Jimma University, College of Natural Sciences

Department of Physics

Mathematical Methods of Physics II (Phys 2032), Assignment II

1. If $\mathbf{A} = (2xy + z^3)\hat{\mathbf{x}} + (x^2 + 2y)\hat{\mathbf{y}} + (3xz^2 - 2)\hat{\mathbf{z}}$, show that (a) $\nabla \times \mathbf{A} = 0$ ($\mathbf{A}$ is irrotational), (b) Find a scalar function $\emptyset$ such that $\mathbf{A} = \nabla\emptyset$.
2. For the position vector $\mathbf{r} = x\,\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$, show that $r^n\mathbf{r}$ is solenoidal only if $n = -3$.
3. An electric dipole moment is located at the origin. The dipole creates an electric potential at $\mathbf{r}$ given by

$$V(\mathbf{r}) = \frac{\mathbf{p}.\mathbf{r}}{4\pi\epsilon_0 r^3}$$

   Find the electrostatic field $\mathbf{E} = -\nabla V$ at $\mathbf{r}$.
4. The velocity of a two-dimensional flow of liquid is given by

$$\mathbf{v} = u(x, y)\hat{\mathbf{x}} - w(x, y)\hat{\mathbf{y}}$$

   If the fluid is incompressible and the flow is irrotational, show that

$$\frac{\partial u}{\partial x} = \frac{\partial w}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial w}{\partial x}$$

5. Show that the vector field

$$V = \frac{-x\,\hat{\mathbf{x}} - y\hat{\mathbf{y}}}{\sqrt{x^2 + y^2}}$$

   is a sink. Give physical interpretation.
6. Classically, orbital angular momentum is given by $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{p}$ is the linear momentum. To go from classical mechanics to quantum mechanics, replace $\mathbf{p}$ by the operator $\frac{\hbar}{i}\nabla$. Show that the quantum mechanical angular momentum operator has Cartesian components,

$$L_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right), \qquad L_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right), \qquad L_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

7. If a vector function $\mathbf{F}$ depends on both space coordinates (x, y, z) and time t, show that

$$d\mathbf{F} = (d\mathbf{r}.\nabla)\mathbf{F} + \frac{\partial \mathbf{F}}{\partial t}dt$$

8. Obtain the expressions of the gradient, divergence and curl in circular cylindrical and spherical polar coordinates.