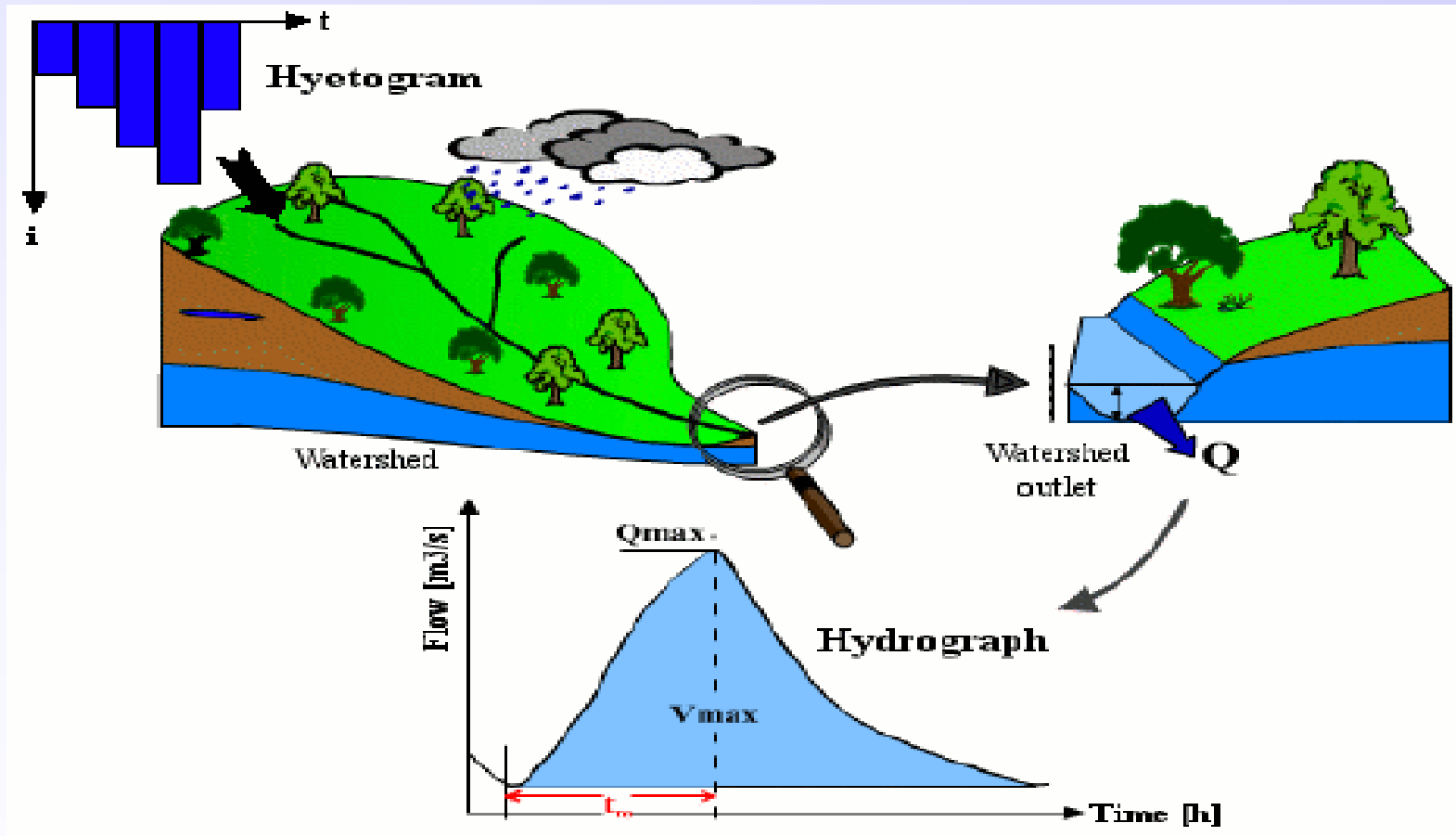


# Stream flow Hydrograph

Prepared by: Birhanu W.

# 2. Stream flow hydrograph



Hydrological response of a catchment

# Hydrograph

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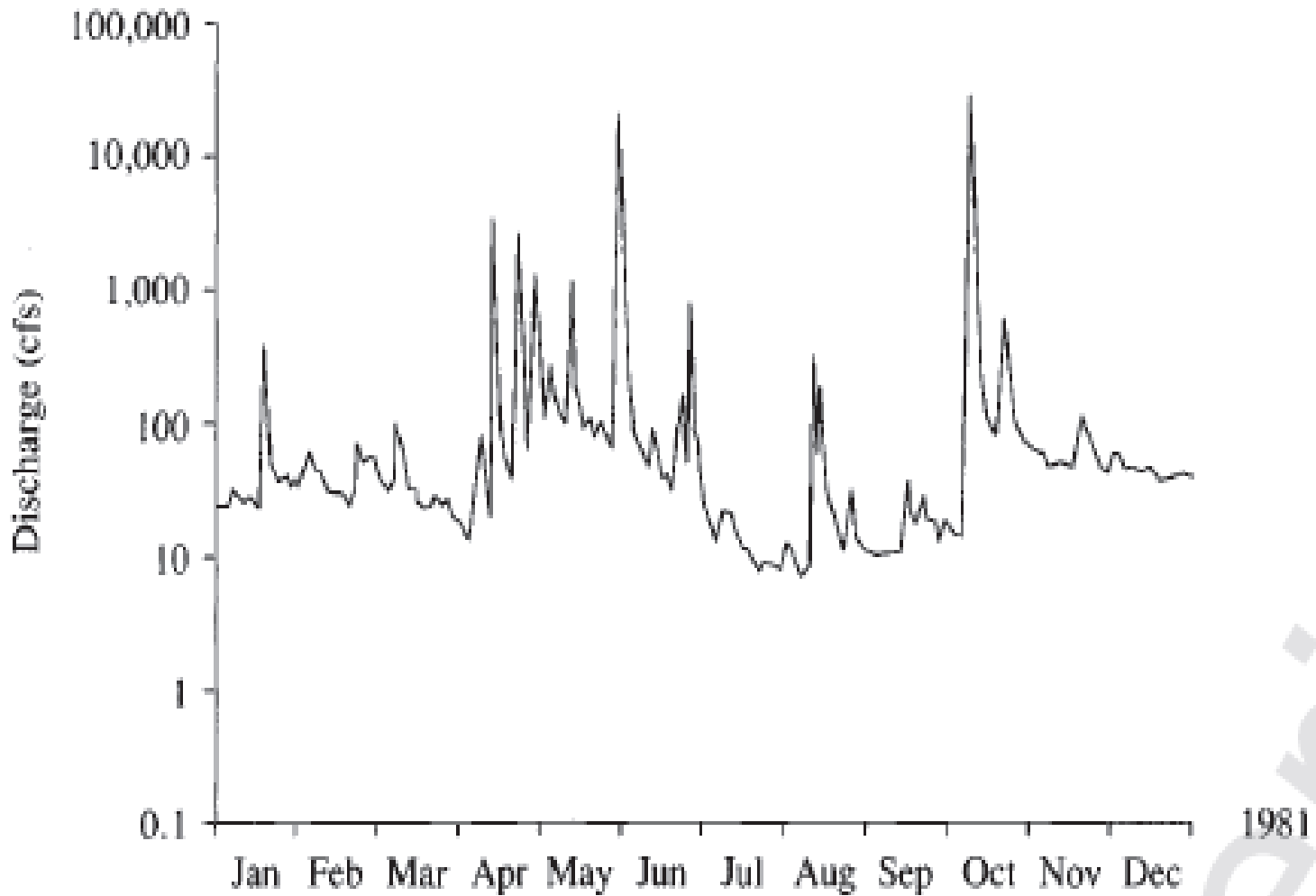
- ✓ Discharge in a stream plotted against time chronologically is called Hydrograph
- *Depending upon the unit of time involved, we have:*
  - I. Annual hydrographs showing the variation of daily or weekly or 10 daily mean flows over a year;
  - II. Monthly hydrographs showing the variation of daily mean flows over a month;
  - III. Seasonal hydrographs depicting the variation of the discharge in a particular season such as the monsoon season or dry season; and
  - IV. Flood hydrographs or storm hydrographs is due to a storm representing stream flow due to a rain fall over a catchment.

# Hydrograph

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- applications, I&II hydrographs are
  - a) calculating the surface water potential of stream,
  - b) reservoir studies and
  - c) drought studies.
- applications, III Flood hydrographs are essential in analyzing stream characteristics associated with floods.
- A study of the annual hydrographs of streams enables one to classify streams into three classes as (i) perennial, (ii) intermittent and (iii) ephemeral.

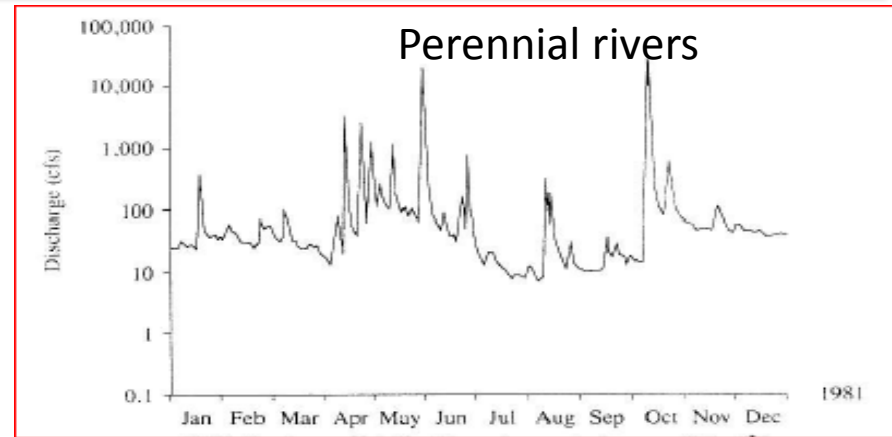
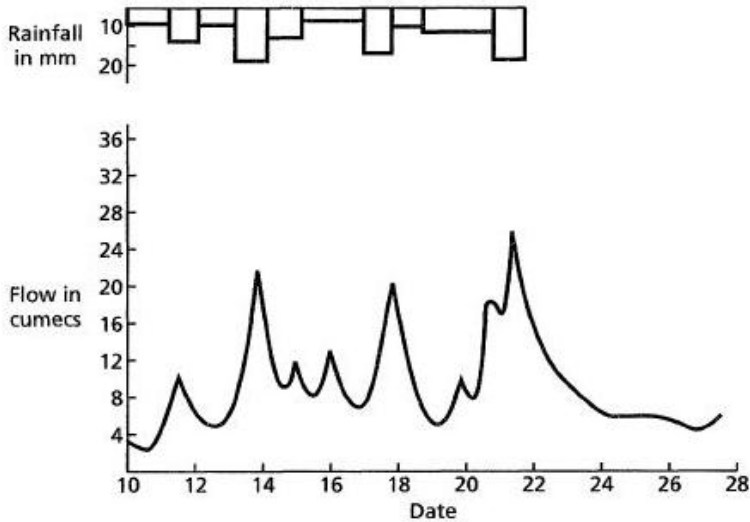
# Cont...



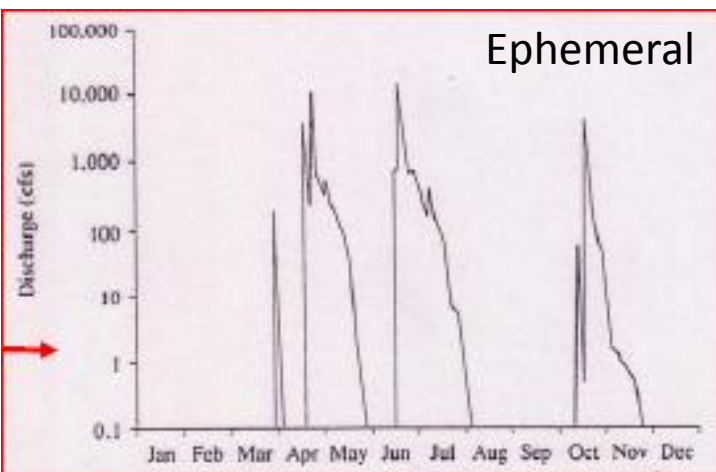
Typical annual Hydrograph

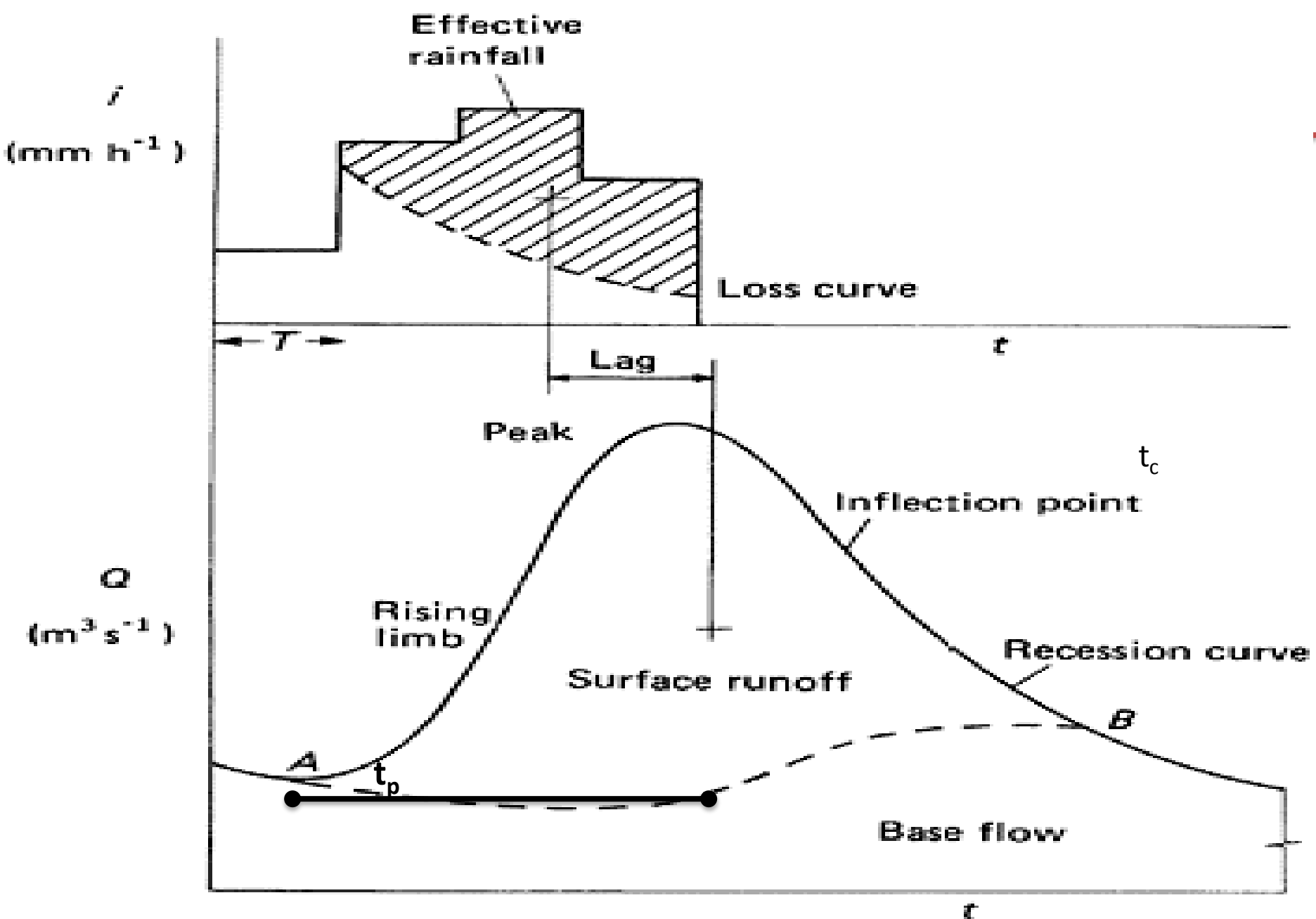
# Types of Streams

**Hydrograph** is a result of a particular effective rainfall hyetograph as modified by basin flow characteristics.



- **Perennial stream** is one which always carries some flow, there is considerable amount of groundwater flow throughout the year.
- Even during dry seasons the water table will be able to reach the bed of the stream.
- **Intermittent stream** has limited contribution from the groundwater.
- During the wet season the water table is above the streambed and there is a contribution of the base flow to the stream flow. which can produce a short-duration flow, the stream remains dry for the most part of the dry months.
- **Ephemeral stream** is one, which does not have any base-flow contribution.
- The annual hydrograph of such a river show series of short duration peaks marking flash flows in response to storms.





# Base Flow Separation

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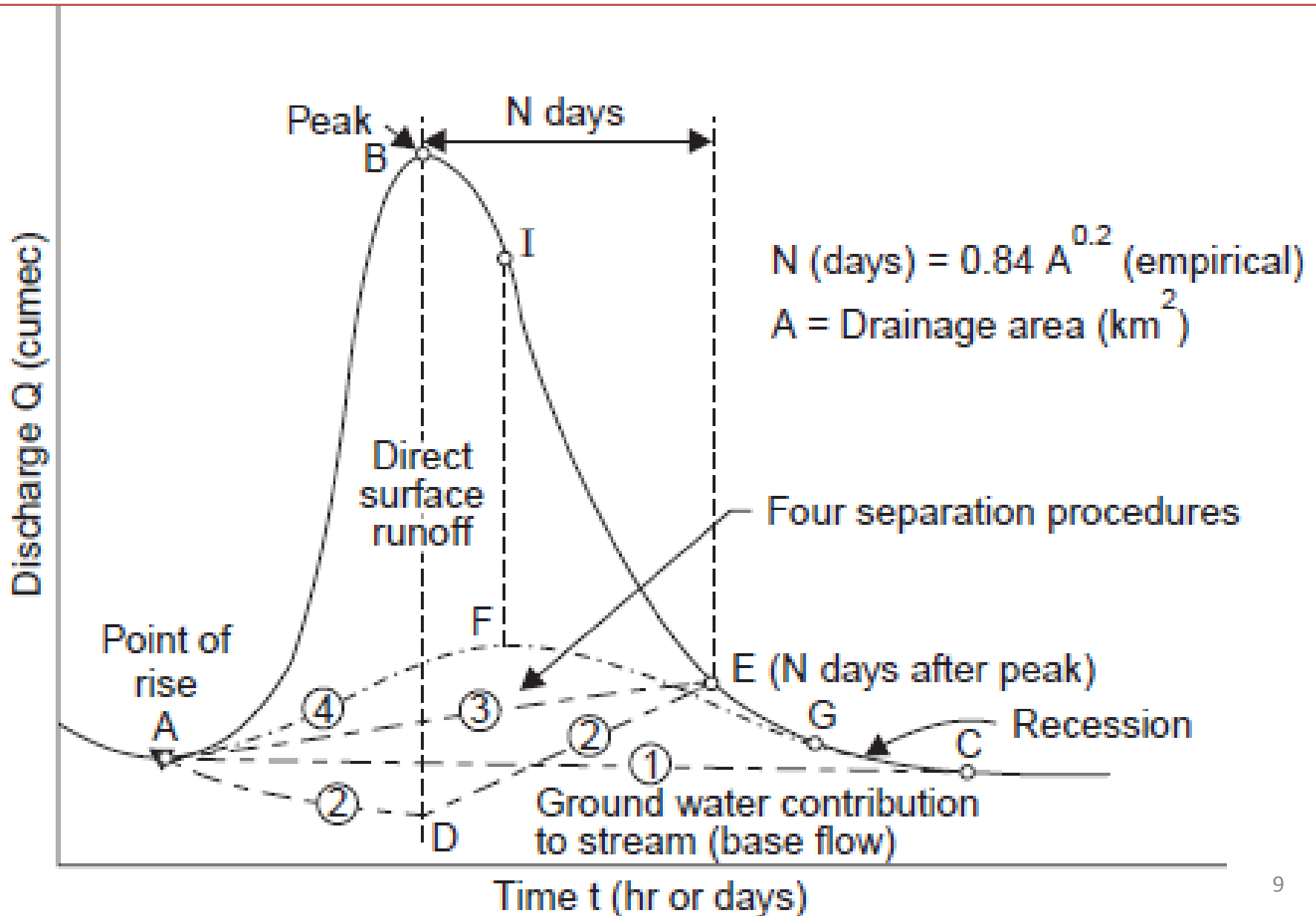
Total runoff consists of direct runoff and base flow.

➤ There are several methods of base flow separation:

- **Straight-line method**
- **Fixed base (Concave )Method**
- **Constant slope method**
- **Variable slope Method IV**



# Method description



# Description of the Methods

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- Method 1- *for preliminary analysis*

Tangential drawing for both rising and recession limb

- Method 2 - mostly used  
$$N = 0.83A^{0.2}$$

Use of the equation to find Point E, and connecting to point D (extension of the initial base flow contribution up to the peak)

- Method 3

Connecting point A and E

- Method 4- *mostly for significant & immediate contributing base flow geology*

I is the inflection point

# Factors Affecting Runoff

---

## **Physical characteristics:**

### **Basin characteristics**

- shape
- size
- slope
- nature of the valley
- elevation
- drainage density

### **Infiltration characteristics**

- land use and cover
- soil type and geological conditions
- lakes, swamps and other storages

## **Climatic Characteristics:**

1. Storm characteristics:  
precipitation: intensity, duration, magnitude and movement of storms
2. Initial losses
3. Evapotranspiration

## **Channel characteristics:**

- cross-section,
- roughness and storage capacity

## *2.1 Unit Hydrograph*

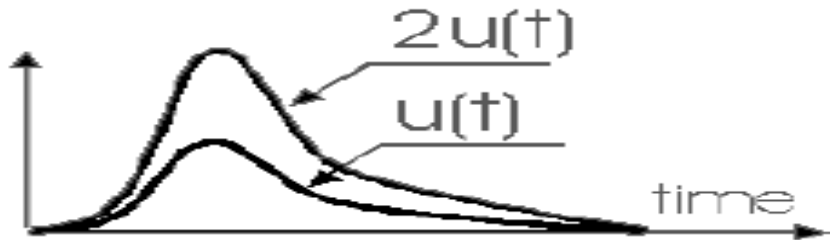
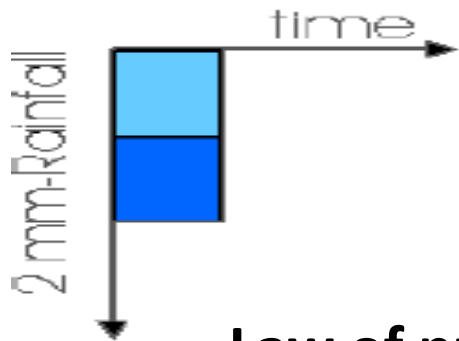
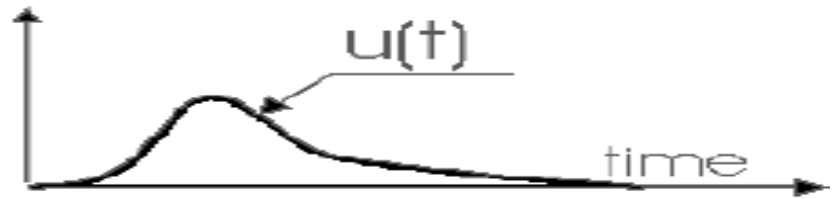
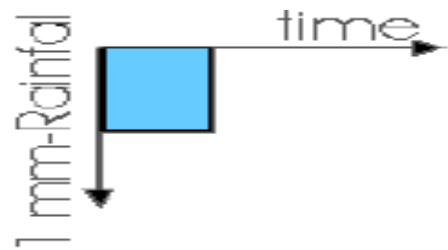
- It is flood hydrograph resulted from unit depth of rain fall excess of certain duration over uniformly distributed rain fall.
- It is a the unit pulse response function of a linear hydrologic system
- First proposed by Sherma (1932)
- Unit hydrograph is simple linear model that can be used to drive the hydrograph resulting from any amount of excess rain fall.
- The unit hydrograph is a “transfer” mechanism for transforming excess precipitation into stream flow.

# Unit Hydrograph Principles

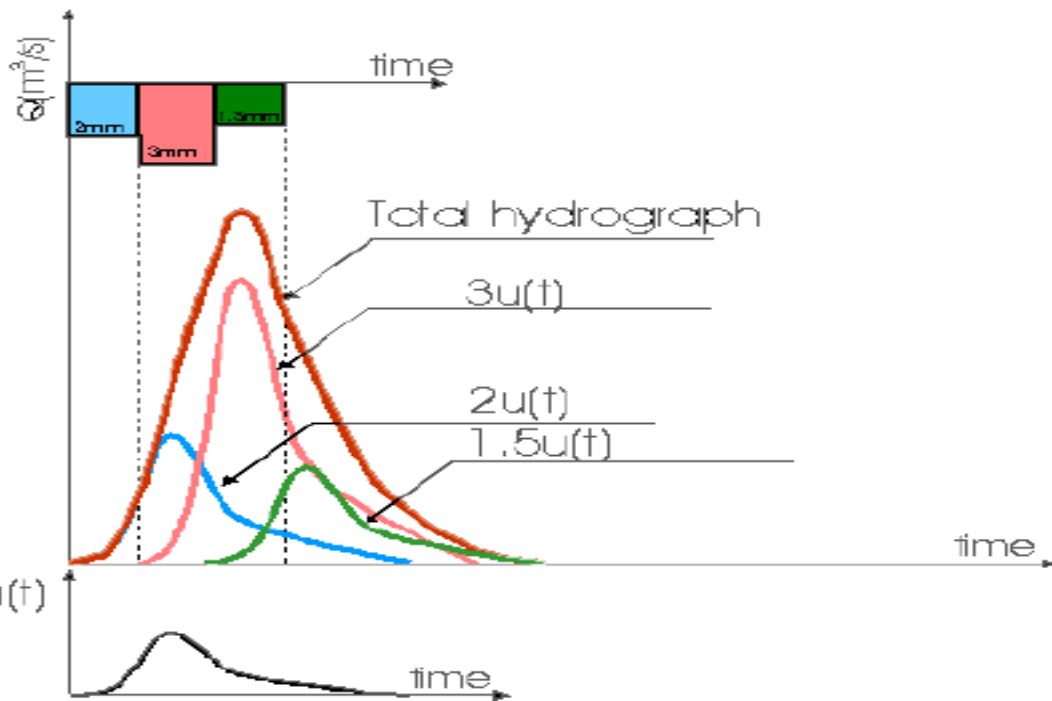
- ✓ Time invariant; the effective rainfall-surface runoff relationship does not change with Time. Irrespective of when it happens,  $D$  hr UH of a catchment is invariant.
- ✓ Linear response the base of the DRH will be the same as that of the unit hydrograph
- ✓ Law of superposition
  - If there is a continuous storm and/or isolated storms of uniform intensity net rain, the ordinates added with the appropriate time lag to get the combined hydrograph.

# The basic assumption constitute the foundation for UHG

- ❖ The excess rain fall has a constant intensity with in the effective duration
- ❖ The effective rainfall is uniformly distributed through out the catchment
- ❖ The base time (duration) of direct runoff hydrograph resulting from an excess rainfall of given duration is constant.
- ❖ The ordinates of all direct runoff hydrographs of a common base time are directly Proportional to the total amount of direct runoff represented by each hydrograph.



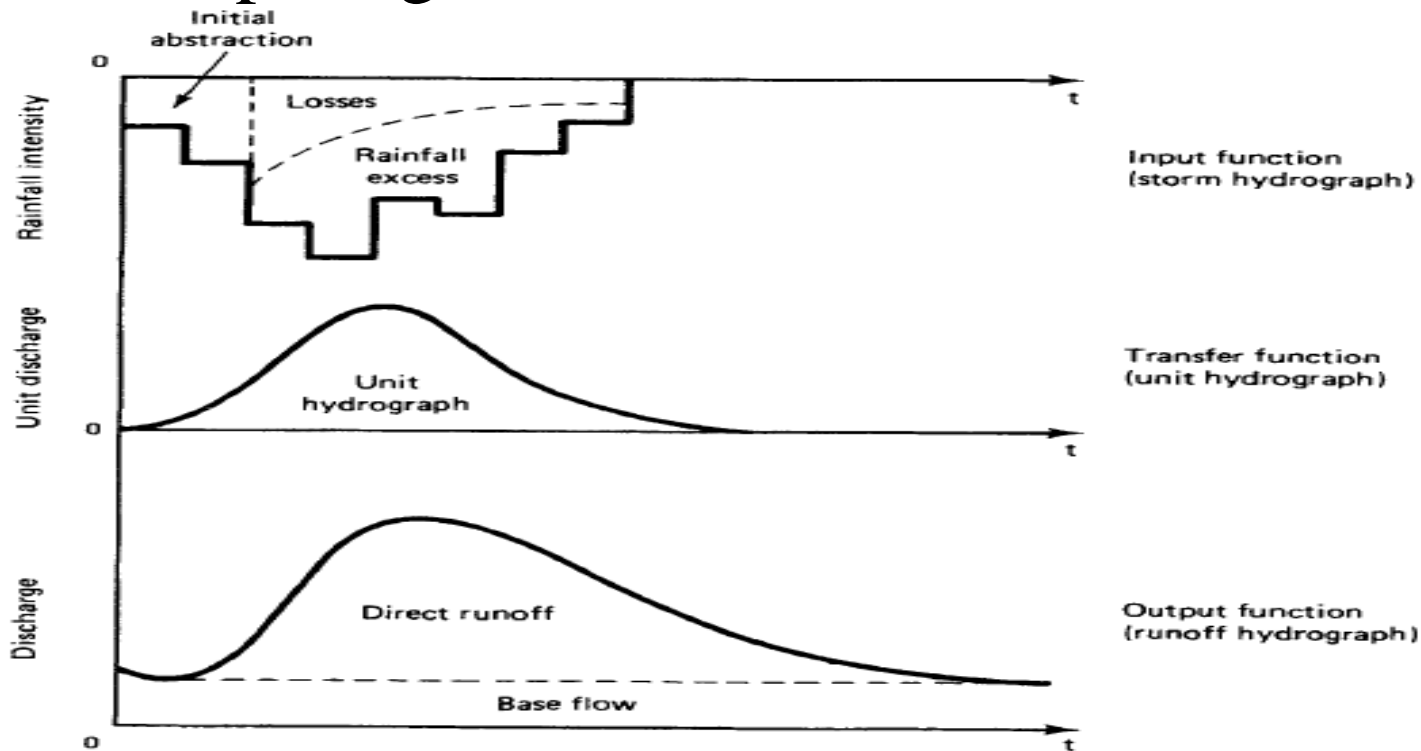
**Law of proportionality**



**Law of superposition**

# Uses of UH

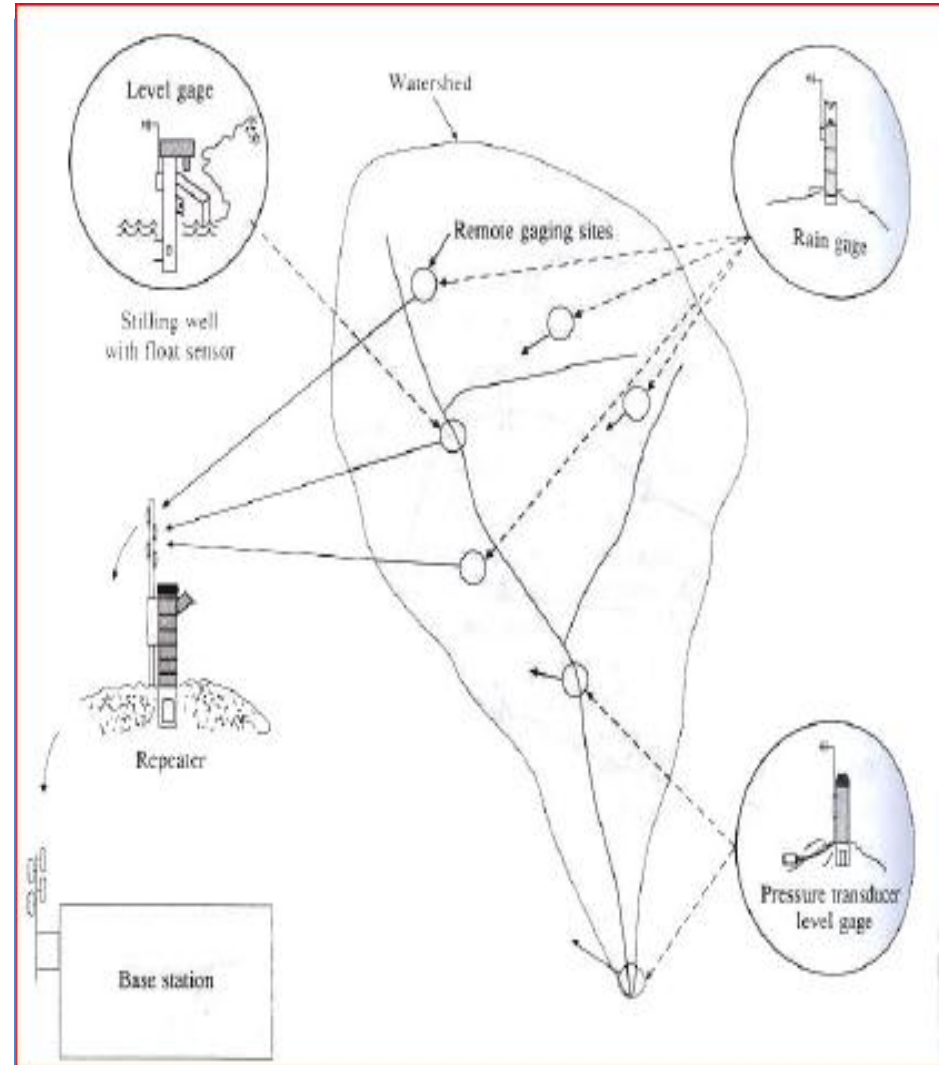
- Computation of flood hydrograph for design of structure(PMP/PMF)
- Flood forecasting and extension of flow records at a site.
- Comparing the catchment characteristic





# Application of UH

- The development of flood hydrograph for extreme rainfall magnitudes for use in design of hydraulic structures
- Extension of flood-flow records based on rainfall records
- The development of flood forecasting and warning systems based on rainfall.
- Collect, transfer, analyze and forecast flood data in order to maximize warning time for occupants



# Application of UH

- From a unit hydrograph of a known duration to obtain a unit hydrograph of the desired duration, either by the S-curve method or by the principle of superposition.
- From the unit hydrograph so derived, to obtain the flood hydrograph corresponding to a single storm or multiple storms.
- For design purposes, a design storm is assumed, which with the help of unit hydrograph, gives a design flood hydrograph.

# Limitation of UH

- The maximum area UH could be applicable is up to  $5000\text{km}^2$ . Above this the basic assumption of UH, having uniformly distributed rain fall over the catchment may be violated.
- The application of UH is not suitable for long basin
- The lower limit for application of UH should be preferably more than  $2\text{ km}^2$
- When larger portion of the basin is covered with snow UH theory should not be applied
- UH should not be derived from a catchment where large storage exist

## 2.2.1 Derivation of UH from single storm

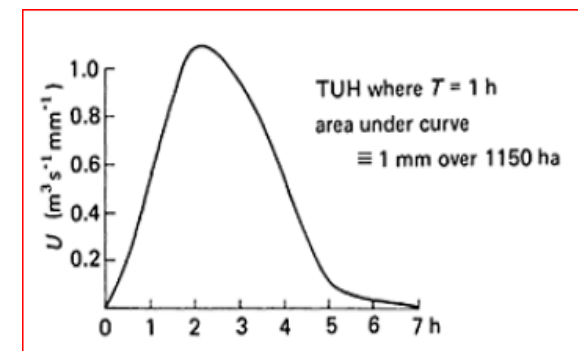
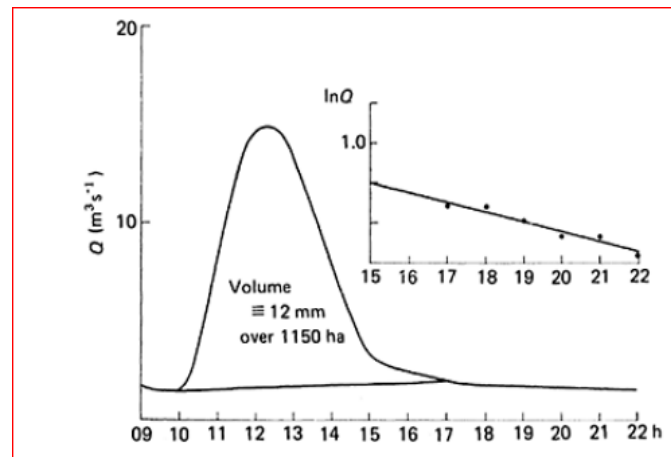
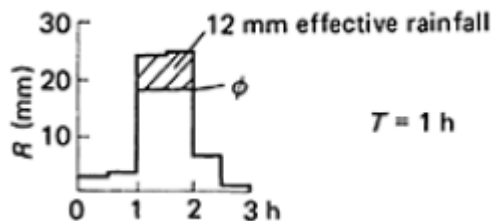
- Ideally UH are derived from simple isolated storm
- Collect the rain fall data preferably have nearly equal duration of storm
- Separate base flow and find DR
- Determine the depth

$$d = \frac{3.6\Delta t \sum Q}{A}$$

- The ordinate for UH is obtain by dividing the ordinate of surface hydrograph to the effective runoff

# Derivation of the TUH from Single Storms

1. the rainfall records are scanned to find a storm of desired duration that gives a fairly **uniform distribution in time and space**. The hyetograph of this storm is constructed using a convenient uniform interval of time.
2. Separate the base flow from the hydrograph
3. determined the depth of the surface runoff as a depth unit , **d**
4. divided the ordinates of the direct runoff hydrograph by the depth **d**
5. check the derived UH area to be unity. (the enclosed volume is equivalent to unit effective rainfall over the area of catchment).



Example: The runoff data at a stream gauging station for a flood are given below. The drainage area is 40 km<sup>2</sup>. The duration of rainfall is 3 hours. Derive the 3-hour unit hydrograph for the basin and plot the same.

<i>Date</i>	<i>Time (hr)</i>	<i>Discharge (cumec)</i>	<i>Remarks</i>
<i>1-3-1970</i>	<i>2</i>	<i>50</i>	
	<i>5</i>	<i>47</i>	
	<i>8</i>	<i>75</i>	
	<i>11</i>	<i>120</i>	
	<i>14</i>	<i>225</i>	
	<i>17</i>	<i>290</i>	← <i>Peak</i>
	<i>20</i>	<i>270</i>	
	<i>23</i>	<i>145</i>	
<i>2-3-1970</i>	<i>2</i>	<i>110</i>	
	<i>5</i>	<i>90</i>	
	<i>8</i>	<i>80</i>	
	<i>11</i>	<i>70</i>	
	<i>14</i>	<i>60</i>	
	<i>17</i>	<i>55</i>	
	<i>20</i>	<i>51</i>	
	<i>23</i>	<i>50</i>	

## Solution

<i>Date</i>	<i>Time (hr)</i>	<i>TRO<sup>1</sup> (cumec)</i>	<i>BFO<sup>2</sup> (cumec)</i>	<i>DRO<sup>2</sup> (3)-(4) (cumec)</i>	<i>UGO<sup>4</sup> (5) + P<sub>net</sub> (cumec)</i>	<i>Time<sup>5</sup> from begin- ning of surface run off (hr)</i>
1	2	3	4	5	6	7
1-3-1970	2	50	50	—	—	—
	5	47	47	0	0	0
	8	75	46	29	1.09	3
	11	120	45	75	2.82	6
	14	225	45	180	6.77	9
	17	290	45	245	9.23	12
	20	270	46	224	8.44	15
	23	145	48	97	3.65	18
2-3-1970	2	110	50	60	2.26	21
	5	90	53	37	1.39	24
	8	80	54	26	0.98	27
	11	70	57	13	0.49	30
	14	60	60	0	0	33
	17	55	55	—	—	—
	20	51	51	—	—	—
	23	50	50	—	—	—
				Σ DRO = 986 cumec		

$$P_{\text{net}} = \frac{\Sigma DRO \cdot t}{A} = \frac{986(3 \times 60 \times 60) \text{m}^3}{40 \times 10^6 \text{m}^2}$$

$$= 0.266 \text{ m} = 26.6 \text{ cm}$$

# *UH from complex storm*

- On events of non availability of isolated storm producing single peaked hydrograph of a basin, the available complex storm have to be used.
- The procedure of deriving a storm hydrograph from an excess rainfall hyetograph using a given unit hydrograph
  - 1) Multiply unit hydrograph ordinates by rainfall excess.
  - 2) Lag by the duration of unit hydrograph.
  - 3) Add.
- Add and lag multiple hydrographs in sequence to obtain storm hydrographs
- The discrete convolution equation allows the computation of direct runoff for excess rainfall  $P_m$  and the unit hydrograph



# Method I . Convolution Method

$$Q_1 = \sum_{j=1}^1 h_j \cdot u_{1-j+1}$$

$$Q_1 = h_1 \cdot u_1$$

$$Q_2 = h_1 \cdot u_2 + h_2 \cdot u_1$$

$$Q_3 = h_1 \cdot u_3 + h_2 \cdot u_2 + h_3 \cdot u_1$$

$$Q_4 = h_1 \cdot u_4 + h_2 \cdot u_3 + h_3 \cdot u_2 \quad (h_4 \cdot u_1 = 0)$$

$$Q_5 = h_1 \cdot u_5 + h_2 \cdot u_4 + h_3 \cdot u_3$$

$$Q_6 = h_1 \cdot u_6 + h_2 \cdot u_5 + h_3 \cdot u_4$$

$$Q_7 = h_1 \cdot u_7 + h_2 \cdot u_6 + h_3 \cdot u_5$$

$$Q_8 = h_1 \cdot u_8 + h_2 \cdot u_7 + h_3 \cdot u_6$$

$$Q_9 = h_2 \cdot u_8 + h_3 \cdot u_7 \quad (h_1 \cdot u_9 = 0 \quad u_9 = 0)$$

$$Q_{10} = h_3 \cdot u_8$$

The total no. of ordinates of the unit hydrograph is  $N-M+1$ ,  
hence  $10-3+1=8$

- Rain fall records  $R_1, R_2, R_3 \dots$  varying excess rain fall but duration of D-hr in (ERH divided it to D block each).
- The rain fall excess in each D-h duration is then operate up on the UH successively to get the various DRH curve)
- Take the complex hydrograph and plot ERH by separating base flow. The discharge is taken at D-h interval  $Q_1, Q_2, Q_3 \dots$
- Let the UH ordinate be  $U_1, U_2, U_3 \dots$

# Example 2.2:

Find the half-hour unit hydrograph using the excess rainfall hyetograph and direct runoff hydrograph given in the table.

## Solution.

The ERH and DRH in table have  $M=3$  and  $N=11$  pulses respectively.

Hence, the number of pulses in the unit hydrograph is  $N-M+1=11-3+1=9$ .

Substituting the ordinates of the ERH and DRH into the equations in table yields a set of 11 simultaneous equations.

Time (1/2hr)	Excess Rainfall (in)	Direct Runoff (cfs)
1	1.06	428
2	1.93	1923
3	1.81	5297
4		9131
5		10625
6		7834
7		3921
8		1846
9		1402
10		830
11		313

$$U_1 = \frac{Q_1}{P_1} = \frac{428}{1.06} = 404 \text{ cfs/in}$$

$$U_2 = \frac{Q_2 - P_2 U_1}{P_1} = \frac{1,928 - 1.93 \times 404}{1.06} = 1,079 \text{ cfs/in}$$

$$U_3 = \frac{Q_3 - P_3 U_1 - P_2 U_2}{P_1} = \frac{5,297 - 1.81 \times 404 - 1.93 \times 1,079}{1.06} = 2,343 \text{ cfs/in}$$

$$U_4 = \frac{9,131 - 1.81 \times 1,079 - 1.93 \times 2,343}{1.06} = 2,506 \text{ cfs/in}$$

$$U_5 = \frac{10,625 - 1.81 \times 2,343 - 1.93 \times 2,506}{1.06} = 1,460 \text{ cfs/in}$$

$$U_6 = \frac{7,834 - 1.81 \times 2,506 - 1.93 \times 1,460}{1.06} = 453 \text{ cfs/in}$$

$$U_7 = \frac{3,921 - 1.81 \times 1,460 - 1.93 \times 453}{1.06} = 381 \text{ cfs/in}$$

$$U_8 = \frac{1,846 - 1.81 \times 453 - 1.93 \times 381}{1.06} = 274 \text{ cfs/in}$$

$$U_9 = \frac{1,402 - 1.81 \times 381 - 1.93 \times 274}{1.06} = 173 \text{ cfs/in}$$

### Unit hydrograph

n	1	2	3	4	5	6	7	8	9
$U_n$ (cfs/in)	404	1,079	2,343	2,506	1,460	453	381	274	173

# Method II. Using matrix

$$\begin{bmatrix}
 h_1 & 0 & 0 & \dots & 0 & \dots & 0 & 0 \\
 h_2 & h_1 & 0 & \dots & 0 & \dots & 0 & 0 \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 h_M & h_{M-1} & h_{M-2} & \dots & h_1 & 0 & \dots & 0 \\
 0 & h_M & h_{M-1} & \dots & h_2 & h_1 & \dots & 0 \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 0 & 0 & 0 & \dots & 0 & \dots & h_M & h_{M-1} \\
 0 & 0 & 0 & \dots & 0 & \dots & 0 & h_M
 \end{bmatrix}
 \times
 \begin{bmatrix}
 u_1 \\
 u_2 \\
 u_3 \\
 \dots \\
 \dots \\
 u_{N-M+1}
 \end{bmatrix}
 =
 \begin{bmatrix}
 Q_1 \\
 Q_2 \\
 \dots \\
 Q_M \\
 Q_{M+1} \\
 \dots \\
 Q_{N-1} \\
 Q_N
 \end{bmatrix}$$

$$[h] \times [u] = [Q]$$

$$[h]^T \cdot [Q] = [h]^T \cdot [h] \cdot [u]$$

$$[u] = ([h]^T \cdot [h])^{-1} \cdot [h]^T \cdot [Q]$$

## 2.3 Changing duration of UH

- There are two methods to change duration of UH

1. By superposition method

2. By S-Curve method

### 1. Super position method

The principle of superposition is applied...

$$U(T_1, t) \Rightarrow U(T_2, t)$$

$U(T_1, t)$  refers(  $1/T_1$  )Unit depth of excess rain fall intensity  
for  $T_1$  duration

$U(T_2, t)$  refers(  $1/T_2$  ) Unit depth of excess rain fall intensity  
for  $T_2$  duration

If  $T_1$  is multiple of  $T_2$

Example if  $T_2=3T_1$  ( $T_1$  is one) then obtain the ordinate by  
superposition of three  $U(T_1, t)$  Shifted/lagged by  
 $T_1$  time

- That is  $U(T_1, t)$ ,  $U(T_1, t-T_1)$ ,  $U(T_1, t-2T_1)$  the cumulative depth will be 3 unit depth for  $T_2$  duration.
- By applying proportionality law divide the depth by 3 unit and obtain unit depth ordinate for  $T_2$

**Example** Use the superposition method to calculate the 2-h and 3-h unit hydrograph of a catchment, based on the following 1-h unit hydrograph.

Time (h)	0	1	2	3	4	5	6
	7	8	9	10	11	12	
Flow ( $m^3/s$ )	0	100	200	400	800	700	600
	500	400	300	200	100	0	

<b>Time(hr)</b>	<b>1-h UH</b>	<b>Lagged 1-h</b>	<b>Lagged 2-h</b>	<b>2-h UH</b>	<b>3-h UH</b>
<b>0</b>	<b>0</b>			<b>0</b>	<b>0</b>
<b>1</b>	<b>100</b>	<b>0</b>		<b>50</b>	<b>33.3333</b>
<b>2</b>	<b>200</b>	<b>100</b>	<b>0</b>	<b>150</b>	<b>100</b>
<b>3</b>	<b>400</b>	<b>200</b>	<b>100</b>	<b>300</b>	<b>233.333</b>
<b>4</b>	<b>800</b>	<b>400</b>	<b>200</b>	<b>600</b>	<b>466.667</b>
<b>5</b>	<b>700</b>	<b>800</b>	<b>400</b>	<b>750</b>	<b>633.333</b>
<b>6</b>	<b>600</b>	<b>700</b>	<b>800</b>	<b>650</b>	<b>700</b>
<b>7</b>	<b>500</b>	<b>600</b>	<b>700</b>	<b>550</b>	<b>600</b>
<b>8</b>	<b>400</b>	<b>500</b>	<b>600</b>	<b>450</b>	<b>500</b>
<b>9</b>	<b>300</b>	<b>400</b>	<b>500</b>	<b>350</b>	<b>400</b>
<b>10</b>	<b>200</b>	<b>300</b>	<b>400</b>	<b>250</b>	<b>300</b>
<b>11</b>	<b>100</b>	<b>200</b>	<b>300</b>	<b>150</b>	<b>200</b>
<b>12</b>	<b>0</b>	<b>100</b>	<b>200</b>	<b>50</b>	<b>100</b>
		<b>0</b>	<b>100</b>	<b>0</b>	<b>33.3333</b>
			<b>0</b>	<b>0</b>	<b>0</b>



# Example 2.4:

<i>Time (hr)</i>	<i>3-hr UGO (cumec)</i>	<i>Time (hr)</i>	<i>3-hr UGO (cumec)</i>
0	0	15	9.4
3	1.5	18	4.6
6	4.5	21	2.3
9	8.6	24	0.8
12	12.0		

- The above table show ordinate of 3-hr unit hydrograph  
prepare 6hr UH for the catchment

<i>Time (hr)</i>	<i>3-hr UGO (cumec)</i>	<i>3-hr UGO (logged) (cumec)</i>	<i>Total (2) + (3) (cumec)</i>	<i>6-hr UGO (4) ÷ 2 (cumec)</i>
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
0	0		0	0
3	1.5	0	1.5	0.7
6	4.5	1.5	6.0	3.0
9	8.6	4.5	13.1	6.5
12	12.0	8.6	20.6	10.3
15	9.4	12.0	21.4	10.7
18	4.6	9.4	14.0	7.0
21	2.3	4.6	6.9	3.4
24	0.8	2.3	3.1	1.5
27		0.8	0.8	0.4

**Do for 9 hr UH for the catchment**

# S-Curve method

- S curve is a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period.
- It is a curve obtained by summation of an infinite series of X hr UH's spaced X-hr
- The curve is very steep initially and reaches equilibrium  $Q_p = 2.78A/D$

The S-curve is computed using the following scheme:

$$S_1 = u_1$$

$$S_2 = u_1 + u_2 = u_2 + S_1$$

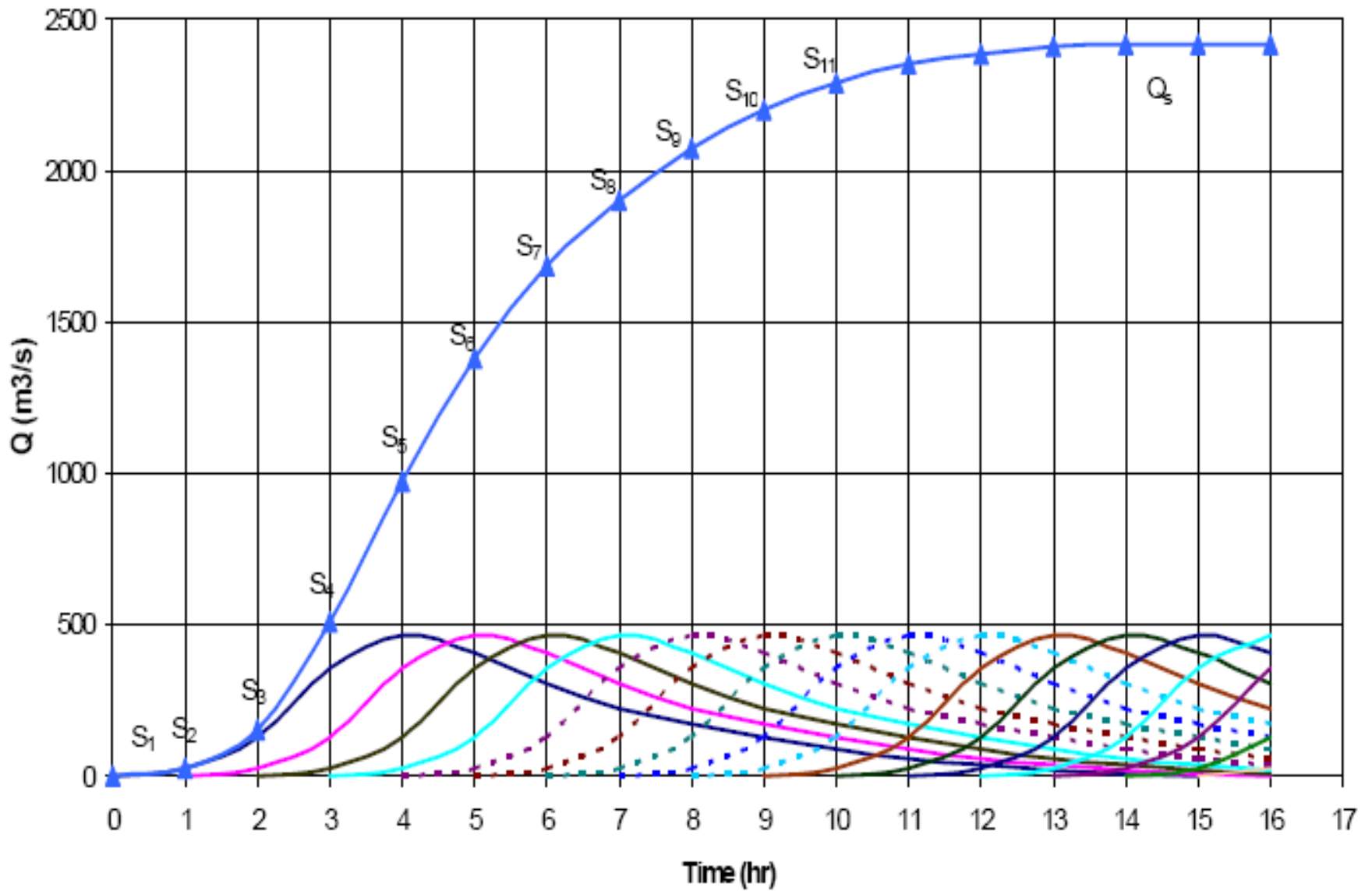
$$S_3 = u_1 + u_2 + u_3 = u_3 + S_2$$

$$S_n = u_1 + u_2 + u_3 + \dots + u_n = u_n + S_{n-1}$$

So, generally;

$$S_i = u_i + S_{i-1} \text{ for } i = 1, \dots, n$$

$$S_i = S_{i-1} \text{ for } i > n$$



# ***S-CURVE***

# Steps to convert $X$ hr UH to $Y$ hr UH

1. Determine  $X$  hr S curve hydrograph. The  $X$  hr S-hydrograph is derived by cumulative of infinite  $X$  hr UH ordinate at interval of equal to  $X$  hr
2. Lag  $X$  hr S-hydrograph by a time interval equal to  $Y$  hr
3. Subtract ordinates of the two previous S-hydrograph
4. Multiply the resulting hydrograph ordinate by  $X/Y$  to obtain the  $Y$ -hr unit hydrograph

# Example 2.5:

- A 4 hr UH ordinate are given below . if 2hr 6 hr and 12 hr of unit hydrograph is required for convolution determine using S- curve

Time	0	2	4	6	8	10	12	14	16	18	20	22	24	2
Q(m <sup>3</sup> /s)	0	30	110	170	210	180	110	80	40	35	30	15	5	0

Time	O-4h	S-curve additio n	S-curve ordinat e(4-hr)	lagg by 2hr	Diff. X(4/2)	lagged by 6hr	Diff. X(4/6)	lagged by 12hr	
0	0		<b>0</b>		0		0.0		
2	30		<b>30</b>	0	60		10.0		
4	110	0	<b>110</b>	30	160		36.7		
6	170	30	<b>200</b>	110	180	0	66.7		
8	210	110	<b>320</b>	200	240	30	96.7		
10	180	200	<b>380</b>	320	120	110	90.0		
12	110	320	<b>430</b>	380	100	200	76.7	0	
14	80	380	<b>460</b>	430	60	320	46.7	30	
16	40	430	<b>470</b>	460	20	380	30.0	110	
18	35	460	<b>495</b>	470	50	430	21.7	200	
20	30	470	<b>500</b>	495	10	460	13.3	320	
22	15	495	<b>510</b>	500	20	470	13.3	380	
24	5	500	<b>510</b>	510	0	495	5.0	430	
		510	<b>510</b>	510	0	500	3.3	460	
		505	<b>510</b>	510	0	510	0.0	470	

<b>Time</b>	<b>O-3h</b>	<b>S-curve addition</b>	<b>S-curve ordinate(3- hr)</b>	<b>lag by 6hr</b>	<b>Diff. X(3/6)</b>
<b>0</b>	<b>0</b>		<b>0</b>		<b>0</b>
<b>3</b>	<b>1.5</b>	<b>0</b>	<b>1.5</b>		<b>0.75</b>
<b>6</b>	<b>4.5</b>	<b>1.5</b>	<b>6</b>	<b>0</b>	<b>3</b>
<b>9</b>	<b>8.6</b>	<b>6</b>	<b>14.6</b>	<b>1.5</b>	<b>6.55</b>
<b>12</b>	<b>12</b>	<b>14.6</b>	<b>26.6</b>	<b>6</b>	<b>10.3</b>
<b>15</b>	<b>9.4</b>	<b>26.6</b>	<b>36</b>	<b>14.6</b>	<b>10.7</b>
<b>18</b>	<b>4.6</b>	<b>36</b>	<b>40.6</b>	<b>26.6</b>	<b>7</b>
<b>21</b>	<b>2.3</b>	<b>40.6</b>	<b>42.9</b>	<b>36</b>	<b>3.45</b>
<b>24</b>	<b>0.8</b>	<b>42.9</b>	<b>43.7</b>	<b>40.6</b>	<b>1.55</b>
<b>27</b>		<b>43.7</b>	<b>43.7</b>	<b>42.9</b>	<b>0.4</b>
<b>30</b>		<b>43.7</b>	<b>43.7</b>	<b>43.7</b>	<b>0</b>
<b>33</b>		<b>43.7</b>	<b>43.7</b>	<b>43.7</b>	<b>0</b>

*Example 2 of superposition*

*Using S curve example of superposition*



# Exercise 1:

- *Ordinate of an 1-hr unit hydrograph at 1-hr interval are 0, 5, 8, 5, 3, and 1 m<sup>3</sup>/s calculate*
- ✓ Watershed area coverage by the UH
- ✓ S-Curve hydrograph derived from this unit hydrograph
- ✓ 3-hr unit hydrograph

## Hint

$$1 \text{ cm} = \frac{\sum UH * t * 60 * 60}{A * 10^4}$$

## Exercise 2:

•The ordinates of a 4-hour unit hydrograph for a particular basin are given below. Derive the ordinates of (i) the S-curve hydrograph, and (ii) the 2-hour unit hydrograph, and plot them, area of the basin is  $630 \text{ km}^2$ .

Time (hr)	Discharge ( $\text{m}^3/\text{s}$ )
0	0
2	25
4	100
6	160
8	190
10	170
12	110
14	70
16	30
18	20
20	6
22	1.5
24	0

## 2.4 Instantaneous Unit Hydrograph

- UH are named according to their duration of rain fall excess.
- As the value of duration decrease the UH become more skewed
- In limiting case the **duration** of rain fall excess of 1cm/mm/in becomes infinitesimally small the resulting UH is called IUH
- IUH is a fictitious, conceptual UH which represent the direct run off from the catchment due to an instantaneous ppt of the 1unit rain fall excess

# Important properties of IUH

1.  $0 \leq u \leq u(t)$  a positive value, for  $t > 0$ ;
2.  $u(t) = 0$  for  $t \leq 0$ ;
3.  $u(t) \rightarrow 0$  for  $t \rightarrow \infty$ ;
4.  $\int_0^{\infty} u(t) dt = \text{unit depth over the catchment}; \text{ and}$
5. Time to peak = time to the centroid of the curve.

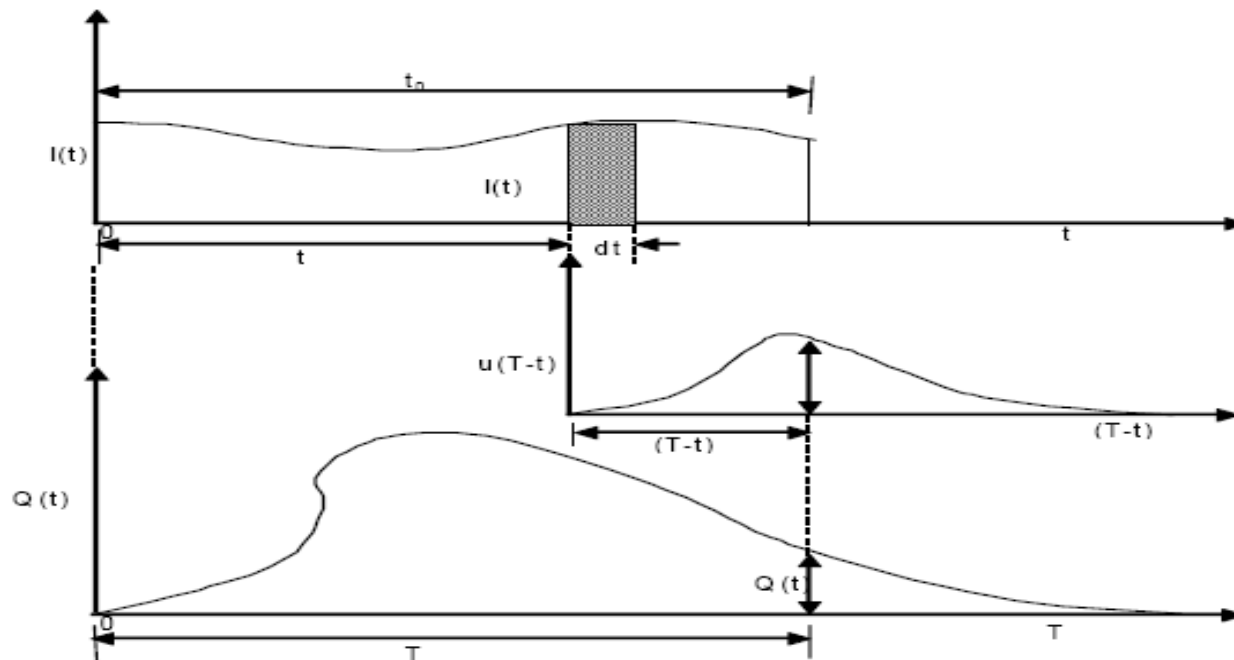


Figure 2.15: Convolution of  $I(t)$  of IUH

## 2.5 Un-gauge catchment

- To develop UH to a catchment require detail information about the rainfall and the resulting flood hydrograph .
- How ever such information may not be available due to technically or economically pertaining reasons.
- In order to construct UH for such area empirical equations w/c relates the salient hydrograph characteristics and basin characteristics are available
- UH derived from such cases are known as *Synthetic UH*

- Deriving a unit hydrograph for an un-gauged requires a relation between the physical geometry of the area and the resulting hydrographs.
- Basin characteristics formulas usually pertain to *time of peak, peak flow, and time base of the unit hydrograph*.
- When these features are established, the hydrograph *can be sketched* to provide the necessary unit volume.
- Synthetic unit hydrographs, once developed for a watershed area, can be used with historical or design rainfalls to produce storm hydrographs at the outlet of the watershed.
- As the *watershed changes over time*, the UH can be *updated* to better represent land use and channel alterations.

# Synthetic-UH

## Unit Hydrograph:

developed from rainfall and stream flow data on a watershed applies only for that watershed and for the point on the stream where the stream flow data were measured.

## Synthetic Unit Hydrograph:

Synthetic unit hydrograph procedures are used to develop unit hydrographs for other locations on the stream in the same watershed or for nearby watersheds of a similar character.

## There are three types of synthetic unit hydrograph:

- Those relating hydrograph characteristics (peak flow rate, base time, etc.) to watershed characteristics. **(Snyder, 1938)**
- Those based on a dimensionless unit hydrograph. **(Soil Conservation Service, 1972)**
- Those based on models of watershed storage. **(Clark, 1943)**

# Synthetic unit hydrograph can be derived

## 1. Snyder's Method

- Based on large number of catchment in Appalachian high land of east United State a set of empirical equation. Developed for those area with some modification these equation can be used for many countries.
- With some modification applicable to many other country. Con
- Snyder 's Synthetic Unit Hydrograph
- *Lag time ; the time difference between centroid of ERH and DRH.*

*The mean travel time of water from all parts of the catchment to the basin outlet for a storm*

- *The Snyder's equation relates the basin lag  $t_p$ , to the of unit hydrograph to the basin characteristic.*
- *ERH duration related to the catchment lag time by*



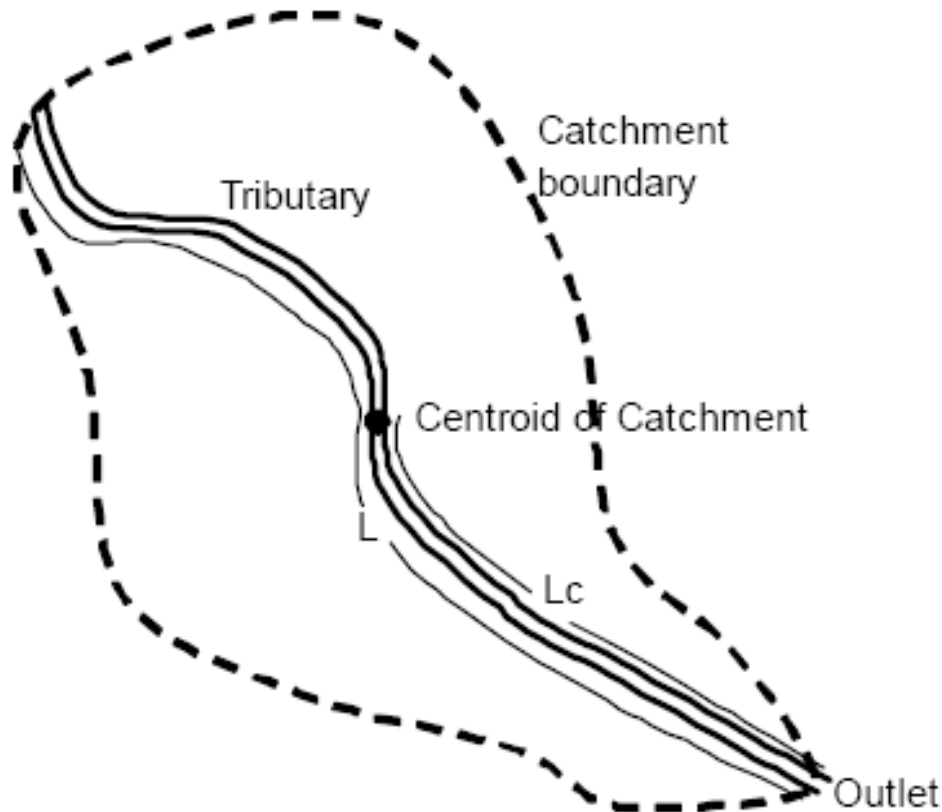
$$t_r = \frac{t_p}{5.5} \quad t_p = C_t (LL_c)^{0.3}$$

$t_r$  = Snyder standard duration

$C_t$  = regional constant representing of basin slope and storage

$L$  = length of the main stream from the outlet to the divide

$L_c$  = length along the main stream to a point nearest the watershed centroid



$$Q_p = \frac{2.78 C_p A}{t_p}$$

$Q_p$  = peak discharge of the unit hydrograph ( $m^3/s$ ),

$A$  = drainage area ( $km^2$ ),

$C_p$  = storage coefficient ranging from 0.4 to 0.8 where the larger

values of  $C_p$  are associated with smaller values of  $C_t$

If non standard duration is considered the lag time is affected so need modification

$$t_p' = t_p + 0.25(t_r' - t_r) \quad \text{then, } Q_p = \frac{2.78 C_p A}{t_p'}$$

**Linsly et al (1958) proposed**

$$t_p = C_{tL} \left( \frac{LL_{ea}}{\sqrt{S}} \right)^n$$

$C_{tL}$  and  $n$  are basin constants. ( $n = 0.38$  and  $C_{tL} = 1.715, 1.03, 0.50$  for mountainous, foot-hill and valley drainages of USA) Standard duration of effective rainfall,  $t_r$  (in hours)

$$t_b = \left( 3 + \frac{t_p'}{8} \right) \text{ days} = (72 + 3t_p') \text{ hours}$$

This equation gives reasonable estimates of time base for large catchments; it may give excessively large values of time base for small catchments. Taylor and Schwartz recommend

$$Tb = \frac{5.556}{q_p}$$

Where Tb is base time and qp is specific discharge

***To assist in the sketching of unit hydrographs,***

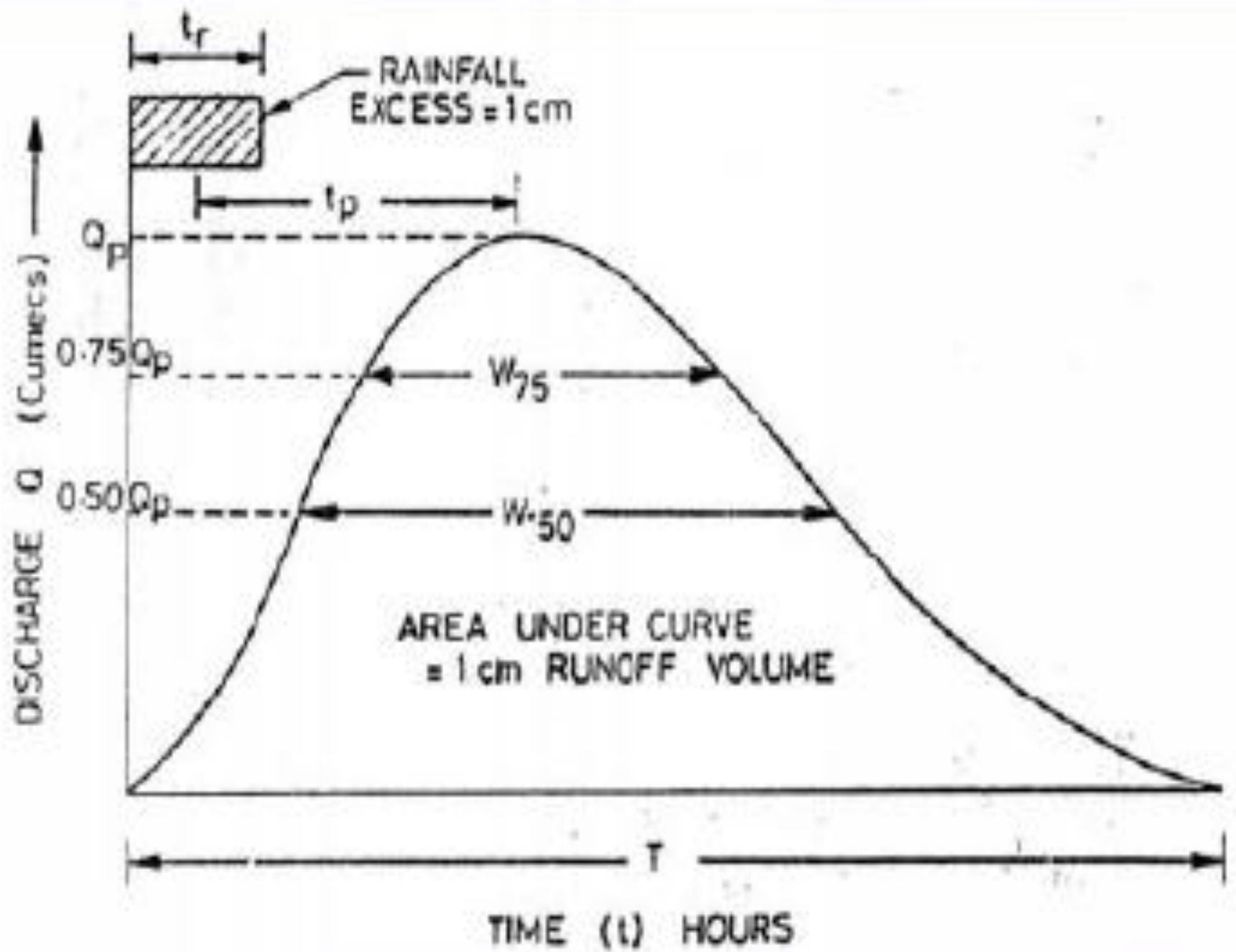
$$W_{50} = \frac{2.14}{q'^{1.08}}$$

$W_{50}$  = width of unit hydrograph in hour at 50% peak.D

$W_{75}$  = width of unit hydrograph in hour at 75% peak .D

$q = Q_p/A =$  peak discharge per unit catchment area in  $m^3/s/km^2$

$$W_{50} = \frac{1.22}{q'^{1.08}}$$



## Example 2.6:

- For a basin of  $198\text{km}^2$ , construct a 4 hr unit hydrograph from the following data.
- The length of the main channel = 21 km
- Length from the centroid to the outlet of the basin along the stream = 12 km
- $C_t$  for the neighboring catchment is found to be 1.5 and take  $C_p$  as 0.59

# Solution

$$t_p = C_t(LL_c)^{0.3} = 1.5 * (21.6 * 11.2)^{0.3} = 7.78 \text{ h}$$

$$t_r = \frac{t_p}{5.5} = 7.8 / 5.5 = 1.41 \text{ h} < 4 \text{ h}$$

Since the desired duration of rain fall excess is 4hr  $t_p'$

$$t_p' = t_p + 0.25(t_r' - t_r) = 7.78 + 0.25(4 - 1.41) = 8.42 \text{ h}$$

$$\text{then, } Q_p = \frac{2.78 C_p A}{t_p'} = (2.78 * 0.59 * 198) / 8.42 = 38.57 \text{ m}^3/\text{s}$$

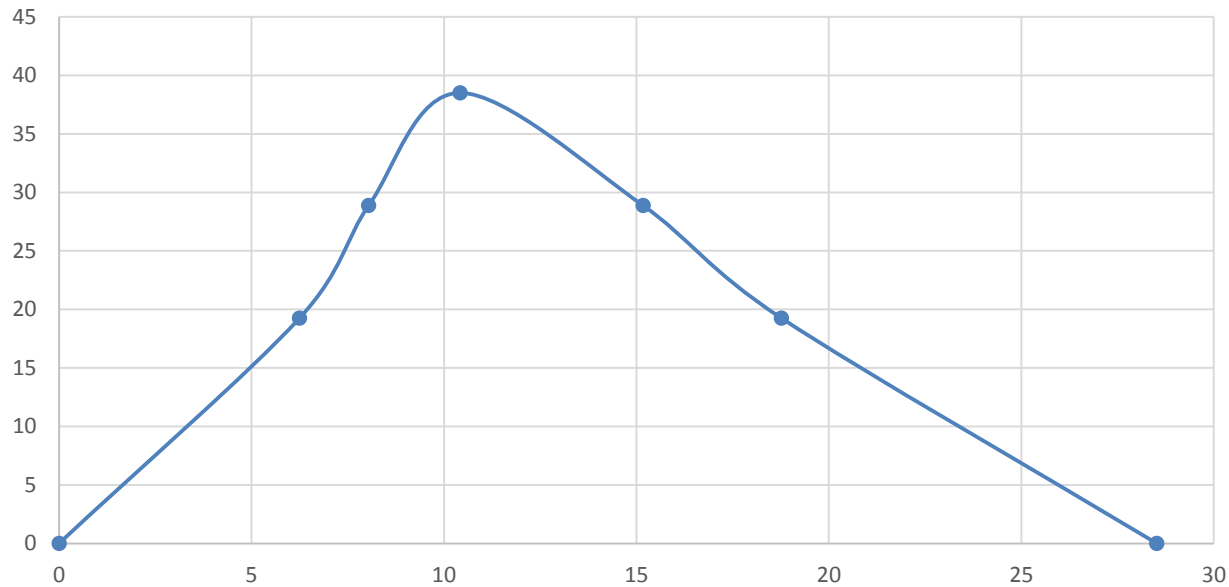
$$q = Q_p/A = 38.57/198 = 0.1948 \text{ m}^3/\text{s}/\text{km}^2$$

$$Tb = \frac{5.556}{q_p} = 28 \text{ h}$$

$$W50 = \frac{2.14}{q'^{1.08}} \Rightarrow 2.14 / (0.1948)^{1.08} = 12 \text{ h}$$

$$W50 = \frac{1.22}{q'^{1.08}} \Rightarrow 1.22 / 1.75 = 7.7 \text{ h}$$

### Snyder's UH



## Example 2.7:

- UH is to be developed for Ungauged catchment for which there is no information of any kind. An adjoining catchment is thoroughly gauged. It has 3-hr UH with peak of  $140\text{m}^3/\text{s}$  appearing 37 hr from the start of rain fall excess. Determine the Snyder's coefficient of the hydrograph to be used for the adjoining un gauged catchment for formulation of 3 hr UH
- Area basin (gauged )  $2718\text{km}^2$
- length along the river  $148\text{km}$
- Centroid length along the river  $76\text{km}$



# Solution

- Rain fall excess  $t_r=3\text{hr}$
- Time to peak from beginning of storm= $37\text{h}$
- $37=t_r/2+t_p \Rightarrow t_p'=35.5\text{h}$

$$t_p' = t_p + 0.25(t_r' - t_r) \Rightarrow 35.5 = t_p + 0.25(3 - \frac{t_p}{5.5})$$

$$= 36.4\text{h}$$

$$t_p = C_t(LL_c)^{0.3}$$

$$C_t = \frac{36.4}{(148 \times 76)^{0.3}} = 2.22$$

$$\text{then, } Q_p = \frac{2.78c_p A}{t_p'}$$

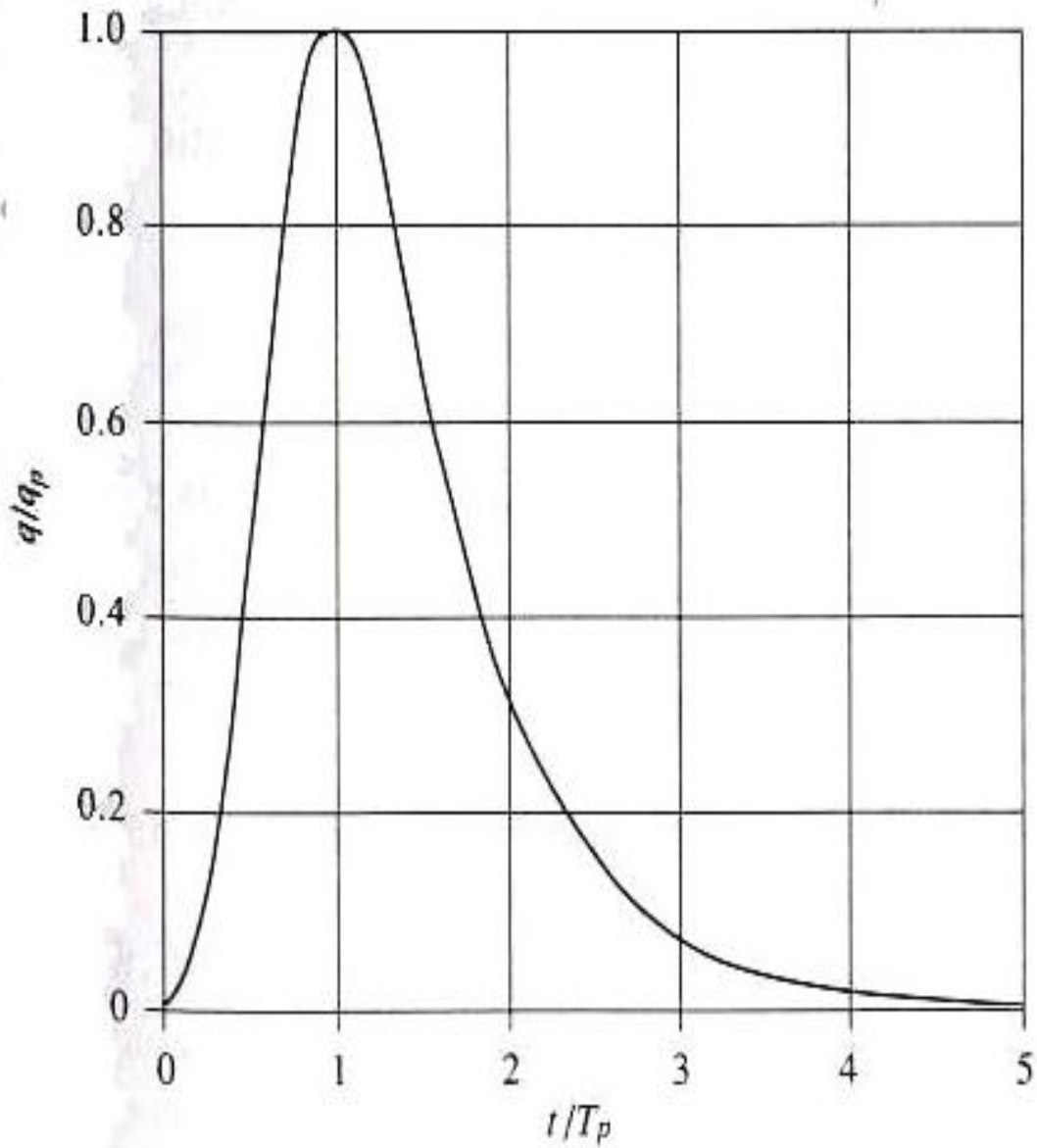
$$c_p = \frac{35.5 \times 140}{2.78 \times 2718} = 0.658$$

**Therefore can be  $C_t=2.22, C_p=0.658$  taken for adjoining un-gauged catchment to drive 3h synthetic UH**

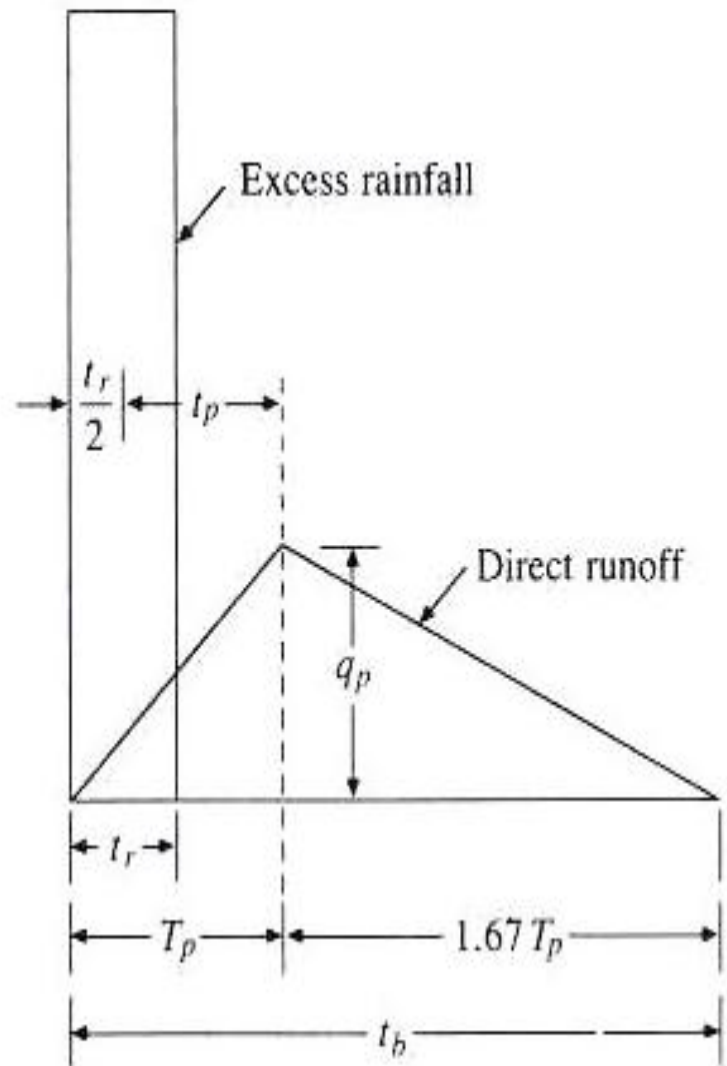
## 2. SCS Dimensionless UH method

- The dimensionless UH used by SCS was developed by *Victor Mockus*
- The unit hydrographs were averaged and the final product was made dimensionless by considering the ratios of  $q/q_p$  (flow/peak flow) on the ordinate axis and  $t/t_p$  (time/time to peak) on the abscissa.
- Thus if  $q_p$  and  $t_p$  are known, the entire UH can be constructed from the standard ratio between  $q/q_p$  &  $t/t_p$

- UH were evaluated for large number of actual water shed and made dimensionless by dividing all discharges ordinate by peak discharge all time by the time to peak. Average of this UH was computed.
- the time base of dimensionless UH was approximately 5 times the time to peak and  $3/8$  of the total volume occurred before the time to peak.
- the inflection point on the recession limb occurred approximately 1.7 times the time to peak
- The curvilinear UH can be approximated by triangular UH that has similar characteristics



(a)



(b)

# Dimensionless UH(SCS)

The figure shows a dimensionless hydrograph, prepared from the unit hydrographs of variety of watersheds. The values of  $q_p$  and  $T_p$  may be estimated using a simplified model of triangular unit hydrograph.

SCS suggests the time of recession may be approximated as  $1.67T_{p(\text{triangular})}$ . As the area under the unit hydrograph should be equal to a direct runoff of 1 cm

$$q_p = \frac{CA}{T_p}$$

$T_p$  = peak time, hr

$q_p$  = peak discharge, m<sup>3</sup>/sec.cm

$c = 2.08$

$A$  = the drainage area, sq.km.

$$T_p = \frac{t_r}{2} + t_p$$

## Example 2.8:

- *Obtain a 30 min UH for a basin of 12sq.km the time concentration can be taken as 2.5 h*

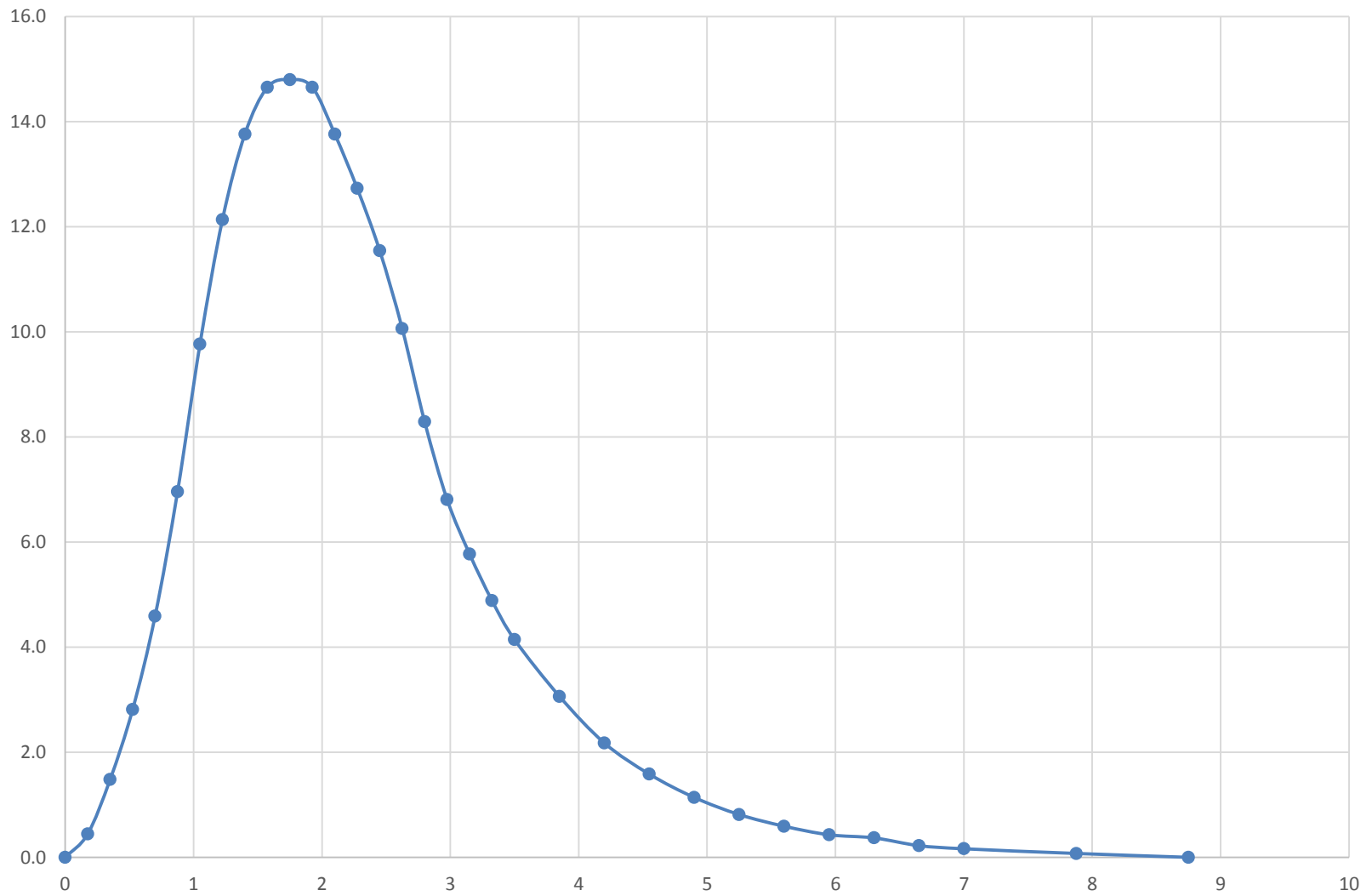
### Solution

- The rain fall excess 30min $\Rightarrow$ 0.5hr
- $T_p = t_r/2 + t_l$
- Where  $t_l = 0.6t_c$  1.5h
- $T_p = 1.75$
- $Q_p = 2.08 * A/t_p = 14.84m^3/s$
- the multiply the peak with the SCS standard values to get the synthetic UH

**TABLE 9-17** Ratios for SCS Dimensionless Unit Hydrograph and Mass Curve

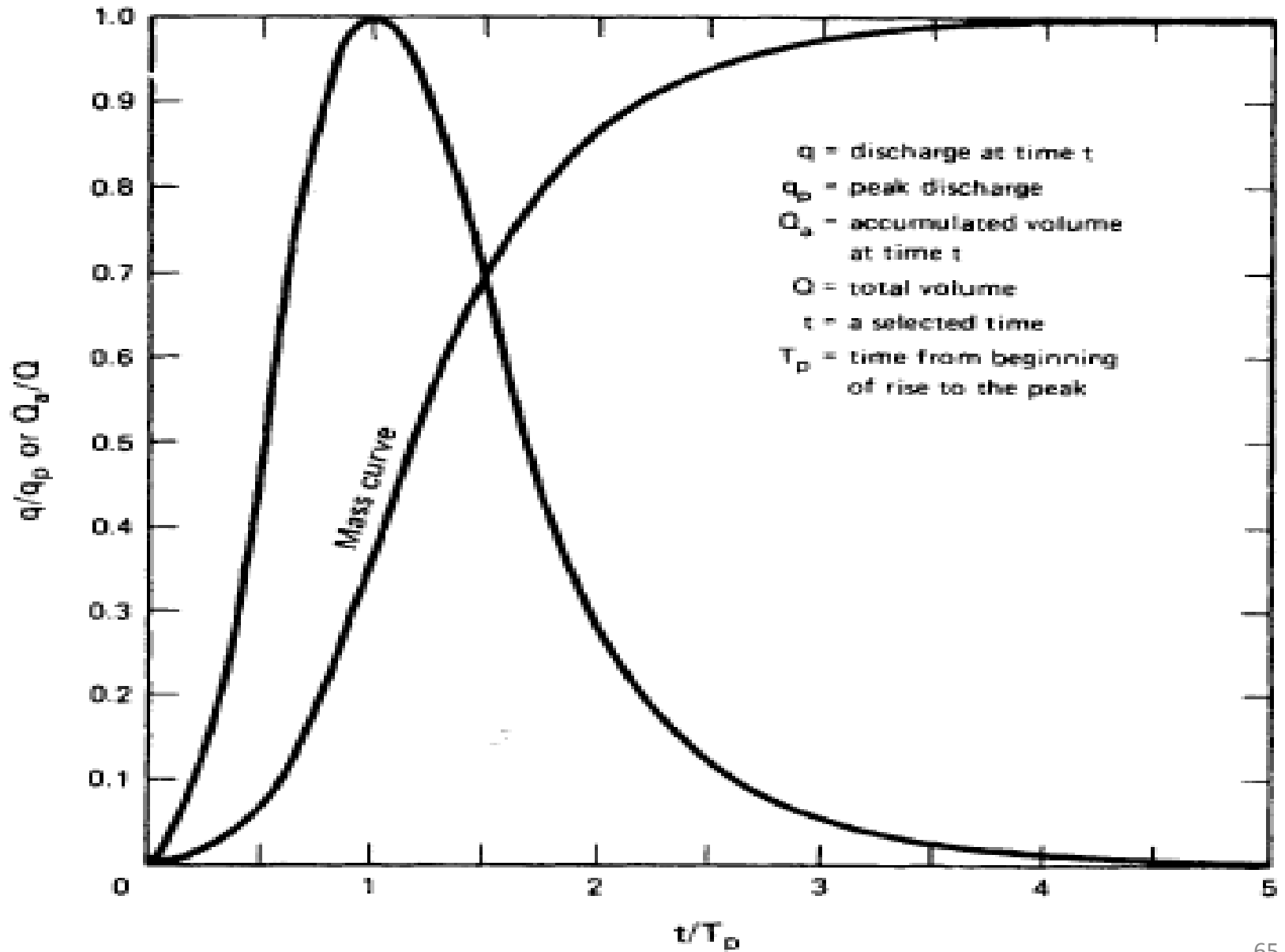
Time Ratios $t/T_p$	Discharge Ratios $q/q_p$	Mass Curve Ratios $Q/Q$
0	0.000	0.000
0.1	0.030	0.001
0.2	0.100	0.006
0.3	0.190	0.012
0.4	0.310	0.035
0.5	0.470	0.065
0.6	0.660	0.107
0.7	0.820	0.163
0.8	0.930	0.228
0.9	0.990	0.300
1.0	1.000	0.375
1.1	0.990	0.450
1.2	0.930	0.522
1.3	0.860	0.589
1.4	0.780	0.650
1.5	0.680	0.700
1.6	0.560	0.751
1.7	0.460	0.790
1.8	0.390	0.822
1.9	0.330	0.849
2.0	0.280	0.871
2.2	0.207	0.908
2.4	0.147	0.934
2.6	0.107	0.967
2.8	0.077	0.953
3.0	0.055	0.977
3.2	0.040	0.984
3.4	0.029	0.989
3.6	0.021	0.993
3.8	0.015	0.995
4.0	0.011	0.997
4.5	0.005	0.999
5.0	0.000	1.000

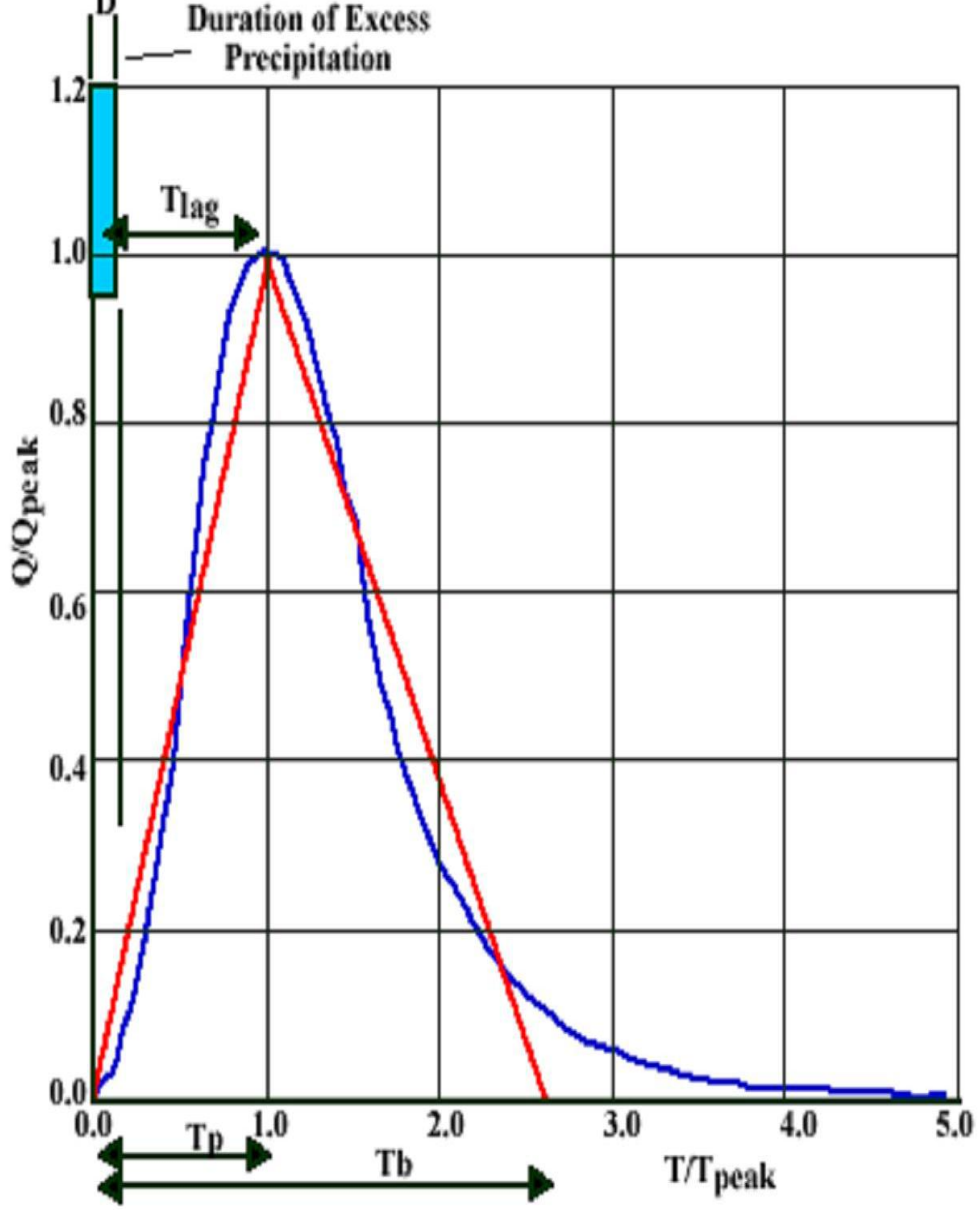
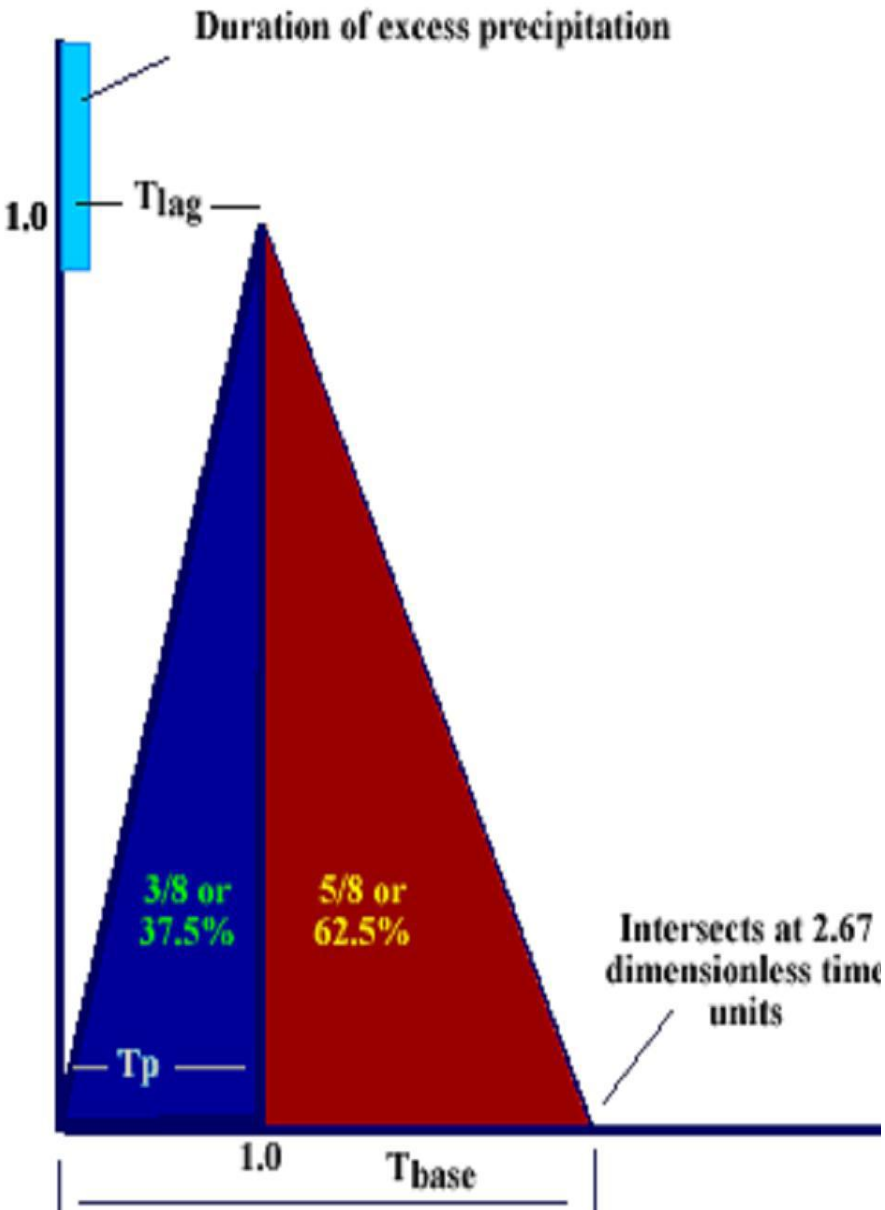
# SCS Dimensionless UH





# Hydrograph Analysis and Synthesis





## SCS Example 2.9:

Let's look at an example of the computations. The pertinent data for a watershed is given in the table below: Develop a 1-hour unit hydrograph using the SCS method and the SCS lag equation for lag time. Use the triangular approach for the final shape.

<b>Item</b>	<b>Value</b>
Drainage Area	90 square miles
Longest drainage path	25,000 feet
Average Curve Number	70
Average Slope	1.34%

- SOLUTION

- The value S is 1000/CN - 10 which is found to be 1000/70 - 10 = **4.28**.

$$T_{lag} = \frac{L^{0.8} \times (S + 1)^{0.7}}{1900 \times (\%slope)^{0.5}}$$

Compute a lag time of **4.8 hours** with the above equation.

The time to peak would now be found from:

where D = the duration which was given as 1 hour.

$$T_p = \frac{D}{2} + T_{lag}$$

Therefore, the time to peak = 1/2 + 4.8 = **5.3 hours**.

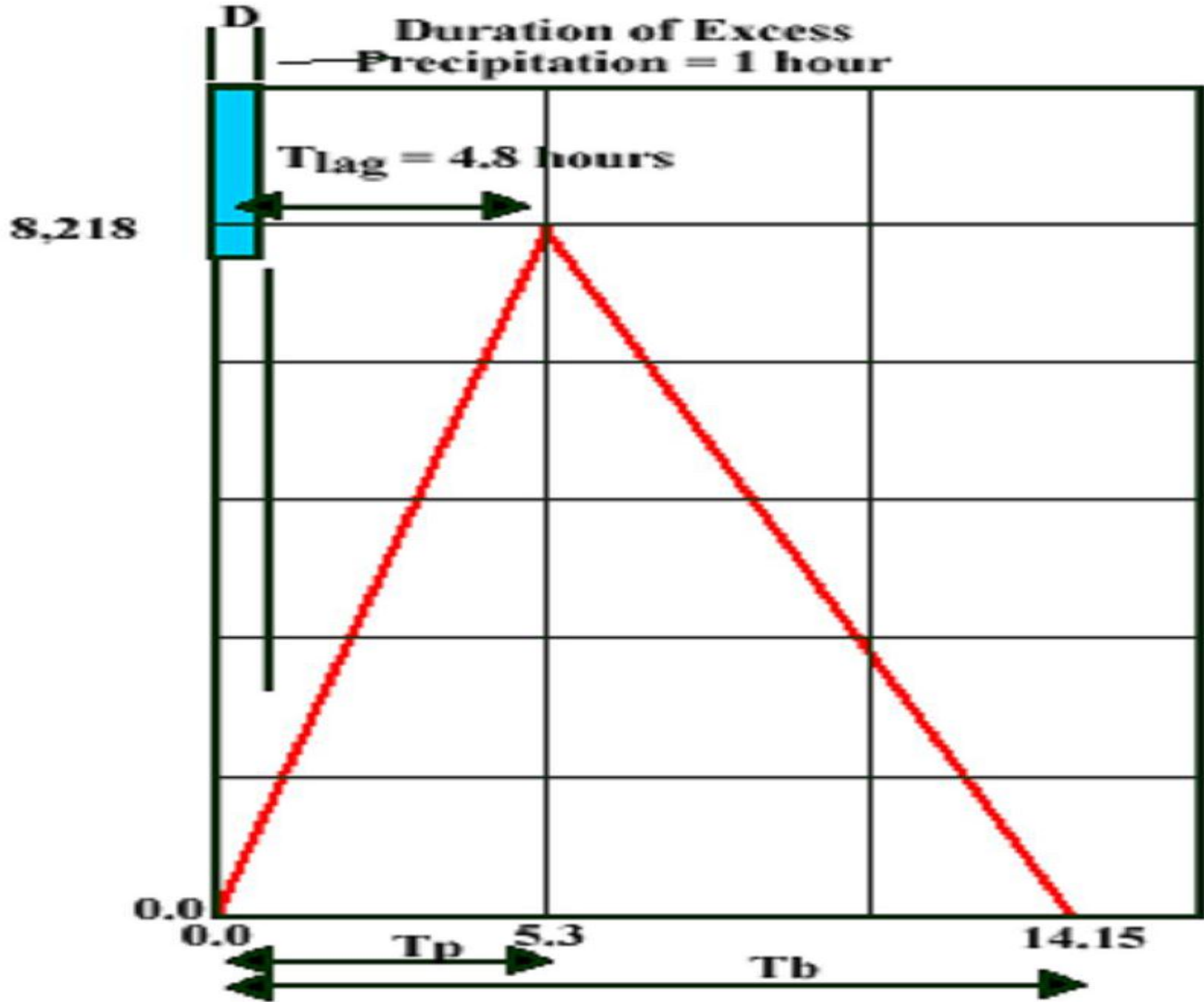
- The peak flow would now be obtained from:

- *And would be computed as **8,218 cfs/inch**.*

$$q_p = \frac{484 \times A \times Q}{\frac{D}{2} + T_{lag}}$$

*Recall that A is the drainage area and Q = 1 inch.*

- The time b  $T_b = 2.67 \times T_p$  7 x 5.3 hours = **14.15 hours**.
- The triangular unit hydrograph would now be:



# Flow Duration Curves(FDC)

Flow-duration curves display the relationship between stream flow and the percentage of time it is exceeded or equaled.

FDC can be used for:

1. assessing the percentage of time a stream flow will provide adequate dilution for industrial wastes or sewage,
2. evaluating the feasibility of hydropower stations
3. evaluating irrigation and water supply capacities
4. investigating environmental flow requirements

# Constructing FDC

Step 1: Sort (rank) average daily discharges for period of record from the largest value to the smallest value, involving a total of n values.

Step 2: Assign each discharge value a rank (M), starting with 1 for the largest daily discharge value.

Step 3: Calculate exceedence probability (P) as follows:

$$P = 100 * [ M / (n + 1) ] \quad \dots \text{plotting position.}$$

P = the probability that a given flow will be equaled or exceeded (% of time)

M = the ranked position on the listing (dimensionless)

n = the number of events for period of record (dimensionless)

# Hydrology of Ungagged Catchments

## Regionalization

**is the process of transferring information from comparable /similar catchments to the catchment of interest.**

### **1. spatial proximity**

this method is based on the rationale that catchments that are close to each other will likely have a similar runoff regime since climate and catchment conditions will often only vary marginally in space. **Can only be used in the presence of a nearby gauged catchment.**

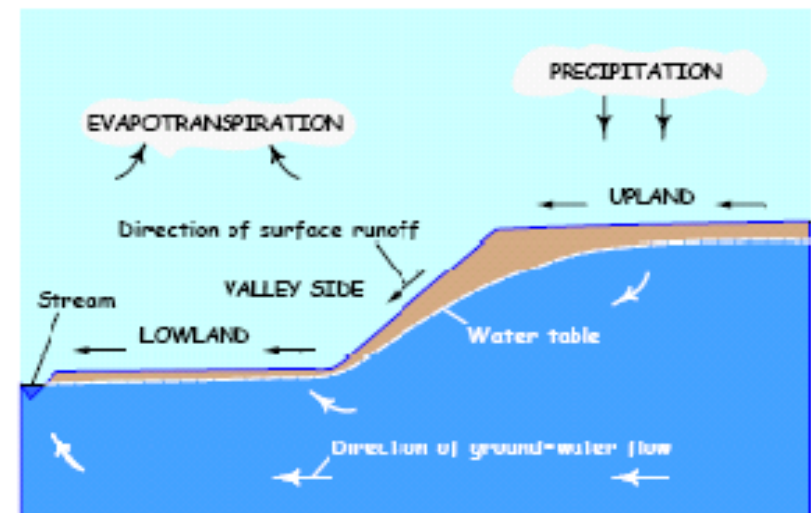
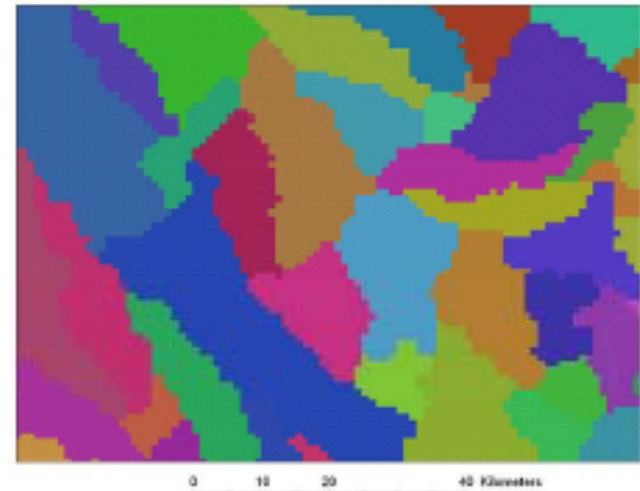
### **2. similarity of catchment characteristic**

(can be used in many range of catchments)



# Factors used to define hydrologic landscape regions

- Climate
  - Mean annual precipitation minus potential evapotranspiration
- Geology
  - Subsurface permeability class
  - Percent sand in soil
- Terrain
  - Slope
  - Percentage and location of flatland in watershed



# Regional Estimation Methods

## 1. Areal Ratios

For ungauged sites, rough estimates of the mean annual flow can be made from nearby gauged sites simply by adjusting for the difference in area,

$$\bar{x}_1 = \bar{x}_2 \left( \frac{A_1}{A_2} \right)^a$$

where

- $x_1$  mean annual flow (volume units) for the ungauged site,  $x_2$  mean annual flow (volume units) for the gauged site,
- $A_1$  and  $A_2$  are the areas of the ungauged and gauged catchments, respectively and,
- $a$  is a coefficient mostly less than 1

# 2. Multiple Regression Analysis

estimating stream flow measures from relationships between gauged data and measures obtained from maps or field measurements.

- Some of the factors which have been considered include: catchment area, channel slope, drainage density, elevation, stream length, mean annual precipitation, mean summer air temperature and percentage of the area covered by forest, swamps, lakes or permeable rock

Many of the factors are intercorrelated, and often the equation is 'boiled down' to a few representative variables such as **catchment area**, **precipitation** and **hydrogeology**.

- A regression analysis provides insight about which factors have the most influence on flow attributes.

The coefficients obtained will be **specific to the hydrologically homogeneous area** from which they were derived. Their reliability for predicting ungauged flow parameters can be tested by collecting a range of actual stream flow measurements at an ungauged the site.

# ...Regression

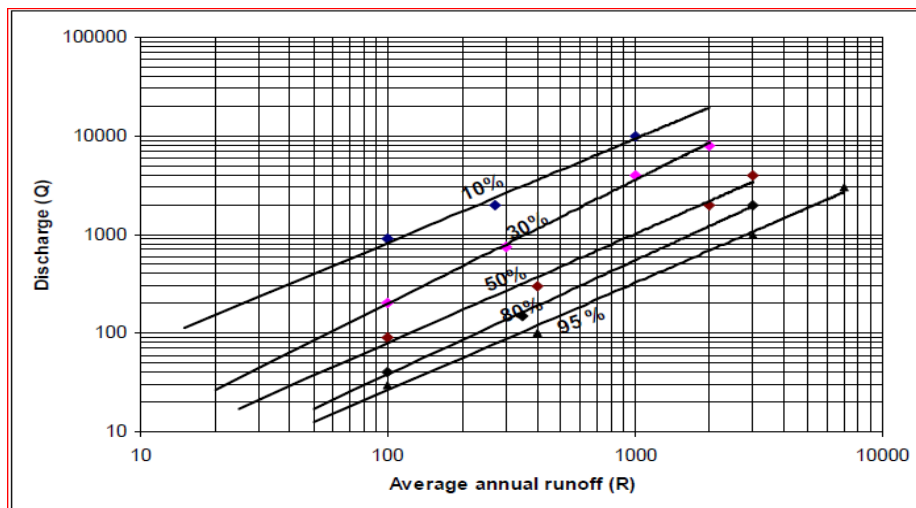
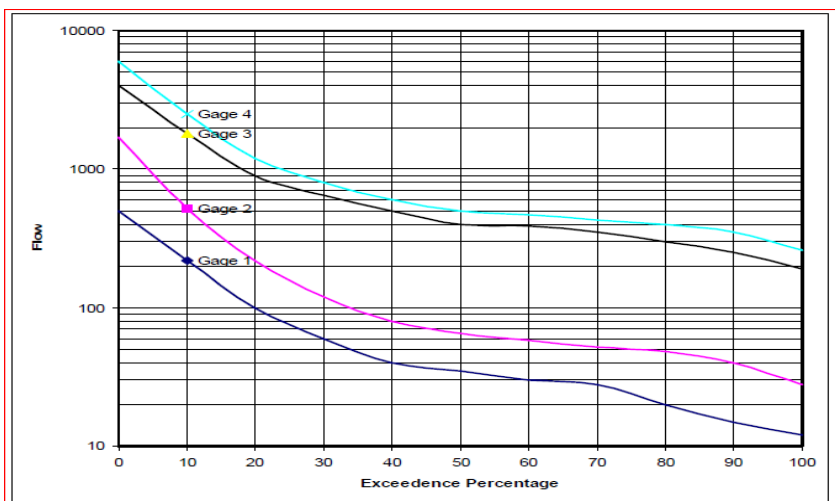
For example, in a study of 81 catchments smaller than 250 km<sup>2</sup> in southeastern Victoria, Australia, Gan et al. (1990) found that a regression equation based on catchment area and mean annual rainfall explained 97% of the variance in mean annual stream flows. The equation they established was

$$Q = 9.3 \times 10^{-6} A^{0.99} R_m^{1.48}$$

# 3. FDC method

1. Plot **flow duration curves** for the “similar” gauged sites.
2. From the flow duration curves, develop **parametric flow duration curves**

(flow is plotted against the average annual runoff,  $R$  or annual discharge,  $Q$  at the respective gages for several exceedence interval percentages. A separate curve is developed for each exceedence interval used. A correlation analysis is then performed to obtain the best-fitting curve for the data taken from the measured records of stream flow.)



# For the ungauged catchment

with the **AVERAGE ANNUAL RUNOFF DISCHARGE** estimate it is possible to enter the parametric flow duration curve and determine values of flow for different exceedence percentages for which the parametric flow duration curve has been developed.

$$\bar{Q} = \frac{k \bar{P} A}{T}$$

## Example 2.10:

A drainage basin has a power plant site located at the mouth of the catchment. An upstream reservoir regulates the flow at the upper portions of the drainage. The area of the hydrologic map representative of the drainage basin below the reservoir has been planimetered and given in table A below. A runoff coefficient for the basin on the annual basis is 0.65. The historic monthly flows of a nearby stream gauge on the downstream side of the stream are presented in table B. The gauge records are considered to be a good representation of seasonal variation of runoff for the un-gauged portion of the river drainage basin. The outflows from the reservoir are given in table C. Using the information provided compute the river flow at its mouth that would be useful for the hydropower study. Scale of the isohyetal map is 1:400,000.

Table a: Values of planimetered areas downstream of the reservoir

Avg value of precipitation between Isohytal lines (mm)	Planimetered Area (mm <sup>2</sup> )
762	11.94
889	26.13
1016	14.45

Table b: Monthly flows for an average year in a representative gauged stream

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
No. of Days	31	28	31	30	31	30	31	31	30	31	30	31
Discharge (m <sup>3</sup> /s)	7.11	7.14	9.88	33.13	80.02	64.31	22.57	11.84	9.40	9.40	9.51	8.44

Table c: Out flow from the upper reservoir

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Discharge (m <sup>3</sup> /s)	1.42	1.27	2.27	2.83	5.66	7.08	7.08	5.66	1.98	1.84	1.70	1.56



# Solution

Step 1: Compute the average annual runoff using NAP

$$\bar{Q} = k \bar{P} A$$

$$\bar{P} = \frac{\sum P A}{\sum A} = \frac{762 * 11.94 + 889 * 26.13 + 1016 * 14.45}{11.94 + 26.13 + 14.45} = 895.07 \text{ mm}$$

$$\bar{Q} = 0.65 * 895.07 / 1000 * 52.52 / (1000 * 1000) * 400,000^2 = 4888941.2 \text{ m}^3 / \text{year}$$

$$\bar{Q} = \frac{4888941.2}{24 * 60 * 60} = 56.58 \text{ m}^3 / \text{sec} / \text{day}$$

Step 2: Compute yearly runoff from the representative gauge

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
No. of Days	31	28	31	30	31	30	31	31	30	31	30	31
Discharge (m3/s)	7.11	7.14	9.88	33.13	80.02	64.31	22.57	11.84	9.40	9.40	9.51	8.44
Run off (m3/s)/(day)	220.41	199.92	306.28	993.9	2480.62	1929.3	699.67	367.04	282	291.4	285.3	261.64
Yearly Total (m3/s)(days)	8317.48											

### Step 3: Compute monthly fraction of runoff

$$q_i = \frac{\text{Runoff for the month}}{\text{Total runoff for the Record period}}, q_i (\text{Jan}) = \frac{220.41}{8317.48} = 0.026$$

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Monthly fraction	0.026	0.024	0.037	0.119	0.298	0.232	0.084	0.044	0.034	0.035	0.034	0.031

### Step 4: Compute flow for the downstream portion

$$Q(\text{Jan}) = \frac{0.026 * 56.58}{31} = 0.05 \text{ m}^3 / \text{sec}$$

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Monthly flow (m <sup>3</sup> /s)	0.05	0.05	0.07	0.23	0.54	0.44	0.15	0.08	0.06	0.06	0.06	0.06

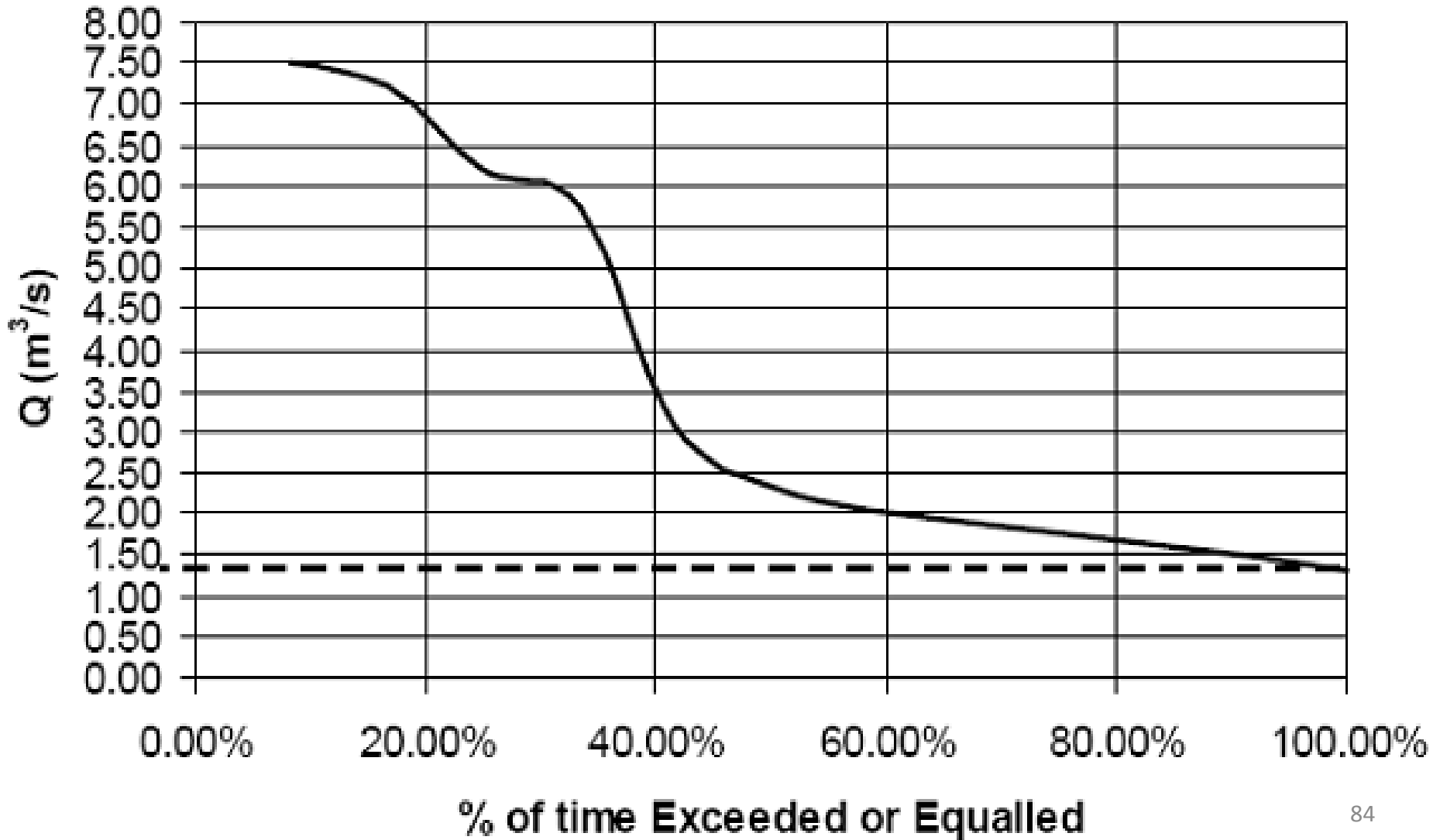
## Step 4 & 5: Compute the flow duration curve

	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Monthly flow (m3/s)	0.05	0.05	0.07	0.23	0.54	0.44	0.15	0.08	0.06	0.06	0.06	0.06
Flow from upper Res(m3/s)	1.42	1.27	2.27	2.83	5.66	7.08	7.08	5.66	1.98	1.84	1.70	1.56
Total Flow	1.47	1.32	2.34	3.06	6.20	7.52	7.23	5.74	2.04	1.90	1.76	1.62

Flow	Flow Descending order	Rank	%Exceeded or Equaled
7.52	7.52	1	8.33%
7.23	7.23	2	16.67%
6.20	6.20	3	25.00%
5.74	5.74	4	33.33%
3.06	3.06	5	41.67%
2.34	2.34	6	50.00%
2.04	2.04	7	58.33%
1.90	1.90	8	66.67%
1.76	1.76	9	75.00%
1.62	1.62	10	83.33%
1.47	1.47	11	91.67%
1.32	1.32	12	100.00%

The firm flow = 1.32 m<sup>3</sup>/sec

### Flow Duration Curve



**Thank You !!!**

