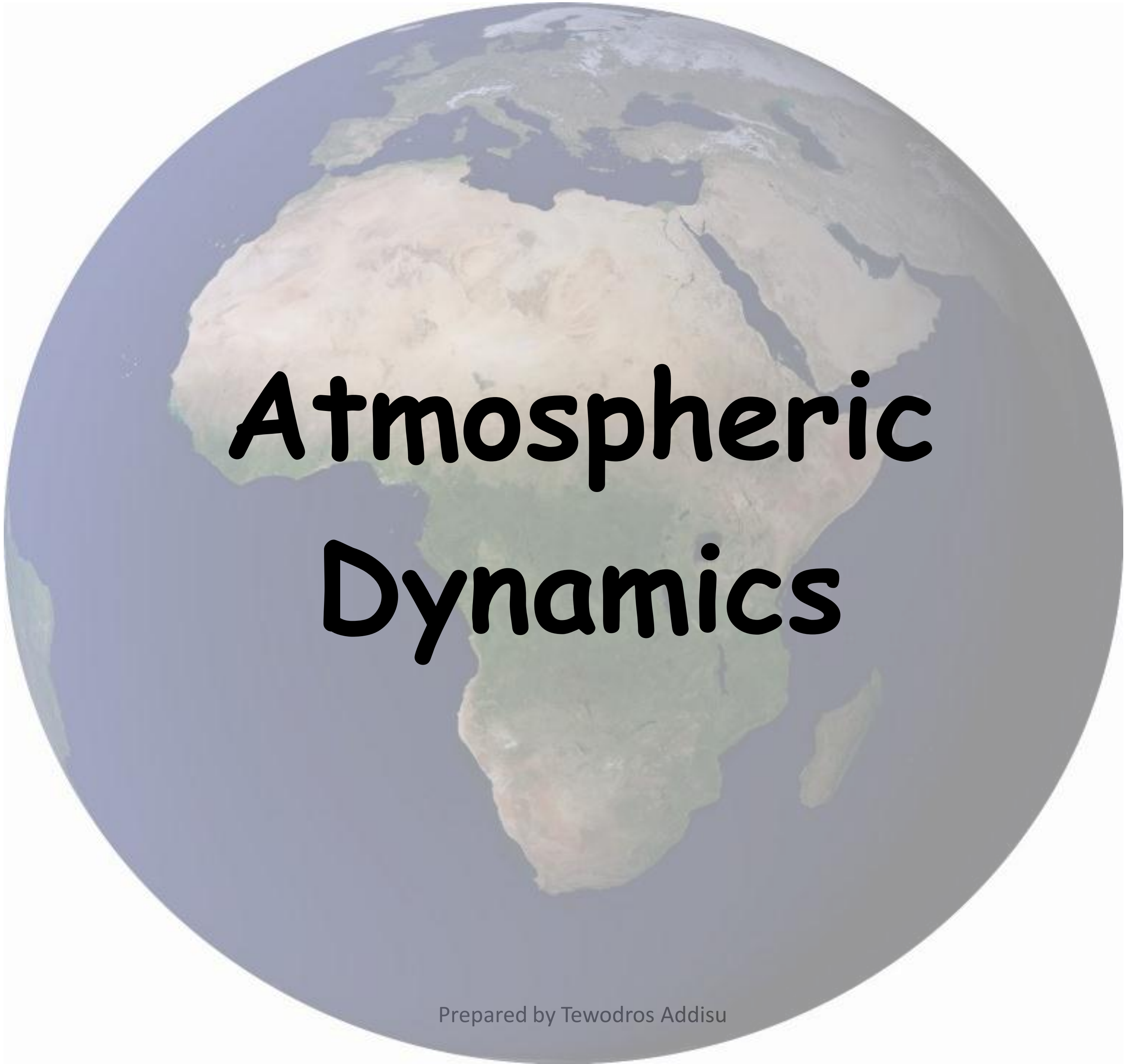


Atmospheric Dynamics

1 Introduction

1.1 The Atmospheric Continuum

- Atmospheric Dynamic is the study of those motions of the atmosphere that are associated with weather and climate.
- For all such motions the discrete molecular nature of the atmosphere can be ignored, and the atmosphere can be regarded as a continuous fluid medium, or continuum.
- A “point” in the continuum is regarded as a volume element that is very small compared with the volume of atmosphere under consideration, but still contains a large number of molecules.

- The expressions **air parcel** and **air particle** are both commonly used to refer to such a point.
- The various physical quantities that characterize the state of the atmosphere (e.g., pressure, density, temperature) are assumed to have unique values at each point in the atmospheric continuum.
- Moreover, these field variables and their derivatives are assumed to be continuous functions of space and time.

- The fundamental laws that govern the motions of the atmosphere satisfy the principle of dimensional homogeneity:

  i.e all terms in the equations have the same physical dimensions.

- International system of units (SI):

SI Base Units

Property	Name	Symbol
Length	Meter (meter)	m
Mass	Kilogram	kg
Time	Second	s
Temperature	Kelvin	K

- SI Derived Units with Special Names:

SI Derived Units with Special Names

Property	Name	Symbol
Frequency	Hertz	Hz (s^{-1})
Force	Newton	N ($kg\ m\ s^{-2}$)
Pressure	Pascal	Pa ($N\ m^{-2}$)
Energy	Joule	J (N m)
Power	Watt	W ($J\ s^{-1}$)

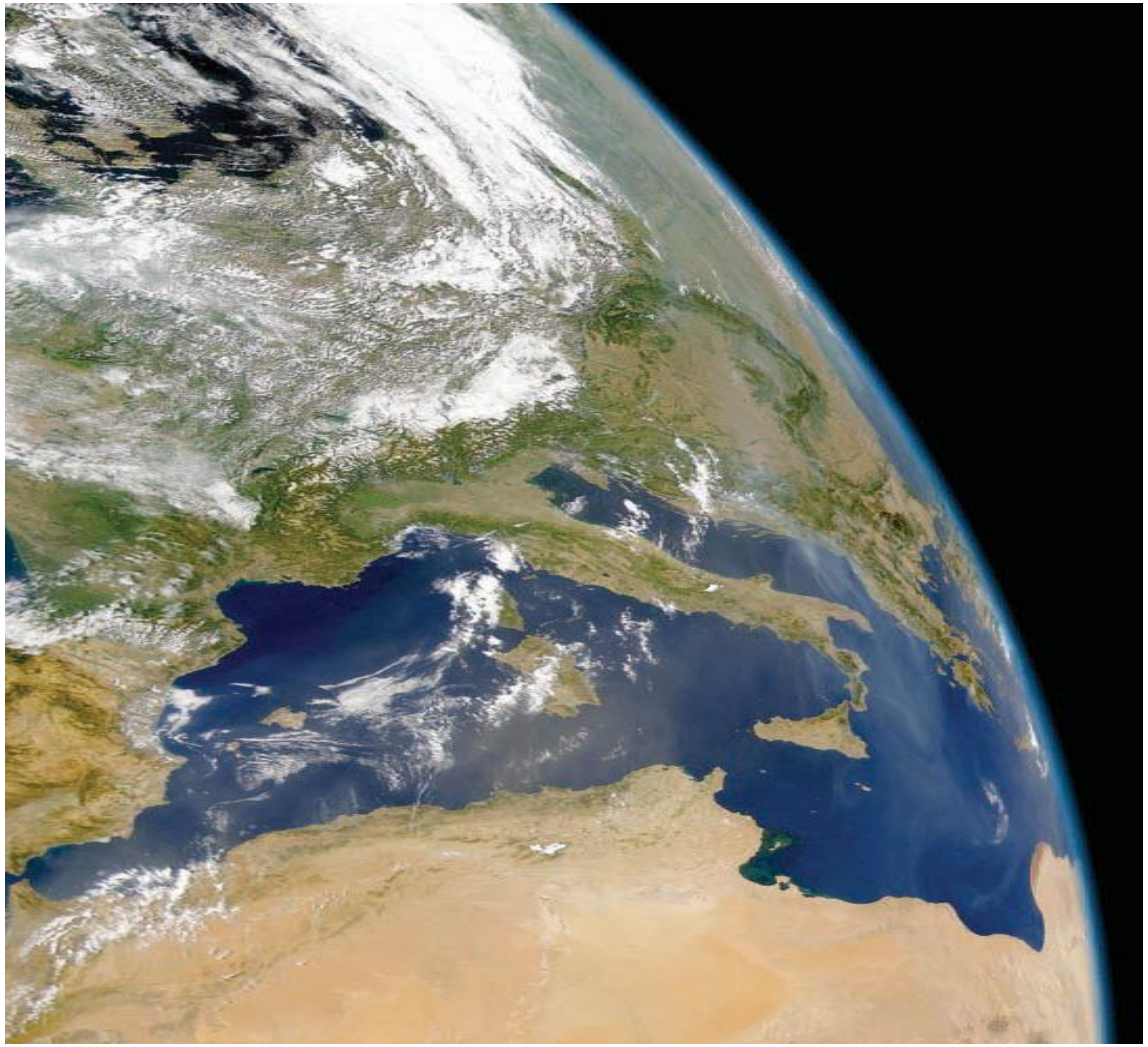
- Not all derived units have special names: for example, velocity has the derived units of meter per second (m /s)

- In order to keep numerical values within convenient limits, it is conventional to use decimal multiples and submultiples of SI units.
- Prefixes used to indicate such multiples and submultiples:

Prefixes for Decimal Multiples and Submultiples of SI Units

Multiple	Prefix	Symbol
10^6	Mega	M
10^3	Kilo	k
10^2	Hecto	h
10^1	Deka	da
10^{-1}	Deci	d
10^{-2}	Centi	c
10^{-3}	Milli	m
10^{-6}	Micro	μ

1.2 Overview and vertical structure



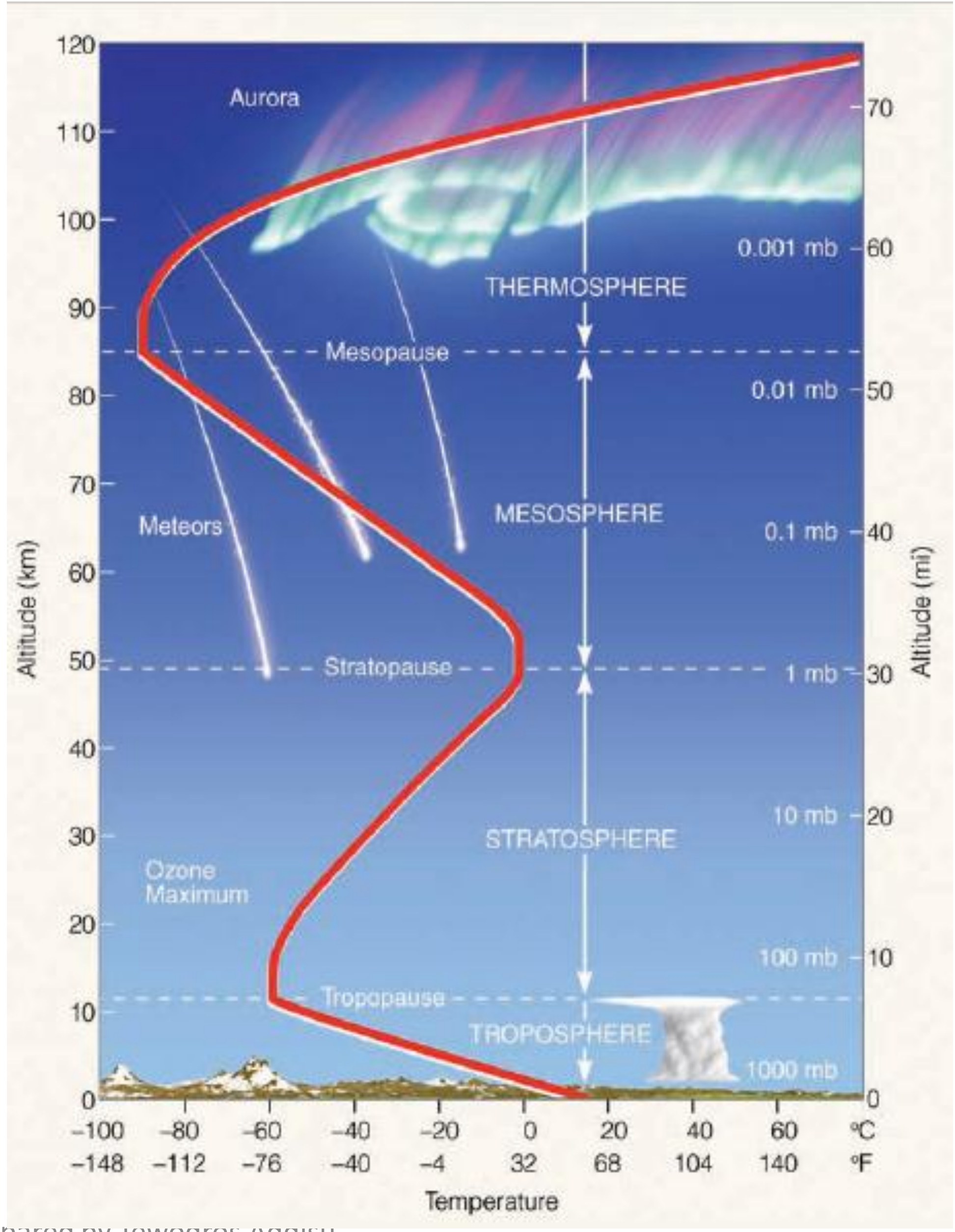
- The earth's atmosphere as viewed from space.
- The atmosphere is the thin **blue region** along the edge of the earth

- It is this thin blanket of air that constantly shields the surface and its inhabitants from the **sun's dangerous ultraviolet** radiant energy
- 99 % of the atmosphere lies within 30 km height

- We can consider the air that surrounds the Earth to be made up of 'columns', rising vertically from each location.
- As we move up through the column, the properties of the air change – its temperature, moisture, cloudiness, chemical constituents, and density all vary.

Temperature

- *Air temperature* is the degree of hotness or coldness of the air and
  it is also a measure of the average speed of the air molecules.

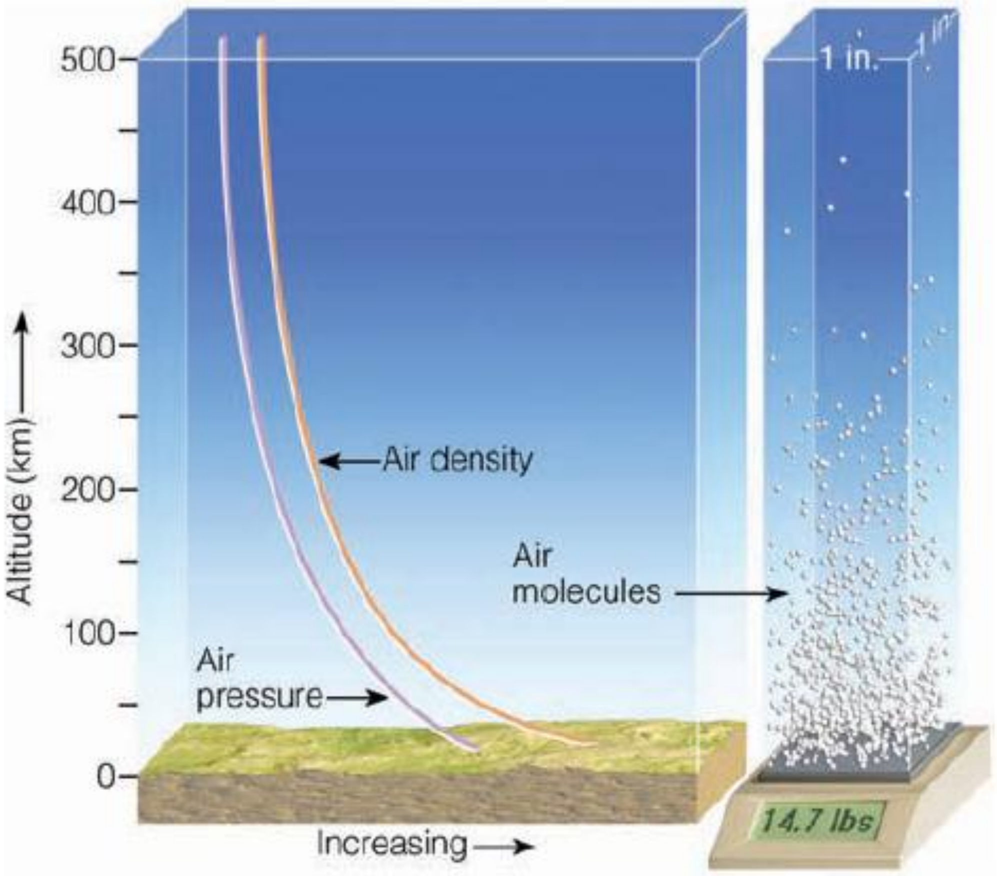


- Air molecules are in constant motion giving a tiny push every time they bounce against an object.
- This small force (push) divided by the area on which it pushes is called **pressure;** thus

$$Pressure = \frac{Force}{Area}$$

- If we weigh a column of air 1 square inch in cross section, extending from the sea level to the “top” of the atmosphere, it would weigh nearly **14.7 pounds.**
- Thus, normal atmospheric pressure near sea level is close to 14.7 pounds per square inch **(1012.83 mb or hpa).**

- *air pressure decreases with increasing altitude*



- At sea level the normal atmospheric pressure is 1012 mb or hp

  (1hp = 100 pa = 1mb)

- Horizontal and vertical variations in pressure give rise to the atmospheric motions.
- Consider some air in a container: the pressure of the air on the walls of the container derives from the momentum of individual molecules as they impact the walls in their random molecular motion.

- If we add more molecules to the container, and that container happens to be a balloon, the difference in pressure between the
  interior and the exterior of the balloon will cause it to expand until a new equilibrium
  is reached

- Similarly, one infinitesimal volume parcel of air exerts pressure on its neighbor, and vice versa, and this force is always perpendicular to the interface between the parcels.

- Hence, the pressure depends not only on the force imparted by the molecules, but also upon the area over which the force is acting.

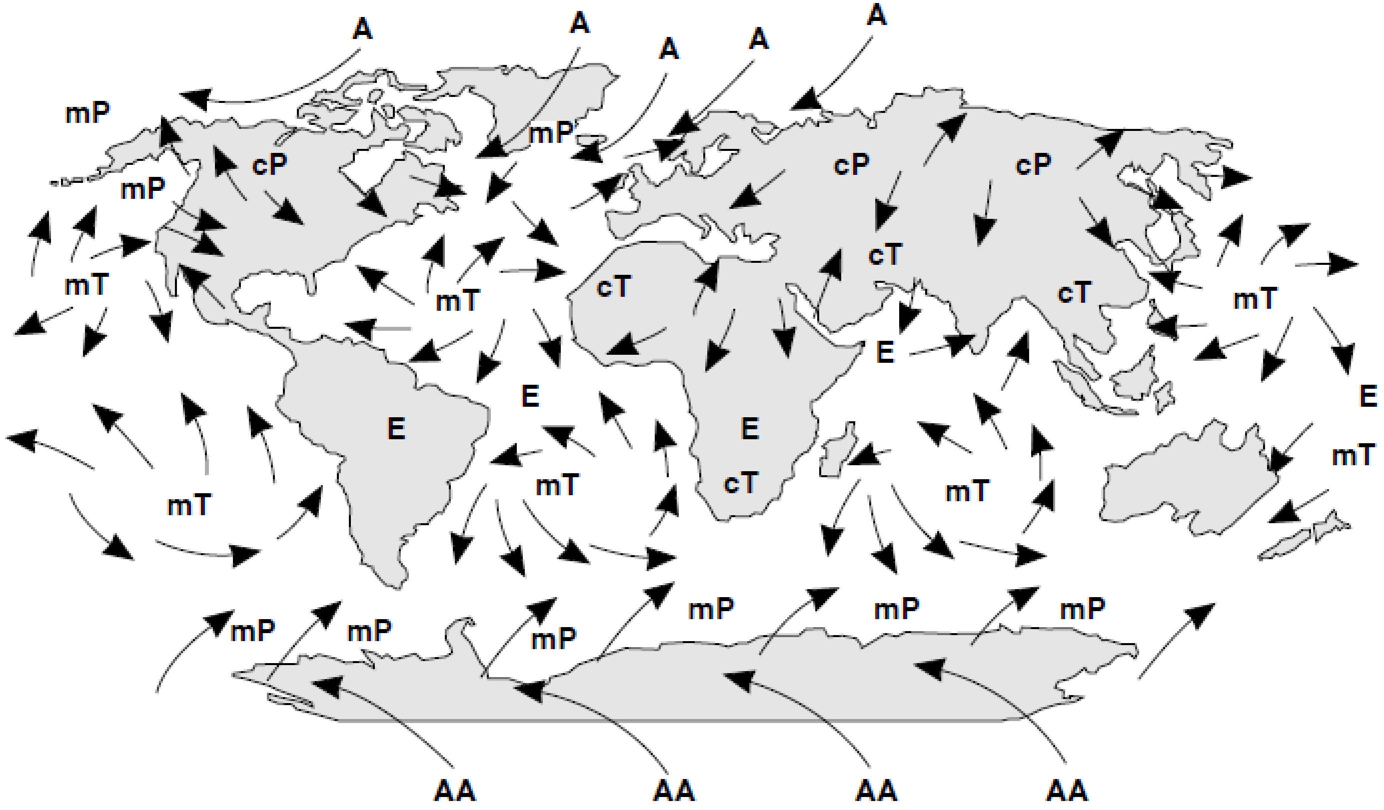
1.3 Air masses and Fronts

Air masses

- An air mass is a large mass of air that has similar characteristics of temperature and humidity within it.
- An air mass acquires these characteristics above an area of land or water (known as its source region).
- The best conditions for the formation of air masses are large areas where air can be in relatively constant conditions long enough to take on quite uniform characteristics.
- When the air mass sits over a region for several days, or longer, it picks up the distinct temperature and humidity characteristics of that region.

- Region where an air mass receives it’s characteristics of temperature and humidity is called the source region.
- The warm, moist tropical oceans and the cold, dry polar land masses are excellent source regions.
- Air masses are classified based on their temperature and humidity characteristics.
- A commonly used classification of air masses, especially in the Northern Hemisphere, is that of Tor Bergeron of the Norwegian School:
    - **Maritime tropical (mT):** moist, warm air mass
    - **Continental tropical (cT):** dry, warm air mass
    - **Maritime polar (mP):** moist, cold air mass
    - **Continental polar (cP):** dry, cold air mass

- Often, additional air masses such as the Arctic (A), Antarctic (AA), and equatorial (E) are added to this list.



A : Arctic – very cold and dry
cP : continental polar – cold and dry
mP : maritime polar – cold and moist
E : equatorial – very warm and moist
mT : maritime tropical – warm and moist
cT : continental tropical – warm and dry
AA : Antarctic – very cold and dry

- Once an air mass moves out of its source region, it begins to be modified as it encounters surface conditions different from those found in the source region.

Fronts

- Fronts are the boundaries that separate different air masses, and are defined by thermodynamic differences across the boundary, and the direction of movement of the boundary.
- Typically fronts separate warm and cold air masses.

- If the cold air mass is advancing and the warm air mass is retreating the boundary is called a cold front.
- If the opposite occurs, with warm air advancing and cold air retreating, the boundary is called a warm front.
- Sometimes the boundary between the two air masses is nearly stationary and this type of front is called a stationary front.
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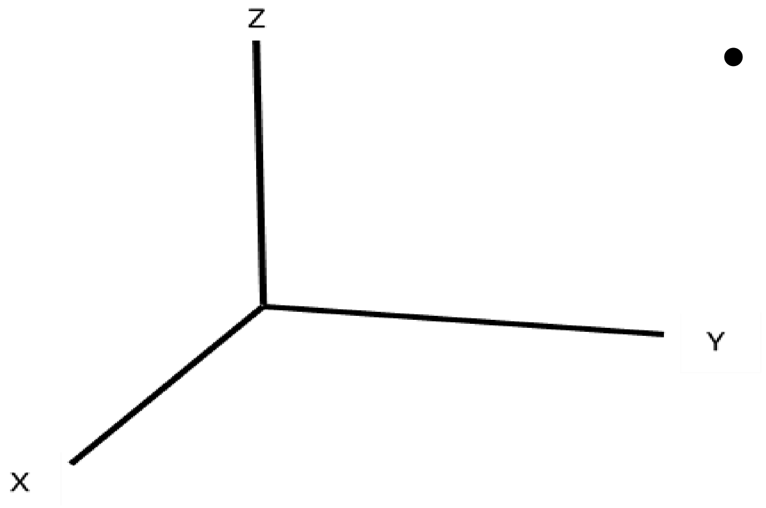
1.4 Coordinate Systems

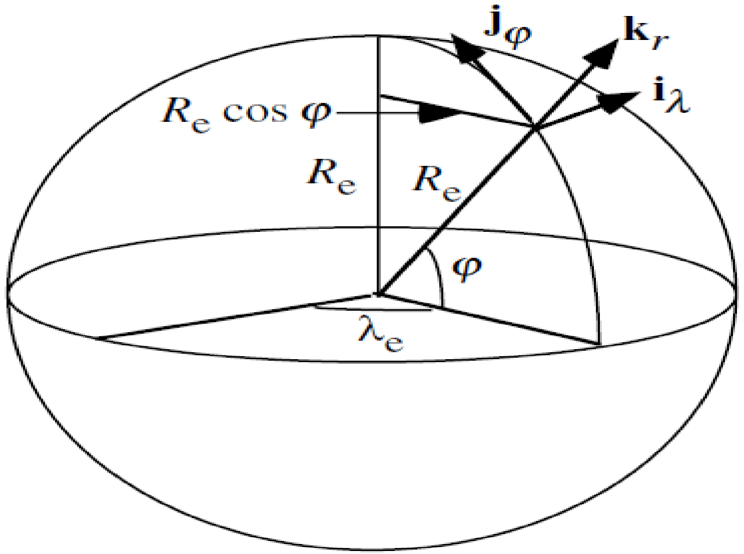
- The most widely used horizontal coordinate system in atmospheric modeling equations are Cartesian (rectangular) and spherical coordinate systems
- Cartesian (rectangular) coordinates are used on the microscale and mesoscale to simulate flow.



Cartesian Coordinate system

- Over long distances, curvature of the earth prevents the accurate division of the earth in to contiguous set of rectangle

- Spherical coordinate system



- Re = Earth's radius
- φ = latitude
- λe = longitude
  - 00 Longitude…Greenwich
  - 00 Latitude….equator

- The spherical coordinate system divides the Earth into longitudes (meridians), which are S-N lines, and latitudes (parallels), which are W-E lines parallel to each other.
- Advantage: it takes curvature of the earth in to account.

- Where as when we see vertical coordinate sytsem: altitude (geometrical height) is the common vertical coordinate for both cartesian and spherical coordinates
- Others include:
    - Pressure Coordinate System
    - Sigma Coordinate System

- Where as when we see vertical coordinate system: altitude (geometrical height) is the common vertical coordinate for both cartesian and spherical coordinates
- Others include:
    - Pressure Coordinate System
    - Sigma Coordinate System

- Pressure as a vertical coordinate system:
  - The thermodynamic state of the atmosphere at any point is determined by the values of pressure, temperature, and density (or specific volume) at that point.
  - These field variables are related to each other by the equation of state for an ideal gas.
  - Letting p, T, ρ, and $\alpha (\equiv 1/\rho)$ denote pressure, temperature, density, and specific volume, respectively, we can express the equation of state for dry air as
  - $p\alpha = RT$ or $p = \rho RT$..........ideal gas law
  - Where R is the gas constant for dry air ($R = 287\ J\ kg^{-1}\ K^{-1}$).

- In the atmosphere in the vertical, pressure is easier to measure than height.
- Often, the location of a parcel of air, or other characteristics of the air, is expressed in terms of the pressure where it is located rather than the height above some surface (as sea level).
- From the hydrostatic equation,

$$\frac{dp}{dz} = -\rho g$$

- It is clear that a single valued monotonic relationship exists between valued monotonic relationship exists between pressure and height in each vertical column of the atmosphere.

2 Fundamental Forces

- To understand the atmospheric dynamics, understanding the different forces responsible for the movement of air parcel is required.

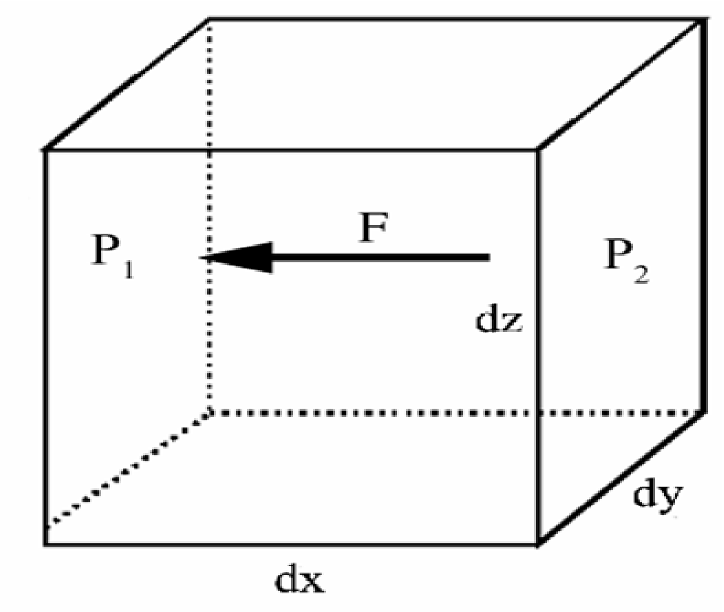
- These forces include
    - Pressure gradient force,
    - Coriolis forces,
    - Centrifugal force,
    - Gravitational force and
    - Frictional force.

Pressure Gradient Force

- The physical concept of pressure is the weight of atmosphere in a column of 1 square inch cross section extending from the required height to the top of the atmosphere.

- A Pressure *Gradient* is a measure of how the pressure is changing as one moves in all direction.
- Pressure gradient force is a force due to horizontal pressure differences.
- Because of this force, water moves from areas of high pressure to areas of low pressure.
- Hence, this force comes with minus sign in different numerical equations.

$$\frac{D\mathbf{v}}{Dt} = \boxed{-\frac{1}{\rho}\nabla p} - 2\mathbf{\Omega} \times \mathbf{v} + \mathbf{g} + \mathbf{F}_r$$

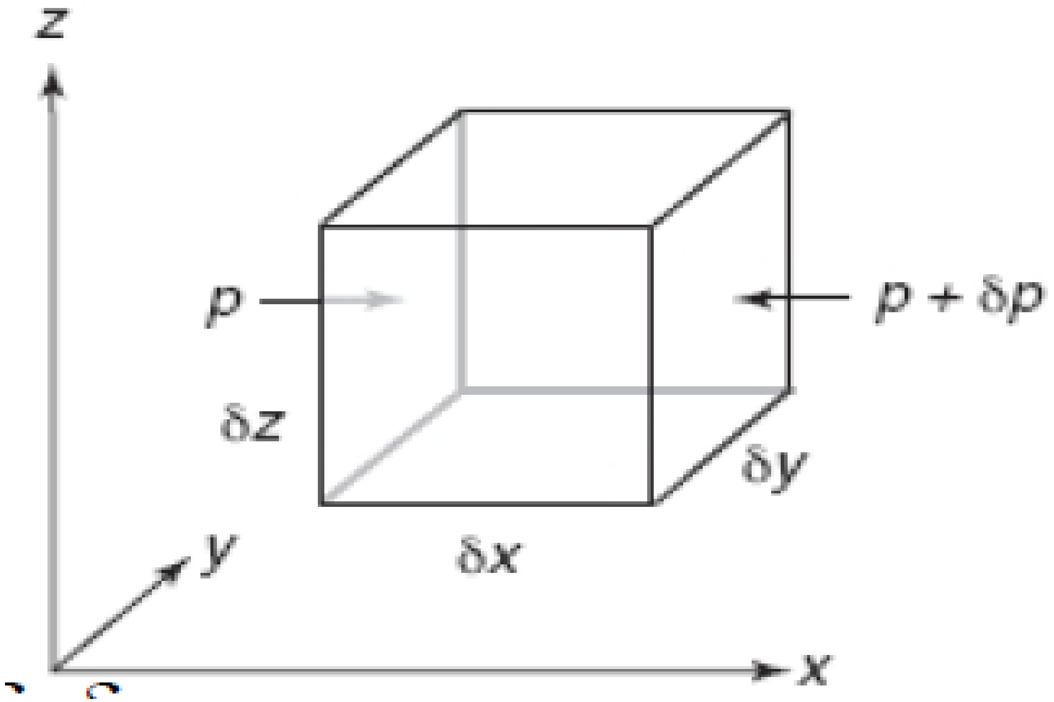


- According to the Fig. above, pressure gradient force between faces 1 and 2 is:

$$F = -(P_2 - P_1)dy.dz = -\Delta P dy.dz.$$

Derivation of PGF:

- Consider a rectangular volume having sides of δx, δy, δz.



- Then the net force in the x-direction due to air pressure is,

$$\Delta F_x = \mathrm{p}\delta y\delta z - (p + \delta p)\delta y\delta z$$

$$\Delta F_x = -\delta p\delta y\delta z \text{ .... } \textbf{\textit{equation 1}}$$

- The small pressure incremental (δp) can be expressed in terms of rate and distance

$$\delta p = \frac{\partial p}{\partial x}\delta x$$

- Thus equation 1 becomes,

$$\Delta F_x = -\frac{\partial p}{\partial x}\delta x\delta y\delta z$$

$$= -\frac{\partial p}{\partial x}\delta V \qquad \text{..................... } \textbf{\textit{equation 2}}$$

- Dividing equation 2 by mass of the fluid in the box, we get

$$\frac{\Delta F_x}{\delta m} = -\frac{\partial p}{\partial x}\frac{\delta V}{\delta m} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

- Forces in y and z directions derived similarly,

$$\frac{\Delta F_y}{\delta m} = -\frac{\partial p}{\partial y}\frac{\delta V}{\delta m} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\Delta F_z}{\delta m} = -\frac{\partial p}{\partial z}\frac{\delta V}{\delta m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

- Therefore the total pressure gradient force per unit mass in all the three direction would be:

$$= -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}i + \frac{\partial p}{\partial y}j + \frac{\partial p}{\partial z}k\right) \quad \ldots \;\; equation\ 4$$

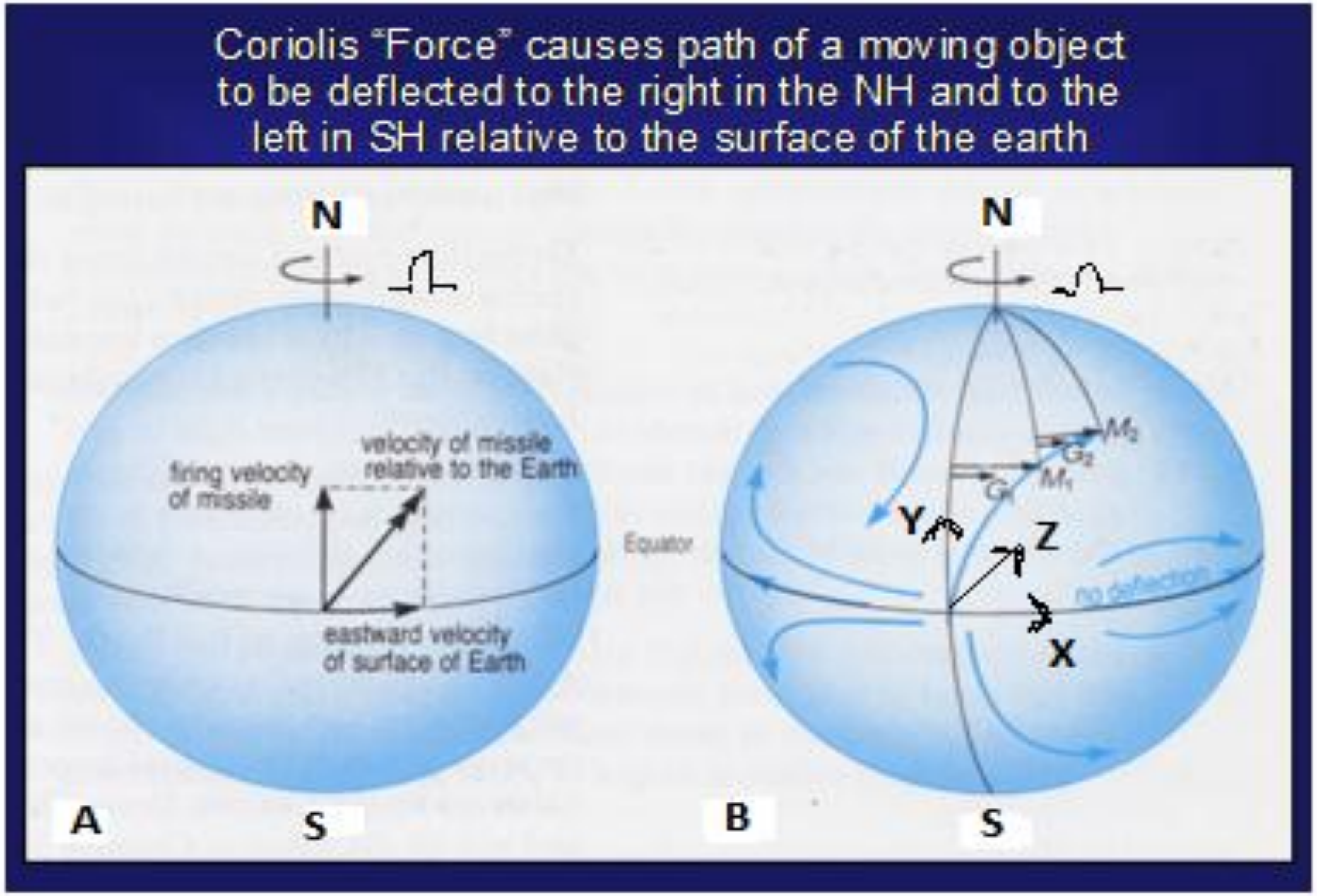
- Using dell operator ($\nabla = \frac{\partial(\quad)}{\partial x}\mathrm{i} + \frac{\partial(\quad)}{\partial y}\mathrm{j} + \frac{\partial(\quad)}{\partial z}k$), equation 4 can be rewritten as:

$$= -\frac{1}{\rho}\nabla p \ldots \text{equation } 5$$

- The minus sign indicates that as the pressure increases in one direction, the pressure force acts in opposite direction.

Coriolis force

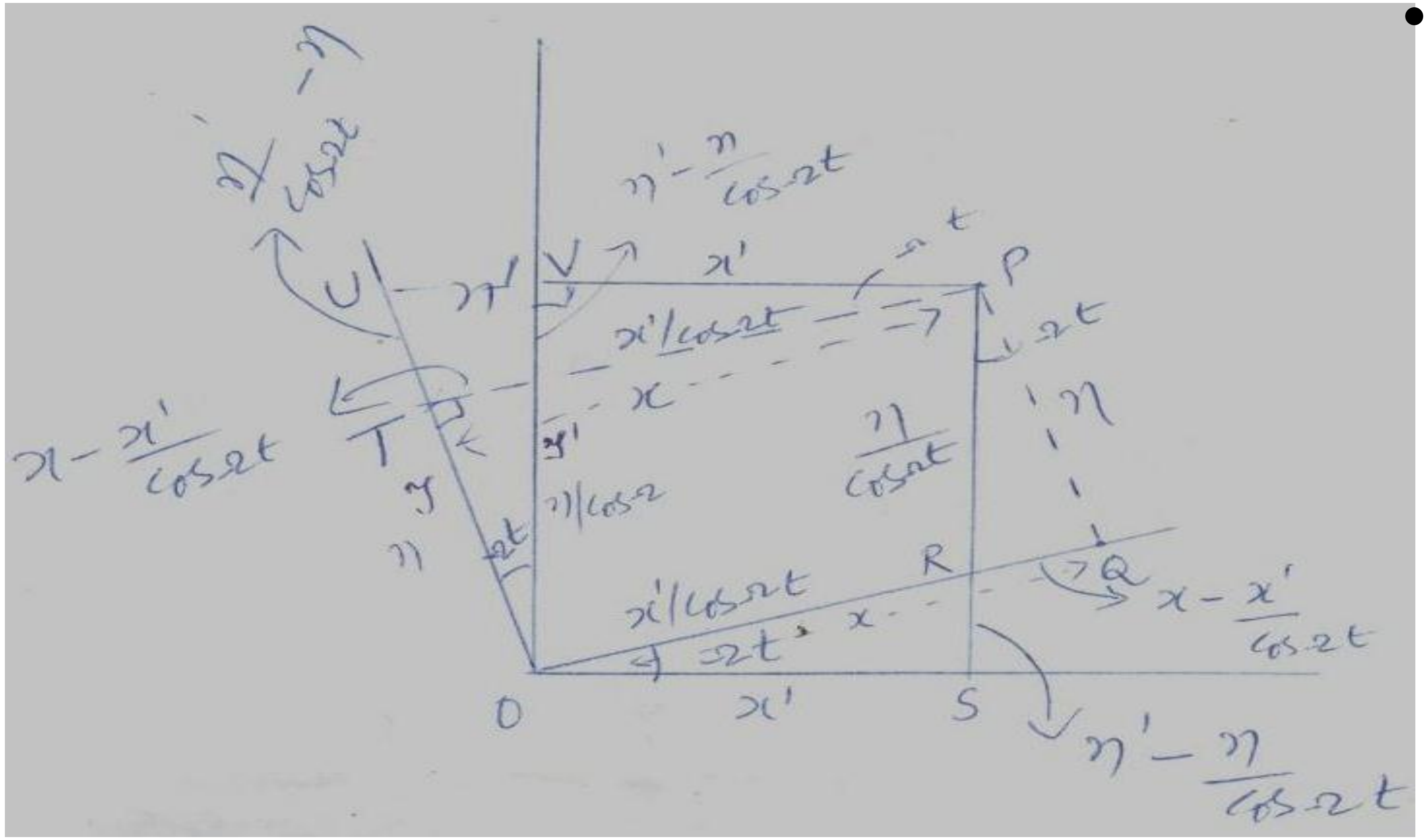
- It is a force acting on a moving object due to the rotation of the earth.
- The effect of this force is proportional to the speed the object
- It deflects moving objects to the right hand side in the Northern Hemisphere, and to the left hand side in the Southern Hemisphere.
- The maganitude of coriolis force is $2\Omega sin\varphi$ where Ω is angular velocity of Earth and φ is the latitude of the object.
- It is biggest at the poles, and goes to zero on the equator, if the motion is parallel to the earth's surface (not vertical).



- The Coriolis force is fundamental for all large-scale motions in ocean and atmosphere..

Derivation of Coriolis force

- Let us consider a coordinate system with the coordinates x', y' and z'as shown in Fig



- Let the coordinate system rotated by an angle Ωt and the new coordinates are x,y and z respectively.

- The new coordinates can be expressed in terms of the old coordinates as (*refer you notes for the derivation*):

$$x' = \mathrm{x}\cos\Omega t - \mathrm{y}\sin\Omega t \ldots \mathrm{eq}\ 6$$
$$y' = x\sin\Omega t + y\cos\Omega t$$
$$z' = z$$

- As the rotation is horizontal, no change along z axis.
- Similarly in place of X, Y, and Z if forces F_x, F_y, and F_z are considered we can write

$$F_x{}' = F_x\cos\Omega t - F_y\sin\Omega t \ldots eq\ 7$$
$$F_y{}' = F_x\sin\Omega t + F_y\cos\Omega t$$
$$F_z{}' = F_z$$

- If we differentiate equation 6 twice with respect to time, we get

$$\ddot{x}' = \cos\Omega t(\ddot{x} - 2\dot{y}\Omega - x\Omega^2) - \sin\Omega t\,(\ddot{y} - 2\dot{x}\Omega - y\Omega^2)$$

- Dividing all sides of equation 7 by mass:

$$\frac{F_x{}'}{m} = \frac{F_x}{m}\cos\Omega t - \frac{F_y}{m}\sin\Omega t$$

- Note that the left side of the above two equations is acceleration.
- Hence, equating the two equations and comparing the coefficients of $\cos\Omega t$ and $\sin\Omega t$, we get:

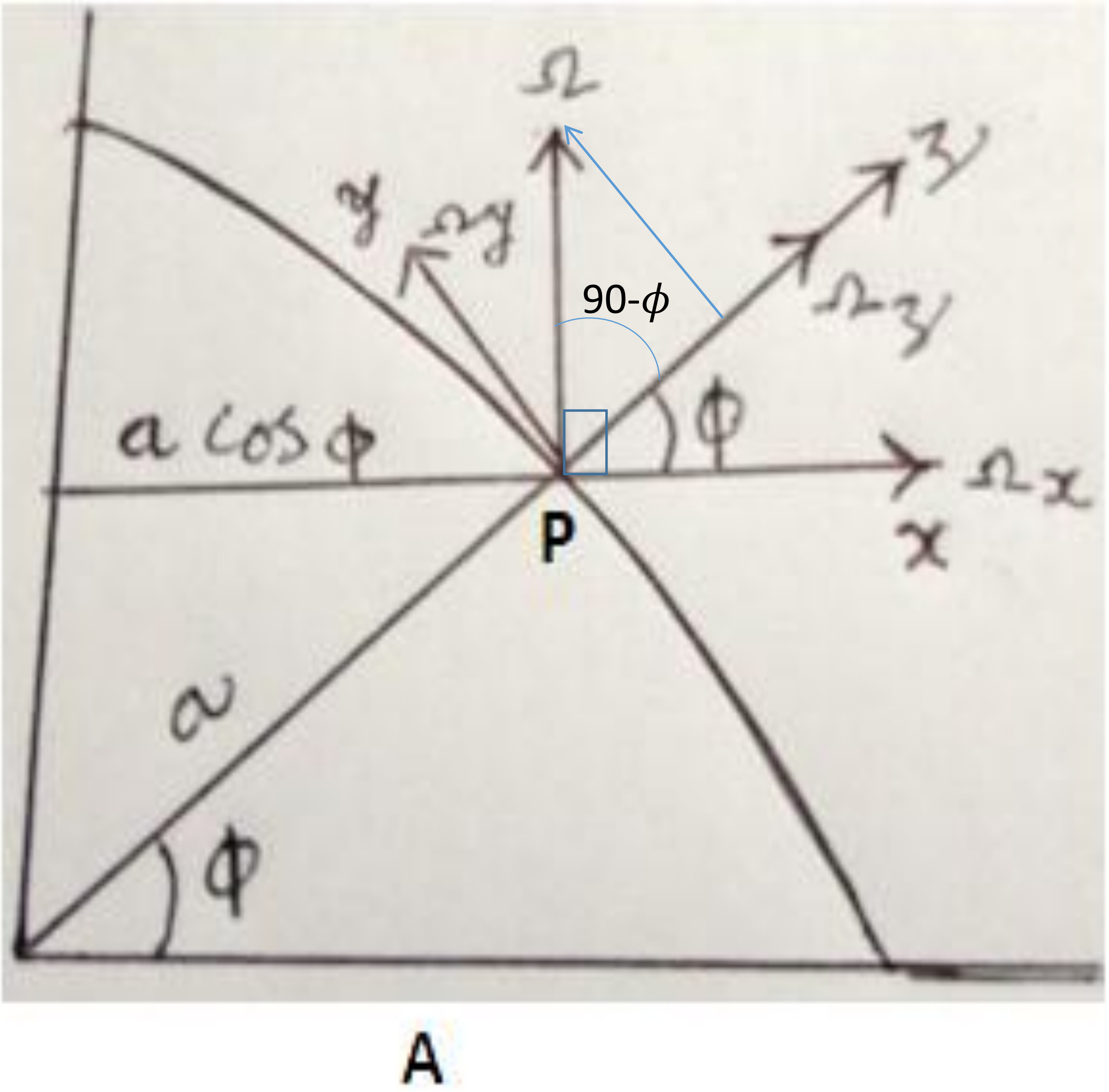
$$\frac{F_x}{m} = \ddot{x} - 2\dot{y}\Omega - x\Omega^2 \ldots \mathrm{eq}\ 8$$

$$\frac{F_y}{m} = \ddot{y} - 2\dot{x}\Omega - y\Omega^2 \ldots eq\ 9$$

- The last of the terms on the right hand side ($x\Omega^2$, $y\Omega^2$) denote the centrifugal accelerations pointing radially outward.
- The second terms on the right hand side ($2\dot{y}\Omega$, $2\dot{x}\Omega$) represent accelerations owing to the combined effects of rotation of the coordinate axes and motion of the particle, relative to the rotating coordinate system.
- These are called Coriolis accelerations.

- We shall now apply these results on a rotating spherical earth, in which x is tangent eastward, y meridionally northward and z radially upward into the atmosphere.

- Let ‘P’ be a point on the earth’s surface which is rotating with the angular velocity Ω, at a distance of ‘$a \cos \varphi$’ from the axis of rotation where ‘a’ is the radius of the earth and ‘φ’ is the latitude.



- From the figure, we see that

$$\Omega_x = \Omega \cos 90 = 0$$
$$\Omega_y = \Omega \sin(90 - \varphi)$$
$$= \Omega \cos \varphi$$
$$\Omega_z = \Omega \cos(90 - \varphi)$$
$$= \Omega \sin \varphi$$

- Considering coriolis acceleration only of eq 8 & 9:

$$\frac{F_x}{m} = -2\dot{y}\Omega$$
$$\frac{F_y}{m} = -2\dot{x}\Omega$$

- This means the coriolis acceleration on the earth can be written as

$$-2\Omega \times V$$

$$c = -2\Omega \times V = -2\Omega \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \end{array} \begin{bmatrix} 0 & \cos\varphi & \sin\varphi \\ u & v & w \end{bmatrix}$$

- Where

$$\Omega = \Omega_x i + \Omega_y j + \Omega_z k$$
$$V = ui + vj + wk$$

- Solving the matrix, we get

$$c_x = -2\Omega w \cos\varphi + 2\Omega v \sin\varphi$$
$$c_y = -2\Omega u \sin\varphi$$
$$c_z = 2\Omega u \cos\varphi$$

- For synoptic scale motions, $2\Omega w \cos\varphi$ and $2\Omega u \cos\varphi$ can be neglected as they are relatively very small compared to the others.

- Finally, we get components of the Coriolis force due to relative motion along a latitude circle as:

$$\left(\frac{Du}{Dt}\right)_{co} = 2\Omega v \sin\varphi = fv$$

$$\left(\frac{Dv}{Dt}\right)_{co} = -2\Omega u \sin\varphi = -fu$$

$$\left(\frac{Dw}{Dt}\right)_{co} = 0$$

Where $f = 2\Omega \sin\varphi$ is the coriolis parameter

- The Coriolis force is negligible for motions with time scales that are very short compared to the period of the earth's rotation.

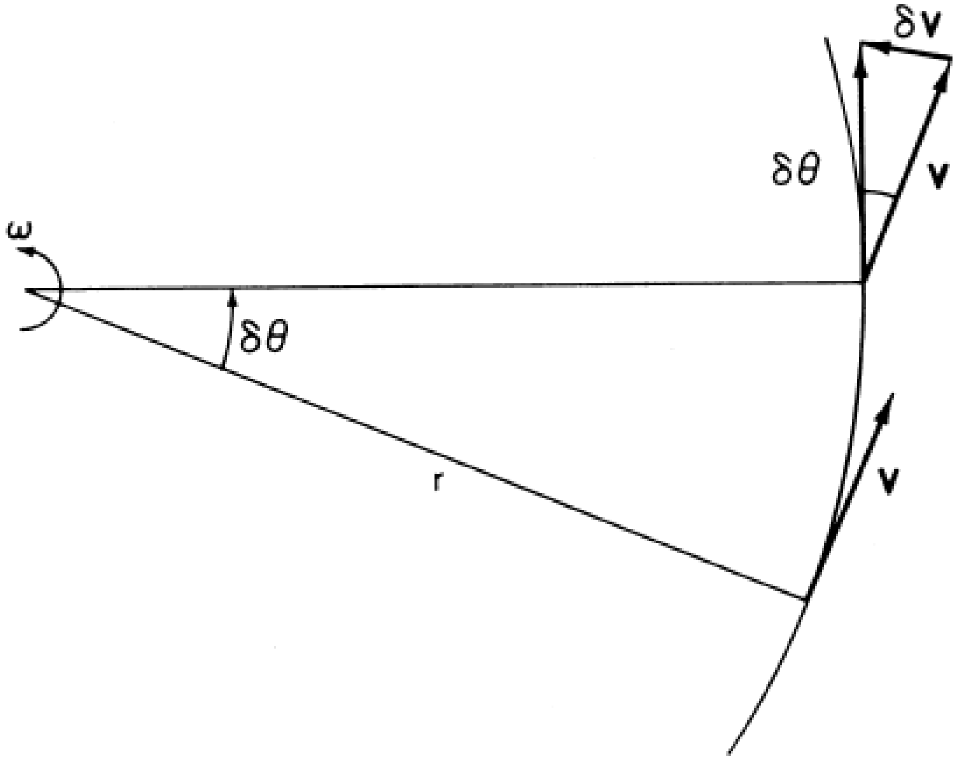
Q.

suppose that a ballistic missile is fired due eastward at 43^0 N Latitude. If the missile travels 1000 km at a horizontal speed u_o=1000 m/s, by how much is the missile deflected from its eastward path by the Coriolis force? ($\Omega = 7.292\times10^{-5}$ rad s^{-1})

Centripetal Acceleration and Centrifugal Force

- Suppose a ball of mass *m* is attached to a string and whirled through a circle of radius *r* at a constant angular velocity ω.
- From the point of view of an observer in inertial space the speed of the ball is constant, but its direction of travel is continuously changing so that its velocity is not constant.

- Suppose δV is change in velocity that occurs for a time increment δt during which the ball rotates through an angle δθ



- Because δθ is also the angle between the vectors **V** and **V** + δ**V**, the magnitude of δ**V** is just |δ**V**| = |**V**| δθ.

- So dividing both sides with δt and converting to derivatives, we get
- $\frac{DV}{Dt} = |V| \frac{D\theta}{Dt} \left(- \frac{\boldsymbol{r}}{r} \right)$

- We know that,

$$|V| = \omega r \text{ and } \frac{D\theta}{Dt} = \omega$$

- Hence, we get

$$\frac{DV}{Dt} = -\omega^2 \mathrm{r}$$

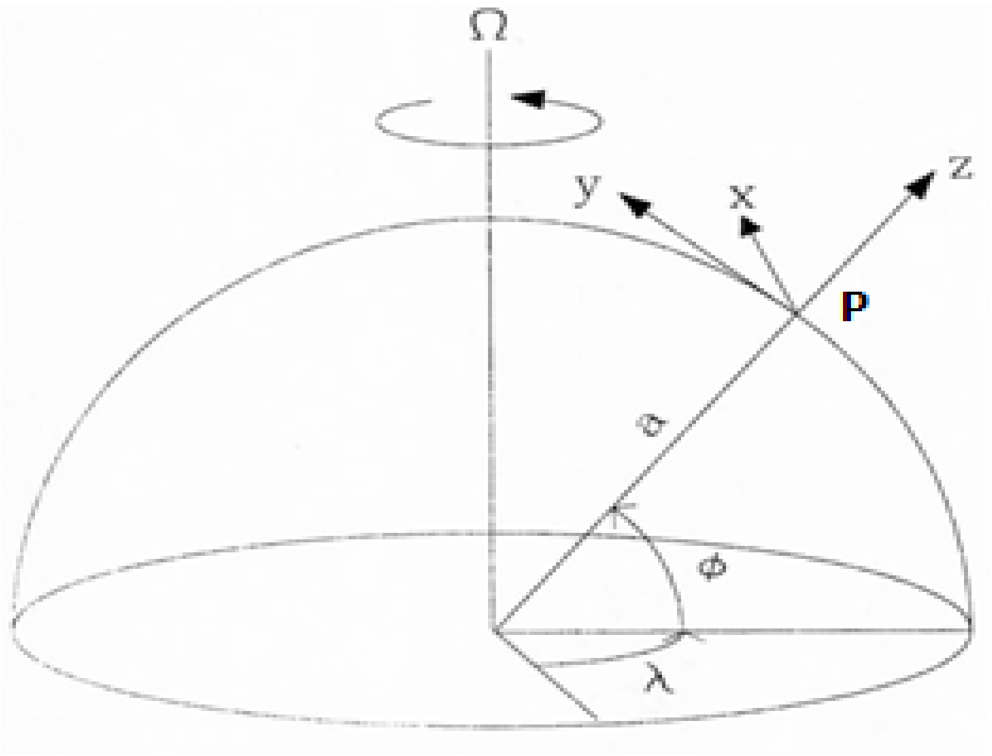
- This acceleration is called *centripetal acceleration* and is caused by the force of the string pulling the ball.
- When observed from a fixed system, the rotating ball undergoes a uniform centripetal acceleration in response to the force exerted by the string.

- Now suppose that we observe the motion in a coordinate system rotating with the ball.
- In this rotating system the ball is stationary, but there is still a force acting on the ball, namely the pull of the string.
- Therefore, in order to apply Newton's second law to describe the motion relative to this rotating coordinate system, we must include an additional apparent force.
- That force is known as centrifugal force and balances the force of the string on the ball.
- It just equal and opposite to the centripetal acceleration:

$$\text{Centrifugal force} = \omega^2 \mathrm{r}$$

- For earth, it would be

$$\text{Centrifugal force} = \Omega^2 a \quad \text{(at the equator)}$$



- where ‘a’ is the distance from the center (radius) and omega (Ω) is the angular velocity such that $\Omega = 2\pi/T$ where T is the time taken to complete one full rotation, $\emptyset$ is the latitude and λ is longitude

- Applying this to the earth: taking the radius of earth (a) = 6000 km, omega (Ω) = 2 π /86400 sec, it comes as:

$$(\Omega)^2 a = 3.17\ cm/sec^2.$$

- This would be the centrifugal force at the equator, where it is maximum. *(Compare this result with acceleration due to gravity : 980 cm/sec²)*

  Q. Suppose a volume water weighs 1000 kg at the pole. How much would it weight if it was moved to the equator

- At the poles, centrifugal force is 0, since the radius to the axis of rotation is zero.
- So, the centrifugal force is given by:

$$\Omega^2 \mathrm{a} \cos \varphi$$

Gravitational Force

- Newton's law of universal gravitation states that any two elements of mass in the universe attract each other with the force proportional to their masses and inversely proportional to the square of the distance separating them.
- Thus, if two mass elements M and m are separated by a distance r ≡ |**r**| (with the vector **r** directed toward *m),* then the force exerted by mass M on mass m due to gravitation is

$$F_g = -\mathrm{G}\frac{Mm}{r^2}\left(\frac{\boldsymbol{r}}{r}\right)$$

where *G* is a universal constant called the gravitational constant.

- Thus, if the earth is designated as mass M and m is a mass element of the sea, then the force per unit mass exerted on the sea by the gravitational attraction of the earth is

$$\frac{\mathbf{F}_g}{m} \equiv \mathbf{g}^* = -\frac{GM}{r^2}\left(\frac{\mathbf{r}}{r}\right)$$