Worked Examples in Basic Electronics

BY

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1. In general, symbols and abbreviations used in the text are those recommended by the British Standards Institution in B.S. 1991, Parts 1 and 6.

2. Specimen answers given to questions from the Graduateship Examination papers of the Institution of Electronic and Radio Engineers are the author's and are not necessarily endorsed by the Institution.

Preface

ONE of the difficulties experienced by students on Engineering Courses is that the time available for formal instruction is very limited. Contact time with their instructors is necessarily devoted to establishing the basic principles of the relevant technology. Too little time is available for the important task of solving problems and obtaining numerical answers.

To rectify this situation this book is published as one of a series devoted entirely to Worked Examples and Problems which will enable the student to follow through a problem step-by-step and then to attempt to solve similar problems with a minimum of supervision. It will be an essential aid to his formal education and training as a potential technician or professional engineer whether on a part-time, sandwich or full-time course.

This book contains a limited amount of material on semiconductors since it is intended to be directly complementary to a book in this series entitled *Worked Examples on Semiconductor Circuits* by Abrahams and Pridham.

N. HILLER

CHAPTER 1

Derivation of Basic Formulae

TWENTY-NINE basic formulae are derived in this chapter, and special note is made of the circuit conditions which must exist if any particular formula is to be valid. Where possible a basic circuit is employed in each section, and either a constant current equivalent circuit, or a constant voltage equivalent circuit, is used to predict the a.c. performance of that circuit, provided the active device is biased on the linear portion of its characteristics, and the alternating input signal is small enough to render amplitude distortion negligible. Hence, when either type of equivalent circuit is used, the fact that the circuit in question is linear is not included under the *Assume* heading of the relevant section—it is understood.

A knowledge of complex numbers and differentiation is sufficient to enable the reader to follow the mathematical steps used to arrive at any result in any of the following sections. Electrical steps are explained in some detail and at the end of each section a *Summary* is included which contains conclusions drawn from the treatment and the derived formulae.

1.1. Voltage Amplification Factor of a Resistance-loaded Triode Valve

BASIC CIRCUIT

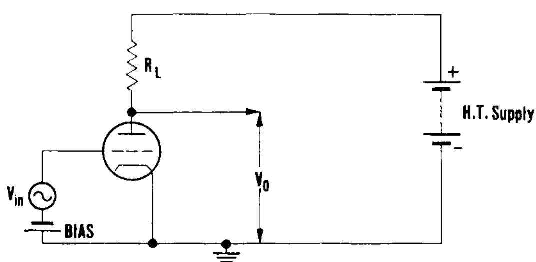


FIG. 1.1

Constant Voltage Equivalent Circuit

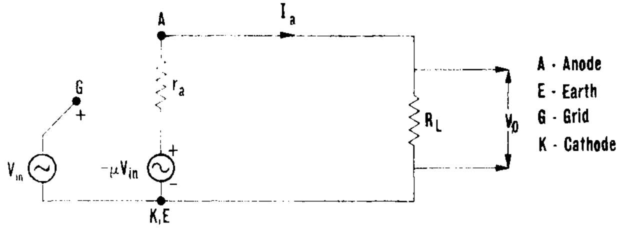


Fig. 1.2

Assume

1. The voltage input $\mathbf{V}_{\text{in}}$ and resulting anode current $\mathbf{I}_a$ vary sinusoidally.
2. The valve operates either on the linear portion of its static characteristic or with constant values of valve parameters μ, g_m and r_a.
3. H.T. and bias supplies fix the operating point of the valve and the values of μ, g_m and r_a. In pentode applications the screen voltage is assumed fixed, and feedback at the signal frequency, negligible.

These comments apply for each constant voltage or constant current equivalent circuit used in this chapter and will not be stated on future occasions.

Proof of the Gain Formula

From the equivalent circuit of Fig. 1.2,

$$\mathbf{I}_a = \frac{-\mu \mathbf{V}_{\text{in}}}{r_a + R_L}$$

$$\mathbf{V}_o = \mathbf{I}_a R_L$$

$$= \frac{-\mu \mathbf{V}_{\text{in}} R_L}{r_a + R_L}$$

The voltage amplification factor V.A.F. is given by V_o/V_{in}, therefore,

$$\text{V.A.F.} = \frac{-\mu R_L}{r_a + R_L}. \tag{1.1}$$

Summary

1. The gain realized by the single valve stage must always be less than the amplification factor μ.
2. The valve acts as a constant voltage generator which produces an e.m.f. μ times larger than the grid input voltage, and 180 deg out of phase with it. This phase reversal is denoted by the minus sign in equation (1.1).
3. The constant voltage generator of the equivalent circuit of Fig. 1.2 has an internal resistance equal to the anode slope resistance of the valve (r_a).
4. If the anode load R_L is not purely resistive then $V_o/V_{in} = -\mu Z_L/(r_a + Z_L)$, where Z_L is the load impedance at the frequency of the sinusoidal input voltage.

1.2. Stage Gain of a Resistance–Capacitance Coupled Voltage Amplifier (Low Audio-frequency Working)

BASIC CIRCUIT

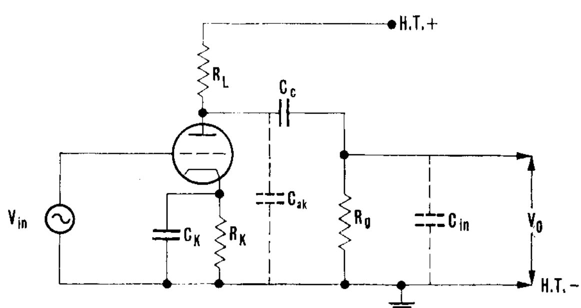


FIG. 1.3

Constant Voltage Equivalent Circuit Correct at all Frequencies

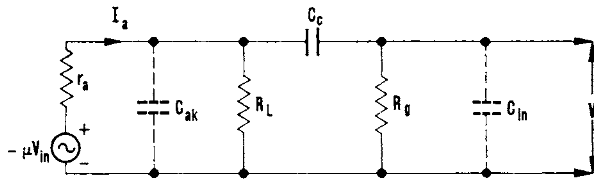


Fig. 1.4

C_c is the coupling capacitor to a following stage.
R_g is the grid leak resistor of a following stage.
C_{ak} is the capacitance between anode and cathode of the valve.
C_{in} is the input capacitance of a following stage, including inter-wiring and stray capacitance effects.

The equivalent circuit of Fig. 1.4 can be simplified over certain limited frequency ranges.

Constant Voltage Equivalent Circuit at Low Frequencies

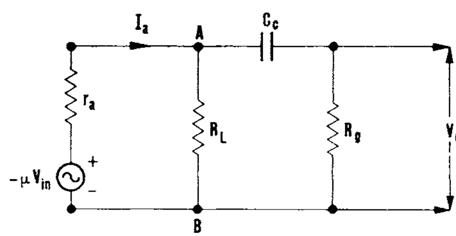


Fig. 1.5

Assume

1. The bias voltage provided by C_K and R_K gives the required working point down to zero frequency with no ripple.

2. The capacitive reactances due to C_{ak} and C_{in} have negligible shunting effect on R_L or R_g at these frequencies.

*Proof of the Gain Formula**

From Fig. 1.5,

$$\mathbf{I}_a = \frac{-\mu \mathbf{V}_{in}}{r_a + Z_{AB}}.$$

Therefore

$$\mathbf{V}_{AB} = \frac{-\mu \mathbf{V}_{in}}{r_a + Z_{AB}} . Z_{AB}$$

And
$$\mathbf{V}_o = \mathbf{V}_{AB} . \frac{R_g}{R_g + \dfrac{1}{j\omega C_c}}$$
$$= \frac{-\mu \mathbf{V}_{in} Z_{AB}}{r_a + Z_{AB}} . \frac{R_g}{R_g + \dfrac{1}{j\omega C_c}}$$

And
$$Z_{AB} = \frac{R_L(1 + j\omega C_c R_g)}{1 + j\omega C_c(R_L + R_g)}$$

Stage gain
$$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}}$$

$$m = \frac{-\mu R_L(1 + j\omega C_c R_g)}{\left[1 + j\omega C_c(R_L + R_g)\right]\left[r_a + R_L \dfrac{(1 + j\omega C_c R_g)}{1 + j\omega C_c(R_L + R_g)}\right]} . \frac{R_g}{\left[\dfrac{1 + j\omega C_c R_g}{j\omega C_c}\right]}$$

$$= \frac{-\mu R_L R_g}{r_a R_L + (r_a + R_L)R_g - j\dfrac{(r_a + R_L)}{\omega C_c}}. \tag{1.2}$$

*An alternative method of deriving equations (1.2) and (1.5) is by the application of Thévenin's theorem to the equivalent circuits of Fig. 1.5 and Fig. 1.8 respectively (see Chapter 2 Example 2.)

The low-frequency voltage gain is 0·707 of its maximum value when real and imaginary terms in the denominator of equation (1.2) are equal.

Hence,
$$r_a R_L + (r_a + R_L) R_g = \frac{(r_a + R_L)}{\omega_1 C_c}$$

where $\omega_1 = 2\pi f_1$ radians per second, and f_1 cycles per second is the frequency at which the gain has reduced to 0·707 of its maximum value.

Therefore
$$f_1 = \frac{1}{2\pi C_c\left[\dfrac{r_a R_L}{r_a + R_L} + R_g\right]} \text{ cycles per second.} \qquad (1.3)$$

Summary

1. It can be seen from equation (1.2) that the stage gain is zero when the frequency of the applied voltage is zero.

2. As the frequency is increased from zero, the imaginary term in the denominator of equation (1.2) reduces until the gain reaches a maximum value ($m_{\max}$) given by
$$m_{\max} = \frac{-\mu R_L R_g}{r_a R_L + (r_a + R_L) R_g}.$$

3. When the low-frequency response is 3 dB down (stage gain $= 0{\cdot}707\, m_{\max}$) the frequency of the applied voltage is given approximately by $f_1 = 1/2\pi C_c R_g$ cycles per second provided $r_a R_L/(r_a + R_L)$ is much less than R_g in equation (1.3).

4. The low-frequency response can be improved by increasing the *time constant* of the coupling components C_c and R_g (see Fig. 1.6). In this way, low frequencies down to zero cycles per second can all be amplified equally.

LOW-FREQUENCY RESPONSE

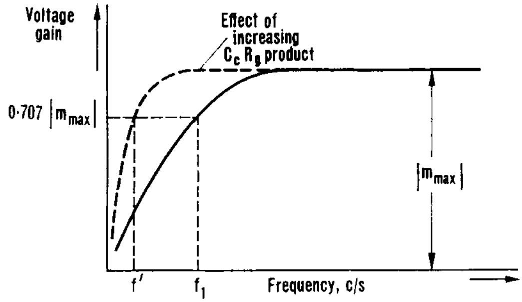


FIG. 1.6

1.3. Stage Gain of a Resistance–Capacitance Coupled Voltage Amplifier (Medium Audio-frequency Working)

BASIC CIRCUIT. See Fig. 1.3.

CONSTANT VOLTAGE EQUIVALENT CIRCUIT AT MEDIUM FREQUENCIES

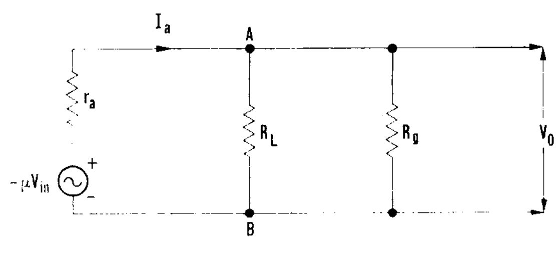


FIG. 1.7

Assume

1. The reactance of the coupling capacitor C_c is negligibly small.
2. The capacitive reactances of C_{ak} and C_{in} have negligible shunting effect on R_L and R_g.

Proof of the Gain Formula

From Fig. 1.7,

$$Z_{AB} = \frac{R_L R_g}{R_L + R_g}$$

Therefore
$$\mathbf{I}_a = \frac{-\mu \mathbf{V}_{in}}{r_a + \dfrac{R_L R_g}{R_L + R_g}}$$

And
$$\mathbf{V}_o = \frac{-\mu R_L R_g \mathbf{V}_{in}}{(R_L + R_g)\left[r_a + \dfrac{R_L R_g}{R_L + R_g}\right]}$$

Therefore stage gain $m = \dfrac{\mathbf{V}_o}{\mathbf{V}_{in}} = \dfrac{-\mu R_L R_g}{r_a R_L + r_a R_g + R_L R_g}$. (1.4)

Summary

1. The gain at medium frequencies is equal to the maximum gain at low frequencies.
2. The gain at medium frequencies is independent of changes in frequency so long as the equivalent circuit of Fig. 1.7 is valid.

1.4. Stage Gain of a Resistance–Capacitance Coupled Voltage Amplifier (High Audio-frequency Working)

BASIC CIRCUIT. See Fig. 1.3.

CONSTANT VOLTAGE EQUIVALENT CIRCUIT AT HIGH FREQUENCIES

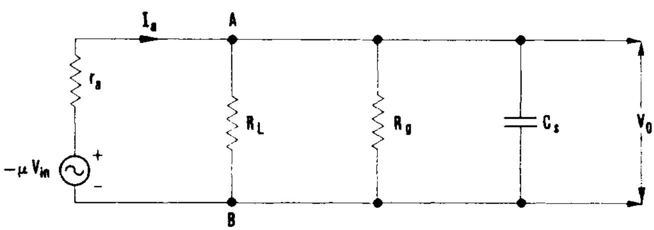


FIG. 1.8

Assume

1. $C_s = C_{ak} + C_{in}$. At high frequencies the reactance of this capacitance begins to shunt $R_L R_g/(R_L + R_g)$ of Fig. 1.8.
2. The reactance of C_c is less than at medium frequencies and again is negligibly small.

Proof of the Gain Formula

From Fig. 1.8,

$$\frac{1}{Z_{AB}} = \frac{1}{R_L} + \frac{1}{R_g} + j\omega C_s$$

$$= \frac{R_g + R_L + j\omega C_s R_L R_g}{R_L R_g}.$$

Therefore

$$Z_{AB} = \frac{R_L R_g}{R_g + R_L + j\omega C_s R_L R_g}.$$

Hence

$$\mathbf{I}_a = \frac{-\mu \mathbf{V}_{in}}{r_a + \dfrac{R_L R_g}{R_g + R_L + j\omega C_s R_L R_g}}$$

And

$$\mathbf{V}_o = \frac{-\mu \mathbf{V}_{in} R_L R_g}{(R_g + R_L + j\omega C_s R_L R_g)\left[r_a + \dfrac{R_L R_g}{R_g + R_L + j\omega C_s R_L R_g}\right]}$$

Therefore

$$\text{stage gain } m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{-\mu R_L R_g}{r_a(R_g + R_L + j\omega C_s R_L R_g) + R_L R_g}.$$

Rearranging gives

$$m = \frac{-\mu R_L R_g}{r_a R_L + r_a R_g + R_L R_g + j\omega C_s R_L R_g r_a} \qquad (1.5)$$

The high-frequency voltage gain is 0·707 of its maximum value when real and imaginary terms in the denominator of equation (1.5) are equal.

Hence $$r_a R_L + r_a R_g + R_L R_g = \omega_2 C_s R_L R_g r_a$$

where $\omega_2 = 2\pi f_2$ radians per second and f_2 cycles per second is the frequency at which the gain has reduced to 0·707 of its maximum value.

Therefore $$f_2 = \frac{1}{2\pi C_s}\left[\frac{1}{r_a} + \frac{1}{R_L} + \frac{1}{R_g}\right] \text{ cycles per second.} \qquad (1.6)$$

Summary

1. It can be seen from equation (1.5) that the stage gain is zero when the frequency of the applied voltage is infinite.

2. When the frequency of the applied voltage is such that the imaginary term in the denominator of equation (1.5) is negligible compared with the real term, the gain has a maximum value of $\mu R_L R_g/(r_a R_L + r_a R_g + R_L R_g)$.

3. As frequency is increased above the medium-frequency range, the gain falls off as the reactance of C_s reduces the value of Z_{AB}.

4. When the high-frequency response is 3 dB down (stage gain $= 0{\cdot}707\, m_{\max}$) the frequency of the applied voltage is given by equation (1.6). It follows that stray capacitance, and input capacitance to a following stage, should be as small as possible to prevent gain falling off at high frequencies.

GAIN-FREQUENCY RESPONSE OF A RESISTANCE–CAPACITANCE COUPLED AMPLIFIER

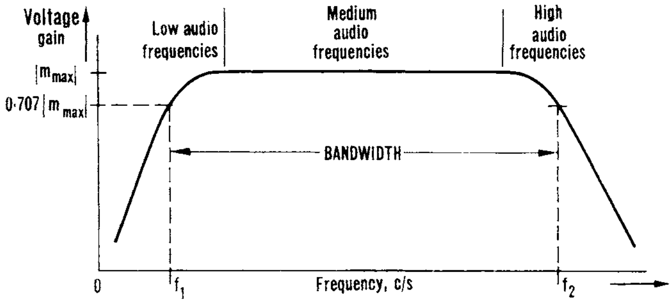


FIG. 1.9

Bandwidth. At frequencies f_1 and f_2, the voltage gain of the amplifier has fallen to 0·707 $m_{\max}$ and the power gain to 0·5 of its maximum value. The difference in frequency between these *half power* points (f_2-f_1), is called the *bandwidth* of the amplifier.

PHASE SHIFT IN A RESISTANCE–CAPACITANCE COUPLED AMPLIFIER

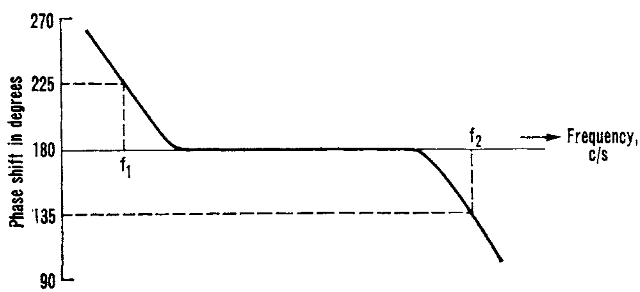


FIG. 1.10

At Low Audio Frequencies

The gain formula has the general form $m = -A-jB$ whence, phase shift (ϕ) of the output voltage ($\mathbf{V}_o$) relative to the input voltage ($\mathbf{V}_{in}$) is given by

$$\phi = \tan^{-1}\frac{-B}{-A}$$

hence, low-frequency phase shift varies between limits of 180° and 270°.

At Medium Audio Frequencies

The gain formula has the general form $m = -A+j0$ and phase shift is 180° due to the action of the valve.

At High Audio Frequencies

The gain formula has the general form $m = -A+jB$.

Therefore $$\phi = \tan^{-1}\frac{B}{-A},$$

hence, high-frequency phase shift varies between limits of 180° and 90°.

1.5. To Show that the Gain–Bandwidth Product is Constant for a Resistance–Capacitance Coupled Pentode Amplifier

BASIC CIRCUIT. See Fig. 1.3.

CONSTANT CURRENT EQUIVALENT CIRCUIT AT MEDIUM FREQUENCIES

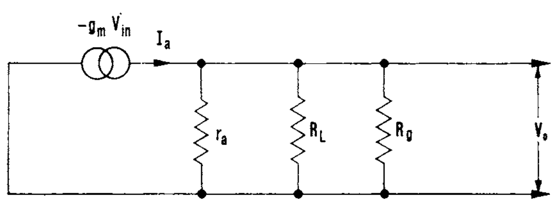


FIG. 1.11

Assume

1. As a pentode is used, $r_a \gg R_L$ and takes negligible current from the constant current generator of Fig. 1.11.
2. R_g is large to improve the low-frequency response, and R_L is low to improve high-frequency response, hence, $R_g \gg R_L$ and also takes negligible current from the constant current generator.
3. The *lower half-power frequency* f_1 is so small that the bandwidth of the stage is approximately given by f_2 from equation (1.6).

To Verify that the Gain–Bandwidth Product is a Constant

As negligible current is taken by r_a and R_g of Fig. 1.11,

$$\mathbf{V}_o = -g_m \mathbf{V}_{in} R_L$$

and
$$|m| = \frac{|V_o|}{|V_{in}|} = g_m R_L. \tag{1.7}$$

Now, from equation (1.6),

$$\text{bandwidth} = f_2 = \frac{1}{2\pi C_s}\left(\frac{1}{r_a}+\frac{1}{R_L}+\frac{1}{R_g}\right) \text{ c/s.}$$

Under the assumptions, r_a and R_g are both very much greater than R_L.

Therefore
$$\text{bandwidth} = \frac{1}{2\pi C_s R_L} \text{ cycles per second.} \tag{1.8}$$

Multiplying equation (1.7) by equation (1.8) gives,

$$\text{gain} \times \text{bandwidth} = g_m R_L \frac{1}{2\pi C_s R_L}$$

$$= \frac{g_m}{2\pi C_s} = K. \tag{1.9}$$

where K is constant for a given type of valve.

This means that in a resistance–capacitance coupled pentode amplifier, it is not possible to increase bandwidth without reduction in gain. Conversely, it is not possible to increase gain without a corresponding reduction in bandwidth.

Note:

The gain expression of equation (1.7) can be derived from equation (1.4) if the necessary assumptions are made. The above treatment is used merely to illustrate how an expression for stage gain can be obtained using the constant current equivalent circuit.

1.6. Stage Gain of a Transformer-coupled Voltage Amplifier (Low Audio-frequency Working)

BASIC CIRCUIT

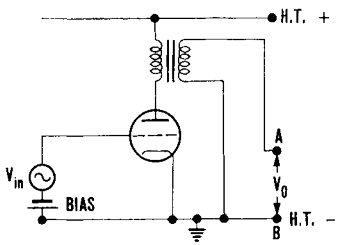


FIG. 1.12

CONSTANT VOLTAGE EQUIVALENT CIRCUIT (CORRECT AT ALL FREQUENCIES)

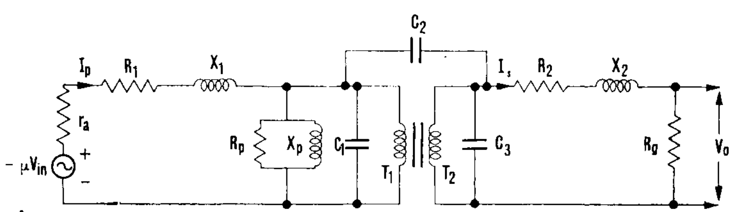


FIG. 1.13

R_1, R_2—primary and secondary leakage resistances.
X_1, X_2—primary and secondary leakage reactances.
R_p—no load parallel resistance.
X_p—no load parallel reactance.
C_1, C_2 and C_3—circuit stray capacitance elements.

The equivalent circuit of Fig. 1.13 can be considerably simplified over certain limited frequency ranges.

CONSTANT VOLTAGE EQUIVALENT CIRCUIT AT LOW FREQUENCIES

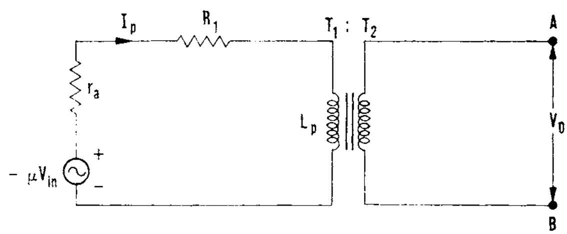


FIG. 1.14

Assume

1. All capacitive effects can be ignored as the frequency is low.
2. Primary and secondary leakage reactances are negligibly small.
3. The no load parallel resistance $R_p \gg X_p$ and has negligible shunting effect on L_p.
4. Secondary current does not flow because AB of Fig. 1.12 is open circuit.

Proof of the Gain Formula

From Fig. 1.14,

$$\text{primary current } \mathbf{I}_p = \frac{-\mu \mathbf{V}_{in}}{r_a + R_1 + j\omega L_p}.$$

And $$\text{voltage across } L_p = j\omega L_p.\mathbf{I}_p$$

$$= \frac{-\mu \mathbf{V}_{in}}{r_a + R_1 + j\omega L_p}.j\omega L_p$$

$$= \frac{-\mu \mathbf{V}_{in}}{1+\left(\dfrac{r_a+R_1}{j\omega L_p}\right)}$$

$$= \frac{-\mu \mathbf{V}_{in}}{1-j\left(\dfrac{r_a+R_1}{\omega L_p}\right)}.$$

Now, $\mathbf{V}_o$ is T_2/T_1 times greater than this, hence, if $a = T_1/T_2$

$$\mathbf{V}_o = \pm\frac{\mu}{a}.\frac{\mathbf{V}_{in}}{1-j\left(\dfrac{r_a+R_1}{\omega L_p}\right)}.$$

Therefore

$$\text{stage gain } m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \pm\frac{\mu}{a}.\frac{1}{1-j\left(\dfrac{r_a+R_1}{\omega L_p}\right)}. \tag{1.10}$$

Summary

1. If an input resistance of a following stage is placed across AB of Fig. 1.12, equation (1.10) will not be affected, because, in general, r_a is much greater than $R_1 + a^2 R_2$.
2. There is no gain when the frequency of the input voltage $\mathbf{V}_{in}$ is zero.
3. As the frequency increases the stage gain m approaches

$$\pm\mu/a.$$

4. Between these extremes, there is a frequency at which the low frequency response is 3 dB down. When the frequency of the

input voltage has this value, the real and imaginary terms in the denominator of equation (1.10) are equal i.e. $\omega_1 = (r_a+R_1)/L_p$. Hence, to improve the low-frequency response (r_a+R_1) should be small and L_p large.

LOW-FREQUENCY RESPONSE

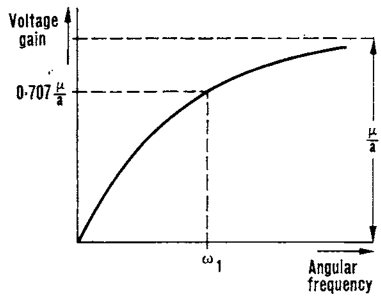


FIG. 1.15

1.7. Stage Gain of a Transformer-coupled Voltage Amplifier (Medium Audio-frequency Working)

BASIC CIRCUIT

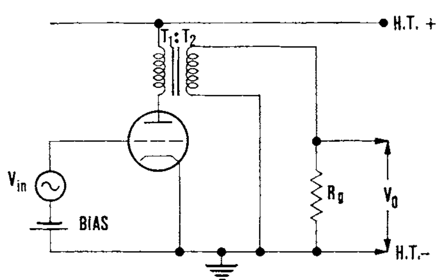


FIG. 1.16

CONSTANT VOLTAGE EQUIVALENT CIRCUIT AT MEDIUM FREQUENCIES

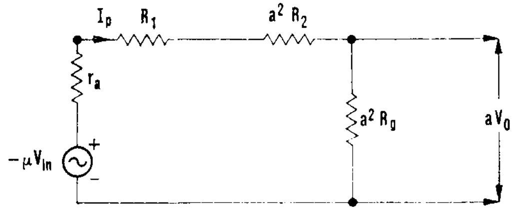


FIG. 1.17

Assume

1. Inductive and capacitive reactance elements are approximately equal and opposite.
2. Grid current due to a following stage does not flow.
3. The no load parallel resistance R_p has negligible effect.

If $a = T_1/T_2$, then a^2R_2 is the secondary leakage resistance referred to the primary, and a^2R_g is the effective value of the grid input resistance of the following stage referred to the primary.

Proof of the Gain Formula

Let $(R_1+a^2R_2)$ of Fig. 1.17 be equal to R_1'. The transformer primary current $\mathbf{I}_p$ may then be written,

$$\mathbf{I}_p = \frac{-\mu\mathbf{V}_{in}}{r_a+R_1'+a^2R_g}$$

And $$a\mathbf{V}_o = \pm\mathbf{I}_p.a^2R_g.$$

Therefore $$\mathbf{V}_o = \pm\frac{\mu}{a}.\frac{1}{1+\dfrac{r_a+R_1'}{a^2R_g}}.$$

Therefore $\quad$ stage gain $m = \dfrac{\mathbf{V}_o}{\mathbf{V}_{in}} = \pm\dfrac{\mu}{a}.\dfrac{1}{1+\dfrac{r_a+R_1'}{a^2R_g}}$ $\qquad$ (1.11)

Summary

1. As R_g tends to infinity, the stage gain approaches a maximum value at medium frequencies of $\pm\mu/a$. This is the case when feeding directly into the grid of the following stage if, as assumed, grid current never flows.
2. Equation (1.11), for gain, is only valid if capacitive and inductive reactance elements are approximately equal and opposite. Hence, frequency does have an effect on gain although this is not immediately evident from the equation.

1.8. Stage Gain of a Transformer-coupled Voltage Amplifier (High Audio-frequency Working)

BASIC CIRCUIT. See Fig. 1.12.

CONSTANT VOLTAGE EQUIVALENT CIRCUIT AT HIGH FREQUENCIES

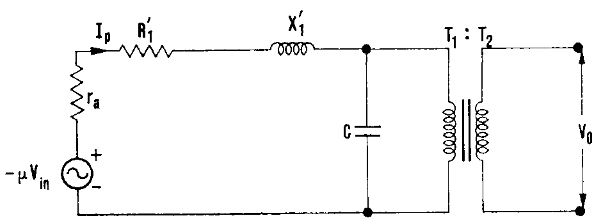


FIG. 1.18

Assume

1. At high frequencies, R_p and X_p of Fig. 1.13 have negligible effect on circuit performance.

2. Capacitance C of Fig. 1.18 accounts for all stray capacitance effects shown in the general equivalent circuit of Fig. 1.13.

Let $R_1' = (R_1 + a^2 R_2)$. This is the effect of the transformer leakage resistance referred to the primary. Let $X_1' = (X_1 + a^2 X_2)$. This is the effect of the transformer leakage reactance referred to the primary. The inductance associated with X_1' is L_1'. Finally, let $R_x = (r_a + R_1')$. This is the total primary resistance.

Proof of the Gain Formula

From Fig. 1.18,

$$\mathbf{I}_p = \frac{-\mu \mathbf{V}_{in}}{\left[R_x + j\left(\omega L_1' - \frac{1}{\omega C}\right)\right]}$$

The voltage $\mathbf{V}_c$ which appears across the transformer primary is given by

$$\mathbf{V}_c = \mathbf{I}_p . \frac{-j}{\omega C}$$

$$= \frac{j\mu \mathbf{V}_{in}}{\omega C\left[R_x + j\left(\omega L_1' - \frac{1}{\omega C}\right)\right]}$$

and $$\mathbf{V}_o = \pm\frac{\mu}{a} . \frac{j\mathbf{V}_{in}}{\omega C\left[R_x + j\left(\omega L_1' - \frac{1}{\omega C}\right)\right]}$$

Therefore

$$\text{stage gain } m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \pm\frac{\mu}{a} . \frac{j}{\omega C\left[R_x + j\left(\omega L_1' - \frac{1}{\omega C}\right)\right]}$$

This has a magnitude $|m|$ given by

$$|m| = \frac{\mu}{a} \cdot \frac{1}{\omega C\left\{(R_x)^2 + \left[\omega L_1' - \frac{1}{\omega C}\right]^2\right\}^{\frac{1}{2}}}$$

$$= K \cdot \frac{1}{\omega C\left\{(R_x)^2 + \frac{1}{(\omega C)^2}[\omega^2 L_1' C - 1]^2\right\}^{\frac{1}{2}}}$$

$$= \frac{K}{\sqrt{\{(\omega C R_x)^2 + (\omega^2 L_1' C - 1)^2\}}} \tag{1.12}$$

Squaring each side of this equation gives

$$|m|^2 = \frac{K_1}{(\omega C R_x)^2 + (\omega^2 L_1' C - 1)^2}.$$

Differentiating with respect to ω gives

$$2|m|\frac{d|m|}{d\omega} = -K_1 \cdot \left\{\frac{2\omega (CR_x)^2 + 4(\omega^2 L_1' C - 1)(\omega L_1' C)}{[(\omega C R_x)^2 + (\omega^2 L_1' C - 1)^2]^2}\right\}$$

and

$$\frac{d|m|}{d\omega} = -\frac{\omega K_1[(CR_x)^2 + 2\omega^2 (L_1' C)^2 - 2L_1' C]}{|m|\,[(\omega C R_x)^2 + (\omega^2 L_1' C - 1)^2]^2},$$

now, the stage gain has a maximum value when $d|m|/d\omega = 0$. Therefore, under these conditions,

$$2L_1' C = (CR_x)^2 + 2\omega^2 (L_1' C)^2$$

whence,

$$\omega^2 = \frac{1}{L_1' C} - \frac{1}{2}\left[\frac{R_x}{L_1'}\right]^2 \text{ radians per second}^2. \tag{1.13}$$

Substituting for ω^2 from equation (1.13) into equation (1.12) gives

$$|m|_{\max} = \frac{2L_1' K}{R_x} \frac{1}{\sqrt{\{4L_1' C - C^2 (R_x)^2\}}}$$

where

$$K = \frac{\mu}{a}.$$

Summary

1. If in equation (1.13), $(R_x)^2/2(L_1')^2 \ll 1/L_1'C$, $\omega^2 \simeq 1/(L_1'C)$. This is the condition for series resonance.
2. If the magnification factor Q of this tuned circuit, comprising (r_a+R_1'), L_1' and C, is too large, additional damping can be achieved by placing a suitable resistor R_g across AB in the basic circuit of Fig. 1.12.
3. As the frequency of the applied voltage increases above $f = 1/2\pi\sqrt{(L_1'C)}$ cycles per second, the voltage appearing across the transformer secondary reduces rapidly.

Gain-frequency Response of a Transformer-coupled Voltage Amplifier

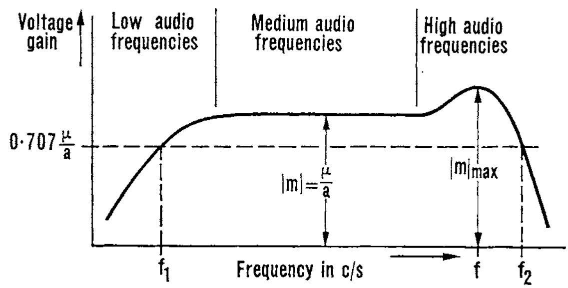


Fig. 1.19

f_1 and f_2 are as previously defined.
f is the frequency at which the gain is maximum and is given approximately by $f = \frac{1}{2}\pi\sqrt{(L_1'C)}$ cycles per second.

General Requirements of a Transformer-coupled Stage

1. The valve used should have a high amplification factor. A pentode would be better than a triode.
2. The transformer should have a high step-up ratio. However, in order to obtain a good low-frequency response and a high

frequency response which is relatively free from unwanted resonances, a 1 : 3 step-up ratio is seldom exceeded.

3. To ensure a good low-frequency response (r_a+R_1) should be small, and $j\omega L_p$ should be large.
4. The total effective capacitance C and transformer leakage inductance L'_1 should both be small if a good high frequency response is to be achieved.

1.9. Stage Gain of a Tuned Radio-frequency Voltage Amplifier at Resonance

Basic Circuit

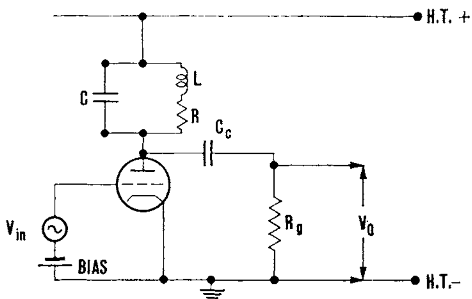


Fig. 1.20

Constant Current Equivalent Circuit

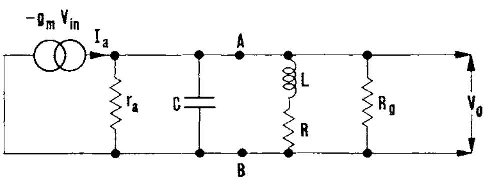


Fig. 1.21

Assume

1. The tuned circuit comprising L, C and R is resonant at the frequency of the input voltage (V_{in}).

2. The magnification factor of the tuned circuit is high.
3. The reactance of the coupling capacitor C_c is negligibly small.
4. The capacitor C includes the anode–cathode capacitance of the valve and the input capacitance to the following stage.

Proof of the Gain Formula

Magnification factor Q_0 of the tuned circuit L, C and R of Fig. 1.21 at resonance is given by

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}. \tag{1.14}$$

The resistance across AB due to the tuned circuit at resonance is termed the *dynamic resistance* R_D of the tuned circuit and is given by

$$R_D = \frac{L}{CR}$$

but $$\frac{1}{CR} = \omega_0 Q_0 \text{ from equation (1.14).}$$

Therefore $$R_D = Q_0 \omega_0 L.$$

Let the effective resistance across AB be R' where

$$\frac{1}{R'} = \frac{1}{r_a} + \frac{1}{R_L} + \frac{1}{Q_0 \omega_0 L}.$$

A simplified version of Fig. 1.21 appears in Fig. 1.22.

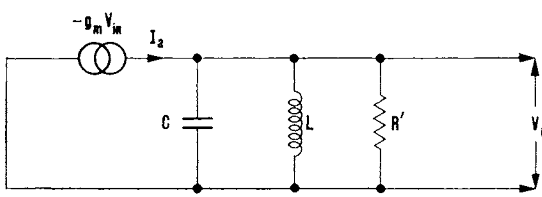


FIG. 1.22

L and C have the same values as in Fig. 1.21.

The magnification factor of the original tuned circuit Q_0 is damped by the presence of r_a and R_g. The modified, reduced magnification factor Q_0', has a value

$$Q_0' = \frac{R'}{\omega_0 L}.$$

Therefore $$R' = Q_0' \omega_0 L. \tag{1.15}$$

At resonance $\omega_0 L = 1/\omega_0 C$ and the tuned circuit is purely resistive. The current from the constant current generator flows through R', hence

$$\mathbf{V}_o = -g_m \mathbf{V}_{in} R'.$$

Therefore $$\text{stage gain } m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = -g_m R'$$

but $R' = Q_0' \omega_0 L$ from equation (1.15).

Therefore $$|\, m \,| = g_m Q_0' \omega_0 L. \tag{1.16}$$

Summary

1. To improve the amplifier gain, the mutual conductance of the valve should be as large as possible.

2. The anode slope resistance should also be as large as possible to reduce tuned circuit damping, for this reason a pentode is preferred to a triode.

3. The dynamic resistance of the tuned circuit should be large.

4. Off resonance, the amplifier gain can be found from the formula $m = -\mu Z_L/(r_a + Z_L)$, where Z_L is the total impedance across AB in Fig. 1.21.

1.10. Stage Gain of a Transformer-coupled Radio-frequency Voltage Amplifier (Tuned Secondary)

BASIC CIRCUIT

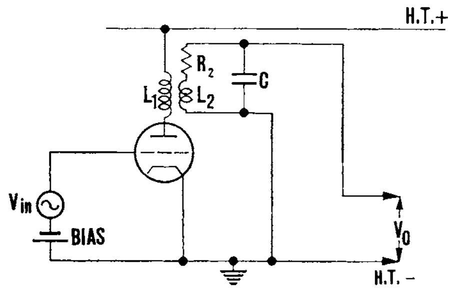


FIG. 1.23

CONSTANT VOLTAGE EQUIVALENT CIRCUIT

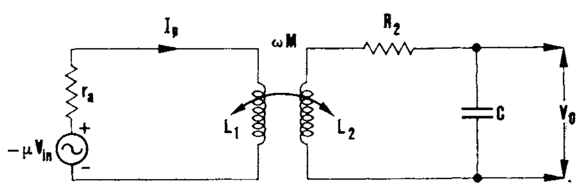


FIG. 1.24

Assume

1. The primary impedance $(R_1 + j\omega L)$ is much less than the anode slope resistance of the valve and can be neglected.
2. Primary capacitance effects can be neglected.
3. The transformer secondary circuit is resonant at the frequency of the input voltage.

4. Capacitor C represents all capacitance effects which appear across the secondary circuit.

5. Grid resistance of a following stage is infinite.

Proof of the Gain Formula

The effective primary resistance R_p of the circuit of Fig. 1.24 at resonance is given by,

$$R_p = r_a + \frac{\omega_0^2 M^2}{R_2},$$

$$\mathbf{I}_p = \frac{-\mu \mathbf{V}_{\text{in}}}{r_a + \dfrac{\omega_0^2 M^2}{R_2}}$$

The secondary induced e.m.f. ($\mathbf{E}_2$) will be

$$\mathbf{E}_2 = \pm j\omega_0 M \mathbf{I}_p$$

$$= \frac{\pm j\omega_0 M \mu \mathbf{V}_{\text{in}}}{r_a + \dfrac{\omega_0^2 M^2}{R_2}}.$$

This e.m.f. supplies a series resonant circuit comprising L_2, C and R_2 as shown in Fig. 1.25.

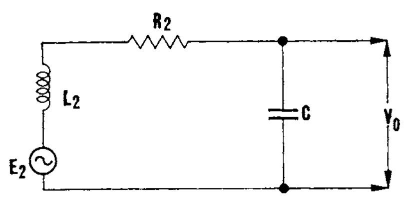


FIG. 1.25

Therefore $\mathbf{V}_o = Q_0 \mathbf{E}_2$

where Q_0 is the magnification factor of the circuit.

Therefore $$\mathbf{V}_o = \pm\frac{\omega_0 M\mu_0 Q_0}{r_a+\dfrac{\omega_0^2 M^2}{R_2}}\mathbf{V}_{in}.$$

And $$|m| = \frac{\omega_0 M\mu_0 Q_0}{r_a+\dfrac{\omega_0^2 M^2}{R_2}}$$

Dividing numerator and denominator by r_a gives

$$|m| = \frac{\omega_0 M g_m Q_0}{1+\dfrac{\omega_0^2 M^2}{r_a R_2}} \tag{1.17}$$

$$= \omega_0 M g_m Q_0'. \tag{1.18}$$

Where Q_0' is the effective magnification factor given by

$$Q_0' = \frac{Q_0}{1+\dfrac{\omega_0^2 M^2}{r_a R_2}}$$

If r_a is very much greater than $\omega_0^2 M^2/R_2$, the stage gain given by equation (1.17) has a value of $|m| = \omega_0 M g_m Q_0$. However, if this is not the case, stage gain has a maximum value when $d|m|/dM = 0$.

Differentiating equation (1.17) with respect to M gives

$$\frac{d|m|}{dM} = \frac{\left[1+\dfrac{\omega_0^2 M^2}{r_a R_2}\right]g_m\omega_0 Q_0 - g_m\omega_0 Q_0 M\left[\dfrac{2\omega_0^2 M}{r_a R_2}\right]}{\left[1+\dfrac{\omega_0^2 M^2}{r_a R_2}\right]^2}.$$

Equating this to zero,

$$\frac{2\omega_0^2 M^2}{r_a R_2} = 1+\frac{\omega_0^2 M^2}{r_a R_2}.$$

Therefore $$M = \frac{\sqrt{(r_a R_2)}}{\omega_0}. \tag{1.19}$$

Substituting this value for M into equation (1.17) gives a maximum value for stage gain,

$$|m|_{\max} = \frac{\omega_0 M g_m Q_0}{2}. \tag{1.20}$$

Summary

1. When the anode slope resistance of the valve is much greater than the transferred impedance of the secondary circuit, the stage gain $|m|$ of the tuned secondary transformer-coupled amplifier is given by $|m| = \omega_0 M g_m Q_0$.
2. When this condition is not satisfied, $|m|$ has a maximum value of $|m|_{\max} = \omega_0 M g_m Q_0/2$.
3. Normally, a pentode is used in conjunction with a step-up transformer. In such cases, the tuned anode arrangement of the previous section will give the higher overall stage gain.

1.11. Miller Effect

In a triode, there is relatively large capacitance between anode and grid (C_{ag}). The feedback from anode to grid via C_{ag} causes the input admittance of the triode to be modified. This is the *Miller effect.*

BASIC CIRCUIT

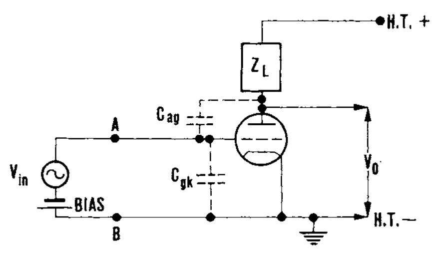


FIG. 1.26

CONSTANT VOLTAGE EQUIVALENT CIRCUIT

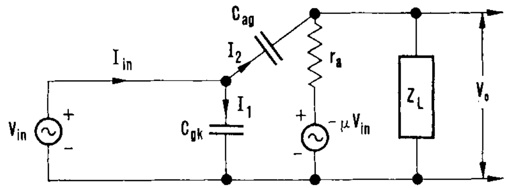


FIG. 1.27

Assume

1. The valve draws current in the grid-cathode circuit due to inter-electrode and inter-wiring capacitance, positive ions collected at the grid from an imperfect vacuum, etc.
2. Anode–cathode capacitance C_{ak} is included in the load impedance, Z_L.

Effect of the Miller Capacitance C_{ag} on Input Admittance

From the equivalent circuit of Fig. 1.27,

$$\mathbf{I}_{in} = (\mathbf{I}_1+\mathbf{I}_2)$$

$$= j\omega C_{gk}\mathbf{V}_{in}+j\omega C_{ag}(\mathbf{V}_{in}-\mathbf{V}_o).$$

Input admittance

$$Y_{in} = \frac{\mathbf{I}_{in}}{\mathbf{V}_{in}}.$$

Therefore

$$Y_{in} = j\omega C_{gk}+j\omega C_{ag}\left(1-\frac{\mathbf{V}_o}{\mathbf{V}_{in}}\right),$$

but $$\frac{\mathbf{V}_o}{\mathbf{V}_{in}} = |m| \angle\theta = |m|(\cos\theta+j\sin\theta)$$

where θ is the phase difference between $\mathbf{V}_o$ and $\mathbf{V}_{in}$ in degrees. Hence,

$$\begin{aligned} Y_{in} &= j\omega C_{gk} + j\omega C_{ag}(1 - |m| \cos\theta - j|m| \sin\theta) \\ &= \omega C_{ag} |m| \sin\theta + j\omega[C_{gk} + C_{ag}(1 - |m| \cos\theta)]. \end{aligned} \tag{1.21}$$

The admittance across terminals A and B is Y_{in} (see Fig. 1.28).

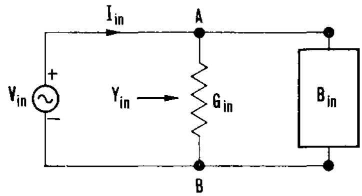


FIG. 1.28

Now, $$Y_{in} = G_{in} + jB_{in} = \frac{1}{R_{in}} + j\frac{1}{X_{in}}. \tag{1.22}$$

By comparing equations (1.21) and (1.22) it can be seen that

$$\frac{1}{R_{in}} = \omega C_{ag} |m| \sin\theta$$

or $$R_{in} = \frac{X_{ag}}{|m| \sin\theta} \tag{1.23}$$

and $$\frac{1}{X_{in}} = \omega[C_{gk} + C_{ag}(1 - |m| \cos\theta)]$$

or $$X_{in} = \frac{1}{\omega[C_{gk} + C_{ag}(1 - |m| \cos\theta)]} \tag{1.24}$$

Summary

1. For a resistive load $\theta = 180°$.

   Hence, $R_{in} = \infty$, $X_{in} = \dfrac{1}{\omega[C_{gk} + C_{ag}(1 + |m|)]}$

   and the input capacitance is $C_{gk} + C_{ag}(1 + |m|)$.

2. For an inductive load $180° < \theta < 270°$. It can be seen from equation (1.23) that R_{in} is negative and the circuit may become unstable. X_{in} is capacitive.

3. For a capacitive load $90° < \theta < 180°$. It can be seen from equation (1.23) that R_{in} is positive and the circuit is stable. X_{in} is capacitive.

1.12. Maximum Theoretical Efficiency of a Transformer-coupled Class A Power Amplifier

BASIC CIRCUIT

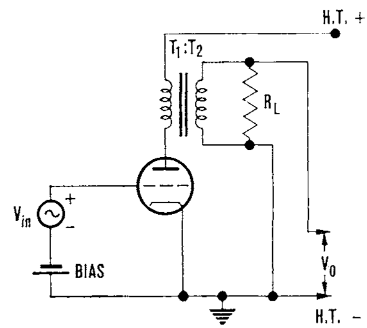


FIG. 1.29

Assume

1. Valve characteristics are absolutely linear, parallel, and equally spaced.

2. Resistance of the transformer primary is negligible.

3. Maximum possible peak to peak sinusoidal input voltage without grid current.

Valve Characteristics

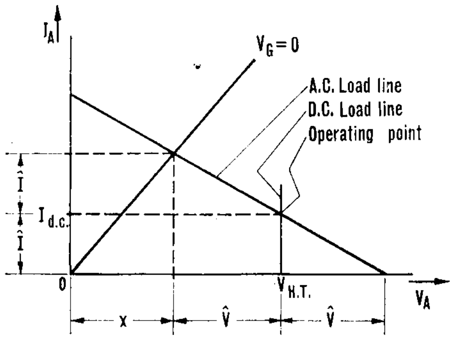


Fig. 1.30

In Fig. 1.30, $V_{H.T.}$ is the H.T. supply voltage and $I_{d.c.}$ is the steady anode current with no signal input voltage.

The power input (P_{in}) to the anode circuit of Fig. 1.29 is

$$P_{in} = I_{d.c.} V_{H.T.} \tag{1.25}$$

r.m.s. anode voltage $= \dfrac{\hat{V}}{\sqrt{2}}$,

r.m.s. anode current $= \dfrac{\hat{I}}{\sqrt{2}}$

The a.c. power output (P_o) is

$$P_o = \frac{\hat{V}\hat{I}}{2}$$

where $\hat{V}$ and $\hat{I}$ are the peak values of anode voltage and anode current respectively.

Now, anode efficiency $\eta = \dfrac{P_o}{P_{in}} \times 100\%$

$$= \frac{\hat{V}\hat{I}}{2 \times I_{d.c.} V_{H.T.}} \times 100\%. \tag{1.26}$$

If the negative swing of the grid input voltage is just sufficient to reduce anode current to zero, $I_{d.c.} = \hat{I}$ and equation (1.26) becomes

$$\eta = \frac{\hat{V}}{2V_{H.T.}} \times 100\%. \tag{1.27}$$

From the valve characteristics of Fig. 1.30 it can be seen that

(a) $$V_{H.T.} = (x+\hat{V}),$$

(b) $$r_a = \frac{x}{2\hat{I}}$$

or $$x = 2\hat{I}r_a,$$

(c) $$R_L' = \frac{2\hat{V}}{2\hat{I}}$$

or $$\hat{V} = \hat{I}R_L',$$

where $$R_L' = \left(\frac{T_1}{T_2}\right)^2 R_L.$$

Using the above expressions, equation (1.27) becomes

$$\eta = \frac{\hat{V}}{2(x+\hat{V})} \times 100\%$$

$$= \frac{\hat{I}R_L'}{2(2\hat{I}r_a+\hat{I}R_L')} \times 100\%$$

$$= \frac{1}{2+4\frac{r_a}{R_L'}} \times 100\%. \tag{1.28}$$

Summary

1. For equation (1.28) to be a maximum, the ratio r_a/R_L' is made as small as possible.
2. Theoretically, the absolute maximum value for efficiency is 50%, i.e. when r_a/R_L' is zero.

1.13. Maximum Power Output of a Transformer-coupled Class A Power Amplifier

BASIC CIRCUIT. See Fig. 1.29.

CONSTANT VOLTAGE EQUIVALENT CIRCUIT

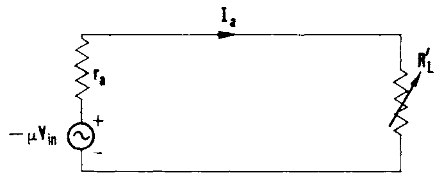


FIG. 1.31

Assume

1. The valve has constant H.T. and bias supplies.
2. The input voltage to the grid is a sine wave of constant amplitude, and is smaller than the value defined in assumption 3 of Section 1.12.
3. R'_L can be varied to obtain maximum power in the load.

To Show that Maximum Power is Transferred to the Load when $r_a = R'_L$

From Fig. 1.31,

$$\mathbf{I}_a = \frac{-\mu \mathbf{V}_{\mathrm{in}}}{r_a + R'_L}.$$

Power output P_o is given by

$$P_o = \mathbf{I}_a^2 \times R'_L$$

$$= \frac{\mu^2 \mathbf{V}_{\mathrm{in}} R'_L}{(r_a + R'_L)^2}. \qquad (1.29)$$

Maximum power is transferred to R'_L, when $dP_o/dR'_L = 0$. Hence,

$$\frac{dP_o}{dR'_L} = \frac{(r_a+R'_L)^2\mu^2\mathbf{V}_{in}{}^2 - \mu^2\mathbf{V}_{in}{}^2R'_L \times 2(r_a+R'_L)}{(r_a+R'_L)^4} = 0.$$

Therefore

$$\mu^2\mathbf{V}_{in}^2(r_a+R'_L)^2 = \mu^2\mathbf{V}_{in}^2R'_L \times 2(r_a+R'_L).$$

Therefore $$r_a+R'_L = 2R'_L$$

and $$r_a = R'_L. \tag{1.30}$$

Substituting for $r_a = R'_L$ from equation (1.30) into equations (1.28) and (1.29) in turn results in

$$\eta = \frac{100}{2+4} = 16{\cdot}7\%$$

and $$P_{o\,max} = \frac{\mu^2\mathbf{V}_{in}^2}{4r_a}. \tag{1.31}$$

Summary

1. Maximum power is transferred to the load when $R'_L = r_a$.
2. The transformation ratio which gives maximum power for a given input voltage is given by $T_1/T_2 = \sqrt{(R_L/r_a)}$.
3. Power output and distortion both increase with increase in input voltage $\mathbf{V}_{in}$.

1.14. Maximum Undistorted Power Output of a Transformer-coupled Class A Power Amplifier

BASIC CIRCUIT

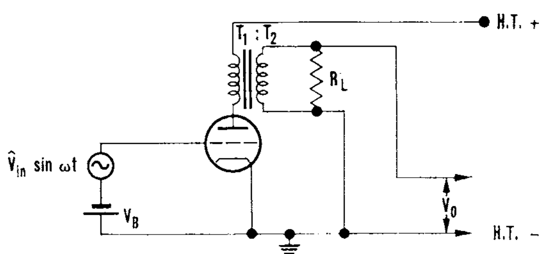


FIG. 1.32

Assume

1. A small triode valve is used whose static anode characteristics above the *bottom bend distortion limit*, are linear, parallel, and equally spaced.
2. For a given valve, the H.T. supply and distortion limits are fixed.
3. As the load resistance is changed, so the amplitude of the sinusoidal input signal and value of grid bias are adjusted to give maximum power output within the distortion limits.
4. The grid bias level V_B is equal to the maximum value of the input sine wave $\hat{V}_{in}$.

VALVE CHARACTERISTICS

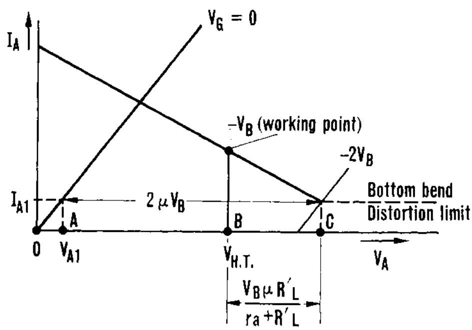


FIG. 1.33

To Show that Maximum Undistorted Power Output is Obtained when $R'_L = 2r_a$

$$\mu = \frac{\delta V_A}{\delta V_G} \text{ with constant anode current.}$$

Hence, it can be seen from Fig. 1.33 that $V_{AC} = 2\mu V_B$ volts,

and $$V_{A1} + 2\mu V_B = V_{OC}. \qquad (1.32)$$

If the peak value of grid input voltage $\hat{V}_{\text{in}} = V_B$ volts, the instantaneous value of anode voltage when the grid input is at its maximum negative value is

$$V_{OC} = V_{\text{H.T.}} + \frac{\mu R'_L V_B}{r_a + R'_L} \tag{1.33}$$

Equating equations (1.32) and (1.33) gives

$$V_{A1} + 2\mu V_B = V_{\text{H.T.}} + \frac{\mu R'_L V_B}{r_a + R'_L}.$$

Therefore
$$V_B = \frac{(V_{\text{H.T.}} - V_{a1})(r_a + R'_L)}{\mu(2r_a + R'_L)} \tag{1.34}$$

Equation (1.34) gives the magnitude of the correct grid bias and the maximum value of grid input voltage $\hat{V}_{\text{in}}$ which may be amplified without distortion.

Constant Voltage Equivalent Circuit

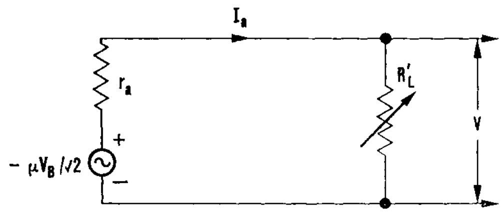


FIG. 1.34

To find the power dissipated in R'_L, r.m.s. values of current and voltage are used. Therefore $\hat{V}_{\text{in}}$ must be expressed in r.m.s. form, i.e. $\hat{V}_{\text{in}}/\sqrt{2}$.

However,
$$\hat{V}_{\text{in}} = V_B$$

Therefore,
$$\frac{\hat{V}_{\text{in}}}{\sqrt{2}} = \frac{V_B}{\sqrt{2}}$$

Hence, the r.m.s. voltage supplying the anode circuit is $-\mu V_B/\sqrt{2}$ as shown in the equivalent circuit of Fig. 1.34.

Power output across R'_L is given by

$$P_o = \frac{V^2}{R'_L} \tag{1.35}$$

and

$$V = \frac{-\mu V_B R'_L}{(r_a+R'_L)\sqrt{2}}. \tag{1.36}$$

Substituting the value for V_B from equation (1.34) into equation (1.36) gives

$$V = \left[\frac{-\mu R'_L}{(r_a+R'_L)\sqrt{2}}\right]\left[\frac{(V_{\text{H.T.}}-V_{A1})(r_a+R'_L)}{\mu(2r_a+R'_L)}\right]$$

Therefore,

$$P_o = \frac{1}{R'_L}\left\{\left[\frac{-\mu R'_L}{(r_a+R'_L)\sqrt{2}}\right]\left[\frac{(V_{\text{H.T.}}-V_{A1})(r_a+R'_L)}{\mu(2r_a+R'_L)}\right]\right\}^2 \tag{1.37}$$

When this is a maximum, $dP_o/dR'_L = 0$.

Differentiating and equating to zero gives,

$$2R'_L(2r_a+R'_L) = (2r_a+R'_L)^2.$$

Therefore,

$$R'_L = 2r_a. \tag{1.38}$$

Now substitute for $R'_L = 2r_a$ in the following equations.

1. *In equation* (1.29)

$$\text{Anode efficiency } \eta = \frac{1}{2+\dfrac{4r_a}{2r_a}} \times 100\%.$$

Therefore,

$$\eta = 25\%. \tag{1.39}$$

2. *In equation* (1.34)

$$\hat{V}_{\text{in}} = V_B = \frac{3}{4}\left[\frac{V_{\text{H.T.}}-V_{A1}}{\mu}\right]. \tag{1.40}$$

3. *In equation* (1.37)

$$P_{o\,max} = \frac{(V_{H.T.} - V_{A1})^2}{16r_a}. \tag{1.41}$$

Summary

1. The results are valid for a small power triode.
2. Maximum power output within the distortion limits is produced when $R_L' = 2r_a = T_1^2 R_L / T_2^2$.
3. If $V_{H.T.}$, V_{A1}, r_a, μ and R_L' are fixed (R_L' being equal to $2r_a$), the peak value of sinusoidal input voltage which can be handled is given by equation (1.40). This equation also gives the value of steady grid bias V_B.
4. Equation (1.41) shows that r_a should be small to increase power output within the distortion limits.
5. The theoretical anode efficiency when $R_L' = 2r_a$, is 25%.

1.15. Amplitude Distortion in a Triode

The *dynamic mutual characteristic* of a triode is not a straight line. If an alternating voltage is fed to the grid of the valve, and the resulting current variation through a resistive anode load is not a true replica of the input voltage which caused it, amplitude distortion occurs due to the non-linearity of the dynamic characteristic.

Assume

1. The triode valve used has an instantaneous anode current (i_a) which is given by

$$i_a = a + bv_g + cv_g^2 \tag{1.42}$$

where a, b and c are constants which depend largely upon the valve used.
2. v_g consists of a steady grid bias voltage V_B, upon which is superimposed an input signal $\hat{V}_{in} \cos \omega t$ volts.

DYNAMIC MUTAL CHARACTERISTIC

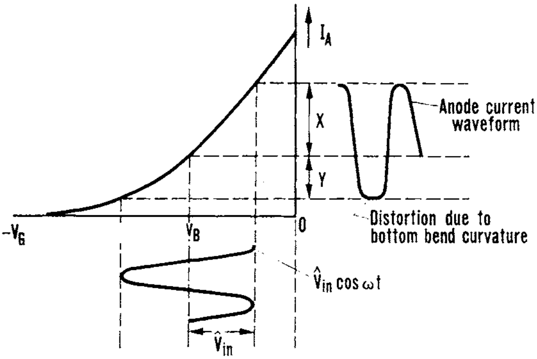


FIG. 1.35

Analysis of the Anode Current Waveform

The instantaneous value of grid voltage v_g is seen from Fig. 1.35 to be

$$v_g = (V_B + \hat{V}_{in} \cos \omega t). \tag{1.43}$$

Substituting for this value for v_g into equation (1.42) gives

$$i_a = a + b(V_B + \hat{V}_{in} \cos \omega t) + c(V_B + \hat{V}_{in} \cos \omega t)^2$$

$$= a + bV_B + b\hat{V}_{in} \cos \omega t + cV_B^2 + c\hat{V}_{in}^2 \cos^2 \omega t + 2c\hat{V}_{in} V_B \cos \omega t.$$

Therefore,

$$i_a = a + bV_B + b\hat{V}_{in} \cos \omega t + cV_B^2 + 2c\hat{V}_{in} V_B \cos \omega t$$
$$+ \frac{c\hat{V}_{in}^2}{2} + \frac{c\hat{V}_{in}^2}{2} \times \cos 2\omega t. \tag{1.44}$$

The anode current wave of equation (1.44) can be broken down into four main components thus:

1. $a + bV_B + cV_B^2$. This defines a value for anode current when no signal is applied.

2. $\frac{1}{2}c\hat{V}_{in}^2$. This is the increase in d.c. level of anode current due to the second harmonic of the input signal.
3. $(b+2cV_B)\hat{V}_{in}\cos\omega t$. This is the component of the anode current which has the same frequency as the alternating grid input.
4. $\frac{1}{2}c\hat{V}_{in}^2\cos 2\omega t$. This is the unwanted second harmonic component of anode current.

If these components are added graphically, the anode current waveform of Fig. 1.35 results.

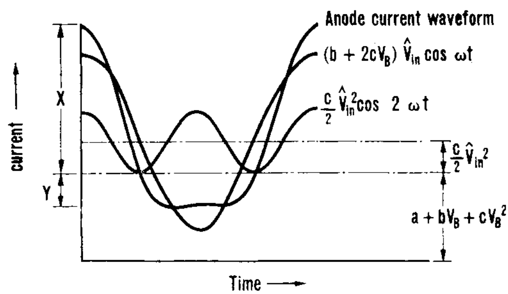


FIG. 1.36

If the maximum value of anode current at signal frequency is given by

$$\hat{I}_1 = \hat{V}_{in}(b+2cV_B) \tag{1.45}$$

and if the maximum value of anode current at the second harmonic of the signal frequency is given by

$$\hat{I}_2 = \frac{c}{2}\hat{V}_{in}^2 \tag{1.46}$$

it can be seen from Fig. 1.36 that

$$X = \hat{I}_1+2\hat{I}_2 \tag{1.47}$$

and

$$Y = \hat{I}_1-2\hat{I}_2 \tag{1.48}$$

Adding equation (1.48) to equation (1.47) gives

$$\hat{I}_1 = \frac{X+Y}{2} \tag{1.49}$$

and subtracting equation (1.48) from equation (1.47) gives

$$\hat{I}_2 = \frac{X-Y}{4} \tag{1.50}$$

The percentage second harmonic distortion D_2

$$= \frac{X-Y}{4} \cdot \frac{2}{X+Y} \times 100\%.$$

Therefore $$D_2 = 50\left(\frac{X-Y}{X+Y}\right)\%. \tag{1.51}$$

X and Y are indicated on the dynamic mutual characteristic of Fig. 1.35.

Summary

1. Second harmonic distortion can be detected by the increase in mean anode current when a signal is applied.
2. The increase in mean anode current when a signal input is applied to the valve, is equal to the maximum value of the second harmonic component.
3. Second harmonic distortion is due to the existence of the cv_g^2 term in equation (1.42).
4. With triodes, only second harmonic distortion is important. Usually, the minimum value of anode current is chosen so that percentage second harmonic distortion D_2 does not exceed 5%.
5. In pentodes, the curvature of the dynamic mutual characteristic for a given valve, depends upon the value of the a.c. load resistance. A high value gives a dynamic mutual

characteristic which may be more curved than that of Fig. 1.35 for a triode. Hence, dv_g^3 and higher degree terms may be present in the expression for anode current (equation 1.42) if a pentode is used. This means that third and higher harmonics may be present in the output waveform.

6. From equation (1.45) it can be seen that the magnitude of anode current depends upon the magnitude of bias voltage V_B.

1.16. Class B Push–Pull Amplifier as a means of Reducing Second Harmonic Distortion

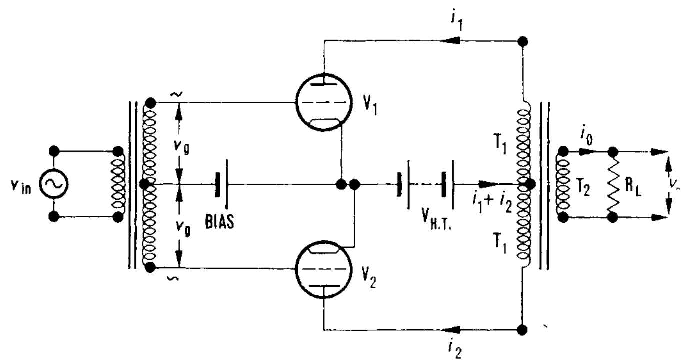


FIG. 1.37

BASIC CIRCUIT

Assume

1. Identical valves for which anode current (i_a) is given by

$$i_a = a+bv_g+cv_g^2+\ldots$$

where $$v_g = \hat{V}_{in} \sin \omega t.$$

2. Valves V_1 and V_2 are biased to operate at projected (extended) cut-off.

3. The alternating input voltages to the grids of V_1 and V_2 are equal in magnitude but 180° out of phase.

4. The valves have anode currents which consist of half sine waves spaced by a half period (see Fig. 1.40).

Now, $$i_1 = a+bv_g+cv_g^2+dv_g^3+ev_g^4+\dots \tag{1.52}$$

and because the input to V_2 is phase displaced by 180° due to the transformer action,

$$\begin{aligned} i_2 &= a+b(-v_g)+c(-v_g)^2+d(-v_g)^3+e(-v_g)^4+\dots \\ &= a-bv_g+cv_g^2-dv_g^3+ev_g^4+\dots . \end{aligned} \tag{1.53}$$

The current which flows through the power supply is (i_1+i_2). Hence, adding equations (1.52) and (1.53) gives

$$(i_1+i_2) = 2(a+cv_g^2+ev_g^4+\dots). \tag{1.54}$$

The induced secondary voltage is proportional to the primary current (i_1-i_2). Hence, subtracting equation (1.53) from equation (1.52) gives

$$(i_1-i_2) = 2(bv_g+dv_g^3+\dots). \tag{1.55}$$

Summary

1. Equation (1.55) reveals that the output contains no even harmonics. This is particularly useful in triode valves where second harmonic distortion is predominant.

2. There is no resultant magnetization of the output transformer, since m.m.f.'s due to i_1 and i_2 cancel.

3. Under certain conditions, it may be possible to provide bias for the circuit using a common cathode resistor with no shunting capacitor. This is because no current flows through the power supply at the fundamental frequency. However, even harmonic currents do flow (see equation (1.54)), and the internal impedance of the power supply should be low.

1.17. Power Output of a Class B Push–Pull Amplifier[1]

BASIC CIRCUIT. See Fig. 1.37.

OUTPUT CIRCUIT

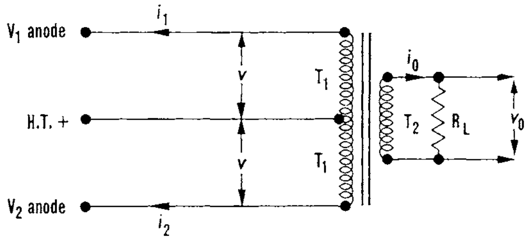


FIG. 1.38

Assume

1. i_1 and i_2 are anode currents of V_1 and V_2 respectively.
2. v is the voltage which appears across T_1 turns due to anode currents i_1 and i_2.
3. The transformer is loss-free and has a turns ratio $a = T_1/T_2$.

Power Output

For an ideal transformer

$$\frac{v_0}{v} = \frac{T_2}{T_1}$$

or
$$v = v_0 T_1/T_2. \tag{1.56}$$

Also, primary and secondary ampere-turns are equal.

Therefore
$$T_1(i_1 - i_2) = T_2\, i_0$$

$$(i_1 - i_2) = \frac{T_2}{T_1}\, i_0. \tag{1.57}$$

Dividing equation (1.56) by equation (1.57) gives,

$$\frac{v}{(i_1 - i_2)} = \left[\frac{T_1}{T_2}\right]^2 . \frac{v_0}{i_0} = a^2 R_L \qquad (1.58)$$

$a^2 R_L$ is termed the *composite resistance*
and

$$\text{COMPOSITE RESISTANCE} = \frac{\text{ANODE-TO-ANODE RESISTANCE}}{4}. \qquad (1.59)$$

Multiplying equation (1.56) by equation (1.57) gives

$$v_0 i_0 = v(i_1 - i_2)$$

However, these are instantaneous values of voltage and current, and r.m.s. values are needed to obtain the power output P_o.

Therefore $$P_o = V(I_1 - I_2) \qquad (1.60)$$

Equation (1.60) gives the power output of the push–pull stage, in terms of the r.m.s. value of primary voltage, and the difference between r.m.s. values of individual primary currents. However, this equation may also be written in a form which enables P_o to be determined directly from the composite characteristics and load line. The modified relationship is

$$P_o = \frac{\hat{V}_0 \times \hat{I}_0}{8} \qquad (1.61)$$

A composite load line is used in Example 6 Chapter 5 to determine the power output and efficiency of a class A push–pull stage.

Summary

The use of a push–pull stage gives twice the power output of a single-valve stage with reduced distortion, or more than twice the power output with the same distortion.

1.18. Efficiency of a Class B Push–Pull Amplifier

BASIC CIRCUIT

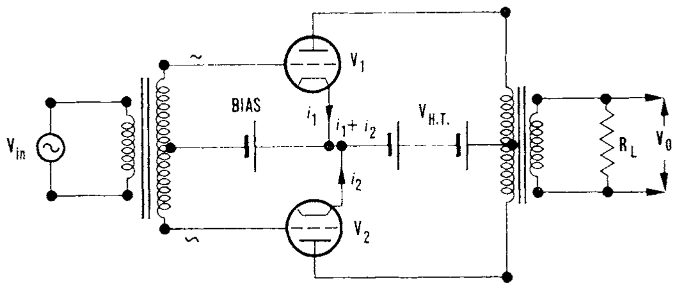


FIG. 1.39

Assume

1. Identical valves with equal and opposite alternating grid inputs.
2. Valves are biased at projected cut-off so that each conducts for one complete half cycle of a sine wave input.

WAVEFORMS

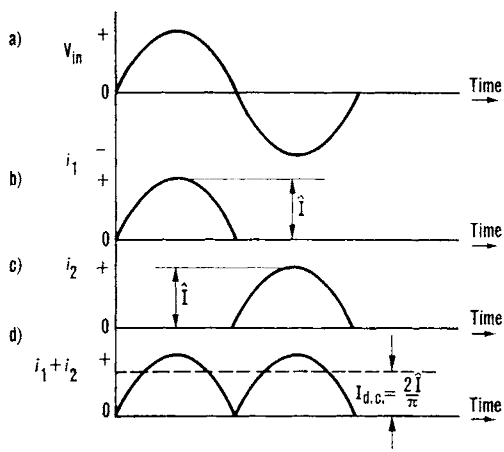


FIG. 1.40

Waveform *a* shows the signal input to the transformer primary.

Waveform *b* shows the current waveform produced when V_1 conducts on the positive half cycle of the input signal.

Waveform *c* shows the current waveform produced when V_2 conducts on the negative half cycle of the input signal.

Waveform *d* shows the current waveform produced in the H.T. supply due to V_1 and V_2 conducting alternately. This waveform is obtained by adding waveforms b and c. The mean supply current ($I_{d.c.}$) can be shown by a Fourier Analysis of this waveform to be $2\hat{I}/\pi$ amperes.

Maximum Theoretical Efficiency of a Class B Push–Pull Amplifier

The steady anode voltage of each valve is $V_{H.T.}$ volts and $I_{d.c.} = 2\hat{I}/\pi$ amperes.

Therefore, the power P_{in} taken from the H.T. supply is given by

$$P_{in} = \frac{2}{\pi}.\hat{I}\times V_{H.T.} \tag{1.62}$$

a.c. power output

$$P_o = \frac{\hat{V}\hat{I}}{2}. \tag{1.63}$$

Hence,

$$\text{efficiency } \eta = \frac{P_o}{P_{in}} = \frac{\hat{V}\hat{I}}{2}\times\frac{\pi}{2}\times\frac{1}{\hat{I}V_{H.T.}}$$

$$= \frac{\pi}{4}\frac{\hat{V}}{V_{H.T.}} \tag{1.64}$$

But $\hat{V}$ is the difference between the steady supply voltage $V_{H.T.}$ and the minimum instantaneous value of anode voltage V_{min}, or

$$\hat{V} = (V_{H.T.} - V_{min}) \tag{1.65}$$

Substituting $\hat{V}$ from equation (1.65) into equation (1.64) gives

$$\eta = \frac{\pi}{4}\left(1-\frac{V_{\min}}{V_{\mathrm{H.T.}}}\right)\times 100\% \tag{1.66}$$

Hence, efficiency has a maximum theoretical value, when $V_{\min}/V_{\mathrm{H.T.}} = 0$, of 78·5%.

Summary

1. The use of class B bias means that there is a reduction in the mean supply current since little valve current flows in the absence of an input signal. The maximum theoretical efficiency is therefore much higher than for a class A power amplifier.

2. If the above treatment is applied to a single-valve class B power amplifier, the maximum theoretical efficiency is still 78·5%.

1.19. Cathode Follower

BASIC CIRCUIT

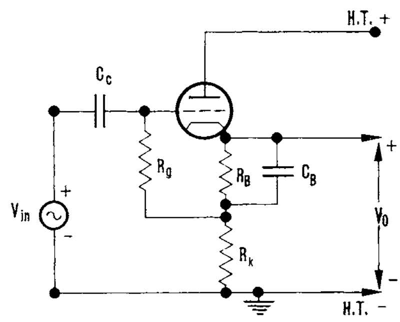


FIG. 1.41

CONSTANT VOLTAGE EQUIVALENT CIRCUIT

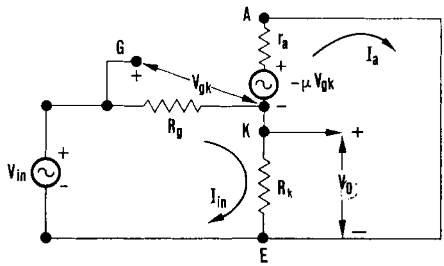


FIG. 1.42

Assume

1. Inter-electrode capacitance effects can be ignored.
2. The reactances of C_c and C_B are negligibly small.
3. Current flow in the input circuit ($\mathbf{I}_{in}$) is much less than the anode current $\mathbf{I}_a$.
4. $\mathbf{V}_{gk}$ is the voltage which appears between grid and cathode of the circuit of Fig. 1.41.

SIMPLIFIED EQUIVALENT CIRCUIT

From the equivalent circuit of Fig. 1.42 it can be seen that

$$\mathbf{V}_{in} = \mathbf{I}_{in}R_g + \mathbf{V}_o, \tag{1.67}$$

but

$$\mathbf{I}_{in}R_g = V_{gk}$$

Hence,

$$\mathbf{V}_{gk} = \mathbf{V}_{in} - \mathbf{V}_o. \tag{1.68}$$

Also,

$$\mathbf{V}_o = -(\mathbf{I}_a - \mathbf{I}_{in})R_k$$

and, as $\mathbf{I}_{in}$ is assumed to be very much less than $\mathbf{I}_a$,

$$\mathbf{V}_o = -\mathbf{I}_a R_k. \tag{1.69}$$

Now

$$-\mu\mathbf{V}_{gk} = \mathbf{I}_a r_a - \mathbf{V}_o. \tag{1.70}$$

Substituting $\mathbf{V}_{gk}$ from equation (1.68) into equation (1.70) and $\mathbf{I}_a$ from equation (1.69) into equation (1.70) gives

$$-\mu(\mathbf{V}_{in}-\mathbf{V}_o) = -\mathbf{V}_o\frac{r_a}{R_k}-\mathbf{V}_o$$

Whence,
$$\mathbf{V}_o = \frac{\mu\mathbf{V}_{in}}{1+\mu+\dfrac{r_a}{R_k}}.$$

Therefore,
$$\mathbf{V}_o = \frac{\mu\mathbf{V}_{in}R_k}{r_a+R_k(1+\mu)} \tag{1.71}$$

The stage gain m is given by

$$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{\mu R_k}{r_a+R_k(1+\mu)} \tag{1.72}$$

Substituting for $\mathbf{V}_o$ from equation (1.69) into equation (1.71) gives

$$\mathbf{I}_a = \frac{-\mu\mathbf{V}_{in}}{r_a+R_k(1+\mu)} \tag{1.73}$$

Figure 1.43 shows a relatively simple circuit which produces this current.

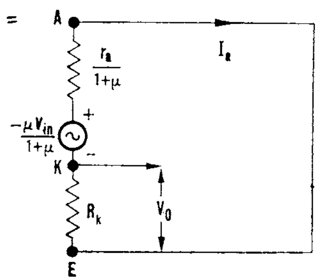


FIG. 1.43

Summary

1. The equivalent circuit of Fig. 1.43 may be used in place of Fig. 1.42 if the assumptions are valid.

2. A rise on the grid of the basic circuit of Fig. 1.41 increases valve conduction. This results in an increase in current flow through R_k and the output voltage $\mathbf{V}_o$ therefore rises. This means that there is no phase shift between $\mathbf{V}_o$ and $\mathbf{V}_{in}$ (see equation (1.71)).
3. The source of e.m.f. in the simplified equivalent circuit is a constant voltage generator of $\mu\mathbf{V}_{in}/(1+\mu)$ volts, which has an internal resistance of $r_a/(1+\mu)$. This is the *output resistance* R_o of the circuit. If $1 \ll \mu$, R_o is approximately equal to $1/g_m$ and is low.
4. The denominator of equation (1.72) is always greater than the numerator, hence the voltage gain of a cathode follower is always less than unity.
5. From equation (1.67), $\mathbf{V}_{in} = (\mathbf{I}_{in}R_g + m\mathbf{V}_{in})$ volts and input resistance $\mathbf{V}_{in}/\mathbf{I}_{in} = R_g/(1-m)$. Thus, the nearer stage gain m is to unity, the larger the input resistance.
6. A cathode follower is often used to match circuits with high output impedance to circuits with low input impedance.
7. If the bias is suitable, the input signal may be very large, approaching the supply voltage in magnitude.

1.20. Grounded Grid Triode Amplifier

BASIC CIRCUIT

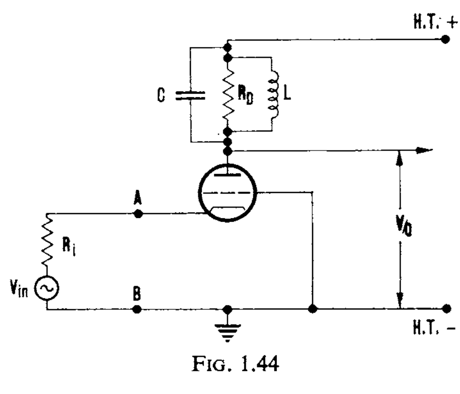


FIG. 1.44

CONSTANT VOLTAGE EQUIVALENT CIRCUIT

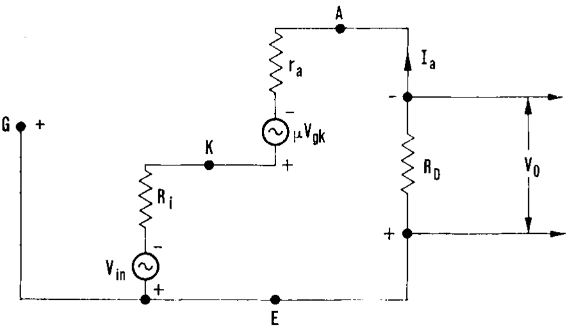


FIG. 1.45

Assume

1. All stray capacitance and inter-electrode capacitance effects are negligible at the frequency of the input voltage.
2. $\mathbf{V}_{gk}$ is the voltage which appears between grid and cathode of the circuit of Fig. 1.45.
3. The tuned circuit of Fig. 1.44 is resonant at the frequency of the input voltage and has a dynamic resistance R_D ohms.

TO OBTAIN EXPRESSIONS FOR GAIN, INPUT RESISTANCE AND OUTPUT RESISTANCE OF THE STAGE

(*a*) *Stage Gain*

Kirchhoff's Second Law states that the algebraic sum of e.m.f.'s around any closed loop is equal to the algebraic sum of voltages dropped across resistance elements which comprise the loop.

For the grid circuit of Fig. 1.45,

$$\mathbf{V}_{in} = \mathbf{I}_a R_i + \mathbf{V}_{gk}$$

and

$$\mathbf{V}_{gk} = \mathbf{V}_{in} - \mathbf{I}_a R_i \tag{1.74}$$

For the anode circuit

$$\mu \mathbf{V}_{gk} + \mathbf{V}_{in} = \mathbf{I}_a(R_i + r_a + R_D). \tag{1.75}$$

Substituting for $\mathbf{V}_{gk}$ from equation (1.74) into equation (1.75) gives,

$$\mu(\mathbf{V}_{in} - \mathbf{I}_a R_i) + \mathbf{V}_{in} = \mathbf{I}_a(R_i + r_a + R_D).$$

Therefore,

$$(\mu + 1)\mathbf{V}_{in} = \mathbf{I}_a[R_i(1+\mu) + (r_a + R_D)]$$

and

$$\mathbf{I}_a = \frac{(1+\mu)\mathbf{V}_{in}}{r_a + R_D + R_i(1+\mu)} \tag{1.76}$$

Now

$$\mathbf{V}_o = \mathbf{I}_a R_D = \frac{(1+\mu)R_D \mathbf{V}_{in}}{r_a + R_D + R_i(1+\mu)} \tag{1.77}$$

Hence,

$$\text{stage gain } m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{(1+\mu)R_D}{r_a + R_D + R_i(1+\mu)} \tag{1.78}$$

An equivalent circuit which will produce the current given by equation (1.76) and the stage gain of equation (1.78) is,

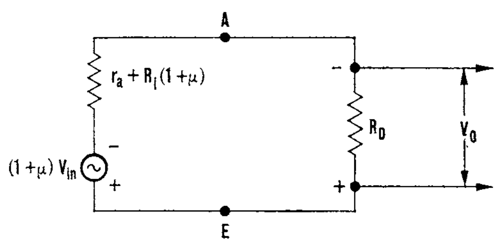


FIG. 1.46

(*b*) *Output Resistance*

This is the resistance across *AE* when the load is open circuit, and the constant voltage generator output is zero (see Fig. 1.46).

Hence, output resistance $= r_a + R_i(1+\mu)$. (1.79)

C

(*c*) *Input Resistance*

The total input resistance of the circuit of Fig. 1.44 consists of the internal resistance R_i of the source, in series with the resistance across AB due to the valve.

Therefore, total input resistance is given by $\mathbf{V}_{\text{in}}/\mathbf{I}_{\text{in}}$,

and
$$\frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_{\text{in}}} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_a} = \frac{r_a + R_D + R_i(1+\mu)}{(1+\mu)} \tag{1.80}$$

$$= R_i + (r_a + R_D)/(1+\mu). \tag{1.81}$$

The input resistance of the valve alone (R_{AB}) is given by

$$R_{AB} = \frac{r_a + R_D}{1+\mu} \tag{1.82}$$

Summary

1. The grounded grid has the effect of considerably reducing the energy transfer between output and input circuits which normally takes place via the anode to grid capacitance of the valve.
2. Although C_{ag} is assumed negligible in the above treatment, it does in fact appear across the anode tuned circuit and slightly modifies its resonant frequency.
3. Pentodes are frequently used in radio-frequency voltage amplifiers. However, pentodes are much noisier than triodes, and in cases where a high signal to noise ratio is required, a grounded grid triode may be preferred to a pentode.
4. The input to the amplifier (across AB of Fig. 1.44) may be derived from a tuned circuit. If this is the case, the grid-cathode capacitance of the valve (C_{gk}) slightly modifies the resonant frequency of the tuned circuit. Also, because the input impedance of a grounded grid amplifier is low, the magnification factor of the input tuned circuit is reduced.
5. If $r_a \gg R_D$ and $\mu \gg 1$, the input resistance of this amplifier is approximately the same as the output resistance of a cathode follower, i.e. $1/g_m$.

6. There is no phase reversal in a grounded grid amplifier. Input and output signals add, giving an effective amplification factor of $(1+\mu)$.

1.21. Negative Feedback in Amplifiers

If a fraction of an amplifier output voltage is fed back to the input circuit in such a manner as to reduce the amplifier input voltage, *negative feedback* is said to be applied to the amplifier.

BLOCK DIAGRAM

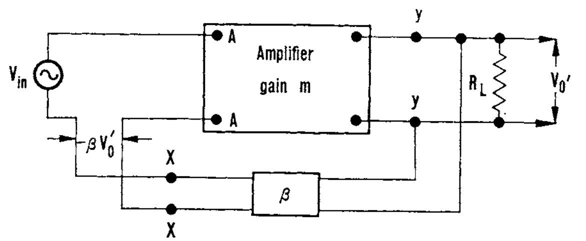


FIG. 1.47

Assume

1. The load, comprising R_L and β in parallel, is purely resistive at the frequency of the input voltage.
2. The frequency of the input voltage is constant.
3. At the frequency of the applied voltage, the amplifier introduces no phase shift, and the feedback network β is connected in such a manner as to introduce 180° phase shift.

Note

The gain m of the amplifier of Fig. 1.47 without feedback, is given by $m = \mathbf{V}_o/\mathbf{V}_{in}$, when $\mathbf{V}_{in}$ is applied across AA, and XX is open circuit.

To Find the Gain m′ *of an Amplifier with Negative Feedback*

The input voltage to the amplifier when negative feedback is applied is $\mathbf{V}_{AA}$, and is given by

$$\mathbf{V}_{AA} = (\mathbf{V}_{\text{in}} - \beta \mathbf{V}'_{\text{o}}).$$

$\mathbf{V}'_{\text{o}}$ is the output voltage across R_L with the feedback loop connected. Hence, after $\mathbf{V}_{AA}$ is amplified through the amplifier,

$$m(\mathbf{V}_{\text{in}} - \beta \mathbf{V}'_{\text{o}}) = \mathbf{V}'_{\text{o}},$$

therefore $$\mathbf{V}'_{\text{o}}(1+\beta m) = m\mathbf{V}_{\text{in}},$$

and stage gain with feedback is given by

$$m' = \frac{\mathbf{V}'_{\text{o}}}{\mathbf{V}_{\text{in}}} = \frac{m}{1+\beta m} \tag{1.83}$$

where m is the gain of the amplifier without feedback and β is the fraction of $\mathbf{V}'_{\text{o}}$ which is fed back.

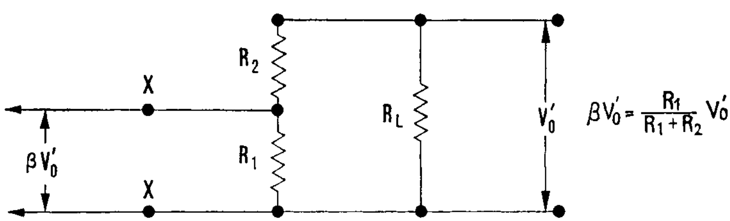


FIG. 1.48

It can be seen from Fig. 1.48 that $\beta = R_1/(R_1+R_2)$ and is real. However, β may become complex with increase in frequency due to the increasing effect of stray capacitance.

Summary

1. Stage gain and phase shift are both reduced when negative feedback is applied.

2. If $m\beta \gg 1$, stage gain with feedback is given approximately by $m' = 1/\beta$. Hence, the amplifier becomes dependent upon the feedback network only. This network is a passive device and can be made with high stability components.
3. Under the conditions of paragraph 2 above, the amplifier gain becomes independent of ageing of valves, variations in supply voltage and changes of frequency.
4. The derivation and application of negative feedback cause changes in input and output impedance of the amplifier.
5. The application of negative feedback reduces hum, distortion and noise produced by the amplifier.

1.22. Effect of Negative Feedback on Amplifier Distortion

GENERAL CIRCUIT

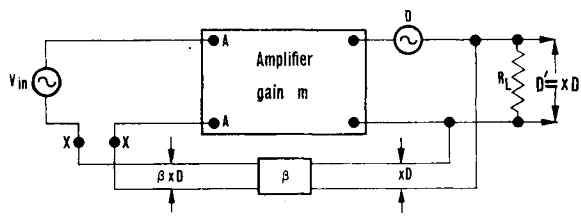


FIG. 1.49

D is the distortion voltage generated by the output stage when no feedback is applied.

Assume

1. Distortion voltage D is a function of the output voltage and can be represented as shown in Fig. 1.49.
2. The frequency of the input voltage V_{in} is fixed.
3. At the frequency of the applied voltage, the amplifier introduces no phase shift, and the feedback network β is connected in such a manner as to introduce 180° phase shift.

Effect of Negative Feedback on Distortion

When negative feedback is applied to the amplifier of Fig. 1.49, the distortion voltage D which appears in the output changes. Let the new distortion voltage be D', where

$$D' = xD. \tag{1.84}$$

Hence, the proportion of this which is fed back is βxD. After amplification this becomes $m\beta xD$, and this voltage is 180° out of phase with the original distortion voltage D. Therefore, the new distortion voltage D' which appears in the output is given by

$$D' = D - m\beta xD. \tag{1.85}$$

Equating values of D' from equations (1.84) and (1.85) gives

$$xD = D - m\beta xD.$$

From which $\quad x(1+m\beta) = 1$

or

$$x = \frac{1}{1+\beta m} \tag{1.86}$$

Substituting this value of x into equation (1.84) gives

$$D' = \frac{D}{1+\beta m} \tag{1.87}$$

Summary

1. The distortion introduced in the output stage of an amplifier without feedback can only be reasonably compared with the distortion introduced by the same output stage when feedback is applied, if the amplifier voltage output is the same in each case. As negative feedback reduces gain, the input to the stage must be increased to maintain the output at its original level.
2. The output stage of the amplifier can be assumed to introduce all the distortion produced by the amplifier. Hence, the input signal to the output stage can be increased to the desired

level by incorporating additional stages before the output stage.

3. A comparison of distortion levels now reveals that, for a given frequency, the application of negative feedback reduces distortion by the same factor as stage gain, i.e. $1/(1+m\beta)$. This is verified by equation (1.87).
4. Equation (1.87) is valid only at the assumed input frequency. At any other frequency, circuit reactance effects may become important causing m and/or β to become complex.

1.23. Tuned Anode Oscillator

BASIC CIRCUIT

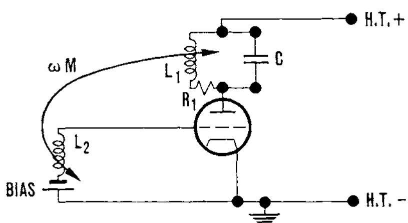


FIG. 1.50

CONSTANT CURRENT EQUIVALENT CIRCUIT

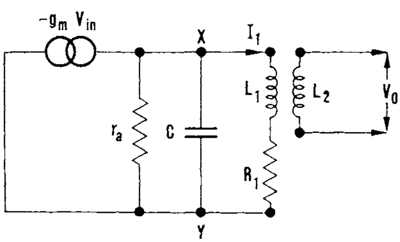


FIG. 1.51

Assume

1. Valve is biased class A.
2. Capacitance C in the tuned circuit includes all stray capacitance effects.
3. Sinusoidal oscillation is possible.

To Determine Conditions of Oscillation

The admittance of the anode circuit of Fig. 1.51 given by

$$Y_{XY} = \frac{1}{r_a} + j\omega C + \frac{1}{R_1 + j\omega L_1}$$

$$= \frac{(R_1 + j\omega L_1) + jr_a\omega C(R_1 + j\omega L_1) + r_a}{r_a(R_1 + j\omega L_1)}.$$

Now,
$$V_{XY} = \frac{-g_m \mathbf{V}_{\text{in}}}{Y_{XY}}$$

$$= -g_m \mathbf{V}_{\text{in}} \times \frac{r_a(R_1 + j\omega L_1)}{r_a + (R_1 + j\omega L_1) + jr_a\omega C(R_1 + j\omega L_1)}$$

and

$$\mathbf{I}_1 = \frac{\mathbf{V}_{XY}}{R_1 + j\omega L_1} = -\frac{g_m r_a \mathbf{V}_{\text{in}}}{r_a + (R_1 + j\omega L_1) + jr_a\omega C(R_1 + j\omega L_1)} \tag{1.88}$$

Now, the induced e.m.f. in the secondary is applied directly to the grid of the valve. In this case the induced e.m.f. is numerically equal to $\mathbf{V}_o$.

Hence,
$$\mathbf{V}_o = \pm j\omega M \mathbf{I}_1 \tag{1.89}$$

substituting the value of $\mathbf{I}_1$ from equation (1.88) into equation (1.89) gives

$$\mathbf{V}_o = \pm \frac{j\omega M g_m r_a \mathbf{V}_{\text{in}}}{r_a + (R_1 + j\omega L_1) + jr_a\omega C(R_1 + j\omega L_1)} \tag{1.90}$$

For continuous oscillations, both phase and magnitude conditions must be satisfied, hence, $\mathbf{V}_o = \mathbf{V}_{in}$ in equation (1.90).

Under these conditions equation (1.90) becomes

$$r_a+(R_1+j\omega L_1)+j\omega Cr_aR_1-\omega^2L_1Cr_a = \pm j\omega Mg_mr_a$$

or, $$r_a+(R_1+j\omega L_1)+j\omega Cr_aR_1-\omega^2L_1Cr_a\pm j\omega Mg_mr_a = 0. \tag{1.91}$$

Equation (1.91) is only satisfied when both real and imaginary terms are zero.

Therefore, equating imaginary terms of equation (1.91) to zero gives,

$$\omega L_1+\omega Cr_aR_1 = \pm\omega Mg_mr_a.$$

Therefore $$\pm M = \frac{L_1+CR_1r_a}{\mu}$$

and if the circuit is assumed to be oscillating

$$M = \frac{L_1+CR_1r_a}{\mu}, \tag{1.92}$$

equating real terms of equation (1.91) to zero gives

$$r_a+R_1 = \omega^2L_1Cr_a.$$

Therefore

$$\omega^2 = \frac{r_a+R_1}{L_1Cr_a}$$

$$f = \frac{1}{2\pi}\left\{\frac{1}{L_1C}\left[1+\frac{R_1}{r_a}\right]\right\}^{\frac{1}{2}} \text{ cycles per second.} \tag{1.93}$$

Summary

1. Although the frequency of oscillations depends mainly upon the values of L_1 and C, it is affected slightly by variations in R_1 and r_a.

2. R_1, L_1 and C vary with changes in temperature.
3. r_a changes as the valve ages or if power supply variations cause the working point to alter.
4. The conditions for maintenance of oscillations are satisfied only if the mutual inductance M has a minimum value given by equation (1.92).

1.24. General Theorem for Tuned Oscillators with no Mutual Coupling

GENERAL CIRCUIT

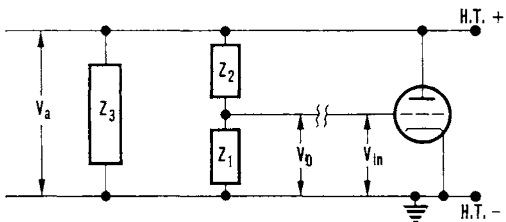


FIG. 1.52

CONSTANT VOLTAGE EQUIVALENT CIRCUIT

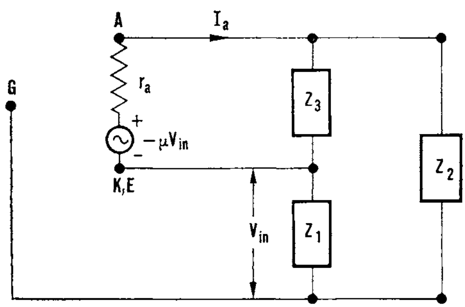


FIG. 1.53

To Determine the Conditions of Oscillation

Z_1, Z_2 and Z_3 are the impedance elements in the circuit of Fig. 1.53.

$$\text{Voltage gain to the anode} = \frac{\mathbf{V}_a}{\mathbf{V}_{in}} = \frac{-\mu Z_L}{(r_a+Z_L)}$$

where
$$Z_L = \frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3},$$

therefore
$$\frac{\mathbf{V}_a}{\mathbf{V}_{in}} = \frac{-\mu(Z_1+Z_2)Z_3}{Z_3(Z_1+Z_2)+r_a(Z_1+Z_2+Z_3)} \tag{1.94}$$

The proportion of anode voltage $\mathbf{V}_a$ which is fed back to the grid is $\mathbf{V}_o$

$$\mathbf{V}_o = \frac{Z_1}{Z_1+Z_2} \times \mathbf{V}_a.$$

Hence, equation (1.94) becomes

$$\mathbf{V}_o = -\frac{\mu Z_1 Z_3 \mathbf{V}_{in}}{Z_3(Z_1+Z_2)+r_a(Z_1+Z_2+Z_3)} \tag{1.95}$$

For continuous oscillations $\mathbf{V}_o = \mathbf{V}_{in}$, and under these conditions equation (1.95) becomes

$$Z_3(Z_1+Z_2)+r_a(Z_1+Z_2+Z_3) = -\mu Z_1 Z_3. \tag{1.96}$$

If it is assumed that the resistive elements associated with Z_1, Z_2 and Z_3 are negligibly small, then

$$Z_1 = jX_1: \quad Z_2 = jX_2: \quad Z_3 = jX_3.$$

(The precise nature of these reactances will become evident later.)

Equation (1.96) can now be modified thus,

$$-X_3(X_1+X_2)+jr_a(X_1+X_2+X_3) = +\mu X_1 X_3. \tag{1.97}$$

When the circuit is oscillating, the real and imaginary parts of equation (1.97) are zero. Hence, equating imaginary parts gives

$$r_a(X_1+X_2+X_3) = 0,$$

but this can only be true if

$$X_1+X_2+X_3 = 0 \tag{1.98}$$

or

$$X_3 = -(X_1+X_2). \tag{1.99}$$

Equating real parts gives,

$$-X_3(X_1+X_2) = \mu X_1 X_3,$$

but $(X_1+X_2) = -X_3$ from equation (1.99),

therefore, $$X_3^2 = \mu X_1 X_3$$

or

$$\mu = \frac{X_3}{X_1} \tag{1.100}$$

Summary

1. For equation (1.100) to be satisfied, X_1 and X_3 must be of the same nature, i.e. either both inductive or both capacitive. Equation (1.100) is termed the *maintenance equation.*

2. From equation (1.98), if X_1 and X_3 are of the same nature, then X_2 must be of opposite kind if the circuit is to oscillate, as the sum of the reactances X_1, X_2 and X_3 must be zero. Equation (1.98) enables the frequency of oscillations to be found.

3. The above conditions are satisfied, for example, in the tuned-anode–tuned-grid oscillator, the tuned-anode–crystal-grid oscillator, and the Colpitt's oscillator, provided that tuned circuit resistance elements are assumed negligible.

1.25. Colpitt's Oscillator

BASIC CIRCUIT

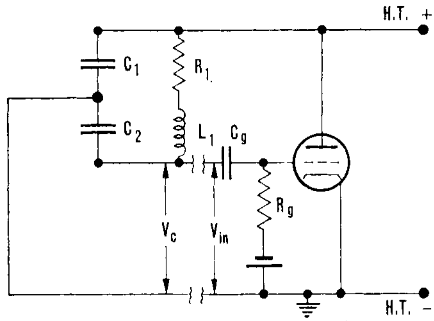


FIG. 1.54

CONSTANT VOLTAGE EQUIVALENT CIRCUIT

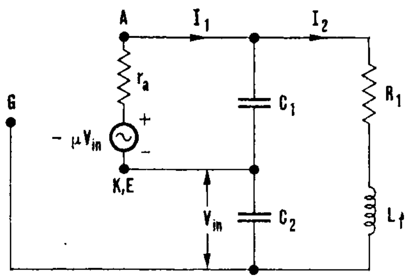


FIG. 1.55

Assume

1. Class A bias is provided by the bias battery shown.
2. C_g is a blocking capacitor which prevents H.T. voltage appearing on the grid, and R_g is a grid leak resistor.
3. The series resistance due to C_1 and C_2 is negligibly small.

To Determine the Conditions of Oscillation

Applying Kirchhoff's Second Law around the $\mathbf{I}_1$ loop of Fig. 1.55 gives

$$-\mu\mathbf{V}_{in} = \mathbf{I}_1 r_a + \frac{\mathbf{I}_1 - \mathbf{I}_2}{j\omega C_1} \tag{1.101}$$

and, if continuous oscillations are to be produced, the voltage across C_2 ($\mathbf{V}_c$) of Fig. 1.54 must be equal to $\mathbf{V}_{in}$.

Hence, $$\mathbf{V}_{in} = \frac{\mathbf{I}_2}{j\omega C_2}$$

and equation (1.96) becomes

$$\mathbf{I}_2\left[\frac{1}{j\omega C_1} - \frac{\mu}{j\omega C_2}\right] = \mathbf{I}_1\left[r_a + \frac{1}{j\omega C_1}\right].$$

Therefore,

$$-\mathbf{I}_2\left[\frac{j\omega C_2 - j\omega C_1\mu}{\omega^2 C_1 C_2}\right] = \mathbf{I}_1\left[\frac{1 + j\omega C_1 r_a}{j\omega C_1}\right]. \tag{1.102}$$

Now, applying Kirchhoff's Second Law around the $\mathbf{I}_2$ loop gives

$$\frac{\mathbf{I}_2 - \mathbf{I}_1}{j\omega C_1} + \mathbf{I}_2(R_1 + j\omega L_1) + \frac{\mathbf{I}_2}{j\omega C_1} = 0.$$

Hence,

$$-\mathbf{I}_2\left[\frac{j\omega C_1 + j\omega C_2 - \omega^2 C_1 C_2 R_1 - j\omega^3 C_1 C_2 L_1}{\omega^2 C_1 C_2}\right] = \frac{\mathbf{I}_1}{j\omega C_1} \tag{1.103}$$

Dividing equation (1.102) by equation (1.103) gives

$$\frac{j\omega C_2 - j\omega C_1\mu}{j\omega C_1 + j\omega C_2 - j\omega^3 C_1 C_2 L_1 - \omega^2 C_1 C_2 R_1} = 1 + j\omega C_1 r_a.$$

This gives,

$$j\omega C_1\mu + j\omega C_1 - j\omega^3 C_1 C_2 L_1 - j\omega^3 C_1^2 C_2 R_1 r_a + \omega^4 C_1^2 C_2 L_1 r_a$$
$$- \omega^2 C_1 C_2 R_1 - \omega^2 C_1^2 r_a - \omega^2 C_1 C_2 r_a = 0. \tag{1.104}$$

Real parts of equation (1.104) give,

$$\omega^2 = \frac{C_1^2 r_a + C_1 C_2 r_a + C_1 C_2 R_1}{C_1^2 r_a C_2 L_1}.$$

Therefore,

$$f = \frac{1}{2\pi}\sqrt{\left[\frac{1}{L_1 C_2} + \frac{1}{L_1 C_1}\left(1 + \frac{R_1}{r_a}\right)\right]} \text{ cycles per second.} \qquad (1.105)$$

Where f is the frequency of oscillations in cycles per second. Imaginary parts of equation (1.104) give

$$C_1(1+\mu) = \omega^2 C_1 C_2 L_1 + \omega^2 C_1^2 C_2 R_1 r_a.$$

Therefore, $$\mu = C_2\omega^2(L_1 + C_1 R_1 r_a) - 1. \qquad (1.106)$$

Summary

1. If all resistance effects in the tuned circuit are negligibly small, $f = 1/(2\pi\sqrt{LC_T})$ where $C_T = C_1C_2/(C_1+C_2)$. If resistance is important, the frequency of oscillations is given by equation (1.105).
2. If oscillations are to be established and maintained, μ must have a value equal to that given by equation (1.106).
3. If any one of the electrical quantities of equation (1.105) varies, the frequency of oscillations must also vary.

1.26. Wein Bridge Resistance–Capacitance Oscillator

BLOCK DIAGRAM

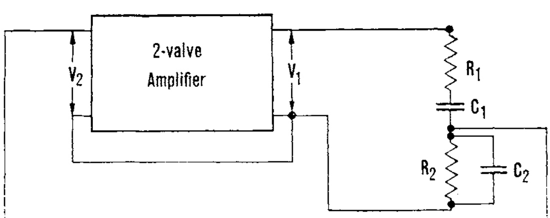


FIG. 1.56

AMPLIFIER LOAD

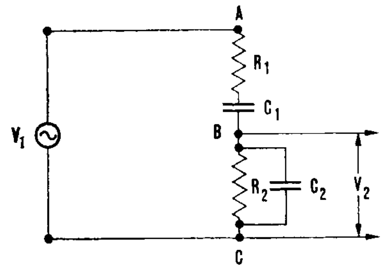


FIG. 1.57

There is one frequency at which the output voltage $\mathbf{V}_2$ of the network shown in Fig. 1.57 is exactly in phase with the input voltage $\mathbf{V}_1$. However, the network attenuates the input voltage so that $|V_2|$ is less than $|V_1|$. In order that oscillatory conditions be established and maintained, this loss must at least be made up by the 2-valve amplifier. Hence, the gain of the amplifier must be at least $|V_1/V_2|$.

Assume

1. The two-valve amplifier of Fig. 1.56 has zero phase shift.
2. The loading effect of the R.C. network on the amplifier is negligible, i.e. the amplifier has a low output impedance.
3. The amplifier input impedance is so high that it does not load the network.

To Determine Conditions of Oscillation

From Fig. 1.57,

$$Z_{AB} = R_1 + \frac{1}{j\omega C_1},$$

$$Z_{BC} = \frac{R_2 \times \dfrac{1}{j\omega C_2}}{R_2 + \dfrac{1}{j\omega C_2}}.$$

Therefore $$Z_{BC} = \frac{R_2}{1+j\omega C_2 R_2}$$

Hence $$\mathbf{V}_2 = \frac{Z_{BC}}{Z_{AB}+Z_{BC}} \times \mathbf{V}_1$$

Therefore $$\mathbf{V}_2 = \frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{R_1+\dfrac{1}{j\omega C_1}+\dfrac{R_2}{(1+j\omega C_2)}} \times \mathbf{V}_1 \qquad (1.107)$$

Simplifying equation (1.107) gives

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{j\omega C_1 R_2}{j\omega C_1 R_1(1+j\omega C_2 R_2)+1+j\omega C_2 R_2+j\omega C_1 R_2}$$

Therefore

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{\dfrac{R_1}{R_2}+\dfrac{C_2}{C_1}+1-j\dfrac{1-\omega^2 C_1 C_2 R_1 R_2}{\omega C_1 R_2}} \qquad (1.108)$$

It has been stated that $\mathbf{V}_2$ must be in phase with $\mathbf{V}_1$ if the circuit is to oscillate. Under these conditions, the imaginary term in the denominator of equation (1.108) is zero.

Therefore $$\frac{1}{\omega C_1 R_2} = \frac{\omega^2 C_1 C_2 R_1 R_2}{\omega C_1 R_2}$$

and $$\omega^2 = \frac{1}{C_1 C_2 R_1 R_2}$$

Therefore $$f = \frac{1}{2\pi\sqrt{(C_1 C_2 R_1 R_2)}} \text{ cycles per second.} \qquad (1.109)$$

For magnitude conditions to be satisfied the gain of the amplifier must be $|V_1/V_2|$ at this frequency. Hence, with the imaginary term of equation (1.108) zero,

$$\text{Amplifier gain} = \left|\frac{V_1}{V_2}\right| = \frac{R_1}{R_2}+\frac{C_2}{C_1}+1 \qquad (1.110)$$

Summary

1. If $R_1 = R_2$ and $C_1 = C_2$, the frequency of oscillation is given by $f = 1/2\pi CR$. Hence, if R_1 and R_2 are fixed values and C_1 and C_2 are ganged together, a variable frequency oscillator is obtained.
2. Under these conditions, the frequency variation is proportional to $1/C$, but in a tuned circuit oscillator, frequency variation is proportional to $1/\sqrt{C}$.
3. Under the conditions of paragraph 1 above, the amplifier gain must be at least 3 if magnitude conditions are to be satisfied.
4. For a good sinusoidal output waveform over the tuning range, negative feedback can be used to keep the gain of the two-stage amplifier at the minimum necessary to maintain oscillations.

1.27. Voltage and Current Gains of a Common-base Transistor Amplifier

BASIC CIRCUIT

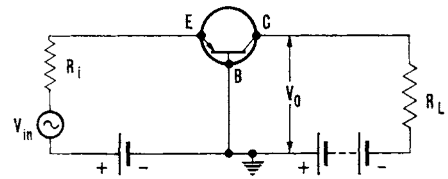


FIG. 1.58

CONSTANT VOLTAGE T-EQUIVALENT CIRCUIT

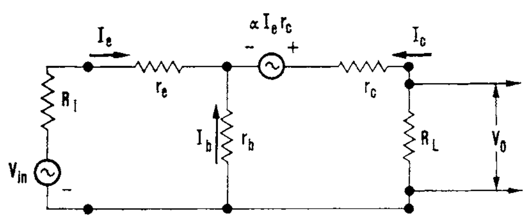


FIG. 1.59

Assume

1. The frequency of the applied voltage $\mathbf{V}_{in}$ is low enough to render effects of circuit capacitance elements negligible.
2. The d.c. conditions set by the bias batteries do not change.
3. The sinusoidal input signal is small.

Voltage Gain of a Common-base Transistor Amplifier

In a transistor,

$$\mathbf{I}_e+\mathbf{I}_b+\mathbf{I}_c = 0. \tag{1.111}$$

Therefore, $\mathbf{I}_b = -(\mathbf{I}_e+\mathbf{I}_c)$.

The current directions indicated on the equivalent circuit of Fig. 1.59 satisfy this equation. Applying Kirchhoff's Second Law around the $\mathbf{I}_e$ loop gives

$$\mathbf{V}_{in} = \mathbf{I}_e(r_e+R_i+r_b)+\mathbf{I}_c r_b, \tag{1.112}$$

and around the $\mathbf{I}_c$ loop,

$$-a\mathbf{I}_e r_c = \mathbf{I}_c(r_c+R_L+r_b)+\mathbf{I}_e r_b$$

or

$$0 = \mathbf{I}_c(r_c+R_L+r_b)+\mathbf{I}_e(r_b+ar_c). \tag{1.113}$$

Now $\mathbf{V}_o = -\mathbf{I}_c R_L$ and Voltage Gain $= \mathbf{V}_o/\mathbf{V}_{in}$

Therefore,

$$\text{voltage gain} = \frac{-\mathbf{I}_c R_L}{\mathbf{V}_{in}}. \tag{1.114}$$

Next, it is necessary to find $\mathbf{I}_c$. This may be done using equations (1.112) and (1.113).

Multiplying equation (1.112) by (r_b+ar_c) and equation (1.113) by $(r_e+R_i+r_b)$ gives

$$\mathbf{V}_{in}(r_b+ar_c) = \mathbf{I}_e(r_e+R_i+r_b)(r_b+ar_c)+\mathbf{I}_c r_b(r_b+ar_c),$$

$$0 = \mathbf{I}_e(r_e+R_i+r_b)(r_b+ar_c)+\mathbf{I}_c(r_e+R_i+r_b)(r_c+R_L+r_b).$$

Subtracting gives,

$$\mathbf{V}_{in}(r_b+ar_c) = -\mathbf{I}_c[(r_e+R_i+r_b)(r_c+R_L+r_b)-r_b(r_b+ar_c)]$$

and

$$\mathbf{I}_c = \frac{-\mathbf{V}_{in}(r_b+ar_c)}{(r_e+R_i+r_b)(r_c+R_L+r_b)-r_b(r_b+ar_c)}. \tag{1.115}$$

Substituting this value of $\mathbf{I}_c$ into equation (1.114) gives

$$\text{voltage gain} = -\mathbf{I}_c \frac{R_L}{\mathbf{V}_{in}}$$

$$= \frac{(r_b + \alpha r_c)R_L}{(r_e + R_i + r_b)(r_c + R_L + r_b) - r_b(r_b + \alpha r_c)} \tag{1.116}$$

However, if r_c is assumed to be much greater than any other resistance in equation (1.116), and α is almost unity,

$$\text{Voltage Gain} = \frac{\alpha R_L}{r_e + R_i + r_b(1 - \alpha)} \tag{1.117}$$

Current Gain of a Common-base Transistor Amplifier

From equation (1.113),

$$-\mathbf{I}_e(r_b + \alpha r_c) = \mathbf{I}_c(r_c + R_L + r_b)$$

and

$$\text{current gain} = \frac{\mathbf{I}_c}{\mathbf{I}_e} = \frac{-(r_b + \alpha r_c)}{r_c + R_L + r_b} \tag{1.118}$$

The minus sign indicates that if $\mathbf{I}_e$ enters the circuit, then $\mathbf{I}_c$ leaves the circuit.

If r_b is much less than αr_c, and R_L is much less than r_c,

$$\frac{\mathbf{I}_c}{\mathbf{I}_e} = -\alpha \tag{1.119}$$

Summary

1. If a sinusoidal input voltage $\mathbf{V}_{in}$ is applied to the common base transistor amplifier as indicated in the Basic Circuit, the resulting voltage across R_L will be in phase with $\mathbf{V}_{in}$.

   When the input voltage swings positively, the effective forward bias of the emitter-base junction is increased, causing an increase in collector current. The p.d. across R_L is increased, and the collector voltage rises towards zero volts.

Hence, voltage gain is accurately given by equation (1.116) as $\mathbf{V}_o/\mathbf{V}_{in}$ has a positive value.

2. The current gain of a common-base transistor amplifier is given by equation (1.118) and has a maximum theoretical value of $-\alpha$ when $R_L = 0$, and r_b is much less than either r_c or αr_c. α is the current amplification factor of a transistor connected in the common-base configuration, i.e. the slope of the I_C/I_B transfer characteristic at any point. It is commonly between 0·95 and 0·99.
3. For large voltage gain, the internal resistance R_i of the constant voltage generator should be small, and the collector load resistance R_L should be large.
4. Power gain is the product of voltage gain and current gain. In this case, power gain is inevitably less than the voltage gain.
5. The output resistance of a common-base transistor is high, and the input resistance low. Hence, if two such stages are connected in cascade, an impedance matching transformer is used.

1.28. Voltage and Current Gains of a Common-emitter Transistor Amplifier

BASIC CIRCUIT

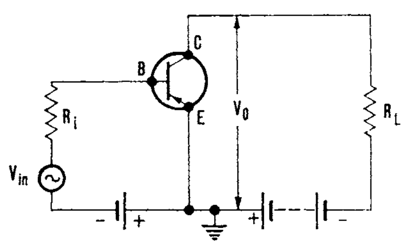


FIG. 1.60

CONSTANT VOLTAGE T-EQUIVALENT CIRCUIT

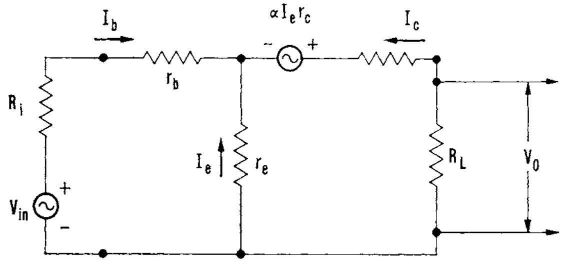


FIG. 1.61

Assume

1. The frequency of the applied voltage $\mathbf{V}_{in}$ is low enough to render effects of circuit capacitance elements negligible.
2. The d.c. conditions set by the bias batteries do not change.
3. The input signal is small.

Voltage Gain of a Common-emitter Transistor Amplifier

From equation (1.111), $\mathbf{I}_e = -(\mathbf{I}_c+\mathbf{I}_b)$, and the current directions shown on the equivalent circuit of Fig. 1.61 satisfy this equation. Applying Kirchhoff's Second Law around the $\mathbf{I}_b$ loop gives

$$\mathbf{V}_{in} = \mathbf{I}_b(r_e+R_i+r_b)+\mathbf{I}_c r_e \tag{1.120}$$

and around the $\mathbf{I}_c$ loop,

$$-\alpha\mathbf{I}_e r_c = \mathbf{I}_c(r_e+R_L+r_c)+\mathbf{I}_b r_e$$

rearranging, and putting $\mathbf{I}_e = -(\mathbf{I}_b+\mathbf{I}_c)$, gives

$$0 = \mathbf{I}_c[r_e+R_L+r_c(1-\alpha)]+\mathbf{I}_b(r_e-\alpha r_c). \tag{1.121}$$

Now, voltage gain $= -\mathbf{I}_c R_L/\mathbf{V}_{in}$ and it is again necessary to find $\mathbf{I}_c$. This may be done using equations (1.120) and (1.121).

Multiplying equation (1.120) by $(r_e - ar_c)$ and equation (1.121) by $(r_e + R_i + r_b)$ gives

$$\mathbf{V}_{\mathrm{in}}(r_e - ar_c) = \mathbf{I}_b(r_e + R_i + r_b)(r_e - ar_c) + \mathbf{I}_c r_e(r_e - ar_c),$$

and

$$0 = \mathbf{I}_b(r_e + R_i + r_b)(r_e - ar_c) + \mathbf{I}_c[r_e + R_L + r_c(1-a)](r_e + R_i + r_b).$$

Subtracting gives

$$\mathbf{V}_{\mathrm{in}}(ar_c - r_e) = \mathbf{I}_c\{r_e(ar_c - r_e) + [r_e + R_L + r_c(1-a)](r_e + R_i + r_b)\}.$$

Whence,

$$\mathbf{I}_c = \frac{(ar_c - r_e)\mathbf{V}_{\mathrm{in}}}{r_e(ar_c - r_e) + [r_e + R_L + r_c(1-a)](r_e + R_i + r_b)}$$

and

$$\text{voltage gain} = \frac{-(ar_c - r_e)R_L}{r_e(ar_c - r_e) + [r_e + R_L + r_c(1-a)](r_e + R_i + r_b)},$$

and if ar_c is much greater than r_e,

$$\text{voltage gain} = -\frac{ar_c R_L}{\{ar_c r_e + [r_e + R_L + r_c(1-a)](r_e + R_i + r_b)\}} \qquad (1.122)$$

Current Gain of a Common Emitter Transistor Amplifier

From equation (1.121)

$$\mathbf{I}_b(ar_c - r_e) = [r_e + R_L + r_c(1-a)]\mathbf{I}_c$$

and $$\text{current gain} = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{ar_c - r_e}{r_e + R_L + r_c(1-a)} \qquad (1.123)$$

If r_e is small compared with $r_c(1-\alpha)$ and $R_L = 0$, equation (1.123) has a maximum value given by

$$\text{current gain } (\alpha') \simeq \frac{r_c}{r_c(1-\alpha)} = \frac{\alpha}{1-\alpha} \tag{1.124}$$

Summary

1. If a sinusoidal input voltage $\mathbf{V}_{in}$ is applied to the common-emitter transistor as indicated in Fig. 1.60, the resulting alternating voltage across the output ($\mathbf{V}_o$) will be 180° out of phase with $\mathbf{V}_{in}$. On the positive-going half cycle of the input voltage, the effective forward bias of the emitter-base junction is decreased, causing a reduction in collector current. The p.d. across R_L is reduced, and the collector voltage falls towards the potential of the negative bias supply. Hence, voltage gain is accurately given by equation (1.122) if $\alpha r_c \gg r_e$. The minus sign denotes the phase reversal due to transistor action.

2. The current gain of a common-emitter transistor is given by equation (1.123). This has a theoretical maximum value which is equal to the current amplification factor of the transistor α'. It can be seen from equation (1.124) that $\alpha' = \alpha/(1-\alpha)$ when $R_L = 0$, and r_e is negligibly small. Hence, the current gain of a common-emitter transistor is much greater than the current gain of the same transistor connected in the common-base configuration.

3. The voltage gain of a common-emitter transistor is about the same as that obtained from the same transistor connected in the common-base configuration.

4. Power gain is most important when choosing the circuit configuration. The common-emitter transistor is used most frequently because its power gain is very high.

5. The common-emitter transistor has a much higher input resistance than a common-base transistor. The output

resistance of a common-emitter amplifier is lower than the output resistance of the corresponding common-base amplifier.

6. Leakage current is much higher in common-emitter than in common-base. Hence, to avoid thermal runaway, bias stabilization is incorporated in practical circuits.

1.29. Deflection Sensitivity of an Electrostatic Cathode-ray Tube

ELECTRIC FIELD STRENGTH (E)

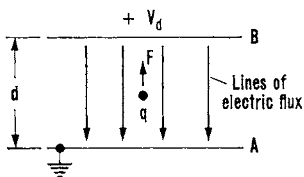


FIG. 1.62

E is the force in newtons on unit charge. Figure 1.62 shows a charge of q coulombs in the space between the two parallel plates d metres apart. It is assumed that the plate spacing is much less than the plate length so that the steady p.d. between the plates (V_d) produces a uniform field with negligible fringing. It is also assumed that the plates are situated in a perfect vacuum.

Now, the magnitude of the force F acting on this charged particle is given by $F = qE$ newtons. If the charged particle is moved from plate A to plate B under the influence of F newtons,

$$\text{mechanical work done} = Fd \text{ newton-metre}$$

$$= qEd \text{ newton-metre.} \qquad (1.125)$$

Now, potential difference is the electrical work done in joules per coulomb. Hence,

$$\text{electrical work done} = qV_d \text{ joules.} \qquad (1.126)$$

If there is no loss of energy, equations (1.125) and (1.126) are equal.

Therefore, $$qV_d = qEd$$

and $$E = V_d/d \text{ volts per metre.} \tag{1.127}$$

Hence, electrical field strength may also be defined as potential gradient in volts per metre.

Velocity of a Charged Particle Moving in an Electric Field

Assume that the charge of q coulombs is initially at rest on the surface of plate A of Fig. 1.62. There is a force of magnitude qE newtons present, attracting it to plate B. The electrical work done in moving the particle from A to B is given by equation (1.126).

At the instant before impact with B, the charge q moves with a velocity of v metres per second. At this instant, the charge possesses kinetic energy (K.E.) given by,

$$\text{K.E.} = \frac{mv^2}{2} \tag{1.128}$$

Now, equating (1.126) to (1.128) gives,

$$qV_d = \frac{mv^2}{2},$$

therefore, $$v = \sqrt{\left(\frac{2V_d q}{m}\right)} \tag{1.129}$$

where v is the final velocity of charge q in metres per second, V_d is the steady voltage on plate A with respect to plate B (in volts), q is the charge on the particle in coulombs, m is the mass of the particle in kilograms, and is assumed here to have a constant value. Relativistic variations are neglected.

Equation (1.129) shows that the final velocity of the charged particle depends only upon the voltage V_d.

ELECTRO-STATIC DEFLECTION IN A CATHODE-RAY TUBE

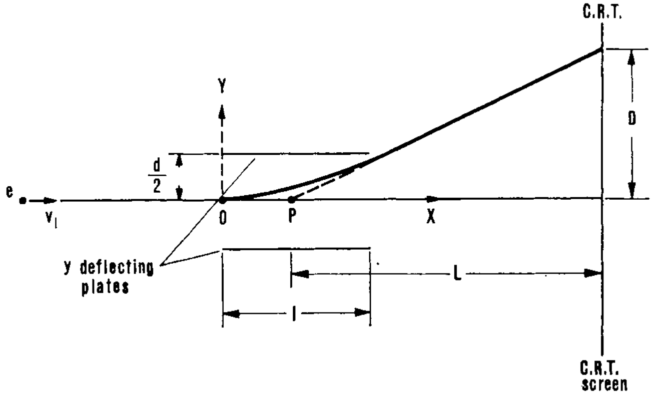


FIG. 1.63

d is the distance between the deflecting plates in metres.

D is the vertical deflection of the spot on the screen in metres.

L is the distance between the centre of the deflecting plates and the screen in metres.

e is the charge on an electron in coulombs ($1{\cdot}602 \times 10^{-19}$ C).

m is the mass of an electron in kilograms ($9{\cdot}106 \times 10^{-31}$ kg).

v_i is the horizontal velocity of the electron (in metres per second) as it enters the deflecting plates.

V_f is the potential of the final accelerating anode in volts and, although not shown in Fig. 1.63, it is this voltage which causes the electron to move at a velocity of v_i metres per second. It is assumed that there is no further horizontal acceleration of the electron after it passes the final anode.

V_d is the potential difference between the plates in volts.

Now, the horizontal distance x travelled by the electron from 0 is given by,

$$x = v_i t \text{ metres,} \tag{1.130}$$

where t is the transit time of the electron between the plates.

In the vertical direction, there is an accelerating force F on the electron due to an electric field strength of V_d/d volts per metre. As the electron has a charge of e coulombs,

$$F = \frac{eV_d}{d} \text{ newtons.}$$

Now,

$$\text{force } F = \text{mass } m \times \text{acceleration } a,$$

hence,

$$a = \frac{F}{m} = \frac{eV_d}{dm} \text{ metres per second per second.} \qquad (1.131)$$

y is the vertical distance travelled in metres, and is given by

$$y = ut + \frac{at^2}{2},$$

where u is the initial vertical velocity in metres per second, a is the vertical acceleration in metres per second per second, t is the time in seconds.

In this case, it is assumed that the initial vertical velocity is zero, whence,

$$y = \frac{at^2}{2} \text{ metres.} \qquad (1.132)$$

Substituting acceleration a from (1.131) into (1.132) gives

$$y = \frac{eV_d}{2dm} \times t^2 \text{ metres.}$$

but, from equation (1.130), $t = x/v_i$ seconds.

Therefore, $$y = \frac{eV_d}{2dmv_i^2} \times x^2 \text{ metres,}$$

but $v_i^2 = 2eV_f/m$ from equation (1.129)

and $$y = \frac{V_d}{4dV_f} \times x^2 \text{ metres.} \qquad (1.133)$$

Now, differentiating equation (1.133) with respect to x gives

$$\frac{dy}{dx} = \frac{2V_d}{4dV_f} \times x. \tag{1.134}$$

When the electron leaves the influence of the deflecting plates, $x = l$, and its path has a slope of dy/dx. Equation (1.134) becomes

$$\frac{dy}{dx} = \frac{l^2 V_d}{4dV_f} \times \frac{1}{l/2}$$

$$= \frac{y_l}{l/2}, \tag{1.135}$$

where y_l is the value of y when $x = l$ (see equation (1.133)). Hence, if the path of the electron is produced backwards, it will pass through point P of Fig. 1.63.

Now, putting $x = l$ in equation (1.134) gives

$$\frac{dy}{dx} = \frac{D}{L} = \frac{lV_d}{2dV_f} \tag{1.136}$$

The *deflection sensitivity* of the cathode-ray tube is the number of volts needed between the deflecting plates to give 1 metre of deflection on the tube face, hence,

$$\text{deflection sensitivity} = \frac{V_d}{D} = \frac{2dV_f}{lL} \tag{1.137}$$

Summary

1. Deflection D is proportional to the deflecting voltage V_d.
2. D is increased if the distance L of Fig. 1.63 is increased. It is worth noting that, in an electrostatic cathode-ray tube, the deflection sensitivity defined by equation (1.137) is less for the Y-plates than for the X-plates.
3. D is increased if the plate length l is increased and is decreased if plate spacing d is increased.
4. The amount of deflection is independent of mass and charge and is the same for all charged particles.

CHAPTER 2

Networks and Circuits

Worked Examples

Example 1

State Kirchhoff's Laws. In the Wheatstone bridge network shown in Fig. 2.1, determine (a) the values of R_1 and R_2, (b) the current flowing in the other resistors.

I.E.R.E., Nov. 1963

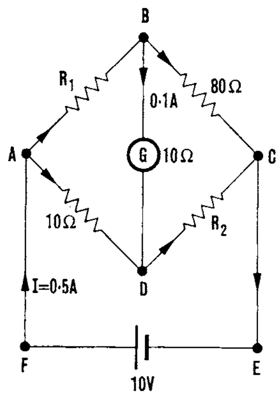


FIG. 2.1. Wheatstone bridge network.

Solution

Kirchhoff's First Law. Kirchhoff's First Law states that the algebraic sum of currents meeting at any point in a circuit is zero.

Hence, in Fig. 2.2,

$$I_1+I_2+(-I_3) = 0.$$

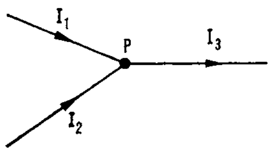


FIG. 2.2. Currents at the junction of a simple circuit.

Kirchhoff's Second Law. Kirchhoff's Second Law states that in any closed circuit the algebraic sum of e.m.f.'s is equal to the algebraic sum of p.d.'s.

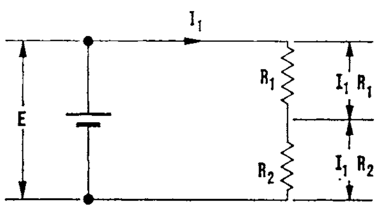


FIG. 2.3. Voltage distribution around a simple closed circuit.

Hence, around the I_1 loop of Fig. 2.3,

$$\text{Supply e.m.f. } E = I_1R_1+I_1R_2.$$

Branch Currents

Assume that the conventional currents in each branch of the network have the directions shown in Fig. 2.1.

Applying Kirchhoff's Second Law around the loop *FADBCE* gives,

$$10 = 10I_{AD}-10\times 0{\cdot}1+80\times I_{BC}$$

and $$11 = 10I_{AD}+80I_{BC}. \qquad (2.1)$$

But, according to Kirchhoff's First Law,

$$I_{AB} = I_{BC}+I_{BD} \quad \text{or} \quad I_{AB}-0{\cdot}1 = I_{BC} \tag{2.2}$$

and

$$I_{AB} = 0{\cdot}5-I_{AD}. \tag{2.3}$$

Now, substituting for I_{AB} from equation (2.3) into equation (2.2), gives

$$I_{BC} = 0{\cdot}4-I_{AD}, \tag{2.4}$$

and using this value of I_{BC} in equation (2.1),

$$11 = 10I_{AD}+80\,(0{\cdot}4-I_{AD}),$$

therefore,

$$21 = 70I_{AD}$$

and

$$I_{AD} = 0{\cdot}3\text{A}. \tag{2.5}$$

Using Kirchhoff's First Law, and I_{AD} from equation (2.5), I_{AB} is given by

$$I_{AB} = I-I_{AD} = 0{\cdot}5-0{\cdot}3 = 0{\cdot}2\text{ A}. \tag{2.6}$$

Similarly,

$$I_{BC} = I_{AB}-I_{BD} = 0{\cdot}2-0{\cdot}1 = 0{\cdot}1\text{ A}. \tag{2.7}$$

and I_{DC} is given by

$$I_{DC} = I_{BD}+I_{AD} = 0{\cdot}1+0{\cdot}3 = 0{\cdot}4\text{ A}. \tag{2.8}$$

Values of Unknown Resistors

Applying Kirchhoff's Second Law around loop *FABCEF* gives

$$10 = I_{AB}R_1+I_{BC}\times 80,$$

but, from equations (2.6) and (2.7) it can be seen that

$$I_{AB} = 0{\cdot}2\text{ A} \quad \text{and} \quad I_{BC} = 0{\cdot}1\text{ A}$$

therefore,

$$10 = 0{\cdot}2R_1+0{\cdot}1\times 80,$$

whence

$$R_1 = 10\ \Omega.$$

Applying the same law around the loop *FADCE* gives

$$10 = 3+I_{DC}R_2.$$

But, from equation (2.8) it can be seen that $I_{DC} = 0{\cdot}4$ A, therefore, $$10 = 3+0{\cdot}4R_2,$$

whence $$R_2 = \frac{7}{0{\cdot}4} = 17{\cdot}5\ \Omega.$$

Example 2

State Thévenin's theorem.

A π section is formed of two 300 Ω resistors as the upright or *shunt* components, and a 2 μF capacitor as the series element. A variable frequency oscillator of output resistance 600 Ω is adjusted to give an open-circuit output of 9 V. It is then connected across one of the two 300 Ω resistors. By means of the above theorem, or otherwise, find the voltage developed across the other 300 Ω resistor at angular frequencies of $\omega = 10$, 10^3 and 10^5 rad/s. I.E.R.E., May 1963

Solution

Thévenin's theorem. Any linear active network with two terminals behaves, as far as a load connected across these terminals is concerned, as if it were a generator of e.m.f. **E** volts which has an internal impedance Z_i. **E** is the voltage measured across the terminals when the load is disconnected, and Z_i is the impedance measured across the same terminals when all voltage sources are generating zero volts.

Thévenin's theorem will be used to solve the numerical part of this problem.

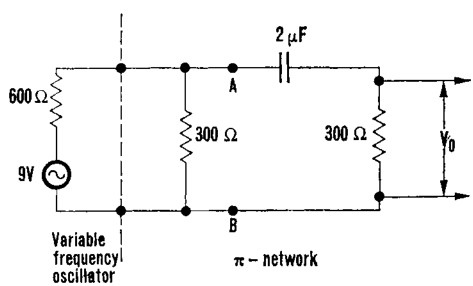


FIG. 2.4. Variable frequency oscillator feeding a π-section.

The open-circuit voltage of the variable frequency oscillator is numerically equal to the e.m.f. of the constant voltage generator of Fig. 2.4, i.e. 9 V.

At $\omega = 10$ rad/s. If Thévenin's theorem is applied across AB in Fig. 2.4, we have

$$\text{open-circuit voltage } \mathbf{V}_{o/c} = \frac{300}{300+600} \times 9 \text{ V}$$
$$= 3 \text{ V}.$$

Hence, the equivalent Thévenin voltage generator produces an output e.m.f. **E** of 3 V.

Now, if the 9 V generator is replaced by its internal resistance, the resistance *looking into* terminals AB is Z_i, and

$$Z_i = \frac{300 \times 600}{300+600} = 200\ \Omega.$$

This means that the circuit to the left of an imaginary vertical line drawn through AB in Fig. 2.4 may be replaced by the simpler circuit of Fig. 2.5.

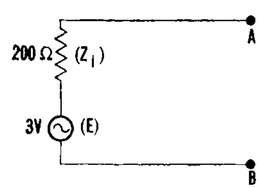


FIG. 2.5. Simplified input circuit.

At $\omega = 10$ rad/s,

$$X_c = \frac{1}{\omega C} = 50 \text{ k}\Omega$$

and $$-jX_c = -j50 \text{ k}\Omega.$$

Hence, an equivalent circuit of Fig. 2.4 employing the Thévenin equivalent generator of Fig. 2.5 would be as shown in Fig. 2.6.

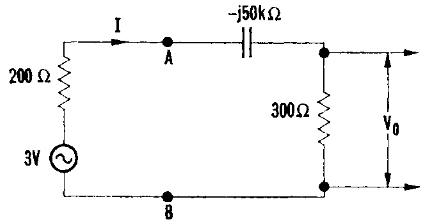


FIG. 2.6. Complete Thévenin equivalent of Fig. 2.4 at $\omega = 10$ rad/s.

From Fig. 2.6,

$$\mathbf{I} = \frac{3}{200+300-j50{,}000}\ \text{A},$$

$$\mathbf{I} = \frac{3}{500-j50{,}000}\ \text{A},$$

And $$\mathbf{V}_o = \mathbf{I}\times 300 = \frac{900}{500-j50{,}000}\ \text{V}. \tag{2.9}$$

It can be seen that the real part of the denominator of equation (2.9) is negligibly small, hence,

$$|\,V_o\,| = 18\ \text{mV}.$$

If required, the phase shift between output and input can be determined from equation (2.9) thus,

$$\mathbf{V}_o = |\,V_o\,|\ \angle\theta = 0{\cdot}018\ \angle\tan^{-1} 100$$
$$\simeq 0{\cdot}018\ \angle 90^\circ$$

At $\omega = 10^3$ rad/s,

$$X_c = \frac{1}{\omega C} = \frac{10^6}{2\times 10^3} = 500\ \Omega$$

and $$-jX_c = -j500\ \Omega.$$

Here again, the Thévenin equivalent generator of Fig. 2.5 is used, this time in conjunction with a modified *CR* load (see Fig. 2.7).

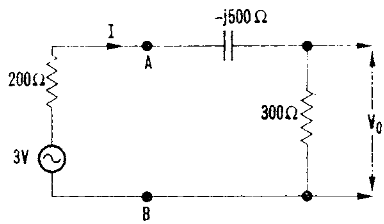


FIG. 2.7. Thévenin equivalent of Fig. 2.4 at $\omega = 10^3$ rad/s.

From Fig. 2.7,

$$\mathbf{I} = \frac{3}{500-j500}\,\text{A}$$

and

$$\mathbf{V}_o = \frac{3\times 300}{500(1-j)} = \frac{900(1+j)}{500\times 2}\,\text{V}$$

$$= 0{\cdot}9(1+j)\ \text{V}. \tag{2.10}$$

From equation (2.10),

$$|V_o| = 0{\cdot}9\sqrt{2} = 1{\cdot}27\ \text{V}$$

and

$$\angle\theta = \tan^{-1}1 = 45°.$$

At $\omega = 10^5$ rad/s

$$X_c = \frac{10^6}{2\times 10^5} = 5\ \Omega$$

and

$$-jX_c = -j5\ \Omega.$$

The Thévenin equivalent generator is again used to simplify the basic circuit of Fig. 2.4.

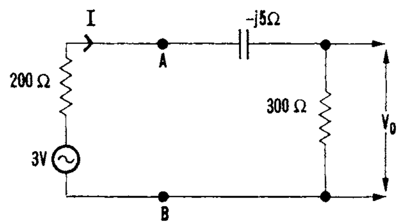


FIG. 2.8. Thévenin equivalent of Fig. 2.4 at $\omega = 10^5$ rad/s.

From Fig. 2.8,

$$\mathbf{I} = \frac{3}{500-j5}\,\mathrm{A}$$

and

$$\mathbf{V}_o = \frac{900}{500-j5}\,\mathrm{V}. \tag{2.11}$$

The imaginary term in the denominator of equation (2.11) is negligible compared with the real term, hence,

$$|V_o| \simeq \frac{900}{500} = 1{\cdot}8\ \mathrm{V}$$

and

$$\angle\theta = \tan^{-1}\frac{1}{100} \simeq 0°.$$

Example 3

A circuit containing R, L and C in series takes 5 mA from a 200 V supply when the frequency is 50 kc/s, and the current falls to 3 mA when the frequency is raised to 100 kc/s, the voltage remaining constant. It is also found that at 50 kc/s the current is in phase with the applied voltage. What are the values of R, L and C? I.E.R.E., May 1963

Solution

Fig. 2.9(a) shows the series *LCR* circuit under consideration and Fig. 2.9(b) gives the vector diagram of the circuit under

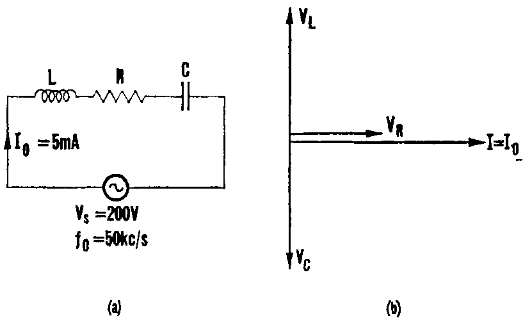


FIG. 2.9. (a) Series circuit at 50 kc/s. (b) Vector diagram of series circuit showing resonance condition (f = 50 kc/s).

series resonance conditions. Supply voltage and current are in phase at 50 kc/s and $\mathbf{V}_L$ and $\mathbf{V}_C$ are equal in magnitude, but opposite in phase. The voltage $\mathbf{V}_R$ across the resistor R at 50 kc/s is equal to the supply voltage, therefore

$$\mathbf{V}_S = \mathbf{V}_R = \mathbf{I}_0 \times R$$

and
$$R = \frac{\mathbf{V}_S}{\mathbf{I}_0} = \frac{200}{5 \times 10^{-3}} = 40\ \text{k}\Omega.$$

At any frequency, the circuit impedance Z is given by $Z = \mathbf{V}_s/\mathbf{I}$, therefore, at 100 kc/s,

$$Z = \frac{200}{3 \times 10^{-3}} = 66{\cdot}7\ \text{k}\Omega$$

and the impedance diagram at this frequency is as shown in Fig. 2.10. X_L will be greater than X_C above resonance and the circuit inductive. It has been shown that $R = 40\ \text{k}\Omega$, and $Z = 66{\cdot}7\ \text{k}\Omega$. Therefore, from Fig. 2.10,

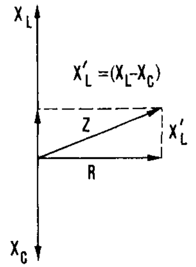


FIG. 2.10. Impedance diagram of series circuit at 100 kc/s.

$$X_L' = X_L - X_C = \sqrt{(Z^2 - R^2)}$$
$$= \sqrt{(66{\cdot}7^2 - 40^2)} = 53{\cdot}4\ \text{k}\Omega. \qquad (2.12)$$

But
$$X_L' = \omega L - \frac{1}{\omega C} \qquad (2.13)$$

and at the series angular resonant frequency ω_0,

$$\omega_0^2 LC = 1 \quad \text{or} \quad C = 1/\omega_0^2 L \qquad (2.14)$$

where $\omega_0 = 2\pi f_0$, and $f_0 = 50$ kc/s.
Now, substituting for $C = 1/\omega_0^2 L$ from (2.14) into (2.13) gives

$$X_L' = \left(\frac{\omega^2 - \omega_0^2}{\omega}\right) L$$

or
$$L = \frac{X_L'}{2\pi}\left(\frac{f}{f^2 - f_0^2}\right). \qquad (2.15)$$

When the frequency f of the input voltage is 100 kc/s, $X_L' = 53{\cdot}4\ \text{k}\Omega$ from equation (2.12), and the circuit inductance may be found by substituting known values into equation (2.15) thus,

$$L = \frac{53{\cdot}4 \times 10^3}{2\pi}\left(\frac{10^5}{(1 - 0{\cdot}25)10^{10}}\right)\ \text{H}.$$

Hence, $L = 113 \cdot 5$ mH.

From equation (2.14)

$$C = \frac{1}{\omega_0^2 L} = \frac{10^3 \times 10^{12}}{4\pi^2 \times 25 \times 10^8 \times 113 \cdot 5} \text{ pF.}$$

$$= 89 \text{ pF.}$$

Example 4

A coil of r.f. resistance 10 Ω is tuned by a capacitor of 318 pF in parallel with it to resonate at 1 Mc/s.

Find (a) the Q of this circuit;

(b) the dynamic resistance of this circuit at resonance;

(c) the impedance of this circuit at a frequency 20% above resonance.

If a 100 kΩ resistance is now connected across the coil, find the new values for (a) and (b).

I.E.R.E., Nov. 1962

Solution

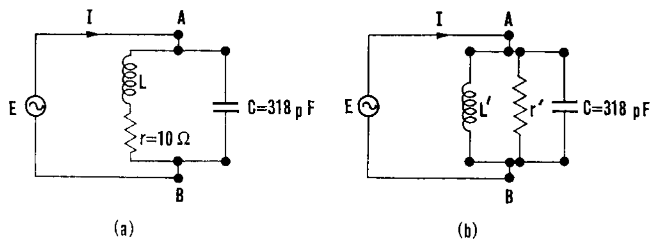


FIG. 2.11. (a) Parallel tuned circuit with series $L.R.$ (b) Parallel tuned circuit with parallel L, R and C. Equivalent of (a).

It will be assumed here that the magnification factor Q of the coil is high (⩾10) so that the supply voltage **E** is in phase with the supply current at the frequency at which the impedance across

AB is maximum. Under these conditions, the series and parallel resonant frequencies are identical and are given by $f_0 = 1/2\pi\sqrt{(LC)}$.

(a) *Q factor.* Consider first the circuit of Fig. 2.11(a). For this circuit,

$$Q_0 = \frac{\omega_0 L}{r} = \frac{1}{\omega_0 Cr}. \tag{2.16}$$

Substituting given values into equation (2.16) gives,

$$Q_0 = \frac{1}{2\pi \times 10^6 \times 318 \times 10^{-12} \times 10} = 50.$$

(b) *Dynamic Resistance.* This is the resistance across AB of Figs. 2.11(a) and (b) when the frequency of the input voltage is the resonant frequency of the parallel tuned circuit. Fig. 2.11(b) shows the inductive branch of Fig. 2.11(a) replaced with its equivalent parallel components L' and r'. The capacitor C is considered loss-free, and its value is the same in each circuit.

At the parallel resonant frequency of the circuit, the reactive effects of L' and C are equal and opposite. The circuit is resistive, the value of the resistance r' being the dynamic resistance of the network.

Now,

$Q_0 = 50$, $\omega_0 = 2\pi \times 10^6$ rad/s, $C = 318$ pF and $Q_0 = \omega_0 Cr'$

for a circuit with parallel L, C and R components,

hence $$r' = \frac{Q_0}{\omega_0 C} = \frac{50 \times 10^{12}}{2\pi \times 10^6 \times 318}\,\Omega$$

$$= 25\ \text{k}\Omega.$$

(c) *Impedance off resonance.* It can be seen from Fig. 2.11(a) that, at any frequency,

$$Z_{AB} = \frac{(r+j\omega L)\dfrac{1}{j\omega C}}{r+j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

and for a high Q coil and loss-free capacitor,

$$Z_{AB} = \frac{L/C}{r+j\left(\omega L-\dfrac{1}{\omega C}\right)}. \tag{2.17}$$

Dividing numerator and denominator of equation (2.17) by r gives,

$$Z_{AB} = \frac{L/Cr}{1+j\left(\dfrac{\omega L}{r}-\dfrac{1}{\omega Cr}\right)} \tag{2.18}$$

but L/Cr is the dynamic resistance r' of the circuit at resonance. Also,

$$\omega_0 L/r = Q_0, \quad \text{and} \quad 1/\omega_0 Cr = Q_0,$$

therefore equation (2.18) may be modified thus,

$$Z_{AB} = \frac{r'}{1+jQ_0\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)}$$

$$= \frac{r'}{1+jQ_0\left(\dfrac{f}{f_0}-\dfrac{f_0}{f}\right)}. \tag{2.19}$$

In this problem $f = 1{\cdot}2f_0$, $Q_0 = 50$, $f_0 = 10^6$ c/s and $r' = 25$ kΩ.

whence,
$$Z_{AB} = \frac{25\times10^3}{1+j50\left(\frac{1{\cdot}2f_0}{f_0}-\frac{f_0}{1{\cdot}2f_0}\right)}\ \Omega$$

$$= \frac{25\times10^3}{1+j50(1{\cdot}2-0{\cdot}833)}\ \Omega$$

$$= \frac{25\times10^3}{1+j18{\cdot}35}\ \Omega \qquad (2.20)$$

and
$$|Z_{AB}| \simeq \frac{25}{18{\cdot}35}\ \text{k}\Omega$$

$$= 1{\cdot}36\ \text{k}\Omega.$$

In equation (2.20), the imaginary term of the denominator is greater than ten times the value of the real term. In such circumstances it is permissible to ignore the real term.

Effect of 100 kΩ *Shunting Resistor*

Fig. 2.12 shows the 100 kΩ damping resistor connected across *AB* of the parallel circuit of Fig. 2.11(b).

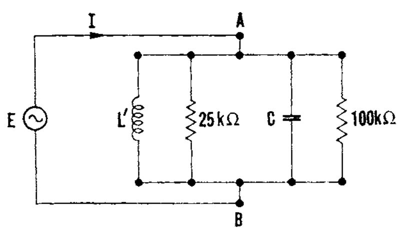


FIG. 2.12. Modified version of Fig. 2.11(b) showing 100 kΩ damping resistor.

From Fig. 2.12 the effective Q factor Q_e is given by

$$Q_e = \omega_0 C R_e \tag{2.21}$$

where ω_0 is the angular resonant frequency of the circuit, and R_e is the effect of r' in parallel with the 100 kΩ damping resistor. In fact, at resonance, it is this resistance R_e which appears across AB of Fig. 2.12 and constitutes the dynamic resistance of the modified circuit.

Therefore,
$$R_e = \frac{25 \times 100}{25 + 100} \text{ k}\Omega$$
$$= 20 \text{ k}\Omega.$$

Hence, Q_e from equation (2.21) becomes,

$$Q_e = 2\pi \times 10^6 \times 318 \times 10^{-12} \times 20 \times 10^3$$
$$= 40.$$

It can be readily seen that the application of the 100 kΩ resistor across AB of the original circuit has two effects. The resistance of the parallel tuned circuit at resonance is reduced from 25 kΩ to 20 kΩ, and the magnification factor of the parallel tuned circuit at resonance is reduced from 50 without the damping resistor, to 40 with it.

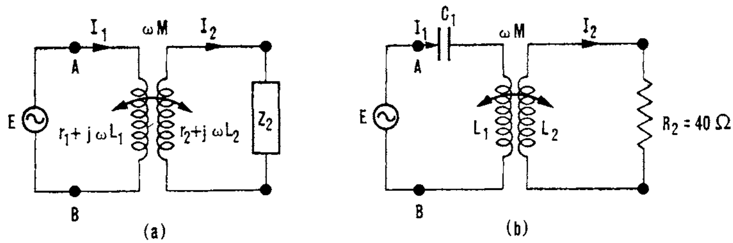
Example 5

The primary and secondary windings of a r.f. transformer have inductances of 80 μH and 20 μH respectively. The mutual inductance between windings is 10 μH, and their resistance is negligible. Across the secondary is connected a 40 Ω resistor.

What impedance would be measured between the primary terminals at an angular frequency of 2×10^6 rad/s?

Find also the value of capacitance which should be connected in series with the primary winding to make the input circuit resonant at this frequency. I.E.R.E., Nov. 1962

Solution



FIGS. 2.13(a) and (b).

Input impedance Z_{AB}. Kirchhoff's Second Law may be used to obtain a general expression for the input impedance across AB of the r.f. transformer shown in Fig. 2.13(a).

Applying this law around the $\mathbf{I}_1$ loop gives

$$\mathbf{E}+j\omega M\mathbf{I}_2 = \mathbf{I}_1(r_1+j\omega L_1) \tag{2.22}$$

and then around the $\mathbf{I}_2$ loop,

$$j\omega M\mathbf{I}_1 = (r_2+j\omega L_2)\mathbf{I}_2+Z_2\mathbf{I}_2$$

whence

$$\mathbf{I}_2 = \frac{j\omega M\mathbf{I}_1}{r_2+j\omega L_2+Z_2} \tag{2.23}$$

Putting $\mathbf{I}_2$ from (2.23) into (2.22) and rearranging gives

$$\mathbf{E} = \left[r_1+j\omega L_1+\frac{\omega^2M^2}{r_2+j\omega L_2+Z_2}\right]\mathbf{I}_1. \tag{2.24}$$

If the effective input impedance across AB is Z_{AB}, then $Z_{AB} = \mathbf{E}/\mathbf{I}_1$.

Also, if

$$Z_1 = r_1+j\omega L_1 \quad \text{and} \quad Z_s = r_2+j\omega L_2+Z_2,$$

from equation (2.24) we have,

$$Z_{AB} = +\mathbf{Z}_s\frac{\omega^2M^2}{Z_1} \tag{2.25}$$

In this problem it is assumed that the resistance elements r_1 and r_2, due to primary and secondary coils respectively, are zero. The secondary load impedance Z_2 is purely resistive.

Therefore
$$Z_{AB} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + R_2} \qquad (2.26)$$

Substituting given values into equation (2.26) gives

$$Z_{AB} = \left(j2\times 10^6 \times 80\times 10^{-6} + \frac{4\times 10^{12}\times 10^2 \times 10^{-12}}{40+j2\times 10^6\times 20\times 10^{-6}}\right)\,\Omega$$

$$= \left(j160+\frac{400}{40+j40}\right)\,\Omega$$

$$= [j160+\tfrac{10}{2}(1-j)]\,\Omega$$

therefore,

$$Z_{AB} = (5+j155)\,\Omega.$$

Series Capacitance C_1

The new set-up is shown in Fig. 2.13(b). Under these conditions equation (2.26) is modified, since the series primary impedance Z_1 now includes the effect of the capacitor C_1. Thus, the new impedance Z_{AB} between terminals A and B is given by

$$Z_{AB} = j\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \frac{\omega^2 M^2}{R_2 + j\omega L_2}$$

$$= \left[j\left(160-\frac{1}{\omega C_1}\right)+5-j5\right]\,\Omega. \qquad (2.27)$$

Equating imaginary terms of equation (2.27) to zero at resonance gives Z_{AB} as a pure resistance of value 5 Ω.

Also
$$+j160-j5 = \frac{1}{\omega C_1}$$

and
$$C_1 = \frac{10^{12}}{2\times 10^6\times 155}\,\text{pF}$$

$$= 3220\ \text{pF}.$$

Example 6

Two coils A and B are connected in parallel across a 10 V source of alternating e.m.f. Coil A has an impedance of $(5+j10)\ \Omega$ and coil B an impedance of $(7+j21)\ \Omega$. The mutual reactance at the frequency of the applied e.m.f. is $\omega M = \pm 10\ \Omega$. Calculate the current flowing in coil B.

Solution

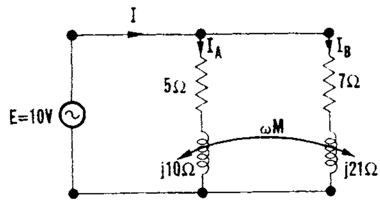


FIG. 2.14

Applying Kirchhoff's Second Law to the network of Fig. 2.14 gives

$$10 \pm j10\mathbf{I}_B = \mathbf{I}_A(5+j10)$$

and

$$10 \pm j10\mathbf{I}_A = \mathbf{I}_B(7+j21).$$

These equations may be rearranged thus,

$$10 = \mathbf{I}_A(5+j10) \pm j10\mathbf{I}_B \tag{2.28}$$

$$10 = \pm j10\mathbf{I}_A + (7+j21)\mathbf{I}_B \tag{2.29}$$

Equations (2.28) and (2.29) may be solved simultaneously. Multiplying equation (2.28) by $\pm j10$ and equation (2.29) by $(5+j10)$ gives

$$\pm j100 = \pm j10(5+j10)\mathbf{I}_A + (\pm j10)^2\mathbf{I}_B \tag{2.30}$$

$$10(5+j10) = \pm j10(5+j10)\mathbf{I}_A + (7+j21)(5+j10)\mathbf{I}_B \tag{2.31}$$

If now equation (2.31) is subtracted from equation (2.30) we have,

$$\begin{aligned} \pm j100-50-j100 &= [(\pm j10)^2-(35-210+j70+j105)]\mathbf{I}_B \\ &= [-100-(-175+j175)]\mathbf{I}_B \\ &= (75-j175)\mathbf{I}_B. \end{aligned}$$

Therefore,

$$\begin{aligned} \mathbf{I}_B &= \frac{-50}{75-j175}\,\mathrm{A} \quad \text{or} \quad \frac{-50-j200}{75-j175}\,\mathrm{A} \\ &= \frac{2\ \angle 180^\circ}{7{\cdot}62\ \angle -66{\cdot}8^\circ}\,\mathrm{A} \quad \text{or} \quad \frac{8{\cdot}25\ \angle 256^\circ}{7{\cdot}62\ \angle -66{\cdot}8^\circ}\,\mathrm{A} \end{aligned}$$

whence,

$$\mathbf{I}_B = 0{\cdot}262\ \angle 246{\cdot}8^\circ\ \mathrm{A}$$

or

$$\mathbf{I}_B = 1{\cdot}08\ \angle 322{\cdot}8^\circ\ \mathrm{A}$$

Which of these currents actually flows through coil B depends upon the sense in which the coil is connected into circuit.

Example 7

Calculate the values of a three-element low-pass constant-k filter having a cut-off frequency of 1·2 kc/s and a terminal impedance of 600 Ω using (a) a T-section, (b) a π-section.

Solution

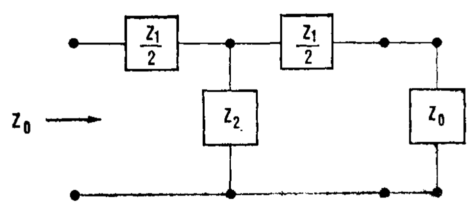


FIG. 2.15. T-section terminated in its characteristic impedance.

(a) *T-section.* For a constant-k filter $Z_1Z_2 = R^2$ where R is the design impedance of the network (in this case $R = 600\ \Omega$). Z_1 and Z_2 are the total series and shunt impedances of the general circuit of Fig. 2.15, and Z_0 is the characteristic impedance of the symmetrical section.

From Fig. 2.15,

$$Z_0 = \frac{Z_1}{2}+\frac{\left(\frac{Z_1}{2}+Z_0\right)Z_2}{\frac{Z_1}{2}+Z_0+Z_2}$$

$$= \frac{Z_1}{2}+\frac{(Z_1+2Z_0)Z_2}{Z_1+2(Z_0+Z_2)}$$

$$= \frac{Z_1[Z_1+2(Z_0+Z_2)]+2Z_2(Z_1+2Z_0)}{2Z_1+4(Z_0+Z_2)}$$

whence

$$Z_0^2+Z_0\left[\frac{Z_1}{2}+Z_2\right] = \frac{Z_1^2}{4}+Z_0\left(\frac{Z_1}{2}+Z_2\right)+Z_1Z_2$$

Simplifying this expression gives,

$$Z_0 = \sqrt{\left\{Z_1Z_2\left[1+\frac{Z_1}{4Z_2}\right]\right\}}. \tag{2.32}$$

If $Z_1 = j\omega L$, $Z_2 = 1/j\omega C$, and $Z_1Z_2 = R^2$, equation (2.32) becomes,

$$Z_0 = R\sqrt{\left(1-\frac{\omega^2LC}{4}\right)}. \tag{2.33}$$

For frequencies at which $\omega^2LC/4 < 1$, equation (2.33) is real. If Z_1 and Z_2 are purely reactive no power is lost in the filter, however, power may be taken from an input source by a load connected across the filter output terminals. Thus frequencies from zero cycles per second to the cut off frequency f_c constitute a pass band. f_c is the frequency at which $\omega^2 LC/4 = 1$.

Therefore $$\omega_c = 2\pi f_c = \frac{2}{\sqrt{LC}}$$

and $$f_c = \frac{1}{\pi\sqrt{LC}} \tag{2.34}$$

Frequencies above f_c are attenuated, and form the attenuation band. Thus, the device described above has the characteristics of a simple low-pass filter. The circuit under consideration is shown in Fig. 2.16.

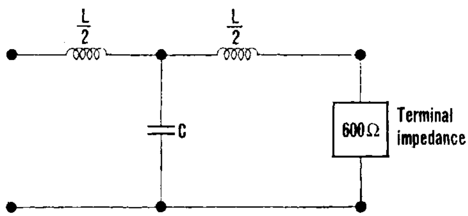


FIG. 2.16

Now, $$Z_1 Z_2 = L/C = R^2$$

whence, $$L = CR^2 \tag{2.35}$$

and from equation (2.34)

$$f_c^2 = \frac{1}{\pi^2 LC} \tag{2.36}$$

Putting $L = CR^2$ from equation (2.35) into equation (2.36) gives

$$C = \frac{1}{\pi R f_c} \tag{2.37}$$

If $R = 600\ \Omega$, and the cut-off frequency is to be 1·2 kc/s

$$C = \frac{1}{600 \times \pi \times 1200}\ \text{F}$$

$$= \frac{10^6}{72 \times 10^4 \times \pi}\ \mu\text{F}$$

therefore $$C = 0{\cdot}442\ \mu\text{F}. \tag{2.38}$$

Using this value of C in equation (2.35) gives,

$$L = CR^2 = 0{\cdot}442 \times 10^{-6} \times 600^2\ \text{H}$$

$$= 0{\cdot}442 \times 10^{-6} \times 36 \times 10^4 \times 10^3\ \text{mH}$$

therefore $$L = 159\ \text{mH}. \tag{2.39}$$

Thus, the series inductors of Fig. 2.16 are each 79·5 mH and the shunt capacitance is 0·442 μF.

(b) *π-section.* Simplifying the impedance network of Fig. 2.17 in the normal way reveals that, for a symmetrical π-network,

$$Z_0 = \sqrt{\left(\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}\right)} \tag{2.40}$$

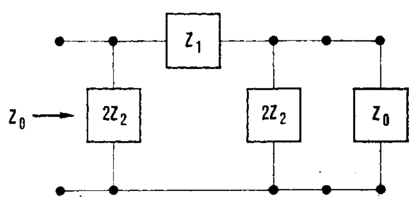


FIG. 2.17. π-section terminated in its characteristic impedance.

If again $Z_1 = j\omega L$, $Z_2 = 1/j\omega C$ and $Z_1 Z_2 = R^2$,

$$Z_0 = \frac{R}{\sqrt{\left(1 - \frac{\omega^2 LC}{4}\right)}} \tag{2.41}$$

It can be seen from equation (2.41) that Z_0 is real at frequencies of zero cycles per second up to the cut-off frequency, but is reactive above this frequency. Thus, the circuit acts as a low-pass filter, and, as before,

$$\frac{\omega_0^2 LC}{4} = 1$$

and $$f_c = (1/\pi)\sqrt{LC}.$$

The component arrangement of the π-filter is shown in Fig. 2.18.

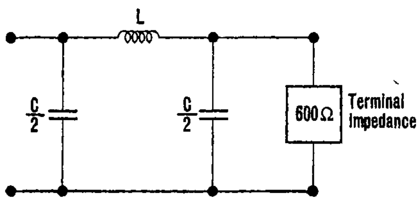


FIG. 2.18

The values of L and C must be those given by expressions (2.38) and (2.39), therefore, in Fig. 2.18,

$$L = 159 \text{ mH}$$

and $$\frac{C}{2} = \frac{0{\cdot}442}{2} = 0{\cdot}221\ \mu\text{F}.$$

PROBLEMS WITH ANSWERS

1. (a) Two cells are connected in parallel, like poles to like. The first has an e.m.f. of 1·5 V and internal resistance 4 Ω and the other an e.m.f. of 1·8 V and internal resistance 5 Ω. What current would pass through a 10 Ω resistor connected across the combination?

(b) Two resistors of 80 kΩ and 60 kΩ are connected in series across a d.c. supply. Across the 60 kΩ resistor is connected a further resistance of 30 kΩ. If the power dissipated in the 60 kΩ resistor is 80/3 mW calculate the supply voltage. What would a voltmeter of resistance 80 kΩ read if it were connected across the 80 kΩ resistor?

0·134 A; 200 V; 133·3 V. I.E.R.E., Nov. 1961

2. (a) State Kirchhoff's laws relating to currents in a network. Use them to establish the equations for currents in the arms of an unbalanced Wheatstone net, and deduce the balance condition.

(b) Three resistors of 200 kΩ, 300 kΩ and 600 kΩ are connected in series to a d.c. supply. A voltmeter reading up to 200 V which takes a current of 1 mA on full-scale deflection is connected across the 200 kΩ resistor. If this voltmeter reads 20 V, calculate (i) the power dissipated in the 300 kΩ resistor, (ii) the supply voltage and (iii) the resistance which, when placed in parallel with the 600 kΩ resistor, would increase the voltmeter reading to 25 V.

12 mW; 200 V; 1·2 MΩ. I.E.R.E., May, 1962

3. For the circuit shown in Fig. 2.19, calculate

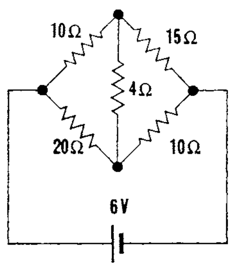


FIG. 2.19

(a) the current in the 4 Ω resistor;
(b) the power dissipated in the 20 Ω resistor;
(c) the equivalent resistance of the network.

0·096 A; 0·56 W; 12·8 Ω. I.E.R.E., May 1961

4. By the use of Thévenin's Theorem or otherwise, derive an expression for the current which flows through the resistance of x ohms in the circuit shown in Fig. 2.20. Hence find the value of x to give the maximum power dissipated in this resistance, and the value of this power.

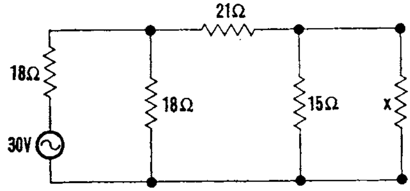


FIG. 2.20

$\left(\frac{5}{10+x}\right)$ A; 10 Ω; 0·62 W. I.E.R.E., Nov. 1962

5. Explain the following terms as applied to alternating currents: mean value, root mean square, power factor, phase angle.

A resistor of 400 Ω is connected in series with a capacitor of 50/3 μF, to a source of alternating voltage of 200 $V_{r.m.s.}$ and frequency $100/\pi$ c/s. Draw a vector diagram and calculate (i) the voltage across the capacitor, (ii) the power dissipated in the resistor, (iii) the capacitance which would have to be substituted for the 50/3 μF capacitor to make the voltage across the resistor $1/\sqrt{2}$ of the supply voltage.

(i) 120 $V_{r.m.s.}$; (ii) 64 W; (iii) 12·5 μF. I.E.R.E., Nov. 1961

6. How would you measure the power factor of a capacitor? The power factor of a 0·001 μF capacitor is 0·002 at 5 Mc/s. Evaluate (a) its equivalent series resistance and (b) its equivalent parallel resistance.

If an alternating voltage of 80 $V_{r.m.s.}$ is applied across the capacitor at this frequency, find the power dissipated as heat in the capacitor.

0·064 Ω; 15·9 kΩ; 0·402 W. I.E.R.E., May 1960

7. In the circuit of Fig. 2.21, *E* is earthed and it may be assumed that both sources have no internal resistance, and that the supplies have been connected long enough for steady state conditions to have been reached. Sketch graphs of the variations with time indicating the peak value and the relative phases (over the same cycle of the a.c. source) of:

(a) the potential of *A*;
(b) the potential of *B* with respect to *D*;
(c) the current through the 10 Ω resistor. I.E.R.E., Nov. 1962.

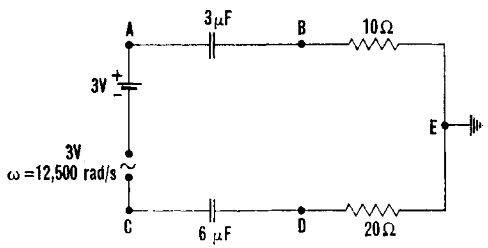


FIG. 2.21

8. Prove that, for maximum power to be developed in a load, the load impedance should be the conjugate of the source impedance.

Two reactors, each of 60 Ω reactance and negligible resistance, are connected in series across an audio-frequency amplifier producing 24 V. A resistance *R* is connected in parallel with one of the reactors. Determine the value of *R* so that the power dissipated is a maximum. Calculate the power dissipated under these conditions.

$R = 30\ \Omega$; $P = 2{\cdot}4$ W. I.E.R.E., May 1963

9. A resistance of 18 Ω is connected in parallel with an inductive reactance of 12 Ω to a supply of unknown frequency. Deduce expressions for and hence calculate the value of the resistance R and reactance X which, when connected in series across the same supply, could replace the parallel circuit without making any change in magnitude or phase relation of the current taken from the supply. Determine the phase relation between the current and the supply voltage.

$R = 5{\cdot}54\ \Omega$, $X = 8{\cdot}3\ \Omega$, $\theta = 56{\cdot}3°$ Lagging. I.E.R.E., May 1963

10. A voltage given by $e = 150 \sin \omega t + 25 \sin (3\omega t + 60°)$ is applied to a circuit containing a resistor of 12 Ω in series with an inductor of 0·02 H and negligible resistance.

Calculate (a) the power dissipated in, and (b) the power factor of, the circuit if the fundamental frequency is 50 c/s.

744·5 W; 0·878 Lagging. I.E.R.E., May 1963

11. A generator of e.m.f. 1 V a.c. and internal impedance $(20+j30)\ \Omega$ has two impedances $(50-j10)\ \Omega$ and $(30+j20)\ \Omega$ connected in parallel across its output terminals.

Calculate the current flowing from the generator and also the current flowing through the $(30+j20)\ \Omega$ impedance.

18·2 mA; 11·4 mA. I.E.R.E., Nov. 1961

12. Find the impedance between terminals A and B of the circuit shown in Fig. 2.22.

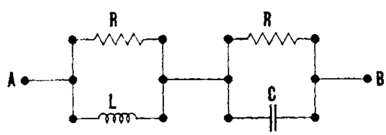


FIG. 2.22

Show that if $L = CR^2$ then this impedance is a pure resistance at all frequencies, and find the value of this resistance. I.E.R.E., Nov. 1960

13. (a) A coil has an inductance of L henrys and a resistance R ohms. It is connected in series with a capacitor of C farads. Derive an expression for the impedance of the circuit. Illustrate, by means of a vector diagram, the resonant condition and state the value of impedance under these conditions.

(b) A resistance of 500 Ω is in series with two capacitors each of 2 μF capacitance and an a.c. supply of E volts frequency f c/s. Determine the value of f for the p.d. across the resistance to be $E/\sqrt{2}$ volts.

$1000/\pi$ c/s. I.E.R.E., Nov. 1962.

14. A coil of inductance 2 H and resistance 300 Ω is connected in series with a capacitor of $3\frac{1}{3}\,\mu$F to an a.c. source of frequency $500/2\pi$ c/s and r.m.s. voltage 100 V. Draw a vector diagram and calculate (a) the supply current, (b) the voltage across the capacitor, (c) the power dissipated and (d) the phase difference between the supply current and voltage. What value of capacitor need be added to the circuit to make the supply current a maximum and how would you connect it?

0·2 $A_{r.m.s.}$; 120 $V_{r.m.s.}$; 12 W; 53° 10′; 5 μF in series.

I.E.R.E., May 1961

15. A coil of resistance 5 Ω is tuned by a parallel capacitor of 100 pF to resonate at 3·18 Mc/s.

Calculate: (a) the inductance of the coil, (b) the Q factor of the tuned circuit, (c) the impedance of the tuned circuit at resonance, (d) the impedance of the tuned circuit at a frequency of 10% above its resonant frequency.

If a 50 kΩ resistor were now connected across the tuned circuit, find (e) the new impedance at resonance, (f) the new Q factor of the circuit.

25 μH; 100; 50 kΩ; 2·6 kΩ; 25 kΩ; 50. I.E.R.E., Nov. 1960

16. A circuit consists of two branches. One is a coil of inductance L and resistance r, and the other a capacitor C in series with a resistance R. The circuit is connected across an a.c. source of voltage E and frequency f c/s. Construct a vector diagram which shows clearly the relative phases of the various current and voltage components. Given $L = 4$ H, $r = 300\ \Omega$, $C = 100/3\ \mu$F, $R = 400\ \Omega$, $E = 100\ V_{r.m.s.}$ and $f = 100/2\pi$ c/s, calculate (i) the voltage across the resistive component in each branch, and their relative phase difference, (ii) the supply current and its phase relative to the supply voltage. By reference to your vector diagram, state the conditions necessary for this phase difference to be zero.

(i) 60 V, 80 V, 90°; (ii) 0·283 A; 8·1° Lagging. I.E.R.E., May 1962

17.

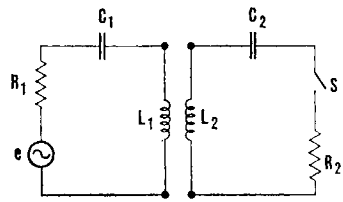


FIG. 2.23

$$e = 20\text{ V at } \omega = 5\times 10^5\text{ rad/s.}$$

$$R_1 = 1\text{ k}\Omega;\ R_2 = 100\ \Omega;\ L_1 = 2\text{ mH};\ L_2 = 5\text{ mH};\ C_2 = 800\text{ pF}.$$

L_1 and L_2 are coupled by a mutual inductance of $M = 0{\cdot}6$ mH. Initially the switch S is open.

Find the value of C_1 needed to obtain the maximum primary current, and the value of this current. C_1 is kept at this value, and the switch S is now closed. Find the new primary current and the p.d. developed across R_2.
200 pF; 20 mA; 10·5 mA, 3·15 V. I.E.R.E., Nov. 1960.

18. A voltage step-up transformer of turns ratio 1 : 5 may be assumed perfect. Across the secondary are connected in parallel a resistor of 500 kΩ and a capacitor of 1000 pF. Find the impedance which the primary will present to an alternating potential at an angular frequency of 1000 rad/s.

In practice such a transformer cannot be assumed to be perfect. What are the more important sources of loss in an audio frequency transformer, and how do they affect the frequency response?
17·9 kΩ. I.E.R.E., Nov. 1961

19. A transformer of turns ratio 1 : 5 has a primary inductance of 20 H. A resistor of 1 MΩ, shunted by a capacitance of 40 pF, is connected across the secondary. Calculate the approximate impedance which the circuit would present when measured between the primary terminals of the transformer at angular frequencies ω of (a) 10^2 and (b) 10^6 rad/s. Find also the frequency at which this input impedance is greatest, and the value of this impedance at this frequency.
2 kΩ; 1 kΩ; 1·1 kc/s; 40 kΩ. I.E.R.E., Nov. 1960

20. Find the values of the T-network which is equivalent to the π-network shown in Fig. 2.24 below, where $Z_1 = (10-j5)$; $Z_2 = (8+j10)$ and $Z_3 = 7-j5$.

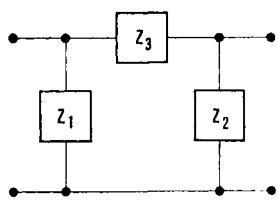


FIG. 2.24

5·2+j2·4; 1·8−j3·4; 4·24+j1·2. I.E.R.E., May 1961

21. Derive the equation $\cosh\gamma = 1+Z_1/2Z_2$ for a T-section, where γ is the propagation constant, Z_1 the total series arm impedance, and Z_2 the shunt arm impedance.

Design from first principles a prototype low-pass filter section having a design impedance 600 Ω and a cut-off frequency of 1 kc/s.
L = 191 mH, C = 0·265 μF. I.E.R.E., May 1956

22. Explain what is meant by the propagation coefficient, p, of the network shown in Fig. 2.25, and prove that $\cosh p = 1+2Z_1/Z_2$. Hence show that if $Z_1 = j\omega L$ ohms, and $Z_2 = -j/\omega C$ ohms, the network may be used as a simple low-pass filter.

State the disadvantages of this simple filter, and outline briefly the design of a composite filter, developed from this basic section, but which does not have the stated disadvantages. I.E.R.E., Nov. 1963

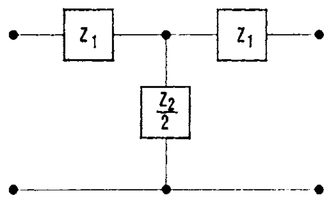


FIG. 2.25

23. An m-derived low-pass filter has the structure shown in Fig. 2.26. The design is based on a prototype section having a design impedance of 600 Ω and a cut-off frequency of 3 kc/s. The frequency response of the composite filter has attenuation peaks at frequencies of 3·4 kc/s and 3·9 kc/s.

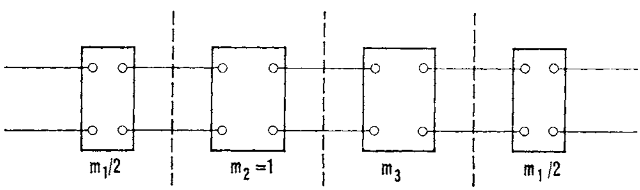


FIG. 2.26

Give a possible structure for each section and calculate the component values. Explain the purpose of each section. I.E.R.E., May 1961

CHAPTER 3

Electron Ballistics

WORKED EXAMPLES

Example 1

Two parallel plates are placed 4 cm apart in a vacuum, the upper plate being at a potential of +40 V with respect to the lower plate. Determine (a) the impact velocity and (b) the transit time between the plates, of an electron leaving the lower plate with negligible velocity.

Solution

It is assumed in this solution that the initial vertical velocity of the charged particle is negligibly small, and that the velocities involved are small enough to ensure that relativistic variation of mass may be neglected. The field between the plates is assumed uniform, and fringing effects are ignored.

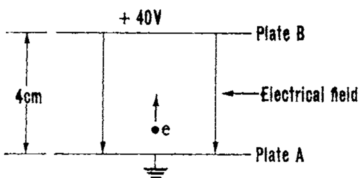


FIG. 3.1. Parallel plates spaced 4 cm apart in a vacuum.

(*a*) *Impact velocity v.* It was shown in Section 1.29, equation (1.129), that if the above assumptions are valid,

$$v = \sqrt{\left(\frac{2V_a e}{m}\right)} \tag{3.1}$$

where v is the velocity (in metres per second) with which the electron e of Fig. 3.1 strikes plate B.

V_d is the p.d. between plate A and plate B in volts (see Fig. 3.1);
e is the charge on an electron ($1{\cdot}602\times10^{-19}$ C);
and m is the constant mass of the electron ($9{\cdot}106\times10^{-31}$ kg).

If given values are substituted into equation (3.1), we have

$$v = \sqrt{\left(\frac{2\times40\times1{\cdot}602\times10^{-19}}{9{\cdot}106\times10^{-31}}\right)} \text{ m/s},$$

$$= \sqrt{(14{\cdot}1\times10^{12})} \text{ m/s},$$

therefore $$v = 3{\cdot}75\times10^{6} \text{ m/s}.$$

(*b*) *Transit time t.* As the electron is assumed to leave plate A of Fig. 3.1 with negligible vertical velocity u, equation (1.132) may be used to determine the transit time;

therefore, $$y = d = \frac{at^2}{2} \tag{3.2}$$

where d is the distance between the plates in metres, a is the acceleration of the electron in metres per second per second, and t is the transit time of the electron between the plates in seconds.

By transposition, equation (3.2) becomes

$$t = \sqrt{\frac{2d}{a}} \tag{3.3}$$

Now, from equation (1.131),

$$a = \frac{eV_d}{dm} = \frac{1{\cdot}76\times10^{11}\times40}{0{\cdot}04} \text{ m/s}^2,$$

therefore $$a = 1{\cdot}76\times10^{14} \text{ m/s}^2. \tag{3.4}$$

Substituting the value for a from equation (3.4) into equation (3.3) gives the time taken for the electron to travel from plate A to plate B.

Therefore
$$t = \sqrt{\left(\frac{2\times 0{\cdot}04}{1{\cdot}76\times 10^{14}}\right)}$$
$$= 2{\cdot}13\times 10^{-8} \text{ seconds.}$$

Example 2

The Y-plates of a cathode-ray tube are 1·8 cm in length and 0·4 cm apart, the centre of the plates being 28 cm from the screen. If the tube operates with a final anode voltage of 1500 V, calculate

(a) the horizontal beam velocity;
(b) the transit time of an electron between the plates;
(c) the vertical velocity acquired when the p.d. between the Y-plates is 30 V;
(d) the maximum value of p.d. between the Y-plates before beam cut-off occurs.

Solution

Here again, it is assumed that a perfect vacuum exists, and that the initial axial velocity of an electron e from the cathode is negligibly small. It is further assumed that relativistic variation of mass may be neglected along with fringing effects.

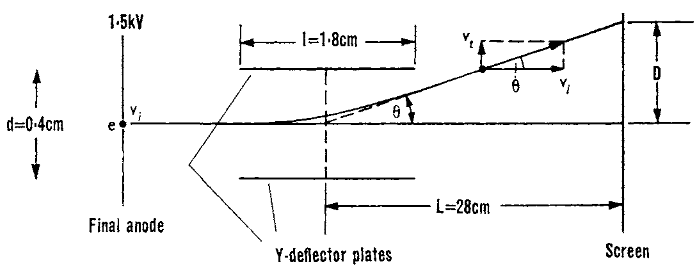


FIG. 3.2. Deflection in an electrostatic C.R.T. Electron e moves with constant velocity after leaving the influence of the final anode.

(*a*) *Horizontal beam velocity* v_i. The horizontal beam velocity v_i may be found by direct substitution in equation (1.129) thus,

$$v_i = \sqrt{(2 \times 1500 \times 1{\cdot}76 \times 10^{11}\ \text{m/s})}$$

therefore $$v_i = \sqrt{(5{\cdot}28 \times 10^{14})} = 22{\cdot}9 \times 10^6\ \text{m/s}. \qquad (3.5)$$

(*b*) *Transit time t*. By reference to Fig. 3.2, it can be readily seen that

$$\text{Plate length } l = \text{axial velocity } v_i \times \text{transit time } t$$

whence, $$t = \frac{l}{v_i}. \qquad (3.6)$$

Substituting $l = 0{\cdot}018$ m, and $v_i = 22{\cdot}9 \times 10^6$ m/s, into equation (3.6) gives the transit time as

$$t = \frac{0{\cdot}018}{22{\cdot}9 \times 10^6} = 7{\cdot}86 \times 10^{-10}\ \text{seconds}. \qquad (3.7)$$

(*c*) *Vertical velocity* v_t. Under the assumptions, normal projectile theory applies, hence,

$$v_t = u + at$$

and, as the initial vertical velocity u is assumed zero,

$$v_t = at \qquad (3.8)$$

where v_t is the final vertical velocity achieved by the electron as it leaves the deflector plates. a and t are as previously defined.

Now, from equation (1.131),

$$a = \frac{eV_d}{dm} = \frac{30 \times 1{\cdot}76 \times 10^{11}}{0{\cdot}004}\ \text{m/s}^2,$$

therefore $$a = 1{\cdot}32 \times 10^{15}\ \text{m/s}^2. \qquad (3.9)$$

v_t can be determined by substituting t from (3.7) and a from (3.9) into equation (3.8) thus,

$$v_t = 1{\cdot}32 \times 10^{15} \times 7{\cdot}86 \times 10^{-10}\ \text{m/s},$$

therefore $$v_t = 1{\cdot}04 \times 10^6\ \text{m/s}.$$

(*d*) *Beam cut-off voltage.* Beam cut-off occurs when the slope of the electron path when leaving the influence of the uniform electric field is greater than d/l, where d is the plate spacing and l is the plate length (see Fig. 3.2).

If the final anode voltage is fixed at +1500 V with respect to cathode, the horizontal velocity and transit time are unaltered. For beam cut-off, the vertical acceleration a of the electron must be increased if a vertical displacement of at least half the plate spacing is to be achieved in a time of $7{\cdot}86 \times 10^{-10}$ seconds. From equation (1.131) $a = eV_d/md$, hence it can be seen that the p.d. between the plates (V_d) must be increased if the required acceleration is to be obtained. As before,

$$y = \frac{at^2}{2}$$

$$= \frac{eV_d}{md} \times \frac{t^2}{2}$$

and if V_d is made the subject of this equation, we have

$$V_d = \frac{2myd}{et^2}. \tag{3.10}$$

However, for beam cut-off $y = d/2$,

therefore, $$V_d = \frac{md^2}{et^2}. \tag{3.11}$$

Substituting known values into equation (3.11) gives

$$V_d = \frac{0{\cdot}004 \times 0{\cdot}004}{1{\cdot}76 \times 10^{11} \times 7{\cdot}86^2 \times 10^{-20}}\ \text{V}$$

$$\simeq \frac{16 \times 10^3}{109}\ \text{V},$$

therefore $V_d = 147$ V.

Example 3

In a cathode-ray tube, a beam of electrons is projected through a transverse magnetic field of density 0·65 mWb/m^2, extending 6 cm in the direction of the beam. What is the accelerating voltage V_f, if the beam is deflected through 15°?

Solution

When a charged particle is moving in, and perpendicular to, a uniform magnetic field, a force F is exerted on the particle which is at right angles to both the direction of motion and the direction of field (see Fig. 3.3). The magnitude of this force is

$$F = Bvq \text{ newtons} \tag{3.12}$$

where B is the uniform flux density in Wb/m^2, v is the speed of the charged particle in metres per second, at right angles to the field, and q is the charge in coulombs. (If the particle is an electron the charge is e coulombs.)

The charged particle describes a circular path, radius r, with a constant speed v, and experiences a centrifrugal force of

$$F = \frac{mv^2}{r} \text{ newtons.} \tag{3.13}$$

Hence, equations (3.12) and (3.13) must be equal,

therefore $$\frac{mv^2}{r} = Bvq$$

and $$r = \frac{mv}{Bq} \text{ metres.} \tag{3.14}$$

The total deflection achieved by an electron travelling through a transverse magnetic field of uniform flux density 0·65 mWb/m^2 and length 0·06 m depends upon the velocity with which the electron enters the field. Equation (1.129) shows that this velocity depends upon the accelerating voltage V_f.

It can be seen from Fig. 3.3 that

$$\sin\theta = \frac{l}{r}. \tag{3.15}$$

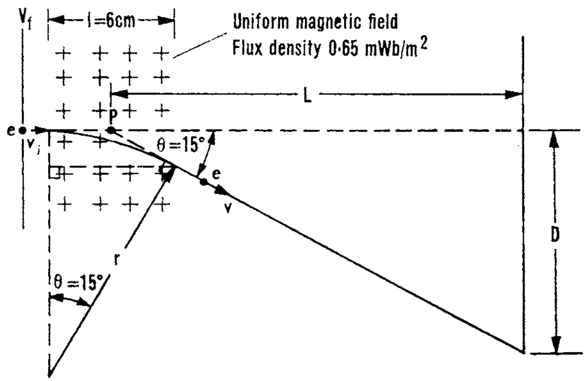


FIG. 3.3. Electron e is accelerated to a velocity v metres per second by V_f volts before being deflected through 15° by the uniform magnetic field.

Transposing equation (3.15) in terms of r, and substituting known values gives,

$$r = \frac{0{\cdot}06}{\sin 15^\circ} = 0{\cdot}232 \text{ m}. \tag{3.16}$$

From equation (3.14)

$$v = \frac{rBe}{m}$$

$$= 0{\cdot}232 \times 0{\cdot}65 \times 10^{-3} \times 1{\cdot}76 \times 10^{11} \text{ m/s}$$

whence $\quad v = 2{\cdot}66 \times 10^{7}$ m/s. $\quad$ (3.17)

v is numerically equal to the final velocity v_i achieved by an electron which leaves the cathode of an electro-magnetic c.r.t. with negligible axial velocity, hence the accelerating voltage V_f may be found using equation (1.129). From this equation,

$$v = \sqrt{\left(\frac{2V_f e}{m}\right)}.$$

Transposing in terms of V_f gives

$$V_f = \frac{v^2 m}{2e} \tag{3.18}$$

$$= \frac{2{\cdot}66 \times 2{\cdot}66 \times 10^{14}}{2 \times 1{\cdot}76 \times 10^{11}} = 2010 \text{ V}.$$

Example 4

Two parallel plates 6 cm long and 2 cm apart have a potential difference of 300 V. An electron projected between them is deflected 5·37 mm while passing between the plates. A magnetic field of 0·5 mWb/m^2, applied simultaneously perpendicular to the electric field, results in zero deflection. Without using values for electronic constants, find

(a) the velocity with which the electron enters the field,

(b) the ratio, charge on an electron : mass of an electron.

Solution

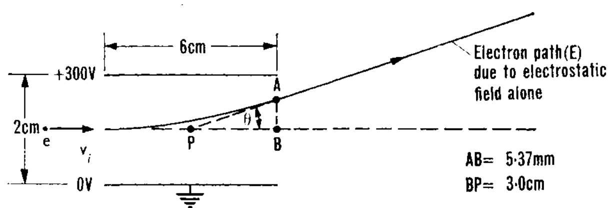


FIG. 3.4. Deflection of an electron due to an electrostatic field.

(*a*) *Velocity* (v_i). Figure 3.4 shows the path of an electron between parallel plates when a p.d. of 300 V exists between them. The vertical force on such an electron is $F = eV_a/d$ newtons.

Figure 3.5 shows the path which would be taken by an electron moving in a purely magnetic field, assuming the same incident velocity v_i and the same overall deflection as in Fig. 3.4.

The vertical force on an electron, of horizontal incident velocity v_i, is given by equation (3.12) as $F = Bev_i$ newtons.

In this problem, the fields of Fig. 3.4 and Fig. 3.5 are applied together as illustrated in Fig. 3.6. The resultant deflection of an

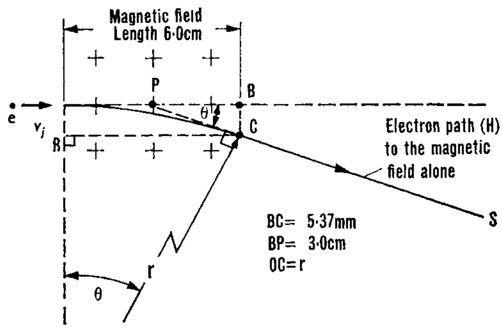


FIG. 3.5. Deflection of an electron due to a magnetic field.

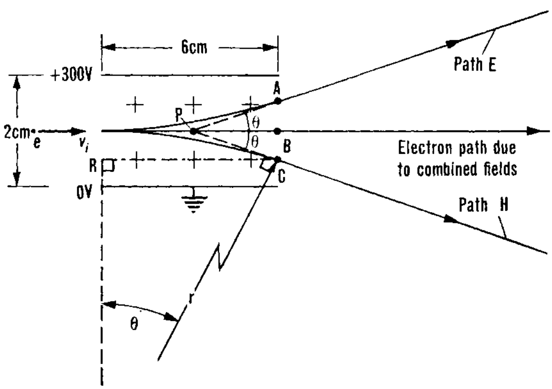


FIG. 3.6. Electron passes undeviated through the combined electromagnetic field.

electron passing through the electromagnetic field is to be zero. This end will be realized if the upwards force due to the electrostatic field is exactly counter-balanced by the downwards force due to the magnetic field.

Hence,
$$\frac{eV_d}{d} = eBv_i$$

whence,
$$v_i = V_d/Bd. \tag{3.19}$$

This means that the incident velocity of the electron into the electromagnetic field, may be determined without knowing the values of the electronic constants e and m. Thus, from equation (3.19),

$$v_i = \frac{300}{0{\cdot}5 \times 10^{-3} \times 0{\cdot}02} \text{ m/s}$$

$$= 3 \times 10^7 \text{ m/s}. \tag{3.20}$$

(*b*) *To find the ratio e/m.* Reference to Fig. 3.6 shows that the deflection due to the magnetic field only is 0·00537 m if the electron is to be undeflected when passing through the electromagnetic field.

It is assumed here that the axial path is short, so that when the path of the electron is projected backwards it passes through point P at the centre of the magnetic field. The straight line PCS is coincident with the circumference of the circle, radius r, at point C, therefore $\angle PCO$ is a right angle, and from geometry of the figure, $\angle COR = \angle\theta$.

Now,
$$\tan\theta = \frac{BC}{BP} = \frac{0{\cdot}00537}{0{\cdot}03} = 0{\cdot}179$$

hence,
$$\angle\theta = 10{\cdot}15°.$$

From equation (3.15) we have,

$$r = \frac{l}{\sin\theta}$$

$$= \frac{0{\cdot}06}{\sin 10{\cdot}15°} = 0{\cdot}34 \text{ m}. \tag{3.21}$$

But equation (3.14) gives the circle radius as

$$r = \frac{mv_t}{Be}$$

or

$$\frac{e}{m} = \frac{v_t}{Br}. \tag{3.22}$$

e/m is found by substituting v_t from (3.20) and r from (3.21) into (3.22) thus,

$$\frac{e}{m} = \frac{3 \times 10^7}{0{\cdot}5 \times 10^{-3} \times 0{\cdot}34}\ \text{C/kg}$$

$$= 1{\cdot}76 \times 10^{11}\ \text{C/kg}.$$

PROBLEMS WITH ANSWERS

1. What do you understand by the terms (a) space charge limited and (b) temperature limited operation of a diode? Give one application of each mode of operation.

The p.d. between the cathode and the anode of a diode is 20 V, and the anode current flowing is 2 mA. Find the number of electrons flowing across the valve in 1 sec and also the velocity with which they arrive at the anode.

Assume the mass of an electron is 9×10^{-31}kg, and the charge on an electron is $1{\cdot}6 \times 10^{-19}$ C.

$1{\cdot}25 \times 10^{16}$; $2{\cdot}66 \times 10^6$ m/s. I.E.R.E., Nov. 1961

2. Two parallel plates are situated 4 cm apart in a vacuum, and the potential difference between them is 200 V.

Calculate the force on an electron situated between the plates. Find also the time taken for an electron leaving the negative plate with zero velocity to reach the positive one, and the velocity with which it arrives. Assume that the charge on an electron is $1{\cdot}6 \times 10^{-19}$ C, and its mass 9×10^{-31} kg.

8×10^{-16} N; $9{\cdot}5 \times 10^{-9}$ s; $8{\cdot}4 \times 10^6$ m/s. I.E.R.E., Nov. 1962

3. The anode and cathode of a diode may be regarded as parallel plates 4 mm apart, the p.d. between them is 20 V, and the current flowing is 10 mA.

If the charge on an electron is $1{\cdot}6 \times 10^{-19}$ C, and its mass is 9×10^{-31} kg, find

(a) the number of electrons arriving at the anode per second;
(b) the velocity with which they arrive;
(c) the force acting on an electron between cathode and anode;
(d) the transit time from cathode to anode.

State any assumptions you make.

6×10^{16}; $2{\cdot}67 \times 10^6$ m/s; 8×10^{-16} N; 3×10^{-9}s.

I.E.R.E., May 1963

4. The deflecting plates of a cathode-ray tube are 2 cm long and 0·5 cm apart. The final anode voltage is 2000 V. Calculate
(a) the time taken for the electron to pass the plates;
(b) the angular deflection of the beam if a p.d. of 150 V is applied between the deflecting plates;
(c) the value of the deflecting voltage to give maximum beam deflection.

$$e = 1{\cdot}6 \times 10^{-19}\ \text{C}$$
$$m = 9{\cdot}11 \times 10^{-13}\ \text{kg}$$

$7{\cdot}55 \times 10^{-10}$ s; 8·55°; 250 V. H.N.C.

5. A cathode-ray tube has parallel deflecting plates 6 cm long and 2 cm apart. The screen is 20 cm from the centre of the deflecting plates. The p.d. between the final anode and cathode is 1·5 kV.

Calculate the length of the line which would be traced on the screen if a sinusoidal alternating voltage of 50 V is applied across the deflector plates.

Derive any formulae used.

2·83 cm. H.N.C.

6. In a cathode-ray tube a beam of electrons is accelerated through a potential difference V_f before passing between a pair of parallel deflecting plates. The distance between the plates is S, and the effective axial length of the plates is l. The centre of the deflecting plates is distance L from the screen. Derive an expression for the deflection of the spot from its mid-position on the screen when a p.d. of V_d is maintained between the plates.

If $l = 3$ cm, $S = 0{\cdot}5$ cm, $L = 20$ cm, $V_f = 3$ kV, calculate
(a) the transit time for an electron between the deflecting plates;
(b) the maximum value of V_d before beam cut-off occurs. $e/m = 1{\cdot}76 \times 10^{11}$ C/kg.

$0{\cdot}92 \times 10^{-9}$ s; 167 V. H.N.C.

7. An electron of charge q coulombs and mass m kilograms is projected with uniform velocity v metres per second into a region in which there is a uniform magnetic field of flux density B webers per square metre acting at right angles to the motion of the electron. Show that the electron will describe an arc of a circle, and derive an expression for the radius of this circle. Describe one practical application of this motion of a charged particle in a magnetic field.

$r = mv/Bq$. I.E.R.E., Nov. 1963

8. The anode potential of a cathode-ray tube is 14·4 kV and the effective length of the scanning coils is 5 cm. What magnetic field strength is required to deflect the beam 17·5°? Derive all formulae used. ($e/m = 1{\cdot}8 \times 10^{11}$C/kg).

$2{\cdot}4 \times 10^{-3}$ Wb/m². I.E.R.E., Nov. 1956

9. An electron beam, moving with velocity v in a cathode-ray tube, may be deflected by either an electric or a magnetic field.

Show that the path of an electron beam in a uniform electric field is parabolic, whilst the path of the beam in a uniform magnetic field is circular. Fringing effects may be ignored.

Calculate the radius of curvature for an electron beam moving with a velocity of 20×10^6 m/s on entering a uniform magnetic field of flux density $0{\cdot}2 \times 10^{-3}$ Wb/m^2.

$e/m = 1{\cdot}77 \times 10^{11}$ C/kg; $r = 0{\cdot}565$ m. H.N.C.

10. An electron of mass m and charge e is projected with a velocity v in the X direction at right angles to a uniform electric field of intensity E directed in the Y direction. Taking the origin of coordinates at the point where the electron enters the field, find the equation of motion of the particle, and show how to find its velocity at any point on the path.

A cathode-ray tube has a final anode voltage of 1000 V, and the distance from the centre of the Y deflector plates to the screen is 0·25 m. If the deflector plates are parallel, 0·04m long, and 0·006m apart, calculate

(a) the deflection sensitivity of the tube;

(b) the maximum usable height of the screen.

(e/m for an electron is $1{\cdot}76 \times 10^{11}$ C/kg.)

0·833 mm/V; 3·75 cm. I.E.R.E., Nov. 1956

CHAPTER 4

Valve and Transistor Characteristics

WORKED EXAMPLES

Example 1

The characteristics of a certain triode are given by the expression

$$I_A = 0{\cdot}00384(V_A + 42{\cdot}5\, V_G)^{3/2}\ \text{mA}. \tag{4.1}$$

Determine the values of the valve parameters r_a, μ and g_m, for the condition $V_A = 300$ V, $V_G = -4$ V.

Solution

Anode Slope Resistance r_a. Transposing equation (4.1) in terms of V_A gives

$$V_A = \left[\frac{I_A}{0{\cdot}00384 \times 10^{-3}}\right]^{2/3} - 42{\cdot}5\, V_G \tag{4.2}$$

r_a may be found by partial differentiation of equation (4.2) thus,

$$r_a = \frac{\partial V_A}{\partial I_A} = \frac{10^6}{5{\cdot}76\sqrt{\{(V_A + 42{\cdot}5\, V_G)\}}}\ \Omega. \tag{4.3}$$

When $V_A = 300$ V, and $V_G = -4$ V,

$$r_a = \frac{10^6}{5{\cdot}76\sqrt{\{(300-170)\}}}\ \Omega$$

$$= 15{\cdot}2\ \text{k}\Omega.$$

Amplification Factor μ. This may be found by partial differentiation of equation (4.2).

Whence, $$\mu = \frac{\partial V_A}{\partial V_G} = -42{\cdot}5.$$

Mutual Conductance g_m. This may be found by partial differentiation of equation (4.1).
Whence,

$$g_m = \frac{\partial I_A}{\partial V_G} = \tfrac{3}{2}\times 3{\cdot}84\times 10^{-6}(V_A+42{\cdot}5\ V_G)^{\frac{1}{2}}\times 42{\cdot}5\ \text{A/V} \quad (4.4)$$

when $V_A = 300\ \text{V}$ and $V_G = -4\ \text{V}$,

$$g_m = 5{\cdot}76\times 10^{-6}\times 42{\cdot}5\sqrt{130}\ \text{A/V}$$
$$= 2{\cdot}8\ \text{mA/V}.$$

Example 2

TABLE 1

	I_A in mA			
V_G	$V_A = 150$ V	$V_A = 200$ V	$V_A = 250$ V	$V_A = 300$ V
0	18	26	34	43
−1 V	8	15	22	32
−2 V	2	6	12	20
−3 V	0	2	5	11

Table 1 shows the static characteristics of a small triode. Draw the I_A/V_A characteristics and the I_A/V_G characteristics. The anode is connected through a 10 kΩ resistor to a +350 V supply.

(a) Find the anode current when the grid is 1·5 V negative with respect to cathode.

(b) If the bias is obtained by a 150 Ω cathode resistor find the current through the valve and the p.d. across it when the grid is connected directly to the negative side of the H.T. supply.

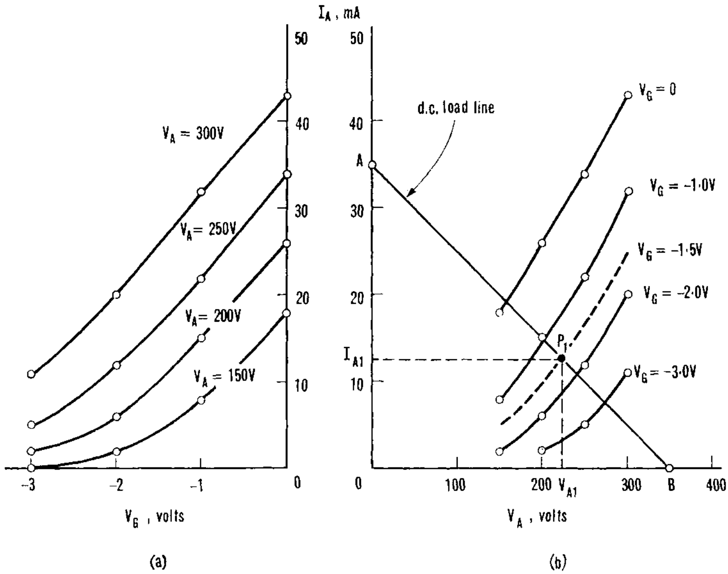


FIG. 4.1(a). Static Mutual Characteristics of a small triode plotted from the readings given in Table 1.

FIG. 4.1(b). Static Anode Characteristics of a small triode plotted from the readings given in Table 1. The intersection of the $V_G = -1{\cdot}5$ V characteristic and the 10 kΩ d.c. load line gives the operating point P_1.

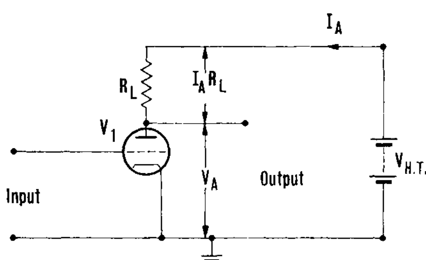


FIG. 4.2. Basic triode amplifier with resistive anode load and earthed cathode.

Solution

The I_A/V_G characteristics are shown in Fig. 4.1(a) and the I_A/V_A characteristics in Fig. 4.1(b).

(a) Applying Kirchhoff's Second Law around the I_A loop of Fig. 4.2 (under d.c. conditions),

$$V_{\text{H.T.}} = I_A R_L + V_A. \tag{4.5}$$

Assume that V_A can have a value of 0 V. Under these conditions, from equation (4.1),

$$I_A = \frac{V_{\text{H.T.}}}{R_L} = \frac{350}{10\times 10^3} = 35 \text{ mA}. \tag{4.6}$$

Now, let the valve be assumed cut-off. Under these conditions, from equation (4.5) with $I_A = 0$ mA,

$$V_A = V_{\text{H.T.}} = 350 \text{ V}. \tag{4.7}$$

Points A and B of Fig. 4.1(b) are given by equation (4.6) and equation (4.7), respectively, and the straight line joining points A and B is called the d.c. load line. This load line shows how anode voltage varies with anode current when a 10 kΩ resistor is used as an anode load (see Fig. 4.2). Various steady values of anode current may be achieved by applying discrete values of negative voltage between grid and cathode of the triode. Hence, in order to determine the anode current when the grid is −1·5 V with respect to cathode, it is merely necessary to note the point of intersection of the −1·5 V characteristic with the 10 kΩ d.c. load line (P_1 of Fig. 4.1(b)), and then to read off the corresponding value of anode current from the vertical scale.

In this case, the −1·5 V characteristic is obtained by interpolation (shown by the dotted line in Fig. 4.1(b)). The steady value of anode current I_{A1} in the absence of an alternating input to the valve, is given by $I_{A1} = 12{\cdot}5$ mA. The anode voltage V_{A1} associated with this value of anode current is $V_{A1} \simeq 223$ V.

Note

1. The d.c. load line of Fig. 4.1(b) may be used to obtain the voltage gain to the anode of the triode of Fig. 4.2 when a low-frequency alternating input is applied between grid and cathode of the valve. For instance, assuming a working point of P_1 in Fig. 4.1(b), and a low-frequency sinusoidal voltage of 1 V peak to peak to be applied to the grid of V_1, a peak to peak variation in anode voltage of approximately 40 V is produced. Hence, the voltage gain of the circuit of Fig. 4.2 is 40.

2. If the output from V_1 of Fig. 4.2 is coupled to a following stage, the coupling components modify the effective anode load of V_1 when a.c. is applied. If the load is purely resistive at the frequency of the input signal, an a.c. load line must be drawn on the valve characteristics. Under the conditions stipulated, i.e. fixed frequency and resistive load, the a.c. load line is a straight line passing through the d.c. working point and having a slope given by

$$\text{Slope of a.c. load line} = -\frac{1}{\text{a.c. load resistance}}.$$

An example of a.c. load line construction is given in Example 4 of Chapter 5.

(b) Applying Kirchhoff's Second Law around the I_A loop of Fig. 4.3 (again considering d.c. operation only) gives

$$V_{\text{H.T.}} = I_A R_L + V_A + I_A R_K$$

hence,
$$V_{\text{H.T.}} = I_A(R_L + R_K) + V_A. \tag{4.8}$$

Equation (4.8) may be used to plot a d.c. load line for the modified circuit of Fig. 4.3. Points A and B are obtained as in part (a) of the solution, but this time,

when
$$V_A = 0, \quad I_A = \frac{V_{\text{H.T.}}}{R_L + R_K} = 34{\cdot}5\ \text{mA}$$

and point A is defined by

$$V_A = 0, \quad I_A = 34{\cdot}5\ \text{mA}.$$

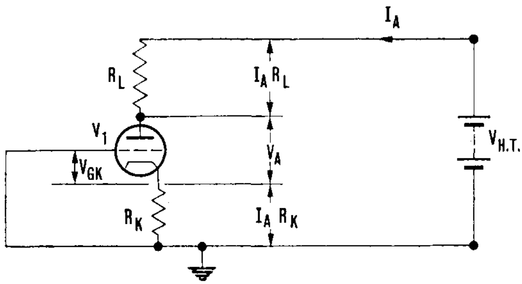


FIG. 4.3. Basic triode amplifier with resistive anode and cathode bias resistor.

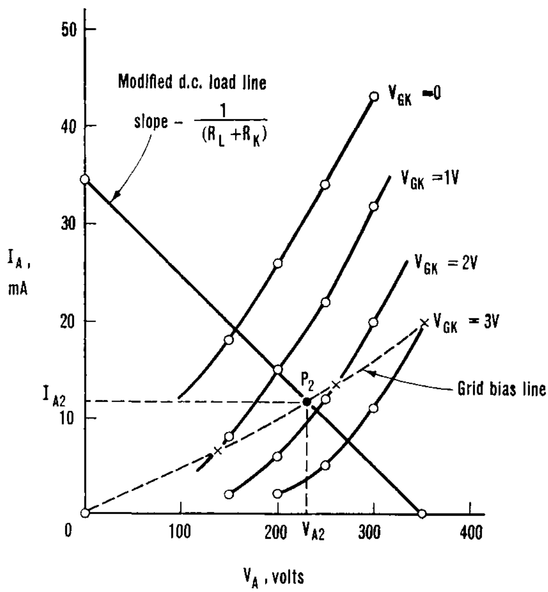


FIG. 4.4. Static Anode Characteristics from Table 1 showing the modified d.c. load line of the circuit of Fig. 4.3 and the grid bias line. The intersection of the modified load line and the grid bias line gives the new operating point P_2.

When $I_A = 0$, equation (4.8) gives

$$V_A = V_{\mathrm{H.T.}} = 350\ \mathrm{V}$$

as before, and point B is defined by

$$V_A = 350\ \mathrm{V}, \quad I_A = 0\ \mathrm{mA}.$$

The straight line joining points A and B in Fig. 4.4 is the modified d.c. load line, and has a slope of $-1/(R_L+R_K)$. If R_K is much smaller than R_L, the slope of the modified d.c. load line is almost the same as that of the original load line of Fig. 4.1(b).

Grid Bias Line

Referring to Fig. 4.3, it may be seen that, as the grid is at earth potential, the bias voltage is provided by the effect of the anode current passing through R_K, hence, the effective negative grid bias V_{GK} is given by

$$V_{GK} = I_A R_K. \tag{4.9}$$

In this case, the grid bias line is plotted by assuming values of grid bias corresponding to the tabulated values of grid voltage, namely, $V_{GK} = 0$ V, 1 V, 2 V, 3 V. As $R_K = 150\ \Omega$, Table 2 may be constructed using equation (4.9) and the indicated results used to plot the *grid bias line* as shown in Fig. 4.4.

The intersection of the grid bias line and the modified d.c. load line gives the new operating point P_2.

TABLE 2

I_A in mA	0	6·67	13·33	20·0
V_{GK} in volts	0	1	2	3

From Fig. 4.4, the new operating point can be seen to be

$$V_{A2} = 230\ \mathrm{V}, \quad I_{A2} = 11{\cdot}5\ \mathrm{mA}.$$

Substituting the above value of anode current and $R_K = 150\ \Omega$

into equation (4.9) gives the grid bias voltage under d.c. conditions.

Hence,

$$V_{GK2} = 11{\cdot}5\times10^{-3}\times0{\cdot}15\times10^{3} = 1{\cdot}725 \text{ V (negative)}.$$

Example 3

Show that the collector current I_C for a transistor in the common-emitter configuration is given by

$$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CO},$$

where α is the common-base current amplification factor, I_B is the base current, and I_{CO} is the collector-base reverse leakage current. Comment on the significance of the term $I_{CO}/(1-\alpha)$.

Solution

When suitable d.c. supply voltages are applied to a *p–n–p* common-base transistor as shown in Fig. 4.5, the resulting current distribution is such that $I_E = I_B + I_C$.

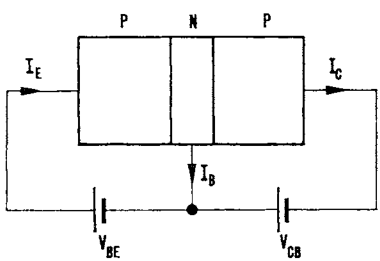


FIG. 4.5. Block diagram of a *p–n–p* transistor connected in the common-base configuration showing biasing arrangement and current distribution.

The emitter current which flows across the forward biased emitter-base junction is in the form of holes. These holes enter the relatively thin base region and find a favourable potential gradient in existence across the reverse biased base-collector junction. A large proportion (α) of holes entering the base region drifts into the collector, and one might expect the collector current to have a value given by $I_C = \alpha I_E$. However, this is not the case, since, in addition to the holes which enter the collector from the emitter, a small amount of additional current (leakage current I_{CO}) flows in the collector due to surface leakage and thermally generated minority holes. The expression for collector current becomes

$$I_C = \alpha I_E + I_{CO}. \tag{4.10}$$

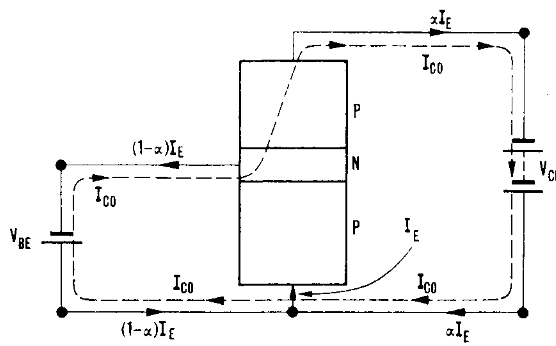


FIG. 4.6. Block diagram of a *p–n–p* transistor connected in the common-emitter configuration showing biasing arrangement and current distribution.

Not all of the holes which enter the base region from the emitter find their way into the collector circuit—some are neutralized in the base by electrons from the external supply. Base current is made up of this portion of emitter current $(I_E - \alpha I_E)$ minus the leakage current component. An expression for base current would be

$$I_B = I_E(1-\alpha) - I_{CO}. \tag{4.11}$$

If the transistor illustrated in Fig. 4.5 is now arranged in the common-emitter configuration with the same relative voltages applied to the transistor junctions, there will be no change in current distribution (see Fig. 4.6).

Now, transposing equation (4.10) in terms of I_E gives

$$I_E = \frac{I_C - I_{CO}}{\alpha} \tag{4.12}$$

and substituting I_E from equation (4.12) into equation (4.11) produces

$$I_B = \frac{I_C - I_{CO}}{\alpha}(1-\alpha) - I_{CO}$$

$$= [I_C(1-\alpha) - I_{CO}] \times \frac{1}{\alpha}$$

whence
$$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CO}. \tag{4.13}$$

When I_B of equation (4.13) is zero,

$$I_C = I'_{CO} = \frac{I_{CO}}{1-\alpha}. \tag{4.14}$$

Equation (4.14) gives the leakage current of the transistor when it is connected in the common-emitter configuration. This leakage current is given the symbol I'_{CO}. When α—the common-base current amplification factor—is, say, 0·98, I'_{CO} is found from equation (4.14) to be

$$I'_{CO} = \frac{I_{CO}}{1-0{\cdot}98} = 50\, I_{CO}.$$

This means that a transistor with a leakage current in common-base of 5 μA, would have a corresponding leakage current in common-emitter of 250 μA. This high value of I'_{CO} may lead to an effect known as thermal runaway.

Example 4

Table 3 gives the collector current in a *p–n–p* transistor for the different values of collector-emitter voltage and of base current.

Plot the output characteristics of the transistor when it is connected in the common-emitter configuration. The quiescent base current is 80 μA when the collector voltage is 4 V.

If the input signal to the base has a peak value of 60 μA and the collector load resistance is 1000 Ω, determine,

(a) the power dissipated at the collector under static conditions;

(b) the a.c. power output and current gain for the dynamic condition.

TABLE 3

V_{CE} (volts)	20	40	60	80	100	120	140	I_B (μA)
−1	0·9	1·8	2·8	3·8	4·8	5·8	6·8	
−2	1·0	2·0	3·0	4·0	5·0	6·0	7·1	
−4	1·1	2·2	3·3	4·3	5·4	6·5	7·7	I_C (mA)
−6	1·25	2·5	3·6	4·6	5·8	7·0	8·3	
−8	1·4	2·8	3·9	5·0	6·2	7·5	8·9	

Solution

The output characteristics for the transistor under consideration are shown plotted in Fig. 4.7.

Point Q of Fig. 4.7 is the operating point given in the problem, and is defined by $V_{CE} = -4$ V, $I_B = 80\ \mu$A.

After marking in point Q, it is necessary to obtain the d.c. load line. The collector load is purely resistive, thus the d.c. load line will be a straight line having a slope of $-1/R_L$ (1 mA/V) drawn through the operating point Q.

(*a*) *Power dissipated at the collector under static conditions.*

Collector dissipation (P_C) = Steady collector current (I_C) × Steady collector voltage (V_{CE}).

when $V_{CE} = -4$ V, $I_C = 4{\cdot}3$ mA.

Therefore

$$P_C = 4 \times 4{\cdot}3 \times 10^{-3} = 0{\cdot}0172 \text{ W} \quad \text{or} \quad 17{\cdot}2 \text{ mW}.$$

(*b*) (*i*) *Current gain for the dynamic condition*

$$\text{Current gain} = \frac{\text{Change in collector current } (\Delta I_C)}{\text{Change in input current } (\Delta I_B)}.$$

From Fig. 4.7, the maximum change in collector current (ΔI_C) due to a change in input current (ΔI_B) of 120 μA (peak to peak value of sinusoidal input current) is seen to be 5·56 mA.

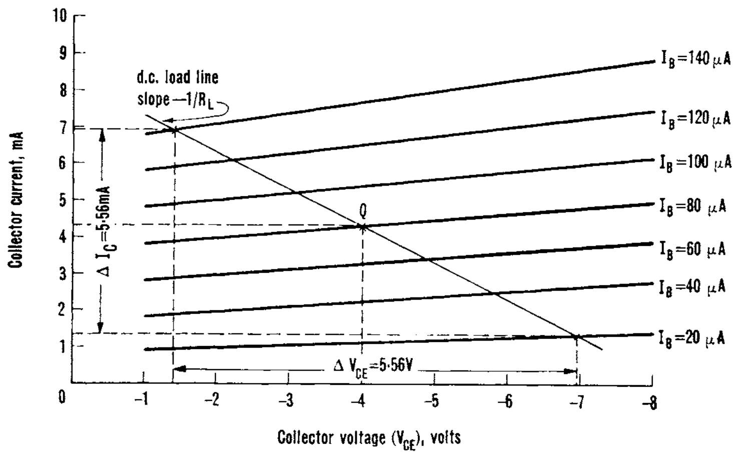


FIG. 4.7. Output characteristics of a common-emitter transistor obtained from the readings given in Table 3 showing the 1 kΩ load line and the operating point Q.

Therefore,

$$\text{current gain} = \frac{\Delta I_C}{\Delta I_B} = \frac{5{\cdot}56 \times 10^{-3}}{120 \times 10^{-6}} = 46{\cdot}4.$$

(*b*) (*ii*) *A.C. power output for the dynamic condition*

a.c. power output ($P_{a.c.}$) = r.m.s. output voltage (V_o)
× r.m.s. output current (I_o)

and $$I_o = \frac{\Delta I_C}{2\sqrt{2}} = \frac{5{\cdot}56 \times 10^{-3}}{2\sqrt{2}}\ \text{A}$$

also $$V_o = \frac{\Delta V_{CE}}{2\sqrt{2}} = \frac{5{\cdot}56}{2\sqrt{2}}\,\text{V}.$$

Hence, $$P_{\text{a.c.}} = I_o V_o = \frac{\Delta I_C \times \Delta V_{CE}}{8}$$
$$= \frac{5{\cdot}56 \times 10^{-3} \times 5{\cdot}56}{8}\,\text{W}$$
$$= 3{\cdot}86 \times 10^{-3}\text{W} \quad \text{or} \quad 3{\cdot}86\,\text{mW}.$$

Example 5

TABLE 4

$I_B = 100\ \mu$A

I_C in mA	1·0	1·05	1·1	1·15	1·2
V_C in volts	0	−10	−20	−30	−40

$I_B = 200\ \mu$A

I_C in mA	3·0	3·05	3·15	3·25	3·35
V_C in volts	0	−10	−20	−30	−40

$I_B = 300\ \mu$A

I_C in mA	4·65	4·8	4·95	5·1	5·25
V_C in volts	0	−10	−20	−30	−40

Using the figures given in Table 4, plot the output characteristics of a common-emitter transistor. If the operating point is defined by $I_B = 200\ \mu$A, $V_{CE} = -20$ V, determine the voltage gain, current gain and power gain of the stage when a sinusoidal input voltage of r.m.s. value 0·212 V is applied to the transistor across an input resistance of 3 kΩ. The collector load resistance is 10 kΩ.

Solution

The alternating input voltage is applied across the input circuit of the transistor as shown in Fig. 4.8.

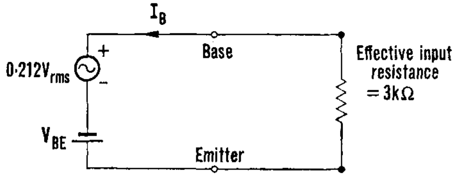


FIG. 4.8

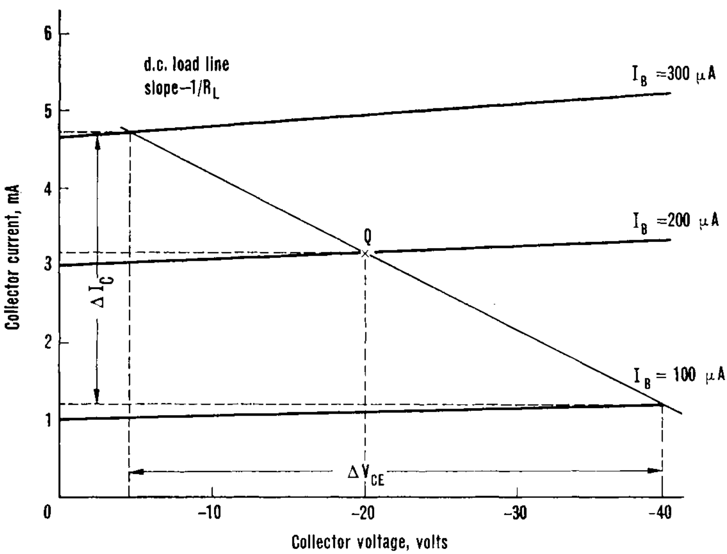


FIG. 4.9. Output characteristics of a common-emitter transistor plotted from the readings given in Table 4 showing the 10 kΩ load line and the operating point Q.

The peak to peak variation of input voltage (ΔV_{in}) about the bias voltage is clearly $2\sqrt{2} \times 0{\cdot}212$ V or 0·6 V. Thus the change in input current (ΔI_B) due to this 0·6 V input variation must be

$$\Delta I_B = \frac{0{\cdot}6}{3 \times 10^3} = 200\,\mu\text{A}.$$

The resulting changes in collector current and collector voltage are ΔI_C and ΔV_{CE} respectively (see Fig. 4.9).

(*a*) *Voltage gain*

$$\text{Voltage gain} = \frac{\Delta V_{CE}}{\Delta V_{\text{in}}}.$$

Now $\Delta V_{\text{in}} = 0{\cdot}6\ \text{V}$

and $\Delta V_{CE} = 35\ \text{V}$ (from Fig. 4.9).

Hence,
$$\text{voltage gain} = \frac{35}{0{\cdot}6} = 58{\cdot}4. \tag{4.15}$$

(*b*) *Current gain*

$$\text{Current gain} = \frac{\Delta I_C}{\Delta I_B}.$$

Now, $\Delta I_B = 200\ \mu\text{A}$

and $\Delta I_C = 3{\cdot}5\ \text{mA}$ (from Fig. 4.9).

Hence,
$$\text{current gain} = \frac{3{\cdot}5 \times 10^{-3}}{200 \times 10^{-6}} = 17{\cdot}5. \tag{4.16}$$

(*c*) *Power gain.* Power gain is obtained by multiplying voltage gain from expression (4.15) by current gain from expression (4.16).

Thus,
$$\begin{aligned}\text{power gain} &= 58{\cdot}4 \times 17{\cdot}5\\ &= 1020.\end{aligned}$$

Problems with Answers

1. If the characteristic curves of a certain triode are given by the expression $I_A = 0{\cdot}0125\,(V_A + 21\,V_G)^{3/2}$ mA, determine the valve parameters for the condition $V_A = 100$ V, $V_G = -2$ V.

-21, $7\ \text{k}\Omega$, $3\ \text{mA/V}$.

2. Plot the I_A/V_G curves for the triode whose characteristics are given in Table 5, and from these obtain values for

(a) the anode slope resistance of the valve,
(b) its amplification factor,
and (c) its mutal conductance.

Assume that the anode voltage is 100 V, and the grid bias voltage is -1 V.

16·7 kΩ; 52·6; 3·15 mA/V.

TABLE 5

V_A = 150 V						
V_G (volts)	−0·5	−1·0	−1·5	−2·0	−2·5	
I_A (mA)	8·4	6·7	5·1	3·4	1·8	
V_A = 100 V						
V_G (volts)	0	−0·5	−1·0	−1·5	−2·0	−2·5
I_A (mA)	6·9	5·3	3·7	2·3	1·1	0

3. Plot the anode current–anode voltage curves for the triode whose characteristics are shown in Table 6. The H.T. supply to the valve is 250 V and the anode load resistance 12·5 kΩ. Determine (a) the quiescent values of anode voltage and current for a steady bias voltage of -3 V, and (b) the gain to the anode of this valve when a sine-wave voltage of amplitude 1 V is superimposed on the assumed bias level.

132 V, 9·3 mA; 11·5.

TABLE 6

V_G = 0						
V_A (volts)	0	25	50	75	100	125
I_A (mA)	0	1·0	3·4	7·8	11·8	16·8
V_G = −2 V						
V_A (volts)	50	75	100	125	150	
I_A (mA)	0·2	2·4	6·0	10·8	15·6	
V_G = −4 V						
V_A (volts)	75	100	125	150	175	200
I_A (mA)	0	1·4	4·7	9·2	13·6	18·2
V_G = −6 V						
V_A (volts)	125	150	175	200	225	
I_A (mA)	0·6	3·2	7·5	11·5	16·0	

4. Plot the I_A/V_A curves of the triode whose characteristics are shown in Table 7. Determine the voltage gain of the valve to its anode when a 25 kΩ resistor is connected between anode and the 300 V H.T. line. Assume a standing bias of −3 V and a sine wave input of 3 V amplitude. What is the a.c. power output of the stage if the amplitude distortion introduced by the triode can be ignored?

25; 112·5 mW.

TABLE 7

$V_G = 0$				
V_A (volts)	50	100	150	200
I_A (mA)	2·5	5·9	9·3	13
$V_G = -3$ V				
V_A (volts)	150	200	250	300
I_A (mA)	0·9	4·2	7·7	11·2
$V_G = -6$ V				
V_A (volts)	250	300	350	400
I_A (mA)	0	2·7	6·0	9·6

5. Table 1 shows the static characteristics of a small triode. Draw the I_A/V_A characteristics and the I_A/V_G characteristics. The anode is connected through an 8 kΩ resistor to a +350 V supply.

(a) Find the anode current when the grid is 0·5 V negative to cathode.
(b) If the bias is obtained by a 120 Ω cathode resistor, find the current through the valve, and the p.d. across it when the grid is connected directly to the negative side of the H.T. supply.

19 mA; 14 mA, 240 V. I.E.R.E., May 1962

6. A valve has the parameters of $r_a = 6$ kΩ and $g_m = 5$ mA/V. Draw accurately the anode voltage–anode current characteristics (assuming them to be linear over the relevant regions) for anode voltages of up to 400 V and for grid voltages of 0, −2 V, −4 V, −6 V and −8 V.

Such an valve has a 15 kΩ resistor connected between its anode and the H.T. supply of +320 V.

(a) Determine the operating point if the grid is maintained at −5·5 V with respect to the cathode.
(b) A 200 Ω resistor is connected between the cathode of the valve and negative rail, and the grid is connected directly to the negative rail. Find the new operating point.

7·5 mA, 210 V; 12 mA, 145 V. I.E.R.E., Nov. 1963

7. A triode has characteristic curves which may be assumed linear, and described by $g_m = 4$ mA/V and $r_a = 5$ kΩ. Draw its anode characteristic for anode voltages up to 300 V and for grid voltages of 0, -3, -6, -9 and -12 V. A resistor of 7·5 kΩ is connected between its anode and the positive of a 300 V d.c. supply, the cathode of the valve being taken to the negative of this supply. The grid has a negative bias of 6 V, in series with which is injected a sinusoidal input of 6 V peak value. Accurately plot the anode voltage and anode current waveforms over a complete cycle of the input showing, wherever convenient, the actual numerical values.

120 V to 264 V, 24 to 5 mA. I.E.R.E., May 1963

8. The equation

$$I = I_0\left(\exp\frac{11600E}{T} - 1\right),$$

where T is the absolute temperature in degrees Kelvin, gives the current I as a function of applied voltage E for an ideal junction diode. Discuss the differences between the volt-ampere characteristic of an actual junction diode and that predicted by the equation.

Explain the mechanisms of avalanche breakdown and Zener breakdown.
I.E.R.E., Nov. 1960

9. (a) Sketch a graph of the voltage–current characteristic of a *p–n* junction diode and give an explanation of its main characteristics.

(b) Sketch the collector current–collector voltage characteristics for a junction transistor in the common-base configuration and explain why the collector current may be controlled by the emitter-base current. H.N.C.

10. Table 8 gives the collector current in a *p–n–p* transistor for the different values of collector-emitter voltage and of base current.

TABLE 8

V_{CE} (volts)	0	0·1	0·2	0·3	0·4	I_B (mA)
−0·5	0·1	3·7	7·6	11·8	14	I_C (mA)
−1·0	0·11	3·8	7·9	12·1	16	
−2·0	0·12	4·0	8·0	12·6	16·7	
−4·0	0·12	4·1	8·2	13·2	18·2	
−6·0	0·12	4·3	8·4	14·0	20	

Draw the common- (or grounded) emitter characteristic of the transistor. Its collector is connected via a 300 Ω resistor to the supply of 6 V. Draw the load line and determine the current gain of the stage.

If it is to be used as a class-A amplifier, determine a suitable value of base bias current, and find the potential difference between collector and emitter when no input signal is present.

40; 0·2 mA; −3·5 V. I.E.R.E., May 1964

11. Table 3 gives the collector current in a *p–n–p* transistor for different values of collector-emitter voltage and of base current. Plot the output characteristics of this transistor when it is connected in the common emitter configuration. If the quiescent base current is 80 μA, and the H.T. supply voltage −8 V, determine the steady values of collector current and collector voltage when the collector load resistance is 1·2 kΩ.

If the input signal to the base has a peak value of 20 μA, determine also

(a) the current gain for the dynamic condition,

(b) the a.c. power output.

4·2 mA, −3 V; 47·5; 426 μW.

12. Using the figures given in Table 4, plot the output characteristics of a common-emitter transistor. If the H.T. supply voltage is −40 V and the quiescent base current 200 μA, determine the working point of the transistor when a collector load resistance of 8 kΩ is used. If the voltage gain of the stage is 50, determine also

(a) power gain of the stage,

(b) the input resistance of the stage if the peak to peak input voltage causes a change in base current of 200 μA.

915; 2·8 kΩ.

13. A *p–n–p* transistor is used as a grounded emitter amplifier. Its h_{fe} (i.e. β) = 40, its output resistance is 10 kΩ, and the collector load is a resistor of 2 kΩ. Draw the collector current against collector voltage family, assuming the characteristics to be linear over the relevant region. Show the load line, determine a suitable operating point, and find the greatest amplitude of a sinusoidally varying current which will be amplified without appreciable distortion. What is the current gain of the stage?

$I_B = 63\ \mu$A, $I_C = 2{\cdot}8$ mA,
$V_C = 4{\cdot}3$ V; 45 μA r.m.s.; 34. I.E.R.E., May 1963

14. The stability factor S of a transistor is defined as

$$S = \frac{dI_C}{dI_{CO}}.$$

Show that if the emitter-base voltage drop is small compared with the voltage drop across R_1, the stability factor of the circuit of Fig. 4.10 is given by

$$S = \frac{1+R_1\left(\frac{1}{R_2}+\frac{1}{R_3}\right)}{1-\alpha+R_1\left(\frac{1}{R_2}+\frac{1}{R_3}\right)}.$$

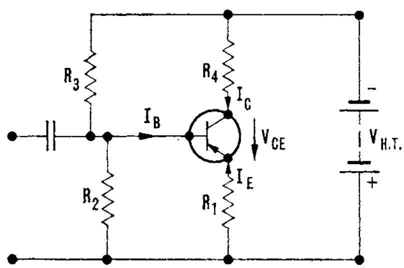


FIG. 4.10

If in this circuit $V_{H.T.} = 12$ V, $R_1 = 1$ kΩ, $\alpha = 0{\cdot}98$ and the operating conditions are to be such that $I_C = -1$ mA and $V_{CE} = 5$ V, calculate values of R_2, R_3 and R_4 so that $S = 5$.

$R_2 = 4{\cdot}85$ kΩ; $R_3 = 52{\cdot}6$ kΩ; $R_4 = 6$ kΩ. I.E.R.E., May 1964

CHAPTER 5

Low-frequency Amplification

Worked Examples

Example 1

A triode valve has parameters $r_a = 20\ \text{k}\Omega$, and $g_m = 2{\cdot}4$ mA/V. What value of anode load resistance R_L will produce a voltage gain of -25?

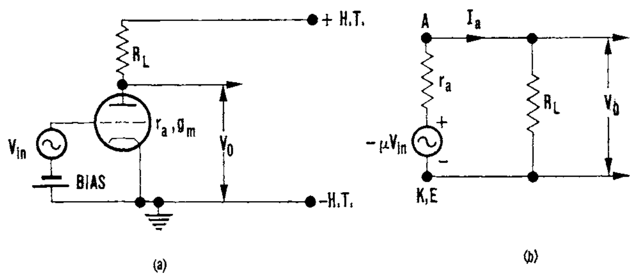


Fig. 5.1(a). Basic circuit.
Fig. 5.1(b). Constant voltage equivalent circuit.

Solution

From the equivalent circuit of Fig. 5.1(b)

$$\mathbf{I}_a = -\frac{\mu \mathbf{V}_{\text{in}}}{r_a+R_L}$$

hence,

$$\mathbf{V}_o = \mathbf{I}_a R_L = -\frac{\mu \mathbf{V}_{\text{in}} R_L}{r_a+R_L}$$

and

$$m = \frac{\mathbf{V}_o}{\mathbf{V}_{\text{in}}} = -\frac{\mu R_L}{r_a+R_L}. \qquad (5.1)$$

The minus sign signifies that 180° phase shift takes place between grid and anode of the valve.

Now, apart from R_L, the amplification factor μ is the only unknown in equation (5.1). This can be found using the relationship $\mu = g_m r_a$.

Hence, $$\mu = 2{\cdot}4 \times 10^{-3} \times 20 \times 10^3 = 48.$$

Substituting given values into equation (5.1) and transposing gives

$$-25(20 \times 10^3 + R_L) = -48R_L$$

and $$500 \times 10^3 = 23R_L,$$

hence $$R_L = 21{\cdot}74\ \text{k}\Omega.$$

Example 2

Determine the stage gain and phase shift for the triode amplifier shown in Fig. 5.2 at frequencies of (a) 31·4 c/s, (b) 300 c/s and (c) 242 kc/s. The valve parameters are $r_a = 10\ \text{k}\Omega$, $\mu = 25$ and $g_m = 2{\cdot}5$ mA/V. What is the bandwidth of the stage? The effect of the biasing components on the a.c. working of the stage may be ignored at all frequencies.

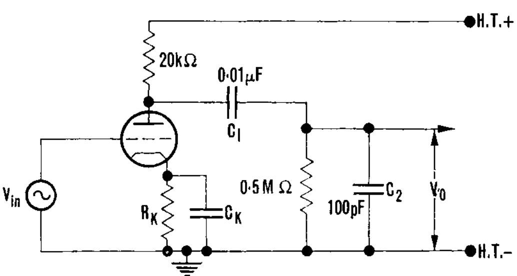


FIG. 5.2

Solution

(a) The low-frequency constant voltage equivalent circuit is shown in Fig. 5.3; however, the calculations involved may be simplified by applying Thévenin's theorem across AB (see Chapter 2). The new, simplified, equivalent circuit is shown in Fig. 5.4.

Now, $$X_{C1} = \frac{1}{\omega C_1} = \frac{1}{2\pi \times 31{\cdot}4 \times 0{\cdot}01 \times 10^{-6}}\ \Omega$$

$$= \frac{10^8}{197} = 507\ \text{k}\Omega$$

hence, $$-jX_{C1} = -j507\ \text{k}\Omega.$$

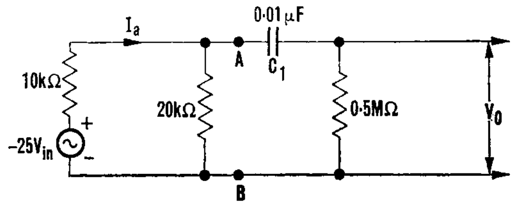


FIG. 5.3. Low-frequency constant voltage equivalent of Fig. 5.2.

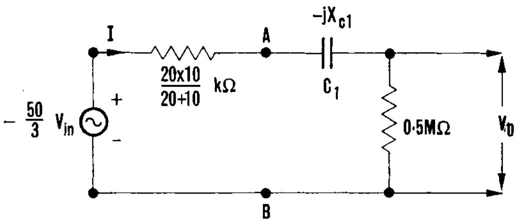


FIG. 5.4

In the following calculation, the unit of current is the milliampere, and the unit of resistance is the kilohm. It follows, therefore, that the unit of $\mathbf{V}_o$ is the volt. Referring to Fig. 5.4 we have

$$\mathbf{V}_o = \mathbf{I}R_L = -\frac{\frac{50}{3} \times 500}{507 - j507}\mathbf{V}_{in}$$

and $$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{16{\cdot}4}{j-1} \quad (5.2)$$

$$= 8{\cdot}2(-1-j) \quad (5.3)$$

From equation (5.2) it can be seen that

$$\left|\frac{V_o}{V_{in}}\right| = \frac{16{\cdot}4}{\sqrt{(1^2+1^2)}} = \frac{16{\cdot}4}{\sqrt{2}} = 11{\cdot}6$$

and from equation (5.3), the phase angle θ is given by

$$\theta = \tan^{-1}\frac{-1}{-1} = 225°$$

(b) At 300 c/s, the capacitor C_1 of Fig. 5.4 has a reactance given by

$$-jX_{C1} = -\frac{j}{\omega C} = -j\frac{10^8}{2\pi \times 300}\,\Omega$$

$$= -j\frac{10^6}{6\pi} = -j53{\cdot}0\ \mathrm{k}\Omega.$$

Hence, it can be seen that the reactive element in the circuit of Fig. 5.4, at this frequency, is very much smaller than the total series resistance and may be neglected. This assumes that the shunting effect of the 100 pF capacitance C_2 on the 500 kΩ resistor at 300 c/s is negligibly small. Thus, the imaginary term in the denominator of equation (5.2) is negligible compared with the real term, and the stage gain m has a maximum value of $-16{\cdot}4$. The minus sign shows that the alternating component of output voltage is 180° out of phase with the grid-cathode voltage.

(c) The high frequency constant voltage equivalent circuit is shown in Fig. 5.5(a), and a slightly simplified version appears in Fig. 5.5(b). The reactance of the coupling capacitor C_1 may be ignored at 242 kc/s, but the shunting effect of the 100 pF capacitor C_2 on the 500 kΩ resistor is extremely important. The calculations involved in this part of the solution may be made easier, if the equivalent circuit of Fig. 5.5(b) is further modified by applying Thévenin's theorem across CD (see Fig. 5.6).

Now, $$X_{C2} = \frac{1}{\omega C_2} = \frac{1}{2\pi \times 242 \times 10^3 \times 10^2 \times 10^{-12}}\ \Omega$$
$$= \frac{10^7}{484\pi} = 6{\cdot}57\ \text{k}\Omega$$

hence, $-jX_{C2}$ of Fig. 5.6 is $-j6{\cdot}57$ kΩ.

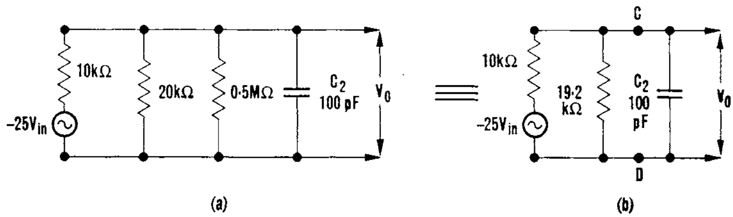


FIG. 5.5(a) and (b). High-frequency constant voltage equivalents of Fig. 5.2.

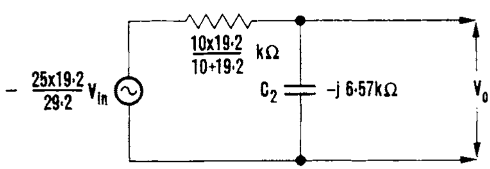


FIG. 5.6

In the following calculation, milliamperes, kilohms, and volts, are the units of current, impedance and p.d. respectively.

$$\mathbf{V}_o = +\frac{25 \times 19{\cdot}2}{29{\cdot}2} \times \frac{j6{\cdot}57\ \mathbf{V}_{in}}{\left[\dfrac{192}{29{\cdot}2} - j6{\cdot}57\right]}\ \text{V}$$

and $$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{25 \times 19{\cdot}2 \times 6{\cdot}57}{(-192 - j192)} = \frac{16{\cdot}4}{(-1-j)} \tag{5.4}$$

$$= 8{\cdot}2(j-1). \tag{5.5}$$

From equation (5.4) it can be seen that

$$\left|\frac{V_o}{V_{in}}\right| = \frac{16{\cdot}4}{\sqrt{(1^2+1^2)}} = 11{\cdot}6$$

and from equation (5.5), the phase angle θ is given by

$$\theta = \tan^{-1}\frac{+1}{-1} = 135^\circ$$

Now, it can be seen that the stage gain m falls to 0·707 of its maximum value at 31·4 c/s and 242 kc/s. Hence, the amplifier bandwidth is the difference between these two frequencies, i.e. 241·9686 kc/s. Little error is introduced by assuming that the circuit bandwidth is equal to the upper 3 dB frequency (242 kc/s.)

Example 3

The mutual conductance g_m of a pentode is 3·0 mA/V. It has an anode load resistance of 20 kΩ, and is R.C. coupled to the next stage. If the effect of the coupling capacitor can be ignored at the frequency of the input voltage $\mathbf{V}_{in}$, determine the approximate gain at mid-band frequencies if the input to the following stage consists of a 0·5 MΩ resistor in parallel with a total capacitive effect equivalent to 100 pF. Find also the upper 3 dB frequency.

Solution

Stage Gain at Mid-band Frequencies

In order to calculate a value for the stage gain of the amplifier of Fig. 5.7(a) at medium (or mid-band) frequencies, it is necessary to make two important assumptions. It is assumed that

(i) the anode slope resistance of the valve is so high that it takes negligible current from the constant-current generator;

(ii) the shunting effect of the 100 pF capacitor on the 0·5 MΩ resistor is negligible.

Using these assumptions, a simplified equivalent circuit, valid at medium frequencies, may be derived. Figure 5.8 shows the equivalent circuit from which an approximate figure for stage gain can be obtained.

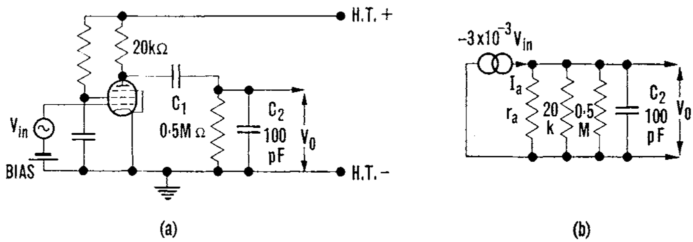


FIG. 5.7(a). Basic circuit.
FIG. 5.7(b). Constant current equivalent of Fig. 5.7(a).

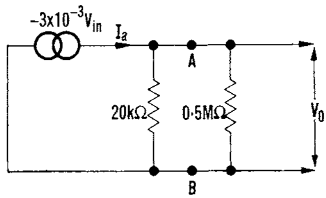


FIG. 5.8. Constant current equivalent of Fig. 5.7(a) valid at medium frequencies.

The effective resistance across AB in Fig. 5.8 is given by

$$R_{AB} = \frac{20\times 500}{20+500}\,\text{k}\Omega$$

$$= 19{\cdot}2\ \text{k}\Omega.$$

As the entire amount of generated anode current is considered to flow through R_{AB},

$$\mathbf{V}_o = \mathbf{I}_a \times R_{AB},$$

and if $\mathbf{I}_a$ is expressed in milliamperes, and R_{AB} in kilohms,

$$\mathbf{V}_o = -3 \times 19{\cdot}2\,\mathbf{V}_{in} \text{ volts}$$

and
$$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = -57{\cdot}6.$$

To Determine the Upper 3 dB Frequency

Figure 5.9 shows the approximate equivalent circuit at high frequencies. r_a is again left out, but the 100 pF capacitance is included because at high frequencies its reactance is low, and effectively reduces the anode load impedance Z_{AB}.

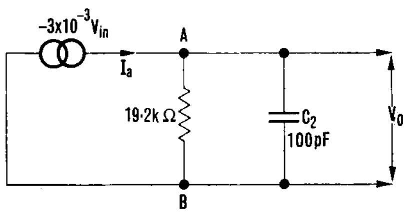


FIG. 5.9. Constant current equivalent of Fig. 5.7(a) valid at high frequencies.

Now,

$$Z_{AB} = \frac{19{\cdot}2 \times 10^3}{j\omega C \times 19{\cdot}2 \times 10^3 + 1}\ \Omega$$

and

$$\mathbf{V}_o = \mathbf{I}_a Z_{AB} = -\frac{3 \times 10^{-3} \times 19{\cdot}2 \times 10^3 \times \mathbf{V}_{in}}{1 + j2\pi f \times 19{\cdot}2 \times 10^3 \times 100 \times 10^{-12}}\ \text{V}$$

hence,

$$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = -\frac{57{\cdot}6}{1 + j120{\cdot}5 \times 10^{-7} f} \tag{5.6}$$

At the upper 3 dB frequency the voltage gain is 0·707 times its mid-band value, and the imaginary term in the denominator of equation (5.6) is equal to the real term.

Therefore $$1 = 120{\cdot}5 \times 10^{-7} \times f$$

and $$f = \frac{10^7}{120{\cdot}5} = 83{,}000 \text{ c/s}$$

$$= 83{\cdot}0 \text{ kc/s}.$$

Example 4

A triode has $r_a = 4\ \mathrm{k\Omega}$ and $g_m = 5$ mA/V, and these may be assumed constant. Draw accurately the I_A/V_A characteristics for voltages up to 500 V.

The valve is to supply a 5 Ω speaker via a transformer of turns ratio 40 : 1, the H.T. is 250 V, and the grid is given a bias of -10 V. Neglecting the resistance of the transformer windings, draw the load line for these conditions on your characteristics, and state the maximum sinusoidal voltage input which may be used for an undistorted output. I.E.R.E., Nov. 1962

Solution

Construction of $I_A - V_A$ characteristics (*Fig.* 5.11)

The anode slope resistance r_a of the valve is given by

$$r_a = \left[\frac{\delta V_A}{\delta I_A}\right] V_G = \text{constant}.$$

However, in this case, r_a and g_m are constant. Therefore, when $V_G = 0$, the anode characteristic is a straight line OA passing through the origin and having a slope $1/r_a$.

Using the relationship $\mu = g_m \times r_a$ (this can be obtained using the definitions of μ, g_m and r_a given in Example 1, Chapter 4), amplification factor μ is found to be 20. Now, if the anode current I_A of the valve is maintained constant, $\mu = \delta V_A/\delta V_G$

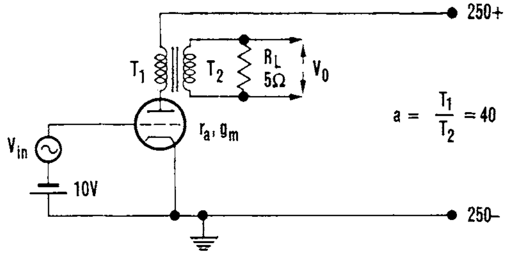


FIG. 5.10. Basic circuit.

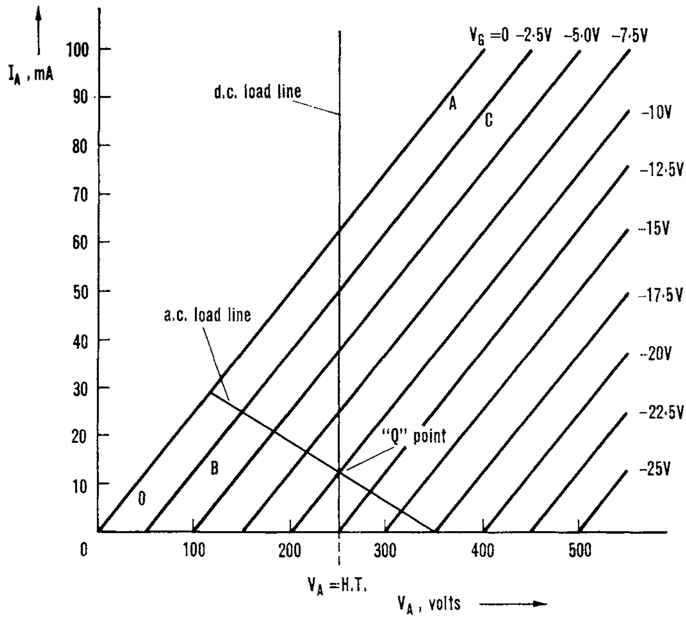


FIG. 5.11. Static anode characteristics of the triode of Fig. 5.10 with a.c. and d.c. load lines.

and $\delta V_A = \mu \delta V_G$. This means that if the anode current is reduced, due to a 2·5 V change in grid bias voltage V_G, the anode voltage V_A must be increased by $20 \times 2{\cdot}5$ V (50 V), in order to re-establish the original anode current.

Hence, when V_G is fixed at $-2{\cdot}5$ V, a straight line *BC* may be drawn to represent the variation of anode current I_A with anode voltage V_A. It is parallel with *OA*, and points of identical anode current are displaced by an anode voltage increment of 50 V, e.g. when $V_G = 0$, $V_A = 0$ and $I_A = 0$; however, when $V_G = -2{\cdot}5$ V, $V_A = 50$ V and $I_A = 0$.

A family of curves may be produced in the above manner of I_A against V_A for bias voltages of $-5{\cdot}0$ V, $-7{\cdot}5$ V, -10 V, $-12{\cdot}5$ V, -15 V, $-17{\cdot}5$ V, -20 V, $-22{\cdot}5$ V and -25 V. The resulting anode characteristics are shown in Fig. 5.11.

To Determine the Operating Point of the Valve

The valve, whose characteristics are illustrated in Fig. 5.11, is used in the basic transformer-coupled amplifier circuit of Fig. 5.10. The d.c. load line is a straight line passing through the point $V_A = V_{\text{H.T.}} = 250$ V, and has a slope of $1/R_{\text{d.c.}}$, where $R_{\text{d.c.}}$ is the d.c. load resistance of the valve. In this case $R_{\text{d.c.}}$ is zero, and a vertical d.c. load line is drawn on the anode characteristics passing through the point $V_A = 250$ V. The point of intersection of this line with the anode characteristic for $V_G = -10$ V gives the working point of the valve. This is sometimes termed the *quiescent point* and is marked *Q point* on Fig. 5.11.

A.C. Load Line

If the transformer is assumed ideal, the secondary load resistance R_L has an effective resistance R_L' when referred to the primary side of the transformer given by $R_L' = a^2 R_L$, where $a = T_1/T_2$. Here, $R_L' = 40^2 \times 5 = 8\ \text{k}\Omega$, and the load line, when a.c. is applied, is a straight line passing through the operating point having a slope $-1/R_L'$. This line is also shown in Fig. 5.11.

Distortionless Output

In order to achieve a sinusoidal output when a sinusoidal input is applied to the stage, grid current must not flow and the valve must never cease to conduct. For the given working point, the maximum sinusoidal input voltage which can produce a distortion-free output across R_L, is seen from Fig. 5.11 to be 7·5 V peak. A larger signal would cause distortion due to the valve cutting off on each negative half cycle of the sinusoidal input.

Example 5

An a.f. transformer, which may be assumed ideal, has a voltage step-up ratio of 4. Its secondary is connected to the grid of a valve which has an impedance equivalent to that of a 1 MΩ resistor in parallel with a 25 pF capacitance.

The primary winding of the transformer forms the anode load of a valve of r_a = 10 kΩ, and g_m = 5 mA/V. Find the voltage gain of the stage at medium frequencies, and also the *upper* 3 *dB* or *half-power* frequency.

Solution

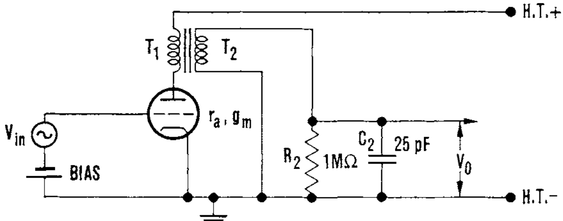


FIG. 5.12. Basic circuit.

Circuit

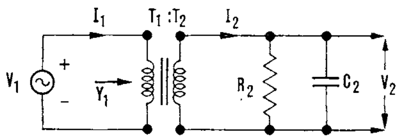


FIG. 5.13

Ideal Transformer

The circuit of an ideal transformer with a R.C. load is shown in Fig. 5.13. The transformer voltages and currents are related to the transformer turns ratio thus,

$$a = \frac{T_1}{T_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}. \tag{5.7}$$

From equation (5.7), it can be seen that,

$$V_1 = \frac{I_2 V_2}{I_1}. \tag{5.8}$$

Now, $$Y_1 = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \mathbf{I}_1 \times \frac{\mathbf{I}_1}{\mathbf{I}_2 \mathbf{V}_2}$$

$$= \frac{\mathbf{I}_1^2}{\mathbf{I}_2 \mathbf{V}_2} \times \frac{\mathbf{I}_2}{\mathbf{I}_2} \tag{5.9}$$

but $$\left[\frac{\mathbf{I}_1}{\mathbf{I}_2}\right]^2 = \frac{1}{a^2}, \quad \text{and} \quad Y_2 = \frac{\mathbf{I}_2}{\mathbf{V}_2} \tag{5.10}$$

Hence, if Y_2 consists of parallel resistive and capacitive elements, it can be seen from equations (5.9) and (5.10) that

$$Y_1 = \frac{Y_2}{a^2} = \frac{G_2}{a^2} + j\frac{B_2}{a^2} \tag{5.11}$$

however, $$Y_1 = G_1 + jB_1 \tag{5.12}$$

and a comparison of equations (5.11) and (5.12) reveals that

$$G_1 = \frac{G_2}{a^2}$$

hence $$R_1 = a^2 R_2 \tag{5.13}$$

and $$B_1 = \frac{B_2}{a^2}$$

hence $$\omega C_1 = \frac{\omega C_2}{a^2}$$

therefore, $$C_1 = \frac{C_2}{a^2} \tag{5.14}$$

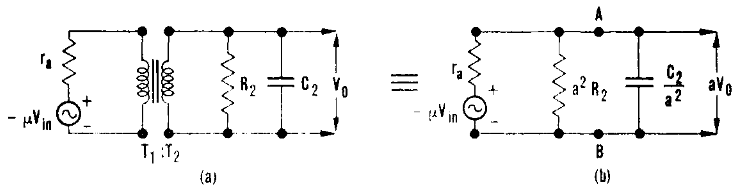


FIG. 5.14(a) and (b).

The circuit of Fig. 5.13 may be simplified by referring the secondary resistance–capacitance load to the primary side of the transformer.

Fig. 5.14(a) shows the constant voltage equivalent of the basic circuit illustrated in Fig. 5.12. Fig. 5.14(b) shows a simplified equivalent circuit with all relevant quantities referred to the primary circuit of the ideal transformer. The voltage appearing across the C.R. network of Fig. 5.14(b) is the voltage across the transformer primary. If $a = T_1/T_2$, the output voltage $\mathbf{V}_o$ is obtained by dividing $a\mathbf{V}_o$ by a. The stage gain of the amplifier shown in Fig. 5.12, at both medium and high frequencies, may be determined using Fig. 5.14(b).

Gain at Medium Frequencies

At medium frequencies, it is assumed that reactive effects are negligible, and the shunting effect of C_2 on R_2 is ignored. This means that the equivalent circuit of Fig. 5.14(b) may be further simplified thus,

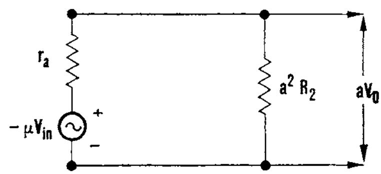


FIG. 5.15. Simplified version of Fig. 5.14(b) valid at medium frequencies.

From Fig. 5.15, $a\mathbf{V}_o$ may be written directly as

$$a\mathbf{V}_o = -\frac{a^2R_2}{r_a+a^2R_2}\times\mu\mathbf{V}_{in}$$

and
$$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \pm\frac{a\mu R_2}{r_a+a^2R_2} \tag{5.15}$$

The sign adopted when using equation (5.15) depends upon the manner in which the transformer is connected.

Now,
$$\mu = g_m r_a$$
$$= 5\times10^{-3}\times10\times10^3 = 50.$$

and
$$a = \frac{T_1}{T_2} = \frac{1}{4}$$

Substituting these, and given quantities, into equation (5.15) gives

$$m = \pm\frac{50\times10^6}{4\left(10\times10^3+\dfrac{10^6}{16}\right)}$$

$$= \pm\frac{50}{0{\cdot}29} = \pm173.$$

Gain at High Frequencies

At high frequencies, the equivalent circuit of Fig. 5.14(b) is valid if it is assumed that all circuit capacitance effects are included in the capacitance C_2/a^2. It will simplify the calculation if Thévenin's theorem is applied across AB in Fig. 5.14(b) thus,

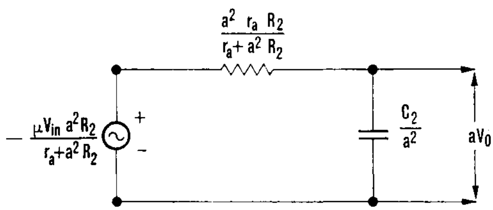


FIG. 5.16. Thévenin equivalent of Fig. 5.14(b).

It can be seen from Fig. 5.16 that

$$aV_o = -\frac{\mu a^2 R_2 V_{in}}{(r_a+a^2R_2)} \times \frac{a^2}{j\omega C_2\left(\dfrac{a^2R_2r_a}{r_a+a^2R_2}+\dfrac{a^2}{j\omega C_2}\right)}$$

$$= -\frac{\mu a^2 R_2 V_{in}}{(r_a+a^2R_2)+j\omega C_2 R_2 r_a}$$

Therefore

$$m = \frac{V_o}{V_{in}} = \pm\frac{\mu a R_2}{(r_a+a^2R_2)+j\omega C_2 R_2 r_a} \tag{5.16}$$

Equation (5.16) has a maximum value equal to the gain at medium frequencies (see equation (5.15)), and at the upper 3 dB or half-power point, the voltage gain given by this equation is 0·707 times its maximum value. The real and imaginary terms in the denominator of equation (5.16) are, therefore, equal.

Hence, $$r_a+a^2R_2 = \omega_2 C_2 R_2 r_a$$

where $\omega_2 = 2\pi f_2$, and f_2 is the high frequency at which the voltage gain is 0·707 times the maximum, mid-band, value.

Thus,
$$f_2 = \frac{r_a + a^2 R_2}{2\pi C_2 R_2 r_a} \qquad (5.17)$$

Substituting known values into equation (5.17) gives

$$f_2 = \frac{10\times 10^3 + 0{\cdot}0625\times 10^6}{2\pi\times 25\times 10^{-12}\times 10^6\times 10^4}\ \text{c/s}$$
$$= \frac{72{\cdot}5\times 10^3}{0{\cdot}5\pi}\ \text{c/s}$$
$$= 46{\cdot}2\ \text{kc/s}.$$

Example 6

Two identical power triodes, having the static characteristics shown in Table 9, are to be operated in class A push–pull. Construct and plot the composite characteristic, showing lines for a signal input voltage of ± 10 V and ± 20 V.

If the peak input signal voltage to each valve is 20 V, and the anode-to-anode load is 6000 Ω, construct the necessary loadline, and hence determine the power output and efficiency of the stage.

TABLE 9.

V_A in volts		80	120	160	200	240	280	320
Anode current in mA for grid voltages of V_G	0	64	104	144				
	−10 V	12	40	76	114			
	−20 V		10	30	60	94		
	−30 V			8	24	48	78	112
	−40 V				6	18	38	66

Quiescent point $V_A = 200$ V; $I_A = 60$ mA; $V_B = -20$ V.

Solution

Construction of the Composite Characteristics. 1. It is necessary to plot two identical sets of anode characteristic curves as shown in Fig. 5.17, using the values given in Table 9. The total base

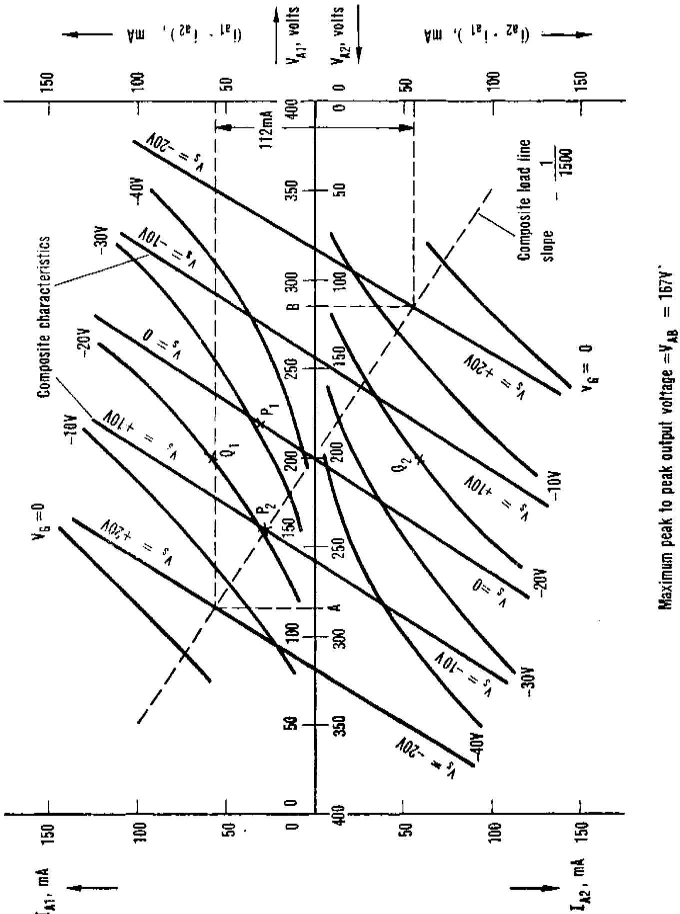


FIG. 5.17. Composite characteristics of a class A push-pull amplifier with composite load line.

line is exactly 2 $V_{H.T.}$, and the two separate scales of V_A(V_{A1} and V_{A2}) are identical at a value equal to the H.T. voltage, i.e. 200 V.

2. A single line composite characteristic may be constructed as follows:

(a) With no a.c. applied to the grids of V_1 and V_2, the grid voltage of each valve is $V_B = -20$ V. Now, assume an identical change in the magnitude of anode voltage of each valve. Let V_{A1} increase by, say, 20 V to 220 V, and let V_{A2} decrease by the same amount to 180 V.

(b) If i_{a1} is noted for the new value of V_{A1} and i_{a2} for the new value of V_{A2}, the first point of the composite characteristic may be obtained by plotting $(i_{a1} - i_{a2})$, i.e. 20 mA, vertically for $V_{A1} = 200$ V (P_1 on Fig. 5.17). Other points are obtained in the same manner. Thus, a single line characteristic may be plotted—the no-signal input line of the composite characteristic.

3. For a signal input voltage of +10 V, the instantaneous grid voltage of V_1 (v_{g1}) is -10 V when V_2 grid voltage (v_{g2}) is -30 V, and a second composite characteristic may be drawn which is the difference between the -10 V anode characteristic of V_1 and the -30 V anode characteristic of V_2. For instance, when $v_{g1} = -10$ V and $V_{A1} = 160$ V, $i_{a1} = 76$ mA, and when $v_{g2} = -30$ V and $V_{A2} = 240$ V, $i_{a2} = 48$ mA. $(i_{a1} - i_{a2})$, i.e. 28 mA, is plotted vertically for $V_{A1} = 160$ V (P_2 of Fig. 5.17). Similarly, characteristics may be constructed for the remaining values of signal input voltage. In all, there are five composite curves and these are shown in Fig. 5.17. It can be seen that the individual anode characteristics of V_1 and V_2 are curved, yet the composite characteristics are almost linear.

To Determine the a.c. Power Output

Before the output power can be ascertained, it is necessary to add a composite load line to the composite characteristics already obtained. The load line is a straight line which passes through the point $V_A = V_{H.T.}$, and has a slope which is numerically equal to the reciprocal of the composite resistance. Now, the composite resistance is 6000/4 = 1500 Ω, and the composite load line shown in Fig. 5.17 has a slope of $-1/1500$. If a single sine wave of amplitude 20 V is applied to the push–pull stage, a total change in current of 112 mA occurs and the corresponding change in anode voltage is 167 V. Both these figures are obtainable directly from Fig. 5.17. It was shown in Section 1.17 that

$$P_o = \frac{\hat{\hat{V}}_o \times \hat{\hat{I}}_o}{8}$$

Hence, using the values obtained from Fig. 5.17, we have

$$P_o = \frac{167 \times 112 \times 10^{-3}}{8}\ \mathrm{W}$$

$$= 2{\cdot}34\ \mathrm{W}.$$

Problems with Answers

1. Give two forms of the equivalent circuit representation of a valve. What are the limitations of these representations?

A valve of $g_m = 5$ mA/V and $r_a = 1$ MΩ has an output capacitance between its anode and earth of 25 pF. The valve is connected as an a.f. amplifier with an anode load of 250 kΩ. Find the voltage gain of the amplifier, at low frequencies and also at 31·8 kc/s.

1000; 707. I.E.R.E., Nov. 1959

2. Sketch the gain-frequency response of a R.C. coupled amplifier and explain what is meant by the gain-bandwidth product.

A resistance-capacitance coupled amplifier uses a triode having an anode load resistance of 18,000 Ω and a coupling capacitor of 0·01 μF. The valve has an amplification factor of 25 and an anode slope resistance of 10,000 Ω. The grid leak resistance is 0·5 MΩ and the total input capacitance is 150 pF.

Calculate the maximum gain, and the frequencies at which the voltage amplification falls to 70·7% of its maximum value, and hence determine the gain-bandwidth of the amplifier.

15·86; 31·4 c/s, 167·3 kc/s; 2·66 Mc/s. I.E.R.E., May 1956

3. A valve of $g_m = 4$ mA/V and $r_a = 20$ kΩ is used as a voltage amplifier with an anode load of 50 kΩ which is coupled to the grid of the next valve by a 0·01 μF capacitor. The grid leak of this next valve is 0·5 MΩ. Find the approximate gain of the stage at mid-band frequencies, and also at an angular frequency of 100 rad/s. Find also the frequency at which the gain of the stage is reduced by 3 dB.

56, 25; 31 c/s. I.E.R.E., Nov. 1960

4. A pentode of mutal conductance 8·0 mA/V and very high anode slope resistance is used with an anode load resistor of 50 kΩ as the first valve of an a.f. amplifier. The grid leak resistor of the next valve is 250 kΩ. Neglecting the reactance of the coupling capacitor, find the gain of the first stage. Sketch the form of the graph of gain against frequency indicating roughly the scales chosen, and account for the shape of the graph.

333. I.E.R.E., May 1960

5. A pentode has a resistive anode load of 2·5 kΩ. Find the maximum permissible value of total capacitance across the output if the upper 3 dB frequency is to be 4 Mc/s. If the input resistance to the next stage is 200 kΩ, find the minimum value of blocking capacitor if the lower 3 dB or half-power-frequency is to be 25 c/s. State any assumptions made.

15·9 pF; 0·032 μF. I.E.R.E., Nov. 1963

6. A pentode of $g_m = 3{\cdot}0$ mA/V and very high r_a has an anode load of 20 kΩ. The output capacitance is 5 pF and the output is fed via a large capacitance to the load which may be regarded as a resistance of 50 kΩ in parallel with 20 pF.

Calculate the maximum gain and the frequencies at which the output has fallen, (a) by 3 dB, and (b) to half its maximum value. Explain how the high-frequency performance of this amplifier may be improved.

42·9; 445 kc/s; 770 kc/s. I.E.R.E., May 1962

7. An a.f. transformer, which may be assumed ideal, has a voltage step-up ratio of 5. Its secondary is connected to the grid of a valve which has an impedance equivalent to that of a 0·5 MΩ resistor in parallel with a capacitance of 20 pF.

The primary winding of the transformer forms the anode load of a valve of $r_a = 10$ kΩ and $g_m = 6$ mA/V. Find the voltage gain of the stage at middle frequencies, and also the *upper 3 dB* or *half-power* frequency.

200; 47·8 kc/s. I.E.R.E., May 1963

8. An amplifier consists of two identical R.C. pentode stages in cascade and has a gain which at 200 kc/s falls to 90% of the mid-band value. If the mutual conductance of each pentode is 5 mA/V and the total capacitance (including strays) from anode to earth is 20 pF for each stage, calculate the load resistances required and the overall mid-band gain.

13·3 kΩ, 73 dB. I.E.R.E., May 1958

9. A class A power amplifier uses a triode valve of anode slope resistance r_a, which is transformer-coupled to a resistive anode load R_L. The value of load resistor R_L is varied, and the values of grid input voltage and grid bias are adjusted to give maximum undistorted power output. The H.T. supply voltage is fixed. Show that the value of R_L which gives the largest output power is given by $2r_a/n^2$, where n is the transformation ratio T_1/T_2. If the power output is to be increased by using two valves, what would be the advantage of push–pull operation? H.N.C.

10. Discuss the factors which affect the choice of the optimum load for a class A transformer-coupled power amplifier using (a) a triode, (b) a pentode.

A triode, which has a constant amplification factor of 11, is used as a transformer-coupled class A amplifier. The transformer primary presents a d.c. resistance of 150 Ω to the anode current, and the effective a.c. load is 1·64 kΩ. The H.T. supply is 200 V, and the grid bias used is −10 V. The anode current–anode voltage characteristic of the triode at zero grid voltage is given in Table 10 below.

Estimate, using a graphical method, the power output and percentage second harmonic distortion, when the sinusoidal input voltage has a peak value of 10 V.

TABLE 10

V_a (volts)	30	50	80	100	120	150
I_a (mA)	20	35	64	85	120	148

1·26 W; 0·95%. I.E.R.E., May 1963

11. Two triodes are used in a class B push–pull amplifier operated from a 400 V supply. When driven, each valve takes a direct current of 31·8 mA, and the a.c. potential difference between each anode and earth is 141·4 V (r.m.s.).

Calculate (a) the total power output, (b) the efficiency and (c) the output transformer turns ratio if the amplifier feeds a 5 Ω load.

10 W; 39·4%; 40 : 1 centre-tapped. I.E.R.E., Nov. 1957

12. Derive an expression for the maximum theoretical efficiency of a class B push–pull amplifier. Two triodes are used in a class B push–pull amplifier. The most suitable load line runs between the points $V_a = 250$ V, $I_a = 0$, and $V_a = 50$ V, $I_a = 50$ mA on the anode characteristics.

Determine (a) the total anode dissipation,
(b) the anode-to-anode load,
(c) the power output,
(d) the efficiency of the amplifier.

3 W; 16 kΩ; 5 W; 63%. I.E.R.E., Nov. 1960

13. Explain the simple principle, and the advantages of push–pull operation in a valve amplifier. Draw the circuit of a phase-splitter and a push–pull power amplifier and explain the action and purpose of the phase-splitter.

Two power triodes, each having characteristics shown in Table 11, operate in class A push–pull. Draw the composite characteristics if the quiescent point is $V_A = 200$ V and $V_B = -20$ V. Draw the composite load line for an anode-to-anode line of 5 kΩ. Determine the power output of this push–pull amplifier when the peak input to each valve is 20 V.

TABLE 11

V_A in volts		40	80	120	160	200	240	280	320
Anode current in mA for grid voltages of V_G	0	13	32	52					
	−10 V			5	15	30	47		
	−20 V					3	9	19	33

1·53 W. H.N.C.

14. Explain what is meant by anode to anode resistance, composite resistance, and composite characteristic, as applied to a transformer-coupled push–pull amplifier.

Two identical power triodes, having static characteristics as in Table 12, are to be operated in Class A push–pull. Construct and plot the composite characteristic, showing lines for a signal input voltage of ± 10 V and ± 20 V.

If the peak input signal voltage to each valve is 20 V, and the anode to anode load is 4·8 kΩ, construct the necessary load line and hence determine the power output.

TABLE 12

Anode volts		80	120	160	200	240	280	320
Anode current in mA for grid volts	0	64	104	144				
	−10 V	12	40	76	114			
	−20 V		10	30	60	94		
	−30 V			8	24	48	78	112
	−40 V				6	18	38	66

Quiescent point $V_A = 200$ V; $I_A = 60$ mA; $V_g = -20$ V.

2·64 W. H.N.C.

15. Determine the maximum collector dissipation of both transistors in a correctly matched class B output stage, given that the quiescent current is I_q, the collector voltage is V_c, and the load R_L.

What is the maximum collector dissipation of both transistors if the maximum output power without distortion due to clipping is 500 mW? Assume that the quiescent current and knee voltages are negligible.

204 mW. I.E.R.E., Nov. 1962

CHAPTER 6

Radio-frequency Amplification

WORKED EXAMPLES

Example 1

Show that the input admittance of a thermionic valve used as an amplifier can be represented by a resistor R and a capacitor C in parallel, where

$$R = \frac{1}{\omega C_{ag} M \sin \theta} \tag{6.1}$$

$$C = C_{gk} + C_{ag}(1 - M \cos \theta). \tag{6.2}$$

$M \angle \theta$ is the ratio of anode voltage to grid voltage $\mathbf{V}_o/\mathbf{V}_{in}$. Calculate the values of C and R for a triode amplifier having an anode load of $(30+j40)$ kΩ.

Assume that

$$\mu = 20, \quad r_a = 10\,\text{k}\Omega, \quad C_{gk} = 4\,\text{pF},$$

$$C_{ag} = 3\,\text{pF}, \quad \omega = 10^5\,\text{rad/s}.$$

Comment on the fact that the resistive component is negative.

H.N.C.

Solution

R and C of Fig. 6.1(b) have values given by equations (6.1) and (6.2) respectively. These formulae were established in Section 1.11, and will not be justified again here.

In order to evaluate C and R, M and $\angle\theta$ must first be determined. If the current taken by the anode-to-grid capacitance C_{ag} is much less than the anode current $\mathbf{I}_a$,

$$m = M\angle\theta = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{-\mu Z_L}{r_a+Z_L}$$

$$= \frac{-20(30+j40)}{10+30+j40}$$

$$= \frac{-15-j20}{1+j} \qquad (6.3)$$

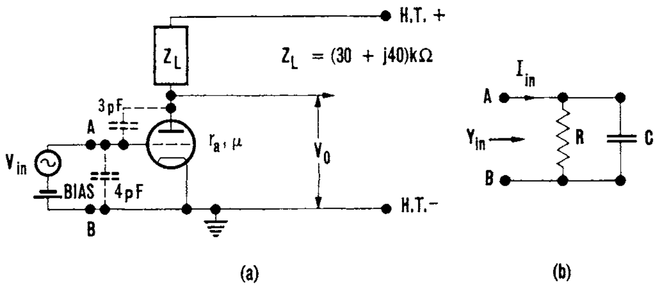


FIG. 6.1(a). Basic circuit showing important inter-electrode capacitance effects.
FIG. 6.1(b). Effective input circuit of Fig. 6.1(a).

Equation (6.3) may be written in polar form thus

$$m = M\angle\theta = \frac{25\angle 233^\circ 9'}{1\cdot414\angle 45^\circ} = 17\cdot7\angle 188^\circ 9'$$

whence

$$M = 17\cdot7 \quad \text{and} \quad \theta = 188^\circ 9'.$$

Now, substituting known values into equation (6.1) gives

$$R = \frac{1}{10^5\times 3\times 10^{-12}\times 17\cdot7\times \sin 188^\circ 9'}\ \Omega$$

$$= -\frac{1}{53\cdot1\times 10^{-7}\times 0\cdot1418\times 10^6}\ \text{M}\Omega$$

$$= -1\cdot33\ \text{M}\Omega,$$

and into equation (6.2) gives

$$C = [4\times10^{-12}+3\times10^{-12}(1-17{\cdot}7\cos 188^\circ\, 9')]\ \text{F.}$$
$$= 4+3[1-17{\cdot}7(-0{\cdot}989)]\ \text{pF.}$$

Therefore

$$C = [4+3(1+17{\cdot}5)]\ \text{pF}$$
$$= (4+55{\cdot}5)\ \text{pF}$$
$$= 59{\cdot}5\ \text{pF.}$$

The input resistance has a value of $-1{\cdot}33$ MΩ; the minus sign signifies that the feedback between anode and grid via C_{ag} is positive, and may cause the circuit to act as an oscillator.

Example 2

A pentode has a tuned anode load consisting of a 120 μH coil of resistance 10 Ω in parallel with a capacitor of 200 pF. If the valve parameters are $\mu = 1000$, and $r_a = 500$ kΩ, determine the resonant frequency of the tuned circuit, its dynamic resistance, and the gain of the stage at resonance.

Solution

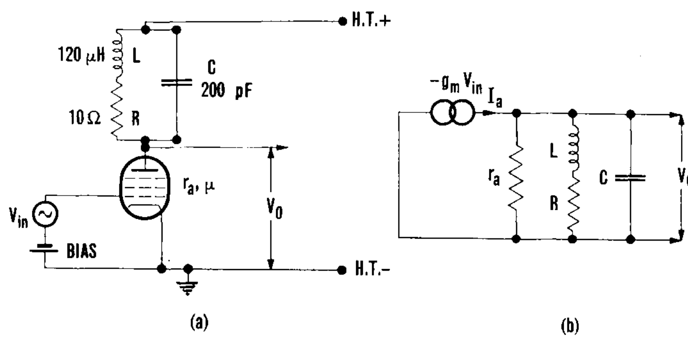


FIG. 6.2(a). Basic circuit.
FIG. 6.2(b). Constant current equivalent of Fig. 6.2(a).

Tuned Circuit Resonant Frequency (f_0)

$$f_0 = \frac{1}{2\pi\sqrt{LC}}, \text{ if the coil } Q \text{ factor is high,}$$

hence, $$f_0 = \frac{1}{2\pi\sqrt{(120\times10^{-6}\times200\times10^{-12})}} \text{ c/s}$$

$$= \frac{10^8}{2\pi\sqrt{240}} \text{ c/s}$$

$$= 1{\cdot}025 \text{ Mc/s.}$$

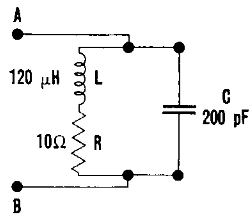


FIG. 6.3. Amplifier tuned circuit load.

Effective Resistance R_D of the Tuned Circuit at Resonance

At any frequency, the impedance of the tuned circuit across AB of Fig. 6.3 is given by Z_{AB} where

$$Z_{AB} = \frac{(j\omega L+R)\dfrac{1}{j\omega C}}{R+j\left(\omega L-\dfrac{1}{\omega C}\right)},$$

however, if the coil Q factor is high ($\omega L \gg R$),

then $$Z_{AB} = \frac{L}{C\left(R+j\left(\omega L-\dfrac{1}{\omega C}\right)\right)}$$

Now, at resonance $\omega_0 L = 1/\omega_0 C$, whence

$$R_D = \frac{L}{CR} \tag{6.4}$$

Equation (6.4) gives value of resistance, which, if substituted for the tuned circuit at resonance, would give exactly the same electrical results as the tuned circuit itself. Hence, at resonance, the equivalent circuit of Fig. 6.2(b) may be simplified as shown in Fig. 6.4.

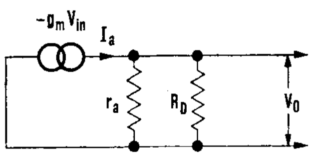


FIG. 6.4

Now, $$g_m = \frac{\mu}{r_a} = \frac{1000}{500\times10^3}\text{ A/V}$$
$$= 2\text{ mA/V}$$

and $$R_D = \frac{L}{CR} = \frac{120\times10^{-6}}{200\times10^{-12}\times10}\ \Omega$$
$$= 60\text{ k}\Omega.$$

Now, the effective load resistance R_e at resonance is given by

$$R_e = \frac{60\times500}{60+500}\text{ k}\Omega$$
$$= 53{\cdot}6\text{ k}\Omega.$$

But, $$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = -g_m R_e$$
$$= -2\times10^{-3}\times53{\cdot}6\times10^3$$

therefore, $$m = -107{\cdot}2.$$

Example 3

In the circuit of Fig. 6.5, the capacitor C_1 may be assumed loss-free, and the reactance of the coupling capacitor C_2, negligible. Calculate

(a) the stage gain at the resonant frequency;

(b) the bandwidth at the 3 dB points.

For part (b), work from first principles, or derive any formulae used.

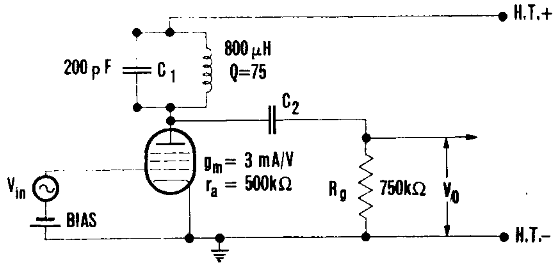


FIG. 6.5

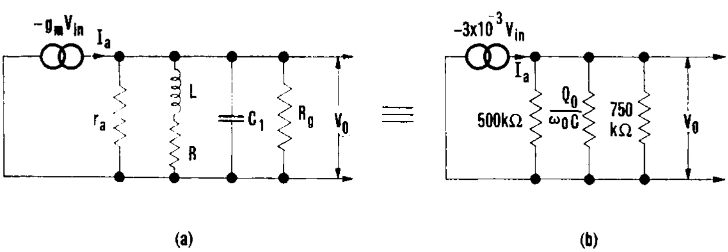


FIG. 6.6

Solution

(*a*) *Stage gain at resonance.* At the resonant frequency f_0, the tuned circuit in the anode of the amplifier of Fig. 6.5 is purely resistive, and its resistance R_D has a value $R_D = Q_0/\omega_0 C$ if the coil Q factor is high. The constant-current equivalent of the amplifier of Fig. 6.5 is shown in Fig. 6.6(a). A simpler version, valid only at resonance, appears in Fig. 6.6(b).

Now, for the tuned circuit at resonance, $\omega_0^2 LC_1 = 1$ since the coil Q factor is high,

therefore, $$\omega_0 = \frac{1}{\sqrt{LC_1}} \text{ rad/s} \tag{6.5}$$

$$= \frac{1}{\sqrt{(800 \times 10^{-6} \times 200 \times 10^{-12})}} \text{ rad/s,}$$

therefore, $$\omega_0 = \frac{10^7}{4} \text{ rad/s.}$$

Now, $$R_D = \frac{Q_0}{\omega_0 C}$$

$$= \frac{75 \times 4}{10^7 \times 200 \times 10^{-12}} \ \Omega$$

$$= 150 \text{ k}\Omega.$$

The effective resistance R_e, across which the output voltage $\mathbf{V}_o$ is developed, is found from Fig. 6.6 and equation (6.6), thus,

$$\frac{1}{R_e} = \left[\frac{1}{500} + \frac{1}{150} + \frac{1}{750}\right] \text{m}\Omega^{-1}, \tag{6.6}$$

whence $$R_e = 100 \text{ k}\Omega$$

and $$m = -g_m R_e = -3 \times 10^{-3} \times 100 \times 10^3$$

$$= -300.$$

Note

The 500 kΩ and 750 kΩ resistors damp the anode tuned circuit of the valve and reduce its effective Q factor. In fact, the load on the valve may be considered as either

(a) a tuned circuit comprising parallel resistive and reactive components R_e, L_p and C_1 (see Fig. 6.7(a)), or

(b) a parallel tuned circuit comprising an inductance L' with its series resistance R', and the original loss-free capacitor C_1 connected across the series LR circuit (see Fig. 6.7(b)).

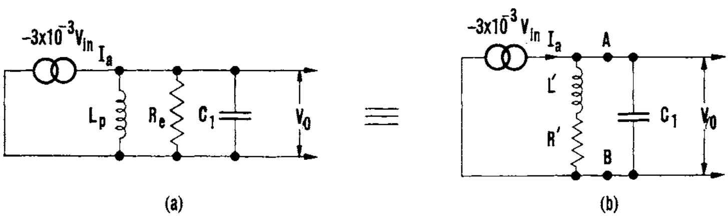


FIG. 6.7(a). Equivalent circuit of Fig. 6.5 showing the effective parallel resistive and reactive components. At resonance, $L_p = R_e/\omega_0 Q_e$, $C_1 = Q_e/\omega_0 R_e$.

FIG. 6.7(b). Alternative equivalent of Fig. 6.5 showing the effective series LR in parallel with the loss-free capacitor C_1. At resonance, $L' = Q_e R'/\omega_0$, $R' = 1/\omega_0 C_1 Q_e$.

From Fig. 6.7(a)

$$Q_e = \omega_0 C_1 R_e.$$

Substituting values gives

$$Q_e = \frac{10^7}{4} \times 200 \times 10^{-12} \times 10^5 = 50.$$

If the series components L' and R' of Fig. 6.7(b) are required, they may be obtained as follows:

$$Q_e = \frac{1}{\omega_0 C_1 R'}$$

hence, $$R' = \frac{1}{\omega_0 C_1 Q_e} = \frac{4}{10^7 \times 200 \times 10^{-12} \times 50}\ \Omega,$$

therefore, $$R' = 40\ \Omega.$$

Also, $$\omega_0 L' = Q_e R' = 50 \times 40 = 2000\ \Omega$$

and $$L' = 2000/\omega_0 = 800\ \mu\text{H}.$$

In the solution to part (b) of this problem R' may, therefore, be assumed to be much less than $\omega L'$.

(*b*) *Bandwidth at* 3 *dB points.* From Fig. 6.7(b) it can be seen that, at any frequency,

$$Z_{AB} = \frac{\left(\dfrac{1}{j\omega C_1}\right)(R'+j\omega L')}{R'+j\left(\omega L'-\dfrac{1}{\omega C_1}\right)}$$

and if $R' \ll \omega L'$

$$Z_{AB} = \frac{L'}{C_1\left[R'+j\left(\omega L'-\dfrac{1}{\omega C_1}\right)\right]} \tag{6.7}$$

Now, dividing numerator and denominator of equation (6.7) by C_1R'

$$Z_{AB} = \frac{R_0}{1+j\dfrac{1}{R'}\left(\omega L'-\dfrac{1}{\omega C_1}\right)} \tag{6.8}$$

where R_0 is the impedance of the circuit of Fig. 6.7(b) at resonance, i.e.

$$R_0 = R_e. \tag{6.9}$$

From equation (6.8),

$$\frac{Z_{AB}}{R_0} = \frac{1}{\left[1+j\dfrac{1}{R'}\left(\omega L'-\dfrac{1}{\omega C_1}\right)\right]} \tag{6.10}$$

now,

$$L' = \frac{Q_eR'}{\omega_0} \quad \text{and} \quad C_1 = \frac{1}{\omega_0R'Q_e}$$

therefore,

$$\frac{Z_{AB}}{R_0} = \frac{1}{1+jQ_e\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)} \tag{6.11}$$

The impedance Z_{AB}, and therefore the output voltage, will have reduced to 0·707 of their maximum values when the denominator of equation (6.11) has values of either $(1+j)$ or $(1-j)$. At these 3 dB points,

$$Q_e\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) = \pm 1$$

and the frequencies at which the response of the tuned circuit is 3 dB down are defined by equation (6.12)

$$\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega} = \pm\frac{1}{Q_e} \tag{6.12}$$

Multiplying through by (ω/ω_0) gives

$$\left(\frac{\omega}{\omega_0}\right)^2 - 1 = \pm\frac{\omega}{\omega_0 Q_e}$$

$$\left(\frac{\omega}{\omega_0}\right)^2 \pm\frac{1}{Q_e}\times\frac{\omega}{\omega_0} - 1 = 0. \tag{6.13}$$

The quadratic equation of (6.13) may be solved in the normal manner.

Hence, $$\frac{\omega}{\omega_0} = \pm\frac{1}{2Q_e}\pm\sqrt{\left[\left(\frac{1}{2Q_e}\right)^2+1\right]} \tag{6.14}$$

Taking the positive solution of equation (6.14) gives

$$\frac{\omega}{\omega_0} = \pm\frac{1}{2Q_e}+\sqrt{\left[\left(\frac{1}{2Q_e}\right)^2+1\right]}$$

therefore, $$\frac{\omega_1}{\omega_0} = \sqrt{\left[\left(\frac{1}{2Q_e}\right)^2+1\right]}-\frac{1}{2Q_e} \tag{6.15}$$

and $$\frac{\omega_2}{\omega_0} = \sqrt{\left[\left(\frac{1}{2Q_e}\right)^2+1\right]}+\frac{1}{2Q_e} \tag{6.16}$$

Subtracting equation (6.15) from equation (6.16) gives

$$\frac{\omega_2-\omega_1}{\omega_0} = \frac{1}{Q_e} \quad\text{or}\quad \frac{f_2-f_1}{f_0} = \frac{1}{Q_e} \tag{6.17}$$

f_1 is the lower 3 dB frequency, f_2 is the upper 3 dB frequency, f_0 is the resonant frequency, and Q_e is the effective Q factor of the equivalent circuit of Fig. 6.7(b) at resonance. Now (f_2-f_1) is the bandwidth of the amplifier, therefore, from equation (6.17),

$$\text{Bandwidth} = \frac{f_0}{Q_e} = \frac{10^7}{8\pi \times 50} = 7{\cdot}96 \text{ kc/s.}$$

Example 4

A pentode is used in a tuned secondary transformer-coupled r.f. transformer. It has parameters $\mu = 1000$, $g_m = 2$ mA/V, and $r_a = 500$ kΩ. The mutual conductance M is 100 μH, and the tuned circuit consists of a 160 μH coil with a series resistance of 8 Ω, in parallel with a capacitor of 220 pF. Find the gain of the stage at resonance.

Solution

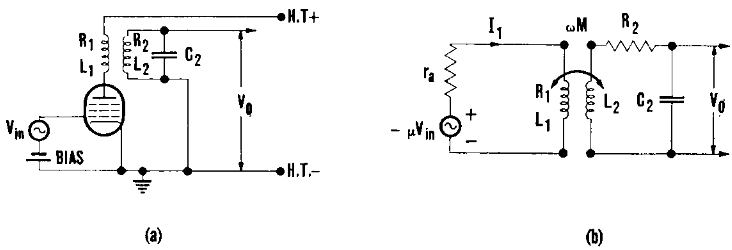


FIG. 6.8(a). Basic circuit.
FIG. 6.8(b). Constant voltage equivalent of Fig. 6.8(a).

The presence of the secondary tuned circuit modifies the primary impedance Z_p of Fig. 6.8(b). It can be shown that the effective primary impedance Z_p' is given by

$$Z_p' = Z_p + \frac{\omega^2 M^2}{Z_s} \tag{6.18}$$

where ω is the angular frequency of the input voltage $\mathbf{V}_{in}$,

M is the mutual inductance,

Z_s is the series impedance of the secondary,

Z_p is the series impedance of the primary.

Now, $Z_p = r_a + R_1 + j\omega_0 L$ at resonance, and Z_s is purely resistive and equal to R_2.

Therefore, assuming that $(R_1 + j\omega_0 L_1)$ is very much less than either r_a or $\omega_0^2 M^2/R_2$, from Fig. 6.8(b),

$$\mathbf{I}_1 = \frac{-\mu \mathbf{V}_{in}}{r_a + \dfrac{\omega_0^2 M^2}{R_2}}. \tag{6.19}$$

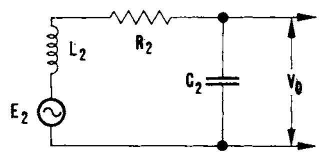


FIG. 6.9

The current given by equation (6.19) is the current which flows through the primary coil of the circuit of Fig. 6.8(b) when the frequency of the input voltage is the resonant frequency f_0 of the secondary series tuned circuit. Fig. 6.19 shows this series tuned circuit which is supplied by a generator of constant e.m.f. $\mathbf{E}_2$ volts, where

$$\mathbf{E}_2 = \pm j\omega_0 M \mathbf{I}_1. \tag{6.20}$$

Now,

$$\omega_0 = \frac{1}{\sqrt{(L_2 C_2)}}$$

$$= \frac{1}{\sqrt{(160 \times 10^{-6} \times 220 \times 10^{-12})}} = \frac{8 \times 10^7}{15} \text{ rad/s.}$$

At this resonant angular frequency ω_0, $\mathbf{V}_o = \mathbf{E}_2 Q_0$, where

$$Q_0 = \frac{\omega_0 L_2}{R_2}$$

whence,
$$Q_0 = \frac{8\times 10^7 \times 160\times 10^{-6}}{15\times 8} = 106{\cdot}7$$

and
$$\mathbf{V}_o = \mathbf{E}_2 \times 106{\cdot}7. \tag{6.21}$$

Substituting $\mathbf{I}_1$ from equation (6.19) into equation (6.20) gives

$$\mathbf{E}_2 = \pm j\frac{\omega_0 M\mu \mathbf{V}_{in}}{r_a + \dfrac{\omega_0^2 M^2}{R_2}} \tag{6.22}$$

Hence, from equations (6.21) and (6.22), we have,

$$\mathbf{V}_o = \pm j\frac{5{\cdot}34\times 10^6 \times 10^{-4}\times 1000\,\mathbf{V}_{in}}{5\times 10^5 + \dfrac{64\times 10^{14}\times 10^{-8}}{225\times 8}}\times 106{\cdot}7$$

$$= \pm j\frac{534\mathbf{V}_{in}}{535{\cdot}6}\times 106{\cdot}7$$

and

$$|\,m\,| = \left|\frac{V_o}{V_{in}}\right| = 0{\cdot}997\times 106{\cdot}7 = 106{\cdot}4.$$

Example 5

A transmitting valve is operated as a class C power amplifier with an efficiency of 76%, and an anode dissipation of 1 kW. Calculate the r.f. power output of the stage, and the peak voltage developed across the capacitor of the anode tuned circuit. The tuned circuit inductor is 120 μH, and works with an effective Q factor of 14. The frequency of operation is 1 Mc/s.

Solution

R.f. power output.

$$\text{Efficiency } \eta = \frac{\text{r.f. power output } (P_{\text{r.f.}})}{\text{d.c. power input } (P_{\text{d.c.}})} \times 100\% \qquad (6.23)$$

and the anode dissipation P_A is given by,

$$P_A = \text{d.c. power input} - \text{r.f. power output} \qquad (6.24)$$

Now, from equation (6.23),

$$P_{\text{d.c.}} = \frac{P_{\text{r.f.}}}{\eta}$$

and for an efficiency of 76%,

$$P_{\text{d.c.}} = \frac{1}{0{\cdot}76} \times P_{\text{r.f.}} \qquad (6.25)$$

Substituting $P_{\text{d.c.}}$ from equation (6.25) into equation (6.24) gives

$$P_A = P_{\text{r.f.}}(1{\cdot}32-1) = 0{\cdot}32P_{\text{r.f.}}$$

But the anode dissipation is 1 kW, therefore

$$P_{\text{r.f.}} = 3{\cdot}12 \text{ kW}.$$

Voltage Across the Output Capacitor

The dynamic resistance R_D of the parallel anode tuned circuit at resonance, may be expressed in terms of the angular resonant frequency ω_0, the circuit parallel inductance L, and the effective Q factor Q_e, thus

$$\frac{R_D}{\omega_0 L} = Q_e \quad \text{and} \quad R_D = \omega_0 L Q_e. \qquad (6.26)$$

Also, $P_{\text{r.f.}}$ may be expressed in terms of R_D, and V_{pk} the peak value of voltage across the parallel L.C.R. network.

Hence,
$$P_{\text{r.f.}} = \left(\frac{V_{pk}}{\sqrt{2}}\right)^2 \times \frac{1}{R_D}. \qquad (6.27)$$

But $R_D/\omega_0 L = Q_e$ from equation (6.27),

therefore,
$$P_{r.f.} = \frac{V_{pk}^2}{2\omega_0 L Q_e}$$

or, as the peak voltage across the capacitor, $\hat{V}_c$, is the same value as the peak voltage across the network,

$$\hat{V}_c = \sqrt{(2\omega_0 L P_{r.f.} Q_e)}. \tag{6.28}$$

Substituting known, and previously calculated values, into equation (6.28) we have,

$$\hat{V}_c = \sqrt{(2\times 2\times \pi\times 10^6\times 120\times 10^{-6}\times 3{\cdot}12\times 10^3\times 14)}\ \text{V},$$

therefore,

$$\hat{V}_c = 8{\cdot}1\ \text{kV}.$$

Example 6

A class C amplifier has a H.T. supply voltage of 1·6 kV. The anode current flows in pulses which can be assumed triangular. The peak value of each of these pulses is 1·7 A, and the angle of flow is 120°. If the current delivered by the tank circuit to a 600 Ω load resistance is 0·707 A (r.m.s.), calculate the efficiency of the amplifier.

Solution

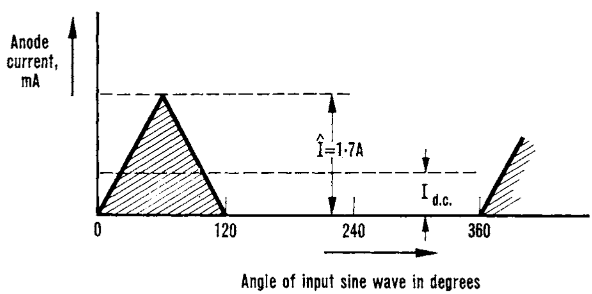


FIG. 6.10. Diagram showing variation of anode current with angle of input signal.

The average value of current $I_{d.c.}$ may be determined from Fig. 6.10 using the relationship

$$I_{d.c.} = \frac{\text{shaded area of Fig. 6.10}}{\text{total base length } (360°)}.$$

Now, the area A of the triangle is given by

$$A = \tfrac{1}{2} \times \frac{120}{1} \times 1{\cdot}7 = 102 \text{ deg-ampere}$$

hence,
$$I_{d.c.} = \frac{102}{360} = 0{\cdot}283 \text{ A}.$$

The d.c. power $P_{d.c.}$ drawn from the H.T. supply is

$$P_{d.c.} = I_{d.c.}V_{d.c.}$$
$$= 0{\cdot}283 \times 1600 \text{ W},$$

therefore,
$$P_{d.c.} = 453 \text{ W}. \tag{6.29}$$

The r.f. power $P_{r.f.}$ delivered to the load is given by

$$P_{r.f.} = I^2_{r.m.s.} \times R$$
$$= 0{\cdot}707 \times 0{\cdot}707 \times 600$$
$$= 300 \text{ W}. \tag{6.30}$$

But efficiency
$$\eta = P_{r.f.}/P_{d.c.} \times 100\%,$$

therefore
$$\eta = \frac{300}{453} \times 100 = 66{\cdot}2\%.$$

Problems with Answers

1. Explain what is meant by "Miller effect". State how it normally arises, and its effect on the input impedance of an amplifier.

A valve used as a voltage amplifier has a grid circuit consisting of an inductance L, which has a resistance R. A capacitance C is connected between anode and grid, and the anode load is such that the stage gain is $-A$, where $A = (a+jb)$. Show that the input impedance becomes infinite if

$$R = \frac{b}{\omega C[(1+a)^2+b^2]} \quad \text{and} \quad L = \frac{(1+a)}{\omega^2 C[(1+a)+b^2]}.$$

H.N.C.

2. A triode valve, having an amplification factor of 28 and an anode slope resistance of 20 kΩ is used as a tuned voltage amplifier. The load impedance at a frequency of 1 Mc/s is 20/45 kΩ. If the inter-electrode capacitance of the valve is 2 pF between anode and grid, and 3 pF between grid and cathode, calculate the effective input capacitance and input resistance of the valve at this frequency.

32·9 pF; −13·8 kΩ.

3. Obtain expressions for the gain, the input admittance, and the output admittance of the arrangement shown in Fig. 6.11. If the high-gain amplifier consists of a pentode having $r_a = 1$ MΩ, $g_m = 2$ mA/V and $Y_i = Y_f = 2 \times 10^{-6}\ \Omega^{-1}$, $Y_g = 0{\cdot}2 \times 10^{-6}\ \Omega^{-1}$, obtain approximate values for the output and input impedances.

1 kΩ; 1 MΩ. I.E.R.E., Nov. 1960

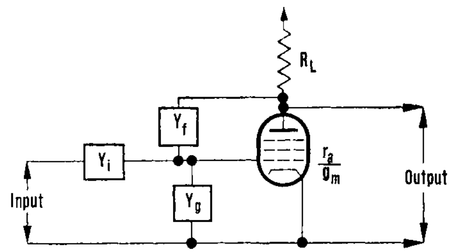


FIG. 6.11

4. An amplifier uses a pentode valve for which $g_m = 5$ mA/V, $C_{g\,k} = 4$ pF, and the inductance of the cathode lead is 0·05 μH.

Calculate the input conductance of the amplifier at a frequency of 100 Mc/s, and comment on the practical significance of the result.

Explain briefly how transit time in valves gives rise to a conductive component of input admittance.

394 $\mu\Omega^{-1}$. I.E.R.E., May 1960

5. A class A earthed-grid amplifier has a tuned circuit anode load whose dynamic impedance is 4000 Ω. Assuming idealized conditions, calculate the input impedance at the cathode. The valve parameters are $g_m = 5$ mA/V and $\mu = 20$.

381 Ω. I.E.R.E., May 1958

6. The earthed-grid amplifier shown in Fig. 6.12 operates at 50 Mc/s and the output circuit has $C = 25$ pF and $Q = 25$. The valve parameters are $\mu = 20$, $r_a = 6$ kΩ. Calculate the voltage gain and the input impedance of the circuit.

Explain why it is usual for an r.f. stage to precede the mixer stage in a v.h.f. receiver. Discuss the relative merits of two types of amplifier circuit commonly used as r.f. input stages.

$m \simeq 7{\cdot}25$; $R_2 \simeq 437\ \Omega$. I.E.R.E., May 1963

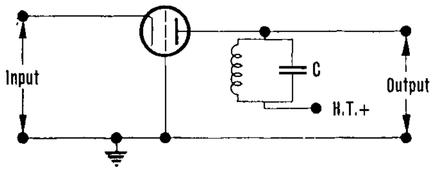


FIG. 6.12

7. A cascode amplifier is to operate at a frequency of 200 Mc/s. The tuned anode load of the second valve has an inductance of 0·1 μH, and a Q factor of 25. If each valve has $\mu = 50$ and $r_a = 10\ \mathrm{k}\Omega$, calculate the theoretical gain of the amplifier. Derive any formulae used.

16. I.E.R.E., Nov. 1960

8. A single-stage tuned anode pentode amplifier uses an inductor of 150 μH which has a Q factor of 60. The resonant frequency of the tuned circuit is 1 Mc/s. The anode slope resistance of the valve is 250 kΩ. Determine the bandwidth of the stage.

21·0 kc/s.

9. A coil of inductance 400 μH and resistance 5 Ω is tuned to resonate at an angular frequency of $\omega = 10^6$ rad/s, by a capacitor in parallel with it.

This circuit is used as the anode load of a pentode of $g_m = 6$ mA/V in a tuned r.f. amplifier. Draw the circuit giving an indication of all component values and calculate the voltage gain (a) at resonance, (b) at a frequency of 5% above the resonant frequency. Find also (c) the bandwidth between the frequencies at which the voltage gain is $1/\sqrt{2}$ of that at resonance.

The effect of the anode slope resistance may be neglected.

192; 23·8; 2 kc/s. I.E.R.E., Nov. 1961

10. Show that, for a series resonant circuit, the frequency bandwidth at the half-power points is given by

$$\text{resonant frequency} \times \frac{1}{Q}.$$

Derive an expression for the stage gain at resonance of a tuned r.f. amplifier, having an untuned primary and tuned secondary coupled by mutual inductance. The slope resistance of the valve may be assumed infinite.

Calculate the stage gain at resonance, and frequency bandwidth for such an amplifier if the valve used has a mutal conductance of 2 mA/V, and the mutal inductance is 300 μH. The tuned secondary comprises 100 pF, 1 mH, and 30 Ω in series.

200; 4·78 kc/s. H.N.C.

11. A tuned r.f. amplifier has an anode load consisting of two identical tuned circuits of magnification factor Q, coupled only by mutual inductance M. The output is taken across the capacitor of the secondary circuit. Show that the stage gain, at the common resonant frequency, is given by:

$$\frac{g_m 2\pi f_0 M}{(1/Q^2)+K^2}$$

The valve anode resistance may be assumed infinite. Hence determine an expression for the value of coupling factor K which gives the maximum value of stage gain at this frequency.

Sketch response curves of the output voltage against frequency if the coupling is (a) greater, (b) smaller than this critical value.

H.N.C.

12. What is meant by *critical inductive coupling?*

A pentode valve with a g_m of 3 mA/V is used in an i.f. amplifier with an anode circuit tuned to 465 kc/s. An identical circuit comprising a coil with a Q factor of 80 and a capacitor of value 450 pF is critically coupled to it. Calculate the overall gain and the coefficient of coupling.

91·2; 0·0125 I.E.R.E., Nov. 1958

13. In a transistor i.f. amplifier, the coupling between two alloy-diffused transistors is by means of a double-tuned i.f. transformer. Draw a circuit diagram adding suitable components necessary for stability, and explain the functioning of the stage.

Determine the value of the mutual inductance required for critical coupling, if the working Q's of both primary and secondary are 70, the i.f. is 500 kc/s and the tuning capacitors are both 500 pF.

2·89 μH. I.E.R.E., May 1962

14. A coil has a Q factor of 500 at 10 Mc/s and is tuned to this frequency by a capacitance of 159 pF. The tuned circuit is then used as the tank circuit of a class C triode amplifier where its Q factor is degraded to 14 by the load.

The H.T. voltage used is 3 kV, the minimum instantaneous anode voltage is 200 V and the mean anode current is 1 A.

Calculate

(a) the anode dissipation;
(b) the power in the load;
(c) the anode efficiency of the triode;
(d) the power loss in the coil.

770 W; 2·17 kW; 0·743; 62·5 W. I.E.R.E., May 1961

15. Why is class C operation normally used in power amplifiers at radio frequencies? Discuss the factors which influence the choice of angle of anode current flow.

The anode current pulse in a class C triode has a maximum instantaneous value of 2 A and an angle of flow of 120°. The H.T. supply is 1·75 kV, the resonant impedance of the tuned load is 2 kΩ and operation may be assumed linear above cut-off.

Calculate the r.f. power delivered to the anode load and the anode efficiency. Justify the method used.

610 W; 80%. I.E.R.E., Nov. 1962

16. Explain briefly the operation of a single ended, class C, r.f. power amplifier stage, showing clearly why the efficiency can be very high.

A class C stage operates with a high tension supply of 1000 V, and the angle of flow of anode current is 120°, the anode current pulses are sine section in shape, and have a maximum value of 600 mA. The minimum anode voltage, and maximum positive grid voltage are both 150 V, the amplification factor of the valve is 5, and the tuned anode load impedance is $(4+j0)$ kΩ.

Calculate

(a) signal power output;

(b) the anode efficiency;

(c) the grid bias voltage;

and (d) the peak signal grid voltage.

80 W; 69%; 380 V, 530 V. I.E.R.E., Nov. 1963

CHAPTER 7

Negative Feedback and Cathode Follower

WORKED EXAMPLES

Example 1

An amplifier without feedback has a gain of 2000 subject to a 10% reduction in use. Determine the percentage reduction when 1% negative feedback is applied.

Solution

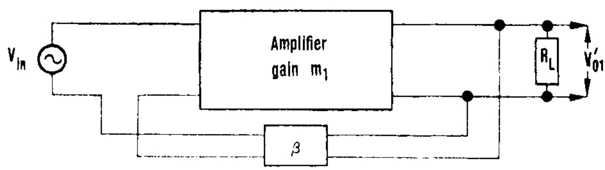


FIG. 7.1

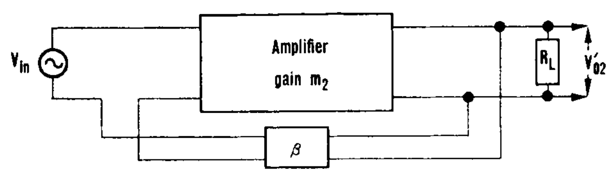


FIG. 7.2

Let the initial gain of the amplifier without feedback be m_1, and let the gain of the amplifier with negative feedback be

m_1'. Fig. 7.1 shows the amplifier with feedback applied when $m_1 = 2000$ and $\beta = 0{\cdot}01$ or 1%.

From equation (1.83),

$$m_1' = \frac{m_1}{1+\beta m_1}$$

$$= \frac{2000}{1+0{\cdot}01\times 2000}.$$

Therefore $$m_1' = 95{\cdot}24.$$

When the amplifier is in use, it is subject to a 10% reduction in gain without feedback. Let the new gain without feedback be m_2, and let the new gain with feedback be m_2'. This condition is shown in Fig. 7.2.

Now, $$m_2 = \frac{90}{100}\times m_1 = 0{\cdot}09\times 2000 = 1800$$

and $$\beta = 0{\cdot}01 \text{ as before.}$$

Hence $$m_2' = \frac{m_2}{1+\beta m_2}$$

$$= \frac{1800}{1+0{\cdot}01\times 1800}$$

$$= 94{\cdot}73.$$

The percentage reduction in gain is

$$\frac{95{\cdot}24-94{\cdot}73}{95{\cdot}24}\times 100 = 0{\cdot}536\%.$$

Example 2

An amplifier with negative feedback has a gain of 35. With no feedback applied, a 0·16 V signal is needed to produce a certain output voltage $\mathbf{V}_0$. With feedback, an input of 3·2 V is needed to produce the same output.

Calculate (a) the amplifier gain without feedback, (b) the feedback factor, β.

Solution

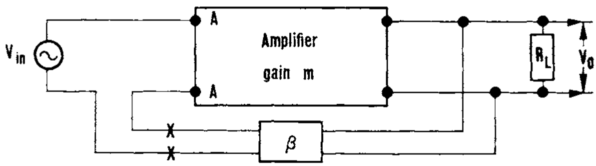


FIG. 7.3

(*a*) *Gain* m *without feedback.* From Fig. 7.3, $m = \mathbf{V}_o/\mathbf{V}_{AA}$ when XX is open circuit, and the amplifier load consists of R_L and β in parallel.

Now $\mathbf{V}_{AA} = 0{\cdot}16$ V.

Therefore, $$m = \frac{\mathbf{V}_o}{0{\cdot}16} \quad \text{or} \quad \mathbf{V}_o = 0{\cdot}16\,m \tag{7.1}$$

With feedback, 3·2 V has to be applied to the amplifier if the same output voltage $\mathbf{V}_o$ is to be obtained. The gain m' of the amplifier of Fig. 7.3 with feedback is given by $m' = \mathbf{V}_o/\mathbf{V}_{in}$.

Hence, $$35 = \mathbf{V}_o/3{\cdot}2 \quad \text{or} \quad \mathbf{V}_o = 35 \times 3{\cdot}2 = 112 \text{ V}. \tag{7.2}$$

As the output voltage is the same in each case, equation (7.1) and (7.2) are equal.

Therefore, $$m = 112/0{\cdot}16 = 700.$$

(*b*) *Feedback factor* β. Now, m' is given by

$$m' = \frac{m}{1+\beta m}$$

Therefore, $$35 = \frac{700}{1+700\beta}$$

and $$1+700\beta = 20,$$

whence $$\beta = 0{\cdot}02714 \quad \text{or} \quad 2{\cdot}714\%.$$

Example 3

Discuss the advantages and disadvantages of applying negative feedback to an amplifier.

An amplifier of open circuit gain 1500, and internal resistance 7500 Ω, has a phase reversal between output and input terminals in the frequency range over which it is used. It is connected in the feedback system of Fig. 7.4.

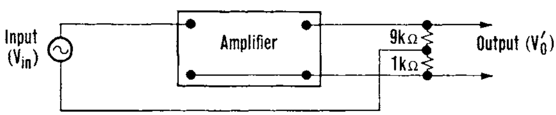


FIG. 7.4

Calculate the gain and output impedance of the feedback amplifier, taking into account the effect of the feedback network.

I.E.R.E., Nov. 1962

Solution

With the feedback network disconnected, the open circuit gain m_0 is given by

$$m_0 = \frac{\text{open circuit voltage}}{\text{input voltage } (\mathbf{V}_{in})} = -1500+j0 = -1500$$

and

$$\text{open circuit voltage } (\mathbf{V}_{o/c}) = -1500\,\mathbf{V}_{in}.$$

However, the open circuit voltage is the effective e.m.f. of the constant voltage output generator of the amplifier. This generator has an internal resistance (R_0) of 7500 Ω (see Fig. 7.5).

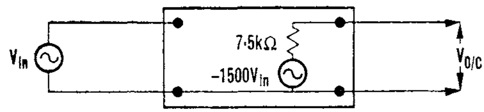


FIG. 7.5

If now the 10,000 Ω series network is connected across the output terminals without feedback being applied, then gain m of the amplifier without feedback can be calculated, if necessary, from equation (7.3). This expression is obtainable directly from Fig. 7.6.

$$\mathbf{V}_o = \frac{m_0 \mathbf{V}_{in} \times 10}{R_0 + 10}. \tag{7.3}$$

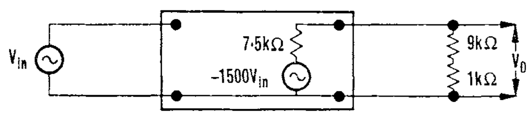


FIG. 7.6

When no feedback is applied, the voltage across the amplifier input terminals is independent of the output voltage $\mathbf{V}_o$; however, when feedback is applied, the voltage which appears across the amplifier input terminals ($\mathbf{V}_{AA}$) does depend upon the output voltage.

From Fig. 7.7,

$$\mathbf{V}_{AA} = \mathbf{V}_{in} + \beta \mathbf{V}_o'$$

when amplified, this becomes

$$m_0 \mathbf{V}_{AA} = m_0 \mathbf{V}_{in} + m_0 \beta \mathbf{V}_o'$$

and
$$\mathbf{V}_o' = (m_0 \mathbf{V}_{in} + m_0 \beta \mathbf{V}_o') \times \frac{10}{R_0 + 10}$$

hence, $$\mathbf{V}_o'(R_0+10) = 10m_0\mathbf{V}_{in}+10m_0\beta\mathbf{V}_o'$$
therefore,
$$\mathbf{V}_o'(R_0+10-10m_0\beta) = 10m_0\mathbf{V}_{in}$$
and
$$\mathbf{V}_o' = \frac{10m_0\mathbf{V}_{in}}{R_0+10(1-\beta m_0)} \tag{7.4}$$

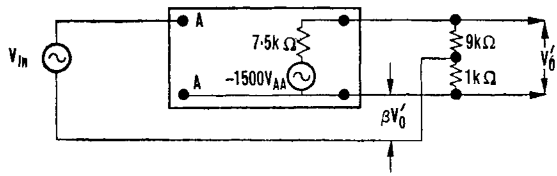


FIG. 7.7

To find m′. From equation (7.4)
$$m' = \frac{\mathbf{V}_o'}{\mathbf{V}_{in}} = \frac{10m_0}{R_0+10(1-\beta m_0)} \tag{7.5}$$
Now,
$$m_0 = -1500, \quad R_0 = 7500\ \Omega, \quad \text{and} \quad \beta = 0{\cdot}1.$$
Substituting these values into equation (7.5) gives
$$m' = \frac{-15{,}000}{7{\cdot}5+10+1500}$$
$$= -\frac{15{,}000}{1517{\cdot}5}$$
$$m' = -9{\cdot}88.$$

To Find R_0'

Equation (7.4) may be re-written thus,
$$\mathbf{V}_o' = \frac{\dfrac{m_0}{1-\beta m_0}\times 10}{\dfrac{R_0}{1-\beta m_0}+10}\times\mathbf{V}_{in}. \tag{7.6}$$

A comparison of equations (7.3) and (7.6) reveals that the e.m.f. and internal resistance of the constant voltage generator, are both effectively reduced by the factor $1/(1+1500\beta)$ when negative feedback is applied (as $m_0 = -1500$). The internal resistance R_0' of the amplifier with feedback is given by the first term in the denominator of the equation (7.6),

hence,
$$R_0' = \frac{R_0}{1+1500\beta}$$

$$= \frac{7{,}500}{1+150}\ \Omega$$

$$= 49{\cdot}6\ \Omega.$$

If the effect of the 10 kΩ resistance network is also considered, the total output resistance looking into the output terminals is R_0' in parallel with 10 kΩ, i.e.

$$\frac{49{\cdot}6\times 10{,}000}{10{,}000+49{\cdot}6} = 49{\cdot}4\ \Omega.$$

Example 4

State briefly the effects of negative feedback on the performance of an amplifier.

The overall gain of a two-stage amplifier is 100, and the second stage has 10% of its output voltage applied as negative feedback. If the gain and second harmonic distortion of the second stage are 150 and 5% respectively without feedback, find

(a) the second harmonic distortion of the amplifier,

(b) the gain of the first stage.

Assume that the first stage introduces negligible distortion.

I.E.R.E., May 1962

Solution

Block Diagram

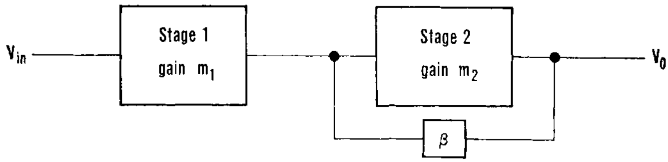


FIG. 7.8

m_1 is the gain of stage 1 without feedback,

m_2 is the gain of stage 2 without feedback,

V_o/V_{in} is the overall stage gain when feedback is applied to stage 2 as shown in Fig. 7.8.

(*a*) *Second harmonic distortion.* Let D_2 be the second harmonic distortion introduced by stage 2.

Using equation (1.87) from Section 1.22,

$$D_2' = \frac{D_2}{1+m_2\beta}$$

D_2' is the second harmonic distortion with feedback applied to stage 2.

β is the feedback factor of the second stage.

Substituting given values into this equation gives,

$$D_2' = \frac{0{\cdot}05}{1+0{\cdot}1\times 150}\text{ per unit}$$

$$= \frac{0{\cdot}05}{16} = 0{\cdot}00313\text{ p.u.} \quad\text{or}\quad 0{\cdot}313\%.$$

Hence, the application of negative feedback is seen to reduce the distortion introduced by the two-stage amplifier from 5% to 0·3125%.

(*b*) *Gain of Stage* 1. Let m'_2 be the gain of the second stage with 10% negative feedback.

From equation (1.83),

$$m'_2 = \frac{m_2}{1+\beta m_2}$$

$$= \frac{150}{1+0\cdot1\times150} = \frac{150}{16}.$$

Therefore $m'_2 = 9\cdot375.$

Now, $\dfrac{\mathbf{V}_o}{\mathbf{V}_{in}} = m_1 \times m'_2.$

Hence $m_1 = \dfrac{100}{9\cdot375} = 10\cdot67.$

Example 5

An amplifier, having a voltage amplification of $m = (3\cdot61+j0\cdot9)$ in the absence of negative feedback, is provided with a non-inductive resistance divider network which taps off 44·6% of the output voltage $\mathbf{V}_o$. This fraction is applied to a feedback network (β) which reduces its value by 50%, and advances the phase by 60°. The resultant voltage fed back is added to $\mathbf{V}_{in}$, the combination being applied to the amplifier m. By calculation and vector diagram, determine the gain of the amplifier with feedback, and state whether the feedback is positive or negative.

Solution

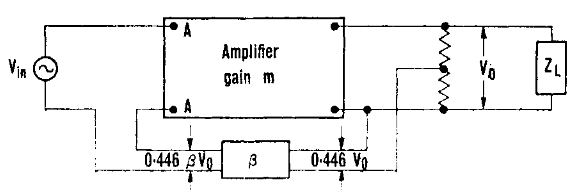


FIG. 7.9

(*a*) *Calculation*

The gain of the amplifier without feedback is given by

$$m = 3{\cdot}61 + j0{\cdot}9$$

and in polar form $\quad m = 3{\cdot}72 \angle 14^\circ$

Feedback fraction $\quad \beta = \frac{1}{2} \angle 60^\circ.$

When feedback is applied, the proportion of the output voltage fed to the feedback network is $0{\cdot}446\ \mathbf{V}_o$.

Now, referring to Fig. 7.9,

$$\mathbf{V}_{AA} = \mathbf{V}_{in} + 0{\cdot}446\beta\mathbf{V}_o$$

And after amplification,

$$\mathbf{V}_o = m\mathbf{V}_{AA} = m(\mathbf{V}_{in} + 0{\cdot}446\beta\mathbf{V}_o)$$

Therefore,

$$m' = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{m}{1 - 0{\cdot}446\beta m}$$

Substituting values gives,

$$m' = \frac{3{\cdot}72 \angle 14^\circ}{1 - 0{\cdot}446(3{\cdot}72 \angle 14^\circ \times 0{\cdot}5 \angle 60^\circ)}$$

$$= \frac{3{\cdot}72 \angle 14^\circ}{1 - 0{\cdot}446 \times 1{\cdot}86 \angle 74^\circ}$$

$$= \frac{3{\cdot}72 \angle 14^\circ}{1 - 0{\cdot}23 - j0{\cdot}8}$$

$$= \frac{3{\cdot}72 \angle 14^\circ}{0{\cdot}77 - j0{\cdot}8}$$

$$= \frac{3{\cdot}72 \angle 14^\circ}{1{\cdot}1 \angle -46^\circ}.$$

Whence $\quad m' = 3{\cdot}38 \angle 60^\circ.$

It can be seen that the application of feedback has the effect of reducing the amplifier gain from 3·72 to 3·38, thus the feedback is negative.

(*b*) *Vector Diagram*

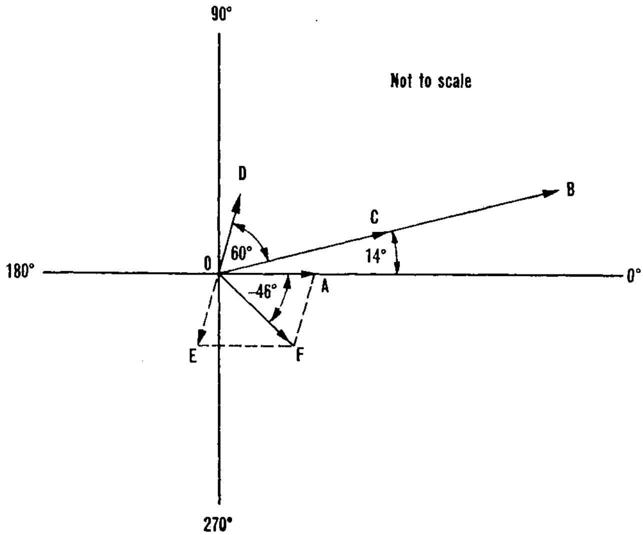


FIG. 7.10

In the diagram of Fig. 7.10

$$OA = \mathbf{V}_{AA} \qquad OD = 0{\cdot}446\beta\mathbf{V}_o$$

$$OB = m\mathbf{V}_{AA} = \mathbf{V}_o \qquad OE = -0{\cdot}446\beta\mathbf{V}_o$$

$$OC = 0{\cdot}446\,\mathbf{V}_o \qquad OF = \mathbf{V}_{in}.$$

The input voltage to the amplifier terminals ($\mathbf{V}_{AA}$) is represented by OA in the vector diagram. This is used as the reference vector. $OB = \mathbf{V}_o$ and shows that $\mathbf{V}_{AA}$ is amplified 3·72 times and phase advanced by 14°. 0·446 $\mathbf{V}_o$ is applied to the feedback network, and this voltage is represented by OC in the diagram.

$\mathbf{V}_{in} = \mathbf{V}_{AA} - 0{\cdot}446\beta\mathbf{V}_o$, hence to obtain a vector for $\mathbf{V}_{in}$, $OD(0{\cdot}446\beta\mathbf{V}_o)$ is rotated through 180° and the resulting vector $OE(-0{\cdot}446\beta\mathbf{V}_o)$ is added to $OA(\mathbf{V}_{AA})$. The resultant of OA and OE is the vector OF which gives the magnitude and phase angle of $\mathbf{V}_{in}$ relative to $\mathbf{V}_{AA}$.

The magnitude $| m' |$ of the amplifier gain is given by

$$| m' | = \left| \frac{V_o}{V_{in}} \right| = \frac{3{\cdot}72}{1{\cdot}1} = 3{\cdot}38.$$

It can be seen from Fig. 7.10 that $\mathbf{V}_o(OB)$ leads $\mathbf{V}_{in}(OF)$ by 60°.

Hence, $\quad m' = | m' | \angle\theta = 3{\cdot}38 \angle 60°$ as before.

Example 6

Calculate $| V_o |/| V_{in} |$ for the amplifier shown in Fig. 7.11. Neglect transformer losses, and assume the reactance of C is negligible at the frequency of the applied voltage $\mathbf{V}_{in}$.

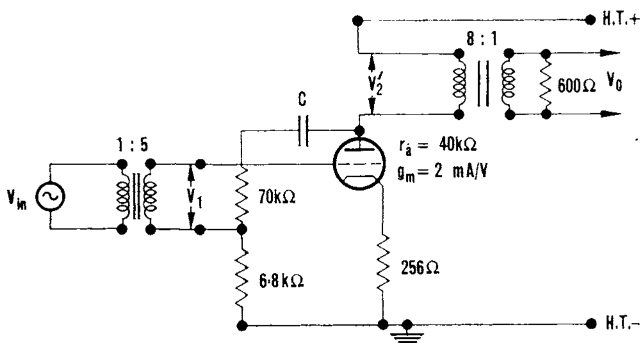


FIG. 7.11

Solution

The anode load resistance R'_L is given by

$$R'_L = \left[\frac{T_1}{T_2}\right]^2 R_L$$

where R'_L is the effect of the secondary load resistance R_L referred to the primary, and T_1/T_2 is the turns ratio of the output transformer.

Hence,

$$R'_L = 64 \times 600 = 38{\cdot}4 \text{ k}\Omega.$$

The circuit of Fig. 7.11 may be modified as shown in Fig. 7.12.

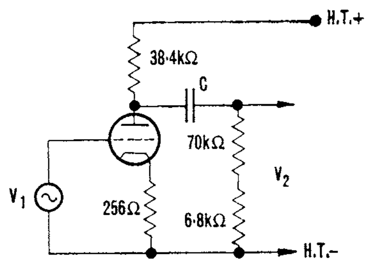


FIG. 7.12

Figure 7.12 does not include an overall transformation ratio between output and input of 5 : 8. The voltage feedback line is disconnected and the 76·8 kΩ series network forms part of the effective anode load. Although current negative feedback is introduced by the cathode resistor, a value for $|V_2|/|V_1|$ can be obtained from Fig. 7.12 and used to determine the gain of the stage $|V'_2|/|V_1|$ when the voltage feedback circuit is connected.

To find $|V_2|/|V_1|$

As the reactance of capacitor C can be ignored, the anode load is actually 38·4 kΩ in parallel with 76·8 kΩ.

Therefore,

$$\text{total anode load resistance} = \frac{38{\cdot}4 \times 76{\cdot}8}{38{\cdot}4 + 76{\cdot}8} = 25{\cdot}6 \text{ k}\Omega$$

and an equivalent circuit of Fig. 7.12 would be

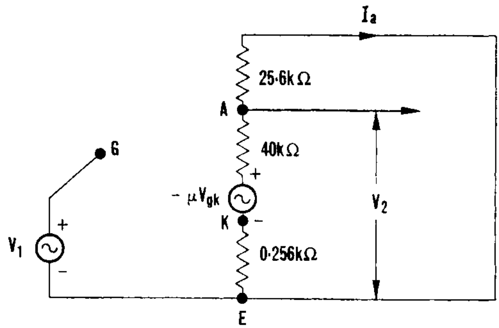


FIG. 7.13

From Fig. 7.13,

$$\mathbf{I}_a = \frac{-\mu \mathbf{V}_{gk}}{40+25{\cdot}6+0{\cdot}256}\ \text{mA} \tag{7.7}$$

But $\qquad \mathbf{V}_1 = \mathbf{V}_{gk}+\mathbf{V}_{ke}$ and $\mathbf{V}_{gk} = \mathbf{V}_1-\mathbf{V}_{ke}$.

And equation (7.7) becomes

$$\mathbf{I}_a = \frac{-\mu(\mathbf{V}_1-\mathbf{V}_{ke})}{40+25{\cdot}6+0{\cdot}256}\ \text{mA}.$$

Now, $\qquad \mathbf{V}_{ke} = -\mathbf{I}_a \times 0{\cdot}256\ \text{V}.$

Therefore,

$$\mathbf{I}_a = \frac{-\mu(\mathbf{V}_1+\mathbf{I}_a\times 0{\cdot}256)}{40+25{\cdot}6+0{\cdot}256}\ \text{mA}$$

$$= \frac{-\mu \mathbf{V}_1}{40+25{\cdot}6+0{\cdot}256(1+\mu)}\ \text{mA}$$

and

$$\mathbf{V}_2 = \mathbf{I}_a R_L = \frac{-\mu \mathbf{V}_1 25{\cdot}6}{40+25{\cdot}6+0{\cdot}256(1+\mu)}\ \text{V}.$$

From which,

$$\text{stage gain } m = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\mu 25{\cdot}6}{40+25{\cdot}6+0{\cdot}256(1+\mu)}$$

This expression may be written in general form thus:

$$m = \frac{-\mu R_L}{r_a + R_L + R_k(1+\mu)} \tag{7.8}$$

Now, $\mu = g_m r_a = 2\times 10^{-3}\times 40\times 10^{3} = 80.$

Therefore
$$m = \frac{-80\times 25{\cdot}6}{65{\cdot}6+20{\cdot}73}$$
$$= -23{\cdot}72$$

and $|m| = |V_2|/|V_1| = 23{\cdot}72.$

The gain of the amplifier of Fig. 7.12 is $|m| = 23{\cdot}72$. The minus sign in equation (7.8) indicates the 180° phase reversal due to the valve.

As the feedback network is purely resistive at the frequency of the applied voltage, there is no phase shift introduced by the network. This means that equation (1.83) can be used to calculate a figure for $|V_2'|/|V_1|$ if β and $|m|$ are taken as $6{\cdot}8/(70+6{\cdot}8)$ and 23·72 respectively.

To find $|V_2'|/|V_1|$

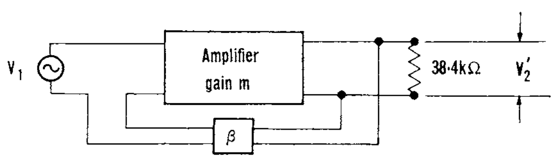


FIG. 7.14

From Fig. 7.11,

$$\beta = \frac{6{\cdot}8}{6{\cdot}8+70} = 0{\cdot}0887$$

and with the feedback loop connected as shown in Fig. 7.14,

$$\left|\frac{V_2'}{V_1}\right| = \frac{23{\cdot}72}{1+0{\cdot}0887\times 23{\cdot}72} = \frac{23{\cdot}72}{3{\cdot}1}$$
$$= 7{\cdot}65.$$

Reference to Fig. 7.11 now reveals that,

$$\left|\frac{V_{in}}{V_1}\right| = \frac{1}{5}, \quad \text{and} \quad \left|\frac{V_2'}{V_o}\right| = \frac{8}{1}$$

Hence,
$$\left|\frac{V_o}{V_{in}}\right| = \frac{5}{8}\times\left|\frac{V_2'}{V_1}\right| = \frac{5}{8}\times 7{\cdot}65 = 4{\cdot}78.$$

Example 7

A triode used in a cathode follower stage has an anode slope resistance of 8 kΩ and an amplification factor of 25. The load resistance in the cathode circuit is 12 kΩ, and the resistance between grid and cathode is 2 MΩ. Deduce expressions for, and hence evaluate, the input and output resistance of the stage.

I.E.R.E., May 1963

Solution

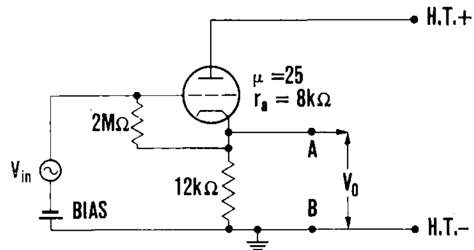


FIG. 7.15. Basic circuit.

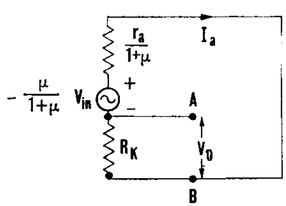


FIG. 7.16. Cathode follower equivalent circuit derived in Section 1.19.

The following points were established in Section 1.19 and should be justified again here as part of this solution. However, to avoid undue repetition, these points will merely be stated without proof.

1. $$\mathbf{V}_{in} = \mathbf{I}_{in}R_g + \mathbf{V}_o$$

where $\mathbf{V}_{in}$ is the r.m.s. value of input voltage;

$\mathbf{I}_{in}$ is the current which flows in the input circuit (not grid current as such);

R_g is the grid leak resistance;

$\mathbf{V}_o$ is the r.m.s. value of output voltage.

2. $$\text{Stage gain } m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{\mu R_k}{r_a + R_k(1+\mu)}$$

where R_k is the cathode load resistance;

μ is the amplification factor of the valve;

and r_a is the anode slope resistance of the valve.

Input Resistance R_i

It was shown in Section 1.19 that

$$\mathbf{V}_{in} = \mathbf{I}_{in}R_g + \mathbf{V}_o$$

but $$\mathbf{V}_o = m\mathbf{V}_{in}$$

Therefore $$\mathbf{V}_{in} = \mathbf{I}_{in}R_g + m\mathbf{V}_{in}$$

and $$\mathbf{V}_{in}(1-m) = \mathbf{I}_{in}R_g$$

Now, input resistance R_i of the circuit of Fig. 7.15 is given by $\mathbf{V}_{in}/\mathbf{I}_{in}$.

Therefore $$R_i = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = \frac{R_g}{1-m} \tag{7.9}$$

From equation (1.72) stage gain m is given by

$$m = \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \frac{\mu R_k}{r_a + R_k(1+\mu)}$$

Substituting values gives

$$m = \frac{25 \times 12}{8 + 12(1+25)} = \frac{300}{320}$$

Therefore $$m = 0{\cdot}9375.$$

And input resistance from equation (7.9) is

$$R_i = \frac{2 \times 10^6}{1 - 0{\cdot}9375}$$

$$= \frac{2 \times 10^6}{0{\cdot}0625}$$

Whence $$R_i = 32\ \text{M}\Omega.$$

Output Resistance R_o

The output resistance R_o of a cathode follower is the resistance which appears across terminals A and B of Fig. 7.16 with R_k disconnected, and the internal e.m.f. $[-\mu\mathbf{V}_{in}/(1+\mu)]$ reduced to zero. Hence, from the simplified equivalent circuit of Fig. 7.16,

$$R_o = \frac{r_a}{1+\mu}$$

Substituting values gives

$$R_o = \frac{8000}{1+25} = 308\ \Omega.$$

Example 8

A triode valve is connected as a cathode follower. The valve parameters are $g_m = 3$ mA/V and $r_a = 15$ kΩ. The anode to grid capacitance C_{ag} is 3 pF, and the capacitance between grid and cathode C_{gk} is 2·5 pF. If the cathode load resistance is 4 kΩ, calculate (a) stage gain, (b) output resistance and (c) input capacitance from first principles.

Solution

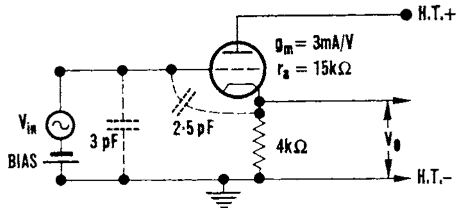


FIG. 7.17. Basic circuit.

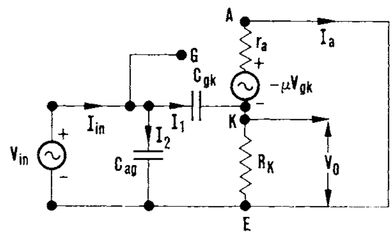


FIG. 7.18. Constant voltage equivalent of Fig. 7.17.

(a) From equation (1.72)*, stage gain is given by

$$m = \frac{\mu R_k}{r_a + R_k(1+\mu)}$$

and $$\mu = g_m r_a = 3\times 10^{-3}\times 15\times 10^3 = 45.$$

Therefore $$m = \frac{45\times 4}{15+4\times 46} = \frac{180}{199} = 0{\cdot}9044.$$

*The expressions obtained in Section 1.19 for m and R_o are not invalidated by the presence of small interelectrode and stray capacitances.

(b) The output resistance R_o is found in the usual way from

$$R_o = \frac{r_a}{1+\mu} = \frac{15{,}000}{1+45}\,\Omega$$

Therefore $R_o = 326\ \Omega$

(c) From Fig. 7.18,

$$\mathbf{I}_{in} = \mathbf{I}_1 + \mathbf{I}_2$$

$$= \mathbf{V}_{in}\omega C_{ag} + \omega C_{gk}(\mathbf{V}_{in} - \mathbf{V}_o).$$

Now, input admittance $Y_{in} = \mathbf{I}_{in}/\mathbf{V}_{in}$

$$= \omega C_{ag} + \omega C_{gk}\left(1 - \frac{\mathbf{V}_o}{\mathbf{V}_{in}}\right).$$

Therefore, $$Y_{in} = \omega C_{ag} + \omega C_{gk}(1-m)$$

However, if C_{ak} can be neglected, and if R_k is resistive,

$$Y_{in} = \omega C_{in}$$

where $$C_{in} = C_{ag} + C_{gk}(1 - |m|). \qquad (7.10)$$

C_{in} from equation (7.10) is the input shunt capacitance. Substituting values into this equation gives,

$$C_{in} = 3 \times 10^{-12} + 2{\cdot}5 \times 10^{-12}(1 - 0{\cdot}9044)\ \mathrm{F}.$$

Therefore

$$C_{in} = 3{\cdot}249\ \mathrm{pF}.$$

It can be seen from equation (7.10) that the larger the stage gain $|m|$, the larger the input impedance and the smaller the input admittance. Also, as $|m| \to 1$, the input shunt capacitance is due almost entirely to C_{ag}.

Problems with Answers

1. An amplifier has a gain of 600 without feedback. What value of input signal voltage will produce a 1 V output if 8% negative feedback is applied?

81·6 mV.

2. A single stage amplifier without feedback has a gain of 10. A second amplifier, operated from the same power supply, has two stages each with a gain of 10, but there is negative feedback reducing the overall gain to 10. Calculate the percentage feedback. As a result of voltage variations, the gain of the first amplifier drops to 9. What is the gain of the second amplifier?

9%; 9·78.

3. An amplifier, in the absence of feedback, has a gain which is liable to fall by 40% of its rated value due to uncontrollable supply variations. If, by application of negative feedback, an amplifier is to be produced with a rated gain of 100, determine the gain before feedback if the gain with feedback never falls below 99.

6000.

4. Draw a simple closed loop diagram, and obtain an expression for the output voltage V'_o of a valve amplifier with 100% negative feed-back, in terms of the input voltage V_{in} and the voltage gain m. What is the input : output ratio if the gain is very high?

$mV_{in}/(1+m)$; 1 : 1.

5. Determine the percentage change of gain of the amplifier of Fig. 7.19 if μ changes from 40 to 35, and at the same time r_a changes from 10 kΩ

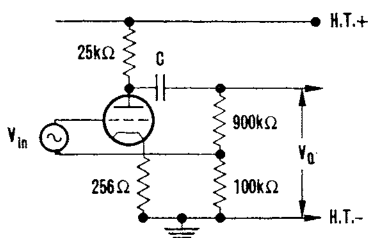


FIG. 7.19

to 12 kΩ. The reactance of C may be assumed negligible, so also may be the shunting effect of the coupling components on the 25 kΩ load resistor.

4·81%.

6. A single-stage amplifier has a gain without feedback of 20, and an output of 105 V with 8% distortion. If feedback is applied, calculate

(a) the value of β necessary to reduce the distortion to 2%;

(b) The output voltage of the amplifier when an input is applied which produces 27 V output voltage from the same amplifier with no feedback.

15%; 6·75.

7. An amplifier has a gain of 125 without feedback and provides 50 V across the output load with 12% second harmonic distortion. What value of feedback fraction β will reduce the distortion to 1%? What must be the gain of an additional stage placed before this amplifier, if the signal voltage is to remain at 50 V?

8·8%; 12.

8. State the conditions for stability as applied to amplifiers using feedback.

An R.C. coupled amplifier consists of a triode for which $g_m = 3$ mA/V, and $r_a = 8$ kΩ. A non-inductive resistance of 480 Ω is connected in the cathode lead. The load resistance in the anode circuit is 12 kΩ, and this is tapped to give a feedback factor β of 0·05. Calculate the effective gain of the stage.

6·2. I.E.R.E., May 1963

9. Discuss the advantages and disadvantages of applying negative feedback to an amplifier.

An amplifier has a gain of $200-j300$ before feedback is applied. What will be the overall gain if 4% of the output is applied in series with the input as negative feedback?

$25{\cdot}9\angle 183{\cdot}45°$. H.N.C.

10. Calculate the stage gain ($|V_o|/|V_{in}|$) and phase shift for the feedback amplifier shown in Fig. 7.20.

4·84; 184° 2′. H.N.C.

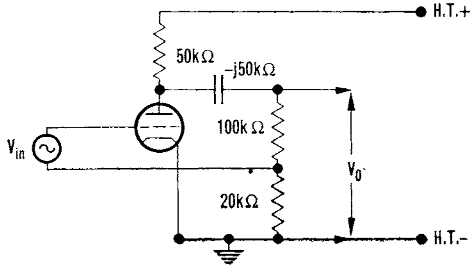


FIG. 7.20

11. Calculate $|V_o|/|V_{in}|$ for the amplifier shown in Fig. 7.21. Neglect transformer losses and assume the reactance of C to be negligible at the frequency of the applied voltage.

7·07.

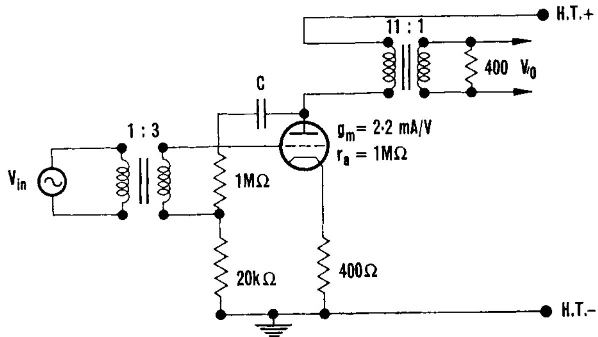


FIG. 7.21

12. A triode valve has an anode slope resistance of 20 kΩ and amplification factor of 25. The input is applied between control grid and negative H.T. and the cathode is connected to the negative H.T. line through a resistance of 1 kΩ. The resistance of the anode load is 10 kΩ. The valve is R.C. coupled to the next stage, the components used being a capacitor of 0·01 μF, and a resistor of 50 kΩ. Calculate the gain and phase shift of the stage if the output is taken across the 50 kΩ resistor and the input sine wave has a frequency of 400 c/s.

$3{\cdot}14\angle 214{\cdot}5°$.

13. (a) An amplifier consists of n identical stages, each of gain m, connected in cascade. If identical negative feedback is applied to each of the n stages such that the overall gain with feedback is m', show that dm'/m' is given by

$$\frac{dm'}{m'} = \frac{n}{(1+\beta m)} \times \frac{dm}{m}.$$

(b) If, instead of applying negative feedback as in part (a) above, feedback is applied between the nth stage and the first stage only, such that the overall gain with feedback is still m', show that dm'/m' is given by

$$\frac{dm'}{m'} = \frac{n}{(1+\beta m^n)} \times \frac{dm}{m}.$$

Which of the above methods is better from the point of view of gain stability?

β is the feedback factor in each of the above cases.

14. Explain what is meant by negative feedback as applied to an amplifier, distinguishing between *current* and *voltage* feedback. What are its advantages and disadvantages?

Derive an expression for the gain at the fundamental frequency of a negative feedback amplifier in terms of the feedback factor and the gain without feedback.

Using this result, or otherwise, determine the gain of a cathode follower in terms of the cathode load R_k and the valve parameters. Deduce that the equivalent constant voltage generator has an e.m.f. of $\mu V_{in}/(1+\mu)$, and an internal resistance of approximately $1/g_m$, where V_{in} is the voltage across the input terminals. H.N.C.

15. Find the impedance of the circuit of Fig. 7.22 "looking into" *AB* in the direction of the arrow.

2·21 kΩ.

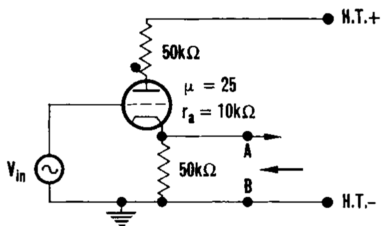


FIG. 7.22

16. Derive expressions for the output voltage and output resistance of a cathode follower stage. A cathode follower employs a triode having $\mu = 45$, and $r_a = 10$ kΩ. If the cathode load resistance is 1·5 kΩ, find the gain of the stage, and also the output impedance.

0·854; 217·5 Ω.

17. The anode load resistor of a triode valve is 50 kΩ and so also is the resistor which connects the valve cathode to the H.T. negative line. If $r_a = 10$ kΩ and $\mu = 25$, find the gain of the stage to the anode. If a 1·5 MΩ grid leak resistor is connected between grid and cathode, find also the input resistance of the stage. Stray capacitance and inter-electrode capacitance effects may be neglected.

−0·918; 18·3 MΩ.

18. A triode valve is connected as a cathode follower. The valve parameters are $g_m = 2{\cdot}5$ mA/V and $r_a = 10$ kΩ. The anode to grid capacitance is 2·6 pF, and the capacitance between grid and cathode C_{gk} is 2·1 pF. If the cathode load resistance is 12 kΩ, calculate from first principles the stage gain, output resistance, and the shunt input capacitance of the stage.

0·932; 384·5 Ω; 2·743 pF.

19. A triode valve operated as a cathode follower has a resistive cathode load R_k and an a.c. input V volts r.m.s. Find an expression for the power developed in the load and determine, in terms of the usual valve parameters, the value of R_k for which this is a maximum. Hence deduce the impedance of the equivalent generator.

A triode of anode slope resistance 20,000 Ω and amplification factor of 19 is operated as a cathode follower, and an input signal of 3 $V_{r.m.s.}$ is applied between grid and earth.

Calculate the value of cathode resistor which will develop maximum power, and calculate, for this condition, the output voltage and power.

1 kΩ; 2·03 mW; 1·425 $V_{r.m.s.}$ H.N.C.

20. The anode load resistor of a triode valve is 60 kΩ, and so also is the resistor connecting the cathode to earth. The input voltage to the triode is applied between grid and earth. A 2 MΩ grid leak is connected between grid and cathode. Outputs are taken from both anode and cathode. The valve has an anode slope resistance of 20 kΩ and an amplification factor of 30. Calculate

(a) the stage gain from grid to anode,

(b) the input impedance.

What would be the stage gain at $\omega = 10^6$ from grid to anode if the cathode resistor is made zero, and a 50 pF capacitor is shunted between anode and earth? Assume that the valve parameters are unaltered.

$-0{\cdot}928$; 27·8 MΩ; $18\angle 143°\ 8'$. H.N.C.

CHAPTER 8

Oscillators

Worked Examples

Example 1

The simplified equivalent circuit of a quartz crystal used in a crystal oscillator is shown in Fig. 8.1. Show that the two resonant frequencies of this crystal are given by

$$\text{(i)} \quad \omega^2 LC_1 = 1$$

and

$$\text{(ii)} \quad \omega^2 LC_1 = \left(1+\frac{C_1}{C_0}\right).$$

Calculate the series resonant frequency from (i), and the anti-resonant frequency from (ii), for a crystal in which $L = 7{\cdot}82$ H, $C_1 = 0{\cdot}02$ pF, and $C_0 = 15$ pF.

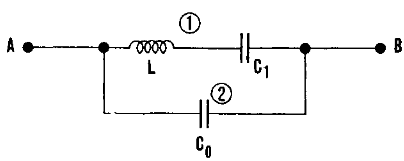


Fig. 8.1

Solution

Referring to Fig. 8.1,

$$\frac{1}{Z_{AB}} = \frac{1}{Z_1}+\frac{1}{Z_2} = \frac{Z_1+Z_2}{Z_1 Z_2}.$$

Therefore $$Z_{AB} = \frac{Z_1 Z_2}{Z_1+Z_2}, \tag{8.1}$$

but $$Z_1 = j\omega L+\frac{1}{j\omega C_1} \tag{8.2}$$

and $$Z_2 = \frac{1}{j\omega C_0} \tag{8.3}$$

Hence, substituting for Z_1 and Z_2 from equations (8.2) and (8.3) into equation (8.1) gives

$$Z_{AB} = \frac{\left(j\omega L+\dfrac{1}{j\omega C_1}\right)\dfrac{1}{j\omega C_0}}{j\omega L+\dfrac{1}{j\omega C_1}+\dfrac{1}{j\omega C_0}}$$

$$= \frac{(\omega^2 LC_1-1)}{\omega^2 C_1 C_0\left[j\omega L+\dfrac{1}{j\omega}\left(\dfrac{1}{C_1}+\dfrac{1}{C_0}\right)\right]}$$

$$= \frac{\omega^2 LC_1-1}{\omega^2 C_1 C_0\left(\dfrac{C_1+C_0-\omega^2 LC_0C_1}{j\omega C_0 C_1}\right)}$$

Whence $$Z_{AB} = \frac{j(\omega^2 LC_1-1)}{\omega(C_1+C_0-\omega^2 LC_0C_1)} \tag{8.4}$$

Now, when the circuit is at series resonance, Z_{AB} is zero as there is no resistance included in the equivalent circuit of Fig. 8.1.

If $Z_{AB} = 0$ in equation (8.4),

$$\omega^2 LC_1-1 = 0$$

therefore, $$\omega^2 LC_1 = 1. \tag{8.5}$$

When the circuit is at parallel resonance Z_{AB} is infinite. However, the denominator of equation (8.4) is zero when Z_{AB} is infinite,

therefore, $$\omega(C_1+C_0-\omega^2 LC_0C_1) = 0$$

and $$\omega^2 LC_1 = \left(1+\frac{C_1}{C_0}\right). \tag{8.6}$$

The series resonant frequency of the crystal given in the problem may be found using equation (8.5).

$$\omega^2 = \frac{1}{LC_1}$$

where $\omega = 2\pi f$ and f is the series resonant frequency.

Hence, $$f = \frac{1}{2\pi\sqrt{(LC_1)}} = \frac{1}{6{\cdot}284\sqrt{7{\cdot}82\times 2\times 10^{-14}}}$$

$$= 402{\cdot}4 \text{ kc/s}. \tag{8.7}$$

The parallel, or anti-resonant frequency, may be found using equation (8.6),

$$f = \frac{1}{2\pi}\left[\frac{1}{LC_1}\left(1+\frac{C_1}{C_0}\right)\right]^{\frac{1}{2}}$$

$$= \frac{1}{2\pi\sqrt{(LC_1)}}\left(1+\frac{C_1}{C_0}\right)^{\frac{1}{2}}.$$

However, from equation (8.7), $1/(2\pi\sqrt{(LC_1)})$ is 402·4 kc/s.

Therefore, $$f = 402{\cdot}4\times 10^3\left(1+\frac{2\times 10^{-14}}{15\times 10^{-12}}\right)^{\frac{1}{2}} \text{ c/s}$$

this may be expanded using the Binomial Theorem thus,

$$f \simeq 402{\cdot}4\times 10^3(1+\tfrac{1}{2}\times 0{\cdot}00133) \text{ c/s}$$

$$= 402{\cdot}4\times 10^3\times 1{\cdot}000665 \text{ c/s},$$

whence, $$f = 402{,}668 \text{ c/s}. \tag{8.8}$$

It can be seen from equations (8.7) and (8.8) that the separation between the series resonant frequency and the parallel resonant frequency is only 268 c/s.

In practice, a resistive component will be present in branch (1) of the equivalent circuit of Fig. 8.1. This means that the impedance of the crystal at series resonance will not be zero, and at parallel resonance it will not be infinite. Figure 8.2 shows how the current through a crystal varies with frequency.

The crystal behaves inductively for frequencies between f_1 and f_2. Figure 8.3 shows this component in the grid circuit of an oscillator. Here, both the crystal and the anode tuned circuit

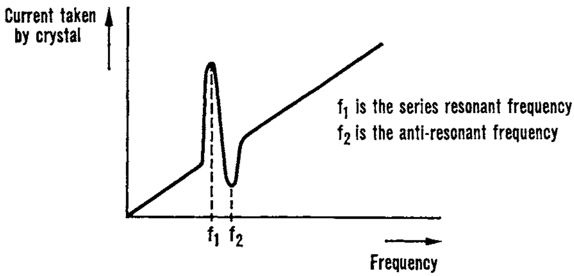


FIG. 8.2

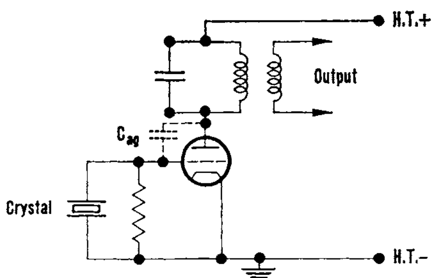


FIG. 8.3

behave inductively, and the capacitive element necessary to complete the oscillator circuit (see Section 1.24) is provided by the anode–grid capacitance of the valve, C_{ag}.

Example 2

Derive expressions giving (a) the minimum value of mutual inductance between anode and grid coils required to maintain oscillations in a tuned grid oscillator, (b) the frequency of oscillations of a tuned grid oscillator.

A certain pentode has a mutual conductance of 4 mA/V. It is used in the circuit of a tuned-grid oscillator which has a grid tuned circuit comprising a 31 mH coil having a series resistance of 250 Ω, and a tuning capacitance of 400 pF. Determine the minimum value of mutual inductance which will enable oscillations to be maintained, and also the frequency of oscillations.

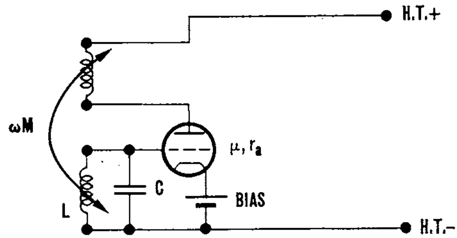


FIG. 8.4. Basic circuit—tuned grid oscillator.

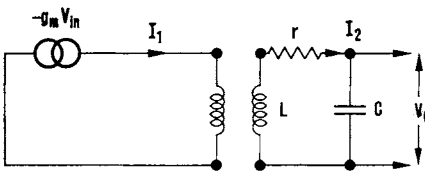


FIG. 8.5. Constant current equivalent of Fig. 8.4.

Solution

Let the input voltage $\mathbf{V}_{in}$ of Fig. 8.5 be applied between grid and cathode of the oscillator valve from an external generator, and let the output voltage $\mathbf{V}_o$ be taken across the tuning capacitor *C*. If continuous oscillations are to be obtained when the external generator is removed, and the capacitor output connected between grid and cathode of the triode, the vector $\mathbf{V}_o$ must have the same argument as $\mathbf{V}_{in}$, and have at least the same magnitude.

The following derivation assumes that

(i) grid current does not flow,
(ii) valve parameters are constant,
(iii) sinusoidal waveforms are obtained,
(iv) r_a is infinite.

Now, under these conditions, the current $\mathbf{I}_1$ which flows through the primary coil is given by

$$\mathbf{I}_1 = -g_m \mathbf{V}_{\text{in}}$$

and the e.m.f. $\mathbf{E}_2$ induced in the secondary winding is, therefore,

$$\mathbf{E}_2 = \pm j\omega M \mathbf{I}_1 = \pm j\omega M g_m \mathbf{V}_{\text{in}}$$

$\mathbf{E}_2$ is the source of e.m.f. which feeds the grid tuned circuit.

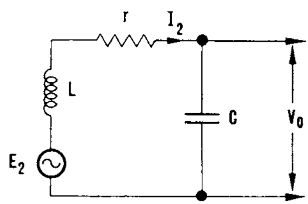


FIG. 8.6

It may be seen from Fig. 8.6, that

$$\mathbf{I}_2 = \frac{\mathbf{E}_2}{Z_2} = \pm \frac{j\omega M g_m \mathbf{V}_{\text{in}}}{r + j\omega L + \dfrac{1}{j\omega C}}$$

and
$$\mathbf{V}_o = \pm \frac{j\omega M g_m \mathbf{V}_{\text{in}}}{\left(r + j\omega L + \dfrac{1}{j\omega C}\right)} \times \frac{1}{j\omega C}$$

hence
$$\frac{\mathbf{V}_o}{\mathbf{V}_{\text{in}}} = \pm \frac{j\omega M g_m}{1 - \omega^2 LC + j\omega Cr}$$

For oscillations, $\mathbf{V}_o$ and $\mathbf{V}_{in}$ must be equal, whence

$$1 = \pm \frac{j\omega M g_m}{1-\omega^2 LC + j\omega Cr}$$

and $$1-\omega^2 LC + j\omega Cr \pm j\omega M g_m = 0. \tag{8.9}$$

Equating the real parts of equation (8.9) to zero gives

$$1 = \omega^2 LC$$

where $\omega = 2\pi f$, and f is the frequency of oscillations,

thus, $$f = \frac{1}{2\pi\sqrt{(LC)}}. \tag{8.10}$$

Equating the imaginary parts of equation (8.9) to zero gives

$$\omega Cr = \pm\omega M g_m,$$

hence, it can be seen that the minimum value of mutual inductance which will allow oscillations to be maintained is

$$\pm M = \frac{Cr}{g_m}. \tag{8.11}$$

However, oscillation will only occur if the sign of M is positive. The minus sign denotes that the coil L may be wound or connected in such a way as to apply negative feedback to the stage. Under these conditions, oscillations could not be maintained.

For the component values given in the problem,

$$f = \frac{1}{2\pi\sqrt{(LC)}} = \frac{1}{2\pi\sqrt{(31\times 10^{-3}\times 4\times 10^{-10})}}$$

$$= \frac{10^6}{2\pi\sqrt{12{\cdot}4}}$$

$$= 45 \text{ kc/s}.$$

and $$M = \frac{Cr}{g_m} = \frac{400\times 10^{-12}\times 250}{4\times 10^{-3}}$$

$$= 25\ \mu\text{H}.$$

Example 3

A parallel tuned circuit has a capacitance of 350 pF, a coil of Q factor 45·5, and a resonant frequency of 50 kc/s. It is used in a tuned anode oscillator with a valve of anode slope resistance 10 kΩ, and amplification factor 30. Find the frequency of oscillation, and the value of M between anode and grid coils for maintained oscillations.

Solution

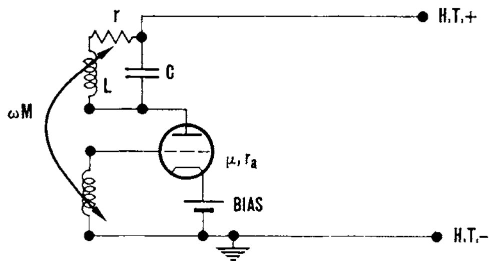


FIG. 8.7

It was shown in Section 1.23 that the frequency of oscillations in a tuned anode oscillator, such as the one shown in Fig. 8.7, is given by

$$f = \frac{1}{2\pi}\sqrt{\left[\frac{1}{LC}\left(1+\frac{r}{r_a}\right)\right]}$$

and that in order to maintain oscillations, the mutual inductance M must have a minimum value given by

$$M = \frac{L+Crr_a}{\mu}.$$

In order to calculate numerical values for both f and M, the inductance L and series resistance r of the anode coil must first be found (see Fig. 8.8).

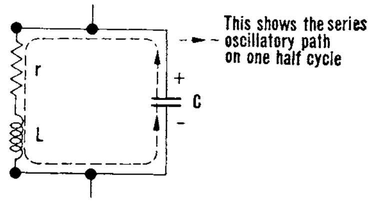


FIG. 8.8

To find r

Figure 8.8 shows a parallel tuned circuit which has a series oscillatory path. Such a circuit was used in Section 1.23 when the above-mentioned formulae for f and M were derived. Now, at resonance,

$$Q = \frac{\omega L}{r} = \frac{1}{\omega C r} \tag{8.12}$$

whence, $$r = \frac{1}{\omega C Q}$$

but $$\omega = 2\pi f = 2\pi \times 50 \times 10^3$$
$$= 314 \times 10^3 \text{ rad/s,}$$

therefore, $$r = \frac{1}{314 \times 10^3 \times 350 \times 10^{-12} \times 45{\cdot}5}\ \Omega$$
$$= 200\ \Omega.$$

To find L

From equation (8.12),

$$L = \frac{Qr}{\omega} = \frac{45{\cdot}5 \times 200}{314 \times 10^3}\ \text{H}$$
$$= \frac{9{\cdot}1}{314} = 29 \text{ mH.}$$

These values for L and r may now be used, along with given quantities, to determine the frequency of oscillations and the mutual inductance.

Frequency of Oscillation

$$f = \frac{1}{2\pi}\sqrt{\left[\frac{1}{LC}\left(1+\frac{r}{r_a}\right)\right]}$$

$$= \frac{1}{6 \cdot 28}\left[\frac{1}{29\times 10^{-3}\times 350\times 10^{-12}}\left(1+\frac{200}{10{,}000}\right)\right]^{\frac{1}{2}} \text{ c/s}$$

$$= \frac{1}{6 \cdot 28}\sqrt{\left(\frac{1 \cdot 02\times 10^{12}}{10 \cdot 16}\right)} \text{ c/s}$$

$$= 50 \cdot 4 \text{ kc/s.}$$

To find M

$$M = \frac{L+Crr_a}{\mu}$$

$$= \frac{29\times 10^{-3}+350\times 10^{-12}\times 200\times 10\times 10^{3}}{30} \text{ H}$$

$$= \frac{29 \cdot 7\times 10^{-3}}{30} \text{ H}$$

$$= 0 \cdot 99 \text{ mH.}$$

Example 4

A sinusoidal voltage $\mathbf{V}_1$ is applied to the input of the network shown in Fig. 8.9. Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{1+6k+5k^2+k^3}$$

where $$k = \frac{Z_1}{Z_2}.$$

A network of this type is used in a phase shift oscillator. Draw a suitable circuit and use the equation above to find the condition for oscillation and the oscillation frequency. State clearly any assumptions made. I.E.R.E., May, 1962

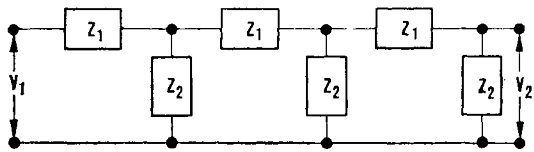


FIG. 8.9

Solution

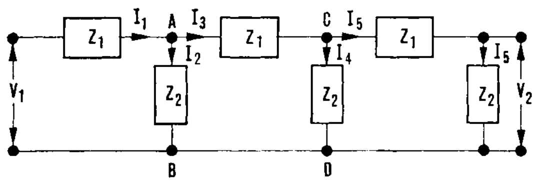


FIG. 8.10

From Fig. 8.10,

$$\mathbf{V}_2 = \mathbf{I}_5 Z_2 \quad \text{or} \quad \mathbf{I}_5 = \frac{\mathbf{V}_2}{Z_2} \tag{8.13}$$

Now, $$\mathbf{I}_4 = \frac{\mathbf{V}_{CD}}{Z_2} = \frac{\mathbf{I}_5(Z_1+Z_2)}{Z_2}$$

but $Z_1/Z_2 = k$, and $\mathbf{I}_5 = \mathbf{V}_2/Z_2$,

therefore, $$\mathbf{I}_4 = \mathbf{I}_5(k+1) = \frac{\mathbf{V}_2(k+1)}{Z_2} \tag{8.14}$$

Also, $$\mathbf{I}_3 = \mathbf{I}_4 + \mathbf{I}_5$$

Therefore, from equations (8.13) and (8.14) we have

$$\mathbf{I}_3 = \frac{\mathbf{V}_2(k+1)}{Z_2} + \frac{\mathbf{V}_2}{Z_2} = \frac{\mathbf{V}_2}{Z_2}(2+k) \tag{8.15}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{AB}}{Z_2} = \frac{\mathbf{I}_3 Z_1 + \mathbf{I}_4 Z_2}{Z_2} = k\mathbf{I}_3 + \mathbf{I}_4.$$

From equations (8.15) and (8.14)

$$\mathbf{I}_2 = k\left[\frac{\mathbf{V}_2}{Z_2}(2+k)\right] + \frac{\mathbf{V}_2}{Z_2}(k+1)$$

$$= \frac{\mathbf{V}_2}{Z_2}(2k+k^2+k+1) = \frac{\mathbf{V}_2}{Z_2}(k^2+3k+1). \tag{8.16}$$

Now, $\mathbf{V}_1 = \mathbf{I}_1 Z_1 + \mathbf{I}_2 Z_2$

and $\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3.$

Therefore,

$$\mathbf{V}_1 = Z_1(\mathbf{I}_2 + \mathbf{I}_3) + \mathbf{I}_2 Z_2$$

$$= \mathbf{I}_2(Z_1 + Z_2) + \mathbf{I}_3 Z_1.$$

Hence, from equations (8.15) and (8.16)

$$\mathbf{V}_1 = \mathbf{V}_2 \frac{(Z_1+Z_2)}{Z_2}(k^2+3k+1) + \mathbf{V}_2 \frac{Z_1}{Z_2}(2+k)$$

$$= \mathbf{V}_2[(k+1)(k^2+3k+1) + 2k + k^2]$$

and $$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{1+6k+5k^2+k^3}. \tag{8.17}$$

Circuit Diagram

It will be assumed in Fig. 8.11 that,

(i) the network input impedance is infinite,
(ii) the input impedance of the valve is also infinite,
(iii) the valve is a pentode,

(iv) the valve parameters are constant,
(v) grid current does not flow,
(vi) sinusoidal oscillations are produced.

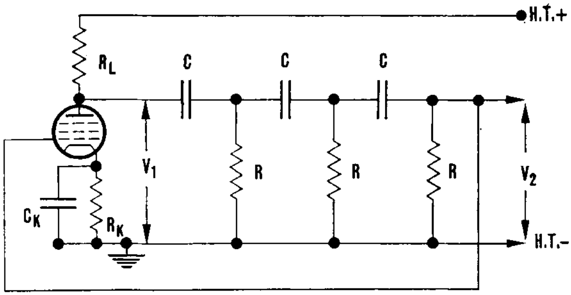


FIG. 8.11

From Fig. 8.11,

$$Z_1 = -\frac{j}{\omega C}$$

and $$Z_2 = R,$$

hence $$k = \frac{Z_1}{Z_2} = -\frac{j}{\omega CR}$$

and equation (8.17) becomes

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{1 - j\dfrac{6}{\omega CR} - \dfrac{5}{(\omega CR)^2} + j\dfrac{1}{(\omega CR)^3}}$$

$$= \frac{1}{1 - \dfrac{5}{(\omega CR)^2} + \dfrac{j}{\omega CR}\left[-6 + \dfrac{1}{(\omega CR)^2}\right]} \tag{8.18}$$

For oscillations, both phase and magnitude conditions must be satisfied. As far as phase shift is concerned, when the circuit oscillates continuously there is 60° phase advance due to each series CR branch of the network, making 180° in all. There is also 180° phase shift due to the valve, thus the phase shift around the loop is 360°.

For magnitude conditions to be satisfied, it is necessary that the attenuation through the phase shift network be made up by amplification through the pentode. Hence, as the loss through the network is $| V_2 / V_1 |$, the pentode gain must be at least $| V_1 / V_2 |$ in magnitude if oscillations are to be maintained.

When the phase shift through the network is 180°, imaginary terms in the denominator of equation (8.18) are zero,

therefore $$6 = \frac{1}{(\omega CR)^2}$$

and $$\omega = \frac{1}{CR\sqrt{6}} \tag{8.19}$$

where $\omega = 2\pi f$, and f is the frequency of oscillation.

Inverting equation (8.18), and ignoring imaginary terms,

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = 1 - \frac{5}{(\omega CR)^2}$$

However, $1/(\omega CR)^2 = 6$,

therefore $$\frac{\mathbf{V}_1}{\mathbf{V}_2} = 1 - 5 \times 6 = -29. \tag{8.20}$$

Hence, oscillations will be maintained at a frequency obtainable from equation (8.19) provided the gain of the pentode amplifier is 29. The minus sign of equation (8.20) denotes the 180° phase shift due to the valve.

Problems with Answers

1. Derive from first principles an expression for the necessary relationship between circuit and valve constants for oscillations to commence, and an expression for the frequency of oscillations produced in a tuned-anode oscillator. Indicate how the stability of the oscillator is affected by the anode slope resistance of the valve.

A parallel tuned circuit has a capacitance of 400 pF, a coil of Q factor 35, and a resonant frequency of 45 kc/s. It is used in a tuned-anode oscillator with a valve of $r_a = 8$ kΩ, and $\mu = 32$. Find the frequency of oscillation, and the value of M between the anode and grid coils for maintained oscillations.

45·8 kc/s; 1 mH. I.E.R.E., May 1956

2. Discuss the importance of the polar plot of the open-loop gain of a feedback amplifier.

Consider the tuned anode oscillator as a feedback amplifier, derive an expression for its open-loop gain and hence obtain the maintenance condition and the oscillation frequency. I.E.R.E., Nov. 1962

3. The anode load of a tuned-anode oscillator has a coil of $Q = 50\pi$ at the resonant frequency in parallel with a 500 pF capacitor. If the alternating anode current has a value of 12 mA peak, and the power dissipated in the coil is 1 W, calculate

(i) the peak current in the coil;
(ii) the resonant frequency of the circuit;
(iii) the dynamic resistance.

20·5 mA; 3·6 Mc/s; 14 kΩ. I.E.R.E., May 1959

4. A parallel tuned circuit has a capacitance of 400 pF, a coil Q factor 40, and a resonant frequency of $500/2\pi$ kc/s. It is used in a tuned-anode oscillator with a valve of $r_a = 10$ kΩ, and $\mu = 40$. Find the frequency of oscillation, and the minimum value of mutual coupling between anode and grid coils for sustained oscillations.

8 kc/s; 262·5 μH.

5. Find approximately the condition for sustained oscillations in a triode with a tuned grid circuit, given that the tuning capacitance is 0·01 μF, and the loss resistance 200 Ω. The valve parameters are $\mu = 10$, and $r_a = 10$ kΩ.

2 mH.

6. For the tuned-grid oscillator of Fig. 8.12, show that

$$\omega^2\left(L_2C_2+\frac{L_1C_2R_2}{r_a}\right) = 1.$$

Show also that oscillations will be maintained if the mutual inductance M has a minimum value given by

$$M = \frac{C_2R_2}{g_m}+\frac{L_1{}^2R_2}{\mu r_a L_2+\mu L_1 R_2}.$$

State clearly any assumptions made.

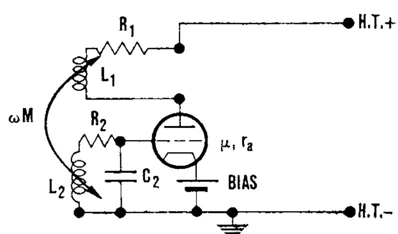


FIG. 8.12

7. The circuit of a Hartley oscillator is given in Fig. 8.13. Show that the circuit will oscillate when the mutual conductance g_m has a value

$$g_m = \frac{L_1}{L_2 r_a}\left[\frac{r_a(R_1+R_2)}{(\omega L_1)^2}+1\right].$$

State clearly any assumptions made.

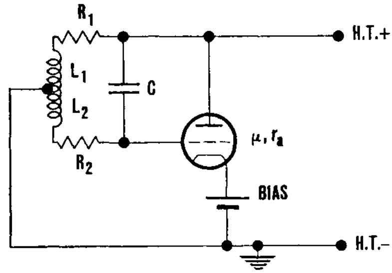


FIG. 8.13

8. The circuit of a Colpitt's oscillator is given in Fig. 8.14. Show that the circuit will oscillate when the amplification factor (μ) of the valve has a value

$$\mu = \frac{C_2}{C_1}+\omega^2 C_1 C_2 R_1 r_a.$$

State clearly any assumptions made.

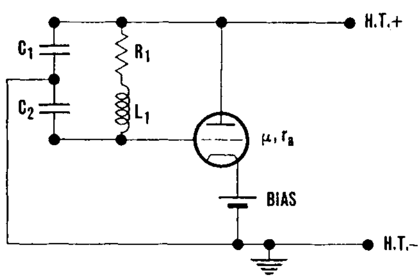


FIG. 8.14

9. The circuit of a negative resistance oscillator is given in Fig. 8.15. Show that the maintenance condition is given by $r_a = L/CR$, and that the frequency of oscillation (f) is given by

$$f = \frac{1}{2\pi}\sqrt{\left[\frac{1}{LC}\left(1-\frac{R}{r_a}\right)\right]}.$$

The tetrode valve used in such an oscillator has an anode slope resistance $r_a = -35$ kΩ. The anode load consists of a coil of inductance 159 μH and resistance 25 Ω. What is the lowest frequency at which the valve will oscillate?

940 kc/s.

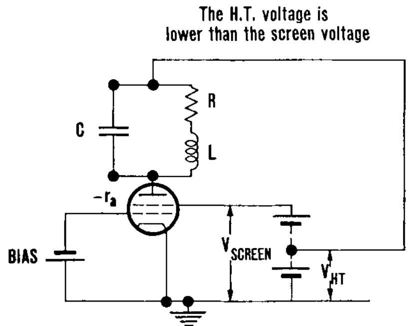


FIG. 8.15

10. Draw the circuit of a R.C. oscillator which employs a three-section ladder phase shifting network consisting of three similar phase retarding sections. If $C = 0{\cdot}0004$ μF, and $R = 120$ kΩ, calculate the frequency of oscillation of the oscillator from first principles, and show that, for oscillations to be maintained, the valve must have a gain of 29. The output of this oscillator may be taken from either the anode or the grid. At which point is the purer waveform obtained? Give reasons for your answer.

3·32 kc/s.

11. Explain the behaviour of a slab of quartz or Rochelle salt as a circuit element in an oscillator, and establish an equation for the equivalent circuit of the element under steady voltage conditions.

In a certain Wein bridge type oscillator, the frequency selective network employs 120 kΩ resistors, and 0·001 μF capacitors. Deduce, from first principles, the frequency of oscillation.

1·325 kc/s. H.N.C.

12. The equivalent circuit of a quartz crystal is shown in Fig. 8.16. Show that the resistive component of such a crystal is R_1 at the series resonant frequency, and $(X_c{}^2 + R_1{}^2)/R_1$ at the parallel resonant frequency.

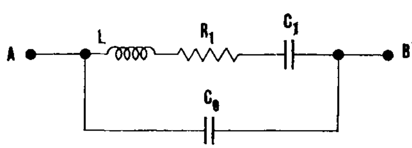


FIG. 8.16

13. Draw the circuit and explain the action of a crystal controlled oscillator. What precautions must be taken in order to obtain the highest possible frequency stability with this circuit?

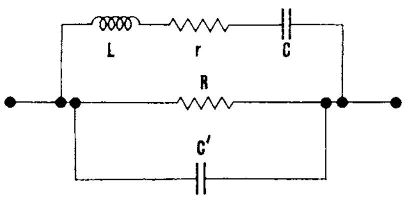


FIG. 8.17

Figure 8.17 shows the equivalent circuit of a quartz crystal which has a minimum impedance of 2 kΩ at 100 kc/s. The impedance reaches a maximum and has a value of 300 kΩ when the frequency is increased by 150 c/s. If the capacitance C', measured at a low frequency is 18 pF, calculate the approximate values of L, r, C and R.

47 H, 2 kΩ, 0·054 pF, 325 kΩ. I.E.R.E., May 1959

14. The electrical constants analogous to mass and compliance of an X-cut piezo-electric plate vibrating in the thickness mode are given by $L = 118\,t^3/lw$ henry, and $C = 0{\cdot}0029\,lw/t$ pF, where l, w and t, denote length, width, and thickness of the plate in cm.

An X-cut having dimensions $l = 2{\cdot}5$ cm, $w = 2$ cm and $t = 0{\cdot}5$ cm, is to be used as a generator of ultrasonic waves. Determine the resonant and anti-resonant frequencies of the plate when it is operated between plane electrodes having the same dimensions for l and w. The permittivity of quartz is 4·5, and the electrodes may be assumed to be in uniform light contact with the faces of the crystal.

555 kc/s; 557·2 kc/s. I.E.R.E., May 1957

15. What properties of a quartz crystal enable it to be used in a valve oscillator? Give a circuit diagram and an explanation of the action of any one type of crystal oscillator. What are its advantages and disadvantages compared with a valve oscillator not using a crystal element? H.N.C.

16. With the aid of a circuit diagram and suitable waveforms, explain the action of a free-running multivibrator. What are the effects of returning one of the grid resistors to a large positive bias? Draw a control grid voltage waveform for this condition. What is the practical advantage of returning both grid resistors to a large positive bias? H.N.C.

CHAPTER 9

Transistors

WORKED EXAMPLES

Example 1

The equivalent circuit for a low-frequency small signal transistor amplifier in the grounded-base configuration is shown in Fig. 9.1. Deduce that

(a) Current gain $= -\dfrac{(ar_c+r_b)}{(r_c+r_b+R_L)}$

(b) Input resistance $= r_e+r_b(1+\text{current gain})$,

(c) Output resistance $= r_b+r_c-\left[\dfrac{r_b(r_b+ar_c)}{R_g+r_e+r_b}\right]$.

H.N.C.

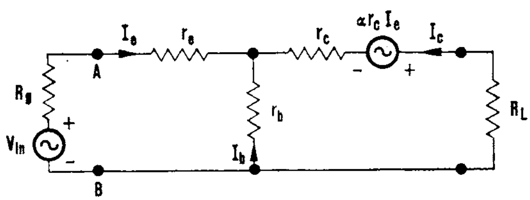


FIG. 9.1

Solution

The voltage generated in the collector circuit of Fig. 9.1 is $ar_c\mathbf{I}_e$ only if $r_b \ll r_c$ and mutual resistance r_m.

(*a*) *Current gain.* See Section 1.27 for the solution to this part of the problem.

(*b*) *Input resistance* R_{in}. The following network equations may be obtained from Fig. 9.1.

$$\mathbf{V}_{in} = \mathbf{I}_e(R_g+r_e+r_b)+\mathbf{I}_c r_b \tag{9.1}$$

$$0 = \mathbf{I}_c(r_b+r_c+R_L)+\mathbf{I}_e(r_b+\alpha r_c). \tag{9.2}$$

From equation (9.2),

$$\mathbf{I}_c = -\frac{(r_b+\alpha r_c)}{r_c+r_b+R_L}\times\mathbf{I}_e. \tag{9.3}$$

Substituting the value of $\mathbf{I}_c$ from equation (9.3) into equation (9.1) gives

$$\mathbf{V}_{in} = \mathbf{I}_e\left[(R_g+r_e+r_b)-\frac{r_b(r_b+\alpha r_c)}{(r_c+r_b+R_L)}\right]$$

and

$$\mathbf{I}_e = \frac{\mathbf{V}_{in}}{R_g+r_e+r_b-\dfrac{r_b(r_b+\alpha r_c)}{(r_c+r_b+R_L)}} \tag{9.4}$$

Now, the input resistance R_{in} of the transistor is that resistance which appears across terminals AB in Fig. 9.1, and

$$\mathbf{I}_e = \frac{\mathbf{V}_{in}}{R_g+R_{AB}} \tag{9.5}$$

Comparing equations (9.4) and (9.5) gives

$$R_{AB} = R_{in} = r_e+r_b-\frac{r_b(r_b+\alpha r_c)}{(r_c+r_b+R_L)}$$

However, the current gain for this circuit is given by

$$\text{Current gain} = -\frac{(\alpha r_c+r_b)}{(r_c+r_b+R_L)}$$

whence, input resistance $R_{in} = r_e+r_b\,(1+\text{current gain})$.

(*c*) *Output resistance* (R_o). From equation (9.2)

$$\mathbf{I}_e = -\frac{(r_c+r_b+R_L)}{(r_b+ar_c)}\times\mathbf{I}_c \tag{9.6}$$

Substituting $\mathbf{I}_e$ from equation (9.6) into equation (9.1) gives,

$$\mathbf{V}_{\text{in}} = -\mathbf{I}_c\left[\frac{(r_c+r_b+R_L)(R_g+r_e+r_b)}{(r_b+ar_c)}-r_b\right]$$

or

$$-\mathbf{I}_c = \frac{\dfrac{(r_b+ar_c)}{(R_g+r_e+r_b)}\times\mathbf{V}_{\text{in}}}{\left[r_c+r_b+R_L-\dfrac{r_b(r_b+ar_c)}{(R_g+r_e+r_b)}\right]}. \tag{9.7}$$

The minus sign in equation (9.7) indicates that the load current ($\mathbf{I}_c$) flows through R_L in the opposite direction to that shown in Fig. (9.1). This current is produced by a generator of e.m.f. $(r_b+ar_c)\mathbf{V}_{\text{in}}/(R_g+r_e+r_b)$ volts and internal resistance $r_c+r_b-r_b(r_b+ar_c)/(R_g+r_e+r_b)$ ohms. This is the output resistance of the generator.

Example 2

A transistor may be used as a small signal amplifier in three useful configurations. Compare the properties of the three possible forms of the amplifier.

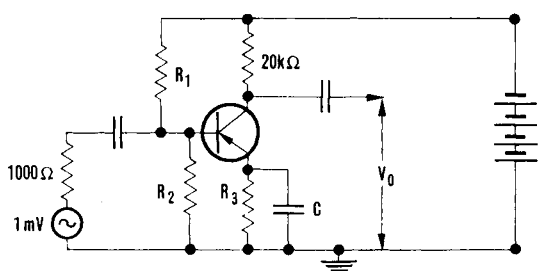


FIG. 9.2

If the parameters of the transistor shown in Fig. 9.2 are $r_e = 20\ \Omega$, $r_b = 500\ \Omega$, $r_c = 1\ \text{M}\Omega$ and $\alpha = 0{\cdot}98$, calculate the signal output voltage $\mathbf{V}_o$. Neglect the effects of the bias and coupling components on the signal, but comment on the bias arrangement shown. I.E.R.E., May 1962

Solution

Equivalent circuit. Assuming that the effects of the bias and coupling components on the signal can be ignored, a *T*-equivalent circuit would be shown as in Fig. 9.3.

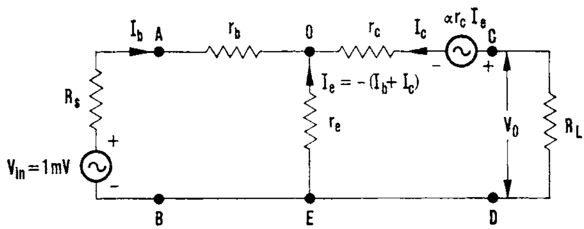


FIG. 9.3

Voltage Gain

Applying Kirchhoff's Second Law around loop *AOEB* of Fig. 9.3 gives,

$$\mathbf{V}_{\text{in}} = \mathbf{I}_b(r_e + R_s + r_b) + \mathbf{I}_c r_e \tag{9.8}$$

and also around the loop *OCDEO*,

$$-\alpha \mathbf{I}_e r_c = \mathbf{I}_c(r_e + R_L + r_c) + \mathbf{I}_b r_e, \tag{9.9}$$

rearranging and putting $\mathbf{I}_e = -(\mathbf{I}_b + \mathbf{I}_c)$ gives

$$0 = \mathbf{I}_c[r_e + R_L + r_c(1-\alpha)] + \mathbf{I}_b(r_e - \alpha r_c). \tag{9.10}$$

Now, voltage gain $= -\mathbf{I}_c R_L/\mathbf{V}_{\text{in}}$, and it is necessary, therefore, to obtain an expression for $\mathbf{I}_c$. This may be done using equations (9.8) and (9.10). Multiplying equation (9.8) by $(r_e - ar_c)$ and equation (9.10) by $(r_e + R_s + r_b)$ gives,

$$\mathbf{V}_{\text{in}}(r_e - ar_c) = \mathbf{I}_b(r_e + R_s + r_b)(r_e - ar_c) + \mathbf{I}_c r_e(r_e - ar_c)$$

and

$$0 = \mathbf{I}_b(r_e + R_s + r_b)(r_e - ar_c) + \mathbf{I}_c[r_e + R_L + r_c(1-a)](r_e + R_s + r_b).$$

Subtracting gives,

$$\mathbf{V}_{\text{in}}(ar_c - r_e) = \mathbf{I}_c\{r_e(ar_c - r_e) + [r_e + R_L + r_c(1-a)](r_e + R_s + r_b)\}$$

whence,

$$\mathbf{I}_c = \frac{(ar_c - r_e)\mathbf{V}_{\text{in}}}{r_e(ar_c - r_e) + [r_e + R_L + r_c(1-a)](r_e + R_s + r_b)}$$

but voltage gain $= -\mathbf{I}_c R_L/\mathbf{V}_{\text{in}}$

$$= -\frac{(ar_c - r_e)R_L}{r_e(ar_c - r_e) + [r_e + R_L + r_c(1-a)](r_e + R_s + r_b)} \quad (9.11)$$

In this case,

$$R_s = 1000\ \Omega, \quad r_e = 20\ \Omega, \quad r_b = 500\ \Omega, \quad r_c = 1\ \text{M}\Omega,$$

$$a = 0{\cdot}98, \quad \text{and} \quad R_L = 20{,}000\ \Omega.$$

When these values are substituted into equation (9.11), r_e will be ignored when compared with ar_c, R_L and $r_c(1-a)$, hence,

voltage gain

$$= \frac{-0{\cdot}98 \times 10^6 \times 20 \times 10^3}{0{\cdot}98 \times 10^6 \times 20 + (20 \times 10^3 + {\cdot}02 \times 10^6)(20 + 1000 + 500)}$$

$$= \frac{-19{\cdot}6 \times 10^3}{19{\cdot}6 + 40 \times 1{\cdot}52}$$

$$= -\frac{19600}{80{\cdot}4} = -244.$$

Therefore, with a 1 mV input signal, the resulting signal output voltage $|V_o|$ is given by

$$|V_o| = 244\,|V_{in}| = 244 \text{ mV}.$$

Due to the inherent properties of a transistor connected in the common-emitter configuration, there is a phase shift of 180° through the device. This phase reversal is indicated by the minus sign in the above solution for voltage gain.

Bias Arrangement

Transistors connected in the common-emitter configuration must be bias stabilized to avoid the possibility of thermal runaway by preventing excessive shift of the d.c. working point. In Fig. 9.2, bias stabilization is achieved by components R_1, R_2 and R_3. Capacitor C is a bypass capacitor which prevents feedback at the signal frequency. The potential divider network (R_1 and R_2) holds the base constant relative to earth. If leakage current rises, the voltage drop across R_3 increases, reducing the amount of forward bias applied to the emitter-base junction. This has the effect of limiting the rise in collector current.

Example 3

Explain briefly what is meant by *n*-type and *p*-type conduction in semi-conductors. Hence, explain the rectifying action of a *p–n* junction.

The hybrid parameters of a *p–n–p* junction transistor used as an amplifier in the common-emitter configuration are: $h'_{11} = 800\ \Omega$; $h'_{21} = 46$; $h'_{22} = 80\times10^{-6}\ \Omega^{-1}$, and $h'_{12} = 5{\cdot}4\times10^{-4}$. If the load resistance is 5 kΩ and the effective source resistance 500 Ω, calculate the voltage and current gains, and the output resistance.

I.E.R.E., May 1959

Solution

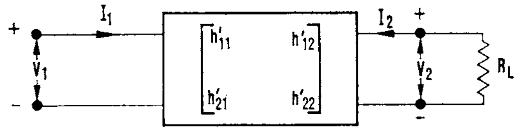


FIG. 9.4. Four-terminal network with hybrid parameters

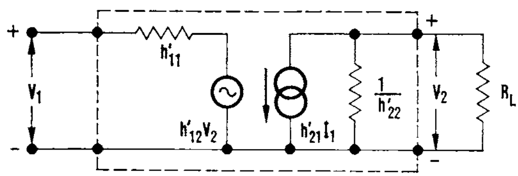


FIG. 9.5. Linear model of a transistor with hybrid parameters.

The network equations from Fig. 9.5 are

$$\mathbf{V}_1 = h'_{11}\mathbf{I}_1 + h'_{12}\mathbf{V}_2 \tag{9.12}$$

$$\mathbf{I}_2 = h'_{21}\mathbf{I}_1 + h'_{22}\mathbf{V}_2 \tag{9.13}$$

and

$$\mathbf{V}_2 = -\mathbf{I}_2 R_L \tag{9.14}$$

Voltage Gain

Multiplying equation (9.12) by h'_{21} and equation (9.13) by h'_{11} gives,

$$h'_{21}\mathbf{V}_1 = h'_{21}h'_{11}\mathbf{I}_1 + h'_{21}h'_{12}\mathbf{V}_2 \tag{9.15}$$

$$h'_{11}\mathbf{I}_2 = h'_{21}h'_{11}\mathbf{I}_1 + h'_{11}h'_{22}\mathbf{V}_2 \tag{9.16}$$

Subtracting equation (9.15) from equation (9.16) gives

$$h'_{11}\mathbf{I}_2 - h'_{21}\mathbf{V}_1 = (h'_{11}h'_{22} - h'_{12}h'_{21})\mathbf{V}_2 \tag{9.17}$$

where $h'_{11}h'_{22} - h'_{12}h'_{21} \equiv \Delta_h$, the determinant of the h matrix. Using $\mathbf{I}_2 = -\mathbf{V}_2/R_L$ from equation (9.14) in equation (9.17) gives

$$-h'_{21}\mathbf{V}_1 = \mathbf{V}_2\left(\Delta_h + \frac{h'_{11}}{R_L}\right) = \frac{\mathbf{V}_2}{R_L}(\Delta_h R_L + h'_{11}).$$

Therefore, $$\text{voltage gain} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{h'_{21}R_L}{h'_{11}+\Delta_h R_L} \tag{9.18}$$

$$= -\frac{46\times 5\times 10^3}{(8\times 10^2\times 80\times 10^{-6}-5{\cdot}4\times 46\times 10^{-4})5\times 10^3+8\times 10^2}$$

$$= -\frac{230{,}000}{198+800} \simeq 230\ \angle 180^\circ.$$

Current Gain

From equations (9.13) and (9.14),

$$\mathbf{I}_2 = h'_{21}\mathbf{I}_1 - h'_{22}R_L\mathbf{I}_2$$

or $$\mathbf{I}_2(1+h'_{22}R_L) = h'_{21}\mathbf{I}_1$$

and $$\text{Current gain} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{h'_{21}}{1+h'_{22}R_L} \tag{9.19}$$

$$= \frac{46}{1+80\times 10^{-6}\times 5000} = \frac{46}{1{\cdot}4} = 33$$

Output Resistance

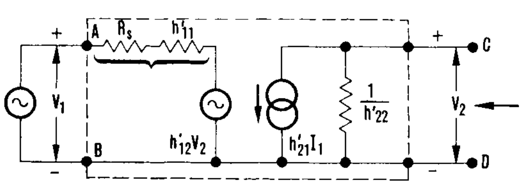


FIG. 9.6. Modified version of Fig. 9.5 including source resistance R_s.

It is assumed in the question that h'_{11} is real at the frequency of the applied voltage $\mathbf{V}_1$, and that the source impedance R_s is resistive. Hence, under these conditions, it is permissible to use the equivalent circuit of Fig. 9.6 in order to compute the output resistance. The new network equations are,

$$\mathbf{V}_1 = (R_s+h'_{11})\mathbf{I}_1+h'_{12}\mathbf{V}_2 \tag{9.20}$$

$$\mathbf{I}_2 = h'_{21}\mathbf{I}_1+h'_{22}\mathbf{V}_2 \tag{9.21}$$

Multiplying equation (9.20) by h'_{21} and equation (9.21) by $(R_s+h'_{11})$ gives,

$$h'_{21}\mathbf{V}_1 = h'_{21}(R_s+h'_{11})\mathbf{I}_1+h'_{21}h'_{12}\mathbf{V}_2 \tag{9.22}$$

$$(R_s+h'_{11})\mathbf{I}_2 = h'_{21}(R_s+h'_{11})\mathbf{I}_1+h'_{22}(R_s+h'_{11})\mathbf{V}_2. \tag{9.23}$$

Now, subtracting equation (9.22) from equation (9.23)

$$\mathbf{I}_2(R_s+h'_{11})-h'_{21}\mathbf{V}_1 = [h'_{22}(R_s+h'_{11})-h'_{12}h'_{21}]\mathbf{V}_2 \tag{9.24}$$

By definition, the output resistance R_o, "looking into" terminals *CD* of Fig. 9.6 in the direction of the arrow, is given by

$$R_o = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}$$

Therefore, putting $\mathbf{V}_1$ equal to zero in equation (9.24) gives,

$$R_o = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{R_s+h'_{11}}{h'_{22}(R_s+h'_{11})-h'_{12}h'_{21}} = \frac{R_s+h'_{11}}{h'_{22}R_s+\Delta_h} \tag{9.25}$$

Substituting given values into equation (9.25) gives,

$$R_o = \frac{500+800}{80\times10^{-6}\times500+0{\cdot}0392} = \frac{1300}{0{\cdot}0792} = 16{\cdot}4\text{ k}\Omega.$$

If the 5 kΩ load is taken into consideration, the effective output impedance is 5 kΩ in parallel with 16·4 kΩ, i.e. 3·94 kΩ.

Example 4

The hybrid parameters for a transistor used in the common emitter configuration are: $h'_{11} = 1{\cdot}5\text{ k}\Omega$, $h'_{12} = 10^{-4}$, $h'_{21} = 70$, and $h'_{22} = 10^{-4}\ \Omega^{-1}$. The transistor has a load resistor of 1 kΩ in the collector circuit, and is supplied from a signal source of resistance 500 Ω. Calculate (a) input resistance and (b) power gain of the stage.

Solution

(*a*) *Input resistance R_i.* As in Example 3, the network equations are

$$\mathbf{V}_1 = h'_{11}\mathbf{I}_1 + h'_{12}\mathbf{V}_2 \tag{9.26}$$

$$\mathbf{I}_2 = h'_{21}\mathbf{I}_1 + h'_{22}\mathbf{V}_2 \tag{9.27}$$

and

$$\mathbf{V}_2 = -\mathbf{I}_2 R_L \tag{9.28}$$

From equations (9.26) and (9.28),

$$\mathbf{V}_1 = h'_{11}\mathbf{I}_1 - h'_{12}R_L\mathbf{I}_2 \tag{9.29}$$

Now, from equation (9.19),

$$\mathbf{I}_2 = \frac{h'_{21}\mathbf{I}_1}{1+h'_{22}R_L}$$

using this value of $\mathbf{I}_2$ in equation (9.29) gives

$$\mathbf{V}_1 = \left[h'_{11} - \frac{h'_{12}h'_{21}R_L}{1+h'_{22}R_L}\right]\mathbf{I}_1$$

Hence

$$R_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = h'_{11} - \frac{h'_{12}h'_{21}R_L}{1+h'_{22}R_L} \tag{9.30}$$

and the effective input resistance including the source resistance R_s is $R_s + h'_{11} - h'_{12}h'_{21}R_L/(1+h'_{22}R_L)$.

Substituting known values into this expression gives,

$$\text{Total input resistance} = 500 + 1500 - \frac{10^{-4} \times 70 \times 10^3}{1 + 10^{-4} \times 10^3}\ \Omega$$

$$= 2000 - \frac{7}{1{\cdot}1}\ \Omega$$

$$\simeq 2\ \text{k}\Omega.$$

(*b*) *Power gain.* It was shown in Example 3 that

$$\text{Voltage gain} = -\frac{h'_{21}R_L}{h'_{11}+\Delta_h R_L}$$

and $$\text{Current gain} = \frac{h'_{21}}{1+h'_{22}R_L}$$

Now, $$\text{Power gain} = \left|\frac{V_2 I_2}{V_1 I_1}\right|$$

$$= \text{voltage gain} \times \text{current gain}$$

$$= \frac{(h'_{21})^2 R_L}{(h'_{11}+R_L\Delta_h)(1+h'_{22}R_L)}$$

In this case,

$$\Delta_h = h'_{11}h'_{22}-h'_{12}h'_{21} = 1{\cdot}5\times 10^3\times 10^{-4}-70\times 10^{-4}$$

$$= 0{\cdot}15-0{\cdot}007$$

$$= 0{\cdot}143$$

and $$\text{Power gain} = \frac{70\times 70\times 10^3}{(1{\cdot}5\times 10^3+10^3\times 0{\cdot}143)(1+0{\cdot}1)}$$

$$= \frac{4{\cdot}9\times 10^6}{10^3(1{\cdot}643)(1{\cdot}1)} = 2710.$$

Problems with Answers

1. Several different systems of equivalent circuits or of parameters may be used to specify the low-frequency properties of transistors. Describe any one system and use it to derive expressions for current gain, and for input impedance of a simple common-emitter amplifier stage with a resistive collector load. I.E.R.E., Nov. 1963

2. A transistor may be used as an amplifier in common-base, common-emitter or common-collector connections. Compare the properties of these three configurations, and give suitable applications for each. Draw an equivalent circuit appropriate to low-frequency operation of transistors, and use it to derive expressions for the input impedance and voltage gain of an emitter-follower circuit. I.E.R.E., May 1963

3. A grounded-base transistor voltage amplifier has a load resistance R_L and is fed by a signal source having an internal impedance R_s.

Show that the voltage gain (A) and the input resistance (r_i) are given by

$$A = \frac{(r_m+r_b)R_L}{(R_s+r_e+r_b)(R_L+r_c+r_b)-r_b(r_b+r_m)}$$

and
$$r_i = r_e+r_b-\left[\frac{r_b(r_b+r_m)}{R_L+r_b+r_c}\right]$$

where r_b, r_c, r_e and r_m are the T-network parameters of the transistor.

I.E.R.E., Nov. 1957

4. Find, from first principles, the current gain, voltage gain and power gain for a low-frequency common-emitter transistor amplifier having parameters: $r_e = 20\ \Omega$, $r_b = 700\ \Omega$, $r_c = 0{\cdot}5\ \text{M}\Omega$, $\alpha = 0{\cdot}96$, $R_L = 25\ \text{k}\Omega$, $R_s = 100\ \Omega$.

10·7; −264; 2820.

5. The T-parameters of a junction transistor connected as an emitter-follower are: $r_e = 20\ \Omega$, $r_c = 2\ \text{M}\Omega$, $r_b = 1\ \text{k}\Omega$ and $\alpha = 0{\cdot}98$. Calculate, from first principles, the input resistance for a resistive load of 10 kΩ.

668 kΩ. I.E.R.E., Nov. 1960

6. The low-frequency equivalent-T presentation of a transistor has the following values:

$$r_e = 30\ \Omega, \quad r_b = 500\ \Omega, \quad r_c = 800\ \text{k}\Omega, \quad \alpha = 0{\cdot}975.$$

The transistor is used as a common- (or grounded) emitter-amplifier with a collector load of 5 kΩ.

Draw the equivalent circuit and use it to find the current gain and the input impedance of the stage.

31·2; 1·466 kΩ. I.E.R.E., May 1964

7. Discuss the phenomenon of current flow in junction transistors in terms of the physical properties of *p*- and *n*-type semi-conductors. A *p–n–p* junction transistor is used as a voltage amplifier in the grounded-base connection, the load resistance being 300 kΩ and the internal resistance of the generator, 200 Ω. Derive an expression for the voltage gain of the amplifier, and calculate its magnitude if the transistor T-network parameters are as follows:

$$r_e = 18\ \Omega, \quad r_b = 700\ \Omega, \quad r_c = 1\ \text{M}\Omega \quad \text{and} \quad r_m = 976\ \text{k}\Omega.$$

$(r_b+r_m)R_L/(R_s+r_e+r_b)(r_c+r_b+R_L)-r_b(r_b+r_m)$: 573.

I.E.R.E., May 1957

8. Identical transistors in the common-emitter connection are used in a two-stage low-frequency amplifier.

Deduce the overall voltage gain, power gain, and the input resistance of the amplifier for small signals if $r_e = 20\ \Omega$, $r_b = 500\ \Omega$, $a = 0{\cdot}98$ and $r_c = 1\ \mathrm{M}\Omega$. Both collector load resistors are $5\ \mathrm{k}\Omega$, the input signal source has zero output impedance and the effect of coupling and bias networks may be neglected.

$4{\cdot}95\times10^3$; 7×10^6; $1{\cdot}45\ \mathrm{k}\Omega$. I.E.R.E., Nov. 1961

9. The common-base hybrid parameters of a transistor amplifier are: $h_{11} = 18\ \Omega$, $h_{12} = 8\times10^{-4}$, $h_{21} = -0{\cdot}98$ and $h_{22} = 1{\cdot}6\times10^{-6}\ \Omega^{-1}$. If the collector load is $5\ \mathrm{k}\Omega$, calculate the power gain of the stage.

216.

10. The common-base hybrid parameters of a transistor are given by $h_{ib} = 40\ \Omega$, $h_{ob} = 0{\cdot}4\ \mu\Omega^{-1}$, $h_{rb} = 5\times10^{-4}$, $h_{fb} = -0{\cdot}98$. The transistor is connected in the common-emitter configuration with a load resistance of $5\ \mathrm{k}\Omega$. Calculate the voltage gain and input resistance of the circuit.

-116; $1{\cdot}93\ \mathrm{k}\Omega$. I.E.R.E., May 1964

11. Define the hybrid parameters of a four-terminal network and prove that the input resistance of such a network when operating into a load of conductance G_L is given by $R_i + h_{11} - h_{12}h_{21}/(h_{22}+G_L)$.

The common-emitter hybrid parameters of a transistor are

$$h'_{11} = 800\ \Omega,\quad h'_{21} = 47,\quad h'_{22} = 80\ \mu\Omega^{-1},\quad \text{and}\quad h'_{12} = 5\times10^{-4}.$$

Calculate the output voltage and output resistance for a common-emitter voltage amplifier using this transistor with a load of $5\ \mathrm{k}\Omega$ and a $10\ \mathrm{mV}$ source of internal resistance $500\ \Omega$.

$1{\cdot}38$ V; $3{\cdot}82\ \mathrm{k}\Omega$ ($16{\cdot}2\ \mathrm{k}\Omega$ in parallel with $5\ \mathrm{k}\Omega$).

I.E.R.E., Nov. 1962

12. Discuss the factors which limit the high-frequency response of junction transistors and explain the methods by which this response may be extended.

A junction transistor has the following hybrid parameters when used in the common-emitter connection: $h'_{11} = 800\ \Omega$, $h'_{22} = 80\times10^{-6}\ \Omega^{-1}$, $h'_{12} = 5\times10^{-4}$, and $h'_{21} = 48$. It is used in this configuration as an amplifier with a load resistance of $8\ \mathrm{k}\Omega$. If this source has an e.m.f. of 100 mV and an internal resistance of $500\ \Omega$, calculate the power developed in the load.

$2{\cdot}08$ mW. I.E.R.E., May 1960

13. Explain the terms "space charge capacitance" and "diffusion capacitance" used in connection with a junction transistor. In Fig. 9.7 is shown a simple form of the equivalent circuit of a transistor valid at relatively low frequencies. The transistor is connected as an amplifier in the common-emitter configuration with a choke as the collector load. Calculate the input admittance for a frequency such that the load impedance may be taken as $j \times 10^5\ \Omega$, and C_c has a reactance of $10^6\ \Omega$.

$(2357-864j)\ \mu\ \Omega^{-1}$. I.E.R.E., May 1961

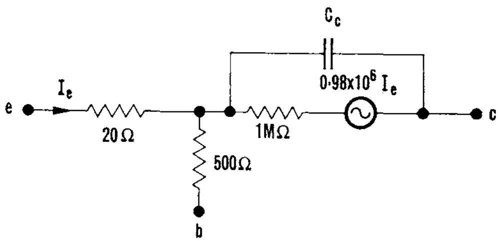


FIG. 9.7

14. Figure 9.8 shows an astable multivibrator in its semiconductor form. Explain in detail how it operates, illustrating your answer by clear sketches

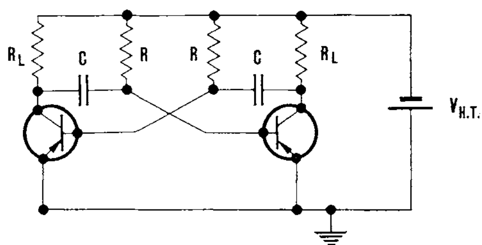


FIG. 9.8

of the appropriate electrode waveforms. Show that, for the circuit of Fig. 9.8, the time period of oscillation T is given by

$$T \simeq 2CR \log_e 2.$$

I.E.R.E., Nov. 1963

15. Sketch typical I_c–V_c curves for a transistor operating in the grounded-emitter connection having a current gain h_{FE} of 20 for collector currents between 0 and 50 mA and collector voltages between 0 and 12 V.

On these curves sketch a typical bi-stable characteristic with a load line and, hence, explain the mechanism of bi-stable circuits and describe the re-setting of each stable state.

A transistor cross-coupled bi-stable has collector loads of 1000 Ω and an H.T. supply of +12 V. Assuming that leakage current is negligible, design the d.c. circuit of the bi-stable for a minimum h_{FE} of 11 and indicate the method of compensating the stored base charge.

$R_{CL} = 1000\ \Omega$ (given), $R_b = 10{,}000\ \Omega$. I.E.R.E., Nov. 1962

16. A transistor has a grounded-emitter frequency of unity current gain (F_1) of 100 Mc/s, a low-frequency grounded-emitter current gain (h_{fe}) of 20, and a grounded-emitter input impedance (h_{ie}) of 500 Ω at 5 mA.

The transistor is to be driven on the base from a source having a negligibly low impedance compared with h_{ie} and it is required that the "device bandwidth" neglecting the load bandwidth should be 20 Mc/s. If the output load has a capacitance of 32 pF and bandwidth of 5 Mc/s, ascertain the value of the degenerative emitter resistor necessary to achieve the design objective and evaluate the low-frequency voltage gain of the complete amplifier.

$R_e = 100\ \Omega$, $A_v \simeq 10$. I.E.R.E., May 1962

17. Sketch typical common-emitter I_c–V_c characteristics for a transistor and then superimpose a family of curves showing the variation of F_1 (unity gain frequency) over the surface of the I_c–V_c characteristic. Draw a typical load line and comment briefly upon the problem of video amplifier design under these conditions.

An uncompensated transistor video amplifier is required to produce a pulse of 15 V amplitude, 10 ms duration, and of 0·05 μs (50 ns) rise and fall time (10–90%) across an output capacity of 50 pF. The output pulse is to be a.c. coupled to the grid of a cathode ray tube, and a droop of 5% is to be allowed.

$$\text{Droop} = \left(\frac{\text{change of amplitude during pulse}}{\text{initial amplitude of pulse}}\right).$$

Determine:

(a) the current capability of the transistor;
(b) the load resistor;
(c) the output coupling capacitor if the grid resistor is 100 kΩ.

48 mA; 450 Ω; 0·2 μF. I.E.R.E., May 1961

18. A transistor, operated in the common-emitter configuration, has the following brief specification:

$$V_c(\max) = 10\text{ V},$$
$$I_c(\max) = 100\text{ mA}.$$

Large signal current gain (h_{FE}) = 40 when: $I_c = 30$ mA and $V_c = -0{\cdot}25$ V.

Maximum stored base charge (Q_s) = 720×10^{-12} C. This transistor is assumed to have a defined saturation characteristic and to be substantially linear over the whole operating region.

(a) Draw on graph paper the I_c–V_c curves appropriate to this specification.
(b) On the transistor curves construct a bi-stable characteristic for the circuit shown in Fig. 9.9, and a suitable load line.

The design should be such that the circuit will function with a minimum h_{FE} of 20 and with a rise time (10% to 90%) of 100 ns, assuming a total output capacitance of 200 pF. The H.T. supply is -6 V. Give reasons for the choice of component values, and state the values of R_c and R_K.

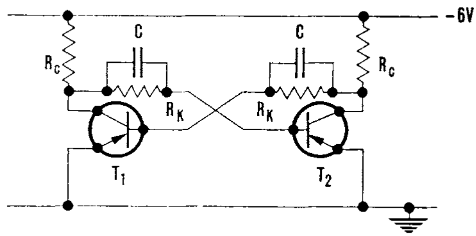


FIG. 9.9

(c) Briefly describe the conditions determining the four regions of the bistable characteristic.
(d) Estimate the minimum change of base current necessary to trigger the circuit assuming that the drive is in the form of positive going pulses and that $h_{FE} = 40$.
(e) Calculate a suitable value for the capacitor C in Fig. 9.9.

220 Ω, 4·3 kΩ; approx. 650 μA; 120 pF. I.E.R.E., Nov. 1960

CHAPTER 10

Electronics Measurements

Worked Examples

Example 1

Describe with the aid of a simple diagram, the principle on which a Q-meter operates.

A coil connected across the *inductor* terminals of a Q-meter is tuned to resonate at different frequencies by the calibrated variable capacitor, and the following results are obtained.

Table 13

Frequency in Mc/s	Capacitance in pF
2	225
2·5	125
4	35
5	10

Find the inductance of the coil and its self capacitance, preferably by a graphical method. I.E.R.E., Nov. 1963

Solution

The block diagram of a Q-meter is given in Fig. 10.1. This composite equipment may be used to measure the magnification factor, self capacitance, and inductance of coils by measuring the voltage drop across a capacitor when it is adjusted to produce series resonance with the test coil.

Q-meter Operation with High-Q Coil

The output meter of the device is calibrated directly in Q-factor (V_o/V_{in} of Fig. 10.1). Some method of monitoring and adjusting V_{in} is necessary, as the input voltage to the LCR network should be the same for each setting of the variable frequency oscillator.

The test coil consists of an inductance L in series with residual resistance r. C_1 is a calibrated tuning capacitor, and C_2 is a trimmer capacitor for incremental adjustment. In addition to the external terminals AB across which the coil under test is

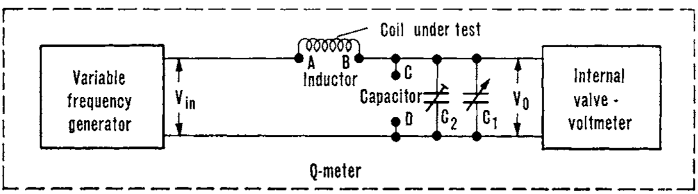


FIG. 10.1. Block diagram of Q-meter.

connected, external terminals CD are also provided for connecting additional capacitance into the circuit. The calibration of C_1 takes into account stray capacitance effects and the input capacitance of the valve-voltmeter.

If the input impedance of the valve-voltmeter indicator is assumed to be infinite and the output impedance of the variable frequency oscillator to be negligible, a series LCR circuit exists which may be tuned to resonance using C_1 (with C_2 set at zero). At resonance, the circulating current and output voltage V_o are both maximum. This condition is indicated by a maximum reading on the output meter of the internal valve-voltmeter.

Now, at resonance,

$$V_o = I_o\omega_0 L = \frac{I_o}{\omega_0 C_1}, \quad \text{and} \quad V_{in} = I_o r.$$

I_o is the circulating current at resonance, and ω_0 is the resonant angular frequency.

Therefore,
$$\frac{V_o}{V_{in}} = \frac{I_o \omega_0 L}{I_o r} = Q$$

and
$$V_o = QV_{in} \tag{10.1}$$

Since the input voltage V_{in} is checked before each reading, the output meter may be calibrated directly in Q-factor.

To Determine Inductance and Self-Capacitance by the Intercept Method

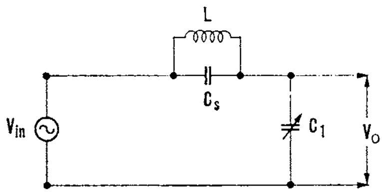


FIG. 10.2. Simplified circuit.

If the resistance of the test coil is negligibly small, and the self capacitance of the coil C_s is distributed as shown in Fig. 10.2, it can be shown that the series resonant frequency f_0 of the circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{\{L(C_1+C_s)\}}}$$

and
$$\frac{1}{f_0^2} = 4\pi^2 L(C_1+C_s). \tag{10.2}$$

Equation (10.2) is the equation of a straight line having slope $4\pi^2 L$.

If a graph is plotted of $1/f^2$ against C_1, and projected so as to cut the horizontal axis OX at P, OP will give the self capacitance of the coil (see Fig. 10.3).

The inductance of the coil is found from the graph using

$$L = \frac{\tan \alpha}{4\pi^2}.$$

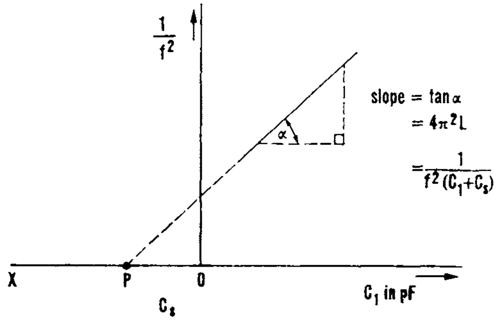


FIG. 10.3. Graph of $1/f^2$ against (C_1+C_s).

In this problem it is necessary to plot a graph of $1/f^2$ against C. Hence, a table is constructed thus,

TABLE 14

$1/f^2$	C in pF
25×10^{-14}	225
16×10^{-14}	125
$6{\cdot}25\times10^{-14}$	35
$4{\cdot}0\times10^{-14}$	10

A graph of $1/f^2$ against C is shown in Fig. 10.4. From this,

$$C_s = OP = 30\text{ pF}$$

$$L = \frac{\tan \alpha}{4\pi^2} = \frac{0{\cdot}00098}{39{\cdot}5} = 24{\cdot}8\ \mu\text{H}.$$

Example 2

If the bridge shown in Fig. 10.5 is balanced, derive expressions for L and r in terms of the other components. When a certain coil is used, balance is obtained with $C_1 = 0{\cdot}1\ \mu F$, $R_2 = 8000\ \Omega$, $C_2 = 0{\cdot}4\ \mu F$, $R_4 = 100\ \Omega$. An additional identical coil is connected in series with this first coil, and mounted close to it. The values now obtained are $C_1 = 0{\cdot}1\ \mu F$, $R_2 = 20{,}000\ \Omega$,

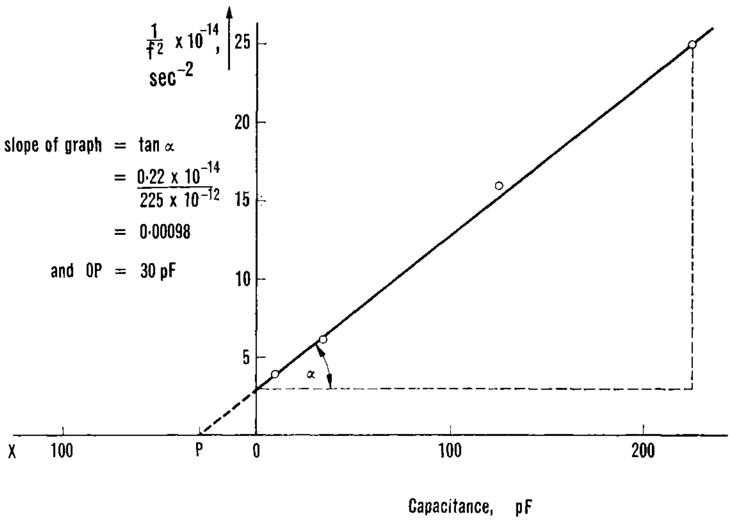


FIG. 10.4

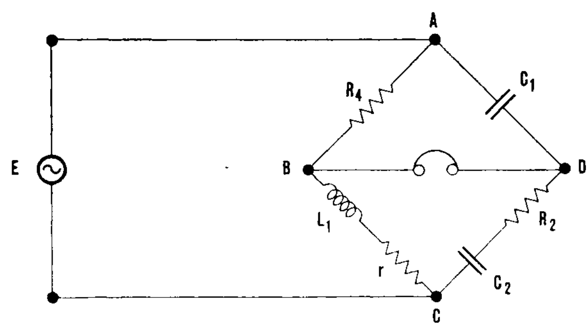


FIG. 10.5

$C_2 = 0{\cdot}2\ \mu F$ and $R_4 = 100\ \Omega$. Find the inductance and resistance of the coils, and the mutual inductance between them.

I.E.R.E., Nov. 1963

Solution

When the bridge is balanced, no alternating current flows through headphones and $Z_{AB}Z_{CD} = Z_{BC}Z_{AD}$.

Hence,
$$R_4\left(R_2+\frac{1}{j\omega C_2}\right) = (r+j\omega L_1)\frac{1}{j\omega C_1}$$

and
$$R_2R_4+\frac{R_4}{j\omega C_2} = \frac{r}{j\omega C_1}+\frac{L_1}{C_1}. \tag{10.3}$$

Equating real parts of equation (10.3) to zero gives

$$R_2R_4 = \frac{L_1}{C_1}$$

therefore,
$$L_1 = R_2R_4C_1 \tag{10.4}$$

Equating imaginary parts of equation (10.3) to zero gives

$$\frac{R_4}{\omega C_2} = \frac{r}{\omega C_1}$$

therefore,
$$r = \frac{C_1R_4}{C_2} \tag{10.5}$$

Now, substituting figures into equation (10.4) gives,

$$L_1 = 0{\cdot}1\times10^{-6}\times8\times10^3\times10^2 = 0{\cdot}08 \text{ H or } 80 \text{ mH},$$

and into equation (10.5) gives,

$$r = \frac{0{\cdot}1\times10^{-6}\times10^2}{0{\cdot}4\times10^{-6}} = 25\ \Omega.$$

When an additional identical coil L_2 is connected in series with L_1 and close to it, the new apparent inductance L_T found from equation (10.4) is

$$L_T = 0{\cdot}1\times10^{-6}\times20\times10^3\times10^2 = 0{\cdot}2 \text{ H or } 200 \text{ mH}.$$

It can be seen that this is more than twice the value of either L_1 or L_2. This implies that the coils are connected in the same sense, mutual inductance M accounting for the balance of the 200 mH. M can be found using the equation,

$$L_T = L_1 + L_2 \pm 2M.$$

However, only the positive solution satisfies the equation here, therefore,

$$200 = 80 + 80 + 2M$$

and $$40 = 2M \quad \text{or} \quad M = 20 \text{ mH}.$$

Example 3

Find the characteristic impedance and the attenuation coefficient of the T-section shown in Fig. 10.6. If the network is used between an a.c. source of internal resistance 20 Ω, and a 20 Ω resistive load, find also the attenuation and insertion loss.

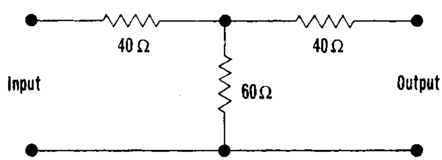


FIG. 10.6

Solution

Characteristic impedance R_o. It can be shown that

$$R_o = \sqrt{(R_{o/c} \times R_{s/c})} \tag{10.6}$$

where $R_{o/c}$ is the resistance measured across the input terminals of Fig. 10.6 with the output terminals open circuit, and $R_{s/c}$ is the resistance measured across the input terminals, this time with the output terminals short circuit.

Now, from Fig. 10.6,

$$R_{o/c} = 40+60 = 100\ \Omega$$

and
$$R_{s/c} = 40+\frac{60\times 40}{60+40} = 64\ \Omega$$

hence
$$R_o = \sqrt{6400} = 80\ \Omega.$$

The resistance across the input terminals is, therefore, 80 Ω when a resistance of 80 Ω is connected across the output terminals (see Fig. 10.7).

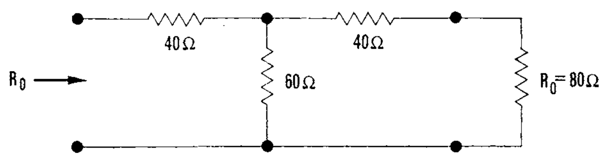


FIG. 10.7

Attenuation Coefficient

This is the attenuation of an attenuator when the source and load resistances are equal to the characteristic impedance R_o of the network.

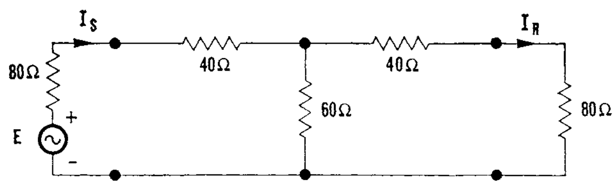


FIG. 10.8

Now,

$$\text{attenuation in dB} = 10 \log_{10} \frac{P_1}{P_2} \qquad (10.7)$$

where P_1 is the power delivered by the generator to the network, and P_2 is the power which is delivered to the load.

Hence, from Fig. 10.8,

$$I_s = \frac{E}{160}$$

and $$P_1 = I_s^2 R_o = \left[\frac{E}{160}\right]^2 \times 80 = \frac{E^2}{320}. \qquad (10.8)$$

Also, $$60(I_s - I_R) = I_R(80+40)$$

whence, $$\frac{I_s}{3} = I_R$$

therefore, $$P_2 = I_R^2 \times 80 = \left[\frac{E}{3\times 160}\right]^2 \times 80 \qquad (10.9)$$

and, $$\text{attenuation in dB} = 10\log_{10}\frac{P_1}{P_2}$$

$$= 10\log_{10}\left[\frac{E^2}{320}\times\frac{160\times 18}{E^2}\right]$$

$$= 10\log_{10} 9 = 9{\cdot}542\ \text{dB}.$$

Attenuation

The attenuator is to be used between a source of resistance 20 Ω and a load of 20 Ω as shown in Fig. 10.9.

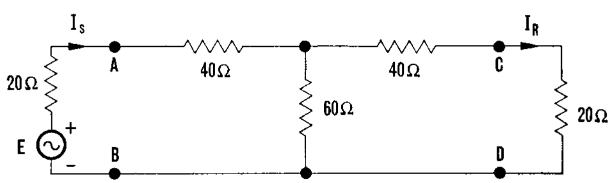


FIG. 10.9

It is again necessary to find P_1 and P_2, but this time using Fig. 10.9.

J**

To find P_1

The resistance across AB due to the network and the load is

$$R_{AB} = 40+\frac{60(40+20)}{60+40+20} = 70\ \Omega,$$

also
$$I_s = \frac{E}{90}$$

therefore,
$$P_1 = \left(\frac{E}{90}\right)^2 \times 70 = \frac{7E^2}{810} \tag{10.10}$$

To find P_2

$$(I_s - I_R)60 = I_R(40+20)$$

whence,
$$I_R = \frac{I_s}{2} = \frac{E}{180}$$

therefore,
$$P_2 = \left(\frac{E}{180}\right)^2 \times 20 = \frac{E^2}{180\times 9} \tag{10.11}$$

and
$$\text{attenuation} = 10 \log_{10} \frac{P_1}{P_2}$$
$$= 10 \log_{10} \frac{7E^2}{810} \times \frac{180\times 9}{E^2}$$
$$= 10 \log_{10} 14 = 11{\cdot}461 \text{ dB}.$$

Insertion Loss

Again referring to Fig. 10.9,

$$\text{Insertion loss in dB} = 10 \log_{10} \frac{P_3}{P_4} \tag{10.12}$$

where P_3 is the power delivered to the load when the load is connected directly across the generator terminals as in Fig. 10.10, and P_4 is the power delivered to the load when the circuit is connected as in Fig. 10.9. (This is also the definition for P_2.)

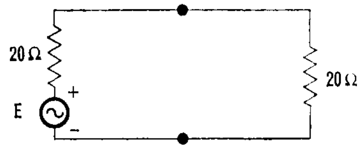


FIG. 10.10

From Fig. 10.10,

$$P_3 = \left(\frac{E}{40}\right)^2 \times 20 = \frac{E^2}{80}$$

and $P_4 = P_2$, hence, from equation (10.11),

$$P_4 = \frac{E^2}{180 \times 9}$$

therefore, insertion loss in dB $= 10 \log_{10} \frac{E^2}{80} \times \frac{180 \times 9}{E^2}$

$$= 13{\cdot}064 \text{ dB}.$$

Example 4

When a high Q coil was connected to the *inductor* terminals of a Q meter, resonance was obtained with a tuning capacitance of 400 pF at 1 Mc/s. With a standard coil connected to the *inductor* terminals it was found that, at a frequency of 9·6 Mc/s, resonance was not affected by connecting the test coil in parallel with the standard coil. Find the self-capacitance and the inductance of the coil under test.

Solution

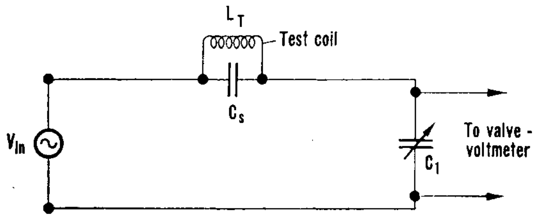


FIG. 10.11. Basic circuit at 1 Mc/s.

When C_1 is adjusted to give a maximum reading on the valve voltmeter,

$$\omega_1^2 L_T(C_1+C_s) = 1$$

or
$$L_T = \frac{1}{\omega_1^2(C_1+C_s)} \tag{10.13}$$

where $\omega_1 = 2\pi f_1$ rad/s and f_1 is 1 Mc/s.

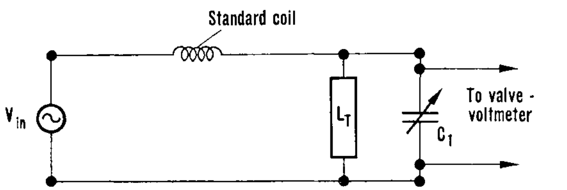


FIG. 10.12. Basic circuit at 9·6 Mc/s.

With the test coil L_T out of circuit, C_1 is adjusted to resonance with L_s, the standard coil (see Fig. 10.12.) If, when L_T is connected, the resonance condition is unaffected, the supply frequency is the natural resonant frequency of the test coil. If this frequency is f_0, then

$$\omega_0^2 L_T C_s = 1$$

or
$$L_T = \frac{1}{\omega_0^2 C_s} \tag{10.14}$$

where $$\omega_0 = 2\pi f_0$$

and $$f_0 = 9{\cdot}6 \text{ Mc/s.}$$

Eliminating L_T from equations (10.13) and (10.14),

$$\omega_1^2(C_1 + C_s) = \omega_0^2 C_s,$$

whence, $$\omega_1^2 C_1 = C_s(\omega_0^2 - \omega_1^2)$$

and $$C_s = \frac{\omega_1^2 C_1}{\omega_0^2 - \omega_1^2} = \frac{f_1^2 C_1}{f_0^2 - f_1^2} \tag{10.15}$$

Substituting values gives

$$C_s = 400 \times 10^{-12} \times \frac{1^2}{(9{\cdot}6^2 - 1^2)} \text{ F}$$

$$= \frac{400}{91{\cdot}2} \text{ pF}$$

$$= 4{\cdot}39 \text{ pF.}$$

Using this value for C_s in equation (10.13) gives

$$L_T = \frac{1}{4 \times \pi^{+2} \times 10^{+12}(404{\cdot}4 \times 10^{-12})} = 62{\cdot}6 \ \mu\text{H.}$$

Example 5

The bridged-T network shown in Fig. 10.13 is to have zero transmission at a frequency of 15·9 kc/s. If $L = 10$ mH, and $r = 5\ \Omega$, find the necessary values of C and R.

Solution

For a certain combination of frequency and component values, the current fed to the detector is zero. In order to obtain the relationships among circuit quantities which produce isolation between output and input of Fig. 10.13, the circuit needs to be simplified. The star circuit comprising C, C, L and r may be transformed into an equivalent delta network using the formula

$$Z_x = Z_1+Z_2+\frac{Z_1Z_2}{Z_3} \tag{10.16}$$

where

$$Z_1 = -jX_c, \quad Z_2 = -jX_c, \quad \text{and} \quad Z_3 = r+j\omega L. \tag{10.17}$$

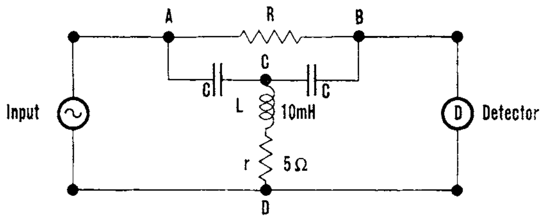


FIG. 10.13

The new circuit is shown in Fig. 10.14.

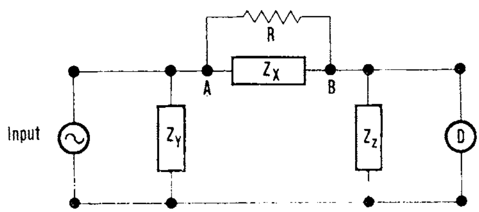


FIG. 10.14

It is possible to obtain values for Z_y and also Z_z using equation (10.16), but these are not needed in the solution of this problem. However, a value for Z_x is required.

Substituting the values for Z_1, Z_2, Z_3 from equation (10.17) into equation (10.16) gives,

$$Z_x = -\frac{j}{\omega C}-\frac{j}{\omega C}+\frac{1}{(j\omega C)^2(r+j\omega L)}$$

$$= -\frac{j2}{\omega C}-\frac{1}{(\omega C)^2(r+j\omega L)} \tag{10.18}$$

Now, from Fig. 10.14,

$$Z_{AB} = \frac{RZ_x}{R+Z_x},$$

and for zero transmission at a particular frequency,

$$R+Z_x = 0.$$

Under these conditions, there would be an effective open circuit between output and input. Using this relationship, and Z_x from equation (10.18),

$$-\frac{j2}{\omega C}-\frac{1}{(r+j\omega L)\omega^2C^2}+R = 0$$

or

$$-\frac{j2}{\omega C}-\frac{r-j\omega L}{[r^2+(\omega L)^2]\omega^2C^2}+R = 0$$

and

$$j\left[\frac{\omega L}{\omega^2C^2(r^2+\omega^2L^2)}-\frac{2}{\omega C}\right]+R-\frac{r}{\omega^2C^2(r^2+\omega^2L^2)} = 0. \qquad (10.19)$$

Equating imaginary parts of equation (10.19) to zero gives,

$$\frac{2}{\omega C} = \frac{\omega L}{\omega^2C^2(r^2+\omega^2L^2)}$$

therefore

$$C = \frac{L}{2(r^2+\omega^2L^2)}. \qquad (10.20)$$

Equating real terms of equation (10.20) to zero gives

$$R = \frac{r}{\omega^2C^2(r^2+\omega^2L^2)}. \qquad (10.21)$$

Equations (10.20) and (10.21) may be used to obtain the numerical values of C and R which will give zero transmission when the frequency of the applied voltage is 15·9 kc/s.

r^2 will be neglected when substituting values, as it is much smaller than ω^2L^2.

Hence, from equation (10.20),

$$C \simeq \frac{10^{-2}}{2(100\times 10^{3}\times 10\times 10^{-3})^{2}}\text{ F}$$
$$= \frac{10^{-8}}{2} = 0{\cdot}005\ \mu\text{F}$$

and, from equation (10.21), using $C = 0{\cdot}005\ \mu$F,

$$R = \frac{5}{10^{10}\times 25\times 10^{-18}\times 10^{6}} = \frac{5}{0{\cdot}25} = 20\ \Omega.$$

Problems with Answers

1.

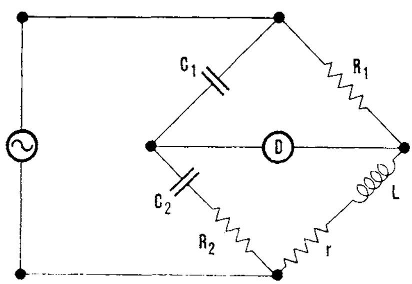


Fig. 10.15

The circuit of Fig. 10.15 shows an a.c. bridge for the measurement of resistance r and inductance L. Derive the condition of balance of this bridge. If $C_1 = 0{\cdot}002\ \mu$F, $C_2 = 0{\cdot}008\ \mu$F, $R_1 = 100\ \Omega$ and $R_2 = 5$ kΩ, find the resistance and inductance of the coil.

25 Ω; 1 mH. I.E.R.E., Nov. 1960

2. A Maxwell bridge consists of four arms joined to form a square, the corners of which are lettered *ABCD*. The components used in the bridge network are as follows:

AB—a calibrated variable resistor P having a range of 10 Ω to 1000 Ω,

BC—a fixed standard 0·1 μF capacitor C in parallel with a calibrated variable resistor R, having a range of 10 Ω to 10 kΩ,

CD—a fixed standard resistor Q of 100 Ω,

DA—coil to be measured.

Derive expressions, in terms of the bridge parameters, for the inductance and resistance of the coil under test. Is the balance condition affected by the frequency of the a.c. supply to the bridge? Using the components listed above, what will be the maximum and minimum values of inductance which can be measured?

$L_x = PQC$; $R_x = PQ/R$; 10,000 μH – 100 μH. H.N.C.

3. The four arms of a Schering bridge are arranged as follows:

AB—a loss-free capacitor C_1,
BC—a loss-free capacitor C_2 in parallel with a resistor R_2,
CD—a pure resistor R_3,
DA—the capacitor under test, which may be taken as an unknown capacitor C_x in parallel with a resistor R_x.

An a.c. supply of 10 kV at 50 c/s is applied across *AC*, and the bridge parameters adjusted until no voltage appears across *BD*. Obtain expressions for the unknowns C_x and R_x in terms of the remaining parameters.

If $C_1 = 100$ pF; $C_2 = 0{\cdot}312\ \mu$F; $R_2 = 125\ \Omega$ and $R_3 = 200\ \Omega$, calculate (a) the loss angle of the unknown capacitor and (b) the voltage across capacitor C_2.

$C_x = C_1R_2 \mid (\omega^2C_2{}^2R_2{}^2R_3+R_3)$; $R_x = 1 \mid \omega^2C_2R_2C_x$; $0°\ 42'$; 3·2 V. H.N.C.

4. Describe fully a method of determining the frequency of a sinusoidal audio-frequency voltage by means of a Wein bridge. Suggest a suitable detector. Derive an expression which may be used to calculate the frequency. What would be the effect on the bridge if the input signal were not sinusoidal, and how might this disadvantage be overcome?

A Wein bridge is balanced when $R_1 = R_2 = 1250\ \Omega$, and $C_1 = C_2 = 0{\cdot}03\ \mu$F. What is the frequency of the supply voltage?

$\omega^2R_1R_2C_1C_2 = 1$; 424·4 c/s. H.N.C.

5. Derive the equations of balance of the bridge network shown in Fig. 10.16. If balance is obtained with $R_1 = 100\ \Omega$, $R_2 = 1000\ \Omega$, $R_3 = 1000\ \Omega$, $r = 500\ \Omega$, $C = 5\ \mu$F, find the values of the two circuit elements which will represent Z in series form.

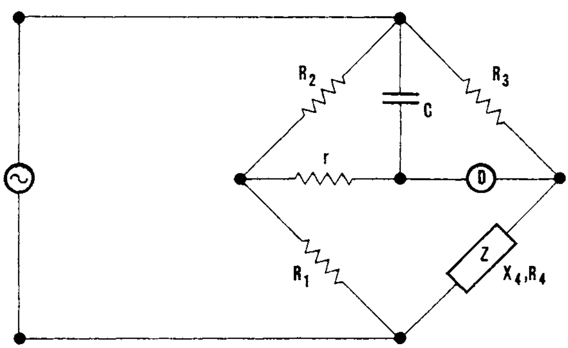


FIG. 10.16

$R_4 = 100\ \Omega$, $L = 3{\cdot}25$ H. I.E.R.E., May 1963

6. Give the names and circuit diagrams of three bridges which are commonly used for the measurement of inductance. Select one of these which is also particularly useful for the measurement of the Q factor of a coil and derive the equation of balance.

A Hay bridge balances at 1 Mc/s with a capacitance of 1000 pF and a resistance of 1·59 Ω in the measuring arm. What is the Q of the unknown coil? What are the probable balance settings at twice the frequency?

100; 250 pF, 1·59 Ω. I.E.R.E., Nov. 1961

7. How would an inductance of about 100 μH be measured
(a) at audio frequencies using a bridge method,
(b) at radio frequencies using a resonance method?
In each case describe in detail the measurements to be made, and how the inductance would be calculated, deriving any formulae used.

I.E.R.E., Nov. 1962

8. What are the essential features of the circuit of a valve-voltmeter consisting of a diode detector and d.c. amplifier? Discuss the factors which limit the highest frequency at which the instrument can be used. H.N.C.

9. Describe the operation of a peak-reading valve-voltmeter and comment on its merits and demerits within its field of application. Calculate the relative reading when measuring a full-wave rectified sine wave if the diode probe is arranged (a) in series, (b) in parallel. Include calculations for reversed diode connections. Assume that calibration has been adjusted to read r.m.s. values on sine wave input.

97·45%, 98·09%. I.E.R.E., May 1963

10. Fig. 10.17 shows the circuit of a d.c. valve-voltmeter. Describe the manner in which the circuit operates and the function of each component. State the main disadvantages of such a circuit. If the valve has $\mu = 20$ and

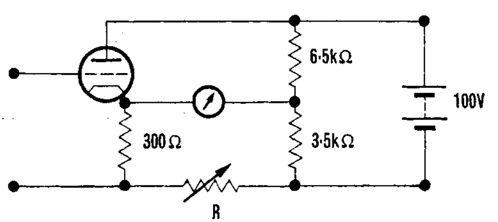


FIG. 10.17

$r_a = 9000$ Ω, R is set at 9700 Ω and the meter resistance is 500 Ω, find (a) the anode current when the meter reads zero, (b) the d.c. input voltage required to produce a current of 1 mA through the meter.

3·5 mA; 0·83 V. I.E.R.E., May 1960

11. Draw a complete circuit diagram of a d.c. valve-voltmeter using four transistors in a balanced arrangement and explain concisely how the circuit functions. Indicate component values, and use these to calculate the input impedance. Compare the performance with a d.c. valve-voltmeter using vacuum valves. I.E.R.E., May 1963

12. The magnification factor of a coil is to be measured with the aid of the following apparatus:

(a) a fixed-frequency, fixed voltage radio-frequency oscillator of negligible internal impedance,

(b) a calibrated variable capacitor, and

(c) an ammeter.

Describe the method fully and show that

$$Q = \frac{C+\Delta C}{\Delta C}\sqrt{(n^2-1)}$$

where C = resonant tuning capacitance,

$C+\Delta C$ = value of tuning capacitance to reduce the current to $1/n$th of the resonant value.

What are the points to be noted if an accurate result is to be obtained?

H.N.C.

13. An e.m.f. of constant voltage at a frequency of 1·2 Mc/s is induced in a coil having a calibrated variable capacitor and ammeter connected in series across the coil. The current is observed to fall to 60% of its maximum value when the capacitance is adjusted to either 140 pF or 145 pF. Neglecting the capacitance of the coil, and the impedance of the ammeter, calculate the effective resistance of the coil at 1·2 Mc/s.

12·23 Ω. I.E.R.E., Nov. 1963

14. In a Q meter, a coil resonates at 2 Mc/s with a capacitance of 320 pF and then shows a Q-factor of 200. When a resistor is connected across the coil, resonance is restored by retuning to a capacitance of 315 pF and the Q factor is then 150. Calculate the value of the resistor and its shunt capacitance.

150 kΩ; 5 pF.

15. If C_1 is the value of capacitor C needed to cause the circuit of Fig. 10.18 to resonate at an input frequency of f_1 c/s and C_2 is the value of capacitor C

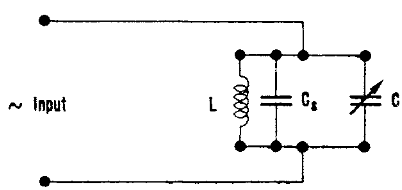


FIG. 10.18

needed to cause the circuit to resonate at an input frequency of f_2 c/s, show that

(a) $C_s = \dfrac{C_1 - n^2 C^2}{n^2 - 1}$ where $n = \dfrac{f_2}{f_1}$

(b) $L = \dfrac{1}{4\pi^2}\left[\dfrac{1}{C_1 - C_2}\right]\left[\dfrac{1}{f_1{}^2} - \dfrac{1}{f_2{}^2}\right]$.

16. An attenuator consists of symmetrical T-sections having series arms each of R_1 ohms and a shunt arm of R_2 ohms. Derive an expression in terms of R_1 and R_2 for the characteristic impedance of this network and the attenuation in decibels per section. If R_1 is 180 Ω, and R_2 is 400 Ω, evaluate these quantities.

420 Ω; 7·96 dB. I.E.R.E., Nov. 1963

17. Find the input impedance and attenuation of the T-attenuator shown in Fig. 10.19.

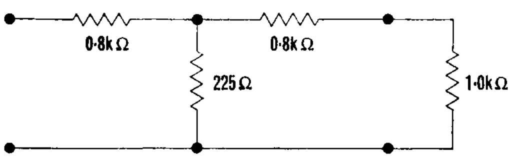


FIG. 10.19

1000 Ω; 19·1 dB.

18. Calculate the impedance of a symmetrical resistive T-network given that the characteristic impedance on short circuit is 420 Ω and the total resistance of the series arm is 600 Ω.

458 Ω. I.E.R.E., Nov. 1957

19. Explain the purpose of a variable attenuator and discuss its electrical requirements and the various types that have been evolved. Calculate the values of a resistive, symmetrical T-pad having a characteristic impedance of 600 Ω and an insertion loss of 20 dB when correctly terminated. What is the attenuation when the termination resistance is changed to 300 Ω?

121·2 Ω; 491 Ω; 23·522 dB. I.E.R.E., May 1963

20. Specify the use of the network shown in Fig. 10.20, and derive the equation of zero transfer admittance. Give the advantages of this type of network over a four-terminal bridge in the same application. Find an approximate value for the admittance of Z for zero transfer if $C_1 = C_2 = 200$ pF, $R = 0{\cdot}125$ MΩ and $\omega = 10^6$ rad/s.

$G = 2\,\mu\,\Omega^{-1}$; $B = -j10^{-4}\,\Omega^{-1}$. I.E.R.E., Nov. 1962

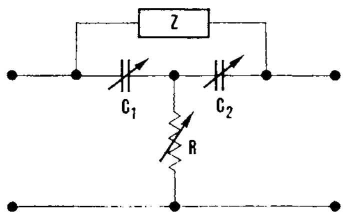


FIG. 10.20

21. Explain a method of measuring the depth of modulation of an amplitude-modulated wave by means of a cathode-ray oscillograph. Show how the appearance of the trace is modified if the modulation level exceeds 100%.

Describe also a method of measuring the frequency deviation of a frequency-modulated wave. H.N.C.

22. Describe how a cathode-ray oscilloscope may be used to monitor an amplitude-modulated wave in order to

(a) determine the depth of modulation,
(b) estimate the group delay, and
(c) detect amplitude distortion.

Sketch the typical waveforms, assuming that the modulating signal is a single sine wave and that it is available for monitoring. The r.m.s. value of current fed to an aerial is 18 A without modulation, and rises to 20·6 A when amplitude modulated with a sine wave. What is the depth of modulation? Derive any formula used.

79%. I.E.R.E., May 1962

23. Derive an expression for the deflectional sensitivity of a cathode-ray tube having electrostatic deflection. With the aid of a sketch explain why vertical and horizontal deflectional sensitivity usually differ.

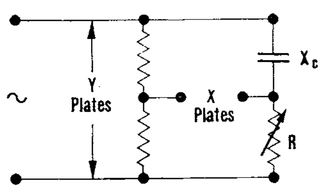


FIG. 10.21

Using the network shown in Fig. 10.21, sketch reasonably to scale the traces obtained (a) when $R = 0$, (b) when $R = X_c$, (c) when $R = \infty$, (d) when $R = \sqrt{3}X_c$. Indicate how the phase difference may be determined.

Assume that the ratio of

$$\frac{\text{vertical deflectional sensitivity}}{\text{horizontal deflectional sensitivity}} = 1{\cdot}5.$$

I.E.R.E., Nov. 1962

24. With the aid of diagrams give concise details of how harmonic content of a waveform may be measured by (a) wave-analyser, (b) distortion factor meter, (c) dynamometer instrument. A dynamometer instrument gave beats of equal amplitude at frequencies of 25, 50 and 75 c/s. The input voltages were 50 V, 6 V and 8 V respectively. What is the total percentage distortion if the frequency response of the dynamometer is 1 dB per octave referred to 50 c/s?

17%. I.E.R.E., May 1963

25. Explain how the frequency of a radio-frequency source may be measured by means of an interpolating oscillator and a crystal oscillator. What are the essential technical requirements of such an interpolating oscillator and crystal oscillator? H.N.C.

26. Give briefly, with the aid of diagrams, two methods by which the selectivity of an a.m. broadcast receiver at 1 Mc/s can be measured.

Draw approximate graphs of the results expected, and compare the relative merits of the two methods. I.E.R.E., Nov. 1962

CHAPTER 11

Transmission Lines

WORKED EXAMPLES

Example 1

A transmission line has the following constants per loop mile: $R = 75\ \Omega$, $L = 1{\cdot}5$ mH, $C = 0{\cdot}1\ \mu$F and G, the conductance, negligible. What impedance is necessary to terminate it correctly at $\omega = 10^4$ rad/s?

If an alternating p.d. of 10 V r.m.s. at this angular frequency is applied to this correctly terminated line, what will be the voltage across the line at a point 5·7 miles from the sending end?

I.E.R.E., Nov. 1959

Solution

For a uniform transmission line to be correctly matched, its termination must be equal to the characteristic impedance Z_0 of the line, where

$$Z_0 = \sqrt{\left(\frac{(R+j\omega L)}{(G+j\omega C)}\right)} \tag{11.1}$$

Substituting given values into equation (11.1) gives,

$$\begin{aligned} Z_0 &= \sqrt{\left(\frac{75+j\times 10^4\times 1{\cdot}5\times 10^{-3}}{j\times 10^4\times 0{\cdot}1\times 10^{-6}}\right)}\ \Omega \\ &= 100\sqrt{\left(\frac{7{\cdot}5+j1{\cdot}5}{j}\right)}\ \Omega \\ &= 100\sqrt{\left(\frac{7{\cdot}64\ \angle 11{\cdot}3^\circ}{1\ \angle 90^\circ}\right)}\ \Omega \\ &= 100\sqrt{(7{\cdot}64\ \angle 11{\cdot}3^\circ-90^\circ)}\ \Omega \\ &= 276\ \angle -39{\cdot}35^\circ\ \Omega. \end{aligned}$$

The general equations for the values of voltage (V_x) and current (I_x) at any distance x from the sending end of any uniform transmission line are respectively,

$$V_x = A \exp(Px) + B \exp(-Px) \tag{11.2}$$

$$I_x = \frac{1}{Z_0}[B \exp(-Px) - A \exp(Px)] \tag{11.3}$$

where A and B are constants which depend upon source and terminal conditions of the line, and P is the propagation constant of the line which is generally complex and given by

$$P = \alpha + j\beta \tag{11.4}$$

$$= \sqrt{[(R+j\omega L)(G+j\omega C)]}. \tag{11.5}$$

Equation (11.2) may be regarded as consisting of two component voltage waves. They are

(a) a forward travelling wave $B \exp(-P_x)$

and (b) a backward travelling wave $A \exp(Px)$.

The forward travelling voltage wave in a line of infinite length approaches zero as x approaches infinity, therefore, from equation (11.2),

$$0 = A \exp(\infty) + 0.$$

Therefore $\quad A = 0$

Thus, for an infinite line,

$$V_x = B \exp(-Px). \tag{11.6}$$

Now, when $x = 0$, $V_x = V_s$, and equation (11.6) under these conditions gives

$$V_x = V_s = B$$

and the voltage at any distance x from the sending end of a uniform transmission line of infinite length is given by

$$V_x = V_s \exp(-Px)$$

$$= V_s \exp[-(\alpha+j\beta)x] = V_s \exp(-\alpha x) \exp(-j\beta x) \tag{11.7}$$

If the voltage applied to the line is assumed sinusoidal, then the voltage at any point is sinusoidal, and equation (11.7) becomes

$$V_x = V_s \exp(-\alpha x)\exp(-j\beta x)\exp(j\omega t) \qquad (11.8)$$

Now, from equation (11.5)

$$\begin{aligned} P &= \sqrt{[(75+j\times 10^4\times 1{\cdot}5\times 10^{-3})(j\times 10^4\times 0{\cdot}1\times 10^{-6})]} \\ &= \sqrt{(76{\cdot}4\ \angle 11{\cdot}3^\circ \times 10^{-3}\ \angle 90^\circ)} \\ &= 0{\cdot}276\ \angle 50{\cdot}65^\circ \\ &= 0{\cdot}276\cos 50{\cdot}65^\circ + j0{\cdot}276\sin 50{\cdot}65^\circ \\ &= 0{\cdot}1745 + j0{\cdot}214. \end{aligned} \qquad (11.9)$$

Comparison of equations (11.9) and (11.4) reveals that the attenuation coefficient α is given by

$$\alpha = 0{\cdot}1745 \text{ neper/mile}$$

and the phase change coefficient β is given by

$$\beta = 0{\cdot}214 \text{ rad/mile.}$$

The voltage 5·7 miles from the sending end has a magnitude of

$$\begin{aligned} V_x &= V_s \exp(-\alpha x) \\ &= 10\exp(-1) \\ &= 3{\cdot}68 \text{ V r.m.s.} \end{aligned}$$

This voltage lags behind the sending end voltage by an angle β radians where

$$\beta x = 0{\cdot}214\times 5{\cdot}7 = 1{\cdot}22 \text{ rad or } 70^\circ.$$

The phase lag would be 2π radians, i.e. 1 wavelength, when

$$\beta x = 2\pi,$$

hence

$$x = \lambda = \frac{2\pi}{\beta} \qquad (11.10)$$

and the velocity of propagation (v) of the voltage waves down the line may be found using

$$v = \lambda f = \frac{2\pi f}{\beta} = \frac{\omega}{\beta} \tag{11.11}$$

In this case, the velocity of propagation

$$v = \frac{10{,}000}{0{\cdot}214} = 4{\cdot}68 \times 10^4 \text{ mile/s.}$$

Problems with Answers

1. What is meant by the term loading when used in connection with telephone cables? Explain the advantages and disadvantages arising from continuous and lumped loading.

A line has the following primary constants per loop mile:

$$R = 30\ \Omega,\quad L = 4\text{ mH},\quad G = 1\ \mu\Omega^{-1},\quad C = 0{\cdot}008\ \mu\text{F}.$$

Calculate, for a frequency of 1592 c/s, the characteristic impedance of the line, the velocity of propagation along it, the phase constant and the attenuation constant.

$Z_0 \simeq 791 \angle -18^\circ\ \Omega$; $v = 1{\cdot}67 \times 10^5$ mile/sec;
$\alpha \simeq 0{\cdot}021$ neper/mile; $\beta \simeq 0{\cdot}06$ rad/mile. I.E.R.E., Nov. 1960

2. The primary coefficients of a cable are:

Resistance per mile	40 Ω.
Capacitance per mile	0·05 μF.
Inductance per mile	1 mH.
Conductance per mile	negligible.

Calculate, for a frequency of 15·9 kc/s, the attenuation, the phase change coefficient, the characteristic impedance, the wavelength, and the phase velocity.

0·127 neper/mile; 0·722 rad/mile; $147 \angle -10{\cdot}9^\circ$; 8·7 miles;
$1{\cdot}38 \times 10^5$ mile/s. I.E.R.E., May 1962

3. Explain what is meant by characteristic impedance and propagation constant of a uniform transmission line. What is the significance of the real and imaginary parts of the propagation constant?

A transmission line 5 km long has a characteristic impedance of 500 $\angle -45^\circ\ \Omega$, and propagation constant $(0{\cdot}06 + j0{\cdot}04)$ per km at 1000 c/s. If the line is terminated at its characteristic impedance and the sending end voltage is 5 V at 1000 c/s, calculate

(a) the receiving end current and its phase relative to the sending end voltage;
(b) the wavelength;
(c) the velocity of propagation.

7·4 $\angle 34{\cdot}5°$ mA; 157 km; $1{\cdot}57 \times 10^5$ km/s. H.N.C.

4. A uniform cable has the following primary constants per loop mile:

Resistance = 90 Ω	Inductance = 1 mH
Capacitance = 0·062 μF	Conductance = 1·5 $\mu\Omega^{-1}$

The cable is 10 miles long, terminated in its characteristic impedance and the power input to the line is 24 mW at 1000 c/s. Calculate the magnitude and phase of the current in, and the voltage across, the line
(a) at the sending end,
(b) at 5 miles from the sending end,
and (c) at the termination.

8·4 $\angle 45°$ mA, 4·03 $\angle 0°$ V; 4·34 $\angle 7{\cdot}2°$ mA, 2·08 $\angle -37{\cdot}8°$ V;
2·4 $\angle -30{\cdot}6°$ mA, 1·15 $\angle -75{\cdot}6°$ V. H.N.C.

5. The sending end impedance Z_g of a loss-free line of electrical length θ degrees and characteristic impedance Z_0 terminated in Z_r is given by:

$$Z_g = Z_0 \frac{(Z_r \cos \theta + jZ_0 \sin \theta)}{(Z_0 \cos \theta + jZ_r \sin \theta)}$$

Derive expressions for sending end impedance Z_g of a line
(a) terminated by an open circuit,
(b) terminated by a short circuit,
and (c) $\lambda/4$ long terminated in Z_r.
Show how a line may be used as a harmonic filter and calculate the dimensions of an open wire line used as a filter to attenuate the third harmonic of 10 Mc/s.

I.E.R.E., Nov. 1962

6. An open wire transmission line of characteristic impedance 400 Ω is to be connected to a purely resistive load of impedance 50 Ω using a quarter wave transformer. Determine the characteristic impedance of the transformer if standing waves are to be avoided on the 400 Ω line.

141·4 Ω.

7. Define (a) characteristic impedance, (b) standing wave ratio as applied to transmission lines.
A loss-free transmission line of 50 Ω characteristic impedance is terminated with a resistance load of 75 Ω. Calculate
(i) the standing wave ratio,
(ii) the reflection coefficient.
Describe one method of producing a standing wave ratio of unity on the line by means of a matching section.

1·5; 1 : 5. I.E.R.E., May 1961

8. The input admittance Y of a length l of low-loss high-frequency transmission line is given by

$$Y = Y_0[1-k\times\exp(-j2\beta l)]/[1+k\times\exp(-j2\beta l)]\ \Omega^{-1}$$

where k is the voltage reflection coefficient at the termination of the line, Y is the characteristic admittance of the line (real), and β radians per unit length is the phase change coefficient of the line.

Show that, for the conductance component of Y to be equal to Y_0, the length of line required is given by

$$\frac{\phi+\cos^{-1}|k|}{2\beta}$$

where ϕ is the angle of the reflection coefficient.

An open wire feeder, the characteristic impedance of which is $(3-j0)$ mΩ^{-1}, is terminated by an aerial which presents a load admittance of $(1-j4)$ mΩ^{-1}.

Calculate the minimum length of a short-circuited matching stub, and the distance of this from the aerial, if the standing waves are to be avoided along the rest of the line. The wavelength of the transmission is 6·28 m.

$l_1 = 2{\cdot}19$ m; $l_2 = 0{\cdot}37$ m. I.E.R.E., Nov. 1963

9. Discuss two methods of producing a standing wave ratio of unity on a concentric feeder terminated with a load which is not equal to its characteristic impedance.

A concentric feeder of characteristic impedance 50 Ω is terminated by a load of $(16+j8)$ mΩ^{-1}.

Unity standing wave ratio is to be achieved by the parallel connection of a short circuit stub of characteristic impedance 50 Ω at a distance $\lambda/4$ from the load. Calculate

(a) the electrical length of the stub,

(b) the current flowing in the load when it dissipates 10 kW of unmodulated power.

$\theta = 116{\cdot}6°$; $I = 14{\cdot}4$ A. I.E.R.E., Nov. 1963

APPENDIX 1

REFERENCES

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APPENDIX 2

SYMBOLS AND ABBREVIATIONS

In general, the symbols and abbrevations used in the text are those recommended by the British Standards' Institution in B.S. 1991, Parts 1 and 6. Some typical examples are given below.

a Acceleration or transformation ratio T_1/T_2.
α Common base current amplification factor.
B Magnetic flux density or susceptance.
β Feedback fraction.
C_c Decoupling capacitor.
C_{ag} Anode–grid capacitance of a triode.
D Per unit or percentage distortion.
δV_g Small change in grid voltage.
E Electric field strength.
$\mathbf{E}$ r.m.s. value of sinusoidally varying induced e.m.f. ($\mathbf{E} = |E| \angle\theta$).
e Charge on an electron ($e = 1{\cdot}6\times10^{-19}$ C).
F Magnitude of force on a charged particle.
f_1 Lower half power frequency.
f_2 Upper half power frequency.
f_0 Resonant frequency of a tuned circuit.
G Conductance.
g_m Mutual conductance.
$h_{11}, h_{12}, h_{21}, h_{22}$ Hybrid parameters of a common-base transistor.
I_A Steady anode current.
$\mathbf{I}_a$ r.m.s. value of sinusoidally varying anode current ($\mathbf{I}_a = |I_a| \angle\theta$).
I_E Steady emitter current.
I_a or $|I_a|$ r.m.s. value of anode current (magnitude only).
i_a Instantaneous value of anode current.
L_p No load parallel inductance of a transformer.
L_1' Transformer leakage inductance referred to the primary.
M or $|m|$ Magnitude of stage gain.
m Stage gain ($m = |m| \angle\theta$) or rest mass of an electron ($m = 9{\cdot}1\times10^{-31}$ kg).
$P_{a.c.}$ a.c. power.
$P_{d.c.}$ d.c. power.
Q_0 Magnification factor of a tuned circuit at resonance.
r_a Anode slope resistance.

r_e	Emitter resistance.
R_L	Load resistance.
R_1'	Transformer leakage resistance referred to the primary.
t	Transit time of an electron between deflection plates.
$\mathbf{V}_o$	r.m.s. value of sinusoidally varying output voltage ($\mathbf{V}_o = \lvert V_o \rvert \angle \theta$).
$\hat{V}_a$	Peak value of anode voltage.
$V_{\text{H.T.}}$	H.T. supply voltage.
v	Velocity or instantaneous anode voltage.
ω_0	Angular resonant frequency.
X_1	Transformer primary leakage reactance.
X_1'	Transformer leakage reactance referred to the primary.
Y_{in}	Input admittance.
μ	Amplification factor.
η	Efficiency.
θ or ϕ	Phase angle.

Index to Worked Examples