General Circulation(GC)

Objectives of this Course to introducing the students the basics of dynamics of global atmospheric flow:

- \checkmark the theoretical models of global circulation
- ✓Why the climate of the entire earth is not uniform?
- \checkmark the deriving mechanism of global circulation
- ✓ oscillations of the atmosphere and their importance in seasonal weather prediction
- ✓ interaction between atmospheric and oceanic circulation teleconnection (link between sea surface change and climate variability for particular place)

Chapter one; Basic General Atmospheric Circulation (GAC)

In this chapter, the following questions will be answered

Why need to know the GAC?

What factors to drive the GAC?

What is the averaged GAC (qualitatively)

How to decompose the GAC (quantitatively)

Introduction:

Some problems in meteorology are so complicated, and involve so many interacting variables and processes, that is impossible to solve.

To solve such problems is important to simplified and idealized by approximation some times called toy model.

A rotating spherical Earth with no oceans is one example of a toy model Toy models are used extensively to study climate change

Atmospheric General Circulation (GAC)

What is GAC?

Some definition of GAC:

✓ Atmospheric circulation is the large-scale movement of air, and the means (together with the smaller ocean circulation) by which thermal energy is distributed on the surface of the Earth.

✓GAC is time averaging sate of the atmosphere with all geographical details.

✓The collection of the permanent and semi-permanent synoptic feature of atmospheric circulation including ITCZ, jeat stream, the major and semi permanent cyclone, anticyclone centers, summer and winter monsoons

✓GAC is the collection of all quantities statistical properties the circulation.

The large-scale structure of the atmospheric circulation varies from year to year, but the basic structure remains fairly constant. Atmospheric circulations are broken-down into different scales bases on

physical size and duration

Macroscale – This is the largest scale, and includes two important sub-scales - Planetary scale – These circulations last for weeks or months, and extend in size from 5000 to 40,000 km

Examples are the Asian monsoon, El Nino, and La Nina

Synoptic scale – These circulations last from days to weeks, and range in size from 100 to 5000 km.

Examples are the high- and low-pressure systems we see on weather maps. Also, hurricanes are synoptic scale phenomena

Mesoscale – These circulations last from minutes to hours, and range in size from 1 to 100 km

Examples are thunderstorms, tornadoes, and land-sea breezes.

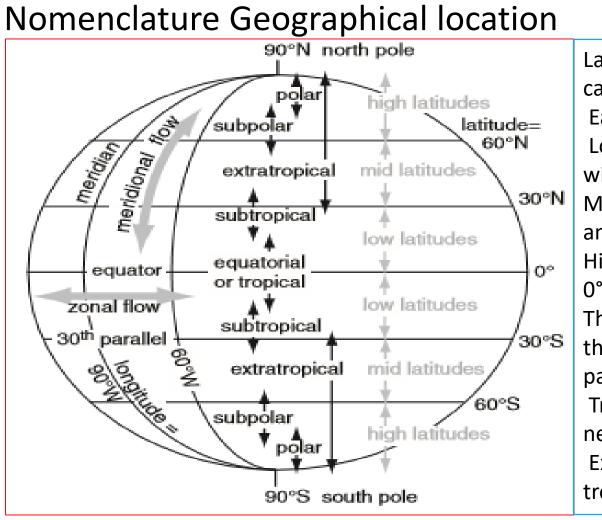
Microscale – These are the smallest circulations, lasting under a few minutes, and being less than 1 km in size

Examples are wind gusts and dust devils

The scales are not independent. A synoptic scale circulation may have mesoscale circulations embedded in it. For example, a hurricane (synoptic scale) contains numerous thunderstorms (mesoscale).

Classification	Horizontal scale (km)	Range of wind speed (m s ⁻¹)
Low/trough Depression Deep depression Cyclonic storm/tropical storm Tropical cyclone (severe cyclonic storm/hurricane/typhoon)	2000-2500 1500-2500 1500-2000 1000-1500 <1000	<8.5 8.5–13.5 14.0–16.5 17.0–31.5 >32.0

Classification of synoptic-scale tropical disturbances



Latitude lines are parallels, and east-west winds are called zonal flow

Each 1° of latitude = 111 km.

Longitude lines are meridians, and north-south winds are called meridional flow.

Mid-latitudes are the regions between about 30° and 60° latitude.

High latitudes are 60° to 90°, and low latitudes are 0° to 30°.

The subtropical zone is at roughly 30° latitude, and the subpolar zone is at 60° latitude, both of which partially overlap mid-latitudes.

Tropics span the equator, and polar regions are near the Earth's poles.

Extratropical refers to everything outside of the tropics: poleward from roughly 30°N and 30°S.

An atmospheric general circulation is concerned with —the dynamics of climate- meaning why it varying?

—with the study of the temporally averaged structures of the fields of wind, temperature, humidity, precipitation and etc

Remember, Why need to know the GAC?

There are various physical quantities that characterize the state of the atmosphere (e.g., pressure, density, temperature).

These are assumed to have unique values at each point in the atmospheric continuum.

The atmosphere can be regarded as a continuous fluid medium, or continuum

Moreover, these field variables and their derivatives are assumed to be continuous functions of space and time

What is field?

Field= is a quantity which is continuously defined over a given

coordinate space.

There are scalar and vector fields.

Scalar Field (or value) is assigned to everywhere in space. Example: temperature field

Vector Field - a vector is assigned to every point in space Example: wind field, gravity field, pressure gradient field.

The air contact with the earth's surface is called planetary boundary layer w/c turbulence take place.

Turbulence can cause exchange of momentum, sensible heat and moisture b/n atmosphere and surface. Moisture upward in to atmosphere via evaporation, momentum via friction.

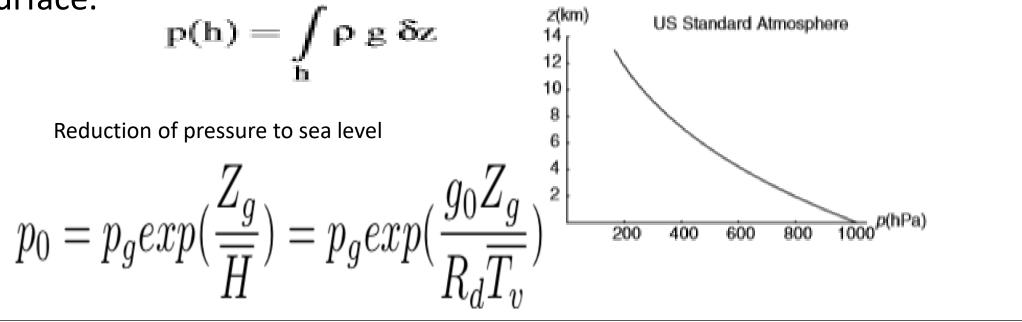
Surface moisture flux is key energy input to general circulation while, surface friction is mechanism that dissipate kinematic energy of general circulation.

Generally, the surface is both source and sink of general circulation. General circulation phenomena's are large scale dynamics like convection, radiative transfer, cloud process, torbulance etc.

Profiles of the atmosphere:

Pressure (p) of the atmosphere at any level is defined as the weight of the overlying column of air per unit area of the surface at that level.

Atmospheric pressure drops off dramatically with height above the surface.



For an atmosphere in hydrostatic equilibrium, the balance of forces in the vertical requires that $\frac{\partial p}{\partial z} = -g\rho$

The negative sign ensures that the pressure decreases with increasing height.

That is, the pressure at height z is equal to the weight of the air in the vertical column of unit cross-sectional area lying above that level.

 $p(z) = \int_z^\infty g\rho\,dz$

Vertical distribution of density:

$$p(z) = p(0)exp(-z/H)$$

Similarly

$$\rho(z) = \rho(0)exp(-z/H)$$

Atmospheric composition:The level of transition from turbulent mixing to molecular diffusion is called The turbopause. The well mixed region below the turbopause is called the Homosphere; the region above is called Heterosphere.

Water vapor: sources and sinks

Main source: evaporation from the earth's surface is the main source of atmospheric water vapor

Condensation which takes place in clouds is the main sink of atmospheric water vapor

Typical "lifetime" of a molecule of water vapor in the atmosphere is only "a week

Ozone: source and sink:

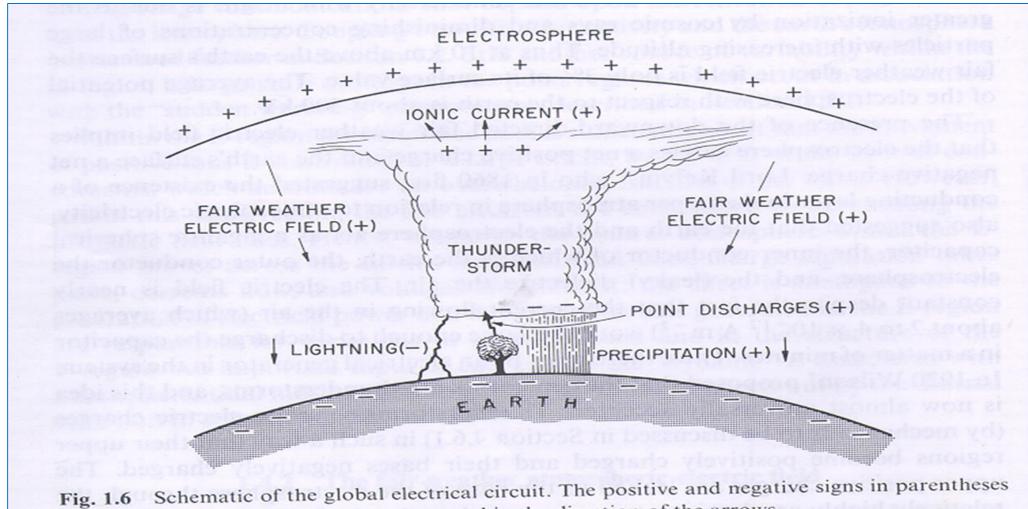
Source: Primarily generated by photochemical reactions in the layer between 20-60 km

Sink: At the earth's surface, reaction with plants and dissolving in water

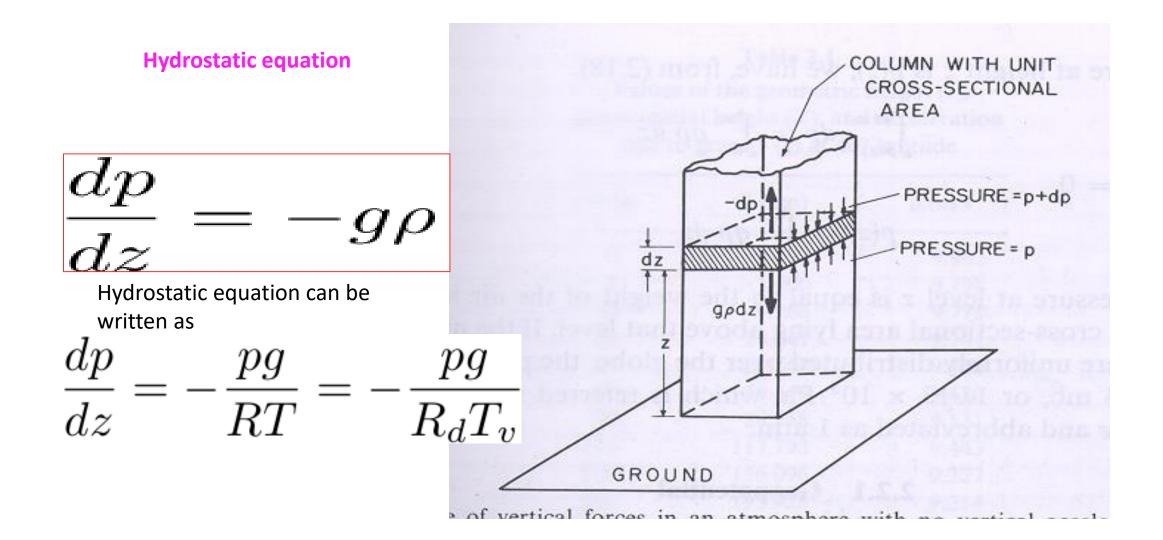
Charged particles: Sources: (1) X-ray and UV radiation from the sun ionizes air molecules.

All the sun's ionizing radiation is absorbed above 60km; (2) Highenergy cosmic rays; (3) Radioactive decay within the earth's crust; (4) Electric charges: separated within clouds.

Play a crucial role in geophysical phenomena: lightening, reflection of radio waves, fluctuations in the geomagnetic field, etc



indicate the signs of the charges transported in the direction of the arrows.



Geopotential height z:

•

Geopotential height Z is used as the vertical Coordinate in most atmospheric applications. (e.g., weather map).

$$Z = \frac{\Phi(z)}{g_0} = \frac{1}{g_0} \int_0^z g dz$$
$$Z_2 - Z_1 = -\frac{R_d}{g_0} \int_{p_2}^{p_1} T_v \frac{dp}{p}$$
hypsometric eqn
scale Height =
$$H = \frac{RT}{g_0} = \frac{R_d T_v}{g_0} = 29.3T_v$$

Individual Assignment one (5%)

submission date megabit 17,2012(geez calendar) or mar26,2020

1. What is the pressure at 5km below the surface in the ocean?

2. Drive density equation for the atmosphere from hydrostatic and state equation.

3. Drive pressure equation for atmosphere at constant temperature(iso-thermal atmosphere).

4. Drive scale height.

5. Calculate the thickness of the layer between the 1000 hPa and 500 hPa pressure surfaces, (a) at a point in the tropics where the mean virtual temperature of the layer is 15°C, and (b) at a point in the polar regions where the mean virtual temperature is −40°C.

The horizontal variation of pressure at the earth's surface is important because it is primarily the horizontal pressure gradient that forces the air to move from a region of high pressure to that of low pressure with an acceleration given by the vector relation.

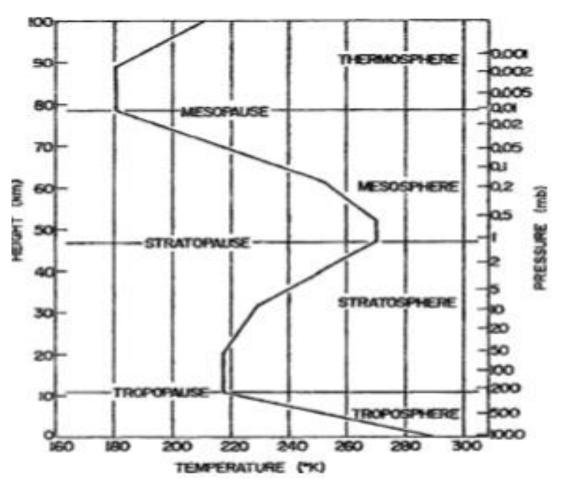
Pressure gradient force= $-\nabla p/\rho$

The fall of pressure with height causes an upward force:

hydrostatic equation

$$(\partial \mathbf{p}/\partial \mathbf{z}) = -\mathbf{\rho}\mathbf{g}$$

Temperature Distribution in the Atmosphere:



Geopotential Surfaces:

If a body of unit mass is raised from the earth's surface to a height z in the atmosphere, the work that must be done against the earth's gravitational field is called its geopotential which is usually denoted by Φ and defined by the relation:

$$\Phi(z) = \int_{0}^{z} g(z) \, \delta z$$

Handy numbers in this course

Radius of the Earth	$6.37 \ge 10^6 m$
Angular velocity of the Earth's rotation	$7.29 \ge 10^{-5} \text{ s}^{-1}$
Latent heat of condensation at 0 °C	$2.52 \text{ x} 10^6 \text{ J kg}^{-1}$
Globally averaged surface air temperature	288 K
Globally averaged precipitable water	$25 \text{ mm} (= 25 \text{ kg m}^{-2})$
Annual mean incident solar radiation	340 W m^{-2}
Global albedo	0.30
Outgoing longwave radiation	$240 \text{ W} \text{m}^{-2}$
Stefan-Boltzman constant	$5.67 \ge 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Globally averaged precipitation rate	3 mm day ⁻¹
c _p for dry air	1000 J kg ⁻¹ K ⁻¹
R for dry air	287 J kg ⁻¹ K ⁻¹
Globally averaged surface pressure	984 mb
Acceleration of Earth's gravity	9.81 m s ⁻²
Density of air near sea level	1.2 kg m ⁻³
Molecular viscosity of air	$1.5 \ge 10^{-5} \ m^2 \ s^{-1}$

- 1.2 Driving Force of Atmospheric General circulation (AGC)
- What factors to drive the GAC?

Main factors to drive the circulation:

- 1. special spatial scale
- 2. solar radiation distributed accordingly by Latitude
- 3. self rotation
- 4. surface un-uniform
- 5. surface friction
- 6. atmospheric internal dynamics and nonlinear interaction

1. Special spatial scale:

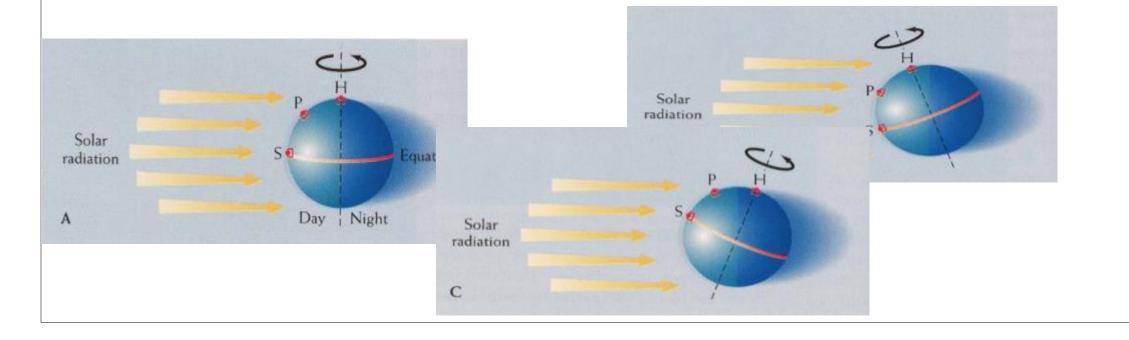
$$Scale: \frac{D}{L} \approx \frac{36}{6371} \square 1$$

Observation suggests that 99% of the total mass of atmosphere is confined within 36km down to the Earth, but its horizontal scale is equivalent to the Earth's radius. So the atmosphere can to some extent be treated as a very thin gaseous skin cover enveloping the Earth when we talk about the atmospheric circulation, which determines that the circulation is quasi-horizontal. Therefore, some standard isobaric level's figures can illustrate the main features of the atmospheric circulation.

Un uniform atmospheric mass distribution can cause to circulation.

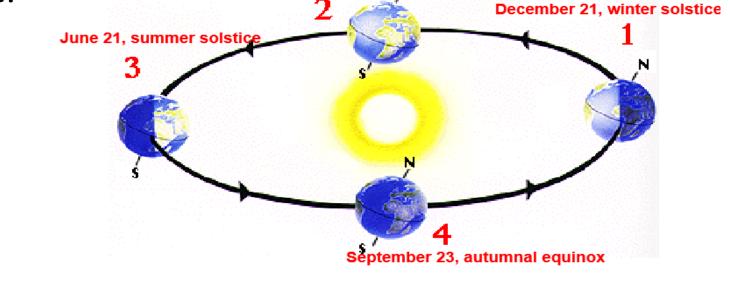
2.solar radiation distributed accordingly by Latitude:

The global atmospheric circulation and its seasonal variability is driven by the uneven solar heating of the Earth's atmosphere and surface.

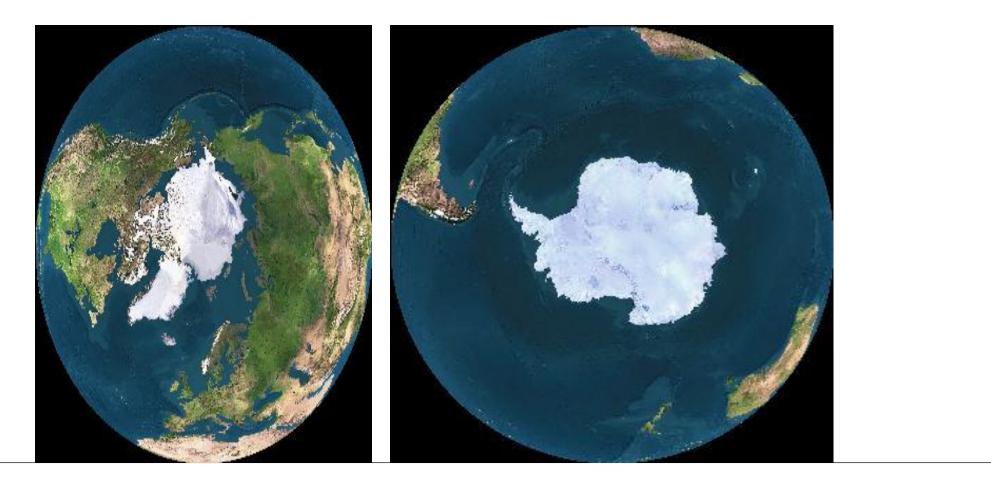


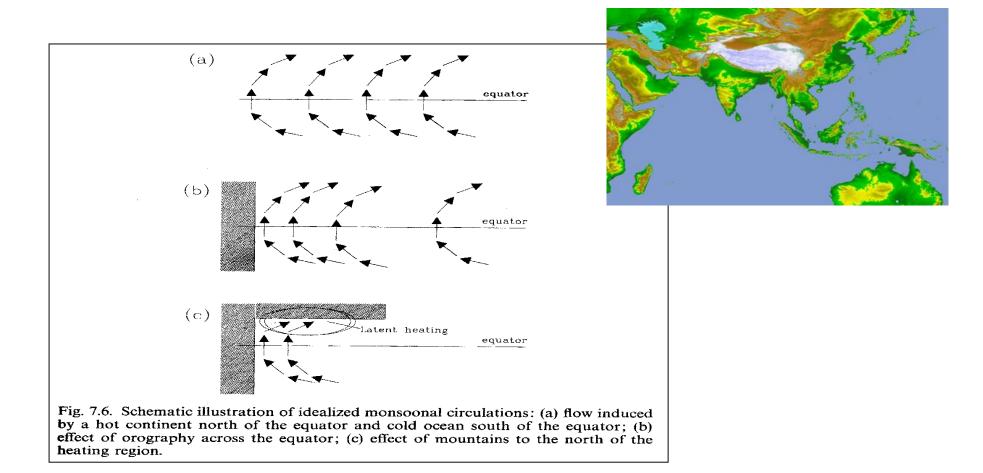
3. Self rotation:

Because Earth's rotation axis is tilted relative to the plane of its orbit around the sun, there is seasonal variability in the geographical distribution of sunshine.



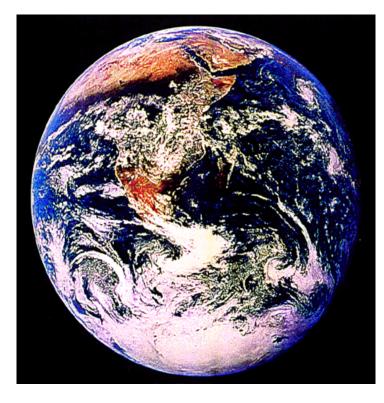
4. Surface un-uniform:

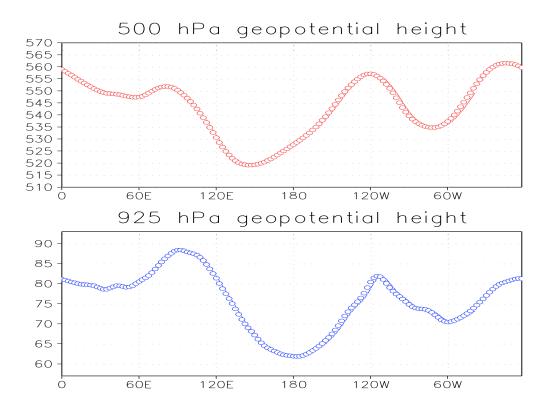




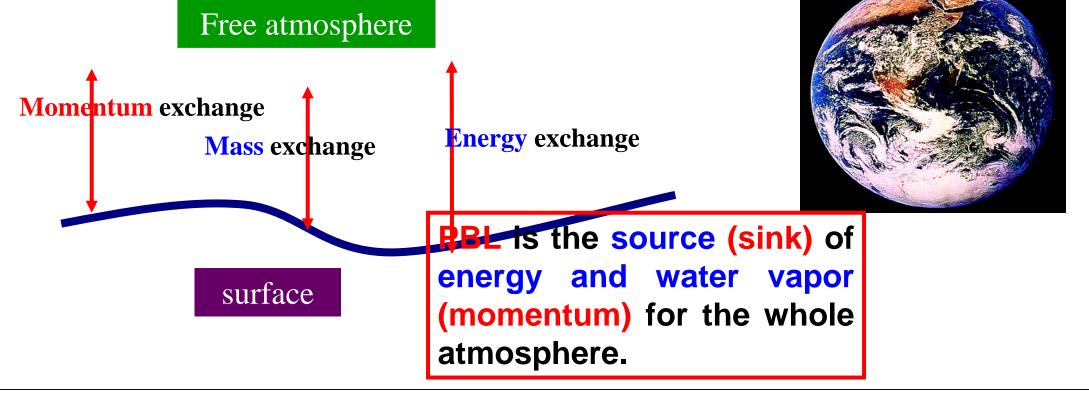
5.Surface friction:

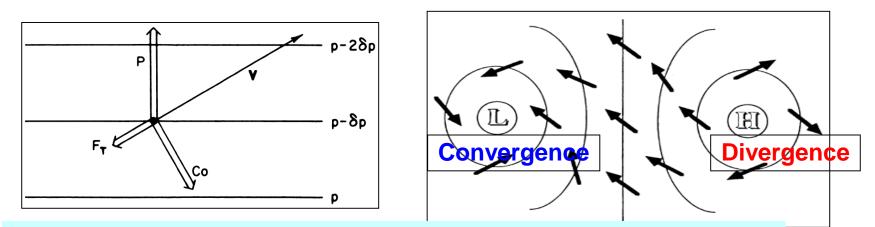
• Play an important role in the energy balance as well as the genesis and balance of the angular momentum in the atmosphere.



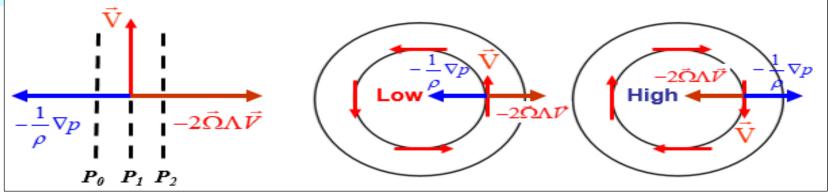


- (1) Planetary Boundary Layer (PBL): 1-1.5Km;
- (2) Through surface friction, westerly (easterly) tends to accelerate (decelerate) the Earth's self rotation.





Balance of forces in the well-mixed planetary boundary layer: *P* designates the pressure gradient force. C_{2} the Coriolis force, and F_{T} the turbulent drag.



6. Atmospheric internal dynamics and nonlinear interaction:

Global 3-D hydrostatic primitive equation set (pressure coordinate $\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \omega\frac{\partial v}{\partial p} - \frac{\partial \Phi}{\partial y} - fu + F_y$

• Momentum equation $\frac{\partial u}{\partial u} = -u \frac{\partial u}{\partial u} - u \frac{\partial u}{\partial u} - \frac{\partial \Phi}{\partial u} = -\frac{\partial \Phi}{\partial u} + \frac{\partial \Phi}{\partial u} = -\frac{\partial \Phi}{$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \omega\frac{\partial u}{\partial p} - \frac{\partial \Phi}{\partial x} + fv + F_x$$

• Thermodynamics equation $\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - \omega\left(\frac{\kappa T}{p} - \frac{\partial T}{\partial p}\right) + \frac{Q}{C_{p}}$

• Moisture equation

$$\frac{\partial q}{\partial t} = -u\frac{\partial q}{\partial x} - v\frac{\partial q}{\partial y} - \omega\frac{\partial q}{\partial p} - P + E$$

• State equation
$$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p}$$

• Continuity equation $\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

1.3 Statistics of Atmospheric Circulation

The atmospheric circulation is highly variable in space and time

The understanding of the general circulation of the atmosphere requires the study of the statistics of various orders for the important meteorological variables.

These statistics can be defined both in space and time and in a mixed space-time domain.

Decomposition of the circulation into temporal mean and transient eddies.

Rules of averaging for decomposed variables: A represents a meteorological variable, suchas temperature, humidity, winds, etc.), one can define the temporal mean:

$$m(A) = \overline{A} = \frac{1}{\tau} \int_0^{\tau} A dt$$
 And the variance $\sigma^2(A) = \frac{1}{\tau} \int_0^{\tau} (A - \overline{A})^2 dt = \frac{1}{\tau} \int_0^{\tau} (A')^2 dt$

 \overline{A} is the average and A' is the deviation from the average. The instantaneous value of A is given by

 $A = \overline{A} + A'$

The average product of two quantities A and B is given by

$$m(AB) = \overline{AB} = \overline{(\overline{A} + A')(\overline{B} + B')} = \overline{A} \ \overline{B} + \overline{A'B'}$$

In this expression, the last term $\overline{A'B'}$ is the covariance of A and B in time, and we define

$$\overline{A'B'} = r(A, B)\sigma(A)\sigma(B)$$

where r is the temporal correlation coefficient and σ the temporal standard deviation. In practice, $\overline{A'B'}$ is always obtained as a residual term:

$$\overline{A'B'} = \overline{AB} - \overline{A} \ \overline{B}$$

Rules of averaging for decomposed variables

Number	Averaging rule	Number	Averaging rule
A1	$\overline{C} = C$	A6	$\overline{\left(\frac{dA}{dt}\right)} = \frac{d\overline{A}}{dt}$
A2	$\overline{(A+B)} = \overline{A} + \overline{B}$	A7	$\overline{\left(\frac{\partial A}{\partial t}\right)} = \frac{\partial \overline{A}}{\partial t}$
A3	$\overline{(CA)} = C\overline{A}$	A8	$\overline{\left(\frac{\partial a'}{\partial t}\right)} = \frac{\partial \overline{a'}}{\partial t}$
A4	$\overline{(A)} = \overline{A}$ and $\overline{(B)} = \overline{B}$	A9	$\overline{\mathcal{A}\left(\frac{\partial \mathscr{A}}{\partial t}\right)} = \overline{\mathcal{A}}\frac{\partial \overline{\mathscr{A}}}{\partial t}$
A5	$\overline{(AB)} = \overline{A} \overline{B}$	A10	$\overline{\left(\frac{\partial(a')^2}{\partial t}\right)} = \frac{\partial\overline{(a')^2}}{\partial t}$

Cont.

Decomposition of a variable into zonal mean and non-symmetric perturbation:

Fields such as wind velocity or temperature are also not uniform in space, varying both as function of latitude and longitude. However meteorological conditions are generally more uniform along a latitude circle than in the north-south direction. It is thus convenient to define zonal-mean values at each latitude circle in order to assess the north south variability. This can be achieved by introducing a zonal-average operator define by:

$$[A]=rac{1}{2\pi}\int_{0}^{2\pi}Ad\lambda$$

and the departure from this average, A*, so that

$$A = [A] + A^*$$

Decomposition of the Circulation

Time and Zonal Decompositions:

 $\overline{A} = \frac{1}{\tau} \int_{0}^{\tau} A dt A = \overline{A} + A' (\overline{A'} = 0)$ time: Stationary/Transient zonal: $[A] = \frac{1}{2\pi} \int_0^{2\pi} A d\lambda A = [A] + A^* ([A^*] = 0)$ **Zonally Symmetric/Eddy Temporal &** $A = [\overline{A}] + \overline{A}^* + [A]' + A''$ space: The zonally symmetric part of the steady time-average quantity, e.g., easterly trade winds at low latitudes and the westerly winds at midlatitudes. \overline{A} The instantaneous fluctuations of symmetric part, such as the fluctuations of the zonalmean circulation (e.g., the index cycle). [A]The asymmetric part of the time-average quantities, such as the monsoon circulations and the longitudinal land-sea temperature contrast. '* The instantaneous, zonally asymmetric part, such as the travaling low- and high-pressure A systems shown on weather maps. $\vec{V} = [\vec{V}] + \vec{V}^* + [\vec{V}]' + \vec{V}'^*$ e.g.:

The Names of Various Components:

- $A = \overline{A}(time mean; stationary; steady) + A'(transient)$
- = [A](zonally mean; zonally symmetric) + A^* (eddy; zonal asymmetric) = $[\overline{A}] + \overline{A}^* + [A]' + A'^*$
 - $A(x, y, z, t) = \overline{A}(x, y, z) + A'(x, y, z, t) = [A](y, z, t) + A^{*}(x, y, z, t)$ $= [\overline{A}](y, z) + \overline{A}^{*}(x, y, z) + [A]'(y, z, t) + A'^{*}(x, y, z, t)$
 - v=mean meridional circulation[\overline{v}](y, z)+stationary eddy $\overline{v}^*(x, y, z)$

+transient meridional circulation[v]'(y, z, t)+transient eddyv^{'*}(x, y, z, t)

1.4 Theoretical Models of AGC (unicellular and three cell circulation)

Temperature differences are key in driving the global atmospheric circulation. Warm air tends to rise because it is light, while cold air tends to sink because it is dense, this sets the atmosphere in motion. The tropical circulation is a good example of this.

Wormer air moves toward pole from equator and colder air move to equator ward from poles.

Tropical zones are areas of heat source while polar zones are areas of heat sink.

Question: If the earth were not rotated what would be the global wind pattern(direction)?

Unicellular circulation model:

This model is called Hadley or —Single cell "model

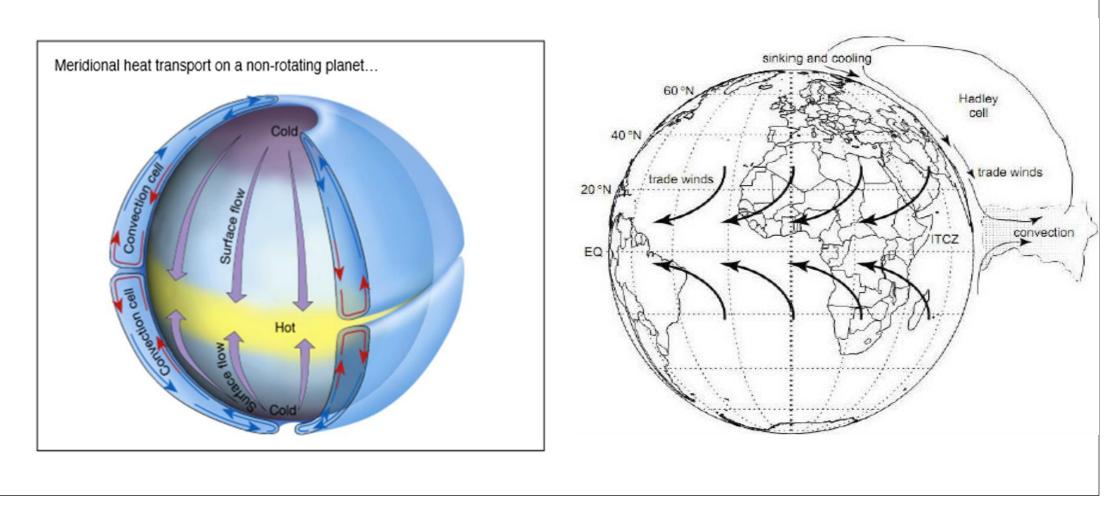
Assumptions:

- 1. The earth's surface is uniformly covered with water only or land only
- 2. The sun is always directly over the equator
- 3. The earth does not rotate

The circulation of this model is called —thermally direct cell.

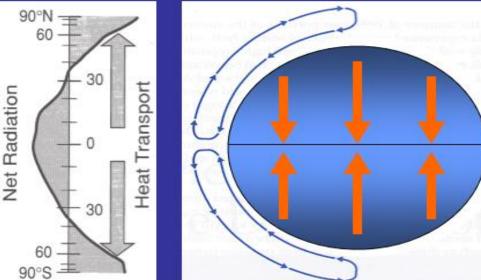
In this model excessive heating of the earth surface over the equatorial region produces surface low pressure (L) and excessive cooling of the earth surface over polar region produces high pressure (H).

Uni- Cellular circulation model (hadely cell/single cell)



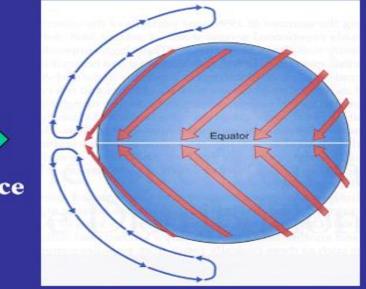
Single-Cell Model: Explains Why There are Tropical Easterlies

Without Earth Rotation





With Earth Rotation



Coriolis Force

Coriolis force causes the wind to deflect to the right of its intent path in the Northern Hemisphere and to the left in the Southern Hemisphere Coriolis Force = $\int V$ where $f = 2*\Omega*Sin(lat)$ and $\Omega=7.292x10^{-5} rad s^{-1}$

The magnitude of Coriolis force depends on (1) the rotation of the Earth, (2) the speed of the moving object, and (3) its latitudinal location.

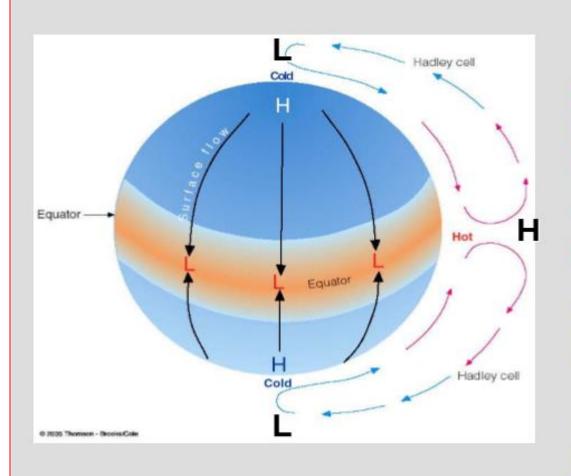
The larger the speed (such as wind speed), the stronger the Coriolis force

The higher the latitude, the stronger the Coriolis force

The Corioils force is zero at the equator

Coriolis force is one major factor that determine weather pattern

General Circulation of Waterworld



HADLEY CELL

A giant convection cell, or thermally direct circulation.

Warm air rises at the equator. Low pressure at surface, high pressure aloft.

Transport of warm air away from the equator aloft.

Cold air sinks at the pole. High pressure at the surface, low pressure aloft.

Transport of cool air toward the equator at the surface.

Path of air through the cell

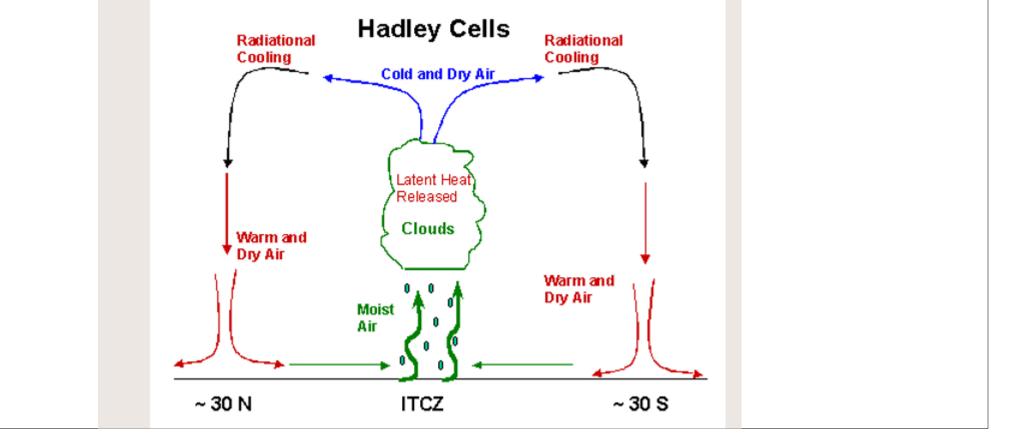
- Equator: Air converges at the surface and rises to form a band of clouds, called the intertropical convergence zone (ITCZ). Releases latent heat
- >Upper level winds transport heat away from the equator
- ➢Air cools by radiating in the longwave to space, air sinks and warms (dry adiabatically) at about 30°latitude, forming a subtropical high.
- Air returns to toward the equator at the surface, causing easterly trade winds

cont. Hadley Cell –also called the meridional cell HADLEY CIRCULATION CELL speeds of a few cm/sec COOL COOL VARM MOIST AI 30°S 30°N EQUATOR

Heated air at low latitudes rises – flows poleward – cools and sinks, with a return flow to low latitudes at the surface. It is limited by the Coriolis force, the effect of the rotation of the earth. Air moving north turns to the right; it cannot go too far north (max \sim 300 lat) with the present rate of rotation. The returning wind at low levels also turns to the right (in the NH), producing the "north-east trade winds".

cont. Warmer stratosphere forces air to move horizontally... where air bunches up at stc (ITCZ) Coriolis force turns northward wind to right ... Air rises in thunderstorms air bunches up " air sinks where Corriolis-turnea EQ Friction-caused outflow from high 30N turns to right, blows toward low

The rising air cools to condensation release latent heat and precipitation formed



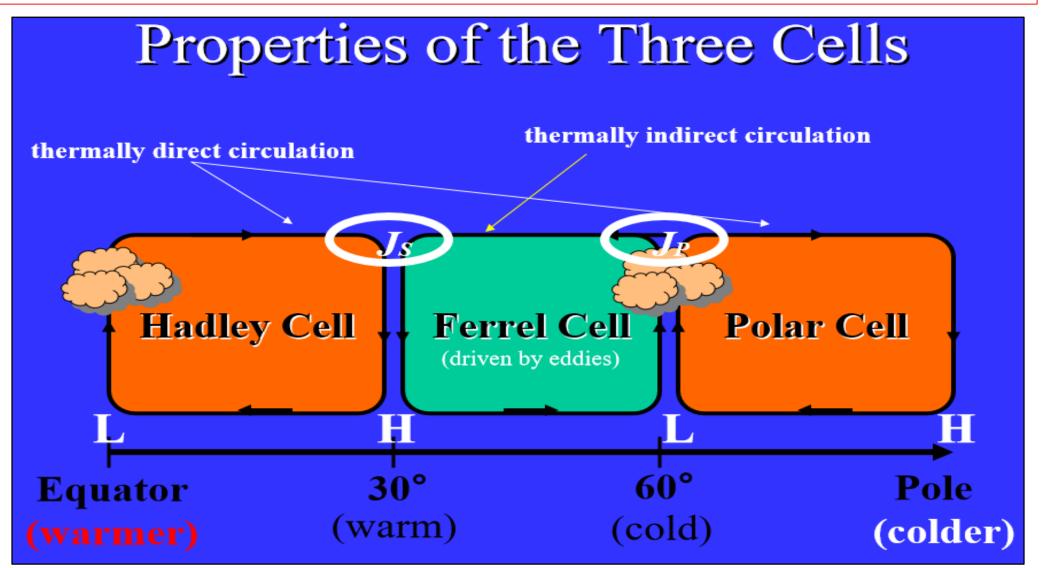
Ferrel Cell

In the middle latitudes the circulation is that of sinking cold air that comes from the poles and the rising warm air that blows from the subtropical high. At the surface these winds are called westerlies and the cell is known as the Ferrel cell.

Polar Cell

At polar latitudes the cold dense air subsides near the poles and blows towards middle latitudes as the polar easterlies. This cell is called the polar cell.

These three cells set the pattern for the general circulation of the atmosphere. The transfer of heat energy from lower latitudes to higher latitudes maintains the general circulation.



Thermally Direct/Indirect Cells

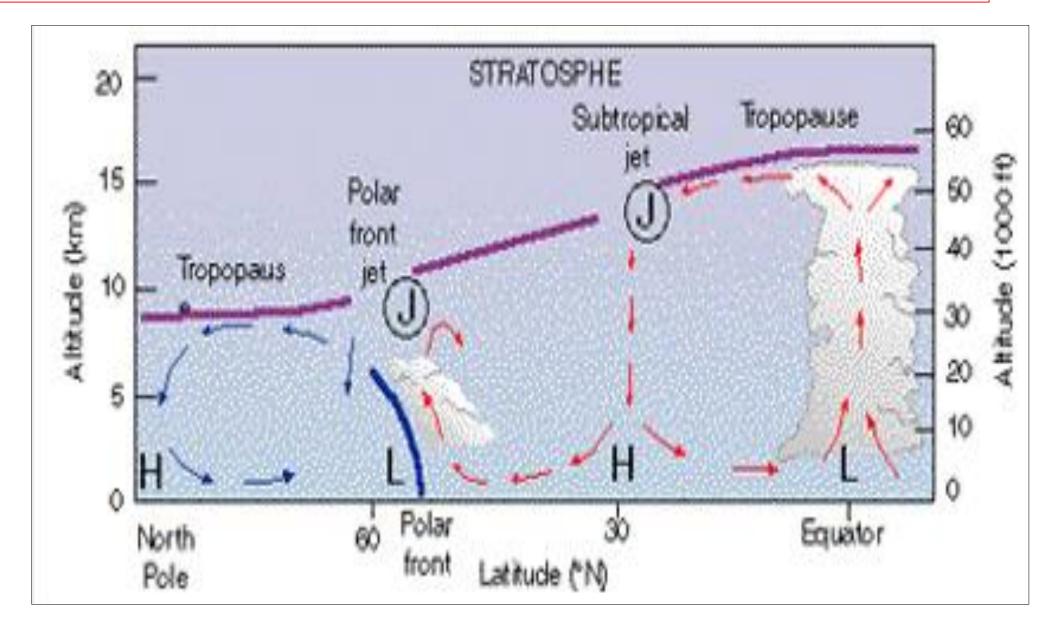
Thermally Direct Cells (Hadley and Polar Cells)

Both cells have their rising branches over warm temperature zones and sinking braches over the cold temperature zone.

Both cells directly convert thermal energy to kinetic energy.

Thermally Indirect Cell (FerrelCell) This cell rises over cold temperature zone and sinks over warm temperature zone.

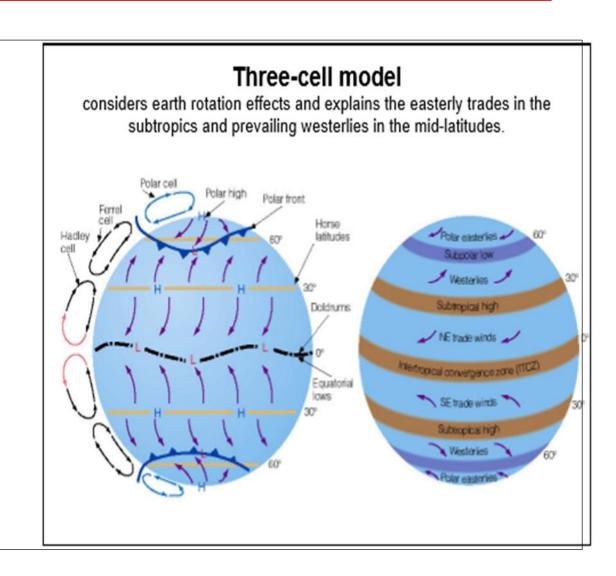
The cell is not driven by thermal forcing but driven by eddy (weather systems) forcing.



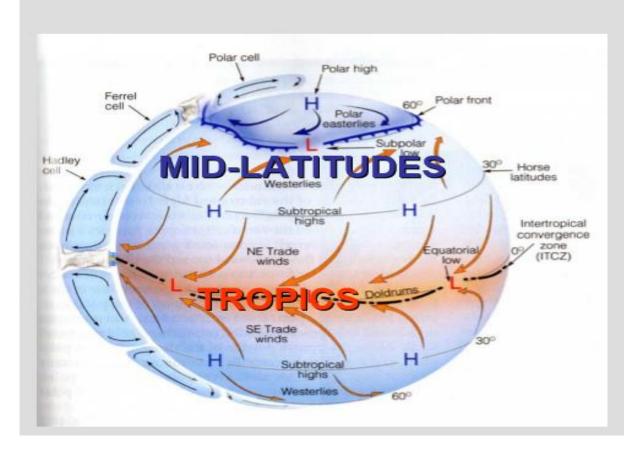
Three cellular circulation model With:

- > earth rotate, uneven earth
- Surface(both water and land)
- ➤The sun not always

over the equator



Three-cell model of general circulation

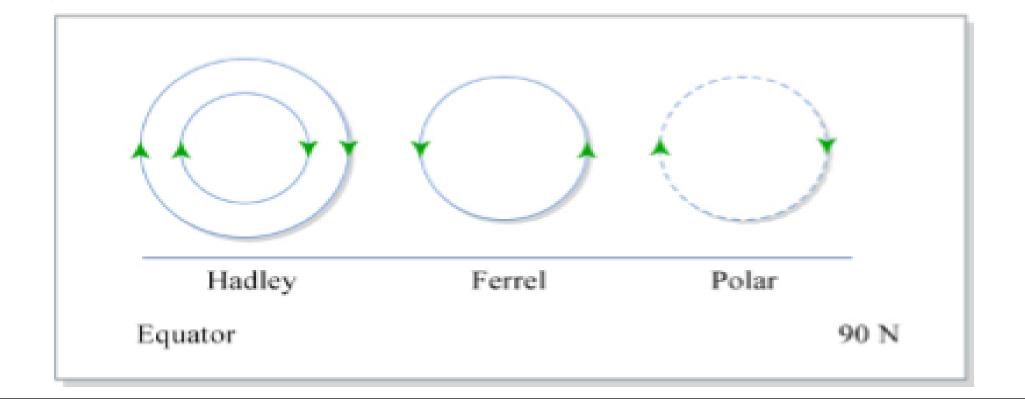


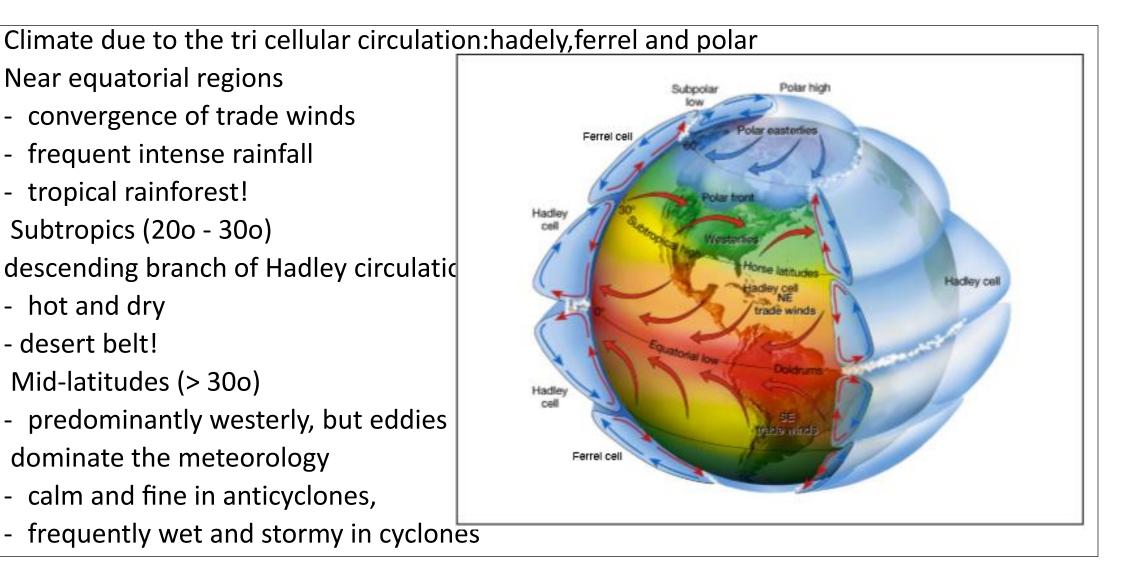
What mechanism transports heat poleward in each of these regions?

Mid-latitudes: 30° to 60° latitude

Tropics: 0 to about 30° latitude

There is generally a three-celled structure in both hemispheres, which are generally referred to as the Hadley cell, the Ferrel cell, and the (very weak) polar cell.



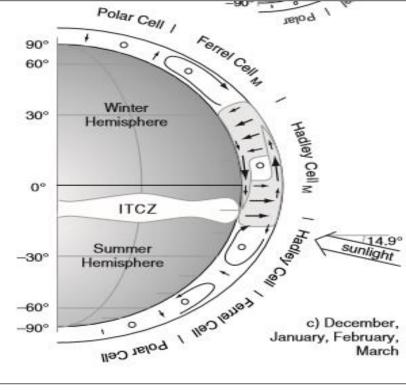


Vertical circulations of warm rising air in the tropics and descending air in the subtropics are called Hadley cells or Hadley circulations.

At the bottom of the Hadley cell are the trade winds. At the top, near the tropopause, are divergent winds. The updraft portion of the Hadley circulation is often filled with thunderstorms and heavy precipitation at the ITCZ. The updraft is often between 0° and 15° latitudes in the summer hemisphere, and has average core vertical velocities of 6 mm/s. The broader downdraft is often found between 10° and 30° latitudes in the winter hemisphere, with average velocity of about –4 mm/s in downdraft centers.

Connecting the up- and downdrafts are meridional wind components of 3 m/s at the cell top and bottom.

The major Hadley cell changes its direction and shifts position between summer and winter. The ITCZ region also vary during summer and winter.



The large-scale atmospheric circulation has a strong influence on precipitation, which is, with temperature, the most important variable in defining the climate of a region.

Along the ITCZ, the cooling of warm and moist surface air during its rising motion leads to condensation and heavy precipitation in this area.

For instance, the western tropical Pacific receives more than 3 m of rainfall per year.

By contrast, the downward motion in the subtropics is associated with the presence of very dry air and very low precipitation rates.

As a consequence, the majority of the large deserts on Earth are located in the sub-tropical belt

1.5 Baroclinic vs. Barotropic

Barotropic		Baroclinic
ρ=ρ(p) only		ρ=ρ(p,T)
 Implications: 1) isobaric and isothermal surfaces coincide 2) no vertical wind shear (thermal wind = 0) 3) no tilt of pressure systems with height 		 Implications: 1) isobaric and isothermal surfaces intersect 2) vertical wind shear (thermal wind ≠ 0) 3) pressure systems tilt with height
Seasons: Geographic:	Atmosphere is most baroclinic in winter. Atmosphere is least baroclinic in summer. Atmosphere is most baroclinic in midlatitudes Atmosphere is least baroclinic in the Tropics	

1.5 Measurement of Rotation

Vorticity and circulation are closely related quantities that describe rotational motion in fluids.

Both are primary measures of rotation in a fluid.

Vorticity describes the rotation at each point while circulation describes rotation over a region.

Circulation, which is a scalar integral quantity, is a macroscopic measure of rotation for a finite area of the fluid.

Vorticity, however, is a vector field that gives a microscopic measure of the rotation at any point in the fluid.

One way to describe an atmospheric flow that involves rotation or curvature is the circulation, which is defined as the line integral of the wind around a closed curve anywhere in the atmosphere.

velo

Circulation is the total "push" you get when going along a path, such as a circle.

The circulation, C, about a closed contour in a fluid is defined as the line

$$C \equiv \oint \mathbf{U} \cdot d\mathbf{I} = \oint |\mathbf{U}| \cos \alpha \, d\mathbf{I}$$

 $C > 0 \rightarrow Counterclockwise$

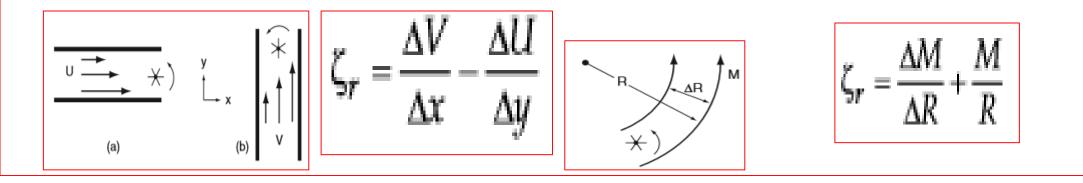
C < 0 ➔ Clockwise

There are four types of Vorticities;

1). Relative Vorticity(ζ_r)

Relative vorticity (ζ_r) is a measure of the rotation of fluids about a vertical axis relative to the Earth's surface. It can be in horizontal or in curvature. It is defined as positive in the counter-clockwise direction. The unit of measurement of vorticity is inverse seconds.

Relative vorticity (eastward, northward) Relative vorticity in curvature



where (U, V) are the (eastward, northward) components of the wind velocity, R is the radius of curvature traveled by a moving air parcel, and M is the tangential speed along that circumference in a counterclockwise direction.

2). Absolute Vorticity(ζa)

Measured with respect to the "fixed" stars, the total vorticity must include the Earth's rotation in addition to the relative vorticity. This sum is called the absolute vorticity ζ_a

$$\zeta_a = \zeta_r + f_c$$

where the Coriolis parameter fc = $2\Omega \cdot sin(\varphi)$ is a measure of the vorticity of the planet, φ is latitude, and where $2\Omega = 1.458 \times 10^{-4} \text{ s}-1$

3). Potential Vorticity(ζ_p)

Potential vorticity ζ_p is defined as the absolute vorticity divided by the depth Δz of the column of air that is rotating: $\zeta_n = \frac{\zeta_r + f_c}{\zeta_n} = \text{constant}$

In the absence of turbulent drag and heating (latent, radiative, etc.), potential vorticity is conserved.

4). Isentropic potential vorticity (IPV)

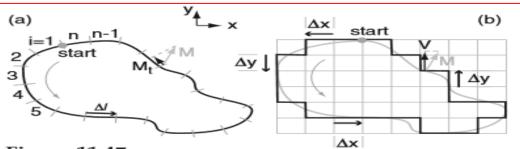
Measured on an isentropic surface (i.e., a $\zeta_{IPV} = \frac{\zeta_r + f_c}{\rho} \cdot \left(\frac{\Delta \theta}{\Delta z}\right)$ necting points of equal potential temperature θ), and ρ is ai

Isentropic potential vorticity is conserved for air moving adiabatically and frictionless along an isentropic surface (i.e., a surface of constant potential temperature).

Horizontal Circulations

The horizontal circulation C is defined as the product the tangential velocity times distance increment, summed over all the increments around the whole perimeter:

$$C = \sum_{i=1}^{n} (M_i \cdot \Delta l)_i \qquad C = \sum_{i=1}^{n} (U \cdot \Delta x + V \cdot \Delta y)_i$$



The sign of Δx is (+) if you travel in the positive x-direction (toward the East), and (–) if opposite. Similar rules apply for Δy (+ toward North).

Consider two more cases .

For a circuit in a constant wind field of any speed, the circulation is C = 0. For a circuit within a region of uniform shear such as $\Delta U/\Delta y$, the circulation is C = $-(\Delta U/\Delta y) \cdot (\Delta y \cdot \Delta x)$.

 $(\Delta y \cdot \Delta x) = A$ is the area enclosed by the circulation.

In general, for uniform U and V shear across a region, the horizontal circulation is: $(\Delta V \ \Delta U)$.

$$C = \left(\frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y}\right) \cdot A$$

This gives an important relationship between horizontal circulation and vorticity:

This is relative circulation (C_r)

$$C = \zeta_r \cdot A$$

An absolute circulation Ca can be defined as

$$C_a = (\zeta_r + f_c) \cdot A$$

For the special case of a frictionless barotropic atmosphere, Kelvin's circulation theorem states that absolute circulation Ca is constant with time.

In general, the circulation around a particular closed curve will be a function of both time and space, since the velocity field is a function of time and space. So, if we wish to determine the rate of change of circulation, we can write.

$$\frac{DC}{Dt} = \oint \frac{D\vec{u}}{Dt} \bullet d\vec{s}$$
$$= \oint -\frac{1}{\rho} \frac{\partial p}{\partial s} ds + \oint \frac{\partial \Phi}{\partial s} ds + \oint \text{ friction}$$

2. Bjerknes' circulation theorem:

For a more realistic baroclinic atmosphere containing horizontal temperature gradients, the Bjerknes circulation theorem:

$$\frac{\Delta C_r}{\Delta t} = -\sum_{i=1}^n \left(\frac{\Delta P}{\rho}\right)_i - f_c \cdot \frac{\Delta A}{\Delta t}$$

Term 1: rate of change of relative circulation The pressure term is called the solenoid term units of $\Delta Cr/\Delta t$. m²·s–2 Term 2: solenoidal term (for a barotropic fluid, the density is a function only of pressure, and the solenoidal term is zero.

Term 3: rate of change of the enclosed area projected on the equatorial plane

1.Application of Bjerknes circulation theorem

For a barotropic fluid, Bjerknes circulation theorem can be integrated following the motion from an initial state Kelvin's circulation theorem integrated following the motion from an initial state to a final state yielding the circulation change.

$$C_2 - C_1 = -2\Omega (A_2 \sin \phi_2 - A_1 \sin \phi_1)$$

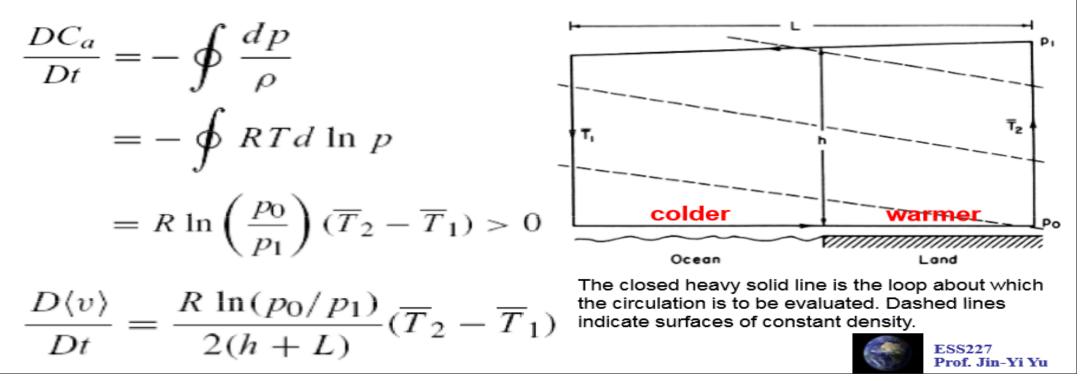
This equation indicates that in a barotropic fluid the relative circulation for p a closed chain of fluid horizontal area enclosed by the loop changes (divergence effect) or the latitude changes(Coriolis effect).

Application of Kelvin's Circulation Theorem: In a barotropic fluid, the solenoid term (Term2) In a barotropic fluid, the solenoid term (Term 2) vanishes.

The absolute circulation (Ca) is conserved following the parcel.

Solenoidal Term in Baroclinic Flow

- In a baroclinic fluid, circulation may be generated by the pressuredensity solenoid term.
- This process can be illustrated effectively by considering the development of a sea breeze circulation,



Absolute Circulation(Ca): consists of both earth and relative circulations;

On the Earth, a component of the circulation around any circuit will be due to the rotation of the frame, that is:

Ca bsolute =C earth+C relative

The circulation due to the rotation of the Earth is then simply

$$\begin{split} C_{earth} &= \oint \vec{u}_{earth} \bullet d\vec{s} \\ &\approx R\Omega \times 2\pi R \sin \phi \\ C_{earth} &\approx 2\pi \Omega R^2 \sin \phi \end{split}$$

Vorticity is the tendency for elements of the fluid to "spin"

Vorticity can be related to the amount of "circulation" or "rotation" (or more strictly, the local angular rate of rotation) in a fluid

Definition:

Absolute Vorticity $\rightarrow \omega_a \equiv \nabla \times \mathbf{U}_a$ Relative Vorticity $\rightarrow \omega \equiv \nabla \times \mathbf{U}$ $\omega = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

Vorticity

Another way to describe the curved motion of fluid parcels without reference to a center of rotation is a quantity known as the vorticity, which is simply the circulation per unit area: $\zeta = \frac{\partial C}{\partial A}$

Since the circulation represents the flux of vorticity through a specified area closed circuit, the circulation is often termed the vortex strength.

For solid body rotation

$$\zeta = \frac{\delta C}{\delta A} = \frac{2\pi\delta r V}{\pi\delta r^2} = \frac{2V}{\delta r} = 2\omega$$

Vorticity of any fluid parcel on the Earth has a component due to the solid body rotation of the Earth, at any point of which and hence $\zeta_{earth} = 2\Omega \sin \phi = f$

It is positive in the Northern Hemisphere and negative in the Southern Hemisphere. Relative vorticity is simply $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

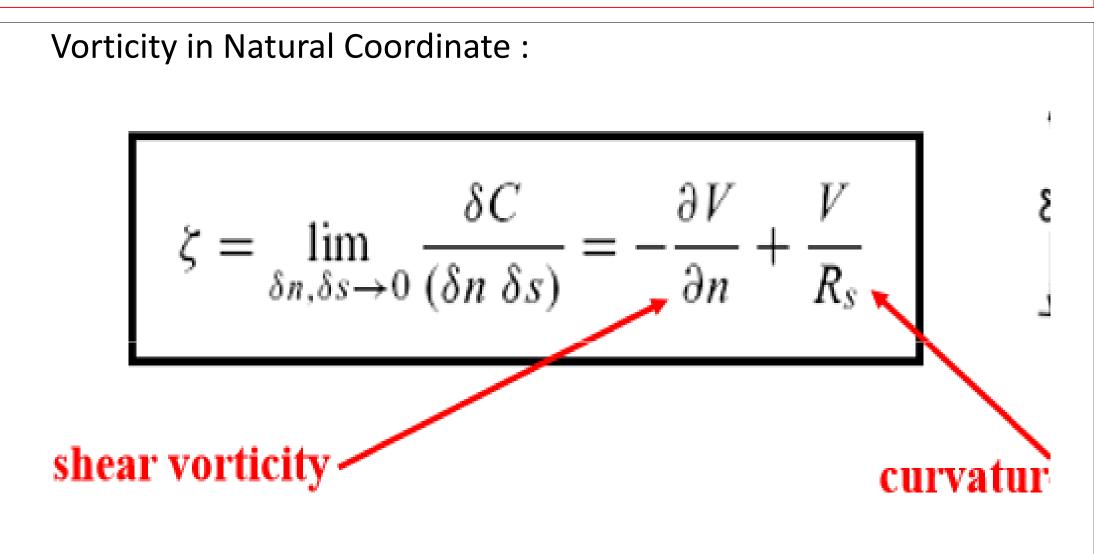
The relative vorticity will be. Therefore, for flopositive for a counterclockwise rotation and will be negative for a clockwise rotationw around a low-pressure center in the Northern Hemisphere, the relative vorticity will be positive. In the Southern Hemisphere the relative vorticity for flow around a low-pressure center is negative

cont.

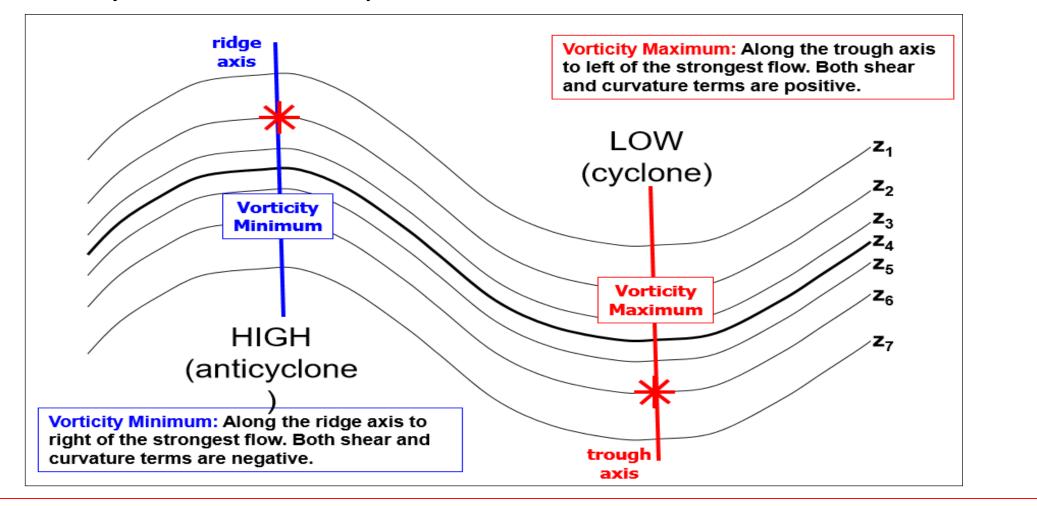
The most important aspect of relative vorticity is not its value at a particular time, but its change in space or time

Temporal changes in relative vorticity tell us about cyclone development, vorticity increases as cyclones spin up, and decreases as they die.

Spatial changes in relative vorticity can indicate the influence of mountains or temperature gradients, Spatial changes in relative vorticity can indicate the influence of mountains or temperature gradients.



Vorticity on weather maps



Dt

 $\frac{DC}{Dt} = -\oint \frac{dp}{\rho} - 2\Omega \frac{DA_e}{Dt}$

Potential Vorticity

We begin with the "circulation equation" (Bjerknes circulation theorem)

We then make use of definitions of potential temperature (Θ) and vorticity (ζ)

(where $Ae = A \sin \Phi$)

$$\theta = T (p_s/p)^{R/c_p} \Rightarrow \rho = p^{c_v/c_p} (R\theta)^{-1} (p_s)^{R/c_p}$$
$$\Rightarrow \oint \frac{dp}{\rho} \propto \oint dp^{(1-c_v/c_p)} = 0$$
$$C \approx \zeta \delta A$$

• We then make use of definitions of potential temperature (Θ) and vorticity (ζ) $\delta A(\zeta_{\theta} + f) = \text{Const}$ (where $f = 2\Omega \sin \phi$)

$$\delta A = -\frac{\delta Mg}{\delta p} = \left(-\frac{\delta\theta}{\delta p}\right) \left(\frac{\delta Mg}{\delta\theta}\right) = \text{Const} \times g\left(-\frac{\delta\theta}{\delta p}\right)$$

$$P \equiv (\zeta_{\theta} + f) \left(-g \frac{\partial \theta}{\partial p} \right) = \text{Const}$$



Why need to know the GAC?

Behind of the climate of a particularly place, there is GAC.There fore understanding of GAC is important for climate prediction.

The relationship between Climate Prediction (CP) and General Atmospheric Circulation GAC).

How do we usually show the products of short-term climate prediction?

(Far) Above Normal

- Normal
- (Far) Below Normal
- Background: Long Term Average

End

