# Unit 4: EXPERIMENTAL DESIGN

## BASIC PRINCIPLES OF EXPERIMENTAL DESIGNS

Professor Fisher has enumerated three principles of experimental designs:

(1) The Principle of Replication;

(2) The Principle of Randomization; and

(3) The Principle of Local Control.

According to **the Principle of Replication**, the experiment should be repeated more than once. Thus, each treatment is applied in many experimental units instead of one. By doing so the statistical accuracy of the experiments is increased. For example, suppose we are to examine the effect of two varieties of rice. For this purpose we may divide the field into two parts and grow one variety in one part and the other variety in the other part. We can then compare the yield of the two parts and draw conclusion on that basis. But if we are to apply the principle of replication to this experiment, then we first divide the field into several parts, grow one variety in half of these parts and the other variety in the remaining parts. We can then collect the data of yield of the two varieties and draw conclusion by comparing the same. The result so obtained will be more reliable in comparison to the conclusion we draw without applying the principle of replication. The entire experiment can even be repeated several times for better results. Conceptually replication does not present any difficulty, but computationally it does. For example, if an experiment requiring a two-way analysis of variance is replicated, it will then require a three-way analysis of variance since replication itself may be a source of variation in the data. However, it should be remembered that replication is introduced in order to increase the precision of a study; that is to say, to increase the accuracy with which the main effects and interactions can be estimated.

**The Principle of Randomization** provides protection, when we conduct an experiment, against the effect of extraneous factors by randomization. In other words, this principle indicates that we should design or plan the experiment in such a way that the variations caused by extraneous factors can all be combined under the general heading of "chance." For instance, if we grow one variety of rice, say, in the first half of the parts of a field and the other variety is grown in the other half, then it is just possible that the soil fertility may be different in the first half in comparison to the other half. If this is so, our results would not be realistic. In such a situation, we may assign the variety of rice to be grown in different parts of the field on the basis of some random sampling technique i.e., we may apply randomization principle and protect ourselves against the effects of the extraneous factors (soil fertility differences in the given case). As such, through the application of the principle of randomization, we can have a better estimate of the experimental error.

**The Principle of Local Control** is another important principle of experimental designs. Under it the extraneous factor, the known source of variability, is made to vary deliberately over as wide a range as necessary and this need to be done in such a way that the variability it causes can be measured and hence eliminated from the experimental error. This means that we should plan the experiment in a manner that we can perform a two-way analysis of variance, in which the total variability of the data is divided into three components attributed to treatments (varieties of rice in our case), the extraneous factor (soil fertility in our case) and experimental error.* In other words, according to the principle of local control, we first divide the field into several homogeneous parts, known as blocks, and then each such block is divided into parts equal to the number of treatments. Then the treatments are randomly assigned to these parts of a block. Dividing the field into several homogenous parts is known as 'blocking'. In general, blocks are the levels at which we hold an extraneous factor fixed, so that we can measure its contribution to the total variability of the data by means of a two-way analysis of variance. In brief, through the principle of local control we can eliminate the variability due to extraneous factor(s) from the experimental error.

**Important Experimental Designs**

Experimental design refers to the framework or structure of an experiment and as such there are several experimental designs. We can classify experimental designs into two broad categories, viz., informal experimental designs and formal experimental designs. Informal experimental designs are those designs that normally use a less sophisticated form of analysis based on differences in magnitudes, whereas formal experimental designs offer relatively more control and use precise statistical procedures for analysis. Important experiment designs are as follows:

**(a) Informal experimental designs:**

(1) Before-and-after without control design.

(2) After-only with control design.

(3) Before-and-after with control design.

**(b) Formal experimental designs:**

(4) Completely randomized design (C.R. Design).

(5) Randomized block design (R.B. Design).

(6) Latin square design (L.S. Design).

(7) Factorial designs.

We may briefly deal with each of the above stated **informal** as well as **formal** experimental designs.

**1. Before-and-after without control design:** In such a design a single test group or area is selected and the dependent variable is measured before the introduction of the treatment. The treatment is then introduced and the dependent variable is measured again after the treatment has been introduced. The effect of the treatment would be equal to the level of the phenomenon after the treatment minus the level of the phenomenon before the treatment. The design can be represented thus:

| | | | |
|---|---|---|---|
| Test area: | Level of phenomenon before treatment (X) | Treatment introduced → | Level of phenomenon after treatment (Y) |
| | Treatment Effect = (Y) – (X) | | |

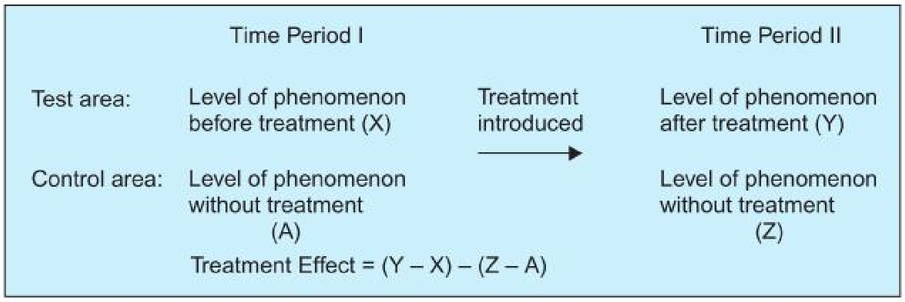
The main difficulty of such a design is that with the passage of time considerable extraneous variations may be there in its treatment effect.

**2. After-only with control design:** In this design two groups or areas (test area and control area) are selected and the treatment is introduced into the test area only. The dependent variable is then measured in both the areas at the same time. Treatment impact is assessed by subtracting the value of the dependent variable in the control area from its value in the test area. This can be exhibited in the following form:

| | | |
|---|---|---|
| Test area: | Treatment introduced | Level of phenomenon after treatment (Y) |
| Control area: | → | Level of phenomenon without treatment (Z) |
| | Treatment Effect = (Y) – (Z) | |

The basic assumption in such a design is that the two areas are identical with respect to their behaviour towards the phenomenon considered. If this assumption is not true, there is the possibility of extraneous variation entering into the treatment effect. However, data can be collected in such a design without the introduction of problems with the passage of time. In this respect the design is superior to before-and-after without control design.

**3. Before-and-after with control design:** In this design two areas are selected and the dependent variable is measured in both the areas for an identical time-period before the treatment. The treatment is then introduced into the test area only, and the dependent variable is measured in both for an identical time-period after the introduction of the treatment. The treatment effect is determined by subtracting the change in the dependent variable in the control area from the change in the dependent variable in test area. This design can be shown in this way:

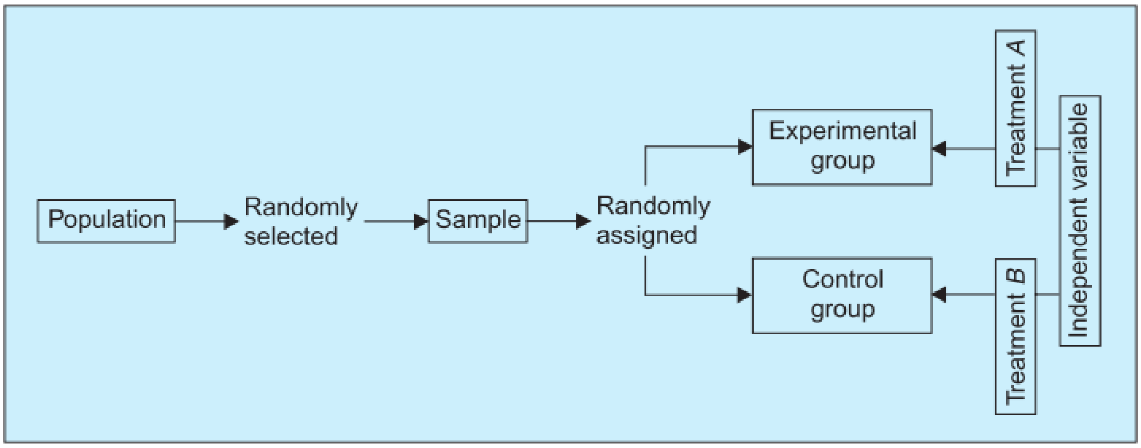



This design is superior to the above two designs for the simple reason that it avoids extraneous variation resulting both from the passage of time and from non-comparability of the test and control areas. But at times, due to lack of historical data, time or a comparable control area, we should prefer to select one of the first two informal designs stated above.

**4. Completely randomized design (C.R. design):** Involves only two principles viz., the principle of replication and the principle of randomization of experimental designs. It is the simplest possible design and its procedure of analysis is also easier. The essential characteristic of the design is that subjects are randomly assigned to experimental treatments (or vice-versa). For instance, if we have 10 subjects and if we wish to test 5 under treatment A and 5 under treatment B, the randomization process gives every possible group of 5 subjects selected from a set of 10 an equal opportunity of being assigned to treatment A and treatment B. One-way analysis of variance is used to analyse such a design. Even unequal replications can also work in this design. It provides maximum number of degrees of freedom to the error. Such a design is generally used when experimental areas happen to be homogeneous. Technically, when all the variations due to uncontrolled extraneous factors are included under the heading of chance variation, we refer to the design of experiment as C.R. design.

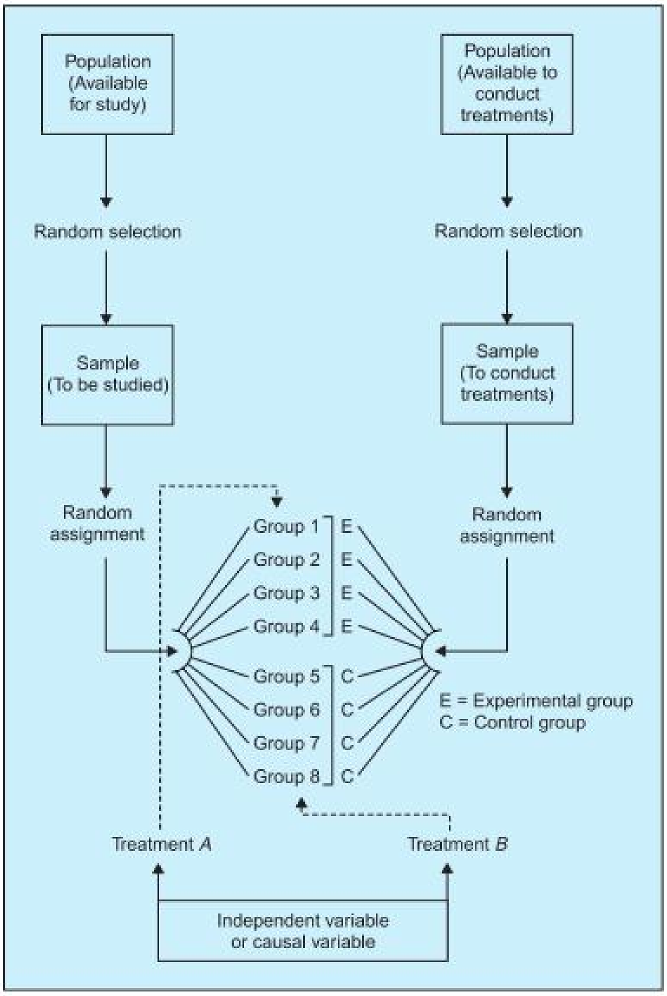
We can present a brief description of the **two forms** of such a design as given below.

**(i) Two-group simple randomized design:** In a two-group simple randomized design, first of all the population is defined and then from the population a sample is selected randomly. Further, requirement of this design is that items after being selected randomly from the population, be randomly assigned to the experimental and control groups (Such random assignment of items to two groups is technically described as principle of randomization). Thus, this design yields two groups as representatives of the population. In a diagram form this design can be shown in this way:




Since in the sample randomized design the elements constituting the sample are randomly drawn from the same population and randomly assigned to the experimental and control groups, it becomes possible to draw conclusions on the basis of samples applicable for the population. The two groups (experimental and control groups) of such a design are given different treatments of the independent variable. This design of experiment is quite common in research studies concerning behavioural sciences. The merit of such a design is that it is simple and randomizes the differences among the sample items. But the limitation of it is that the individual differences among those conducting the treatments are not eliminated, i.e., it does not control the extraneous variable and as such the result of the experiment may not depict a correct picture. This can be illustrated by taking an example. Suppose the researcher wants to compare two groups of students who have been randomly selected and randomly assigned. Two different treatments viz., the usual training and the specialised training are being given to the two groups. The researcher hypothesises greater gains for the group receiving specialised training. To determine this, he tests each group before and after the training, and then compares the amount of gain for the two groups to accept or reject his hypothesis. This is an illustration of the two-groups randomized design, wherein individual differences among students are being randomized. But this does not control the differential effects of the extraneous independent variables (in this case, the individual differences among those conducting the training programme).

**(ii) Random replications design:** The limitation of the two-group randomized design is usually eliminated within the random replications design. In the illustration just cited above, the teacher differences on the dependent variable were ignored, i.e., the extraneous variable was not controlled. But in a random replications design, the effect of such differences are minimised (or reduced) by providing a number of repetitions for each treatment. Each repetition is technically called a 'replication'. Random replication design serves two purposes viz., it provides controls for the differential effects of the extraneous independent variables and secondly, it randomizes any individual differences among those conducting the treatments. Diagrammatically we can illustrate the random replications design thus:




From the diagram it is clear that there are two populations in the replication design. The sample is taken randomly from the population available for study and is randomly assigned to, say, four experimental and four control groups. Similarly, sample is taken randomly from the population available to conduct experiments (because of the eight groups eight such individuals be selected) and the eight individuals so selected should be randomly assigned to the eight groups. Generally, equal number of items is put in each group so that the size of the group is not likely to affect the result of the study. Variables relating to both population characteristics are assumed to be randomly distributed among the two groups. Thus, this random replication design is, in fact, an extension of the two-group simple randomized design.

**5. Randomized block design (R.B. design)** is an improvement over the C.R. design. In the R.B. design the principle of local control can be applied along with the other two principles of experimental designs. In the R.B. design, subjects are first divided into groups, known as blocks, such that within each group the subjects are relatively homogeneous in respect to some selected variable. The variable selected for grouping the subjects is one that is believed to be related to the measures to be obtained in respect of the dependent variable. The number of subjects in a given block would be equal to the number of treatments and one subject in each block would be randomly assigned to each treatment. In general, blocks are the levels at which we hold the extraneous factor fixed, so that its contribution to the total variability of data can be measured. The main feature of the R.B. design is that in this each treatment appears the same number of times in each block. The R.B. design is analysed by the two-way analysis of variance technique.

Let us illustrate the R.B. design with the help of an example. Suppose four different forms of a standardised test in statistics were given to each of five students (selected one from each of the five I.Q. blocks) and following are the scores which they obtained.

| | Very low I.Q. | Low I.Q. | Average I.Q. | High I.Q. | Very high I.Q. |
|---|---|---|---|---|---|
| | Student A | Student B | Student C | Student D | Student E |
| Form 1 | 82 | 67 | 57 | 71 | 73 |
| Form 2 | 90 | 68 | 54 | 70 | 81 |
| Form 3 | 86 | 73 | 51 | 69 | 84 |
| Form 4 | 93 | 77 | 60 | 65 | 71 |

If each student separately randomized the order in which he or she took the four tests (by using random numbers or some similar device), we refer to the design of this experiment as a R.B. design. The purpose of this randomization is to take care of such possible extraneous factors (say as fatigue) or perhaps the experience gained from repeatedly taking the test.

**6. Latin square design (L.S. design)** is an experimental design very frequently used in agricultural research. The conditions under which agricultural investigations are carried out are different from those in other studies for nature plays an important role in agriculture. For instance, an experiment has to be made through which the effects of five different varieties of fertilizers on the yield of a certain crop, say wheat, it to be judged. In such a case the varying fertility of the soil in different blocks in which the experiment has to be performed must be taken into consideration; otherwise the results obtained may not be very dependable because the output happens to be the effect not only of fertilizers, but it may also be the effect of fertility of soil. Similarly, there may be impact of varying seeds on the yield. To overcome such difficulties, the L.S. design is used when there are two major extraneous factors such as the varying soil fertility and varying seeds. The Latin-square design is one wherein each fertilizer, in our example, appears five times but is used only once in each row and in each column of the design. In other words, the treatments in a L.S. design are so allocated among the plots that no treatment occurs more than once in any one row or any one column. The two blocking factors may be represented through rows and columns (one through rows and the other through columns). The following is a diagrammatic form of such a design in respect of, say, five types of fertilizers, viz., A, B, C, D and E and the two blocking factor viz., the varying soil fertility and the varying seeds:

FERTILITY LEVEL

Seeds differences

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| $X_1$ | A | B | C | D | E |
| $X_2$ | B | C | D | E | A |
| $X_3$ | C | D | E | A | B |
| $X_4$ | D | E | A | B | C |
| $X_5$ | E | A | B | C | D |

The above diagram clearly shows that in a L.S. design the field is divided into as many blocks as there are varieties of fertilizers and then each block is again divided into as many parts as there are varieties of fertilizers in such a way that each of the fertilizer variety is used in each of the block (whether columnwise or row-wise) only once. The analysis of the L.S. design is very similar to the two-way ANOVA technique.

The merit of this experimental design is that it enables differences in fertility gradients in the field to be eliminated in comparison to the effects of different varieties of fertilizers on the yield of the crop. But this design suffers from one limitation, and it is that although each row and each column represents equally all fertilizer varieties, there may be considerable difference in the row and column means both up and across the field. This, in other words, means that in L.S. design we must assume that there is no interaction between treatments and blocking factors. This defect can, however, be removed by taking the means of rows and columns equal to the field mean by adjusting the results. Another limitation of this design is that it requires number of rows, columns and treatments to be equal. This reduces the utility of this design. In case of (2 × 2) L.S. design, there are no degrees of freedom available for the mean square error and hence the design cannot be used. If treatments are 10 or more, than each row and each column will be larger in size so that rows and columns may not be homogeneous. This may make the application of the principle of local control ineffective. Therefore, L.S. design of orders (5 × 5) to (9 × 9) is generally used.

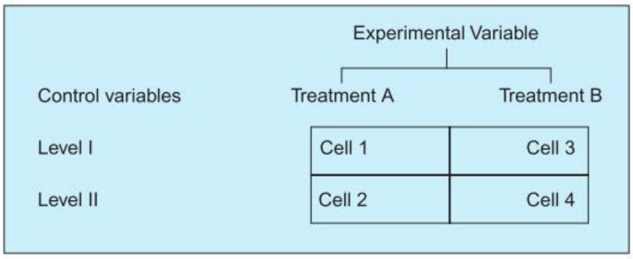
**7. Factorial designs:** Factorial designs are used in experiments where the effects of varying more than one factor are to be determined. They are specially important in several economic and social phenomena where usually a large number of factors affect a particular problem. Factorial designs can be of two types: (i) simple factorial designs and (ii) complex factorial designs.

(i) Simple factorial designs: In case of simple factorial designs, we consider the effects of varying two factors on the dependent variable, but when an experiment is done with more than two factors, we use complex factorial designs. Simple factorial design is also termed as a 'two-factor-factorial design', whereas complex factorial design is known as 'multifactor-factorial design.' Simple factorial design may either be a 2 × 2 simple factorial design, or it may be, say, 3 × 4 or 5 × 3 or the like type of simple factorial design. We illustrate some simple factorial designs as under:

***Illustration 1:*** (2 × 2 simple factorial design).

A 2 × 2 simple factorial design can graphically be depicted as follows:

**2 × 2 SIMPLE FACTORIAL DESIGN**




In this design the extraneous variable to be controlled by homogeneity is called the control variable and the independent variable, which is manipulated, is called the experimental variable. Then there are two treatments of the experimental variable and two levels of the control variable. As such there are four cells into which the sample is divided. Each of the four combinations would provide one treatment or experimental condition. Subjects are assigned at random to each treatment in the same manner as in a randomized group design. The means for different cells may be obtained along with the means for different rows and columns. Means of different cells represent the mean scores for the dependent variable and the column means in the given design are termed the main effect for treatments without taking into account any differential effect that is due to the level of the control variable. Similarly, the row means in the said design are termed the main effects for levels without regard to treatment. Thus, through this design we can study the main effects of treatments as well as the main effects of levels. An additional merit of this design is that one can examine the interaction between treatments and levels, through which one may say whether the treatment and levels are independent of each other or they are not so. The following examples make clear the interaction effect between treatments and levels. The data obtained in case of two (2 × 2) simple factorial studies may be as given below.

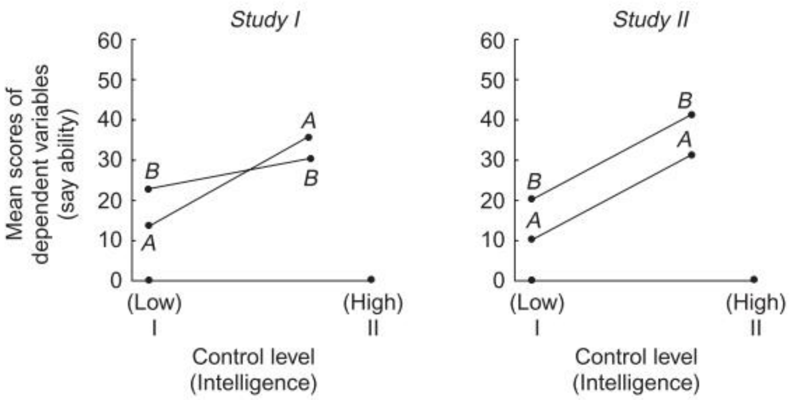
**STUDY I DATA**

| | | Training | | |
|---|---|---|---|---|
| | | Treatment A | Treatment B | Row Mean |
| Control (Intelligence) | Level I (Low) | 15.5 | 23.3 | 19.4 |
| | Level II (High) | 35.8 | 30.2 | 33.0 |
| | Column mean | 25.6 | 26.7 | |

**STUDY II DATA**

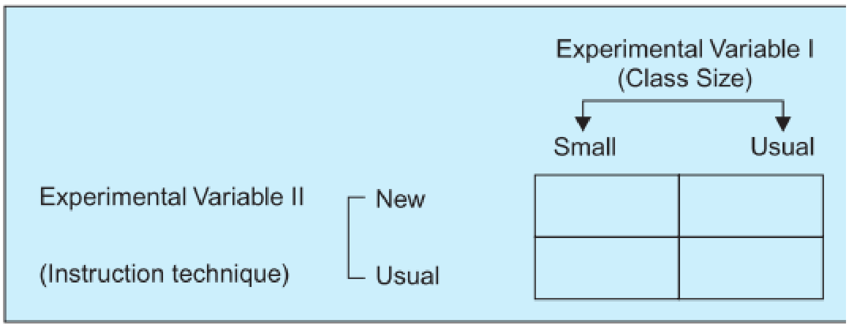
| | | Training | | |
|---|---|---|---|---|
| | | Treatment A | Treatment B | Row Mean |
| Control (Intelligence) | Level I (Low) | 10.4 | 20.6 | 15.5 |
| | Level II (High) | 30.6 | 40.4 | 35.5 |
| | Column mean | 20.5 | 30.5 | |

All the above figures (the study I data and the study II data) represent the respective means. Graphically, these can be represented as shown in Fig. 3.10.



The graph relating to Study I indicates that there is an interaction between the treatment and the level which, in other words, means that the treatment and the level are not independent of each other. The graph relating to Study II shows that there is no interaction effect which means that treatment and level in this study are relatively independent of each other.

The 2 × 2 design need not be restricted in the manner as explained above i.e., having one experimental variable and one control variable, but it may also be of the type having two experimental variables or two control variables. For example, a college teacher compared the effect of the class size as well as the introduction of the new instruction technique on the learning of research methodology. For this purpose he conducted a study using a 2 × 2 simple factorial design. His design in the graphic form would be as follows:




But if the teacher uses a design for comparing males and females and the senior and junior students in the college as they relate to the knowledge of research methodology, in that case we will have a 2 × 2 simple factorial design wherein both the variables are control variables as no manipulation is involved in respect of both the variables.

*Illustration 2:* (4 × 3 simple factorial design).

The 4 × 3 simple factorial design will usually include four treatments of the experimental variable and three levels of the control variable. Graphically it may take the following form:

**4 × 3 SIMPLE FACTORIAL DESIGN**

| Control Variable | Experimental Variable | | | |
|---|---|---|---|---|
| | Treatment A | Treatment B | Treatment C | Treatment D |
| Level I | Cell 1 | Cell 4 | Cell 7 | Cell 10 |
| Level II | Cell 2 | Cell 5 | Cell 8 | Cell 11 |
| Level III | Cell 3 | Cell 6 | Cell 9 | Cell 12 |

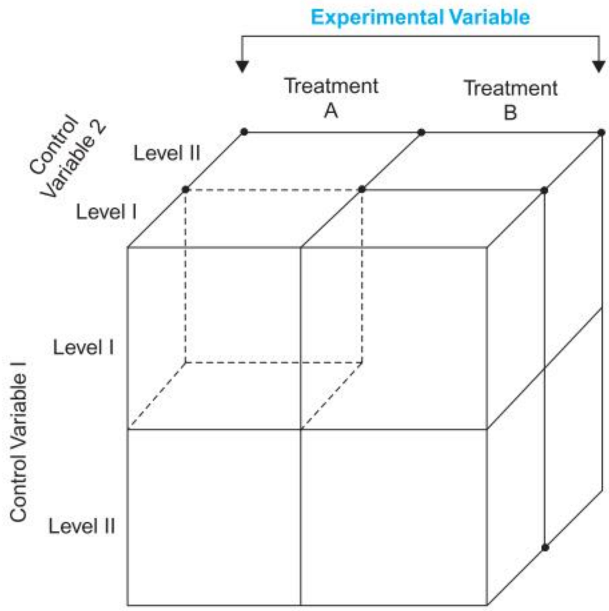
This model of a simple factorial design includes four treatments viz., A, B, C, and D of the experimental variable and three levels viz., I, II, and III of the control variable and has 12 different cells as shown above. This shows that a 2 × 2 simple factorial design can be generalised to any number of treatments and levels. Accordingly we can name it as such and such (–×–) design. In such a design the means for the columns provide the researcher with an estimate of the main effects for treatments and the means for rows provide an estimate of the main effects for the levels. Such a design also enables the researcher to determine the interaction between treatments and levels.

(ii) Complex factorial designs: Experiments with more than two factors at a time involve the use of complex factorial designs. A design which considers three or more independent variables simultaneously is called a complex factorial design. In case of three factors with one experimental variable having two treatments and two control variables, each one of which having two levels, the design used will be termed 2 × 2 × 2 complex factorial design which will contain a total of eight cells as shown below:

## 2 × 2 × 2 COMPLEX FACTORIAL DESIGN

| | | Experimental Variable | | | |
|---|---|---|---|---|---|
| | | Treatment A | | Treatment B | |
| | | Control Variable 2 Level I | Control Variable 2 Level II | Control Variable 2 Level I | Control Variable 2 Level II |
| Control Variable 1 | Level I | Cell 1 | Cell 3 | Cell 5 | Cell 7 |
| | Level II | Cell 2 | Cell 4 | Cell 6 | Cell 8 |

In Fig. 3.14 a pictorial presentation is given of the design shown below.




The dotted line cell in the diagram corresponds to Cell 1 of the above stated 2 × 2 × 2 design and is for Treatment A, level I of the control variable 1, and level I of the control variable 2. From this design it is possible to determine the main effects for three variables i.e., one experimental and two control variables. The researcher can also determine the interactions between each possible pair of variables (such interactions are called 'First Order interactions') and interaction between variable taken in triplets (such interactions are called Second Order interactions). In case of a 2 × 2 × 2 design, the further given first order interactions are possible:

Experimental variable with control variable 1 (or EV × CV 1);

Experimental variable with control variable 2 (or EV × CV 2);

Control variable 1 with control variable 2 (or CV1 × CV2);

Three will be one second order interaction as well in the given design (it is between all the three variables i.e., EV × CV1 × CV2).

To determine the main effects for the experimental variable, the researcher must necessarily compare the combined mean of data in cells 1, 2, 3 and 4 for Treatment A with the combined mean of data in cells 5, 6, 7 and 8 for Treatment B. In this way the main effect for experimental variable, independent of control variable 1 and variable 2, is obtained. Similarly, the main effect for control variable 1, independent of experimental variable and control variable 2, is obtained if we compare the combined mean of data in cells 1, 3, 5 and 7 with the combined mean of data in cells 2, 4, 6 and 8 of our 2 × 2 × 2 factorial design. On similar lines, one can determine the main effect for the control variable 2 independent of experimental variable and control variable 1, if the combined mean of data in cells 1, 2, 5 and 6 are compared with the combined mean of data in cells 3, 4, 7 and 8.

To obtain the first order interaction, say, for EV × CV1 in the above stated design, the researcher must necessarily ignore control variable 2 for which purpose he may develop 2 × 2 design from the 2 × 2 × 2 design by combining the data of the relevant cells of the latter design as shown below:

| | | Experimental Variables | |
|---|---|---|---|
| | | Treatment A | Treatment B |
| Control Variable 1 | Level I | Cells 1, 3 | Cells 5, 7 |
| | Level II | Cells 2, 4 | Cells 6, 8 |

Similarly, the researcher can determine other first order interactions. The analysis of the first order interaction, in the manner described above, is essentially a sample factorial analysis as only two variables are considered at a time and the remaining one is ignored. But the analysis of the second order interaction would not ignore one of the three independent variables in case of a 2 × 2 × 2 design. The analysis would be termed as a complex factorial analysis.

It may, however, be remembered that the complex factorial design need not necessarily be of 2 × 2 × 2 type design, but can be generalised to any number and combination of experimental and control independent variables. Of course, the greater the number of independent variables included in a complex factorial design, the higher the order of the interaction analysis possible. But the overall task goes on becoming more and more complicated with the inclusion of more and more independent variables in our design.

Factorial designs are used mainly because of the two advantages. (i) They provide equivalent accuracy (as happens in the case of experiments with only one factor) with less labour and as such are a source of economy. Using factorial designs, we can determine the main effects of two (in simple factorial design) or more (in case of complex factorial design) factors (or variables) in one single experiment. (ii) They permit various other comparisons of interest. For example, they give information about such effects which cannot be obtained by treating one single factor at a time. The determination of interaction effects is possible in case of factorial designs.

*The researcher must decide in advance of collection and analysis of data as to which design would prove to be more appropriate for his research project. He must give due weight to various points such as the type of universe and its nature, the objective of his study, the resource list or the sampling frame, desired standard of accuracy and the like when taking a decision in respect of the design for his research project.*

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