# Mekelle University <br> College of Business and Economics <br> <br> Department of Accounting and Finance 

 <br> <br> Department of Accounting and Finance}


Course Name - Investment Analysis and Portfolio Management
Course Code - AcFn 3201

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| :---: | :---: | :---: |
| Course Information |  |  |
| Course code |  | AcFn3201 |
| Course Title |  | Investment and Portfolio Management |
| Degree Program |  | BA Degree in Accounting and Finance |
| Module |  | Project and Investment Analysis |
| Module no and code |  | M20; AcFn-M3201 |
| Module Coordinator |  | Dr.Bereket |
| Lecturer |  | Dr.Bereke and Haftom, |
| ETCTS Credits |  | 3 |
| Contact Hours (per week) |  | 2 |
| Course Objectives |  | The course will enable students to understand different investment avenues and aware of the risk return of different investment alternatives and estimate the value of securities so as to make valuable investment decisions. |
| Course Description |  | This course provides an overview of the field of investment .it explains basic concepts and methods useful in investment. The course also tries to imitate the valuation of bond and stocks. It also covers fundamental and technical analysis as well as portfolio construction and portfolio managements. |
| WEEK S | Course Contents | 2.5. Measuring expected risk and return |
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|  | 2. Risk and return <br> 2.1. Return <br> 2.2. Risk <br> 2.3. Measuring historical risk <br> 2.4. Measuring historical return | e. Rating of bonds <br> f. Analysis of convertible bonds |

## 4. Stock and equity valuation

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7.1. Portfolio performance evaluation
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7.3. Risk management and hedging
7.4. Active portfolio management
7.5. International portfolio management


## Chapter One

## Introduction to investment

### 1.1 What is an Investment?

When current income exceeds current consumption desires, people tend to save the excess. They can do any of several things with these savings. One possibility is to put the money under a mattress or bury it in the backyard until some future time when consumption desires exceed current income. When they retrieve their savings from the mattress or backyard, they have the same amount they saved.

Another possibility is that they can give up the immediate possession of these savings for a future larger amount of money that will be available for future consumption. This tradeoff of present consumption for a higher level of future consumption is the reason for saving. What you do with the savings to make them increase over time is investment

Those who give up immediate possession of savings (that is, defer consumption) expect to receive in the future a greater amount than they gave up. Conversely, those who consume more than their current income (that is, borrow) must be willing to pay back in the future more than they borrowed.

The rate of exchange between future consumption (future dollars) and current consumption (current dollars) is the pure rate of interest. Both people's willingness to pay this difference for borrowed funds and their desire to receive a surplus on their savings give rise to an interest rate referred to as the pure time value of money. This interest rate is established in the capital market by a comparison of the supply of excess income available (savings) to be invested and the demand for excess consumption (borrowing) at a given time. If you can exchange $\$ 100$ of certain income today for $\$ 104$ of certain income one year from today, then the pure rate of exchange on a risk-free investment (that is, the time value of money) is said to be 4 percent (104/100-1).

The investor who gives up $\$ 100$ today expects to consume $\$ 104$ of goods and services in the future. This assumes that the general price level in the economy stays the same. For instance in US the price stability has rarely been the case during the past several decades when inflation rates have varied from 1.1 percent in 1986 to 13.3 percent in 1979, with an average of about 5.4 percent a year from 1970 to 2001. If investors expect a change in prices, they will require a higher rate of return to compensate for it. For example, if an investor expects a rise in prices (that is, he or she expects inflation) at the rate of 2 percent during the period of investment, he or she will increase the required interest rate by 2 percent. In our example, the investor would require $\$ 106$ in the future to defer the $\$ 100$ of consumption during an inflationary period (a 6 percent nominal, risk-free interest rate will be required instead of 4 percent).

Further, if the future payment from the investment is not certain, the investor will demand an interest rate that exceeds the pure time value of money plus the inflation rate. The uncertainty of the payments from an investment is the investment risk. The additional return added to the nominal, risk-free interest rate is called a risk premium. In our previous example, the investor would require more than $\$ 106$ one year from today to compensate for the uncertainty. As an example, if the required amount were $\$ 110, \$ 4$, or 4 percent, would be considered a risk premium.

From our discussion, we can specify a formal definition of investment. Specifically, an investment is the current commitment of dollars for a period of time in order to derive future payments that will compensate the investor for
i. the time the funds are committed,
ii. the expected rate of inflation, and
iii. The uncertainty of the future payments.

The "investor" can be an individual, a government, a pension fund, or a corporation. Similarly, this definition includes all types of investments, including investments by corporations in plant and equipment and investments by individuals in stocks, bonds, commodities, or real estate. The investor is trading a known dollar amount today for some expected future stream of payments that will be greater than the current outlay.

Why people invest and what they want from their investments. They invest to earn a return from savings due to their deferred consumption. They want a rate of return that compensates them for the time, the expected rate of inflation, and the uncertainty of the return. This return, the investor's required rate of return, is discussed throughout this course. A central question of this course is how investors select investments that will give them their required rates of return.

### 1.2 MEASURES OF RETURN AND RISK

A return is the ultimate objective for any investor. But a relationship between return and risk is a key concept in finance. As finance and investments areas are built upon a common set of financial principles, the main characteristics of any investment are investment return and risk. However to compare various alternatives of investments the precise quantitative measures for both of these characteristics are needed.

With most investments, an individual or business spends money today with the expectation of earning even more money in the future. The concept of return provides investors with a convenient way of expressing the financial performance of an investment.

Many investments have two components of their measurable return:

- a capital gain or loss;
- Some form of income.

The rate of return is the percentage increase in returns associated with the holding period:

### 1.2.1 Measures of Historical Rates of Return:

If you commit $\$ 200$ to an investment at the beginning of the year and you get back $\$ 220$ at the end of the year, what is your return for the period? The period during which you own an investment is called its holding period, and the return for that period is the holding period return (HPR).

In this example, the HPR is 1.10 , calculated as follows:

$$
\begin{aligned}
\text { HPR } & =\frac{\text { Ending Value of Investment }}{\text { Beginning Value of Investment }} \\
& =\frac{\$ 220}{\$ 200}=1.10
\end{aligned}
$$

This value will always be zero or greater-that is, it can never be a negative value. A value greater than 1.0 reflects an increase in your wealth, which means that you received a positive rate of return during the period. A value less than 1.0 means that you suffered a decline in wealth, which indicates that you had a negative return during the period. An HPR of zero indicates that you lost all your money.

Although HPR helps us express the change in value of an investment, investors generally evaluate returns in percentage terms on an annual basis. This conversion to annual percentage rates makes it easier to directly compare alternative investments that have markedly different characteristics. The first step in converting an HPR to an annual percentage rate is to derive a percentage return, referred to as the holding period yield (HPY). The HPY is equal to the HPR minus 1 .

$$
\mathrm{HPY}=\mathrm{HPR}-1
$$

$$
\begin{aligned}
\mathrm{HPY}=1.10-1 & =0.10 \\
& =10 \%
\end{aligned}
$$

To derive an annual HPY, you compute an annual HPR and subtract 1. Annual HPR is found by:

$$
\text { Annual } \mathrm{HPR}=\mathrm{HPR}^{1 / n}
$$

where:

## $n=$ number of years the investment is held

Consider an investment that cost $\$ 250$ and is worth $\$ 350$ after being held for two years:

$$
\begin{aligned}
\text { HPR } & =\frac{\text { Ending Value of Investment }}{\text { Beginning Value of Investment }}=\frac{\$ 350}{\$ 250} \\
& =1.40
\end{aligned}
$$

$$
\begin{aligned}
\text { Annual HPR } & =1.40^{1 / n} \\
& =1.40^{1 / 2} \\
& =1.1832 \\
\text { Annual HPY } & =1.1832-1=0.1832 \\
& =18.32 \%
\end{aligned}
$$

If you experience a decline in your wealth value, the computation is as follows:

$$
\begin{aligned}
& \mathrm{HPR}=\frac{\text { Ending Value }}{\text { Beginning Value }}=\frac{\$ 400}{\$ 500}=0.80 \\
& \mathrm{HPY}=0.80-1.00=-0.20=-20 \%
\end{aligned}
$$

A multiple year loss over two years would be computed as follows:

$$
\mathrm{HPR}=\frac{\text { Ending Value }}{\text { Beginning Value }}=\frac{\$ 750}{\$ 1,000}=0.75
$$

$$
\begin{aligned}
\text { Annual HPR } & =(0.75)^{1 / n}=0.75^{1 / 2} \\
& =0.866
\end{aligned}
$$

$$
\text { Annual HPY }=0.866-1.00=-0.134=-13.4 \%
$$

In contrast, consider an investment of $\$ 100$ held for only six months that earned a return of $\$ 12$ :

$$
\begin{aligned}
\text { HPR } & =\frac{\$ 112}{\$ 100}=1.12(n=0.5) \\
\text { Annual } \mathrm{HPR} & =1.12^{1 / .5} \\
& =1.12^{2} \\
& =1.2544 \\
\text { Annual HPY } & =1.2544-1=0.2544 \\
& =25.44 \%
\end{aligned}
$$

### 1.2.2 Computing Mean Historical Returns

Single Investment Given a set of annual rates of return (HPYs) for an individual investment, there are two summary measures of return performance. The first is the arithmetic means return, the second the geometric mean return. To find the arithmetic mean (AM), the sum of annual HPYs is divided by the number of years ( $n$ ) as follows:

$$
\mathrm{AM}=\Sigma \mathrm{HPY} / n
$$

where:

## $\Sigma H P Y=$ the sum of annual holding period yields

An alternative computation, the geometric mean (GM), is the $n$th root of the product of the HPRs for $n$ year minus one
$\mathrm{GM}=[\pi \mathrm{HPR}]^{1 / n}-1$
where:
$\pi=$ the product of the annual holding period returns as follows:

$$
\left(\mathrm{HPR}_{1}\right) \times\left(\mathrm{HPR}_{2}\right) \cdots\left(\mathrm{HPR}_{n}\right)
$$

To illustrate these alternatives, consider an investment with the following data:

|  | Beginning | Ending |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Year | Value | Value | HPR | HPY |
| 1 | 100.0 | 115.0 | 1.15 | 0.15 |
| 2 | 115.0 | 138.0 | 1.20 | 0.20 |
| 3 | 138.0 | 110.4 | 0.80 | -0.20 |

$$
\begin{aligned}
\mathrm{AM} & =[(0.15)+(0.20)+(-0.20)] / 3 \\
& =0.15 / 3 \\
& =0.05=5 \% \\
\mathrm{GM} & =[(1.15) \times(1.20) \times(0.80)]^{1 / 3}-1 \\
& =(1.104)^{1 / 3}-1 \\
& =1.03353-1 \\
& =0.03353=3.353 \%
\end{aligned}
$$

Investors are typically concerned with long-term performance when comparing alternative investments. GM is considered a superior measure of the long-term mean rate of return because it indicates the compound annual rate of return based on the ending value of the investment versus its beginning value. Specifically, using the prior example, if we compounded 3.353 percent for three years, (1.03353), we would get an ending wealth value of 1.104 .

Although the arithmetic average provides a good indication of the expected rate of return for an investment during a future individual year, it is biased upward if you are attempting to measure an asset's long-term performance. This is obvious for a volatile security. Consider, for example, a security that increases in price from $\$ 50$ to $\$ 100$ during year 1 and drops back to $\$ 50$ during year 2 .

The annual HPYs would be:

|  | Beginning | Ending |  |  |
| :---: | :---: | :---: | :---: | ---: |
| Year | Value | Value | HPR | HPY |
| 1 | 50 | 100 | 2.00 | 1.00 |
| 2 | 100 | 50 | 0.50 | -0.50 |

This would give an AM rate of return of:

$$
\begin{aligned}
{[(1.00)+(-0.50)] / 2 } & =.50 / 2 \\
& =0.25=25 \%
\end{aligned}
$$

This investment brought no change in wealth and therefore no return, yet the AM rate of return is computed to be 25 percent.

The GM rate of return would be:

$$
\begin{aligned}
(2.00 \times 0.50)^{1 / 2}-1 & =(1.00)^{1 / 2}-1 \\
& =1.00-1=0 \%
\end{aligned}
$$

This answer of a 0 percent rate of return accurately measures the fact that there was no change in wealth from this investment over the two-year period. When rates of return are the same for all years, the GM will be equal to the AM. If the rates of return vary over the years, the GM will always be lower than the AM. The difference between the two mean values will depend on the year-to-year changes in the rates of return. Larger annual changes in the rates of return-that is, more volatility-will result in a greater difference between the alternative mean values.

An awareness of both methods of computing mean rates of return is important because published accounts of investment performance or descriptions of financial research will use both the AM and the GM as measures of average historical returns.

### 1.2.3 Calculating Expected Rates of Return

Risk is the uncertainty that an investment will earn its expected rate of return. In the examples in the prior section, we examined realized historical rates of return. In contrast, an investor who is evaluating a future investment alternative expects or anticipates a certain rate of return.

The investor might say that he or she expects the investment will provide a rate of return of 10 percent, but this is actually the investor's most likely estimate, also referred to as a point estimate. Pressed further, the investor would probably acknowledge the uncertainty of this point estimate return and admit the possibility that, under certain conditions, the annual rate of return on this investment might go as low as -10 percent or as high as 25 percent. The point is, the specification of a larger range of possible returns from an investment reflects the investor's uncertainty regarding what the actual return will be. Therefore, a larger range of possible returns implies that the investment is riskier.

An investor determines how certain the expected rate of return on an investment is by analyzing estimates of possible returns. To do this, the investor assigns probability values to all possible returns. These probability values range from zero, which means no chance of the return, to one, which indicates complete certainty that the investment will provide the specified rate of return. These probabilities are typically subjective estimates based on the historical performance of the investment or similar investments modified by the investor's expectations for the future.

As an example, an investor may know that about 30 percent of the time the rate of return on this particular investment was 10 percent. Using this information along with future expectations regarding the economy, one can derive an estimate of what might happen in the future.

The expected rate of return from an investment is: is the sum of the products of each possible outcome times its associated probability-it is a weighted average of the various possible outcomes, with the weights being their probabilities of occurrence:

$$
\begin{gathered}
\text { Expected Return }=\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right) \sum_{i=1}^{n}(\text { Probability of Return }) \times(\text { PossibleReturn }) \\
\text { Expected rate of return }=\hat{\mathrm{r}}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Pir}_{\mathrm{i}} .
\end{gathered}
$$

- Where the number of possible outcomes is virtually unlimited, continuous probability distributions are used in determining the expected rate of return of the event.
- The tighter, or more peaked, the probability distribution, the more likely it is that the actual outcome will be close to the expected value, and, consequently, the less likely it is
that the actual return will end up far below the expected return. Thus, the tighter the probability distribution, the lower the risk assigned to a stock.

Risk aversion: The assumption that most investors will choose the least risky alternative, all else being equal and that they will not accept additional risk unless they are compensated in the form of higher return
> Most investors are risk averse. This means that for two alternatives with the same expected rate of return, investors will choose the one with the lower risk.
$>\quad$ No investment will be undertaken unless the expected rate of return is high enough to compensate the investor for the perceived risk of the investment.
Illustration- Expected rate of return: Let us begin our analysis of the effect of risk with an example of perfect certainty wherein the investor is absolutely certain of a return of 5 percent.

Let us begin our analysis of the effect of risk with an example of perfect certainty wherein the investor is absolutely certain of a return of 5 percent. Exhibit 1.1 illustrates this situation.

Perfect certainty allows only one possible return, and the probability of receiving that return is 1.0. Few investments provide certain returns and would be considered risk-free investments.

In the case of perfect certainty, there is only one value for PiRi:

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=(1.0)(0.05)=0.05=5 \%
$$

## Probability Distribution

a probability distribution gives all the possible outcomes with the chance, or probability, each outcome will occur

Figure 1.1 Probability distribution for risk free investment


In an alternative scenario, suppose an investor believed an investment could provide several different rates of return depending on different possible economic conditions. As an example, in a strong economic environment with high corporate profits and little or no inflation, the investor
might expect the rate of return on common stocks during the next year to reach as high as 20 percent. In contrast, if there is an economic decline with a higher-than-average rate of inflation, the investor might expect the rate of return on common stocks during the next year to be -20 percent. Finally, with no major change in the economic environment, the rate of return during the next year would probably approach the long-run average of 10 percent.

The investor might estimate probabilities for each of these economic scenarios based on past experience and the current outlook as follows:

## Economic Conditions

Probability
Rate of
Return

| Strong economy, no inflation | 0.15 | 0.20 |
| :--- | :---: | :---: |
| Weak economy, above-average inflation | 0.15 | $\mathbf{- 0 . 2 0}$ |
| No major change in economy | 0.70 | 0.10 |

This set of potential outcomes can be visualized as shown in Exhibit 1.2.
The computation of the expected rate of return $\left[\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)\right]$ is as follows:

$$
\begin{aligned}
\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)= & {[(0.15)(0.2)+(0.15)(-0.2)+(0.7)(0.1)] } \\
& =0: 07
\end{aligned}
$$

Obviously, the investor is less certain about the expected return from this investment than about the return from the prior investment with its single possible return.

A third example is an investment with 10 possible outcomes ranging from-40 percent to 50 percent with the same probability for each rate of return. A graph of this set of expectations would appear as shown in Exhibit 1.3.

In this case, there are numerous outcomes from a wide range of possibilities. The expected rate of return $\left[E\left(R_{i}\right)\right]$ for this investment would be:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=[(0.10)(-0.4)+(0.10)(-0.3)+(0.1)(-0.2)+(0.10)(-0.1)+(0.10)(0.00)+(0.1)(0.1) \\
& +(0.10)(0.20)+(0.1)(0.3)+(0.1)(0.4)+(0.10)(0.50)] \\
& =[(-0.04)+(-0.03)+(-0.02)+(-0.01)+(0.00)+(0.01)+(0.02)+(0.03)+(0.04)+(0.05) \\
& =0.05=5 \%
\end{aligned}
$$

The expected rate of return for this investment is the same as the certain return discussed in the first example; but, in this case, the investor is highly uncertain about the actual rate of return. This would be considered a risky investment because of that uncertainty.

We would anticipate that an investor faced with the choice between this risky investment and the certain (risk-free) case would select the certain alternative. This expectation is based on the belief that most investors are risk averse, which means that if everything else is the same, they will select the investment that offers greater certainty (i.e., less risk).

Figure 1.2: Probability Distribution for Risky Investment with Three Possible Rates of Return


Figure 1.3: Probability Distribution for Risky Investment with 10 Possible Rates of Return


### 1.2.4 Measures the Risk of the Expected Rates of Return

We have shown that we can calculate the expected rate of return and evaluate the uncertainty, or risk, of an investment by identifying the range of possible returns from that investment and assigning each possible return a weight based on the probability that it will occur. Although the graphs help us visualize the dispersion of possible returns, most investors want to quantify this dispersion using statistical techniques. These statistical measures allow you to compare the return and risk measures for alternative investments directly. Two possible measures of risk (uncertainty) have received support in theoretical work on portfolio theory: the variance and the standard deviation of the estimated distribution of expected returns.

Variance $=\sum_{i=1}^{n}\left(\right.$ probability $(\text { possible rate of return-expected rate of return })^{2}$
Standard Deviation is the square root of the variance
Std $=$

$$
\begin{gathered}
\sqrt{\sum_{i=1}^{n} P_{i}\left[\mathbb{R}_{i}-E\left(R_{i}\right)\right]^{2}} \\
S t d=\sqrt{\sigma^{2}}
\end{gathered}
$$

.The standard deviation is a probability-weighted average deviation from the expected value, and it gives you an idea of how far above or below the expected value the actual value is likely to be.

Question: Compute the variance and Standard deviation for the three examples above
Ans: 1.The variance for the perfect-certainty example would be: 0
2.

| Probability of <br> Possible Return $\left(\boldsymbol{P}_{\boldsymbol{i}}\right)$ | Possible Return <br> $\left(\boldsymbol{R}_{\boldsymbol{i}}\right)$ | $\boldsymbol{P}_{\boldsymbol{i}} \boldsymbol{R}_{\boldsymbol{i}}$ |
| :---: | :---: | :---: |
| 0.15 | 0.20 | 0.03 |
| 0.15 | -0.20 | -0.03 |
| 0.70 | 0.10 | 0.07 |
|  |  | $\sum=0.07$ |

This gives an expected return $[E(R i)]$ of 7 percent. The dispersion of this distribution as measured by variance is:

| Probability $\left(\boldsymbol{P}_{i}\right)$ | $\operatorname{Return}\left(\boldsymbol{R}_{i}\right)$ | $\boldsymbol{R}_{i}-E\left(\boldsymbol{R}_{i}\right)$ | $\left[\boldsymbol{R}_{i}-E\left(\boldsymbol{R}_{i}\right)\right]^{2}$ | $P_{i}\left[\boldsymbol{R}_{i}-E\left(\boldsymbol{R}_{i}\right)\right]^{2}$ |
| :---: | :---: | :---: | :---: | ---: |
| 0.15 | 0.20 | 0.13 | 0.0169 | 0.002535 |
| 0.15 | -0.20 | -0.27 | 0.0729 | 0.010935 |
| 0.70 | 0.10 | 0.03 | 0.0009 | 0.000630 |
|  |  |  |  | $\sum=0.014100$ |

The variance $\left(\sigma^{2}\right)$ is equal to 0.0141 . The standard deviation is equal to the square root of the variance:

$$
\text { Standard Deviation }\left(\sigma^{2}\right)=\sqrt{\sum_{i=1}^{n} P_{i}\left[R_{i}-E\left(R_{i}\right)\right]^{2}}
$$

Consequently, the standard deviation for the preceding example would be:

$$
\sigma_{i}=\sqrt{0.0141}=0.11874
$$

In this example, the standard deviation is approximately 11.87 percent. Therefore, you could describe this distribution as having an expected value of 7 percent and a standard deviation of 11.87 percent.

## Coefficient of Variation

The variance and standard deviation are absolute measures of dispersion. That is, they can be influenced by the magnitude of the original numbers. To compare series with greatly different values, you need a relative measure of dispersion. A measure of relative dispersion is the coefficient of variation, which is defined as: Because lower risk is preferred to higher risk and higher return is preferred to lower return, an investment with a lower coefficient of variation, CV , is preferred to an investment with a higher CV

## Coefficient of Variation $(C V)=\frac{\text { Standard Deviation of Returns }}{\text { Expected Rate of Return }}$

A larger value indicates greater dispersion relative to the arithmetic mean of the series. For the previous example, the $C V$ would be:
$C V_{1}=\frac{0.0756}{0.0400}=1.89$

It is possible to compare this value to a similar figure having a markedly different distribution. As an example, assume you wanted to compare this investment to another investment that had an average rate of return of 10 percent and a standard deviation of 9 percent. The standard deviations alone tell you that the second series has greater dispersion ( 9 percent versus 7.56 percent) and might be considered to have higher risk. In fact, the relative dispersion for this second investment is much less.

$$
\begin{aligned}
& C V_{1}=\frac{0.0756}{0.0400}=1.89 \\
& C V_{2}=\frac{0.0900}{0.1000}=0.90
\end{aligned}
$$

Considering the relative dispersion and the total distribution, most investors would probably prefer the second investment.

### 1.2.5 Measuring the Risk of Historical Rates of Return

In many instances, you might want to compute the variance or standard deviation for a historical series in order to evaluate the past performance of the investment. To measure the risk for a series of historical rates of returns, we use the same measures as for expected returns (variance and standard deviation) except that we consider the historical holding period yields (HPYs) as follows:

$$
\sigma^{2}=\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[H P Y_{\mathrm{i}}-\mathrm{E}(\mathrm{HPY})^{2 / \mathrm{n}}\right.
$$

Assume that you are given the following information on annual rates of return (HPY) for common stocks listed on the New York Stock Exchange (NYSE):

| Year | Annual Rate <br> of Return |
| :---: | :---: |
| 2003 | 0.07 |
| 2004 | 0.11 |
| 2005 | -0.04 |
| 2006 | 0.12 |
| 2007 | -0.06 |

In this case, we are not examining expected rates of return but actual returns. Therefore, we assume equal probabilities, and the expected value (in this case the mean value, $R$ ) of the series is the sum of the individual observations in the series divided by the number of observations, or 0.04 (0.20/5). The variances and standard deviations are:

| Year | $R_{i}$ | $R_{i}-\bar{R}$ | $\left(\boldsymbol{R}_{i}-\bar{R}\right)^{2}$ |  |
| :--- | ---: | ---: | ---: | :---: |
| 2003 | 0.07 | 0.03 | 0.0009 | $\sigma^{2}=0.0286 / 5$ |
| 2004 | 0.11 | 0.07 | 0.0049 | $=0.00572$ |
| 2005 | -0.04 | -0.08 | 0.0064 |  |
| 2006 | 0.12 | 0.08 | 0.0064 | $\sigma=\sqrt{0.00572}$ |
| 2007 | -0.06 | -0.10 | $\Sigma=\frac{0.0110}{0.0286}$ | $=0.0756$ |
|  |  |  | $\Sigma$ |  |

We can interpret the performance of NYSE common stocks during this period of time by saying that the average rate of return was 4 percent and the standard deviation of annual rates of return was 7.56 percent.

### 1.3 DETERMINANTS OF REQUIRED RATES OF RETURN

Recall that the selection process of an investment involves finding securities that provide a rate of return that compensates you for: (1) the time value of money during the period of investment, (2) the expected rate of inflation during the period, and (3) the risk involved.

The summation of these three components is called the required rate of return. This is the minimum rate of return that you should accept from an investment to compensate you for deferring consumption. Because of the importance of the required rate of return to the total investment selection process, this section contains a discussion of the three components and what influences each of them.

The analysis and estimation of the required rate of return are complicated by the behavior of market rates over time. First, a wide range of rates is available for alternative investments at any time. Second, the rates of return on specific assets change dramatically over time. Third, the difference between the rates available (that is, the spread) on different assets changes over time.

The real risk-free rate (RRFR) is the basic interest rate, assuming no inflation and no uncertainty about future flows. An investor in an inflation-free economy who knew with certainty what cash flows he or she would receive at what time would demand the RRFR on an investment. Earlier, we called this the pure time value of money, because the only sacrifice the investor made was deferring the use of the money for a period of time. This RRFR of interest is the price charged for the exchange between current goods and future goods.

Two factors, one subjective and one objective, influence this exchange price. The subjective factor is the time preference of individuals for the consumption of income. When individuals give up $\$ 100$ of consumption this year, how much consumption do they want a year from now to compensate for that sacrifice? The strength of the human desire for current consumption influences the rate of compensation required. Time preferences vary among individuals, and the market creates a composite rate that includes the preferences of all investors. This composite rate
changes gradually over time because it is influenced by all the investors in the economy, whose changes in preferences may offset one another.

The objective factor that influences the RRFR is the set of investment opportunities available in the economy. The investment opportunities are determined in turn by the long-run real growth rate of the economy. A rapidly growing economy produces more and better opportunities to invest funds and experience positive rates of return. A change in the economy's long-run real growth rate causes a change in all investment opportunities and a change in the required rates of return on all investments. Just as investors supplying capital should demand a higher rate of return when growth is higher; those looking for funds to invest should be willing and able to pay a higher rate of return to use the funds for investment because of the higher growth rate. Thus, a positive relationship exists between the real growth rate in the economy and the RRFR.

## Factors Influencing the Nominal Risk-Free Rate (NRFR)

when we discuss rates of interest, we need to differentiate between real rates of interest that adjust for changes in the general price level, as opposed to nominal rates of interest that are stated in money terms. That is, nominal rates of interest that prevail in the market are determined by real rates of interest, plus factors that will affect the nominal rate of interest, such as the expected rate of inflation and the monetary environment.

Nominal RFR $=(1+$ Real RFR $) x(1+$ Expected Rate of Inflation $)-1$
Rearranging the formula, you can calculate the RRFR of return on an investment as follows:

$$
\left[\frac{(1+\text { Nominal RFR })}{(1+\text { Rate of Inflation })}\right]-1
$$

Example: assume that the nominal return on U.S. government T-bills was 9 percent during a given year, when the rate of inflation was 5 percent. What would be the RRFR of return on this T-bills?

$$
\begin{aligned}
\mathrm{RRFR} & =[(1+0.09) /(1+0.05)]-1 \\
& =1.038-1 \\
& =0.038=3.8 \%
\end{aligned}
$$

## Risk Premium

A risk-free investment was defined as one for which the investor is certain of the amount and timing of the expected returns. The returns from most investments do not fit this pattern. An investor typically is not completely certain of the income to be received or when it will be received. Investments can range in uncertainty from basically risk-free securities, such as T-bills,
to highly speculative investments, such as the common stock of small companies engaged in high-risk enterprises.

Most investors require higher rates of return on investments if they perceive that there is any uncertainty about the expected rate of return. This increase in the required rate of return over the NRFR (Nominal Risk Free Rate) is the risk premium (RP). Although the required risk premium represents a composite of all uncertainty, it is possible to consider several fundamental sources of uncertainty.

## Major Sources of Uncertainty:

- business risk,
- financial risk (leverage),
- liquidity risk,
- exchange rate risk, and
- Country (political) risk.
. The standard deviation of returns is referred to as a measure of the security's total risk, which considers the individual stock by itself-that is, it is not considered as part of a portfolio.

Risk Premium $=f$ (Business Risk, Financial Risk, Liquidity Risk, Exchange Rate Risk, Country Risk)
Markowitz and Sharpe indicated that investors should use an external market measure of risk. Under a specified set of assumptions, all rational, profit maximizing investors want to hold a completely diversified market portfolio of risky assets, and they borrow or lend to arrive at a risk level that is consistent with their risk preferences. Under these conditions, the relevant risk measure for an individual asset is its co movement with the market portfolio. This co movement, which is measured by an asset's covariance with the market portfolio, is referred to as an asset's systematic risk, the portion of an individual asset's total variance attributable to the variability of the total market portfolio. In addition, individual assets have variance that is unrelated to the market portfolio (that is, it is nonmarket variance) that is due to the asset's unique features. This nonmarket variance is called unsystematic risk, and it is generally considered unimportant because it is eliminated in a large, diversified portfolio. Therefore, under these assumptions, the risk premium for an individual earning asset is a function of the asset's systematic risk with the aggregate market portfolio of risky assets.

The measure of an asset's systematic risk is referred to as its beta:

## Risk Premium $=f$ (Systematic Market Risk)

## Fundamental Risk versus Systematic Risk

Some might expect a conflict between the market measure of risk (systematic risk) and the fundamental determinants of risk (business risk, and so on). A number of studies have examined the relationship between the market measure of risk (systematic risk) and accounting variables used to measure the fundamental risk factors, such as business risk, financial risk, and liquidity risk.

The authors of these studies have generally concluded that a significant relationship exists between the market measure of risk and the fundamental measures of risk.

Therefore, the two measures of risk can be complementary. This consistency seems reasonable because, in a properly functioning capital market, the market measure of the risk should reflect the fundamental risk characteristics of the asset. As an example, you would expect a firm that has high business risk and financial risk to have an above average beta.

It is possible that a firm that has a high level of fundamental risk and a large standard deviation of return on stock can have a lower level of systematic risk because its variability of earnings and stock price is not related to the aggregate economy or the aggregate market. Therefore, one can specify the risk premium for an asset as:

Risk Premium $=f$ (Business Risk, Financial Risk, Liquidity Risk, Exchange Rate Risk, Country Risk)

> or
> Risk Premium $=f($ Systematic Market Risk $)$

### 1.4 RELATIONSHIP BETWEEN RISK AND RETURN

Exhibit 1.4 graphs the expected relationship between risk and return. It shows that investors increase their required rates of return as perceived risk (uncertainty) increases. The line that reflects the combination of risk and return available on alternative investments is referred to as the security market line (SML). The SML reflects the risk-return combinations available for all risky assets in the capital market at a given time. Investors would select investments that are consistent with their risk preferences; some would consider only low-risk investments, whereas others welcome high-risk investments. Beginning with an initial SML, three changes can occur. First, individual investments can change positions on the SML because of changes in the perceived risk of the investments. Second, the slope of the SML can change because of a change in the attitudes of investors toward risk; that is, investors can change the returns they require per unit of risk. Third, the SML can experience a parallel shift due to a change in the RRFR or the expected rate of inflation-that is, a change in the NRFR. These three possibilities are discussed in this section.

## Movements along the SML

Investors place alternative investments somewhere along the SML based on their perceptions of the risk of the investment. Obviously, if an investment's risk changes due to a change in one of its risk sources (business risk, and such), it will move along the SML. For example, if a firm increases its financial risk by selling a large bond issue that increases its financial leverage, investors will perceive its common stock as riskier and the stock will move up the SML to a higher risk position. Investors will then require a higher rate of return. As the common stock becomes riskier, it changes its position on the SML. Any change in an asset that affects its fundamental risk factors or its market risk (that is, its beta) will cause the asset to move along the SML as shown in Exhibit. Note that the SML does not change, only the position of assets on the SML.

## Exhibit 1.4 Relationship between risk and return

RELATIONSHIP BETWEEN RISK AND RETURN


## Changes in the Slope of the SML

The slope of the SML indicates the return per unit of risk required by all investors. Assuming a straight line, it is possible to select any point on the SML and compute a risk premium (RP) for an asset through the equation:
$R P_{i}=E\left(R_{i}\right)-\mathrm{NRFR}$
where:
$R P_{i}=$ risk premium for asset $i$
$E\left(R_{i}\right)=$ the expected return for asset $i$
NRFR $=$ the nominal return on a risk-free asset

If a point on the SML is identified as the portfolio that contains all the risky assets in the market (referred to as the market portfolio), it is possible to compute a market RP as follows:

$$
R P_{m}=E\left(R_{m}\right)-\mathrm{NRFR}
$$

where:

$$
\begin{aligned}
R P_{m} & =\text { the risk premium on the market portfolio } \\
E\left(R_{m}\right) & =\text { the expected return on the market portfolio } \\
\text { NRFR } & =\text { the nominal return on a risk-free asset }
\end{aligned}
$$

This market RP is not constant because the slope of the SML changes over time. Although we do not understand completely what causes these changes in the slope, we do know that there are changes in the yield differences between assets with different levels of risk even though the inherent risk differences are relatively constant.

These differences in yields are referred to as yield spreads, and this yield spreads change over time. As an example, if the yield on a portfolio of Aaa-rated bonds is 7.50 percent and the yield on a portfolio of Baa-rated bonds is 9.00 percent, we would say that the yield spread is 1.50 percent. This 1.50 percent is referred to as a credit risk premium because the Baa-rated bond is considered to have higher credit risk-that is, greater probability of default. This Baa-Aaa yield spread is not constant over time. For an example of changes in a yield spread, note the substantial changes in the yield spreads on Aaa-rated bonds and Baa-rated bonds shown in Exhibit 1.5

## Exhibit 1.5

Changes in the required rate of return due to movements along the sml



Although the underlying risk factors for the portfolio of bonds in the Aaa-rated bond index and the Baa-rated bond index would probably not change dramatically over time, it is clear from the time-series plot in Exhibit that the difference in yields (i.e., the yield spread) has experienced changes of more than 100 basis points ( 1 percent) in a short period of time (for example, see the yield spread increase in 1974 to 1975 and the dramatic yield spread decline in 1983 to 1984). Such a significant change in the yield spread during a period where there is no major change in the risk characteristics of Baa bonds relative to Aaa bonds would imply a change in the market RP. Specifically, although the risk levels of the bonds remain relatively constant, investors have changed the yield spreads they demand to accept this relatively constant difference in risk.

This change in the RP implies a change in the slope of the SML. Such a change is shown in Exhibit. The exhibit assumes an increase in the market risk premium, which means an increase in the slope of the market line. Such a change in the slope of the SML (the risk premium) will affect the required rate of return for all risky assets. Irrespective of where an investment is on the original SML, its required rate of return will increase, although its individual risk characteristics remain unchanged.

## Changes in Capital Market Conditions or Expected Inflation

The graph in Exhibit shows what happens to the SML when there are changes in one of the following factors:
(1) Expected real growth in the economy,
(2) Capital market conditions, or
(3) The expected rate of inflation.

For example, an increase in expected real growth, temporary tightness in the capital market, or an increase in the expected rate of inflation will cause the SML to experience a parallel shift upward. The parallel shift occurs because changes in expected real growth or in capital market conditions or a change in the expected rate of inflation affect all investments, no matter what their levels of risk are.

CAPITAL MARKET CONDITIONS, EXPECTED INFLATION, AND THE SECURITY MARKET LINE


## Chapter two: investment valuation

## Stock valuation

The value of an asset is the present value of its expected returns. Specifically, you expect an asset to provide a stream of returns during the period of time you own it. To convert this estimated stream of returns to a value for the security, you must discount this stream at your required rate of return. This process of valuation requires estimates of
(1) The stream of expected returns and
(2) The required rate of return on the investment.

An estimate of the expected returns from an investment encompasses not only the size but also the form, time pattern, and the uncertainty of returns, which affect the required rate of return.
Form of Returns :The returns from an investment can take many forms, including earnings, cash flows, dividends, interest payments, or capital gains (increases in value) during a period. We will consider several alternative valuation techniques that use different forms of returns. As an example, one common stock valuation model applies a multiplier to a firm's earnings, whereas another valuation model computes the present value of a firm's operating cash flows, and a third model estimates the present value of dividend payments. Returns or cash flows can come in many forms, and you must consider all of them to evaluate an investment accurately.

Time Pattern and Growth Rate of Returns: You cannot calculate an accurate value for a security unless you can estimate when you will receive the returns or cash flows. Because money has a time value, you must know the time pattern and growth rate of returns from an investment. This knowledge will make it possible to properly value the stream of returns relative to alternative investments with a different time pattern and growth rate of returns or cash flows.

Uncertainty of Returns (Cash Flows) You : required rate of return on an investment is determined by (1) the economy's real risk-free rate of return, plus (2) the expected rate of inflation during the holding period, plus (3) a risk premium that is determined by the uncertainty of returns. All investments are affected by the risk-free rate and the expected rate of inflation because these two variables determine the nominal risk-free rate. Therefore, the factor that causes a difference in required rates of return is the risk premium for alternative investments. In turn, this risk premium depends on the uncertainty of returns or cash flows from an investment. We can identify the sources of the uncertainty of returns by the internal characteristics of assets or by market-determined factors. Earlier, we subdivided the internal characteristics for a firm into business risk (BR), financial risk (FR), liquidity risk (LR), exchange rate risk (ERR), and country risk (CR). The market-determined risk measures are the systematic risk of the asset, its beta, or its multiple APT factors.

To ensure that you receive your required return on an investment, you must estimate the intrinsic value of the investment at your required rate of return and then compare this estimated intrinsic value to the prevailing Market price. You should not buy an investment if its market price exceeds your estimated value because the difference will prevent you from receiving your required rate of return on the investment. In contrast,
If the estimated value of the investment exceeds the market price, you should buy the investment. In summary:
If Estimated Value > Market Price, Buy
If Estimated Value $<$ Market Price, Don't Buy

For example, assume you read about a firm that produces athletic shoes and its stock is listed on the NYSE. Using one of the valuation models we will discuss and making estimates of earnings, cash flow, and growth based on the company's annual report and other information, you estimate the company's stock value using your required rate of return as $\$ 20$ a share. After estimating this value, you look in the paper and see that the stock is currently being traded at $\$ 15$ a share. You would want to buy this stock because you think it is worth $\$ 20$ a share and you can buy it for $\$ 15$ a share. In contrast, if the current market price was $\$ 25$ a share, you would not want to buy the stock because, based upon your valuation, it is overvalued.
The theory of value provides a common framework for the valuation of all investments. Different applications of this theory generate different estimated values for alternative investments because of the different payment streams and characteristics of the securities. The interest and principal payments on a bond differ substantially from the expected dividends and future selling price for a common stock. The initial discussion that follows applies the discounted cash flow method to bonds, preferred stock, and common stock. This presentation demonstrates that the same basic model is useful across a range of investments. Subsequently, because of the difficulty in estimating the value of common stock, we consider two general approaches and numerous techniques for the valuation of stock.

## Approaches to Equity Valuation

Because of the complexity and importance of valuing common stock, various techniques for accomplishing this task have been devised over time. These techniques fall into one of two general approaches:
(1) the discounted cash flow valuation techniques, where the value of the stock is estimated based upon the present value of some measure of cash flow, including dividends, operating cash flow, and free cash flow; and
(2) The relative valuation techniques, where the value of a stock is estimated based upon its current price relative to variables considered to be significant to valuation, such as earnings, cash flow, book value, or sales.

## 1. Discounted Cash Flow Techniques(absolute valuation model)

- Present Value of Dividends (DDM)
- Present Value of Operating Free Cash Flow
- Present Value of Free Cash Flow to Equity


## 2. Relative Valuation Techniques

- Price/Earnings Ratio (P/E)
- Price/Cash Flow Ratio (P/CF)
- Price/Book Value Ratio (P/BV)
- Price/Sales Ratio (P/S)

An important point is that both of these approaches and all of these valuation techniques have several common factors.
-First, all of them are significantly affected by the investor's required rate of return on the stock because this rate becomes the discount rate or is a major component of the discount rate.
-Second, all valuation approaches are affected by the estimated growth rate of the variable used in the valuation technique-for example, dividends, earnings, cash flow, or sales. both of these critical variables must be estimated. As a result, different analysts using the same valuation techniques will derive different estimates of value for a stock because they have different estimates for these critical variable inputs. The following discussion of equity valuation techniques considers the specific models and the theoretical and practical strengths and weaknesses of each of them. Notably, the authors' intent is to present these two approaches as complementary, not competitive, approaches - that is, you should learn and use both of them.
Why and When to Use the Discounted Cash Flow Valuation Approach
These discounted cash flow valuation techniques are obvious choices for valuation because they are the epitome of how we describe value-that is, the present value of expected cash flows. The major difference between the alternative techniques is how one specifies cash flow-that is, the measure of cash flow used.
The cleanest and most straightforward measure of cash flow is dividends because these are clearly cash flows that go directly to the investor, which implies that you should use the cost of equity as the discount rate. However, this dividend technique is difficult to apply to firms that do not pay dividends during periods of high growth, or that currently pay very limited dividends because they have high rate of return investment alternatives available. On the other hand, an advantage is that the reduced form of the dividend discount model (DDM) is very useful when discussing valuation for a stable, mature entity where the assumption of relatively constant growth for the long term is appropriate.
The second specification of cash flow is the operating free cash flow, which is generally described as cash flows after direct costs (cost of goods and S, G \& A expenses) and before any payments to capital suppliers. Because we are dealing with the cash flows available for all capital suppliers, the discount rate employed is the firm's weighted average cost of capital (WACC).

This is a very useful model when comparing firms with diverse capital structures because you determine the value of the total firm and then subtract the value of the firm's debt obligations to arrive at a value for the firm's equity.
The third cash flow measure is free cash flow to equity, which is a measure of cash flows available to the equity holder after payments to debt holders and after allowing for expenditures to maintain the firm's asset base. Because these are cash flows available to equity owners, the appropriate discount rate is the firm's cost of equity.
Beyond being theoretically correct, these models allow a substantial amount of flexibility in terms of changes in sales and expenses that implies changing growth rates over time. Once you understand how to compute each measure of cash flow, you can estimate cash flow for each year by constructing a pro forma statement for each year or you can estimate overall growth rates for the alternative cash flow values as we will demonstrate with the DDM.
A potential difficulty with these cash flow techniques is that they are very dependent on the two significant inputs-(1) the growth rates of cash flows (both the rate of growth and the duration of growth) and (2) the estimate of the discount rate. As we will show in several instances, small change in either of these values can have a significant impact on the estimated value. This is a critical realization when using any theoretical model: Everyone knows and uses the same model, but it is the inputs that are critical-GIGO: garbage in, garbage out!

## Why and When to Use the Relative Valuation Techniques

As noted, a potential problem with the discounted cash flow valuation models is that it is possible to derive intrinsic values that are substantially above or below prevailing prices depending on how you adjust your estimated inputs to the prevailing environment.
An advantage of the relative valuation techniques is that they provide information about how the market is currently Valuing stock at several levels-that is, the aggregate market, alternative industries, and individual stocks within industries. Following this chapter, which provides the background for these two approaches, we will demonstrate the alternative relative valuation ratios for the aggregate market, for an industry relative to the market, and for an individual company relative to the aggregate market, to its industry, and to other stocks in its industry.
The good news is that this relative valuation approach provides information on how the market is currently valuing securities. The bad news is that it is providing information on current valuation. The point is, the relative valuation approach provides this information on current valuation, but it does not provide guidance on whether these current valuations are appropriate-that is, all valuations at a point in time could be too high or too low. For example, assume that the market becomes significantly overvalued. For example, if you compare the value for an industry to the very overvalued market, you might contend based on such a comparison that an industry is undervalued relative to the market. Unfortunately, your judgment may be wrong because of the benchmark you are using-that is, you might be comparing a fully valued industry to a very overvalued market. Put another way, the relative valuation techniques are appropriate to consider under two conditions:

1. You have a good set of comparable entities-that is, comparable companies that are similar in terms of industry, size, and, it is hoped, risk.
2. The aggregate market and the company's industry are not at a valuation extreme-that is, they are not either seriously undervalued or overvalue.

## Discounted Cash Flow Valuation Techniques

All of these valuation techniques are based on the basic valuation model, which asserts that the value of an asset is the present value of its expected future cash flows as follows:

$$
\mathrm{V}=\sum_{\mathrm{I}=1}^{\mathrm{n}} C F /(1+k) \mathrm{t}
$$

Where:
$\mathbf{V}=$ value of stock j
$\mathrm{n}=$ life of the asset
$\mathrm{CF}=$ cash flow in period t
$\mathrm{k}=$ the discount rate that is equal to the investors' required rate of return for asset j , which is determined by the uncertainty (risk) of the stock's cash flows
As noted, the specific cash flows used will differ between techniques. They range from dividends (the best-known model) to operating free cash flow and free cash flow to equity. We begin with a fairly detailed presentation of the present-value-of-dividend model, referred to as the dividend discount model (DDM), because it is intuitively appealing and is the best-known model. Also, its general approach is similar to the other discounted cash flow models.

## The Dividend Discount Model (DDM):

The dividend discount model assumes that the value of a share of common stock is the present value of all future dividends as follows:

$$
\mathrm{Vi}=\mathrm{D} 1 /(1+\mathrm{k})+\mathrm{d} 2 /(\mathrm{k}+1)^{2}+\mathrm{D} 3 /(1+\mathrm{k})^{2}+\ldots . .+\mathrm{Dn} /(1+\mathrm{k})^{\mathrm{n}}=\sum_{\mathrm{I}=1}^{\mathrm{n}} \underline{\mathrm{Dt}}(1+\mathrm{k})^{\mathrm{n}}
$$

Where:
$\mathbf{V j}=$ value of common stock j
$\mathrm{Dt}=$ dividend during period t
$\mathrm{k}=$ required rate of return on stock
What happens when the stock is not held for an infinite period? A sale of the stock at the end of Year 2 would imply the following formula:
$\mathrm{Vj}=\mathrm{D} 1 /(1+\mathrm{k})+\mathrm{D} 2 /(1+\mathrm{k}) 2+\mathrm{SPt} 2 /(1+\mathrm{k}) 2$

## One-Year Holding Period model

Assume an investor wants to buy the stock, hold it for one year, and then sell it. To determine the value of the stock-that is, how much the investor should be willing to pay for it-using the DDM, we must estimate the dividend to be received during the period, the expected sale price at the end of the holding period, and the investor's required rate of return. To estimate the dividend for the coming year, adjust the current dividend for expectations regarding the change in the dividend during the year. Assume the company we are analyzing earned $\$ 2.50$ a share last year and paid a dividend of $\$ 1$ a share. Assume further the firm has been fairly consistent in maintaining this 40 percent payout over time. The consensus of financial analysts is that the firm will earn about $\$ 2.75$ during the coming year and will raise its dividend to $\$ 1.10$ per share.

A crucial estimate is the expected selling price for the stock a year from now. You can estimate this expected selling price by either of two alternative procedures. In the first, you can apply the dividend discount model where you estimate the specific dividend payments for a number of years into the future and calculate the value of the stock from these estimates. In the second procedure, the earnings multiplier model, you multiply the future expected earnings for the stock by an earnings multiple, which you likewise estimate, to find an expected sale price. We will discuss the earnings multiple model in a later section of the chapter. For now, assume you prefer the DDM. Applying this model, you project the sale price of this stock a year from now to be \$22.

Finally, you must determine the required rate of return. As discussed before, the nominal riskfree rate is determined by the real risk-free rate and the expected rate of inflation.
A good proxy for this rate is the promised yield on one-year government bonds because your investment horizon (expected holding period) is one year. You estimate the stock's risk premium by comparing its risk level to the risk of other potential investments. In later chapters, we discuss how you can estimate this risk. For the moment, assume that one-year government bonds are yielding 10 percent, and you believe that a 4 percent risk premium over the yield of these bonds is appropriate for this stock. Thus, you specify a required rate of return of 14 percent.
In summary, you have estimated the dividend at $\$ 1.10$ (payable at year end), an ending sale price of $\$ 22$, and a required rate of return of 14 percent. Given these inputs, you would estimate the value of this stock as follows.

$$
V_{1} \underset{(1+0.14)}{=\$ 1.1}+\frac{\$ 22}{(1+0.14)}=0.09+19.30=20.26 \%
$$

Note that we have not mentioned the current market price of the stock. This is because the market price is not relevant to you as an investor except as a comparison to the independently derived value based on your estimates of the relevant variables. Once we have calculated the stock's value as $\$ 20.26$, we can compare it to the market price and apply the investment decision
rule: If the stock's market price is more than $\$ 20.26$, do not buy; if it is equal to or less than $\$ 20.26$, buy.

## Multiple-Year Holding Period model

If you anticipate holding the stock for several years and then selling it, the valuation estimate is harder. You must forecast several future dividend payments and estimate the sale price of the stock several years in the future.

The difficulty with estimating future dividend payments is that the future stream can have numerous forms. The exact estimate of the future dividends depends on two projections.
The first is your outlook for earnings growth because earnings are the source of dividends.
The second projection is the firm's dividend policy, which can take several forms. A firm can have a constant percent payout of earnings each year, which implies a change in dividend each year, or the firm could follow a step pattern in which it increases the dividend rate by a constant dollar amount each year or every two or three years. The easiest dividend policy to analyze is one where the firm enjoys a constant growth rate in earnings and maintains a constant dividend payout. This set of assumptions implies that the dividend stream will experience a constant growth rate that is equal to the earnings growth rate.
Assume the expected holding period is three years, and you estimate the following dividend payments at the end of each year:

| year | Dividend per share |
| :--- | :--- |
| 1 | $\$ 1.1$ |
| 2 | $\$ 1.20$ |
| 3 | $\$ 1.35$ |

The next estimate is the expected sale price (SP) for the stock three years in the future. Again, if we use the DDM for this estimate, you would need to project the dividend growth pattern for this stock beginning three years from now. Assume an estimated sale price using the DDM of $\$ 34$. The final estimate is the required rate of return on this stock during this period. Assuming the 14 percent required rate is still appropriate, the value of this stock is

$$
\begin{aligned}
& V=\frac{1.10}{(1.14)}+\frac{1.20}{(1.14) 2}+\frac{1.35}{(1.14) 3}+\frac{34}{(1.14) 3} \\
& =0.96+0.92+0.91+22.95=25.74
\end{aligned}
$$

Again, to make an investment decision, you would compare this estimated value for the stock to its current market price to determine whether you should buy.
At this point, you should recognize that the valuation procedure discussed here is similar to that used in corporate finance when making investment decisions, except that the cash flows are from dividends instead of returns to an investment project. Also, rather than estimating the scrap value or salvage value of a corporate asset, we are estimating the ending sale price for the stock. Finally, rather than discounting cash flows using the firm's cost of capital; we use the individual's required rate of return on the company's equity. In both cases, we are looking for excess present value, which means that the present value of expected cash inflows-that is, the estimated intrinsic value of the asset-exceeds the present value of cash outflows, which is the market price of the asset.

## Infinite Period Model

We can extend the multiple period models by extending our estimates of dividends 5,10 , or 15 years into the future. The benefits derived from these extensions would be minimal, however, and you would quickly become bored with this exercise. Instead, we will move to the infinite period dividend discount model, which assumes investors estimate future dividend payments for an infinite number of periods.

Needless to say, this is a formidable task! We must make some simplifying assumptions about this future stream of dividends to make the task viable.

## A. Constant Dividend growth model(Gordon Valuation Model)

The first assumption is that the future dividend stream will grow at a constant rate for an infinite period. This is a rather heroic assumption in many instances, but where it does hold, we can use the model to value individual stocks as well as the aggregate market and alternative industries. This model is generalized as follows
$\mathrm{Vj}=\frac{\mathrm{Do}(1+\mathrm{g})}{(1+\mathrm{k})}+\frac{\mathrm{Do}(1+\mathrm{g})^{2}}{(1+\mathrm{k})^{2}}+\ldots+\frac{\mathrm{Do}(1+\mathrm{g})^{\mathrm{n}}}{(1+\mathrm{k})^{\mathrm{n}}}$

Where:
$\mathrm{Vj}=$ the value of stock j
D0=the dividend payment in the current period
$\mathrm{g}=$ the constant growth rate of dividends
$\mathrm{k}=$ the required rate of return on stock j
$\mathrm{n}=$ the number of periods, which we assume to be infinite
In the appendix to this chapter, we show that with certain assumptions, this infinite period constant growth rate model can be simplified to the following expression:

$$
\mathrm{Vj}=\mathrm{D} 1 /(\mathrm{k}-\mathrm{g})
$$

You will probably recognize this formula as one that is widely used in corporate finance to estimate the cost of equity capital for the firm-that is, $\mathrm{k}=\mathrm{D} /(\mathrm{V}+\mathrm{g})$.
To use this model for valuation, you must estimate (1) the required rate of return (k) and (2) The expected constant growth rate of dividends (g). After estimating $\mathbf{g}$, it is a simple matter to estimate D1, because it is the current dividend (D0) times ( $\mathbf{1}+\mathbf{g}$ ).
Consider the example of a stock with a current dividend of $\$ 1$ a share, which you expect to rise to $\$ 1.09$ next year. You believe that, over the long run, this company's earnings and dividends will continue to grow at 9 percent; therefore, your estimate of $\mathbf{g}$ is 0.09 . For the long run, you expect the rate of inflation to decline, so you set your long-run required rate of return on this stock at 13 percent; your estimate of k is 0.13 . To summarize the relevant estimate:

$$
\begin{gathered}
\mathrm{g}=0.09, \mathrm{k}=0.13 \\
\mathrm{D} 1=1.09(\$ 1.00 \times 1.09)=1.09 \\
\mathrm{~V}=\frac{1.09}{(0.13-0.09)}=27.25
\end{gathered}
$$

Eg 2.Suppose an investor is considering the purchase of a share of the SA-AN Company. The stock will pay a $\$ 3$ dividend a year from today. This dividend is expected to grow at 10 percent per year ( $\mathrm{g}=10 \%$ ) for the foreseeable future. The investor thinks that the required return (r) on this stock is 15 percent. What is the value of a share of SA-AN Company's stock?
Using the constant growth formula the value is $\$ 60$ :

$$
V=3 /(0.15-0.10)=\$ 60
$$

A small change in any of the original estimates will have a large impact on $\mathbf{V}$,as shown by the following examples:

1. $\mathrm{g}=0.09 ; \mathrm{k}=0.14 ; \mathrm{D} 1=\$ 1.09$. (We assume an increase in k .)
$\mathrm{V}=\$ 1.09 /(0.14-.09)=\$ 21.80$
2. $\mathrm{g}=0.10 ; \mathrm{k}=0.13 ; \mathrm{D} 1=\$ 1.10$. (We assume an increase in g .)
$\mathrm{V}=\$ 1.10(0.13-0.1)=1.1 / 0.03=\$ 36.67$

These examples show that as small a change as 1 percent in either g or k produces a large difference in the estimated value of the stock. The crucial relationship that determines the value of the stock is the spread between the required rate of return (k) and the expected growth rate of dividends (g). Anything that causes a decline in the spread will cause an increase in the computed value, whereas any increase in the spread will decrease the computed value of the stock.

## B. Zero growth rate

The formula for the present value of a stock with zero growth is dividends per period divided by the required return per period. Since future cash flows are constant, the value of a zero growth stock is the present value of perpetuity.

It Assumes dividends will remain at the same level forever

$$
\begin{aligned}
& \mathrm{V}=\mathrm{D} 1 /(1+\mathrm{k})^{1}+\mathrm{D} 2 /(1+\mathrm{k})^{2}+\mathrm{D} 3 /(1+\mathrm{k})^{3} \\
& \mathrm{~V}=\mathrm{D} / \mathrm{K} \\
& \text { Where } \mathbf{K}=\text { require rate of return } \\
& \mathrm{D}=\text { dividend }
\end{aligned}
$$

Eg, suppose an investor is considering the purchase of a share of the DINO Company. The company paid a $\$ 4$ dividend a year this dividend is expected to remain constant for the foreseeable future. The investor thinks that the required return (r) on this stock is 15 percent.

$$
\mathrm{V}=\mathrm{D} / \mathrm{k}=\$ 4 / 0.15=\$ 26.67
$$

## C. Infinite Period DDM and Growth Companies

As noted in the appendix, the infinite period DDM has the following assumptions:

1. Dividends grow at a constant rate.
2. The constant growth rate will continue for an infinite period.
3. The required rate of return (k) is greater than the infinite growth rate (g).If it is not, the model gives meaningless results because the denominator becomes negative.
What is the effect of these assumptions if you want to use this model to value the stock of growth companies Growth companies are firms that have the opportunities and abilities to earn rates of return on investments that are consistently above their required rates of return.
. To exploit these outstanding investment opportunities, these growth firms generally retain a high percentage of earnings for reinvestment, and their earnings will grow faster than those of the typical firm. A firm's sustainable growth is a function of its retention rate and its return on equity (ROE).
First, the infinite period DDM assumes dividends will grow at a constant rate for an infinite period. This assumption seldom holds for companies currently growing at above average rates. . It is unlikely that they can maintain such extreme rates of growth because of the inability to continue earning the ROEs implied by this growth for an infinite period in an economy where other firms will compete with them for these high rates of return.
Second, during the periods when these firms experience abnormally high rates of growth, their rates of growth probably exceed their required rates of return. There is no automatic relationship between growth and risk; a high-growth company is not necessarily a high-risk company. In fact, a firm growing at a high constant rate would have lower risk (less uncertainty) than a low-growth firm with an unstable earnings pattern.
In summary, some firms experience periods of abnormally high rates of growth for some finite periods of time. The infinite period DDM cannot be used to value these true growth firms because these high-growth conditions are temporary and therefore inconsistent with the assumptions of the DDM. In the following section, we discuss how to supplement the DDM to value a firm with temporary supernormal growth .
Valuation with Temporary Supernormal Growth (Variable-Growth Mode)

Thus far, we have considered how to value a firm with different growth rates for short periods of time (one to three years) and how to value a stock using a model that assumes a constant growth rate for an infinite period. As noted, the assumptions of the model make it impossible to use the infinite period constant growth model to value true growth companies. A company cannot permanently maintain a growth rate higher than its required rate of return because competition will eventually enter this apparently lucrative business, which will reduce the firm's profit margins and therefore its ROE and growth rate. Therefore, after a few years of exceptional growth-that is, a period of temporary supernormal growth - a firm's growth rate is expected to decline. Eventually its growth rate is expected to stabilize at a constant level consistent with the assumptions of the infinite period DDM.

To determine the value of a temporary supernormal growth company, you must combine the previous models. In analyzing the initial years of exceptional growth, you examine each year individually. If the company is expected to have two or three stages of supernormal growth, you must examine each year during these stages of growth. When the firm's growth rate stabilizes at a rate below the required rate of return, you can compute the remaining value of the firm assuming constant growth using the DDM and discount this lump-sum constant growth value back to the present. The technique should become clear as you work through the following example.
The Bourke Company has a current dividend (D0) of $\$ 2$ a share. The following are the expected annual growth rates for dividends.

| year | Dividend growth rate |
| :--- | :--- |
| $1-3$ | $25 \%$ |
| $4-6$ | $20 \%$ |
| $7-9$ | $15 \%$ |
| 10 on | $9 \%$ |

Computation of value for the stock of a company with temporary supernormal growth

| year | dividend | Discount factor <br> $(14 \%)$ | Present value |
| :--- | :--- | :--- | :--- |
| 1 | 2.5 | 0.8772 | $\$ 2.193$ |


| 2 | 3.12 | 0.7695 | 2.401 |
| :--- | :--- | :--- | :--- |
| 3 | 3.91 | 0.6750 | 2.639 |
| 4 | 4.69 | 0.5921 | 2.777 |
| 5 | 5.63 | 0.5194 | 2.924 |
| 6 | 6.76 | 0.4556 | 3.080 |
| 7 | 7.77 | 0.3996 | 3.105 |
| 8 | 8.94 | 0.3506 | 3.134 |
| 9 | 10.28 | 0.3075 | 3.161 |
| 10 | 11.21 | 224.20 | $\underline{68.941}$ |

Value of dividend stream for Year 10 and all future dividends (that is, $\$ 11.21 /(0.14-0.09)$ $=\$ 224.20$ ).

The discount factor is the ninth-year factor because the valuation of the remaining stream is made at the end of Year 9 to reflect the dividend in Year 10 and all future dividends.
The computations in the above table indicate that the total value of the stock is $\$ 94.36$. As before, you would compare this estimate of intrinsic value to the market price of the stock when deciding whether to purchase the stock. The difficult part of the valuation is estimating the supernormal growth rates and determining how long each of the growth rates will last.
To summarize this section, the initial present value of cash flow stock valuation model considered was the dividend discount model (DDM). After explaining the basic model and the derivation of its reduced form, we noted that the infinite period DDM cannot be applied to the valuation of stock for growth companies because the abnormally high growth rate of earnings for the growth company is inconsistent with the assumptions of the infinite period constant growth DDM model. Subsequently we modified the DDM model to evaluate companies with temporary supernormal growth. In the following sections, we discuss the other present value of cash flow techniques assuming a similar set of scenarios.

## Present Value of Operating Free Cash Flows

In this model, you are deriving the value of the total firm because you are discounting the operating free cash flows prior to the payment of interest to the debt holders but after deducting funds needed to maintain the firm's asset base (capital expenditures). Also, because you are discounting the total firm's operating free cash flow, you would use the firm's weighted average
cost of capital (WACC) as your discount rate. Therefore, once you estimate the value of the total firm, you subtract the value of debt, assuming your goal is to estimate the value of the firm's equity. The total value of the firm is equal to:

$$
\mathrm{Vj}=\sum_{\mathrm{I}=1}^{\mathrm{n}} \underline{\underline{\text { OFCF }}}
$$

## Where:

$\mathrm{Vj}=$ value of firm j
$\mathrm{n}=$ number of periods assumed to be infinite
$\mathrm{OFCFt}=$ the firm's operating free cash flow in period t .
WACC $j=$ firm $j$ 's weighted average cost of capital.

Similar to the process with the DDM, it is possible to envision this as a model that requires estimates for Different period of time as
A , one year holding period
B , multiple year holding periods
C, infinite period. $\downarrow$, Zero growth rate

- Constant growth rate
- Supper normal growth rate

Example1. The following cash flows for the next five years are as follow
WACC=9.94\%
Value of debt=\$800
What is value of equity?

| year | OFCF |
| :--- | :--- |
| 1 | $\$ 90$ |
| 2 | 100 |
| 3 | 108 |
| 4 | 116.2 |
| 5 | 123.49 |
| Terminal cash flow | $2,363.08$ |

If assume that the OFCF has stable growth; you can adapt the infinite period constant growth DDM model as follows.
$\mathrm{VJ}=\frac{\mathrm{OFCF}}{\mathrm{WACC}-\mathrm{g}_{\text {OFCF }}}$

Where:
OFCF1=operating free cash flow in period 1 equal to OFCF0 ( $1+\mathrm{g}$ OFCF)
g OFCF $=$ long-term constant growth rate of operating free cash flow
Alternatively, assuming that the firm is expected to experience several different rates of growth for OFCF, these estimates can be divided into three or four stages, as demonstrated with the temporary supernormal dividend growth model. Similar to the dividend model, the analyst must estimate the rate of growth and the duration of growth for each of these periods of supernormal growth as follows e.g. assume
$\mathrm{OFCF} 0=\$ 50$
WACC=9\%

| YEAR | OFCF GROWTHRATE |
| :--- | :--- |
| $1-4$ | $20 \%$ |
| $5-7$ | 16 |
| $8-10$ | 12 |
| 11 on | 7 |

the calculations of the value would estimate the present value specific OFCFs for each year through Year 10 Based on the expected growth rates, but you would use the infinite growth model estimate when the growth rate reached stability after Year 10.

## Present Value of Free Cash Flows to Equity

The third discounted cash flow technique deals with "free" cash flows to equity, which would be derived after operating free cash flows, have been adjusted for debt payments (interest and principle). Also, these cash flows precede dividend payments to the common stockholder. Such cash flows are referred to as "free" because they are what is left after meeting all obligations to other capital suppliers (debt and preferred stock) and after providing the funds needed to maintain the firm's asset base (similar to operating free cash flow).
Notably, because these are cash flows available to equity owners, the discount rate used is the Firm's cost of equity (k) rather than the firm's WACC

$$
\mathrm{Vj}=\sum^{\mathrm{n}} \quad \underline{\text { FCFEt }}
$$

```
I=1 (1+Kj)
```

Where:
$\mathrm{Vj}=$ value of the stock of firm j
$\mathrm{n}=$ number of periods assumed to be infinite
FCFEt $=$ the firm's free cash flow to equity in period $t$.
Example: the free cash flow to equity (FCFE) for the next five years and the terminal cash flow as follow and cost of equity is $13,625 \%$

| Year | FCFE |
| :--- | :--- |
| 1 | $\$ 50$ |
| 2 | 60 |
| 3 | 68 |
| 4 | 76.5 |
| 5 | 83.49 |
| Terminal cash flow | $1,603.03$ |

The present value calculated using multiple year holding period model
Again, how an analyst would implement this general model depends upon the firm's position in its life cycle. That is, if the firm is expected to experience stable growth, analysts can use the infinite growth model. In contrast, if the firm is expected to experience a period of temporary supernormal growth, analysts should use the multistage growth model similar to the process used with dividends and for operating free cash flow.

## RELATIVE VALUATION TECHNIQUES

In contrast to the various discounted cash flow techniques that attempt to estimate a specific value for a stock based on its estimated growth rates and its discount rate, the relative valuation techniques implicitly contend that it is possible to determine the value of an economic entity (i.e., the market, an industry, or a company) by comparing it to similar entities on the basis of several relative ratios that compare its stock price to relevant variables that affect a stock's value, such as earnings, cash flow, book value, and sales. Therefore, in this section, we discuss the following relative valuation ratios:
(1) price/earnings (P/E), (2) price/cash flow (P/CF), (3) price/book value (P/BV), and price/sales (P/S). We begin with the P/E ratio, also referred to as the earnings multiplier model, because it is the most popular relative valuation ratio. In addition, we will show that the P/E ratio can be directly related to the DDM in a manner that indicates the variables that affect the $\mathrm{P} / \mathrm{E}$ ratio

## 1. Earnings Multiplier Model

As noted, many investors prefer to estimate the value of common stock using an earnings multiplier model. The reasoning for this approach recalls the basic concept that the value of any investment is the present value of future returns. In the case of common stocks, the returns that investors are entitled to receive are the net earnings of the firm. Therefore, one way investors can estimate value is by determining how many dollars they are willing to pay for a dollar of expected earnings (typically represented by the estimated earnings during the following 12month period). For example, if investors are willing to pay 10 times expected earnings, they would value a stock they expect to earn $\$ 2$ a share during the following year at $\$ 20$. You can compute the prevailing earnings multiplier, also referred to as the price/earnings ( $\mathrm{P} / \mathrm{E}$ ) ratio, as follows

Earnings Multiplier Price / Earnings Ratio
$=$ Current Market Price
Expected 12-Month Earnings

This computation of the current earnings multiplier ( $\mathrm{P} /$ ratio) indicates the prevailing attitude of investors toward a stock's value. Investors must decide if they agree with the prevailing P/E ratio (that is, is the earnings multiplier too high or too low?) based upon how it compares to the P/E ratio for the aggregate market, for the firm's industry, and for similar firms and stocks.
To answer this question, we must consider what influences the earnings multiplier ( $\mathrm{P} / \mathrm{E}$ ratio) over time. For example, over time the aggregate stock market $\mathrm{P} / \mathrm{E}$ ratio, as represented by the S\&P Industrials Index has varied from about 6 times earnings to about 30 times earnings. The infinite period dividend discount model can be used to indicate the variables that should determine the value of the P/E ratio as follows:

$$
\mathrm{P}=\mathrm{D} / \mathrm{k}-\mathrm{g}
$$

If we divide both sides of the equation by E1(expected earnings during the next 12 months), the result is $\mathrm{Pi} / \mathrm{Ei}=\underline{\mathrm{Dt} / \mathrm{Et}}$

K-G
Thus, the $\mathrm{P} / \mathrm{E}$ ratio is determined by

1. The expected dividend payout ratio (dividends divided by earnings)
2. The estimated required rate of return on the stock (k)
3. The expected growth rate of dividends for the stock (g)

Required Rate of Return (k) =risk free rate of return + inflation + risk premium
Growth rate $(\mathbf{g})=($ Retention Rate $) \times($ Return on Equity $)=\mathrm{RR} \times \mathrm{ROE}$
Payout ratio $=100 \%$-blow back (retention) ratio
E.g. if pay out $40 \%$ of earnings as dividends and $\mathrm{ROE}=20 \%$, what is g ?

Retention rate $(R R)=100 \%$ - payout ratio $=100 \%-40 \%=60 \%$
$\mathrm{g}=\mathrm{RR} \times \mathrm{ROE}=.60^{*} .20=12 \%$
As an example, if we assume a stock has an expected dividend payout of 50 percent, a required rate of return of 12 percent, and an expected growth rate for dividends of 8 percent, this would imply the following:
$\mathrm{D} / \mathrm{E}=0.05, \mathrm{k}=0.12, \mathrm{~g}=0.08$
$\mathrm{P} / \mathrm{E}=0.05 /(0.12-0.08)=12.5$

Again, a small difference in either $\mathbf{k}$ or $\mathbf{g}$ or both will have a large impact on the earnings multiplier. The spread between $\mathbf{k}$ and $\mathbf{g}$ is the main determinant of the size of the $\mathrm{P} / \mathrm{E}$ ratio. Although the dividend payout ratio has an impact, we are generally referring to a firm's long run target payout, which is typically rather stable with little effect on year-to-year changes in the P/E ratio (earnings multiplier).
After estimating the earnings multiple, you would apply it to your estimate of earnings for the next year (E1) to arrive at an estimated value. In turn, E1 is based on the earnings for the current Year (E0) and your expected growth rate of earnings. Using these two estimates, you would compute an estimated value of the stock and compare this estimated value to its market price. Consider the following estimates for an example firm:
$\mathrm{D} / \mathrm{E}=\mathbf{0 . 5 0} \quad \mathrm{k}=\mathbf{0 . 1 2} \quad \mathrm{g}=\mathbf{0 . 0 9} \quad \mathbf{E}_{0}=\mathbf{\$ 2 . 0 0}$
Using these estimates, you would compute earnings multiple of:
$\mathrm{P} / \mathrm{E}=0.05 / 90.12-0.09$ ) $=0.05 / 0.03=17.7$
Given current earnings ( $\mathrm{E}_{0}$ ) of $\$ 2.00$ and a g of 9 percent, you would expect E to be $\$ 2.18$.
Therefore, you would estimate the value (price) of the stock as
$\mathrm{V}=16.7 \times \$ 2.18==\$ 36.4$
This estimated value of the stock to its current market price to decide whether you should invest in it. This estimate of value is referred to as a "two-step process" because it requires you to estimate future earnings (E1) and a P/E ratio based on expectations of $\mathbf{k}$ and $\mathbf{g}$

## 2. The Price/Cash Flow Ratio

The growth in popularity of this relative valuation ratio can be traced to concern over the propensity of some firms to manipulate earnings per share, whereas cash flow values are generally less prone to manipulation. Also, as noted, cash flow values are important in fundamental valuation (when computing the present value of cash flow), and they are critical when doing credit analysis where "cash is king." The price to cash flow ratio is computed as follows:

$$
\mathrm{P} / \mathrm{CFj}==\mathrm{P} /(\mathrm{CFjt}+1)
$$

Where:
$\mathrm{P} / \mathrm{CFj}=$ the price/cash flow ratio for firm j
$\mathrm{Pt}=$ the price of the stock in period t
$\mathrm{CFt}+1=$ the expected cash flow per share for firm j

Regarding what variables affect this valuation ratio, the factors are similar to the P/Eratio. Specifically, the main variables should be: (1) the expected growth rate of the cash flow variable used, and (2) the risk of the stock as indicated by the uncertainty or variability of the cash flow series over time. The specific cash flow measure used is typically EBITDA, but the measure will vary depending upon the nature of the company and industry and which cash flow specification (for example, operating cash flow or free cash flow) is the best measure of performance for this industry. An appropriate ratio can also be affected by the firm's capital structure

## 3. The Price/Book Value Ratio

The price/book value ( $\mathrm{P} / \mathrm{BV}$ ) ratio has been widely used for many years by analysts in the banking industry as a measure of relative value. The book value of a bank is typically considered as good indicator of intrinsic value because most bank assets, such as bonds and commercial loans, have a value equal to book value. This ratio gained in popularity and credibility as a relative valuation technique for all types of firms based upon a study by Fama and French that indicated a significant inverse relationship between $\mathrm{P} / \mathrm{BV}$ ratios and excess rates of return for a cross section of stocks.
The $\mathrm{P} / \mathrm{BV}$ ratio is specified as follows
$\mathrm{P} / \mathrm{Bj}=\mathrm{Pt} / \mathrm{BVt}+1$
Where
$\mathrm{P} / \mathrm{BVj}=$ =the price/book value ratio for firm j
$\mathrm{Pt}=$ the price of the stock in period t
$\mathrm{BVt}+1=$ the estimated end-of-year book value per share for firm j
As with other relative valuation ratios, it is important to match the current price with the future book value that is expected to prevail at the end of the year. The difficulty is that this future book value is not generally available. One can derive an estimate of the end-of-year book value based upon the historical growth rate for the series or use the growth rate implied by the sustainable growth formula : $\mathrm{g}=(\mathrm{ROE})$ (Retention Rate).

Regarding what factors determine the size of the P/BV ratio, it is a function of ROE relative to the firm's cost of equity since the ratio would be one if they were equal-that is, if the firm earned its required return on assets. In contrast, if the ROE is much larger, it is a growth company and investors are willing to pay a premium over book value for the stock.

## 4. The Price/Sales Ratio

The price/sales (P/S) ratio has a volatile history. It was a favorite of Phillip Fisher, a well-known money manager in the late 1950s, his son, and others. Recently, the P/S ratio has been suggested as useful by Martin Leibowitz, a widely admired stock and bond portfolio manager.
These advocates consider this ratio meaningful and useful for two reasons. First, they believe that strong and consistent sales growth is a requirement for a growth company. Although they note the importance of an above-average profit margin, they contend that the growth process
must begin with sales. Second, given all the data in the balance sheet and income statement, sales information is subject to less manipulation than any other data item.
The specific P/S ratio is:
$\mathrm{P} / \mathrm{sj}=\mathrm{Pt} / \mathrm{st}+1$
Where:
$\mathrm{P} / \mathrm{Sj}=$ the price to sales ratio for firm j
$\mathrm{Pt}=$ the price of the stock in period t
$\mathrm{St}+1=$ the expected sales per share for firm j
Again, it is important to match the current stock price with the firm's expected sales per share, which may be difficult to derive for a large cross section of stocks. Two caveats are relevant to the price to sales ratio. First, this particular relative valuation ratio varies dramatically by industry. For example, the sales per share for retail firms, such as Kroger or Wal-Mart, are typically much higher than sales per share for computer or microchip firms. The second consideration is the profit margin on sales. The point is, retail food stores have high sales per share, which will cause a low $\mathbf{P / S}$ ratio, which is considered good until one realizes that these firms have low net profit margins. Therefore, your relative valuation analysis using the P/S ratio should be between firms in the same or similar industries.

## Implementing the Relative Valuation Technique

As noted, the relative valuation technique considers several valuation ratios-such as $\mathrm{P} / \mathrm{E}$, $\mathrm{P} / \mathrm{BV}$ - to derive a value for a stock. To properly implement this technique, it is essential to compare the various ratios but also to recognize that the analysis needs to go beyond simply comparing the ratios-it is necessary to understand what factors affect each of the valuation ratios and, therefore, know why they should differ. The first step is to compare the valuation ratio (e.g., the P/E ratio) for a company to the comparable ratio for the market, for the stock's industry, and to other stocks in the industry to determine how it compares-that is, is it similar to these other P/Es, or is it consistently at a premium or discount? Beyond knowing the overall relationship to the market, industry, and competitors, the real analysis is involved in understanding why the ratio has this relationship or why it should not have this relationship and the implications of this mismatch. Specifically, the second step is to explain the relationship. To do this, you need to understand what factors determine the specific valuation ratio and then compare these factors for the stock versus the same factors for the market, industry, and other stocks.

To illustrate this process, consider the following example wherein you want to value the stock of a pharmaceutical company and, to help in this process, you decide to employ the $\mathrm{P} / \mathrm{E}$ as a relative valuation technique. Assume that you compare the $\mathrm{P} / \mathrm{E}$ ratios for this firm over time (e.g. the last 15 years) to similar ratios for the S\&P Industrials, the pharmaceutical industry, and competitors. The results of this comparison indicate that the company P/E ratios are consistently
above all the other sets. The obvious question leads you into the second part of the analysis whether the fundamental factors that affect the $\mathrm{P} / \mathrm{E}$ ratio (i.e., the firm's growth rate and required rate of return) justify the higher P/E.A positive scenario would be that the firm had a historical and expected growth rate that was substantially above all the comparables and a lower required rate of return. This would indicate that the higher P/E ratio is justified; the only question that needs to be considered is, how much higher should the P/E ratio be? Alternatively, the negative scenario would be if the company's expected growth rate was equal to or lower than the industry and competitors while the required $\mathbf{k}$ was higher than for the industry and competitors. This would signal a stock that is apparently overpriced based on the fundamental factors that determine a stock's $\mathrm{P} / \mathrm{E}$ ratio.

## Chapter Four

## Fixed income securities

## (Assignment)

a. Bond characteristic
b. Bond price
c. Bond yield
d. Risks in bond
e. Rating of bonds
f. Analysis of convertible bonds

## Chapter Four

## Security Analysis

### 4.1 INTRODUCTION

Security analysis is the basis for rational investment decisions. If a security's estimated value is above its market price, the security analyst will recommend buying the stock. If the estimated value is below the market price, the security should be sold before its price drops. However, the values of the securities are continuously changing as news about the securities becomes known. The search for the security pricing involves the use of fundamental analysis. Under fundamental analysis, the security analysts studies the fundamental facts affecting a stock's values, such as company's earnings, their management, the economic outlook, the firm's competition, market conditions etc.

### 4.2 Fundamental analysis

Fundamental analysis is primarily concerned with determining the intrinsic value or the true value of a security. For determining the security's intrinsic value the details of all major factors (GNP, industry sales, firm sales and expense etc) is collected or an estimates of earnings per share may be multiplied by a justified or normal prices earnings ratio. After making this determination, the intrinsic value is compared with the security's current market price. If the market price is substantially greater than the intrinsic value the security is said to be overpriced. If the market price is substantially less than the intrinsic value, the security is said to be under priced. However, fundamental analysis comprises.

1. Economic Analysis
2. Industry Analysis
3. Company Analysis

## 1. ECONOMIC ANALYSIS

For the security analyst or investor, the anticipated economic environment, and therefore the economic forecast, is important for making decisions concerning both the timings of an
investment and the relative investment desirability among the various industries in the economy. The key for the analyst is that overall economic activities manifest itself in the behavior of the stocks in general. That is, the success of the economy will ultimately include the success of the overall market. For studying the Economic Analysis, the Macro Economic Factors and the Forecasting Techniques are studied in following paragraphs.

### 1.1 MACRO ECONOMIC FACTORS

The macro economy is the study of all the firms operates in economic environment. The key variables to describe the state of economy are explained as below:

## 1. Growth rate of Gross Domestic Product (GDP):

GDP is a measure of the total production of final goods and services in the economy during a year. It is indicator of economic growth. It consists of personal consumption expenditure, gross private domestic investment, government expenditure on goods and services and net export of goods and services. The firm estimates of GDP growth rate are available with a time lag of one or two years. Generally, GDP growth rate ranges from 6-8 percent. The growth rate of economy points out the prospects for the industrial sector and the returns investors can expect from investment in shares. The higher the growth rate of GDP, other things being equal, the more favorable it is for stock market.

## 2. Savings and investment:

Growth of an economy requires proper amount of investments which in turn is dependent upon amount of domestic savings. The amount of savings is favorably related to investment in a country. The level of investment in the economy and the proportion of investment in capital market is major area of concern for investment analysts. The level of investment in the economy is equal to: Domestic savings + inflow of foreign capital - investment made abroad. Stock market is an important channel to mobilize savings, from the individuals who have excess of it, to the individual or corporate, who have deficit of it. Savings are distributed over various assets like equity shares, bonds, small savings schemes, bank deposits, mutual fund units, real estates, bullion etc. The demand for corporate securities has an important bearing on stock prices
movements. Greater the allocation of equity in investment, favorable impact it have on stock prices.

## 3. Industry Growth rate:

The GDP growth rate represents the average of the growth rate of agricultural sector, industrial sector and the service sector. The current contribution of industry sector in GDP in the year 2004-05 is 6.75 percent approximately. Publicly listed company play a major role in the industrial sector. The stock market analysts focus on the overall growth of different industries contributing in economic development. The higher the growth rate of the industrial sector, other things being equal, the more favorable it is for the stock market.

## 4. Price level and Inflation:

If the inflation rate increases, then the growth rate would be very little. The increasingly inflation rate significantly affect the demand of consumer product industry. The industry which have a weak market and come under the purview of price control policy of the government may lose the market, like sugar industry. On the other hand the industry which enjoy a strong market for their product and which do not come under purview of price control may benefit from inflation. If there is a mild level of inflation, it is good to the stock market but high rate of inflation is harmful to the stock market.

## 5. Agriculture and monsoons:

Agriculture is directly and indirectly linked with the industries. Hence increase or decrease in agricultural production has a significant impact on the industrial production and corporate performance. Companies using agricultural raw materials as inputs or supplying inputs to agriculture are directly affected by change in agriculture production. For example- Sugar, Cotton, Textile and Food processing industries depend upon agriculture for raw material. Fertilizer and insecticides industries are supplying inputs to agriculture. A good monsoon leads to higher demand for inputs and results in bumper crops. This would lead to buoyancy in stock market. If the monsoon is bad, agriculture production suffers and cast a shadow on the share market.

## 6. Interest Rate:

Interest rates vary with maturity, default risk, inflation rate, productivity of capital etc. The interest rate on money market instruments like Treasury Bills are low, long dated government securities carry slightly higher interest rate and interest rate on corporate debenture is still higher. With the deregulation interest rates are softened, which were quite high in regulated environment. Interest rate affects the cost of financing to the firms. A decrease in interest rate implies lower cost of finance for firms and more profitability and it finally leads to decline in discount rate applied by the equity investors, both of which have a favorable impact on stock prices. At lower interest rates, more money at cheap cost is available to the persons who do business with borrowed money, this leads to speculation and rise in price of share.

## 7. Government budget and deficit:

Government plays an important role in the growth of any economy. The government prepares a central budget which provides complete information on revenue, expenditure and deficit of the government for a given period. Government revenue come from various direct and indirect taxes and government made expenditure on various developmental activities. The excess of expenditure over revenue leads to budget deficit. For financing the deficit the government goes for external and internal borrowings. Thus, the deficit budget may lead to high rate of inflation and adversely affects the cost of production and surplus budget may results in deflation. Hence, balanced budget is highly favorable to the stock market.

## 8. The tax structure:

The business community eagerly awaits the government announcements regarding the tax policy in March every year. The type of tax exemption has impact on the profitability of the industries. Concession and incentives given to certain industry encourages investment in that industry and have favorable impact on stock market.

## 9. Balance of payment, forex reserves and exchange rate:

Balance of payment is the record of all the receipts and payment of a country with the rest of the world. This difference in receipt and payment may be surplus or deficit. Balance of payment is a measure of strength of birr on external account. The surplus balance of payment augments forex
reserves of the country and has a favorable impact on the exchange rates; on the other hand if deficit increases, the forex reserve depletes and has an adverse impact on the exchange rates. The industries involved in export and import are considerably affected by changes in foreign exchange rates. The volatility in foreign exchange rates affects the investment of foreign institutional investors in Stock Market. Thus, favorable balance of payment renders favorable impact on stock market.

## 10. Infrastructural facilities and arrangements:

Infrastructure facilities and arrangements play an important role in growth of industry and agriculture sector. A wide network of communication system, regular supply or power, a well developed transportation system (railways, transportation, road network, inland waterways, port facilities, air links and telecommunication system) boost the industrial production and improves the growth of the economy. Banking and financial sector should be sound enough to provide adequate support to industry and agriculture. The government has liberalized its policy regarding the communication, transport and power sector for foreign investment. Thus, good infrastructure facilities affect the stock market favorable.

## 11. Demographic factors:

The demographic data details about the population by age, occupation, literacy and geographic location. These factors are studied to forecast the demand for the consumer goods. The data related to population indicates the availability of work force. The cheap labour force in India has encouraged many multinationals to start their ventures. Population, by providing labour and demand for products, affects the industry and stock market.

## 12. Sentiments:

The sentiments of consumers and business can have an important bearing on economic performance. Higher consumer confidence leads to higher expenditure and higher business confidence leads to greater business investments. All this ultimately leads to economic growth. Thus, sentiments influence consumption and investment decisions and have a bearing on the aggregate demand for goods and services.

## 2. INDUSTRY ANALYSIS

The mediocre firm in the growth industry usually out performs the best stocks in a stagnant industry. Therefore, it is worthwhile for a security analyst to pinpoint growth industry, which has good investment prospects. The past performance of an industry is not a good predictor of the future- if one look very far into the future. Therefore, it is important to study industry analysis. For an industry analyst- industry life cycle analysis, characteristics and classification of industry is important. All these aspects are enlightened in following sections:

### 2.1 INDUSTRY LIFE CYCLE ANALYSIS

Many industrial economists believe that the development of almost every industry may be analyzed in terms of following stages.

1. Pioneering stage: During this stage, the technology and product is relatively new. The prospective demand for the product is promising in this industry. The demand for the product attracts many producers to produce the particular product. This lead to severe competition and only fittest companies survive in this stage. The producers try to develop brand name, differentiate the product and create a product image. This would lead to non-price competition too. The severe competition often leads to change of position of the firms in terms of market share and profit.
2. Rapid growth stage: This stage starts with the appearance of surviving firms from the pioneering stage. The companies that beat the competition grow strongly in sales, market share and financial performance. The improved technology of production leads to low cost and good quality of products. Companies with rapid growth in this stage, declare dividends during this stage. It is always a disable to invest in these companies.
3. Maturity and stabilization stage: After enjoying above-average growth, the industry now enters in maturity and stabilization stage. The symptoms of technology obsolescence may appear. To keep going, technological innovation in the production process should be introduced. A close monitoring at industries events are necessary at this stage.
4. Decline stage: The industry enters the growth stage with satiation of demand, encroachment of new products, and change in consumer preferences. At this stage the earnings of the industry
are started declining. In this stage the growth of industry is low even in boom period and decline at a higher rate during recession. It is always advisable not to invest in the share of low growth industry.

### 2.2 CLASSIFICATION OF INDUSTRY

Industry means a group of productive or profit making enterprises or organizations that have a similar technically substitute goods, services or source of income. Besides Standard Industry Classification (SIC), industries can be classified on the basis of products and business cycle i.e. classified according to their reactions to the different phases of the business cycle. These are classified as follows:

## 1. Growth Industries:

These industries have special features of high rate of earnings and growth in expansion, independent of the business cycle. The expansion of the industry mainly depends on the technological change or an innovative way of doing or selling something. For example-in present scenario the information technology sector have higher growth rate. There is some growth in electronics, computers, cellular phones, engineering, petro-chemicals, telecommunication, energy etc.

## 2. Cyclical Industries:

The growth and profitability of the industry move along with the business cycle. These are those industries which are most likely to benefit from a period of economic prosperity and most likely to suffer from a period of economic recession. These especially include consumer goods and durables whose purchase can be postponed until persona; financial or general business conditions improve. For example-Fast Moving Consumer Goods (FMCG) commands a good market in the boom period and demand for them slackens during the recession.

## 3. Defensive Industries:

These are industries that have little sensitivity to the business cycle. Defensive industries are those, such as the food processing industry, which hurt least in the period of economic downswing. For example- the industries selling necessities of consumers withstands recession and depression. The stock of defensive industries can be held by the investor for income earning
purpose. Consumer nondurable and services, which in large part are the items necessary for existence, such as food and shelter, are products of defensive industry.

## 4. Cyclical-growth Industries:

These possess characteristics of both a cyclical industry and a growth industry. For example, the automobile industry experiences period of stagnation, decline but they grow tremendously. The change in technology and introduction of new models help the automobile industry to resume their growing path.

## 3. Company Analysis and Stock Valuation

Company analysis should occur in the context of the prevailing economic and industry conditions. We discuss some competitive strategies that can help firms maximize returns in an industry's competitive environment. We demonstrate cash flow models and relative valuation ratios that can be used to determine a stock's intrinsic value and identify undervalued stocks.

This portion of the chapter is titled "Company Analysis and Stock Valuation" to convey the idea that the common stocks of good companies are not necessarily good investments. The point is, after analyzing a company and deriving an understanding of its strengths and risks, you need to compute the fundamental intrinsic value of the firm's stock and compare the intrinsic value of a stock to its market value to determine if the company's stock should be purchased.

The stock of a wonderful firm with superior management and strong performance measured by sales and earnings growth can be priced so high that the intrinsic value of the stock is below its current market price and should not be acquired. In contrast, the stock of a company with less success based on its sales and earnings growth may have a stock market price that is below its intrinsic value. In this case, although the company is not as good, its stock could be the better investment. The classic confusion in this regard concerns growth companies versus growth stocks. The stock of a growth company is not necessarily a growth stock. Recognition of this difference is absolutely essential for successful investing.

## Growth Companies and Growth Stocks

Observers have historically defined growth companies as those that consistently experience above-average increases in sales and earnings. This definition has some limitations because many firms could qualify due to certain accounting procedures, mergers, or other external events. In contrast, financial theorists define a growth company as a firm with the management ability and the opportunities to make investments that yield rates of return greater than the firm's required rate of return.

This required rate of return is the firm's weighted average cost of capital (WACC). As an example, a growth company might be able to acquire capital at an average cost of 10 percent and yet have the management ability and the opportunity to invest those funds at rates of return of 15 to 20 percent. As a result of these investment opportunities, the firm's sales and earnings grow faster than those of similar risk firms and the overall economy.

In addition, a growth company that has above-average investment opportunities should, and typically does, retains a large portion of its earnings to fund these superior investment projects (i.e., they have low dividend payout ratios). Growth stocks are not necessarily shares in growth companies. A growth stock is a stock with a higher rate of return than other stocks in the market with similar risk characteristics. The stock achieves this superior risk-adjusted rate of return because at some point in time the market undervalued it compared to other stocks. Although the stock market adjusts stock prices relatively quickly and accurately to reflect new information, available information is not always perfect or complete. Therefore, imperfect or incomplete information may cause a given stock to be undervalued or overvalued at a point in time.

If the stock is undervalued, its price should eventually increase to reflect its true fundamental value when the correct information becomes available. During this period of price adjustment, the stock's realized return will exceed the required return for a stock with its risk, and, during this period of adjustment, it will be considered a growth stock. Growth stocks are not necessarily limited to growth companies. A future growth stock can be the stock of any type of company; the stock need only be undervalued by the market.

The fact is, if investors recognize a growth company and discount its future earnings stream properly, the current market price of the growth company's stock will reflect its future earnings
stream. Those who acquire the stock of a growth company at this correct market price will receive a rate of return consistent with the risk of the stock, even when the superior earnings growth is attained. In many instances, overeager investors tend to overestimate the expected growth rate of earnings and cash flows for the growth company and, therefore, inflate the price of a growth company's stock. Investors who pay the inflated stock price will earn a rate of return below the risk-adjusted required rate of return, despite the fact that the growth company experiences the above-average growth of sales and earnings. Several studies that have examined the stock price performance for samples of growth companies have found that their stocks performed poorly-that is, the stocks of growth companies have generally not been growth stocks.

Defensive Companies and Stock Defensive companies are those whose future earnings are likely to withstand an economic downturn. One would expect them to have relatively low business risk and not excessive financial risk. Typical examples are public utilities or grocery chains-firms that supply basic consumer necessities.

There are two closely related concepts of a defensive stock. First, a defensive stock's rate of return is not expected to decline during an overall market decline, or decline less than the overall market. Second, our CAPM discussion indicated that an asset's relevant risk is its covariance with the market portfolio of risky assets-that is, an asset's systematic risk. A stock with low or negative systematic risk (a small positive or negative beta) may be considered a defensive stock according to this theory because its returns are unlikely to be harmed significantly in a bear market.

Cyclical Companies and Stock A cyclical company's sales and earnings will be heavily influenced by aggregate business activity. Examples would be firms in the steel, auto, or heavy machinery industries. Such companies will do well during economic expansions and poorly during economic contractions. This volatile earnings pattern is typically a function of the firm's business risk (both sales volatility and operating leverage) and can be compounded by financial risk.

A cyclical stock will experience changes in its rates of return greater than changes in overall market rates of return. In terms of the CAPM, these would be stocks that have high betas. The
stock of a cyclical company, however, is not necessarily cyclical. A cyclical stock is the stock of any company that has returns that are more volatile than the overall market - that is, high-beta stocks that have high correlation with the aggregate market and greater volatility.

## Speculative Companies and Stocks

A speculative company is one whose assets involve great risk but that also has a possibility of great gain. A good example of a speculative firm is one involved in oil exploration. A speculative stock possesses a high probability of low or negative rates of return and a low probability of normal or high rates of return. Specifically, a speculative stock is one that is overpriced, leading to a high probability that during the future period when the market adjusts the stock price to its true value, it will experience either low or possibly negative rates of return. Such an expectation might be the case for an excellent growth company whose stock is selling at an extremely high price/earnings ratio-i.e., it is substantially overvalued.

## Value versus Growth Investing

Some analysts also divide stocks into "growth" stocks and "value" stocks. As we have discussed, growth stocks are companies that will have positive earnings surprises and above-average risk adjusted rates of return because the stocks are undervalued.

If the analyst does a good job in identifying such companies, investors in these stocks will reap the benefits of seeing their stock prices rise after other investors identify their earnings growth potential. Value stocks are those that appear to be undervalued for reasons other than earnings growth potential. Value stocks are usually identified by analysts as having low price-earning or price-book value ratios. Notably, in these comparisons between growth and value stocks, the specification of a growth stock is not consistent with our preceding discussion.

In these discussions, a growth stock is generally specified as a stock of a company that is experiencing rapid growth of sales and earnings (e.g., Intel and Microsoft). As a result of this company performance, the stock typically has a high $\mathrm{P} / \mathrm{E}$ and price-book-value ratio. Unfortunately, the specification does not consider the critical comparison between intrinsic value and market price.

### 4.3 Technical Analysis

The market reacted yesterday to the report of a large increase in the short interest on the NYSE. Although the market declined today, it was not considered bearish because of the light volume. The market declined today after three days of increases due to profit taking by investors.

These and similar statements appear daily in the financial news. All of them have as their rationale one of numerous technical trading rules. Technical analysts develop technical trading rules from observations of past price movements of the stock market and individual stocks. The philosophy behind technical analysis is in sharp contrast to the efficient market hypothesis that we studied, which contends that past performance has no influence on future performance or market values. It also differs from what we learned about fundamental analysis, which involves making investment decisions based on the examination of the economy, an industry, and he market reacted yesterday to the report of a large increase in the short interest on the NYSE.

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## Underlying assumptions of technical analysis

Technical analysts base trading decisions on examinations of prior price and volume data to determine past market trends from which they predict future behavior for the market as a whole and for individual securities. Several assumptions lead to this view of price movements:

1. The market value of any good or service is determined solely by the interaction of supply and demand.
2. Supply and demand is governed by numerous rational and irrational factors. Included in these factors are those economic variables relied on by the fundamental analyst as well as opinions, moods, and guesses. The market weighs all these factors continually and automatically.
3. Disregarding minor fluctuations, the prices for individual securities and the overall value of the market tend to move in trends, which persist for appreciable lengths of time.
4. Prevailing trends change in reaction to shifts in supply and demand relationships. These shifts, no matter why they occur, can be detected sooner or later in the action of the market itself.

Certain aspects of these assumptions are controversial, leading fundamental analysts and advocates of efficient markets to question their validity. Those aspects are emphasized in the preceding list.

## EXHIBIT 16.1

technical view of price adjustment to new information


The first two assumptions are almost universally accepted by technicians and non technicians alike. Almost anyone who has had a basic course in economics would agree that, at any point in time, the price of a security (or any good or service) is determined by the interaction of supply and demand. In addition, most observers would acknowledge that supply and demand are governed by many variables. The only difference in opinion might concern the influence of the irrational factors. Certainly, everyone would agree that the market continually weighs all these factors.

A stronger difference of opinion arises over the assumption about the speed of adjustment of stock prices to changes in supply and demand. A technical analyst expect stock prices to move in trends that persist for long periods because they believe that new information does not come to the market at one point in time but, rather, enters the market over a period of time.

This pattern of information access occurs because of different sources of information or because certain investors receive the information or perceive fundamental changes earlier than others. As various groups ranging from insiders to well-informed professionals to the average investor receive the information and buy or sell a security accordingly, its price moves gradually toward the new equilibrium. Therefore, technicians do not expect the price adjustment to be as abrupt as fundamental analysts and efficient market supporters do, but expect a gradual price adjustment to reflect the gradual flow of information.

Exhibit 16.1 shows this process wherein new information causes a decrease in the equilibrium price for a security, but the price adjustment is not rapid. It occurs as a trend that persists until the stock reaches its new equilibrium. Technical analysts look for the beginning of a movement from one equilibrium value to a new equilibrium value. Technical analysts do not attempt to predict the new equilibrium value. They look for the start of a change so that they can get on the bandwagon early and benefit from the move to the new equilibrium by buying if the trend is up or selling if the trend is down. Obviously, if there is a rapid adjustment of prices, as expected by those who espouse an efficient market, it would keep the ride on the bandwagon so short that investors could not get on board and benefit from the ride.

## Chapter Five <br> Portfolio Theory

## Part I: Portfolio Theory

## Background Assumptions

- As an investor, you want to maximize return for a given level of risk.
- Your portfolio includes all of your assets and liabilities, not just your traded securities.
- The relationship between the returns of the assets in the portfolio is important.
- A good portfolio is not simply a collection of individually good investments.


## Risk Aversion

Portfolio theory also assumes that investors are basically risk averse, meaning that, given a choice between two assets with equal rates of return, they will select the asset with the lower level of risk.
$\checkmark$ Evidence that most investors are risk averse is that they purchase various types of insurance, including life insurance, car insurance, and health insurance. Buying insurance basically involves an outlay of a given amount to guard against an uncertain, possibly larger outlay in the future. When you buy insurance, this implies that you are willing to pay the current known cost of the insurance policy to avoid the uncertainty of a potentially large future cost related to a car accident or a major illness.
$\checkmark$ Further evidence of risk aversion is the difference in promised yield (the required rate of return) for different grades of bonds that supposedly have different degrees of credit risk. Specifically, the promised yield on bonds increases as you go from AAA (the lowest-risk class) to AA to A, and so on-that is, investors require a higher rate of return to accept higher risk.

This does not imply that everybody is risk averse or that investors are completely risk averse regarding all financial commitments. The fact is, not everybody buys insurance for everything. Some people have no insurance against anything, either by choice or because they cannot afford it. In addition, some individuals buy insurance related to some risks such as auto accidents or illness, but they also buy lottery tickets and gamble at race tracks or in casinos, where it is known that the expected returns are negative, which means that participants are willing to pay for the excitement of the risk involved. This combination of risk preference and risk aversion can be explained by an attitude toward risk that depends on the amount of money involved. Friedman and Savage speculate that this is the case for people who like to gamble for small amounts (in lotteries or slot machines) but buy insurance to protect themselves against large potential losses, such as fire or accidents.

While recognizing this diversity of attitudes, our basic assumption is that most investors committing large sums of money to developing an investment portfolio are risk averse. Therefore, we expect a positive relationship between expected return and expected risk. Notably, this is also what we generally find in terms of long-run historical results-that is, there is
generally a positive relationship between the rates of return on various assets and their measures of risk.

## Assumptions Underlying MARKOWITZ PORTFOLIO THEORY

1. Investors consider each investment alternative as being represented by a probability distribution of expected returns over some holding period.
2. Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.
3. Investors estimate the risk of the portfolio on the basis of the variability of expected returns.
4. Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance (or standard deviation) of returns only.
5. For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk.

Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

## Expected Return and Risk

## 1. Expected Return Individual Asset/ Single Asset

- For an individual asset - sum of the possible returns multiplied by the corresponding probability of the return occurring
- Multiply each possible outcome by its probability of occurrence \& then sum these products (=weighted average of possible outcomes).

$$
\text { Expected rate of return }=\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{P}_{\mathrm{i}} \mathrm{R}_{\mathrm{i}}
$$

Example: Compute the expected return of an asset with the following returns \& associated probabilities.

| Probability | Possible Rate of Return |
| :--- | :--- |
| $35 \%$ | $8 \%$ |
| $30 \%$ | $10 \%$ |
| $20 \%$ | $12 \%$ |
| $15 \%$ | $14 \%$ |

Ans:

$$
\begin{aligned}
E\left(R_{\text {Security }}\right) & =\sum_{i=1}^{N} P_{i} R_{i} \\
& =(.35)(8)+(.30)(10)+(.20)(12)+(.15)(14) \\
& =10.30 \%
\end{aligned}
$$

Variance (Standard Deviation) of Returns for an Individual Investment)

$$
\text { Variance }\left(\sigma^{2}\right)=\sum_{i=1}^{n} \mathrm{P}_{\mathrm{i}}\left(\mathrm{R}_{\mathrm{i}}-\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)\right)^{2}
$$

## Where:

$P_{i}$ is the probability of $R_{i}$ occurring
$R_{i}$ is the $i^{\text {th }}$ rate of return
Standard deviation $(\sigma)=\sqrt{\sum_{i=1}^{n} \mathrm{P}_{\mathrm{i}}\left(\mathrm{R}_{\mathrm{i}}-\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)\right)^{2}}$
Question: Calculate the variance \& standard deviation for the asset in example above

$$
\begin{aligned}
\operatorname{Variance}\left(\sigma^{2}\right) & =\sum_{i=1}^{n} \mathrm{P}_{\mathrm{i}}\left(\mathrm{R}_{\mathrm{i}}-\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)\right)^{2} \\
& =0.35(8-10.3)^{2}+0.30(10-10.3)^{2}+0.20(12-10.3)^{2}+0.15(14-10.3)^{2} \\
& =4.51
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{Standard} \text { deviation }(\sigma) & =\sqrt{\sum_{i=1}^{n} \mathrm{P}_{\mathrm{i}}\left(\mathrm{R}_{\mathrm{i}}-\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)\right)^{2}} \\
& =\sqrt{0.35(8-10.3)^{2}+0.30(10-10.3)^{2}+0.20(12-10.3)^{2}+0.15(14-10.3)^{2}} \\
& =\sqrt{4.51} \\
& =2.12 \%
\end{aligned}
$$

## 2. Expected Return of Portfolio

- The return on the risky asset portfolio is calculated as weighted average of the expected rates of return for the individual investments in the portfolio
- Weights are the market values of each asset divided by the total market value of the portfolio
$\mathrm{E}\left(\mathrm{R}_{\text {portfoio } \mathrm{i}}\right)=\sum_{i=1}^{n} \mathrm{~W}_{i} \mathrm{R}_{i}$
where:
$\mathrm{W}_{\mathrm{i}}=$ the percent of the portfolioin asset i
$E\left(R_{i}\right)=$ the expected rate of return for asset ${ }_{63}$

Example: Compute the expected return of a portfolio composes of four assets with the following weights \& associated expected returns.

| Weight (\% of Portfolio) | Expected Return (Asset i) |
| :--- | :--- |
| $20 \%$ | $10 \%$ |
| $30 \%$ | $11 \%$ |
| $30 \%$ | $12 \%$ |
| $20 \%$ | $13 \%$ |

Ans:

$$
\begin{aligned}
E\left(R_{\text {Portfolio }}\right) & =\sum_{i=1}^{N} W_{i} R_{i} \\
& =(.20)(10 \%)+(.30)(11 \%)+(.30)(12 \%)+(.20)(13 \%) \\
& =11.50 \%
\end{aligned}
$$

## Calculating Portfolio Risk: Two Risky Assets

- The risk of a single risky asset is calculated as its standard deviation
- When there are two or more risky assets in a portfolio, we must also incorporate how the individual assets move in relation to each other
- Thus we need to understand covariance \& correlation
- Unlike returns, the risk of a portfolio, $\sigma \mathrm{p}$, is generally not the weighted average of the standard deviations of the individual assets in the portfolio.
- The portfolio risk will be smaller than the weighted average of the assets' $\sigma$ 's.
- Theoretically, possible to combine two stocks that are individually risky as measured by $\sigma$ to form a portfolio completely riskless, with $\sigma p=0$.


## Covariance of Returns

- A measure of the degree to which two variables "move together" relative to their individual mean values over time
- If both returns are typically above their respective means at the same time, the covariance will be positive
- If one return is typically above its mean when the other return is below its mean, covariance will be negative
For two assets, i and j , the covariance of their returns is defined as:

$$
\operatorname{Cov}_{\mathrm{ij}}=\mathrm{E}\left\{\left[\mathrm{R}_{\mathrm{i}}-\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)\right]\left[\mathrm{R}_{\mathrm{j}}-\mathrm{E}\left(\mathrm{R}_{\mathrm{j}}\right)\right]\right\}
$$

| COMPUTATION OF MONTHLY RATES OF RETURN: 2001 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | COCA-COLA |  |  | HOME DEPOT |  |  |
| Date | Closing Price | Dividend | Rate of <br> Return (\%) | Closing Price | Dividend | Rate of Return (\%) |
| Dec-00 | 60.938 |  |  | 45.688 |  |  |
| Jan-01 | 58.000 |  | -4.82 | 48.200 |  | 5.50 |
| Feb-01 | 53.030 |  | -8.57 | 42.500 |  | -11.83 |
| Mar-01 | 45.160 | 0.18 | -14.50 | 43.100 | 0.04 | 1.51 |
| Apr-01 | 46.190 |  | 2.28 | 47.100 |  | 9.28 |
| May-01 | 47.400 |  | 2.62 | 49.290 |  | 4.65 |
| Jun-01 | 45.000 | 0.18 | -4.68 | 47.240 | 0.04 | -4.08 |
| Jul-01 | 44.600 |  | -0.89 | 50.370 |  | 6.63 |
| Aug-01 | 48.670 |  | 9.13 | 45.950 | 0.04 | -8.70 |
| Sep-01 | 46.850 | 0.18 | -3.37 | 38.370 |  | -16.50 |
| Oct-01 | 47.880 |  | 2.20 | 38.230 |  | -0.36 |
| Nov-01 | 46.960 | 0.18 | -1.55 | 46.650 | 0.05 | 22.16 |
| Dec-01 | 47.150 |  | 0.40 | 51.010 |  | 9.35 |
|  |  |  | $=-1.81$ |  |  | mot) $=1.47$ |

TIME SERIES OF MONTHLY RATES OF RETURN FOR COCA-COLA: 2001


TIME SERIES OF MONTHLY RATES OF RETURN FOR HOME DEPOT: 2001


COMPUTATION OF COVARIANCE OF RETURNS FOR COCA-COLA AND HOME DEPOT: 2001

| Date | Monthly Return |  | Coca-Cola$R_{t}-E\left(R_{1}\right)$ | Home Depot$R_{J}-E\left(R_{J}\right)$ | Coca-Cola$\left[R_{,}-E\left(R_{1}\right)\right]$ | $\times$ | Home Depot$\left[R_{s}-E\left(R_{\jmath}\right)\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coca-Cola ( $R_{1}$ ) | Home Depot ( $R_{\text {J }}$ ) |  |  |  |  |  |
| Jan-01 | -4.82 | 5.50 | -3.01 | 4.03 |  |  | -12.13 |
| Feb-01 | -8.57 | -11.83 | -6.76 | -13.29 |  |  | 89.81 |
| Mar-01 | -14.50 | 1.51 | -12.69 | 0.04 |  |  | -0.49 |
| Apr-01 | 2.28 | 9.28 | 4.09 | 7.81 |  |  | 31.98 |
| May-01 | 2.62 | 4.65 | 4.43 | 3.18 |  |  | 14.11 |
| Jun-01 | -4.68 | -4.08 | -2.87 | -5.54 |  |  | 15.92 |
| Jul-01 | -0.89 | 6.63 | 0.92 | 5.16 |  |  | 4.76 |
| Aug-01 | 9.13 | -8.70 | 10.94 | -10.16 |  |  | -111.16 |
| Sep-01 | -3.37 | -16.50 | -1.56 | -17.96 |  |  | 27.97 |
| Oct-01 | 2.20 | -0.36 | 4.01 | -1.83 |  |  | -7.35 |
| Nov-01 | -1.55 | 22.16 | 0.27 | 20.69 |  |  | 5.52 |
| Dec-01 | 0.40 | 9.35 | 2.22 | 7.88 |  |  | 17.47 |
|  | i) $=-1.81$ | $E\left(R_{j}\right)=1.47$ |  |  | Sum $=$ |  | 76.42 |

$\operatorname{Cov}_{i j}=76.42 / 12=6.37$

| Coca-Cola |  |  | Home Depot |  |
| :---: | :---: | :---: | :---: | :---: |
| Date | $R_{t}-E\left(R_{J}\right)$ | $\left[R_{,}-E(R)\right]^{2}$ | $R_{\text {, }}-E\left(R_{\text {J }}\right)$ | $\left[R_{s}-E\left(R_{\text {J }}\right)\right]^{2}$ |
| Jan-01 | -3.01 | 9.05 | 4.03 | 16.26 |
| Feb-01 | -6.76 | 45.65 | -13.29 | 176.69 |
| Mar-01 | -12.69 | 161.01 | 0.04 | 0.00 |
| Apr-01 | 4.09 | 16.75 | 7.81 | 61.06 |
| May-01 | 4.43 | 19.64 | 3.18 | 10.13 |
| Jun-01 | -2.87 | 8.24 | -5.54 | 30.74 |
| Jul-01 | 0.92 | 0.85 | 5.16 | 26.61 |
| Aug-01 | 10.94 | 119.64 | -10.16 | 103.28 |
| Sep-01 | -1.56 | 2.42 | -17.96 | 322.67 |
| Oct-01 | 4.01 | 16.09 | -1.83 | 3.36 |
| Nov-01 | 0.27 | 0.07 | 20.69 | 428.01 |
| Dec-01 | 2.22 | 4.92 | 7.88 | 62.08 |
| Sum $=404.34$ |  |  | Sum $=1240.90$ |  |
| Variance $_{j}=404.34 / 12=$ |  | 33.69 | Variance $_{j}=240.90 / 12=$ | 103.41 |
| Standard | $=(33.69)^{1 / 2}$ | 5.80Standard Deviation ${ }_{j}=(103.41)^{1 / 2}=$ |  | 10.17 |

$$
\begin{aligned}
& r_{i j}=\frac{\operatorname{Cov}_{i j}}{\sigma_{i} \sigma_{j}}=\frac{6.37}{(5.80)(10.17)}=\frac{6.37}{58.99}=0.108 \\
& \operatorname{Cov}_{\mathrm{ij}}=r_{i j} \sigma_{i} \sigma_{j}(.108)(5.80)(10.17)=6.37
\end{aligned}
$$

## Portfolio Standard Deviation Formula

$$
\sigma_{\mathrm{part}}=\sqrt{\sum_{i=1}^{n} w_{i}^{2} \sigma_{i}^{2}+\sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{i=1}^{n} w_{i} w_{j} \operatorname{Cov}_{i j}}
$$

where:
$\sigma_{\text {port }}=$ the standard deviation of the portfolio
$w_{i}=$ the weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio
$\sigma_{i}^{2}=$ the variance of rates of return for assets $i$
$\operatorname{Cov}_{i j}=$ the covariance between the rates of return for assets $i$ and $j$, where $\operatorname{Cov}_{i j}=r_{i j} \sigma_{i} \sigma_{j}$

- This formula indicates that the standard deviation for a portfolio of assets is a function of the weighted average of the individual variances (where the weights are squared),plus the weighted covariance between all the assets in the portfolio.
- In a portfolio with a large number of securities, this formula reduces to the sum of the weighted covariance
- what happens in a large portfolio with many assets. Specifically, what happens to the portfolio's standard deviation when you add a new security to such a portfolio?
$>$ The important factor to consider when adding an investment to a portfolio that contains a number of other investments is not the investment's own variance but its average covariance with all the other investments in the portfolio
$>$ Any asset of a portfolio may be described by two characteristics:
$>$ The expected rate of return
$>$ The expected standard deviations of returns
$>$ The correlation, measured by covariance, affects the portfolio standard deviation
$>$ Low correlation reduces portfolio risk while not affecting the expected return
$>$ Negative correlation reduces portfolio risk
$>$ Combining two assets with -1.0 correlation reduces the portfolio standard deviation to zero only when individual standard deviations are equal


## time patterns of returns for two assets with perfect negative CORRELATION



Illustration 1: Measuring Portfolio Return \& Risk: 2 Risky Assets

- Equal Risk and Return-Changing correlations Consider first the case in which both assets have the same expected return and expected standard deviation of return.
- As an example, let us assume

| Asset | $\mathrm{E}\left(\mathbf{R}_{\mathrm{i}}\right)$ | $\sigma_{\mathrm{i}}$ |
| :---: | :---: | :---: |
| 1 | 0.2 | 0.1 |
| 2 | 0.2 | 0.1 |

- To show the effect of different covariance, assume different levels of correlation between the two assets.
- Consider the following examples where the two assets have equal weights in the portfolio (W1=0.50; W2=0.50).
- Therefore, the only value that changes in each example is the correlation between the returns for the two assets

| Case | Corr. Coefficient | Covariance( | ) |
| :---: | :---: | :---: | :--- |
| a | +1.00 | .010 | 0.10 |
| b | +0.50 | .005 | 0.0868 |
| c | 0.00 | .000 | 0.0707 |
| d | -0.50 | -.005 | 0.05 |
| e | -1.00 | -.010 | 0.00 |

$\sigma_{\text {port }}=\sqrt{w_{1}^{2} \sigma_{1}^{2}+w_{2}^{2} \sigma_{2}^{2}+2 w_{1} w_{2} r_{1,2} \sigma_{1} \sigma_{2}}$

## Note That:

- In the case $a$, where the returns for the two assets are perfectly positively correlated ( $\mathrm{r} 1,2=1.0$ ), the standard deviation for the portfolio is, in fact, the weighted average of the individual standard deviations. The important point is that we get no real benefit from combining two assets that are perfectly correlated; they are like one asset already because their returns move together.
- The only term that changed from Case $a$ to case $b$ is the last term, Cov1,2, which changed from 0.01 to 0.005 . As a result, the standard deviation of the portfolio declined by about 13 percent, from 0.10 to 0.0868 . Note that the expected return did not change because it is simply the weighted average of the individual expected_returns; it is equal to 0.20 in both cases.


## * What would happen to portfolio expected return and risk when we change the weights and the correlations

To illustrate consider the following example

| Asset | $E(R)$ | Weight | Variance | Std dev |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 0.10 | 0.50 | 0.0049 | 0.07 |
| 2 | 0.20 | 0.50 | 0.0100 | 0.15 |

PORTFOLIO RISK-RETURN PLOTS FOR DIFFERENT WEIGHTS WHEN $r_{i, j}=+1.00$; +0.50; 0.00; -0.50; - 1.00


## The Efficient Frontier

The envelope curve that contains the best of all these possible combinations is referred to as the efficient frontier. Specifically, the efficient frontier represents that set of portfolios that has the maximum rate of return for every given level of risk, or the minimum risk for every level of return.



Portfolio A in Exhibit dominates Portfolio C because it has an equal rate of return but substantially less risk. Similarly, Portfolio B dominates Portfolio C because it has equal risk but a higher expected rate of return. Because of the benefits of diversification among imperfectly correlated assets, we would expect the efficient frontier to be made up of portfolios of investments rather than individual securities. Two possible exceptions arise at the end points, which represent the asset with the highest return and that asset with the lowest risk.

As an investor, you will target a point along the efficient frontier based on your utility function and your attitude toward risk. No portfolio on the efficient frontier can dominate any other portfolio on the efficient frontier. All of these portfolios have different return and risk measures, with expected rates of return that increase with higher risk.

## SELECTING AN OPTIMAL RISKY PORTFOLIO ON THE EFFICIENT FRONTIER



An individual investor's utility curves specify the trade-offs he or she is willing to make between expected return and risk. In conjunction with the efficient frontier, these utility curves determine
which particular portfolio on the efficient frontier best suits an individual investor. Two investors will choose the same portfolio from the efficient set only if their utility curves are identical.

The optimal portfolio is the portfolio on the efficient frontier that has the highest utility for a given investor. It lies at the point of tangency between the efficient frontier and the curve with the highest possible utility. A conservative investor's highest utility is at point $X$ in Exhibit, where the curve U2 just touches the efficient frontier. A less-risk-averse investor's highest utility occurs at point $Y$, which represents a portfolio with a higher expected return and higher risk than the portfolio at $X$.

## Part II: Capital Market Theory

## Background for Capital Market Theory

Assumptions of Capital Market Theory
Because capital market theory builds on the Markowitz portfolio model, it requires the same assumptions, along with some additional ones:

1. All investors are Markowitz efficient investors who want to target points on the efficient frontier. The exact location on the efficient frontier and, therefore, the specific portfolio selected will depend on the individual investor's risk-return utility function.
2. Investors can borrow or lend any amount of money at the risk-free rate of return $(R F R)$. Clearly, it is always possible to lend money at the nominal risk-free rate by buying risk free securities such as government T-bills. It is not always possible to borrow at this risk free rate, but we will see that assuming a higher borrowing rate does not change the general results.
3. All investors have homogeneous expectations; that is, they estimate identical probability distributions for future rates of return. Again, this assumption can be relaxed. As long as the differences in expectations are not vast, their effects are minor.
4. All investors have the same one-period time horizon such as one month, six months, or one year. The model will be developed for a single hypothetical period, and its results could be affected by a different assumption. A difference in the time horizon would require investors to derive risk measures and risk-free assets that are consistent with their investment horizons.
5. All investments are infinitely divisible, which means that it is possible to buy or sell fractional shares of any asset or portfolio. This assumption allows us to discuss investment alternatives as continuous curves. Changing it would have little impact on the theory.
6. There are no taxes or transaction costs involved in buying or selling assets. This is a reasonable assumption in many instances. Neither pension funds nor religious groups have to pay taxes, and the transaction costs for most financial institutions are less than 1 percent on most financial instruments. Again, relaxing this assumption modifies the results, but it does not change the basic thrust.
7. There is no inflation or any change in interest rates, or inflation is fully anticipated. This is a reasonable initial assumption, and it can be modified.
8. Capital markets are in equilibrium. This means that we begin with all investments properly priced in line with their risk levels.

Development of Capital Market Theory
The major factor that allowed portfolio theory to develop into capital market theory is the concept of a risk-free asset. Following the development of the Markowitz portfolio model, several authors considered the implications of assuming the existence of a risk-free asset, that is, an asset with zero variance. Such an asset would have zero correlation with all other risky assets and would provide the risk-free rate of return $(R F R)$. It would lie on the vertical axis of a portfolio graph.

This assumption allows us to derive a generalized theory of capital asset pricing under conditions of uncertainty from the Markowitz portfolio theory. This achievement is generally attributed to William Sharpe, for which he received the Nobel Prize, but Lintner and Mossin derived similar theories independently. Consequently, you may see references to the Sharpe-Lintner- Mossin (SLM) capital asset pricing model.

## Risk-Free Asset

We have defined a risky asset as one from which future returns are uncertain, and we have measured this uncertainty by the variance, or standard deviation, of expected returns. Because the expected return on a risk-free asset is entirely certain, the standard deviation of its expected return is zero $(\sigma \mathrm{RF}=0)$. The rate of return earned on such an asset should be the risk-free rate of return $(R F R)$ should equal the expected long-run growth rate of the economy with an adjustment for short-run liquidity.

The covariance of the risk-free asset with any risky asset or portfolio of assets will always equal zero. Similarly, the correlation between any risky asset $i$, and the risk-free asset, RF, would be zero.

## Combining a Risk-Free Asset with a Risky Portfolio

What happens to the average rate of return and the standard deviation of returns when you combine a risk-free asset with a portfolio of risky assets such as those that exist on the Markowtz efficient frontier?

$$
\begin{aligned}
& E\left(R_{\mathrm{porr}}\right)=w_{\mathrm{RF}}(R F R)+\left(1-w_{\mathrm{RF}}\right) E\left(R_{i}\right) \\
& \sigma_{\mathrm{port}}^{2}=w_{1}^{2} \sigma_{1}^{2}+w_{2}^{2} \sigma_{2}^{2}+2 w_{1} w_{2} r_{1,2} \sigma_{1} \sigma_{2}
\end{aligned}
$$

Substituting the risk-free asset for Security 1, and the risky asset portfolio for Security 2, this formula would become

$$
\sigma_{\mathrm{port}}^{2}=w_{\mathrm{RF}}^{2} \sigma_{\mathrm{RF}}^{2}+\left(1-w_{\mathrm{RF}}\right)^{2} \sigma_{i}^{2}+2 w_{\mathrm{RF}}\left(1-w_{\mathrm{RF}}\right) r_{\mathrm{RF} i} \sigma_{\mathrm{RF}} \sigma_{i}
$$

We know that the variance of the risk-free asset is zero, that is, $\sigma^{2}{ }_{\text {RF }} \square \square 0$. Because the correlation between the risk-free asset and any risky asset, $i$, is also zero, the factor $r_{\mathrm{RF}, i}$ in the preceding equation also equals zero. Therefore, any component of the variance formula that has either of these terms will equal zero. When you make these adjustments, the formula becomes

$$
\begin{aligned}
\sigma_{\mathrm{port}}^{2} & =\left(1-w_{\mathrm{RF}}\right)^{2} \sigma_{i}^{2} \\
\sigma_{\mathrm{port}} & =\sqrt{\left(1-w_{\mathrm{RF}}\right)^{2} \sigma_{i}^{2}} \\
& =\left(1-w_{\mathrm{RF}}\right) \sigma_{i}
\end{aligned}
$$

Therefore, the standard deviation of a portfolio that combines the risk-free asset with risky assets is the linear proportion of the standard deviation of the risky asset portfolio.

The Risk-Return Combination Because both the expected return and the standard deviation of return for such a portfolio are linear combinations, a graph of possible portfolio returns and risks looks like a straight line between the two assets. (see Exhibit below)

You can attain any point along the straight line $R F R$-A by investing some portion of your portfolio in the risk-free asset $w_{\mathrm{RF}}$ and the remainder ( $1-w_{\mathrm{RF}}$ ) in the risky asset portfolio at Point A on the efficient frontier. This set of portfolio possibilities dominates all the risky asset portfolios on the efficient frontier below Point A because some portfolio along Line RFR-A has equal variance with a higher rate of return than the portfolio on the original efficient frontier.
Likewise, you can attain any point along the Line $R F R$-B by investing in some combination of the risk-free asset and the risky asset portfolio at Point B. Again, these potential combinations dominate all portfolio possibilities on the original efficient frontier below Point B (including Line $R F R-A$ ).

You can draw further lines from the $R F R$ to the efficient frontier at higher and higher points until you reach the point where the line is tangent to the frontier, which occurs in Exhibit at Point M. The set of portfolio possibilities along Line $R F R$-M dominates all portfolios below Point M. For example, you could attain a risk and return combination between the $R F R$ and Point M (Point C) by investing one-half of your portfolio in the risk-free asset (that is, lending money at the $R F R$ ) and the other half in the risky portfolio at Point M.

## PORTFOLIO POSSIBILITIES COMBINING THE RISK-FREE ASSET

 AND RISKY PORTFOLIOS ON THE EFFICIENT FRONTIER

Risk-Return Possibilities with Leverage An investor may want to attain a higher expected return than is available at Point M in exchange for accepting higher risk. One alternative would be to invest in one of the risky asset portfolios on the efficient frontier beyond Point M such as the portfolio at Point D. A second alternative is to add leverage to the portfolio by borrowing money at the risk-free rate and investing the proceeds in the risky asset portfolio at Point M. What effect would this have on the return and risk for your portfolio?

If you borrow an amount equal to 50 percent of your original wealth at the risk-free rate, $w \mathrm{RF}$ will not be a positive fraction but, rather, a negative 50 percent ( $w$ RF $\square \square-0.50$ ). The effect on the expected return for your portfolio is:

$$
\begin{aligned}
E\left(R_{\mathrm{port}}\right) & =w_{\mathrm{RF}}(R F R)+\left(1-w_{\mathrm{RF}}\right) E\left(R_{\mathrm{M}}\right) \\
& =-0.50(R F R)+[1-(-0.50)] E\left(R_{\mathrm{M}}\right) \\
& =-0.50(R F R)+1.50 E\left(R_{\mathrm{M}}\right)
\end{aligned}
$$

The return will increase in a linear fashion along the Line $R F R-\mathrm{M}$ because the gross return increases by 50 percent, but you must pay interest at the $R F R$ on the money borrowed. For example, assume that $E(R F R) \square \square .06$ and $E(R M) \square \square .12$. The return on your leveraged portfolio would be:

$$
\begin{aligned}
E\left(R_{\text {port }}\right) & =-0.50(0.06)+1.5(0.12) \\
& =-0.03+0.18 \\
& =0.15
\end{aligned}
$$



The effect on the standard deviation of the leveraged portfolio is similar.

$$
\begin{aligned}
\sigma_{\text {port }} & =\left(1-w_{\mathrm{RF}}\right) \sigma_{\mathrm{M}} \\
& =[1-(-0.50)] \sigma_{\mathrm{M}}=1.50 \sigma_{\mathrm{M}}
\end{aligned}
$$

where:

## $\sigma_{\mathrm{M}}=$ the standard deviation of the $M$ portfolio

Therefore, both return and risk increase in a linear fashion along the original Line RFR-M, and this extension dominates everything below the line on the original efficient frontier. Thus, you have a new efficient frontier: the straight line from the $R F R$ tangent to Point M. This line is referred to as the capital market line (CML)

Our discussion of portfolio theory stated that, when two assets are perfectly correlated, the set of portfolio possibilities falls along a straight line. Therefore, because the CML is a straight line, it implies that all the portfolios on the CML are perfectly positively correlated. This positive correlation appeals to our intuition because all these portfolios on the CML combine the risky asset Portfolio M and the risk-free asset. You either invest part of your portfolio in the risk-free asset
(i.e., you lend at the $R F R$ ) and the rest in the risky asset Portfolio M , or you borrow at the riskfree rate and invest these funds in the risky asset portfolio. In either case, all the variability comes from the risky asset M portfolio. The only difference between the alternative portfolios on the CML is the magnitude of the variability, which is caused by the proportion of the risky asset portfolio in the total portfolio.

## The Market Portfolio

Because Portfolio M lies at the point of tangency, it has the highest portfolio possibility line, and everybody will want to invest in Portfolio M and borrow or lend to be somewhere on the CML. This portfolio must, therefore, include all risky assets. If a risky asset were not in this portfolio in
which everyone wants to invest, there would be no demand for it and therefore it would have no value.
Because the market is in equilibrium, it is also necessary that all assets are included in this portfolio in proportion to their market value. If, for example, an asset accounts for a higher proportion of the M portfolio than its market value justifies, excess demand for this asset will increase its price until its relative market value becomes consistent with its proportion in the M portfolio.

This portfolio that includes all risky assets is referred to as the market portfolio. It includes not only U.S. common stocks but all risky assets, such as non-U.S. stocks, U.S. and non-U.S. bonds, options, real estate, coins, stamps, art, or antiques. Because the market portfolio contains all risky assets, it is a completely diversified portfolio, which means that all the risk unique to individual assets in the portfolio is diversified away. Specifically, the unique risk of any single asset is offset by the unique variability of all the other assets in the portfolio.

This unique (diversifiable) risk is also referred to as unsystematic risk. This implies that only systematic risk, which is defined as the variability in all risky assets caused by macroeconomic variables, remains in the market portfolio. This systematic risk, measured by the standard deviation of returns of the market portfolio, can change over time if and when there are changes in the macroeconomic variables that affect the valuation of all risky assets. Examples of such macroeconomic variables would be variability of growth in the money supply, interest rate volatility, and variability in such factors as industrial production, corporate earnings, and corporate cash flow.

## How to Measure Diversification

As noted earlier, all portfolios on the CML are perfectly positively correlated, which means that all portfolios on the CML are perfectly correlated with the completely diversified market Portfolio M. This implies a measure of complete diversification. Specifically, a completely diversified portfolio would have a correlation with the market portfolio of $\square 1.00$. This is logical because complete diversification means the elimination of all the unsystematic or unique risk. Once you have eliminated all unsystematic risk, only systematic risk is left, which cannot be diversified away. Therefore, completely diversified portfolios would correlate perfectly with the market portfolio because it has only systematic risk.

An important point to remember is that, by adding stocks to the portfolio that are not perfectly correlated with stocks in the portfolio, you can reduce the overall standard deviation of the portfolio but you cannot eliminate variability. The standard deviation of your portfolio will eventually reach the level of the market portfolio, where you will have diversified away all unsystematic risk, but you still have market or systematic risk. You cannot eliminate the variability and uncertainty of macroeconomic factors that affect all risky assets.

## NUMBER OF STOCKS IN A PORTFOLIO AND THE STANDARD DEVIATION of PORTFOLIO RETURN

Standard Deviation of Return


## The CML and the Separation Theorem

The CML leads all investors to invest in the same risky asset portfolio, the M portfolio. Individual investors should only differ regarding their position on the CML, which depends on their risk preferences.

In turn, how they get to a point on the CML is based on their financing decisions. If you are relatively risk averse, you will lend some part of your portfolio at the $R F R$ by buying some riskfree securities and investing the remainder in the market portfolio of risky assets. For example, you might invest in the portfolio combination at Point A in Exhibit below. In contrast, if you prefer more risk, you might borrow funds at the $R F R$ and invest everything (all of your capital plus what you borrowed) in the market portfolio, building the portfolio at Point B. This financing decision provides more risk but greater returns than the market portfolio. As discussed earlier, because portfolios on the CML dominate other portfolio possibilities, the CML becomes the efficient frontier of portfolios, and investors decide where they want to be along this efficient frontier. Tobin called this division of the investment decision from the financing decision the separation theorem.

Specifically, to be somewhere on the CML efficient frontier, you initially decide to invest in the market Portfolio M, which means that you will be on the CML. This is your investment decision. Subsequently, based on your risk preferences, you make a separate financing decision either to borrow or to lend to attain your preferred risk position on the CML.

## CHOICE OF OPTIMAL PORTFOLIO COMBINATIONS ON THE CML



## THE CAPITAL ASSET PRICING MODEL: EXPECTED RETURN AND RISK

Up to this point, we have considered how investors make their portfolio decisions, including the significant effects of a risk-free asset. The existence of this risk-free asset resulted in the derivation of a capital market line (CML) that became the relevant efficient frontier. Because all investors want to be on the CML, an asset's covariance with the market portfolio of risky assets emerged as the relevant risk measure.
Now that we understand this relevant measure of risk, we can proceed to use it to determine an appropriate expected rate of return on a risky asset. This step takes us into the capital asset pricing model (CAPM), which is a model that indicates what should be the expected or required rates of return on risky assets. This transition is important because it helps you to value an asset by providing an appropriate discount rate to use in any valuation model. Alternatively, if you have already estimated the rate of return that you think you will earn on an investment, you can compare this estimated rate of return to the required rate of return implied by the CAPM and determine whether the asset is undervalued, overvalued, or properly valued.
To accomplish the foregoing, we demonstrate the creation of a security market line (SML) that visually represents the relationship between risk and the expected or the required rate of return on an asset. The equation of this SML, together with estimates for the return on a risk-free asset and on the market portfolio, can generate expected or required rates of return for any asset based on its systematic risk. You compare this required rate of return to the rate of return that you estimate that you will earn on the investment to determine if the investment is undervalued or
overvalued. After demonstrating this procedure, we finish the section with a demonstration of how to calculate the systematic risk variable for a risky asset.

## The Security Market Line (SML)

We know that the relevant risk measure for an individual risky asset is its covariance with the market portfolio (Covi,M). Therefore, we can draw the risk-return relationship as shown in Exhibit below with the systematic covariance variable ( $\mathrm{Covi} i, \mathrm{M}$ ) as the risk measure.

## GRAPH OF SECURITY MARKET LINE



The return for the market portfolio $(R M)$ should be consistent with its own risk, which is the covariance of the market with itself. If you recall the formula for covariance, you will see that the covariance of any asset with itself is its variance, Covi,i$\square \square \sigma^{2} i$. In turn, the covariance of the market with itself is the variance of the market rate of return CovM, $M \square \square \sigma^{2}$ м. Therefore, the equation for the risk-return line in Exhibit is:

$$
\begin{aligned}
E\left(R_{i}\right) & =R F R+\frac{R_{\mathrm{M}}-R F R}{\sigma_{\mathrm{M}}^{2}}\left(\operatorname{Cov}_{i, \mathrm{M}}\right) \\
& =R F R+\frac{\operatorname{Cov}_{i, \mathrm{M}}}{\sigma_{\mathrm{M}}^{2}}\left(R_{\mathrm{M}}-R F R\right)
\end{aligned}
$$

Defining Covi, $\mathrm{M} / \sigma^{2}{ }_{\mathrm{m}}$ (Covariance between security and market returns divided by variance of market returns) as beta, ( $\beta i$ ), this equation can be stated:
$E\left(R_{i}\right)=R F R+\beta_{i}\left(R_{\mathrm{M}}-R F R\right)$
Beta can be viewed as a standardized measure of systematic risk. Specifically, we already know that the covariance of any asset $i$ with the market portfolio $(\operatorname{Covi} i \mathrm{M})$ is the relevant risk measure.

Beta is a standardized measure of risk because it relates this covariance to the variance of the market portfolio. As a result, the market portfolio has a beta of 1 . Therefore, if the $\beta i$ for an asset is above 1.0, the asset has higher normalized systematic risk than the market, which means that it is more volatile than the overall market portfolio.
Given this standardized measure of systematic risk, the SML graph can be expressed as shown in Exhibit below. This is the same graph as in Exhibit above, except there is a different measure of risk. Specifically, the graph in Exhibit below replaces the covariance of an asset's returns with the market portfolio as the risk measure with the standardized measure of systematic risk (beta), which is the covariance of an asset with the market portfolio divided by the variance of the market portfolio.

## GRAPH OF SML WITH NORMALIZED SYSTEMATIC RISK



## Determining the Expected Rate of Return for a Risky Asset

The last equation and the graph in Exhibit above tell us that the expected (required) rate of return for a risky asset is determined by the $R F R$ plus a risk premium for the individual asset. In turn, the risk premium is deter- mined by the systematic risk of the asset ( $\beta i$ ), and the prevailing market risk premium $(R \mathrm{M}-R F R)$. To demonstrate how you would compute the expected or required rates of return, consider the following example stocks assuming you have already computed betas:

| Stock | Beta |
| :---: | :---: |
| A | 0.70 |
| B | 1.00 |
| C | 1.15 |
| D | 1.40 |
| E | -0.30 |

Assume that we expect the economy's $R F R$ to be 6 percent ( 0.06 ) and the return on the market portfolio $(R M)$ to be 12 percent (0.12). This implies a market risk premium of 6 percent ( 0.06 ). With these inputs, the SML equation would yield the following expected (required) rates of return for these five stocks:

$$
\begin{aligned}
E\left(R_{i}\right) & =R F R+\beta_{i}\left(R_{\mathrm{M}}-R F R\right) \\
E\left(R_{\mathrm{A}}\right) & =0.06+0.70(0.12-0.06) \\
& =0.102=10.2 \% \\
E\left(R_{\mathrm{B}}\right) & =0.06+1.00(0.12-0.06) \\
& =0.12=12 \% \\
E\left(R_{\mathrm{C}}\right) & =0.06+1.15(0.12-0.06) \\
& =0.129=12.9 \% \\
E\left(R_{\mathrm{D}}\right) & =0.06+1.40(0.12-0.06) \\
& =0.144=14.4 \% \\
E\left(R_{\mathrm{E}}\right) & =0.06+(-0.30)(0.12-0.06) \\
& =0.06-0.018 \\
& =0.042=4.2 \%
\end{aligned}
$$

As stated, these are the expected (required) rates of return that these stocks should provide based on their systematic risks and the prevailing SML.

Stock A has lower risk than the aggregate market, so you should not expect (require) its return to be as high as the return on the market portfolio of risky assets. You should expect (require) Stock A to return 10.2 percent. Stock B has systematic risk equal to the market's (beta $\square \square 1.00$ ), so its required rate of return should likewise be equal to the expected market return ( 12 percent). Stocks C and D have systematic risk greater than the market's, so they should provide returns consistent with their risk. Finally, Stock E has a negative beta (which is quite rare in practice), so its required rate of return, if such a stock could be found, would be below the $R F R$.
In equilibrium, all assets and all portfolios of assets should plot on the SML. That is, all assets should be priced so that their estimated rates of return, which are the actual holding period rates of return that you anticipate, are consistent with their levels of systematic risk. Any security
with an estimated rate of return that plots above the SML would be considered underpriced because it implies that you estimated you would receive a rate of return on the security that is above its required rate of return based on its systematic risk. In contrast, assets with estimated rates of return that plot below the SML would be considered overpriced. This position relative to the SML implies that your estimated rate of return is below what you should require based on the asset's systematic risk.
In an efficient market in equilibrium, you would not expect any assets to plot off the SML because, in equilibrium, all stocks should provide holding period returns that are equal to their required rates of return. Alternatively, a market that is "fairly efficient" but not completely efficient may misprice certain assets because not everyone will be aware of all the relevant information for an asset.

A superior investor has the ability to derive value estimates for assets that are consistently superior to the consensus market evaluation. As a result, such an investor will earn better rates of return than the average investor on a risk-adjusted basis.

## Identifying Undervalued and Overvalued Assets

Now that we understand how to compute the rate of return one should expect or require for a specific risky asset using the SML, we can compare this required rate of return to the asset's estimated rate of return over a specific investment horizon to determine whether it would be an appropriate investment. To make this comparison, you need an independent estimate of the return outlook for the security based on either fundamental or technical analysis techniques.

Let us continue the example for the five assets discussed in the previous section. Assume that analysts in a major trust department have been following these five stocks. Based on extensive fundamental analysis, the analysts provide the expected price and dividend estimates contained in Exhibit below. Given these projections, you can compute the estimated rates of return the analysts would anticipate during this holding period.

PRICE, DIVIDEND, AND RATE OF RETURN ESTIMATES

| Current Price | Expected Price <br> $\left(P_{t+1}\right)$ | Expected Dividend <br> $\left(D_{t+1}\right)$ | Estimated Future Rate <br> of Return (Percent) |  |
| :---: | :---: | :---: | :---: | :---: |
| Stock | $\left(P_{t}\right)$ | 27 | 0.50 | $10.0 \%$ |
| A | 25 | 42 | 0.50 | 6.2 |
| B | 40 | 39 | 1.00 | 21.2 |
| C | 33 | 65 | 1.10 | 3.3 |
| D | 64 | 54 | - | 8.0 |
| E | 50 |  |  |  |

## COMPARISON OF REQUIRED RATE OF RETURN TO ESTIMATED RATE OF RETURN

|  |  | Required Return | Estimated <br> Return | Estimated Return <br> Minus $E\left(R_{i}\right)$ | Evaluation |
| :---: | :---: | :---: | :---: | :---: | :--- |
| Stock | Beta | $E\left(R_{i}\right)$ | 10.0 | -0.2 | Properly valued |
| A | 0.70 | 10.2 | 6.2 | -5.8 | Overvalued |
| B | 1.00 | 12.0 | 21.2 | 8.3 | Undervalued |
| C | 1.15 | 12.9 | 3.3 | -11.1 | Overvalued |
| D | 1.40 | 14.4 | 8.0 | 3.8 | Undervalued |
| E | -0.30 | 4.2 |  |  |  |

## PLOT OF ESTIMATED RETURNS ON SML GRAPH



This difference between estimated return and expected (required) return is sometimes referred to as a stock's alpha or its excess return. This alpha can be positive (the stock is undervalued) or negative (the stock is overvalued). If the alpha is zero, the stock is on the SML and is properly valued in line with its systematic risk.

Assuming that you trusted your analyst to forecast estimated returns, you would take no action regarding Stock A, but you would buy Stocks C and E and sell Stocks B and D. You might even sell Stocks B and D short if you favored such aggressive tactics.

## Chapter Six

## Portfolio Performance Evaluation

Chapter overview:

- A portfolio manager analyzes the investment opportunities and decides on what to invest in...then forms the portfolio
- After some time, need way to measure how well the portfolio did and how good the manager's decisions were
- A measure of performance must take into account both the return earned and the level of the risk


## Portfolio Performance Measures

Portfolio performance evaluation involves determining periodically how the portfolio performed in terms of not only the return earned, but also the risk experienced by the investor. For portfolio evaluation appropriate measures of return and risk as well as relevant standards (or "benchmarks") are needed. In general, the market value of a portfolio at a point of time is determined by adding the markets value of all the securities held at that particular time.

The essential idea behind performance evaluation is to compare the returns which were obtained on portfolio with the results that could be obtained if more appropriate alternative portfolios had been chosen for the investment. Such comparison portfolios are often referred to as benchmark portfolios. In selecting them investor should be certain that they are relevant, feasible and known in advance. The benchmark should reflect the objectives of the investor.

## I. Simple benchmark index(peer group comparison)

This is probably the most commonly used approach to appraising the performance of a portfolio and involves a simple comparison between the portfolio's return and the return on a benchmark index that has (or is assumed to have) similar risk to the portfolio being measured. For example, a well-diversified portfolio of domestic shares might be benchmarked against the S\&P500 Index, which measures the performance of the shares in the 500 largest companies listed on the NYSE.

The advantages associated with using this approach to performance appraisal are that it is easy to implement and to understand. The main problem with this approach is that it implies that the risk of the portfolio is identical to the risk of the benchmark index, whereas, with the exception of socalled passive funds, which are specifically established to mimic (or track) the performance of benchmark indices, this will rarely be the case.

As noted, peer group comparisons are potentially flawed in the sense that they do not make explicit adjustments for the risk of the portfolios in the comparison.

## II. Risk Adjusted Performance Measures

Investment performance measures combine the return measure of the fund with the risk measure of the fund. There are four well-established portfolio performance measures (Risk Adjusted Performance Measures) used widely in practice:

We now consider four commonly used ways of measuring the performance of a portfolio. Each of these measures has a different approach to trying to determine the 'expected' performance of the benchmark portfolio in order to determine whether the portfolio has met, exceeded or failed to meet expectations.
The four commonly used measures are:

1. Sharpe Ratio
2. Treynor Ratio
3. Jensen's Alpha
4. Information Ratio

To understand how these measures are calculated and what they mean, let's consider a hypothetical situation. Specifically, suppose that you must assess the investment performance of a group of portfolio managers over a given period of time. In executing this task, you will be using a historical data set consisting of ' N ' periodic observations on the following variables:

$$
\mathrm{R}_{\mathrm{pt}}=\text { the period } \mathrm{t} \text { return to the } \mathrm{p} \text {-th portfolio; }
$$

$\mathrm{R}_{\mathrm{mt}}=$ the period t return to a proxy for the market portfolio ;
$R F_{t}=$ the period $t$ return to a risk-free security (i.e., a T-bill).

## 1. Sharpe Ratio:

This measure ranks investment performance on the basis of the portfolio's risk premium earned per unit of risk, where risk is measured by the standard deviation of the set of historical returns (i.e., $\sigma p$ ). That is, for the p-th portfolio calculate:

$$
\begin{gathered}
S_{p}=\frac{\left(R_{p}-R F\right)}{\sigma_{p}}
\end{gathered}
$$

where the numerator is the difference between the historical average periodic returns to the portfolio and the risk-free rate, respectively. In practice, the denominator can be calculated as either the standard deviation of the actual portfolio returns or as the standard deviation of the excess portfolio returns (i.e., the portfolio returns net of the risk-free rate).

The Sharpe ratio is then used to establish an ordinal ranking of managerial performance by listing the values corresponding to each portfolio from highest to lowest.

Example: Assume the market return is $14 \%$ with a standard deviation of $20 \%$, and risk-free rate is $8 \%$. The average annual returns for Managers D, E, and F are $13 \%, 17 \%$, and $16 \%$ respectively. The corresponding standard deviations are $18 \%, 22 \%$, and $23 \%$. What are the Sharpe measures for the market and managers?

Ans:
The Sharpe Measures

$$
\begin{aligned}
& S_{M}=(14 \%-8 \%) / 20 \%=0.300 \\
& S_{D}=(13 \%-8 \%) / 18 \%=0.273 \\
& S_{E}=(17 \%-8 \%) / 22 \%=0.409 \\
& S_{F}=(16 \%-8 \%) / 23 \%=0.348
\end{aligned}
$$

The D portfolio had the lowest risk premium return per unit of total risk, failing even to perform as w0ell as the aggregate market portfolio. In contrast, Portfolio E and F performed better than the aggregate market: Portfolio E did better than Portfolio F.
$>$ An advantage of the Sharpe ratio is that it is relatively easy to compute and widely used. The disadvantages are that it is difficult to interpret and does not permit precise statistical comparisons between portfolios.

## 2. Treynor Ratio:

Like the Sharpe ratio, the Treynor measure assesses performance on the basis of a ratio of average excess return to risk. The difference is that Treynor considers only the systematic component of a portfolio's risk to be relevant. Letting $\beta$ p be the portfolio's beta coefficient, the Treynor ratio is calculated as follows:

$$
\mathrm{Tp}=\frac{\left(\mathrm{R}_{\mathrm{p}}-\mathrm{RF}\right)}{\beta_{\mathrm{p}}}
$$

Example: Assume the market return is $14 \%$ and risk-free rate is $8 \%$. The average annual returns for Managers $\mathrm{W}, \mathrm{X}$, and Y are $12 \%, 16 \%$, and $18 \%$ respectively. The corresponding betas are $0.9,1.05$, and 1.20. What are the the Treynor ratio ( T values) for the market and managers?

Ans:

$$
\mathrm{T}_{\mathrm{M}}=(14 \%-8 \%) / 1=6 \%
$$

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{W}}=(12 \%-8 \%) / 0.9=4.4 \% \\
& \mathrm{~T}_{\mathrm{X}}=(16 \%-8 \%) / 1.05=7.6 \% \\
& \mathrm{~T}_{\mathrm{Y}}=(18 \%-8 \%) / 1.20=8.3 \%
\end{aligned}
$$

Like the Sharpe measure, Tp produces an ordinal ranking of performance. (In fact, if all the portfolios being ranked are fully diversified, the Sharpe and Treynor indexes will create the same ranking.)

The Treynor ratio is not as easy to compute as the Sharpe ratio (i.e., it requires the calculation of the portfolio's beta coefficient) but is based on a widely accepted measure of risk. Similar to $S_{p}$, the disadvantages of Treynor's measure are that it is difficult to interpret and does not permit precise statistical comparisons between portfolios.

## 3. Jensen's Alpha:

Unlike the previous two measures, which summarize the historical return data by taking simple averages, the Jensen procedure estimates the coefficients of the following time-series regression for each portfolio:

$$
\left(\mathrm{R}_{\mathrm{pt}}-\mathrm{RF}_{\mathrm{t}}\right)=\alpha_{\mathrm{p}}+\beta_{\mathrm{p}}\left(\mathrm{R}_{\mathrm{mt}}-\mathrm{RF}_{\mathrm{t}}\right)+\varepsilon_{\mathrm{t}} ; \mathrm{t}=1, \ldots, \mathrm{~N}
$$

In this procedure, $\alpha$ p is the performance index. According to the CAPM, Jensen's alpha should be equal to zero. Thus, if it is significantly above (below) zero, you can conclude that the portfolio manager has significantly outperformed (underperformed) the market, after adjusting for the risk of his or her investment.

There are three advantages to Jensen's alpha as a performance measure: (i) since it is a byproduct of a regression, its statistical validity can be established directly, (ii) it can be interpreted as the level of return that the manager generated in excess (deficient) of what he or she should have earned given the risk of the investment, and (iii) it can be adapted to other models of estimating expected returns besides the CAPM (e.g., Fama-French three-factor model).

## 4. Information Ratio

The information ratio (also known as an appraisal ratio) measures a portfolio's average return in excess of that of a comparison or benchmark portfolio divided by the standard deviation of this excess return. Formally, the information ratio (IR) is calculated as:

Information Ratio $=\alpha_{p} / s\left(e_{p}\right)$

$$
\mathrm{R}_{\mathrm{p}, \mathrm{t}}-\mathrm{r}_{\mathrm{f}, \mathrm{t}}=\alpha_{p}+\beta_{p}\left(\mathrm{R}_{\mathrm{M}, \mathrm{t}}-r_{f, t}\right)+\varepsilon_{p, t}
$$

$\checkmark$ Note that : Nonsystematic risk could, in theory, be eliminated by diversification

Exercise: Consider the following data for a particular sample period:

|  | Portfolio $P$ | Market $M$ |
| :--- | :--- | :--- |
| Average return | $35 \%$ | $28 \%$ |
| Beta | 1.20 | 1.00 |
| Standard deviation | $42 \%$ | $30 \%$ |
| Nonsystematic risk, $\sigma(e)$ | $18 \%$ | 0 |

The T-bill rate during the period was $6 \%$.

Required : Compute the Sharp ratio Treynor ration, Jensens alpha and Information ratio for portfolio P and Market M and evaluate the performance of the portfolio with the market

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