### 4.3 Renewable resources

## Fisheries

3.1 Biological dimension of fisheries

- Fisheries are renewable resource.
- Because fish are living creatures that reproduce, grow, and die.
- The following illustration shows this.

- Where $B=$ number of birth; $D=$ number of death
- Population is measured in biomass (weight)

G (growth) = B - D.

- If $\mathbf{G}>0==>$ Population goes up.
- If $\mathbf{G}=0==>$ Population remains fixed.
- If $\mathbf{G}<0==>$ Population declines
$\rightarrow$ Distinguish between stocks and flows in a fishery.
- The stock or population of fish is the aggregate weight of the fish population measured at any point in time.
- The flow is the change in the stock over an interval of time,
- The change results from
- Biological factors, such as the entry of new fish into the population through birth (called recruitment),
- Growth of existing members of the population
- Natural death; and
- Economic factors such as harvesting the fish. This section will illustrate three important concepts:
- The biological mechanics of fishery
- How harvesting affects the stock size
- A comparison of the private property harvest with the open access harvest


## 1. The Biological Dimensions

## Biological Model of Fishery

The growth rate of fish stock depends on its size.

- Small stock size $\rightarrow$ large food supply $\rightarrow$ births outnumber deaths
- As the stock size increases $\rightarrow$ food per fish decreases $\rightarrow$ deaths begin to rise $\rightarrow$ growth rate will decline.
- Ultimately the stock gets so large $\rightarrow$ deaths will equal births $\rightarrow$ the growth rate falls to zero.
- Figure 6.1 shows the relationship between growth rate and the stock of fish.

Figure 6.1 shows the relationship between growth rate and the stock of fish.


- $X=$ the stock of fish in the fishery
- the aggregate weight of the fish population
- $F(X)=$ the rate of growth of the stock.
- The stock size $X$ is represented on the horizontal axis and the growth of the stock on the vertical axis.
- As the graph in Figure 6.1 shows,
- Starting at a small stock size, the stock at first grows rapidly.
- Growth reaches a maximum at $\mathrm{F}^{*}(\mathrm{X})$, then declines and eventually falls to zero when the stock size reaches $\mathrm{K} / \mathrm{X}_{\text {max }}$.
- This is called logistic growth function,
- Which yields a parabola when the growth $F(X)$ is plotted against the stock size X .
- Each point on the growth curve represents a sustainable yield of fish for a given stock of fish, X.
- A sustainable yield is one that can be maintained forever.
- As long as the stock size $X$ remains constant, the growth $F(X)$ will remain constant as well.
- $X_{\text {MSY }}$ is known as the maximum sustainable yield stock
$-X_{M S Y}$ is the stock of fish $X$ at which the growth $F^{*}(X)$ is maximized.
- $\mathrm{F}^{*}(\mathrm{X})$ is maximum sustained yield for the fishery.
- $\mathrm{X}_{\text {MSY }}$ occurs at exactly one-half of $\mathrm{K}\left(\mathrm{X}_{\max }\right)$
- $K\left(X_{\max }\right)$ is the carrying capacity of the fishery.
- $\mathrm{K}\left(\mathrm{X}_{\max }\right)$ represents the maximum stock size that would exist in the absence of human intervention.
- If no harvest takes place in the fishery, we would expect to find $\mathrm{K}\left(\mathrm{X}_{\max }\right)$ stock size at each point of time.
- This is known as the carrying capacity of the fishery. That is, the maximum sustainable stock that the environment can support given the available resources.
- For stock between 0 to $X_{M S Y}$, growth increases to the maximum sustainable yield, $\mathrm{F}^{*}(\mathrm{X})$.
- For stock size $>X_{M S Y}$, the stock continues to grow but the rate of growth becomes smaller.
- As the stock approaches to the carrying capacity, K $\left(X_{\max }\right)$, growth declines to zero.
- If the stock of fish $X>K\left(X_{\text {max }}\right)$, the stock will decline to $K\left(X_{\text {max }}\right)$ because growth is negative
- If the stock of fish $X<K\left(X_{\text {max }}\right)$, the stock will increase to $\mathrm{K}\left(\mathrm{X}_{\text {max }}\right)$. This occurs because growth is positive.
- The growth in the stock can be identical at different levels of the stock.
- For example, a growth rate of $\mathrm{F}_{1}(\mathrm{X})$ can be obtained with a small stock $X_{1}$ or a larger stock $X_{2}$.
- At $X_{1}$, births greatly outnumber deaths because stock is small and food is ample.
- At $X_{2}$, births slightly outnumber deaths, and the average size of the stock is large.
- Biological equilibrium for the fishery
- A biological equilibrium is the size of the fish stock $X$ for which there is no growth in the stock, i.e. $F(X)$ is equal to zero.
- There are two possible values of $X$ for which there is no growth in the stock - at $X=0$ and $X=$ $K\left(X_{\max }\right)$.
- If $X$ is equal to zero, there are no fish and therefore no growth occurs.
- An interesting equilibrium is where the growth curve crosses the X axis at the point labeled K $\left(X_{\max }\right)$.
- K is the carrying capacity of the fishery.
-The fish will be in a biological equilibrium where $X=K$.


## 2. Economics of Fisheries

### 2.1 Equilibrium between harvests and stock levels

- We will now examine the role played by the economic activity of harvesting to derive an equilibrium that combines the biological mechanics with economic activity.
- Let H denote harvest (catch).
- Relationship between stock $X$, growth $F(X)$ and harvest (H)

Growth ( $\mathrm{F}(\mathrm{X})$ ) Harvest (H)


- Each point along the growth curve represents equilibrium between harvest and stock.
- Change in fish stock over time is given by the difference between biological growth, $\mathrm{F}(\mathrm{X})$ and level harvest, H
- When $H=F(X)$, a sustainable yield is attained
- Sustainable yield is equilibrium between H and X where the fish stock does not change
- Two different rates of harvest: $\mathbf{H}_{1}$ and $\mathbf{H}_{2}$
- At $\mathrm{H}_{1}$ rate of harvest,
- The stock stabilizes at $X_{M S Y}$, the stock size where growth $F(X)$ is at a maximum.
- At $X_{\text {MSY }}$ (the maximum sustainable yield stock), the largest sustainable harvest can occur
- For stock size $(X)$ between K and $\mathrm{X}_{\text {MSY }}$, the stock gradually falls to $\mathrm{K} / 2$ or XMSY because rate of harvest $\left(\mathrm{H}_{1}\right)>$ growth $(\mathrm{F}(\mathrm{X}))$ in this range.
- At $\mathrm{H}_{2}$ rate of harvest,
- There are two possible equilibriums for the fishery: $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$, where $\mathrm{H}_{2}$ intersects the fishery growth function, $\mathrm{F}(\mathrm{X})$.
- Which one is likely to occur?
- If stock is initially at K , and $\mathrm{H}_{2}$ is harvested, equilibrium is attained at stock level $\mathrm{X}_{2}$. The fishery yields $\mathrm{H}_{2}$ and the stock size is stabilized at $\mathrm{X}_{2} \mathrm{~b} / \mathrm{s} \mathrm{H}_{2}>\mathrm{F}(\mathrm{X})$.
- If the stock is initially between $X_{1}$ and $X_{2}$ and $H_{2}$ is harvested, the stock increases until the equilibrium at $X_{2}$ is reached $b / s H_{2}<F(X) b / n X_{1}$ and $\mathrm{X}_{2}$
- Any stock size > $X_{1}$ ultimately yields equilibrium at $X_{2}$. We call $X_{2}$ a stable equilibrium.
- If the stock $<\mathrm{X}_{1}$ and $\mathrm{H}_{2}$ is harvested, the fish will go to extinction and the fishery will cease to function. $\mathrm{B} / \mathrm{s}$ for stock $<X_{1}, H_{2}>F(X)$ and the stock decreases to zero.
- If the stock is $X_{1}$ and $H_{2}$ is harvested, $X_{1}$ will be equilibrium. But this is unstable equilibrium since a slight movement of the stock to the right or to the left of level $X_{1}$ will lead to a new equilibrium
- The change in the fish stock over time is influenced by the difference between the biological growth function $F(X)$ and the amount of harvesting H .
- When $F(X)=H$ there will be no change in the size of the stock over time and a sustainable yield is attained.
- A harvest level is said to represent a sustainable yield whenever it equals the growth rate of the stock (i.e. $\mathrm{H}=$ $F(X)$ ), since it can be maintained forever.
- As long as the stock size remains constant the growth rate and hence the harvest will remain constant as well.


## 2. ECONOMICS OF FISHERY

Let $\mathrm{H}=$ harvest
$X=$ stock size
$F(X)=$ growth in stock size
$E=$ effort
E - Number of boats deployed to catch (harvest) fish.

- Harvest $(H)$ depends on effort (E) and stock size (X),

$$
H=f(E, X) .
$$

- Harvest and effort
- The smaller the stock of fish (X), the greater the effort required to catch any amount of fish $(\mathrm{H})$ than if $X$ is large.
- For a given stock size, $\uparrow E \rightarrow \uparrow H$
- There is diminishing marginal product of effort (E)
- As E increases on fixed stock of fish, H increases at a decreasing rate
- Harvest and stock size
- The larger the size of the stock, more fish are caught with a given level of effort
- For a given level of effort, $\uparrow \mathrm{X} \rightarrow \uparrow \mathrm{H}$
- Figure 6.3: Relationship between stock (X), growth ( $\mathrm{F}(\mathrm{X})$ ), harvest (H) and effort (E)

- We can now determine the sustainable equilibrium - where there is no change in the stock size.
- This requires that the rate of harvest be equal to the growth in the stock size (i.e. $\mathrm{H}=\mathrm{F}(\mathrm{X})$ ).
- Graphically, the equilibrium is defined where the harvest function, $H$, intersects the growth function, $F(X)$.
- For a given stock size $(X)$, a sustainable equilibrium yields a harvest rate $\mathrm{H}=\mathrm{F}(\mathrm{X})$.
- As long as the stock size remains constant, $F$ $(\mathrm{X})$ and hence H remain the same as well.
- Such a stock size is said to be sustainable yield stock and can be sustained indefinitely.
$\rightarrow$ What happens when we increase the level of effort by increasing the number of boats?
- As the level of effort increases the rate of harvest, H , pivots upwards
- In Figure 6.3, $\mathrm{EO} \mathrm{E}_{1}, \mathrm{E}_{2}, \mathrm{E}_{3}$, and $\mathrm{E}_{4}$ represent different levels of effort - E0< $E_{1},<E_{2},<E_{3}<E_{4}$
- At $\mathrm{E}_{0}$ level of effort, $\mathrm{H}_{0}$ amount of fish is harvested $\left(\mathrm{H}_{0}=\right.$ $F\left(X_{4}\right)$ ).
- As Effort increases from $E_{0}$ to $E_{1}, E_{2}$, and $E_{3}$, harvest rises from $H_{0}$ to $H_{1}, H_{2}$, and $\mathrm{H}_{3}$ but stock size declines from $X_{4}$ to $X_{3}, X_{2}$ and $X_{1}=X_{M S Y}$
- i.e., additional effort means more harvest, but lower stocks
- As the level of effort increases, harvest also increases, but at a decreasing rate.
- Total harvest increases but harvest per unit of effort (boat) declines
- B/s when more and more boats enter the fishery, many boats will have to compete for a fixed stock of fish.


## $\rightarrow$ What happens when effort increases to $E_{4}$ ?

- When $E_{4}$ level of effort is put into fishing
- The rate of harvest $\mathrm{H}_{2}$ intersects the biological growth function $F(X)$ at stock size $X_{0}$ to the left of the $\mathrm{X}_{M S Y}$ stock
- However, $\mathrm{H}_{2}$ level of harvest can be attained with only $E_{2}$ level effort at a stock size of $X_{2}$
$-E_{2}$ represents lower level of effort than $E_{4}$ and
- the stock size $X_{0}$ is much lower than stock $X_{2}$
- Thus, when the sock size is larger, less effort is required to harvest the same number of fish (compare $E_{2}$ with $E_{4}$ and $X_{2}$ with $X_{0}$ ).
- As the level of effort increases, total harvest initially rises at a decreasing rate and eventually it declines
- This reflects diminishing marginal product of effort
- As effort increases, the marginal harvest (marginal product) of each additional unit of effort declines and hence the average harvest of each unit of effort decreases.
- An $\uparrow E \rightarrow \uparrow H$ until the sustainable equilibrium where $X=$ $\mathrm{X}_{\mathrm{MSY}}$
- Thereafter, an $\uparrow E \rightarrow \downarrow \mathrm{H}$
- In a static model, it is economically inefficient for a fishing industry to operate to the left of $X_{M S Y} \mathrm{~b} / \mathrm{s}$ more effort than necessary is used to catch a given amount of fish
- Increasing effort (the number of boats) therefore ultimately leads to lower harvests and lower stocks
- Using this information, we can show the relationship between fishing effort and harvest.
- Consider the same effort levels as in Figure 6.3.
- Note that effort, E, is now on the x - axis.

Figure 6.4: Relationship between effort level and harvest


Low Effort
High Stock

High Effort
Low Stock

- As effort increases (from left to right), the harvest increases up to the maximum sustainable yield
- But as effort increases beyond $E_{3}$, harvest begins to decline.
- The top of the curve is the point of maximum sustainable yield and $E_{3}$ level of effort would provide this rate of harvest.
$\rightarrow$ We can now introduce cost, price, and profit
- $W=$ per unit cost of effort (i.e. the cost of a boat)
- $\mathrm{P}=$ price per unit of fish harvested, H .
- If H is in Kg ., P is price per k.g.
- Both W and $P$ are assumed to be constant.
- Profit $(\pi)=$ TR - TC
- TR = P X H = Price X Harvest
- $\mathrm{TC}=\mathrm{W} X E=$ Cost per Boat $X$ the number of Boats
- Figure 6.5: Relationship between TR and TC



## Total Revenue (TR)

- The shape of the TR curve is the same as that of the growth curve $F$ $(\mathrm{X}) \mathrm{b} / \mathrm{s}$ it expresses the relationship between harvest and revenue.
- As effort increases, harvests and hence TR increases up to the point of $\mathrm{E}_{M S}$.
- After that harvest and hence TR decreases.
- TR is just price times harvests. Since price is assumed to be constant, TR is a constant price times the level of harvest.
- AR is TR/E, while MR is dTR/dE. Both AR and MR are negatively sloped, with AR lying above MR.


## Total Cost (TC)

- TC is a straight line (linear) due to the relationship between the cost of each boat and the number of boats.
- We assume that the same type of boats are entering the fishery so that the cost for each boat is the same, i.e. W is constant.
- With constant unit cost, marginal cost (MC) equals W and is also equal to average cost (AC).
- MC and AC are constant, or flat.
- As the number of boats increases, total cost (TC) increases by the cost of each boat (W) times the number of additional boats.
- The slope of the TC is therefore constant and equal to W.
- Optimal harvest under private property rights
- Efficient level of effort $E_{P}$, where $M R=M C$
- Harvest under open access
- At $E_{O}$, where $A R=M C$ (TR=TC)
$-\pi=0$,i.e. scarcity rent is dissipated $b / s$ no owner can appropriate it
- Compared to efficient operation under private property rights, open access
- Leads to higher level of effort is applied - $\mathrm{E}_{\mathrm{O}}>$ $E_{P}$
- Occurs at low stock size, i.e. the fishery is overexploited


## Exercises

Consider a fishing firm that has a private property right on a fishery. The firm operates in a competitive fishing industry and takes all prices and factor cost as given. Suppose the sustainable catch (harvest) and the total revenue functions of the firm are given by:

$$
\begin{aligned}
& T R(E)=100 E-5 E^{2} \\
& H(E)=20 E
\end{aligned}
$$

Where H is harvest in tons of fish, TR is total revenue in Birr and $E$ is fishing effort measured by number of fishing boats. The marginal cost of effort is constant and equal to Birr 50.
(a) Compute the private property level of effort, harvest and profit.
(b) Assuming the fishery is turned to an open access resource, compute the open access level of effort harvest and profit

Suppose the growth function of the stock of fish in the fishery is given by

$$
G(S)=S\left(1-\frac{S}{320}\right)
$$

where is growth in fish stock and is fish stock.
(c) Using the given growth function, calculate the carrying capacity and the maximum sustainable yield (MSY) stock size of the fishery.
(d) Assuming the manager of the fishery decides to harvest an amount equal to the maximum sustainable yield (MSY) stock size, compute the required level of effort and the amount of profit the manager would earn. (4 points)

Consider a fishing firm that has a private property right on a fishery. The firm operates in a competitive fishing industry and takes all prices and factor cost as given. Suppose the sustainable catch and the total revenue functions of the firm are given by:

$$
\begin{aligned}
& H(E)=0.6 E-0.01 E^{2} \\
& T R(E)=580 E-5 E^{2}
\end{aligned}
$$

- Where H is harvest in tons of fish, TR is total revenue in Birr and E is fishing effort measured by number of fishing boats. The marginal cost of effort is constant and equal to Birr 540. The price of fish is also constant at Birr 1000 per ton.
Given this information compute:
- The private property level of effort.
- The private property level of harvest
- The private property level of profit or loss, whichever is appropriate.
- The open access level of effort, assuming the fishery is turned to an open access.


## Forest Resources

## 1. Economic value of a forest

- Forests products provide a wide variety of inputs
- Raw materials for housing and other lumber products
- Pulp and paper products
- A source of fuel wood
- Trees also play a major role in the environment
- Trees cleanse the air by absorbing carbon dioxide and adding oxygen
- Trees play a major role in watersheds that provide much of our drinking water.
- Trees provide shelter for wildlife, protecting the ecological and genetic diversity.


## Total value of a forest



- We are interested in timber value of a forest only


## 2. Biological Basics of Trees and Forests

- Trees grow at a relatively slow rate, often taking over 100 years to reach their maximum volume.
- The time horizon involved is often lengthy
- The forestry growth function
- Growth of a stand of trees is a function of time, not a function of stock and growth is measured in volume (cubic feet)
$-\mathrm{V}(\mathrm{t})$ - volume of timber (e.g. in cubic feet) at any time t
- Timber volume is a function of time
- Biological model of timber growth
$\mathrm{V}(\mathrm{t})$ Volume of timber

- Any timber-harvesting plan must consider the nature of biological growth of trees.
- At first, the volume increases at an increasing rate for very young trees. The growth rate is at its peak at $T_{p g}$
- Then growth of volume slows and increases at a decreasing rate. The forest reaches maturity at $T_{m}$ where growth rate equals zero.
- Finally, the trees get very old and begin to have negative growth as they rot, decay, and become subject to disease and pests.
- For $\mathrm{t}>\mathrm{t}_{\text {max }}$ volume of timber declines
- The volume of a stand of trees is maximized at time $\mathrm{T}_{\max }$, with a volume $\mathrm{V}\left(\mathrm{T}_{\max }\right)$
- The growth rate of the volume of a stand of trees is maximized at time $T_{p g}$ with a volume $\mathrm{V}\left(\mathrm{T}_{\mathrm{pg}}\right)$
- Growth rate (change in timber volume) of tree over time
- Growth is given by the derivative of timber volume with respect to time
$\frac{d V(t)}{t}$


## Terminology

- DEFINITION: The Current Annual Increment (CAI) is the annual increase in the volume of the stand in the preceding decade.
- In economic terms this is the marginal product of the tree stand.

$$
C A I=\frac{d V(t)}{d t}=\text { marginal product of tree stand }
$$

- DEFINITION: The Mean Annual Increment (MAI) is the total volume of the stand divided by the number of years the stand has been growing.
- In economic terms, this is the average product of the tree stand.

$$
M A I=\frac{V(t)}{t}=\text { average product of tree stand }
$$

- Forest management practices typically focus on MAI rather than CAI.
- DEFINITION: The Culmination of the Mean Annual Increment (CMAI) is the maximum MAI.
- Rotation - length of time between the planting of a timber stand and its harvest

Series of rotations of length $\mathbf{R}$


| Age <br> (years) | Volume <br> (cubic feet) | MAl <br> (cubic feet) | CAl <br> (cubic feet) |
| ---: | ---: | ---: | ---: |
| 10 | 694 | 69.4 | 69.4 |
| 20 | 1912 | 95.6 | 121.8 |
| 30 | 3558 | 118.6 | 164.6 |
| 40 | 5536 | 138.4 | 197.8 |
| 50 | 7750 | 155.0 | 221.4 |
| 60 | 10104 | 168.0 | 235.4 |
| 70 | 12502 | 178.6 | 239.8 |
| 80 | 14848 | 185.6 | 234.6 |
| 90 | 17046 | 189.4 | 219.8 |
| 100 | 19000 | 190.0 | 195.4 |
| 110 | 20614 | 187.4 | 161.4 |
| 120 | 21792 | 184.6 | 117.8 |
| 130 | 22438 | 172.6 | 64.4 |
| 135 | 22514 | 166.8 | 11.6 |

When to harvest the stand of trees?

## Biological decision rule

- The biological decision rule is to harvest at CMAI when MAI is maximum.
- Determining the biologically optimal age of harvest for the stand
- Biological optimality requires harvesting the maximum sustainable yield (MSY).
- MSY occurs where MAI is at its maximum
- To determine the biological optimal age of harvest

$$
\text { maximize MAI }-\quad M A I=\frac{V(t)}{t}
$$

- Max MAI $\rightarrow$ FOC : $\frac{d(t)}{d t}=0$

$$
\frac{V^{\prime}(t) t-V(t)}{t^{2}}=0 \rightarrow V^{\prime}(t) t=V(t) \rightarrow \frac{V(t)}{t}=V^{\prime}(t)
$$

- Thus, biological optimal age of harvest occurs where the average product of the forest equals the marginal product
- i.e. where $\mathrm{MAI}=\mathrm{CAI}$
- In forest biology, this concept is known as Maximizing the Mean Annual Increment
- In economic sense, what is wrong with this rule?
- It ignores the costs of production

The Economics of Forest Harvesting

- The economic decision rule: maximize the present value of net benefits.
- The Single Rotation Case

Let:

- $\mathrm{V}(\mathrm{t})$ denote the volume of wood at time t
- P denote the price of wood per unit volume
- C denote the cost of planting the trees
- h denote the cost of harvesting the trees per unit volume
- T denote the optimal harvesting time
- Profits are given by $\begin{aligned} & =(P-h) V(t)-C\end{aligned}$
- When should the stand be harvested?
- When the present value of profits are maximized
- Present value of profit:

$$
\begin{aligned}
& \pi=e^{-r t}(P-h) V(t)-C \\
& \frac{d \pi}{d t}=(P-h) V^{\prime}(t) e^{-r t}+(P-h) V(t) e^{-r t}(-r)=0 \\
& (P-h) V^{\prime}(t) e^{-r t}=r(P-h) V(t) e^{-r t} \\
& (P-h) V^{\prime}(t)=r(P-h) V(t)
\end{aligned}
$$

- MB of waiting (value of new growth) = MC of waiting (lost interest on TR)

$$
r=\frac{V^{\prime}(t)}{V(t)}
$$

- If the forest manager delays the harvest, she will not earn interest on revenues .

$$
(p-h) V^{\prime}(t)
$$

- If the forest manager delays the harvest, she will gain the value of new growth.

$$
(p-h) V(t)
$$

- The optimality condition requires that:

$$
r=\frac{V^{\prime}(t)}{V(t)}
$$

- If $\frac{V^{\prime}(t)}{V(t)}>, r$ then the forest is increasing in value
quicker than market investments and the manager should delay the harvest decision.
- If $\frac{V^{\prime}(t)}{V(t)}<r$,then market investments are
increasing in value quicker than the growth in value of the forest (harvesting should have already occurred)


## Exercise

1. The yield function of a stand of trees of the same species with uniform growth characteristics is given by

$$
V(t)=5 t+t^{2}-0.05 t^{3} \quad \text { where } V(t) \quad \text { is timber }
$$

volume of the stand in cubic meters and $\mathbf{t}$ is the age of the stand in years.
(a) Assuming the forest manager decides to harvest all the timber from the stand and considers only timber value in his harvesting decisions; compute the biologically optimal age of harvest for the stand.
(b) Suppose the forest manager has a plan to allocate the land for a different purpose 50 years after the stand was established. Given the above yield function, compute the number of rotations the manager could have if he follows the biological harvesting decision rule.
(c) Suppose very young trees of the type in the stand are highly demanded in the Christmas week as they are widely used as Christmas trees. Using the yield function given above, calculate the timber volume the forest manger could harvest from the stand if he decides to clear cut the entire tree stands at the age of 2 years and sell it in the form of Christmas trees.

Suppose the yield function of a stand of trees of a certain species with uniform growth characteristics is given by $V(t)=45 t-0.15 t^{3}$, where $V(t)$ is timber volume of
the stand in cubic meters and $\boldsymbol{t}$ is the age of the stand in years.
(d) Assuming the discount rate is $0 \%$, compute the economically optimal age of harvest for the stand.
(e) Assume the planting (regeneration) cost paid when the stand was established is Birr 5000, the cost of harvesting is constant at Birr 50 per cubic meter, the price of timber is constant and equal to Birr 100 per cubic meter and the discount rate is $0 \%$. Using the given yield function [i.e., $V(t)=45 t-0.15 t^{3}$ ] calculate the net present value of the
stand if the manager decides to harvest the entire tree stands at the economically optimal age of harvest.

