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Prior Analytics A8–22 in Predicate Logic

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ARISTOTLE'S MODAL PROOFS

Prior Analytics A8–22 in Predicate Logic

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Thec hapters below carry through on a project begun in a series of articles (Rini 1995, 1996, 1998, 2000, 2003, 2007). There are two people without whose help and insight this book could not have been written: Robin Smith and Max Cresswell. Smith's (1989) translation of the *Prior Analytics* and his extensive commentary on the syllogistic have helped to make Aristotle's technical works more easily accessible and help to identify and isolate the stages, methods and problems that characterise the modal syllogistic. Cresswell kept reminding me of the importance of asking what *logical* point is being made in a portion of Aristotle's text. I also thank an anonymous reader from Springer whose thoughtful comments and criticisms prompted substantial improvements in the manuscript. I am grateful to Helga Kocurek for English translations of passages from Ebert and Nortmann's (2007) German translation and commentary on Aristotle's *Prior Analytics*. Finally, I thank Springer's editorial and production staff for their courteousness and helpfulness at all stages.

Readers familiar with the literature on Aristotle's *Prior Analytics* will appreciate the influence that earlier studies by Paul Thom, Ulrich Nortmann and others have had on my thinking. While we are not always agreed on our readings I am greatly in their debt.

Note on the translations:

Introduction

Aristotle invented logic. He was the first to devise a way to study not just examples of human reasoning but the very patterns and structures of human reasoning. This invention is the subject of the early chapters of Aristotle's *Prior Analytics*. Aristotle is keenly aware that he has created a new and useful tool – he himself sees his system of *syllogistic* as the foundation of all our scientific reasoning about the world, and he takes great care to show how the syllogistic can accommodate the kind of *necessity* and *possibility* that together form the basic building blocks of his science. But his exploration of syllogisms involving necessity and possibility is generally thought to cast a shadow over his otherwise brilliant innovation. Until recently the standard view of Aristotle's syllogistic has been that it separates into two parts. First, there is the basic syllogistic set out in *Prior Analytics* A1–7, made up of fourteen argument forms. This was the traditional logic taught in universities right up until it was replaced by modern formal logic in the nineteenth century, and this basic system of syllogistic is what today is often called *non-modal* (or sometimes the *assertoric*) syllogistic. There is a clarity and simplicity to the non-modal syllogistic that is easy to see – and anyone who thinks logic is beautiful will likely find that beauty alive in Aristotle's non-modal syllogistic too. By and large a modern reader can approach it with an easy familiarity. It looks in the main how we expect a logic to look. We can see what Aristotle is doing, and we can see why it is good. The second part is the *modal* syllogistic, *Prior Analytics* A8–22, in which Aristotle studies syllogisms about necessity and possibility. While the invention of the simple system of syllogistic logic – i.e., the non-modal syllogistic – is almost always rated as one of mankind's greatest accomplishments, the modal syllogistic has been infamously labelled 'a failure', 'incoherent', 'a realm of darkness'.

This book is written in the conviction that whether or not the modal syllogistic *is* a realm of darkness is a question of *logic*. The *Prior Analytics* is after all a work of logic. That is why this must be a logic book. It is a logic book in the tradition of work by McCall (1963), Johnson (1989), Thomason (1993, 1997), Patterson (1995), Nortmann (1996), Thom (1996), Schmidt (2000), Malink (2006), and others, who offer formal modellings of Aristotle's modal syllogistic. Many of those who produce a formal modelling for the modal syllogistic present a set of axioms or principles from which they derive as theorems formal representations of those and only those syllogisms that they consider Aristotle accepted, or ought to have accepted. But the modal syllogistic is more than just a list of valid forms. In it Aristotle tries to explain just why modal conclusions really do follow from modal premises. I have made it my project to present and evaluate his proof methods. I have tried to show that there is a simple logical structure to this part of the *Analytics*. The first and most noticeable respect in which the present work differs from many of these other modern interpretations is in the fact that it is concerned with a logical representation of the ways in which Aristotle *himself* tries to *prove* his modal syllogisms.

A second respect in which the present work differs from others is that it uses standard predicate logic translations with only *de re* modality. The reason for preferring predicate logic is philosophically important. Certainly, part of what I want to illustrate in the course of the following chapters is that when our project is *interpreting* Aristotle's *Prior Analytics* A8–22, then we need something which gives a precise representation by formulae which have standard and well-understood meanings. The simplest way to achieve that, in my view, is to make use of modern modal predicate logic. Of the recent scholarly interpretations mentioned above, only Nortmann (1996) and Schmidt (2000) study Aristotle using modern modal predicate logic. But their interpretations include in addition to *de re* modals, also *de dicto* modals, and this saddles them with more sophisticated formal techniques than the present study requires. Perhaps most scholars who work on the subject will disagree with this predicate logic approach. Some resist it strongly. They object to the introduction of such powerful tools as the individual variable and all that it affords. And, to be sure, Aristotle does not have the individual variable and the associated combination of unrestricted quantifiers and truth-functional connectives. But because logicians and philosophers know how to interpret predicate logic this makes it an especially useful tool. Clearly it is a far more powerful tool than we want to attribute to Aristotle, but that is a different point altogether.

Another major need in any interpretation of Aristotle's syllogistic is close adherence to the text of the *Prior Analytics*. I have based my discussion on Smith's (1989) translation, and I have consulted other translations and referred to the original Greek whenever this has seemed necessary. In fact the problems in interpreting *An.Pr.* A8–22 are primarily logical and philosophical, and are only peripherally illuminated by the nuances of Aristotle's Greek. I have of course paid close attention to those cases where the Greek itself is vital. The question of adherence to the text takes another form also. There is a well-known need for an account of how Aristotle's system of logic relates to the rest of his philosophy. A careful and conscientious scholar looks to identify and investigate links between individual parts of the *Prior Analytics* and other parts of Aristotle's works. There is not anyone who disputes this – it is one of the big questions within Aristotle scholarship. We all want the links between the *Prior Analytics* and the rest explained, and eventually that must be done. But before we can study the links, we first need an account of what is going on in *An.Pr.* A8–22 – an account that deals with the modal syllogistic itself, that closely respects Aristotle's textual discussion in those chapters. Whether we can relate the logic to other parts of Aristotle's philosophy is an important consideration, but it should not be allowed to get in the way of the more basic project of explaining the modal syllogistic as it is set out in *An.Pr.* A8–22.

One criticism of the 'logical' approach may be that it is going to distort Aristotle. Undoubtedly there is much truth in that. Predicate logic translations are one distortion. In this study I allow what might appear to be a second distortion: I distinguish different kinds of terms which I label 'red' and 'green'. Red terms are terms like 'horse', 'plant' or 'man'. They name things in virtue of features those things *must* have. Green terms are terms like 'moving', which name things in virtue of their non-necessary features; since a horse might be moving, but need not be moving. I put this distinction to work in Part II, showing how it helps to make sense of syllogisms involving necessity. It is a clumsy distinction, and Part III shows the need for refinement in order to accommodate all of the claims Aristotle makes in *An.Pr.* about possibility.

This view of the syllogistic is contentious. Barnes (2007, p. 133), for example, thinks any such approach wrong-headed:

> Of course, Aristotle's syllogistic is essentially tied to the concept of predication; for the argument forms which it examines are fixed by a certain logical structure, namely the subject–predicate structure. But nothing in the syllogistic requires, or even suggests, any classification of predicates: that a predicate is substantial or qualitative, relational or a matter of *habitus* – all that is of supreme indifference to the syllogistic.

Barnes is undoubtedly right for the *non-modal* syllogistic. The machinery Aristotle develops in the non-modal syllogistic is what I later call a colour-blind system – it does not take account of any difference between kinds of terms. Red and green are not at play in the non-modal syllogistic. We can certainly take Barnes's comments that way. But what is important to consider is that when Aristotle investigates the *modal* syllogistic, there is a straightforward way of understanding him according to which he is showing that if you put colour into his basic syllogistic machinery then you get colour out of his basic syllogistic machinery.

The *Prior Analytics* is a technical work, and ultimately it is Aristotle's technical tools and logical language that need to be interpreted and explained. Part I sets the foundation, explaining the scholarly tradition, and explaining the basic building blocks of Aristotle's system. Part II focuses on Aristotle's syllogistic involving necessity – the *apodeictic* syllogistic. I will show that Aristotle's apodeictic syllogistic in *Prior Analytics* A8–11 requires nothing more than the simple non-modal syllogistic system of *An.Pr.* A1–7, together with a general restriction. The restriction depends on a principle that I call the Substance Principle.

> ***The Substance Principle (SP)***
> If φ is a red term then φ is equivalent to necessarily-φ.

Any time a red term applies to something at all it also applies by necessity. There are two ways we might understand this. We might take SP to describe two different *terms*, 'man' and 'necessary man' – one a non-modal term and one a modal term. Or we might take SP to describe two different *relations* between an individual and a single term – for we might say that this same term can apply or can apply by necessity. SP is intended to be neutral on this question. All it insists is that if a term is red then if it applies at all it applies by necessity.

The restriction applies to one of Aristotle's most basic proof methods in the syllogistic – what he calls conversion. In establishing the validity of a syllogistic schema Aristotle often converts the terms – that is, he switches the order of the subject term and the predicate term. Such conversions are straightforward in the non-modal syllogistic. Aristotle moves easily from 'some men are moving' to 'some moving things are men'. But when modals are involved the conversions sometimes need to be carefully restricted. Aristotle does not give much guidance about restrictions in the *Prior Analytics*, but he seems to need something like the restriction he describes in a passage in *Posterior Analytics* A22. In that passage Aristotle explains that the only way to *genuinely predicate* is to predicate something of a subject which is identified by a term for something with an essential nature. Take for example a proposition (i):

(i) All men are necessary-animals

Proposition (i) is an example of a genuine predication – the subject term 'man' is a red term. It names a substance in virtue of its essential nature. Consider another example (ii):

(ii) All moving things are necessary-animals

Proposition (ii) is not genuine. *Moving thing* is not a red term.[1] Aristotle seems to have something like genuine predication in mind in the *Prior Analytics* when he describes certain modal conversion principles.

> ***The Genuineness Requirement***
> In certain modal conversions the 'input' proposition must be genuine — its subject must be a red term.

For example, when we try to convert propositions (i) and (ii) we get different results: (i) 'All men are necessary-animals' converts to give us 'Some animals are necessary-men.' But (ii) 'All moving things are necessary-animals' does not validly convert – i.e., we do not get 'Some animals are necessary-movers.'

[1]In *An.Post.* A22, Aristotle suggests that even though a term like 'moving thing' might be the *grammatical* subject of the proposition, it is not always appropriate to treat it as the real *logical* subject. Even so, in the *Prior Analytics*, Aristotle often uses propositions such as (ii) as premises in the logic, but not where he requires modal conversion.

In the case of what I call LE-conversion, in which we convert a privative universal proposition, Aristotle requires a *negative* form of the Substance Principle:

> ***Neg(SP)***
> Where φ is a red term then not only is φ equivalent to necessarily-φ, but also not-φ is equivalent to necessarily-not-φ.

Aristotle does not appear to realize that Neg(SP) does not follow from the Substance Principle.

The function of the Substance Principle together with the Genuineness Requirement is to guide our choice of syllogistic terms, making sure that we fit the right Aristotelian terms into the right locations when we are constructing and proving syllogisms about necessity. The semantic constraints, however, are not made explicit in the *Prior Analytics*. If we suppose that Aristotle really is a good logician, then the need in the apodeictic syllogistic for some kind of implicit guiding principles becomes obvious. One consequence of this approach is that the semantic restrictions are doing the real work and the apodeictic syllogistic does not depend upon principles of *modal* logic. There are historical antecedents for this, particularly in the work of the medieval philosophers Averröes and Kilwardby, who distinguish modal and non-modal syllogisms according to the type of terms occurring in them. (See Thom 2003, and Knuuttila 2008.)

Part III deals with syllogisms involving possibility. This part is traditionally known as the *problematic* syllogistic. It is the subject of *An.Pr.* A13–22. In Part III there are two different methods at work: *ampliation* and *realization*.

> ***Ampliation***
> Aristotle makes a distinction (32b24–37) between the following kinds of construction involving possibility:
>
> (i) Some *B* is a possible *A*
> (ii) Some possible *B* is a possible *A*
>
> The second, where both the *A* and *B* term involve possibility, is, in the medieval jargon, an ampliated proposition. By contrast, (i) is unampliated: only the *A* term – i.e., the predicate term – involves possibility. Using ampliation many of the syllogisms in the problematic syllogistic turn out to be substitution instances of non-modal syllogisms, and are therefore trivial. Here, too, it emerges that principles of *modal* logic are not involved.

In cases where ampliation is not available, Aristotle introduces a new method. This is easiest to explain with an example. An acorn can become an oak or remain a non-oak. So although an oak is a necessary oak (the Substance Principle guarantees this), a non-oak, for instance an acorn, is *not* a necessary-non-oak, for an acorn *could become an oak* – its potentiality could be *realized*.

> ***Realization***
> Given a premise that something is possible, assume that the possibility is realized, and then reason non-modally. Any non-modal proposition obtained in this way may then be concluded to be possible.

There are undoubtedly places where Aristotle seems not quite as sure footed as we would like him to be. Often, for instance, Aristotle seems to get into difficulty because of his views about negation. Another difficulty, which arises in the problematic syllogistic, is how he handles the tension between realization and the negative form of the Substance Principle. But even acknowledging such problems, what I have tried to tell in the chapters that follow is how to find a clarity and beauty, and even a surprising simplicity, in Aristotle's modal syllogistic.

Note on the formal notation:

I use standard symbols of the Lower Predicate Calculus (LPC) throughout (i.e. $\sim$, $\vee$, &, $\supset$, $\equiv$, $\forall$, and $\exists$), with L for 'it is necessary that' and M for 'it is possible that' (in the sense in which $M\varphi \equiv \sim L\sim\varphi$) and Q for 'it is contingent that', where $Q\varphi$ implies both $M\varphi$ and $M\sim\varphi$. The letters L, M and Q, along with X, are also used to indicate the modal status of a proposition, and in that case they are not italicised. I follow McCall's system for classifying the modal status of a syllogism or schema, as described on p. 45. Where authors use a different notation, as for instance N for necessity and P for possibility, I have translated their notation into my own, unless I am explicitly commenting on notational matters.

Part I

MODERN METHODS FOR ANCIENT LOGIC

Chapter 1
The Non-Modal Syllogistic: *An.Pr.* A1–7

This chapter provides a brief outline of what is usually called the *assertoric syllogistic*. This is the part of Aristotle's system that deals specifically with syllogisms from non-modal premises. Aristotle discusses these in *Prior Analytics* A1–7. The assertoric syllogistic provides the foundation for the entire syllogistic system. Aristotle's methods in the assertoric syllogistic are generally clear, easy to understand, and, as Aristotle scholarship goes, the methods here are relatively uncontroversial. The material in this chapter provides the backdrop for the modal syllogisms and does not, I trust, introduce any substantive new controversy. The only potential controversy comes from my preference for using modern Lower Predicate Calculus (LPC) to represent and analyze Aristotle's logical idioms. The use of LPC will be defended in Chapter 2.

The building blocks of the syllogistic are simple propositions – declarative sentences which are capable of being true or false. Aristotle describes these propositions as either *affirmative* (sometimes *positive*) or *privative* (*An.Pr.* 24a16). 'Grammar belongs to all men' *affirms* something (being grammatical) of a subject. 'Grammar belongs to no horses' is *privative* because it denies something (being grammatical) of a subject. The 'something' that is affirmed or denied is the *predicate* of the proposition.

In Aristotle's syllogistic propositions, the subject and predicate terms name things within certain Aristotelian categories, and propositions composed of such general terms are traditionally called *categorical sentences*. Predicates can be affirmed or denied either of the whole of a subject or only of part of a subject. When a predicate is affirmed of the whole of a subject, Aristotle calls the proposition a *universal affirmative*. 'Animal belongs to all men' is a (true) universal affirmative (because all men are animals). When a predicate is denied of the whole of a subject, Aristotle calls the proposition a *universal privative* – for example, 'Animal belongs to no horses' (false) or 'Grammar belongs to no horses' (true). When a predicate is affirmed of part of a subject, the result is a *particular affirmative* – e.g., 'White belongs to some men.' And a predicate denied of a part of a subject results in a *particular privative* proposition – e.g., 'White does not belong to some horses.'

Aristotle uses *variables* to stand for the subject and predicate terms. Commentators use the vowels A, E, I, and O to represent Aristotle's four types of categorical sentences as follows:

(A)	'*A* belongs to all *B*'	::	*universal affirmative*
(E)	'*A* belongs to no *B*'	::	*universal privative*
(I)	'*A* belongs to some *B*'	::	*particular affirmative*

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(O) '*A* does not belong to some *B*' :: *particular privative*

The most important of Aristotle's proof methods in the syllogistic is what he calls *conversion*. Aristotle explains in *An.Pr.* A2 how conversion works: 'It is necessary for a universal privative premise of belonging to convert with respect to its terms' (25a6). I will follow custom and call this E-Conversion:

E-Conversion '*A* belongs to no *B*' converts to '*B* belongs to no *A*.'

'And the positive premise necessarily converts, though not universally but in part' (a8). This gives A-conversion:

A-Conversion '*A* belongs to all *B*' converts to '*B* belongs to some *A*.'

'Among the particular premises [I and O], the affirmative [I] must convert partially...' (a10):

I-Conversion '*A* belongs to some *B*' converts to '*B* belongs to some *A*'

Conversion effectively changes the order of the subject and predicate terms. This device plays a crucial role in Aristotle's syllogistic proof method. The conversions must therefore be logically valid principles. And, in 25a14–25, Aristotle gives proofs to establish their validity. But as he explains, in 25a12, there is no O-Conversion: 'the privative premise [O] need not [convert].' He gives a counter-example to show why: 'for it is not the case that if man does not belong to some animal, then animal will not belong to some man.' It is true that man does not belong to some animal – a horse, for example, is just such an animal since for a thing to be a horse excludes its being a man. But animal will always belong to every man, since being an animal is part of what it is to be a man. So when *A* is man and *B* is animal, converting '*A* does not belong to some *B*' takes us from a true proposition to a false proposition.

Man does not belong to some animal	T
Animal does not belong to some man	F

And Aristotle's point is that this 'conversion' is illegitimate: O-Conversion cannot be guaranteed. In modern parlance, O-Conversion is not valid.

Aristotle takes pairs of categorical sentences which share exactly one term in common and then asks whether from a given premise pair a conclusion relating the other two terms must follow. The term in common to the premises may be the subject of one premise and the predicate of the other (Aristotle calls this combination the *first figure*), the term in common may be the predicate of both premises (the *second figure*), or the term in common may be the subject of both premises (the *third figure*).[1] Aristotle describes the three figures in *An.Pr.* A4–6, and considers each premise pair in turn in order to determine whether a conclusion follows:

First figure	Second figure	Third figure
predicate-subject	predicate-subject	predicate-subject
A-B	A-B	A-C
B-C	A-C	B-C
A-C	B-C	A-B

If a conclusion does follow, Aristotle constructs a proof. If a conclusion does not follow, he offers a counter-example to show that true premises may sometimes yield a false conclusion. Modern logicians describe such deductive arguments as valid or invalid. But the words *valid* and *invalid* are not part of Aristotle's vocabulary. Where there is a valid deduction, Aristotle says 'there is a syllogism.' Where there is not a valid deduction, he says 'there is no syllogism.' So strictly speaking an Aristotelian syllogism is always a *valid* deductive argument. Table 1 lists plain English versions of all of Aristotle's (valid) non-modal syllogisms in the three figures, using their traditional medieval names.

[1]In the non-modal logic Aristotle considers what commentators call a 'fourth figure'. The fourth figure moods are obtained by converting the conclusion of a first figure syllogism. Aristotle does not offer any detailed investigation of these in the modal logic.

Table 1 Non-Modal Syllogisms

First Figure:	Second Figure:	Third Figure:
Barbara A belongs to every B B belongs to every C A belongs to every C	Cesare A belongs to no B A belongs to every C B belongs to no C	Darapti A belongs to every C B belongs to every C A belongs to some B
Celarent A belongs to no B B belongs to every C A belongs to no C	Camestres A belongs to every B A belongs to no C B belongs to no C	Felapton A belongs to no C B belongs to every C A does not belong to some B
Darii A belongs to every B B belongs to some C A belongs to some C	Festino A belongs to no B A belongs to some C B does not belong to some C	Datisi A belongs to every C B belongs to some C A belongs to some B
Ferio A belongs to no B B belongs to some C A does not belong to some C	Baroco A belongs to every B A does not belong to some C B does not belong to some C	Disamis A belongs to some C B belongs to every C A belongs to some B
		Bocardo A does not belong to some C B belongs to every C A does not belong to some B
		Ferison A belongs to no C B belongs to some C A does not belong to some B

Medieval scholars concocted a system of mnemonics to remember how to complete the various syllogistic proofs. In most commentaries today the medieval mnemonics are used as a convenient way to label the syllogisms. The names 'Barbara', 'Celarent', etc., encode instructions for Aristotle's proofs. Smith (1989, pp. 229–230) explains these ideas, and I paraphrase Smith here:

- A, E, I, and O are the vowels that occur in the names for the syllogisms. The three As in the name Barbara indicate that the syllogism of that name has three universal affirmative propositions; that is, each of the two premises and the conclusion is an A-type proposition. The name Cesare describes a syllogism in which the first premise is an E-type, the second is an A-type, and the conclusion is an E-type proposition.

- The first letter of each name (B, C, D, or F) tells which of the first figure syllogisms is needed in the proof; for example, the D in Darapti tells that the proof is based on the first figure syllogism called Darii.

- An S after an E or an I indicates that the E or I proposition must be 'converted' in order to complete the proof. (Early scholars called conversions from E and I propositions *conversio simplex*.)

- Aristotle's proofs sometimes require the conversion of A propositions. In the medieval codes, the need for A-Conversion is indicated by the letter P after an A – as in the names Darapti and Felapton. In terms of the medieval codes, this conversion always takes an A proposition and converts it into an I proposition – that is, the conversion goes from a universal affirmative, to a particular affirmative. (Such conversion into a particular is sometimes described as *conversio per accidens*.)

In Aristotle's logic first figure syllogisms are taken as axiomatic – Aristotle's word is *teleion* (25b35), usually translated as 'perfect' or 'complete'. Conversion is important because it provides the principal method by which syllogisms in the other figures are proved. Consider, for example, the third figure Datisi in Table 1. By conversion of the second premise we get

(1) *A* belongs to every *C*
<u>*C* belongs to some *B*</u>
A belongs to some *B*

and of course (1) is just Darii with *B* for *C* and *C* for *B*.

Recent scholarship has added to the syllogistic jargon by introducing schematic representations of Aristotle's categorical propositions. These schematic representations can be confusing because there is no single standard. Take an I-type proposition – that is, a particular affirmative proposition. Depending on the commentator's preferences for ordering subject term, predicate term, and propositional vowel, Aristotle's '*A* belongs to some *B*' can be represented as *Iab*, *Iba*, *aIb* or *bIa*. Surveying the literature a reader might encounter all of these and need to flit between each author's own preferred style.[2] This can be even more difficult than it might first appear, because Aristotle grounds so much of his syllogistic on the conversion principles which reverse the order of the subject term and predicate term. The I proposition '*A* belongs to some *B*' *converts* to '*B* belongs to some *A*.' One author might represent '*A* belongs to some *B*' as *Iab* and then convert this to *Iba*, while another author might represent the initial proposition '*A* belongs to some *B*' as *Iba* which then converts to *Iab*. A reader always has to keep in mind whether an author's *Iab* (or *Iba* or *aIb* or *bIa*) is intended to represent '*A* belongs to some *B*' or '*B* belongs to some *A*.' The abbreviated representations are useful because they help to emphasize the formal structure of Aristotle's system. But there is a danger that any formal representation might look like an attempt to impose an interpretation, and so we need to proceed with some caution. What we want is minimal: a way to represent the categorical propositions which makes the logical-grammatical role of each term immediately identifiable. One way to achieve this is simply to adopt the language of the Lower Predicate Calculus (LPC). Aristotle's '*A* belongs to some *B*' would be represented by $\exists x(Bx \;\&\; Ax)$, where the *B* term is subject and the *A* term is predicate. What this formula says is that there is an x which is a *B*, and that that x is also an *A*. Alternatively, following Hintikka (2004, p. 23), you can think of a game played between 'myself' and 'nature' where I am allowed to choose an x (more correctly I choose an individual to be the value of x). If I can choose one which satisfies both *B* and *A*, I win the game, and if I cannot then nature wins. Either way $\exists x(Bx \;\&\; Ax)$ will be true if and only if there is a *B* of which *A* is true. Aristotle's Greek expressions do not have a variable like x, but it is not hard to see that our LPC formula says just what he does. For Aristotle's '*A* can be predicated of every *B*' we can write $\forall x(Ax \supset Bx)$. This says that no matter what x may be, if it satisfies *B* then it also satisfies *A*, or in Hintikka's account, nature is allowed to pick an x, and it is up to myself to show that if it satisfies *B* then it also satisfies *A*.[3] The full LPC translations of Aristotle's categorical propositions would be as follows:

A	*A* belongs to every *B*	$\forall x(Bx \supset Ax)$
E	*A* belongs to no *B*	$\forall x(Bx \supset {\sim}Ax)$
I	*A* belongs to some *B*	$\exists x(Bx \;\&\; Ax)$
O	*A* does not belong to some *B*	$\exists x(Bx \;\&\; {\sim}Ax)$

The LPC translations of Aristotle's conversion principles are:

[2]Patzig clearly means to help when he suggests that we abandon the *Iba* representation for the *Iab* representation. His reason for this is that the latter more closely reflects the order of terms in Aristotle's '*A* belongs to some *B*.' Note, however, that Aristotle also uses the equivalent expression 'Some *B* is *A*.'

[3]An LPC formula like $\forall x(Ax \supset Bx)$ uses a quantifier $\forall x$, which refers to an unrestricted domain of 'things'. This is almost certainly unAristotelian, so it is important to remember that all the formulae which represent the propositions of the Aristotelian syllogistic, have a restriction on the quantifiers.

I-Conversion: $\exists x(Bx \,\&\, Ax)$ converts to $\exists x(Ax \,\&\, Bx)$.

E-Conversion: $\forall x(Bx \supset {\sim}Ax)$ converts to $\forall x(Ax \supset {\sim}Bx)$.

A-Conversion: $\forall x(Bx \supset Ax)$ converts to $\exists x(Ax \,\&\, Bx)$.

Aristotle's '*A* belongs to every *B*' should always be understood as presupposing the existence of some *B*s. (Cf. Crivelli 2004, Chapter 5.) If no terms are empty then $\forall x(Bx \supset Ax)$ is never merely trivially, or vacuously, true. And so, no situation arises in which $\forall x(Bx \supset Ax)$ is true, but $\exists x(Ax \,\&\, Bx)$ is false. So, $\forall x(Bx \supset Ax)$ can always validly convert to $\exists x(Ax \,\&\, Bx)$, as for example the third figure syllogism Darapti requires.

The syllogisms of Table 2 are the LPC equivalents of the non-modal syllogisms of Table 1. Of course the LPC interpretations are subject to the restriction to non-empty terms.

Table 2 Non-Modal Syllogisms in LPC

First Figure	Second Figure	Third Figure
Barbara $\forall x(Bx\supset Ax)$ $\underline{\forall x(Cx\supset Bx)}$ $\forall x(Cx\supset Ax)$	Cesare $\forall x(Bx\supset\sim Ax)$ $\underline{\forall x(Cx\supset Ax)}$ $\forall x(Cx\supset\sim Bx)$	Darapti $\forall x(Cx\supset Ax)$ $\underline{\forall x(Cx\supset Bx)}$ $\exists x(Bx\& Ax)$
Celarent $\forall x(Bx\supset\sim Ax)$ $\underline{\forall x(Cx\supset Bx)}$ $\forall x(Cx\supset\sim Ax)$	Camestres $\forall x(Bx\supset Ax)$ $\underline{\forall x(Cx\supset\sim Ax)}$ $\forall x(Cx\supset\sim Bx)$	Felapton $\forall x(Cx\supset\sim Ax)$ $\underline{\forall x(Cx\supset Bx)}$ $\exists x(Bx\&\sim Ax)$
Darii $\forall x(Bx\supset Ax)$ $\underline{\exists x(Cx\& Bx)}$ $\exists x(Cx\& Ax)$	Festino $\forall x(Bx\supset\sim Ax)$ $\underline{\exists x(Cx\& Ax)}$ $\exists x(Cx\&\sim Bx)$	Datisi $\forall x(Cx\supset Ax)$ $\underline{\exists x(Cx\& Bx)}$ $\exists x(Bx\& Ax)$
Ferio $\forall x(Bx\supset\sim Ax)$ $\underline{\exists x(Cx\& Bx)}$ $\exists x(Cx\&\sim Ax)$	Baroco $\forall x(Bx\supset Ax)$ $\underline{\exists x(Cx\&\sim Ax)}$ $\exists x(Cx\&\sim Bx)$	Disamis $\exists x(Cx\& Ax)$ $\underline{\forall x(Cx\supset Bx)}$ $\exists x(Bx\& Ax)$
		Bocardo $\exists x(Cx\&\sim Ax)$ $\underline{\forall x(Cx\supset Bx)}$ $\exists x(Bx\&\sim Ax)$
		Ferison $\forall x(Cx\supset\sim Ax)$ $\underline{\exists x(Cx\& Bx)}$ $\exists x(Bx\&\sim Ax)$

The syllogisms in Tables 1 and 2 are the deductive schemas Aristotle accepts as valid. These are the 'syllogisms'. What makes each a syllogism is the guarantee that a conclusion of the required form follows from the given premise combination. When Aristotle wants to reject an invalid deductive schema, he takes a premise combination and shows that that combination does not, in itself, guarantee any conclusion. He does this by showing that from true premises, first, we do not get a negative conclusion and, second, neither do we get an affirmative conclusion. So in each case Aristotle gives double-barrelled counter-examples. He gives two sets of terms for each rejected premise combination. Each set is an (ordered) triple of terms ⟨A,B,C⟩, and each set makes the premises true. The difference comes with the conclusion. The first set of terms shows that the conclusion will not always be privative. The terms that make this clear Aristotle calls 'terms for belonging' – from these terms an affirmative conclusion (about belonging) does follow. The second set of terms shows that the conclusion will not always be affirmative. He calls these the terms 'for not belonging'. Often Aristotle effects the difference between belonging and not belonging by changing a single term.

To see how this works, consider an example. At *An.Pr.* 26a2–9, Aristotle wants to make clear that no conclusion logically follows from the first figure premise pair 'all *B*s are *A*' and 'no *C*s are *B*':

> ...nothing necessary results in virtue of these things being so. For it is possible for the first extreme [the A term] to belong to all as well as to none of the last [the C term]. Consequently, neither a particular nor a universal deduction becomes necessary; and, since nothing is necessary because of these, there will not be a deduction. Terms for belonging are animal, man, horse; for belonging to none, animal, man, stone. (26a4–9)

From the first triple ⟨animal, man, horse⟩ we get as our premises

> All men are animals
> No horses are men

but it is clear that from these we cannot get a negative conclusion: we cannot have either 'no horse is an animal' or 'some horse is not an animal' because all horses are animals. So neither an E proposition nor an O proposition follows. So, a privative conclusion is ruled out.

The second set of terms ⟨animal, man, stone⟩ gives the premises

> All men are animals
> No stones are men

but from these we cannot guarantee a universal affirmative conclusion ('all stones are animals' is false) nor can we guarantee a particular affirmative conclusion (it is false that 'some stones are animals'). So, neither an A proposition nor an I proposition follows from the premises. So, an affirmative conclusion is ruled out.

But these exhaust the possibilities. When we begin with premises of the form 'All *B*s are *A* s' and 'No *C*s are *B*s', then *whatever* form we try to give the conclusion – that is, whether we try to make it negative or affirmative, universal or particular (A, E, I, or O) – we can give terms that make the premises true and the conclusion false. And so, no conclusion follows 'of necessity'; so no valid syllogism results from these premises.

In concluding this sketch of the non-modal syllogistic, something should be said about a use of 'necessity' which applies to the syllogistic quite generally. Aristotle defines a syllogism as follows:

> A *deduction* [sullogismos] is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because these things are so. By 'because these things are so' I mean 'resulting through them,' and by 'resulting through them' I mean 'needing no further term from outside in order for the necessity to come about'. (*An. Pr.* A1, 24b18–22)

What is significant here is the phrase 'of necessity', since that seems clearly to be a modal notion. Normally this would be unimportant, but in a study of Aristotle's modal syllogistic it is important to appreciate that this use of 'necessity' does *not* refer to the modal status of any propositions. Some interpreters explain this by appeal to what is called 'hypothetical' or 'relative' necessity. Patzig (1968) explains that this is not a special kind of necessity attaching to the conclusion as a single proposition, but simply refers to the necessity – that is, the validity – of the whole inference. And Smith (1989, p. 122f), commenting on this passage, notes that:

> [Aristotle's] doctrine on this point is alien to modern logicians: Aristotle takes the sentence 'If A, then necessarily B' as attributing a kind of necessity ('hypothetical' necessity) to B. The point which he makes in the present case actually applies to any assertoric deduction, since (according to his definition) the conclusion of a deduction follows of necessity from the premises: thus, the conclusion of any deduction is necessary-if-the-premises-are-true. (Cf. the note on 27a16–18.)

The important point is that Aristotle is here describing a *process* not a proposition. Calling this process 'hypothetical' necessity is fine, provided that it is remembered that it is just the relation of the premises to the conclusion of *any* valid syllogism. When Aristotle comes to discuss modal syllogisms he is well aware that the claim that a conclusion itself is a necessary proposition is quite different from the claim that it is a valid consequence from certain premises.

Chapter 2
The Assertoric Syllogistic in LPC

As the previous chapter indicates I am going to rely on modern lower predicate calculus (LPC) to represent Aristotle's syllogistic and proof methods. This way of representing Aristotle is often regarded with suspicion. It should not be, and I want to take this chapter to explain why it should not. Developing a modern LPC representation of Aristotle's logic is straightforward, and so the reasons that are usually offered against this approach need to be carefully examined. When we look closely it becomes clear that every one of the criticisms is simply irrelevant and is no reason against a modern analysis. We do not need to get hung up about them. This chapter is about establishing why we don't. Recall from Chapter 1 the LPC translations of Aristotle's categorical propositions:

(A)	A belongs to every B	$\forall x(Bx \supset Ax)$
(E)	A belongs to no B	$\forall x(Bx \supset {\sim}Ax)$
(I)	A belongs to some B	$\exists x(Bx \;\&\; Ax)$
(O)	A does not belong to some B	$\exists x(Bx \;\&\; {\sim}Ax)$

There is an important distinction to bear in mind when we introduce modern formal logic: the distinction between what Aristotle *means* when he says, for example, 'A belongs to all B' and how we *represent* this meaning. I stress this because there is a tradition in modern formal logic which says that this simply cannot be right. Underlying this tradition are some basic assumptions about what we mean by 'logic'. If you think of logic as *purely formal* then the way it is represented will be the principal issue. The question will naturally arise about whether *our* formal representation coincides with *Aristotle's* formal representation. For example, Aristotle typically says 'A belongs to every B' or 'A is predicated of all B', and sometimes 'every B is A'. By contrast in modern formal logic we say $\forall x(Bx \supset Ax)$, which we interpret as meaning something like 'pick any individual you choose and no matter what you choose, if that individual is a B then that individual is an A.' In an important sense these are no more than *various ways of saying the same thing*. And a criticism of any such representation as being unAristotelian is misplaced.

The fact that these LPC representations involve what logicians call *individual variables* – here, the letter x – is of no consequence and does not interfere with our ability to use modern notation to represent Aristotle's meaning. 'No Bs are As' does not involve any individual variable in its surface structure, but its lower predicate calculus *translation* does use them. In using LPC to represent syllogistic premises, we do not attribute to Aristotle the use of individual variables. All that need be claimed is that the LPC representations express what Aristotle means. To be sure, the use of variables and other devices of LPC enables it to express far more than is ever needed for the syllogistic. For that purpose all that is ever needed is a very restricted fragment of LPC.

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
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For instance only one-place predicates are required, and only one individual variable x is ever required; which explains the prevalence of formalisms without individual variables. As I explained in the introduction my reason for using LPC is that its formulae are in standard use, and are easily understood by a reader with any knowledge of modern formal logic.

More important perhaps is that Aristotle thinks of logic as a tool which can be *applied*, and in this respect may not please those who view logic as free from any interpretation.[1] Łukasiewicz (1951) seems to be bothered by what he calls 'inexactitudes' in Aristotle's logic:

> Aristotle constantly uses different phrases for the same thoughts. I shall give only a few examples of this kind. He begins his syllogistic with the words '*A* is predicated of all *B*', but shortly he changes these words into the phrase '*A* belongs to all *B*', which seems to be regular. The words 'is predicated' and 'belongs' are frequently omitted, sometimes even the important sign of the quantity 'all' is dropped.... Although these inexactitudes have no bad consequences for the system, they contribute in no way to its clearness or simplicity.
>
> This procedure of Aristotle is probably not accidental, but seems to derive from some preconceptions. Aristotle says occasionally that we ought to exchange equivalent terms, words for words and phrases for phrases. (p. 18)

This is an awkward passage. It is awkward because Łukasiewicz seems to be saying that Aristotle would have done better if he had represented his intended meanings more strictly, with greater care about the actual *formal* representation. But even if Łukasiewicz is bothered by this, he is correct when he says that 'these inexactitudes have no bad consequences for the system'. So *unless we are concerned with rigidly formal representation* – a weirdly formal representation – we have no worry about such 'inexactitudes'. Aristotle clearly has no such worry – the variety of his own equivalent logical idioms and expressions shows that he is not rigidly formal himself. Where Aristotle says '*A* is predicated of all *B*' or '*A* belongs to all *B*' – or even more cryptically 'the *AB* premise is affirmative and universal' – I have represented his meaning with the lower predicate calculus formula $\forall x(Bx \supset Ax)$. If such formulae respect Aristotle's meaning, the question of how we *represent* his meaning is of no real

[1] Recall, for example, how Alonzo Church in his *Introduction to Mathematical Logic* (Church 1956), kept anything to do with semantics in a tiny font, keeping semantic notions as separate as possible from his mathematical – i.e., *formal* – logic.

consequence. I do not mean that the form of the LPC representation never matters. For instance, I describe I-Conversions as:

$$\exists x(Bx \,\&\, Ax) \equiv \exists x(Ax \,\&\, Bx)$$

Each side of this formula is logically equivalent in LPC to the other. But the left hand side represents Aristotle's 'some *B* is *A*', where *B* is the subject, while the right hand side represents 'some *A* is *B*', where *A* is the subject.

One of the chief tools in Aristotle's study is the logical variable. He is credited with the invention of the logical variable. Specifically, he is credited with the invention of the term variable. Łukasiewicz (pp. 7– 8) puts it this way:

> The introduction of variables into logic is one of Aristotle's greatest inventions. It is almost incredible that till now, as far as I know, no one philosopher or philologist has drawn attention to this most important fact. I venture to say that they must have been bad mathematicians, for every mathematician knows that the introduction of variables into arithmetic began a new epoch in that science. It seems that Aristotle regarded his invention as entirely plain and requiring no explanation, for there is nowhere in his logical works any mention of variables. It was Alexander who first said explicitly that Aristotle presents his doctrine in letters, *stoicheia*, in order to show that we get the conclusion not in consequence of the matter of the premises, but in consequence of their form and combination; the letters are marks of universality and show that such a conclusion will follow and for any term we may choose.

Aristotle's insight is to use variable letters – e.g., *A*, *B*, *C* – to represent what he calls terms. Among the many examples of terms in the *Prior Analytics,* we find man, horse, white, sleeping, raven, swan, musical, sitting, line, science. Aristotle uses the variable letters in three-line syllogistic schemas. An example helps to illustrate the nature of Aristotle's approach and this is crucially important to showing just how formal Aristotle's basic system is.

Take a simple deduction: (This example is not one of Aristotle's own.)

(1)	All men are mortal
(2)	<u>All bachelors are men</u>
(3)	All bachelors are mortal

Aristotle notices that when general terms like our mortal, man, and bachelor are uniformly replaced by variables *A*, *B*, and *C*, the *structure* of the original deduction is preserved.

(4)	All *B* are *A*
(5)	<u>All *C* are *B*</u>
(6)	All *C* are *A*

This of course glosses over any details about the substitution of predicate variables for terms. In the syllogistic Aristotle does not give any kind of account of what he takes to be the role of his variables. He readily uses variables, and uses them with a careful precision, but he does not explain them, and he does not announce their use as something new. The situation with respect to his ordinary language terms is much the same – again Aristotle tells us surprisingly little about their precise nature. His own choices of terms have tended to beguile his interpreters. Some down-play the importance of the terms. Jeroen van Rijen, for example, argues that the 'striking carelessness of [Aristotle's use of terms in constructing counter-examples] witnesses the relative unimportance of this part of the theory's systematics'. (van Rijen 1989, p. 201)

Some interpreters bemoan Aristotle's use of ordinary language terms altogether, not because of a carelessness about them, but because of a conviction that terms simply do not belong in any formal logic, that they are inappropriate in formal logic. Łukasiewicz is an obvious example. Quoting liberally from his work helps to show some of the real force of this view. Łukasiewicz (p. 2) gives us the following examples:

(7) If all men are mortal
and all Greeks are men,
then all Greeks are mortal.

(8) If all broad-leaved plants are deciduous
and all vines are broad-leaved plants,
then all vines are deciduous.

But he seems to think there is something *wrong* with these and with other examples like them:

> All these syllogisms, whether Aristotelian or not, are only examples of some logical forms, but do not belong to logic, because they contain terms not belonging to logic, such as 'man' or 'vine'. Logic is not a science about men or plants, it is simply applicable to these objects just as to any others. In order to get a syllogism within the sphere of pure

> logic, we must remove from the syllogism what may be called its matter, preserving only its form. This was done by Aristotle, who introduced letters instead of concrete subjects and predicates. Putting in (8) the letter *A* for ‘deciduous’, the letter *B* for ‘broad-leaved plant’, the letter *C* for ‘vine’, and using, as Aristotle does, all these terms in the singular, we get the following syllogistic form:
>
> (9) If all *B* is *A*
> and all *C* is *B*,
> then all *C* is *A*.

This view would have to disallow any restrictions on our choice of terms in a purely formal logic. Restrictions just do not make sense – there is no role for them in a purely formal system. Łukasiewicz (p. 7) specifically rules out any way of restricting syllogistic terms:

> It is essential for the Aristotelian syllogistic that the same term may be used as a subject and as a predicate without any restriction.

And Łukasiewicz, taking this to be the case, offers glowing praise for Aristotle’s *Prior Analytics*:

> This purely logical work is entirely exempt from any philosophic contamination. (p. 6)

Terms from ordinary language are most obviously at work in Aristotle’s counter-examples. Aristotle’s regular method for establishing the invalidity of an argument form involves constructing a counter-example using terms, as we saw on p. 19 above. Ross gives a lengthy and passionate explanation of his dissatisfaction with Aristotle’s use of counter-examples:

> ...it is not a completely satisfactory way of proving the invalidity of invalid combinations; for instead of appealing to their form as the source of their invalidity, he appeals to our supposed knowledge of certain particular propositions in each case. Whereas in dealing with the valid moods he works consistently with ABC for the first figure, MNX for the second, PRS for the third, and, by taking propositional functions denoted by pairs of letters, not actual propositions about particular things, makes it plain that validity depends upon form, and thus becomes the originator of formal logic, he discovers the invalidity of

> the invalid moods simply by trial and error. (Ross 1957, pp. 28–29)

Ross adheres to a view according to which Aristotle's logic, like all logic, is formal in the strictest sense. Since to introduce terms and propositions is to introduce extra-logical, and hence, irrelevant, information, Ross sees Aristotle's counter-examples as really something of an embarrassment. This is a position about the nature of logic which logicians once subscribed to more widely.[2] And while Ross's view of logic may seem quaint and old-fashioned to logicians today, this old view of the formality of logic plays an important role in the history of interpreting Aristotle's logic. Ross's idea that the syllogistic is purely formal appears to be driven by a conviction that insofar as the syllogistic is logic, then it *ought to be* purely formal. Semantics, or questions about meaning and interpretation, because they are not purely formal, are irrelevant. Or, Ross seems to want to say, at least, they *ought to be* irrelevant. Ross's complaint misses its mark. Certainly it misses insofar as he directs it against the use of counter-examples. All that those show is just that the syllogistic is not purely formal in Ross's weirdly strict sense. That weird sense is not what we mean when we say that a logic is purely formal. What we mean is that the form alone determines validity – that is, when 'there is a syllogism' we know that no instance of this form can have true premises and a false conclusion. What we don't know is the modal status of the conclusion, and it is that that I will claim is a non-formal issue. Aristotle uses ordinary language terms – such as man and white and whistles – when he wants to show that a syllogistic schema is *invalid* – that is, when he wants to show that 'there is not a syllogism.' That is, when he wants to show that you *can* have true premises and a false conclusion. The counter-example is a case where you *do* have true premises and a false conclusion. In modern formal logic validity is defined as the absence of any counter-example.[3]

Günther Patzig is one of the first of the modern interpreters to suggest that there might be any question about the formalness of Aristotle's logic:

> In mathematical logic the proposition "The *A* belongs to all *B*" has the

[2]While it is easy to see that Ross takes this too far, the effects of this view on philosophy are often subtle and easy to overlook. They are part of our modern heritage. And they really only began to be challenged later in the twentieth century, when logic took a 'semantic turn'. The semantic turn in logic is frequently dated from the definition of truth in Tarski (1936). In modal logic it is marked by the development of 'possible worlds semantics' of Prior, Hintikka, Kanger, Kripke, and others.

[3]In the notes to his translation Smith (p. 114) makes exactly this point in response to criticisms that Ross, Geach, and Łukasiewicz have about this feature of Aristotle's method. Quoting Smith: 'Ross complains that the use of counter-examples is not "completely satisfactory" because it introduces extra-logical knowledge. But there is nothing logically flawed in Aristotle's procedure: in fact, countermodels are the paradigmatic means of proving invalidity for modern logicians.'

> form $\forall x(Bx \supset Ax)$.[4] In Aristotle's view, the subject of this proposition is not the universal class of individuals but the class of individuals of which B holds, in short the class B; and the predicate of the proposition is the predicate A. Thus Aristotle's conception differs from that of mathematical logic in that he restricts the 'universe of discourse' to those objects of which B holds. It follows from this restriction that, although Aristotle's proposition is never true when that of mathematical logic is false, in certain cases it is neither true nor false – that is, it is meaningless – when on the mathematical interpretation it is true. This occurs whenever an individual is substituted for x which does *not* belong to the class B. Aristotle's proposition says nothing at all about this case; in mathematical logic, on the other hand, the proposition remains true: it asserts only that the predicate A belongs to x *if* the predicate B belongs to it. (Patzig 1968, pp. 37–38)

Patzig is concerned with the fact that in modern formal logic

(10) $\forall x(Bx \supset Ax)$

is trivially true if there are no *B*s. This is a worry if we use (10) to represent Aristotle's '*A* belongs to all *B*.' In modern formal logic, in order to guarantee $\forall x(Bx \supset Ax) \supset \exists x(Ax \; \& \; Bx)$, we have to add $\exists xBx$. As noted on p. 17, that B not be empty is required to validate A-conversion, so that we have to stipulate that 'there is at least one B' in order to accommodate that feature of modern logic which makes $\forall x(Bx \supset Ax)$ trivially true if B is empty. That is one way to accommodate the restriction that Patzig describes. To put it simply in English, for any predicate we choose, say B, there is at least one thing which actually is B.[5] There is a sense in which this is not a purely formal restriction since it depends upon a predicate actually holding of at least one subject. And that introduces 'extra-logical' contaminations. Łukasiewicz, adhering to a purely formal view of logic, struggles to give an explanation:

> Nothing is said in the *Prior Analytics* about the terms. A definition of the universal and the singular terms is given only in the *De*

[4] I am modernizing his notation. Patzig actually gives 'the A belongs to all B' as having the form $(x)(Bx \supset Ax)$. There is of course no difference in the meaning here.

[5] Robin Smith (1995, pp. 43—44) explains like this: 'Aristotle tacitly employs certain assumptions about the *existential import* of terms. The simplest way to preserve his results is to suppose that *all* terms have existential import (in which case the syllogistic can be interpreted as a theory of the relations of non-empty classes). This has intuitive support: if I say "All my daughters are brilliant," you will conclude that I have daughters.'

> *Interpretatione*, where a term is called universal if it is of such a nature as to be predicated of many subjects, e.g. 'man'; a term which does not have this property is called singular, e.g. 'Callias'. Aristotle forgets that a non-universal term need not be singular, for it may be empty, like the term 'goat-stag' cited by himself a few chapters before. In building up his logic Aristotle did not take notice either of singular or of empty terms. (p. 4)[6]

But this is not evidence that Aristotle 'forgets' empty terms. It may be simply that Aristotle is assuming that syllogistic logic involves no terms which are empty. Of course, this is a semantic restriction, and Łukasiewicz does not want to allow semantic restrictions. But Aristotle's syllogistic logic is *about* certain things – namely, things which *are*, things which *exist*. If Aristotle's terms are restricted to non-empty terms, then the move from $\forall x(Bx \supset Ax) \supset \exists x(Ax \,\&\, Bx)$ is guaranteed because if terms only range over 'what there is' then we know that *there is* at least one *B*. If this is what Aristotle is doing, then even the assertoric syllogistic is not purely formal. Content matters – it matters insofar as it is restricted to terms which signify *about* the world. Of course, if that is what is going on in the non-modal syllogistic, then even the non-modal syllogistic is informed by broader philosophical concerns. So even the 'successful' part of the syllogistic does not fit the view of logic as purely formal.

There is a surprising amount of scholarly debate about how best to formalize Aristotle's syllogisms. Consider Łukasiewicz again:

> It must be said emphatically that no syllogism is formulated by Aristotle as an inference with the word 'therefore' (ἄρα), as is done in the traditional logic. Syllogisms of the form:
>
> All *B* is *A*;
> all *C* is *B*
> therefore
> all *C* is *A*
>
> are not Aristotelian. We do not meet them until Alexander. This transference of the Aristotelian syllogisms from the implicational forms into the inferential is probably due to the influence of the Stoics. (1957, p. 21)

Both Patzig and Bocheński follow Łukasiewicz and interpret the syllogisms as

[6]The complex term 'goat-stag' is not instantiated by anything in the world. As Aristotle explains in *On Interpretation* I.16a16: 'even *goat-stag* signifies something but not, as yet, anything true or false – unless 'is' or 'is not' is added.'

conditional propositions. Smiley (1973) and Corcoran (1974a, b) treat the syllogisms as deductions. And Corcoran develops an interpretation according to which the syllogistic can be formally modelled as a system of natural deduction. I will side-step such issues. For the crucial point is that we capture the *meaning* Aristotle intends, and the method by which we represent that meaning is less important.[7]

One respect in which LPC certainly treats matters differently from the way Aristotle does is the phenomenon known as *scope*, and it will be helpful to look at the notion of logical scope in the context of the present discussion. When we talk about scope, we are talking about scope *of* one or another logical qualifier – that is, we are talking about what a qualifier qualifies or 'ranges over'. Take a simple example. In

(11) $\sim\forall x(Bx \supset Ax)$

the negation '~' has scope over the proposition $\forall x(Bx \supset Ax)$. In

(12) $\forall x(Bx \supset \sim Ax)$

the negation has scope only over the predicate term A. Obviously (11) and (12) are different formulae. In modern logic we might discuss the scope of negation (e.g., 'not', 'non-', 'it is not the case'). We might discuss the scope of a quantifier (e.g., 'every', 'all', 'some', 'at least one', 'few', 'most'). In modern logic, each of these – negation and quantification – raises questions about scope. And in modern logic we use precise formation rules in order to make scope clear and unambiguous, so that we can distinguish between different meanings.

When we represent Aristotle's syllogistic using modern formal logic we have to be especially careful about how we deal with matters of scope in order not to commit Aristotle to notions which may not be at play in his logic. In this section I want to consider the evidence against attributing to Aristotle a notion of scope.[8] First, let's look at negation from a modern point of view and at negation in Aristotle's logic. Modern logic allows us to treat negation as a propositional operator. We express this with a simple truth table, where α is any proposition, and the tilde '~' represents negation:

---

[7]There is one respect in which a natural deduction account may be preferable. On p. 20, I noted that Aristotle has a notion of necessity which simply signals the validity of a syllogism. If you follow Łukasiewicz and interpret the syllogisms as conditional propositions it is easy to think of necessity as a modal operator applying to such a proposition, and there is little evidence that Aristotle ever thinks of relative necessity in this way. If you think in terms of natural deduction then this 'relative necessity' applies to a process and not to a proposition, and so there is less danger of confusing it with any kind of propositional necessity.

[8]This point is perhaps well known, but should be better known. Kneale and Kneale (1962) offer an especially helpful discussion. So too does Horn (1989).

$\sim$	α
F	T
T	F

Aristotle does not have any such truth table. He does however have affirmation and denial. He also describes what has come to be called a 'square of opposition'. *On Interpretation* 7 has the following picture:

(A_1)	Every man is white	(E_1)	No man is white
(I_1)	Some man is white	(O_1)	Some man is not white

(A_1) and (O_1) are contradictories. And (I_1) and (E_1) are contradictories. It might help to look at the relations in the square using the 'belongs to' construction more common in the *Prior Analytics*. Then the square reads like this:

(A_2)	White belongs to all men	(E_2)	White does not belong to any man
(I_2)	White belongs to some men	(O_2)	White does not belong to some man

Looked at this way, the (E) and (O) propositions involve simple term negation. When we take an A-proposition but then *deny* the predicate term of the subject, we get an E. When we take an I and deny the predicate of the subject, we get an O.

When we represent this in lower predicate calculus the square of opposition looks like this:

(A)	$\forall x(Bx \supset Ax)$	(E)	$\forall x(Bx \supset \sim Ax)$
(I)	$\exists x(Bx \;\&\; Ax)$	(O)	$\exists x(Bx \;\&\; \sim Ax)$

The introduction of individual variables lets logicians distinguish between internal and external negation by the alternative placement of the *same* sentential negation operator. E.g., $\forall x(Bx \supset \sim Ax)$ is equivalent to $\sim\exists x(Bx \;\&\; Ax)$. They both mean 'every B is not an A' or, of course equivalently, 'no B is an A.' Similarly, $\sim\forall x(Bx \supset Ax)$ is equivalent to $\exists x(Bx \;\&\; \sim Ax)$. Here, both mean 'not every B is an A.' The use of individual variables is crucial to this modern way of doing logic. In the truth table, the $\sim$ is defined as a sentential operator on a well-formed formula α. When we add an individual variable to a predicate term we create an atomic formula – e.g., Ax – which functions exactly like the formula α in the truth table.

Aristotle, of course, does not think this way at all since he does not have individual variables, *but all of these same distinctions are available to him through his square of opposition*. In LPC the principles of the square of opposition might be expressed as:

(13) $\sim\forall x(Bx \supset Ax) \equiv \exists x(Bx \mathbin{\&} \sim Ax)$
$\sim\exists x(Bx \mathbin{\&} Ax) \equiv \forall x(Bx \supset \sim Ax)$
$\sim\exists x(Bx \mathbin{\&} \sim Ax) \equiv \forall x(Bx \supset Ax)$
$\sim\forall x(Bx \supset \sim Ax) \equiv \exists x(Bx \mathbin{\&} Ax)$

In all of these equivalences the left-hand side contains a ~ outside the scope of the quantifier, while the right-hand side is either an affirmative proposition and has no ~, or has the ~ preceding the predicate term. However, our modern representation stays closer to Aristotle if we understand his square not as (13) but as distinguishing between negation and denial, and represent the square as:

(14) To deny $\forall x(Bx \supset Ax)$ is to affirm $\exists x(Bx \mathbin{\&} \sim Ax)$
To deny $\exists x(Bx \mathbin{\&} Ax)$ is to affirm $\forall x(Bx \supset \sim Ax)$
To deny $\exists x(Bx \mathbin{\&} \sim Ax)$ is to affirm $\forall x(Bx \supset Ax)$
To deny $\forall x(Bx \supset \sim Ax)$ is to affirm $\exists x(Bx \mathbin{\&} Ax)$

In (14) the only negation operator operates directly on predicates, which represent the *terms* in the propositions of an Aristotelian syllogism. So that (14) can be regarded as treating negation in a categorical proposition as a term qualifier. Therefore (14), rather than (13), seems the more faithful way to understand the propositions which appear in Aristotle's syllogisms. When I say that Aristotle did not have the notion of the scope of an operator I mean that he seems unaware that the equivalences in (14) can be expressed in terms of a different placement of a single operator – as in (13).

Nothing so far in this chapter depends upon modal notions. However, if we add to our present account Aristotle's own considerations about what cannot be otherwise and about what can be otherwise, then we move even further from a purely formal logic.

Chapter 3
A Realm of Darkness

There is a tradition that says that the modal syllogistic is logically incoherent. Even Aristotle's translators sometimes seem close to despair:

> In recent years, interpreters have expended enormous energy in efforts to find some interpretation of the modal syllogistic that is consistent and nevertheless preserves all (or nearly all) of Aristotle's results; generally, the outcomes of such attempts have been disappointing. I believe this simply confirms that Aristotle's system is incoherent and that no amount of tinkering can rescue it. (Of course, this still leaves us with the knotty problem of why Aristotle should have developed such a system.) Fortunately for the student of Aristotle, the modal syllogistic is largely self-contained: hardly anything in Aristotle's other works, even including the *Analytics,* appears to take notice of it. (Smith 1995, p. 45)

Striker (2009) seems to share the same frustration:

> One can hardly avoid the conclusion that the system of modal syllogisms as it stands is logically incoherent. (p. 115)

And so the question arises 'Why think that the modal syllogistic is incoherent?' This chapter provides an outline of some of the most influential accounts. Albrecht Becker argues that the modal syllogistic falls apart because Aristotle makes a crucial mistake involving two different uses of *necessity*. (Becker 1933, pp. 41–43) Becker notices that Aristotle's modal syllogisms appear to require that necessity acts as a qualifier on terms. Take an example:

(1) All men are necessary-animals
(2) <u>All moving things are men</u>
(3) All moving things are necessary-animals

Here, as the hyphen indicates, it seems that the only way the syllogism (1)–(3) makes sense requires that necessity qualifies the term 'animal'. The syllogism's validity demands it. In fact (1)–(3) has the same structure as Aristotle's most basic non-modal syllogism:

(4) All *B* are *A*
<u>All *C* are *B*</u>
All *C* are *A*

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
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If we take the non-modal syllogism (4) as a starting-point, we obtain the modal syllogism (1)–(3) simply by making *A* 'necessary-animal', *B* 'man', and *C* 'mover'.

But, Becker tells us, some of Aristotle's proof methods do not work when the modal qualifies a term. Instead, Becker explains, these proof methods require something different – they require that necessity acts as a qualifier on *propositions*. Becker singles out conversion. In simple conversion Aristotle reverses the order of the subject and predicate terms. Subject and predicate are transposed or 'turned around'. Thus, 'some *B* is *A*' converts to 'some *A* is *B*.' Becker notices that the kind of necessity that we appealed to above in our example (1)–(3) does not get the conversion to work. We can easily convert a non-modal proposition into another non-modal proposition. We can convert

(5)	Some moving things are animals	T

into

(6)	Some animals are moving things	T

Such non-modal conversion is unproblematic. The trouble comes with the addition of necessity. We *cannot* convert 'some *B* is a necessary-*A*' into 'some *A* is a necessary-*B*.' To see why we cannot, let *A* be animal and *B* be moving thing. If we choose these as our *A* and *B*, then the conversion is not valid:

(7)	Some moving things are necessary-animals	T

to

(8)	Some animals are necessarily-moving things	F

For while it may be true that some moving things are necessary-animals, it is not correct to say that some animals are necessary-movers. ('Moving' is one of Aristotle's stock examples of a predicate which may hold of a subject, but not by necessity. Animals may have the capacity to move but, according to Aristotle, no animal moves of necessity, 30a30–33.) Becker thinks that Aristotle's necessary conversion can be saved if we use a *different kind of necessity*. Becker's analysis of this modal conversion then works like this. If it is true that

(9)	Necessarily (Some *B* are *A*)	T

then by the conversion of (9), it must also be true that

(10)	Necessarily (Some *A* are *B*)	T

In each of (9) and (10), necessity qualifies the entire (bracketed) proposition. If this is the correct analysis of the modal conversion, then necessity in (9) and (10), usually referred to as necessity *de dicto*, functions differently than necessity in (1)–(3), necessity *de re*. This is clear from the fact that when we have necessity qualifying a proposition, then we get wrong results in the syllogisms. Rather than a valid syllogism (1)–(3) we get a result which is plainly invalid, since, assuming that in fact only men are moving, we have:

(11)	Necessarily (All men are animals)	T
(12)	<u>All moving things are men</u>	T
(13)	Necessarily (All moving things are animals)	F

If (11) and (12) are true, then it is clearly true that

(14)	All moving thing are animals	T

But (14) is not a modal conclusion. The modal proposition (13) is false because it is not *necessarily* the case that all moving things are animals. Some philosophers sometimes struggle with the logical distinction between *de dicto* and *de re* modals and want to argue that *de re* interpretations of propositions about necessity do not always make sense. Following Quine, they point out that the following *de dicto* modal proposition is true:

(15) It is necessarily the case that all bachelors are unmarried

whereas the *de re* reading is false:

(16) All bachelors are necessarily-unmarried.

The *de dicto* proposition (15) is true because the simple proposition ‘all bachelors are unmarried’ is itself *analytically true*. The *de dicto* ‘necessity’ qualifies a true analytic proposition, so the modal proposition (15) itself is also true. But such a modal proposition – even if true – is not ever an Aristotelian syllogistic premise. It is good to bear in mind, here, that Quine’s original complaint against *de re* modality was a complaint against what he called Aristotelian essentialism; it was a complaint specifically *against* what Quine took to be Aristotle’s demand for *de re* necessity. Quine’s *de dicto* analysis of modals is not an *alternative* to Aristotle’s *de re* modals – Quine means it to be a flat rejection of Aristotle’s essentialist metaphysics.[1] But, here, since we are in the business of interpreting Aristotle’s modals, we needn’t bother much

[1]Quine (1960, p. 199f)

about Quine's complaint. If Quine is right that *de re* modals do in fact reflect Aristotle's essentialist metaphysics and since a *de re* analysis of modal propositions seems to more closely represent the propositions that Aristotle describes, then whatever Quine's own complaints we at least have reason to prefer a *de re* analysis here. But (16) of course is a *de re* proposition and (16) is simply false. The important point about (16) is that it is not a good example of an Aristotelian modal premise. In (16) the 'necessity' qualifies the predicate 'unmarried'. The problem comes about because being unmarried is very definitely the kind of thing that can be otherwise – it becomes otherwise every time a bachelor (or a maid) marries – and so for Aristotle being unmarried is simply not the kind of thing that is *necessary* to anyone. It is this fact which guarantees the falsity of (16) and makes it not a good syllogistic premise even though it is a *de re* modal proposition.

Becker thinks the trouble for the modal syllogistic comes from the fact that Aristotle actually requires both *de re* and *de dicto* necessity, but that Aristotle fails to see that they are not the same. If Becker is correct then the modal syllogistic is not a coherent system. Of course if Becker is correct then there should be evidence that Aristotle *uses* both kinds of necessity in his syllogistic. One claim throughout the interpretation in this book is that Aristotle does not have both kinds of necessity at work in the way Becker describes.

Jaakko Hintikka (1973) identifies another source of incoherence. Hintikka focuses on the way Aristotle links time and modality in what is called the *Principle of Plenitude*[2]:

(17) If it is possible that p, then at some time it is the case that p
(18) If it is always the case that p, then it is necessary that p

Hintikka argues that the principle of plenitude is one of the basic axioms of Aristotle's philosophy, and so Hintikka thinks that plenitude surely must be at work in the modal syllogistic. But as Hintikka explains, if plenitude applies to the syllogisms about necessity then Aristotle forfeits the coherence of the logic:

> As a consequence [of the principle of plenitude], whatever is always true is true necessarily according to Aristotle.
>
> Now, Aristotle also insists that universal assertoric (non-modal) premises, i.e. premises of the form
>
> (I) *A* applies to all *B*,
>
> have to be understood with no limitation with respect to time, for instance so as to be restricted to the present moment. (See *An.Pr.* I 15,

[2]See also Mignucci (1972) and Crivelli (2004, p. 21). Also Waterlow (1982).

> 34b7–18.) What this means is that premises like (I) will have to take in all individuals, past, present, and future. From the principle of plenitude it therefore follows that if (I) is true, it is necessarily true. (Hintikka 1973, pp. 136–137)

If Hintikka is right about how to interpret an ordinary non-modal premise of the form

(I) *A* applies to all *B*,

then there can then be no real difference in the syllogistic between a true non-modal proposition and a true modal proposition about necessity. But Aristotle's syllogistic system demands such a distinction in order to separate valid from invalid schemas. For example, Aristotle clearly distinguishes between the following:

(19)	All *B* are necessarily-*A*	
	All *C* are *B*	
	All *C* are necessarily-*A*	(*An.Pr.* 30a17–23)

(20)	All *B* are *A*	
	All *C* are necessarily-*B*	
	All *C* are necessarily-*A*	(*An.Pr.* 30a23–32)

In Aristotle's modal system, (19) is a syllogism – that is, (19) is valid. It is the same schema as (4), above. But (20) is not a syllogism – that is, (20) is invalid – the conclusion does not logically follow from the premises. Aristotle himself establishes the invalidity of the schema (20) by constructing a counter-example. He substitutes the terms moving, animal, and man for the variables *A*, *B*, and *C*:

(21)	All animals are moving	T
	All men are necessary animals	T
	All men are necessarily moving	F

Aristotle's explanation of the invalidity of (20) via the counter-example (21) can be found in *An.Pr.* 30a23–32.

In Aristotle's treatment the difference between (19) and (20) is the difference between validity and invalidity – or, in more Aristotelian vocabulary, the difference is between having a syllogism and not having a syllogism. But if we take plenitude to work in the way that Hintikka describes, then it seems that Aristotle cannot distinguish between (19) and (20) at all.[3] And in fact, as the quotation from Hintikka (1973) shows, the situation is even worse. If Aristotle adheres to plenitude then he forfeits any distinction between necessary and non-modal premises.[4]

These criticisms from each of Becker and Hintikka focus on problems about how to give a formal analysis of Aristotle's use of necessity. Striker (1993) has a different focus. She points to specific examples in Aristotle's own discussion. Consider the following propositions:

(22) Animal necessarily belongs to some white things
(23) Animal belongs to nothing white

Aristotle offers each of (22) and (23) as examples of *true* propositions. We find (22) at *An.Pr.* A9, 30b5–6. We find (23) at 30b35. But, obviously (22) and (23) cannot both be true. Striker reminds us that any attempt to give a formal analysis of Aristotle's use of necessity has to adhere closely to the text. The real problem, in Striker's view, is that the text itself cannot be rendered entirely consistent – some of Aristotle's examples are just plain bad. If the text contains inconsistencies such as (22) and (23), then there is no good way for an interpreter to get around the problem, and Striker's point is that no amount of logical or interpretive manoeuvring will help.

These are just some of the problems said to undermine the modal syllogistic. It is certainly not a pretty picture, but it not clear that it is a truly hopeless picture either. Plainly Aristotle's modal logic has earned him a lot of bad press. As a result, Aristotle's real accomplishments in logic are often overlooked and sometimes they are simply dismissed by Aristotle scholars. G.E.M. Anscombe is one of the more dismissive.

> Part of [Aristotle's] fame as a philosopher rests upon his having started the science of logic. He understood his own claim to greatness on this account... Nevertheless an account of Aristotle's formal logic would, it seems to me, be of only scholarly interest. (1961, p. 5)

Without a neat and defensible interpretation of the modal syllogistic it is not at all clear what we ought to say about it.

So should Aristotle have stopped 'while he was ahead' – at just the non-modal syllogistic? This chapter has focused on the kinds of general interpretive problems that give the modal syllogistic a bad reputation. The only way to bring light to the 'realm

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[3]Using the medieval names for the syllogisms, with McCall's modal notation (for which see p. 45 below), (19) is Barbara LXL, and (20) is Barbara XLL. For more on what is often known as 'The Problem of the Two Barbaras' see also McCall (1963, pp. 15—18), Thom (1991), and Patterson (1989).

[4]See Rini (2003).

of darkness' is to produce a coherent account of the modal syllogistic which will accurately reflect Aristotle's text. The provision of such an account is the task of Parts II and III.

Chapter 4
Technicolour Terms

Aristotle has a metaphysical view according to which facts about the world around us divide into two: those things which *cannot be otherwise* and those things which *can be otherwise* than they are. In *Posterior Analytics* A2 Aristotle is especially emphatic about the importance of what cannot be otherwise, and he links what cannot be otherwise with our ability to understand – that is, he links it with the possibility of having genuine knowledge:

> We think we understand a thing *simpliciter* (and not in the sophistic fashion accidentally) whenever we think we are aware both that the explanation because of which the object is is its explanation, and that it is not possible for this to be otherwise. It is clear, then, that to understand is something of this sort; for both those who do not understand and those who do understand – the former think they are themselves in such a state, and those who do understand actually are. Hence that of which there is understanding *simpliciter* cannot be otherwise. (*An.Post.* A2, 71b9–16)

The point is elaborated in *An.Post.* A4, where Aristotle links the necessity of what cannot be otherwise directly with our ability to syllogize:

> Since it is impossible for that of which there is understanding *simpliciter* to be otherwise, what is understandable in virtue of demonstrative understanding will be necessary (it is demonstrative if we have it by having a demonstration). Demonstration, therefore, is deduction from what is necessary. (*An. Post.* 73a21–27)

But not everyone agrees that there are such links to be found between Aristotle's metaphysics and his logic. Anscombe, plainly does not. She tells us that 'Aristotle's doctrine of substance is integral to most of his philosophical work, not, however, to his strictly formal logic – a discipline which he inaugurated.' (Anscombe 1961, p. vi) Whether or not Anscombe is correct it certainly seems clear that Aristotle thought that there could be no *scientific* syllogizing about what could be otherwise.

> Since, then, if a man understands demonstratively, it must belong from necessity, it is clear that he must have his demonstration through a middle term that is necessary too; or else he will not understand either why or that it is necessary for that to be the case, but either he will think but not know it (if he believes to be necessary what is not necessary) or he will not even think it (equally whether he knows the fact through

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
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middle terms or the reason why actually through immediates).

> Of accidentals which do not belong to things in themselves in the way in which things belonging in themselves were defined, there is not demonstrative understanding. For one cannot prove the conclusion from necessity; for it is possible for what is accidental not to belong – for that is the sort of accidental I am talking about.... Since in each kind what belongs to something in itself and as such belongs to it from necessity, it is evident that scientific demonstrations are about what belongs to things in themselves, and depend on such things. For what is accidental is not necessary, so that you do not necessarily know why the conclusion holds – not even if it should always be the case but not in itself (e.g., deductions through signs, οἱ διὰ σημείων συλλογισμοί). For you will not understand why it holds. (To understand why is to understand through the explanation.) Therefore the middle term must belong to the third, and the first to the middle, because of itself. (*An. Post.* 73a12–37)

Although this book is not a work about Aristotle's metaphysics, we should at least indicate how his metaphysics might be influencing the syllogistic. A very simplified account of the metaphysical distinction between essence and accident goes something like this. There are things in the world which are what they are because they have an essential nature. To say that a thing *A* has an essential nature is, for Aristotle, to say that there is a 'what-it-is-to-be *A*' which is necessary to any *A*. If a thing is an *A* then it cannot be otherwise than an *A*. Not all that we see in the world around us has any such an essence about it. Sometimes what we see *only happens to be the way it is*. Anything like that is not essential because it 'falls short of necessity'. We often talk about the weather in this way. It might rain tomorrow or it might not. Both are possibilities; but neither is necessary. All of this is very basic, if grossly simplified, Aristotle. The important point to take from it is that Aristotle's view of the world demands a distinction between what *cannot* be otherwise and what *can* be otherwise.[1]

The burden of the present chapter is to motivate a case for applying the distinction between what cannot be otherwise and what can be otherwise to the terms that we use in that part of the syllogistic which deals with necessity. A paradigm

[1]One respect in which what I have said is over simple is this. Matthews (1982) and Lewis (1991) have suggested that when a white man is picked out as a 'white' what is picked out is a 'kooky object' (Matthews) or 'accidental compound' (Lewis). All I want to stress in this book is that this debate, important though it undoubtedly is in our understanding of Aristotle, does not really affect the modal syllogistic, where it seems that when Aristotle speaks of a 'white' he means a white thing, and that is why 'white' is a green term.

example of something which cannot be otherwise is a substance – e.g., a man, a horse. A paradigm example of something which can be otherwise is an Aristotelian accident – e.g., white, moving, sitting. For simplicity, I will sometimes call things which cannot be otherwise *essential* and things which can be otherwise *accidental*. But bear in mind that I am making no distinction at this stage between substance terms, differentia, or *propria* – so far, the only distinction we have is the one, guided by *An.Post.* A2, between what cannot be otherwise and what can be otherwise. Taking the *An.Post.* passage as a guide, I want to propose a simple condition: *Any syllogistic term that we might choose belongs to one and only one of these two kinds.*[2]

Perhaps it is best to use a neutral terminology. Call the essential terms red. And call the accidental terms green. Consider red terms. Red terms refer to what *cannot be otherwise* than it is. Things named by red terms are things that have essential natures, and the terms refer to those natures. They are *necessary*. Man, horse, swan, animal are examples of red terms. Green terms are different. Green terms name what *can be otherwise* than it is. They fall short of necessity but they are real possibilities. And they include different kinds of possibilities. In the first instance, green terms name simple accidental features of the world around us. Walking, moving, sleeping, being white are this sort of green, accidental terms.[3]

This simple colour-coding helps to make a lot of interpretive puzzles melt away. If we can distinguish between terms for things which cannot be otherwise (red) and terms for things which can be otherwise than they are (green), then we can do Aristotle's modal syllogistic by choosing the right coloured terms. This of course is contrary to the views of many interpreters, who assume that there are *no* restrictions or rules of any sort governing terms in the syllogistic.[4] But as Chapter 3 illustrates, the end result of this approach is a syllogistic plagued with interpretive problems and at best only tenuously connected to broader Aristotelian philosophy. The colour-coding device provides an easy and clear way to introduce a fundamental distinction at work in

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[2]In Part III we will see the need for refinement. For instance, Aristotle sometimes speaks of a term like 'white' which is accidental to some things, humans, but essential to others, like swans. The premise 'all swans are white by necessity' which we find in *An.Pr.* A16, might, perhaps, anticipate developments in Aristotle's metaphysics. A medieval example might be 'laughs', where a man may or may not be laughing, but 'no horse is laughing' might be true by necessity.

[3]There is another way in which things around us are said to be possible. It might be that even though, in the normal course of the world, a thing *has the potential to become A*, it just has not done so yet, and in fact it might not ever actually become *A*. Take an acorn as a convenient example. Given the right conditions an acorn can, in the natural course of things, grow into an oak tree, but until it actually has done so, it cannot really be said to *be* an oak. It is just an acorn with a potential to become an oak. This potency is a kind of possibility, but it is only a possibility, because it, too, falls short of necessity. Part III of this book looks at this latter sort of possibility in greater detail. Potencies can fudge the distinction between essence and accident which works so well in the apodeictic syllogistic.

[4]See for example Barnes (2007, especially pp. 109, 133). See also p. 3 above.

Aristotle's modal syllogistic. Of course, if you are colour-blind and do not make this distinction between colours of terms, then all terms look the same – say, gray – and there is no difference between terms for things which cannot be otherwise and terms for things which can be. If you approach Aristotle as though you are colour-blind with respect to the terms, then you are trying to give an interpretation for what turns out to be a fundamentally different body of data. The results of the colour-blind approach indicate that Aristotle's modal syllogistic is incoherent, inconsistent, and – my favourite – 'a realm of darkness' – painted in gray.[5]

If the colour-blind approach makes a mess of the modal syllogistic then that in itself is good reason to try a technicolour approach. So rather than treating the terms as though they are all the same kind, in my treatment of syllogistic terms I will adhere closely to Aristotle's distinction between what cannot be otherwise and what can be otherwise than it is. This kind of distinguishing between terms is *not* ever needed in Aristotle's non-modal, assertoric syllogistic. It is important to stress that the non-modal syllogistic in *An.Pr.* A4–6 is colour-blind. And the hypothesis here is that because it is colour-blind it is not precise enough for Aristotle's purposes. Such a colour-blind syllogistic is ill-suited to Aristotle's scientific theories because such a syllogistic does not reveal anything of the structure of his metaphysical world which is, at heart, a world of essence and accident. If Aristotle has it in mind to link his logic and his science then it is not surprising that he should try to refine the syllogistic logic so that it better and more explicitly reflects his own metaphysical world-view and the foundations of his science. If his logic is going to be any use to him at all then it should at least be shown to accommodate this much.

Distinguishing red and green terms gives a straightforward response to Becker's criticism described in the last chapter on pp. 32–35. Becker's criticism is that Aristotle mistakenly conflates two different kinds of necessity. One kind of necessity qualifies terms, and this is the kind of necessity required to validate the modal syllogisms, as in our earlier example, where 'necessary' acts as a qualifier on the term 'animal':

(1) All men are necessary animals
(2) <u>All moving things are men</u>

[5]Among modern authors Englebretsen (1988) and perhaps to a lesser extent Malink (2006) come closest to the interpretation offered here. Malink (2006, p. 97) argues for a distinction 'between substantial and non-substantial essential predication' and tells us that 'the distinction between the ten categories... is of importance also for the logical purposes of modal syllogistic.' Malink is here disagreeing with Patterson (1995, p. 41) but his remarks would seem also to apply to Barnes (2007, p. 133). Englebretsen requires *de dicto* modal operators. Malink does not include *de dicto* modals (Malink 2006, p. 96), but stands out in 'giving up the logical distinction between syllogistic terms and zero-order individuals' (p. 97). Patterson (1995, p. 41) and Thom (1996, p. 5), like Barnes, downplay distinctions about Aristotelian categories in the modal syllogistic. Others have noticed the need for restrictions in other works in Aristotle's *Organon*. See for instance Anscombe (1961), van Rijen (1989), Striker (1993) and Cresswell (2004). See also Crivelli (2004, p. 16) on the connection between predication and the categories.

(3) All moving things are necessary animals

As we saw in Chapter 3, Becker thought that Aristotle sometimes puts the necessity qualifier on the predicate term of a syllogistic premise, and other times on the entire premise itself. The former is the *de re* modal qualifier. The latter is the *de dicto* modal qualifier, and a crucial claim in this book is that Aristotle *never* requires a *de dicto* modal premise in the syllogistic. A *de re* interpretation is much closer to what Aristotle describes, and using *de re* qualifiers on terms gives a clear and convenient way of representing an apodeictic proposition – that is, a way of representing an Aristotelian necessary proposition. However, if necessity acts as a qualifier on terms in modal conversions then it seems that Aristotle's conversions are invalid. We want to rule out invalid instances of modal conversions – such as the following:

(4)	Some moving thing is a necessary animal	T
(5)	Some animal is a necessarily-moving thing	F

According to the red/green distinction, animal is a red term – it is a term which signifies what cannot be otherwise. All of this suggests an easy solution. It suggests that the conversion might go from (4) to (6) as follows:

(4) Some moving thing is a necessary animal
(6) Some necessary animal is a moving thing

This is certainly valid and it is easy, but it is not up to the job Aristotle requires of modal conversion. In a sense, the conversion from (4) to (6) is not really a *modal* conversion – it has exactly the same structure as Aristotle's non-modal conversion from 'some *B* is *A*' to 'some *A* is *B*.' But, as we shall see, Aristotle sometimes needs conversion principles in which the modal really does shift from qualifying one term to qualifying the other term. Thus:

(7) Some *B* is necessarily *A*
(8) Some *A* is necessarily *B*

According to Aristotle (*An.Pr.* A3, 25a32), (7) and (8) should be equivalent. Now (4)(5) is an instance of (7)(8), and if you look at (4) you will see that the *B* term is not red but green. If we rule that out as a possible choice then the conversion might still be valid – albeit, restrictedly valid. If some *B* is a necessary *A* then certainly some *A* is a *B*, so that if *B* is a red term then some *A* is a *necessary B*. This suggests that we might employ such a restriction on the choice of terms in the apodeictic syllogistic. We can require, for example, that whenever we want to convert 'some *B* is necessarily *A*' then whatever we choose as our *B* must be a red term. If the *B* term is red, then the passage from (7) to (8) will be valid. The restriction blocks out the possibility of counter-examples such as (4)(5). Such counter-examples cannot arise because the restriction prevents the choice of an accidental, green, *B* term such as 'mover'.

The restriction we are considering is a restriction on the subject term of any proposition that acts as the input to modal conversion. Such subject terms must be red. I don't mean to suggest that Aristotle *explicitly* assumes such a restriction as part of his formal logic. But I do want to suggest that because it is an *applied* logic the modal status of the conclusion will depend on the choice of terms, and that the restriction in modal conversion to red terms is needed to validate syllogisms which depend on this conversion. At this stage I do not want to try to defend the restriction. Right now what I want to emphasize is that the restriction to red subject terms is enough to guarantee the validity of the conversions described above. This helps to answer Becker because this way of interpreting Aristotle does not require that the necessity in a modal conversion is a *de dicto* operator. Instead, the necessity is a consequence of what it is for a term to be a red term.

We shall see in Part II that the use of red terms in conversions enables a simple account of most of the apodeictic syllogistic. However the red/green distinction is not sensitive enough on its own to handle the problematic syllogistic, and we shall see in Part III how it will need refinement.

Chapter 5
Representing the Modals

The standard current system of classifying the modal syllogisms is that used by McCall (1963). In this system an assertoric (non-modal) proposition is denoted by X, a proposition about necessity by L, and a proposition about possibility by either M or Q depending on the kind of possibility involved. Thus, Barbara LLL is Barbara with premises and conclusion all necessary propositions, Barbara LXL is Barbara with the first premise a necessary proposition, the second premise assertoric, and the conclusion necessary, and so on. I will use this classification system in what follows.

In Chapter 2, I argued for the use of LPC in representing Aristotle's logic. When it comes to modality LPC needs to be augmented by *modal operators*. In modern modal logic (see for instance Hughes and Cresswell, 1996) necessity is represented by an operator L (or $\Box$) attached to a formula φ, with $M\varphi$ (possibility) defined as $\sim L\sim\varphi$, and with $Q\varphi$ (contingency or sometimes 'two way possibility') defined as $M\varphi \;\&\; M\sim\varphi$. In particular, in the same way as $\sim Ax$ means that x is not A, so LAx means that x is by necessity A. As we saw in Chapter 3, a choice must be made whether to represent Aristotle's modal propositions by *de dicto* formulae or by *de re* formulae. It is easy to represent the distinction in LPC using the scope of the modal operators. For the affirmative cases the simplest and most obvious *de re* readings are:

LA	It is necessary for A to belong to every B:	$\forall x(Bx \supset LAx)$
LI	It is necessary for A to belong to some B:	$\exists x(Bx \;\&\; LBx)$

LA says that if anything satisfies B then it satisfies A by necessity, or 'A can be predicated by necessity of every B'. LI says that there is something, x, which satisfies B and it, x, satisfies A by necessity, or 'A can be predicated by necessity of some of the Bs'. The *de dicto* readings are:

LA′	It is necessary for A to belong to every B:	$L\forall x(Bx \supset Ax)$
LI′	It is necessary for A to belong to some B:	$L\exists x(Bx \;\&\; Bx)$

LA′ says that it is necessary that if anything satisfies B then it also satisfies A, as in, it is necessary that whatever satisfies 'bachelor' also satisfies 'unmarried', where there is no question of either term applying to anything by necessity. LI′ says that it is necessary that something satisfies both B and A. *De dicto* particular propositions are harder to come by – the closest is a game in which there must be a winner, but in which no one wins by necessity.

As noted in the introduction, the use of predicate logic is even today considered controversial. Storrs McCall shows one way to represent the traditional modal syllogisms about necessity as part of an axiomatic system which he calls 'the *L-X-M*

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
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calculus'[1] – but McCall limits his project carefully. He focuses on syllogisms involving necessity and on certain aspects of Aristotelian possibility. McCall says little about the parts of the modal syllogistic that deal with contingency. And he does not offer any kind of interpretation for the axiomatic system he represents. But what he does offer is a way of approaching Aristotle's syllogistic about necessity which at least avoids the level of outright incoherence that Becker (1933) and Hintikka (1973) claim to discover in their interpretations. McCall manages to avoid the incoherence because he does not give an interpretation. Even if McCall's project is limited, it marks an important advance. It is important because by giving an uninterpreted, formal representation of a part of the modal syllogistic, McCall helps to distinguish the actual formal structure of Aristotle's logic from its interpretation. And by developing the axiomatic system *L-X-M*, McCall helps to address the traditional criticisms. His work suggests that there might be a greater level of coherence to the modal syllogistic than others have seen. But because his representation is an uninterpreted system, we need an interpretation of McCall just as much as we need an interpretation of Aristotle. For this reason McCall's formal representation is a first step and only that.

Fred Johnson (1989) and S.K. Thomason (1993) each offer a semantics for McCall's axiomatization of the modal syllogistic. Both take Aristotle's modal operators as operators on terms, and both offer a consistent formal model. But they do so at a cost. Their formal models depend upon the introduction of certain formal principles of logic that are hard to square with other basic principles of Aristotle's philosophy. Examples of two such formal principles are (1) and (2). Both of these examples are from Thomason[2]:

(1) $\exists x(Bx \,\&\, Ax) \supset \exists x(Bx \,\&\, LAx)$

(2) $\forall x(Bx \supset LAx) \supset \forall x(L{\sim}Ax \supset L{\sim}Bx)$

These plainly don't sit well with some of Aristotle's discussions about necessity and possibility. For instance, with 'white' for *A* and 'man' for *B*, (1) would allow the move from 'some man is white' to 'some man is necessarily white.' For Aristotle 'some man is white' is true but 'some man is necessarily white' is false. Despite such problems, Johnson and Thomason do much to advance the study of the modal syllogistic because

[1]McCall (1963) identifies Ross (1957) as a main source of data, and Ross himself is influenced by Becker (1933). McCall refers to the table in Ross (facing p. 286) as providing evidence about which syllogistic schemas are valid and which are invalid. Some difficulties arise where Ross's data seems to differ from the account we get from Aristotle in *Prior Analytics*. See Rini (2000). The point is picked up in Chapters 7 and 8 of the present work.

[2]Thomason (1993, pp. 120, 123). I use standard modal LPC translations Johnson and Thomason do not. They give direct set theoretic interpretations and rules, but their interpretations give precisely (1) and (2).

they provide very strong and clear evidence to support interpreting Aristotle's modals as qualifiers on terms. In seeking a formal model for McCall's representation, the results they offer help to resolve Becker's complaints about Aristotle's supposed confusion. Their interpretations do not involve a conflation of different senses of necessity. However the difficulties that their formal methods introduce put a limit on their persuasiveness. Because their interpretations are not always closely supported by Aristotle's text, classical scholars have tended to ignore their results.

More classically minded interpreters have sought to define the scholarly landscape without appeal to modern formal methods. Patterson (1995) tries to keep the formalism to a minimum and develops an analysis in which the modal attaches to the copula. What is crucial, he tells us, is 'the manner of the predicate's applying to the subject' (p. 8). Patterson goes on to say that 'It is the copula or linking expression between the terms to which Aristotle, in the *Prior Analytics*, ordinarily attaches his modal operators' (pp. 8–9). There are two ways we might construe Patterson's claim. Either a subject is a thing and the copula links a term with this thing, or a subject is a term and the copula links one term with another term.[3] In fact Patterson does have two different copulae, one for each of these kinds of cases. Patterson, in elaborating his own copulative reading of Aristotle's modals, seems close to the spirit of Becker, finding ambiguities in Aristotle's modal notions.

> ...the modal copula reading of Aristotelian necessity (and other modalities) is itself already ambiguous between two interpretations. One sort of *de copula* (or *cop*, for short) reading asserts a definitional relation either of entailment or exclusion between its subject and predicate *terms*, where (Aristotelian) definitions are accounts of the natures or essences signified by such terms rather than of the meanings of linguistic subject and predicate. On the other *cop* reading, a necessity proposition asserts a necessary relation between its own predicate term

[3]A term-term link appears to be suggested by Vilkko and Hintikka (2006, p. 364 f). They say

> If we tried to think anachronistically of Aristotle's syllogistic premises in terms of quantifiers ranging over certain entities, we would have to say that their values are some sorts of possible individuals. However this is not how Aristotle looked upon his syllogistic premises. For him they represented primarily relations of the forms expressed by the subject and the predicate. A premise like 'every *B* is *A*' says that it is a fact about the form expressed by the term *B* that it is always accompanied by the form expressed by *A*. And since a premise like 'every *B* is *A*' is thus thought of as dealing in the first place with relations of forms, the sets of entities instantiating these forms become largely irrelevant.

Vilkko and Hintikka are not of course talking of the modal syllogistic here, but we saw on p. 35 that Hintikka's espousal of the principle of plenitude leads him to interpret an LA proposition as a universally quantified temporal proposition, and this certainly suggests a *de dicto* attitude to modality.

> and the items referred to by its subject term, where those two terms themselves may or may not bear anything more than an accidental relation to one another. (Patterson 1995, p. 11)

Patterson calls these different copulative readings of necessity 'strong *cop*' and 'weak *cop*'. In his discussion of contingency [two-way possibility] Patterson again finds an ambiguity: "But just as in the case of necessity, so here we find an ambiguity in all two way possibility [contingent] propositions. One reading has to do simply with relations between the natures signified by the terms *A* and *B*, whereas the other takes account of the identity of the actual *B*'s" (Patterson 1995, p. 128). On this basis Patterson makes a distinction between what he calls 'term-term relations' and 'term-thing relations'. Patterson thinks that the modal syllogistic requires both term-term and term-thing relations, and he notes the obvious links between his analysis and standard *de dicto* and *de re* interpretations. As Brennan (1997, p. 230) and Malink (2006, p. 111) observe, when Patterson's method is unpacked, it is clear that his modal copulative readings are equivalent to *de dicto* and *de re* modals. Insofar as Patterson finds two modal readings at work in the syllogistic, Patterson's analysis shares much in common with Becker's analysis, since Becker certainly thinks that Aristotle is confused and that his logic requires both *de dicto* and *de re* modals. This feature of their respective analyses most clearly distinguishes Patterson and Becker from other modern interpreters. It is a feature which Striker also preserves in her 2009 commentary.

In LPC the modal operators are like negation, and the difference between the *de re* and *de dicto* readings is indicated by a difference in the scope of a univocal operator. As I noted in Chapter 2, Aristotle does not appear to have a notion of scope, and the connection between modality and negation is an interesting one. We saw there that Aristotle does not have a propositional negation operator, and treats phenomena of scope in terms of his square of opposition.[4] Aristotle's square of opposition serves him

[4]Does Aristotle have any propositional modal operator? Becker clearly thinks he does, since Becker reads modal conversion as, for example, 'Necessarily (Some *B* is *A*) converts to Necessarily (Some *A* is *B*).' If we take Aristotle's syllogisms as inferential schemas and not deductive schemas then we could describe the validity of a syllogism as the necessary truth of the implicative statement that if the premises are true then so is the conclusion. Aristotle is certainly aware of this but, as noted on p. 20, there is no reason to suppose that he *confuses* this with the necessity of a single proposition. Aristotle is certainly aware of the difference between the necessity of a syllogism and the necessity of its conclusion. Thus at *An. Pr.* 30b38—40 we have

> Consequently, the conclusion will be necessary when these things are so, but not necessary without qualification (ὥστε τούτων μὲν ὄντων ἀναγκαῖον ἔσται τὸ συμπέρασμα, ἁπλῶς δ᾽οὐκ ἀναγκαῖον).

What is perhaps more helpful is to note that Aristotle does have a meta-logical use of propositional necessity:

> It must first be explained that if it is necessary for *B* to be when *A* is, then when *A* is possible *B* will of necessity also be possible. (*An.Pr.* 34a5—7)

> Next, one must not take 'when *A* is, *B* is' as if it meant that *B* will be when some single thing *A*

well in the case of non-modal affirmatives and privatives. But it does cause trouble in the modals where negation and modality are combined. The trouble arises because we can formulate combinations of modal and negative in two ways. Using L as a necessity operator we can see that there is a difference between ${\sim}LAx$ and $L{\sim}Ax$. If we want to say 'no man is white by necessity' we might mean either of the following:

(3) $\forall x(Bx \supset L{\sim}Ax)$
(4) $\forall x(Bx \supset {\sim}LAx)$

Aristotle himself seems to be aware of the two constructions. He recognises the difference at *An.Pr.* A3, where he explains two ways we speak of possible:

> ...the first of these of necessity does not belong, while the other does not necessarily belong. (A3, 25b7–8)

Clearly, Aristotle has different *constructions*, and it is also clear that he understands a difference in *meaning* – for he uses both constructions when he describes certain modal equivalences in *On Int* 13, 22a14–b28. Ackrill illustrates these modal equivalences as follows: (I have added the modern notation.)

I		II	
not necessary not to be	$[{\sim}L{\sim}]$	necessary not to be	$[L{\sim}]$
possible to be	$[M]$	not possible to be	$[{\sim}M]$
not impossible to be	$[{\sim}{\sim}M]$	impossible to be	$[{\sim}M]$

III		IV	
not necessary to be	$[{\sim}L]$	necessary to be	$[L]$
possible not to be	$[M{\sim}]$	not possible not to be	$[{\sim}M{\sim}]$
not impossible not to be	$[{\sim}{\sim}M{\sim}]$	impossible not to be	$[{\sim}M{\sim}]$

There are some important points to notice about these modal expressions. The expressions in column I are contradictories of the expressions in column II. The expressions in column III are contradictories of column IV. Column II and III are contraries. And Aristotle explains the precise relationship between $L{\sim}$ and ${\sim}L$ at *On Int* 13, 22a39:

> is. For nothing is of necessity when a single thing is, but instead only if at least two things are, that is, when the premises are so related as was stated concerning deductions. (34a16–19)

While these *Prior Analytics* passages certainly demand careful interpretation, they cannot be cited as clear evidence of confusion about *de dicto* and *de re* uses of necessity.

> For the negation of 'necessary not to be' is not 'not necessary to be'. For both may be true of the same thing, since the necessary not to be is not necessary to be. The reason why these do not follow in the same way as the others is that it is when applied in a contrary way that 'impossible' and 'necessary' have the same force. For if it is impossible *to be* it is necessary for this (not, *to be*, but) *not to be*; and if it is impossible not to be it is necessary for this to be. (Ackrill's translation)

In *Prior Analytics* Aristotle uses a different word order to capture two common ways in which negation and necessity combine. In LPC the $\forall x(Bx \supset \sim LAx)$ construction is what we would expect in the square of opposition. But it is far from obvious that Aristotle has a strong grasp of the logical difference between $\forall x(Bx \supset L\sim Ax)$ and $\forall x(Bx \supset \sim LAx)$. In *An.Pr.* when Aristotle comes to syllogize, he interprets universal privative propositions about necessity such as 'no man is white by necessity' as $\forall x(Bx \supset L\sim Ax)$. While Aristotle does not use the $\sim L$ translations in the apodeictic syllogistic, he does have their $M\sim$ equivalents, and in fact the modal interchange principles enable any formula of the L/M modal syllogistic to be written with $\sim$ applying only to predicates.

David Charles (2000) is among those who favour a copulative approach, and he provides some important detail about how this approach has to work. Charles initially finds both term-term (p. 379) and term-thing (p. 385) copulative relations in Aristotle, though when Charles begins to sketch formation rules for the copulative propositions the term-term relation seems to drop out of his discussion. Charles notices that what he calls the "order of sentence construction" is crucial – for we get a different proposition depending upon whether, e.g., negation or modality is added first. Here is how Charles explains:

> Aristotle's remarks in *De Interpretatione* 21b26ff. appear to follow this pattern. He envisages the following order of sentence construction:
>
> white, man
>
> [Addition 1] is, is not
>
> [Addition 2] possible, not possible
>
> Addition 1 is made to the terms 'white' and 'man', Addition 2 to 'is' or 'is not', now taken itself 'as a subject'. In Addition 1, 'is' acts as an indicator of the way in which man and white are connected... In Addition 2, 'is' is like a subject because... it is modified by 'possible' and 'not possible'. (Charles 2000, pp. 381–382)

The order of the additions to the copulative construction provides the way to distinguish, for example, between (3) and (4). In (3), 'is not' is added 'at the level defined by Addition 1' – i.e., before the modal is added. In (4), the modal 'necessity' is added first, before 'is not' is added. If Aristotle is thinking of modal copulae in the way that Charles describes, then it is not surprising that Aristotle has to take great care about whether, e.g., in an LE premise the negation or the modal is added first. For the order would be the only way to distinguish between (3) and (4). After giving the sketch of such formation rules, Charles also begins to sketch a semantics for his copulative interpretation of Aristotle's modal propositions, but Charles does not give a full semantics to account for all the modal combinations. In particular, he leaves necessary privatives and contingent privatives largely unexplained. But certainly Charles' distinction between the levels of Additions 1 and 2 can be elaborated to accommodate all that is required. What is important to remember in trying to explain Aristotle's syllogistic in this way is that in the modal premises the additions at the level defined by Addition 1 will always come before the additions at the level defined by Addition 2. That is, when constructing syllogistic premises Aristotle *always* adds 'is' or 'is not' before he adds the modals. Adhering always to this order of construction is what is important. It is less important whether we represent the addition of the modal as explicitly modifying the copula itself (as Charles wants to) or whether we represent the addition of the modal (L, M, or Q) in predicate logic as, e.g., $\forall x(Bx \supset L{\sim}Ax)$, $\forall x(Bx \supset M{\sim}Ax)$, $\forall x(Bx \supset Q{\sim}Ax)$. The predicate logic translations of whatever notation Charles might use to represent the modal copulative propositions will be just my ordinary *de re* interpretations.

Similar comments apply to the discussion of the red terms which figure in what I called on p. 3 the Substance Principle: that if φ is a red term then φ is equivalent to necessarily-φ. One way of taking the substance principle makes it look as though for a red term A that A and LA are the very same term, and there is a lot in Aristotle that makes that plausible. Scholars who claim that necessity is not a property of the term but a property of the copula may be suspicious of my treatment of SP. But SP can easily accommodate a modal copula. On a modal copula reading what SP says is that when the term is red, if it can be applied at all then it can be applied by necessity. SP holds even if we treat the necessity operator as applying to the copula rather than to a term. The LPC representations are intended to be neutral on whether in LAx the LA marks a complex *term* or whether it modifies a (copulative) connection between x and the same term A. As far as I can tell nothing in the syllogistic hinges on which way this is taken. It is perhaps helpful to look at the difference as that between, on the one hand, a style of representation that involves uniform substitution of modal *copulae* for non-modal *copulae*, and, on the other hand, a style of representation that involves uniform substitution of modal *terms* for non-modal *terms*. But the important point is that when we are representing the modal syllogisms in predicate logic then a modal copula is represented by a *de re* modal attaching to the predicate term.[5]

In view of the discussion so far I will assume the following LPC representations of Aristotle's modal propositions involving necessity:

LA	It is necessary for *A* to belong to every *B*	$\forall x(Bx \supset LAx)$
LI	It is necessary for *A* to belong to some *B*	$\exists x(Bx \,\&\, LBx)$
LE	It is necessary for *A* to belong to no *B*	$\forall x(Bx \supset L{\sim}Ax)$
LO	It is necessary for *A* not to belong to some *B*	$\exists x(Bx \,\&\, L{\sim}Ax)$

The principal test of these representations is to account for the modal conversions, since it is here that scholars have found difficulty with the *de* re interpretations that I am assuming. For this reason I want to look closely at various methods scholars have used to explain modal conversion. In order to highlight the differences between other explanations and my own I will translate these other explanations of conversion into modal predicate logic. But first, look at the account of modal conversion foreshadowed in the last chapter. In *Prior Analytics* A3, Aristotle describes modal conversion principles which correspond to the non-modal conversions discussed on pp. 12 and 17:

> It will also be the same way in the case of necessary premises: the universally privative premise converts universally, while each kind of affirmative premise converts partially. For if it is necessary for A to belong to no B, then it is necessary for B to belong to no A (for if it is possible for it to belong to some, then it would be possible for A to belong to some B). And if A belongs to every or to some B of necessity, then it is necessary for B to belong to some A (for if it is not necessary, then neither would A belong to some B of necessity). But a particular privative premise does not convert, for the same reason as that which we also stated earlier. (25a27–36)

Using the LPC representations given above we have

LE-Conversion: $\forall x(Bx \supset L{\sim}Ax) \equiv \forall x(Ax \supset L{\sim}Bx)$
'it is necessary for *A* to belong to no *B*' converts to
'it is necessary for *B* to belong to no *A*'

[5]Hintikka has protested in many places at what he calls the 'Frege-Russell' thesis of the ambiguity of 'is'.(See for instance Hintikka (2004, p.3)). Hintikka explains that the Frege-Russell thesis is not part of Aristotle's thinking, and so urges caution in the use in the use of LPC representations. As far as LPC representations of the modal syllogistic are concerned the important ambiguity is between *de re* and *de dicto*.

LI-Conversion: $\exists x(Bx \,\&\, LAx) \equiv \exists x(Ax \,\&\, LBx)$
'*A* belongs to some *B* of necessity' converts to
'it is necessary for *B* to belong to some *A*'

LA-Conversion: $\forall x(Bx \supset LAx) \supset \exists x(Ax \,\&\, LBx)$
'*A* belongs to every *B* of necessity' converts to
'it is necessary for *B* to belong to some *A*'

Aristotle plainly intends these modal conversions to be valid, but as they stand they are not valid. They are subject to counter-examples like (4)(5) on p. 43. On pp. 43–44, I noted that the validity of LI-conversion can be ensured by placing a restriction on the input to modal conversion. If the *B* term is a red term then it is already implicitly a 'necessary' term, and so putting *B* within the scope of a necessity operator (*L*) plainly does not affect the truth value of a proposition. It only makes the implicit necessity explicit. The substance principle (see p. 3) will always guarantee that this is trouble-free because it guarantees that where a term is a red term, we can add or remove *L*s without any change in truth value. So in the present case, if *B* is required to be a red term then $LB \equiv B$. In fact, Aristotle describes just such a restriction on predication in *Posterior Analytics* A22. He describes what he calls 'genuine (*haplos*) predication'. The proposition 'the white thing is a log' is not an example of genuine predication. It is not, because 'white' identifies a subject indirectly, or accidentally. Genuine predication does not allow picking out a subject in this way; the only way to genuinely predicate is to predicate something of a subject which is identified by what we have been calling a red term. We can apply Aristotle's genuineness requirement to conversion. If we restrict the input into LI-conversion to instances of genuine predication, then our subject term is guaranteed to fit in the scope of a necessity operator, and so the LI-conversion itself is valid.

Topics I.8, 103b7ff has much the same spirit and deals specifically with conversion:

> For necessarily, whenever one thing is predicated of another, it either counterpredicates with the subject or it does not. And if it does counterpredicate, then it must be a definition or a unique property.

Definitions and unique properties belong to their subjects necessarily.

Making all of the implicit necessity explicit in the surface structure, genuine LI-conversion would really look like this:

Genuine LI-conversion: $\exists x(LBx \,\&\, LAx) \supset \exists x(LAx \,\&\, LBx)$

So, genuine LI-conversion is simply a substitution instance of non-modal I-conversion with LAx for Ax and LBx for Bx. Of course, the problem with (4)(5) on p. 43 above is that it is not an instance of *genuine LI* conversion. It is not because 'moving' (the *B* term) is not a red term.

LA, LI, and LE-conversions (i.e., *de re* L-conversions generally) are validated by the genuineness requirement.[6] Making all of the implicit *L*s explicit, LE and LA conversions look like this:

Genuine LE conversion: $\forall x(LBx \supset L{\sim}Ax) \equiv \forall x(LAx \supset L{\sim}Bx)$
Genuine LA conversion: $\forall x(LBx \supset LAx) \supset \exists x(LAx \,\&\, LBx)$

I will follow Aristotle's example and will not as a rule make the necessity of the subject term part of the formal representation of L-conversion.

By way of support for this account of modal conversion I will compare alternative approaches to modal conversion. In this section I will focus most specifically on explanations of LI conversion because it is where some important differences emerge. I will consider how LI conversion is explained in Patterson (1995), Nortmann (1996), Ebert and Nortmann (2007), Thom (1996), Malink (2006), and Striker (2009). It should be clear from the account above that the tools I use to validate modal conversion are:

(i) the simple *de re* LPC translations of Aristotle's modal propositions,
(ii) the Substance Principle, and
(iii) the Genuineness Requirement.

The textual passages I cite in support of semantic restrictions on conversion are not usually cited as evidence of restrictions on conversion because no other modern interpreter has considered whether restricted conversion might explain the purported logical problems. Most modern interpreters fall into either of two main camps. The first follows Becker and finds Aristotle's text ambiguous about modals. The second camp introduces complex logical representations to explain conversion.

Consider Patterson first. Patterson is definitely in the first camp. Not only does Patterson (1995) not use predicate logic representations of Aristotle's logical propositions, Patterson rejects modal predicate logic readings as not thoroughly Aristotelian. Patterson stresses an ambiguity in Aristotle's modal propositions. It is an

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[6]This is because of the validity of the following corollaries:
(C1) $[\forall x(Bx \supset L{\sim}Ax) \,\&\, \forall x({\sim}Bx \supset L{\sim}Bx)] \supset \forall x(Ax \supset L{\sim}Bx)$
(C2) $[\exists x(Bx \,\&\, LAx) \,\&\, \forall x(Bx \supset LBx)] \supset \exists x(Ax \,\&\, LBx)$

ambiguity which Patterson tells us interpreters should respect because, he argues, the ambiguity is part of Aristotle's own thinking, and so modern interpreters who aim to disambiguate Aristotle's text are going to miss Aristotle's own way of thinking about modals. In this way Patterson strives in his interpretation to preserve the ambiguity which he thinks is part of Aristotle's way of thinking. But Patterson himself *disambiguates* with his strong *cop* and weak *cop* readings, as mentioned on p. 48 above. According to Patterson, it is strong *cop* which is needed to explain conversion. Recall that the strong *cop* reading links a term with a term. So for Patterson conversion is straightforward: from 'Some A is necessarily B' we can convert to 'Some B is necessarily A.' And precisely because the strong modal *cop* is a link between two terms, Patterson's conversion rules translate into *de dicto* modals in predicate logic. But the weak *cop* reading is what Patterson uses in the syllogisms. Weak cop translates into *de re*. The effect is that Patterson retains old-fashioned Becker-style problems about *de dicto* and *de re*. Patterson is not alone in this approach. In her 2009 Clarendon translation Gisela Striker seems to share Patterson's affinity for ambiguity. This puts Striker squarely in the first camp. Striker herself is focused on translation, not interpretation, though she notes that she finds Becker's diagnosis "still the most convincing" (Striker 2009, p. xvi). That is, Striker, like Patterson, finds evidence of both *de dicto* and *de re* modals at work in the syllogistic. The syllogisms require *de re* modals, while the conversion rules appear to require *de dicto* modals. This is why she thinks that the modal syllogistic is logically incoherent (Striker 2009, p.115). Neither Patterson nor Striker explores a modern predicate logic approach in any detail. If we follow Patterson and Striker it seems we have to say (i) that Aristotle requires both *de dicto* and *de re* modals and (ii) that Aristotle does not notice his inconsistency or its devastating effect on the modal syllogistic. Patterson and Striker are modern-day proponents of the 'realm of darkness' interpretation.

Nortmann (1996), and Ebert and Nortmann (2007), take a different approach. They readily turn to modern predicate logic to help explain Aristotle's modal syllogistic. The extent to which they actually *use* the power of modern modal predicate logic is particularly evident in the treatment of modal conversion. In order to capture the essentialism in Aristotle's syllogisms, Nortmann uses *de re* necessity. In order to validate conversion, he uses *de dicto* necessity. But Nortmann, unlike Becker and Patterson and Striker, does not produce *separate* accounts for the necessity of the syllogisms and the necessity of the conversion rules; instead, Nortmann's approach is doubly modal. It combines *de dicto* and *de re* necessity in unified translations. Again, let's use LI-conversion as our example. Nortmann (1996, p.115) and Ebert and Nortmann (2007, p. 426) translate an LI-proposition as $\exists xL(Bx \& LAx)$ – putting a *de dicto* modal after the quantifier, and a *de re* modal attached to the predicate term A. Treated Nortmann's way LI-conversion is $\exists xL(Bx \& LAx) \equiv \exists xL(Ax \& LBx)$. Nortmann does not give a proof of LI-conversion but the proof would seem to go something like the following:

(1)	$\exists xL(Bx \mathbin{\&} LAx)$
(2)	$\exists x(LBx \mathbin{\&} LLAx)$
(3)	$\exists x(LBx \mathbin{\&} LAx)$
(4)	$\exists x(LAx \mathbin{\&} LBx)$
(5)	$\exists xL(Ax \mathbin{\&} LBx)$

The move from (2) to (3) is legitimate in S4. If (1)–(5) is what Nortmann means by LI-conversion then notice that whenever we have a true LI-proposition, we have two red terms – i.e., both *A* and *B* must be red. Ebert and Nortmann represent an LE proposition as $L\forall x(Bx \supset L{\sim}Ax)$, and give a proof of LE-conversion (Ebert and Nortmann 2007, pp. 252–259).[7] The proof of LE-conversion requires S5:

(1)	$L\forall x(Bx \supset L{\sim}Ax)$	
(2)	$\forall xL(Bx \supset L{\sim}Ax)$	[Barcan Formula (BF)]
(3)	$\forall x(MBx \supset L{\sim}Ax)$	[S5]
(4)	$\forall x(MBx \supset {\sim}MAx)$	[Modal Interchange]
(5)	$\forall x(MAx \supset {\sim}MBx)$	[PC]
(6)	$\forall xL(Ax \supset L{\sim}Bx)$	[S5]
(7)	$L\forall x(Ax \supset L{\sim}Bx)$	[S5]

Because they find more obvious textual evidence for taking (1) – rather than (2) – as an LE proposition, Ebert and Nortmann need the Barcan Formula [in the form $\forall xL\varphi \equiv L\forall x\varphi$] in order to get from (1) to (2).[8] Again there is a cost. Relying on the Barcan Formula and S5 takes us some long way from the kinds of tools that Aristotle had to hand. There is another worry too. Let *A* be white and *B* be man. The proof of LE-conversion in (1)–(7) takes us from (1), an ordinary LE proposition, e.g., 'no man is white by necessity', to (3) 'all *possible men* are necessary non-whites' and to (5) 'all *possible whites* are not *possible men*.' Aristotle tells us in *An.Pr.* A3 that conversion will 'be the same way in the case of necessary premises' as conversion in the non-

[7] See also Nortmann (2002, pp. 256–258), who credits the idea of representing an LE proposition by $\forall x(MBx \supset L{\sim}Ax)$ to Angelleli (1979), (see footnote 4 on p. 125 below) and Schmidt (1989).

[8] For some remarks on LA-conversion see Ebert and Nortmann (2007, p. 425). If LA-conversion is $L\forall x(Bx \supset LAx) \supset \exists xL(Ax \mathbin{\&} LBx)$, a proof can be given as follows – it also requires the power of S5:

(1)	$L\forall x(Bx \supset LAx)$
(2)	$\forall xL(Bx \supset LAx)$
(3)	$\exists xL(Bx \mathbin{\&} LAx)$
(4)	$\exists x(LBx \mathbin{\&} LAx)$
(5)	$\exists x(LAx \mathbin{\&} LBx)$
(6)	$\exists xL(Ax \mathbin{\&} LBx)$

modal cases. But non-modal E-conversion and modal LE-conversion can't 'be the same way' if LE-conversion changes a subject from B to MB. Non-modal E-conversion will never do that. Further, as we shall see in footnote 4 on p. 75, Nortmann's translations cause some problems in the representation of first figure mixed apodeictic syllogisms. The powerful methods of logic at work in Nortmann (1996) cause Striker to wonder how far this logic is from Aristotle. Nortmann is cautious about this same point and does not claim that Aristotle is working in the modal system S5, only that these modern tools might help to explain the logical puzzles in the modal syllogistic.

The recent analyses by Thom (1996) and Malink (2006) share a common approach to the interpretation of an Aristotelian LI-proposition. Both give a 'disjunctive' interpretation of an LI-proposition. Let's look first at Thom's account. Thom does not represent the syllogistic in predicate logic, but when we express Thom's semantics in predicate logic what we find is that where Aristotle says 'some *B*s are necessarily *A*s' Thom means something like

(5) $\exists x(Bx \,\&\, LAx) \vee \exists x(Ax \,\&\, LBx)$.[9]

What Thom gains by this disjunctive interpretation of an LI-proposition is a trivially valid LI-conversion. Conversion reverses the order of the subject and predicate, and this reversal is already built in to Thom's interpretation of the LI proposition. If we begin with a disjunctive LI-proposition (5) conversion gets us another disjunctive LI-proposition

(6) $\exists x(Ax \,\&\, LBx) \vee \exists x(Bx \,\&\, LAx)$.

And (6) of course follows from (5) by commutation of $\vee$. In Thom's disjunctive propositions, it doesn't matter whether the terms A and B are red or green, and there need be no restrictions whatsoever on the terms. If it is true that some white thing is necessarily a man, then it will be true that some man is necessarily white. Why? Because, according to Thom, Aristotle must be interpreting 'some white thing is necessarily a man' as

(7) Either some white thing is a necessary man or some man is necessarily white

and (7) is equivalent to

[9] Thom's notation for an LI-proposition is Lab^i. Thom sets out this part of his semantics on pp. 142—151, and the semantics for Lab^i is given in 22.1.7 on p. 146, and his gloss is "Either some *a* is a necessary *b* or some *b* is a necessary *a*."

(8) Either some man is necessarily white or some white thing is a necessary man

and (8), then, *means* the same as 'some man is necessarily white.' This technique certainly gets this part of the logic to work – i.e., it validates LI-conversion. But it raises the question: does Aristotle have anything in mind like this disjunctive interpretation of an ordinary LI proposition? We shall see on pp. 92–94 that the disjunctive interpretation can cause trouble in some of the syllogisms.

The disjunctive interpretation of LI propositions has another, more recent defender in Marko Malink. Malink (2006, p. 109) acknowledges that his treatment of LI-propositions follows Thom, but Malink differs from Thom, who relies on set theoretical explanations. Malink prefers 'a kind of mereological framework' to a set theoretical explanation, and he claims (p. 95) that 'the reason that attempts at consistently reconstructing modal syllogistic have failed up to now lies not in the modal syllogistic itself, but in the inappropriate application of modern modal logic and extensional set theory to the modal syllogistic.' So while Malink's analysis involves a disjunctive LI proposition, the likeness to Thom does not go far. On the other hand, Malink follows Patterson in favouring a modal copulative interpretation. But unlike Patterson, Malink (pp. 107–108) eschews individuals in his model. He describes Aristotle's syllogistic as 'a pure term logic that does not recognize an extra syntactic category of individual symbols besides syllogistic terms' (p. 95). This 'pure term logic' emerges as very different from what Patterson describes, since Malink envisages a system without any distinction between objects and their features:

> ... in Aristotle's syllogistic the domain of quantification consists of entities of the same type as syllogistic terms and... there is no logical distinction between individuals on the one hand and syllogistic terms on the other hand. It is in this sense that we claim that the syllogistic is a pure term logic in which there is no room for individuals... [Aristotle] does not recognize an extra category of singular terms as opposed to general terms. (Malink 2006 p. 107)

Malink offers as an advantage of his system the fact that it does not require any kind of *de dicto* or *de re* modals:

> ...there is no distinction between *de re* and *de dicto* readings in Aristotle's modal syllogistic; for there are no modal sentential operators which could be applied to *dicta*, and no zero-level individuals which could serve as the *res* of *de re* modalities... There is no need to put a lot of effort into avoiding the *de re-de dicto* ambiguity; it simply vanishes

> once we have settled on a pure term logic and given up the logical distinction between first-order predicates and zero-level individuals. (Malink 2006, p. 111)

Despite Malink's claims it's not clear that the *de re*/*de dicto* issue can be avoided so easily. Malink's representation of an LI-proposition is $\exists z((\Upsilon bz \,\&\, \hat{E}az) \vee (\Upsilon az \,\&\, \hat{E}bz))$. When translated into second-order predicate logic, this becomes:

$$(9) \qquad \exists\varphi((\forall x(\varphi x \supset Bx) \,\&\, \forall x(\varphi x \supset LAx)) \vee (\forall x(\varphi x \supset Ax) \,\&\, \forall x(\varphi x \supset LBx)))$$

(9) is in fact logically equivalent to (5), Thom's disjunctive LI proposition, given second-order logic with non-empty predicates. Since Malink's interpretation can be paraphrased neatly enough in modal predicate logic, it's not entirely clear that this amounts to a significant difference. What is more significant is the difference between the complex disjunctive interpretation and Aristotle's own very simple modal expressions – for both Thom's (5) and Malink's (9) are supposed to be interpretations of Aristotle's 'some *B*s are necessarily *A*s.'

Striker (2009, p. xvi) comments on recent interpretations:

> Both Nortmann's and Thom's interpretations remain, of course, counterfactual – they can at best show what Aristotle might have done if he had been aware of the *de re-de dicto* distinction, or if he had been able to use the powerful tools of modern mathematical logic. Furthermore, though it is plausible to think that Aristotle, given his metaphysical views, would have preferred a *de re*-interpretation of modal propositions to a *de dicto* version, it seems rash to assume that he would have made no changes in the rules for his modal logic. While there is a lot to be learned from these experiments, the reader of Aristotle's treatise has to deal with the original version.

Striker seems right about this. We are playing different games if we attribute a *de dicto*/*de re* distinction to Aristotle, or if we suppose he was able to use the powerful tools of modern mathematical logic. Most scholars today do accept that *de re* modals are a better fit for the metaphysics than *de dicto* modals. But while it may be rash to make any kinds of suppositions about changes in the rules for modal logic, given that Aristotle provided no clear explanation about how to interpret those rules, it does fall to his interpreters to try to offer plausible suppositions, plausible interpretations. Striker in the end does make some suggestions of her own. As noted earlier she follows Becker and supposes that Aristotle's use of modals is inconsistent – sometimes *de dicto* (i.e., in the modal conversions), other times *de re* (i.e., in the syllogisms). (See Striker 2009 pp. xvi–xvii, also p. 71.) Certainly, getting the logic to work is not necessarily the same project as making sense of the text, and I share Striker's bias in favour of the latter. It seems to me to be by far the more important and philosophically interesting project, but it also brings home the need for textual justification for the kinds of rules and techniques that Nortmann, Thom and other logicians use to explain Aristotle. In this respect I sympathize with Striker. Nevertheless I think it is helpful to be able to give a line-by-line symbolization of Aristotle's proofs in a framework that is more powerful that anything Aristotle has at hand, provided that the extra power is never *used* except for the clarity it provides over natural language. Some classical scholars seem to suggest that there is no legitimacy whatsoever about *any* logical representations of Aristotle, though of course this does not follow. If modal predicate logic representations are only that – i.e., merely representations – then there is no real problem. The proof of my claims can of course only be given by the line-by-line LPC analysis of Aristotle's text, and that is the task which will occupy the remainder of the book.

Part II

NECESSITY IN THE SYLLOGISTIC
An. Pr. A8–12

Chapter 6
Syllogizing in Red: Trivializing the Modals

I noted in Chapter 4 that in the *Posterior Analytics* Aristotle clearly links science with what cannot be otherwise. He tells us many times that there is no science of the accidental. So let us suppose that Aristotle's science concerns only red terms, terms for what cannot be otherwise and, so, terms which have a certain necessity about them. This chapter is about how such a reading affects the apodeictic syllogisms of the *Prior Analytics*. Scholars have emphasized the *Posterior Analytics* passages, citing them as evidence that Aristotle's science demands syllogisms about necessity. When this evidence is brought to bear on the syllogistic laid out in the *Prior Analytics*, then the obvious explanation is that the scientific syllogisms which lead to demonstrative knowledge must in fact be exactly Aristotle's modal apodeictic syllogisms. And so it would seem that there is a prima facie case for limiting the discussion of Aristotle's scientific syllogisms to those which involve what I have been calling red terms, since it is red terms which are necessary and which cannot be otherwise. For convenience, let us call such a restricted syllogistic a *red syllogistic*. Whether a red syllogistic is the right approach to Aristotle's apodeictic syllogistic is a question we shall need to consider carefully.

In fact, there is much to recommend a red syllogistic. A red syllogistic clearly sits well with Aristotle's explanation of pure apodeictic syllogisms in *An.Pr.* A8. The complete chapter A8 is given here:

> Since to belong and to belong of necessity and to be possible to belong are different (for many things belong, but nevertheless not of necessity, while others neither belong of necessity nor belong at all, but it is possible for them to belong), it is clear that there will also be different deductions of each and that their terms will not be alike: rather, one deduction will be from necessary terms, one from terms which belong, and one from possible terms.
>
> In the case of necessary premises, then, the situation is almost the same as with premises of belonging: that is, there either will or will not be a deduction with the terms put in the same way, both in the case of belonging and in the case of belonging or not belonging of necessity, except that they will differ in the addition of 'belonging (or not belonging) of necessity' to the terms (for the privative premise converts in the same way, and we can interpret 'being in as a whole' and 'predicated of all' in the same way).
>
> In the other cases, then, the conclusion will be proved to be necessary through conversion in the same way as in the case of

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
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> belonging. But in the middle figure, when the universal is affirmative and the particular is privative, and again in the third figure, when the universal is positive and the particular privative, the demonstration is not possible in the same way. Instead, it is necessary for us to set out that part to which each term does not belong and produce the deduction about this. For it will be necessary in application to each of these; and if it is necessary of what is set out, then it will be necessary of some part of that former term (for what is set out is just a certain 'that'). Each of these deductions occurs in its own figure. (*An.Pr.* A8, 29b29–30a14)

This chapter is usually understood to be about pure apodeictic syllogisms. One clear message is that for every non-modal syllogism (for all syllogisms about mere 'belonging') there is a corresponding modal syllogism about necessity (about 'belonging or not belonging of necessity'). Aristotle does not give any more precise and detailed analysis of how this is supposed to work. He does not tell whether he means the pure apodeictic syllogistic to be what we are here calling a red syllogistic. He does however claim that the syllogisms that require proving are all proved 'in the same way as in the case of belonging'.[1] So let's consider why a red syllogistic might be right and let's consider how exactly might a red syllogistic work.

If we are entitled to suppose that science deals with red terms then the pure apodeictic syllogisms simply become trivial. If we restrict syllogisms to those involving only red terms, the difference between a modal syllogism and its non-modal correlate will only amount to a difference in terms, or, if you prefer, a difference in the way the term applies to an individual, but not a difference in logical structure. Let us start with some of Aristotle's simple non-modal or assertoric syllogisms. First consider non-modal Barbara:

(1) Every *B* is *A*
<u>Every *C* is *B*</u>
Every *C* is *A*

(1) is valid. So of course we get a valid instance even when we put in straightforwardly accidental terms – for example, when we make *A* laughing, *B* moving, and *C* happy:

(2) Every moving thing is laughing
<u>Every happy thing is moving</u>
Every happy thing is laughing

[1] With two exceptions – one from the second figure (Baroco LLL) and one from the third figure (Bocardo LLL). These are discussed in Chapter 8 in the context of modal conversion.

In the language of red and green terms, all of these terms are green. Laughing, moving, and being happy are not red terms – i.e., they are not terms about necessity – because anything that is laughing or moving or happy can be otherwise. It is because they can be otherwise that we call them green terms. But such green terms do not affect validity here. The reason these terms do not affect it is because the logical structure of the syllogistic schema Barbara guarantees that *every instance* is valid – every instance is valid no matter what we choose for our terms. Since (2) is just an instance of Barbara, then plainly (2) is valid. And so Aristotle will say that it is a syllogism. But if his science demands syllogisms about necessity, then (2) – even though it is a valid syllogism – is not a *scientific* syllogism.

Next, let us restrict the terms in (1) so that *A*, *B*, and *C* are all and only red terms – i.e., necessary terms, terms for what cannot be otherwise. Let's call this Red Barbara. We get an instance of Red Barbara when, for example, we make *A* animal, *B* mammal, and *C* man:

(3) Red Barbara
Every mammal is an animal
<u>Every man is a mammal</u>
Every man is an animal

(3) is of course simply another instance of (1). And, again, since the logical structure of the schema Barbara (1) guarantees that every instance is valid, (3) is then plainly valid. Furthermore, because all the terms are red and name what cannot be otherwise, (3) looks like it might be a good candidate for an Aristotelian scientific syllogism.[2]

Such a red syllogistic has a lot in its favour. It fits the demands that *An. Post.* requires of scientific demonstrations. And, formally at least, a red syllogistic works exactly the same way as the non-modal syllogistic works. If we are guaranteed the validity of the non-modal syllogisms, then with a restriction to red terms, we are guaranteed the red apodeictic syllogisms as well. Often in the *Prior Analytics* when Aristotle uses syllogistic premises about necessity, his own locutions make the necessity explicit. When he describes apodeictic syllogisms, he explicitly includes a modal qualifier such as 'necessary' or 'necessarily' in each of the syllogistic premises and in the conclusion. While we have red terms – which of course are themselves necessary terms – in (3), we do not have any explicit 'necessity' qualifier in either the

[2]Jeroen van Rijen, in his 1989 book, looks to *Posterior Analytics* A4–6 for an account of how to choose terms in Aristotle's science. van Rijen considers how a restriction to what he calls 'homogeneous' terms might affect Aristotle's modal syllogistic. This bears a close connection to the red syllogistic that I am suggesting here but van Rijen's homogeneity requirement is I think a stronger restriction than a simple restriction to what I call red terms. See van Rijen, Chapters 8 and 9, and especially pp. 178–179 and 205–209.

premises or the conclusion. Of course since we are using only red terms, the Substance Principle entitles us to put an explicit modal qualifier into the red propositions. In a purely red syllogistic we are certainly entitled to put an explicit necessity qualifier on every term. Doing so, we get from (3) to (4):

(4)	Every necessary-mammal is a necessary-animal
	<u>Every necessary-man is a necessary-mammal</u>
	Every necessary-man is a necessary-animal

Suppose that what Aristotle means in *An.Pr.* A8 is that he wants us to add necessity to *all* of the syllogistic terms. That is, suppose he means that in the LLL syllogisms *all* of the terms are *red*. If that is what he means then the apodeictic propositions should be analysed as follows, with *L*s attached to subject and predicate terms:

(LA_1)	'A belongs of necessity to all B'	::	$\forall x(LBx \supset LAx)$
(LE_1)	'A belongs of necessity to no B'	::	$\forall x(LBx \supset L{\sim}Ax)$
(LI_1)	'A belongs of necessity to some B'	::	$\exists x(LBx \,\&\, LAx)$
(LO_1)	'A does not belong of necessity to some B'	::	$\exists x(LBx \,\&\, L{\sim}Ax)$

The LLL syllogisms themselves would then be exactly the syllogisms in Table 2, but with all red terms, and hence with two *L*s in every syllogistic proposition. Table 3 below shows the effect of this:

Table 3
Red L+L Syllogisms in Three Figures (*An.Pr.* A8)
(with $LA_1 - LO_1$)

First Figure	Second Figure	Third Figure
Barbara LLL $\forall x(LBx \supset LAx)$ $\underline{\forall x(LCx \supset LBx)}$ $\forall x(LCx \supset LAx)$	Cesare LLL $\forall x(LBx \supset L{\sim}Ax)$ $\underline{\forall x(LCx \supset LAx)}$ $\forall x(LCx \supset L{\sim}Bx)$	Darapti LLL $\forall x(LCx \supset LAx)$ $\underline{\forall x(LCx \supset LBx)}$ $\exists x(LBx \& LAx)$
Darii LLL $\forall x(LBx \supset LAx)$ $\underline{\exists x(LCx \& LBx)}$ $\exists x(LCx \& LAx)$	Camestres LLL $\forall x(LBx \supset LAx)$ $\underline{\forall x(LCx \supset L{\sim}Ax)}$ $\forall x(LCx \supset L{\sim}Bx)$	Felapton LLL $\forall x(LCx \supset L{\sim}Ax)$ $\underline{\forall x(LCx \supset LBx)}$ $\exists x(LBx \& L{\sim}Ax)$
Celarent LLL $\forall x(LBx \supset L{\sim}Ax)$ $\underline{\forall x(LCx \supset LBx)}$ $\forall x(LCx \supset L{\sim}Ax)$	Festino LLL $\forall x(LBx \supset L{\sim}Ax)$ $\underline{\exists x(LCx \& LAx)}$ $\exists x(LCx \& L{\sim}Bx)$	Datisi LLL $\forall x(LCx \supset LAx)$ $\underline{\exists x(LCx \& LBx)}$ $\exists x(LBx \& LAx)$
Ferio LLL $\forall x(LBx \supset L{\sim}Ax)$ $\underline{\exists x(LCx \& LBx)}$ $\exists x(LCx \& L{\sim}Ax)$	Baroco LLL $\forall x(LBx \supset LAx)$ $\underline{\exists x(LCx \& L{\sim}Ax)}$ $\exists x(LCx \& L{\sim}Bx)$	Disamis LLL $\exists x(LCx \& LAx)$ $\underline{\forall x(LCx \supset LBx)}$ $\exists x(LBx \& LAx)$
		Bocardo LLL $\exists x(LCx \& L{\sim}Ax)$ $\underline{\forall x(LCx \supset LBx)}$ $\exists x(LBx \& L{\sim}Ax)$
		Ferison LLL $\forall x(LCx \supset L{\sim}Ax)$ $\underline{\exists x(LCx \& LBx)}$ $\exists x(LBx \& L{\sim}Ax)$

The syllogisms in Table 3 are all *substitution instances* of the syllogisms in Table 2, with modally qualified red terms. What I mean is this. The syllogisms in Table 2 are valid whatever the predicates are, so, in particular, whatever holds for every *A*, *B* and *C*, certainly holds for every red *A*, *B* and *C* and for every (red) ~*A*, ~*B* and ~*C*. But for red terms the Substance Principle ensures that *A*, *B* and *C* are, respectively, equivalent to *LA*, *LB* and *LC*; and ~*A*, ~*B* and ~*C* are, respectively, equivalent to *L*~*A*, *L*~*B* and *L*~*C*. So that, in a sense, *with red terms* all of Table 3 just *is* Table 2, since for red terms the *L* can be added or dropped without affecting validity. Note that *even without restriction to red terms* first and third figure Table 3 syllogisms are all valid as instances of the syllogisms in Table 2.[3]

If Table 3 represents what Aristotle means in *An.Pr.* A8, then the LLL syllogisms are not really about *modal logic*. In *The Development of Logic* (1962), William and Martha Kneale make this the basis of a complaint about the syllogistic. They say that if this is what is going on, then the modal syllogistic is not really *modal* logic involving special modal principles; instead, it is just a logic of modally qualified terms:

> If modal words modify predicates, there is no need for a special theory of *modal* syllogisms. For there are only ordinary assertoric [non-modal] syllogisms of which the premises have peculiar predicates. (p. 91)

If Table 3 is correct, then so are the Kneales.

But such a closely restricted red syllogistic is not the *only* way we might understand Aristotle's instructions in *An.Pr.* A8. Even where Aristotle makes the necessity of a proposition explicit he does not ever indicate that this might involve *two* occurrences of the necessity operator. Each of the *propositions* in Table 3 includes two occurrences of necessity – one for each red term. But Aristotle never quite tells us so much. And so there is a question about whether this purely red syllogistic is right. There is a question because even an explicitly modal statement about necessity immediately gives rise to interpretive questions since the 'necessity' qualifier can go in different places in a modal proposition.

Let's consider *An.Pr.* A8 in a different light. Notice that Aristotle says that when we add 'belonging of necessity' or 'not belonging of necessity' to a non-modal syllogism, then we get a syllogism about necessity. This suggests that necessity is linked only to the predicate term. If this is the way to understand Aristotle's instructions in *An.Pr.* A8 about the LLL syllogisms, then his necessary A, E, I, and O propositions should be analysed, as on p. 52, as

(LA_2)	'A belongs of necessity to all B'	::	$\forall x(Bx \supset LAx)$
(LE_2)	'A belongs of necessity to no B'	::	$\forall x(Bx \supset L{\sim}Ax)$

[3]The case of the affirmatives is easy. Simply substitute uniformly LA for A, LB for B and LC for C, since whatever holds for every A, B and C, also holds for every LA, LB and LC. (Of course, when I say that LA is substituted for A I need not be taking sides on whether LA is a complex term applying to an individual x, or whether LA indicates that A applies to x in a certain way. For Aristotle, applying by necessity is certainly applying, and so if a principle holds of every application of every predicate A to x then it certainly holds of every apodeictic application of A to x.) The case of the privatives is slightly more complex, for here we need to substitute $L{\sim}A$ for ${\sim}A$, $L{\sim}B$ for ${\sim}B$ and $L{\sim}C$ for ${\sim}C$, and these latter are not atomic formulae. What we do is substitute ${\sim}L{\sim}A$ for A, ${\sim}L{\sim}B$ for B and ${\sim}L{\sim}C$ for C to get ${\sim\sim}L{\sim}A$, ${\sim\sim}L{\sim}B$ and ${\sim\sim}L{\sim}C$. Then use the principle of double negation ($\varphi \equiv {\sim\sim}\varphi$) to get $L{\sim}A$, $L{\sim}B$ and $L{\sim}C$. Thus, substitution of ${\sim}L{\sim}A$ for A in a formula like $\forall x(Bx \supset {\sim}Ax)$ gives $\forall x(Bx \supset {\sim\sim}L{\sim}Ax)$ which is equivalent to $\forall x(Bx \supset L{\sim}Ax)$.

(LI_2)	'A belongs of necessity to some B'	::	$\exists x(Bx \mathbin{\&} LAx)$
(LO_2)	'A does not belong of necessity to some B'	::	$\exists x(Bx \mathbin{\&} L{\sim}Ax)$

This gives Table 4:

Table 4
L+L Syllogisms in Three Figures (*An.Pr.* A8)
(with $LA_2 - LO_2$)

First Figure	Second Figure	Third Figure
Barbara LLL $\forall x(Bx \supset LAx)$ $\underline{\forall x(Cx \supset LBx)}$ $\forall x(Cx \supset LAx)$	Cesare LLL $\forall x(Bx \supset L{\sim}Ax)$ $\underline{\forall x(Cx \supset LAx)}$ $\forall x(Cx \supset L{\sim}Bx)$	Darapti LLL $\forall x(Cx \supset LAx)$ $\underline{\forall x(Cx \supset LBx)}$ $\exists x(Bx \mathbin{\&} LAx)$
Darii LLL $\forall x(Bx \supset LAx)$ $\underline{\exists x(Cx \mathbin{\&} LBx)}$ $\exists x(Cx \mathbin{\&} LAx)$	Camestres LLL $\forall x(Bx \supset LAx)$ $\underline{\forall x(Cx \supset L{\sim}Ax)}$ $\forall x(Cx \supset L{\sim}Bx)$	Felapton LLL $\forall x(Cx \supset L{\sim}Ax)$ $\underline{\forall x(Cx \supset LBx)}$ $\exists x(Bx \mathbin{\&} L{\sim}Ax)$
Celarent LLL $\forall x(Bx \supset L{\sim}Ax)$ $\underline{\forall x(Cx \supset LBx)}$ $\forall x(Cx \supset L{\sim}Ax)$	Festino LLL $\forall x(Bx \supset L{\sim}Ax)$ $\underline{\exists x(Cx \mathbin{\&} LAx)}$ $\exists x(Cx \mathbin{\&} L{\sim}Bx)$	Datisi LLL $\forall x(Cx \supset LAx)$ $\underline{\exists x(Cx \mathbin{\&} LBx)}$ $\exists x(Bx \mathbin{\&} LAx)$
Ferio LLL $\forall x(Bx \supset L{\sim}Ax)$ $\underline{\exists x(Cx \mathbin{\&} LBx)}$ $\exists x(Cx \mathbin{\&} L{\sim}Ax)$	Baroco LLL $\forall x(Bx \supset LAx)$ $\underline{\exists x(Cx \mathbin{\&} L{\sim}Ax)}$ $\exists x(Cx \mathbin{\&} L{\sim}Bx)$	Disamis LLL $\exists x(Cx \mathbin{\&} LAx)$ $\underline{\forall x(Cx \supset LBx)}$ $\exists x(Bx \mathbin{\&} LAx)$
		Bocardo LLL $\exists x(Cx \mathbin{\&} L{\sim}Ax)$ $\underline{\forall x(Cx \supset LBx)}$ $\exists x(Bx \mathbin{\&} L{\sim}Ax)$
		Ferison LLL $\forall x(Cx \supset L{\sim}Ax)$ $\underline{\exists x(Cx \mathbin{\&} LBx)}$ $\exists x(Bx \mathbin{\&} L{\sim}Ax)$

With (LA_2), (LE_2), (LI_2) and (LO_2) as our LPC translations, consider what is needed to validate the first and third figure syllogisms in Table 4. *All* that is needed to validate the first figure LLLs is a principle which guarantees that for any term φ, $L\varphi \supset \varphi$. And of course $L\varphi \supset \varphi$ is valid no matter what we choose as our φ. That is, whatever is necessarily-so is so. This makes proofs of all of the first figure LLLs of Table 4 simple and straightforward. Consider our earlier non-modal syllogism (3). Instead of belonging, we add belonging of necessity. Thus

(5)	$\forall x(Bx \supset LAx)$	All mammals are necessarily animals
	$\underline{\forall x(Cx \supset LBx)}$	<u>All men are necessarily mammals</u>
	$\forall x(Cx \supset LAx)$	All men are necessarily animals

The result is valid. Anything that is necessarily a mammal is a mammal, and so we can syllogize.

When we look at the second figure syllogisms in Table 4, we can see that these are not quite so simple. The syllogisms in the second and third figures typically require proof by conversion, and the conversions that we need are complicated by the addition of 'belonging or not belonging of necessity'. In *Prior Analytics* A8 Aristotle is clearly thinking of constructing proofs by conversion, for he remarks that in these 'other cases', too, 'the conclusion will be proved to be necessary through conversion in the same way as in the case of belonging.' Let's take the second figure Cesare LLL as an example to see what Aristotle might have in mind:

(6) Cesare LLL

$\forall x(Bx \supset L{\sim}Ax)$

$\underline{\forall x(Cx \supset LAx)}$

$\forall x(Cx \supset L{\sim}Bx)$

In the non-modal XXX syllogistic, Aristotle proves Cesare XXX is valid by converting the initial premise. That is, $\forall x(Bx \supset {\sim}Ax)$ converts to $\forall x(Ax \supset {\sim}Bx)$, by ordinary, non-modal E-conversion. In the modal case that we are considering, if the proof is supposed to proceed in the same way, then we need a *modal* conversion. That is, we need to get from $\forall x(Bx \supset L{\sim}Ax)$ to $\forall x(Ax \supset L{\sim}Bx)$:

(7) $$\forall x(Bx \supset L{\sim}Ax) \supset \forall x(Ax \supset L{\sim}Bx).$$

We noted in the last chapter that such conversions can be validated provided that the *B* term is red. A more general restriction would be to the effect that in order to validate the conversion *both* the *A* and the *B* terms must be red terms. This would allow both terms to fit in the scope of a modal *L*.[4] The restriction to red terms does validate the

[4] In his discussion of LE-propositions, Malink says 'both argument terms... are required to be substance terms.' See Malink (2006, p. 110). In order to preserve a modal square of opposition Malink in fact requires a conjunctive form of an LE-proposition. His version of an LE-proposition (i.e., his $\mathbb{N}^e ab$) when translated into LPC with *de re* modals becomes: $\forall x(Ax \equiv LAx)$ & $\forall x(Bx \equiv LBx)$ & ${\sim}\exists x(Ax \& Bx)$. Both terms are red, and LE-conversion is valid.

modal conversion (7) above. And so if we have restricted conversions and $L\varphi \supset \varphi$ (i.e., the modal logician's *T*-principle), then we *can* validate all of the syllogisms in Table 4. The question at this stage is whether that is what we *ought* to do if our object is to give a good interpretation of Aristotle's instructions in *An.Pr.* A8. We should, I think, be especially cautious about how to interpret A8 because Aristotle tells us very little at all about the details of the LLL proofs. He merely suggests that the proofs will involve conversion, but he does not do the proofs for us, and so he does not show us *how* to do the modal conversions or what those conversions involve. In short, what we have, here, are *two ways* to interpret Aristotle's instructions; we just cannot say for certain which is the way that Aristotle himself has in mind. He might have in mind an analysis, such as the one according to which the pure apodeictic syllogisms are those given in Table 3. Or he might have in mind Table 4.

The problem about how to interpret the pure apodeictic syllogisms described in *Prior Analytics* A8 comes from the fact that the chapter on its own does not give enough detail to decide between the two different analyses – i.e., in Tables 3 and 4. There is, however, evidence from other parts of the *Prior Analytics* which supports Table 3 as the better way to capture what Aristotle has in mind for the LLLs.[5] The evidence comes from what is called the QQQ syllogistic. As we shall see in Chapter 11, below, Aristotle's treatment of QQQ syllogisms makes QQQ syllogisms only trivial instances of the non-modal syllogisms in Table 2.[6] In the case of the QQQ syllogisms we have Aristotle's very explicit discussion about how *both* the subject and predicate terms in QQQ premises are modally qualified. In a method known as *ampliation*, Aristotle explicitly puts a possibility qualifier on the subject terms in all QQQ syllogisms. This method of *ampliation* is discussed in detail in *Prior Analytics* A13, and in Chapter 11, below. For the purposes of our present discussion, it is perhaps enough to note that the parallel between the LLLs and the QQQs is strong and lends greater weight to the supposition that the LLLs and QQQs alike turn out to be semantically restricted but otherwise trivial instances of XXXs. So if Aristotle had stopped at XXX and LLL and QQQ syllogisms then the Kneales would have a very persuasive argument about the triviality of Aristotle's logic of terms. But as we shall see, Aristotle does not stop there. His treatment of mixed apodeictic (LXL and XLL) does not permit him a purely red syllogistic.

[5]Günther Patzig (pp. 61—67) offers some helpful insight. Patzig gives a very careful and detailed account of the ambiguity present in Aristotle's discussion of LLLs. Patzig himself is concerned to explain the basis of 'perfection' in a modal syllogism as a consequence of the identity of terms. He readily counts *LAx* as an Aristotelian term. And he offers LA_1, LE_1, LI_1, LO_1 and LA_2, LE_2, LI_2, LO_2 as the different ways of analysing Aristotle's L-propositions.

[6]Aristotle counts all first and third figure QQQ schemas as valid. He rules out second figure QQQ schemas in *An.Pr.* A17.

Chapter 7
First Figure Mixed Apodeictic Syllogisms

Aristotle's study of syllogisms involving necessity follows a precise structure. As we saw in the previous chapter, *Prior Analytics* A8 gives a brief but sweeping description of all syllogisms from two premises about necessity (apodeictic premises). The focus of the next three chapters is Aristotle's discussion of necessity in *An.Pr.* A9–11, where he studies syllogisms in which one premise is necessary and the other is assertoric (or non-modal). These are L+X and X+L premise combinations. Aristotle devotes great care to explaining these. Later chapters of *An.Pr.* deal with syllogisms from one possible and one necessary premise, but I will follow Aristotle and treat these as part of the syllogisms about possibility. It is often convenient to distinguish the pure apodeictic L+L premise combinations from combinations involving different modes, and so I sometimes refer to 'mixed modals'. The syllogisms discussed in *An.Pr.* A9–11 are among the mixed modals. *An.Pr.* A9 deals with L+X and X+L combinations in the first figure, *An.Pr.* A10 with the second figure, and *An.Pr.* A11 with the third figure. Aristotle clearly treats the non-modal XXX syllogisms as axioms. No syllogism about necessity will be valid that is not an instance of an XXX syllogism. So whenever we add Ls to premises, we know that at the very least an X-conclusion will always follow. For every valid XXX syllogism this means there are only LLL, LXL, and XLL combinations to consider.

One crucially important feature of the mixed modal LXL and XLL syllogisms is that in the mixed cases Aristotle plainly analyses apodeictic L-premises as LA_2, LE_2, LI_2, and LO_2, on p. 68–69 above. He does *not* use the alternative LA_1, LE_1, LI_1 and LO_1 analyses on p. 66 when he discusses mixed modal premise pairs. This creates some awkwardness for any interpretation, but the textual evidence for this throughout *Prior Analytics* A9–11 is perfectly clear, and so we need to reflect this difference in our representation of Aristotle. Explaining the difference is another matter since Aristotle himself does not comment on the difference, but we can trace developments in the syllogistic which help to explain. *Prior Analytics* A8 shows how Aristotle's first steps towards a modal syllogistic appear to involve restricting the non-modals of Table 2 to all red terms. That results in only a trivial variant of the ordinary syllogistic, as illustrated by Table 3. But a logic which is restricted solely to red terms cannot make reference to non-scientific propositions. We might conjecture that Aristotle sees this – and, perhaps, is frustrated and unsatisfied by it in the same way the Kneales find it frustrating and unsatisfying. At any rate, he pushes onwards and begins a study of mixed modals in which he does *not* assume all terms are of the same kind. In fact, he assumes that terms are not all the same kind.

In order to capture Aristotle's LXL and XLL syllogisms as they are described in *Prior Analytics* A9–11, we need to take the non-modal syllogisms of Table 2 as our starting points. To get an apodeictic premise about necessity, we follow the method that

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led in Chapter 6 to the syllogisms of Table 4. Aristotle offers detailed proofs of the validity of each of the LXL and XLL syllogisms. He works through these taking each figure in turn.

The discussion of the first figure LXL syllogisms comes in *An.Pr.* A9. These syllogisms are Barbara LXL, Celarent LXL, Darii LXL, and Ferio LXL. And these LXLs turn out to be substitution instances of XXX syllogisms.[1] In Table 5, below, the LXL syllogisms are listed in the right-most column for comparison with the XXX and LLL correlates:

Table 5
Comparison of First Figure XXX, LLL, and LXL Syllogisms

XXX Syllogisms (A4)	LLL Syllogisms (A8) Table 3	LLL Syllogisms (A8) Table 4	LXL Syllogisms (A9)
Barbara XXX	Barbara LLL	Barbara LLL	Barbara LXL
$\forall x(Bx \supset Ax)$	$\forall x(LBx \supset LAx)$	$\forall x(Bx \supset LAx)$	$\forall x(Bx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$	$\underline{\forall x(LCx \supset LBx)}$	$\underline{\forall x(Cx \supset LBx)}$	$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset Ax)$	$\forall x(LCx \supset LAx)$	$\forall x(Cx \supset LAx)$	$\forall x(Cx \supset LAx)$
Celarent XXX	Celarent LLL	Celarent LLL	Celarent LXL
$\forall x(Bx \supset \sim Ax)$	$\forall x(LBx \supset L\sim Ax)$	$\forall x(Bx \supset L\sim Ax)$	$\forall x(Bx \supset L\sim Ax)$
$\underline{\forall x(Cx \supset Bx)}$	$\underline{\forall x(LCx \supset LBx)}$	$\underline{\forall x(Cx \supset LBx)}$	$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset \sim Ax)$	$\forall x(LCx \supset L\sim Ax)$	$\forall x(Cx \supset L\sim Ax)$	$\forall x(Cx \supset L\sim Ax)$
Darii XXX	Darii LLL	Darii LLL	Darii LXL
$\forall x(Bx \supset Ax)$	$\forall x(LBx \supset LAx)$	$\forall x(Bx \supset LAx)$	$\forall x(Bx \supset LAx)$
$\underline{\exists x(Cx \& Bx)}$	$\underline{\exists x(LCx \& LBx)}$	$\underline{\exists x(Cx \& LBx)}$	$\underline{\exists x(Cx \& Bx)}$
$\exists x(Cx \& Ax)$	$\exists x(LCx \& LAx)$	$\exists x(Cx \& LAx)$	$\exists x(Cx \& LAx)$
Ferio XXX	Ferio LLL	Ferio LLL	Ferio LXL
$\forall x(Bx \supset \sim Ax)$	$\forall x(LBx \supset L\sim Ax)$	$\forall x(Bx \supset L\sim Ax)$	$\forall x(Bx \supset L\sim Ax)$
$\underline{\exists x(Cx \& Bx)}$	$\underline{\exists x(LCx \& LBx)}$	$\underline{\exists x(Cx \& LBx)}$	$\underline{\exists x(Cx \& Bx)}$
$\exists x(Cx \& \sim Ax)$	$\exists x(LCx \& L\sim Ax)$	$\exists x(Cx \& L\sim Ax)$	$\exists x(Cx \& L\sim Ax)$

It is clear from Table 5 that the LXLs are even more straightforward than the corresponding LLLs. (This is because the surface structure of the Table 4 LLLs involves both an *LB* and a *B* term, whereas the surface structure of the LXLs only involves a simple *B* term. And so, in the LXLs we do not even need to use $L\varphi \supset \varphi$.) *All* that is needed to get from an XXX to an LXL is the simple substitution of *LA* for *A*, or *~L~A* for *A*, as explained in footnote 3 on p. 68.

[1] In this respect the LXL syllogisms are very like the LLLs of Table 3, in Chapter 5 above.

Barbara LXL, Celarent LXL, Darii LXL, and Ferio LXL are the only valid syllogisms that come from combining an X and an L premise in the first figure. Even though their validity is more obvious than that of the first figure LLLs, Aristotle does not treat the LXLs as trivial.[2]

> It sometimes results that the deduction becomes necessary when only one of the premises is necessary (not whatever premise it might be, however, but only the premise in relation to the major extreme [here, the A-term].) (30a15−17)

In this way Aristotle distinguishes the LXLs as the only L+X or X+L premise pairs which produce first figure syllogisms. He also offers evidence in support of their validity. First, he defends the validity of Barbara and Celarent LXL:

> For instance, if A has been taken to belong or not to belong of necessity to B, and B merely to belong to C: for if the premises have been taken in this way, then A will belong or not belong to C of necessity. For since A belongs or does not belong of necessity to every B and C is some of the Bs,[3] it is evident that one or the other of these will also apply to C of necessity. (30a17–23)

Barbara LXL	
$\forall x(Bx \supset LAx)$	A belongs of necessity to every B
$\underline{\forall x(Cx \supset Bx)}$	and B merely belongs to C
$\forall x(Cx \supset LAx)$	then A will belong to C of necessity
Celarent LXL	
$\forall x(Bx \supset L{\sim}Ax)$	A does not belong of necessity to every B
$\underline{\forall x(Cx \supset Bx)}$	and B merely belongs to C
$\forall x(Cx \supset L{\sim}Ax)$	then A will not belong to C of necessity

[2]Because these are so obviously valid, Aristotle could certainly have skipped the detail here. But the detail is perhaps explained by a bit of history: Aristotle's friend and student Theophrastus thought there could never be a valid LXL or XLL syllogism – he argued that a conclusion about necessity could not follow unless *both* premises were also about necessity. It might be that Aristotle is trying to answer his contemporary critics in *An.Pr.* A9.

[3]Interpreters agree that the syllogisms here are Barbara LXL and Celarent LXL. Therefore the somewhat curious phrase 'C is some of the Bs' must mean that every C is a B. To Smith this phrase suggests that ecthesis is somehow involved. At any rate, the logic of Barbara LXL and Celarent LXL is straightforward.

Darii and Ferio LXL are explained similarly:

> In the case of particular deductions, if the universal is necessary then the conclusion will also be necessary... First, then, [Darii LXL] let the universal be necessary, and let A belong to every B of necessity, but let B merely belong to some C. Then it is necessary for A to belong to some C of necessity (for C is under B, and A belonged to every B of necessity). It will also be similar if the deduction is privative [Ferio LXL], for the demonstration will be the same. (30a34–b2)

Darii LXL

$\forall x(Bx \supset LAx)$	A belongs to every B of necessity
$\underline{\exists x(Cx \;\&\; Bx)}$	B merely belongs to some C
$\exists x(Cx \;\&\; LAx)$	then A to belongs to some C of necessity

Ferio LXL

$\forall x(Bx \supset L{\sim}Ax)$	A does not belong to every B of necessity
$\underline{\exists x(Cx \;\&\; Bx)}$	B merely belongs to some C
$\exists x(Cx \;\&\; L{\sim}Ax)$	then A does not belong to some C of necessity

Each of these is a trivial substitution instance of a non-modal syllogism. That makes their validity obvious.[4] But because these are trivial in this sense they leave us still with a lingering question about the difference between pure apodeictic syllogisms and mixed apodeictic L+X syllogisms. That is, there is still a question about whether we should take the LLL syllogisms to be those listed in Table 3 or those listed in Table 4. The point is easier to see with an example. We need to consider whether to count something like the following as an instance of Barbara LLL with true premises:

Every man is a necessary animal
<u>Everything walking is a necessary man</u>
Everything walking is a necessary animal.

On the face of it, this looks like an instance of Barbara LLL since each premise is

---

[4]This is certainly so when the modal propositions are represented as on p. 52. However other translations have been proposed. For instance, Nortmann (1996, p. 126), for reasons connected with the justification of conversion, as described above on pp. 55–57, represents Barbara LXL as

$\forall Lx(Bx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset LAx)$.

This is valid, but requires translating an LA-proposition differently depending on its place in a syllogism. (For a helpful discussion of this see Thom 1991, pp. 431–433.)

apodeictic. But without a way to decide between a pure red apodeictic syllogistic represented in Table 3, or an apodeictic syllogistic represented in Table 4, then we cannot decide with perfect certainly that the syllogism above is *not* an instance of Barbara LLL because not all terms are red ('walking' is not). This syllogism would be Barbara LLL if the pure apodeictic syllogisms were the schemas listed in Table 4. The syllogism above *is*, however, an instance of the valid Barbara LXL *and* both of its premises are necessary. As we shall see, in setting out counter-examples Aristotle plainly does allow necessary premises with accidental subjects.

In the apodeictic syllogistic Aristotle approaches the question of validity as a matter of determining whether an L conclusion (a conclusion about necessity) follows from the given premises. Only L+X premise pairs yield valid syllogisms in the first figure. These are Barbara, Celarent, Darii, and Ferio LXL. Aristotle offers proof of invalidity for all the remaining first figure X+L combinations. *An.Pr.* A9 explains why there are no first figure XLL syllogisms. The XLL combinations which he rejects are listed in Table 6. The rejected conclusions are marked with a *.

Table 6
Invalid First Figure X+L Combinations

	Barbara XLL
	$\forall x(Bx \supset Ax)$
	$\underline{\forall x(Cx \supset LBx)}$
*	$\forall x(Cx \supset LAx)$
	Darii XLL
	$\forall x(Bx \supset Ax)$
	$\underline{\exists x(Cx \;\&\; LBx)}$
*	$\exists x(Cx \;\&\; LAx)$
	Celarent XLL
	$\forall x(Bx \supset {\sim}Ax)$
	$\underline{\forall x(Cx \supset LBx)}$
*	$\forall x(Cx \supset L{\sim}Ax)$
	Ferio XLL
	$\forall x(Bx \supset {\sim}Ax)$
	$\underline{\exists x(Cx \;\&\; LBx)}$
*	$\exists x(Cx \;\&\; L{\sim}Ax)$

In each case Aristotle rejects an *L* conclusion relating a subject *C* to a predicate *A*. Since

Aristotle reserves the name 'syllogism' strictly for the valid premise-conclusion combinations, this means that in his terminology 'there are *no* [first figure XLL] syllogisms'. He accounts for each such possible combination, constructing counter-examples to show that in each case a conclusion about necessity cannot be guaranteed. First he explains why Barbara and Celarent XLL are not valid syllogisms:

> It is, moreover, also evident from terms that the conclusion can fail to be necessary, as for instance, if A were motion, B animal, and C stood for man. For a man is of necessity an animal, but an animal does not move of necessity, nor does a man. It would also be similar if AB were privative (for the demonstration is the same). (30a28–33)

If a conclusion can fail to be necessary then we cannot syllogize to an *L* conclusion. Putting the terms in place as Aristotle describes we get the following counter-examples:

Barbara XLL	(30a28–33)	
$\forall x(Bx \supset Ax)$	All animals are moving	T
$\underline{\forall x(Cx \supset LBx)}$	All men are necessary animals	T
$\forall x(Cx \supset LAx)$	All men are necessarily moving	F

Since moving is only accidental to man it is false to say a man moves of necessity. So an L-conclusion does not follow. Of course an X-conclusion does follow. That is, Barbara XLX is valid, but not Barbara XLL. Aristotle also extends the point to the privative: Celarent XLL is invalid. The same terms show why.

Celarent XLL	(30a28–33)	
$\forall x(Bx \supset \sim Ax)$	All animals are not moving	T
$\underline{\forall x(Cx \supset LBx)}$	All men are necessary animals	T
$\forall x(Cx \supset L\sim Ax)$	All men are necessarily not moving	F

Again, an X-conclusion is fine: if the premises are true, then it follows that all men are not moving, $\forall x(Cx \supset \sim Ax)$. Note that we would *not have a counter-example* if the conclusion were $\forall x(Cx \supset \sim LAx)$. This is in line with the remarks made on p. 49.

In accounting for the invalidity of Darii and Ferio XLL, Aristotle again introduces terms:

> ...if the particular premise is necessary, the conclusion will not be necessary (for nothing impossible results), just as it was not in the case of universal deductions; and similarly also in the case of privatives. Terms are motion, animal, white. (30b2–6)

Darii XLL

$\forall x(Bx \supset Ax)$	All animals are moving	T
$\underline{\exists x(Cx \; \& \; LBx)}$	<u>Some white thing is a necessary animal</u>	T
$\exists x(Cx \; \& \; LAx)$	Some white thing is necessarily moving	F

Ferio XLL

$\forall x(Bx \supset {\sim}Ax)$	No animals are moving	T
$\underline{\exists x(Cx \; \& \; LBx)}$	<u>Some white thing is a necessary animal</u>	T
$\exists x(Cx \; \& \; L{\sim}Ax)$	Some white thing is necessarily not moving	F

Aristotle explains why Darii and Ferio XLL are invalid by setting out terms of which two are accidents – ‘white’ and ‘moving’. Only one accident appears in any premise, but both feature in the conclusions. In Barbara XLL and Celarent XLL, the L-conclusions are rejected apparently because *no* thing moves of necessity. This same reason makes the (particular) L-conclusions of Darii and Ferio XLL unacceptable, too. Again note that if the privative conclusions were about ‘what is not necessarily A’ – that is, if the privative conclusions above were given as $\forall x(Cx \supset {\sim}LAx)$ or $\exists x(Cx \; \& \; {\sim}LAx)$ – then Celarent XLL and Ferio XLL would be *valid*. The fact that Aristotle counts these as invalid and offers terms for constructing counter-examples indicates that we must take the conclusions to be about ‘what is necessarily not *A*’ – that is, they must be $\forall x(Cx \supset L{\sim}Ax)$ and $\exists x(Cx \; \& \; L{\sim}Ax)$, as represented in the schemas above. Also note that the use of accidental terms to pick out subjects can be avoided in Darii and Ferio by taking different terms. For example, let A be moving, B be animal, and C be man. Nevertheless, the fact that here Aristotle does use accidents in subject position of a true L-premise is important because it seems that such accidental subjects cause worries. These problems come up most specifically with the second figure, which we turn to in the next chapter.

Chapter 8
Modal Conversion in the Apodeictic Syllogistic: *An.Pr.* A9–11

This chapter is concerned with mixed apodeictic syllogisms in the second and third figures. Table 7 below lists a family of the mixed LXL syllogisms that Aristotle counts as valid, as described in *Prior Analytics* A9–11.

Table 7
L+X Syllogisms in Three Figures
with Unconverted Conclusions (*An.Pr.* A9–11)

First Figure (A9)	Second Figure (A10)	Third Figure (A11)
Barbara LXL (30a17–23)	Cesare LXL (30b9–13)	Darapti LXL (31a24–30)
$\forall x(Bx \supset LAx)$	$\forall x(Bx \supset L{\sim}Ax)$	$\forall x(Cx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$	$\underline{\forall x(Cx \supset Ax)}$	$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset LAx)$	$\forall x(Cx \supset L{\sim}Bx)$	$\exists x(Bx \& LAx)$
Darii LXL (30a37–b2)	Festino LXL (31a5–10)	Felapton LXL (31a33–37)
$\forall x(Bx \supset LAx)$	$\forall x(Bx \supset L{\sim}Ax)$	$\forall x(Cx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \& Bx)}$	$\underline{\exists x(Cx \& Ax)}$	$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Cx \& LAx)$	$\exists x(Cx \& L{\sim}Bx)$	$\exists x(Bx \& L{\sim}Ax)$
Celarent LXL (30a17–23)		Datisi LXL (31b19–20)
$\forall x(Bx \supset L{\sim}Ax)$		$\forall x(Cx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$		$\underline{\exists x(Cx \& Bx)}$
$\forall x(Cx \supset L{\sim}Ax)$		$\exists x(Bx \& LAx)$
Ferio LXL (30a37–b2)		Ferison LXL (31b35–37)
$\forall x(Bx \supset L{\sim}Ax)$		$\forall x(Cx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \& Bx)}$		$\underline{\exists x(Cx \& Bx)}$
$\exists x(Cx \& L{\sim}Ax)$		$\exists x(Bx \& L{\sim}Ax)$

Chapter 7 introduced the first figure L+X syllogisms listed in the left hand column of Table 7. Consider the second figure apodeictic syllogisms in Table 7. The most obvious comment is that, as they stand, neither is valid.[1] So why does Aristotle recognise them as syllogisms? Proofs of second figure syllogisms, generally, require at least one conversion. Where the proposition which must be converted is itself a modally qualified L-proposition, then the proof will require a *modal conversion.* LA, LI, and LE-conversions (i.e., *de re* L-conversions generally) are validated by the genuineness requirement – that the *B*-term be red, and so *B* is interchangeable with *LB*. Notice that

[1] A counter-example to Cesare LXL is: *A* = horses, *B* = moving and *C* = white, where only horses are white, and only men are moving. Nor will interpreting the L-propositions as *de dicto* help. Use *A* = married, *B* = bachelor and *C* = my neighbours, where all my neighbours are in fact married.

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the genuineness restriction is carefully limited. It only applies to propositions used as the *input* into the modal L-conversion. It does not apply to necessary propositions generally. It does not apply more generally because Aristotle often uses necessary propositions which do not meet the genuineness requirement. For example, we saw on p. 78 that he uses ‘some white thing is a necessary animal’ as a true premise in the counter-examples he constructs for each of Darii XLL and Ferio XLL. ‘Some white thing is a necessary animal’ is a true proposition, but it is not an instance of genuine predication. And so it does not convert, but Darii and Ferio are first figure syllogisms and their proofs do not require any conversion. This has what might seem an awkward consequence. We cannot tell just by looking at the premises of a syllogism whether terms will be restricted because we cannot tell in advance whether conversion will be used.

The genuineness restriction on conversion validates the second figure syllogisms in Table 7 – Cesare LXL and Festino LXL. Of course, validity in these cases means validity for appropriately restricted terms. The proofs are as follows.

Cesare LXL (30b9–13)

(1)	$\forall x(Bx \supset L{\sim}Ax)$	
(2)	$\forall x(Cx \supset Ax)$	
(3)	$\forall x(Ax \supset L{\sim}Bx)$	Genuine LE-conversion (1) (To keep the steps to a minimum, I leave the necessity on the subject terms implicit only.)
(4)	$\forall x(Cx \supset L{\sim}Bx)$	Celarent LXL (2)(3) (Table 7, with *A* and *B* transposed)

Festino LXL (31a5–10)

(1)	$\forall x(Bx \supset L{\sim}Ax)$	
(2)	$\exists x(Cx \,\&\, Ax)$	
(3)	$\forall x(Ax \supset L{\sim}Bx)$	Genuine LE-conversion (1)
(4)	$\exists x(Cx \,\&\, L{\sim}Bx)$	Ferio LXL (2)(3) (Table 7, with *A* and *B* transposed)

The proofs of these are sufficient to guarantee the validity of each of the corresponding second figure LLLs in Table 4: Cesare LLL and Festino LLL. Of course as we saw in Chapter 6, in LLL syllogisms it might be that Aristotle really means to require that *all* terms be red terms. This would make the LLL syllogisms those listed in Table 3. If Table 3 is what Aristotle has in mind for the LLLs then, here in the mixed L+X syllogisms, any restrictions needed for genuine L-conversion are automatically satisfied.

As mentioned in footnote 1 on p.64, there is a second figure LLL syllogism that

Aristotle proves by LE-conversion. It is Baroco LLL, in Table 4. Aristotle picks out Baroco LLL and the third figure Bocardo LLL for special consideration. He notes in *An.Pr.* A8, 30a6–14 that these cannot be proved by straightforward conversion. In this same passage he claims that Baroco LLL can be proven by a method he calls 'setting out (*ekthesis*)' but he does not carry out the proof for this case. Patterson (1995) and Thom (1996) give accounts of how *ekthesis* does the job. What is especially interesting about their accounts is that they involve a *B* subject in an LE-conversion. Here is how Patterson and Thom explain *ekthesis*.

Suppose
$\exists x(Cx \,\&\, L{\sim}Ax)$.

So, there is something that is both *C* and *L~A*. What this means is that we may choose a term, say *D*, to designate that part of the *C*s which are *L~A*s. So by *ekthesis* we have

$\forall x(Dx \supset Cx)$

and

$\forall x(Dx \supset L{\sim}Ax)$.

This is a powerful tool – it creates two universal propositions from a single existential.

Patterson's proof (1995, p. 73) of Baroco LLL works like this:

Baroco LLL

(1)	$\forall x(Bx \supset LAx)$	Given
(2)	$\exists x(Cx \,\&\, L{\sim}Ax)$	Given
(3)	$\forall x(Dx \supset Cx)$	Ekthesis, 2
(4)	$\forall x(Dx \supset L{\sim}Ax)$	Ekthesis, 2
(5)	$\forall x(Bx \supset L{\sim}Dx)$	Cesare LLL, 4,1
(6)	$\forall x(Dx \supset L{\sim}Bx)$	LE-conv, 5
(7)	$\exists x(Cx \,\&\, Dx)$	XA-conv, 3
(8)	$\exists x(Cx \,\&\, L{\sim}Bx)$	Ferio LXL, 6,7

Patterson sidesteps the issue of the validity of the LE-conversion in line (6). He says "whether or not that conversion is valid is obviously beside the present point." But plainly we are now able to explain its validity. The conversion in line (6) is valid under the genuineness requirement. This means that the *B* term must be a red term. So, Baroco LLL is valid.

Thom's proof (1996, p. 50) uses *ekthesis* together with Camestres LLL, and the LE proposition that converts in Thom's proof has *B* as the subject term. So, again, the genuineness requirement on modal conversion guarantees that *B* is a red term, and so, this method too shows that Baroco LLL is valid.

Aristotle's usual method for establishing invalidity is to offer counter-examples showing that true premises are not a guarantee of a true conclusion of the required form. In the case of one invalid second figure schema Aristotle gives a formal proof, as well as a straightforward counter-example. This is to establish the invalidity of Cesare XLL, at 30b20–24. In *An.Pr.* A10, Aristotle rejects each of the following second figure schemas as invalid (Table 8). An asterisk marks the purported false conclusions.

Table 8
Invalid Second Figure L+X, X+L Forms

	Cesare XLL	(30b20–40)[2]
	$\forall x(Cx \supset \sim Ax)$	All white things are not animals
	$\underline{\forall x(Bx \supset LAx)}$	All men are necessary animals
*	$\forall x(Bx \supset L\sim Cx)$	All men are necessarily not white
	Festino XLL	
	$\forall x(Bx \supset \sim Ax)$	
	$\underline{\exists x(Cx \,\&\, LAx)}$	(no terms given, no specific discussion)
*	$\exists x(Cx \,\&\, L\sim Bx)$	
	Baroco LXL	('the same terms will serve,' 31a10–15)
	$\forall x(Bx \supset LAx)$	All men are necessary animals
	$\underline{\exists x(Cx \,\&\, \sim Ax)}$	Some white thing is not an animal
*	$\exists x(Cx \,\&\, L\sim Bx)$	Some white thing is necessarily not a man
	Baroco XLL	('through the same terms,' 31a15–17)
	$\forall x(Bx \supset Ax)$	All men are animals
	$\underline{\exists x(Cx \,\&\, L\sim Ax)}$	Some white thing is necessarily not an animal
*	$\exists x(Cx \,\&\, L\sim Bx)$	Some white thing is necessarily not a man

I'm taking the discussion of Cesare XLL to occur in 30b20–40, but the passage causes problems – so much so that most recent commentaries and interpretations (my own included (Rini, 1998)) take the passage to be an argument about the invalidity of a different schema, Camestres LXL.[3]

[2]The premises are transposed and not in their usual order.

[3]Thus Ross (1957) in the chart facing p. 286 lists the passage as a proof of Camestres LXX, as does Smith (1989, p. 231). Striker (2009, p. 120) claims that "Aristotle discusses only the case of CamestresNXN, but since Cesare and Camestres differ only in their premiss order, and both are reduced

Camestres LXL
$\forall x(Bx \supset LAx)$
$\forall x(Cx \supset {\sim}Ax)$
$\forall x(Cx \supset L{\sim}Bx)$.

The precise steps of Aristotle's reasoning are not entirely clear, and the best way to approach the difficulties is by going to the text. Ross (1957, p. 321f) divides the passage into three proofs:

(α) 30b20–24;
(β) 30b24–31; and
(γ) 30b31–40.

This is a natural division and I follow it here because it helps to clarify the separate moves within Aristotle's larger proof. The entire passage is quoted below, with Smith's translation, but with Ross's divisions indicated in the margin, and my numberings for later reference:

<u>*An.Pr.* A10, 30b18–40:</u>
But if the positive premise is necessary, the conclusion will not be necessary.

(α) For (1) let A belong to every B of necessity (2) but merely belong to no C. Then, when the privative premise is converted (3), it becomes the first figure (4); and it has been proved that in the first figure, when a privative premise in relation to the major term is not necessary, the conclusion will not be necessary either. Consequently, neither will it be of necessity in this case.

(β) Moreover, (5) if the conclusion is necessary, (6) it results that C of necessity does not belong to some A. For if (5) B belongs of necessity to no C, then (7) C will also belong to no B of necessity. But, in fact, (8) it would be necessary for B to belong to some A, given that (1) A belonged to every B of necessity. Consequently, (6) it would be necessary for C not to belong to some A. (9) But nothing prevents the A having been chosen in such a way that it is possible for C to belong to all of it.

(γ) And moreover, it would be possible to prove by setting out terms that the conclusion is not necessary without qualification, but only necessary when these things are so. For instance, let A be animal, B

to Celarent, his argument covers CesareXNN as well." (Striker's N is just our L.)

man, C white, and let the premises have been taken in the same way (for it is possible for animal to belong to nothing white). Then, man will not belong to anything white either, but not of necessity: for it is possible for a man to become white, although not so long as animal belongs to nothing white. Consequently, the conclusion will be necessary when these things are so, but not necessary without qualification.

Before I look at Camestres LXL it is worth considering how to understand the passage if we assume that it is about Cesare XLL. Considered on its own (α) 30b20–24 provides a straightforward formal explanation of the invalidity of Cesare XLL. I have numbered the steps in the quotation above. The point of (α) is to claim that Cesare XLL would have to be based on Celarent XLX.

Cesare XLX (30b20–23, the formal method, with premises transposed)

(1)	$\forall x(Bx \supset LAx)$	A belongs to every B of necessity
(2)	$\forall x(Cx \supset \sim Ax)$	but merely belongs to no C
(3)	$\forall x(Ax \supset \sim Cx)$	then, when the privative is converted
(4)	$\forall x(Bx \supset \sim Cx)$	it becomes the first figure (i.e., Celarent XLX, 30a28–33)

Aristotle then reminds us that the conclusion of the first-figure Celarent XLX will *not* be necessary. His point is that since Celarent XLX provides the basis for the proof of Cesare XLX, then here, too, in Cesare XLX the conclusion will not be necessary. So Aristotle's proof of Cesare XLX explains why only an X but not an L conclusion follows. That is, it explains why we do not get

(7) $\forall x(Bx \supset L\sim Cx)$

but only (4), i.e., it explains why Cesare XLL is not valid.

Cesare XLL

(2) $\forall x(Cx \supset \sim Ax)$
(1) $\underline{\forall x(Bx \supset LAx)}$
(7) $\forall x(Bx \supset L\sim Cx)$

Putting Aristotle's terms into the Cesare XLL schema we get the following:

(10)	No white things are animals	T
(11)	All men are necessary animals	T

(12) All men are necessarily not white F

Certainly the conclusion here must be false. So, assuming (10) that no animals are white, the terms Aristotle gives in (γ) serve as a counter-example to Cesare XLL. To understand the (β) passage as an invalidity proof for Cesare XLL, suppose that we had indeed deduced (7) from (1) and (2). Then we could continue as follows:

(8) $\exists x(Ax \,\&\, LBx)$ LA conversion (1)
(6) $\exists x(Ax \,\&\, L{\sim}Cx)$ Ferio LLL (7)(8)

While premise (2) requires, by E-conversion, that no *A* be *C*, it does not require that this is so by necessity, and so leaves open the possibility that

(9) nothing prevents *A* having been chosen in such a way that it is possible for *C* to belong to all of *A*. (30b30–31)

So the truth of (1) and (2) is compatible with (9). But (9) is the contradictory of something which could be derived from (7), i.e., (6), and so (7) cannot be derived from (1) and (2). The (γ) passage would then provide terms which have been so chosen. 'Animal' has been so chosen that it is possible for all animals to be white.

This makes the passage a proof of the invalidity of Cesare XLL. Note that the LA-conversion used in getting (8) from (1) depends on the *B* term being red, and in Aristotle's example it is 'man'. So far so good, but, as I said, the whole (α)(β)(γ) passage is thought to be a rejection of Camestres LXL. Here is why. At step (5) in the (β) passage, Aristotle supposes that 'the conclusion is necessary.' From the steps taken so far, it would seem that he means to suppose that (4) is necessary. Supposing (4) is necessary would give

(7) $\forall x(Bx \supset L{\sim}Cx)$.

Instead, we find that Aristotle supposes

(5) $\forall x(Cx \supset L{\sim}Bx)$

i.e., the conclusion of Camestres LXL. The only difference between Camestres LXL, and Cesare XLL is in the conclusion. If you replace (5) by (7) in Camestres LXL you get Cesare XLL with transposed premises. The reason the passage has been thought to be about Camestres LXL is probably because of the sentence at 30b26–28, where Aristotle appears to claim that (5) and (7) are equivalent, presumably by LE-conversion. As noted above Aristotle's argument in (β) requires that the *B* term must be red. And

(9) requires that the C term be something which does not apply to the As, but could, i.e., the C term must be green. And indeed in Aristotle's example that is so, since B is 'man' and C is 'white'.

We have to consider whether, in the (γ) passage, Aristotle means to give a counter-example to Cesare XLL or a counter-example to Camestres LXL. The conclusion of Cesare XLL is (7), but (7) is not the conclusion Aristotle seems to describe in (γ). He seems instead to be thinking again of the converted conclusion (5). Putting his terms into (5) gives the conclusion described at 30b35:

(13) All white things are necessarily not men

Aristotle takes (13) to be false, and claims that only an assertoric, non-modal conclusion follows:

(14) All white things are not men

For he tells us at 30b34–38 that 'man will not belong to anything white either, but not of necessity: for it is possible for man to become white, though not so long as animal belongs to nothing white.' This would seem to be an explanation of the falsity of (13).

But if we take LE-conversion to be subject to the genuineness requirement there is a serious problem about why Aristotle should take (5) to be false. Look closely at the steps in his reasoning. The formal explanation (α) shows that we can (validly) syllogize to the assertoric conclusion (4) $\forall x(Bx \supset \sim Cx)$. Taking the terms from the (γ) passage this means we *can* syllogize to

(15) All men are not white.

And since an L conclusion does not follow, we *cannot* conclude

(12) All men are necessarily not white

That is what it means to say that Cesare XLL is not a syllogism (is invalid) but that Cesare XLX is a (valid) syllogism. But if Aristotle means to give a counter-example to Camestres LXL he has to argue that because (12) is false, so is (13). It seems that Aristotle wants to use LE-conversion to get from the falsity of (7) to the falsity of (5), which, in the present example would mean getting from the falsity of (12) to the falsity of (13). And this could be used to invalidate Cesare LXL (Table 7, 30b9–13). Let the terms be horse, man, and white:

Cesare LXL (30b9–13)	
$\forall x(Bx \supset L{\sim}Ax)$	All men are necessarily not horses
$\underline{\forall x(Cx \supset Ax)}$	<u>All white things are horses</u>
$\forall x(Cx \supset L{\sim}Bx)$	All white things are necessarily not men

Aristotle tells us Cesare LXL is valid. Yet it cannot be valid if from the true premises above we get 'all white things are necessarily not men' as a false conclusion, on the assumption that the only white things are horses. We are faced here with a clear inconsistency, but it is a tractable and explicable inconsistency. The source of the problem is not in the account of Cesare LXL – we have to count that as valid and the instance above must be a valid instance. The problem is about the invalidity of the conversion from (7) to (5).

Since (12) is a *genuine predication*, it might seem that LE-conversion does apply. But if so the mistake is subtle – and I will call it the *Subtle Mistake*. Although we can, of course, state LE-conversion as an equivalence

$$\forall x(Bx \supset L{\sim}Ax) \equiv \forall x(Ax \supset L{\sim}Bx)$$

it is better from Aristotle's point of view to think of it as a rule which can be used in both directions. In that case we need not require that *both* terms be red terms, but we must remember that when it is used in one direction, it is the A term that must be a red term; in the other direction it is the B term that must be a red term. In arguing from falsity to falsity, Aristotle is in fact using a contraposed form of the rule. And in the (γ) passage (30b32–40), he is mistaken about which term must be a red term. In going from the falsity of 'all men are necessarily not white' to the falsity of 'all white things are necessarily not men' we are in fact using a rule which would take us from the *truth* of (13) to the *truth* of (12). This rule (LE-conversion) demands that white be a red term, and it clearly is not. What is going on? In the case of the *valid* second figure moods, such as the passage from (1) to (3) in Cesare LXL, Aristotle uses LE-conversion to go from *truth* to *truth*. Here in the (γ) passage the mistake is to use LE-conversion to go from *falsity* to *falsity*.

This was my explanation in Rini (1998). Do be aware that the subtle mistake is not the mistake of supposing that a false antecedent guarantees a false consequent. See *Topics* VIII.12, 162b12–15 for evidence that Aristotle is aware of *that* mistake.[4] The subtle mistake is different. It is subtle. The propositions (7) and (5) are both LE-propositions. All LE-propositions convert, subject to the genuineness requirement. Aristotle knows that LE-conversion is an equivalence, and he knows that while you cannot reason from the falsity of the antecedent in a true *implication* to the falsity of the consequent, you *can* reason in a true *equivalence* from the falsity of either side to

[4]I thank an anonymous reader from Springer for reminding me of the *Topics* passage.

the falsity of the other. That in itself is no problem. The problem is that the validity of LE-conversion depends on the genuineness restriction on the *subject term,* and even in an equivalence the *subject* of the proposition on one side of the equivalence is different from the subject of the proposition on the other side of the equivalence. So that when you are using the equivalence to go from left to right it is the subject term of the *left hand side* that must be genuine, while if you are going from right to left it is the subject of the *right hand side* that must be genuine.[5] And it is here that going from falsity to falsity makes a difference. For going from the falsity of the left hand side to the falsity of the right hand side, even in an equivalence, is the same as going from the truth of the left hand side to the truth of the right hand side.

Aristotle concludes his discussion by reminding us at 30b39–40 that because you can get an X conclusion you can of course obtain a conclusion which is 'necessary when these things are so, but not necessary without qualification'. There is a scholarly tradition according to which this passage is best explained by the distinction mentioned on p. 20 – between 'relative' or 'hypothetical' necessity, and 'absolute' necessity.[6] Whatever may be said about this tradition it is no more than the assertion that Aristotle thinks that an X conclusion follows but an L conclusion does not, and that is hardly controversial.

The subtle mistake affects some other invalid schemas too. In *An.Pr.* A 10, the subtle mistake affects Aristotle's account of Baroco LXL and Baroco XLL. These two are listed among the invalid L+X and X+L Forms in Table 8, above. Aristotle claims that the same terms he uses in the (γ) passage can be used to invalidate Baroco LXL and XLL. In the (γ) passage, he interprets

(13) All white things are necessarily not men F

as false. When he comes to Baroco LXL he rejects an existential conclusion of the same form. So, since (13) is supposed to be false,

(16) Some white things are necessarily not men F

[5]Note the point made on p. 23 that different, but logically equivalent, formulae of LPC can represent different Aristotelian propositions where the subject of one proposition is the predicate of the other.

[6]The Kneales give a detailed explanation of this view (Kneale and Kneale 1962, pp. 92—94), linking it to passages in *On Interpretatione* 9 (18a39—18b1; 19a25—27) and more importantly to *De Sophisticis Elenchis* 4 (166a22—30). Though even the Kneales are cautious: "But this use, though natural, may give rise to confusion, because statements of relative necessity or possibility are often made elliptically and may for this reason be misunderstood as statements about absolute necessity or possibility... the distinction between absolute and relative possibility, although at times clearly recognized, does, as we have seen, cause Aristotle difficulty in practice. And he never says explicitly that relative necessity and possibility involve absolute necessity and possibility." On this issue see also Thom (1991, p.54, n. 4).

must be false for the very same reason. The purported counter-examples against Baroco LXL and XLL are included in Table 8. In each of these, the premises are true, but a conclusion (16) is rejected as false.

We have to suspend judgement about the invalidity of Camestres LXL and Baroco LXL. Aristotle claims these are invalid. But if indeed he is trying to show that Camestres LXL is invalid, he makes the subtle mistake with his LE-conversion. The subtle mistake makes the conversion invalid, and the counter-example does not establish the schema's invalidity. Because Baroco LXL is exactly Camestres LXL with privative existential propositions in place of universal privatives, the subtle mistake applies to Baroco LXL too. Aristotle also appears to count a related schema, Baroco XLL, listed in Table 8, as invalid. As evidence of its invalidity he explains 'the demonstration is through the same terms' – these would appear to be the same terms offered for Camestres LXL and Baroco LXL. The purported counter-example is as follows:

	Baroco XLL	(31a15–17)
(17)	$\forall x(Bx \supset Ax)$	All men are animals
(18)	$\underline{\exists x(Cx \mathbin{\&} L{\sim}Ax)}$	Some white thing is necessarily not an animal
(19)	$\exists x(Cx \mathbin{\&} L{\sim}Bx)$	Some white thing is necessarily not a man

Notice that the conclusion is the same as the conclusion rejected in Baroco LXL. It seems clear that Aristotle intends to describe an instance through terms with true premises but a false conclusion. And that would make Baroco XLL *invalid.* There is however, some direct textual evidence that Aristotle really wants to say Baroco XLL is valid. The evidence for the *validity* of Baroco XLL comes from three features of his discussion. First, in an earlier passage in *An.Pr.* A10, Aristotle sets out a general rule about second figure X+L, L+X syllogisms:

> ... in the case of the second figure, if the privative premise is necessary, then the conclusion will also be necessary; but if the positive premise is, the conclusion will not be necessary. (30b7–9)

This would make Baroco XLL *valid* because the privative premise (18) is necessary. If that is right then the counter-example in Table 8 (repeated above as (17) (18) (19)) is not really a counter-example. This leads to a second reason to think Aristotle is badly confused: his counter-example to Baroco XLL is just plain fishy since in it

(18)	Some white thing is necessarily not an animal	T

is supposed to be true, but

(19) Some white thing is necessarily not a man F

is supposed to be false. (18) must be true if it is to serve as a premise in the counter-example to Baroco XLL. (19) is the false conclusion which makes Baroco XLL invalid. But (18) and (19) should stand or fall together since the predicate terms ‘animal’ and ‘man’ are both red terms. One might think that if Aristotle’s counter-example invalidates Baroco XLL then it also invalidates Baroco LLL (in Table 4). There is no suggestion in the text that Aristotle notices this. The evidence for the validity of Baroco LLL comes entirely from *Prior Analytics* A8, which indicates that for every XXX syllogism there is a corresponding LLL syllogism. Since Baroco XXX is valid, so then is Baroco LLL. However, as we noted in Chapter 6, in the LLL cases it might be that *all* terms are required to be red. However, the terms are not all red in the present counter-example, and so this example only counts against Baroco XLL, and *not* against Baroco LLL. We have no real way to tell what Aristotle’s response to the subtle mistake would be, but plainly it causes him troubles about Baroco XLL.[7]

The second figure invalid schemas – Festino XLL and Cesare XLL in Table 8 – are not implicated by the Subtle Mistake. Aristotle offers no discussion of Festino XLL. It reduces by straightforward non-modal conversion to a first figure syllogism with an X but not an L conclusion. Festino XLX and Cesare XLX are valid, but not Festino and Cesare XLL.

The focus of this next section is the third figure apodeictic syllogisms (A11). Table 9 lists the valid third figure L+X combinations.

[7]Thom says the ‘rejection of Baroco XLL and Bocardo LXL must be put down to carelessness’ (Thom 1996, p. 135).

Table 9
L+X Syllogisms in the Third Figure (A11)

Darapti LXL (31a24–30)
$\forall x(Cx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \,\&\, LAx)$

Felapton LXL (31a33–37)
$\forall x(Cx \supset L{\sim}Ax)$
$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \,\&\, L{\sim}Ax)$

Datisi LXL (31b19–20)
$\forall x(Cx \supset LAx)$
$\underline{\exists x(Cx \,\&\, Bx)}$
$\exists x(Bx \,\&\, LAx)$

Ferison LXL (31b33–37)
$\forall x(Cx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \,\&\, Bx)}$
$\exists x(Bx \,\&\, L{\sim}Ax)$

All the syllogisms in Table 9 depend simply on Uniform Substitution (of *LA* for *A*, or *L~A* for *~A*, and so on, see footnote 3 on p. 68), together with ordinary non-modal conversion, though in the case of Datisi LXL a slight textual problem arises. Datisi LXL in Table 9 is valid. But Aristotle actually discusses a syllogism with *different* premises – that is, not with a premise combination that gives us a Datisi. The passage in question is *An.Pr.* A11, 31b16–20. There the first premise is still 'all Cs are necessary As' – that is the same as in Datisi. But Aristotle gives the second premise by saying 'B is below (*hupo*) C.' For Aristotle this sometimes means that *every* B is a C and sometimes that *some* B is a C. If it is a particular premise then it would have to be $\exists x(Bx \,\&\, Cx)$. So we really have a valid *first figure* syllogism, with an L-conclusion:

31b16–20
$\forall x(Cx \supset LAx)$
$\underline{\exists x(Bx \,\&\, Cx)}$
$\exists x(Bx \,\&\, LAx)$

And that means we do not really find in Aristotle's text any statement that he counts the third figure Datisi LXL as valid. But in fact, both versions are valid.

Aristotle's third figure invalids are listed in Table 10.

Table 10
Invalid Third Figure L+X Forms (A11)

	Felapton XLL	(31a37–b10)
	$\forall x(Cx \supset \sim Ax)$	All horses are not awake
	$\underline{\forall x(Cx \supset LBx)}$	All horses are necessarily animals
*	$\exists x(Bx \,\&\, L{\sim}Ax)$	Some animals are necessarily not awake
	Datisi XLL	(31b20–31)
	$\forall x(Cx \supset Ax)$	All animals are wakeful
	$\underline{\exists x(Cx \,\&\, LBx)}$	Some animal is necessarily a biped
*	$\exists x(Bx \,\&\, LAx)$	Some biped is necessarily wakeful
	Bocardo XLL	(31b40–32a1)
	$\exists x(Cx \,\&\, \sim Ax)$	Some man is not wakeful
	$\underline{\forall x(Cx \supset LBx)}$	All men are necessarily animals
*	$\exists x(Bx \,\&\, L{\sim}Ax)$	Some animal is necessarily not wakeful
	Ferison XLL	(32a1–4)
	$\forall x(Cx \supset \sim Ax)$	All animals are not wakeful
	$\underline{\exists x(Cx \,\&\, LBx)}$	Some animal is necessarily white
*	$\exists x(Bx \,\&\, L{\sim}Ax)$	Some white thing is necessarily not wakeful

In all of these in Table 10, above, Aristotle is able to give counter-examples in which the premises are genuine, in the sense that they have subject terms that are red terms.[8] Such counter-examples will block a proof by modal conversion in the following way. A term that 'begins life' as the predicate of a non-modal (X) assertoric premise is for Aristotle a term of 'mere belonging.' Mere belonging must be able to cover accidental or green predicate terms; terms about mere belonging cannot be restricted to red terms. In the third figure invalids in which the A term begins life as the predicate of an assertoric premise, the A term clearly cannot be guaranteed to be a red term, and therefore, cannot be guaranteed to satisfy the restriction on genuine modal conversion.

Thom (1996, p. 94) objects to Aristotle's counter-example to Datisi XLL.

[8] In Ferison XLL the conclusion has a green, accidental subject, but as in the case of the first figure invalids, a completely genuine counter-example could be given. Let A be wakeful, B be animal, and C be man.

	Datisi XLL (31b20–31)			
	(1)	$\forall x(Cx \supset Ax)$	All animals are wakeful	T
	(2)	$\exists x(Cx \,\&\, LBx)$	Some animal is necessarily a biped	T
*	(3)	$\exists x(Bx \,\&\, LAx)$	Some biped is necessarily wakeful	F

This is because on Thom's analysis (see p. 57 above) 'some biped is necessarily awake' means

> 'some biped is necessarily awake, or something awake is necessarily a biped.'

And even though the left disjunct is false, the right disjunct is true here. So Thom (1996, p. 94) attributes to Aristotle a mistake:

> But [Aristotle] forgets that since [LI-propositions] are convertible, some biped will be necessarily awake if something awake is necessarily a biped. And, something awake *is* necessarily biped given that all men are both awake and necessarily biped (Darapti XLL).[9] So it seems that this counter-example is unsuccessful by Aristotelian standards.

Clearly Thom's claim hangs on his disjunctive analysis of LI-propositions. Thom (1996, p. 94) suggests that 'biped' be replaced by 'approaching'. This would give:

(1) All animals are wakeful
(4) Some animal is necessarily approaching
(5) Something approaching is necessarily wakeful

On the simple-minded interpretation, (2) is true, but (4) is not true. With Thom's disjunctive interpretation, the counter-example becomes:

(1)	All animals are wakeful	T
(6)	Some animal is necessarily approaching or something approaching is necessarily an animal	T
(7)	Something approaching is necessarily wakeful or something wakeful is necessarily approaching	F

[9] Thom's disjunctive approach validates Darapti XLL:

$\forall x(Cx \supset Ax)$
$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \,\&\, LAx) \vee \exists x(Ax \,\&\, LBx)$.

On the interpretation adopted in this book, Darapti XLL is not valid. See pp. 95—97 below.

The choice here is whether to take Aristotle's text as evidence against the disjunctive analysis, or whether the disjunctive analysis gives evidence that Aristotle has made a mistake.[10] According to the simple interpretation in the present book, Datisi XLL is straightforwardly invalid just as Aristotle's counter-example illustrates. Nothing is necessarily approaching. Approaching is only an accidental feature of any subject. Approaching is a green term and when a proposition predicates 'necessarily approaching' that proposition is false.

[10]Malink follows Thom's disjunctive approach and lists Datisi XLL among the invalid schemas (Malink 2006, pp. 117, 131).

Chapter 9
Against the Canonical Listings

The tables and discussion so far differ from those traditionally offered. This chapter is about what is missing and why. The relevant sections of the *Prior Analytics* are mainly A10 and A11. And I will follow Aristotle's text closely in order to best explain the omissions I make – for there are differences between what Aristotle's text requires and what an interpretive tradition requires. All the way through *An.Pr.* A9–11, Aristotle gives a new and separate proof of each modal syllogism. He does *not* take the mixed syllogisms as entirely trivial and obvious. He tries to explain them and to establish their validity. And that is where the differences arise. When we look closely at Aristotle's proofs in the modal syllogistic, we discover that in some cases there are differences between the modal and non-modal 'correlates'. The mnemonics do not make the differences apparent. The purpose of this chapter is to show how and where some of the problems and inconsistencies traditionally levelled against Aristotle's modal syllogistic only arise because the textual proofs and the mnemonics are assumed to be the same even in those cases where they are not.

Traditional accounts of the modal syllogistic list two additional valid third figure apodeictic syllogisms which I leave out of Table 7. These are usually called Darapti XLL and Disamis XLL. In 31a19–20 Aristotle makes some preliminary blanket remarks which include claims about families of valid schemas, and which appear to cover Darapti XLL and Disamis XLL. But when Aristotle is producing specific proofs he does not in fact produce proofs of Darapti XLL or Disamis XLL. There is a tension in the text, since on the one hand Aristotle appears to count them as valid, but on the other hand he does not actually make a case for their validity. As we shall see there is in each case a good reason *not* to attribute validity to these. First, consider Darapti XLL. The mnemonic tradition dictates that Darapti XLL must be as follows:

Darapti XLL

(1)	$\forall x(Cx \supset Ax)$
(2)	$\underline{\forall x(Cx \supset LBx)}$
(3)	$\exists x(Bx \,\&\, LAx)$

This is typically counted as valid, and it is said to be the syllogism set out in *An.Pr.* 31a31–33. But before we look at the text, let's consider just what this Darapti XLL involves. It purports to involve a modal operator (L) which shifts from scope over the B term in (2), to scope over the A term in (3).[1] Darapti XLL is easy to invalidate with a counter-example. Let A be moving, B be animal, and C be man. If we suppose the premises are true, the conclusion can still come out false:

[1] Of course, L-conversion regularly does the same, but L-conversion is restricted to genuine conversion, in which both subject and predicate terms are red terms, and so validity is preserved.

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
DOI 10.1007/978-94-007-0050-5_9,

Every man is moving	T
<u>Every man is a necessary animal</u>	T
Some animal is necessarily moving	F

Now look at what we find in the text. In *An.Pr.* A11, 31a19–33, Aristotle is discussing two closely related syllogistic schemas. The first is a straightforward Darapti LXL (31a24–30), listed as valid in Table 7. The second is the schema which according to tradition is the valid syllogism Darapti XLL (31a31–33):

> In the last figure, when the terms are universal in relation to the middle and both the premises are positive, then if either one is necessary the conclusion will also be necessary. (31a18–21)

> For let both premises first be positive, and let both A and B belong to every C, but let AC be necessary. [Darapti LXL] Then, since B belongs to every C, C will also belong to some B because the universal converts into a particular. Consequently, if A belongs to every C of necessity, and C belongs to some B, then it is also necessary for A to belong to some B (for B is under C). The first figure therefore comes about. (31a24–30)

> And it will also be proved in the same way if BC is necessary. For C converts to some A; consequently, if B belongs to every C of necessity, then it will also belong to some A of necessity. (31a31–33)

The passage 31a31–33 is *not* a proof of the validity of Darapti XLL. Aristotle is discussing a syllogism with the same premises as in Darapti XLL, ('let both A and B belong to every C', 31a25; and make BC necessary, a31). But Aristotle is concerned with a different conclusion. His proof proceeds as follows:

(1) $\forall x(Cx \supset Ax)$ 'A belongs to every C' (but not of necessity)
(2) $\forall x(Cx \supset LBx)$ 'B belongs to every C of necessity'

convert (1) to

(3) $\exists x(Ax \,\&\, Cx)$ I-conversion (1)

'Consequently, if (2) B belongs to every C of necessity, then it will also belong to some A of necessity,' (31a32–33); that is,

(4) $\exists x(Ax \,\&\, LBx)$ 'B will belong to some A of necessity'

And Aristotle stops there. (1)–(4), however, is not Darapti XLL. It would *become* Darapti XLL if Aristotle were to convert (4), but in the text he does not convert it. There is a good reason for this. As Aristotle gives the first premise, 'A belongs to every C', it is not a modal premise. It is not '*of necessity*,' so we have to take (1) to be about mere belonging. If A merely belongs to every C, then according to Aristotle 'it is possible (*endechetai*) that A belongs to C' (31b29–30).[2] So Aristotle clearly leaves open the case for an accidental A predicate in (1). If A is an accident, then, though (4) is an LI-proposition, it is not a *genuine* LI-proposition. So Aristotle is not entitled to convert, and he does not.[3] This means that we do not get a proof from Aristotle that Darapti XLL is valid, and so Darapti XLL does not clearly belong in Table 7 among the valid third figure apodeictic syllogisms. Next, consider another syllogistic schema which I have omitted from Table 7: Disamis XLL.

Disamis XLL
$\exists x(Cx \,\&\, Ax)$
$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \,\&\, LAx)$

A first comment is that the terms Aristotle uses for Felapton XLL at 31b4–10 (see Table 10) can be used to give a (genuine) counter-example to Disamis XLL:

Some horses are awake	T
All horses are necessary animals	T
Some animals are necessarily awake	F

Yet *An.Pr.* A11, 31b12–19 is typically assumed to contain a proof of Disamis XLL:

> ...if one term is universal, the other is particular, and both are positive, then the conclusion will be necessary whenever the universal is necessary. The demonstration is the same as the previous one, for the positive particular also converts. Thus, (1) if it is necessary for B to

[2]We find a similar point in the discussion of the invalid Barbara XLL. If it is true that A belongs to every B but not of necessity, then 'B may be such that it is possible for A to apply to no B' (30a27–28). That is, A might be an accidental term, something which possibly does not belong to B.

[3]Using the terms in the counter-examples above – moving thing for A, man for B – the conversion would be from 'some mover is a necessary man' to 'some man is necessarily moving.' And this is invalid – the truth of 'some mover is a necessary man' does not guarantee the truth of 'some man is necessarily moving.'

> belong to every C and (2) A is below C, (3) then it is necessary for B to belong to some A. But if it is necessary for B to belong to some A, then (4) it is also necessary for A to belong to some B (for it converts).

Here is how the proof is traditionally taken to go:

Disamis XLL

(1)	$\forall x(Cx \supset LBx)$	if it is necessary for B to belong to every C
(2a)	$\exists x(Cx \mathbin{\&} Ax)$	and A belongs to some C
(3)	$\exists x(Ax \mathbin{\&} LBx)$	then it is necessary for B to belong to some A
(4)	$\exists x(Bx \mathbin{\&} LAx)$	it is also necessary for A to belong to some B

Note that the order of the premises makes no difference to the validity of a schema. But there is of course a problem about this traditional interpretation. Aristotle's premise (2) 'A is below C' cannot mean the same as (2a) 'A belongs to some C.' All the evidence we have about Aristotle's use of the word 'below (*hupo*)' in *An.Pr.* indicates that 'A is below C' must here mean 'C belongs to some A'.[4] Sometimes Aristotle uses 'below' (*hupo*) in a universal proposition – e.g., man is below animal. This cannot mean $\forall x(Cx \supset Ax)$ where man is A and animal is C. That is, 'man is below animal' cannot mean 'anything that is an animal is a man.' The latter is plainly false, since, for instance, Dixie the Bernese Mountain Dog is an animal but is not a man. 'Man is below animal' should be understood to mean that man is a *less* general category than animal: anything which is a man belongs as well to the *wider* category animal. Returning to the matter of how to interpret 'below' in (2), it is easy to see that the same holds when 'below' is used in an existential proposition. (2) 'A is below C' must mean (2b) 'some A is C', where A is the subject term and C the predicate. The traditional interpretation (2a) inappropriately reverses the terms; whereas, (2b) preserves the deeper logical structure. And so we must use (2b) here not (2a). This makes the syllogism at 31b12–19 in Aristotle's text:

(1)	$\forall x(Cx \supset LBx)$	if it is necessary for B to belong to every C
(2b)	$\exists x(Ax \mathbin{\&} Cx)$	and A is below C,
(3)	$\exists x(Ax \mathbin{\&} LBx)$	then it is necessary for B to belong to some A
(4)	$\exists x(Bx \mathbin{\&} LAx)$	it is also necessary for A to belong to some B

[4]For an obsessively detailed justification of taking the second premise to be $\exists x(Ax \& Cx)$ see Rini (2000).

This has been missed by modern interpreters who typically suppose the premise to have the form given in (2a).[5] Probably they read it this way with an eye to finding, here, a modal version of the schema of Disamis XXX in Table 2:

Disamis XXX
$\exists x(Cx \;\&\; Ax)$
$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \;\&\; Ax)$

In Disamis XXX, the mnemonic code dictates that the A term is the *predicate* in the non-modal premise. But the mnemonics encode proofs for the assertoric syllogistic and plainly do not capture the differences that occur in Aristotle's discussion in the modal syllogistic.

Consider the effect this interpretation has on the proof. Of course (2a) and (2b) are equivalent in LPC, but they represent different Aristotelian propositions. In (2b), A represents the *subject* term of an assertoric premise. In (2a), A represents the *predicate* of an assertoric premise. If A is the predicate of an assertoric premise then the possibility is always open that A might be only an accidental term. If A is only an accidental term in line (2a) then it is again only an accident in (3). That is not a problem so far as it goes, since in (3) the A term is only the subject of an L proposition. But in order to reach a standard third figure conclusion, (3) must be converted, and this is where the trouble starts. If A is only an accidental term, then (3) cannot be genuinely converted since genuine LI conversion requires that the input always have an essential term as subject. If A is an accident then the conversion is invalid.

On the other hand, if we take the valid (1)(2b)(3)(4) to be what Aristotle is describing in 31b12–19, then we should not call *that* syllogism Disamis XLL, since what Aristotle describes does not fit the traditional mnemonics. Any Disamis would have to be in the third figure because of the form of each of the premises. What Aristotle describes, however, is a first figure syllogism with a converted conclusion – that is, he has set out a proof of a *fourth* figure apodeictic syllogism at 31b12–19. It is

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[5]See Rini (2000). Ebert and Nortmann (2007, p. 456), also reject Disamis XLL. Thom's disjunctive approach will validate Disamis XLL and reject Disamis LXL:

Disjunctive Disamis XLL
$\exists x(Cx \;\&\; Ax)$
$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \;\&\; LAx) \vee \exists x(Ax \;\&\; LBx)$

Disjunctive Disamis LXL
Some animal is necessarily approaching or something approaching is a necessary animal.
All animals are wakeful.
Something wakeful is necessarily approaching or something approaching is necessarily wakeful.

a first figure syllogism Darii LXL with a converted conclusion, and the conversion of the conclusion is valid provided that A is an essential term, thus guaranteeing that the conversion is genuine.

This clears up several potentially serious problems since if Aristotle did argue that Darapti XLL and Disamis XLL were valid, then he would be landed at once with invalid modal conversions and with two apodeictic schemas which are open to easy counter-examples. But Aristotle does not make any invalid conversions here in the *Prior Analytics*. And he does not argue that Darapti XLL and Disamis XLL are valid. If Aristotle did offer proofs that Darapti XLL and Disamis XLL were valid, then he would have to allow accidents inside the scope of necessity. We have seen that Aristotle has specific ways of identifying accidental terms: he describes them as predicates that 'merely belong' or that belong 'but not of necessity'. In the apodeictic syllogistic Aristotle never attempts to put such terms inside the scope of necessity in a proposition he counts as true. In fact, his own counter-examples in the rest of the apodeictic syllogistic rule this out – in those counter-examples, we produce a falsehood every time by taking a term φ which merely or accidentally belongs and putting φ inside the scope of necessity. Such terms are only green, accidental predicates, not red, necessary predicates.

As the proof at 31b12–19 shows, Aristotle handles subject terms differently. In the mixed apodeictics, a term φ which starts life as the subject of a syllogistic premise may be either green or red – that is, either an accidental or an essential term – it does not matter whether the premise is assertoric or necessary. But if the premise is necessary and is later the input to L-conversion, then the subject φ must be a red, essential term. If φ is a green, accidental term then the validity of the L-conversion is not guaranteed, since such a term φ may not belong of necessity and so may not fit inside the scope of an *L*.

Table 10 leaves out two schemas which are traditionally included among the invalid third figure schemas, Disamis LXL and Bocardo LXL. The schema rejected at 31b31–33 is assumed to be Disamis LXL, and the terms used are assumed to be those given for Datisi XLL:

	Disamis LXL	(31b31–33)
	$\exists x(Cx \,\&\, LAx)$	Some animal is necessarily a biped
	$\underline{\forall x(Cx \supset Bx)}$	All animals are wakeful
*	$\exists x(Bx \,\&\, LAx)$	Something wakeful is a necessary biped

For Bocardo we have

> When the privative premise is particular and necessary, the terms are biped, man, animal (with animal the middle). (32a4–5)

	Bocardo LXL	(32a4–5)
	$\exists x(Cx \,\&\, L{\sim}Ax)$	Some animal is necessarily not a biped
	$\underline{\forall x(Cx \supset Bx)}$	All animals are moving
*	$\exists x(Bx \,\&\, L{\sim}Ax)$	Some moving thing is necessarily not a biped

Both these syllogisms are valid in LPC, since they are substitution instances of non-modal Disamis and Bocardo. Yet in each case, Aristotle suggests a counter-example, taking us from true premises to a false conclusion. The conclusions he rejects as false are 'something wakeful is a necessary biped' and 'some moving thing is necessarily not a biped.' There is a question about why these conclusions should be counted as falsehoods. The reason cannot be that nothing is a necessary biped, or that nothing is necessarily not a biped, because the true premises in these counter examples tell us clearly that some animals are. Given what we have seen so far, the conclusions in question would appear to be true and Disamis LXL and Bocardo LXL valid.[6] So, what is Aristotle doing? In order to see what he is doing, consider his counter-example to Datisi XLL, where he gives wakeful, biped, and animal for *A*, *B*, and *C*. This gives us the following:

Datisi XLL (31b20–31)

	(1)	$\forall x(Cx \supset Ax)$	All animals are wakeful	T
	(2)	$\underline{\exists x(Cx \,\&\, LBx)}$	Some animal is necessarily a biped	T
*	(3)	$\exists x(Bx \,\&\, LAx)$	Some biped is necessarily wakeful	F

In the next several lines, 31b31–33, Aristotle says that Disamis LXL can be shown to be invalid 'through the same terms', and these are taken to be the terms Aristotle gives for Datisi XLL.[7] Here is how the argument would go:

Disamis LXL (31b31–33)

	(2)	$\exists x(Cx \,\&\, LAx)$	Some animal is necessarily a biped	T
	(1)	$\forall x(Cx \supset Bx)$	All animals are wakeful	T
	(4)	$\exists x(Ax \,\&\, LCx)$	Some biped is necessarily an animal	T
*	(5)	$\exists x(Ax \,\&\, LBx)$	Some biped is necessarily wakeful	F
*	(6)	$\exists x(Bx \,\&\, LAx)$	Something wakeful is a necessary biped	F

[6] Commentators are agreed that there are problems with what Aristotle says at 31b31—33. See Ross (1957, p. 324) and Striker (2009, p. 125).

[7] He cannot mean that A is wakeful, B is biped, and C is animal, because those would make both premises in Disamis LXL false. So while the same terms will work in both Darapti XLL and Disamis LXL, Aristotle clearly does not mean that they should be taken in the same order.

Genuine LI conversion of (2) gives us (4). And (4) and (1) give us the premises of Darii XLL which is invalid, so the L proposition (5) is false. (5) is the same as (3), above in Datisi, and (contra Thom) Aristotle has already established in Datisi XLL that (3) 'some biped is necessarily wakeful' is false. Two lines later, when he comes to Disamis LXL, he wants to have (6) 'something wakeful is necessarily a biped' false. This looks like he is making the subtle mistake again. For this reason I have not included Disamis LXL in Table 10, which lists the invalid third figure schemas. Bocardo LXL falls with Disamis LXL. Notice that Bocardo is simply Disamis with *L~A* in place of *LA*. This makes Bocardo and Disamis LXL just different substitution instances. And if Disamis LXL involves the subtle mistake, the same problem affects Bocardo LXL. Again Aristotle seems confused. For this reason Bocardo LXL, like Disamis LXL, is not included in Table 10.

The discussion in these last two chapters is assuming that modal conversion in the apodeictic syllogistic is governed by the genuineness requirement. I do not intend to suggest that Aristotle is explicitly aware of this, and if he is not, then the subtle mistake would be a very easy one for him to make. It may be that the evidence for genuineness is indirect in that it is as Striker (2009, p. xvi) says 'counterfactual', but as argued in Chapter 5, the alternatives are either to give up hope of a coherent account, or to resort to more complex analyses of the kind found in Thom or Malink. I have included tables below in order to help classify families of schemas. Table 11 lists Aristotle's L+X and X+L syllogisms whose proofs require converted conclusions. And Table 12 shows how to extend this same method in order to obtain the remaining fourth figure apodeictics which are not actually detailed in the *Prior Analytics*. Table 13 gives the traditional lists of valid L+X and X+L syllogisms. I've marked those I claim should not be there by ✕. Table 14 gives the schemas whose proofs are affected by Aristotle's subtle mistake.

Table 11
L+X, X+L Syllogisms with Converted Conclusions
Located in *An.Pr.*

From the First Figure (to Fourth Figure)	From the Second Figure (A10)	From the Third Figure (A11)
aii-LXL (A11, 31b12—19)	Camestres XLL (30b14—18)	None, because a 3rd figure
$\forall x(Bx \supset LAx)$	$\forall x(Bx \supset Ax)$	conclusion always relates a B
$\exists x(Cx \& Bx)$	$\forall x(Cx \supset L{\sim}Ax)$	subject to an A predicate, but
$\underline{\exists x(Cx \& LAx)}$ Darii LXL	$\underline{\forall x(Bx \supset L{\sim}Cx)}$ Celarent LXL	A and B both start life as
$\exists x(Ax \& LCx)$ LI conv	$\forall x(Cx \supset L{\sim}Bx)$ LE-conv	predicate terms. In an L+X,
		X+L syllogism this means *at*
		most one of A or B is
	aee-LXL (30b20—40)	guaranteed to be an essential
	$\forall x(Bx \supset LAx)$	term, since the other starts life
	$\forall x(Cx \supset {\sim}Ax)$	as a predicate of mere
	$\underline{\forall x(Cx \supset L{\sim}Bx)}$ Cesare LXL	belonging in an X premise.
	$\forall x(Bx \supset L{\sim}Cx)$ LE-conv	

Table 12
Putative L+X Syllogisms
First Figure Conclusions Converted to Fourth Figure

From the First Figure (to Fourth Figure)

aai-LXL
$\forall x(Bx \supset LAx)$
$\forall x(Cx \supset Bx)$
$\underline{\forall x(Cx \supset LAx)}$ Barbara LXL
$\exists x(Ax \& LCx)$ Genuine LA conv

eae-LXL
$\forall x(Bx \supset L{\sim}Ax)$
$\forall x(Cx \supset Bx)$
$\underline{\forall x(Cx \supset L{\sim}Ax)}$ Celarent LXL
$\forall x(Ax \supset L{\sim}Cx)$ Genuine LE-conv

Table 13
L+X, X+L Syllogisms: The Traditional Lists

First Figure

Barbara LXL (30a17—23)
$\forall x(Bx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset LAx)$

Darii LXL (30a37—b2)
$\forall x(Bx \supset LAx)$
$\underline{\exists x(Cx \& Bx)}$
$\exists x(Cx \& LAx)$

Celarent LXL (30a17—23)
$\forall x(Bx \supset L{\sim}Ax)$
$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset L{\sim}Ax)$

Ferio LXL (30a37—b2)
$\forall x(Bx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \& Bx)}$
$\exists x(Cx \& L{\sim}Ax)$

Second Figure

Cesare LXL (30b9—13)
$\forall x(Bx \supset L{\sim}Ax)$
$\underline{\forall x(Cx \supset Ax)}$
$\forall x(Cx \supset L{\sim}Bx)$

Camestres XLL (30b14—18)
$\forall x(Bx \supset Ax)$
$\underline{\forall x(Cx \supset L{\sim}Ax)}$
$\forall x(Cx \supset L{\sim}Bx)$

Festino LXL (31a5—10)
$\forall x(Bx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \& Ax)}$
$\exists x(Cx \& L{\sim}Bx)$

Third Figure

Darapti LXL (31a24—30)
$\forall x(Cx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \& LAx)$

Darapti XLL (31a31—33)
$\forall x(Cx \supset Ax)$
$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \& LAx)$ $\times$

Felapton LXL (31a33—37)
$\forall x(Cx \supset L{\sim}Ax)$
$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \& L{\sim}Ax)$

Datisi LXL (31b19—20)
$\forall x(Cx \supset LAx)$
$\underline{\exists x(Cx \& Bx)}$
$\exists x(Bx \& LAx)$

Disamis XLL (31b12—19)
$\exists x(Cx \& Ax)$
$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \& LAx)$ $\times$

Ferison LXL (31b35—37)
$\forall x(Cx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \& Bx)}$
$\exists x(Bx \& L{\sim}Ax)$

Table 14
Schemas Affected by the Subtle Mistake

First Figure	Second Figure	Third Figure
None	Baroco LXL	Bocardo LXL(32a4–5)
	$\forall x(Bx \supset LAx)$	$\exists x(Cx \,\&\, L{\sim}Ax)$
	$\underline{\exists x(Cx \,\&\, {\sim}Ax)}$	$\underline{\forall x(Cx \supset Bx)}$
	$\exists x(Cx \,\&\, L{\sim}Bx)$	$\exists x(Bx \,\&\, L{\sim}Ax)$
	Baroco XLL	
	$\forall x(Bx \supset Ax)$	
	$\underline{\exists x(Cx \,\&\, L{\sim}Ax)}$	
	$\exists x(Cx \,\&\, L{\sim}Bx)$	
Fourth Figure (First + Converted Conclusion)	Second Figure + Converted Conclusion	Third Figure + Converted Conclusion
None	Camestres LXL	Disamis LXL (31b31–33)
	$\forall x(Bx \supset LAx)$	$\exists x(Cx \,\&\, LAx)$
	$\underline{\forall x(Cx \supset {\sim}Ax)}$	$\underline{\forall x(Cx \supset Bx)}$
	$\forall x(Cx \supset L{\sim}Bx)$	$\exists x(Bx \,\&\, LAx)$

Chapter 10
Apodeictic Possibility

Part III of this book will be about how possibility features in syllogisms and about how syllogisms about possibility link with well known principles of Aristotle's metaphysics. Before we look at that, we need to look first at what I call *apodeictic possibility* where M is simply an abbreviation for $\sim L \sim$, and where the principles that govern the apodeictic syllogistic still apply.

We saw in Chapter 6 how the pure apodeictic LLL syllogisms can be made trivially valid by restricting all terms to red terms. The effect of this approach is to make the syllogisms in Table 3 the right representation of the LLL syllogisms. And the LLL syllogisms are then simply restricted instances of the non-modal assertoric syllogisms. One important point to notice is that this same approach will also give us trivially valid instances of pure MMM syllogisms. When all terms are red, then anywhere we have a valid LLL syllogism, we will also have a valid MMM syllogism. And these MMMs, just like the LLLs, will always be restricted instances of non-modal assertoric syllogisms. Recall the following example from Chapter 6:

Every necessary-mammal is a necessary-animal
<u>Every necessary-man is a necessary-mammal</u>
Every necessary-man is a necessary-animal

This is an instance of what in Chapter 6 we called 'Red Barbara'. It is a Barbara LLL. Consider the effect of putting the same red terms in a Barbara form in which the modal is M-possibility, rather than L-necessity. Then we get the following:

Every possible-mammal is a possible-animal
<u>Every possible-man is a possible-mammal</u>
Every possible-man is a possible-animal

This too is a Red Barbara, but it is a Red Barbara about M-possibility. Any such Red Barbara is trivially valid. Any pure LLL or MMM syllogism with all red terms will be trivially valid, a restricted instance of a non-modal syllogism. And so such syllogisms are really only trivially modal – the simple form of the non-modal assertoric syllogism is what guarantees validity. The 'modality' is not a feature of the logic but of the terms, which since they are red can be said to be necessary. This makes the logic uninteresting. And it also raises an important question about whether we really can make meaningful distinctions between LLL and MMM syllogisms. This is the question to which we must now turn.

Aristotle treats true apodeictic propositions as the spring-board from which to launch his discussion of possibility. This has some curious consequences. Most

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important of these is that even in the discussion of M-possibility the substance principle (SP) is at play. Recall that the substance principle governs our substitution of modal and non-modal terms in the apodeictic syllogistic. Since the apodeictic syllogistic provides the basis for the discussion of M-possibility, the substance principle governs our substitution of terms here too. Where we have a true apodeictic proposition, we have a predicate which is a red term, and we have a concomitant proposition about M-possibility. This M-proposition is true whenever the corresponding L-proposition is true – that is, when the predicate is a red term.

It will help to have a schematic representation to show how this works. First consider what the substance principle tells us. Where φ is a red term then the substance principle guarantees that $L\varphi \equiv \varphi$, and so, substituting $\sim\varphi$ for φ, we have $L{\sim}\varphi \equiv {\sim}\varphi$. For example: anything whose essence excludes being a horse, say, is also the sort of thing which is not necessarily a horse, and vice versa. When we deny $L{\sim}\varphi$, then, with double negation, we get the following equivalences:

Where φ *is a red term*, ${\sim}L{\sim}\varphi \equiv {\sim}{\sim}\varphi \equiv \varphi$.

Since Aristotle takes $M\varphi$ to be equivalent to ${\sim}L{\sim}\varphi$, then in his system a red term φ in the scope of an M must also make a true M-proposition. The upshot of this can be expressed as follows:

Where φ *is a red term*, $L\varphi \equiv M\varphi \equiv {\sim}L{\sim}\varphi \equiv {\sim}{\sim}\varphi \equiv \varphi$.

In Chapter 6, we noticed that there are two separate but equally viable ways to represent the LLLs. We might use Table 3, putting *L*s onto all of the syllogistic terms. Or we might use Table 4, putting *L*s on the predicate terms of the premises. The validity of the syllogistic forms in Table 3 is immediate since these just are the non-modal syllogisms restricted to red terms. And we can treat MMM syllogisms exactly the same way as we treated LLL syllogisms in Table 3. If on the other hand our LLL syllogisms follow the forms given in Table 4, then we should expect our MMM syllogisms to mirror the forms in Table 4. But giving proofs of the LLL syllogisms in Table 4 was not immediate – giving proofs required an additional principle $L\varphi \supset \varphi$. The proofs of the similar MMMs should then also require some additional principle about M-possibility. And as we have seen, the substance principle guarantees $M\varphi \supset \varphi$. And so, at least for red terms, $M\varphi \supset \varphi$ can do the job in the MMM problematic syllogistic that $L\varphi \supset \varphi$ does in the LLL apodeictic syllogistic in Table 4.

The upshot of this is that just as there are two ways to represent the LLLs, there are two ways to represent the MMMs. These different ways are given in Tables 15 and 16 below.

Table 15
Red M+M Syllogisms in Three Figures (*An.Pr.* A8)

First Figure

Barbara MMM
$\forall x(MBx \supset MAx)$
$\underline{\forall x(MCx \supset MBx)}$
$\forall x(MCx \supset MAx)$

Darii MMM
$\forall x(MBx \supset MAx)$
$\underline{\exists x(MCx \& MBx)}$
$\exists x(MCx \& MAx)$

Celarent MMM
$\forall x(MBx \supset M{\sim}Ax)$
$\underline{\forall x(MCx \supset MBx)}$
$\forall x(MCx \supset M{\sim}Ax)$

Ferio MMM
$\forall x(MBx \supset M{\sim}Ax)$
$\underline{\exists x(MCx \& MBx)}$
$\exists x(MCx \& M{\sim}Ax)$

Second Figure

Cesare MMM
$\forall x(MBx \supset M{\sim}Ax)$
$\underline{\forall x(MCx \supset MAx)}$
$\forall x(MCx \supset M{\sim}Bx)$

Camestres MMM
$\forall x(MBx \supset MAx)$
$\underline{\forall x(MCx \supset M{\sim}Ax)}$
$\forall x(MCx \supset M{\sim}Bx)$

Festino MMM
$\forall x(MBx \supset M{\sim}Ax)$
$\underline{\exists x(MCx \& MAx)}$
$\exists x(MCx \& M{\sim}Bx)$

Baroco MMM
$\forall x(MBx \supset MAx)$
$\underline{\exists x(MCx \& M{\sim}Ax)}$
$\exists x(MCx \& M{\sim}Bx)$

Third Figure

Darapti MMM
$\forall x(MCx \supset MAx)$
$\underline{\forall x(MCx \supset MBx)}$
$\exists x(MBx \& MAx)$

Felapton MMM
$\forall x(MCx \supset M{\sim}Ax)$
$\underline{\forall x(MCx \supset MBx)}$
$\exists x(MBx \& M{\sim}Ax)$

Datisi MMM
$\forall x(MCx \supset MAx)$
$\underline{\exists x(MCx \& MBx)}$
$\exists x(MBx \& MAx)$

Disamis MMM
$\exists x(MCx \& MAx)$
$\underline{\forall x(MCx \supset MBx)}$
$\exists x(MBx \& MAx)$

Bocardo MMM
$\exists x(MCx \& M{\sim}Ax)$
$\underline{\forall x(MCx \supset MBx)}$
$\exists x(MBx \& M{\sim}Ax)$

Ferison MMM
$\forall x(MCx \supset M{\sim}Ax)$
$\underline{\exists x(MCx \& MBx)}$
$\exists x(MBx \& M{\sim}Ax)$

Table 16
M+M Syllogisms in Three Figures (*An.Pr.* A8)
(with $M\varphi \supset \varphi$)

First Figure	Second Figure	Third Figure
Barbara MMM $\forall x(Bx \supset MAx)$ $\underline{\forall x(Cx \supset MBx)}$ $\forall x(Cx \supset MAx)$	Cesare MMM $\forall x(Bx \supset M{\sim}Ax)$ $\underline{\forall x(Cx \supset MAx)}$ $\forall x(Cx \supset M{\sim}Bx)$	Darapti MMM $\forall x(Cx \supset MAx)$ $\underline{\forall x(Cx \supset MBx)}$ $\exists x(Bx \& MAx)$
Darii MMM $\forall x(Bx \supset MAx)$ $\underline{\exists x(Cx \& MBx)}$ $\exists x(Cx \& MAx)$	Camestres MMM $\forall x(Bx \supset MAx)$ $\underline{\forall x(Cx \supset M{\sim}Ax)}$ $\forall x(Cx \supset M{\sim}Bx)$	Felapton MMM $\forall x(Cx \supset M{\sim}Ax)$ $\underline{\forall x(Cx \supset MBx)}$ $\exists x(Bx \& M{\sim}Ax)$
Celarent MMM $\forall x(Bx \supset M{\sim}Ax)$ $\underline{\forall x(Cx \supset MBx)}$ $\forall x(Cx \supset M{\sim}Ax)$	Festino MMM $\forall x(Bx \supset M{\sim}Ax)$ $\underline{\exists x(Cx \& MAx)}$ $\exists x(Cx \& M{\sim}Bx)$	Datisi MMM $\forall x(Cx \supset MAx)$ $\underline{\exists x(Cx \& MBx)}$ $\exists x(Bx \& MAx)$
Ferio MMM $\forall x(Bx \supset M{\sim}Ax)$ $\underline{\exists x(Cx \& MBx)}$ $\exists x(Cx \& M{\sim}Ax)$	Baroco MMM $\forall x(Bx \supset MAx)$ $\underline{\exists x(Cx \& M{\sim}Ax)}$ $\exists x(Cx \& M{\sim}Bx)$	Disamis MMM $\exists x(Cx \& MAx)$ $\underline{\forall x(Cx \supset MBx)}$ $\exists x(Bx \& MAx)$
		Bocardo MMM $\exists x(Cx \& M{\sim}Ax)$ $\underline{\forall x(Cx \supset MBx)}$ $\exists x(Bx \& M{\sim}Ax)$
		Ferison MMM $\forall x(Cx \supset M{\sim}Ax)$ $\underline{\exists x(Cx \& MBx)}$ $\exists x(Bx \& M{\sim}Ax)$

The main evidence that Aristotle counts the MMM syllogisms as valid comes from *Prior Analytics* A8. It is part of the passage we looked at in Chapter 6 on p. 63:

> Since to belong and to belong of necessity and to be possible to belong are different (for many things belong, but nevertheless not of necessity, while others neither belong of necessity nor belong at all, but it is possible for them to belong), it is clear that there will also be different deductions of each and that their terms will not be alike: rather, one deduction will be from necessary terms, one from terms which belong, and one from possible terms. (*An.Pr.* A8, 29b29–35)

We met this passage in our discussion of Aristotle's LLL apodeictic syllogisms. Recall from the principles laid out earlier in the present chapter that for M-possibility we know that when φ is a red term, then $M\varphi \equiv \varphi$, and $M{\sim}\varphi \equiv {\sim}\varphi$.

If we take MMM syllogisms as anything more than simple substitution instances of LLL syllogisms, and if we want to be able to give independent proofs of the MMMs, then we need validity-preserving principles for the *M*-conversions. Second and third figure MMMs will require *M*-conversion. In Chapter 6 it was claimed that if in any LLL syllogism all terms must be red then an LLL syllogism is just a special case of an XXX syllogism. If all terms are red then an MMM syllogism is in exactly the same situation. For mixed apodeictic syllogisms (LXLs and XLLs) it was necessary to look very closely at modal L-conversion, because in the mixed cases not all terms are red terms. And the solution put forward in Chapter 8 is that in the mixed apodeictic syllogistic when L-conversion is needed, then the terms involved in the actual conversion are in fact restricted to red terms. Is there anything analogous to be said about M-conversions? Aristotle's *Prior Analytics* A3 makes clear that he does in fact have M-conversion principles:

> When it comes to being possible, since 'to be possible' is said in several ways (i.e., we say it [a] of what is necessary, [b] of what is not necessary, and [c] of what is potential that it is possible), the situation with respect to conversion will be the same in all these cases with the affirmatives [MA and MI]. For if it is possible for A to belong to every or to some B, then it will be possible for B to belong to some A[1]: for if it is possible for it to belong to none, then neither will it be possible for A to belong to any B (this has been shown earlier). (25a37–53)

This clearly gives conversion principles for both MA- and MI-premises. But how, precisely, should we represent them?

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[1]These conversions, so far, must include conversions involving M-possibility. In later chapters, I look at another sense of possibility – this is possibility in the sense of contingency. There is I think no explicit mention of contingency in the passages quoted so far from A3. All that we are told is:

(a) what is necessary is said to be possible,
(b) what is not necessary is said to be possible, and
(c) what is potential is said to be possible.

The sense of possibility given in (a) is plainly inconsistent with contingency. And neither (b) or (c) expressly implicates 'contingency'.

Some interpreters find contingency at play here. Paul Thom reads this passage as saying that A-conversion and I-conversion 'hold in all senses of possible.' He then takes contingent to be one *sense* of possible and finds in 25a37—41 an argument for conversions involving contingency:

(d) if every b is contingently a, then some a is contingently b,
(e) if some b is contingently a, then some a is contingently b. (Thom 1996, p. 38)

Ross isn't clear about where *exactly* 'contingency' comes into the discussion, but he, too, wants to find conversions involving contingency in the present passage. (See Ross 1957, pp. 296—297.)

Again Aristotle models his principles about M-possibility on his principles about L-necessity. He wants the affirmative M-conversions to follow the patterns of LA- and LI-conversions since he says 'the situation with respect to conversion' is the same in all affirmative cases. In the account of the apodeictic conversion in Chapter 8 we found the following (affirmative) L-conversions:

(LA-conversion) $\forall x(Bx \supset LAx) \supset \exists x(Ax \ \& \ LBx)$
(LI-conversion) $\exists x(Bx \ \& \ LAx) \equiv \exists x(Ax \ \& \ LBx)$.

If Aristotle's M-conversions follow the same pattern, then substituting MB for LB we get:

(MA-conversion) $\forall \mathrm{x}(Bx \supset MAx) \supset \exists x(Ax \ \& \ MBx)$
(MI-conversion) $\exists \mathrm{x}(Bx \ \& \ MAx) \equiv \exists x(Ax \ \& \ MBx)$.

But now we are faced with a rather pressing question. What would make these M-conversions true? We know from Chapter 8 that at least in the mixed cases apodeictic conversions depend upon certain restrictions. Take a true LI premise. It has the logical form $\exists \mathrm{x}(Bx \ \& \ LAx)$. And take LI-conversion: $\exists x(Bx \ \& \ LAx) \equiv \exists x(Ax \ \& \ LBx)$. For a start:

(1) For the LI premise to be true, the *A* term – that is, the predicate term – must be a red term since it is in the scope of an *L* in a true proposition.

But second is the genuineness requirement:

(2) For any modal premise to convert, then the *B* term – that is, the subject term – must be restricted to red terms.

Both (1) and (2) are required in the apodeictic syllogistic in any LI-conversion which has a true input. The net effect of (1) and (2) is that in such conversions *both* the subject term and the predicate term are red terms. Now, in the M-syllogistic Aristotle wants MA- and MI-conversion to follow the pattern of LA- and LI-conversion. For this to work then in M-conversions, just as in apodeictic L-conversions, *both* the subject and the predicate terms must be red. Probably Aristotle is thinking that they are. If so, then his opening move in the problematic conversions is to simply carry over from the apodeictic syllogistic all of his baggage about *L*s into his analysis of *M*s.

There is, however, this difference between L-propositions and M-propositions. In the case of an L-proposition, one can 'read off' the fact that its predicate is red from its truth. This is obvious for affirmative L-propositions. For example:

(3) All men are necessarily animals $\forall x(man\ x \supset L\ animal\ x)$

(3) is clearly true. But next consider

(4) All men are necessarily white $\forall x(man\ x \supset L\ white\ x)$

(4) is clearly false. The very same terms do *not* make a statement about *possibility* false:

(5) All men are possibly white $\forall x(man\ x \supset M\ white\ x)$

This is not a falsehood, and so even if we still have a red subject term – i.e., what (2) above requires – nothing here guarantees that the *predicate* term is red. That is, the truth of an M premise does not ensure that the predicate term is a red term. One might think that there is nothing corresponding to (1) for negative propositions since

(6) No man is necessarily white

seems to be true. But is (6) true? There's a sense in which it is true – since white is only an accidental property of any man, then we can say of any man whatsoever that he is not necessarily white. On this analysis (6) would have to be

(7) $\forall x(man\ x \supset {\sim}L\ white\ x)$.

Let's call this the $\sim L$ analysis. But this is not the only sense of (6) and not the only way to analyze it. In fact it is not the way Aristotle analyses it when he is syllogizing. But this needs some explaining. As noted on pp. 49 and 77, when Aristotle is analyzing syllogistic premises he interprets propositions like (6) as

(8) $\forall x(Bx \supset L{\sim}Ax)$.

Let's call (8) the $L\sim$ analysis. Notice that with 'white' for A then (8) is just as false as $\forall x(Bx \supset LAx)$. In fact, when a predicate term A is a green term like 'white', then LAx and $L{\sim}Ax$ would both seem to be false, and if they are false, LA and $L{\sim}A$ are empty terms and, therefore, are not legitimate terms in the syllogistic. If so, then no legitimate L-premise can contain such a green predicate term. In the apodeictic syllogistic Aristotle's own examples never do. His privative premises about necessity always take the $L\sim$ form and so require red predicate terms.

In Chapter 5, I noted that it is a consequence of Charles's analysis of predication that Aristotle treats negation as *inside* the scope of a modal operator. While this may rule out the $\sim L$ analysis it does not rule out the equivalent $M\sim$ analysis. Whether Aristotle has in mind a $\sim L$ (i.e., an $M\sim$) analysis or an $L\sim$ analysis affects whether he takes (6) to be true. (6) is false in the apodeictic syllogistic where M is not involved and so is always given an $L\sim$ analysis, as in (8). But (6) can be taken as a true proposition about M-possibility. So there is an obvious need to be cautious about the differences between L-propositions and M-propositions. If terms are not restricted to red then differences between L-propositions and M-propositions begin to emerge. The examples (3)– (8) above and the question of the $\sim L$ translation or the $L\sim$ translation are part of the evidence for this. One point to note about the discussion so far is that it suggests that differences between L-propositions and M-propositions arise when there is negation involved or when some modal proposition is treated as false.

Aristotle is certainly aware that *something* different is going on in his discussion of necessity and possibility. We saw above how Aristotle describes MA- and MI-conversion in *An.Pr.* A3. In the discussion of ME-conversion he gives an example which shows there is some trouble about how to select terms about possibility. The full passage is A3, 25b3–13:

> It is not the same way in the case of the negatives, though it is similar for those which are said to be possible in virtue of belonging of necessity or not of necessity not belonging, as, for example, if someone were to say that it is possible for a man not to be a horse or for white to belong to no coat: the first of these of necessity does not belong, while the other does not necessarily belong, and the premise converts similarly. (For if it is possible for horse to belong to no man, then it is possible for man to belong to no horse; and if it is possible for white to belong to no coat, then it is possible for coat to belong to nothing white. For if belonging to some is necessary, then white belonging to some coat will be of necessity: for this has been proved earlier.) (25b3)

Consider how to represent the conversion that this passage describes. If ME-conversion is modelled on LE-conversion, then ME-conversion would have to be as follows:

ME-conversion $\quad \forall x(Bx \supset M{\sim}Ax) \supset \forall x(Ax \supset M{\sim}Bx)$

If both terms are restricted to red terms then – just like LE-conversion – ME-conversion is valid. Proof of its validity depends upon the definition of M-possibility together with the substance principle and the genuineness requirement. A restriction to red terms takes care of all of that. And if ME-conversion is supposed to be valid then a restriction to red terms is what we would expect. But Aristotle gives examples in the passage above to illustrate his ME-conversion, and one of the examples indicates that there are problems. The first example is trouble-free:

(9) If it is possible for horse to belong to no man, then it is possible for man to belong to no horse.

Both the subject term and the predicate term are red so there is nothing difficult about it. The second example is the troublesome one. The conversion described works like this:

(10) It is possible for white to belong to no coat [$\forall x(Bx \supset M{\sim}Ax)$] converts to
(11) it is possible for coat to belong to nothing white [$\forall x(Ax \supset M{\sim}Bx)$]

Suppose 'coat' is red, in accordance with the genuineness requirement. Then (10) is true. That is, all coats are possibly not white. But from this it doesn't necessarily follow that all white things are possibly not coats.

Seeing the trouble is easier if we take a look back at the situation so far. Any true L-proposition that gets converted involves a red term φ for *both* subject and predicate. The restriction to red terms for the subject is the genuineness requirement, and in the present case that would be the requirement that 'coat' be a red term. But the truth of (10) does not require that 'white' is a red term – and here is where the difference between *L* and *M* becomes important. In the case of converting a true L-proposition both terms will be red. And, so we are always able to move from a true L-proposition to a true M-proposition, because where φ is a red term we have $L\varphi \equiv M\varphi$. But look at the terms involved, now, in the conversion from proposition (10) to proposition (11). 'White' – the original predicate term – is not a red term; it is an accidental, green term. But (10) is, of course, a true proposition, and Aristotle clearly takes it to be true, here, in this passage. A proposition about possibility – such as (10) – does not become false if an accidental term falls inside the scope of possibility. An L-proposition, on the other hand, does become false if an accident falls inside the scope of necessity. If we replace 'possible' in (10) with 'necessary', this becomes obvious:

(12) It is necessary for white to belong to no coat [$\forall x(Bx \supset L{\sim}Ax)$]

Plainly (12) is false. So the parallel between L-conversion and M-conversion cannot be sustained, because true M-propositions admit green predicate terms in ways that L-propositions do not, and this affects conversion. Assume that Aristotle notices all this. What does he do? Does he say 'Oops, wrong semantic restriction'? Not immediately. What he does is set the *M*-syllogistic aside, and he considers his account of possibility all over again, beginning with a new description of 'possibility' in *An.Pr.*

A13. Why think that he sets the *M*-syllogistic aside? Nowhere does he offer any MMM, MXM, or XMM proofs. Of course, we can try to construct them ourselves by adhering to his instructions for L-apodeictic syllogisms; but we do not find any specific instances of *M*-possibility syllogisms in the *Prior Analytics*. This should not be surprising. If *M*-premises are always restricted to essential terms then the *M*-syllogisms are completely trivial variants of L-syllogisms. Probably somewhere along the line Aristotle notices this. The omission of any *M*-syllogisms is then easily explained. They do not generate a separate syllogistic; they are simply redundant. Wherever we have an LLL syllogism, we also have an MMM; wherever we have an LXL syllogism, we have an MXM; wherever we have an XLL, we have an XMM.

Part III

CONTINGENCY IN THE SYLLOGISTIC
An. Pr. A13–22

Chapter 11
Contingency (A13, A14)

Throughout the apodeictic syllogistic, Aristotle works with a definition of possibility according to which 'what is possibly so' is the same as 'what is not necessarily not so'. This is the sense of possibility used throughout Part II. It is what I labelled *apodeictic possibility* or *M-possibility*, after the modal logician's definition of possibility: $M\varphi =_{\mathrm{df}} {\sim}L{\sim}\varphi$.[1] As we saw in Chapter 10, Aristotle seems to have only a minor, derivative interest in this sense of possibility. He introduces a *new* sense of possibility when he begins his detailed study of syllogisms from premises about possibility – the part of the *Prior Analytics* which is called the *problematic syllogistic*. I label the new sense *Q-contingency*, after the modal logician's definition of contingency: $Q\varphi =_{\mathrm{df}} {\sim}L\varphi \,\&\, {\sim}L{\sim}\varphi$. That is, what is contingent is neither necessary nor impossible. Aristotle calls both senses of possibility by the same name – i.e., 'possible' – but as a rule he is careful to specify which way we are to understand 'possible'. Here is his new definition:

> I use the expressions 'to be possible (*endechestai*)' and 'what is possible (*to endechomenon*)' in application to something if it is not necessary but nothing impossible will result if it is put as being the case (for it is only equivocally that we say that what is necessary is possible). (A13, 32a19–22).

At 33a24–25 Aristotle tells us that this is a new definition (*horismos*). He often describes this sense of possible as 'according to the determination' (33b25–33). This new sense of possibility raises new questions about how to construct and how to validate syllogisms about Q-contingency. One might think that we could discuss the problematic syllogistic by doing no more than defining Q in terms of L and M and using all the same principles we have found so helpful in Part II – e.g., treating QQQ syllogisms like LLL and MMM syllogisms. Unfortunately this would trivialise the problematic syllogistic. This is because, if we accept SP and Neg(SP) for red terms, then $L\varphi \equiv \varphi$ and $L{\sim}\varphi \equiv {\sim}\varphi$, and so $Q\varphi \equiv {\sim}L\varphi \,\&\, {\sim}L{\sim}\varphi \equiv {\sim}\varphi \,\&\, \varphi$. So if φ is red, then $Q\varphi$ would always be empty. On the other hand, if, instead of using red terms in the QQQ syllogistic, we make all terms green, then it looks like we include too much, because it seems that any green term φ is *both* ${\sim}L\varphi$ *and* ${\sim}L{\sim}\varphi$. For example, suppose φ is 'mover'. Anything that is a mover is *not a necessary mover* (because being a mover is not necessary to anything) and it is *not necessarily not a mover* (because anything that is a mover can move). If terms in the QQQ syllogistic must either be empty or too full, then we are never going to get a non-trivial QQQ syllogistic.

So it is clear that we should expect some real logical differences between the

[1] As in Parts I and II, I am here using φ as a metalogical variable for a formula such as Bx.

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apodeictic and the problematic syllogisms, especially when red terms are involved. The aim of this chapter is to see how far we can get in following the method used in the LLL syllogisms in Table 3, and the MMM syllogisms in Table 15, where the syllogisms were no more than substitution instances of non-modal syllogisms. So we might try to construct a syllogistic about Q-contingency along the lines of the schemas in Tables 3 and 15, and leave until later chapters the question of just how *Q* terms are to be interpreted. That is to say, before we consider the problems about what *Q* means, that is, before we try to explain how to *interpret Q*, it will help to take the kind of approach used in Chapter 6, and for the moment simply assume that *Q* does have some meaning. Proceeding in this way will allow us to focus initially on how *Q* appears to function in the syllogisms. In the case of QQQs the result would look something like the following two Tables:

Table 17
Ampliated Q+Q Syllogisms in Three Figures
(with $Q{\sim}Ax$)

First Figure	Second Figure	Third Figure
Barbara QQQ	Cesare QQQ	Darapti QQQ
$\forall x(QBx\supset QAx)$	$\forall x(QBx\supset Q{\sim}Ax)$	$\forall x(QCx\supset QAx)$
$\underline{\forall x(QCx\supset QBx)}$	$\underline{\forall x(QCx\supset QAx)}$	$\underline{\forall x(QCx\supset QBx)}$
$\forall x(QCx\supset QAx)$	$\forall x(QCx\supset Q{\sim}Bx)$	$\exists x(QBx\,\&\,QAx)$
Darii QQQ	Camestres QQQ	Felapton QQQ
$\forall x(QBx\supset QAx)$	$\forall x(QBx\supset QAx)$	$\forall x(QCx\supset Q{\sim}Ax)$
$\underline{\exists x(QCx\,\&\,QBx)}$	$\underline{\forall x(QCx\supset Q{\sim}Ax)}$	$\underline{\forall x(QCx\supset QBx)}$
$\exists x(QCx\,\&\,QAx)$	$\forall x(QCx\supset Q{\sim}Bx)$	$\exists x(QBx\,\&\,Q{\sim}Ax)$
Celarent QQQ	Festino QQQ	Datisi QQQ
$\forall x(QBx\supset Q{\sim}Ax)$	$\forall x(QBx\supset Q{\sim}Ax)$	$\forall x(QCx\supset QAx)$
$\underline{\forall x(QCx\supset QBx)}$	$\underline{\exists x(QCx\,\&\,QAx)}$	$\underline{\exists x(QCx\,\&\,QBx)}$
$\forall x(QCx\supset Q{\sim}Ax)$	$\exists x(QCx\,\&\,Q{\sim}Bx)$	$\exists x(QBx\,\&\,QAx)$
Ferio QQQ	Baroco QQQ	Disamis QQQ
$\forall x(QBx\supset Q{\sim}Ax)$	$\forall x(QBx\supset QAx)$	$\exists x(QCx\,\&\,QAx)$
$\underline{\exists x(QCx\,\&\,QBx)}$	$\underline{\exists x(QCx\,\&\,Q{\sim}Ax)}$	$\underline{\forall x(QCx\supset QBx)}$
$\exists x(QCx\,\&\,Q{\sim}Ax)$	$\exists x(QCx\,\&\,Q{\sim}Bx)$	$\exists x(QBx\,\&\,QAx)$
		Bocardo QQQ
		$\exists x(QCx\,\&\,Q{\sim}Ax)$
		$\underline{\forall x(QCx\supset QBx)}$
		$\exists x(QBx\,\&\,Q{\sim}Ax)$
		Ferison QQQ
		$\forall x(QCx\supset Q{\sim}Ax)$
		$\underline{\exists x(QCx\,\&\,QBx)}$
		$\exists x(QBx\,\&\,Q{\sim}Ax)$

In Table 17 the syllogisms in the first and third figures are all substitution instances of corresponding non-modal syllogisms.[2] The second figure syllogisms are not. In fact, the second figure comes out invalid in Table 17. Look for example at what the substitution would need to be in order to get Cesare QQQ from Cesare XXX. We would need ${\sim}QAx$ in place of $Q{\sim}Ax$. But ${\sim}QAx$ and $Q{\sim}Ax$ are *not* equivalent in Aristotle's system. As we

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[2]Remembering that for the negative cases we have to substitute ${\sim}Q{\sim}A$ for ${\sim}A$, and then use double negation to get from ${\sim\sim}Q{\sim}A$ to $Q{\sim}A$, and so on.

shall see $Q{\sim}Ax \neq {\sim}QAx$ because according to Aristotle $Q{\sim}Ax \equiv QAx$.

Aristotle does sometimes have difficulties when privatives and modals combine, and so we should at least consider the different possible interpretations. In Table 17, a negative proposition such as 'No *B* is contingently *A*' has been formalized as

$$\forall x(QBx \supset Q{\sim}Ax).$$

Let's suppose instead that we formalize 'No *B* is contingently *A*' as

$$\forall x(QBx \supset {\sim}QAx).$$

This would give us Table 18.

Table 18
Ampliated Q+Q Syllogisms in Three Figures
(with $\sim QAx$)

First Figure	Second Figure	Third Figure
Barbara QQQ	Cesare QQQ	Darapti QQQ
$\forall x(QBx \supset QAx)$	$\forall x(QBx \supset \sim QAx)$	$\forall x(QCx \supset QAx)$
$\underline{\forall x(QCx \supset QBx)}$	$\underline{\forall x(QCx \supset QAx)}$	$\underline{\forall x(QCx \supset QBx)}$
$\forall x(QCx \supset QAx)$	$\forall x(QCx \supset \sim QBx)$	$\exists x(QBx \& QAx)$
Darii QQQ	Camestres QQQ	Felapton QQQ
$\forall x(QBx \supset QAx)$	$\forall x(QBx \supset QAx)$	$\forall x(QCx \supset \sim QAx)$
$\underline{\exists x(QCx \& QBx)}$	$\underline{\forall x(QCx \supset \sim QAx)}$	$\underline{\forall x(QCx \supset QBx)}$
$\exists x(QCx \& QAx)$	$\forall x(QCx \supset \sim QBx)$	$\exists x(QBx \& \sim QAx)$
Celarent QQQ	Festino QQQ	Datisi QQQ
$\forall x(QBx \supset \sim QAx)$	$\forall x(QBx \supset \sim QAx)$	$\forall x(QCx \supset QAx)$
$\underline{\forall x(QCx \supset QBx)}$	$\underline{\exists x(QCx \& QAx)}$	$\underline{\exists x(QCx \& QBx)}$
$\forall x(QCx \supset \sim QAx)$	$\exists x(QCx \& \sim QBx)$	$\exists x(QBx \& QAx)$
Ferio QQQ	Baroco QQQ	Disamis QQQ
$\forall x(QBx \supset \sim QAx)$	$\forall x(QBx \supset QAx)$	$\exists x(QCx \& QAx)$
$\underline{\exists x(QCx \& QBx)}$	$\underline{\exists x(QCx \& \sim QAx)}$	$\underline{\forall x(QCx \supset QBx)}$
$\exists x(QCx \& \sim QAx)$	$\exists x(QCx \& \sim QBx)$	$\exists x(QBx \& QAx)$
		Bocardo QQQ
		$\exists x(QCx \& \sim QAx)$
		$\underline{\forall x(QCx \supset QBx)}$
		$\exists x(QBx \& \sim QAx)$
		Ferison QQQ
		$\forall x(QCx \supset \sim QAx)$
		$\underline{\exists x(QCx \& QBx)}$
		$\exists x(QBx \& \sim QAx)$

If we use Table 18, then *all* of the QQQ syllogisms would be instances of valid non-modals – including the second figure. But it is better to translate 'No *B* is contingently *A*' as $\forall x(QBx \supset Q{\sim}Ax)$, *not* as $\forall x(QBx \supset \sim QAx)$. This is in line with the remarks made in Chapter 5, which suggest that in his privative modal premises Aristotle always puts negation inside the modal operator. There is in fact a large amount of textual evidence for this in Aristotle's discussion of privative *Q* propositions – not least being that when he comes to the second figure in *An.Pr.* A17 he claims that there are *no* second figure Q+Qs. This is something we shall look at closely in Chapter 16 below. But it makes one thing especially clear in our present discussion. It makes Table 17 rather than Table 18 a better representation of the Q+Qs. In what follows I will highlight the evidence against the Table 18 constructions as it arises through this and subsequent chapters.

One special point should be noted about the data in Table 17. Syllogisms about contingency that take these forms are traditionally called *ampliated* syllogisms. An ampliated premise is just a regular *Q*-premise whose *subject* term is qualified by a modal. Consider an example: 'every *B* is contingently *A*.' The regular *unampliated* reading is

$$\forall x(Bx \supset QAx)$$

or its equivalent,

$$\forall x(Bx \supset Q{\sim}Ax).$$

In both of these the only term that is modally qualified is the predicate term. The ampliated reading puts a separate modal on the subject term as well. As for example:

$$\forall x(QBx \supset QAx)$$
$$\forall x(QBx \supset Q{\sim}Ax)$$

Why do we need ampliation at all? Paul Thom explains succinctly:

> We need a separate class of ampliated contingency-forms because (*An.Pr.* A13), 32b23–32 notes a syntactic ambiguity in the expression '*kath' hou to B, to endechesthai*'. It appears from that text that contingency-propositions may be either ampliated or unampliated. (Thom 1996, p. 9)

Aristotle is less succinct:

> Now, the expression 'it is possible for this to belong to that' may be understood in two ways: it may mean either 'to that to which this belongs' or 'to that to which it is possible for this to belong.' For 'of what B is true, it is possible that A' signifies one or the other of the following: (1) 'of what B is said' or (2) 'of what it is possible for B to be said'. But 'it is possible that A <is said> of what B is' is no different from 'it is possible for A to belong to every B.' Therefore, it is evident that 'it is possible for A to belong to every B' might have two meanings. (32b25–32)

Following Aristotle's instructions, the steps marked (1) and (2) in this passage would give us the following, using *P* to stand for the equivocal usage of 'possible', i.e., for *Q* or *M*:

(1) $\forall x(Bx \supset QAx)$
(2) $\forall x(PBx \supset QAx)$

(1) illustrates an *unampliated* modal structure. (2) illustrates an ampliated structure.[3] It is important to appreciate that ampliation is introduced as a distinction between two ways in which something may be said to be possible – in other words ampliation can only occur when the predicate is a *Q* or an *M* predicate. The point is important because our LPC representations allow wff like

(3) $\forall x(QBx \supset Ax)$
(4) $\forall x(QBx \supset LAx)$

Neither (3) nor (4) is a way of analysing Aristotle's 'it is possible for this to belong to that'. Since this fact will be important in later chapters I shall state the *Restricted Ampliation Principle*:

RAP No proposition in the premises or conclusion of any (valid) syllogism can contain an ampliated subject with an assertoric or an apodeictic predicate.[4]

As stated RAP may seem unmotivated, but that is not in fact so. Think of it this way.

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[3]The examples in the text are of 'ampliation to the contingent'. We might also 'ampliate to the possible' (i.e., to an *M*) and give $\forall x(MBx \supset QAx)$ and $\forall x(MBx \supset Q{\sim}Ax)$.Thom investigates both ampliation to the possible and to the contingent. (See Thom 1996, p. 212ff.) Ampliating to an *M* works much the same as to a *Q*:

$\forall x(MBx \supset QAx)$
$\forall x(MCx \supset QBx)$
$\forall x(MCx \supset QAx)$

Aristotle might have such ampliation in mind, since from the definition of Q-contingency $QBx \supset MBx$. Nowhere in his discussion of ampliation does Aristotle tell us *which* kind of ampliation he has in mind. He merely notices that possible premises may be understood as ampliated or not, and he leaves it at that. Malink (2006, p. 115) offers an interpretation in which there is no meaningful distinction between ampliated and unampliated propositions. In Malink's system *A*: "there is no separate doubly modalized or ampliated possibility such as is drawn upon by many commentators...".

[4]RAP is in direct contrast to Angelleli (1979) and Thom (1996) who distinguish 'qualified' and 'unqualified' assertoric propositions. In modal LPC a qualified assertoric XA-proposition is just our $\forall x(Bx \supset Ax)$. An unqualified assertoric has an ampliated subject term, and so (3) $\forall x(QBx \supset Ax)$ is allowed on the Angelelli-Thom interpretation. Thom refers to *An.Pr.* A15, 34b7—9 as textual support for the unqualified premise. But see below, p. 148, for reasons against this. (See also Footnote 7 on p. 56.)

A possibility proposition (whether M or Q) has a modal operator, and the question is whether that operator applies only to the predicate term, or to both the predicate term and the subject term. In the case of a Q proposition the question is therefore whether the Q is put before the predicate only, as in $\forall x(Bx \supset QAx)$, or before both subject and predicate as in $\forall x(QBx \supset QAx)$. In an assertoric proposition there is no modal operator, and so the question of whether it applies to the predicate only or to the subject and the predicate does not arise. In the case of an apodeictic proposition the analogous question is whether the L applies to the predicate alone or to both the subject and the predicate, and this is a question that was raised in Chapter 6, and exhibited in the difference between Tables 3 and 4 on pp. 67 and 69.

The traditional view is that ampliation is needed to validate all QQQ syllogisms. A look at the logic makes clear why it is needed. Barbara QQQ without ampliated premises is not valid:

Barbara QQQ (Unampliated)

(5) $\forall x(Bx \supset QAx)$

(6) $\underline{\forall x(Cx \supset QBx)}$

(7) $\forall x(Cx \supset QAx)$

Formally, we need a principle that takes us validly from QB to B in order to validate the unampliated Barbara QQQ (5) (6) (7). The situation here is similar to that which arises about each the interpretation of LLL syllogisms and the interpretation of MMM syllogisms. In the case of LLL and MMM syllogisms, we appealed to the substance principle to guarantee that $L\varphi \equiv M\varphi \equiv \varphi$. But the substance principle is subject to a semantic restriction on terms – it only applies to *red* terms. M-possibility is governed by the substance principle, and $M\varphi \supset \varphi$ *fails* if φ is a green term like 'moving'. Unampliated Barbara QQQ would be validated by the addition of a principle $Q\varphi \supset \varphi$. If we had such a principle then we could use it to do the work in the contingent syllogistic that the substance principle does in the syllogisms about necessity and M-possibility. But, if Q were subject to the substance principle $Q\varphi \supset \varphi$ would be trivially valid if φ is red, since $Q\varphi$ would then be always empty – and Aristotle does not allow empty terms. For green terms $Q\varphi \supset \varphi$ is not valid. If φ is green, then $Q\varphi$ includes *both* $\sim L\varphi$ *and* $\sim L\sim\varphi$. But if φ is green, then neither $\sim L\varphi \supset \varphi$ nor $\sim L\sim\varphi \supset \varphi$ is valid. And, in any case there is no reason to allow $Q\varphi \supset \varphi$, since it is not plausible that everything that is contingently φ is *actually* φ. A horse may be a contingent mover because it might or might not move, but it does not follow from this that the horse is *actually moving*. Since something can be contingently φ without being actually φ, we do not have $Q\varphi \supset \varphi$, and without $Q\varphi \supset \varphi$ to put to work in the unampliated syllogism Barbara QQQ, then even the first figure syllogism fails, and the Barbara above – i.e., (5) (6) (7) – comes out invalid.

Ampliating the premises avoids this problem. Here is Barbara QQQ with premises ampliated to the contingent:

$$\begin{array}{l} \forall x(QBx \supset QAx) \\ \underline{\forall x(QCx \supset QBx)} \\ \forall x(QCx \supset QAx) \end{array}$$

That is valid, trivially. It is just non-modal Barbara with terms QA, QB, and QC. Robin Smith, in the notes to his translation, looks for some explanation of the ‘rather surprising’ new twist that ampliation brings to the syllogistic:

> One consideration may be the need to have a single middle term in a deduction. If we regard ‘it is possible that A to B’ as attributing the predicate ‘possibly A’ to B, and likewise ‘it is possible that B to C’ as attributing the predicate ‘possibly B’ to C, then these two premises appear to contain four terms: ‘possibly A,’ ‘B,’ ‘possibly B,’ and ‘C.’ On the interpretation which Aristotle advocates, there are only three terms, but they are ‘possibly A,’ ‘possibly B,’ and ‘possibly C.’ (Smith 1989, p. 128)

Recall how in the case of the pure apodeictic LLL Red Barbara the terms are in effect ‘necessarily A’, ‘necessarily B’ and ‘necessarily C’. It makes sense to suppose that when Aristotle comes to discuss pure contingent QQQ syllogisms he adopts the same pattern, now, with ‘contingently A’, ‘contingently B’, and ‘contingently C’.

We can at least try to lighten the burden of ampliation by minimizing its use. Again, look at Barbara QQQ without ampliation:

$$\begin{array}{l} \forall x(Bx \supset QAx) \\ \underline{\forall x(Cx \supset QBx)} \\ \forall x(Cx \supset QAx) \end{array}$$

Barbara QQQ would be formally valid if even only the first premise were ampliated to $\forall x(QBx \supset QAx)$:

$$\begin{array}{l} \forall x(QBx \supset QAx) \\ \underline{\forall x(Cx \supset QBx)} \\ \forall x(Cx \supset QAx) \end{array}$$

When Barbara QQQ is taken this way, not all terms are in the scope of a Q. The C term is not qualified by a modal. Aristotle tells us that ampliated and unampliated Q-propositions might have different meanings, 32b25–32. The difference would seem, therefore, to be due to the subject term. The unampliated proposition 'all moving things are contingently white' would seem to be about all *actually* moving things. The ampliated proposition

(8) All contingently moving things are contingently white

is a stronger statement since it is about all the things that are actually moving (if there are any) *and* all the things that *could be* moving (but maybe aren't). Perhaps the way to try to understand an ampliated premise such as 'all contingently moving things are contingently white' is as saying that everything that is a contingent mover (e.g., every man, every horse, etc.) is also contingently white.[5]

In addition to ampliation, the present chapter also explores another main theme involving Aristotle's treatment of syllogisms about contingency. It is a consequence of the definition of Q-contingency that anything that is contingently φ is also contingently not φ. For example, anything that contingently moves could equally well not move, so it is a contingent mover and a contingent non-mover. Similarly, anything that is contingently white could also be not white, and so is contingently not white. Aristotle notices that his account of Q-contingency allows as much. He describes this feature as a kind of conversion (32a30). Ross gives it the name 'complementary conversion' in order to distinguish it from ordinary conversion principles (Ross 1957, p. 298). Much of the focus of this and the next several chapters concerns what Aristotle has to say about this new kind of conversion (CC) and how he puts it to use in his proofs. In the present chapter it will help to give a precise form for the complementary conversions that Aristotle describes. I will label the complementary conversion of an *A* proposition (CC_A) and complementary conversion of an *I* proposition (CC_I):

$$(CC_A) \qquad \forall x(Bx \supset QAx) \equiv \forall x(Bx \supset Q{\sim}Ax)$$
$$(CC_I) \qquad \exists x(Bx \,\&\, QAx) \equiv \exists x(Bx \,\&\, Q{\sim}Ax).$$

We would expect that formally these are equivalences, and I give them as equivalences here. Complementary conversion licenses the move from QA to $Q{\sim}A$, and from $Q{\sim}A$ to

[5] Malink (2006) sets a more restricted project than the textual study attempted here – for he omits the method of ampliation and the proofs that require it. See especially Malink's discussion on pp. 115, 122. Malink offers a theorem according to which $\forall z(\Upsilon bz \supset \Pi az) \vdash_A \forall z(\Pi bz \supset \Pi az)$, where Υab is a 'plain accidental predication' with b as the subject term, and where $\Pi ab =_{df} \neg(\Sigma a \& \Sigma b) \,\&\, \neg \mathrm{E}ab \,\&\, \neg \mathrm{E}ba \,\&\, ((\Sigma a \vee \Sigma b) \supset \exists z(\Upsilon az \& \Upsilon bz))$, where $\mathrm{E}ab$ is 'substantial essential predication' and where $\Sigma a =_{df} \exists z \mathrm{E}za$. In other words, in Malink's system *A*, there is no need to formally distinguish between an ampliated and an unampliated proposition. Aristotle certainly makes the distinction, though, as Malink rightly points out (p. 115), Aristotle 'fails to make it clear where he uses which'. In the present work I use ampliation only when needed, preserving the simple structure of the syllogistic proofs.

QA, but as we shall see later, *QA* and *Q~A,* according to Aristotle, do not have different meanings; they are equivalent ways of saying the same thing. In Part II, where our focus was the apodeictic syllogisms, one of the distinguishing features of the first figure syllogisms is that they require no conversions of any kind. In the apodeictic syllogistic only second and third figure syllogisms require conversion. In the problematic Q-syllogistic some of the *first* figure QQQ syllogisms require complementary conversion.

All of the QQQ syllogisms in the first figure require ampliation, though Aristotle does not say so explicitly. And he does not tell us *how much* ampliation he envisages. In the first figure QQQ syllogisms I have ampliated only the first premise. Throughout the tables and discussion in this book I use as little ampliation as the logic will allow. This helps make clear where exactly ampliation is required, and makes clear where it is not needed. Aristotle discusses the first figure QQQ syllogisms in *An. Pr.* A14. These are listed in Table 19.

Table 19
QQQ Syllogisms in the First Figure (A14)

Barbara QQQ (32b38–33a1)
$\forall x(QBx \supset QAx)$
$\underline{\forall x(Cx \supset QBx)}$
$\forall x(Cx \supset QAx)$

Celarent QQQ (33a1–5)
$\forall x(QBx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$
$\forall x(Cx \supset Q{\sim}Ax)$

Darii QQQ (33a23–25)
$\forall x(QBx \supset QAx)$
$\underline{\exists x(Cx \;\&\; QBx)}$
$\exists x(Cx \;\&\; QAx)$

Ferio QQQ (33a25–27)
$\forall x(QBx \supset Q{\sim}Ax)$
$\underline{\exists x(Cx \;\&\; QBx)}$
$\exists x(Cx \;\&\; Q{\sim}Ax)$

CC-Barbara QQQ (33a5–12)
$\forall x(QBx \supset QAx)$
$\underline{\forall x(Cx \supset Q{\sim}Bx)}$
$\forall x(Cx \supset QAx)$

CC-Celarent QQQ (33a12–17)
$\forall x(QBx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset Q{\sim}Bx)}$
$\forall x(Cx \supset QAx)$

CC-Darii QQQ (33a27–34)
$\forall x(QBx \supset QAx)$
$\underline{\exists x(Cx \;\&\; Q{\sim}Bx)}$
$\exists x(Cx \;\&\; QAx)$

The syllogisms in the left-hand column are all of a kind. Their validity depends on the assertoric first figure syllogisms with simple substitution of modal for non-modal terms, *QA* for *A*, *Q~A* for *~A*. Aristotle usually describes syllogisms in the first figure as complete (*teleios*) and obvious (*phaneros*) – that is, they need nothing further to illustrate their validity. If so then we could call a first figure modal syllogism 'complete' if it is a substitution instance of a first figure XXX syllogism. Ampliation enables the first figure QQQs in the left hand column to be shown to be complete.[6]

The syllogisms in the right-hand column all depend additionally on the fact that for Aristotle what is contingently φ is also contingently not-φ, and vice versa. As Aristotle explains the syllogism at 33a5–12, which I have called CC-Barbara QQQ – since it is Barbara QQQ with one of the premises converted by CC – what is needed is conversion 'in accordance with possibility,' and it is clear that at this stage in his discussion, 'possibility' means what we now call 'contingency', and conversion in accordance with possibility means complementary conversion. The most serious problem in *An.Pr.* A14 concerns Aristotle's treatment of negatives – for in this part of the syllogistic the fact that Aristotle lacks any proper notion of scope leaves him unable to attain any satisfying clarity and precision where modals and privatives combine. This becomes an especially serious problem because his notion of complementary conversion blurs the distinction between affirmation and denial. (Aristotle's discussion of syllogisms like these provides a further indication that Table 18 is not what he has in mind.)

Aristotle gives proofs for each of the syllogisms on the right-hand side of Table 19:

> When (1) it is possible for *A* to belong to every *B* and (2) possible for *B* to belong to no *C*, then no deduction comes about through the premises taken; but if premise *BC* (2) is converted in accordance with possibility, then the same deduction comes about as previously. For since (2) it is possible for *B* to belong to no *C*, (3) it is also possible for it to belong to every *C* (this was stated earlier). Thus, if *B* to every *C* and *A* to every *B*, the same deduction (4) comes about again.[7] (33a5–12)

(1)	$\forall x(QBx \supset QAx)$	Given (ampliated)
(2)	$\forall x(Cx \supset Q{\sim}Bx)$	Given
(3)	$\forall x(Cx \supset QBx)$	CC (2)
(4)	$\forall x(Cx \supset QAx)$	Barbara (3)(1)

The conversion needed to get from step (2) to step (3) is (CC_A): $\forall x(Cx \supset Q{\sim}Bx) \supset$

[6]Later in A15 Aristotle considers 'incomplete' first figure syllogisms. (See p. 148 below.)

[7]Note that this final summing up is described in purely non-modal language: 'Thus, if *B* to every *C* and *A* to every *B*, the same deduction comes about again.' He is using a simple non-modal Barbara to justify his proof. This means of course that QAx is substituted for Ax, and QBx is substituted for Bx.

$\forall x(Cx \supset QBx)$.

The proof of the syllogism at 33a12–17 requires complementary conversion of each of its two premises:

> It is also similar if a negation is added along with ‘is possible’ in both of the premises (I mean, for instance, if it is possible for A to belong to none of the Bs and B to none of the Cs). For no deduction comes about through the premises taken, but if they are converted it will again be the same deduction as before. (33a12–17)

(1)	$\forall x(QBx \supset Q{\sim}Ax)$	Given (ampliated)
(2)	$\forall x(Cx \supset Q{\sim}Bx)$	Given
(3)	$\forall x(QBx \supset QAx)$	CC (1)
(4)	$\forall x(Cx \supset QBx)$	CC (2)
(5)	$\forall x(Cx \supset QAx)$	Barbara (4)(3)

Aristotle’s point seems to be that without complementary conversion we would not be able to syllogize from (1) and (2), because at the level of surface structure, at least, (1) and (2) are both privatives, and there are no non-modal syllogisms from two privative premises. In order to syllogize we bring out their affirmative equivalences, (3) and (4), and from these a simple non-modal Barbara follows, again with QBx for Bx, and QAx for Ax.

The syllogism at 33a27–34 requires only complementary conversion of the minor CB premise:

> But if the particular is taken as privative, the universal premise as affirmative, and they are similarly related in position (i.e., (1) it is possible for A to belong to every B and (2) it is possible for B not to belong to some C), then an evident deduction does not come about through the premises taken; but when the particular premise (2) is converted (3) and it is put that it is possible for B to belong to some C, then there will also be the same conclusion (4) as before, just as in the initial cases. (33a27–34)

(1)	$\forall x(QBx \supset QAx)$	Given (ampliated)
(2)	$\exists x(Cx \;\&\; Q{\sim}Bx)$	Given
(3)	$\exists x(Cx \;\&\; QBx)$	CC (2)
(4)	$\exists x(Cx \;\&\; QAx)$	Darii (1)(3)

These are all the first figure QQQs. This completes the discussion of the valid first figure QQQ syllogisms of *An.Pr.* A14. Notice that because of Aristotle's treatment of negative *Q* propositions as *Q~* propositions *all* first figure Q+Q schemes with universal premises are versions of Barbara, and *all* first figure Q+Q schemas with the BA premise as universal and the CB premise as particular are instances of Darii. This leaves only Q+Q schemas in which the major is particular and the minor is universal to be considered. And, Aristotle tells us that there is no deduction in such cases.

> If the premise in relation to the major extreme [the BA premise] is taken as particular and the premise in relation to the minor [the CB premise] as universal, then whether both are put as affirmatives, or both as privatives, or they are not put as the same in form, or both as indeterminates or particulars, there will not be a deduction in any way. (33a34–38)

This cryptic passage needs to be unpacked. What Aristotle claims here is that for each of the following first figure premise combinations, there is no conclusion:

(1)	Some *B* is *QA*	(I)
	All *C* are *QB*	(A)
(2)	Some *B* is *Q~A*	(O)
	All *C* are *Q~B*	(E)
(3)	Some *B* is *QA*	(I)
	All *C* are *Q~B*	(E)
(4)	Some *B* is *Q~A*	(O)
	All *C* are *QB*	(A)

In this expanded form, (1)–(4) seem to present a large amount of new data, but as the original passage, 33a34–38, suggests Aristotle wants to handle it altogether in one swoop. He is able to do so because of the power of his complementary conversions. All the premises in (1)–(4) are Q-premises. And all Q-premises admit complementary conversion. So we can use complementary conversion to convert a QI premise into a QO premise, a QO premise into a QI, a QA premise into a QE premise, and a QE premise into a QA premise. None of these changes brings a change in meaning. Perhaps the easiest way to think of this is that Aristotle's complementary conversions allow the addition as well as the removal of a negation sign in the scope of a Q-contingency operator – all of which occurs without any change in meaning. The effect here is that the premise pairs (2), (3), and (4), listed above, can be conveniently reduced to the first premise pair (1) which combines a simple affirmative QI premise and a QA premise. In fact, (1)–(4) are all *equivalent*. Anything we prove about the QI+QA combination will hold also for the others. This effect of complementary conversion is made explicit in the 33a34–38 passage, where Aristotle is careful to include all these possible combinations. But of course Aristotle wants to show that these are invalid and do not produce syllogisms. In order to show this, because of the complementary equivalences, all he needs to establish is that the first QI+QA combination is invalid – that is, he wants to show that the premise pair (1) does not yield a syllogism.

So let's consider (1) above. Aristotle's proofs of validity for syllogisms about Q-contingency all appeal to syllogisms established earlier in *Prior Analytics*. We have already seen that in the case of the L and M syllogistic, there can be no modal syllogism except where there is an XXX syllogism. In the non-modal case corresponding to (1) there is no syllogism from the X-premises

Some *B* is *A*
Every *C* is *B*.

For we may put all the *C*s into that part of the *B*s which are not *A*, and in (1) we may put the *C*s into that part of the *B*s which are not *QA* (33a38–b3). To get a counter-example we need to find an *A* such that some of the *B*s are *QA*s but not all of them are *QA*s. That is to say, *A* has to be a predicate which is contingent of some of the *B*s, but non-contingent of others. Ordinary green terms like 'moving' are not happy choices, since they seem to be contingent of everything. As we shall see Aristotle himself seems to have trouble selecting appropriate terms in this part of the syllogistic, presumably for that reason. Aristotle offers two sets of terms to invalidate the first figure QQQ schemas.

> Terms in common for all cases, for belonging of necessity, are animal, white, man; for not being possible, animal white, coat. (33b7–8)

Then the first set of terms – animal, white, man – is supposed to give true premises but a false conclusion:

Some white is contingently an animal	T
All men are contingently white	T
Some men are contingently animals	F

The second set of terms – animal, white, coat – gives:

Some white is contingently an animal	T
All coats are contingently white	T
Some coats are contingently animals	F

The problem in each of these cases is of course the first premise, since animal is a red term. But although animal is a red term perhaps Aristotle has it in mind that there are some white things (seeds, perhaps) which could, though need not, become animals. This causes a tension with the simple-minded red/green distinction which served us so well in Part II. In the next chapter, we shall see that Aristotle may nevertheless be thinking of something like this. To see how he handles the kinds of problems this raises we look in Chapter 12 at a new method that he introduces in his discussion of mixed syllogisms in which only one premise is about possibility.

Chapter 12
Realizing Possibilities

We have noted in Part II how helpful the red/green distinction from Chapter 4 proves to be in the apodeictic syllogistic. But, as we saw in Chapter 11, choosing red terms seems to make Q-propositions empty and so would make QQQs trivial in a very undesirable way. It is undesirable because it leaves us always trying to syllogize from false premises, and whatever modern logic has to say about that, it does not suit Aristotle's scientific method. His way forward out of aporia involves a new method for syllogizing about possibility.

It is a method that has no precedent in any part of our study so far – it is a wholly new technique and one that is unique to the study of possibility. I call it 'realizing' or sometimes 'actualizing'. In syllogisms about possibility Aristotle routinely and very carefully explains to us that what is possible *can* be actual – and nothing impossible results from supposing that a possibility is realized. If it is possible then it can be actual. Any possibility can be *realized*. So even though something *false* may result from supposing a possibility is actual, *no impossibility* will result from supposing a possibility is actual. This is at the heart of Aristotle's new approach to scientific deductions about what can be otherwise. So let's return to the text and consider more explicitly the passages from *An.Pr.* A13 in which the various notions of possibility are introduced:

> I use the expressions 'to be possible' and 'what is possible' in application to something if it is not necessary but nothing impossible will result if it is put as being the case (for it is only equivocally that we say that what is necessary is possible). (32a18–22)

> Having made these distinctions, let us next explain that 'to be possible' has two meanings. (32b4–5)

> One meaning is what happens for the most part and falls short of necessity, as for a man to turn gray or grow or shrink, or in general what is natural to belong (for this does not have continuous necessity because a man does not always exist; however, when there is a man, it is either of necessity or for the most part). (32b5–10)

> The other meaning is the indefinite, which is capable of being thus as well as not thus, as, for instance, for an animal to walk or for there to be an earthquake while one is walking, or, in general, what comes about by chance (for it is no more natural for this to happen in one way than in the opposite). (32b10–13)

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
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> Now, each of these kinds of possible premises also converts in relation to its opposite premise, but not, however, in the same way. A premise concerning what is natural converts because it does not belong of necessity (for it is in this way that it is possible for a man not to turn gray), whereas a premise concerning what is indefinite converts because it is no more this way than that. (32b13–18)

> Science and demonstrative deduction are not possible concerning indefinite things because the middle term is disorderly; they are possible concerning what is natural, however, and arguments and inquiries would likely be about what is possible in this sense. A deduction might possibly arise about the former, but it is, at any rate, not usually an object of inquiry. (32b18–22)

32a18–22 is just the definition of Q-contingency: $Q\varphi =_{\mathrm{df}} {\sim}L\varphi \,\&\, {\sim}L{\sim}\varphi$. But at 32b5–10, Aristotle suddenly introduces an entirely new distinction between two different meanings of Q-contingency. According to this passage, one meaning is

> what happens for the most part,
> what is natural, but
> what falls short of necessity.

The second meaning is described in 32b10–13, where possible means

> what is indefinite,
> capable of being thus as well as not thus,
> or, in general, what comes about by chance.

Aristotle's examples of possibility in the second passage are straightforward, paradigm examples of Q-contingency. The first of these involves a green, accidental predicate term. It is possible for an animal to walk. Of course walking belongs to an animal only accidentally. So 'some animal possibly walks' is true, but 'some animal walks of necessity (or not of necessity)' will always be false. This is a standard example that illustrates how a green, accidental term fits in the scope of a Q but not in the scope of an unnegated L.

Aristotle explains at 32b13–14 that a premise about such possibility converts. The conversion here is complementary conversion (CC). So for example, the conversion goes from

(1) Some animal possibly walks

to

(2) Some animal possibly does not walk.

Both (1) and (2) are true propositions about possibility. At 32b19 Aristotle explores a difficulty presented by premises like (1) and (2). The predicate term 'walk', he tells us, is 'disorderly' (ἄτακτον). Aristotle seems to mean that having the capacity to walk is the same as having the capacity not to walk. Let 'walk' be *A*. Anything that is *QA* is also *Q~A*, and vice versa. (Cf *Metaphysics* E2.)

The second example of possibility is more complex. It is possible for there to be an earthquake while one is walking. (32b12) But clearly this possibility, too, is Q-contingency. It is possible for there to be an earthquake and the possibility is a matter of chance, no more this way than that. In fact it is just as possible for there to be an earthquake while one is walking home as it is for there not to be one. The passages from Aristotle that I have quoted have long been recognised as justifying the need to distinguish between possibility as 'not necessarily not' (one-way, or *M*-possibility) and possibility as contingency (two-way possibility). But just as important is to see that the consideration of potentiality suggests that contingency itself may have various senses, and may play different roles in the syllogistic.

Let's consider how well these various examples of possibility sit with the definition of *Q*. It is possible for a man to turn gray. It is possible for a man to grow. It is possible for a man to shrink. All of these are possible because they are natural. In fact they are natural processes or happenings. Turning gray even happens for the most part, or happens to most men. But of course none of these things happens necessarily. Not all men turn gray and not all men shrink. But all men have the natural capacity to do so. This suggests that the possibility described in 32b5–10 is about some of the kinds of *potentialities* that Aristotle discusses in *Metaphysics*. And so Aristotelian notions of potency, potentiality, and natural capacities are brought to bear in the contingent syllogistic.

Aristotle clearly links what is potential with contingency because what is potential might happen or might not happen. The examples he gives at 32b6–7 all involve what we are calling green predicate terms – graying, growing, shrinking. This makes the changes and processes under discussion all examples of accidental change. It may be in a thing's nature to grow or gray or shrink, but growing, graying or shrinking do not involve any change in a thing's nature. It is only accidental change because the underlying substance remains the same. Aristotle tells us at 32b8 that the natural capacities he mentions 'do not have continuous necessity'. A man has the potential to turn gray, and this potential is natural to him. In a sense *having the capacity* to turn gray is part of his nature and so part of his essence. However, 'a man does not always exist' so the potential does not always exist. But 'when there is a man, [when the potential to turn gray is actualized] it is either *of necessity* or for the most part.' The italics are mine. The necessity here is awkward and needs to be explained, but it is plainly explicable. Aristotle is trying to make clear that such a capacity has a certain metaphysical naturalness. It is a capacity that is tied to the nature or essence of a thing. Not everything has the capacity to turn gray – a tree, for example, is not the kind of thing that ever turns gray. A tree does not have the potential to turn gray. But a man does have the potential, and indeed most men do turn gray. In this sense, a natural capacity is 'of necessity or for the most part'. But the realization or actualization of such a capacity is not necessary – 'it falls short of necessity' because nothing about the way the world is guarantees that any particular capacity or potentiality is ever realized.

This would explain how natural capacities can be involved in accidental change about what is Q-contingent. But Aristotle has two kinds of change – accidental and substantial. What about substantial change then? Aristotle does not include examples of substantial change in these discussions, but if pressed to, what might he have to say? Consider an example from *Metaphysics* Θ 8. Aristotle is explaining that 'actuality is prior to potentiality':

> I mean that the matter and the seed and that which is capable of seeing, which are potentially a man and corn and seeing, but not yet actually so, are prior in time to this particular man who now exists actually, and to the corn and to the seeing subject... (1049b19–21)

An actual man is prior to any potential man. Actual corn is prior to any seed. A certain bit of matter might turn into a man, a seed might turn into corn. In this sense these substantial changes involve potentialities. The seed has within it a natural potency which for the most part causes seed to change and to grow into corn. Possibilities of this sort should be about Q-contingency. There isn't the certainty about them that statements about necessity require – the seed might not grow into corn, so here in this case, too, the potential falls short of necessity. These potencies seem to be like the possibilities described at 32b5. But these kinds of natural potentialities cause problems with some of the principles that ground the apodeictic syllogistic. Most especially, if a seed is contingently corn, or matter is contingently a man, then all of the sudden Q-contingency does not sit well with the substance principle from p. 3.

What we need to consider is what happens when we take as our starting point a *true* premise about possibility such as:

> 'Some seed is potentially corn', or
> 'Some acorn is potentially an oak tree.'

Recall that the substance principle has two parts. The first part is about affirmatives and tells us that where a term φ is a red term, then $L\varphi \equiv \varphi$. The second part is about privatives and tells us that where φ is an essential term, then $L{\sim}\varphi \equiv {\sim}\varphi$. It will help to use an example to see what these two parts commit Aristotle to, so let 'oak tree' be φ.

> The affirmative part of the substance principle (SP) gives us this:
> Since 'oak tree' names a thing with an essential nature, anywhere we have 'oak tree' we can substitute 'necessary oak tree' [$\varphi \supset L\varphi$], and vice versa [$L\varphi \supset \varphi$]. So where φ is an essential term, $L\varphi \equiv \varphi$.
>
> The privative part of the substance principle, Neg(SP), gives us this:
> Since 'oak tree' names a thing with an essential nature, anywhere we have something that is 'not an oak tree (a non-oak tree)' we have something whose nature excludes its being an oak tree, and so we have something which is 'necessarily not an oak tree' [${\sim}\varphi \supset L{\sim}\varphi$]. And, anything which is necessarily not an oak tree is not an actual oak tree [$L{\sim}\varphi \supset {\sim}\varphi$]. So where φ is an essential term, $L{\sim}\varphi \equiv {\sim}\varphi$.

Now an acorn is not an oak tree. So by Neg(SP) an acorn is *by necessity* not an oak tree. That is, an acorn is an *L~oak tree*. But an acorn is potentially an oak tree. That is, an acorn is a *Q oak tree*. But then an acorn is *not* an *L~oak tree* – i.e., an acorn is a *~L~oak tree*. But this is a contradiction. The affirmative version of the substance principle expresses the plausible principle that *once* something is an oak tree it is one by necessity. But the negative version of the substance principle stops something that is *not already* an oak tree from becoming one. And that seems wrong.

It is important to see the nature of what is going on here because it has far-reaching consequences in the modal syllogistic. If an acorn is a potential oak tree, then an acorn is contingently an oak tree. It is neither a necessary oak tree nor necessarily not an oak tree. It might become an oak. It is in its nature to become one, but that nature might be frustrated in a variety of ways, and so there can be no guarantee that the potential is ever realized. In other words, it is not necessary nor is it impossible for the acorn to become an oak. This gives us a true *Q* proposition:

(3) All acorns are *Q* oak trees

But the potential does not work in the other direction. On the assumption that an oak tree cannot become an acorn – because if it did it would cease to be an oak – we have a true L proposition:

(4) All oak trees are *L~* acorns.

(4) is a true LE-proposition. So, since an LE-proposition *does* convert, we should also

have (5):

(5) All acorns are *L*~ oak trees.

According to LE-conversion (4) should entail (5), but consider whether that can be right. (4) should count as true since anything that is an oak tree is not an acorn even if it was *once* an acorn. And the privative part of the substance principle would seem to guarantee that anything that is not an acorn is by necessity not an acorn. But the result of LE-conversion of (4) gives (5), and (5) *must* be false because (3) is true. (3) tells us that all acorns *are possible* oak trees; (5) tells us that all acorns *are not possible* oak trees. Once an acorn's potential to become an oak tree is actualized, then there is no longer an acorn. What we have is a tension between, on the one hand, what SP and Neg(SP) require, and, on the other hand, what is involved in realizing a natural potency. Realizing a natural potency describes a substantial change in a subject and so it would seem to involve red terms. But red terms are governed by SP and Neg(SP) in the syllogistic. We shall see in later chapters that this tension appears not entirely clear to Aristotle, and makes for some uncertainty in places about what precisely he thinks is going on. McCall (1963, p. 75) wonders whether Aristotle might in fact have in mind a notion of possibility which is primitive and which cannot be defined in terms of *L*s and *M*s:

> ...there would seem to be a good case for giving up the attempt to define Aristotle's concept of contingency in terms of other notions, and instead to try to reconstruct Aristotle's system of contingent syllogisms using a logical operator *Q* that is primitive.

McCall may be on the right track, provided that at least one sense of *Q* preserves the incompatibility of *L* and *L*~ with *Q*, since Aristotle routinely relies on such contradictions in his proofs and counter-examples.

One way to reconcile SP and Neg(SP) with natural potencies would be to say that they must hold only at a moment. That is, *when* the acorn is still an acorn, it *is* by necessity not an oak. But it could eventually become one. The example above shows how an account of actualizing natural potencies requires reference to time. What is an acorn at time t_1 has at t_1 the potential to become an oak tree at some later time t_2. While there is an oak tree, from time t_2 until some t_n when the oak tree goes out of existence, then the oak tree is *by necessity* an oak tree. Once a natural potential is realized – as when an acorn becomes an oak – there is no going back. Paraphrasing Aristotle's discussion in *An.Pr.* 32b8–10, when there is an oak tree it is of necessity. Or, perhaps more naturally, once there is an actual oak it is a necessary oak. At these different times different modal qualifiers hold.

t_1 (acorn, ~oak tree, *Q*-oak tree, (~*L* oak tree & ~*L*~ oak tree))
t_2 (~acorn, oak tree, *L*-oak tree, *M*-oak tree)
.
.
.
t_n (~oak tree)

Considerations about time and modality are at play in Aristotle's science and they are at play in his logic.[1] A possibility does not have to happen, because it falls short of necessity, but when it does happen it happens in time.

We find Aristotle making much the same point in another work, *De Caelo*. Consider the following passage from *De Caelo* I.12:

> A man has, it is true, the capacity at once of sitting and of standing, because when he possesses the one he also possesses the other; but it does not follow that he can at once sit and stand, only that at another time he can do the other also. But if a thing has for infinite time more than one capacity, another time is impossible and the times must coincide. Thus if anything which exists for infinite time is destructible, it will have the capacity of not being. Now if it exists for infinite time let this capacity be actualized; and it will be in actuality at once existent and non-existent. (*De Caelo* I.12, 281b16–23)

But, as Aristotle goes on to explain, *this* is impossible – nothing can both be and not be at the same time. And so anything that exists for an infinite time cannot be destructible. Suppose Socrates is actually sitting at t_1. He still has (at t_1) the capacity to stand even while he is sitting. That capacity is unrealized at t_1 but might be realized at some other time t_2. So at t_1 possibly Socrates is standing, even though he is actually sitting. That is:

(6) [(Possibly not-sitting) at t_1] & [(sitting) at t_1]

But in the *De Caelo* passage Aristotle is worrying about a thing's having a capacity for an infinite time – 'if a thing has for infinite time more than one capacity, another time is impossible and the times must coincide.' So 'sitting' in (6) needs to be upgraded to 'always sitting' to get (7):

[1] In the language of modern modal logic we might say that for Aristotle realizing a possibility takes place sequentially but in the same world.

(7) [(Possibly not-sitting) at t_1] & [always sitting]

Aristotle appears to take (7) as false.

Commentators have noticed this and have offered a number of interpretations to explain what is going on here. These interpretations fall into two categories. Either Aristotle has made a bad mistake (one which, presumably, he would retract if anyone were to point it out to him), or Aristotle is here in *De Caelo* reasoning according to certain principles which correctly entail the falsity of (7). Jaakko Hintikka and Sarah Waterlow argue for the latter. Waterlow (1982) attributes to Aristotle a notion of 'relative temporalized possibility' which she finds at work here. Hintikka (1973) finds what he calls the principle of plenitude at work here. According to the principle of plenitude, anything that is possible is actual at some time. If Aristotle believes all possibilities will be realized at some time, then (7) really is impossible. This is because according to plenitude at some time t_2 Socrates will not sit, and yet he always sits. Plenitude takes us from (7) to (8):

(8) [Not sitting at t_2] & [always sitting].

But (8) is impossible. The problem with this answer has been noted by others: this cannot be a *proof* of plenitude, because it *assumes* plenitude.

C. J. F. Williams and Lindsay Judson accuse Aristotle of varying degrees of logical errors. Williams (1965) suggests that Aristotle's mistake is to read (6) as though it were (9):

(6) [(Possibly not-sitting) at t_1] & [(sitting) at t_1]
(9) Possibly (not-sitting at t_2 & sitting at t_1)

This leads him to read (7) as though it were (10):

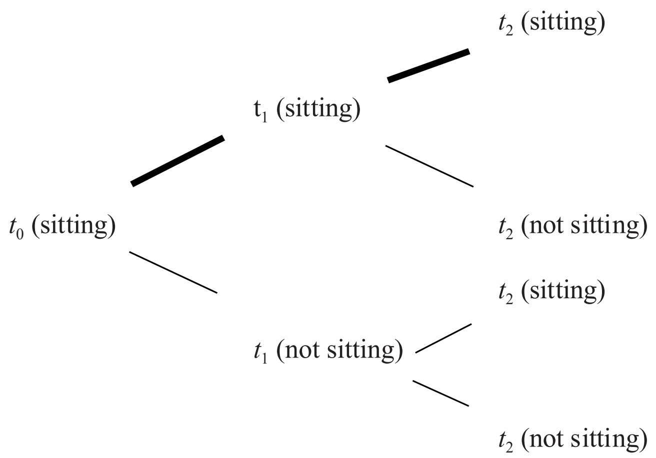
(7) [(Possibly not-sitting) at t_1] & [always sitting]
(10) Possibly (not-sitting at t_2 & always sitting)

(9) could be true, but (10) could not be.[2]

If you think of time as a line that unrolls, then you certainly might say that anything possible will happen at some point *on that line*. And you might explain future

[2]Patterson (1995) sees a similar problem with Aristotle's treatment of some modal principles. Patterson singles out a mistake about arguing from [(possibly p) & (possibly q)] to [possibly (p&q)]. This is very close to what is going on in our (9), however, note that in (9) Aristotle allows that p and q can be realized at different times. The addition of the temporal qualifications helps make clear exactly where the problem arises – Aristotle only gets into trouble when one capacity is realized at all times, or *haplos*.

possibilities by saying that there are 'branching futures' – that is, there are various ways the future might happen, various possible time lines into the future, only one of which will be realized. We then say that you have a *capacity* to do something at a time if you do it in a *possible future* of that time. On the branching futures model, Socrates could be sitting at t_1 in one branch and not sitting at t_1 in another branch. But if there is any branch on which he is always sitting then his capacity to stand would not be realized on that branch.



In the diagram above, the actualized possibilities are linked by a bold line. On the bold line, Socrates is *always* sitting, but *even on the bold line* he has the capacity to not sit because he does not sit at t_1 and at t_2 *on other lines*. One way of taking the matter then is to suppose that Aristotle understands the possible as something that can be realized (for no contradiction results from realizing a possibility, *An.Pr.* 34a25 – 27). This makes good sense on a branching futures model.

Our distinction between red and green terms can be brought to bear in this discussion. Maybe we can say that once a potentiality involving a red term has been realized, then it becomes necessary – i.e., true on every branch and true forever (or for at least as long as the thing exists). For example, when an acorn realizes its potential by becoming an oak tree, then all future branches must be 'oak tree' branches. For a green term, the case is different. Socrates could be sitting at t_0 and could be not sitting at t_1 and then sitting again at t_2. On the other hand, when an acorn's potentiality to become an oak is realized between t_1 and t_2 then there can be no going back at t_2 because the nature of the subject has changed. What was once an acorn is no longer an acorn but is now an oak. Where red terms are involved, then the realizing appears to involve a substantial change. This analysis has much in common with Aristotle's discussions of potentiality and actuality in *Metaphysics* Θ. The following passage from *Met* Θ7 emphasizes the role of time in the actualizing of such change:

> What and what sort of thing the actual is may be taken as explained by these and similar considerations. But we must distinguish when a thing is potentially and when it is not; for it is not at any and every time. E.g. is *earth* potentially a man? No – but rather when it has already become *seed*, and perhaps not even then, as not everything can be healed by the medical art or by chance,... And in the cases in which the source of the becoming is in the very thing which suffers change, all those things are said to be potentially something else, which will be it of themselves if nothing external hinders them. E.g. the seed is not yet potentially a man; for it must further undergo a change in a foreign medium. But when through its own motive principle it has already got such and such attributes, in this state it is already potentially a man... (1048b35–1049a16)

All of this raises some important points about red and green terms. First, necessary terms are red. They are terms that signify what cannot be otherwise. Terms for possibility are *either green or red*. Green terms signify what can be otherwise. And for anything that can be otherwise, plainly we can say that it is possible for it to be otherwise. A red term for possibility is different. It can only represent an Aristotelian potency. Such a potency can involve *substantial change* – as occurs, for example, in the natural development of a seed into a man, or of an acorn into an oak. These kinds of substantial changes are, according to Aristotle, clearly within the purview of syllogistic and of scientific reasoning. But when an Aristotelian potency is realized and made actual, that requires a different time, for this kind of natural development happens over time. What is at one time an acorn might at some later time be no longer an acorn but now an oak. The introduction of time indicates that the red/green distinction requires careful handling. Consider a green term like sitting. A man might sit or not sit, and nothing about the nature of man dictates which of these he satisfies. And even when he realizes the capacity to sit, he still has the capacity to not sit. An acorn over time can cease to be an acorn and become an oak, but being an acorn is part of its essential nature since an acorn is essentially a potential oak. This kind of change over time is just what Aristotelian natural change does require. And this kind of change requires a refinement of the basic red/green (cannot be/can be otherwise) distinction that we have so far relied on. 'Acorn' seems to be a straightforward example of a red term insofar as an acorn has an essential nature. But even if an acorn is something which 'can be otherwise' insofar as it can become an oak, *this kind* of 'can be otherwise' is very different from the can be otherwise that characterizes an accident like 'sit'. The red/green (cannot be/can be otherwise) distinction works well in the apodeictic syllogistic, and the distinction continues to work in the syllogisms about possibility but we have to appreciate that 'natural changes' allow red terms to enter into statements of potentiality.

This is an important point. The red/green distinction itself is a distinction between terms about necessity and terms about possibility. That would suggest that syllogisms about necessity involve red terms, not green terms; and that syllogisms about possibility involve green terms, not red terms. But that turns out to be too simplistic and not a good fit for Aristotle, since Aristotle plainly offers syllogisms about possibility which involve red terms. So it emerges that for Aristotle there are two ways to realize a possibility. One way involves realizing an accidental property. Another way involves realizing an Aristotelian potentiality.

Chapter 13
Barbara XQM[1]

Aristotle studies first figure Q+X and X+Q premise pairs in *Prior Analytics* A15. And he studies first figure Q+L and L+Q premise pairs in *An.Pr.* A16. Some of the tools he has at hand for these discussions get carried over from his earlier discussions of the apodeictic syllogistic. Some of the tools at hand are specific to his discussion of possibility. For the most part Aristotle is very clear and cautious about exactly which sense of possibility he uses at every stage of his discussion. There are, however, some tricky passages where we face interpretive difficulties about Aristotle's precise handling of the distinction between *M*-possibility and *Q*-contingency. We look closely at these passages in the next chapter. One theme that has emerged in the discussions of contingency is that terms about possibility need to be carefully qualified. In *An.Pr.* A15 and A16, we begin to see how in his treatment of contingency Aristotle accommodates these differences.

The valid first figure Q+X and X+Q syllogisms of *An.Pr.* A15 divide into three sorts. Aristotle distinguishes the first sort as follows:

> If one of the premises is taken as belonging and the other as possible, then, when the premise in relation to the major extreme [the BA-premise] signifies being possible, all the deductions will be both complete and of being possible according to the stated determination. (33b25–28)

This tells us that Barbara QXQ, Celarent QXQ, Darii QXQ, and Ferio QXQ are valid. The syllogisms are listed below in Table 20.

[1]Much of the material in this chapter is a lightly revised version of Rini (2003, 2007).

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
DOI 10.1007/978-94-007-0050-5_13, © Springer Science+Business Media B.V. 2011

Table 20
First Figure Q+X Valid Syllogisms with Major Q-premise (A15, 33b25–28)

Barbara QXQ
$\forall x(Bx \supset QAx)$
$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset QAx)$

Celarent QXQ
$\forall x(Bx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset Bx)}$
$\forall x(Cx \supset Q{\sim}Ax)$

Darii QXQ
$\forall x(Bx \supset QAx)$
$\underline{\exists x(Cx \,\&\, Bx)}$
$\exists x(Cx \,\&\, QAx)$

Ferio QXQ
$\forall x(Bx \supset Q{\sim}Ax)$
$\underline{\exists x(Cx \,\&\, Bx)}$
$\exists x(Cx \,\&\, Q{\sim}Ax)$

The syllogisms in Table 20 are the easy ones and they are not particularly interesting since they are all trivially valid. And they do not require any ampliation. These trivial cases are simple substitution instances of non-modal syllogisms, and the introduction of Q-contingency has little real effect in these.

Matters get more complicated and a great deal more interesting with the second sort of mixed-modal first figure premises. The first of these is known as Barbara XQM. Aristotle's treatment of Barbara XQM marks the first clear textual evidence in the syllogistic of the special new method that we discussed in Chapter 12 – this is the method of realizing or actualizing a possibility:

> when something false but not impossible is assumed, then what results through that assumption will also be false but not impossible. (34a25–27)

Aristotle's point is that if you wish to reason from one possibility to another, you are entitled to do so by assuming the first possibility true – that is to say, actualized – and reasoning that in that case the second possibility will also be true – that is to say, actualized. When we realize a possibility we shift from a premise about mere possibility (i.e., a Q) to a non-modal X-premise. And in some cases doing so is what allows us to syllogize. Aristotle reasons that if something is possible then it can be actual. That is, if it is possible then it can be realized. The result of realizing a possibility may be false, but it will never be impossible. This realization method gets put to use in a large number of syllogisms. The rest of the present chapter concentrates on how realization helps to explain Barbara XQM.

(1) Barbara XQM
Every *B* is *A*
Every *C* is contingently-*B*
Every *C* is possibly-*A*

In LPC this would be:

(2) Barbara XQM
$\forall x(Bx \supset Ax)$
$\underline{\forall x(Cx \supset QBx)}$
$\forall x(Cx \supset MAx)$

Barbara XQM would be trivially valid if ampliation were permitted on the first premise:

(3) Ampliated Barbara XQM
$\forall x(QBx \supset Ax)$
$\forall x(Cx \supset QBx)$
$\forall x(Cx \supset Ax)$ [non-modal Barbara]
$\forall x(Cx \supset MAx)$ [$\varphi \supset M\varphi$]

However, the first premise of (3) has to have an ampliated subject with an assertoric predicate, and this is ruled out by RAP on p. 125. Aristotle himself realises that Barbara XQM cannot be proved by uniform substitution in non-modal Barbara, since he describes it as 'incomplete' (which can be understood in the sense described on p. 130). A 'complete' syllogism in this case would be a substitution instance of Barbara XXX, as (3) is. So Barbara XQM is (2) rather than (3). Aristotle also specifies that the conclusion is an *M* and not a *Q*. Aristotle explains that in the first figure when the *minor* CB premise is the Q-premise, then the conclusion is 'not of what is possible according the determination':

> However, when the premise in relation to the minor extreme is possible, then not only are the deductions all incomplete, but also the privative deductions[2] are not of what is possible according to the determination, but rather of what belongs of necessity to none, or not to every (for if something belongs of necessity to none, or not to every, we also say it is possible for it to belong to none or not to every). (33b28–33)

A conclusion about what is possible according to the determination is a conclusion about Q-contingency. But a conclusion about 'what belongs of necessity to none, or not to every' is a conclusion about M-possibility. So Aristotle's point is that he is now considering cases where X+Q premise pairs logically entail conclusions about M-possibility, rather than conclusions according the usual determination.

Yet (2) appears subject to an easy counter-example. If we take our terms *A*, *B*, and *C* to be horse, in the paddock, and man, then we get the following:

(4)	Everything in the paddock is a horse	T
	<u>Every man could be in the paddock</u>	T
	Every man could be a horse	F

So it would seem that Barbara XQM is invalid. Aristotle, however, counts Barbara XQM as valid. He explains that when the premises are like this, the fact that there will be a deduction must be proved through an impossibility, (34a2–3). Aristotle's method raises several questions. It will help to look at how he tries to prove Barbara XQM, since in his account Barbara sets the standard for subsequent proofs. Here is the explanation we find in the text:

> Now, with these determinations made, (5) let A belong to every B and (6) let it be possible for B to belong to every C. Then (7) it is necessary for it to be possible for A to belong to every C. (8) For let it not be possible, and (9) put B as belonging to every C (this is false although not impossible). Therefore, if (8) it is not possible for A to belong to every C[3] and (9) B belongs to every C, then (10) it will not be possible

[2]Tredennick explains that 'this is a mistake on Aristotle's part; the qualification applies equally well to the affirmative syllogisms' (Tredennick 1938, p. 266). This point is generally accepted. From the point of view of the logic Aristotle's claim would make sense if by 'privative deductions' he means 'proofs that proceed through impossibility', since any such proof begins with a denial. See also Smith (1989, p. 131) at 33b25—33n.

[3]Aristotle's text suggests that he thinks that the reductio hypothesis is really a universal. From a modern point of view, this is not correct. But more interestingly, it is not obvious why Aristotle should think it is correct since his own logical principles do not take (7) and (*) 'Every *C* is not possibly-*A*' to be

for A to belong to every B (for a deduction comes about through the third figure). But it was assumed that it is possible for A to belong to every B. Therefore, it is necessary for it to be possible for A to belong to every C (for when something false but not impossible was supposed, the result is impossible). (*An.Pr.* A15, 34a34–b2)

(5) Every *B* is *A*
(6) <u>Every *C* is contingently-*B*</u>
(7) Every *C* is possibly-*A*
Suppose
(8) Some *C* is not possibly-*A*
(9) Every *C* is *B*
Then
(10) Some *B* is not possibly-*A*

But (10) and (5) cannot both be true, so Aristotle wants to say the reductio shows we can syllogize to (7). That is, Aristotle counts Barbara XQM or (5) (6) (7) as valid.

But this is awkward because (4) appears to be a counter-example. And this suggests that Aristotle has made a mistake. The problem is particularly easy to see when we put in place the terms from our counter-example (4):

(11) Everything in the paddock is a horse
(12) Every man could be in the paddock
(13) Every man could be a horse
Suppose
(14) Some man could not be a horse
(15) Every man is in the paddock
Then
(16) Something in the paddock could not be a horse

contradictories. In the square of opposition in *On Interpretation* Aristotle is careful to distinguish contrary from contradictory. The contradictory of an *A* proposition is an *O* proposition. The contrary of an *A* is an *E*. This is what he gets exactly right in his non-modal reductios. But here in *An.Pr.* A15, where he is doing a modal reductio proof and so needs modal contradictories, he only gives contraries – that is, he gives (7) and (*). In the non-modal assertoric syllogistic, Aristotle uses proof through impossibility and he has no difficulty about it there. Aristotle mentions such non-modal reductio proofs frequently, though most often he only mentions them in parenthetic remarks. In fact, there are only two passages in which he shows us how to use the reductio method in the proofs of non-modal syllogisms. He shows us how to use a reductio to prove Baroco at 27a36—b3, and to prove Bocardo at 28b17—21. In all other cases (27a9—15, 28a17—26, 28a26—30, 28b11—15) he mentions that we can use such a method, but he leaves the details to his reader. At A7, 29a35—39 he states a general explanation of the method. Smith's comments on 27a14—15 about (non-modal) proof through impossibility are particularly insightful. In the discussion of the present passage I give the reductio hypothesis as a particular. For another passage see footnote 1 on p. 172 below.

As Aristotle explains, the proof requires *actualizing the possibility* in (6). This is captured in the move from (6)/(12) to (9)/(15). And when we use the terms as given above, then actualizing the possibility in (6)/(12) creates a problem. In supposing (6)/(12) is actualized Aristotle seems to have forgotten that that changes the truth value of our initial premise (5)/(11); for in supposing that every man *is* in the paddock we are *denying* that everything in the paddock is a horse. The terms I'm using here are mine – not Aristotle's. But if this is supposed to establish Barbara XQM's *validity* then Aristotle really has goofed.

Lindsay Judson (1983) in his discussion of *De Caelo* finds Aristotle committing a 'monstrous error.' And the monstrous error he identifies seems strikingly similar to the situation we have here in *Prior Analytics* with Barbara XQM. Judson calls the *De Caelo* mistake the '*insulated realization manœuvre*,' or 'IR manœuvre' for short:

> Aristotle here [*De Caelo* 281b2–25] seems to think that his test can be applied to a candidate for possibility *without regard to whether the supposition of its holding requires changes in what else can be taken to be true*... the realization of the possibility (or exercise of the capacity) is supposed in complete insulation – causal and logical – from anything else which is taken to hold. (I do not, of course, mean to suggest that Aristotle calmly thought out this manœuvre as I have characterized it, and took it to be a perfectly sound analytical tool; rather, it is simply what his fallacious way of arguing here amounts to.) (Judson 1983, p. 230)

As Judson makes abundantly clear, it is a terrible error for Aristotle, or for any logician, to make.

There is, however, some reason to believe that Aristotle is *not* in fact making such a mistake – at least not in our passage. Whatever may be going on in *De Caelo*, in *An.Pr.* A15 Aristotle is trying to tell us how to *avoid* making such a mistake by telling us *not* to choose terms as I have here. Here are his own instructions:

> One must take 'belonging to every' without limiting it with respect to time (μὴ κατὰ χρόνον), e.g., 'now' or 'at this time', but rather without qualification (ἁπλῶς). For it is also by means of these sorts of premises that we produce deductions, since there will not be a deduction if the premise is taken as holding only at a moment (κατά τὸ νῦν). For perhaps nothing prevents man from belonging to everything in motion at some time (for example, if nothing else should be moving), and it is possible for moving to belong to every horse, but yet it is not possible for man to belong to any horse. Next, let the first term be

> animal, the middle term moving, the last term man. The premises will be in the same relationship, then, but the conclusion will be necessary, not possible (for a man is of necessity an animal). It is evident, then, that the universal should be taken as holding without qualification, and not as determined with respect to time. (*An.Pr.* A15, 34b7–18)[4]

The terms that give us premise (11) 'Everything in the paddock is a horse' violate Aristotle's instructions here. Premise (11) is only true at a time and not true without qualification. What results is not a syllogism. Premise (11) changes its meaning – it becomes false *when* premise (12) is realized. Aristotle's instructions in *An.Pr.* A15 are a caution against choosing terms in such a way.

Aristotle emphasizes the point by offering his own sets of terms to illustrate how the choice of terms affects our ability to syllogize. First, at 34b11–14 he takes as his A, B, and C the terms 'animal', 'moving', 'man'. When we put these into the schema Barbara XQM, we get the following:

(17)	All moving things are animals	T
	<u>All men are possible moving things</u>	T
	All men are possible animals	F

Putting these terms in place we get a false Q-conclusion. It is false because no men are Q-animals – because all men are necessarily animals. So one thing the terms show us is that Barbara XQQ is not valid. Of course 'possible is said in many ways' and one way is that possible is said of what is necessary. So we certainly have a true M-conclusion because the terms give an L-proposition – but all that means is that the conclusion is possible but not according to the usual determination.

Aristotle offers a second set of terms. Let 'man', 'moving', and 'horse' be A, B, and C:

(18)	All moving things are men	T
	<u>All horses are possible moving things</u>	T
	All horses are possible men	F

In this case, not only do we have a false Q-conclusion; we even have a false M-conclusion. It is false that all horses are possible men, because all horses are necessarily

[4]Some interpreters are suspicious about the authenticity of *An.Pr.* 34b7–18. See Patterson (1995, pp. 167–174) and Malink (2006, p. 102, n. 19). Thom (1996, p. 78) takes 34b7–9 to be evidence of an 'unqualified' i.e., ampliated assertoric premise of the form $\forall x(QBx \supset Ax)$. But this is disallowed by RAP (p. 125 above), and by Aristotle's insistence that Barbara XQM is 'incomplete' as explained above p. 148).

not men. Clearly, this is a counter-example and Aristotle means it to be a counter-example. It works just the same as our (4). The problem arises because of the *B* term – 'moving'. The first premise is about a mere happenstance fact and it changes from true to false when the possibility in the second premise is realized – i.e., the first premise becomes false when all horses are moving. If Aristotle allows this then he is guilty of what Judson calls the IR manoeuvre. But Aristotle is not clearly guilty here. In the case of Barbara XQM, Aristotle's instructions suggest that he is aware of the problem that comes of choosing terms in this way. If we have a premise that holds only at a moment and if that premise is made false by actualizing a possibility, then we are no longer arguing from two true premises, and then we are not able to use them in a scientific deduction. So it is clear that these terms won't do and clear that we must select our terms more carefully if we want to syllogize. Notice that both of Aristotle's counter-examples (17) and (18) and also my own counter-example (4) all make the *B* term a green accidental term. It is perhaps worth observing that subject terms which hold 'only at a moment' would be less natural with an ampliated subject. Whether or not a thing has the *ability* to move seems a more or less permanent feature of its nature. It is whether or not it is exercising that ability which holds or fails to hold 'at a moment'. So it may not be surprising that the realization method, which is invoked when ampliation is unavailable, brings with it the restrictions that Aristotle mentions here.

We need to choose terms in order to avoid mere happenstance truths which are liable to change from moment to moment. Scientific premises should hold without qualification. In order to get a scientific deduction we must restrict our choice of terms to terms which hold without qualification. And in fact restricting the *B* term to red terms is needed in order to preserve the validity of the schema. The result is of course a restricted validity, but it is appropriate to scientific deductions. Consider Barbara XQM with red terms:

Red Barbara XQM
(19) Every man is an animal
(20) <u>Every seed is contingently-man</u>
(21) Every seed is possibly-animal
Suppose
(22) Some seed is not possibly-animal
(23) Every seed is (has become) a man
Then
(24) Some man is not possibly-animal

Step (23) involves supposing the possibility in (20) is actualized. But (23) leads to a conclusion (24) which contradicts our initial premise (19). But there is no problem about the truth of premise (19) in this instance. Premise (19) is clearly true and will always be true. And so we have a proof of *validity*. Our reductio assumption (22) leads to a contradiction.

But look at Aristotle's counter-example (18). The reason why (18) works as a counter-example is that actualizing the possibility that horses move *does* change the truth of the first premise since it says that all moving things are men. Moving is a green accidental term and so the things that move can and do change – that is precisely what makes a term an accident. *Essences*, on the other hand, do not change. When we restrict *B* to a red essential term, the truth value of the first premise is unaffected, and so we generate a contradiction. When the *B* term is an accident, then there is no contradiction; there is just a change in truth value.

With this in mind, consider another instance of Barbara XQM with all red terms:

Another Red Barbara XQM

(25)	Every oak is a deciduous tree	T
(26)	<u>Every acorn could be an oak</u>	T
(27)	Every acorn could be a deciduous tree	T

Now, in (25)(26)(27) we have true premises, provided that (26) is understood to mean that every acorn could (by its nature) *become* an oak, and the (true) conclusion plainly follows from the premises. Realizing (26) cannot affect (25) since (25) is a necessary truth. So, (25)(26)(27) appears to be valid. It seems that changing the terms makes a difference. But could that be right?

If we insist that Aristotle's syllogistic logic puts no restrictions on allowable terms then there is only one legitimate answer: (5) (6) (7) is an invalid schema. (25)(26)(27) is just a true instance of it. But I have suggested that Aristotle *restricts* certain logical principles in order to reflect basic facts about his view of the world – a view which is reflected in the distinction between red and green terms. We have already noted that when (20) is actualized to (23) the truth of (19) is not affected and the IR manoeuvre does not apply. Nor does it apply in (25)(26)(27).[5]

Can we give a proof that Red Barbara XQM – that is, (5) (6) (7) when *B* is red – is in fact a valid syllogistic schema?

(5) Every *B* is *A*
(6) <u>Every *C* is possibly-*B*</u>

[5]Although certain authors produce their own systems according to which Barbara XQM is valid (see for instance Thom, 1996, esp. pp. 251–255, Malink, 2006), the theme in this book is that Aristotle might in fact have special semantic restrictions in mind, and so the crucial question for a textual interpreter is what sense to make of Aristotle's own instructions in *An.Pr.* A15 about Barbara XQM. (See also Rini, 2007.)

(7) Every *C* is possibly-*A*

Start with the conclusion (7). The only way to have a false conclusion here is to produce a *C* which *cannot possibly be A*. In other words, for (7) to be false *A* must be necessary and cannot be an accident. So, *A* as well as *B* must be a red term and cannot be a green term. But *A* occurs as a term in premise (5), and so premise (5) will be affected too. If *A* must be a red term then premise (5) is not really an ordinary assertoric, non-modal premise – (5) is really an apodeictic premise involving necessity. Next consider premise (6). Aristotle's proof of (5) (6) (7) depends upon being able to realize the possibility expressed in premise (6). When we actualize 'every *C* is possibly-*B*' we get 'every *C* is *B*'. In *De Caelo* I.12, it is just such a procedure that generates the monstrous error that Judson calls the IR-manoeuvre. But it is an error that can only occur when *B* is an accidental green term.

This distinction comes into play when we actualize (6). If *B* signifies something that is possible because it is a matter of mere chance or coincidence, then premise (5) only happens to be true at a given time. For example, if *B* is 'in the paddock' and *A* is a red term 'horse', then premise (5) will be 'everything in the paddock is a horse.' Or, following Aristotle's example, if *B* is 'moving' and *A* is 'horse', then premise (5) will be 'everything moving is a horse.' These premises hold *at a time*; they are true at a moment. But they are not true always, without respect to a time. The nature of the *B* term guarantees this because in these examples the *B* term signifies something whose possibility arises because of chance, happenstance, mere coincidence. Aristotle tells us not to choose premises like this. They get us into trouble and make a mess of syllogistic. The reason they cause trouble is that they can change too easily from true to false, and in fact they *do* change from true to false when the possibility in premise (6) is realized. If we actualize the possibility 'all men are possibly in the paddock', we get 'all men are in the paddock' and then it is not true that everything in the paddock is a horse. This makes premise (5) false when (6) is actualized. This is the IR-manoeuvre and if Aristotle were to allow it then he would be making a monstrous error. But Aristotle does not allow it. He tries to signal this in *An.Pr.* A15, 34b7–18, where he tells us not to choose premises which hold only at a moment. To put it simply, (5) (6) (7) is guaranteed to be valid provided the *B* term is restricted to red terms. In the case of our example (25) (26) (27), the first premise (25) is a necessary truth, and so no realizing of a possibility can change that.

Notice however that when an Aristotelian potency is realized and made actual, that requires a different time, for this kind of natural development happens over time. What is at one time an acorn might at some later time be no longer an acorn but now an oak. And once it is an oak it is necessarily one. An acorn over time can cease to be an acorn and become an oak, but being an acorn is part of its essential nature since an acorn is essentially a potential oak. This kind of change over time is just what

Aristotelian natural change does require.[6] 'Acorn' seems to be a straightforward example of a red term insofar as an acorn has an essential nature. But even if an acorn is something which 'can be otherwise' insofar as it can become an oak, *this kind* of 'can be otherwise' is very different from the 'can be otherwise' that characterizes an accident like 'sit'. This is why we have to appreciate that 'natural changes' allow red terms to enter into statements of potentiality.

[6] Consider *Metaphysics* θ, 1049b19–24: '...the matter and the seed and that which is capable of seeing, which are potentially a man and corn and seeing, but not yet actually so, are prior in time to this particular man who now exists actually, and to the corn and to the seeing subject; but they are posterior in time to other actually existing things, from which they were produced.'

Chapter 14
First Figure X+Q (A15)

This chapter considers other mixed modal syllogisms involving Q-contingency. All of the first figure X+Q syllogisms, including Barbara XQM, are listed in Table 21.

Table 21
Proof Through Impossibility in the First Figure X+Q (A15)

<u>Universals:</u>

Barbara XQM (34a34–b2)
$\forall x(Bx \supset Ax)$
$\underline{\forall x(Cx \supset QBx)}$
$\forall x(Cx \supset MAx)$

CC-Barbara XQM (35a3–11)
$\forall x(Bx \supset Ax)$
$\underline{\forall x(Cx \supset Q{\sim}Bx)}$
$\forall x(Cx \supset MAx)$

Celarent XQM (34b19–35a2)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$
$\forall x(Cx \supset M{\sim}Ax)$

CC-Celarent XQM (35a11–20)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\forall x(Cx \supset Q{\sim}Bx)}$
$\forall x(Cx \supset M{\sim}Ax)$

<u>Particulars:</u>

Darii XQM (35a35–b1)
$\forall x(Bx \supset Ax)$
$\underline{\exists x(Cx \;\&\; QBx)}$
$\exists x(Cx \;\&\; MAx)$

CC-Darii XQM (35b2–8)
$\forall x(Bx \supset Ax)$
$\underline{\exists x(Cx \;\&\; Q{\sim}Bx)}$
$\exists x(Cx \;\&\; MAx)$

Ferio XQM (35a35–b1)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\exists x(Cx \;\&\; QBx)}$
$\exists x(Cx \;\&\; M{\sim}Ax)$

CC-Ferio XQM (35b2–8)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\exists x(Cx \;\&\; Q{\sim}Bx)}$
$\exists x(Cx \;\&\; M{\sim}Ax)$

A feature to note about the syllogisms in Table 21 is that their conclusions are about M-possibility (see the passage, 33b28–33, quoted on p. 149.)[1]

[1]These syllogisms number among those first figure syllogisms which Aristotle says are valid but which he says are nonetheless *incomplete* (*atelēs*, see p. 148). As he explains: ‘it will also be clear that they are incomplete, since the proof is not from the premises taken.’ (34a3–5) Each of the first figure syllogisms in Table 21 requires something more in order to complete its proof. The proofs are not from the premises taken, and in this respect they are very like ordinary second and third figure syllogisms which are typically proven by bringing them ‘back to the first figure.’ And in a sense this is what the syllogisms in Table 21 also require. Aristotle offers proofs of these by bringing them back to some syllogism whose validity has been previously established.

A. Rini, *Aristotle’s Modal Proofs*, The New Synthese Historical Library 68,
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Immediately after Barbara XQM, Aristotle turns to consider a closely related syllogism, Celarent XQM, found at A15, 34b19–22. And his treatment of Celarent XQM mirrors his treatment of Barbara XQM. In particular, his discussion, again comes in three separate parts. First, Aristotle declares Celarent XQM valid (34b19–22); second, he gives a formal proof (34b19–26); and then, third, he gives two sets of terms (34b33–b39). In this lengthy discussion, we find some of the same issues as those that arise with Barbara XQM, but now in Celarent XQM they come with a twist. The twist arises from the combination of a privative proposition with a modal operator.

First, consider Aristotle's statement of Celarent XQM's validity:

> Next, let AB be a universally privative premise, and let A be taken to belong to no B, but let it be possible for B to belong to every C. With these put as premises, then, it is necessary for it to be possible for A to belong to no C. (34b19–22)

Aristotle repeats the point that the kind of possibility involved in the conclusion is 'not of what is possible according to our determination, but rather of what belongs to none of necessity'. (34b27–28) The possibility in the conclusion is not the kind of possibility that we capture with Q-contingency, but rather with M-possibility. And so we can represent the schema Celarent XQM as follows:

Celarent XQM
(1) $\forall x(Bx \supset {\sim}Ax)$
(2) $\underline{\forall x(Cx \supset QBx)}$
(3) $\forall x(Cx \supset M{\sim}\mathrm{Ax})$

The formal proof of validity works just the same as the formal reductio proof that he uses for Barbara XQM. The proof of Celarent XQM goes like this:

> For (4) let [the conclusion] not be possible, and (5) let B be put as belonging to C, just as before. Then, (6) it is necessary for A to belong to some B (for a deduction comes about through the third figure). But this is impossible; consequently, (3) it would be possible for A to belong to no C (for when that was put as false, the result was impossible). (34b19–26)

From premises (1) and (2), we want to establish that (3) follows.

(3) $\forall x(Cx \supset M{\sim}\mathrm{Ax})$

So, we suppose that the conclusion is not possible. That is, we suppose that some *C* is necessarily *A*:

(4) $\exists x(Cx \,\&\, LAx)$[2]

The next step in the proof requires that we actualize the possibility in (2). This is exactly the same supposition that Aristotle explains in his earlier proof of Barbara XQM. And so, in the present case, when we actualize the possibility in premise (2), we arrive at the following:

(5) $\forall x(Cx \supset Bx)$

When we take (4) and (5) together as premises we can syllogize through the third figure to (6)[3]:

(6) $\exists x(Bx \,\&\, LAx)$

But (6) contradicts the initial premise (1) $\forall x(Bx \supset \sim Ax)$. So, we have a reductio proof that (1)(2)(3) is valid. We supposed that (3) $\forall x(Cx \supset M{\sim}\mathrm{Ax})$ is false: '...for when that was put as false, the result (6) was impossible' (34b26). That is to say, the reductio assumption $\exists x\,(Cx \,\&\, LAx)$ results in a contradiction, so our original schema (1)(2)(3) is valid.

In the final part of his discussion of Celarent XQM, Aristotle offers two sets of terms for us to consider. But before we look at the terms that Aristotle offers, consider the following counter-example to Celarent XQM, using our own terms. Let terms be animal, moving, and man.

(7)	No moving thing is an animal	T
	Every man could move	T
	No man could be an animal	F

[2]Smith (1989) comments on Aristotle's use of 'let it not be possible' in 34b22 as a way of denying the purported conclusion: "Aristotle immediately equates this denial of 'Possibly A to no C' with 'Necessarily A to some C'." Smith's diagnosis clearly captures what Aristotle has in mind.

[3]The proof (4)(5)(6) is through Disamis LXL. Here in A15, Aristotle correctly supposes Disamis LXL is valid, though in A11 he seems to suppose he has a counter-example (which as we saw in Chapter 9 seems to invole the subtle mistake). See pp. 100–102.

This counter-example is constructed in the same way as our counter-examples against Barbara XQM, and in the same way as Aristotle's own second counter-example against Barbara XQM, at 34b7–18. The crucial feature here in Celarent XQM, as in Barbara XQM, is that the middle term is a green term – 'moving'. Because it is a green term and names something which *can* be otherwise, this makes the first premise something that is true in a happenstance kind of way, or true only at a moment. 'No moving thing is an animal' is true when nothing that is an animal is moving. But of course animals *can* move. Men, for instance, *can* move. When we suppose that the possibility in the second premise is actualized – that is, when we actualize the possibility that every man *could* move, and suppose that every man *is* moving – this changes the truth of the first premise. It now becomes false. So, again, we do not have true premises, and so we are not able to syllogize. The lesson from the earlier discussion of Barbara XQM is that we can block the difficulties these inappropriate terms generate. In order to talk about validity we need choose premises that hold without qualification, and not merely at a moment. And as we saw in Chapter 13 in the discussion of Barbara XQM, we can have premises that hold without qualification if we restrict our *B* term to red terms.

As in the case of Barbara XQM, Aristotle himself offers terms which purport to prove Celarent XQM invalid. The terms he offers are raven, reasoner, man (34b33–34); and moving, science, man (b38). The *B* terms 'reasoner' and 'science' are curious choices here. Aristotle uses such terms in earlier chapters of the non-modal syllogistic, but they do not often feature in the counter-examples for the modal syllogisms. When we put these two sets of terms into the schema as Aristotle instructs, then he claims that we get the following:

(8)	Every reasoner is not a raven	T
	<u>Every man is a possible reasoner</u>	T
	Every man is possibly not a raven	F
(9)	Every scientific thing is not moving	T
	<u>Every man is possibly a scientific thing</u>	T
	Every man is possibly not moving	F

The problem with both of these does not lie in the premises, which indeed seem true. The problem is that the conclusion in each case also seems true. In (8), the first premise seems not to be the troublesome kind of happenstance premise of our earlier examples. Nothing that reasons is a raven. This premise is true and its truth is not subject to change from time to time. So when we use the terms Aristotle describes here, there is no danger of an IR-manoeuvre. Realizing the possibility in the second premise poses no danger since it is irrelevant to the truth value of the first premise. And of course, the second premise is straightforwardly true since reasoning is something any man *can* do.

So reasoning is possible to every man. In other words, every man is contingently a reasoner. But what does Aristotle have in mind about the conclusion in (8)?

Aristotle tells us that 'it is evident from terms that the conclusion will not be a possible one' (34b32). This is exquisitely ambiguous. Aristotle could mean simply that the conclusion will not be a *Q*-proposition but rather will be an *M*-proposition. That is, it *will be possible* but not according to the determination. This makes a good deal of sense and it fits with his earlier proof of Barbara XQM where the conclusion is very definitely an *M*-proposition. And it seems to be Aristotle's point in the passage quoted on p. 149 above, 33b28–33. If this is what Aristotle means then the conclusion should have the form $\forall x(Cx \supset M{\sim}Ax)$, since the ~ normally goes inside Aristotle's modal operator, and since here we are translating Aristotle's 'it is possible for *A* to belong *to no C*'. There is, however, something different that Aristotle might mean by 'it is evident from terms that the conclusion will not be a possible one' at 34b32. He might mean here in Celarent that the conclusion will be a proposition about *what is not possible*. That is, that the conclusion will have the form $\forall x(Cx \supset {\sim}MAx)$, in which the ~ is outside the modal operator. But even if this is what Aristotle means the conclusion is still true. It is equivalent to the true LE-proposition 'every man is necessarily not a raven.' (8) seems to be a first part of a proof that Celarent XQQ is not a syllogism.

Is (9) any clearer? Here, 'moving' – the *A* term in (3) – is a green term. And in (9) the conclusion is unambiguously rejected. 'Every man is possibly not moving' is false. It is false because 'it is not necessary that no man be moving, nor necessary that some one be [moving]', 34b40. If that is his meaning then he is interpreting the conclusion as $\forall x(Cx \supset {\sim}MAx)$. But this is $\forall x(Cx \supset L{\sim}Ax)$ and so all we would have is a counter-example to Celarent XQL. There is a suggestion that Aristotle himself is unhappy with these results. For he remarks at 35a1 that 'the terms should be better chosen'. The example (7) on p. 159, is one in which the terms *have* been better chosen – the terms in (7) are animal, moving, man, and while they are not Aristotle's own, they do nonetheless produce a clear counter-example against Celarent XQM when 'happenstance' green terms are allowed.

Aristotle claims that the four schemas on the right-hand side of Table 21 are syllogisms, and he appeals to complementary conversion in order to justify them:

CC-Barbara XQM (35a3–11)

(10) $\forall x(Bx \supset Ax)$

(11) $\underline{\forall x(Cx \supset Q{\sim}Bx)}$

(12) $\forall x(Cx \supset MAx)$

At 35a3–11, Aristotle explains how the first of these works:

If the privative is put in relation to the minor extreme and signifies

> being possible, then there will be no deduction from the actual premises taken, but there will be one if the possible premise is converted accordingly, just as in the previous cases.
>
> For (10) let *A* belong to every *B* and (11) let it be possible for *B* to belong to none of the *C*s. Now, when the terms are in this relationship, nothing will be necessary. However, when (11) *BC* is converted and (13) it is taken to be possible for *B* to belong to every *C*, then a deduction comes about just as before (for the terms are similarly related in position).

Complementary conversion of (11) $\forall x(Cx \supset Q{\sim}Bx)$ gets

(13) $\forall x(Cx \supset QBx)$.

And we can syllogize through (10) and (13) to get (12). In fact (10) (13) (12) is just Barbara XQM. The other schemas on the right-hand side of Table 21 are justified in much the same way.

At 35a11–20, Aristotle explains the schema marked CC-Celarent XQM in Table 21:

CC-Celarent XQM (35a11–20)
(14) $\forall x(Bx \supset {\sim}Ax)$
(15) $\underline{\forall x(Cx \supset Q{\sim}Bx)}$
(16) $\forall x(Cx \supset M{\sim}Ax)$

> And in the same manner, also, when both intervals are privative, if *AB* signifies not belonging and *BC* signifies being possible to belong to none. For a necessity in no way comes about from the actual premises taken, but when the possible premise is accordingly converted there will be a deduction. For (14) let *A* have been taken to belong to no *B* and (15) *B* to be possible to belong to no *C*. Through these, then, nothing is necessary; but if (17) it is taken to be possible for *B* to belong to every *C* (which is true) and premise *AB* is the same, then there will again be the same deduction.

Again, complementary conversion of (15) gets

(17) $\forall x(Cx \supset QBx)$.

And we can syllogize through (14) and (17) to get (16). In fact (14) (17) (16) is just

Celarent XQM. The proofs Aristotle offers of CC-Barbara XQM (35a3–11) and CC-Celarent XQM (35a11–20) show that he is now taking Barbara XQM and Celarent XQM as part of his basic proof method. Up to now complementary conversion and realization have not featured together in a proof, but here Aristotle combines them. He does not combine *overtly*. CC-Barbara XQM and CC-Celarent XQM involve complementary conversion. Complementary conversion turns CC-Barbara XQM into Barbara XQM, and turns CC-Celarent XQM into Celarent XQM. Both Barbara and Celarent XQM have been proven by realization.

Barbara XQM and Celarent XQM – and the complementary XQM syllogisms CC-Barbara XQM and CC-Celarent XQM – all involve universal premises. Aristotle counts the first figure schemas with a particular *CB* premise as valid syllogisms also. These are Darii XQM and Ferio XQM. And he counts the complementary CC-Darii XQM and CC-Ferio XQM syllogisms as valid also. These make up the bottom half of Table 21. Aristotle approaches these four in the same way that he approaches the syllogisms from universal premises that make up the top half of Table 21. The explanation in the text (35a35–b2) is as follows:

> And when the interval in relation to the major extreme is universal, but belonging [X] rather than possible, and the other interval is particular and possible, then whether both premises are put as negative [CC-Ferio XQM], or both affirmative [Darii], or one negative and the other affirmative [Ferio, CC-Darii XQM], in all these ways there will be an incomplete deduction. Some deductions, however, will be proved through an impossibility, and others through the conversion of a premise expressing possibility, just as in the previous cases.

Aristotle does not give a detailed explanation of how the proofs work when the *CB* premise is about possibility and is particular. But plainly Darii XQM and Ferio XQM must require proof through impossibility and realization like their universal counterparts Barbara XQM and Celarent XQM. He does explain that the complementary syllogisms require complementary conversion of the *BC* premise. The explanation of these two syllogisms is in 35b2–8:

> There will be a deduction through conversion when the universal premise is put in relation to the major extreme and signifies belonging or not belonging and the particular premise is privative and takes something to be possible, for instance, if *A* belongs or does not belong to every *B* and it is possible for *B* not to belong to some *C*. For a deduction comes about when *BC* is converted in accordance with possibility.

So, by complementary conversion of the second premise, CC-Darii XQM becomes Darii XQM. By complementary conversion of the second premise, CC-Ferio XQM becomes Ferio XQM. Darii and Ferio are valid when it is a red term which is realized, but invalid when green terms are permitted. But CC-Darii XQM and CC-Ferio XQM would involve both complementary conversion and realization, and so I group them on the right-hand of Table 21, since these are all proved similarly.

The eight syllogisms in Table 21 are the only first figure XQM syllogisms. Aristotle then moves on to Q+X combinations in the first figure. In order to establish that the remaining Q+X premise combinations do not produce syllogisms, Aristotle gives terms to illustrate their invalidity. The next example in *An.Pr.* A15 for which Aristotle offers terms is as follows:

> But if it is put that B does not belong to every C, and not just that it is possible for it not to belong, then there will in no way be a deduction, whether the premise AB is privative or positive. (35a21–23)

From these instructions, we have the following premise combinations:

(QA)	$\forall x(Bx \supset QAx)$	(QE)	$\forall x(Bx \supset Q{\sim}\mathrm{A}x)$
(E)	$\forall x(Cx \supset {\sim}Bx)$	(E)	$\forall x(Cx \supset {\sim}Bx)$

The difference between these two premise pairs is in the first premise, what Aristotle here calls the premise *AB*. The relation of the *A* and *B* terms can be either positive (and therefore, a *QA*-premise) or privative (and therefore, a *QE*-premise). Since for any term φ, $Q\varphi \equiv Q{\sim}\varphi$, ordinary complementary conversion of either a *QA* or a *QE* premise immediately gets us *both* a *QA* and a *QE* premise. So anything that holds for the left holds for the right, and vice versa. The assertoric schema corresponding to QA+E is of course:

(A)	$\forall x(Bx \supset Ax)$
(E)	$\forall x(Cx \supset {\sim}Bx)$

As we learned in the assertoric syllogistic, there is no syllogism from such premises. A counter-example to QA+E has to involve an *A* term such that all *B*s are contingently *A* but no *C*s are. So *A* must be a term which is contingent of some things but non-contingent of other things.

It is easy to illustrate the invalidity of QA+E with a counter-example. We have to show that neither an *M* nor an *M~* conclusion is available. Let *B* be 'acorn', *A* 'oak', and *C* 'stone'. Then we have the following:

(18)	Every acorn is a *Q* oak	T
	<u>No stone is an acorn</u>	T
	Every stone is an *M* oak	F

Both premises are true, but the conclusion is clearly false. These terms rule out an M conclusion. Next, let terms be oak, acorn, and deciduous tree. Then we have:

(19)	Every acorn is a *Q* oak	T
	<u>No deciduous tree is an acorn</u>	T
	Every deciduous tree is an *M*~ oak	F

These terms rule out a syllogism to an *M*~ conclusion.

In fact Aristotle himself offers terms: ‘Terms in common for belonging of necessity are (i) white, animal, snow; terms for not being possible are (ii) white, animal, pitch’ (a23). The first set of terms gives us the following:

(20) All animals are *Q* white
(21) <u>All snow is not an animal</u>
(22) All snow is *M*~ white

If we actualize the possibility in (20), the result is a non-modal premise:

(23) All animals are white

But from (23) and (21), we do not have any non-modal syllogism, so we cannot syllogize to any conclusion.[4] In the modal case (20) (21) (22), by offering terms as he does, Aristotle seems to be saying that in fact (22) ‘all snow is possibly not white’ is a false proposition – these terms give a proposition about belonging of necessity. That is, he seems to be saying that (22) is false because snow is *necessarily* white. If (22) is

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[4]Here is how Aristotle explains the non-modal case earlier in *An.Pr.* A4, 26a2—8:

> However, if the first extreme follows all the middle and the middle belongs to none of the last, there will not be a deduction of the extremes, for nothing necessary results in virtue of these things being so. For it is possible for the first extreme to belong to all as well as to none of the last. Consequently, neither a particular nor a universal conclusion becomes necessary; and, since nothing is necessary because of these, there will not be a deduction.

Aristotle’s discussion in the non-modal assertoric syllogistic shows that no conclusion follows from a first figure A+E premises combination: ‘Terms for belonging to every are animal, man, horse; for belonging to none, animal, man, stone.’ That is, animal, man, horse (26a9) give a universal affirmative *A*-proposition; animal, man, stone (a10) give a universal privative *E*-proposition. Neither of the two sets of terms he offers in the non-modal counter-example involves any green terms; in the non-modal counter-examples all of Aristotle’s terms are red.

false then of course we can never syllogize from true premises to (22).

There are, however, difficulties about what to say about whether snow is necessarily white, and about whether there can be *necessarily white* things. These difficulties come from a tension between Aristotle's treatment of accidental properties and his treatment of necessity. But here in his discussion of contingency in the syllogistic, Aristotle clearly is not questioning this, but is simply assuming that snow is necessarily white. And on this assumption, (22) is false. The tension surrounding any such necessary white is what gives rise to Striker's complaint (1993) about inconsistency in Aristotle's methods. We noticed Striker's complaint in Chapter 3. The problem she identifies involving necessarily white things is not a lone problem. Aristotle also assumes in this part of the *Prior Analytics* that swans are necessarily white, that ravens are necessarily black, and that pitch is necessarily black. Striker can point to any one of these assumptions; each leads to an inconsistency of the sort she finds in Aristotle. Each is inconsistent with the notion that black and white are incidental properties of things, and therefore only ever hold accidentally, never necessarily. As noted earlier, Jeroen van Rijen explains that Aristotle is not very careful about his counter-examples. And certainly the second set of terms Aristotle offers for QA+E are no improvement – they give the following:

(24) All animals are *Q* white
(25) <u>All pitch is not an animal</u>
(26) All pitch is not *M* white

If (26) is supposed to be false Aristotle is telling us that some things – pitch – might fail to be white *by necessity*. There are undoubtedly problems here, but maybe all it shows is that the red/green distinction requires refinement. Or perhaps it is just a fact that it *is* in the nature of pitch that it cannot become white, and in the nature of snow that it *must* be white.

Aristotle's next move is to consider what might happen if the privative premise is particular. At A15, 35b8–11, still discussing the first figure, Aristotle gives more terms and tells us:

> But when the premise put as particular takes something not to belong, there will not be a deduction. Terms for belonging are white, animal, snow; for not belonging, white, animal, pitch (the demonstration must be gotten through the indeterminate).

He has in mind the following premise pairs:

(QA) $\forall x(Bx \supset QAx)$ (QE) $\forall x(Bx \supset Q{\sim}\mathrm{A}x)$
(O) $\exists x(Cx \ \& \ {\sim}Bx)$ (O) $\exists x(Cx \ \& \ {\sim}Bx)$

There will not be a syllogism from either of these. This is already obvious from the fact that Aristotle has just rejected the QA+E schema and the QE+E schema (35a21–23). Here in 35b8–11 we are considering the QA+O schema and the QE+O schema. When we replace a universal privative E-premise with a particular privative O-premise, the proofs work in much the same way. For an M conclusion we have:

Every acorn is a *Q* oak	T
<u>Some stone is not an acorn</u>	T
Some stone is an *M* oak	F

In fact every stone is an *L~oak* and so the conclusion is false. For an *M~* conclusion we have:

Every acorn is a *Q* deciduous tree	T
<u>Some oak is not an acorn</u>	T
Some oak is an *M~* deciduous tree	F

The conclusion is plainly false. So the schema can be falsified. However, as before, Aristotle's own counter-examples involve necessarily white snow and necessarily black pitch.

(27) Every animal is *Q* white
(28) <u>Some snow is not an animal</u>
(29) Some snow is *M~* white

(30) Every animal is *Q* white
(31) <u>Some pitch is not an animal</u>
(32) Some pitch is *M* white

When we put his terms into the modal schemas we get once more what Aristotle claims are true premises but false conclusions.

At A15, 35b12–14 Aristotle sets out another family of X+Q schemas which do not allow deductions:

> If the universal is put in relation to the minor extreme and the particular in relation to the major, then there will in no way be a deduction, regardless of whether either term is privative or positive, possible or belonging.

A few lines later he gives 'common terms' for these and other obviously invalid schemas (e.g, schemas involving two particular or two indeterminate premises):

> Common terms for belonging of necessity are animal, white, man; terms for not being possible are animal, white, coat. (35b20)

These give the following:

Some white is an animal	Some white is an animal	T
<u>Every man is *Q* not white</u>	<u>Every coat is *Q* white</u>	T
Some man is not an animal	Some coat is an animal	F

Neither a privative nor an affirmative conclusion follows. So 'there will in no way be a deduction'.

The point to notice is that Aristotle's standard way of invalidating these schemes involves adapting the terms from the earlier non-modal counter-examples, now, to the modal case. When he does adapt the earlier counter-examples, he uses a term which is construed as red when applied to some things, but green when applied to other things. But in each case we have seen that genuine counter-examples can be given. It is becoming clear as well that even while there are tensions emerging in the contingent syllogistic about the precise meaning of Q, Aristotle nonetheless relies on the incompatibility of $Q\varphi$ and $L{\sim}\varphi$ in his proofs. These tensions become more apparent in the L+Q combinations, as we shall see in the next chapter.

Chapter 15
First Figure L+Q, Q+L (A16)

In *Prior Analytics* A16 Aristotle describes first figure syllogisms in which one premise is an apodeictic L-proposition and the other premise is a proposition about Q-contingency.

Table 22a
L+Q and Q+L Universal Syllogisms in the First Figure (A16)

L+Q:	**Q+L:**
Barbara LQM (35b36–36a2)	Barbara QLQ (36a2–7)
$\forall x(Bx \supset LAx)$	$\forall x(Bx \supset QAx)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset LBx)}$
$\forall x(Cx \supset MAx)$	$\forall x(Cx \supset QAx)$
Celarent LQX (36a7–17)	Celarent QLQ (36a17–25)
$\forall x(Bx \supset L{\sim}Ax)$	$\forall x(Bx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset LBx)}$
$\forall x(Cx \supset {\sim}Ax)$	$\forall x(Cx \supset Q{\sim}Ax)$
CC-Barbara LQM (35b27–30; 36a25–27)	
$\forall x(Bx \supset LAx)$	
$\underline{\forall x(Cx \supset Q{\sim}Bx)}$	
$\forall x(Cx \supset MAx)$	

Aristotle tells us that the proof of Barbara LQM is not immediate – he calls it an 'incomplete deduction'. Here is his explanation at 35b37–36a2:

> It is evident, then, that a necessary conclusion does not come about when the terms are affirmative. For (1) let A belong to every B of necessity, and (2) let it be possible for B to belong to every C. There will then be an incomplete deduction that (3) it is possible for A to belong to every C. That it is incomplete is clear from the demonstration; it will be proved in the same way as in the previous cases.

'The same way as in the previous cases' can only refer to Barbara XQM and related schemas whose validity depends upon realizing the possibility in the Q-premise. In such cases, Aristotle is quite clear that only an *M* conclusion and not a *Q* conclusion follows.

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
DOI 10.1007/978-94-007-0050-5_15,

Barbara LQM

(1)	$\forall x(Bx \supset LAx)$
(2)	$\underline{\forall x(Cx \supset QBx)}$
(3)	$\forall x(Cx \supset MAx)$

So the proof for Barbara LQM proceeds as follows:

(4)	$\exists x(Cx \ \& \ L{\sim}Ax)$	Reductio Assumption
(5)	$\forall x(Cx \supset Bx)$	Realization (2)
(6)	$\exists x(Bx \ \& \ {\sim}Ax)$	Bocardo LXX (4)(5)

But (6) contradicts (1), and so (3) follows.

Of course, the truth of premise (1) requires that the *A* term be red. And so in the conclusion (3) the *A* term is red, but a red *A* can be M-possible. As with Barbara XQM, the LQM syllogism is valid provided the *B* term is red. If the *B* term is not red, then (7) provides a counter-example:

(7)	Everything in the paddock is a necessary horse	T
	<u>Every man could be in the paddock</u>	T
	Every man could be a horse	F

Compare this with an example in which *B* is a red term:

(8)	Every oak is by necessity a deciduous tree
	<u>Every acorn could become an oak</u>
	Every acorn could become a deciduous tree

The syllogism at 35b27–30; 36a25–26 needs complementary conversion to turn it into Barbara LQM.

CC-Barbara LQM

$\forall x(Bx \supset LAx)$

$\underline{\forall x(Cx \supset Q{\sim}Bx)}$

$\forall x(Cx \supset MAx)$

Aristotle's discussion of the proof is very brief. In 36a25–26 he tells us that 'there will be a deduction through conversion.' All that Aristotle says about the CC-Barbara LQM conclusion is that it is 'as in the previous cases' which must mean it is an M-conclusion. The proof then involves the (covert) combination of complementary conversion with realization, as in the previous chapter.

At 36a3–6 Aristotle discusses Barbara QLQ. He tells us that Barbara QLQ is valid:

> Next, (9) let it be possible for A to belong to every B and (10) let B belong to every C of necessity. There will then be a deduction that (11) it is possible for A to belong to every C, but not that it belongs; and it will be complete rather than incomplete (for it is brought to completion at once by means of the initial premises).

And so Barbara QLQ works just the same as Barbara XXX in Table 2 and Barbara QXQ in Table 20.

	Barbara QLQ
(9)	$\forall x(Bx \supset QAx)$
(10)	$\underline{\forall x(Cx \supset LBx)}$
(11)	$\forall x(Cx \supset QAx)$

The proof is almost immediate: ‘it is brought to completion at once by means of the initial premises.’ All that is required is the substitution of QAx for Ax in Barbara XXX, and the principle $L\varphi \supset \varphi$, but Aristotle regards this as so obvious that it does not prevent the proof being completed ‘at once’.

Celarent LQX (36a7–17) needs special comment – for it has an assertoric X conclusion where we might expect an M conclusion. Celarent LQM of course trivially follows from Celarent XQM, and in fact can easily be proved by realization:

(12)	$\forall x(Bx \supset L{\sim}Ax)$	
	$\forall x(Cx \supset QBx)$	
	| $\forall x(Cx \supset Bx)$	Realization hypothesis
	| $\forall x(Cx \supset L{\sim}Ax)$	Celarent LXL
	$\forall x(Cx \supset M{\sim}Ax)$	discharge

But Aristotle does not follow this method when he singles out Celarent L+Q for a lengthy discussion. It will help to look closely at his explanation:

> If the premises are not the same in form, first let the privative premise be necessary, and (13) let it not be possible for A to belong to any B, but (14) let it be possible for B to belong to every C. Then (15) it is necessary for A to belong to no C. For let A be put as belonging either

> to every or (16) to some C.[1] But it was assumed (13) to be possible for A to belong to no B. Therefore, since the privative (13) converts, (17) neither is it possible for B to belong to any A. But A was put as belonging either to every or (16) to some C. Consequently, it (18) would not be possible for B to belong to any C, or to every C: but it was initially assumed (14) to be possible for B to belong to every C. And it is evident that a deduction of (19) being possible not to belong also comes about, given that there is one of not belonging. (36a7–17)

Some of the structure of the proof is provided by Aristotle's preliminary remarks at the start of A16.

> If one [premise] is affirmative and the other privative, then... when the privative is necessary, then the conclusion is both of being possible not to belong and of not belonging... There will be no deduction of not belonging of necessity (for 'does not of necessity belong' is different from 'of necessity does not belong'). (35b29–36)

As Ross explains: "a conclusion of the form 'C is necessarily not A' can never be drawn" (Ross, p. 345). Ross takes this to mean that (15) describes an assertoric proposition

(15) $\forall x(Cx \supset {\sim}Ax)$

I.e., the syllogism is

Celarent LQX
(13) $\forall x(Bx \supset L{\sim}Ax)$
(14) $\underline{\forall x(Cx \supset QBx)}$
(15) $\forall x(Cx \supset {\sim}Ax)$

Ross's proof (1957, p. 47) – translated into modal predicate logic – proceeds as follows:

(16)	$\exists x(Cx \,\&\, Ax)$	Reductio assumption
(17)	$\forall x(Ax \supset L{\sim}Bx)$	LE-conversion (13)
(18)	$\exists x(Cx \,\&\, L{\sim}Bx)$	Ferio LXL (16)(17)

[1] In footnote 3 on p. 149, I mentioned Aristotle's uncertainty about modal reductios. In the present discussion Aristotle appears to recognise the problem explicitly, since he gives both a universal and a particular as the reductio assumption.

(18) contradicts (14), so (13)(14)(15) is valid – i.e., Celarent LQX is valid. Ross also explains that since $\varphi \supset M\varphi$, then since (15) is a true conclusion, so too is

(19) $\forall x(Cx \supset M{\sim}Ax)$

So Celarent LQM is valid. This is Ross's explanation, and clearly it makes good sense of 36a7–17, provided we take (15) 'it is necessary for *A* to belong to no *C*' to be referring to the 'relative necessity' of a conclusion given the premises, as described on p. 20. One of the oddities of Aristotle's proof at 37a7–17 is that it involves LE-conversion of (13) into (17). It is the first time Aristotle explicitly appeals to LE-conversion in a proof of a first figure syllogism. (Aristotle does not remark on this – maybe he thinks that a direct (reductio) proof is preferable to proof through realization.)

There are according to Ross, eight moods in which Aristotle claims an assertoric conclusion from L+Q or Q+L premises. These are Celarent LQX, Ferio LQX, Cesare LQX, Camestres QLX, Festino LQX, Felapton LQX, Bocardo LQX, and Ferison LQX. According to Ross, Aristotle claims, in addition to an X conclusion, an M conclusion for each Celarent LQM, Cesare LQM, Camestres QLM. Aristotle does clearly claim X conclusions, and so it will be helpful to refer to the cases in which he does make this claim. I will call it the LQX phenomenon.

The situation is simpler with the next syllogism: Celarent QLQ is trivial and immediate.

Celarent QLQ (36a18)
$\forall x(Bx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset LBx)}$
$\forall x(Cx \supset Q{\sim}Ax)$

The proof works like the proof for Barbara QLQ at 36a3.

These syllogisms in Table 22a are 'the universals', Aristotle tells us 'The situation will also be the same in the case of the particular deductions'. (36a31) This should give us the syllogisms listed in Table 22b:

Table 22b
Q+L and L+Q Particular Syllogisms in the First Figure (A16)

<u>L+Q</u>:	**<u>Q+L</u>:**
Darii LQM (36a40–b2) $\forall x(Bx \supset LAx)$ $\underline{\exists x(Cx \,\&\, QBx)}$ $\exists x(Cx \,\&\, MAx)$	Darii QLQ (35b23–28) $\exists x(Bx \,\&\, QAx)$ $\underline{\forall x(Cx \supset LBx)}$ $\exists x(Cx \,\&\, QAx)$
Ferio LQX (36a34–39) $\forall x(Bx \supset L{\sim}Ax)$ $\underline{\exists x(Cx \,\&\, QBx)}$ $\exists x(Cx \,\&\, {\sim}Ax)$	Ferio QLQ (36a39–b2) $\forall x(Bx \supset Q{\sim}Ax)$ $\underline{\exists x(Cx \,\&\, LBx)}$ $\exists x(Cx \,\&\, Q{\sim}Ax)$
CC-Darii LQM (35b28–30) $\forall x(Bx \supset LAx)$ $\underline{\exists x(Cx \,\&\, Q{\sim}Bx)}$ $\exists x(Cx \,\&\, MAx)$	

If we take Aristotle's general remark at 36a31 at face value, and suppose that the particulars work 'the same' as the universals, then Table 22b is correct. Darii LQM and its complementary CC-Darii LQM are proved like Barbara LQM and its complementary CC-Barbara LQM. Darii and Ferio QLQ are immediate like Barbara QLQ. Ferio LQX is like Celarent LQX and needs special comment.

Ferio LQX involves the LQX phenomenon. Aristotle wants an X conclusion and an M conclusion. As in the case of Celarent LQM it is easy to give a proof through realization.

Ferio LQM (by realization)

$\forall x(Bx \supset L{\sim}Ax)$	
$\exists x(Cx \,\&\, QBx)$	
$\quad\mid\quad \exists x(Cx \,\&\, Bx)$	Realization
$\quad\mid\quad \exists x(Cx \,\&\, L{\sim}Ax)$	Ferio LXL
$\exists x(Cx \,\&\, M{\sim}Ax)$	discharge

But Aristotle does not proceed this way. As in the case of Celarent LQM he uses a reductio with LE-conversion.

> For example, if (1) it is not possible for A to belong to any of the Bs and (2) possible for B to belong to some of the Cs, then (3) it is

> necessary for A not to belong to some of the Cs. For if (4) it belongs to every C but (1) it is not possible for A to belong to any B, then (5) neither is it possible for B to belong to any A. Consequently, if (4) A belongs to every C, then (6) it is not possible for B to belong to any of the Cs. But it was assumed to be possible to some. (36a34–9)

If the remarks at 35b29–35 are a guide, then the LQX phenomenon is at play. Following Ross, let's take the necessity at step (3) in Aristotle's discussion to mean that a conclusion follows of the form 'Some C is not A.'

(1)	$\forall x(Bx \supset L{\sim}Ax)$	
(2)	$\exists x(Cx \,\&\, QBx)$	
(3)	$\exists x(Cx \,\&\, {\sim}Ax)$	
Proof by reductio:		
(4)	$\forall x(Cx \supset Ax)$	Reductio hypothesis
(5)	$\forall x(Ax \supset L{\sim}Bx)$	LE-conversion (1)
(6)	$\forall x(Cx \supset L{\sim}Bx)$	First figure (4)(5)

Since this is a reductio proof the contradiction would have to be between (6) and (2).

Also in A16 Aristotle wants to establish that there are no other first figure L+Q or Q+L syllogisms. It is important to note right away that any rejected Q+L schema discussed here *uses the same terms* as the corresponding Q+X schemas in the last chapter and indeed each of those Q+X combinations was also a Q+L. And in fact there are cases where even after realizing a *Q* possibility we still cannot syllogize. Look for instance at the premise pair described in 36a27.

(36a27)
$\forall x(Bx \supset QAx)$
$\forall x(Cx \supset L{\sim}Bx)$
'no deduction'

Aristotle tells us there will not be a deduction from such premises. He does not give a detailed explanation about why there will not be. But there is no syllogism from the corresponding *non-modal* premises:

$\forall x(Bx \supset Ax)$
$\forall x(Cx \supset {\sim}Bx)$
'no deduction'

Aristotle explains the non-modal case in a much earlier passage, *An.Pr.* A4, 26a3–5, where he gives the following sets of terms:

> animal, man, horse;
> animal, man, stone.

If there is no way to syllogize non-modally then we cannot complete the modal proof through realizing. Aristotle offers terms for the Q+L case as well. He tells us terms for belonging are white, animal, snow; terms for not belonging are white, animal, pitch (36a30). These are *the same terms* as those he gave for Q+X in A15 and so involve the same issues as are discussed there.

The invalid schemas involving a particular premise are similar. That is, in these cases, too, there is no assertoric conclusion. Aristotle highlights this point about an X-conclusion, remarking that 'there will not be a deduction of belonging':

> And when the particular affirmative premise in the privative deduction (that is, BC) is necessary (20), or the universal premise in the positive deduction (that is, AB) (21), then there will not be a deduction of belonging. (The demonstration is the same as that in the previous cases.) (36a39–b2)

The invalid schemas Aristotle describes are as follows:

(20) Ferio QLX

$\forall x(Bx \supset Q{\sim}Ax)$
$\underline{\exists x(Cx \,\&\, LBx)}$
$\exists x(Cx \,\&\, {\sim}Ax)$

(21) Darii LQX

$\forall x(Bx \supset LAx)$
$\underline{\exists x(Cx \,\&\, QBx)}$
$\exists x(Cx \,\&\, Ax)$

Aristotle is concerned to establish that we cannot syllogize from these to a conclusion of belonging. That is, there is no assertoric X-conclusion from these premise combinations. That is, he does *not* allow Ferio QLX or Darii LQX (*pace* Smith and Tredennick.

The next schemas he considers are those described in A16, 36b3–6, and the point there is again the same – there is no assertoric conclusion from the schemas in question:

And if the universal premise, whether affirmative (22) or privative (23), is put as possible in relation to the minor extreme, and the particular premise (24) as necessary in relation to the major extreme, there will not be a deduction. (Terms for belonging of necessity are animal, white, man; for not being possible, animal, white, coat.)[2]

(24)	$\exists x(Bx \;\&\; LAx)$	(24)	$\exists x(Bx \;\&\; LAx)$
(22)	$\forall x(Cx \supset QBx)$	(23)	$\forall x(Cx \supset Q{\sim}Bx)$

Of course, (22) ≡ (23) by complementary conversion, so either (24) (22) or (24) (23) both yield a conclusion or neither does. Aristotle wants to show that no deduction of any kind comes about from these premises. In the corresponding non-modal X+X case, at A4, 26a31–35 Aristotle makes it clear that there is no deduction in either case:

> But if the universal is put in relation to the minor extreme (whether positive or privative), then there will not be a deduction... terms for belonging are good, condition, wisdom; terms for not belonging, good, condition, ignorance. (A4, 26a31–35)

In the corresponding modal case X+Q, at A15, 35b12–14 Aristotle makes it clear that there is no deduction. He gives 'common terms' for this and several related cases:

> Common terms for belonging of necessity are animal, white, man; terms for not being possible are animal, white, coat. (35b20)

In the case of L+Q, at A16, 36b3–6, Aristotle again gives terms for constructing counter-examples. He, again, describes these as 'terms for belonging of necessity' and 'for not being possible'. And the terms here are exactly the same as the terms he offered earlier in the X+Q case: animal, white, man; animal, white, coat. When we put these terms into the L+Q premises, it is clear that there is no syllogism.

[2]Ross suggests an excision of the phrase 'in relation to the major extreme' at 36b5. Smith notes a similarity between the language of this passage and the language used earlier in A4, 26a18–19 and 26a39–b1.

Some white is an *L* animal	Some white is an *L* animal	T
<u>Every man is *Q* not white</u>	<u>Every coat is *Q* white</u>	T
Some man is an *M*~ animal	Some coat is an *M* animal	F

I have given the weakest possible privative and affirmative conclusions with Aristotle's terms, and neither a privative nor an affirmative conclusion follows. 'Some man is an *M*~ animal' is false because men are necessary animals. 'Some coat is an *M* animal' is false because coats are by nature not animals. So, plainly, in neither case will there be a deduction.

We can summarize Aristotle's first figure counter-examples about contingency from A14, A15 and A16 by the following table. Table 23 lists the different terms Aristotle offers regarding each Q+Q, X+Q, Q+X, Q+L, and L+Q premise combinations. Notice that each set of terms involves at least one green term. The green terms in Table 23 are highlighted by small capitals.

Table 23
Terms in the Problematic Syllogistic

Modality:	Chapter:	line:	terms for belonging of necessity	terms for not possibly belonging
			First Figure	
Q+Q	A14	33b8	⟨animal, WHITE, man⟩	⟨animal, WHITE, coat⟩
X+Q	A15	34b11–18	⟨animal, MOVING, man⟩	⟨man, MOVING, horse⟩
		34b33–35a2	⟨MOVING, science, man⟩	⟨raven, REASONING, man⟩
Q+X		35a20–24	⟨WHITE, animal, snow⟩	⟨WHITE, animal, pitch⟩
		35b8–14	⟨WHITE, animal, snow⟩	⟨WHITE, animal, pitch⟩
		35b15–22	⟨animal, WHITE, man⟩	⟨animal, WHITE, coat⟩
Q+L	A16	36a27–31	⟨WHITE, animal, snow⟩	⟨WHITE, animal, pitch⟩
		36b3–7	⟨animal, WHITE, man⟩	⟨animal, WHITE, coat⟩
		36b7–10	⟨animal, WHITE, raven⟩	⟨animal, WHITE, pitch⟩
		36b11–12	⟨animal, WHITE, swan⟩	⟨animal, WHITE, snow⟩
		36b12–18	⟨animal, WHITE, man⟩	⟨animal, WHITE, inanimate⟩

Chapter 16
Contingency in the 2nd Figure (A17–19)

When we turn to consider *An.Pr.* A17 where Aristotle discusses Q+Qs in the second figure, we find something very different right at the start. There are not any valid second figure Q+Q syllogisms. Aristotle rejects *all* Q+Q premise combinations in the second figure:

> In the second figure, when both premises take something to be possible, there will be no deduction, whether they are put as positive or as privative, as universal or as particular. (36b26–29)

All but one of his second figure deductions depend explicitly on E-conversions. In a second figure Q+Q premise combination, the E-conversion would have to be QE-conversion. But there is no QE-conversion. Privative universal premises about possibility do not convert. So when both premises involve Q-contingency, we cannot use E-conversion to establish validity. Aristotle's first objective in A17 is to prove that QE-premises do not convert:

> First, then, it must be proved that a privative premise of possibility does not convert: that is, if it is possible for A to belong to no B, it is not also necessary for it to be possible for B to belong to no A. For let this be assumed, and let it be possible for B to belong to no A. Then, since affirmations in being possible convert with their denials (contraries as well as oppositions), and it is possible for B to belong to no A, then evidently it would also be possible for B to belong to every A. But this is incorrect: for if it is possible for this to belong to every that, it is not necessary for it to be possible for that to belong to every this. Consequently, a privative universal does not convert. (36b35–37a3)

Aristotle's proof is elegant. He shows that we do not have QE-conversion by showing what would happen if the conversion were allowed. Suppose, for example, the following instance of conversion:

$$(1) \qquad \forall x(Bx \supset Q{\sim}Ax) \supset \forall x(Ax \supset Q{\sim}Bx)$$

Given (1): 'Then, since affirmations in being possible convert with their denials' (36b40), by complementary conversion, we would also have

$$(2) \qquad \forall x(Bx \supset QAx) \supset \forall x(Ax \supset QBx).$$

But (2) is 'incorrect'. (2) is not valid because 'if it is possible for this [A] to belong to

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every that [B], it is not necessary for it to be possible for that [B] to belong to every this [A]' (37a2–3). Consider what happens when we take the modals out of (2) and look at its non-modal, assertoric form. This would be (3):

$$(3) \qquad \forall x(Bx \supset Ax) \supset \forall x(Ax \supset Bx).$$

(3) is not even valid assertorically. This is because Aristotle's A-premises only convert *in part* – that is, an A-premise always converts to an I-premise, never to another A-premise. Since (3) is rejected for assertorics, (2) is rejected for Qs. Since (1) is equivalent to (2) by complementary conversion, (1) must also be rejected. So, the fact that complementary conversion turns a QE-premise into a QA-premise is what ultimately rules out the QE-conversions. This passage provides further evidence that Aristotle expresses 'no B is QA' as $\forall x(Bx \supset Q{\sim}Ax)$ and not as $\forall x(Bx \supset {\sim}QAx)$.

The consequences for the syllogisms are easy to see. Recall that in the second figure non-modal syllogisms, all but Baroco depend straightforwardly on E-conversion. In the case of the second figure QQQ syllogisms all but Baroco depend on modal QE-conversion. Since Aristotle rejects any QE-conversion, he counts as invalid all of the second figure Q+Q syllogisms that require QE-conversion. But this leaves Aristotle with the question of what to say about Baroco QQQ? Earlier in the non-modal syllogistic in A5, Aristotle establishes the validity of Baroco XXX with a non-modal reductio proof which proceeds as follows:

Baroco XXX (A5, 27a36–b1)[1]

$(4) \qquad \forall x(Bx \supset Ax)$

$(5) \qquad \exists x(Cx \mathbin{\&} {\sim}Ax)$

$(6) \qquad \exists x(Cx \mathbin{\&} {\sim}Bx)$

He asks us to suppose:

$(7) \qquad \forall x(Cx \supset Bx).$

Then, from (7) and (4), we would have:

$(8) \qquad \forall x(Cx \supset Ax).$

But (8) and (5) contradict, so (6) must in fact be true, and the non-modal syllogism (4) (5) (6) is valid. Consider the weakest version of a Q+Q Baroco:

[1] In fact, Aristotle uses different term letters in A5, but I have regularized them to the usual term letters *A*, *B*, and *C*.

Baroco QQM

(9) $\forall x(Bx \supset QAx)$
(10) $\underline{\exists x(Cx \ \&\ Q{\sim}Ax)}$
(11) $\exists x(Cx \ \&\ M{\sim}Bx)$

Suppose we try to prove Baroco QQM through impossibility in the manner of proof of Baroco XXX, above. Then we begin by denying (11). That is, we suppose

(12) $\forall x(Cx \supset LBx)$.

If we take (12) as true, then by Barbara QLQ we get

(13) $\forall x(Cx \supset QAx)$.

But here there is a difference. While (8) contradicted (5) in Baroco XXX, here in Baroco QQM (13) does not contradict (10). In fact, (10) is equivalent to

(14) $\exists x(Cx \ \&\ QAx)$,

and (14) is certainly compatible with (13). So Baroco QQM is invalid and not a syllogism. In fact, (13) entails (14). So, although the proof of Baroco XXX does not involve conversion, the method Aristotle uses in arguing against QE-conversion can still be used in disallowing Baroco QQM.

Aristotle's claim in A17 that in the second figure there are no syllogisms when both premises are Q-premises answers the question raised earlier in Chapter 11 about whether Table 17 or 18 is closer to Aristotle's text. The absence of second figure QQQs is evidence that Table 18 cannot be right. Recall that the analysis at work in Table 18 is the version which makes the second figure QQQs come out valid. Since Aristotle says there are no valid Q+Q syllogisms in the second figure, then we cannot approach the contingent QQQ syllogisms in the same way that *Prior Analytics* A8 tells us to approach LLL and MMM syllogisms. That is, we cannot in all cases obtain valid QQQ syllogisms by uniform substitution of modal for non-modal terms, and that is why Table 18 is wrong.

Next, in *An.Pr.* A18, Aristotle considers second figure combinations in which one premise is non-modal and one is a Q-proposition. These are proved by conversion into mixed first figure syllogisms whose proofs depend on the realization method. This of course brings with it a restriction to red terms in certain cases. In the second figure all combinations involve one affirmative and one privative premise. Aristotle begins his A18 study of the mixed X+Q cases with a general comment: if the positive premise is non-modal and the privative premise is the Q-premise then there is never a deduction, 37b19–21. This is due to the fact that each of the second figure syllogisms Cesare, Camestres, Festino requires E-conversion, so if the E-premise is also the Q-premise then we cannot convert – because, as we saw in A17, there is no QE-conversion. Where the privative premise is particular, we have the second figure Baroco. And Baroco XQM and QXM both fail for the same reason that Baroco QQM fails.

Table 24 lists the valid second figure X+Q and Q+X syllogisms discussed in A18.

Table 24
Second Figure Syllogisms about Q-Contingency
X+Q and Q+X (A18)

<u>*Universals:*</u>

Cesare XQM (37b24–28)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\forall x(Cx \supset QAx)}$
$\forall x(Cx \supset M{\sim}Bx)$

CC-Cesare XQM (37b29–35)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\forall x(Cx \supset Q{\sim}Ax)}$
$\forall x(Cx \supset M{\sim}Bx)$

Camestres QXM (37b29)
$\forall x(Bx \supset QAx)$
$\underline{\forall x(Cx \supset {\sim}Ax)}$
$\forall x(Cx \supset M{\sim}Bx)$

CC-Camestres XQM (37b29–35)
$\forall x(Bx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset {\sim}Ax)}$
$\forall x(Cx \supset M{\sim}Bx)$

<u>*Particulars:*</u>

Festino XQM (38a3–4)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\exists x(Cx \,\&\, QAx)}$
$\exists x(Cx \,\&\, M{\sim}Bx)$

CC-Festino XQM (38a4–7)
$\forall x(Bx \supset {\sim}Ax)$
$\underline{\exists x(Cx \,\&\, Q{\sim}Ax)}$
$\exists x(Cx \,\&\, M{\sim}Bx)$

The first syllogism Aristotle discusses is Cesare XQM, at 37b24–28:

> For (15) let A have been taken to belong to no B and (16) to be possible to belong to every C. Then, if the privative premise (15) is converted, (17) B will belong to no A. But it was possible for A to belong to every C (16), so a deduction that (18) it is possible for B to belong to no C comes about through the first figure.

Cesare XQM

(15)	$\forall x(Bx \supset {\sim}Ax)$	
(16)	$\forall x(Cx \supset QAx)$	
(17)	$\forall x(Ax \supset {\sim}Bx)$	E-conversion (15)

(18) $\forall x(Cx \supset M{\sim}Bx)$ Celarent XQM (17)(16) [Table 21]

'And similarly also if the privative premise is put in relation to C.' (37b29) This must be Camestres QXM, and the proof would require two separate conversions: one non-modal conversion and a final ME-conversion:

Camestres QXM

(19)	$\forall x(Bx \supset QAx)$	
(20)	$\forall x(Cx \supset {\sim}Ax)$	
(21)	$\forall x(Ax \supset {\sim}Cx)$	E-conversion (20)
(22)	$\forall x(Bx \supset M{\sim}Cx)$	Celarent XQM (21)(19)
(23)	$\forall x(Cx \supset M{\sim}Bx)$	ME-conversion (22)

As we saw in Chapter 10, ME-conversion requires red terms. So, since Camestres QXM requires ME-conversion, Camestres QXM is restrictedly valid. Furthermore, since both Cesare XQM and Camestres QXM depend on Celarent XQM, then (covert) realization is part of the proofs. Aristotle also validates the complementary syllogisms to Cesare XQM and Camestres QXM, CC-Cesare XQM and CC-Camestres QXM.

> But if both premises are privative and one signifies not belonging and the other being possible, then nothing necessary results through the actual premises taken, but when the possible premise is accordingly converted, a deduction that it is possible for B to belong to no C comes about, as in the previous cases (for it will again be the first figure). (37b29–35)

In order to syllogize here, we must first use complementary conversion in order to turn the 'privative' Q-premise into an 'affirmative' one. Then a deduction comes about – one which has already been established.

CC-Cesare XQM

(24)	$\forall x(Bx \supset {\sim}Ax)$	
(25)	$\forall x(Cx \supset Q{\sim}Ax)$	
(26)	$\forall x(Cx \supset QAx)$	CC (25)
(27)	$\forall x(Cx \supset M{\sim}Bx)$	Cesare XQM (24)(26)

CC-Camestres QXM

(28)	$\forall x(Bx \supset Q{\sim}Ax)$	
(29)	$\forall x(Cx \supset {\sim}Ax)$	
(30)	$\forall x(Bx \supset QAx)$	CC (24)
(31)	$\forall x(Cx \supset M{\sim}Bx)$	Camestres QXM (30)(29)

The next passage to discuss begins at 37b36. Aristotle explains that there is no X+Q or Q+X syllogism in the second figure from two affirmative premises. Of course there is no second figure syllogism from two affirmative premises in the non-modal syllogistic either,[2] but because of complementary conversion, contingent premises are unruly and can sometimes lead to new combinations, and so Aristotle's usual practice in the contingent syllogistic is to consider each premise combination in turn. Here, he gives terms to generate counter-examples against any conclusion:

> But if both premises are put as positive, there will not be a deduction. Terms for belonging are health, animal, man; terms for not belonging are health, horse, man. (37b35–38)

The possible Q+X and X+Q affirmative combinations include each of the following:

Every animal is *Q* healthy	T
<u>Every man is healthy</u>	T
Some man is an *M*~ animal	F
Every animal is healthy	T
<u>Every man is *Q* healthy</u>	T
Some man is an *M*~ animal	F
Every horse is *Q* healthy	T
<u>Every man is healthy</u>	T
Some man is an *M* horse	F
Every horse is healthy	T
<u>Every man is *Q* healthy</u>	T
Some man is an *M* horse	F

In each case I have given the weakest affirmative or the weakest privative conclusion using Aristotle's terms. There is no syllogism from two affirmatives in the second figure.

This completes Aristotle's treatment of the second figure X+Q universals. He

[2] In the non-modal syllogistic of A5, Aristotle uses terms to establish the invalidity of the corresponding non-modal schema. In A5, 27a18—20, 'terms for belonging are substance, animal, man; for not belonging, substance, animal, number (the middle is substance).'

has, however, left some explanations out. He does not give any explanation or counter-examples to demonstrate the failure of Q+X and X+Q universals in which the privative premise is Q-contingent. He stipulates that these are not valid in his opening remarks to A18, but he never explains why. Probably he takes it for granted that a reader will by this stage readily recognise that *Q* is equivalent to *Q*~. In any case, we do not find in A18 a discussion of Cesare QXM, which Aristotle probably thinks must be invalid because the proof of Cesare depends upon E-conversion and where the E-premise is also a Q-premise, there can be no conversion. As we shall see, later in A19, Aristotle gives counter-examples to establish the invalidity of Cesare QLM (A19, 38a27b5).

He goes on to explain, next in A18, that where we have universal deductions, we also have particular deductions: 'the situation will also be the same in the case of the particular deductions. (37b39)

> For when the affirmative premise is put as belonging, whether taken as universal or as particular, there will be no deduction (this is also proved similarly to the previous cases, and through the same terms). (37b40–38a2)

So, when the affirmative premise is a non-modal X-premise, there is never an X+Q or a Q+X syllogism. But 'when the privative premise is belonging [i.e., when the privative is non-modal], there will be a deduction through conversion, as in the previous cases.' (38a3–4) That is, when the privative premise is the non-modal X-premise, then the particular syllogism is like the universal syllogism. Along with the universal Cesare XQM, we have the particular Festino XQM. The particular proof through conversion would proceed as follows:

Festino XQM

(1)	$\forall x(Bx \supset {\sim}Ax)$	
(2)	$\exists x(Cx \ \& \ QAx)$	
(3)	$\forall x(Ax \supset {\sim}Bx)$	E-conversion (1)
(4)	$\exists x(Cx \ \& \ M{\sim}Bx)$	Ferio XQM (3)(2) [Table 21]

Similarly, there can be particular syllogisms involving complementary conversion:

> Next, if both intervals are taken as privative and the interval of not belonging [i.e., if the X-premise] is universal, then there will not be a necessity from the actual premises, but when the premise of being possible is converted as in the previous cases, there will be a deduction. (38a5–7)

CC-Festino XQM

(1)	$\forall x(Bx \supset \sim Ax)$	
(2)	$\exists x(Cx \;\&\; Q\sim Ax)$	
(3)	$\exists x(Cx \;\&\; QAx)$	CC (2)
(4)	$\exists x(Cx \;\&\; M\sim Bx)$	Festino XQM (1)(3)

In all other X+Q and Q+X cases Aristotle explains that there will not be a deduction. 'The demonstration [of the particulars] is the same and through the same terms [as the universals]' (38a12). In particular, Baroco QXM cannot be proved by conversion – and the proof through impossibility runs into the same snag that we noticed in the case of Baroco QQM. So the syllogisms in Table 24 are his only valid second figure X+Q and Q+X syllogisms.

The last of the second figure combinations that Aristotle considers are those in A19, in which one premise is about Q-contingency and the other is a necessary L-premise. Aristotle begins A19 with an overview.

> If one of the premises signifies belonging of necessity and the other premise signifies being possible to belong, then when the privative premise is necessary there will be a deduction, not only that it is possible for something not to belong, but also that it does not belong; but when the affirmative premise is necessary, there will not be a deduction. (38a13–16)

The point here in A19 about the L+Q and Q+L combinations links with the point Aristotle made at the start of A18 concerning X+Q and Q+X combinations, and at the start of A16 concerning LE+Q and Q+LE combinations. The schemas in A18 and A19 share this in common: when the privative premise is not the Q-premise, but is either X or L, then there is a syllogism. When the privative premise is the Q-premise then, because there is no QE-conversion (and no QO-conversion), we cannot syllogize. The schemas in A16 and A19 also share something in common: when the privative premise is the L-premise then there will be both an X assertoric and an M conclusion. This is the LQX phenomenon. The valid L+Q and Q+L syllogisms discussed in A19 are listed below in Table 25:

Table 25
Second Figure Syllogisms about Q-Contingency
L+Q and Q+L (A19)

Universals:

Cesare LQX (38a16–20)	CC-Cesare LQX (38b6–12)
$\forall x(Bx \supset L{\sim}Ax)$	$\forall x(Bx \supset L{\sim}Ax)$
$\underline{\forall x(Cx \supset QAx)}$	$\underline{\forall x(Cx \supset Q{\sim}Ax)}$
$\forall x(Cx \supset {\sim}Bx)$	$\forall x(Cx \supset {\sim}Bx)$
Camestres QLX (38a25–26)	CC-Camestres QLX (38b6–12)
$\forall x(Bx \supset QAx)$	$\forall x(Bx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset L{\sim}Ax)}$	$\underline{\forall x(Cx \supset L{\sim}Ax)}$
$\forall x(Cx \supset {\sim}Bx)$	$\forall x(Cx \supset {\sim}Bx)$

Particulars:

Festino LQX (38b24–27)	CC-Festino LQX (38b32–34)
$\forall x(Bx \supset L{\sim}Ax)$	$\forall x(Bx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \,\&\, QAx)}$	$\underline{\exists x(Cx \,\&\, Q{\sim}Ax)}$
$\exists x(Cx \,\&\, {\sim}Bx)$	$\exists x(Cx \,\&\, {\sim}Bx)$

The discussion of these second figure syllogisms begins at 38a16:

> For (1) let A be put as belonging of necessity to no B and (2) as possible to belong to every C. Then, if the privative premise is converted, (3) neither will B belong to any A. But it was possible for A to belong to every C, so a deduction that it is possible for B to belong to no C (4) comes about again through the first figure.

Cesare LQX (38a16–20)

(1)	$\forall x(Bx \supset L{\sim}Ax)$	
(2)	$\forall x(Cx \supset QAx)$	
(3)	$\forall x(Ax \supset L{\sim}Bx)$	LE-conversion (1)
(4)	$\forall x(Cx \supset {\sim}Bx)$	Celarent LQX [Table 22a]

In his opening remarks in A19, 38a13–16, Aristotle signals that the LQX phenomenon affects second figure syllogisms. Certainly if we have Cesare LQX then LQM follows trivially. More significantly, Aristotle's proof of Celarent LQX given on p. 172 above in fact includes a proof (without conversion) of Cesare LQX:

Cesare LQX (without conversion)

(1)	$\forall x(Bx \supset L{\sim}Ax)$	
(2)	$\forall x(Cx \supset QAx)$	
(4)	$\forall x(Cx \supset {\sim}Bx)$	
Proof by reductio:		
(5)	$\exists x(Cx \;\&\; Bx)$	Reductio hypothesis
(6)	$\exists x(Cx \;\&\; L{\sim}Ax)$	Ferio LXL (1)(5)

(6) contradicts (2), so Cesare LQX is valid. And *this* proof does not involve any LE-conversion, and so applies to all terms without restriction. In fact, rather than Cesare LQX depending on Celarent LQX, it seems that Celarent LQX depends on Cesare LQX. And this is odd because Aristotle's proofs of syllogisms in other figures usually depend on first figure syllogisms, and typically involve conversion. It is in fact easy to see why Cesare LQX should be valid if $L{\sim}Bx$ and QBx contradict each other. For (1) says that all the *B*s are necessary not-*A*s, while (2) says that all the *C*s are contingent *A*s. So it follows that nothing can be both *B* and *C*. It is also clear why there is no analogous proof of Cesare XQX, since although a *Q* might contradict an *L*~ it need not contradict a plain ~.

We also have Camestres QLX as valid. Aristotle sees that Camestres is parallel to Cesare. He says 'It can also be proved in the same way if the privative is put in relation to C.' (38a25–26) The proof, which involves two separate conversions, would go as follows:

Camestres QLX

(1)	$\forall x(Bx \supset QAx)$	
(2)	$\forall x(Cx \supset L{\sim}Ax)$	
(3)	$\forall x(Ax \supset L{\sim}Cx)$	LE-conversion (2)
(4)	$\forall x(Bx \supset {\sim}Cx)$	Celarent LQX (3)(1)
(5)	$\forall x(Cx \supset {\sim}Bx)$	E-conversion (4)

Again, the LQX phenomenon will give both an X and an M conclusion. So Camestres QLM is also valid. And as with Cesare a conversion-free proof of Camestres LQX is available:

(6)	$\exists x(Cx \;\&\; Bx)$	Reductio hypothesis
(7)	$\exists x(Cx \;\&\; QAx)$	Darii QXQ (1)(6)

And (7) contradicts (2). By contrast Aristotle's proof, if it requires conversion, in fact requires two conversions – one to get from the first figure to the second, and another to get back.

At 38b6–12 Aristotle explains that complementary conversion gets the additional universal syllogisms listed on the right-hand side of Table 25. The LQX phenomenon mentioned at 38a13–17 gives an X conclusion and an M conclusion. Several lines later in A19, he explains that he has particular second figure L+Q syllogisms also. These are listed at the bottom of Table 25. As Aristotle explains Festino:

> It will be similar in the case of the particular deductions. For whenever the privative premise is both universal and necessary, there will always be a deduction both of being possible and of not belonging (the demonstration is through conversion). (38b24–27)

Festino LQX

(1)	$\forall x(Bx \supset L{\sim}Ax)$	
(2)	$\exists x(Cx \,\&\, QAx)$	
(3)	$\forall x(Ax \supset L{\sim}Bx)$	LE-conversion (1)
(4)	$\exists x(Cx \,\&\, {\sim}Bx)$	Ferio LQX (2)(3)

Aristotle's proof of Festino LQX is through Ferio LQX. As with Cesare LQX Aristotle speaks of both an X and an M conclusion. These follow as before, and a conversion-free proof is available here too, though not suggested in the text.

This completes Aristotle's discussion of the second figure L+Q and Q+L syllogisms considered in A19. They are all listed in Table 25. Aristotle seems to want to claim that no other L+Q or Q+L schemas are valid. At 38a27 he appears to say that Cesare Q+L does not give a syllogism. The schema he offers is as follows:

$$\begin{array}{l} \forall x(Bx \supset Q{\sim}Ax) \\ \underline{\forall x(Cx \supset LAx)} \\ \forall x(Cx \supset Q{\sim}Bx) \end{array}$$

Certainly this schema is not a syllogism, and Aristotle gives terms to show it. The first set is white, man, swan (38a31). The second set is motion, animal, awake (38a41). The first set of terms give us the following:

All men are *Q* not white	T
All swans are *L* white	T
All swans are *Q* not men	F

Recall that in this part of the syllogistic Aristotle routinely assumes that swans are white by necessity. In the present case, that means that we have true premises. Aristotle explains the falsity of the conclusion: 'It is evident, then, that there is no deduction of being possible, for what is of necessity was not possible.' For 'man belongs to no swan of necessity.' The second set of terms gives:

All animals are *Q* not moving	T
All wakeful things are *L* moving	T
All wakeful things are *Q* not animals	F

The conclusion is false because all wakeful things are necessarily animals. The conclusion is an L-proposition, but of course an L-conclusion cannot be guaranteed: 'neither is there a deduction of a necessary conclusion' (38a36). What is interesting about Aristotle's discussion of Cesare Q+L is that his counter-examples do not address Cesare QLX:

Cesare QLX
(1) $\forall x(Bx \supset Q{\sim}Ax)$
(2) $\underline{\forall x(Cx \supset LAx)}$
(3) $\forall x(Cx \supset {\sim}Bx)$

This is certainly valid in LPC, and the terms above would give an instance: if all animals are possibly not moving, and all wakeful things are necessarily moving, then all wakeful things are not animals. In fact, Cesare QLX can be validated by reductio.

(4)	$\exists x(Cx \& Bx)$	Reductio hypothesis
(5)	$\exists x(Cx \& Q{\sim}Ax)$	Ferio QXQ (4)(1)

A contradiction arises between (5) and (2). So we are entitled to conclude (3) – i.e., Cesare QLX is valid. Normally after a proof such as this Aristotle also explains that we are also entitled to an M~ conclusion. That is, by the LQX phenomenon, if Cesare QLX is valid, so too is Cesare QLM. But there is no evidence that Aristotle realizes this. He does not recognize Cesare QLX as a syllogism and so he does not apply the LQX phenomenon and so he does not recognize Cesare QLM as a syllogism.

Patterson (1995, pp. 195–196) looks at the particular case of Cesare QLX, but as Patterson notes, this is not the only case where Aristotle's normal proof methods will validate second figure schemas that Aristotle appears to reject. In a passage at 38b14ff, Aristotle explains that there is no Q+L deduction from two affirmative premises in the second figure.

> However, if the premises are put as positive, there will not be a deduction. For it is evident that there will not be one of not belonging or of not belonging of necessity, because a privative premise has not been taken, either as expressing belonging or as expressing belonging of necessity. But neither will there be a deduction of being possible not to belong... Nor, indeed, will there be a deduction of the opposite affirmations, since B has been shown as of necessity not belonging to C. Therefore, no deduction comes about at all. (38b14–23)

Both Patterson (1995, pp. 197–198) and Thom (1996, pp. 128–130) conclude that Aristotle has made a mistake here about second figure Q+L affirmatives and point out that the fact that there is no corresponding non-modal syllogism is a weak justification in the case of Q syllogisms. Take for instance the syllogism that Thom calls LaQaXe-2, i.e.,

(1) $\forall x(Bx \supset LAx)$
(2) $\underline{\forall x(Cx \supset QAx)}$
(3) $\forall x(Cx \supset {\sim}Bx)$

The reductio proof is as before:

(4) $\exists x(Cx \,\&\, Bx)$ Reductio hypothesis
(5) $\exists x(Cx \,\&\, LAx)$ Darii LXL (1)(4)

And a contradiction arises between (5) and (2), since a Q rules out not only an $L{\sim}$, but also an L. Yet Aristotle is quite emphatic that no deduction comes about at all, and so we need an explanation. Perhaps it goes something like this. In A13, 32a30–b2, Aristotle tells us that premises about what is $Q\varphi$ or $Q{\sim}\varphi$ are 'positive and not privative'. He clearly regards 'all *B*s are contingently not *A*s' and 'some *B*s are contingently not *A*s' as affirmative propositions. He explains the point carefully, and he takes it as obvious in his discussions of what I have called complementary syllogisms – those listed, e.g., on the right of Tables 24 and 25. But while Aristotle reasons that all Q propositions are affirmative, he doesn't, perhaps, appreciate that the same reason could also explain why all Q propositions are also privative. Whether Aristotle does not see this or rejects this, it perhaps explains why he limits his study of contingency in the second figure and does not consider deductions from two 'affirmative' premises. Since he does not apply his usual reductio method to second figure affirmatives, he does not allow any second figure affirmative LQX or QLX syllogisms. And since he doesn't allow those, he doesn't allow the corresponding LQM or QLM syllogisms either.

Aristotle gives some brief comments on the corresponding particular premise combinations. And he tells us ‘it will be similar in the case of the particular deductions.’ (38b24) Festino LQM is valid (38b25–26). Baroco L+Q does not give a syllogism, (38b28). Two particular ‘affirmative’ premises do not give a syllogism (b30). Finally, at 38b38–39a3, he summarizes the situation of L+Q and Q+L combinations in the second figure. Second figure combinations involve one privative premise, and at least one premise must be universal. Further, ‘when it is a privative universal premise which is put as necessary, a deduction always comes about’ – i.e., whenever there is an LE-premise, as in Cesare and Festino L+Q and Camestres Q+L, then there is a deduction. Also, ‘a deduction never comes about when it is the affirmative premise that is put as necessary.’ (38b41)

Chapter 17
Contingency in the 3rd Figure (A20−22)

Aristotle begins Chapter A20 with a sweeping overview about contingency in the third figure. There are, he tells us, deductions from Q+Q premise combinations (A20). There are deductions from X+Q and Q+X premise combinations (A21). In all these cases we reach a conclusion about possibility. And there are third figure deductions from L+Q and Q+L premise combinations. These L+Q and Q+L combinations are the subject of A22, and some involve the LQX phenomenon.

The valid QQQ syllogisms in the third figure work similarly to the valid first figure QQQs. The third figure QQQ syllogisms of Chapter A20 are listed below in Table 26. In constructing the table I have used the minimum amount of ampliation. I have done so in order to avoid extra clutter in the tables and discussion. Of course, nothing is lost by ampliating all the premises in Table 26, and, as we have seen in Chapter 11, ampliated premises are often what Aristotle has in mind for QQQs. The important point, as Table 26 shows, is not about whether to ampliate the Q+Q premises. The important point is that all QQQ syllogisms in the third figure must have ampliated conclusions. (Since all third figure conclusions are particular, the ampliated proposition is weaker than the unampliated one, and an unampliated conclusion cannot always be guaranteed.[1])

[1] As *An. Pr.* A21 and A22 show this is only true of the QQQs. Mixed third figure contingency syllogisms can have unampliated conclusions.

A. Rini, *Aristotle's Modal Proofs*, The New Synthese Historical Library 68,
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Table 26
QQQ Syllogisms in the Third Figure (A20)

Darapti QQQ (39a14–19)	CC-Darapti QQQ (39a26–28)
$\forall x(Cx \supset QAx)$	$\forall x(Cx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset Q{\sim}Bx)}$
$\exists x(QBx \ \& \ QAx)$	$\exists x(QBx \ \& \ QAx)$
Felapton QQQ (39a19–23)	
$\forall x(Cx \supset Q{\sim}Ax)$	
$\underline{\forall x(Cx \supset QBx)}$	
$\exists x(QBx \ \& \ Q{\sim}Ax)$	
Datisi QQQ (39a31–35)	CC-Datisi QQQ (39a38–b2)
$\forall x(Cx \supset QAx)$	$\forall x(Cx \supset Q{\sim}Ax)$
$\underline{\exists x(Cx \ \& \ QBx)}$	$\underline{\exists x(Cx \ \& \ Q{\sim}Bx)}$
$\exists x(QBx \ \& \ QAx)$	$\exists x(QBx \ \& \ QAx)$
Disamis QQQ (39a35–36)	CC-Disamis QQQ (39a38–b2)
$\exists x(Cx \ \& \ QAx)$	$\exists x(Cx \ \& \ Q{\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset Q{\sim}Bx)}$
$\exists x(QBx \ \& \ QAx)$	$\exists x(QBx \ \& \ QAx)$
Bocardo QQQ (39a36–38)	
$\exists x(Cx \ \& \ Q{\sim}Ax)$	
$\underline{\forall x(Cx \supset QBx)}$	
$\exists x(QBx \ \& \ Q{\sim}Ax)$	
Ferison QQQ (39a36–38)	
$\forall x(Cx \supset Q{\sim}Ax)$	
$\underline{\exists x(Cx \ \& \ QBx)}$	
$\exists x(QBx \ \& \ Q{\sim}Ax)$	

The syllogisms in the left-hand column of Table 26 do not require any complementary conversions. Aristotle certainly is aware of this – consider, for instance, his account of Darapti QQQ:

> Let the premises first be possible, then, and let it be possible for A and B to belong to every C. Then, since the affirmative premise converts in part and it is possible for B to belong to every C, it would also be

> possible for C to belong to some B. Consequently, if it is possible for A to belong to every one of the Cs and C to some B, then it is also necessary for it to be possible for A to belong to some of the Bs (for the first figure comes about). (39a14–19)

Aristotle is converting a Q premise but he makes no mention of complementary conversion. He is using ordinary A- and I-conversions. Further, they are simple substitution instances of ordinary non-modal A- and I-conversion.

The proof of Darapti proceeds as follows:

Darapti QQQ (39a14–19)

(1)	$\forall x(Cx \supset QAx)$	Given
(2)	$\forall x(Cx \supset QBx)$	Given
(3)	$\exists x(QBx \,\&\, Cx)$	A-conv (2)
(4)	$\exists x(QBx \,\&\, QAx)$	Darii (3)(1)

Felapton QQQ is exactly the same as Darapti, but with $Q{\sim}Ax$ in place of QAx. Aristotle describes the proof:

> And if it is possible for A to belong to no C and for B to belong to every C, then it is necessary for it to be possible for A not to belong to some B[2] (for it will be the first figure again through conversion). (39a19–23)

This gives us the following steps:

Felapton QQQ (39a19–23)

(1)	$\forall x(Cx \supset Q{\sim}Ax)$	Given
(2)	$\forall x(Cx \supset QBx)$	Given
(3)	$\exists x(QBx \,\&\, Cx)$	A-conv (2)
(4)	$\exists x(QBx \,\&\, Q{\sim}Ax)$	Ferio (3)(1)

Datisi QQQ is discussed at 39a31–35:

> For let it be possible for A to belong to every C and B to some C. Then it will be the first figure again if the particular premise is converted. For if it is possible for A to belong to every C and C to some of the Bs, then it is possible for A to belong to some B. (39a31–35)

[2] I have corrected Smith's typo, putting Aristotle's 'some B' for Smith's 'some C'.

Formally we have:

Datisi QQQ (39a31–35)

(1)	$\forall x(Cx \supset QAx)$	Given
(2)	$\exists x(Cx \,\&\, QBx)$	Given
(3)	$\exists x(QBx \,\&\, Cx)$	I-conv (2)
(4)	$\exists x(QBx \,\&\, QAx)$	Darii (3)(1)

Following on from the proof of Datisi QQQ, Aristotle briefly indicates that Disamis QQQ is valid. All he says is: 'Likewise if the universal is put in relation to BC' (39a35–36). The proof of Disamis QQQ clearly involves the conversion of *ampliated* Q-propositions. This is evident in the move from (4) to (5), below:

Disamis QQQ (39a35–36)

(1)	$\exists x(Cx \,\&\, QAx)$	Given
(2)	$\forall x(Cx \supset QBx)$	Given
(3)	$\exists x(QAx \,\&\, Cx)$	I-conv (1)
(4)	$\exists x(QAx \,\&\, QBx)$	Darii (3)(2)
(5)	$\exists x(QBx \,\&\, QAx)$	Ampliated I-conv (4)

Ampliated I-conversion is yet another substitution instance of ordinary assertoric I-conversion with modal terms for non-modal terms. Here, *both* the *A* and *B* are replaced by modally qualified terms, *QA* and *QB*:

Ampliated I-conversion $\quad \exists x(QAx \,\&\, QBx) \equiv \exists x(QBx \,\&\, QAx)$

The conversion is valid and unproblematic.

Bocardo QQQ is immediate from Disamis QQQ using complementary conversion, and Ferison QQQ is exactly Datisi QQQ, above, with $Q{\sim}Ax$ in place of QAx. Aristotle's explanation of Ferison QQQ is brief but simple:

> And similarly also if AC should be privative and BC affirmative (for it will again be the first figure through conversion). (39a36–38)

Ferison QQQ (39a36)

(1)	$\forall x(Cx \supset Q{\sim}Ax)$	Given
(2)	$\exists x(Cx \,\&\, QBx)$	Given
(3)	$\exists x(QBx \,\&\, Cx)$	I-conv (2)
(4)	$\exists x(QBx \,\&\, Q{\sim}Ax)$	Ferio (3)(1)

This completes Aristotle's proofs of all the syllogisms on the left-hand side of Table 26. In each case validity requires a substitution instance of a non-modal conversion principle.

Let's turn next to the syllogisms on the right-hand side of Table 26. These all involve complementary conversions. Aristotle gives a general comment about proving these three syllogisms:

> And if both premises are put as privative, then there will not be a necessary result from the actual premises taken, but there will be a deduction when the premises have been converted, as in the previous case. (39a22–25)

The passage 39a26–28 describes CC-Darapti QQQ in Table 26:

> For if it is possible for A and B not to belong to C, then if 'is possible to belong' is substituted, it will again be the first figure through conversion. (39a26–28)

If 'is possible to belong' is substituted for 'is possible not to belong,' then the syllogism becomes the first figure. This substitution is just complementary conversion. The proof of the syllogism at 39a26–28, then, proceeds as follows:

CC-Darapti QQQ

(1)	$\forall x(Cx \supset Q{\sim}Ax)$	Given
(2)	$\forall x(Cx \supset Q{\sim}Bx)$	Given
(3)	$\forall x(Cx \supset QAx)$	CC (1)
(4)	$\forall x(Cx \supset QBx)$	CC (2)
(5)	$\exists x(QBx \,\&\, Cx)$	A-conv (4)
(6)	$\exists x(QBx \,\&\, QAx)$	Darii (5)(3)

The last two syllogisms, CC-Datisi QQQ and CC-Disamis QQQ in Table 26, are proven in much the same way:

> But if both premises should be put as privative, one as universal and the other as particular, then there will not be a deduction through the actual premises taken, but when they are converted there will be, as in the previous cases. (39a38–b2)

When the CA premise is universal and CB is particular, we have CC-Datisi QQQ:

CC-Datisi QQQ		
(1)	$\forall x(Cx \supset Q{\sim}Ax)$	Given
(2)	$\exists x(Cx \ \&\ Q{\sim}Bx)$	Given
(3)	$\forall x(Cx \supset QAx)$	CC (1)
(4)	$\exists x(Cx \ \&\ QBx)$	CC (2)
(5)	$\exists x(QBx \ \&\ Cx)$	I-conv (4)
(6)	$\exists x(QBx \ \&\ QAx)$	Darii (5)(3)

When the CB premise is universal and CA is particular, we have CC-Disamis QQQ:

CC-Disamis QQQ		
(1)	$\exists x(Cx \ \&\ Q{\sim}Ax)$	Given
(2)	$\forall x(Cx \supset Q{\sim}Bx)$	Given
(3)	$\exists x(Cx \ \&\ QAx)$	CC (1)
(4)	$\forall x(Cx \supset QBx)$	CC (2)
(5)	$\exists x(QAx \ \&\ Cx)$	I-conv (3)
(6)	$\exists x(QAx \ \&\ QBx)$	Darii (5)(4)
(7)	$\exists x(QBx \ \&\ QAx)$	Ampliated I-conv (6)

The complementary conversions used so far have only a minor affect in the syllogistic. When we look at Table 26, and when we compare the syllogisms on the left-hand side with the syllogisms on the right-hand side, we can see that the two sides differ only slightly – on the left the CB premise is always an *affirmative* Q-premise; on the right the CB premise is a *privative* Q-premise. Complementary conversion can be used to turn a *privative* Q-premise into an equivalent but *affirmative* Q-premise. In fact what we find in Aristotle's own proofs, as described above, is that he uses complementary conversion in each case to bring us back to syllogisms we have already proven. The syllogisms on the right-hand side of Table 26 are not *new* syllogisms, they are ways of 'converting' given premises into more familiar syllogisms whose validity we have already established; that is, they are completed 'as in the previous cases'.

Aristotle also mentions in Chapter A20 that 'there will not be a deduction' from particular premises.

> When both premises are taken as indeterminate or as particular, there will not be a deduction (for it is necessary for A to belong to every as well as to no B). (39b2–4)

Of course there is no X+X syllogism from two particular premises, but here Aristotle nonetheless looks at the Q+Q case, and he gives terms to illustrate invalidity: 'terms for belonging are animal, man, white; for not belonging, horse, man, white (white is the middle term).' The rejected schema is this:

(1)	$\exists x(Cx \mathbin{\&} QAx)$
(2)	$\underline{\exists x(Cx \mathbin{\&} QBx)}$
(3)	$\exists x(QBx \mathbin{\&} QAx)$

All this might seem trivial, but in a subsequent discussion of invalid Q-contingent schemas in Chapter A21, 40a1–3, Aristotle repeats exactly the explanation he gives here here in A20, 39b2–4. And he claims in the later chapter that 'the demonstration [of invalidity] is the same' and proceeds through 'the same terms'. So let's look closely at how the counter-examples work in the present case. When we put in the terms we get the following:

(4)	Some white thing is a *Q* animal	T
(5)	<u>Some white thing is a *Q* man</u>	T
(6)	Some *Q* man is a *Q* animal	F
(7)	Some white thing is a *Q* horse	T
(8)	<u>Some white thing is a *Q* man</u>	T
(9)	Some *Q* man is a *Q* horse	F

These two examples are supposed to serve as *counter-examples*. If, for example, a child is a *Q* man, then 'some *Q* man is an *L* animal' and 'some *Q* man is an *L*~horse' are both *true* L-propositions. So, (6) and (9) are false. (In the second example each *Q* can be weakened to *M*, assuming that nothing can be both a possible man and a possible horse.)

The remaining chapters of the modal syllogistic, *An.Pr.* A21 and A22, are brief and Aristotle often seems to leave out the details of his proofs. This means we find less explicit direction than usual, and an interpreter has to try to fill in the detail. In some cases there is more than one possible proof method to explain the same results. In A21 Aristotle considers mixed X+Q and Q+X combinations. He explains that 'when the terms are in the same relationships as in the previous cases' then there is a Q-conclusion (39b9–10). He deals with the valid universal syllogisms first. These are listed below in Table 27a, though as we shall see there is some question about the correct form of Darapti and Felapton XQM.

Table 27a
Q+X and X+Q Syllogisms in the Third Figure (A21)

<u>Universals:</u>

Darapti XQM (39b10–16)	Darapti QXQ (39b17)
$\forall x(Cx \supset Ax)$	$\forall x(Cx \supset QAx)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \;\&\; MAx)$	$\exists x(Bx \;\&\; QAx)$
Felapton XQM (39b17–22)	Felapton QXQ (39b17–22)
$\forall x(Cx \supset {\sim}Ax)$	$\forall x(Cx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \;\&\; M{\sim}Ax)$	$\exists x(Bx \;\&\; QAx)$

Consider Aristotle's explanation of Darapti XQM (39b10–16):

> ... let the terms be positive, and let *A* belong to every *C*, but let it be possible for *B* to belong to every *C*. Then, when *BC* is converted, it will be the first figure, and the conclusion will be that it is possible for *A* to belong to some of the *B*s.

It would help to have more detail about just how Aristotle works this proof. As it stands, this passage leaves open the possibility of two different methods of proof. It is clear what our premises are supposed to be:

(1) $\forall x(Cx \supset Ax)$
(2) $\forall x(Cx \supset QBx)$

But when Aristotle says '*BC* is converted', he could mean $\exists x(QBx \;\&\; Cx)$ or he could mean $\exists x(Bx \;\&\; Cx)$. These give different proofs. First, consider what we get if we take Aristotle to mean $\exists x(QBx \;\&\; Cx)$:

Darapti XQM (39b10–16)

(1)	$\forall x(Cx \supset Ax)$	Given
(2)	$\forall x(Cx \supset QBx)$	Given
(3)	$\exists x(QBx \;\&\; Cx)$	'*BC* is converted' [A-conv of (2)]
(4)	$\exists x(QBx \;\&\; Ax)$	'it will be the first figure' [Darii]

This last step (4) appears to have an 'ampliated' subject (*QBx*) but an assertoric predicate (Ax). But this is precisely what is ruled out by RAP on p. 125. And in the case of Barbara XQM on p. 148 it was clear that an ampliated syllogism is not what Aristotle envisaged. So it would seem to be inappropriate to give (4) as the form of the conclusion of this Darapti. There is a simple fix. Since with $\varphi \supset M\varphi$ we can get from (4) to:

(5) $\exists x(QBx \mathbin{\&} MAx)$

and since (5) is an ampliated proposition about possibility, we saddle Aristotle with less baggage if we represent the conclusion of Darapti as (5) and treat the syllogism as Darapti XQM. We have met valid XQM syllogisms before – e.g., in *An.Pr.* A15 – where Aristotle very clearly intends M-conclusions and defends their use. An XQM here in Darapti gets the logic to work, and it is in keeping with Aristotle's demonstrated methods. But Darapti XQM raises questions about Aristotle's understanding of ampliation, and if (1)–(5) is what explains the text then we want to know what governs Aristotle's uses of possibility and ampliation? This way of proving Darapti XQM must involve the principle $\varphi \supset M\varphi$ since the predicate in (1) is assertoric, and must involve step (4). I shall refer to the following consequence of RAP as the *Restricted Ampliation Manœuvre*:

RAM Where Aristotle provides a proof involving a proposition with an ampliated subject and an assertoric (or apodeictic) predicate, he always downgrades the predicate to an M predicate in the conclusion.

If (1)–(5) is a model of Aristotle's method then we shall see RAM at work in a number of syllogisms.[3]

But (1)–(5) is only one way we might try to fill in the detail that Aristotle leaves missing. Instead of (1)–(5) we might suppose that Aristotle is thinking here of a proof through realization, and that '*BC* is converted' describes a consequence of the realization: $\exists x(Bx \mathbin{\&} Cx)$. In that case, there is no need for RAM. The proof through realization would proceed as follows:

Darapti XQM (39b10–16) [through realization]

(1)	$\forall x(Cx \supset Ax)$		Given
(2)	$\forall x(Cx \supset QBx)$		Given
(3)	|	$\forall x(Cx \supset Bx)$	Realization (2)
(4)	|	$\exists x(Bx \mathbin{\&} Cx)$	'*BC* is converted' (3)

[3] One might try to avoid the use of RAM by ampliating (2) to read $\forall x(QCx \supset QBx)$. However (1) is assertoric, and RAP requires that its C be unampliated.

(5)	|	$\exists x(Bx \,\&\, Ax)$	'it will be the first figure' [Darii (1)(4)]
(6)	$\exists x(Bx \,\&\, MAx)$		discharge

There is little in the text to decide between these two proof methods – the one with RAM and the other through realization. If we convert premise (2) then we get subject terms in the scope of a *Q* – but there is no evidence of ampliated subjects in this part of the text. And since there is no textual evidence that Aristotle has *Q*s on subjects in anything but ampliated propositions about possibility, then we need RAM to get us back to a proposition about possibility – in this case, $\exists x(QBx \,\&\, MAx)$. If, on the other hand, we realize the possibility in premise (2), then the proof proceeds in the same way as earlier XQM syllogisms. But there is no textual evidence that Aristotle does realize (2). Realization might seem closer to Aristotle's earlier proof methods, but RAM gives conclusions closer in form to the ampliated conclusions of the QQQs in A20.

The proof of the next syllogism in Table 27a – Darapti QXQ – is straightforward. Aristotle says only that the case is similar 'if *BC* signifies belonging and *AC* signifies being possible'. The steps in the proof are easy to fill in:

Darapti QXQ (39b17)

(1)	$\forall x(Cx \supset QAx)$	Given
(2)	$\forall x(Cx \supset Bx)$	Given
(3)	$\exists x(Bx \,\&\, Cx)$	A-conv (2)
(4)	$\exists x(Bx \,\&\, QAx)$	Darii (1)(3)

Aristotle describes the next two syllogisms – Felapton QXQ and Felapton XQM – together:

> ... if *AC* is privative and *BC* positive (and either one is belonging), in both ways the conclusion will be possible. For the first figure comes about again... (39b17–22)

Aristotle does not give detailed proofs, but again it is clear that these are valid syllogisms. Felapton QXQ, for example, is just Darapti QXQ with uniform substitution of $Q{\sim}Ax$ for QAx. And complementary conversion of the privative premise turns Felapton into Darapti:

Felapton QXQ

(1)	$\forall x(Cx \supset Q{\sim}Ax)$	Given
(2)	$\forall x(Cx \supset Bx)$	Given
(3)	$\forall x(Cx \supset QAx)$	CC (1)
(4)	$\exists x(Bx \,\&\, QAx)$	Darapti QXQ

Of course, (4) is equivalent to $\exists x(Bx \,\&\, Q{\sim}Ax)$. The conclusion is particular and contingent. And so, using the medieval A, E, I, O codes, the conclusion can be represented as either a QI or as a QO proposition, with no difference in meaning. The medieval mnemonic codes give no natural way of capturing this. The next proof in Table 27a – Felapton XQM – proceeds from an XE+QA premise combination. It is clear that we can syllogize from these premises, but Aristotle does not explain the details of the proof and, just as with Darapti XQM above, there are two ways we might explain Felapton XQM. We can convert the Q premise and use RAM, or we can realize the Q premise and then when we close the realization assumption, we downgrade to an M conclusion.

Felapton XQM	[with RAM]	
(1)	$\forall x(Cx \supset {\sim}Ax)$	Given
(2)	$\forall x(Cx \supset QBx)$	Given
(3)	$\exists x(QBx \,\&\, Cx)$	A-conv (2)
(4)	$\exists x(QBx \,\&\, {\sim}Ax)$	Ferio (1)(3)
(5)	$\exists x(QBx \,\&\, M{\sim}Ax)$	RAM, $\varphi \supset M\varphi$

Non-modal A-conversion takes us from (2) to (3). And (non-modal) Ferio gets us (1)(3)(4). But, as in the case of Darapti XQM above, (4) is not an acceptable form, and RAM must be used to give an ampliated conclusion about possibility (5). The proof through realization works as follows:

Felapton XQM	[through realization]		
(1)	$\forall x(Cx \supset {\sim}Ax)$		Given
(2)	$\forall x(Cx \supset QBx)$		Given
(3)	|	$\forall x(Cx \supset Bx)$	Realization (2)
(4)	|	$\exists x(Bx \,\&\, Cx)$	A-conv (3)
(5)	|	$\exists x(Bx \,\&\, {\sim}Ax)$	Ferio (1)(4)
(6)	$\exists x(Bx \,\&\, M{\sim}Ax)$		discharge

The text does not provide enough detail to decide between these two methods.

In one swoop at 39b26–31 Aristotle describes a cluster of valid third figure syllogisms involving particulars:

> And if one of the premises is universal and the other is particular, then when both premises are positive, or when the universal premise is privative and the particular premise is affirmative, the manner of the deductions will be the same. For they all come to a conclusion through the first figure; consequently, it is evident that the deduction will be of

being possible and not of belonging.

This gets all the A+I, I+A, E+I, and I+E combinations – that is, Datisi, Disamis and Ferison as above. The conclusion in each will be of being possible [M] and not of belonging – that is, in no case is there an X conclusion. Table 27b lists Aristotle's valid Q+X and X+Q syllogisms involving particulars. As in Table 27a, I have in Table 27b given the XQM syllogisms unampliated M conclusions:

Table 27b
Q+X and X+Q Syllogisms in the Third Figure (A21)

Particulars:

Datisi XQM (39b6–31)	Datisi QXQ (39b26–31)
$\forall x(Cx \supset Ax)$	$\forall x(Cx \supset QAx)$
$\underline{\exists x(Cx \,\&\, QBx)}$	$\underline{\exists x(Cx \,\&\, Bx)}$
$\exists x(Bx \,\&\, MAx)$	$\exists x(Bx \,\&\, QAx)$
Disamis XQM (39b26–31)	Disamis QXQ (40a39–b2)
$\exists x(Cx \,\&\, Ax)$	$\exists x(Cx \,\&\, QAx)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \,\&\, MAx)$	$\exists x(Bx \,\&\, QAx)$
	Bocardo QXM (39b33–39)
	$\exists x(Cx \,\&\, Q{\sim}Ax)$
	$\underline{\forall x(Cx \supset Bx)}$
	$\exists x(Bx \,\&\, M{\sim}Ax)$
Ferison XQM (39b27–31)	Ferison QXQ (39b27–31)
$\forall x(Cx \supset {\sim}Ax)$	$\forall x(Cx \supset Q{\sim}Ax)$
$\underline{\exists x(Cx \,\&\, QBx)}$	$\underline{\exists x(Cx \,\&\, Bx)}$
$\exists x(Bx \,\&\, M{\sim}Ax)$	$\exists x(Bx \,\&\, Q{\sim}Ax)$

The proofs for these are the same as before – 'the manner of the deductions will be the same.' The only problem with this is that we cannot say with absolute certainty what in each case the precise manner of the deductions is supposed to be. Some proofs are ambiguous between conversion with RAM and realization. Nonetheless, I am going to suppose that Aristotle has proof through realization in mind, and I will show how the relevant proofs can be completed through realization. Of course, anytime there is a proof though realization from an LE+Q or LO+Q premise pair, there is also a corresponding proof using RAM, to an ampliated conclusion.

Datisi XQM [through realization]

(1)	$\forall x(Cx \supset Ax)$		
(2)	$\exists x(Cx \;\&\; QBx)$		
(3)	|	$\exists x(Cx \;\&\; Bx)$	Realization (2)
(4)	|	$\exists x(Bx \;\&\; Cx)$	I-conversion (3)
(5)	|	$\exists x(Bx \;\&\; Ax)$	Darii (1)(4)
(6)	$\exists x(Bx \;\&\; MAx)$		discharge

Disamis XQM proceeds similarly:

Disamis XQM [through realization]

(1)	$\exists x(Cx \;\&\; Ax)$		
(2)	$\forall x(Cx \supset QBx)$		
(3)	|	$\forall x(Cx \supset Bx)$	Realization (2)
(4)	|	$\exists x(Bx \;\&\; Cx)$	A-conversion (3)
(5)	|	$\exists x(Bx \;\&\; Ax)$	Darii (1)(4)
(6)	$\exists x(Bx \;\&\; MAx)$		discharge

Ferison XQM is just Disamis XQM with $\sim A$ for A.

Aristotle singles out the O+A combination for separate discussion. The proof for Bocardo QXM is 'through an impossibility' – i.e., it is a reductio proof. Aristotle explains at 39b33–39:

> For (2) let *B* belong to every *C*, and (1) let it be possible for *A* not to belong to some *C*. Then (3) it must be possible for *A* not to belong to some *B*. For (4) if *A* belongs of necessity to every *B* and (2) *B* is put as belonging to every *C*, then (5) *A* will belong to every *C* of necessity (this was proved earlier): but (1) it was assumed to be possible for *A* not to belong to some *C*.

Bocardo QXM (39b33–39)

(1) $\exists x(Cx \;\&\; Q{\sim}Ax)$
(2) $\forall x(Cx \supset Bx)$
(3) $\exists x(Bx \;\&\; M{\sim}Ax)$

Suppose (3) is false. That is, suppose (4):

(4)	$\forall x(Bx \supset LAx)$	Reductio hypothesis
(5)	$\forall x(Cx \supset LAx)$	Barbara LXL (4)(2)

(5) and (1) contradict. So (1)(2)(3) is valid.[4]

Aristotle does not discuss Bocardo XQM. The schema would be:

Bocardo XQM
$\exists x(Cx \mathbin{\&} {\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$
$\exists x(Bx \mathbin{\&} M{\sim}Ax)$

But Bocardo XQM cannot be validated by Aristotle's methods. When premise (2) is converted, the result is a particular proposition, and no conclusion follows from two particular premises. This is a point we find reinforced when Aristotle explains in three short lines why there are some invalid third figure X+Q and Q+X combinations:

> When both premises are taken as indeterminate or as particular, there will not be a deduction. The demonstration is the same one as in the previous cases and through the same terms. (40a1–3)

The explanation is formulaic and repeats exactly the explanation of invalids given earlier in A20, 39b2–4. Aristotle it would appear does not want to labour the point – there are *no* third figure syllogisms from two particular premises. So there are no third figure syllogisms from two particular premises when contingency is involved (A20, 39b2–4; A21, 40a1–2). In the next Chapter A22, in which Aristotle considers Q+L and L+Q combinations in the third figure, he simply leaves out any mention of combinations from two particular premises. They disappear altogether from Aristotle's discussion – presumably because their invalidity is at this stage taken to be obvious.

Tables 28a and 28b list the valid Q+L and L+Q syllogisms in the third figure. These are discussed in *An.Pr.* A22.

[4]While Aristotle only establishes in this passage that Bocardo QXM is valid, he is entitled to Bocardo QXQ, though the reductio assumption requires that we look at *both* ways to falsify a conclusion of the form $\exists x(Bx \mathbin{\&} Q{\sim}Ax)$. If we assume $\forall x(Bx \supset L{\sim}Ax)$ we obtain $\forall x(Cx \supset L{\sim}Ax)$, and this also contradicts (1).

Table 28a
Q+L and L+Q Syllogisms in the Third Figure (A22)

Universals:

Darapti LQM (40a11–16)	Darapti QLQ (40a16–18)
$\forall x(Cx \supset LAx)$	$\forall x(Cx \supset QAx)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \,\&\, MAx)$	$\exists x(Bx \,\&\, QAx)$
Felapton LQX (40a25–32)	Felapton QLQ (40a18–25)
$\forall x(Cx \supset L{\sim}Ax)$	$\forall x(Cx \supset Q{\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$	$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \,\&\, {\sim}Ax)$	$\exists x(Bx \,\&\, Q{\sim}Ax)$
CC-Darapti LQM (40a33–35)	
$\forall x(Cx \supset LAx)$	
$\underline{\forall x(Cx \supset Q{\sim}Bx)}$	
$\exists x(Bx \,\&\, MAx)$	

Consider Aristotle's explanation of Darapti LQM at A22, 40a11–16:

> First, then, let the terms be positive, and (1) let *A* belong to every *C* of necessity, but (2) let it be possible for *B* to belong to every *C*. Then, since (1) it is necessary that *A* belongs to every *C* and it is possible for *C* to belong to some *B*, then (6) also that *A* belongs to some [*B*] will be possible, but not belonging (for that is the way it turned out in the case of the first figure).

Tredennick links the remark about 'the way it turned out in the case of the first figure' to Aristotle's discussion in *An.Pr.* A16, 35b38–36al. The A16 passage is discussed in Chapter 15 above – it is about the syllogism Barbara LQM, in Table 22. And Aristotle's proof of Barbara LQM relies upon realizing a possibility of the form $\forall x(Cx \supset QBx)$ to get a non-modal $\forall x(Cx \supset Bx)$ from which we reason through to an M-conclusion. The proof of Darapti LQM proceeds similarly:

Darapti LQM

(1)	$\forall x(Cx \supset LAx)$		Given
(2)	$\forall x(Cx \supset QBx)$		Given
(3)	|	$\forall x(Cx \supset Bx)$	Realization (2)
(4)	|	$\exists x(Bx \mathbin{\&} Cx)$	A-conversion (2)
(5)	|	$\exists x(Bx \mathbin{\&} LAx)$	Darii LXL (1)(4)
(6)	$\exists x(Bx \mathbin{\&} MAx)$		discharge

If this is the way to interpret Aristotle then it is easy to see why the conclusion 'will be possible, but not belonging'. In Barbara LQM only an M conclusion follows, not an X conclusion. Of course Darapti LQM can be proved with RAM – in which case we would be able to derive $\exists x(QBx \mathbin{\&} LAx)$, and then use RAM to downgrade to $\exists x(QBx \mathbin{\&} MAx)$. The same method would be available for all L+Q syllogisms. This is important in L+Q and Q+L syllogisms because of the LQX phenomenon. The LQX phenomenon applies to unampliated conclusions, which are therefore not subject to RAM. The only time the LQX phenomenon comes in is when there is a privative LE premise together with a Q premise, and there is no privative LE premise in either Darapti LQM or Barbara LQM. Only an M conclusion results in these affirmative cases. I will assume that Aristotle has only unampliated subjects in mind because this seems to me closer to the text.

Aristotle tells us that 'If *BC* is put as necessary and *AC* as possible, it can also be proved similarly. (40a17–18) Aristotle is not clear about what this means, and interpreting the text depends upon what 'proved similarly' means. Aristotle might mean that Darapti QLQ is valid, in which case all that is needed is LA-conversion and $L\varphi \supset \varphi$.

Darapti QLQ

(1)	$\forall x(Cx \supset QAx)$	
(2)	$\forall x(Cx \supset LBx)$	
(3)	$\exists x(Bx \mathbin{\&} LCx)$	LA-conv (2)
(4)	$\exists x(Bx \mathbin{\&} QAx)$	Darii QLQ (1)(3)

Or he might mean that Darapti QLM is valid, in which case the proof will proceed through realization, i.e., it will be proved similarly to Barbara XQM.

Darapti QLM

(1)	$\forall x(Cx \supset QAx)$		
(2)	$\forall x(Cx \supset LBx)$		
(3)	|	$\forall x(Cx \supset Ax)$	Realization (1)
(4)	|	$\exists x(Bx \mathbin{\&} LCx)$	LA-conv (2)
(5)	|	$\exists x(Bx \mathbin{\&} Ax)$	Darii XLX (3)(4)
(6)	$\exists x(Bx \mathbin{\&} MAx)$		discharge

Aristotle describes another syllogism at 40a18–25.

> Next, let one term be positive, the other privative, and the positive necessary, that is, (1) let it be possible for A to belong to no C and (2) let B belong to every C of necessity. Now, it will again be the first figure; and since the privative premise signifies being possible, it is therefore evident that the conclusion will be possible (for when the premises are like this in the first figure, the conclusion was also possible).

But again the text is ambiguous. It could mean that Felapton QLQ is valid, or it could mean that Felapton QLM is valid. Probably Aristotle notices that Felapton Q+L is the same as Darapti Q+L with $Q{\sim}A$ for QA, and so thinks Felapton Q+L gives a valid syllogism. The syllogism Aristotle describes would then be either of the following:

Felapton QLQ
(1) $\forall x(Cx \supset Q{\sim}Ax)$
(2) $\underline{\forall x(Cx \supset LBx)}$
(3) $\exists x(Bx \,\&\, Q{\sim}Ax)$

Felapton QLM
(1) $\forall x(Cx \supset Q{\sim}Ax)$
(2) $\underline{\forall x(Cx \supset LBx)}$
(3) $\exists x(Bx \,\&\, M{\sim}Ax)$

The proofs would be similar to the proofs of Darapti QLQ and QLM, just given.

Felapton LQX is especially interesting. Aristotle could prove Felapton LQM by realization of the Q premise:

Felapton LQM

(1)	$\forall x(Cx \supset L{\sim}Ax)$		
(2)	$\forall x(Cx \supset QBx)$		
(3)	|	$\forall x(Cx \supset Bx)$	Realization (2)
(4)	|	$\exists x(Bx \,\&\, Cx)$	A-conv (3)
(5)	|	$\exists x(Bx \,\&\, L{\sim}Ax)$	Ferio LXL (4)(1)
(6)	$\exists x(Bx \,\&\, M{\sim}Ax)$		discharge

But Aristotle does not prove Felapton LQM through realization. Here is his proof:

> But if the privative premise is necessary, then the conclusion will be both that it is possible not to belong to some, and that it does not belong. For (1) let A be put as not belonging to C of necessity, and (2) let it be possible for B to belong to every C. Then, when the affirmative BC is converted (3), it will be the first figure (4) with the privative premise necessary. But when the premises are like this, it turned out both that (5) it is possible for A not to belong to some B and (6) that it does not belong, so that also necessarily A does not belong to some B.

On the face of it Aristotle begins as follows:

Felapton LQX
(1) $\forall x(Cx \supset L{\sim}Ax)$
(2) $\forall x(Cx \supset QBx)$
(3) $\exists x(QBx \,\&\, Cx)$
(4) $\exists x(QBx \,\&\, L{\sim}Ax)$

But (4) is without precedent, and clearly not what Aristotle has in mind as he continues. For, as Ross notes (1957, p. 368), he seems to think that conversion of (2) gets Ferio L+Q premises, from which both (5), an M-conclusion, and (6), an X-conclusion, follow. Certainly Aristotle uses a reductio proof for Ferio LQX and LQM, and a reductio would make sense here in Felapton. But then it is the *privative* LE-premise which needs to be converted. The steps of the proof would then go as follows:

Felapton LQX
(1) $\forall x(Cx \supset L{\sim}Ax)$
(2) $\forall x(Cx \supset QBx)$
(7) $\exists x(Bx \,\&\, {\sim}Ax)$
Proof by reductio:

(8)	$\forall x(Bx \supset Ax)$	Reductio hypothesis
(9)	$\forall x(Ax \supset L{\sim}Cx)$	LE-conversion (1)
(10)	$\forall x(Bx \supset L{\sim}Cx)$	Celarent LXL (8)(9)
(11)	$\forall x(Cx \supset L{\sim}Bx)$	LE-conversion (10)

(11) and (2) contradict, and so we are entitled to conclude (7) $\exists x(Bx \,\&\, {\sim}Ax)$. And, as Ross points out, $\varphi \supset M\varphi$, so we can also conclude $\exists x(Bx \,\&\, M{\sim}Ax)$. That is, both Felapton LQX and Felapton LQM are valid.

The last of the syllogisms in Table 28a is CC-Darapti LQM:

> But when the privative is put in relation to the minor extreme, then if

> it is possible there will be a deduction when the premise is replaced, as in the previous cases... (40a33–35)

When the privative premise is 'replaced' using complementary conversion, then the proof proceeds as follows:

CC-Darapti LQM (40a33–35)

(1)	$\forall x(Cx \supset LAx)$	
(2)	$\forall x(Cx \supset Q{\sim}Bx)$	
(3)	$\forall x(Cx \supset QBx)$	CC (2)
(4)	$\exists x(Bx \mathbin{\&} MAx)$	Darapti LQM (1)(3)

This completes the proofs of the universal syllogisms in Table 28a. Let's look next at Aristotle's comments about third figure Q+L and L+Q syllogisms involving particulars. Table 28b provides a list of the syllogisms Aristotle counts as valid:

Table 28b
Q+L and L+Q Syllogisms in the Third Figure (A22)

Particulars:

Datisi LQM (40a39–b3)
$\forall x(Cx \supset LAx)$
$\underline{\exists x(Cx \mathbin{\&} QBx)}$
$\exists x(Bx \mathbin{\&} MAx)$

Datisi QLQ (40a39–b3)
$\forall x(Cx \supset QAx)$
$\underline{\exists x(Cx \mathbin{\&} LBx)}$
$\exists x(Bx \mathbin{\&} QAx)$

Disamis LQM (40a39–b3)
$\exists x(Cx \mathbin{\&} LAx)$
$\underline{\forall x(Cx \supset QBx)}$
$\exists x(Bx \mathbin{\&} MAx)$

Disamis QLQ (40a39–b3)
$\exists x(Cx \mathbin{\&} QAx)$
$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \mathbin{\&} QAx)$

Bocardo LQX (40b3–8)
$\exists x(Cx \mathbin{\&} L{\sim}Ax)$
$\underline{\forall x(Cx \supset QBx)}$
$\exists x(Bx \mathbin{\&} {\sim}Ax)$

Bocardo QLQ (40b2–3)
$\exists x(Cx \mathbin{\&} Q{\sim}Ax)$
$\underline{\forall x(Cx \supset LBx)}$
$\exists x(Bx \mathbin{\&} Q{\sim}Ax)$

Ferison LQX (40b3–8)
$\forall x(Cx \supset L{\sim}Ax)$
$\underline{\exists x(Cx \mathbin{\&} QBx)}$
$\exists x(Bx \mathbin{\&} {\sim}Ax)$

Ferison QLQ (40ab2–3)
$\forall x(Cx \supset Q{\sim}Ax)$
$\underline{\exists x(Cx \mathbin{\&} LBx)}$
$\exists x(Bx \mathbin{\&} Q{\sim}Ax)$

Aristotle's comments are again very brief, but some points are very clear. Again, there are Q syllogisms about particulars in which contingency is only trivially included. These are valid substitution instances of non-modal syllogisms. And these include Datisi, Disamis, Bocardo and Ferison QLQ. Aristotle's discussion is too brief and sweeping to provide a sure guide, but it appears from A22, 4039–b3 that he means to count not only the QLQs in the right hand column, but also Datisi LQM and Disamis LQM as valid:

> It will also be similar if one of the terms is universal and the other particular in relation to the middle. For when both are affirmative, then the deduction will be of being possible, but not of belonging; and also when one is taken as privative and the other as affirmative and the affirmative is necessary. (40a39–b2)

At 40b3–8, Aristotle describes Ferison LQX and Bocardo LQX:

> But when the privative is necessary, the conclusion will also be of not belonging. For the manner of proof will be the same whether the terms are universal or not universal (for the deductions must be completed through the first figure; consequently, it necessarily turns out just the same way in these cases as it did in those).

Since he claims both an X and an M conclusion, we should expect reductio proofs for these, together with the LQX phenomenon.

Ferison LQX

(1)	$\forall x(Cx \supset L{\sim}Ax)$	
(2)	$\exists x(Cx \ \& \ QBx)$	
(3)	$\exists x(Bx \ \& \ {\sim}Ax)$	

Proof by reductio

(4)	$\forall x(Bx \supset Ax)$	Reductio hypothesis
(5)	$\forall x(Ax \supset L{\sim}Cx)$	LE-conversion (1)
(6)	$\forall x(Bx \supset L{\sim}Cx)$	Celarent LXL (5)(4)
(7)	$\forall x(Cx \supset L{\sim}Bx)$	LE-conversion (6)

(7) contradicts (2), so we can conclude (3) $\exists x(Bx \ \& \ {\sim}Ax)$, and, because $\varphi \supset M\varphi$, we can also conclude $\exists x(Bx \ \& \ M{\sim}Ax)$. So both Ferison LQX and Ferison LQM are valid.

Notice that Ferison requires two conversions. In the case of Bocardo conversion is not available, because the LO premise does not convert. Aristotle can, however, use a reductio proof through ecthesis to validate Bocardo LQX:

Bocardo LQX

(1)	$\exists x(Cx \,\&\, L{\sim}Ax)$	
(2)	$\underline{\forall x(Cx \supset QBx)}$	
(3)	$\exists x(Bx \,\&\, {\sim}Ax)$	
Proof by Reductio:		
(4)	$\forall x(Bx \supset Ax)$	Reductio hypothesis
(5)	$\forall x(Dx \supset Cx)$	Ecthesis (1)
(6)	$\forall x(Dx \supset QBx)$	Barbara QXQ (2)(5)
(7)	$\forall x(Dx \supset L{\sim}Ax)$	Ecthesis (1)
(8)	$\forall x(Ax \supset L{\sim}Dx)$	LE conversion (6)
(9)	$\forall x(Bx \supset L{\sim}Dx)$	Celarent LXL (8)(4)
(10)	$\forall x(Dx \supset L{\sim}Bx)$	LE conversion (9)

There is a contradiction between (6) and (10), so we can conclude (3) $\exists x(Bx \,\&\, {\sim}Ax)$. Aristotle can also use a proof by realization to obtain Bocardo LQM:

Bocardo LQM [by realization]

(1)	$\exists x(Cx \,\&\, L{\sim}Ax)$		
(2)	$\underline{\forall x(Cx \supset QBx)}$		
(3)	$\exists x(Bx \,\&\, M{\sim}Ax)$		
Proof by Reductio:			
(4)	$\forall x(Bx \supset Ax)$		Reductio hypothesis
(5)	|	$\forall x(Cx \supset Bx)$	Realization (2)
(6)	|	$\forall x(Cx \supset Ax)$	Barbara (4)(5)
(7)	$\forall x(Cx \supset MAx)$		Discharge

There is a contradiction between (7) and (1), so we can conclude (3) $\exists x(Bx \,\&\, M{\sim}Ax)$.

Throughout Chapters A20, A21 and A22 Aristotle is less reliant on counter-examples to illustrate invalidity. In A22 he gives only one set of Q counter-examples. These are described at 40a33–38. Aristotle is discussing why we cannot syllogize from a Q+L premise combination

$$\forall x(Cx \supset QAx)$$
$$\forall x(Cx \supset L{\sim}Bx)$$

to a Q conclusion. He gives two sets of unusual terms: ‘Terms for belonging to all are sleep, sleeping horse, man; for belonging to none, sleep, waking horse, man.’ (40a37) Such complex terms are unprecedented in the modal syllogistic. Of course, complex terms are not without precedent in other parts of Aristotle’s philosophical works. Recall his discussion of ‘white man’ and ‘cloak’ in *Metaphysics* Z4, 1029b23–1030a7. But in fact the complex terms are something of a red herring, here, in *Prior Analytics* A22 in the modal syllogistic. And terms can be better chosen to make the simple point that the logic demands. Aristotle wants to show that no Q conclusion follows from the given premise combination. Let terms for belonging be white, raven, man. Then we get the following:

All men are *Q* white	T
All men are *L* not ravens	T
All ravens are *Q* white	F

There cannot be a Q conclusion because terms for belonging give a true LE proposition

All ravens are *L*~white.

Let terms for belonging to none be white, swan, man:

All men are *Q* white	T
All men are *L* not swans	T
All swans are *Q* not white	F

There cannot be a Q conclusion because terms for belonging to none give a true LA proposition

All swans are *L* white.

Note in these cases we also fail to get an *M* conclusion since we have:

All ravens are *M* white	F
All swans are *M* not white	F

Aristotle does not give any other counter-examples to illustrate invalidity in A22. In fact Aristotle offers no counter-example of any schema about particulars in the third figure L+Q and Q+L schemas. That means that he does not give general *proofs* that LQL and LQX schemas are invalid.

A22 brings to an end Aristotle's discussion of the modal syllogistic. I have tried to show how treating Aristotle's modal syllogistic as simple applied logic enables a coherent picture to be given of his thinking. If there remain some areas where he seems to have a less than perfect grasp of what is happening, we should perhaps remember that he has created logic from nothing. He explains the situation in the last passages of *On Sophistical Refutations*.

Of the present inquiry... it was not the case that part of the work had been thoroughly done before, while part had not. Nothing existed at all.... on the subject of deduction we had absolutely nothing else of an earlier date to mention, but were kept at work for a long time in experimental researches. If, then, it seems to you after inspection that, such being the situation as it existed at the start, our investigation is in a satisfactory condition compared with the other inquiries that have been developed by tradition, there must remain for all of you, our students, the task of extending us your pardon for the shortcomings of the inquiry, and for the discoveries thereof your warm thanks. (*SE* 34, 183b35–184b7)

Chapter 18
Summary and Conclusion

The approach to the modal syllogistic presented in these pages has been guided by the need for a precise way to represent Aristotle's own explanations and proofs while at the same time keeping the modern technical devices to a minimum and as transparent as possible. For these reasons I prefer to use the lower predicate calculus (LPC) to represent Aristotle's propositions rather than the kinds of notation usually encountered in discussions of Aristotle's logic. I follow McCall's system for annotating the medieval mnemonics. An assertoric (non-modal) proposition is denoted by X, a proposition about necessity by L, and a proposition about possibility by either M or Q depending on the kind of possibility involved.

Part I is concerned to show that LPC is suited to the task. Such an accounting is necessary because some scholars find the use of LPC problematic. Aristotle certainly did not use LPC, and for all the advantages it affords there is always a danger that the LPC representations might cloud the discussion. I have tried to explain how little this need worry a reader because in fact I use very little of the power of modern predicate logic. A reader will discover no funny business hidden in fancy notation – just a small fragment of LPC together with a few restrictions which I make explicit. As explained in Chapter 2, some seem to think, for example, that because Aristotle doesn't have anything like the individual variable, it must be inappropriate for us to represent his logic by using LPC, which does have the individual variable. Of course there is a sense in which it *is* inappropriate – as, for instance, it would be if by representing Aristotle's logic in LPC we meant thereby to attribute to him some notion of the individual variable. We would have to say the same about the combination of unrestricted quantifiers and truth-functional connectives – Aristotle doesn't have any of these and it would be inappropriate to attribute them to him. But though I use LPC to represent Aristotle's logic I do not attribute these tools to him. In using LPC we are simply availing ourselves of a more precise and powerful logical language than Aristotle's own semi-technical Greek. Modern LPC is a language in which we can express what Aristotle expresses. That is not a claim that Aristotle can express all that we do in LPC, and it is not a claim that attributes to him all the power of our modern logical devices.

Most of Aristotle's modern interpreters avoid LPC and offer various alternative representations. But these alternatives can frustrate a reader. There are several reasons why. There is no one preferred style which a student might master; instead we are faced with a swarm of idiosyncratic representations. These vary in different ways, but perhaps the most crucial difference concerns the placement of subject and predicate terms. As noted in Chapter 1, one common representation of Aristotle's (particular affirmative categorical proposition) 'some *B* is *A*' is *Iab*, in which *a* is the subject and *b* is the predicate. But another common representation of the very same proposition is *Iba*, where *b* is the subject and *a* is the predicate. What emerges as an important advantage of the LPC representations is the way they allow us to neatly track the subject and

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predicate terms through Aristotle's premises and proofs. This is important because Aristotle frequently 'converts' premises, turning the subject term into the predicate term – and vice versa. In certain situations – as, for example, in modal conversions – what plays the subject role and what plays the predicate role can affect the logic, and so it helps to have a neat and unambiguous way to track the ways these can and do shift in Aristotle's own proofs.

LPC representations work especially neatly in the non-modal syllogistic, where all that is needed is a simple restriction against empty terms. As we saw in Chapter 1, this restriction is needed because the LPC version of Aristotle's 'All Bs are As' is $\forall x(Bx \supset Ax)$ and this is trivially true when B is empty. Because Aristotle converts 'All Bs are As' to get 'Some As are Bs', we need to make sure that our translations make such conversion valid. So we can never allow a situation in which $\forall x(Bx \supset Ax)$ is true but $\exists x(Ax \mathbin{\&} Bx)$ is false. The restriction against empty terms prevents such a situation arising. Chapter 2 illustrates how, bearing in mind the restriction against empty terms, we can approach the non-modal syllogistic as a purely formal system. What this means in the context of the syllogistic is that so long as we start off with a valid schema then we can choose anything we want as our terms A, B, C – the result will always be a valid instance of the schema. In the non-modal syllogistic the restriction against empty terms is the only restriction. In the non-modal, except for this, there is no way in which the choice of terms affects our ability to syllogize, and so the non-modal syllogistic does appear to be a purely formal system. If this is correct, then there is no reason to suppose that Aristotle's metaphysical theories are at work in the non-modal syllogistic. In particular, if the non-modal syllogistic is purely formal, then non-modal conversions are straightforward and unproblematic.

Chapter 2 begins a discussion that carries through several chapters about how a modern reader acquainted with logical scope should understand Aristotle who did not have such a notion. This affects our treatment of both the non-modal and the modal syllogistic. It affects the non-modal syllogistic because in LPC $\sim$ is an operator with scope. For example, one way to describe the difference between the two equivalent wff $\forall x(Bx \supset {\sim}Ax)$ and ${\sim}\exists x(Bx \mathbin{\&} Ax)$ is in terms of the scope of $\sim$. In the first it has narrow scope. In the latter it has wide. LPC allows us to explain this, but Aristotle has a different explanation. There is not any evidence to show that Aristotle has any construction like ${\sim}\exists x(Bx \mathbin{\&} Ax)$. If anything, he appears to avoid constructions which today we would describe as treating negation as a propositional operator. But Aristotle suffers no real loss of expressive power because he takes each of affirmation and denial as basic – for example, to deny that 'some B is not A' is to affirm that 'every B is A'. Or in LPC, to deny $\exists x(Bx \mathbin{\&} {\sim}Ax)$ is to affirm $\forall x(Bx \supset Ax)$. In the medieval jargon that is to say that to deny an O-proposition is to affirm an A-proposition. I have tried to highlight the evidence from the *Prior Analytics* that shows why it is inappropriate to attribute any kind of propositional operator to Aristotle.

Chapter 3 looks at how some scholars have sought to explain why we should think that Aristotle's modal syllogistic is a failure. In particular, Chapter 3 looks at criticisms recently put forward by each of Becker, Hintikka, and Striker. Becker assumes that Aristotle confuses two separate notions of necessity, and thinks that there is an ambiguity present in Aristotle's account of modals. If so, then it is one which an interpreter must either preserve or try to disambiguate. But Chapter 3 explains how, when any such ambiguity about modals does form part of our background assumptions about the modal syllogistic, then we are working on an assumption that itself immediately undermines Aristotle's modal logic, and that makes it appear that Aristotle is incoherent. Hintikka's criticism of the modal syllogistic hinges on whether the principle of plenitude is at work in the logic. Hintikka notes that several passages in Aristotle's works suggest that he links necessity with what is always the case, and links possibility with what is sometimes the case. This is the principle of plenitude. Hintikka thinks it is a fundamental part of Aristotle's treatment of modality, but when plenitude is brought to bear on the modal syllogistic, Hintikka points out that Aristotle's distinction between necessary and merely assertoric syllogisms must be forfeited. Striker points out inconsistencies in Aristotle's own examples and suggests that no subtleties of interpretation can render the text consistent.

Chapter 4 introduces one of the respects in which my approach differs from most others on offer. For I argue that in the *modal* syllogistic, the kind of term involved plays an important role. To understand this role I introduce, in Chapter 4, some new but neutral terminology: I call some terms 'red' and I call other terms 'green'. Red and green are exclusive; no term is both. Red terms refer to what *cannot be otherwise* than it is. Green terms name what *can be otherwise* than it is. The red/green distinction reflects a distinction Aristotle makes in his metaphysics between essence and accident. Chapter 4 begins the project of putting the distinction between red and green terms to work in the modal syllogistic. If something like the red/green distinction is part of Aristotle's thinking in the modal syllogistic, then the modal syllogistic is different from the non-modal syllogistic, since that appears to be a purely formal system which takes no account of any differences between kinds of terms. Chapter 4 shows how the red/green distinction helps to answer the specific criticisms of Becker, Hintikka, and Striker.

Chapter 5 has two main themes. First, it is concerned with how to represent Aristotle's modals. Becker's criticism of Aristotle can be expressed in LPC. Aristotle's '*A* can be predicated by necessity of every *B*' may be translated in two ways. In modal LPC one of these is the *de re* formula $\forall x(Bx \supset LAx)$. The other is *de dicto* $L\forall x(Bx \supset Ax)$. A principal claim of this book is that *all* representations of Aristotle's modal propositions involve a *de re* interpretation. Becker notes that a *de re* translation is needed to explain the validity of the syllogisms – for example, Barbara LXL:

$$\begin{array}{ll} (1) & \forall x(Bx \supset LAx) \\ & \underline{\forall x(Cx \supset Bx)} \\ & \forall x(Cx \supset LAx) \end{array}$$

But Becker thinks that a *de dicto* translation is needed in order to validate Aristotle's conversions, so, for example, in LPC, *LI* conversion would be $L\exists x(Bx \,\&\, Ax) \supset L\exists x(Ax \,\&\, Bx)$. Becker does think Aristotle is confused about these two different kinds of necessity, but not all interpreters agree. Many scholars think that Aristotle's categorical propositions have a tripartite structure of term+copula+term. When the tripartite copulative structure gets carried over to the study of the modal syllogistic then it looks like Aristotle's modals act as qualifiers on the copula. Because I use modal LPC to represent Aristotle's modal propositions, in my representations the modals look like qualifiers on predicate terms – as, e.g., in $\forall x(Bx \supset LAx)$. This difference – whether the modal qualifies a copula or a predicate – has suggested to some that a copulative reading is really in tension with LPC. Chapter 5 explains why this is only an apparent tension. (A more technically precise explanation of this is included in a separate Appendix.) Showing why there is no real tension between a modal copulative reading and modal LPC helps to reinforce my claim that a simple *de re* representation of Aristotle's modals is all that is needed in the syllogistic. This chapter introduces various scope distinctions involving the modal operators. This is needed because ~ and *L* have different scopes in the modal expressions $L{\sim}Ax$ and ${\sim}LAx$. Following David Charles, I argue that Aristotle's modal propositions have to be represented in a way which never puts ~ anywhere but immediately in front of the predicate. When Aristotle wants to represent ${\sim}L$ he uses a modal square of opposition and switches to $M{\sim}$. Similarly, the *L* has different scope in $\forall x(Bx \supset LAx)$ and $L\forall x(Bx \supset Ax)$. Aristotle's treatment of negation and his treatment of modals do I think indicate that he is not working with a notion of a propositional operator. This means that we should not attribute to Aristotle anything like a notion of *de dicto* modals.

A second and closely related theme in Chapter 5 concerns modal conversion. Modal conversion provides a crucial test of any interpretation of the modal syllogistic. Since Aristotle counts *LI* conversion as valid, we cannot allow the following as an instance of *LI* conversion: 'if some white thing is necessarily a man, then some man is necessarily white.' The problem is that, here, the antecedent can be true while the consequent is false, since no man is white by necessity. I introduce the *genuineness requirement* in order to restrict such modal conversions to valid instances. This requirement demands that the input into conversion be a genuine predication – i.e., in the language of red and green terms, genuineness requires a *red* subject term. The chapter explains how Patterson, Thom, Nortmann, and Malink each explain modal conversion of 'some *B* is necessarily *A*' – an *LI* premise. In order to make the differences in their treatments of *LI* conversion more easily accessible to a reader who is not interested in mastering all the technical machinery that these interpreters employ, I have 'translated' their explanations into modal LPC. The chapter then compares their various approaches to conversion with my simple restricted *de re* LPC conversion.

Part II studies the apodeictic syllogistic – the syllogistic involving premises about what is necessary. In Part II it begins to become clearer why this must be a logic book. Aristotle frequently provides only a sketch or outline of a proof, leaving the details to the reader to work out. Sometimes the logic allows two different interpretations, and there is little direct textual evidence to show which of them Aristotle has in mind. You can of course, follow Becker, Striker and others who think that the result is a realm of darkness, but if that leaves you unsatisfied, then you have got to find an interpretation of the logic which makes Aristotle's system plausible. Thom and Nortmann, for example, do try to find such an interpretation without restricting terms to red and green. Striker, however, takes them to task, pointing out that their interpretations are artificial, for they demand what appear to be very sophisticated modern tools. Red and green terms provide an alternative. Bringing in Aristotle's distinction between what cannot be otherwise (red) and what can be otherwise (green) means that we do not need complex translation mechanisms. Instead, we can use a straightforward translation method and we can represent Aristotle's modal propositions using simple restricted LPC formulae. What's remarkable is how simple and comprehensible the red/green distinction makes the modal syllogistic. To be sure, Aristotle in the *Prior Analytics* does not announce anything like the red/green distinction – if he had, then the problem of interpretation would not be the long-standing problem that it has been, and commentators would not have described the modal syllogistic as a realm of darkness.

Chapter 6 focuses specifically on various ways we might interpret Aristotle's description of syllogisms from two necessary premises. Aristotle's account of these is in *An.Pr.* A8. His discussion in *An. Post.* A2 shows how he has in mind to link his scientific deductions with 'what cannot be otherwise'. And this discussion in *An. Post.* provides good reason to suppose that Aristotle thinks syllogisms from premises about what cannot be otherwise – i.e., syllogisms from premises about necessity – are the right sort of syllogisms for proper science. Chapter 6 considers how – in the light of this – the red/green distinction might be brought to bear on Aristotle's apodeictic LLL syllogistic. But the introduction of the red/green distinction still leaves open the question of whether all terms are modally qualified, or only some terms. Using modal LPC, we might, for example, represent Aristotle's first figure syllogism Barbara LLL this way, putting every term in the scope of an *L* operator:

(2)
$$\begin{array}{l} \forall x(LBx \supset LAx) \\ \underline{\forall x(LCx \supset LBx)} \\ \forall x(LCx \supset LAx) \end{array}$$

This would make Barbara LLL a substitution instance of the non-modal syllogism Barbara XXX.

$$(3) \qquad \begin{array}{l} \forall x(Bx \supset Ax) \\ \underline{\forall x(Cx \supset Bx)} \\ \forall x(\mathrm{C}x \supset Ax) \end{array}$$

The modal syllogism would be just the non-modal syllogism but with modally qualified *red* terms – i.e., Barbara LLL (2) is the same as (3) Barbara XXX but with LA for A, LB for B, LC for C. I call such an LLL Barbara 'Red Barbara' and I call such a treatment of the apodeictic syllogistic a red syllogistic. But a red syllogistic is not the *only* way we might understand Aristotle's instructions in *An.Pr.* A8. Rather than putting a modal qualifier on every term, another natural way we might understand Aristotle would be, in LPC, to put only the predicate terms in the scope of an L operator. In this case the following seems a better way to represent Barbara LLL:

$$(4) \qquad \begin{array}{l} \forall x(Bx \supset LAx) \\ \underline{\forall x(Cx \supset LBx)} \\ \forall x(\mathrm{C}x \supset LAx) \end{array}$$

In order to validate (4) we need a principle which guarantees that for any term φ, $L\varphi \supset \varphi$. And of course $L\varphi \supset \varphi$ is valid no matter what we choose as our φ. But the crucial difference between representing Barbara LLL as (2) or as (4) is that in (2) if the premises are true then all the terms are restricted to red terms. In (4) only the A and B terms are so restricted, because, as the predicate terms, only they are in the scope of an L operator. In (4) the C term is unrestricted and so can be a green term. Aristotle does not offer specific proofs of the LLL syllogisms and so there is little direct textual evidence to help decide between these two readings.

Chapter 7 begins the study of syllogisms whose premises are 'mixed'. In this chapter we look at schemas involving one necessary and one non-modal premise – that is, we look at syllogisms from L+X and X+L premise pairs. Aristotle gives syllogisms for each of the first figure L+X premise pairs, but he rejects as invalid all first figure X+L combinations. It emerges that in the first figure mixed syllogisms, when we are translating Aristotle's logical expressions into LPC using the translations defended in Chapter 5, then, e.g., 'A belongs of necessity to every B' goes to $\forall x(Bx \supset LAx)$, and not to $\forall x(LBx \supset LAx)$. That is, in LPC translations Aristotle's modal qualifier attaches to the predicate term alone. The mixed modals make clear what the pure LLLs don't – in the mixed cases there is no reason to put Ls on subject terms. This has the effect that all of Aristotle's first figure mixed L+X syllogisms turn out to be trivial substitution instances of non-modal syllogisms: the difference between a non-modal first figure

XXX syllogism and a mixed LXL syllogism can be seen as the difference between substituting a 'modal term' *LA* for a non-modal predicate term *A*. The chapter shows how we can translate into *de re* LPC Aristotle's proofs of the validity of each of Barbara LXL, Celarent LXL, Darii LXL and Ferio LXL. Aristotle offers counter-examples to establish the invalidity of all other first figure schemas. He clearly allows the terms in a non-modal X premise to be green terms about what can be otherwise. Chapter 7 shows how the success of Aristotle's own counter-examples hinges on being able to generate a falsehood by putting a green term in the scope of necessity. So as far as syllogisms in the first figure go, the restriction to red terms and the genuineness requirement play no role. These first figure mixed modal syllogisms turn out to be unrestrictedly valid. That is why the genuineness requirement *only* applies to the input to a modal conversion.

Chapter 8 studies the mixed apodeictic syllogisms in the second and third figures. Unlike the first figure modal syllogisms, second and third figure modal syllogisms require various conversions. The chapter begins by explaining how genuine modal conversion works in the second figure. Second figure Cesare LXL and Festino LXL are shown to work with simple restricted, genuine conversion. Each of these syllogisms involves the conversion of an *LE* premise $\forall x(Bx \supset L{\sim}Ax)$, and in order to preserve validity, genuineness requires that the *B* term is red. If you look at Cesare LXL you can see how to construct a counter-example by using green terms – that is, you can construct counter-examples by *not* restricting according to genuineness. For example, suppose that no animals are moving. Then the premises are true, but the conclusion is false:

	Cesare LXL		
	(1)	$\forall x(Bx \supset L{\sim}Ax)$	All moving things are necessarily not animals
	(2)	$\underline{\forall x(Cx \supset Ax)}$	All men are animals
*	(3)	$\forall x(Cx \supset L{\sim}Bx)$	All men are necessarily not moving

The conclusion is false because, of course, men *can* move even if they are not moving. This counter-example does assume a *de re* translation. *De dicto*, however, fares no better. See footnote 1, p. 79. The problem in (1)(2)(3) is the *B* term. If we allow a green *B* term, here, then we generate a counter-example. In Cesare LXL the modal *LE* premise (1) must be converted, and the conversion is only guaranteed if the input is restricted by genuineness which requires that *B* is *not* green. If a green *B* term is blocked by restricted, genuine conversion, then validity is restored, and Aristotle's explanation works as he describes – Cesare LXL is valid. Whether or not modal conversion is used is of course a feature of how Aristotle *proves* the syllogism in question, and may not be predictable from the syllogism itself. This is why this book has to be about his *proofs*. Another theme in Chapter 8 concerns Aristotle's proof of

Baroco LLL which involves a technique called *ekthesis*. Chapter 8 shows that when *ekthesis* is unpacked in modal LPC, then it too turns out to depend upon restricted, genuine modal conversion. I use Patterson's and Thom's explanations of *ekthesis* to show how this works.

As a matter of routine, Aristotle offers counter-examples to establish invalidity, and Chapter 8 looks at his own counter-examples to second figure mixed apodeictics. In particular, the relation between Camestres LXL and Cesare XLL requires close attention. The difference between Camestres LXL and Cesare XLL is in the conclusion. Both conclusions are *LE* propositions. Aristotle offers the following counter-example to Cesare XLL:

	Cesare XLL	
(1)	$\forall x(Cx \supset \sim Ax)$	All white things are not animals
(2)	$\underline{\forall x(Bx \supset LAx)}$	All men are necessary animals
(3)	$\forall x(Bx \supset L\sim Cx)$	All men are necessarily not white

The conclusion clearly is false since being white or not is only accidental to a man. But when we put these same terms into Camestres LXL, we need an explanation.

	Camestres LXL	
(2)	$\forall x(Bx \supset LAx)$	All men are necessary animals
(1)	$\underline{\forall x(Cx \supset \sim Ax)}$	All white things are not animals
(4)	$\forall x(Cx \supset L\sim Bx)$	All white things are necessarily not men

Since 'man' is a red term (4) would be true, given (2) and (1). If we replace (4) by (3) in Camestres LXL then we get Cesare XLL with transposed premises. Since an *LE*-proposition $\forall x(Cx \supset L\sim Bx)$ converts to another *LE*-proposition $\forall x(Bx \supset L\sim Cx)$, and vice versa, Aristotle seems to think of *LE*-conversion an equivalence. In modal LPC it would be: $\forall x(Cx \supset L\sim Bx) \equiv \forall x(Bx \supset L\sim Cx)$. So, if *LE*-conversion is valid, then since (3) is false, (4) must be false also. But the validity of *LE*-conversion depends on the genuineness restriction, and that requires that the subject of the input be a red term. Of course, 'white', the subject in (4), is not a red term. So we cannot replace (4) with (3) because we cannot meet the genuineness requirement on conversion. When Aristotle treats *LE*-conversion as an equivalence which works in either direction – rather than as a rule which, whenever it is applied, is applied in one direction – he seems to forget that the genuineness requirement must then apply to *both* subject terms; otherwise it is invalid. In Chapter 8, I call this the *Subtle Mistake*, and I trace its effect through the modal syllogistic. Although Aristotle claims that Camestres LXL is invalid, in fact he does not establish what he claims.

Chapter 8 also discusses third figure mixed modals. They work much like the first figure mixed modals. This is because, although conversion is used in the proofs of valid third figure syllogisms, it is not specifically *modal* conversion and so is not subject to the genuineness requirement. Consider Darapti LXL for example:

Darapti LXL
$\forall x(Cx \supset LAx)$
$\underline{\forall x(Cx \supset Bx)}$
$\exists x(Bx \;\&\; LAx)$

Darapti LXL is an instance of Darapti XXX with uniform substitution of *LA* for *A*, and the proof of Darapti LXL requires only ordinary non-modal *A*-conversion of $\forall x(Cx \supset Bx)$ into $\exists x(Bx \;\&\; Cx)$. In the case of the third figure mixed invalid schemas, Aristotle is able to give counter-examples.

I have tried to make Aristotle's own textual discussions the main source of my data, and I have tried to make it my job to offer an interpretation of Aristotle using modal LPC. As Chapters 7 and 8 illustrate, on the whole that works smoothly and without major difficulties if you can accommodate the genuineness requirement and the subtle mistake. There are, however, places in the *Prior Analytics* where it is not altogether clear just what syllogism Aristotle takes himself to be describing. Chapter 9 looks closely at such cases, and particularly at those cases where we cannot be confident that Aristotle gives a complete proof. A consequence of this is that in some cases I have not accepted, as Aristotelian, schemas which are conventionally counted as ones he accepts.

Chapter 10 focuses on what I call *apodeictic possibility*. By this I mean that sense of possibility according to which what is possible is what is not-necessarily-not the case. This is the logician's *M,* an abbreviation for ~*L*~. When Aristotle discusses this sense of *M*-possibility, the principles that govern his apodeictic syllogistic still apply. Because of this the MMM syllogisms will work like the LLL syllogisms, raising all the same questions. But Aristotle seems not to be especially interested in a careful study of syllogisms from premises about apodeictic possibility – perhaps because he sees that such a study will turn out to be like his earlier study of ordinary apodeictic *necessity*. He does, however, notice that necessity and possibility don't work the same when combined with negation, but he does not develop the point in much detail. And in fact he offers no proofs at all for any MMM, MXM, or XMM syllogisms. Instead, he shifts his discussion to consider a different sense of possibility – and this discussion makes up what is called the problematic syllogistic.

Part III is about the problematic syllogistic – the syllogistic involving a new sense of possibility. Initially Aristotle offers what appears to be a new definition of possibility as contingency, according to which what is possible is what is neither necessary nor impossible. Aristotle explains that anything that is contingently φ is also contingently not φ. In LPC the natural way to represent this sense of possibility is with the logician's Q, where $Q\varphi =_{df} \sim L\varphi \ \& \sim L{\sim}\varphi$. There is, however, a question about whether in fact Aristotle might be working here with a primitive notion of contingency – one which cannot really be explained in terms of *L*s and *M*s. This question is first raised in Chapter 12 and carries through subsequent chapters. The difficulty is this. If we simply adopt the definition of Q and *also* the principle that for red terms φ and $L\varphi$, and $\sim\varphi$ and $L{\sim}\varphi$ are equivalent then in the resulting syllogistic $Q\varphi$ will be empty when φ is red, and this will contravene the restriction against empty terms. Whatever the analysis, Aristotle introduces what is known as complementary conversion to accommodate the fact that in the syllogistic any time you have a proposition about what is $Q\varphi$, you also have a proposition about what is $Q{\sim}\varphi$, and vice versa. For example, if it is possible that a man sits, then it is also possible that he not sit, and vice versa. To have the capacity to sit is also to have the capacity to not sit. Aristotle appears to think of $Q\varphi$ and $Q{\sim}\varphi$ as equivalent. Yet he also treats complementary conversion as a way of generating additional premises which in turn sometimes generate 'new' syllogisms. Aristotle explains complementary conversion carefully, but he does not treat it as trivial and instead offers step-by-step proofs of each of the 'new' syllogisms. In his proofs he shows how to bring them back to more familiar syllogisms whose validity he has already established.

In the problematic syllogistic we face questions about how precisely to interpret Aristotle's problematic propositions about contingency. We have to be able to say where the Q-operators go. In his syllogistic about Q-contingency, Aristotle introduces two entirely new techniques. The first is known as *ampliation*. This technique allows him to make a distinction between kinds of Q-propositions. In LPC the difference is between whether we interpret 'All B are QA' as $\forall x(QBx \supset QAx)$ or as $\forall x(Bx \supset QAx)$. The first is ampliated, which is to say that both the subject and the predicate term are in the scope of a Q operator. On the other hand, $\forall x(Bx \supset QAx)$ is unampliated. Why the distinction? It would seem that Aristotle notices that he needs some sure way of validating QQQ syllogisms, and ampliation provides that way. Consider Barbara QQQ. If the premises are unampliated, there is no syllogism:

Barbara QQQ (Unampliated)
(5) $\forall x(Bx \supset QAx)$
(6) $\underline{\forall x(Cx \supset QBx)}$
(7) $\forall x(Cx \supset QAx)$

This is invalid because there is no way to get from QB to B. We cannot go from being a merely possible B to being a B. But Aristotle sees that if the premises are ampliated, then there is a syllogism. So Aristotle ampliates. Barbara QQQ is then as follows:

Barbara QQQ (Ampliated)
(8) $\forall x(QBx \supset QAx)$
(9) $\underline{\forall x(QCx \supset QBx)}$
(10) $\forall x(QCx \supset QAx)$

The results that depend upon ampliation are clearly similar to the results we would have in a red syllogistic as discussed in Chapter 6.

A second technique, also new to the problematic syllogistic, is *realization*. Aristotle makes the point that if something is possible, then supposing that it is actual can perhaps result in a falsehood, but never in an impossibility. The discussion of realization begins in Chapter 12 with a particularly perplexing syllogism, Barbara XQM.

(11) Barbara XQM
$\forall x(Bx \supset Ax)$
$\underline{\forall x(Cx \supset QBx)}$
$\forall x(Cx \supset MAx)$

Aristotle says that Barbara XQM is valid, but (11) seems susceptible of easy counter-examples. If we take our terms *A*, *B*, and *C* to be horse, in the paddock, and man, then we get the following:

(12)	Everything in the paddock is a horse	T
	<u>Every man could be in the paddock</u>	T
	Every man could be a horse	F

So it would seem that Barbara XQM is invalid. It would be trivially valid if ampliation were permitted on the first premise:

(13) Ampliated Barbara XQM
$\forall x(QBx \supset Ax)$
$\forall x(Cx \supset QBx)$
$\forall x(Cx \supset Ax)$ [non-modal Barbara]
$\forall x(Cx \supset MAx)$ [$\varphi \supset M\varphi$]

However, the first premise of (13) has to have an ampliated subject with an assertoric predicate. Aristotle's discussion of ampliation provides clear textual evidence to support the use of our LPC formulae $\forall x(Bx \supset QAx)$ and $\forall x(QBx \supset QAx)$, and of course the LPC formulae allow constructions such as $\forall x(QBx \supset Ax)$ and $\forall x(MBx \supset Ax)$, but Chapter 13 explains the evidence against these latter formulae. I introduce what I call the *Restricted Ampliation Principle*:

> RAP No proposition in the premises or conclusion of any (valid) syllogism can contain an ampliated subject with an assertoric or an apodeictic predicate.

In LPC, RAP rules out $\forall x(QBx \supset Ax)$ and $\forall x(QBx \supset LAx)$ (also $\forall x(MBx \supset Ax)$ and $\forall x(MBx \supset Ax)$) as Aristotelian premises. An ampliated Barbara XQM would be trivially valid, but Aristotle proceeds differently. He puts realization to work in the problematic syllogistic by allowing the possibilities described in modal premises to be realized. Realization is typically used as a step in a reductio proof. For example, Aristotle proves Barbara XQM and Celarent XQM using realization and reductio. Commentators have sometimes thought that Aristotle gets into serious trouble in proofs through realization. Judson (1983) and Rini (2003) accuse Aristotle of sometimes inappropriately realizing a possibility 'in complete insulation' from other background facts and truths. Judson calls this the '*insulated realization manœuvre*,' or 'IR manœuvre'. Chapter 13 explains how Aristotle avoids the dangers of the IR manœuvre by carefully restricting terms in accord with the red/green distinction. Chapters 13 and 14 explain Aristotle's proofs of first figure syllogisms involving one non-modal premise and one premise about *Q*-contingency.

Chapter 15 studies Aristotle's proofs of first figure syllogisms involving one premise about necessity and one premise about *Q* contingency. In this chapter we see that Aristotle does not always use his realization method where he might, and this explains some curious turns in the problematic syllogistic. For example, it seems clear that Aristotle *could have* used realization to establish the validity of Celarent LQM. We would expect a proof through realization to proceed as follows:

$\forall x(Bx \supset L{\sim}Ax)$		
$\forall x(Cx \supset QBx)$		
|	$\forall x(Cx \supset Bx)$	Realization hypothesis
|	$\forall x(Cx \supset L{\sim}Ax)$	Celarent LXL
$\forall x(Cx \supset M{\sim}Ax)$		discharge

His logical tools make this explanation available to him. But the text makes very clear that this is not how Aristotle proceeds – he uses a reductio proof and he uses modal conversion, but he does not use any realization. Instead, he first sets out to establish via reductio that a non-modal X conclusion, $\forall x(Cx \supset {\sim}Ax)$, follows from the premises. Then since $\varphi \supset M\varphi$, a true *M* conclusion, $\forall x(Cx \supset M{\sim}Ax)$, also follows. So Aristotle includes Celarent LQX and Celarent LQM among the valid modal syllogisms. He uses this method regularly and so allows many cases where L+Q premises and Q+L premises produce *both* an X conclusion and an M conclusion. I call this the LQX phenomenon, and I trace its use through the problematic syllogistic. The LQX phenomenon is at work in Aristotle's proofs of each of Celarent LQX, Ferio LQX, Cesare LQX, Camestres QLX, Festino LQX, Felapton LQX, Bocardo LQX, and Ferison LQX. Many of the syllogisms about contingency are really very simple and require nothing more than the substitution of QAx for Ax for the proof to work. Chapter 15 includes a list of all the terms Aristotle offers for constructing counter-examples involving contingency. Each set of terms includes at least one green term. In the problematic syllogistic Aristotle relies on the fact that $L{\sim}\varphi$ and $Q\varphi$ contradict, and $L\varphi$ and $Q\varphi$ contradict.

Chapter 16 investigates contingency in the second figure. Aristotle rejects *all* Q+Q premise combinations in the second figure. His explanation makes clear that he rejects them because he rejects QE-conversion. In LPC, a QE-premise might be $\forall x(Bx \supset Q{\sim}Ax)$. If this were to convert to $\forall x(Ax \supset Q{\sim}Bx)$, then complementary conversion would take us to $\forall x(Bx \supset QAx) \supset \forall x(Ax \supset QBx)$. But this conversion is not valid – if we convert $\forall x(Bx \supset QAx)$ we get only a particular, not a universal. So, QE-conversion itself must be rejected and all the second figure syllogisms which would require QE-conversion must be rejected also. The fact that Aristotle rejects all second figure QQQs indicates that he does not suppose QQQs, LLLs, and MMMs are isomorphic. In second figure syllogisms from one non-modal X-premise and one Q-contingent premise, Aristotle's proofs involve conversion into first figure syllogisms which depend upon realization and so involve a restriction to red terms. Some also require complementary conversion. Aristotle's proofs of second figure syllogisms from one L-premise and one Q-premise work much the same as the proofs from X+Q premise pairs. The failure of *QE*-conversion limits the valid combinations. Among the valid second figure syllogisms, when the privative premise is the L-premise then the LQX phenomenon comes into play, guaranteeing both an X and an M conclusion. Some combinations also require complementary conversion. Aristotle appears to reject some L+Q and Q+L second figure schemas, such as Cesare QLX and Cesare QLM, which his usual methods would allow him to validate. Chapter 16 suggests an explanation. Consider Cesare QLX:

Cesare QLX
(1) $\forall x(Bx \supset Q{\sim}Ax)$
(2) $\underline{\forall x(Cx \supset LAx)}$
(3) $\forall x(Cx \supset {\sim}Bx)$

The *QE*-premise (1) is of course equivalent to $\forall x(Bx \supset QAx)$ by complementary conversion. But this leaves us two affirmative premises in a third figure schema, and Aristotle tells us there are no third figure syllogisms from two affirmative premises.

There is a tension that I do not think Aristotle ever completely resolves in the modal syllogistic about just how we are to understand $Q\varphi$ and $Q{\sim}\varphi$. He tells us that premises about what is $Q\varphi$ and $Q{\sim}\varphi$ are 'positive and not privative'. But he does not seem to see that, by parity of reasoning, he might just as well count premises about what is $Q\varphi$ and $Q{\sim}\varphi$ as *privative* and not positive. Complementary conversion would seem to allow this, and this makes Aristotle's claim that both are positive seem arbitrary. The second figure syllogisms make sense of the curiosity of the LQX phenomenon. Take

Cesare LQX
$\forall x(Bx \supset L{\sim}Ax)$
$\underline{\forall x(Cx \supset QAx)}$
$\forall x(Cx \supset {\sim}Bx)$

It is easy to see why Cesare LQX should be valid. For (1) says that all the *B*s are necessary not-*A*s, while (2) says that all the *C*s are contingent *A*s. So it follows that nothing can be both *B* and *C*.

Chapter 17 studies contingency in the third figure. The third figure QQQ syllogisms are substitution instances of ordinary non-modal XXX syllogisms. All QQQ syllogisms in the third figure must have ampliated conclusions, and when we represent the proofs in LPC it is clear that the QQQ syllogisms depend on simple non-modal *A*- and *I*-conversions. Some also use complementary conversion. Aristotle allows syllogisms from third figure X+Q and Q+X premise combinations, but sometimes he leaves off the detail needed for constructing proofs. Still, we want to be able to explain. One way we might explain involves what I call the *Restricted Ampliation Manœuvre* – it is a consequence of RAP.

RAM Where Aristotle provides a proof involving a proposition with an ampliated subject and an assertoric (or apodeictic) predicate, he always downgrades the predicate to an *M* predicate in the conclusion.

RAM helps if for example we reach a conclusion $\exists x(QBx \ \& \ Ax)$ which violates RAP. RAM downgrades this to $\exists x(QBx \ \& \ MAx)$, which does not violate RAP. Here is how RAM works in Darapti XQM:

Darapti XQM

(1)	$\forall x(Cx \supset Ax)$	Given
(2)	$\forall x(Cx \supset QBx)$	Given
(3)	$\exists x(QBx \ \& \ Cx)$	'*BC* is converted' [A-conv of (2)]
(4)	$\exists x(QBx \ \& \ Ax)$	'it will be the first figure' [Darii]
(5)	$\exists x(QBx \ \& \ MAx)$	RAM (4)

Here is an alternative that works without RAM:

Darapti XQM [through realization]

(1)	$\forall x(Cx \supset Ax)$		Given
(2)	$\forall x(Cx \supset QBx)$		Given
(3)	|	$\forall x(Cx \supset Bx)$	Realization (2)
(4)	|	$\exists x(Bx \,\&\, Cx)$	'*BC* is converted' (3)
(5)	|	$\exists x(Bx \,\&\, Ax)$	'it will be the first figure' [Darii (1)(4)]
(6)	$\exists x(Bx \,\&\, MAx)$		discharge

These two alternatives give the syllogism a different conclusion and hence a different form, and there is nothing in Aristotle's text to decide between them. This interpretive dilemma affects a family of third figure mixed contingent syllogisms, and Chapter 17 explains when such different interpretations are possible through either RAM or realization. Not all third figure mixed contingent syllogisms present such interpretive difficulties – some syllogisms are substitution instances of non-modal syllogisms, as for example Darapti QXQ, in which contingency is only trivially included. Some syllogisms clearly depend upon the LQX phenomenon. Some syllogistic schemas get left out of Aristotle's discussion, such as Bocardo XQM and LQM.

Most interpreters of Aristotle's modal syllogistic want to offer a formal model. I have not sought to impose a formal modelling. Instead I have focused on explaining and justifying Aristotle's own step-by-step proofs. The principle theme that emerges from this study is that, unlike the non-modal syllogistic, Aristotle's modal syllogistic has to be understood as an applied logic. I say this because where the modal syllogistic is more than a trivial version of the non-modal syllogistic, we find evidence that Aristotle does restrict his terms in order to reflect his basic distinction between what cannot be otherwise and what can be – between what I call red terms and green terms. And this would seem to point to a clear link between his views of science and demonstration.

Appendix: The LPC Framework

This book has studied the modal syllogistic using the tools of modern modal predicate logic. The purpose of this appendix is to set out modal LPC, and make a few remarks about some formal frameworks in the literature which do not base themselves on LPC. Formulae (sometimes called *well-formed* formulae, wff) of LPC are built up from atomic wff of the form Ax, where A is a (one-place) predicate variable, and x is an individual variable.[1] Aristotle typically uses A, B and C (A, B, Γ) in his syllogisms, though sometimes he uses R, P and S (P, Π and Σ) or other letters. Aristotle does not have individual variables, and part of the purpose of this appendix is to explain precisely why this does not matter.

Atomic wff can be combined into complexes using the operators ~, ⊃, and &.[2] If φ and ψ are both wff so are ~φ, φ ⊃ ψ and φ & ψ. The meanings of these operators are given by their truth tables:

~ φ	φ ⊃ ψ	φ & ψ
F T	T T T	T T T
T F	T F F	T F F
	F T T	F F T
	F T F	F F F

~φ is true if φ is false, and false if φ is true, φ ⊃ ψ is true in all cases except when φ is true and ψ is false, while φ & ψ is only true when both φ and ψ are true.

For non-modal LPC we only need to add the quantifiers ∀ and ∃. In LPC a formula is evaluated with respect to a *domain* (or 'Universe of Discourse') D, and an assignment of values to the predicates. Each predicate A has, as its value, a (non-empty[3]) set of individuals from D. Where we have a fixed interpretation in mind we use the italicised A (or B or C) for the predicate, and the unitalicised A (or B or C) for the set of individuals which is the predicate's value in the interpretation in question. To interpret an atomic wff Ax we also need an individual to be assigned to x, and then Ax is true if and only if (iff) the individual assigned to x is in the set A which is the value of the predicate A. We can then use the truth tables for ~, ⊃, and & to determine the truth or falsity of any wff made up by these operators from atomic wff. Where φ is such a wff $\forall x\varphi$ will be true iff φ itself is true no matter what x may be (strictly, no matter

[1]Aristotle's syllogistic has no need of n-place predicates for $n > 1$, though these are important in modern LPC, which, unlike Aristotle, is concerned also with relations like 'is larger than'.

[2]Other symbols used in this book are ∨ and ≡, but these are not normally used in a formula which represents a proposition in the syllogistic.

[3]The requirement that A be non-empty is not one imposed in LPC generally, but, as stated on p. 17, it is a requirement that Aristotle assumes in the syllogistic.

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which individual is assigned to the variable x) and $\exists x\varphi$ will be true iff for some individual in D, φ is true when x is (assigned) that individual. We can illustrate this by two wff which represent Aristotelian syllogistic propositions – the A-proposition $\forall x(Bx \supset Ax)$ and the O-proposition $\exists x(Bx\ \&\ {\sim}Ax)$. The former is true iff $Bx \supset Ax$ is true no matter what x may be, and this in turn is true iff no individual in D is in B but not in A. The latter is true iff there is an x in D for which $Bx\ \&\ {\sim}Ax$ is true, and this is so provided there is an individual in D which is in B but not in A.

To get *modal* LPC we need to interpret L and M (and later Q). Assuming M is defined as ${\sim}L{\sim}$ we only need to interpret L. In modal logic this is normally interpreted by introducing a set W of *possible worlds* of which one is the actual world. I'll call this w_a. The other worlds represent ways things could have been, from the point of view of w_a. In a modal interpretation each predicate A has as its value at each world w a set, A_w, of individuals from D, which are thought of as the things which satisfy A in w. We say that a wff $L\varphi$ is true in w_a iff φ is true in every world w, i.e., every $w \in$ W, either w_a itself or some other w, and $M\varphi$ is true in w_a iff φ is true in some world $w \in$ W.

This book has imposed significant constraints on the wff of modal LPC which represent the propositions of Aristotle's syllogistic, of which the most important are that

(i) ~ occurs *only* in front of an atomic wff

and

(ii) L or M occurs only in front of an atomic wff or its negation.

(i) and (ii) rule out *de dicto* wff like LA′ on p. 45, and rule out wff in which one L occurs in the scope of another as in Nortmann's (1) on p. 56. So that in evaluating the truth of a modal wff at the actual world w_a we never need to consider the truth or falsity of *modal* wff at any world other than w_a.[4] When I speak of truth or falsity simpliciter I mean truth or falsity in w_a.

Given such an interpretation we can, for any predicate A, form the following six sets of individuals:

A is the set of individuals which satisfy A in w_a
A^L is the set of individuals which satisfy A in *every* world[5]

[4] Among other things this means that for these wff the difference between different modal systems, which exercises much of Nortmann's work, makes no difference to the *de re* wff that represent Aristotle's modal propositions. In a relational model the W used here would represent the worlds accessible from w_a. Since there are no modal wff to be evaluated in any other w the question of what worlds are accessible from *them* will never arise. See also Charles (2000, pp. 385—387).

[5] It's worth pointing out that A^L = A and $A^{L\sim}$ = $A^{\sim}$ for any *red* A, and so for red A, A^L and $A^{L\sim}$ will only be empty if A is. For a green term A on the other hand, even though A is required to be non-empty yet A^L and $A^{L\sim}$ may well be empty. (On this see Thom 1996, p. 192.) This is not a problem for the view defended

A^M is the set of individuals which satisfy *A* in *at least one* world
$A^{\sim}$ is the set of individuals which do not satisfy *A* in w_a
$A^{L\sim}$ is the set of individuals which do not satisfy *A* in *any* world
$A^{M\sim}$ is the set of individuals which fail to satisfy *A* in *at least one* world

These sets are connected with the truth of atomic wff as follows:

Ax is true iff the individual assigned to x is in A
LAx is true iff the individual assigned to x is in A^L
MAx is true iff the individual assigned to x is in A^M
$\sim Ax$ is true iff the individual assigned to x is in $A^{\sim}$
$L{\sim}Ax$ is true iff the individual assigned to x is in $A^{L\sim}$
$M{\sim}Ax$ is true iff the individual assigned to x is in $A^{M\sim}$

From this it easily follows that, where $\subseteq$, $\cap$ and $\varnothing$ are the usual set-theoretical operations[6] on subsets of D, we have

$\forall x(Bx \supset Ax)$ is true iff $B \subseteq A$
$\forall x(Bx \supset {\sim}Ax)$ is true iff $B \cap A = \varnothing$
$\forall x(Bx \supset LAx)$ is true iff $B \subseteq A^L$
$\forall x(Bx \supset L{\sim}Ax)$ is true iff $B \subseteq A^{L\sim}$
$\forall x(Bx \supset MAx)$ is true iff $B \subseteq A^M$
$\forall x(Bx \supset M{\sim}Ax)$ is true iff $B \subseteq A^{M\sim}$
$\exists x(Bx \,\&\, Ax)$ is true iff $B \cap A \neq \varnothing$
$\exists x(Bx \,\&\, {\sim}Ax)$ is true iff $B \cap A^{\sim} \neq \varnothing$ (alternatively $B \not\subseteq A$.)
$\exists x(Bx \,\&\, LAx)$ is true iff $B \cap A^L \neq \varnothing$
$\exists x(Bx \,\&\, L{\sim}Ax)$ is true iff $B \cap A^{L\sim} \neq \varnothing$
$\exists x(Bx \,\&\, MAx)$ is true iff $B \cap A^M \neq \varnothing$
$\exists x(Bx \,\&\, M{\sim}Ax)$ is true iff $B \cap A^{M\sim} \neq \varnothing$

What is significant about this way of looking at things is that the semantics of the restricted class of wff of modal LPC needed to express Aristotelian propositions can be given simply in terms of the sets A, $A^{\sim}$, A^L, $A^{L\sim}$, A^M and $A^{M\sim}$. So, although the possible-worlds modelling may have been introduced as the basis for modal LPC semantics, once A, $A^{\sim}$, A^L, $A^{L\sim}$, A^M and $A^{M\sim}$ have been defined, they are enough to do the job of

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in this book, since, in such a case Aristotle would simply not syllogize.

[6] $B \subseteq A$ means that every member of B is also in A, $A \cap B$ is the set whose members are precisely those individuals in both A and B, and $\varnothing$ is the empty set, with no members. For an individual $u \in A$ means that u is a member of A, and $w \in W$ means that w is a member of W.

interpreting Aristotle's modal propositions on their own, and the possible worlds can drop out of the picture.

Of the formal accounts of the syllogistic not based on LPC one of the most thorough and comprehensive is that of Thom (1996), who builds on the work of Johnson (1989) and Thomason (1993). If we use Thom's notation as definitional abbreviations of LPC wff we have:

$\forall x(Bx \supset Ax)$:	ab^a is true iff $\mathrm{B} \subseteq \mathrm{A}$
$\forall x(Bx \supset {\sim}Ax)$:	ab^e is true iff $\mathrm{B} \cap \mathrm{A} = \varnothing$
$\forall x(Bx \supset LAx)$:	Lab^a is true iff $\mathrm{B} \subseteq \mathrm{A}^L$
$\forall x(Bx \supset L{\sim}Ax)$:	Lab^e is true iff $\mathrm{B} \subseteq \mathrm{A}^{L\sim}$
$\forall x(Bx \supset MAx)$:	Mab^a is true iff $\mathrm{B} \subseteq \mathrm{A}^M$
$\forall x(Bx \supset M{\sim}Ax)$:	Mab^e is true iff $\mathrm{B} \subseteq \mathrm{A}^{M\sim}$
$\exists x(Bx \,\&\, Ax)$:	ab^i is true iff $\mathrm{B} \cap \mathrm{A} \neq \varnothing$
$\exists x(Bx \,\&\, {\sim}Ax)$:	ab^o is true iff $\mathrm{B} \cap \mathrm{A}^{\sim} \neq \varnothing$
$\exists x(Bx \,\&\, LAx)$:	Lab^i is true iff $\mathrm{B} \cap \mathrm{A}^L \neq \varnothing$
$\exists x(Bx \,\&\, L{\sim}Ax)$:	Lab^o is true iff $\mathrm{B} \cap \mathrm{A}^{L\sim} \neq \varnothing$
$\exists x(Bx \,\&\, MAx)$:	Mab^i is true iff $\mathrm{B} \cap \mathrm{A}^M \neq \varnothing$
$\exists x(Bx \,\&\, M{\sim}Ax)$:	Mab^o is true iff $\mathrm{B} \cap \mathrm{A}^{M\sim} \neq \varnothing$

So far of course the interpretation has reflected LA-LO on p. 52, and, as noted in that chapter, other authors offer interpretations which are different from that offered there. Consider Thom's semantics for LI propositions. Thom (1996, p. 139) uses f_1, f_2, f_3 ...etc., as subsets of D which interpret the predicates a, b, c, ... etc. This is in line with my use of A, B, and C to interpret A, B and C. For each such f there is a 'star-set' f^* which represents those things which are f by necessity. Thom also has a 'sun-set' $f^{¤}$. In the notation used here f^* corresponds to A^L and $f^{¤}$ corresponds to $\mathrm{A}^{L\sim}$. For Thom the non-modal propositions are evaluated as follows:

ab^a is true iff $f_2 \subseteq f_1$
ab^e is true iff $f_2 \cap f_1 = \varnothing$
ab^i is true iff $f_2 \cap f_1 \neq \varnothing$
ab^o is true iff $f_2 \not\subseteq f_1$

For Lab^a we have (in Thom's 21.1.5 on p. 146) that Lab^a is true iff $f_2 \subseteq f_1^*$, which is the same as my semantics above. Thom has a disjunctive interpretation of Lab^i according to which (21.1.7) Lab^i is true iff $f_1 \cap f_2^* \neq \varnothing$ or $f_2 \cap f_1^* \neq \varnothing$. In LPC this means that Lab^i is true iff either $\mathrm{B} \cap \mathrm{A}^L \neq \varnothing$ or $\mathrm{A} \cap \mathrm{B}^L \neq \varnothing$, which is precisely the condition that

$$\exists x(Bx \,\&\, LAx) \vee \exists x(Ax \,\&\, LBx)$$

is true, as I claimed on p. 57.

Let's look briefly at the copulative approach in Patterson (1995). This appendix is not the place to discuss Patterson's views on Aristotle's theory of predication. It is sufficient to show how to express the *de re*/*de dicto* distinction by means of a copulative relation between terms. Imagine the following relation between subsets A and B of D:

$$\text{B } N_{\text{weak}} \text{ A iff B} \subseteq \text{A}^{L}$$

This corresponds to the weak cop of Patterson (1995, p. 12). Strong cop is a little more complex. The relation of *de dicto* necessity, as for instance the relation between 'bachelor' and 'unmarried' is not a relation between *sets* of individuals. For even if it should happen that the bachelors are precisely those who live in Wellington, and the unmarried are precisely those who are happy, it need not follow that there is any relation of necessity between living in Wellington and being happy. In possible worlds terms the meaning of a predicate *A* is a function from worlds to sets of individuals. (I'll call it A*, though it must not be confused with Thom's star sets.) Then strong cop may be defined as

$$\text{B}^* \; N_{\text{strong}} \; \text{A}^* \text{ iff for every } w \text{ in W, B}^*(w) \subseteq \text{A}^*(w).$$

In these terms weak cop would be defined as

$$\text{B}^* \; N_{\text{weak}} \; \text{A}^* \text{ iff B}^*(w_a) \subseteq (\text{A}^*(w_a))^{L}.$$

It is not difficult to see that these 'copulative' relations correspond exactly to *de re* and *de dicto* interpretations of universal affirmative propositions, as in LA and LA′ on p. 45 above. And the case is similar with the other Aristotelian modals.

It is not even necessary to assume that the As, Bs and so on are sets of individuals – even though that is the way they have been defined in this appendix – provided we can make use of the operations $\subseteq$, $\cap$ $\varnothing$ and the like, which can, for instance, be interpreted mereologically as suggested by Malink (2006, p. 107). Some of the scholars who eschew LPC notation do so because they feel that predicates of individuals distort Aristotle's own text too much.[7] Questions like this, however

[7]Vilkko and Hintikka (2006, p. 374) point out: "For instance, when 'every man is mortal' is expressed as '$(\forall x)(x$ is a man $\supset x$ is mortal)' the phrase 'every man' disappears altogether as a whole. 'Every' goes into '$(\forall x)$' and 'man' becomes the predicate term of the antecedent." See also Malink (2006, p. 106). Malink is able to avoid reference to individuals by extensive use of quantification over terms, which from the point of view of LPC amounts to second order quantification. Thus one can analyse $\exists x(Bx \;\&\; Ax)$, which appears to involve reference to individuals, as $\exists$C(C $\subseteq$ B & C $\subseteq$ A). This enables Malink to analyse

important they may be to understanding Aristotle's views about predication, seem to me to go far beyond the simple logical structure of the modal syllogistic.

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cases like 'some white thing is a log' (see p. 53 above) in terms of a relation Υab which is like my $B \subseteq A$ (Malink, p. 98). In Malink's notation $\exists x(Bx \,\&\, Ax)$ would be represented as $\exists z(\Upsilon az \wedge \Upsilon bz)$. There is of course a sense in which Malink's theory *is* set out in LPC. For one can consider it as a first order theory whose primitive predicates include Υ, $\mathbf{E}$ and $\tilde{\mathbf{E}}$.

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