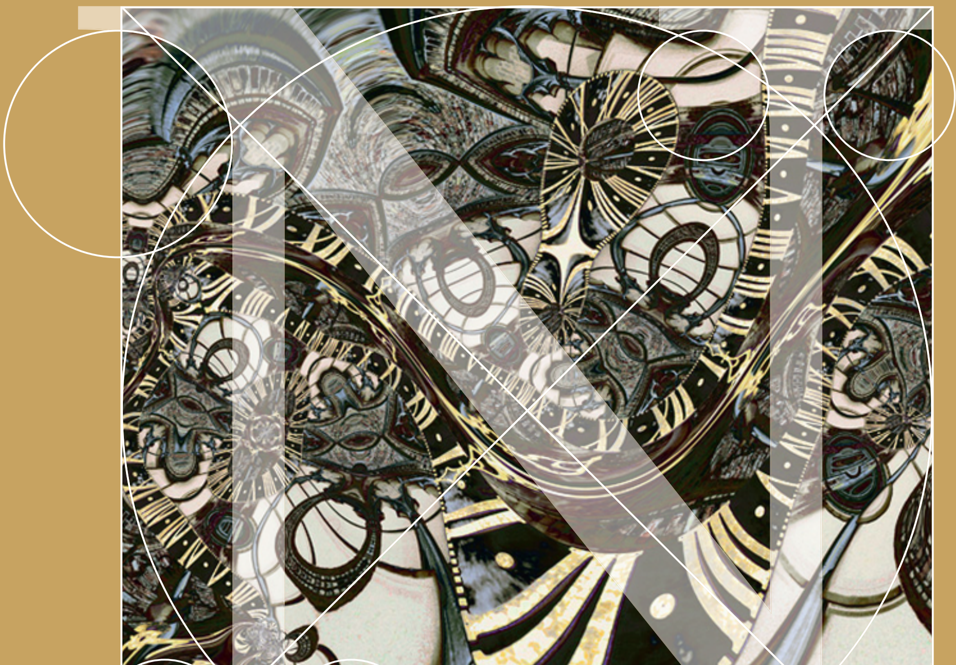
Nexus Network Journal

Volume 14
Number 1
2012
ISSN 1590-5896

Architecture and Mathematics



Birkhäuser

NEXUS NETWORK JOURNAL Architecture and Mathematics

Aims and Scope

Founded in 1999, the ***Nexus Network Journal*** (NNJ) is a peer-reviewed journal for researchers, professionals and students engaged in the study of the application of mathematical principles to architectural design. Its goal is to present the broadest possible consideration of all aspects of the relationships between architecture and mathematics, including landscape architecture and urban design.

Editorial Office

Cover
Clock cathedral of Ypres 2010 #3

Instructions for Authors

Authorship

Submission of a manuscript implies:
- that the work described has not been published before;
- that it is not under consideration for publication elsewhere;
- that its publication has been approved by all coauthors, if any, as well as by the responsible authorities at the institute where the work has been carried out;
- that, if and when the manuscript is accepted for publication, the authors agree to automatically transfer the copyright to the publisher; and
- that the manuscript will not be published elsewhere in any language without the consent of the copyright holder.

Exceptions of the above have to be discussed before the manuscript is processed. The manuscript should be written in English.

Submission of the Manuscript

Material should be sent to Kim Williams
via e-mail to: kwb@kimwilliamsbooks.com
or via regular mail to: Kim Williams Books,
Corso Regina Margherita, 72, 10153 Turin (Torino), Italy

Please include a cover sheet with name of author(s), title or profession (if applicable), physical address, e-mail address, abstract, and key word list.

Contributions will be accepted for consideration to the following sections in the journal: research articles, didactics, viewpoints, book reviews, conference and exhibits reports.

Final PDF files

Authors receive a pdf file of their contribution in its final form. Orders for additional printed reprints must be placed with the Publisher when returning the corrected proofs. Delayed reprint orders are treated as special orders, for which charges are appreciably higher. Reprints are not to be sold.

Articles will be freely accessible on our online platform SpringerLink two years after the year of publication.

Nexus Network Journal

Architecture, Systems Research and Computational Sciences

Volume 14, Number 1
Spring 2012

Kim Williams Books

Nexus Network Journal
Vol. 14
No. 1
Pp. 1-194
ISSN 1590-5896

Contents

Kim Williams

Kim Williams Books
Corso Regina Margherita, 72
10153 Turin (Torino) ITALY
kwb@kimwilliamsbooks.com

Letter from the Editor

Architecture, Systems Research and Computational Sciences

Abstract. *NNJ* editor-in-chief Kim Williams introduces the papers in *NNJ* vol. 14, no. 1 (Spring 2012).

One of the most interesting developments in the Nexus community since its inception in 1996 involves the way information technology has entered both the theory and the practice of architecture. When Nexus began, most of the research involved what I have now begun to think of as "traditional" relationships between architecture and mathematics: geometrical constructions and systems of proportions, for example. At Nexus 2010 in Porto, where the first two groups of papers in this present issue were presented, it was abundantly clear that today computer science is an integral part of even strictly historical investigations, such as those concerning the construction of vaults, where the computer is used to survey the existing building, analyse the data and draw the ideal solution. What the papers in this issue make especially evident is that information technology has had an impact at a much deeper level as well: architecture itself can now be considered as a manifestation of information and as a complex system. This is the fulfillment of prophecies made by voices crying in the wilderness as far back in time as the 1930s and 1940s.

This issue opens with Gonçalo Furtado's introduction to "Architecture, Systems Research and Computational Sciences", in which he outlines some of the ideas that he has been working with for a number of years, and focuses especially on the little known work of Gordon Pask. Pau de Solà-Morales examines "Information, Architecture, Complexity", showing how the study of patterns of relationships can provide the key for understanding of architecture as organization. In "After the Paradigm of Contemporary Physics in Architecture: Spatial Possibilities and Variations" Lora Dikova discusses the representation of informational flows via organizing spatial systems and the transformation of different informational modes into spatial structures. Suzanne Strum takes us back to a crucial and fascinating period in the relatively recent history of architectural theory. In "Informational Architectures of the SSA and Knud Lönberg-Holm" she shows the genesis of ideas about architecture as organization.

Nexus 2010 also included a roundtable discussion moderated by Celestino Soddu, concerning new methods of design. As an outgrowth of that discussion, here Marie-Pascale Corcuff presents a discussion of "Modularity and Proportions in Architecture and their Relevance to a Generative Approach to Architectural Design", while contemporary artist Philip Van Loocke uses his own work as a vehicle for discussing "Counterpoint in the Visual Arts".

Two of the papers presented at Nexus 2010 concerned the very important theme of didactics: the question of what and how to teach students about relationships between architecture about mathematics. Maycon Ricardo Sedrez and Alice T. Cybis Pereira describe a project entitled "Fractal Architecture" aimed at introducing students to the concepts of self-similarity and iterative processes while they are becoming familiar with computer-aided design and rendering. Mathematician Jürgen Bokowski and architect Heike Matcha joined forces for "Möbius Strip Segmented into Flat Trapezoids: Design-

Nexus Netw J 14 (2012) 1–2

DOI 10.1007/s00004-011-0092-2; *published online* 26 January 2012

Build Project by the Departments of Architecture and Mathematics of the Technische Universität Darmstadt", in which students from the two departments collaborated to conceive, design and carry through to completion a kiosk in the form of a Möbius strip.

This issue also includes four other research papers. Ute Poerschke presents "Architecture as a Mathematical Function: Reflections on Gottfried Semper", in which she discusses the attempt made by the nineteenth-century thinker Semper (1803-1879) to connect architecture with infinitesimal calculus, his mathematical background, and his desire to give architecture a scientific foundation through methods of systematic comparison and classification. In "From Drawing to Technical Drawing" Adriana Rossi looks at how progress in drawing techniques and the capacity to conceive forms before they were build led to the emergence of a new role for the architect in the Middle Ages. Two papers, "Domes in the Islamic Architecture of Cairo City: A Mathematical Approach" by Ahmed Ali Elkhateeb and "The Vault of the Chapel of the Presentation in Burgos Cathedral: Divine Canon? No, Cordovan Proportion" by Tomás Gil-López, form an interesting counterpoint, in that each takes a different approach to a similar problem: the analysis of systems for covering space (domes and vaults).

The papers presented at Nexus 2010 have formed the basis of the past four issues of the *NNJ*. Now we are looking forward to Nexus 2012, to take place 11-14 June of this year in Milan, hosted by the Department of Industrial Design, Art, Comunication and Fashion (INDACO) and the Department of Mathematics of the Politecnico di Milano. I am looking very forward to seeing what new developments this ninth edition of the Nexus conference will show us. Stay tuned!

About the author

Kim Williams is the director of the conference series "Nexus: Relationships Between Architecture and Mathematics" and the founder and editor-in-chief of the *Nexus Network Journal*.

Gonçalo Furtado

Oporto University
School of Architecture
Rua do Gólgota 215
4150 Porto PORTUGAL
gmfcl@hotmail.com

Keywords: systems research, architectural design, information systems, computational sciences, cybernetics, artificial intelligence

Research

Dealing with Information, Complex Dynamics and Organizations: Notes on Architecture, Systems Research and Computational Sciences

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010.

Abstract. Similarities can certainly be found between systems research and computational sciences, and architecture and design. The first pair deals with information, complex dynamics and organizations; the second pair is often understood as synthetic and systemic. Postwar history recalls a sequence of exchanges between these fields; the aim of this paper is to highlight the relevance of some exchanges and their contemporary legacy. In this connection, the first part briefly outlines the meaning and history of the former disciplines, highlighting the strict circular models and how first-order cybernetics evolved towards a second order. The second part points to some exchanges between systems research, computational sciences and art forms, as well as to its architectural legacy. To a large extent, the current architectural interest in new sciences of emergence and complexity is rooted in the early systems research approach. Both areas are possible root sources of a future, effective built environment.

Introduction

The present text was prepared as an introductory presentation to the session at Nexus 2010 entitled "Architecture, Systems Research and Computational Sciences".[1] The session was dedicated to exploring the exchanges between architecture and the fields of systems research and computational sciences. The session's main areas also included: second-order cybernetics, architectural morphogenesis and sustainability. Presentations focusing on topics such as complex systems, self-organization, emergence, topology, Cad-Cam, virtual environments and cyberspace, as well as on architects, designs and buildings which illustrate the relationship between architecture and mathematics, were included.

Regarding the present paper, it should be acknowledged that the computational connection with telecommunications, which led to cyberspace, has constituted an architectural challenge at the level of urban building and design practice. More recently, architecture's interest in the new sciences of emergence and complexity, which Jencks even associated to a 'New Paradigm in Architecture' has became noticeable. However, the current digital architectural culture is rooted in early systems approach; an area which tended to view 'organization' from an approach of complexity.

Nexus Netw J 14 (2012) 3–15
DOI 10.1007/s00004-011-0093-1; *published online* 13 January 2012

Notes on systems research, and computational sciences: from the circular model to a second-order

The term "Cybernetics" was first used in 1948 by Norbert Wiener to define "the science of control and communication in the animal and the machine" [Wiener 1948]. It evolved into a discipline dedicated to the study of systems, offering tools for dealing with complex dynamics and organizations. I recall, in this connection, that Claude Shannon and Warren Weaner's [1949] Information Theory and the interdisciplinary efforts of the American Macy's Foundation Conferences (1946-53),[2] chaired by McCulloch, were crucial to cybernetics' early development. Also important since 1954 were Von Bertalanffy's Society for General Systems, and the broad trans-disciplinary emphasis of Systems Theory is mentioned by Heylighen and Josyln [1992].

In fact, after World War II, cybernetic notions such as information, control, system (i.e., an assemblage of interdependent interacting entities) and feedback (i.e., the idea of reintroduction of outputs in the system), became extremely influential. And in the 1960/70s the field of systems research operated by abstracting interconnected components and goals, actions and feedbacks, into a descriptive scheme of an organized complexity. It enabled, for example, Forrester's attempt to model the world, which influenced Meadows' research on the global problem and its ongoing evolution, followed by the ambitious Club of Rome planetary ecological systemic attempt [Forrester 1961].

With continued advances, around the early 1970s, a second-order cybernetics arose. This trend acknowledged the observer's participation and developed as a kind of applied epistemology. More recently, new sciences of complexity emerged; but to a certain extent they continued to have their roots in cybernetics and systems thought.

In parallel, it is also of importance to recall that the scientific concern with complex dynamics and organizations was also paralleled by a broader intellectual shift from determinacy to the acknowledgement of "indeterminacy" or uncertainty. Scientific occurrences of such a history include the astronomic "three-body problem", developments of quantum physics or the areas of emergent complexities, of which brief descriptions were provided, for instance, by Rosmorduc and L'Echat [2004]. The acknowledgement of "uncertainty" had a broader cultural meaning, as highlighted by David Peat, for it constituted "…a major transformation in human thinking … [and] while our millennium may no longer offer certainty, it does hold a new potential for growth, change, discovery and creativity in all walks of life" [Peat 2002].

It is also important to recall the parallel history of computation and "artificial intelligence", which progressively developed its capacities becoming increasingly available.

Broader accounts of computation are included in books such as that of José Terceiro [1997]. These histories include some well known events. First there was the ENIAC, which was the first electronic computer created by J. Presper Eckert and John Mauchly in 1943-46. It was used for the calculation of flight paths of rockets and missiles and later for weather forecasts. Second came Von Neumann's crucial idea of "software", that is, the idea of a stored computer program, which was introduced in 1945 with the EDVAC.[3]

It is also important to point to the first commercially available computer, the UNIVAC 1, created in 1951 by Remington Rand.[4] In this connection, I highlight that the transistor was invented by Shockley, Brattain and Bardeen in 1948, and that the microprocessors dated from the early 1970s; these achievements went some way towards the dissemination of computation in society.

In fact, the 1970s were an exciting decade for more than disco and postmodernism. Despite the oil and economic crises, it was also a decade marked by strong technological progress. The effects of developments in computation were strongly felt, for instance in the areas of automatization and robotics (see Jasia Reichardt's account of robots in this respect [1978]). In addition, a look at popular books for collectors of the 1970s, such as the memorabilia of Higgins, demonstrates the explosion of electronic gadgets in that decade. They are filled with achievements and artifacts in the areas of electronics computation and the like, including the 1970 IBM floppy disc, the 1972 LCD watches, the 1972 IBM laser printers, the mid-1970s' video games, the 1979 Sony Walkman, etc. As Higgins states, the 1970s were linked to the dissemination of microelectronics and computation:

> The 1970s saw an explosion in electronics innovation…. At the forefront of change was the world's first microprocessor, the legendary 4004, launched by America's Intel Corporation in 1971. Many could hardly believe that a chip the size of a thumbnail was as powerful as the world's first electronic computer, ENIAC (1946), which had filled an entire room. As less expensive, more powerful chips followed, so did electronic gadgets [Higgins 2001: 44-47].

The Intel 8008 microprocessor dating from 1971 was followed by the launch of the first personal computer, the Altair 8800, in 1975. That same year, microcomputers went on sale in the US; the PC prototype of Bill Gates (who later founded Microsoft) was developed; and Apple, the first computer with a plastic case and color graphics came out; whereas the IBM PC was only announced in 1981.

In short, we understand that electronics enabled microprocessors and the accessibility of personal computers, after the mid-1970s and especially in the late 1970s. These advancements were largely promoted by popular magazines in the 1980s.[5] Today, we have undoubtedly ended up living in an Information Society. In 2003, computational scientist and architect John Frazer stated: "I toast the new millennium and declare myself post-digital … " [Frazer 2005: 43].

Let it be recalled that a particular area that benefited from computational sciences and the cybernetic equation of man-machine was Artificial Intelligence (AI).

A historical overview of AI, its aims and areas of development, is provided for instance in Simons' *Introducing Artificial Intelligence* [1984]. Several important phases can be recalled. For example, Alan Turing conceived his evaluation test of machine intelligence in 1950. In the same year, at the Dartmouth conference, in which many pioneers participated, McCarthy coined the term "AI". Other historical moments in the "Automata Studies" were Shannon's 1956 publication, as well as Von Neumann's work. The latter's work was extended to areas of computer sciences such as AI; he worked with self-replicating systems and conceived the "Cellular automata" concept (see especially [Von Neumann 1966]. In fact, Pierre Marchal [1998] highlights Von Neumann's role as "the Founding Father of Artificial Life"; and his cellular automata concept later inspired John Conway's "Game of Life" [Gardner 1970]. In addition, Oliver Selfridge and Marvin Minsky, of course, also conducted well known research at MIT's outstanding AI Laboratory. (The Laboratory was founded in 1959; see Minsky's account on "Computation: Finite and Infinite Machines" [1967].)

Regarding AI, two important languages were McCarthy's "Lisp", from 1960, and Alain Colmerauer's "Prolog", from 1972. Regarding AI research methods, these included parallel processing, neural networks, fuzzy and genetic procedures.

Currently, the polemic debate on AI continues. In this connection, I would like to state that the concern of a new paradigm in AI, which took into account the evolutionary character of human beings and the observer, was acknowledged by Pedro Medina Martins (see [Medina-Martin and Rocha 1992]).

As mentioned in the introduction, systems research and computational sciences deal with information and complex dynamics and organizations.

At this juncture I would like to point out to the distinction between the perspective of second-order cybernetics and Artificial Intelligence or first-order cybernetics.

At the outset, it should be recalled that after the discipline of cybernetics's boom in the 1960s, it experienced a certain decline in the 1970s. In relation to the systems perspective, accounts even emerge that provide a critical review of it, such as Lilienfeld's *Rise of Systems Theory: An Ideological Analysis* [1978]. However, progressively and in what concerned cybernetics, a second-order-cybernetics was arising, which acknowledged observer participation in the observed system, shifting the focus from control to interaction. Later on, the new sciences of emergence acknowledged and explored the system's recreation of boundaries and so on. Putting it briefly, Second-order cybernetics denominates a shift within the discipline. It goes back to ideas such as Mead's *"Cybernetics of Cybernetics",* or Von Foerster's *"Observing Systems"*, which advanced the notion that the system also included observer participation [Von Foerster 1974, 1981]. Putting it briefly, there was an awareness about the question concerning self-referentiality and the idea that the researcher is part of the system that he or she constructs in order to investigate or to relate himself to something.

Heylighen and Josyln provided a historical account and distinction between first and second-order-cybernetics, highlighting the development of the discipline in the post-war period. They stated that second-order cybernetics arose in the 1970s from a trend that wished to distinguish itself from a more reductive climate or mechanistic orientations:

> Cybernetics had from the beginning been interested in the similarities between autonomous, living systems and machines. …In the post-war, the fascination with the new control and computer technologies tended to focus attention on the engineering approach, where it is the system designer who determines what the system will do. However after the control engineering approach had become fully independent, the remaining cyberneticians felt the need to clearly distinguish themselves from these mechanistic approaches, by emphasizing autonomy, self-organization, cognition, and the role of the observer in modeling a system. In the early 1970s this movement became known as second-order cybernetics. …[A] 'first-order cyberneticist', will study a system as … an objectively given 'thing' … A second-order cybernetician … recognizes that system as an agent in its own right, interacting with another agent, the observer… [Heylighen and Joslyn 2001].

To a large extent, it can be understood that second-order-cybernetics have a philosophical stance, and that this approach can be associated with challenges to more conventional scientific positions.

In fact, embracing the presence of the subject has a huge impact on science and on life, to a broader extent. In this connection, Glanville's paper "Chasing the Blame" [1995] not only provided a clear explanation of what "cybernetics of cybernetics" is but also added to its deep human meaning. In his own words:

> What characterizes cybernetics of cybernetics is the inclusion of the agent that is determining the system under consideration. …When cybernetics considers its own subject matter cybernetically, it is being cybernetic. Then we have cybernetics of cybernetics.

The involved explanation provided by Glanville further on enables one to understand its deep human meaning:

> The cybernetics of cybernetics is, as its name suggests, full of circles. Circularity is one of its major characteristics…, and the cybernetics of cybernetics is full of involvement. By this, I mean the involvement of the observer in his observing…, of the knower in his knowing…, of the conversationalist in his conversing, of that which is alive in his living. …What we can understand from this is that the observer is responsible for both his observing and its frozen version, which we like to call observation…. By not accepting that our observation is ours, we exteriorize and reject it. Thus, we make cause and effect, for cause is the mechanism that explains why we are not (often, in general) responsible for the effect (that is, excuses our willingness to accept our responsibility) [Glanville 1995].

Exchanges with the art forms and its architectural legacy

The paper's first part briefly outlined the meaning and history of Systems Research and computational sciences. In the following part, I would like to point to some exchanges between the aforementioned fields and art forms, as well as to their architectural legacy.

In terms of what concerns the history of post-war architecture, an important moment was the establishment of modern architecture In this connection several accounts could be highlighted: Legault and Goldhagen [2001] provided an understanding of the Modern Movement's that is worthy of note, pointing to its intrinsic complexity and its process of evolution ([Montaner 2001] alluded to the idea of "the Overcoming of Modernity"). To a certain extent, designers were anxious to overcome the rigid planning and architecture of modernism by acknowledging the role of users and representing the dynamics of time; as described in Ewan Branda's paper "Programming the Utopia of the Present" [2003]. Many events and protagonists contributed to the so-called "overcoming" of modernity. For example, Rouillard's *Superarchitecture: Le Future de l'Archietcture 1950-70* [2004] described three tendencies that resulted in the crisis of modern architecture, while MoMA's *The Changing of the Avant-garde* exhibited occurrences that constituted "… the roots sources of our architecture today" [Riley 2002: 14].

At this juncture, the existence of a specific atmosphere in the 1960s and 1970s should be highlighted (see the catalogue of the Pompidou's exhibition "Les Anées Pop" [Francis 2001] and the March-April 2005 issue of *Architectural Design* edited by Hardingham which focused on the following decade). The atmosphere was characterized by a desire for flexibility and technological optimism; such desires were highlighted at the time in several magazines, especially in *Architectural Design*'s monthly section on "Cosmorama" published in the 1960-70s, and the *Archigram Magazine* published between 1961-1974. In relation to these magazines, the occurrence of many important avant-garde architectural practices as well as events should also be acknowledged. Among the radical practices developed in Europe, we could highlight the relevance of *Archigram* in the UK, GEAM in France, Metabolists in Japan, Superstudio in Italy, among others. Regarding events, I could recall the International Dialogue of Experimental Architecture, which took place in Folkestone in June 1966, and the World Design Science Decade inspired by Fuller, which took place in the 1970s. To a large extent the late-modern "Megastructural" movement, analyzed by Reyner Banham [1976], was fuelled by such desires of flexibility and technology.

More important, is that the modernist crisis led to a postmodern shift, which must be identified in conceptual-philosophical terms, rather than merely associated with a historical style. In this connection, many accounts could be referred to, including Hal Foster's outstanding book *Postmodern Culture* [1985], which provides a multidisciplinary discussion of the postmodern shift. Architecture's theoretical agenda from 1943 to 1995 has also been covered by a sequence of anthologies authored by Joan Ockman [1993], K. Michael Hays [1998] and Kate Nesbitt [1996].

Concerning the exchanges between systems research and architecture, I would like to highlight the relevance of British cybernetician Gordon Pask.

Pask was an important figure of second-order cybernetics and a seminal promoter of cybernetics within art forms. In this connection, I would like to note that he developed interactive art pieces beginning in the 1950s (for example, "Musicolour", "The Colloquium of Mobiles"), and he worked in the architectural milieu (for example, as a consultant to Cedric Price's Fun Place and the Architectural Association). He has also published original pieces in the 1960s and 1970s in *Architectural Design*.[6] In 1969, in an article in *Architectural Design*, Pask described "The Architectural Relevance of Cybernetics" at various levels. This outstanding article appeared in an issue edited by Landau, which included articles such as Greene's "Cybernetic Forest", M.I.T.'s "Computer Aided Design" and Negroponte's "Architecture Machine".[7] Later Pask wrote about "Complexity and Limits" [1972], establishing a parallel relationship between the topics of language, codification and the observer, and architecture.

In addition, I would like to recall other important protagonists and contributions. For example, the importance of Christopher Alexander's *Notes on the Synthesis of Forms* [1964], which revealed his work on design patterns; Haissman's papers on "Five Dimensional Concepts and Architecture" [1967], and Abel's 1973-1974 conception of the design device "Architrainer" at MIT, should also be recognized. Another important issue of *Architectural Design* was that edited by Andrew Rabeneck in 1976 (vol. 46, no. 5, May 1976) which raised the alarming question "Whatever Happened to Systems Approach?" at a time when architectural postmodernism was on the rise and included articles on Price, Seagal and Fuller. In the editorial introduction to Rabeneck's article (pp. 298-303) it explicitly stated that:

> The system approach once held to be the key to a panacea for all humanity's ailments, has recently been discarded in the latest quest to improve the quality of life – this time by less overtly technical means. With the help of editorial consultant Andrew Rabeneck, this issue of *AD*

asks 'Whatever Happened to Systems Approach?' and find answers for the architects featured (p. 267).

I would like to pause here and state that from the late 1960s – especially in the 1970s and 1980s – another important research platform on computation was Negroponte's "Architecture Machine Group" in the USA [Negroponte 1970, 1975]. Pask was among those who interacted with Negroponte's group.[8] I have detailed this elsewhere [Furtado 2009b], as well as the fact that the elevation of "Conversation Theory" to a unifying status within architectural thought was attempted in detail by Pask [ca. 1980s]. His papers from the 1980s, including "Architectural Systems" [1982], "Architecture of Knowledge" [1984]], "Space Time Frames" and "The Reality of History" [1985], expressed other important topics that related to his future work.

In addition, it should also be mentioned that in the mid-1980s, Pask and architect Cedric Price developed another project – Japan Net – as a competition entry for the city of Kawasaki. Elsewhere I have provided a complete account based on the material available at the date of my research visit to the Cedric Price Archives at the Canadian Centre for Architecture [Furtado 2009c]. I noted that the character of the proposal for Kawasaki reflects Price and Pask's architectural and second-order cybernetics stances.

Regarding the exchanges between architecture and computational sciences, I would also like to recall the accessibility of personal microcomputers since the 1970s, and their connection with 1990s telecommunications leading to cyberspace. Digital space and life constituted a huge architectural challenge at the level of the city's building and design practice,

Nowadays, we have undoubtedly ended up living in an Information Society. There are many definitions and accounts of IS; for example, Flusser's [1998] definition of I.S. expresses the ubiquity of information.[9] It has become mankind's "Third Environment" according to Echeverria [1999], turning the prognostics of McLuhan's "global village" [1962] and Gibson's vision of "cyberspace" into a reality [1884]. In this context, we can state that our lives are supported by the "Internet" and the "virtual". The virtual has its own complex nature, as analyzed by Quéau [1995] and Maldonado [1994]. The same applies to the Internet, as referred by Trejo [1996].

Let me note that the earliest architectural approaches to digital space date back to the early 1990s. Michael Benedikt's seminal multidisciplinary book, *Cyberspace: First Steps* [1991], and the 1993 *ANY* issue on "Electroculture" edited by Taylor, must be acknowledged here. As Michael Hensel pointed out, "architecture and media are both ... shapers of environments" [Lootsma and Rijken 1998: 80]. In such a context, a new series of practices concerning digital architecture emerged, as stated by Peter Zellner in *Hybrid Space* [1999]. This new stance of digital architecture followed the architectural engagement with Derrida's deconstruction and Deleuze's fold.

In addition to the impact of the digital in the urban environment, the manifold applications of computers also parallel their impact on the practice of design and building. There were many protagonists and the developments focused on the relationship between computation and design, as well as on computation and building. Regarding the former, Pellegrino, for example, has researched the relations between informatics and architectural design [1999], and developments in the design field were described in Morgan and Zampi's *Virtual Architecture* [1995]. Regarding the latter, for example, an approach to the field of intelligent buildings is provided in *Arquitectura i Maquina* [Florensa 1996].

I would like to note that architecture's technological engagement goes on further to encompass other technologies. For example, Prof. Neil Spiller (at Bartlett's Unit 19), among others, conducted some leading research on responsive and "reflexive" environments, attending to nano and bio technologies. The same happens with the new contemporary architectural interests in the new sciences of emergence and complexity. To a large extent, it expresses architecture's new desire for an evolving environment.

In fact, it is noticeable that in recent years contemporary professionals (such as Gausa), have privileged "processes" rather than "occurrences" in the search for a more dynamic "understanding of architecture" [Moreira dos Santos 2003: 54]. Accordingly, new sciences also impact architecture. I highlight the importance of the 1997 *Architectural Design* issue edited by Jencks, consisted of revealing articles on the impact of "New Science" on a "New Architecture". There, Cecil Balmond stated that "the paradigm is one of emergence" [Balmond 1997: 88], and Jencks provided a definition of "complexity" pointing out that in such a process "… quality emerges spontaneously as self-organization …." [Jencks 1997: 8].

In addition, I would like to mention Abel's outstanding "Visible and Invisible Complexities" [1996], which criticizes the common tendency for an interest in a merely superficial, visual and formal complexity, while in contrast, the era of information technology and complexity' affects Architecture, from conception to production, at a distinct level. The importance of Raoul Bunschoten's outstanding architectural event on "Chaos and Order" and others at the AA should be acknowledged.

As an aside, my hero Gordon Pask was also involved in occurrences and events close to the topic during the 1990s. He was involved in Raoul's "Risk and Transgression", and intended to organize an event on the time-architecture relationship with Farhad Toussi alluding to architecture's organization of time and to "an architecture of temporality" [Pask 1991 ca]. Pask also supervised and examined some academic works on the issues mentioned, at a time when it did not coincide with mainstream interests. For example, he supervised James Bradburne's 1988 thesis on "The Strange Attraction of Chaos" (see [Pask 1988]), and reviewed a 1993 thesis on "Chaos, Architecture and the Strange Attractor".[10] Pask pointed out that "… great architects … know that space is a malleable … commodity … ." Elsewhere [Furtado 2007] I have noted and described that at the AA school of architecture a complex perspective on architecture continued to be promoted by Pask throughout the last years of his life. For example, Pask contributed to Frazer's AA unit, which developed outstanding achievements and exhibitions.

A summarized description of the work developed by Frazer's AA unit was provided in a 2005 *Architectural Design* issue [Frazer 2005] and was detailed in a 2001 *Kybernetes*. Frazer mentioned Pask and Price's contribution to the unit in interviews that I conducted with him on 22 March and 5 October 2005. Regarding his achievements, I suggest Frazer's paper co-authored with Rastogi and Graham [1995], which explains the "Interactivator" data structure, cellular growth, and genetic search of the model in *Architects in Cyberspace*. The interactive version of the evolutionary model is on-line at Ellipsis [Frazer 1995]. In regards to the exhibitions, there are several reviews. The one by Barrie Evans highlighted its complexity and benefits [1995]. Another review in the *AA Files* provided a critique of Frazer's vision saying "… Evolutionary Architecture may just excel in the very thing that it does not profess to do: the modeling of solutions to particular architectural problems …" [Bettum 1995]. Interestingly, the second phase of Frazer's unit's experiment, concerned with real-world situations, consisted of the development of an urban model. It was described by John and Julia Frazer in the sequel to the magazine *Architects in Cyberspace* [Frazer and Frazer 1998]. In particular, Frazer and Rastogi's text "The New Canvas" [1998], included in the aforementioned publication, described aspects of the evolutionary approach and models of co-evolving environments, such as a city. Regarding Pask, as I have stated elsewhere, he expressed his intention to conceive texts and designs expressing his later ideas on an evolving informational environment.

Concluding notes

Similarities exist between systems research, computational sciences, architecture and design. The aim of this present paper was to highlight the relevance of some exchanges between those areas and its contemporary legacy. The text's first part briefly outlined the meaning and history of the former disciplines; and the last part pointed to some exchanges and their architectural legacy.

As described, systems research goes back to the war period, and it is concerned with "organization" from an approach of complexity. The history of the field recalls an expansion of attention at the level of the planet with the Club of Rome's systemic approach to the global problem (see especially [Rosnay 1978]). However, dealing with issues of complexity was also paralleled by the acknowledgement of "uncertainty".

Systems research, like cybernetics, became influential throughout the post-war period, and it embraced a wide field of application. There was, undoubtedly, a later 'backlash' against the latter; however, as Scott has mentioned in his obituary of Pask [1996], its concepts permeated such areas as AI, systems and emergence Sciences. Moreover, a "second-order cybernetics" arose around the 1970s, acknowledging the presence of the observer in Systems, and leading to theoretical developments such as "Autopoiesis", "Conversation Theory", etc. At the time, computation became ubiquitous, and its later connection with telecommunications led to cyberspace and to the Information Society in which we now live.

At an early date, systems research, cybernetics and computational sciences went on to interact with the fields of arts and architecture. Early occurrences included the work of Schoffer, Pask, Jones, Alexander and Negroponte, and they were fuelled by a desire to overcome the rigid architecture and planning of modernism, by representing the dynamics of time. Progressively, digital space and life also constituted an architectural challenge at the level of the city's building and design practice. The earliest of such approaches date back to the early 1990s, and more advanced explorations were made by architects such as Novak (on "transvergence") and Frazer (on "evolutionary architecture"). To a certain extent, the current digital architectural culture is rooted in cybernetics, and the systems approach enables a systemic focus of contemporary cities, and the global ecological problem. Today, architecture's desire for a more evolving environment is leading to an interest in the new sciences of emergence and complexity, which Jencks even calls a "New Paradigm in Architecture". In our opinion, both areas are the roots sources of a future, effective built environment that could evolve.

Notes

1. This text is based in part of my Ph.D. [Furtado 2007]; a summary was published in [Furtado 2009a]. Short short excerpts from my paper presented at the last colloquium of WOSC; see

[Furtado 2009b]. My Ph.D. and research were supported by a grant from the Fundação para a Ciência e Tecnologia/POCI 2010. I am grateful to Amanda Heitler, who gave me permission to conduct research in her father Gordon Pask's archive in 2005 and publish the findings. It is possible that some of these materials will be found today at the University of Vienna, where an archive was recently established.
2. As an aside, Foerster was editor of the proceedings of several conferences on cybernetics. The first volume that he edited was that of the sixth conference [1949]. Those that followed were edited by Foerster with H.L. Tuber and Margaret Mead: the seventh in 1950, the eighth in 1951, the ninth in1953 and the tenth in 1955.
3. I digress slightly to refer that a biographical account of Von Neumann is provided by J. A. N. Lee [2002]. The Papers of John von Neumann on Computers and Computing Theory were edited, with an introduction, by Arthur Burkes [Von Neumann 1986]. Lee's biographical account of Von Neumann describes that Von Neumann, like Gödel, was encouraged to move to the US by Morganstern, being appointed Professor at Princeton in 1933. Years later, in 1945, Von Neumann signed the "First Draft of a Report on the EDVAC" [1945] introducing the basis of the first stored program computer. Interestingly, it is also asserted that he knew Turing's 1934 paper on the universal machine, since Turing studied in Princeton between 1936-38, and then moved to the UK and worked at Bletchley Park. After the war, Von Neumann continued to have exchange with Los Alamos relating to computation and the conception of the hydrogen bomb [Lee 2002].
4. SYSTEM RESEARCH LTD, 1962-80. Visitors Book. Gordon Pask Archive. My archival research was conducted at Amanda Heitler's house.
5. For example, Gordon Pask kept some issues of *The Home Computer Course: Mastering your Home Computer in 24 Weeks* (London: Orbis Publications).
6. The importance of this magazine in the dissemination of new ideas ought to be acknowledged, for as Frazer stated: "Whilst mainstream architectural practice was still concerned with modernism, these new preoccupations [of indeterminacy and interactivity] found fertile ground in schools of architecture and by the mid-60s… Architectural Design … was carrying increasing coverage of these new ideas" [Frazer 1993: 44].
7. The issue includes articles by Pask, Green, Negroponte; it also includes an article by Cedric Price. See [Pask 1969; Green 1969; Negroponte 1969].
8. See for example [Pask 1975], "Artificial Intelligence: A Preface and a Theory", which was originally prepared for Negroponte's *Machine Intelligence in Design* [1973]; it became *Soft Architecture Machine* [1975], in which Pask published "Aspects of Machine Intelligence".
9. Aguadero [1997], among others, has provided a summary account of this theme, and several political programs have tried to implement it. Following the "American technological plan" of 1993, the European Union created the "Delors Plan" and Bangeman report (both 1994).
10. Evidence exists in the archive that Pask reviewed the unidentified student's paper titled "Chaos, Architecture and the Strange Attractor" in May 1993.

References

ABEL, C., 1996. Visible and Invisible complexities. *Architectural Review* 199, 1188: 76-83.
AGUADERO, F. 1997. *Sociedad de la Información*. Madrid: Acento Editorial.
ALEXANDER, C. 1964. *Notes on the Synthesis of Form*. Cambridge, MA: University Press.
BALMOND, C. 1997. The New Structure and the Informal. *Architectural Design* 67, 9/10: 88-96.
BANHAM, R. 1976. *Megastructure. Urban Futures of the Recent Past*. London: Thames and Hudson.
BENEDIKT, M. 1991. *Cyberspace: First Steps*. Cambridge, MA: MIT Press.
BETTUM, J. 1995. An Evolutionary Architecture. *AA Files* 30: 70-73.
BRANDA, E. 2003. Programming The Utopia of the Present. UCLA.
BUTTERY, L. 2000. *Bletchley Park – Home of Station X*. Hampshire: Pitkin Guide.
ECHEVERRIA, J. 1999. *Los Señores del Aire : Teléopolis y el Tercer Entorno*, Barcelona: Destino.
EVANS, B. 1995. Architecture seen as Artificial Life. *Architects Journal* 201, 7: 43.
FLORENSA, R. 1996. *Arquitectura i Màquina*. Barcelona: Ediciones UPC.

FLUSSER, V. 1998. Agrupamento ou Interconecção. Pp. 21-28 in *Ars Telemática*, C. Giannetti, ed. Lisbon: Edicões Relógio d'Água.
FORRESTER, J. 1961. *Industrial Dynamics*. Cambridge, MA: MIT Press.
———. 1971. *World Dynamics*. Cambridge, MA: Wright and Allen.
FOSTER, H., ed. 1985. *Posmodern Culture*. London: Pluto Press.
FRANCIS, M., ed., 2001. *Les Anneés Pop 1956-1968*. Exhibition catalogue. Paris: Centre George Pompidou.
FRAZER, J. 1993. The Architectural Relevance of Cybernetics. *Systems Research* **10**, 3: 43-48.
———. 1995. *An Evolutionary Architecture*. London: Architectural Association School of Architecture. http://www.aaschool.ac.uk/publications/ea/intro.html).
———. 2005. Computing Without Computers. *Architectural Design* **75**, 2 (March-April 2005): 34-43.
FRAZER, John and Julia FRAZER. 1998. The Groningen Experiment: Architecture as an Artificial Form: Materialisation Phase II. *Architectural Design* **68**, 11/12: 12-15.
FRAZER, J. and M. RASTOGI. 1998. The New Canvas. *Architectural Design* **68**, 11-12: 8-11.
FRAZER, J. M. RASTOGI and P. GRAHAM. 1995. The Intereactivator. *Architectural Design* **65**, 11-12: 80-81.
FURTADO, Gonçalo (Gonçalo Miguel Furtado Cardoso Lopes). 2007. Envisioning an Evolving Environment: Encounters of Gordon Pask, Cedric Price and John Frazer. Ph.D. dissertation, University College of London.
———. 2009a. Envisioning an Evolving Environment: Encounters of Gordon Pask, Cedric Price and John Frazer. Pp. 118-199 in *New Realities: Being Syncretic*, R. Ascott, et al., eds. Vienna: Ed. Angewandt, 2009.
———. 2009b. Gordon Pask: Exchanges Between Cybernetics and Architecture and the Envisioning of the I.E. *Kybernetes* **38**, 7-8: 1317-1331.
———. 2009c. Cedric Price and Gordon Pask: The Japan Net Experiment (1986). Pp. 138-143 in *EMCSR 2008*, R. Trappl, ed. Vienna: Austrian Society for Cybernetic Studies.
GARDNER, M., 1970. The Fantastic Combinations of John Conway's New Solitaire Game Life. *Scientific American* 223: 120-123.
GIBSON, W. 1984. *Neuromancer*. New York: Penguin-Putnam.
GLANVILLE, R. 1995. Chasing the Blame. http://www.univie.ac.at/constructivism/papers/glanville/glanville95-chasing.pdf (last accessed 6 June 2011).
GREENE, D. 1969. Cybernetic Forest. *Architectural Design* 9: 506-507.
HARDINGHAM, S., ed. 2005. The 1970s is Here and Now. Special issue, *Architectural Design* **75**, 2 (March-April 2005).
HAYS, K. 1998. *Architecture Theory Since 1968*. Cambridge, MA: MIT Press.
LOOTSMA, B. and D. RIJKEN, eds. 1998. *Media and Architecture*. Amsterdam: Berlage Institute of Architecture.
HEYLIGHEN, F. and C. JOSYLN. 1992. What is Systems Theory? In *Principia Cybernetica Web*, F. Heylighen, C. Joslyn and V. Turchin, eds. Available at: http://pespmc1.vub.ac.be/systheor.html (last accessed 6 June 2011).
———. 2001. Cybernetics and Second Order Cybernetics. Pp. 155-170 in *Encyclopaedia of Physical Science and Technology*, vol. 4, R. Meyers, ed. New York: Academy Press.
HIGGINS, K. 2001. *Miller's Collecting the 70s*. London: Mitchel Beazley.
JENCKS, C. 1997. Nonlinear Architecture: New Science = New Architecture? *Architectural Design* **67**, 9/10: 6-7.
LEE, J.A.N. 2002. John Louis von Neumann. http://ei.cs.vt.edu/~history/VonNeumann.html (last accessed 6 June 2011).
LEGAULT, R. and S. GOLDHAGEN, eds. 2001. *Anxious Modernisms: Experimentation in Postwar Architectural Culture*. Cambridge, MA: MIT Press and CCA.
LILIENFELD, R. 1978. *Rise of Systems Theory: An Ideological Analysis*. New York: Wiley-Interscience.
MALDONADO, T. 1994. *Lo Real y lo Virtual*. Barcelona: Geodisa editorial.

MARCHAL, P. 1998. John von Neumann: The Founding Father of Artificial Life. *Artificial Life* 4, 3: 229-235.
MARTINS, P. 1991-92 ca. The In and Out: A Cybernetic Approach (Ars Electronica - Linz). Gordon Pask Archive.
MCLUHAN, M. 1962. *The Gutenberg Galaxy*. London: Routledge & Kegan Paul.
MEADOWS, Donella, Dennis MEADOWS, Jørgen RANDERS and William W. BEHRENS III. 1972. *Limits of Growth*. New York: Signet.
MEDINA-MARTINS, P. and L. ROCHA. 1992.The in and the out: an evolutionary approach. Pp. 681-689 in *Cybernetics and Systems 92*, R. Trappl, ed. Singapore: World Scientific Press.
MINSKY, M., 1967. *Computation: Finite and Infinite Machines*. New Jersey: Prentice Hall.
MONTANER, J. 2001. *A Modernidade Superada: Arquitectura Arte e Pensamento do Século XX*. Barcelona: GG.
MOREIRA DOS SANTOS, Inés. 2003. Da investigaçao a pratica (intervista a Manuel Gausa). *Arquitectura e Vida* 38: 50-57.
MORGAN, C. and G. ZAMPI. 1995. *Virtual Architecture*. London: B.T.Batsford.
NEGROPONTE, N. 1969. Architecture Machine. *Architectural Design* 39, 9: 510-514.
———. 1970. *The Architecture Machine: Towards a More Human Environment*. Cambridge, MA: MIT Press.
———. 1973. *Machine Intelligence in Design*. Cambridge, MA: MIT Press.
———. 1975. *Soft Architecture Machines*. Cambridge, MA: MIT Press.
NESBIT, K., ed. 1996. *Theorizing a New Agenda for Architectural Theory, 1965-95*. New York: Princeton Architectural Press.
OCKMAN, J. 1993. *Architecture Culture 1943-1968: A Documentary Anthology*. New York: Rizzoli International Publications.
PASK, G. 1969. The Architectural Relevance of Cybernetics. *Architectural Design* 9: 494-496.
———. 1972. Complexity and Limits. *Architectural Design* 42, 10: 622-624.
———. 1976. Artificial Intelligence: A Preface and a Theory. Pp. 6-31 in *Soft Architecture Machines*, N. Negroponte, ed. Cambridge, MA: MIT Press.
———. ca.1980s. An Initial Essay – Towards an Unification of Architectural Theories. Gordon Pask Archive.
———. 1982. Architectural Systems: Some Thoughts. In *Proceedings: Seminar Technological Design Strategies*, School of Architecture-North East London Polytechnic, February 1982.
———. 1984. The Architecture of Knowledge and Knowledge of Architecture. In *Cybernetics and System Research 2. Proceedings of the Seventh European Meeting in Cybernetics and Systems Research*, Vienna, 24-27 April 1984, R. Trappl, ed. Amsterdam: North-Holland.
———. 1985. The Reality of History. *Across Architecture* (AA) 5.
———. 1988. Letter to James Bradburne's on the "The Strange Attraction of Chaos", 26 May 1988. Gordon Pask Archive.
———. 1991 ca. Letter to Farhad Toussi made on the abstract "The time of Architecture". Gordon Pask Archive.
PEAT, F. David. 2002. *From Certainty to Uncertainty: The Story of Sciences and Ideas in the Twentieth Century*. Washington: Joseph Henry Press.
PELLEGRINO, P. and D. CORAY, at al. 1999. *Arquitectura e Informática*. Barcelona: Gustavo Gili.
QUÉAU, P. 1995. *Lo Virtual: Virtudes y Vértigos*. Barcelona: Piados.
REICHARDT, J. 1978. *Robots, Facts + Predictions*. London: Thames & Hudson.
RILEY, T., ed. 2002. *The Changing of the Avant-garde: Visionary Architectural Drawings from the Howard Gilman Collection*. New York: MoMA.
ROSMORDUC, J. and D. L'ELCHAT. 2004. 25 *Mots Clés de la Culture Scientifique*. Paris: Marabout.
ROSNAY, J. 1978. History of Cybernetics and Systems Science. In *Principia Cybernetica Web* (http://pespmc1.vub.ac.be/systheor.html), F. Heylighen, C. Joslyn and V. Turchin, eds.
ROUILLARD, D. 2004. *Superarchitecture: Le Future de l' Architecture 1950-70*. Paris: Ed. de la Villete.
SCOTT, B. 1996. Obituary for Professor Gordon Pask. *The Independent*. http://www2.venus.co.uk/gordonpask/gpaskobit.htm (last accessed 6 June 2011).

SHANNON, C., ed. 1956. *Automata Studies*. Princeton: University of Princeton University.
SHANNON, C. and W. WEANER. 1949. *The Mathematical Theory of Communication*. Urbana: University of Illinois Press.
SIMONS, G.L. 1984. *Introducing Artificial Intelligence*. Chichester: Wiley.
TAYLOR, M., ed. 1993. ANY, N.3.
TERCEIRO, J., 1997. *Socied@de Digit@l: do Homo Sapiens ao Homo Digitalis*. Lisbon: Relógio d'Água (Portuguese trans. of *Socied@d Digit@l: Del homo sapiens al homo digitalis* (Madrid: Alianza Editorial, 1996).
TREJO, R. 1996. *La Nueva Algombra Mágica*. Spain: Ed. Fundesco.
TURING, A. 1950. Computing Machinery and Intelligence. *Mind* 59, 236: 433-460.
VON FOERSTER, H. ed. 1949. *Cybernetics: Transactions of the Sixth Conference*. New York: Josiah Macy Jr. Foundation.
———, ed. 1974. *Cybernetics of Cybernetics*. Urbana: University of Illinois.
———. 1981. *Observing Systems*. Seaside-CA: Intersystem.
VON NEUMANN, J., 1945. First Draft of a Report on the EDVAC. University of Pennsylvania, Moore School of Electrical Engineering.
———. 1961-1963. *The Collected Works of John von Neumann*. A.W. Taub and A.H. Taub, eds. 6 vols. Oxford: Pergamon Press.
———. 1966. Theory of Automata: Construction, Reproduction, Homogeneity. Pp. 29-87 in *Theory of Self-Reproducing Automata [by] John von Neumann*, A.W. Burks, ed. Urbana: University of Illinois Press, 1966.
———. 1986. *Papers of John von Neumann on Computers and Computing Theory*. A. W. Burkes, ed. Cambridge, MA: MIT Press.
WIENER, N. 1948. *Cybernetics: Or, Control and Communication in the Animal and in the Machine*. New York: John Wiley & Sons.
ZELLNER, P. 1999. *Hybrid Space: New Forms in Digital Architecture*. London: Thames and Hudson.

About the author

Gonçalo Furtado graduated in architecture (Oporto University, Portugal), earned a Master's in architecture (UPC, Spain), and a Ph.D. in theory and history of architecture (UCL, England). He has won prizes such as the "Florêncio de Carvalho Award" (1999), a "Highly Commended-paper" at the last World Organisation of Systems and Cybernetics, and scholarships from the FCT and the Luso-British Foundation. Furtado teaches in the Master and Ph.D. programs at Oporto University and ing the master's program in industrial design of FEUP. He has been involved in the organization of many events and exhibitions, and has lectured in Portugal, the United Kingdom, the United States, Spain, Brazil, Poland, Germany, Colombia, Austria, Mexico, Germany and Austria. His publications include *Arquitectura: Prótese do Corpo* (Furtado et al., eds., 2002); *Architecture and Information Society* (Furtado and Braz, eds., 2002); *Notes on the Space of Digital Technique* (2002); *Marcos Cruz: Unpredictable Flesh* (2004); *Off Fourm: Postglobal City and Marginal Design Discourses* (Furtado and Hernandez, eds., 2004); *Interferencias: Conformação, Implementação e Futuro da Cultura* Digital (2005); *The Construction of the Critical Project* (2005); *Architecture: Machine and Body* (Furtado and Braz, eds., 2006); *Generator and Beyond: Encounters of Cedric Price and John Frazer* (2008).

Pau de Solà-Morales
Avenir 1, ppal-1
08006 Barcelona, SPAIN
pau.desolamorales@urv.cat

Keywords: design theory, computer-based representations of architecture, CAAD, CAD, information, complexity, patterns, pattern language

Research

Information, Architecture, Complexity

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010.

Abstract. The study of the relationship between information, architecture and complexity can be accomplished through the study of patterns of relationships, opening up the field for the understanding of architecture as organization.

Introduction

The study of organization did not become part of the program of modern science until the discovery of systems of particles and the development of thermodynamics. The understanding of complex systems is parallel to the definition of two major concepts related to organization: on the one hand, the concept of entropy (S) is a measure of the order or disorder in the internal structure of systems of objects, and therefore of its internal complexity; on the other hand, the notion of "information" (loosely derived by Claude E. Shannon from the concept of entropy) expresses the internal organization of a communication system, or the internal order or disorder of a message. The relationship of entropy to information must not be underestimated, for it is the key to grasp the little details at play in the game of complex systems.[1]

While the universe and its constituents tend to a state of maximum entropy or disorder (following the second law of thermodynamics) there are local enclaves where there is a limited and temporary tendency for organization to increase. Complex adaptive systems, typical of living organisms, are some of these enclaves: non-linear, dynamic and open, these systems exchange matter and energy (hence information) with their environment, acting as rudimentary information systems. While reacting to the uncertainties of their environment (noise), they increase their internal organization to absorb and respond to it: it is the phenomenon that Henri Atlan described as "order out of noise" [Atlan 1979]. In so doing, they defy the second law, creating organization instead of evolving towards a greater state of disorder.

Architecture and complexity

Architecture can be defined as the technological production of built structures, from their inception or design to their construction. Architecture has traditionally been seen as a system of juxtaposed elements – objects above objects, objects next to objects, objects supporting other objects – but its organization has usually been disregarded. The focus, as with modern science, has been on the elements, not on how these objects are arranged in respect to each other, and to the whole of the system.

However, architecture is one of those phenomena that can be regarded as the creation of a complex, organized system of parts, the act of creating order out of disorder, of transforming plain raw materials into an organized whole that acts coherently, and maintaining this organization (i.e., the building) against all odds. The process of design, and the later construction of the projected building or town results in structure and organization where there was previously neither order nor structure, only *noise*. Considered as a complex system, then, architecture represents the transformation of a

Nexus Netw J 14 (2012) 17–24
DOI 10.1007/s00004-011-0094-0; *published online* 13 January 2012

certain state of the environment into a more organized one, using energy and raw materials, and the exchange of information with the environment: a transformation of information into organization.

But how can we account for the type and amount of organization in a building? How can we get hold of it and study it? What can we learn from other disciplines that have found in the study of organization a leap forward in the resolution of such problems?

All these questions are important and will be answered soon, but they must be preceded by a more fundamental question: how can we equate architecture to a complex system?

Patterns of relationsips

A complex system is much more than just a set of many different parts, more than a multiplicity of elements which are considered together. A complex system also considers the many *relationships* among these elements, the different ways in which the elements of the system interact. But that is not all: the organization of complex systems begins with an economy of elements, so that these fall into a limited number of categories. These elements, in turn, are themselves composed of systems of smaller elements, in a very particular pattern: systems nested within systems.

In a building we have different types of elements: supported and supporting, vertical and horizontal, solids and voids, and structural, dividing connecting elements. But this classification does not give us a clue about how these elements interact with each other.

First observed and proposed by Christopher Alexander, *patterns of relationships* are a way to view the built environment from an organizational perspective. In his seminal book, *The Timeless Way of Building* [1979], Alexander makes it evident that for a study of organization the elements alone are not sufficient, but that it is also necessary to account for the invariants that these elements show:

> ... *it is very puzzling to realize that the "elements", which seem like elementary building blocks, keep varying, and are different every time that they occur.*
> ...
> Since every church is different, the so called element we call "church" is not constant at all. Giving it a name only deepens the puzzle. If every church is different, what is it that remains the same, from church to church, that we call "church"?
> When we say that matter is made of electrons, protons and so forth, this is a satisfying way of understanding things, because these electrons seem, indeed, to be the same each time that they occur, and it therefore makes sense to show how matter can be built up from combinations of these "elements", because the elements are truly elementary.
> But if the so-called elements of which a building or a town is made—the houses, the streets, windows, doors—are merely names, and the underlying things which they refer to keep on changing, then we have no solidity at all in our picture, and we need to find some other elements which truly are invariant throughout the variation, in a way that we can understand a building or a town as a structure made up by combination of these elements [1979: 84-85].

The finding of Alexander points in the correct direction: there can be no understanding of organization without first defining the set of *atoms of the* *environmental structure*. Having found that the elements are not the kind of stable structures that he needs to define organization, he will turn to relationships in search of these invariants:

> *Let us therefore look more carefully at the structure of the space from which a building or a town is made, to find out what it really is that is repeating there.*
> We may notice at first that over and above the elements, there are relationships between the elements which keep repeating too, just as the elements themselves repeat…
> Beyond its elements, each building is defined by certain patterns of relationships among the elements.
> …
> *Evidently, then, a large part of the "structure" of a building or a town consists of patterns of relationships* [1979:85-87].

Alexander finds in the relationships – and not in the elements themselves – the truly invariant *atoms* that are constant and allow us to identify the elements:

Indeed, in our discipline we use names for certain elements ("wall", "room", "stair", "ceiling", "beam", etc.) to indicate what are in fact particular arrangements of elements. For example, it would be difficult to define the element "column". Columns come in different flavors, and may show many different shapes and forms, making it inadequate for an invariant object. Nonetheless, the "particular arrangement of a supporting vertical shaft transmitting a load from a supported element to a bearing bottom" (that is, a *pattern of relationships*) is what we *always* – invariantly – call "column".

> *At first sight it seems as though these patterns of relationships are separate from the elements.*
> …
> *When we look closer, we realize that these relationships are not extra, but necessary to the elements, indeed a part of them.*
> …
> … *it is not merely true that the relationships are attached to the elements: the fact is that the elements* themselves *are patterns of relationships* [1979: 87-88].

From this particular point of view, buildings are made of determinate arrangements of certain secondary elements (such as spaces or rooms), each one being bound to – but also *separated* or *divided* by – other, even smaller elements (a particular pattern of relationships among walls, floors, ceilings, etc.) arranged in particular ways. Depending on this arrangement, we distinguish between rooms, corridors, staircases or porches. In turn, each of these tertiary elements are composed of other smaller elements, and so on.

Although only briefly sketched here, fully understanding the concept of *pattern of relationships* —and all its consequences— is the first step towards understanding the organization of the built environment, and will prepare the terrain for an architecture of organization. Patterns of relationships offer the clue to identify organized structures and distinguish them from bare materials, as we are able to distinguish a cell from its surrounding molecules.

Patterns in information technology

Alexander's amazing insight was followed by by another book entitled *A Pattern Language* [1977], which contained a list of the most desirable patterns in architecture, according to its author. At the time of publication, *patterns* (the most common and recognized good solution to a design problem) were documented in text and drawing, and compiled in a book: each pattern was described by a name, a descriptive entry, and some cross-references – much like a dictionary entry – and explained why that solution may be considered a good one for that problem in a given context. All the patterns documented together formed a pattern *language*.

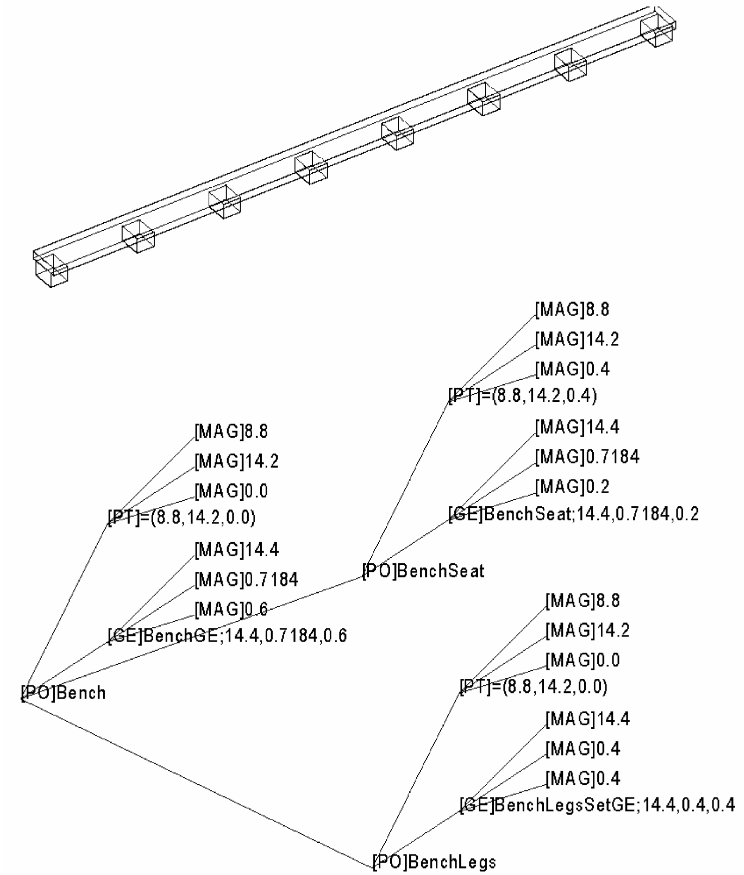


Fig. 1. A geometric rendering of a simple element (top), and its initial representation as "patterns of relationships", showing a top-down approach (PO, physical object; PT, point; GE, geometry; MAG, magnitude)

This was acclaimed as a new design methodology, and was widely used around the world to design buildings and foster public participation in the design of social environments. However, the methodology did not yield the expected results, and was not accepted by the architecture intelligentsia, and was finally abandoned as another failed experiment in the history of architecture.

But in the late 1990s software programmers rediscovered patterns as they began to implement software systems with the new programming languages based on objects[2] – mostly object-oriented interfaces. They too found that software systems tended to repeat, again and again, certain basic processes (such as opening a file, responding to interaction, modifying visual cues, etc.) that could be identified as *patterns*, and for which there also existed a good design schema. Inspired by Alexander's insights (the idea of "patterns of relationships", not "pattern languages") certain software designers began describing and documenting *software patterns.*[3] They also documented the elements (software objects) and the set of relationships (members, methods, etc.) that composed the pattern.

But this time something was different. Computers had been around for a certain length of time, and were the natural medium of programmers, therefore patterns were not documented with loose, fragmentary and ambiguous descriptions: this time object data could be stored, together with their relationships, in a consistent data structure: the patterns were physically implemented, physically stored, and physically reused – if we can call computer software something "physical".

Architectural patterns revisited

In any case, software design patterns have given us the clue to the implementation of architectural patterns in computer systems, so that we can capture the organization of designed objects, analyze them and take the most out of them.

By coupling the flexibility of object-oriented database management systems, or OODBMS [Bertino and Martino 1993], to which objects and relationships are native, with a well designed data model, architecture can be captured and represented, reproducing closely the insights of Alexander [Sola-Morales 2000]. In this way, we have been capable to make several breakthroughs.

In the first place, we have been able to represent architecture in its complexity (i.e., architecture represented as a complex system), increasing the amount of information that is stored about an architectural object. These representations become semantically richer, overcoming the problems of most CAD systems, which can only be used, at best, to represent the form of architectural entities.

In the second place, we have been able to capture design decisions: every time a designer makes a decision, he is merely creating certain relationships between the parts (for example, the alignment of some objects, the covering or the enclosing of a space). These relationships are physically captured in a database, so that we can later trace the designer's intentions. The designs are filled with meaningful descriptions.

Thirdly, we envision that we will be able, with a relative amount of data, to analyze and understand the organization of design objects and processes, so that we can advance in the translation of architecture to the world of organization. And by doing so, we will also enter the real information era of architecture, for organization and information, as we have seen, are closely and intimately interrelated.

Finally, we will be able to define a library of architectural patterns, much in the way Alexander did in *A Pattern Language*, but with a digital support. Yet, digital architecture patterns are formless, not subject to the rigidity of geometry (design patterns do not rely on geometry, only on objects and relationships). With this function, and freed from the difficulties of geometric drawing, we can "copy and paste" patterns, and reuse design solutions wherever we like, without the need of actually copying a plan or 3D model and adapting it to its new context.

Patterns of relationships managed by computer offer an affordable method for capturing the organization of architectural objects, understanding and decoding their internal structure, and upgrading the study of the built environment to the new paradigm of complexity and information.

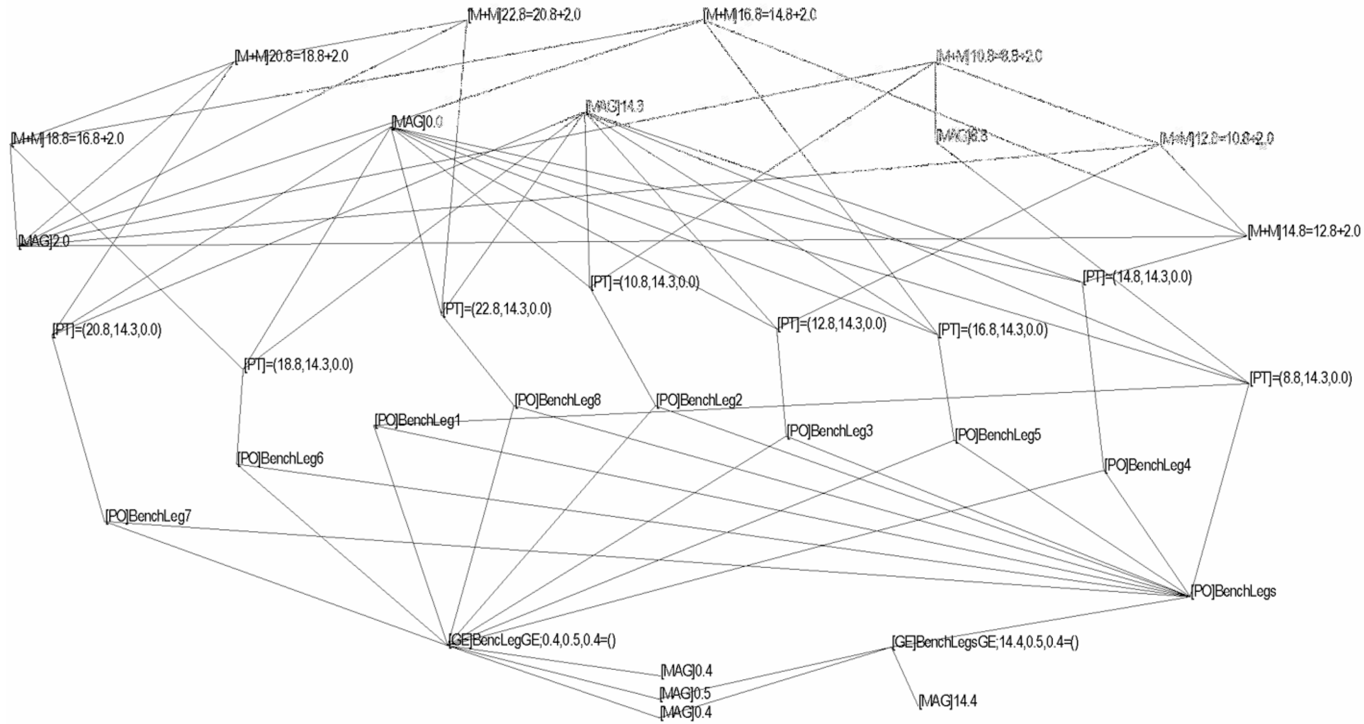


Fig. 2. The same object as in fig.1, whose representation is now completed with all the possible relationships. Each 'link' denotes a design decision. Note in particular the operations between magnitudes

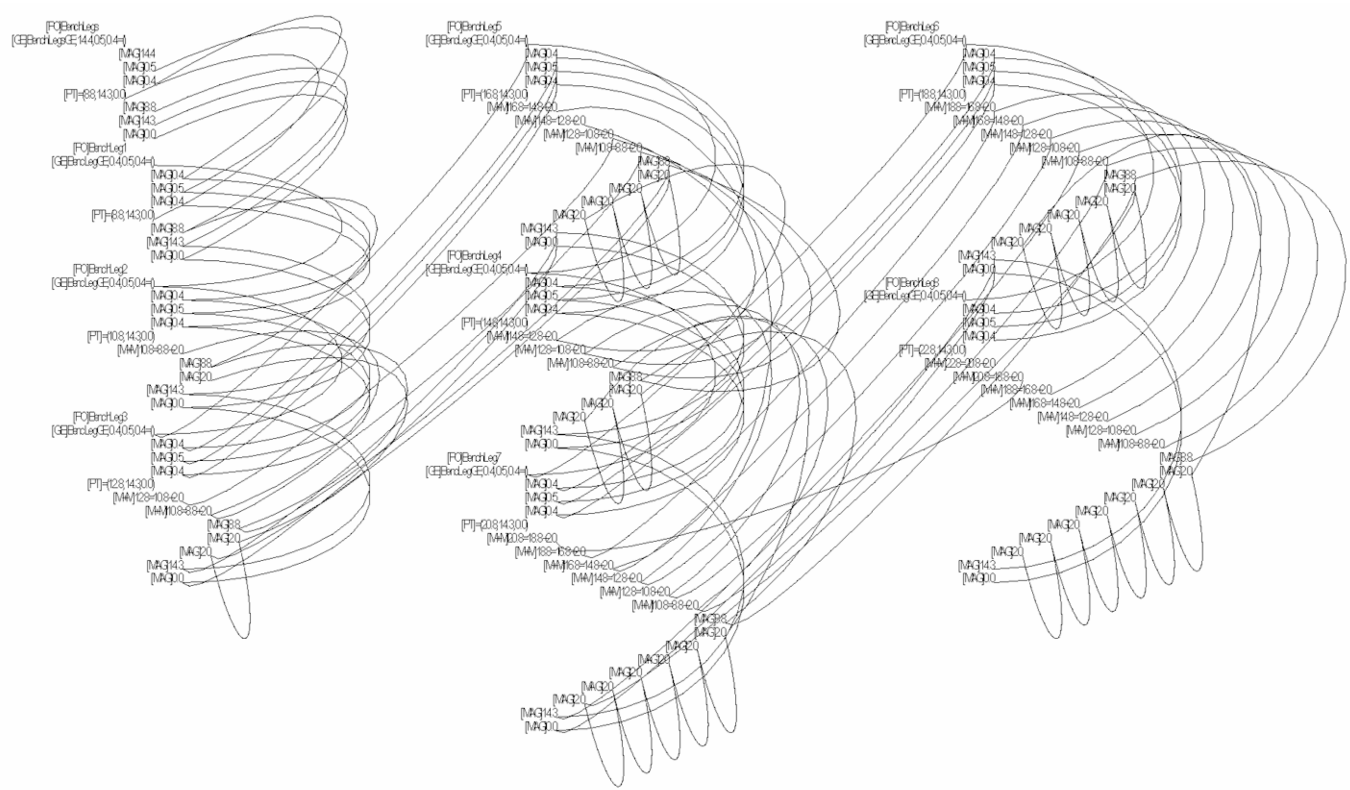


Fig. 3. The same set of nodes and relationships as in fig.2 but in a different layout, showing elements and their complete list of components. Lines connect shared or repeated elements

Notes

1. Many accounts have been given of the importance of entropy for the paradigm shift in science and the advent of complexity. See for example [Prigogine and Stengers 1997]; [Taylor 2001]; [Capra 1996].
2. Programming languages of this type are called "object-oriented".
3. See for example the pioneering books by Erich Gamma, et al. [1995] and Martin Fowler [1997].

Bibliography

ALEXANDER, Christopher. 1977. *A Pattern Language: Towns, Buildings, Construction*. New York: Oxford University Press.

———. 1979. *The Timeless Way of Building*. New York: Oxford University Press.

ATLAN, Henri. 1979. *Entre le cristal et la fumée: essai sur l'organisation du vivant*. Paris: Seuil.

BERTINO, Elisa, and Lorenzo MARTINO. 1993. *Object-Oriented Database Systems: Concepts and Architectures.* International computer science series. Boston: Addison-Wesley.

CAPRA, Fritjof. 1996. *The Web of Life: A New Scientific Understanding of Living Systems*. New York: Anchor Books.

GAMMA, Erich, Richard HELM, Ralph JOHNSON and John VLISSIDES. 1995. *Design Patterns: Elements of Reusable Object-Oriented Software*. Boston: Addison-Wesley.

FOWLER, Martin. 1997. *Analysis Patterns: Reusable Object Models*. Boston: Addison-Wesley.

PRIGOGINE, Ilya and Isabelle STENGERS. 1997. *The End of Certainty: Time, Chaos, and the New Laws of Nature*. NewYork: Free Press.

SOLA-MORALES, Pau. 2000. Representation in Architecture: A Data Model for Computer-Aided Architectural Design. Ph.D. thesis, Harvard Graduate School of Design, Cambridge, Massachusetts.

TAYLOR, Mark C. 2001. *The Moment of Complexity: Emerging Network Culture*. Chicago: University of Chicago Press.

About the author

Pau de Solà-Morales was born in Barcelona, where he graduated in architecture from the ETSAB in 1993. He received a doctoral degree from Harvard University (2000). His research focuses on the relationships between architectural theory and the use and application of information technologies in design, in particular on the different approaches to architecture from the sciences of complexity, about which he has written several articles and participated in various international conferences. He has been visiting professor at the Harvard Graduate School of Design and the Accademia di Architettura, Mendrisio, Switzerland (USI), as well as in other international schools. At present he is professor of architectural theory and of informatics at the Architecture School at Reus (URV).

Lora Dikova

943 Steamboat Drive #204
Cincinnati, OH 45244
USA
loradikova@gmail.com

Keywords: information, string theory, M-theory, spacetime, morphologies

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010

Research

After the Paradigm of Contemporary Physics in Architecture: Spatial Possibilities and Variations

Abstract. Architecture in the information age becomes a mixture of spatial and temporal processes that are directly linked to changes in science, technology and culture. In the digital era, when information becomes a matter of philosophy, methods of informational transitions provide multiple possibilities for conceptualizing space, and thus opening new horizons to architecture. This research explores algorithms of analogy between string theory, M-theory and architecture. The mathematical spacetime concepts of movement, dimension and topology in physics are studied as tools for achieving informational transitions in the design elements and their structures. As a result this methodology opens numerous possibilities for interpretation and creation of new design morphologies.

Fig. 1 Untying Form into Shadow

The function of Art is to imitate Nature in her manner of operation. Our understanding of "her manner of operation" changes according to advances in the sciences

John Cage [1969: 31]

Architecture in space and time

Architecture is not only a spatial statement, but is also a philosophical one. Design transforms data into spatial structures by organizing physical, technical, cultural, functional, aesthetic and economic information, by means of abstract connections. Therefore, architects participate in the design process by analyzing restrictions in space, time and culture. Later, they convert them into a non-accidental spatial artifact. This process of repetitive organization, namely defining the multitude of elements and their connections, requires particular methodology. This paper explores a possible approach for achieving such design methodology by using spatial concepts from the area of modern physics. Employing an indirect analogy with string theory and M-theory, the paper encompasses a variety of questions, bringing architecture, philosophy, and science together, to investigate how the interconnectedness of these questions could affect contemporary design, providing different modes for organization of the design elements in space, time, and context.

To focus on why I bring architecture, philosophy, and science together I would like to introduce a question asked by the physical chemist Ilya Prigogine, when referring to the organization of the embryo: "How can an inert mass, even a Newtonian mass

DOI 10.1007/s00004-011-0095-z ; *published online* 13 January 2012

animated by the forces of gravitational interaction, be the starting point for organized active local structures?"[1]

Representing a philosophical paradox in the sciences, namely the mystery of the act of processing organization, the question might just as well be ascribed to architecture, because even though the final result from design is a spatial artifact, that artifact is not a direct product of spatial elements, merely put together. In spite of its peculiar attitude towards physical, technical, cultural, functional, aesthetic, and economic restrictions, initially, each design process deals with structuring pure states of information about objects and their possible relations. Therefore, the primary cause of any designed artifact is a transformation of information; this transformation is in fact "the starting point for organized active local structures".

Moreover, it is precisely such informational transitions that link the facts which are configured in systems by means of science. It is also these transitions which philosophy makes interpretations of. Prigogine stated that, "It is science, not its results, that is the subject of philosophy" [Prigogine and Stengers 1984: 88]. The diversity of the tendencies in philosophy frames our culture. Therefore, by processing the information about how science and culture relate, new patterns of organizational structures could be derived, giving multiple and challenging answers to Prigogine's question.

Living in the twenty-first century we experience the impact of the informational revolution, marked by rapid alterations in science and technology, and consequently in culture. These alterations affect the way we make and comprehend architecture. Inevitably, they impose change in the design process, because our appreciation of space alters as well. Space has become the result of a diverse mixture of the developing scientific, technological, and cultural activities; space has turned into a spatial and temporal layering of information, constantly accumulating and altering the very way space itself is comprehended. Therefore, in order to make and comprehend spaces in the twenty-first century, we need to frame a language of relevant organization of the design elements, precisely because the alterations in the science and culture change the philosophical meaning of architecture. As mentioned, designers, in particular architects, react to these alterations by transforming spatial and temporal information. Architects such as Marcos Novak, Kas Oosterhuis and Stephen Perrella have already undertaken their quests for such a shift in the attitude towards the relation between science and culture.

In relation to the beginning of a new approach towards space and time this paper investigates the possibilities for designers to take advantage of the various methods for transforming cultural and scientific informational patterns into spatial artifacts. To do so, I specifically explore how algorithms of analogies, which are initially derived from the spacetime framework in modern physics, could then be transferred into design methodologies. I consider the main concepts of movement, dimension, and topology, as they are particularly framed in string theory and M-theory, and convert them into design tools for mediating transformations of information in a spatial way. To do that I investigate two ways these concepts could contribute to the design process: first, providing a conceptual ground for organizing a "cause and effect" structure of design elements; and second, exploring the possibilities for form creation.

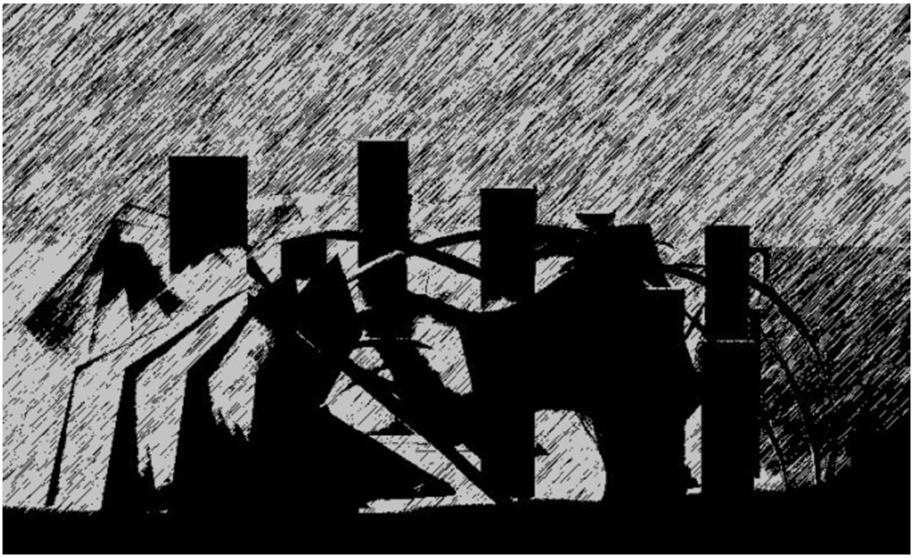


Fig. 2. The Topological City

Even though the indirect analogy with contemporary physics represents one of the many ways to form particular organizational structures in design, its concepts correspond to architecture both by providing a particular method of organization of spatial elements in a system, and by providing tools for manipulating the organization of complex dynamic systems. The theories used in this analysis, string theory and M-theory, are recent examples of ongoing scientific attempts to combine the theory of quantum mechanics and the theory of general relativity into a unified theory of everything.[2] The main concept is that particles, considered point-like in classical physics, are in fact minute strings, vibrating in a sub-quantum world of mathematical multidimensional spaces, thus producing the properties of bigger particles. In M-theory, a successor to string theory, these strings surpass the dimensions of the sub-atomic scale and form huge membranes, "branes" for short, which collide and produce a multitude of new universes. Highly controversial, because they cannot be tested by experiment, and radically strange, because they digress from our general understanding of the physical world, these concepts cannot be applied to the design process directly. However, the inner logic for organization of spatial and temporal elements defined by these theories could successfully be applied in design as a way to transform spatial and temporal information, precisely because they offer challenging connections among the concepts of movement, dimension, and topology. These concepts, derived from modern physics but interconnected with mathematics, art and philosophy, could be applied in architecture as design tools for organizing spatial and temporal information.

Architecture, following the paradigm of modern physics, could therefore result not only in a new morphology, but also in the creation of new contexts. Design might turn into algorithms of informational patterns, which change the *x*, *y*, *z* time variables of objects. Such a change would influence the subjects in the environment of these objects. Therefore, this change of the variables of the objects would help achieve a diversity of spatial and temporal events.

Architecture and movement

The architect Kostas Terzidis stated that, “motion involves time as a measurement of change” [2003: 33]. In string theory and M-theory, “the wild vibrations of the different strings” [Green 2006: 226], which are different from ordinary vibrations because they account for the quantum fluctuations at that scale, could still be considered as a form of alteration in time, on the condition that we accept time, even without being able to measure it in the quantum world. Moreover, the distinguishable difference among the elementary particles, tested by classical physics, results from the difference in the specific vibrations of the strings in multi-dimensional spaces, according to string theory and M-theory. Therefore, such a spacetime framework represents not only a repetitive movement and energy exchange, but also allows the concept of movement to serve as a tool to form informational transitions.

To transform this concept of movement in architecture, we should consider both the dynamic possibilities of the design elements as well as the option of tracing motion in a static form. The first would imply the use of movement as a tool for transforming information about space and time in architecture; the second implies its use as a tool for imposing a form onto a designed artifact.

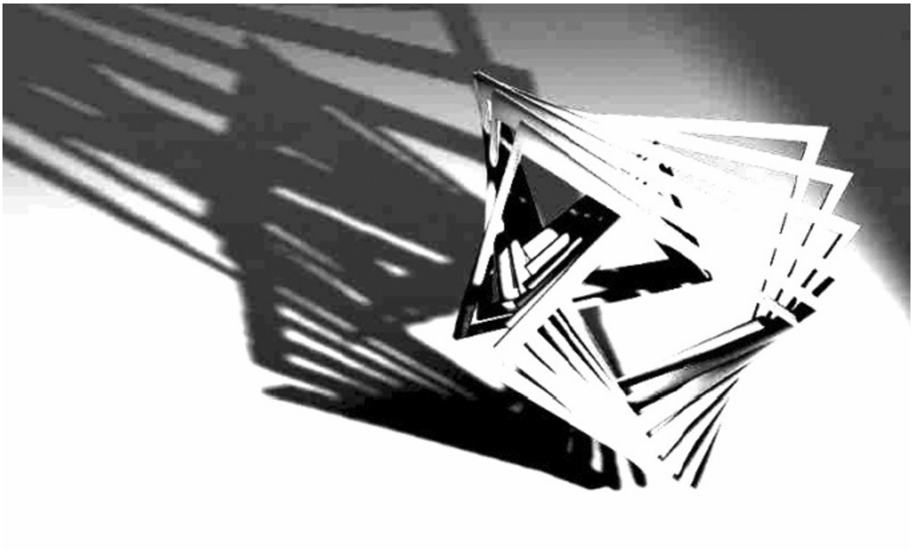


Fig. 3. Temporal Sequence of Form over Time

As an organizational tool, movement in *n*-dimensions could be applied both to objects and subjects in the design process either to force or to imply a particular structure of connectivity. Such a structure would inevitably create a specific frame of information. The awareness of this specific type of communication between objects and subjects provides opportunities for the designers. For example, in his design “Articulated Cloud”, the artist Ned Kahn takes advantage of this by designing a four-dimensional movement (three spatial and one temporal dimensions) in space and time. The dynamic façade of the building, made of flexible, wind sensitive and reflective exterior panels, alters naturally with the change of wind and the dynamics of the reflected sky, thus communicating any change in the spatial and temporal information of the immediate building environment.

Such attempts at animating architectural objects, done by introducing movement to unconventional elements for those purposes, make it possible to assign a new identification to the architectural object. In relation to this the architect Kas Oosterhuis stated that the architectural object turns into a "building body" [2002a: 38], encompassing all design elements, objects and subjects, by forming a homogenous environment of body structures which respond to common informational flows.

However, if we consider the architectural artifact as an overall static structure, excluding movable elements, like doors, windows, equipment, but rather referring to the massing of the architectural object, the tool of movement also provides possibilities for informational transitions, as a result of "cause and effect" design organization. Kas Oosterhuis, for example, treats the classic house as a "vectoral body whose direction is frontally oriented to arrival and departure" [2002b: 114]. Thus, if we consider the dual nature of architecture, being static, and yet imposing dynamic vectoral orientations in space, we could investigate, following the analogy with string theory, movement in *n*-dimensional spacetime as a mode of dynamic organization of static objects. The multi-dimensional aspect of the theory does not denote that we impose these dimensions literally, since additional dimensions merely mean additional states of information. Moreover, that aspect allows for more connections between the static and dynamic states of the objects. In four-dimensional space and time, for example, the architect Zaha Hadid developed such an approach in her early designs, applying, however, predominantly visual organizational patterns to limited elements and using collage design, rather than producing austerely articulated static places in movement.

Movement in multiple dimensions could also allow us to create a particular form in architecture. Even though the architect Kostas Terzidis stated that, "form itself does not involve time" [2003: 33], it could capture the change of time in spatial configurations, depicting the consequence of movement through time and space. Form, as a consequence of the movement in multiple dimensions through spacetime, does not necessarily demand an "animated building", as Greg Lynn would argue, but rather implies the inevitability of tracing movement onto the integrity of the form itself. For example, the Casa Guardiola designed by the architect Peter Eisenman is a result of the rotation, translation, cutting, mirroring, unification and duplication of a cube in space. The final architectural form and structure of the house communicate the notion of these processes of violation of the integrity of space, achieved by the movement of geometrical structures in time. Therefore, the form of the house is "animated" in the sense that it preserves traces of the series of movements in spacetime as geometrical projections, articulated in the final form. Therefore, a form could be considered a spatial memory of certain movement in space-time. Moreover, Michael Leyton argues that shape is memory storage and vice versa. When discussing the two-dimensional structure of paintings, he states that memory storage is also a reflection of the information about the past [Leyton 2006: 1-5]. Thus, we could consider the potential of a form as a consequence of *n*-dimensional movement, to provide informational transitions in space and time, being aware that

> [t]he designer is a stylist of the entire flux through the building body. However, the modern designer is more than that. (S)he also gives shape to the flux of the physical building body itself. The designer shapes the building body that will eventually change its shape and content in real time [Oosterhuis 2002b: 30].

Architecture and dimension

Dimension is the measure of the information on the location of elements in spacetime. Therefore, an element in the design system of four dimensions, relevant to our classical physical reality, is identified in space and time by one temporal and three spatial measures. However, according to string theory and M-theory, there are respectively ten and eleven dimensions in our physical world; therefore, more complex informational structures exist among the elements in these theories. Even though we design in three dimensions and time, we should consider and take advantage of the consequences of the possibility of higher-dimensional spaces, resulting in the creation of more states of information about objects, therefore, more options for them to relate technically or contextually in intriguing manners.

The significance of string theory and M-theory as tools for design methodology in architecture, is that the mathematical meaning of dimension is not isolated on its own, but it is woven into the structure of space and time. Thus the tool of dimension sets very particular conditions on how space and time operate together. "Now although it is hard to picture in more than three dimensions, this conclusion – more dimensions mean more vibrational patterns – is general", states the physicist Brian Greene [2004: 370]. Therefore, the concepts of dimension and movement are interconnected. How would such an analogy refract into the design process? As mentioned before, the concept of movement in architecture, even in static structures, proliferates with design possibilities for meaning and form. Therefore, by altering the concept of dimension, we disturb the tool of movement in the design process, changing the connectivity of the designed elements as well as altering their form. That inevitable dual transformation allows controlling the design in a more flexible way and also provides additional modes of framing information into spatial structures. The artist Tony Robbin, for example, used the concepts of movement and dimension in his sculpture for the Center of Art, Science and Technology at Denmark's Technical University. Working on the geometry of quasicrystals, "three-dimensional projections of higher-dimensional objects", Tony Robbin appreciates the "richness of four-dimensional geometry" by investigating its possibility for "multiple objects in the same place and time, objects appearing and disappearing by rotation, objects passing through one another without interference". Describing his sculpture, Robbin emphasizes the importance of combining concepts of movement and dimension to "allow the viewer to pass under, over, around, and through the work, and to happen upon the many and unexpected occurrences of fivefold, threefold, and twofold symmetry" [Robbin 1997: 434-435], caused by the change of light and the viewer's position.

Since the concepts of movement and dimensions are interconnected, as discussed above, and since the influence of movement as a design tool could shape a particular form in architecture, we can assume that the use of dimension can also indirectly achieve architectural form. Moreover, knowing that a common way to experience a form is by processing visual information, technical manipulation of multi-dimensional projections could serve as a tool to generate form. Tony Robbin expressed his awareness of that fact by referring to Henri Poincaré, who "repeatedly suggested that successive models of the projections of four dimensional figures when seen in sequence could lead to a vision of the fourth dimension" [Robbin 1997: 430]. Therefore, the possibility of framing whole structures, for example, in building sections or on urban level, would provide an immediate tool for generating form in the design process. Moreover, a form generated in this way would differ from other morphologies in architecture by providing a new appreciation of spaces, namely the visual experience of n-dimensionality.

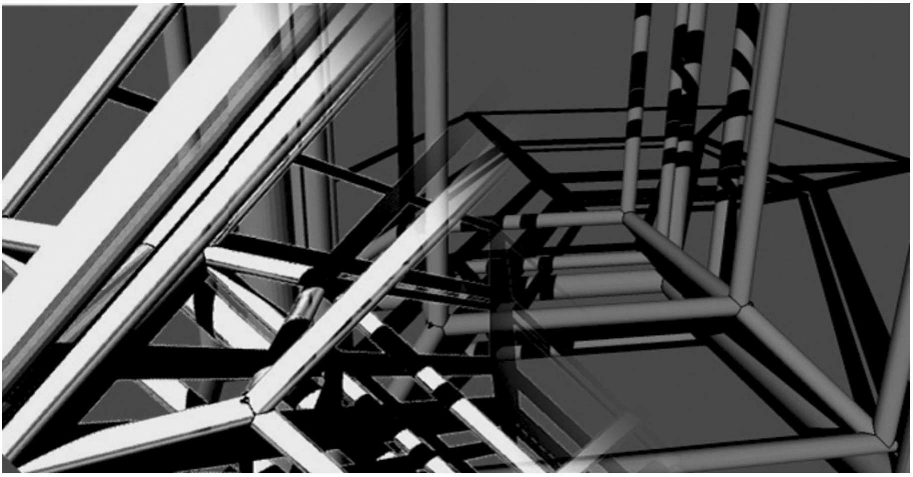


Fig. 4. N-Dimensional Variations on Tracing Form

However, because we are used to the restraints of our four-dimensional world, any representation of *n*-dimensionality could pose a challenge. Treated as a structure of connectivity among elements, *n*-dimensionality could take another direction in influencing the design process, namely by increasing the possibilities for informational transitions. As Brian Greene would argue, "… since a string's vibrational pattern determines its mass and charge," in string theory and M-theory, "this means that the extra dimensions play a pivotal role in determining particle properties" [Greene 2004: 371]. In architecture, the concept of dimension, considered as the link among all design elements in a system, might be investigated in context, not necessarily by technical geometrical projections. The architect Stephen Perrella, for example, tests such contextualism in his studies of hyperspaces, which are richer in informational transitions. Moreover, for him, hypersurfaces "are thought to render a more complex notion of spacetime information" than the mathematically defined ones, because in the hypersurfaces "the abstractness of these mathematical dimensions is shifting, defecting or devolving into our lived cultural context" [Emmer 2004: 57]. Unlike Stephen Perrella, Marcos Novak considers the concept of dimension in M-theory as a way to identify the design elements. Therefore, he suggests that dimension serves not as a tool to combine elements together in a system, but to differentiate their typology:

> …both are manifolds, the difference between hyperspace and hypersurface of a hyperspace of (n)-dimensions is a submanifold of (n-1) dimensions. Thus the hypersurface of a hyperspace of four spatial dimensions is a space of three spatial dimensions, produced by an act of projection or section or screening [Novak 1998: 85].

Architecture and topology

Deriving a form from topological spaces is the predominantly direct way to structure what Kenneth Powell calls "sculptural drama" [Powell 1993: 7]. Intriguing results of the application of the concept of topological forms in the arts are found in the two-dimensional art of M. C. Escher, where spacetime is visually curved, as well as in the sculptures of Eva Hild, improvising with the pressure, curvature, and intensity of a form. Moreover, topology has been already widely utilized in architectural theory by architects such as Greg Lynn, Peter Eisenman and Kostas Terzidis. The “weak form” of Peter Eisenman and the “fold” of Greg Lynn have tested topological structures and the way they relate to morphology in architecture. Greg Lynn implements topology as a tool to investigate flexibility and continuum in a form as well as its signification, affected by “programmatic, structural, economic, aesthetic, political and contextual influences” [Terzidis 2003: 23]. In similar way, UN studio, for example, transfer the Möbius strip directly from topology into architecture in their design for the Möbius House with the concept of infinite interaction of the users’ activities.

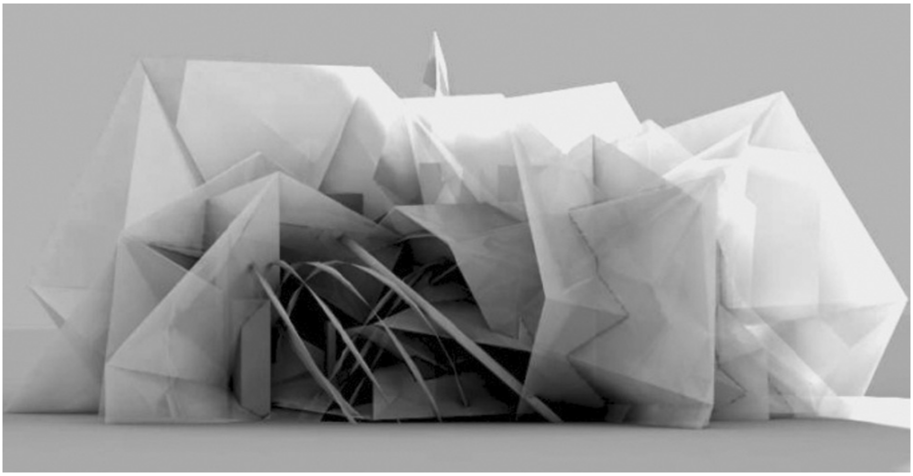


Fig. 5. Topological Invariants on Analytical Cubism

However, the mathematical approach taken by string theory and M-theory towards the concepts of topology and *n*-dimensionality suggests an intriguing relation between topology, dimension, and movement. This relation provides various algorithms for organizing elements in structures. Specifically, it bonds in a unique way the spatial character of the elements with the temporal feature of their structure. The strings vibrate in the sub-quantum world, in six-dimensional mathematical spaces called the Calabi-Yau manifolds, thus forming the properties of the various particles. Brian Greene explained further that the “vibrational patterns of the strings are influenced by the twists and turns in the geometry of the extra six dimensions” [2004: 371]. Therefore, the tools of topology, movement and dimension become interconnected and self-regulatory, given that topology controls deformations of form, such as stretching, twisting, folding, and scaling, while preserving the form integrity. Moreover, the architect Kostas Terzidis argues that form, subject to topological deformations, would reveal particular properties, as it would “allow time to be imprinted on form”, as well as be an “object in disguise” because the object could be “composed of the topology of the one object and the geometry of the other” [2003: 16, 24]. This implies that by applying topological transformations to the design elements, we change the concepts of movement and dimension. Therefore, topology could be used to control the design mechanisms and organization in their unity. Kas Oosterhuis, for example, suggested that, “[t]he tension between the dimensions can be made visible and tangible by stretching them almost physically, like a baby in a bunting bag” [Oosterhuis 2002b: 237]. Similarly, movement, as a design tool, could also be transformed, stretched, folded or scaled, which would result in a change of form and connectivity of the design elements. Therefore, topology provides rich possibilities for manipulation of the design elements in form and context.

Dealing with the possible relationships among the elements in architecture, topology could serve as a tool not only for morphological and contextual, but also for programmatic organization of the relations between objects and subjects. Therefore, topology could constitute a powerful instrument for how architects transform the data of all design restrictions into spatial and temporal structures, emphasizing specific characteristics about such an organization. Dealing with deformations, preserving the object's integrity, but defining boundaries – "in and out" conditions – topological transformations could be applied in the design process, for example, as a spatial solution to border contexts. In a single building unit, or on urban level scale, exploring the elements as sets of typologies might serve as a tool to set categories of identities. This could therefore help to deal with a multitude of objects and subjects at once, by introducing them as part of a single topological structure, giving the advantage of controlling complexity because:

> [r]ather than assuming a continuous behavior that governs all finite elements, the behavior of each finite element contributes toward a generalized behavior. Finite elements can be regarded not as arbitrary units but rather as localized samples. In this context, general principles are derived from particular instances [Terzidis 2003: 50].

Architecture after the concepts of modern physics

"We have our own discoveries to make, based on the mathematics and physics of our own time", stated the artist Tony Robbin [1997: 437]. Therefore, if we go back to the question of the organization of the embryo, posed by Ilya Prigogine, we might find that in each time and space we accept different answers to that question. Our way of thinking as artists, scientists or philosophers who organize given information to achieve a developed structure of interconnected elements, is subject to change according to "our own discoveries". This paper presents a way to frame architectural theory through a reflection on a scientific analysis. By applying an indirect analogy with concepts from string theory and M-theory, the paper investigates possibilities for interpretation of information in the design process. The analysis of the tools of movement, dimension, and topology provide opportunities for both, interpretation and generation of design artifacts. It also reveals the importance of the spacetime framework in string theory and M-theory, taken as a design tool for dealing with complex dynamic systems, such as architecture, because unlike acknowledged concepts in classical physics, like phase space,[3] for example, string theory and M-theory link the concept of dynamics with the concept of spatial deformation topologically as well as temporally. Someday string theory and M-theory might prove to be wrong about how they define our physical space and time. However, that would not repudiate any of the places we might have designed as "animated through multi-dimensional spacetime form", exuberant in the diversity of their topological structures On the contrary, it would only be one of the various ways to answer Prigogine's question – one more time, in one of our many understandings of spacetime.

Acknowledgment

All images are by the author.

Notes

1. [Prigogine and Stengers 1984: 82]. Prigogine discusses the relationship between physics and philosophy; in this case referring to the viewpoint of the French philosopher Denis Diderot.
2. The theory of general relativity, including gravity, is mathematically irreconcilable with the theory of quantum mechanics, including electromagnetism, the strong and weak forces. However, scientists assume that under conditions, similar to the ones they suppose formed in the Big Bang or in black holes, these forces should be unified. String theory and its follower M theory represent a way to do it. However, they are only theoretical and due to technological restrictions, unverifiable.
3. Phase space is used for analyzing dynamic systems by introducing six coordinates to an object, defining its location by three spatial coordinates and their respective momentum, relating to the movement of the object.

References

CAGE, John. 1969. *A Year from Monday: New Lectures and Writing*. Middletown, CN: Wesleyan University Press.

EMMER, Michele. 2004. *Mathland: From Flatland to Hypersurfaces*. Basel: Birkhäuser.

GREENE, Brian. 2004. *The Fabric of the Cosmos.* New York: Alfred A. Knopf.

LEYTON, Michael. 2006. *The Structure of Paintings.* Vienna: Springer-Verlag/Wien.

NOVAK, Marcos. 1998. Transarchitecture and Hypersurfaces: Operations of Transmodernity. *Architectural Design* **68**, 5-6: 84-93.

OOSTERHUIS, Kas. 2002a. *Hyperbodies: Towards an E-motive Architecture.* Basel: Birkhäuser.

———. 2002b. *Architecture Goes Wild.* Rotterdam: 010 Publishers.

POWELL, Kenneth. 1993. Unfolding Folding. In: Architectural Design Profile 102: *Folding in Architecture. Architectural Design* **63**, 3-4: 6-7.

PRIGOGINE, Ilya and Isabelle STENGERS. 1984. *Order Out of Chaos: Man's New Dialogue with Nature*. Boulder: New Science Library.

ROBBIN, Tony. 1997. Quasicrystal Architecture: The Space of Experience. Pp. 434-435 in *Beyond the Cube. The Architecture of Space Frames and Polyhedra*, J. Francois Gabriel, ed. New York: John Wiley & Sons.

TERZIDIS, Kostas. 2003. Expressive *Form. A Conceptual Approach to Computational Design.* London and New York: Spon Press.

Young, John Wesley. 1930. *Projective Geometry.* Chicago: Open Court Pub.

About the author

Lora Dikova is a Bulgarian architect. She received her two professional accredited Master of Architecture degrees from the University of Architecture, Civil Engineering and Geodesy, Sofia, Bulgaria in 2006 and from Miami University, USA in 2010. She has published several studies on interdisciplinary design methodologies, focusing on the relation between architecture and urban design. She has received numerous honours and awards for her design work in the area of Commercial Building Design, and her projects were exhibited on the XI World Architectural Triennale in Sofia, 2006, Bulgaria and on the European Awards for Bulgarian Architecture, Sofia, 2007. Currently, she is a representative to the International Union of Architects on behalf of the Union of Architects in Bulgaria.

Suzanne Strum

Consolat del Mar, 43 apt. 3
08003 Barcelona SPAIN
strum@coac.net

Keywords: Structural Study Associates, SSA, Knud Lönberg-Holm, Car Theodore Larsen, information technology and organization

Research

Informational Architectures of the SSA and Knud Lönberg-Holm

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010.

Abstract. This paper offers an account of the unusual proto-systemic and informational approach to architecture that emerged in the early 1930′s within the largely unknown Structural Study Associates (SSA), a circle of architects around Buckminster Fuller. It examines the design of a dynamic information system for architects by SSA members Knud Lönberg-Holm and Carl Theodore Larson and compares them to pre- and post-war knowledge indexing systems and world projects. This study also explores the systems-oriented positions outlined by these architects. This provides a view of American architecture of the era that counters the one presented by the canonical International Style Exhibition of 1932, which essentially edited out the more variegated approaches to modern architecture and technology as represented by the SSA and especially Lönberg-Holm.

Introduction

In 1932 a loosely knit coterie of productivist architects known as the Structural Study Associates, or SSA, rallied behind Buckminster Fuller's short lived editorial project in the magazine, *Shelter: A Correlating Medium for the Forces of Architecture*.[1] They propagated a radical technologist and productivist manifesto that anticipated the systems and communication theory that emerged in the post-war era. Their position regarding advanced technology and information has a contemporary resonance. Already in the early 1930s, the SSA introduced such seemingly postmodernist terminologies as *performance, emergence, emergency, ephemeralization, biologic design, networks, mobility, flows, decentralization, ecology* and *entropy*. They promoted a research based, macro-scale systems approach, modeled at the time on the flow of energy.

With the motto: "Don't fight forces use them", they insisted on the term *environmental controls* instead of architecture. Their vision of an advanced techno-society promoted a nomadic, tensile, metallic dwelling unit as well as a data-driven design culture predicated on free divulgation of continually updated information. The SSA promoted architecture as instruments, the outcome of the service-minded mediation of technology for the benefit of the community.

Some of the later projects put forth by SSA members might be seen as particular case studies of a proto-history of systems and informational architectures that emerged in the context of the magazine *Shelter* and the economic and social crisis of the American Depression. *Shelter* acted as publicity platform for promoting Buckminster Fuller's Dymaxion house as "Universal Architecture". But beyond his appartent protagonism, beginning in the 1930s Knud Lönberg-Holm (1895–1972) and Carl Theodore Larson (1903-1988), two architects who emerged from the SSA, became information specialists, and Fuller's fellow travelers. They began an unusual collaborative project to create a

DOI 10.1007/s00004-011-0096-y ; *published online* 13 January 2012

dynamic, constantly evolving database of information related to community design and environmental controls. They sought to use postwar CIAM as an international platform.

Their wartime publication *Planning for Productivity* [1940] and the post-war *Development Index* [1953] (fig. 1) are also essentially knowledge indexing proposals, which can be compared to a selection of similar pre-war and post-war encyclopedic and world projects based on the retrieval, centralization and transmission of knowledge, including Buckminster Fuller's Geoscope of 1962. Within these projects they reformulated design according to unsentimental scientific criteria: as the effective transformation and arrangement of energy into flow patterns for productive use. They also theorized information flow (fig. 2).

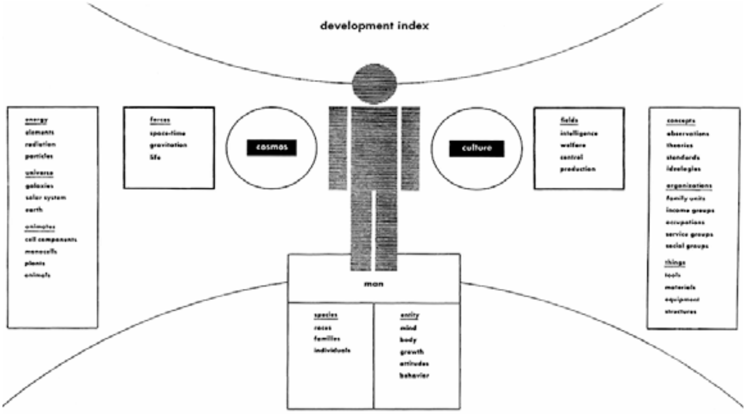
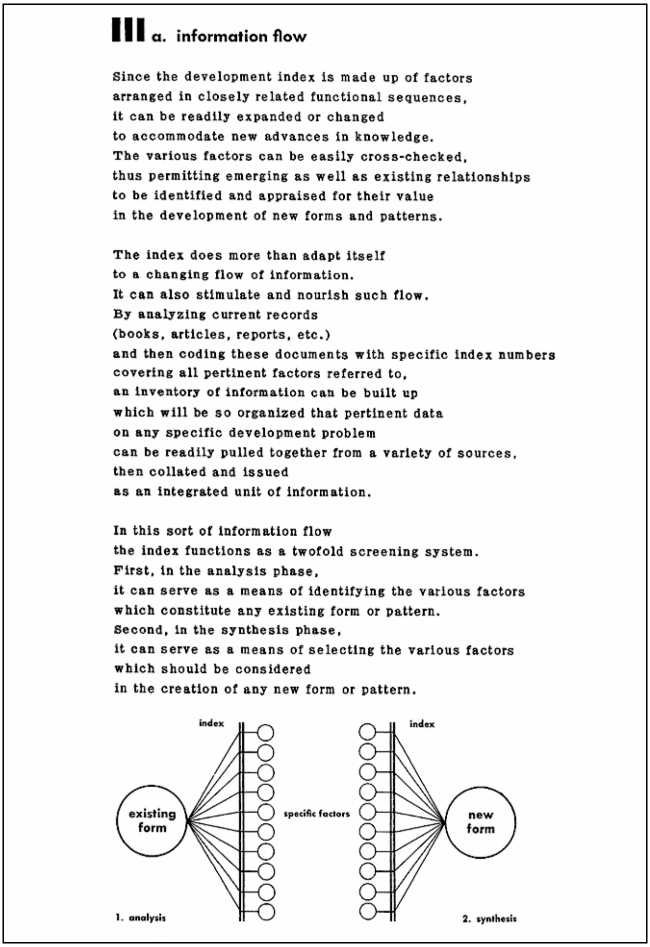


Fig. 1. "Man, Cosmos, and Culture" from *Development Index* [1953]

As a class of projects that processes data and indexes knowledge, Lönberg-Holm and Larson's proposals share an affinity with early modern *World projects*, such as the World Brain, the World Encyclopedia, the World City, World Auxiliary Languages and the International System of Typographic Picture Education. Such systems were put forward by diverse figures in the inter-war period including novelist H. G. Wells, Wilhelm Ostwald, winner of the 1909 Nobel prize in chemistry, Paul Otlet, and Otto Neurath.

Such world projects, aimed at the centralizing and standardizing of <u>all</u> knowledge in one single location and unifying norms for international use, find their precursors in incipient global networks of standards and conventions.[2]

In their pre-war manifestation, these projects point to the impact of Taylorization on knowledge systems – the storage, classification and retrieval of complex data – as information specialists theorized the management and dissemination of publicly available data, the development of institutions and unifying systems in a pre-digital era. Information technologies were themselves subject to the processes of scientific management. *Planning for Productivity* and the *Development Index* might be situated somewhere between those modernist World Indexing projects, including Buckminster Fuller's broadcast conning station that appeared in *Shelter*,[3] and his later, and more multimedia Geoscope mentioned earlier, a proposal for a floating informational globe, with constantly updated real-time data transmission about the earth. The projects of Lönberg-Holm and Larson carried forward the common credo and positions outlined in the depression-era in *Shelter* magazine.



III a. information flow

Since the development index is made up of factors
arranged in closely related functional sequences,
it can be readily expanded or changed
to accommodate new advances in knowledge.
The various factors can be easily cross-checked,
thus permitting emerging as well as existing relationships
to be identified and appraised for their value
in the development of new forms and patterns.

The index does more than adapt itself
to a changing flow of information.
It can also stimulate and nourish such flow.
By analyzing current records
(books, articles, reports, etc.)
and then coding these documents with specific index numbers
covering all pertinent factors referred to,
an inventory of information can be built up
which will be so organized that pertinent data
on any specific development problem
can be readily pulled together from a variety of sources,
then collated and issued
as an integrated unit of information.

In this sort of information flow
the index functions as a twofold screening system.
First, in the analysis phase,
it can serve as a means of identifying the various factors
which constitute any existing form or pattern.
Second, in the synthesis phase,
it can serve as a means of selecting the various factors
which should be considered
in the creation of any new form or pattern.

Fig. 2. "Information flow" from *Development Index* [1953]

The Structural Study Associates (SSA) and Shelter

The non-commercial *Shelter* magazine was in founded in 1930 by Beaux Arts trained George Howe, an architect with William Lescaze, renowned for his Philadelphia Savings Fund Bank, the first skyscraper in the US designed in the International Style. Starting as the very local *T-Square Club Journal* in Philadelphia, in its two years of existence, it went through three changes in title, design and editorial direction, ending with a national projection and the radical technologist position of the SSA. Its seventeen issues performed a special role in the debates surrounding modern architecture in the United States. Divisive viewpoints often coalesced in the same issue, between architects in the same camp, those promoting a modern architecture. Contributors included traditionalists, housing reformers, and internationalists who presented modern architecture as a received European style, and the SSA, who looked to native technological sources.

In 1932, the Structural Study Associates joined Buckminster Fuller in taking over *Shelter,* rebuking the formalist, aesthetic agenda and classifications of the International

Style Exhibition held at MoMA, curated by Henry Russell Hitchcock and Philip Johnson, then also associate editors of *Shelter.*[4] Instead the SSA decisively called for an architecture predicated on the latest technological advances in materials and structures.

While placing the emergency situation of the American Depression in the foreground, they remapped the country as an abstract, networked, national space based on mobility, energy, resource systems, communication and transportation infrastructures. With extra-architectural imagery – electric power lines and tensile engineering – their magazine was dense with charts, diagrams, news clippings, scientific information, technical reports and economic data. They held that the evolutionary nature of technology would have a transformative social value, representing one of the most extreme versions of such thinking in architecture at the time, a critical moment when modern architecture was being promoted in the US and when the term and modern conception of "technology" first came into general and widespread usage. By the 1930s technology could be thought of as an autonomous agent of progress, an independent mode of thought, instrumentality in and of itself.[5]

1932 was statistically one of the worst years of the Great Depression, the last year of Herbert Hoover's presidency and the prelude to Franklin Delano Roosevelt's propelling the New Deal, At the time, Technocracy Incorporated, a group of engineers, scientists, economists and architects, had attracted an enormous popular national following. This ideological movement arose out of a think tank, Technical Alliance, commissioned to create an energy survey of the US.[6] In 1930 Buckminster Fuller had met the group, whose leader was Howard Scott, based in Columbia University's engineering department.[7] But he never joined. Technocracy INC, believed in autocratic rule by technical specialists to bring the United States productive capacity to optimal level. The founders were disciples of the economist Thorstein Veblen, whose book *The Engineers and the Price System* [1921] had suggested that a "Soviet of Technicians" should rebel against absentee factory owners, to bring the country to full production inherent in the machine age.[8]

The SSA, a group mainly formed of editors of professional magazines with access to information about the building industry, were clearly Fuller's "Soviet of Architects" and they drew on Veblen's writings in their aim to overhaul the obsolete building industry and their belief in "industrial emancipation". They were also clearly indebted to the extra large scale framework of the contemporary Technocracy Incorporated, drawing on their protocols involving resource and energy surveys, economic indexes, thermodynamics, charts and correlations of industrial production [Technocracy Incorporated 1934]. In the post-war period Fuller would be assimilated into ecologist movements, but the SSA's systems paradigm, circa 1932, had its origins in the hierarchical and centralized national electric grid. The SSA put their emphasis on environmental controls, defining design as the harnessing and deployment of flows of energy.

Knud Lönberg-Holm: architect of information

Some of the SSA members played an important role in promoting a modern architecture in the US, based on advanced technology. Such is the case of the little known Danish émigré architect Knud Lönberg-Holm, who completely abandoned practice to transform the outmoded building industry, a central aim of the SSA in *Shelter*. His professional trajectory, a myriad of simultaneous and largely anonymous collaborations, concentrated on organizational modes, communication and information technologies from within a special field. He was involved in the refinement and designing of instrumental architectures and systems transforming quotidian practice in the 1930s as Beaux Arts compositional modes were supplanted by modernist diagrammatic practices and objective scientific protocols.

Lönberg-Holm arrived in the US via Germany in the early 1920s, where he had been affiliated with De Stijl and international constructivist collectives. Respected by his peers, his career is mostly forgotten, appearing mainly in the footnotes of studies on the period. His unsubmitted Chicago Tribune Tower of 1922 was published by both Walter Gropius and Le Corbusier in Europe.[9] He contributed sixteen photographs of cityscapes and high rise steel construction to Erich Mendelsohn's *Amerika, Biderbuch eines Architekten* [1926], although the images were not credited until the book's sixth edition. The extreme vantage points introduced by the Danish architect had an impact on later images by Rodchenko and El Lissitzky.[10]

He represented the US in the first exhibition of modern architecture held in America in 1927, organized by the *Little Review* and focused on constructivist architecture [Heap 1927]. In 1924 he taught at the University of Michigan, thanks to the introduction by Eliel Saarinen, promoting De Stijl elementarist principles in his design studio.

His projects and essays throughout the period offered European colleagues an incisive critical portrait of American architecture and urbanity [Lönberg-Holm 1924, 1928]. Lönberg-Holm maintained close ties to vanguards in Europe as the east coast delegate to CIAM from 1928-1959, along with Richard Neutra, his west coast counterpoint. With *Shelter* magazine as a platform of dissemination, he acted as a transatlantic conduit. He elaborated the Functional City plans of Detroit presented by Cornelis van Eesteren at the fourth CIAM congress in 1933 and developed the theme of the post-war renewal congress held in Bridgewater, England, in 1947. His insistence on collectivity, cooperative group research, scientific and analytical models, central planning, and the role of the technician in the cultural sphere can be correlated back to his longstanding involvement with CIAM, where he held on to the organization's foundational premise focused on building, technology and standardization.

Already in the 1920s, Lönberg-Holm was involved in refining and designing instrumental architectures, organizational systems for architects' data, the consolidation of normative modernist practices and information infrastructures. His anti-formalist position closely resembled that of the radical materialist Swiss ABC group, which included communist architects Mart Stam and Hannes Meyer. Like them, he sought the scientization of architecture, and viewed standardization as a radical premise in a mass technological society.[11] Lönberg-Holm merged Buckminster Fuller's interest in native vernacular technologies and the American roadside culture of mobility with a position very close to that of Hannes Meyer. Architecture was an apparatus responding to the needs of body and mind. Building was a biological process, the organization of function and new building materials in a constructive whole based on economic principles that were to be determined by life rather that art. But his positions were veiled within the technical news and research departments of America's capitalist architectural corporations.

As a technical editor at *Architectural Record* magazine from 1929, Lönberg-Holm reformatted the magazine according to the tenets of New Typography, promoting a modern architecture based on scientific innovation and criteria of performance.[12] He contributed to the whole ambit of information systems developed in tandem with that magazine, including *Sweets Catalog*, an essential compilation of product information of evolving building materials, which paradoxically played a critical role in the corporization of everyday practice.[13] His aim had been to make it possible to create radical designs of the 1920s, such as Mies van der Rohe's glass skysrapers with standardized manufactured components.

As the director of design and research at *Sweets* from 1932 to 1960, he reconfigured this industrial system with Czech graphic designer Ladislav Sutnar, creating a theory based formulation for applying modernist techniques to knowledge management, which goes beyond typography, to pioneer *information design*.[14] His most significant contribution to *Shelter* magazine critiqued the mass, weight, deadload permanence and immobility of empty skyscrapers in the early 1930s as monuments of the collapse of the acquisitive capitalistic system in the Depression, in favor of architecture as instruments [Lönberg-Holm 1932].

Before he was hired in 1929, *Architectural Record* editors rejected his essay "Architecture in the Industrial Age" [1967][15] as far too radical for the professional architectural press. The text took a holistic, anti-aesthetic approach to the problems of building, community, new technologies, advances in science, and the transformation of the profession in the face of such changes. His essay urged city planning based on the organic functions of a community and its culturally based space organizing process. It charged that science had changed man's relation to nature and to society and that new needs would lead to the reorganization of life and society to reflect that new reality.

He called for the creation of an economically independent research institute to deal with architecture in an industrial age where collective problems could be collectively investigated. The objective of such an institution was to "act as a clearing house for individual research"; to foment "the research work-analysis of problems, the determination and definition of types and norms" [Lönberg-Holm 1967: 22]. This program would later be proposed by him in the context of post-war CIAM and within the informational systems that he designed with SSA member Carl Theodore Larson.

Design for enviornmental controls and the technician on the cultural front

Within the *Sweets* research department, Lönberg-Holm developed a series of independent collaborations with Larson that present a radically scientific definition of design and aimed to create a comprehensive non-corporate information system for architects in the international arena. The Harvard-educated Larson served as an associate editor at *Architectural Record* from 1930-1936 and became a member of the SSA in 1932. In 1948, as a professor at the University of Michigan, he founded a research laboratory dedicated to prefabricated housing, one of the SSA's key concerns. Over the course of their collaborations, which lasted until the 1970s, Lönberg-Holm and Larson kept as much abreast of evolution in information technologies as in material production.

In 1936, the two architects had been effectively demoted to *Sweets Catalog* from *Architectural Record*, based on the controversy surrounding two co-authored texts which labeled them as red suspects, advocates of the Soviet system.[16] "Design for Environmental Controls" [1936a] and "The Technician on the Cultural Front" [1936b] are effectively technological manifestos with affinities to Buckminster Fuller and their former involvement in *Shelter* magazine. They approached design based on the interrelation of invisible forces of energy beyond any tradition of architecture. These articles formed the conceptual base for their later projects and offer a systems approach to design.

Lönberg-Holm and Larson analyzed enviornmental forces in terms of two classifications of motion or energy: as human activities that included biologic and social forces; and as matter in the form of solids, liquids, gases, and electromagnetic radiation. Invisible forces had transformed human shelter from protection into controls of environmental forces [Lönberg-Holm and Larson 1953]. The purpose was to increase the life of human organization. Changeable forms of energy constituted the very materials of design, defined as the effective transformation and arrangement of energy into flow patterns for productive use: "New and unthought of forms impossible with traditional means of production are thus implied" [Lönberg-Holm and Larson 1936b: 155].

Planning for Productivity and the Development Index

As an alternative to the corporatism of *Sweets*, Lönberg-Holm and Larson also postulated a dynamic, comprehensive system of research and information management dealing with all aspects of environmental controls. *Planning for Productivity* [1940] was rejected by *Sweets*, but instead found an outlet with the International Industrial Relations Organization founded in the 1920s by Mary van Kleek, a social feminist and advocate of Soviet socialism who was engaged in promoting international planning as the solution for putting resources in the service of all the world's people, productivity as a means of social progress, and technology as a source of abundance.[17] The book proposed centralized planning, and cycles of performance for materials and buildings. The authors sought a rehaul of the building industry with centralized research and information systems to allow for advanced productivity.

Their proposal was essentially a skeletal schema of all topics related to environmental controls, specific flow patterns that conform to the changing needs of man, and a "Production Index" dedicated to continually updated information. This project was presented to CIAM, which would then have an impact on Lönberg-Holm and Larson's later collaboration on the *Development Index* [1953]. Their reformulation of design paradigms was based on a wholly scientific conception of dynamic interrelated forces, energy flows, networks and productivity.

The *Development Index* [Lönberg-Holm and Larson 1953] studied the interaction of human activity, environmental relations and communication and would appear to be more closely aligned with synergetics, the dynamic geometry and whole systems approach that Buckminster Fuller developed in the postwar era, here adapted to a proposal for a dynamic communication index of concepts related to the built environment. It is a diagrammatic outline of correlations between cosmos, man and culture, a pattern for the organization of knowledge, a non-static indexing system; a master switchboard for data flow and a tool of research to be updated constantly in line with "new expansions in human knowledge".

Larsen and Lönberg-Holm saw advanced productivity as leading to a surplus of leisure and based once again on the cycles of: 1) Research (analysis); 2) Design (synthesis), 3) Production (formation); 4) Distribution (dispersion), 5) Utilization (performance); and 6) Elimination (termination) (fig. 3). Emergent needs would be met with new forms and patterns (fig. 4).

b. development cycle

The development of any new form or activity pattern
can be analyzed as a process
comprising six characteristic and interacting phases:

1. research (analysis)
2. design (synthesis)
3. production (formation)
4. distribution (dispersion)
5. utilization (performance)
6. elimination (termination)

To achieve a rhythmic and balanced continuity in development,
there must be a progressive elimination of the old
along with the emergence of the new.
Such continuity requires a close correlation between
the research and elimination phases of the development cycle.

This definition of development does not imply
any deliberate destruction of the old merely because it is old,
nor does it demand the creation of something new
solely for the sake of novelty or as a change in "fashion".
So long as the old serves a need,
it clearly should continue in use.
The objective in developing new forms and patterns
is to satisfy those emerging needs of man
which cannot be met adequately by existing forms and patterns.

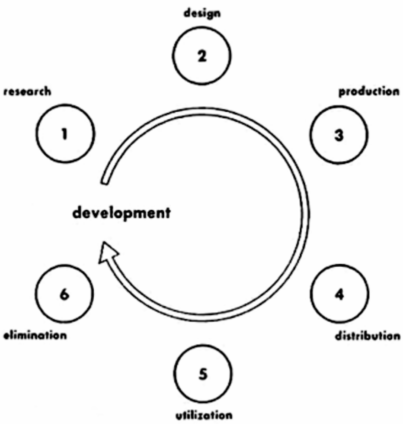
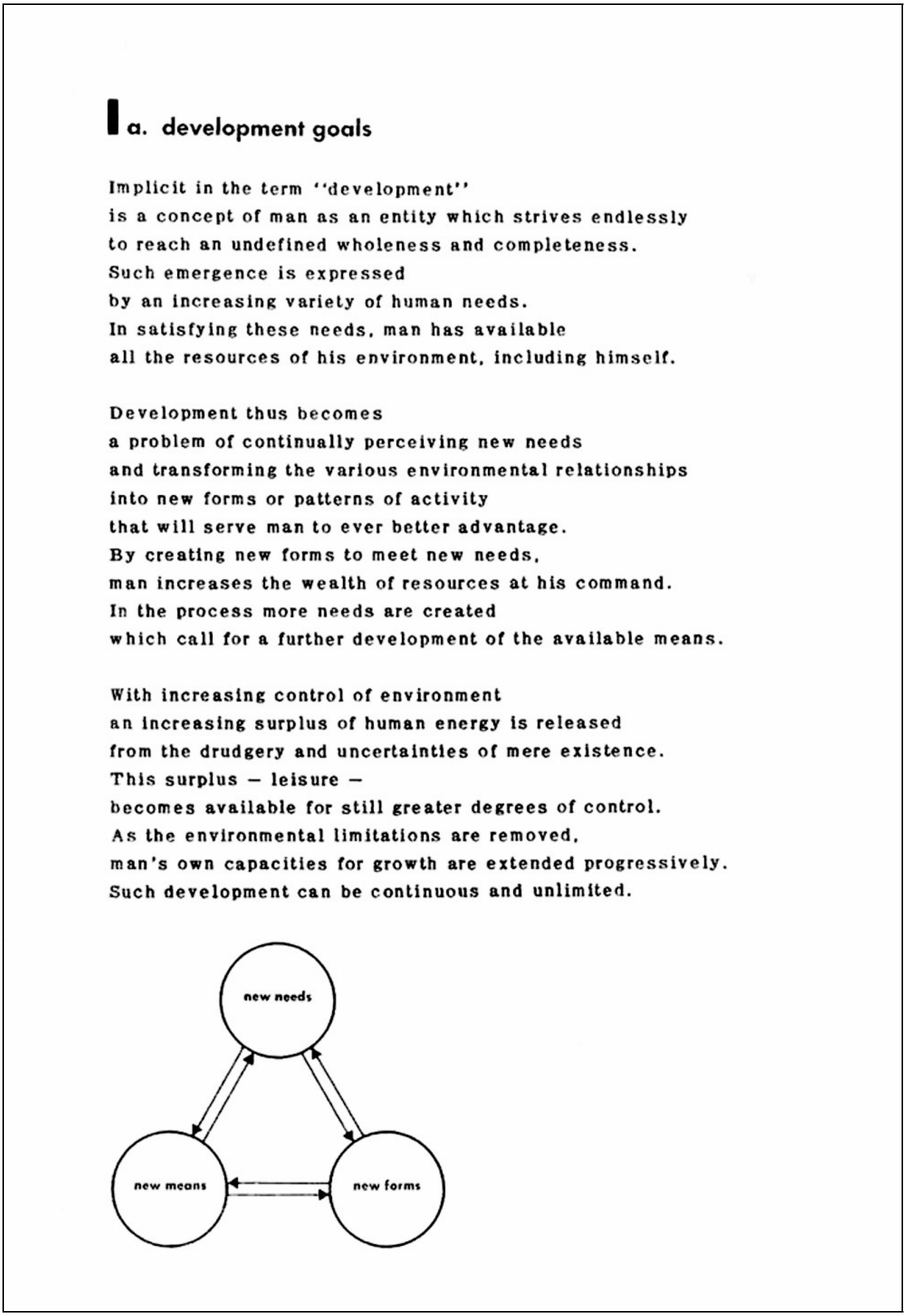


Fig. 3. "Development cycle" as part of the chapter on "Man and Development" in Lönberg-Holm and Larson, *Development Index* [1953]



I a. development goals

Implicit in the term "development"
is a concept of man as an entity which strives endlessly
to reach an undefined wholeness and completeness.
Such emergence is expressed
by an increasing variety of human needs.
In satisfying these needs, man has available
all the resources of his environment, including himself.

Development thus becomes
a problem of continually perceiving new needs
and transforming the various environmental relationships
into new forms or patterns of activity
that will serve man to ever better advantage.
By creating new forms to meet new needs,
man increases the wealth of resources at his command.
In the process more needs are created
which call for a further development of the available means.

With increasing control of environment
an increasing surplus of human energy is released
from the drudgery and uncertainties of mere existence.
This surplus — leisure —
becomes available for still greater degrees of control.
As the environmental limitations are removed,
man's own capacities for growth are extended progressively.
Such development can be continuous and unlimited.

Fig. 4. "Development goals" as part of the chapter on "Man and Development" in Lönberg-Holm and Larson, *Development Index* [1953]

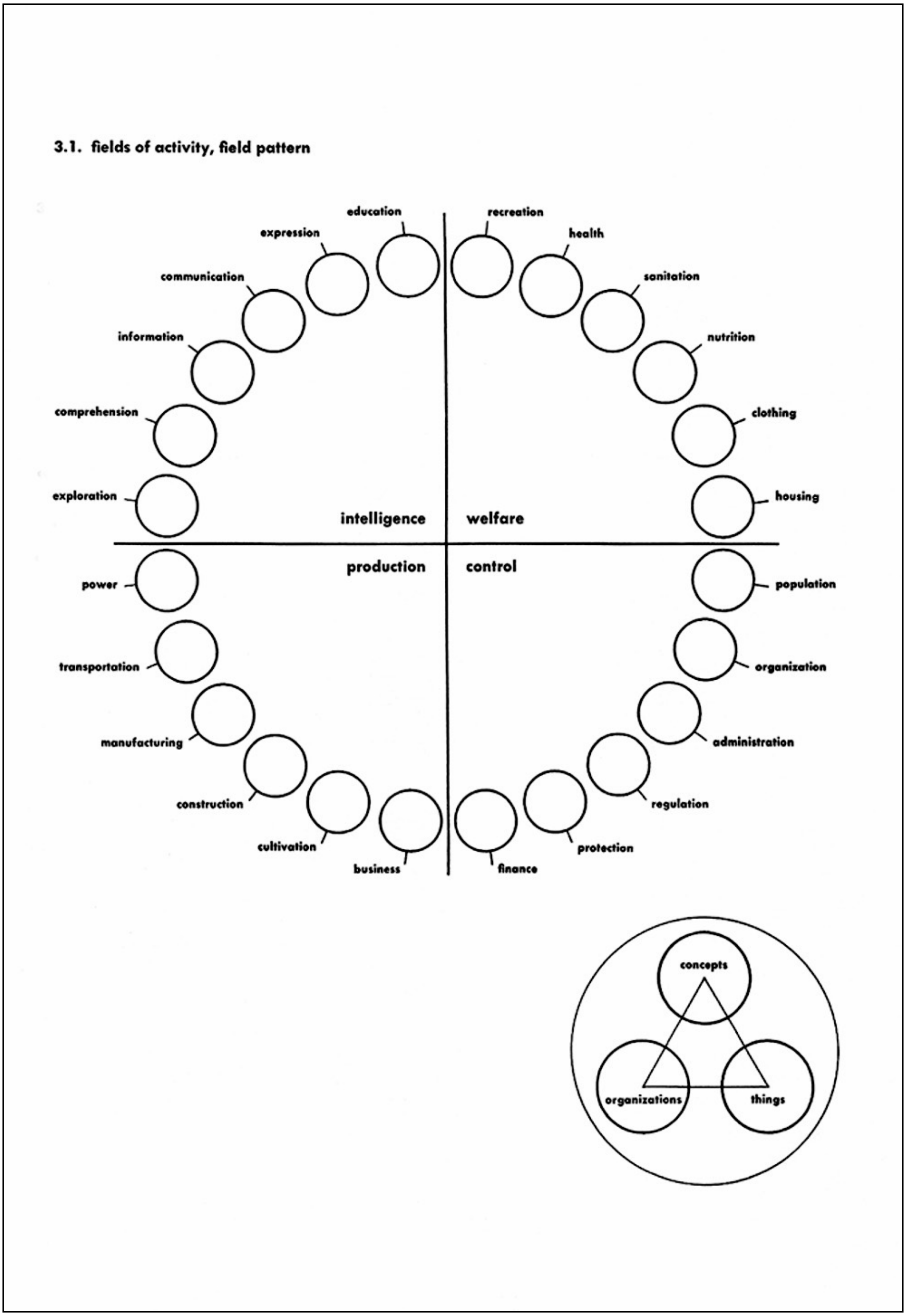


Fig. 5. "Fields of activity, field pattern", from *Development Index* [1953]

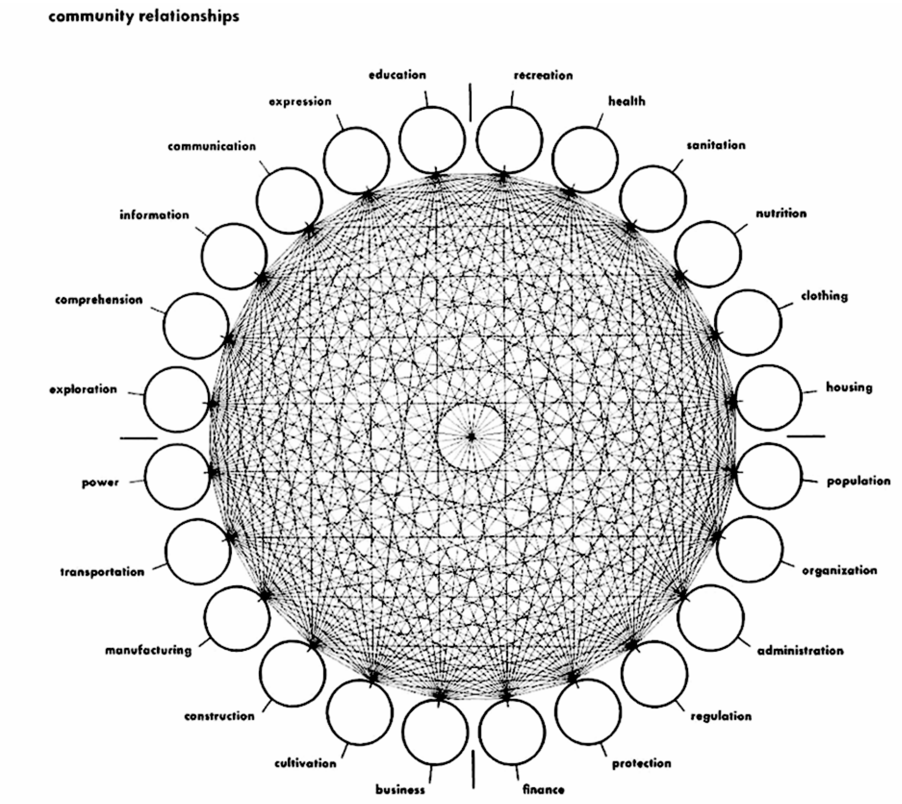


Fig. 6. "Community relationships" from *Development Index* [1953]

Development Index offered a complex relational system; an attempt to identify and organize into operational unity all the factors to be considered in the development of forms and patterns that will further man's growth. It was meant to act as a twofold screening system for managing information flow and servicing relevant data, where a central collection and dissemination department would coordinate the organization of development information, alongside of decentralized units. This would make use of state-of-the-art media such as microfilm, microprints, and electronics.

World Indexing Projects

Planning for Productivity and *Development Index* are essentially diagrammatic organizational outlines for modern knowledge apparatuses that bridge the pre- and post-war years. These projects were not inconsistent with the context of CIAM, and the organization's attempt to distinguish itself from existing planning institutions and define an international role in planning. In this context, Lönberg-Holm offered a global specialized information system. The comprehensiveness of the outline and the compression of the texts drew on his experiences in CIAM's organizational culture of questionnaires and checklists to establish broad thematics. He had been promoting the idea of an institution to centralize information about building and aid in architectural research since 1929 and he now saw it as essential within the context of CIAM's post-war reconstruction efforts. and new planning developments.

Lönberg-Holm and Larson's schematic formulation to create a constantly evolving database of all information related to environmental controls is also in line with the ambitions of World Encyclopedic projects. Modern era internationalization had truly begun with transportation paradigms – global networked infrastructures such as railroads, steam ships and telegraphy – in the mid-nineteenth century. The first decade of the twentieth century held the promise of Nikola Tesla's World System for wireless telegraphy (1910) and the creation of norms and standard arrangements, ranging from the first international meeting for world traffic conventions, held in Rome 1908, to a World Palace to house all of the knowledge in the world. Originally, international organizations before the League of Nations dealt with technological interdependence and were organized by engineers. Concern for the unification and standardization of knowledge was theorized by proponents of newly-established world organizations. Such *World Projects* made an inflationary use of their prefix and expressed a totality of range, taking the scope of the projects to the maximum, which therein held the seeds of inevitable incompleteness.

The advocates overlapped around 1930. A World Encyclopedia was proposed by Paul Otlet but also by Otto Neurath, father of ISOTYPE , and again by H. G. Wells, who called such a mechanism for the systematic ordering of human thought, the "World Brain" (1938). These projectors imagined an apparatus capable of receiving, sorting, summarizing, digesting, indexing and networking primary materials. Lönberg-Holm was familiar with Otto Neurath and the "Mundaneum" from CIAM. *Shelter* magazine had published texts by H.G.Wells, as well as C.K. Ogden's *Basic English*.

For his "Mundaneum", the unrealized architectural embodiment of an all-world knowledge database designed by Le Corbusier, Otlet developed the first modern systematic organization of information, based on his own universal decimal system and a mechanical data base that resulted in twelve million cross-referenced 3 x 5 index cards.[18] In his 1933 futuristic novel *The Shape of Things to Come*, H. G. Wells envisioned a central collection bureau located in Barcelona where all forms of knowledge would be snipped and edited by experts into an international information mechanism. Wells's project for a World Brain was to be controlled by a World State and proposed the editing, distilling, sorting and collecting of data, with totalitarian overtones.[19]

Ogden's Basic English would be the lingua franca of the new ruling elites. Charles K. Orden (1889-1957) was a British linguist educated at Cambridge University and a disciple of Victoria Lady Welby. He began promoting Basic English in 1930, as part of International Language Reform, one of several World auxiliary languages such as Esperanto, conceived to promote world communication and universal understanding. Ogden took a technological approach to language and its operational context, applying functionalism. He reduced the 25,000 English words found in any standard dictionary into basic concepts that could be achieved with only 850 words, some of which acted as *accelerators, lubricants, accessories* or *gadgets*.. Ogden was influenced by the English philosopher of law and utilitarianism and the inventor of the panopticon, Jeremy Bentham (1748-1832), whose 1812 *Theory of Fictions* he published in 1932. Bentham had been the inventor of the terms *international, utilitarianism, minimize* and *maximize.* Ogden based his language on what he called the *panoptic affect*, meaning that the words could be understood at a glance. The language attracted the attention of world leaders after World War I, including Winston Churchill and Franklin Delano Roosevelt, as well as the SSA in *Shelter*. Lönberg-Holm wrote in Basic English, and so did Neurath.

On a more mundane level Wilhelm Ostwald formulated projects for a single currency or World Money, a World Auxiliary Language, Ido (a variant of Esperanto), the World Format, a universal system of paper sizes as well as a standardized color system. Neurath, the first non-architect member of CIAM, proposed ISOTYPE, a universal sign system as an international picture language, based on icons that could be understood across cultures.

But new technologies, communication and information theory emerging out of World War II consigned such paper-based World projects to oblivion and radically transformed data collection and storage media from newspaper clipping agencies and index card catalogues to memex, microfilm, incipient computerization and other electronic documents, as described by Vannever Bush in his expose "As We May Think" [1945], a seminal post-war text on the future of information processing. This represented a move from a modernist desire to centralize and monumentalize knowledge to a post-war conception of globalized networks.

Epitome of Navigation

On another level, Fuller's influence might also be felt in "*Planning for Productivity"* and *"Development Index",* based on his involvement in systems theory in the post-war period. Fuller had conceived of *Shelter* as the standard reference on par with the American Practical Navigator, a manual for mariners. Initially published in 1803 and continually updated since, the book was authored by Nathaniel Bowditch, a self taught mathematician, a student of celestial bodies and a Unitarian seaman from Salem Massachusetts. "Bowditch", or the "Epitome of Navigation" as it was known, could be found on every US Navy vessel and was the western hemisphere's shipping industry standard for more than 150 years. The original text included several novel solutions to the spherical triangle problem, as well as extensive formulae and tables for navigation. Fuller's experience in the Navy was decisive in shaping his world view and gave him the experience of working within a closed system that was capable of processing and deploying vast quantities of information, resources and new technologies. Maritime standard manuals such as "Bowditch" emphasized the gathering and interpretation of positioning data as essential to the seafarer.

Indeed, for Fuller and any sailor, the world itself was a complex information system, and the "Comprehensive Anticipatory Designer" was a nomadic roamer who could process information from industry and other technocratic sources while remaining an outsider. Such an elite figure would be "a Harvester of potentials" peripheral to rationalized bureaucratic industrial systems, yet able to analyze and deploy the products of technocracy to serve the world's needs. The Geoscope and World Game of the 1960s were concerned with whole systems analysis and continually updated information exchange.

On a more personal level, Fuller's Chronofile, begun in 1915 with closure upon his death, is a chronologically organized archive of Guinea Pig B's personal development correlated to major world events and to scientific and technological inventions. The Chronofile and the Dymaxion Index were inspired by Fuller's Navy experience following World War I, when he had to compile secret records in chronological order, a problem of information indexing. Fuller, a proponent of lightness and ephemeralization, amassed a time capsule of private and world information that eventually weighed in at 45 tons or 90,000 pounds of paper, scraps and clippings that had to be sorted, indexed, bound and carted about throughout his nomadic entrepreneurial existence.

Role of the mass media of information

The more holistic outlook of *Development Index* had benefited from Lönberg-Holm's involvement in CIAM and the influence of the younger generation on community, collectivity, and the idea of Habitat as an environment to accommodate the total and harmonious, spiritual, intellectual and physical fulfilment of the inhabitants. It also carried forward the aims of CIAM to develop universal standards for community developments for the benefit of man. *Development Index* was informed by Larson's connection from the early 1950s on to Kenneth E. Boulding of the Department of Economics at the University of Michigan from 1949-1967, a leading figure in evolutionary economics and general systems theory, and the organizer of an interdisciplinary seminar on the Integration of the Social Sciences, which included a session on the "Theory of Information and Communication".[20] Through this affiliation, Larson may have been first introduced to "General Systems Theory" by Ludwig von Bertalanaffy and Claude Shannon's "Communication Theory".[21]

The final incarnation of this unusual collaborative effort in information indexing was updated in 1972 due to new advances in technologies. "Role of the Mass Media of Information" [Larson and Lönberg-Holm 1972], published by Martinus Nijhoff in the European Cultural Foundation's Plan Europe 2000, was one of seventeen prospective studies for *The Future is Tomorrow*. The essay cited Claude Shannon and Marshall McLuhan projecting the beneficial role that globalized mass media could have including possibilities of mass education through distance learning via television and computers, and advances in art such as Robert Rauschenberg's collaboration with MIT's Experiments in Art and Technology. It explored the obsolescence of paper-based and microfilm systems for the computer, this time evoking Teilhard de Chardin's notion of super intelligence arising from accelerated technological progress.

Lönberg-Holm and Larson's ongoing collaboration in information systems, for the coordinated deployment of up-to-date data essential to community design are case studies that propagate a notion of building as environmental controls, conceived as the deliberate organization of the processes of life where building ceases to be monument and becomes instrument. Surprisingly, their interest in systems and information emerged within the context of the Great Depression and developed throughout World War II. They represent an anomalous and extreme position that originated in the 1930s with their collaboration with Buckminster Fuller in *Shelter,* one that has been obscured by the predominance of Hitchcock and Johnson's streamlined account of the architecture movements of the time.

Notes

1. The SSA was formed by Knud Lönberg-Holm, Carl Theodore Larson, Douglass Haskell, Charter Secretary of AUDAC and the Nation's architectural critic, a contributor to the *New Republic* and *Architectural Record*; *Architectural Forum* editor Roger Sherman; Peter Stone of *American Contractor*, formerly of *General Building Contractor*; the practitioners Henry Churchill and Simon Brienes; Frederick Kiesler; the Austrian born theatrical designer who Fuller knew from avant-garde circles in Greenwich Village; Eugene Schoen, founder of AUDAC and creator of Macy's interiors; Howard Robertson of the Architectural Association London; Henry Wright, community planner, War Department and City Housing Corporation, NYC, Radburn and Buhl Foundation, connected to Lewis Mumford; Dr. Alvin Johnson, political economist, editor of the *New Republic*, and president of the New School of Social Research; William Adams Delano, a Columbia professor and designer of public buildings in Washington; Electus Litchfield, an architect of public housing; A. Lawrence

Kocher, director of *Architectural Record*'s editorial policy and designer of Williamsburg, Virginia; his partner, Swiss-born architect Albert Frey; Maxwell Levinson, technical editor of *Shelter*, and his brother Leon Levinson, managing editor.

2. For a further examination of such knowledge indexing projects, see [Rayward 2008] and [Krajewski 2006].
3. See [SSA (Anonymous) 1932]. Fuller's description of the tower states:

   > There could be mechanical hook-ups of industrial unit production headquarters by teletype, telephoto and television with central publishing headquarters of industrial units, who in turn would be tactically hooked up in like manner with information sources such as Bureaus of Standards, Navigation, Department of Commerce, etcetera or corporations such as Standard Statistics, Consumers' Research, Science News Service, etcetera, as well as university hook-ups [SSA 1932: 64].

4. As George Howe removed himself from *Shelter* magazine, Maxwell Levinson, the technical editor sought financing. Buckminster Fuller cashed out his life insurance policy to be able to take over the magazine, but by then Levinson had already procured financing from Philip Johnson as well.
5. See [Marx 1994]. In this essay, Marx discusses the "invention" of the term technology derived from the Greek *techne*, meaning "art" or "craft" and its availability since the seventeenth century. At the turn of the century, Thorstein Veblen would introduce the term "machine technology" into modern discourse in the United States. Leo Marx also traces the assimilation of the word into general usage in the 1930s.
6. Organized by Howard Scott, the 1918 think tank Technical Alliance was commissioned by the International Workers of the World to conduct an energy resource survey of the United States with the aim of industrial restructuring. The engineer Charles Proteus Steinmetz and Thorstein Veblen were listed as members of the coterie, along with economist Stuart Chase, the New York architects Frederick Lee Ackerman, Robert Kohn, and Charles H. Whitaker, and the forester Benton MacKaye (all founding members of the Regional Planning Association of America), and engineer Bassett Jones, among others. The group dissolved in 1921, without ever publishing findings.
7. Howard Scott, Harold Loeb, and Walter Rautenstrauch, a Columbia professor of industrial engineering, all formed splinter groups: Technocracy Incorporated, Committee on Technocracy, and Continental Committee on Technocracy, respectively. At the onset of the Depression they received much attention in the media, until Howard Scott's professional credentials were called into question
8. Tosten Bunde Veblen (1857-1929) was born in Wisconsin to Norwegian immigrant parents. He became Thorstein Veblen, an economist who applied Charles Darwin's Theory of Evolution to the analysis of modern industrial systems. He was the founder of institutional economics in America; John Kenneth Galbraith, among others, is considered to be a later disciple. Writing at the time of the creation of mergers and trusts among American corporations, he became one of the first critics of finance capitalism and investment banking manipulations. His work studied the dynamics of market competition and processes and sought to overcome material scarcity. See [Veblen 2006].
9. His unsubmitted Chicago Tribune competition entry, 1922 was influential in Europe, published by Le Corbusier in the *Almanach d'architecture moderne* (1925) where it was juxtaposed with towers by Mies van der Rohe and with a residential skyscraper by Auguste Perret. It also appeared in Walter Gropius's article in *Internationale Architecktur Bauhausbücher*, no1 (Munich, 1925).
10. For a further explanation of Lönberg-Holm's images included in Mendelsohn's *Amerika* see [Cohen 1995, 1992]. In this French edition all of Lönberg-Holm's photographs are identified.
11. The ABC group included pro-communist architects from Basel: Hannes Meyer, Hans Wittwer, Emil Roth, Hans Schmidt and Paul Artaria and the Zurich architects Werner Moser, Max Ernst Haefeli and Rudolf Steiger. The group produced a journal from 1924 to 1928. Of the eight issues, the first and second served as tributes to Lissitzky. For a detailed study of the ABC group's publication and built work see [Ingberman 1994].

12. Lönberg-Holm's articles for the Technical News and Research Section of *Architectural Record* iIncluded the following articles: "New Theatres in Europe" (May 1930: 490-496); "The Gasoline Filling and Service Station" (June 1930: 561-584); "Heating, Cooling and Ventilating the Theatre" (July 1930: 93-94); "The Week-end House" (August 1930: 175-192; "Glass" (October 1930: 327-358); "Recent Technical Developments; Reducing dead load, saving time and increasing control" (December 1930: 473-482); "Planning the Retail Store" (June 1931: 495-514); "Trends in Lighting" (October 1931: 279-302); "Technical Developments" (January 1932: 59-72); "City Planning, Survey of Detroit, Michigan with Otto Sen and S. Washizuka" (March 1933: 148-149).
13. *Sweets Catalog*, an interface between architects, builders and manufacturers, has been taking up an enormous amount of shelf space since 1906. It still exists as a voluminous compendium of trade catalogs by fabricators of building material, currently used monthly by 200,000 American construction professionals in the field – architects, engineers and contractors – and is organized according to the same categories used in the writing of architectural specifications. *Sweets*, a clearing house for standardized industrial building parts and a binder for collecting and classifying commercial trade catalogues of products, was made available to architects just at the moment that the building industry in the United States was converting from small-scale localized trades to national business structures based on advertising and consumer culture.
14. The most comprehensive source of information about Sutnar, both his early career in Prague and his emigre years in New York, can be found in the exhibition catalogue [Janáková 2003].
15. See [Lönberg-Holm 1967]. This was originally written in 1929 for *Architectural Record* but rejected. The article itself gives an explanation of the protracted publication time.
16. This is revealed in a letter from Lönberg-Holm to Larson dated April 22, 1965 in the Larson Papers.
17. The International Industrial Relations Organization lasted from 1928-1948 and its aim was to improve the life of workers. Van Kleek emerged as an early reformer for women workers. She visited the Soviet Union in 1932 and advocated social and economic planning.
18. Paul Otlet (1868-1944) was a Belgian lawyer and a pioneer in information science and documentation. He created the universal decimal classification system. As a visionary he conceived a number of World projects including the *Palais Mondial* or World Palace (later called the Mundaneum) and the *Cité Mondial* or World City, which would house all of the world's international organizations. Otlet promoted the global diffusion of information and experimented with new media such as microfilm developed in coordination with engineer Robert Goldschmidt. After World War II Otlet and his work faded into oblivion, to be recently resurrected as a precursor to the World Wide Web.
19. H. G. Wells (1866-1946) was a prolific writer known for science fiction and utopian novels and a member of the socialist Fabian society. *The Shape of Things to Come*, 1933 envisioned the world being taken over by a world council of scientists. He developed the idea of the World Brain in *The Idea of a Permanent World Encyclopedia* in 1938. See [Rayward 1999].
20. From a prospectus in the Larson Papers (1954 March 24). The first seminar dealt with a theory of competition and cooperation and brought to together academics in ecology, forestry, psychology, social psychology, group dynamics, sociology and economics; the second covered theory of individual behaviour; the third general theory of growth; the fourth dealt with theory of Information and communication.
21. In the early 1960s they collaborated on a project for windowless classrooms with the anthropologist Edward T. Hall, author of the *Silent Language* and the *Hidden Dimension*. The project was directed by Larson as a collaboration between the Department of Architecture at the University of Michigan and the Ford Foundation's Educational Facilities Lab, Inc. It was an interdisciplinary collaboration that involved insight from psychologist, sociologists and others (from a Memo of the Department of Architecture March 1962, in the Larson Papers). Larson also corresponded with Hall regarding his work on " A Systems Approach to the Performance Concept in Building" (from a Letter dated February 21, 1966 from Larson to Edward T. Hall in the Larson Papers).

References

BUSH, Vannevar. 1945. As We May Think. In *Atlantic Monthly* (July 1945). Available at: http://www.the atlantic.com/doc/194507/bush. Last accessed 3 October 2011.

COHEN, Jean Louis. 1992. Postface. In Erich Mendelsohn, *Amerika: livre d'image dún architecte.* Marianne Brausch, trans. France: Les Editions du Demi-Cercle.

———. 1995. The Moderns Discover America: Mendelsohn, Neutra, and Maiakovsky. Pp. 85-105 in *Scenes of the World to Come: European Architecture and the American Challenge.* Paris: Flammarion.

INGBERMAN, Sima. 1994. *ABC: International Constructivist Architecture, 1922-1939.* Cambridge MA: MIT Press.

HEAP, Jane. 1927. The Machine Age Exposition. *The Little Review* 12, 1 Spring Supplement.

JANÁKOVÁ, Helena, ed. 2003. *Ladislav Sutnar: Prague, New York, Design in Action*. Prague: Museum of Decorative Arts In Prague, 2003

KRAJEWSKI, Markus. 2006. *Restlosigkeit: Weltprojekte um 1900* (World Projects around 1900). Franfurt am Main: Fischer Taschenbuch Verlag (in German).

LARSON PAPERS. C. Theodore Larson papers 1930-1985 (bulk 1951-1974). Bentley Historical Library, University of Michigan. http://quod.lib.umich.edu/b/bhlead/umich-bhl-9438?rgn=main;view=text.

LARSON, Carl Theodore and Knud LÖNBERG-HOLM. 1972. Role of Mass Media of Information and Communication. Pp 543-585 in *Future is Tomorrow*. The Hague: Martinus Nijhoff.

LÖNBERG-HOLM, Knud. 1928. Reflections: America. *i10 International Revue* 15, 1928: 49-55.

———. 1924. Amerika. *ABC Beiträge zum Bauen* 1.

———. 1932. Monuments and Instruments. *Shelter: A Correlating Medium for the Forces of Architecture* 2, 4 (May 1932): 5-7.

———. 1967. Architecture in the Industrial Age. *Arts and Architecture*, April 1967: 22.

LÖNBERG-HOLM, Knud and Carl Theodore LARSON. 1936a. Design for Enviornmental Controls. *Architectural Record,* August 1936: 154-159.

———. 1936b. The Technician on the Cultural Front. *Architectural Record,* December 1936: 472-473.

———. 1940. *Planning for Productivity*. The Hague: International Industrial Relations Institute.

———. 1953. *Development Index: A proposed pattern for organizing and facilitating the flow of information needed by man to further in his own development with particular reference to the development of building, communities and other forms of environmental controls.* Ann Arbor: University of Michigan Press.

MARX, Leo. 1994. The Idea of "Technology" and Postmodern Pessimism. Pp. 237-258 in *Does Technology Drive History*, Merrit Roe Smith and Leo Marx, eds. Cambridge, MA: MIT Press.

MENDELSOHN, Erich. 1926. *Amerika, Biderbuch eines Architekten*. Berlin: Rudolf Mosse.

RAYWARD, W. Boyd. 1999. H.G. Wells' Idea of a World Brain: A Critical Re-assessment. In *The American Society of Information Science* 50 (May 15, 1999): 557-579.

———, ed. 2008. *European Modernism and the Information Society: Informing the Present, Understanding the Past.* Farnham: Ashgate, 2008.

SSA (Anonymous). 1932. Conning Tower. *Shelter: A Correlating Medium for the Forces of Architecture* 2, 5 (November 1932): 64-65.

TECHNOCRACY, Inc. 1934. *Technocracy Study Course: An Outline of Those Elements of Science and Technology Essential to Understanding Our Social Mechanism.* New York: Technocracy Inc.

VEBLEN, Thorstein. 2006. *The Engineers and the Price System* (1921). New York: Cosimo Classics.

About the author

Suzanne Strum has a Ph.D. in Theory and History of Architecture from the Universitat Politècnica de Catalunya UPC. She is a graduate of the Columbia University Graduate School of Architecture, Planning and Preservation in New York. She also holds a Master's from the Metropolis Program in

Architecture and Urban Culture, UPC, where she is currently Head of Studies. She is a professor at Elisava Design School, Pompeu Fabra University and adjunct professor for Texas A&M University´s Barcelona program, as well as collaborating in other international architecture programs. She is the author of *Barcelona: A Guide to Recent Architecture* (Ellipsis/Chrysalis, 2002) and co-editor of *Metropolis: Ciudades, Redes, Paisaje* (GG, 2004).

Marie-Pascale Corcuff

15 rue Yves Mayeuc
35000 Rennes
FRANCE
m-p.c@wanadoo.fr

Keywords: modularity, proportion, architectural design, generative design, Le Corbusier, Hans van der Laan, golden number, plastic number

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010

Research

Modularity and Proportions in Architecture and their Relevance to a Generative Approach to Architectural Design

Abstract. Modularity and proportions have been at stake in architecture since at least the «orders» in classical antiquity, throughout centuries, and well into our time, as highlighted by architects Rudolph M. Schindler, Hans Van der Laan, and Le Corbusier. Not all architectural modular and proportioning systems are founded on the same assumptions. Some systems, which are truly modular, are based on a unit, and deal only with integer multiples and dividers of this unit (and therefore rational ratios), others imply an irrational division of a chosen length. While both are based on proportions, music and architecture do not relate in the same way to space and time. They involve different issues regarding physics, perception and dimensions. The generative approach to architectural design means the use of transformation rules, which often involve ratios. The use of transformation rules, which relates art to the way nature generates forms, permits events like "hybridization": how may those operations contribute to a search for "resonance"?

1 Modularity and proportions in architecture

During the twentieth century, the most important modular and/or proportional systems that have been invented are those of architects Rudolf M. Schindler (1887-1953), Le Corbusier (1887-1967), and Hans van der Laan (1904-1991). Though born very nearly at the same time, these three architects have known very different destinies and reputations, as well as their systems which are called, respectively, the *Schindler Frame*, the *Modulor* and the *Plastic number*. Though his book, *Le nombre plastique* [Van der Laan 1960] was published in 1960, van der Laan worked as early as 1930 on his system; Le Corbusier invented the *Modulor* around 1943, and Schindler first published his frame in 1945 [Schindler 1947]. We know that Schindler knew about, and was opposed to, Le Corbusier's *Modulor*, but we don't know if Le Corbusier knew about Schindler's system, nor if van der Laan knew either one. Dom Hans van der Laan, being a monk as well as an architect, and much less famous (and having a less extensive body of work, consisting only of a few religious buildings for his community) was probably not known by the two other architects. The three systems will not be exhaustively examined here; to find more developed explanations one can refer to the bibliography.

1.1 Measure

When devising a modular or proportional system, architects do not deal with *space* as such, or even with *size* (or *magnitude*) as perceived, but with *measure*. Le Corbusier emphasizes this term: *L'homme mis ici en question est architecte et peintre, pratiquant depuis quarante-cinq années un art où* tout est mesure ("The man here in question is an architect and a painter, who for the past forty-five years has practised an art in which *all is measure*"; Le Corbusier refers to himself) [Le Corbusier 1951: 25; 1954: 25], and van

DOI 10.1007/s00004-011-0097-x ; *published online* 13 January 2012

der Laan insists upon that too: *L'architecte, nul n'en disconviendra, est un homme sans cesse occupé de mesures et de nombres. Ses dessins sont couverts de chiffres, et la règle graduée est toujours à portée de sa main* ("The architect, no one will disagree, is a man constantly taken up with measures and numbers. His drawings are covered with numbers, and the ruler is always within his reach") [Van der Laan 1960: 1]. This latter statement leads us to question that of Le Corbusier's because most painters are surely not as obsessed with measures, or anyway with numbers, as architects are. Obviously, one does not need a ruler to draw something which has got a measure. Anything we draw has a measure, even if we don't know its value. Sculptors and painters can work without using numbers, and we can still say that in their work *all is measure*, but only in the sense that everything they do has a size, and that those different sizes, being seen together, infer relationships, and proportions.

But architecture has to be drawn before being built, and those drawings will be used by workers to do the actual building, measures will be taken by them on these documents in order to know which size they have to give to the actual elements of the building. Le Corbusier remarks that musical scales had to be invented in order to transmit, and therefore to write, music [1951: 15], and regrets that architecture has not done the same (in which he is wrong, because measuring units of length do exist, and have been used for a long time). Hans van der Laan insists a lot on the necessity to get and give measures all along the chain of transmission that starts from the drawing and finishes in the building [Van der Laan 1960: 2-5]. Architects have to think their design with numbers because they know that numbers will be necessary to build what they have conceived.

All measures are based upon a unit: measuring consists in counting how many units there are in some magnitude. What is measured in architecture is *length*, or distance, and there have been units of length for measuring from ancient times. We could even say that measuring any other phenomenon, at least in the first stages of measuring instruments, is done by referring it to a measure of length (as temperature is measured by the height of mercury in a tube). The measure of length is linked to the idea of number as such: we identify numbers with points on a straight line.

In most measuring systems, there is not only one unit, since this unit is divided, or multiplied, to furnish other units for other orders of magnitude. For instance, we measure time in seconds, minutes, hours, days, weeks, months, years. The numbers applied in these groupings are 60, 24 (or 12), 7, 30 (or 27, 28, 31), and 365 (or 366). The Imperial system uses at least three units, *inch*, *foot*, and *yard*, related by numbers 12 and 3. The metric system itself may be considered in such a way. The metre is divided into 10 decimetres and 10 metres make 1 decametre. Actually we don't use those units a lot, though centimetres, millimetres and kilometres are in common use. The great difference is that the divisions are all the same (10). The choice of 10 may be debatable, its major drawback being that it has only 2 and 5 as divisors (while 12 has got 2, 3, 4 and 6). You cannot express 1/3 meter as an integer number of centimetres, but one must add that you cannot either express 1/5 foot by an integer number of inches. And you cannot express 1/7, for instance, by an integer number of sub-units in either system (while for van der Laan, that ratio is fundamental). The great advantage of the metric system is that it is coherent with our way of writing numbers, based upon 10. That makes it possible to express the multiplications and divisions of units very simply by shifting the floating point (1 cm = 0.01 m, for instance). Why the decimal system of writing numbers has become predominant is a question that is largely beyond the scope of this paper, but that's a fact. Though the Imperial system is at core a duodecimal system, measures must be written with the 10 symbols of the decimal system.

1.2 Module

One of the issues of a modular system is to define a *new unit* (called *module*) for measuring lengths. Our three architects reject the metric system, even if they have to deal with it (and to express their new unit in it), but don't stick to the Imperial system either (Le Corbusier intended to reconcile both systems, without much success). The metre is considered as too abstract, because it is not based upon concrete measurements like old European systems, and the Imperial system. One may object to that assumption because, if it is true that the meter was defined at first as 1/10000000 of the distance between the North Pole and the Equator (which is actually not very easy to apprehend), it is easily and simply relied for instance to the common size of a human being (at least a male one) which can be expressed as 1.75 m (or 1¾ m if one wants to insist on the use of simple fractions). It is true that the names of units in other systems are more evocative, but when you are used to the metric system, you know as well what concretely represents 1 m, or 5 cm, or 1 km. Schindler proposed a module (new unit) of 4 ft (or 48 in). It was not maybe his preoccupation but one can remark that the common size of a (male American) human being is expressed as 1 ½ module (6 ft) in that case. The average size of American males is actually 1.75 m, but it is commonly given as 6 ft, not because of any excessive pride, but only because it is easily expressed in the Imperial system. Though his system is not modular, and does not define a new unit, Le Corbusier started from the height of a human being. He settled first on 1.75 m, but then chose 1.83 m, which corresponds again to 6 ft.

Having defined a unit or module, one has to decide how to divide, and possibly multiply it. What is at stake is the issue of orders of magnitude. In the metric system, one goes from one power of ten to the next, as has been illustrated in the film *Powers of ten* (Charles and Ray Eames, 1977). Though his system is not modular, van der Laan thought a lot about this question of orders of magnitude. He concludes that the ratio 1:7 is the one that leads from one order of magnitude to another. He could have deduced from that a modular system in which a unit would be divided into 7 sub-units, and so on, going from one power of seven to the next, but he went further to build a much more complicated sequence. Schindler defined a sub-unit of 4 in, which is 1/12 of his module; in that way he stuck to the tradition of the Imperial system.

A modular system is very close to any common measuring system; it is only based upon another unit. A modular system allows to express any magnitude as a multiple of this unit, or *module*. This leads to an arithmetical sequence of numbers. This module may be divided and multiplied to get a sequence of units: if the factor is always the same, this sequence of units is a geometrical sequence.

A modular system leads to a grid, which is a useful tool for architects. Le Corbusier's and van der Laan's systems do not allow to establish such a grid, which is maybe the major drawback of those systems.

1.3 Proportions

A measuring system, or a modular one, is an arithmetical sequence; it even can be assimilated to the sequence of integer numbers. This sequence leads to any rational number by dividing two of its elements. This allows to establish a proportion between two numbers, and even to link four numbers in a relationship ($a : b = c : d$). This is obviously very often used in the scaled drawings made by architects: from the point of view of proportions, the scaled drawing is similar to the actual plan of the building.

Weirdly, neither Le Corbusier nor van der Laan mentions the issue of scaled drawings. Their systems do not allow us to choose a scaling ratio very easily. One can hardly draw at a ratio belonging to the Modulor sequence, and if one chooses any arbitrary ratio (like 1:100) then the measurements on the drawing do not belong to the Modulor sequence... In the case of van der Laan, it is even more paradoxical, as he proposes a set of sticks for experimenting with his system, and those sticks are clearly not at an architectural scale.

Architects consider only lengths, as we said, but they look for a relationship between lengths, either in the same direction, or moreover in different directions and, more specifically, in the directions of the Cartesian coordinate system (length, width, height). Those relationships between numbers are proportions, or ratios.

Architects seem to like rational proportions, and even to prefer simple fractions. If that was understandable in old times, it is weird in an age of electronic devices. Moreover, if earlier architects (as well as painters) preferred some proportions, it is in a large part because those are easy to arrive at through a construction with compass and straightedge; that allowed them to include the irrational $\sqrt{2}$, which is very simple to construct (Palladio, for instance, proposed those proportions: 1:1, $\sqrt{2}$:1, 4:3, 3:2, 5:3, and 2:1). It is surprising that none of the three architects we consider mentions geometrical constructions. Or, to be more precise, Le Corbusier considers a geometrical construction, but he is looking for *le lieu de l'angle droit* ("the place of the right angle") where we can be certain he will not find it.

According to Lionel March [1993a], Schindler uses approximations of $\sqrt{2}$ like 7:5 ($=1.4 \approx 1.414\ldots$) or of $\sqrt{3}$ like 5:3 ($=1.666\ldots \approx 1.732\ldots$, which is actually closer to the golden mean...), but it is more in token of classical tradition than by resorting to a geometrical construction. He definitely prefers simple ratios among those that his module, divisible into 12 sub-units, permits.

Le Corbusier's and van der Laan's systems are not modular. They may be described as *sequences* of measures which are not arithmetic (as in a modular system); they could have been geometric sequences, though actually both architects settled on sequences of integer numbers which approximate such sequences. Their two systems are very much related, the *plastic number* being an equivalent in three dimensions of the *golden number* on which the *Modulor* is founded. Hans van der Laan and Le Corbusier both were looking for a "magic" number, an ideal proportion. Le Corbusier chose a well known one, φ, the "golden mean" (=1.61803...). Dom van der Laan, even if he never expressed it as such, relied on another one, the "plastic number", p (=1.32472...).

Before seeing how Le Corbusier and van der Laan actually built their systems, we shall describe the characteristics of those two numbers, which are very interesting from the point of view of mathematics. While φ and the Fibonacci sequence are common knowledge, p and the Padovan sequence (named after Richard Padovan, who ascribed it to van der Laan [Padovan 1994, 2002]) are less known; we will present them in parallel. φ verifies a relationship between the length and the width of a rectangle (a, b):

$$\frac{(a+b)}{b}=\frac{b}{a}$$, or, for a rectangle $(1, \varphi)$:

$$\frac{1+\varphi}{\varphi}=\frac{\varphi}{1}$$

$$1+\varphi=\varphi^2 \text{ or } \varphi^2-\varphi-1=0$$

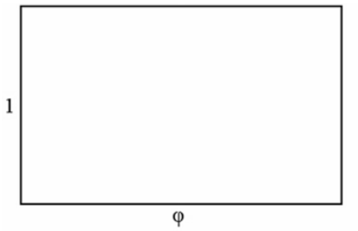


Fig. 1. The golden rectangle

p verifies a relationship between the length, the width, and the height of a parallelepiped (a, b, c):

$$\frac{(a+b)}{b}=\frac{c}{a} \text{ and } \frac{b}{a}=\frac{c}{b}$$

or, for a parallelepiped $(1, p, p^2)$:

$$\frac{(1+p)}{p}=\frac{p^2}{1}$$

$$1+p=p^3$$

or

$$p^3-p=0$$

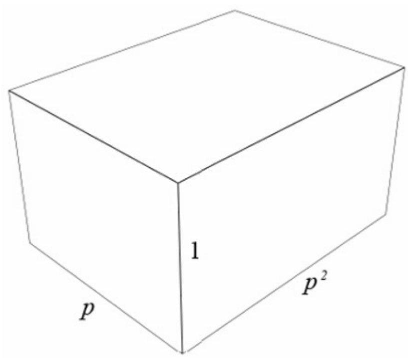


Fig. 2. The plastic parallelepiped

So, those numbers are zeros of equations $x^2-x-1=0$ and $x^3-x-1=0$, respectively; as such, they are called *morphic* numbers, and it has been demonstrated that they are the only ones [Aarts, Fokkink and Kruijtzer 2001].

$x^2-x-1=0$ is not very difficult to solve:

$\Delta=1+4=5$

and the roots of the equation are:

$$x=\left(1\pm\sqrt{5}\right)/2,$$

of which we take the positive one:

$$x=\left(1+\sqrt{5}\right)/2=1.61803\ldots\ .$$

$x^3-x-1=0$ is much more difficult to solve ...

In an equation of this type (x^3+px+q), we get one real solution (and two complex ones) only if

$$D=4p^3+27q^2>0\ .$$

In that case $D=23>0$ and then the solution is expressed by:

$$x = \sqrt[3]{\frac{-q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{\frac{-q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

$$x = 1.32472\ldots.$$

φ can be considered as the ratio of an irrational geometric sequence:

φ^0	φ^1	φ^2	φ^3	φ^4	φ^5	φ^6	φ^7	φ^8
1	1.61803…	2.61803…	4.23607…	6.8541…	11.0902…	17.9443…	29.0344…	46.9787…

As expected, $\varphi^2 - \varphi^1 = 1$ (and $\varphi^3 - \varphi^2 = \varphi$, and so on…).

In the same way, p can be considered as the ratio of an irrational geometric sequence:

p^0	p^1	p^2	p^3	p^4	p^5	p^6
1.0	1.32472…	1.75488…	2.32472…	3.07962…	4.07963…	5.40437…

Here, $p^3 - p^1 = 1$ (and $p^4 - p^2 = p$, p and so on…).

Those “geometric” sequences (of numbers) have their “geometrical” counterpart:

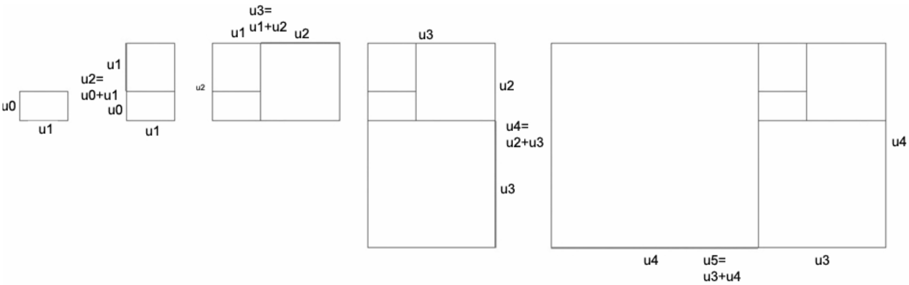


Fig. 3. Rectangle growing in a geometric progression

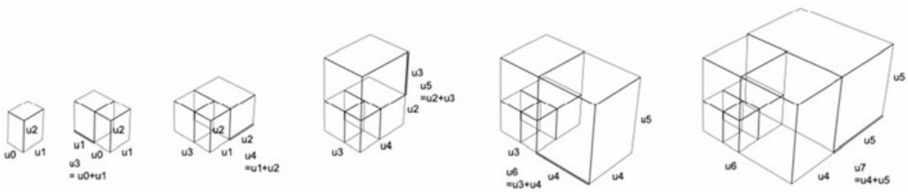


Fig. 4. Parallelepiped growing in a geometric progression

But φ and p are also the limits of the ratios of terms of integer sequences, called the Fibonacci sequence and the Padovan sequence, respectively.

Fibonacci sequence: $u_{n+1} = u_n + u_{n-1}$

0	1	1	2	3	5	8	13	21	34	55	89	…

The sequence of ratios of successive terms (rational numbers), tends towards irrational number φ = 1.61803...

1/1	2/1	3/2	5/3	8/5	13/8	21/13	34/21	55/34	89/55	…
1	2	1.5	1.666…	1.6	1.625	1.615…	1.619…	1.617…	1.618…	…

Padovan sequence: $u_{n+2} = u_n + u_{n-1}$

0	1	1	1	2	2	3	4	5	7	9	12	16	…

The sequence of ratios of successive terms (rational numbers), tends towards irrational number $p = 1.32472\ldots$.

1/1	1/1	2/1	2/2	3/2	4/3	5/4	7/5	9/7	12/9	16/12	…
1	1	2	1	1.5	1.333…	1.25	1.4	1.285…	1.333…	1.333…	…

Those sequences have also their geometrical counterpart.

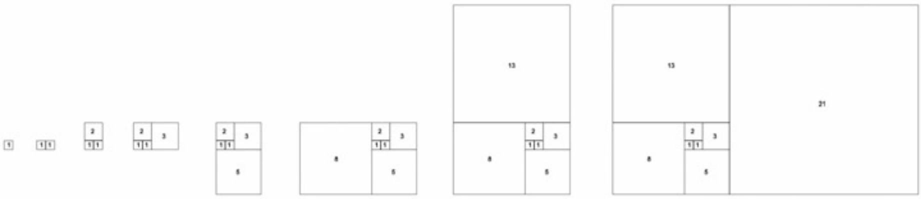


Fig. 5. Rectangle growing in a Fibonacci progression

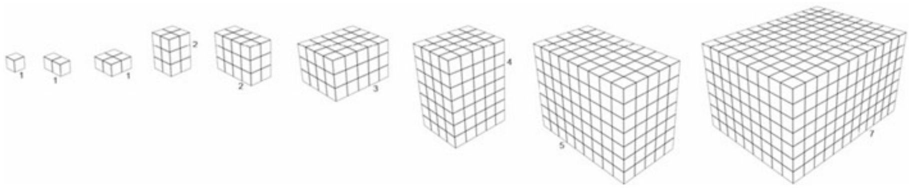


Fig. 6. Rectangle growing in a Padovan progression

Both irrationals numbers φ and p are approximated by such ratios: $\varphi \approx 8/5$, for instance, and $p \approx 4/3$. This is probably why those proportions may be "discovered" in drawings or designs from times when those numbers were not actually known.

In brief, we have two types of sequences:

- a geometric sequence of ratio φ (resp. p) the *irrational* terms of which obey $u_{n+1} = u_n + u_{n-1}$ (resp. $u_{n+2} = u_n + u_{n-1}$);
- an *integer* sequence the terms of which obey $u_{n+1} = u_n + u_{n-1}$ (resp. $u_{n+2} = u_n + u_{n-1}$), the *rational* ratios of successive terms tending to φ (resp. p).

Le Corbusier and van der Laan both had some difficulty trying to relate those integer and irrational sequences.

According to a famous drawing of the Modulor, Le Corbusier settles on this sequence (red series, in mm):

6	9	15	24	39	63	102	165	267	432	698	1130	1829

This is almost the Fibonacci sequence issued from 6 and 9:

6	9	15	24	39	63	102	165	267	432	**699**	**1131**	**1830**

However, he claims to use the golden section (φ) to make it.

Van der Laan settles on this sequence (divided in three groups):

2	2.5	3.5	4.5	6	8	10.5	14
14	18.5	24.5	32.5	43	57	75.5	100
100	132.5	175.5	232.5	308	408	540.5	716

This is actually the Padovan sequence issued from 4, 5 and 9 (whose terms have been divided by 2):

4	.5	7	9	12	16	21	28
28	37	49	65	86	114	151	200
200	265	351	465	616	816	1081	1432

He claims to find constant ratios, like:

$$2.5/2 = 18.5/14 = 132.5/100 = 4/3$$

which is obviously wrong, as:

$$2.5/2 = 1.25$$

$$18.5/14 = 1.321...$$

$$132.5/100 = 1.325...$$

He also claims that his numbers are in a geometric progression [Van der Laan 1960: 26, 81], which is again obviously wrong.

The proprieties of the golden mean are well known, and its use widespread, although perhaps not to as large an extent as some authors would like. The Fibonacci sequence is common knowledge too. Those were more or less known by Le Corbusier, notwithstanding his mistakes regarding mathematics, and even geometry.

On the other hand, the so-called plastic number and the Padovan sequence are much more esoteric. Van der Laan did not know in advance that some number would have the proprieties of the plastic number. He doesn't even express this number as such. Neither is he aware that he is building a Padovan sequence. He builds his sequence intuitively, by first establishing a fundamental ratio, 7:1, chosen as being what constitutes a change of order of magnitude. Then he looks for intermediate values that have other ratios which he wants to introduce, involving the arithmetic mean and the geometric mean, which are 7:4, 7:3 and 4:3. This leads him, after a rather hard work, to insert 4/3, 7/4, 7/3, 3 and 4 between 1 and 7. As he has already settled on the number of the components of his initial sequence (eight, including first one 1 and last one 7), and even given them their names, he needs another intermediate number between 4 and 7 and settles on 21/4.

small element	great element	small piece	great piece	small part	great part	small whole	great whole
1	4/3	7/4	7/3	3	4	21/4	7

But then he reconsiders his work, and introduces some "margins", which leads him to this new sequence:

2	2.5	3.5	4.5	6	8	10.5	14,

which would be, divided by 2, and expressed in simple fractions:

1	5/4	7/4	9/4	3	4	21/4	7,

where we see that the "great element" and the "great piece" have changed. Which is a problem, because in the first stage, van der Laan had very clearly stated some fundamental relationships, such as the great element relates to the small element according to the fundamental ratio which is 4:3 [Van der Laan 1960: 66].

What is very weird is that van der Laan does not use the mechanism of a Padovan sequence ($u_{n+2} = u_n + u_{n-1}$) yet arrives at one all the same.

Another issue concerning those sequences is their absolute values. One must first remark that, their rejection of the metric system notwithstanding, both architects must express their measures in some conventional system and do that in the metric system. If Le Corbusier and van der Laan had settled on a geometric sequence, they could have chosen any initial number. But they established sequences of integer numbers (or a sequence of integer numbers divided by two in the case of van der Laan) and so they are not absolutely free in their choice.

Le Corbusier chose 1829 mm as a fundamental measure, from which the entire sequence is derived. By the way, had he been more feminist, he could have chosen 1618 mm, which is close to the average height of French women (1625 mm); he would then have introduced the golden number itself in his sequence. Anyway, the measure he chose is very abstract, as it is completely dependent on the choice of the metre itself. For van der Laan, an inconspicuous choice is made – *Commençons par le petit élément du système I et attribuons-lui la valeur 100* ("Let us start from the small element of system I and let us give it the value 100") [Van der Laan 1960: 76] – for which he never provides an explanation. One might see in this arbitrary choice an atavism of the metric system, but what is not so clear is what unit these numerical values are expressed in. He is not very interested in that point, as for him it is the ratios that matter. In his abacus, constituted of sticks, the manipulation of which is supposed to give the sensation of proportions, he must give concrete magnitudes, and those are different from the above sequence. He does not give details, but mentions 3.7 mm, 26.5 mm and 189.7 mm, the diameter of the cylindrical sticks being constant and equal to 10 mm [Van der Laan 1960: 91]. We can complete the sequences: a geometric sequence of ratio p and first element 3.7 indeed reaches 26.5. With rounded measurements (let us not forget that those are mm!), we obtain:

small element	great element	small piece	great piece	small part	great part	small whole	great whole
3.7	4.9	6.5	8.6	11.4	15.1	20	26.5
26.5	35.1	46.5	61.6	81.6	108.1	143.2	189.7

Obviously, there are only two systems in the abacus, because if we go on, we reach values which could not enter into a small box, but which begin to be interesting for architecture (rounded in mm):

190	251	333	441	584	774	1025	1358	
1358	1799	2383	3157	4182	5540	7340	9723	…

We find 1799 mm on our way, which could represent the height of a man, but in any case, those are not the same numbers than in the sequence mentioned above. It confirms that, in contrast to Le Corbusier, it is not the absolute value of measures that interests van der Laan.

2 Relationships between architecture and music

Much has been said about relationships between architecture and music. With few exceptions, these are the two most abstract arts, and don't have figurative aims, while for a long time painting and sculpture had to represent the real world. This lack of figurative aim, at least, relates music and architecture. This famous and often repeated quote, "Architecture is a frozen music", attributed to Goethe or von Schelling, refers to aesthetics, to a very subjective perception of music and architecture, and not to the poetics, the making of music and architecture, which is what we are more interested in here. What is at stake in music and architecture, what musicians and architects deal with, is what will be approached, especially concerning modules and proportions. The relationship of music and architecture to mathematics is the other, more positive way in which architecture and music are related.

2.1 Spatial magnitudes and sounds

Any physical phenomenon is perceived through our senses as a perceptive *continuum*. Henri Poincaré [1913] explains this through the example of the weight (as we feel it by hefting): suppose we make a continuous chain of weights, all of which are different, and suppose that we cannot distinguish one weight from another, though we can very easily distinguish between the first and the last ones of this chain. Poincaré then establishes, through the concept of *cut* (*coupure*) and by recursion, that such a continuum has a *dimension* (or *number of dimensions*). Weight, temperature, length, and so forth, are one-dimensional, because one point of the continuum cuts it, but colour, for instance, is three-dimensional, because we need a surface (itself two-dimensional) to cut it.

Poincaré intended to prove that space, as perceived by human beings, is three-dimensional. This is not an abstract quality of space; space, as perceived by other creatures, could be two-, three- or four-dimensional, or even higher. After all, some birds have four types of colour receptors, instead of our three, and perceive colour as four-dimensional. In his famous novel *Flatland* [2006] Edwin Abbott imagined creatures whose space was two-dimensional (and were very astounded by the visit of three-dimensional creatures).

Lengths, or distances, constitute a one-dimension space. Sound is a complex phenomenon which involves at least pitch, intensity and timbre, and most of the time it is not pure. But from this complexity, at least in a musical sound, we can extract a component that is the *pitch* and which also constitutes a one-dimension space. However, those two kinds of one-dimensional spaces are not configured in the same way. We cannot imagine a length, a distance, which would be less than nothing (less than zero, but let's remember that we are considering a phenomenon that is perceived, not measured), while, even if we cannot concretely encompass it, we can imagine a length longer than any other, or rather, we cannot imagine that there is a length which is the longest of all. Our intuitive image of the space of lengths is therefore a half-line. On the other hand, even if our audition limits our hearing to a part of the possible pitches, we can imagine sounds that are lower-pitched or higher-pitched than those we can hear. In addition, there is a very specific event in the space of pitches: some sounds seem very alike without being at the same pitch (the same note at different octaves). The one-dimensional space of pitches can then be represented by an helix with a vertical axis, the same notes being at the same angle but at different heights.

Spatial magnitudes, i.e., lengths and distances, are perceived through our visual and kinaesthetic senses, while sounds are perceived through our hearing sense. There is a relationship between sound and length, though: a vibrating cord of some length will make a sound that we perceive as being at the same pitch, an octave lower, than a cord of half its length. The terms "low" or "high" refer to spatial qualities too; they are not only metaphoric (as with "warm" or "cold" for colours) as we actually feel that a "higher" pitch comes from a higher place in our body.

The first stage of evaluating is comparing, as we do in the experiment proposed by Poincaré. We can say that something is longer that something else when we see them near each other and in the same direction, but otherwise we shall have to use another device to which both can be compared as, for instance, a string is used to compare the nearness of two boules to the jack in the game of *pétanque*. On the other hand, comparing two pitches is a very subjective task. There is no objective way, as in comparing two lengths, to prove that such a sound is higher-pitched than another. But it seems that most people can recognize the same note at two different octaves, and that there are some chords that sound harmonious for most ears.

Any continuum needs to be discretized to be worked with. We do this through language by giving names to some stages of a continuum, which is an intuitive way of *sampling*, of *scanning* (in the first meaning of this term, which comes from Latin *scandere*, and applies to verse) it, as we do by naming colours, for instance, and as has been done in music by choosing notes and giving them names:

C D E F G A B C …

or

do re mi fa sol la ti do …

Modular systems are attempts to discretize lengths in architecture. Le Corbusier clearly relies his research to musical notation at the beginning of *Le Modulor*:

> *Le son est un événement continu, conduisant sans rupture du grave à l'aigu. … Il fallait le représenter par des éléments saisissables, par conséquent découper le continu selon une certaine convention et en faire du gradué* ("Sound is a continuous phenomenon, an uninterrupted transition from low to high. … It was necessary to represent sound by elements which could be grasped, breaking up a continuous whole in accordance with a certain convention and making from it a series of progressions") [Le Corbusier 1951: 15; 1954: 15].

Referring to the establishment of the different scales up to the well-tempered one, Le Corbusier wonders: *Sait-on qu'en ce qui concerne les choses visuelles, les longueurs, nos civilisations n'ont pas encore franchi l'étape accomplie par la musique?* ("How many of us know that in the visual sphere – in the matter of *lengths* – our civilizations have not yet come to the stage they have reached in music?") [1951: 16; 1954: 16].

Defining units of measure is a way of giving names to specific lengths. Schindler defined his module (48″ or 4′), and its subunit (4″), naming, or rather, qualifying them respectively as *rhythm* and *texture*. Those names, like the names of measure units, refer to orders of magnitude, the elements of each order are not named (they are expressed as numbers).

Hans van der Laan does not refer much to music, but evokes "the musical scale which Plato explains in detail in the Timaeus", and remarks that:

> in music, however, nothing is fixed but the intervals, and every piece of music can be played at different pitches; the actual pitch of the notes must be established 'from outside' … As soon as one tone is fixed, so too are all the others, and all intermediate tones are false (from "Architectonic space", 1989, in [Ferlenga and Verde 2001: 196]).

He however went the furthest in an analogy with music by giving names to the *elements* of his systems (each system being numbered and corresponding to an order of magnitude), in a way very similar to the musical scale:

small element	great element	small piece	great piece	small part	great part	small whole	great whole		System II			
							=					
			System I				small element	great element	small piece	great piece	small part	...

It may be a coincidence that he uses the same number of distinct grades as in the (diatonic) scale, i.e., seven (the "great whole" being equal to the "small element" of the next system), or in the colours as defined by Newton. Hans van der Laan doesn't link his choice to either musical notes or colours. The seven notes of the diatonic scale are not completely arbitrary: fa – do – sol – re – la – mi – ti or F – C – G – D – A – E - B is a sequence of perfect fifths, considered the most consonant interval. But Newton's seven colours – red, orange, yellow, green, blue, indigo and violet – are more subjective and induced by the diatonic scale. Newton had first split the spectrum into five principal colours and inserted orange and indigo to be able to link colours and notes: violet/D – indigo/E – blue/F – green/G – yellow/A – orange/B – red/C. As we now know, any splitting of the spectrum into more than three primitive colours is completely subjective.

2.2 Modules, proportions, and musical scales

The issue of modules and proportions in architecture and their relationships to musical scales is a very crucial and controversial point. It questions the use of numbers in each discipline and the types of perception they involve.

Musical scales relate to acoustics, and therefore to frequencies. Frequencies, and the way we perceive them, lead to ratios, the most simple of them being 2:1. Two notes with fundamental frequencies in a ratio of 2:1 (or any power of two) are perceived as very similar. When frequency ratios are simple fractions (3:2, 5:4, etc.), the combination of the notes will sound *consonant*. When they are near to a simple fraction, but not exact, a physical phenomenon called "beating" is produced, and the combination is perceived as *dissonant*. So there are physical and perceptual reasons to use exact ratios in music. Those reasons may be in part cultural as well. When Europeans hear music from other parts of the world, they sometimes feel that it is not harmonious; and some contemporary works of music that explore other possibilities can hurt our ears. However, it is possible that we are simply not used to them.

Although sound is a vibration (or rather a combination of vibrations) which has some frequency, we are not aware of this when we hear it. The frequency is the scientific measure of the pitch, but what we hear in a very subjective way is the pitch. We don't *know* the ratio between two frequencies, we *feel* that the combination is consonant or dissonant. Moreover, a *geometric* sequence of frequencies (like the one that is used to tune a piano) is felt as an *arithmetic* sequence of intervals (we feel that there are equal intervals between two adjacent keys).

Concerning the perception of lengths, we could say that we can evaluate their proportions directly. When a length is 2/3 of another one, for instance, we are able to see that, at least if the smallest one is included in the largest. But proportions are most often used for lengths in different directions (typically for the length and the width of a room). In plan it is still possible to evaluate their ratio (especially if a grid is drawn that allows us to count the units in each direction), but when we are in that room, without any measuring device, and considering that what we see is in perspective, are we still able to evaluate their ratio? Some might say that, as with the pitches, we don't have to measure or even to evaluate, that "harmonious" proportions of lengths are *felt* in the same way as consonant chords. But there are actually no rational or physical groundings for such an affirmation. Palladio wrote:

> *Le proporzioni delle voci sono armonia delle orrecchie, cosi' quelle delle misure sono armonia degli occhi nostri, la quale secondo il suo costume sommamente diletta senza sapersi perche' fuori che da quelli che studiano di sapere le ragioni delle cose* ("Just as the proportions of voices are harmony to the ears, so those of measurement are harmony to the eyes, which according to their habit delights [in them] to a great degree, without it being known why, save by those who study to know the reasons of things") [Puppi 1988: 123].

Unfortunately it is not sure that any one knows.

This objection is no reason to exclude any attempt to use modules and ratios in architecture. The architect must decide on measurements and proportions, because, anyway, they *are* in his drawing, and then in his building, even if he does not think about them. If a modular system is established, which leads to a grid, it is obvious that simple ratios will appear. Lionel March sees in ratios chosen by Schindler a relation to musical ratios [March 1993b], and he is very convincing. But must it not happen as soon as simple ratios are chosen? Schindler was not the first to use modules and simple ratios, and even the "row" [Park 2006] and its correlation to musical harmonies is present in, for instance, Bragdon's theory [2010].

Le Corbusier and van der Laan opted for irrational ratios (though they did not settle on a geometric sequence, as seen before) without relationships to musical ratios. Is this a reason to reject their choice as contrary to harmony, as we cannot establish rationally the foundations of harmony in spatial magnitudes? Supporters of the golden mean (and there are many of them) will affirm that a golden rectangle is the most harmonious of all. Perhaps some research will prove that our brain is able to perceive that in such a rectangle the length is to the width as the sum of both to the length, or that if you draw a square on the smaller edge what remains is again a golden rectangle... But we could just as well promote $\sqrt{2}$:1 which has the great advantage that such a rectangle, when cut into two equal parts, leads to two rectangles of the same proportions: which is satisfying for the mind, and very useful, and has been used to define the ISO216 standard for paper size (A0, A1, A2, and so on).

2.3 The equivalent of counterpoint (*contrappunto*) in architecture

Any transposition from one form of art to another is in a great part arbitrary. Indeed, some artists claim to do such transpositions very naturally, and even to experience a kind of synaesthesia, among them Olivier Messiaen or Wassily Kandinsky. Those synaesthesiae mostly involve colours, sounds and graphemes (letters and numbers); there is no reference to a correspondence between musical sounds and proportions of a rectangle. Anyway, there is no need to invoke synaesthesia to look for equivalents between two forms of art. That is a legitimate concern, as any that can lead us to renew our conception.

In music, counterpoint is the relationship between two or more voices that are independent in contour and rhythm and are harmonically dependant. It involves the writing of musical “lines” that sound very different and move independently from each other but sound harmonious when played together. Through the simultaneity of two or more different lines, some harmonies may (and must) occur, though those “vertical” features are considered secondary and almost incidental in counterpoint.

Music occurs in time: a musical “line” is a sequence of notes that develops in time. One may notice that pitch and time are transcribed in musical notation into spatial positions: along a vertical axis for the pitch, and a horizontal one for the occurrence in time; this spatial representation is combined with an arbitrary code for the duration of the notes. As we saw for the continuum of pitches, time (also a one-dimensional space) is discretized into bars or “measures”, which are themselves divided into beats. As with pitches, this discretization is relative. There can be a metronome mark, but in most cases, music may be played at a slightly different tempo.

Architecture is essentially a distribution, a composition, of solids and voids in space. This distribution does not change in time, though we perceive it in time by moving inside and around it. Music can also be considered as a distribution or composition of solids and voids (sounds and silences, notes and rests), not in space (though it can also intervene), but in time. Architecture and music have therefore very different relationships to space and time.

One way to link them is to consider musical notation, and transpose time into a line in one direction of space. Some architectural features may then be considered as “lines”, equivalent to musical lines, even if those lines do not develop in time. A colonnade, a row of windows, or the vertical divisions in windows (as in the C*ouvent de la Tourette*, by Le Corbusier and musician-architect Iannis Xenakis), and so on, are opportunities of linear sequences, the terms of which can be related to pitch or rhythm. In that case, we might wonder how two or more such lines may be “played” together. Hans van der Laan developed four ways in which two measures of length can interact: they may be superposed or juxtaposed, and in each case they may be in the same orientation or in the opposite. This creates the potential for an approach towards one kind of architectural equivalent to counterpoint.

But we don’t want to restrict ourselves to linear occurrences in architecture, and other possibilities have to be explored. Time may be considered in terms of perception, or as the frame of a generative process.

3 Generative processes

For Philip Galanter,

> Generative art refers to any art practice where the artist creates a process, such as a set of natural language rules, a computer program, a machine, or

> other procedural invention, which is then set into motion with some degree of autonomy contributing to or resulting in a completed work of art (from http://www.philipgalanter.com/generative_art/index.html).

More precisely, generative art is based upon transformation processes, which include cellular automata, formal grammars, L-systems, fractal processes, iterated function systems (IFS), genetic algorithms, artificial life, and so on; what is interesting is that the same types of processes may be used to create music, architecture, or any other kind of art, as participants in Generative Art conferences (http://www.generativeart.com/) demonstrate each year. This is a new way to consider the relationship between music and architecture, without either of them being indebted to the other, as happens when musical ratios are borrowed by architecture. IFS have been chosen to illustrate this issue. In the world of generative processes, IFS are correlated with space, since they work with geometrical transformations, but musicians have used them too, to make music [Gogins 1991] as well as analyse it [Meloon and Sprott 1997].

3.1 Attractors and time

IFS have been introduced and theorized mainly by Michael Barnsley [1998], and are a very efficient way to generate fractals, or rather, self-similar forms. A simple example of IFS is the half-way transformation: one defines a reference point, and then imagines starting from any other point and going half way towards the reference point, and going half way again from this new place, and so on. The limit of the process is the reference point, which is no surprise, and is also a kind of illustration of one of Zeno's paradoxes. But now let's take two reference points, and, starting from any other point, let's *choose at random* one of the two reference points and go half way towards it, and do the same thing from this new place and so on. What is called the *attractor* of this new process is the segment between the two reference points; which is maybe a little more surprising. If we do the same with three reference points (again choosing at random among the reference points the one towards which we are going) we obtain a famous fractal figure, a Sierpinski triangle (or gasket); if the three points are not the vertices of an equilateral triangle, it may be an irregular one. We can carry on with more reference points, the result is that we get self-similar figures, some of them fractals, others which are not fractal but are anyway self-similar (like the segment or the filled square, if the reference points are the vertices of a square).

The key point of IFS is that fractals, or self-similar forms in general, are attractors of a set of transformations, applied iteratively. The use of randomness is not mandatory, it is only one way of writing an algorithm. One can also apply systematically all the transformations of the set on the result of the previous step. Starting from any set of pixels, if you apply systematically and iteratively the set of transformations that corresponds to the Sierpinski triangle, you will obtain the Sierpinski triangle. Any process may be described by an initial stage and transformation rules, but concerning IFS and attractors, the initial stage does not matter.

Time is an important dimension of generative processes. Time is discretized, and most generative processes work step by step. Ideally, time is considered infinite, though for obvious reasons we never let a process work infinitely. Some processes converge more or less rapidly towards an attractor, while other ones evolve erratically. IFS converge towards fractal attractors, as we said, but we can also use their first steps to see how an initial form (which in this case matters) is transformed. Each step makes its mark on the form. The result is the trace of all the steps of the process. It is a way to include time inside the design itself, and not only in the perception of it; dynamics is used in the process and the result reflects this dynamics. It is perhaps a new way to consider architecture as frozen music. The experiments shown in §3.3 below are based upon this hypothesis.

3.2 Self-similarity and proportions

Many kinds of transformations may be used in IFS, as long as they are "contracting". Mostly the transformations are scalings (homotheties) combined with rotations and translations. It is then crucial to choose ratios for the homotheties. Those ratios may be irrational (though they will be approximated in this case) or rational numbers. Those transformations reflect the self-similarity of the attractor.

Proportions have do to with geometric similarity: two forms are similar if they have the same proportions. Self-similarity is a stronger requirement than proportions: sides of a rectangle may be in a given proportion, but a self-similar form is composed of parts that are each similar to the whole form.

Van der Laan reminds us that symmetry in ancient times meant more than what we refer to nowadays [Van der Laan 1960: 8-15]. It did not refer only to the axial symmetry, nor even to the other symmetries between parts, but to a relationship, a ratio, a proportion, between the parts and the whole. In "Disposition of the town" [1960: 177-181] he shows sketches that are strikingly self-similar. He analyses Hagia Sophia in Istanbul [1960: 115-116] and again proposes a sketch that makes evident the self-similarity of elements of the plan. Obviously, he doesn't use the term "self-similarity", which he didn't probably know, but his conception of measures, and especially of orders of magnitude, had a lot to do with that geometrical concept.

Self-similar forms are also said to be scale-invariant. That could be a drawback as far as architecture is concerned. Architecture has a lot to do with scale: scale is very important in relation to its users and context. But what scale-invariance means is that at any of many scales, we find the same structure. This series of scales is not continuous. On the contrary, we are able to define ranges of scales according to the needs of our design. Besides, we are not compelled to pursue the process of diminishing scales very far.

3.3 A search for counterpoint in architecture: experiments

In order to illustrate my hypothesis on the use of IFS in a search for counterpoint in architecture, I carried out experiments on basic schemes involving the golden mean φ. Those experiments do not pretend to be architectural, though some results could be interpreted in such terms, but they do deal with a distribution of solids and voids, which is a primary concern in architecture.

The basic schemes consist in placing four squares of ratio $1/\varphi$ (and $1/\varphi^2$) either at the corners of a square, or at the centre of its edges (Fig. 7).

The use of the golden mean is relevant because as we can see for instance in the first scheme, the squares of the first ratio intersect to get those of the second one (because of the fundamental relationship: $\varphi^0 + \varphi^1 = \varphi^2$). The basic schemes provide a division of the square into squares and rectangles, among which are golden rectangles (in grey in Fig. 7). Divisions of the square were explored by Le Corbusier [1951: figs. 39-44, pp. 95-101] though most of his are not so symmetric as mine.

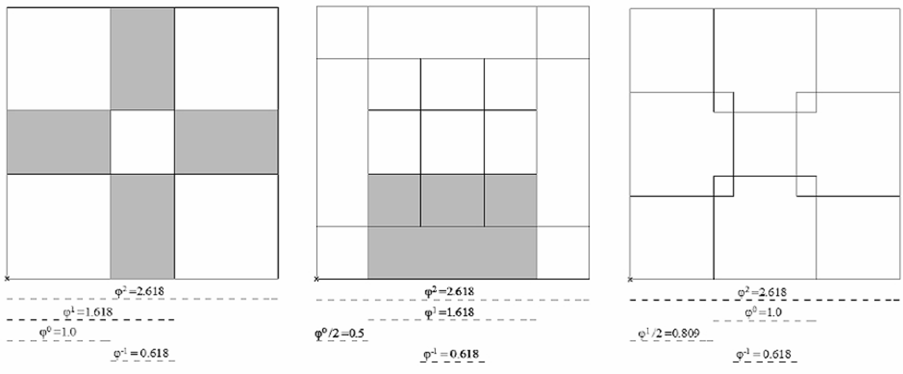


Fig. 7. Basic schemes

IFS 1	scaling	rotation	translation x	translation y
w1	1/φ	0°	– (1–1/φ)/2	0
w2	1/φ	0°	(1–1/φ)/2	0
w3	1/φ	0°	0	– (1–1/φ)/2
w4	1/φ	0°	0	(1–1/φ)/2

Table 1. IFS1

IFS 2	scaling	rotation	translation x	translation y
w1	1/φ	0°	– (1–1/φ)/2	(1–1/φ)/2
w2	1/φ	0°	(1–1/φ)/2	– (1–1/φ)/2
w3	1/φ	0°	– (1–1/φ)/2	– (1–1/φ)/2
w4	1/φ	0°	(1–1/φ)/2	(1–1/φ)/2

Table 2. IFS2

IFS 3	scaling	rotation	translation x	translation y
w1	$1/\varphi^2$	0°	– (1/φ)/2	0
w2	$1/\varphi^2$	0°	(1/φ)/2	0
w3	$1/\varphi^2$	0°	0	–(1/φ)/2
w4	$1/\varphi^2$	0°	0	(1/φ)/2

Table 3. IFS3

IFS 4	scaling	rotation	translation x	translation y
w1	$1/\varphi^2$	0°	– (1/φ)/2	(1/φ)/2
w2	$1/\varphi^2$	0°	(1/φ)/2	– (1/φ)/2
w3	$1/\varphi^2$	0°	– (1/φ)/2	– (1/φ)/2
w4	$1/\varphi^2$	0°	(1/φ)/2	(1/φ)/2

Table 4. IFS 4

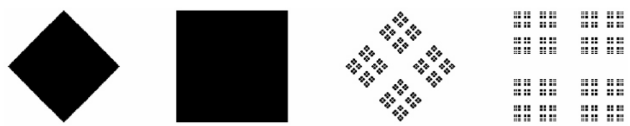


Fig. 8. Attractors of IFS1, IFS2, IFS3, IFS4

Four IFS have been built, each constituted of four very simple transformations: scalings (homotheties) of ratio $1/\varphi$ (IFS1 and IFS2) or $1/\varphi^2$ (IFS3 and IFS4) combined with translations that put the scaled squares at the middle of the edges of the first square (IFS1 and IFS 3) or at its corners (IFS2 and IFS 4).

Attractors of those IFS (Fig. 8) are not what interests me here. The IFS are used with a systematic algorithm on a white and black bitmap, starting from a solid black square, but with a slight change: at each step, white and black pixels are switched. The bitmaps are then interpreted as 3D forms, white corresponding to solid, black to void.

I "hybridize" [Corcuff 2008] the IFS pairwise, in two ways. In the first hybridization, the algorithm chooses between the two IFS (the four transformations) at each step (Fig. 9-14); in the second one, it chooses between the transformations of each IFS at each step. In the first case, one can show all results of three steps of the process; in the second case, there are too many possibilities, so one must randomize the process to show only some of them (Fig. 15, 16).

The first hybridizations maintain the square symmetry of the basic schemes. This symmetry is counteracted by the second type of hybridization.

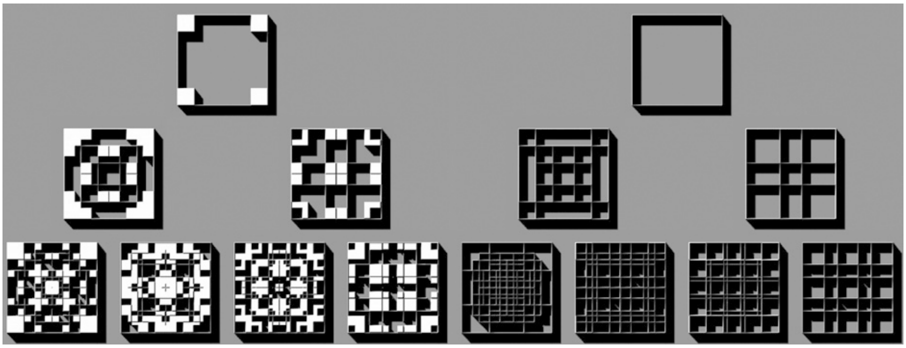


Fig. 9. First hybridization of IFS1 and IFS2

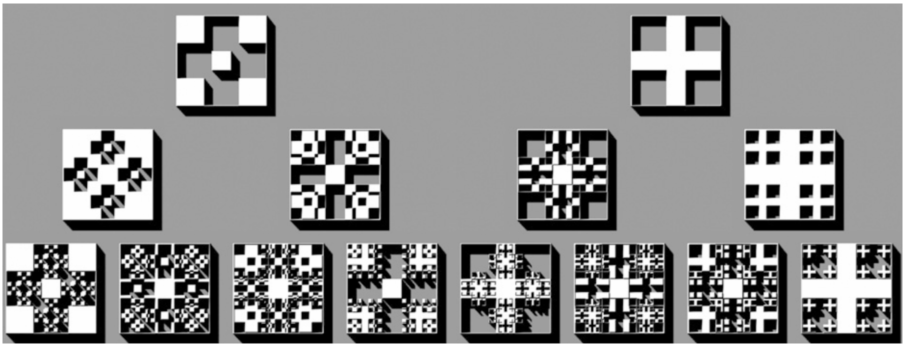


Fig. 10. First hybridization of IFS3 and IFS4

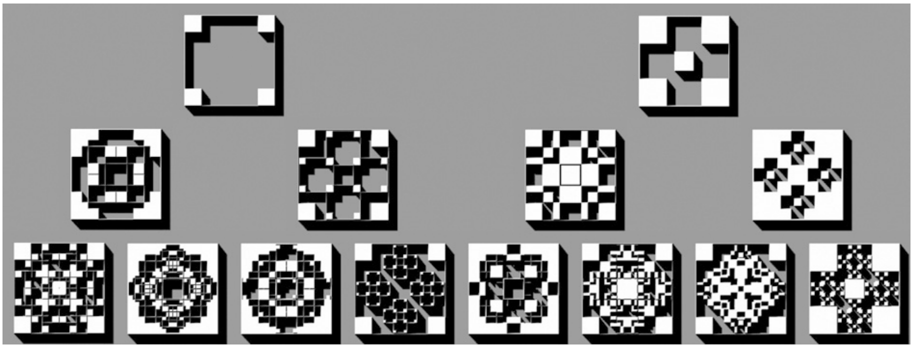


Fig. 11. First hybridization of IFS1 and IFS3

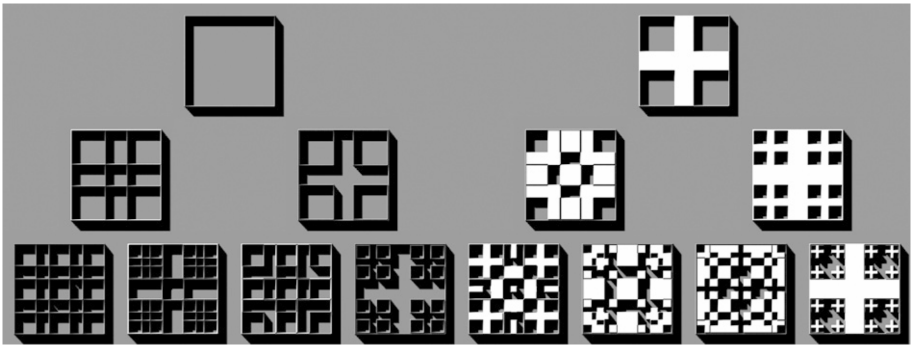


Fig. 12. First hybridization of IFS2 and IFS4

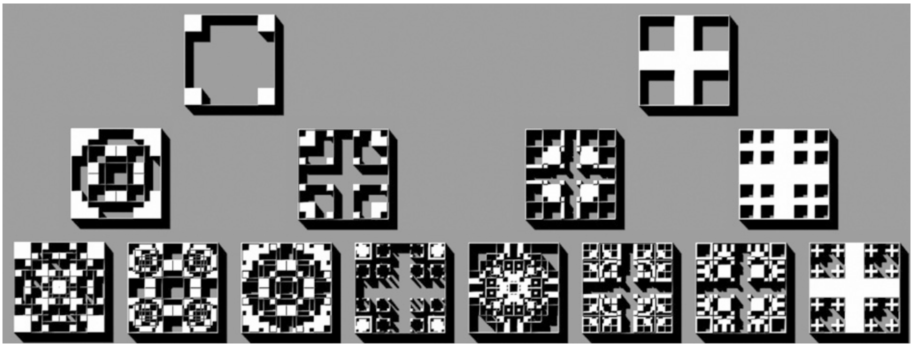


Fig. 13. First hybridization of IFS1 and IFS4

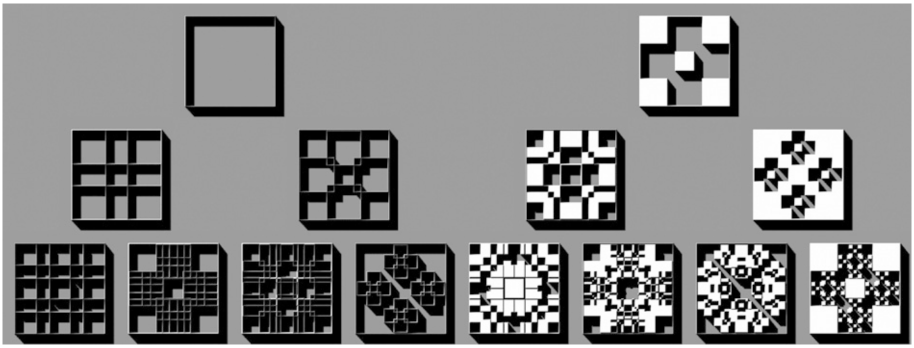


Fig. 14. First hybridization of IFS2 and IFS3

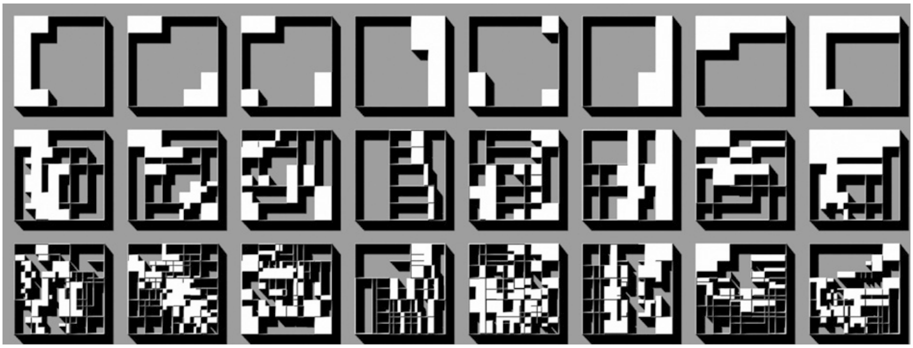


Fig. 15. Second hybridization of IFS1 and IFS2

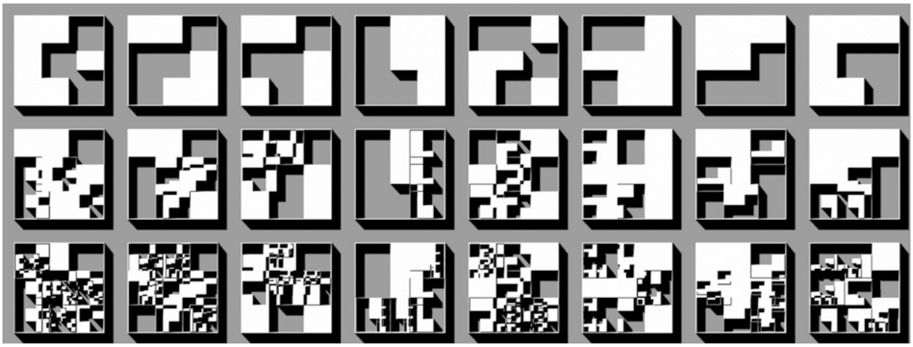


Fig. 16. Second hybridization of IFS3 and IFS4

Generative processes permit us to work a little like nature itself. We can imagine individual processes, but also alter them through "mutation", or combine two or more of them through "hybridization", as counterpoint implies two or more musical lines. My interpretation of counterpoint in these experiments is the assimilation of the simultaneity of different lines of music to the simultaneous transformations at each step of the process, and the lines themselves to the evolution of the process. The harmonic dependence of the "lines" is attained by the use of the golden ratio. That is obviously only one way of envisaging the notion of counterpoint.

I did not try to translate a piece of music into architecture, or even ornament. There are a lot of different ways to search for an equivalent for counterpoint in architecture; my contribution is only one of them.

References

AARTS, Jan, Robbert FOKKINK, Godfried KRUIJTZER. 2001. Morphic numbers. *Nieuw Archief voor Wiskunde* 5/2 (2001). Available at: http://www.nieuwarchief.nl/serie5/deel02/mrt2001/pdf/archi.pdf. Last accessed 17 Sept 2011.

ABBOT, Edwin A. 2006. *Flatland* (1884). Oxford : Oxford University Press.

BARNSLEY, Michael F. 1988. *Fractals everywhere*. New York : Academic Press.

BRAGDON, Claude Fayette. 2010. *The Beautiful Necessity*. Lenox, MA: Hardpress Publishing. (First serialized in the architectural press in 1902, and published in complete form in 1910.)

CORCUFF, Marie-Pascale. 2008. Chance and Generativity. Pp. 189-199 in *GA2008*, Proceedings of the 11th Generative Art Conference, Politecnico di Milano, 2008. Milan : Domus Argenia Publisher. Available at : http://www.generativeart.com/, last accessed 6 September 2011.

FERLENGA, Alberto and Paola VERDE. 2001. *Dom Hans van der Laan Works and words*. Amsterdam: Architectura & Natura.

GOGINS, Michael. 1991. Iterated Functions Systems Music. *Computer Music Journal* 15, 1 (Spring 1991) : 40-48.

LE CORBUSIER. 1951. *Le Modulor*. Paris : Editions de l'Architecture d'aujourd'hui. (1st ed. 1946.)

———. 1954. *The Modulor.* Peter de Francia and Anna Bostock, trans. London: Faber and Faber.

MARCH, Lionel. 1993a. Proportion is an alive and expressive tool. Pp. 88-101 in *R. M. Schindler, Composition and Construction*, Lionel March and Judith Sheine, eds. London: Academy.

———. 1993b. Dr How's magical Music Box. Pp. 124-145 in *R. M. Schindler, Composition and Construction*, Lionel March and Judith Sheine, eds. London: Academy.

MELOON, Brian and Julien C. SPROTT. 1997. Quantification of Determinism in Music using Iterated Function Systems. *Empirical Studies of the Arts* 15 : 3-13.

PADOVAN, Richard. 1994. *Dom Hans van der Laan: Modern Primitive*. Amsterdam: Architectura & Natura Press.

———. 2002. Dom Hans Van Der Laan and the Plastic Number. Pp. 181-193 in *Nexus IV: Architecture and Mathematics,* Kim Williams and Jose Francisco Rodrigues, eds. Fucecchio (Florence): Kim Williams Books.

PALLADIO, Andrea. 1988. *Scritti sull'architettura, 1554-1579*. Lionello Puppi, ed. Vicenza: Neri Pozza.

PARK, Jin-Ho. 2006. Rudolph M. Schindler : Proportion, Scale and the "Row". *Nexus Network Journal* 5, 2: 60-72.

POINCARÉ, Henri. 1913. *Dernières pensées*. Paris : Flammarion.

SCHINDLER, Rudolf M. 1947. The Schindler Frame. *Architectural Record* 101 (May 1947) : 143-146. (First written in 1943.)

VAN DER LAAN, Hans. 1960. *Le Nombre plastique, quinze leçons sur l'ordonnance architectonique*. Leiden : E.J. Brill.

About the author

Marie-Pascale Corcuff is a French architect living and working in Rennes, France. She teaches computer science in the Ecole Nationale Supérieure d'Architecture de Bretagne. She is the director of research group GRIEF (Groupe de Recherche sur l'Invention et l'Evolution des Formes) at ENSAB. She received her diploma (architecte DPLG) in Strasbourg in 1982, and a Doctorat in 2007 (title of thesis : « Penser l'espace et les formes »). She participates in international conferences, especially Generative Art in Milan, on the topics of generative processes (fractals, cellular automata, etc.) applied to art and architecture.

Philip Van Loocke

Sashoekstraat 2
8630 Wulveringem, BELGIUM
info@philipvanloocke.com
philip.vanloocke@ugent.be

Keywords: counterpoint, string theory, higher dimensional geometry, consciousness, quasicrystals, golden section, tiling the plane

Research

Counterpoint in the Visual Arts. A Perspective Based on my Graphical and Sculptural Work

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010.

Abstract. Counterpoint is a concept which has its roots in music, where it refers, loosely speaking, to the simultaneous presence of musical lines which complement one another, at times converging and at other moments evolving into a relationship of contrast. In this paper I use the notion of counterpoint in a visual context. I conjecture that such an extended use is meaningful, and explain this for graphical work as well as for sculptures. I take the freedom to illustrate this with some of my own artwork.

Introduction

Counterpoint is a concept that originated in music. The transfer of this concept to the context of visual art broadens its meaning. If the meaning of 'counterpoint' is stretched too far, one risks to make the concept almost trivial. But there are forms of visual art for which a meaning of sufficient specificity can be maintained, and for which the analogy with the musical context is reasonably clear. I will illustrate this for graphical art as well as for sculptures, and will take the freedom to take my own artwork as a basis for my explanation.

The graphical works which I create have a relation with string theory. This relation is stronger than a weak metaphor, since different works correspond to different higher dimensional vibrations of a picture. When the same picture is used as a starting point, the works belong together and the visual representations have relations of both similarity and contrast, and may express complementary meanings of a deeper underlying theme. Further, my sculptures are often created in pairs or groups, and a play of convergence and divergence between individual works can enter the process of creation. The relation between interior and exterior in my sculptural work creates additional aspects of counterpoint.

Graphical work and counterpoint

In this section I will explain how I perceive counterpoint movements in my graphical art. I will do this in three steps. First, I will explain why different levels of meaning are present in a single work. I will refer to higher dimensions and will sketch the philosophical and historical context of my method. In the second subsection I will be somewhat more specific on the technique that I have elaborated. These considerations will allow me to come to the subject of counterpoint in the third subsection, which is the section with the illustrations.

Nexus Netw J 14 (2012) 75–96

DOI 10.1007/s00004-011-0098-9; *published online* 13 January 2012

Levels of meaning in a single work

My graphical work takes images of the environment as a starting point. I transform these images by defining additional dimensions, which means that I (a) de-contextualize the images and (b) at the same time add new context.

(a) My transformations take a building or person out of the familiar context. This separation from context occurs because the transformations are based on particular physical and mathematical methods. Recent physics teaches us that higher dimensions and non-Euclidean geometry are indispensible for describing both the smallest and the largest scales of reality. These concepts play a central role in the computer simulations that I use to transform objects that we perceive on our scale. Our cognitive capacities have evolved in three dimensions. I have often observed that transformations that pass through higher dimensions result in a visual language suitable for expressing poetry, but such that our daily concepts are not apt to nail down the specific nature of this poetry. This disconnection makes my technique suited to working around particular deep themes connected with the human condition (I comment on this below).

(b) While these transformations take objects out of their familiar frameworks, the transformation also adds visual elements and new context to an image. More specifically, in each work, my goal is to achieve one of the following three aims:

1. The image of an object of high particularity, such as a building, can be transformed into an image that displays a universal symbolism. There are many views on what good art is. I often find myself attracted by artwork that for some reason tends to have a universal appeal. In some of my works, I express a shift toward universality by constructing transformations that yield motifs referencing universal core themes of myths or religions. These themes allude to conceptions of reality as a whole. As noted above, my transformations involve concepts that are central to physical descriptions of, among other things, the largest scale of reality. In my artwork, these recently developed concepts resonate with old concepts that are associated with this largest scale.
2. The visual elements involved in a transformation may give a building additional connotations, which are linked with the place where it is found, or with the meaning the place has for the artist.
3. In several works, I specify the transformation in such a way that the result expresses a theme or property of the worldview associated with the transformation. These transformations are about additional dimensions and non-Euclidean geometry. The context of such concepts is recent physics and they are associated with a non-classical worldview. These include, for instance, perspectives on time, which I express in the visual language enabled by the transformations.

The processes of de-contextualization and re-contextualization imply that an artwork has different layers of meaning. There is the starting picture with its 'ordinary' meaning, for instance a picture of a clock. This is transformed into a visual composition that has a direct visual interpretation, for instance an anthropomorphic interpretation if the deformation is such that a face or a person can be recognized. There is also an interpretation at a deeper level, when the face or person formed by the clock has an expression corresponding to a deep property of time, or, more generally, when the composition is reflecting such a property.

Subjecting the same picture to different higher-dimensional vibrations leads to different artworks. If these works are put together, counterpoint appears on the level of visual composition as well as on the level of deeper meaning: time has several connotations, both from the perspective of daily life and from the perspective of recent physics, which allows for counterpoint relations on a deeper level. An example will clarify what I mean. Bur before turning to it, I will elaborate on the method which I use and on its philosophical context.

Higher dimensions and basics of the method

Let me describe in general terms how I transform a flat photograph into a higher-dimensional surface. To begin, I define an origin in the plane of the photograph. Suppose that, in each point of the plane, a piece of rope is stretched that connects this point with the origin. On the plane I place a sphere that touches the plane at the origin. Now suppose that we wrap around the sphere the rope connecting a point with the origin, keeping fixed the endpoint of the rope that is located at the origin. The other endpoint of the rope then defines a point in three dimensions. Mathematically, we can also put a four-dimensional hypersphere on the plane of the photograph. By wrapping the rope around the hypersphere, we map each point in the plane of the photograph onto a four-dimensional point. This procedure can be defined for hyperspheres, but also for more complicated geometrical forms of arbitrary dimensionality.

By the process that I described graphically in terms of wrapping a rope, I map each point P of the plane of the photograph onto a higher-dimensional point, so that the plane is transformed into a higher-dimensional surface. Then, I project this point back onto the plane of the photograph, which leads to the identification of a point Q in that plane.[1] I transfer the colour values of the latter point Q to the former point P. Given that with an appropriate choice of geometry, the higher-dimensional surface is folded several times above the plane of the photograph, this procedure results in an image that contains several reduced copies of the original picture.

Colors in my artwork have usually shifted relative to the original photograph. I use two techniques to accomplish this transformation. First, I translate the curvature of the geometry into a color shift. The most straightforward way is the application of a redshift, in agreement with the concept in general relativity that curvature implies this phenomenon. There is also another technique that I use to determine the colors in my works. By placing point-sources in space from which solarization waves originate, I emphasize particular places in the image, or I determine their role in the composition (I am using the concept 'solarization' according to its meaning in photography, where it refers to the phenomenon in which a color moves over into its complementary color). I obtain still more freedom for working with color by using colored lightsources and by combining different layers into the final image. In practice, both the determination of an appropriate geometry and the determination of appropriate colorization are the most time-intensive aspects of my technique.

There is a reason why I mention some more technical concepts in this section. The concepts that are combined into my method originate in fundamental mathematical and physical models. Thus, there may be a connotation of 'heaviness' in two respects: it is heavy in the sense of being technically difficult, but it is also heavy in the sense that reflecting on deeper reality carries an existential load; an attitude of brooding comes close sometimes. Precisely in order to balance this 'heavy' aspect, in several works I use a visual language that is deliberately fairy-like: deeper reflection then resonates in a light visual language with a touch of humor. The artworks illustrate that deeper reality, or thinking about it, must not necessarily lead to some kind of somberness. Instead, from it sprouts a poetic language of forms confirming a pleasant spirit. It is the ambition of my art to appeal aesthetically to people who are not familiar with the method with which the art is created. But those who are more or less familiar with it will recognize an additional layeredness, even in works that were created in an intentionally fairy-like style.

Higher dimensions in art

Around the end of the nineteenth century, the idea of higher dimensions began to affect art and culture, ranging from literature to theology. In 1884, Edwin Abbott published his satirical novel *Flatland: A Romance of Many Dimensions*, which is still being reprinted.[2] His main character is a square who lives in a two-dimensional world. In this world, the social status of a person is determined by his number of sides. Triangles are considered less intelligent, and priests are polygons with several sides. Women in Flatland are lines and have a single side. One day, the main character is lifted out of the plane by a three-dimensional sphere. He acquires the notion that there are more than two dimensions. When he talks about this idea in Flatland, his notions are considered subversive, and he is put in prison.

Abbott not only aimed at social satire, but also criticized the educational system, which suppressed creativity instead of encouraging it (being a schoolmaster himself, he was in a good position to make this diagnosis). In the art world, which took a position independent of formal schooling at many decisive moments, the idea of higher dimensions soon firmly took root.[3] For instance, in his Paris time, Picasso became familiar with a book on higher-dimensional geometry by the mathematician Esprit Jouffret [1903]. The book has several schemas visualizing projections of hypercubes, which can be recognized in Picasso's 1910 paintings in which he initiated cubism (see [Robbin 2006]). Also several of his fellow cubists created paintings visualizing how the world would look from a four-dimensional perspective. Higher dimensions also influenced futurism and expressionism. Whereas cubism considered an additional spatial dimension, futurism tried to represent the idea of a four-dimensional space-time, which implies that different temporal perspectives are integrated into a single painting. Expressionists often worked with a more symbolic representation of higher dimensions. One example is Dali's *Crucifixion (Corpus Hypercubus)*, in which Christ hangs on a three-dimensional unfolding of a four-dimensional hypercube.

At the current moment, about a century after the cubist movement originated, physical theories have become more sophisticated, but higher dimensions have remained a core theme in physics. String theory (and its successor, M-theory) evolved into an eleven-dimensional theory, covering ten spatial dimensions and one time dimension. In addition to the three spatial dimensions of our ordinary perception, there are six spatial dimensions that are strongly curved in all versions of the theory. The tenth spatial dimension has different properties in different variations of the theory.[4] In one of these, it is extended infinitely and offers room for an infinity of parallel universes.

With respect to the application of higher dimensions in art, the current moment differs in several ways from the first half of the twentieth century. Because recent theories deal with domains in which much higher energies are considered, the curvatures that are presently contemplated are stronger and more complicated. Correspondingly, I use geometries that sometimes result in fractal images. But higher dimensions also emerged in another domains, such as crystallography. As was found in the 1980s, quasicrystals can be considered as projections of parts of high-dimensional grids of hypercubes. There are artists who based graphical works or paintings on this science [Robbin 2006]. As I explain below, I apply these concepts in my sculptures. Another change with art from the first half of the twentieth century follows from the fact that, at this moment, we have computers. We can simulate higher-dimensional models and in this way reflect on the implications of higher dimensions with exactitude, whereas the artists of the cubist movements largely depended on intuition only.

Philosophical comments

Our physical reality includes sixty powers of ten. What does that mean? Take the smallest size that appears in physics, which is the size of a string in string theory. Multiply this length by ten. This length is again multiplied by ten, and so on, until sixty multiplications have been performed. At that point, we have reached the scale of the largest cosmic structures that we have observed. As humans, we find ourselves at the middle point of these orders of magnitude.[5] At our order of magnitude, and for energies that confront us in daily circumstances, we can describe reality with models that work with a three-dimensional Euclidean space and an independent time dimension.

This is not the case for smaller or larger scales, or for energies that are significantly larger than those we encounter in our typical daily environment. The first physical theory to be formulated in four dimensions was the theory of restricted relativity, in which space and time became a four-dimensional whole. Non-Euclidean dimensions were incorporated into the development of general relativity. In the course of the past century, many trials have targeted the addition of more dimensions, with the aim of obtaining a unified theory for the forces of nature [Van Loocke 2008: ch. 6; Halpern 2004]. Only string theory has come close to achieving this ambition in a mathematically coherent manner.

String theory is also being applied in cosmology, as so-called string cosmology. Among cosmologists, however, the inflation model of the cosmos is most popular at that scale. In the course of the 1980s this model led to a reformulation of classical big-bang cosmology. Because this reformulation is a profound one in different respects, several authors have written in terms of a 'revolution' in cosmology [Van Loocke 2008: ch. 7; Guth 1997]. The inflation model integrates general relativity with some central concepts of particle physics (a central concept being the so-called inflation field, formulated by analogy with the Higgs field in particle physics). This model implies that our universe is immensely larger than our visible universe. In addition, our universe is one among many 'islands universes' in the inflation vacuum. For an observer who stands inside an island universe, such a universe is infinitely large and has a unique moment of origination, the moment of the big bang. For an observer standing outside the island universe, the big bang lasts infinitely long and takes place at the boundary of an island universe of finite size. Such facts are consequences of the way in which inflation cosmology integrates general relativity.

At the very beginning of our island universe, the largest and the smallest scales coincided, simply because the largest level was as small as the smallest level. A fraction of a second after the beginning, however, the inflationary phase began. During this phase, quantum fluctuations were magnified up to macroscopic orders of magnitude. This resulted in inhomogeneities in the distribution of matter and energy, leading to the structures we presently see in the universe, like galaxies and galaxy clusters. Because of the role of quantum fluctuations in the origination of cosmic structures, a common refrain is the proposition that the same uncertainty principle that describes the behavior of the electron was responsible for the formation of the galaxies. Without galaxies or stars, there would, of course, be no people. Recent cosmology thus emphasizes the very early stage of the universe and the quantum fluctuations present at that stage as significant for everything that would follow. This very early stage can only be described with higher-dimensional, non-Euclidean geometry. Our existence became possible because cosmic inflation magnified the properties of this stage.

Above, I used the term 'de-contextualization'. The transformations which I construct take an object out of its familiar, three-dimensional Euclidean context – the context that is typical and familiar for our order of magnitude. On the basis of the above observations, this de-contextualization is open to a broader interpretation: it refers not to the order of magnitude, but also to the moment in cosmic history. Higher dimensions are a reference to the earliest stage of everything that exists.

Scientific experiments or technological realizations aside, the question is whether or not we, at this moment of cosmic history, are locked into our present scale. For me, this is an open question. I refer to something that everyone experiences, namely consciousness. Consciousness at present is not explained. I am aware that for some people, this statement is emotionally loaded but evolutionary theory cannot explain consciousness either.[6] Since the mid-1980s, it has become common to draw a distinction between 'hard' and 'easy' problems of consciousness. The 'easy' problems concern the question of which neural structures correlate with consciousness as well as questions concerning their function. The 'hard' problem concerns the question why this correlation even exists. As far as consciousness is concerned, current science can identify correlations, but that is profoundly different from establishing underlying causation.

Some scientists propose that reference to the smallest level of reality is necessary before progress can be made on the hard problem of consciousness.[7] Such an achievement would yield a captivating picture. If that were the case, then matter, at moments at which consciousness is correlating with it, would be described using models encompassing concepts that are also central to the theories describing the origins of our universe. Many religions and old myths propose such a relationship between human consciousness and the origination of the universe. It is impossible to predict whether or not this scientific integration I describe here will actually transpire: perhaps yes, perhaps, no.

But it is normal practice for artists to find inspiration in deep, open questions. As I noted above, I like to work with the shifts from particularity to universality. The core themes of myths are a gratifying beginning to the realization of this movement. Identifying the themes I incorporate into a particular work is not always an easy task. One reason is simply the fact that knowledge of these themes is not widespread or that a theme is communicated from a single perspective only. Take, for instance, the theme of the Fall. At first sight, this choice seems to be only an archaic theme from the Old Testament, hardly an inspiration for contemporary art. Only if we can look beyond the Judeo-Christean-Islamic complex can we grasp the universality behind this theme. In the Old Testament, God exiles people from Paradise. In African myths, we often hear the opposite: The people repel the gods by breaking the harmony of the world [Van Loocke 2008: ch. 2; Sprout 1991]. The deeper common theme is that a particular type of serenity, acceptance, or humility is valued more highly than an egocentric orientation or conceit that people use to try to escape their human condition. This underlying theme is more abstract than the concrete, visual language of the myths. In different works, I have at times intertwined such themes, both abstract and more concrete.

Space-time, evanescence of being, and non-verbal poetry

The higher-dimensional geometry I apply is non-Euclidean and therefore curved. As a result, an object (or parts thereof) can appear several times. For clarification, I refer to a gravitational lens. Consider a star located behind a galaxy. The gravitation of the galaxy is equivalent to a curvature of space-time, which has a lens effect on the light sent out by the star. Therefore, it is possible that we see different images of a single star. By analogy, one can observe that the original image often reappears several times in my works.

With an appropriate choice of geometry, it is possible to create an image that consists of several part-images, or that seems to scatter into fragments, in each of which the whole can be recognized. In this way, I work with a complexity that, although aesthetic, has a certain degree of capriciousness. But even an image that seems to be fragmenting retains its poetry. Because of the relationship between fragmentation and evanescence, I regularly use this technique of fragmentation to express the poetry of evanescence.

The transitoriness of being is usually associated with agony. Life has moments that make it almost impossible to see the poetry of evanescence. In different works, I try to reflect a visual glimpse of that poetry. But I have another reason I work with this theme in my art. Einstein proposed that the agony associated with the fragility of being was relative from a four-dimensional perspective anyway. The real arena of physical processes is four-dimensional space-time as a whole.[8] A moment of consciousness correlates with a small area in space-time. This restriction of consciousness burdens us with the illusion of evanescence. To Einstein, this insight was relevant to his sense of life, and he tried to comfort survivors of deceased friends with this insight.

Whether Einstein was on the right track with this attitude is a matter of discussion.[9] There are many other ways to cope with the fragility of being. For instance, recent insights on personal identity are relevant.[10] People are discontinuous processes to a significantly higher extent than introspection suggests. If we do not have a truly essentialistic self, there is no such self that can disappear or evanesce. Still, I share with Einstein the view that insights on the fundamental structure of the universe do have implications for our sense of life.

I return to another point that I made in the first section. I use the higher-dimensional perspective to add meaning. Nevertheless, the visual poetry of the resulting artwork may be explicable only in part with verbal concepts. Our aesthetic, emotional, and cognitive intuition have evolved in three dimensions. In my works, one recognizes the presence of a visual poetry. But because our usual concepts are fitted for a three-dimensional universe, verbal descriptions of appreciation of works that refer to higher dimensions are subject to restriction. The clear presence of poetry, without words available to articulate it, is one of the facts we mean when we use the word 'sublime' in an artistic context. Higher dimensions therefore offer a suitable context for working with the sublime.

Graphical work and counterpoint

After these remarks we are ready to consider an illustration. Consider the photograph in fig. 1. It shows the clock on the cathedral of Ypres (Ypres is a city in Belgium famous for its commemoration of the first world war). We let the image vibrate, and do not confine ourselves to the third dimension but we let the deformation extend to higher dimensions. We proceed with the projection to the plane of the photograph in accordance with the explanation given higher. This we do three times. After appropriate colorization, the works shown in figs. 2a-c result.

Fig. 1. Picture of the clock on the cathedral of Ypres

Fig. 2a. Clock cathedral of Ypres, 2010 #5

Fig. 2b. Clock cathedral of Ypres, 2010 #6

Fig. 2c. Clock cathedral of Ypres 2010 #3

As is exemplified in these illustrations, in most of my works one recognizes motifs with an easy interpretation, such as eyes and mouth, heads and bodies, mythical birds, and so on. This is achieved by proper construction of the higher dimensional geometric model. I choose it in a way so that the motifs combine into a composition that accords with my inspiration. Different compositions express different connotations of time. Along with easily interpretable motifs, one recognizes themes surrounding the deep problems of time, which strengthen counterpoint relations between different works, since different deeper themes may be articulated in different works.

Sculptures and counterpoint

Kepler and the golden section

Higher dimensions play a central role in my sculptural work as well. There are two historical lines of mathematical work which can be seen as precursors to the method I am using. The first one is Kepler's geometry. Kepler was one of the brightest mathematicians of his time. One of his points of interest was the mathematics of the golden section, or as he called it, 'division into extreme and mean ratio'. In the field of algebra he discovered how the number Φ=1.618 appears in the Fibonacci sequence. He also discovered polyhedra in which is a defining number, such as the triacontahedron. It has thirty identical faces, each of which is the golden rhombus (in a golden rhombus the ratio of the long diagonal to the short one is equal to Φ). It is a structure which reappears in recent mathematics from a new viewpoint: the triacontahedron can be obtained as a combination of the basic cells of quasicrystals. The latter are described at this moment with help of projections from higher dimensional structures onto three-dimensional space. Partial triacontahedra can be seen in several of my sculptures.

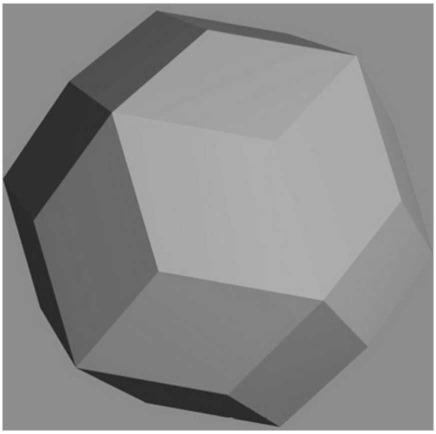


Fig. 3. Kepler's triacontahedron

Because the golden section played such a remarkable role in his mathematical reflections, Kepler stated: 'Geometry has two great treasures. One is the theorem of Pythagoras; the other the division of a line into extreme and mean ratio. The first we may compare to a measure of gold, the second we may name a precious jewel'. From our perspective this reads like an exaggerated statement. But Kepler's worldview was religious-Platonic, complemented with thoughts we would associate with magic or overly literalistically conceived mythology.

Kepler was not only a mathematician and an astronomer, he also was an astrologist, devising his own astrological technique on the basis of planet configurations. During the Renaissance, several philosophers valued astrology more highly than astronomy. Kepler shifted this order. Since the late Middle Ages the idea that God's thoughts are rational had gained support. Inspired by this, Kepler was convinced that these thoughts could be reconstructed with geometry and numbers. Astronomy was a better context for such an endeavor than astrology. But in spite of this shift, part of his thought remained rooted in older traditions. For instance, it can be conjectured that he came to the concept of elliptical planetary motion by taking inspiration from the egg form, which in Orphic mythology defines the initial form of the universe. The historical context of sculptures in which the golden rhombus plays a central role is therefore an appropriate context to link recent scientific and older mythical themes.

As I mentioned, Kepler was one of the brightest mathematicians of his days. But there is a domain in which he had been surpassed by Arab mathematicians and architects of the Middle Ages. It is a domain in which the golden section also plays a role, and that is properly understood only since the 1980s. In his work *Harmonices mundi*, Kepler considers tilings of the plane, that is, he considers various sets of polygons which can pieced together to cover the plane without gaps or overlaps. In Arab mosque art, this type of problem was considered earlier, since Islamic art excluded images of humans or the Prophet, and the Arabs often made recourse to geometrical motifs. Compositions of motifs were used to cover large walls or portals. The question therefore arose of how to tile the plane in a way that is of sufficient beauty and sophistication to do justice to the dignity of a religious building, while at the same time remaining practically feasible. It was recently discovered that in certain Arab patterns, tilings can be identified which were constructed by Roger Penrose in the 1970s (see [Lu and Steinhardt 2007], or, for more illustrations, [Prange 2009]). The Arabs appear to have found tilings with a special property: they are non-periodic. In the next section I explain why this is remarkable, and how this is related with my sculptural work.

The golden section, tilings and projections from higher dimensions

In the 1970s, Roger Penrose was searching for non-periodic tilings of the plane. In such a tiling, the whole pattern cannot be obtained by translation of subpatterns. The reason for his search had to do with a fundamental problem in mathematics: if such a tiling exists, then a particular type of mathematical problem could be proven to be unsolvable in principle. Mathematics has problems for which a solution would mean that all of mathematics would be inconsistent. This would entail, for instance, that 1 equals 2. Therefore, it is posited that such problems are unsolvable in principle. Around 1973 Penrose had indeed found different non-periodic tilings, one of which is shown in figs. 4a-b.

Consider the two rhombi in fig. 4a. The acute angles of the rhombi measure 36° and 72°, respectively; these are the angles which appear in the golden triangle. These rhombi tile the plane non-periodically as shown in fig. 4b. It can be observed that there are different points at which one can recognize motifs with partial pentagonal symmetry. Because of the prominence of the golden section in the regular pentagon, this adds to the prominence of the golden section in the planar quasicrystal tiling of fig. 4b. There is another fact that emphasizes the relation of this structure with the golden section, which is related to a remarkable insight achieved by the Dutch mathematician Nicolaas De Bruyn in 1981.

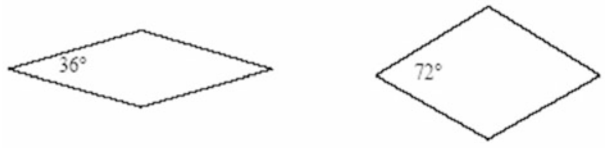


Fig. 4a. The two rhombi which are combined in fig. 4b

Fig. 4b. Non-periodic tiling of the plane

An ordinary crystal has translation symmetry: its molecular structure has sub-patterns which, after translation, reappear in the entire structure. A quasicrystal is also composed of a limited number of types of cells, but their combination is non-periodic. Because of applications in material science and in design of electronic components, quasicrystals are widely studied. Quasicrystals have been fabricated in the laboratory, and it has recently been discovered that they occur in nature as well. De Bruyn's insight was that two-dimensional quasicrystals (tilings of the plane like in fig. 4b) are projections of part of a five-dimensional grid of hypercubes. Three-dimensional quasicrystals, which play an important role in my sculptural work, result as projections of part of a grid of six-dimensional hypercubes onto three-dimensional space.

At first sight this may seem surprising, since a grid of cubes, whatever their dimensionality, is perfectly periodic. Therefore it is remarkable that a projection of such a grid results in non-periodic structures. The point which explains this fact is that only part of the grid is projected. This is easy to explain when we consider the projection of a two-dimensional lattice on a line (which is a one-dimensional structure). Fig. 5 shows a grid of squares, a yellow and a green line. The yellow line is the line onto which part of the grid is projected. The projection is confined to points on the grid located between the green line and the yellow line (these points are marked by red circles in fig. 5).

The angle between the horizontal lines of the grid and the green and yellow line is defined such that its tangent equals $1/\Phi$. The projection of the points marked with red circles leads to a division of the yellow line into line segments of two lengths, denoted L and S, respectively (L standing for long, S for short). The ratio of the lengths of these segments is equal to Φ. The pattern LSLLSL… that results is non-periodic, and can be recoded in such a way that the numbers of the Fibonacci series appear. By a similar procedure, two-dimensional and three-dimensional quasicrystals are obtained as projections of grids of five-dimensional and six-dimensional hypercubes, respectively.[11]

$$\tan(\theta) = 1/\Phi \quad \frac{|L|}{|K|} = \varphi$$

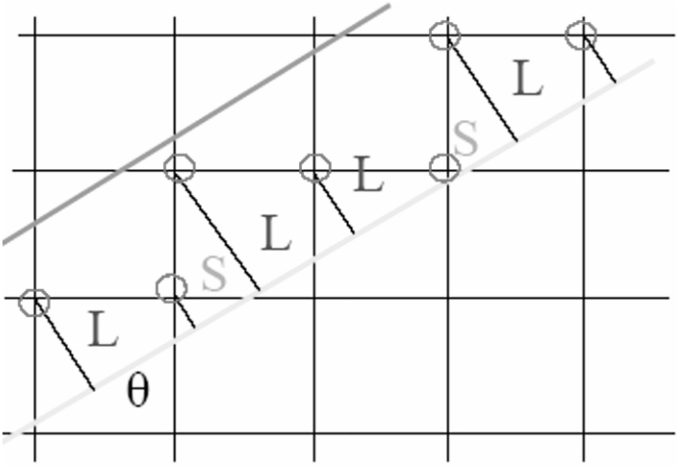


Fig. 5. Projection of part of a two-dimensional grid leads to a non-periodic one-dimensional structure on the yellow line

In two dimensions, quasicrystals can be build by assembling the two rhombi in fig. 4a. In three dimensions, we obtain quasicrystals by combining the two golden rhombohedra (fig. 6). The golden rhombohedra have six faces, each of which is the golden rhombus. With these golden rhombohedra, different familiar polyhedra can be composed. Twenty rhombohedra (ten of each type), for instance, if correctly assembled, result in Kepler's triacontahedron.

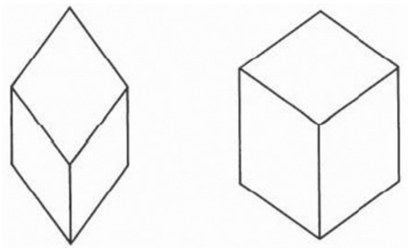


Fig. 6. The golden rhombohedra

Initially, I explored this world of forms with help of software I had written for this purpose (there are many open questions on the combinatorics of the golden rhombohedra; mathematics constrains the combinatorics of the basic cells less stringently than in the two-dimensional case). Fig. 7 shows a simple example in which two partial triacontahedra can be recognized at the top. This software-based approach is not always practical, since a full sculpture (meaning a sculpture with an interior that is completely filled with golden rhombohedra) easily contains many more rhombohedra than there are, for instance, in a triacontahedron. For me, the best way is to work on the basis of intuition. I make a large set of golden rhombi and combine them into a sculpture that fits my inspiration. Because by far not all combinations of rhombi result in an extended surface without holes, this takes much practice and experience-based spatial insight; that is a precondition for this kind of art. In some instances working on the basis of intuition leads me beyond the original context of quasicrystals. For some of my sculptures I verified that it is impossible to fill them with quasicrystals, even though all angles between the golden rectangles involved are familiar from quasicrystals. Other works can be filled completely with the golden rhombohedra.

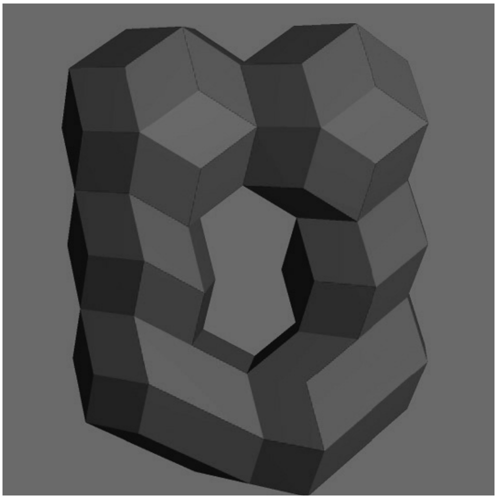


Fig. 7. Form resulting from combination of the golden rhombohedra

Sculptures tiled with mirrors

I make my sculptures open; therefore they have an exterior and an interior. I tile the interior completely with mirrors of a golden rhombus shape. Sculptures with mirrors have different antecedents in contemporary art. For instance, Robert Smithson created sculptures in which he combined different mirrors, and in which a limited number of mirror-reflection iterations are visible.[12] In comparison to such installations and constructions, my work has a more intricate complexity of light reflection. In this sense it is related with video-feedback art, in which loops are created by a videocamera which is directed at a screen. The image registrated by the camera is transmitted to the screen, captured by the camera, sent to the screen, and so on. As a result, the screen displays fractal patterns. A variation of this type of loop appears in the interior of my sculptures, but without the intervention of a camera and electronics. An incoming ray of light is reflected several times on different mirrors before reaching the eye of an observer. Since different rays reflect on different series of mirrors, an image with fractal depth results.

Because the mirrors have the shape of the golden rhombus, they create images which at times have a cubistic feel. The image changes every time the observer changes position. The environment, as well as the observer, are drawn into the reflection process and become part of the artwork. Below I include three examples of sculptures (figs. 8a-c).

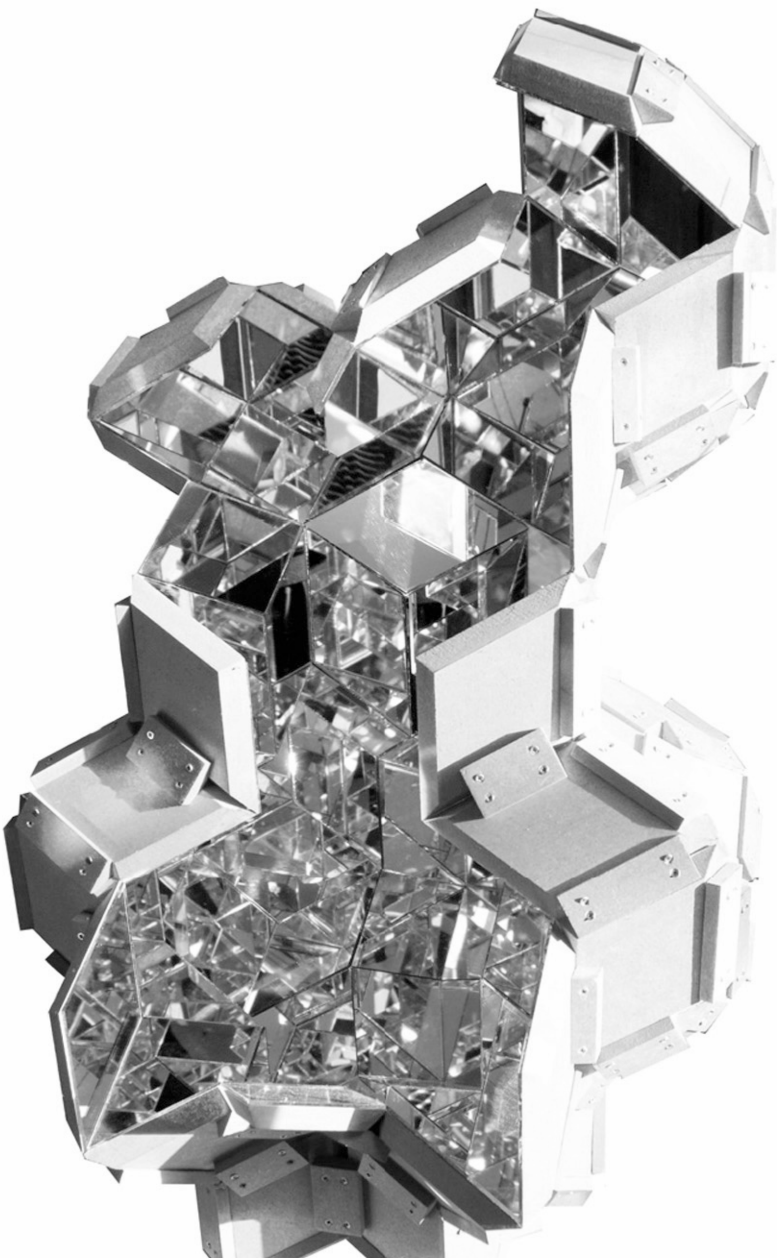


Fig. 8a. First example of a sculpture (Ph. Van Loocke 2010, Sculpture #1)

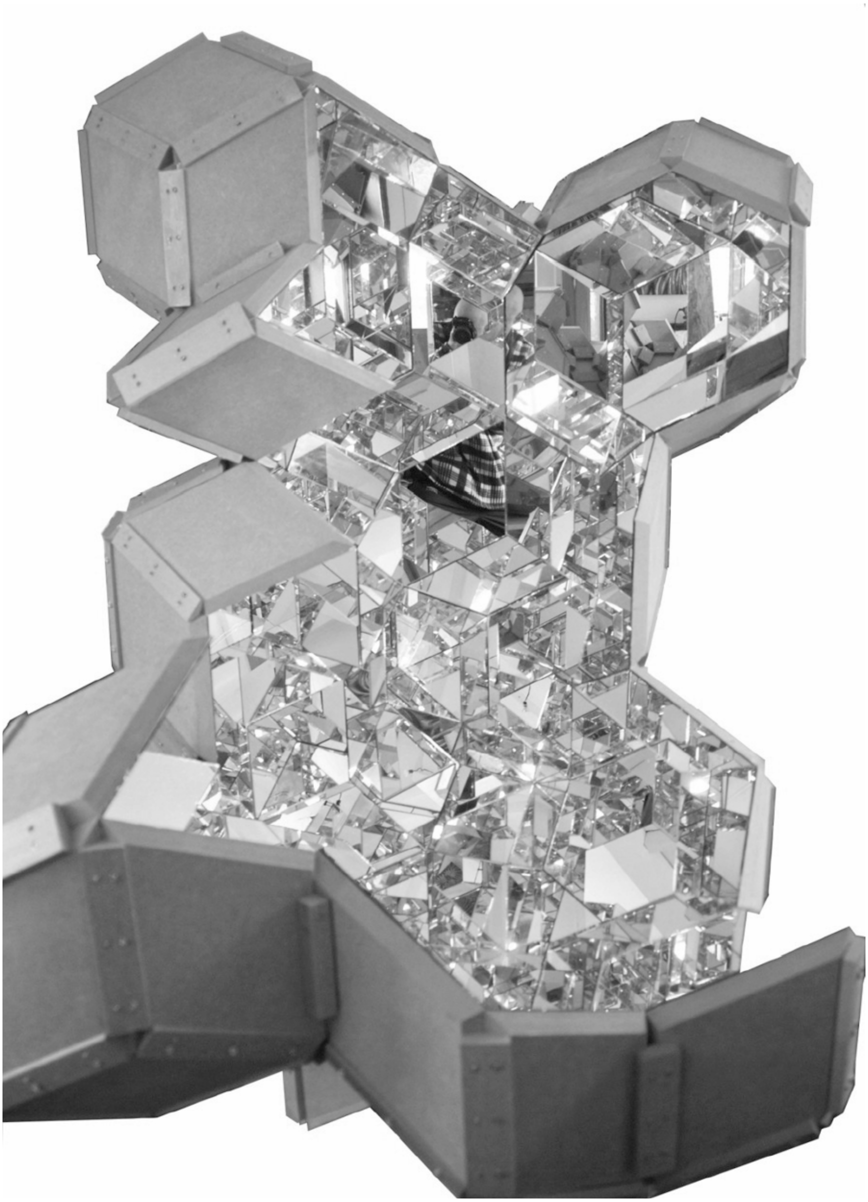


Fig. 8b. Second example of a sculpture (Ph. Van Loocke 2010, Large sculpture #1)

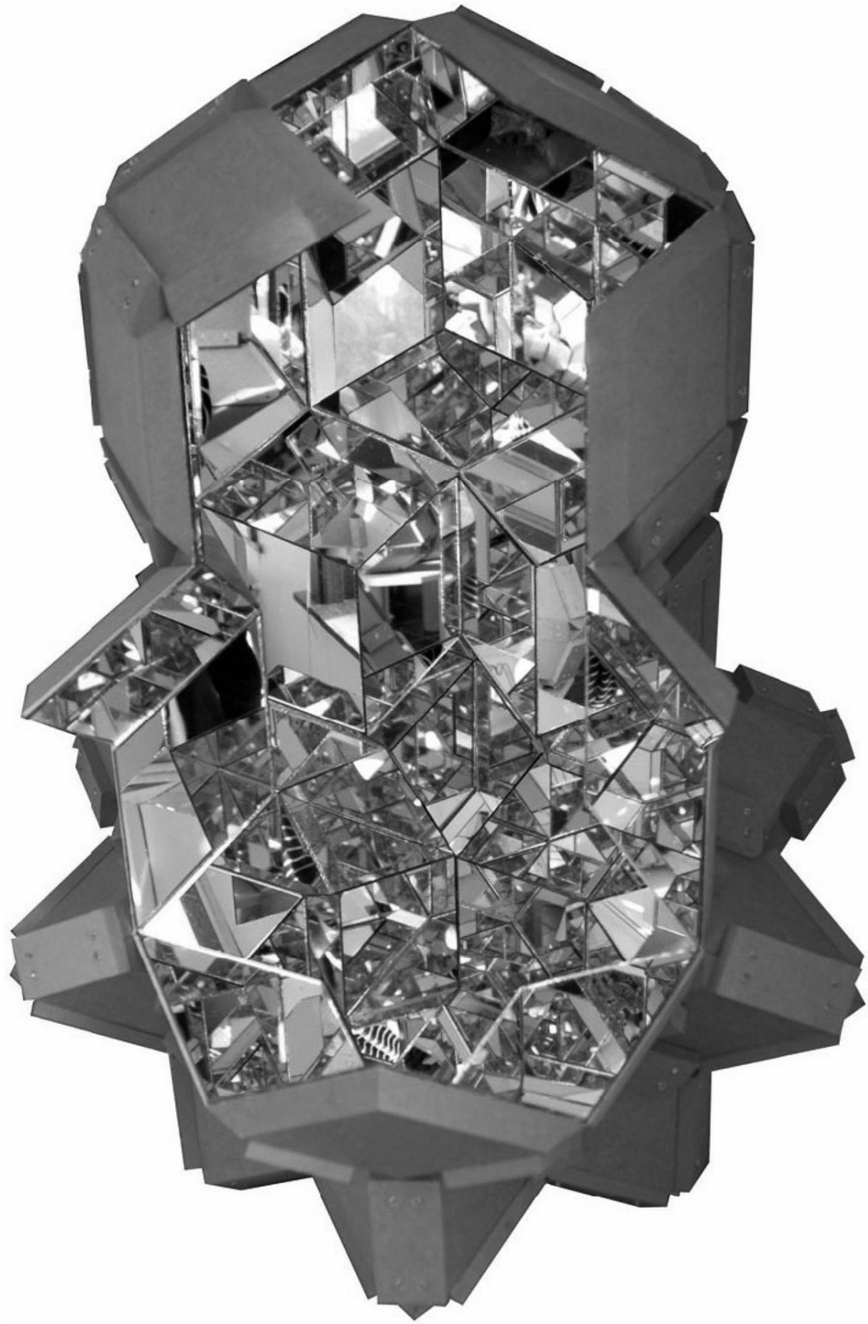


Fig. 8c. Third example of a sculpture (Ph. Van Loocke 2010, Sculpture #2)

Mirror based photography

When I zoom in with a photo camera on the interior of my sculptures, images result in which light weaves a pattern of fragments that belong together, but which nevertheless seem disparate. Figs. 9a-b show two examples. In course of the reflection process some fragments receive more light than others, and the sharpness with which they are present in the image is inevitably subject to variation (focussing on large fragments implies that smaller fragments become more vague and vice versa: although the physical distance between the mirrors and the camera is not subject to strong variation, the lengths of trajectories of light rays differ strongly as a function of the number of reflections to which they are subject).

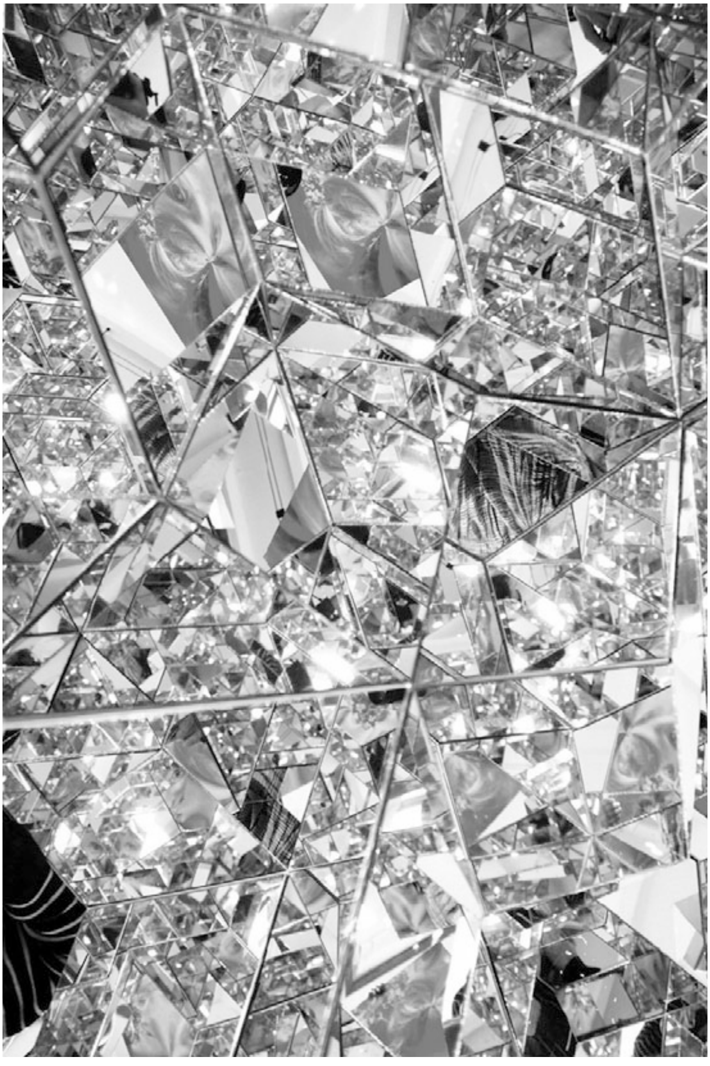


Fig. 9a. Ph. Van Loocke 2010, mirror based photography #1

Fig. 9b. Ph. Van Loocke 2010, mirror based photography #6

I want to make two additional metaphysical comments in relation to this art, which helps to understand the point that I am about to make concerning counterpoint. First I make a remark on the mirrors, which reflect one another in such a way that in a sculpture with a limited number of faces, a much vaster complexity appears. The theme of mirrors which mirror one another is an old theme in Eastern philosophy. Think, for instance, of the mirrors or 'pearls' of Indra, which mirror one another in an infinite regression, so that in each pearl a pattern of infinite detail is created. This theme entered Western philosophy thanks to Leibniz. Leibniz's view strongly differs from Descartes's atomism, according to which reality is composed of infinitely small atoms. In interplanetary space these revolve around the sun and like minuscule stones they drag the planets around the sun in their vortexes. In Leibniz's view, reality is composed of monads, which are infinitely small mirrors that reflect one another. For instance, the mirror in our brain which is mirroring best acquires dominance, and it determines the content of our consciousness. Leibniz was influenced by Eastern philosophy.[13] For several centuries, he had few followers in Western philosophy. But I like finding affinity between my art and the great stories, especially when the latter extend over different cultures. It gives inspiration and it may relate art of the present with world views of the future.

The second example of a metaphysical relation that I give at this place concerns 'intellectual' kabbala. I use the adjective 'intellectual' in order to make it clear that I am not referring to new-age kabbala but to the kabbala of Isaac Luria, who in the sixteenth century proposed a cosmogenesis with different stages. At the beginning there was only the infinity of God. In this infinity there is no place for a world like ours, since God fills everything. At a certain instant, God selects a special point in this infinity. He retracts from this point and allows it to expand.[14] Thus comes into existence the place where our world can originate. But since God had to retract from this place in order to enable our world, his qualities are no longer permeating it. Therefore, evil can exist. But if we take an open, relaxed attitude, then we still perceive traces of the goodness of God. Fragments of the world contain traces of the initial divine light, and it is up to people to connect these fragments and to increase their relative weight. The theme of fragments, and the play of the light connecting them, is visible in the photographic work that I make on basis of my sculptures. Acceptance of Luria's cosmogenesis is not a necessary condition for appreciating this art form, but familiarity with this metaphysical context can add another layer of meaning, which may increase the subjective value of an artwork.

Counterpoint and sculptural work

As in case of my graphical work, different semantic levels are present in a single work. In case of sculptures, this holds for the interior as well as for the exterior. In the exterior and at a figurative level, certain motifs, such as biomorphic themes, abstract people, and so forth, can be recognized. The higher-dimensional projections which describe the shape of the exterior have a physical and metaphysical context that I have outlined when I discussed my graphical work. In addition, the golden section, which is prevalent, has its own long tradition with ramifications in Platonism and beyond. The interior of a sculpture, and the mirror based photography to which it leads, can be related to Eastern metaphysics, with Leibniz or with kabbala.

This often creates a counterpoint in an individual sculpture. The finiteness of the exterior communicates with the sheer infinity of the interior, and the metaphysical themes associated with the exterior complements the mirror-and-light metaphysics associated with the interior.

In the case of my graphical work, a single photograph leads to different works if different higher-dimensional deformations or 'vibrations' are applied to it. In the case of my sculptural work, the same grid of six-dimensional hypercubes leads to different sculptures if different parts of the grid are projected (although some sculptures fall outside the range of quasicrystals, as I explained before). In some instances I create my sculptures in small groups, and I orient the interiors of the sculptures toward one another, so that the reflecting mirror systems in different sculptures increase the complexity of the reflection pattern in each of them. In this case, the counterpoint is not only involved through the complementary relations between interior and exterior in individual sculptures, but is also observed in the relation between different sculptures.

Conclusion

The round table session at Nexus 2010 entitled "Generative Architectural Codesness" included two interventions in which counterpoint is discussed and explained from a broad perspective. Moderator Celestino Soddu explained how counterpoint is a major theme in his architectural work. The dynamics of convergence and divergence of several forms is evident in the method that he designed, in which algorithmic variation is combined with an artist-based selection process. I took the freedom to discuss the concept of counterpoint in relation with my own artwork. Both in my graphical and in my sculptural work it has a natural place.

Notes

1. The point Q is located in the plane of the photograph, but may fall outside the area covered by the latter. In order to associate colour values with Q in such a case, I tile the plane caleidoscopically with the photograph. This means that each tile has a copy of the photograph, with orientation in such a way that neighbouring tiles contain images which are mirrored along the shared side.
2. Note that this was before the restricted theory of relativity had been formulated. The concept of higher dimensions was studied in mathematics and from there found its way into art.
3. A large number of famous artists from the first half the twentieth century were inspired by the concept of higher dimensions. The classical reference work on this subject is [Henderson 1983].
4. For an overview, see [Van Loocke 2008].
5. For the scales in our universe and for the cosmological concepts to which I am referring below, see [Primack and Abrams 2006].
6. For a keen discussion, see [Blackmore 2003]. For my position on consciousness, see [Van Loocke 2008: ch. 8].
7. See, for instance, [Penrose 1997]. In a statistical analysis of the opinions of scientists, the stance that Penrose defends would appear to have the support of a minority. The history of science and the observation of the sociological mechanisms at play suggest that one cannot draw very firm conclusions from this fact.
8. 'Arena' is a strong word: from Einstein's perspective, all change is illusion and only static structures in four-dimensional space-time exist – otherwise, for instance, our past could change.
9. Einstein's opinion implies that the future is fixed, which is too deterministic in view of quantum mechanics. The discussion referenced is whether small or large variations of Einstein's view are tenable.
10. These recent scientific insights come close to old Buddhist propositions: see [Van Loocke 2008: ch. 9] and [Blackmore 2003].
11. For a more extended description, see [Dunlap 1997].
12. See for instance the work 'Four Sided Vortex' at the beginning of the official site for Robert Smithson's work on www.robertsmithson.com
13. Through Anastasius Kircher (who had contact with Jesuits who had traveled to the East) Leibniz had become familar with Eastern concepts. Leibniz's cosmology, like the Eastern cosmologies he knew, is a correlative cosmology instead of a causal cosmology. Monads do not interact causally (like in classical physics one process influences another one by exerting a force on it or by bouncing onto it), but are rather synchronized beforehand by the creator of the universe, which explains the coherence of our world. The hierarchical relation between monads also agrees with Eastern philosophy [Van Loocke 2008].
14. One may wonder how far the analogy goes between Luria's cosmology and present scientific cosmology, since both approaches share the theme of an expanding universe. A more detailed comparison shows that there are indeed various similarities; see [Primack and Abrams 2006].

References

BLACKMORE, S. 2003. *Consciousness: An Introduction*. London: Hodder and Stoughton.

DUNLAP, R. 1997. *The golden ratio and Fibonacci numbers*. Signapore: World Scientific Publishing

HENDERSON, L. 1983. *The Fourth Dimension and Non-Euclidean Geometry in Modern Art*. Princeton: Princeton University Press.

JOUFFRET, E. 1903. *Traité élémentaire de géométrie à quatre dimensions.* Paris: Gauthier-Villars.

LU, Peter and Paul STEINHARDT. 2007. Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture. *Science* **315**: 1106-1110

PENROSE, R. 1997. *The Large, the Small and the Human Mind*. Cambridge: Cambridge University Press.

PRANGE, S. 2009. The tiles of infinity. *Saudi Aramco World* **60**, 5 (September/October 2009). Available at: http://www.saudiaramcoworld.com/issue/200905/the.tiles.of.infinity.htm. Last accessed 21 September 2011.

PRIMACK, J. R. and N. E. ABRAMS. 2006. *The View from the Center of the Universe. Discovering our Extraordinary Place in the Cosmos*. New York: Riverhead books.

ROBBIN, T. 2006. *Shadows of reality. The fourth dimension in relativity, cubism and modern thought.* New Haven: Yale University Press

SPROUT, B. 1991. *Primal Myths. Creation Myths Around the World*. New York: Harper Collins

VAN LOOCKE, Philip. 2008. *Het wereldbeeld van de wetenschap*. Antwerp: Garant (in Dutch).

About the author

Philip Van Loocke was born in 1963 in Bruges. He is an artist/philosopher/mathematician. He is also a professor at the University of Ghent, currently associated with the department of the Rector as Art/Science Liaison Officer. His website is http://www.philipvanloocke.com.

Maycon Ricardo Sedrez*
*Corresponding author
Rua Paraíba, 115 - Sala 01
Centro - Blumenau - SC - Brazil
CEP 89012 130
mayconsedrez@gmail.com

Alice T. Cybis Pereira
Universidade Federal de Santa Catarina
PósARQ – CTC – Caixa Postal 476
Trindade – Florianópolis – SC - BRAZIL
CEP 88040-900
acybis@gmail.com

Didactics

Fractal Shape

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010.

Abstract. This present paper deals with the fractal geometry applied in architecture. The generative rules for fractals can be used to develop the students' knowledge about shape grammar. Students use that knowledge to increase their own shape vocabulary in the early stages of architectural design.

Keywords: fractal geometry, shape grammar, architectural design

Introduction

What do fractals have to offer to contemporary architecture? Since Peter Eisenman's 1985 project "Moving Arrows, Eros and other Errors," some architects have continually used fractal geometry as a shape design instrument. Previously, architects included aspects such as fractal recursiveness, to take one example, in an intuitive way. Nowadays some studies have pointed out appropriate models for the potential use of fractals in architecture.

This experiment presented here was carried out with architecture students and attempted to bring fractal geometry to computer-aided architectural design (CAAD). We elaborated a program directed to architecture called "Fractal Shape," concentrating on the generative characteristics of fractals. This content provides an overview of fractal architecture, presents methods for using fractals in the design process, and proposes activities for the early stages of design. Thus, the fractal order is explored through generative exercises that deal with concepts of scale, recursivity, self-similarity and iteration.

The aim of the study is to facilitate comprehension of fractal geometry as a shape grammar, using scale transformations which provide generative rules. When these experiments are applied in the early stages of design, it increases the students' knowledge of shape vocabulary. The student identifies patterns, symmetries and design resources during the transformations of shapes and scales. The idea is to allow them to develop their own shape vocabulary well before they advance to the definition of the design.

We developed a program that relates fractal grammar and architectural design, which we established through pedagogical criteria and brought to the students through on-line virtual learning. The hypertext activities are about the creation of guiding regulatory frames for fractals, and then the transformation of the two-dimensional objects into three-dimensional objects. To do those exercises the students use addition and/or subtraction of elements, shape configuration and grouping and, especially, the extrusion of fractal objects viewed in plan or section. During these procedures the student selects and creates shapes, working on ambiguous models. They then set a proportion and a utility for the object, in a process called scale and function emergence.

Nexus Netw J 14 (2012) 97–107

DOI 10.1007/s00004-011-0099-8; *published online* 14 January 2012

The study of a fractal shape grammar is interesting from a pedagogical point of view when it combines the areas of interest of the students involved in this research: contemporary architecture content, the shape grammar exercises and the computer-aided architectural design technology. Students are also interested in learning subjects that are closely related to the reality of contemporary architecture. Finally, for the design professor, we see fractal geometry as one of the available tools which helps instigate students' creativity.

Fractal shape

Fractal geometry arose at the end of the 1970s from the research of Benoît Mandelbrot [1983, 1998]. Despite the creation of architectural works that have fractal concepts, long before the establishment of that theory, it was Peter Eisenman who first used that geometry to justify a project [Eisenman 1988]. The inclusion of fractal geometry in the architecture design grew increasingly popular in many ways in the years that followed. Our study presents a teaching methodology to introduce that content and the results of using fractals as a tool for generative shapes in the early stages of design. We believe that before the students come into contact with the complexity of architectural design, they should handle shapes, creating their own visual vocabulary. So the union of fractals with contemporary architecture as a generative tool is the approach that supports this research.

The authors have developed the course "Creativity and CAAD" which was first offered online to undergraduate architecture students at the Santa Catarina Federal University (UFSC) in the second half of 2008. The course content was made available through a virtual learning object, using hypertext[1] that was produced for the Virtual Learning Environment for Architecture and Design (AVAAD in Portuguese). This environment is developed and maintained by the lab Hiperlab from UFSC and used by students who follow the courses through chapters of hyperbooks.[2] Pereira [2007] explains that virtual learning environments make content accessible through computers, thus serving as basis for various media that compose the teaching strategies of this research. In this context it is believed that the AVAAD contributes to the teaching of fractal architecture due to its characteristics of providing resources to support diverse teaching-learning processes.

Fractals

Before discussing methodology, we will briefly explain the characteristics of fractals that make its study relevant to architects. Fractal shapes are extremely irregular and discontinuous. According to Mandelbrot [1998], such forms are natural and chaotic, and fractal geometry can be represented by rough, porous or fragmented objects. Fractals are irregular and self-similar, have infinite complexity, are developed through iterations, are dependent on an initial condition, and are common in nature [Lorenz 2002].

We can say that fractal geometric objects are generated by recursive[3] processes where an initiator and a generator are iterated[4] an infinite number of times. Thus a part of a fractal is equal, similar or similar in all. This is possible because these objects are formed by a process of repetition. According Lorenz, a self-similar structure is changed by modifying the structure by the same factor of scale: "the new form can be smaller, larger, rotated and/or reversed, but the shape remains similar" [2002: 10].

The idea behind fractals is the iteration of simple mathematical expressions with a rigid order specified in origin, which produces a very complex and irregular behavior that seems random [Espanés 2001: 144]. There are several ways to consider fractals in architecture: through its recursive patterns, as generative patterns, as tools of scale perception. Translating the characteristics of fractal geometry in order to apply them in architecture allows the students to create a shape vocabulary. As Barbosa [2002] explains, the use of fractal geometry is important in the classroom because of its connection with various sciences, the dissemination and access to computer technology, the existence of beauty in fractals and the possibility to awaken and develop an aesthetic sense in the student. For Montenegro [1987], to use fractal geometry in didactics is to work with the aesthetic sense, the beauty of shape, the organization or the order as a concept of beauty, and therefore requires the student's intuition. Barbosa and Montenegro both cite the focus on the intuitive and aesthetic appearance of fractals; in architecture we discuss these issues when fractal geometry suggests a generative system of shapes.

This characteristic of fractals to behave as a generative system developing morphologies has been used recently, especially in the creative context. Celani [2003] explains that generative design is a process of generating shapes from rules, and argues that fractal geometry is an example of a generative system with creative attributes. We can say that the potential of fractals in the generation of shapes begins to be exploited in architecture. Espanés believes that the use of fractal geometry "in the creative process contributes to translate the essential idea in architectural form" [2003: 118]. When multiple solutions are listed through a generative process, we open a space for individual creativity.

Fractal architecture

In confirming the existence of architecture with fractal attributes, we can name many projects that intuitively make use of fractal forms, even in periods prior to the creation of the term fractal, such as Hindu temples, Gothic cathedrals and decorative patterns in Africa. Some contemporary architects clearly show inspiration of the fractal in their designs. The fractal architecture defined by Jencks [2002] is diverse in forms; architects work with curves, lines, planes or volumes that refer to fractals. The virtual learning object (hyperbook) developed for our course cites the main buildings of fractal architecture, based on indications provided by Jencks [2002], Baier and Sedrez [2001], Ostwald [2001], Sedrez [2009] and other buildings that have recently used fractals in the composition, for example, pavillions developed by the London AA School students.

Other research projects have looked into the creation of methods to apply fractal geometry in architecture; these can be divided into three types:

1. Conceptual methods, which use fractal geometry and its concepts as a guiding element to its theories (Eisenman [1988]; Salingaros [2005]; Haggard, Cooper and Gyovai [2006]). The conceptual methods provide theoretical solutions that ultimately influence the final form, for example, Haggard, Cooper and Gyovai [2006] used holistic characteristics and endless scales aiming at the creation of sustainable architecture.

2. Geometric-mathematical methods, which use the scheme of counting squares to calculate the fractal dimension (Bovill [1996]; Sala [2000], Lorenz [2002], Capo [2004]) and which use computer calculations for fractal simulation (Çagdas, Edïz and Gözübüyük [2006, 2005]). Mathematical methods are useful for an analysis of existing buildings. But the experiment of Çagdas, Edïz and Gözübüyük [2006] goes further, using a shape generative algorithm based on fractals for the early

stages of design; the architect inserts data of the topography, a fractal dimension and the inicial shape; a software then searches a library of architectural shapes and generates alternatives.

3. Geometric-intuitive methods, which use the geometry as inspiration for creative expression (Espanés [2003]; Architectural Association School of Architecture [Hunter 2006]). We selected the geometric-intuitive method of Espanés [2003] as the first pedagogical strategy, because it happens to be the most appropriate for the main object: teaching CAAD with a creative approach from fractals as shape generative system.

The method of Espanés arose from research to develop "geometrical and morphological guidelines for design applying fractal geometry and to determine the creative relations between the ideas contained within this new order and architectural forms" [2003: 13]. Compositional and design activities are contextualized by Espanés, who uses modeling techniques with materials such as wood and paper to demonstrate fractal aspects that might produce architecture. The shape vocabulary of fractals is used to create shapes where the observer/designer decides what aesthetic he/she wants to highlight or achieve.

Espanés makes an analysis of fractal morphology applied to architecture, suggests compositional and design activities without regard for the constructive, functional and scale aspects. Therefore, the conjunction of Espanés' method with that of Baier and Sedrez [2007] proved to be relevant and was adopted as a second teaching strategy. Based on a mathematical approach to the history of architecture, Baier and Sedrez advance to the use of software for drawing geometric shapes, providing an alternative to the teaching of mathematics aimed at CAAD using software that builds three-dimensional graphics.

According to Baier and Sedrez, "the beautiful and complex shapes of the graphics inspire the development of sketches of architectural shapes, contributing to the architect's formation" [2007]. In the phase when they come into contact with the mathematical knowledge, the architecture students have little knowledge about architectural program, building technologies and other aspects of architecture. Therefore, there is still a vital creative freedom with regard to the shapes, resulting in proposals based on each student's empirical knowledge.

When the methods of Espanés and Baier and Sedrez are combined, they bring out another aspect of the fractal architecture, especially because it is studied using CAAD. We have developed other learning strategies that encourage students' creative thought while learning to use Google SketchUp. Through techniques for creation of shapes, students understand the functions and commands of the software, understand the mathematical relations of the fractal geometry generative system and learn about the creative potential of fractal architecture. The virtual learning environment features synchronous tools and, especially, asynchronous tools for communication: e-mail, links to web sites, forums, document sharing, and databases, all of which are provided by AVAAD simply and affordably.

In keeping with the characteristics of the method of Baier and Sedrez, activities with specific, rigidly defined architectural programs were not imposed on students; we used function emergence as the pedagogical line of thought. The final generated shapes of the suggested fractals were freely adapted to the programs by the student's creativity. So after a series of geometric, aesthetic and creative decisions, with the final volumetric shape set the student can choose the appropriate size for that object, adding model dolls or figures[5] and proposing architectural information through doors/windows, landscaping, furniture and surfaces, finishes, colors.

Structure of the discipline

The course lasted eighteen weeks. In the first two weeks extracurricular activities from the architecture course were developed, aimed at familiarizing the student with the virtual environment – AVAAD. This environment mediated between the processes of teaching and learning. Students were asked to install the CAAD software in use, Google SketchUp. This software is free, and has a language that is easily understood by all students. In a first step we introduced historical and creative aspects of CAAD and the main controls of the software through exercises that explore creativity. This stage of the course was adapted from the master's thesis by Bruno Ribeiro Fernandes [2006] and serves as the introduction to Google SketchUp. The second stage of the course was developed for Maycon Sedrez master's thesis [2009] and is the focus of this article. This presents the historical and mathematical aspects of fractal geometry, the evolution of the applications of fractals in architecture, key projects and methods, and a model with practical activities using fractals as a shape generative tool.

Using Bloom's Taxonomy of Educational Objectives [1972] we have elaborated Learning Objectives for the initial familiarization with fractal geometry and with fractal architecture, as shown in fig. 1.

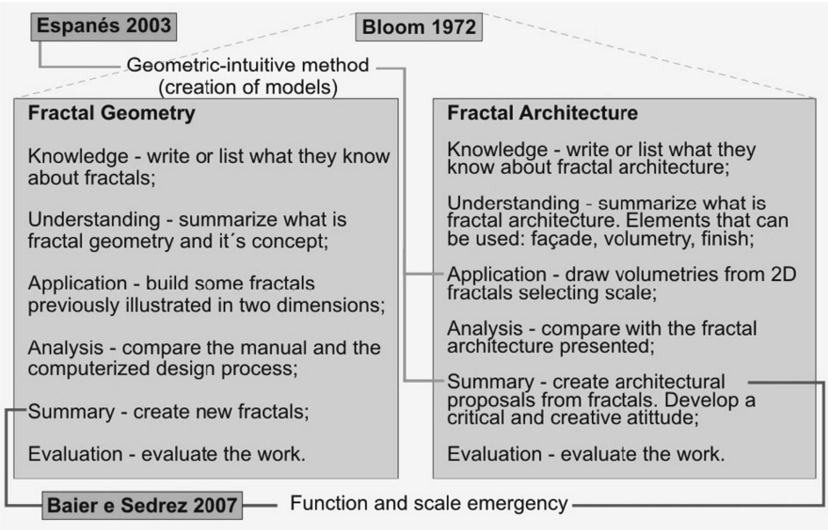


Fig. 1. Pedagogical strategies map [Sedrez 2009]

These two modules (familiarization with fractal geometry and fractal architecture) form the content of Fractal Shape and were taught at a distance through AVAAD. Based on the fractal objects drawn in the first stage seen in plan or section, the students are guided to the creation of three-dimensional objects. At this moment, doing the composition activities, they already have knowledge about the recursive behavior of fractals and how it generates shapes. They also know about contemporary fractal architecture and how architects incorporate these iterative patterns. Finally the design activities are proposed as follows.

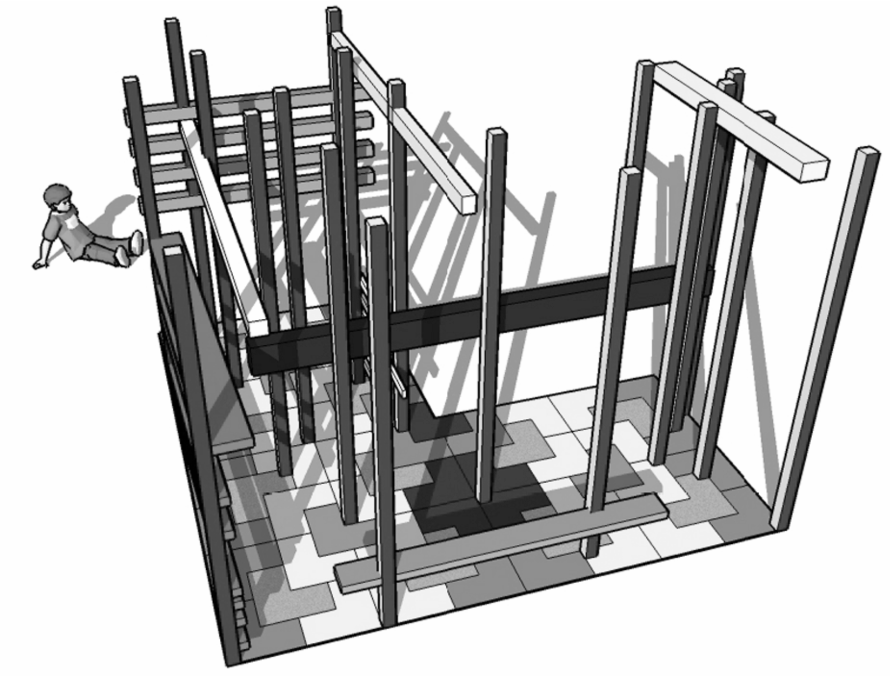


Fig. 2. Design by Luciano Santana Portella [Sedrez 2009]

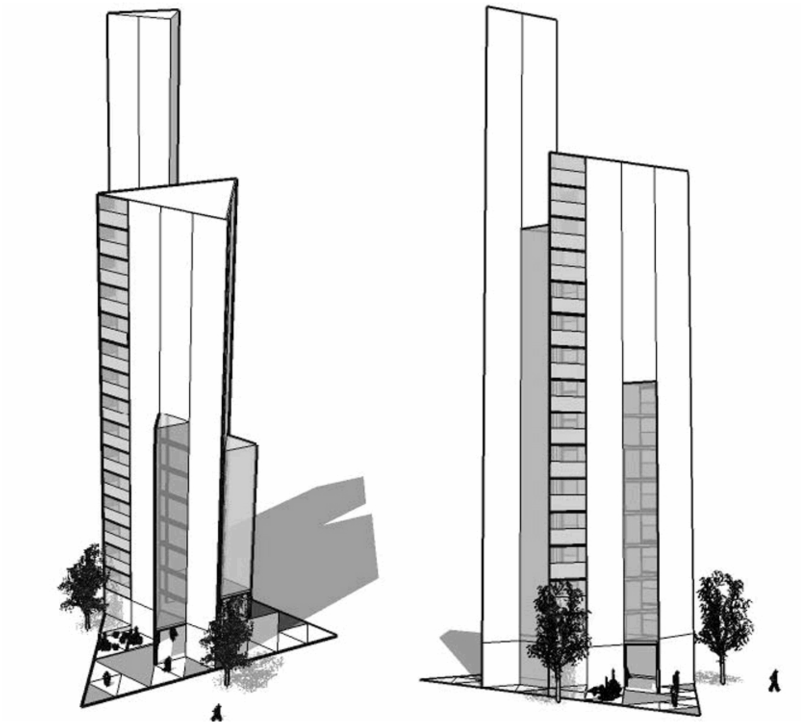


Fig. 3. Design by Luiz Henrique Fernandes [Sedrez 2009]

Regulatory outline. The student must assemble a combination of vertical and horizontal elements whose arrangement is governed by the floor plan using the fractal "L" as base. The choices of the type of object, the dimensions and materials are at the discretion of the student, who can compose the elements randomly, following a numerical pattern, or symmetrically. Luciano created a composition for a playground, observing the appropriate scale (fig. 2). The regulatory outline defines a complex volume that is apparently irregular. This type of order is useful as experimentation and as a resource for the architectural design.

Horizontal extruded section. The objective is to extrude parts or the whole of the subdivided plans of any fractal. The distribution can follow rules of symmetry or proportion, calling on students' sense of composition and scale. The student Luiz preferred to use Sierpinski triangles to model an office building with sixteen floors (fig. 3). This activity asks students to work with aesthetic appearance through colors, elements and styles. Students understand how to work with unusual angles on Google SketchUp. The horizontal extruded section leads to the understanding of how an architectural composition can be developed with the addition or subtraction of elements through a complex order.

Two-dimensional fractal modules. We use the Hilbert curve, which can be rotated, inverted and mirrored to allow further configurations. The section of the fractal must be transformed into a volume, then copied several times. The modules are extruded and juxtaposed through the aggregation of elements. Students regulate the composition, creating a rhythm of elements. Some indications of architecture are also required for the definition of scale, for example, the addition of human figures and stairs, windows, landscaping, and so forth. Letícia proposed a residential building with twin units (fig. 4).

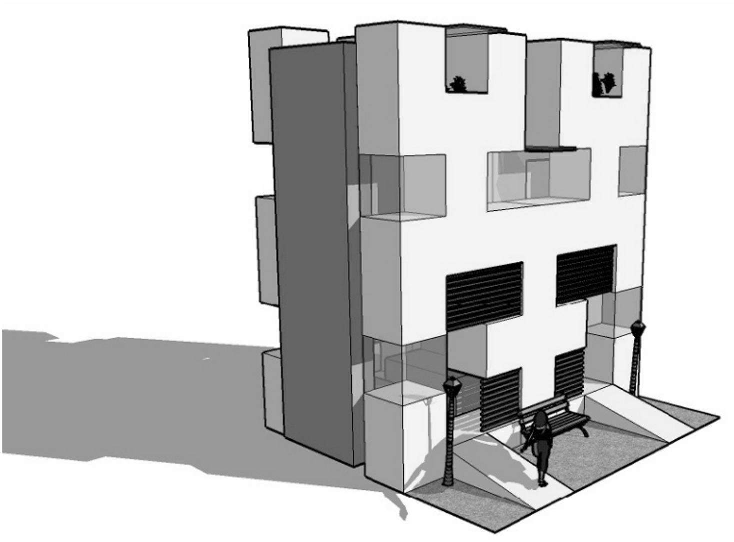


Fig. 4. Design by Letícia Longo Triches [Sedrez 2009]

Vertical extruded section. From a replicated linear fractal, we build up different objects, and then put them together and re-adapt them. Students start with abstract manipulation moving towards architectural forms. They can use a Hilbert curve or another object designed earlier and make the extrusion of the object into three parts, representing sections. The definition of the scale should be improved, moving increasingly closer to that of an architectonic design model. The student Patricia created a multifunctional building for parties and meetings (fig. 5).

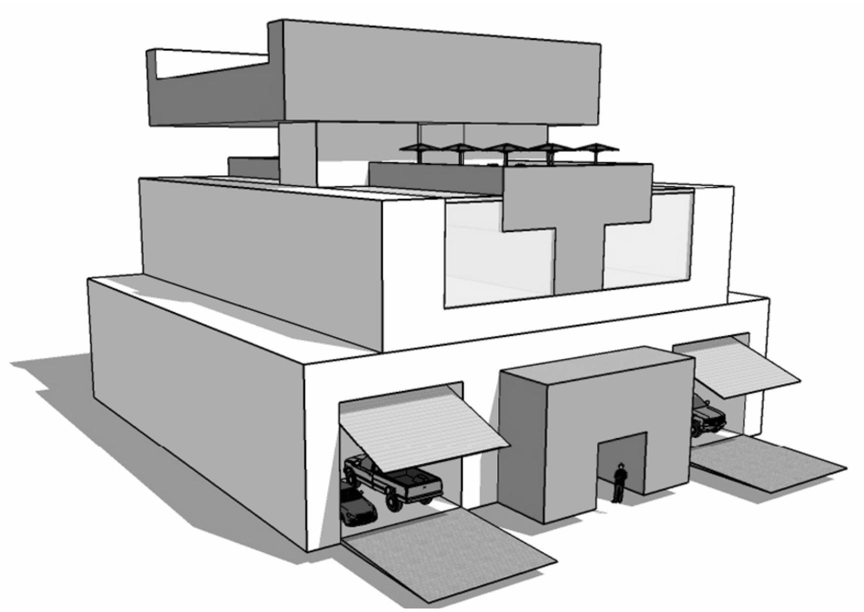


Fig. 5. Design by Patricia Ramos [Sedrez 2009]

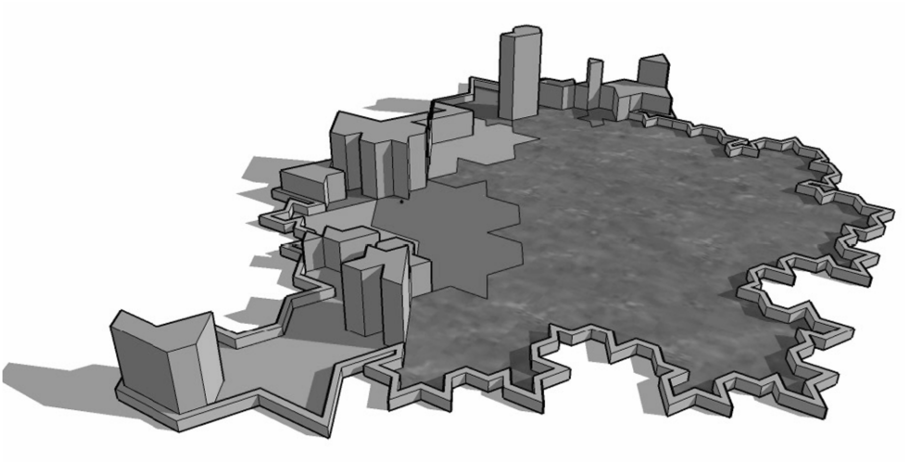


Fig. 6. Design by Fernando Carneiro Pires [Sedrez 2009]

Scale superposition. The Koch curve and Sierpinski triangle are exemplified for this activity. A fractal that is re-scaled and superposed several times creates in its intersections buildings, squares and a territory. The student is asked to multiply the object at three different scales, superposing them. This activity aims to extend the student's attention to the issue of urban scale and how buildings can be placed in context. Fernando created a set of buildings on the edge of a lake or reservoir, in this activity, most studies showed only volumetric intentions (fig. 6).

Conclusion

This paper aims to provide architecture students with tools for manipulating shapes, before deepening their knowledge of architectural design. At this stage students can explore morphologies with greater freedom. The "Fractal Shape" hyperbook activities used the concepts of function and scale emergence [Baier and Sedrez 2007; Mayer and Turkienicz 2008]. We felt that students received the content and the methods in a positive way. We have noted that the forms produced starting from the same basic fractal objects were completely different for each student. These issues were also reported by students in the comments of activities.

The students developed their knowledge of the software and complex shapes, improving their visual vocabulary. Students were taught useful shortcuts and commands in the design process of fractals that can be used in any other project. Decisions regarding the use of Google SketchUp to generate shapes and select the scales were intuitive, that is, made naturally without requiring much effort. Proposals reflected the harmony and symmetry that are characteristic of fractals, but because these were initial compositions, students did not have to completely resolve issues such as combination of colors and materials or aspects of the architectural program.

We highlight students' perception of fractal geometry as a regulatory framework that facilitates creative composition. There is therefore a diversity of fractal architecture and it can be incorporated into any project level. This feature is supported by a study prepared by the AA School [Hunter 2006], whose students, guided by teachers from different areas, developed and built a small pavilion.

The good level of content understanding and creative results highlighted the personal approach of each student. The digital models produced were not detailed, and this is a step that is intended for the development of the course. The next step will be a discussion and evaluation of the proposal to teach fractals in architecture.

We perceived the necessity of more time for design activities. Another issue that can also be addressed is the complete elaboration of one activity, where the student must present a more detailed design for a real site. This research showed that the most important quality of fractal geometry for architecture lies in its generative system of shapes.

All activities can include other fractal objects and especially improve the use of nonlinear fractals. For this we need to advance research on available computer programs that do a reading of fractal objects with a high number of iterations. We intend to continue to create activities with fractals as architectural shapes. In order to continue this research, we want to create a set of composition activities for architecture students to develop before going into the technical aspects of architecture design.

Acknowledgment

We thank the students who participated in this course and all who contributed to the work.

Notes

1. "Hypertext" is the group of nodes linked by connections; the nodes can be words, pictures, graphics, sounds, documents [Filatro 2004].
2. "Hyperbook" is the grouping of hypertext in units (or subunits) that allows linear and random navigation.
3. "Recursive" is the procedure used to generate a fractal, the successive application of the same routine.
4. An "iteration" occurs each time a fractal is fed with a self-similar shape through a recursive process.
5. In Brazil we call a small models, doll or figure *calunga*, representations of the human figure.

References

BAIER, Tânia and Maycon R. SEDREZ. 2001. Geometria fractal e arquitetura contemporânea – Kisho Kurokawa. In: *Salão de Iniciação Científica* 13. Porto Alegre: UFRGS.

———. 2007. Uso de gráficos de funções matemáticas, construídos com recurso computacional, na elaboração de projetos arquitetônicos: uma proposta pedagógica. In: *Congresso Nacional de Matemática Aplicada e Computacional*, Florianópolis: SBMAC.

BARBOSA, Rui M. 2002. *Descobrindo a geometria fractal para sala de aula*. Autêntica: Belo Horizonte.

BEHRENS, Marilda Aparecida. 2006. *Paradigma da complexidade: metodologia de projetos, contratos didáticos e portfólios*. Petrópolis: Vozes.

BLOOM, Benjamin Samuel. 1972. *Taxionomia de objetivos educacionais. Vol. 1 Domínio Cognitivo.* Porto Alegre: Globo.

BOVILL, Carl. 1996. *Fractal Geometry in Architecture and Design*. Boston: Birkhäuser.

ÇAGDAS, Gulen, Gaye GÖZÜBÜYÜK and Özgur EDİZ. 2006. Fractal based design model for different architectural languages. Pp. 280-286 in *GameSetandMatch II*. International Conference on Computer Games, Advanced Geometries and Digital Technologies, Delft University of Technology, Delft, Netherlands, 29 March – 1 April 2006. K. Oosterhuis, L. Feireiss, eds. Rotterdam: episode publishers.

———. 2005. Fractal based generative design for harmony between old and new. Pp. 150-159 in *GA 2005*, Proceedings of the 8th International Conference on Generative Art, Generative Art 2005, Milan, Politecnico di Milano. http://www.generativeart.com/on/cic/papers2005/14.GayeG%F6z%FCb%FCy%FCk.htm. Last accessed 13 June 2011.

CAPO, Daniele. 2004. The Fractal Nature of the Architectural Orders. *Nexus Network Journal* 6, 1: 30-40.

CELANI, Maria Gabriela. 2003. *CAD criativo: exercícios para desenvolver a criatividade de arquitetos na era digital*. São Paulo: Campus.

EISENMAN, Peter. 1988. Eisenmanesie. *Architecture + Urbanism*, Extra ed. (August 1988): 70.

ESPANÉS, Inés Moisset de. 2003. *Fractales y formas arquitectónicas*. Córdoba: I+P División Editorial.

———. 2001. El desafio de los fractales. *Summa+* 51 (outubro e novembro 2001): 144.

FERNANDES, Bruno Ribeiro. 2006. Estratégias pedagógicas de uso de técnicas de computação gráfica como instrumento de apoio ao processo criativo de projeto de arquitetura. Master's thesis, Programa de Pós-graduação em Arquitetura, Universidade Federal de Santa Catarina, Florianópolis.

FILATRO, Andrea. 2004. *Design instrucional contextualizado: educação e tecnologia*. São Paulo: Ed. SENAC.

HAGGARD, Kenneth, Polly COOPER and Christine GYOVAI. 2006. *Fractal Architecture: Design for Sustainability*. North Charleston: BookSurge Publishing.

HUNTER, Will. 2006. On the Cutting Room Floor. *Building Design* (23 June 2006). http://www.bdonline.co.uk/story.asp?sectioncode=453& storycode=3069350. Last accessed 13 June 2011.

JENCKS, Charles. 2002. *The New Paradigm in Architecture: The Language of Post-Modernism*. New Haven: Yale University Press.

LORENZ, Wolfgang E. 2002. Fractals and fractal architecture. Master's thesis, Vienna University of Technology.

MANDELBROT, Benoit B. 1983. *The Fractal Geometry of Nature*. New York: W. H. Freeman.

———. 1998. *Objetos fractais: forma, acaso e dimensão seguido de panorama da linguagem fractal*. 2 ed. Lisbon: Gradiva, 1998 (Portuguese trans. of *Les objects fractals*).

MAYER, Rosirene and Benamy TURKIENICZ. 2008. The cognitive studio: exercises in design learning. In: Third International Conference on Design Computing and Cognition DCC08. Atlanta: DCC08.

MONTENEGRO, Gildo A. 1987. *A invenção do projeto: a criatividade aplicada em desenho industrial, arquitetura, comunicação visual*. São Paulo: E. Blucher.

OSTWALD, Michael J. 2001. "Fractal Architecture": Late Twentieth-Century Connections Between Architecture and Fractal Geometry. *Nexus Network Journal* **3**, 1 (Winter 2001): 73-83.

PEREIRA, Alice Cybis, ed. 2007. *Ambientes virtuais de aprendizagem em diferentes contextos*. Rio de Janeiro: Ciência Moderna.

SALA, Nicoletta. 2000. Fractal Models in Architecture: A Case of Study. Pp. 18-23 in *Proceeding of the International Conference on "Mathematics for Living,"* Jordan, November 2000. http://math.unipa.it/~grim/Jsalaworkshop.PDF Last accessed 13 June 2011.

SALINGAROS, Nikos A. 2005. *Principles of Urban Structure*. Amsterdam: Techne Press.

SEDREZ, Maycon R. 2009. Forma fractal no ensino de projeto arquitetônico assistido por computador. Master's thesis, Programa de Pós-graduação em Arquitetura e Urbanismo, Universidade Federal de Santa Catarina, Florianópolis.

About the authors

Alice Theresinha Cybis Pereira graduated in architecture and urbanism from the Federal University of Rio Grande do Sul (1982), specialized in Housing at the same University (1983) and received a Ph.D in Architecture and Computer Graphics from the Faculty of Architectural Studies at the University of Sheffield, England (1992). Currently, she is a professor at the Universidade Federal de Santa Catarina, teaching Design, Architecture and Knowledge Engineering and Management. She has experience in Architecture, with emphasis on CAAD, in virtual learning environments, and hypermedia.

Maycon Ricardo Sedrez graduated in architecture and urbanism from the University of Blumenau (2003), and earned a master's in architecture and urbanism from the Federal University of Santa Catarina (2009). He works as a collaborator at the University of Blumenau. He is an associate architect of the M2 Architecture Studio in Blumenau, and he continues his research in fractals and architecture. He has experience in residential and commercial architeture and interior design. He is also an artist and illustrator.

Jürgen Bokowski*
*Corresponding author
Technical University Darmstadt
Department of Mathematics
64289 Darmstadt GERMANY
juergen.bokowski@googlemail.com

Heike Matcha
Technical University Darmstadt
Department of Architecture
Institute of Design
64289 Darmstadt GERMANY
hm@hmgb.net

Keywords: Möbius strip, parametric design, didactics

Didactics

Möbius Strip Segmented into Flat Trapezoids: Design-Build Project by the Departments of Architecture and Mathematics of the Technische Universität Darmstadt

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010.

Abstract. The Möbius strip trade fair stand was designed and built by students of the Faculty of Architecture of Darmstadt Technical University in collaboration with students of the Faculty of Mathematics. The project focused on two main issues: to introduce computer-driven parametric design and production strategies, techniques and technologies into architectural education, and to build and operate the final result. This required knowledge of different software packages and readiness to become familiar quickly with unknown data formats, flexibility in the time schedule and disposition to transport the material to the machinery. It introduced students to the possibilities of designing with parametric modeling software and familiarized them with the initial difficulties and ample payback in actual production.

1. Introduction: the Möbius strip as a trade fair stand

The Möbius strip trade fair stand is part of a series of three trade fair stands that were designed and built by students of the Faculty of Architecture of Darmstadt Technical University (fig. 1). This series of 1:1 design and build projects focused on two main issues: firstly, to introduce computer-driven parametric design and production strategies, techniques and technologies into architectural education; secondly, to not just design a project, but to actually build and operate it.

Parametric design and production enables architects to construct their designs from elements that are neither completely identical nor totally different – as was dictated by the principles of serialized mass production. Instead, elements can be similar and varied, like members of a family. This permits greater complexity and diversification within the built structures and promises more appropriate designs that can respond better to the specific individual characteristics of users, program, site, etc. Most educational projects in architecture schools never leave the drawing board, so to speak. While this is understandable with large-scale projects like large cultural buildings or residential complexes, it excludes from architectural education the very important development a design project undergoes as it is built: ideas have to adapt and change according to the restraints and possibilities of budget, availability of material, manufacturing and building machines and issues of transportation. In this development, quite different skills are needed than the conceptualising and presentation skills necessary for designing. Instead, organization, management, craftsmanship and aptitude with materials and tools become essential. The annual Hobit fair of the Technical University Darmstadt, where prospective students become acquainted with the university and its faculties and

Nexus Netw J 14 (2012) 109–118

DOI 10.1007/s00004-011-0100-6; *published online* 13 January 2012

departments, provided a perfect occasion to conduct a course that merged our two educational ambitions: the architecture faculty needed a fair stand that would provide information about the courses and, through its sculptural qualities, would also communicate what the study of architecture is about. The Möbius strip was chosen as basis for the stand's shape because it inherently displays fascinating aspects of geometry, and because constructing it on the scale of a fair stand requires geometrical and constructive effort that would illustrate important issues of the study of architecture.

Fig. 1. Series of three trade fair stands clockwise: Möbius strip, Parametric Origami and Cardboard Wave. Photos top and bottom right by Stefan Daub; photo bottom left by Thomas Ott

Moreover, the Faculty of Mathematics had expressed its interest in collaborating on the stand, and the Möbius strip is one of the most iconic and well-known mathematical objects. The collaboration between the two faculties would also become an important part of the exhibition, as collaboration with different disciplines is an important part of architectural work.

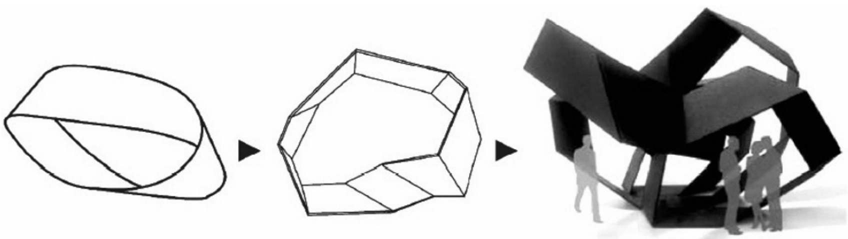


Fig. 2. Transformation of Möbius strip to segmentation into flat trapezoids. Images courtesy of Technische Universität Darmstadt

We chose to segment the Möbius strip into several flat trapezoids because it would provide us with different possibilities of use: it could become a counter, a shelf, a table and other things. As it turned out, the segmentation was much more difficult than expected (figs. 2, 3).

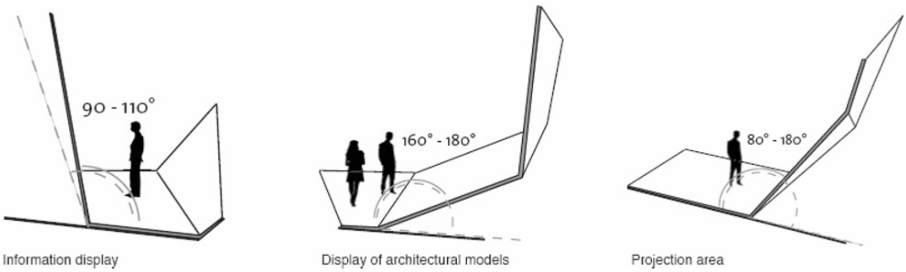


Fig. 3. Flat trapezoids provide different possibilities of use.
Images courtesy of Technische Universität Darmstadt

2. Mathematical problem: constructing a Möbius strip out of regular flat trapezoids

We introduced the topic of Möbius strips through a talk from a mathematical point of view. Its focus, though, lay not in a discussion of topological issues – an area that would usually be opened up through such an introduction. Instead, we focussed on geometrical issues, partly because of the task at hand, and partly because past experiences with students of architecture have shown that projective geometry, let alone higher-dimensional aspects are not a very suitable topic for them. This time, though, a higher-dimensional model came into being naturally from the very beginning. The final mathematical way to see the problem can be written as follows. We prepare a rectangular paper strip with endpoints A,B,C, and D, say in counter-clockwise order. We identify or we glue line segment AB with line segment CD, such that A equals C and B equals D. The resulting surface with boundary becomes non-orientable, and we obtain a Möbius strip. Now we draw a regular zig-zag with end points on the boundary BC and AD of the Möbius strip such that the strip will be partitioned into isosceles triangles. We next use a remaining smaller strip by cutting off another strip parallel along the Möbius strip´s boundary. Thus from each triangle we are left with a trapezoid that we than require to be of flat shape. The edges of adjacent trapezoids are considered to function as hinges. Of course, we require to have these trapezoids without self-intersections (fig. 4).

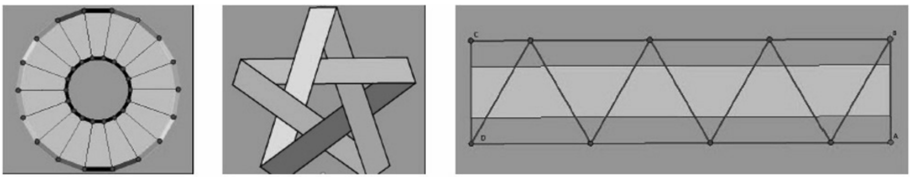


Fig. 4. Möbius strip when antipodal line segments identified (left), knotted Möbius strip (middle), Möbius strip partitioned into isomorphic trapezoids (right).
Images courtesy of Technische Universität Darmstadt

Mathematical questions immediately arose that were close to what the architects were interested in. Under which conditions does such a model exist? There is, for example, a triangulation of the Möbius strip by Ulrich Brehm that cannot be realized with flat triangles and without self-intersections. Of course, the students in our "trapezoidulation problem" already had a lot of models showing their existence, didn`t they? However, when it comes to calculating the angles between adjacent trapezoids, we cannot trust the paper versions, can we? When the trapezoids become in a certain limit rectangles, we are sure that a solution cannot exist. And what happens near such a rectangular limit case? Let us assume that we have n trapezoids and therefore n angles between adjacent ones. Mathematicians introduce for each angle between adjacent trapezoids a dimension and they describe the problem as an n -dimensional one. The conditions A=C and B=D reduce the dimension n of the realization space by five: three conditions for the three coordinates of the requirement A=C, and only two for the condition B=D because the distance from A to B equals the distance from C to D. The resulting manifold of dimension n-5 that describes our solutions in this n-dimensional space is not connected. We have to assume in general that there are solutions with knots. A knot solution cannot be transformed continuously into an unknotted version. Has the reader noticed that n has to be odd and that we cannot choose n to be very small? Any suggested solution of a polyhedral Möbius strip of the architects with certain angles measured from their models is very unlikely to be a correct point on our (n-5)-dimensional solution manifold for polyhedral Möbius strips. Can we use nevertheless such an "angle point" of the architects in n-space to find in the vicinity of it a true solution on the solution manifold? Can we somehow project this point onto this manifold? It was good for the architects to have some mathematical advice for realizing their 1:1 model. The following formulation might tell the non-mathematician what the problem looked like for a mathematician. For simplicity, we assume for that the exact solution manifold is a curve in space, i.e., a one-dimensional manifold. In other words, only points on this curve can be accepted for a model. The students then suggested only "points" near this curve and the mathematical problem was to correct their data in order to get an acceptable solution point near the given one on the solution curve.

3. Designing and constructing the trade fair stand

The students had three months to design, manufacture and build the stand. We started with a design competition that produced a large number of alternative designs, all based on Möbius strips, and all accomodating different situations of use, like information desk and various display possibilities (fig. 5). One of these alternatives was elected by a jury for its use of parametrics and its promise in terms of buildability.

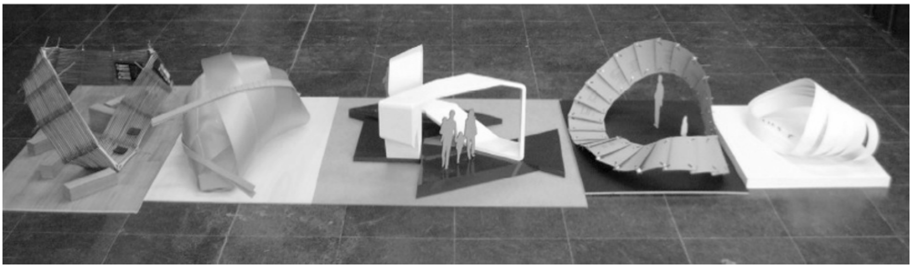


Fig. 5. Various Möbius strip designs for the trade fair stand.
Photo courtesy of Technische Universität Darmstadt

The students then worked as a group to develop the winning project further. Physical models were produced and tested as to their qualities as fair stands. The parametrics in this design stage consisted of varying the number and precise positioning of trapezoids to produce different shapes and possibilities for use. An important issue in producing and discussing these studies and their qualities consisted of making implicit factors explicit. Architects very often unconsciously set themselves a set of implicit rules that govern their designs but which are also often broken, leading to much confusion. We urged the students to make their implicit rules explicit so that their being broken woud surface and their productive qualities be discussed. To be able to actually build the Möbius strip, the trapezoids were further broken up into straight ribs. This was to resolve the complex 3D joint of the trapezoids, and the ribs provided additional means of display. The ribs were to be manufactured of glass and wood. The joints between the different strips or parts of the original trapezoids were designed to be adjustable so that the geometry of the Möbius strip closing in on itself could be realized on site. Without a mathematical solution or procedure, segmenting the strip geometry into flat trapezoids became very difficult and cumbersome as the last neighbouring trapezoids never matched up. The collaboration with mathematicians proved extremely fruitful as they provided the architects with an interactive software application that could generate many variations of perfectly segmented strips and export them in a file format that the architects' CAD packages were able to use, which will be discussed in the next section.

4. An interactive application that offers solutions to the mathematical / geometrical problem

Without an attempt to explain the mathematical background of the problem that we tried to describe in Section 2, the mathematical help for the architects was provided by a mathematics student, Sebastian Stammler. He used the mathematical software package MATLAB's built-in optimization software and wrote corresponding software parts in close collaboration with the architects. Wikipedia provides the following description:

> MATLAB (matrix laboratory) is a numerical computing environment and fourth-generation programming language. Developed by MathWorks, MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.

The set of angles between adjacent trapezoids served as the input of the software. An animation visualized how the starting strip could be moved to its final position as a polyhedral Möbius strip. This helped a lot to imagine what the precise solution would finally look like (fig. 6).

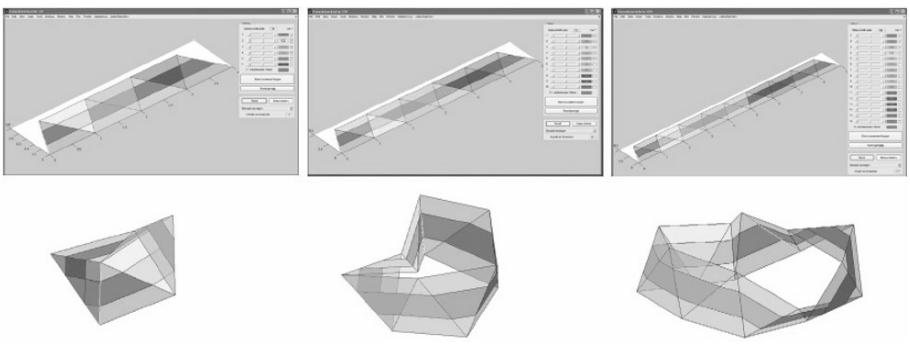


Fig. 6. Examples of configurations of Möbius strips with varying number of trapezoids.
Images courtesy of Technische Universität Darmstadt

Furthermore, all spatial information for manufacturing the trapezoids were produced and the interfaces for other production software were provided. It was possible to play with parameters like the number of trapezoids and the shape of the trapezoids defined via its angles or the width of the strip (fig. 7).

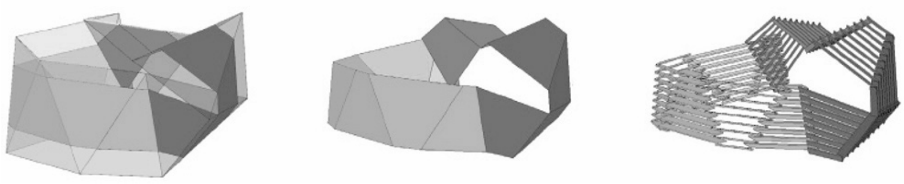


Fig. 7. The interactive MATLAB programme provides different segmentations.
Images courtesy of Technische Universität Darmstadt

5. Building and operating the trade fair stand

Building the trade fair stand with students made it necessary to limit the crafts involved to no more than three or four. Furthermore, since the university does not provide CNC tools or CAAM machinery that can produce objects larger than simple models, the students had to find regional companies that would provide machinery and material.

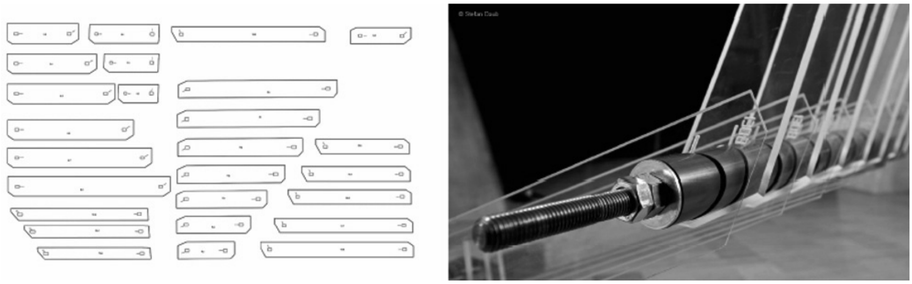


Fig. 8. Cut-out ribs of plexiglass and timber (left) and hinge detail made from PVC spacers and steel (right). Image left courtesy of Technische Universität Darmstadt; photo right by Stefan Daub

The two-dimensional segments made from plexiglass and timber were cut by a local joiner's CNC laser cutting machine. The three-dimensional spacing elements between the ribs were cut from a simple PVC pipe (fig. 8). The specific geometry for the spacers was produced by five-dimensional CNC milling. The data for both elements was generated in Rhino. The transfer to the laser cutter was handled through AutoCAD; the transfer to the milling machine through the mechanical engineering file-format Parasolid.

Whenever possible, the students operated the machines themselves and handled transportation and storage of the construction components. The assembly of the parts was tested twice: first before manufacturing the components, in a small-scale model (fig. 9), and a second time a few days before the opening of the fair in a kind of dress rehearsal at the architecture faculty over the weekend (fig. 10). Both tests proved invaluable and provided information that influenced both the construction details and the final assembly procedure on site.

After the final assembly in the spaces of the fair, the students took shifts to operate the stand, to explain its construction and principles to the visiting high-school students.

Fig. 9. Physical model in scale 1:5 (left) and pre-assembly of segmented parts (right). Photos courtesy of Technische Universität Darmstadt

Fig. 10. Pre-assembly of the Möbius strip. Photos courtesy of Technische Universität Darmstadt

Fig. 11. Möbius strip made from ribs of plexiglass and timber. Photo by Stefan Daub

Fig. 12. Display detail (left) and group of representatives at the Hobit fair (right). Photo courtesy of Technische Universität Darmstadt

6. Conclusion

The idea of the design of the Möbius strip consisted not only of one specific object: parameterizing the idea meant that a whole family of related objects was designed. Together with a fully integrated digital production chain, this would in principle allow for the production of a large number of fair stands, all stemming from the same design, but adjusted to situational specifics like place, size, arrangement of different functions. Realizing design-build projects in a non-CAAM environment constitutes a big challenge for all participants: knowledge of different software packages and readiness to become familiar quickly with unknown data formats, flexibility in the time schedule and disposition to transport the material to the machinery. The project not only introduced the students to the possibilities of designing with parametric modeling software, but also familiarized them with the initial difficulties and the following large payback in employing them for actual production. The finished stands went far beyond what they initially had believed possible. Employing the Möbius strip geometry proved indeed to communicate issues of geometry and the fascination of solving geometrical and technical problems and challenges. As we had hoped, the fair stand in itself became a showcase for architecture and mathematics. Although the stand contained a number of different exhibition objects, the Möbius strip geometry became an exhibit in itself.

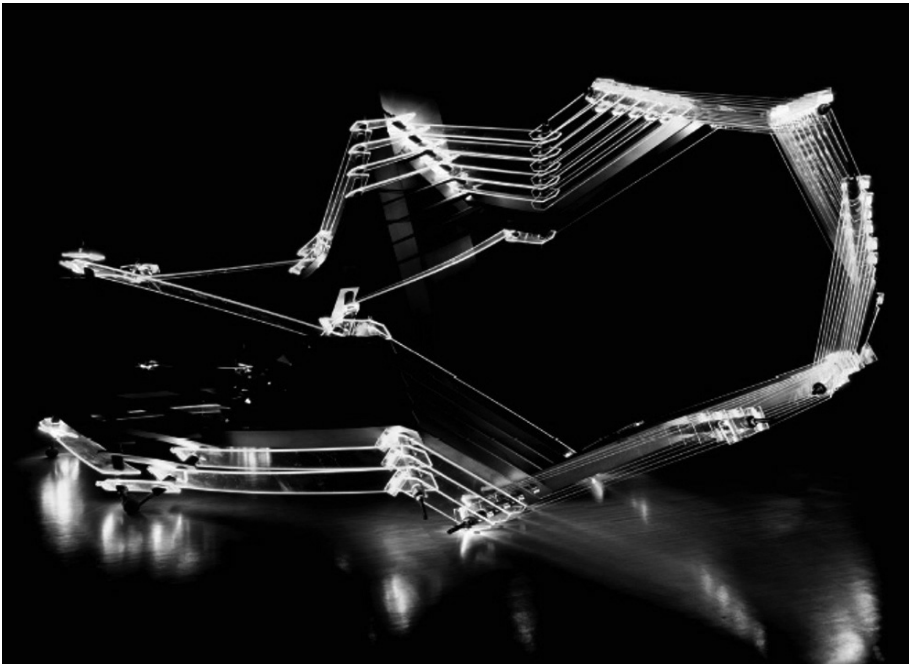


Fig. 13. Lighting installation of Möbius strip. Photo by Stefan Daub

Acknowledgments

The project was conducted in cooperation with Rüdiger Karzel, Institute of Design & Industrial Construction Methods of Professor Moritz Hauschild. Thanks to Sebastian Stammler for programming the Möbius strip geometry and to Jochen Stahl for the structural consulting and prototype testing. Thanks to our partners from the building industry for their support and to our department for funding our research projects. And of course many thanks to all the students who participated in the project.

Further information

Movie showing segmented Möbius strip:
http://hmgb.net/files/hmgb-expo15-2-film-matlab-1.mov
Movie showing interactive programme:
http://hmgb.net/files/hmgb-expo15-2-film-matlab-2.mov
Movie showing set-up of the built Möbius strip:
http://hmgb.net/files/hmgb-expo15-2-film-aufbau.mov

About the authors

Jürgen Bokowski, Electrical Engineer, Berlin 1964, studied mathematics, Ph.D. at Technical University Berlin, 1972. Habilitation at University Bochum 1979, Professor of Mathematics at Technical University of Darmstadt since 1981, Guest Professor at Université 6, Paris, 2004, and at Universidad National Autónoma de México, 2008/2009. Teaching activities included descriptive geometry for architects. For research interests, mainly convexity and combinatorial geometry, see his book entitled *Computational Oriented Matroids, Equivalence classes of matrices within a natural framework* (Cambridge University Press, 2006), which also shows many self-made mathematical pottery models.

Heike Matcha is an architect based in Berlin. After studying in Stuttgart and Aachen, she worked for practices in London and Berlin before setting up her own practice HMGB Architects in Berlin (www.hmgb.net) together with Günter Barczik. As Assistant Professor she taught at the Technical University of Braunschweig andDarmstadt. She researches new possibilities for architectural design and construction through parametric modelling and customized mass production. Results of her research have been presented in several conferences and publications.

Ute Poerschke
The Pennsylvania State University
429 Stuckeman Family Building
University Park, PA 16802 USA
uxp10@psu.edu

Keywords: Gottfried Semper, functionalism, nineteenth century, algorithms, calculus, differential equations, infinity

Research

Architecture as a Mathematical Function: Reflections on Gottfried Semper

Abstract. In a lecture of 1853, the architect and architectural theorist Gottfried Semper (1803-1879) explained a work of art as a mathematical function. The lecture was published for the first time in 1884 and the equation for the work of art was presented there as $Y = F(x, y, z$, etc.). Since then, several widely differing manuscripts, translations, and interpretations have appeared. The following essay describes Semper's equation, the variations and explanations he gave in his writings, and the interpretations by others that have followed up to the present. It discusses Semper's attempts to connect architecture with infinitesimal calculus, his mathematical background, and his desire to give architecture a scientific foundation through methods of systematic comparison and classification.

Introduction

The nineteenth-century discussion of architecture was shaped by the theme of the organic as contrasted with the mechanical, in reference to natural history, evolving biology, Romantic philosophy, and linguistics. The architect and architectural theorist Gottfried Semper (1803–79) found inspiration in these fields, too, especially in natural history. Deeply impressed by Georges Cuvier's and Alexander von Humboldt's developments of comparative methods in natural history, he wished to write a *Vergleichende Baulehre* (Comparative Architectural Theory).

In contrast to this frequently interpreted relationship between natural history and architecture in Semper's oeuvre,[1] the present text attempts to link architecture with mathematics and more specifically with infinitesimal calculus. It will address Semper's fascination with this relatively new mathematical field, which had only been established in the early eighteenth century, and the role it played in Semper's architectural theory in relation to questions of style and the classification of architecture.

Gottfried Semper's equation

In 1884 Hans and Manfred Semper published the manuscripts of their father's lectures under the title *Kleine Schriften* (Minor Writings), including a lecture Semper gave in English in London on November 11, 1853, which Hans Semper translated under the title "Entwurf eines Systemes der vergleichenden Stillehre" (Outline for a System of Comparative Style Theory). For roughly a century, Semper theorists used this translation as the basis for their interpretations of his mathematical understanding of architecture, until in the 1980s Wolfgang Herrmann and Harry Francis Mallgrave pointed out in a number of publications that Semper's son had translated the English lecture incorrectly, combining and editing two different manuscripts, and hence distorted the lecture and, more specifically, his explanation of architecture as a mathematical function, published as $Y = F$ (x, y, z etc.) [Herrmann 1981: 154; Mallgrave, Rykwert and Semper 1983; Mallgrave 1996: 5].

Nexus Netw J 14 (2012) 119–134

DOI 10.1007/s00004-011-0101-5; *published online* 13 January 2012

To help understand the course of the interpretations, therefore, this passage is quoted first in its German version:

> *Jedes Kunstwerk ist ein Resultat, oder, um mich eines mathematischen Ausdruckes zu bedienen, eine Funktion einer beliebigen Anzahl von Agentien oder Kräften, welche die variablen Koefficienten ihrer Verkörperung sind.*
>
> $Y = F\ (x, y, z \text{ etc.})$
>
> *In dieser Formel steht* Y *für das Gesamtresultat und* x, y, z *etc. stellen ebensoviele verschiedene Agentien dar, welche in irgend welcher Richtung zusammen oder aufeinander wirken, oder voneinander abhängig sind. Die Art dieser gegenseitigen Beeinflussung oder Abhängigkeit ist hier durch das Zeichen* F *(Funktion) ausgedrückt. …*
>
> *Es sind zwei Klassen von Einflüssen zu unterscheiden, welche bei der Entstehung eines Kunstwerkes bestimmend einwirken. Die erste derselben umfaßt diejenigen Anforderungen, welche in dem Kunstwerke selbst begründet sind und auf gewissen Gesetzen der Natur und des Bedürfnisses beruhen, die zu allen Zeiten und unter allen Umständen sich gleich bleiben.*
>
> *Diese Klasse von Einflüssen bezeichnen wir mit dem Buchstaben* F.
>
> *Die zweite Klasse umfaßt diejenigen Einflüsse, die wir als von außen her auf die Entstehung eines Kunstwerkes wirkend bezeichnen dürfen. Ihnen entsprechen in der oben angewendeten allgemeinen Formel die Buchstaben* x, y, z *etc.* [Semper (1884) 1979: 267-269].

"*F (Funktion)*" is described as the way the agents *x, y, z*, etc. influence or depend on each other, later also explained as motive (*Motiv*) [(1884) 1979: 269] and purpose (*Zweck*) [(1884) 1979: 274]. In particular, the term "purpose" raised questions about the quality of the translation, since Gottfried Semper didn't use the word "purpose" in any of his original manuscripts, but spoke instead of "the use of the things" [Mallgrave, Rykwert and Semper 1983: 13].[2] In the German translation, *F* is explained with the example of a drinking cup, which will generally look the same regardless of its national origin, time, material, or fashion [Semper (1884) 1979: 269]. In contrast, *x, y, z* etc., the external influences on a work of art, are unlimited in number but can be broken down in three groups: first, the materials and the ways of making; second, local and ethnologic influences, as well as climate, religion, politics and other national conditions; third, all personal influences of the client, artist or craftsman [Semper (1884) 1979: 271].

In 1983 Mallgrave published the two manuscripts originally written in English, which are held in the gta-Archiv at the ETH Zurich (Ms. 122 and Ms. 124). In Ms. 122, which according to Mallgrave is the version of the lecture that Semper delivered, the corresponding passage reads as follows:

> Every work of art is a *result*, or, using a Mathematical Term, it is a *Function* of an indefinite number of quantities or powers, which are the variable coefficients of the embodiment of it.
>
> $U = C\ x, y, z, t, v, w.$
>
> … We must distinguish two different kinds of influences, which act upon the embodiment of an artistic work.

The first class comprises the exigencies, of the work itself and which are based upon certain laws of nature and of necessity, which are the same at all times and under every circumstance.

The second class comprises such vehicles, which we may call outward influences acting upon the performance of a work of art.

That part of the Doctrine of Style, which treats of the first class, embraces the elementary Ideas or what the Artist calls the Motives of the things, and the early forms, in which these fundamental Ideas have been clothed.

… The second part of the Doctrine of Style comprehends chiefly local and personal Influences, such as the climate and physical constitution of a country, the political and religious institutions of a nation, the person or the corporation by whom a work is ordered, the place for which it is destined, and the Occasion on which it was produced. Finally also the individual personality of the Artist [Mallgrave, Rykwert and Semper 1983: 11-12].[3]

The equation reads differently here. The two "influences" or "classes" described are not assigned in any clear way to C, on the one hand, and to x, z, y, t, v, w, on the other. Moreover, materials and the ways of making are not assigned to any class, and the example of the drinking cup is not mentioned. In particular, however, it is completely unclear whether C stands for the function notation (the expression of a function) or a variable. Mallgrave explained the differences between Ms. 122 and the translation thus:

Hans Semper no doubt found his father's explanation of the formula vague or puzzling, for after presenting the mathematical analogy in his translation he rejected the text of this manuscript (Ms. 122) and turned to an explanation for the formula given in an earlier draft of the lecture (Ms. 124) [Mallgrave 1996: 218].

In Ms. 124, the passage reads as follows:

Every work of art is a *result*, or, using a mathematical term, it is a Function of an undefined number of *agents* or *powers*, which are the variable coefficients of the Embodiment of it.

$Y = C\ (x, y, z, t, v, w, \ldots)$

In this formula, Y stands for the general Result, and x, y, z, t, v, w represent as many different agents, which work together in a certain way which way is expressed here by the Greek letter C or function.

… By the letter C we may understand the exigencies of the work of industry or art in itself, which are based upon certain laws of nature and of necessity which is the same at all times and under every circumstance. A drinking cup for instance will be the same in its general feature for all nations and at all times; it will be in principle the same, if executed in wood, in Earthenware in glass in metal or whatever other material it may be no matter.

The elementary Idea of a work of art which is based upon its use and destination is independent upon fashion upon material and upon local conditions. The artists call this the motive of an object of art.

… We come now to those coefficients of our result which I signified in the general formula by the letters *x, v, t, w* etc. Those are the different vehicles which act upon the embodiment of our hands, and modify the appearances of the Elementary ideas. … Their number is undefined, but they can be grouped into three distinct classes. Among these agents is one group which consists of the materials and the modes of execution, or the processes which come in question for their execution. The second Group comprises the local and ethnological influences upon artistical performances, the influences of Clime, religious and political institutions and other national conditions. The third Group is that, which includes all the personal influences … from those who are the commanders of the works, or from the artists and the practical performers of the last [Mallgrave, Rykwert and Semper 1983: 18-19].

From Ms. 124 it is clear that C is to be understood as a function notation (an algorithm). Thus it could be said that Ms. 124 sheds light on Ms. 122, and that would explain why Hans Semper introduced this passage into his translation. This, however, might raise the question why Gottfried Semper did not write *F* rather than *C*, since the formula $Y=F(x, y, \ldots)$ would have made it perfectly clear that $F(\ )$ was a function notation.

In addition to Mss. 122 and 124, there is another manuscript at the ETH Zurich (Ms. 179, "Theorie des Formell-Schönen", Eng. trans: The Attributes of Formal Beauty), which was cited by Mallgrave as evidence that Semper's son misinterpreted the equation. It was written upwards of 1855 and published in its original German version in 1981 and in English in 1984, in both cases by Herrmann. Being very close to the German original, the English version is quoted here:

However, there is also a stylistic conception of what is beautiful in art – this considers the object … as a unit, as the uniform *result* or *function* of several variable values that unite in certain combinations and form the coefficients of a general equation; by giving these variables the values appropriate to the particular case, one will arrive at the solution of the problem:

$U = C\,(x, y, z, t, v, w \ldots.)$

… What are these variable coefficients, these elements of the general formula whose result we deem to be a work of art? Their number is indeterminable; we shall touch upon only some of the most important ones.

They can be divided into two distinct classes: first, those elements that are contained, as it were, in the work itself and that comply with certain compelling natural and physical laws that are the same under all circumstances and at all times; second, those elements that have an influence on the genesis of the work of art from the outside.

To the first class belongs above all the *purpose* of the object … . Also belonging to this class is the material … . Third, the utensils with which the work is to be done and the various processes for treating the material … .

The extrinsic coefficients of the artistic form of representation are more varied. To be taken into account first are local and personal influences and factors, such as climate, topography, national education, political-religious and social institutions, historical memories and traditions, local environment …, the person or group who commission the work … . Among numerous other influences there are also incidental circumstances that have an effect on the work in hand. Finally, the artist's hand, his individual taste and artistic attitude as essential factors in the creation of a work of art, are among the extrinsic influences [Hermann 1984: 241-242].

This quote presents a fourth version of the equation, again slightly different from the others. And again it is not completely clear whether *C* and (*x, y, z, t, v, w*. . . .) are the representatives of the two classes named. If they were, why would the first class be identified only by the letter *C* and the second by many letters? Moreover, Semper says neither that C represents the function notation, as he did in Ms. 124, nor that it is a variable. It is also interesting to see how the position of material and its treatment changes: in the son's translation and in Ms. 124 they belong to the second class; in Ms. 122 they are not assigned; and in Ms. 179 they are included in the first class. This changing position is an example of how permeated Semper's theoretical work was by the question of whether material and its treatment represent internal or external qualities of a work, that is, whether they determine the form or merely modify it. The organization of his magnum opus, *Der Stil in den technischen und tektonischen Künsten* (Style in the Technical and Tectonic Arts) of 1860-63, raises this question, too, since the chapters "Textile Kunst" (Textile Art), "Keramik" (Ceramics), "Tektonik" (Tectonics), "Stereotomie" (Stereotomy), and "Metallotechnik" (Metallurgy) are not focused on materials, but on means of production that admit of several materials.

When all four texts are compared, the problem arises that the equations can be interpreted in two different ways, resulting from the fact that it is not clear whether the formula is a general equation such as $y = f(x)$ or a specific equation such as $y = ax^2$. In the first case, Semper's C would be a function notation or the correspondence rule between variables; in the second, it would be a variable. Semper's son seems to have assumed the former, apparently basing that on the reading of Ms. 124, and thus changed the letter C to F. Mallgrave, in contrast, chose the latter based on Ms. 179 and consequently had to conclude that Mss. 122 and 124 are "incompatible" [Mallgrave 1996: 218]. In what follows the question of what speaks for the one reading and what for the other will be addressed.

With regard to the first reading, one could ask how a motive or a design idea could serve to define a function or correspondence of variables. To better understand this, we can refer to a discussion in a text on logic by Gottlob Frege, "Funktion und Begriff" (Function and Concept) written in 1891, although it was not published until almost forty years after Semper's lecture. Frege argued that functions and concepts have corresponding meanings. He offered as an example the concept "capital" for which he defined "the capital of *x*" as "the expression of a function." If "German Empire" is input for *x*, the result is "Berlin" [Frege (1891) 1980: 31-32]. Thus, Frege's "concept" or functional expression states how the variables and the result relate to one another. Yet a concept is never a concrete thing. Viewed as a function, it is "incomplete, in need of supplementation, or unsaturated" [Frege (1891) 1980: 24], that is, it requires values that complete it to a concrete object, which can then be experienced through the senses. In mathematics, the incompleteness of a function *F*(*x*), or in Frege's words, "the need of the function for supplementation" is represented by the parentheses, "the space between these is meant to receive the sign for the argument" [Frege (1891) 1980: 27-28]. Relating this thought back to Semper's equation, his "motive" can be interpreted in the sense of Frege's "concept," in that both the motive and the concept are incomplete. One could easily conclude from Semper's use of the parentheses that his notation C(…) is meant as an unsaturated functional expression and the letters in the brackets as the values needed to complete the motive.

One might come to a different conclusion when reading Semper's *Ueber die bleiernen Schleudergeschosse der Alten* (On the Leaden Slingshot Missiles of the Ancients) [1859][4] in which he made heavy use of equations. That treatise, in which Semper attempted to describe the forms of projectiles mathematically, was published in 1859, but it was written at the same time as the aforementioned lecture and was completed in 1855, after he had asked mathematicians to read it and then revised it. In that text Semper formulated the equation $W = d.N.,$ where W is the effect (*Wirkung*), d the density of the medium, and N the "other coefficients of the value for W, whatever their nature" [Semper 1859: 10]. Like all the other formulas in this text, all of the letters on the right side signify coefficients, and none of them expresses the functional correspondence. In its generality, the formula is very similar to the one in the 1853 lecture. It is an example of Semper formulating equations in which all letters signify coefficients rather than a functional sign; however, no brackets are used in this equation.

Other Semper texts do not feature an equation but do have "coefficients" and "influences" and, depending on how they are classified, it is possible to argue for one of the two readings.[5] Semper wrote as early as 1843 that "architecture too is based on certain normal forms that – conditioned by an original idea – permit in constant reappearance an infinite variety depending on special purposes and on further determining circumstances" [Herrmann 1976: 216-217; also quoted in Ms. 55 in Herrmann 1981: 184]. Semper distinguished here between an "original idea" and "special purposes," and this can be connected to two lines of thought. First, this distinction supports Mallgrave's criticism that Semper's son wrongly equated "idea" and "purpose" and thus abetted Gottfried Semper's reception as a materialist. It could be objected, however, that Semper himself did not in fact always clearly separate the concepts of "idea" and "purpose."[6] It seems rather that Semper distinguished between two different kinds of purposes: on the one hand, traditional, culturally evolved purposes that determine the idea; on the other hand, current, practical, more specified purposes. In this view, a fundamental purpose thus belongs to the idea or the motive of an object – as a vessel, for example – and should be distinguished from a more special purpose – for example, as a bucket or scoop in flowing water.[7]

Second, in this quotation Semper listed not just two but three conditions for a work, namely, the "original idea," "special purposes," and other "circumstances." In *Wissenschaft, Industrie und Kunst* (Science, Industry, and Art [(1852) 1989]) as well, Semper explicitly treats "three parts": first, the "the primordial motives (*Urmotiven*) and the primary forms derived from them"; second, the "means" and the "material"; and third, the "local, temporal, and personal influences on form extrinsic to the work of art." A stylistic work is composed of these three parts, since "Style means giving emphasis and artistic significance to the basic idea and to all intrinsic and extrinsic coefficients that modify the embodiment of the theme in a work of art" [Semper (1852) 1989: 136-138].

If this understanding of three parts is to be associated with the formula, one has to ask where the "original idea," "primordial motive" or "primary form" is located in the equation. If C and x, y, z are taken by the internal and external coefficients, respectively, then one would have to conclude that the idea, motive or primary form is the equation itself. That definition of style can only be reconciled with the second reading presented above. That is the conclusion to which Mallgrave arrived.

The mathematical parallel is found in *Style in the Technical and Tectonic Arts* as well. As an "empirical theory of art," the text

> will not apprehend and explain the works of art of different periods and countries as facts but rather it will *expand upon them*, as it were, by identifying in each the necessarily different values of a function composed of many variables. It will do this primarily with the intention of revealing the inner law governing the world of the art-form, just as it governs the world of nature. … Nothing is arbitrary; everything is conditioned by circumstances and relations [(1860) 2004: 71-72].[8]

Function can be understood as the inner law of becoming that remains to be discovered.

The place of mathematics in Semper's life

Is Semper's equation proof of his taking a materialistic and deterministic view of art, as he was repeatedly accused of doing?[9] As a way of approaching this question, it may be helpful to examine what Semper actually knew about mathematics and see if his life experiences provide indications of his understanding of the connection between infinitesimal calculus and architecture.

Semper, born in 1803, showed a talent for mathematics as a student. The director of his school recommended that he study under the mathematicians Carl Friedrich Gauss and Bernhard Friedrich Thibaut at the University of Göttingen, and initially Semper followed this advice (see [Mallgrave 1996: 12; Herrmann 1990]. In a letter to his parents dated August 1824, he stated that he had been preoccupied with mathematics for eight months [Mallgrave 1996: 13]. Surviving loan cards from the university library prove that Semper had borrowed books on mathematics, mechanics, and military science. At the same time, Semper also confessed to having lost his taste for the subject and preferring to dedicate himself to more practical tasks [Mallgrave 1996: 14]. Beginning around this time he began to search for new career goals and, in addition to a military career, he seriously considered hydraulic engineering, for which he would have needed extensive knowledge of mathematics used in fluid mechanics as well as the mathematical analysis of motion and acceleration.[10] During the next two decades, Semper studied architecture in Paris from late 1826 onward, made a long journey to Italy, Sicily and Greece from 1830 to 1833, and taught as a professor and was increasingly active as an architect from 1834 to 1849. His interest in mathematics, and especially his knowledge of hydraulic engineering, did not become evident again until the publication in 1859 of his *Ueber die bleiernen Schleudergeschosse der Alten*. That work clearly reveals Semper's passion of mathematics. Its subtitle, *Ein Versuch die dynamische Entstehung gewisser Formen in der Natur und in der Kunst nachzuweisen* (An Attempt to Demonstrate the Dynamic Origin of Certain Forms in Nature and Art), testifies to Semper's desire to analyze optimal forms – from nature as well as from art – scientifically. He introduced the text by stating his conviction "that every true artistic form must be the expression of a certain law of innermost necessity, just as is surely the case for natural forms" [Semper 1859: 1-2].

The concept of infinitesimal calculus, for example in the descriptions of tangents and areas of curves, can already be found in antiquity [Jahnke 1999: 5-6]. Semper's enormous interest in antiquity led him to speculate, in his text on leaden slingshot projectiles, about whether the ancients possessed knowledge of infinitesimal mathematics – that is, "whether these Projectiles are the Result of an instinctive feeling of their makers for fitness or if they are proofs of the high State of Mechanical Science with the Greeks" [Mallgrave, Rykwert and Semper 1983: 13-14]. To be clear: Semper explicitly rejected the view that artistic forms could be calculated. In Ms. 124 and in the 1884 German translation of Semper's London lecture – but not in Ms. 122, the lecture he actually delivered – he referred to the formula as a "crutch" and stated that "results in fine arts are hardly obtainable by calculation" [Mallgrave, Rykwert and Semper 1983: 18]. In the slingshot text he also stated that it should "not be claimed that the Greeks constructed their forms on the basis of mathematical formulas." However, in the same sentence, he referred to infinitesimal terminology by stating that the Greeks "clearly recognized the law of nature that nature observes the extreme limits within her formations and lets tensions prevail everywhere" [Semper 1859: 60].[11]

As early as 1834, he declared that architecture was based "on indeterminable but for that reason no less certain and solid laws …. Although we are convinced of the existence of these laws, nevertheless we cannot determine them a priori with mathematics" [Semper 1834 (1991): 224]. Semper concluded that science was not yet advanced enough to express these complex laws of beautiful architecture but that an architect can feel them and achieve them through this feeling and must therefore strive to practice "the only criterion of their existence—the feeling for their excellence" [Semper 1834 (1991): 224]. In particular, he claimed that science lacked knowledge of the "vital force" (*Lebenskraft*)[12] – that is, how life evolves and grows. According to Semper, the most interesting forms in nature are created by this force. And again, since science is not there yet, "we have no other Guides than our own natural feeling assisted by a right study of natural history and of that of art" [Mallgrave, Rykwert and Semper 1983: 14].[13] He described this feeling as an unconscious differential and integral calculation by someone gifted with an aesthetic sense, since, as he says in "Ueber die formelle Gesetzmäßigkeit des Schmuckes und dessen Bedeutung als Kunstsymbol," "the impression that the form makes on our sense of beauty is at first founded on an unconscious measuring, weighing, and integrating of functions that are too complicated for our science and whose solutions can be achieved by it alone" [Semper (1884) 1979: 326].[14] It is very interesting to read that Semper sees integration better achieved by the sense of beauty than by calculation. The unfortunate aspect of this is, according to Semper, that

> [the] more we advance in civilization and science the more it seems that that instinctive feeling … loses its strength, while Science has not yet attained to the point, of compensating us for this loss. It very often happens, that we are led back by science and calculation to such forms, which were observed heretofore only by savages, and semibarbarians [Mallgrave, Rykwert and Semper 1983: 13].

Hence, all these texts on the subject served to demonstrate that an artistic feeling educated by science and history leads to more precise and more beautiful results than the scientific methods that had been developed by that time.

Semper also criticized the laws that had been proposed by earlier architects as insufficient, too schematic, and too rigid. In particular he accused Jean-Nicolas-Louis Durand of getting lost in tables, formulas, and mechanical ways of connecting things [Mallgrave, Rykwert and Semper 1983: 9]. In his text on leaden slingshot projectiles, Semper described his participation in a meeting of the Royal Institute of British Architects in London in which one member presented his findings on ancient laws of proportions, which in Semper's view were too schematic, simple, and absolute. "Does nature, does the artist's sense create like a wood turner according to a template?" he asked and replied that the physically beautiful depends on a wide variety of external conditions and is "only under certain circumstances truly beautiful and proportional, while under others it is – though its form and color remain unchanged – indifferent or ugly" [Semper 1859: 2-3]. In this context, he emphasizes the superiority of mathematical analysis of constant change and infinity as opposed to simple mathematics and the laws of proportion:

> These determining circumstances are, however, subject to infinite variations, and therefore universally valid *numerical* rules of proportion for the beautiful cannot be specified. The formulas in which the true laws of beauty would be defined, (if it is possible at all to formulate the latter) may, in any case, only be treated as equations in which variable and constant values interact in the most diverse ways" [Semper 1859: 3].

So, whereas the simple laws of proportion could only give mechanical answers, for Semper infinitesimal calculus seemed to permit "organic" – that is, vital and complex – insights.

This is not to say that rules of proportion, symmetry, harmony, and ornamentation lost their authority in Semper's understanding of architecture. Quite the opposite, since these rules were developed in the practical arts long before architecture [Mallgrave, Rykwert and Semper 1983: 9], they made their way into architecture as "motives," thus bringing us back to the question where in the functional equation the motives are represented: in the function sign C (as interpreted by Hans Semper) or in the equation itself (as interpreted by Mallgrave). Regarding these rules as basic motives, Semper is able to explain them as valid in an abstract way, while they become concrete in measurable things only if confronted with the "coefficients" of specific purpose, material, place, artist, and all the other influences.[15]

Semper was driven by the effort to let science enter into architecture, but he did not try to do so in a schematic way, but rather by acknowledging the developments of history, culture, languages, and other influences. For that reason, Semper's concern when introducing science into art was not to simplify design or offer a recipe for design – which is how he regarded Durand's writings – but rather to understand and reveal the richness of design processes more precisely. Hence Semper seemed to guard against a clearer formulation of his equation, which after all could only have been employed too mechanistically. One could also claim that Semper described his formula as a "crutch" not because he did not believe in a mathematical explanation of form but rather because he believed its complexity could not be formulated. Semper's text on leaden slingshot projectiles has to be evaluated in that context, as he was inspired to write it in order

> to demonstrate with the simplest possible example that the Greeks did not merely observe the laws of nature and try to imitate forms evolved from them but rather had truly studied these laws and based on them, independently of all imitation, created their own forms, which coincided with those of nature only in terms of the commonality of the law [Semper 1859: 6].

Mathematical functions

The purpose of the following, more detailed discussion of the mathematical concept of function is to speculate about whether further insights for art and architecture can be derived from Semper's equation.

Infinitesimal calculus and the concept of function are, along with analytic geometry, the outstanding achievements of mathematics in the seventeenth century, authored by Isaac Newton and Gottfried Wilhelm Leibniz. The notation $f(x)$ derived from Leonhard

Euler and was first published in 1740. In his famous *Introductio in Analysin Infinitorum* (Introduction to Analysis of the Infinite, E101) of 1748, Euler explained that "analysis is concerned with variable quantities and functions of such variables" [Euler 1888: vi], and his *Institutiones Calculi Differentialis* (Foundations of Differential Calculus, E212) of 1755 offered this definition:

> Those quantities that depend on others in this way, namely, those that undergo a change when others change, are called *functions* of these quantities. This definition applies rather widely and includes all ways in which one quantity can be determined by others. Hence, if x designates the variable quantity, all other quantities that in any way depend on x or are determined by it are called its functions [Euler 2000: vi].

The development of mathematical analysis made possible the transition from calculation with constants to calculation with variables. Mathematicians have often regarded the variable as the true core of a function, whereby an input variable (what Semper calls "coefficient") is the independent variable, and the output variable (what Semper calls "result" or "work of art") is the dependent variable. A function F is a correspondence rule, that is, a rule that produces an explicit relationship between an input variable x and precisely one output variable y; in other words: from x follows y, but x does not follow from y. Transferred directly to Semper's formula, the "coefficients" determine the work of art but, conversely, a work of art can be traced back to various input variables (for example, a different climate, history, artist). It was only the fact that a mathematical function is dynamic when coefficients come into play that made it interesting for Semper's reflections on style. For Semper, different styles resulted from the variable coefficients: materials, national events, and individual architects or artists lead to material styles ("Wood Style"), national styles ("Egyptian Style"), styles of patrons ("Style of Louis XIV"), and artist's styles ("Style of Raphael").[16] In short, what he was attempting with his equation was nothing less than a model to explain his theory of style, according to which both the temporary coefficients and the timeless motives and early forms are necessary to create a style. In that spirit, in 1869 Semper wrote in "Ueber Baustile" (On Architectural Styles) that style "is the accord of an art object with its genesis, and with all the preconditions and circumstances of its becoming. When we consider the object from a stylistic point of view, we see it not as something absolute but as a result" [Semper (1869) 1989: 269]. The motives and early forms alone cannot be style. They are the forms that remain when the "coefficients" are removed – that is, everything on which all cultures are based. They are the timeless, the placeless, the artistless. From this it follows that "motive," "early form," and "design idea" are not variables but can only be the function itself. This understanding is what Ms. 124 explains and was therefore merged into the German translation by Hans Semper:

> As soon as one or some of these coefficients vary, the result must vary likewise, if x becomes $(x+a)$ the result will be U, quite a different one from that, which we call now Y; but it will in the principle remain identical to the last, being connected with it by a common relation which is expressed by the letter C. Likewise, if x, y, z, t etc. remain the same but if C changes, the Y will change in an other manner than before. It will be fundamentally different from what it was before the change took place, although the coefficients x, y, z, t, v etc. have undergone no change [Mallgrave, Rykwert and Semper 1983: 18].

Functions or correspondence rules specify how to calculate with variables. One of the advantages of this calculation is that one can follow what happens when different input values are entered for a variable. For example, the value "climate a" or "climate b" (if such a value could be determined at all) can be input into a variable and the different results compared. In other words, this process is very well suited to the comparative method. Infinitesimal calculus as a comparative method is one of Semper's extremely interesting ideas, but note that it should be understood as merely a theoretical conception, since in his view the variables were too complex and also impossible to determine numerically.

One core idea of infinitesimal calculus – and perhaps the most productive theme in its application to architecture and art – is the relationship of differentiation and integration of functions. From a historical perspective, this is about the connection between determining tangents and determining area: differentiation involves finding a tangent to a point on a curve, and integration involves finding the area contained within a curve. Leibniz and Newton had already recognized that there was an inverse connection between the two. The notion that integration could be derived from the inversion of differentiation survived into the nineteenth century [Wussing 2008: 1, 429 and 461-468], while Johann Bernoulli had already aptly formulated the problem of integration as the inverse operation of differentiation in 1691-92:

> But as easy it is to find the differential of any one provided quantity, so difficult is it conversely to specify the integral of any differential, so that we sometimes cannot even claim with certainty whether the integral can be formed of the provided quantity or not [Bernoulli 1914: 8].

If this theme is compared with Semper's two main architectural problems – namely, the classifications of works of architecture and the invention of them – it suggests the hypothesis that in Semper's way of thinking classification corresponds to differentiation and invention to integration. His writings can be interpreted to the effect that the operation of differentiation (classification) is intended to draw conclusions for integration (invention). This hypothesis is supported by Semper's remark that a comparative (and hence classifying) method similar to Cuvier's could

> form the base of a doctrine of *Style*, and of a Sort of topic or Method, how to invent, which may guide us to find out the natural way of invention which would be more than could be allowed to the great Naturalist to do for his sublime science.[17]

Seen in this way, his equation can be read in two directions. Read horizontally, the artist aims at the work of art/result (that is, the left side of the equation) and the theorist at the variables (the right side of the equation). Read vertically, the equation contains the search for style by means of integration and the search for motives and internal laws by means of differentiation. Semper studied works of art or results, and in the process determined the variables and ultimately derived the motives from them. In other words, Semper read the equation backward, with the result as the known quantity. Semper probably recognized the motives first in the four elements – hearth, roof, enclosure, and mound[18] – and later in the manufacturing processes of textile art, ceramics, tectonics, stereotomy, and metallurgy. Yet Semper, much like Bernoulli, recognized integration as a much greater challenge than differentiation. He reasoned that mathematics

> can certainly calculate differentials of very complex functions but it rarely succeeds at integration, especially in physical cases where forces interact in

complex ways according to laws that still need to be defined. But at least mathematics *attempts* such integration; indeed, it sees it as its most important task [Semper (1860) 2004: 80].

From what has been said thus far it should have become clear that Semper compared things in nature and works of art and made an analogy between them. Very much analogously to motives and coefficients in art, he wrote that Georges Cuvier's naturalist collection shows

> progressing nature, with all its variety and immense richness, most sparing and economical in its fundamental forms and Motives; we see the same skeleton repeating itself continually, but with innumerable varieties, modified by gradual developments of the Individuals and by the conditions of existence which they had to fulfill [Mallgrave, Rykwert and Semper 1983: 8].

He concluded with the question: "may we not by Analogy assume, that it will be nearly the same with the creations of our hands, with the works of industrial art?" [Mallgrave, Rykwert and Semper 1983: 8].

Correspondingly, Semper's reflections on mathematics did not stop when observing nature. The mathematics of constant change, infinity, and variables permitted a dynamic perspective on nature whose complexity enabled it to serve as a model for art:

> Nature works not like a turner after working drawings or what they call templates, its forms are altogether dynamical productions, and it is only by the way of that science, which treats of the mutual actions and reactions of forces, that we may hope to find the keys for some of the simplest material forms. What is true in Nature, has its application also for artistical forms [Mallgrave, Rykwert and Semper 1983: 15].

Semper's attempt to give a more scientific account of the factors that determine form was his modern contribution to the contemporaneous discourse on art as the imitation of nature.

One could add here that Semper's application of the concept of mathematical functions to architectural theory had its counterparts among the works of the theorists of natural history, who provided literal and metaphorical comparisons of organisms and equations of curves. For example, Georges Cuvier compared the relationship between a tooth and the other parts of a body to the way "the equation of a curve regulates all its other properties" [Cuvier 1818: 102], while Gottfried Leibniz wrote that there are special points of change in the life of a person which are part of a general path like "the distinctive points of a curve can be determined by its general nature or its equation" [Leibniz 1989: 658-659].

Art and functional dependency

The discussion has shown that both Hans Semper's and Harry Francis Mallgrave's readings of Gottfried Semper's equation result in productive theses. The question of which is the "correct" reading will be left open. In general, however, it can be concluded from both readings that "function" refers to a process, an active synthesis of all of the "influences" Semper described. Thus the equation testifies to Semper's central concern: understanding this synthesis, which is the richness of the design process, or, in Semper's words, *to explore within individual cases the regularity and order that become apparent in artistic phenomena during the creative process of becoming and to deduce from that the general principles, the fundamentals of an empirical theory of art* [Semper (1860) 2004: 71].

While a specific equation will never be found and the "power of a genius may subconsciously achieve such nature-like creation," he nevertheless asserted that it should be the task of a contemplating architect "to seek and pursue the rise and development of basic ideas and to reduce to its simplest expression the law that lies hidden within the artistic covering" [Semper (1855) 1984: 194].

Neither reading of the formula leads to a primitive determinism or materialism, for it is neither possible to determine the equation nor to provide a complete set of variables. Semper always fundamentally and vehemently opposed materialism. If Semper is regarded, on the basis of this equation, as a precursor to High Modernism, it is also necessary to acknowledge in his theory the internalization of history, artistic originality, symbolic motives, national interests, and the distinction between traditional/idea-founding purposes and current/specific purposes. The biggest criticism aimed at Semper for his equation was that the artist was degraded to just one variable among many. Herrmann in particular has described it as part of Semper's maturation process that in his later writings he attributed increasing significance to the creative human being [Hermann 1990: 80]. For example, in 1869, in "On Architectural Styles," Semper observed that

> the free will of the creative human spirit is the first and most important factor in the question of the origin of architecture styles, although, of course, man's creative power is confined by certain higher laws of tradition, demand, and necessity. Yet man appropriated these laws and made them subservient, as it were, to his free, objective interpretation and exploitation [Semper (1869) 1989: 268].

This does not, however, bring down the equation, since a weighting of the different "coefficients" that the increasing significance of the artist would express is by no means the object of the equation. In the prolegomena to *Style in the Technical and Tectonic Arts*, Semper managed in one breath both to offer a mathematical-functional analogy and to emphasize "the sense and the purely human impulse of *being creative as an end to itself* and the gift – so indispensable to the artist as well as to people receptive to art – of *direct intuitive thinking"* [Semper (1860) 2004: 73]. Semper had always emphasized the superiority of artistic feeling and aimed at integrating poetic and scientific activity. Thus his formula should be interpreted as an attempt to grasp the world – including art and science – as a unity.

Acknowledgments

I would like to thank Hans Christian Poerschke for his insights and support concerning infinitesimal calculus. I would also like to thank Steven Lindberg for the main translation and the discussions about appropriate translations, which is a major question of this essay. All translations of original quotations provided in the text are by Steven Lindberg and Ute Poerschke unless otherwise noted.

Notes

1. See, for example, [Hauser 1985]. See also the discussion of the "comparative method" in architecture, anatomy, and linguistics in [Hvattum 2004: 114-136].
2. The original manuscripts are published in full in [Mallgrave, Rykwert and Semper 1983].

3. When reading the manuscript, one should keep in mind that Gottfried Semper's knowledge of English was limited at the time. The unusual capitalization in Mallgrave's publication of the text follows the manuscript.
4. There is no published English translation of Semper's *Ueber die bleiernen Schleudergeschosse der Alten ....* . An introduction to the text can be found in [Mallgrave 1996: 222-224]. See also [Herrmann 1984: 74].
5. In addition to the texts described, see "Bemerkungen zu des M. Vitruvius Pollio zehn Büchern der Baukunst" (1856), in [Semper (1884) 1979], esp. 203-204.
6. In Ms. 179, for example, Semper wrote of *Zweck* (purpose) and *Zweckeinheit* (unity of purpose), but then crossed out the relevant paragraphs and wrote anew of *Idee* (idea) and *Inhaltsangemessenheit* (appropriateness of content). See [Herrmann 1981: 226]. See also Mari Hvattum's comment on the "ambiguous role of 'purpose'" in Semper's work [Hvattum 2004: 223, n. 95].
7. Semper offers this example in "Plan eines idealen Museums," in [Semper 1966: 72-79].
8. Later, in *Style in the Technical and Tectonic Arts*, Semper orders the coefficients again differently. The "technical arts" will be explored "from the following two points of view: 1. The work as a result of the *material service* or *use* that is intended … . 2. The work as a result of the *material* used to produce it, as well as of the *tools* and *procedures* applied. The use of any technical product remains essentially the same at all times. It is based on universal human needs and on natural principles seeking formal expression that are valid everywhere and at all times. But the materials used to make this product, and particularly how they are treated, change radically over time according to local and all other possible circumstances. It is thus appropriate to link more general formal-aesthetic considerations to the question of purpose and to link considerations of the history of style to materials. But we cannot expect to apply this principle with full consistency" [(1860) 2004: 107].
9. Ernst Stockmeyer discussed various interpretations; see [1939: 8].
10. See [Wussing 2008: I, 432]. Wussing emphasizes that master builder in hydraulics and fortifications, shipbuilders, mining and structural engineers profited from infinitesimal calculus
11. The complicated German sentence reads as follows: *Mit dieser Bemerkung … soll nicht behauptet sein, dass die Griechen ihre Formen nach mathematischen Formeln construirten, welches in der Kunst anzunehmen absurd wäre, sondern dass sie das Gesetz der Natur, wonach diese bei ihren Formgebungen die extremen Grenzen beobachtet und überall Spannungen herrschen lässt, nicht bloss dunkel ahnten, sondern klar erkannten* [Semper 1859: 60].
12. See [Semper 1859: 4]. See also [Semper (1860) 2004: 90]. In Ms. 122 Semper named it "the power of animal and vegetable Life" [Mallgrave, Rykwert and Semper 1983: 14].
13. See also [Semper 1859: 4]: "Here we find ourselves in an area where at first no guide accompanies us other than our own artistic sense, supported by a healthy study of nature and art history."
14. See also Semper's commentary on Vitruvius, "Bemerkungen zu des M. Vitruvius Pollio zehn Büchern der Baukunst", in which he asserted that the sense of beauty alone is able "to integrate the most difficult and most transcendental dynamic problems and expressions from which our science shies away" [Semper (1884) 1979: 206].
15. See also Semper's remarks on Vitruvius [(1884) 1979: 203-204], in which he explains the relationship of the basic rules of symmetry, proportion, and direction, on the one hand, and the external coefficients, on the other hand, through a "logarithmic analogy" (*logarithmischen Gleichnisses*).
16. See, for example, [Mallgrave, Rykwert and Semper 1983: 12] and "Ueber die formelle Gesetzmäßigkeit des Schmuckes und dessen Bedeutung als Kunstsymbol" [Semper (1884) 1979: 343].
17. See [Mallgrave, Rykwert and Semper 1983: 9]. See also Ms. 55, "Influence of Historical Research on Trends in Contemporary Architecture": "It will certainly be important to trace these standard forms and the idea inherent in them. Not only will the overall view and the understanding of what exists be made easier, but it will also be possible to derive an architectural theory of design and inventiveness" [Hermann 1984: 195].

18. See "The Four Elements of Architecture" [Semper (1851) 1989]. Here the influences are not yet subdivided or worked out. He writes very generally: "According to how different human societies developed under the varied influences of climate, natural surroundings, social relations, and different racial dispositions, the combinations in which the four elements of architecture were arranged also had to change, with some elements becoming more developed while others receded into the background" [(1851) 1989: 103].

References

BERNOULLI, Johann. 1914. *Die erste Integralrechnung: eine Auswahl aus Johann Bernoullis mathematischen Vorlesungen über die Methode der Integrale und anderes*. Gerhard Kowalewski, ed. and trans. Leipzig: Engelmann.

BÖRSCH-SUPAN, Eva, et al. 1976. *Gottfried Semper und die Mitte des 19.Jahrhunderts*. Basel: Birkhäuser.

CUVIER, Georges. 1818. *Essay on the Theory of Earth*. Robert Jameson, trans. New York: Kirk & Mercein.

EULER, Leonhard. 1988. *Introduction to Analysis of the Infinite*. John D. Blanton, trans. New York: Springer.

———. 2000. *Foundations of Differential Calculus*. John D. Blanton, trans. New York: Springer.

FREGE, Gottlob. (1891) 1980. *Funktion und Begriff.* Eng. trans: Function and Concept. Peter Geach and Max Black, trans. Pp. 21-41 in *Translations from the Philosophical Writings of Gottlob Frege*, 3rd ed., Peter Geach and Max Black, eds. Oxford: Blackwell.

HAUSER, Andreas. 1985. Der 'Cuvier der Kunstwissenschaft': Klassifizierungsprobleme in Gottfried Sempers 'Vergleichender Baulehre.' Pp. 97-114 in *Grenzbereiche der Architektur*, Thomas Bolt, ed. Basel: Birkhäuser.

HERRMANN, Wolfgang. 1976. Semper und Eduard Vieweg. Pp. 199-237 in Eva Börsch-Supan, et al., *Gottfried Semper und die Mitte des 19. Jahrhunderts*. Basel: Birkhäuser.

———. 1978. *Gottfried Semper im Exil*. Basel: Birkhäuser.

———. 1981. *Gottfried Semper: Theoretischer Nachlass an der ETH Zürich; Katalog und Kommentare*. Basel: Birkhäuser.

———. 1984. *Gottfried Semper: In Search of Architecture*. Cambridge, MA: MIT Press.

———. 1990. Sempers Weg von der Mathematik zur vergleichenden Baulehre. Pp. 73-81 in *Bildfälle: Die Moderne im Zwielicht*, Beat Wyss, ed. Zurich: Artemis.

HVATTUM, Mari. 2001. Gottfried Semper: Between Poetics and Practical Aesthetics. *Zeitschrift für Kunstgeschichte* **64**: 537-46.

———. 2004. *Gottfried Semper and the Problem of Historicism*. Cambridge, UK: Cambridge Univ. Press.

JAHNKE, Hans Niels. 1999. *Geschichte der Analysis*. Heidelberg: Spektrum.

LAUDEL, Heidrun. 1991. *Gottfried Semper: Architektur und Stil*. Dresden: Verlag der Kunst.

LEIBNIZ, Gottfried Wilhelm. 1989. *Philosophical Papers and Letters*, 2nd ed. Leroy E. Loemker, ed. and trans. Dordrecht: Kluwer.

MALLGRAVE, Harry Francis. 1996. *Gottfried Semper: Architect of the Nineteenth Century*. New Haven, CT: Yale University Press.

MALLGRAVE, Harry Francis, Joseph Rykwert, and Gottfried Semper. 1983. London Lecture of November 11, 1853. *RES: Anthropology and Aesthetics* **6**: 5-31.

SEMPER, Gottfried. (1834) 1991. Dresdner Antrittsvorlesung 1834. Pp. 221-34 in Heidrun Laudel, *Gottfried Semper: Architektur und Stil*. Dresden: Verlag der Kunst.

———. (1851) 1989. Die vier Elemente der Baukunst. Eng. trans: The Four Elements of Architecture. Pp. 74-129 in Gottfried Semper, *The Four Elements of Architecture and Other Writings*. Harry Francis Mallgrave and Wolfgang Herrmann, trans. Cambridge: Cambridge University Press.

———. (1852) 1989. Wissenschaft, Industrie und Kunst: Vorschläge zur Anregung nationalen Kunstgefühles. Eng. trans: Science, Industry, and Art. Pp. 130-167 in Gottfried Semper, *The Four Elements of Architecture and Other Writings*, Harry Francis Mallgrave and Wolfgang Herrmann, trans. Cambridge: Cambridge University Press.

———. (1855) 1984. Theorie des Formell-Schönen. Eng. trans: The Attributes of Formal Beauty. Pp. 219-244 in Gottfried Semper, *In Search of Architecture*. Wolfgang Hermann, ed. Cambridge, MA: MIT Press.

———. 1859. *Ueber die bleiernen Schleudergeschosse der Alten und über zweckmässige Gestaltung der Wurfkörper im Allgemeinen: Ein Versuch die dynamische Entstehung gewisser Formen in der Natur und in der Kunst nachzuweisen*. Frankfurt: Verlag für Kunst und Wissenschaft.

———. (1860) 2004. *Der Stil in den technischen und tektonischen Künsten; oder, Praktische Aesthetik; Ein Handbuch für Techniker, Künstler und Kunstfreunde* Vol. 1: *Textile Kunst*. Eng. trans: *Style in the Technical and Tectonic Arts; or, Practical Aesthetics*. Harry Francis Mallgrave and Michael Robinson, trans., Harry Francis Mallgrave, ed. Los Angeles: Getty Research Institute.

———. (1869) 1989. Ueber Baustile. Eng. trans: On Architectural Styles. Pp. 264-284 in Gottfried Semper, *The Four Elements of Architecture and Other Writings*. Harry Francis Mallgrave and Wolfgang Herrmann, trans. Cambridge: Cambridge University Press.

———. (1884) 1979. *Kleine Schriften*. Ed. Hans and Manfred Semper. Mittenwald: Mäander.

———. 1966. *Wissenschaft, Industrie und Kunst und andere Schriften über Architektur, Kunsthandwerk und Kunstunterricht*. Hans M. Wingler, ed. Berlin: Kupferberg.

———. 1989. *The Four Elements of Architecture and Other Writings*. Harry Francis Mallgrave and Wolfgang Herrmann, trans. Cambridge: Cambridge University Press.

STOCKMEYER, Ernst. 1939. *Gottfried Sempers Kunsttheorie*. Zurich: Rascher.

WUSSING, Hans. 2008. *6000 Jahre Mathematik: Eine kulturgeschichtliche Zeitreise*, 2 vols. Berlin: Springer.

About the author

Ute Poerschke has been an Associate Professor in the Department of Architecture at The Pennsylvania State University, USA, since 2006. She taught at the Technical Universities of Berlin and Munich, Germany (1999-2005), and completed her doctoral degree in architectural theory about interpretations of functionalism at the Technical University of Cottbus in 2005. Her teaching areas include architectural design, technical systems integration, and architectural theory. She has been a principal of the architectural firm Friedrich-Poerschke-Zwink Architekten, Munich, since 2004. Her research interests are in the fields of theory, practice and teaching of architectural design as a process of integration.

Adriana Rossi

Department of Civil Engineering
Second University of Naples
Real casa dell'Annunziata
Via Roma, 29
81031 Aversa (Caserta) ITALY
adriana.rossi@unina2.it

Keywords: architectural drawing, technical drawing, golden section, Villard de Honnecourt, Leonardo Pisano, Fibonacci, ratio, proportion

Research

From Drawing to Technical Drawing

Abstract. It is in the early Middle Ages that the demonstrative effectiveness of drawing begins to be correlated to mathematical description, thus laying the foundations for the geometrical conceptualization that technical drawing makes measurable. The biunivocal procedures that make it possible to transcribe three-dimensional shapes and to trace from these their exact collocation in space, their real size and their real shape, arose from the experience of specialized building site craftsmen and from the study of classical works. The new figure emerged that of the master mason who could draw and interpret designs. The need to show or to see what the end product would look like before it was actually built brought about the refinement of a drawing system from which the professional figure of the architect emerged.

It is in the early Middle Ages that the demonstrative effectiveness of drawing begins to be correlated to mathematical description, thus laying the foundations for the geometrical conceptualization that technical drawing makes measurable. The biunivocal procedures that make it possible to transcribe three-dimensional shapes and to trace from these their exact collocation in space, their real size and their real shape, arose from the experience of specialized building site craftsmen and from the study of classical works which were consulted more often than one would be inclined to think at the monastery libraries and the cultural centers that were beginning to flourish around the most enlightened courts in Europe [Le Goff 1981].

Eugène-Emmanuel Viollet-le-Duc (1814-1879) was the first to point out that mediaeval building site practice not only followed a set of rules to dimension spaces, but also a method according to which parts were ordered in succession. In complete disagreement with his contemporaries, who believed that Gothic art lacked symmetry,[1] the French scholar proved that Gothic cathedrals were not the result of an empirically improvised building practice, but were in fact remarkable buildings that were well proportioned in every part, if one knew how to look at them in the light of the fixed and constant rules of the renewed aesthetics [Antolini 1817]. These rules were based on the teachings of Thomas Aquinas (1225-1274), who suggested conformity of object to intellect a guideline towards the pursuit of *veritas* [Eco 1956,1959,1987,1970].[2] In the context of the mediaeval building site this approach meant that solutions were sought bearing in mind the nature of the landscape and place and the opportunities they offered: pre-existing elements were harmonized with vaults and arches, leftover material was re-used and the "areas" (*saltus*) in which the voids and the planes were to be distributed were built using "dividers" (*rigores*) [Lenza 2002]. Ratios and proportions were calculated substituting the fixed point identified by the reference pickets with numbers. The transcription criterion was derived from the teachings of Leonardo Pisano (1170-1240), the mathematician who introduced the Hindu-Arabic place-value decimal system and the use of Arabic numerals into Europe.[3] Fibonacci, as he was known, was the son of a wealthy merchant, Guglielmo Fibonacci. He used fractions (from the Latin *fractus*) to teach how to obtain measurements to those who, like his father, needed to measure things which were not only or always directly related to the length of the "cane," a sort of

Nexus Netw J 14 (2012) 135–149
DOI 10.1007/s00004-011-0102-4; *published online* 13 January 2012

ruler graded according to the length of the palm, foot and arm of a person of average height. In a chapter of his 1202 *Liber Abaci* concerning the division of the cane, Leonardo Pisano introduces classes of equivalence relations capable of "breaking down" lengths and thus solving "practical geometry" problems on the building site. This system allowed for a re-interpretation of the mean proportional theory ascribed to Pythagoras and brought to a sophisticated level of abstraction by his friend Archytas of Tarentum (428-347 B.C.), a disciple of Philolaus.[4] In the *Timaeus*, widely known in the Middle Ages thanks to an abridged and commented translation by Calcidius,[5] Plato claimed that it was impossible to combine two things well without a third. A link was needed between them and there was no better link than the one that connects them to one another as a whole.

This, which according to the philosopher represented the nature of proportion [Wittkower 1962: 101ff], had been studied by Archytas, who analyzed consonant intervals and proposed a new division of the tetrachords (enharmonic, chromatic and diatonic) to which a new definition of three different types of relationships was linked. The algorithms needed to obtain, through a finite number of steps, the properties that link three quantities in succession (a, b, c) were transcribed by Porphyry of Tyre[6] (233-305 A.D.) and referred to by Eudemus of Rhodes (370-300 BC) in his Commentary on Aristotle's Physics:[7]

- *the arithmetical relationship* in which the difference between the terms is constant:

$$a - b = b - c$$

- *the geometrical relationships* obtained when the quotients are constant:

$$a : b = b : c$$

- *the harmonic relationships* obtained by the inverse in arithmetical proportion:

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$$

$$(b - a) : a = (c - b) : c$$

From the respective definitions the means are derived:

- *Arithmetical* (6: 9: 12), when the terms exceed each other so that the first exceeds the second as the second exceeds the third

$$b = (a + c)/2\,;$$

- *Geometrical* (6: 12: 24), obtained when the terms are such that the first is to the second as the second is to the third. Here the interval of the greater terms is equal to the one of the lesser terms

$$b = \sqrt{ac}\ ;$$

- *Harmonic* (6: 8: 12), also called *subcontrary* as they derive from the musical octave in which the second term exceeds the first as the as it exceeds the third

$$b = \frac{2ac}{(a + c)}\,.$$

How to determine mean proportionals with numbers or letters had become widespread knowledge, thanks also to a sort of manual by Johannes de Sacrobosco, entitled *Algorismus vulgaris* or *Algorismus de integris* (1250), which taught how to perform calculations using addition, subtraction, multiplication, division, square and cube roots, or the inverse elevation to a power. The groups of numbers, or as we would say today, the fields of algebraic existence, prompted the comparison of solutions which left the commensuration criteria unchanged if one substituted a concrete modulus with an abstract one represented by a number. The analytical procedure allowed for the quantification of the mean of Phidias, called Φ after the Greek sculptor of the Parthenon who is reputed to have used the diameter of the column to determine the rhythmic division of the temple according to the "right harmonic proportions". The use of measurements systems such as that of the "cane," which divided lengths so that the sum of the first two quantities remained constant, a procedure theoretically formalized by Fibonacci himself, paved the way for the computation of harmonic relationships. How to divide a segment in mean and extreme ratio was a problem which had been solved from a geometric point of view many centuries before: proposition XXX of Book IV of Euclid's *Elements* fixed a point on a segment "so that it was to the greater as the greater was to the lesser". A proportion between the parts could be now calculated by solving a second degree equation in x.

The total length of a, segment being known, the calculation:

$$a : x = x : (a - x)$$

leads to a standard formula:

$$x^2 + ax - a^2 = 0$$

The actual concrete problem allowed for one possible solution. For $a = 1$ the positive value of the root is a number whose characteristics cannot be transcribed as a rational fraction

$$x = \frac{\sqrt{5} - 1}{2} = 0.6180339\ldots$$

Assuming that there exists a positive real number such that:

$$\frac{1}{x} = 1 + x$$

We get the initial expression whereby its positive root x=0.6180339… defines the number Φ :

$$\frac{1}{0.6180339} = 1 + 0.6180339 = 1.6180339\ldots = \Phi\,.$$

This is believed by many to be a recurrent constant in the proportions of the temple dedicated to Athena Parthenos. It can be shown that this quantity can be obtained as an approximation of an arithmetic succession. By studying the problem of how many rabbits could be begotten in a year starting with one pair,[8] Fibonacci manages to prove that given the succession (sequence) of numbers:

(1), 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377...

The ratio between two consecutive terms approximates to the constant:

1/1; 2/1; 3/2; 5/3; 8/5; 13/8; 21/13; 34/21; 55/34, 89/55; 144/89; …

1 ; 2; 1.5; 1.666; 1.6; 1.625; 1.615; 1.619; 1.617; 1.6181; 1.6180; …

It is observed that

$$\Phi^2 = 1 + \Phi$$

$$\frac{1}{\Phi} = \frac{\sqrt{5} - 1}{2} = 0.6180339\ldots$$

The first expression shows that Φ is a solution of the second degree equation

$$\Phi^2 - \Phi - 1 = 0\,,$$

so one returns to the original expression, considered capable of guaranteeing aesthetic and therefore functional and static reliability in that it was derived from the laws of organic differentiation. Quite familiar with the mathematical concepts that during the Middle Ages made Euclidean geometry computable, Viollet-le-Duc tried to empirically de-construct the surveyed buildings to seek in the configuration of the elevation and plans the necessary criteria to re-trace the design work. Substituting the building site tools with compass and ruler the scholar examined the drawing of the naves considering the rectangles and the squares subtending their spans as sums of right triangles, equilateral and isosceles triangles or, as Viollet would have called them, Egyptian triangles as they were believed to be derived from the properties of the monuments built in the Valley of Kings[9] (fig.1).

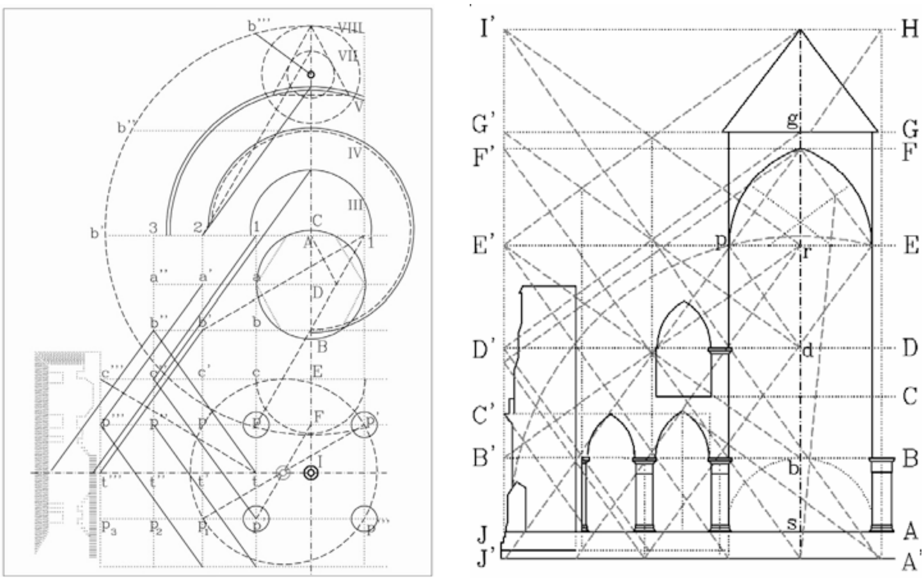


Fig. 1. Viollet–le-Duc. Plan and section of Notre Dame de Paris. Re-elaboration in CAD by F.S. Golia

According to Herodotus (*Histories* II, 124; 5) the matrix-section of the pyramid of Cheops is an isosceles triangle that can be divided by its height into two mirror-symmetric right angle triangles (fig.2) [Taylor 1859; Smyth 1978].

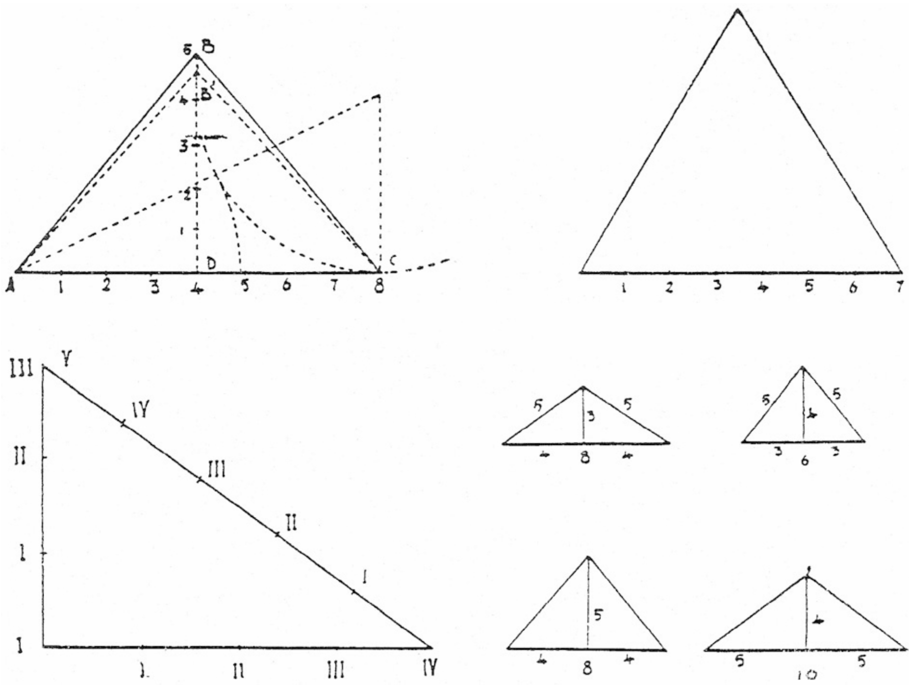


Fig. 2. Geometries widely used on medieval building sites derived from the model of the Pyramid of Cheops. Harmonic triangles: stable (base 8, height 5) and Egyptian (base 8, height 4.94432)

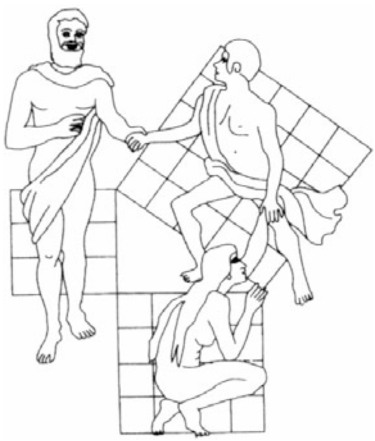


Fig. 3. The myth of Isis, Osiris and Horus. This representation of the Pythagorean theorem is attributed to Euclid

Attention is drawn to the sides of the triangle which, when endowed with the lengths of the perfect or sacred triangle (3 or 4 units), represented the mythical union of Osiris (god of the dead and of fertility) and Isis mother of Horus, the god who unified upper and lower Egypt.[10] According to the legend, the triangle having sides 3, 4 and 5 is linked to the proof of Pythagoras' theorem. The area of the square built on the hypotenuse, 5 units in length, represented by Horus the child in the myth, is obtained as the sum of the areas of the squares on the other sides. This icon is, for Plato, the symbol of marital union (fig. 3).[11]

Applying algebraic knowledge to Euclid's theorems the master masons were able to calculate the geometry of the pyramid (fig. 4).[12] From the direct and indirect formulas the relationship between the apothem of a side and that of the square base could be obtained. For certain values this was shown to be equal to the square of the height of the volume.[13] For the height h of the pyramid and the half side of the base a, one obtains:

$$h \times a = h'^2 .$$

Applying Pythagoras theorem, the following expression is obtained:

$$h \times a = h^2 + a^2 .$$

Having divided by a the problem is reduced to the second degree equation previously used to calculate Phidias's mean, which is about

$$\left(\frac{h}{a}\right)^2 - \left(\frac{h}{a}\right) - 1 = 0 ,$$

an expression that calculates the golden ratio as a positive solution of the equation which coincides as shown with

$$x^2 - x - 1 = 0 ,$$

that is,

$$\Phi^2 - \Phi - 1 = 0 .$$

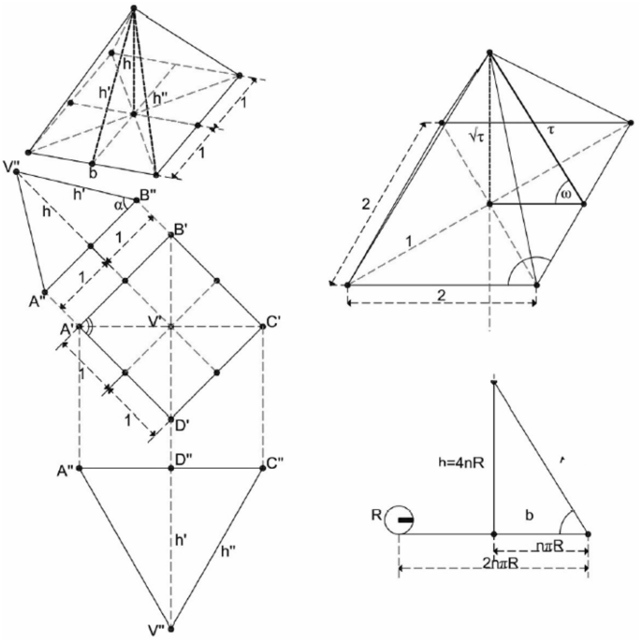
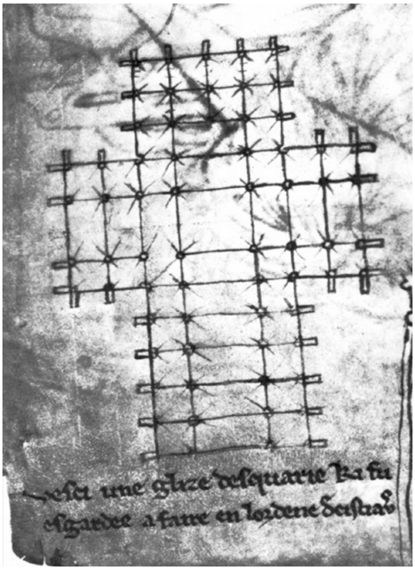


Fig. 4. The geometry of the "Egyptian" pyramid. Re-elaboration in CAD by A. Erario

Whether or not the craftsman responsible for building walls was able to compute and solve the problems encountered during work in progress was a very debated issue at the beginning of the twentieth century.

Amongst the documents that attest to the craftsmen's technical expertise one can undoubtedly include Villard de Honnencourt's *Livre de portraiture* [1235]. Villard was a master mason who, like all of his colleagues in the profession, had travelled to wherever work could be found.



The booklet, considered by some a construction site or guild book [Frankl 1960: 36], by others a *Carnet de voyage*, travel book [Barnes 1989: 211], or a long pondered work is generally thought to be "an exceptional testimony" [Erlande-Branderburg 1987: 9] on the art of building walls and the technical jargon of cathedral building [Bechmann 1993: 313-314]. Confirmation of this is given by figure shown on fol. 14v in which the Latin cross plan of a church is described: *Vesci une glize d'esquarie, ki fu esgardée a faire en l'ordene de Cistiaus* (Here is a church with a square plan, which it was thought to build for the Cistercian order) says the caption [Villard de Honnecourt 2009: 93] (fig. 5).

Fig. 5. Plan of a Cistercian church from the sketchbook of Villard de Honnecourt, fol. 14v.

The drawing in the perspective of the work carried out by the school of Roland Bechmann (1880-1968) is a guide in the composition of series of vault spans. Just as eloquent is the drawing on fol. 18v included amongst the pages dedicated to geometrical research drawing where a 'squared' face is represented (fig. 6).

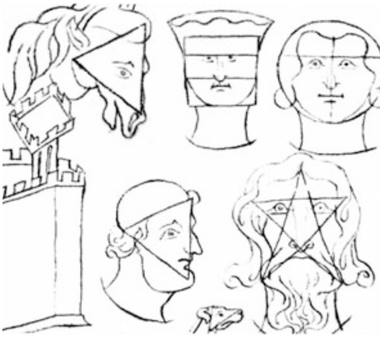


Fig. 6a. Geometric schemes for designing figures, from the sketchbook of Villard de Honnecourt, fol. 18v (detail)

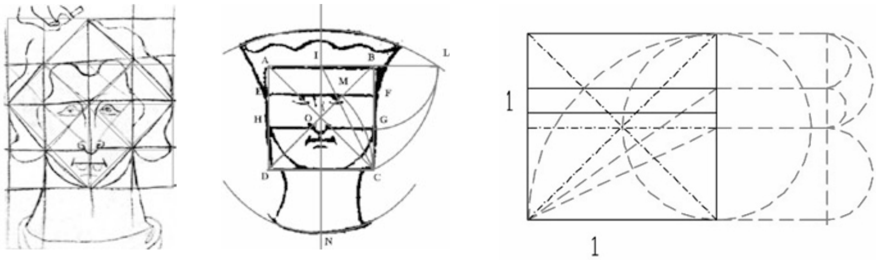


Fig. 6b. left) sketch from the sketchbook of Villard de Honnecourt, fol. 19v; centre and right) graphic reconstruction on a face drawn by Villard (fol. 18v) of the method given by Vitruvius for drawing the human face according to classical proportions, after [Tani 2002].

The apparently naïve structure teaches those who have the means to understand how to divide a segment in mean and extreme ratio (Euclid, *Elements*, Bk. II, prop. XI). An alternative way to divide a given segment into proportional parts which has endured the test of time for its aesthetic qualities is the "Golden number" for dividing lengths according to Matyla C. Ghyka's definition ("golden section") [1931] (fig. 7).

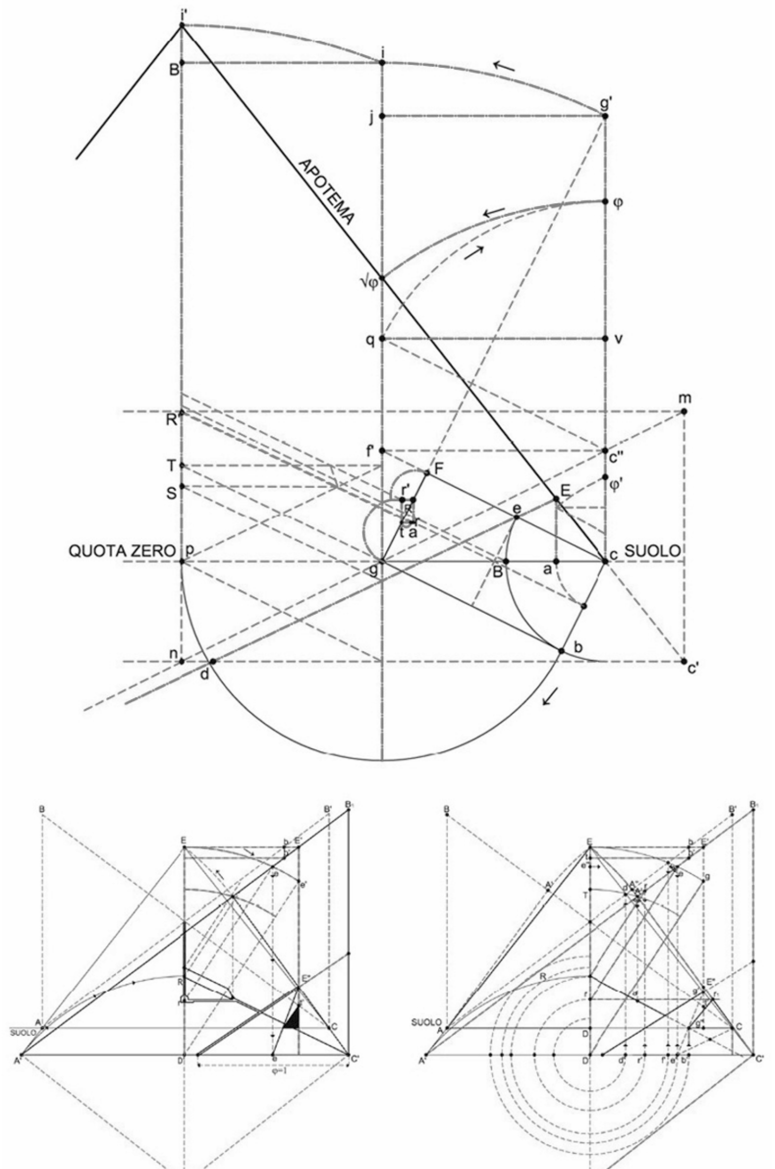


Fig. 7. Decodification of the drawing by Matila Ghyka used to determine the golden section in the Great Pyramid of Giza. Re-elaboration in CAD by A. Erario

The need to find in buildings ratios and proportions which were functionally and statically correct and as well as aesthetically pleasing can also be seen in the properties of the basic shapes obtained using pairs of plumb bobs. Villard de Honnecourt adopts as a model for cloister passage ways a circle between an inscribed square and a circumscribed circle: “In this way one obtains a cloister that has the same surface area as the lawn” (*Par chu fait om on clostre autretant es voies com el prael*) as the caption of figure 20r says. The drawing tends to hint at, rather than display, the geometric configuration, if one is to give credit to the page titles concerning the *Ars geometrie* in which it is placed. In this perspective the subsequent drawing is also quite revealing: a rhombus inscribed in a square bisected by the medians and diagonals, which is the oldest of known icons and which bears the caption: “how to divide a stone so that the two halves are square” *(Par chu partis om one pirre que les moitiés sont quareies* (fig. 8).

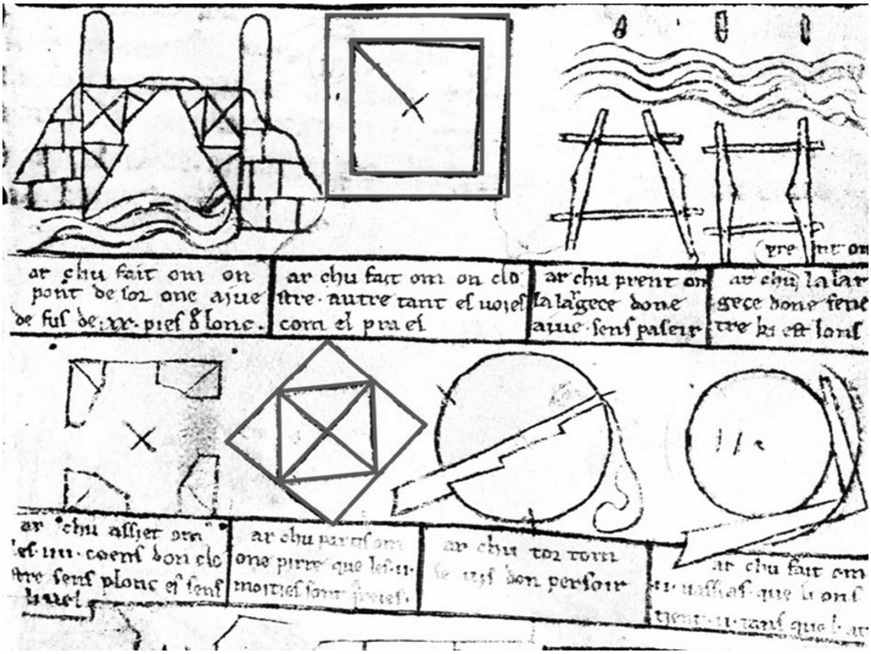


Fig. 8. Villard de Honnecourt, fol. 20r (detail). Above) *Par chu fait om on clostre autretant es voies com el prae* (By this [means] one makes a cloister equal in it walkways as in its garth); Below) *Par chu partis om one pirre que les IJ. moitiés sont quareies* (By this [means] one divides one stone [so] that the two halves are square). See [Villard de Honnecourt 2009: 136]

The figure shows the easiest and fastest way to halve and double areas in a set perimeter, a problem tackled by Plato in his dialogues, who cited Eratosthenes (276-194 B.C.). In order to save the city from the plague the oracle of Delos had ordered the altar of his temple to be “doubled”, the Athenians made a mistake and “octupled” the cube that served as a sacrifical altar.[14] The main point of the anecdote mentioned by Plato, i.e., the importance of geometric logic, is also made in the dialogue between Socrates and Meno, who, under the master’s guidance, learns how to calculate the length of the side and the diagonal of a square (Plato, *Meno*, 84d-85b). In the introduction to Book IX Vitruvius refers to Plato as the inventor of methods for laying out enclosures, angles and distances on the ground (fig. 9).

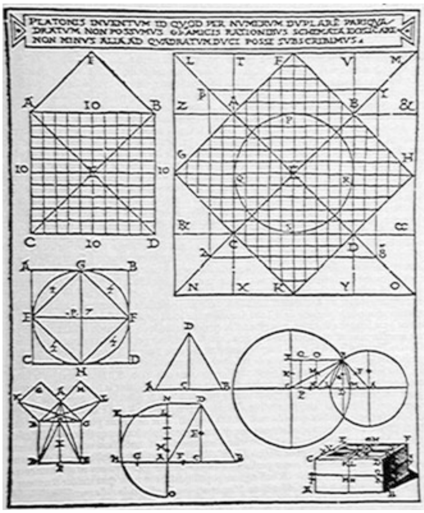


Fig. 9. Cesare Cesariano, methods for doubling the square
[Vitruvius1521: Liber Nonus, CXXXXIIIIr]

Compared to the drawings that illustrate the content, the exegesis behind the decoding of the Vitruvian text is a more significant example of how mathematical thought evolved in time. If one considers the secrets unveiled in the table included in Roriczer's *Büchlein von der Fialen Gerechtigkeit* (*Booklet Concerning Pinnacle Correctitude*) [1486], the metamorphosis started when on the building site the reduction of masonry sections was checked on the basis of the height from the ground (fig. 10).

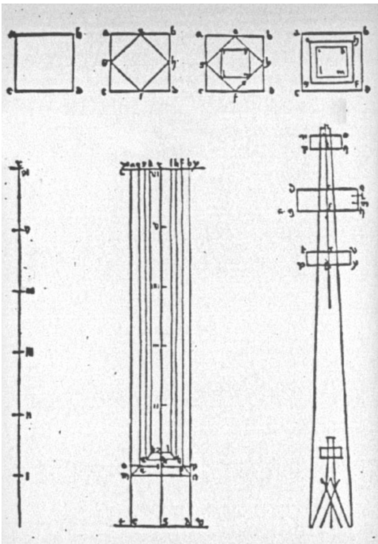


Fig. 10. Plate from Mathis Roriczer, *Büchlein von der Fialen Gerechtigkeit* [1486]

For those who are familiar with descriptive geometry, the construction unveiled by Mathis Roriczer and Hans Schmuttermayer (1435-1495) with the permission of the authorities, when the new guild interests had already arisen, betrays the homology correspondences (in particular homothety)[15] that are established between a horizontal plane and a vertical plane leaving fixed the modular relationships between plans and elevations [Borsi 1967]. A useful choice from a figurative point of view as well as can be seen by drawing the geometrical matrixes of the mapping points or by re-tracing the color pattern of the marble tiles [Mandelli 1983: 116-117]. The possibility that there might be some technical content allegorically hidden in the *Livre de portraiture* is supported by information concerning its author.

According to some of Villard's biographers, it is possible that he started his apprenticeship by studying the texts that were in the Honnecourt-sur-Escaut abbey, which was the epicentre of several schools or similar structures in the area [Tani 2002]. Le Goff identified about twenty-two such schools [Bechmann 1993: 17; see also Le Goff 1993].

However, not all scholars consider Villard de Honnecourt a learned man. According to James Ackerman he, was "not an architect or master mason, but an artison with more limited capacities" [Ackerman 2002: 34]. In the American scholar's opinion the vertical section of Reims cathedral, the one that appears on fol. 32v of the *Sketchbook* (fig. 11), cannot be a 'real life' drawing because at the time the cathedral had not reached its final height, but that it is "conceptually highly sophisticated" compared to the other drawings collected in his notes [Ackerman 2002: 36].

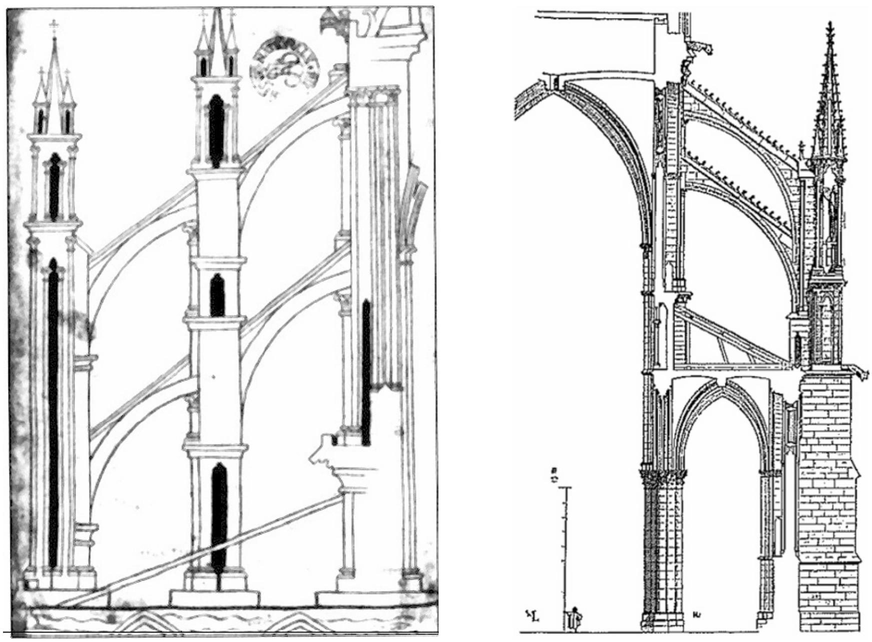


Fig. 11. Cathedral of Reims, view of the buttresses and section of the nave. A, left) from the sketchbook of Villard de Honnecourt, fol. 32v; b, right) Viollet-le-Duc, *Dictionnaire raisonné* [1856, p. 318]

The possibility that the sketchbook might include sketches by different authors was considered also by Ronald Bechmann's disciples: this hypothesis however does not in any way alter the value of the sketchbook. What James Ackerman found of particular interest is "the demonstration of the capacity to represent on one plane cuts at several levels" [Ackerman 2002: 40].

A new figure emerged: that of the a different kind of master mason who could draw and interpret designs who used the workshop where he kept his scale drawings, engraved on a plaster covered floor [Lenza 2002: 60-61]. The need to show or to see what the end product would look like before it was actually built brought about the refinement of a drawing system from which the professional figure of the architect emerged.

In great building works it is customary to have a main master who manages and conducts the work by word only and rarely – or perhaps never – actually performs any practical task. It was a situation that the construction site workers resented, as one can easily understand: "The masters of the masons, carrying a baguette and gloves … worked not at all, although they received a larger payment; it is this way with many modern prelates" [Gimpel 1961: 136]. Borsi described the rise of a new professional figure: "Nothing could be further from the truth than speaking of the choral nature of the Gothic cathedral; the opposite is true, a single mind directed the work … and thought of everything" [Borsi 1965: 45 (my translation)]. It is in the pre-Renaissance period, claims Le Goff, that the art of building becomes a science and the architect a scientist [Le Goff 1981: 235].

The times were just right for Brunelleschi to prepare for his countrymen, on the door of Santa Maria del Fiore, an application which made it possible to describe the spatiality of objects starting from two-dimensional points of view, It would take two centuries for the biunivocal procedures to develop into a complex disciplinary corpus around which design know-how would evolve, a science in continuous evolution in direct correlation with the metamorphosis of ideas [Docci and Migliari 1996: 9-11].

Notes

1. See [Viollet-le-Duc, 1854-1868, Tome VII, Proportion, sub voce]: *On doit entendre par proportions, les rapports entre le tout et les parties, rapports logiques, nécessaires, et tels qu'ils satisfassent en même temps la raison et les yeux. Â plus forte raison, doit-on établir une distinction entre les proportions et les dimensions. Les dimensions indiquent simplement des hauteurs, largeurs et surfaces, tandis que les proportions sont les rapports relatifs entre ces parties suivant une loi. «L'idée de proportion, dit M. Quatremère de Quincy dans son Dictionnaire d'Architecture (Tomo II, p.318), renferme celle de rapports fixes, nécessaires, et constamment les mêmes, et réciproques entre des parties qui ont une fin déterminée». Le célèbre académicien nous paraît ne pas saisir ici complètement la valeur du mot proportion. Les proportions, en architecture, n'impliquent nullement des rapports fixes, constamment les mêmes entre des parties qui auraient une fin déterminée, mais au contraire des rapports variables, en vue d'obtenir une échelle harmonique.*
2. *Praeterea, veritas est adaequatio rei et intellectus. Sed haec adaequatio non potest esse nisi in intellectu. Ergo nec veritas est nisi in intellectu* (Truth is the conformity of thing and intellect. But since this conformity can be only in the intellect, truth is only in the intellect) [Thomas Aquinas, *Quaestiones disputatae de veritate* q. 1, a. 2, s.c. 2].
3. Leonardo Pisano, *Liber Abaci,* 1202: *Nouem Figure indorum he sunt. Cum his itaque nouem figures, et cum hoc signo 0, quod arabice zephirum appellatur, scribitur quilibet numerus, ut inferius demonstratur* (The nine Indian figures are: 9 8 7 6 5 4 3 2 1. With these nine figures and with the sign 0, which the Arabs call zephir, any number whatsoever is written, as is demonstrated below) [Leonardo Pisano 2002: 17].
4. See [Bagni 1996, vol. I]. The separation of geometrical science and mathematical science is ascribed to the distinctions introduced by Archytas.

5. There were also other translations available, such as the one by Cicero, but our knowledge of it is indirect and fragmentary.
6. Porphyry of Tyre, commentary on *Ptolemy's Harmonics* (*Eis ta Harmonika Ptolemaiou hypomnēma*) [Camerano: Capo. 1, http://www.pitagorismo.it/?p=157].
7. Eudemus of Rhodes, *Physica*, fr. 30 (see also Simplicius, *On Aristotle's Physics* 4, 67, 26).
8. The series was obtained as an answer to the following question: "How many pairs of rabbits can be produced from a pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?"
9. From the history of mathematics we know that the Egyptians knew how to trace a right triangle by using a rope divided into twelve equal parts.
10. "The upright, therefore, may be likened to the male, the base to the female, and the hypotenuse to the child of both, and so Osiris [the father] may be regarded as the origin, Isis [the mother] as the recipient, and Horus [the son] as perfected result. Three is the first perfect odd number: four is a square whose side is the even number two; but five is in some ways like to its father, and in some ways like to its mother, being made up of three and two" [Plutarch 1936: 136-137].
11. The geometrical demonstration seems to be ascribed to Euclid, it is certain, however, that Plato chose it as an emblem of his Republic; see Plato, *Republic* VIII, 546.
12. The isosceles triangle has the following property: the angle opposite the base is a 90° angle. It can therefore be obtained by inscribing it in a semi-circumference. The properties of an equilateral triangle are referred to the entire circumference . The median point of the triangle is in relation to the circumference and therefore can be traced to the calculation of the mean of Phidias.
13. The height of the pyramid is equal to the tangent line of the acute angle formed with the base. For $\varpi = 54°54'$ hypotenuse $\phi \cong 1.618$.
14. In a letter addressed to Ptolemy III, cited seven hundred later by Eutocius, in his commentary on Archimedes' *On the sphere and cylinder* Eratosthenes told the King that the legendary Minos wished to build a tomb for Glaucus in the shape of a cube. In a different context the same problem is posed and linked to Plato's the famous sentence "Let no one ignorant of geometry enter".
15. Homothety a linear transformation that involves no rotation, the composition of a translation and a central dilatation. It is of the form $x' = kx$, $y' = ky$, and is a stretching if $k > 1$, and a shrinking if $0 < k < 1$ [quoted from http://www.maplesoft.com/support/help/AddOns/view.aspx?path=Definition/homothety].

Bibliography

ACKERMAN, J. S. 2002. *Origins, Imitation, Conventions*, Cambridge, MA: MIT Press.

ANTOLINI, Giovanni A. 1817. *Osservazioni ed aggiunte ai principi di architettura civile di F. Milizia.* Milan: Stella.

BAGNI, G. T. 1996. *Storia della Matematica,* 2 vols. Bologna: Pitagora.

BARNES, C. 1989. *Les batisseur des cathédrales gothique*. Strasbourg: Editions Les Musées De La Ville De Strasbourg.

BECHMANN, R. 1993. *Villard de Honnecourt. La pensée technique au XIII^{e} siècle et sa communication*. Paris: Picard.

BORSI, F. 1965. *Cultura e Disegno*. Florence: Libreria Editrice Fiorentina.

———. 1967. *Mattheus Roriczer. Il libro della costruzione esatta dei pinnacoli.* Quaderno n°2, Cattedra di Disegno dell'Architettura. Florence: Facoltà di Architettura di Firenze.

CAMERANO, Rigo. n.d. Pitagorismo ed epicureismo a Taranto in età classica ed ellenistica. http://www.pitagorismo.it/?p=157. Last accessed 15 October 2011.

DOCCI, M. and R. MILGLIARI. 1996. *Scienza della Rappresentazione*. Rome: La nuova Italia Scientifica.

ECO, U. 1956. *Sul problema estetico in San Tommaso*. Torino: Edizioni di Filosofia.

———. 1959. Sviluppo dell'estetica medievale. In *Momenti e problemi di storia dell'estetica*, Milan: Marzorati.

———. 1987. *Arte e bellezza nell'estetica medievale*. Milan: Bompiani.

———. 1970. *Il problema estetico in Tommaso d'Aquino*. Milan: Bompiani.

ERLANDE-BRANDERBURG, A. 1987. Introduzione. Pp. 7-9 in *Villard de Honnecourt. Disegni*, A. Erlande-Branderburg, R. Pernoud, J. Gimpel and R. Bechmann, eds. Milan: Jaca Book.

FRANKL, P. 1960. *The Gothic Literary Sources and Interpretations through Eight Centuries*, Princeton: Princeton University Press.

GHYKA, M. G. 1931. *Le nombre d'or.* Paris: Gallimard.

GIMPEL, J. 1961. *The Cathedral Builders.* London: Evergreen Profile Book.

———. 1975. *La révolution industrielle du Moyen Age*. Paris : Éditions du Seuil.

LE GOFF, J. 1981. *La civiltà dell'Occidente Medievale.* Torino: Einaudi.

LENZA, C. 2002. Architettura medievale: etica, estetica e tecnica.In *Dal Pantheon a Brunelleschi*, A. Gambardella, ed. Naples: Edizioni Scientifiche Italiane.

LEONARDO Pisano (Fibonacci). 1202. *Liber Abaci.* In *Scritti di Leonardo Pisano matematico del secolo decimoterzo pubblicati da Baldassarre Boncompagni, Volume I: Il Liber Abbaci di Leonardo Pisano*. Roma: Tipografia delle Scienze Matematiche e Fisiche, 1857-1862.

———. 1220. *Practica Geometriae*. In *Scritti di Leonardo Pisano matematico del secolo decimoterzo pubblicati da Baldassarre Boncompagni, Volume II: Leonardi Pisani Practica geometriae ed opuscoli*. Roma: Tipografia delle Scienze Matematiche e Fisiche, 1857-1862.

———. 2002. *Fibonacci's Liber Abaci: a Translation into Modern English of Leonardo Pisano's Book of Calculation*. L. E. Sigler, trans. New York: Springer-Verlag New York.

———. 2008. *Fibonacci's De practica geometrie*. Barnabas Hughes, ed. New York: Springer Science + Business Media.

MANDELLI, M. 1983. Lettura di un disegno: la pergamena di Siena*. Studi di documenti di architettura* 11 (giugno 1983): 116-117.

PIAZZI SMYTH, C. 1978. *The Great Pyramid: Its Secrets and Mysteries Revealed*. New York: Gramercy.

PLUTARCH. 1936. On Isis and Osiris. Frank Cole Babbit, trans. In Plutarch, *Moralia,* vol. 5. Loeb Classical Library.

QUATREMÈRE DI QUINCY, Antoine Chrysostome. 1832. Proportion, sub voce. *Dictionnaire Historique de l'architecture*, Tomo II, p. 317-321.Paris: D'Adrien Le Clere et C^ie^.

RORICZER, Mathis. 1486. *Büchlein von der Fialen Gerechtigkeit*. English translation: *Booklet Concerning Pinnacle Correctitude*, Lon Shelby, trans. In *Gothic Design Techniques: The Fifteenth-Century Design Booklets by Mathes Roriczer and Hans Schmuttermayer*. Carbondale and Edwardsville: Southern Illinois University Press, 1977.

TANI, C. 2002. Villard de Honnecourt: divulgatore di cultura scientifica? Ricerca di tracce degli Elementi di Euclide nel "Taccuino"*. Heliopolis. Culture e Civiltà* (Quadrimestrale dell'Istituto Politeia) 1, 1: 107-123.

TAYLOR, J. 1859. *The Great Pyramid: Why Was It Built and Who Built It?* London: J. Taylor.

VILLARD DE HONNECOURT. 1235. *Livre de portraiture*. Ms Fr 19093, Bibliothèque nationale de France, Paris.

———. 1988.*Villard de Honnecourt. Disegni*, A. Erlande-Branderburg, R. Pernoud, J. Gimpel and R. Bechmann, eds. Milan: Jaca Book.

———. 2009. *The Portfolio of Villard de Honnecourt. A New Critical Edition and Color Facsimile*. Carl F. Barnes, Jr., ed. Surrey, UK and Burlington, USA: Ashgate Publishing.

VIOLLET-LE-DUC, E-E. 1856. *Dictionnaire raisonné de l'architecture française du XIe au XVIe siècle*. Paris: A. Morel.

VITRUVIUS, Marco Pollione. 1521. *Di Lucio Vitruvio Pollione de Architectura libri dece : traducti de latino in vulgare, affigurati, commentati & con mirando ordine insigniti*. Cesare Cesariano, trans. Como : Gotardus de Ponte.

———. 2002. *De Architectura, Libri X*. Franca Bossalino and Vilma Nazzi, trans. and eds. Rome: Edizioni Kappa.

WITTKOWER, R. 1962. *Architectonical Principles in the Age of Humanism*. City: Publisher.

About the author

Adriana Rossi received her degree in architecture with honors from the University of Naples Federico II. She was qualified as an architect in 1985. That same year she began to collaborate with the University of Naples in the department of “Configurazione e attuazione dell'architettura”. She earned her Ph.D. in 1992 in “Survey and representation of built work”, and was awarded a post-doctoral grant from the Dipartimento di Rappresentazione of the University of Palermo. She joined the Second University of Naples as a researcher in 1995, becoming part of the Department of “Cultura del Progetto” in 1996. Today she is an Associate Professor there, teaching “Drawing and techniques for design”. Among the books she has authored are *I Campi Flegrei* (Officina Edizioni, 2006), *DisegnoDesign* (Officina Edizioni, reprinted in 2008) and *Suoni di pietra* (Deputació de Barcellona, 2009).

Ahmed Ali Elkhateeb

King Abdulaziz University
Department of Architecture
Faculty of Environmental Design
Jeddah, Saudia Arabia
aelkhateeb@kau.edu.sa

Keywords: domes, domes mathematics, Islamic architecture, Cairo, pendentives, squinches, muquarnas

Research

Domes in the Islamic Architecture of Cairo City: A Mathematical Approach

Abstract. This work analyzes mathematically and graphically the two methods used historically in the transitional zone between the circular base of the dome and the square top of the cube where the dome is supported. The time frame of this work is the distinguished historical buildings of Islamic Cairo built between the ninth and eighteenth centuries. Ten samples were chosen out of a total of thirty. A set of mathematical expressions has been derived to relate the different parts of the squinches/pendentives to the cube with a side of length l. The equations derived were validated twice, first by generating 3D graphical sequences for both squinches and pendentives for the selected domes using CAD software based on the values obtained from the driven equations, and second by executing physical models using a 3D printer for two examples of squinches and pendentives.

1 Introduction

Domes are one of the most distinguished architectural elements; their antiquity (which may date back to more than five thousand years ago) increases their ambiguity and charm, and their high flexibility in covering the very wide range of different spans from few meters to hundreds of meters increases their modernity and power. From the greatness of history and the strength of the present, domes acquired their originality and modernity, thus it deserve to be the king of all roofs. Historically there isn't any difficulty in constructing walls as boundaries for a space to fulfill the requirements of both privacy and security, as construction is one of the oldest human activities, but the real problem was how to cover or roof the space. This may be one of the reasons that limited the spans of spaces for a long time. In the various phases of construction, the dome was one of the inventions for covering large span spaces that emerged in response to the requirements of the newest functions, such as temples, palaces and the like.

Fig. 1. The oldest and the most famous domes in Islamic Architecture. a, left) The Holy Dome of the Rock, Jerusalem; b, right) The Taj Mahal, Agra

Nexus Netw J 14 (2012) 151–176

DOI 10.1007/s00004-011-0103-3 ; *published online* 13 January 2012

In Islamic architecture, the subject of this present work, the dome has played a leading role since the early beginning of Islamic era. The Holy Dome of the Rock (fig. 1a), built by the Umayyad ruler Abdol Malek Ibn Mrwan between 687 and 691 A.D. in Jerusalem, may be the oldest dome in Islamic Architecture. Subsequently, Muslim architects admired the dome and used it in excellently, thus the dome applied widely with different forms as a cover for various types of buildings in the Islamic era: Masjids (Mosques), palaces, baths, mausoleums and more. With its onion dome, the mausoleum of Taj Mahal in Agra, India, built between 1632-1648 A.D. by Shah Jahan (the Mughal ruler) as a mausoleum for his favourite wife, is one of the most famous domes in Islamic architecture (fig. 1b).

1.1 Problem definition

While there are numerous research works examining the architectural features and aesthetical values of Islamic architecture, research works concerning this architecture from the analytical and technical points of view are still limited (see, for example, [Cipriani 2005; Harmsen 2006; Harmsen et al. 2007; Krömker 2010; Takahashi]). It is a well-known historical fact that the domes were supported even on squinches or pendentives [Abouseif-Behrens 1996]. This present work investigates mathematically and geometrically the types of forms that were utilized to support the domes covering the cuboid spaces (square plans). More precisely, it tries to answer two questions:

- What are the forms that were used to generate both the squinches and pendentives?
- What are the mathematical formulae that can be used to generate such forms?

1.2 Objectives

The main purpose of this work is to derive sets of mathematical formulae that express the different forms used in the transitional zone between the circular base of the dome and the square top of the cube where this dome rests. These formulae will contribute to:

- Greater understanding of the two terms *squinches* and *pendentives*, the only two methods used historically to support the domes in architecture generally and in Islamic architecture in particular;
- Differentiate and distinguish architecturally and mathematically between the formative features of the terms squinches and pendentives;
- Generate accurate drawings for both forms, which are a prerequisite for more advanced technical analysis such as acoustics and lighting that utilize simulation software and require accurate drawings for the spaces under consideration.

1.3 Methodology

For the purpose of this work, a comprehensive historical survey was conducted on the most distinguished domes of the historical buildings in Islamic Cairo in order to determine the various shapes of the historical domes, and to collect the main different forms that were utilized in both squinches and pendentives in the transitional zone.

Based on the historical survey [Elkhateeb and Soliman 2009], ten Masjids were chosen out of thirty Masjids that were built in Cairo between the *Tulunid dynasty* (868-905) and the end of *Ottoman* period in Egypt (1517-1798). Finally, the collected forms were analyzed using the mathematical principles of solid bodies and analytic geometry to derive the required formulae. The formulae thus derived were verified twice, first by producing a 3D graphical sequence for such forms based on the values obtained from the derived equations using AutoCAD (ver2008), and second by generating physical models for one example that uses squinches and one that uses pendentives, utilizing a 3D printer.

Fig. 2 summarizes the methodology applied in this work.

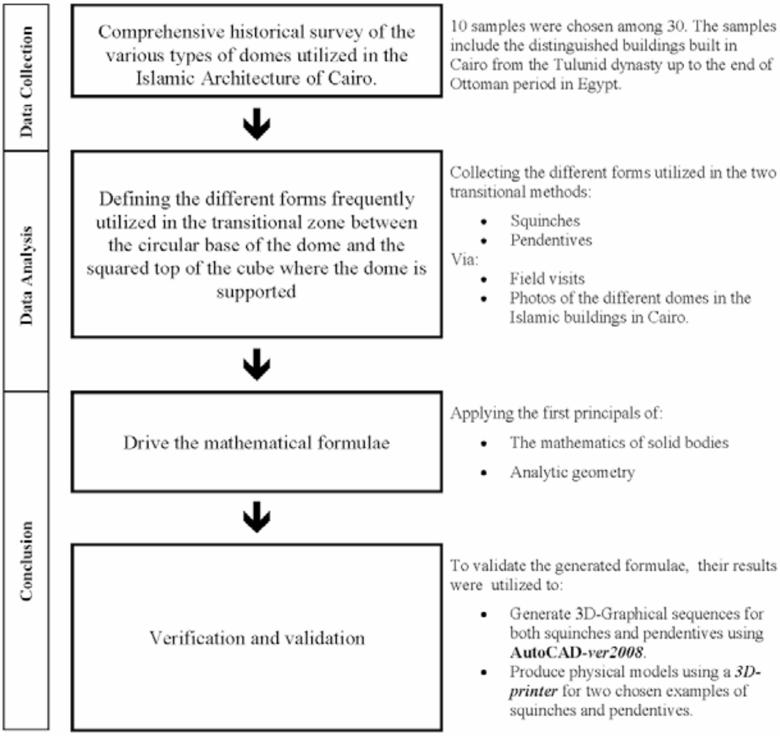


Fig. 2. Methodology

2 The origin of the domes and their types

In architecture, the dome is a vaulted roof having a circular, polygonal, or elliptical base. Since it is difficult to count the various types of domes, the next part will discuss only the most famous types of domes used in Cairo's Islamic architecture. From the architectural point of view, there are two types of domes that were used frequently almost in all of Cairo's Islamic buildings, the spherical dome (based on a perfect sphere) and the elliptical dome (based on a spheroid).

Under these two main categories, many other types of domes can be classified, such as onion and shallow (saucer) domes. Mathematically, both spherical and prolate, or vertically elliptical, domes originate from a complete rotation (360°) of a segment of an arch around its vertical axis. The type of the dome (spherical or elliptical with their different types) will be determined according to the rotated part of the arch and consequently, the position of the rotation axis Y and the horizontal axis X and the relation of both with the origin (0,0) (fig. 3a). In this context, there are two main possibilities, within which are three others, as listed as follows:

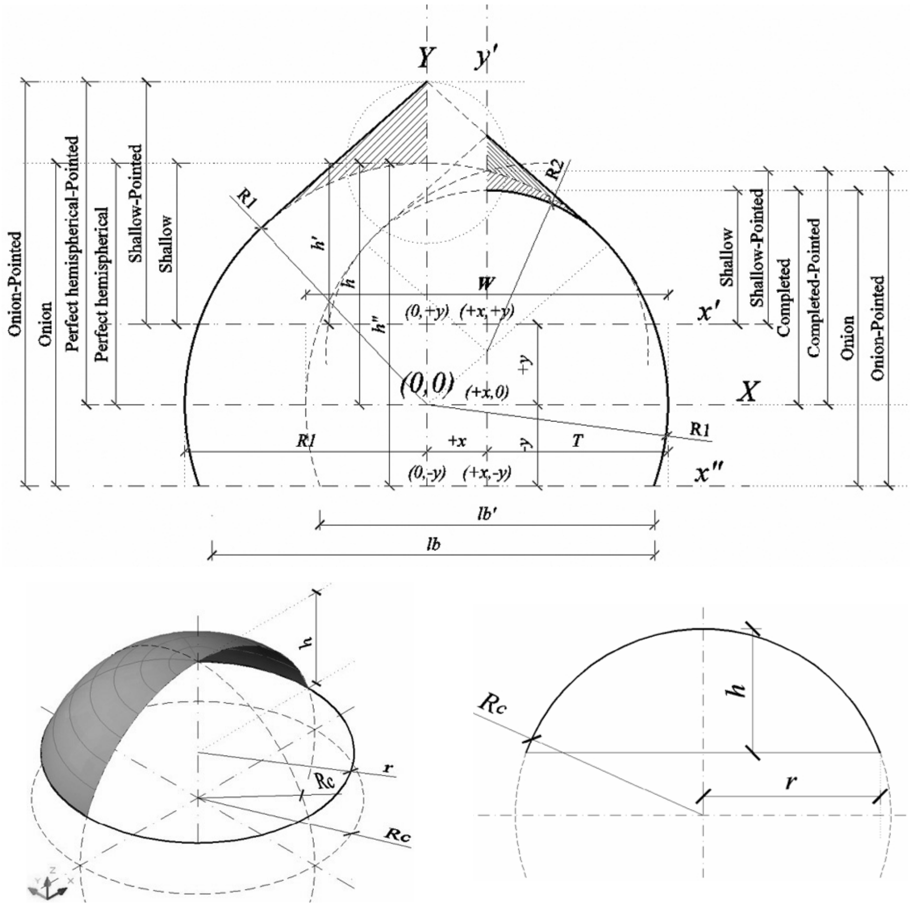


Fig. 3. The origin of the domes and their types. a, above) The origin of the most famous domes; b, lower left) Spherical dome, perspective; c, lower right) Spherical dome, mathematical values

1. The first possibility: the center of the rotated arch and the rotational axis *Y* is the origin (0,0). This is the *spherical dome*. Under this category and according to the location of the horizontal axis *X*, the other three possibilities are:
   - The horizontal axis *X* passes through the origin (0,0). This is the perfect hemispherical dome with its two types (flat or pointed);
   - The horizontal axis *X* moves to the location *x′* to pass through the point (0,+*y*). This is the shallow (saucer) dome with its two types (flat or pointed);
   - The horizontal axis *X* moves to the location *x″* to pass through the point (0,–*y*). This is the onion dome with its two types (flat or pointed).
2. The second possibility: the center of the rotated arch is the origin (0,0) but the rotational axis *Y* is moved to the location (+*x*,0) to be at the location *y′*. This is the *elliptical dome*. Within this category and according to the shape of the rotating part and the location of the horizontal axis *X*, the other three possibilities are:

- The horizontal axis X passes through the location $(+x,0)$. This is the perfect half elliptical dome with its two types (flat or pointed);
- The horizontal axis X moves to the location x' to pass through the point $(+x,+y)$. This is the shallow (saucer) elliptical dome with its two types (flat or pointed);
- The horizontal axis X moves to the location x'' to pass through the point $(+x,-y)$. This is the onion elliptical dome with its two types (flat or pointed).

In shallow (saucer) domes (either spherical or elliptical), the height of the rotated part h' is less than the arch radius $R1$, whereas in the onion dome (either spherical or elliptical) the height of the rotated part h'' is greater than the arch half span $R1$ or T. In other words, the diameter of the dome base lb or lb' is less than its width $2R1$ or W respectively. Table 1 summarizes the previous classification.

Dome	Classification		Location of Arch center	Location of rotation Axis Y	Location of the X Axis	Remarks
spherical	Shallow	Pointed	*0,0*	*Y (0,0)*	*x' (0,+y)*	Dome height **h'< its radius**
		Flat	*0,0*	*Y (0,0)*	*x' (0,+y)*	
	Perfect	Pointed	*0,0*	*Y (0,0)*	*X (0,0)*	Dome height **h= its radius**
		Flat	*0,0*	*Y (0,0)*	*X (0,0)*	
	Onion	Pointed	*0,0*	*Y (0,0)*	*X'' (0,-y)*	Dome height **h'' its radius**
		Flat	*0,0*	*Y (0,0)*	*X'' (0,-y)*	
Elliptical	Shallow	Pointed	*0,0*	*y' (+x,0)*	*x' (+x,+y)*	Dome height **h'< its radius**
		Flat	*0,0*	*y' (+x,0)*	*x' (+x,+y)*	
	Perfect	Pointed	*0,0*	*y' (+x,0)*	*X (+x,0)*	The X axis passes the arch center
		Flat	*0,0*	*y' (+x,0)*	*X (+x,0)*	
	Onion	Pointed	*0,0*	*y' (+x,0)*	*X''(+x,-y)*	The diameter of dome base **lb< its width W**
		Flat	*0,0*	*y' (+x,0)*	*X'' (+x,-y)*	

Table 1. The architectural classification of the most famous historical domes in Cairo

Historically, onion domes have a limited application in Cairo's Islamic architecture, but it is well known and widely used in the Far East, Russia, India and Turkey. Still, there are a limited number of examples in Mamluk architecture. In Cairo, the other types of domes have been used widely, because of their importance in the context of this present research, the following parts will discuss them in some details.

2.1 Spherical domes

From the mathematical point of view, the spherical domes can be completely described via its height h and radius of curvature R_c (see figs. 3a-c), where:

$$R_c = \frac{r^2 + h^2}{2h} \tag{1}$$

The volume of this type of domes can be also calculated by knowing its height $\boldsymbol{h}$ and radius of curvature R_c, or the radius of dome base r [Gieck and Gieck 2006]:

$$V_s = \frac{1}{3}\pi h^2(3R_c - h) = \frac{1}{6}\pi h(3r^2 + h^2) \tag{2}$$

The total surface area of this dome can be calculated:

$$S_a = 2\pi h R_c = \pi(h^2 + r^2) \tag{3}$$

2.2 Elliptical domes

Elliptical domes have wide applications in architecture; they are especially useful in covering the rectangular spaces. The oblate, or horizontal elliptical, dome utilized in particular where there is a need to limit the excessive height of the space that accompanies the use of the spherical domes. The prolate, or vertical elliptical, dome, is the one most often applied from the architectural point of view, as the majority of the most famous domes in architectural history, including Islamic architecture (as will be seen later, see §4), are related to this type of elliptical dome. Usually, the mathematical description of the elliptical domes originates from the mathematical description of the spheroid itself. As the mathematical description of the elliptical domes is more complicated than that of spherical ones, this part will not be considered here, and only its applied architectural part will be considered.

3 Methods of supporting domes in Islamic architecture

Because the form of the dome initially originated from the arch, it exerts thrusts all around its perimeter, and the earliest monumental examples that utilize the dome as a cover required heavy continuous circular (or polygonal) walls to offer continuous support to withstand these forces safely. This may explain those massive walls used, for example, in the Pantheon, which allowed the engineers of the temple to create some decorative niches in the walls, aimed at decreasing its weight. To be sure, these massive walls permitted few openings (and had to be rounded or polygonal to give the required continuous support), limiting for a long time the possibilities to incorporate the domes in complex buildings that consist of many adjacent spaces required connection and continuity, especially when adjacent spaces are vaulted. And this may explain again the reason for the simplicity in the form of the early buildings that utilized domes as a cover.

The next milestone in covering a space with a dome was the transformation from the circular to the square plan (see fig. 4). By this transformation, the incorporation of domes in the building became easier, important and distinguished. It is not an exaggeration to say that with very few exceptions the most valuable historical domes in this context cover a square plans; this is clear at least in historical Islamic Cairo where most of the buildings are either square or semi-square. Historically, the main difficulty that has to be overcome by engineers is how to fill the four voids emerging between the circular base of the dome and square top of the cuboid space where the dome rests (see fig. 4). These voids were known in the different historical references as the "transitional zone" (see a perspective view in fig. 8a). For the purpose of this work, this will be called the "corner blocks".

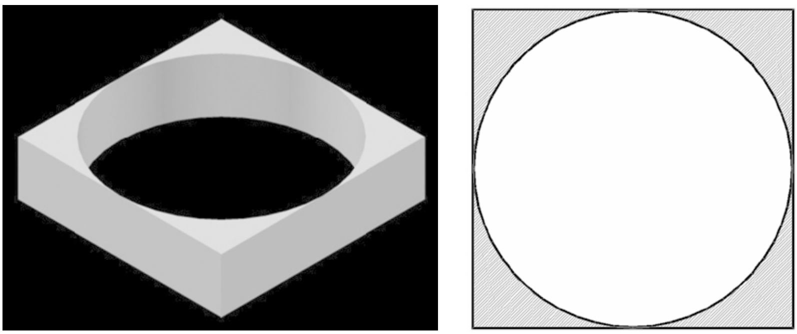


Fig. 4. The transitional zone between square and circle. a, left) perspective; b, right) plan

With their form, these blocks were not appreciated by the architects of the different eras for many reasons: they were unattractive if they were seen in such form inside the room, they were so heavy without any necessity or reason, and unsafe from the structural point of view. The transmission paths of loads coming from the dome and its weight do not require such heavy blocks. To overcome this difficulty, the architects of those days turned to *hollowing* the unwanted parts of these massive blocks. The concept of hollowing the corner blocks was a solution that was at once both aesthetic and functional, as it first makes it possible to decrease the loads of these blocks, keeping them safe to support the dome, thus it functions well structurally and second, provides unlimited possibilities to invent aesthetic methods for this hollowing, leading ultimately to beautiful and innovative forms.

Two methods were utilized to hollow those corner blocks. These methods distinguish between the ways of hollowing, and between the way the loads of the dome are transimitted to the cuboid:

1. The first method: using the squinch(es). Originally, it is a support carried across the corner of a room under a superimposed mass. In architecture, squinches can be one of any of several devices by which a square or polygonal room has its upper corners filled in to form a support for a dome. In its simplest form used in Cairo's Islamic architecture, it is a four diagonal niches, one in each corner of the room transforming the square plan of the room to an octagonal one.
2. The second method: using the pendentive(s). Originally one of the concave triangular members that support a dome over a square space, in architecture, pendentives are triangular segments of a sphere, taper to points at the bottom and spread at the top to establish the continuous circular or elliptical base needed to support a dome.

The invention of methods of supporting a dome over a cuboid space and consequently the need to hollow the four corner blocks which appear in the transitional zone is not an absolute Islamic invention. In fact, it preceded Islam itself by many centuries. However, while the the idea of hollowing itself cannot be be credited to Islamic civilization, Islamic architecture demonstates its superiority in innovating new methods for this hollowing. Successive eras of the Islamic civilization in Egypt (specially the *Burgi Mamluks*, 1382-1517) added the methods to manipulate the exterior part of the transitional zone sculpturally as well. This means that the architectural manipulation of the corner block is not limited to the interior of the space, but can be done internally and externally simultaneously. Some examples are shown in in fig. 5.

Modern structural theories indicate that a slope of 1:2 or higher in corbels (see fig. 6) transmits the loads smoothly and safely (see, for example, [American Concrete Institute 2004]. According to this rule, the height of the corner blocks has to fulfill at least the following equations:

$$r_c = \frac{l}{\sqrt{2}} \tag{4}$$

$$C_l = \frac{l}{\sqrt{2}} - \frac{l}{2} \tag{5}$$

$$C_h = \frac{2l}{\sqrt{2}} - l = 0.4142\, l \tag{6}$$

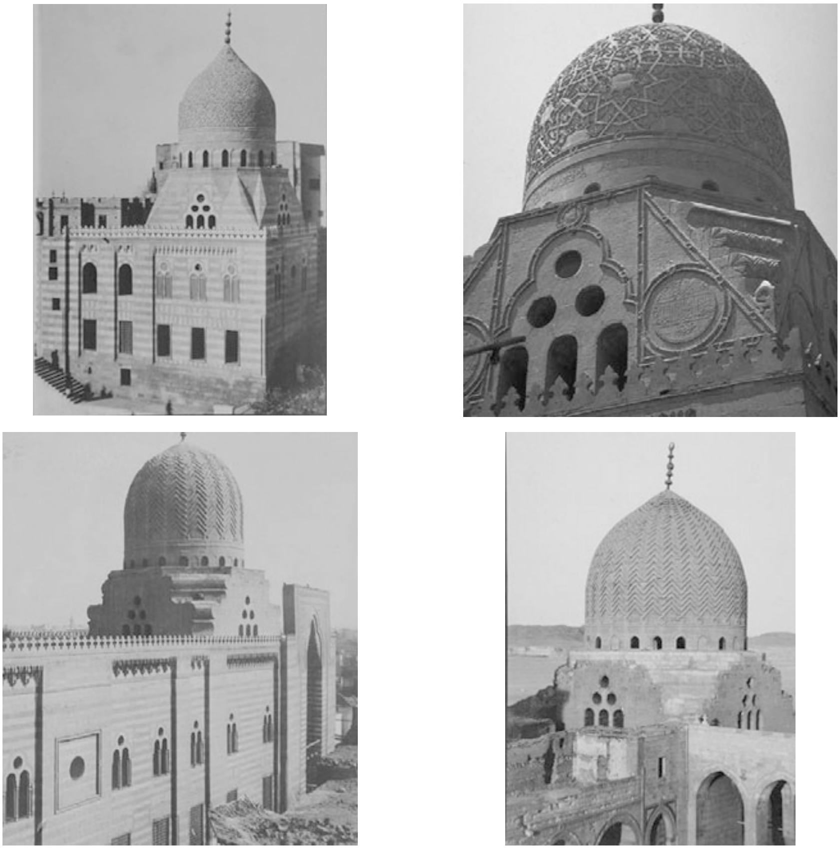


Fig. 5. Examples for the transitional zone formation in chronological order. a, upper left) Amir Qanibay al-Rammah, 1503 A.D.; b, upper right) Sultan Qaytbay, 1475 A.D.; c, lower left) Sultan al-Mu'ayyad Shaykh, 1420 A.D.; d, lower right) Sultan Farag Ibn Barquq, 1411 A.D.

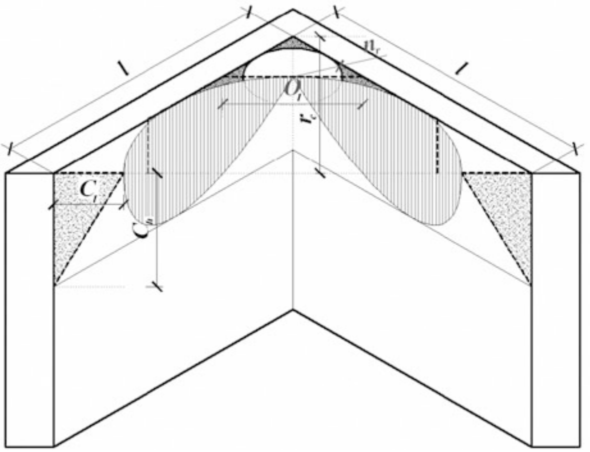


Fig. 6. The corbel length C_l, height C_h and the cube side length l

where l is the cube side length, C_l is the maximum corbel length and C_h is its height (fig. 6). Although this rule was not applied in some of the cases studied, as shown clearly in the squinches of Firouzabad Palace (the squinches of conical vault, see fig. 7), it seems to have been applied in some other examples, especially the pendentives.

3.1 The mathematical analysis of squinches

The squinch was a primitive solution to hollow the corner blocks in the transitional zone. It is believed that it was originally developed, almost simultaneously, by the Roman builders of the late Imperial period and the Sasanian in Persia, where it was used in the palace of Ardeshir, the Emperor of the Persian Empire, near Firouzabad, Iran. Later it was used by the Byzantine architects in their domed buildings, and finally by the architects of the Islamic era. It seems that Islamic architecture developed its own squinches, borrowing from the Sasanian civilization, the nearest to it in place and time.

"Squinch" is defined in the *Encyclopedia Britanica* in the following way:

> **squinch**, in architecture, any of several devices by which a square or polygonal room has its upper corners filled in to form a support for a dome: by corbelling out the courses of masonry, each course projecting slightly beyond the one below; by building one or more arches diagonally across the corner; by building in the corner a niche with a half dome at its head [see fig. 8]; or by filling the corner with a little conical vault that has an arch on its outer diagonal face and its apex in the corner [see fig. 7]
>
> (see http://www.britannica.com/EBchecked/topic/561801/squinch).

From this definition it seems that the word squinch can be considered a category more than a terminology; this category includes every method used to fill in the transitional zone except the pendentives. Again, according to *Britannica's* definition, the main features of the squinches can be summarized as follows:

- Transformation of the square top of the cube first to a polygon (usually an octagon, where the four corner blocks, one at each corner, will appear) and then to a circle. The octagon side length *OL* can be calculated according to Eq. 7;
- The dome rests on the polygon that rests on the square;
- The lower part of the squinch must be a line, not a point, in contrast to pendentives (as will be seen later);
- The corner block is hollowed by the niches or any other means, as previously mentioned.

The length of octagon side O_L (fig. 6) can be calculated from:

$$O_L = l \tan 22.5 = 0.4142\, l \qquad (7)$$

In Cairo's Islamic architecture, the squinches started in abstracted form, only four niches with a half domes at their heads, one at each corner, as used in the Mihrab (niche) dome of Al-Hakim Masjid (1013 A.D.) from the Fatimid period. Here it is clear that the dome rests on the octagon that rests on the square (see fig. 8). Later, and over the course of many decades, squinches developed more complicated forms, for example, using the stone corbel as in the Masjid and school of Sultan Baybars Al-Jashankir (1310 A.D.), using the spherical-pointed vault as in Al-Fadawiyya Dome (1479 A.D.), and others. It seems that the conical vault (see fig. 7) used in the Sasanian architecture was not used in Islamic architecture in Cairo, as I have been unable to find any similarity, but the other types have been widely utilized in buildings of Islamic Cairo.

The squinches of a conical vault, the simplest form of the squinches, originate from the intersection between a diagonal right circular half-conical vault, its apex in the corner, and the corner block. The radius of the cone is equal to the height of the corner blocks and its height can be calculated from Eq. 5.

Fig. 7 represents in an analytical series the method for generating the kind of squinch used in Firouzabad Palace. The squinches of the niche with a half-dome originate from the intersection between the positive form of the niche with its half-dome and the corner block. Fig. 8 represents in an analytical series the method for generating the kind of squinch used in Al-Hakim Masjid. The height of the niche is determined according to Eq. 6 whereas its radius n_r (see fig. 6) can be calculated from:

$$n_r = 0.14644l \qquad (8)$$

The stone corbel squinches originate by corbelling out the courses of stone: each course projects gradually and slightly beyond the one below until it reaches the dome's circular base. In Islamic architecture, stone corbels were known as "stalactite vaults" (see §3.2). In the last type of squinch where the concept of stalactites is used, the way of hollowing the corner blocks has various and different forms depending on the architect's imagination and his ability to invent new stalactite forms which, when complete, compose the stone corbel. Fig. 9 represents in an analytical series the method for generating the kind of squinch used in Masjid Sultan Baybars Al-Jashankir.

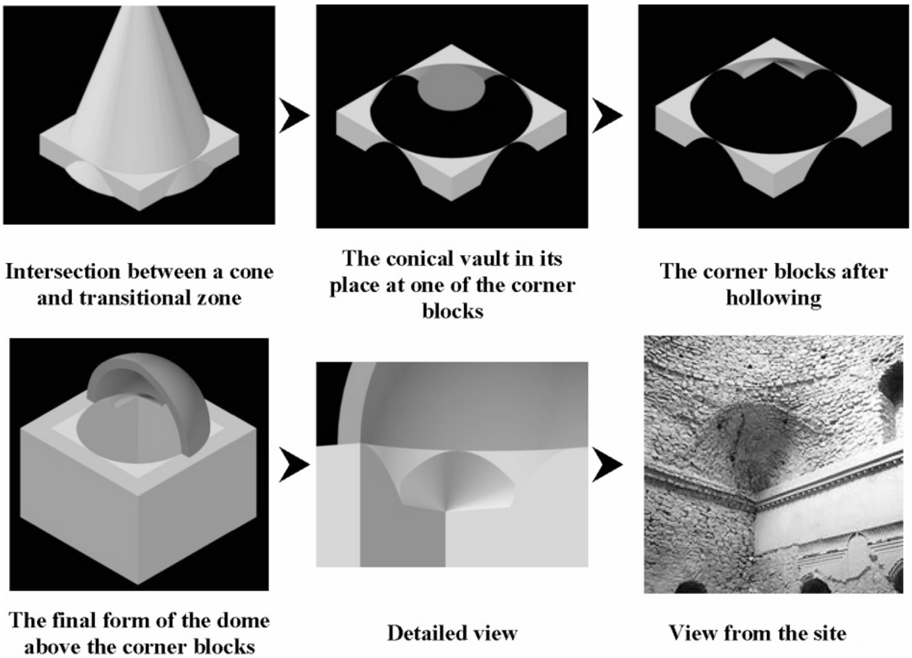


Fig. 7. The conical vault squinch used in the Palace of Ardeshir, Sasanian architecture

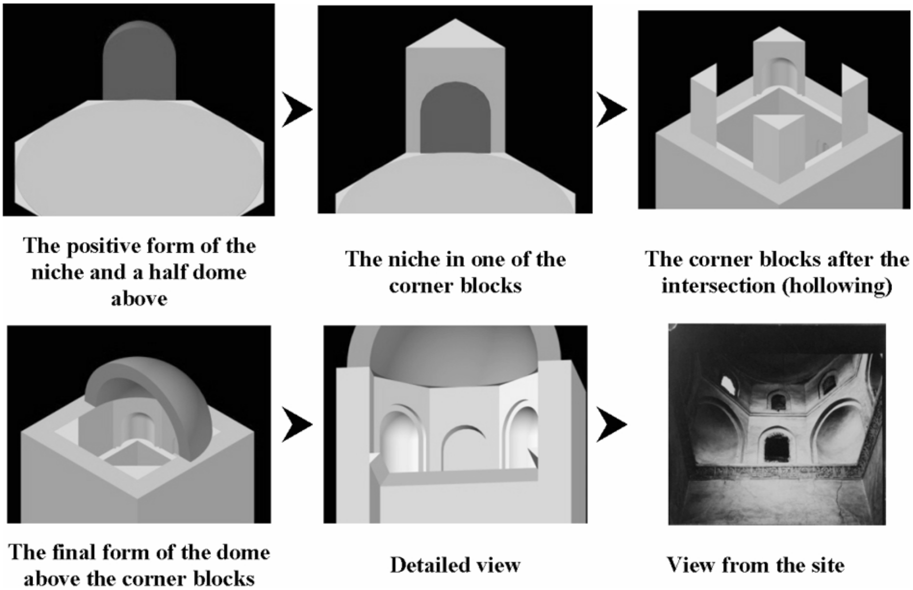


Fig. 8. The niche with a half-dome squinch used in Al-Hakim Masjid (Fatimid period)

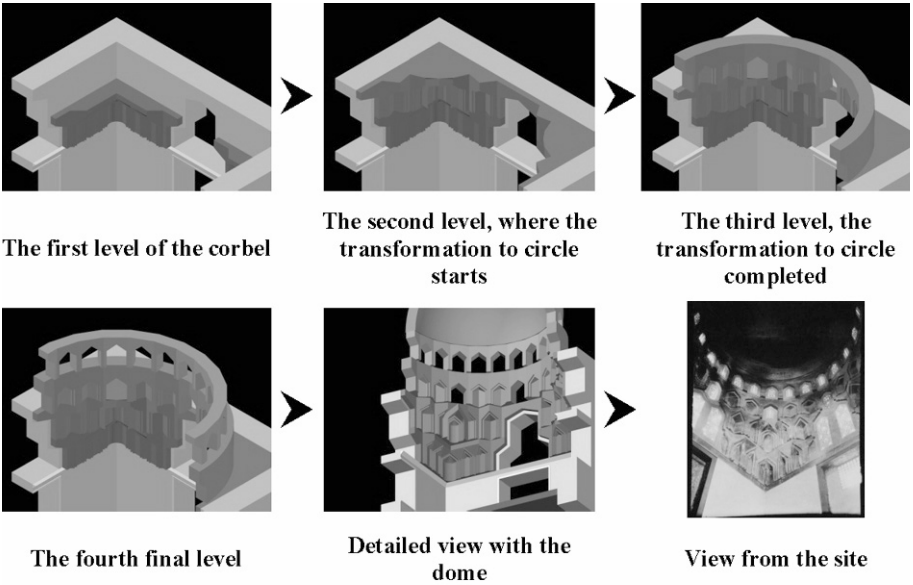


Fig. 9. The stone corbel (stalactite vault) squinch used in Sultan Baybars Al-Jashankir Masjid (Bahari Mamluks period)

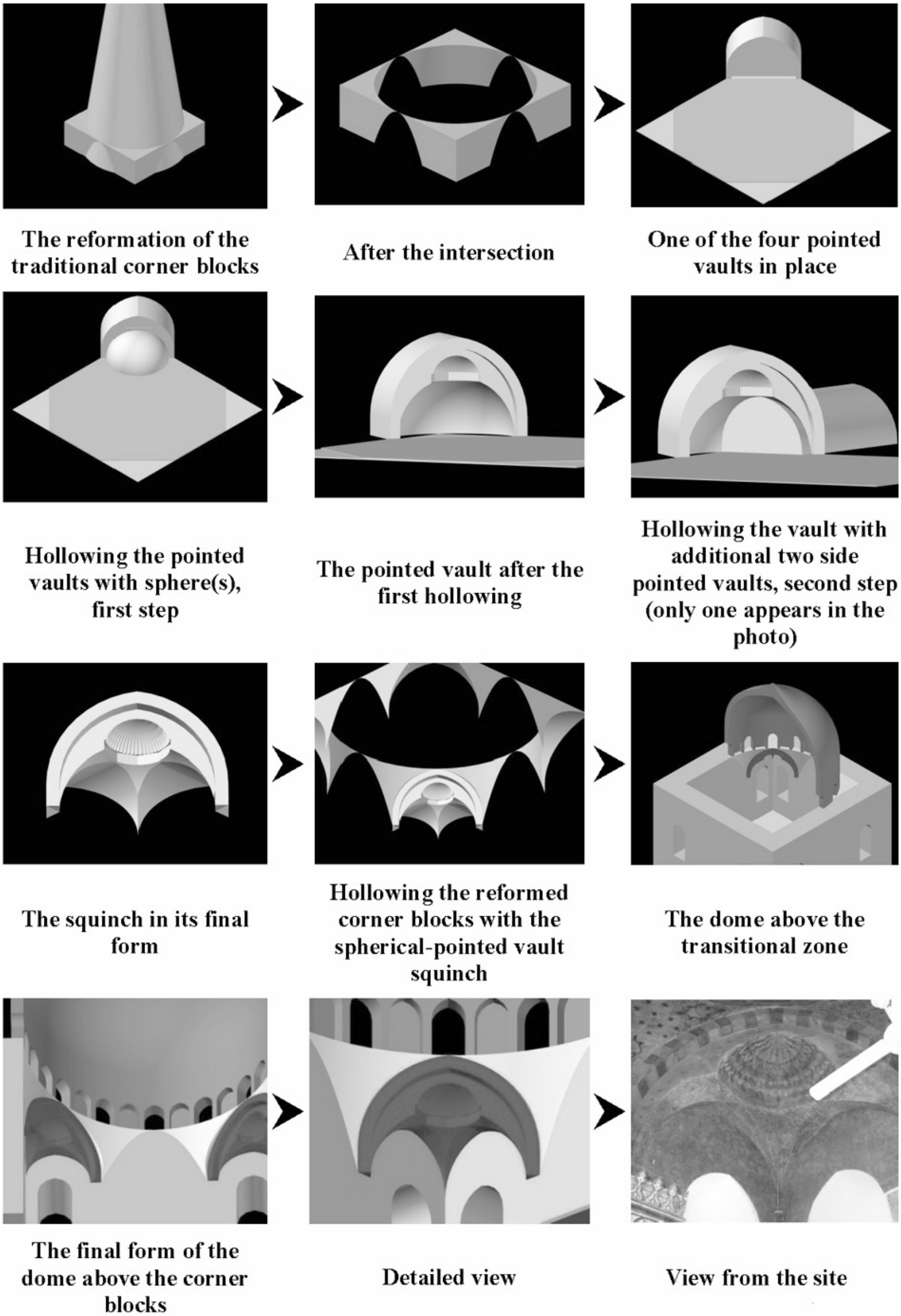


Fig. 10. The spherical-pointed vault squinch used in Al-Fadawiyya Dome Masjid (Burgi Mamluks period)

On the other hand, the most complicated type of squinch is the spherical-pointed vault. This type of squinch has been used near the end of the second Mamluks era (Burji Mamluks, 1382-1517) in Al-Fadawiyya Dome Masjid in Cairo. Later it was utilized typically in many examples up to the end of the Ottoman period in Egypt (see, for example, Masjid Sinan Pasha, 1571 A.D., and Masjid Muhammad Bey Abu al-Dhahab, 1774 A.D.). In such special type squinch, the corner blocks are firstly reformed by intersecting a frustum cone (its height is given by Eq. 6) with the traditional corner blocks. The upper and lower radii of the frustum cone (r_u, r_l respectively) can be calculated from:

$$r_l = r_u + 0.04119l \tag{9}$$

$$r_u = l/2 \tag{10}$$

$$r_l = 0.54119l \tag{11}$$

The resultant corner blocks are then hollowed in two steps:

1. The first, by using spherical forms (the maximum radius of this sphere is $0.2071l$).
2. The second, by intersecting the resultant hollow blocks with eight pointed perpendicular vaults, two per each block.

Fig. 10 represents in an analytical series the method for generating the kind of spherical-pointed squinch used in Al-Fadawiyya Dome Masjid.

3.2 The mathematical analysis of pendentives

Byzantine architects invented a new solution – one is considered again a milestone in the long history of domes – by constructing domes on piers instead of the massive continuous cylindrical walls. The transition from the square top of the cube to the circle was achieved by four inverted spherical triangles called pendentives, which are masses of masonry curved both horizontally and vertically. Their apexes rested on the four piers, to which they conducted the forces of the dome; their sides joined to form arches over openings in four faces of the cube; their bases met in a complete circle to form the dome foundation. The pendentive dome could either rest directly on this foundation, or upon a cylindrical wall, called a drum, inserted between the two to increase height.

Two main features distinguish pendentives. The first is that their lower parts are points (not lines as in the case of the squinches). The second is that they support the dome directly on the top of the cube without using the octagon as a transitional phase between the square and the circle (as in the case of the squinches). Instead, the volume of the transitional zones here is filled with the pendentives.

Pendentives originated from the intersection of a sphere (or pendentives sphere, as it will be called later) and a virtual cube (of side length l), where the center of the sphere is located on the axis of the virtual cube. The resulting pendentives depend on the type of the sphere (i.e., sphere or spheroid), its diameter D and the relation between its center and the center of the virtual cube. In case of the perfect sphere, there will be four possibilities to form the pendentives:

1. The two centers (of the sphere and the cube) are identical (located in the same point), the diameter of the sphere D and the diagonal of the faces of the cube $l\sqrt{2}$ are equal. This means that:

$$D = l\sqrt{2} \tag{12}$$

This case results in four pendentives intersecting in four points directly under the dome (see fig. 11) to form the apexes of four arches on the four faces of the cube as previously mentioned. This is the case that has been used in Hagia Sophia for example (see fig. 17b). Also, the height of the pendentives h_p resulting from such intersection equals $l/2$ (see fig. 15, Eq. 16).

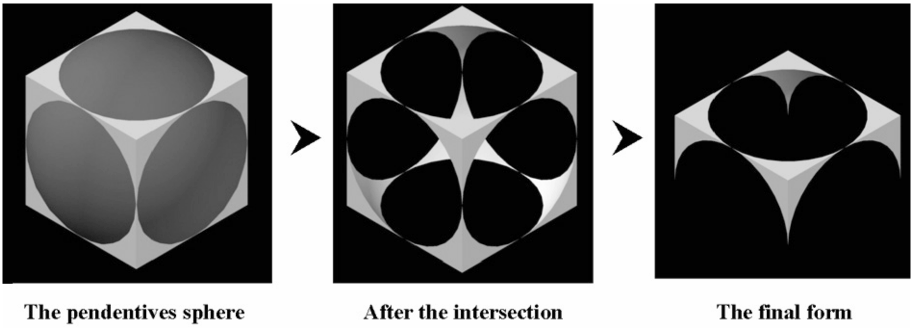


Fig. 11. Case One: the two centers are identical, and the diameter of the sphere and the diagonal of the faces of the cube are equal

It is important to mention here that in the simplest type of this form, the pendentives are part of the dome itself, and in this case the curvature of the dome R_c (see §2.1) must be equal to the vale $l\sqrt{2}$, but this is not a common case in Cairo's Islamic architecture due to the limited height of the dome.

2. The two centers are identical and the diameter of the sphere D is larger than the diagonal of the faces of the cube. Tthis means that:

$$D > l\sqrt{2} \tag{13}$$

This case gives the common form of the pendentives in Cairo, where the pendentives bases are separated (see fig. 12) and the height of the pendentives h_p is less than $l/2$.

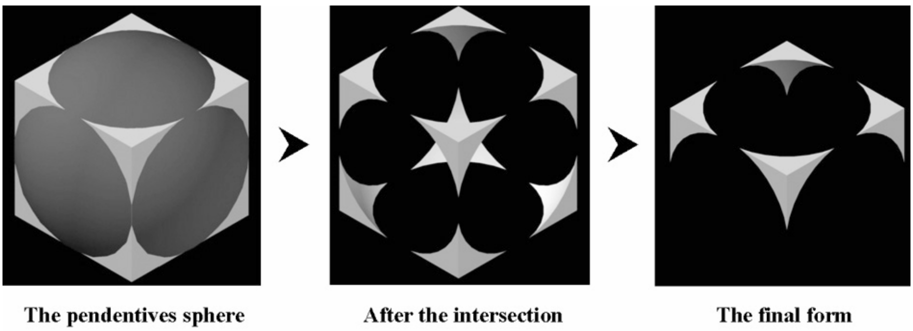


Fig. 12. Case Two: the two centers are identical and the diameter of the sphere D is larger than the diagonal of the faces of the cube

3. The center of the sphere is shifted from the center of the cube with a distance S (see fig. 15a), where S fulfills the condition:

$$l/2 + S > l/2 \tag{14}$$

In this case,

$$D = 2\sqrt{\frac{l^2}{2} + S^2 + lS} \tag{15}$$

This case results in pendentives similar to the first case where the pendentives bases all intersect in four points directly under the dome (see fig. 13). But the height of the pendentives h_p – calculated according to Eq. 16 – will be less than $l/2$.

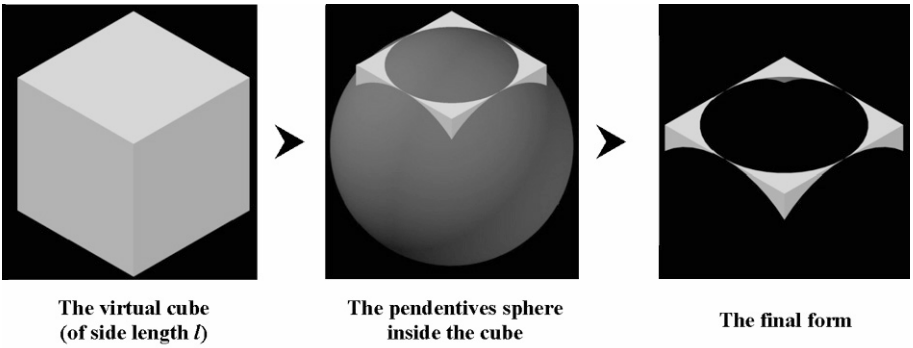


Fig. 13. Case Three: the center of the sphere is shifted from the center of the cube by a distance S, the diameter of the sphere is calculated according to eq. 14

4. The two centers are shifted and the diameter D is larger than the value resulting from Eq. 15. This case gives pendentives similar to the second case (see fig. 14) where the pendentives bases are separated and the height h_p is also less than $l/2$.

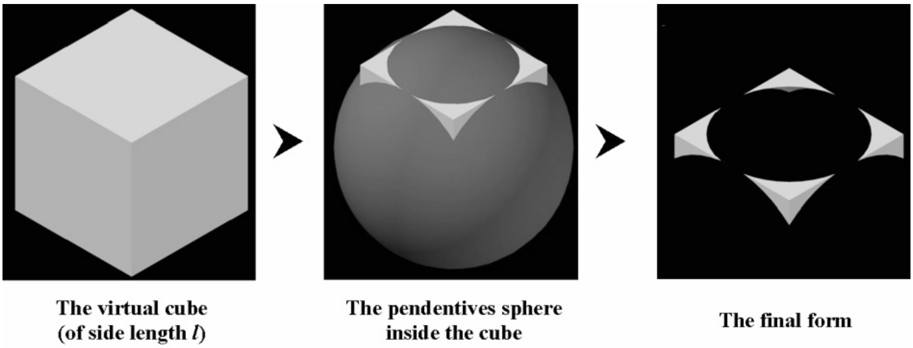


Fig. 14. Case Four: the two centers are shifted and the diameter D is larger than the value calculated according to eq. 14

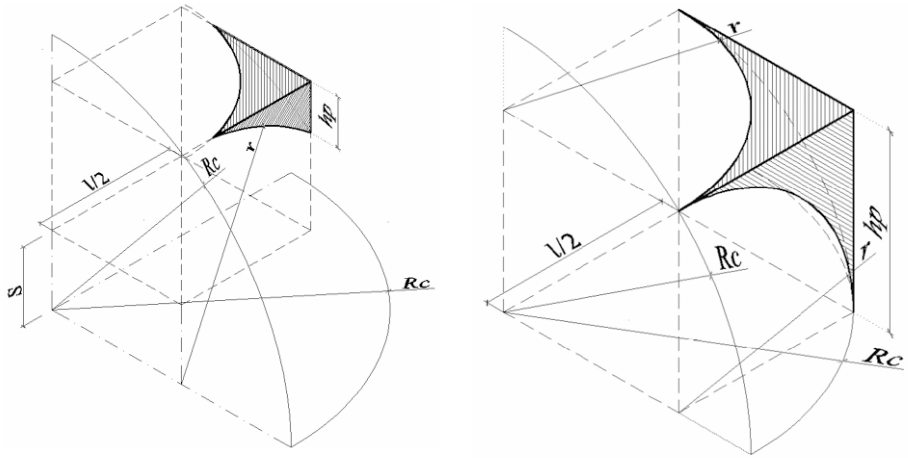


Fig. 15. The relation between pendentives height h_p , radius of curvature r_c , the virtual cube side length l and the radius of the intersected circle with the side faces of the cube r (the common case). (r of the side faces circles is equal to the top circle in the special case). a, left) the height h_p (the common case); the two centers are shifted with distance S; b, right) the height h_p (the special case); the two centers are identical and S=0

The height of the pendentives h_p (which must fulfill also Eq. 6) can be calculated by knowing the radius of curvature R_c (see Eq. 1), the virtual cube side length l, the radius of the intersected circle with the side faces of the cube r and the shift S (see fig. 15) according to the equation:

$$h_p = (l/2 + S) - \sqrt{r^2 - \frac{l^2}{4}} \qquad (16)$$

where r can be calculated by knowing R_c and l according to:

$$r = \sqrt{R_c{}^2 - \frac{l^2}{4}} \qquad (17)$$

The case of the identical centers (of sphere and cube) can be considered a special case in Eq. 16, where the shift S equals 0 and the radii of the intersected circles with the virtual cube faces are all equal.

It is also possible to generate the pendentives using the spheroid instead of the perfect sphere. This case generates the separated pendentives (see fig. 16) as previously mentioned in the second and fourth cases, but the intersection of the spheroid with the top face of the cube (where the dome rests) will remain a circle. The height of the pendentives h_p in this case will be also less than $l/2$. Although this is a possible method, it is beyond the scope of this present research.

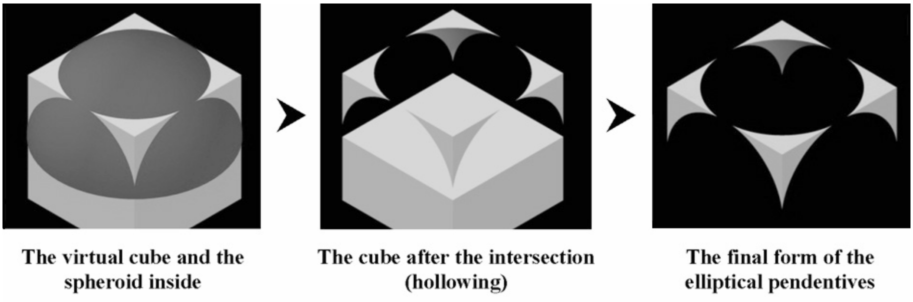


Fig. 16. Generating the pendentives using a spheroid instead of a perfect sphere

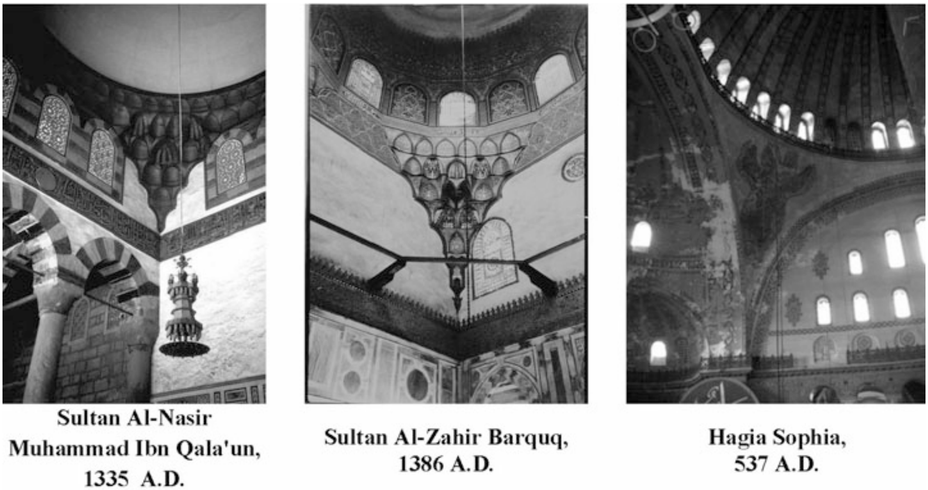


Fig. 17. Pendentives from Byzantine architecture to Islamic architecture. a, left and centre) Islamic architecture, muqarnas pendentives; b, right) Byzantine architecture, abstracted pendentives

What we see in Cairo's Islamic architecture is not the common type of pendentives (see figs. 11-14 and 16). Instead, it is a unique modified version in Islamic architecture. The architects of that era developed their own style of pendentives. They did not stop at the abstracted form of pendentives as originally invented, but they merged the concept of *muqarnas* (the Arabic word for stalactites vaults) with pendentives (see fig. 17a). A muqarnas is a special three–dimensional ornamentation consisting of tiers (layers), which themselves consist of niche-like elements. Muqarnas can be used to make a smooth transition from a rectangular basis to a vaulted ceiling. There is a great variety of these elements, yet most of them can be more or less deduced from a small set of basic elements [Dold-Samplonius and Harmsen 2004; Harmsen 2006; Hoeven and Veen 2010].

Almost all the pendentives utilized in Islamic Cairo consist of rows of muqarnas. In order to visualize how far this corbel may reach, it is sufficient to consider the dome (made of wooden beams) of the Mausoleum of Sultan Hassan (1356-1362 A.D.), which has a radius of more than 20 m [OICC 1992]. This radius results a corbel length (calculated according to Eq. 6) of more than 4 m, a long corbel that seems critical even today, with all the modern tools available.

Muqarnas pendentives – this terminology may be more expressive than the abstracted term "pendentives" – in Islamic architecture can be generated through the intersection between a *group of circles* located on the surface of the pendentives' sphere and the corner blocks as previously mentioned (see fig. 4), where the face of this block will be divided into a serious of sequential rows each of which represents one row of the muqarnas. To do that, it is important first to define the radii of the circles group. These circles must 1) be located on the surface of the virtual sphere that composes the pendentives; 2) have a radius equals to the radius of the corresponding muqarnas row.

From the mathematical point of view, all the circles that are tangential to the surface of the pendentives' sphere must be located on a diagonal circle in the sphere; its radius r_d is equal to the radius of that sphere R_c. Given the value of r_d and the height of each muqarnas row measured from the center of the sphere h_m (see fig. 18), the chords l_c can be calculated according to the equation:

$$l_c = 2\sqrt{{r_d}^2 - h_m^2} \tag{18}$$

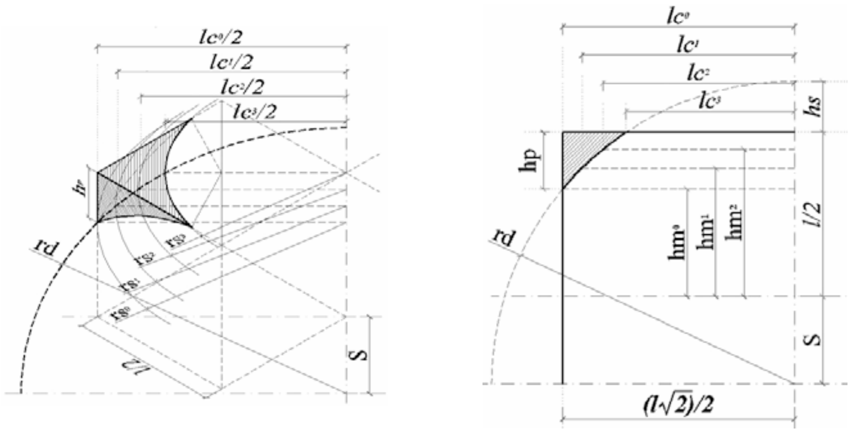


Fig. 18. The relation between the pendentives, the diagonal circle and muqarnas rows' circles.
a, left) Perspective represents the pendentives and its relation with the diagonal circle;
b, right) Diagonal section in the virtual cube

These chords represent in the third dimension the radii of the circles required to divide the pendentives into a sequential rows of muqarnas. In the special case where ($D = l\sqrt{2}$) the last formula can be rewritten to be:

$$l_c = 2\sqrt{\frac{l}{2}^2 - h_m^2} \tag{19}$$

Given the arc height h_s, the lengths l_c can be calculated according to:

$$l_c = 2\sqrt{2r_d h_s - {h_s}^2} \tag{20}$$

where h_m must fulfill the equation:

$$h_m \le l/2 \tag{21}$$

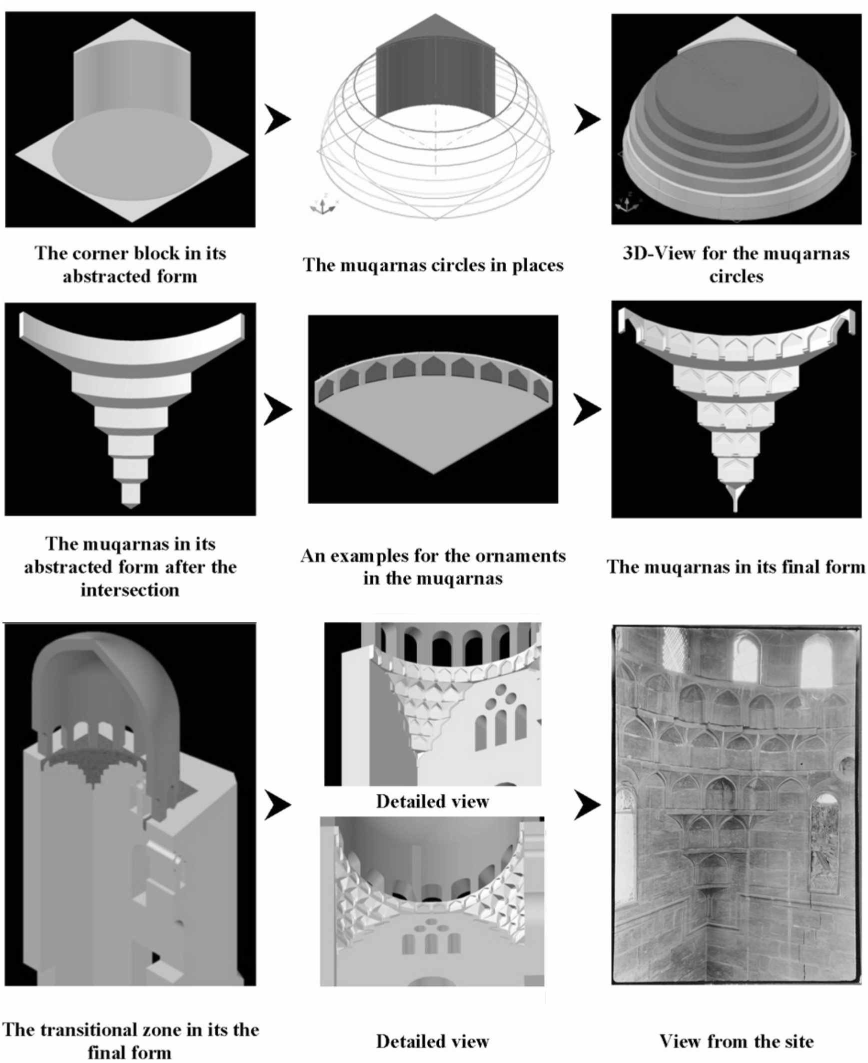


Fig. 19. Generating the muqarnas pendentives used in the dome of the Mausoleum of Sultan Al-Ashraf Barsbay (Burgi Mamluks period)

l	S	h_s	$r_d=D/2$	r	h_p	n*	R_h**
6.00	0.00	1.24	4.24	3.00	3.00	6	0.50
h_m							
0.00	0.50	1.00	1.50	2.00	2.50	3.00	—
$l_c=r_s$							
4.24	4.21	4.12	3.97	3.74	3.43	3.00	—

* Number of Rows

** Height of each row

Table 2. The numerical values (calculated from eqs. 12 to 21) used to generate the muqarnas pendentives in the Mausoleum of Masjid Sultan Al-Ashraf Barsbay (values in m)

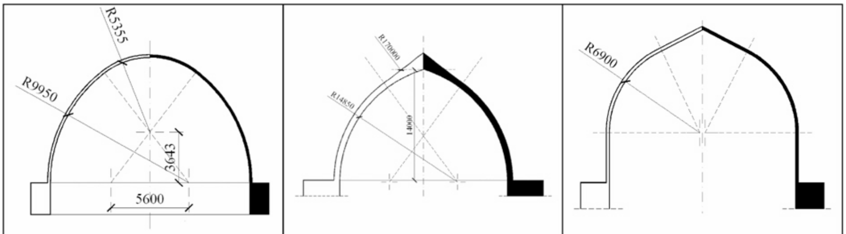
By calculating the radii of the muqarnas circles (applying the Eqs. 12 to 21), the rows of muqarnas can be generated. It is clear that the values of both h_m and l_c are determined according to the number of the muqarnas rows. Again, the design, form and arrangement of the muqarnas will depend on the designer himself, thus it may be difficult to count for all of the available forms. Fig. 19 represents in an analytical series an example for a direct application of the previous equations (values are given in Table 2) to generate the muqarnas pendentives as used in the Mausoleum of Masjid Sultan Al-Ashraf Barsbay (1425 A.D.) which, according to the historical references [OICC 1992] and the available photos for this dome, is a square room of dimensions 6 x 6 m.

4 Some common forms of the domes used in Cairo's Islamic architecture

In order to know the most common types of domes in Cairo's Islamic architecture, a sample of thrity Masjids built between 868 A.D. (the Tuluned Period) and 1798 A.D. (the end of Ottoman period in Egypt) [Elkhateeb and Soliman 2009] has been examined, of which ten were chosen. The chosen Masjids are those whose domes are the most distinguished architecturally and the most dominant visually. As a result, the following analysis does not include, for example, the domes used to cover the arcades of the Masjids.

Among the various types of domes, elliptical domes with their different forms are the most applicable in Islamic Cairo; all of the chosen samples are elliptical. Of the ten chosen, five are pointed perfect, three are flat perfect (looking from inside), one is pointed onion, and one is flat onion.

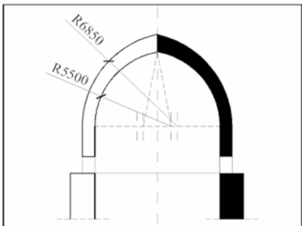
Not one of the examined samples has a spherical dome. The utilization of the squinches is also the most common; it has been used in seven of the chosen samples. The spans vary widely as well, starting from a minimum of 6 m in El-Hakim Masjid (Fatimid era) and increasing considerably, exceeded 20 m in the Masjid and school of Sultan Hassan (Bahari Mamluks era). Four of the domes chosen were used to cover a mausoleum in the Masjid, four were used to cover the space of the Masjid either partially or completely, one was placed above the niche (Mihrab) at the end of the crossed aisle and one covers the ablution fountain. Figs. 20 and 21 represent the domes that were studied.



	Sultan Al-Naser Muhammad Ibn Qalaun (Bahari Mamluks) 1335 AD	Sultan Al-Nasir Hassan (Bahari Mamluks) 1362 AD	Sultan Farag Ibn Barquq (Burgi Mamluks) 1411 AD
Dome Location	The mausoleum	The mausoleum	The mausoleum
Dome Type	Flat-Perfect-Elliptical	Pointed-Perfect - Elliptical	Pointed- Perfect-Elliptical
Span (m)	14.30	21.10	14.20
C_l (m)§	2.96	4.37	2.94
C_h (m)¤	5.92	8.74	5.88
Transitional Zone	Stalactite Pendentives	Stalactite Pendentives	Stalactite Pendentives

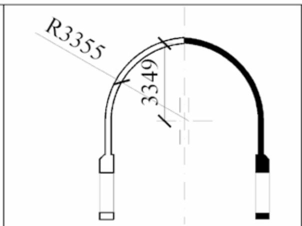
§,¤ Calculated according to Eqs. 5 and 6

Fig. 20. Domes supported on muqarnas (stalactite) pendentives in a chronological order



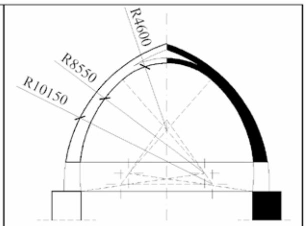
Ahmed Ibn Tulun (Tulunid) 879 A.D.

Dome Location	Ablution Fountain
Dome Type	Pointed-Perfect-Elliptical
Span (m)	10.80
C_l (m)§	2.235
C_h (m)¤	4.47
Transitional Zone	Squinches, Corbel



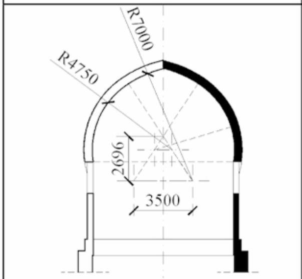
El-Hakim (Fatimid) 1013 A.D.

Dome Location	Above the niche (Mihrab)
Dome Type	Pointed-Perfect - Elliptical
Span (m)	6.00
C_l (m)§	1.24
C_h (m)¤	2.485
Transitional Zone	Squinches, niches with half dome above



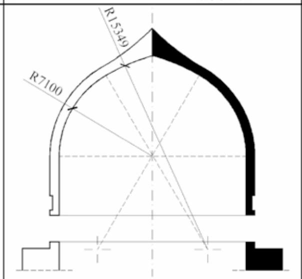
Sutan Qalaun (Bahari Mamluks) 1285 A.D.

Dome Location	The mausoleum
Dome Type	Flat-Onion-Elliptical (from inside)
Span (m)	11.72
C_l (m)§	2.425
C_h (m)¤	4.85
Transitional Zone	Squinches, niches with half dome above



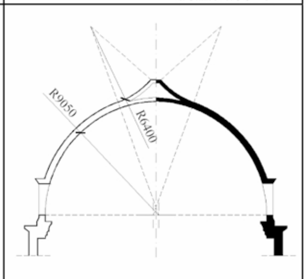
Amir Sorgotmosh (Bahari Mamluks) 1362 A.D.

Dome Location	The Qibla Iwan
Dome Type	Pointed-Onion-Elliptical
Span (m)	8.55
C_l (m)§	1.77
C_h (m)¤	3.54
Transitional Zone	Squinches, Corbel



Al-Fadawiyya Dome (Burgi Mamluks) 1479 A.D.

Dome Location	The Masjid
Dome Type	Pointed-Perfect - Elliptical
Span (m)	14.20
C_l (m)§	2.94
C_h (m)¤	5.88
Transitional Zone	Squinches, Spherical-Pointed Vaults



Sinan Pasha (Ottoman) 1571 A.D. Identical in Muhammad Bey Abu al-Dahab (Ottoman, 1774), span 15.50m

Dome Location	The Masjid
Dome Type	Flat-Perfect-Elliptical (from inside)
Span (m)	17.65
C_l (m)§	3.655
C_h (m)¤	7.31
Transitional Zone	Squinches, Spherical-Pointed Vaults

Fig. 21. Domes supported on squinches in a chronological order

5 Validation

Based on the derived mathematical expressions stated previously, AutoCAD 3D drawings were generated. From among the drawings generated, two drawings (one for squinches and the other for the pendentives) were chosen for building physical models using a 3D printer. Both domes belong to the Burgi Mamluks period. They are:

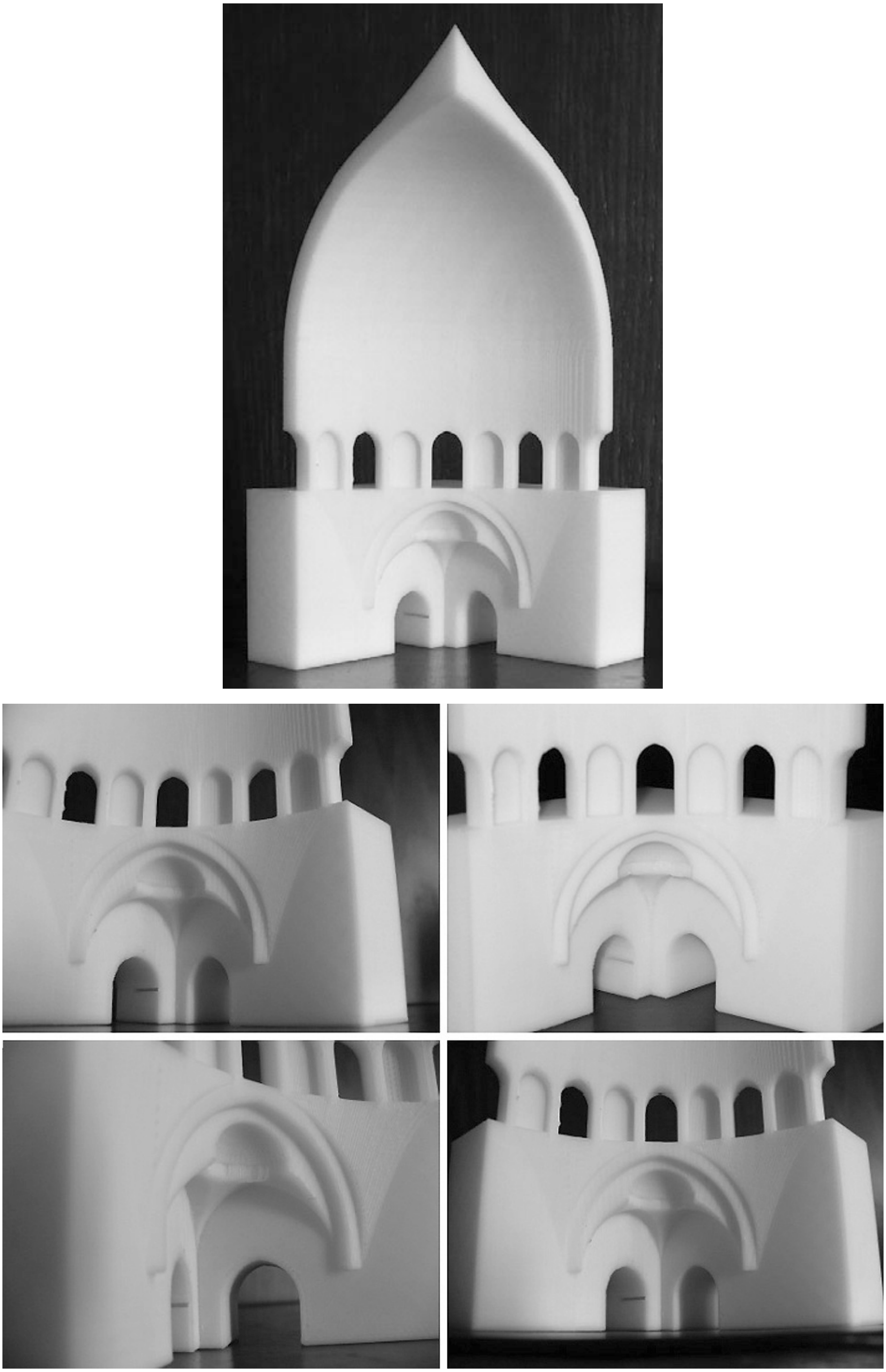


Fig. 22. The spherical pointed vault squinches in Al-Fadawiyya Dome Masjid, photos from the model. a) The dome above the squinch; b) detailed views

– Squinches model (see fig. 22): Al-Fadawiyya Dome Masjid (1479 A.D.), a square room of dimensions (14.20 x 14.20 m). The dome is used to cover the Masjid. The total room height under dome is 24.56 m [OICC 1992].
– Pendentives model (see fig. 23): the Mausoleum of Sultan Al-Ashraf Barsbay Masjid (1425 A.D.), a square room of dimensions (6.00x6.00m). The total room height under dome is about 28 m [OICC 1992].

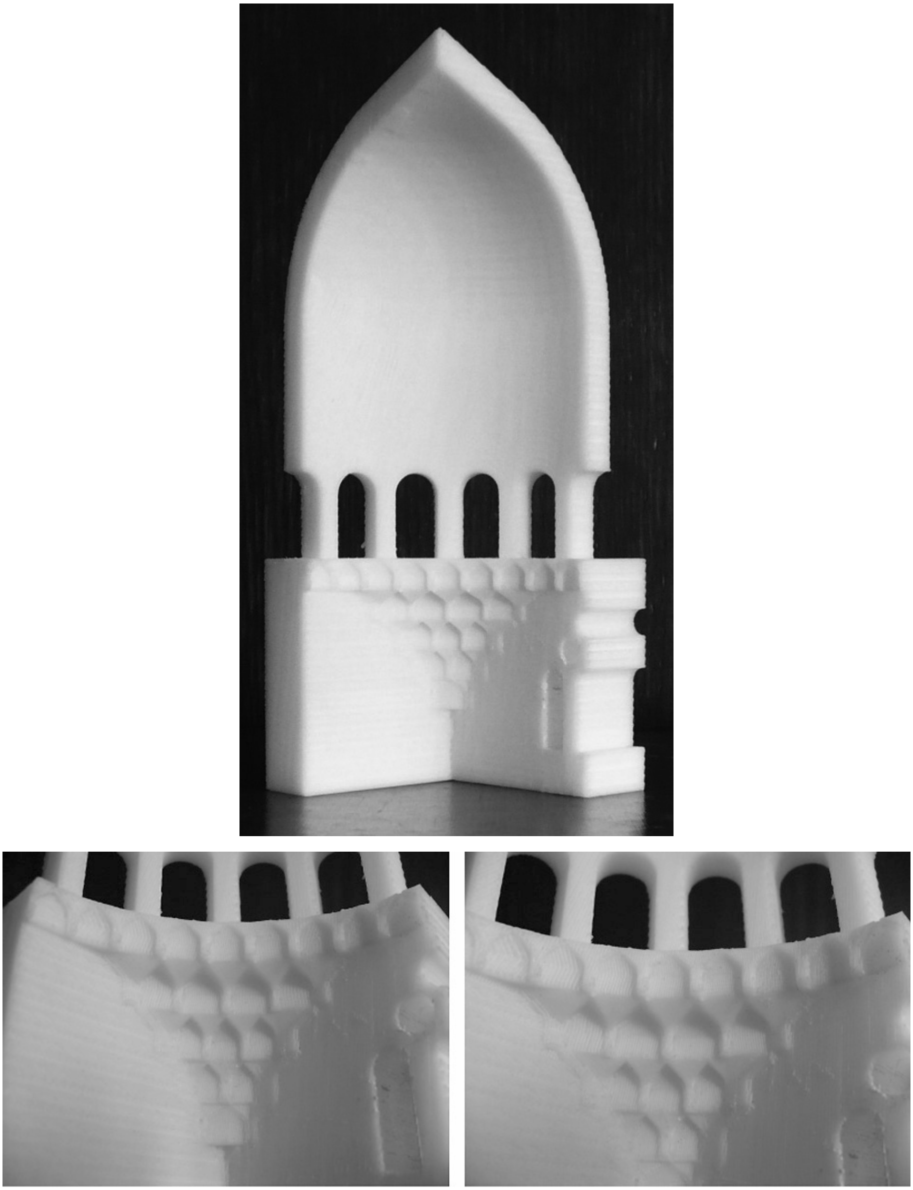


Fig. 23. The muqarnas pendentives in the Mausoleum of Sultan Al-Ashraf Barsbay, photos from the model. a) The dome above the pendentive; b) detailed views

6 Summary

Among the different methods to cover a space, the dome is the oldest and most distinguished. Although its development began some sixty centuries ago, it continues to develop more and more. This development enables it to cover a diversity of spaces and spans from few to hundreds of meters. One of the most important problems that faced the architects of the old days was how to transform the square top of the cube to a circle where the dome rested, which is known as the transitional zone. For structural and aesthetical reasons, the architects hollowed the transitional zone, applying one of two techniques: squinches and pendentives. The first is a support carried across the corner of a room under a superimposed mass. In architecture, squinches can be one of any of several devices by which a square or polygonal room has its upper corners filled in to form a support for a dome. In Cairo's Islamic architecture, they first appeared as four diagonal niches (with a half dome at their heads), one in each corner of the room to transform its square plan to an octagonal one where the dome was supported. The pendentive is originally one of the concave triangular members that support a dome over a square space. In architecture, the pendentives are triangular segments of a sphere, taper to points at the bottom and spread at the top to establish the continuous circular or elliptical base needed to support a dome.

Through the previous work a set of mathematical formulae has been derived to express the most famous forms for both squinches and pendentives. These formulae relate the different parts of the squinch or pendentive to the cube side length *l* where the dome will ultimately rest. Figs. 24 and 25 are graphical representations of the formulae stated previously.

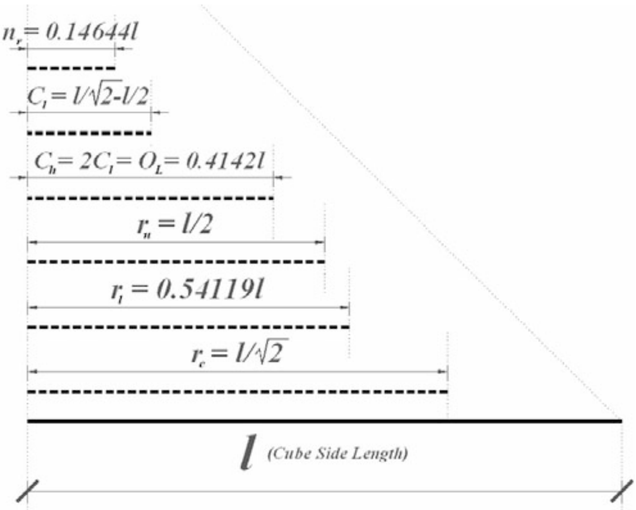


Fig. 24. The relation between the cube side length *l* and the different parts of the squinch

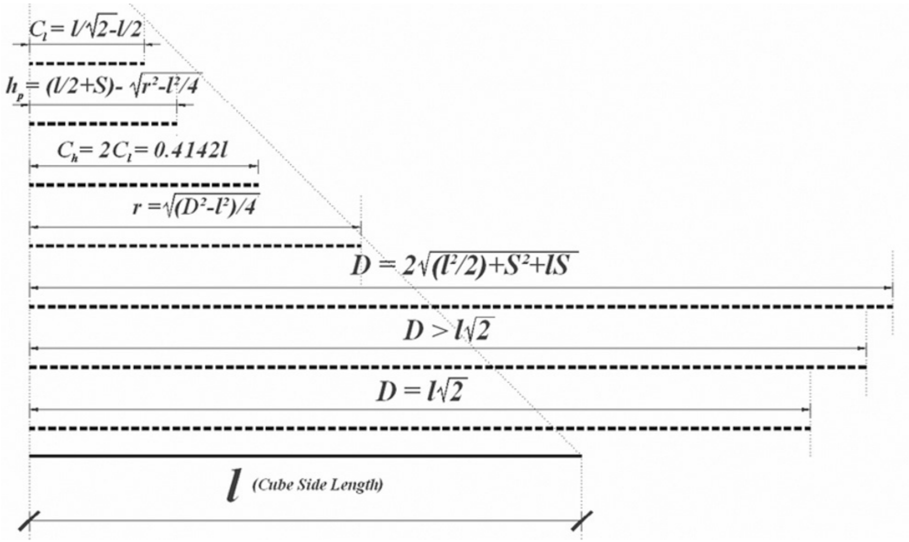


Fig. 25. The relation between the cube side length l and the different parts of the pendentive

Although modern materials and construction systems can accommodate various methods for supporting a dome over a square room, many architects in the Islamic world still prefer and appreciate the traditional methods (squinches and pendentives) in the transitional zone even though it may be constructed with new materials (i.e., reinforced concrete). Fig. 26 shows two examples that utilize squinches and pendentives in contemporary Masjids.

Fig. 26. Although the various methods available now to support a dome, the squinches and pendentives are still favorable for the architects of the Islamic world. a, left) Spherical-pointed vault squinches in Qeba'a Masjid, Almadena Almonawara, Saudi Arabia; b, right) pendentives in Elhosaree Masjid, 6th of October City, Egypt

Acknowledgments

The author would like to express his deep thanks and sincere appreciation to the colleagues who help in doing this work: Arch. Zinub M. Najeeb, Dr. Sabah S. Soliman, Dr. Khalid Hilal, Eng. M. Zahran, Dr. Tamer Elnady and his the team in LAB. Finally, thanks also go to Mrs. Elkhateeb for her effort in editing and revising the text.

References

ABOUSEIF-BEHRENS, D. 1996 *Islamic Architecture in Cairo-an Introduction.* Cairo: The American University in Cairo Press.

AMERICAN CONCRETE INSTITUTE. 2004. *Building code requirements for structural concrete (ACI 318-02) and commentary (ACI 318R-02) reported by ACI Committee.* Farmington Hill.

CIPRIANI, B. 2005. *Development of Construction techniques in the Mamluks Domes of Cairo.* Master's thesis, Architecture Studies, MIT.

DOLD-SAMPLONIUS, Y. and S. Harmsen. 2004. Muqarnas: Construction and Reconstruction. Pp. 69-78 in *Nexus V: Architecture and Mathematics*, Kim Williams and Francisco J. Delgado Cepeda, eds. Fucecchio (Florence): Kim Williams Books.

ELKHATEEB, A. and S. SOLIMAN. 2009. The Numerical Description of The Historical Masjids In Cairo. *Ain Shams Journal of Architectural Engineering (ASJAE)* 2 (Nov. 2009): 65-88.

GIECK, K and R. GIECK. 2006. *Engineering Formulas.* 8th ed. McGraw Hill Professional.

HARMSEN, S. 2006. Algorithmic Computer Reconstructions of Stalactite Vaults – Muqarnas – in Islamic Architecture. Ph.D. Thesis, University of Heidelberg. http://archiv.ub.uniheidelberg.de/volltextserver/volltexte/2007/7047/pdf/diss.pdf.

HARMSEN, S., JUNGBLUT, D. and KRÖMKER, S. 2007. Seljuk Muqarnas along the Silk Road. *Portal* (September 2007): 1-11. Interdisziplinäres Zentrum für Wissenschaftliches Rechnen (IWR), University of Heidelberg.

HOEVEN, S., and VEEN, M. 2010. Muqarnas: Mathematics in Islamic Arts. Seminar Mathematics in Islamic Arts, Utrecht University, Faculty of Science, Department of Mathematics. http://www.wiskuu.nl/muqarnas/Muqarnas_english8.pdf. Last accessed 27 Oct. 2011.

KRÖMKER, S. n.d. *Muqarnas Visualization in the Numerical Geometry Group.* http://www.iwr.uni-heidelberg.de/groups/ngg/Muqarnas/. Last accessed 27 Oct. 2011.

OICC. 1992. *Principles of Architectural Design and Urban Planning During Different Islamic Eras: Analytical Study for Cairo*. Jeddah: Organization of Islamic Capitals and Cites (OICC).

TAKAHASHI, S. n.d. Muqarnas: A Three-Dimensional Decoration of Islam Architecture. http://www.tamabi.ac.jp/idd/shiro/muqarnas/. Last accessed 27 Oct. 2011.

About the author

Ahmed Ali Elkhateeb graduated in 1990 from the Department of Architecture, Faculty of Engineering of Ain Shams University (Cairo, Egypt). He worked in the same department until summer 2011. He completed his M.Sc. and Ph.D. in the field of architectural acoustics, which is his primary area of interest, in addition to mathematics and its relation with architecture. As of fall 2011, he is a Professor (A) of architecture and building science in the Department of Architecture in Faculty of Environmental Design of King Abdul Aziz University, Jeddah, Kingdom of Saudi Arabia.

Tomás Gil-López

Madrid School of Architecture.
Madrid Polytechnic University
Madrid, Spain
tomas.gill@upm.es

Keywords: Chapel of the Presentation, Burgos Cathedral, octagonal Gothic vault, geometric analysis, Cordovan proportion, photogrammetry

Research

The Vault of the Chapel of the Presentation in Burgos Cathedral: "Divine Canon? No, Cordovan Proportion"

Abstract. The "Cordovan proportion" was discovered by chance in 1973, during an attempt to demonstrate the validity of the golden ratio in buildings in the city of Cordova. Far from being a local exception, later research have shown its sphere of application to be universal. However, few authors have researched the influence of the Cordovan triangle on the layout of architectural elements, and particularly on octagonal Gothic vaults. The present paper presents an analysis of the Cordovan proportion in the curved ribbing of Gothic vaults. Specifically, it consists of a geometric study of the octagonal vault of the Chapel of the Presentation in Burgos Cathedral. The results show that when the side of the octagon is taken as a modulus, the Cordovan triangles drawn by the layout of the vault form a geometric succession whose ratio is the Cordovan number.

Introduction

The Chapel of the Presentation in Burgos Cathedral

Burgos Cathedral in Spain, which was declared a World Heritage Building by UNESCO [1984], has four octagonal Gothic vaults: the Cimborio, the Constable Chapel, the Chapel of the Presentation and the old Chapter House. The best known vault is the one covering the Constable Chapel. However, the most complex vault regarding the layout of the ribs is the Chapel of the Presentation, due to the incorporation of curved ribbing.

The vault of the Chapel of the Presentation (fig. 1) was the work of the master mason Juan de Matienzo, between 1519 and 1522 [Gomez 1998], inspired by the work of Simón de Colonia and especially by the Constable Chapel. The vault in question comprises Gothic ribbing of a regular octagonal plant. The central severies are openwork, possibly influenced by Almoravid models, such as the Mosque of Tremecen in Algeria.

The stereotomy of the Gothic vault always has its origin in the horizontal projection [Palacios 2009; Rabasa 2000: 39]. The plan becomes totally relevant in the design of a vault right from the very beginning. Therefore, a geometric analysis must focus on the horizontal projection of the vault.

The Cordovan proportion: Cordovan rectangle vs. Cordovan triangle

A reprint of the essay entitled "The Cordovan Proportion" written in 1973 by the architect Rafael de la Hoz Arderius, has recently been announced, under the title of: "Divine Canon? No, Cordovan Proportion".[1] In the work, he demonstrated how, by chance, when attempting to demonstrate the validity of the golden ratio in buildings in the city of Cordova, a canon appeared, unknown until then, which was different from the harmonic rectangle. Suddenly, apparently anarchical distributions took on a logical composition and a pattern appeared which endowed the whole with order.

DOI 10.1007/s00004-011-0104-2; *published online* 13 January 2012

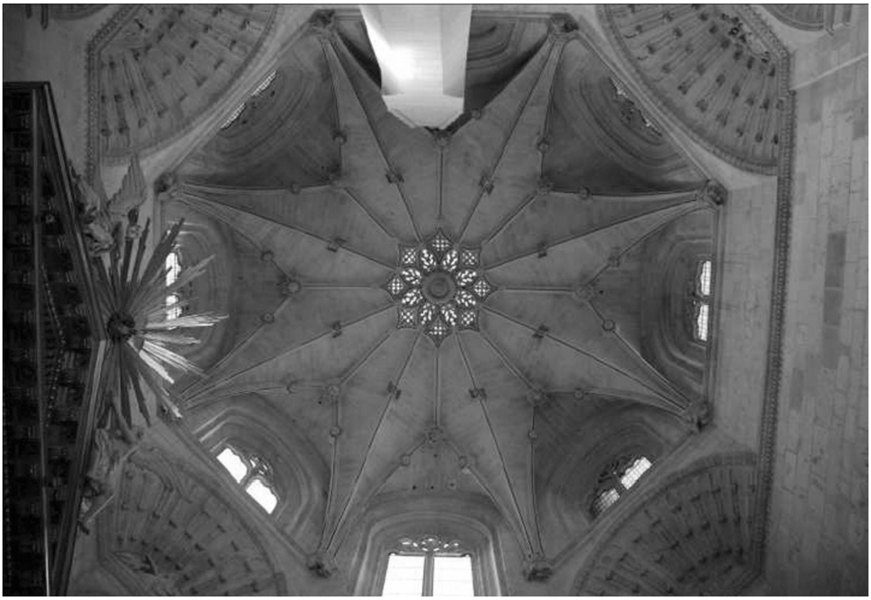


Fig. 1. Vault of the Chapel of the Presentation in Burgos Cathedral

Therefore, since the golden ratio is the ratio of the side to the radius of a regular decagon [Bonell 2000: 25; Huntley 1970: 25], the Cordovan ratio is defined as the ratio of the side to the radius of a regular octagon (fig. 2). Its value is

$$c = \frac{r}{l} = \frac{1}{\sqrt{2-\sqrt{2}}} = 1.30656296487\ .^{2}$$

A Cordovan rectangle is defined as that which maintains the Cordovan ratio between its longest and shortest side.

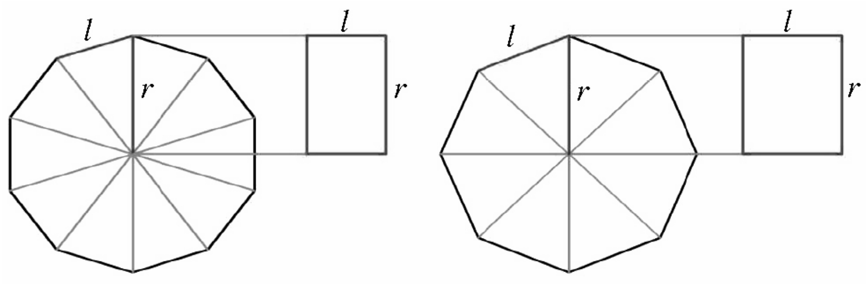


Fig. 2. left) Golden ratio, based on a decagon; right) Cordovan ratio, based on an octagon

It first it was thought to be a local exception, but various pieces of subsequent research have proven its scope of application to be wider. According to Hoz [2002], the Cordovan rectangle is not only to be found in the arcades of the Mosque of Cordoba but also in the following examples: Hispanic contexts, like the Aqueduct of Segovia and the Alcala Gate (Puerta de Alcalá) in Madrid; European contexts, like Agrippa's Pantheon in Rome, the Basilica of Maxentius, and in contexts even more distant from the point of discovery, like the Pyramids of Egypt, the Pyramid of the Moon in Teotihuacán or the Church of the Company of Jesus in Cordoba (Argentina).

Moreover, the side and radius of a regular octagon can form an isosceles triangle with a longer base *l* and sides *r*, called a Cordovan triangle [Redondo, et al. 2008a, 2008b] (fig. 3). The rectangle circumscribing two Cordovan triangles touching at the vertex is known as a silver rectangle[3] (fig. 4).

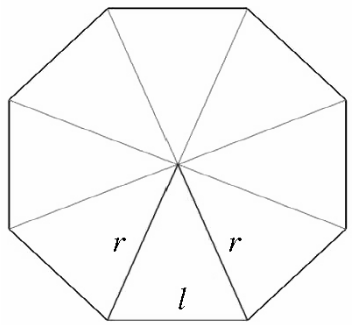


Fig. 3. Cordovan triangle

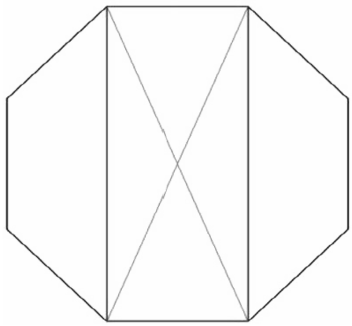


Fig. 4. Silver rectangle

Since architectural elements (mainly elevations) have been found that observe the ratio of the Cordovan rectangle, among which are those mentioned previously, there have been few authors who have researched the influence of the Cordovan triangle [Huylebrouck, et al. 2009], which, as will be seen in this research, is what defines the geometric layout of the studied vault.

Geometric Analysis

Data acquisition

Since we are dealing with a vault that is difficult to access, making direct measurements is highly complex. For this reason a photogrammetric method was used. Eight stations were chosen that were conveniently situated so as to avoid any areas of shadow in the vault, from which various shots were taken with a high resolution digital camera using a calibrated lens.

Using the *Photomodeler* computer program the same spatial points were referenced in different photographs.

They were then projected onto a horizontal plane in order to obtain the layout of the vault (fig. 5).

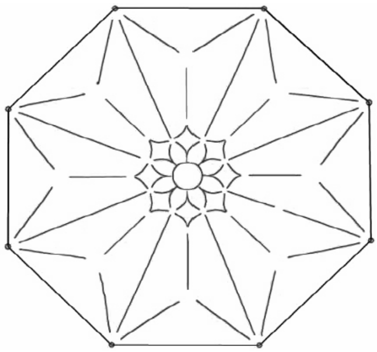


Fig. 5. Data obtained by the photogrammetric method

Layout of the vault

In an octagonal vault, the horizontal projection of the **polar keystone** usually coincides with the centre of the polygon. In this particular case, it is substituted by a circular layout rib to which the curved ribs are attached.

The **secondary keystones** are obtained by drawing the Cordovan triangles whose vertices are situated at the midpoint of the side of the octagon (fig. 6). These keystones are to be found at the midpoint of the base of the Cordovan triangle, which is denominated as T_1.

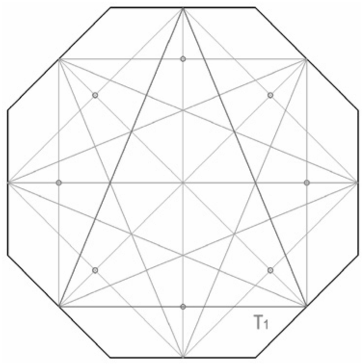


Fig. 6. Secondary keystones and Cordovan triangle T_1

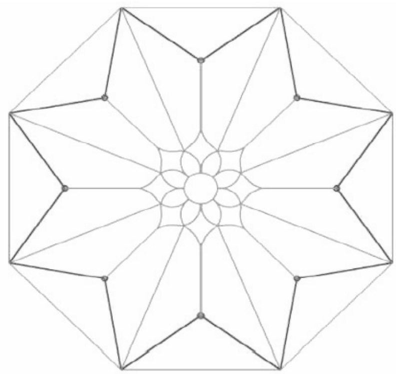


Fig. 7. Position of the secondary keystones and tierceron ribs of the vault

Having situated the secondary keystones, the tierceron ribs of the vault are now defined (fig. 7).

The position of the **tertiary keystones** will determine the layout of the diagonal and ridge ribs as well as the curved ones. There are sixteen keystones of this type positioned in two concentric circumferences. The eight keystones situated on the circumference of largest radius are called *outer tertiary keystones* (fig. 8), and it is to them that the ridge and curved ribs are attached; the remaining eight, situated on the circumference of smallest radius are called *inner tertiary keystones* (fig. 9), and to these are attached the diagonal and curved ribs.

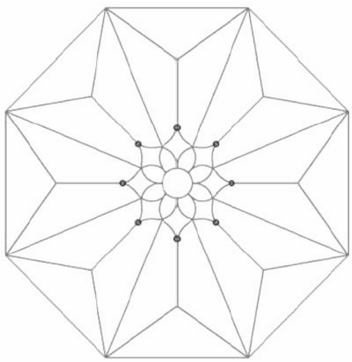


Fig. 8. Position of the outer tertiary keystones

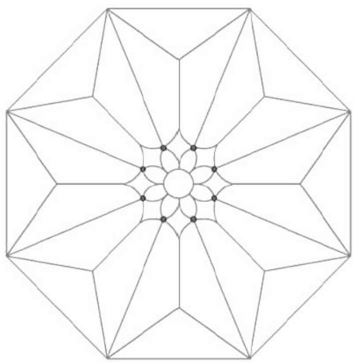


Fig. 9. Position of the inner tertiary keystones

The position of the *outer tertiary keystones* is determined by the construction shown in fig. 10. Their position coincides with the intersections of the longest sides of a Cordovan triangle, which is denominated as T_2, when rotated around the centre of the octagon.

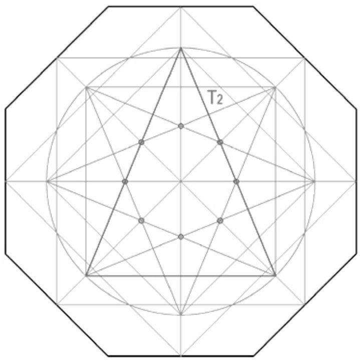


Fig. 10. Outer tertiary keystones and Cordovan triangle T_2

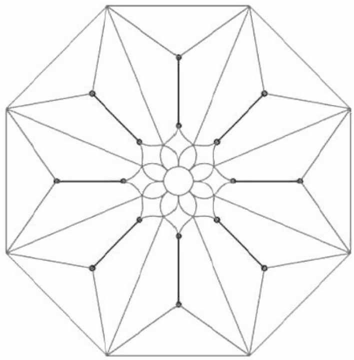


Fig. 11. Ridge ribs of the vault

Having defined the secondary keystones and the outer tertiary keystones, the ridge ribs of the vault are also determined (fig. 11).

The curved ribs are formed by arcs belonging to the two types of circumferences. Depending on the size of their radius they are called *greater circumferences* (fig. 12) and *lesser circumferences* (fig. 13). In order to define them, the centres and radii of each of the two types are determined.

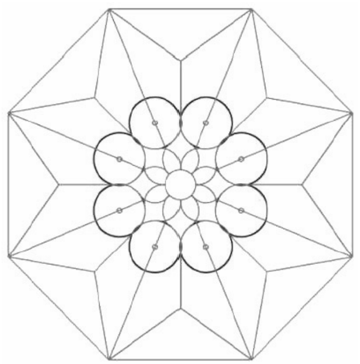


Fig. 12. Greater circumferences

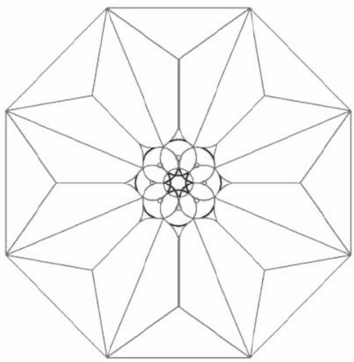


Fig. 13. Lesser circumferences

The centres of the greater circumferences are obtained using the construction performed in fig. 14. These coincide with the intersections of the longest sides of the Cordovan triangle T_1 and a Cordovan triangle, denominated as T_3, whose longest side is the radius of the octagon and whose shortest side is the side of the octagon.

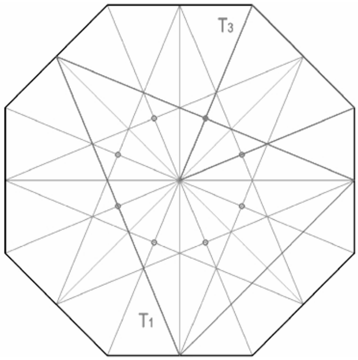


Fig. 14. Centres of the greater circumferences and Cordovan triangles T_1 and T_3

Another way of finding the centres is illustrated in fig. 15. The points looked for coincide with two of the vertices of a Cordovan triangle, denominated as T_4. The length of its sides is half the length of the sides of T_1 (fig. 16).

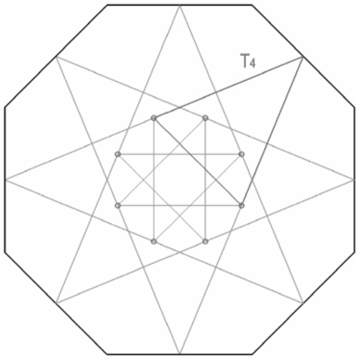


Fig. 15. Centres of the greater circumferences and Cordovan triangle T_4

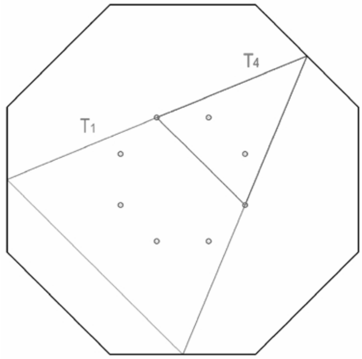


Fig. 16. Cordovan triangles T_1 and T_4

The radius of the circumference is determined by the distance between its centre and the closest outer tertiary keystones, found previously (fig. 17).

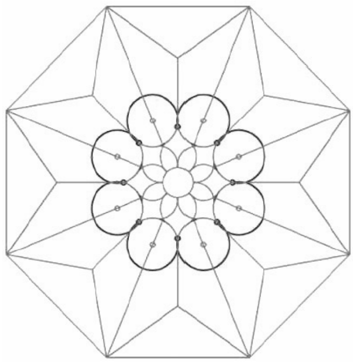


Fig. 17. Radius of the greater circumferences

The centre of the lesser circumferences is determined according to the construction set out in fig. 18. These centres coincide with the midpoint of the base of triangle T_4.

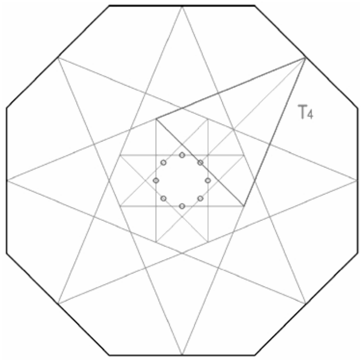


Fig. 18. Centres of the lesser circumferences and Cordovan triangle T_4

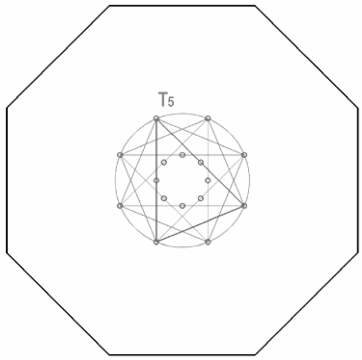


Fig. 19. Ratio of the greater to the lesser circumferences and Cordovan triangle T_5

The ratio of the greater to the lesser circumferences is shown in fig. 19. The vertices of a Cordovan triangle, denominated T_5, are the centres of the greater circumferences, and the midpoint of their longest sides is the centre of the circumferences of smaller radius.

The radius of the lesser circumferences is the distance between their centres and the closest apothem of the octagon (fig. 20). The position of the *inner tertiary keystones* is fixed by the intersection of the lesser circumferences.

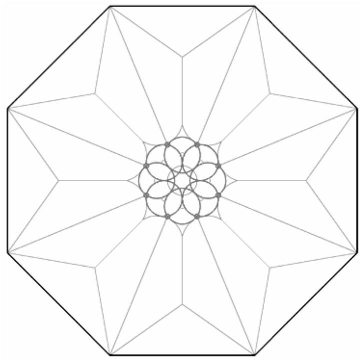


Fig. 20. Radius of the lesser circumferences and inner tertiary keystones

Once the inner tertiary keystones are known, the diagonal ribs (fig. 21) and the curved ribs (fig. 22) are also determined.

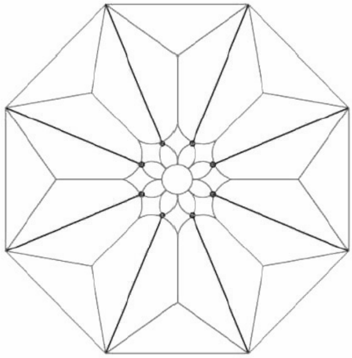


Fig. 21. Diagonal ribs of the vault

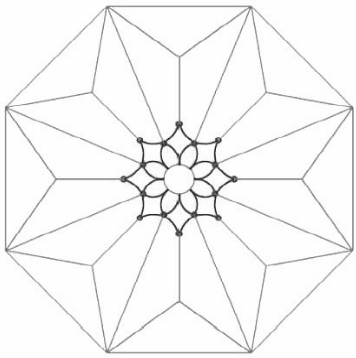


Fig. 22. Curved ribs of the vault

It is of particular interest that the greater circumferences do not pass through the inner tertiary keystones. As a result, the sections of curved ribs between the outer and inner tertiary keystones are formed by two arcs of circumference, secants of each other, and the point of intersection does not coincide with any keystone. This change of curvature can be easily appreciated in the following photograph (fig. 23).

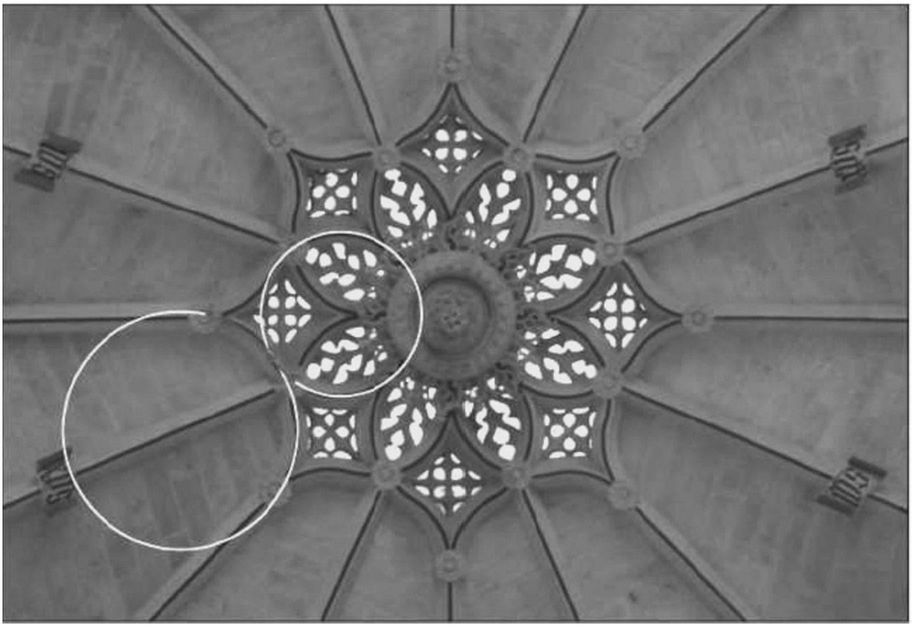


Fig. 23. Detail of the curved ribs of the vault

Having defined the position of the keystones and the geometry of the ribs, the layout of the vault is also determined. However, as fig. 1 illustrates, there are two **decorative elements** in its ribbing that remain to be situated: *false keystones* in the tierceron ribs (fig. 24) and some *bosses* on the diagonal ribs (fig. 25).

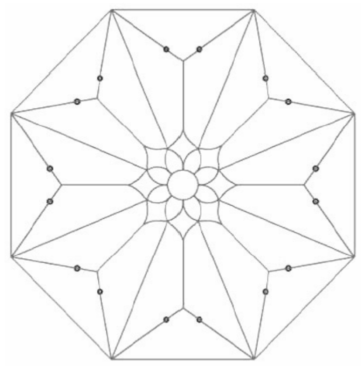


Fig. 24. Position of the *false keystones* on the tierceron ribs

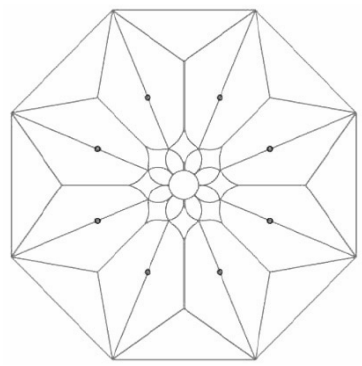


Fig. 25. Position of the *bosses* on the diagonal ribs

The position of the *false keystones* on the tierceron ribs is determined by the construction shown in fig. 26. That is, the intersections of the longest sides of the Cordovan triangle T_1 with the tierceron ribs define the points that are sought.

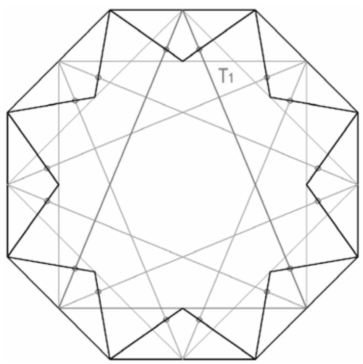


Fig. 26. *False keystones* in the tierceron ribs and Cordovan triangle T_1

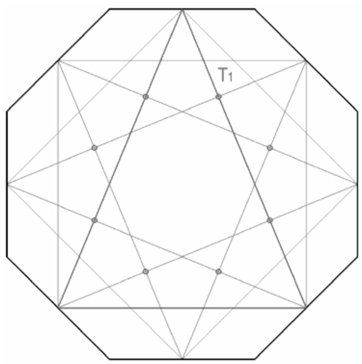


Fig. 27. *Bosses* on the diagonal ribs

The position of the *bosses* on the diagonal ribs is obtained by intersecting the longest sides of the Cordovan triangles T_1 (fig. 27).

Results

The Cordovan triangles denominated as T_1, T_2, T_3, T_4 and T_5, which serve as a pattern (fig. 28) for finding the layout of the vault of the Chapel of the Presentation, are interrelated. To be precise, the relationship between each with the previous one is the Cordovan number *c*.

As pointed out previously, the ratio of the longest to the shortest side of the Cordovan triangle is *c*. The relationship between the shortest "l_i" (or longest "L_i") sides of the cited triangles is also the Cordovan number; that is, the shortest side of one of them is the longest side of another and so on successively, as fig. 29 shows.

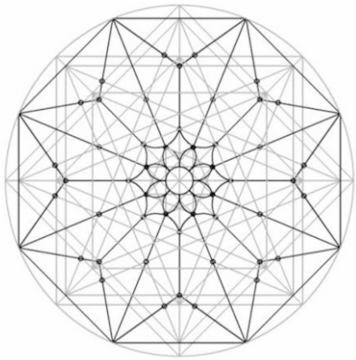
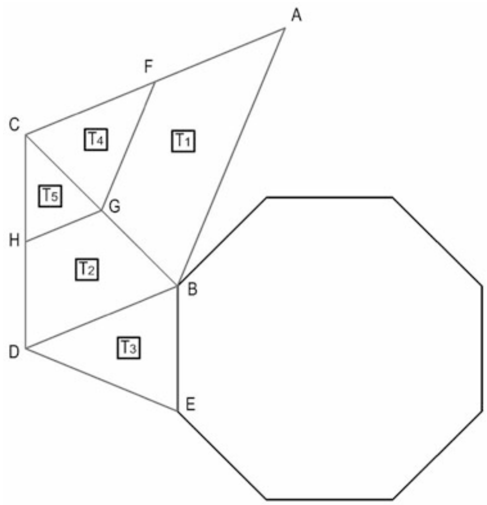


Fig. 28. Cordovan triangles that generate the layout of the vault



$$L_1 = AB = AC$$

$$l_1 = L_2 = BC = CD$$

$$l_2 = L_3 = BD = DE$$

$$l_3 = BE$$

$$L_4 = \frac{L_1}{2} = CF = FG$$

$$l_4 = L_5 = \frac{l_1}{2} = CG = CH$$

$$l_5 = \frac{l_2}{2} = GH$$

Fig. 29. Geometric relation between Cordovan triangles

In the above figure, it can be seen that they not only are the triangles related to one another but that, in addition, the length of their sides depends on the side of the octagon l, exactly in proportion to the Cordovan number:

$$l_3 = l$$

$$l_2 = L_3 = l_3 \times c = l \times c$$

$$l_1 = L_2 = l_2 \times c = l \times c \times c = l \times c^2$$

$$L_1 = l_1 \times c = l \times c^2 \times c = l \times c^3$$

$$l_5 = \frac{l_2}{2} = \frac{l}{2} \times c$$

$$l_4 = L_5 = l_5 \times c = \frac{l}{2} \times c \times c = \frac{l}{2} \times c^2$$

$$L_4 = l_4 \times c = \frac{l}{2} \times c^2 \times c = \frac{l}{2} \times c^3$$

The sides of the triangles form two geometric successions *a* and *b*, the ratio of which coincides with the Cordovan number *c*:

$$a_n = l \times c^n \text{ and } b_n = \frac{l}{2} \times c^n$$

(where *l* is the side of the octagon), as table 1 shows.

			***T*₁**	
			shortest side	longest side
		***T*₂**		
		shortest side	longest side	
	***T*₃**			
	shortest side	longest side		
$a_n = l \times c^n$	$l \times c^0$	$l \times c^1$	$l \times c^2$	$l \times c^3$
$b_n = \frac{l}{2} \times c^n$	$\frac{l}{2} \times c^0$	$\frac{l}{2} \times c^1$	$\frac{l}{2} \times c^2$	$\frac{l}{2} \times c^3$
		shortest side	longest side	
		***T*₅**		
			shortest side	longest side
			***T*₄**	

Table 1. Geometric successions of ratio *c*

The positions of the centres of the circumferences defining the layout of the curved ribs are also interrelated. The circumference whose diameter coincides with the apothem of the octagon passes through the centres of the greater circumferences (fig. 30). The circumference whose diameter is equal to the distance between the centre of the polygon and the centre of the greater circumference passes through the centres of the lesser circumferences (fig. 31).

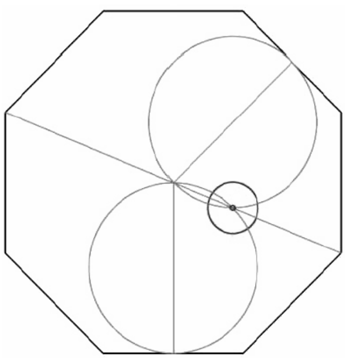


Fig. 30. Relationship between the centre of the greater circumferences and the octagon

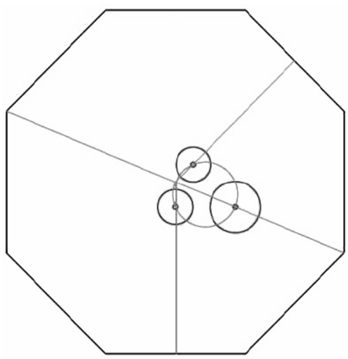


Fig. 31. Relationship between the centres of the greater and lesser circumferences

The relationship between both constructions becomes apparent in fig. 32. In this figure, two right triangles are marked ("arcs capables" of the circumferences). The vertices of the largest triangle coincide with: the centre of the octagon, the midpoint of the polygon side and the centre of the greater circumference. The vertices of the smallest triangle are: the centre of the octagon, the centre of the greater circumference and the centre of the lesser circumference. Thus, both triangles are related to one another since the hypotenuse of the smaller one coincides with one of the catheti of the larger one. The relationship between the sides of both triangles is not arbitrary, but is exactly *2c*.

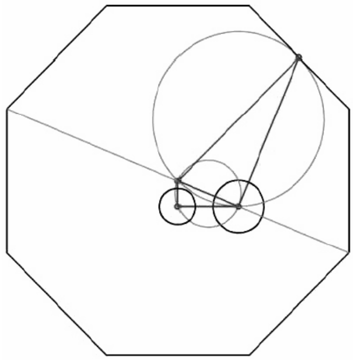


Fig. 32. Relationship between the centres of the circumferences and the octagon

Conclusions

As is clearly shown in this work, the canon of the octagonal vault of the Chapel of the Presentation of the Cathedral of Burgos is the Cordovan proportion. The superposition of the proposed layout and that obtained using data acquisition (fig. 33) shows the small differences existing between them (possible deformations and/or imprecisions during the construction).

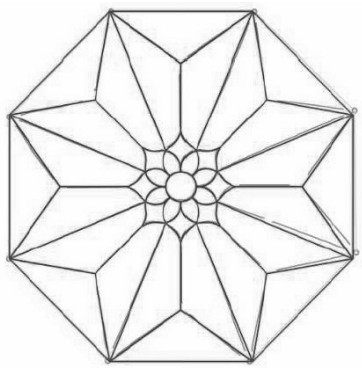


Fig. 33. Superposition of the proposed layout (red) and that obtained using data acquisition (grey)

The layout of the analysed vault was obtained from a Cordovan triangle, whose shortest side is the side of the octagon, together with other similar, whose sides are multiple of the Cordovan number *c*. The superposition of these triangles determines not only the position of the keystones but also the geometry of their ribbing and even the layout of the curved ribs formed by two arcs of circumference.

Although it is not known if the master mason of this vault knew the Cordovan proportion, this research is the first to prove its compliance with the layout of the curved ribbing.

Acknowledgments

The author would like to express his gratitude to the Metropolitan Chapter of Burgos Cathedral and the staff of the cathedral's Historic Archives for their kind assistance.

Notes

1. The article "¿Canon divino? No, cordobés" ("Divine Canon? No, Cordovan") was published in the Spanish newspaper *ABC* in February 2010 [Roso 2010].
2. Given the very slight difference between the Cordovan number (1.3065…) and the sesquitercian number (1.3333…) [Pacioli 1987: 81], extremely high precision is required in its study in order not to commit any errors.
3. A rectangle is said to be a "silver rectangle" if on cutting two equal squares whose sides are equal to the shortest side of the rectangle, the resulting polygon is a rectangle similar to the original. The ratio of its longest to its shortest side is $1+\sqrt{2}$.

References

BONELL, Carmen. 2000. *La divina proporción. Las formas geométricas*. Barcelona: Edicions UPC.

GOMEZ, Javier. 1998. *El gótico español de la Edad Moderna: Bóvedas de crucería*. Valladolid: Universidad de Valladolid.

HOZ , Rafael de la. 2002. *La proporción cordobesa*. Córdoba: Colegio Oficial de Arquitectos de Córdoba.

HUNTLEY, Ernest. 1970. *The Divine Proportion: A Study in Mathematical Beauty*. New York: Dover Publications, Inc.

HUYLEBROUCK, Dirk, Antonia REDONDO and Encarnación REYES. 2009. Octagonal Geometry of the Cimborio in Burgos Cathedral. *Nexus Network Journal* **13**, 1: 195-203.

PACIOLI, Luca. 1987. *La divina proporción*. With an Introduction to the Edition by Antonio M. González. Madrid: Akal.

PALACIOS, Jose Carlos. 2009. La construcción de la bóveda de crucería. El Arte de la Piedra*. Teoría y Práctica de la Cantería*. Madrid: Fundación Universitaria San Pablo CEU: 27-51.

RABASA, Enrique. 2000. *Forma y construcción en piedra: de la cantería medieval a la estereotomía del siglo XIX*. Madrid: AKAL.

REDONDO, Antonia and Encarnación REYES. 2008a. The Cordovan Proportion: Geometry, Art and Paper Folding. *Hyperseeing. Proceedings of Seventh Interdisciplinary Conference ISAMA*. Valencia (Spain). May-June 2008: 107-114.

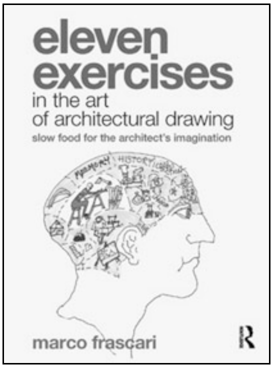
———. 2008b. The Geometry of the Cordovan Polygons. *Visual Mathematics* **10**, 4.

ROSO, M. 2010. ¿Canon Divino? No, Cordobés. *ABC* (Córdoba) 21_2_2010: 74. http://hemeroteca.abc.es/nav/Navigate.exe/hemeroteca/cordoba/abc.cordoba/2010/02/21/076.html

UNESCO. 1984. Convention concerning the protection of the world cultural and natural heritage. SC/84/CONF.004/9 C.F.R.

About the author

Tomás Gil-López earned his Ph. D. in Architecture in 2003. He is a Professor at the Madrid School of Architecture in Madrid Polytechnic University, Spain. He is an instructor in courses on Descriptive Geometry, CAD and Technological Innovation in Construction at the bachelor and master level in Madrid Polytechnic University (2004-2011). Since the late 1990s, he has devoted his time to research in descriptive geometry and its applications in architecture and education. His contributions at international congress are focused on the domain of geometric analysis in Gothic vaults. He is the author of *Superficies Cuádricas y sus Combinaciones (I). Superficies de Curvatura Simple* (Madrid, 2006) and *Influencia de la Configuración del Borde Público-Privado. Parámetros de Diseño* (Madrid, 2007). He has also completed several works of architecture and urban design.



Keywords: Marco Frascari, architectural drawing, slow food, design

Book Review

Marco Frascari

Eleven Exercises in the Art of Architectural Drawing: Slow Food for the Architect's Imagination

Oxon, UK: Routledge, 2011

Reviewed by Sylvie Duvernoy

Via Benozzo Gozzoli, 26
50124 Florence ITALY
syld@kimwilliamsbooks.com

In the process of architectural design, the drawings that are intended to represent the ideas stemming from the designer's imagination fall into two main categories: conceptual sketches, related mainly to the early research phases; and communication drawings, related to the phase of divulgation of the results of the research. Conceptual sketches relate emotions, while communication drawings mostly report information. The communication drawing, in order to be efficient, must adhere to the standard codes of two- and three-dimensional representation, and therefore takes the familiar forms of plan, section, elevation, perspective, digital rendering, and so forth. It can be done by assistants or collaborators. Marco Frascari calls this kind of drawing "trivial drawing". On the other hand, the "non-trivial drawing", which describes the idea rather than the form, may take any graphic form whatsoever, and may be done with any kind of graphic tool. The quality of the conceptual sketch, done by no one other than the designers themselves, depends heavily on their ability to express their feelings on paper with pencils, markers, colors, and all sorts of graphic techniques.

Marco Frascari, of Italian origin, has been teaching architectural design for many years in the United States and Canada. This new book is totally dedicated to the "non-trivial drawing", inquiring into the relationship between drawing and designing, or better, between drawing and thinking. Although the title of the volume is *Eleven Exercises in the Art of Architectural Drawing*, the book is not meant only for students but also for colleagues engaged in teaching drawing and/or working in design studios, and of course, for anyone interested in the meaning of drawing. More than a textbook, it is an essay. The subtitle gives us the key to understanding the author's thesis and the purpose of the eleven exercises: "slow food for the architect's imagination".

The arguments that Frascari discusses in his essay are so many and so diverse that it is impossible to comment on them all. Among them, three main themes that recur in the various chapters of the book have caught my attention: the question of slow vs. fast, the opposition between digital and non-digital, and – last but not least – the relationship between design and drawing tools.

DOI 10.1007/s00004-011-0105-1; *published online* 17 January 2012

slow/fast

When I started teaching architectural drawing in Italy more than ten years ago, the first-year drawing class lasted for the full year: two semesters. Today, in nearly all Italian universities, the drawing class lasts only one semester: from September to January, sometimes from September to December. It must also encompass descriptive geometry, which used to be a separate course before the "reform" of the curricula. Is it really surprising that the colleagues teaching the second-semester design studios complain that the students enrolled in their classes are not able to express their ideas on paper? Are the drawing teachers to blame, or the pace at which we are supposed to train them? The students have no time to digest all the information with which we feed them. Fast, fat food … no assimilation time. I really wish I was still given another semester in order to contribute more to the progress of their drawing skills, which goes along with the growth of their maturity and … which takes time. Maybe one would argue that it is not positive to separate drawing from designing, and that the drawing class should not be so strongly oriented toward the "trivial drawing". I could even agree to some extent. France has a different approach to these didactic problems, but speed is a feature common to all architecture curricula.

First-year students however do not complain. The quicker the hand-drawing course is over, the quicker they switch to digital drawing, which thrills them much more because they feel more professional as soon as they approach computer technology.

digital/non-digital

I still do not understand why digital drawing is sometimes referred to – in Italian – as *disegno automatico*, and why it is considered to be quicker than hand drawing. There is nothing automatic in digital drawing. The computer only does what the operator tells it to do: if the input is lousy, the output is lousy. No increase in intellectual or artistic value is to be expected from a machine that was not handled by a highly skilled (and long trained) operator. But sophisms die hard. My students are still convinced that the computer is going to produce a perspective view better and quicker than what they could do by hand. This is why I get, after weeks of efforts, awkward black and white 3D views, printed on shiny white paper, that are a caricature of the architectural object that they had in mind. It is much harder to embed pathos in a digital drawing than in a hand drawing.

Among the eleven exercises suggested by Marco Frascari, I particularly enjoy the first one:

> Instead of dipping your nibs or your brushes in store-bought inks and paints, use exclusively liquids, pastes, juices or powders that you normally eat, drink or use to spice and flavour your food.

I enjoy it when the act of drawing involves sensations that go beyond the mere sight. I enjoy touching the many kinds of paper, I have fun getting my hands and clothes dirty. I enjoy the smell of some media. Even taste… I remember a fellow student drawing watercolours by licking constantly her brush to wet it (true watercolour pastes are made from honey and are not toxic!). Hand drawing can be very fast: it does not take much time to spread color on paper. And since the procedure is fast, in case the first result is not satisfactory, the idea of starting over and trying again is much less discouraging.

In the late eighties and early nineties of the past century, when CAD first appeared in professional practice, and simultaneously started to be taught in architecture schools, students and newly-graduated architects were hired by big firms because of their familiarity with the modern computer technology that the senior architects of that time had not mastered. The salary would be directly proportional to the juniors' ability to use the CAD programs that were spreading throughout the business world. Now that CAD has become the most widespread drafting tool, both at school and at work, I hope to witness the reverse tendency someday: students and junior architects will be hired for their ability to sketch and draw by hand, a skill that seniors will have lost because the excitement about computer technology was paramount in their youth.

design/drawing tools

Among Marco Frascari's eleven exercises, three are about the importance and influence of the drawing tools and representation techniques on the design process and result. This is an argument that has been discussed at length – and still will be – in the pages of the *Nexus Network Journal*: how and how much do the means of drawing (hands, 2D, 3D, CAD…) influence the shape and form of architecture? Frascari suggests trying to draw with crooked rulers, non-straightedges, bent squares, etc. Build new tools to find new forms. Play with the representation techniques: imagine, for example, that a reverse perspective is the natural view of a true space.

We would like to think that new tools are constructed only when the need for them arises, technology thus being a mere consequence of imagination, the necessary step to make desire come true. This theory is pleasant because it puts mind above hands, ideas above mechanics. But only geniuses have a totally inventive imagination. Most of us are subject to the technical possibilities that are offered to us: the instruments acting as a motor for inspiration. This is why experimenting freely with available tools can suggest gracious results. When the tool is eventually modified according to chosen rules, then a two-way relationship is settled between drawing and design.

The ancient Greek mathematicians had defined three basic right-angled triangles that sufficed to build the Platonic solids representing the beauty of the elemental world: the half square, the half equilateral triangle, and the triangle of golden proportion. What if we could buy today in any nearby stationery shop all three different little plastic squares (one of angles 45°-45°-90°, one of -30°-60°-90° and one of angles -33°-57°-90°) instead of the only first two being available? We surely would have many more monuments with golden proportions.

Few books on architectural drawing approach the question from such a "non-trivial" standpoint. Frascari's essay is full of suggestions for architects and professors in architecture schools. Each reader is going to be interested in some precise point and focus on some peculiar issue that will trigger his or her own curiosity or suggest some controversy. Reading Frascari's words forces the readers to define and justify their own opinions on the subject. Which is a sufficient reason to read the book.

About the reviewer

Sylvie Duvernoy is the book editor of the *Nexus Network Journal*, and the author of the recently published book on "trivial" drawing *Elementi di disegno. 12 lezioni di disegno dell'architettura* – with English text (Florence: Le Lettere, 2011).

Subscription information
ISSN print edition 1590-5896
ISSN electronic edition 1522-4600

Subscription rates
For information on subscription rates please contact:
Springer Customer Service Center GmbH
The Americas (North, South, Central America and the Caribbean)
journals-ny@springer.com
Outside the Americas: subscriptions@springer.com

Orders and inquiries
The Americas (North, South, Central America and the Caribbean)
Springer Journal Fulfillment
P.O. Box 2485, Secaucus
NJ 07096-2485, USA
Tel.: 800-SPRINGER (777-4643)
Tel.: +1-201-348-4033 (outside US and Canada)
Fax: +1-201-348-4505
e-mail: journals-ny@springer.com

Outside the Americas
via a bookseller or
Springer Customer Service Center GmbH
Haberstrasse 7, 69126 Heidelberg, Germany
Tel.: +49-6221-345-4304
Fax: +49-6221-345-4229
e-mail: subscriptions@springer.com
Business hours: Monday to Friday
8 a.m. to 8 p.m. local time and on German public holidays

Cancellations must be received by September 30 to take effect at the end of the same year.

Changes of address: Allow six weeks for all changes to become effective. All communications should include both old and new addresses (with postal codes) and should be accompanied by a mailing label from a recent issue.

According to § 4 Sect. 3 of the German Postal Services Data Protection Regulations, if a subscriber's address changes, the German Post Office can inform the publisher of the new address even if the subscriber has not submitted a formal application for mail to be forwarded. Subscribers not in agreement with this procedure may send a written complaint to Customer Service Journals,within 14 days of publication of this issue.

Back volumes: Prices are available on request.

Microform editions are available from: ProQuest. Further information available at: http://www.il.proquest.com/umi/

Electronic edition
An electronic edition of this journal is available at springerlink.com

Advertising
Ms Raina Chandler
Springer, Tiergartenstraße 17
69121 Heidelberg, Germany
Tel.: +49-62 21-4 87 8443
Fax: +49-62 21-4 87 68443
springer.com/wikom
e-mail: raina.chandler@springer.com

Instructions for authors
Instructions for authors can now be found on the journal's website: birkhauser-science.com/NNJ

Production
Springer, Petra Meyer-vom Hagen
Journal Production, Postfach 105280,
69042 Heidelberg, Germany
Fax: +49-6221-487 68239
e-mail: petra.meyervomhagen@springer.com
Typesetter: Scientific Publishing Services (Pvt.) Limited, Chennai, India
Springer is a part of
Springer Science+Business Media
springer.com