# JIMMA University

#### College of Natural Sciences

### Department of Mathematics

Course tittle: Linear Algebra II Course code: Math 326 Credit hours:3 Prerequisite: Math325 Course category: Compulsory

Tutorial :2

#### Aims

The course intends to develop further concepts in Linear Algebra providing a foundation for studies in a number of other areas of mathematics and related fields

#### **Course description**

This course covers the characteristic equation of matrix, orthogonality, matrix factorizations, canonical forms, direct sum decomposition of vector spaces, bilinear, quadratic and positive definite forms.

#### **Course Objective**

On the completion of the course, successful students will be able to:

- Find eigenvalues and eigenvectors of a square matrix.
- Identify similar matrices
- Diogonilize a matrix when this is possible
- Difine inner product space
- ➢ Find and apply LU factorization of matrix
- Understand the Gram-Schmidt process
- ➢ Find an orthogonal basis for a subspace
- > Find an orthogonal complement of subspace
- Recognize and invert orthogonal matrices
- Comprehend the three canonical forms of matrices

#### **Course outline**

# Chapter 1: The characteristic equation of a matrix

- ✓ Eigenvalues and eigenvectors
- $\checkmark$  The characteristic polynomial
- ✓ Similarity of matrices and characteristic polynomial
- $\checkmark$  The spectral radius of a matrix
- ✓ Diagonalization
- ✓ Decomposable matrices
- ✓ Minimal polynomial and Cayley-Hamilton theorem

# Chapter2: Orthogonality

- $\checkmark$  The inner product
- $\checkmark$  Inner product space
- $\checkmark$  Orthonormal sets
- ✓ The Gram-Schmidt orthogonalization process
- ✓ Cauchy-Schwartz and triangular inequalities
- ✓ The dual space
- ✓ Adjoint of linear operators
- ✓ Self-adjoint linear operators
- ✓ Isometry
- $\checkmark$  Normal operators and the spectral theorem

- ✓ Factorization of a matrix (LU, cholesky, QR)
- $\checkmark$  Singular value decomposition

### **Chapter 3: Canonical forms**

- Elementary row and column operations on matrices
  Equivalence of matrices of polynomials
- ✓ Smith canonical forms and invariant factors
- ✓ Similarity of matrices and invariant factors
- $\checkmark$  The rational canonical forms
- ✓ Elementary divisors
- $\checkmark$  The normal and Jordan canonical forms

# Chapter 4: Bilinear and quadratic forms

- ✓ Bilinear forms and matrices
- ✓ Alternating bilinear forms
- ✓ Symmetric bilinear forms and quadratic forms
- ✓ Real symmetric bilinear forms

# Chapter 5: Direct sum decomposition of vectors spaces

- $\overline{\checkmark}$  Definition of a direct sum of vector spaces
- $\checkmark$  Projection and invariant subspaces of a linear operator
- ✓ Primary decomposition theorem

# **Teaching-learning methods**

Three contact hours of lectures and two hours of tutorials per week. Students do home assignments.

Assignments /quizzes/ 20%

Mid semester examination 30%

Final examination 50%

#### **Teaching materials**

# **Textbooks:**

- Serge Lang, Linear Algebra
- Schaum's Outline in Linear Algebra

**References:** 

- S. Lipschitz, Theory and problems of Linear Algebra, second Ed., McGraw-Hill1991
- Larson /Edwards, Elementary Linear Algebra, D.C. Heath and company, Lexington, 1988
- > J.N. Sharma and et al, Linear Algebra, Krishna prakashan Media(p) Ltd., 2003
- Isaak and Manougian, Basic Concept of Linear Algebra, 1<sup>st</sup> ed., George J.McLead Limited,1976
- > Otto Bretscher, Linear algebra with application, 3<sup>rd</sup> ed., Prentice Hall, 2005
- Howard Anton, Elementary linear algebra, 8th ed., John Wiley, 2000
- ▶ K. Hoffman and R. kunze, Linear Algebra, 2<sup>nd</sup> ed., prentice Hall INC., 1971