.75 \text{ kN} = 3.75 \text{ kN} \uparrow$$

For the member *AB*:

$$\uparrow \Sigma F_y = 0: \qquad -(3) - V_B = 0$$

$$V_B = -3 \text{ kN} = 3 \text{ kN} \uparrow$$

$$\circlearrowleft \Sigma M_B = 0: \qquad M_B + (3)(1.5) = 0$$

$$M_B = -4.5 \text{ kN}\cdot\text{m} = 4.5 \text{ kN}\cdot\text{m} \circlearrowright$$

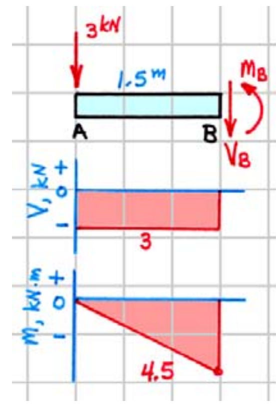
For the member *CD*:

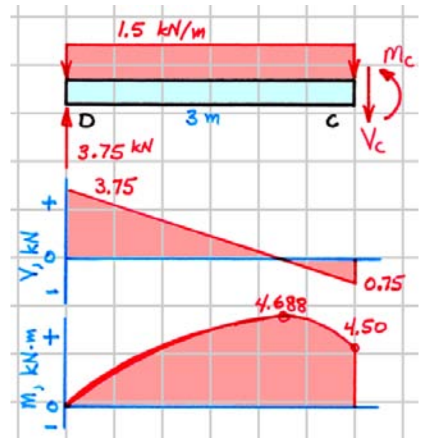
$$\uparrow \Sigma F_y = 0: \qquad (3.75) - (1.5\times 3) - V_C = 0$$

$$V_C = -0.75 \text{ kN} = 0.75 \text{ kN} \uparrow$$

$$\circlearrowleft \Sigma M_C = 0: \qquad M_C + (1.5\times 3)(1.5) - (3.75)(3) = 0$$

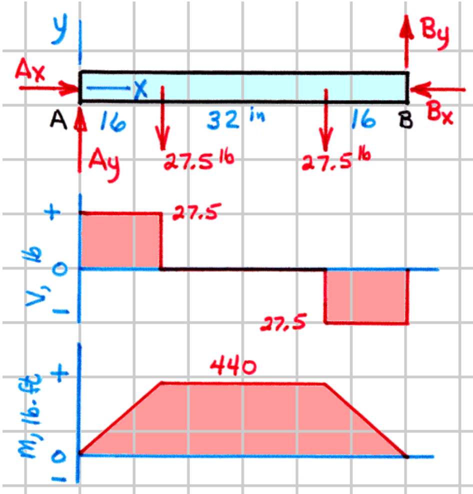
$$M_C = +4.5 \text{ kN}\cdot\text{m} = 4.5 \text{ kN}\cdot\text{m} \circlearrowleft$$





7-67*

(a) For pipe *AB*:



$$\circlearrowleft \Sigma M_A = 0: \qquad 64B_y - (16)(27.5) - (48)(27.5) = 0$$

$$B_y = 27.5 \text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = 0: \qquad A_y - 27.5 - 27.5 + 27.5 = 0$$

$$A_y = 27.5 \text{ lb} \uparrow$$

From the moment diagram: $M_{max} = 440 \text{ lb} \cdot \text{in.}$

From Table B-13 for a $\frac{1}{2}$-in. diameter pipe: $S = 0.041 \text{ in.}^3$

Therefore

$$\sigma = \frac{M}{S} = \frac{440}{0.041} = 10,732 \text{ psi}$$

$\sigma \cong 10.73$ ksi (T, bottom; C top) .... **Ans.**

(b) $B_y = F_{BC} \sin\theta \qquad \theta = \tan^{-1}(35/64) = 28.673°$

$$F_{BC} = B_y / \sin\theta = 27.5/\sin 28.673° = 57.314 \text{ lb}$$

$$A_{BC} = \pi(3/16)^2/4 = 0.02761 \text{ in.}^2$$

$\sigma_{BC} = \dfrac{F_{BC}}{A_{BC}} = \dfrac{57.314}{0.02761} = 2076 \text{ psi} \cong 2.08 \text{ ksi (T)}$ .... **Ans.**

(c) $\rightarrow \Sigma F_x = 0: \qquad A_x - 57.314\cos 28.673° = 0$

$$A_x = +50.29 \text{ lb} = 50.29 \text{ lb} \rightarrow$$

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(50.29)^2 + (27.5)^2} = 57.32 \text{ lb}$$

$$A_A = \pi(1/4)^2/4 = 0.04909 \text{ in.}^2$$

$\tau_A = \dfrac{F_A}{A_A} = \dfrac{57.32}{0.04909} = 584 \text{ psi}$ .... **Ans.**

7-68

$\circlearrowleft \Sigma M_D = 0: \quad 1.5R_E - [(40)(1.5)/2](2.5) = 0 \qquad R_E = 50 \text{ kN} \uparrow$

$\uparrow \Sigma F_y = 0: \quad R_D + 50 - [(40)(1.5)/2] = 0$

$R_D = -20 \text{ kN} = 20 \text{ kN} \downarrow$

$\circlearrowleft \Sigma M_C = 0: \quad 2R_B - [(80)(1.5)](2.75)$

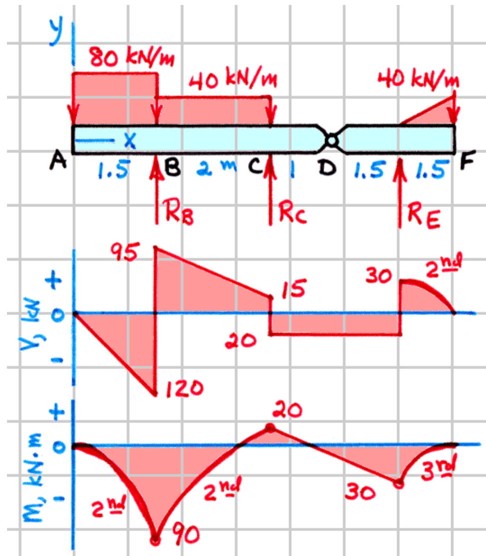
$-[(40)(2)](1) - (20)(1) = 0$

$R_B = 215 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_B = 0: \quad 2R_C + [(80)(1.5)](0.75)$

$-[(40)(2)](1) + (20)(3) = 0$

$R_C = -35 \text{ kN} = 35 \text{ kN} \downarrow$



(a) Shear force and bending moment graphs are shown to the right.

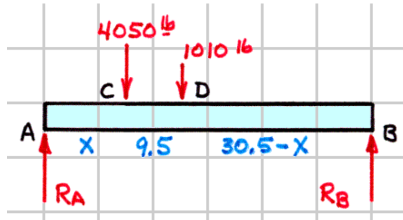
(b) From the moment diagram: $M_{max} = -90 \text{ kN} \cdot \text{m}$

From Table B-4 for an S457×81 section : $S = 1465(10^3) \text{ mm}^3$

$$\sigma_{max} = \frac{M}{S} = \frac{(90,000)}{(1465 \times 10^{-6})} = 61.4(10^6) \text{ N/m}^2$$

$\sigma_{max} = 61.4 \text{ MPa (T, top; C, bottom)}$ .... **Ans.**

7-69



$\circlearrowleft \Sigma M_B = 0:$

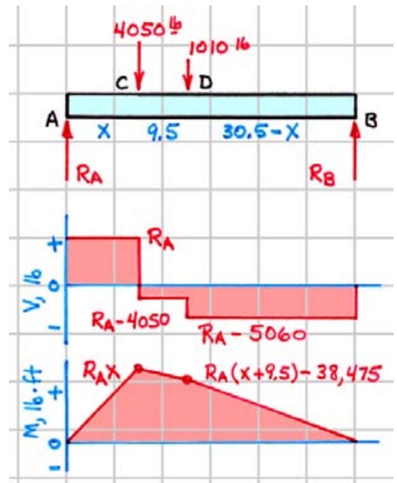
$$(4050)(40-x)+(1010)(30.5-x)-40R_A = 0$$

$$R_A = (4820-126.5x) \text{ lb} \uparrow$$

$\circlearrowleft \Sigma M_A = 0:$

$$40R_B-(4050)(x)-(1010)(x+9.5) = 0$$

$$R_B = (239.9+126.5x) \text{ lb} \uparrow$$



Note that the maximum moment occurs under one of the wheels – probably under the rear wheels (C) but possibly under the front wheels (D). Finding the position x which gives the maximum moment under the rear wheels,

$$M_C = R_A x = (-126.5x^2+4820x) \text{ lb}\cdot\text{ft}$$

$$\frac{dM_C}{dx} = (-253x+4820) = 0 \qquad x = 19.05 \text{ ft}$$

$$M_{C\max} = M_{C19.05} = -126.5(19.05)^2+4820(19.05)$$

$$M_{C\max} = 45{,}914 \text{ lb}\cdot\text{ft} \cong 45.9 \text{ kip}\cdot\text{ft}$$

Finding the position x which gives the maximum moment under the front wheels,

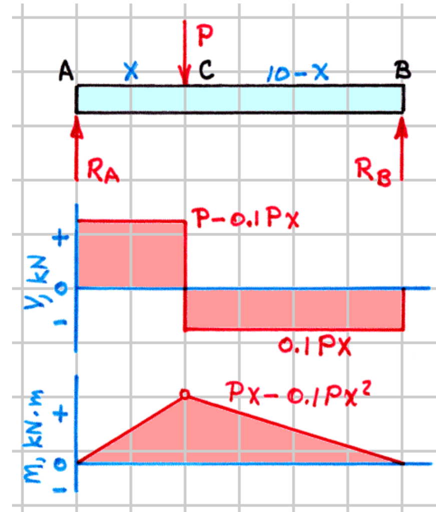
$$M_D = R_B(30.5-x) = (239.9+126.5x)(30.5-x) = (-126.5x^2+3618x+7317) \text{ lb}\cdot\text{ft}$$

$$\frac{dM_D}{dx} = (-253x+3618) = 0 \qquad x = 14.30 \text{ ft}$$

$$M_{D\max} = M_{D14.30} = -126.5(14.30)^2+3618(14.30)+7317 = 33{,}186 \text{ lb}\cdot\text{ft}$$

$M_{\max} = 45{,}914 \text{ lb}\cdot\text{ft} \cong 45.9 \text{ kip}\cdot\text{ft}$ .... **Ans.**

7-70*



(a) $P = mg = 1500(9.81) = 14,715 \text{ N} = 14.715 \text{ kN}$

$\circlearrowleft \Sigma M_B = 0: \qquad P(10-x) - R_A(10) = 0$

$$R_A = (P - 0.1Px) \text{ kN}$$

$\circlearrowleft \Sigma M_A = 0: \qquad R_B(10) - Px = 0$

$$R_B = (0.1Px) \text{ kN}$$

(b) $M = R_A x = (Px - 0.1Px^2) \text{ kN} \cdot \text{m}$

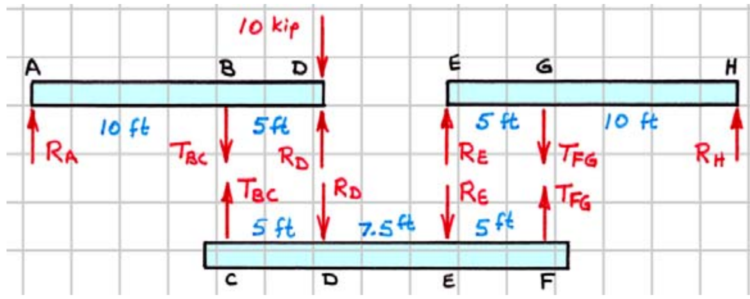
$$\frac{dM}{dx} = P - 0.2Px = 0$$

$x = 5.00$ m .... **Ans.**

$$M_{max} = M_5 = (14.715)(5) - 0.1(14.715)(5)^2 = 36.8 \text{ kN} \cdot \text{m}$$

7-71

For the combined system of beams:



$$\circlearrowleft \Sigma M_H = 0: \quad 22.5(10) - 37.5 R_A = 0$$

$$R_A = 6.00 \text{ kip} = 6.00 \text{ kip} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: \quad 37.5 R_H - 15(10) = 0$$

$$R_H = 4.00 \text{ kip} = 4.00 \text{ kip} \uparrow$$

For the beam *ABD*:

$$\circlearrowleft \Sigma M_D = 0: \quad 5T_{BC} - 15(6) = 0$$

$$T_{BC} = 18.00 \text{ kip} = 18.00 \text{ kip (T)}$$

$$\uparrow \Sigma F_y = 0: \quad 6.00 - 18.00 - 10 + R_D = 0$$

$$R_D = 22.00 \text{ kip} = 22.00 \text{ kip} \uparrow$$

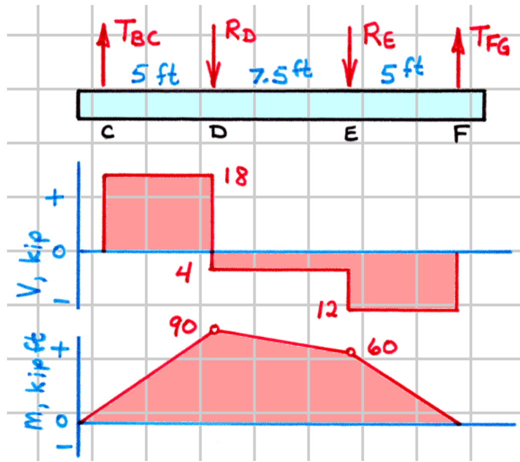
For the beam *EGH*:

$$\circlearrowleft \Sigma M_E = 0: \quad 15(4) - 5T_{FG} = 0$$

$$T_{FG} = 12.00 \text{ kip} = 12.00 \text{ kip (T)}$$

$$\uparrow \Sigma F_y = 0: \quad 4.00 - 12.00 + R_E = 0$$

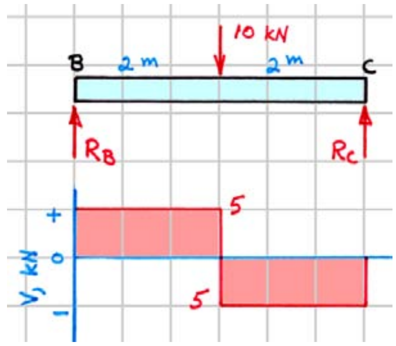
$$R_E = 8.00 \text{ kip} = 8.00 \text{ kip} \uparrow$$



7-72*

$\circlearrowleft \Sigma M_C = 0: \quad (10)(2) - 4R_B = 0 \quad R_B = 5 \text{ kN} = 5 \text{ kN} \uparrow$

$\uparrow \Sigma F_y = 0: \quad 5 - V_A = 0 \quad V_A = 5.00 \text{ kN} = 5.00 \text{ kN} \downarrow$



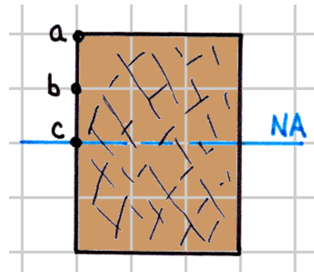
$$I = \frac{(150)(200)^3}{12} = 100.0(10^6) \text{ mm}^4$$

$Q_a = 0 \qquad \tau_a = 0 \text{ MPa}$ .... **Ans.**

$$Q_b = y_C A = 75(150 \times 50) = 562.5(10^3) \text{ mm}^3$$

$$\tau_b = \frac{VQ}{It} = \frac{(5000)(562.5 \times 10^{-6})}{(100.0 \times 10^{-6})(0.150)} = 187.5(10^3) \text{ N/m}^2$$

$\tau_b = 187.5 \text{ kPa}$ .... **Ans.**



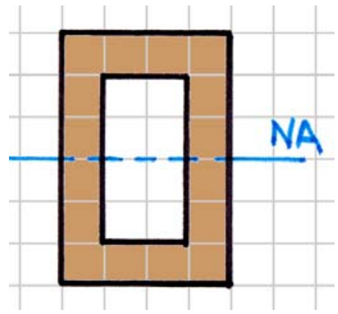
$$Q_c = y_C A = 50(150 \times 100) = 750.0(10^3) \text{ mm}^3$$

$$\tau_b = \frac{VQ}{It} = \frac{(5000)(750.0 \times 10^{-6})}{(100.0 \times 10^{-6})(0.150)} = 250(10^3) \text{ N/m}^2$$

$\tau_b = 250 \text{ kPa}$ .... **Ans.**

7-73*

$$I = \frac{(8)(12)^3}{12} - \frac{(4)(8)^3}{12} = 981.3 \text{ in.}^4$$



(a) $Q_2 = y_C A = 5(8\times 2) = 80.0 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(7000)(80.0)}{(981.3)(4)} = 142.7 \text{ psi}$$ .... **Ans.**

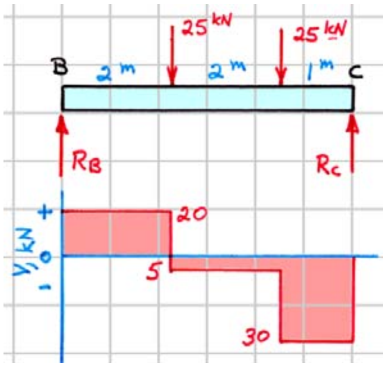
(b) $Q_3 = y_C A = 5(8\times 2) + 3.5(4\times 1) = 94.0 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(7000)(94.0)}{(981.3)(4)} = 167.6 \text{ psi}$$ .... **Ans.**

(c) $Q_{NA} = 5(8\times 2) + 2(4\times 4) = 112.0 \text{ in}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(7000)(112.0)}{(981.3)(4)} = 199.7 \text{ psi (at neutral axis)}$$ .... **Ans.**

7-74



$\circlearrowleft \Sigma M_C = 0: \qquad (25)(3) + (25)(1) - 5R_B = 0$

$R_B = 20.0 \text{ kN} = 20.0 \text{ kN} \uparrow$

$\uparrow \Sigma F_y = 0: \qquad 20 - 25 - V_A = 0$

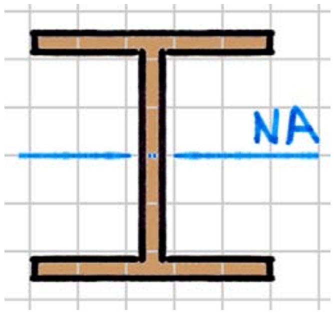
$V_A = -5.00 \text{ kN} = 5.00 \text{ kN} \uparrow$

From Table B-2 for a $\text{W}\,254 \times 89$ section: $I = 142(10^6)\text{ mm}^4$

$d = 2c = 260 \text{ mm} \qquad t_w = 10.7 \text{ mm}$

$w_f = 256 \text{ mm} \qquad t_f = 17.3 \text{ mm}$

(a) $Q_{NA} = y_C A = 121.35(256 \times 17.3) + 56.35(112.7 \times 10.7) = 605.4(10^3)\text{ mm}^3$



$$\tau_{\max} = \frac{VQ}{It} = \frac{(5000)(605.4 \times 10^{-6})}{(142 \times 10^{-6})(0.0107)} = 1.992(10^6)\text{ N/m}^2$$

$\tau_{\max} = 1.992 \text{ MPa}$ ....**Ans.**

(b) $A_w = [260 - 2(17.3)](10.7) = 2412 \text{ mm}^2$

$$\tau_{avg} = \frac{V}{A_w} = \frac{5000}{2412(10^{-6})} = 2.07(10^6)\text{ N/m}^2$$

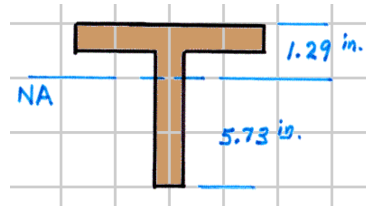
$\tau_{\max} = 2.07 \text{ MPa}$ ....**Ans.**

7-75*

$$R_B = R_C = (1000 \times 8)/2 = 4000 \text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = 0: \qquad 4000 - (1000 \times 2) - V_r = 0$$

$$V_r = 2000 \text{ lb}$$



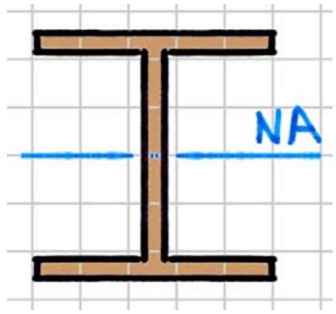
From Table B-11 for a $\text{WT}\, 7 \times 34$ section: $d = 7.020$ in.

$t_s = 0.415$ in. $\quad I = 32.6$ in.^{4} $\quad y_C = 1.29$ in.

$$Q_3 = y_C A = 2.865(5.73 \times 0.415) = 6.813 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(2000)(6.813)}{(32.6)(0.415)} = 1007 \text{ psi (at neutral axis)}$$ .... **Ans.**

7-76



$$I = \frac{(180)(200)^3}{12} - \frac{(140)(120)^3}{12} = 99.84(10^6)\ \text{mm}^4$$

$$Q_J = y_C A = 80(40 \times 180) = 576(10^3)\ \text{mm}^3$$

$$Q_{NA} = y_C A = 80(40 \times 180) + 30(40 \times 60) = 648(10^3)\ \text{mm}^3$$

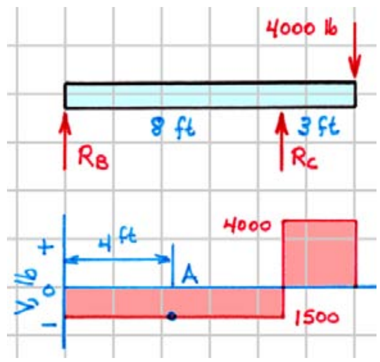
$$V_{\max} = R_A = R_B = (6)(3.5)/2 = 10.5\ \text{kN}$$

(a) $$\tau_J = \frac{VQ}{It} = \frac{(10{,}500)(576 \times 10^{-6})}{(99.84 \times 10^{-6})(0.040)} = 1.514(10^6)\ \text{N/m}^2$$

$\tau_J = 1.514$ MPa .... **Ans.**

(b) $$\tau_{NA} = \frac{(10{,}500)(648 \times 10^{-6})}{(99.84 \times 10^{-6})(0.040)} = 1.721(10^6)\ \text{N/m}^2 = 1.721\ \text{MPa}$$ .... **Ans.**

7-77



$\circlearrowleft \Sigma M_C = 0: \quad -8R_B - 3(4000) = 0$

$R_B = -1500 \text{ lb} = 1500 \text{ lb} \downarrow$

$\uparrow \Sigma F_y = 0: \quad -1500 - V_A = 0$

$V_A = -1500 \text{ lb} = 1500 \text{ lb} \uparrow$

From Table B-1 for a $W10 \times 30$ section : $I = 170 \text{ in.}^4$

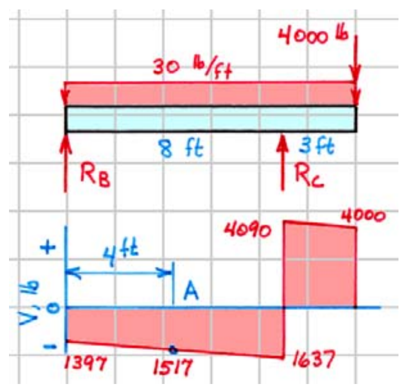
$d = 2c = 10.47 \text{ in.} \qquad t_w = 0.300 \text{ in.}$

$w_f = 5.810 \text{ in.} \qquad t_f = 0.510 \text{ in.}$

(a) $Q_{NA} = y_C A = 4.980(5.810 \times 0.510) + 2.3625(4.725 \times 0.300) = 18.105 \text{ in}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(1500)(18.105)}{(170)(0.300)} = 533 \text{ psi (at neutral axis)}$$ .... **Ans.**

(b) With the weight of the beam included:



$w = 30 \text{ lb/ft}$

$\circlearrowleft \Sigma M_C = 0: \quad 2.5(30 \times 11) - 8R_B - 3(4000) = 0$

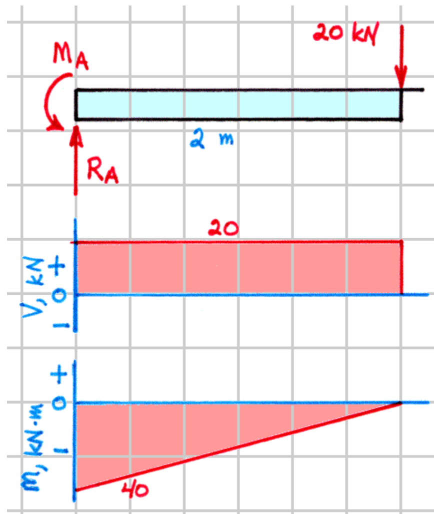
$R_B = -1397 \text{ lb} = 1397 \text{ lb} \downarrow$

$\uparrow \Sigma F_y = 0: \quad -1397 - (30 \times 4) - V_A = 0$

$V_A = -1517 \text{ lb} = 1517 \text{ lb} \uparrow$

$$\tau_{max} = \frac{VQ}{It} = \frac{(1517)(18.105)}{(170)(0.300)} = 539 \text{ psi (at neutral axis)}$$ .... **Ans.**

7-78*



$\circlearrowleft \Sigma M_A = 0: \qquad M_A - 2(20) = 0$

$$M_A = 40.0 \text{ kN} \cdot \text{m} \circlearrowleft$$

$\uparrow \Sigma F_y = 0: \qquad R_A - 20 = 0$

$$R_A = 20 \text{ kN} = 20 \text{ kN} \uparrow$$

From Table B-2 for a $\text{W } 203 \times 60$ section :

$$d = 2c = 210 \text{ mm} \qquad t_w = 9.1 \text{ mm}$$

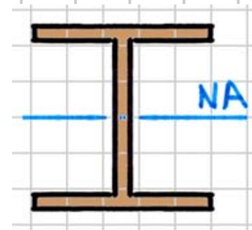
$$w_f = 205 \text{ mm} \qquad t_f = 14.2 \text{ mm}$$

$$I = 60.8\left(10^6\right) \text{ mm}^4 \qquad S = 582\left(10^3\right) \text{ mm}^3$$

From the shear-force and bending-moment diagrams

$$V_{\max} = 20 \text{ kN (full length of the beam)}$$

$$M_{\max} = 40.0 \text{ kN} \cdot \text{m (at the wall)}$$



$$\sigma_{\max} = \frac{M_r}{S} = \frac{(40,000)}{\left(582 \times 10^{-6}\right)} = 68.7\left(10^6\right) \text{ N/m}^2$$

$\sigma_{\max} = 68.7$ MPa (T, top; C bottom) .... **Ans.**

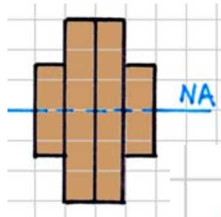
$$Q_{NA} = y_C A = 97.9(205 \times 14.2) + 45.4(90.8 \times 9.1) = 322.5\left(10^3\right) \text{ mm}^3$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(20,000)\left(322.5 \times 10^{-6}\right)}{\left(60.8 \times 10^{-6}\right)(0.0091)} = 11.66\left(10^6\right) \text{ N/m}^2$$

$\tau_{\max} = 11.66$ MPa .... **Ans.**

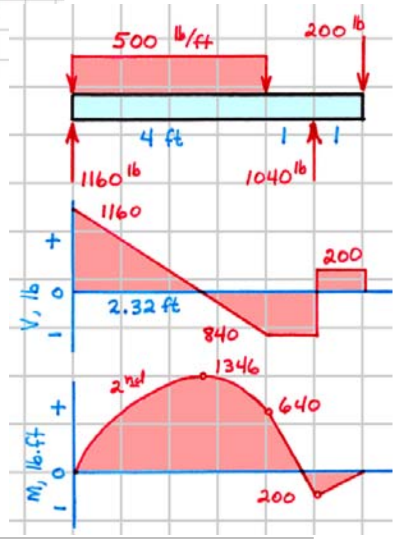
7-79*

From the shear-force and bending-moment diagrams:



$$V_{max} = 1160 \text{ lb} \qquad M_{max} = 1346 \text{ lb} \cdot \text{ft}$$

$$I = \frac{bh^3}{12} = \frac{(2)(6)^3}{12} + \frac{(2)(3)^3}{12} = 40.5 \text{ in.}^4$$



(a) At the bottom of the beam 2.32 ft from the left support:

$$\sigma_{max} = \frac{-M_r y}{I} = \frac{-(1346 \times 12)(-3)}{(40.5)} = +1196 \text{ psi}$$

$\sigma_{max} = 1196$ psi (T) .... **Ans.**

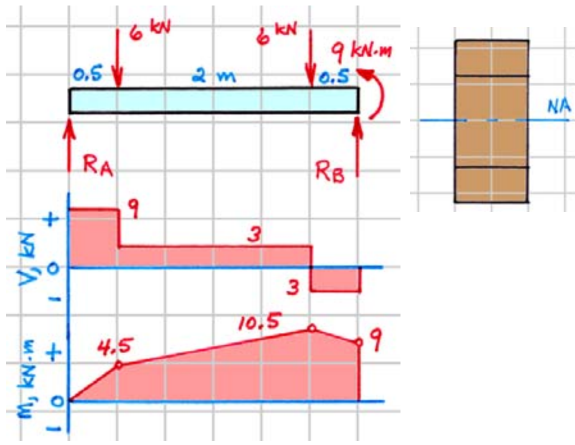
(b) At the left support:

$$\tau_{NA} = \frac{VQ}{It} = \frac{(1160)\left[0.75(1.5 \times 2) + 1.5(3 \times 2)\right]}{(40.5)(4)} = 80.6 \text{ psi}$$

$$\tau_{1.5} = \frac{VQ}{It} = \frac{(1160)\left[2.25(1.5 \times 2)\right]}{(40.5)(4)} = 96.7 \text{ psi}$$

$\tau_{max} = \tau_{1.5} = 96.7$ psi (1.5 in. above and below NA) .... **Ans.**

7-80



$\circlearrowleft \Sigma M_B = 0: \qquad 9 + 2.5(6) + 0.5(6) - 3R_A = 0$

$R_A = 9.0 \text{ kN} \uparrow$

$\uparrow \Sigma F_y = 0: \qquad 9 - 6 - 6 + R_B = 0$

$R_B = 3.0 \text{ kN} \uparrow$

From the shear-force and bending-moment diagrams

$V_{max} = 9.0 \text{ kN} \qquad M_{max} = 10.5 \text{ kN} \cdot \text{m}$

$$I = \frac{(100)(240)^3}{12} = 115.2(10^6) \text{ mm}^4$$

(a) $Q_J = y_C A = 95(100 \times 50) = 475(10^3) \text{ mm}^3$

$$\tau_J = \frac{VQ}{It} = \frac{(9000)(475 \times 10^{-6})}{(115.2 \times 10^{-6})(0.100)} = 371(10^3) \text{ N/m}^2 = 371 \text{ kPa}$$ .... **Ans.**

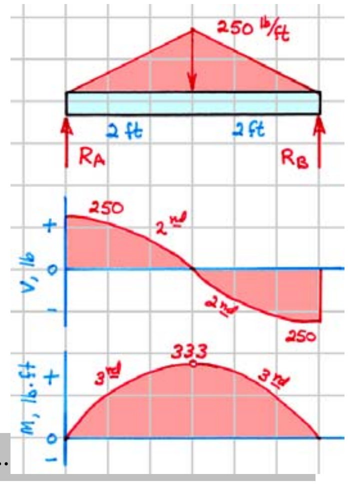
(b) $Q_{NA} = y_C A = 60(100 \times 120) = 720(10^3) \text{ mm}^3$

$$\tau_{NA} = \frac{VQ}{It} = \frac{(9000)(720 \times 10^{-6})}{(115.2 \times 10^{-6})(0.100)} = 563(10^3) \text{ N/m}^2 = 563 \text{ kPa}$$ .... **Ans.**

(c) $$\sigma_{max} = \frac{M_{max} c}{I} = \frac{(10,500)(0.120)}{(115.2 \times 10^{-6})} = 10.94(10^6) \text{ N/m}^2$$

$\sigma_{max} = 10.94$ MPa (T, bottom; C, top) .... **Ans.**

7-81



$\circlearrowleft \Sigma M_B = 0: \quad \left[(250)(4)/2\right](2) - 4R_A = 0$

$R_A = R_B = 250 \text{ lb} \uparrow$

From the shear-force and bending-moment diagrams

$V_{max} = 250 \text{ lb} \qquad M_{max} = 333.3 \text{ lb} \cdot \text{ft}$

$$I = \frac{bh^3}{12} = \frac{(8)(0.5)^3}{12} = 0.08333 \text{ in.}^4$$

$$\sigma_{max} = \frac{M_r c}{I} = \frac{(333.3 \times 12)(0.25)}{(0.08333)} = 11{,}999 \text{ psi}$$

$\sigma_{max} \cong 12.00 \text{ ksi (T, bottom; C, top)}$ ....

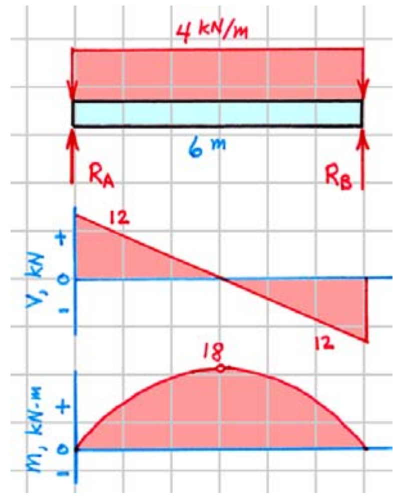
$Q_{NA} = y_C A = (0.125)(8 \times 0.25) = 0.25 \text{ in.}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(250)(0.25)}{(0.08333)(8)} = 93.8 \text{ psi (at neutral axis)}$$ .... **Ans.**

The results for τ_{max} are worthless since $w/d = 16$ and Eq. 7-12 gives useful results only when $w/d < 1$.

7-82*

$$R_A = R_B = (4\times 6)/2 = 12.0 \text{ kN}\uparrow$$



From the shear-force and bending-moment diagrams

$$V_{\max} = 12.0 \text{ kN} \qquad V_{0.5} = 12.0 - 4(0.5) = 10.0 \text{ kN}$$

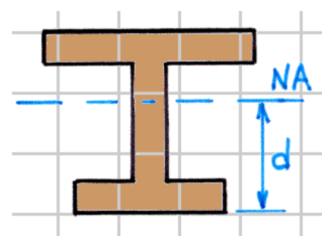
$$M_{\max} = 18.0 \text{ kN}\cdot\text{m} \qquad V_{1.0} = 12.0 - 4(1) = 8.0 \text{ kN}$$

$$d = \frac{(225)\left[(250)(50)\right]+(125)\left[(50)(150)\right]+(25)\left[(150)(25)\right]}{\left[(250)(50)\right]+\left[(50)(150)\right]+\left[(150)(25)\right]}$$

$$= 143.18 \text{ mm}$$

$$I_{NA} = \frac{(150)(143.18)^3}{3} - \frac{(100)(93.18)^3}{3}$$

$$+\frac{(250)(106.82)^3}{3} - \frac{(200)(56.82)^3}{3} = 209.1(10^6) \text{ mm}^4$$



(a) $Q_{TJ} = y_C A = 81.82(250\times 50) = 1022.8(10^3) \text{ mm}^3$

$$\tau_{TJ} = \frac{VQ}{It} = \frac{(8000)(1022.8\times 10^{-6})}{(209.1\times 10^{-6})(0.050)}$$

$\tau_{TJ} = 783(10^3) \text{ N/m}^2 = 783 \text{ kPa}$ ....**Ans.**

(b) $Q_{BJ} = 118.18(150\times 50) = 886.4(10^3) \text{ mm}^3$

$\tau_{BJ} = \dfrac{(10{,}000)(886.4\times 10^{-6})}{(209.1\times 10^{-6})(0.050)} = 848(10^3) \text{ N/m}^2 = 848 \text{ kPa}$ .... **Ans.**

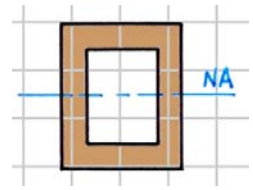
(c) $Q_{NA} = 81.82(250\times 50) + 28.41(56.82\times 50) = 1103.5(10^3) \text{ mm}^3$

$\tau_{\max} = \dfrac{(12{,}000)(1103.5\times 10^{-6})}{(209.1\times 10^{-6})(0.050)} = 1267(10^3) \text{ N/m}^2 = 1267 \text{ kPa}$ .... **Ans.**

(d) $\sigma_{\max} = \dfrac{-M_{\max} c}{I} = \dfrac{-(18{,}000)(-0.14318)}{(209.1\times 10^{-6})} = +12.33(10^6) \text{ N/m}^2$

$\sigma_{\max} = 12.33 \text{ MPa (T)}$ .... **Ans.**

7-83



$\circlearrowleft \Sigma M_B = 0: \qquad (200\times6)(6)-(600)(3)-9R_A = 0$

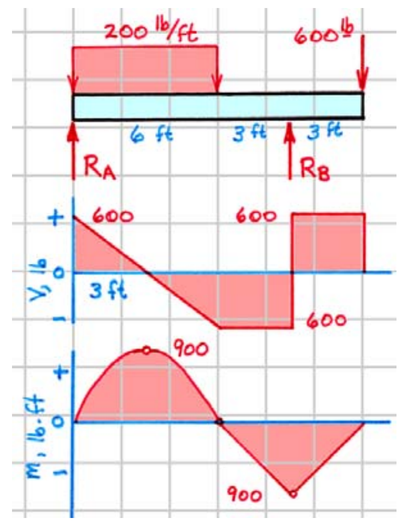
$R_A = 600 \text{ lb} = 600 \text{ lb} \uparrow$

$\uparrow \Sigma F_y = 0: \qquad (600)-(200\times6)+R_B-600 = 0$

$R_B = 1200 = 1200 \text{ lb} \uparrow$

From the shear-force and bending-moment diagrams:

$V_{\max} = \pm600 \text{ lb} \qquad M_{\max} = \pm900 \text{ lb}\cdot\text{ft}$



$$I = \frac{(5)(6)^3}{12} - \frac{(3)(4)^3}{12} = 74.0 \text{ in.}^4$$

(a) $Q_J = y_C A = 2.5(5\times1) = 12.5 \text{ in}^3$

$\tau_J = \dfrac{VQ}{It} = \dfrac{(600)(12.5)}{(74.0)(2)} = 50.7 \text{ psi}$ .... **Ans.**

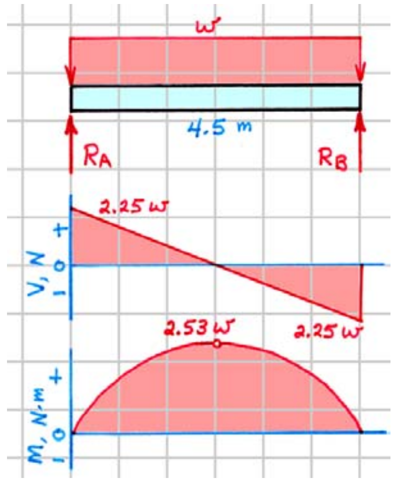
(b) $Q_{NA} = 2.5(5\times1) + 2\left[1(2\times1)\right] = 16.5 \text{ in}^3$

$\tau_{\max} = \dfrac{(600)(16.5)}{(74.0)(2)} = 66.9 \text{ psi (at NA)}$ .... **Ans.**

(c) $\sigma_{\max} = \dfrac{M_r c}{I} = \dfrac{(900\times12)(3)}{(74.0)} = 438 \text{ psi}$

$\sigma_{\max} = 438 \text{ psi (T \& C)}$ .... **Ans.**

7-84



$$R_A = R_B = (4.5w)/2 = (2.25w)\ \text{N}\uparrow$$

From the shear-force and bending-moment diagrams

$$V_{\max} = (2.25w)\ \text{N} \qquad M_{\max} = (2.53w)\ \text{N}\cdot\text{m}$$

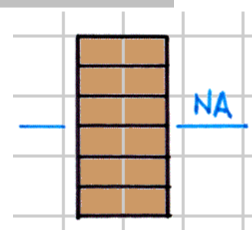
$$I = \frac{(150)(300)^3}{12} = 337.5(10^6)\ \text{mm}^4$$

$$Q_{NA} = y_C A = 75(150\times150) = 1687.5(10^3)\ \text{mm}^3$$

(a) $$\tau_{\max} = \frac{(2.25w)(1687.5\times10^{-6})}{(337.5\times10^{-6})(0.150)} = 75.00w$$

$$\tau_{\max} = 75.00w = 825(10^3)\ \text{N/m}^2$$

$w = 11{,}000\ \text{N/m} = 11.00\ \text{kN/m}$ .... **Ans.**



(b) $$V_2 = R_A - wx = 2.25(11{,}000) - 11{,}000(0.6) = 18{,}150\ \text{N}$$

$$Q_2 = 125(150\times50) = 937.5(10^3)\ \text{mm}^3$$

$$\tau_H = \frac{(18{,}150)(937.5\times10^{-6})}{(337.5\times10^{-6})(0.150)} = 336(10^3)\ \text{N/m}^2$$

$\tau_H = 336\ \text{kPa}$ .... **Ans.**

(c) $$M_{\max} = 2.53(11{,}000) = 27{,}830\ \text{N}\cdot\text{m}$$

$$\sigma_{\max} = \frac{-M_r y}{I} = \frac{-(27{,}830)(-0.150)}{(337.5\times10^{-6})} = 12.37(10^6)\ \text{N/m}^2$$

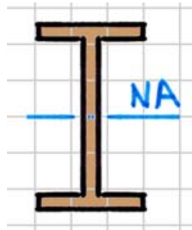
$\sigma_{\max} = 12.37\ \text{MPa}$ .... **Ans.**

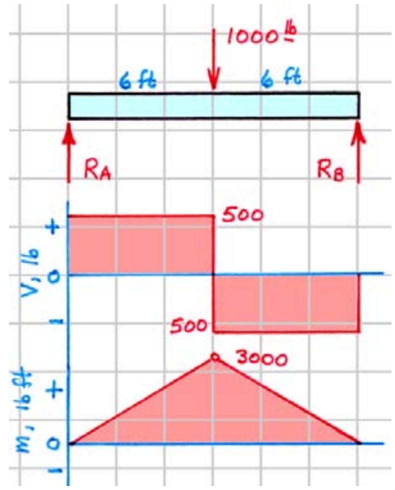
7-85*

$$R_A = R_B = 1000/2 = 500 \text{ lb} = 500 \text{ lb} \uparrow$$

$$V = \text{constant} = 500 \text{ lb}$$

$$I = \frac{(6)(12)^3}{12} - \frac{(4)(8)^3}{12} = 693.3 \text{ in.}^4$$





(a) $$Q_J = y_C A = 5(6 \times 2) = 60.0 \text{ in}^3$$

$$\tau_J = \frac{VQ}{It} = \frac{(500)(60.0)}{(693.3)(2)} = 21.636 \text{ psi}$$

$$F_J = \tau_J A_J = (21.636)(12 \times 2) = 519.3 \text{ lb}$$

$F_J \cong 519$ lb .... **Ans.**

(b) $N = F_J / F_N = 519.3/100 = 5.19$ nails $s = 12/5.19 = 2.13$ in.

Use 6 nails spaced 2 in. apart in a 12-in. length. .... **Ans.**

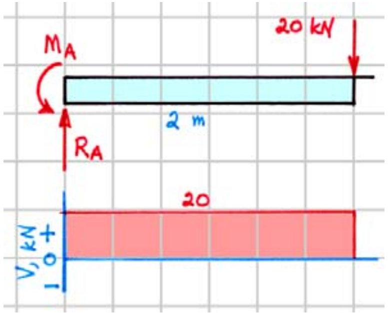
7-86*

From Table B-6 for a $C\,457 \times 86$ channel :

$$I = 7.41\left(10^6\right)\ \text{mm}^4 \qquad A = 11{,}030\ \text{mm}^2$$

$$x_C = 21.9\ \text{mm} \qquad t_w = 17.8\ \text{mm}$$

For the beam:



$$I = 2\left[7.41\left(10^6\right) + 11{,}030\left(21.9\right)^2\right] = 25.40\left(10^6\right)\ \text{mm}^4$$

$$Q_{NA} = y_C A = 21.9\left(11{,}030\right) = 241.6\left(10^3\right)\ \text{mm}^3$$

$$V = \text{constant} = 20\ \text{kN}$$

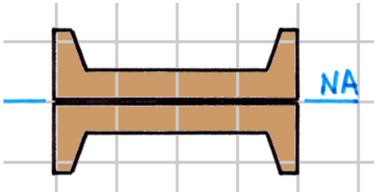
(a) $$\tau_{NA} = \frac{VQ}{It} = \frac{\left(20{,}000\right)\left(241.6 \times 10^{-6}\right)}{\left(25.40 \times 10^{-6}\right)\left(0.4572\right)}$$

$$\tau_{NA} = 416\left(10^3\right)\ \text{N/m}^2 = 416\ \text{kPa}$$

$$F_B = \tau_{NA} A_S/2 = \left(416.1 \times 10^3\right)\left(0.4572 \times 0.300\right)/2 = 28.5\left(10^3\right)\ \text{N}$$

$F_B = 28.5\ \text{kN}$ .... **Ans.**

(b) $$\tau = \frac{F_B}{A_B} = \frac{28{,}540}{\pi d^2/4} = 60\left(10^6\right)\ \text{N/m}^2$$



$$d = 0.02461\ \text{m} = 24.61\ \text{mm}$$

$$\sigma_b = \frac{F_B}{dt_w} = \frac{28{,}540}{d\left(0.0178\right)} = 125\left(10^6\right)\ \text{N/m}^2$$

$$d = 0.01283\ \text{m} = 12.83\ \text{mm}$$

Therefore

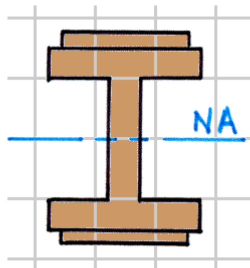
$d_{\min} = 24.6\ \text{mm}$ .... **Ans.**

7-87

From Table B-1 for a $W18\times97$ section : $I = 1750 \text{ in.}^4$

$$d = 2c = 18.59 \text{ in.} \qquad w_f = 11.145 \text{ in.}$$

For the beam:



$$V = \frac{\Delta M}{\Delta x} = \frac{4600 - 2300}{20} = 115 \text{ kip}$$

$$I = 1750 + 2\left[\frac{(10)(0.5)^3}{12} + (10\times0.50)(9.545)^2\right] = 2661 \text{ in.}^4$$

$$Q_J = y_C A = 9.545(10\times0.50) = 47.73 \text{ in}^3$$

$$\tau_J = \frac{VQ}{It} = \frac{(115)(47.73)}{(2661)(10)} = 0.2063 \text{ ksi} = 206.3 \text{ psi}$$

$$F_J = \tau_J A_J = (206.3)(20\times10) = 41{,}260 \text{ lb}$$

$$N = \frac{F_J}{F_W} = \frac{41{,}260}{2(2400)} = 8.596$$

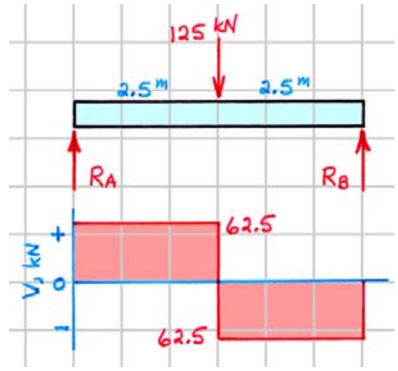
Use 5 welds on each side. .... **Ans.**

7-88

$$R_A = R_B = 125/2 = 62.5 \text{ kN}\uparrow \qquad V = \text{constant} = 62.5 \text{ kN}$$

From Table B-6 for a $C305\times 45$ channel: $I = 67.4\left(10^6\right) \text{ mm}^4$

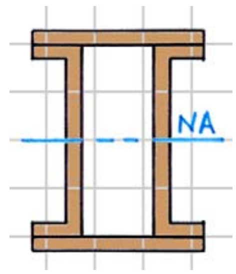
$$d = 2c = 304.8 \text{ mm} \qquad w_f = 80.5 \text{ mm}$$



For the beam:

$$I = 2\left[\frac{(260)(15)^3}{12} + (260\times 15)(159.9)^2\right] + 2\left(67.4\times 10^6\right) = 334.4\left(10^6\right) \text{ mm}^4$$

$$Q_J = y_C A = 159.9(260\times 15) = 623.6\left(10^3\right) \text{ mm}^3$$



$$\tau_J = \frac{(62,500)\left(623.6\times 10^{-6}\right)}{\left(334.4\times 10^{-6}\right)(2\times 0.0805)} = 723.9\left(10^3\right) \text{ N/m}^2$$

$$F_J = \tau_J A_J = \left(723.9\times 10^3\right)(2\times 0.0805s) = 116.55s\left(10^3\right) \text{ N}$$

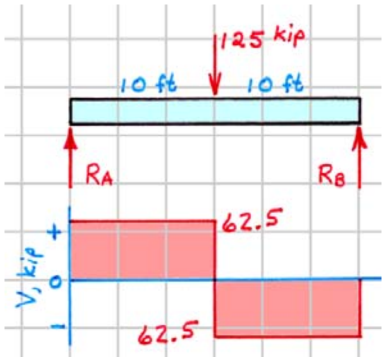
$$F_B = \tau_B A_B = 2\left(150\times 10^6\right)\left[\pi(0.020)^2/4\right] = 94.248\left(10^3\right) \text{ N}$$

Since $F_B = F_J$

$$s = \frac{94.248}{116.55} = 0.809 \text{ m} = 809 \text{ mm}$$ .... **Ans.**

7-89*

$$R_A = R_B = 125/2 = 62.5 \text{ kip} = 62.5 \text{ kip} \uparrow \qquad V = \text{constant} = 62.5 \text{ kip}$$



From Table B-1 for a W 21×101 section: $I = 2420 \text{ in.}^4$

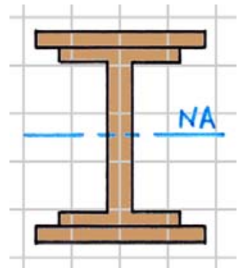
$$d = 2c = 21.36 \text{ in.} \qquad w_f = 12.290 \text{ in.}$$

For the beam:

$$I = 2420 + 2\left[\frac{(16)(0.75)^3}{12} + (16 \times 0.75)(11.055)^2\right] = 5354 \text{ in.}^4$$

$$Q_J = y_C A = 11.055(16 \times 0.75) = 132.66 \text{ in}^3$$

$$\tau_J = \frac{VQ}{It} = \frac{(62{,}500)(132.66)}{(5354)(12.290)} = 126.01 \text{ psi}$$



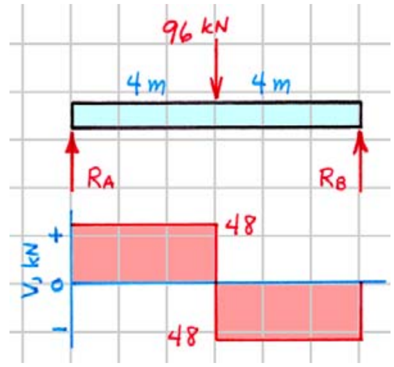
$$F_J = \tau_J A_J = (126.01)(12.290s) = (1548.6s) \text{ lb}$$

$$F_B = \tau_B A_B = 2(17{,}500)\left[\pi (0.75)^2 / 4\right] = 15{,}463 \text{ lb}$$

Since $F_B = F_J$ $\quad s = \dfrac{15{,}463}{1548.6} = 9.985 \text{ in.} \cong 10.00 \text{ in.}$ ....**Ans.**

7-90

$R_A = R_B = 96/2 = 48 \text{ kN} \uparrow$ $V = \text{constant} = 48 \text{ kN}$



From Table B-6 for a C381×74 channel: $I = 4.58(10^6) \text{ mm}^4$

$A = 9485 \text{ mm}^2$ $x_C = 20.3 \text{ mm}$ $t_w = 18.2 \text{ mm}$

From Table B-2 for a W356×122 section: $I = 367(10^6) \text{ mm}^4$

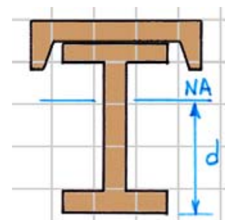
$A = 15,550 \text{ mm}^2$ $d = 2c = 363 \text{ mm}$ $w_f = 257 \text{ mm}$

For the beam:

$$d = \frac{M_x}{A} = \frac{(360.9)(9485) + (181.5)(15,550)}{(9485) + (15,550)} = 249.5 \text{ mm}$$

$$I = \left[367(10^6) + 15,550(68)^2\right] + \left[4.58(10^6) + 9485(111.4)^2\right] = 561.2(10^6) \text{ mm}^4$$

$$Q_J = 111.4(9485) = 1056.6(10^3) \text{ mm}^3$$



(a) $$\tau_J = \frac{VQ}{It} = \frac{(48,000)(1056.6\times10^{-6})}{(561.2\times10^{-6})(0.257)} = 351.6(10^3) \text{ N/m}^2$$

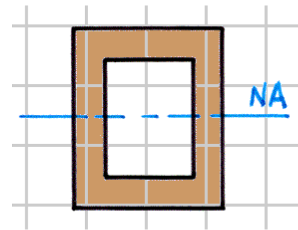
$$F_B = \tau_J A_S/2 = (351.64\times10^3)(0.257\times0.500)/2 = 22,593 \text{ N}$$

$F_B = 22.6 \text{ kN}$ .... **Ans.**

(b) $$\tau = \frac{F_B}{A_B} = \frac{22,593}{\pi d^2/4} = 60(10^6) \text{ N/m}^2$$ $d = 0.02190 \text{ m}$

$d_{\min} = 21.9 \text{ mm}$ .... **Ans.**

7-91



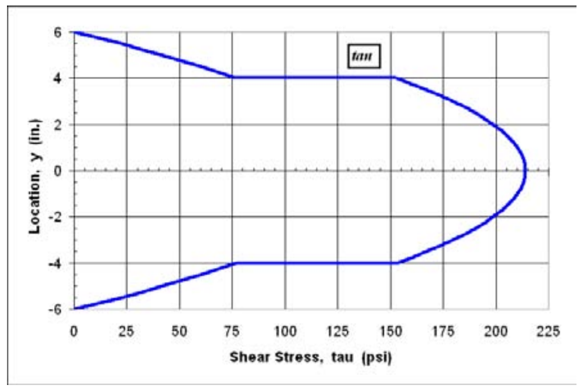
$$I = \frac{(8)(12)^3}{12} - \frac{(4)(8)^3}{12} = 981.33 \text{ in.}^4$$

$-4 \text{ in.} \le y \le +4 \text{ in.}:$ $\qquad t = 4 \text{ in.}$

$$Q = y_C A = 5(8\times 2) + 2\left[\frac{(4+y)}{2}(2)(4-y)\right] = (112 - 2y^2) \text{ in.}^3$$

$$\tau = \frac{VQ}{It} = \frac{(7500)(116-2y^2)}{(981.33)(4)}$$

$$= 3.8213(56 - y^2) \text{ psi}$$



$-6 \text{ in.} \le y \le -4 \text{ in.}$ and $4 \text{ in.} \le y \le 6 \text{ in.}:$

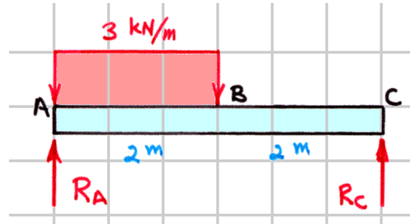
$t = 8 \text{ in.}$

$$Q = y_C A = \left[\frac{(6+y)}{2}(8)(6-y)\right]$$

$$= (144 - 4y^2) \text{ in.}^3$$

$$\tau = \frac{VQ}{It} = \frac{(7500)(144-4y^2)}{(981.33)(8)} = 3.8213(36 - y^2) \text{ psi}$$

7-92



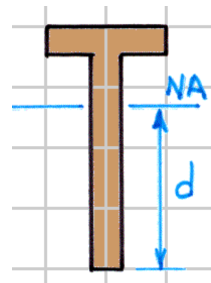
$$\circlearrowleft \Sigma M_B = 0: \qquad (3\times 2)(3) - 4R_A = 0$$

$$R_A = 4.5 \text{ kN}$$

$$\uparrow \Sigma F_y = 0: \qquad 4.5 - (3\times .5) - V_r = 0$$

$$V_r = 3.0 \text{ kN}$$

$$d = \frac{M_x}{A} = \frac{(100)\left[(200)(25)\right] + (212.5)\left[(100)(25)\right]}{(200)(25)+(100)(25)} = 137.5 \text{ mm}$$



$$I = \frac{(25)(137.5)^3}{3} + \frac{(100)(87.5)^3}{3} - \frac{(75)(62.5)^3}{3}$$

$$= 37.89\left(10^6\right) \text{ mm}^4$$

$$-137.5 \text{ mm} \le y \le +62.5 \text{ mm}: \qquad t = 25 \text{ mm}$$

$$Q = y_C A = \frac{137.5 + y}{2}(25)(137.5 - y) = 12.5\left(137.5^2 - y^2\right) \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{(3000)\left[(12.5)\left(137.5^2 - y^2\right)\left(10^{-9}\right)\right]}{\left(37.89\times 10^{-6}\right)(0.025)}$$

$$= 39.59\left(137.5^2 - y^2\right) \text{ N/m}^2$$

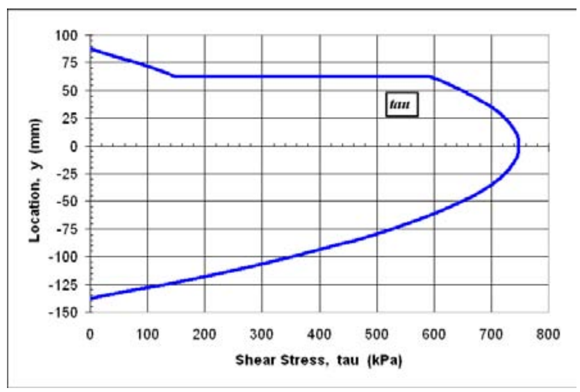
$$+62.5 \text{ mm} \le y \le +87.5 \text{ mm}: \qquad t = 100 \text{ mm}$$

$$Q = y_C A = \frac{87.5 + y}{2}(100)(87.5 - y)$$

$$= 50\left(87.5^2 - y^2\right) \text{ mm}^3$$

$$\tau = \frac{(3000)\left[(50)\left(87.5^2 - y^2\right)\left(10^{-9}\right)\right]}{\left(37.89\times 10^{-6}\right)(0.100)}$$

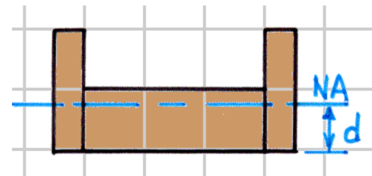
$$= 39.59\left(87.5^2 - y^2\right) \text{ N/m}^2$$



7-93

$$d = \frac{M_x}{A} = \frac{(1)[(6)(2)]+(2)[(4)(1)]+(2)[(4)(1)]}{(6)(2)+(4)(1)+(4)(1)} = 1.400 \text{ in.}$$

$$I = \frac{(8)(1.4)^3}{3} + \frac{(6)(0.6)^3}{3} + \frac{(1)(2.6)^3}{3} + \frac{(1)(2.6)^3}{3} = 19.467 \text{ in.}^4$$

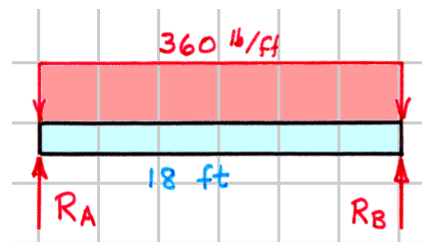


By symmetry, each support carries half the total load

$$R_A = R_B = (360\times 18)/2 = 3240 \text{ lb}\uparrow$$

$$\uparrow \Sigma F_y = 0: \qquad 3240 - (360\times 2) - V_r = 0$$

$$V_r = 2520 \text{ lb}$$



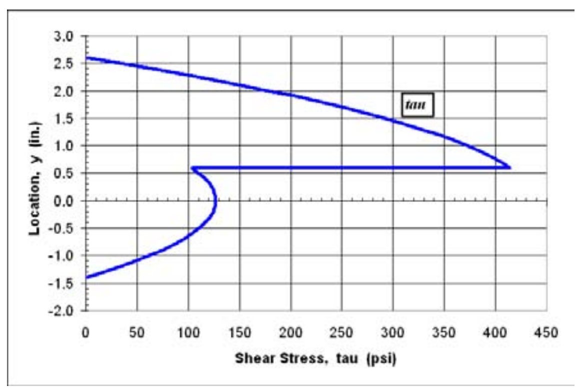
$-1.4 \text{ in.} \le y \le +0.6 \text{ in.}: \qquad t = 8 \text{ in.}$

$$Q = y_C A = \left[\frac{(1.4+y)}{2}(8)(1.4-y)\right]$$

$$= 4(1.4^2 - y^2) \text{ in.}^3$$

$$\tau = \frac{VQ}{It} = \frac{(2520)[(4)(1.4^2-y^2)]}{(19.467)(8)}$$

$$= 64.725(1.4^2 - y^2) \text{ psi}$$



$+0.6 \text{ in.} \le y \le +2.6 \text{ in.} \qquad t = 2 \text{ in.}$

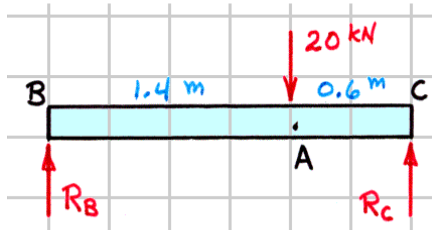
$$Q = y_C A = 2\left[\frac{(2.6+y)}{2}(1)(2.6-y)\right]$$

$$= (2.6^2 - y^2) \text{ in.}^3$$

$$\tau = \frac{VQ}{It} = \frac{(2520)(2.6^2-y^2)}{(19.467)(2)}$$

$$= 64.725(2.6^2 - y^2) \text{ psi}$$

7-94*

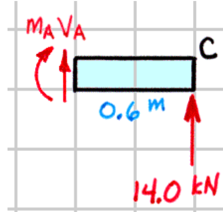


$\circlearrowleft \Sigma M_B = 0: \qquad 2R_C - (20)(1.4) = 0$

$R_C = 14.0$ kN

$\uparrow \Sigma F_y = 0: \qquad V_A + (14) = 0$

$V_A = -14 \text{ kN} = 14 \text{ kN} \downarrow$



$\circlearrowleft \Sigma M_{cut} = 0: \qquad -M_A + (14)(0.6) = 0$

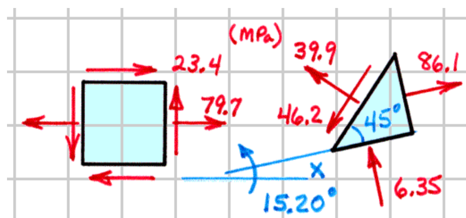
$M_A = +8.40 \text{ kN} \cdot \text{m}$

$$I = \frac{(80)(120)^3}{12} - \frac{(75)(100)^3}{12} = 5.270(10^6) \text{ mm}^4$$

$$\sigma_A = \frac{-M_r y}{I} = \frac{-(8400)(-0.050)}{(5.270 \times 10^{-6})} = +79.70(10^6) \text{ N/m}^2 = 79.70 \text{ MPa (T)}$$

$$Q_A = y_C A = 55(80 \times 10) = 44.0(10^3) \text{ mm}^3$$

$$\tau_A = \frac{VQ}{It} = \frac{(14,000)(44.0 \times 10^{-6})}{(5.270 \times 10^{-6})(0.005)} = 23.38(10^6) \text{ N/m}^2 = 23.38 \text{ MPa}$$



$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{79.70 + 0}{2} \pm \sqrt{\left(\frac{79.70 - 0}{2}\right)^2 + (23.38)^2}$$

$$= 39.85 \pm 46.20 \text{ MPa}$$

$\sigma_{p1} = 39.85 + 46.20 = 86.05 \text{ MPa} \cong 86.1 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = 39.85 - 46.20 = -6.350 \text{ MPa} \cong 6.35 \text{ MPa (C)}$ .... **Ans.**

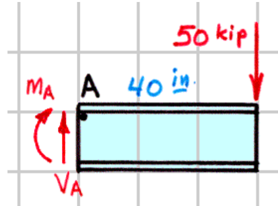
$\sigma_{p3} = 0$ MPa .... **Ans.**

Since σ_{p1} and σ_{p2} have opposite signs:

$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 46.2$ MPa .... **Ans.**

$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(23.38)}{(79.70)-(0)} = +15.20°, \quad -74.80°$ .... **Ans.**

7-95*

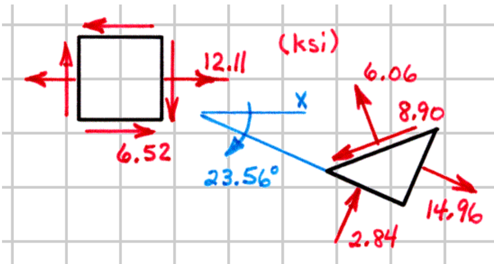


$\uparrow \Sigma F_y = 0: \qquad V_A - 50 = 0 \qquad V_A = +50 \text{ kip} = +50 \text{ kip} \uparrow$

$\circlearrowleft \Sigma M_{cut} = 0: \qquad -M_A - (50)(40) = 0 \qquad M_A = -2000 \text{ kip} \cdot \text{in.}$

$$I = \frac{(10)(15)^3}{12} - \frac{(9.5)(13)^3}{12} = 1073.2 \text{ in.}^4$$

$$\sigma_A = \frac{-M_r y}{I} = \frac{-(2000)(6.5)}{(1073.2)} = +12.113 \text{ ksi} = 12.113 \text{ ksi (T)}$$



$$Q_A = y_C A = 7(10 \times 1) = 70.0 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(50.0)(70.0)}{(1073.2)(0.5)} = 6.523 \text{ ksi}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{12.113 + 0}{2} \pm \sqrt{\left(\frac{12.113 - 0}{2}\right)^2 + (-6.523)^2} = 6.057 \pm 8.901 \text{ ksi}$$

$\sigma_{p1} = 6.057 + 8.901 = 14.958 \text{ ksi} \cong 14.96 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = 6.057 - 8.901 = -2.844 \text{ ksi} \cong 2.84 \text{ ksi (C)}$ .... **Ans.**

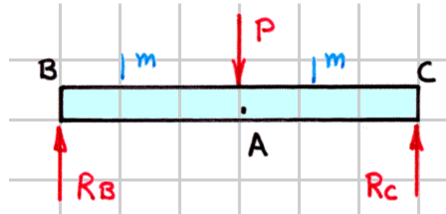
$\sigma_{p3} = 0 \text{ ksi}$ .... **Ans.**

Since σ_{p1} and σ_{p2} have opposite signs:

$\tau_{\max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 8.90 \text{ ksi}$ .... **Ans.**

$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(-6.523)}{(12.113)-(0)} = -23.56°, \quad +66.44°$ .... **Ans.**

7-96

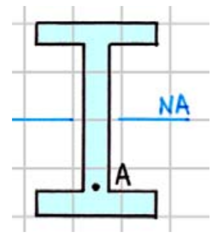


$$R_B = R_C = 0.5P$$

On a section through A: $V_A = (0.5P)\ \text{N}\downarrow$

$$M_A = (0.5P)\ \text{N}\cdot\text{m}$$

$$I = \frac{(250)(350)^3}{12} - \frac{(235)(300)^3}{12} = 364.5(10^6)\ \text{mm}^4$$



$$\sigma_A = \frac{-M_r y}{I} = \frac{-(0.5P)(-0.150)}{(364.5\times10^{-6})} = (205.8P)\ \text{N/m}^2$$

$$Q_A = y_C A = 162.5(250\times25) = 1015.6(10^3)\ \text{mm}^3$$

$$\tau_A = \frac{VQ}{It} = \frac{(0.5P)(1015.6\times10^{-6})}{(364.5\times10^{-6})(0.015)} = (92.88P)\ \text{N/m}^2$$

$$\sigma_{\max} = \sigma_{p1} = \frac{205.8P+0}{2} + \sqrt{\left(\frac{205.8P-0}{2}\right)^2 + (92.88P)^2}$$

$$= 241.52P \le 120(10^6)\ \text{N/m}^2 \qquad P \le 497(10^3)\ \text{N}$$

$$\tau_{\max} = \tau_p = \sqrt{\left(\frac{205.8P-0}{2}\right)^2 + (92.88P)^2} = 138.62P \le 75(10^6)\ \text{N/m}^2$$

$$P \le 541(10^3)\ \text{N}$$

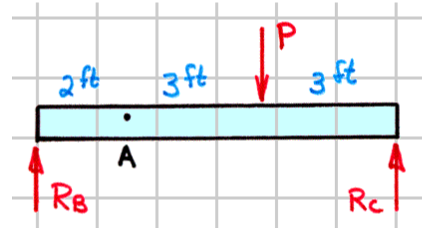
Therefore $P_{\max} = 497\ \text{kN}$ .... **Ans.**

7-97

$\circlearrowleft \Sigma M_C = 0: \quad 3P - 8R_B = 0 \quad R_B = 0.375P \text{ lb}$

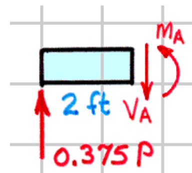
$\uparrow \Sigma F_y = 0: \quad 0.375P - V_A = 0 \quad V_A = 0.375P \text{ lb}$

$\circlearrowleft \Sigma M_{cut} = 0: \quad M_A - (0.375P)(2) = 0 \quad M_A = 0.75P \text{ lb} \cdot \text{ft}$



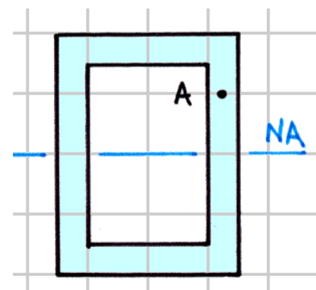
$$I = \frac{(6)(8)^3}{12} - \frac{(4)(6)^3}{12} = 184.0 \text{ in.}^4$$

$$\sigma_A = \frac{-M_r y}{I} = \frac{-(0.75P \times 12)(+2)}{(184.0)} = -0.09783 \text{ psi}$$



$$Q_A = y_C A = 3.5(6\times1) + 2[2.5(1\times1)] = 26.0 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(0.375P)(26.0)}{(184.0)(2)} = 0.02649P \text{ psi}$$



$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-0.09783 + 0}{2} \pm \sqrt{\left(\frac{-0.09783 - 0}{2}\right)^2 + (0.02649P)^2}$$

$$= -0.04892P \pm 0.05563P \text{ psi}$$

$$\sigma_{p1} = -0.04892P + 0.05563P \text{ psi} = +0.00671P \text{ psi (T)}$$

$$\sigma_{p2} = -0.04892P - 0.05563P \text{ psi} = -0.10455P \text{ psi} \le -400 \text{ psi}$$

from which: $P \le 3826 \text{ lb}$

Since σ_{p1} and σ_{p2} have opposite signs:

$$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 0.05563P \text{ psi} \le 200 \text{ psi}$$

from which: $P \le 3595 \text{ lb}$

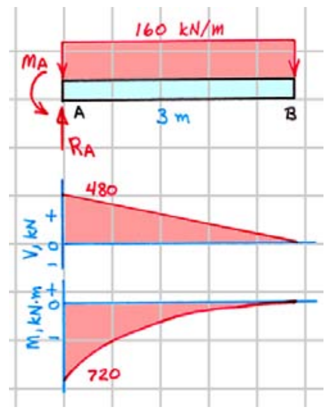
Therefore $P_{max} = 3595 \text{ lb} \cong 3.60 \text{ kip}$ .... **Ans.**

7-98*

From Table B-2 for a $\text{W}\,610 \times 155$ section :

$$I = 1290\left(10^6\right)\text{ mm}^4 \qquad d = 2c = 611\text{ mm} \qquad t_w = 12.7\text{ mm}$$

$$S = 4230\left(10^3\right)\text{ mm}^3 \qquad w_f = 324\text{ mm} \qquad t_f = 19.1\text{ mm}$$



$$\uparrow \Sigma F_y = 0: \qquad R_A - (160 \times 3) = 0$$

$$R_A = +480\text{ kN} = 480\text{ kN}\uparrow$$

$$\circlearrowleft \Sigma M_A = 0: \qquad M_A - (160 \times 3)(1.5) = 0$$

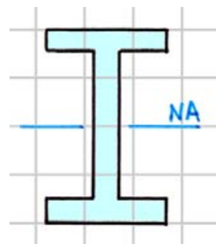
$$M_A = +720\text{ kN}\cdot\text{m} = 720\text{ kN}\cdot\text{m}\circlearrowleft$$

From the shear-force and bending-moment diagrams:

$$V_{\max} = 480\text{ kN} \qquad M_{\max} = 720\text{ kN}\cdot\text{m}$$

At the top of the beam (at the support):

$$\sigma_{\max} = \frac{M}{S} = \frac{\left(720 \times 10^3\right)}{\left(4230 \times 10^{-6}\right)} = 170.21\left(10^6\right)\text{ N/m}^2 = 170.21\text{ MPa (T)}$$



$$\tau_{\max} = \sigma_{\max}/2 = 170.21/2 = 85.11\text{ MPa}$$

At the junction of the flange and the web (at the support):

$$Q_J = y_C A = 296(324 \times 19.1) = 1832\left(10^3\right)\text{ mm}^3$$

$$\sigma = \frac{My}{I} = \frac{\left(720 \times 10^3\right)(0.2864)}{\left(1290 \times 10^{-6}\right)} = 159.85\left(10^6\right)\text{ N/m}^2 = 159.85\text{ MPa (T)}$$

$$\tau = \frac{VQ}{It} = \frac{(480{,}000)\left(1832 \times 10^{-6}\right)}{\left(1290 \times 10^{-6}\right)(0.0127)} = 53.67\left(10^6\right)\text{ N/m}^2 = 53.67\text{ MPa}$$

$$\sigma_{\max} = \sigma_{p1} = \frac{159.85 + 0}{2} \pm \sqrt{\left(\frac{159.85 - 0}{2}\right)^2 + (53.67)^2} = 79.93 + 96.27 = 176.2\text{ MPa (T)}$$

$$\tau_{\max} = \tau_p = 96.27\text{ MPa}$$

At the neutral axis (at the support):

$$Q_{NA} = 296(324 \times 19.1) + 143.2(286.4 \times 12.7) = 2353\left(10^3\right)\text{ mm}^3$$

$$\sigma_{\max} = \tau_{\max} = \frac{(480{,}000)\left(2353 \times 10^{-6}\right)}{\left(1290 \times 10^{-6}\right)(0.0127)} = 68.94\left(10^6\right)\text{ N/m}^2 = 68.94\text{ MPa}$$

Therefore, the maximum stresses occur at the junction of the flange and the web (at the support):

$\sigma_{\max} = 176.2\text{ MPa (T)}$ .... **Ans.**

$\tau_{\max} = 96.3\text{ MPa}$ .... **Ans.**

7-99*

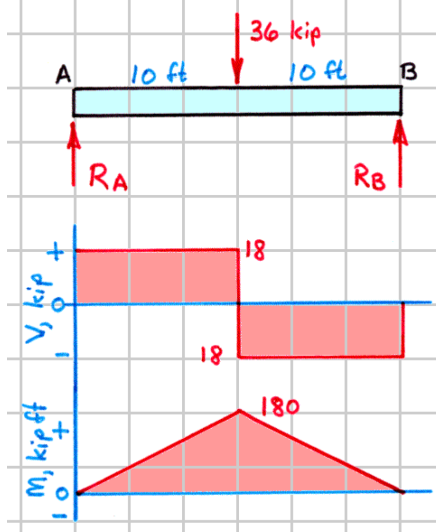
From Table B-1 for a $W18\times 60$ section:

$$I = 984 \text{ in.}^4 \qquad d = 2c = 18.24 \text{ in.} \qquad t_w = 0.415 \text{ in.}$$
$$S = 108 \text{ in.}^3 \qquad w_f = 7.555 \text{ in.} \qquad t_f = 0.695 \text{ in.}$$

From symmetry:

$$R_A = R_B = 36/2 = 16 \text{ kip} = 16 \text{ kip} \uparrow$$

From the shear-force and bending-moment diagrams:



$$V_{\max} = 18 \text{ kip} \qquad M_{\max} = 180 \text{ kip}\cdot\text{ft}$$

At the bottom of the beam (at midspan):

$$\sigma_{\max} = \frac{M}{S} = \frac{(180\times 12)}{(108)} = +20.0 \text{ ksi} = 20.0 \text{ ksi (T)}$$

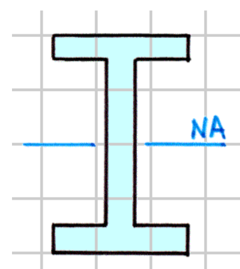
$$\tau_{\max} = \sigma_{\max}/2 = 20.0/2 = 10.00 \text{ ksi}$$

At the junction of the flange and the web (at midspan):

$$Q_J = y_C A = 8.773(7.555\times 0.695) = 46.06 \text{ in.}^3$$

$$\sigma = \frac{-My}{I} = \frac{-(180\times 12)(-8.425)}{(984)} = +18.494 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(18)(46.06)}{(984)(0.415)} = 2.030 \text{ ksi}$$



$$\sigma_{\max} = \sigma_{p1} = \frac{18.494+0}{2} \pm \sqrt{\left(\frac{18.494-0}{2}\right)^2 + (2.030)^2}$$
$$= 9.247 + 9.467 = 18.71 \text{ ksi}$$

$$\tau_{\max} = \tau_p = 9.467 \text{ ksi}$$

At the neutral axis (at midspan):

$$Q_{NA} = 8.773(7.555\times 0.695) + 4.213(8.425\times 0.415) = 60.79 \text{ in.}^3$$

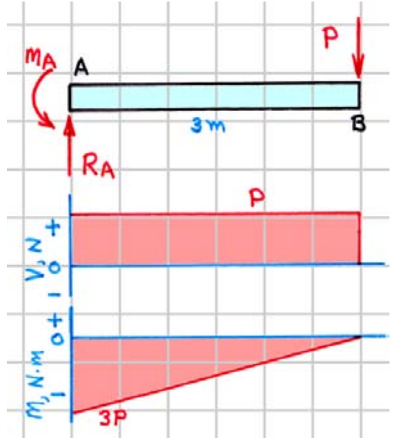
$$\sigma_{\max} = \tau_{\max} = \frac{(18)(60.79)}{(984)(0.415)} = 2.680 \text{ ksi}$$

Therefore, the maximum stresses occur at the top and bottom surfaces (at midspan):

$\sigma_{\max} = 20.0$ ksi (T, bottom; C, top) .... **Ans.**

$\tau_{\max} = 10.00$ ksi .... **Ans.**

7-100



$\uparrow \Sigma F_y = 0: \qquad R_A - P = 0$

$$R_A = +(P)\ \text{N} = (P)\ \text{N} \uparrow$$

$\circlearrowleft \Sigma M_A = 0: \qquad M_A - 3P = 0$

$$M_A = +(3P)\ \text{N}\cdot\text{m} = (3P)\ \text{N}\cdot\text{m} \circlearrowleft$$

From the shear-force and bending-moment diagrams:

$$V_{max} = (P)\ \text{N} \qquad M_{max} = (3P)\ \text{N}\cdot\text{m}$$

From Table B-2 for a $W\,305\times 97$ section : $d = 2c = 308$ mm

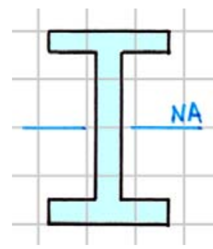
$$I = 222(10^6)\ \text{mm}^4 \qquad t_w = 9.9\ \text{mm}$$

$$S = 1440(10^3)\ \text{mm}^3 \qquad w_f = 305\ \text{mm} \qquad t_f = 15.4\ \text{mm}$$

At the top of the beam (at the support):

$$\sigma_{max} = \frac{M}{S} = \frac{(3P)}{(1440\times 10^{-6})} = (2083P)\ \text{N/m}^2 \qquad \tau_{max} = \frac{\sigma_{max}}{2} = \frac{2083P}{2} = (1042P)\ \text{N/m}^2$$

At the junction of the flange and the web (at the support):



$$Q_J = y_C A = 146.3(305\times 15.4) = 687.1(10^3)\ \text{mm}^3$$

$$\sigma = \frac{My}{I} = \frac{(3P)(0.1386)}{(222\times 10^{-6})} = (1873P)\ \text{N/m}^2$$

$$\tau = \frac{VQ}{It} = \frac{(P)(687.1\times 10^{-6})}{(222\times 10^{-6})(0.0099)} = (312.6P)\ \text{N/m}^2$$

$$\sigma_{max} = \sigma_{p1} = \frac{1873P+0}{2} \pm \sqrt{\left(\frac{1873P-0}{2}\right)^2 + (312.6P)^2} = (1924P)\ \text{N/m}^2$$

$$\tau_{max} = \tau_p = \sqrt{\left(\frac{1873P-0}{2}\right)^2 + (312.6P)^2} = (987P)\ \text{N/m}^2$$

At the neutral axis (at the support):

$$Q_{NA} = 146.3(305\times 15.4) + 69.3(138.6\times 9.9) = 782.3(10^3)\ \text{mm}^3$$

$$\sigma_{max} = \tau_{max} = \frac{(P)(782.3\times 10^{-6})}{(222\times 10^{-6})(0.0099)} = (356P)\ \text{N/m}^2$$

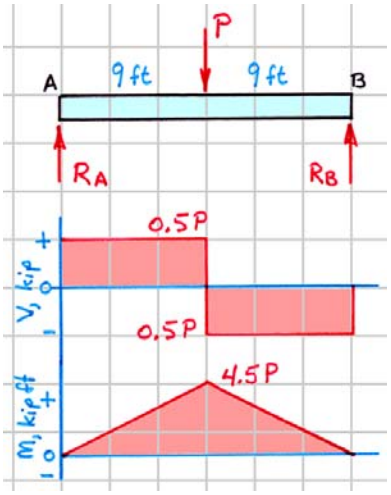
Therefore, at the top (at the support):

$$\sigma_{max} = (2083P)\ \text{N/m}^2 \le 125(10^6)\ \text{N/m}^2 \qquad P \le 60.0(10^3) = 60.0\ \text{kN}$$

$$\tau_{max} = 1042P\ \text{N/m}^2 \le 75(10^6)\ \text{N/m}^2 \qquad P \le 72.0(10^3) = 72.0\ \text{kN}$$

$P_{max} = 60.0$ kN .... **Ans.**

7-101



From symmetry:

$$R_A = R_B = P/2 = (0.5P) \text{ kip} \uparrow$$

From the shear-force and bending-moment diagrams:

$$V_{\max} = (0.5P) \text{ kip}$$

$$M_{\max} = (4.5P) \text{ kip}\cdot\text{ft} = (54P) \text{ kip}\cdot\text{in.}$$

From Table B-1 for a W 24×62 section:

$I = 1550 \text{ in.}^4$ $S = 131 \text{ in.}^3$

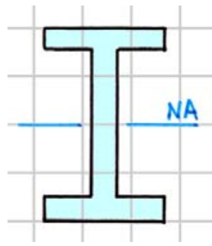
$d = 2c = 23.74 \text{ in.}$ $t_w = 0.430 \text{ in.}$

$w_f = 7.040 \text{ in.}$ $t_f = 0.590 \text{ in.}$

At the bottom of the beam (at midspan):

$$\sigma_{\max} = \frac{M}{S} = \frac{(54P)}{(131)} = (0.4122P) \text{ ksi} \qquad \tau_{\max} = \frac{\sigma_{\max}}{2} = \frac{0.4122P}{2} = (0.2061P) \text{ ksi}$$

At the junction of the bottom flange and the web (at midspan):



$$Q_J = y_C A = 11.575(7.040 \times 0.590) = 48.08 \text{ in.}^3$$

$$\sigma = \frac{-My}{I} = \frac{-(54P)(-11.28)}{(1550)} = (0.3930P) \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(0.5P)(48.08)}{(1550)(0.430)} = (0.03607P) \text{ ksi}$$

$$\sigma_{\max} = \sigma_{p1} = \frac{0.3930P + 0}{2} \pm \sqrt{\left(\frac{0.3930P - 0}{2}\right)^2 + (0.03607P)^2} = (0.3963P) \text{ ksi}$$

$$\tau_{\max} = \tau_p = \sqrt{\left(\frac{0.3930P - 0}{2}\right)^2 + (0.03607P)^2} = (0.1998P) \text{ ksi}$$

At the neutral axis (at midspan):

$$Q_{NA} = 11.575(7.040 \times 0.590) + 5.64(11.28 \times 0.430) = 75.43 \text{ in.}^3$$

$$\sigma_{\max} = \tau_{\max} = \frac{(0.5P)(75.43)}{(1550)(0.430)} = (0.05659P) \text{ ksi}$$

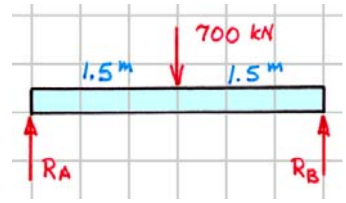
Therefore:

$\sigma_{\max} = (0.4122P) \text{ ksi} \le 18 \text{ ksi}$ $P \le 43.7 \text{ kip}$

$\tau_{\max} = (0.2061P) \text{ ksi} \le 10 \text{ ksi}$ $P \le 48.5 \text{ kip}$

$P_{\max} = 43.7 \text{ kip}$ .... **Ans.**

7-102*



(a) $R_A = R_B = 700/2 = 350 \text{ kN}\uparrow$

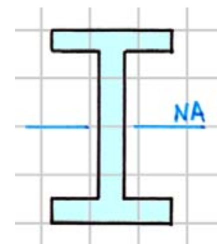
$V_{\max} = 350 \text{ kN} \qquad M_{\max} = (350\times1.5) = 525 \text{ kN}\cdot\text{m}$

From Table B-2 for a $\text{W}356\times179$ section : $d = 2c = 368 \text{ mm}$

$I = 574(10^6) \text{ mm}^4 \qquad S = 3115(10^3) \text{ mm}^3$

$t_w = 15.0 \text{ mm} \qquad w_f = 373 \text{ mm} \qquad t_f = 23.9 \text{ mm}$

At the bottom of the beam (at midspan):



$$\sigma_{\max} = \frac{M}{S} = \frac{(525\times10^3)}{(3115\times10^{-6})} = 168.54(10^6) \text{ N/m}^2 = 168.54 \text{ MPa (T)}$$

$$\tau_{\max} = \sigma_{\max}/2 = 168.54/2 = 84.27 \text{ MPa}$$

At the junction of the bottom flange and the web (at midspan):

$$Q_J = y_C A = 172.05(373\times23.9) = 1533.8(10^3) \text{ mm}^3$$

$$\sigma = \frac{-My}{I} = \frac{-(525\times10^3)(-0.1601)}{(574\times10^{-6})} = 146.43(10^6) \text{ N/m}^2 = 146.43 \text{ MPa (T)}$$

$$\tau = \frac{VQ}{It} = \frac{(350,000)(1533.8\times10^{-6})}{(574\times10^{-6})(0.0150)} = 62.35(10^6) \text{ N/m}^2 = 62.35 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{p1} = \frac{146.43+0}{2} \pm \sqrt{\left(\frac{146.43-0}{2}\right)^2 + (62.34)^2} = 169.38 \text{ MPa (T)}$$

$$\tau_{\max} = \tau_p = \sqrt{\left(\frac{146.43-0}{2}\right)^2 + (62.34)^2} = 96.17 \text{ MPa}$$

At the neutral axis (at midspan):

$$Q_{NA} = 172.05(373\times23.9) + 80.05(160.1\times15) = 1726.0(10^3) \text{ mm}^3$$

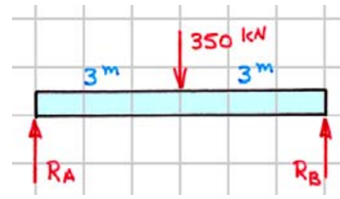
$$\sigma_{\max} = \tau_{\max} = \frac{(350,000)(1726.0\times10^{-6})}{(574\times10^{-6})(0.0150)} = 70.16(10^6) \text{ N/m}^2 = 70.16 \text{ MPa}$$

Therefore, the maximum stresses occur at the junction of the flange and the web (at midspan):

$\sigma_{\max} = 169.4 \text{ MPa (T \& C)}$ .... **Ans.**

$\tau_{\max} = 96.2 \text{ MPa}$ .... **Ans.**

7-102 (cont.)



(b) $R_A = R_B = 350/2 = 175 \text{ kN} \uparrow$

$V_{max} = 175 \text{ kN}$ $\qquad M_{max} = (175 \times 3) = 525 \text{ kN} \cdot \text{m}$

At the bottom of the beam (at midspan):

$$\sigma_{max} = \frac{M}{S} = \frac{(525 \times 10^3)}{(3115 \times 10^{-6})} = 168.54(10^6) \text{ N/m}^2 = 168.54 \text{ MPa (T)}$$

$$\tau_{max} = \sigma_{max}/2 = 168.54/2 = 84.27 \text{ MPa}$$

At the junction of the bottom flange and the web (at midspan):

$$\sigma = \frac{-(525 \times 10^3)(-0.1601)}{(574 \times 10^{-6})} = 146.43(10^6) \text{ N/m}^2 = 146.43 \text{ MPa (T)}$$

$$\tau = \frac{(175{,}000)(1533.8 \times 10^{-6})}{(574 \times 10^{-6})(0.0150)} = 31.17(10^6) \text{ N/m}^2 = 31.17 \text{ MPa}$$

$$\sigma_{max} = \sigma_{p1} = \frac{146.43 + 0}{2} \pm \sqrt{\left(\frac{146.43 - 0}{2}\right)^2 + (31.17)^2} = 152.8 \text{ MPa (T)}$$

$$\tau_{max} = \tau_p = \sqrt{\left(\frac{146.43 - 0}{2}\right)^2 + (31.17)^2} = 79.57 \text{ MPa}$$

At the neutral axis (at midspan):

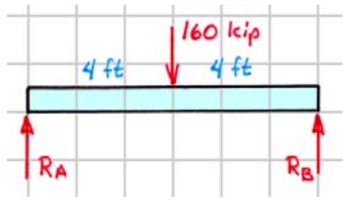
$$\sigma_{max} = \tau_{max} = \frac{(175{,}000)(1726.0 \times 10^{-6})}{(574 \times 10^{-6})(0.0150)} = 35.08(10^6) \text{ N/m}^2 = 35.08 \text{ MPa}$$

Therefore, the maximum stresses occur at the top and bottom surfaces at midspan:

$\sigma_{max} = 168.5 \text{ MPa (T, bottom; C, top)}$ .... **Ans.**

$\tau_{max} = 84.3 \text{ MPa}$ .... **Ans.**

7-103



(a) $R_A = R_B = 160/2 = 80 \text{ kip}\uparrow$

$V_{\max} = 80 \text{ kip} \qquad M_{\max} = (80 \times 4) = 320 \text{ kip}\cdot\text{ft}$

From Table B-1 for a $W14 \times 120$ section :

$I = 1380 \text{ in.}^4 \qquad S = 190 \text{ in.}^3$

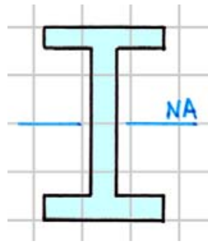
$d = 2c = 14.48 \text{ in.} \qquad t_f = 0.940 \text{ in.}$

$t_w = 0.590 \text{ in.} \qquad w_f = 14.67 \text{ in.}$

At the bottom of the beam (at midspan):

$$\sigma_{\max} = \frac{M}{S} = \frac{(320 \times 12)}{(190)} = 20.21 \text{ ksi (T)} \qquad \tau_{\max} = \frac{\sigma_{\max}}{2} = \frac{20.21}{2} = 10.11 \text{ ksi}$$

At the junction of the bottom flange and the web (at midspan):



$$Q_J = y_C A = 6.77(14.67 \times 0.940) = 93.36 \text{ in.}^3$$

$$\sigma = \frac{-My}{I} = \frac{-(320 \times 112)(-6.30)}{(1380)} = 17.53 \text{ ksi (T)}$$

$$\tau = \frac{VQ}{It} = \frac{(80)(93.36)}{(1380)(0.590)} = 9.173 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{p1} = \frac{17.530 + 0}{2} \pm \sqrt{\left(\frac{17.530 - 0}{2}\right)^2 + (9.173)^2} = 21.45 \text{ ksi (T)}$$

$$\tau_{\max} = \tau_p = \sqrt{\left(\frac{17.530 - 0}{2}\right)^2 + (9.173)^2} = 12.687 \text{ ksi}$$

At the neutral axis (at midspan):

$$Q_{NA} = 6.77(14.67 \times 0.940) + 3.15(6.30 \times 0.590) = 105.07 \text{ in.}^3$$

$$\sigma_{\max} = \tau_{\max} = \frac{(80)(105.07)}{(1380)(0.590)} = 10.324 \text{ ksi}$$

Therefore, the maximum stresses occur at the junction of the flange and the web (at midspan):

$\sigma_{\max} = 21.5 \text{ ksi (T \& C)}$ .... **Ans.**

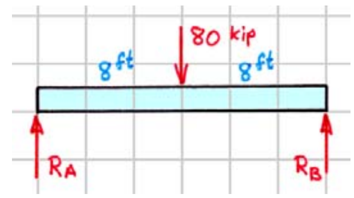
$\tau_{\max} = 12.69 \text{ ksi}$ .... **Ans.**

7-103 (cont.)

(b) $R_A = R_A = 80/2 = 40 \text{ kip} \uparrow$

$V_{\max} = 40 \text{ kip}$

$M_{\max} = (40 \times 8) = 320 \text{ kip} \cdot \text{ft}$



At the bottom of the beam (at midspan):

$$\sigma_{\max} = \frac{M}{S} = \frac{(320 \times 12)}{(190)} = 20.21 \text{ ksi (T)} \qquad \tau_{\max} = \frac{\sigma_{\max}}{2} = \frac{20.21}{2} = 10.11 \text{ ksi}$$

At the junction of the bottom flange and the web (at midspan):

$$\sigma = \frac{-(320 \times 12)(-6.30)}{(1380)} = 17.53 \text{ ksi (T)} \qquad \tau = \frac{(40)(93.36)}{(1380)(0.590)} = 4.587 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{p1} = \frac{17.530 + 0}{2} \pm \sqrt{\left(\frac{17.530 - 0}{2}\right)^2 + (4.587)^2} = 18.66 \text{ ksi (T)}$$

$$\tau_{\max} = \tau_p = \sqrt{\left(\frac{17.530 - 0}{2}\right)^2 + (4.587)^2} = 9.893 \text{ ksi}$$

At the neutral axis (at midspan):

$$\sigma_{\max} = \tau_{\max} = \frac{(40)(105.07)}{(1380)(0.590)} = 5.162 \text{ ksi}$$

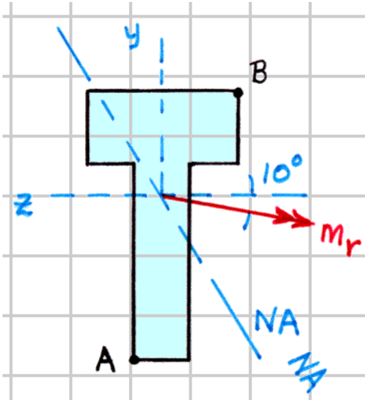
Therefore, the maximum stresses occur at the top and bottom surfaces at midspan:

$\sigma_{\max} = 20.2 \text{ ksi (T \& C)}$ .... **Ans.**

$\tau_{\max} = 10.11 \text{ ksi}$ .... **Ans.**

7-104*

$$I_Z = I_z = \frac{(90)(90)^3}{3} - \frac{(60)(30)^3}{3} + \frac{(30)(150)^3}{3} = 55.08(10^6)\ \text{mm}^4$$



$$I_Y = I_y = \frac{(60)(90)^3}{12} + \frac{(180)(30)^3}{12} = 4.05(10^6)\ \text{mm}^4$$

$$M_{ry} = -20\sin 10° = -3.473\ \text{kN}\cdot\text{m}$$

$$M_{rz} = -20\cos 10° = -19.696\ \text{kN}\cdot\text{m}$$

(a) $$\tan\beta = \frac{M_{rY} I_Z}{M_{rZ} I_Y} = \frac{(-3.473)(55.08)}{(-19.696)(4.05)} = +2.398$$

$\beta = +67.36° = 67.36°$ ↻ .... **Ans.**

(b) At point A ($y = -150$ mm and $z = +15$ mm):

$$\sigma_A = \frac{-M_{rZ} Y_A}{I_Z} + \frac{M_{rY} Z_A}{I_Y} = \frac{-(-19,696)(-0.150)}{(55.08\times10^{-6})} + \frac{(-3473)(+0.015)}{(4.05\times10^{-6})}$$

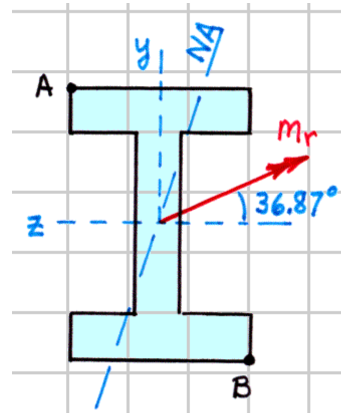
$\sigma_A = -66.5(10^6)\ \text{N/m}^2 = 66.5$ MPa (C) .... **Ans.**

At point B ($y = +90$ mm and $z = -45$ mm):

$$\sigma_B = \frac{-M_{rZ} Y_B}{I_Z} + \frac{M_{rY} Z_B}{I_Y} = \frac{-(-19,696)(+0.090)}{(55.08\times10^{-6})} + \frac{(-3473)(-0.045)}{(4.05\times10^{-6})}$$

$\sigma_B = +70.8(10^6)\ \text{N/m}^2 = 70.8$ MPa (T) .... **Ans.**

7-105*



$$I_Z = I_z = \frac{(4)(6)^3}{12} - \frac{(3)(4)^3}{12} = 56.0 \text{ in}^4$$

$$I_Y = I_y = \frac{(2)(4)^3}{12} + \frac{(4)(1)^3}{12} = 11.0 \text{ in.}^4$$

$$M_{ry} = 10\sin 36.87° = 6.00 \text{ kip}\cdot\text{in.}$$

$$M_{rz} = -10\cos 36.87° = -8.00 \text{ kip}\cdot\text{in.}$$

(a) $$\tan\beta = \frac{M_{rY} I_Z}{M_{rZ} I_Y} = \frac{(6.00)(56.0)}{(-8.00)(11.0)} = -3.818$$

$\beta = -75.32° = 75.32°$ ↺ .... **Ans.**

(b) At point A ($y = +3$ in. and $z = +2$ in.):

$$\sigma_A = \frac{-M_{rZ} Y_A}{I_Z} + \frac{M_{rY} Z_A}{I_Y} = \frac{-(-8.00)(3)}{(56.0)} + \frac{(6.00)(+2)}{(11.0)}$$

$\sigma_A = +1.519 \text{ ksi} = 1.519 \text{ ksi (T)}$ .... **Ans.**

At point B ($y = -3$ in. and $z = -2$ in.):

$$\sigma_B = \frac{-M_{rZ} Y_B}{I_Z} + \frac{M_{rY} Z_B}{I_Y} = \frac{-(-8.00)(-3)}{(56.0)} + \frac{(6.00)(-2)}{(11.0)}$$

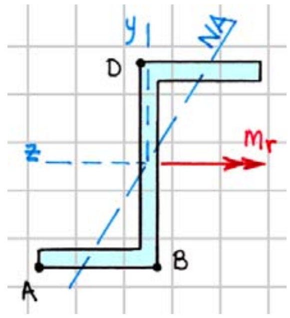
$\sigma_B = -1.519 \text{ ksi} = 1.519 \text{ ksi (C)}$ .... **Ans.**

7-106

(a) At point A ($y = -90$ mm and $z = +85$ mm):

$$\sigma_A = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(4.02)(-90)-(-6.05)(85)}{(4.02)(16.64)-(-6.05)^2}\right]\left[\frac{10^{-9}}{10^{-12}}\right](-8000)$$

$\sigma_A = +40.3(10^6)$ N/m^2 = 40.3 MPa (T) ....**Ans.**



(b) $$\tan\beta = \frac{I_{yz}}{I_y} = \frac{(-6.05)}{(4.02)} = -1.5050$$

$\beta = -56.40° = 56.40°$ ↺ ....**Ans.**

(c) At point B ($y = -90$ mm and $z = -5$ mm):

$$\sigma_B = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(4.02)(-90)-(-6.05)(-5)}{(4.02)(16.64)-(-6.05)^2}\right]\left[\frac{10^{-9}}{10^{-12}}\right](-8000)$$

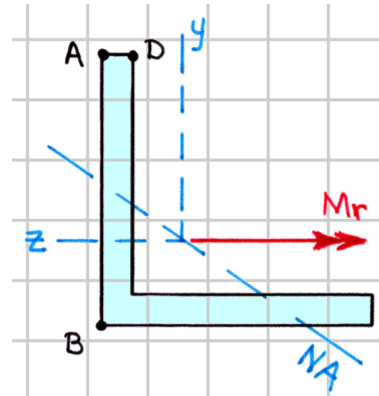
$\sigma_B = -103.5(10^6)$ N/m^2 = 103.5 MPa (C) .... **Ans.**

At point D ($y = +90$ mm and $z = +5$ mm):

$$\sigma_D = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(4.02)(+90)-(-6.05)(+5)}{(4.02)(16.64)-(-6.05)^2}\right]\left[\frac{10^{-9}}{10^{-12}}\right](-8000)$$

$\sigma_D = +103.5(10^6)$ N/m^2 = 103.5 MPa (T) .... **Ans.**

7-107



(a) At point A ($y = ++5.63 \text{ in.}$ and $z = +2.37 \text{ in.}$):

$$\sigma_A = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz}$$

$$= -\left[\frac{(89.0)(+5.63) - (52.5)(+2.37)}{(89.0)(89.0) - (52.5)^2}\right](-7.50 \times 12)$$

$\sigma_A = +6.56 \text{ ksi} = 6.56 \text{ ksi (T)}$ .... **Ans.**

(b) $$\tan\beta = \frac{I_{yz}}{I_y} = \frac{(52.5)}{(89.0)} = +0.5899$$

$\beta = +30.54° = 30.54°$ ↻ .... **Ans.**

(c) At point B ($y = -2.37 \text{ in.}$ and $z = +2.37 \text{ in.}$):

$$\sigma_B = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(89.0)(-2.37) - (52.5)(+2.37)}{(89.0)(89.0) - (52.5)^2}\right](-7.50 \times 12)$$

$\sigma_B = -5.84 \text{ ksi} = 5.84 \text{ ksi (C)}$ .... **Ans.**

At point D ($y = +5.63 \text{ in.}$ and $z = +1.37 \text{ in.}$):

$$\sigma_D = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(89.0)(+5.63) - (52.5)(+1.37)}{(89.0)(89.0) - (52.5)^2}\right](-7.50 \times 12)$$

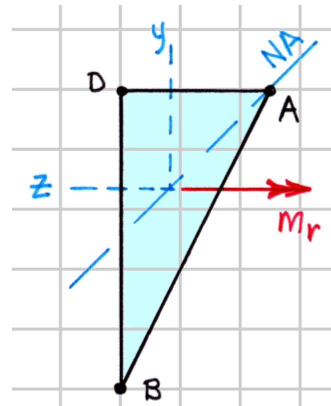
$\sigma_D = +7.48 \text{ ksi} = 7.48 \text{ ksi (T)}$ .... **Ans.**

7-108*

$$I_y = \frac{(180)(90)^3}{36} = 3.645(10^6)\text{ mm}^4 \qquad I_z = \frac{(90)(180)^3}{36} = 14.580(10^6)\text{ mm}^4$$

$$I_{yz} = \frac{-(180)^2(90)^2}{72} = -3.645(10^6)\text{ mm}^4$$

(a) At point *A* ($y = +60$ mm and $z = -60$ mm):



$$\sigma_A = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz}$$

$$= -\left[\frac{(3.645)(+60) - (-3.645)(-60)}{(3.645)(14.580) - (-3.645)^2}\right]\left[\frac{10^{-9}}{10^{-12}}\right](-2000)$$

$\sigma_A = 0\text{ N/m}^2 = 0\text{ MPa}$ .... **Ans.**

(b) $$\tan\beta = \frac{I_{yz}}{I_y} = \frac{(-3.645)}{(3.645)} = -1.000$$

$\beta = -45.00° = 45.00°$ ↺ .... **Ans.**

(c) At point *B* ($y = -120$ mm and $z = +30$ mm):

$$\sigma_B = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(3.645)(-120) - (-3.645)(+30)}{(3.645)(14.580) - (-3.645)^2}\right]\left[\frac{10^{-9}}{10^{-12}}\right](-2000)$$

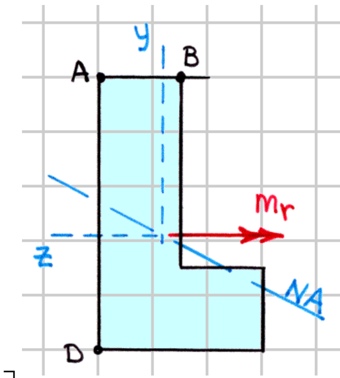
$\sigma_B = -16.46(10^6)\text{ N/m}^2 = 16.46\text{ MPa (C)}$ .... **Ans.**

At point *D* ($y = +60$ mm and $z = +30$ mm):

$$\sigma_D = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(3.645)(+60) - (-3.645)(+30)}{(3.645)(14.580) - (-3.645)^2}\right]\left[\frac{10^{-9}}{10^{-12}}\right](-2000)$$

$\sigma_D = +16.46(10^6)\text{ N/m}^2 = 16.46\text{ MPa (T)}$ .... **Ans.**

7-109*



$$I_y = \frac{(6)(1.5)^3}{3} + \frac{(4)(0.5)^3}{3} + \frac{(2)(2.5)^3}{3} = 17.33 \text{ in.}^4$$

$$I_z = \frac{(2)(3.5)^3}{3} + \frac{(4)(2.5)^3}{3} - \frac{(2)(0.5)^3}{3} = 49.33 \text{ in.}^4$$

$$I_{yz} = (2\times 4)(1.5)(0.5) + (2\times 4)(-1.5)(-0.5) = 12.00 \text{ in.}^4$$

(a) At point *A* ($y = +3.50$ in. and $z = +1.50$ in.):

$$\sigma_A = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(17.33)(+3.50) - (12.00)(+1.50)}{(17.33)(49.33) - (12.00)^2}\right](-50)$$

$\sigma_A = +3.00 \text{ ksi} = 3.00 \text{ ksi (T)}$ .... **Ans.**

(b) $$\tan\beta = \frac{I_{yz}}{I_y} = \frac{(12.00)}{(17.33)} = +0.6924$$

$\beta = +34.70° = 34.70°$ ↻ .... **Ans.**

(c) At point *B* ($y = +3.50$ in. and $z = -0.50$ in.):

$$\sigma_B = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(17.33)(+3.50) - (12.00)(-0.50)}{(17.33)(49.33) - (12.00)^2}\right](-50)$$

$\sigma_B = +4.69 \text{ ksi} = 4.69 \text{ ksi (T)}$ .... **Ans.**

At point *D* ($y = -2.50$ in. and $z = +1.50$ in.):

$$\sigma_D = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(17.33)(-2.50) - (12.00)(+1.50)}{(17.33)(49.33) - (12.00)^2}\right](-50)$$

$\sigma_D = -4.31 \text{ ksi} = 4.31 \text{ ksi (C)}$ .... **Ans.**

7-110

$$I_Z = \frac{(50)(300)^3}{12} = 112.50\left(10^6\right) \text{ mm}^4 \qquad I_Y = \frac{(300)(50)^3}{12} = 3.125\left(10^6\right) \text{ mm}^4$$

For symmetric bending: $$\sigma = \frac{-M_r y}{I} = \frac{-M(-0.150)}{\left(112.50\times 10^{-6}\right)} = (1333.3M) \text{ N/m}^2$$

For non-symmetric bending:

$$\sigma = \frac{-M_{rZ}Y}{I_Z} + \frac{M_{rY}Z}{I_Y} = \frac{-(M\cos 3°)(-0.150)}{\left(112.50\times 10^{-6}\right)} + \frac{(M\sin 3°)(+0.025)}{\left(3.125\times 10^{-6}\right)} = (1750.2M) \text{ N/m}^2$$

$$\Delta\sigma = \frac{1750.2-1333.3}{1333.3}(100) = 31.3\%$$ .... **Ans.**

7-111

$$\sigma = \frac{-M_{rZ}Y}{I_Z} + \frac{M_{rY}Z}{I_Y} \qquad I_Y = \frac{hb^3}{12} \qquad I_Z = \frac{bh^3}{12}$$

$$M_{rY} = M\sin\alpha \qquad M_{rZ} = -M\cos\alpha$$

$$\sigma_{\max} = \frac{-(M\cos\alpha)(-h/2)}{(bh^3/12)} + \frac{(M\sin\alpha)(b/2)}{(hb^3/12)} = \frac{6M}{b^2h^2}(b\cos\alpha + h\sin\alpha)$$

$$\frac{d\sigma_{\max}}{d\alpha} = \frac{6M}{b^2h^2}(-b\sin\alpha + h\cos\alpha) = 0$$

$$b\sin\alpha = h\cos\alpha$$

$\alpha = \tan^{-1}\dfrac{h}{b} = \tan^{-1}\dfrac{12}{6} = 63.43°$ .... **Ans.**

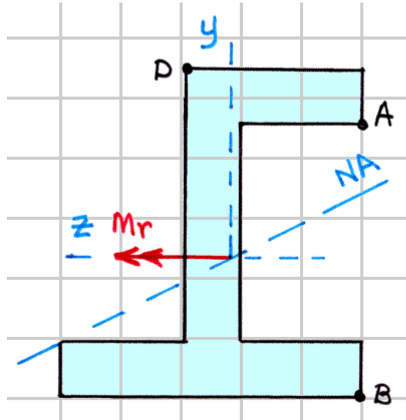
For $\sigma = 2000$ psi :

$$M = \frac{b^2h^2\sigma}{6(b\cos\alpha + h\sin\alpha)} = \frac{(6)^2(12)^2(2000)}{6(6\cos 63.43° + 12\sin 63.43°)}$$

$M = 128.8(10^3)$ lb·in. $= 128.8$ kip·in. .... **Ans.**

7-112*

$$A = (140)(20) + (120)(20) + (80)(20) = 6800 \text{ mm}^2$$



$$I_y = \frac{(20)(140)^3}{12} + \frac{(120)(20)^3}{12} + \frac{(20)(70)^3}{3} + \frac{(20)(10)^3}{3} - 6800(7.06)^2 = 6.61(10^6) \text{ mm}^4$$

$$I_z = \frac{(140)(80)^3}{3} - \frac{(120)(60)^3}{3} + \frac{(80)(80)^3}{3} - \frac{(60)(60)^3}{3} - 6800(12.36)^2 = 23.5(10^6) \text{ mm}^4$$

$$I_{yz} = 0 - (60\times 20)(70)(40) - (6800)(7.06)(12.36) = -3.95(10^6) \text{ mm}^4$$

$$\sigma = -\left[\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2}\right] M_{rz} = -\left[\frac{(6.61)y - (-3.95)z}{(6.61)(23.5) - (-3.95)^2}\right]\left[\frac{10^{-9}}{10^{-12}}\right](20,000)$$

$$= \left[-946.1(10^6)y - 565.4(10^6)z\right] \text{ N/m}^2 = \left[-946.1y - 565.4z\right] \text{ MPa}$$

(a) At point A ($y = +72.36$ mm and $z = -62.94$ mm):

$$\sigma_A = \left[-946.1(72.36) - 565.4(-62.94)\right] = -32.9 \text{ MPa} = 32.9 \text{ MPa (C)}$$ .... **Ans.**

(b) $$\tan\beta = \frac{I_{yz}}{I_y} = \frac{(-3.95)}{(6.61)} = -0.5976$$

$$\beta = -30.86° = 30.86° \circlearrowleft$$ .... **Ans.**

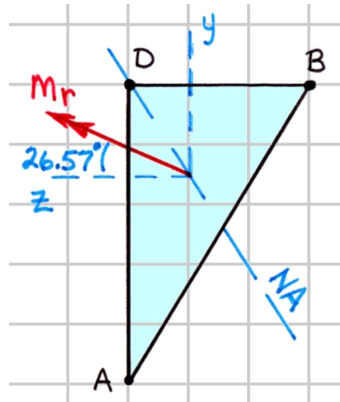
(b) At point B ($y = -67.64$ mm and $z = -62.94$ mm):

$$\sigma_B = \left[-946.1(-67.64) - 565.4(-62.94)\right] = +99.6 \text{ MPa} = 99.6 \text{ MPa (T)}$$ .... **Ans.**

At point D ($y = +92.36$ mm and $z = +17.06$ mm):

$$\sigma_B = \left[-946.1(92.36) - 565.4(+17.06)\right] = -97.0 \text{ MPa} = 97.0 \text{ MPa (C)}$$ .... **Ans.**

7-113*



$$I_y = \frac{(6)(3)^3}{36} = 4.50 \text{ in.}^4 \qquad I_z = \frac{(3)(6)^3}{36} = 18.00 \text{ in.}^4$$

$$I_{yz} = \frac{-(6)^2(3)^2}{72} = -4.50 \text{ in.}^4$$

$$M_{ry} = 10\sin 26.57° = 4.473 \text{ kip} \cdot \text{in.}$$

$$M_{rz} = 10\cos 26.57° = 8.944 \text{ kip} \cdot \text{in.}$$

$$\sigma = -\left[\frac{M_{rz}I_y + M_{ry}I_{yz}}{I_yI_z - I_{yz}^2}\right]y + \left[\frac{M_{ry}I_z + M_{rz}I_{yz}}{I_yI_z - I_{yz}^2}\right]z$$

$$\sigma = -\left[\frac{(8.944)(4.50)+(4.473)(-4.50)}{(4.50)(18.00)-(-4.50)^2}\right]y + \left[\frac{(4.473)(18.00)+(8.944)(-4.50)}{(4.50)(18.00)-(-4.50)^2}\right]z$$

$$= (-0.3312y + 0.6628z) \text{ ksi}$$

(a) At point A ($y = -4$ in. and $z = +1$ in.):

$\sigma_A = (-0.3312)(-4) + (0.6628)(+1) = +1.988 \text{ ksi} = 1.988 \text{ ksi (T)}$ .... **Ans.**

At point B ($y = +2$ in. and $z = -2$ in.):

$\sigma_B = (-0.3312)(+2) + (0.6628)(-2) = -1.988 \text{ ksi} = 1.988 \text{ ksi (C)}$ .... **Ans.**

At point D ($y = +2$ in. and $z = +1$ in.):

$\sigma_D = (-0.3312)(+2) + (0.6628)(+1) = +0.0004 \text{ ksi} \cong 0 \text{ ksi}$ .... **Ans.**

(b) $$\tan\beta = \frac{M_{ry}I_z + M_{rz}I_{yz}}{M_{rz}I_y + M_{ry}I_{yz}} = \frac{(4.473)(18.00)+(8.944)(-4.50)}{(8.944)(4.50)+(4.473)(-4.50)} = +2.001$$

$\beta = +63.45° = 63.45°$ ↻ .... **Ans.**

7-114

(a) $\sigma = \dfrac{-M_{rZ}Y}{I_Z} + \dfrac{M_{rY}Z}{I_Y}$ $\qquad I_Y = \dfrac{hb^3}{12}$ $\qquad I_Z = \dfrac{bh^3}{12}$

$M_{rY} = M\sin\alpha \qquad M_{rZ} = -M\cos\alpha$

$$\sigma_{\max} = \frac{-(M\cos\alpha)(-h/2)}{(bh^3/12)} + \frac{(M\sin\alpha)(b/2)}{(hb^3/12)} = \frac{6M}{b^2h^2}(b\cos\alpha + h\sin\alpha)$$ .... **Ans.**

(b) $\dfrac{d\sigma_{\max}}{d\alpha} = \dfrac{6M}{b^2h^2}(-b\sin\alpha + h\cos\alpha) = 0 \qquad b\sin\alpha = h\cos\alpha$

Therefore $\alpha = \tan^{-1}(h/b)$ .... **Ans.**

(c) When $h = 2b$: $\qquad \alpha = \tan^{-1}(2) = +63.43° = +63.43°$ ↻

$I_Y = \dfrac{hb^3}{12} = \dfrac{(2b)b^3}{12} = \dfrac{b^4}{6} \qquad I_Z = \dfrac{bh^3}{12} = \dfrac{b(2b)^3}{12} = \dfrac{2b^4}{3}$

$\tan\beta = \dfrac{I_Z}{I_Y}\tan\alpha = \dfrac{2b^4/3}{b^4/6}\tan 63.43° = 8.00$

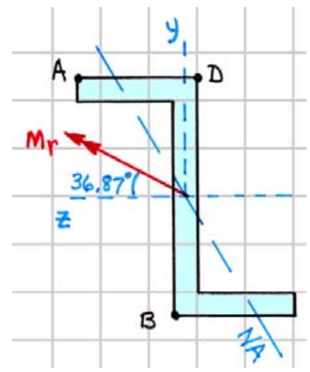
$\beta = \tan^{-1} 8.00 = +82.87° = 82.87°$ ↻ .... **Ans.**

7-115

$$M_{ry} = 20\sin 36.87° = 12.00 \text{ kip}\cdot\text{in.}$$

$$M_{rz} = 20\cos 36.87° = 16.00 \text{ kip}\cdot\text{in.}$$

$$\sigma = -\left[\frac{M_{rz}I_y + M_{ry}I_{yz}}{I_yI_z - I_{yz}^2}\right]y + \left[\frac{M_{ry}I_z + M_{rz}I_{yz}}{I_yI_z - I_{yz}^2}\right]z$$



$$\sigma = -\left[\frac{(16.00)(8.83)+(12.00)(11.3)}{(8.83)(25.4)-(11.3)^2}\right]y + \left[\frac{(12.00)(25.4)+(16.00)(11.3)}{(8.83)(25.4)-(11.3)^2}\right]z$$

$$= (-2.8665y + 5.0273z) \text{ ksi}$$

(a) At point *A* ($y = +3$ in. and $z = +3.3125$ in.):

$$\sigma_A = (-2.8665)(+3) + (5.0273)(+3.3125) = +8.05 \text{ ksi} = 8.05 \text{ ksi (T)}$$ .... **Ans.**

(b) $$\tan\beta = \frac{M_{ry}I_z + M_{rz}I_{yz}}{M_{rz}I_y + M_{ry}I_{yz}} = \frac{(12.00)(25.4)+(16.00)(11.3)}{(16.00)(8.83)+(12.00)(11.3)} = +1.7538$$

$$\beta = +60.31° = 60.31° \circlearrowright$$ .... **Ans.**

(c) At point *B* ($y = -3$ in. and $z = +0.1875$ in.):

$$\sigma_B = (-2.8665)(-3) + (5.0273)(+0.1875) = +9.54 \text{ ksi} = 9.54 \text{ ksi (T)}$$ .... **Ans.**

At point *D* ($y = +3$ in. and $z = -0.1875$ in.):

$$\sigma_D = (-2.8665)(+3) + (5.0273)(-0.1875) = -9.54 \text{ ksi} = 9.54 \text{ ksi (C)}$$ .... **Ans.**

7-116*

$$\sigma = \frac{K_t Mc}{I} \qquad \frac{w}{h} = \frac{75}{60} = 1.25 \qquad \frac{r}{h} = \frac{6}{60} = 0.10$$

From Fig. 7-34: $K_t = 1.70$

$$I = \frac{(20)(60)^3}{12} = 360\left(10^3\right)\ \text{mm}^4$$

$$M = \frac{\sigma I}{K_t c} = \frac{\left(80\times10^6\right)\left(360\times10^{-9}\right)}{(1.70)(0.030)} = 565\ \text{N}\cdot\text{m}$$ .... **Ans.**

7-117*

$$\frac{w}{h} = \frac{3/8}{1/4} = 1.50$$

For $K_t = 1.40$ Fig. 7-34 gives $r/h = 0.25$

$r = 0.25(1/4) = 0.0625$ in. .... **Ans.**

7-118

$$\sigma = \frac{Mc}{I} = \frac{M(D/2)}{\pi D^4/64} = \frac{32M}{\pi D^3} \qquad M = \frac{\sigma \pi D^3}{32}$$

$$\sigma = K_t \frac{M_R(d/2)}{\pi d^4/64} = \frac{32 M_R K_t}{\pi d^3} \qquad M_R = \frac{\sigma \pi d^3}{32 K_t}$$

$$\frac{D}{d} = \frac{100}{80} = 1.25 \qquad \frac{r}{d} = \frac{8}{80} = 0.10$$

From Fig. 7-34: $K_t = 1.90$

$$\%R = \left[\frac{M - M_R}{M}\right](100) = \left[\frac{(\sigma \pi D^3/32) - (\sigma \pi d^3/32K_t)}{(\sigma \pi D^3/32)}\right](100)$$

$$= \left[\frac{D^3 - (d^3/K_t)}{D^3}\right](100) = \left[1 - \frac{(d/D)^3}{K_t}\right](100)$$

$$\%R = \left[1 - \frac{(1/1.25)^3}{1.90}\right](100) = 73.0\%$$ .... **Ans.**

7-119

$$\sigma = \frac{Mc}{I} = \frac{M(h/2)}{Bh^3/12} = \frac{6M}{Bh^2} \qquad M = \frac{\sigma Bh^2}{6}$$

$$\sigma = K_t \frac{M_R(h/2)}{bh^3/12} = \frac{6M_R K_t}{bh^2} \qquad M_R = \frac{\sigma bh^2}{6K_t}$$

$$\frac{d}{r} = \frac{1/16}{1/16} = 1.00 \qquad \frac{r}{b} = \frac{1/16}{5/8} = 0.10$$

From Fig. 7-34: $K_t = 2.30$

$$\%R = \left[\frac{M - M_R}{M}\right](100) = \left[\frac{(\sigma Bh^2/6) - (\sigma bh^2/6K_t)}{(\sigma Bh^2/6)}\right](100)$$

$$= \left[\frac{B - (b/K_t)}{B}\right](100) = \left[1 - \frac{(b/B)}{K_t}\right](100)$$

$$\%R = \left[1 - \frac{(0.625/0.75)}{2.30}\right](100) = 63.8\%$$ .... **Ans.**

7-120*

$$\sigma = \frac{Mc}{I} = \frac{M(h/2)}{bh^3/12} = \frac{6M}{bh^2} \qquad M = \frac{\sigma bh^2}{6}$$

$$\sigma = K_t \frac{M_R(h/2)}{(b-d)h^3/12} = \frac{6M_R K_t}{(b-d)h^2} \qquad M_R = \frac{\sigma(b-d)h^2}{6K_t}$$

$$\frac{h}{d} = \frac{200}{25} = 8.00 \qquad \frac{d}{b} = \frac{25}{150} = 0.17$$

From Fig. 7-34: $K_t = 2.55$

$$\%R = \left[\frac{M - M_R}{M}\right](100) = \left[\frac{b-(b-d)/K_t}{b}\right](100) = \left[1 - \frac{b-d}{bK_t}\right](100)$$

$$\%R = \left[1 - \frac{150-25}{150(2.55)}\right](100) = 67.3\%$$ .... **Ans.**

7-121

For 04%C hot-rolled steel: $\sigma_y = 53 \text{ ksi}$ $\sigma_{all} = \sigma_y / FS = 53/3 = 17.667 \text{ ksi}$

At the wall: $I = \dfrac{\pi (1.5)^4}{4} = 3.976 \text{ in.}^4$ $M = (22P) \text{ kip} \cdot \text{in.}$

$$\sigma = \frac{Mc}{I} = \frac{(22P)(1.5)}{3.976} \le 17.667 \text{ ksi} \qquad P \le 2.13 \text{ kip}$$

At the reduced section: $I = \dfrac{\pi (1.365)^4}{4} = 2.727 \text{ in.}^4$ $M = (12P) \text{ kip} \cdot \text{in.}$

$$\frac{w}{h} = \frac{3}{2.73} = 1.10 \qquad \frac{r}{h} = \frac{0.25}{2.73} = 0.09$$

From Fig. 7-34: $K_t = 1.50$

$$\sigma = K_t \frac{Mc}{I} = (1.50)\frac{(12P)(1.365)}{2.727} \le 17.667 \text{ ksi} \qquad P \le 1.961 \text{ kip}$$

Therefore $P_{\max} = 1.961 \text{ kip}$ .... **Ans.**

7-122*

For 04%C hot-rolled steel: $\sigma_y = 360\text{ MPa}$ $\sigma_{all} = \sigma_y / FS = 360/4 = 90\text{ MPa}$

$$\sigma = K_t \frac{Mc}{I} = K_t \frac{M(h/2)}{(b-d)(h)^3/12} = \frac{6K_t M}{(b-d)h^2}$$

Rearranging gives $$\frac{b-d}{K_t} = \frac{6M}{\sigma h^2} = \frac{6(50{,}000)}{(90\times10^6)(0.200)^2} = 0.08333\text{ m} \quad \text{(a)}$$

$$\frac{h}{d} = \frac{200}{25} = 8.00 \qquad \frac{d}{b} = \frac{25}{b}$$

Solve by trial and error.

Guess that $K_t \cong 2.50$.

Then Eq. (a) gives $b - d = 0.208\text{ m}$ $b = 208 + 25 = 233\text{ mm}$

Then from Fig. 7-34: $d/b = 0.107$ $K_t = 2.70$

Guess that $K_t \cong 2.70$.

Then Eq. (a) gives $b - d = 0.225\text{ m}$ $b = 212 + 25 = 250\text{ mm}$

Then from Fig. 7-34: $d/b = 0.100$ $K_t = 2.70$

Therefore $b_{\min} \cong 250\text{ mm}$ .... **Ans.**

7-123

$$I = \frac{(2-0.625)(4)^3}{12} = 7.333\text{ in.}^4 \qquad M = \left(\frac{P}{2}\right)\left(\frac{L}{3}\right) = \left(\frac{PL}{6}\right)\text{ kip}\cdot\text{in.}$$

$$\sigma = K_t\frac{Mc}{I} = K_t\frac{(PL/6)c}{I} \qquad \text{gives} \qquad L = \frac{6\sigma I}{K_t cP}$$

$$\frac{h}{d} = \frac{4}{0.625} = 6.40 \qquad \frac{d}{b} = \frac{0.625}{2} = 0.3125$$

From Fig. 7-34: $K_t \cong 2.35$

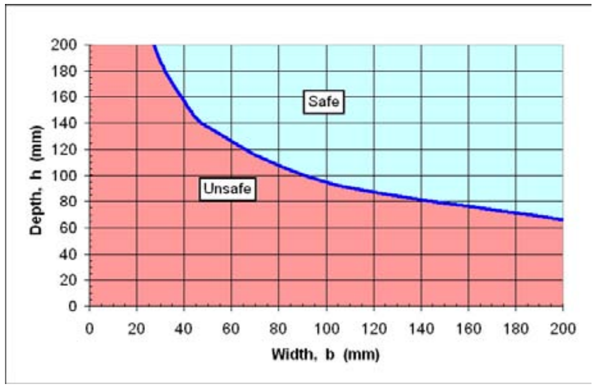
$$L = \frac{6(20)(7.333)}{(2.35)(2)(5)} = 37.445\text{ in.} = 3.12\text{ ft}$$ .... **Ans.**

7-124

$$\sigma = K_t \frac{Mc}{I} = K_t \frac{M(h/2)}{(b-d)h^3/12} = \frac{6K_t M}{(b-d)h^2} \le 200 \text{ MPa}$$

$$h^2 \ge \frac{6K_t(10\times10^3)}{(b-d)(200\times10^6)} = \frac{K_t(300\times10^{-6})}{(b-0.010)} \text{ m}^2$$

d/b	b (mm)	K_t	h (mm)
0.05	200	2.80	66
0.10	100	2.70	95
0.20	50	2.50	137
0.25	40	2.45	157
0.30	33	2.37	176
0.35	29	2.30	191
0.40	25	2.25	212



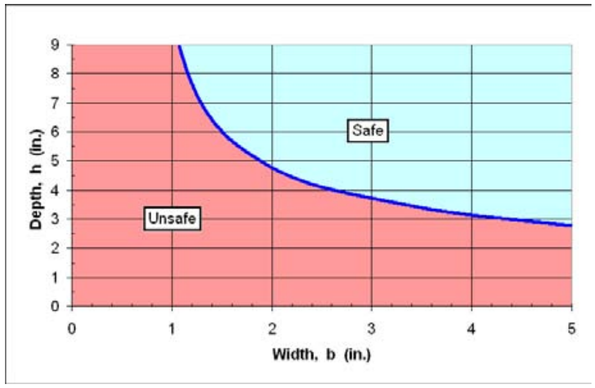
7-125

$$M = (P/2)(L/3) = (2.25)(14) = 31.5 \text{ kip}\cdot\text{in.}$$

$$\sigma = K_t \frac{Mc}{I} = K_t \frac{M(h/2)}{(b-d)h^3/12} = \frac{6K_t M}{(b-d)h^2} \le 15 \text{ ksi}$$

$$h^2 \ge \frac{6K_t(31.5)}{(b-d)(15)} = \frac{12.60K_t}{(b-0.75)} \text{ in.}^2$$

d/b	b (in.)	K_t	h (in.)
0.15	5.00	2.60	2.78
0.20	3.75	2.50	3.24
0.30	2.50	2.35	4.11
0.40	1.88	2.25	5.00
0.50	1.50	2.17	6.04
0.60	1.25	2.10	7.27
0.70	1.07	2.03	8.94



7-126*

For a W 203×50 section $d = 2c = 210$ mm $t_w = 9.1$ mm

$w_f = 205$ mm $t_f = 14.2$ mm $S = 582\left(10^3\right)$ mm^3

$$M_e = \sigma_y S = \left(250\times 10^6\right)\left(582\times 10^{-6}\right)$$

$M_e = 145.5\left(10^3\right)\text{ N}\cdot\text{m} = 145.5\text{ kN}\cdot\text{m}$ .... **Ans.**

$$M_p = 2\left(250\times 10^6\right)\left[\left(0.0979\right)\left(0.205\times 0.0142\right)\right]$$
$$+ 2\left(250\times 10^6\right)\left[\left(0.0454\right)\left(0.0908\times 0.0091\right)\right]$$

$M_p = 142.49\left(10^3\right) + 18.76\left(10^3\right) = 161.3\left(10^3\right)\text{ N}\cdot\text{m} = 161.3\text{ kN}\cdot\text{m}$ .... **Ans.**

7-127*

For a W 33×201 section $d = 2c = 33.68$ in. $t_w = 0.715$ in.

$w_f = 15.745$ in. $t_f = 1.150$ in. $S = 684$ in.^{3}

$M_e = \sigma_y S = (36)(684) = 24,624$ kip·in. $\cong 24,600$ kip·in. .... **Ans.**

$M_p = 2(36)\left[(16.265)(15.745\times 1.150)\right] + 2(36)\left[(7.845)(15.69\times 0.715)\right]$

$M_p = 21,204 + 6337 = 27,541$ kip·in. $\cong 27,500$ kip·in. .... **Ans.**

7-128

For a W 762×196 section $d = 2c = 770 \text{ mm}$ $t_w = 15.6 \text{ mm}$

$w_f = 268 \text{ mm}$ $t_f = 25.4 \text{ mm}$ $S = 6225\left(10^3\right) \text{ mm}^3$

$$M_e = \sigma_y S = \left(250\times10^6\right)\left(6225\times10^{-6}\right)$$

$$M_e = 1556\left(10^3\right) \text{ N}\cdot\text{m} = 1556 \text{ kN}\cdot\text{m}$$ .... **Ans.**

$$M_p = 2\left(250\times10^6\right)\left[\left(0.3723\right)\left(0.268\times0.0254\right)\right] + 2\left(250\times10^6\right)\left[\left(0.1798\right)\left(0.3596\times0.0156\right)\right]$$

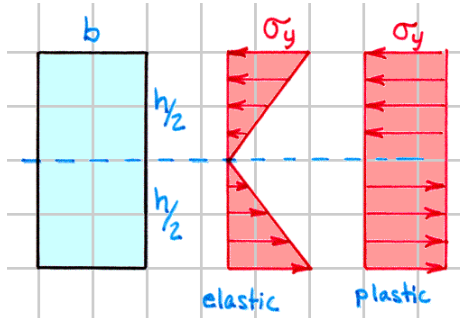
$$M_p = 1267.2\left(10^3\right) + 504.3\left(10^3\right) = 1772\left(10^3\right) \text{ N}\cdot\text{m} = 1772 \text{ kN}\cdot\text{m}$$ .... **Ans.**

7-129*

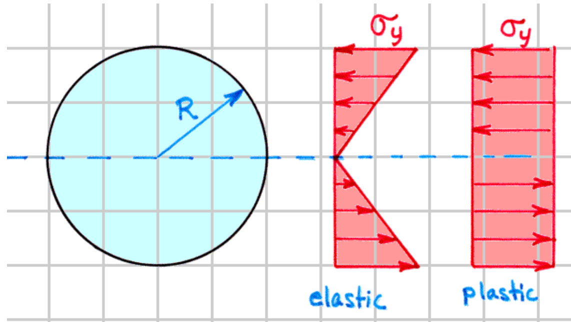
$$M_e = \frac{\sigma_y I}{c} = \frac{\sigma_y \left(bh^3/12\right)}{\left(h/2\right)} = \frac{bh^2\sigma_y}{6}$$

$$M_p = 2\sigma_y \left(\frac{bh}{2}\right)\left(\frac{h}{4}\right) = \frac{bh^2\sigma_y}{4}$$

$$\frac{M_p}{M_e} = \frac{1/4}{1/6} = 1.5$$ .... **Ans.**



7-130



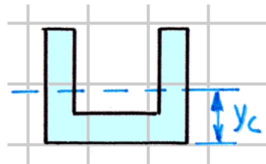
$$M_e = \frac{\sigma_y I}{c} = \frac{\sigma_y \left(\pi R^4/4\right)}{(R)} = \frac{\pi R^3 \sigma_y}{4}$$

$$M_p = 2\sigma_y \left(\frac{\pi R^2}{2}\right)\left(\frac{4R}{3\pi}\right) = \frac{4R^3 \sigma_y}{3}$$

$$\frac{M_p}{M_e} = \frac{4/3}{\pi/4} = 1.698$$ .... **Ans.**

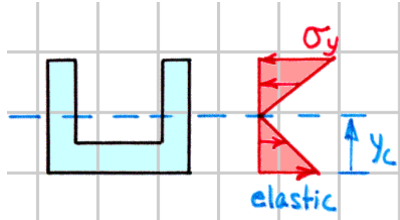
7-131*

$$y_C = \frac{M_x}{A} = \frac{(5)\left[(2)(10)\right]+(1)\left[(8)(2)\right]+(5)\left[(2)(10)\right]}{(2)(10)+(8)(2)+(2)(10)} = 3.857 \text{ in.}$$



$$I = \frac{(12)(3.857)^3}{3} - \frac{(8)(1.857)^3}{3} + \frac{(4)(6.143)^3}{3} = 521.5 \text{ in.}^4$$

Elastic action:



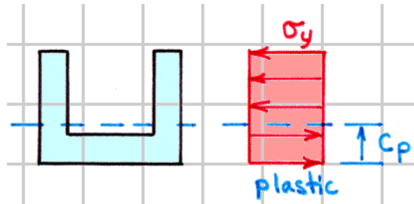
$$M_e = \frac{\sigma_y I}{c} = \frac{\sigma_y (521.5)}{6.143} = 84.89\sigma_y$$

For fully plastic action:

$$A_T = A_C = 56/2 = 28 \text{ in.}^2$$

$$4(10 - c_p) = 28 \text{ in.}^2$$

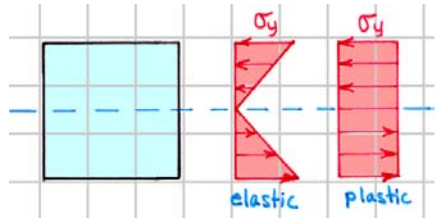
$$c_p = 3.00 \text{ in.}$$



$$M_p = \left[(1.5)(12\times3) - (0.5)(8\times1) + (3.5)(4\times7)\right]\sigma_y$$
$$= 148.0\sigma_y$$

$$\frac{M_p}{M_e} = \frac{148.0}{84.89} = 1.743$$ .... **Ans.**

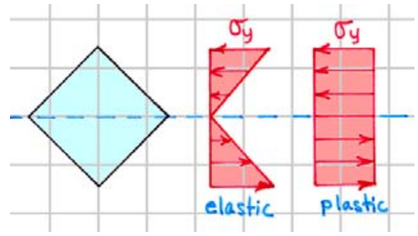
7-132*



(a) $$M_e = \frac{\sigma_y I}{c} = \frac{\sigma_y\left(a^4/12\right)}{\left(a/2\right)} = \frac{a^3\sigma_y}{6}$$

$$M_p = 2\sigma_y\left(\frac{a^2}{2}\right)\left(\frac{a}{4}\right) = \frac{a^3\sigma_y}{4}$$

$$\frac{M_p}{M_e} = \frac{1/4}{1/6} = 1.500$$ .... **Ans.**



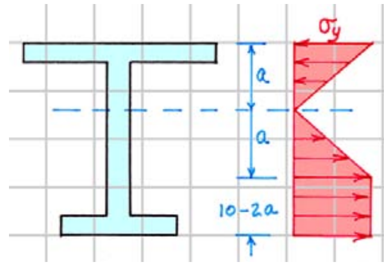
(b) $$M_e = \frac{\sigma_y I}{c} = \frac{\sigma_y\left(a^4/12\right)}{\left(a\sin 45°\right)} = 0.11785a^3\sigma_y$$

$$M_p = 2\sigma_y\left(\frac{a^2}{2}\right)\left(\frac{a\sin 45°}{3}\right) = 0.23570a^3\sigma_y$$

$$\frac{M_p}{M_e} = \frac{0.23570}{0.11785} = 2.00$$ .... **Ans.**

7-133

(a) Assume that the lower flange will be fully plastic.



$$\sigma_{a-1} = \left[36(a-1)/a\right] \text{ ksi}$$

$$F_C = \left[\frac{(36)(10\times a)}{2}\right] - \left[\frac{(36)(a-1)}{a}\right]\left[\frac{(9)(a-1)}{2}\right]$$

$$F_T = \frac{(36)(1\times a)}{2} + (36)(1)(9-2a) + (36)(1)(6)$$

Equating F_C to F_T gives: $72a^2 - 216a - 162 = 0$

From which $a = 3.621 \text{ in.} \cong 3.62 \text{ in.}$ .... **Ans.**

$$10 - 2a = 10 - 2(3.621) = 2.758 \text{ in.} > 1 \text{ in.}$$

Therefore, the lower flange will be fully plastic as was assumed.

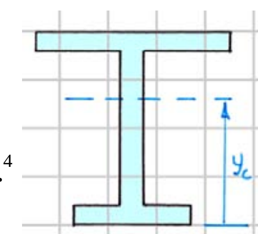
(b) $\sigma_{a-1} = \left[36(3.621-1)/3.621\right] = 26.06 \text{ ksi}$

$$M = \left[\frac{(36)(10\times 3.621)}{2}\right]\left[\frac{2(3.621)}{3}\right] - \left[\frac{(26.06)(9\times 2.621)}{2}\right]\left[\frac{2(2.621)}{3}\right]$$

$$+\left[\frac{(36)(1\times 3.621)}{2}\right]\left[\frac{2(3.621)}{3}\right] + (36)(1\times 1.758)(4.50) + (36)(1\times 6)(5.879)$$

$M = 2748 \text{ kip}\cdot\text{in.} \cong 2750 \text{ kip}\cdot\text{in.}$ .... **Ans.**

(c) $$y_C = \frac{M_x}{A} = \frac{(9.5)(10\times1)+(5)(8\times1)+(0.5)(6\times1)}{(10\times1)+(8\times1)+(6\times1)} = 5.75 \text{ in.}$$



$$I = \frac{(10)(4.25)^3}{3} - \frac{(9)(3.25)^3}{3} + \frac{(6)(5.75)^3}{3} - \frac{(5)(4.75)^3}{3} = 354.5 \text{ in.}^4$$

$$M_e = \frac{\sigma I}{c} = \frac{(36)(354.5)}{(5.75)} = 2219 \text{ kip}\cdot\text{in.}$$

For fully plastic action: $A_T = A_C = 24/2 = 12 \text{ in.}^2$

$A_T = (6\times1) + (1)(c_p - 1) = 12 \text{ in.}^2$ $c_p = 7.00 \text{ in.}$

$$M_p = (36)(6\times1)(6.5) + (36)(6\times1)(3) + (36)(2\times1)(1) + (36)(10\times1)(2.5) = 3024 \text{ kip}\cdot\text{in.}$$

$\frac{M_p}{M_e} = \frac{3024}{2219} = 1.363$ .... **Ans.**

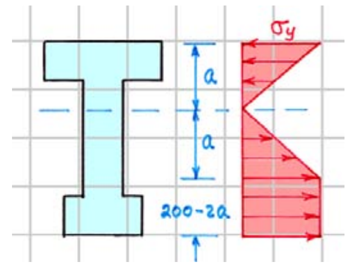
7-134

(a) Assume that the lower flange will be only partially plastic.

$$\sigma_{a-40} = \left[240(a-40)/a\right] \text{ MPa}$$

$$F_C = \left[\frac{(240)(120\times a)}{2}\right] - \left[\frac{(240)(a-40)}{a}\right]\left[\frac{(80)(a-40)}{2}\right]$$

$$F_T = \frac{(240)(80\times a)}{2} - \left[\frac{(240)(160-a)}{a}\right]\left[\frac{(40)(160-a)}{2}\right] + (240)(80)(200-2a)$$



Equating F_C to F_T gives: $a^2 - 120a + 2800 = 0$

From which $a = 88.28 \text{ mm} \cong 88.3 \text{ mm}$ .... **Ans.**

$$200 - 2a = 200 - 2(88.28) = 23.44 \text{ mm} < 40 \text{ mm}$$

Therefore, the lower flange will be only partially plastic as was assumed.

(b) $\sigma_{a-40} = \left[240(88.28-40)/88.28\right] = 131.26 \text{ MPa}$

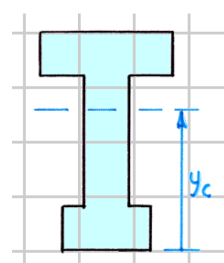
$$\sigma_{160-a} = \left[240(160-88.28)/88.28\right] = 194.98 \text{ MPa}$$

$$M = \left[\frac{(240)(120\times 88.28)}{2}\right]\left[\frac{2(0.08828)}{3}\right] - \left[\frac{(131.26)(80\times 48.28)}{2}\right]\left[\frac{2(0.04828)}{3}\right] + \left[\frac{(240)(80\times 88.28)}{2}\right]\left[\frac{2(0.08828)}{3}\right] - \left[\frac{(194.98)(40\times 71.72)}{2}\right]\left[\frac{2(0.07172)}{3}\right] + (240)(80\times 23.44)(0.100)$$

$M = 148.2(10^3) \text{ N}\cdot\text{m} \cong 148.2 \text{ kN}\cdot\text{m}$ .... **Ans.**

(c) $$y_C = \frac{M_x}{A} = \frac{(20)(80\times 40) + (100)(40\times 120) + (180)(120\times 40)}{(80\times 40) + (40\times 120) + (120\times 40)} = 110 \text{ mm}$$

$$I = \frac{(120)(90)^3}{3} - \frac{(80)(50)^3}{3} + \frac{(80)(110)^3}{3} - \frac{(40)(70)^3}{3} = 56.75(10^6) \text{ mm}^4$$

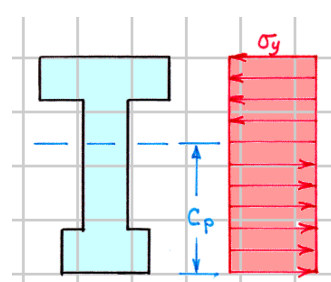


$$M_e = \frac{\sigma I}{c} = \frac{(240)(56.75)}{(0.110)} = 123.82(10^3) \text{ N}\cdot\text{m} = 123.82 \text{ kN}\cdot\text{m}$$

For fully plastic action: $A_T = A_C = 12{,}800/2 = 6400 \text{ mm}^2$

$$A_T = (40\times 80) + (40)(c_p - 40) = 6400 \text{ mm}^2$$

$$c_p = 120 \text{ mm}$$



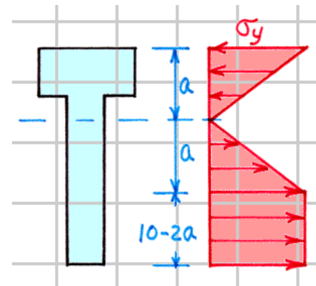
7-134 (cont.)

$$M_p = (240)(40\times 80)(0.100) + (240)(40\times 80)(0.040) + (240)(40\times 120)(0.060) + (240)(40\times 40)(0.020)$$

$$= 184.32(10^3)\text{ N}\cdot\text{m} = 184.32\text{ kN}\cdot\text{m}$$

$$\frac{M_p}{M_e} = \frac{184.32}{123.82} = 1.489$$ .... **Ans.**

7-135*



(a) $\sigma_{a-2} = \left[36(a-2)/a\right]$ ksi

$$F_C = \left[\frac{(36)(4\times a)}{2}\right] - \left[\frac{(36)(a-2)}{a}\right]\left[\frac{(2.5)(a-2)}{2}\right]$$

$$F_T = \frac{(36)(1.5\times a)}{2} + (36)(1.5)(10-2a) = 540 - 81a$$

Equating F_C to F_T gives: $108a^2 - 360a - 180 = 0$

From which $a = 3.775\text{ in.} \cong 3.78\text{ in.}$ .... **Ans.**

(b) $\sigma_{a-2} = \left[36(3.775-2)/3.775\right] = 16.927$ ksi

$$M = \left[\frac{(36)(4\times 3.775)}{2}\right]\left[\frac{2(3.775)}{3}\right] - \left[\frac{(16.927)(2.5\times 1.775)}{2}\right]\left[\frac{2(1.775)}{3}\right]$$
$$+\left[\frac{(36)(1.5\times 3.775)}{2}\right]\left[\frac{2(3.775)}{3}\right] + (36)(1.5\times 2.45)(5)$$

$M = 1558$ kip·in. .... **Ans.**

(c) $$y_C = \frac{M_x}{A} = \frac{(4)(8\times 1.5)+(9)(4\times 2)}{(8\times 1.5)+(4\times 2)} = 6.00\text{ in.}$$

$$I = \frac{(1.5)(6)^3}{3} + \frac{(4)(4)^3}{3} - \frac{(2.5)(2)^3}{3} = 186.67\text{ in.}^4$$

$$M_e = \frac{\sigma I}{c} = \frac{(36)(186.67)}{(6)} = 1120.0\text{ kip}\cdot\text{in.}$$

For fully plastic action: $A_T = A_C = 20/2 = 10\text{ in.}^2$

$A_T = (1.5)(c_p) = 10\text{ in.}^2$ $c_p = 6.667$ in.

$$M_p = (36)(1.5\times 6.667)(3.333) + (36)(1.5\times 1.333)(0.667)$$
$$+(36)(4\times 2)(2.333) = 1919.9\text{ kip}\cdot\text{in.}$$

$$\frac{M_p}{M_e} = \frac{1919.9}{1120.0} = 1.714$$ .... **Ans.**

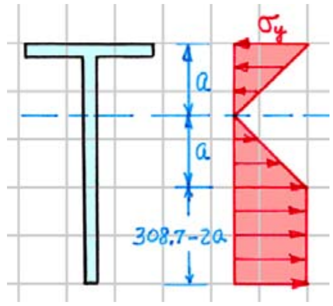
7-136

For a WT 305×70 section

$$d = 2c = 308.7 \text{ mm} \qquad t_w = 13.1 \text{ mm}$$

$$w_f = 230.3 \text{ mm} \qquad t_f = 22.2 \text{ mm}$$

$$A = 8905 \text{ mm}^2 \qquad S = 333(10^3) \text{ mm}^3$$



(a) $\sigma_{a-22.2} = [250(a-22.2)/a]$ MPa

$$F_C = \left[\frac{(250)(230.3\times a)}{2}\right] - \left[\frac{(250)(a-22.2)}{a}\right]\left[\frac{(217.2)(a-22.2)}{2}\right]$$

$$F_T = \frac{(250)(13.1\times a)}{2} + (250)(13.1)(308.7-2a)$$

Equating F_C to F_T gives: $a^2 + 29.69a - 2043 = 0$

From which $a = 32.73 \text{ mm} \cong 32.7 \text{ mm}$ .... **Ans.**

$$308.7 - 2a = 308.7 - 2(32.73) = 243.24 \text{ mm}$$

(b) $\sigma_{a-22.2} = [250(32.73-22.2)/32.73] = 80.44$ MPa

$$M = \left[\frac{(250)(230\times 32.73)}{2}\right]\left[\frac{2(0.03273)}{3}\right] - \left[\frac{(80.44)(217.2\times 10.53)}{2}\right]\left[\frac{2(0.01053)}{3}\right]$$

$$+\left[\frac{(250)(13.1\times 32.73)}{2}\right]\left[\frac{2(0.03273)}{3}\right] + (250)(13.1\times 243.24)(0.15435)$$

$M = 144.04(10^3) \text{ N}\cdot\text{m} \cong 144.0 \text{ kN}\cdot\text{m}$ .... **Ans.**

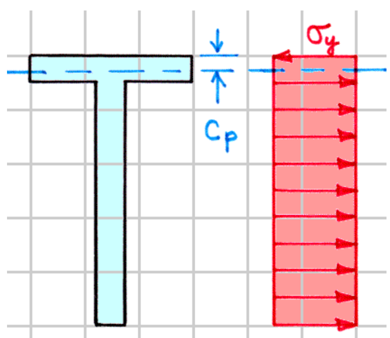
(c) $M_e = \sigma S = (250)(333) = 83.25(10^3) \text{ N}\cdot\text{m} = 83.25 \text{ kN}\cdot\text{m}$

For fully plastic action:

$$A_T = A_C = 8905/2 = 4453 \text{ mm}^2$$

$$A_C = (230.3)(c_p) = 4453 \text{ mm}^2$$

$$c_p = 19.34 \text{ mm} < 22.2 \text{ mm}$$



Therefore, the neutral axis for fully plastic action is in the flange.

$$M_p = (250)(230.3\times 19.34)(0.00967)$$
$$+ (250)(230.3\times 2.86)(0.00143)$$
$$+ (250)(13.1\times 286.5)(0.14611)$$
$$= 148.10(10^3) \text{ N}\cdot\text{m} = 148.10 \text{ kN}\cdot\text{m}$$

$\dfrac{M_p}{M_e} = \dfrac{148.10}{83.25} = 1.779$ .... **Ans.**

7-137*

$$\varepsilon = \frac{\varepsilon_c y}{c} = \frac{\varepsilon_c y}{(h/2)} = \frac{2\varepsilon_c y}{h} \qquad \sigma = K\varepsilon^{1/2} = K\left(\frac{2\varepsilon_c y}{h}\right)^{1/2}$$

$$M = \int y\sigma dA = 2\int_0^{h/2} K\left(\frac{2\varepsilon_c}{h}\right)^{1/2} y^{3/2} b dy = \frac{4}{5}Kb\left(\frac{2\varepsilon_c}{h}\right)^{1/2}\left[y^{5/2}\right]_0^{h/2}$$

$$= \frac{4}{5}b\left(\frac{h}{2}\right)^{5/2}\left(\frac{\sigma}{y^{1/2}}\right)$$

Therefore $\sigma = \dfrac{5\sqrt{2}\,M}{bh^2}\left(\dfrac{y}{h}\right)^{1/2}$ .... **Ans.**

7-138

Since $\sigma_{max} = 99.3 \text{ MPa}$ $\varepsilon_{max} = 3.5(10^{-3})$

$$\sigma = 2792\varepsilon^{0.59} \text{ MPa}$$

(a) $\varepsilon = \dfrac{\varepsilon_c y}{c} = \dfrac{0.0035y}{(c)}$

$$\sigma = 2792\left(\frac{0.0035y}{c}\right)^{0.59} = 99.29\left(\frac{y}{c}\right)^{0.59} \text{ MPa} = 99.29(10^6)\left(\frac{y}{c}\right)^{0.59} \text{ N/m}^2$$

$$F_C = \int_0^{0.050} 99.29(10^6)\left(\frac{y}{c}\right)^{0.59}(0.100)\,dy$$

$$F_T = \int_0^{c} 99.29(10^6)\left(\frac{y}{c}\right)^{0.59}(0.025)\,dy$$

Since $F_C = F_T$: $\displaystyle\int_0^{0.050} 4\left(\frac{y}{c}\right)^{0.59} dy = \int_0^{c}\left(\frac{y}{c}\right)^{0.59} dy$

$$0.02148 = \frac{c^{1.59}}{1.59}$$

Therefore $c = 0.11957 \text{ m} \cong 119.6 \text{ mm}$ .... **Ans.**

(b) $\displaystyle M = \int y\sigma\,dA = \int_0^{0.05} 99.29(10^6)\left(\frac{y}{c}\right)^{0.59} y(0.100)\,dy$

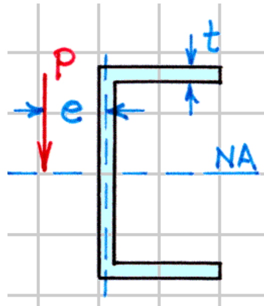
$$+\int_0^{0.11957} 99.29(10^6)\left(\frac{y}{c}\right)^{0.59} y(0.025)\,dy$$

$$= 34.76(10^6)\int_0^{0.05} y^{1.59}dy + 8.69(10^6)\int_0^{0.11957} y^{1.59}dy$$

Therefore $M = 19.43(10^3) \text{ N}\cdot\text{m} = 19.43 \text{ kN}\cdot\text{m}$ .... **Ans.**

7-139*

$$I = \frac{(4)(7)^3}{12} - \frac{(3.75)(7-2t)^3}{12} = \left[114.33 - \frac{3.75(7-2t)^3}{12}\right] \text{ in.}^4$$

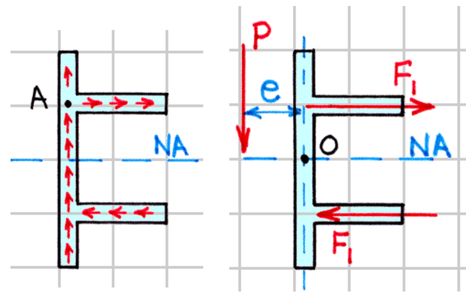


For a channel section:

$$e = \frac{b^2h^2t}{4I} = \frac{(3.875)^2(7-t)^2 t}{4\left[114.33 - \dfrac{3.75(7-2t)^3}{12}\right]} = 1.50 \text{ in.}$$

Solving by trial and error yields: $t = 0.250$ in. .... **Ans.**

7-140*



$$I = \frac{(2.5)(100)^3}{12} + 2(2.5 \times 50)(25)^2$$

$$= 364.6(10^3) \text{ mm}^4$$

(a) $Q_A = (25)(50 \times 2.5) = 3125 \text{ mm}^3$

$$\tau_A = \frac{VQ_A}{It_f} = \frac{(2500)(3125 \times 10^{-9})}{(364.6 \times 10^{-9})(0.0025)} = 8.571(10^6) \text{ N/m}^2$$

$$F_1 = (\tau_f A_f / 2) = (8.571 \times 10^6)(0.0025 \times 0.050)/2 = 535.7 \text{ N}$$

$\circlearrowleft \Sigma M_O = 0:$ $\quad Pe - F_1 d = 0$

$e = \dfrac{(535.7)(50)}{2500} = 10.71 \text{ mm}$ .... **Ans.**

(b) $Q_O = (25)(50 \times 2.5) + (25)(50 \times 2.5) = 6250 \text{ mm}^3$

$$\tau_O = \frac{VQ_O}{It_w} = \frac{(2500)(6250 \times 10^{-9})}{(364.6 \times 10^{-9})(0.0025)} = 17.14(10^6) \text{ N/m}^2$$

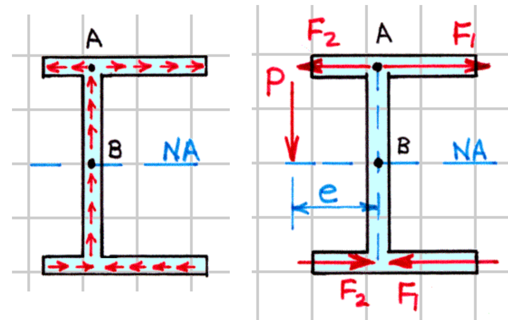
$\tau_O = 17.14 \text{ MPa}$ .... **Ans.**

7-141

$Q_{A1} = (6.5)(7 \times 1) = 45.5 \text{ in.}^3$ $Q_{A2} = (6.5)(3 \times 1) = 19.5 \text{ in.}^3$

$Q_{Aw} = 45.5 + 19.5 = 65.0 \text{ in.}^3$ $Q_B = (65.0) + (3)(6 \times 1) = 83.0 \text{ in.}^3$

$$I = \frac{bh^3}{12} = \frac{(10)(14)^3}{12} - \frac{(9)(12)^3}{12} = 990.7 \text{ in.}^4$$



(a) $$\tau_{A1} = \frac{VQ_{A1}}{It_f} = \frac{(100)(45.5)}{(990.7)(1)} = 4.593 \text{ ksi}$$

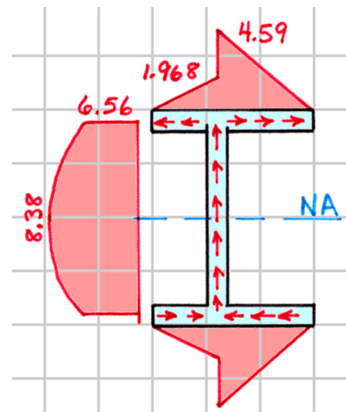
$$F_1 = (\tau_{A1} A_1/2) = (4.593)(7 \times 1)/2 = 16.076 \text{ kip}$$

$$\tau_{A2} = \frac{VQ_{A2}}{It_f} = \frac{(100)(19.5)}{(990.7)(1)} = 1.968 \text{ ksi}$$

$$F_2 = (\tau_{A2} A_2/2) = (1.968)(3 \times 1)/2 = 2.952 \text{ kip}$$

$\circlearrowleft \Sigma M_B = 0: \qquad Pe - (F_1 - F_2)d = 0$

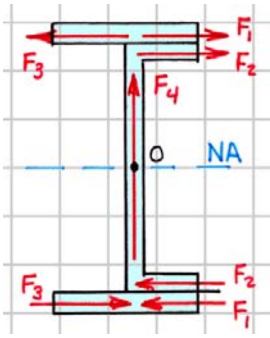
$$e = \frac{(16.076 - 2.952)(13)}{100} = 1.706 \text{ in.}$$ .... **Ans.**



(b) $$\tau_{Aw} = \frac{VQ_{Aw}}{It_w} = \frac{(100)(65.0)}{(990.7)(1)} = 6.56 \text{ ksi}$$

$$\tau_B = \frac{VQ_B}{It_w} = \frac{(100)(83.0)}{(990.7)(1)} = 8.38 \text{ ksi}$$

7-142



$$F = \tau_{avg} A = \frac{VQA}{2It}$$

(a) $$F_1 = \frac{V}{2I}\left[\frac{(253)(93\times 14)}{14}\right](93\times 14) = \frac{V}{I}\left(15.317\times 10^6\right)$$

$$F_2 = \frac{V}{2I}\left[\frac{(222)(93\times 16)}{16}\right](93\times 16) = \frac{V}{I}\left(15.360\times 10^6\right)$$

$$F_3 = \frac{V}{2I}\left[\frac{(253)(b-93)(14)}{14}\right](b-93)(14) = \frac{V}{I}(b-93)^2\left(1.7710\times 10^3\right)$$

$$\circlearrowleft \Sigma M_O = 0: \qquad F_3 h_3 - F_1 h_1 - F_2 h_2 = 0$$

$$\left[\frac{V}{I}(b-93)^2\left(1.7710\times 10^3\right)\right](474) = \left[\frac{V}{I}\left(15.317\times 10^6\right)\right](474) + \left[\frac{V}{I}\left(15.360\times 10^6\right)\right](444)$$

$$(b-93)^2 = 16.773\left(10^3\right)$$

$$b^2 - 186b - 8124.0 = 0$$

$b = 222.5 \text{ mm} \cong 223 \text{ mm}$ .... **Ans.**

(b) $$Q_{NA} = (237)(223\times 14) + (222)(93\times 16) + (111)(222\times 14) = 1.4152\left(10^6\right)\ \text{mm}^3$$

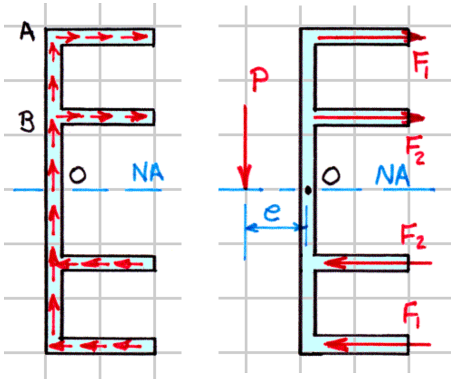
$$I = 2(223\times 14)(237)^2 + 2(93\times 16)(222)^2 + \frac{(16)(444)^3}{12}$$

$$= 614.1\left(10^6\right)\ \text{mm}^4$$

$$\tau_{\max} = \frac{VQ_{NA}}{It_w} = \frac{(40{,}000)\left(1.4152\times 10^{-3}\right)}{\left(614.1\times 10^{-6}\right)(0.014)} = 6.58\left(10^6\right)\ \text{N/m}^2$$

$\tau_{\max} = 6.58 \text{ MPa}$ .... **Ans.**

7-143*



$$Q_A = (6)(4\times 0.25) = 6.0 \text{ in.}^3$$

$$Q_B = (3)(4\times 0.25) = 3.0 \text{ in.}^3$$

$$I \cong \frac{(0.25)(12)^3}{12} + 2(4\times 0.25)(6)^2 + 2(4\times 0.25)(3)^2 = 126.0 \text{ in.}^4$$

(a) $$F_1 = \frac{\tau_A A_F}{2} = \left[\frac{V(6)}{(126.0)(0.25)}\right]\frac{(4\times 0.25)}{2} = 0.09524V$$

$$F_2 = \frac{\tau_B A_F}{2} = \left[\frac{V(3)}{(126.0)(0.25)}\right]\frac{(4\times 0.25)}{2} = 0.04762V$$

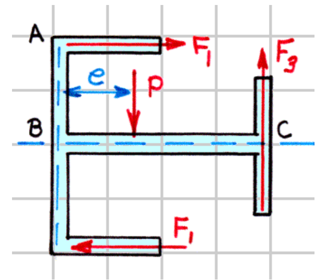
$\circlearrowleft \Sigma M_O = 0$: $\quad Pe - F_1(12) - F_2(6) = 0$

Since $V = P$: $\quad e = 0.09524(12) + 0.04762(6) = 1.429$ in. .... **Ans.**

(b) $$Q_{NA} = (6)(4\times 0.25) + (3)(4\times 0.25) + (3)(6\times 0.25) = 13.5 \text{ in.}^3$$

$$\tau_O = \frac{VQ_{NA}}{It_w} = \frac{(1500)(13.5)}{(126.0)(0.25)} = 643 \text{ psi}$$ .... **Ans.**

7-144*



$$Q_A = (90)(90\times 6) = 48.6(10^3)\ \text{mm}^3$$

$$Q_B = (90)(90\times 6) + (45)(90\times 6) = 72.9(10^3)\ \text{mm}^3$$

$$Q_C = (30)(60\times 6) = 10.8(10^3)\ \text{mm}^3$$

$$I \cong \frac{(6)(180)^3}{12} + \frac{(6)(120)^3}{12} + 2(90\times 6)(90)^2 = 12.528(10^6)\ \text{mm}^4$$

$$\tau_A = \frac{VQ_A}{It} = \frac{(6000)(48.6\times 10^{-6})}{(12.528\times 10^{-6})(0.006)} = 3.879(10^6)\ \text{N/m}^2 \cong 3.88\ \text{MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{(6000)(72.9\times 10^{-6})}{(12.528\times 10^{-6})(0.006)} = 5.819(10^6)\ \text{N/m}^2 \cong 5.82\ \text{MPa}$$

$$\tau_C = \frac{VQ_C}{It} = \frac{(6000)(10.8\times 10^{-6})}{(12.528\times 10^{-6})(0.006)} = 0.8621(10^6)\ \text{N/m}^2 \cong 0.862\ \text{MPa}$$

Therefore: $\tau_{max} = \tau_B = 5.82\ \text{MPa}$ .... **Ans.**

$$F_1 = (\tau_A A_1/2) = (3.879\times 10^6)(0.090\times 0.006)/2 = 1047.3\ \text{N}$$

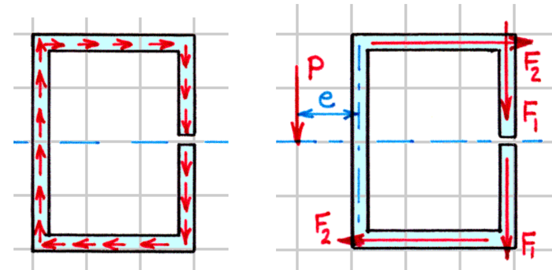
$$F_3 = (\tau_C A_3/2) = (0.8621\times 10^6)(0.120\times 0.006)/2 = 413.8\ \text{N}$$

$$\circlearrowleft \Sigma M_B = 0: -Pe - F_1(180) + F_3(180) = 0$$

Since $V = P$:

$$e = \frac{(413.8)(180) - (1047.3)(180)}{6000} = -19.00\ \text{mm} = 19.00\ \text{mm} \leftarrow$$ .... **Ans.**

7-145



$$I \cong 2\left[\frac{th^3}{12}\right] + 2\left[(bt)\left(\frac{h}{2}\right)^2\right] = \frac{th^2}{6}(h+3b)$$

$$\tau_1 = \frac{V}{It}\int_0^s yt\,dy = \frac{Vs^2}{2I}$$

$$F_1 = \int \tau_1\,dA = \int_0^{h/2}\frac{Vs^2}{2I}(t\,ds) = \frac{Vh^3t}{48I}$$

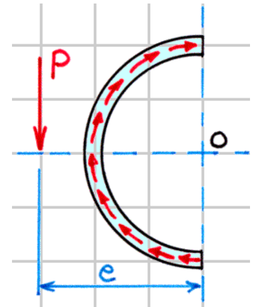
$$\tau_2 = \tau_1\big|_{s=h/2} + \frac{V}{It}\int_0^s\left(\frac{h}{2}\right)(t\,dx) = \frac{Vh^2}{8I} + \frac{Vhs}{2I}$$

$$F_2 = \int \tau_2\,dA = \int_0^b\left(\frac{Vh^2}{8I} + \frac{Vhs}{2I}\right)(t\,ds) = \frac{Vhbt}{8I}(h+2b)$$

$\circlearrowleft \Sigma M_O = 0:$ $\quad Pe - 2F_1 b - F_2 h = 0$

Since $V = P$: $\quad e = \dfrac{h^2 tb}{12I}(2h+3b) = \dfrac{b(2h+3b)}{2(h+3b)}$ .... **Ans.**

7-146*



(a) $$I = \int y^2\, dA = \int_0^{\pi} \left(R\cos\theta\right)^2 \left(tR\, d\theta\right) = \frac{\pi R^3 t}{2}$$

$$\tau = \frac{V}{It}\int_0^s yt\, ds = \frac{V}{I}\int_0^{\theta}\left(R\cos\theta\right)^2\left(R\, d\theta\right) = \frac{VR^2}{I}\sin\theta$$

$$T = \int R\, dF = \int R\tau\, dA = \int_0^{\pi} R\left(\frac{VR^2}{I}\sin\theta\right)\left(tR\, d\theta\right)$$

$$= \frac{VR^4 t}{I}\int_0^{\pi}\sin\theta\, d\theta = \frac{2VR^4 t}{I} = \frac{2VR^4 t}{\pi R^3 t/2} = \frac{4VR}{\pi}$$

$\circlearrowleft \Sigma M_O = 0:$ $\qquad Pe - T = 0 \qquad Pe - 4VR/\pi = 0$

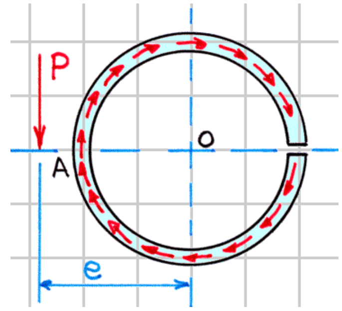
Since $V = P$: $\qquad e = 4R/\pi$ .... **Ans.**

(b) $$I = \pi R^3 t/2 = \pi(25)^3(2.5)/2 = 61.359\left(10^3\right) \text{ mm}^4$$

$$\tau_A = \frac{VR^2}{I}\sin\theta_A = \frac{(440)(0.025)^2}{\left(61.359\times 10^{-9}\right)}\sin 90° = 4.48\left(10^6\right) \text{ N/m}^2$$

$\tau_A = 4.48$ MPa .... **Ans.**

7-147*



(a) $I = \int y^2\, dA = \int_0^{2\pi} (R\sin\theta)^2 (tR\, d\theta) = \pi R^3 t$

$$\tau = \frac{V}{It}\int_0^s yt\, ds = \frac{V}{I}\int_0^\theta (R\sin\theta)^2 (R\, d\theta) = \frac{VR^2}{I}(1-\cos\theta)$$

$$T = \int R\, dF = \int R\tau\, dA = \int_0^{2\pi} R\left[\frac{VR^2}{I}(1-\cos\theta)\right](tR\, d\theta)$$

$$= \frac{VR^4 t}{I}\int_0^{2\pi}(1-\cos\theta)\, d\theta = \frac{2\pi VR^4 t}{I} = \frac{2\pi VR^4 t}{\pi R^3 t} = 2VR$$

$\circlearrowleft \Sigma M_O = 0:$ $\quad Pe - T = 0 \quad Pe - 2VR = 0$

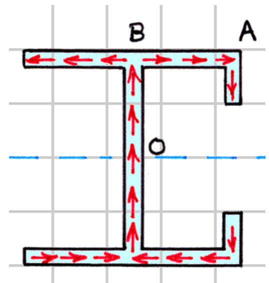
Since $V = P$: $\quad e = 2R$ .... **Ans.**

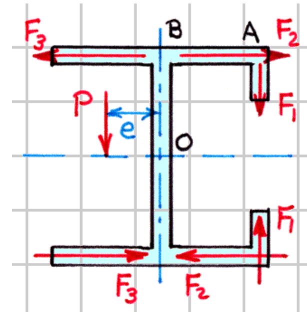
(b) $I = \pi R^3 t = \pi(2)^3(0.1) = 2.51327 \text{ in.}^4$

$$\tau_A = \frac{VR^2}{I}(1-\cos\theta_A) = \frac{(110)(2)^2}{(2.51327)}(1-\cos 180°) = 350 \text{ psi}$$ .... **Ans.**

7-148

Let $a = 20$ mm





$$Q_A = (1.5a)(at) = 1.5a^2t$$

$$Q_{B2} = 1.5a^2t + (2a)(2at) = 5.5a^2t$$

$$Q_{B3} = (2a)(2at) = 4a^2t$$

$$Q_{Bw} = 5.5a^2t + 4a^2t = 9.5a^2t$$

$$Q_O = 9.5a^2t + (a)(2at) = 11.5a^2t$$

$$I = \frac{(2t)(4a)^3}{12} - \frac{(t)(2a)^3}{12} + 2(4a \times t)(2a)^2 = 42a^3t$$

(a) $$\tau_1 = \frac{V}{It}\int_0^s y(t\,ds) = \frac{V}{It}\int_0^s (a+s)t\,ds = \frac{V}{I}\left(as + \frac{s^2}{2}\right)$$

$$F_1 = \int \tau_1\,dA = \int_0^a \frac{V}{I}\left(as + \frac{s^2}{2}\right)(t\,ds) = \frac{2Va^3t}{3I} = \frac{2Va^3t}{3(42a^3t)} + \frac{V}{63}$$

$$F_2 = \tau_{avg}A_2 = \frac{VQ_{avg}}{It}(2at) = \frac{V(3.5a^2t)}{(42a^3t)(t)}(2at) = \frac{V}{6}$$

$$F_3 = \tau_{avg}A_3 = \frac{VQ_{avg}}{It}(2at) = \frac{V(2a^2t)}{(42a^3t)(t)}(2at) = \frac{V}{10.5}$$

$\circlearrowleft \Sigma M_O = 0$: $$Pe - 2F_1(2a) - (F_2 - F_3)(4a) = 0$$

$$Pe - 2(V/63)(2a) - (V/6)(4a) + (V/10.5)(4a) = 0$$

Since $V = P$: $$e = \frac{4a}{63} + \frac{4a}{6} - \frac{4a}{10.5} = \frac{22a}{63} = \frac{22(20)}{63} = 6.98 \text{ mm}$$ .... **Ans.**

7-149

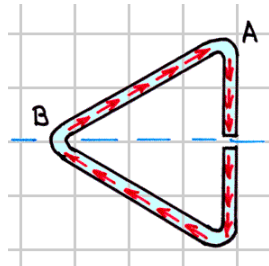
$$Q_A = (1)(2\times 0.125) = 0.25 \text{ in.}^3$$

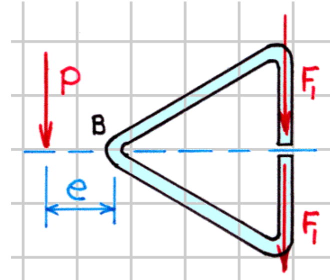
$$Q_B = 0.25 + (1)(4\times 0.125) = 0.75 \text{ in.}^3$$

$$I = \frac{(0.375)(4)^3}{12} = 2.00 \text{ in.}^4$$

$$\tau_A = \frac{VQ_A}{It} = \frac{(300)(0.25)}{(2.00)(0.125)} = 300 \text{ psi}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{(300)(0.75)}{(2.00)(0.125)} = 900 \text{ psi}$$





Therefore:

$$\tau_{max} = \tau_B == 900 \text{ psi}$$ .... **Ans.**

$$F_1 = \frac{\tau_A A_1}{3} = \frac{(300)(2\times 0.125)}{3} = 25.00 \text{ lb}$$

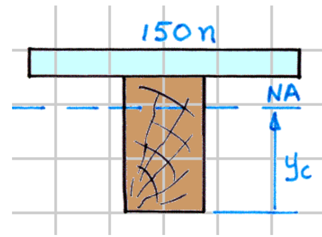
$$\circlearrowleft \Sigma M_B = 0: \qquad Pe - 2F_1(4\cos 30°) = 0$$

Therefore:

$$e = \frac{2(25.00)(4\cos 30°)}{(300)} = 0.577 \text{ in.}$$ .... **Ans.**

7-150*

$$n = \frac{E_s}{E_w} = \frac{200}{10} = 20$$

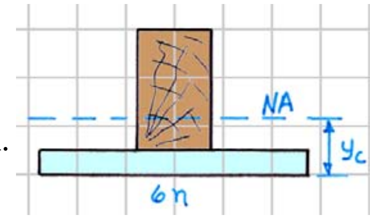


$$y_C = \frac{(175)\left[(150)(350)\right] + (357.5)\left[(3000)(15)\right]}{(150)(350) + (3000)(15)} = 259.2 \text{ mm}$$

$$\sigma_w = \frac{-c_w}{c_s}\left(\frac{\sigma_s}{n}\right) = \frac{-259.2}{105.8}\left(\frac{75}{20}\right) = -9.19 \text{ MPa}$$

$\sigma_w = 9.19$ MPa (C) .... **Ans.**

7-151*



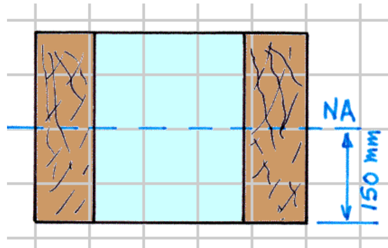
$$n = \frac{E_s}{E_w} = \frac{30,000}{1500} = 20$$

$$y_C = \frac{M_x}{A} = \frac{(6.5)\left[(6)(12)\right] + (0.25)\left[(120)(0.5)\right]}{(6)(12) + (120)(0.5)} = 3.659 \text{ in.}$$

$$\sigma_s = \frac{-c_s}{c_w}(n\sigma_w) = \frac{-3.659}{8.841}(20)(-1250) = +10,347 \text{ psi}$$

$\sigma_s \cong 10.35$ ksi (T) .... **Ans.**

7-152



$$n = \frac{E_a}{E_w} = \frac{73}{8} = 9.125$$

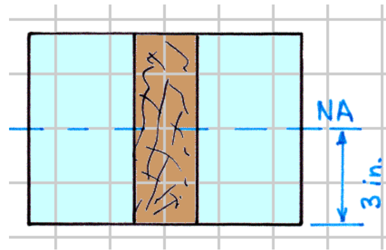
$$M = \left(\frac{P}{2}\right)\left(\frac{L}{2}\right) = \frac{PL}{4} = \frac{(30)(4)}{4} = 30 \text{ kN}\cdot\text{m}$$

$$I_T = \frac{(200 + 228.1)(300)^3}{12} = 963.2\left(10^6\right) \text{ mm}^4$$

$$\sigma_w = \frac{Mc}{I_T} = \frac{(30{,}000)(0.150)}{\left(963.2\times 10^{-6}\right)} = 4.672\left(10^6\right) \text{ N/m}^2 \cong 4.67 \text{ MPa (T)}$$ .... **Ans.**

$$\sigma_a = n\sigma_w = (9.125)(4.672) = 42.6 \text{ MPa (T)}$$ .... **Ans.**

7-153



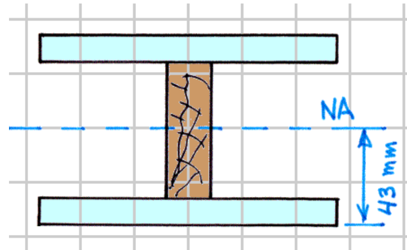
$$n = \frac{E_s}{E_w} = \frac{29,000}{1600} = 18.125$$

$$I_T = \frac{(4 + 18.125)(6)^3}{12} = 398.25 \text{ in.}^4$$

$$\sigma_w = \frac{Mc}{I_T} = \frac{(10,000 \times 12)(3)}{(398.25)} = 903.95 \text{ psi} \cong 904 \text{ psi (T)}$$ .... **Ans.**

$$\sigma_s = n\sigma_w = (18.125)(903.95) = 16,384 \text{ psi} \cong 16,380 \text{ psi (T)}$$ .... **Ans.**

7-154*



$$n = \frac{E_a}{E_w} = \frac{70}{10} = 7.00$$

$$\sigma_w = \frac{My_w}{I_T} = \frac{(3000)(0.040)}{I_T} \le 15(10^6) \text{ N/m}^2$$

$$I_T \ge 8.000(10^{-6}) \text{ m}^4$$

$$\sigma_a = \frac{nMy_a}{I_T} = \frac{7(3000)(0.043)}{I_T} \le 135(10^6) \text{ N/m}^2 \qquad I_T \ge 6.689(10^{-6}) \text{ m}^4$$

$$I_T = \frac{(7w)(86)^3}{12} - \frac{(7w-50)(80)^3}{12} \ge 8.000(10^6) \text{ mm}^4$$

$$72{,}366w \ge 5.8667(10^6)$$

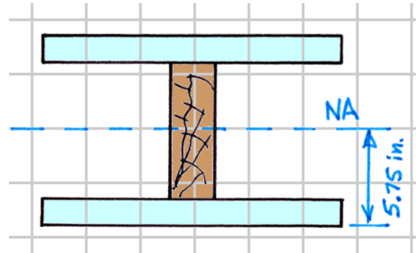
$w \ge 81.1$ mm .... **Ans.**

7-155

$$n = \frac{E_s}{E_w} = \frac{30,000}{1600} = 18.75$$

$$M = PL = (4000)(6) = 24,000 \text{ lb} \cdot \text{ft}$$

$$I_T = \frac{(75)(11.5)^3}{12} - \frac{(71)(10)^3}{12} = 3589 \text{ in.}^4$$

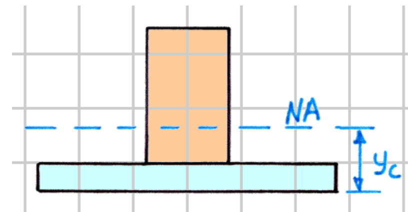


$$\sigma_w = \frac{My_w}{I_T} = \frac{(24,000 \times 12)(5)}{(3598)} = 401.2 \text{ psi} \cong 401 \text{ psi (T \& C)}$$ .... **Ans.**

$$\sigma_s = \frac{nMy_s}{I_T} = \frac{18.75(24,000 \times 12)(5.75)}{(3598)} = 8651 \text{ psi} \cong 8650 \text{ psi (T \& C)}$$ .... **Ans.**

7-156

$$n = \frac{E_b}{E_p} = \frac{100}{1.4} = 71.43$$



$$\sigma_p = \frac{My_p}{I_T} = \frac{M(131 - y_C)}{I_T}$$

$$\sigma_b = \frac{nMy_b}{I_T} = \frac{71.43My_C}{I_T}$$

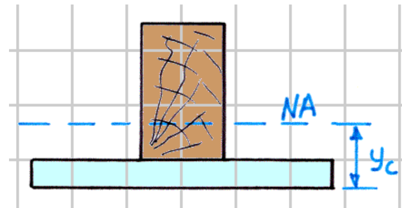
For $\sigma_b = 10\sigma_p$: $71.43y_C = 10(131 - y_C)$

$$y_C = 16.088 \text{ mm}$$

$$y_C = \frac{(3)[(6)(71.43w)] + (68.5)[(50)(125)]}{(6)(71.43w) + (50)(125)} = 16.088 \text{ mm}$$

$5609w = 327{,}575$ $w = 58.4$ mm .... **Ans.**

7-157*



$$n = \frac{E_s}{E_w} = \frac{30,000}{1600} = 18.75$$

$$M = \frac{wL^2}{8} = \frac{w(16)}{8} = 32w$$

$$y_C = \frac{M_x}{A} = \frac{(8)\left[(8)(15)\right]+(0.25)\left[(150)(0.5)\right]}{(8)(15)+(150)(0.5)} = 5.019 \text{ in.}$$

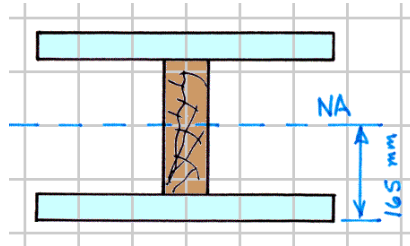
$$I_T = \frac{(8)(10.481)^3}{3} + \frac{(150)(5.019)^3}{3} - \frac{(142)(4.519)^3}{3} = 5024 \text{ in.}^4$$

$$\sigma_w = \frac{My_w}{I_T} = \frac{(32w\times 12)(10.481)}{(5024)} \le 1600 \text{ psi} \qquad w \le 1997.3 \text{ lb/ft}$$

$$\sigma_s = \frac{nMy_s}{I_T} = \frac{18.75(32w\times 12)(5.019)}{(5024)} \le 18,000 \text{ psi} \qquad w \le 2502.5 \text{ lb/ft}$$

Therefore: $w_{max} = 1997$ lb/ft .... **Ans.**

7-158*



$$n = \frac{E_s}{E_w} = \frac{200}{13} = 15.38$$

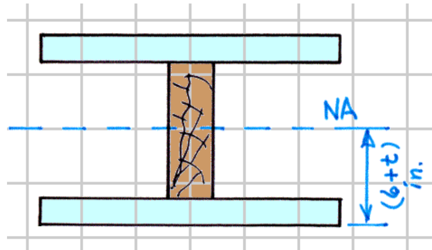
$$M = \frac{wL^2}{8} = \frac{(20)(5)^2}{8} = 62.50 \text{ kN}\cdot\text{m}$$

$$I_T = \frac{(2307)(330)^3}{12} - \frac{(2157)(300)^3}{12} = 2056(10^6) \text{ mm}^4$$

$$\sigma_w = \frac{My_w}{I_T} = \frac{(6250\times10^3)(0.150)}{(2056\times10^{-6})} = 4.560(10^6) \text{ N/m}^2 \cong 4.56 \text{ MPa (T)}$$ .... **Ans.**

$$\sigma_a = \frac{y_s}{y_w}(n\sigma_w) = \frac{165}{150}(15.38)(4.560) = 77.1 \text{ MPa (T)}$$ .... **Ans.**

7-159



$$n = \frac{E_s}{E_w} = \frac{30,000}{1500} = 20$$

$$M = (2.50)(10) = 25.0 \text{ kip} \cdot \text{ft}$$

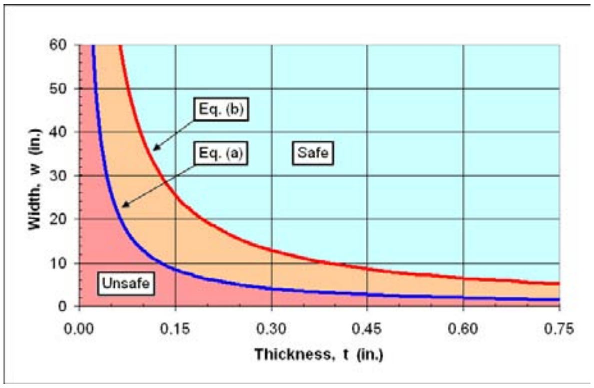
$$I_T = \frac{(6)(12)^3}{12} + \frac{(20w)(t)^3}{12} + (20wt)(6+0.5t)^2$$

$$\sigma_w = \frac{My_w}{I_T} = \frac{(25.0 \times 12)(6)}{I_T} \le 1 \text{ ksi} \qquad I_T \ge 1800 \text{ in.}^4$$

$$I_T = 864 + 1.66667wt^3 + 20wt(6+0.5t)^2 \ge 1800 \text{ in.}^4 \qquad \text{(a)}$$

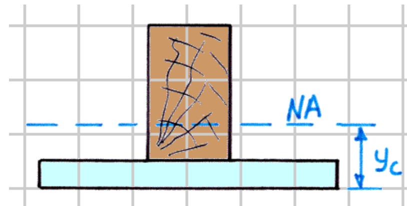
$$\sigma_s = \frac{nMy_s}{I_T} = \frac{20(25.0 \times 12)(6+t)}{I_T} \le 10 \text{ ksi} \qquad I_T \ge 600(6+t) \text{ in.}^4$$

$$I_T = 864 + 1.66667wt^3 + 20wt(6+0.5t)^2 \ge 600(6+t) \text{ in.}^4 \qquad \text{(b)}$$



7-160

$$n = \frac{E_s}{E_w} = \frac{200}{12} = 16.667$$



$$M = 2(P/2) = (P) \text{ N}\cdot\text{m}$$

$$y_C = \frac{M_x}{A} = \frac{(8)\left[(3333)(16)\right] + (191)\left[(200)(350)\right]}{(3333)(16) + (200)(350)}$$

$$= 111.87 \text{ mm}$$

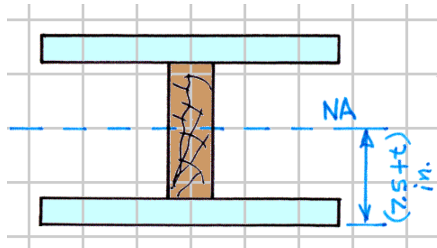
$$I_T = \frac{(3333)(111.87)^3}{3} - \frac{(3133)(95.87)^3}{3} + \frac{(200)(254.13)^3}{3} = 1729.4\left(10^6\right) \text{ mm}^4$$

$$\sigma_w = \frac{My_w}{I_T} = \frac{(P)(0.25413)}{\left(1729.4\times10^{-6}\right)} \le 10\left(10^6\right) \text{ N/m}^2 \qquad P \le 68.05\left(10^3\right) \text{ N}$$

$$\sigma_s = \frac{nMy_s}{I_T} = \frac{16.667(P)(0.11187)}{\left(1729.4\times10^{-6}\right)} \le 75\left(10^6\right) \text{ N/m}^2 \qquad P \le 69.56\left(10^3\right) \text{ N}$$

Therefore: $P_{max} = 68.1$ kN .... **Ans.**

7-161



(a) $$I_w = \frac{bh^3}{12} = \frac{(8)(15)^3}{12} = 2250 \text{ in.}^4$$

$$M = \frac{\sigma_w I_w}{c_w} = \frac{(2.4)(2250)}{7.5}$$

$$M_{\max} = 720 \text{ kip} \cdot \text{in.}$$ .... **Ans.**

(b) $$n = \frac{E_s}{E_w} = \frac{30{,}000}{1600} = 18.75$$

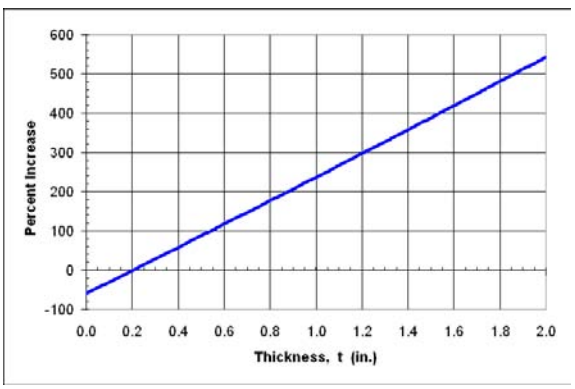
$$I_T = \frac{(8)(15)^3}{12} + 2\left[\frac{(150)(t)^3}{12} + (150t)(7.5 + 0.5t)^2\right]$$

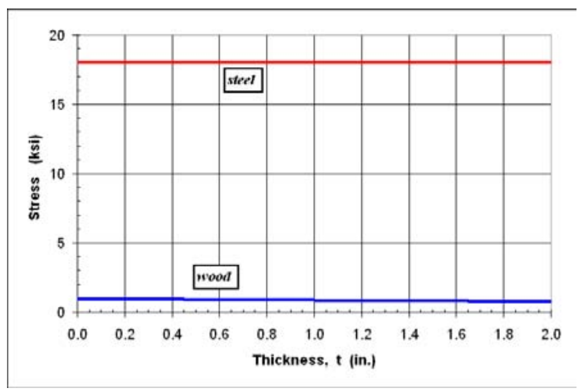
$$= \left[2250 + 25t^3 + 300t(7.5 + 0.5t)^2\right] \text{ in.}^4$$

Either: $$M = \frac{\sigma_w I_T}{c_w} = \frac{(2.4)(I_T)}{7.5}$$ or $$M = \frac{\sigma_s I_T}{n c_s} = \frac{(18)(I_T)}{18.75(7.5 + t)}$$

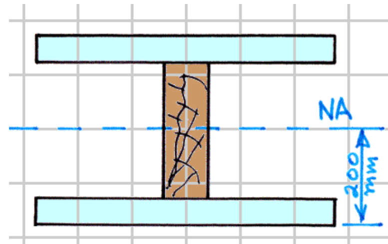
Choose smaller M and compute $$\Delta M = \frac{M - 720}{720}(100)$$

$$\sigma_w = \frac{M c_w}{I_T} = \frac{M(7.5)}{I_T}$$ $$\sigma_s = \frac{n M c_s}{I_T} = \frac{18.75 M(7.5 + t)}{I_T}$$





7-162



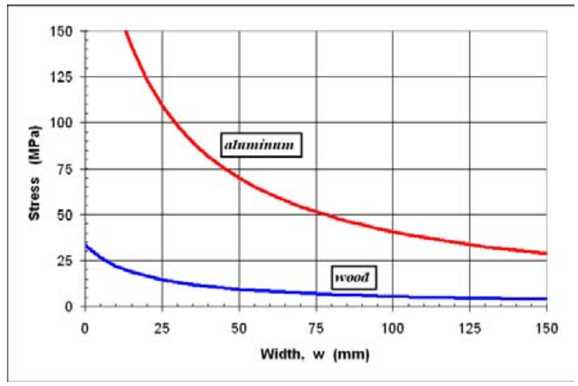
(a) $n = \frac{E_a}{E_w} = \frac{73}{13} = 5.615$

$$I_T = \frac{(150)(300)^3}{12} + 2\left[\frac{(nw)(50)^3}{12} + (50nw)(150+25)^2\right]$$

$$= \left[337.5\left(10^6\right) + 3.083\left(10^6\right)nw\right] \text{ mm}^4$$

$$\sigma_w = \frac{Mc_w}{I_T} = \frac{\left(75\times10^3\right)(0.150)}{I_T}$$

$$\sigma_a = \frac{nMc_a}{I_T} = \frac{5.615\left(75\times10^3\right)(0.200)}{I_T}$$

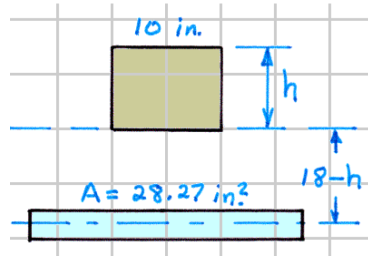


(b) For $\sigma_w = 15$ MPa $\quad w = 23.8$ mm

For $\sigma_a = 135$ MPa $\quad w = 16.54$ mm

Therefore $w_{\min} = 23.8$ mm .... **Ans.**

7-163*



$$A_s = 3\left[\pi(1)^2/4\right] = 2.356 \text{ in.}^2$$

$$A_T = nA_s = 12(2.356) = 28.27 \text{ in.}^2$$

$$10h(h/2) = 28.27(18 - h)$$

$$5h^2 + 28.27h - 508.86 = 0$$

From which: $h = 7.650$ in. $18 - h = 10.350$ in.

$$I_T = \frac{(10)(7.650)^3}{3} + (28.27)(10.350)^2 = 4521 \text{ in.}^4$$

$$\sigma_c = \frac{My_c}{I_T} = \frac{(M \times 12)(7.650)}{(4521)} \le 1000 \text{ psi}$$

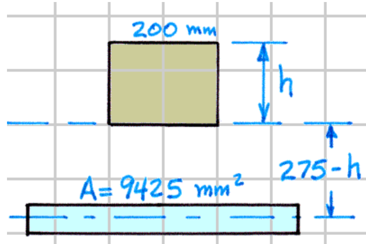
$$M \le 49,248 \text{ lb} \cdot \text{ft}$$

$$\sigma_s = \frac{nMy_s}{I_T} = \frac{12(M \times 12)(10.350)}{(4521)} \le 18,000 \text{ psi}$$

$$M \le 54,601 \text{ lb} \cdot \text{ft}$$

Therefore: $M_{\max} = 49.2 \text{ kip} \cdot \text{ft}$ .... **Ans.**

7-164*



$$n = \frac{E_s}{E_c} = \frac{200}{15} = 13.333$$

$$A_s = 4\left[\pi(15)^2/4\right] = 706.9 \text{ mm}^2$$

$$A_T = nA_s = 13.333(706.9) = 9425 \text{ mm}^2$$

$$200h(h/2) = 9425(275 - h)$$

$$h^2 + 94.25h - 25{,}919 = 0$$

From which: $h = 120.62$ mm $\quad 275 - h = 154.38$ mm

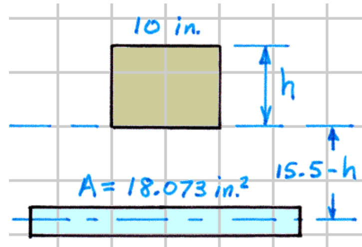
$$I_T = \frac{(200)(120.62)^3}{3} + (9425)(154.38)^2 = 341.6(10^6) \text{ mm}^4$$

$$\sigma_c = \frac{My_c}{I_T} = \frac{(15{,}000)(0.12062)}{(341.6\times10^6)} = 5.30(10^6) \text{ N/m}^2$$

$\sigma_c = 5.30$ MPa (C) .... **Ans.**

$\sigma_s = \dfrac{y_s}{y_c}(n\sigma_c) = \dfrac{(154.38)(13.333\times5.297)}{(120.62)} = 90.4$ MPa (T) .... **Ans.**

7-165



$$n = \frac{E_s}{E_c} = \frac{30,000}{2200} = 13.636$$

$$M_{\max} = \frac{wL^2}{8} = \frac{(820)(13)^2}{8} = 17,322 \text{ lb} \cdot \text{ft}$$

$$A_s = 3\left[\pi(0.75)^2/4\right] = 1.3254 \text{ in.}^2$$

$$A_T = nA_s = 13.636(1.3254) = 18.073 \text{ in.}^2$$

$$10h(h/2) = 18.073(15.5 - h)$$

$$h^2 + 3.615h - 56.02 = 0$$

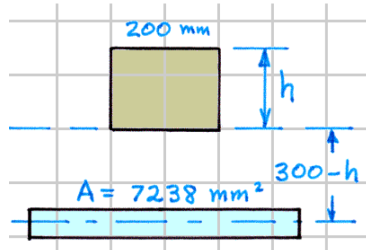
From which: $h = 5.892$ in. $15.5 - h = 9.608$ in.

$$I_T = \frac{(10)(5.892)^3}{3} + (18.073)(9.608)^2 = 2350 \text{ in.}^4$$

$$\sigma_c = \frac{My_c}{I_T} = \frac{(17.322 \times 12)(5.892)}{(2350)} = 0.5212 \text{ ksi} \cong 0.521 \text{ ksi (C)}$$ .... **Ans.**

$$\sigma_s = \frac{y_s}{y_c}(n\sigma_c) = \frac{(9.608)(13.636 \times 0.5212)}{(5.892)} = 11.59 \text{ ksi (T)}$$ .... **Ans.**

7-166*



$$n = \frac{E_s}{E_c} = 12$$

$$M_{\max} = \frac{3wL^2}{32} = \frac{3w(4)^2}{32} = (1.5w) \text{ N}\cdot\text{m}$$

$$A_s = 3\left[\pi(16)^2/4\right] = 603.2 \text{ mm}^2$$

$$A_T = nA_s = 12(603.2) = 7238 \text{ mm}^2$$

$$200h(h/2) = 7238(300-h)$$

$$h^2 + 72.38h - 21{,}714 = 0$$

From which: $h = 115.55$ mm $\quad 300 - h = 184.45$ mm

$$I_T = \frac{(200)(115.55)^3}{3} + (7238)(184.45)^2 = 349.1(10^6) \text{ mm}^4$$

$$\sigma_c = \frac{My_c}{I_T} = \frac{(1.5w)(0.11555)}{(349.1\times10^{-6})} \le 6.5(10^6) \text{ N/m}^2 \qquad w \le 13.092(10^3) \text{ N/m}$$

$$\sigma_s = \frac{nMy_s}{I_T} = \frac{12(1.5w)(0.18445)}{(349.1\times10^{-6})} \le 120(10^6) \text{ N/m}^2 \qquad w \le 12.618(10^3) \text{ N/m}$$

Therefore: $w_{\max} = 12.62$ kN/m .... **Ans.**

7-167

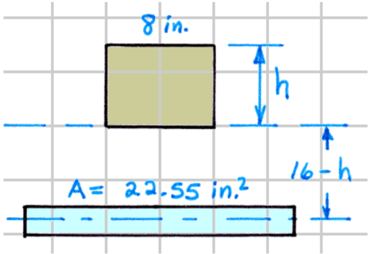
$$n = \frac{E_s}{E_c} = \frac{30,000}{2400} = 12.5 \qquad M_{max} = \frac{wL^2}{8} = \frac{w(12)^2}{8} = (18w)\ \text{lb}\cdot\text{ft}$$

$$A_s = 3\left[\pi(0.875)^2/4\right] = 1.8040\ \text{in.}^2$$

$$A_T = nA_s = 12.5(1.8040) = 22.55\ \text{in.}^2$$

$$8h(h/2) = 22.55(16-h)$$

$$4h^2 + 22.55h - 360.8 = 0$$



From which: $h = 7.088$ in. $\qquad 16 - h = 8.912$ in.

$$I_T = \frac{(8)(7.088)^3}{3} + (22.55)(8.912)^2 = 2741\ \text{in.}^4$$

$$\sigma_c = \frac{My_c}{I_T} = \frac{(18w\times 12)(7.088)}{(2741)} \le 1000\ \text{psi} \qquad w \le 1790.3\ \text{lb/ft}$$

$$\sigma_s = \frac{nMy_s}{I_T} = \frac{12.5(18w\times 12)(8.912)}{(2741)} \le 16,000\ \text{psi} \qquad w \le 1822.6\ \text{lb/ft}$$

Therefore: $w_{max} = 1790$ lb/ft .... **Ans.**

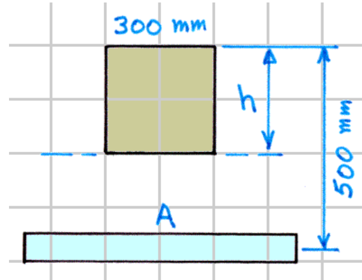
7-168*

$$n = \frac{E_s}{E_c} = \frac{198}{16.5} = 12$$

$$300h(h/2) = 12A_s(500 - h)$$

$$A_s = \frac{12.5h^2}{500 - h}$$

$$\sigma_c = \frac{My_c}{I_T} = \frac{Mh}{I_T} \le 7(10^6) \text{ N/m}^2 \qquad \text{(a)}$$



$$\sigma_s = \frac{nMy_s}{I_T} = \frac{12M(500 - h)}{I_T} \le 125(10^6) \text{ N/m}^2 \qquad \text{(b)}$$

From Eqs. (a) and (b): $84(500 - h) = 125h$ $\qquad h = 200.95$ mm

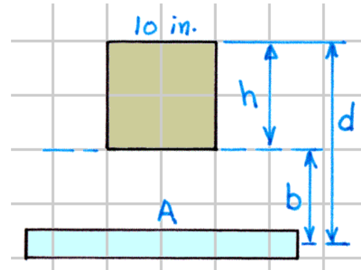
(a) $A_s = \dfrac{12.5h^2}{500 - h} = \dfrac{12.5(200.95)^2}{500 - 200.95} = 1688 \text{ mm}^2$ .... **Ans.**

(b) $I_T = \dfrac{(300)(200.95)^3}{3} + 12(1688)(299.05)^2 = 2623(10^6) \text{ mm}^4$

$$M_{max} = \frac{\sigma_c I_T}{h} = \frac{(7\times 10^6)(2623\times 10^{-6})}{(0.20095)} = 91,370 \text{ N}\cdot\text{m}$$

$M_{max} = 91.4 \text{ kN}\cdot\text{m}$ .... **Ans.**

7-169



$$n = \frac{E_s}{E_c} = \frac{30,000}{2400} = 12.5$$

$$M_{\max} = \frac{wL^2}{8} = \frac{(1000)(16)^2}{8} = 32,000 \text{ lb} \cdot \text{ft}$$

$$\sigma_c = \frac{My_c}{I_T} = \frac{(32,000 \times 12)h}{I_T} \le 800 \text{ psi} \qquad \text{(a)}$$

$$\sigma_s = \frac{nMy_s}{I_T} = \frac{12.5(32,000 \times 12)b}{I_T} \le 16,000 \text{ psi} \qquad \text{(b)}$$

From Eqs. (a) and (b): $I_T = 480h = 300b$ $\qquad h = 0.625b$

$$10h(h/2) = nA_s b = 12.5A_s b \qquad A_s = \frac{5h^2}{12.5b} = \frac{5(0.625b)^2}{12.5b} = 0.15625b$$

$$I_T = \frac{(10)(h)^3}{3} + nA_s(b)^2 = \frac{(10)(0.625b)^3}{3} + 12.5(0.15625b)(b)^2 = 2.767b^3$$

Therefore $I_T = 2.767b^3 = 300b$ $\qquad b = 10.413$ in.

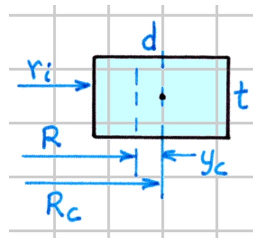
$$h = 0.625b = 0.625(10.413) = 6.508 \text{ in.}$$

(a) $A_s = 0.15625b = 0.15625(10.413) = 1.627 \text{ in.}^2$ .... **Ans.**

(b) $d = h + b = 6.508 + 10.413 = 16.92$ in. .... **Ans.**

7-170

From the curved beam formula:



$$R = \frac{r_o - r_i}{\ln(r_o/r_i)} = \frac{11d - 10d}{\ln(1.1)} = 10.49206d$$

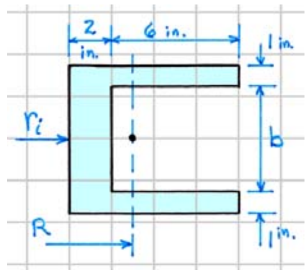
$$y_C = R - R_C = 10.49206d - 10.5d = -0.00794d$$

$$\sigma_{\max} = \frac{My_i}{r_i A y_C} = \frac{M(0.49206d)}{(10d)(td)(-0.00794d)} = \frac{-6.197M}{td^2}$$

From the flexure formula: $$\sigma_{\max} = \frac{-Mc}{I} = \frac{-M(d/2)}{(td^3/12)} = \frac{-6M}{td^2}$$

$$Error = \frac{6.197 - 6}{6.197}(100) = 3.18\%$$ .... **Ans.**

7-171*



$$\sigma_o = -\sigma_i = \frac{M\left(R-r_o\right)}{r_o A y_C} = \frac{-M\left(R-r_i\right)}{r_i A y_C}$$

$$r_o\left(R-r_i\right) = r_i\left(r_o - R\right)$$

$$14\left(R-6\right) = 6\left(14-R\right)$$

Which gives $R = 8.400 \text{ in.}$

$$A = 2\left(8\times 1\right) + \left(b\times 2\right) = \left(2b+16\right) \text{ in.}^2 \qquad A = R\int \frac{dA}{\rho}$$

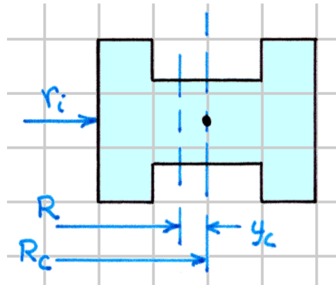
Using Table B-20

$$\int \frac{dA}{\rho} = \left[\left(b\right)\ln\frac{8}{6}\right] + 2\left[\left(1\right)\ln\frac{14}{6}\right] = 0.28768b + 1.69750$$

$$2b + 16 = \left(8.400\right)\left(0.28768b + 2.26996\right)$$

$b = 4.24$ in. .... **Ans.**

7-172



$$R_C = 450 + 100 = 550 \text{ mm}$$

$$A = 2(50 \times 150) + (75 \times 100) = 22{,}500 \text{ mm}^2$$

$$A = R\int \frac{dA}{\rho}$$

Using Table B-20

$$\int \frac{dA}{\rho} = (150)\ln\frac{500}{450} + (75)\ln\frac{600}{500} + (150)\ln\frac{650}{600} = 41.484 \text{ mm}$$

$$22{,}500 = R(41.484) \qquad R = 542.4 \text{ mm}$$

$$y_C = R - R_C = 542.4 - 550 = -7.600 \text{ mm}$$

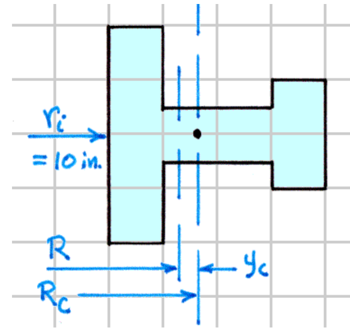
$$\sigma_i = \frac{M(R - r_i)}{r_i A y_C} = \frac{(20{,}000)(0.5424 - 0.450)}{(0.450)(0.0225)(-0.00760)}$$

$\sigma_i = -24.0(10^6) \text{ N/m}^2 = 24.0 \text{ MPa (C)}$ .... **Ans.**

$$\sigma_o = \frac{M(R - r_o)}{r_o A y_C} = \frac{(20{,}000)(0.5424 - 0.650)}{(0.650)(0.0225)(-0.00760)}$$

$\sigma_o = +19.36(10^6) \text{ N/m}^2 = 19.36 \text{ MPa (T)}$ .... **Ans.**

7-173*



$$A = (8\times 2) + (4\times 2) + (4\times 2) = 32 \text{ in.}^2$$

$$R_C = \frac{(11)(8\times 2) + (14)(4\times 2) + (17)(4\times 2)}{32} = 13.25 \text{ in.}$$

$$A = R\int \frac{dA}{\rho}$$

Using Table B-20

$$\int \frac{dA}{\rho} = (8)\ln\frac{12}{10} + (2)\ln\frac{16}{12} + (4)\ln\frac{18}{16} = 2.5051 \text{ in.}$$

$$32 = R(2.5051) \qquad R = 12.774 \text{ in.}$$

$$y_C = R - R_C = 12.774 - 13.25 = -0.4760 \text{ in.}$$

$$\sigma_i = \frac{M(R - r_i)}{r_i A y_C} = \frac{(-30\times 12)(12.774 - 10)}{(10)(32)(-0.4760)} = +6.56 \text{ ksi} = 6.56 \text{ ksi (T)}$$ .... **Ans.**

$$\sigma_o = \frac{M(R - r_o)}{r_o A y_C} = \frac{(-30\times 12)(12.774 - 18)}{(18)(32)(-0.4760)} = -6.86 \text{ ksi} = 6.86 \text{ ksi (C)}$$ .... **Ans.**

7-174*

$R_C = 75 + 150 = 225$ mm $\quad A = (250\times 300) - (150\times 200) = 45{,}000\ \text{mm}^2$

Using Table B-20

$$\int \frac{dA}{\rho} = (250)\ln\frac{125}{75} + 2(50)\ln\frac{325}{125} + (250)\ln\frac{375}{325} = 259.0\ \text{mm}$$

$$A = R\int\frac{dA}{\rho} \qquad 45{,}000 = R(259.0) \qquad R = 173.75\ \text{mm}$$

$$y_C = R - R_C = 173.75 - 225 = -51.25\ \text{mm}$$

$$\sigma_i = \frac{M(R - r_i)}{r_i A y_C} = \frac{M(0.17375 - 0.075)}{(0.075)(0.0450)(-0.05125)} \le -140(10^6)\ \text{N/m}^2$$

$$M \le 245(10^3)\ \text{N}\cdot\text{m}$$

$$\sigma_o = \frac{M(R - r_o)}{r_o A y_C} = \frac{M(0.17375 - 0.375)}{(0.375)(0.0450)(-0.05125)} \le 35(10^6)\ \text{N/m}^2$$

$$M \le 150.4(10^3)\ \text{N}\cdot\text{m}$$

Therefore: $M_{max} = 150.4\ \text{kN}\cdot\text{m}$ .... **Ans.**

7-175

$$A = (1.5 \times 0.75) + (1.5 \times 3) = 5.625 \text{ in.}^2$$

$$A = R\int \frac{dA}{\rho} = R\left[\int_{2.25}^{3}\left(-2 + \frac{4\rho}{3}\right)\frac{d\rho}{\rho} + \int_{3}^{6}\left(3 - \frac{\rho}{3}\right)\frac{d\rho}{\rho}\right]$$

$$= R\left[-2\ln\left(\frac{3}{2.25}\right) + \frac{4}{3}(3 - 2.25) + 3\ln\left(\frac{6}{3}\right) - \frac{1}{3}(6 - 3)\right]$$

$$= 1.50408R = 5.625 \text{ in.}^2$$

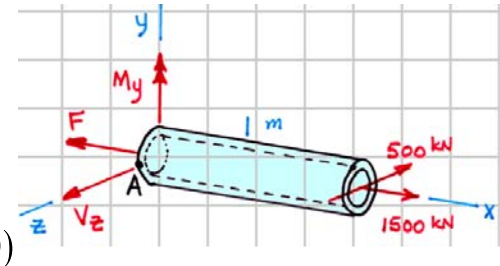
$$R = \frac{5.625}{1.50408} = 3.7398 \text{ in.}$$

$$y_C = R - R_C = 3.7398 - 4.00 = -0.2602 \text{ in.}$$

$$\sigma_i = \frac{M(R - r_i)}{r_i A y_C} = \frac{(-70)(3.7398 - 2.25)}{(2.25)(5.625)(-0.2602)} = +31.7 \text{ ksi} = 31.7 \text{ ksi (T)}$$ .... **Ans.**

$$\sigma_o = \frac{M(R - r_o)}{r_o A y_C} = \frac{(-70)(3.7398 - 6.00)}{(6.00)(5.625)(-0.2602)} = -18.02 \text{ ksi} = 18.02 \text{ ksi (C)}$$ .... **Ans.**

7-176*



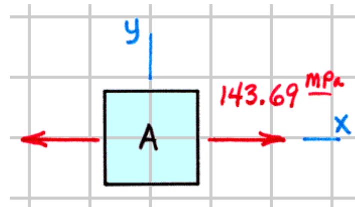
$F = 1500 \text{ kN (T)} \qquad V_z = 500 \text{ kN}$

$M_y = -(500 \times 1) = -500 \text{ kN} \cdot \text{m}$

$$\sigma_{xA} = \frac{F}{A} + \frac{Mc}{I}$$

$$= \frac{1500(10^3)}{\pi(0.400^2 - 0.300^2)/4} + \frac{(500 \times 10^3)(0.200)}{\pi(0.200^4 - 0.150^4)/4}$$

$$= 27.28(10^6) + 116.41(10^6) \text{ N/m}^2 = 143.69 \text{ MPa (T)}$$



$\sigma_{yA} = \tau_{xyA} = 0 \text{ MPa}$

$\sigma_{p1} = 143.7 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_{p3} = 0 \text{ MPa}$ .... **Ans.**

$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = (143.69 - 0)/2 = 71.8 \text{ MPa}$ .... **Ans.**

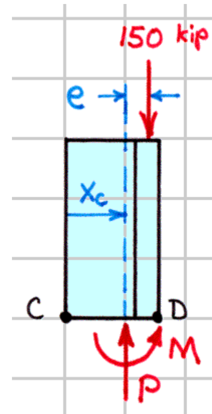
7-177*

$P = 150 \text{ kip (C)} \qquad M = (150)(2) = 300 \text{ kip} \cdot \text{in.}$

$A = 2(2 \times 6) = 24 \text{ in}^2$

$$x_C = \frac{M_y}{A} = \frac{(3)[(2)(6)] + (7)[(6)(2)]}{2[(6)(2)]} = 5.00 \text{ in.}$$

$$I_y = \frac{(2)(5)^3}{3} + \frac{(6)(3)^3}{3} - \frac{(4)(1)^3}{3} = 136.00 \text{ in.}^4$$



$$\sigma_{yC} = \frac{-P}{A} + \frac{Mc}{I} = \frac{-150}{24} + \frac{(300)(5)}{136} = 4.78 \text{ ksi (T)}$$ .... **Ans.**

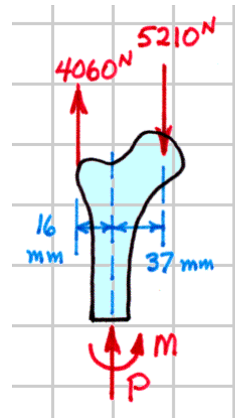
$$\sigma_{yD} = \frac{-P}{A} - \frac{Mc}{I} = \frac{-150}{24} - \frac{(300)(3)}{136} = -12.87 \text{ ksi} = 12.87 \text{ ksi (C)}$$ .... **Ans.**

7-178

$\uparrow \Sigma F_y = 0: \qquad P + 4060 - 5210 = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \qquad M - (4060)(0.016) - (5210)(0.037) = 0$

$P = 1150 \text{ N (C)} \qquad M_y = 257.73 \text{ N}\cdot\text{m}$



(a) Solid: $A = \pi(27)^2/4 = 572.6 \text{ mm}^2$

$I = \pi(27)^4/64 = 26.09(10^3) \text{ mm}^4$

$$\sigma_{xA} = \frac{-F}{A} \pm \frac{Mc}{I} = \frac{-1150}{572.6(10^{-6})} \pm \frac{(257.3)(0.0135)}{26.09(10^{-9})}$$

$$= (-2.0084 \pm 133.36)(10^6) \text{ N/m}^2$$

$\sigma_{T\max} = +131.4 \text{ MPa} = 131.4 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{C\max} = -135.4 \text{ MPa} = 135.4 \text{ MPa (C)}$ .... **Ans.**

(b) Hollow: $A = \pi(27^2 - 16^2)/4 = 371.5 \text{ mm}^2$

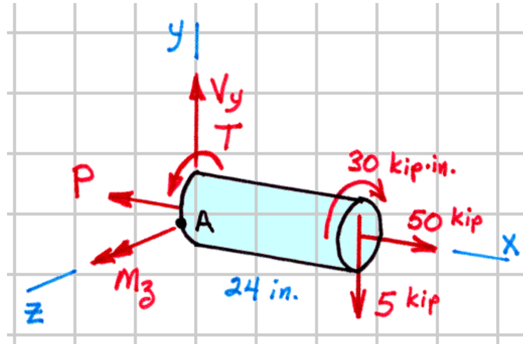
$I = \pi(27^4 - 16^4)/64 = 22.87(10^3) \text{ mm}^4$

$$\sigma_{xA} = \frac{-1150}{371.5(10^{-6})} \pm \frac{(257.3)(0.0135)}{22.87(10^{-9})} = (-3.096 \pm 152.14)(10^6) \text{ N/m}^2$$

$\sigma_{T\max} = +149.0 \text{ MPa} = 149.0 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{C\max} = -155.2 \text{ MPa} = 155.2 \text{ MPa (C)}$ .... **Ans.**

7-179*



$P = 50 \text{ kip (T)} \qquad V_y = 5 \text{ kip}$

$M_z = (5)(24) = 120 \text{ kip}\cdot\text{in.}$

$T = 30 \text{ kip}\cdot\text{in.}$

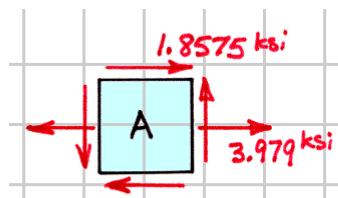
$I_z = \pi(4)^4/64 = 12.5664 \text{ in.}^4$

$J = 2I_z = 25.1327 \text{ in.}^4$

$$Q = y_C A = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right) = \frac{2r^3}{3} = \frac{2(2)^3}{3} = 5.3333 \text{ in.}^3$$

$$\sigma_{xA} = \frac{P}{A} = \frac{(50)}{\pi(4)^2/4} = 3.979 \text{ ksi (T)} \qquad \sigma_{yA} = 0 \text{ ksi}$$

$$\tau_{xyA} = \frac{Tc}{J} - \frac{V_y Q}{I_z t} = \frac{(30)(2)}{(25.1327)} - \frac{(5)(5.3333)}{(12.5664)(4)} = 1.8575 \text{ ksi}$$



$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3.979 + 0}{2} \pm \sqrt{\left(\frac{3.979 - 0}{2}\right)^2 + (1.8575)^2}$$

$\sigma_{p1} = 1.9895 + 2.7218 = +4.7113 \text{ ksi} \cong 4.71 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = 1.9895 - 2.7218 = -0.7323 \text{ ksi} \cong 0.732 \text{ ksi (C)}$ .... **Ans.**

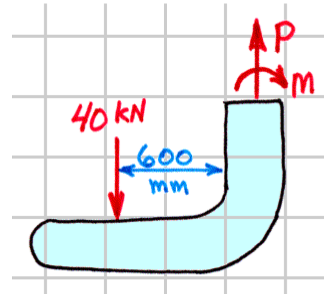
$\sigma_{p3} = 0 \text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = (4.7113 + 0.7323)/2 = 2.72 \text{ ksi}$ .... **Ans.**

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\tan^{-1}\frac{2(1.8575)}{(3.979) - 0} = 21.52°$$ .... **Ans.**

7-180

$P = 40\text{ kN (T)}$ $\qquad M = (40)(0.65919) = 26.37\text{ kN}\cdot\text{m}$



$$A = (100\times 50) + (30\times 80) + (60\times 20) = 8600\text{ mm}^2$$

$$x_C = \frac{(25)(100\times 50) + (90)(30\times 80) + (140)(60\times 20)}{8600} = 59.19\text{ mm}$$

$$I_y = \left[\frac{100(50)^3}{12} + (100\times 50)(34.19)^2\right]$$

$$+\left[\frac{30(80)^3}{12} + (30\times 80)(30.81)^2\right] + \left[\frac{60(20)^3}{12} + (60\times 20)(80.81)^2\right]$$

$$= 18.321(10^6)\text{ mm}^4$$

$$\sigma_{CD} = \frac{P}{A} + \frac{Mc}{I} = \frac{40,000}{8600(10^{-6})} + \frac{(26.37)(0.09081)}{18.321(10^{-6})}$$

$\sigma_{CD} = 89.845(10^6)\text{ N/m}^2 = 89.8\text{ MPa (T)}$ .... **Ans.**

$$\sigma_{EF} = \frac{P}{A} - \frac{Mc}{I} = \frac{40,000}{8600(10^{-6})} - \frac{(26.37)(0.05919)}{18.321(10^{-6})}$$

$\sigma_{EF} = -126.1(10^6)\text{ N/m}^2 = 126.1\text{ MPa (C)}$ .... **Ans.**

7-181

$$P = 15 \text{ kip (C)}$$

$$M = (15)(12) = 180 \text{ kip} \cdot \text{in.}$$

$$A = (5 \times 12) = 60 \text{ in}^2$$

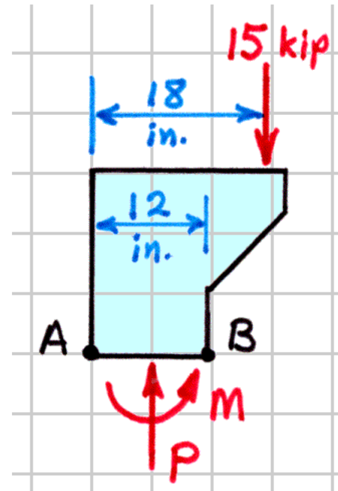
$$I = (5)(12)^3 / 12 = 720 \text{ in.}^4$$

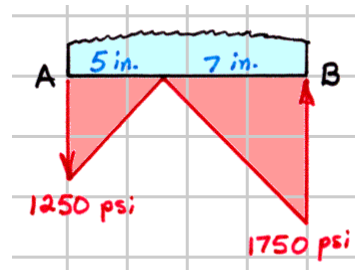
$$\sigma_A = \frac{-P}{A} + \frac{Mc}{I} = \frac{-15}{60} + \frac{(180)(6)}{720} = 1.250 \text{ ksi (T)}$$

$\sigma_A = 1.250$ ksi (T) .... **Ans.**

$$\sigma_B = \frac{-P}{A} - \frac{Mc}{I} = \frac{-15}{60} - \frac{(180)(6)}{720}$$

$\sigma_B = -1.750$ ksi $= 1.750$ ksi (C) .... **Ans.**

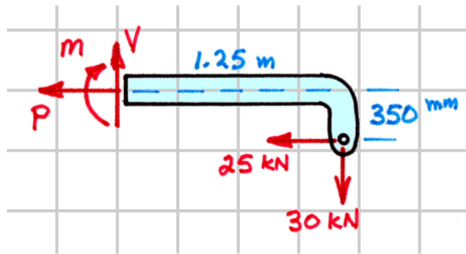




7-182*

$\rightarrow \Sigma F_x = 0: \quad -P - 25 = 0 \quad P = -25 \text{ kN} = 25 \text{ kN (C)}$

$\uparrow \Sigma F_y = 0: \quad V - 30 = 0 \quad V = 30 \text{ kN}$



$\circlearrowleft \Sigma M_{cut} = 0: \quad -M - (25)(0.350) - (30)(1.25) = 0$

$$M = -46.25 \text{ kN}\cdot\text{m} = 46.25 \text{ kN}\cdot\text{m} \circlearrowleft$$

$$A = (100 \times 150) = 15{,}000 \text{ mm}^2$$

$$I = (100)(150)^3/12 = 28.13(10^6) \text{ mm}^4$$

$$\sigma_{top} = \frac{P}{A} + \frac{Mc}{I} = \frac{(-25{,}000)}{15.00(10^{-3})} + \frac{(46{,}250)(0.075)}{28.13(10^{-6})} = +121.64(10^6) \text{ N/m}^2$$

$$\sigma_{bottom} = \frac{P}{A} - \frac{Mc}{I} = \frac{(-25{,}000)}{15.00(10^{-3})} - \frac{(46{,}250)(0.075)}{28.13(10^{-6})} = -124.98(10^6) \text{ N/m}^2$$

Therefore: $\sigma_{\max} = 125.0$ MPa (C) .... **Ans.**

7-183*

$P = 450 \text{ lb (T)} \qquad M = (450)(3) = 1350 \text{ lb}\cdot\text{in.}$

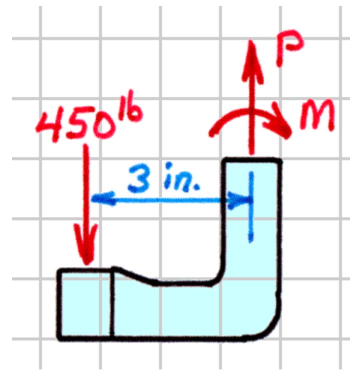
$A = (0.5 \times h) = (0.5h) \text{ in}^2$

$I = (0.5)(h)^3/12 = (h^3/24) \text{ in.}^4$

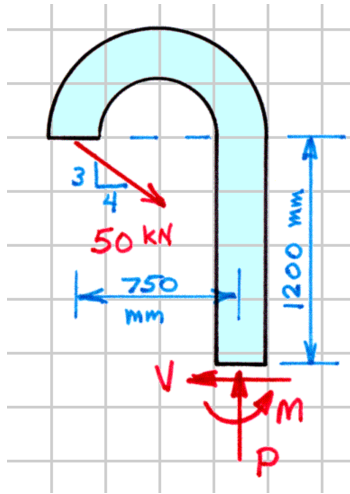
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{450}{0.5h} + \frac{(1350)(h/2)}{h^3/24} \le 16,000 \text{ psi}$$

$16,000h^2 - 900h - 16,200 \ge 0$

$h \ge 1.035$ in. .... **Ans.**



7-184



$\rightarrow \Sigma F_x = 0: \quad P - 30 = 0 \quad P = 30 \text{ kN (C)}$

$\uparrow \Sigma F_y = 0: \quad 40 - V = 0 \quad V = 40 \text{ kN}$

$\circlearrowleft \Sigma M_{cut} = 0: \quad M + (30)(0.750) - (40)(1.20) = 0$

$$M = 25.5 \text{ kN}\cdot\text{m} \circlearrowleft$$

$$A = 2(50 \times 150) + (100 \times 75) = 22{,}500 \text{ mm}^2$$

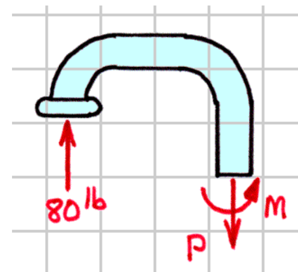
$$I = \frac{(150)(200)^3}{12} - \frac{(75)(100)^3}{12} = 93.75(10^6) \text{ mm}^4$$

$$\sigma_{BB} = \frac{-P}{A} \pm \frac{Mc}{I} = \frac{-(30{,}000)}{22.5(10^{-3})} \pm \frac{(25{,}500)(0.100)}{93.75(10^{-6})}$$

Therefore: $\sigma_L = \sigma_{\max T} = +25.9(10^6) \text{ N/m}^2 = 25.9 \text{ MPa (T)}$ .... **Ans.**

$\sigma_R = \sigma_{\max C} = -28.5(10^6) \text{ N/m}^2 = 28.5 \text{ MPa (C)}$ .... **Ans.**

7-185



$$P = 80 \text{ lb (T)}$$

$$M = (80)(3) = 240 \text{ lb}\cdot\text{in.}$$

$$A = (0.5 \times 0.1875) = 0.093750 \text{ in}^2$$

$$I = (0.1875)(0.5)^3 / 12 = 1.9531(10^{-3}) \text{ in.}^4$$

$$\sigma = \frac{P}{A} \pm \frac{Mc}{I} = \frac{80}{0.09375} + \frac{(240)(0.25)}{1.9531(10^{-3})} = (853.33 \pm 30,720) \text{ psi}$$

Therefore: $\sigma_L = \sigma_{\max T} = +31.6 \text{ ksi} = 31.6 \text{ ksi (T)}$ .... **Ans.**

$\sigma_R = \sigma_{\max C} = -29.9 \text{ ksi} = 29.9 \text{ ksi (C)}$ .... **Ans.**

7-186*

$P = 780 \text{ kN (T)} \qquad V_y = 10 \text{ kN}$

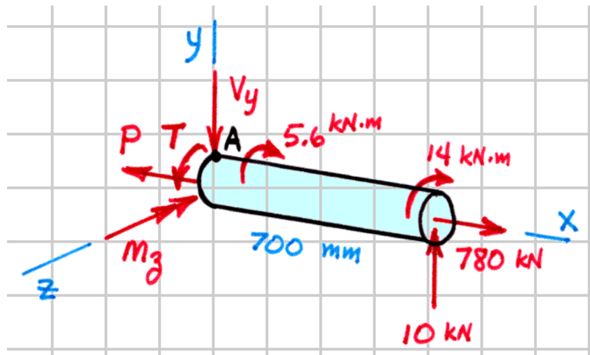
$M_z = (10 \times 0.700) = 7.00 \text{ kN}\cdot\text{m}$

$T = 14.0 + 5.6 = 19.6 \text{ kN}\cdot\text{m}$

$A = \pi(150)^2/4 = 17{,}671 \text{ mm}^2$

$I = \pi(150)^4/64 = 24.85(10^6) \text{ mm}^4$

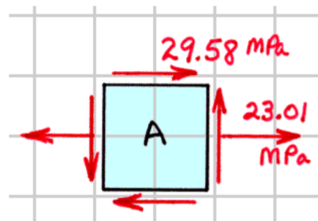
$J = 2I = 49.70(10^6) \text{ mm}^4$



$$\sigma_{xA} = \frac{P}{A} - \frac{Mc}{I} = \frac{780(10^3)}{17.671(10^{-3})} - \frac{(7000)(0.075)}{24.850(10^{-6})}$$

$$= 44.14(10^6) - 21.13(10^6) \text{ N/m}^2 = 23.01 \text{ MPa (T)}$$

$\sigma_{yA} = 0 \text{ MPa}$



$$\tau_{xyA} = \frac{Tc}{J} + \frac{V_y Q}{I_z t} = \frac{(19{,}600)(0.075)}{49.70(10^{-6})} + 0$$

$$= 29.58(10^6) \text{ N/m}^2 = 29.58 \text{ MPa}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{23.01 + 0}{2} \pm \sqrt{\left(\frac{23.01 - 0}{2}\right)^2 + (29.58)^2}$$

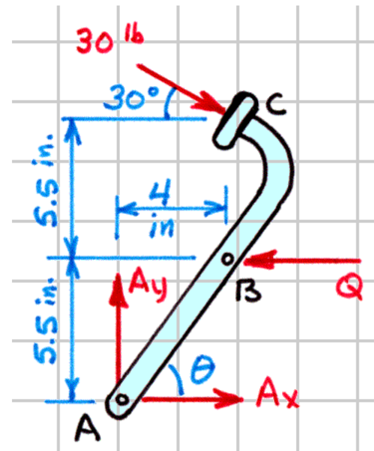
$\sigma_{p1} = 11.505 + 31.74 = 43.245 \text{ MPa} \cong 43.2 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = 11.505 - 31.74 = -20.235 \text{ MPa} \cong 20.2 \text{ MPa (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ MPa}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(43.245) - (-20.235)}{2} = 31.7 \text{ MPa}$ .... **Ans.**

7-187*



$\rightarrow \Sigma F_x = 0: \quad A_x - Q + 30\cos 30° = 0$

$\uparrow \Sigma F_y = 0: \quad A_y - 30\sin 30° = 0$

$\circlearrowleft \Sigma M_A = 0: \quad 5.5Q - (11)(30\cos 30°) - (4)(30\sin 30°) = 0$

$Q = 62.87 \text{ lb}$

$A_x = 36.89 \text{ lb} \qquad A_y = 15.00 \text{ lb}$

$\theta = \tan^{-1}(5.5/4) = 53.97°$

Z $\Sigma F_n = 0: \quad 36.89\cos\theta + 15.00\sin\theta - N = 0$

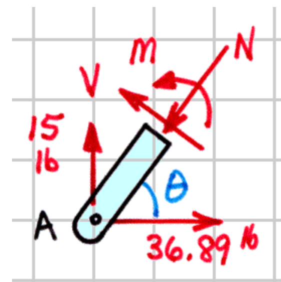
^ $\Sigma F_t = 0: \quad V + 15.00\cos\theta - 36.89\sin\theta = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \quad M + (36.89)(2.75) - (15.00)(2) = 0$

$N = 33.83 \text{ lb} \qquad V = 21.01 \text{ lb}$

$M = -71.45 \text{ lb}\cdot\text{in.} = 71.45 \text{ lb}\cdot\text{in.} \circlearrowright$

$A = (0.18750 \times 1) = 0.18750 \text{ in.}^2$



$I = (0.18750)(1)^3/12 = 0.015625 \text{ in.}^4$

$$\sigma_{top} = \frac{-N}{A} + \frac{Mc}{I} = \frac{-(33.83)}{0.18750} + \frac{(71.45)(0.500)}{0.015625} = +2106 \text{ psi}$$

$$\sigma_{bottom} = \frac{-N}{A} - \frac{Mc}{I} = \frac{-(33.83)}{0.18750} - \frac{(71.45)(0.500)}{0.015625} = -2467 \text{ psi}$$

Therefore: $\sigma_{top} = \sigma_{\max T} = +2.11 \text{ ksi} = 2.11 \text{ ksi (T)}$ .... **Ans.**

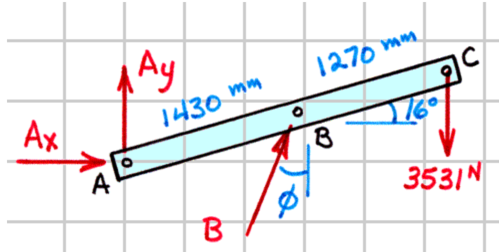
$\sigma_{bottom} = \sigma_{\max C} = -2.47 \text{ ksi} = 2.47 \text{ ksi (C)}$ .... **Ans.**

7-188

$a = 1430\cos 16° = 1374.6 \text{ mm}$ $\qquad W = mg = (360)(9.807) = 3531 \text{ N}$

$b = 1430\sin 16° = 394.2 \text{ mm}$

$$\phi = \tan^{-1}\frac{1374.6 - 890}{394.2 + 900} = 20.53°$$



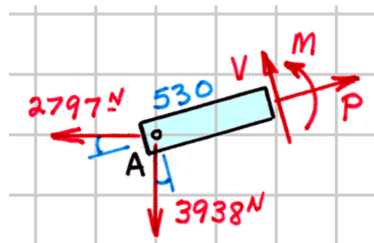
$\rightarrow \Sigma F_x = 0: \qquad A_x + B\sin 20.53° = 0$

$\uparrow \Sigma F_y = 0: \qquad A_y + B\cos 20.53° - 3531 = 0$

$\circlearrowleft \Sigma M_A = 0: \quad (B\cos 20.53°)(1374.6) - (B\sin 20.53°)(394.2) - (3531)(2700\cos 16°) = 0$

$B = 7976 \text{ N}$

$A_x = -2797 \text{ N} \qquad A_y = -3938 \text{ N}$



Z $\Sigma F_n = 0: \qquad P - 2797\cos 16° - 3938\sin 16° = 0$

^ $\Sigma F_t = 0: \qquad V + 2797\sin 16° - 3938\cos 16° = 0$

$P = 3774 \text{ N} \qquad V = 3014 \text{ N}$

$\circlearrowleft \Sigma M_{cut} = 0: \quad M + (3938)(0.530\cos 16°) - (2797)(0.530\sin 16°) = 0$

$M = -1597.7 \text{ N}\cdot\text{m} = 1597.7 \text{ N}\cdot\text{m} \circlearrowright$

$A = (100\times 100) - (60\times 60) = 6400 \text{ mm}^2$

$$I = \frac{(100)(100)^3}{12} - \frac{(60)(60)^3}{12} = 7.253(10^6) \text{ mm}^4$$

$$\sigma_{top} = \frac{P}{A} + \frac{Mc}{I} = \frac{(3774)}{6400(10^{-6})} + \frac{(1597.7)(0.050)}{7.253(10^{-6})} = +11.60(10^6) \text{ N/m}^2$$

$\sigma_{top} = \sigma_{\max T} = 11.60 \text{ MPa (T)}$ .... **Ans.**

$$\sigma_{bottom} = \frac{P}{A} - \frac{Mc}{I} = \frac{(3774)}{6400(10^{-6})} - \frac{(1597.7)(0.050)}{7.253(10^{-6})} = -10.42(10^6) \text{ N/m}^2$$

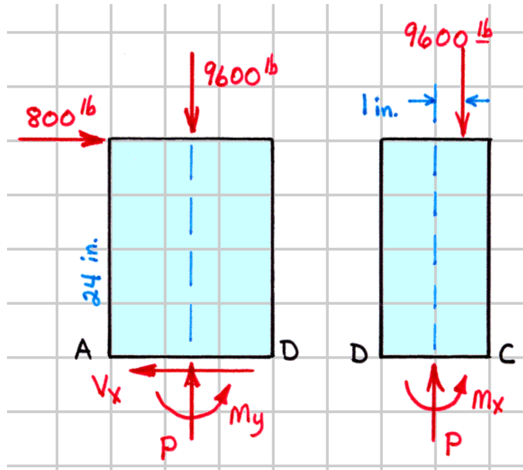
$\sigma_{bottom} = \sigma_{\max C} = 10.42 \text{ MPa (C)}$ .... **Ans.**

7-189*

$$A = (6 \times 4) = 24 \text{ in.}^2$$

$$I_x = \frac{(6)(4)^3}{12} = 32 \text{ in.}^4$$

$$I_y = \frac{(4)(6)^3}{12} = 72 \text{ in.}^4$$



$P = 9600$ lb (C) $\qquad V_x = 800$ lb

$$M_x = (9600)(1) = 9600 \text{ lb} \cdot \text{in.}$$

$$M_y = (800)(24) = 19{,}200 \text{ lb} \cdot \text{in.}$$

$$\sigma_P = \frac{P}{A} = \frac{9600}{24} = 400 \text{ psi (C)}$$

$$\sigma_{M_x} = \frac{M_x c}{I_x} = \frac{(9600)(2)}{32} = 600 \text{ psi (T \& C)}$$

$$\sigma_{M_y} = \frac{M_y c}{I_y} = \frac{(19{,}200)(3)}{72} = 800 \text{ psi (T \& C)}$$

$\sigma_A = -400 + 600 + 800 = +1000$ psi $= 1000$ psi (T) .... **Ans.**

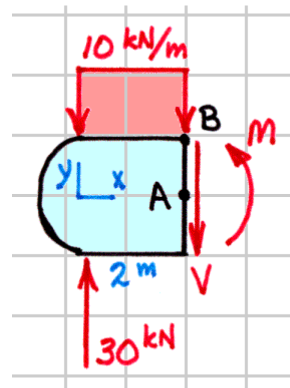
$\sigma_B = -400 - 600 + 800 = -200$ psi $= 200$ psi (C) .... **Ans.**

$\sigma_C = -400 - 600 - 800 = -1800$ psi $= 1800$ psi (C) .... **Ans.**

$\sigma_D = -400 + 600 - 800 = -600$ psi $= 600$ psi (C) .... **Ans.**

7-190

By symmetry, each support carries half of the total weight. Then,



$$V = 10 \text{ kN}$$

$$M = (30)(2) - (10 \times 2)(1) = 40.00 \text{ kN}\cdot\text{m}$$

$$I = \frac{\pi (604)^4}{4} - \frac{\pi (600)^4}{4} = 2741(10^6) \text{ mm}^4$$

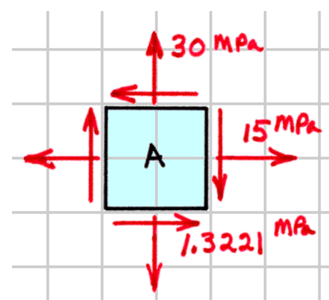
$$Q = y_C A = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right) = \frac{2r^3}{3} = \frac{2(604)^3}{3} - \frac{2(600)^3}{3}$$

$$= 2.899(10^6) \text{ mm}^3$$

Stresses due to the internal pressure:

$$\sigma_x = \sigma_{axial} = \frac{pr}{2t} = \frac{(200 \times 10^3)(0.600)}{2(0.004)} = 15.00(10^6) \text{ N/m}^2 \text{ (T)}$$

$$\sigma_y = \sigma_{hoop} = 2\sigma_{axial} = 30.00(10^6) \text{ N/m}^2 \text{ (T)}$$



At *A*:

$$\sigma_x = 15.00 \text{ MPa} \qquad \sigma_y = 30.00 \text{ MPa}$$

$$\tau_{xy} = \frac{-VQ}{It} = \frac{-(10{,}000)(2.899 \times 10^{-3})}{(2741 \times 10^{-6})(0.008)}$$

$$= -1.3221(10^6) \text{ N/m}^2 = -1.3221 \text{ MPa}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{15+30}{2} \pm \sqrt{\left(\frac{15-30}{2}\right)^2 + (-1.3221)^2}$$

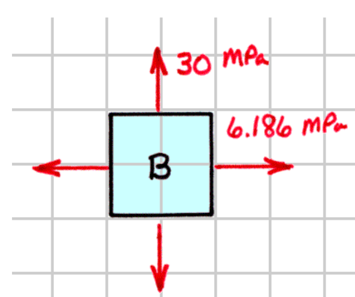
$\sigma_{p1} = 22.500 + 7.616 = +30.116 \text{ MPa} \cong 30.1 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = 22.500 - 7.616 = +14.884 \text{ MPa} \cong 14.88 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ MPa}$ .... **Ans.**

$\tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{(30.116) - 0}{2} = 15.06 \text{ MPa}$ .... **Ans.**

At *B*:



$$\sigma_x = \frac{pr}{2t} - \frac{Mc}{I} = 15(10^6) - \frac{(40{,}000)(0.604)}{(2741 \times 10^{-6})}$$

$$= +6.186(10^6) \text{ N/m}^2 = 6.186 \text{ MPa (T)}$$

$$\sigma_y = 30.00 \text{ MPa} \qquad \tau_{xy} = 0 \text{ MPa}$$

7-190 (cont.)

Since $\tau_{xy} = 0\text{ MPa}$ these are principal stresses and

$\sigma_{p1} = \sigma_y = +30.0\text{ MPa} = 30.0\text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = \sigma_x = +6.186\text{ MPa} \cong 6.19\text{ MPa (T)}$ .... **Ans.**

$\sigma_{p3} = 0\text{ MPa}$ .... **Ans.**

$\tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{(30.0) - 0}{2} = 15.00\text{ MPa}$ .... **Ans.**

7-191

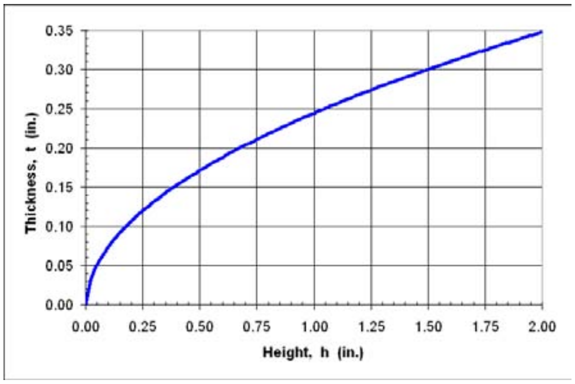
$$A = (1)(t) = (t) \text{ in}^2 \qquad I = \frac{bh^3}{12} = \frac{(1)(t)^3}{12} = (0.08333t^3) \text{ in.}^4$$

$$\sigma = E\varepsilon = \frac{P}{A} + \frac{Mc}{I}$$

$$(10.6\times10^6)(1000\times10^{-6}) = \frac{110}{t} + \frac{(110h)(t/2)}{0.08333t^3}$$

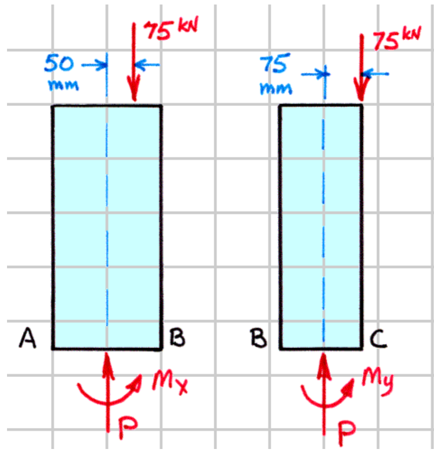
which gives $1767t^2 - 18.33t = 110h$

h (in.)	t (in.)
0	0.0000
0.25	0.1246
0.50	0.1713
0.75	0.2110
1.00	0.2444
1.25	0.2738
1.50	0.3004
1.75	0.3249
2.00	0.3477



7-192*

$$I_x = \frac{(200)(150)^3}{12} = 56.25\left(10^6\right)\text{ mm}^4 \qquad A = (200\times 150) = 30{,}000\text{ mm}^2$$



$$I_y = \frac{(150)(200)^3}{12} = 100.0\left(10^6\right)\text{ mm}^4$$

$$P = 75\text{ kN (C)}$$

$$M_x = (75)(0.075) = 5.625\text{ kN}\cdot\text{m}$$

$$M_y = (75)(0.050) = 3.750\text{ kN}\cdot\text{m}$$

$$\sigma_P = \frac{P}{A} = \frac{75{,}000}{30\left(10^{-3}\right)} = 2.50\left(10^6\right)\text{ N/m}^2\text{ (C)}$$

$$\sigma_{M_x} = \frac{M_x c}{I_x} = \frac{(5625)(0.075)}{56.25\left(10^{-6}\right)} = 7.50\left(10^6\right)\text{ N/m}^2\text{ (T \& C)}$$

$$\sigma_{M_y} = \frac{M_y c}{I_y} = \frac{(3750)(0.100)}{100.0\left(10^{-6}\right)} = 3.75\left(10^6\right)\text{ N/m}^2\text{ (T \& C)}$$

$\sigma_A = -2.50 + 7.50 + 3.75 = +8.75\text{ MPa} = 8.75\text{ MPa (T)}$ .... **Ans.**

$\sigma_B = -2.50 + 7.50 - 3.75 = +1.25\text{ MPa} = 1.25\text{ MPa (T)}$ .... **Ans.**

$\sigma_C = -2.50 - 7.50 - 3.75 = -13.75\text{ MPa} = 13.75\text{ MPa (C)}$ .... **Ans.**

$\sigma_D = -2.50 - 7.50 + 3.75 = -6.25\text{ MPa} = 6.25\text{ MPa (C)}$ .... **Ans.**

7-193

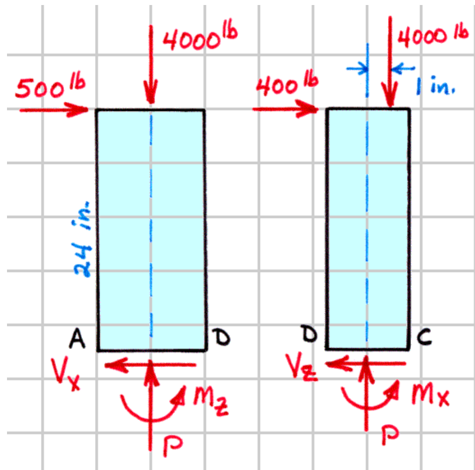
$$I_x = (6)(4)^3/12 = 32 \text{ in.}^4 \qquad A = (6\times 4) = 24 \text{ in.}^2$$

$$I_y = (4)(6)^3/12 = 72 \text{ in.}^4$$

$$P = 4000 \text{ lb (C)}$$

$$M_x = (400)(24) + (4000)(1) = 13{,}600 \text{ lb}\cdot\text{in.}$$

$$M_z = (500)(24) = 12{,}000 \text{ lb}\cdot\text{in.}$$



Neither V_x nor V_z contributes to the stresses at the corners.

$$\sigma_P = \frac{P}{A} = \frac{4000}{24} = 166.67 \text{ psi (C)}$$

$$\sigma_{M_x} = \frac{M_x c}{I_x} = \frac{(13{,}600)(2)}{32} = 850 \text{ psi (T \& C)}$$

$$\sigma_{M_z} = \frac{M_z c}{I_z} = \frac{(12{,}000)(3)}{72} = 500 \text{ psi (T \& C)}$$

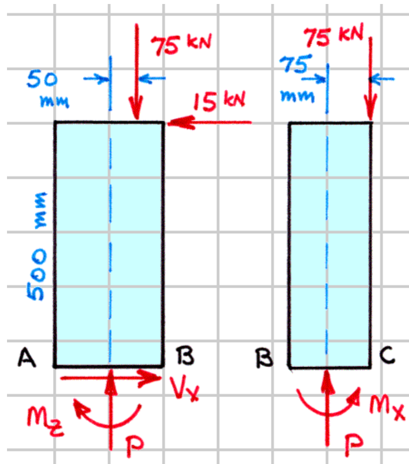
$\sigma_A = -166.67 + 850 + 500 = +1183 \text{ psi} = 1183 \text{ psi (T)}$ .... **Ans.**

$\sigma_B = -166.67 - 850 + 500 = -517 \text{ psi} = 517 \text{ psi (C)}$ .... **Ans.**

$\sigma_C = -166.67 - 850 - 500 = -1517 \text{ psi} = 1517 \text{ psi (C)}$ .... **Ans.**

$\sigma_D = -166.67 + 850 - 500 = +183.3 \text{ psi} = 183.3 \text{ psi (T)}$ .... **Ans.**

7-194*



$$A = (200 \times 150) = 30,000 \text{ mm}^2$$

$$I_x = \frac{(200)(150)^3}{12} = 56.25(10^6) \text{ mm}^4$$

$$I_z = \frac{(150)(200)^3}{12} = 100.0(10^6) \text{ mm}^4$$

$$P = 75 \text{ kN (C)}$$

$$M_x = (75)(0.075) = 5.625 \text{ kN}\cdot\text{m}$$

$$M_z = (15)(0.5) - (75)(0.050) = 3.750 \text{ kN}\cdot\text{m}$$

$$\sigma_P = \frac{P}{A} = \frac{75,000}{30(10^{-3})} = 2.50(10^6) \text{ N/m}^2 \text{ (C)}$$

$$\sigma_{M_x} = \frac{M_x c}{I_x} = \frac{(5625)(0.075)}{56.25(10^{-6})} = 7.50(10^6) \text{ N/m}^2 \text{ (T \& C)}$$

$$\sigma_{M_z} = \frac{M_z c}{I_z} = \frac{(3750)(0.100)}{100.0(10^{-6})} = 3.75(10^6) \text{ N/m}^2 \text{ (T \& C)}$$

$\sigma_A = -2.50 + 7.50 - 3.75 = +1.25 \text{ MPa} = 1.25 \text{ MPa (T)}$ .... **Ans.**

$\sigma_B = -2.50 + 7.50 + 3.75 = +8.75 \text{ MPa} = 8.75 \text{ MPa (T)}$ .... **Ans.**

$\sigma_C = -2.50 - 7.50 + 3.75 = -6.25 \text{ MPa} = 6.25 \text{ MPa (C)}$ .... **Ans.**

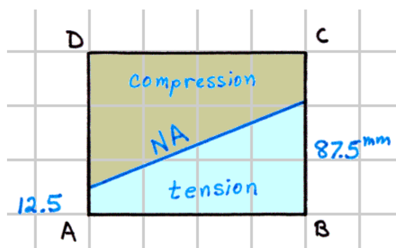
$\sigma_D = -2.50 - 7.50 - 3.75 = -13.75 \text{ MPa} = 13.75 \text{ MPa (C)}$ .... **Ans.**

$$\frac{1.250}{z'} = \frac{13.25}{150 - z'}$$

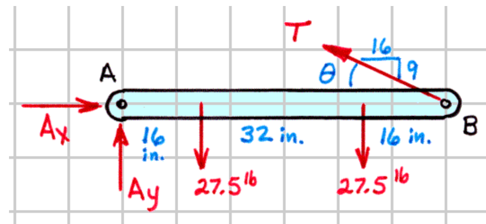
$$z' = 12.5 \text{ mm}$$

$$\frac{8.75}{z''} = \frac{6.25}{150 - z''}$$

$$z'' = 87.5 \text{ mm}$$



7-195



$$\theta = \tan^{1}(9/16) = 29.357°$$

$\circlearrowleft \Sigma M_A = 0:$

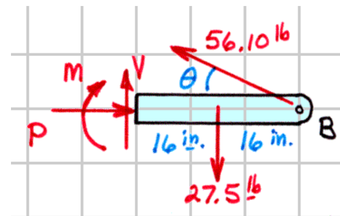
$$(T\sin\theta)(64) - (27.5)(16) - (27.5)(48) = 0$$

$$T = 56.10 \text{ lb}$$

$\rightarrow \Sigma F_x = 0: \qquad P - 56.10\cos\theta = 0$

$\uparrow \Sigma F_y = 0: \qquad V - 27.5 + 56.10\sin\theta = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \qquad -M - (27.5)(16) + (56.10\sin\theta)(32) = 0$



$$P = 48.89 \text{ lb} \qquad V = 0 \text{ lb} \qquad M = 440 \text{ lb}\cdot\text{in.}$$

(a) $$\sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{-48.89}{(1\times 2)} - \frac{(440)(0.5)}{(2)(1)^3/12} = -24.445 - 1320.0$$

$$\sigma_x = -1344.4 \text{ psi} \cong 1344 \text{ psi (C)} \qquad \sigma_y = \tau_{xy} = 0 \text{ lb}$$

Since $\tau_{xy} = 0$ these are principal stresses and

$\sigma_{p2} = 1344$ psi (C) .... $\sigma_{p1} = \sigma_{p3} = 0$ lb .... **Ans.**

$$\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{0 - (-1344.4)}{2} = 672 \text{ psi}$$ .... **Ans.**

(b) $$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = -24.445 + 1320.0 = +1295.6 \text{ psi} \cong 1296 \text{ psi (T)} \qquad \sigma_y = \tau_{xy} = 0 \text{ lb}$$

Since $\tau_{xy} = 0$ these are principal stresses and

$\sigma_{p1} = 1296$ psi (T) .... $\sigma_{p2} = \sigma_{p3} = 0$ lb .... **Ans.**

$$\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{1295.6 - 0}{2} = 648 \text{ psi}$$ .... **Ans.**

(c) $$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = -24.445 + 0 = -24.445 \text{ psi} \cong 24.4 \text{ psi (C)} \qquad \sigma_y = 0 \text{ lb}$$

Because $V = 0$, $\tau_{xy} = 0$ and these are principal stresses

$\sigma_{p2} = 24.4$ psi (C) .... $\sigma_{p1} = \sigma_{p3} = 0$ lb .... **Ans.**

$$\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{0 - (24.445)}{2} = 12.22 \text{ psi}$$ .... **Ans.**

7-196*

$$A = \pi(100)^2 / 4 = 7854 \text{ mm}^2$$

$$I = \pi(100)^4 / 64 = 4.909(10^6) \text{ mm}^4$$

$$\sigma_1 = E\varepsilon_1 = (210\times10^9)(-200\times10^{-6}) = -42.0(10^6) \text{ N/m}^2 = 42.0 \text{ MPa (C)}$$

$$\sigma_2 = E\varepsilon_2 = (210\times10^9)(820\times10^{-6}) = 172.2(10^6) \text{ N/m}^2 = 172.2 \text{ MPa (T)}$$

$$\sigma_3 = E\varepsilon_3 = (210\times10^9)(600\times10^{-6}) = 126.0(10^6) \text{ N/m}^2 = 126.0 \text{ MPa (T)}$$

$$\sigma_4 = E\varepsilon_4 = (210\times10^9)(-420\times10^{-6}) = -88.2(10^6) \text{ N/m}^2 = 88.2 \text{ MPa (C)}$$

$$\sigma_1 = \frac{P}{A} - \frac{M_z c}{I} \qquad \sigma_2 = \frac{P}{A} + \frac{M_y c}{I}$$

$$\sigma_3 = \frac{P}{A} + \frac{M_z c}{I} \qquad \sigma_4 = \frac{P}{A} - \frac{M_y c}{I}$$

Therefore: $$P = \frac{(\sigma_1 + \sigma_3)A}{2} = \frac{(-42.0 + 126.0)(10^6)(7854\times10^{-6})}{2} = 329.9(10^3) \text{ N}$$

$P \cong 330$ kN .... **Ans.**

Similarly: $$M_y = \frac{(\sigma_2 - \sigma_4)I}{2c} = \frac{(172.2 - 88.2)(10^6)(4.909\times10^{-6})}{2(0.050)} = 12.783(10^3) \text{ N}\cdot\text{m}$$

$M_y \cong 12.78 \text{ kN}\cdot\text{m}$ .... **Ans.**

$$M_z = \frac{(\sigma_3 - \sigma_1)I}{2c} = \frac{(126.0 + 42.0)(10^6)(4.909\times10^{-6})}{2(0.050)} = 8.247(10^3) \text{ N}\cdot\text{m}$$

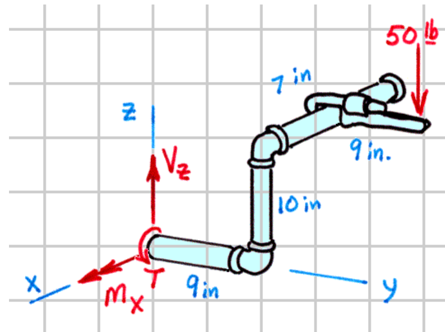
$M_z \cong 8.25 \text{ kN}\cdot\text{m}$ .... **Ans.**

7-197*

$$I = \frac{\pi(1)^4}{64} - \frac{\pi(0.75)^4}{64} = 0.03356 \text{ in.}^4 \qquad J = 2I = 0.06711 \text{ in.}^4$$

$$M_x = (50)(18) = 900 \text{ lb}\cdot\text{in.}$$

$$T = (50)(7) = 350 \text{ lb}\cdot\text{in.} \qquad V_z = 50 \text{ lb}$$



$$\sigma = \frac{Mc}{I} = \frac{(900)(0.5)}{0.003356} = 13,409 \text{ psi (T, top; C, bottom)}$$

$$\tau = \frac{Tc}{J} = \frac{(350)(0.50)}{0.06711} = 2608 \text{ psi}$$

On the top of the pipe:

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{13,409+0}{2} \pm \sqrt{\left(\frac{13,409-0}{2}\right)^2 + (2608)^2}$$

$\sigma_{p1} = 6705 + 7194 = +13,899 \text{ psi} \cong 13.90 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = 6705 - 7194 = -489 \text{ psi} \cong 0.489 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(13,899) - (-489)}{2} = 7194 \text{ psi} \cong 7.19 \text{ ksi}$ .... **Ans.**

On the bottom of the pipe:

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-13,409+0}{2} \pm \sqrt{\left(\frac{-13,409-0}{2}\right)^2 + (2608)^2}$$

$\sigma_{p1} = -6705 + 7194 = +489 \text{ psi} \cong 0.489 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = -6705 - 7194 = -13,899 \text{ psi} \cong 13.90 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(489) - (-13,899)}{2} = 7194 \text{ psi} \cong 7.19 \text{ ksi}$ .... **Ans.**

7-198

$$I = \frac{\pi(120)^4}{64} = 10.179(10^6)\ \text{mm}^4 \qquad J = 2I = 20.358(10^6)\ \text{mm}^4$$

$$Q_b = \frac{(120)^3}{12} = 144.0(10^3)\ \text{mm}^3 \qquad T_A = (30)(0.250) - (5)(0.250) = 6.25\ \text{kN}\cdot\text{m}$$

For the horizontal plane:

$$\Sigma M_B = 0: \quad R_C(2800) - (35)(800) = 0 \qquad R_C = 10.00\ \text{kN}\ \text{Z}$$

$$\Sigma M_C = 0: \quad (35)(2000) - R_B(2800) = 0 \qquad R_B = 25.00\ \text{kN}\ \text{Z}$$

For the horizontal plane:

$$\Sigma M_B = 0: \quad R_C(2800) - (35)(2000) = 0 \qquad R_C = 25.00\ \text{kN}\ \uparrow$$

$$\Sigma M_C = 0: \quad (35)(2000) - R_B(800) = 0 \qquad R_B = 10.00\ \text{kN}\ \uparrow$$

From the shear-force and bending-moment diagrams:

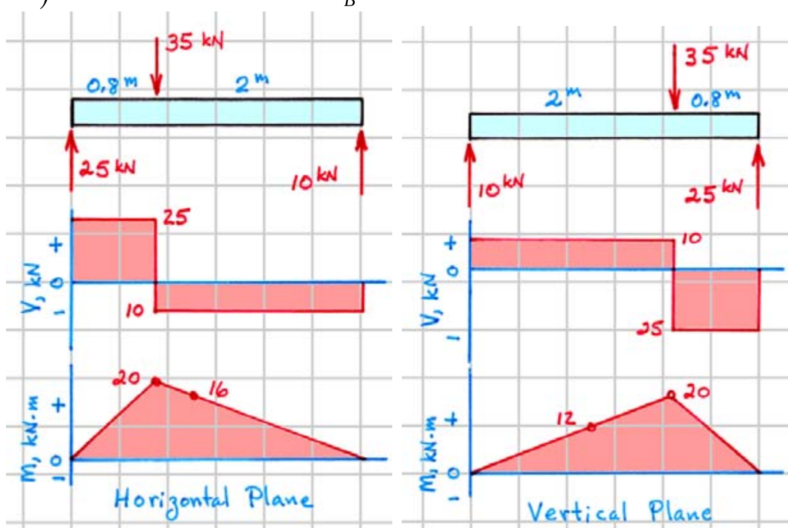
$$V_{Ax} = -10\ \text{kN}$$

$$M_{Ax} = 12\ \text{kN}\cdot\text{m}$$

$$V_{Az} = +10\ \text{kN}$$

$$M_{Az} = 16\ \text{kN}\cdot\text{m}$$

(Note that neither V_{Az} nor M_{Az} affect the stresses at *A*.)



Therefore:

$$\sigma_x = \frac{-Mc}{I} = \frac{-(12{,}000)(0.060)}{10.179(10^{-6})} = -70.73(10^6)\ \text{N/m}^2 = 70.73\ \text{MPa (C)}$$

$$\tau_{xy} = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(6250)(0{,}060)}{20.358(10^{-6})} + \frac{(10{,}000)(144\times10^{-6})}{(10.179\times10^{-6})(0.120)} = 19.60(10^6)\ \text{N/m}^2 = 19.60\ \text{MPa}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-70.73 + 00}{2} \pm \sqrt{\left(\frac{-70.73 - 0}{2}\right)^2 + (19.60)^2}$$

$\sigma_{p1} = -35.365 + 40.433 = +5.068\ \text{MPa} \cong 5.07\ \text{MPa (T)}$ .... **Ans.**

$\sigma_{p2} = -35.365 - 40.433 = -75.798\ \text{MPa} \cong 76.0\ \text{MPa (C)}$ .... **Ans.**

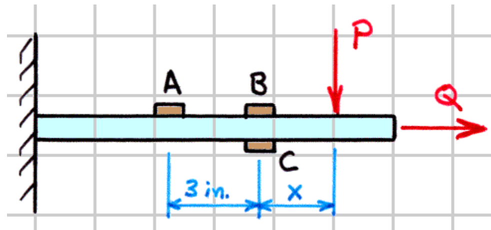
$\sigma_{p3} = 0\ \text{MPa}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(5.068) - (-75.798)}{2} = 40.4\ \text{MPa}$ .... **Ans.**

7-199*

$$A = \pi(1)^2/4 = 0.7854 \text{ in.}^2$$

$$I = \pi(1)^4/64 = 0.04909 \text{ in.}^4$$



$$\sigma_A = E\varepsilon_A = (10\times10^6)(550\times10^{-6}) = 5500 \text{ psi}$$

$$\sigma_B = E\varepsilon_B = (10\times10^6)(400\times10^{-6}) = 4000 \text{ psi}$$

$$\sigma_C = E\varepsilon_C = (10\times10^6)(-300\times10^{-6}) = -3000 \text{ psi}$$

$$\sigma_A = E\varepsilon_A = \frac{Q}{A} + \frac{[P(x+3)]c}{I} = \frac{Q}{0.7854} + \frac{P(x+3)(0.50)}{0.04909} = 5500 \text{ psi} \qquad \text{(a)}$$

$$\sigma_B = E\varepsilon_B = \frac{Q}{A} + \frac{(Px)c}{I} = \frac{Q}{0.7854} + \frac{Px(0.50)}{0.04909} = 4000 \text{ psi} \qquad \text{(b)}$$

$$\sigma_C = E\varepsilon_C = \frac{Q}{A} - \frac{(Px)c}{I} = \frac{Q}{0.7854} - \frac{Px(0.50)}{0.04909} = -3000 \text{ psi} \qquad \text{(c)}$$

From adding Eqs. (b) and (c): $Q = 392.7 \text{ lb} \cong 393 \text{ lb}$ .... **Ans.**

Then from Eq. (b): $Px = 343.63$ lb

and from Eq. (a): $Px + 3P = 490.90$ lb

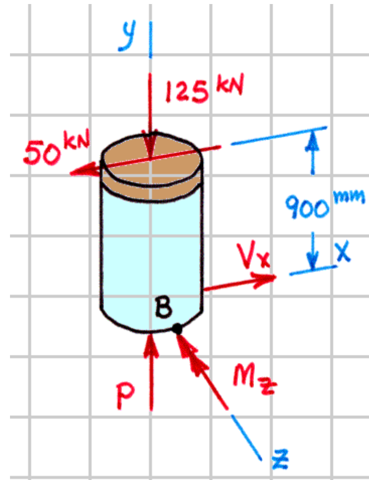
Combining these last two equations gives: $P = 49.09 \text{ lb} \cong 49.1 \text{ lb}$ .... **Ans.**

and $x = 7.00$ in. .... **Ans.**

7-200

$P = 125\ \text{kN (C)} \qquad V_x = 50\ \text{kN}$

$M_z = (50 \times 0.900) = 45.0\ \text{kN} \cdot \text{m}$



$$A = \frac{\pi(265)^2}{4} - \frac{\pi(250)^2}{4} = 6067.2\ \text{mm}^2$$

$$I = \frac{\pi(265)^4}{4} - \frac{\pi(250)^4}{4} = 805.27(10^6)\ \text{mm}^4$$

$$Q = \frac{2r^3}{3} = \frac{2(265)^3}{3} - \frac{2(250)^3}{3} = 1989.75(10^3)\ \text{mm}^3$$

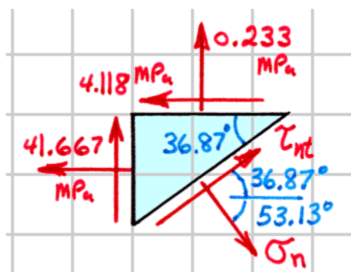
$$\sigma_P = \frac{P}{A} = \frac{125(10^3)}{6067.2(10^{-6})} = 20.60(10^6)\ \text{N/m}^2\ \text{(C)}$$

$$\sigma_{axial} = \frac{pr}{2t} = \frac{(2500 \times 10^3)(0.250)}{2(0.015)} = 20.833(10^6)\ \text{N/m}^2\ \text{(T)}$$

$$\sigma_{hoop} = \frac{pr}{t} = 41.667(10^6)\ \text{N/m}^2\ \text{(T)}$$

$$\tau_V = \frac{VQ}{It} = \frac{(50{,}000)(1989.75 \times 10^{-6})}{(805.27 \times 10^{-6})(0.030)} = 4.118(10^6)\ \text{N/m}^2$$

Note that both *A* and *B* are on the neutral axis for bending and the bending moment does not affect the stress at either *A* or *B*. The affect of the other stresses is the same at both *A* and *B*.



$\sigma_x = 41.667\ \text{MPa (T)}$

$\sigma_y = 20.833 - 20.60 = 0.233\ \text{MPa (T)}$

$\tau_{xy} = -4.118\ \text{MPa}$

7-200 (cont.)

(a) $\theta_n = -53.13°$

$$\sigma_n = \sigma_x \cos^2\theta_n + \sigma_y \sin^2\theta_n + 2\tau_{xy}\sin\theta_n\cos\theta_n$$
$$= (41.667)\cos^2(-53.13°) + (0.233)\sin^2(-53.13°) + 2(-4.118)\sin(-53.13°)\cos(-53.13°)$$

$\sigma_n = +19.10 \text{ MPa} = 19.10 \text{ MPa (T)}$ .... **Ans.**

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$
$$= -[(41.667) - (0.233)]\sin(-53.13°)\cos(-53.13°) + (-4.118)[\cos^2(-53.13°) - \sin^2(-53.13°)]$$

$\tau_{nt} = +21.0 \text{ MPa}$ .... **Ans.**

(b) $$\sigma_{p1,p2} = \frac{41.667 + 0.233}{2} \pm \sqrt{\left(\frac{41.667 - 0.233}{2}\right)^2 + (-4.118)^2}$$

$\sigma_{p1} = 20.950 + 21.122 = 42.072 \text{ MPa} \cong 42.1 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = 20.950 - 21.122 = -0.172 \text{ MPa} \cong 0.172 \text{ MPa (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ MPa}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(42.072) - (-0.172)}{2} = 21.1 \text{ MPa}$ .... **Ans.**

7-201

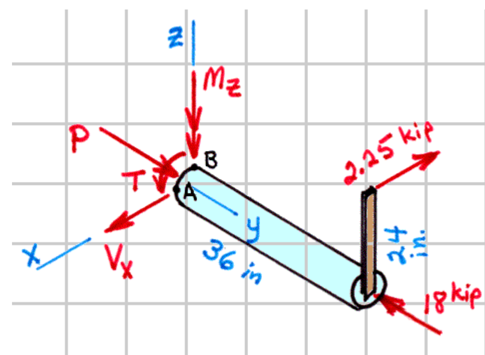
$$A = \pi(4)^2/4 = 12.566 \text{ in.}^2 \qquad I_z = \pi(4)^4/64 = 12.566 \text{ in.}^4$$

$$Q = 2r^3/3 = 2(2)^3/3 = 5.333 \text{ in.}^3 \qquad J = 2I = 25.133 \text{ in.}^4$$

$$P = 18 \text{ kip (C)} \qquad V_x = 2.25 \text{ kip}$$

$$M_z = (2.25 \times 36) = 81.0 \text{ kip}\cdot\text{in.}$$

$$T = (2.25 \times 24) = 54.0 \text{ kip}\cdot\text{in.}$$



$$\sigma_P = \frac{P}{A} = \frac{18}{12.566} = 1.4324 \text{ ksi (C)}$$

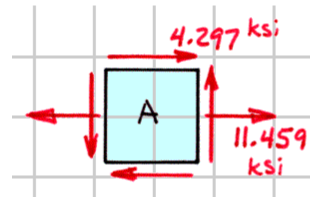
$$\sigma_{M_z} = \frac{M_z c}{I_z} = \frac{(81)(2)}{12.566} = 12.892 \text{ ksi (T, at } A)$$

$$\tau_T = \frac{Tc}{J} = \frac{(54)(2)}{25.133} = 4.297 \text{ ksi}$$

$$\tau_V = \frac{V_x Q}{I_z t} = \frac{(2.25)(5.333)}{(12.566)(4)} = 0.2387 \text{ ksi}$$

7-201 (cont.)

Therefore, at *A*:



$$\sigma_x = 12.892 - 1.4324 = 11.459 \text{ ksi}$$

$$\sigma_y = 0 \text{ ksi}$$

$$\tau_{xy} = 4.297 \text{ ksi}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{11.459 + 0}{2} \pm \sqrt{\left(\frac{11.459 - 0}{2}\right)^2 + (4.297)^2}$$

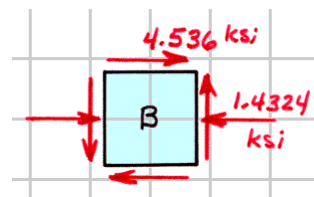
$\sigma_{p1} = 5.730 + 7.162 = +12.892 \text{ ksi} \cong 12.89 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = 5.730 - 7.162 = -1.432 \text{ ksi} \cong 1.432 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(12.892) - (-1.432)}{2} = 7.16 \text{ ksi}$ .... **Ans.**

And at *B*:



$$\sigma_x = -1.4324 \text{ ksi} = 1.4324 \text{ ksi (C)}$$

$$\sigma_y = 0 \text{ ksi}$$

$$\tau_{xy} = 4.297 + 0.2387 = 4.536 \text{ ksi}$$

$$\sigma_{p1,p2} = \frac{-1.4324 + 0}{2} \pm \sqrt{\left(\frac{-1.4324 - 0}{2}\right)^2 + (4.536)^2}$$

$\sigma_{p1} = -0.7162 + 4.592 = +3.876 \text{ ksi} \cong 3.88 \text{ ksi (T)}$ .... **Ans.**

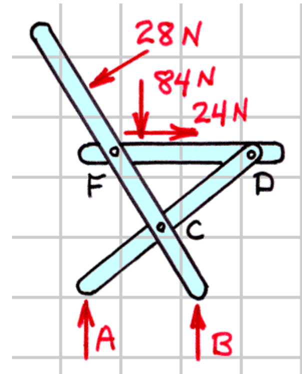
$\sigma_{p2} = -0.7162 - 4.592 = -5.308 \text{ ksi} \cong 5.31 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(3.876) - (-5.308)}{2} = 4.59 \text{ ksi}$ .... **Ans.**

7-202*

$$\theta = \tan^{-1}(3/5) = 30.96°$$



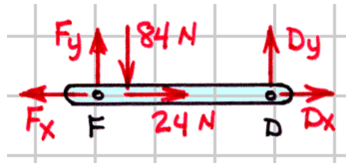
For the entire chair:

$$\uparrow \Sigma F_y = 0: \qquad A + B - 84 - 28\sin\theta = 0$$

$$\circlearrowleft \Sigma M_B = 0: \qquad (0.2)(84) - (0.5)(24) - (0.4)A + \left(0.3 + \frac{0.5}{\cos\theta}\right)(28) = 0$$

$$A = 73.82 \text{ N}$$

$$B = 24.58 \text{ N}$$



For member *DF*:

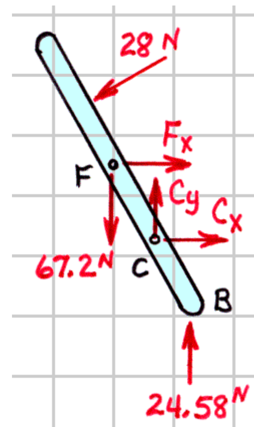
$$\rightarrow \Sigma F_x = 0: \qquad D_x - F_x + 24 = 0$$

$$\uparrow \Sigma F_y = 0: \qquad F_y + D_y - 84 = 0$$

7-202* (cont.)

$\circlearrowleft \Sigma M_D = 0: \quad (0.4)(84) - (0.5)F_y = 0 \qquad F_y = 67.2 \text{ N}$

and $\quad D_x = (F_x - 24) \text{ N} \qquad D_y = 16.80 \text{ N}$



For member *BCF*:

$\rightarrow \Sigma F_x = 0: \quad F_x + C_x - 28\cos\theta = 0$

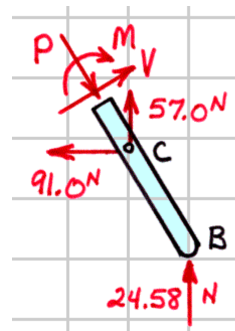
$\uparrow \Sigma F_y = 0: \quad 24.58 + C_y - 67.2 - 28\sin\theta = 0$

$$\circlearrowleft \Sigma M_D = 0: \quad \left(0.3 + \frac{0.1667}{\sin\theta}\right)(28) + (0.1333)(24.58) + (0.1667)(67.2) - (0.2777)F_x = 0$$

$F_x = 115.1 \text{ N} \qquad C_x = -91.0 \text{ N} \qquad C_y = 57.0 \text{ N}$

On a section midway between pins *C* and *F*:

$a + b = 500 \text{ mm} \qquad \dfrac{400}{a} = \dfrac{500}{b}$



Therefore $\quad a = 222.2 \text{ mm} \qquad b = 277.8 \text{ mm} \qquad b/2 = 138.9 \text{ mm}$

$c = 133.3 \text{ mm} \qquad d = 166.7 \text{ mm} \qquad d/2 = 83.4 \text{ mm}$

$\wedge \ \Sigma F_n = 0: \quad 24.6\cos 30.96° + 57.0\cos 30.96° + 91.0\sin 30.96° - P = 0$

$\circlearrowleft \Sigma M_{cut} = 0: \quad M + (91.0)(138.9) - (57.0)(83.4) - (24.6)(216.7) = 0$

$M = -2555 \text{ N}\cdot\text{mm} = 2.555 \text{ N}\cdot\text{m} \circlearrowleft$

$A = (10 \times 30) = 300 \text{ mm}^2 \qquad I = \dfrac{(10)(30)^3}{12} = 22.5(10^3) \text{ mm}^4$

$$\sigma_{top} = \frac{-P}{A} + \frac{Mc}{I} = \frac{-(116.79)}{(300\times10^{-6})} + \frac{(2.555)(0.015)}{(22.5\times10^{-9})}$$

$\sigma_{top} = +1.3140(10^6) \text{ N/m}^2 \cong 1.314 \text{ MPa (T)}$ .... **Ans.**

$$\sigma_{bottom} = \frac{-P}{A} - \frac{Mc}{I} = \frac{-(116.79)}{(300\times10^{-6})} - \frac{(2.555)(0.015)}{(22.5\times10^{-9})}$$

$\sigma_{top} = -2.093(10^6) \text{ N/m}^2 \cong 2.09 \text{ MPa (C)}$ .... **Ans.**

7-203*

$$A = \frac{\pi(4)^2}{4} = 12.566 \text{ in.}^2 \qquad G = \frac{E}{2(1+\nu)} = \frac{29,000}{2(1+0.30)} = 11{,}154 \text{ ksi}$$

$$I = \frac{\pi(4)^4}{64} = 12.566 \text{ in.}^2 \qquad J = 2I = 25.133 \text{ in.}^2$$

$$Q = \frac{2r^3}{3} = \frac{2(2)^3}{3} = 5.333 \text{ in.}^3$$

At gage *A*: $\tau_{xy} = \frac{Tc}{J} - \frac{VQ}{It} = \sigma_{45°} = -\sigma_{-45°}$

$$\varepsilon_{45°} = \frac{\sigma_{45°} - \nu\sigma_{-45°}}{E} = \frac{1+\nu}{E}\left[\frac{Tc}{J} - \frac{VQ}{It}\right] = \varepsilon_A$$

At gage *C*: $\tau_{xy} = \frac{Tc}{J} + \frac{VQ}{It} = \sigma_{45°} = -\sigma_{-45°}$

$$\varepsilon_{45°} = \frac{\sigma_{45°} - \nu\sigma_{-45°}}{E} = \frac{1+\nu}{E}\left[\frac{Tc}{J} + \frac{VQ}{It}\right] = \varepsilon_C$$

Therefore: $\varepsilon_A + \varepsilon_C = 2\left(\frac{1+\nu}{E}\right)\left(\frac{Tc}{J}\right)$

which gives $T = \frac{EJ(\varepsilon_A + \varepsilon_C)}{2(1+\nu)c} = \frac{(29,000)(25.133)(450+550)(10^{-6})}{2(1+0.30)(2)}$

$T = 140.16 \text{ kip}\cdot\text{in.} \cong 140.2 \text{ kip}\cdot\text{in.}$ .... **Ans.**

Also $\varepsilon_C - \varepsilon_A = 2\left(\frac{1+\nu}{E}\right)\left(\frac{VQ}{It}\right)$

which gives $V = \frac{EIt(\varepsilon_C - \varepsilon_A)}{2(1+\nu)Q} = \frac{(29,000)(12.566)(4)(550-450)(10^{-6})}{2(1+0.30)(5.333)}$

$V = 10.513 \text{ kip} \cong 10.51 \text{ kip}$ .... **Ans.**

At gage *B*: $\tau_{xy} = \frac{Tc}{J} = \frac{(140.16)(2)}{(25.133)} = 11.153 \text{ ksi}$

$$\sigma = \frac{-Mc}{I} = \frac{-M(2)}{12.566} = (-0.159160M) \text{ ksi}$$

$$\varepsilon_B = \varepsilon_x \cos^2 45° + \varepsilon_y \sin^2 45° + \gamma_{xy} \sin 45° \cos 45°$$

$$= (\sigma_x/E)(0.5) + (-\nu\sigma_x/E)(0.5) + (\tau_{xy}/G)(0.5)$$

$$= \left[\sigma_x(1-\nu) + \tau_{xy}2(1+\nu)\right]/2E$$

$$2(29,000)(325\times10^{-6}) = (-0.159160M)(1-0.30) + 2(11.154)(1+0.30)$$

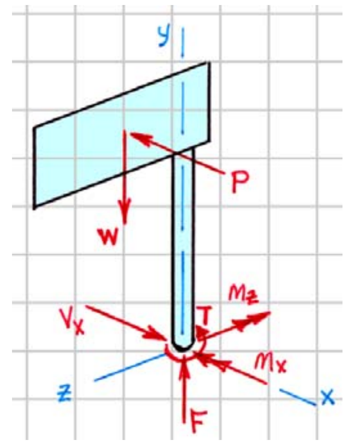
$M = 91.1 \text{ kip}\cdot\text{in.}$ .... **Ans.**

7-204

$$A = \pi\left(450^2 - 440^2\right) = 27,960 \text{ mm}^2 \qquad Q = \frac{2\left(450^3 - 440^3\right)}{3} = 3960.7\left(10^3\right) \text{ mm}^3$$

$$I = \frac{\pi\left(450^4 - 440^4\right)}{4} = 2768.8\left(10^6\right) \text{ mm}^4 \qquad J = 2I = 5537.5\left(10^6\right) \text{ mm}^4$$

$$F = W = mg = (250)(9.81) = 2453 \text{ N (C)}$$



$$V_x = P = pA = (1500)(8\times 3) = 36,000 \text{ N}$$

$$M_x = W(3) = (2453)(3) = 7359 \text{ N}\cdot\text{m}$$

$$M_z = P(9) = (36,000)(9) = 324\left(10^3\right) \text{ N}\cdot\text{m}$$

$$T = P(3) = (36,000)(3) = 108,000 \text{ N}\cdot\text{m}$$

$$\sigma_F = \frac{F}{A} = \frac{2453}{27,960\left(10^{-6}\right)} = 0.08773\left(10^6\right) \text{ N/m}^2$$

$$= 0.08773 \text{ MPa (C)}$$

$$\sigma_{M_x} = \frac{M_x c}{I} = \frac{(7359)(0.450)}{2768.8\left(10^{-6}\right)} = 1.1959\left(10^6\right) \text{ N/m}^2 = 1.1959 \text{ MPa}$$

$$\sigma_{M_z} = \frac{M_z c}{I} = \frac{\left(324\times 10^3\right)(0.450)}{2768.8\left(10^{-6}\right)} = 52.65\left(10^6\right) \text{ N/m}^2 = 52.65 \text{ MPa}$$

$$\tau_T = \frac{Tc}{J} = \frac{(108,000)(0.450)}{\left(5537.5\times 10^{-6}\right)} = 8.776\left(10^6\right) \text{ N/m}^2 = 8.776 \text{ MPa}$$

$$\tau_{V_x} = \frac{V_x Q}{It} = \frac{(36,000)\left(3960.7\times 10^{-6}\right)}{\left(2768.8\times 10^{-6}\right)(0.020)} = 2.575\left(10^6\right) \text{ N/m}^2 = 2.575 \text{ MPa}$$

At *A*: $\sigma_x = 0$ MPa $\qquad \tau_{xy} = \frac{Tc}{J} + \frac{V_x Q}{It} = 8.776 + 2.575 = 11.351$ MPa

$$\sigma_y = \frac{F}{A} + \frac{M_x c}{I} = 0.08773 + 1.1959 = 1.2838 \text{ MPa (C)}$$

7-204 (cont.)

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{0 - 1.2838}{2} \pm \sqrt{\left(\frac{0 + 1.2838}{2}\right)^2 + (11.351)^2}$$

$\sigma_{p1} = -0.6418 + 11.369 = +10.7272 \text{ MPa} \cong 10.73 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = -0.6418 - 11.369 = -12.0108 \text{ MPa} \cong 12.01 \text{ MPa (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ MPa}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(10.7272) - (-12.0108)}{2} = 11.37 \text{ MPa}$ .... **Ans.**

At *B*: $\sigma_x = 0 \text{ MPa}$ $\tau_{xy} = \dfrac{Tc}{J} = 8.776 \text{ MPa}$

$$\sigma_y = \frac{F}{A} + \frac{M_z c}{I} = -0.08773 + 52.65 = 52.56 \text{ MPa (T)}$$

$$\sigma_{p1,p2} = \frac{0 + 52.56}{2} \pm \sqrt{\left(\frac{0 - 52.56}{2}\right)^2 + (8.776)^2}$$

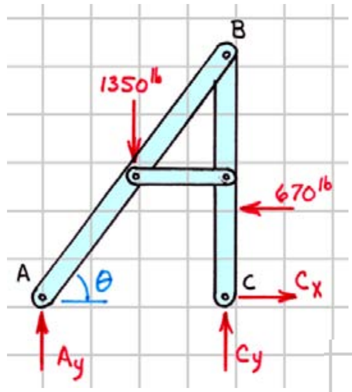
$\sigma_{p1} = 26.28 + 27.71 = +53.99 \text{ MPa} \cong 54.0 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = 26.28 - 27.71 = -1.430 \text{ MPa} \cong 1.430 \text{ MPa (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ MPa}$ .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(53.99) - (-1.430)}{2} = 27.7 \text{ MPa}$ .... **Ans.**

7-205



$$\circlearrowleft \Sigma M_C = 0: \quad (1350)(5) + (670)(5) - 10R_A = 0$$

$$R_A = 1010 \text{ lb}$$

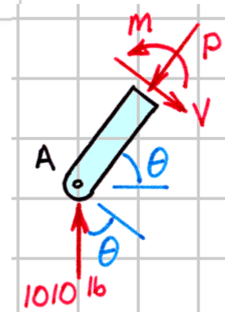
$$\theta = \tan^{-1}(13/10) = 52.431°$$

At the section containing *G* and *H*:

Z $\Sigma F_n = 0: \quad 1010\sin\theta - P = 0$

^ $\Sigma F_t = 0: \quad 1010\cos\theta - V = 0$

$$\circlearrowleft \Sigma M_{cut} = 0: \quad M - (1010\cos\theta)(3.5) = 0$$



$$P = 800.6 \text{ lb} \qquad V = 615.8 \text{ lb}$$

$$M = 2155.3 \text{ lb}\cdot\text{ft}$$

$$A = (4\times 4) = 16 \text{ in.}^2 \qquad Q_H = y_C A = (1.5)(4\times 1) = 6.00 \text{ in.}^3$$

$$I = \frac{(4)(4)^3}{12} = 21.333 \text{ in.}^4$$

At point *G*:

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{-800.6}{16} - \frac{(2155.3\times 12)(2)}{21.333} = -2474.8 \text{ psi} \qquad \tau = 0$$

$\sigma_{p1} = \sigma_{p3} = 0$ psi .... $\sigma_{p2} = -2474.8$ psi $\cong 2470$ psi C .... **Ans.**

$$\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{(0) - (-2474.8)}{2} = 1237 \text{ psi}$$ .... **Ans.**

At point *H*:

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{-800.6}{16} + \frac{(2155.3\times 12)(1)}{21.333} = +1162.3 \text{ psi}$$

$$\tau = \frac{VQ}{It} = \frac{(615.8)(6.00)}{(21.333)(4)} = 43.30 \text{ psi}$$

$$\sigma_{p1,p2} = \frac{1162.3 + 0}{2} \pm \sqrt{\left(\frac{1162.3 - 0}{2}\right)^2 + (43.30)^2}$$

$\sigma_{p1} = 581.15 + 582.76 = +1163.91$ psi $\cong 1164$ psi (T) .... **Ans.**

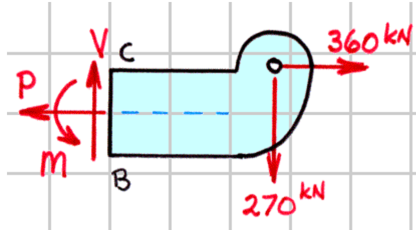
$\sigma_{p2} = 581.15 - 582.76 = -1.61$ psi $\cong 1.61$ psi (C) .... **Ans.**

$\sigma_{p3} = 0$ psi .... **Ans.**

$$\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{(1163.91) - (-1.61)}{2} = 583 \text{ psi}$$ .... **Ans.**

7-206

(a)

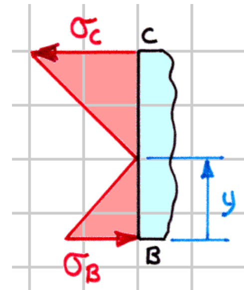


$$P = 360 \text{ kN} \qquad V = 270 \text{ kN}$$

$$M = (360)(0.150) + (270)(0.450) = 175.5 \text{ kN}\cdot\text{m}$$

$$A = 2(120\times 30) + (30\times 180) = 12{,}600 \text{ mm}^2$$

$$I = \frac{(120)(240)^3}{12} - \frac{(90)(180)^3}{12} = 94.50\left(10^6\right) \text{ mm}^4$$



$$\sigma_x = \frac{P}{A} + \frac{My}{I} = \frac{360\left(10^3\right)}{12{,}600\left(10^{-6}\right)} + \frac{\left(175.5\times 10^3\right)y}{\left(94.50\times 10^{-6}\right)}$$

$$= (28.571 + 1857.14y)\left(10^6\right) \text{ N/m}^2$$

$$\sigma_C = 28.571 + (1857.14)(0.120) = 251.43 \text{ MPa (T)}$$

$$\sigma_B = 28.571 - (1857.14)(0.120) = 194.29 \text{ MPa (C)}$$

$$Q = y_C A = \frac{(120+y)}{2}(120)(120-y) = 60\left(120^2 - y^2\right) \text{ mm}^3 \qquad 90 \le y \le 120 \text{ mm}$$

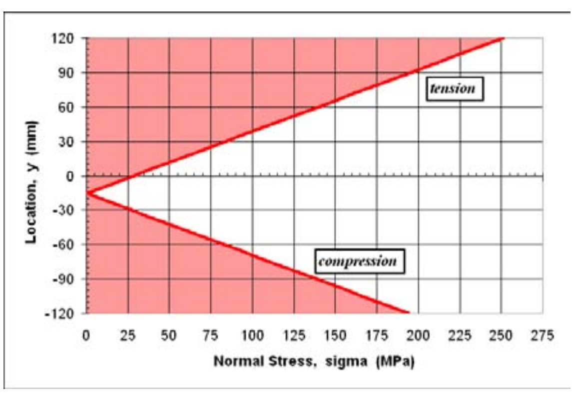
$$Q = (105)(120\times 30) + \frac{(90+y)}{2}(30)(90-y)$$

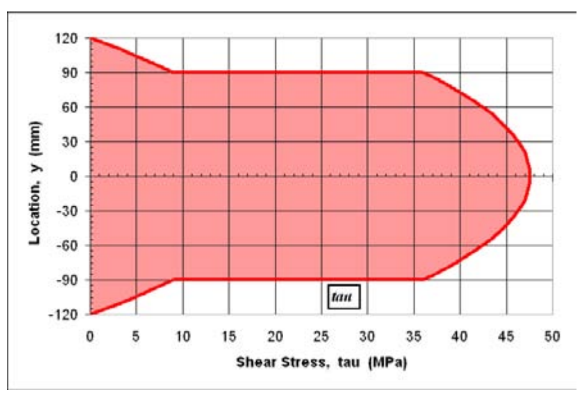
$$= \left[378{,}000 + 15\left(90^2 - y^2\right)\right] \text{ mm}^3 \qquad -90 \le y \le 90 \text{ mm}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{\left(270\times 10^3\right)Q}{\left(94.50\times 10^{-6}\right)t}$$

$t = 120$ mm $\qquad 90 \le y \le 120$ mm

$t = 30$ mm $\qquad -90 \le y \le 90$ mm





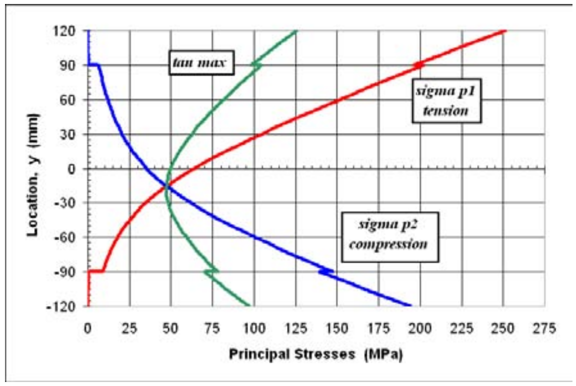
7-206 (cont.)

(b) $$\frac{y}{\sigma_B} = \frac{240}{\sigma_B + \sigma_C}$$

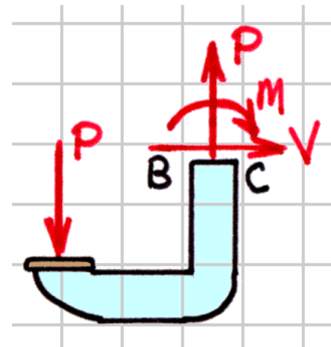
$$y = 240\frac{194.29}{194.29 + 251.43} = 104.62 \text{ mm}$$

(c) $$\sigma_{p1,p2} = \frac{\sigma_x + 0}{2} \pm \sqrt{\left(\frac{\sigma_x - 0}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2}$$



7-207



$$A=(1\times 0.25)=0.25 \text{ in}^2$$

$$I=\frac{bh^3}{12}=\frac{(0.25)(1)^3}{12}=0.02083 \text{ in.}^4$$

$$M=P(3)=(3P) \text{ lb}$$

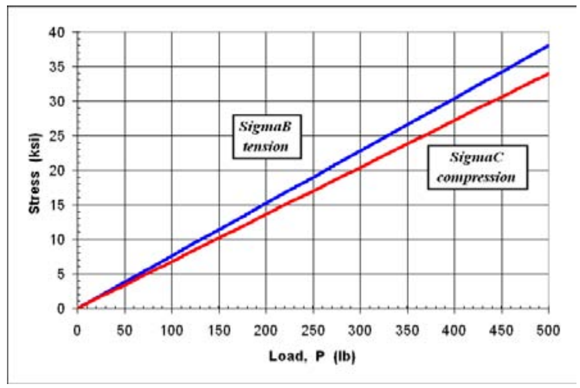
(a) $$\sigma_B=\frac{P}{A}+\frac{Mc}{I}=\frac{P}{0.25}+\frac{(3P)(0.5)}{0.02083}=(76.01P) \text{ psi}$$

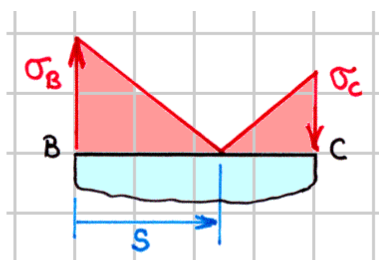
$$\sigma_C=\frac{P}{A}-\frac{Mc}{I}=\frac{P}{0.25}-\frac{(3P)(0.5)}{0.02083}=(-68.01P) \text{ psi}$$

(b) $$\frac{s}{\sigma_B}=\frac{1-s}{\sigma_C}$$

$$s=\frac{\sigma_B}{\sigma_B+\sigma_C}=\frac{76.01}{76.01+68.01}=0.5278 \text{ in.}$$

The distance s is independent of P.





7-208*

By symmetry each support carries half of the total load:

$$A = B = 6700 \text{ N} \uparrow$$

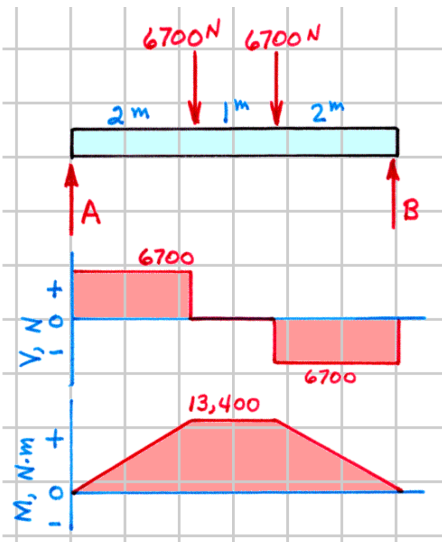
From the shear-force and bending-moment diagrams:

$$V_{\max} = 6700 \text{ N} \qquad M_{\max} = 13{,}400 \text{ N}\cdot\text{m}$$

Then $$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{13{,}400}{S} \le 9\left(10^6\right) \text{ N/m}^2$$

$$S \ge \frac{13{,}400}{9\left(10^6\right)} = 1488.9\left(10^{-6}\right) \text{ m}^3$$

$$= 1488.9\left(10^3\right) \text{ mm}^3$$



Try a 203×254-mm timber with

$$S = 1850\left(10^3\right) \text{ mm}^3 \qquad A = 46{,}000 \text{ mm}^2$$

$$I = 223\left(10^6\right) \text{ mm}^4 \qquad m = 29.4 \text{ kg/m}$$

When the weight of the beam is included,
the maximum moment is

$$M = M_{load} + M_{weight}$$

$$= 13{,}400 + \frac{(29.4 \times 9.81)(5)^2}{8} = 14{,}301 \text{ N}\cdot\text{m}$$

$$S \ge \frac{14{,}301}{9\left(10^6\right)} = 1589\left(10^{-6}\right) \text{ m}^3 = 1589\left(10^3\right) \text{ mm}^3$$

which is still okay. Next, check the shear stress,

$$V_{\max} = V_{load} + V_{weght} = 6700 + \frac{(29.4 \times 9.81)(5)}{2} = 7421 \text{ N}$$

$$\tau_{\max} = 1.5\frac{V_{\max}}{A} = \frac{1.5(7421)}{46{,}000} = 242\left(10^3\right) \text{ N/m}^2 = 242 \text{ kPa}$$

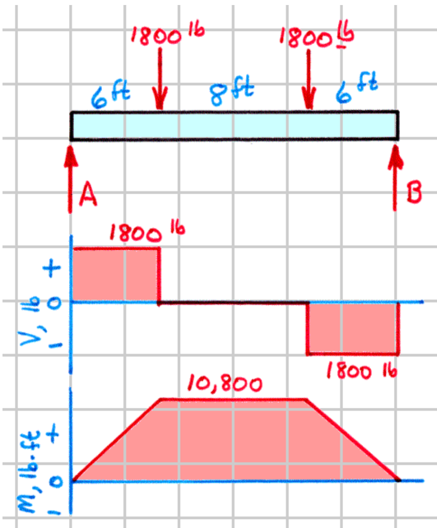
which is much less than the allowable shear stress of 600 kPa. Therefore, this design is okay.

Use a 203×254-mm timber .... **Ans.**

7-209*

By symmetry each support carries half of the total load: $A = B = 1800 \text{ lb} \uparrow$

From the shear-force and bending-moment diagrams:



$$V_{\max} = 1800 \text{ lb} \qquad M_{\max} = 10,800 \text{ lb} \cdot \text{ft}$$

Then $$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{10,800 \times 12}{S} \le 1900 \text{ psi}$$

$$S \ge \frac{10,800 \times 12}{1900} = 68.21 \text{ in.}^3$$

Try an 8×8-in. timber with

$$S = 70.3 \text{ in.}^3 \qquad A = 56.3 \text{ in.}^2$$
$$I = 264 \text{ in.}^4 \qquad w = 15.6 \text{ lb/ft}$$

When the weight of the beam is included,
the maximum moment is

$$M = M_{load} + M_{weight} = 10,800 + \frac{(15.6)(20)^2}{8} = 11,580 \text{ lb} \cdot \text{ft}$$

$$S \ge \frac{11,580 \times 12}{1900} = 73.14 \text{ in.}^3$$

which is bigger than the section modulus of the chosen timber. Next, try a 4×12-in. timber with

$$S = 79.9 \text{ in.}^3 \qquad I = 459 \text{ in.}^4 \qquad A = 41.7 \text{ in.}^2 \qquad w = 11.6 \text{ lb/ft}$$

When the weight of the beam is included, the maximum moment is now

$$M = 10,800 + \frac{(11.6)(20)^2}{8} = 11,380 \text{ lb} \cdot \text{ft} \qquad S \ge \frac{11,380 \times 12}{1900} = 71.87 \text{ in.}^3$$

which is still okay. Next, check the shear stress,

$$V_{\max} = V_{load} + V_{weght} = 1800 + \frac{(11.6)(20)}{2} = 1916 \text{ lb}$$

$$\tau_{\max} = 1.5\frac{V_{\max}}{A} = \frac{1.5(1916)}{41.7} = 68.92 \text{ psi}$$

which is less than the allowable shear stress of 90 psi. Therefore, this design is okay.

Use a 4×12-in. timber .... **Ans.**

7-210

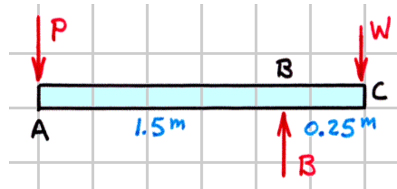
$\circlearrowleft \Sigma M_B = 0: \quad 1.5P - (0.25)(275 \times 9.807) = 0$ $\qquad P = 449.5 \text{ N}$

$\circlearrowleft \Sigma M_C = 0: \quad 1.75(449.5) - 0.25B = 0$ $\qquad B = 3147 \text{ N}$

$$M_{max} = (449.5)(1.5) = 674.25 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{674.25}{S} \le 135(10^6) \text{ N/m}^2$$

$$S \ge \frac{674.25}{135(10^6)} = 4.994(10^{-6}) \text{ m}^3 = 4.994(10^3) \text{ mm}^3$$

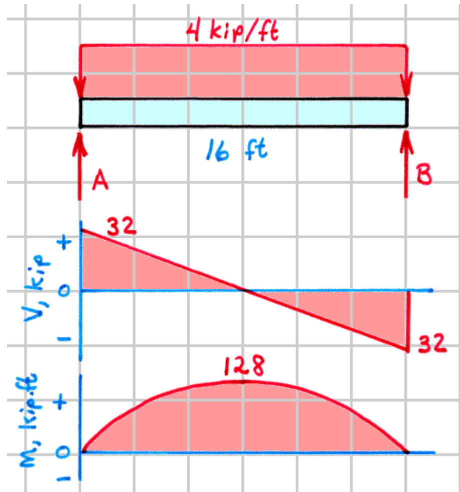


Therefore Use a 38-mm diameter standard weight pipe .... **Ans.**

7-211*

By symmetry each support carries half of the total load:

$$A = B = (4\times16)/2 = 32 \text{ kip} \uparrow$$



From the shear-force and bending-moment diagrams:

$$V_{\max} = 32 \text{ kip} \qquad M_{\max} = 128 \text{ kip}\cdot\text{ft}$$

Then $\sigma_{\max} = \dfrac{M_{\max}}{S} = \dfrac{128\times12}{S} \le 22 \text{ ksi}$

$$S \ge \frac{128\times12}{22} = 69.82 \text{ in.}^3$$

Try a W 18×60 section with

$$S = 108 \text{ in.}^3 \qquad d = 2c = 18.24 \text{ in.}$$

$$t_w = 0.415 \text{ in.} \qquad w = 60 \text{ lb/ft}$$

When the weight of the beam is included, the maximum moment is

$$M = M_{load} + M_{weight} = 128 + \frac{(0.060)(16)^2}{8} = 129.92 \text{ lb}\cdot\text{ft}$$

$$S \ge \frac{129.92\times12}{22} = 70.87 \text{ in.}^3$$

which is still okay. Next, check the shear stress,

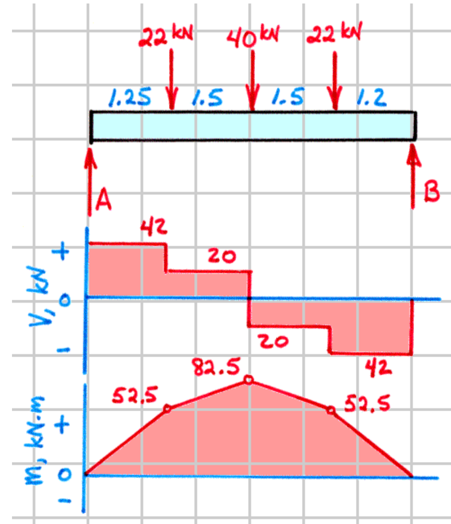
$$V_{\max} = V_{load} + V_{weght} = 32 + \frac{(0.060)(16)}{2} = 32.48 \text{ kip}$$

$$\tau_{\max} = \frac{V_{\max}}{A_{web}} = \frac{32.48}{0.415\times18.24} = 4.29 \text{ ksi}$$

which is less than the allowable shear stress of 14.5 ksi. Therefore, this design is okay.

Use a W 18×60 section .... **Ans.**

7-212*



By symmetry each support carries half of the total load:

$$A = B = 42 \text{ kN} \uparrow$$

From the shear-force and bending-moment diagrams:

$$V_{\max} = 42 \text{ kN} \qquad M_{\max} = 82.5 \text{ kN}\cdot\text{m}$$

Then
$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{82{,}500}{S} \le 152\left(10^6\right) \text{ N/m}^2$$

$$S \ge \frac{82{,}500}{152\left(10^6\right)} = 542.8\left(10^{-6}\right) \text{ m}^3 = 542.8\left(10^3\right) \text{ mm}^3$$

Try an S 305×47 section with

$$S = 596\left(10^3\right) \text{ mm}^3 \qquad d = 2c = 304.8 \text{ mm}$$

$$t_w = 8.9 \text{ mm} \qquad m = 47 \text{ kg/m}$$

When the weight of the beam is included, the maximum moment is

$$M = M_{load} + M_{weight} = 82{,}500 + \frac{(47\times 9.81)(5.5)^2}{8} = 84{,}243 \text{ N}\cdot\text{m}$$

$$S \ge \frac{84{,}243}{152\left(10^6\right)} = 554.2\left(10^{-6}\right) \text{ m}^3 = 554.2\left(10^3\right) \text{ mm}^3$$

which is still okay. Next, check the shear stress,

$$V_{\max} = V_{load} + V_{weght} = 42{,}000 + \frac{(47\times 9.81)(5.5)}{2} = 43{,}268 \text{ N}$$

$$\tau_{\max} = \frac{V_{\max}}{A_{web}} = \frac{43{,}268}{0.0089\times 0.3048} = 15.95\left(10^6\right) \text{ N/m}^2 = 15.95 \text{ MPa}$$

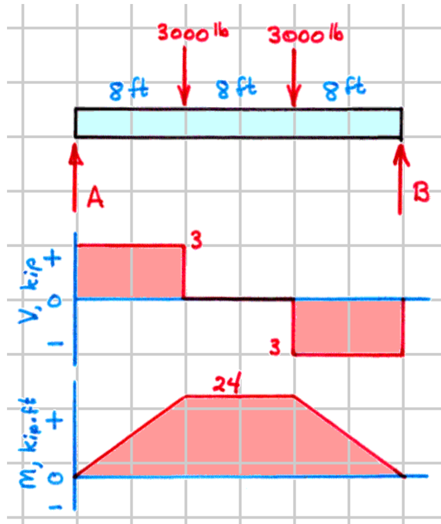
which is much less than the allowable shear stress of 100 MPa. Therefore, this design is okay.

Use an S 305×47 section .... **Ans.**

7-213

By symmetry each support carries half of the total load: $A = B = 3000 \text{ lb} \uparrow$

From the shear-force and bending-moment diagrams:



$$V_{\max} = 3 \text{ kip} \qquad M_{\max} = 24 \text{ kip} \cdot \text{ft}$$

Then $$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{24 \times 12}{S} \le 22 \text{ ksi}$$

$$S \ge \frac{24 \times 12}{22} = 13.091 \text{ in.}^3$$

Try a W 6×25 section with

$$S = 16.7 \text{ in.}^3 \qquad d = 2c = 6.38 \text{ in.}$$

$$t_w = 0.320 \text{ in.} \qquad w = 25 \text{ lb/ft}$$

When the weight of the beam is included, the maximum moment is

$$M = 24{,}000 + \frac{(25)(24)^2}{8} = 25{,}800 \text{ lb} \cdot \text{ft}$$

$$S \ge \frac{25.80 \times 12}{22} = 14.07 \text{ in.}^3$$

which is still okay. Next, check the shear stress,

$$V_{\max} = V_{load} + V_{weght} = 3000 + \frac{(25)(24)}{2} = 3300 \text{ lb}$$

$$\tau_{\max} = \frac{V_{\max}}{A_{web}} = \frac{3300}{0.320 \times 6.35} = 1616.4 \text{ psi}$$

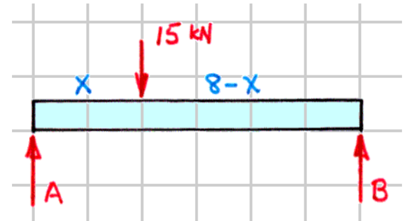
which is much less than the allowable shear stress of 14.5 ksi. Therefore, this design is okay.

Use a W 6×25 section .... **Ans.**

7-214*

$\circlearrowleft \Sigma M_B = 0: \quad (15)(8-x) - 8A = 0 \qquad A = (15 - 1.875x) \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_A = 0: \quad 8B - 15x = 0 \qquad B = (1.875x) \text{ kN} \uparrow$



For any location x, the maximum moment in the beam occurs under the load and is equal to

$$M_C = Ax = (15x - 1.875x^2) \text{ kN}\cdot\text{m}$$

The location x that makes M_C a maximum is found from

$$\frac{dM_C}{dx} = (15 - 3.75x) = 0$$

which gives $x = 4.00$ m $\qquad M_{\max} = 30 \text{ kN}\cdot\text{m}$

Then $$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{30,000}{S} \le 152(10^6) \text{ N/m}^2$$

$$S \ge \frac{30,000}{152(10^6)} = 197.4(10^{-6}) \text{ m}^3 = 197.4(10^3) \text{ mm}^3$$

Try an S 203×27 section with

$$S = 236(10^3) \text{ mm}^3 \qquad d = 2c = 203.2 \text{ mm} \qquad t_w = 6.9 \text{ mm} \qquad m = 27 \text{ kg/m}$$

When the weight of the beam is included, the maximum moment is

$$M = M_{load} + M_{weight} = 34.386 \text{ kN}\cdot\text{m}$$

$$S \ge \frac{34,386}{152(10^6)} = 226.2(10^{-6}) \text{ m}^3 = 226.2(10^3) \text{ mm}^3$$

which is still okay. Next, check the shear stress, The maximum shear stress occurs when the load is near one of the supports $(x = 0 \text{ or } x = 8 \text{ m})$ and is equal to

$$V_{\max} = V_{load} + V_{weght} = 15,000 + \frac{(27 \times 9.81)(8)}{2} = 16,059 \text{ N}$$

$$\tau_{\max} = \frac{V_{\max}}{A_{web}} = \frac{16,059}{0.0069 \times 0.2032} = 11.45(10^6) \text{ N/m}^2 = 11.45 \text{ MPa}$$

which is much less than the allowable shear stress of 100 MPa. Therefore, this design is okay.

Use an S 203×27 section .... **Ans.**

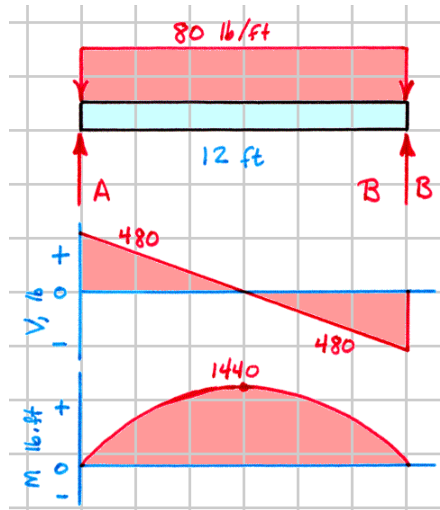
7-215

Since the joists are spaced 16 in. (1.3333 ft) apart, the uniformly distributed load is

$$w = (60)(1.3333) = 80 \text{ lb/ft}$$

By symmetry each support carries half of the total load:

$$A = B = (12\times 80)/2 = 480 \text{ lb} \uparrow$$



From the shear-force and bending-moment diagrams:

$$V_{\max} = 480 \text{ lb} \qquad M_{\max} = 1440 \text{ lb}\cdot\text{ft}$$

Then $\sigma_{\max} = \dfrac{M_{\max}}{S} = \dfrac{1440\times 12}{S} \le 1200 \text{ psi}$

$$S \ge \frac{1440\times 12}{1200} = 14.40 \text{ in.}^3$$

Try a 2×8-in.timber with

$$S = 15.3 \text{ in.}^3 \qquad I = 57.1 \text{ in.}^4 \qquad A = 12.2 \text{ in.}^2 \qquad w = 3.39 \text{ lb/ft}$$

When the weight of the beam is included, the maximum moment is

$$M = 1440 + \frac{(3.39)(12)^2}{8} = 1501 \text{ lb}\cdot\text{ft} \qquad S \ge \frac{1501\times 12}{1200} = 15.01 \text{ in.}^3$$

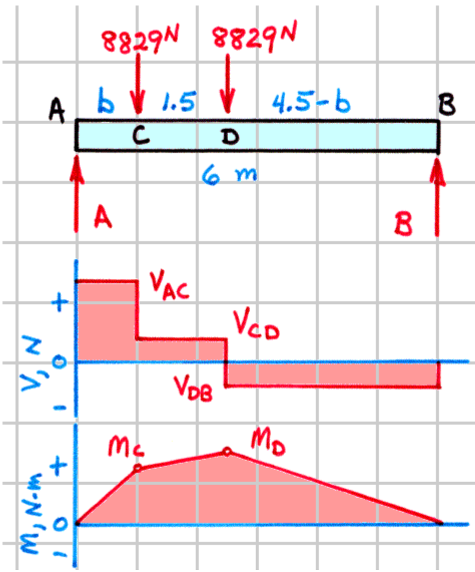
which is still okay. Next, check the shear stress,

$$V_{\max} = 480 + \frac{(3.39)(12)}{2} = 500.3 \text{ lb} \qquad \tau_{\max} = 1.5\frac{V_{\max}}{A} = \frac{1.5(500.3)}{12.2} = 61.51 \text{ psi}$$

which is much less than the allowable shear stress of 120 psi. Therefore, this design is okay.

Use a 2×8-in. timber section .... **Ans.**

7-216



$$\circlearrowleft \Sigma M_A = 0: \quad 6B - 8829b - (8829)(b + 1.5) = 0$$

$$B = (2943b + 2207) \text{ N}$$

$$\uparrow \Sigma F_y = 0: \quad A + B - 2(8829) = 0$$

$$A = (15{,}451 - 2943b) \text{ N}$$

Then $V_{AC} = A = (15{,}451 - 2943b) \text{ N}$

$$V_{CD} = V_{AC} - 8829 = (6622 - 2943b) \text{ N}$$

$$M_C = V_{AC} b = (15{,}451b - 2943b^2) \text{ N} \cdot \text{m}$$

$$M_D = M_C + V_{CD}(1.5)$$

$$= (9933 + 11{,}036b - 2943b^2) \text{ N} \cdot \text{m}$$

Clearly, the maximum moment occurs under one of the wheels. The location b which gives the maximum for these moments is found from

$$\frac{dM_C}{db} = (15{,}451 - 5886b) = 0$$

$$b = 2.625 \text{ m} \qquad M_C = 20{,}280 \text{ N} \cdot \text{m} \qquad M_D = 18{,}624 \text{ N} \cdot \text{m}$$

$$\frac{dM_D}{db} = (11{,}036 - 5886b) = 0$$

$$b = 1.875 \text{ m} \qquad M_C = 18{,}624 \text{ N} \cdot \text{m} \qquad M_D = 20{,}280 \text{ N} \cdot \text{m}$$

Therefore, the maximum bending moment occurs under the wheel closest to the center of the beam

$$M_{max} = 20{,}280 \text{ N} \cdot \text{m}$$

The minimum section modulus required is:

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{20{,}280}{165(10^6)} = 122.9(10^{-6}) \text{ m}^3 = 122.9(10^3) \text{ mm}^3$$

Try a W 127 × 24 section with

$$S = 139(10^3) \text{ mm}^3 \qquad d = 2c = 127 \text{ mm}$$

$$t_w = 6.1 \text{ mm} \qquad m = 24 \text{ kg/m}$$

Next, check the shear stress, The maximum shear stress occurs when the load is near one of the supports $(x = 0 \text{ or } x = 6 \text{ m})$

$$\tau_{max} = \frac{V_{max}}{A_{web}} = \frac{15{,}451}{0.0061 \times 0.127} = 19.94(10^6) \text{ N/m}^2 = 19.94 \text{ MPa}$$

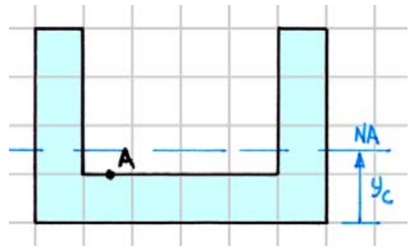
which is much less than the allowable shear stress of 100 MPa.
Therefore, this design is okay.

Use a W 127 × 24 section .... **Ans.**

7-217*

$$y_C = \frac{M_x}{A} = \frac{2\left[(4)(2\times 8)\right]+(1)(8\times 2)}{2(2\times 8)+(8\times 2)} = 3.00\text{ in.}$$

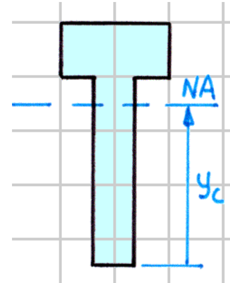
$$I = 2\frac{(2)(5)^3}{3} + \frac{(12)(3)^3}{3} - \frac{(8)(1)^3}{3} = 272.00\text{ in.}^4$$



(a) $\sigma_{\max} = (5/-1)\sigma_A = (-5)(2000) = -10,000\text{ psi} = 10\text{ ksi (C)}$ .... **Ans.**

(b) $M_r = \frac{-\sigma_A I}{y_A} = \frac{-(2000)(272.00)}{(-1)} = +544\left(10^3\right)\text{ lb}\cdot\text{in.} = 544\text{ kip}\cdot\text{in.}$ .... **Ans.**

7-218*



$$y_C = \frac{M_x}{A} = \frac{(100)(37.5\times 200)+(225)(100\times 50)}{(37.5\times 200)+(100\times 50)} = 150.0 \text{ mm}$$

$$I = \frac{(37.5)(150)^3}{3} + \frac{(100)(100)^3}{3} - \frac{(62.5)(50)^3}{3} = 72.92\left(10^6\right) \text{ mm}^4$$

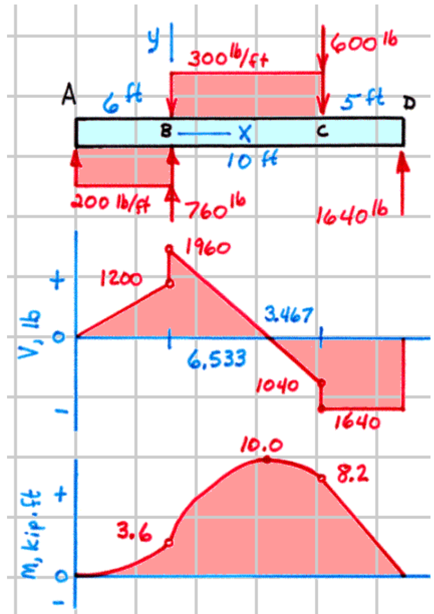
$$\sigma_{bottom} = \frac{-M_r y}{I} = \frac{-\left(100\times 10^3\right)(-0.150)}{\left(72.92\times 10^{-6}\right)}$$

$\sigma_{bottom} = +206\left(10^6\right) \text{ N/m}^2 = 206 \text{ MPa (T)}$ .... **Ans.**

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-\left(100\times 10^3\right)(+0.100)}{\left(72.92\times 10^{-6}\right)}$$

$\sigma_{top} = -137.1\left(10^6\right) \text{ N/m}^2 = 137.1 \text{ MPa (C)}$ .... **Ans.**

7-219



For $0 \text{ ft} \le x \le 10 \text{ ft}$

$$V = (200 \times 6) + 760 - 300x$$

$$V = (-300x + 1960) \text{ lb}$$ .... **Ans.**

$$M = (200 \times 6)(x+3) + 760x - (300x)(x/2)$$

$$M = (-150x^2 + 1960x + 3600) \text{ lb} \cdot \text{ft}$$ .... **Ans.**

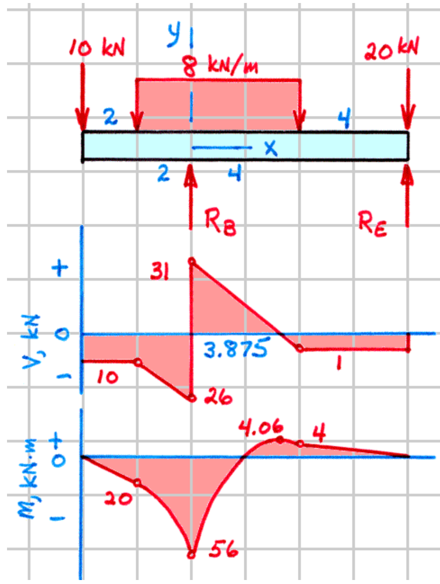
7-220*

$$\circlearrowleft \Sigma M_E = 0: \quad (10)(12) + (8\times 6)(7) - 8R_B = 0$$

$$R_B = 57 \text{ kN}$$

$$\uparrow \Sigma F_y = 0: \quad R_B + R_E - 10 - (8\times 6) - 20 = 0$$

$$R_E = 21 \text{ kN}$$



For $0 \text{ m} \le x \le 4 \text{ m}$

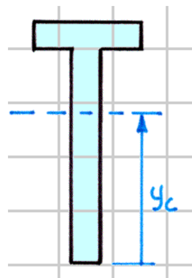
$$V = -(10) - (8)(x+2) + (57)$$

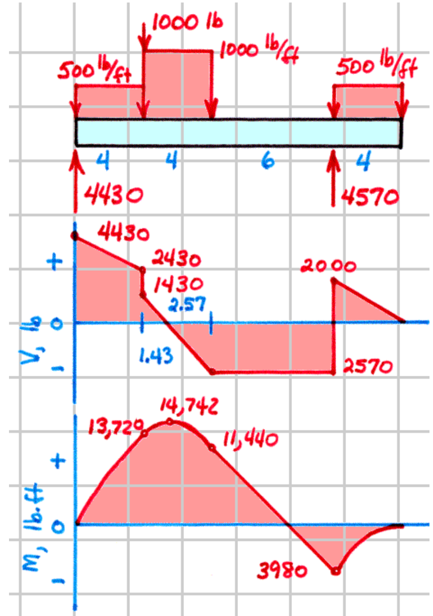
$$V = (31 - 8x) \text{ kN}$$ .... **Ans.**

$$M = -(10)(x+4) - [(8)(x+2)]\frac{(x+2)}{2} + (57x)$$

$$M = (-4x^2 + 31x - 56) \text{ kN}\cdot\text{m}$$ .... **Ans.**

7-221





$$y_C = \frac{(4)(1\times 8)+(8.5)(4\times 1)}{(1\times 8)+(4\times 1)}$$

$$= 5.50 \text{ in.}$$

$$I = \frac{(4)(3.5)^3}{3} - \frac{(3)(2.5)^3}{3} + \frac{(1)(5.5)^3}{3}$$

$$= 97.00 \text{ in.}^4$$

At $M = +14.742 \text{ kip}\cdot\text{ft}$

$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(14.742\times 12)(+3.5)}{(97.00)} = -6.38 \text{ ksi}$$

$$\sigma_{bottom} = \frac{-M_r y}{I} = \frac{-(14.742\times 12)(-5.5)}{(97.00)} = +10.03 \text{ ksi}$$

At $M = -3.980 \text{ kip}\cdot\text{ft}$

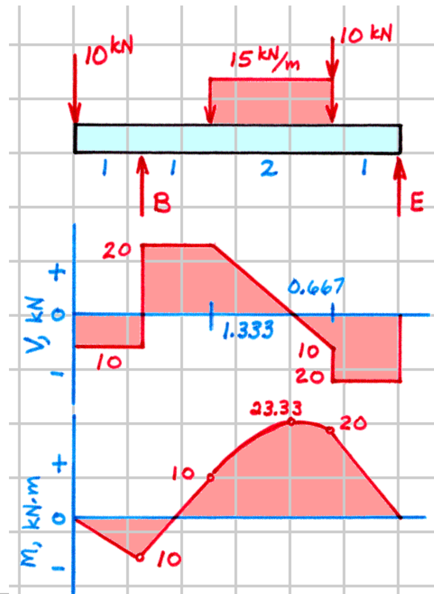
$$\sigma_{top} = \frac{-M_r y}{I} = \frac{-(-3.980\times 12)(+3.5)}{(97.00)} = +1.723 \text{ ksi}$$

$$\sigma_{bottom} = \frac{-M_r y}{I} = \frac{-(-3.980\times 12)(-5.5)}{(97.00)} = -2.71 \text{ ksi}$$

$\sigma_T(\max) = 10.03$ ksi (T) .... **Ans.**

$\sigma_C(\max) = 6.38$ ksi (C) .... **Ans.**

7-222



$$\circlearrowleft \Sigma M_E = 0: \quad (10)(5) - 4B + (15\times 2)(2) + (10)(1) = 0$$

$$B = 30.0 \text{ kN}$$

$$\circlearrowleft \Sigma M_B = 0: \quad 4E + (10)(1) - (15\times 2)(2) - (10)(3) = 0$$

$$E = 20.0 \text{ kN}$$

For a $\text{WT } 305\times 70$ section : $d = 2c = 308.7 \text{ mm}$

$y_C = 75.9 \text{ mm}$ $\quad t_w = 13.1 \text{ mm}$

$w_f = 230.3 \text{ mm}$ $\quad t_f = 22.2 \text{ mm}$

$I = 77.4\left(10^6\right) \text{ mm}^4$

(a) At the bottom of the beam 2.33 m from the left support:

$$\sigma_{bottom} = \frac{-Mc}{I} = \frac{-\left(23.33\times 10^3\right)(-0.2328)}{\left(77.4\times 10^{-6}\right)}$$

$\sigma_{\max T} = +70.2\left(10^6\right) \text{ N/m}^2 = 70.2 \text{ MPa (T)}$ .... **Ans.**

(b) At the bottom of the beam at the left support:

$$\sigma_{bottom} = \frac{-Mc}{I} = \frac{-\left(-10.00\times 10^3\right)(-0.2328)}{\left(77.4\times 10^{-6}\right)}$$

$\sigma_{\max C} = -30.1\left(10^6\right) \text{ N/m}^2 = 30.1 \text{ MPa (C)}$ .... **Ans.**

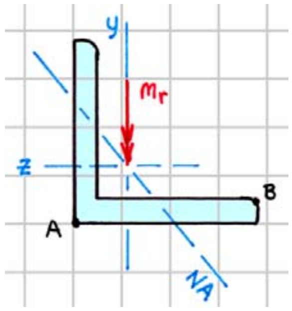
(c) $Q_{NA} = y_C A = 116.4(232.8\times 13.1) = 355.0\left(10^3\right) \text{ mm}^3$

$\tau_{\max} = \dfrac{\left(20\times 10^3\right)\left(355\times 10^{-6}\right)}{\left(77.4\times 10^{-6}\right)(0.0131)} = 7.00\left(10^6\right) \text{ N/m}^2 = 7.00 \text{ MPa}$ .... **Ans.**

(d) $Q_J = y_C A = 64.8(230.3\times 22.2) = 331.3\left(10^3\right) \text{ mm}^3$

$\tau_J = \dfrac{\left(20\times 10^3\right)\left(331.3\times 10^{-6}\right)}{\left(77.4\times 10^{-6}\right)(0.0131)} = 6.53\left(10^6\right) \text{ N/m}^2 = 6.53 \text{ MPa}$ .... **Ans.**

7-223*



$$M_{ry} = -6.00 \text{ kip}\cdot\text{ft} = -72.00 \text{ kip}\cdot\text{in.} \qquad M_{rz} = 0$$

$$\tan\beta = \frac{M_{ry}I_z + M_{rz}I_{yz}}{M_{rz}I_y + M_{ry}I_{yz}} = \frac{I_z}{I_{yz}} = \frac{89.0}{52.5} = 1.69524$$

$$\beta = +59.46° = 59.46° \circlearrowright$$

At point *A* ($y = -2.37$ in. and $z = +2.37$ in.):

$$\sigma_A = -\left[\frac{M_{rz}I_y + M_{ry}I_{yz}}{I_yI_z - I_{yz}^2}\right]y + \left[\frac{M_{ry}I_z + M_{rz}I_{yz}}{I_yI_z - I_{yz}^2}\right]z = \frac{M_{ry}\left(-I_{yz}y + I_z z\right)}{I_yI_z - I_{yz}^2}$$

$$\sigma_A = \frac{(-72.00)\left[-(52.5)(-2.37)+(89.0)(+2.37)\right]}{(89.0)^2 - (52.5)^2}$$

$\sigma_A = -4.68 \text{ ksi} = 4.68 \text{ ksi (C)}$ .... **Ans.**

At point *B* ($y = -1.37$ in. and $z = -5.63$ in.):

$$\sigma_B = \frac{(-72.00)\left[-(52.5)(-1.37)+(89.0)(-5.63)\right]}{(89.0)^2 - (52.5)^2}$$

$\sigma_B = +5.98 \text{ ksi} = 5.98 \text{ ksi (T)}$ .... **Ans.**

7-224

$$I = \frac{(30)(75)^3}{12} = 1.05469\left(10^6\right)\ \text{mm}^4$$

$$\frac{h}{d} = \frac{75}{20} = 3.75 \qquad \frac{d}{b} = \frac{20}{50} = 0.4$$

From Fig. 7-34: $K_t \cong 2.25$

$$\sigma = K_t \frac{Mc}{I} = (2.25)\frac{(1400)(0.0375)}{1.05469\left(10^{-6}\right)} = 112.0\left(10^6\right)\ \text{N/m}^2 = 112.0\ \text{MPa}$$ .... **Ans.**

7-225*

For a W 14×120 section $\quad d = 2c = 14.48$ in. $\quad t_w = 0.590$ in.

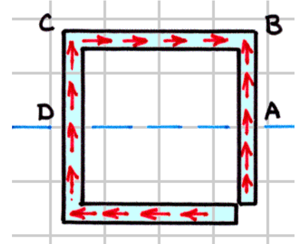
$w_f = 14.670$ in. $\quad t_f = 0.940$ in. $\quad S = 190$ in.^{3}

$M_e = \sigma_y S = (36)(190) = 6840$ kip·in. .... **Ans.**

$$M_p = 2(36)\left[(14.670 \times 0.940)(6.77)\right] + 2(36)\left[(6.30 \times 0.590)(3.15)\right]$$

$M_p = 7564.7$ kip·in. $\cong 7560$ kip·in. .... **Ans.**

7-226*

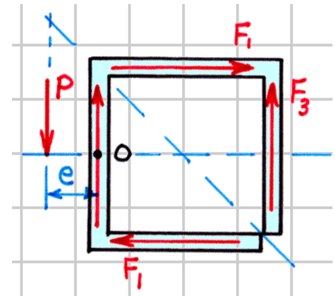


$$Q_A = (25)(50 \times 3) = 3.75(10^3)\ \text{mm}^3 \qquad Q_B = 0\ \text{mm}^3$$

$$Q_C = (50)(100 \times 3) = 15.00(10^3)\ \text{mm}^3$$

$$Q_D = (50)(100 \times 3) + (25)(50 \times 3) = 18.75(10^3)\ \text{mm}^3$$

$$I \cong \frac{(6)(100)^3}{12} + 2(100 \times 3)(50)^2 = 2.00(10^6)\ \text{mm}^4$$



$$\tau_A = \frac{VQ_A}{It} = \frac{(2500)(3.75 \times 10^{-6})}{(2.00 \times 10^{-6})(0.003)} = 1.5625(10^6)\ \text{N/m}^2$$

$$\tau_C = \frac{VQ_C}{It} = \frac{(2500)(15.00 \times 10^{-6})}{(2.00 \times 10^{-6})(0.003)} = 6.25(10^6)\ \text{N/m}^2$$

$$\tau_D = \frac{VQ_D}{It} = \frac{(2500)(18.75 \times 10^{-6})}{(2.00 \times 10^{-6})(0.003)} = 7.81(10^6)\ \text{N/m}^2 \qquad \tau_B = 0$$

Therefore: $\tau_{max} = \tau_D = 7.81$ MPa .... **Ans.**

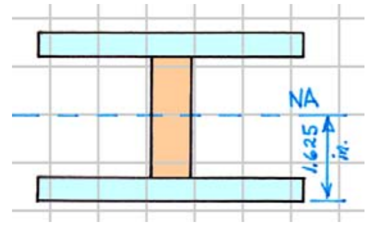
$$F_1 = (\tau_C A_1/2) = (6.25 \times 10^6)(0.100 \times 0.003)/2 = 937.5\ \text{N}$$

$$F_3 = 2(\tau_A A_3)/3 = 2(1.5625 \times 10^6)(0.100 \times 0.003)/3 = 312.5\ \text{N}$$

$\circlearrowleft \Sigma M_O = 0: \qquad Pe - F_1(100) + F_3(100) = 0$

$$e = \frac{(937.5)(100) - (312.5)(100)}{2500} = 25.00\ \text{mm} = 25.00\ \text{mm} \leftarrow$$ .... **Ans.**

7-227



$$n = \frac{E_a}{E_p} = \frac{10,000}{300} = 33.33$$

$$\sigma_p = \frac{My_p}{I_T} = \frac{(10)(1.5)}{I_T} \le 1 \text{ ksi} \qquad I_T \ge 15.00 \text{ in.}^4$$

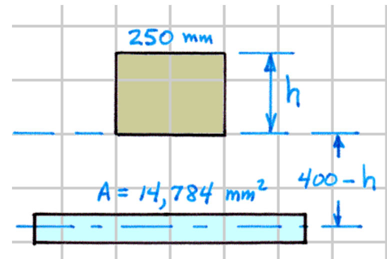
$$\sigma_a = \frac{nMy_a}{I_T} = \frac{33.33(10)(1.625)}{I_T} \le 20 \text{ ksi} \qquad I_T \ge 27.08 \text{ in.}^4$$

$$I_T = \frac{(33.33w)(3.25)^3}{12} - \frac{(33.33w-2)(3)^3}{12} \ge 27.08 \text{ in.}^4$$

$20.36w \ge 22.58$ $\qquad$ $w \ge 1.109$ in. .... **Ans.**

7-228

$$n = \frac{E_s}{E_c} = \frac{200}{17} = 11.765$$



$$A_s = 4\left[\pi(20)^2/4\right] = 1256.6 \text{ mm}^2$$

$$A_T = nA_s = 11.765(1256.6) = 14,784 \text{ mm}^2$$

$$250h(h/2) = 14,784(400-h)$$

$$h^2 + 118.27h - 47,310 = 0$$

From which: $h = 166.27 \text{ mm}$ $400 - h = 233.7 \text{ mm}$

$$I_T = \frac{(250)(166.27)^3}{3} + (14,784)(233.7)^2 = 386.5(10^6) \text{ mm}^4$$

$\sigma_c = \dfrac{My_c}{I_T} = \dfrac{(62,500)(0.16627)}{(386.5\times10^6)} = 26.89(10^6) \text{ N/m}^2 \cong 26.9 \text{ MPa (C)}$ .... **Ans.**

$\sigma_s = (y_s/y_c)(n\sigma_c) = (233.7/166.27)(11.765\times26.89) = 445 \text{ MPa (T)}$ .... **Ans.**

7-229*

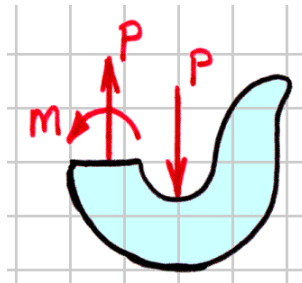
$$A = (2.25\times 3) + (2.5\times 1) = 9.25\text{ in.}^2$$

$$R_C = \frac{(3.5)(2\times 1) + (11/3)\left[(1\times 1)/2\right] + (5.5)(1.5\times 3) + (5)\left[(1.5\times 3)/2\right]}{9.25} = 4.8468\text{ in.}$$

$$A = R\int \frac{dA}{\rho} = R\left[\int_3^4 (-1+\rho)\frac{d\rho}{\rho} + \int_4^7 (5-0.5\rho)\frac{d\rho}{\rho}\right]$$

$$= R\left[(-1)\ln(4/3) + (4-3) + (5)\ln(7/4)4 - (0.5)(7-4)\right]$$

$$= 2.01040R = 9.25\text{ in.}^2$$



$$R = \frac{9.25}{2.01040} = 4.6011\text{ in.}$$

$$y_C = R - R_C = 4.6011 - 4.8468 = -0.2457\text{ in.}$$

$$M = R_C P = (-4.8468P)\text{ kip}\cdot\text{in.}$$

$$\sigma_i = \frac{P}{A} + \frac{M(R-r_i)}{r_i A y_C} = \frac{P}{9.25} + \frac{(-4.8468P)(4.6011-3)}{(3)(9.25)(-0.2457)} \le 12\text{ ksi}$$

$$P \le 9.63\text{ kip}$$

$$\sigma_o = \frac{P}{A} + \frac{M(R-r_o)}{r_o A y_C} = \frac{P}{9.25} + \frac{(-4.8468P)(4.6011-7)}{(7)(9.25)(-0.2457)} \le -16\text{ ksi}$$

$$P \le 25.7\text{ kip}$$

$P_{max} = 9.63$ kip .... **Ans.**

7-230

$$P = (320)(3/5) = 192 \text{ kN}$$

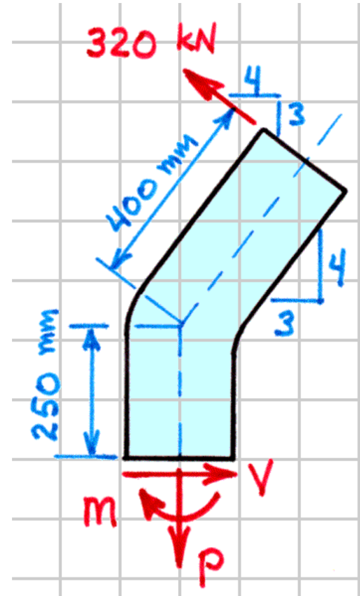
$$V = (320)(4/5) = 256 \text{ kN}$$

$$M = (256)(0.250 + 0.320) + (192)(0.240)$$
$$= 192 \text{ kN}\cdot\text{m}$$

$$A = (75 \times 200) = 15{,}000 \text{ mm}^2$$

$$I = \frac{bh^3}{12} = \frac{(75)(200)^3}{12} = 50.0(10^6) \text{ mm}^4$$

$$Q = y_C A = (75)(50 \times 75) = 281.3(10^3) \text{ mm}^3$$



At point A: $\sigma_{xA} = 0$ MPa

$$\sigma_{yA} = \frac{P}{A} + \frac{Mc}{I} = \frac{192(10^3)}{15(10^{-3})} + \frac{(192 \times 10^3)(0.050)}{50.0 \times 10^{-6}}$$
$$= 204.8(10^6) \text{ N/m}^2 = 204.8 \text{ MPa}$$

$$\tau_{xyA} = \frac{VQ}{It} = \frac{(256 \times 10^3)(281.3 \times 10^{-6})}{(50.0 \times 10^{-6})(0.075)} = 19.203(10^6) \text{ N/m}^2 = 19.203 \text{ MPa}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{0 + 204.8}{2} \pm \sqrt{\left(\frac{0 - 204.8}{2}\right)^2 + (19.203)^2}$$

$\sigma_{p1} = 102.4 + 104.19 = 206.6 \text{ MPa} \cong 207 \text{ MPa (T)}$ .... **Ans.**

$\sigma_{p2} = 102.4 - 104.19 = -1.790 \text{ MPa} \cong 1.790 \text{ MPa (C)}$ .... **Ans.**

$\sigma_{p3} = 0$ MPa .... **Ans.**

$\tau_{max} = \tau_p = \dfrac{\sigma_{p1} - \sigma_{p2}}{2} = \dfrac{(206.6) - (-1.790)}{2} = 104.2 \text{ MPa}$ .... **Ans.**

7-231

From symmetry, each of the supports carries half of the total load

$$R_B = R_C = 2900 \text{ lb}$$

At point *A*:

$$M_A = 5800 \text{ lb}\cdot\text{ft} = 69.6 \text{ kip}\cdot\text{in.}$$

$$T = (2500)(12) - (400)(12)$$

$$= 25{,}200 \text{ lb}\cdot\text{in.} = 25.20 \text{ kip}\cdot\text{in.}$$

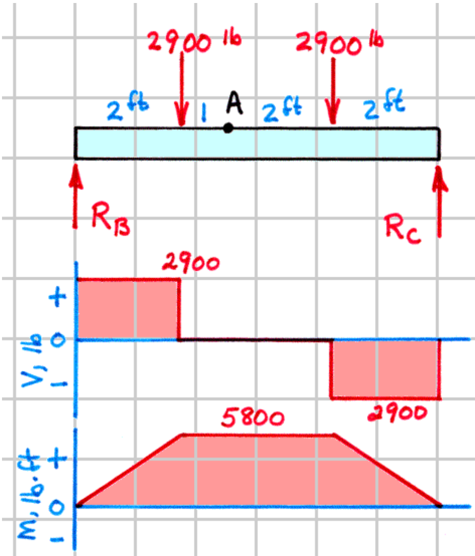
$$I = \pi d^4/64 = \pi(4)^4/64 = 12.566 \text{ in.}^4$$

$$J = 2I = 25.133 \text{ in.}^4$$

$$\sigma_x = \frac{-Mc}{I} = \frac{-(69.6)(2)}{12.566} = -11.078 \text{ ksi}$$

$$\sigma_y = 0 \text{ ksi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(25.2)(2)}{25.133} = 2.005 \text{ ksi}$$



$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-11.078 + 0}{2} \pm \sqrt{\left(\frac{-11.078 - 0}{2}\right)^2 + (2.005)^2}$$

$\sigma_{p1} = -5.539 + 5.891 = 0.352 \text{ ksi} = 0.352 \text{ ksi (T)}$ .... **Ans.**

$\sigma_{p2} = -5.539 - 5.891 = -11.430 \text{ ksi} = 11.43 \text{ ksi (C)}$ .... **Ans.**

$\sigma_{p3} = 0 \text{ ksi}$ .... **Ans.**

$\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{(0.352) - (-11.430)}{2} = 5.89 \text{ ksi}$ .... **Ans.**

7-232*

From the bending-moment diagram:

$$M_{max} = 7.5 \text{ kN}\cdot\text{m}$$

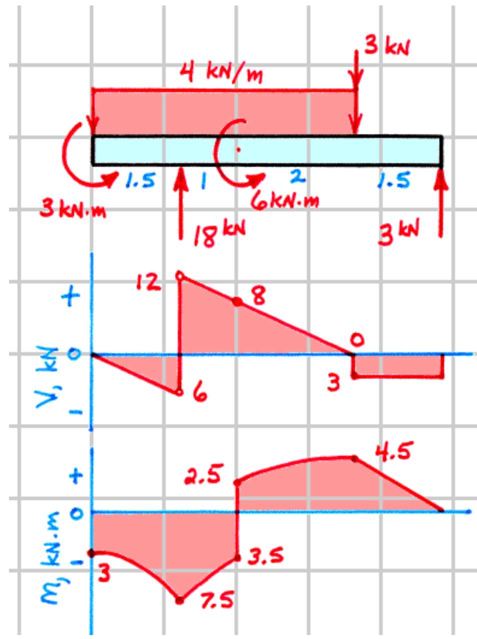
$$S = \frac{M}{\sigma} = \frac{7500}{60\left(10^6\right)} = 125.0\left(10^{-6}\right) \text{ m}^3$$

$$= 125.0\left(10^3\right) \text{ mm}^3$$

For each angle:

$$S = \frac{125\left(10^3\right)}{2} = 62.5\left(10^3\right) \text{ mm}^3$$

Use two L 178×102×9.5-mm angles .... **Ans.**



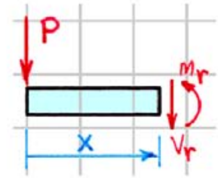
8-1*

(a) $EIv'' = M_r = -Px$

Boundary Conditions:

$EIv' = \frac{-Px^2}{2} + C_1$ At $x = L$, $v' = 0$: $C_1 = \frac{PL^2}{2}$

$EIv = \frac{-Px^3}{6} + C_1 x + C_2$ At $x = L$, $v = 0$: $C_2 = \frac{-PL^3}{3}$



$v' = \frac{P}{2EI}\left(-x^2 + L^2\right)$

$v = \frac{P}{6EI}\left(-x^3 + 3L^2 x - 2L^3\right)$ .... **Ans.**

(b) $\delta_A = v_{x=0} = \frac{P}{6EI}\left(0 + 0 - 2L^3\right) = \frac{-PL^3}{3EI} = \frac{PL^3}{3EI} \downarrow$ .... **Ans.**

(c) $\theta_A = v'_{x=0} = \frac{P}{2EI}\left(0 + L^2\right) = \frac{PL^2}{2EI} = \frac{PL^2}{2EI}$ ∡ .... **Ans.**

8-2*

From overall equilibrium:

$$R_A = wL \uparrow \qquad\qquad M_A = \left(wL^2/2\right) \circlearrowleft$$

(a) $$EIv'' = M_r = wLx - \frac{wL^2}{2} - \frac{wx^2}{2}$$

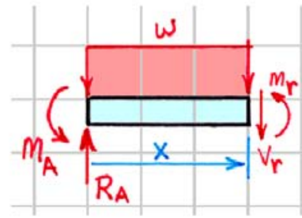
Boundary Conditions:

$$EIv' = \frac{wLx^2}{2} - \frac{wL^2x}{2} - \frac{wx^3}{6} + C_1$$

At $x = 0$, $v' = 0$: $C_1 = 0$

$$EIv = \frac{wLx^3}{6} - \frac{wL^2x^2}{4} - \frac{wx^4}{24} + C_1x + C_2$$

At $x = 0$, $v = 0$: $C_2 = 0$



$$v' = \frac{w}{6EI}\left(-x^3 + 3Lx^2 - 3L^2x\right)$$

$$v = \frac{w}{24EI}\left(-x^4 + 4Lx^3 - 6L^2x^2\right)$$ ....**Ans.**

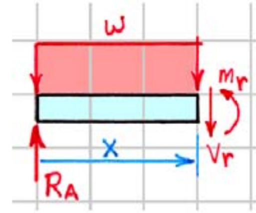
(b) $$\delta_B = v_{x=L} = \frac{w}{24EI}\left(-L^4 + 4L^4 - 6L^4\right) = \frac{-wL^4}{8EI} = \frac{wL^4}{8EI} \downarrow$$ .... **Ans.**

(c) $$\theta_B = v'_{x=L} = \frac{w}{6EI}\left(-L^3 + 3L^3 - 3L^3\right) = \frac{-wL^3}{6EI} = \frac{wL^3}{6EI} \circlearrowright$$ .... **Ans.**

8-3

From overall equilibrium:

$$R_A = R_B = (wL/2)\uparrow$$



(a) $$EIv'' = M_r = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$EIv' = \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$ $C_1 = \frac{-wL^3}{24}$

$$v' = \frac{w}{24EI}\left(-4x^3 + 6Lx^2 - L^3\right)$$

$$v = \frac{w}{24EI}\left(-x^4 + 2Lx^3 - L^3x\right)$$ .... **Ans.**

(b) $$\delta_M = v_{x=L/2} = \frac{w}{24EI}\left(\frac{-L^4}{16} + \frac{L^4}{4} - \frac{L^4}{2}\right) = \frac{-5wL^4}{384EI} = \frac{5wL^4}{384EI}\downarrow$$ .... **Ans.**

(c) $$\theta_A = v'_{x=0} = \frac{w}{24EI}\left(0 + 0 - L^3\right) = \frac{-wL^3}{24EI} = \frac{wL^3}{24EI}$$ ↻ .... **Ans.**

8-4*

(a) $EIv'' = M_r = \dfrac{-wx^3}{6L}$

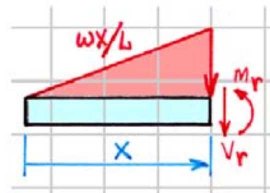
Boundary Conditions:

$EIv' = \dfrac{-wx^4}{24L} + C_1$

At $x = L$, $v' = 0$: $C_1 = \dfrac{wL^3}{24}$

$EIv = \dfrac{-wx^5}{120L} + C_1 x + C_2$

At $x = L$, $v = 0$: $C_2 = \dfrac{-wL^4}{30}$



$v' = \dfrac{w}{24EIL}\left(-x^4 + L^4\right)$

$v = \dfrac{w}{120EIL}\left(-x^5 + 5L^4 x - 4L^5\right)$ ....**Ans.**

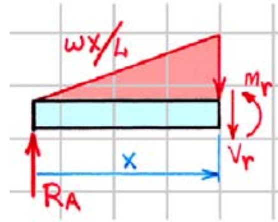
(b) $\delta_A = v_{x=0} = \dfrac{w}{120EIL}\left(0 + 0 - 4L^5\right) = \dfrac{-wL^4}{30EI} = \dfrac{wL^4}{30EI} \downarrow$ .... **Ans.**

(c) $\theta_A = v'_{x=0} = \dfrac{w}{24EIL}\left(0 + L^4\right) = \dfrac{+wL^3}{24EI} = \dfrac{wL^3}{24EI}$ ∡ .... **Ans.**

8-5

From overall equilibrium:

$R_A = (wL/6)\uparrow \qquad R_B = (wL/3)\uparrow$



(a) $EIv'' = M_r = \dfrac{wLx}{6} - \dfrac{wx^3}{6L}$

$EIv' = \dfrac{wLx^2}{12} - \dfrac{wx^4}{24L} + C_1$

Boundary Conditions:

$EIv = \dfrac{wLx^3}{36} - \dfrac{wx^5}{120L} + C_1x + C_2$

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$: $C_1 = \dfrac{-7wL^3}{360}$

$v' = \dfrac{w}{360EIL}\left(-15x^4 + 30L^2x^2 - 7L^4\right)$

$v = \dfrac{w}{360EIL}\left(-3x^5 + 10L^2x^3 - 7L^4x\right)$ .... **Ans.**

(b) $\delta_M = v_{x=L/2} = \dfrac{w}{360EIL}\left(\dfrac{-3L^5}{32} + \dfrac{10L^5}{8} - \dfrac{7L^5}{2}\right) = \dfrac{-5wL^4}{768EI} = \dfrac{5wL^4}{768EI}\downarrow$ .... **Ans.**

(c) $\theta_B = v'_{x=L} = \dfrac{w}{360EIL}\left(-15L^4 + 30L^4 - 7L^4\right) = \dfrac{+wL^3}{45EI} = \dfrac{7wL^3}{45EI}$ ∡ .... **Ans.**

8-6

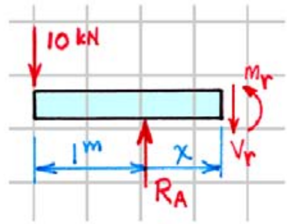
$R_A = R_B = 10\text{ kN}\uparrow \qquad E = 200\text{ GPa} \qquad I = 32.0\left(10^6\right)\text{ mm}^4$

$M_r = 10x - 10(x+1) = -10\text{ kN}\cdot\text{m}$

$EIv'' = M_r = (-10)\text{ kN}\cdot\text{m}$

$EIv' = (-10x + C_1)\text{ kN}\cdot\text{m}^2$

$EIv = \left(-5x^2 + C_1x + C_2\right)\text{ kN}\cdot\text{m}^3$



Boundary Conditions:

At $x = 0$, $\quad v = 0$: $\quad C_2 = 0$

At $x = 2\text{ m}$, $\quad v = 0$: $\quad C_1 = +10\text{ kN}\cdot\text{m}^2$

$EIv = \left(-5x^2 + 10x\right)\text{ kN}\cdot\text{m}^3$

At $x = 1\text{ m}$:

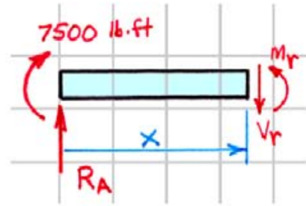
$EIv = -(5)(1)^2 + (10)(1) = 5\text{ kN}\cdot\text{m}^3 = 5000\text{ N}\cdot\text{m}^3$

$v = \dfrac{5000}{\left(200\times10^9\right)\left(32.0\times10^{-6}\right)} = +0.000781\text{ m} = 0.781\text{ mm}\uparrow$ .... **Ans.**

8-7*

$R_A = R_B = 0$ kip ↑ $\qquad E = 30{,}000$ ksi $\qquad I = 32.1$ in.^{4}

$M_r = 7.5 - R_A x = 7.50$ kip·ft



$EIv'' = M_r = (7.5)$ kip·ft

$EIv' = (7.5x + C_1)$ kip·ft^2

$EIv = (3.75x^2 + C_1 x + C_2)$ kip·ft^3

Boundary Conditions:

At $x = 0$, $\quad v = 0$: $\quad C_2 = 0$

At $x = 12$ ft, $\quad v = 0$: $\quad EI(0) = 3.75(12)^2 + C_1(12) + 0$

: $\quad C_1 = -45$ kip·ft^2

$EIv = (3.75x^2 - 45x)$ kip·ft^3

At $x = 6$ ft:

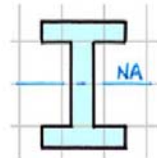
$EIv = (3.75)(6)^2 - (45)(6) = -135.0$ kip·ft$^3 = -233{,}280$ kip·in.^{3}

$$v = \frac{-233{,}280}{(30{,}000)(32.1)} = -0.242 \text{ in.} = 0.242 \text{ in.}\downarrow$$ .... **Ans.**

8-8

$E = 70$ GPa

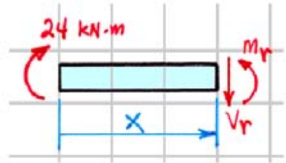
$$I = \frac{(120)(180)^3}{12} - \frac{(90)(120)^3}{12} = 45.36 \text{ mm}^4$$



$EIv'' = M_r = (24)$ kN·m

$EIv' = (24x + C_1)$ kN·m²

$EIv = (12x^2 + C_1 x + C_2)$ kN·m³



Boundary Conditions:

At $x = 2$ m, $v' = 0$: $C_1 = -48 \text{ kN}\cdot\text{m}^2$

At $x = 2$ m, $v = 0$: $EI(0) = 12(2)^2 + (-48)(2) + C_2$

$C_2 = +48 \text{ kN}\cdot\text{m}^3$

$EIv = (12x^2 - 48x + 48)$ kN·m³

At $x = 0$ m:

$EIv = 0 - 0 + (48 \times 10^3) \text{ N}\cdot\text{m}^3 = (48{,}000) \text{ N}\cdot\text{m}^3$

$$v = \frac{48{,}000}{(70 \times 10^9)(45.36 \times 10^{-6})} = +0.01512 \text{ m} = 15.12 \text{ mm} \uparrow$$ .... **Ans.**

8-9

(a) $EIv'' = M_r = \dfrac{-wLx}{2} - \dfrac{wx^2}{2}$

$EIv' = \dfrac{-wLx^2}{4} - \dfrac{wx^3}{6} + C_1$

$EIv = \dfrac{-wLx^3}{12} - \dfrac{wx^4}{24} + C_1 x + C_2$

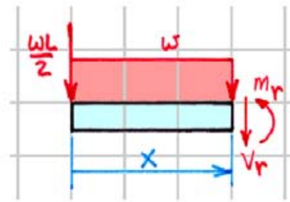
Boundary Conditions:

At $x = L$, $v' = 0$: $C_1 = \dfrac{5wL^3}{12}$

At $x = L$, $v = 0$: $C_2 = \dfrac{-7wL^4}{24}$

$$v' = \frac{w}{24EI}\left(-4x^3 - 6Lx^2 + 10L^3\right)$$

$$v = \frac{w}{24EI}\left(-x^4 - 2Lx^3 + 10L^3 x - 7L^4\right) \text{ .... Ans.}$$



(b) $\delta_A = v_{x=0} = \dfrac{w}{24EI}\left(-0 - 0 + 0 - 7L^4\right) = \dfrac{-7wL^4}{24EI} = \dfrac{7wL^4}{24EI} \downarrow$

For a W 10×30 section $I = 170 \text{ in.}^4$

$L = 10 \text{ ft} = 120 \text{ in.}$ $w = 2 \text{ kip/ft} = (2/12) \text{ kip/in.}$

$$\delta_A = \frac{7wL^4}{24EI} = \frac{7(2/12)(120)^4}{24(29{,}000)(170)} = 2.04 \text{ in} \downarrow \text{ .... Ans.}$$

8-10*

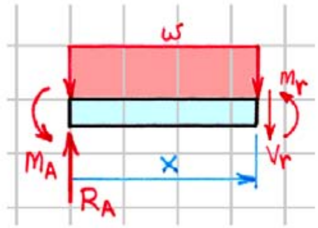
$R_A = wL \uparrow \qquad\qquad M_A = \left(wL^2/8\right) \circlearrowleft$

(a) $EIv'' = M_r = wLx - \dfrac{wL^2}{8} - \dfrac{wx^2}{2}$

Boundary Conditions:

$EIv' = \dfrac{wLx^2}{2} - \dfrac{wL^2x}{8} - \dfrac{wx^3}{6} + C_1$ At $x = 0$, $v' = 0$: $C_1 = 0$

$EIv = \dfrac{wLx^3}{6} - \dfrac{wL^2x^2}{16} - \dfrac{wx^4}{24} + C_1x + C_2$ At $x = 0$, $v = 0$: $C_2 = 0$



$v' = \dfrac{w}{24EI}\left(-4x^3 + 12Lx^2 - 3L^2x\right)$

$v = \dfrac{w}{48EI}\left(-2x^4 + 8Lx^3 - 3L^2x^2\right)$ .... **Ans.**

(b) The maximum deflection will occur either where $v' = 0$ or at $x = L$

$v' = 0$ when $-4x^3 + 12Lx^2 - 3L^2x = 0$

$x = 0,\ 0.2753L,$ and $2.725L$ (not on the beam)

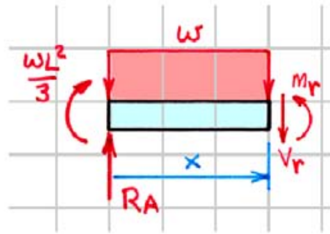
$\delta_1 = v_{x=0.2753L} = -0.001379\left(wL^4/EI\right) \qquad \delta_B = v_{x=L} = +\left(wL^4/16EI\right)$

$\delta_{\max} = \delta_B = \dfrac{(1500)(3)^4}{16\left(210\times10^9\right)\left(2.5\times10^{-6}\right)} = 0.01446 \text{ m} = 14.46 \text{ mm} \uparrow$ .... **Ans.**

8-11*

From overall equilibrium: $R_A = (wL/6)\uparrow$

(a) $EIv'' = M_r = \dfrac{wL^2}{3} + \dfrac{wLx}{6} - \dfrac{wx^2}{2}$



$$EIv' = \frac{wL^2 x}{3} + \frac{wLx^2}{12} - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{wL^2 x^2}{6} + \frac{wLx^3}{36} - \frac{wx^4}{24} + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$: $EI(0) = \dfrac{wL^4}{6} + \dfrac{wL^4}{36} - \dfrac{wL^4}{24} + C_1 L + 0$ $\qquad C_1 = \dfrac{-11wL^3}{72}$

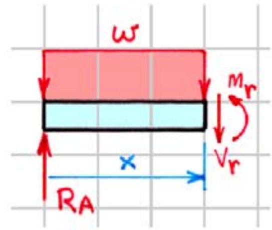
$v = \dfrac{w}{72EI}\left(-3x^4 + 2Lx^3 + 12L^2x^2 - 11L^3x\right)$ .... **Ans.**

(b) $\delta_M = v_{x=L/2} = \dfrac{w}{72EI}\left(\dfrac{-3L^4}{16} + \dfrac{L^4}{4} + 3L^4 - \dfrac{11L^4}{2}\right) = \dfrac{-13wL^4}{384EI} = \dfrac{13wL^4}{384EI}\downarrow$ .... **Ans.**

8-12

From overall equilibrium: $R_A = (7wL/16)\uparrow$

(a) $EIv'' = M_r = \dfrac{7wLx}{16} - \dfrac{wx^2}{2}$



$$EIv' = \frac{7wLx^2}{32} - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{7wLx^3}{96} - \frac{wx^4}{24} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$: $EI(0) = \dfrac{7wL^4}{96} - \dfrac{wL^4}{24} + C_1L + 0$

$$C_1 = \frac{-wL^3}{32}$$

$v = \dfrac{w}{96EI}\left(-4x^4 + 7Lx^3 - 3L^3x\right)$ .... **Ans.**

(b) $\delta_M = v_{x=L/2} = \dfrac{w}{96EI}\left(\dfrac{-L^4}{4} + \dfrac{7L^4}{8} - \dfrac{3L^4}{2}\right) = \dfrac{-7wL^4}{768EI} = \dfrac{7wL^4}{768EI}\downarrow$ .... **Ans.**

8-13*

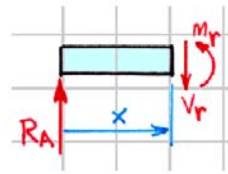
By symmetry: $R_A = R_B = (P/2)\uparrow$ and $v' = 0$ at $x = L/2$

(a) $EIv'' = M_r = \dfrac{Px}{2}$

Boundary Conditions:

$EIv' = \dfrac{Px^2}{4} + C_1$ At $x = \dfrac{L}{2}$, $v' = 0$: $C_1 = \dfrac{-PL^2}{16}$

$EIv = \dfrac{Px^3}{12} + C_1 x + C_2$ At $x = 0$, $v = 0$: $C_2 = 0$



$$v' = \frac{P}{16EI}\left(4x^2 - L^2\right)$$

$$v = \frac{P}{48EI}\left(4x^3 - 3L^2 x\right)$$ .... **Ans.**

(b) $\theta_A = v'_{x=0} = \dfrac{-PL^2}{16EI} = \dfrac{PL^2}{16EI}$ ↘ .... $\theta_B = \dfrac{PL^2}{16EI}$ ↙ .... **Ans.**

(c) $\delta_M = v_{x=L/2} = \dfrac{P}{48EI}\left(\dfrac{4L^3}{8} - \dfrac{3L^3}{2}\right) = \dfrac{-PL^3}{48EI} = \dfrac{PL^3}{48EI}\downarrow$ .... **Ans.**

8-14*

$R_A = 7\text{ kN}\uparrow \qquad M_A = 14\text{ kN}\cdot\text{m}\circlearrowleft \qquad I = (100)(300)^3/12 = 225(10^6)\text{ mm}^4$

(a) $EIv'' = M_r = (7x - 14)\text{ kN}\cdot\text{m}$

Boundary Conditions:

At $x = 0$, $v' = 0$: $C_1 = 0$

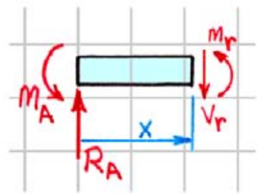
At $x = 0$, $v = 0$: $C_2 = 0$

$EIv' = (3.5x^2 - 14x + C_1)\text{ kN}\cdot\text{m}^2$

$EIv = (1.16667x^3 - 7x^2 + C_1x + C_2)\text{ kN}\cdot\text{m}^3$

$EIv' = (3.5x^2 - 14x)\text{ kN}\cdot\text{m}^2$

$EIv = (1.16667x^3 - 7x^2)\text{ kN}\cdot\text{m}^3$



$$\delta_B = v_{x=2} = \frac{\left[1.16667(2)^3 - 7(2)^2\right](10^3)}{(8\times10^9)(225\times10^{-6})}$$

$\delta_B = -0.01037\text{ m} = 10.37\text{ mm}\downarrow$ .... **Ans.**

(b) $$\theta_B = v'_{x=2} = \frac{\left[3.5(2)^2 - 14(2)\right](10^3)}{(8\times10^9)(225\times10^{-6})} = -0.00778\text{ rad}$$

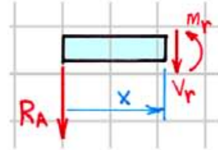
$\delta_C = \delta_B + \theta_B(1.5) = (-0.01037) + (-0.007778)(1.5)$

$\delta_C = -0.0220\text{ m} = 22.0\text{ mm}\downarrow$ .... **Ans.**

8-15

From overall equilibrium: $R_A = (M/2L)\downarrow$ $R_B = (M/2L)\uparrow$

$0 \le x \le L$:



$$EIv_1'' = M_r = \frac{-Mx}{2L}$$

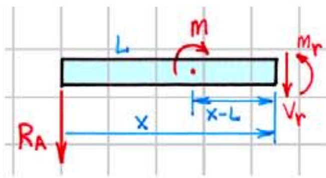
$$EIv_1' = \frac{-Mx^2}{4L} + C_1$$

$$EIv_1 = \frac{-Mx^3}{12L} + C_1 x + C_2$$

Boundary Condition:

At $x = 0$, $v_1 = 0$: $C_2 = 0$

$L \le x \le 2L$:



$$EIv_2'' = M_r = M - \frac{Mx}{2L}$$

$$EIv_2' = Mx - \frac{Mx^2}{4L} + C_3$$

$$EIv_2 = \frac{Mx^2}{2} - \frac{Mx^3}{12L} + C_3 x + C_4$$

Boundary Condition:

At $x = 2L$, $v_2 = 0$: $EI(0) = 2ML^2 - \dfrac{2ML^3}{3L} + C_3(2L) + C_4$ (a)

Matching Conditions:

At $x = L$, $v_1' = v_2'$: $\dfrac{-ML^2}{4L} + C_1 = ML - \dfrac{ML^2}{4L} + C_3$ (b)

At $x = L$, $v_1 = v_2$: $\dfrac{-ML^3}{12L} + C_1 L + (0) = \dfrac{ML^2}{2} - \dfrac{ML^3}{12L} + C_3 L + C_4$ (c)

Solving Eqs. (a), (b), and (c) gives: $C_1 = \dfrac{ML}{12}$ $C_3 = \dfrac{-11ML}{12}$ $C_4 = \dfrac{ML^2}{2}$

$$v_1 = \frac{M}{12EIL}\left(-x^3 + L^2 x\right) \qquad v_2 = \frac{M}{12EIL}\left(-x^3 + 6x^2 L - 11L^2 x + 6L^3\right)$$

The maximum deflection occurs when $v_1' = 0$ which gives

$$-3x^2 + L^2 = 0 \qquad x = L\sqrt{1/3} = 0.5774L$$

$\delta_{max} = v_{1,x=0.5774L} = \dfrac{+0.03208ML^2}{EI} = \dfrac{0.03208ML^2}{EI}\uparrow$ (in left half) .... **Ans.**

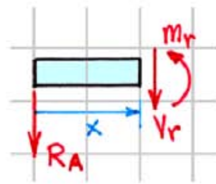
Note that $\delta_M = v_{1,x=L} = \dfrac{M}{12EIL}\left(-L^3 + L^3\right) = 0$

and $\delta_{max} = \dfrac{-0.03208ML^2}{EI} = \dfrac{0.03208ML^2}{EI}\downarrow$ (in right half) .... **Ans.**

8-16*

From overall equilibrium: $R_A = 1471.5 \text{ N} \downarrow$

$R_B = 2060.1 \text{ N} \uparrow$



$0 \text{ m} \le x \le 0.6 \text{ m}$:

$$EIv_1'' = M_r = (-1471.5x) \text{ N}\cdot\text{m}$$

$$EIv_1' = (-735.75x^2 + C_1) \text{ N}\cdot\text{m}^2$$

$$EIv_1 = (-245.25x^3 + C_1 x + C_2) \text{ N}\cdot\text{m}^3$$

Boundary Conditions:

At $x = 0$, $v_1 = 0$: $C_2 = 0$

At $x = 0.6 \text{ m}$, $v_1 = 0$ $C_1 = 88.290$

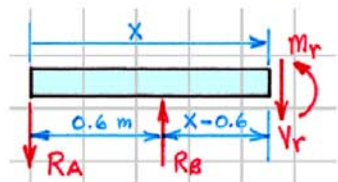
Then, at $x = 0.6 \text{ m}$ $EIv_1' = -735.75(0.6)^2 + 88.290 = -176.58 \text{ N}\cdot\text{m}^2$

$0.6 \text{ m} \le x \le 2.1 \text{ m}$:

$$EIv_2'' = M_r = (588.35x - 1236) \text{ N}\cdot\text{m}$$

$$EIv_2' = (294.175x^2 - 1236x + C_3) \text{ N}\cdot\text{m}^2$$

$$EIv_2 = (98.058x^3 - 618x^2 + C_3 x + C_4) \text{ N}\cdot\text{m}^3$$



Boundary Conditions:

At $x = 0.6 \text{ m}$, $v_2 = 0$: $EI(0) = 98.058(0.6)^3 - 618(0.6)^2 + C_3(0.6) + C_4$ (a)

At $x = 0.6 \text{ m}$, $v_2' = v_1'$: $294.175(0.6)^2 - 1236(0.6) + C_3 = -176.58$ (b)

Solving Eqs. (a) and (b) gives: $C_3 = 459.12$ $C_4 = -74.171$

The maximum deflection occurs at the right end of the board

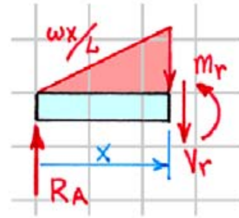
$$EIv_{2,x=2.1} = 98.058(2.1)^3 - 618(2.1)^2 + 459.12(2.1) - 74.171 = (-927.284) \text{ N}\cdot\text{m}^3$$

where $E = 70 \text{ GPa}$ $I = \dfrac{(300)(40)^3}{12} = 1.600(10^6) \text{ mm}^4$

$$\delta_{\max} = v_{2,x=2.1} = \frac{-927.284}{(10\times10^9)(1.600\times10^{-6})} = -0.0579 \text{ m} = 57.9 \text{ mm} \downarrow$$ .... **Ans.**

8-17

By symmetry: $R_A = R_B = (wL/2)\uparrow$ and $v' = 0$ at $x = L$



$$EIv'' = M_r = \frac{wLx}{2} - \frac{wx^3}{6L}$$

Boundary Conditions:

$$EIv' = \frac{wLx^2}{4} - \frac{wx^4}{24L} + C_1$$

At $x = L$, $v' = 0$: $C_1 = \frac{-5wL^3}{24}$

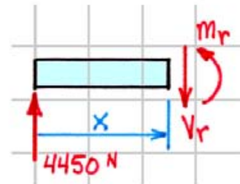
$$EIv = \frac{wLx^3}{12} - \frac{wx^5}{120L} + C_1 x + C_2$$

At $x = 0$, $v = 0$: $C_2 = 0$

$$v = \frac{w}{120EIL}\left(-x^5 + 10L^2x^3 - 25L^4x\right)$$

$$\delta_{max} = v_{x=L} = \frac{w}{120EIL}\left(-L^5 + 10L^5 - 25L^5\right) = \frac{-2wL^4}{15EI} = \frac{2wL^4}{15EI}\downarrow$$ .... **Ans.**

8-18



By symmetry: $R_A = R_B = 4450 \text{ N} \uparrow$

and $v' = 0$ at $x = 2.75$ m

(a) $EIv'' = M_r = (4450x) \text{ N}\cdot\text{m}$

Boundary Conditions:

$EIv' = (2225x^2 + C_1) \text{ N}\cdot\text{m}^2$ At $x = 2.75$ m, $v' = 0$: $C_1 = -16{,}826.6$

$EIv = (741.667x^3 + C_1x + C_2) \text{ N}\cdot\text{m}^3$ At $x = 0$, $v = 0$: $C_2 = 0$

$EIv' = (2225x^2 - 16{,}826.6) \text{ N}\cdot\text{m}^2$ $EIv = (741.667x^3 - 16{,}826.6x) \text{ N}\cdot\text{m}^3$

$$I = \frac{(150)(300)^3}{12} = 337.5(10^6) \text{ mm}^4$$

$$\theta_A = v'_{x=0} = \frac{-16{,}826.6}{(10\times10^9)(337.5\times10^{-6})} = -4.9857(10^{-3}) \text{ rad} = 4.9857(10^{-3}) \text{ rad} ⬊$$

As a result of symmetry $\theta_B = -\theta_A = +4.9857(10^{-3}) \text{ rad} = 4.9857(10^{-3}) \text{ rad} ⬋$

and $\delta_P = \theta_B(1.5) = (4.9857\times10^{-3})(1.5) = 0.00748 \text{ m} = 7.48 \text{ mm} \uparrow$ .... **Ans.**

(b) $$\delta_{max} = v_{x=2.75} = \frac{741.667(2.75)^3 - 16{,}826.6(2.75)}{(10\times10^9)(337.5\times10^{-6})} = -0.00914 \text{ m} = 9.14 \text{ mm} \downarrow$$

$\delta_{max} = -0.00914 \text{ m} = 9.14 \text{ mm} \downarrow$ .... **Ans.**

8-19*

$0 \le x \le L$:

$$EIv_1'' = M_r = -Px$$

$$EIv_1' = \frac{-Px^2}{2} + C_1$$

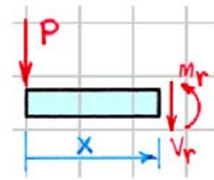
$$EIv_1 = \frac{-Px^3}{6} + C_1 x + C_2$$

$L \le x \le 2L$:

$$E(2I)v_2'' = M_r = -Px$$

$$2EIv_2' = \frac{-Px^2}{2} + C_3$$

$$2EIv_2 = \frac{-Px^3}{6} + C_3 x + C_4$$



Boundary Conditions:

(None)

Boundary Conditions:

At $x = 2L$, $v_2' = 0$: $C_3 = 2PL^2$

At $x = 2L$, $v_2 = 0$: $C_4 = -8PL^3/3$

Matching Conditions:

At $x = L$, $v_1' = v_2'$: $\frac{-PL^2}{2} + C_1 = \frac{1}{2}\left[\frac{-PL^2}{2} + 2PL^2\right]$ $\quad C_1 = \frac{5PL^2}{4}$

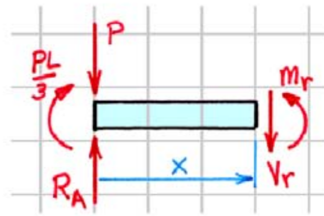
At $x = L$, $v_1 = v_2$: $\frac{-PL^3}{6} + C_1 L + C_2 = \frac{1}{2}\left[\frac{-PL^3}{6} + 2PL^3 - \frac{8PL^3}{3}\right]$ $\quad C_2 = \frac{-3PL^3}{2}$

Therefore $\quad v_1 = \frac{P}{12EI}\left(-2x^3 + 15L^2 x - 18L^3\right)$

and at A $(x = 0)$: $\quad \delta_A = v_{1,x=0} = \frac{P}{12EI}\left(0 + 0 - 18L^3\right) = \frac{-3PL^3}{2EI} = \frac{3PL^3}{2EI} \downarrow$ .... **Ans.**

8-20

From overall equilibrium: $R_A = (2P/3)\uparrow$



Replace rigid lever with equivalent force $(P\downarrow)$ and moment $(PL/3\ \circlearrowleft)$

(a) $EIv'' = M_r = \dfrac{PL}{3} - \dfrac{Px}{3}$

$EIv' = \dfrac{PLx}{3} - \dfrac{Px^2}{6} + C_1$

Boundary Conditions:

$EIv = \dfrac{PLx^2}{6} - \dfrac{Px^3}{18} + C_1 x + C_2$

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$: $C_1 = \dfrac{-PL^2}{9}$

$$v' = \frac{P}{18EI}\left(-3x^2 + 6Lx - 2L^2\right)$$

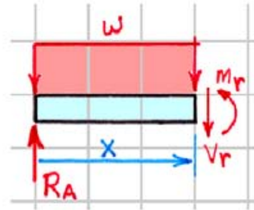
$$v = \frac{P}{18EI}\left(-x^3 + 3Lx^2 - 2L^2 x\right)$$ .... **Ans.**

(b) $\theta_B = v'_{x=L} = \dfrac{P}{18EI}\left(-3L^2 + 6L^2 - 2L^2\right) = \dfrac{+PL^2}{18EI} = \dfrac{PL^2}{18EI}$ ∡ .... **Ans.**

(c) $\delta_M = v_{x=L/2} = \dfrac{P}{18EI}\left(-\dfrac{L^3}{8} + \dfrac{3L^3}{4} - L^3\right) = \dfrac{-PL^3}{48EI} = \dfrac{PL^3}{48EI}\downarrow$ .... **Ans.**

8-21

From overall equilibrium:



$$R_A = R_B = (wL/2)\uparrow$$

(a) $$EIv'' = M_r = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$EIv' = \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$ $C_1 = -wL^3/24$

$$v' = \frac{w}{24EI}\left(-4x^3 + 6Lx^2 - L^3\right) \qquad \theta_B = v'_{x=L} = \frac{+wL^3}{24EI} = \frac{wL^3}{24EI}$$

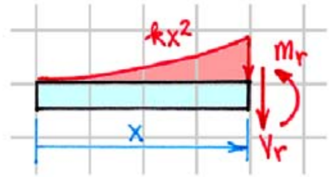
$$v = \frac{w}{24EI}\left(-x^4 + 2Lx^3 - L^3 x\right) \qquad \delta_{max} = v_{x=L/2} \text{ (from symmetry)}$$

$$\delta_{max} = v_{x=L/2} = \frac{w}{24EI}\left(\frac{-L^4}{16} + \frac{L^4}{4} - \frac{L^4}{2}\right) = \frac{-5wL^4}{384EI} = \frac{5wL^4}{384EI}\downarrow$$ .... **Ans.**

(b) $$\delta_C = \theta_B\left(\frac{L}{2}\right) = \frac{+wL^3}{24EI}\left(\frac{L}{2}\right) = \frac{+wL^4}{48EI} = \frac{wL^4}{48EI}\uparrow$$ .... **Ans.**

8-22*

$$M(x) = -\left[\frac{(kx^2)(x)}{3}\right]\left(\frac{x}{4}\right) = \frac{-kx^4}{12}$$



At $x = L$, $w = kL^2$ Therefore $k = w/L^2$

(a) $EIv'' = M_r = \frac{-kx^4}{12} = \frac{-wx^4}{12L^2}$

Boundary Conditions:

$EIv' = \frac{-wx^5}{60L^2} + C_1$ At $x = L$, $v' = 0$: $C_1 = \frac{wL^3}{60}$

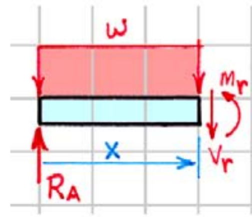
$EIv = \frac{-wx^6}{360L^2} + C_1 x + C_2$ At $x = L$, $v = 0$: $C_2 = \frac{-5wL^4}{360}$

$v = \frac{w}{360EIL^2}\left(-x^6 + 6L^5 x - 5L^6\right)$ .... **Ans.**

(b) $\delta_A = v_{x=0} = \frac{w}{360EIL^2}\left(0 + 0 - 5L^6\right) = \frac{-5wL^4}{360EI} = \frac{wL^4}{72EI} \downarrow$ .... **Ans.**

8-23*

From overall equilibrium: $R_A = (3wL/4)\uparrow$



(a) $$EIv'' = M_r = \frac{3wLx}{4} - \frac{wx^2}{2}$$

$$EIv' = \frac{3wLx^2}{8} - \frac{wx^3}{6} + C_1$$

Boundary Conditions:

$$EIv = \frac{3wLx^3}{24} - \frac{wx^4}{24} + C_1 x + C_2$$

At $x = 0$, $v = 0$: $C_2 = 0$

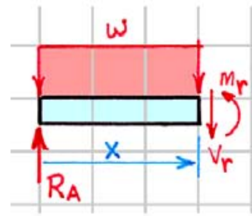
At $x = L$, $v = 0$: $C_1 = -wL^3/12$

$$v = \frac{w}{24EI}\left(-x^4 + 3Lx^3 - 2L^3 x\right)$$ .... **Ans.**

(b) $$\delta_M = v_{x=L/2} = \frac{w}{24EI}\left(-\frac{L^4}{16} + \frac{3L^4}{8} - L^4\right) = \frac{-11wL^4}{384EI} = \frac{11wL^4}{384EI}\downarrow$$ .... **Ans.**

8-24

From overall equilibrium: $R_A = \dfrac{7wL}{16}\uparrow$



(a) $EIv'' = M_r = \dfrac{7wLx}{16} - \dfrac{wx^2}{2}$

$EIv' = \dfrac{7wLx^2}{32} - \dfrac{wx^3}{6} + C_1$

Boundary Conditions:

$EIv = \dfrac{7wLx^3}{96} - \dfrac{wx^4}{24} + C_1x + C_2$

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$: $C_1 = \dfrac{-wL^3}{32}$

$v = \dfrac{w}{96EI}\left(-4x^4 + 7Lx^3 - 3L^3x\right)$ .... **Ans.**

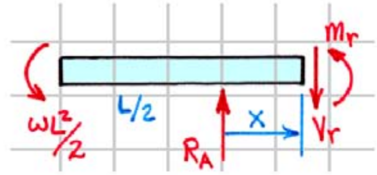
(b) For a WT 203×37 Tee section: $I = 17.6\left(10^6\right)\ \text{mm}^4$

$$\delta_M = v_{x=L/2} = \frac{w}{96EI}\left(-\frac{4L^4}{16} + \frac{7L^4}{8} - \frac{3L^4}{2}\right) = \frac{-7wL^4}{768EI}$$

$\delta_M = \dfrac{-7(5500)(3.5)^4}{768\left(200\times10^9\right)\left(17.6\times10^{-6}\right)} = -0.00214\ \text{m} = 2.14\ \text{mm}\downarrow$ .... **Ans.**

8-25

From overall equilibrium: $R_A = \dfrac{3wL}{8}\uparrow$



(a) $EIv'' = M_r = \dfrac{-wL^2}{2} + \dfrac{3wLx}{8}$

$EIv' = \dfrac{-wL^2x}{2} + \dfrac{3wLx^2}{16} + C_1$

Boundary Conditions:

$EIv = \dfrac{-wL^2x^2}{4} + \dfrac{3wLx^3}{48} + C_1x + C_2$

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L$, $v = 0$: $C_1 = \dfrac{3wL^3}{16}$

$v = \dfrac{w}{16EI}\left(Lx^3 - 4L^2x^2 + 3L^3x\right)$ .... **Ans.**

(b) For a W 8×40 wide-flange section: $I = 146\left(10^6\right)$ in.^{4}

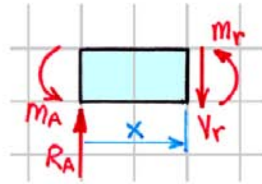
$\delta_M = v_{x=L/2} = \dfrac{w}{16EI}\left(\dfrac{L^4}{8} - \dfrac{4L^4}{4} + \dfrac{3L^4}{2}\right) = \dfrac{+5wL^4}{128EI}$

$\delta_M = \dfrac{+5(240/12)(16\times12)^4}{128\left(29\times10^6\right)(146)} = +0.251\text{ in.} = 0.251\text{ in.}\uparrow$ .... **Ans.**

8-26*

From overall equilibrium:

$R_A = 2P \uparrow \qquad M_A = 3PL \circlearrowleft$



$0 \le x \le L$:

$E(2I)v_1'' = M_r = 2Px - 3PL$

$2EIv_1' = Px^2 - 3PLx + C_1$

$2EIv_1 = \frac{Px^3}{3} - \frac{3PLx^2}{2} + C_1 x + C_2$

Boundary Conditions:

At $x = 0$, $v_1' = 0$: $C_1 = 0$

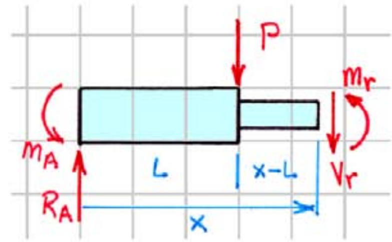
At $x = 0$, $v_1 = 0$: $C_2 = 0$

$L \le x \le 2L$:

$EIv_2'' = M_r = 2Px - 3PL - P(x - L)$

$EIv_2' = Px^2 - 3PLx - \frac{P(x-L)^2}{2} + C_3$

$EIv_2 = \frac{Px^3}{3} - \frac{3PLx^2}{2} - \frac{P(x-L)^3}{6} + C_3 x + C_4$



Matching Conditions:

At $x = L$, $v_1' = v_2'$: $\frac{1}{2}\left[PL^2 - 3PL^2\right] = PL^2 - 3PL^2 + C_3 \qquad C_3 = PL^2$

At $x = L$, $v_1 = v_2$: $\frac{1}{2}\left[\frac{PL^3}{3} - \frac{3PL^3}{2}\right] = \frac{PL^3}{3} - \frac{3PL^3}{2} + C_3 L + C_4 \qquad C_4 = \frac{-5PL^3}{12}$

$$v_1 = \frac{P}{12EI}\left(2x^3 - 9Lx^2\right) \qquad 0 \le x \le L:$$

$$v_2 = \frac{P}{12EI}\left[4x^3 - 18Lx^2 - 2(x-L)^3 + 12L^2 x - 5L^3\right] \qquad L \le x \le 2L:$$

(a) $\delta_B = v_{1,x=L} = \frac{P}{12EI}\left(2L^3 - 9L^3\right) = \frac{-7PL^3}{12EI} = \frac{7PL^3}{12EI} \downarrow$ .... **Ans.**

(b) $\delta_C = v_{2,x=2L} = \frac{P}{12EI}\left[4(2L)^3 - 18L(2L)^2 - 2(L)^3 + 12L^2(2L) - 5L^3\right]$

$\delta_C = \frac{-23PL^3}{12EI} = \frac{23PL^3}{12EI} \downarrow$ .... **Ans.**

8-27*

From overall equilibrium: $R_A = P \uparrow$ $M_A = Pa \circlearrowleft$

(a) $EIv'' = M_r = Px - Pa$

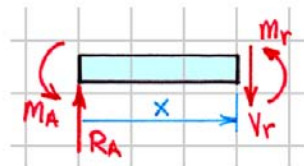
Boundary Conditions:

$$EIv' = \frac{Px^2}{2} - Pax + C_1$$ At $x = 0$, $v' = 0$: $C_1 = 0$

$$EIv = \frac{Px^3}{6} - \frac{Pax^2}{2} + C_1 x + C_2$$ At $x = 0$, $v = 0$: $C_2 = 0$

$$v = \frac{P}{6EI}\left(x^3 - 3ax^2\right)$$ .... **Ans.**

(b) $$\delta_B = v_{x=L} = \frac{P}{6EI}\left(L^3 - 3aL^2\right)$$



For $a = a_{\min} = \dfrac{L}{4}$: $\delta_B = \dfrac{+PL^3}{24EI} = \dfrac{PL^3}{24EI} \uparrow$

For $a = a_{\max} = \dfrac{3L}{4}$: $\delta_B = \dfrac{-5PL^3}{24EI} = \dfrac{5PL^3}{24EI} \downarrow$ $a = \dfrac{3L}{4}$: .... **Ans.**

(c) $v = 0$ when $L^3 - 3aL^2 = 0$ $a = L/3$ .... **Ans.**

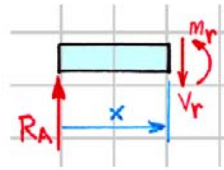
8-28

By symmetry: $R_A = R_B = (P/2)\uparrow$ and $v' = 0$ at $x = 3L/2$

$0 \le x \le L$:

$$EIv_1'' = M_r = Px/2$$

$$EIv_1' = \frac{Px^2}{4} + C_1$$



Boundary Condition:

$$EIv_1 = \frac{Px^3}{12} + C_1 x + C_2$$

At $x = 0$, $v_1 = 0$: $C_2 = 0$

$L \le x \le 3L/2$:

$$E(2I)v_2'' = M_r = Px/2$$

Boundary Condition:

$$2EIv_2' = \frac{Px^2}{4} + C_3$$

At $x = 3L/2$, $v_2' = 0$: $C_3 = \frac{-9PL^2}{16}$

$$2EIv_2 = \frac{Px^3}{12} + C_3 x + C_4$$

Matching Conditions:

At $x = L$, $v_1' = v_2'$: $\frac{PL^2}{4} + C_1 = \frac{1}{2}\left[\frac{PL^2}{4} - \frac{9PL^2}{16}\right]$ $\qquad C_1 = \frac{-13PL^2}{32}$

At $x = L$, $v_1 = v_2$: $\frac{PL^3}{12} - \frac{13PL^3}{32} = \frac{1}{2}\left[\frac{PL^3}{12} - \frac{9PL^3}{16} + C_4\right]$ $\qquad C_4 = \frac{-PL^3}{6}$

$$v_1 = \frac{P}{96EI}\left(8x^3 - 39L^2 x\right) \qquad 0 \le x \le L$$

$$v_2 = \frac{P}{96EI}\left(4x^3 - 27L^2 x - 8L^3\right) \qquad L \le x \le 3L/2$$

(a) $\delta_B = v_{1,x=L} = \frac{P}{96EI}\left(8L^3 - 31L^3\right) = \frac{-31PL^3}{96EI} = \frac{31PL^3}{96EI}\downarrow$ .... **Ans.**

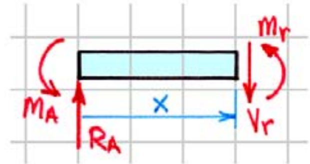
(b) $\delta_{\max} = v_{2,x=3L/2} = \frac{P}{96EI}\left[4\left(\frac{27L^3}{8}\right) - 27L^2\left(\frac{3L}{2}\right) - 8L^3\right]$

$\delta_{\max} = \frac{-70PL^3}{192EI} = \frac{35PL^3}{96EI}\downarrow$ .... **Ans.**

8-29*

From overall equilibrium:

$R_A = (wL/2)\uparrow \qquad M_A = (2wL^2/3) \circlearrowleft$



$0 \le x \le L$:

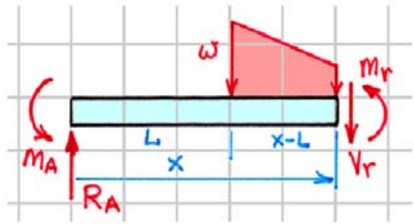
$$EIv_1'' = M_r = \frac{wLx}{2} - \frac{2wL^2}{3}$$

Boundary Conditions:

$$EIv_1' = \frac{wLx^2}{4} - \frac{2wL^2x}{3} + C_1 \qquad \text{At } x = 0, \quad v_1' = 0: \quad C_1 = 0$$

$$EIv_1 = \frac{wLx^3}{12} - \frac{wL^2x^2}{3} + C_1x + C_2 \qquad \text{At } x = 0, \quad v_1 = 0: \quad C_2 = 0$$

$L \le x \le 2L$:



$$EIv_2'' = M_r = \frac{wLx}{2} - \frac{2wL^2}{3} - \frac{w(x-L)^2}{2} + \frac{w(x-L)^3}{6L}$$

$$EIv_2' = \frac{wLx^2}{4} - \frac{2wL^2x}{3} - \frac{w(x-L)^3}{6} + \frac{w(x-L)^4}{24L} + C_3$$

$$EIv_2 = \frac{wLx^3}{12} - \frac{2wL^2x^2}{6} - \frac{w(x-L)^4}{24} + \frac{w(x-L)^5}{120L} + C_3x + C_4$$

Matching Conditions:

At $x = L$, $\quad v_1' = v_2'$: $\qquad C_3 = C_1 = 0$

At $x = L$, $\quad v_1 = v_2$: $\qquad C_4 = C_2 = 0$

$L \le x \le 2L$:

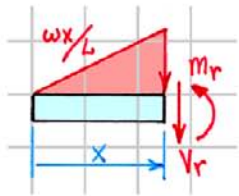
$$v_2 = \frac{w}{120EIL}\left[10L^2x^3 - 40L^3x^2 - 5L(x-L)^4 + (x-L)^5\right] \qquad L \le x \le 2L:$$

$$\delta_{max} = v_{2,x=2L} = \frac{-7wL^4}{10EI} = \frac{7wL^4}{10EI}\downarrow$$ .... **Ans.**

8-30

From overall equilibrium:

$$R_B = R_C = (3wL/4)\uparrow$$

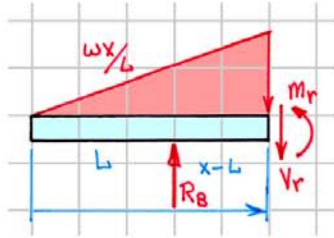


$0 \le x \le L$:

$$EIv_1'' = M_r = \frac{-wx^3}{18L}$$

$$EIv_1' = \frac{-wx^4}{72L} + C_1$$

$$EIv_1 = \frac{-wx^5}{360L} + C_1 x + C_2$$



$L \le x \le 3L$:

$$EIv_2'' = M_r = \frac{-wx^3}{18L} + \frac{3wL(x-L)}{4}$$

$$EIv_2' = \frac{-wx^4}{72L} + \frac{3wL(x-L)^2}{8} + C_3$$

$$EIv_2 = \frac{-wx^5}{360L} + \frac{3wL(x-L)^3}{24} + C_3 x + C_4$$

Boundary Conditions:

At $x = L$, $v_1 = 0$: $\quad C_1 L + C_2 = \dfrac{wL^5}{360L}$ (a)

At $x = L$, $v_2 = 0$: $\quad C_3 L + C_4 = \dfrac{wL^5}{360L}$ (b)

At $x = 3L$, $v_2 = 0$: $\quad C_3 3L + C_4 = \dfrac{-117wL^4}{360}$ (c)

Matching Condition:

At $x = L$, $v_1' = v_2'$: $\quad \dfrac{-wL^3}{72} + C_1 = \dfrac{-wL^3}{72} + C_3$ (d)

Solving Eqs. (a), (b), (c), and (d) gives $\quad C_1 = C_3 = \dfrac{-59wL^3}{360} \qquad C_2 = C_4 = \dfrac{wL^4}{6}$

(a) $0 \le x \le L$: $\quad v_1 = \dfrac{w}{360EIL}\left(-x^5 - 59L^4 x + 60L^5\right)$

$$\delta_A = v_{1,x=0} = \frac{+60wL^5}{360EIL} = \frac{wL^4}{6EI}\uparrow$$ .... **Ans.**

(b) $L \le x \le 3L$: $\quad v_2 = \dfrac{w}{360EIL}\left[-x^5 + 45L^2(x-L)^3 - 59L^4 x + 60L^5\right]$

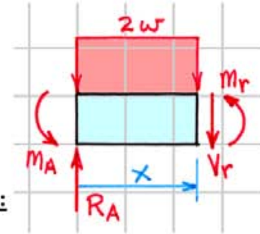
$$\delta_M = v_{2,x=2L} = \frac{w}{360EIL}\left[-32L^5 + 45L^5 - 118L^5 + 60L^5\right]$$

$$\delta_M = \frac{-wL^4}{8EIL} = \frac{wL^4}{8EI}\downarrow$$ .... **Ans.**

8-31

From overall equilibrium:

$R_A = 3wL \uparrow \qquad M_A = 3wL^2 \circlearrowleft$



$0 \le x \le L$:

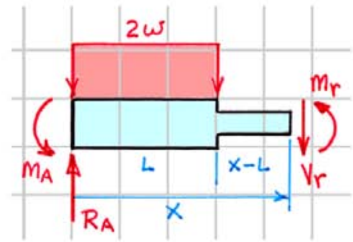
$E(4I)v_1'' = M_r = 3wLx - 3wL^2 - wx^2$

Boundary Conditions:

$4EIv_1' = \dfrac{3wLx^2}{2} - 3wL^2x - \dfrac{wx^3}{3} + C_1$ At $x = 0$, $v_1' = 0$: $C_1 = 0$

$4EIv_1 = \dfrac{wLx^3}{2} - \dfrac{3wL^2x^2}{2} - \dfrac{wx^4}{12} + C_1x + C_2$ At $x = 0$, $v_1 = 0$: $C_2 = 0$

$L \le x \le 2L$:



$EIv_2'' = M_r = 3wLx - 3wL^2 - wx^2 + w(x-L)^2$

$EIv_2' = \dfrac{3wLx^2}{2} - 3wL^2x - \dfrac{wx^3}{3} + \dfrac{w(x-L)^3}{3} + C_3$

$EIv_2 = \dfrac{wLx^3}{2} - \dfrac{3wL^2x^2}{2} - \dfrac{wx^4}{12} + \dfrac{w(x-L)^4}{12} + C_3x + C_4$

Matching Conditions:

At $x = L$, $v_1' = v_2'$: $\dfrac{1}{4}\left[\dfrac{3wL^3}{2} - 3wL^3 - \dfrac{wL^3}{3}\right] = \dfrac{3wL^3}{2} - 3wL^3 - \dfrac{wL^3}{3} + C_3 \qquad C_3 = \dfrac{33wL^3}{24}$

At $x = L$, $v_1 = v_2$: $\dfrac{1}{4}\left[\dfrac{wL^4}{2} - \dfrac{3wL^4}{2} - \dfrac{wL^4}{12}\right] = \dfrac{wL^4}{2} - \dfrac{3wL^4}{2} - \dfrac{wL^4}{12} + C_3L + C_4 \qquad C_4 = \dfrac{-27wL^4}{48}$

(a) $v_1 = \dfrac{w}{48EI}\left(-x^4 + 6Lx^3 - 18L^2x^2\right)$

$\delta_B = v_{1,x=L} = \dfrac{w}{48EI}\left(-L^4 + 6L^4 - 18L^4\right) = \dfrac{-13wL^4}{48EI} = \dfrac{13wL^4}{48EI} \downarrow$ .... **Ans.**

(b) $v_2 = \dfrac{w}{48EI}\left[-4x^4 + 24Lx^3 - 72L^2x^2 + 4(x-L)^4 + 66L^3x - 27L^4\right]$

$\delta_C = v_{2,x=2L} = \dfrac{w}{48EI}\left[-4(16L^4) + 24(8L^4) - 72(4L^4) + 4(L^4) + 66(2L^4) - 27L^4\right]$

$\delta_C = \dfrac{-51wL^4}{48EI} = \dfrac{17wL^4}{16EI} \downarrow$ .... **Ans.**

8-32*

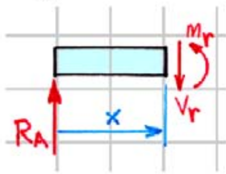
By symmetry: $R_A = R_B = (wa/2)\uparrow$ and $v' = 0$ at $x = 3a/2$

$0 \le x \le a$:

$$EIv_1'' = M_r = \frac{wax}{2}$$

$$EIv_1' = \frac{wax^2}{4} + C_1$$

$$EIv_1 = \frac{wax^3}{12} + C_1 x + C_2$$



$a \le x \le 2a$:

$$EIv_2'' = M_r = \frac{wax}{2} - \frac{w(x-a)^2}{2}$$

$$EIv_2' = \frac{wax^2}{4} - \frac{w(x-a)^3}{6} + C_3$$

$$EIv_2 = \frac{wax^3}{12} - \frac{w(x-a)^4}{24} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $v_1 = 0$: $C_2 = 0$

At $x = \frac{3a}{2}$, $v_2' = 0$: $\frac{9wa^3}{16} - \frac{wa^3}{48} + C_3 = 0$ $C_3 = \frac{-13wa^3}{24}$

Matching Conditions:

At $x = a$, $v_1' = v_2'$: $\frac{wa^3}{4} + C_1 = \frac{wa^3}{4} + C_3$ (a)

At $x = a$, $v_1 = v_2$: $\frac{wa^4}{12} + C_1 a = \frac{wa^4}{12} + C_3 a + C_4$ (b)

Solving Eqs. (a) and (b) gives $C_1 = C_3 = \frac{-13wa^3}{24}$ $C_4 = C_2 = 0$

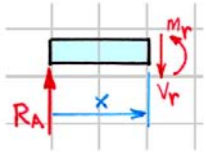
$a \le x \le 2a$: $v_2 = \frac{w}{24EI}\left[2ax^3 - (x-a)^4 - 13a^3 x\right]$

$$\delta_M = v_{2,x=3a/2} = \frac{w}{24EI}\left[\frac{54a^4}{8} - \frac{a^4}{16} - \frac{39a^4}{2}\right] = \frac{-205wa^4}{384EI} = \frac{205wa^4}{384EI}\downarrow$$ .... **Ans.**

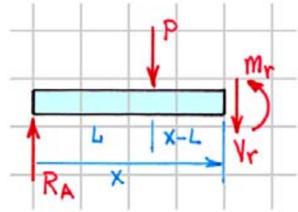
8-33*

By symmetry: $R_A = R_B = (P)\uparrow$ and $v' = 0$ at $x = 3a/2$

$0 \le x \le L$:



$L \le x \le 2L$:



$$EIv_1'' = M_r = Px$$

$$EIv_1' = \frac{Px^2}{2} + C_1$$

$$EIv_1 = \frac{Px^3}{6} + C_1 x + C_2$$

$$EIv_2'' = M_r = Px - P(x-L)$$

$$EIv_2' = \frac{Px^2}{2} - \frac{P(x-L)^2}{2} + C_3$$

$$EIv_2 = \frac{Px^3}{6} - \frac{P(x-L)^3}{6} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $v_1 = 0$: $C_2 = 0$

At $x = \frac{3L}{2}$, $v_2' = 0$: $\frac{9PL^2}{8} - \frac{PL^2}{8} + C_3 = 0$ $C_3 = -PL^2$

Matching Conditions:

At $x = L$, $v_1' = v_2'$: $\frac{PL^2}{2} + C_1 = \frac{PL^2}{2} + C_3$ (a)

At $x = L$, $v_1 = v_2$: $\frac{PL^3}{6} + C_1 L = \frac{PL^3}{6} + C_3 L + C_4$ (b)

Solving Eqs. (a) and (b) gives $C_1 = C_3 = -PL^2$ $C_4 = C_2 = 0$

(a) $0 \le x \le L$: $v_1' = \frac{P}{2EI}\left[x^2 - 2L^2\right]$

$\theta_A = v'_{1,x=0} = \frac{-PL^2}{EI} = \frac{PL^2}{EI}$ ↻ .... **Ans.**

(b) $L \le x \le 2L$: $v_2 = \frac{P}{6EI}\left[x^3 - (x-L)^3 - 6L^2 x\right]$

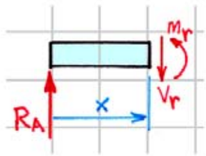
$\delta_M = v_{2,x=3L/2} = \frac{P}{6EI}\left[\frac{27L^3}{8} - \frac{L^3}{8} - \frac{18L^3}{2}\right] = \frac{-46PL^3}{48EI} = \frac{23PL^3}{24EI}\downarrow$ .... **Ans.**

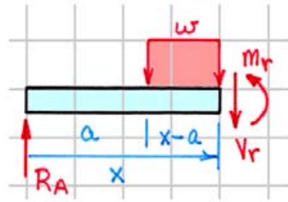
8-34

From overall equilibrium:

$R_A = (wL/4)\uparrow$ $\qquad R_B = (3wL/4)\uparrow$

$0 \le x \le L$: $\qquad L \le x \le 2L$:





$$EIv_1'' = M_r = \frac{wLx}{4} \qquad EIv_2'' = M_r = \frac{wLx}{4} - \frac{w(x-L)^2}{2}$$

$$EIv_1' = \frac{wLx^2}{8} + C_1 \qquad EIv_2' = \frac{wLx^2}{8} - \frac{w(x-L)^3}{6} + C_3$$

$$EIv_1 = \frac{wLx^3}{24} + C_1 x + C_2 \qquad EIv_2 = \frac{wLx^3}{24} - \frac{w(x-L)^4}{24} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $\quad v_1 = 0$: $\qquad C_2 = 0$

At $x = 2L$, $\quad v_2 = 0$: $\qquad \frac{8wL^4}{24} - \frac{wL^4}{24} + 2LC_3 + C_4 = 0 \qquad$ (a)

Matching Conditions:

At $x = L$, $\quad v_1' = v_2'$: $\qquad \frac{wL^3}{8} + C_1 = \frac{wL^3}{8} + C_3 \qquad$ (b)

At $x = L$, $\quad v_1 = v_2$: $\qquad \frac{wL^4}{24} + C_1 L = \frac{wL^4}{24} + C_3 L + C_4 \qquad$ (c)

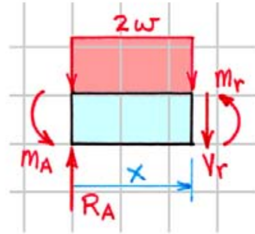
Solving Eqs. (a), (b), and (c) gives $\qquad C_1 = C_3 = -(7wL^3/48) \qquad C_4 = C_2 = 0$

$0 \le x \le L$: $\qquad v_1 = \frac{w}{48EI}\left[2Lx^3 - 7L^3 x\right]$

$$\delta_M = v_{1,x=L} = \frac{w}{48EI}\left[2L^4 - 7L^4\right] = \frac{-5wL^4}{48EI} = \frac{5wL^4}{48EI}\downarrow$$ .... **Ans.**

8-35

From overall equilibrium:



$$R_A = 3wL \uparrow \qquad M_A = \left(5wL^2/2\right) \circlearrowleft$$

$0 \le x \le L$:

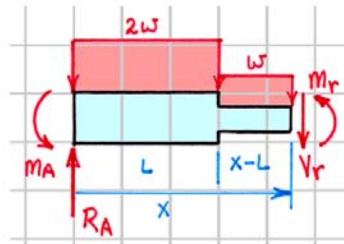
$$E(4I)v_1'' = M_r = 3wLx - \frac{5wL^2}{2} - wx^2$$

Boundary Conditions:

$$4EIv_1' = \frac{3wLx^2}{2} - \frac{5wL^2x}{2} - \frac{wx^3}{3} + C_1 \qquad \text{At } x = 0, \quad v_1' = 0: \quad C_1 = 0$$

$$4EIv_1 = \frac{wLx^3}{2} - \frac{5wL^2x^2}{4} - \frac{wx^4}{12} + C_1x + C_2 \qquad \text{At } x = 0, \quad v_1 = 0: \quad C_2 = 0$$

$L \le x \le 2L$:



$$EIv_2'' = M_r = 3wLx - \frac{5wL^2}{2} - wx^2 + \frac{w(x-L)^2}{2}$$

$$EIv_2' = \frac{3wLx^2}{2} - \frac{5wL^2x}{2} - \frac{wx^3}{3} + \frac{w(x-L)^3}{6} + C_3$$

$$EIv_2 = \frac{wLx^3}{2} - \frac{5wL^2x^2}{4} - \frac{wx^4}{12} + \frac{w(x-L)^4}{24} + C_3x + C_4$$

Matching Conditions:

$$\text{At } x = L, \quad v_1' = v_2': \quad \frac{1}{4}\left[\frac{3wL^3}{2} - \frac{5wL^3}{2} - \frac{wL^3}{3}\right] = \frac{3wL^3}{2} - \frac{5wL^3}{2} - \frac{wL^3}{3} + C_3$$

$$\text{At } x = L, \quad v_1 = v_2: \quad \frac{1}{4}\left[\frac{wL^4}{2} - \frac{5wL^4}{4} - \frac{wL^4}{12}\right] = \frac{wL^4}{2} - \frac{5wL^4}{4} - \frac{wL^4}{12} + C_3L + C_4$$

Therefore: $\qquad C_3 = wL^3 \qquad C_4 = -3wL^4/8$

(a) $\qquad v_1 = \dfrac{w}{48EI}\left(-x^4 + 6Lx^3 - 15L^2x^2\right)$

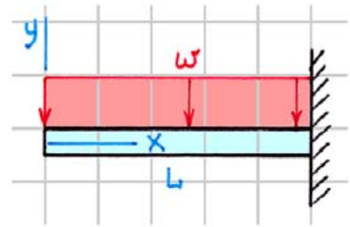
$$\delta_B = v_{1,x=L} = \frac{w}{48EI}\left(-L^4 + 6L^4 - 15L^4\right) = \frac{-10wL^4}{48EI} = \frac{5wL^4}{24EI} \downarrow$$ .... **Ans.**

(b) $\qquad v_2 = \dfrac{w}{24EI}\left[-2x^4 + 12Lx^3 - 30L^2x^2 + (x-L)^4 + 24L^3x - 9L^4\right]$

$$\delta_C = v_{2,x=2L} = \frac{w}{24EI}\left[-2\left(16L^4\right) + 12\left(8L^4\right) - 30\left(4L^4\right) + \left(L^4\right) + 24\left(2L^4\right) - 9L^4\right]$$

$$\delta_C = \frac{-16wL^4}{24EI} = \frac{2wL^4}{3EI} \downarrow$$ .... **Ans.**

8-36*



$$EIv'''' = -w$$

$$EIv''' = -wx + C_1$$

$$EIv'' = \frac{-wx^2}{2} + C_1 x + C_2$$

$$EIv' = \frac{-wx^3}{6} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-wx^4}{24} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $V = EIv''' = 0$: $C_1 = 0$

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $v' = 0$: $C_3 = wL^3/6$

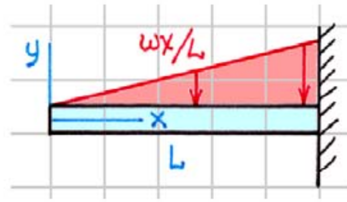
At $x = L$, $v = 0$: $C_4 = -wL^4/8$

(a) $v = \frac{w}{24EI}\left(-x^4 + 4L^3 x - 3L^4\right)$ .... **Ans.**

(b) $\delta_{\max} = v_{x=0} = \frac{w}{24EI}\left(-0 + 0 - 3L^4\right) = \frac{-3wL^4}{24EI} = \frac{wL^4}{8EI} \downarrow$ .... **Ans.**

8-37*

$$EIv'''' = -wx/L$$



$$EIv''' = \frac{-wx^2}{2L} + C_1$$

$$EIv'' = \frac{-wx^3}{6L} + C_1 x + C_2$$

$$EIv' = \frac{-wx^4}{24L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-wx^5}{120L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $V = EIv''' = 0$: $C_1 = 0$

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $v' = 0$: $C_3 = wL^3/24$

At $x = L$, $v = 0$: $C_4 = -wL^4/30$

(a) $v = \dfrac{w}{120EIL}\left(-x^5 + 5L^4 x - 4L^5\right)$ .... **Ans.**

(b) $\delta_{\max} = v_{x=0} = \dfrac{w}{120EIL}\left(-0 + 0 - 4L^5\right) = \dfrac{-4wL^4}{120EI} = \dfrac{wL^4}{30EI} \downarrow$ .... **Ans.**

8-38

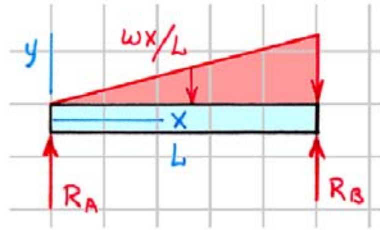
$$EIv'''' = -wx/L$$

$$EIv''' = \frac{-wx^2}{2L} + C_1$$

$$EIv'' = \frac{-wx^3}{6L} + C_1x + C_2$$

$$EIv' = \frac{-wx^4}{24L} + \frac{C_1x^2}{2} + C_2x + C_3$$

$$EIv = \frac{-wx^5}{120L} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4$$



Boundary Conditions:

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $M = EIv'' = 0$: $C_1 = wL/6$

At $x = 0$, $v = 0$: $C_4 = 0$

At $x = L$, $v = 0$: $C_3 = -7wL^3/360$

(a) $v = \frac{w}{360EIL}\left(-3x^5 + 10L^2x^3 - 7L^4x\right)$ .... **Ans.**

(b) $\delta_M = v_{x=L/2} = \frac{w}{360EIL}\left(\frac{-3L^5}{32} + \frac{10L^5}{8} - \frac{7L^5}{2}\right) = \frac{-5wL^4}{768EI} = \frac{5wL^4}{768EI}\downarrow$ .... **Ans.**

8-39*

$$EIv'''' = \frac{-wx^3}{L^3}$$

$$EIv''' = \frac{-wx^4}{4L^3} + C_1$$

$$EIv'' = \frac{-wx^5}{20L^3} + C_1 x + C_2$$

$$EIv' = \frac{-wx^6}{120L^3} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-wx^7}{840L^3} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $V = EIv''' = 0$: $C_1 = 0$

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $v' = 0$: $C_3 = wL^3/120$

At $x = L$, $v = 0$: $C_4 = -wL^4/140$

(a) $v = \frac{w}{840EIL^3}\left(-x^7 + 7L^6 x - 6L^7\right)$ .... **Ans.**

(b) $\delta_{\max} = v_{x=0} = \frac{w}{840EIL^3}\left(-0 + 0 - 6L^7\right) = \frac{-6wL^4}{840EI} = \frac{wL^4}{140EI}\downarrow$ .... **Ans.**

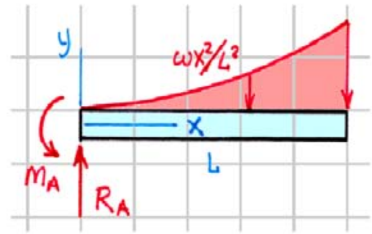
(c) $R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[\frac{-wL}{4} + 0\right] = \frac{+wL}{4} = \frac{wL}{4}\uparrow$ .... **Ans.**

$M_B = M_{x=L} = EIv''_{x=L} = \left[\frac{-wL^2}{20} + 0 + 0\right] = \frac{-wL^2}{20} = \frac{wL^2}{20}\circlearrowright$ .... **Ans.**

8-40*

$$EIv'''' = -wx^2/L^2$$

$$EIv''' = \frac{-wx^3}{3L^2} + C_1$$



$$EIv'' = \frac{-wx^4}{12L^2} + C_1 x + C_2$$

$$EIv' = \frac{-wx^5}{60L^2} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-wx^6}{360L^2} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $v = v' = 0$: $C_3 = C_4 = 0$

At $x = L$, $V = EIv''' = 0$: $C_1 = wL/3$

At $x = L$, $M = EIv'' = 0$: $C_2 = -wL^2/4$

(a) $v = \frac{w}{360EIL^2}\left(-x^6 + 20L^3x^3 - 45L^4x^2\right)$ .... **Ans.**

(b) $\delta_{max} = v_{x=L} = \frac{w}{360EIL^2}\left(-L^6 + 20L^6 - 45L^6\right) = \frac{-26wL^4}{360EI} = \frac{13wL^4}{180EI} \downarrow$ .... **Ans.**

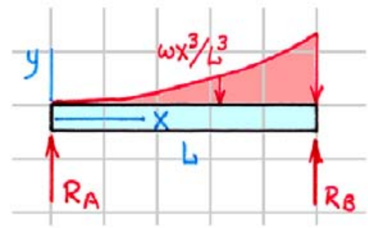
8-41

$$EIv'''' = \frac{-wx^3}{L^3}$$

$$EIv''' = \frac{-wx^4}{4L^3} + C_1$$

$$EIv'' = \frac{-wx^5}{20L^3} + C_1 x + C_2$$

$$EIv' = \frac{-wx^6}{120L^3} + \frac{C_1 x^2}{2} + C_2 x + C_3$$



$$EIv = \frac{-wx^7}{840L^3} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $M = EIv'' = 0$: $C_1 = wL/20$

At $x = 0$, $v = 0$: $C_4 = 0$

At $x = L$, $v = 0$: $C_3 = -wL^3/140$

(a) $v = \frac{w}{840EIL}\left(-x^7 + 7L^4x^3 - 6L^6x\right)$ .... **Ans.**

(b) $\delta_M = v_{x=L/2} = \frac{w}{840EIL^3}\left(\frac{-L^7}{128} + \frac{7L^7}{8} - 3L^7\right) = \frac{-13wL^4}{5120EI} = \frac{13wL^4}{5120EI}\downarrow$ .... **Ans.**

(c) $v' = \frac{w}{840EIL}\left(-7x^6 + 21L^4x^2 - 6L^6\right)$ δ_{max} when $v' = 0$

$-7x^6 + 21L^4x^2 - 6L^6 = 0$ $x = 0.5424L$

$\delta_{max} = v_{x=0.5424L} = -0.00256\frac{wL^4}{EI} = 0.00256\frac{wL^4}{EI}\downarrow = \frac{13.11wL^4}{5120EI}\downarrow$ .... **Ans.**

(d) $R_A = V_{x=0} = EIv'''_{x=0} = \left[0 + \frac{wL}{20}\right] = \frac{+wL}{20} = \frac{wL}{20}\uparrow$ .... **Ans.**

$R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[\frac{-wL}{4} + \frac{wL}{20}\right] = \frac{+4wL}{20} = \frac{wL}{5}\uparrow$ .... **Ans.**

8-42*

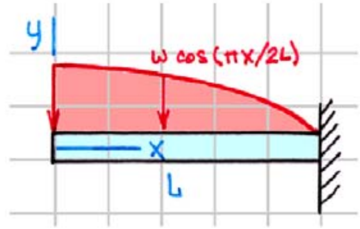
$$EIv'''' = -w\cos\frac{\pi x}{2L}$$

$$EIv''' = \frac{-2wL}{\pi}\sin\frac{\pi x}{2L} + C_1$$

$$EIv'' = \frac{4wL^2}{\pi^2}\cos\frac{\pi x}{2L} + C_1 x + C_2$$

$$EIv' = \frac{8wL^3}{\pi^3}\sin\frac{\pi x}{2L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-16wL^4}{\pi^4}\cos\frac{\pi x}{2L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$



Boundary Conditions:

At $x = 0$, $V = EIv''' = 0$: $C_1 = 0$

At $x = 0$, $M = EIv'' = 0$: $C_2 = -4wL^2/\pi^2$

At $x = L$, $v' = 0$: $C_3 = \frac{4wL^3}{\pi^3}(\pi - 2)$

At $x = L$, $v = 0$: $C_4 = \frac{2wL^4}{\pi^3}(4 - \pi)$

(a) $v = \frac{w}{2\pi^4 EI}\left[-32L^4\cos\frac{\pi x}{2L} - 4\pi^2 L^2 x^2 + 8(\pi - 2)\pi L^3 x + 4\pi(4 - \pi)L^4\right]$ .... **Ans.**

(b) $\delta_A = v_{x=0} = \frac{w}{2\pi^4 EI}\left[-32L^4 + 4\pi(4 - \pi)L^4\right] = -0.1089\frac{wL^4}{EI} = 0.1089\frac{wL^4}{EI}\downarrow$ .... **Ans.**

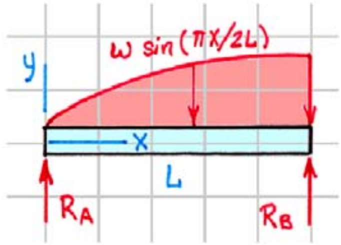
(c) $R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[\frac{-2wL}{\pi}\right] = \frac{+2wL}{\pi} = \frac{2wL}{\pi}\uparrow$ .... **Ans.**

$M_B = M_{x=L} = EIv''_{x=L} = \left[\frac{-4wL^2}{\pi^2}\right] = \frac{-4wL^2}{\pi^2} = \frac{4wL^2}{\pi^2}$ ↻ .... **Ans.**

8-43*

$$EIv'''' = -w\sin\frac{\pi x}{2L}$$

$$EIv''' = \frac{2wL}{\pi}\cos\frac{\pi x}{2L} + C_1$$



$$EIv'' = \frac{4wL^2}{\pi^2}\sin\frac{\pi x}{2L} + C_1 x + C_2$$

$$EIv' = \frac{-8wL^3}{\pi^3}\cos\frac{\pi x}{2L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-16wL^4}{\pi^4}\sin\frac{\pi x}{2L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $M = EIv'' = 0$: $C_1 = -4wL/\pi^2$

At $x = 0$, $v = 0$: $C_4 = 0$

At $x = L$, $v = 0$: $C_3 = \frac{2wL^3}{3\pi^4}\left(24 + \pi^2\right)$

(a) $v = \frac{2w}{3\pi^4 EI}\left[-24L^4\sin\frac{\pi x}{2L} - \pi^2 L x^3 + \left(24 + \pi^2\right)L^3 x\right]$ .... **Ans.**

(b) $\delta_M = v_{x=L/2} = \frac{2w}{3\pi^4 EI}\left[-24L^4\sin\frac{\pi}{4} - \frac{\pi^2 L^4}{8} + \left(\frac{24+\pi^2}{2}\right)L^4\right]$

$\delta_M = -0.00869\frac{wL^4}{EI} = 0.00869\frac{wL^4}{EI}\downarrow$ .... **Ans.**

(c) $v' = \frac{2w}{3\pi^4 EI}\left[-12\pi L^3\cos\frac{\pi x}{2L} - 3\pi^2 L x^2 + \left(24 + \pi^2\right)L^3\right]$ $\delta_{\max}$ when $v' = 0$

$-12\pi L^3\cos\frac{\pi x}{2L} - 3\pi^2 L x^2 + \left(24 + \pi^2\right)L^3 = 0$ $x \cong 0.5154L$

$\delta_{\max} = v_{x=0.5154L} = -0.00870\frac{wL^4}{EI} = 0.00870\frac{wL^4}{EI}\downarrow$ .... **Ans.**

(d) $\theta_A = v'_{x=0} = \frac{2w}{3\pi^4 EI}\left[-12\pi L^3 + \left(24 + \pi^2\right)L^3\right]$

$\theta_A = -0.0262\frac{wL^3}{EI} = 0.0262\frac{wL^3}{EI}$ ↘ .... **Ans.**

(e) $R_A = V_{x=0} = EIv'''_{x=0} = \left[\frac{2wL}{\pi} + \frac{4wL}{\pi^2}\right] = \frac{2wL}{\pi^2}(\pi - 2) = \frac{2wL}{\pi^2}(\pi - 2)\uparrow$ .... **Ans.**

$R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[\frac{-4wL}{\pi^2}\right] = \frac{+4wL}{\pi^2} = \frac{4wL}{\pi^2}\uparrow$ .... **Ans.**

8-44

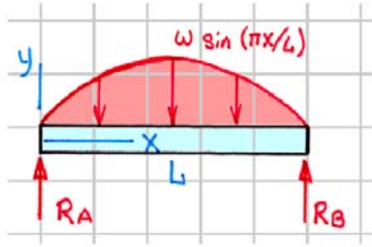
$$EIv'''' = -w\sin\frac{\pi x}{L}$$

$$EIv''' = \frac{wL}{\pi}\cos\frac{\pi x}{L} + C_1$$

$$EIv'' = \frac{wL^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

$$EIv' = \frac{-wL^3}{\pi^3}\cos\frac{\pi x}{L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-wL^4}{\pi^4}\sin\frac{\pi x}{L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$



Boundary Conditions:

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $M = EIv'' = 0$: $C_1 = 0$

At $x = 0$, $v = 0$: $C_4 = 0$

At $x = L$, $v = 0$: $C_3 = 0$

(a) $v = \frac{-wL^4}{\pi^4 EI}\sin\frac{\pi x}{L}$ .... **Ans.**

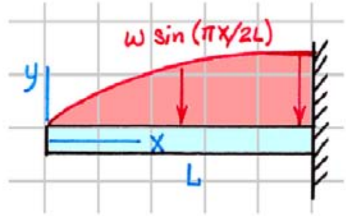
(b) $\delta_M = v_{x=L/2} = \frac{-wL^4}{\pi^4 EI} = \frac{wL^4}{\pi^4 EI}\downarrow$ .... **Ans.**

(c) $\theta_A = v'_{x=0} = \frac{-wL^3}{\pi^3 EI} = \frac{wL^3}{\pi^3 EI}$ ↻ .... **Ans.**

(d) $R_A = V_{x=0} = EIv'''_{x=0} = \frac{wL}{\pi} = \frac{wL}{\pi}\uparrow$ .... **Ans.**

$R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[\frac{-wL}{\pi}\right] = \frac{+wL}{\pi} = \frac{wL}{\pi}\uparrow$ .... **Ans.**

8-45



$$EIv'''' = -w\sin\frac{\pi x}{2L}$$

$$EIv''' = \frac{2wL}{\pi}\cos\frac{\pi x}{2L} + C_1$$

$$EIv'' = \frac{4wL^2}{\pi^2}\sin\frac{\pi x}{2L} + C_1 x + C_2$$

$$EIv' = \frac{-8wL^3}{\pi^3}\cos\frac{\pi x}{2L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-16wL^4}{\pi^4}\sin\frac{\pi x}{2L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $V = EIv''' = 0$: $C_1 = -2wL/\pi$

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $v' = 0$: $C_3 = wL^3/\pi$

At $x = L$, $v = 0$: $C_4 = \frac{2wL^4}{3\pi^4}\left(24 - \pi^3\right)$

(a) $$v = \frac{w}{3\pi^4 EI}\left[-48L^4\sin\frac{\pi x}{2L} - \pi^3 L x^3 + 3\pi^3 L^3 x + 2\left(24 - \pi^3\right)L^4\right]$$ .... **Ans.**

(b) $$\delta_A = v_{x=0} = \frac{w}{3\pi^4 EI}\left[2\left(24 - \pi^3\right)L^4\right] = -0.0480\frac{wL^4}{EI} = 0.0480\frac{wL^4}{EI}\downarrow$$ .... **Ans.**

(c) $$v' = \frac{w}{\pi^3 EI}\left[-8L^3\cos\frac{\pi x}{2L} - \pi^2 L x^2 + \pi^2 L^3\right]$$

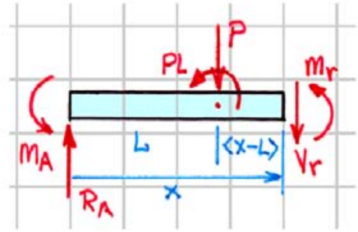
$$\theta_A = v'_{x=0} = \frac{w}{\pi^3 EI}\left[-8L^3 + \pi^2 L^3\right] = +0.0603\frac{wL^3}{EI} = 0.0603\frac{wL^3}{EI}$$ ∡ .... **Ans.**

(d) $$R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[\frac{-2wL}{\pi}\right] = \frac{+2wL}{\pi} = \frac{2wL}{\pi}\uparrow$$ .... **Ans.**

$$M_B = M_{x=L} = EIv''_{x=L} = \left[\frac{4wL^2}{\pi^2} - \frac{2wL^2}{\pi}\right] = \frac{2wL^2}{\pi^2}(\pi - 2)$$ ↻ .... **Ans.**

8-46*

From overall equilibrium:



$$R_A = 2P \uparrow \qquad M_A = 2PL \circlearrowleft$$

$$EIv'' = M_r = -2PL + 2Px - PL\langle x-L\rangle^0 - P\langle x-L\rangle^1$$

$$EIv' = -2PLx + Px^2 - PL\langle x-L\rangle^1 - \frac{P\langle x-L\rangle^2}{2} + C_1$$

$$EIv = -PLx^2 + \frac{Px^3}{3} - \frac{PL\langle x-L\rangle^2}{2} - \frac{P\langle x-L\rangle^3}{6} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $\qquad C_1 = 0$

At $x = 0$, $v = 0$: $\qquad C_2 = 0$

Therefore $\qquad v = \frac{P}{6EI}\left[2x^3 - 6Lx^2 - 3L\langle x-L\rangle^2 - \langle x-L\rangle^3\right]$

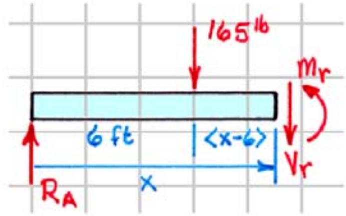
(a) $\delta_B = v_{x=L} = \frac{P}{6EI}\left[2L^3 - 6L^3\right] = \frac{-2PL^3}{3EI} = \frac{2PL^3}{3EI}\downarrow$ .... **Ans.**

(b) $\delta_C = v_{x=2L} = \frac{P}{6EI}\left[16L^3 - 24L^3 - 3L^3 - L^3\right] = \frac{-2PL^3}{EI} = \frac{2PL^3}{EI}\downarrow$ .... **Ans.**

8-47*

From overall equilibrium:

$R_A = 66.0 \text{ lb} \uparrow \qquad R_B = 99.0 \text{ lb} \uparrow$



$$EIv'' = M_r = \left[66.0x - 165\langle x-6\rangle^1\right] \text{ lb}\cdot\text{ft}$$

$$EIv' = \left[33.0x^2 - 82.5\langle x-6\rangle^2 + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[11.0x^3 - 27.5\langle x-6\rangle^3 + C_1x + C_2\right] \text{ lb}\cdot\text{ft}^3$$

Boundary Conditions:

At $x = 0$ ft, $v = 0$: $\quad C_2 = 0$

$$I = \frac{(12)(2)^3}{12} = 8.00 \text{ in.}^4$$

At $x = 10$ ft, $v = 0$: $\quad C_1 = -924.0 \text{ lb}\cdot\text{ft}^2$

Therefore $\quad EIv = \left[11.0x^3 - 27.5\langle x-6\rangle^3 - 924.0x\right] \text{ lb}\cdot\text{ft}^3$

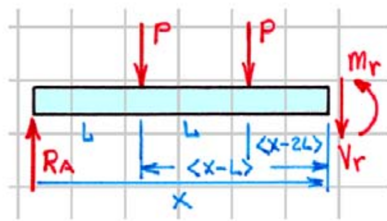
$$EIv_{x=6} = \left[11.0(6)^3 - 0 - 924.0(6)\right] = -3168.0 \text{ lb}\cdot\text{ft}^3 = -5474.3 \text{ kip}\cdot\text{in.}^3$$

$$\delta_B = v_{x=6} = \frac{-5474.3}{(1700)(8)} = -0.403 \text{ in.} = 0.403 \text{ in.}\downarrow$$ .... **Ans.**

8-48

From overall equilibrium:

$$R_A = R_D = P \uparrow$$



$$EIv'' = M_r = Px - P\langle x-L\rangle^1 - P\langle x-2L\rangle^1$$

$$EIv' = \frac{Px^2}{2} - \frac{P\langle x-L\rangle^2}{2} - \frac{P\langle x-2L\rangle^2}{2} + C_1$$

$$EIv = \frac{Px^3}{6} - \frac{P\langle x-L\rangle^3}{6} - \frac{P\langle x-2L\rangle^3}{6} + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 3L$, $v = 0$: $C_1 = -PL^2$

Therefore $$v = \frac{P}{6EI}\left[x^3 - \langle x-L\rangle^3 - \langle x-2L\rangle^3 - 6L^2 x\right]$$

(a) $\delta_B = v_{x=L} = \dfrac{P}{6EI}\left[L^3 - 6L^3\right] = \dfrac{-5PL^3}{6EI} = \dfrac{5PL^3}{6EI}\downarrow$ .... **Ans.**

(b) $\delta_M = v_{x=3L/2} = \dfrac{P}{6EI}\left[\dfrac{27L^3}{8} - \dfrac{L^3}{8} - 9L^3\right] = \dfrac{-23PL^3}{24EI} = \dfrac{23PL^3}{24EI}\downarrow$ .... **Ans.**

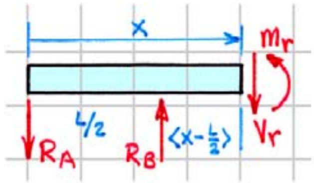
(c) $\delta_C = v_{x=2L} = \dfrac{P}{6EI}\left[8L^3 - L^3 - 12L^3\right] = \dfrac{-5PL^3}{6EI} = \dfrac{5PL^3}{6EI}\downarrow$ .... **Ans.**

(Or by symmetry $\delta_B = \delta_C$.)

8-49*

From overall equilibrium:

$$R_A = 2P \downarrow \qquad R_B = 3P \uparrow$$



$$EIv'' = M_r = -2Px + 3P\left\langle x - \frac{L}{2}\right\rangle^1$$

$$EIv' = -Px^2 + \frac{3P}{2}\left\langle x - \frac{L}{2}\right\rangle^2 + C_1$$

$$EIv = \frac{-Px^3}{3} + \frac{P}{2}\left\langle x - \frac{L}{2}\right\rangle^3 + C_1x + C_2$$

Boundary Conditions: At $x = 0$, $v = 0$: $C_2 = 0$

At $x = L/2$, $v = 0$: $C_1 = PL^2/12$

Therefore $$v = \frac{P}{12EI}\left[-4x^3 + 6\left\langle x - \frac{L}{2}\right\rangle^3 + L^2x\right]$$

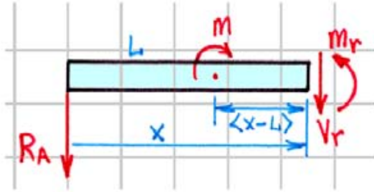
(a) $$\delta_C = v_{x=3L/2} = \frac{P}{12EI}\left[\frac{-108L^3}{8} + 6L^3 + \frac{3L^3}{2}\right] = \frac{-PL^3}{2EI} = \frac{PL^3}{2EI}\downarrow$$ .... **Ans.**

(b) $$\delta_M = v_{x=L/4} = \frac{P}{12EI}\left[\frac{-L^3}{16} + \frac{L^3}{4}\right] = \frac{+PL^3}{64EI} = \frac{PL^3}{64EI}\uparrow$$ .... **Ans.**

8-50*

From overall equilibrium:

$$R_A = M/2L \downarrow \qquad R_C = M/2L \uparrow$$



$$EIv'' = M_r = \frac{-Mx}{2L} + M\langle x-L\rangle^0$$

$$EIv' = \frac{-Mx^2}{4L} + M\langle x-L\rangle^1 + C_1$$

$$EIv = \frac{-Mx^3}{12L} + \frac{M\langle x-L\rangle^2}{2} + C_1 x + C_2$$

Boundary Conditions: At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 2L$, $v = 0$: $C_1 = ML/12$

Therefore $v = \dfrac{M}{12EIL}\left[-x^3 + 6L\langle x-L\rangle^2 + L^2 x\right]$

$$\delta_M = v_{x=L} = \frac{M}{12EIL}\left[-L^3 + L^3\right] = 0$$ .... **Ans.**

8-51

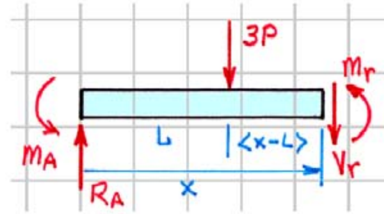
From overall equilibrium:

$$R_A = 3P \uparrow \qquad M_A = PL \circlearrowleft$$

$$EIv'' = M_r = -PL + 3Px - 3P\langle x-L\rangle^1$$

$$EIv' = -PLx + \frac{3Px^2}{2} - \frac{3P\langle x-L\rangle^2}{2} + C_1$$

$$EIv = \frac{-PLx^2}{2} + \frac{Px^3}{2} - \frac{P\langle x-L\rangle^3}{2} + C_1x + C_2$$



Boundary Conditions:

At $x = 0$, $v' = 0$: $\qquad C_1 = 0$

At $x = 0$, $v = 0$: $\qquad C_2 = 0$

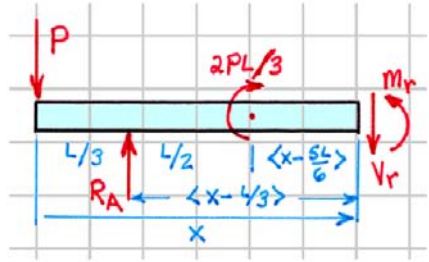
Therefore $\qquad v = \frac{P}{2EI}\left[x^3 - Lx^2 - \langle x-L\rangle^3\right]$

(a) $\delta_B = v_{x=L} = \frac{P}{2EI}\left[L^3 - L^3\right] = 0$ .... **Ans.**

(b) $\delta_C = v_{x=3L/2} = \frac{P}{2EI}\left[\frac{27L^3}{8} - \frac{9L^3}{4} - \frac{L^3}{8}\right] = \frac{+PL^3}{2EI} = \frac{PL^3}{2EI}\uparrow$ .... **Ans.**

8-52

From overall equilibrium:



$$R_A = 2P/3 \uparrow \qquad R_B = P/3 \uparrow$$

$$EIv'' = M_r = -Px + \frac{2P}{3}\left\langle x - \frac{L}{3}\right\rangle^1 + \frac{2PL}{3}\left\langle x - \frac{5L}{6}\right\rangle^0$$

$$EIv' = \frac{-Px^2}{2} + \frac{P}{3}\left\langle x - \frac{L}{3}\right\rangle^2 + \frac{2PL}{3}\left\langle x - \frac{5L}{6}\right\rangle^1 + C_1$$

$$EIv = \frac{-Px^3}{6} + \frac{P}{9}\left\langle x - \frac{L}{3}\right\rangle^3 + \frac{PL}{3}\left\langle x - \frac{5L}{6}\right\rangle^2 + C_1x + C_2$$

Boundary Conditions:

At $x = L/3$, $v = 0$: $\qquad C_1 = 7PL^2/36$

At $x = 4L/3$, $v = 0$: $\qquad C_2 = -19PL^3/324$

Therefore $$v = \frac{P}{324EI}\left[-54x^3 + 36\left\langle x - \frac{L}{3}\right\rangle^3 + 108L\left\langle x - \frac{5L}{6}\right\rangle^2 + 63L^2x - 19L^3\right]$$

(a) $$\delta_L = v_{x=0} = \frac{P}{324EI}\left[-19L^3\right] = \frac{-19PL^3}{324EI} = \frac{19PL^3}{324EI} \downarrow$$ .... **Ans.**

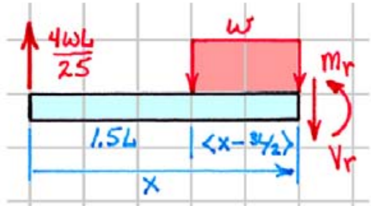
(b) $$\delta_M = v_{x=5L/6} = \frac{+PL^3}{48EI} = \frac{PL^3}{48EI} \uparrow$$ .... **Ans.**

8-53*

$$EIv'' = M_r = \frac{4wLx}{25} - \frac{w}{2}\left\langle x - \frac{3L}{2}\right\rangle^2$$

$$EIv' = \frac{2wLx^2}{25} - \frac{w}{6}\left\langle x - \frac{3L}{2}\right\rangle^3 + C_1$$

$$EIv = \frac{2wLx^3}{75} - \frac{w}{24}\left\langle x - \frac{3L}{2}\right\rangle^4 + C_1 x + C_2$$



Boundary Conditions:

At $x = 5L/2$, $v' = 0$: $C_1 = -wL^3/3$

At $x = 5L/2$, $v = 0$: $C_2 = 11wL^4/24$

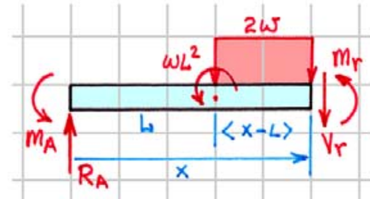
Therefore $$v = \frac{w}{600EI}\left[16Lx^3 - 25\left\langle x - \frac{3L}{2}\right\rangle^4 - 200L^3x + 275L^4\right]$$

$$\delta_A = v_{x=0} = \frac{w}{600EI}\left[275L^4\right] = \frac{+275wL^4}{600EI} = \frac{11wL^4}{24EI}\uparrow$$ .... **Ans.**

8-54*

From overall equilibrium:

$$R_A = 2wL \uparrow \qquad M_A = 2wL^2 \circlearrowleft$$



$$EIv'' = M_r = 2wLx - 2wL^2 - wL^2\langle x-L\rangle^0 - w\langle x-L\rangle^2$$

$$EIv' = wLx^2 - 2wL^2x - wL^2\langle x-L\rangle^1 - \frac{w\langle x-L\rangle^3}{3} + C_1$$

$$EIv = \frac{wLx^3}{3} - wL^2x^2 - \frac{wL^2\langle x-L\rangle^2}{2} - \frac{w\langle x-L\rangle^4}{12} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $\qquad C_1 = 0$

At $x = 0$, $v = 0$: $\qquad C_2 = 0$

Therefore $\qquad v = \frac{w}{12EI}\left[4Lx^3 - 12L^2x^2 - 6L^2\langle x-L\rangle^2 - \langle x-L\rangle^4\right]$

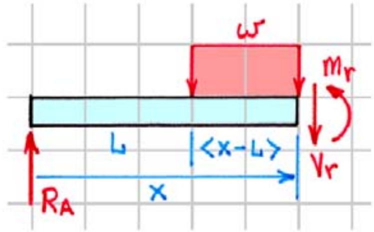
(a) $\qquad \delta_B = v_{x=L} = \frac{w}{12EI}\left[4L^4 - 12L^4\right] = \frac{-8wL^4}{12EI} = \frac{2wL^4}{3EI}\downarrow$ .... **Ans.**

(b) $\qquad \delta_C = v_{x=2L} = \frac{w}{12EI}\left[32L^4 - 48L^4 - 6L^4 - L^4\right] = \frac{-23wL^4}{12EI} = \frac{23wL^4}{12EI}\downarrow$ .... **Ans.**

8-55

From overall equilibrium:

$$R_A = \frac{2wL}{3}\uparrow \qquad R_B = \frac{4wL}{3}\uparrow$$



$$EIv'' = M_r = \frac{2wLx}{3} - \frac{w\langle x-L\rangle^2}{2}$$

$$EIv' = \frac{wLx^2}{3} - \frac{w\langle x-L\rangle^3}{6} + C_1$$

$$EIv = \frac{wLx^3}{9} - \frac{w\langle x-L\rangle^4}{24} + C_1x + C_2$$

Boundary Conditions: At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 3L$, $v = 0$: $C_1 = \frac{-7wL^3}{9}$

Therefore $v = \frac{w}{72EI}\left[8Lx^3 - 3\langle x-L\rangle^4 - 56L^3x\right]$

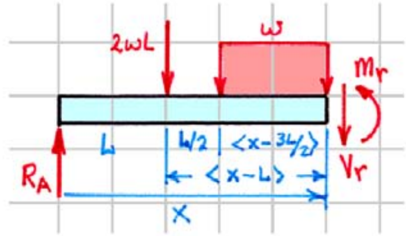
(a) $\delta_L = v_{x=L} = \frac{w}{72EI}\left[8L^4 - 56L^4\right] = \frac{-48wL^4}{72EI} = \frac{2wL^4}{3EI}\downarrow$ .... **Ans.**

(b) $\delta_M = v_{x=3L/2} = \frac{w}{72EI}\left[27L^4 - \frac{3L^4}{16} - 84L^4\right] = \frac{-305wL^4}{384EI} = \frac{305wL^4}{384EI}\downarrow$ .... **Ans.**

8-56*

From overall equilibrium:

$$R_A = 7wL/5 \uparrow \qquad R_B = 8wL/5 \uparrow$$



$$EIv'' = M_r = \frac{7wLx}{5} - 2wL\langle x-L\rangle^1 - \frac{w}{2}\left\langle x - \frac{3L}{2}\right\rangle^2$$

$$EIv' = \frac{7wLx^2}{10} - wL\langle x-L\rangle^2 - \frac{w}{6}\left\langle x - \frac{3L}{2}\right\rangle^3 + C_1$$

$$EIv = \frac{7wLx^3}{30} - \frac{wL\langle x-L\rangle^3}{3} - \frac{w}{24}\left\langle x - \frac{3L}{2}\right\rangle^4 + C_1x + C_2$$

Boundary Conditions: At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 5L/2$, $v = 0$: $C_1 = -119wL^3/120$

Therefore $$v = \frac{w}{120EI}\left[28Lx^3 - 40L\langle x-L\rangle^3 - 5\left\langle x - \frac{3L}{2}\right\rangle^4 - 119L^3x\right]$$

(a) $$\delta_L = v_{x=L} = \frac{w}{120EI}\left[28L^4 - 119L^4\right] = \frac{-91wL^4}{120EI} = \frac{91wL^4}{120EI}\downarrow$$ .... **Ans.**

(b) $$\delta_{3L/2} = v_{x=3L/2} = \frac{w}{120EI}\left[\frac{189L^4}{2} - 5L^4 - \frac{357L^4}{2}\right] = \frac{-89wL^4}{120EI} = \frac{89wL^4}{120EI}\downarrow$$ .... **Ans.**

8-57

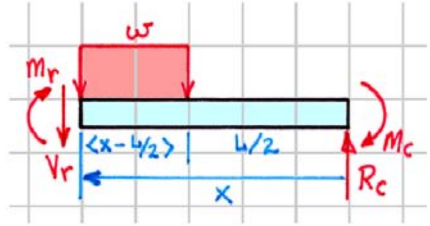
From overall equilibrium:

$R_C = wL \uparrow \qquad M_C = wL^2 \circlearrowleft$

$$EIv'' = M_r = wLx - wL^2 - \frac{w}{2}\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EIv' = \frac{wLx^2}{2} - wL^2x - \frac{w}{6}\left\langle x - \frac{L}{2}\right\rangle^3 + C_1$$

$$EIv = \frac{wLx^3}{6} - \frac{wL^2x^2}{2} - \frac{w}{24}\left\langle x - \frac{L}{2}\right\rangle^4 + C_1x + C_2$$



Boundary Conditions: At $x = 0$, $v' = 0$: $C_1 = 0$

At $x = 0$, $v = 0$: $C_2 = 0$

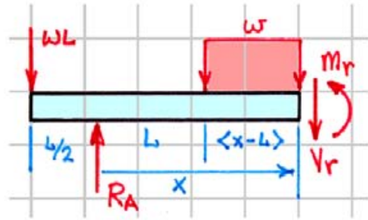
Therefore $$v = \frac{w}{24EI}\left[4Lx^3 - 12L^2x^2 - \left\langle x - \frac{L}{2}\right\rangle^4\right]$$

$$\delta_A = v_{x=3L/2} = \frac{w}{24EI}\left[\frac{27L^4}{2} - 27L^4 - L^4\right] = \frac{-29wL^4}{48EI} = \frac{29wL^4}{48EI}\downarrow$$ .... **Ans.**

8-58

From overall equilibrium:

$R_A = 11wL/8 \uparrow \qquad R_B = wL/8 \uparrow$



$$EIv'' = M_r = \frac{-wL^2}{2} + \frac{3wLx}{8} + \frac{wL\langle x-L\rangle^1}{8} - \frac{w\langle x-L\rangle^2}{2}$$

$$EIv' = \frac{-wL^2x}{2} + \frac{3wLx^2}{16} + \frac{wL\langle x-L\rangle^2}{16} - \frac{w\langle x-L\rangle^3}{6} + C_1$$

$$EIv = \frac{-wL^2x^2}{4} + \frac{wLx^3}{16} + \frac{wL\langle x-L\rangle^3}{48} - \frac{w\langle x-L\rangle^4}{24} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $\qquad C_2 = 0$

At $x = L$, $v = 0$: $\qquad C_1 = 3wL^3/16$

Therefore $\qquad v = \dfrac{w}{48EI}\left[3Lx^3 - 12L^2x^2 + L\langle x-L\rangle^3 - 2\langle x-L\rangle^4 - 9L^3x\right]$

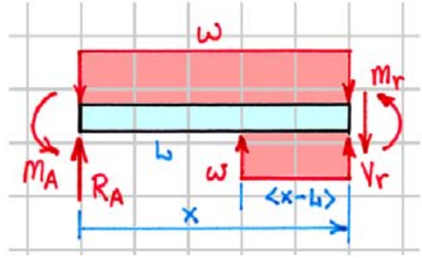
(a) $\quad \delta_C = v_{x=3L/2} = \dfrac{w}{48EI}\left[\dfrac{81L^4}{8} - 27L^4 + \dfrac{L^4}{8} - \dfrac{L^4}{8} + \dfrac{27L^4}{2}\right] = \dfrac{-9wL^4}{128EI} = \dfrac{9wL^4}{128EI}\downarrow$ .... **Ans.**

(b) $\quad \delta_M = v_{x=L/2} = \dfrac{w}{48EI}\left[\dfrac{3L^4}{8} - 3L^4 + \dfrac{9L^4}{2}\right] = \dfrac{+5wL^4}{128EI} = \dfrac{5wL^4}{128EI}\uparrow$ .... **Ans.**

8-59*

From overall equilibrium:

$$R_A = 7wL/8 \uparrow \qquad M_A = wL^2/4 \circlearrowleft$$



$$EIv'' = M_r = \frac{7wLx}{8} - \frac{wL^2}{4} - \frac{wx^2}{2} + \frac{w\langle x-L\rangle^2}{2}$$

$$EIv' = \frac{7wLx^2}{16} - \frac{wL^2x}{4} - \frac{wx^3}{6} + \frac{w\langle x-L\rangle^3}{6} + C_1$$

$$EIv = \frac{7wLx^3}{48} - \frac{wL^2x^2}{8} - \frac{wx^4}{24} + \frac{w\langle x-L\rangle^4}{24} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $\qquad C_1 = 0$

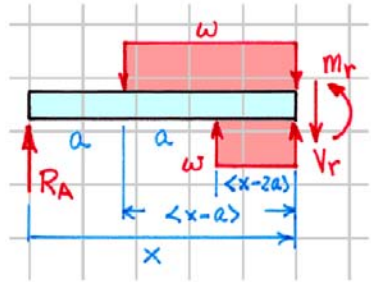
At $x = 0$, $v = 0$: $\qquad C_2 = 0$

Therefore $\qquad v = \dfrac{w}{48EI}\left[-2x^4 + 7Lx^3 - 6L^2x^2 + 2\langle x-L\rangle^4\right]$

$$\delta_C = v_{x=2L} = \frac{w}{48EI}\left[-32L^4 + 56L^4 - 24L^4 + 2L^4\right] = \frac{+2wL^4}{48EI} = \frac{wL^4}{24EI} \uparrow$$ .... **Ans.**

8-60*

From overall equilibrium:



$$R_A = R_B = wa/2 \uparrow$$

$$EIv'' = M_r = \frac{wax}{2} - \frac{w\langle x-a\rangle^2}{2} + \frac{w\langle x-2a\rangle^2}{2}$$

$$EIv' = \frac{wax^2}{4} - \frac{w\langle x-a\rangle^3}{6} + \frac{w\langle x-2a\rangle^3}{6} + C_1$$

$$EIv = \frac{wax^3}{12} - \frac{w\langle x-a\rangle^4}{24} + \frac{w\langle x-2a\rangle^4}{24} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 3a$, $v = 0$: $C_1 = -13wa^3/24$

Therefore $$v = \frac{w}{24EI}\left[2ax^3 - \langle x-a\rangle^4 + \langle x-2a\rangle^4 - 13a^3x\right]$$

(a) $$\delta_L = v_{x=a} = \frac{w}{24EI}\left[2a^4 - 13a^4\right] = \frac{-11wa^4}{24EI} = \frac{11wa^4}{24EI}\downarrow$$ .... **Ans.**

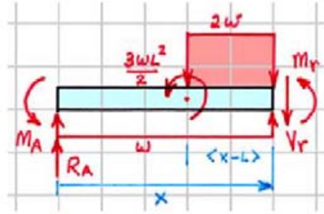
(b) $$\delta_M = v_{x=3a/2} = \frac{w}{24EI}\left[\frac{54a^4}{8} - \frac{a^4}{16} - \frac{39a^4}{2}\right] = \frac{-205wa^4}{384EI} = \frac{205wa^4}{384EI}\downarrow$$ .... **Ans.**

(c) $$\delta_R = v_{x=2a} = \frac{w}{24EI}\left[16a^4 - a^4 - 26a^4\right] = \frac{-11wa^4}{24EI} = \frac{11wa^4}{24EI}\downarrow$$ .... **Ans.**

8-61

From overall equilibrium:

$$R_A = wL \uparrow \qquad M_A = wL^2 \circlearrowleft$$



$$EIv'' = M_r = wLx - wL^2 + \frac{wx^2}{2} - \frac{3wL^2\langle x-L\rangle^0}{2} - w\langle x-L\rangle^2$$

$$EIv' = \frac{wLx^2}{2} - wL^2x + \frac{wx^3}{6} - \frac{3wL^2\langle x-L\rangle^1}{2} - \frac{w\langle x-L\rangle^3}{2} + C_1$$

$$EIv = \frac{wLx^3}{6} - \frac{wL^2x^2}{2} + \frac{wx^4}{24} - \frac{3wL^2\langle x-L\rangle^2}{4} - \frac{w\langle x-L\rangle^4}{8} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $\qquad C_1 = 0$

At $x = 0$, $v = 0$: $\qquad C_2 = 0$

Therefore $\qquad v = \frac{w}{24EI}\left[x^4 + 4Lx^3 - 12L^2x^2 - 18L^2\langle x-L\rangle^2 - 3\langle x-L\rangle^4\right]$

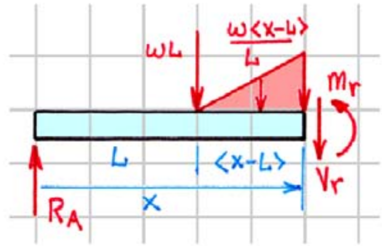
(a) $\qquad \delta_B = v_{x=L} = \frac{w}{24EI}\left[L^4 + 4L^4 - 12L^4\right] = \frac{-7wL^4}{24EI} = \frac{7wL^4}{24EI}\downarrow$ .... **Ans.**

(b) $\qquad \delta_C = v_{x=2L} = \frac{w}{24EI}\left[16L^4 + 32L^4 - 48L^4 - 18L^4 - 3L^4\right] = \frac{-21wL^4}{24EI} = \frac{7wL^4}{8EI}\downarrow$ .... **Ans.**

8-62

From overall equilibrium:

$$R_A = 7wL/12 \uparrow \qquad R_B = 11wL/12 \uparrow$$



$$EIv'' = M_r = \frac{7wLx}{12} - wL\langle x-L\rangle - \frac{w\langle x-L\rangle^3}{6L}$$

$$EIv' = \frac{7wLx^2}{24} - \frac{wL\langle x-L\rangle^2}{2} - \frac{w\langle x-L\rangle^4}{24L} + C_1$$

$$EIv = \frac{7wLx^3}{72} - \frac{wL\langle x-L\rangle^3}{6} - \frac{w\langle x-L\rangle^5}{120L} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $\quad C_2 = 0$

At $x = 2L$, $v = 0$: $\quad C_1 = -217wL^3/720$

Therefore $\quad v = \frac{w}{720EIL}\left[70L^2x^3 - 120L^2\langle x-L\rangle^3 - 6\langle x-L\rangle^5 - 217L^4x\right]$

(a) $\quad \delta_M = v_{x=L} = \frac{w}{720EIL}\left[70L^5 - 217L^5\right] = \frac{-147wL^4}{720EI} = \frac{147wL^4}{720EI}\downarrow$ .... **Ans.**

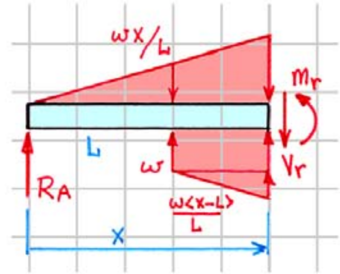
(b) $\quad v' = \frac{w}{720EIL}\left[210L^2x^2 - 360L^2\langle x-L\rangle^2 - 30\langle x-L\rangle^4 - 217L^4\right]$

δ_{max} when $v' = 0$ $\qquad x \cong 1.0168L$

$\delta_{max} = v_{x=1.0168L} = \frac{-0.2043wL^4}{EI} = \frac{0.2043wL^4}{EI}\downarrow = \frac{147.06wL^4}{720EI}\downarrow$ .... **Ans.**

8-63*

From overall equilibrium:



$$R_A = wL/3 \uparrow \qquad R_B = wL/6 \uparrow$$

$$EIv'' = M_r = \frac{wLx}{3} - \frac{wx^3}{6L} + \frac{w\langle x-L\rangle^2}{2} + \frac{w\langle x-L\rangle^3}{6L}$$

$$EIv' = \frac{wLx^2}{6} - \frac{wx^4}{24L} + \frac{w\langle x-L\rangle^3}{6} + \frac{w\langle x-L\rangle^4}{24L} + C_1$$

$$EIv = \frac{wLx^3}{18} - \frac{wx^5}{120L} + \frac{w\langle x-L\rangle^4}{24} + \frac{w\langle x-L\rangle^5}{120L} + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 2L$, $v = 0$: $C_1 = -41wL^3/360$

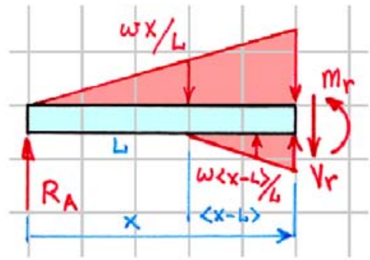
Therefore $v = \frac{w}{360EIL}\left[20L^2x^3 - 3x^5 + 15L\langle x-L\rangle^4 + 3\langle x-L\rangle^5 - 41L^4x\right]$

$\delta_M = v_{x=L} = \frac{w}{360EIL}\left[20L^5 - 3L^5 - 41L^5\right] = \frac{-24wL^4}{360EI} = \frac{wL^4}{15EI} \downarrow$ .... **Ans.**

8-64

From overall equilibrium:

$$R_A = 7wL/12 \uparrow \qquad R_B = 11wL/12 \uparrow$$



$$EIv'' = M_r = \frac{7wLx}{12} - \frac{wx^3}{6L} + \frac{w\langle x-L\rangle^3}{6L}$$

$$EIv' = \frac{7wLx^2}{24} - \frac{wx^4}{24L} + \frac{w\langle x-L\rangle^4}{24L} + C_1$$

$$EIv = \frac{7wLx^3}{72} - \frac{wx^5}{120L} + \frac{w\langle x-L\rangle^5}{120L} + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $\qquad C_2 = 0$

At $x = 2L$, $v = 0$: $\qquad C_1 = -187wL^3/720$

Therefore $\qquad v = \frac{w}{720EIL}\left[70L^2x^3 - 6x^5 + 6\langle x-L\rangle^5 - 187L^4x\right]$

(a) $\qquad \delta_M = v_{x=L} = \frac{w}{720EIL}\left[70L^5 - 6L^5 - 187L^5\right] = \frac{-123wL^4}{720EI} = \frac{41wL^4}{240EI}\downarrow$ .... **Ans.**

(b) $\qquad v' = \frac{w}{720EIL}\left[210L^2x^2 - 30x^4 + 30\langle x-L\rangle^4 - 187L^4\right]$

δ_{max} when $v' = 0$ $\qquad x \cong 1.0233L$

$\delta_{max} = v_{x=1.0233L} = \frac{-0.1709wL^4}{EI} = \frac{0.1709wL^4}{EI}\downarrow = \frac{41.02wL^4}{240EI}\downarrow$ .... **Ans.**

8-65

$$I = \frac{(18)(2)^3}{12} = 12.00 \text{ in.} \qquad EI = (1.8\times10^6)(12.00) = 21.60(10^6) \text{ lb}\cdot\text{in.}^2$$

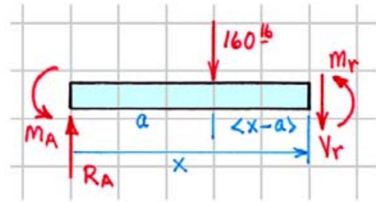
From overall equilibrium:

$$R_A = 160 \text{ lb}\uparrow \qquad M_A = (160a) \text{ lb}\cdot\text{ft} \circlearrowleft$$

$$EIv'' = M_r = \left[160x - 160a - 160\langle x-a\rangle^1\right] \text{ lb}\cdot\text{ft}$$

$$EIv' = \left[80x^2 - 160ax - 80\langle x-a\rangle^2 + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{80x^3}{3} - 80ax^2 - \frac{80\langle x-a\rangle^3}{3} + C_1x + C_2\right] \text{ lb}\cdot\text{ft}^3$$



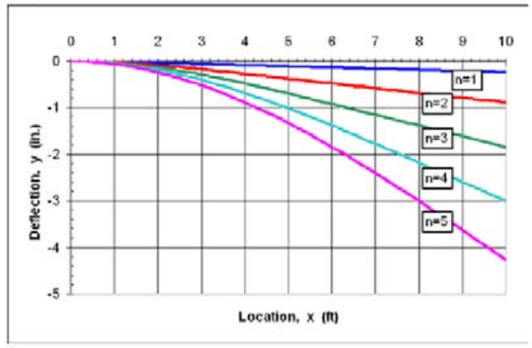
Boundary Conditions:

At $x = 0$, $v' = 0$: $\qquad C_1 = 0$

At $x = 0$, $v = 0$: $\qquad C_2 = 0$

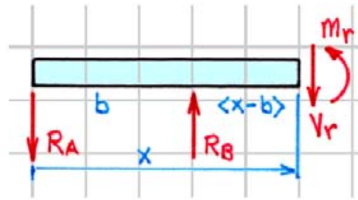
Therefore

$$3EIv = \left[80x^3 - 240ax^2 - 80\langle x-a\rangle^3\right] \text{ lb}\cdot\text{ft}^3$$



8-66

$$W = (70)(9.81) = 686.7 \text{ N}$$



From overall equilibrium:

$$R_A = \frac{W(3-b)}{b} \text{ N}\downarrow \qquad R_B = (3W/b) \text{ N}\uparrow$$

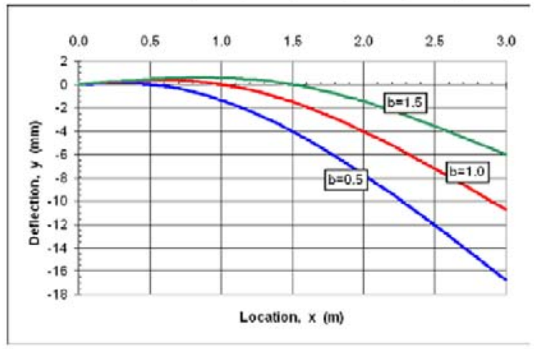
$$I = \frac{(500)(80)^3}{12} = 21.33(10^6) \text{ mm}^4$$

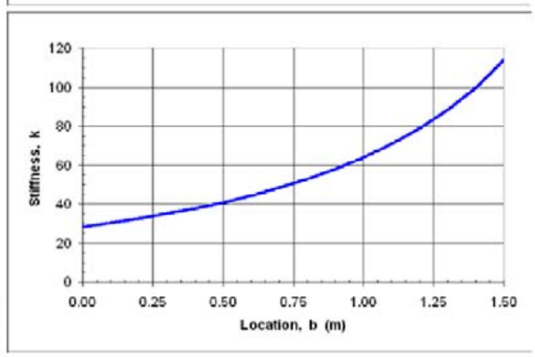
$$EI = (12\times10^9)(21.33\times10^{-6}) = 255{,}960 \text{ N}\cdot\text{m}^2$$

$$EIv'' = M_r = -R_A x + R_B\langle x-b\rangle^1$$

$$EIv' = \frac{-R_A x^2}{2} + \frac{R_B\langle x-b\rangle^2}{2} + C_1$$

$$EIv = \frac{-R_A x^3}{6} + \frac{R_B\langle x-b\rangle^3}{6} + C_1 x + C_2$$





Boundary Conditions:

At $x = 0$, $v = 0$: $\quad C_2 = 0$

At $x = b$, $v = 0$: $\quad C_1 = R_A b^2/6$

$$v = \frac{1}{6EI}\left[-R_A x^3 + R_B\langle x-b\rangle^3 + R_A b^2 x\right]$$

$$= \frac{W}{6EIb}\left[-(3-b)x^3 + 3\langle x-b\rangle^3 + (3-b)b^2 x\right]$$

$$v_{x=3} = \frac{W}{6EIb}\left[-(3-b)(3)^3 + 3(3-b)^3 + (3-b)b^2(3)\right]$$

$$k = \frac{W}{v_{x=3}} = \frac{2EIb}{-9(3-b)+(3-b)^3+(3-b)b^2}$$

Therefore $\quad k$ is independent of W .... **Ans.**

$$v_{x=3} = \frac{W}{6EIb}\left[-(3-b)(3)^3 + 3(3-b)^3 + (3-b)b^2(3)\right]$$

$$k = \frac{W}{v_{x=3}} = \frac{2EIb}{-9(3-b)+(3-b)^3+(3-b)b^2}$$

Therefore $\quad k$ is independent of W .... **Ans.**

8-67

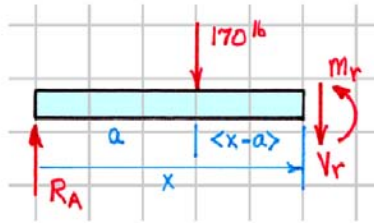
From overall equilibrium:

$$R_A = 17(10-a)\ \text{lb}\uparrow \qquad R_B = (17a)\ \text{lb}\uparrow$$

$$EIv'' = M_r = \left[R_A x - 170\langle x-a\rangle^1\right]\ \text{lb}\cdot\text{ft}$$

$$EIv' = \left[\frac{R_A x^2}{2} - \frac{170\langle x-a\rangle^2}{2} + C_1\right]\ \text{lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - \frac{170\langle x-a\rangle^3}{6} + C_1 x + C_2\right]\ \text{lb}\cdot\text{ft}^3$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

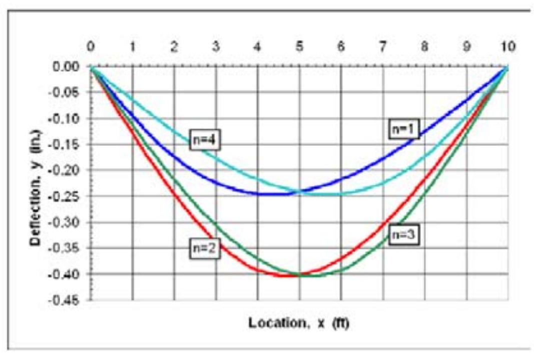
At $x = 10$ ft, $v = 0$: $C_1 = \dfrac{-100R_A}{6} + \dfrac{17(10-a)^2}{6}$

Therefore

$$6EIv = \left[R_A x^3 - 170\langle x-a\rangle^3 - 100R_A x + 17(10-a)^3 x\right]$$
$$= 17\left[(10-a)x^3 - 10\langle x-a\rangle^3 - 100(10-a)x + (10-a)^3 x\right]\ \text{lb}\cdot\text{ft}^3$$

$$I = \frac{(12)(2)^3}{12} = 8.00\ \text{in.}^4$$

$$EI = (1.8\times10^6)(8.00)$$
$$= 14.40(10^6)\ \text{lb}\cdot\text{in.}^2$$



8-68

From overall equilibrium:

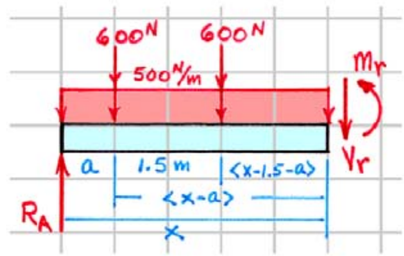
$$R_A = (2270 - 240a)\ \text{N}\uparrow \qquad R_D = (240a + 1430)\ \text{N}\uparrow$$

$$EIv'' = M_r = \left[R_A x - 250x^2 - 600\langle x-a\rangle^1 - 600\langle x-a-1.5\rangle^1\right]\ \text{N}\cdot\text{m}$$

$$EIv' = \left[\frac{R_A x^2}{2} - \frac{250x^3}{3} - 300\langle x-a\rangle^2 - 300\langle x-a-1.5\rangle^2 + C_1\right]\ \text{N}\cdot\text{m}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - \frac{125x^4}{6} - 100\langle x-a\rangle^3 - 100\langle x-a-1.5\rangle^3 + C_1 x + C_2\right]\ \text{N}\cdot\text{m}^3$$

Boundary Conditions:



At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 5$ m, $v = 0$:

$$C_1 = R_A b^2/6\,EIv = \frac{-R_A(5)^2}{6} + \frac{125(5)^3}{6} + 20(5-a)^3 + 20(3.5-a)^3$$

$$v = \frac{1}{6EI}\Big[(2270-240a)(x^3-25x) + 125(125x - x^4) - 600\langle x-a\rangle^3 - 600\langle x-a-1.5\rangle^3 + 120(5-a)^3 x + 120(3.5-a)^3 x\Big]\ \text{N}\cdot\text{m}^3$$

(a) The deflection will be maximum at the middle of the span when the cart is at the middle of the bridge. Therefore:

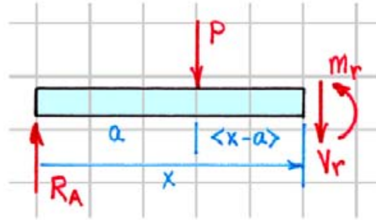
$x = 2.5$ m $\qquad a = 1.75$ m $\qquad R_A = R_D = 1850$ N

8-69

$L = 8\text{ ft} \qquad P = 75\text{ lb}$

For each cub scout, from overall equilibrium:

$$R_A = \frac{P(L-a)}{L}\text{ lb}\uparrow \qquad R_E = \left(\frac{Pa}{L}\right)\text{ lb}\uparrow$$



$$EIv'' = M_r = \left[R_A x - P\langle x-a\rangle^1\right]\text{ lb}\cdot\text{ft}$$

$$EIv' = \left[\frac{R_A x^2}{2} - \frac{P\langle x-a\rangle^2}{2} + C_1\right]\text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - \frac{P\langle x-a\rangle^3}{6} + C_1 x + C_2\right]\text{ lb}\cdot\text{ft}^3$$

Boundary Conditions: At $x = 0$, $v = 0$: $\qquad C_2 = 0$

At $x = L$, $v = 0$: $\qquad C_1 = \frac{-R_A L^2}{6} + \frac{P(L-a)^3}{6L}$

Therefore
$$6EILv = \left[R_A x^3 - P\langle x-a\rangle^3 - R_A L^2 x + P(L-a)^3 x\right]$$
$$= P\left[(L-a)x^3 - L\langle x-a\rangle^3 - L^2(L-a)x + (L-a)^3 x\right]\text{ lb}\cdot\text{ft}^3$$

$$I = \frac{(12)(1.5)^3}{12} = 3.375\text{ in.}^4$$

$$EI = \left(1.8\times10^6\right)(3.375) = 6.075\left(10^6\right)\text{ lb}\cdot\text{in.}^2$$

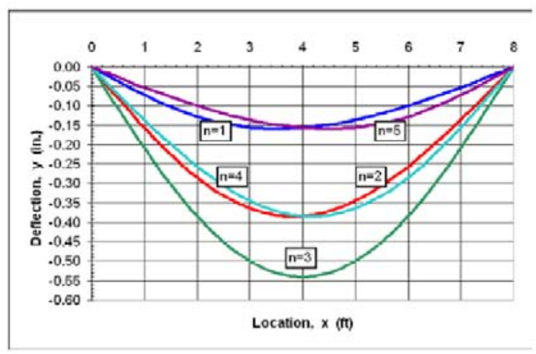
Case 1: $\quad v = v(P,2)$

Case 2: $\quad v = v(P,2) + v(P,4)$

Case 3: $\quad v = v(P,2) + v(P,4) + v(P,6)$

Case 4: $\quad v = v(P,4) + v(P,6)$

Case 5: $\quad v = v(P,6)$

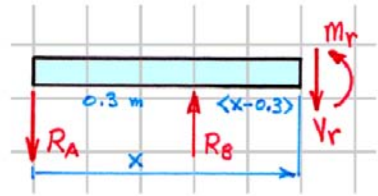


8-70

$$I = \frac{(30)(10)^3}{12} = 2500 \text{ mm}^4 \qquad EI = \left(12\times10^9\right)\left(2500\times10^{-12}\right) = 30.00 \text{ N}\cdot\text{m}^2$$

From overall equilibrium:

$$R_A = 233.333 \text{ N}\downarrow \qquad R_B = 333.333 \text{ N}\uparrow$$



(a) $$EIv'' = M_r = \left[-233.333x + 333.333\langle x-0.3\rangle^1\right] \text{ N}\cdot\text{m}$$

$$EIv' = \left[-116.667x^2 + 166.667\langle x-0.3\rangle^2 + C_1\right] \text{ N}\cdot\text{m}^2$$

$$EIv = \left[-38.8889x^3 + 55.5556\langle x-0.3\rangle^3 + C_1x + C_2\right] \text{ N}\cdot\text{m}^3$$

Boundary Conditions: At $x=0$, $v=0$: $C_2 = 0$

At $x = 0.3$ m, $v = 0$: $C_1 = 3.500 \text{ N}\cdot\text{m}^2$

Therefore

$$v = \left[-1.29630x^3 + 1.85185\langle x-0.3\rangle^3 + 0.116667x\right] \text{ m}$$

(b) Use $$EI\frac{d^2v/dx^2}{\left[1+(dv/dx)^2\right]^{3/2}} = M_r$$ and integrate numerically

Let $y_1 = v$ and $y_2 = v'$

Then $y_1' = v' = y_2$

and $$y_2' = v'' = \frac{M_r}{EI}\left[1+(dv/dx)^2\right]^{3/2} = \left(-7.77778x + 11.11111\langle x-0.3\rangle^1\right)\left[1+(dv/dx)^2\right]^{3/2}$$

Integration (iteration) scheme:

$$x^{(n+1)} = x^{(n)} + \Delta x$$

$$y_1^{(n+1)} = y_1^{(n)} + y_1'^{(n)}\Delta x = y_1^{(n)} + y_2^{(n)}\Delta x$$

$$y_2^{(n+1)} = y_2^{(n)} + y_2'^{(n)}\Delta x$$

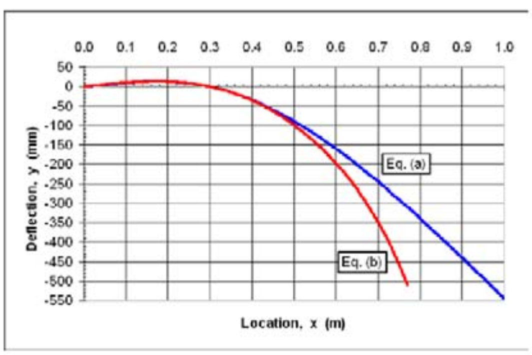
$$L^{(n+1)} = L^{(n)} + \sqrt{(\Delta x)^2 + (\Delta y_1)^2}$$

Use initial values:

$$x^{(0)} = L^{(0)} = 0$$

$$y_1^{(0)} = v(x=0) = 0$$

$$y_2^{(0)} = v'(x=0) = 0.1$$

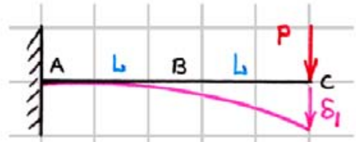


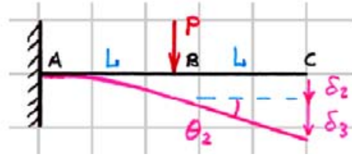
Adjust initial value of y_2 until $v = 0$ at $x = 0.30$ m $\qquad \left[y_2^{(0)} = v'(x=0) = 0.10613\right]$

Integrate until $L^{(n)} \cong 1.00$ m $\qquad$ [occurs about $x \cong 0.770$ m]

8-71*

Using the solution for case 1 in Table B-19:





$$v_1 = \frac{-Px^2}{6EI}\left[3(2L)-x\right] \qquad \delta_2 = \frac{-PL^3}{3EI}$$

$$v_1' = \frac{-P}{6EI}\left[12Lx-3x^2\right] \qquad \theta_2 = \frac{-PL^2}{2EI}$$

(a) $$\delta_B = v_{1B} + v_{2B} = v_{1,x=L} + \delta_2 = \frac{-PL^2}{6EI}\left[3(2L)-L\right] - \frac{PL^3}{3EI}$$

$$\delta_B = \frac{-7PL^3}{6EI} = \frac{7PL^3}{6EI}\downarrow$$ .... **Ans.**

$$\theta_B = v_{1B}' + v_{2B}' = v_{1,x=L}' + \theta_2 = \frac{-P}{6EI}\left[12L^2-3L^2\right] - \frac{PL^2}{2EI} = \frac{-2PL^2}{EI} = \frac{2PL^2}{EI}$$ ↻ .... **Ans.**

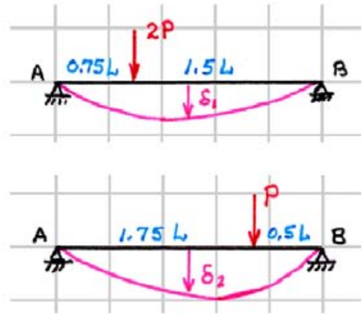
(b) $$\delta_C = v_{1C} + v_{2C} = \delta_1 + (\delta_2 + \theta_2 L) = \frac{-P(2L)^3}{3EI} + \left[\frac{-PL^3}{3EI} - \frac{PL^2}{2EI}(L)\right]$$

$$\delta_C = \frac{-21PL^3}{6EI} = \frac{7PL^3}{2EI}\downarrow$$ .... **Ans.**

$$\theta_C = v_{1C}' + v_{2C}' = \theta_1 + \theta_2 = \frac{-P(2L)^2}{2EI} - \frac{PL^2}{2EI} = \frac{-5PL^2}{2EI} = \frac{5PL^2}{2EI}$$ ↻ .... **Ans.**

8-72*

Using the solution for case 5 in Table B-19 with $P_1 = 2P$ and $P_2 = P$



$$\delta = v_{P1} + v_{P2}$$

$$= \frac{-2P(3L/4)\left[3(9L/4)^2 - 4(3L/4)^2\right]}{48EI}$$

$$- \frac{P(L/2)\left[3(9L/4)^2 - 4(L/2)^2\right]}{48EI}$$

$$= \frac{-53PL^3}{96EI}$$

$$\delta = \frac{-53(13{,}500)(3)^3}{96\left(200\times10^9\right)\left(80\times10^{-6}\right)} = -0.01258 \text{ m} = 12.58 \text{ mm} \downarrow$$ .... **Ans.**

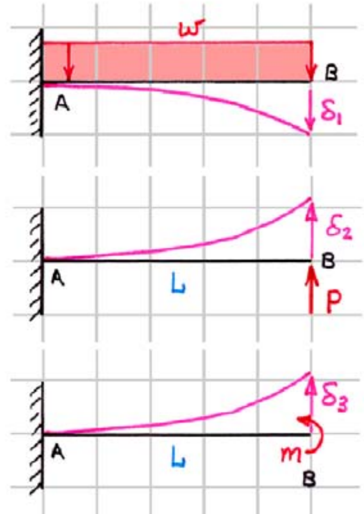
8-73

Using the solutions for cases 1, 2, and 4 in Table B-19 with $P = wL$ and $M = wL^2/2$

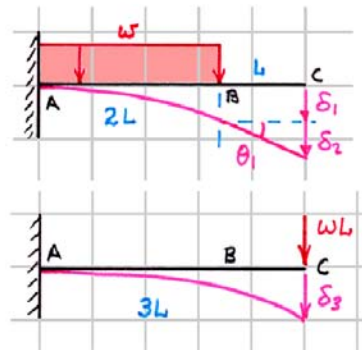
$$\delta_B = v_w + v_P + v_M$$

$$= \frac{-wL^4}{8EI} + \frac{(wL)(L)^3}{3EI} + \frac{(wL^2/2)(L)^2}{2EI}$$

$$\delta_B = \frac{+11wL^4}{24EI} = \frac{11wL^4}{24EI} \uparrow$$ .... **Ans.**



8-74*



Using the solutions for cases 1 and 2 in Table B-19 with $P = wL$

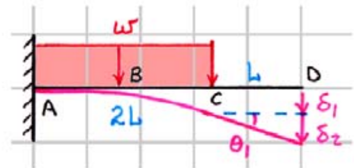
$$\delta_C = v_w + v_P = v_{Bw} + \theta_{Bw} L + v_{CP}$$

$$= \frac{-w(2L)^4}{8EI} - \frac{w(2L)^3}{6EI}(L) - \frac{(wL)(3L)^3}{3EI}$$

$$\delta_C = \frac{-37wL^4}{3EI} = \frac{37wL^4}{3EI} \downarrow$$ .... **Ans.**

8-75

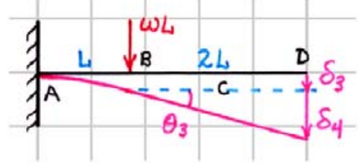
Using the solutions for cases 1 and 2 in Table B-19 with $P = wL$



$$\delta_D = v_w + v_P = \left[v_{Cw} + \theta_{Cw} L\right] + \left[v_{BP} + \theta_{BP}(2L)\right]$$

$$= \frac{-w(2L)^4}{8EI} - \frac{w(2L)^3}{6EI}(L)$$

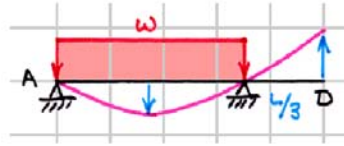
$$- \frac{(wL)(L)^3}{3EI} - \frac{(wL)(L)^2}{2EI}(2L)$$

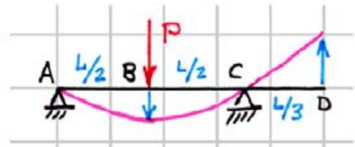


$$\delta_D = \frac{-14wL^4}{3EI} = \frac{14wL^4}{3EI} \downarrow$$ .... **Ans.**

8-76

Using the solutions for cases 6 and 7 in Table B-19 with $P = wL$





(a) $\delta_B = v_{Bw} + v_{BP}$

$$= \frac{-5wL^4}{384EI} - \frac{(wL)L^3}{48EI} = \frac{-13wL^4}{384EI} = \frac{13wL^4}{384EI}\downarrow$$ .... **Ans.**

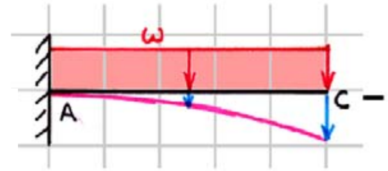
(b) $\delta_D = \theta_C (L/3) = [\theta_{Cw} + \theta_{CP}](L/3)$

$$= \left[\frac{wL^3}{24EI} + \frac{(wL)(L)^2}{16EI}\right]\left(\frac{L}{3}\right) = \frac{+5wL^4}{144EI} = \frac{5wL^4}{144EI}\uparrow$$ .... **Ans.**

8-77*

Using the solutions for cases 1 and 2 in Table B-19

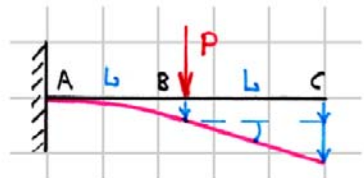
(a) $\delta_B = v_{Bw} + v_{BP}$



$$= \frac{-wL^2}{24EI}\left[L^2 - 4(2L)(L) + 6(2L)^2\right] - \frac{PL^3}{3EI}$$

$$\delta_B = \frac{-17wL^4}{24EI} - \frac{PL^3}{3EI} = \left[\frac{17wL^4}{24EI} + \frac{PL^3}{3EI}\right]\downarrow \text{ .... Ans.}$$

(b) $\delta_C = v_{Cw} + v_{CP} = v_{Cw} + \left[v_{BP} + \theta_{BP}L\right]$



$$= \frac{-w(2L)^4}{8EI} - \frac{PL^3}{3EI} - \left(\frac{PL^2}{2EI}\right)(L)$$

$$\delta_C = \frac{-2wL^4}{EI} - \frac{5PL^3}{6EI} = \left[\frac{2wL^4}{EI} + \frac{5PL^3}{6EI}\right]\downarrow \text{ .... Ans.}$$

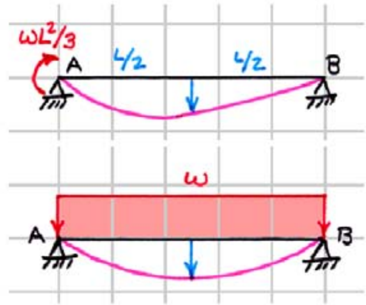
8-78

Using the solutions for cases 7 and 8 in Table B-19 with $M = wL^2/3$

$$\delta_M = v_M + v_w$$

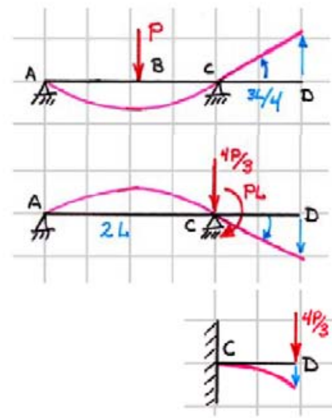
$$= \frac{-\left(wL^2/3\right)\left(L\right)^2}{16EI} - \frac{5wL^4}{384EI}$$

$$\delta_M = \frac{-13wL^4}{384EI} = \frac{13wL^4}{384EI} \downarrow$$ .... **Ans.**



8-79*

Using the solutions for cases 1, 6, and 8 in Table B-19 with $M = PL$

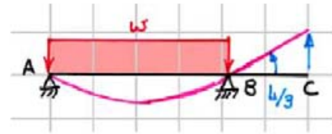


$$\delta_D = \nu_{DP} + \theta_D\left(3L/4\right) = \nu_{DP} + \left[\theta_{DP} + \theta_{DM}\right]\left(3L/4\right)$$

$$= \left[\frac{P(2L)^2}{16EI} - \frac{(PL)(2L)}{3EI}\right]\left(\frac{3L}{4}\right) - \frac{(4P/3)(3L/4)^3}{3EI}$$

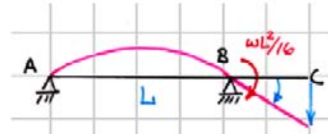
$$\delta_D = \frac{-PL^3}{2EI} = \frac{PL^3}{2EI}\downarrow$$ .... **Ans.**

8-80*



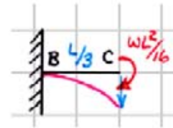
Using the solutions for cases 4, 7, and 8 in Table B-19 with $M = -wL^2/16$

$$\delta_C = v_{CM} + \theta_B\left(L/3\right) = v_{CM} + \left[\theta_{Bw} + \theta_{BM}\right]\left(L/3\right)$$



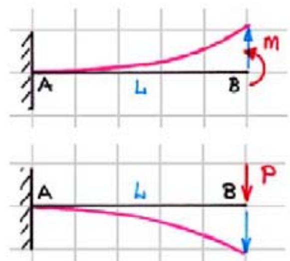
$$= \frac{-\left(wL^2/16\right)\left(L/3\right)^2}{2EI} + \left[\frac{wL^3}{24EI} - \frac{\left(wL^2/16\right)\left(L\right)}{3EI}\right]\left(\frac{L}{3}\right)$$

$$\delta_C = \frac{-wL^4}{288EI} + \frac{wL^4}{144EI} = \frac{wL^4}{288EI} = \frac{wL^4}{288EI}\uparrow$$ .... **Ans.**



8-81

Using the solutions for cases 1 and 4 in Table B-19 with $P = W$ and $M = WL/2$



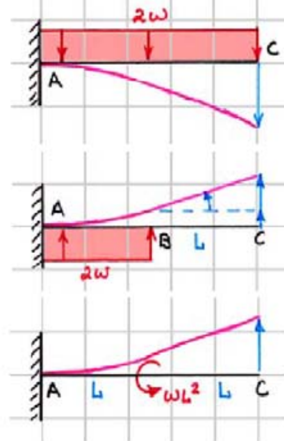
$$\delta_B = v_{BM} + v_{BP} = \frac{(WL/2)(L)^2}{2EI} - \frac{WL^3}{3EI} = \frac{-WL^3}{12EI} = \frac{WL^3}{12EI} \downarrow$$

$$\theta_B = \theta_{BM} + \theta_{BP} = \frac{(WL/2)(L)}{EI} - \frac{WL^2}{2EI} = 0$$

Since the slope of the beam AB is zero at B, the arm BC does not rotate and the vertical movement of point C is the same as the vertical movement of point B.

$$\delta_C = \delta_B = \frac{-WL^3}{12EI} = \frac{-(5)(2)^3}{12(100)} = -0.0333 \text{ in.} = 0.0333 \text{ in.} \downarrow$$ .... **Ans.**

8-82*



Using the solutions for cases 2 and 4 in Table B-19 with $M = wL^2$

$$\delta_C = v_{Cwt} + v_{Bwb} + v_{BM} + \left[\theta_{Bwb} + \theta_{BM}\right](L)$$

$$= \frac{-(2w)(2L)^4}{8EI} + \frac{(2w)(L)^4}{8EI} + \frac{(wL^2)(L)^2}{2EI} + \left[\frac{(2w)(L)^3}{6EI} + \frac{(wL^2)(L)}{EI}\right](L)$$

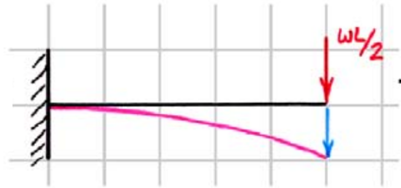
$$= \frac{-4wL^4}{EI} + \frac{wL^4}{4EI} + \frac{wL^4}{2EI} + \frac{wL^4}{3EI} + \frac{wL^4}{EI} = \frac{-23wL^4}{12EI}$$

$$\delta_C = \frac{-23(7000)(1.8)^4}{12(200\times10^9)(130\times10^{-6})} = -0.00542\text{ m} = 5.42\text{ mm}\downarrow$$ .... **Ans.**

8-83
Using the solutions for cases 1 and 2 in Table B-19 with $P = wL/2$

$$\delta_B = v_w + v_P = \left[v_{Bw} + \theta_{Bw}(3L/4)\right] + v_P$$

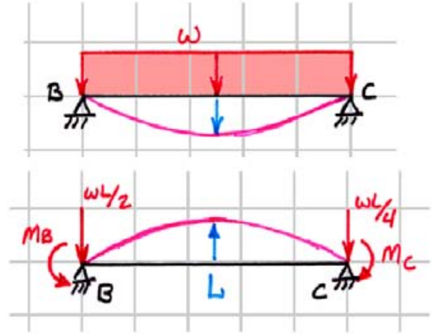
$$= \left[\frac{-wL^4}{8EI} - \frac{wL^3}{6EI}\right]\left(\frac{3L}{4}\right) - \frac{(wL/2)(7L/4)^3}{3EI}$$



$$\delta_C = \frac{-439wL^4}{384EI} = \frac{439wL^4}{384EI}\downarrow$$ .... **Ans.**

8-84

Using the solutions for cases 7 and 8 in Table B-19 with $M_B = wL^2/8$ and $M_C = wL^2/4$



$$\delta_M = v_w + v_{M_B} + v_{M_C}$$

$$= \frac{-5wL^4}{384EI} + \frac{(wL^2/8)(L)^2}{16EI} + \frac{(wL^2/4)(L)^2}{16EI}$$

$$\delta_M = \frac{+wL^4}{96EI} = \frac{wL^4}{96EI} \uparrow$$ .... **Ans.**

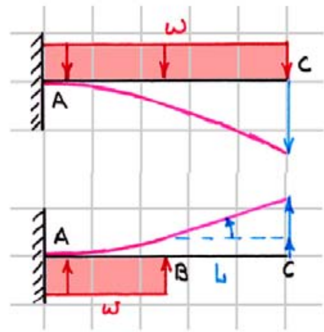
8-85*

Using the solution for case 2 in Table B-19

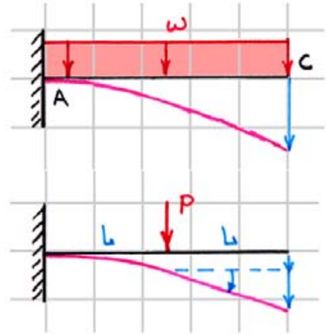
$$\delta_C = v_{wt} + v_{wb} = v_{Cwt} + \left[v_{Bwb} + \theta_{Bwb} L\right]$$

$$= \frac{-w(2L)^4}{8EI} + \left[\frac{wL^4}{8EI} + \left(\frac{wL^3}{6EI}\right)(L)\right]$$

$$\delta_C = \frac{-41wL^4}{24EI} = \frac{41wL^4}{24EI} \downarrow$$ .... **Ans.**



8-86



Using the solutions for cases 1 and 2 in Table B-19 with $P = wL/2$

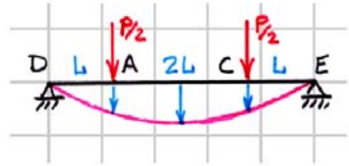
$$\delta_C = v_w + v_P = v_{Cw} + \left[v_{BP} + \theta_{BP} L\right]$$

$$= \frac{-w(2L)^4}{8EI} - \frac{(wL/2)L^3}{3EI} - \frac{(wL/2)L^2}{2EI}(L)$$

$$\delta_C = \frac{-29wL^4}{12EI} = \frac{29wL^4}{12EI} \downarrow$$ .... **Ans.**

8-87*

Using the solutions for cases 5 and 6 in Table B-19

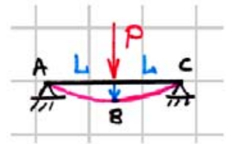


$$\delta_B = \delta_{A/D} + \delta_{B/A}$$

$$\delta_{A/D} = v_{A,P1} + v_{A,P2}$$

$$= \frac{-(P/2)(L)^2(3L)^2}{3EI(4L)} - \frac{(P/2)(L)(L)}{6EI(4L)}\left[(4L)^2 - (L)^2 - \right](L)^2$$

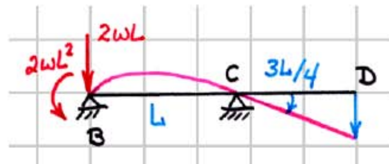
$$= \frac{-2PL^3}{3EI}$$



$$\delta_B = \frac{-2PL^3}{3EI} - \frac{P(2L)^3}{48EI} = \frac{-5PL^3}{6EI} = \frac{5PL^3}{6EI}\downarrow$$ .... **Ans.**

8-88*

Using the solution for case 8 in Table B-19 with $M = 2wL^2$

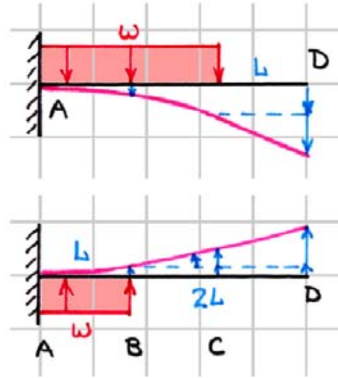


$$\delta_C = \theta_C\left(3L/4\right) = \frac{-\left(2wL^2\right)L}{6EI}\left(3L/4\right)$$

$$\delta_C = \frac{-wL^4}{4EI} = \frac{wL^4}{4EI}\downarrow$$ .... **Ans.**

8-89

Using the solution for case 2 in Table B-19



$$\delta = v_{wt} + v_{wb}$$

(a) $$\delta_B = \frac{-w(L)^2}{24EI}\left[(L)^2 - 4(2L)(L) + 6(2L)^2\right] + \frac{wL^4}{8EI}$$

$$\delta_B = \frac{-7wL^4}{12EI} = \frac{7wL^4}{12EI} \downarrow$$ .... **Ans.**

(b) $$\delta_C = \frac{-w(2L)^4}{8EI} + \left[\frac{wL^4}{8EI} + \frac{wL^3}{6EI}(L)\right]$$

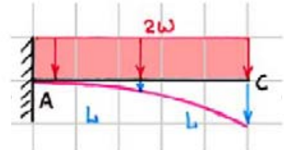
$$\delta_C = \frac{-41wL^4}{24EI} = \frac{41wL^4}{24EI} \downarrow$$ .... **Ans.**

(c) $$\delta_C = \left[\frac{-w(2L)^4}{8EI} - \frac{w(2L)^3}{6EI}(L)\right] + \left[\frac{wL^4}{8EI} + \frac{wL^3}{6EI}(2L)\right]$$

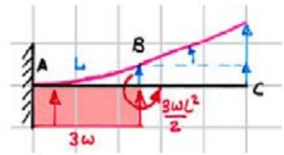
$$\delta_C = \frac{-69wL^4}{24EI} = \frac{23wL^4}{8EI} \downarrow$$ .... **Ans.**

8-90*

Using the solutions for cases 2 and 4 in Table B-19 with $M = 3wL^2/2$



$$\delta = v_{wt} + v_{wb} + v_M$$



(a) $$\delta_B = \frac{-(2w)(L)^2}{24EI}\left[(L)^2 - 4(2L)(L) + 6(2L)^2\right] + \frac{(3w)L^4}{8EI} + \frac{(3wL^2/2)L^2}{2EI} = \frac{-7wL^4}{24EI}$$

$$\delta_B = \frac{-7(7500)(3)^4}{24(200\times10^9)(180\times10^{-6})} = -0.00492 \text{ m} = 4.92 \text{ mm}\downarrow \text{ .... Ans.}$$

(b) $$\delta_C = \frac{-(2w)(2L)^4}{8EI} + \left[\frac{(3w)L^4}{8EI} + \frac{(3w)L^3}{6EI}(L)\right] + \left[\frac{(3wL^2/2)L^2}{2EI} + \frac{(3wL^2/2)L}{EI}(L)\right] = \frac{-7wL^4}{8EI}$$

$$\delta_C = \frac{-7(7500)(3)^4}{8(200\times10^9)(180\times10^{-6})} = -0.01477 \text{ m} = 14.77 \text{ mm}\downarrow \text{ .... Ans.}$$

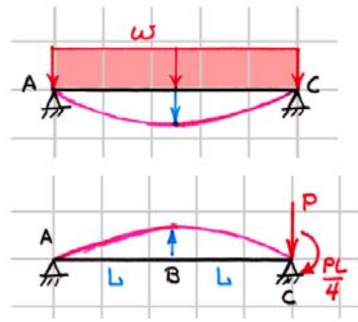
8-91

Using the solutions for cases 7 and 8 in Table B-19 with $M = PL/4$

$$\delta_M = v_w + v_M$$

$$= \frac{-5w(2L)^4}{384EI} + \frac{(PL/4)(2L)^2}{16EI}$$

$$\delta_M = \frac{-80wL^4 + 24PL^3}{384EI}$$ .... **Ans.**



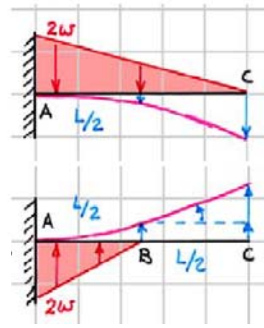
8-92

Using the solution for case 3 in Table B-19

$$\delta = v_{wt} + v_{wb}$$

(a) $$\delta_B = \frac{-(2w)(L/2)^2}{120EI(L/2)}\left[10(L)^3 - 10(L)^2\left(\frac{L}{2}\right) + 5(L)\left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^3\right] + \frac{(2w)(L/2)^4}{30EI}$$

$$\delta_B = \frac{-41wL^4}{1920EI} = \frac{41wL^4}{1920EI}\downarrow$$ .... **Ans.**

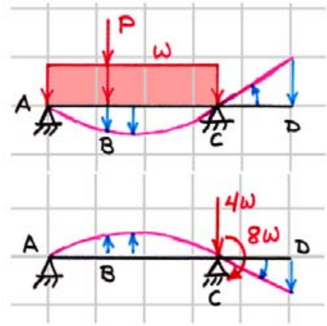


(b) $$\delta_C = \frac{-(2w)(L)^4}{30EI} + \left[\frac{(2w)(L/2)^4}{30EI} + \frac{(2w)(L/2)^3}{24EI}\left(\frac{L}{2}\right)\right]$$

$$\delta_C = \frac{-11wL^4}{192EI} = \frac{11wL^4}{192EI}\downarrow$$ .... **Ans.**

8-93*

Using the solutions for cases 2, 5, 7, and 8 in Table B-19 with $M = 8w = 3200\ \text{lb}\cdot\text{ft}$



(a) $\delta_M = v_w + v_M + v_P$

$$= \frac{-5(400/12)(120)^4}{384(350\times10^6)} + \frac{(3200\times12)(120)^2}{16(350\times10^6)} - \frac{(2000)(36)\left[3(120)^2 - 4(36)^2\right]}{48(350\times10^6)}$$

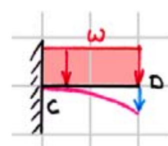
$\delta_M = -0.321\text{ in.} = 0.321\text{ in.}\downarrow$ .... **Ans.**

(b) $\delta_D = \theta_C L + v_{Dw} = \left(\theta_{Cw} + \theta_{CM} + \theta_{CP}\right) L + v_{Dw}$

$$\delta_D = \left[\frac{(400/12)(120)^3}{24(350\times10^6)} - \frac{(3200\times12)(120)}{3(350\times10^6)} + \frac{(2000)(36)\left[120^2 - 36^2\right]}{6(350\times10^6)(120)}\right](48) - \frac{(400/12)(48)^4}{8(350\times10^6)}$$

$\delta_D = -0.235\text{ in.} = 0.235\text{ in.}\downarrow$ .... **Ans.**

(c) $\delta_B = v_w + v_M + v_P$

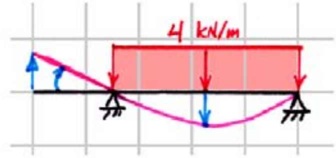


$$= \frac{-(400/12)(36)}{24(350\times10^6)}\left[(36)^3 - 2(120)(36)^2 + (120)^3\right] + \frac{(3200\times12)(36)}{6(350\times10^6)(120)}\left[(120)^2 - (36)^2\right] - \frac{(2000)(84)^2(36)^2}{3(350\times10^6)(120)}$$

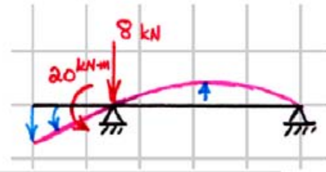
$\delta_B = -0.283\text{ in.} = 0.283\text{ in.}\downarrow$ .... **Ans.**

8-94

$$EI = (200\times10^9)(90\times10^{-6}) = 18(10^6)\ \text{N}\cdot\text{m}^2$$



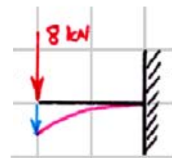
Using the solutions for cases 1, 7, and 8 in Table B-19 with $M = 20\ \text{kN}\cdot\text{m}$



(a) $\delta_M = v_w + v_M$

$$= \frac{-5(4000)(5)^4}{384(18\times10^6)} + \frac{(20,000)(5)^2}{16(18\times10^6)}$$

$\delta_M = -72.3(10^{-6})\ \text{m} = 0.0723\ \text{mm}\downarrow$ .... **Ans.**



(b) $\delta_A = \theta_A L + v_{AP} = (\theta_{Aw} + \theta_{AM})L + v_{AP}$

$$\delta_A = \left[\frac{(4000)(5)^3}{24(18\times10^6)} - \frac{(20,000)(5)}{3(18\times10^6)}\right](2.5) - \frac{(8000)(2.5)^3}{3(18\times10^6)}$$

$\delta_A = -0.00405\ \text{m} = 4.05\ \text{mm}\downarrow$ .... **Ans.**

(c) $\delta_2 = v_w + v_M$

$$= \frac{-(4000)(2)}{24(18\times10^6)}\left[(2)^3 - 2(5)(2)^2 + (5)^3\right] + \frac{(20,000)(3)}{6(18\times10^6)(5)}\left[(5)^2 - (3)^2\right]$$

$\delta_2 = +55.6(10^{-6})\ \text{m} = 0.0556\ \text{mm}\uparrow$ .... **Ans.**

8-95*

$$I = \frac{\pi d^4}{64} = \frac{\pi (4)^4}{64} = 4\pi \text{ in.}^4 \qquad Q_{NA} = \frac{4(d/2)}{3\pi}\left[\frac{\pi d^2}{8}\right] = \frac{d^3}{12}$$

$$dv = \frac{VQ}{ItG}dx = \frac{-P(d^3/12)\,dx}{(\pi d^4/64)(d)(G)} = \frac{-16P\,dx}{3\pi d^2 G}$$

$$v_s = \int_0^L dv = \int_0^L \frac{-16P\,dx}{3\pi d^2 G} = \frac{-16PL}{3\pi d^2 G} = \frac{-16(1200)(4\times 12)}{3\pi (4)^2 (11\times 10^6)} = -0.0005556 \text{ in.}$$

$$v_f = \frac{-PL^3}{3EI} = \frac{-(1200)(4\times 12)^3}{3(29\times 10^6)(4\pi)} = -0.12139 \text{ in.}$$

$$Increase = \frac{0.0005556}{0.12139} = 0.00458 = 0.458\%$$ .... **Ans.**

8-96*

For a rectangular cross section:

$$v_s = \frac{-3wL^2}{4AG} = \frac{-3(5000)(1.5)^2}{4(0.050 \times 0.100)(28 \times 10^9)} = -0.06027(10^{-3})\ \text{m}$$

$$I = \frac{(50)(100)^3}{12} = 4.167(10^6)\ \text{mm}^4$$

$$v_f = \frac{-wL^4}{8EI} = \frac{-(5000)(1.5)^4}{8(73 \times 10^9)(4.167 \times 10^{-6})} = -10.402(10^{-3})\ \text{m}$$

$$Increase = \frac{0.06027}{10.402} = 0.00579 = 0.579\%$$ .... **Ans.**

8-97

$$I = \frac{(3)(5)^3}{12} - \frac{(2)(4)^3}{12} = 20.58 \text{ in.}^4$$

$$Q_{NA} = \left[2.25(3\times 0.5)\right] + 2\left[1(2\times 0.5)\right] = 5.375 \text{ in.}^3$$

$$v_s = \int_0^{L/2} dv = \int_0^{L/2} \frac{-VQ\,dx}{ItG} = \frac{-VQ(L/2)}{ItG} = \frac{-(2000)(5.375)(4\times 12)}{(20.58)(1)(11\times 10^6)} = -0.002279 \text{ in.}$$

$$v_f = \frac{-PL^3}{48EI} = \frac{-(4000)(8\times 12)^3}{48(29\times 10^6)(20.58)} = -0.12353 \text{ in.}$$

$$Increase = \frac{0.002279}{0.12353} = 0.01845 = 1.845\%$$ .... **Ans.**

8-98

For a W 203×60 wide-flange section: $I = 60.8(10^6)\ \text{mm}^4$

$d = 2c = 210$ mm $\quad w_f = 205$ mm $\quad t_f = 14.2$ mm $\quad t_w = 9.1$ mm

$$Q_{NA} = [97.9(205 \times 14.2)] + [45.4(90.8 \times 9.1)] = 322.5(10^3)\ \text{mm}^3$$

$$v_s = \int_0^{L/2} dv = \int_0^{L/2} \frac{-VQ\,dx}{ItG} = \frac{-Q}{ItG}\int_0^{L/2}\left[\frac{wL}{2} - wx\right]dx$$

$$= \frac{-wL^2Q}{8ItG} = \frac{-(20{,}000)(4)^2(322.5 \times 10^{-6})}{8(60.8 \times 10^{-6})(0.0091)(76 \times 10^9)} = -0.0003068\ \text{m}$$

$$v_f = \frac{-5wL^4}{384EI} = \frac{-5(20{,}000)(4)^3}{384(200 \times 10^9)(60.8 \times 10^{-6})} = -0.005482\ \text{mm}$$

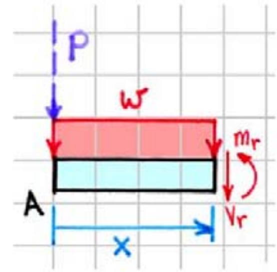
$$Increase = \frac{0.0003068}{0.005482} = 0.0560 = 5.60\%$$ .... **Ans.**

8-99*

$$M_r = \frac{-wx^2}{2} - Px \qquad \frac{\partial M_r}{\partial P} = -x$$

$$\delta_A = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial P}dx = \frac{1}{EI}\int_0^L \left(\frac{-wx^2}{2}\right)(-x)\,dx$$

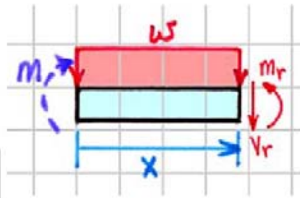
$$\delta_A = \left[\frac{wx^4}{8EI}\right]_0^L = \frac{+wL^4}{8EI} = \frac{wL^4}{8EI}\downarrow$$ ....**Ans.**



$$M_r = M - \frac{wx^2}{2} \qquad \frac{\partial M_r}{\partial M} = 1$$

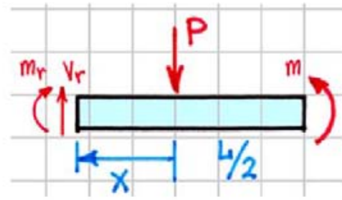
$$\theta_A = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial M}dx = \frac{1}{EI}\int_0^L \left(\frac{-wx^2}{2}\right)(1)\,dx$$

$$\theta_A = \left[\frac{-wx^3}{6EI}\right]_0^L = \frac{-wL^3}{6EI} = \frac{wL^3}{6EI}$$ ↻ ....**Ans.**



8-100*

$$M_r = M - Px \qquad \frac{\partial M_r}{\partial P} = -x$$



$$\delta_P = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial P} dx = \frac{1}{EI}\int_0^L (M - Px)(-x)\, dx$$

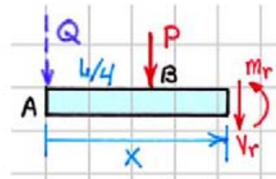
$$= \left[\frac{-Mx^2}{2EI} + \frac{Px^3}{3EI}\right]_0^L = \frac{-ML^2}{2EI} + \frac{PL^3}{3EI}$$

With $M = PL$ $\qquad \delta_P = \frac{-(PL)L^2}{2EI} + \frac{PL^3}{3EI} = \frac{-PL^3}{6EI} = \frac{PL^3}{6EI}\uparrow$ .... **Ans.**

8-101

$$M_{r1} = -Qx \qquad \frac{\partial M_{r1}}{\partial Q} = -x$$

$$M_{r2} = -Qx - P\left(x - \frac{L}{4}\right) \qquad \frac{\partial M_{r2}}{\partial Q} = -x$$



With $Q = 0$:

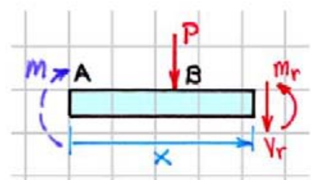
$$\delta_A = \frac{1}{EI}\int_0^{L/4} M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_{L/4}^{L} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx = \frac{1}{EI}\int_{L/4}^{L}\left(Px^2 - \frac{PLx}{4}\right)dx$$

$$\delta_A = \frac{1}{EI}\left[\frac{Px^3}{3} - \frac{PLx^2}{8}\right]_{L/4}^{L} = \frac{1}{EI}\left[\frac{PL^3}{3} - \frac{PL^3}{192} - \frac{PL^3}{8} + \frac{PL^3}{128}\right]$$

$$\delta_A = \frac{+27PL^3}{128EI} = \frac{27PL^3}{128EI}\downarrow$$ .... **Ans.**

$$M_{r1} = M \qquad \frac{\partial M_{r1}}{\partial M} = 1$$

$$M_{r2} = M - P\left(x - \frac{L}{4}\right) \qquad \frac{\partial M_{r2}}{\partial M} = 1$$



With $M = 0$:

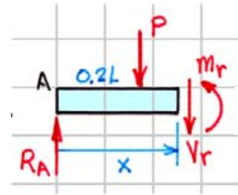
$$\theta_A = \frac{1}{EI}\int_0^{L/4} M_{r1}\frac{\partial M_{r1}}{\partial M}dx + \frac{1}{EI}\int_{L/4}^{L} M_{r2}\frac{\partial M_{r2}}{\partial M}dx = \frac{1}{EI}\int_{L/4}^{L}\left(-Px + \frac{PL}{4}\right)dx$$

$$= \frac{1}{EI}\left[\frac{-Px^2}{2} + \frac{PLx}{4}\right]_{L/4}^{L} = \frac{1}{EI}\left[\frac{-PL^2}{2} + \frac{PL^2}{32} + \frac{PL^2}{4} - \frac{PL^2}{16}\right]$$

$$\theta_A = \frac{-9PL^2}{32EI} = \frac{9PL^2}{32EI}$$ ↺ .... **Ans.**

8-102*

From overall equilibrium $R_A = 0.8P\uparrow$ $R_B = 0.2P\uparrow$



$$M_{r1} = R_A x = 0.8Px \qquad \frac{\partial M_{r1}}{\partial P} = 0.8x$$

$$M_{r2} = R_A x - P(x - 0.2L) = -0.2P(x - L) \qquad \frac{\partial M_{r2}}{\partial P} = -0.2(x - L)$$

$$\delta_P = \frac{1}{EI}\int_0^{0.2L} M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_{0.2L}^{L} M_{r2}\frac{\partial M_{r2}}{\partial P}dx$$

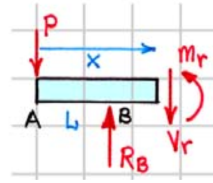
$$= \frac{1}{EI}\int_0^{0.2L}\left(\frac{4Px}{5}\right)\left(\frac{4x}{5}\right)dx + \frac{1}{EI}\int_{0.2L}^{L}\left[\frac{P(x-L)}{5}\right]\left[\frac{(x-L)}{5}\right]dx$$

$$= \frac{16P}{25EI}\left[\frac{x^3}{3}\right]_0^{L/5} + \frac{P}{25EI}\left[\frac{(x-L)^3}{3}\right]_{L/5}^{L} = \frac{16P}{75EI}\left(\frac{L^3}{125}\right) + \frac{P}{75EI}\left(\frac{64L^3}{125}\right)$$

$$\delta_P = \frac{+80PL^3}{9375EI} = \frac{16PL^3}{1875EI}\downarrow$$ .... **Ans.**

8-103

(a) From overall equilibrium $R_B = 3P/2 \uparrow$ $R_C = P/2 \downarrow$



$$M_{r1} = -Px \qquad \partial M_{r1}/\partial P = -x$$

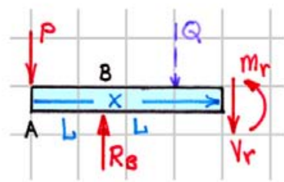
$$M_{r2} = -Px + \frac{3P}{2}(x-L) = \frac{P}{2}(x-3L) \qquad \frac{\partial M_{r2}}{\partial P} = \frac{x-3L}{2}$$

$$\delta_A = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_L^{3L} M_{r2}\frac{\partial M_{r2}}{\partial P}dx$$

$$= \frac{1}{EI}\int_0^L (Px^2)dx + \frac{1}{EI}\int_L^{3L}\frac{P}{4}(x-3L)^2 dx = \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^L + \frac{P}{4EI}\left[\frac{(x-3L)^3}{3}\right]_L^{3L}$$

$$\delta_A = \frac{+12PL^3}{12EI} = \frac{PL^3}{EI}\downarrow$$ .... **Ans.**

(b) From overall equilibrium $R_B = \frac{3P}{2} + \frac{Q}{2}\uparrow$ $R_C = \frac{P}{2} - \frac{Q}{2}\downarrow$



$$M_{r1} = -Px \qquad \partial M_{r1}/\partial Q = 0$$

$$M_{r2} = \frac{P}{2}(x-3L) + \frac{Q}{2}(x-L) \qquad \frac{\partial M_{r2}}{\partial Q} = \frac{x-L}{2}$$

$$M_{r3} = \frac{P}{2}(x-3L) - \frac{Q}{2}(x-3L) \qquad \frac{\partial M_{r3}}{\partial Q} = \frac{-(x-3L)}{2}$$

With $Q = 0$:

$$\delta_M = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_L^{2L} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx + \frac{1}{EI}\int_{2L}^{3L} M_{r3}\frac{\partial M_{r3}}{\partial Q}dx$$

$$= 0 + \frac{1}{EI}\int_L^{2L}\frac{P}{4}(x^2 - 4Lx + 3L^2)dx - \frac{1}{EI}\int_{2L}^{3L}\frac{P}{4}(x-3L)^2 dx$$

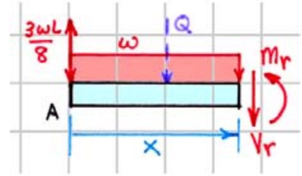
$$= 0 + \frac{P}{4EI}\left[\frac{x^3}{3} - 2Lx^2 + 3L^2x\right]_L^{2L} - \frac{P}{4EI}\left[\frac{(x-3L)^3}{3}\right]_{2L}^{3L}$$

$$\delta_M = \frac{-3PL^3}{12EI} = \frac{PL^3}{4EI}\uparrow$$ .... **Ans.**

8-104*

$$M_{r1} = \frac{3wLx}{8} - \frac{wx^2}{2} \qquad \frac{\partial M_{r1}}{\partial Q} = 0$$

$$M_{r2} = \frac{3wLx}{8} - \frac{wx^2}{2} - Q\left(x - \frac{L}{2}\right) \qquad \frac{\partial M_{r2}}{\partial Q} = -\left(x - \frac{L}{2}\right)$$



With $Q = 0$:

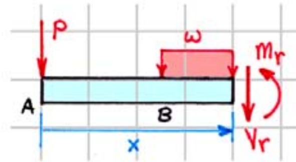
$$\delta_P = \frac{1}{EI}\int_0^{L/2} M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_{L/2}^{L} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx = 0 + \frac{1}{EI}\int_{L/2}^{L}\left[\frac{3wLx}{8} - \frac{wx^2}{2}\right]\left[-x + \frac{L}{2}\right]dx$$

$$= \frac{P}{EI}\left[\frac{-3wLx^3}{24} + \frac{3wL^2x^2}{32} + \frac{wx^4}{8} - \frac{wLx^3}{12}\right]_0^{L/2}$$

$$\delta_M = \frac{+wL^4}{192EI} = \frac{wL^4}{192EI}\downarrow$$ .... **Ans.**

8-105*

$$M_{r1} = -Px \qquad \frac{\partial M_{r1}}{\partial P} = -x$$



$$M_{r2} = -Px - \frac{w}{2}\left(x - \frac{3L}{2}\right)^2 \qquad \frac{\partial M_{r2}}{\partial P} = -x$$

$$\delta_A = \frac{1}{EI}\int_0^{3L/2} M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_{3L/2}^{5L/2} M_{r2}\frac{\partial M_{r2}}{\partial P}dx$$

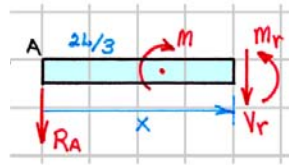
$$= \frac{1}{EI}\int_0^{3L/2}(-Px)(-x)\,dx + \frac{1}{EI}\int_{3L/2}^{5L/2}\left[-Px(-x) - \frac{w}{2}\left(x - \frac{3L}{2}\right)^2(-x)\right]dx$$

$$= \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^{3L/2} - \frac{1}{EI}\left[\frac{Px^3}{3} + \frac{wx^4}{8} - \frac{wLx^3}{2} + \frac{9L^2x^2}{16}\right]_{3L/2}^{5L/2} = \frac{125PL^3}{24EI} + \frac{3wL^4}{8EI}$$

With $P = \dfrac{4wL}{25}$ $\qquad \delta_A = \dfrac{+29PL^3}{24EI} = \dfrac{29PL^3}{24EI}\downarrow$ .... **Ans.**

8-106

From overall equilibrium $R_A = M/L \downarrow$ $R_B = M/L \uparrow$



$$M_{r1} = -R_A x = \frac{-Mx}{L} \qquad \frac{\partial M_{r1}}{\partial M} = \frac{-x}{L}$$

$$M_{r2} = -R_A x + M = \frac{-Mx}{L} + M \qquad \frac{\partial M_{r2}}{\partial M} = \frac{-(x-L)}{L}$$

$$\theta_M = \frac{1}{EI}\int_0^{2L/3} M_{r1}\frac{\partial M_{r1}}{\partial M}dx + \frac{1}{EI}\int_{2L/3}^{L} M_{r2}\frac{\partial M_{r2}}{\partial M}dx$$

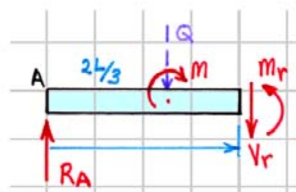
$$= \frac{1}{EI}\int_0^{2L/3}\left(\frac{-Mx}{L}\right)\left(\frac{-x}{L}\right)dx + \frac{1}{EI}\int_{2L/3}^{L}\left[\frac{-M(x-L)}{L}\right]\left[\frac{-(x-L)}{L}\right]dx$$

$$= \frac{M}{EIL^2}\left[\frac{x^3}{3}\right]_0^{2L/3} + \frac{M}{EIL^2}\left[\frac{(x-L)^3}{3}\right]_{2L/3}^{L} = \frac{M}{3EIL^2}\left(\frac{8L^3}{27}\right) + \frac{M}{3EIL^2}\left(\frac{L^3}{27}\right)$$

$$\theta_M = \frac{+ML}{9EI} = \frac{ML}{9EI}$$ ↺ .... **Ans.**

From overall equilibrium $R_A = \left(\frac{Q}{3} - \frac{M}{L}\right)\uparrow$ $R_B = \left(\frac{2Q}{3} + \frac{M}{L}\right)\uparrow$

$$M_{r1} = -R_A x = \frac{-Mx}{L} + \frac{Px}{3} \qquad \frac{\partial M_{r1}}{\partial Q} = \frac{x}{3}$$



$$M_{r2} = R_A x + M - Q\left(x - \frac{2L}{3}\right)$$

$$= \frac{-M(x-L)}{L} - \frac{2Q(x-L)}{3} \qquad \frac{\partial M_{r2}}{\partial Q} = \frac{-2(x-L)}{3}$$

With $Q = 0$

$$\delta_M = \frac{1}{EI}\int_0^{2L/3} M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_{2L/3}^{L} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx$$

$$= \frac{1}{EI}\int_0^{2L/3}\left(\frac{-Mx}{L}\right)\left(\frac{x}{3}\right)dx + \frac{1}{EI}\int_{2L/3}^{L}\left[\frac{-M(x-L)}{L}\right]\left[\frac{-2(x-L)}{3}\right]dx$$

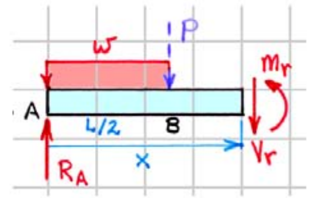
$$= \frac{-M}{3EIL}\left[\frac{x^3}{3}\right]_0^{2L/3} + \frac{2M}{3EIL}\left[\frac{(x-L)^3}{3}\right]_{2L/3}^{L} = \frac{-M}{9EIL}\left(\frac{8L^3}{27}\right) + \frac{2M}{9EIL}\left(\frac{L^3}{27}\right)$$

$$\delta_M = \frac{-2ML^2}{81EI} = \frac{2ML^2}{81EI}\uparrow$$ .... **Ans.**

8-107*

From overall equilibrium $R_A = \left(\frac{P}{2} + \frac{3wL}{8}\right)\uparrow$ $R_C = \left(\frac{P}{2} + \frac{wL}{8}\right)\uparrow$

$$M_{r1} = R_A x - \frac{wx^2}{2} = \frac{Px}{2} + \frac{3wLx}{8} - \frac{wx^2}{2} \qquad \frac{\partial M_{r1}}{\partial P} = \frac{x}{2}$$



$$M_{r2} = R_A x - \frac{wx^2}{2} - P\left(x - \frac{L}{2}\right)$$

$$= \frac{-P(x-L)}{2} + \frac{3wLx}{8} - \frac{wx^2}{2} \qquad \frac{\partial M_{r2}}{\partial P} = -x$$

With $P = 0$

$$\delta_B = \frac{1}{EI}\int_0^{L/2} M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_{L/2}^{L} M_{r2}\frac{\partial M_{r2}}{\partial P}dx$$

$$= \frac{1}{EI}\int_0^{L/2}\left(\frac{3wLx}{8} - \frac{wx^2}{2}\right)\left(\frac{x}{2}\right)dx + \frac{1}{EI}\int_{L/2}^{L}\left[\frac{-wL(x-L)}{8}\right]\left[\frac{-(x-L)}{2}\right]dx$$

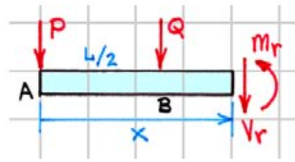
$$= \frac{w}{EI}\left[\frac{3Lx^3}{48} - \frac{x^4}{16}\right]_0^{L/2} + \frac{wL}{16EI}\left[\frac{(x-L)^3}{3}\right]_{L/2}^{L} = \frac{3wL^4}{768EI} + \frac{wL^4}{384EI}$$

$$\delta_B = \frac{+5wL^4}{768EI} = \frac{5wL^4}{768EI}\downarrow$$ .... **Ans.**

8-108

$$M_{r1} = -Px$$

$$\partial M_{r1}/\partial P = -x \qquad \partial M_{r1}/\partial Q = 0$$



$$M_{r2} = -Px - Q\left(x - \frac{L}{2}\right)$$

$$\frac{\partial M_{r2}}{\partial P} = -x \qquad \frac{\partial M_{r2}}{\partial Q} = -\left(x - \frac{L}{2}\right)$$

Deflection at A:

$$\delta_A = \frac{1}{EI}\int_0^{L/2} M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_{L/2}^{L} M_{r2}\frac{\partial M_{r2}}{\partial P}dx$$

$$= \frac{1}{EI}\int_0^{L/2}(-Px)(-x)\,dx + \frac{1}{EI}\int_{L/2}^{L}\left[-Px(-x) - Q\left(x - \frac{L}{2}\right)(-x)\right]dx$$

$$= \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^{L/2} + \frac{1}{EI}\left[\frac{Px^3}{3} + \frac{Qx^3}{3} - \frac{QLx^2}{4}\right]_{L/2}^{L}$$

$$\delta_A = \frac{+(16P + 5Q)L^3}{48EI} = \frac{(16P + 5Q)L^3}{48EI}\downarrow$$ .... **Ans.**

Deflection at B:

$$\delta_B = \frac{1}{EI}\int_0^{L/2} M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_{L/2}^{L} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx$$

$$= 0 + \frac{1}{EI}\int_{L/2}^{L}\left[Px\left(x - \frac{L}{2}\right) + Q\left(x - \frac{L}{2}\right)^2\right]dx = \frac{1}{EI}\left[\frac{Px^3}{3} - \frac{PLx^2}{4} + \frac{Q}{3}\left(x - \frac{L}{2}\right)^3\right]_{L/2}^{L}$$

$$\delta_B = \frac{+(5P + 2Q)L^3}{48EI} = \frac{(5P + 2Q)L^3}{48EI}\downarrow$$ .... **Ans.**

8-109

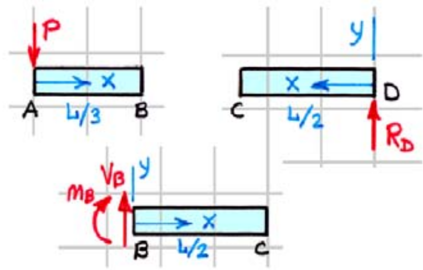
From overall equilibrium $\quad R_B = \left(\dfrac{4P}{3} - \dfrac{M}{L}\right)\uparrow \qquad R_D = \left(\dfrac{M}{L} - \dfrac{P}{3}\right)\uparrow$

$$M_{r1} = -Px \qquad \frac{\partial M_{r1}}{\partial P} = -x$$

$$M_{r2} = M_B + V_B x = \frac{-PL}{3} + \frac{Px}{3} - \frac{Mx}{L}$$

$$\frac{\partial M_{r2}}{\partial P} = \frac{1}{3}(x - L)$$

$$M_{r3} = R_D x = \frac{Mx}{L} - \frac{Px}{3} \qquad \frac{\partial M_{r3}}{\partial P} = \frac{-x}{3}$$



With $M = 2PL$:

$$\delta_A = \frac{1}{EI}\int_0^{L/3} M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_0^{L/2} M_{r2}\frac{\partial M_{r2}}{\partial P}dx + \frac{1}{EI}\int_0^{L/2} M_{r3}\frac{\partial M_{r3}}{\partial P}dx$$

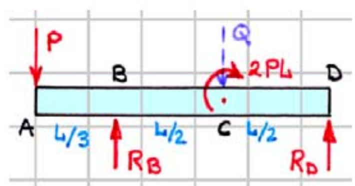
$$= \frac{P}{EI}\int_0^{L/3} x^2 dx + \frac{P}{EI}\int_0^{L/2}\left[\frac{1}{9}(x-L)^2 - \frac{2x^2}{3} + \frac{2Lx}{3}\right]dx + \frac{P}{9EI}\int_0^{L/2}\left(-5x^2\right)dx$$

$$= \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^{L/3} + \frac{P}{EI}\left[\frac{(x-L)^3}{27} - \frac{2x^3}{9} + \frac{2Lx^2}{6}\right]_0^{L/2} + \frac{P}{EI}\left[\frac{-5x^3}{27}\right]_0^{L/2}$$

$\delta_A = \dfrac{+13PL^3}{324EI} = \dfrac{13PL^3}{324EI}\downarrow$ .... **Ans.**

From overall equilibrium (with $M = 2PL$)

$$R_B = \left(\frac{Q}{2} - \frac{2P}{3}\right)\uparrow \qquad R_D = \left(\frac{Q}{2} + \frac{5P}{3}\right)\uparrow$$

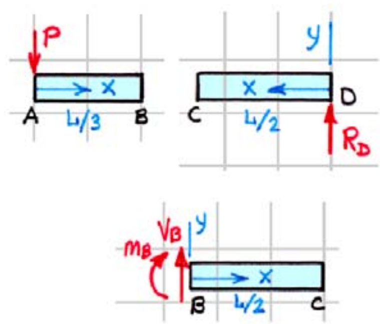


$$M_{r1} = -Px \qquad \frac{\partial M_{r1}}{\partial Q} = 0$$

$$M_{r2} = M_B + V_B x = \frac{-PL}{3} - \frac{5Px}{3} + \frac{Qx}{2} \qquad \frac{\partial M_{r2}}{\partial Q} = \frac{x}{2}$$

$$M_{r3} = R_D x = \frac{5Px}{3} + \frac{Qx}{2} \qquad \frac{\partial M_{r3}}{\partial Q} = \frac{x}{2}$$

With $Q = 0$:



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Problem 8-109 continued

$$\delta_C = \frac{1}{EI}\int_0^{L/3} M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_0^{L/2} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx + \frac{1}{EI}\int_0^{L/2} M_{r3}\frac{\partial M_{r3}}{\partial Q}dx$$

$$= 0 + \frac{P}{EI}\int_0^{L/2}\left(\frac{-Lx}{6} - \frac{5x^2}{6}\right)dx + \frac{P}{EI}\int_0^{L/2}\left(\frac{5x^2}{6}\right)dx$$

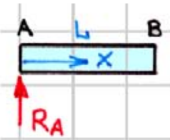
$$= \frac{P}{EI}\left[\frac{-Lx^2}{12} - \frac{5x^3}{18}\right]_0^{L/2} + \frac{P}{EI}\left[\frac{5x^3}{18}\right]_0^{L/2}$$

$$\delta_C = \frac{-PL^3}{48EI} = \frac{PL^3}{48EI}\uparrow$$ .... **Ans.**

8-110*

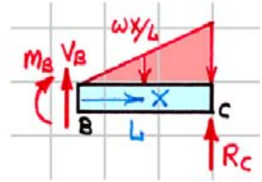
From overall equilibrium $\quad R_A = \left(\dfrac{P}{2} + \dfrac{wL}{12}\right)\uparrow \qquad R_C = \left(\dfrac{P}{2} + \dfrac{5wL}{12}\right)\uparrow$

$$M_{r1} = R_A x = \frac{Px}{2} + \frac{wLx}{12}$$



$$\frac{\partial M_{r1}}{\partial P} = \frac{x}{2}$$

$$M_{r2} = M_B + V_B x - \frac{wx^3}{6L} = \frac{PL}{2} + \frac{wL^2}{12} - \frac{Px}{2} + \frac{wLx}{12} - \frac{wx^3}{6L}$$



$$\frac{\partial M_{r2}}{\partial P} = \frac{-x}{2} + \frac{L}{2}$$

With $P = wL$

$$\delta_B = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_0^L M_{r2}\frac{\partial M_{r2}}{\partial P}dx$$

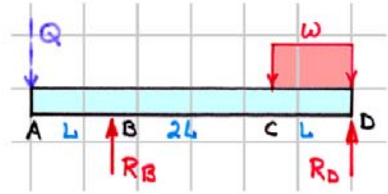
$$= \frac{1}{EI}\int_0^L \left(\frac{7wLx}{12}\right)\left(\frac{x}{2}\right)dx + \frac{1}{EI}\int_0^L \left[\frac{-5wLx}{12} + \frac{7wL^2}{12} - \frac{wx^3}{6L}\right]\left[\frac{L}{2} - \frac{x}{2}\right]dx$$

$$\delta_B = \frac{7wL}{24EI}\left[\frac{x^3}{3}\right]_0^L + \frac{w}{EI}\left[\frac{-5L^2x^2}{48} + \frac{7L^3x}{24} - \frac{x^4}{48} + \frac{5Lx^3}{72} - \frac{7L^2x^2}{48} + \frac{x^5}{60L}\right]_0^L$$

$$\delta_B = \frac{+49wL^4}{240EI} = \frac{49wL^4}{240EI}\downarrow$$ .... **Ans.**

8-111*

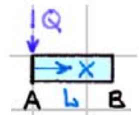
From overall equilibrium

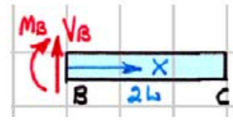


$$R_B = \left(\frac{4Q}{3} + \frac{wL}{6}\right)\uparrow \qquad R_D = \left(\frac{5wL}{6} - \frac{Q}{3}\right)\uparrow$$

$$M_{r1} = -Qx \qquad \frac{\partial M_{r1}}{\partial Q} = -x$$

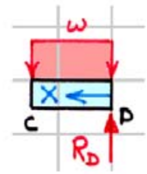
$$M_{r2} = M_B + V_B x = -QL + \frac{Qx}{3} + \frac{wLx}{6}$$





$$\frac{\partial M_{r2}}{\partial Q} = \frac{x}{3} - L$$

$$M_{r3} = R_D x - \frac{wx^2}{2} = \frac{5wLx}{6} - \frac{Qx}{3} - \frac{wx^2}{2} \qquad \frac{\partial M_{r3}}{\partial Q} = \frac{-x}{3}$$



With $Q = 0$:

$$\delta_A = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_0^{2L} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx + \frac{1}{EI}\int_0^L M_{r3}\frac{\partial M_{r3}}{\partial Q}dx$$

$$= 0 + \frac{1}{EI}\int_0^{2L}\left(\frac{wLx}{6}\right)\left(\frac{x}{3} - L\right)dx + \frac{1}{EI}\int_0^L\left(\frac{5wLx}{6} - \frac{wx^2}{2}\right)\left(\frac{-x}{3}\right)dx$$

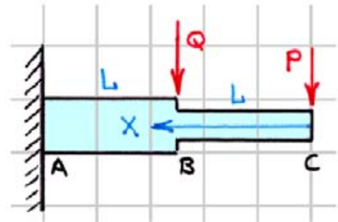
$$= \frac{w}{EI}\left[\frac{Lx^3}{54} - \frac{L^2x^2}{12}\right]_0^{2L} + \frac{w}{EI}\left[\frac{-5Lx^3}{54} + \frac{x^4}{24}\right]_0^L$$

$$\delta_A = \frac{-17wL^4}{72EI} = \frac{17wL^4}{72EI}\uparrow$$ .... **Ans.**

8-112

$M_{r1} = -Px$ $\qquad \partial M_{r1}/\partial P = -x$

$M_{r2} = -Px - Q(x-L)$ $\qquad \partial M_{r2}/\partial P = -x$



With $Q = P$:

$$\delta_C = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{2EI}\int_L^{2L} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx$$

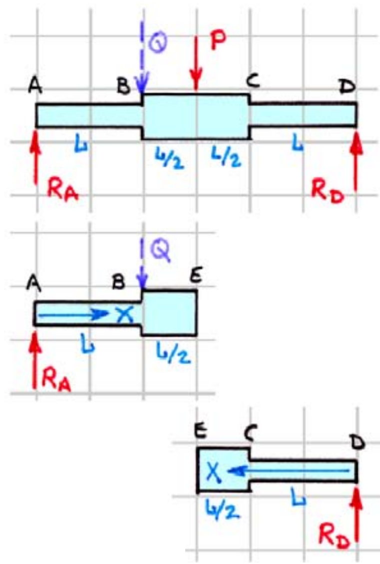
$$= \frac{1}{EI}\int_0^L (-Px)(-x)\,dx + \frac{1}{2EI}\int_L^{2L}\left[(-2Px)(-x)+(PL)(-x)\right]dx$$

$$= \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^L + \frac{P}{2EI}\left[\frac{2x^3}{3} - \frac{Lx^2}{2}\right]_L^{2L}$$

$$\delta_C = \frac{+23PL^3}{12EI} = \frac{23PL^3}{12EI}\downarrow$$ .... **Ans.**

8-113

From overall equilibrium



$$R_A = \left(\frac{2Q}{3} + \frac{P}{2}\right)\uparrow \qquad R_D = \left(\frac{Q}{3} + \frac{P}{2}\right)\uparrow$$

$$M_{r1} = R_A x = \frac{2Qx}{3} + \frac{Px}{2} \qquad \frac{\partial M_{r1}}{\partial Q} = \frac{2x}{3}$$

$$M_{r2} = R_A x - Q(x-L) = \frac{2Qx}{3} + \frac{Px}{2} - Q(x-L)$$

$$\frac{\partial M_{r2}}{\partial Q} = \frac{2x}{3} - (x-L)$$

$$M_{r3} = R_D x = \frac{Qx}{3} + \frac{Px}{2} \qquad \frac{\partial M_{r3}}{\partial Q} = \frac{x}{3}$$

With $Q = 0$:

$$\delta_B = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{2EI}\int_L^{3L/2} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx$$

$$+\frac{1}{EI}\int_0^L M_{r3}\frac{\partial M_{r3}}{\partial Q}dx + \frac{1}{2EI}\int_L^{3L/2} M_{r3}\frac{\partial M_{r3}}{\partial Q}dx$$

$$= \frac{1}{EI}\int_0^L\left(\frac{Px^2}{3}\right)dx + \frac{1}{2EI}\int_L^{3L/2}\left(\frac{-Px^2}{6} + \frac{PLx}{2}\right)dx$$

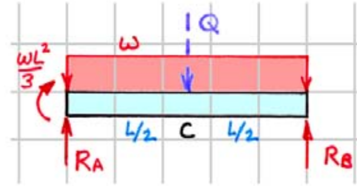
$$+\frac{1}{EI}\int_0^L\left(\frac{Px^2}{6}\right)dx + \frac{1}{2EI}\int_L^{3L/2}\left(\frac{Px^2}{6}\right)dx$$

$$= \frac{1}{EI}\left[\frac{Px^3}{9}\right]_0^L + \frac{1}{2EI}\left[\frac{-Px^3}{18} + \frac{PLx^2}{4}\right]_L^{3L/2} + \frac{1}{EI}\left[\frac{Px^3}{18}\right]_0^L + \frac{1}{2EI}\left[\frac{Px^3}{18}\right]_L^{3L/2}$$

$$\delta_B = \frac{+31PL^3}{96EI} = \frac{31PL^3}{96EI}\downarrow$$ .... **Ans.**

8-114

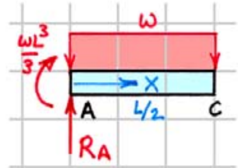
From overall equilibrium

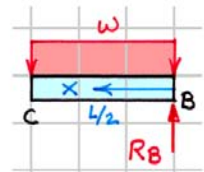


$$R_A = \left(\frac{wL}{6} + \frac{Q}{2}\right)\uparrow \qquad R_B = \left(\frac{5wL}{6} + \frac{Q}{2}\right)\uparrow$$

$$M_{r1} = R_A x + \frac{wL^2}{3} - \frac{wx^2}{2} = \frac{wLx}{6} + \frac{Qx}{2} + \frac{wL^2}{3} - \frac{wx^2}{2}$$

$$\frac{\partial M_{r1}}{\partial Q} = \frac{x}{2}$$





$$M_{r2} = R_B x - \frac{wx^2}{2} = \frac{5wLx}{6} + \frac{Qx}{2} - \frac{wx^2}{2}$$

$$\frac{\partial M_{r2}}{\partial Q} = \frac{x}{2}$$

With $Q = 0$:

$$\delta_C = \frac{1}{EI}\int_0^{L/2} M_{r1}\frac{\partial M_{r1}}{\partial Q}dx + \frac{1}{EI}\int_L^{L/2} M_{r2}\frac{\partial M_{r2}}{\partial Q}dx$$

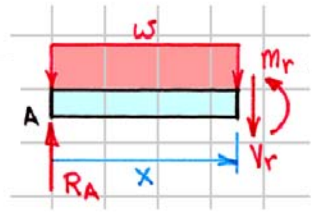
$$= \frac{1}{EI}\int_0^{L/2}\left(\frac{wLx^2}{12} + \frac{wL^2x}{6} - \frac{wx^3}{4}\right)dx + \frac{1}{EI}\int_L^{L/2}\left(\frac{5wLx^2}{12} - \frac{wx^3}{4}\right)dx$$

$$= \frac{1}{EI}\left[\frac{wLx^3}{36} + \frac{wL^2x^2}{12} - \frac{wx^4}{16}\right]_0^{L/2} + \frac{1}{EI}\left[\frac{5wLx^3}{36} - \frac{wx^4}{16}\right]_L^{L/2}$$

$$\delta_C = \frac{+13PL^3}{384EI} = \frac{13PL^3}{384EI}\downarrow$$ .... **Ans.**

8-115*

$$EIv'' = M_r = R_A x - \frac{wx^2}{2}$$



$$EIv' = \frac{R_A x^2}{2} - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{R_A x^3}{6} - \frac{wx^4}{24} + C_1 x + C_2$$

Boundary Conditions:

At, $x = 0$ $\quad v = 0:$ $\qquad C_2 = 0$

At $x = L$, $\quad v' = 0:$ $\qquad \frac{R_A L^2}{2} - \frac{wL^3}{6} + C_1 = 0$

At $x = L$, $\quad v = 0:$ $\qquad \frac{R_A L^3}{6} - \frac{wL^4}{24} + C_1 L = 0$

Solving simultaneously gives: $\qquad C_1 = \frac{-wL^3}{48}$

and $\qquad R_A = \frac{3wL}{8} = \frac{3wL}{8} \uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

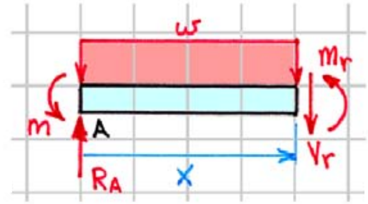
$\uparrow \Sigma F_y = 0:$ $\qquad R_A - wL + R_B = 0$ $\qquad R_B = \frac{5wL}{8} = \frac{5wL}{8} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0:$ $\qquad \frac{wL^2}{2} - R_A L - M_B = 0$ $\qquad M_B = \frac{wL^2}{8} = \frac{wL^2}{8} \circlearrowleft$ .... **Ans.**

8-116*

$$EIv'' = M_r = -M + R_A x - \frac{wx^2}{2}$$

$$EIv' = -Mx + \frac{R_A x^2}{2} - \frac{wx^3}{6} + C_1$$



8-116* (cont.)

$$EIv = \frac{-Mx^2}{2} + \frac{R_A x^3}{6} - \frac{wx^4}{24} + C_1 x + C_2$$

Boundary Conditions:

At, $x = 0$ $\quad v = 0$: $\quad C_2 = 0$

At $x = 0$, $\quad v' = 0$: $\quad C_1 = 0$

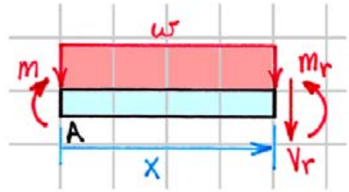
At $x = L$, $\quad v = 0$: $\quad \frac{-ML^2}{2} + \frac{R_A L^3}{6} - \frac{wL^4}{24} = 0$

At $x = L$, $\quad v' = 0$: $\quad -ML + \frac{R_A L^2}{2} - \frac{wL^3}{6} = 0$

Solving simultaneously gives:

$R_A = \frac{wL}{2} = \frac{wL}{2} \uparrow$ .... $M = \frac{wL^2}{12} = \frac{wL^2}{12}$ ↺ .... **Ans.**

8-117



$$EIv'' = M_r = M - \frac{wx^2}{2}$$

$$EIv' = Mx - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{Mx^2}{2} - \frac{wx^4}{24} + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $C_1 = 0$

At $x = L$, $v' = 0$: $ML - \frac{wL^3}{6} = 0$

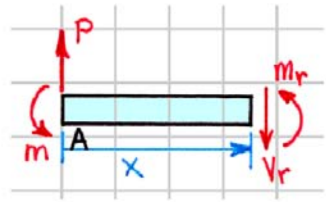
$M = \frac{wL^2}{6} = \frac{wL^2}{6}$ ↻ .... **Ans.**

8-118*

$$EIv'' = M_r = Px - M$$

$$EIv' = \frac{Px^2}{2} - Mx + C_1$$

$$EIv = \frac{Px^3}{6} - \frac{Mx^2}{2} + C_1 x + C_2$$



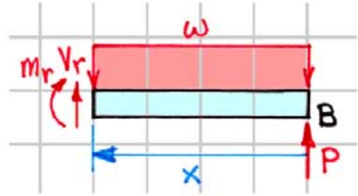
Boundary Conditions:

At $x = 0$, $v' = 0$: $C_1 = 0$

At $x = L$, $v' = 0$: $PL^2/2 - ML = 0$

$M = \frac{PL}{2} = \frac{PL}{2}$ ↺ .... **Ans.**

8-119



$$EIv'' = M_r = Px - \frac{wx^2}{2}$$

$$EIv' = \frac{Px^2}{2} - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{Px^3}{6} - \frac{wx^4}{24} + C_1x + C_2$$

Boundary Conditions:

At, $x = 0$ $\quad v = 0$: $\quad C_2 = 0$

At $x = L$, $\quad v' = 0$: $\quad \frac{PL^2}{2} - \frac{wL^3}{6} + C_1 = 0$

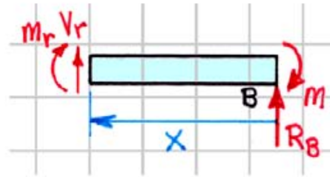
At $x = L$, $\quad v = 0$: $\quad \frac{PL^3}{6} - \frac{wL^4}{24} + C_1L = 0$

Solving simultaneously gives: $\quad C_1 = \frac{-wL^3}{48}$

and $\quad P = \frac{3wL}{8} = \frac{3wL}{8}\uparrow$ .... **Ans.**

8-120

$$EIv'' = M_r = R_B x - M$$



$$EIv' = \frac{R_B x^2}{2} - Mx + C_1$$

$$EIv = \frac{R_B x^3}{6} - \frac{Mx^2}{2} + C_1 x + C_2$$

Boundary Conditions:

At, $x = 0$ $\quad v = 0$: $\quad C_2 = 0$

At $x = L$, $\quad v' = 0$: $\quad \frac{R_B L^2}{2} - ML + C_1 = 0$

At $x = L$, $\quad v = 0$: $\quad \frac{R_B L^3}{6} - \frac{ML^2}{2} + C_1 L = 0$

Solving simultaneously gives: $\quad C_1 = \frac{ML}{4}$

and $\quad R_B = \frac{3M}{2L} = \frac{3M}{2L} \uparrow$ .... **Ans.**

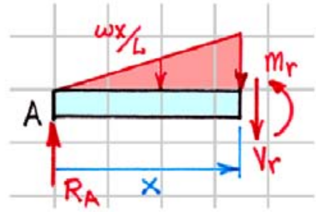
Then the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0$: $\quad R_A + R_B = 0$ $\quad R_A = \frac{-3M}{2L} = \frac{3M}{2L} \downarrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0$: $\quad R_B L - M_A - M = 0$ $\quad M_A = \frac{M}{2} = \frac{M}{2} \circlearrowright$ .... **Ans.**

8-121*

$$EIv'' = M_r = R_A x - \frac{wx^3}{6L}$$



$$EIv' = \frac{R_A x^2}{2} - \frac{wx^4}{24L} + C_1$$

$$EIv = \frac{R_A x^3}{6} - \frac{wx^5}{120L} + C_1 x + C_2$$

Boundary Conditions:

At, $x = 0 \quad v = 0: \qquad C_2 = 0$

At $x = L, \quad v' = 0: \qquad \frac{R_A L^2}{2} - \frac{wL^3}{24} + C_1 = 0$

At $x = L, \quad v = 0: \qquad \frac{R_A L^3}{6} - \frac{wL^4}{120} + C_1 L = 0$

Solving simultaneously gives: $\qquad C_1 = \frac{-wL^3}{120}$

and $\qquad R_A = \frac{+wL}{10} = \frac{wL}{10} \uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0: \qquad R_A - \frac{wL}{2} + R_B = 0 \qquad R_B = \frac{2wL}{5} = \frac{2wL}{5} \uparrow$ .... **Ans.**

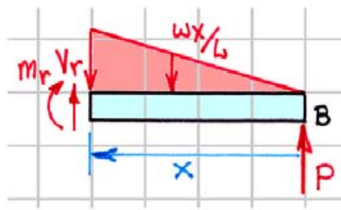
$\circlearrowleft \Sigma M_B = 0: \qquad \frac{wL^2}{6} - M_B - R_A L = 0 \qquad M_B = \frac{wL^2}{15} = \frac{wL^2}{15} \circlearrowright$ .... **Ans.**

8-122*

$$EIv'' = M_r = Px - \frac{wx^3}{6L}$$

$$EIv' = \frac{Px^2}{2} - \frac{wx^4}{24L} + C_1$$

$$EIv = \frac{Px^3}{6} - \frac{wx^5}{120L} + C_1 x + C_2$$



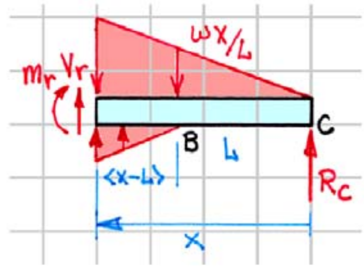
Boundary Conditions:

At $x = 0$, $v' = 0$: $C_1 = 0$

At $x = L$, $v' = 0$: $\frac{PL^2}{2} - \frac{wL^3}{24} = 0$

$P = \frac{wL}{12} = \frac{wL}{12} \uparrow$ .... **Ans.**

8-123



$$EIv'' = M_r = R_C x - \frac{wx^3}{6L} + \frac{2w\langle x-L\rangle^3}{6L}$$

$$EIv' = \frac{R_C x^2}{2} - \frac{wx^4}{24L} + \frac{w\langle x-L\rangle^4}{12L} + C_1$$

$$EIv = \frac{R_C x^3}{6} - \frac{wx^5}{120L} + \frac{w\langle x-L\rangle^5}{60L} + C_1 x + C_2$$

Boundary Conditions:

At, $x=0$ $\quad v=0$: $\qquad C_2 = 0$

At $x=2L$, $\quad v'=0$: $\qquad \frac{R_C(2L)^2}{2} - \frac{w(2L)^4}{24L} + \frac{wL^4}{12L} + C_1 = 0$

At $x=2L$, $\quad v=0$: $\qquad \frac{R_C(2L)^3}{6} - \frac{w(2L)^5}{120L} + \frac{wL^5}{60L} + C_1(2L) = 0$

Solving simultaneously gives: $\qquad C_1 = \frac{-5wL^3}{48}$

and $\qquad R_C = \frac{11wL}{32} = \frac{11wL}{32}\uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

$\uparrow\Sigma F_y = 0$: $\qquad R_A - wL + R_C = 0$ $\qquad R_A = \frac{21wL}{32} = \frac{21wL}{32}\uparrow$ .... **Ans.**

$\circlearrowleft\Sigma M_A = 0$: $\qquad M_A + R_C(2L) - wL^2 = 0$ $\qquad M_A = \frac{5wL^2}{16} = \frac{5wL^2}{16}\circlearrowleft$ .... **Ans.**

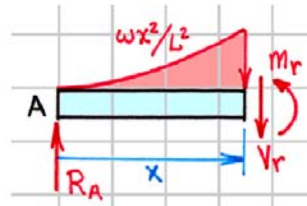
8-124*

$$EIv'''' = \frac{-wx^2}{L^2}$$

$$EIv''' = \frac{-wx^3}{3L^2} + C_1$$

$$EIv'' = \frac{-wx^4}{12L^2} + C_1 x + C_2$$

$$EIv' = \frac{-wx^5}{60L^2} + \frac{C_1 x^2}{2} + C_2 x + C_3$$



$$EIv = \frac{-wx^6}{360L^2} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Boundary Conditions:

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = 0$, $v = 0$: $C_4 = 0$

By symmetry: $M_B = \frac{5wL^2}{24}$ ↻ .... **Ans.**

(b) $v = \frac{w}{240EIL}\left(20L^2x^3 - 25L^3x^2 - 2x^5\right)$

$\delta_M = v_{x=L} = \frac{w}{240EIL}\left(20L^5 - 25L^5 - 2L^5\right) = \frac{-7wL^4}{240EI} = \frac{7wL^4}{240EI}$ ↓ .... **Ans.**

8-125

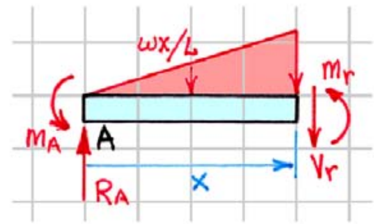
(a) By symmetry: $v' = 0$ at $x = L$

and $R_A = R_B = (wL/2)\uparrow$ .... **Ans.**

$$EIv'' = M_r = \frac{wLx}{2} - M_A - \frac{wx^3}{6L}$$

$$EIv' = \frac{wLx^2}{4} - M_A x - \frac{wx^4}{24L} + C_1$$

$$EIv = \frac{wLx^3}{12} - \frac{M_A x^2}{2} - \frac{wx^5}{120L} + C_1 x + C_2$$



Boundary Conditions:

At, $x = 0$ $\quad v = 0$: $\quad C_2 = 0$

At $x = 0$, $\quad v' = 0$: $\quad C_1 = 0$

At $x = L$, $\quad v' = 0$: $\quad \frac{wL^3}{4} - M_A L - \frac{wL^3}{24} = 0$

gives $M_A = \frac{+5wL^2}{24} = \frac{5wL^2}{24}$ ↺ .... **Ans.**

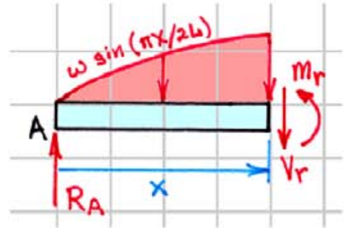
By symmetry: $M_B = \frac{5wL^2}{24}$ ↻ .... **Ans.**

(b) $$v = \frac{w}{240EIL}\left(20L^2x^3 - 25L^3x^2 - 2x^5\right)$$

$$\delta_M = v_{x=L} = \frac{w}{240EIL}\left(20L^5 - 25L^5 - 2L^5\right) = \frac{-7wL^4}{240EI} = \frac{7wL^4}{240EI}\downarrow$$ .... **Ans.**

8-126

$$EIv'''' = -w\sin\left(\frac{\pi x}{2L}\right)$$



$$EIv''' = \frac{2wL}{\pi}\cos\left(\frac{\pi x}{2L}\right) + C_1$$

$$EIv'' = \frac{4wL^2}{\pi^2}\sin\left(\frac{\pi x}{2L}\right) + C_1x + C_2$$

$$EIv' = \frac{-8wL^3}{\pi^3}\cos\left(\frac{\pi x}{2L}\right) + \frac{C_1x^2}{2} + C_2x + C_3$$

$$EIv = \frac{-16wL^4}{\pi^4}\sin\left(\frac{\pi x}{2L}\right) + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4$$

Boundary Conditions: At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = 0$, $v = 0$: $C_4 = 0$

At $x = L$, $v' = 0$: $\frac{C_1L^2}{2} + C_3$

At $x = L$, $v = 0$: $\frac{-16wL^4}{\pi^4} + \frac{C_1L^3}{6} + C_3L = 0$

Solving simultaneously gives: $C_1 = \frac{-48wL}{\pi^4}$ $C_3 = \frac{24wL^3}{\pi^4}$

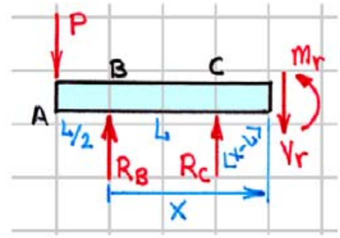
$$EIv''' = \frac{2wL}{\pi}\cos\left(\frac{\pi x}{2L}\right) - \frac{48wL}{\pi^4} \qquad EIv'' = \frac{4wL^2}{\pi^2}\sin\left(\frac{\pi x}{2L}\right) - \frac{48wLx}{\pi^4}$$

$R_A = V_{x=0} = EIv'''_{x=0} = \left(\frac{2wL}{\pi} - \frac{48wL}{\pi^4}\right)\uparrow$ .... **Ans.**

$R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[0 - \frac{48wL}{\pi^4}\right] = \frac{48wL}{\pi^4}\uparrow$ .... **Ans.**

$M_B = M_{x=L} = EIv''_{x=L} = \left(\frac{4wL^2}{\pi^2} - \frac{48wL^2}{\pi^4}\right) \circlearrowleft$ .... **Ans.**

8-127*



$$EIv'' = M_r = R_B x - P\left(x + \frac{L}{2}\right) + R_C \langle x - L \rangle$$

$$EIv' = \frac{R_B x^2}{2} - \frac{P}{2}\left(x + \frac{L}{2}\right)^2 + \frac{R_C \langle x - L \rangle^2}{2} + C_1$$

$$EIv = \frac{R_B x^3}{6} - \frac{P}{6}\left(x + \frac{L}{2}\right)^3 + \frac{R_C \langle x - L \rangle^3}{6} + C_1 x + C_2$$

Boundary Conditions:

At, $x = 0$ $\quad v = 0$: $\quad \dfrac{-PL^3}{48} + C_2 = 0 \qquad C_2 = \dfrac{PL^3}{48}$

At $x = L$, $\quad v = 0$: $\quad \dfrac{R_B L^3}{6} - \dfrac{27PL^3}{48} + C_1 L + \dfrac{PL^3}{48} = 0 \qquad$ (a)

At $x = 2L$, $\quad v = 0$: $\quad \dfrac{8R_B L^3}{6} - \dfrac{125PL^3}{48} + \dfrac{R_C L^3}{6} + 2C_1 L + \dfrac{PL^3}{48} = 0 \qquad$ (b)

and the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0$: $\quad R_B + R_C + R_D = P \qquad$ (c)

$\circlearrowleft \Sigma M_D = 0$: $\quad 2R_B L + R_C L = 5PL/2 \qquad$ (d)

Solving Eqs. (a) – (d) simultaneously gives:

$$C_1 = \frac{13PL^2}{48} \qquad C_2 = \frac{PL^3}{48}$$

and $\quad R_B = \dfrac{+13P}{8} = \dfrac{13P}{8}\uparrow$ .... **Ans.**

$R_C = \dfrac{-3P}{4} = \dfrac{3P}{4}\downarrow$ .... **Ans.**

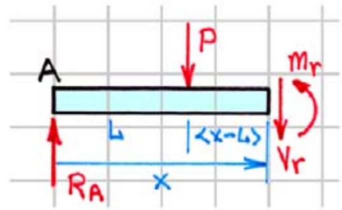
$R_D = \dfrac{+P}{8} = \dfrac{P}{8}\uparrow$ .... **Ans.**

8-128*

(a) $$EIv'' = M_r = R_A x - P\langle x - L\rangle$$

$$EIv' = \frac{R_A x^2}{2} - \frac{P\langle x-L\rangle^2}{2} + C_1$$

$$EIv = \frac{R_A x^3}{6} - \frac{P\langle x-L\rangle^3}{6} + C_1 x + C_2$$



Boundary Conditions:

At, $x = 0$ $\quad v = 0:$ $\quad C_2 = 0$

At $x = 2L$, $\quad v' = 0:$ $\quad \frac{R_A(2L)^2}{2} - \frac{PL^2}{2} + C_1 = 0$

At $x = 2L$, $\quad v = 0:$ $\quad \frac{R_A(2L)^3}{6} - \frac{PL^3}{6} + C_1(2L) = 0$

Solving simultaneously gives: $\quad C_1 = \frac{-PL^2}{8}$

and $\quad R_A = \frac{+5P}{16} = \frac{5P}{16}\uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

$\uparrow\Sigma F_y = 0:$ $\quad R_A - P + R_B = 0$ $\quad R_B = \frac{+11P}{16} = \frac{11P}{16}\uparrow$ .... **Ans.**

$\circlearrowleft\Sigma M_B = 0:$ $\quad PL - R_A(2L) - M_B = 0$ $\quad M_B = \frac{+3PL}{8} = \frac{3PL}{8}\circlearrowright$ .... **Ans.**

(b) $$v = \frac{P}{96EI}\left[5x^3 - 16\langle x-L\rangle^3 - 12L^2 x\right]$$

$\delta_M = v_{x=L} = \frac{P}{96EI}\left[5L^3 - 12L^3\right] = \frac{-7PL^3}{96EI} = \frac{7PL^3}{96EI}\downarrow$ .... **Ans.**

8-129

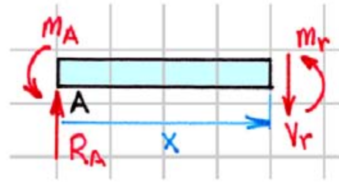
(a) By symmetry: $v' = 0$ at $x = L/2$

and $R_A = R_B = (P/2)\uparrow$ .... **Ans.**

$$EIv'' = M_r = Px/2 - M_A$$

$$EIv' = \frac{Px^2}{4} - M_A x + C_1$$

$$EIv = \frac{Px^3}{12} - \frac{M_A x^2}{2} + C_1 x + C_2$$



Boundary Conditions:

At, $x = 0$ $\quad v = v' = 0:$ $\quad C_1 = C_2 = 0$

At $x = \dfrac{L}{2}$, $\quad v' = 0:$ $\quad \dfrac{PL^2}{16} - \dfrac{M_A L}{2} = 0$

Therefore $M_A = \dfrac{+PL}{8} = \dfrac{PL}{8}$ ↺ .... **Ans.**

and by symmetry $M_B = \dfrac{PL}{8}$ ↻ .... **Ans.**

(b) $v = \dfrac{P}{48EI}\left[4x^3 - 3Lx^2\right]$

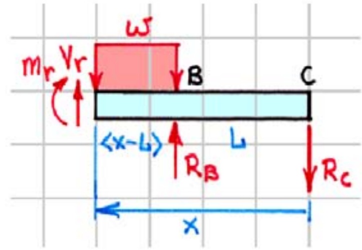
$\delta_M = v_{x=L/2} = \dfrac{P}{48EI}\left[\dfrac{L^3}{2} - \dfrac{3L^3}{4}\right] = \dfrac{-PL^3}{192EI} = \dfrac{PL^3}{192EI}\downarrow$ .... **Ans.**

8-130*

$$EIv'' = M_r = -R_C x + R_B \langle x-L \rangle - \frac{w\langle x-L \rangle^2}{2}$$

$$EIv' = \frac{-R_C x^2}{2} + \frac{R_B \langle x-L \rangle^2}{2} - \frac{w\langle x-L \rangle^3}{6} + C_1$$

$$EIv = \frac{-R_C x^3}{3} + \frac{R_B \langle x-L \rangle^3}{3} - \frac{w\langle x-L \rangle^4}{24} + C_1 x + C_2$$



Boundary Conditions:

At, $x = 0$ $\quad v = 0$: $\quad C_2 = 0$

At, $x = L$ $\quad v = 0$: $\quad \frac{-R_C L^3}{6} + C_1 L = 0$ (a)

At $x = 2L$, $\quad v = 0$: $\quad \frac{-R_C (2L)^3}{6} - \frac{R_B L^3}{6} - \frac{wL^4}{24} + C_1 (2L) = 0$ (b)

and the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0$: $\quad R_A + R_B - R_C = wL$ (c)

$\circlearrowleft \Sigma M_A = 0$: $\quad R_B L - R_C (2L) = wL^2/2$ (d)

Solving Eqns. (a) – (d) simultaneously gives: $\quad C_1 = wL^3/96$

and $R_A = \frac{+7wL}{16} = \frac{7wL}{16} \uparrow$ .... **Ans.**

$R_B = \frac{+5wL}{8} = \frac{5wL}{8} \uparrow$ .... **Ans.**

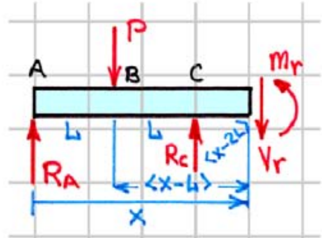
$R_C = \frac{+wL}{16} = \frac{wL}{16} \downarrow$ .... **Ans.**

8-131*

$$EIv'' = M_r = R_A x - P\langle x-L\rangle + R_C\langle x-2L\rangle$$

$$EIv' = \frac{R_A x^2}{2} - \frac{P\langle x-L\rangle^2}{2} + \frac{R_C\langle x-2L\rangle^2}{2} + C_1$$

$$EIv = \frac{R_A x^3}{3} - \frac{P\langle x-L\rangle^3}{3} + \frac{R_C\langle x-2L\rangle^3}{3} + C_1 x + C_2$$



Boundary Conditions:

At, $x = 0$ $\quad v = 0:$ $\qquad C_2 = 0$

At, $x = 2L$ $\quad v = 0:$ $\qquad \frac{4R_A L^3}{3} - \frac{PL^3}{6} + 2C_1 L = 0$ (a)

At $x = 3L$, $\quad v' = 0:$ $\qquad \frac{9R_A L^2}{2} - \frac{4PL^2}{2} + \frac{R_C L^2}{2} + C_1 = 0$ (b)

At $x = 3L$, $\quad v = 0:$ $\qquad \frac{9R_A L^3}{2} - \frac{4PL^3}{3} + \frac{R_C L^3}{6} + 3C_1 L = 0$ (c)

Solving Eqns. (a) – (c) simultaneously gives: $\qquad C_1 = -7PL^2/44$

and $\quad R_A = \frac{+4P}{11} = \frac{4P}{11}\uparrow$ .... **Ans.**

$R_C = \frac{+23P}{22} = \frac{23P}{22}\uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0:$ $\qquad R_A + R_C + R_D = P$

$\circlearrowleft \Sigma M_D = 0:$ $\qquad M_D - R_A(3L) + P(2L) - R_C L = 0$

$R_D = \frac{-9P}{22} = \frac{9P}{22}\downarrow$ .... **Ans.**

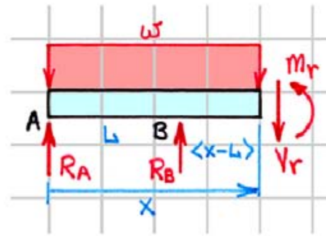
$M_D = \frac{+3PL}{22} = \frac{3PL}{22}\ \circlearrowleft$ .... **Ans.**

8-132

$$EIv'' = M_r = R_A x - \frac{wx^2}{2} + R_B \langle x - L \rangle$$

$$EIv' = \frac{R_A x^2}{2} - \frac{wx^3}{6} + \frac{R_B \langle x - L \rangle^2}{2} + C_1$$

$$EIv = \frac{R_A x^3}{6} - \frac{wx^4}{24} + \frac{R_B \langle x - L \rangle^3}{6} + C_1 x + C_2$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At , $x = L$ $v = 0$: $\frac{R_A L^3}{6} - \frac{wL^4}{24} + C_1 L = 0$ (a)

At $x = 2L$, $v' = 0$: $2R_A L - \frac{4wL^3}{3} + \frac{R_B L^2}{2} + C_1 = 0$ (b)

At $x = 2L$, $v = 0$: $\frac{4R_A L^3}{3} - \frac{2wL^4}{3} + \frac{R_B L^3}{6} + 2C_1 L = 0$ (c)

Solving Eqns. (a) – (c) simultaneously gives: $C_1 = -wL^3/42$

and $R_A = \frac{+11wL}{28} = \frac{11wL}{28} \uparrow$ .... **Ans.**

$R_B = \frac{+8wL}{7} = \frac{8wL}{7} \uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0$: $R_A + R_B + R_C = 2wL$

$\circlearrowleft \Sigma M_C = 0$: $M_C - R_A(2L) - R_B L = 2wL^2$

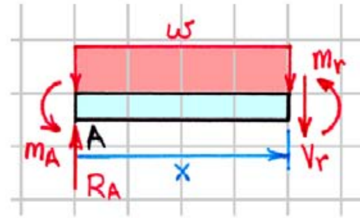
$R_C = \frac{+13wL}{28} = \frac{13wL}{28} \uparrow$ .... **Ans.**

$M_C = \frac{-wL^2}{14} = \frac{wL^2}{14} \circlearrowright$ .... **Ans.**

8-133*

(a) By symmetry: $v' = 0$ at $x = L$

and $R_A = R_C = wL \uparrow$ .... **Ans.**



$$EIv'' = M_r = wLx + M_A - \frac{wx^2}{2}$$

$$EIv' = \frac{wLx^2}{2} + M_A x - \frac{wx^3}{6} + C_1$$

$$EIv = \frac{wLx^3}{6} + \frac{M_A x^2}{2} - \frac{wx^4}{24} + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

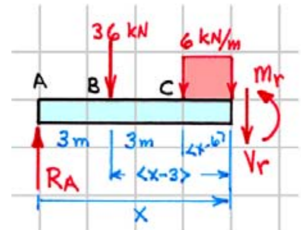
At $x = 0$, $v' = 0$: $C_1 = 0$

At $x = L$, $v' = 0$: $\frac{wL^3}{2} + M_A L - \frac{wL^3}{6} = 0$

gives $M_A = \frac{+wL^2}{3} = \frac{wL^2}{3}$ ↺ .... **Ans.**

By symmetry: $M_C = \frac{wL^2}{3}$ ↻ .... **Ans.**

8-134



(a) $$EIv'' = M_r = \left[R_A x - 36\langle x-3\rangle - 3\langle x-6\rangle^2 \right] \text{ kN}\cdot\text{m}$$

$$EIv' = \left[\frac{R_A x^2}{2} - 18\langle x-3\rangle^2 - \langle x-6\rangle^3 + C_1 \right] \text{ kN}\cdot\text{m}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - 6\langle x-3\rangle^3 - 0.25\langle x-6\rangle^4 + C_1 x + C_2 \right] \text{ kN}\cdot\text{m}^3$$

Boundary Conditions:

At $x = 0$, $v = 0$: $\quad C_2 = 0$

At $x = 12$, $v' = 0$: $\quad 72R_A - 1674 + C_1 = 0 \quad$ (a)

At $x = 12$, $v = 0$: $\quad 288R_A - 4698 + 12C_1 = 0 \quad$ (b)

Solving Eqns. (a) and (b) simultaneously gives: $\quad C_1 = -249.75 \text{ kN}\cdot\text{m}^2$

and $\quad R_A = 26.72 \text{ kN} \cong 26.7 \text{ kN} \uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0$: $\quad R_A - 36 - (6\times 6) + R_D = 0 \qquad R_D = 45.28 \text{ kN} \cong 45.3 \text{ kN} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_D = 0$: $\quad -M_D + (6\times 6)(3) + (36)(9) - 12R_A = 0$

$M_D = 111.36 \text{ kN}\cdot\text{m} \cong 111.4 \text{ kN}\cdot\text{m} \circlearrowright$ .... **Ans.**

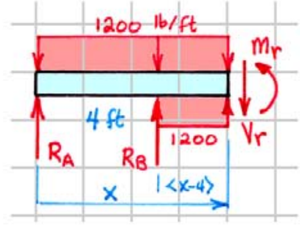
(b) $$EIv = \left[4.4531x^3 - 6\langle x-3\rangle^3 - 0.25\langle x-6\rangle^2 - 249.75x \right] \text{ kN}\cdot\text{m}^3$$

At $x = 3$ m: $\quad EIv_{x=3} = \left[4.4531(3)^3 - 249.75(3) \right] = -629 \text{ kN}\cdot\text{m}^3$

$$\delta_B = v_{x=3} = \frac{-629\times 10^3}{(200\times 10^9)(350\times 10^{-6})} = -0.00899 \text{ m} = 8.99 \text{ mm} \downarrow$$ .... **Ans.**

8-135

(a) $EIv'' = M_r = \left[R_A x - 600x^2 + R_B \langle x-4 \rangle + 600 \langle x-4 \rangle^2 \right] \text{ lb}\cdot\text{ft}$



$$EIv' = \left[\frac{R_A x^2}{2} - 200x^3 + \frac{R_B \langle x-4 \rangle^2}{2} + 200 \langle x-4 \rangle^3 + C_1 \right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - 50x^4 + \frac{R_B \langle x-4 \rangle^3}{6} + 50 \langle x-4 \rangle^4 + C_1 x + C_2 \right] \text{ lb}\cdot\text{ft}^3$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 4$ ft, $v = 0$: $(32R_A/3) + 4C_1 = 12{,}800$ (a)

At $x = 10$ ft, $v = 0$: $(500R_A/3) + 36R_B + 10C_1 = 435{,}200$ (b)

and the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0$: $R_A + R_B + R_D - (1200 \times 4) - 2000 = 0$ (c)

$\circlearrowleft \Sigma M_D = 0$: $(1200 \times 4)(8) - 10R_A - 6R_B - (2000)(2) = 0$ (d)

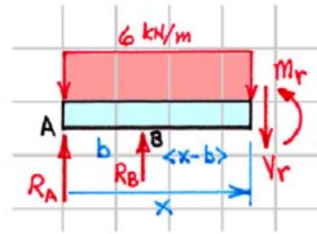
Solving Eqns. (a) – (d) simultaneously gives: $C_1 = -3360 \text{ lb}\cdot\text{ft}^2$

and $R_A = 2460 \text{ lb} = 2460 \text{ lb}\uparrow$ .... **Ans.**

$R_B = 1633 \text{ lb} = 1633 \text{ lb}\uparrow$ .... **Ans.**

$R_D = 2710 \text{ lb} = 2710 \text{ lb}\uparrow$ .... **Ans.**

8-136



$$EIv'' = M_r = \left[R_A x - 3x^2 + R_B\langle x-b\rangle\right]\ \text{kN}\cdot\text{m}$$

$$EIv' = \left[\frac{R_A x^2}{2} - x^3 + \frac{R_B\langle x-b\rangle^2}{2} + C_1\right]\ \text{kN}\cdot\text{m}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - \frac{x^4}{4} + \frac{R_B\langle x-b\rangle^3}{6} + C_1 x + C_2\right]\ \text{kN}\cdot\text{m}^3$$

Boundary Conditions:

At $x = 0$, $v = 0$: $\quad C_2 = 0$

At $x = b$, $v = 0$: $\quad C_1 b = \dfrac{b^4}{4} - \dfrac{R_A b^3}{6}$ (a)

At $x = 12$, $v = 0$: $\quad 10C_1 = \dfrac{(10)^4}{4} - \dfrac{R_A(10)^3}{6} - \dfrac{R_B(10-b)^3}{6}$ (b)

and from overall equilibrium:

$\circlearrowleft \Sigma M_C = 0$: $\quad (6\times 10)(5) - 10R_A - (10-b)R_B = 0$ (c)

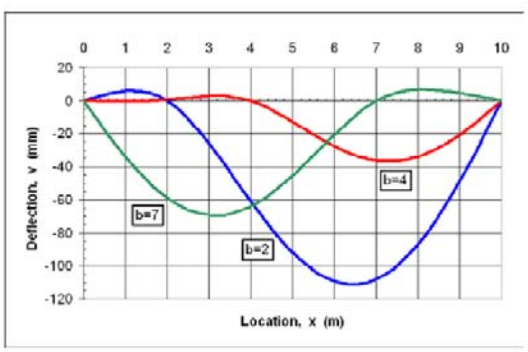
Solving Eqns. (a) – (c) simultaneously gives:

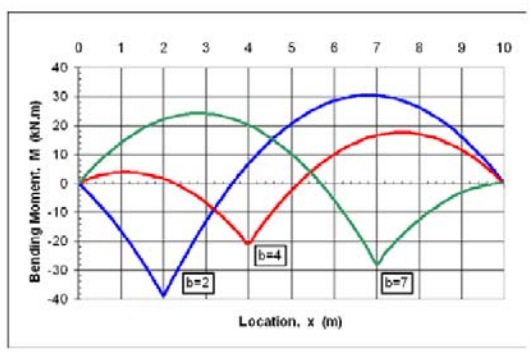
$$R_A = \frac{30b^2 + 600b^2 - 12{,}000b + 30{,}000}{40b^2 - 400b}$$

$$R_B = \frac{300 - 10R_A}{10-b} \qquad\qquad C_1 = \frac{3b^2 - 2R_A b^2}{12}$$

(a) $EIv = \left[\dfrac{4R_A x^3 - 6x^4 + 4R_B\langle x-b\rangle^3 + 24C_1 x}{24}\right]\ \text{kN}\cdot\text{m}^3$ .... **Ans.**

(b) $M_r = \left[R_A x - 3x^2 + R_B\langle x-b\rangle\right]\ \text{kN}\cdot\text{m}$ .... **Ans.**





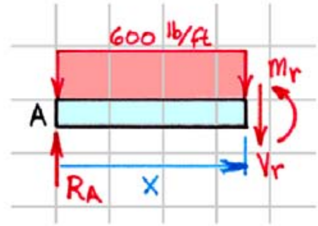
8-137

(a) By symmetry: $R_A = R_D = (wL/2) = (600 \times 30/2) = (9000 \text{ lb})\uparrow$

$$EIv'' = M_r = \left[9000x - 300x^2\right] \text{ lb}\cdot\text{ft}$$

$$EIv' = \left[4500x^2 - 100x^3 + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[1500x^3 - 25x^4 + C_1 x + C_2\right] \text{ lb}\cdot\text{ft}^3$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 30$ ft, $v = 0$: $C_1 = 675,000 - 1,350,000 = -675,000 \text{ lb}\cdot\text{ft}^2$

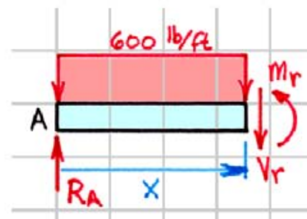
$EIv = \left[1500x^3 - 25x^4 - 675,000x\right] \text{ lb}\cdot\text{ft}^3$ .... **Ans.**

(b) By symmetry: $R_A = R_B = (wL/2) = (600 \times 15/2) = (4500 \text{ lb})\uparrow$

$$EIv'' = M_r = \left[4500x - 300x^2\right] \text{ lb}\cdot\text{ft}$$

$$EIv' = \left[2250x^2 - 100x^3 + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[750x^3 - 25x^4 + C_1 x + C_2\right] \text{ lb}\cdot\text{ft}^3$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

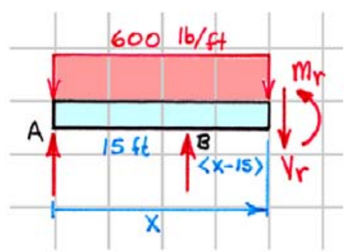
At $x = 15$ ft, $v = 0$: $C_1 = 84,375 - 168,750 = -84,375 \text{ lb}\cdot\text{ft}^2$

$EIv = \left[750x^3 - 25x^4 - 84,375x\right] \text{ lb}\cdot\text{ft}^3$ .... **Ans.**

(c) $$EIv'' = M_r = \left[R_A x - 300x^2 + R_B\langle x-15\rangle\right] \text{ lb}\cdot\text{ft}$$

$$EIv' = \left[\frac{R_A x^2}{2} - 100x^3 + \frac{R_B\langle x-15\rangle^2}{2} + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - 25x^4 + \frac{R_B\langle x-15\rangle^3}{6} + C_1 x + C_2\right] \text{ lb}\cdot\text{ft}^3$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 15$ ft, $v = 0$: $15C_1 = 25(15)^4 - \dfrac{R_A(15)^3}{6}$ (a)

Continued on next slide

Problem 8-137 continued

At $x = 30$ ft , $v = 0$: $$30C_1 = 25(30)^4 - \frac{R_A(30)^3}{6} - \frac{R_B(15)^3}{6} \quad \text{(b)}$$

and from overall equilibrium:

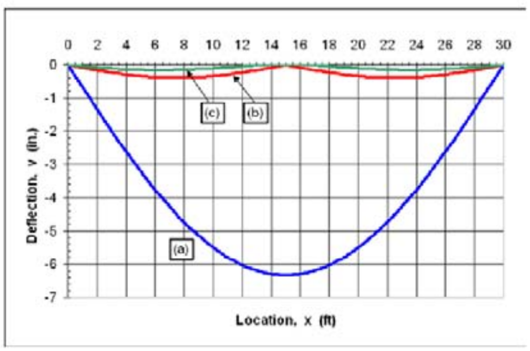
$$\circlearrowleft \Sigma M_D = 0: \quad (600\times 30)(15) - 30R_A - 15R_B = 0 \quad \text{(c)}$$

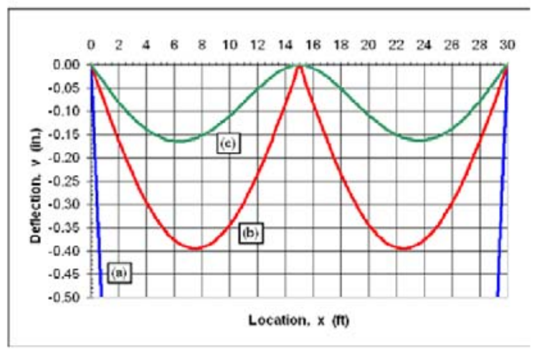
Solving Eqns. (a) – (c) simultaneously gives: $C_1 = -42,213 \text{ lb}\cdot\text{ft}^2$

$R_A = 3376 \text{ lb} = 3376 \text{ lb}\uparrow$ $\qquad R_B = 11,249 \text{ lb} = 11,249 \text{ lb}\uparrow$

$$EIv = \left[\frac{R_A x^3}{6} - 25x^4 + \frac{R_B\langle x-15\rangle^3}{6} + C_1 x\right] \text{lb}\cdot\text{ft}^3$$ .... **Ans.**

(Graphs are on the next page…)





8-138

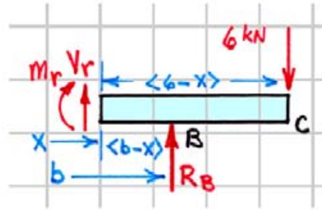
For a WT 178×51 section: $c_t = 32.8$ mm $c_b = 145.5$ mm

$$I = 13.6\left(10^6\right) \text{ mm}^4 \qquad EI = \left(200\times10^9\right)\left(13.6\times10^{-6}\right) = 2720 \text{ kN}\cdot\text{m}^2$$

$$EIv'' = M_r = \left[R_B\langle b-x\rangle - P\langle 6-x\rangle\right] \text{ kN}\cdot\text{m} \qquad P = 6 \text{ kN}$$

$$EIv' = \left[\frac{-R_B\langle b-x\rangle^2}{2} + \frac{P\langle 6-x\rangle^2}{2} + C_1\right] \text{ kN}\cdot\text{m}^2$$

$$EIv = \left[\frac{R_B\langle b-x\rangle^3}{6} - \frac{P\langle 6-x\rangle^3}{6} + C_1x + C_2\right] \text{ kN}\cdot\text{m}^3$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = \dfrac{-R_Bb^3}{6} + \dfrac{P(6)^3}{6}$

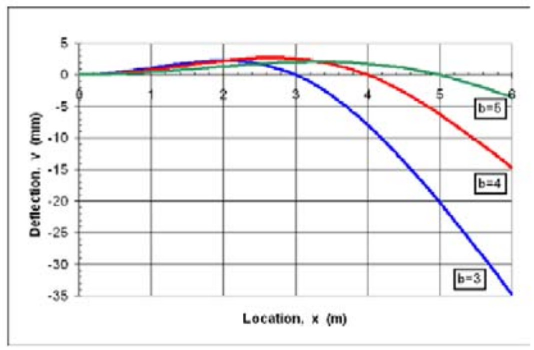
At $x = 0$, $v' = 0$:

$$C_1 = \frac{R_Bb^2}{2} - \frac{P(6)^2}{2}$$

At $x = b$, $v = 0$:

$$0 = \frac{-P(6-b)^3}{6} + C_1b + C_2$$

$$R_B = \frac{(6-b)^3 + 108b - 216}{2b^3}P$$



(a) $$EIv = \left[\frac{R_B\langle b-x\rangle^3 - P\langle 6-x\rangle^3 + 6C_1x + 6C_2}{6}\right] \text{ kN}\cdot\text{m}^3$$ .... **Ans.**

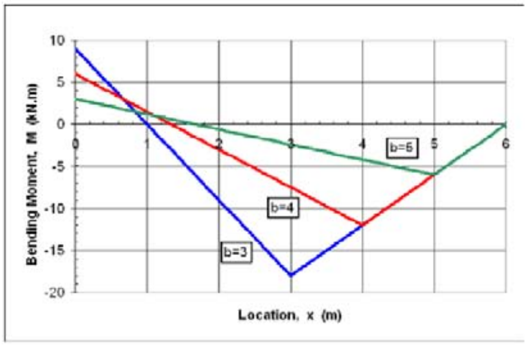
(b) $M_r = \left[R_B\langle b-x\rangle - P\langle 6-x\rangle\right] \text{ kN}\cdot\text{m}$ .... **Ans**

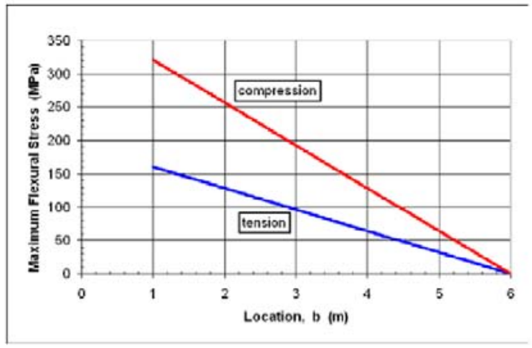
Continued on next slide

Problem 6-137 continued

(c) For any value of b the maximum positive bending moment occurs at $x = 0$ and the maximum negative bending moment occurs at $x = b$. The maximum tensile stress occurs on the bottom of the beam at the maximum positive bending moment and the maximum compressive stress occurs on the bottom of the beam at the maximum negative bending moment

$\sigma = Mc/I$ .... **Ans.**





8-139

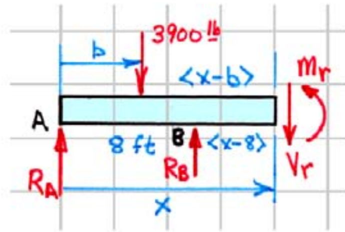
For a W 4×13 section: $I = 11.3 \text{ in.}^4$ $c = 2.08 \text{ in.}$

$$EI = (29\times10^6)(11.3) = 327.7(10^6) \text{ lb}\cdot\text{in.}^2$$

$$EIv'' = M_r = [R_A x + R_B\langle x-8\rangle - P\langle x-b\rangle] \text{ lb}\cdot\text{ft}$$ $P = 3900$ lb

$$EIv' = \left[\frac{R_A x^2}{2} + \frac{R_B\langle x-8\rangle^2}{2} - \frac{P\langle x-b\rangle^2}{2} + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{R_A x^3}{6} + \frac{R_B\langle x-8\rangle^3}{6} - \frac{P\langle x-b\rangle^3}{6} + C_1 x + C_2\right] \text{ lb}\cdot\text{ft}^3$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 8$ ft, $v = 0$: $8C_1 = \dfrac{P\langle 8-b\rangle^3}{6} - \dfrac{R_A(8)^3}{6}$ (a)

At $x = 16$ ft, $v = 0$: $16C_1 = \dfrac{P\langle 16-b\rangle^3}{6} - \dfrac{R_A(16)^3}{6} - \dfrac{R_B(8)^3}{6}$ (b)

and from overall equilibrium:

$\circlearrowleft\Sigma M_C = 0$: $P(16-b) - 16R_A - 8R_B = 0$ (c)

Solving Eqs. (a) – (b) simultaneously gives: $R_A = \dfrac{(16-b)^3 - 64(16-b) - 2\langle 8-b\rangle^3}{2048}P$

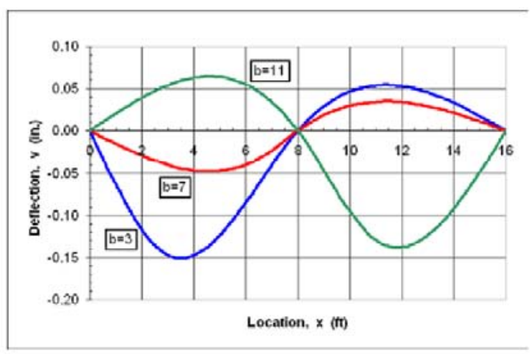
$R_B = \dfrac{P(16-b) - 16R_A}{8}$ $C_1 = \dfrac{P(8-b) - 512R_A}{48}$

(a) $EIv = \left[\dfrac{R_A x^3 + R_B\langle x-8\rangle^3 - P\langle x-b\rangle^3 + 6C_1 x}{6}\right] \text{ lb}\cdot\text{ft}^3$ .... **Ans.**

(b) $M_r = [R_A x + R_B\langle x-8\rangle - P\langle x-b\rangle] \text{ lb}\cdot\text{ft}$ .... **Ans.**

(c) Since the distance to the top and bottom of the beam is the same, the maximum flexural stress will occur at the location where the bending moment is a maximum.

$\sigma = Mc/I$ .... **Ans.**



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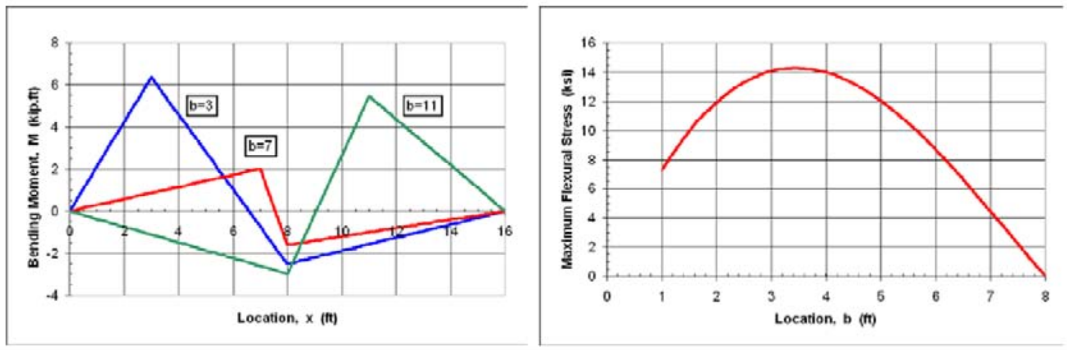
Problem 8-139 continued



8-140

For an S 457×104 section: $I = 358(10^6)\ \text{mm}^4$ $\qquad c = 457.2/2 = 228.6\ \text{mm}$

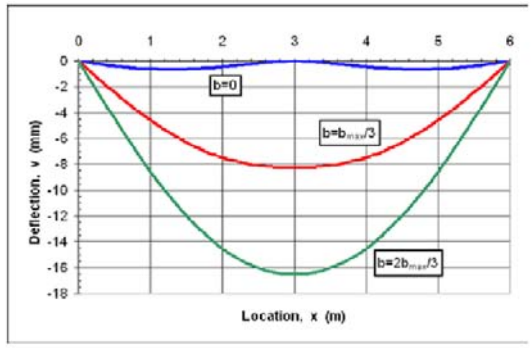
$$EI = (190\times10^9)(358\times10^{-6}) = 68.02(10^6)\ \text{N}\cdot\text{m}^2$$

(a) If no center support $(R_B = 0)$, then

$$b_{max} = \frac{5wL^4}{384EI} = \frac{5(100\times10^3)(6)^4}{384(68.02\times10^6)}$$

$b_{max} = 0.02481\ \text{m} \cong 24.8\ \text{mm}$ ....**Ans.**

$$R_A = R_C = wL/2 = (100\times6)/2 = 300\ \text{kN}$$



(b) If $b = 0$, then

$$\frac{R_B L^3}{48EI} = \frac{R_B(6)^3}{48(68.02\times10^6)} = 0.02481\ \text{m}$$

$R_B = 375.0(10^3)\ \text{N} = 375\ \text{kN}$ $\qquad R_A = R_C = (600-375)/2 = 112.5\ \text{kN}$

If $b = b_{max}/3 = 0.02481/3 = 0.00827\ \text{m}$, then

$$\frac{R_B L^3}{48EI} = \frac{R_B(6)^3}{48(68.02\times10^6)} = 0.02481 - 0.00827 = 0.01654\ \text{m}$$

$R_B = 250.0(10^3)\ \text{N} = 250\ \text{kN}$ $\qquad R_A = R_C = (600-250)/2 = 175.0\ \text{kN}$

If $b = 2b_{max}/3 = 2(0.02481)/3 = 0.01654\ \text{m}$, then

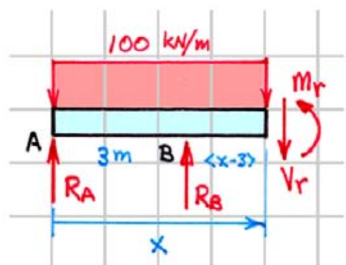
$$\frac{R_B L^3}{48EI} = \frac{R_B(6)^3}{48(68.02\times10^6)} = 0.02481 - 0.01654 = 0.00827\ \text{m}$$

$R_B = 125.0(10^3)\ \text{N} = 125\ \text{kN}$ $\qquad R_A = R_C = (600-125)/2 = 237.5\ \text{kN}$

$$EIv'' = M_r = \left[R_A x + R_B\langle x-3\rangle - 50x^2\right]\ \text{kN}\cdot\text{m}$$

$$EIv' = \left[\frac{R_A x^2}{2} + \frac{R_B\langle x-3\rangle^2}{2} - \frac{50x^3}{3} + C_1\right]\ \text{kN}\cdot\text{m}^2$$

$$EIv = \left[\frac{R_A x^3}{6} + \frac{R_B\langle x-3\rangle^3}{6} - \frac{25x^4}{6} + C_1 x + C_2\right]\ \text{kN}\cdot\text{m}^3$$



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Problem 8-140 continued

Boundary Conditions: At $x = 0$, $v = 0$: $C_2 = 0$

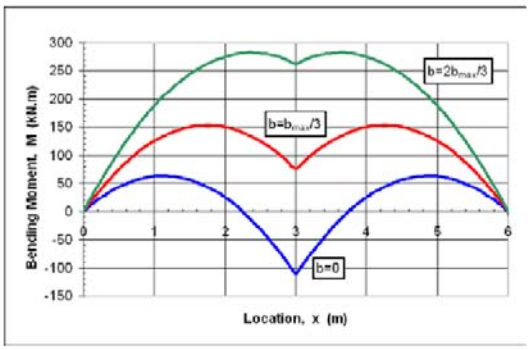
At $x = 3$ m, $v = -b$: $-EIb = \left[\frac{R_A(3)^3}{6} - \frac{(25)(3)^4}{6} + 3C_1 \right]$ kN·m³

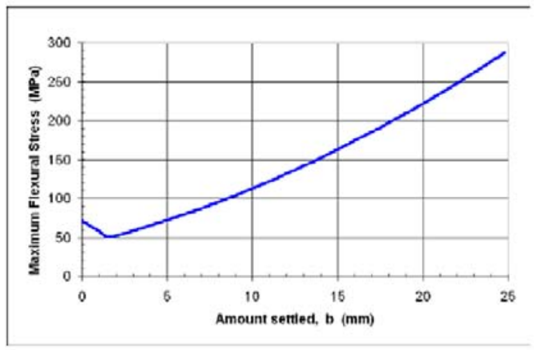
At $x = 6$ m, $v = 0$: $C_1 = 900 - 6R_A - 0.75R_B$

$$EIv = \left[\frac{R_A x^3}{6} + \frac{R_B \langle x-3 \rangle^3}{6} - \frac{25x^4}{6} + C_1 x \right] \text{ kN} \cdot \text{m}^3$$ .... **Ans.**

(b) $M_r = \left[R_A x + R_B \langle x-3 \rangle - 50x^2 \right]$ kN·m .... **Ans.**

(c) $\sigma = Mc/I$ .... **Ans.**





8-141

For an S 12×35 section: $I = 229$ in.^{4} $c = 6$ in.

$$EI = (29\times10^6)(229) = 6.641(10^9)\ \text{lb}\cdot\text{in.}^2 = 46.12(10^6)\ \text{lb}\cdot\text{ft}^2$$

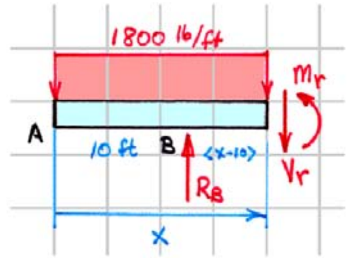
(a) If no left support $(R_A = 0)$, then from equilibrium

$R_C = 0$ and $R_B = wL = 1800\times20 = 36{,}000$ lb

$$EIv'' = M_r = \left[R_B\langle x-10\rangle - wx^2/2\right]\ \text{lb}\cdot\text{ft}$$

$$EIv' = \left[\frac{R_B\langle x-10\rangle^2}{2} - \frac{wx^3}{6} + C_1\right]\ \text{lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{R_B\langle x-10\rangle^3}{6} - \frac{wx^4}{24} + C_1x + C_2\right]\ \text{lb}\cdot\text{ft}^3$$



Boundary Conditions:

At $x = 10$ ft, $v = 0$: $10C_1 + C_2 = (1800)(10)^4/24 = 750{,}000$

At $x = 20$ ft, $v = 0$: $20C_1 + C_2 = \dfrac{(1800)(20)^4}{24} - \dfrac{R_B(10)^3}{6}$

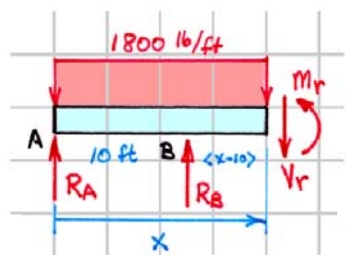
Solving yields $C = 525{,}000$ lb·ft^2 $C_2 = -4{,}500{,}000$ lb·ft^3

$$a_{\max} = -v_{x=0} = \frac{-C_2}{EI} = \frac{-(-4{,}500{,}000)}{46.12(10^6)} = 0.0975\ \text{ft} = 1.1709\ \text{in.}$$ .... **Ans.**

(b) If the support A settles a distance a $(v_{x=0} = -a)$, then

$$EIv'' = M_r = \left[R_Ax + R_B\langle x-10\rangle - wx^2/2\right]\ \text{lb}\cdot\text{ft}$$

$$EIv' = \left[\frac{R_Ax^2}{2} + \frac{R_B\langle x-10\rangle^2}{2} - \frac{wx^3}{6} + C_1\right]\ \text{lb}\cdot\text{ft}^2$$



$$EIv = \left[\frac{R_Ax^3}{6} + \frac{R_B\langle x-10\rangle^3}{6} - \frac{wx^4}{24} + C_1x + C_2\right]\ \text{lb}\cdot\text{ft}^3$$

Boundary Conditions:

At $x = 0$, $v = -a$: $C_2 = -EIa = \left[-(46.12\times10^6)a\right]$ lb·ft^3

At $x = 10$ ft, $v = 0$: $10C_1 + C_2 = \dfrac{(1800)(10)^4}{24} - \dfrac{R_A(10)^3}{6}$ (a)

At $x = 20$ ft, $v = 0$: $20C_1 + C_2 = \dfrac{(1800)(20)^4}{24} - \dfrac{R_A(20)^3}{6} - \dfrac{R_B(10)^3}{6}$ (b)

and from overall equilibrium:

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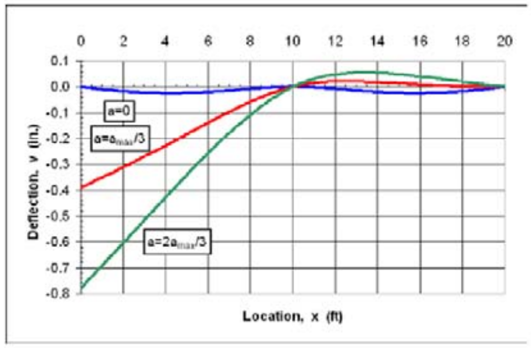
Problem 8-141 continued

$\circlearrowleft \Sigma M_C = 0: \qquad 20R_A + 10R_B = (1800\times 20)(10) \qquad$ (c)



Solving Eqs. (a) – (c) simultaneously gives:

$$R_A = \left[\frac{(864\times 10^6)+192C_2}{128{,}000}\right] \text{ lb}$$

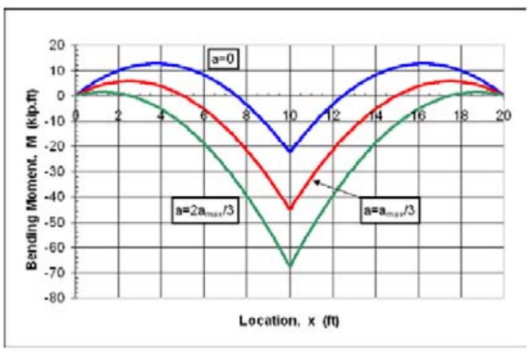
$$R_B = (36{,}000 - 2R_A) \text{ lb}$$

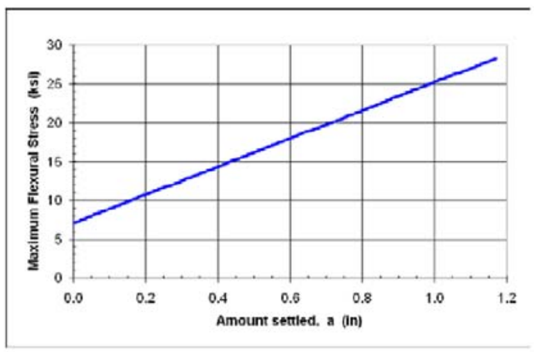
$$C_1 = \left(75{,}000 - \frac{50R_A}{3} - \frac{C_2}{10}\right) \text{ lb}\cdot\text{ft}^2$$

and $EIv = \left[\dfrac{R_A x^3}{6} + \dfrac{R_B\langle x-10\rangle^3}{6} - 75x^4 + C_1 x + C_2\right] \text{ lb}\cdot\text{ft}^3$ .... **Ans.**

(b) $M_r = \left[R_A x + R_B\langle x-10\rangle - 900x^2\right] \text{ lb}\cdot\text{ft}$ .... **Ans.**

(c) $\sigma = M_r c/I = M_r(6)/(29)$ .... **Ans.**

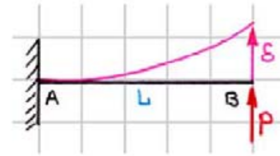




8-142*

Using the solutions for cases 1 and 4 in Table B-19

$$\delta_B = v_M + v_{R_B} = \frac{-ML^2}{2EI} + \frac{R_B L^3}{3EI} = 0$$



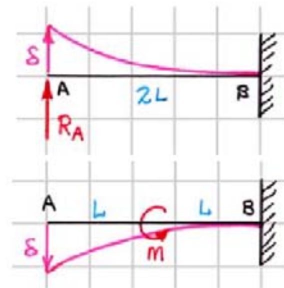
$$R_B = \frac{3M}{2L} = \frac{3M}{2L}\uparrow$$ ....Ans.

Then from equilibrium

$\uparrow \Sigma F_y = 0:$ $\quad R_A + \dfrac{3M}{2L} = 0$ $\quad R_A = \dfrac{-3M}{2L} = \dfrac{3M}{2L}\downarrow$ .... Ans.

$\circlearrowleft \Sigma M_A = 0:$ $\quad \dfrac{3M}{2L}(L) - M_A - M = 0$ $\quad M_A = \dfrac{+M}{2} = \dfrac{M}{2} \circlearrowright$ .... Ans.

8-143*



Using the solutions for cases 1 and 4 in Table B-19

$$\delta_A = v_{R_A} + v_M + \theta_M L = \frac{R_A (2L)^3}{3EI} - \frac{ML^2}{2EI} - \frac{ML}{EI}(L) = 0$$

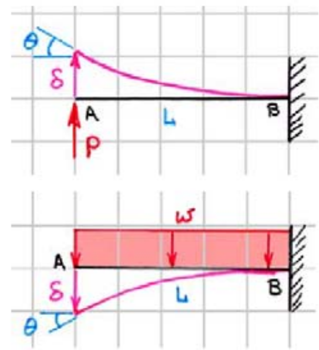
$$R_A = \frac{9M}{16L} = \frac{9M}{16L} \uparrow$$ .... **Ans.**

Then from equilibrium

$\uparrow \Sigma F_y = 0:$ $\qquad \frac{9M}{16L} + R_B = 0$ $\qquad R_B = \frac{-9M}{16L} = \frac{9M}{16L} \downarrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0:$ $\qquad M_B + M - \frac{9M}{16L}(2L) = 0$ $\qquad M_B = \frac{+M}{8} = \frac{M}{8} \circlearrowleft$ .... **Ans.**

8-144

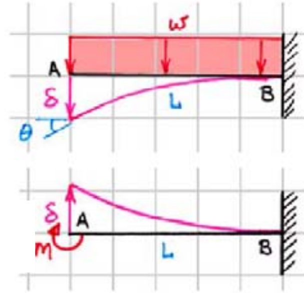


Using the solutions for cases 1 and 2 in Table B-19

$$\theta_A = \theta_P + \theta_w = \frac{P(L)^3}{2EI} - \frac{wL^3}{6EI} = 0$$

$P = \dfrac{wL}{3}$ ....**Ans.**

8-145*



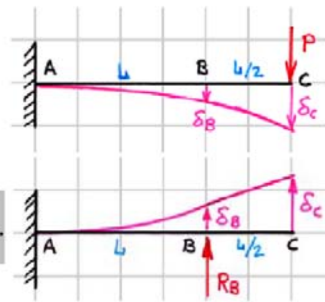
Using the solutions for cases 2 and 4 in Table B-19

$$\delta_A = v_M + v_w = \frac{M(L)^2}{2EI} - \frac{wL^4}{8EI} - = 0$$

$$M = \frac{wL^2}{4}$$ ....**Ans.**

8-146

Using the solution for case 1 and 4 in Table B-19



$$\delta_B = v_{R_B} + v_P = \frac{R_B(L)^3}{3EI} - \frac{PL^2}{6EI}\left[3\left(\frac{3L}{2}\right) - (L)\right] = 0$$

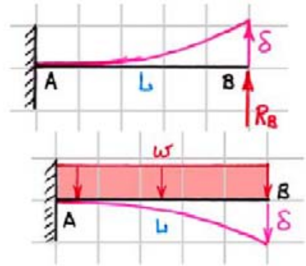
$$R_A = \frac{7P}{4} = \frac{7P}{4}\uparrow$$ ....**Ans.**

Then from equilibrium

$\uparrow \Sigma F_y = 0$: $\quad R_A + \frac{7P}{4} - P = 0 \quad$ $R_A = \frac{-3P}{4} = \frac{3P}{4}\downarrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0$: $\quad \frac{7P}{4}(L) - M_A - P\left(\frac{3L}{2}\right) = 0 \quad$ $M_A = \frac{PL}{4} = \frac{PL}{4} \circlearrowright$ .... **Ans.**

8-147



Using the solutions for cases 1 and 2 in Table B-19

$$\delta_B = v_{R_B} + v_w = \frac{R_B L^3}{3EI} - \frac{wL^4}{8EI} = 0$$

$$R_B = \frac{3wL}{8} = \frac{3wL}{8} \uparrow$$ ....**Ans.**

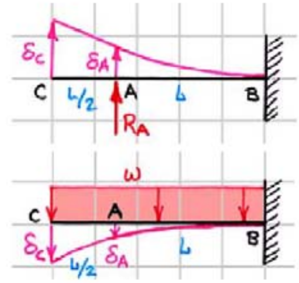
Then from equilibrium

$\uparrow \Sigma F_y = 0:$ $\quad R_A - wL + \frac{3wL}{8} = 0$ $\qquad R_A = \frac{5wL}{8} = \frac{5wL}{8} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0:$ $\quad M_A - \frac{wL^2}{2} + \frac{3wL}{8}(L) = 0$ $\qquad M_A = \frac{wL^2}{8} = \frac{wL^2}{8} \circlearrowleft$ .... **Ans.**

8-148*

Using the solutions for cases 1 and 2 in Table B-19



$$\delta_A = v_{R_A} + v_w = \frac{R_A L^3}{3EI} - \frac{wL^2}{24EI}\left[L^2 - 4\left(\frac{3L}{2}\right)(L) + 6\left(\frac{3L}{2}\right)\right] = 0$$

$$R_A = \frac{17wL}{16} = \frac{17wL}{16} \uparrow$$ ....**Ans.**

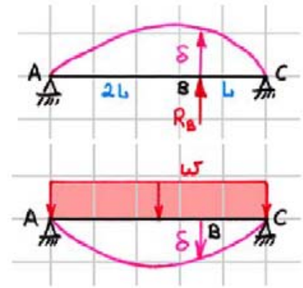
Then from equilibrium

$\uparrow \Sigma F_y = 0:$ $\quad \dfrac{17wL}{16} - \dfrac{3wL}{2} + R_B = 0$ $\quad R_B = \dfrac{7wL}{16} = \dfrac{7wL}{16} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0:$ $\quad \dfrac{3wL}{2}\left(\dfrac{3L}{4}\right) - \dfrac{17wL}{16} - M_B = 0$ $\quad M_B = \dfrac{wL^2}{16} = \dfrac{wL^2}{16} \circlearrowright$ .... **Ans.**

8-149

Using the solutions for cases 1 and 4 in Table B-19



$$\delta_B = v_{R_B} + v_w$$

$$= \frac{R_B(L)(2L)}{6EI(3L)}\left[(3L)^2 - (L)^2 - (2L)^2\right]$$

$$- \frac{w(2L)}{24EI}\left[(2L)^3 - 2(3L)(2L)^2 + (3L)^3\right] = 0$$

$$R_B = \frac{33wL}{16} = \frac{33wL}{16}\uparrow$$ .... **Ans.**

Then from equilibrium

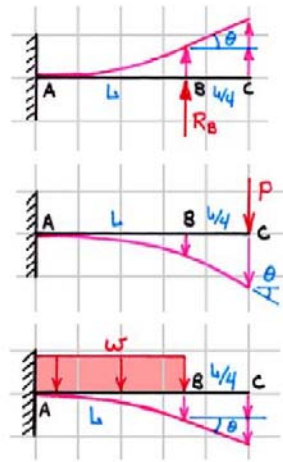
$$\circlearrowleft \Sigma M_C = 0: \quad (3wL)\left(\frac{3L}{2}\right) - R_A(3L) - \frac{33wL}{16}(L) = 0$$

$$R_A = \frac{13wL}{16} = \frac{13wL}{16}\uparrow$$ .... **Ans.**

$$\uparrow \Sigma F_y = 0: \frac{13wL}{16} + \frac{33wL}{16} + R_C - 3wL = 0$$

$$R_C = \frac{2wL}{16} = \frac{wL}{8}\uparrow$$ .... **Ans.**

8-150*



Using the solutions for cases 1 and 2 in Table B-19

$$\delta_B = v_{R_B} + v_w + v_P = \frac{R_B L^3}{3EI} - \frac{wL^4}{8EI} - \frac{PL^2}{6EI}\left[3\left(\frac{5L}{4}\right) - (L)\right] = 0$$

$$\theta_C = \theta_{R_B} + \theta_w + \theta_P = \frac{R_B L^2}{2EI} - \frac{wL^3}{6EI} - \frac{P(5L/4)^2}{2EI} = 0$$

Solving simultaneously gives

(a) $P = \dfrac{2wL}{9}$ .... **Ans.**

(b) $R_B = \dfrac{49wL}{72} = \dfrac{49wL}{72}\uparrow$ .... **Ans.**

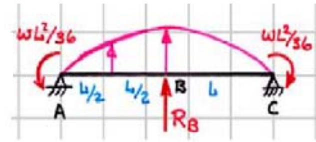
Then from equilibrium

$\uparrow \Sigma F_y = 0: \quad R_A - wL + \dfrac{49wL}{72} - \dfrac{2wL}{9} = 0$ $\qquad R_A = \dfrac{13wL}{24} = \dfrac{13wL}{24}\uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0: \quad M_A - \dfrac{wL^2}{2} + \dfrac{49wL}{72}(L) - \dfrac{2wL}{9}\left(\dfrac{5wL}{4}\right) = 0$

$M_A = \dfrac{7wL^2}{72} = \dfrac{7wL^2}{72} \circlearrowleft$ .... **Ans.**

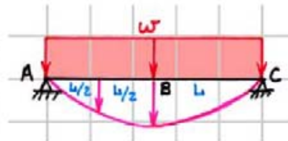
8-151*



(a) Using the solutions for cases 6, 7, and 8 in Table B-19
with $M_A = M_C = wL^2/36$

$$\delta_B = v_{M_A} + v_{M_C} + v_w + v_{R_B} = 0$$

$$\frac{(wL^2/36)(2L)^2}{16EI} + \frac{(wL^2/36)(2L)^2}{16EI} - \frac{5w(2L)^4}{384EI} + \frac{R_B(2L)^3}{48EI} = 0$$



$$R_B = \frac{7wL}{6} = \frac{7wL}{6}\uparrow$$ .... **Ans.**

and from symmetry

$$R_A = R_C = \frac{2wL - R_B}{2} = \frac{5wL}{12}\uparrow$$ .... **Ans.**

(b) $$\delta_M = v_{x=L/2} = v_{M_A} + v_{M_C} + v_w + v_{R_B}$$

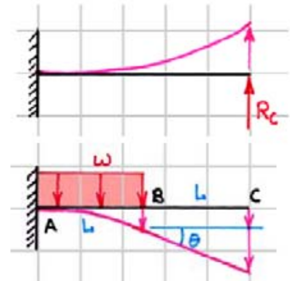
$$= \frac{(wL^2/36)(L/2)}{6EI(2L)}\left[(2L)^2 - \left(\frac{L}{2}\right)^2\right] + \frac{(wL^2/36)(3L/2)}{6EI(2L)}\left[(2L)^2 - \left(\frac{3L}{2}\right)^2\right]$$

$$- \frac{w(L/2)}{24EI}\left[\left(\frac{L}{2}\right)^3 - 2(2L)\left(\frac{L}{2}\right)^2 + (2L)^3\right] + \frac{(7wL/6)(L/2)}{48EI}\left[3(2L)^2 - 4\left(\frac{L}{2}\right)^2\right]$$

$$\delta_M = \frac{-5wL}{1152EI} = \frac{5wL}{1152EI}\downarrow$$ .... **Ans.**

8-152

(a) Using the solutions for cases 1 and 2 in Table B-19



$$\delta_C = v_{R_C} + v_{Bw} + \theta_{Bw} L = \frac{R_C (2L)^3}{3EI} - \frac{wL^4}{8EI} - \frac{wL^3}{6EI}(L) = 0$$

$$R_C = \frac{7wL}{64} = \frac{7wL}{64} \uparrow$$ .... **Ans.**

Then from equilibrium

$\uparrow \Sigma F_y = 0:$ $\quad R_A - wL + \dfrac{7wL}{64} = 0$ $\qquad R_A = \dfrac{57wL}{64} = \dfrac{57wL}{64} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0:$ $\quad M_A + \dfrac{7wL}{64}(2L) - \dfrac{wL^2}{2} = 0$ $\qquad M_A = \dfrac{9wL^2}{32} = \dfrac{9wL^2}{32} \circlearrowleft$ .... **Ans.**

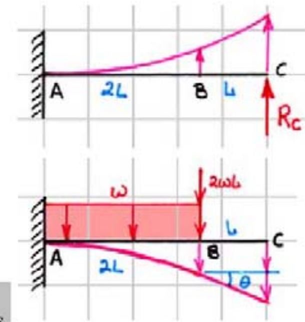
(b) $\delta_B = v_{x=L} = v_{R_C} + v_w$

$$= \frac{(7wL/64)(L)^2}{6EI}\left[3(2L) - (L)\right] - \frac{wL^4}{8EI}$$

$$\delta_B = \frac{-13wL}{384EI} = \frac{13wL}{384EI} \downarrow$$ .... **Ans.**

8-153*

(a) Using the solutions for cases 1 and 2 in Table B-19



$$\delta_C = v_{R_C} + v_{Bw} + \theta_{Bw}L + v_{BP} + \theta_{wP}L$$

$$= \frac{R_C(3L)^3}{3EI} - \frac{w(2L)^4}{8EI} - \frac{w(2L)^3}{6EI}(L)$$

$$- \frac{(2wL)(2L)^3}{3EI} - \frac{(2wL)(2L)^2}{2EI}(L) = 0$$

$R_C = \dfrac{38wL}{27} = \dfrac{38wL}{27}\uparrow$ ....**Ans.**

Then from equilibrium

$\uparrow\Sigma F_y = 0: \quad R_A - 2wL - 2wL + \dfrac{38wL}{27} = 0$ $\quad R_A = \dfrac{70wL}{27} = \dfrac{70wL}{27}\uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0: \quad M_A - (2wL)(L) - (2wL)(2L) + \dfrac{38wL}{27}(3L) = 0$

$M_A = \dfrac{16wL^2}{9} = \dfrac{16wL^2}{9}\ \circlearrowleft$ .... **Ans.**

(b) $\delta_B = v_{x=2L} = v_{R_B} + v_w + v_P$

$$= \frac{(38wL/27)(2L)^2}{6EI}\left[3(3L)-(2L)\right] - \frac{w(2L)^4}{8EI} - \frac{(2wL)(2L)^3}{3EI}$$

$\delta_B = \dfrac{-62wL^4}{81EI} = \dfrac{62wL^4}{81EI}\downarrow$ .... **Ans.**

8-154

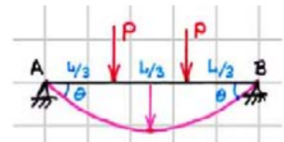
By symmetry: $M_A \circlearrowleft = M_B \circlearrowright$

and $R_A = R_B = P \uparrow$ .... **Ans.**

(a) Using the solutions for cases 5 and 8 in Table B-19

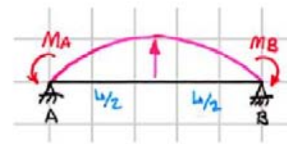
$$\theta_A = \theta_{AP_1} + \theta_{AP_2} + \theta_{AM_1} + \theta_{AM_2}$$

$$= \frac{-P(L/3)(8L^2/9)}{6EIL} - \frac{P(2L/3)(5L^2/9)}{6EIL} + \frac{M_A L}{3EI} + \frac{M_B L}{6EI} = 0$$



$M_A = \frac{2PL}{9} = \frac{2PL}{9} \circlearrowleft$ .... **Ans.**

$M_B = \frac{2PL}{9} \circlearrowright$ .... **Ans.**

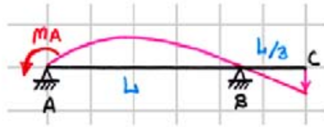


(b) $\delta_M = v_{AP_1} + v_{AP_2} + v_{AM_1} + v_{AM_2}$

$$= \frac{-P(L/3)(23L^2/9)}{48EI} - \frac{P(L/3)(23L^2/9)}{48EI} + \frac{(2PL/9)(L)^2}{16EI} + \frac{(2PL/9)(L)^2}{16EI}$$

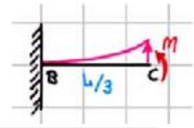
$\delta_M = \frac{-5PL^3}{648EI} = \frac{5PL^3}{648EI} \downarrow$ .... **Ans.**

8-155
Using the solutions for cases 4 and 8 in Table B-19 with $M_A = PL/3$ and $M_B = M$



(a) $\theta_C = \theta_B + \theta_{CM} = \theta_{BM_A} + \theta_{BM_B} + \theta_{CM}$

$$= \frac{-(PL/3)(L)}{6EI} + \frac{ML}{3EI} + \frac{M(L/3)}{EI} = 0$$



$M = \frac{PL}{12} = \frac{PL}{12}$ ↺ .... **Ans.**

(b) $\delta_C = \theta_B(L/3) + \delta_{CM} = \theta_{BM_A}(L/3) + \theta_{BM_B}(L/3) + \delta_{CM}$

$$= \frac{-(PL/3)(L)}{6EI}\left(\frac{L}{3}\right) + \frac{ML}{3EI}\left(\frac{L}{3}\right) + \frac{M(L/3)^2}{2EI} = 0$$

$M = \frac{PL}{9} = \frac{PL}{9}$ ↺ .... **Ans.**

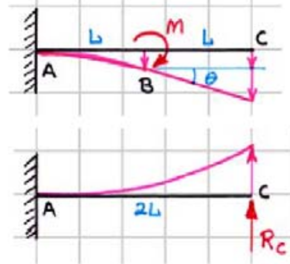
8-156*

Using the solutions for cases 1 and 4 in Table B-19

$$\delta_C = v_{CR_C} + v_{BM} + \theta_{BM} L$$

$$= \frac{R_C (2L)^3}{3EI} - \frac{ML^2}{2EI} - \frac{ML}{EI}(L) = 0$$

$$R_C = \frac{9M}{16L} = \frac{9M}{16L} \uparrow \text{ .... Ans.}$$



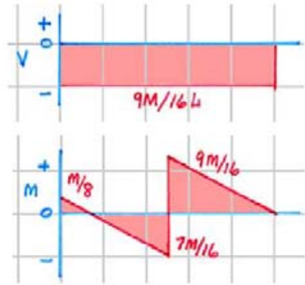
Then from equilibrium

$$\uparrow \Sigma F_y = 0: \qquad R_A + \frac{9M}{16L} = 0$$

$$R_A = \frac{-9M}{16L} = \frac{9M}{16L} \downarrow \text{ .... Ans.}$$

$$\circlearrowleft \Sigma M_A = 0: \quad M_A - M + \frac{9M}{16L}(2L) = 0$$

$$M_A = \frac{-M}{8} = \frac{M}{8} \circlearrowright \text{ .... Ans.}$$



$$\theta_A = \theta_{AR_A} + \theta_{AM} + \theta_{Aw} = -\frac{R_A L^2}{2EI} + \frac{M_A L}{EI} + \frac{wL^3}{24EI} = 0$$

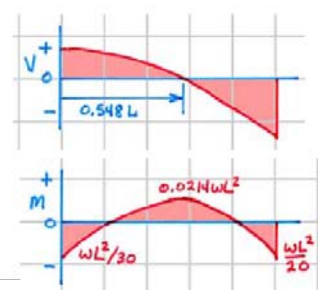
Solving simultaneously gives

$$R_A = \frac{3wL}{20} = \frac{3wL}{20} \uparrow \text{ .... Ans.}$$

$$M_A = \frac{wL^2}{30} = \frac{wL^2}{30} \circlearrowleft \text{ .... Ans.}$$

Then from equilibrium

$$\uparrow \Sigma F_y = 0: \quad \frac{3wL}{20} - \frac{wL}{2} + R_B = 0$$



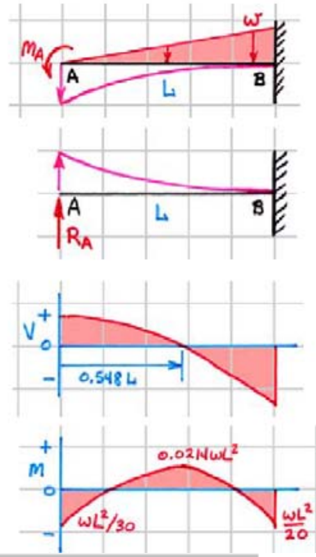
$$R_B = \frac{7wL}{20} = \frac{7wL}{20} \uparrow \text{ .... Ans.}$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{wL}{2}\left(\frac{L}{3}\right) + \frac{wL^2}{30} - \frac{3wL}{20}(L) - M_B = 0$$

$$M_B = \frac{wL^2}{20} = \frac{wL^2}{20} \circlearrowright \text{ .... Ans.}$$

8-157

Using the solutions for cases 1, 3, and 4 in Table B-19



$$\delta_A = v_{AR_A} + v_{AM} + \delta_{Aw} = \frac{R_A L^3}{3EI} - \frac{M_A L^2}{2EI} - \frac{wL^4}{30EI} = 0$$

$$\theta_A = \theta_{AR_A} + \theta_{AM} + \theta_{Aw} = -\frac{R_A L^2}{2EI} + \frac{M_A L}{EI} + \frac{wL^3}{24EI} = 0$$

Solving simultaneously gives

$$R_A = \frac{3wL}{20} = \frac{3wL}{20} \uparrow$$ .... **Ans.**

$$M_A = \frac{wL^2}{30} = \frac{wL^2}{30} \circlearrowleft$$ .... **Ans.**

Then from equilibrium

$$\uparrow \Sigma F_y = 0: \quad \frac{3wL}{20} - \frac{wL}{2} + R_B = 0$$

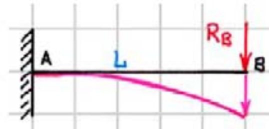
$$R_B = \frac{7wL}{20} = \frac{7wL}{20} \uparrow$$ .... **Ans.**

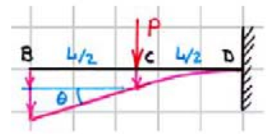
$$\circlearrowleft \Sigma M_B = 0: \quad \frac{wL}{2}\left(\frac{L}{3}\right) + \frac{wL^2}{30} - \frac{3wL}{20}(L) - M_B = 0$$

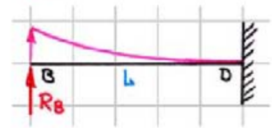
$$M_B = \frac{wL^2}{20} = \frac{wL^2}{20} \circlearrowright$$ .... **Ans.**

8-158*

Using the solution for case 1 in Table B-19







$$\delta_{B/A} = \delta_{B/D}$$

$$\delta_{BR_B} = \delta_{CP} + \theta_{CP}\left(L/2\right) + \delta_{BR_B}$$

$$\frac{-R_B L^3}{3EI} = \frac{-P\left(L/2\right)^3}{3EI} - \frac{P\left(L/2\right)^2}{2EI}\left(\frac{L}{2}\right) + \frac{R_B L^3}{3EI}$$

For beam AB: $R_B = \frac{5P}{32} = \frac{5P}{32} \downarrow$ ....**Ans.**

$\uparrow \Sigma F_y = 0:$ $R_A - \frac{5P}{32} = 0$ $R_A = \frac{5P}{32} = \frac{5P}{32} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0:$ $M_A - \frac{5P}{32}(L) = 0$ $M_A = \frac{5PL}{32} = \frac{5PL}{32} \circlearrowleft$ .... **Ans.**

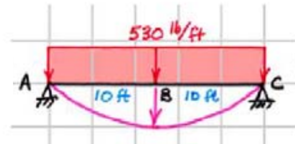
For beam CD: $R_B = \frac{5P}{32} = \frac{5P}{32} \uparrow$ .... **Ans.**

$\uparrow \Sigma F_y = 0:$ $\frac{5P}{32} - P + R_D = 0$ $R_D = \frac{27P}{32} = \frac{27P}{32} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_D = 0:$ $P\left(\frac{L}{2}\right) - \frac{5P}{32}(L) - M_D = 0$ $M_D = \frac{11PL}{32} = \frac{11PL}{32} \circlearrowright$ .... **Ans.**

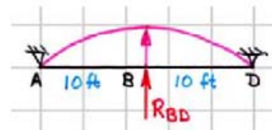
8-159*

$$EI_{beam} = (29\times10^6)(120) = 3.480(10^9)\ \text{lb}\cdot\text{in.}^2$$



$$EA_{post} = (1.5\times10^6)(6\times6) = 54.00(10^6)\ \text{lb}$$

Using the solutions for cases 6 and 7 in Table B-19



$$\delta_{beam} = v_{Bw} + v_{BR_{BD}} = \delta_{post}$$

$$\frac{-5wL^4}{384EI} + \frac{R_{BD}L^3}{48EI} = \frac{-R_{BD}L_{BD}}{A_{BD}E_{BD}}$$

$$\frac{-5(530/12)(20\times12)^4}{384(3.480\times10^9)} + \frac{R_{BD}(20\times12)^3}{48(3.480\times10^9)} = \frac{-R_{BD}(20\times12)}{(54.00\times10^6)}$$

$R_{BD} = 6287\ \text{lb} \cong 6.29\ \text{kip}$ .... **Ans.**

8-160

$$EI_{beam} = \left(70\times10^9\right)\left(30\times10^{-6}\right) = 2.100\left(10^6\right)\ \text{N}\cdot\text{m}^2$$

Using the solutions for cases 1 and 2 in Table B-19

$$\delta_{beam} = v_{BF_s} + v_{Bw} = \delta_{spring}$$

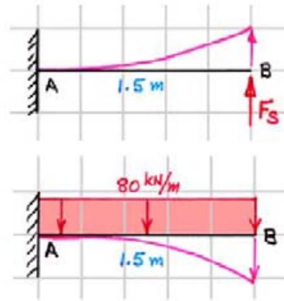
$$\frac{F_S L^3}{3EI} - \frac{wL^4}{8EI} = \frac{-F_S}{k}$$

$$\frac{F_S\left(1.50\right)^3}{3\left(2.100\times10^6\right)} - \frac{\left(80,000\right)\left(1.50\right)^4}{8\left(2.100\times10^6\right)} = \frac{-F_S}{900\left(10^3\right)}$$

$$F_S = 14,639\ \text{N}$$

$$F_S = k\delta_S = 900\left(10^3\right)\delta_S = 14,639\ \text{N}$$

$\delta_S = 0.01627\ \text{m} = 16.27\ \text{mm}$ .... **Ans.**



8-161*

For an S 4×9.5 section: $d = 4.00$ in. $I = 6.79$ in.^{4} $S = 3.39$ in.^{3}

(Beam *AB*) $t_w = 0.326$ in. $t_f = 0.293$ in. $w_f = 2.796$ in.

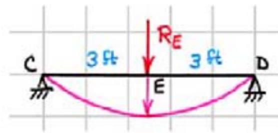
$$EI = \left(29\times10^6\right)(6.79) = 196.91\left(10^6\right)\ \text{lb}\cdot\text{in.}^2$$

For an S 5×14.75 section: $d = 5.00$ in. $I = 15.2$ in.^{4} $S = 6.09$ in.^{3}

(Beam *CD*) $t_w = 0.494$ in. $t_f = 0.326$ in. $w_f = 3.284$ in.

$$EI = \left(29\times10^6\right)(15.2) = 440.80\left(10^6\right)\ \text{lb}\cdot\text{in.}^2$$

Using the solutions for cases 1 and 6 in Table B-19



$$\delta_{E/B} = \delta_{E/D}$$

$$\delta_{ER_E} + \delta_{EP} = \delta_{ER_E}$$

$$\frac{R_E L_{AB}^3}{3(EI)_{AB}} - \frac{Px^2}{6(EI)_{AB}}(3L_{AB} - x) = \frac{-R_E L_{CD}^3}{48(EI)_{CD}}$$

$$\frac{R_E(36)^3}{3\left(196.91\times10^6\right)} - \frac{(1600)(36)^2}{6\left(196.91\times10^6\right)}\left[3(54)-(36)\right] = \frac{-R_E(6\times12)^3}{48\left(440.80\times10^6\right)}$$

$$R_E = 2289 \text{ lb}$$

For beam *AB*: $R_E = 2289$ lb ↑

$\uparrow\Sigma F_y = 0$: $2289 - 1600 + R_B = 0$ $R_B = -689 \text{ lb} = 689 \text{ lb}\downarrow$

$\circlearrowleft\Sigma M_B = 0$: $1600(4.5) - 2289(3) - M_B = 0$ $M_B = 333\ \text{lb}\cdot\text{ft} = 333\ \text{lb}\cdot\text{ft}\ \circlearrowright$

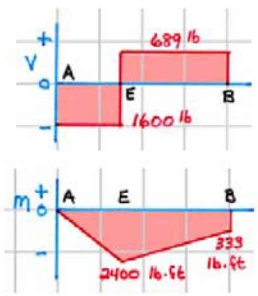
From the shear-force and bending-moment diagrams:

$V_{max} = 1600$ lb $M_{max} = 2400\ \text{lb}\cdot\text{ft}$

Continued on next slide

Problem 8-161 continued

$$Q = 1.8535(2.796 \times 0.293) + 0.8535(1.707 \times 0.326)$$
$$= 1.9936 \text{ in.}^3$$



(a) $\sigma_{max} = \dfrac{M}{S} = \dfrac{(2400 \times 12)}{3.39} = 8496$ psi

$\sigma_{max} = 8.50$ ksi (T, top; C, bottom) .... **Ans.**

(b) $\tau_{max} = \dfrac{VQ}{It} = \dfrac{(1600)(1.9936)}{(6.79)(0.326)} = 1441$ psi

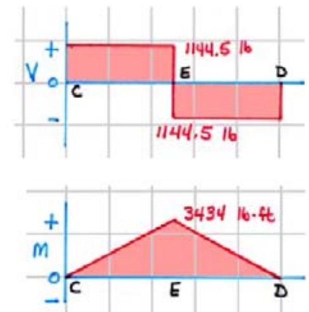
$\tau_{max} = 1.441$ ksi .... **Ans.**

For beam *CD*: $R_E = 2289$ lb ↓

From symmetry $R_C = R_D = 1144.5$ lb ↑

From the shear-force and bending-moment diagrams:

$$V_{max} = 1144.5 \text{ lb}$$
$$M_{max} = 3434 \text{ lb} \cdot \text{ft}$$
$$Q = 2.337(3.284 \times 0.326) + 1.087(2.174 \times 0.494)$$
$$= 3.669 \text{ in.}^3$$



(a) $\sigma_{max} = \dfrac{M}{S} = \dfrac{(3434 \times 12)}{6.09} = 6767$ psi $= 6.77$ ksi (T, bottom; C, top) .... **Ans.**

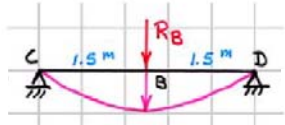
(b) $\tau_{max} = \dfrac{VQ}{It} = \dfrac{(1144.5)(3.669)}{(15.2)(0.494)} = 559$ psi .... **Ans.**

8-162

$$I_{AB} = I_{BC} = (100)(100)^3/12 = 8.333(10^6)\ \text{mm}^4 \qquad (EI)_{AB} = (EI)_{BC} = EI$$

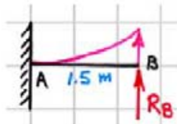
Using the solutions for cases 1 and 6 in Table B-19

$$\delta_{B/A} = \delta_{B/D}$$

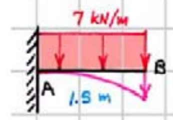


$$\delta_{BR_B} + \delta_{Bw} = \delta_{BR_B}$$

$$\frac{R_B L_{AB}{}^3}{3(EI)_{AB}} - \frac{wL_{AB}^4}{8(EI)_{AB}} = \frac{-R_B (L_{CD})^3}{48(EI)_{CD}}$$



$$\frac{R_B(1.5)^3}{3EI} - \frac{(7000)(1.5)^4}{8EI} = \frac{-R_B(3)^3}{48EI} \qquad R_B = 2625.0\ \text{N}$$



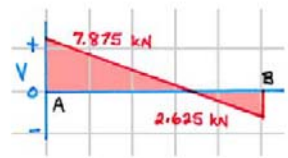
For beam AB: $\qquad R_B = 2625.0\ \text{N}\uparrow$

$\uparrow\Sigma F_y = 0: \quad R_A - (7000)(1.5) + 2625 = 0 \qquad R_A = 7875.0\ \text{N}\uparrow$

$\circlearrowleft\Sigma M_A = 0: \quad M_A - (7000\times1.5)(0.75) + (2625)(1.5) = 0$

$$M_A = 3937.5\ \text{N}\cdot\text{m} = 3937.5\ \text{N}\cdot\text{m}\ \circlearrowleft$$

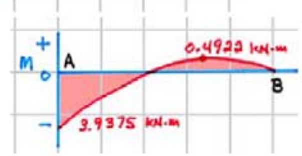
From the shear-force and bending-moment diagrams:



$$V_{max} = 7875\ \text{N} \qquad M_{max} = -3937.5\ \text{N}\cdot\text{m}$$

$$Q = 25(50\times100) = 125.00(10^{-3})\ \text{mm}^3$$

(a) $$\sigma_{max} = \frac{Mc}{I} = \frac{(3937.5)(0.050)}{8.333(10^{-6})} = 23.63(10^6)\ \text{N/m}^2$$



σ_{max} = 23.6 MPa (T, bottom; C, top) .... **Ans.**

(b) $$\tau_{max} = \frac{VQ}{It} = \frac{(7875)(125\times10^{-6})}{(8.333\times10^{-6})(0.100)} = 1.1813(10^6)\ \text{N/m}^2$$

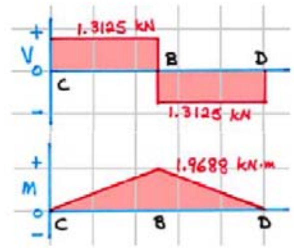
τ_{max} = 1.181 MPa .... **Ans.**

Continued on next slide

Problem 8-162 continued

For beam *CD*: $R_B = 2625.0 \text{ N} \downarrow$

From symmetry $R_C = R_D = 1312.5 \text{ N} \uparrow$



From the shear-force and bending-moment diagrams:

$V_{max} = 1312.5 \text{ N}$ $M_{max} = 1968.8 \text{ N} \cdot \text{m}$

$Q = 25(50 \times 100) = 125.00(10^{-3}) \text{ mm}^3$

(a) $\sigma_{max} = \dfrac{Mc}{I} = \dfrac{(1968.8)(0.050)}{8.333(10^{-6})} = 11.813(10^6) \text{ N/m}^2$

$\sigma_{max} = 11.81 \text{ MPa}$ (T, bottom; C, top) .... **Ans.**

(b) $\tau_{max} = \dfrac{VQ}{It} = \dfrac{(1312.5)(125 \times 10^{-6})}{(8.333 \times 10^{-6})(0.100)} = 0.19688(10^6) \text{ N/m}^2$

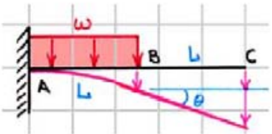
$\tau_{max} = 0.1969 \text{ MPa}$ .... **Ans.**

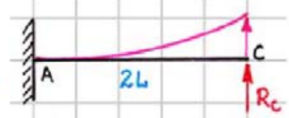
8-163

Using the solutions for cases 1, 2, and 4 in Table B-19

$$\theta_C = \theta_{Cw} + \theta_{CM} + \theta_R = 0$$

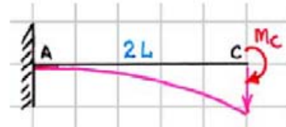
$$\frac{-wL^3}{6EI} - \frac{M_C(2L)}{EI} + \frac{R_C(2L)^2}{2EI} = 0$$





$$\delta_C = \delta_{Cw} + \delta_{CM} + \delta_R = 0$$

$$\left[\frac{-wL^4}{8EI} - \frac{wL^3}{6EI}(L)\right] - \frac{M_C(2L)^2}{2EI} + \frac{R_C(2L)^3}{3EI} = 0$$



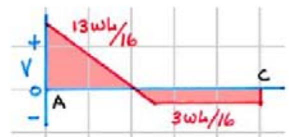
Solving simultaneously gives:

$R_C = \dfrac{+3wL}{16} = \dfrac{3wL}{16} \uparrow$ .... $M_C = \dfrac{5wL^2}{48} = \dfrac{5wL^2}{48}$ ↻ .... **Ans.**

Then from equilibrium

$$\uparrow \Sigma F_y = 0: \quad R_A - wL + \frac{3wL}{16} = 0$$

$R_A = \dfrac{+13wL}{16} = \dfrac{13wL}{16} \uparrow$ ....**Ans.**



$$\circlearrowleft \Sigma M_A = 0: \quad M_A - \frac{wL^2}{2} + \frac{3wL}{16}(2L) - \frac{5wL^2}{48} = 0$$

$M_A = \dfrac{+11wL^2}{48} = \dfrac{11wL^2}{48}$ ↺ ....**Ans.**

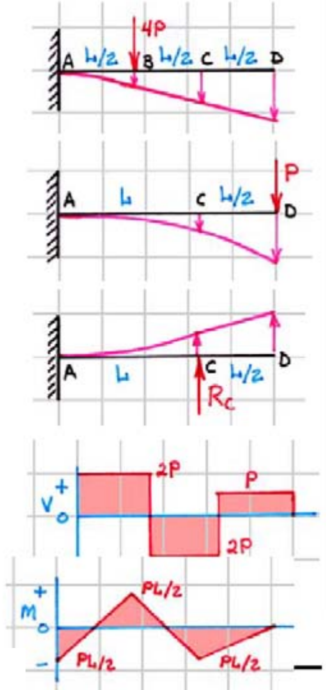
8-164*

Using the solutions for case 1 in Table B-19

$$\delta_C = \delta_{4P} + \delta_P + \delta_{R_C} = 0$$

$$\left[\frac{-(4P)(L/2)^3}{3EI} - \frac{(4P)(L/2)^2}{2EI}\left(\frac{L}{2}\right)\right] - \frac{P(L)^2}{6EI}\left[3\left(\frac{3L}{2}\right) - L\right] + \frac{R_C(L)^3}{3EI} = 0$$

$R_C = 3P \uparrow$ .... **Ans.**



Then from equilibrium

$\uparrow \Sigma F_y = 0: \qquad R_A - 4P + 3P - P = 0$

$R_A = 2P = 2P \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0: \quad M_A - (4P)\left(\frac{L}{2}\right) + (3P)(L) - (P)\left(\frac{3L}{2}\right) = 0$

$M_A = \frac{+PL}{2} = \frac{PL}{2} \circlearrowleft$ .... **Ans.**

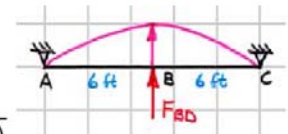
8-165*

$$I_{AB} = I_{BC} = \frac{(4)(6)^3}{12} = 72 \text{ in.}^4 \qquad A_{BD} = \frac{\pi(0.5)^2}{4} = 0.196350 \text{ in.}^2$$

Using the solutions for cases 6 and 7 in Table B-19

$$\frac{5wL_{AC}^4}{384(EI)_{AC}} - \frac{F_{BD}L_{AC}^{\ 3}}{48(EI)_{AC}} = \frac{F_{BD}(L_{BD})}{(EA)_{BD}}$$

(a) For a 1/2-in. diameter steel $(E = 30,000 \text{ ksi})$ rod:



$$\frac{5(400/12)(12\times12)^4}{384(1.8\times10^6)(72)} - \frac{F_{BD}(12\times12)^3}{48(1.8\times10^6)(72)} = \frac{F_{BD}(16\times12)}{(0.196350)(30\times10^6)}$$

$F_{BD} = 2809$ lb

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{2809}{0.196350} = 14,306 \text{ psi} \cong 14.31 \text{ ksi}$$ .... **Ans.**

(b) For a 1/2-in. diameter aluminum alloy $(E = 10,000 \text{ ksi})$ rod:

$$\frac{5(400/12)(12\times12)^4}{384(1.8\times10^6)(72)} - \frac{F_{BD}(12\times12)^3}{48(1.8\times10^6)(72)} = \frac{F_{BD}(16\times12)}{(0.196350)(10\times10^6)}$$

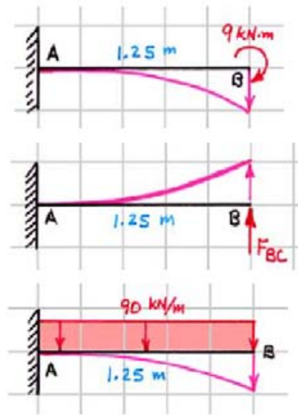
$F_{BD} = 2492$ lb

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{2492}{0.196350} = 12,690 \text{ psi} \cong 12.69 \text{ ksi}$$ .... **Ans.**

8-166

$$EI_{AB} = \left(70\times10^{9}\right)\left(40\times10^{-6}\right) = 2.800\left(10^{6}\right)\ \text{N}\cdot\text{m}^2$$

$$EA_{BC} = \left(70\times10^{9}\right)\left(100\times10^{-6}\right) = 7.000\left(10^{6}\right)\ \text{N}\cdot\text{m}^2$$



Using the solutions for cases 1, 2, and 4 in Table B-19

$$\delta_{Bw} + \delta_{BF_{BC}} = \delta_{rod} + \delta_{spring} + clearance$$

$$\frac{wL_{AB}^4}{8(EI)_{AB}} + \frac{ML_{AB}^2}{2(EI)_{AB}} - \frac{F_{BC}L_{AB}^3}{3(EI)_{AB}} = \frac{F_{BC}L_{BC}}{(EA)_{BC}} + \frac{F_{BC}}{k} + 0.0025$$

$$\frac{(90,000)(1.25)^4}{8\left(2.800\times10^6\right)} + \frac{(9000)(1.25)^2}{2\left(2.800\times10^6\right)} - \frac{F_{BC}(1.25)^3}{3\left(2.800\times10^6\right)}$$

$$= \frac{F_{BC}(1.25)}{\left(7.000\times10^6\right)} + \frac{F_{BC}}{\left(1000\times10^3\right)} + 0.0025$$

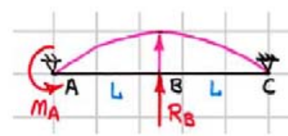
$$F_{BC} = 6959.4\ \text{N}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{6959.4}{100\times10^{-6}} = 69.6\left(10^6\right)\ \text{kN/m}^2 = 69.6\ \text{MPa}$$ .... **Ans.**

8-167

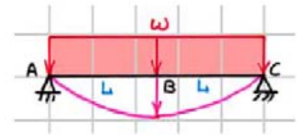
Using the solutions for cases 6, 7, and 8 in Table B-19

$$\theta_A = \theta_{AR_B} + \theta_{AM} + \theta_{Aw} = \frac{R_B(2L)^2}{16EI} + \frac{M_A(2L)}{3EI} - \frac{w(2L)^3}{24EI} = 0$$



$$3R_B L + 8M_A - 4wL^2 = 0$$

$$\delta_B = v_{BR_B} + v_{BM} + \delta_{Bw} = \frac{R_B(2L)^3}{48EI} + \frac{M_A(2L)^2}{16EI} - \frac{5w(2L)^4}{384EI} = 0$$



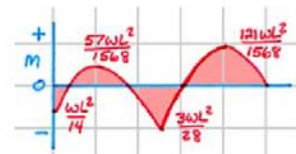
$$4R_B L + 6M_A - 5wL^2 = 0$$

Solving simultaneously gives

$$R_B = \frac{+8wL}{7} = \frac{8wL}{7} \uparrow \text{ .... } \textbf{Ans.}$$

$$M_A = \frac{+wL^2}{14} = \frac{wL^2}{14} \circlearrowleft \text{ .... } \textbf{Ans.}$$

Then from equilibrium



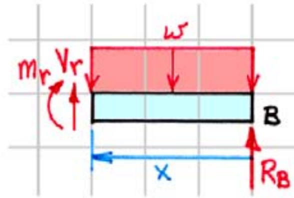
$$\circlearrowleft \Sigma M_A = 0: \quad \frac{wL^2}{14} - (2wL)(L) + \frac{8wL}{7}(L) + R_C(2L) = 0$$

$$R_C = \frac{+11wL}{28} = \frac{11wL}{28} \uparrow \text{ .... } \textbf{Ans.}$$

$$\uparrow \Sigma F_y = 0: \quad R_A - 2wL + \frac{8wL}{7} + \frac{11wL}{28} = 0 \qquad R_A = \frac{+13wL}{28} = \frac{13wL}{28} \uparrow \text{ .... } \textbf{Ans.}$$

8-168*

$$M_r = R_B x - \frac{wx^2}{2} \qquad \frac{\partial M_r}{\partial R_B} = x$$



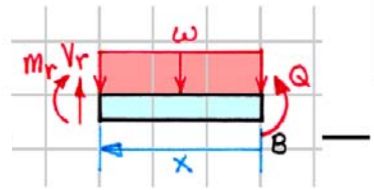
$$\delta_B = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial R_B} dx = \frac{1}{EI}\int_0^L \left(R_B x - \frac{wx^2}{2} \right)(x)\, dx$$

$$= \frac{1}{EI}\left[\frac{R_B x^3}{3} - \frac{wx^4}{8} \right]_0^L = \frac{L^3}{24EI}(8R_B - 3wL) = 0$$

$R_B = \frac{+3wL}{8} = \frac{3wL}{8} \uparrow$ .... **Ans.**

8-169*

$$M_r = Q - \frac{wx^2}{2} \qquad \frac{\partial M_r}{\partial Q} = 1$$



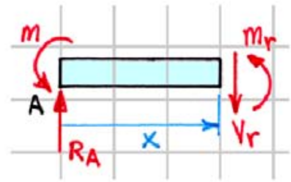
$$\theta_B = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial Q} dx = \frac{1}{EI}\int_0^L \left(Q - \frac{wx^2}{2}\right)(1)\,dx$$

$$= \frac{1}{EI}\left[Qx - \frac{wx^3}{6}\right]_0^L = \frac{L}{6EI}\left(6Q - wL^2\right) = 0$$

$Q = \dfrac{+wL^2}{6} = \dfrac{wL^2}{6}$ ↺ .... **Ans.**

8-170

$M_r = R_A x - M \qquad \partial M_r / \partial R_A = x$



$$\delta_B = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial R_A} dx = \frac{1}{EI}\int_0^L (R_A x - M)(x)\,dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{Mx^2}{2}\right]_0^L = \frac{L^2}{6EI}(2R_A L - 3M) = 0$$

$R_A = \frac{+3M}{2L} = \frac{3M}{2L} \uparrow$ .... **Ans.**

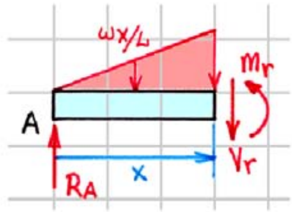
Then from equilibrium

$\uparrow \Sigma F_y = 0: \quad \frac{3M}{2L} + R_B = 0 \qquad R_B = \frac{-3M}{2L} = \frac{3M}{2L} \downarrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0: \quad M_B + M - \frac{3M}{2L}(L) = 0 \qquad M_B = \frac{+M}{2} = \frac{M}{2} \circlearrowleft$ .... **Ans.**

8-171*

$$M_r = R_A x - \frac{wx^3}{6L} \qquad \frac{\partial M_r}{\partial R_A} = x$$



$$\delta_A = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial R_A} dx = \frac{1}{EI}\int_0^L \left(R_A x - \frac{wx^3}{6L} \right)(x)\, dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^5}{30L} \right]_0^L = \frac{L^3}{30EI}(10R_A - wL) = 0$$

$$R_A = \frac{+wL}{10} = \frac{wL}{10} \uparrow$$ .... **Ans.**

Then from equilibrium

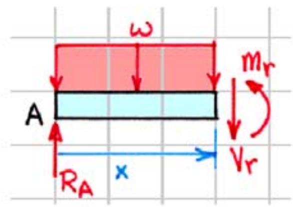
$\uparrow \Sigma F_y = 0: \quad \frac{wL}{10} - \frac{wL}{2} + R_B = 0$ $\qquad R_B = \frac{+4wL}{10} = \frac{2wL}{5} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0: \quad \frac{wL}{2}\left(\frac{L}{3}\right) - \frac{wL}{10}(L) - M_B = 0$ $\qquad M_B = \frac{+wL^2}{15} = \frac{wL^2}{15} \circlearrowright$ .... **Ans.**

8-172

From symmetry $R_A = R_C$ and $\frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial R_A} dx = \frac{2}{EI}\int_0^{L} M_r \frac{\partial M_r}{\partial R_A} dx$

$$M_r = R_A x - \frac{wx^2}{2} \qquad \frac{\partial M_r}{\partial R_A} = x$$



$$\delta_A = \frac{2}{EI}\int_0^L M_r \frac{\partial M_r}{\partial R_A} dx = \frac{2}{EI}\int_0^L \left(R_A x - \frac{wx^2}{2} \right)(x)\,dx$$

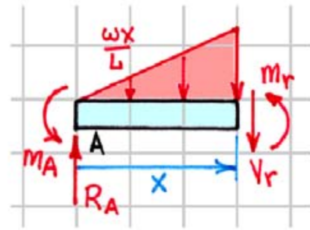
$$= \frac{2}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{8} \right]_0^L = \frac{2L^3}{24EI}(8R_A - 3wL) = 0$$

$R_A = R_C = \frac{+3wL}{8} = \frac{3wL}{8} \uparrow$ .... **Ans.**

and from equilibrium

$\uparrow \Sigma F_y = 0: \quad \frac{3wL}{8} + R_B + \frac{3wL}{8} - 2wL = 0 \qquad R_B = \frac{+10wL}{8} = \frac{5wL}{4} \uparrow$ .... **Ans.**

8-173



$$M_r = R_A x - M_A - \frac{wx^3}{6L} \qquad \frac{\partial M_r}{\partial R_A} = x \qquad \frac{\partial M_r}{\partial M_A} = -1$$

$$\delta_A = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial R_A} dx = \frac{1}{EI}\int_0^L \left(R_A x - M_A - \frac{wx^3}{6L} \right)(x)\,dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{M_A x^2}{2} - \frac{wx^5}{30L} \right]_0^L = 0$$

$$10R_A L - 15M_A - wL^2 = 0 \qquad \text{(a)}$$

$$\theta_A = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial M_A} dx = \frac{1}{EI}\int_0^L \left(R_A x - M_A - \frac{wx^3}{6L} \right)(-1)\,dx$$

$$= \frac{-1}{EI}\left[\frac{R_A x^2}{2} - M_A x - \frac{wx^4}{24L} \right]_0^L = 0$$

$$12R_A L - 24M_A - wL^2 = 0 \qquad \text{(b)}$$

Solving Eqs. (a) and (b) simultaneously gives

$$R_A = \frac{+3wL}{20} = \frac{3wL}{20} \uparrow$$ .... **Ans.**

$$M_A = \frac{wL^2}{30} = \frac{wL^2}{30} \circlearrowleft$$ .... **Ans.**

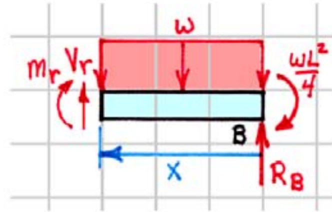
Then from equilibrium

$$\uparrow \Sigma F_y = 0: \qquad \frac{3wL}{20} - \frac{wL}{2} + R_B = 0$$

$$R_B = \frac{+7wL}{20} = \frac{7wL}{20} \uparrow$$ .... **Ans.**

8-174*

$$M_r = R_B x - \frac{wx^2}{2} - \frac{wL^2}{4} \qquad \frac{\partial M_r}{\partial R_B} = x$$



$$\delta_B = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial R_B} dx = \frac{1}{EI}\int_0^L \left(R_B x - \frac{wx^2}{2} - \frac{wL^2}{4} \right)(x)\,dx$$

$$= \frac{1}{EI}\left[\frac{R_B x^3}{3} - \frac{wx^4}{8} - \frac{wL^2 x^2}{8} \right]_0^L = \frac{L^3}{24EI}(8R_B - 6wL) = 0$$

$R_B = \dfrac{+6wL}{8} = \dfrac{3wL}{4} \uparrow$ .... **Ans.**

Then from equilibrium

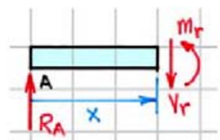
$\uparrow \Sigma F_y = 0: \qquad R_A - wL + \dfrac{3wL}{4} = 0 \qquad\qquad R_A = \dfrac{+wL}{4} = \dfrac{wL}{4} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0: \qquad M_A - wL\left(\dfrac{L}{2}\right) + \dfrac{3wL}{4}(L) - \dfrac{wL^2}{4} = 0 \qquad\qquad M_A = 0$ .... **Ans.**

8-175*

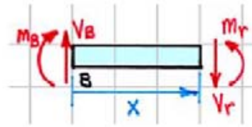
(a) From overall equilibrium $\qquad R_A = \left(\frac{P}{3} - \frac{2R_B}{3}\right)\uparrow \qquad R_D = \left(\frac{2P}{3} - \frac{R_B}{3}\right)\uparrow$

$$M_{r1} = R_A x = \frac{Px}{3} - \frac{2R_B x}{3} \qquad \frac{\partial M_{r1}}{\partial R_B} = \frac{-2x}{3}$$

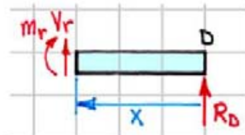


$$M_{r2} = M_B + V_B x = \frac{PL}{3} - \frac{2R_B L}{3} + \frac{Px}{3} + \frac{R_B x}{3}$$

$$\frac{\partial M_{r2}}{\partial R_B} = \frac{1}{3}(x - 2L)$$



$$M_{r3} = R_D x = \frac{2Px}{3} - \frac{R_B x}{3} \qquad \frac{\partial M_{r3}}{\partial R_B} = \frac{-x}{3}$$



$$\delta_B = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial R_B}dx + \frac{1}{EI}\int_0^L M_{r2}\frac{\partial M_{r2}}{\partial R_B}dx + \frac{1}{EI}\int_0^L M_{r3}\frac{\partial M_{r3}}{\partial R_B}dx$$

$$= \frac{1}{9EI}\int_0^L\left[-4R_B x^2 - 2Px^2\right]dx + \frac{1}{9EI}\int_0^L\left[R_B x^2 - 2Px^2\right]dx$$

$$+ \frac{1}{9EI}\int_0^L\left(R_B x^2 - 4R_B Lx + 4R_B L^2 + Px^2 - PLx - 2PL^2\right)dx$$

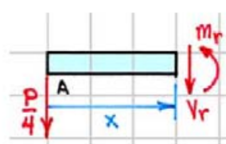
$$= \frac{1}{18EI}\left[4R_B x^3 - 4R_B Lx^2 + 8R_B L^2 x - 2Px^3 - PLx^2 - 4PL^2 x\right]_0^L = 0$$

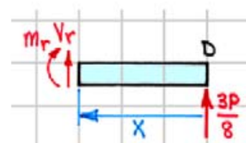
$R_B = \frac{+7P}{8} = \frac{7P}{8}\uparrow$ .... **Ans.**

$R_A = \frac{P}{3} - \frac{2R_B}{3} = \frac{P}{3} - \frac{2}{3}\left(\frac{7P}{8}\right) = \frac{-6P}{24} = \frac{P}{4}\downarrow$ .... **Ans.**

$R_D = \frac{2P}{3} - \frac{1}{3}\left(\frac{7P}{8}\right) = \frac{+9P}{24} = \frac{3P}{8}\uparrow$ .... **Ans.**

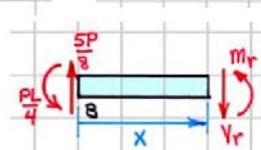
(b) $M_{r1} = \frac{-Px}{4} \qquad \frac{\partial M_{r1}}{\partial P} = \frac{-x}{4}$





$M_{r2} = \frac{-PL}{4} + \frac{5Px}{8} \qquad \frac{\partial M_{r2}}{\partial P} = \frac{5x}{8} - \frac{L}{4}$

$M_{r3} = \frac{3Px}{8} \qquad \frac{\partial M_{r3}}{\partial P} = \frac{3x}{8}$



$$\delta_P = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial P}dx + \frac{1}{EI}\int_0^L M_{r2}\frac{\partial M_{r2}}{\partial P}dx + \frac{1}{EI}\int_0^L M_{r3}\frac{\partial M_{r3}}{\partial P}dx$$

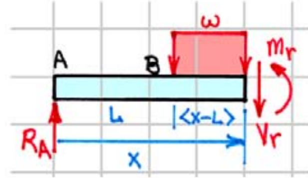
$$= \frac{1}{EI}\int_0^L\left[\frac{Px^2}{16}\right]dx + \frac{1}{EI}\int_0^L\left[\frac{25Px^2}{64} - \frac{5PLx}{16} + \frac{PL^2}{16}\right]dx + \frac{1}{EI}\int_0^L\left(\frac{9Px^2}{64}\right)dx$$

Continued on next slide

$$\delta_P = \frac{1}{EI}\left[\frac{19Px^3}{96} - \frac{5PLx^2}{32} + \frac{PL^2x}{16}\right]_0^L$$

$$\delta_P = \frac{+5PL^3}{48EI} = \frac{5PL^3}{48EI}\downarrow$$ .... **Ans.**

8-176



$$M_r = R_A x - \frac{w\langle x-L\rangle^2}{2} \qquad \frac{\partial M_r}{\partial R_A} = x$$

$$\delta_A = \frac{1}{EI}\int_0^{3L} M_r \frac{\partial M_r}{\partial R_A}\,dx$$

$$= \frac{1}{EI}\int_0^{L}\left(R_A x^2\right)dx + \frac{1}{EI}\int_L^{3L}\left(R_A x^2 - \frac{wx^3}{2} + wLx^2 - \frac{wL^2 x}{2}\right)dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3}\right]_0^L + \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{8} + \frac{wLx^3}{3} - \frac{wL^2x^2}{4}\right]_L^{3L} = 0$$

$R_A = \frac{+10wL}{27} = \frac{10wL}{27}\uparrow$ .... **Ans.**

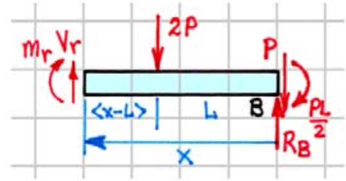
Then from equilibrium

$\uparrow\Sigma F_y = 0: \quad \frac{10wL}{27} - 2wL + R_C = 0$ $\qquad R_C = \frac{+44wL}{27} = \frac{44wL}{27}\uparrow$ .... **Ans.**

$\circlearrowleft\Sigma M_C = 0: \quad 2wL^2 - \frac{10wL}{27}(3L) - M_C = 0$ $\qquad M_C = \frac{+8wL^2}{9} = \frac{8wL^2}{9}\ \circlearrowright$ .... **Ans.**

8-177

$$M_r = R_B x - \frac{PL}{2} - Px - 2P\langle x - L\rangle \qquad \frac{\partial M_r}{\partial R_B} = x$$



$$\delta_A = \frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial R_B}dx$$

$$= \frac{1}{EI}\int_0^{L}\left(R_B x^2 - Px^2 - \frac{PLx}{2}\right)dx + \frac{1}{EI}\int_L^{2L}\left(R_B x^2 - 3Px^2 + \frac{3PLx}{2}\right)dx$$

$$= \frac{1}{EI}\left[\frac{R_B x^3}{3} - \frac{Px^3}{3} - \frac{PLx^2}{4}\right]_0^L + \frac{1}{EI}\left[\frac{R_B x^3}{3} - Px^3 - \frac{3PLx^2}{4}\right]_L^{2L} = 0$$

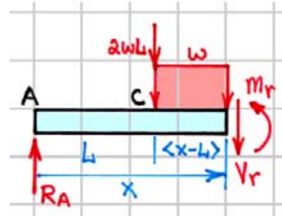
$R_B = +2P = 2P\uparrow$ .... **Ans.**

Then from equilibrium

$\uparrow\Sigma F_y = 0: \quad R_A - 2P + 2P - P = 0$ $\qquad$ $R_A = +P = P\uparrow$ .... **Ans.**

$\circlearrowleft\Sigma M_A = 0: \quad M_A - (2P)L + (2P)(2L) - P(5L/2) = 0$ $\qquad$ $M_A = PL/2 \circlearrowleft$ .... **Ans.**

8-178*



$$M_r = R_A x - 2wL\langle x-L\rangle - \frac{w\langle x-L\rangle^2}{2} \qquad \frac{\partial M_r}{\partial R_A} = x$$

$$\delta_A = \frac{1}{EI}\int_0^{3L} M_r \frac{\partial M_r}{\partial R_A}dx$$

$$= \frac{1}{EI}\int_0^{L}\left(R_A x^2\right)dx + \frac{1}{EI}\int_L^{3L}\left(R_A x^2 - \frac{wx^3}{2} - wLx^2 + \frac{3wL^2x}{2}\right)dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3}\right]_0^L + \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{8} - \frac{wLx^3}{3} + \frac{3wL^2x^2}{4}\right]_L^{3L} = 0$$

$R_A = \dfrac{+38wL}{27} = \dfrac{38wL}{27}\uparrow$ .... **Ans.**

Then from equilibrium

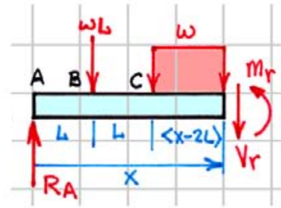
$\uparrow \Sigma F_y = 0: \quad \dfrac{38wL}{27} - 2wL - 2wL + R_B = 0$ $\qquad R_B = \dfrac{+70wL}{27} = \dfrac{70wL}{27}\uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0: \quad 2wL^2 + (2wL)(2L) - \dfrac{38wL}{27}(3L) - M_B = 0$

$M_B = \dfrac{16wL^2}{9} = \dfrac{16wL^2}{9} \circlearrowright$ .... **Ans.**

8-179

$$M_r = R_A x - wL\langle x-L\rangle - \frac{w\langle x-2L\rangle^2}{2} \qquad \frac{\partial M_r}{\partial R_A} = x$$



$$\delta_A = \frac{1}{EI}\int_0^{4L} M_r \frac{\partial M_r}{\partial R_A} dx$$

$$= \frac{1}{EI}\int_0^{L}\left(R_A x^2\right)dx + \frac{1}{EI}\int_L^{2L}\left(R_A x^2 - wLx^2 + wL^2 x\right)dx$$

$$+ \frac{1}{EI}\int_{2L}^{4L}\left(R_A x^2 - \frac{wx^3}{2} + wLx^2 - wL^2 x\right)dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3}\right]_0^L + \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wLx^3}{3} + \frac{wL^2x^2}{2}\right]_L^{2L} + \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{8} + \frac{wLx^3}{3} - \frac{wL^2x^2}{2}\right]_{2L}^{4L} = 0$$

$R_A = \dfrac{+109wL}{128} = \dfrac{109wL}{128}\uparrow$ .... **Ans.**

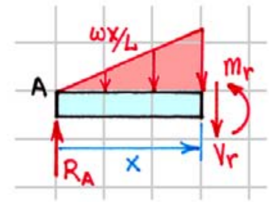
Then from equilibrium

$\uparrow\Sigma F_y = 0: \quad \dfrac{109wL}{128} - wL - 2wL + R_D = 0$ $\qquad R_D = \dfrac{+275wL}{128} = \dfrac{275wL}{128}\uparrow$ .... **Ans.**

$\circlearrowleft\Sigma M_D = 0: \quad 2wL^2 + (wL)(3L) - \dfrac{109wL}{128}(4L) - M_D = 0$ $\qquad M_D = \dfrac{51wL^2}{32}\ \circlearrowright$ .... **Ans.**

8-180*

From overall equilibrium $R_A = \left(\dfrac{M_B}{L} + \dfrac{wL}{6}\right)\uparrow$



Then $M_r = R_A x - \dfrac{wx^3}{6L} = \dfrac{M_B x}{L} + \dfrac{wLx}{6} - \dfrac{wx^3}{6L}$ $\qquad \dfrac{\partial M_r}{\partial M_B} = \dfrac{x}{L}$

$$\theta_B = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial M_B}\,dx = \frac{1}{EI}\int_0^L \left(\frac{M_B x^2}{L^2} + \frac{wx^2}{6} - \frac{wx^4}{6L^2}\right)dx$$

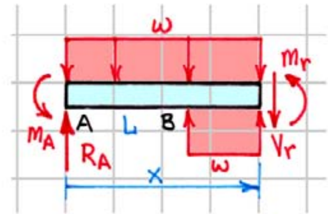
$$= \frac{1}{EI}\left[\frac{M_B x^3}{3L^2} + \frac{wx^3}{18} - \frac{wx^5}{30L^2}\right]_0^L = \frac{-wL^3}{24EI}$$

$$M_B = \frac{-23wL^2}{120} = \frac{23wL^2}{120} \circlearrowright$$

and $R_A = \dfrac{M_B}{L} + \dfrac{wL}{6} = \dfrac{-23wL}{120} + \dfrac{wL}{6} = \dfrac{-wL}{40} = \dfrac{wL}{40}\downarrow$ .... **Ans.**

8-181*

$$M_r = R_A x - M_A - \frac{wx^2}{2} + \frac{w\langle x-L\rangle^2}{2}$$



$$\frac{\partial M_r}{\partial R_A} = x \qquad \frac{\partial M_r}{\partial M_A} = -1$$

$$\delta_A = \frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial R_A} dx$$

$$= \frac{1}{EI}\int_0^{L}\left(R_A x^2 - M_A x - \frac{wx^3}{2}\right)dx + \frac{1}{EI}\int_L^{2L}\left(R_A x^2 - M_A x - wLx^2 + \frac{wL^2 x}{2}\right)dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{M_A x^2}{2} - \frac{wx^4}{8}\right]_0^L + \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{M_A x^2}{2} - \frac{2wLx^3}{6} + \frac{wL^2x^2}{4}\right]_L^{2L} = 0$$

$$4R_A L - 3M_A = 41wL^2/16 \qquad \text{(a)}$$

$$\theta_A = \frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial M_A} dx$$

$$= \frac{1}{EI}\int_0^{L}\left(R_A x - M_A - \frac{wx^2}{2}\right)(-1)\,dx + \frac{1}{EI}\int_L^{2L}\left(R_A x - M_A - wLx + \frac{wL^2}{2}\right)(-1)\,dx$$

$$= \frac{-1}{EI}\left[\frac{R_A x^2}{2} - M_A x - \frac{wx^3}{6}\right]_0^L + \frac{-1}{EI}\left[\frac{R_A x^2}{2} - M_A x - \frac{wLx^2}{2} + \frac{wL^2 x}{2}\right]_L^{2L} = 0$$

$$R_A L - M_A = 7wL^2/12 \qquad \text{(b)}$$

Solving Eqs. (a) and (b) simultaneously gives

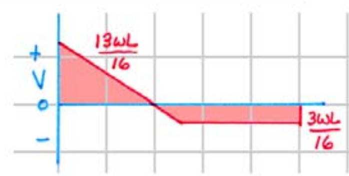
$R_A = \dfrac{+13wL}{16} = \dfrac{13wL}{16}\uparrow$ .... **Ans.**

$M_A = \dfrac{+11wL^2}{48} = \dfrac{11wL^2}{48}$ ↺ .... **Ans.**

Then from equilibrium

$$\uparrow\Sigma F_y = 0: \qquad \frac{13wL}{16} - wL + R_C = 0$$

$R_C = \dfrac{+3wL}{16} = \dfrac{3wL}{16}\uparrow$ .... **Ans.**



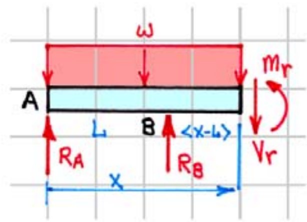
$$↺\Sigma M_A = 0: \qquad \frac{11wL^2}{48} - \frac{wL^2}{2} + \frac{3wL}{16}(2L) - M_C = 0$$

$M_C = \dfrac{+5wL^2}{48} = \dfrac{5wL^2}{48}$ ↻ .... **Ans.**

8-182

$$M_r = R_A x - \frac{wx^2}{2} + R_B \langle x - L \rangle$$

$$\frac{\partial M_r}{\partial R_A} = x \qquad \frac{\partial M_r}{\partial R_B} = \langle x - L \rangle$$



$$\delta_A = \frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial R_A} dx$$

$$= \frac{1}{EI}\int_0^{L}\left(R_A x^2 - \frac{wx^3}{2}\right)dx + \frac{1}{EI}\int_L^{2L}\left[R_A x^2 - \frac{wx^3}{2} + R_B\left(x^2 - Lx\right)\right]dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{8}\right]_0^L + \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{8} + \frac{R_B x^3}{3} - \frac{R_B L x^2}{2}\right]_L^{2L} = 0$$

$$16R_A - 5R_B = 12wL \qquad \text{(a)}$$

$$\delta_B = \frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial R_B} dx$$

$$= 0 + \frac{1}{EI}\int_L^{2L}\left[R_A x\left(x - L\right) - \frac{wx^2}{2}\left(x - L\right) + R_B\left(x - L\right)^2\right]dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{R_A L x^2}{2} - \frac{wx^4}{8} + \frac{wLx^3}{6} + \frac{R_B\left(x - L\right)^3}{3}\right]_L^{2L} = 0$$

$$20R_A + 8R_B = 17wL \qquad \text{(b)}$$

Solving Eqs. (a) and (b) simultaneously gives

$R_A = \dfrac{+11wL}{28} = \dfrac{11wL}{28} \uparrow$ .... **Ans.**

$R_B = \dfrac{+8wL}{7} = \dfrac{8wL}{7} \uparrow$ .... **Ans.**

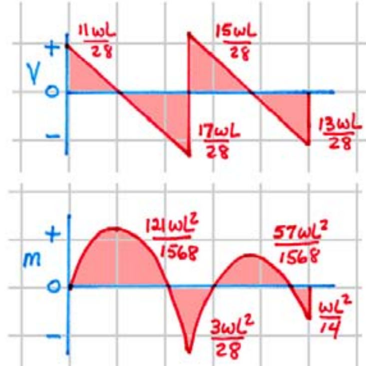
Then from equilibrium

$\uparrow \Sigma F_y = 0: \quad \dfrac{11wL}{28} + \dfrac{8wL}{7} - 2wL + R_C = 0$

$R_C = \dfrac{+13wL}{28} = \dfrac{13wL}{28} \uparrow$ .... **Ans.**

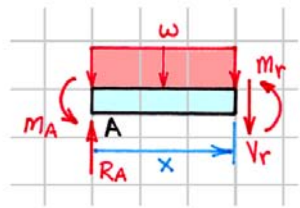
$\circlearrowleft \Sigma M_C = 0: \quad 2wL^2 - \dfrac{11wL}{28}(2L) - \dfrac{8wL}{7}(L) - M_C = 0$

$M_C = +\dfrac{wL^2}{14} = \dfrac{wL^2}{14} \circlearrowleft$ .... **Ans.**



8-183*

From symmetry $\qquad R_A = R_C \qquad M_A = M_C$



and $\qquad \dfrac{1}{EI}\displaystyle\int_0^{2L} M_r \dfrac{\partial M_r}{\partial R_A}dx = \dfrac{2}{EI}\displaystyle\int_0^{L} M_r \dfrac{\partial M_r}{\partial R_A}dx$

$$M_r = R_A x - M_A - \frac{wx^2}{2} \qquad \frac{\partial M_r}{\partial R_A} = x \qquad \frac{\partial M_r}{\partial M_A} = -1$$

$$\delta_A = \frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial R_A}dx = \frac{2}{EI}\int_0^{L}\left(R_A x - M_A - \frac{wx^2}{2}\right)(x)\,dx$$

$$= \frac{2}{EI}\left[\frac{R_A x^3}{3} - \frac{M_A x^2}{2} - \frac{wx^4}{8}\right]_0^L = \frac{2L^2}{24EI}\left(8R_A L - 12M_A - 3wL^2\right) = 0 \qquad \text{(a)}$$

$$\theta_A = \frac{1}{EI}\int_0^{2L} M_r \frac{\partial M_r}{\partial M_A}dx = \frac{2}{EI}\int_0^{L}\left(R_A x - M_A - \frac{wx^2}{2}\right)(-1)\,dx$$

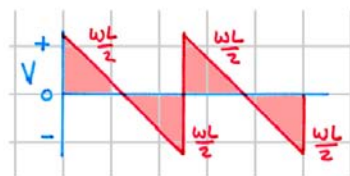
$$= \frac{-2}{EI}\left[\frac{R_A x^2}{2} - M_A x - \frac{wx^3}{6}\right]_0^L = \frac{-2L^2}{6EI}\left(3R_A L - 6M_A - wL^2\right) = 0 \qquad \text{(b)}$$

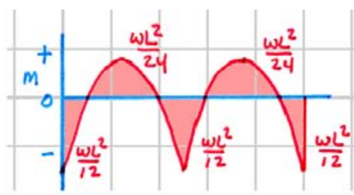
Solving Eqs. (a) and (b) simultaneously gives

$R_A = R_C = \dfrac{+wL}{2} = \dfrac{wL}{2}\uparrow$ .... **Ans.**

$M_A = \dfrac{+wL^2}{12} = \dfrac{wL^2}{12}\ \circlearrowleft$ .... **Ans.**

$M_C = \dfrac{wL^2}{12}\ \circlearrowright$ .... **Ans.**





and from equilibrium

$$\uparrow \Sigma F_y = 0: \qquad \frac{wL}{2} - 2wL + R_B + \frac{wL}{2} = 0$$

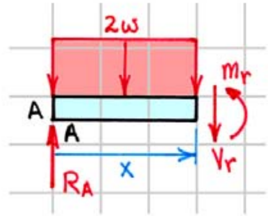
$R_B = wL = wL\uparrow$ .... **Ans.**

8-184

(a) From overall equilibrium

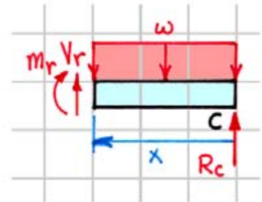
$$\circlearrowleft \Sigma M_B = 0: \qquad R_C(3L) - 3wL(3L/2) + 4wL(L) - R_A(2L) = 0$$

$$R_C = \frac{2R_A}{3} + \frac{wL}{6}$$



$$M_{r1} = R_A x - wx^2 \qquad \frac{\partial M_{r1}}{\partial R_A} = x$$

$$M_{r2} = R_C x - \frac{wx^2}{2} = \frac{2R_A x}{3} + \frac{wLx}{6} - \frac{wx^2}{2} \qquad \frac{\partial M_{r2}}{\partial R_A} = \frac{2x}{3}$$



$$\delta_A = \frac{1}{EI}\int_0^{2L} M_{r1}\frac{\partial M_{r1}}{\partial R_A}dx + \frac{1}{EI}\int_0^{3L} M_{r2}\frac{\partial M_{r2}}{\partial R_A}dx$$

$$= \frac{1}{EI}\int_0^{2L}\left[R_A x^2 - wx^3\right]dx + \frac{1}{EI}\int_0^{3L}\left[\frac{4R_A x^2}{9} + \frac{wLx^2}{9} - \frac{wx^3}{3}\right]dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{4}\right]_0^{2L} + \frac{1}{EI}\left[\frac{4R_A x^3}{27} + \frac{wLx^3}{27} - \frac{wx^4}{12}\right]_0^{3L}$$

$$= \frac{1}{EI}\left[\frac{8R_A L^3}{3} - 4wL^4 + 4R_A L^3 + wL^4 - \frac{27wL^4}{4}\right] = 0$$

$R_A = \dfrac{+117wL}{80} = \dfrac{117wL}{80}\uparrow$ .... **Ans.**

$R_C = \dfrac{2R_A}{3} + \dfrac{wL}{6} = \dfrac{2}{3}\left(\dfrac{117wL}{80}\right) + \dfrac{wL}{6} = \dfrac{137wL}{120}\uparrow$ .... **Ans.**

and from equilibrium

$$\uparrow \Sigma F_y = 0: \qquad \frac{117wL}{80} - 4wL + R_B - 2wL + \frac{137wL}{120} = 0$$

$R_B = \dfrac{211wL}{48}\uparrow$ .... **Ans.**

Continued on next slide

Problem 8-184 continued

(b) For an S 178×30 section:

$d = 177.8$ mm $\qquad w_f = 98.0$ mm

$t_w = 11.4$ mm $\qquad t_f = 10.0$ mm

$S = 198(10^3)\ \text{mm}^3 \qquad I = 17.6(10^6)\ \text{mm}^4$

From the shear-force and bending-moment diagrams:

$V_{max} = -50.75$ kN $\qquad M_{max} = -43.00$ kN·m

$$Q = 83.9(98 \times 10) + 39.45(78.9 \times 11.4) = 117.71(10^3)\ \text{mm}^3$$

$$\sigma_{max} = \frac{M}{S} = \frac{43,000}{198(10^{-6})} = 217(10^6)\ \text{N/m}^2$$

$\sigma_{max} = 217$ MPa (T, top; C, bottom) .... **Ans.**

$$\tau_{max} = \frac{VQ}{It} = \frac{(50,750)(117.71 \times 10^{-6})}{(17.6 \times 10^{-6})(0.0114)} = 29.8(10^6)\ \text{N/m}^2$$

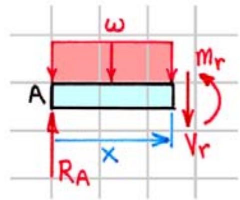
$\tau_{max} = 29.8$ MPa .... **Ans.**

8-185

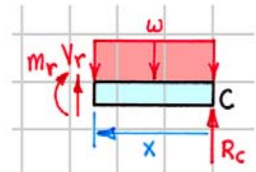
(a) From overall equilibrium

$$\circlearrowleft \Sigma M_B = 0: \quad R_C(2L) - 3wL(L/2) - R_A(L) = 0 \qquad R_C = \frac{R_A}{2} + \frac{3wL}{4}$$

$$M_{r1} = R_A x - \frac{wx^2}{2} \qquad \frac{\partial M_{r1}}{\partial R_A} = x$$



$$M_{r2} = R_C x - \frac{wx^2}{2} = \frac{R_A x}{2} + \frac{3wLx}{4} - \frac{wx^2}{2} \qquad \frac{\partial M_{r2}}{\partial R_A} = \frac{x}{2}$$



$$\delta_A = \frac{1}{EI}\int_0^L M_{r1}\frac{\partial M_{r1}}{\partial R_A}dx + \frac{1}{EI}\int_0^{2L} M_{r2}\frac{\partial M_{r2}}{\partial R_A}dx$$

$$= \frac{1}{EI}\int_0^L \left[R_A x^2 - \frac{wx^3}{2}\right]dx + \frac{1}{EI}\int_0^{2L}\left[\frac{R_A x^2}{4} + \frac{3wLx^2}{8} - \frac{wx^3}{4}\right]dx$$

$$= \frac{1}{EI}\left[\frac{R_A x^3}{3} - \frac{wx^4}{8}\right]_0^L + \frac{1}{EI}\left[\frac{R_A x^3}{12} + \frac{wLx^3}{8} - \frac{wx^4}{16}\right]_0^{2L}$$

$$= \frac{1}{EI}\left[\frac{R_A L^3}{3} - \frac{wL^4}{8} + \frac{2R_A L^3}{3} + wL^4 - wL^4\right] = 0$$

$$R_A = \frac{+wL}{8} = \frac{wL}{8}\uparrow$$ .... **Ans.**

$$R_C = \frac{R_A}{2} + \frac{3wL}{4} = \frac{1}{2}\left(\frac{wL}{8}\right) + \frac{3wL}{4} = \frac{13wL}{16}\uparrow$$ .... **Ans.**

and from equilibrium

$$\uparrow \Sigma F_y = 0: \quad \frac{wL}{8} - 3wL + R_B + \frac{13wL}{16} = 0 \qquad R_B = \frac{33wL}{16}\uparrow$$ .... **Ans.**

(b) For an S 6×17.25 section:

$d = 6.00$ in. $\quad t_w = 0.465$ in. $\quad w_f = 3.565$ in.

$I = 25.3$ in.^{4} $\quad S = 8.77$ in.^{3} $\quad t_f = 0.359$ in.

Continued on next slide

Problem 8-185 continued

From the shear-force and bending-moment diagrams:

$$V_{max} = \frac{19wL}{16} = \frac{19(500)(6)}{16} = 3562.5 \text{ lb}$$

$$M_{max} = \frac{-3wL^2}{8} = \frac{-3(500)(6)^2}{8} = -6750 \text{ lb} \cdot \text{ft}$$

$$Q = 2.8205(3.565 \times 0.359) + 1.3205(2.641 \times 0.465) = 5.231 \text{ in.}^3$$

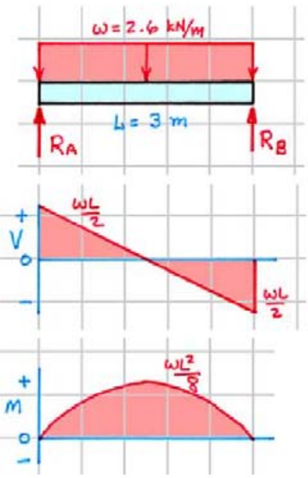
$$\sigma_{max} = \frac{M}{S} = \frac{(6750 \times 12)}{8.77} = 9236 \text{ psi}$$

$\sigma_{max} \cong 9.24$ ksi (T, top; C, bottom) .... **Ans.**

$$\tau_{max} = \frac{VQ}{It} = \frac{(3562.5)(5.231)}{(25.3)(0.465)} = 1584 \text{ psi}$$

$\tau_{max} \cong 1.584$ ksi .... **Ans.**

8-186*



From symmetry $R_A = R_B = \dfrac{wL}{2} = \dfrac{(2600)(3)}{2} = 3900\text{ N}$

From the shear-force and bending-moment diagrams:

$$V_{max} = \frac{wL}{2} = 3900\text{ N}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(2600)(3)^2}{8} = 2925\text{ N}\cdot\text{m}$$

The properties required by the three specifications are:

$$\sigma_{max} = \frac{M}{S} \qquad S = \frac{2925}{8(10^6)} = 365.6(10^{-6})\text{ m}^3 = 365.6(10^3)\text{ mm}^3$$

$$\tau_{max} = 1.5\frac{V_{max}}{A} \qquad A = \frac{1.5(3900)}{0.7(10^6)} = 8357(10^{-6})\text{ m}^2 = 8357\text{ mm}^2$$

$$\delta_{max} = \frac{5wL^4}{384EI} \qquad I = \frac{5(2600)(3)^4}{384(13\times10^9)(0.010)} = 21.09(10^{-6})\text{ m}^4 = 21.09(10^6)\text{ mm}^4$$

Try a 51×254-mm timber section with:

$$m = 6.38\text{ kg/m} \qquad A = 9880\text{ mm}^2 \qquad I = 48.3(10^6)\text{ mm}^4 \qquad S = 400(10^3)\text{ mm}^3$$

With the weight of the beam included:

$$w = 2600 + (6.38\times9.81) = 2663\text{ N/m}$$

$$V_{max} = \frac{wL}{2} = \frac{(2663)(3)}{2} = 3994\text{ N} \qquad M_{max} = \frac{wL^2}{8} = \frac{(2663)(3)^2}{8} = 2995\text{ N}\cdot\text{m}$$

Then the normal stress, the shear stress, and the deflection are:

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{2995}{400(10^{-6})} = 7.49(10^6)\text{ N/m}^2 = 7.49\text{ MPa} < \sigma_{all} = 8\text{ MPa}$$

$$\tau_{max} = 1.5\frac{V_{max}}{A} = 1.5\frac{3994}{9880(10^{-6})} = 0.606(10^6)\text{ N/m}^2 = 0.606\text{ MPa} < \tau_{all} = 0.7\text{ MPa}$$

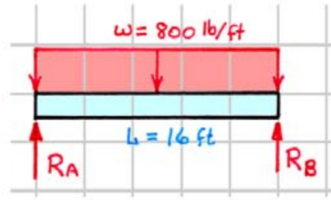
$$\delta_{max} = \frac{5wL^4}{384EI} = \frac{5(2662)(3)^4}{384(13\times10^9)(48.3\times10^{-6})} = 0.00447\text{ m} = 4.47\text{ mm} < \delta_{all} = 10\text{ mm}$$

Therefore Use a 51×254-mm timber section .... **Ans.**

8-187*

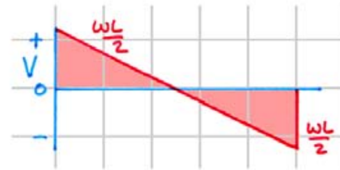
From symmetry $R_A = R_B = \dfrac{wL}{2} = \dfrac{(800)(16)}{2} = 6400$ lb

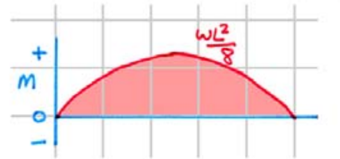
From the shear-force and bending-moment diagrams:



$$V_{max} = \frac{wL}{2} = 6400 \text{ lb}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(800)(16)^2}{8} = 25,600 \text{ lb}\cdot\text{ft}$$





The properties required by the three specifications are:

$$\sigma_{max} = \frac{M_{max}}{S} \qquad S = \frac{(25,600\times 12)}{1200} = 256 \text{ in.}^3$$

$$\tau_{max} = 1.5\frac{V_{max}}{A} \qquad A = \frac{1.5(6400)}{90} = 107.7 \text{ in.}^2$$

$$\delta_{max} = \frac{5wL^4}{384EI} \qquad I = \frac{5(800/12)(16\times 12)^4}{384(1900\times 10^3)(0.50)} = 1242 \text{ in.}^4$$

Try an 8×16-in. timber section with:

$w = 32$ lb/ft $A = 116$ in.^{2} $I = 2327$ in.^{4} $S = 300$ in.^{3}

With the weight of the beam included:

$$w = 800 + 32 = 832 \text{ lb/ft}$$

$$V_{max} = \frac{wL}{2} = \frac{(832)(16)}{2} = 6656 \text{ lb} \qquad M_{max} = \frac{wL^2}{8} = \frac{(832)(16)^2}{8} = 26,624 \text{ lb}\cdot\text{ft}$$

Then the normal stress, the shear stress, and the deflection are:

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{(26,624\times 12)}{300} = 1065 \text{ psi} < \sigma_{all} = 1200 \text{ psi}$$

$$\tau_{max} = 1.5\frac{V_{max}}{A} = 1.5\frac{6656}{116} = 86.1 \text{ psi} < \tau_{all} = 90 \text{ psi}$$

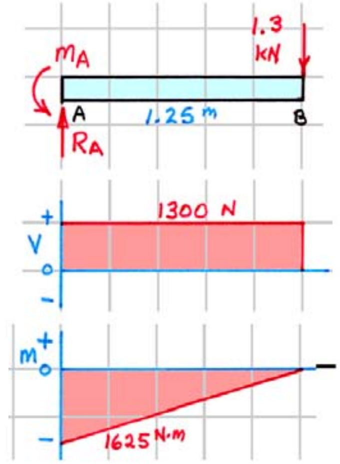
$$\delta_{max} = \frac{5wL^4}{384EI} = \frac{5(832/12)(16\times 12)^4}{384(1900\times 10^3)(2327)} = 0.277 \text{ in.} < \delta_{all} = 0.50 \text{ in.}$$

Therefore Use an 8×16-in. timber section .... **Ans.**

8-188

From overall equilibrium: $R_A = 1300 \text{ N} \uparrow$

and $M_A = (1300)(1.25) = 1625 \text{ N}\cdot\text{m} \circlearrowleft$



From the shear-force and bending-moment diagrams:

$$V_{max} = 1300 \text{ N} \qquad M_{max} = 1625 \text{ N}\cdot\text{m}$$

The properties required by the two specifications are:

$$\sigma_{max} = \frac{M}{S} \qquad S = \frac{1625}{150(10^6)} = 10.83(10^{-6}) \text{ m}^3$$

$$= 10.83(10^3) \text{ mm}^3$$

$$\delta_{max} = \frac{PL^3}{3EI} \qquad I = \frac{(1300)(1.25)^3}{3(200\times 10^9)(0.005)} = 0.8464(10^{-6}) \text{ m}^4$$

$$= 0.8464(10^6) \text{ mm}^4$$

Try a 76-mm diameter standard weight pipe with:

$$m = 11.27 \text{ kg/m} \qquad A = 1437 \text{ mm}^2$$

$$I = 1.256(10^6) \text{ mm}^4 \qquad S = 28.25(10^3) \text{ mm}^3$$

With the weight of the pipe included:

$$w = (11.27\times 9.81) = 110.56 \text{ N/m}$$

$$V_{max} = R_A = 1300 + (110.56)(1.25) = 1438.2 \text{ N}$$

$$M_{max} = M_A = 1625 + \frac{(110.56)(1.25)^2}{2} = 1711.4 \text{ N}\cdot\text{m}$$

Then the normal stress and the deflection are:

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{1711.4}{28.25(10^{-6})} = 60.6(10^6) \text{ N/m}^2$$

$$= 60.6 \text{ MPa} < \sigma_{all} = 150 \text{ MPa}$$

$$\delta_{max} = \frac{PL^3}{3EI} + \frac{wL^4}{8EI} = \frac{(1300)(1.25)^3}{3(200\times 10^9)(1.256\times 10^{-6})} + \frac{(110.56)(1.25)^4}{8(200\times 10^9)(1.256\times 10^{-6})}$$

$$= 0.00350 \text{ m} = 3.50 \text{ mm} < \delta_{all} = 5 \text{ mm}$$

Therefore Use a 76-mm diameter standard weight pipe .... **Ans.**

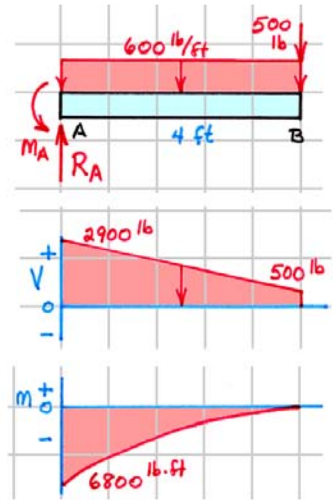
8-189

From overall equilibrium: $R_A = 500 + (600)(4) = 2900 \text{ lb} \uparrow$

and $M_A = (500)(4) + (600 \times 4)(2) = 6800 \text{ lb} \cdot \text{ft} \circlearrowleft$

From the shear-force and bending-moment diagrams:

$V_{max} = 2900 \text{ lb}$ $\qquad M_{max} = 6800 \text{ lb} \cdot \text{ft}$



The properties required by the three specifications are:

$$\sigma_{max} = \frac{M_{max}}{S} \qquad S = \frac{(6800 \times 12)}{1300} = 62.77 \text{ in.}^3$$

$$\tau_{max} = 1.5\frac{V_{max}}{A} \qquad A = \frac{1.5(2900)}{80} = 54.38 \text{ in.}^2$$

$$\delta_{max} = \frac{PL^3}{3EI} + \frac{wL^4}{8EI} \qquad I = \frac{8PL^3 + 3wL^4}{24E\delta_{max}}$$

$$I = \frac{8(500)(4 \times 12)^3 + 3(600/12)(4 \times 12)^4}{24(1200 \times 10^3)(0.20)}$$

$$= 215.0 \text{ in.}^4$$

Try an 8×8-in. timber section with:

$w = 15.6$ lb/ft $\qquad A = 56.3 \text{ in.}^2$

$I = 264 \text{ in.}^4$ $\qquad S = 70.3 \text{ in.}^3$

With the weight of the beam included:

$$w = 600 + 15.6 = 615.6 \text{ lb/ft}$$

$$V_{max} = R_A = 500 + (615.6)(4) = 2962 \text{ lb}$$

$$M_{max} = M_A = (500)(4) + \frac{(615.6)(4)^2}{2} = 6925 \text{ lb} \cdot \text{ft}$$

Then the normal stress, the shear stress, and the deflection are:

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{(6925 \times 12)}{70.3} = 1182 \text{ psi} < \sigma_{all} = 1300 \text{ psi}$$

$$\tau_{max} = 1.5\frac{V_{max}}{A} = 1.5\frac{2962}{56.3} = 78.9 \text{ psi} < \tau_{all} = 80 \text{ psi}$$

$$\delta_{max} = \frac{PL^3}{3EI} + \frac{wL^4}{8EI}$$

$$= \frac{(500)(4 \times 12)^3}{3(1200 \times 10^3)(264)} + \frac{(615.4/12)(4 \times 12)^4}{8(1200 \times 10^3)(264)} = 0.1656 \text{ in.} < \delta_{all} = 0.20 \text{ in.}$$

Therefore Use an 8×8-in. timber section .... **Ans.**

8-190*

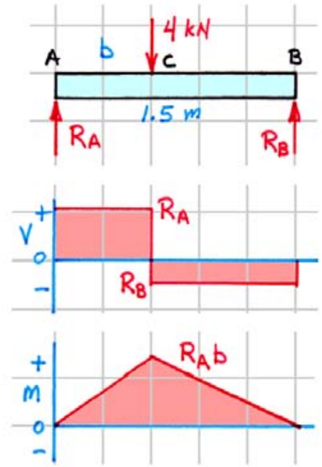
From overall equilibrium:

$$\circlearrowleft \Sigma M_A = 0: \qquad R_B(1.5) - (4)(b) = 0$$

$$R_B = (2.667b) \text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \qquad R_A - 4 + R_B = 0$$

$$R_A = (4 - 2.667b) \text{ kN} \uparrow$$



The maximum bending moment is $M_{max} = R_A b$ and occurs when

$$\frac{dM_C}{db} = 4 - 5.334b = 0$$

$$b = 0.750 \text{ m}$$

When $b = 0.750$ m the maximum shear-force and bending-moment are

$$V_{max} = 2000 \text{ N} \qquad M_{max} = 1500 \text{ N} \cdot \text{m}$$

The minimum diameter pipe which satisfies the flexural stress requirement is

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{M_{max}c}{\pi c^4/4} = \frac{4M_{max}}{\pi c^3}$$

$$c^3 = \frac{4M_{max}}{\pi \sigma_{all}} = \frac{4(1500)}{\pi(152\times 10^6)} = 12.595(10^{-6}) \text{ m}^3$$

$$c = 0.02325 \text{ m} = 23.25 \text{ mm}$$

$$d = 2c = 46.50 \text{ mm}$$

For a 50-mm diameter shaft the shear stress and deflection are:

$$Q = y_C A = \left[\frac{4(25)}{3\pi}\right]\left[\frac{\pi(25)^2}{2}\right] = 10.417(10^3) \text{ mm}^3$$

$$I = \frac{\pi c^4}{4} = \frac{\pi(25)^4}{4} = 0.3068(10^6) \text{ mm}^4$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(2000)(10.417\times 10^{-6})}{(0.3068\times 10^{-6})(0.050)} = 1.358(10^6) \text{ N/m}^2$$

$$= 1.358 \text{ MPa} < \tau_{all} = 100 \text{ MPa}$$

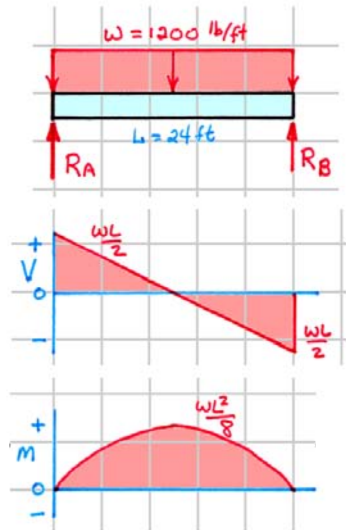
$$\delta_{max} = \frac{PL^3}{48EI} = \frac{(4000)(1.5)^3}{48(200\times 10^9)(0.3068\times 10^{-6})} = 0.00458 \text{ m}$$

$$= 4.58 \text{ mm} < \delta_{all} = 5 \text{ mm}$$

Therefore Use a 50-mm diameter shaft.. **Ans.**

8-191

From symmetry $R_A = R_B = \dfrac{wL}{2} = \dfrac{(1200)(24)}{2} = 14,400 \text{ lb}\uparrow$



$$V_{\max} = \frac{wL}{2} = 14.40 \text{ kip}$$

$$M_{\max} = \frac{wL^2}{8} = \frac{(1200)(24)^2}{8} = 86,400 \text{ lb}\cdot\text{ft} = 86.4 \text{ kip}\cdot\text{ft}$$

$$\delta_{\max} = (24\times 12)/360 = 0.800 \text{ in.}$$

The properties required by the three specifications are:

$$\sigma_{\max} = \frac{M_{\max}}{S} \qquad S = \frac{(86.4\times 12)}{24} = 43.2 \text{ in.}^3$$

$$\tau_{\max} = \frac{V_{\max}}{A_w} \qquad A_w = \frac{14.4}{14} = 1.0286 \text{ in.}^2$$

$$\delta_{\max} = \frac{5wL^4}{384EI} \qquad I = \frac{5(1200/12)(24\times 12)^4}{384(29,000\times 10^3)(0.80)} = 386.1 \text{ in.}^4$$

Try an S 15×42.9 section with:

$d = 15.00$ in. $\qquad t_w = 0.411$ in.

$I = 447$ in.^{4} $\qquad S = 59.6$ in.^{3}

With the weight of the beam included: $\qquad w = 1200 + 42.9 = 1242.9$ lb/ft

$$V_{\max} = \frac{wL}{2} = \frac{(1242.9)(24)}{2} = 14,915 \text{ lb} = 14.915 \text{ kip}$$

$$M_{\max} = \frac{wL^2}{8} = \frac{(1242.9)(24)^2}{8} = 84,489 \text{ lb}\cdot\text{ft} = 84.489 \text{ kip}\cdot\text{ft}$$

Then the normal stress, the shear stress, and the deflection are:

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{(89.489\times 12)}{59.6} = 18.02 \text{ ksi} < \sigma_{all} = 24 \text{ ksi}$$

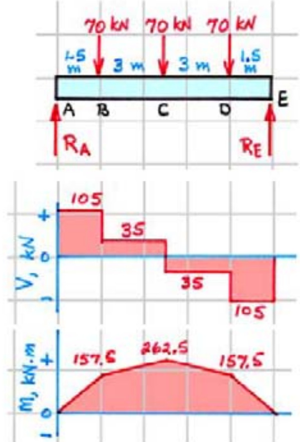
$$\tau_{\max} = \frac{V_{\max}}{A_w} = \frac{14.915}{(15.00\times 0.411)} = 2.42 \text{ ksi} < \tau_{all} = 14 \text{ psi}$$

$$\delta_{\max} = \frac{5wL^4}{384EI} = \frac{5(1242.9/12)(24\times 12)^4}{384(29,000\times 10^3)(447)} = 0.716 \text{ in.} < \delta_{all} = 0.80 \text{ in.}$$

Therefore $\qquad$ Use an S 15×42.9 section .... **Ans.**

8-192*

From symmetry $\quad R_A = R_B = \dfrac{3(70)}{2} = 105 \text{ kN} \uparrow$



From the shear-force and bending-moment diagrams:

$$V_{\max} = 105 \text{ kN} \qquad M_{\max} = 262.5 \text{ kN}\cdot\text{m}$$

The properties required by the three specifications are:

$$\sigma_{\max} = \frac{M}{S} \qquad S = \frac{262.5\left(10^3\right)}{165\left(10^6\right)} = 1591\left(10^{-6}\right) \text{ m}^3 = 1591\left(10^3\right) \text{ mm}^3$$

$$\tau_{\max} = \frac{V_{\max}}{A_w} \qquad A_w = \frac{105\left(10^3\right)}{100\left(10^6\right)} = 1050\left(10^{-6}\right) \text{ m}^2 = 1050 \text{ mm}^2$$

$$\delta_{\max} = \frac{2Pb\left(3L^2 - 4b^2\right)}{48EI} + \frac{PL^3}{48EI} \qquad \delta_{\max} = \frac{9}{360} = 0.025 \text{ m} = 25 \text{ mm}$$

$$I = \frac{2Pb\left(3L^2 - 4b^2\right) + PL^3}{48E\delta_{\max}} = \frac{2(70{,}000)(1.5)\left[3(9)^2 - 4(1.5)^2\right] + (70{,}000)(9)^3}{48\left(200\times 10^9\right)(0.025)}$$

$$= 417.4\left(10^{-6}\right) \text{ m}^4 = 417.4\left(10^6\right) \text{ mm}^4$$

Try a W 533×92 section with: $\qquad w = (92\times 9.81) = 902.5$ N/m

$$d = 533 \text{ mm} \qquad t_w = 10.2 \text{ mm} \qquad I = 554\left(10^6\right) \text{ mm}^4 \qquad S = 2080\left(10^3\right) \text{ mm}^3$$

With the weight of the beam included:

$$V_{\max} = 105\left(10^3\right) + \frac{wL}{2} = 105\left(10^3\right) + \frac{(902.5)(9)}{2} = 109.06\left(10^3\right) \text{ N}$$

$$M_{\max} = 262.5\left(10^3\right) + \frac{wL^2}{8} = 262.5\left(10^3\right) + \frac{(902.5)(9)^2}{8} = 271.6\left(10^3\right) \text{ N}\cdot\text{m}$$

Then the normal stress, the shear stress, and the deflection are:

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{271.6\left(10^3\right)}{2080\left(10^{-6}\right)} = 130.6\left(10^6\right) \text{ N/m}^2 = 130.6 \text{ MPa} < \sigma_{all} = 165 \text{ MPa}$$

$$\tau_{\max} = \frac{V_{\max}}{A_w} = \frac{109.06\left(10^3\right)}{(0.533\times 0.0102)} = 20.1\left(10^6\right) \text{ N/m}^2 = 20.1 \text{ MPa} < \tau_{all} = 100 \text{ MPa}$$

$$\delta_{\max} = \frac{2Pb\left(3L^2 - 4b^2\right)}{48EI} + \frac{PL^3}{48EI} + \frac{5wL^4}{384EI}$$

$$= \frac{2(70{,}000)(1.5)\left[3(9)^2 - 4(1.5)^2\right]}{48\left(200\times 554\times 10^3\right)} + \frac{(70{,}000)(9)^3}{48\left(200\times 554\times 10^3\right)} + \frac{5(902.5)(9)^4}{384\left(200\times 554\times 10^3\right)}$$

$$= 0.01953 \text{ m} = 19.53 \text{ mm} < \delta_{all} = 25 \text{ mm}$$

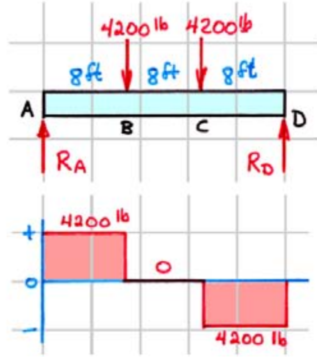
Therefore $\qquad$ Use a W 533×92 section.. **Ans.**

8-193

From symmetry $\quad R_A = R_B = \dfrac{2(4200)}{2} = 4200 \text{ lb} \uparrow$

From the shear-force and bending-moment diagrams:

$$V_{\max} = 4200 \text{ lb} \qquad\qquad M_{\max} = 33{,}600 \text{ lb} \cdot \text{ft}$$



The properties required by the three specifications are:

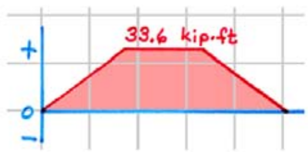
$$\sigma_{\max} = \frac{M_{\max}}{S} \qquad S = \frac{(33{,}600 \times 12)}{1900} = 212.2 \text{ in.}^3$$

$$\tau_{\max} = 1.5\frac{V_{\max}}{A} \qquad A = \frac{1.5(4200)}{85} = 74.12 \text{ in.}^2$$

$$\delta_{\max} = \frac{2Pb\left(3L^2 - 4b^2\right)}{48EI} \qquad \delta_{\max} = \frac{24 \times 12}{360} = 0.800 \text{ in.}$$

$$I = \frac{2Pb\left(3L^2 - 4b^2\right)}{48E\delta_{\max}} = \frac{2(4200)(96)\left[3(288)^2 - 4(96)^2\right]}{48\left(1900 \times 10^3\right)(0.800)}$$

$$= 2343 \text{ in.}^4$$



Try a 10×16-in. timber section with:

$$w = 40.9 \text{ lb/ft} \qquad A = 147 \text{ in.}^2$$

$$I = 2948 \text{ in.}^4 \qquad S = 380 \text{ in.}^3$$

With the weight of the beam included:

$$V_{\max} = 4200 + \frac{wL}{2} = 4200 + \frac{(40.9)(24)}{2} = 4691 \text{ lb}$$

$$M_{\max} = 33{,}600 + \frac{wL^2}{8} = 33{,}600 + \frac{(40.9)(24)^2}{8} = 36{,}545 \text{ lb} \cdot \text{ft}$$

Then the normal stress, the shear stress, and the deflection are:

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{(36{,}545 \times 12)}{380} = 1154 \text{ psi} < \sigma_{all} = 1900 \text{ psi}$$

$$\tau_{\max} = 1.5\frac{V_{\max}}{A} = 1.5\frac{4691}{147} = 47.9 \text{ psi} < \tau_{all} = 85 \text{ psi}$$

$$\delta_{\max} = \frac{2Pb\left(3L^2 - 4b^2\right)}{48EI} + \frac{5wL}{384EI}$$

$$= \frac{2(4200)(96)\left[3(288)^2 - 4(96)^2\right]}{48\left(1900 \times 10^3\right)(2948)} + \frac{5(40.9/12)(24 \times 12)^4}{384\left(1900 \times 10^3\right)(2948)}$$

$$= 0.690 \text{ in.} < \delta_{all} = 0.80 \text{ in.}$$

Therefore $\quad$ Use a 10×16-in. timber section .... **Ans.**

8-194*

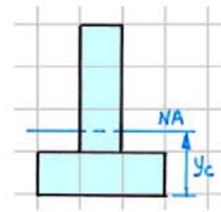
From Eq. 8-2: $$\frac{1}{\rho} = \frac{M}{EI}$$

Then $$\sigma_{max} = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = \frac{Ec}{\rho} = \frac{E(t/2)}{\rho}$$

$$t = \frac{2\sigma_{max}\rho}{E} = \frac{2(15\times10^6)(5)}{(10\times10^9)} = 0.01500 \text{ m} = 15.00 \text{ mm}$$ .... **Ans.**

8-195*

(a) $$y_C = \frac{(0.5)(3\times1)+(2.5)(1\times3)}{2(3\times1)} = 1.500 \text{ in.}$$

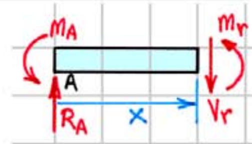


$$I = \frac{(1)(2.5)^3}{3} + \frac{(3)(1.5)^3}{3} - \frac{(2)(0.5)^3}{3} = 8.50 \text{ in.}^4$$

From Eq. 8-2: $$\frac{1}{\rho} = \frac{M}{EI}$$

$$\rho = \frac{EI}{M} = \frac{(30{,}000)(8.50)}{(3.0\times12)} = 7083 \text{ in.} \cong 7080 \text{ in.}$$ .... **Ans.**

(b) From overall equilibrium:



$R_A = 0$ kip $\qquad M_A = 3000 \text{ lb}\cdot\text{ft} = 36.0 \text{ kip}\cdot\text{in.}$ ↺

$M_r = R_A x - M_A = (-36) \text{ kip}\cdot\text{in.}$

$EIv'' = M_r = (-36) \text{ kip}\cdot\text{in.}$

$EIv' = (-36x + C_1) \text{ kip}\cdot\text{in.}^2$

$EIv = (-18x^2 + C_1 x + C_2) \text{ kip}\cdot\text{in.}^3$

Boundary Conditions:

At $x = 0$, $\qquad v = v' = 0$: $\qquad C_1 = C_2 = 0$

$EIv = (-18x^2) \text{ kip}\cdot\text{in.}^3$

At $x = 5 \text{ ft} = 60 \text{ in.}$:

$EIv = (-18)(60)^2 = -64{,}800 \text{ kip}\cdot\text{in.}^3$

$$v = \frac{-64{,}800}{(30{,}000)(8.5)} = -0.254 \text{ in.} = 0.254 \text{ in.}\downarrow$$ .... **Ans.**

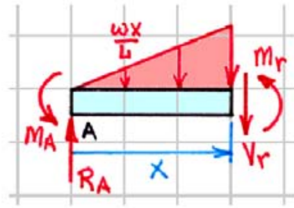
At $x = 3 \text{ ft} = 36 \text{ in.}$:

$EIv = (-18)(36)^2 = -23{,}328 \text{ kip}\cdot\text{in.}^3$

$$v = \frac{-23{,}328}{(30{,}000)(8.5)} = -0.0915 \text{ in.} = 0.0915 \text{ in.}\downarrow$$ .... **Ans.**

8-196

From overall equilibrium:



$\uparrow \Sigma F_y = 0:$ $\quad R_A - (wL/2) = 0$

$\circlearrowleft \Sigma M_A = 0:$ $\quad M_A - \left(\frac{wL}{2}\right)\left(\frac{2L}{3}\right) = 0$

$R_A = (wL/2)\uparrow \qquad M_A = (wL^2/3) \circlearrowleft$

(a) $$EIv'' = M_r = \frac{wLx}{2} - \frac{wL^2}{3} - \frac{wx^3}{6L}$$

$$EIv' = \frac{wLx^2}{4} - \frac{wL^2x}{3} - \frac{wx^4}{24L} + C_1$$

$$EIv = \frac{wLx^3}{12} - \frac{wL^2x^2}{6} - \frac{wx^5}{120L} + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $\quad v' = 0: \quad C_1 = 0$

At $x = 0$, $\quad v = 0: \quad C_2 = 0$

$v = \frac{w}{120EIL}\left(-x^5 + 10L^2x^3 - 20L^3x^2\right)$ .... **Ans.**

(b) $$v' = \frac{w}{24EIL}\left(-x^4 + 6L^2x^2 - 8L^3x\right)$$

$\theta_B = v'_{x=L} = \frac{w}{24EIL}\left(-L^4 + 6L^4 - 8L^4\right) = \frac{-3wL^4}{24EIL} = \frac{wL^3}{8EI}$ ↘ .... **Ans.**

(c) $\delta_B = v_{x=L} = \frac{w}{120EIL}\left(-L^5 + 10L^5 - 20L^5\right) = \frac{-11wL^5}{120EIL} = \frac{11wL^4}{120EI}\downarrow$ .... **Ans.**

8-197*

Using the solution for case 3 in Table B-19

$$\delta_A = v_B + \theta_B (L/2) \quad = \frac{-wL^4}{30EI} - \frac{wL^3}{24EI}\left(\frac{L}{2}\right)$$

$$\delta_A = \frac{-13wL^4}{240EI} = \frac{13wL^4}{240EI} \downarrow$$ .... **Ans.**

8-198

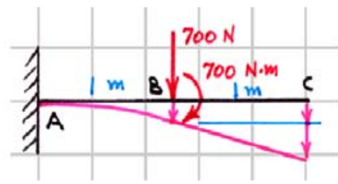
For a C 127×10 channel section:

$A = 1270 \text{ mm}^2 \qquad I_{Y-Y} = 0.199(10^6) \text{ mm}^4 \qquad w_f = 44.5 \text{ mm}$

$x_C = 12.3 \text{ mm} \qquad c_{BC} = w_f - x_C = 44.5 - 12.3 = 32.2 \text{ mm}$

$$I_{AB} = 2\left[\left(0.199\times10^6\right) + 1270(12.3)^2\right] = 0.782\left(10^6\right) \text{ mm}^4$$

Using the solutions for cases 1 and 4 in Table B-19 with $P = wL$ and $M = wL^2/2$



$$\delta_B = v_{BP} + v_{BM} = \frac{-PL^3}{3EI} - \frac{ML^2}{2EI}$$

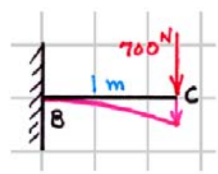
$$= \frac{-(700)(1)^3}{3\left(200\times10^9\right)\left(0.782\times10^{-6}\right)} - \frac{(700)(1)^2}{2\left(200\times10^9\right)\left(0.782\times10^{-6}\right)}$$

$$= -0.003733 \text{ m} = 3.733 \text{ mm} \downarrow$$

$$\theta_B = \theta_{BP} + \theta_{BM} = \frac{-PL^2}{2EI} - \frac{ML}{EI}$$

$$= \frac{-(700)(1)^2}{2\left(200\times10^9\right)\left(0.782\times10^{-6}\right)} - \frac{(700)(1)}{\left(200\times10^9\right)\left(0.782\times10^{-6}\right)}$$

$$= -0.006714 \text{ rad} = 0.006714 \text{ rad}$$



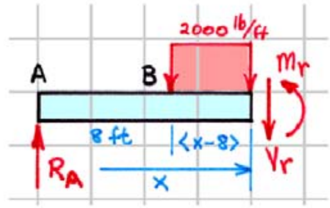
$$\delta_C = \delta_B + \theta_B L_{BC} + \delta_{C/B} = \delta_B + \theta_B L_{BC} - \frac{PL^3}{3EI}$$

$$= -0.003733 - 0.006714(1) - \frac{(700)(1)^3}{3\left(200\times10^9\right)\left(0.199\times10^{-6}\right)}$$

$\delta_C = -0.01258 \text{ m} = 12.58 \text{ mm} \downarrow$ .... **Ans.**

8-199

From overall equilibrium:



$$R_A = \frac{wL}{4} = \frac{(2000)(8)}{4} = 4000 \text{ lb}$$

$$R_B = (2000)(8) - 4000 = 12,000 \text{ lb} \uparrow$$

$$EIv'' = M_r = \left[4000x - 1000\langle x-8\rangle^2\right] \text{ lb}\cdot\text{ft}$$

$$EIv' = \left[2000x^2 - \frac{1000\langle x-8\rangle^3}{3} + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[\frac{2000x^3}{3} - \frac{250\langle x-8\rangle^4}{3} + C_1x + C_2\right] \text{ lb}\cdot\text{ft}^3$$

Boundary Conditions: At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 16$ ft, $v = 0$: $C_1 = -149,333 \text{ lb}\cdot\text{ft}^2$

Therefore $$EIv = \left[\frac{2000x^3}{3} - \frac{250\langle x-8\rangle^4}{3} - 149,333x\right] \text{ lb}\cdot\text{ft}^3$$

δ_{max} occurs where $v' = 0$

$$EIv' = \left[2000x^2 - \frac{1000\langle x-8\rangle^3}{3} - 149,333\right] = 0 \qquad x = 8.643 \text{ ft}$$

$$EIv_{x=8.643} = -861,270 \text{ lb}\cdot\text{ft}^3 = -1.486546(10^9) \text{ lb}\cdot\text{in.}^3 \le EI(0.200 \text{ in.})$$

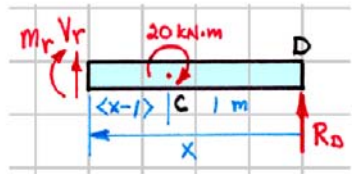
$$I_{min} = \frac{1.486546(10^9)}{(29\times10^6)(0.200)} = 256.3 \text{ in.}^4$$

$$M_{max} = (M_r)_{x=8.643} = 34,159 \text{ lb}\cdot\text{ft} \qquad S_{min} = \frac{M_{max}}{\sigma_{max}} = \frac{(34,159\times12)}{10,000} = 40.99 \text{ in.}^3$$

The lightest beam satisfying both requirements is $W\ 16\times40$ wide-flange section......................... **Ans.**

8-200*

From overall equilibrium:



$$R_B = 4.6 \text{ kN} \uparrow \qquad R_D = 4.4 \text{ kN} \uparrow$$

$$EIv'' = M_r = \left[4.4x - 20\langle x-1\rangle^0\right] \text{ kN}\cdot\text{m}$$

$$EIv' = \left[2.2x^2 - 20\langle x-1\rangle^1 + C_1\right] \text{ kN}\cdot\text{m}^2$$

$$EIv = \left[\frac{2.2x^3}{3} - 10\langle x-1\rangle^2 + C_1 x + C_2\right] \text{ kN}\cdot\text{m}^3$$

Boundary Conditions: At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 2.5$ m, $v = 0$: $C_1 = 4.41667 \text{ kN}\cdot\text{m}^2$

Therefore $$EIv = \left[\frac{2.2x^3}{3} - 10\langle x-1\rangle^2 + 4.41667x\right] \text{ kN}\cdot\text{m}^3$$

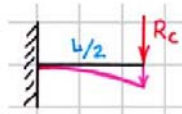
and at the middle of span BD $(x = 1.25 \text{ m})$

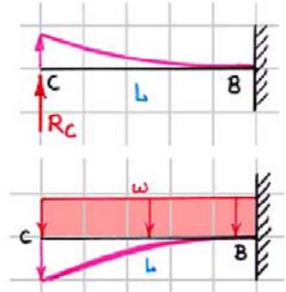
$$\left(70\times10^9\right)\left(20\times10^{-6}\right)v = \left[\frac{2.2(1.25)^3}{3} - 10(0.25)^2 + 4.41667(1.25)\right]\left(10^3\right)$$

$$v_{x=1.25} = \frac{6328.13}{1.400\left(10^6\right)} = 0.00452 \text{ m} = 4.52 \text{ mm} \uparrow$$ .... **Ans.**

8-201

Using the solutions for cases 1 and 2 in Table B-19 with $I_{BC} = 2I_{AC}$ and $E_{BC} = 2E_{AC}$





$$\delta_{C/A} = \delta_{C/B}$$

$$\delta_{CR_C} = \delta_{Cw} + \delta_{CR_C}$$

$$\frac{-R_C(L/2)^3}{3EI} = \frac{-wL^4}{8(2E)(2I)} + \frac{R_C L^3}{3(2E)(2I)}$$

$$R_C = \frac{+wL}{4}$$ .... **Ans.**

For beam AC:

$\uparrow \Sigma F_y = 0: \quad R_A - R_C = 0 \qquad R_A = R_C = \dfrac{+wL}{4} = \dfrac{wL}{4} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_A = 0: \quad M_A - R_C(L/2) = 0 \qquad M_A = \dfrac{R_C L}{2} = \dfrac{wL^2}{8} \circlearrowleft$ .... **Ans.**

For beam BC:

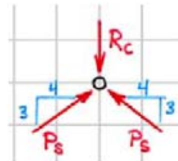
$\uparrow \Sigma F_y = 0: \quad R_C - wL + R_B = 0 \qquad R_B = wL - \dfrac{wL}{4} = \dfrac{3wL}{4} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_B = 0: \quad \dfrac{wL^2}{2} - R_C L - M_B = 0 \qquad M_B = \dfrac{wL^2}{2} - \dfrac{wL^2}{4} = \dfrac{wL^2}{4} \circlearrowright$ .... **Ans.**

8-202*

$$(EI)_{AB} = (200\times10^9)(25\times10^{-6}) = 5.00(10^6)\ \text{N}\cdot\text{m}^2$$

$$(EA)_S = (10\times10^9)(6400\times10^{-6}) = 64(10^6)\ \text{N}$$



For the two struts:

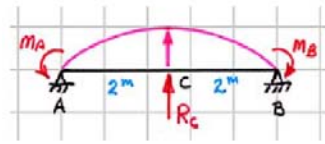
$$\uparrow \Sigma F_y = 0: \qquad 2P_S(3/5) - R_C = 0 \qquad\qquad R_C = 6P_S/5 \qquad \text{(a)}$$

Also, when the center of the beam moves down a distance δ_C each strut will shorten an amount

$$\delta_S = 3\delta_C/5$$

For the beam AB:

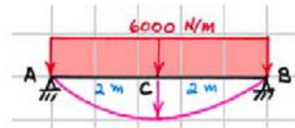
By symmetry: $\quad M_A = M_B = M \quad$ and $\quad R_A = R_B$



Using the solutions for cases 6, 7, and 8 in Table B-19

$$\theta_A = \theta_{AM_A} + \theta_{AM_B} + \theta_{Aw} + \theta_{AR_C}$$

$$= \frac{M(4)}{3(EI)_{AB}} + \frac{M(4)}{6(EI)_{AB}} - \frac{(6000)(4)^3}{24(EI)_{AB}} + \frac{R_C(4)^2}{16(EI)_{AB}} = 0$$



gives $\qquad 2M + R_C = 16{,}000\ \text{N} \qquad$ (b)

$$\delta_C = \delta_{CM_A} + \delta_{CM_B} + \delta_{Cw} + \delta_{CR_C}$$

$$= \frac{M(4)^2}{16(EI)_{AB}} + \frac{M(4)^2}{16(EI)_{AB}} - \frac{5(6000)(4)^4}{384(EI)_{AB}} + \frac{R_C(4)^3}{48(EI)_{AB}} = \frac{6M - 60{,}000 + 4R_C}{3(EI)_{AB}}$$

$$\delta_S = \frac{P_S(2.5)}{(EA)_S} = \frac{-3\delta_C}{5} = \frac{-3(6M - 60{,}000 + 4R_C)}{15(EI)_{AB}} \qquad \text{(c)}$$

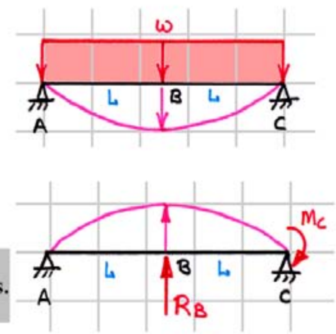
Solving Eqs. (a), (b), and (c) simultaneously gives

$P_S = 5513\ \text{N} \cong 5.51\ \text{kN (C)}$ .... **Ans.**

$R_C = 6616\ \text{N} \qquad$ and $\qquad M = 4692\ \text{N}\cdot\text{m}$

8-203*

(a) Using the solutions for cases 6, 7, and 8 in Table B-19 with $M_C = wL^2/6$



$$\delta_B = v_{M_C} + v_w + v_{R_B} = -wL^4/12EI$$

$$\frac{(wL^2/6)(2L)^2}{16EI} - \frac{5w(2L)^4}{384EI} + \frac{R_B(2L)^3}{48EI} = \frac{-wL^4}{12EI}$$

$R_B = \dfrac{+wL}{2} = \dfrac{wL}{2} \uparrow$ ....**Ans.**

Then from equilibrium

$$\circlearrowleft \Sigma M_C = 0: \qquad 2wL^2 - R_A(2L) - \frac{wL}{2}(L) - \frac{wL}{3}\left(\frac{L}{2}\right) = 0$$

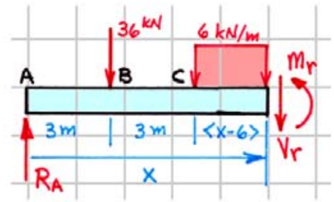
$R_A = \dfrac{+2wL}{3} = \dfrac{2wL}{3} \uparrow$ .... **Ans.**

$$\uparrow \Sigma F_y = 0: \qquad \frac{2wL}{3} - 2wL + \frac{wL}{2} + R_C - \frac{wL}{3} = 0$$

$R_C = \dfrac{+7wL}{6} = \dfrac{7wL}{6} \uparrow$ .... **Ans.**

8-204

(a) $$EIv'' = M_r = \left[R_A x - 36\langle x-3\rangle - 3\langle x-6\rangle^2 \right] \text{ kN}\cdot\text{m}$$



$$EIv' = \left[\frac{R_A x^2}{2} - 18\langle x-3\rangle^2 - \langle x-6\rangle^3 + C_1 \right] \text{ kN}\cdot\text{m}^2$$

$$EIv = \left[\frac{R_A x^3}{6} - 6\langle x-3\rangle^3 - 0.25\langle x-6\rangle^4 + C_1 x + C_2 \right] \text{ kN}\cdot\text{m}^3$$

Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 0$

At $x = 12$, $v' = 0$: $72R_A - 1674 + C_1 = 0$ (a)

At $x = 12$, $v = 0$: $288R_A - 4698 + 12C_1 = 0$ (b)

Solving Eqns. (a) and (b) simultaneously gives: $C_1 = -249.75 \text{ kN}\cdot\text{m}^2$

and $R_A = 26.72 \text{ kN} \cong 26.7 \text{ kN} \uparrow$ .... **Ans.**

Then the overall equilibrium equations give:

$\uparrow \Sigma F_y = 0$: $R_A - 36 - (6\times 6) + R_D = 0$

$R_D = 45.28 \text{ kN} \cong 45.3 \text{ kN} \uparrow$ .... **Ans.**

$\circlearrowleft \Sigma M_D = 0$: $-M_D + (6\times 6)(3) + (36)(9) - 12R_A = 0$

$M_D = 111.36 \text{ kN}\cdot\text{m} \cong 111.4 \text{ kN}\cdot\text{m} \circlearrowright$ .... **Ans.**

(b) $$EIv = \left[4.4531x^3 - 6\langle x-3\rangle^3 - 0.25\langle x-6\rangle^2 - 249.75x \right] \text{ kN}\cdot\text{m}^3$$

At $x = 3$ m: $EIv_{x=3} = \left[4.4531(3)^3 - 249.75(3) \right] = -629 \text{ kN}\cdot\text{m}^3$

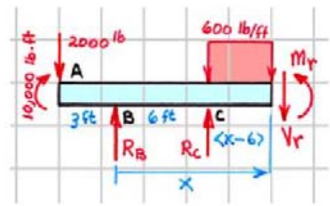
$$\delta_B = v_{x=3} = \frac{-629\times 10^3}{(200\times 10^9)(350\times 10^{-6})} = -0.00899 \text{ m} = 8.99 \text{ mm} \downarrow$$ .... **Ans.**

8-205

$$EIv'' = M_r = \left[10{,}000 - 2000(x+3) + R_B x + R_C\langle x-6\rangle - 300\langle x-6\rangle^2\right] \text{ lb}\cdot\text{ft}$$

$$EIv' = \left[10{,}000x - 1000(x+3)^2 + \frac{R_B x^2}{2} + \frac{R_C\langle x-6\rangle^2}{2} - 100\langle x-6\rangle^3 + C_1\right] \text{ lb}\cdot\text{ft}^2$$

$$EIv = \left[5000x^2 - \frac{1000(x+3)^3}{3} + \frac{R_B x^3}{6} + \frac{R_C\langle x-6\rangle^3}{6} - 25\langle x-6\rangle^4 + C_1 x + C_2\right] \text{ lb}\cdot\text{ft}^3$$



Boundary Conditions:

At $x = 0$, $v = 0$: $C_2 = 9000 \text{ lb}\cdot\text{ft}^3$

At $x = 6$ ft, $v = 0$: $6R_B + C_1 = 9000$ (a)

At $x = 14$ ft, $v = 0$: $2744R_B + 512R_C + 84C_1 = 4{,}506{,}400$ (b)

and the overall equilibrium equations give:

$\uparrow\Sigma F_y = 0$: $R_B + R_C + R_D - (600\times10) - 2000 = 0$ (c)

$\circlearrowleft\Sigma M_D = 0$: $(600\times10)(3) - 14R_B - 8R_C + (2000)(17) - 10{,}000 = 0$ (d)

Solving Eqns. (a) – (d) simultaneously gives: $C_1 = 4257.14 \text{ lb}\cdot\text{ft}^2$

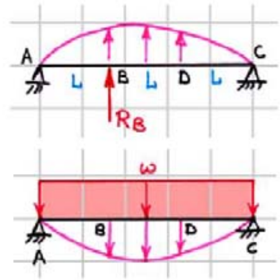
and $R_B = 790.48 \text{ lb} \cong 790 \text{ lb}\uparrow$ .... **Ans.**

$R_C = 3866.67 \text{ lb} \cong 3870 \text{ lb}\uparrow$ .... **Ans.**

$R_D = 3342.86 \text{ lb} \cong 3340 \text{ lb}\uparrow$ .... **Ans.**

8-206*

(a) Using the solutions for cases 5 and 7 in Table B-19



$$\delta_B = v_{BR_B} + v_{Bw}$$

$$\frac{R_B(L)(2L)}{6EI(3L)}\left[(3L)^2 - (L)^2 - (2L)^2\right]$$

$$-\frac{w(L)}{24EI}\left[(L)^3 - 2(3L)(L)^2 + (3L)^3\right] = 0$$

$$R_B = \frac{+33wL}{16} = \frac{33wL}{16}\uparrow$$ .... **Ans.**

Then from equilibrium

$$\circlearrowleft \Sigma M_C = 0: \quad (3wL)\left(\frac{3L}{2}\right) - R_A(3L) - \frac{33wL}{16}(2L) = 0$$

$$R_A = \frac{wL}{8} = \frac{wL}{8}\uparrow$$ .... **Ans.**

$$\uparrow \Sigma F_y = 0: \quad \frac{wL}{8} - 3wL + \frac{33wL}{16} + R_C = 0$$

$$R_C = \frac{13wL}{16} = \frac{13wL}{16}\uparrow$$ .... **Ans.**

(b) The deflection at point D located midway between the supports at B and C is:

$$\delta_D = v_{DR_B} + v_{Dw}$$

$$= \frac{(33wL/16)(L)(L)}{6EI(3L)}\left[(3L)^2 - (L)^2 - (L)^2\right]$$

$$-\frac{w(2L)}{24EI}\left[(2L)^3 - 2(3L)(2L)^2 + (3L)^3\right]$$

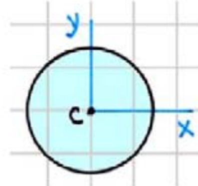
$$\delta_D = \frac{-11wL}{96EI} = \frac{11wL}{96EI}\downarrow$$ .... **Ans.**

9-1*

$A = \pi d^2/4 = \pi(1)^2/4 = 0.7854 \text{ in.}^2$

$I = \pi d^4/64 = \pi(1)^4/64 = 0.04909 \text{ in.}^4$

$r = \sqrt{I/A} = \sqrt{0.04909/0.7854} = 0.250 \text{ in.}$



(a) $L/r = 40/0.250 = 160$ .... **Ans.**

(b) $\dfrac{L}{r} = \sqrt{\dfrac{\pi^2 E}{\sigma_y}} = \sqrt{\dfrac{\pi^2(29{,}000)}{(36)}} = 89.2$ .... **Ans.**

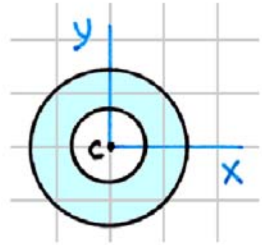
(c) $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(29{,}000)(0.04909)}{(40)^2} = 8.78 \text{ kip}$ .... **Ans.**

9-2*

$$A = \frac{\pi\left(d_o^2 - d_i^2\right)}{4} = \frac{\pi\left(125^2 - 100^2\right)}{4} = 4418 \text{ mm}^2$$

$$I = \frac{\pi\left(d_o^4 - d_i^4\right)}{64} = \frac{\pi\left(125^4 - 100^4\right)}{64} = 7.075\left(10^6\right) \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7.075\left(10^6\right)}{4418}} = 40.02 \text{ mm}$$



(a) $L/r = 6000/40.02 = 149.9$ .... **Ans.**

(b) $\sigma_{cr} = \sigma_y$ $\qquad \frac{L}{r} = \sqrt{\frac{\pi^2 E}{\sigma_y}} = \sqrt{\frac{\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)}} = 88.9$ .... **Ans.**

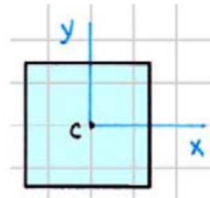
(c) $P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2\left(200\times10^9\right)\left(7.075\times10^{-6}\right)}{(6)^2} = 388\left(10^3\right) \text{ N} = 388 \text{ kN}$ .... **Ans.**

9-3

$$A = bh = (4)(4) = 16.00 \text{ in.}^2$$

$$I = \frac{bh^3}{12} = \frac{(4)(4)^3}{12} = 21.33 \text{ in.}^4$$

$$r = \sqrt{I/A} = \sqrt{21.33/16.00} = 1.1546 \text{ in.}$$



(a) $L/r = (10\times 12)/1.1546 = 103.9$ .... **Ans.**

(b) $\sigma_{cr} = \sigma_y$ $\quad \dfrac{L}{r} = \sqrt{\dfrac{\pi^2 E}{\sigma_y}} = \sqrt{\dfrac{\pi^2(1900)}{(6.4)}} = 54.1$ .... **Ans.**

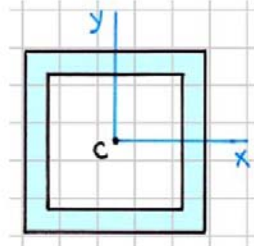
(c) $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(1900)(21.33)}{(120)^2} = 27.8 \text{ kip}$ .... **Ans.**

9-4*

$$A = (150)^2 - (100)^2 = 12,500 \text{ mm}^2$$

$$I = \frac{(150)^4 - (100)^4}{12} = 33.85(10^6) \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{33.85(10^6)}{12,500}} = 52.04 \text{ mm}$$



(a) $L/r = 5000/52.04 = 96.1$ .... **Ans.**

(b) $P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (14\times10^9)(33.85\times10^{-6})}{(5)^2} = 187.09(10^3) \text{ N} \cong 187.1 \text{ kN}$ .... **Ans.**

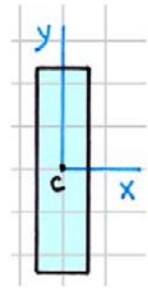
(c) $\sigma = \frac{P}{A} = \frac{187.09(10^3)}{12.5(10^{-3})} = 14.97(10^6) \text{ N/m}^2 = 14.97 \text{ MPa}$ .... **Ans.**

9-5

$$A = bh = \left(\frac{5}{32}\right)\left(\frac{9}{8}\right) = 0.17578 \text{ in.}^2$$

$$I = \frac{bh^3}{12} = \frac{(9/8)(5/32)^3}{12} = 357.6\left(10^{-6}\right) \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{357.6\left(10^{-6}\right)}{0.17578}} = 0.04510 \text{ in.}$$



(a) $L/r = 36/0.0451 = 798$ .... **Ans.**

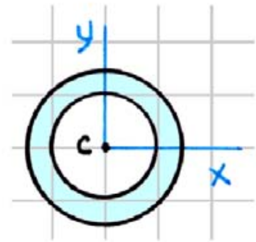
(b) $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(1900)\left(357.6\times10^{-6}\right)}{(36)^2} = 0.00517 \text{ kip} = 5.17 \text{ lb}$ .... **Ans.**

9-6

From Table B-18: $\sigma_y = 250 \text{ MPa}$ $E = 200 \text{ GPa}$

For an 89-mm diameter pipe:

$A = 1729 \text{ mm}^2$ $r = 34.0 \text{ mm}$



$$L/r = 5000/34.0 = 147.06$$

$$P_{max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EA}{(FS)(L/r^2)} = \frac{\pi^2 (200\times 10^9)(1729\times 10^{-6})}{(2)(147.06)^2}$$

$$= 78.91\times 10^3 \text{ N}$$

$P_{max} = 78.9 \text{ kN}$ .... **Ans.**

9-7

From Table B-17: $\sigma_y = 4.6$ ksi $\qquad E = 1800$ ksi

$$A = bh = (5)(5) = 25 \text{ in.}^2 \qquad I = bh^3/12 = (5)(5)^3/12 = 52.08 \text{ in.}$$

$$r = \sqrt{I/A} = \sqrt{52.08/25} = 1.4433 \text{ in.} \qquad L/r = 18\times12/1.4433 = 149.66$$

$$P_{\max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EI}{(FS)L^2} = \frac{\pi^2(1800)(52.08)}{(3)(18\times12)^2} = 6.61 \text{ kip}$$ .... **Ans.**

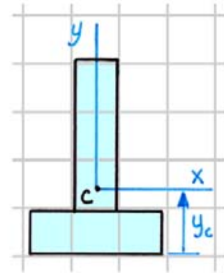
9-8*

$$A = 2(150 \times 50) = 15{,}000 \text{ mm}^2$$

$$y_C = \frac{M_x}{A} = \frac{(25)\big[(150)(50)\big] + (125)\big[(50)(150)\big]}{2\big[(50)(150)\big]} = 75 \text{ mm}$$

$$I_x = \frac{(50)(125)^3}{3} + \frac{(150)(75)^3}{3} - \frac{(100)(25)^3}{3} = 53.13\left(10^6\right) \text{ mm}^4$$

$$I_y = \frac{(50)(150)^3}{12} + \frac{(150)(50)^3}{12} = 15.625\left(10^6\right) \text{ mm}^4$$



$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{15.625\left(10^6\right)}{15{,}000}} = 32.27 \text{ mm}$$

(a) $\dfrac{L}{r} = \dfrac{2500}{32.27} = 77.5$ .... **Ans.**

(b) $\dfrac{L}{r} = \sqrt{\dfrac{\pi^2 E}{\sigma_y}} = \sqrt{\dfrac{\pi^2\left(13 \times 10^9\right)}{\left(44 \times 10^6\right)}} = 54.0$ .... **Ans.**

(c) $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2\left(13 \times 10^9\right)\left(15.625 \times 10^{-6}\right)}{(2.5)^2} = 321\left(10^3\right) \text{ N} = 321 \text{ kN}$ .... **Ans.**

9-9*

From Table B-11: $A = 10.6 \text{ in.}^2$ $r_{\min} = 1.48 \text{ in.}$ $I_{\min} = 23.2 \text{ in.}^4$

From Table B-17: $E = 29,000 \text{ ksi}$ $\sigma_y = 36 \text{ ksi}$

(a) $\frac{L}{r} = \frac{15 \times 12}{1.48} = 121.6$ .... **Ans.**

(b) $\frac{L}{r} = \sqrt{\frac{\pi^2 E}{\sigma_y}} = \sqrt{\frac{\pi^2 (29,000)}{(36)}} = 89.2$ .... **Ans.**

(c) $P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29,000)(23.2)}{(15 \times 12)^2} = 205 \text{ kip}$ .... **Ans.**

9-10

From Table B-12: $A = 11,355 \text{ mm}^2$ $r_{\min} = 39.9 \text{ mm}$ $I_{\min} = 18.1(10^6) \text{ mm}^4$

From Table B-18: $E = 200 \text{ GPa}$ $\sigma_y = 250 \text{ MPa}$

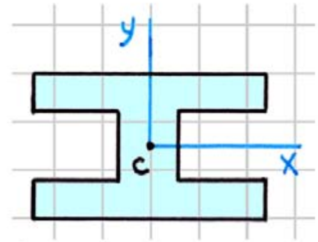
(a) $\dfrac{L}{r} = \dfrac{6000}{39.9} = 150.4$ .... **Ans.**

(b) $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2 (200\times 10^9)(18.1\times 10^{-6})}{(6)^2} = 992.44(10^3) \text{ N} \cong 992 \text{ kN}$ .... **Ans.**

(c) $\sigma = \dfrac{P}{A} = \dfrac{992.44(10^3)}{11,355(10^{-6})} = 87.4(10^6) \text{ N/m}^2 = 87.4 \text{ MPa}$ .... **Ans.**

9-11

$$A = 2(5\times 0.75) + (2\times 1) = 9.50 \text{ in.}^2$$



$$I_x = \frac{(5)(3.5)^3}{12} - \frac{(4)(2)^3}{12} = 15.198 \text{ in.}^4$$

$$I_y = \frac{(1.5)(5)^3}{12} + \frac{(2)(1)^3}{12} = 15.792 \text{ in.}^4$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{15.198}{9.5}} = 1.2648 \text{ in.} \qquad \frac{L}{r} = \frac{(10\times 12)}{1.2648} = 94.88$$

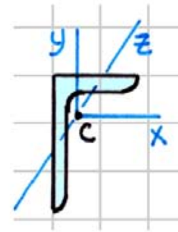
$$P_{max} = \frac{P_{cr}}{FS} = \frac{\pi^2 E I_{min}}{(FS)L^2} = \frac{\pi^2 (10,000)(15.198)}{(2.25)(10\times 12)^2} = 46.3 \text{ kip}$$ .... **Ans.**

9-12

From Table B-18: $E = 200$ GPa $\sigma_y = 250$ MPa

From Table B-10: $I_x = 3.93(10^6)$ mm^4 $I_y = 1.07(10^6)$ mm^4

$A = 2420$ mm^2 $r_{min} = 16.5$ mm $x_C = 19.1$ mm

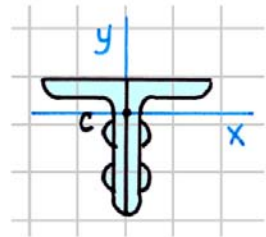


(a) $I_{min} = r_{min}^2 A = (16.5)^2(2420) = 0.6588(10^6)$ mm^4

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200\times 10^9)(0.6588\times 10^{-6})}{(4.5)^2} = 64,223 \text{ N}$$

But there are two angles, so

$P_{max} = 2(64.223) = 128.4$ kN ....**Ans.**



(b) For the pair of angles fastened together

$I_x = 2\left[3.93(10^6)\right] = 2\left[7.96(10^6)\right]$ mm^4

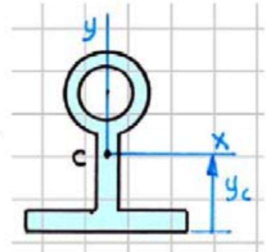
$I_y = 2\left[1.07(10^6) + 2420(19.1)^2\right] = 3.906(10^6)$ mm^4

$$P_{cr} = \frac{\pi^2 EI_{min}}{L^2} = \frac{\pi^2(200\times 10^9)(3.906\times 10^{-6})}{(4.5)^2} = 381(10^3) \text{ N}$$

$P_{max} = 381$ kN ....**Ans.**

9-13*

$$A = (8\times 1) + (4\times 1) + \pi\left(2^2 - 1.5^2\right) = 17.498 \text{ in.}^2$$



$$y_C = \frac{M_x}{A} = \frac{(0.5)(8\times 1) + (3)(4\times 1) + (7)\left[\pi\left(2^2 - 1.5^2\right)\right]}{17.498} = 3.114 \text{ in.}$$

$$I_x = \frac{(8)(3.114)^3}{3} - \frac{(7)(2.114)^3}{3} + \frac{(1)(1.886)^3}{3} + \frac{\pi\left(2^4 - 1.5^4\right)}{4} + \pi\left(2^2 - 1.5^2\right)(3.886)^2 = 152.33 \text{ in.}^4$$

$$I_y = \frac{(1)(8)^3}{12} + \frac{(4)(1)^3}{12} + \frac{\pi\left(2^4 - 1.5^4\right)}{64} = 51.59 \text{ in.}^4$$

$$P_{max} = \frac{P_{cr}}{FS} = \frac{\pi^2 E I_{min}}{(FS)L^2} = \frac{\pi^2 (10{,}000)(51.59)}{(2.50)(10\times 12)^2} = 141.4 \text{ kip}$$ .... **Ans.**

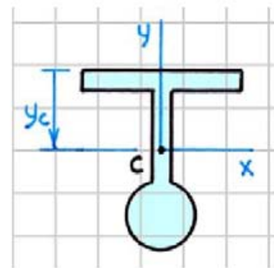
9-14

$$A = (250 \times 25) + (150 \times 25) + \pi(50)^2 = 17{,}854 \text{ mm}^2$$

$$y_C = \frac{M_x}{A} = \frac{(12.5)(250 \times 25) + (100)(150 \times 25) + (225)\left[\pi(50)^2\right]}{17{,}854} = 124.36 \text{ mm}$$

$$I_x = \frac{(250)(124.36)^3}{3} - \frac{(225)(99.36)^3}{3} + \frac{(25)(50.64)^3}{3}$$

$$+ \frac{\pi(50)^4}{4} + \pi(50)^2(100.64)^2 = 172.24(10^6) \text{ mm}^4$$



$$I_y = \frac{(25)(250)^3}{12} + \frac{(150)(25)^3}{12} + \frac{\pi(50)^4}{4} = 37.66(10^6) \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L^2} = \frac{\pi^2 (200 \times 10^9)(37.66 \times 10^{-6})}{(6.5)^2} = 1759.47(10^3) \text{ N}$$

$P_{max} = \dfrac{P_{cr}}{FS} = \dfrac{1759.47}{1.92} = 916$ kN .... **Ans.**

9-15*

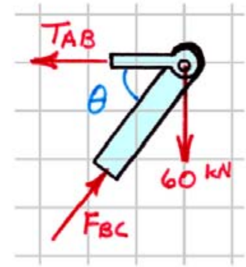
$A = \pi d^2/4 = \pi(2)^2/4 = 3.1412 \text{ in.}^2$ $\qquad I = \pi d^4/64 = \pi(2)^4/64 = 0.7854 \text{ in.}^4$

$$\delta = \left(\frac{\sigma}{E} + \alpha\Delta T\right)L = \left(\frac{-P_{cr}}{AE} + \alpha\Delta T\right)L = \left(\frac{-\pi^2 I}{AL^2} + \alpha\Delta T\right)L = 0$$

$$\Delta T = \frac{\pi^2 I}{\alpha A L^2} = \frac{\pi^2(0.7854)}{(12.5\times10^{-6})(3.142)(10\times12)^2} = 13.71\ ^\circ\text{F}$$ .... **Ans.**

9-16*

$$\theta = \tan^{-1}(2/1.5) = 53.13°$$



$$\rightarrow \Sigma F_x = 0: \quad F_{BC}\cos\theta - T_{AB} = 0$$

$$\uparrow \Sigma F_y = 0: \quad F_{BC}\sin\theta - 60 = 0$$

$$F_{BC} = 75.0 \text{ kN (C)}$$

$$T_{AB} = 45.0 \text{ kN (T)}$$

$$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{45{,}000}{\pi(0.030)^2/4} = 63.662(10^6) \text{ N/m}^2$$

$$FS_{AB} = \frac{360}{63.662} = 5.66 \quad \text{(yielding)}$$

$$\sigma_{BC} = \frac{T_{BC}}{A_{BC}} = \frac{75{,}000}{\pi(0.080^2 - 0.050^2)/4} = 24.485(10^6) \text{ N/m}^2$$

$$FS_{BC} = \frac{280}{24.485} = 11.43 \quad \text{(yielding)}$$

$$I = \frac{\pi(80^4 - 50^4)}{64} = 1.7038(10^6) \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(73\times10^9)(1.7038\times10^{-6})}{(2.5)^2} = 196.409(10^3) \text{ N}$$

$$FS_{BC} = \frac{196.409}{75.0} = 2.62 \quad \text{(buckling)}$$

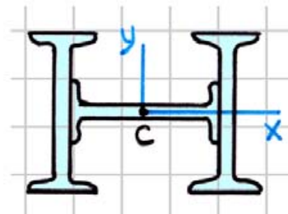
$FS = 2.62$ .... **Ans.**

9-17

From Table B-3 for an S 10×25.4 section:

$I_x = 124 \text{ in.}^4$ $\quad I_y = 6.79 \text{ in.}^4$ $\quad A = 7.46 \text{ in.}^2$

$d = 10.00 \text{ in.}$ $\quad t_w = 0.311 \text{ in.}$



From Table B-17: $\quad E = 29{,}000 \text{ ksi}$ $\quad \sigma_y = 36 \text{ ksi}$

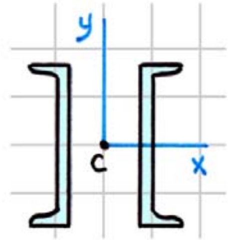
For the riveted column: $\quad I_x = 2(124) + 6.79 = 254.79 \text{ in.}^4$

$$I_x = 124 + 2\left[6.79 + 7.46(5.156)^2\right] = 534.22 \text{ in.}^4$$

$$P_{\max} = P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29{,}000)(254.79)}{(20 \times 12)^2} = 1266 \text{ kip} \text{ .... } \textbf{Ans.}$$

9-18*

From Table B-6 for a C 229×30 section:



$$I_x = 25.3(10^6)\ \text{mm}^4 \qquad I_y = 1.01(10^6)\ \text{mm}^4$$

$$A = 3795\ \text{mm}^2 \qquad x_C = 14.8\ \text{mm}$$

From Table B-18: $E = 200$ GPa $\sigma_y = 250$ MPa

For the latticed channels: $I_x = 2\left[25.3(10^6)\right] = 50.6(10^6)\ \text{mm}^4$

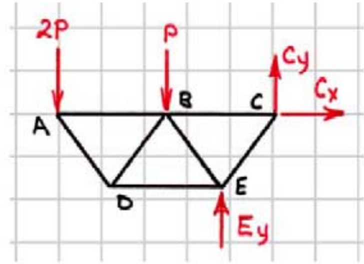
$$I_y = 2\left[1.01(10^6) + 3795(75+14.8)^2\right] = 63.226(10^6)\ \text{mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200\times10^9)(50.6\times10^{-6})}{(12)^2} = 694(10^3)\ \text{N}$$

$P_{\max} = 694$ kN .... **Ans.**

9-19

From the overall free-body diagram:



$\circlearrowleft \Sigma M_C = 0: \qquad 2P(36) + P(18) - R_E(9) = 0$

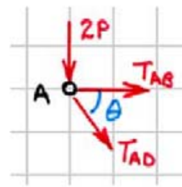
$R_E = 10P$

From a free-body diagram of pin A:

$\theta = \tan^{-1}(12/9) = 53.13°$

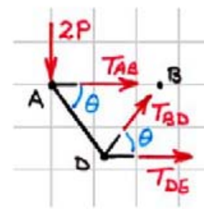
$\uparrow \Sigma F_y = 0: \qquad -T_{AD}\sin 53.13° - 2P = 0$

$T_{AD} = -2.50P = 2.50P \text{ (C)}$



$\rightarrow \Sigma F_x = 0: \qquad T_{AB} + T_{AD}\cos 53.13° = 0$

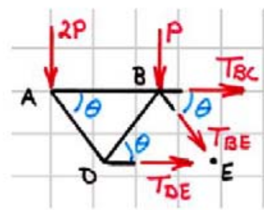
$T_{AB} = +1.50P = 1.50P \text{ (T)}$



From a free-body diagram of the part of the truss to the left of pin B:

$\uparrow \Sigma F_y = 0: \qquad T_{BD}\sin 53.13° - 2P = 0$

$T_{BD} = +2.50P = 2.50P \text{ (T)}$



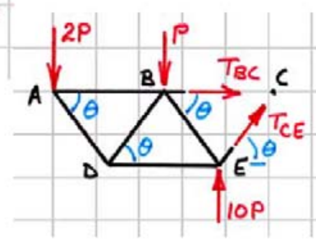
From a free-body diagram of the part of the truss to the left of pin E:

$\uparrow \Sigma F_y = 0: \qquad -T_{BE}\sin 53.13° - 3P = 0$

$T_{BE} = -3.75P = 3.75P \text{ (C)}$

$\circlearrowleft \Sigma M_B = 0: \qquad T_{DE}(12) + 2P(18) = 0$

$T_{DE} = -3.00P = 3.00P \text{ (C)}$



From a free-body diagram of the part of the truss to the left of pin C:

$\uparrow \Sigma F_y = 0: \qquad T_{CE}\sin 53.13° + 10P - 3P = 0 \qquad T_{CE} = -8.75P = 8.75P \text{ (C)}$

$\circlearrowleft \Sigma M_E = 0: \qquad 2P(27) + P(9) - T_{BC}(12) = 0 \qquad T_{BC} = +5.25P = 5.25P \text{ (T)}$

From Table B-5 for a C 10×30 section: $A = 8.82 \text{ in.}^2 \qquad x_C = 0.649 \text{ in.}$

$I_x = 103 \text{ in.}^4 \qquad I_y = 3.94 \text{ in.}^4$

From Table B-17: $E = 29{,}000 \text{ ksi} \qquad \sigma_y = 36 \text{ ksi}$

For member BC: $F_{max} = \sigma_y A = (36)(2 \times 8.82) = 635.0 \text{ kip}$

$$T_{BC} = 5.25P \le \frac{F_{max}}{FS} = \frac{635.0}{1.75} \qquad P \le 69.1 \text{ kip}$$

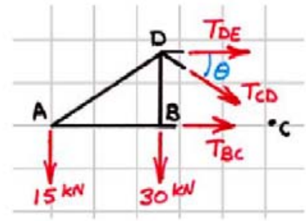
For the bolted channels: $I_y = I_{min} = 2\left[3.94 + 8.82(0.649)^2\right] = 15.31 \text{ in.}^4$

For member CE: $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(29{,}000)(15.31)}{(15 \times 12)^2} = 135.25 \text{ kip}$

$$T_{CE} = 8.75P \le \frac{P_{cr}}{FS} = \frac{135.25}{4} \qquad P \le 3.86 \text{ kip}$$

Therefore $P_{max} = 3.86 \text{ kip}$ .... **Ans.**

9-20

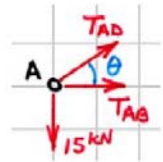


$\circlearrowleft \Sigma M_C = 0: \quad 15(8) + 30(4) - T_{DE}(3) = 0$

$T_{DE} = +80 \text{ kN} = 80 \text{ kN (T)}$

From a free-body diagram of pin *A*:

$\theta = \tan^{-1}(3/4) = 36.87°$



$\uparrow \Sigma F_y = 0: \quad T_{AD} \sin 36.87° - 15 = 0$

$T_{AD} = +25 \text{ kN} = 25 \text{ kN (T)}$

$\rightarrow \Sigma F_x = 0: \quad T_{AB} + 25 \cos 36.87° = 0$

$T_{AB} = -20 \text{ kN} = 20 \text{ kN (C)}$

From a free-body diagram of pin *B*:

$\uparrow \Sigma F_y = 0: \quad T_{BD} - 30 = 0 \qquad T_{BD} = +30 \text{ kN} = 30 \text{ kN (T)}$

$\rightarrow \Sigma F_x = 0: \quad T_{BC} + 20.0 = 0 \qquad T_{BC} = -20 \text{ kN} = 20 \text{ kN (C)}$

$\uparrow \Sigma F_y = 0: \quad -T_{CD} \sin 36.87° - 15 - 30 = 0 \qquad T_{CD} = -75 \text{ kN} = 75 \text{ kN (C)}$

From Table B-12 for a WT 102 × 43 section: $A = 5515 \text{ mm}^2 \qquad I_x = I_{min} = 3.80(10^6) \text{ mm}^4$

From Table B-18: $E = 200 \text{ GPa} \qquad \sigma_y = 250 \text{ MPa}$

(a) For member *DE*: $P_{max} = \sigma_y A = (250 \times 10^6)(5515 \times 10^{-6}) = 1379(10^3) \text{ N} = 1379 \text{ kN}$

$FS = \dfrac{P_{max}}{T_{DE}} = \dfrac{1379}{80} = 17.24$ .... **Ans.**

(b) For member *CD*: $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(200 \times 10^9)(3.80 \times 10^{-6})}{(5)^2} = 300.0(10^3) \text{ N} = 300.0 \text{ kN}$

$FS = \dfrac{P_{cr}}{T_{CD}} = \dfrac{300.0}{75} = 4.00$ .... **Ans.**

9-21*

$$L' = 0.5L = 0.5(10) = 5 \text{ ft} \qquad I_{\min} = (3.5)(1.625)^3/12 = 1.2515 \text{ in.}^4$$

$$P_{\max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EI}{(FS)(L')^2} = \frac{\pi^2 (1600)(1.2515)}{(3)(5\times 12)^2} = 1.830 \text{ kip}$$ .... **Ans.**

9-22*

$$L' = 0.7L = 0.7(3) = 2.10 \text{ m}$$

From Table B-10 for an $\text{L } 102 \times 76 \times 6.4\text{-mm}$ section:

$$r_{\min} = 16.5 \text{ mm} \qquad A = 1090 \text{ mm}^2 \qquad L'/r_{\min} = 2100/16.5 = 127.27$$

$$P_{\max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EA}{(FS)(L'/r)^2} = \frac{\pi^2 (70 \times 10^9)(1090 \times 10^{-6})}{(1.75)(127.27)^2}$$

$P_{\max} = 26,566 \text{ N} \cong 26.6 \text{ kN}$ .... **Ans.**

9-23

$$L' = 2L = 2(10) = 20 \text{ ft}$$

From Table B-1 for a W 8×15 section: $I_{\min} = 3.41 \text{ in.}^4$

$$P_{\max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EI}{(FS)(L')^2} = \frac{\pi^2 (29,000)(3.41)}{(2)(20\times12)^2} = 8.47 \text{ kip}$$ .... **Ans.**

9-24

$L' = 0.7L = 0.7(2.5) = 1.75\text{ m}$ $\qquad A = (50 \times 75) = 3750\text{ mm}^2$

$I_{min} = (75)(50)^3 / 12 = 0.7813(10^6)\text{ mm}^4$

$$P_{max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EI}{(FS)(L')^2} = \frac{\pi^2 (73 \times 10^9)(0.7813 \times 10^{-6})}{(3)(1.75)^2}$$

$P_{max} = 61.3(10^3)\text{ N} = 61.3\text{ kN}$ .... **Ans.**

9-25*

$L' = 0.7L = 0.7(20) = 14$ ft $\qquad I_{\min} = (6)(6)^3/12 = 108$ in.^{4}

$$P_{cr} = \frac{\pi^2 EI}{(L')^2} = \frac{\pi^2(1900)(108)}{(14\times 12)^2} = 71.756 \text{ kip}$$

$FS = P_{cr}/P = 71.756/40 = 1.794$ .... **Ans.**

9-26*

From Table B-2 for a W 254×33 section:

$$A = 4185 \text{ mm}^2 \qquad r_x = 108 \text{ mm} \qquad r_y = 33.8 \text{ mm}$$

For axis x–x: $L' = L = 6.00$ m $\qquad L'/r = 6000/108 = 55.56$

For axis y–y: $L' = 0.5L = 0.5(6) = 3.00$ m $\qquad L'/r = 3000/33.8 = 88.76$

$$P_{max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EA}{(FS)(L'/r)^2} = \frac{\pi^2 (200 \times 10^9)(4185 \times 10^{-6})}{(1.9)(88.76)^2}$$

$P_{max} = 552(10^3)$ N $= 552$ kN .... **Ans.**

9-27

From Table B-1 for a W 10×22 section:

$$A = 6.49 \text{ in.}^2 \qquad r_x = 4.27 \text{ in.} \qquad r_y = 1.33 \text{ in.}$$

For axis x–x: $L' = 0.7L = 0.7(20 \times 12) = 168$ in. $\qquad L'/r = 168/4.27 = 39.34$

For axis y–y: $L' = 0.5L = 0.5(20 \times 12) = 120$ in. $\qquad L'/r = 120/1.33 = 90.23$

$$P_{\max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EA}{(FS)(L'/r)^2} = \frac{\pi^2 (29{,}000)(6.49)}{(3)(90.23)^2} = 76.1 \text{ kip}$$ .... **Ans.**

9-28

From Table B-2 for a W 127×15 section: $A = 1895 \text{ mm}^2$

$$I_{X-X} = 5.12(10^6) \text{ mm}^4 \qquad I_{Y-Y} = 0.508(10^6) \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(L')^2} \qquad L'_x = \sqrt{\frac{\pi^2 EI}{P_{cr}}} = \sqrt{\frac{\pi^2 (200 \times 10^9)(5.12 \times 10^{-6})}{(60 \times 10^3)}} = 12.978 \text{ m} > 12 \text{ m}$$

Therefore, no braces are required to prevent buckling about the x-axis.

$$L'_y = \sqrt{\frac{\pi^2 EI}{P_{cr}}} = \sqrt{\frac{\pi^2 (200 \times 10^9)(0.508)}{(60 \times 10^3)}} = 4.088 \text{ m} < 12 \text{ m}$$

(a) Two braces are required to prevent buckling about the y-axis. .... **Ans.**

(b) With the two braces installed: $L'_y = 4.00 \text{ m}$

$$P_{max} = P_{cr} = \frac{\pi^2 EI}{(L')^2} = \frac{\pi^2 (200 \times 10^9)(0.508 \times 10^{-6})}{(4.00)^2}$$

$P_{max} = 62.7(10^3) \text{ N} = 62.7 \text{ kN}$ .... **Ans.**

9-29*

From Table B-11 for a WT 7×24 section: $I_{X-X} = I_{\min} = 24.9 \text{ in.}^4$

$$P_{\max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EI}{(FS)(L')^2} = \frac{\pi^2 (29,000)(24.9)}{(2)(L')^2} = \frac{3563}{(L')^2} \text{ kip}$$

(a) $L' = L = (20 \times 12) = 240 \text{ in.}$ $P_{\max} = \frac{3563}{(240)^2} = 61.9 \text{ kip}$ .... **Ans.**

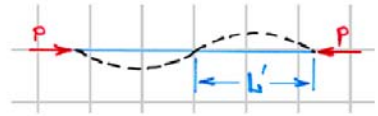
(b) $L' = 2L = 2(20 \times 12) = 480 \text{ in.}$ $P_{\max} = \frac{3563}{(480)^2} = 15.47 \text{ kip}$ .... **Ans.**

(c) $L' = 0.7L = 0.7(20 \times 12) = 168 \text{ in.}$ $P_{\max} = \frac{3563}{(168)^2} = 126.3 \text{ kip}$ .... **Ans.**

(d) $L' = 0.5L = 0.5(20 \times 12) = 120 \text{ in.}$ $P_{\max} = \frac{3563}{(120)^2} = 247 \text{ kip}$ .... **Ans.**

9-30*

$$I = \frac{\pi D^4}{64} \qquad L' = \frac{L}{2}$$



$$P_{cr} = \frac{\pi^2 EI}{(L')^2} = \frac{\pi^2 E(\pi D^4/64)}{(L/2)^2} = \frac{\pi^3 ED^4}{16L^2}$$ .... **Ans.**

9-31

$$A = \frac{\pi d^2}{4} = \frac{\pi (1.0)^2}{4} = 0.7854 \text{ in.}^2$$

$$I = \frac{\pi d^4}{64} = \frac{\pi (1.0)^4}{64} = 0.04909 \text{ in.}^4$$

$$L' = L = 6 \text{ ft} = 72 \text{ in.}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.04909}{0.7854}} = 0.2500 \text{ in.}$$

$$L'/r = 72/0.2500 = 288 \quad \text{(slender)}$$

$$P_{max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EI}{(FS)(L')^2} = \frac{\pi^2 (29{,}000)(0.04909)}{(2.5)(72)^2} = 1.084 \text{ kip}$$ .... **Ans.**

9-32*

$$A = \pi d^2/4 = \pi(50)^2/4 = 1963.5 \text{ mm}^2$$

$$I = \pi d^4/64 = \pi(50)^4/64 = 306.8(10^3) \text{ mm}^4$$

$$L' = 2L = 2(5) = 10 \text{ m} \qquad r = \sqrt{I/A} = \sqrt{306.8(10^3)/1963.5} = 12.500 \text{ mm}$$

$$L'/r = 10(10^3)/12.500 = 800 \quad \text{(slender)}$$

$$P_{\max} = \frac{P_{cr}}{FS} = \frac{\pi^2 EI}{(FS)(L')^2} = \frac{\pi^2(200\times10^9)(0.3068\times10^{-6})}{(2)(10)^2} = 3028 \text{ N} \cong 3.03 \text{ kN}$$ .... **Ans.**

9-33

$A = \pi d^2/4 = \pi(2)^2/4 = 3.142 \text{ in.}^2$

$I = \pi d^4/64 = \pi(2)^4/64 = 0.7854 \text{ in.}^4$

$L' = L/3 = (24/3) = 8 \text{ ft}$ $r = \sqrt{I/A} = \sqrt{0.7854/3.142} = 0.500 \text{ in.}$

$L'/r = 96/0.500 = 192$ (slender)

$$P_{cr} = \frac{\pi^2 EI}{(L')^2} = \frac{\pi^2(10,000)(0.7854)}{(8\times12)^2} = 8.411 \text{ kip}$$

$FS = P_{cr}/P = 8.411/4 = 2.10$ .... **Ans.**

9-34

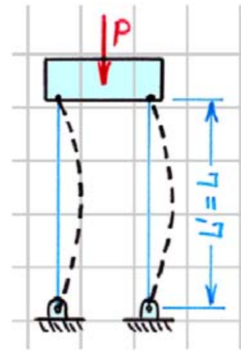
$A = \pi d^2/4 = \pi(25)^2/4 = 490.9 \text{ mm}^2$

$I = \pi d^4/4 = \pi(25)^4/4 = 19.175(10^3) \text{ mm}^4$

$L' = L = 3 \text{ m}$

$r = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{19.175(10^3)}{490.9}} = 6.250 \text{ mm}$

$P_{cr} = \dfrac{\pi^2 EI}{(L')^2} = \dfrac{\pi^2(200\times10^9)(19.175\times10^{-9})}{(3)^2} = 4206 \text{ N}$

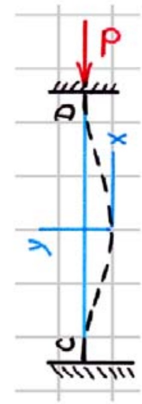


$FS = \dfrac{2P_{cr}}{W} = \dfrac{2P_{cr}}{mg} = \dfrac{2(4206)}{(300)(9.81)} = 2.86$ .... **Ans.**

9-35

$$EIv'' = M_D - Pv$$

$$v'' + \frac{P}{EI}v = \frac{M_D}{EI}$$



which has the solution

$$v = A\sin kx + B\cos kx + \frac{M_D}{P} \qquad \text{where } k^2 = \frac{P}{EI}$$

$$v' = Ak\cos kx - Bk\sin kx$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $A = 0$

At $x = 0$, $v = 0$: $B = -M_D/P$

At $x = L/2$, $v' = 0$: $\sin\frac{kL}{2} = 0$

If $\sin\frac{kL}{2} = 0$, then $\frac{kL}{2} = \sqrt{\frac{P}{EI}}\frac{L}{2} = n\pi$ or $P = \frac{4n^2\pi^2 EI}{L^2}$

The smallest value of P is obtained with $n = 1$.

Therefore:

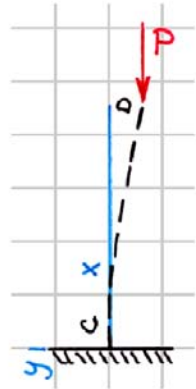
$$P = \frac{4\pi^2 EI}{L^2} = \frac{\pi^2 EI}{(L/2)^2}$$

$L' = L/2$ .... **Ans.**

9-36

$$EIv'' = P(\delta - v)$$

$$v'' + \frac{P}{EI}v = \frac{P\delta}{EI}$$



which has the solution

$$v = A\sin kx + B\cos kx + \delta \qquad \text{where } k^2 = \frac{P}{EI}$$

$$v' = Ak\cos kx - Bk\sin kx$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $A = 0$

At $x = 0$, $v = 0$: $B = -\delta$

At $x = L$, $v = \delta$: $\cos kL = 0$

If $\cos kL = 0$, then $kL = \sqrt{\frac{P}{EI}}L = (2n+1)\frac{\pi}{2}$ or $P = \frac{(2n+1)^2\pi^2 EI}{4L^2}$

The smallest value of P is obtained with $n = 0$.

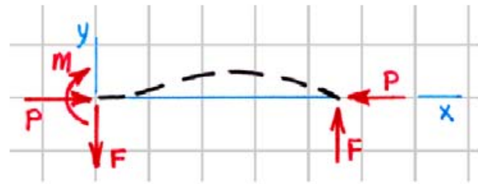
Therefore:

$$P = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 EI}{(2L)^2}$$

$L' = 2L$ .... **Ans.**

9-37

$$F = M/L$$



$$EIv'' = M - Pv - \frac{Mx}{L}$$

$$v'' + \frac{P}{EI}v = \frac{M}{EI} - \frac{Mx}{EIL}$$

which has the solution

$$v = A\sin kx + B\cos kx + \frac{M}{P} - \frac{Mx}{PL} \qquad \text{where } k^2 = \frac{P}{EI}$$

$$v' = Ak\cos kx - Bk\sin kx - \frac{M}{PL}$$

Boundary Conditions:

At $x = 0$, $\quad v' = 0$: $\quad A = M/PLk$

At $x = 0$, $\quad v = 0$: $\quad B = -M/P$

At $x = L$, $\quad v = 0$: $\quad \tan kL = kL$

If $\tan kL = kL$, then $\quad kL = \sqrt{\frac{P}{EI}}L = 1.430\pi$ or $\quad P = \frac{2.045\pi^2 EI}{L^2}$

Therefore:

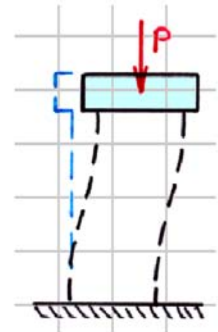
$$P = \frac{2.045\pi^2 EI}{L^2} = \frac{\pi^2 EI}{(0.6993L)^2}$$

$L' = 0.6693L \cong 0.7L$ .... **Ans.**

9-38

$$EIv'' = M - Pv$$

$$v'' + \frac{P}{EI}v = \frac{M}{EI}$$



which has the solution

$$v = A\sin kx + B\cos kx + \frac{M}{P} \qquad \text{where } k^2 = \frac{P}{EI}$$

$$v' = Ak\cos kx - Bk\sin kx$$

Boundary Conditions:

At $x = 0$, $v' = 0$: $A = 0$

At $x = 0$, $v = 0$: $B = -M/P$

At $x = L$, $v' = 0$: $\sin kL = 0$

If $\sin kL = 0$, then $kL = \sqrt{\frac{P}{EI}}L = n\pi$ or $P = \frac{n^2\pi^2 EI}{L^2}$

The smallest value of P is obtained with $n = 1$.

Therefore:

$$P = \frac{\pi^2 EI}{L^2}$$

$L' = L$ .... **Ans.**

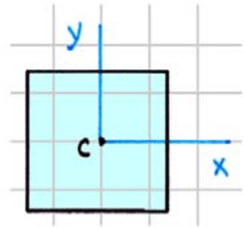
9-39*

$$A = bd = (3 \times 3) = 9.0 \text{ in.}^2$$

$$k = 0.671\sqrt{\frac{E}{F_C}} = 0.671\sqrt{\frac{1800}{4.6}} = 13.273$$

$$\frac{L'}{d} = \frac{(5 \times 12)}{3} = 20.0 > k$$

$$\sigma_{all} = \frac{0.30E}{(L/d)^2} = \frac{0.3(1800)}{(20.0)^2} = 1.350 \text{ ksi}$$

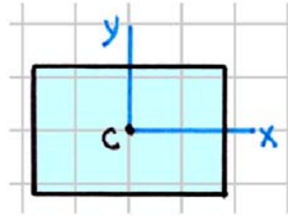


$P_{max} = \sigma_{all} A = (1.350)(9) = 12.15$ kip .... **Ans.**

9-40*

$$A = bd = (300 \times 200) = 60(10^3) \text{ mm}^2$$

$$k = 0.671\sqrt{\frac{E}{F_C}} = 0.671\sqrt{\frac{11{,}000}{7.6}} = 25.53$$



(a) $$\frac{L'}{d} = \frac{2000}{200} = 10.0 < 11$$

$$P_{\max} = \sigma_{all} A = (7.6 \times 10^6)(0.060) = 456(10^3) \text{ N} = 456 \text{ kN}$$ .... **Ans.**

(b) $$\frac{L'}{d} = \frac{4000}{200} = 20.0 < 25.53$$

$$\sigma_{all} = F_C\left[1 - \frac{1}{3}\left(\frac{L/d}{k}\right)^4\right] = 7.6\left[1 - \frac{1}{3}\left(\frac{20}{25.53}\right)^4\right] = 6.646 \text{ MPa}$$

$$P_{\max} = \sigma_{all} A = (6.646 \times 10^6)(0.060) = 399(10^3) \text{ N} = 399 \text{ kN}$$ .... **Ans.**

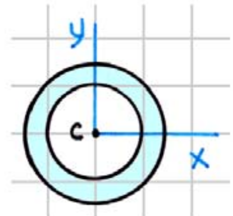
(c) $$\frac{L'}{d} = \frac{6000}{200} = 30.0 > 25.53$$

$$\sigma_{all} = \frac{0.30E}{(L/d)^2} = \frac{0.3(11 \times 10^9)}{(30)^2} = 3.667(10^6) \text{ N/m}^2 = 3.667 \text{ MPa}$$

$$P_{\max} = \sigma_{all} A = (3.667 \times 10^6)(0.060) = 220(10^3) \text{ N} = 220 \text{ kN}$$ .... **Ans.**

9-41

From Table B-13 (for a 2.5-in. diameter pipe):



$$A = 1.704 \text{ in.}^2 \qquad r = 0.95 \text{ in.}$$

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2 (29{,}000)}{36} = 15{,}901$$

$$C_c = 126.10$$

$$\frac{L}{r} = \frac{(8 \times 12)}{0.95} = 101.05 < 126.10$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{101.05}{126.10}\right) - \frac{1}{8}\left(\frac{101.05}{126.10}\right)^3 = 1.903$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.903}\left[1 - \frac{1}{2}\left(\frac{101.05}{126.10}\right)^2\right] = 12.843 \text{ ksi}$$

$P_{\max} = \sigma_{all} A = (12.843)(1.704) = 21.9$ kip .... **Ans.**

9-42*

From Table B-2 (for a W 254×89 section):

$$A = 11{,}355 \text{ mm}^2 \qquad r_{X-X} = 112 \text{ mm} \qquad r_{Y-Y} = 65.3 \text{ mm}$$

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15{,}791$$

$$C_c = 125.66$$

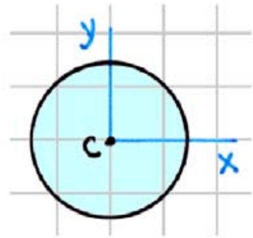
$$\frac{L'}{r_{\min}} = \frac{3000}{65.3} = 45.94 < 125.66$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{45.94}{125.66}\right) - \frac{1}{8}\left(\frac{45.94}{125.66}\right)^3 = 1.7977$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L'/r}{C_c}\right)^2\right] = \frac{250}{1.7977}\left[1 - \frac{1}{2}\left(\frac{45.94}{125.66}\right)^2\right] = 129.77 \text{ MPa}$$

$$P_{\max} = \sigma_{all} A = \left(129.77\times10^6\right)\left(11{,}355\times10^{-6}\right) = 1474\left(10^3\right) \text{ N} = 1474 \text{ kN}$$ .... **Ans.**

9-43



$$A = \frac{\pi d^2}{4} = \frac{\pi(3)^2}{4} = 7.069 \text{ in.}^2$$

$$I = \frac{\pi d^4}{64} = \frac{\pi(3)^4}{64} = 3.976 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.976}{7.069}} = 0.750 \text{ in.}$$

$$\frac{L}{r} = \frac{30}{0.750} = 40.0 < 66$$

$$\sigma_{all} = 20.2 - 0.126(L/r) = 20.2 - 0.126(40.0) = 15.16 \text{ ksi}$$

$$P_{\max} = \sigma_{all} A = (15.16)(7.069) = 107.2 \text{ kip}$$ .... **Ans.**

9-44

$$A = \frac{\pi\left(100^2 - 80^2\right)}{4} = 2827 \text{ mm}^2 \qquad I = \frac{\pi\left(100^4 - 80^4\right)}{64} = 2.898\left(10^6\right) \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.898\left(10^6\right)}{2827}} = 32.02 \text{ mm}$$

$$\frac{L}{r} = \frac{1000}{32.02} = 31.23 < 55$$

$$\sigma_{all} = 212 - 1.585\left(L/r\right) = 212 - 1.585\left(31.23\right) = 162.50 \text{ MPa}$$

$$P_{\max} = \sigma_{all} A = \left(162.50 \times 10^6\right)\left(2827 \times 10^{-6}\right) = 459\left(10^3\right) \text{ N} = 459 \text{ kN}$$ .... **Ans.**

9-45*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

(a) $A = (4\times 1) = 4.0\text{ in.}^2 \qquad I = (4)(1)^3/12 = 0.3333\text{ in.}^4$

$$r = \sqrt{I/A} = \sqrt{0.3333/4} = 0.2887\text{ in.}$$

$$\frac{L'}{r} = \frac{0.5(8\times 12)}{0.2887} = 162.26 > 126.10$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29,000)}{1.92(162.26)^2} = 5.662\text{ ksi}$$

$P_{max} = \sigma_{all} A = (5.662)(4) = 22.6\text{ kip}$ .... **Ans.**

(b) $I_{min} = 2\left[\frac{(1)(4)^3}{12}\right] + \frac{(4)(1)^3}{12} = 11.00\text{ in.}^4$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.00}{3(4)}} = 0.9574\text{ in.}$$

$$\frac{L'}{r} = \frac{0.5(8\times 12)}{0.9574} = 50.14 < 126.10$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{50.14}{126.10}\right) - \frac{1}{8}\left(\frac{50.14}{126.10}\right)^3 = 1.8079$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.8079}\left[1 - \frac{1}{2}\left(\frac{50.14}{126.10}\right)^2\right] = 18.338\text{ ksi}$$

$P_{max} = \sigma_{all} A = (18.338)(3\times 4) = 220\text{ kip}$ .... **Ans.**

9-46*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$

$$A = \frac{\pi\left(80^2 - 50^2\right)}{4} = 3063 \text{ mm}^2 \qquad I = \frac{\pi\left(80^4 - 50^4\right)}{64} = 1.7038\left(10^6\right) \text{ mm}^4$$

$$r = \sqrt{I/A} = \sqrt{1,703,800/3063} = 23.58 \text{ mm} \qquad L' = L = 3.5 \text{ m}$$

(a)

$$\frac{L'}{r} = \frac{3500}{23.58} = 148.43 > 125.66$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\left(L'/r\right)^2} = \frac{\pi^2\left(200\times10^9\right)}{1.92\left(148.43\right)^2} = 46.66\left(10^6\right) \text{ N/m}^2$$

$$P_{\max} = \sigma_{all} A = \left(46.66\times10^6\right)\left[3\left(3063\times10^{-6}\right)\right] = 429\left(10^3\right) \text{ N} = 429 \text{ kN}$$ .... **Ans.**

(b)

$$A = 3\left[\frac{\pi\left(80^2 - 50^2\right)}{4}\right] = 9189 \text{ mm}^2$$

$$I = 3\left[\frac{\pi\left(80^4 - 50^4\right)}{64}\right] + 2\left[\frac{\pi\left(80^2 - 50^2\right)}{4}\left(40^2\right)\right] = 14.913\left(10^6\right) \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{14,913,000}{9189}} = 40.29 \text{ mm} \qquad L' = L = 3.5 \text{ m}$$

$$\frac{L'}{r} = \frac{3500}{40.29} = 86.87 < 125.66$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{86.87}{125.66}\right) - \frac{1}{8}\left(\frac{86.87}{125.66}\right)^3 = 1.884$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.884}\left[1 - \frac{1}{2}\left(\frac{86.87}{125.66}\right)^2\right] = 100.99 \text{ MPa}$$

$$P_{\max} = \sigma_{all} A = \left(100.99\times10^6\right)\left(9189\times10^{-6}\right) = 928\left(10^3\right) \text{ N} = 928 \text{ kN}$$ .... **Ans.**

9-47

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

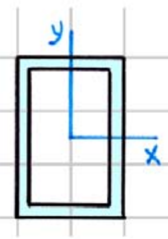
From Table B-5 (for a $C\ 10\times 15.3$ channel): $A = 4.49 \text{ in.}^2$

$r_{Y-Y} = 0.713$ in. $\qquad w_f = 2.600$ in.

$x_C = 0.634$ in. $\qquad r_{X-X} = 3.87$ in.

$$L' = 0.7L = 0.7(12) = 8.4 \text{ ft}$$

For the cross section:



$$r_x = 3.87 \text{ in.}$$

$$r_y = \sqrt{(0.713)^2 + (2.600 - 0.634)^2} = 2.091 \text{ in.}$$

$$\frac{L'}{r_{min}} = \frac{(8.4\times 12)}{2.091} = 48.21 < 126.10$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{48.21}{126.10}\right) - \frac{1}{8}\left(\frac{48.21}{126.10}\right)^3 = 1.803$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.803}\left[1 - \frac{1}{2}\left(\frac{48.21}{126.10}\right)^2\right] = 18.508 \text{ ksi}$$

$P_{max} = \sigma_{all} A = (18.508)\left[2(4.49)\right] = 166.2$ kip .... **Ans.**

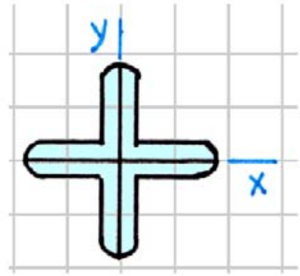
9-48

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$

From Table B-8 (for an L $76\times76\times12.7$ angle): $A = 1775 \text{ mm}^2$

$r_{X-X} = r_{Y-Y} = 22.8$ mm $\qquad x_C = 23.7$ mm

For the cross section:



$$r_x = r_y = \sqrt{(22.8)^2 + (23.7)^2} = 32.89 \text{ mm}$$

$$L' = 0.5L = 0.5(5) = 2.5 \text{ m}$$

$$\frac{L'}{r} = \frac{2500}{32.89} = 76.01 < 125.66$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{76.01}{125.66}\right) - \frac{1}{8}\left(\frac{76.01}{125.66}\right)^3 = 1.866$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.866}\left[1 - \frac{1}{2}\left(\frac{76.01}{125.66}\right)^2\right] = 109.47 \text{ MPa}$$

$P_{max} = \sigma_{all} A = \left(109.47\times10^6\right)\left[4\left(1775\times10^{-6}\right)\right] = 777\left(10^3\right) \text{ N} = 777 \text{ kN}$ .... **Ans.**

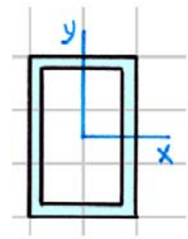
9-49*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

From Table B-9 (for an L $4 \times 3 \times 3/8$ angle): $A = 2.48 \text{ in.}^2$

$r_{Y-Y} = 0.879$ in. $\qquad x_C = 0.782$ in.

For the cross section:



$$r_{min} = \sqrt{(0.879)^2 + (3 - 0.782)^2} = 2.386 \text{ in.}$$

$$L' = L = 11 \text{ ft}$$

$$\frac{L'}{r_{min}} = \frac{(11 \times 12)}{2.386} = 55.32 < 126.10$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{55.32}{126.10}\right) - \frac{1}{8}\left(\frac{55.32}{126.10}\right)^3 = 1.821$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.821}\left[1 - \frac{1}{2}\left(\frac{55.32}{126.10}\right)^2\right] = 17.871 \text{ ksi}$$

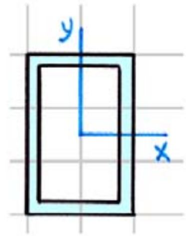
$P_{max} = \sigma_{all} A = (17.871)\left[4(2.48)\right] = 177.3$ kip .... **Ans.**

9-50

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$

From Table B-10 (for an L $102\times76\times9.5$ angle): $A = 1600\text{ mm}^2$

$r_{Y-Y} = 22.3\text{ mm}$ $\qquad x_C = 19.9\text{ mm}$



$$r_y = r_{min} = \sqrt{(22.3)^2 + (38-19.9)^2} = 28.72\text{ mm} \qquad L' = L = 7\text{ m}$$

$$\frac{L'}{r} = \frac{7000}{28.72} = 243.7 > 125.66$$

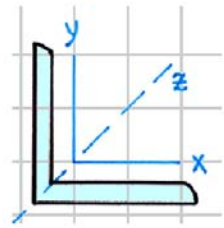
$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2\left(200\times10^9\right)}{1.92(243.7)^2} = 17.311\left(10^6\right)\text{ N/m}^2$$

$P_{max} = \sigma_{all} A = \left(17.311\times10^6\right)\left[2\left(1600\times10^{-6}\right)\right] = 55.4\left(10^3\right)\text{ N} = 55.4\text{ kN}$ .... **Ans.**

9-51

From Table B-7 (for an L $5 \times 5 \times 3/4$ angle): $A = 6.94 \text{ in.}^2$ $r_{\min} = 0.975$ in.

Assume slender range $(L/r > 55)$:



$$P_{all} = \sigma_{all} A = \frac{54,000(A)}{(L'/r)^2} = \frac{54,000(6.94)}{(L'/r)^2} = 120 \text{ kip}$$

$$L'/r = 55.88 > 55$$

Therefore: $L' = 55.88 r_{\min} = 55.88(0.975) = 54.48$ in.

$L = 54.48/0.7 = 77.83 \text{ in.} \cong 6.49 \text{ ft}$ .... **Ans.**

9-52*

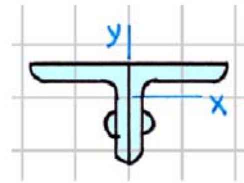
From Table B-8 (for an L $127\times127\times19.1$ angle): $A = 4475 \text{ mm}^2$

$r_{X-X} = r_{Y-Y} = 38.4$ mm $\qquad x_C = 38.6$ mm

$$r_x = r_{X-X} = 38.4 \text{ mm} \qquad r_y = \sqrt{(38.4)^2 + (38.6)^2} = 54.45 \text{ mm}$$

$$\frac{L'_x}{r_x} = \frac{2100}{38.4} = 54.69 < 55$$

$$\frac{L'_y}{r_y} = \frac{3000}{54.45} = 55.10 > 55$$



$$\sigma_{all} = \frac{372(10^3)}{(L/r)^2} = \frac{372(10^3)}{(55.10)^2} = 122.53 \text{ MPa}$$

$$P_{max} = \sigma_{all} A = (122.53\times10^6)\left[2(4475\times10^{-6})\right]$$

$$P_{max} = 1097(10^3) \text{ N} = 1097 \text{ kN}$$ .... **Ans.**

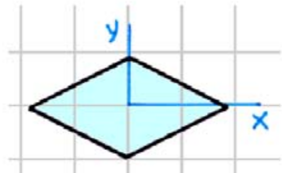
9-53*

$$A = \frac{(2d)(d)}{2} = d^2 \qquad I_{min} = 2\frac{(2d)(d/2)^3}{12} = \frac{d^4}{24}$$

$$r_{min} = \sqrt{I/A} = 0.2041d$$

$$L' = 0.7L = 0.7(4\times 12) = 33.6 \text{ in.}$$

$$\frac{L'}{r_{min}} = \frac{33.6}{0.2041d} = \frac{164.63}{d}$$



Assume that $9.5 \le (L/r) \le 66$

Then $\sigma_{all} = 20.2 - 0.126(L/r)$

$$\sigma_{all} = \frac{P}{A} = \frac{165}{d^2} = 20.2 - 0.126\left(\frac{164.63}{d}\right) = 20.2 - \frac{20.74}{d}$$

which gives ` $d = 3.417$ in.

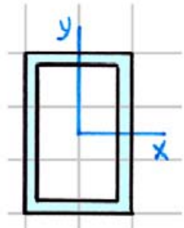
Check (L/r): $\frac{L'}{r} = \frac{164.63}{d} = \frac{164.63}{3.417} = 48.18$ is within $9.5 \le (L/r) \le 66$

Therefore $d = 3.42$ in. .... **Ans.**

9-54

$$r_x = \sqrt{(48.3)^2 + (23.9)^2} = 53.89 \text{ mm} \qquad \frac{L'_x}{r_x} = \frac{4750}{53.89} = 88.14 > 55$$

$$r_y = \sqrt{(24.1)^2 + (30.0)^2} = 38.48 \text{ mm} \qquad \frac{L'_y}{r_y} = \frac{3250}{38.48} = 84.46 > 55$$



$$\sigma_{all} = \frac{372(10^3)}{(L/r)^2} = \frac{372(10^3)}{(88.14)^2} = 47.88 \text{ MPa}$$

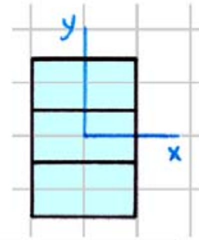
$$P_{\max} = \sigma_{all} A = (47.88\times10^6)\left[2(2910\times10^{-6})\right]$$

$$P_{\max} = 279(10^3) \text{ N} = 279 \text{ kN}$$ .... **Ans.**

9-55

$A = 3bd = 3(2\times 4) = 24.0\text{ in.}^2$ $\qquad L' = 0.7L = 0.7(8) = 5.60\text{ ft}$

$$k = 0.671\sqrt{\frac{E}{F_C}} = 0.671\sqrt{\frac{1,600,000}{1100}} = 25.59$$



$$\frac{L'}{d} = \frac{(5.60\times 12)}{4} = 16.80 < k$$

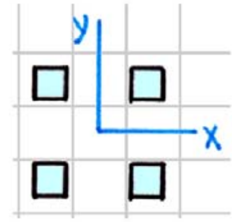
$$\sigma_{all} = F_C\left[1-\frac{1}{3}\left(\frac{L/d}{k}\right)^4\right] = 1100\left[1-\frac{1}{3}\left(\frac{16.80}{25.59}\right)^4\right] = 1031.9\text{ psi}$$

$P_{max} = \sigma_{all}A = (1031.9)(24) = 24.8(10^3)\text{ lb} = 24.8\text{ kip}$ .... **Ans.**

9-56

$$A = 4bd = 4(200 \times 250) = 200(10^3)\ \text{mm}^2 \qquad L' = 0.5L = 0.5(4.5) = 2.25\ \text{m}$$

$$k = 0.671\sqrt{\frac{E}{F_C}} = 0.671\sqrt{\frac{13(10^9)}{9(10^6)}} = 25.5$$



$$\frac{L'}{d} = \frac{(2250)}{200} = 11.25 < k$$

$$\sigma_{all} = F_C\left[1 - \frac{1}{3}\left(\frac{L/d}{k}\right)^4\right] = 9\left[1 - \frac{1}{3}\left(\frac{11.25}{25.5}\right)^4\right] = 8.886\ \text{MPa}$$

$$P_{max} = \sigma_{all}A = (8.886 \times 10^6)(200 \times 10^{-3}) = 1777(10^3)\ \text{N} = 1777\ \text{kN}$$ .... **Ans.**

9-57*

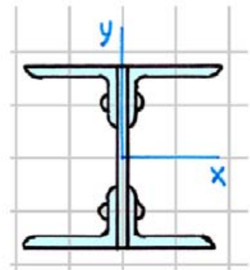
$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29{,}000)}{36} = 15{,}901 \qquad C_c = 126.10$$

From Table B-9 (for an L $5\times3\frac{1}{2}\times1/2$ angle): $A = 4.00 \text{ in.}^2$

$r_{X-X} = 1.58$ in. $\quad r_{Y-Y} = 1.01$ in. $\quad x_C = 1.66$ in.

For the cross section:

$$A_T = (10\times0.5) + 4(4.00) = 21.00 \text{ in.}^2$$



$$I_{\min} = I_y = \frac{(10)(0.5)^3}{12} + 4(4)\left(1.58^2 + 1.91^2\right) = 98.42 \text{ in.}^2$$

$$r_{\min} = r_y = \sqrt{I/A_T} = \sqrt{98.42/21.00} = 2.165 \text{ in.}$$

(a) $L' = L = 25$ ft

$$\frac{L'}{r_{\min}} = \frac{(25\times12)}{2.165} = 138.57 > 126.10$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29{,}000)}{1.92(138.57)^2} = 7.764 \text{ ksi}$$

$P_{\max} = \sigma_{all} A_T = (7.764)(21.00) = 163.0$ kip .... **Ans.**

(b) $L' = 0.7L = 0.7(25) = 17.5$ ft $\qquad \dfrac{L'}{r_{\min}} = \dfrac{(17.5\times12)}{2.165} = 97.0 < 126.10$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{97.0}{126.10}\right) - \frac{1}{8}\left(\frac{97.0}{126.10}\right)^3 = 1.898$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.898}\left[1 - \frac{1}{2}\left(\frac{97.0}{126.10}\right)^2\right] = 13.356 \text{ ksi}$$

$P_{\max} = \sigma_{all} A = (13.356)(21.00) = 280$ kip .... **Ans.**

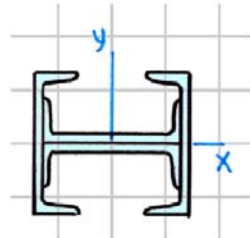
9-58*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$

From Table B-6 (for a C 178×22 channel): $d = 177.8$ mm $\quad A = 2795\ \text{mm}^2$

$r_{X-X} = 63.8$ mm $\quad r_{Y-Y} = 14.3$ mm

$t_w = 10.6$ mm $\quad x_C = 13.5$ mm



$$I_x = 2(2795)(63.8)^2 + 2(2795)(14.3)^2 + 2(2795)(13.5)^2$$
$$= 24.92\left(10^6\right)\ \text{mm}^4$$

$$I_y = 2(2795)(63.8)^2 + 2(2795)(14.3)^2 + 2(2795)(86.0)^2$$
$$= 65.24\left(10^6\right)\ \text{mm}^4$$

$$r_x = \sqrt{\frac{24.92\left(10^6\right)}{4(2795)}} = 47.21\ \text{mm} \qquad \frac{L'_x}{r_x} = \frac{7500}{47.21} = 158.86 > 125.66$$

$$r_y = \sqrt{\frac{65.24\left(10^6\right)}{4(2795)}} = 76.39\ \text{mm} \qquad \frac{L'_y}{r_y} = \frac{0.7(12,000)}{76.39} = 109.96 < 125.66$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2\left(200\times10^9\right)}{1.92(158.86)^2} = 40.74\left(10^6\right)\ \text{N/m}^2$$

$P_{\max} = \sigma_{all} A = \left(40.74\times10^6\right)\left[4\left(2795\times10^{-6}\right)\right] = 455\left(10^3\right)\ \text{N} = 455\ \text{kN}$ .... **Ans.**

9-59

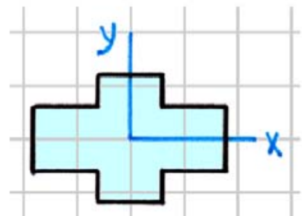
$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29{,}000)}{145} = 3948 \qquad C_c = 62.83$$

$$A_T = 2(1\times 0.5) + (3\times 1) = 4.00 \text{ in.}^2$$

$$I_x = \frac{(2)(1)^3}{12} + \frac{(1)(2)^3}{12} = 0.8333 \text{ in.}^2 \qquad r_x = \sqrt{0.8333/4} = 0.4564 \text{ in.}$$

$$I_y = \frac{(1)(1)^3}{12} + \frac{(1)(3)^3}{12} = 2.3333 \text{ in.}^2 \qquad r_x = \sqrt{2.3333/4} = 0.7637 \text{ in.}$$

$$\frac{L'_x}{r_x} = \frac{36}{0.4564} = 78.88 > 62.83 \qquad \frac{L'_y}{r_y} = \frac{60}{0.7637} = 78.56 > 62.83$$



$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29{,}000)}{1.92(78.88)^2} = 23.96 \text{ ksi}$$

$P_{\max} = \sigma_{all} A_T = (23.96)(4.00) = 95.8$ kip ....**Ans.**

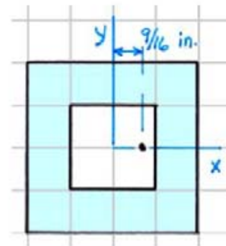
9-61*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

$$A = 5^2 - 4^2 = 9.00 \text{ in.}^2 \qquad I_x = I_y = \frac{5^4 - 4^4}{12} = 30.75 \text{ in.}^4$$

$$r = \sqrt{I/A} = \sqrt{30.75/9.00} = 1.8484 \text{ in.} \qquad L' = L = 10 \text{ ft}$$

$$\frac{L'}{r} = \frac{(10 \times 12)}{1.8484} = 64.92 < 126.10$$



$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{64.92}{126.10}\right) - \frac{1}{8}\left(\frac{64.92}{126.10}\right)^3 = 1.749$$

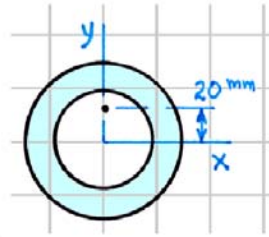
$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.749}\left[1 - \frac{1}{2}\left(\frac{64.92}{126.10}\right)^2\right] = 17.868 \text{ ksi}$$

$$\frac{P/A}{\sigma_a} + \frac{(Pec)/I}{\sigma_b} = \frac{P/9.00}{17.868} + \frac{P(0.5625)(2.5)/(30.75)}{24} \le 1$$

$P \le 123.1$ kip $\qquad P_{max} = 123.1$ kip .... **Ans.**

9-62*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$



$$A = \frac{\pi\left(150^2 - 120^2\right)}{4} = 6362 \text{ mm}^2$$

$$I_x = I_y = \frac{\pi\left(150^4 - 120^4\right)}{64} = 14.672\left(10^6\right) \text{ mm}^4$$

$$r_x = r_y = \sqrt{\frac{14.672\left(10^6\right)}{6362}} = 48.02 \text{ mm} \qquad \frac{L'}{r} = \frac{4000}{48.02} = 83.30 < 125.66$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{83.30}{125.66}\right) - \frac{1}{8}\left(\frac{83.30}{125.66}\right)^3 = 1.879$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.879}\left[1 - \frac{1}{2}\left(\frac{83.30}{125.66}\right)^2\right] = 103.82 \text{ MPa}$$

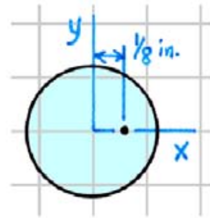
$$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{6362\left(10^{-6}\right)} + \frac{P(0.020)(0.075)}{14.672\left(10^{-6}\right)} \le \sigma_{all} = 103.82\left(10^6\right) \text{ N/m}^2$$

$P \le 400\left(10^3\right)$ N $\qquad P_{max} = 400$ kN .... **Ans.**

9-63

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29{,}000)}{36} = 15{,}901 \qquad C_c = 126.10$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(2)^2}{4} = 3.142 \text{ in.}^2$$



$$I_x = I_y = \frac{\pi d^4}{64} = \frac{\pi(2)^4}{64} = 0.7854 \text{ in.}^4$$

$$r = \sqrt{I/A} = \sqrt{0.7854/3.142} = 0.500 \text{ in.}$$

$$\frac{L'_x}{r} = \frac{75}{0.500} = 150.0 > 126.10 \qquad \frac{L'_y}{r} = \frac{50}{0.500} = 100.0 < 126.10$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29{,}000)}{1.92(150.0)^2} = 6.625 \text{ ksi} = \sigma_a$$

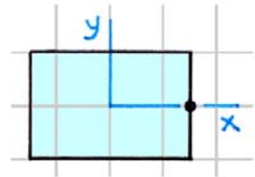
$$\frac{P/A}{\sigma_a} + \frac{(Pec)/I}{\sigma_b} = \frac{P/3.142}{6.625} + \frac{P(0.125)(1.0)/(0.7854)}{24} \le 1$$

$P \le 18.29$ kip $\qquad P_{max} = 18.29$ kip .... **Ans.**

9-64

$$A = bh = (150 \times 100) = 15{,}000 \text{ mm}^2$$

$$L' = 2L = 2(2) = 4 \text{ m}$$



$$I_y = \frac{bh^3}{12} = \frac{(100)(150)^3}{12} = 28.125(10^6) \text{ mm}^4$$

$$k = 0.671\sqrt{\frac{E}{F_c}} = 0.671\sqrt{\frac{12(10^9)}{9(10^6)}} = 24.50 \qquad \frac{L'}{d} = \frac{4000}{100} = 40.0 > k$$

$$\sigma_{all} = \frac{0.30E}{(L'/d)^2} = \frac{0.30(12 \times 10^9)}{(40.0)^2} = 2.25(10^6) \text{ N/m}^2 = 2.25 \text{ MPa}$$

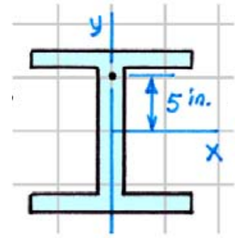
$$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{15{,}000(10^{-6})} + \frac{P(0.075)(0.075)}{28.125(10^{-6})} \le \sigma_{all} = 2.25(10^6) \text{ N/m}^2$$

$P \le 8.44(10^3)$ N $\qquad P_{max} = 8.44$ kN .... **Ans.**

9-65*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

$$L' = L = 20 \text{ ft}$$



From Table B-1 (for a W 14×82 section): $A = 24.1 \text{ in.}^2$

$I_{X-X} = 882 \text{ in.}^4$ $I_{Y-Y} = 148 \text{ in.}^4$

$r_{X-X} = 6.05 \text{ in.}$ $r_{Y-Y} = 2.48 \text{ in.}$

$$\frac{L'}{r_x} = \frac{(20\times 12)}{6.05} = 39.67 < 126.10 \qquad \frac{L'}{r_y} = \frac{(20\times 12)}{2.48} = 96.77 < 126.10$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{96.77}{126.10}\right) - \frac{1}{8}\left(\frac{96.77}{126.10}\right)^3 = 1.881$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.881}\left[1 - \frac{1}{2}\left(\frac{96.77}{126.10}\right)^2\right] = 13.503 \text{ ksi}$$

(a) $$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{24.1} + \frac{P(5)(7.155)}{882} \le \sigma_{all} = 13.503 \text{ ksi}$$

$P \le 164.6 \text{ kip}$ $P_{\max} = 164.6 \text{ kip}$ .... **Ans.**

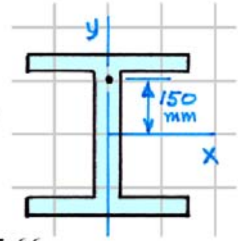
(b) $$\frac{P/A}{\sigma_a} + \frac{(Pec)/I}{\sigma_b} = \frac{P/24.1}{13.503} + \frac{P(5)(7.155)/(882)}{24} \le 1$$

$P \le 210 \text{ kip}$ $P_{\max} = 210 \text{ kip}$ .... **Ans.**

9-66*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2 (200\times10^9)}{(250\times10^6)} = 15,791 \qquad C_c = 125.66$$

$$L' = 0.5L = 0.5(7) = 3.5 \text{ m}$$



From Table B-2 (for a W 356×64 section): $A = 8130 \text{ mm}^2$

$I_{X-X} = 178(10^6) \text{ mm}^4$ $\qquad I_{Y-Y} = 18.8(10^6) \text{ mm}^4$

$r_{X-X} = 148 \text{ mm}$ $\qquad r_{Y-Y} = 48.0 \text{ mm}$

$$\frac{L'}{r_x} = \frac{(3500)}{148} = 23.65 < 125.66 \qquad \frac{L'}{r_y} = \frac{(3500)}{48.0} = 72.92 < 125.66$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{72.92}{125.66}\right) - \frac{1}{8}\left(\frac{72.92}{125.66}\right)^3 = 1.860$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.860}\left[1 - \frac{1}{2}\left(\frac{72.92}{125.66}\right)^2\right] = 111.78 \text{ MPa}$$

(a) $$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{8130(10^{-6})} + \frac{P(0.150)(0.1735)}{178(10^{-6})} \le \sigma_{all} = 111.78(10^6) \text{ N/m}^2$$

$P \le 415 \text{ kN}$ $\qquad P_{max} = 415 \text{ kN}$ .... **Ans.**

(b) $$\frac{P/A}{\sigma_a} + \frac{(Pec)/I}{\sigma_b} = \frac{P/(8130\times10^{-6})}{111.78(10^6)} + \frac{P(0.150)(0.1735)/(178\times10^{-6})}{160(10^6)} \le 1$$

$P \le 497(10^3) \text{ N}$ $\qquad P_{max} = 497 \text{ kN}$ .... **Ans.**

9-67

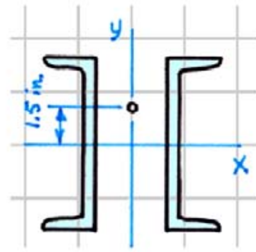
$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29{,}000)}{36} = 15{,}901 \qquad C_c = 126.10 \qquad L' = L = 25 \text{ ft}$$

From Table B-5 (for a C 8×18.75 channel): $A = 5.51 \text{ in.}^2$ $\quad d = 8.00$ in.

$I_{X-X} = 44.0 \text{ in.}^4$ $\quad r_{X-X} = 2.82$ in. $\quad r_{Y-Y} = 0.599$ in. $\quad x_C = 0.565$ in.

For the latticed channels: $\quad r_y = \sqrt{(0.599)^2 + (1.5 + 0.565)^2} = 2.15$ in.

$$\frac{L'}{r_x} = \frac{(25\times 12)}{2.82} = 106.38 < 126.10 \qquad \frac{L'}{r_y} = \frac{(25\times 12)}{2.15} = 139.53 > 126.10$$



$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r_y)^2} = \frac{\pi^2(29{,}000)}{1.92(139.53)^2} = 7.657 \text{ ksi} = \sigma_a$$

$$\frac{P/A}{\sigma_a} + \frac{(Pec)/I}{\sigma_b} = \frac{P/(2\times 5.51)}{7.657} + \frac{P(1.5)(4.00)/(2\times 44.0)}{24} \le 1$$

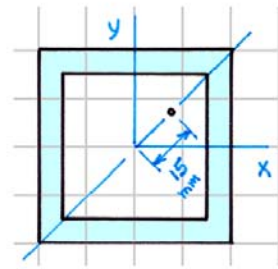
$P \le 68.1$ kip $\qquad P_{\max} = 68.1$ kip .... **Ans.**

9-68

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times 10^9\right)}{\left(250\times 10^6\right)} = 15,791 \qquad C_c = 125.66$$

$$A = (100)^2 - (70)^2 = 5100 \text{ mm}^2 \qquad L' = L = 4 \text{ m}$$

$$I = \frac{(100)^4}{12} - \frac{(70)^4}{12} = 6.333\left(10^6\right) \text{ mm}^4 \quad \text{(all axes)}$$



$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{6.333\left(10^6\right)}{5100}} = 35.24 \text{ mm}$$

$$\frac{L'}{r} = \frac{(4000)}{35.24} = 113.51 < 125.66$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{113.51}{125.66}\right) - \frac{1}{8}\left(\frac{113.51}{125.66}\right)^3 = 1.913 \qquad c = \sqrt{(50)^2 + (50)^2} = 70.71 \text{ mm}$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.913}\left[1 - \frac{1}{2}\left(\frac{113.51}{125.66}\right)^2\right] = 77.37 \text{ MPa}$$

$$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{5100\left(10^{-6}\right)} + \frac{P(0.015)(0.07071)}{6.333\left(10^{-6}\right)} \le \sigma_{all} = 77.37\left(10^6\right) \text{ N/m}^2$$

$P \le 213\left(10^3\right)$ N $\qquad P_{max} = 213$ kN .... **Ans.**

9-69*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10 \qquad L' = L = 10 \text{ ft}$$

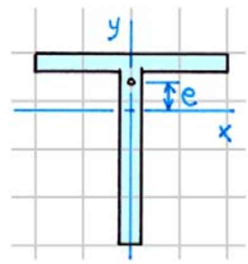
From Table B-11 (for a WT 8×25 tee section): $A = 7.37\text{ in.}^2$ $\quad c = y_C = 1.89\text{ in.}$

$I_{X-X} = 42.3\text{ in.}^4 \qquad I_{Y-Y} = 18.6\text{ in.}^4 \qquad r_{X-X} = 2.40\text{ in.} \qquad r_{Y-Y} = 1.59\text{ in.}$

$$\frac{L'}{r_x} = \frac{(10\times12)}{2.40} = 50.0 < 126.10 \qquad \frac{L'}{r_y} = \frac{(10\times12)}{1.59} = 75.47 < 126.10$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{75.47}{126.10}\right) - \frac{1}{8}\left(\frac{75.47}{126.10}\right)^3 = 1.8643$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.8643}\left[1 - \frac{1}{2}\left(\frac{75.47}{126.10}\right)^2\right] = 15.852 \text{ ksi}$$



$$P_{all} = \sigma_{all} A = (15.852)(7.37) = 116.83 \text{ kip} > P = 100 \text{ kip}$$

$$\frac{P/A}{\sigma_a} + \frac{(Pec)/I}{\sigma_b} = \frac{100/7.37}{15.852} + \frac{(100)(e)(1.89)/(42.3)}{24} \le 1$$

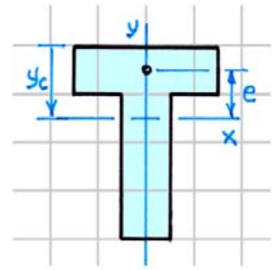
$e \le 0.774$ in. $\qquad e_{max} = 0.774$ in. .... **Ans.**

9-70*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$

$$A = 2\left(50\times150\right) = 15,000 \text{ mm}^2 \qquad L' = L = 5 \text{ m}$$

$$y_C = \frac{(25)(50\times150)+(125)(50\times150)}{15,000} = 75 \text{ mm}$$



$$I_x = \frac{(150)(75)^3}{3} - \frac{(100)(25)^3}{3} + \frac{(50)(125)^3}{3} = 53.125\left(10^6\right) \text{ mm}^4$$

$$I_y = \frac{(50)(150)^3}{12} + \frac{(150)(50)^3}{12} = 15.625\left(10^6\right) \text{ mm}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{53.125\left(10^6\right)}{15,000}} = 59.51 \text{ mm} \qquad \frac{L'}{r_x} = \frac{5000}{59.51} = 84.02 < 125.66$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{15.625\left(10^6\right)}{15,000}} = 32.27 \text{ mm} \qquad \frac{L'}{r_y} = \frac{5000}{32.27} = 154.94 > 125.66$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\left(L'/r\right)^2} = \frac{\pi^2\left(200\times10^9\right)}{1.92(154.94)^2} = 42.83\left(10^6\right) \text{ N/m}^2$$

$$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{15,000\left(10^{-6}\right)} + \frac{P(0.050)(0.075)}{53.125\left(10^{-6}\right)} \le \sigma_{all} = 42.83\left(10^6\right) \text{ N/m}^2$$

$P \le 312\left(10^3\right)$ N $\qquad P_{max} = 312$ kN .... **Ans.**

9-71

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10 \qquad L' = L = 15 \text{ ft}$$

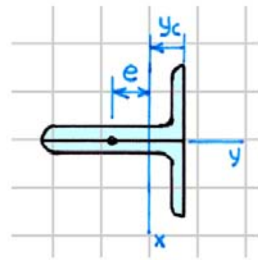
From Table B-9 (for an L $6\times 3\frac{1}{2}\times\frac{1}{2}$ section): $A = 4.50 \text{ in.}^2$ $x_C = 0.833$ in.

$I_{X-X} = 16.6 \text{ in.}^4$ $r_{X-X} = 1.92$ in. $r_{Y-Y} = 0.972$ in. $y_C = 2.08$ in.

(a) For the welded angles: $r_y = \sqrt{(0.972)^2 + (0.8333)^2} = 1.280$ in.

$$\frac{L'}{r_x} = \frac{(12\times 12)}{1.92} = 75.0 < 126.10$$

$$\frac{L'}{r_y} = \frac{(12\times 12)}{1.28} = 112.5 < 126.10$$



$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{112.5}{126.10}\right) - \frac{1}{8}\left(\frac{112.5}{126.10}\right)^3 = 1.912$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.912}\left[1 - \frac{1}{2}\left(\frac{112.5}{126.10}\right)^2\right] = 11.335 \text{ ksi}$$

$P = \sigma_{all} A = (11.335)\left[2(4.5)\right] = 102.0$ kip .... **Ans.**

(b) For $c = 2.08$ in.:

$$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{50}{9.00} + \frac{(50)(e)(2.08)}{2(16.6)} \le \sigma_{all} = 7.538 \text{ ksi}$$

$e \le 0.633$ in. $d_{min} = 2.08 - 0.633 = 1.447$ in.

For $c = 6 - 2.08 = 3.92$ in.:

$$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{50}{9.00} + \frac{(50)(e)(3.92)}{2(16.6)} \le \sigma_{all} = 7.538 \text{ ksi}$$

$e \le 0.336$ in. $d_{max} = 2.08 + 0.336 = 2.42$ in.

Therefore $1.447 \text{ in.} \le d \le 2.42$ in. .... **Ans.**

9-72

For the section: $A = 2050\ \text{mm}^2$ $r_x = 30.5\ \text{mm}$ $r_y = 20.8\ \text{mm}$ $L' = 1.25\ \text{m}$

$$\frac{L'}{r_x} = \frac{1250}{30.5} = 40.98 < 55 \qquad \frac{L'}{r_y} = \frac{1250}{20.8} = 60.10 > 55$$

$$\sigma_{all} = \frac{372\left(10^3\right)}{\left(L'/r\right)^2} = \frac{372\left(10^3\right)}{\left(60.10\right)^2} = 102.99\left(10^6\right)\ \text{N/m}^2$$

(a) $P_{all} = \sigma_{all} A = \left(102.99\times10^6\right)\left(2050\times10^{-6}\right) = 211\left(10^3\right)\ \text{N} = 211\ \text{kN}$ .... **Ans.**

(b) $$\frac{P}{A} + \frac{Mc}{I} = \frac{175\left(10^3\right)}{2050\left(10^{-6}\right)} + \frac{M\left(0.0282\right)}{\left(2050\times10^{-6}\right)\left(30.5\times10^{-6}\right)} \le \sigma_{all} = 102.99\left(10^6\right)\ \text{N/m}^2$$

$M \le 6.96\left(10^3\right)\ \text{N}\cdot\text{m} = 6.96\ \text{kN}\cdot\text{m}$ .... **Ans.**

9-73*

From Table B-17: $\sigma_y = 36$ ksi $\qquad E = 29,000$ ksi

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{36}{1.667} = 21.596 \text{ ksi} \qquad A_{\min} = \frac{P}{\sigma_{all}} = \frac{70.0}{21.596} = 3.241 \text{ in.}^2$$

First, try a 5-in. diameter standard weight pipe (Table B-13) with:

$$A = 4.30 \text{ in.}^2 \qquad r = 1.88 \text{ in.} \qquad L/r = (10\times 12)/1.88 = 63.83$$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

Therefore, the column is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{63.83}{126.10}\right) - \frac{1}{8}\left(\frac{63.83}{126.10}\right)^3 = 1.840$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.840}\left[1 - \frac{1}{2}\left(\frac{63.83}{126.10}\right)^2\right] = 17.059 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (17.059)(4.30) = 73.354 \text{ kip} > P = 70 \text{ kip}$$

Check to see if a 4-in. diameter standard weight pipe would work:

$$A = 3.174 \text{ in.}^2 \qquad r = 1.51 \text{ in.} \qquad L/r = (10\times 12)/1.51 = 79.47$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{79.47}{126.10}\right) - \frac{1}{8}\left(\frac{79.47}{126.10}\right)^3 = 1.862$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.862}\left[1 - \frac{1}{2}\left(\frac{79.47}{126.10}\right)^2\right] = 15.495 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (15.495)(3.174) = 49.179 \text{ kip} < P = 70 \text{ kip}$$

Therefore Use a 5-in. diameter standard weight pipe. .... **Ans.**

9-74*

From Table B-18: $\sigma_y = 250$ MPa $\qquad E = 200$ GPa

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{250}{1.667} = 150 \text{ MPa} \qquad A_{\min} = \frac{P}{\sigma_{all}} = \frac{200(10^3)}{150(10^6)} = 1333(10^{-6}) \text{ m}^2$$

First, try a 76-mm diameter standard weight pipe (Table B-14) with:

$$A = 1437 \text{ mm}^2 \qquad r = 29.5 \text{ mm} \qquad L/r = 4000/29.5 = 135.6$$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(200\times10^9)}{(250\times10^6)} = 15,791 \qquad C_c = 125.66$$

Therefore, the column is in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(200\times10^9)}{1.92(135.6)^2} = 55.91(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (55.91\times10^6)(1437\times10^{-6}) = 80.34(10^3) \text{ N} = 80.3 \text{ kN} < P = 200 \text{ kN}$$

This allowable load is much less than the design load;
therefore, a pipe with a much larger area and a larger
radius of gyration must be used.

Next, try a 102-mm diameter standard weight pipe with:

$$A = 2048 \text{ mm}^2 \qquad r = 38.4 \text{ mm} \qquad L/r = 4000/38.4 = 104.17$$

Since $L/r = 104.2 < 125.66$ the column is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{104.17}{125.66}\right) - \frac{1}{8}\left(\frac{104.17}{125.66}\right)^3 = 1.906$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.906}\left[1 - \frac{1}{2}\left(\frac{104.17}{125.66}\right)^2\right] = 86.10 \text{ MPa}$$

$$P_{all} = \sigma_{all}A = (86.10\times10^6)(2048\times10^{-6}) = 176.3(10^3) \text{ N} = 176.3 \text{ kN} < P = 200 \text{ kN}$$

This allowable load is close to the design load; therefore, the next larger size pipe should be satisfactory.

For a 127-mm diameter standard weight pipe with:

$$A = 2774 \text{ mm}^2 \qquad r = 47.8 \text{ mm} \qquad L/r = 4000/47.8 = 83.68$$

Since $L/r = 83.68 < 125.66$ the column is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{83.68}{125.66}\right) - \frac{1}{8}\left(\frac{83.68}{125.66}\right)^3 = 1.879$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.879}\left[1 - \frac{1}{2}\left(\frac{83.68}{125.66}\right)^2\right] = 103.6 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = \left(103.6 \times 10^6\right)\left(2774 \times 10^{-6}\right) = 287\left(10^3\right) \text{ N} = 287 \text{ kN} > P = 200 \text{ kN}$$

Therefore Use a 127-mm diameter standard weight pipe. .... **Ans.**

9-75

From Table B-17: $\sigma_y = 36$ ksi $\qquad E = 29,000$ ksi

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{36}{1.667} = 21.596 \text{ ksi} \qquad A_{\min} = \frac{P}{\sigma_{all}} = \frac{200.0}{21.596} = 9.261 \text{ in.}^2$$

First, try a W 8×40 section (Table B-1) with:

$$A = 11.7 \text{ in.}^2 \qquad r_{\min} = 2.04 \text{ in.} \qquad L/r = (12\times12)/2.04 = 70.59$$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

Therefore, the column is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{70.59}{126.10}\right) - \frac{1}{8}\left(\frac{70.59}{126.10}\right)^3 = 1.855$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.855}\left[1 - \frac{1}{2}\left(\frac{70.59}{126.10}\right)^2\right] = 16.366 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (16.366)(11.7) = 191.480 \text{ kip} < P = 200 \text{ kip}$$

This allowable load is less than the design load; therefore, a section with a larger area and possibly a larger radius of gyration must be used.

Next, try a W 14×43 section with:

$$A = 12.6 \text{ in.}^2 \qquad r_{\min} = 1.89 \text{ in.} \qquad L/r = (12\times12)/1.89 = 76.19$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{76.19}{126.10}\right) - \frac{1}{8}\left(\frac{76.19}{126.10}\right)^3 = 1.866$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.866}\left[1 - \frac{1}{2}\left(\frac{76.19}{126.10}\right)^2\right] = 15.771 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (15.771)(12.6) = 198.7 \text{ kip} \cong P = 200 \text{ kip}$$

Therefore Use a W 14×43 section. .... **Ans.**

9-76

From Table B-18: $\sigma_y = 250$ MPa $\quad E = 200$ GPa

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{250}{1.667} = 150 \text{ MPa} \qquad A_{\min} = \frac{P}{\sigma_{all}} = \frac{400(10^3)}{150(10^6)} = 2667(10^{-6}) \text{ m}^2$$

First, try a W 203×22 section (Table B-2) with:

$$A = 2865 \text{ mm}^2 \qquad r_{\min} = 22.3 \text{ mm} \qquad L/r = 7000/22.3 = 313.9$$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(200\times 10^9)}{(250\times 10^6)} = 15,791 \qquad C_c = 125.66$$

Therefore, the column is in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(200\times 10^9)}{1.92(313.9)^2} = 10.434(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (10.434\times 10^6)(2865\times 10^{-6})$$

$$= 29.88(10^3) \text{ N} = 29.88 \text{ kN} = P = 400 \text{ kN}$$

This allowable load is much less than the design load; therefore, a section with a much larger area and a larger radius of gyration must be used.

Next, try a W 305×74 section with:

$$A = 9485 \text{ mm}^2 \qquad r_{\min} = 49.8 \text{ mm} \qquad L/r = 7000/49.8 = 140.6$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(200\times 10^9)}{1.92(140.6)^2} = 52.01(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (52.01\times 10^6)(9485\times 10^{-6})$$

$$= 493(10^3) \text{ N} = 493 \text{ kN} > P = 400 \text{ kN}$$

This allowable load is larger than the design load; therefore, a slightly smaller section may be satisfactory.

Try a W 356×64 section with:

$$A = 8130 \text{ mm}^2 \qquad r_{\min} = 48.0 \text{ mm} \qquad L/r = 7000/48.0 = 145.8$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(200\times 10^9)}{1.92(145.8)^2} = 48.36(10^6) \text{ N/m}^2$$

Continued on next slide

Problem 9-76 continued

$$P_{all} = \sigma_{all} A = (48.36 \times 10^6)(8130 \times 10^{-6})$$
$$= 393(10^3) \text{ N} = 393 \text{ kN} < P = 400 \text{ kN}$$

Finally, try a W 254×67 section with:

$A = 8580 \text{ mm}^2$ $\quad r_{min} = 51.1 \text{ mm}$ $\quad L/r = 7000/51.1 = 137.0$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2 (200 \times 10^9)}{1.92(137.0)^2} = 54.78(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all} A = (54.78 \times 10^6)(8580 \times 10^{-6})$$
$$= 470(10^3) \text{ N} = 470 \text{ kN} > P = 400 \text{ kN}$$

Therefore Use a W 254×67 section. .... **Ans.**

9-77*

Assume that $L/r > 55$ Then from Code 2: $\sigma_{all} = \dfrac{54,000}{(L/r)^2}$ ksi

For a square cross section:

$$A = (t \times t) = t^2 \qquad I = (t)(t)^3/12 = t^4/12 \qquad r = \sqrt{I/A} = 0.2887t$$

$$\frac{L}{r} = \frac{(12\times 12)}{0.2887t} = \frac{498.8}{t} \qquad \sigma_{all} = \frac{54,000}{(L/r)^2} = \frac{54,000}{(498.8/t)^2} = \frac{P}{A} = \frac{20}{t^2}$$

Solving for t yields: $t = 3.098 \text{ in.} \cong 3.10 \text{ in.}$

Check that $L/r > 55$: $\dfrac{L}{r} = \dfrac{498.8}{t} = \dfrac{498.8}{3.098} = 161.0 > 55$

Therefore Use a 3.10×3.10-in. cross section with $A = 9.61 \text{ in.}^2$ .... **Ans.**

9-78*

Assume that $L/r > 55$ Then from Code 2: $\sigma_{all} = \dfrac{372,000}{(L/r)^2}$ MPa $= \dfrac{372(10^9)}{(L/r)^2}$ N/m^2

For a rectangular cross section:

$$A = (w \times t) = (2t)(t) = 2t^2 \qquad I_{min} = \frac{(2t)(t)^3}{12} = \frac{t^4}{6} \qquad r = \sqrt{\frac{I}{A}} = 0.2887t$$

$$\frac{L}{r} = \frac{4}{0.2887t} = \frac{13.855}{t} \qquad \sigma_{all} = \frac{372(10^9)}{(L/r)^2} = \frac{372(10^9)}{(13.855/t)^2} = \frac{P}{A} = \frac{15,000}{2t^2}$$

Solving for t yields: $t = 0.04435 \text{ m} \cong 44.4 \text{ mm}$ $w = 2t = 88.8 \text{ mm}$

Check that $L/r > 55$: $\dfrac{L}{r} = \dfrac{13.855}{t} = \dfrac{13.855}{0.04435} = 313 > 55$

Therefore Use a 44.4×88.8-mm cross section with $A = 3943 \text{ mm}^2$ .... **Ans.**

9-79

Assume that $L/d < 11$

Then from Code 4: $\sigma_{all} = F_C = 1350 \text{ psi}$ $A_{min} = \dfrac{P}{\sigma_{all}} = \dfrac{60{,}000}{1350} = 44.44 \text{ in.}^2$

First, try an 8×8-in. timber (Table B-15) with: $A = 56.3 \text{ in.}^2$ $d = 7.5 \text{ in.}$

$$k = 0.671\sqrt{\frac{E}{F_C}} = 0.671\sqrt{\frac{1.8\left(10^6\right)}{1350}} = 24.50 \qquad \frac{L}{d} = \frac{14\times 12}{7.5} = 22.4 < k = 24.50$$

Therefore, the column is in the intermediate range for which:

$$\sigma_{all} = F_C\left[1 - \frac{1}{3}\left(\frac{L/d}{k}\right)^4\right] = (1350)\left[1 - \frac{1}{3}\left(\frac{22.4}{24.50}\right)^4\right] = 1035.6 \text{ psi}$$

$$P_{all} = \sigma_{all}A = (1035.6)(56.3) = 58{,}302 \text{ lb} \cong P = 60 \text{ kip}$$

Next, try an 8×10-in. timber with: $A = 71.3 \text{ in.}^2$ $d = 7.5 \text{ in.}$

$k = 24.50$ $L/d = 22.4$ $\sigma_{all} = 1035.6 \text{ psi}$

(all same as before). Therefore,

$$P_{all} = \sigma_{all}A = (1035.6)(71.3) = 73{,}838 \text{ lb} > P = 60 \text{ kip}$$

Finally, try a 6×12-in. timber with: $A = 63.3 \text{ in.}^2$ $d = 6 \text{ in.}$

$$(L/d) = (14\times 12)/6 = 28.0 > k = 24.50$$

Therefore, the column is in the slender range for which:

$$\sigma_{all} = \frac{0.30E}{(L/d)^2} = \frac{0.30\left(1.8\times 10^6\right)}{(28.0)^2} = 688.8 \text{ psi}$$

$$P_{all} = \sigma_{all}A = (688.8)(63.3) = 43{,}599 \text{ lb} < P = 60 \text{ kip}$$

Therefore Use an 8×8-in. timber with $A = 56.3 \text{ in.}^2$ .... **Ans.**

9-80

Assume that $L/d < 11$

Then from Code 4: $\sigma_{all} = F_C = 9.3 \text{ MPa}$

$$A_{min} = \frac{P}{\sigma_{all}} = \frac{100(10^3)}{9.3(10^6)} = 10.75(10^{-3}) \text{ m}^2 = 10,750 \text{ mm}^2$$

First, try a $102 \times 152\text{-mm}$ timber (Table B-16) with: $A = 13,200 \text{ mm}^2$ $d = 92 \text{ mm}$

$$k = 0.671\sqrt{\frac{E}{F_C}} = 0.671\sqrt{\frac{12(10^9)}{9.3(10^6)}} = 24.10 \qquad \frac{L}{d} = \frac{4000}{92} = 43.48 > k = 24.10$$

Therefore, the column is in the slender range for which:

$$\sigma_{all} = \frac{0.30E}{(L/d)^2} = \frac{0.30(12\times10^9)}{(43.48)^2} = 1.904(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (1.904\times10^6)(13,200\times10^{-6}) = 25,133 \text{ N} < P = 100 \text{ kN}$$

Next, try a $152 \times 152\text{-mm}$ timber with: $A = 19,600 \text{ mm}^2$ $d = 140 \text{ mm}$

$$(L/d) = (4000)/140 = 28.57 > k = 24.10$$

Therefore, this column is also in the slender range for which:

$$\sigma_{all} = \frac{0.30E}{(L/d)^2} = \frac{0.30(12\times10^9)}{(28.57)^2} = 4.410(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (4.41\times10^6)(19,600\times10^{-6}) = 86,436 \text{ N} < P = 100 \text{ kN}$$

This allowable load is only slightly less than the design load; therefore, a timber with a slightly larger area will probably be satisfactory.

Try a $152 \times 203\text{-mm}$ timber with: $A = 26,700 \text{ mm}^2$ $d = 140 \text{ mm}$

$$(L/d) = (4000)/140 = 28.57 > k = 24.10$$

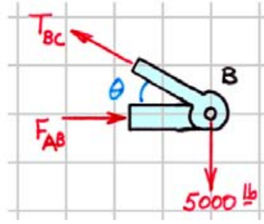
Therefore, this column is also in the slender range for which:

$$\sigma_{all} = \frac{0.30E}{(L/d)^2} = \frac{0.30(12\times10^9)}{(28.57)^2} = 4.410(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (4.41\times10^6)(26,700\times10^{-6}) = 117,747 \text{ N} > P = 100 \text{ kN}$$

Therefore Use a $152 \times 203\text{-mm}$ timber with $A = 26,700 \text{ mm}^2$ .... **Ans.**

9-81*



$$\tan\theta = (5/9) \qquad \theta = 29.055°$$

$$\rightarrow \Sigma F_x = 0: \qquad F_{AB} - T_{BC}\cos\theta = 0$$

$$\uparrow \Sigma F_y = 0: \qquad T_{BC}\sin\theta - 5000 = 0$$

$$F_{AB} = 9000 \text{ lb} \qquad T_{BC} = 10,296 \text{ lb}$$

From Table B-17: $\sigma_y = 36$ ksi $\qquad E = 29,000$ ksi

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{36}{1.667} = 21.596 \text{ ksi} \qquad A_{\min} = \frac{P}{\sigma_{all}} = \frac{9.0}{21.596} = 0.4167 \text{ in.}^2$$

First, try a 1-in. diameter standard weight pipe (Table B-13) with:

$$A = 0.494 \text{ in.}^2 \qquad r = 0.42 \text{ in.} \qquad L/r = (9\times 12)/0.42 = 257.1$$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$

Therefore, the column is in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29,000)}{1.92(257.1)^2} = 2.255 \text{ ksi}$$

$$P_{all} = \sigma_{all}A = (2255)(0.494) = 1114 \text{ lb} = \; P = 9000 \text{ lb}$$

This allowable load is much less than the design load; therefore, a pipe with a much larger area and a larger radius of gyration must be used.

Next, try a 2-in. diameter standard weight pipe with:

$$A = 1.075 \text{ in.}^2 \qquad r = 0.79 \text{ in.} \qquad L/r = (9\times 12)/0.79 = 136.7$$

Therefore, the column is again in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29,000)}{1.92(136.7)^2} = 7.977 \text{ ksi}$$

$$P_{all} = \sigma_{all}A = (7977)(1.075) = 8575 \text{ lb} < P = 9000 \text{ lb}$$

This allowable load is close to the design load; therefore, the next larger size pipe should be satisfactory.

Next, try a $2\frac{1}{2}$-in. diameter standard weight pipe with:

$$A = 1.704 \text{ in.}^2 \qquad r = 0.95 \text{ in.} \qquad L/r = (9\times 12)/0.95 = 113.68$$

Continued on next slide

Problem 9-81 continued

Therefore, the column is now in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{113.68}{126.10}\right) - \frac{1}{8}\left(\frac{113.68}{126.10}\right)^3 = 1.913$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.913}\left[1 - \frac{1}{2}\left(\frac{113.68}{126.10}\right)^2\right] = 11.170 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (11.170)(1.704) = 19.03 \text{ kip} > P = 9000 \text{ lb}$$

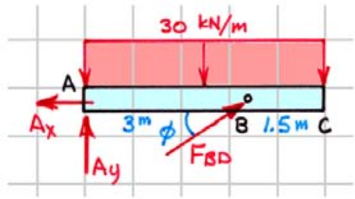
Therefore Use a $2\frac{1}{2}$-in. diameter standard weight pipe. .... **Ans.**

9-82*

$\tan\phi = (2/3) \qquad \phi = 33.690°$

$\circlearrowleft \Sigma M_A = 0: \quad (F_{BD}\sin 33.690°)(3) - [(30)(4.5)](2.25) = 0$

$F_{BD} = P = 182.53 \text{ kN}$



From Table B-18: $\sigma_y = 250$ MPa $\qquad E = 200$ GPa

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{250}{1.667} = 150 \text{ MPa} \qquad A_{\min} = \frac{P}{\sigma_{all}} = \frac{182.53(10^3)}{150(10^6)} = 1217(10^{-6}) \text{ m}^2$$

First, try a 76-mm diameter standard weight pipe (Table B-14) with:

$A = 1437 \text{ mm}^2 \qquad r = 29.5 \text{ mm} \qquad L/r = 3606/29.5 = 122.24$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(200\times10^9)}{(250\times10^6)} = 15,791 \qquad C_c = 125.66$$

Since $L/r = 122.24 < 125.66$ the column is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{122.24}{125.66}\right) - \frac{1}{8}\left(\frac{122.24}{125.66}\right)^3 = 1.916$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.916}\left[1 - \frac{1}{2}\left(\frac{122.24}{125.66}\right)^2\right] = 68.74 \text{ MPa}$$

$$P_{all} = \sigma_{all}A = (68.74\times10^6)(1437\times10^{-6}) = 98.8(10^3) \text{ N} = 98.8 \text{ kN} < P = 182.5 \text{ kN}$$

This allowable load is much less than the design load; therefore, a section with a much larger area and a larger radius of gyration must be used.

Next, try a 102-mm diameter standard weight pipe with:

$A = 2048 \text{ mm}^2 \qquad r = 38.4 \text{ mm} \qquad L/r = 3606/38.4 = 93.91$

Therefore, the column is still in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{93.91}{125.66}\right) - \frac{1}{8}\left(\frac{93.91}{125.66}\right)^3 = 1.895$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.895}\left[1 - \frac{1}{2}\left(\frac{93.91}{125.66}\right)^2\right] = 95.09 \text{ MPa}$$

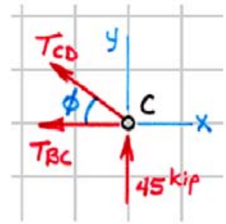
$$P_{all} = \sigma_{all}A = (95.09\times10^6)(2048\times10^{-6}) = 194.7(10^3) \text{ N} = 194.7 \text{ kN} > P = 182.5 \text{ kN}$$

This allowable load is close to the design load; any smaller size pipe would not be satisfactory.

Therefore Use a 102-mm diameter standard weight pipe. .... **Ans.**

9-83

By symmetry, $R_A = R_C = (90/2) = 45 \text{ kip} \uparrow$



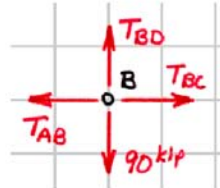
For joint *C*: $\phi = \tan^{-1}(7.5/10) = 36.87°$

$\uparrow \Sigma F_y = 0$: $45 + T_{CD}\sin 36.87° = 0$

$T_{CD} = T_{AD} = -75.0 \text{ kip} = 75.0 \text{ kip (C)}$

$\rightarrow \Sigma F_x = 0$: $-T_{BC} - T_{CD}\cos 36.87° = 0$

$T_{BC} = T_{AB} = +60.0 \text{ kip} = 60.0 \text{ kip (T)}$



For joint *B*:

$\uparrow \Sigma F_y = 0$: $T_{BD} - 90 = 0$

$T_{BD} = +90.0 \text{ kip} = 90.0 \text{ kip (T)}$

Only members *AD* and *CD* are in compression and $T_{AD} = T_{CD} = 75.0 \text{ kip (C)}$

From Table B-17: $\sigma_y = 36 \text{ ksi}$ $E = 29{,}000 \text{ ksi}$

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{36}{1.667} = 21.596 \text{ ksi} \qquad A_{min} = \frac{P}{\sigma_{all}} = \frac{75.0}{21.596} = 3.473 \text{ in.}^2$$

First, try a W 4×13 section (Table B-1) with:

$A = 3.83 \text{ in.}^2$ $r_{min} = 1.00 \text{ in.}$ $L/r = (12.5\times12)/1.00 = 150.0$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29{,}000)}{36} = 15{,}901 \qquad C_c = 126.10$$

Therefore, the column is in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29{,}000)}{1.92(150.0)^2} = 6.625 \text{ ksi}$$

$$P_{all} = \sigma_{all}A = (6.625)(3.83) = 25.4 \text{ kip} < P = 75 \text{ kip}$$

This allowable load is much less than the design load; therefore,
a section with a larger area and a larger radius of gyration must be used.

Next, try a W 8×24 section with:

$A = 7.08 \text{ in.}^2$ $r_{min} = 1.61 \text{ in.}$ $L/r = (12.5\times12)/1.61 = 93.17$

This section is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{93.17}{126.10}\right) - \frac{1}{8}\left(\frac{93.17}{126.10}\right)^3 = 1.893$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.893}\left[1 - \frac{1}{2}\left(\frac{93.17}{126.10}\right)^2\right] = 13.827 \text{ ksi}$$

Continued on next slide

Problem 9-83 continued

$$P_{all} = \sigma_{all} A = (13.827)(7.08) = 97.9 \text{ kip} > P = 75 \text{ kip}$$

This allowable load is greater than the design load; therefore, a slightly smaller section may also work.

Next, try a W 10×22 section with:

$A = 6.49 \text{ in.}^2$ $\qquad r_{min} = 1.33 \text{ in.}$ $\qquad L/r = (12.5 \times 12)/1.33 = 112.8$

This section is also in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{112.8}{126.10}\right) - \frac{1}{8}\left(\frac{112.8}{126.10}\right)^3 = 1.913$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.913}\left[1 - \frac{1}{2}\left(\frac{112.8}{126.10}\right)^2\right] = 11.289 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (11.289)(6.49) = 73.3 \text{ kip} < P = 75 \text{ kip}$$

This allowable load is close to (but smaller than) the design load.

Therefore Use a W 8×24 section .... **Ans.**

9-84

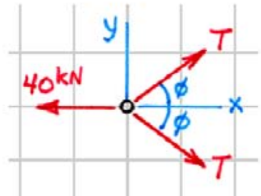
$$\phi = \tan^{-1}(0.75/2) = 20.56°$$

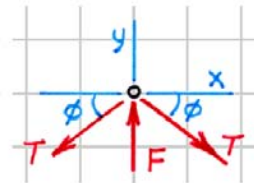
$\rightarrow \Sigma F_x = 0:\qquad 2(T\cos 20.56°) - 40 = 0$

$$T = +21.36 \text{ kN} = 21.36 \text{ kN (T)}$$

$\uparrow \Sigma F_y = 0:\qquad F - 2(T\sin 20.56°) = 0$

$$F = +15.003 \text{ kN} = 15.003 \text{ kN (C)}$$





From Table B-18: $\sigma_y = 250$ MPa $\qquad E = 200$ GPa

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{250}{1.667} = 150 \text{ MPa} \qquad A_{\min} = \frac{F}{\sigma_{all}} = \frac{15.003(10^3)}{150(10^6)} = 100.02(10^{-6}) \text{ m}^2$$

First, try a 13-mm diameter standard weight pipe (Table B-14) with:

$$A = 161.3 \text{ mm}^2 \qquad r = 6.6 \text{ mm} \qquad L/r = 1500/6.6 = 227.3$$

For structural steel:

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(200\times10^9)}{(250\times10^6)} = 15,791 \qquad C_c = 125.66$$

Since $L/r = 227.3 > 125.66$ the column is in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(200\times10^9)}{1.92(227.3)^2} = 19.90(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (19.90\times10^6)(161.3\times10^{-6}) = 3.21(10^3) \text{ N} < F = 15.003 \text{ kN}$$

This allowable load is much less than the design load; therefore, a pipe with a much larger area and a larger radius of gyration must be used.

Next, try a 25-mm diameter standard weight pipe with:

$$A = 318.7 \text{ mm}^2 \qquad r = 10.7 \text{ mm} \qquad L/r = 1500/10.7 = 140.2$$

Since $L/r = 140.2 > 125.66$ the column is in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(200\times10^9)}{1.92(140.2)^2} = 52.30(10^6) \text{ N/m}^2$$

$$P_{all} = \sigma_{all}A = (52.30\times10^6)(318.7\times10^{-6}) = 16.67(10^3) \text{ N} > F = 15.003 \text{ kN}$$

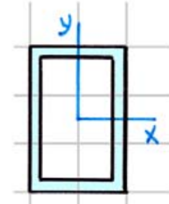
This allowable load is only slightly larger than the design load; it is unlikely that any smaller pipe would work.

Therefore $\qquad$ Use a 25-mm diameter standard weight pipe. .... **Ans.**

9-85*

$$A = (6\times 10) - (2\times 6) = 48.0 \text{ in.}^2 \qquad I_{min} = \frac{(10)(6)^3}{12} - \frac{(6)(2)^3}{12} = 176.0 \text{ in.}^4$$

$$r = \sqrt{I/A} = \sqrt{176.0/48.0} = 1.915 \text{ in.}$$



(a) $L/r = (20\times 12)/1.915 = 125.3$ .... **Ans.**

(b) $\dfrac{L}{r} = \sqrt{\dfrac{\pi^2 E}{\sigma_y}} = \sqrt{\dfrac{\pi^2(1200)}{(2.4)}} = 70.3$ .... **Ans.**

(c) $P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(1200)(176.0)}{(20\times 12)^2} = 36.19 \text{ kip} \cong 36.2 \text{ kip}$ .... **Ans.**

(d) $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{36.19}{48.0} = 0.754 \text{ ksi} = 754 \text{ psi (C)}$ .... **Ans.**

9-86*

From Table B-12 for a WT 178×51 section: $A = 6445\ \text{mm}^2$ $I_{min} = 13.6\left(10^6\right)\ \text{mm}^4$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2\left(200\times10^9\right)\left(13.6\times10^{-6}\right)}{(8)^2} = 419.5\left(10^3\right)\ \text{N} = 419.5\ \text{kN}$$

$$P_{max} = \frac{P_{cr}}{FS} = \frac{419.5}{1.92} = 218\ \text{kN}$$ .... **Ans.**

9-87

From Table B-1 for a W 36×160 section: $A = 47.0\text{ in.}^2$ $I_{\min} = 295\text{ in.}^4$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(29{,}000)(295)}{(30\times12)^2} = 651.5\text{ kip}$$

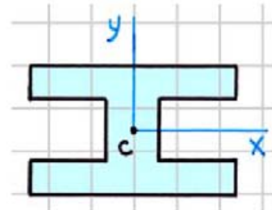
$$P_{\max} = \frac{P_{cr}}{FS} = \frac{651.5}{2.24} = 291\text{ kip}$$ .... **Ans.**

9-88*

$$A = 2(125\times 20)+(25\times 50) = 6250\ \text{mm}^2$$

$$I_x = \frac{(125)(90)^3}{12} - \frac{(100)(50)^3}{12} = 6.552(10^6)\ \text{mm}^4$$

$$I_y = \frac{(40)(125)^3}{12} + \frac{(50)(25)^3}{12} = 6.576(10^6)\ \text{mm}^4$$



$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{6.552(10^6)}{6250}} = 32.38\ \text{mm}$$

(a) $L/r = 3000/32.38 = 92.6$ .... **Ans.**

(b) $\sigma_{cr} = \sigma_y$ $\quad \frac{L}{r} = \sqrt{\frac{\pi^2 E}{\sigma_y}} = \sqrt{\frac{\pi^2(13\times 10^9)}{(35\times 10^6)}} = 60.6$ .... **Ans.**

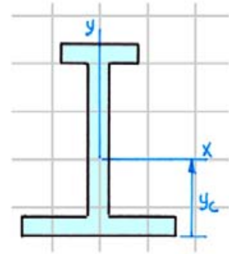
(c) $P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(13\times 10^9)(6.552\times 10^{-6})}{(3)^2} = 93.41(10^3)\ \text{N} \cong 93.4\ \text{kN}$ .... **Ans.**

(d) $\sigma = \frac{P_{cr}}{A} = \frac{93.41(10^3)}{6250(10^{-6})} = 14.95(10^6)\ \text{N/m}^2 = 14.95\ \text{MPa (C)}$ .... **Ans.**

9-89

$$A = 2(8\times1)+(4\times1) = 20 \text{ in.}^2$$

$$y_C = \frac{(0.5)(8\times1)+(5)(8\times1)+(9.5)(4\times1)}{20} = 4.10 \text{ in.}$$



$$I_y = I_{min} = \frac{(1)(8)^3}{12}+\frac{(1)(4)^3}{12}+\frac{(8)(1)^3}{12} = 48.67 \text{ in.}^4$$

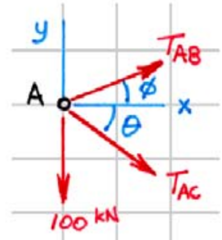
$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{48.67}{20}} = 1.560 \text{ in.} \qquad \frac{L}{r} = \frac{(12\times12)}{1.560} = 92.31 \quad \text{(slender)}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(10,600)(48.67)}{(12\times12)^2} = 245.55 \text{ kip}$$

$$P_{max} = \frac{P_{cr}}{FS} = \frac{245.55}{2.25} = 109.1 \text{ kip}$$ .... **Ans.**

9-90

$\phi = \tan^{-1}(2.5/6) = 22.62°$ $\qquad\qquad$ $\theta = \tan^{-1}(4.5/6) = 36.87°$



$\rightarrow \Sigma F_x = 0:$ $\qquad$ $T_{AB}\cos 22.62° + T_{AC}\cos 36.87° = 0$

$\uparrow \Sigma F_y = 0:$ $\qquad$ $T_{AB}\sin 22.62° - T_{AC}\sin 36.87° - 100 = 0$

Solving yields:

$T_{AB} = 92.86\text{ kN}$ $\qquad\qquad$ $T_{AC} = -107.14\text{ kN}$

$$A_{AB} = \frac{\pi(25)^2}{4} = 490.9\text{ mm}^2 \qquad\qquad A_{AC} = \frac{\pi\left(150^2 - 100^2\right)}{4} = 9817\text{ mm}^2$$

(a) $$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{92.86\left(10^3\right)}{490.9\left(10^{-6}\right)} = 189.16\left(10^6\right)\text{ N/m}^2 = 189.16\text{ MPa (T)}$$

$FS_{AB} = \dfrac{250}{189.16} = 1.322$ .... **Ans.**

$$\sigma_{AC} = \frac{T_{AC}}{A_{AC}} = \frac{-107.14\left(10^3\right)}{9817\left(10^{-6}\right)} = -10.914\left(10^6\right)\text{ N/m}^2 = 10.914\text{ MPa (C)}$$

$FS_{AC} = \dfrac{250}{10.914} = 22.9$ .... **Ans.**

(b) $$I_{AC} = \frac{\pi\left(150^4 - 100^4\right)}{64} = 19.942\left(10^6\right)\text{ mm}^4$$

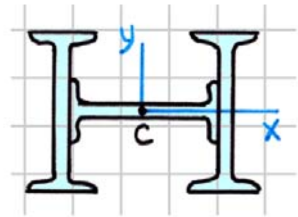
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{19.942\left(10^6\right)}{9817}} = 45.07 \qquad\qquad \frac{L}{r} = \frac{7500}{45.07} = 166.41 \quad \text{(slender)}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2\left(200\times 10^9\right)\left(19.942\times 10^{-6}\right)}{(7.5)^2} = 699.8\left(10^3\right)\text{ N} \cong 699.8\text{ kN}$$

$FS_{AC} = \dfrac{699.8}{107.14} = 6.53$ .... **Ans.**

9-91*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29,000)}{36} = 15,901 \qquad C_c = 126.10$$



From Table B-3 (for an S 10×35 section): $A = 10.3$ in.^{2}

$I_{X-X} = 147$ in.^{4} $\quad I_{Y-Y} = 8.36$ in.^{4} $\quad t_w = 0.594$ in.

For the column: $A = 3(10.3) = 30.9$ in.^{2}

$$I_x = 2(147) + (8.36) = 302.4 \text{ in.}^4 \qquad L'_x = 20 \text{ ft}$$

$$I_y = (147) + 2\left[(8.36) + (10.3)(5.491)^2\right] = 784.8 \text{ in.}^4 \qquad L'_y = 0.7(30) = 21.0 \text{ ft}$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{302.4}{30.9}} = 3.128 \text{ in.} \qquad \frac{L'}{r_x} = \frac{(20\times12)}{3.128} = 76.73 < 126.10$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{784.8}{30.9}} = 5.040 \text{ in.} \qquad \frac{L'}{r_y} = \frac{(21.0\times12)}{5.040} = 50.0 < 126.10$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{76.73}{126.10}\right) - \frac{1}{8}\left(\frac{76.73}{126.10}\right)^3 = 1.867$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.867}\left[1 - \frac{1}{2}\left(\frac{76.73}{126.10}\right)^2\right] = 15.710 \text{ ksi}$$

$P_{max} = P_{all}A = (15.710)(30.9) = 485$ kip .... **Ans.**

9-92*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$

From Table B-2 (for a W 356×122 section):

$$A = 15,550\text{ mm}^2 \qquad r_{X-X} = 154\text{ mm} \qquad r_{Y-Y} = 63.0\text{ mm}$$

(a) $$\frac{L'}{r_y} = \frac{9000}{63.0} = 142.86 > 125.66$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\left(L'/r\right)^2} = \frac{\pi^2\left(200\times10^9\right)}{1.92\left(142.86\right)^2} = 50.37\left(10^6\right)\text{ N/m}^2$$

$P_{max} = \sigma_{all} A = \left(50.37\times10^6\right)\left(15,550\times10^{-6}\right) = 783\left(10^3\right)\text{ N} = 783\text{ kN}$ .... **Ans.**

(b) $$\frac{L'}{r_y} = \frac{6000}{63.0} = 95.24 < 125.66 \qquad \frac{L'}{r_x} = \frac{6000}{154} = 58.44 < 125.66$$

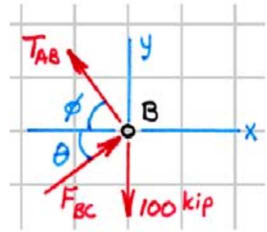
$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{95.24}{125.66}\right) - \frac{1}{8}\left(\frac{95.24}{125.66}\right)^3 = 1.896$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.896}\left[1 - \frac{1}{2}\left(\frac{95.24}{125.66}\right)^2\right] = 93.98\text{ MPa}$$

$P_{max} = \sigma_{all} A = \left(93.98\times10^6\right)\left(15,550\times10^{-6}\right) = 1461\left(10^3\right)\text{ N} = 1461\text{ kN}$ .... **Ans.**

9-93

$\phi = \tan^{-1}(4/3) = 53.13°$ $\qquad \theta = \tan^{-1}(3/4) = 36.87°$



$\rightarrow \Sigma F_x = 0: \qquad F_{BC}\cos 36.87° - T_{AB}\cos 53.13° = 0$

$\uparrow \Sigma F_y = 0: \qquad F_{BC}\sin 36.87° + T_{AB}\sin 53.13° - 100 = 0$

Solving yields:

$T_{AB} = 80.0 \text{ kip (T)} \qquad F_{BC} = 60.0 \text{ kip (C)}$

From Table B-17: $\qquad \sigma_y = 36 \text{ ksi} \qquad E = 29{,}000 \text{ ksi}$

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29{,}000)}{36} = 15{,}901 \qquad C_c = 126.10$$

$L'_x = L = 200 \text{ in.} \qquad L'_y = 2L = 400 \text{ in.}$

If L/r is small: $\qquad FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{36}{1.667} = 21.596 \text{ ksi} \qquad A_{min} = \frac{P}{\sigma_{all}} = \frac{60.0}{21.596} = 2.778 \text{ in.}^2$$

First, try a WT 5×30 section (Table B-11) with:

$A = 8.82 \text{ in.}^2 \qquad r_{X-X} = 1.21 \text{ in.} \qquad r_{Y-Y} = 2.57 \text{ in.}$

$$\frac{L'_x}{r_x} = \frac{200}{1.21} = 165.29 > 126.10 \qquad \frac{L'_y}{r_y} = \frac{400}{2.57} = 155.64 > 126.10$$

Therefore, the strut is in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29{,}000)}{1.92(165.29)^2} = 5.456 \text{ ksi}$$

$$P_{all} = \sigma_{all}A = (5.456)(8.82) = 48.12 \text{ kip} < F_{BC} = 60.0 \text{ kip}$$

This allowable load is much less than the design load; therefore,
a section with a larger area and a larger radius of gyration must be used.

Next, try a WT 7×34 section with:

$A = 9.99 \text{ in.}^2 \qquad r_{X-X} = 1.81 \text{ in.} \qquad r_{Y-Y} = 2.46 \text{ in.}$

$$\frac{L'_x}{r_x} = \frac{200}{1.81} = 110.49 < 126.10 \qquad \frac{L'_y}{r_y} = \frac{400}{2.46} = 162.60 > 126.10$$

This strut is also in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29{,}000)}{1.92(162.60)^2} = 5.638 \text{ ksi}$$

$$P_{all} = \sigma_{all}A = (5.638)(9.99) = 56.32 \text{ kip} < F_{BC} = 60.0 \text{ kip}$$

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Problem 9-93 continued

The next heavier section is a WT 6×36 section with:

$A = 10.6 \text{ in.}^2$ $\quad r_{X-X} = 1.48 \text{ in.}$ $\quad r_{Y-Y} = 3.04 \text{ in.}$

$$\frac{L'_x}{r_x} = \frac{200}{1.48} = 135.14 > 126.10 \qquad \frac{L'_y}{r_y} = \frac{400}{3.04} = 131.58 > 126.10$$

This strut is also in the slender range for which:

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L'/r)^2} = \frac{\pi^2(29{,}000)}{1.92(135.14)^2} = 8.163 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (8.163)(10.6) = 86.53 \text{ kip} > F_{BC} = 60.0 \text{ kip}$$

Therefore Use a WT 6×36 section .... **Ans.**

9-94

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15,791 \qquad C_c = 125.66$$

From Table B-2 (for a W 254×67 section):

$$A = 8580 \text{ mm}^2 \qquad r_{X-X} = 110 \text{ mm} \qquad r_{Y-Y} = 51.1 \text{ mm}$$

(a) $$\frac{L'}{r_x} = \frac{12,000}{110} = 109.09 < 125.66 \qquad \frac{L'}{r_y} = \frac{6000}{51.1} = 117.41 < 125.66$$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{117.41}{125.66}\right) - \frac{1}{8}\left(\frac{117.41}{125.66}\right)^3 = 1.915$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250}{1.915}\left[1 - \frac{1}{2}\left(\frac{117.41}{125.66}\right)^2\right] = 73.56 \text{ MPa}$$

$P_{max} = \sigma_{all} A = \left(73.56\times10^6\right)\left(8580\times10^{-6}\right) = 631\left(10^3\right) \text{ N} = 631 \text{ kN}$ .... **Ans.**

(b) $$\frac{L'}{r_y} = \frac{12,000}{51.1} = 234.8 > 125.66$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\left(L'/r\right)^2} = \frac{\pi^2\left(200\times10^9\right)}{1.92\left(234.8\right)^2} = 18.648\left(10^6\right) \text{ N/m}^2$$

$P_{max} = \sigma_{all} A = \left(18.648\times10^6\right)\left(8580\times10^{-6}\right) = 160.0\left(10^3\right) \text{ N} = 160.0 \text{ kN}$ .... **Ans.**

9-95*

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2 (29,000)}{36} = 15,901 \qquad C_c = 126.10$$

From Table B-1 (for a W 8×40 section):

$A = 11.7$ in.^{2} $\quad d = 8.25$ in.

$I_{X-X} = 146$ in.^{4} $\quad r_{X-X} = 3.53$ in. $\quad r_{Y-Y} = 2.04$ in.

$$\frac{L'}{r_x} = \frac{(25 \times 12)}{3.53} = 84.99 < 126.10 \qquad \frac{L'}{r_y} = \frac{(25 \times 12)}{2.04} = 147.06 > 126.10$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L'/r_y)^2} = \frac{\pi^2 (29,000)}{1.92 (147.06)^2} = 6.893 \text{ ksi} = \sigma_a$$

$$\sigma_b = 0.66 \sigma_y = 0.66(36) = 23.76 \text{ ksi}$$

$$\frac{P/A}{\sigma_a} + \frac{Mc/I}{\sigma_b} = \frac{P/(11.7)}{6.893} + \frac{(10 \times 12)(4.125)/(146)}{23.76} \le 1$$

$P \le 69.1$ kip $\quad$ $P_{max} = 69.1$ kip .... **Ans.**

9-96

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2\left(200\times10^9\right)}{\left(250\times10^6\right)} = 15{,}791 \qquad C_c = 125.66$$

From Table B-6 (for a C 229×30 channel): $A = 3795\text{ mm}^2$ $\quad x_C = 14.8\text{ mm}$

$I_{X-X} = 25.3\left(10^6\right)\text{ mm}^4 \qquad d = 228.6\text{ mm}$

$r_{X-X} = 81.8\text{ mm} \qquad r_{Y-Y} = 16.3\text{ mm}$

For the latticed channels: $r_y = \sqrt{(16.3)^2 + (60.0+14.8)^2} = 76.6\text{ mm}$

$$\frac{L'}{r_x} = \frac{10{,}000}{81.8} = 122.25 < 125.66 \qquad \frac{L'}{r_y} = \frac{10{,}000}{76.6} = 130.55 > 125.66$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\left(L'/r_y\right)^2} = \frac{\pi^2\left(200\times10^9\right)}{1.92(130.55)^2} = 60.32\left(10^6\right)\text{ N/m}^2$$

$$\frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2\left(3795\times10^{-6}\right)} + \frac{P(0.050)(0.1143)}{2\left(25.3\times10^{-6}\right)} \le \sigma_{all} = 60.32\left(10^6\right)\text{ N/m}^2$$

$P \le 247\left(10^3\right)\text{ N}$ $\qquad P_{max} = 247\text{ kN}$ .... **Ans.**

9-97

From Table B-17: $\sigma_y = 36$ ksi $\qquad E = 29{,}000$ ksi

$$C_c^2 = \frac{2\pi^2 E}{\sigma_y} = \frac{2\pi^2(29{,}000)}{36} = 15{,}901 \qquad C_c = 126.10$$

$$L'_x = L = 20 \text{ ft} \qquad L'_y = 0.5L = 0.5(20) = 10 \text{ ft}$$

If L/r is small: $FS \cong 5/3 = 1.667$

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{36}{1.667} = 21.596 \text{ ksi} \qquad A_{\min} = \frac{P}{\sigma_{all}} = \frac{110}{21.596} = 5.093 \text{ in.}^2$$

First, try a W 10×22 section (Table B-1) with: $A = 6.49$ in.^{2}

$r_{X-X} = 4.27$ in. $\qquad r_{Y-Y} = 1.33$ in.

$$\frac{L'_x}{r_x} = \frac{(20 \times 12)}{4.27} = 56.21 < 126.10 \qquad \frac{L'_y}{r_y} = \frac{(10 \times 12)}{1.33} = 90.23 < 126.10$$

Therefore, the column is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{90.23}{126.10}\right) - \frac{1}{8}\left(\frac{90.23}{126.10}\right)^3 = 1.889$$

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.889}\left[1 - \frac{1}{2}\left(\frac{90.23}{126.10}\right)^2\right] = 14.18 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (14.18)(6.49) = 92.0 \text{ kip} < P = 110 \text{ kip}$$

This allowable load is less than the design load; therefore, a section with a larger area and a larger radius of gyration must be used.

Next, try a W 12×30 section with: $A = 8.79$ in.^{2}

$r_{X-X} = 5.21$ in. $\qquad r_{Y-Y} = 1.52$ in.

$$\frac{L'_x}{r_x} = \frac{(20 \times 12)}{5.21} = 46.07 < 126.10 \qquad \frac{L'_y}{r_y} = \frac{(10 \times 12)}{1.52} = 78.95 < 126.10$$

Therefore, the column is in the intermediate range for which:

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L'/r}{C_c}\right) - \frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3} + \frac{3}{8}\left(\frac{78.95}{126.10}\right) - \frac{1}{8}\left(\frac{78.95}{126.10}\right)^3 = 1.871$$

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Problem 9-97 continued

$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1-\frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.871}\left[1-\frac{1}{2}\left(\frac{78.95}{126.10}\right)^2\right] = 15.472 \text{ ksi}$$

$$P_{all} = \sigma_{all}A = (15.472)(8.79) = 136.0 \text{ kip} > P = 110 \text{ kip}$$

Next, try an $S\ 10\times 25.4$ section (Table B-3) with: $A = 7.46 \text{ in.}^2$

$r_{X-X} = 4.07$ in. $\quad r_{Y-Y} = 0.954$ in.

$$\frac{L'_x}{r_x} = \frac{(20\times 12)}{4.07} = 58.97 < 126.10 \qquad \frac{L'_y}{r_y} = \frac{(10\times 12)}{0.954} = 125.8 < 126.10$$

This section is in the intermediate range for which:

$$FS = \frac{5}{3}+\frac{3}{8}\left(\frac{L'/r}{C_c}\right)-\frac{1}{8}\left(\frac{L'/r}{C_c}\right)^3 = \frac{5}{3}+\frac{3}{8}\left(\frac{125.8}{126.10}\right)-\frac{1}{8}\left(\frac{125.8}{126.10}\right)^3 = 1.917$$

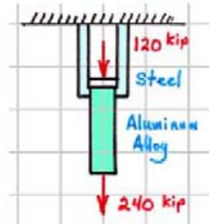
$$\sigma_{all} = \frac{\sigma_y}{FS}\left[1-\frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.917}\left[1-\frac{1}{2}\left(\frac{125.8}{126.10}\right)^2\right] = 9.438 \text{ ksi}$$

$$P_{all} = \sigma_{all}A = (9.438)(7.46) = 70.4 \text{ kip} < P = 110 \text{ kip}$$

There are no other American Standard sections lighter than the $W\ 12\times 30$ section.

Therefore Use a $W\ 12\times 30$ section .... **Ans.**

10-1*



$$A_s = \frac{\pi\left(6^2 - 4.5^2\right)}{4} = 12.370 \text{ in.}^2$$

$$A_a = \frac{\pi(4)^2}{4} = 12.566 \text{ in.}^2$$

(a) $$U_s = \frac{P_s^2 L_s}{2A_s E_s} = \frac{(360)^2(3\times 12)}{2(12.370)(29{,}000)} = 6.503 \text{ kip}\cdot\text{in.} \cong 6.50 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

(b) $$U_a = \frac{P_a^2 L_a}{2A_a E_a} = \frac{(240)^2(4\times 12)}{2(12.566)(10{,}600)} = 10.378 \text{ kip}\cdot\text{in.} \cong 10.38 \text{ kip}\cdot\text{in.}$$ .... **Ans.**

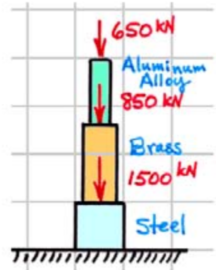
(c) $U_{total} = U_s + U_a = 6.503 + 10.378 = 16.88 \text{ kip}\cdot\text{in.}$ .... **Ans.**

10-2*

$$A_a = \frac{\pi(100)^2}{4} = 7854 \text{ mm}^2$$

$$A_b = \frac{\pi(150^2 - 100^2)}{4} = 9817 \text{ mm}^2$$

$$A_s = \frac{\pi(200^2 - 125^2)}{4} = 19{,}144 \text{ mm}^2$$



(a) $$U_a = \frac{P_a^2 L_a}{2A_a E_a} = \frac{(650{,}000)^2(1.0)}{2(7854\times10^{-6})(73\times10^9)} = 368.5 \text{ N}\cdot\text{m} \cong 369 \text{ N}\cdot\text{m}$$ .... **Ans.**

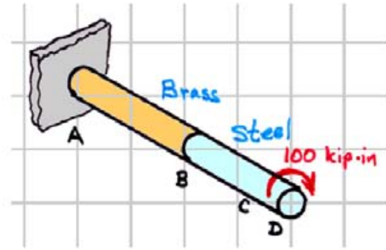
$$U_b = \frac{P_b^2 L_b}{2A_b E_b} = \frac{(1{,}500{,}000)^2(1.25)}{2(9817\times10^{-6})(100\times10^9)} = 1432.5 \text{ N}\cdot\text{m} \cong 1433 \text{ N}\cdot\text{m}$$ .... **Ans.**

$$U_s = \frac{P_s^2 L_s}{2A_s E_s} = \frac{(3{,}000{,}000)^2(0.75)}{2(19{,}144\times10^{-6})(210\times10^9)} = 839.5 \text{ N}\cdot\text{m} \cong 840 \text{ N}\cdot\text{m}$$ .... **Ans.**

(b) $$U_{total} = U_a + U_b + U_s = 368.5 + 1432.5 + 839.5 = 2640.5 \text{ N}\cdot\text{m} \cong 2.64 \text{ kN}\cdot\text{m}$$ .... **Ans.**

10-3

$$J_b = J_s = \frac{\pi r^4}{2} = \frac{\pi (2)^4}{2} = 25.13 \text{ in.}^4$$



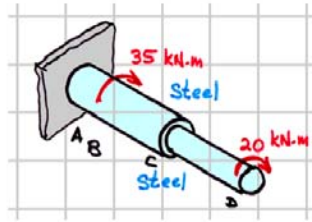
(a) $$U_b = \frac{T_b^2 L_b}{2 G_b J_b} = \frac{(100)^2 (6 \times 12)}{2(5000)(25.13)}$$

$U_b = 2.865 \text{ kip} \cdot \text{in.} \cong 2.87 \text{ kip} \cdot \text{in.}$ .... **Ans.**

$U_s = \dfrac{T_s^2 L_s}{2 G_s J_s} = \dfrac{(100)^2 (4 \times 12)}{2(12{,}000)(25.13)} = 0.7959 \text{ kip} \cdot \text{in.} \cong 796 \text{ lb} \cdot \text{in.}$ .... **Ans.**

(b) $U_{total} = U_b + U_s = 2.865 + 0.7959 = 3.66 \text{ kip} \cdot \text{in.}$ .... **Ans.**

10-4*



$$J_{AB} = J_{BC} = \pi r^4/2 = \pi(80)^4/2 = 64.34(10^6)\ \text{mm}^4$$

$$J_{CD} = \pi r^4/2 = \pi(50)^4/2 = 9.817(10^6)\ \text{mm}^4$$

(a) $$U_{AB} = \left(\frac{T^2 L}{2GJ}\right)_{AB} = \frac{(55{,}000)^2(1.0)}{2(80\times10^9)(64.34\times10^{-6})} = 293.8\ \text{N}\cdot\text{m}$$

$U_{AB} \cong 294\ \text{N}\cdot\text{m}$ .... **Ans.**

$$U_{BC} = \left(\frac{T^2 L}{2GJ}\right)_{BC} = \frac{(20{,}000)^2(1.5)}{2(80\times10^9)(64.34\times10^{-6})} = 58.28\ \text{N}\cdot\text{m}$$

$U_{BC} \cong 58.3\ \text{N}\cdot\text{m}$ .... **Ans.**

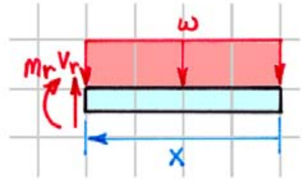
$$U_{CD} = \left(\frac{T^2 L}{2GJ}\right)_{CD} = \frac{(20{,}000)^2(2.0)}{2(80\times10^9)(9.817\times10^{-6})} = 509.3\ \text{N}\cdot\text{m}$$

$U_{CD} \cong 509\ \text{N}\cdot\text{m}$ .... **Ans.**

(b) $U_{total} = U_{AB} + U_{BC} + U_{CD} = 293.8 + 58.28 + 509.3 = 861\ \text{N}\cdot\text{m}$ .... **Ans.**

10-5

$$M_r = \frac{-wx^2}{2} \qquad I = \frac{bh^3}{12}$$



$$U = \int_0^L \frac{M_r^2\,dx}{2EI} = \int_0^L \frac{\left(wx^2/2\right)^2 dx}{2EI} = \frac{w^2}{8EI}\int_0^L x^4\,dx$$

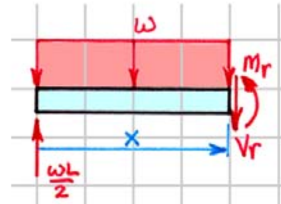
$$U = \frac{w^2L^5}{40EI} = \frac{w^2L^5}{40E\left(bh^3/12\right)} = \frac{3w^2L^5}{10Ebh^3}$$ .... **Ans.**

10-6

$$M_r = \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w\left(Lx - x^2\right)}{2}$$

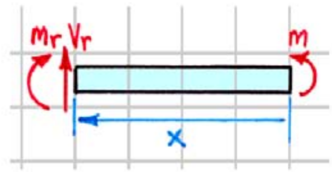
$$U = \int_0^L \frac{M_r^2\,dx}{2EI} = \frac{w^2}{8EI}\int_0^L \left(Lx - x^2\right)^2 dx$$

$$= \frac{w^2}{8EI}\int_0^L \left(L^2x^2 - 2Lx^3 + x^4\right) dx$$



$$U = \frac{w^2}{8EI}\left[\frac{L^2x^3}{3} - \frac{Lx^4}{2} + \frac{x^5}{5}\right]_0^L = \frac{w^2L^5}{240EI}$$ .... **Ans.**

10-7*



$$U_{beam} = \int_0^L \frac{M_r^2\,dx}{2EI} = \left[\frac{M^2 x}{2EI}\right]_0^L = \frac{M^2 L}{2EI}$$

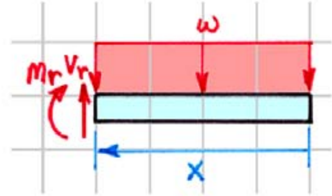
$$\sigma_{max} = \frac{Mc}{I} \qquad M = \frac{\sigma_{max} I}{c}$$

$$I = \frac{b(2c)^3}{12} = \frac{2bc^3}{3} \qquad U_{beam} = \frac{M^2 L}{2EI} = \frac{\sigma_{max}^2 IL}{2Ec^2} = \frac{\sigma_{max}^2 bcL}{3E}$$

$$U_{bar} = \frac{\sigma_{max}^2 AL}{2E} = \frac{\sigma_{max}^2 (2bc)L}{2E} = \frac{\sigma_{max}^2 bcL}{E}$$

Therefore $U_{bar} = 3U_{beam}$ .... **Ans.**

10-8*



$$V = wx \qquad I = \frac{bh^3}{12}$$

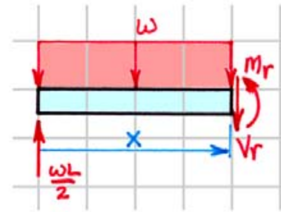
$$U = \int_0^L \frac{V^2}{2GI^2}\left(\int_A \frac{Q^2}{t^2}dA\right)dx$$

$$\int_A \frac{Q^2}{t^2}dA = \int_{-h/2}^{+h/2} \frac{b}{4}\left[\frac{h^2}{4} - y^2\right]^2 dy = \frac{b}{4}\left[\frac{h^4 y}{16} - \frac{h^2 y^3}{6} + \frac{y^5}{5}\right]_{-h/2}^{+h/2} = \frac{bh^5}{120}$$

$$U = \int_0^L \frac{w^2x^2}{2GI^2}\left(\frac{bh^5}{120}\right)dx = \frac{w^2bh^5}{240G\left(bh^3/12\right)^2}\int_0^L x^2\, dx$$

$$U = \frac{3w^2}{5Gbh}\left[\frac{x^3}{3}\right]_0^L = \frac{w^2L^3}{5Gbh}$$ .... **Ans.**

10-9



$$V = \frac{wL}{2} - wx \qquad I = \frac{bh^3}{12}$$

$$U = \int_0^L \frac{V^2}{2GI^2}\left(\int_A \frac{Q^2}{t^2}\,dA\right)dx$$

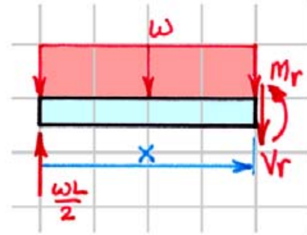
$$\int_A \frac{Q^2}{t^2}\,dA = \int_{-h/2}^{+h/2} \frac{b}{4}\left[\frac{h^2}{4} - y^2\right]^2 dy = \frac{b}{4}\left[\frac{h^4 y}{16} - \frac{h^2 y^3}{6} + \frac{y^5}{5}\right]_{-h/2}^{+h/2} = \frac{bh^5}{120}$$

$$U = \int_0^L \frac{w^2}{2GI^2}\left(\frac{L}{2} - x\right)^2\left(\frac{bh^5}{120}\right)dx = \frac{w^2bh^5}{240G\left(bh^3/12\right)^2}\int_0^L\left(\frac{L^2}{4} - Lx + x^2\right)dx$$

$$U = \frac{3w^2}{5Gbh}\left[\frac{L^2x}{4} - \frac{Lx^2}{2} + \frac{x^3}{3}\right]_0^L = \frac{w^2L^3}{20Gbh}$$ .... **Ans.**

10-10

$$M_r = \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w\left(Lx - x^2\right)}{2}$$



$$U = \int_0^L \frac{M_r^2\, dx}{2EI} = \frac{w^2}{8EI}\int_0^L \left(Lx - x^2\right)^2 dx$$

$$= \frac{w^2}{8EI}\int_0^L \left(L^2x^2 - 2Lx^3 + x^4\right)dx$$

$$= \frac{w^2}{8EI}\left[\frac{L^2x^3}{3} - \frac{Lx^4}{2} + \frac{x^5}{5}\right]_0^L = \frac{w^2L^5}{240EI}$$

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{\left(wL^2/8\right)c}{I} = \frac{wL^2c}{8I} \qquad w = \frac{8\sigma_{\max}I}{L^2c}$$

$$I = \frac{b\left(2c\right)^3}{12} = \frac{2bc^3}{3} \qquad U_{beam} = \frac{w^2L^5}{240EI} = \frac{4\sigma_{\max}^2 IL}{15Ec^2} = \frac{8\sigma_{\max}^2 bcL}{45E}$$

$$U_{bar} = \frac{\sigma_{\max}^2 AL}{2E} = \frac{\sigma_{\max}^2\left(2bc\right)L}{2E} = \frac{\sigma_{\max}^2 bcL}{E}$$

Therefore $U_{bar} = \left(45/8\right)U_{beam} = 5.63U_{beam}$ .... **Ans.**

10-11*

$$J_{AB} = J_{CD} = \pi c^4/2 = \pi(1)^4/2 = 1.5708 \text{ in.}^4$$

$$T_{CD} = 2T_{AB} = 2(740) = 1480 \text{ lb}\cdot\text{ft}$$

$$U_{AB} = \left(\frac{T^2L}{2GJ}\right)_{AB} = \frac{(740\times12)^2(48)}{2(11\times10^6)(1.5708)} = 109.5 \text{ lb}\cdot\text{in.}$$ .... **Ans.**

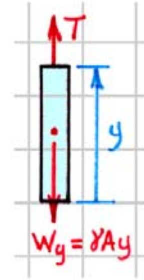
$$U_{CD} = \left(\frac{T^2L}{2GJ}\right)_{CD} = \frac{(1480\times12)^2(36)}{2(11\times10^6)(1.5708)} = 329 \text{ lb}\cdot\text{in.}$$ .... **Ans.**

10-12*

$$\sigma = \frac{T}{A} = \frac{\gamma A y}{A} = \gamma y$$

$$dU = \frac{\sigma^2}{2E}(A\,dy) = \frac{\gamma^2 A}{2E}(y^2\,dy)$$

$$U = \int_0^L \frac{\gamma^2 A}{2E}(y^2\,dy) = \frac{\gamma^2 A}{2E}\left[\frac{y^3}{3}\right]_0^L = \frac{\gamma^2 A L^3}{6E}$$ ....**Ans.**

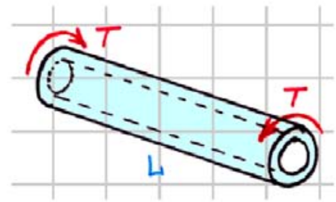


10-13*

$$J = \frac{\pi\left[d^4 - (d/2)^4\right]}{32} = \frac{15\pi d^4}{512}$$

$$A = \frac{\pi\left[d^2 - (d/2)^2\right]}{4} = \frac{3\pi d^2}{16}$$

$$G = \frac{E}{2(1+\nu)}$$



$$\sigma_{\max} = \tau_{\max} = \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{\sigma_{\max} J}{c}$$

$$U_{shaft} = \frac{T^2 L}{2GJ} = \frac{\sigma_{\max}^2 JL}{c^2(2G)} = \frac{\sigma_{\max}^2 \left(15\pi d^4/512\right)L}{(d/2)^2\left[E/(1+\nu)\right]} = \frac{15(1+\nu)\pi d^2 L\sigma_{\max}^2}{128E}$$

$$U_{bar} = \frac{\sigma_{\max}^2 AL}{2E} = \frac{\sigma_{\max}^2\left(3\pi d^2/16\right)L}{2E} = \frac{3\pi d^2 L\sigma_{\max}^2}{32E}$$

Therefore $U_{bar} = (5/4)(1+\nu)U_{shaft}$ .... **Ans.**

10-14

$T = qx \qquad J = \pi c^4/2$

$$dU = \frac{T^2\,dx}{2GJ} = \frac{q^2x^2\,dx}{2G\left(\pi c^4/2\right)} = \frac{q^2x^2\,dx}{\pi c^4 G}$$

$$U = \int_0^L \frac{q^2x^2\,dx}{\pi c^4 G} = \frac{q^2}{\pi c^4 G}\left[\frac{x^3}{3}\right]_0^L = \frac{q^2L^3}{3\pi c^4 G}$$ .... **Ans.**

10-15*

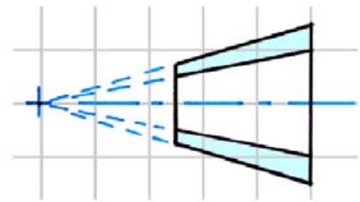
$$T = \frac{1}{2}\left(\frac{qx}{L}\right)x = \frac{qx^2}{2L} \qquad J = \frac{\pi c^4}{2}$$

$$dU = \frac{T^2\,dx}{2GJ} = \frac{\left(qx^2/2L\right)^2 dx}{2G\left(\pi c^4/2\right)} = \frac{q^2x^4\,dx}{4\pi c^4 L^2 G}$$

$$U = \int_0^L \frac{q^2x^4\,dx}{4\pi c^4 G L^2} = \frac{q^2}{4\pi c^4 G L^2}\left[\frac{x^5}{5}\right]_0^L = \frac{q^2 L^3}{20\pi c^4 G}$$ .... **Ans.**

10-16

$$\rho_o = \frac{rx}{L} \qquad \rho_i = \frac{rx}{2L}$$



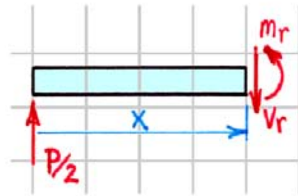
$$J = \frac{\pi\left(r_o^4 - r_i^4\right)}{2} = \frac{15\pi r^4 x^4}{32L^4}$$

$$dU = \frac{T^2\,dx}{2GJ} = \left(\frac{16T^2L^4}{15\pi r^4 G}\right)\frac{dx}{x^4}$$

$$U = \int_L^{2L}\left(\frac{16T^2L^4}{15\pi r^4 G}\right)\frac{dx}{x^4} = \frac{16T^2L^4}{15\pi r^4 G}\left[\frac{-1}{3x^3}\right]_L^{2L} = \frac{14T^2L}{45\pi r^4 G}$$ .... **Ans.**

10-17

By symmetry, each support carries half of the total load $\left(R_A = R_C = P/2\right)$ and the strain energy of the entire beam is twice the strain energy in either half.



$$M_r = Px/2$$

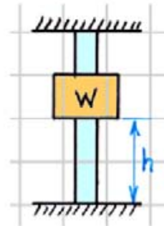
$$U = 2\int_0^{L/2} \frac{M_r^2\,dx}{2EI} = \frac{P^2}{4EI}\int_0^{L/2} x^2\,dx = \frac{P^2}{4EI}\left[\frac{x^3}{3}\right]_0^{L/2}$$

$$U = \frac{P^2L^3}{96EI}$$ .... **Ans.**

10-18*

$$\sigma = \frac{W}{A} + \sqrt{\left(\frac{W}{A}\right)^2 + \frac{2WhE}{AL}}$$

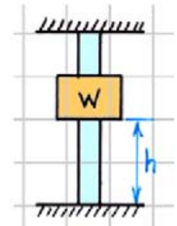
$$W = mg = (20)(9.81) = 196.20\ \text{N}$$



$$L = \frac{2WhE}{\sigma^2 A - 2\sigma W} = \frac{2(196.20)(1.0)(100\times10^9)}{(70\times10^6)^2(2500\times10^{-6}) - 2(70\times10^6)(196.20)}$$

$L = 3.21$ m .... **Ans.**

10-19*



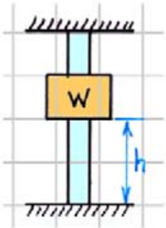
$$\sigma = \frac{W}{A} + \sqrt{\left(\frac{W}{A}\right)^2 + \frac{2WhE}{AL}}$$

$$A = \frac{2W\sigma L + 2WhE}{\sigma^2 L} = \frac{2(30)(25,000)(50) + 2(30)(40)(30 \times 10^6)}{(25,000)^2 (50)}$$

$$= 2.3064 \text{ in.}^2$$

$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(2.3064)}{\pi}} = 1.714$ in. .... **Ans.**

10-20



$$\sigma = \frac{W}{A} + \sqrt{\left(\frac{W}{A}\right)^2 + \frac{2WhE}{AL}}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (25)^2}{4} = 490.9 \text{ mm}^2$$

(a) $h = 600 \text{ mm} = 0.600 \text{ m}$

$$W = \frac{\sigma^2 AL}{2\sigma L + 2hE} = \frac{(250\times10^6)^2 (490.9\times10^{-6})(0.900)}{2(250\times10^6)(0.900) + 2(0.600)(200\times10^9)} = 114.839 \text{ N}$$

$$m = \frac{W}{g} = \frac{114.839}{9.81} = 11.71 \text{ kg}$$ .... **Ans.**

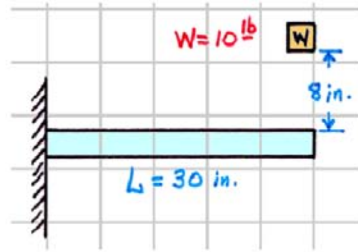
(b) $h \to 0 \text{ m}$

$$W = \frac{\sigma^2 AL}{2\sigma L + 0} = \frac{(250\times10^6)^2 (490.9\times10^{-6})(0.900)}{2(250\times10^6)(0.900) + 0} = 61{,}362.5 \text{ N}$$

$$m = \frac{W}{g} = \frac{61{,}362.5}{9.81} = 6255 \text{ kg} \cong 6260 \text{ kg}$$ .... **Ans.**

10-21

$$I = (2)(3)^3 / 12 = 4.5 \text{ in.}^4$$



(a) $$\Delta = \frac{PL^3}{3EI} \qquad P = \frac{3EI\Delta}{L^3}$$

$$W(h+\Delta) = \frac{P\Delta}{2} = \frac{3EI\Delta^2}{2L^3}$$

$$10(8+\Delta) = \frac{3(30\times10^6)(4.5)\Delta^2}{2(30)^3} = 7500\Delta^2$$

$$7500\Delta^2 - 10\Delta - 80 = 0$$

$$\Delta = \frac{10+\sqrt{(10)^2+4(7500)(80)}}{2(7500)} = 0.103948 \text{ in.} \cong 0.1039 \text{ in.}$$ .... **Ans.**

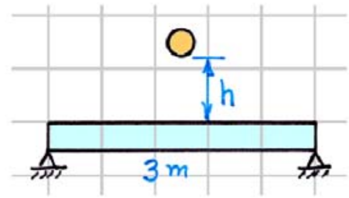
(b) $$P = \frac{3EI\Delta}{L^3} = \frac{3(30\times10^6)(4.5)(0.103948)}{(30)^3} = 1559.2 \text{ lb} \cong 1559 \text{ lb}$$ .... **Ans.**

(c) $$\sigma = \frac{Mc}{I} = \frac{(1559.2)(30)(1.5)}{4.5} = 15{,}592 \text{ psi} \cong 15.59 \text{ ksi}$$ .... **Ans.**

10-22*

$$\sigma = \frac{Mc}{I} = \frac{(PL/4)c}{I} \qquad I = \frac{bh^3}{12} = \frac{(150)(100)^3}{12} = 12.50(10^6)\ \text{mm}^4$$

$$P = \frac{4\sigma I}{Lc} = \frac{4(10\times10^6)(12.50\times10^{-6})}{(3.0)(0.050)} = 3333\ \text{N}$$



$$\Delta = \frac{PL^3}{48EI} = \frac{(3333)(3.0)^3}{48(8.2\times10^9)(12.50\times10^{-6})} = 0.018291\ \text{m}$$

$$mg(h+\Delta) = P\Delta/2$$

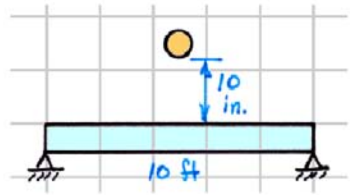
$$h = \frac{P\Delta}{2mg} - \Delta = \frac{(3333)(0.018291)}{2(14)(9.81)} - 0.018291 = 0.204\ \text{m} = 204\ \text{mm}$$ .... **Ans.**

10-23

(a) $\Delta = \dfrac{PL^3}{48EI}$ $\qquad P = \dfrac{48EI\Delta}{L^3}$ $\qquad I = \dfrac{bh^3}{12} = \dfrac{(6)(4)^3}{12} = 32.0 \text{ in.}^4$

$$W(h+\Delta) = \frac{P\Delta}{2} = \frac{24EI\Delta^2}{L^3}$$

$$(30)(10+\Delta) = \frac{24(1.2\times10^6)(32.0)\Delta^2}{(120)^3} = 533.3\Delta^2$$



$$533.3\Delta^2 - 30\Delta - 300 = 0$$

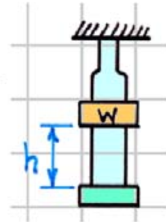
$$\Delta = \frac{30+\sqrt{(30)^2+4(533.3)(300)}}{2(533.3)} = 0.7787 \text{ in.} \cong 0.779 \text{ in.}$$ .... **Ans.**

(b) $$P = \frac{48EI\Delta}{L^3} = \frac{48(1.2\times10^6)(32.0)(0.7787)}{(120)^3} = 828.48 \text{ lb} \cong 828 \text{ lb}$$ .... **Ans.**

(c) $$\sigma = \frac{Mc}{I} = \frac{(P/2)(L/2)c}{I} = \frac{PLc}{4I} = \frac{(828.48)(120)(2.0)}{4(32.0)} = 1553 \text{ psi}$$ .... **Ans.**

10-24*

When $\sigma_{top} = 200 \text{ MPa}$, $\sigma_{bottom} = 100 \text{ MPa}$ $\qquad U = W(h+\delta)$



$$U = \sum \frac{\sigma^2 AL}{2E} = \frac{(200\times10^6)^2(250\times10^{-6})(0.5)}{2(200\times10^9)} + \frac{(100\times10^6)^2(500\times10^{-6})(1.5)}{2(200\times10^9)}$$

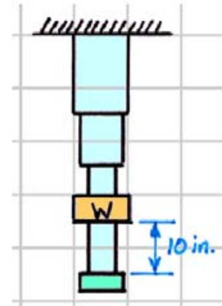
$$= 31.25 \text{ N}\cdot\text{m}$$

$$\delta = \sum \frac{\sigma L}{E} = \frac{(200\times10^6)(0.5)}{(200\times10^9)} + \frac{(100\times10^6)(1.5)}{(200\times10^9)} = 0.00125 \text{ m}$$

$$h = \frac{U - W\delta}{W} = \frac{U - mg\delta}{mg} = \frac{(31.25)-(2\times9.81)(0.00125)}{(2\times9.81)} = 1.592 \text{ m}$$ .... **Ans.**

10-25*

(a) When $\sigma_{bottom} = 18 \text{ ksi}$, $\sigma_{mid} = 9 \text{ ksi}$, and $\sigma_{top} = 6 \text{ ksi}$



$$U = \sum \frac{\sigma^2 AL}{2E} = \frac{(18{,}000)^2 (2)(20)}{2(30\times10^6)} + \frac{(9000)^2 (4)(10)}{2(30\times10^6)} + \frac{(6000)^2 (6)(15)}{2(30\times10^6)}$$

$$= 324 \text{ lb}\cdot\text{in.}$$

$$\delta = \sum \frac{\sigma L}{E} = \frac{(18{,}000)(20)}{30\times10^6} + \frac{(9000)(10)}{30\times10^6} + \frac{(6000)(15)}{30\times10^6} = 0.018 \text{ in.}$$

$$W = \frac{U}{h+\delta} = \frac{324}{10+0.018} = 32.3 \text{ lb}$$ .... **Ans.**

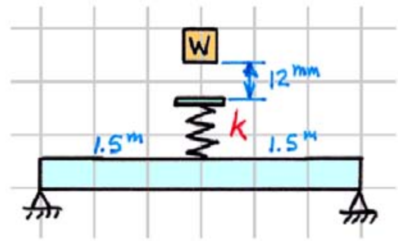
(b) $$U = \frac{\sigma^2 AL}{2E} = \frac{(18{,}000)^2 (2)(45)}{2(30\times10^6)} = 486 \text{ lb}\cdot\text{in.}$$

$$\delta = \frac{\sigma L}{E} = \frac{(18{,}000)(45)}{30\times10^6} = 0.027 \text{ in.}$$

$$W = \frac{U}{h+\delta} = \frac{486}{10+0.027} = 48.5 \text{ lb}$$ .... **Ans.**

10-26

$$I = \frac{bh^3}{12} = \frac{(75)(25)^3}{12} = 97.656(10^3)\ \text{mm}^4$$



$$\delta_{st} = \delta_{st,beam} + \delta_{st,spring} = \frac{WL^3}{48EI} + \frac{F_{spring}}{k}$$

$$\delta_{st} = \frac{(240)(3)^3}{48(70\times10^9)(97.656\times10^{-9})} + \frac{240}{36{,}000}$$

$$= 0.026416\ \text{m}$$

$$IF = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{2(0.012)}{0.026416}} = 2.3815$$

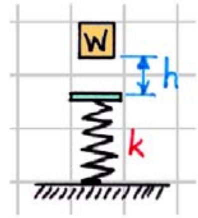
$$P = W(IF) = (240)(2.3815) = 571.56\ \text{N}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{(P/2)(L/2)c}{I} = \frac{PLc}{4I} = \frac{(571.56)(3.0)(0.0125)}{4(97.656\times10^{-9})} = 54.9(10^6)\ \text{N/m}^2$$

$\sigma_{max} = 54.9$ MPa .... **Ans.**

10-27*

(a) $U = W(h+\delta) = \frac{1}{2}(k\delta)(\delta) = \frac{k\delta^2}{2}$



$$\delta^2 - \frac{2W\delta}{k} - \frac{2Wh}{k} = 0$$

$$\delta = \frac{W}{k} + \sqrt{\left(\frac{W}{k}\right)^2 + \frac{2Wh}{k}} = \frac{50}{100} + \sqrt{\left(\frac{50}{100}\right)^2 + \frac{2(50)(4)}{100}}$$

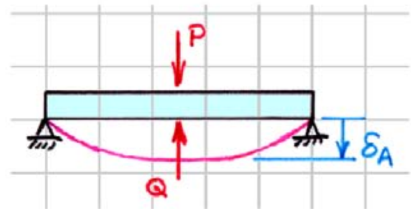
$\delta = 2.562$ in. $\cong 2.56$ in. .... **Ans.**

(b) $W = k\delta = (100)(2.562) \cong 256$ lb .... **Ans.**

10-28

$$W = mg = 22(9.81) = 215.82 \text{ N} \qquad I = \frac{bh^3}{12} = \frac{(75)(25)^3}{12} = 97.656(10^3) \text{ mm}^4$$

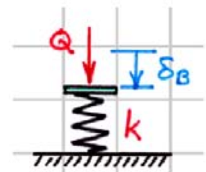
$$\delta_A = \delta_B = \delta$$



$$\delta = \frac{(P-Q)L^3}{48EI} = \frac{Q}{k}$$

$$\frac{(P-Q)(3.0)^3}{48(70\times10^9)(97.656\times10^{-9})} = \frac{Q}{18,000}$$

$$P = 1.67515Q \qquad Q = 0.59696P$$



$$\delta = \frac{Q}{k} = \frac{0.59696P}{18,000} = 33.165(10^{-6})P$$

$$W(h+\delta) = P\delta/2$$

$$(215.82)(0.050) + (215.82)\delta = P\delta/2 \qquad \text{where} \quad \delta = 33.165(10^{-6})P$$

Therefore: $16.583P^2 - 7157.7P - 10.791(10^6) = 0$

$$P = 1050.86 \text{ N} \qquad Q = 627.32 \text{ N}$$

(a) $P = W(IF)$ $\qquad IF = P/W = 1050.86/(215.82) = 4.869 \cong 4.87$ .... **Ans.**

(b) $IF = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$ $\qquad 4.869 = 1 + \sqrt{1 + \frac{2(0.050)}{\delta_{st}}}$

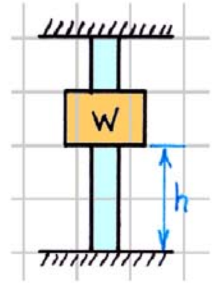
$\delta_{st} = 0.00716 \text{ m} = 7.16 \text{ mm}$ .... **Ans.**

(c) $R_A = R_C = (P-Q)/2 = 211.77 \text{ N}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{(211.77)(1.5)(0.0125)}{97.656(10^{-9})} = 40.7(10^6) \text{ N/m}^2$$

$\sigma_{max} = 40.7 \text{ MPa}$ .... **Ans.**

10-29



Eq. (10-14) $\sigma = \frac{W}{A}\left[1+\sqrt{1+\frac{2hAE}{WL}}\right] = \frac{W}{A}\left[1+\sqrt{1+\frac{2h}{\delta_{st}}}\right]$

For $h ? \delta_{st}$ $\quad 1+\sqrt{1+\frac{2h}{\delta_{st}}} \rightarrow \sqrt{\frac{2h}{\delta_{st}}}$

Therefore $\quad \sigma = \frac{W}{A}\sqrt{\frac{2h}{\delta_{st}}}$ for $h ? \delta_{st}$ .... **Ans.**

Eq. (14-16) $\delta = \frac{WL}{AE}\left[1+\sqrt{1+\frac{2hAE}{WL}}\right] = \delta_{st}\left[1+\sqrt{1+\frac{2h}{\delta_{st}}}\right]$

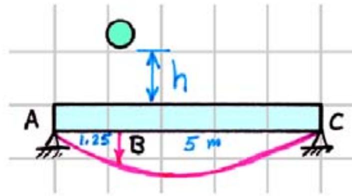
As before: $\quad 1+\sqrt{1+\frac{2h}{\delta_{st}}} \rightarrow \sqrt{\frac{2h}{\delta_{st}}}$ for $h ? \delta_{st}$

Therefore $\quad \delta = \delta_{st}\sqrt{\frac{2h}{\delta_{st}}}$ for $h ? \delta_{st}$ .... **Ans.**

10-30*

$$W = mg = (10)(9.81) = 98.1 \text{ N}$$

$$I = \frac{bh^3}{12} = \frac{(75)(75)^3}{12} = 2.637(10^6) \text{ mm}$$



$$\Delta_{st} = \frac{\partial U}{\partial W} = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial W}dx$$

$$= \frac{1}{EI}\int_0^{L/5} \frac{4Wx}{5}\left(\frac{4x}{5}\right)dx + \frac{1}{EI}\int_0^{4L/5} \frac{Wx}{5}\left(\frac{x}{5}\right)dx$$

$$= \frac{16W}{25EI}\left[\frac{x^3}{3}\right]_0^{L/5} + \frac{W}{25EI}\left[\frac{x^3}{3}\right]_0^{4L/5} = \frac{16WL^3}{1875EI}$$

(a) $$\Delta = \frac{16PL^3}{1875EI} \qquad P = \frac{1875EI\Delta}{16L^3}$$

$$W(h+\Delta) = \frac{P\Delta}{2} = \frac{1875EI\Delta^2}{32L^3}$$

$$98.1(0.100+\Delta) = \frac{1875(200\times10^9)(2.639\times10^{-6})\Delta^2}{32(6.25)^3}$$

$$126{,}576\Delta^2 - 98.1\Delta - 9.81 = 0$$

$$\Delta = \frac{98.1+\sqrt{(98.1)^2+4(126{,}576)(9.81)}}{2(126{,}576)} = 0.009198 \text{ m} \cong 9.20 \text{ mm}$$ .... **Ans.**

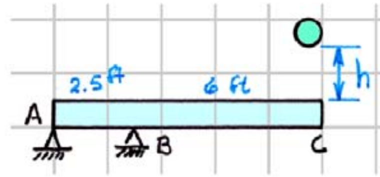
(b) $$P = \frac{1875EI\Delta}{16L^3} = \frac{1875(200\times10^9)(2.637\times10^{-6})(0.009198)}{16(6.25)^3} = 2328 \text{ N}$$

$$\sigma = \frac{Mc}{I} = \frac{(4P/5)(1.25)c}{I} = \frac{[4(2328)/5](1.25)(0.0375)}{2.637(10^{-6})}$$

$$\sigma = 33.10(10^6) \text{ N/m}^2 = 33.10 \text{ MPa}$$ .... **Ans.**

10-31*

$$I = \frac{bh^3}{12} = \frac{(24)(2)^3}{12} = 16 \text{ in.}^4$$

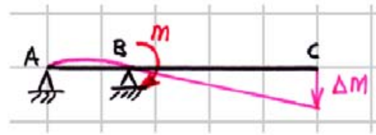


By superposition (with $M = 72P$):

$$\Delta = \Delta_M + \Delta_P$$

$$= \frac{(72P)(30)}{3(1.8\times10^6)(16)}(72) + \frac{P(72)^3}{3(1.8\times10^6)(16)} = 0.00612P$$

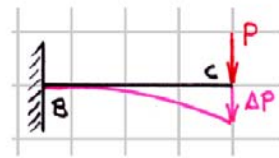
or $P = 163.40\Delta$



$$W(h+\Delta) = P\Delta/2 = 81.70\Delta^2$$

$$145(24+\Delta) = 81.70\Delta^2$$

$$81.70\Delta^2 - 145\Delta - 3480 = 0$$

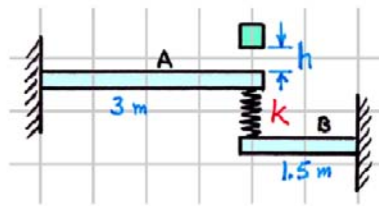


$$\Delta = \frac{145 + \sqrt{(145)^2 + 4(81.70)(3480)}}{2(81.70)} = 7.4739 \text{ in.}$$

$$P = 163.40\Delta = 163.40(7.4739) = 1221.2 \text{ lb}$$

$$\sigma = \frac{Mc}{I} = \frac{(72)(1221.2)(1.0)}{16} = 5495.4 \text{ psi} \cong 5.50 \text{ ksi}$$ .... **Ans.**

10-32



$$I = \frac{bh^3}{12} = \frac{(100)(25)^3}{12} = 130.2(10^3)\ \text{mm}^4$$

$$P = 4W = 4mg = 4(5)(9.81) = 192.20\ \text{N}$$

$$\Delta_{SP} = \frac{R}{k} = \frac{R}{20{,}000} = 50(10^{-6})R$$

$$\Delta_A = \frac{(P-R)L^3}{3EI} = \frac{(P-R)(3.0)^3}{3(70\times10^9)(130.2\times10^{-9})} = 987.49(10^{-6})(P-R)$$

$$\Delta_B = \frac{RL^3}{3EI} = \frac{R(1.5)^3}{3(70\times10^9)(130.2\times10^{-9})} = 123.44(10^{-6})R$$

$$\Delta_A - \Delta_B = \Delta_{SP} \quad \text{gives} \quad 987.49(P-R) - 123.44R = 50R$$

$$R = 0.8506P = 0.8506(196.20) = 166.89\ \text{N}$$

$$P - R = 196.20 - 166.89 = 29.31\ \text{N}$$

$$\Delta_{SP} = 50(10^{-6})R = 50(10^{-6})(166.89) = 0.008344\ \text{m}$$

$$\Delta_A = 987.49(10^{-6})(P-R) = 987.49(10^{-6})(29.31) = 0.02894\ \text{m}$$

$$\Delta_B = 123.44(10^{-6})R = 123.44(10^{-6})(166.89) = 0.02060\ \text{m}$$

$$\Delta = \Delta_A = 0.02894\ \text{m} = 28.94\ \text{mm}$$

(a) $W(h+\Delta) = P\Delta/2$

$$h = \frac{P\Delta}{2W} - \Delta = \frac{(4W)\Delta}{2W} - \Delta = \Delta = 28.9\ \text{mm}$$ .... **Ans.**

(b) $$\sigma_{\max,A} = \frac{Mc}{I} = \frac{(P-R)Lc}{I} = \frac{(29.31)(3)(0.0125)}{130.2(10^{-9})}$$

$$\sigma_{\max,A} = 8.44(10^6)\ \text{N/m}^2 = 8.44\ \text{MPa}$$ .... **Ans.**

$$\sigma_{\max,B} = \frac{Mc}{I} = \frac{(RL)c}{I} = \frac{(166.89)(1.5)(0.0125)}{130.2(10^{-9})}$$

$$\sigma_{\max,B} = 24.0(10^6)\ \text{N/m}^2 = 24.0\ \text{MPa}$$ .... **Ans.**

10-33

$\circlearrowleft \Sigma M_B = 0: \quad P_S(L/2) - PL = 0 \qquad P_S = 2P$

$$\Delta_b = \frac{PL^3}{3EI} \qquad \Delta_S = \frac{P_S}{k} = \frac{2P}{6EI/L^3} = \frac{PL^3}{3EI}$$

$$\Delta = \Delta_b + 2\Delta_S = \frac{PL^3}{3EI} + \frac{2PL^3}{3EI} = \frac{PL^3}{EI} \qquad P = (IF)W = 4W$$

$$W(h+\Delta) = P\Delta/2 = (4W)\Delta/2 = 2W\Delta$$

$$h = \frac{2W\Delta}{W} - \Delta = \Delta = \frac{PL^3}{EI} = \frac{4WL^3}{EI}$$ .... **Ans.**

10-34

$$I = \frac{bh^3}{12} = \frac{(150)(50)^3}{12} = 1.5625\left(10^6\right)\ \text{mm}^4$$

$$\sigma = \frac{Mc}{I} = \frac{RLc}{I} \qquad R = \frac{\sigma I}{Lc} = \frac{\left(8\times10^6\right)\left(1.5625\times10^{-6}\right)}{(1.5)(0.025)} = 333.33\ \text{N}$$

$$\Delta_B = \frac{RL^3}{3EI} = \frac{(333.33)(1.5)^3}{3\left(8\times10^9\right)\left(1.5625\times10^{-6}\right)} = 0.0300\ \text{m} = 30.0\ \text{mm}$$

$$\Delta_B = \Delta_A = \frac{(P-R)L^3}{3EI}$$

$$P = \frac{3EI\Delta_A}{L^3} + R = \frac{3\left(8\times10^9\right)\left(1.5625\times10^{-6}\right)(0.0300)}{(3)^3} + 333.33 = 375.0\ \text{N}$$

(a) $W(h+\Delta) = P\Delta/2$

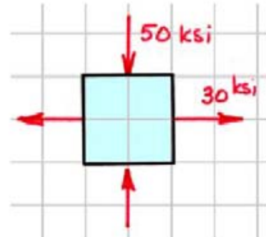
$$h = \frac{P\Delta}{2W} - \Delta = \frac{P\Delta}{2mg} - \Delta = \frac{(375.0)(0.0300)}{2(9\times9.81)} - 0.0300$$

$h = 0.0337\ \text{m} = 33.7\ \text{mm}$ .... **Ans.**

(b) $IF = \dfrac{P}{W} = \dfrac{P}{mg} = \dfrac{375.0}{(9\times9.81)} = 4.25$ .... **Ans.**

10-35*

$\sigma_f = 60$ ksi $\sigma_{p1} = 30$ ksi (T) $\sigma_{p2} = 0$ ksi $\sigma_{p3} = 50$ ksi (C)



Maximum-Normal-Stress:

$$\sigma_{max} = \sigma_{p3} = 50 \text{ ksi} < \sigma_f = 60 \text{ ksi}$$

No failure...**Ans.**

Maximum-Shear-Stress:

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (30 + 50)/2 = 40 \text{ ksi}$$

$$\tau_f = \sigma_f/2 = 60/2 = 30 \text{ ksi}$$

$\tau_{max} = 40 \text{ ksi} > \tau_f = 30 \text{ ksi}$ **Failure** .... **Ans.**

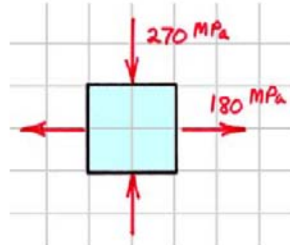
Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sqrt{(30)^2 - (30)(-50) + (-50)^2} = 70 \text{ ksi}$$

$\sigma_e = 70 \text{ ksi} > \sigma_f = 60 \text{ ksi}$ **Failure** .... **Ans.**

10-36*

$\sigma_f = 380$ MPa $\sigma_{p1} = 180$ MPa (T) $\sigma_{p2} = 0$ MPa $\sigma_{p3} = 270$ MPa (C)



Maximum-Normal-Stress:

$$\sigma_{max} = \sigma_{p3} = 270 \text{ MPa} < \sigma_f = 380 \text{ MPa}$$

No failure..**Ans.**

Maximum-Shear-Stress:

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (180 + 270)/2 = 225 \text{ MPa}$$

$$\tau_f = \sigma_f/2 = 380/2 = 190 \text{ MPa}$$

$$\tau_{max} = 225 \text{ MPa} > \tau_f = 190 \text{ MPa}$$

Failure.. **Ans.**

Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sqrt{(180)^2 - (180)(-270) + (-270)^2} = 392.3 \text{ MPa}$$

$$\sigma_e = 392.3 \text{ MPa} > \sigma_f = 380 \text{ MPa}$$

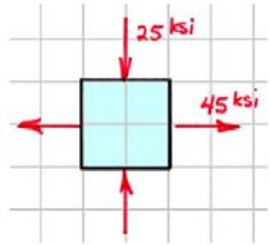
Failure.. **Ans.**

10-37

$\sigma_{p1} = 45$ ksi (T) $\qquad \sigma_{p2} = 0$ ksi $\qquad \sigma_{p3} = 25$ ksi (C)

Maximum-Normal-Stress:

$\sigma_{PL} > \sigma_{max} = \sigma_{p1} = 45$ ksi .... **Ans.**



Maximum-Shear-Stress:

$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (45 + 25)/2 = 35$ ksi

$\sigma_{PL} > 2\tau_f = 2\tau_{max} = 2(35) = 70$ ksi .... **Ans.**

Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sqrt{(45)^2 - (45)(-25) + (-25)^2} = 61.4 \text{ ksi}$$

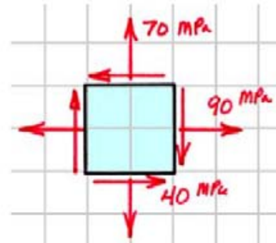
$\sigma_{PL} > \sigma_e = 61.4$ ksi .... **Ans.**

10-38*

$\sigma_f = 250$ MPa $\quad \sigma_x = 90$ MPa $\quad \sigma_y = 70$ MPa $\quad \tau_{xy} = -40$ MPa

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{90+70}{2} \pm \sqrt{\left(\frac{90-70}{2}\right)^2 + (-40)^2} = 80 \pm 41.231$$



$$\sigma_{p1} = 80 + 41.231 = 121.231 \text{ MPa}$$

$$\sigma_{p2} = 80 - 41.231 = 38.769 \text{ MPa}$$

$$\sigma_{p3} = 0 \text{ MPa}$$

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (121.231 - 0)/2 = 60.6 \text{ MPa}$$

Maximum-Normal-Stress:

$\sigma_{max} = \sigma_{p1} = 121.2 \text{ MPa} < \sigma_f = 250 \text{ MPa}$ **No failure.. Ans.**

Maximum-Shear-Stress:

$$\tau_f = \sigma_f/2 = 250/2 = 125 \text{ MPa}$$

$\tau_{max} = 60.6 \text{ MPa} < \tau_f = 125 \text{ MPa}$ **No failure .... Ans.**

Maximum-Distortion-Energy:

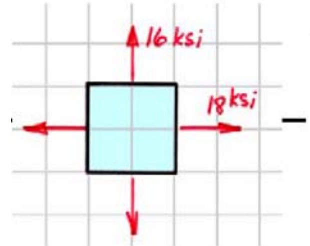
$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2} = \sqrt{(121.231)^2 - (121.231)(38.769) + (38.769)^2} = 107.2 \text{ MPa}$$

$\sigma_e = 107.2 \text{ MPa} < \sigma_f = 250 \text{ MPa}$ **No failure .... Ans.**

10-39

$\sigma_f = 36$ ksi $\qquad \sigma_{p1} = 18$ ksi (T) $\qquad \sigma_{p2} = 16$ ksi (T) $\qquad \sigma_{p3} = 0$ ksi

Maximum-Normal-Stress:



$$FS = \frac{\sigma_f}{\sigma_{max}} = \frac{\sigma_f}{\sigma_{p1}} = \frac{36}{18} = 2.00$$ ....**Ans.**

Maximum-Shear-Stress:

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (18 - 0)/2 = 9 \text{ ksi}$$

$$\tau_f = \sigma_f/2 = 36/2 = 18 \text{ ksi}$$

$$FS = \frac{\tau_f}{\tau_{max}} = \frac{18}{9} = 2.00$$ .... **Ans.**

Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2} = \sqrt{(18)^2 - (18)(16) + (16)^2} = 17.09 \text{ ksi}$$

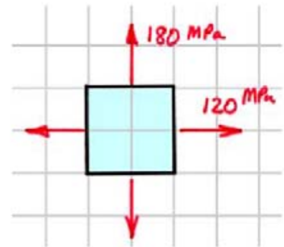
$$FS = \frac{\sigma_f}{\sigma_e} = \frac{36}{17.09} = 2.11$$ .... **Ans.**

10-40

$\sigma_{p1} = 120$ MPa (T) $\qquad \sigma_{p2} = 0$ MPa $\qquad \sigma_{p3} = 180$ MPa (C)

Maximum-Normal-Stress:

$\sigma_{PL} > \sigma_{max} = \sigma_{p3} = 180$ MPa ....**Ans.**



Maximum-Shear-Stress:

$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (120 + 180)/2 = 150.0$ MPa

$\sigma_{PL} > 2\tau_f = 2\tau_{max} = 2(150.0) = 300$ MPa ....**Ans.**

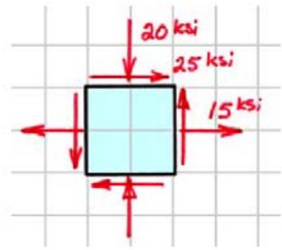
Maximum-Distortion-Energy:

$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sqrt{(120)^2 - (120)(-180) + (-180)^2} = 264$ MPa

$\sigma_{PL} > \sigma_e = 264$ MPa .... **Ans.**

10-41*

$\sigma_f = 60$ ksi $\qquad \sigma_x = 15$ ksi $\qquad \sigma_y = -20$ ksi $\qquad \tau_{xy} = 25$ ksi



$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{15-20}{2} \pm \sqrt{\left(\frac{15+20}{2}\right)^2 + (25)^2} = -2.50 \pm 30.516$$

$$\sigma_{p1} = -2.50 + 30.516 = 28.016 \text{ ksi}$$

$$\sigma_{p3} = -2.50 - 30.516 = -33.016 \text{ ksi}$$

$$\sigma_{p2} = 0 \text{ ksi} \qquad \tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (28.016 + 33.016)/2 = 30.516 \text{ ksi}$$

Maximum-Normal-Stress:

$$FS = \frac{\sigma_f}{\sigma_{max}} = \frac{\sigma_f}{\sigma_{p3}} = \frac{60}{33.016} = 1.817$$ .... **Ans.**

Maximum-Shear-Stress:

$$\tau_f = \sigma_f/2 = 60/2 = 30 \text{ ksi}$$

$$FS = \frac{\tau_f}{\tau_{max}} = \frac{30}{30.516} = 0.983 \quad \text{(failure)}$$ .... **Ans.**

Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sqrt{(28.016)^2 - (28.016)(-33.016) + (-33.016)^2} = 52.914 \text{ ksi}$$

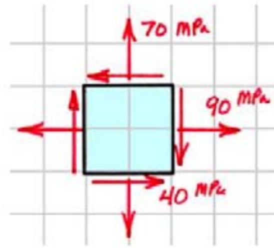
$$FS = \frac{\sigma_f}{\sigma_e} = \frac{60}{52.914} = 1.134$$ .... **Ans.**

10-42

$\sigma_f = 250\text{ MPa}$ $\sigma_x = 140\text{ MPa}$ $\sigma_y = 95\text{ MPa}$ $\tau_{xy} = -80\text{ MPa}$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{140+95}{2} \pm \sqrt{\left(\frac{140-95}{2}\right)^2 + (-80)^2} = 117.5 \pm 83.10$$



$\sigma_{p1} = 117.5 + 83.10 = 200.60\text{ MPa}$

$\sigma_{p2} = 117.5 - 83.10 = 34.40\text{ MPa}$

$\sigma_{p3} = 0\text{ MPa}$

$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (200.60 - 0)/2 = 100.3\text{ MPa}$

Maximum-Normal-Stress:

$$FS = \frac{\sigma_f}{\sigma_{max}} = \frac{\sigma_f}{\sigma_{p1}} = \frac{250}{200.6} = 1.246$$ .... **Ans.**

Maximum-Shear-Stress:

$\tau_f = \sigma_f/2 = 250/2 = 125\text{ MPa}$

$$FS = \frac{\tau_f}{\tau_{max}} = \frac{125}{100.3} = 1.246$$ .... **Ans.**

Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2} = \sqrt{(200.6)^2 - (200.6)(34.4) + (34.4)^2} = 185.8\text{ MPa}$$

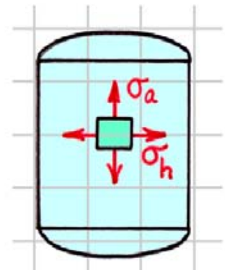
$$FS = \frac{\sigma_f}{\sigma_e} = \frac{250}{185.8} = 1.346$$ .... **Ans.**

10-43*

$$\sigma_h = \sigma_{p1} = \frac{pr}{t} = \frac{p(30)}{1.5} = 20p \qquad \sigma_r = \sigma_{p3} = 0 \qquad \text{(Outside surface)}$$

$$\sigma_a = \sigma_{p2} = \frac{pr}{2t} = \frac{p(30)}{2(1.5)} = 10p \qquad \sigma_r = \sigma_{p3} = -p \qquad \text{(Inside surface)}$$

(a) Maximum-Shear-Stress $\tau_f = \sigma_f/2 = 36/2 = 18$ ksi



Outside surface:

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (20p - 0)/2 = 10p$$

$$p = \frac{\tau_{max}}{10} = \frac{\tau_f}{10} = \frac{18}{10} = 1.800 \text{ ksi} = 1800 \text{ psi}$$

Inside surface:

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (20p + p)/2 = 10.5p$$

$$p = \frac{\tau_{max}}{10.5} = \frac{\tau_f}{10.5} = \frac{18}{10.5} = 1.714 \text{ ksi} = 1714 \text{ psi}$$ .... **Ans.**

(b) Maximum-Distortion-Energy

Outside surface: $\sigma_f = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2}$

$$36 \text{ ksi} = \sqrt{(20p)^2 - (20p)(10p) + (10p)^2} = 17.321p$$

$$p = 2.078 \text{ ksi} = 2078 \text{ psi}$$

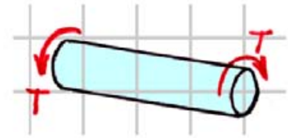
Inside surface: $2\sigma_f^2 = (\sigma_{p1} - \sigma_{p2})^2 + (\sigma_{p2} - \sigma_{p3})^2 + (\sigma_{p3} - \sigma_{p1})^2$

$$2(36)^2 = (20p - 10p)^2 + (10p + p)^2 + (-p - 20p)^2 = 662p^2$$

$$p = 1.979 \text{ ksi} = 1979 \text{ psi}$$ .... **Ans.**

10-44*

$$\sigma_{p1} = -\sigma_{p3} = \tau_{\max} = \tau_{xy} = \frac{Tc}{J} = \frac{Tc}{\pi c^4/2} = \frac{2T}{\pi c^3} = \frac{2T}{\pi(0.050)^3} = 5093T$$



(a) Maximum-Shear-Stress: $\tau_f = \sigma_f/2 = 400/2 = 200$ MPa

$$T = \frac{\tau_f}{5093} = \frac{200(10^6)}{5093} = 39.3(10^3)\ \text{N}\cdot\text{m} = 39.3\ \text{kN}\cdot\text{m}$$ .... **Ans.**

(b) Maximum-Distortion-Energy:

$$\sigma_f = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2}$$

$$400(10^6) = \sqrt{(5093T)^2 - (5093T)(-5093T) + (-5093T)^2} = 8821T$$

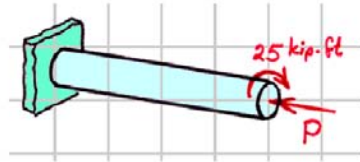
$$T = \frac{400(10^6)}{8821} = 45.3(10^3)\ \text{N}\cdot\text{m} = 45.3\ \text{kN}\cdot\text{m}$$ .... **Ans.**

10-45

$$\sigma_x = \frac{-P}{A} = \frac{-P}{\pi c^2} = \frac{-P}{\pi(3)^2} = -0.3183P$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{Tc}{\pi c^4/2} = \frac{2T}{\pi c^3} = \frac{2(25{,}000 \times 12)}{\pi(3)^3} = 7074 \text{ psi}$$

$$\sigma_y = 0$$



Maximum-Shear-Stress: $$\tau_f = \frac{1}{2}\left(\frac{\sigma_f}{FS}\right) = \frac{60}{2(3.0)} = 10 \text{ ksi}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-0.3183P - 0}{2}\right)^2 + (7074)^2} = 10.00 \text{ ksi}$$

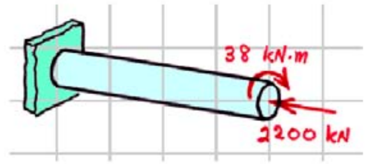
$P = 44.4$ kip .... **Ans.**

10-46

$$\sigma_x = \frac{-P}{A} = \frac{-P}{\pi c^2} = \frac{-\left(2200\times10^3\right)}{\pi\left(0.075\right)^2} = -124.49\left(10^6\right)\text{ N/m}^2 = 124.49\text{ MPa (C)}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{Tc}{\pi c^4/2} = \frac{2T}{\pi c^3} = \frac{2\left(38,000\right)}{\pi\left(0.075\right)^3} = 57.34\left(10^6\right)\text{ N/m}^2 = 57.34\text{ MPa}$$

$$\sigma_y = 0$$



$$\sigma_{p1,p3} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \frac{-124.49+0}{2} \pm \sqrt{\left(\frac{-124.49-0}{2}\right)^2+\left(57.34\right)^2} = -62.25 \pm 84.63$$

$$\sigma_{p1} = -62.25+84.63 = 22.38\text{ MPa} \qquad \sigma_{p2} = 0\text{ MPa}$$

$$\sigma_{p3} = -62.25-84.63 = -146.88\text{ MPa}$$

$$\tau_{max} = \left(\sigma_{max}-\sigma_{min}\right)/2 = \left(22.38+146.88\right)/2 = 84.63\text{ MPa}$$

Maximum-Shear-Stress: $\tau_f = \sigma_f/2 = 360/2 = 180\text{ MPa}$

$$FS = \frac{\tau_f}{\tau_{max}} = \frac{180}{84.63} = 2.13$$ .... **Ans.**

Maximum-Distortion-Energy:

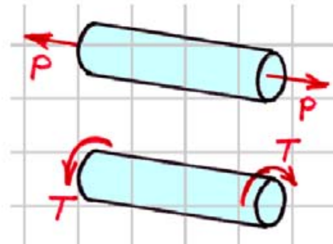
$$\sigma_e = \sqrt{\sigma_{p1}^2-\sigma_{p1}\sigma_{p3}+\sigma_{p3}^2} = \sqrt{\left(22.38\right)^2-\left(22.38\right)\left(-146.88\right)+\left(-146.88\right)^2} = 159.25\text{ MPa}$$

$$FS = \frac{\sigma_f}{\sigma_e} = \frac{360}{159.25} = 2.26$$ .... **Ans.**

10-47

For the tension test: $\sigma_{max} = \sigma_{p1} = \sigma_x = \sigma_f = \sigma_y$ $\quad (\sigma_y \;-\; \text{yield strength})$

$\tau_{max} = \sigma_{max}/2 = \sigma_f/2 = \sigma_y/2$



For the torsion test: $\sigma_{max} = \sigma_{p1} = -\sigma_{p3} = \tau_{max} = \tau_{xy} = \tau_f = \tau_y$

(a) Maximum-Normal-Stress: $\sigma_{max} = \sigma_f = \sigma_y$

Therefore: $\tau_y = \sigma_y$ .... **Ans.**

(b) Maximum-Shear-Stress: $\tau_{max} = \sigma_f/2 = \sigma_y/2$

Therefore: $\tau_y = \sigma_y/2 = 0.5\sigma_y$ .... **Ans.**

(c) Maximum-Distortion-Energy:

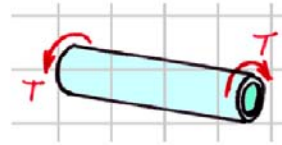
$$\sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sigma_f = \sigma_y$$

$$\sqrt{(\tau_y)^2 - (\tau_y)(-\tau_y) + (-\tau_y)^2} = \sigma_y \qquad 1.7321\tau_y = \sigma_y$$

Therefore: $\tau_y = \sigma_y/1.7321 = 0.577\sigma_y$ .... **Ans.**

10-48*

$$\tau_{max} = \frac{1}{2}\left(\frac{\sigma_y}{FS}\right) = \frac{250}{2(1.5)} = 83.33 \text{ MPa}$$



$$\tau_{max} = \tau_{xy} = \frac{Tc}{J} = \frac{(40{,}000)(0.075)}{J} = \frac{3000}{J} = 83.33\left(10^6\right) \text{ N/m}^2$$

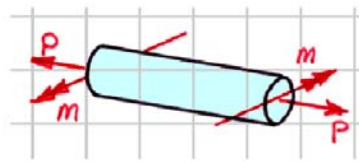
$$J = \frac{\pi\left[(0.150)^4 - d_i^4\right]}{32} = \frac{3000}{83.33\left(10^6\right)}$$

$d_i = 0.1087 \text{ m} = 108.7 \text{ mm}$ .... **Ans.**

10-49

$$\sigma_{max} = \frac{\sigma_y}{FS} = \frac{36}{2} = 18 \text{ ksi} \qquad \tau_{max} = \frac{1}{2}\left(\frac{\sigma_y}{FS}\right) = \frac{36}{2(2)} = 9 \text{ ksi}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{10}{\pi c^2} + \frac{5c}{\pi c^4/4} = \frac{10}{\pi c^2} + \frac{20}{\pi c^3} = 18 \text{ ksi}$$



$$18\pi c^3 - 10c - 20 = 0$$

$$c = 0.7902 \text{ in.}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1}{2}\left[\frac{10}{\pi c^2} + \frac{20}{\pi c^3}\right] = 9 \text{ ksi}$$

$$18\pi c^3 - 10c - 20 = 0$$

(But this is the same cubic equation and it has the same solution.)

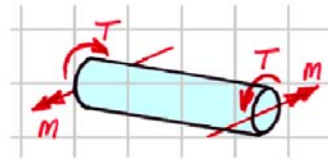
Therefore:

$d_{min} = 2c = 2(0.7902) = 1.580$ in. .... **Ans.**

10-50

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} \qquad \tau_{xy} = \frac{Tc}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{\sigma_y}{2(FS)}$$



Therefore: $\frac{\sigma_y}{2(FS)} = \frac{2}{\pi d^3}\sqrt{(8M)^2 + (8T)^2}$

Solving for d yields: $d = \left[\frac{32(FS)}{\pi \sigma_y}\sqrt{M^2 + T^2}\right]^{1/3}$ .... **Ans.**

10-51

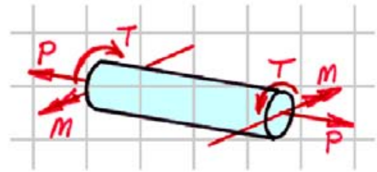
$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{\pi d^2/4} + \frac{M(d/2)}{\pi d^4/64} = \frac{4P}{\pi d^2} + \frac{32M}{\pi d^3}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3}$$

$$\tau_{max} = \frac{1}{2}\left[\frac{\sigma_y}{(FS)}\right] = \frac{\sigma_y}{2(FS)}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{2P}{\pi d^2} + \frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{\sigma_y}{2(FS)}$$



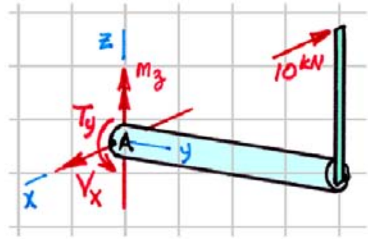
Therefore: $$\frac{\sigma_y}{2(FS)} = \left[\frac{2}{\pi d^3}\sqrt{(Pd + 8M)^2 + (8T)^2}\right]$$ .... **Ans.**

10-52*

$$V_x = 10 \text{ kN}$$

$$T_y = (10)(0.6) = 6.0 \text{ kN}\cdot\text{m}$$

$$M_z = -(10)(0.4) = -4.0 \text{ kN}\cdot\text{m}$$



$$\sigma_{xA} = \frac{Mc}{I} = \frac{(4000)(D/2)}{\pi D^4/64} = \frac{40{,}744}{D^3}$$

$$\tau_{xyA} = \frac{Tc}{J} = \frac{(6000)(D/2)}{\pi D^4/32} = \frac{30{,}558}{D^3}$$

$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{40{,}744}{2D^3} \pm \sqrt{\left(\frac{40{,}744}{2D^3}\right)^2 + \left(\frac{30{,}558}{D^3}\right)^2} = \frac{20{,}372}{D^3} \pm \frac{36{,}726}{D^3}$$

$$\sigma_{p1} = \frac{20{,}372}{D^3} + \frac{36{,}726}{D^3} = \frac{57{,}098}{D^3} \qquad \sigma_{p3} = \frac{20{,}372}{D^3} - \frac{36{,}726}{D^3} = \frac{-16{,}354}{D^3}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1}{2}\left[\left(\frac{57{,}098}{D^3}\right) - \left(\frac{-16{,}354}{D^3}\right)\right] = \frac{36{,}726}{D^3} \qquad \sigma_{p2} = 0 \text{ MPa}$$

(a) Maximum-Shear-Stress:

$$\tau_{max} = \frac{36{,}726}{D^3} = \frac{\sigma_f}{2(FS)} = \frac{360(10^6)}{2(2.0)}$$

$D = 0.0742 \text{ m} = 74.2 \text{ mm}$ .... **Ans.**

(b) Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \frac{\sigma_f}{(FS)} = \frac{360}{2.0} = 180 \text{ MPa}$$

$$\sqrt{\left(\frac{57{,}098}{D^3}\right)^2 - \left(\frac{57{,}098}{D^3}\right)\left(\frac{-16{,}354}{D^3}\right) + \left(\frac{-16{,}354}{D^3}\right)^2} = \frac{66{,}794}{D^3} = 180(10^6) \text{ N/m}^2$$

$D = 0.0719 \text{ m} = 71.9 \text{ mm}$ .... **Ans.**

10-53*

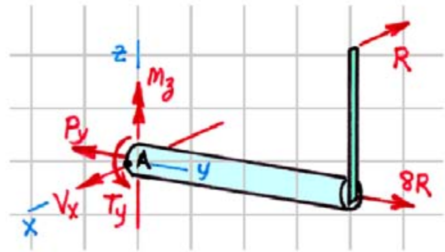
$$A = \pi c^2 = \pi(2)^2 = 4\pi \text{ in.}^2 \qquad I = \pi c^4/4 = \pi(2)^4/4 = 4\pi \text{ in.}^4$$

$$J = \frac{\pi c^4}{2} = \frac{\pi(2)^4}{2} = 8\pi \text{ in.}^4 \qquad \tau_{\max} = \frac{1}{2}\left(\frac{\sigma_f}{FS}\right) = \frac{\sigma_y}{2FS} = \frac{48}{2(2)} = 12 \text{ ksi}$$

$$P_y = (8R) \text{ kip} \qquad V_x = (R) \text{ kip}$$

$$T_y = (R)(20) = (20R) \text{ kip}\cdot\text{in.}$$

$$M_z = -(R)(2\times 12) = (-24R) \text{ kip}\cdot\text{in.}$$



$$\sigma_{xA} = \frac{P}{A} + \frac{Mc}{I} = \frac{8R}{4\pi} + \frac{(24R)(2)}{4\pi} = (4.4563R) \text{ ksi}$$

$$\tau_{xyA} = \frac{Tc}{J} = \frac{(20R)(2)}{8\pi} = (1.5915R) \text{ ksi}$$

$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4.4563R}{2} \pm \sqrt{\left(\frac{4.4563R}{2}\right)^2 + (1.5915R)^2} = 2.228R \pm 2.738R$$

$$\sigma_{p1} = 2.228R + 2.738R = (4.966R) \text{ ksi}$$

$$\sigma_{p3} = 2.228R - 2.738R = (-0.510R) \text{ ksi}$$

$$\sigma_{p2} = 0 \text{ ksi} \qquad \tau_{\max} = (\sigma_{\max} - \sigma_{\min})/2 = (4.966R + 0.510R)/2 = (2.738R) \text{ ksi}$$

(a) Maximum-Shear-Stress:

$$\tau_{\max} = (2.738R) \text{ ksi} = 12 \text{ ksi}$$

$$R = \frac{12}{5.476} = 4.38 \text{ kip}$$ .... **Ans.**

(b) Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sigma_y/FS = 48/2.0 = 24 \text{ ksi}$$

$$\sqrt{(4.966R)^2 - (4.966R)(-0.510R) + (-0.510R)^2} = 5.2396R = 24 \text{ ksi}$$

$$R = \frac{24}{5.2396} = 4.58 \text{ kip}$$ .... **Ans.**

10-54

$$\sigma_h = \sigma_{p1} = \frac{pr}{t} = \frac{(5.5\times10^6)(0.150)}{t} = \frac{825,000}{t}$$

$$\sigma_{p3} = -p = -5.5 \text{ MPa}$$

$$\sigma_a = \sigma_{p2} = 0$$

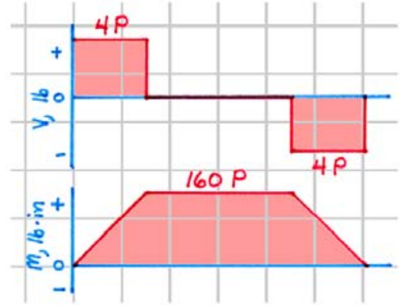
$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_f}{2(FS)} = \frac{250}{2(4)} = 31.25 \text{ MPa}$$

$$\frac{1}{2}\left[\frac{825,000}{t} - \left(-5.5\times10^6\right)\right] = 31.25\left(10^6\right) \text{ N/m}^2$$

$$t = \frac{825,000}{57\left(10^6\right)} = 0.01447 \text{ m} = 14.47 \text{ mm}$$ .... **Ans.**

10-55*

$$I = \frac{\pi c^4}{4} = \frac{\pi (2)^4}{4} = 4\pi \text{ in.}^4 \qquad J = \frac{\pi c^4}{2} = \frac{\pi (2)^4}{2} = 8\pi \text{ in.}^4$$



$$\sigma_x = \frac{Mc}{I} = \frac{(160P)(2)}{4\pi} = (25.465P) \text{ ksi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(24P)(2)}{8\pi} = (1.910P) \text{ ksi}$$

$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{25.465P}{2} \pm \sqrt{\left(\frac{25.465P}{2}\right)^2 + (1.910P)^2} = 12.733P \pm 12.875P$$

$$\sigma_{p1} = 12.733P + 12.875P = (25.608P) \text{ ksi}$$

$$\sigma_{p3} = 12.733P - 12.875P = (-0.142P) \text{ ksi} \qquad \sigma_{p2} = 0 \text{ ksi}$$

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (25.608P + 0.142P)/2 = (12.875P) \text{ ksi}$$

(a) Maximum-Shear-Stress:

$$\tau_{max} = \tau_f / (FS) = 23/2.5 = 9.20 \text{ ksi}$$

$$P = \frac{9.20}{12.875} = 0.715 \text{ kip} = 715 \text{ lb}$$ .... **Ans.**

(b) Maximum-Distortion-Energy:

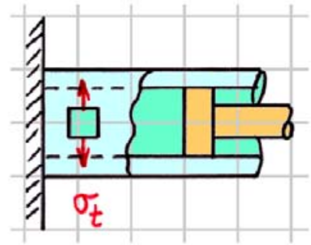
$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p3} + \sigma_{p3}^2} = \sigma_y / FS = 40/2.5 = 16.00 \text{ ksi}$$

$$\sqrt{(25.608P)^2 - (25.608P)(-0.142P) + (-0.142P)^2} = (25.68P) \text{ ksi}$$

$$P = \frac{16.00}{25.68} = 0.623 \text{ kip} = 623 \text{ lb}$$ .... **Ans.**

10-56*

$$\sigma_t = \sigma_{p1} = \frac{a^2 p}{b^2 - a^2}\left[1 + \frac{b^2}{a^2}\right] = \frac{b^2 + a^2}{b^2 - a^2} p \qquad \sigma_r = \sigma_{p3} = -p$$



$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{1}{2}\left[\frac{b^2 + a^2}{b^2 - a^2} p + p\right] = \frac{b^2 p}{b^2 - a^2} = \frac{\sigma_y}{2(FS)}$$

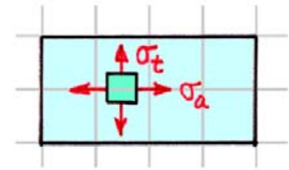
$$\frac{b^2\left(50\times 10^6\right)}{b^2 - (0.075)^2} = \frac{275\left(10^6\right)}{2(2)}$$

$$b = 0.14361 \text{ m} = 143.61 \text{ mm}$$

$d_o = 2b = 2(143.61) = 287$ mm .... **Ans.**

10-57

$$\sigma_t = \sigma_{p1} = \frac{a^2 p}{b^2 - a^2}\left[1 + \frac{b^2}{a^2}\right] = \frac{(3)^2 p}{(5)^2 - (3)^2}\left[1 + \frac{(5)^2}{(3)^2}\right] = 2.1250p$$



$$\sigma_a = \sigma_{p2} = \frac{P}{A} = \frac{p\pi a^2}{\pi\left(b^2 - a^2\right)} = \frac{p\pi(3)^2}{\pi\left(5^2 - 3^2\right)} = 0.5625p$$

$$\sigma_r = \sigma_{p3} = -p$$

$$\left(\sigma_{p1} - \sigma_{p2}\right)^2 + \left(\sigma_{p2} - \sigma_{p3}\right)^2 + \left(\sigma_{p3} - \sigma_{p1}\right)^2 = 2\left(\frac{\sigma_y}{FS}\right)^2 = 2\left(\frac{80}{2.5}\right)^2 = 2048$$

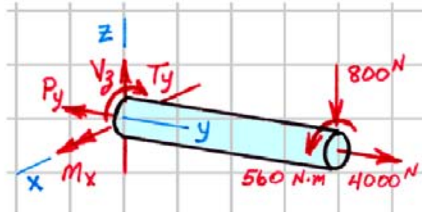
$$\left(2.1250p - 0.5625p\right)^2 + \left(0.5625p + p\right)^2 + \left(-p - 2.1250p\right)^2 = 14.648p^2 = 2048$$

$p = 11.82$ ksi .... **Ans.**

10-58

$P_y = 4000\text{ N} \qquad V_z = 800\text{ N}$

$T_y = 560\text{ N}\cdot\text{m} \qquad M_x = (800)(0.200) = 160\text{ N}\cdot\text{m}$



$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{\pi d^2/4} + \frac{M(d/2)}{\pi d^4/64} = \frac{4P}{\pi d^2} + \frac{32M}{\pi d^3}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} \qquad \tau_{\max} = \frac{1}{2}\left[\frac{\sigma_y}{(FS)}\right] = \frac{\sigma_y}{2(FS)}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{2P}{\pi d^2} + \frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{\sigma_y}{2(FS)}$$

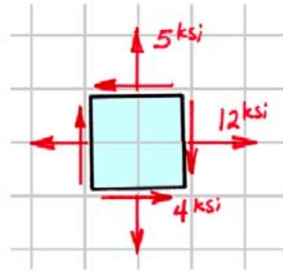
$$\frac{\sigma_y}{2(FS)} = \frac{2}{\pi d^3}\sqrt{(Pd + 8M)^2 + (8T)^2}$$

$$\frac{330(10^6)}{2(2.5)} = \frac{2}{\pi d^3}\sqrt{\left[4000d + 8(160)\right]^2 + \left[8(560)\right]^2}$$

$d = 0.0356\text{ m} = 35.6\text{ mm}$ .... **Ans.**

10-59*

$\sigma_x = 12$ ksi $\qquad \sigma_y = 5$ ksi $\qquad \tau_{xy} = -4$ ksi



$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{12+5}{2} \pm \sqrt{\left(\frac{12-5}{2}\right)^2 + (-4)^2} = 8.5 \pm 5.315$$

$\sigma_{p1} = 8.5 + 5.315 = 13.815$ ksi

$\sigma_{p2} = 8.5 - 5.315 = 3.185$ ksi $\qquad \sigma_{p3} = 0$ ksi

$$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{13.815}{26} - \frac{0}{97} = 0.531 \le 1$$

Safe .... **Ans.**

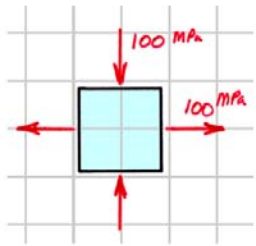
10-60*

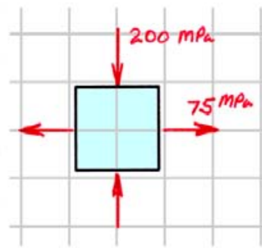
(a) $$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{100}{152} - \frac{-100}{572} = 0.833 \le 1$$

Safe ....Ans.

(b) $$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{75}{152} - \frac{-200}{572} = 0.843 \le 1$$

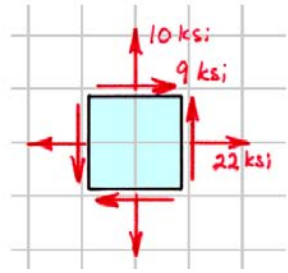
Safe ....Ans.





10-61

$\sigma_x = 22$ ksi $\qquad \sigma_y = 10$ ksi $\qquad \tau_{xy} = 9$ ksi



$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{22+10}{2} \pm \sqrt{\left(\frac{22-10}{2}\right)^2 + (9)^2} = 16.00 \pm 10.817$$

$$\sigma_{p1} = 16.00 + 10.817 = 26.817 \text{ ksi}$$

$$\sigma_{p2} = 16.00 - 10.817 = 5.183 \text{ ksi} \qquad \sigma_{p3} = 0 \text{ ksi}$$

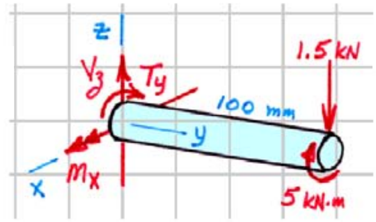
$$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{26.817}{30} - \frac{0}{108} = 0.894 \le 1$$

Safe.. **Ans.**

10-62*

$M_x = (1500)(0.1) = 150 \text{ N}\cdot\text{m}$ $\qquad T_y = 5000 \text{ N}\cdot\text{m}$ $\qquad V_z = 1500 \text{ N}$

$\sigma_x = \dfrac{Mc}{I} = \dfrac{(150)c}{\pi c^4/4} = \left(\dfrac{190.99}{c^3}\right) \text{ N/m}^2$ $\qquad \sigma_y = 0 \text{ N/m}^2$



$\sigma_{p3} = \dfrac{70.03}{c^3} - \dfrac{168.07}{c^3} = \left(\dfrac{-98.04}{c^3}\right) \text{ ksi}$

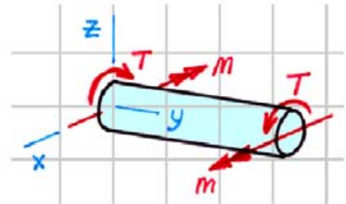
$\dfrac{\sigma_{p1}}{\sigma_{ut}} - \dfrac{\sigma_{p3}}{\sigma_{uc}} = \dfrac{238.10/c^3}{43} - \dfrac{-98.04/c^3}{140} = \dfrac{6.237}{c^3} \le 1$

$c = 1.8407$ in. $\qquad d_{min} = 2c = 3.68$ in. .... **Ans.**

10-63

$$\sigma_x = \frac{Mc}{I} = \frac{(110)c}{\pi c^4/4} = \left(\frac{140.06}{c^3}\right) \text{ ksi} \qquad \sigma_y = 0 \text{ ksi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{240c}{\pi c^4/2} = \left(\frac{152.79}{c^3}\right) \text{ ksi}$$



$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{70.03}{c^3} \pm \sqrt{\left(\frac{70.03}{c^3}\right)^2 + \left(\frac{152.79}{c^3}\right)^2} = \frac{70.03}{c^3} \pm \frac{168.07}{c^3}$$

$$\sigma_{p1} = \frac{70.03}{c^3} + \frac{168.07}{c^3} = \left(\frac{238.10}{c^3}\right) \text{ ksi} \qquad \sigma_{p2} = \sigma_z = 0 \text{ ksi}$$

$$\sigma_{p3} = \frac{70.03}{c^3} - \frac{168.07}{c^3} = \left(\frac{-98.04}{c^3}\right) \text{ ksi}$$

$$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{238.10/c^3}{43} - \frac{-98.04/c^3}{140} = \frac{6.237}{c^3} \le 1$$

$c = 1.8407$ in. $\qquad d_{min} = 2c = 3.68$ in. .... **Ans.**

10-64

$$\sigma_h = \sigma_{p1} = \frac{pr}{t} = \frac{p(0.150)}{0.005} = 30.00p$$

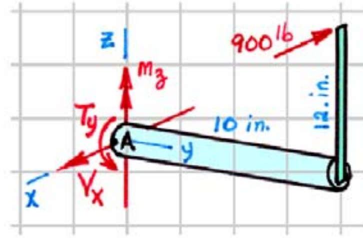
$$\sigma_a = \sigma_{p2} = \frac{pr}{2t} = \frac{p(0.150)}{2(0.005)} = 15.00p \qquad \sigma_r = \sigma_{p3} = -p$$

$$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{30p}{276} - \frac{-p}{340} = 0.11164p \le 1$$

$p \le 8.96$ MPa .... **Ans.**

10-65*

$A = \pi c^2 \qquad I = \pi c^4/4 \qquad J = \pi c^4/2$



$V_x = 900$ lb

$T_y = (900)(12) = 10{,}800$ lb·in.

$M_z = (900)(10) = 9000$ lb·in.

$$\sigma_x = \frac{Mc}{I} = \frac{(9000)c}{\pi c^4/4} = \left(\frac{11{,}459}{c^3}\right) \text{ psi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(10{,}800)c}{\pi c^4/2} = \left(\frac{6875}{c^3}\right) \text{ psi} \qquad \sigma_y = 0$$

$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{5730}{c^3} \pm \sqrt{\left(\frac{5730}{c^3}\right)^2 + \left(\frac{6875}{c^3}\right)^2} = \frac{5730}{c^3} \pm \frac{8949}{c^3}$$

$$\sigma_{p1} = \frac{5730}{c^3} + \frac{8949}{c^3} = \left(\frac{14{,}679}{c^3}\right) \text{ psi} \qquad \sigma_{p2} = 0$$

$$\sigma_{p3} = \frac{5730}{c^3} - \frac{8949}{c^3} = \left(\frac{-3219}{c^3}\right) \text{ psi}$$

$$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{14.679/c^3}{36.5} - \frac{-3.219/c^3}{124} = \frac{0.4281}{c^3} \le 1$$

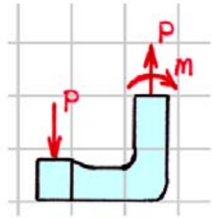
$c = 0.7537$ in.

$d_{min} = 2c = 1.507$ in. .... **Ans.**

10-66

$A = (12 \times 25) = 300 \text{ mm}^2 \qquad I = (12)(25)^3 / 12 = 15{,}625 \text{ mm}^4$

$M = P(0.075) \text{ N}\cdot\text{m}$



$$\sigma_i = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{300(10^{-6})} + \frac{(0.075P)(0.0125)}{15{,}625(10^{-12})}$$

$$= (63{,}333P) \text{ N/m}^2$$

$$\sigma_o = \frac{P}{A} - \frac{Mc}{I} = \frac{P}{300(10^{-6})} - \frac{(0.075P)(0.0125)}{15{,}625(10^{-12})}$$

$$= (-56{,}667P) \text{ N/m}^2$$

$$\frac{\sigma_{p1}}{\sigma_{ut}} = \frac{63{,}333P}{180(10^6)} \le 1 \qquad P \le 2842 \text{ N}$$

$$\frac{\sigma_{p3}}{\sigma_{uc}} = \frac{56{,}667P}{670(10^6)} \le 1 \qquad P \le 11{,}823 \text{ N}$$

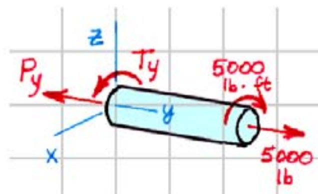
$P_{\max} = 2842 \text{ N} \cong 2.84 \text{ kN}$ .... **Ans.**

10-67

$$A = \frac{\pi d^2}{4} = \frac{\pi(4)^2}{4} = 4\pi \text{ in.}^2 \qquad J = \frac{\pi d^4}{32} = \frac{\pi(4)^4}{32} = 8\pi \text{ in.}^4$$

(The maximum torque occurs in the right segment of the shaft.)

$P_y = 5000$ lb $\qquad T_y = 5000$ lb·ft



$$\sigma_x = \frac{P}{A} = \frac{5000}{4\pi} = 397.9 \text{ psi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(5000\times 12)(2)}{8\pi} = 4775 \text{ psi}$$

$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{397.9}{2} \pm \sqrt{\left(\frac{397.9}{2}\right)^2 + (4775)^2} = 198.95 \pm 4779$$

$\sigma_{p1} = 198.95 + 4779 = 4978$ psi $\qquad \sigma_{p2} = 0$ ksi

$\sigma_{p3} = 198.95 - 4779 = -4580$ psi

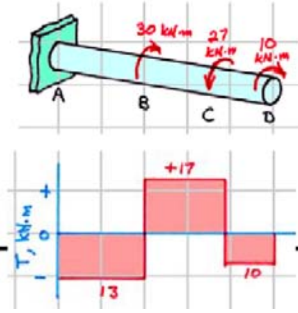
$$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{4.978}{22} - \frac{-4.580}{82} = 0.282 \le 1$$

Safe.. **Ans.**

10-68*

$T_{AB} = -13 \text{ kN} \cdot \text{m}$ $\quad$ $T_{BC} = +17 \text{ kN} \cdot \text{m}$ $\quad$ $T_{CD} = -10 \text{ kN} \cdot \text{m}$

$$J = \pi d^4 / 32 = \pi (100)^4 / 32 = 9.817(10^6) \text{ mm}^4$$



(a) $$U_{AB} = \left(\frac{T^2 L}{2GJ}\right)_{AB} = \frac{(-13,000)^2 (2.0)}{2(28 \times 10^9)(9.817 \times 10^{-6})} = 614.8 \text{ N} \cdot \text{m}$$

$U_{AB} \cong 615 \text{ N} \cdot \text{m}$ .... **Ans.**

$$U_{BC} = \left(\frac{T^2 L}{2GJ}\right)_{BC} = \frac{(17,000)^2 (1.5)}{2(80 \times 10^9)(9.817 \times 10^{-6})} = 276.0 \text{ N} \cdot \text{m}$$

$U_{BC} \cong 276 \text{ N} \cdot \text{m}$ .... **Ans.**

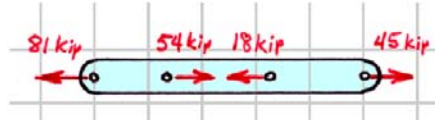
$$U_{CD} = \left(\frac{T^2 L}{2GJ}\right)_{CD} = \frac{(-10,000)^2 (1.0)}{2(80 \times 10^9)(9.817 \times 10^{-6})} = 63.7 \text{ N} \cdot \text{m}$$

$U_{CD} \cong 63.7 \text{ N} \cdot \text{m}$ .... **Ans.**

(b) $U_{total} = U_{AB} + U_{BC} + U_{CD} = 614.8 + 276.0 + 63.7 = 955 \text{ N} \cdot \text{m}$ .... **Ans.**

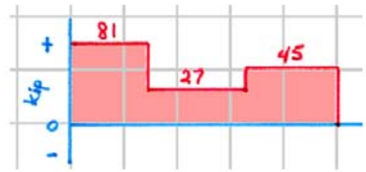
10-69*

$P_{AB} = 81\text{ kip}$ $\qquad P_{BC} = 27\text{ kip}$ $\qquad P_{CD} = 45\text{ kip}$



(a) $U_{AB} = \left(\frac{P^2L}{2AE}\right)_{AB} = \frac{(81)^2(30)}{2(3)(30,000)} = 1.0935\text{ kip}\cdot\text{in.}$

$U_{AB} \cong 1.094\text{ kip}\cdot\text{in.}$ .... **Ans.**



$U_{BC} = \left(\frac{P^2L}{2AE}\right)_{BC} = \frac{(27)^2(45)}{2(3)(30,000)} = 0.1823\text{ kip}\cdot\text{in.}$

$U_{BC} \cong 0.1823\text{ kip}\cdot\text{in.}$ .... **Ans.**

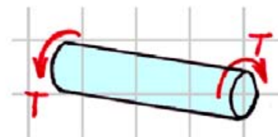
$U_{CD} = \left(\frac{P^2L}{2AE}\right)_{CD} = \frac{(45)^2(40)}{2(3)(30,000)} = 0.4500\text{ kip}\cdot\text{in.}$ .... **Ans.**

(b) $U_{total} = U_{AB} + U_{BC} + U_{CD} = 1093.5 + 182.3 + 450.0 = 1726\text{ lb}\cdot\text{in.}$ .... **Ans.**

10-70

$$\sigma_{\max} = \tau_{\max} = \tau_{xy} = Tc/J \qquad T = \sigma_{\max} J/c \qquad E = 2(1+\nu)G$$

$$U_{shaft} = \frac{T^2 L}{2GJ} = \frac{\sigma_{\max}^2 JL}{c^2 (2G)} = \frac{\sigma_{\max}^2 \left(\pi d^4/32\right) L}{(d/2)^2 \left[E/(1+\nu)\right]} = \frac{(1+\nu)\pi d^2 L \sigma_{\max}^2}{8E}$$



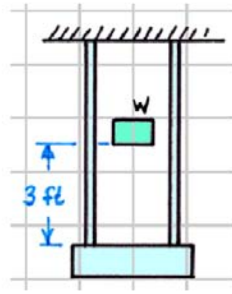
$$U_{bar} = \frac{P^2 L}{2AE} = \frac{\sigma_{\max}^2 AL}{2E} = \frac{\sigma_{\max}^2 \left(\pi d^2/4\right) L}{2E} = \frac{\pi d^2 L \sigma_{\max}^2}{8E}$$

Therefore: $U_{shaft} = (1+\nu) U_{bar}$ .... **Ans.**

10-71*

(a) $\delta_{st} = \frac{PL}{AE} = \frac{(40)(8\times 12)}{(2\times 2.5)(30\times 10^{6})} = 2.56\left(10^{-5}\right)$ in.

$$IF = 1+\sqrt{1+\frac{2h}{\delta_{st}}} = 1+\sqrt{1+\frac{2(3\times 12)}{\left(2.56\times 10^{-5}\right)}} = 1678.1 \cong 1678 \text{ .... } \textbf{Ans.}$$



(b) $\sigma_{max} = \sigma_{st}(IF) = \frac{W}{2A}(IF) = \frac{40}{2(2.5)}(1678.1)$

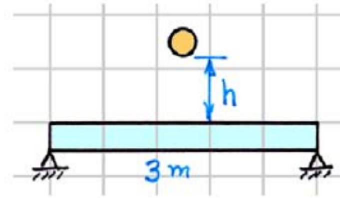
$$\sigma_{max} = 13{,}424 \text{ psi} \cong 13.42 \text{ ksi} \text{ .... } \textbf{Ans.}$$

(c) $\delta_{max} = \delta_{st}(IF) = \left(2.56\times 10^{-5}\right)(1678.1) = 0.0430$ in. .... **Ans.**

10-72

For an S 305×74 section: $I = 127(10^6)\ \text{mm}^4$ $c = 152.4\ \text{mm}$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{(PL/4)c}{I} = \frac{PLc}{4I}$$



$$P = \frac{4I\sigma_{max}}{Lc} \qquad M = \frac{Px}{2} = \frac{2I\sigma_{max}}{Lc}x$$

$$\delta = \frac{PL^3}{48EI} = \frac{\sigma_{max}L^2}{12Ec} = \frac{(120\times10^6)(3.0)^2}{12(200\times10^9)(0.1524)} = 0.002953\ \text{m}$$

$$U = 2\int_0^{L/2}\frac{M^2dx}{2EI} = \frac{2I\sigma_{max}^2}{EL^2c^2}\int_0^{L/2}x^2dx = \frac{\sigma_{max}^2 IL}{12Ec^2} = \frac{(120\times10^6)^2(127\times10^{-6})(3)}{12(200\times10^9)(0.1524)^2} = 98.43\ \text{N}\cdot\text{m}$$

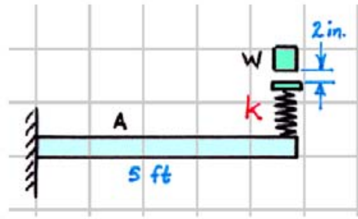
$$W(h+\delta) = U \qquad W = \frac{U}{h+\delta} = \frac{98.43}{(0.75)+(0.002953)} = 130.7\ \text{N}$$ .... **Ans.**

10-73

$$I = \frac{bh^3}{12} = \frac{(3)(1)^3}{12} = 0.250 \text{ in.}^4 \qquad \Delta_{beam} = \frac{PL^3}{3EI}$$

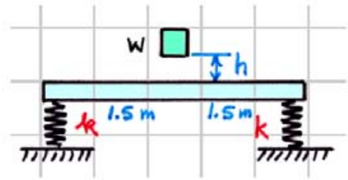
$$P = \frac{3EI\Delta_{beam}}{L^3} = \frac{3(29\times10^6)(0.250)(2.4)}{(5\times12)^3} = 241.7 \text{ lb}$$

$$IF = \frac{P}{W} = \frac{241.7}{90} = 2.69$$ .... **Ans.**



10-74*

$$I = \frac{bh^3}{12} = \frac{(75)(25)^3}{12} = 97.66\left(10^3\right)\ \text{mm}^4$$



$$\delta = \frac{mg}{2k} + \frac{mgL^3}{48EI} = \frac{(5\times 9.81)}{2\left(10\times 10^3\right)} + \frac{(5\times 9.81)(3)^3}{48\left(80\times 10^9\right)\left(97.66\times 10^{-9}\right)}$$

$$\delta = 0.005982\ \text{m} = 5.982\ \text{mm} \qquad \Delta = 4\delta = 4(5.982) = 23.928\ \text{mm}$$

$$P = 4mg = 4(5\times 9.81) = 196.20\ \text{N}$$

$$mg(h+\Delta) = P\Delta/2 \qquad (5\times 9.81)(h+0.023928) = (196.20)(0.023928)/2$$

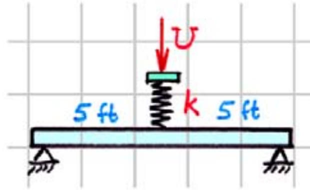
$h = 0.0239\ \text{m} = 23.9\ \text{mm}$ .... **Ans.**

10-75*

$$\Delta_{spring} = \frac{P}{k} = \frac{P}{200} = 0.005P \qquad I = \frac{bh^3}{12} = \frac{(3)(2)^3}{12} = 2.00 \text{ in.}^4$$

$$U_{spring} = \frac{P\Delta_{spring}}{2} = \frac{P(0.005P)}{2} = 720 \text{ lb}\cdot\text{in.}$$

$$P = 536.7 \text{ lb}$$

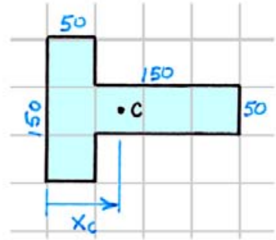


$$\Delta_{beam} = \frac{PL^3}{48EI} = \frac{(536.7)(10\times 12)^3}{48(30\times 10^6)(2)} = 0.3220 \text{ in.}$$

$$U_{beam} = \frac{P\Delta_{beam}}{2} = \frac{(536.7)(0.3220)}{2} = 86.4 \text{ lb}\cdot\text{in.}$$ .... **Ans.**

10-76

$$A = 2(50 \times 150) = 15{,}000 \text{ mm}^2$$



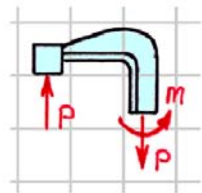
$$x_C = \frac{(25)(50 \times 150) + (125)(150 \times 50)}{15{,}000} = 75.00 \text{ mm}$$

$$I = \frac{(150)(75)^3}{3} + \frac{(50)(125)^3}{3} - \frac{(100)(25)^3}{3} = 53.125(10^6) \text{ mm}^4$$

$P = 100$ kN $\qquad M_z = (100)(0.350 + 0.075) = 42.5 \text{ kN} \cdot \text{m}$

$$\sigma_i = \frac{P}{A} + \frac{Mc}{I} = \frac{100(10^3)}{15(10^{-3})} + \frac{(42{,}500)(0.075)}{53.125(10^{-6})} = 66.67(10^6) \text{ N/m}^2 = 66.67 \text{ MPa}$$

$$\sigma_o = \frac{P}{A} - \frac{Mc}{I} = \frac{100(10^3)}{15(10^{-3})} - \frac{(42{,}500)(0.125)}{53.125(10^{-6})} = -93.33(10^6) \text{ N/m}^2 = -93.33 \text{ MPa}$$



Since the state of stress is uniaxial at both the inside and outside edges,

(a) Maximum-Shear-Stress:

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (0 + 93.33)/2 = 46.67 \text{ MPa}$$

$$\tau_f = \sigma_f/2 = 220/2 = 110 \text{ MPa}$$

$$FS = \frac{\tau_f}{\tau_{max}} = \frac{110}{46.67} = 2.36$$ .... **Ans.**

(b) Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2}$$

$$= \sqrt{(0)^2 - (0)(-93.33) + (-93.33)^2} = 93.33 \text{ MPa}$$

$$FS = \frac{\sigma_f}{\sigma_{max}} = \frac{220}{93.33} = 2.36$$ .... **Ans.**

10-77*

(a) $\sigma_f = 36$ ksi $\quad \sigma_{p1} = 20$ ksi $\quad \sigma_{p2} = 12$ ksi $\quad \sigma_{p3} = 0$ ksi

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (20 - 0)/2 = 10.00 \text{ ksi}$$

Maximum-Normal-Stress:

$$FS = \frac{\sigma_f}{\sigma_{max}} = \frac{\sigma_f}{\sigma_{p1}} = \frac{36}{20} = 1.800$$ .... **Ans.**

Maximum-Shear-Stress:

$$\tau_f = \sigma_f/2 = 36/2 = 18 \text{ ksi}$$

$$FS = \frac{\tau_f}{\tau_{max}} = \frac{18}{10} = 1.800$$ .... **Ans.**

Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2} = \sqrt{(20)^2 - (20)(12) + (12)^2} = 17.436 \text{ ksi}$$

$$FS = \frac{\sigma_f}{\sigma_e} = \frac{36}{17.435} = 2.06$$ .... **Ans.**

(b) $\sigma_f = 36$ ksi $\quad \sigma_{p1} = 20$ ksi $\quad \sigma_{p2} = 0$ ksi $\quad \sigma_{p3} = -12$ ksi

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2 = (20 + 12)/2 = 16.00 \text{ ksi}$$

Maximum-Normal-Stress:

$$FS = \frac{\sigma_f}{\sigma_{max}} = \frac{\sigma_f}{\sigma_{p1}} = \frac{36}{20} = 1.800$$ .... **Ans.**

Maximum-Shear-Stress:

$$\tau_f = \sigma_f/2 = 36/2 = 18 \text{ ksi}$$

$$FS = \frac{\tau_f}{\tau_{max}} = \frac{18}{16} = 1.125$$ .... **Ans.**

Maximum-Distortion-Energy:

$$\sigma_e = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2} = \sqrt{(20)^2 - (20)(-12) + (-12)^2} = 28.00 \text{ ksi}$$

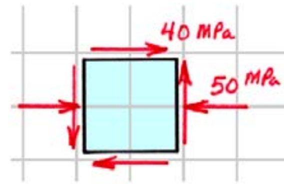
$$FS = \frac{\sigma_f}{\sigma_e} = \frac{36}{28} = 1.286$$ .... **Ans.**

10-78

$\sigma_x = -50 \text{ MPa} \qquad \sigma_y = 0 \text{ MPa} \qquad \tau_{xy} = 40 \text{ MPa}$

$$\sigma_{p1,p3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-50+0}{2} \pm \sqrt{\left(\frac{-50-0}{2}\right)^2 + (40)^2} = -25 \pm 47.170$$



$\sigma_{p1} = -25 + 47.170 = 22.17 \text{ MPa} \qquad \sigma_{p2} = 0 \text{ MPa}$

$\sigma_{p3} = -25 - 47.170 = -72.17 \text{ MPa}$

$$\frac{\sigma_{p1}}{\sigma_{ut}} - \frac{\sigma_{p3}}{\sigma_{uc}} = \frac{22.17}{68} - \frac{-72.17}{206} = 0.676 \le 1$$

Safe.. **Ans.**