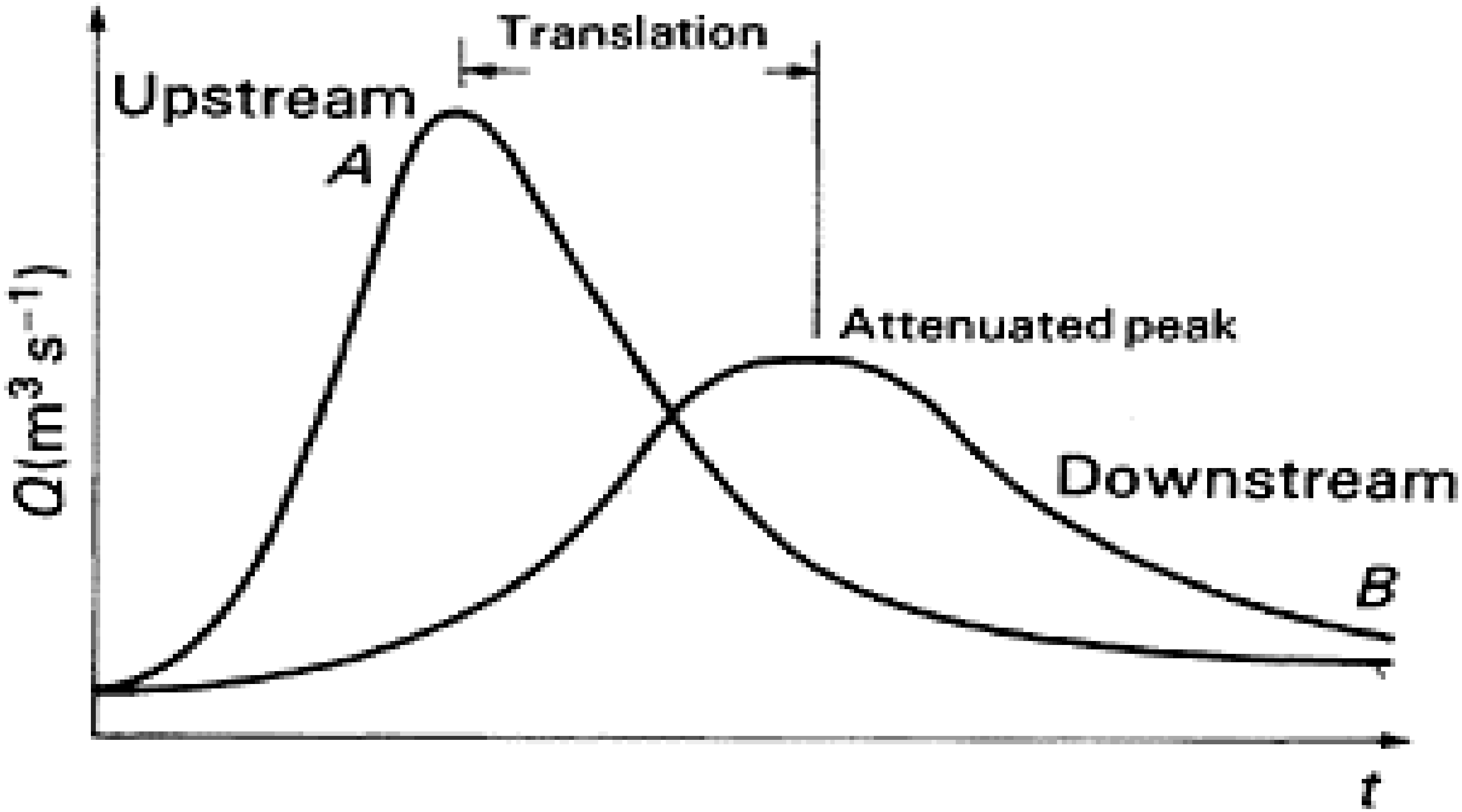
4. Flood Routing



Prepared by: Birhanu W.

Flood Routing

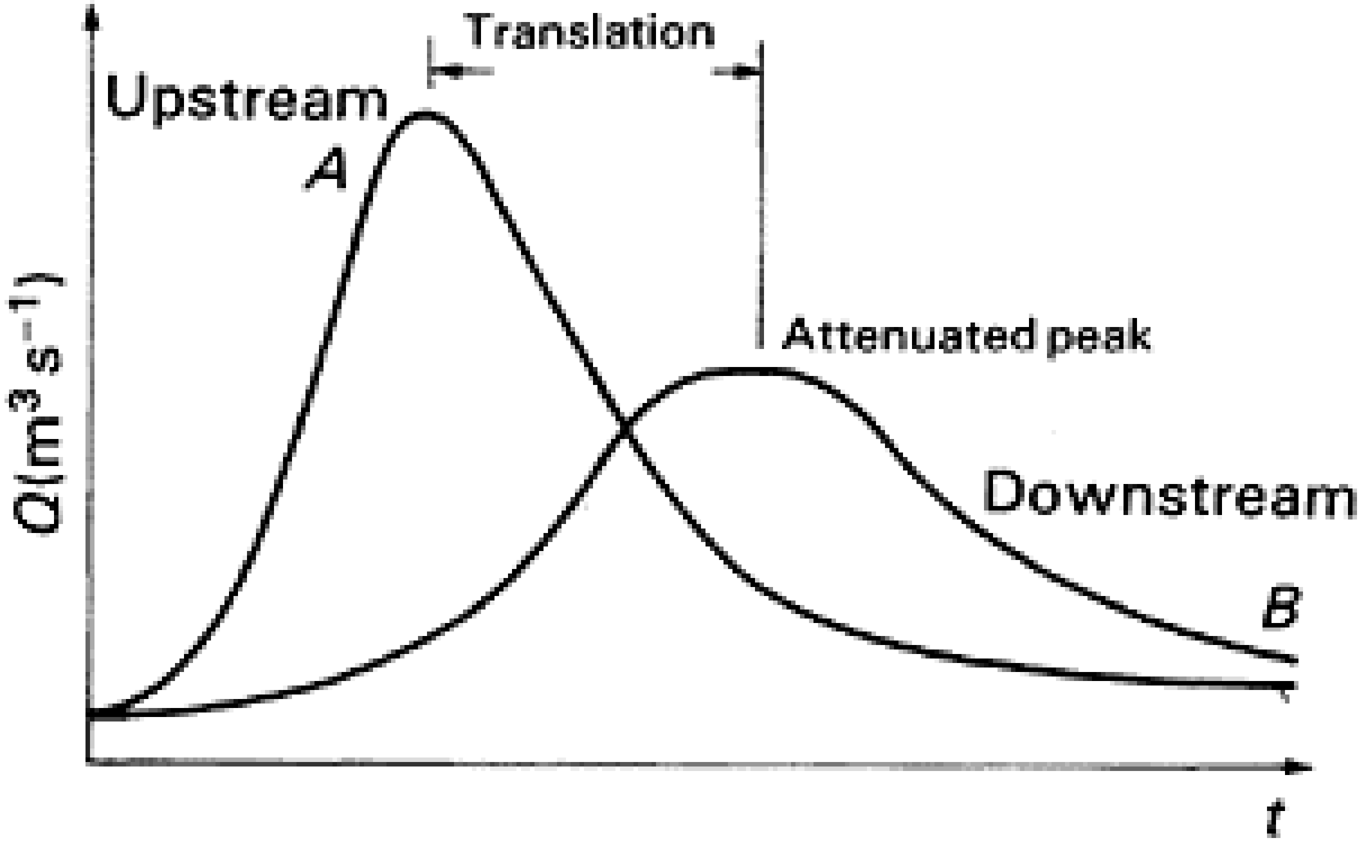
- ✓ What is flood routing and describe its application in different engineering fields?
- ✓ Point out the basic difference between different methods of flood routing? or
- ✓ How is reservoir routing different from Channel routing?
- ✓ Explain the basic methods of reservoir routing.
- ✓ Explain the basic methods of channel routing

4. Flood Routing

- Estimation of the hydrograph of a river at any given point on the river during the course of a flood event.
- **Flood Routing** is the process of following the behavior of a flood hydrograph upstream or downstream from one point to another point on the river.
- A flood hydrograph is modified in two ways as the storm water flows down-stream.

1. The time of the peak rate of flow occurs later at downstream points. This is known as ***translation***,

2. The magnitude of the peak rate of flow is diminished at downstream points, the shape of the hydrograph flattens out, and the volume of flood water takes longer to pass a lower section. This modification to the hydrograph is called ***attenuation***.



Applicable in river development works such as flood protection, reservoir and spillway designs, determining bridge span,…

4.1 Methods of flood Routing approaches) :

Flood routing methods may be divided into two main categories differing in their

fundamental approaches to the problem.

1.Hydrologic Routing methods using the *principle of continuity* and a relationship between *discharge* and the *temporary storage* of excess volumes of water during the flood period between two reaches.
(The calculations are relatively simple and reasonably accurate)

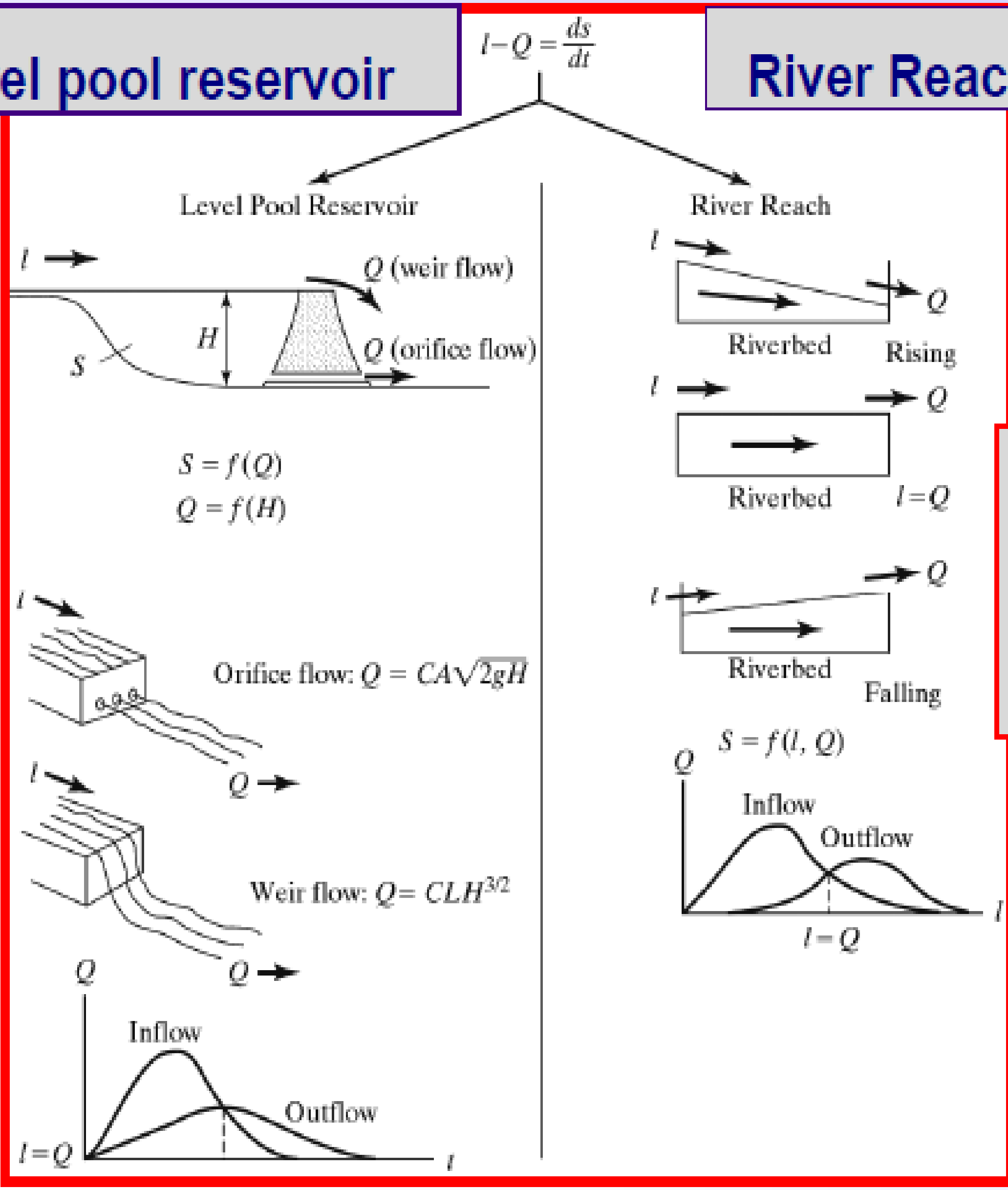
2. **Hydraulic Routing** ,method that adopt more rigorous equations of motion for unsteady flow in open channels,
(assumptions and approximations are often necessary to obtain solutions)

Types of Flood Routing

1. **Reservoir Routing**
2. **Channel Routing**

Level pool reservoir

River Reach



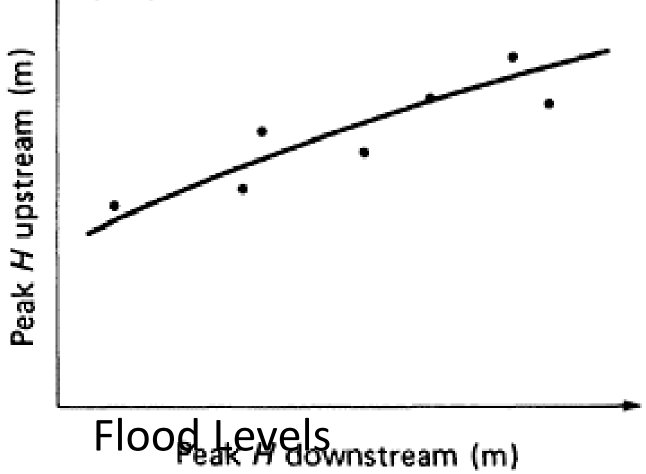
Comparisons: River vs. Reservoir Routing

4.2 Simple Routing Methods

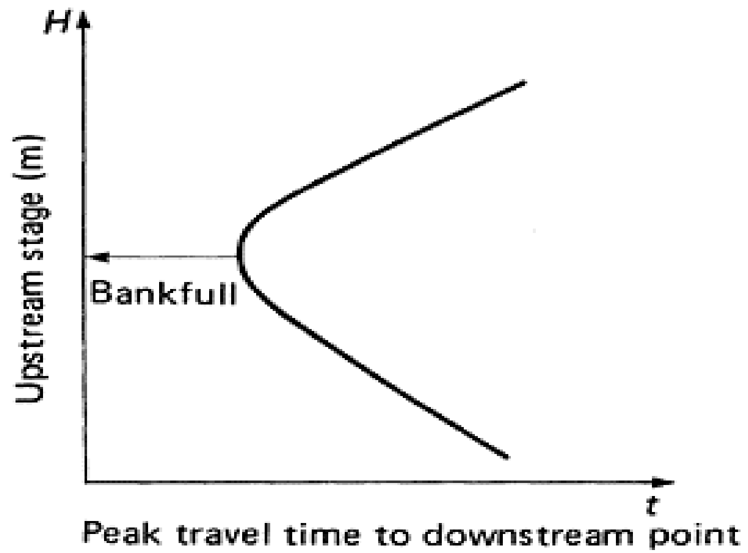
***In the case less or no data are available*:**

- *Points taken from experience of a number of past flood events*
- ***No tributaries are assumed here*.**

- The principal advantages is ,it can be developed from simple stage discharge relation and with no rating Curve .
- it is easy to apply and quick and give warning of impending of flood inundation



Flood Levels



Travel Time

Advantage:
1. quick and easy
2. developed for stations with only stage measurements

4.3 Storage Routing

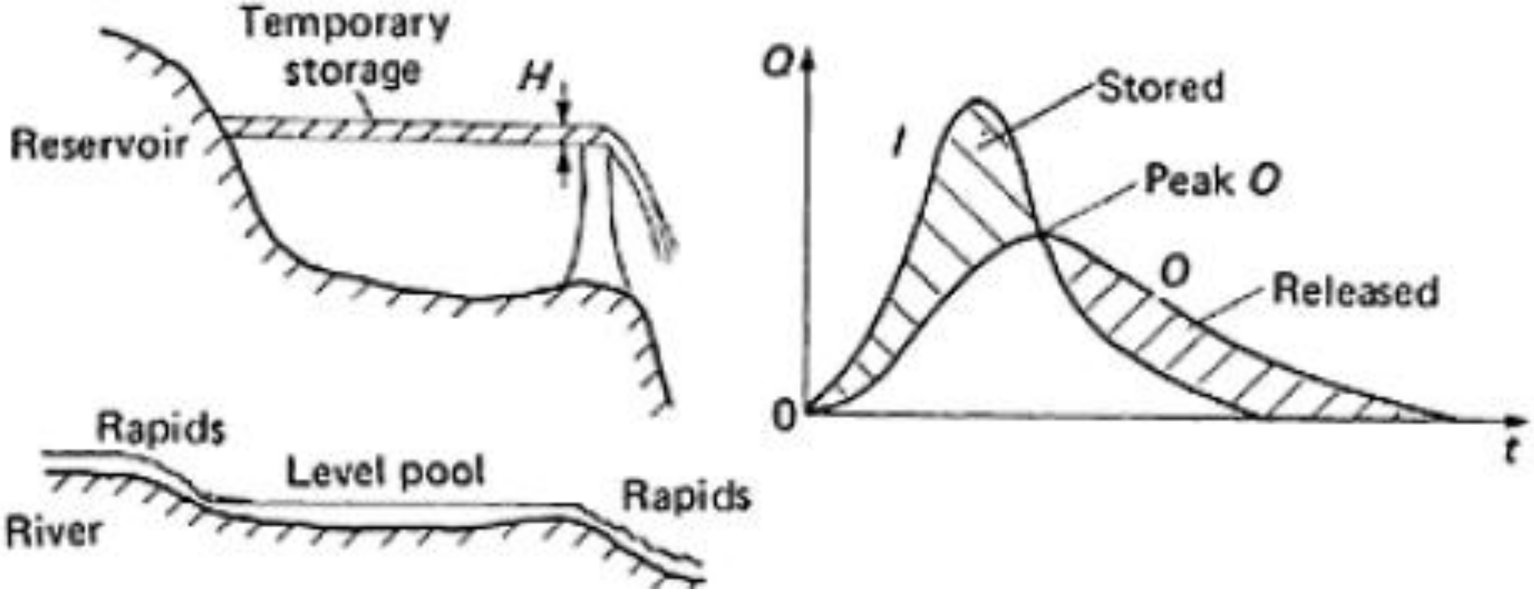
When a storm event occurs,

- at the beginning of the reach the flood hydrograph is (above normal flow) is given as ***I***, the inflow, during the period of the flood, T1, the channel reach has received the flood volume given by the area under the inflow hydrograph.
- Similarly, at the lower end of the reach, with an outflow hydrograph ***O***, the flood is given by the area under the curve.
- In a flood situation relative quantities may be such that lateral and tributary inflows can be *neglected*, By the principle of continuity, the volume of inflow equals the volume of outflow

$$\frac{dS}{dt} = I - O$$

Storage/reservoir Routing

This, *the continuity equation*, forms the basis of all the storage routing methods. The routing problem consists of finding *O* as a function of time,



From Conservation of Mass

$$\frac{dS}{dt} = I_t - Q_t$$

$$\bar{I}\Delta t - \bar{Q}\Delta t = \Delta S$$

or

$$\frac{S_{i+1} - S_i}{\Delta t} = \frac{1}{2}\left(I_{i+1} + I_i\right) - \frac{1}{2}\left(Q_{i+1} + Q_i\right)$$

Input data for reservoir routing:

- Field data, which is to establish relationship between elevation, outflow and storage of the reservoir.

✓ Inflow hydrograph, I (t)

✓ Initial values of storage, outflow and inflow which is recorded at the time of start of flow (t=0)

- The following two methods are commonly used for reservoir routing:

a. Level Pool Routing

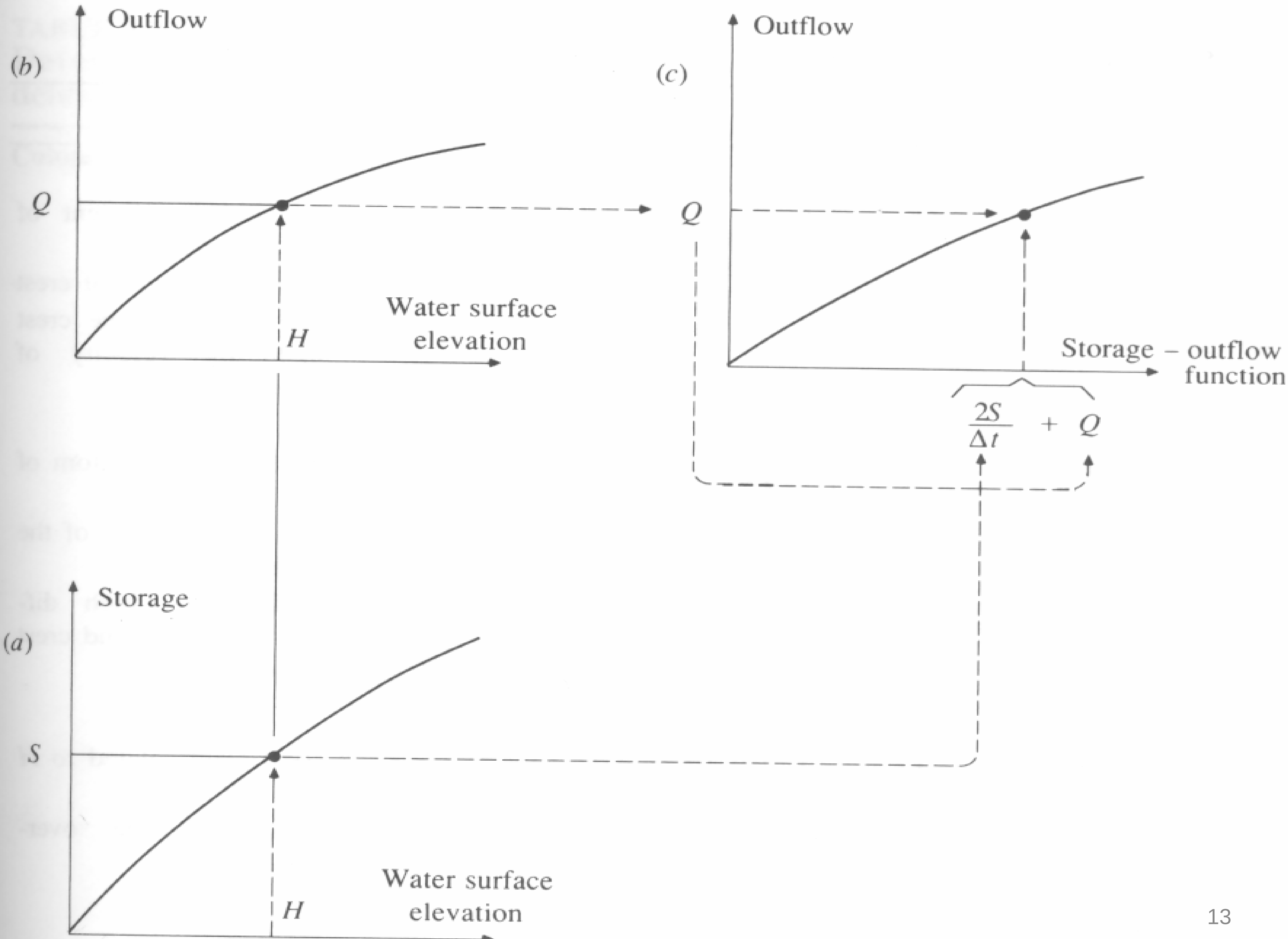
b. Goodrich Method

Level Pool Routing

.

The following data are required:

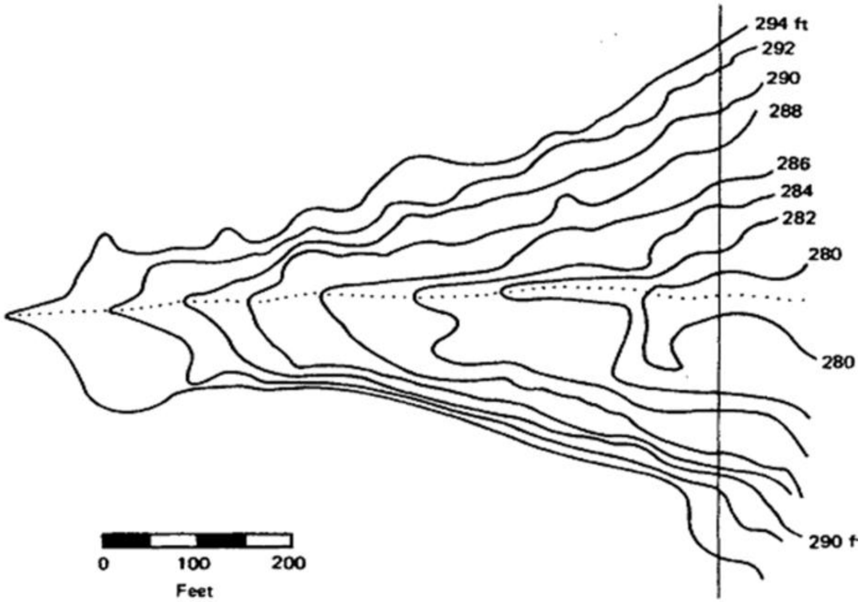
1. Storage volume versus elevation for the reservoir
2. Water surface elevation versus out flow and hence storage versus outflow discharge
3. Inflow hydrograph, I= I(t); and
4. Initial values of S, I and O at time t

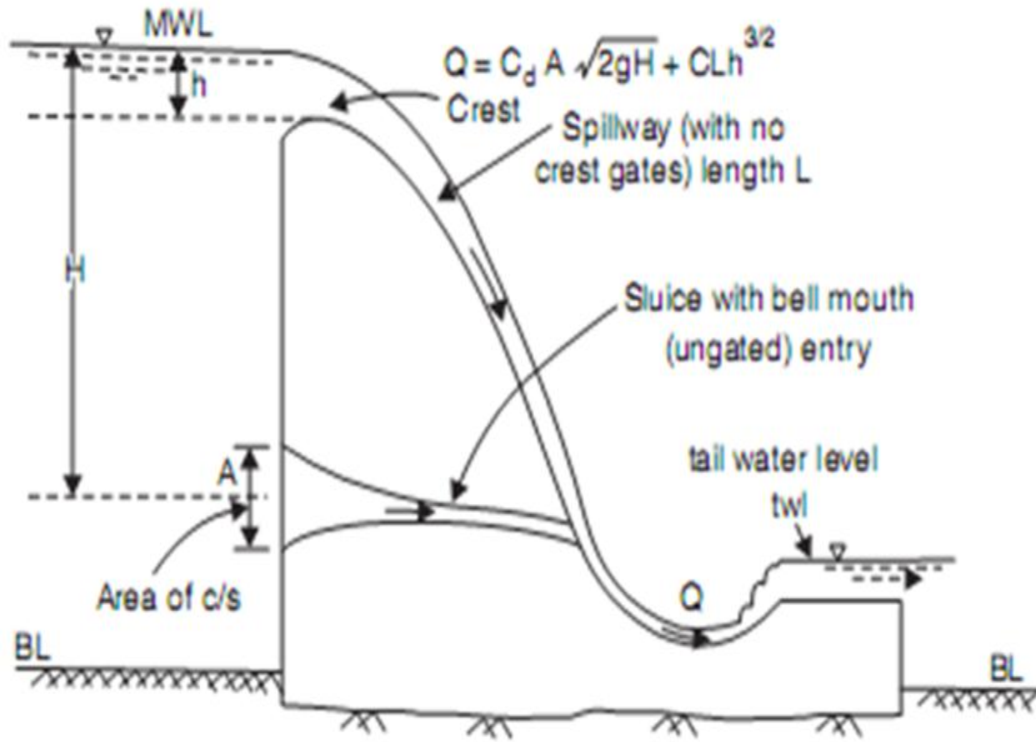

Outflow
(b)
Q
H
Water surface
elevation
Outflow
(c)
Q
Storage – outflow
function
2S/Δt + Q
Storage
(a)
S
H
Water surface
elevation

For a level pool:

. The temporary storage, *S,* is a function of *H,*

- The discharge from the 'pool' Q is also a function *H,*





$$\Delta S_{i+1} = \frac{1}{2}\left(A_{i+1} + A_i\right)\Delta H \, trapezoidal\ formula$$

$$\Delta S_{i+1} = \frac{1}{3}\left(A_{i+1} + A_i + \left(A_{i+1}A_i\right)^{\frac{1}{2}}\right)\Delta H \, cone\ form$$

then *S* is indirectly but uniquely a function of *Q*.

$$S_{j+1} - S_j = \frac{I_j + I_{j+1}}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t$$

$$\left(\frac{2S_{j+1}}{\Delta t} + Q_{j+1}\right) = (I_j + I_{j+1}) + \left(\frac{2S_j}{\Delta t} - Q_j\right)$$

$$\left(\frac{2S_{j+1}}{\Delta t} - Q_{j+1}\right) = \left(\frac{2S_{j+1}}{\Delta t} + Q_{j+1}\right) - 2Q_{j+1}$$

Example 8,2.1. A reservoir for detaining flood flows is one acre in horizontal area, has vertical sides, and has a 5-ft diameter reinforced concrete pipe as the outlet structure. The headwater-discharge relation for the outlet pipe is given io columns I and 2 of Table Use the level pool routing method to calculate the reservoir outflow from the inflow hydrograph given in columns 2 and 3 of Table below. Assume that the reservoir is initially empty

$$\left(\frac{2S_2}{\Delta t} + Q_2\right) = (I_1 + I_2) + \left(\frac{2S_1}{\Delta t} - Q_1\right)$$

$$= 60 + 0$$

$$= 60 \text{ cfs}$$

$$\left(\frac{2S_2}{\Delta t} - Q_2\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right) - 2Q_2$$

$$= 60 - 2 \times 2.4$$

$$= 55.2 \text{ cfs}$$

Column:	1 Elevation H (ft)	2 Discharge Q (cfs)	3 Storage S (ft³)	4 $(2S/\Delta t)^* + Q$ (cfs)
	0.0	0	0	0
	0.5	3	21,780	76
	1.0	8	43,560	153
	1.5	17	65,340	235
	2.0	30	87,120	320
	2.5	43	108,900	406
	3.0	60	130,680	496
	3.5	78	152,460	586
	4.0	97	174,240	678
	4.5	117	196,020	770
	5.0	137	217,800	863
	5.5	156	239,580	955
	6.0	173	261,360	1044
	6.5	190	283,140	1134
	7.0	205	304,920	1221
	7.5	218	326,700	1307
	8.0	231	348,480	1393
	8.5	242	370,260	1476
	9.0	253	392,040	1560
	9.5	264	413,820	1643
	10.0	275	435,600	1727

*Time interval $\Delta t = 10$ min.

$$\left(\frac{2S_3}{\Delta t} + Q_3\right) = (I_2 + I_3) + \left(\frac{2S_2}{\Delta t} - Q_2\right)$$

$$= 180 + 55.2$$

$$= 235.2 \text{ cfs}$$

Column: 1 Time index j	2 Time (min)	3 Inflow (cfs)	4 $I_j + I_{j+1}$ (cfs)	5 $\frac{2S_j}{\Delta t} - Q_j$ (cfs)	6 $\frac{2S_{j+1}}{\Delta t} + Q_{j+1}$ (cfs)	7 Outflow (cfs)
1	0	0		0.0		0.0
2	10	60	60	55.2	60.0	2.4
3	20	120	180	201.1	235.2	17.1
4	30	180	300	378.9	501.1	61.1
5	40	240	420	552.6	798.9	123.2
6	50	300	540	728.2	1092.6	182.2
7	60	360	660	927.5	1388.2	230.3
8	70	320	680	1089.0	1607.5	259.3
9	80	280	600	1149.0	1689.0	270.0
10	90	240	520	1134.3	1669.0	267.4
11	100	200	440	1064.4	1574.3	254.9
12	110	160	360	954.1	1424.4	235.2
13	120	120	280	820.2	1234.1	206.9
14	130	80	200	683.3	1020.2	168.5
15	140	40	120	555.1	803.3	124.1
16	150	0	40	435.4	595.1	79.8
17	160		0	338.2	435.4	48.6
18	170			272.8	338.2	32.7
19	180			227.3	272.8	22.8
20	190			194.9	227.3	16.2
21	200			169.7	194.9	12.6
22	210				169.7	9.8

Goodrich Method

Procedure for solution Goodrich's equation:

1) Select time increment Δt, which is equal to about 20 to 40% of the time rise of the inflow hydrograph. Then, compute $(\frac{2S}{\Delta t}+Q)$ by using field data.
2) Plot the graph of $(\frac{2S}{\Delta t}+Q)$ Vs reservoir elevation and Discharge (Q) Vs reservoir elevation on the same graph paper. And, compute the values of outflow (Q), reservoir elevation and $(\frac{2S}{\Delta t}+Q)$ for initial time (t=0), then by using these values calculate $\left(\frac{2S_1}{\Delta t}-Q_1\right)$ by subtracting 2Q from $(\frac{2S}{\Delta t}+Q)$.
3) Calculate $\left(\frac{2S_2}{\Delta t}-Q_2\right)$ for the first time interval by using equation3.4.
4) From procedure 2, find reservoir elevation and Q for $\left(\frac{2S_2}{\Delta t}-Q_2\right)$ from the plot, and calculate $(\frac{2S}{\Delta t}+Q)$ for the next time increment.
5) Keep repeating the procedure till the entire flood is routed.

A water reservoir has the following characteristics:

Elevation (m)	100	101	102	102.5	103
Storage, S (Mm^3)	3.5	4	5	5.5	6
Outflow, Q (m^3/s)	0	10	60	90	125

When the water level in the reservoir was 101m, a flood with the following input hydrograph entered the reservoir.

Time (hr)	0	6	12	18	24	30	36	42
Discharge (m^3/s)	10	15	25	40	28	18	13	11

Solution:

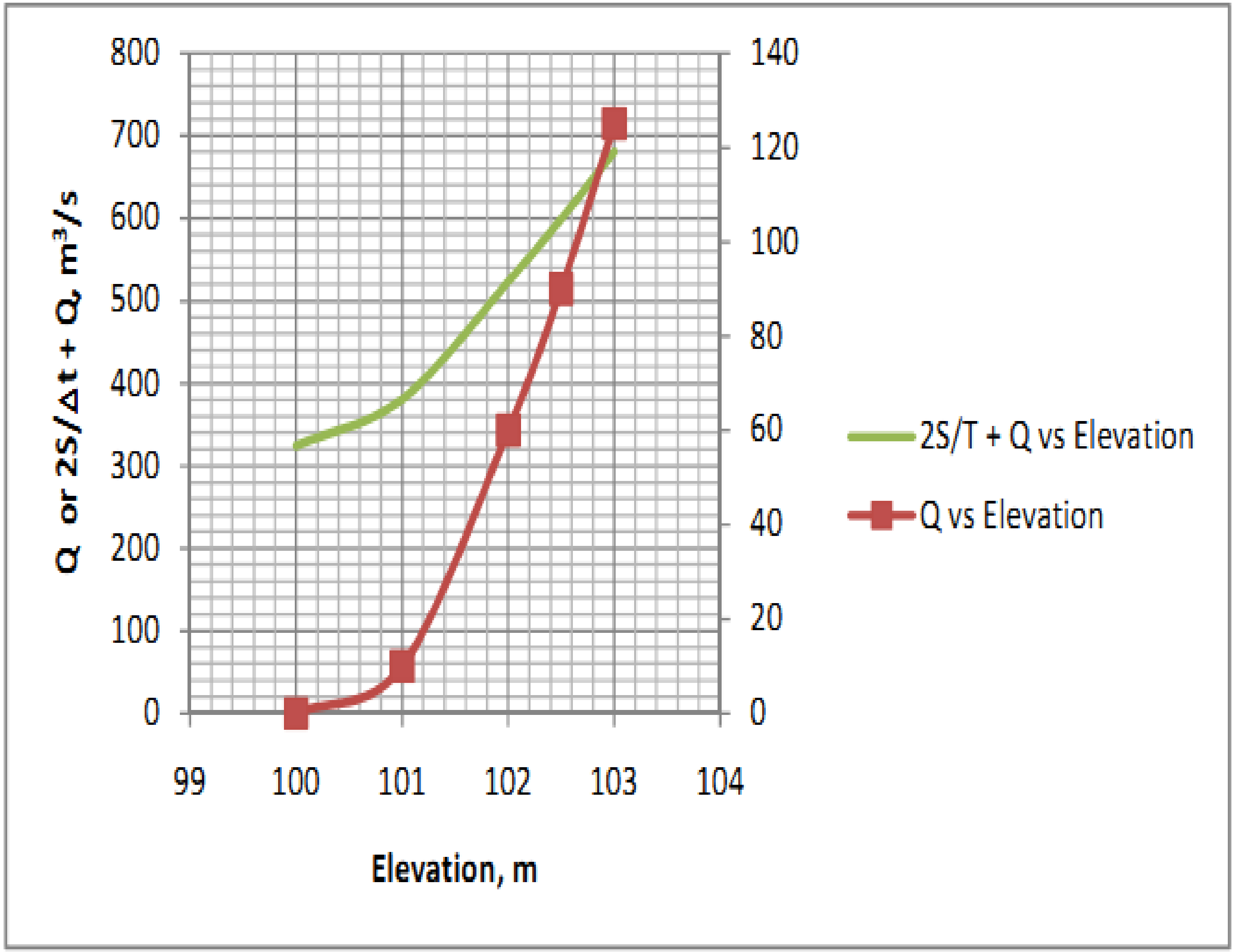
a) Goodrich Method

The increment of Δt = 6hr = 0.0216Ms is taken and the table below is prepared.

Table: Channel routing by Goodrich method

Elevation	Discharge, Q	Storage, S Mm3	$\frac{2S}{\Delta t} + Q$
100	0	3.5	324.0755
101	10	4	380.372
102	60	5	522.965
102.5	90	5.5	599.2615
103	125	6	680.558

Plot a graph of Q vs Elevation and $\frac{2S}{\Delta t} + Q$ vs Elevation on same graph



When routing started, reservoir elevation was 101m. Then, Q = 10m3/s and $\frac{2S}{\Delta t} + Q = 380.36 m3/s$ are read from the above graph for Elevation = 101m.

Hence, $\frac{2S_1}{\Delta t} - Q_1 = \left(\frac{2S_2}{\Delta t} + Q_2\right) - 2Q_2 = 380.36 - 2 \times 10 = 360.36 \frac{m^3}{s}$

For the first time interval $\Delta t = 6hr$ we have $I_1 = 10, I_2 = 15$ *and* $Q_1 = 10$.

$$(I_1 + I_2) + \left(\frac{2S_1}{\Delta t} - Q_1\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right)$$

$$\left(\frac{2S_2}{\Delta t} + Q_2\right) = \left(\frac{2S_1}{\Delta t} - Q_1\right) + (I_1 + I_2) = 360.36 + (10 + 15) = 385.36\, \mathrm{m^3/s}$$

From the above graph:

For $\left(\frac{2S_2}{\Delta t} + Q_2\right) = 385.36\, \mathrm{m^3/s}$, Elevation = 101.05m and from same graph for Elevation = 101.05m, Q = $12\, \mathrm{m^3/s}$

Therefore, $\left(\frac{2S_1}{\Delta t} - Q_1\right) = 385.36 - 2 \times 12 = 361.36\, \mathrm{m^3/s}$

The rest of calculation is shown in the table given below

Table: Channel routing by Goodrich method

Time (hr)	Inflow	$I_1 + I_2$	$\frac{2S_1}{\Delta t} - Q_1$	$\frac{2S_2}{\Delta t} + Q_2$	Elevation	Q_2	$2Q_2$
0	10				101	10	20
		25	360.36	385.36			
6	15				101.05	12	24
		40	361.36	401.36			
12	25				101.1	14	28
		65	373.36	438.36			
18	40				101.3	22	44
		68	394.3	462.36			
24	28				101.5	28	56
		46	406.36	452.36			
30	18				101.35	26	52
		31	384.36	415.36			
36	13				101.2	18	36
		24	379.36	403.36			
42	11				101.1	14	28
			375.36				

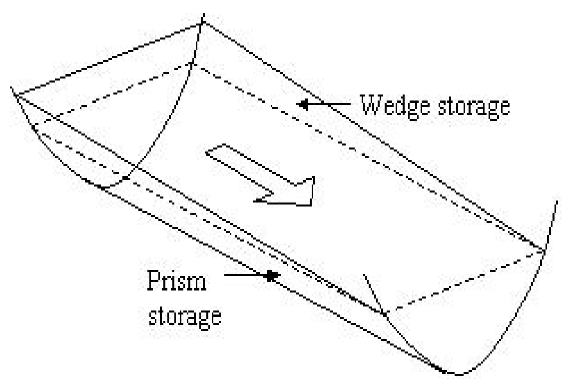
Finally, Attenuation in the peak flow rate = 40 – 28 = 12m³/s. and Lag in peak flow time = 18 – 24 = 6hr

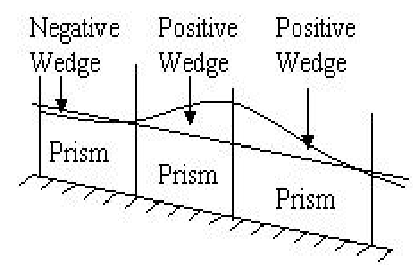
Channel /river Routing

- In reservoir routing the storage is a function of the out flow.
- In Channel/River routing the storage is a function of both outflow and inflow discharges and hence a different routing method is needed.
- For a river reach where the water surface cannot be considered parallel to the river bottom during the passage of a flood wave, the storage in the reach may be split up in two parts:

(i) prism storage and

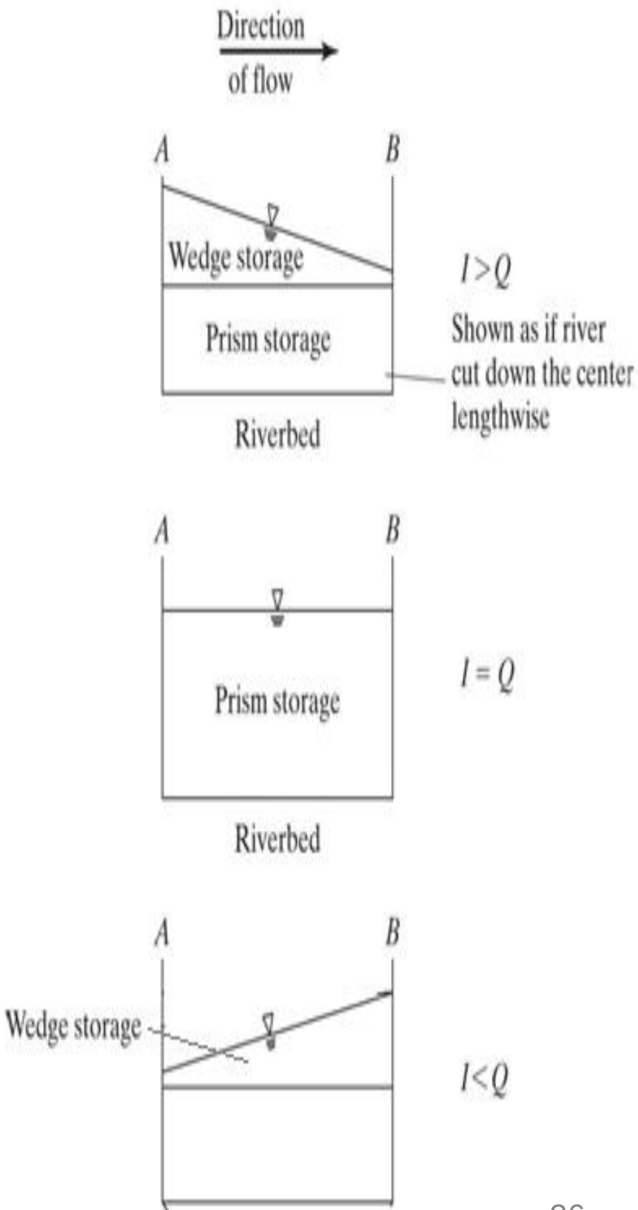
(ii) wedge storage.





Wedge and Prism storage

- **Prism storage:** it is the volume that would exist if uniform flow occurred at the downstream depth, i.e. the volume formed by an imaginary plane parallel to the channel bottom drawn at the outlet section water surface. Storage is a function of Q.
- **Wedge storage:** it is the wedge-like volume formed between the actual water surface profile and the top surface of the prism storage. It exists because the inflow I differs from the outflow Q and hence is assumed to a function of the difference I-Q; during the rising stages of the flood I>Q and the wedge storage is positive, whereas during the falling stages when I<Q it is negative.



$$S=f_1(Q)+f_2(I-Q)$$

Muskingum method of Routing

$$S=f_1(Q)+f_2(I-Q)$$

- In the application of the method by *McCarthy in 1938 on the Muskingum River* he assumed that both f_1 and f_2 are linear functions of the type:

$$f_1(Q)=KQ \quad \text{and} \quad f_2(I-Q)=b(I-Q)$$

So that: $$S=KQ+b(I-Q)=bI+(K-b)Q$$

$$\Rightarrow S=K\left(\frac{b}{K}I+\left(1-\frac{b}{K}\right)\right)Q$$

Let $x=\frac{b}{K}$

then $$S=K\left(xI+(1-x)Q\right)$$

- Assuming linear relation exponent constant is 1

- where x is a weighting factor between 0 and 0.5 that says something about how inflow and outflow vary within a given reach, and K is the travel time of the flood wave.

- The storage discharge equation is written in a finite difference form:

$$1/2\ (I_1 + I_2) - 1/2\ (Q_1 + Q_2) = (S_2 - S_1)/\Delta t$$

Equation 1

- The Muskingum routing procedure itself uses this form combined wit $S = K[xI + (1-x)Q]$

In the form:

$$S_2 - S_1 = K[x(I_2 - I_1) + (1-x)(Q_2 - Q_1)]$$

Equation 2

- Combining eqn 1 and eqn 2 the Muskingum outflow equation becomes:

$$Q_2 = C_1 I_1 + C_2 I_2 + C_3 Q_1$$

Getting rid of S

Muskingum equation coefficients

$$Q_2 = C_1 I_1 + C_2 I_2 + C_3 Q_1$$

$$c_1 = \frac{\Delta t + 2Kx}{2K(1-x) + \Delta t}$$

$$c_2 = \frac{\Delta t - 2Kx}{2K(1-x) + \Delta t}$$

$$c_3 = \frac{2K(1-x) - \Delta t}{2K(1-x) + \Delta t}$$

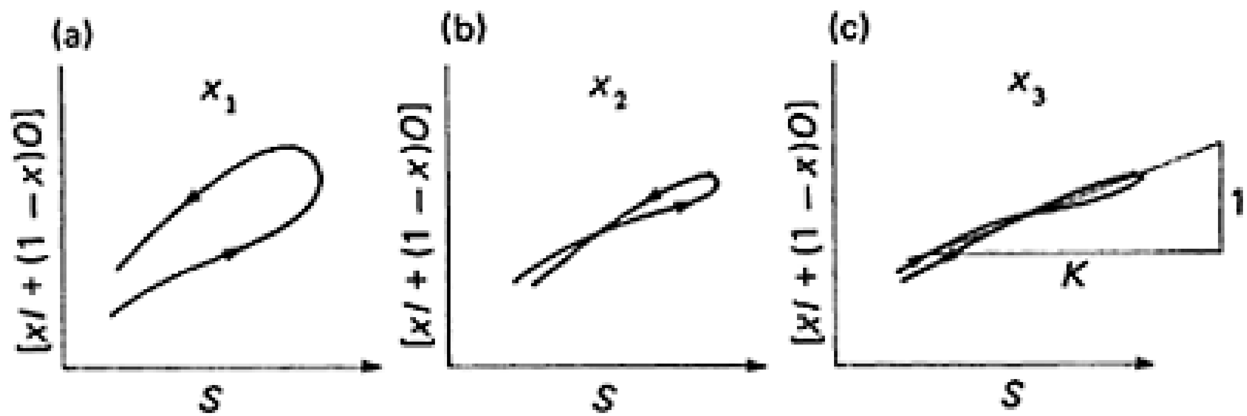
- It is easily verified that $c_1+c_2+c_3=1$ and further that Δt and K should have the same time units. The use of the method requires choices on Δt, K and x.

Routing interval:

- the routing interval should be less than K and is advised to be taken as ***1/2 to 1/4 of K***.
- It has been found that for best results the routing interval Δt should be chosen that ***K>Δt>2Kx. If Δt<2Kx, the coefficient C_2 will be negative.***

Parameters K and x

- Using recorded hydrographs of a flood at the beginning and end of the river reach, trial values of *x* are taken, and for each trial the weighted flows in the reach, $[xI+(1-x)Q]$, are plotted against actual storages determined from the inflow and outflow hydrographs as indicated in the following figure.
- The correct value for *x* will be the one giving the best approximation to a straight-line plot. When the looping plots of the weighted discharges against storages have been narrowed down



- The Muskingum coefficients *c*1, *c*2 and c3 are next evaluated. Then beginning with the initial inflow and outflow, sequential values of *Q* are computed

Procedure of Determining K and X:

1) Choose trial value of X
2) Plot a graph between storage (S) vs. [XI+ (1-X) O] and the assumed value of X is correct when this graph is very near to straight line.
3) Storage time constant K is the reciprocal of slope of the line formed for the selected values of X.

Example

- In the catchment of a natural stream, an isolated storm occurred. The following hydrograph of the runoff was measured at a point at intervals of 6 h.

T (hr)	0	6	12	18	24	30	36	42
Runoff (m3/s)	10	12	20	40	28	18	13	11

Determine the ordinate of the hydrograph at another point, after flood wave has traveled for 12 hr. Assume the weighting factor, x= 0.23 and the flow on the steam the inflow of the storm runoff is 10 m^3/s.

Example

For a river reach K=28hr, X=0.25 and take $O_1 = I_1$ for the beginning of time step. Route the following inflow hydrograph and compute values of attenuation and translation of the peak.

Time (hr.)	0	6	12	18	24	30	36	42	48	54	60
Inflow (m^3/sec)	30	62	242	170	114	78	56	44	38	34	30

Example.1

The data in the following Table represent observed inflow and outflow flood hydrographs for a channel reach. Derived Muskingum constants for the reach K and x [redacted] Using the derived constants in the Muskingum equations, route the observed inflow hydrograph through the reach and plot the calculated outflow hydrograph obtained, comparing it with the observed hydrograph.

Time (hr)	0	12	24	36	48	60	72	84	96	108	120	132
I (m3/s)	22	35	103	109	86	59	39	28	22	20	19	18
Q (m3/s)	22	21	34	55	75	85	80	64	44	30	22	20

Solution

Step -1 **Determination of K and X**

Given

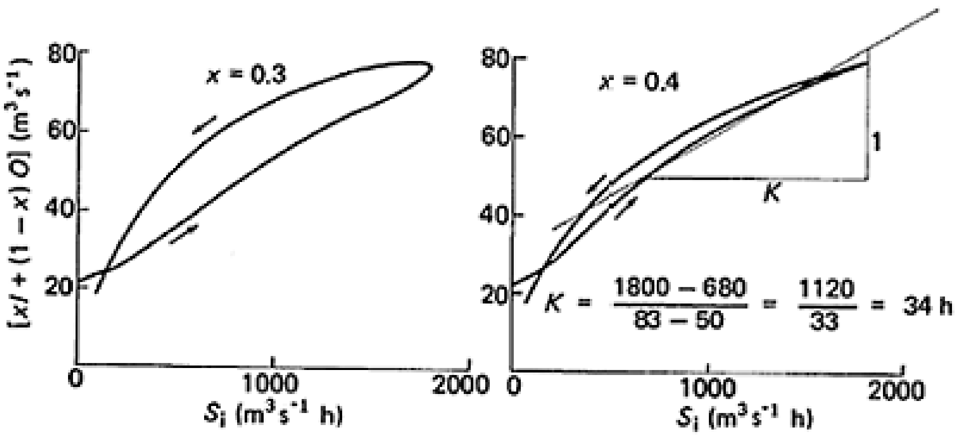
T	I	Q	I-Q	ΔS (m^3)	S_i	S x=0.3	S x =0.4
0	22	22	0	0	0	22	22
12	35	21	14	168	168	25.2	26.6
24	103	34	69	828	996	54.7	61.6
36	109	55	54	648	1644	71.2	76.6
48	86	75	11	132	1776	78.3	79.4
60	59	85	-26	-312	1464	77.2	74.6
72	39	80	-41	-492	972	67.7	63.6
84	28	64	-36	-432	540	53.2	49.6
96	22	44	-22	-264	276	37.4	35.2
108	20	30	-10	-120	156	27	26
120	19	22	-3	-36	120	21.1	20.8
132	18	20	-2	-24	96	19.4	19.2

Assume x and plot S Vs computed S, the slope of the "line" is k.

$$\Delta S=(I-Q)\Delta t$$

$$S_i=\Delta S_i+\Delta S_{i+1}+\ldots+\Delta S_n$$

$$S = K[xI + (1-x)Q]$$



…continued

$$c_1 = \frac{\Delta T + 2Kx}{\Delta T + 2K - 2Kx} = \frac{12 + 2 \times 34 \times 0.4}{12 + 2 \times 34 - 2 \times 34 \times 0.4} = \frac{39.2}{52.8} = 0.74$$

$$c_2 = \frac{\Delta T - 2Kx}{\Delta T + 2K - 2Kx} = \frac{12 - 2 \times 34 \times 0.4}{12 + 2 \times 34 - 2 \times 34 \times 0.4} = \frac{-15.2}{52.8} = -0.29$$

$$c_3 = 1 - c_1 - c_2 = 1 - 0.74 + 0.29 = 0.55$$

$$\left(\text{Check } c_3 = \frac{-\Delta T + 2K - 2Kx}{\Delta T + 2K - 2Kx} = \frac{28.8}{52.8} = 0.55\right)$$

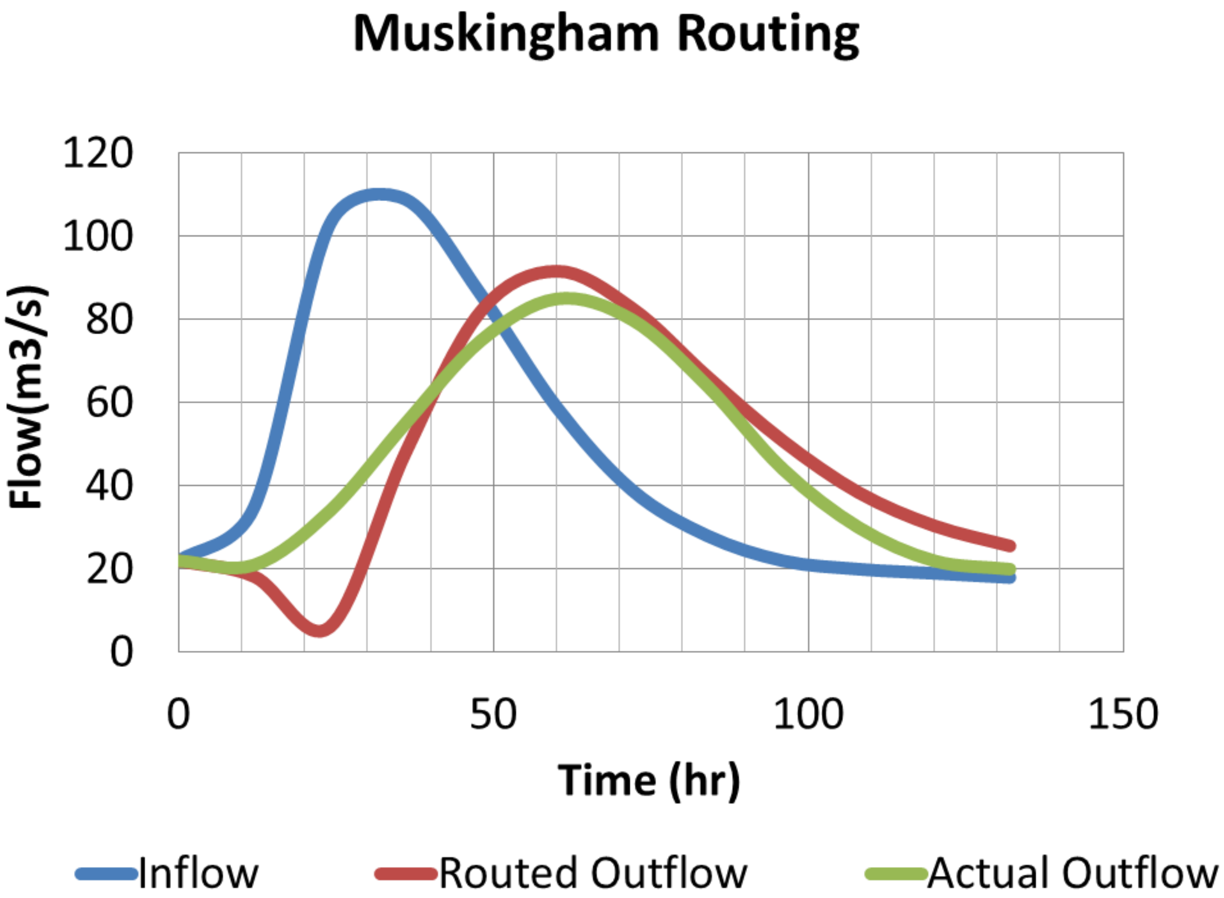
Hence using

$$Q_2 = 0.74 I_1 - 0.29 I_2 + 0.55 Q_1$$

With a sequential computation:

T	0	12	24	36	48	60	72	84	96	108	120	132
I	22	35	103	109	86	59	39	28	22	20	19	18
Q	22	18.23	6.057	47.94	82.09	91.68	82.77	66.27	50.79	38.41	30.42	25.57

…continued

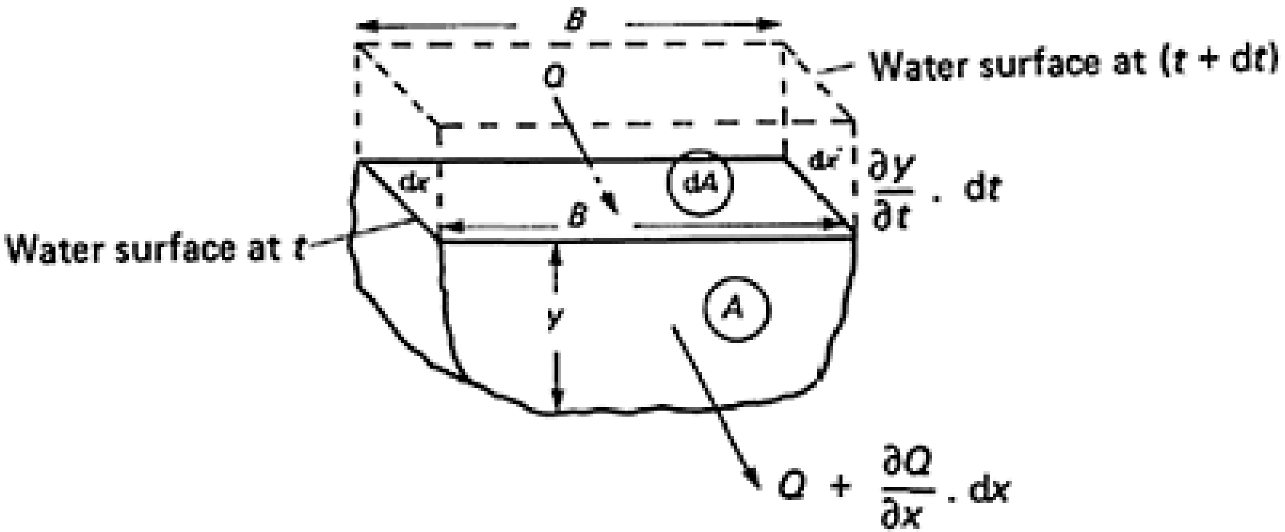


Muskingham Routing
120
100
80
60
40
20
0
Flow(m3/s)
0
50
100
150
Time (hr)
Inflow
Routed Outflow
Actual Outflow

Hydraulic Routing

- The hydrologic method is based on continuity and S vs Q/I (empirical approximation)
- The hydraulic methods of flood routing are based on the solution of the two basic differential equations governing gradually varying non-steady flow in open channels (the Saint Venant equations).

1. Continuity equation
2. Momentum equation



Continuity equation

$$\frac{\partial S}{\partial t} = Q - \left(Q + \frac{\partial Q}{\partial x}\,\mathrm{d}x\right)$$

S=A.dx

$$\frac{\partial S}{\partial t} = \frac{\partial A}{\partial t}\,\mathrm{d}x = -\frac{\partial Q}{\partial x}\,\mathrm{d}x$$

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

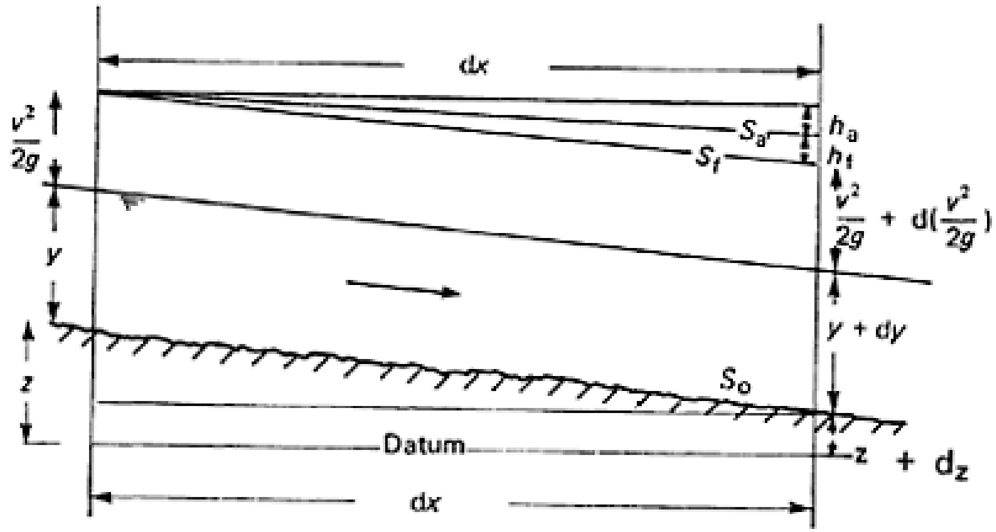
Q=A.V

$$A\frac{\partial v}{\partial x} + v\frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} = 0$$

A=By, Assuming B constant

$$A\frac{\partial v}{\partial x} + Bv\frac{\partial y}{\partial x} + B\frac{\partial y}{\partial t} = 0$$

Dynamic equation



$h_f = S_f \mathrm{d}x$, the head loss due to friction $h_a = S_a \mathrm{d}x = \frac{1 \partial v}{g \partial t} \mathrm{d}x$, the head loss due to acceleration.

$$H = z + y + \frac{v^2}{2g}$$

$$\frac{\mathrm{d}H}{\mathrm{d}x} = -S_f - S_a = \frac{\mathrm{d}}{\mathrm{d}x}\left[z + y + \frac{v^2}{2g}\right]$$

If it is assumed that the channel bed slope is small and the vertical component of the acceleration force is negligible, then the combined loss of head is *(h_f+h_a)*. Using the Bernoulli expression for total head, H:

…Continued

$$S_f = \frac{-\partial z}{\partial x} - \frac{\partial y}{\partial x} - \frac{v \partial v}{g \partial x} - \frac{1 \partial v}{g \partial t}$$

$$\Rightarrow \quad S_f = S_0 - \frac{\partial y}{\partial x} - \frac{v \partial v}{g \partial x} - \frac{1 \partial v}{g \partial t}$$

The friction slope can be computed from Chezy:

$$S_f = \frac{v^2}{C^2 R}$$

Thank You !!!