

# Debre Berhan University College of Natural and Computational Science Department of Mathematics

Curriculum for M.Sc. Program in Differential Equations

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# 1. Degree Nomenclature

Upon successful completion of the program, the degree would be awarded as

THE DEGREE OF Master of Science in Differential Equations

# 2. Course Requirement:

There are two categories of courses, namely: compulsory and concentration area courses of total 39 credit hours.

# 2.1.List of Compulsory Courses

		Credit	Tutorial/Lab.
Course No.	Course Title	Hours	hrs
Math 621	Algebra I	3	2
Math 681	Selected topics in DE	3	2
Math 632	Computational Mathematic	3	2
Math 664	Functional Analysis I	3	2
Math 661	Real analysis I	3	2
Math 666	Real analysis II	3	2
Math 673	Topology	3	2
Math 622	Algebra II	3	2
Math 702	Thesis/Project	6	
Math 701	Graduate seminar	1	
Sub total	ı	31	

Differential Equations Courses			
Math 682	1. Ordinary differential Equations	3	2
Math 783	2. Partial Differential Equations	3	2
Math 745	3. Mathematical Modeling	3	2
Math 788	4. Distribution Theory and Function Spaces	3	2
TO	OTAL CREDIT HOURS:	12	

# 3. Course Break down

# YEAR ONE SEMESTER I

Course No.	Course Title	Credit Hours
Math 621	Algebra I	3
Math 661	Real Analysis I	3
Math 673	Topology	3
Math 681	Selected topics in DE	3
Total		12

# YEAR ONE SEMESTER II

Course No.	Course Title	Credit Hours
Math 632	Computational Mathematics	3
Math 664	Functional Analysis I	3
Math 666	Real Analysis II	3
Math 622	Algebra II	3
Total		12

## YEAR TWO SEMESTER I

Course No.	Course Title	Credit
		Hours
Math 783	Partial Differential Equations	3
Math 745	Mathematical Modeling	3
Math 788	Distribution Theory and Function Spaces	3
Math 701	Graduate seminar *	1
Total		10

# YEAR TWO SEMESTER II (Differential equations Stream)

Course No.	Course Title	Credit
		Hours
Math 702	Thesis /Project	6
Total		6

\*: the students will submit the title of their seminar at the end of first year second semester to his/her respective advisor and present the title to the department graduate committee and get approval.

# 4. Course Description

Course title: Algebra I

Course code: Math 621

Credit Hr.: 3 Cr. Hrs.

Tutorial Hr.: 2 hours per week

**Prerequisite: None** 

Learning objectives: After the completion of this course students will be able to

- Define the concept group and give examples
- Classify groups upto isomorphism
- Obtain complete structure theorems for various restricted class of groups
- Prove Krull-Schmidt theorem
- State symmetric ,free abelian, nilpotent and solvable groups
- Define ring and give examples
- construct examples of rings, ideals, and homomorphisms etc. that satisfy or fail
- state the first and the third isomorphism theorem for quotient rings and the correspondence theorem for ideals in quotient rings, and uses these theorems to determine the structure of a quotient ring.

# **Course Descriptions**

Binary operations, permutations, homomorphisms and subgroups, cosets and counting, Lagrange's theorem, the first isomorphism theorem, the second isomorphism theorem, the third isomorphism theorem, symmetric, alternating and Dihedral groups, free groups, generators and relations, free Abelian groups, finitely generated Abelian groups, Krull-Schmidt theorem, the action of a group on a set, the sylow theorems, classification of finite groups, nilpotent and solvable groups, normal and subnormal series, rings and homomorphisms, ideals, factorization in

commutative rings, rings of quotients and localization, rings of polynomials, factorization in polynomial rings.

Course Delivery: Lectures, Group Assignments.

# **Assessment Techniques**

- Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...) 50%
- Final Examination: 50%

Text Book : Algebra , Thomas W. Hungerford ,Springer Verlag, New York Inc. Reference

- 1. Hungerford, T.H.: Algebra, Springer-Verlag, 1974.
- 2. Lang, S: Algebra, Addison-Wesley, 1970.
- 3. Goldstein. L.J: Abstract Algebra: A First Course, Prentice-Hall Inc., 1973

**Course Title: Functional Analysis I** 

Course Code: Math 664

Credit Hour: 3 Cr. Hrs.

**Tutorial: 2 hours per week** 

Prerequisite: none

This course extends the ideas studied in Analysis and Topology. Many of the topics studied in the course have applications in Approximation theory, operator's theory and other areas of mathematics

Learning Objectives: Upon completion of the course, the student should be able to:

- study the details of Banach and Hilbert Spaces and to introduce Banach algebras
- describe definitions and relations in the theory of functional analysis and to use these in problem solving
- Interpret, communicate and argue using mathematical notions.

 Apply the studied theories within one area in applied mathematics, science and engineering.

## **Course description:**

Metric Spaces, Completion of Metric Spaces, Normed Spaces, Finite Dimensional Normed Spaces, Bounded Linear operators, Linear Functional, Dual Space, Inner Product Spaces and Hilbert Spaces, Riesz Representation Theorem, Hilbert Adjoint Operator; Self-Adjoint, Unitary and Normal Operators; Hahn-Banach Theorem, Uniform Boundedness Principle, Strong and Weak Convergence, Open Mapping and Closed Graph Theorems.

## **Course Delivery Methods**

Lectures, Tutorial, Group Assignments

Assessment Techniques

- Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...) 50%
- Final Examination: 50%

Text book: K. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons New York, 1978

#### References:

- 1. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966
- 2. N. Dunford and J. T. Schwartz, Linear Operators, Part I, Interscience, New York, 1958
- 3. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987
- 4. P. K. Jain, O. P. Ahuja and Khalil Ahmad, Functional Analysis, New Age International (P) Ltd. Wiley Eastern Ltd. N. Delhi- 1997
- 5. R. B. Holmes, Geometric Functional Analysis and its Applications, Springer-Verlag, 1975
- 6. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.

- 7. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw -Hill Co., New York, 1963.
- 8. A. E. Taylor, Introduction to Functional Analysis, John Wiley and Sons, New York, 1958
- 9. K. Yosida, Functional Analysis, 3rd edition Springer Verlag, New York 1971.
- 10. J. B. Conway, A course in functional analysis, Springer-Verlag, New York 199012.
- 11. Any Functional Analysis Book

**Course Title: Real Analysis I** 

**Course Code: Math 661** 

Credit Hour: 3 Cr. Hrs.

**Tutorial: 2 hours per week** 

**Prerequisite: none** 

Learning Objectives: At the end of this course students will be able to

- Define and state basic concepts and properties of Real number system
- Relate different mathematical concepts like metric spaces, continuity and compactness
- Distinguish point wise and uniform convergence
- State and apply fundamental properties of sequences and series, superior and inferior
- State and prove theorems of Riemann, Stielgets theorems
- Integrate and differentiate higher level integrals like, Riemann and Stielgets integrals
- Solve applicable problems using the above concepts and properties

# Course Description

• Background: Basic properties of Real number system (Countability, sequence and series etc)

• Metric spaces, continuity and compactness

• Basic limits; like infinite limits, limit at infinity, superior and inferior

• Sequences and series of functions, convergence 9point wise and uniform

convergence)

• Integration and differentiation; properties of integrals, Riemann Stieljets

integral, Lebesgue integral, convergence and integration

Course delivery methods:

Active lecture, Tutorials, Group assignments

Assessment Techniques

• Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...)

50%

• Final Examination: 50%

References

• Goldberg Methods of Real Analysis

• Principles of Analysis by Walter Rudin

• Real analysis by H. L. Royden

**Course Title: Computational Mathematics** 

**Course Code: Math.632** 

**Credit hours: 3** 

Lab: - 2 hours per week

**Prerequisite: None** 

Course Description:

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Computational science is a blend of applications, computations and mathematics. It is a mode of scientific investigation that supplements the traditional laboratory and theoretical methods of acquiring knowledge. This is done by formulating mathematical models whose solutions are approximated by computer simulations. Learning Objectives:

Computational Mathematics will introduce some key ideas and techniques associated with the numerical solution of differential equations, ranging from theoretical questions about the accuracy of finite difference schemes and the efficiency of algorithms, through to implementation in computer codes. The course therefore provides a foundation for postgraduate study and research in many fields that rely on numerical modeling. The course is also devoted to computer programming for scientific and engineering applications. We will write programs using a subset of C, C++, Matlab, Mathimatica, FORTRAN 95/, and introduce a few standard software development tools under Linux/Windows.

In addition to learning the mathematical content of the course also write computer programs and be introduced to some standard numerical libraries. See how the performance of a practical code depends on the efficient implementation of stable and accurate numerical algorithms.

# Course Delivery Method:

The lectures focus on scientific computing, then on numerical methods for problems in linear algebra and ordinary differential equations. This material lays the foundation for an introduction to the numerical solution of partial differential equations. Then it covers numerical algorithms with computer programs. In the tutorials and labs, work on many small problems, developing the skill set needed to piece together a complete numerical simulation.

# Assessment Techniques

• Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...) 50%

• Final Examination: 50%

#### References:

- 1. Computational Mathematics by K. Thangavel, Alpha Science International, Ltd.
- 2. Introduction to Computational Mathematics by Xin-She Yang.
- 3. Computational Mathematics, Modeling and Algorithm by J. C. Misra, Narosa Pub. House, New Delhi.

**Course Title: Real Analysis II** 

**Course Code: Math 666** 

Contact Hr. 3 Cr. hrs

**Tutorial: 2 hours per week** 

Prerequisite: Real Analysis I

Course description

- Outer Measures, measurable sets and Functions
- Integration
- The Lebesgue Measures
- Product Measures, Sign Measures
- Lp– Spaces
- Positive Linear Functional

Course delivery methods:

Active lecture, class activities, Tutorials, Group assignments

Assessment methods:

Continuous assessment (quizzes, tests, activities, assignments etc)

#### Final examination 50%

#### References

- 1. Real Analysis I by H. L. Royden
- 2. Real and Complex analysis by Walter Riudin

**Course Title: Topology** 

Course code: Math 673

Credit Hours: 3 Cr.

**Tutorial: - 2 hours per week** 

**Pre-requisite: None** 

Learning objectives: After the completion of this course students will be able to

- Differentiate finite and infinite sets.
- Prove the existence of the choice function.
- Give examples of topological spaces
- Determine whether a collection of subsets of a set determine a topology.
- Determine whether a collection of subsets of a set determine a basis for topology
- Define the product topology
- Define the subspace topology
- Define closed sets and limit points and give examples
- State conditions for a topological space to be Hausdorff
- Define and construct continuous functions
- Compare the box and product topology
- Define a metric topology and give examples
- Define a connected space
- State properties of connected spaces of the real line

• Define compact spaces and prove various properties of a compact space

• State properties of compact subspaces of the real line

• State different formulations of the notions of compactness

**Course Descriptions** 

Finite and infinite sets, the axiom of choice, topological spaces, the subspace topology, closed and limit points, continuous functions, metric spaces, connectedness and compactness.

Course Delivery Methods

Lectures, Tutorial, Group Assignments

**Assessment Techniques** 

• Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...)

50%

• Final Examination: 50%

Text Book: Topology, James R. Munkres Prentice –Hall India private limited,

New, Delhi.

References

1. Fred H. Croom, Principles of Topology, Saunders College Publishing,

Philadelphia,

New York, Chicago. ISBN 03-012813-7

2. K.D. Joshi, General Topology, Wiley Eastern

3. J.L. Kelley, General Topology, Van Nostrand

**Course Title: Thesis /Project** 

Course Code: Math 702

Credit Hour: 6 Cr. Hrs.

Learning Objectives: At the end of this course students will be able to

Plan for scientific research work

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- Work out research projects at higher level
- Solve physical and community problems through scientific study
- Produce published scientific articles that could solve societal problems
- Create mathematical models that could solve physical problems
- Participate in scientific research team work
- Create abstract assumptions and model them for possible solution
- Produce original and review articles through scientific procedures

# **Course Description**

The following areas are assumed that post graduate students of mathematics could possibly do their research projects.

- Local assumptions related to & solve societal problems showing the application of mathematics
- Mathematics applicability in other fields like; Statistics, Computer Sciences, Physics, technology, Chemistry, Geography, Health related (Medicine, Environmental, Pharmacy, Laboratory Technicians etc)
- In line with concentration areas which will be more of document analysis and review articles around the areas
- Original work in mathematics, applied or pure, like problem solving and creating a model for possible solutions.

Course delivery methods: Totally student centered supported by guidance, follow up and supervision of advisor(s)

**Evaluation methods** 

• Continuous assessment starting right from title/problem selection,

proposal development and its approval through presentation, data

collection process, analysis and report writing; which constitutes

50% by the advisor(s).

• Examination through rigorous presentation by two examiners; one

external (if possible) the second internal, and the advisor(s). This

will count the rest 50%.

The passing grade will be at least 60% on aggregate with

equivalent letter grade of minimum B.

\* Remark: Even though students will register in the second year second

semester, the course will began in the second year first semester with its

continuous assessment.

**Course Title:- Ordinary differential Equations** 

Course code: MATH 682

Credit hours: 3 Cr. Hrs.

Tutorial / Lab: 2 hours per week

Prerequisite -----

Learning Objective:

At the end of this course students will be able to:

define the solution of an ordinary differential equation

learn mathematical methods to solve Higher Order

differential Equations and apply to dynamical problems

• define and solve initial value problems

differentiate linear from non linear O.D.E

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- find differential equations that model certain problems
- state different types of functions

## **Course Description**

General remarks on solutions of differential equations &Families of curves, Modeling with ODE, Initial Value Problems, Linear Differential Equations of Higher Order, Fixed Point Theorems, Existence and uniqueness theorems, Linear systems, Nonlinear differential equations, Stability and asymptotic behavior of solutions, Lyapunov functions, Legender polynomials, Bessels functions, Gamma functions & Hermite polynomials, Sturm-Liouville Problems and generalized Fourier series, Green's Functions, Dirac delta functions, Bessel's differential equations, Runge-Kutta methods.

Mode of course delivery

Giving explanation for the topics that need clarification and elaboration with active learning method. Certain topics or units may be given as an assignment.

## **Assessment Techniques**

- Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...) 50%
- Final Examination: 50%

#### References

- 1. S. G. Deo, Ragavendra, 'Ordinary Differential Equations and Stability Theory', Tata McGraw-Hill Publishing Company Ltd. (1980)
- 2. W. W. Bell, 'Special functions for Scientists and Engineers', D.Van Nostrand Company Ltd.(1968).
- 3. E. A. Coddington: Theory of Ordinary Differential Equations, McGrawal-Hill Book Company, New York, 1984
- 4. E. Hairer, S.P. Norsett and G. Wanner: Solving Differential Equation I
- 5. Antole K. & Boris H.: Introduction to Modern Theory of Dynamical Systems
- 6. Ross & Weinberger: Introduction to Differential Equations,

7. Ritger-Rose & Dennmeyer: Differential Equations, Mc Grawal-Hill, USA, 1968

8. George F. Simmons, 'Differential Equations with Applications and Historical

Notes

**Course Title: - Partial Differential Equations** 

Course code: MATH .783

Credit hours: 3crhs.

Tutorial/Lab: 2 hours per week

**Prerequisite: Ordinary differential Equations** 

**Learning Objectives:** 

At the end of the course the student will be able to:

• define partial differential equations

• formulate problems that lead to partial differential equations

• classify partial differential equations as elliptic, parabolic and hyperbolic

• apply classical solution methods for heat and wave equations

**Course description:** 

General remark on partial differential equations, linear and Non-linear partial differential equations, Cauchy's method of characteristic, Compatible systems of , Chapit's method of solution, Solutions satisfying given conditions, Definition and Examples of second order linear equations, origin of second order partial differential equations, Classification of higher order partial differential equations with constant coefficients (Jordan decomposition and quadratic forms), Cauchy problems and characteristics, Classical method for the wave and heat equations, D' Alembert's method, Fourier's method and the Laplace equations, harmonic and sub-harmonic functions.

Mode of course delivery

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Giving explanation for the topics that need clarification and elaboration with active learning method. Certain topics or units may be given as an assignment.

## Assessment Techniques

- Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...) 50%
- Final Examination: 50%

#### **References:**

- 1. Zafar Ahsan: Differential equations and their applications,
- 2. Sneddon L.N. Elements of partial differential equations
- 3. J.N. Sharma and Kehar Singh, Partial Differential equations for engineers and scientists I, Narosa publishing house, New delhi, 2000

**Course Title: Mathematical Modeling** 

**Course Code: Math 745** 

**Credit hours: 3** 

Tutorial / Lab. 2 hours per week

**Prerequisite: differential Equations** 

# **Learning Objectives:**

At the end of the course the student is expected to:

- define mathematical modeling
- identify techniques of mathematical modeling
- identify different types of mathematical models
- develop and analyze some simple mathematical models

# **Course description:**

Mathematical modeling: Some situations requiring mathematical modeling, Techniques of mathematical modeling, Classifications of mathematical models, Some characteristics of mathematical models, Limitations of mathematical modeling, Some simple illustrations, Mathematical modeling through ordinary differential equations of first order: Mathematical modeling through ordinary differential equations, Linear growth and decay models, Non-linear growth and decay models, Compartment models, Mathematical modeling in dynamics through ordinary differential equations of first order, Mathematical modeling of geometrical problems through ordinary differential equations of first order.

Course Delivery: Lectures, Group Assignments

**Assessment Techniques** 

• Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...) 50%

• Final Examination: 50%

#### **References:**

1. D. N. Burghes, Mathematical modeling in the social Management and life science, Ellis Herwood and John Wiley.

2. Edward A. Bender, An Introduction to Mathematical Modeling.

3. J. N. Kapure, Mathematical modeling, Wiley Eastern.

**Course Title:- Distribution Theory and Function Spaces** 

Course code: MATH .788

Credit hours: 3crhs.

Tutorial / Lab. 2 hours per week

Prerequisite -----

**Learning Objectives:** 

At the end of this course students will be able to:

- define different function spaces distribution
- find fundamental solutions of differential operators in the space of distributions
- use generalized integral transforms to solve PDEs

## **Course Description**

Introduction, the spaces S and S', the spaces D and D', the concept of distributions, test functions and distributions, the spaces of test functions D and distributions D, Properties of functions from D and D, Regular and Singular distributions, Differentiation of distributions, Examples, Direct product and convolution of distributions, Some applications, Tempered distributions and Fourier transform, Structure Theorem for Distribution, Fourier-Lap lace Transform, Partial Differential Equations.

Mode of course delivery

Giving explanation for the topics that need clarification and elaboration with active learning method. Certain topics or units may be given as an assignment.

**Assessment Techniques** 

- Continuous Assessment.: (Quizzes ,Test, Project, Assignments, activities,...) 50%
- Final Examination: 50%

#### **REFERENCES:**

- J. Kevorkian: Partial Differential Equations: Analytical Solution Techniques,
   2nd Edition, Springer-Verlag, New York 2000
- 2. P.R. Garbadin: Partial Differential Equations, John Wiley & Sons, New York, 1964
- 3. Ross & Weinberger: Introduction to Differential Equations, Blaisdell Publishing Company, a division of Ginn and Company.
- 4. Ritger-Rose & Dennmeyer: Differential Equations, McGrawal-Hill, USA, 1968

- 5. E.Zaunderer: Partial Differential Equations of Applied Mathematics, John Wiley
- & Sons, New York, 2006.
- 6. Mark A. Pinsky: Partial Differential Equations and Boundaryvalue Problems with Appl., McGrawal-Hill, 1998.

# 5. Mode of Course Delivery

The main teaching-learning methods employed in the curriculum include:

- Conducting lecture based on active participation of students
- Tutorial classes where students will discuss and solve problems
- Discussion and presentation in group
- Reading assignment
- Practical work (computer Lab.)
- Project work
- Modeling of practical problems
- Class activities

# 6. Quality Assurance

The Department of Mathematics will monitor and maintain the quality of the program according to the quality assurance standards set by the university. To this effect, the department will:

- ensure that contents of the courses are covered.
- ensure that exams, tests, assignments and projects are properly set and conducted.
- ensure that appropriate technology is employed in the teachinglearning process.
- conduct short-term courses and seminars for staff members in order to make use of modern methodology and IT.

- make sure that tutorial classes are well-organized, relevant exercises, home works and assignments are carefully set to enhance and strengthen the students' ability to solve problems and understand the underlying theory.
- ensure that appropriate and recent text books are used for the courses.
   provide enough reference books for each course.
- evaluate the courses at the end of each semester based on the feed back obtained from the instructors, the tutors and the students, so as to make the courses more relevant.
- ensure that the lectures are conducted by an appropriate instructors

## 7. References

- 1. M.Sc., in Mathematics Curriculum, Addis Ababa University, Ethiopia.
- 2. M.Sc., in Mathematics Curriculum, Bahirdar University, Ethiopia.
- 3. M.Sc. in Mathematics Curriculum, Haromaya University, Ethiopia.
- 4. M.Sc. in Mathematics Curriculum, Jimma University, Ethiopia.
- 5. Adama Science and Technology University, Ethiopia.