ARBA MINCH UNIVERSITY WATER TECHNOLOGY INSTITUTE

Faculty Of Water Supply And Environmental Engineering

Course title: *Water Quality Modelling and Management*

Course code: *WSEE 5272*

Target groups: *5th year Water Supply and Environmental Engineering students*

Academic year 2019/20, semester II

Instructor: *Zemed Menberu(MSc)*

Course syllabus

Introduction to water quality modeling and management

- *Environmental Statistics*
- *Why environmental modeling?*
- *Basic types of model*
- *Application of Environmental Modeling*
- *Statistical Performance Criteria*
- *Water quality and remote sensing*

Lake water quality modeling

- *Mass Balance for a Well-Mixed Lake*
- *Assimilation factor of lake*
- *Temperature effects of reaction on modelling*
- *Transfer Functions and Residence Time*
- *Non-steady State Solution for Two Lakes in Series*

Course syllabus

River water quality modelling

- *Ideal Reactors*
- *Continuously stirred tank reactor (CSTR):*
- *Plug-flow Reactor (PFR)*
- *Reaction rate constant k*
- *Comparison of CSTR and PFR*
- *Application of PFR Model to Streams*
- *Dissolved oxygen in rivers and modeling*
- *Oxygen sag curve*
- Mass Balance for Well-Mixed System
- Advection-Dispersion Model for Large Rivers and Estuaries

Eutrophication and lake morphometry

- *Introduction*
- *Eutrophication Problem & Nutrients*
- *Measurement Techniques for eutrophication*
- *Nutrients causing eutrophication*
- *Sources of Nutrients in Eutrophication Problems*
- *Major effects of eutrophication*
- *Total phosphorus model*
- *Lake Morphometry*
- *Importance and measurement of lake morphometry*

Course syllabus

River and lake water quality management

- *River and Lake Pollution Control Method*
- *Management of Agricultural Sources of Water Pollution*
- *Drinking water quality management*
- *Corrosion, Biofilm and Sludge Management*
- *Risk management for various water sources (surface and ground water)*

Evaluation

- Test and quiz------------- -------15%
- Laboratory report----------------10%
- Mid exam---------------------------25%
- Final exam--------------------------50%

Chapter one

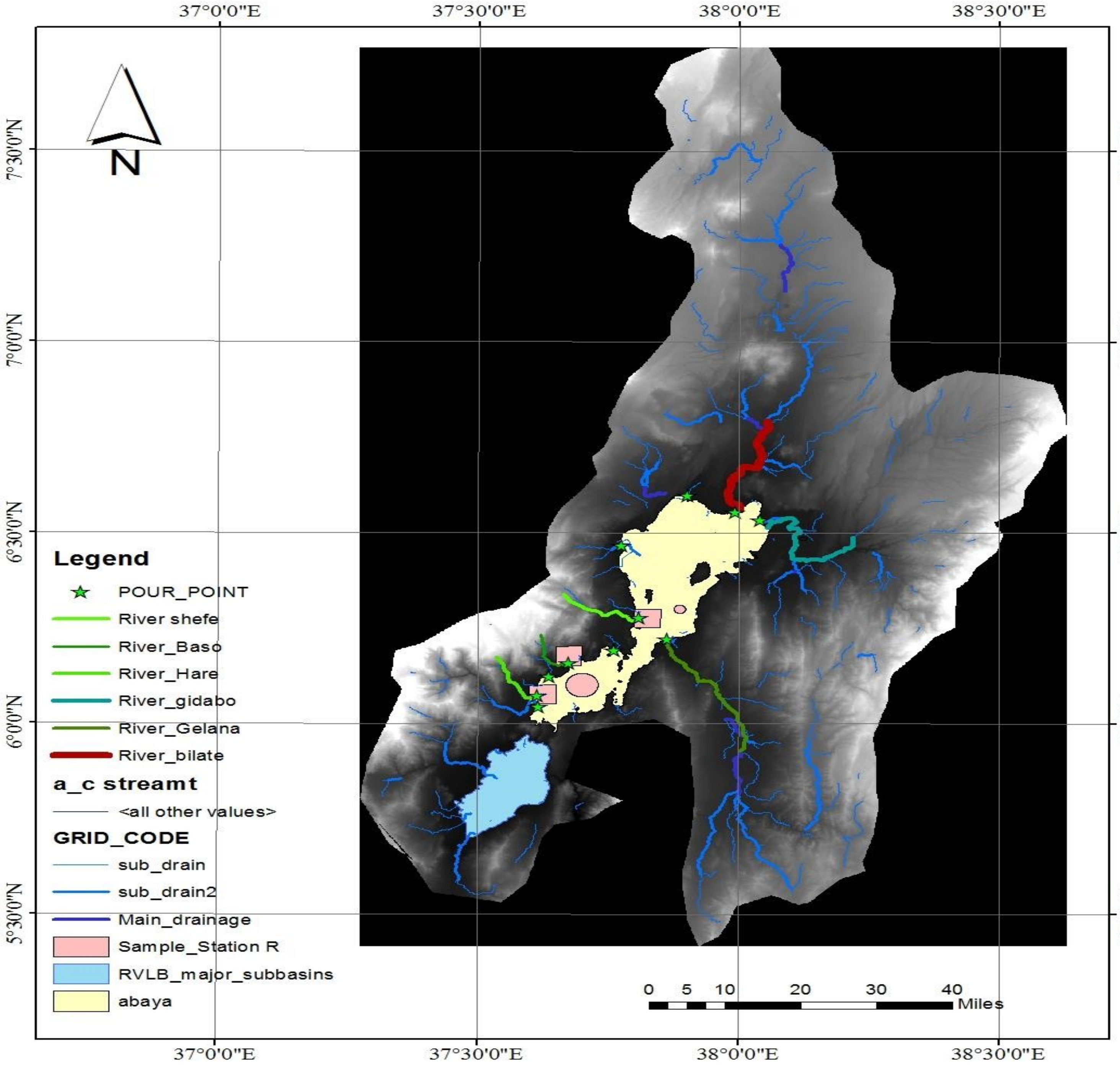
Introduction to Water Quality Modelling and Management

What is modeling?

- Modeling is the process of application of fundamental knowledge or experience to simulate or describe the performance of a real system to achieve certain goals
- Modeling has long been an integral component in organizing, synthesizing, and rationalizing observations of and measurements from real systems and in understanding their causes and effects

Why environmental modeling?

- Gain a better understanding of and clean insight into environmental processes and their influence on the fate and transport of contaminants in the environment.
- Determine short- and long-term chemical concentrations in the various compartments of the ecosphere for use in regulatory enforcement and in the assessment of exposures, impacts, and risks of existing as well as proposed chemicals
- **Predict future** environmental concentrations of pollutants under various waste loadings and/or management alternatives
- **Satisfying regulatory** and statutory requirements relating to environmental emissions, discharges, transfers, and releases of controlled pollutants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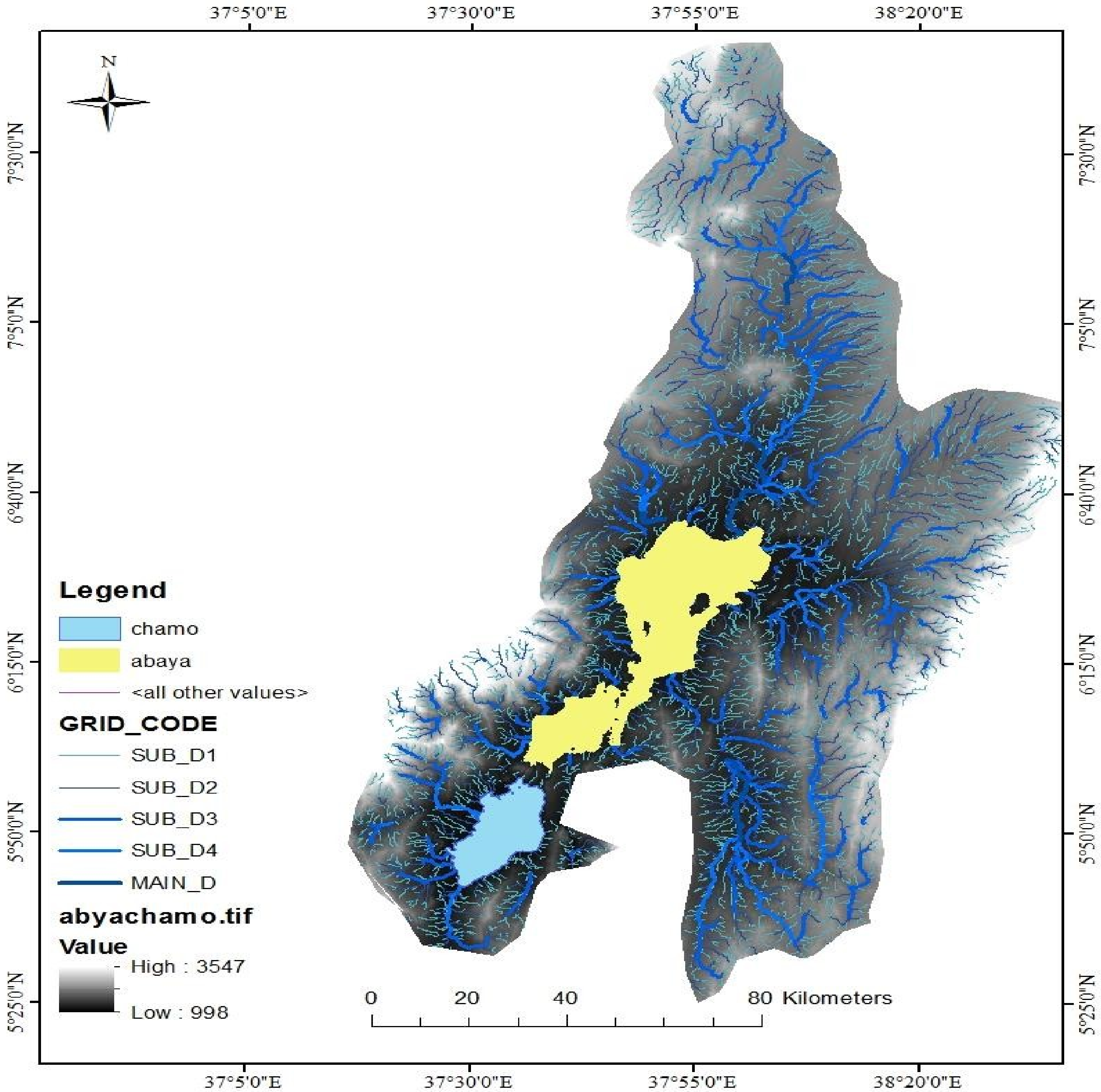
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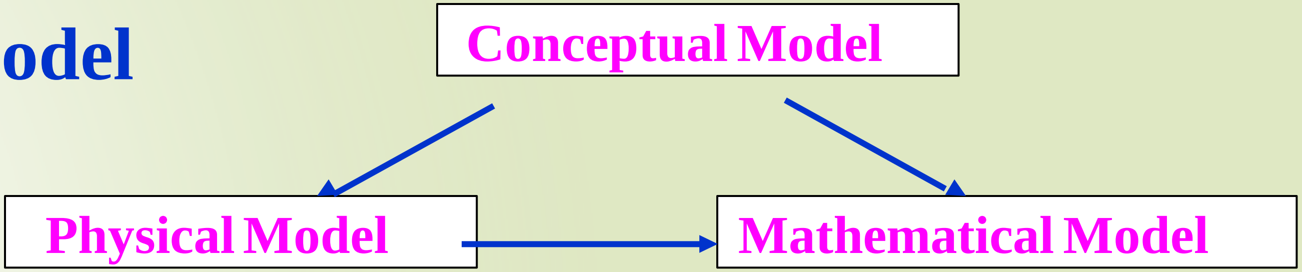
Why environmental modeling?

- Use in hypothesis testing relating to processes, pollution control alternatives, etc.
- Implement in the design, operation, and optimization of reactors, processes, pollution control alternatives, etc.
- Simulate complex systems at real, compressed, or expanded time horizons that may be too dangerous, too expensive, or too elaborate to study under real conditions
- Generate data for post-processing, such as statistical analysis, visualization, and animation, for better understanding, communication, and dissemination of scientific information
- Use in environmental impact assessment of proposed new activities that are currently nonexistent

Basic types of model



Conceptual Model

Physical Model

Mathematical Model

Conceptual model

- a mental picture of the system processes (e.g. Maps of places, ideas, relationships; Flow charts, graphs; Hypotheses, theories)
- Qualitative, usually based on graphs ==> the first step in any successful modeling efforts
- Represent important system:
  - ➢ components
  - ➢ processes
  - ➢ linkages
  - ➢ Interactions

When should the conceptual models be used?

- ➢ As an initial step –

  For hypothesis testing

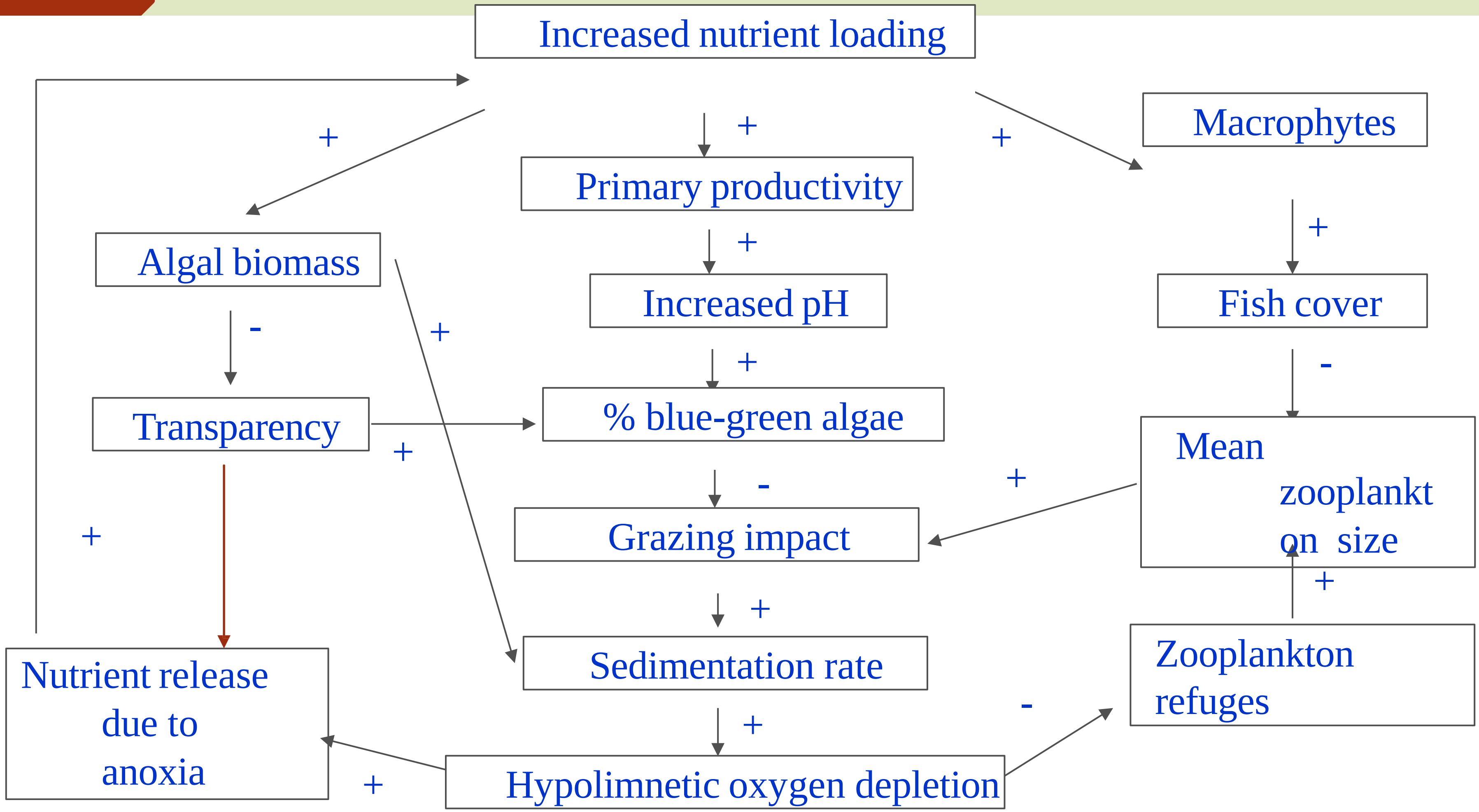
  For mathematical model development
- ➢ As a framework –

  For future monitoring, research, and management actions at a site

How can the conceptual models be used?

- Design field sampling and monitoring programs: Ensure that all important system attributes are measured
- Determine causes of environmental problems: Identify system linkages and possible cause and effect relationships
- Identify potential conflicts among management objectives
- Anticipate the full range of possible system responses to management actions, including potential negative effects

Conceptual Model Example



Increased nutrient loading

Macrophytes

Primary productivity

Algal biomass

Increased pH

Fish cover

Transparency

% blue-green algae

Mean zooplankton size

Grazing impact

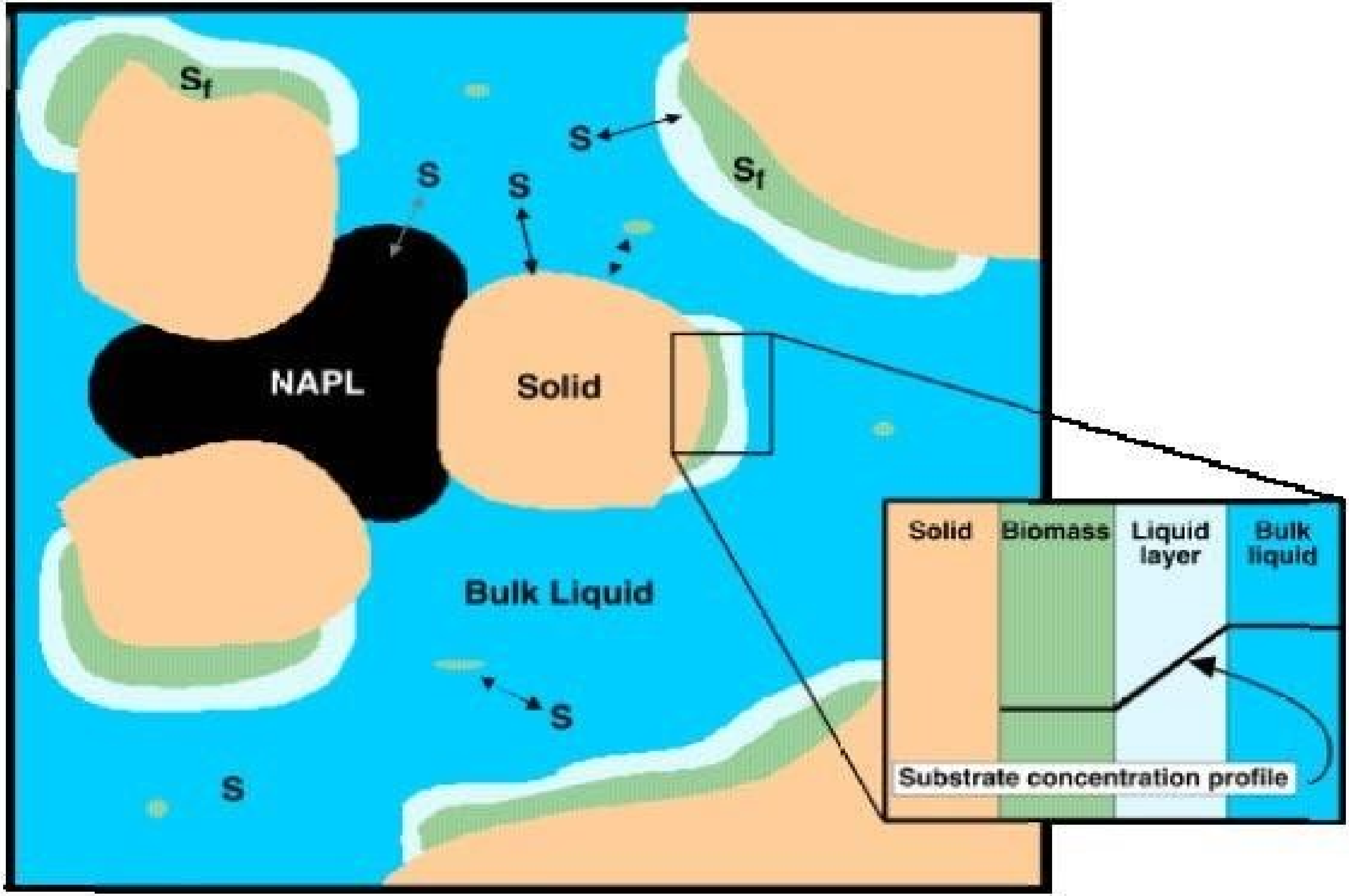
Sedimentation rate

Zooplankton refuges

Nutrient release due to anoxia

Hypolimnetic oxygen depletion

Conceptual Model Example



subsurface biodegradation process

Physical model

- Small scale physical representations which retain certain key features of larger natural system ==> Laboratory simulation of the processes and systems under investigation ==> explore the behavior of the system or test the mathematical models


Physical model

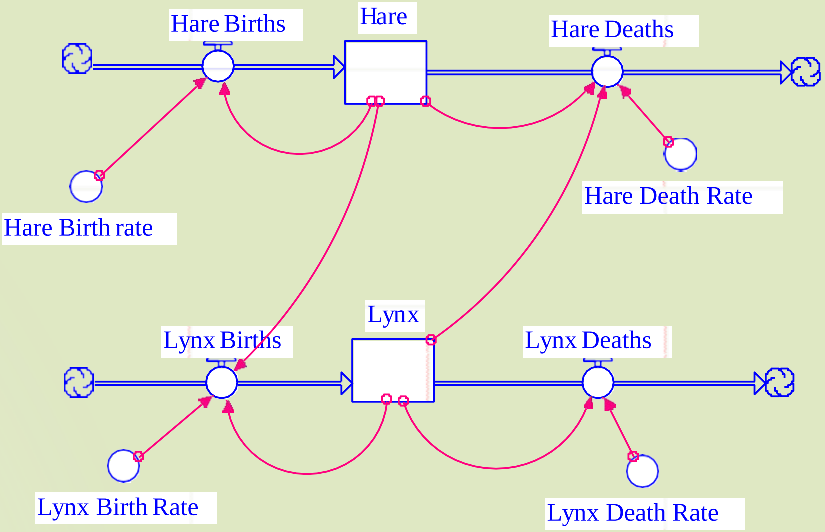
Mathematical model

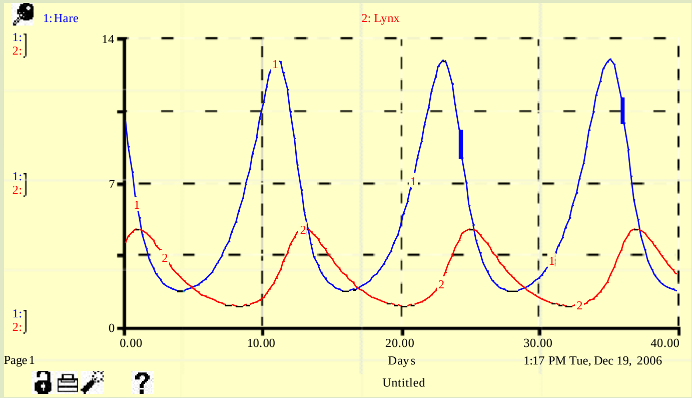
- Mathematical equations that translate a conceptual understanding of a systemor process into quantitative terms ==> the most common modeling tool ==> various levels from very simple algebraic models to lumped parameter models composed of ordinary partial equations to dynamic spatially distributed models described by partial differential equations.

$$\frac{dX}{dt} = aX - bXY$$

$$\frac{dY}{dt} = cXY - eY$$

Example: a simple predator-prey model



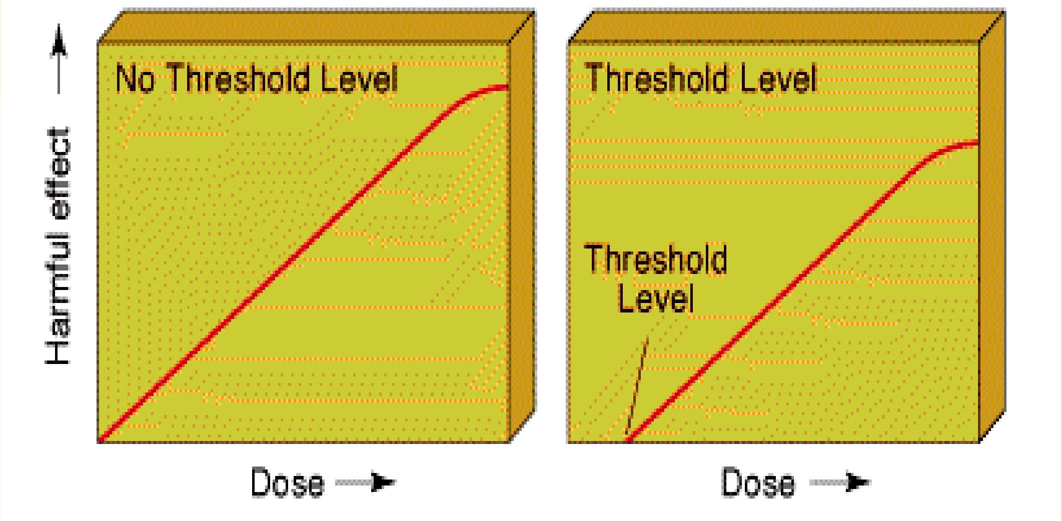


Models and relationships among variables

$$Y = f\,(X_1, X_2, \ldots, X_n)$$

- Independent variables X_i (e.g. cause, stimulus, forcing functions)
- Dependent variable Y (e.g. effect, response, state variables)

e.g. Dose-response curve



Examples of mathematical modeling

(a) $\frac{dC}{dt} = -kC$ if $C = C_0$ at $t = 0$ and k = constant.

(b) $\frac{dC}{dt} + k_1 C = k_2$ if $C = C_0$ at $t = 0$ and k_1 and k_2 are constant

How are the mathematical models used?

- Diagnosis (e.g., what is the cause of reduced water clarity in a lake?)
- Prediction (e.g., how long will it take for lake water quality to improve, once controls are in place?)

Categories of Mathematical Models

<table>
<tr><th colspan="2">Type</th></tr>
<tr><td>Empirical
Based on data analysis</td><td>Mechanistic
Mathematical descriptions based on theory</td></tr>
<tr><th colspan="2">Time Factor</th></tr>
<tr><td>Static or steady-state
Time-independent</td><td>Dynamic
Describe or predict system behavior over time</td></tr>
<tr><th colspan="2">Treatment of Data Uncertainty and Variability</th></tr>
<tr><td>Deterministic
Do not address data variability</td><td>Stochastic
Address variability/uncertainty</td></tr>
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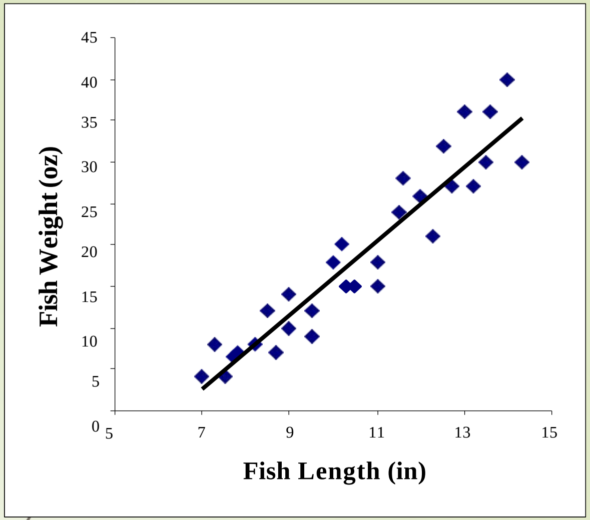
Empirical vs. Mechanistic?

- ***Empirical model*****:** based on statistical relationships between parameters of interest and other variables The relationships are usually defined by regression and correlation analyses.



e.g. Fish Weight = 4.48*Length -28.722

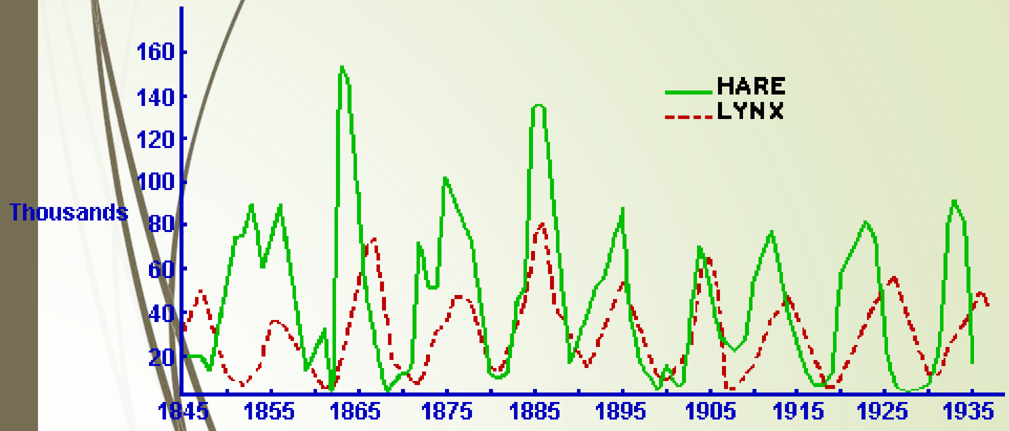
- ***Mechanistic Model*****:** rely on mathematical equations to describe the chemical, physical and biological processes governing the system and system prediction
  - apply scientific principles such as the conservation of mass, momentum and energy.
  - Mechanistic models include kinetic, thermodynamic equilibrium and mass transport models.
  - Parameter estimates (i.e. chemical reaction rates, mass transfer coefficients) have a critical role in mechanistic modeling

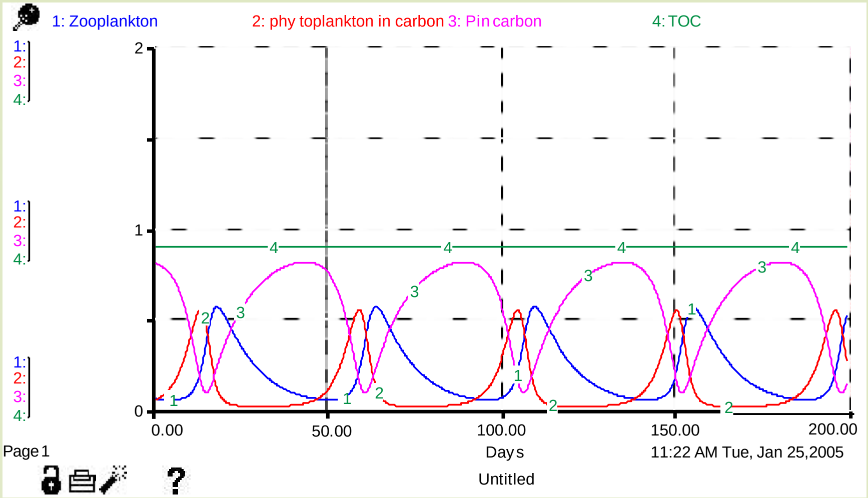
Procedures of development of mechanistic models:

- use fundamental knowledge of the interactions between process variables to define the model structure
- perform experiments to determine the parameters of the model
- collect data from the process to validate the model
- if the model is not satisfactory, go to step (a) and re-examine process knowledge

Static vs. Dynamic?

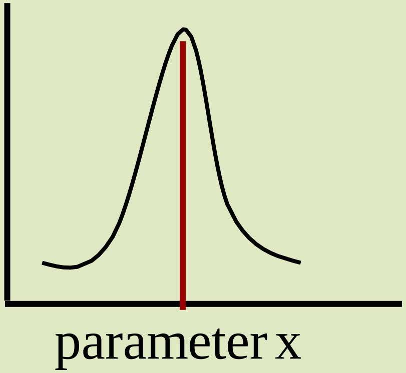
- Static models do not depend on time.
- Dynamic models predict how state variables change with time.





Deterministic vs. Stochastic?

- Deterministic: each component and input is determined exactly by mathematical equations
- Stochastic: A mathematical model which contains random (stochastic) components or inputs □ model output variables are known only in terms of probability distributions



When should you not use a model?

- If you do not understand the problem or system well enough to express it in concise, quantitative terms
- If the model has not been tested and verified for situations and conditions similar to your resource

Some truths about models

- All models are wrong, some are useful
- No model is ever complete
- We have no choice but to model
- Models do not substitute for logical thinking or other types of data collection and analysis
- Models should not be more complicated than is necessary for the task at hand

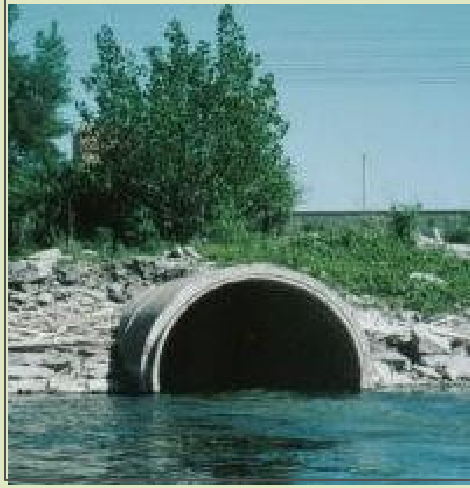
Application of Environmental Modeling

Environmental Medium	Issues/Concerns	Uses of Models in
Atmosphere	Hazardous air pollutants; air emissions; toxic release; acid rain; smog; CFCs; particulates; health concerns; global warming	Concentration profiles; exposure; design and analysis of control processes and equipment; evaluation of management actions; environmental impact assessment of new projects; compliance with regulations
Surface Water	Wastewater treatment plant discharges; industrial discharges; agricultural/urban runoff; storm water discharges; potable water sources; eutrophication; food chain	Fate and transport of pollutants; concentration plumes; design and analysis of control processes and equipment; waste load allocations; evaluation of management actions; environmental impact assessment of new projects; compliance with regulations
Groundwater	Leaking underground storage tanks; land application of solid and hazardous wastes; leachates from landfills and agriculture; injection; potable water source	Fate and transport of pollutants; concentration plumes; design and analysis of remedial actions; drawdowns; evaluation of management actions; compliance with regulations
Ocean	Sludge disposal; spills; outfalls; food chain	Fate and transport of pollutants; plume concentration; design and analysis of control processes; evaluation of management actions

Examples of Lake Water Quality Modeling:

- **Use Lake Models to Ask:**
  - What is the lake’s present water quality?
  - Development:
    - ✓ What was the lake’s water quality before development?
    - ✓ How will future watershed development affect water quality?
  - Nutrients:
    - ✓ What are the most important sources of nutrients to the lake?
    - ✓ What level of nutrient loading can the lake tolerate before it develops algae problems?
  - Nutrient management:

- ✓ How much must nutrients be reduced to eliminate nuisance algal blooms?
- ✓ How long will it take for lake water quality to improve once controls are in place?
- ✓ How successful will restoration be, based on water quality management goals?
- ✓ Are proposed lake management goals realistic and cost effective?



Environmental Modeling Steps

Developing Conceptual Models → **Developing Mathematical Models** → **Calibrating and Verifying Models** → **Sensitivity and Uncertainty Analysis**

(1) Developing Conceptual Models

- Define the problem
- Define the evaluative environment
- Define the relevant contaminant properties and characteristics
- Define the relevant fate and transport phenomena
- Define the "state" of the system.

(2) Developing Mathematical Models (i.e. a set of mathematical equations to describe the conceptual model)

- Clearly define a control volume
- Consider which transport phenomena control contaminant inputs and outputs across the model boundaries
- Consider reaction kinetics within the control volume
- Develop mathematical model

(3) Calibrating and Verifying Models

- Compare field data to model results
- Calibrate the model by "tuning" model input parameters to obtain a close fit to field data
- Develop performance criteria with which to judge the model
- Verify the model by testing the calibrated model with a second, independent set of field data

(4) Sensitivity and Uncertainty Analysis

- Sensitivity analysis: determination of the effect of a small change in model input parameters on the model results
- Uncertainty analysis: determination of the uncertainty in the model results due to uncertainty in the model input parameters

Statistical Performance Criteria

A. Chi-Square Goodness-of-Fit Criteria

$$\chi^2 = \sum_{i=1}^{n} \frac{(\text{observed value}_i - \text{simulated value}_i)^2}{\text{simulated value}_i}$$

In order to **accept the model results as a good fit** ➔

$$P(\chi^2 \le \chi^2_0) = 1 - \alpha$$

Where α is confidence level, 1- α is significance level

B. Linear regression of paired data for model predictions and field observations at the same time

$$s_{x,y} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

Sample covariance

$$s_x = \sqrt{\frac{\sum_{i=1}^{n} (x - \bar{x})^2}{n-1}}$$

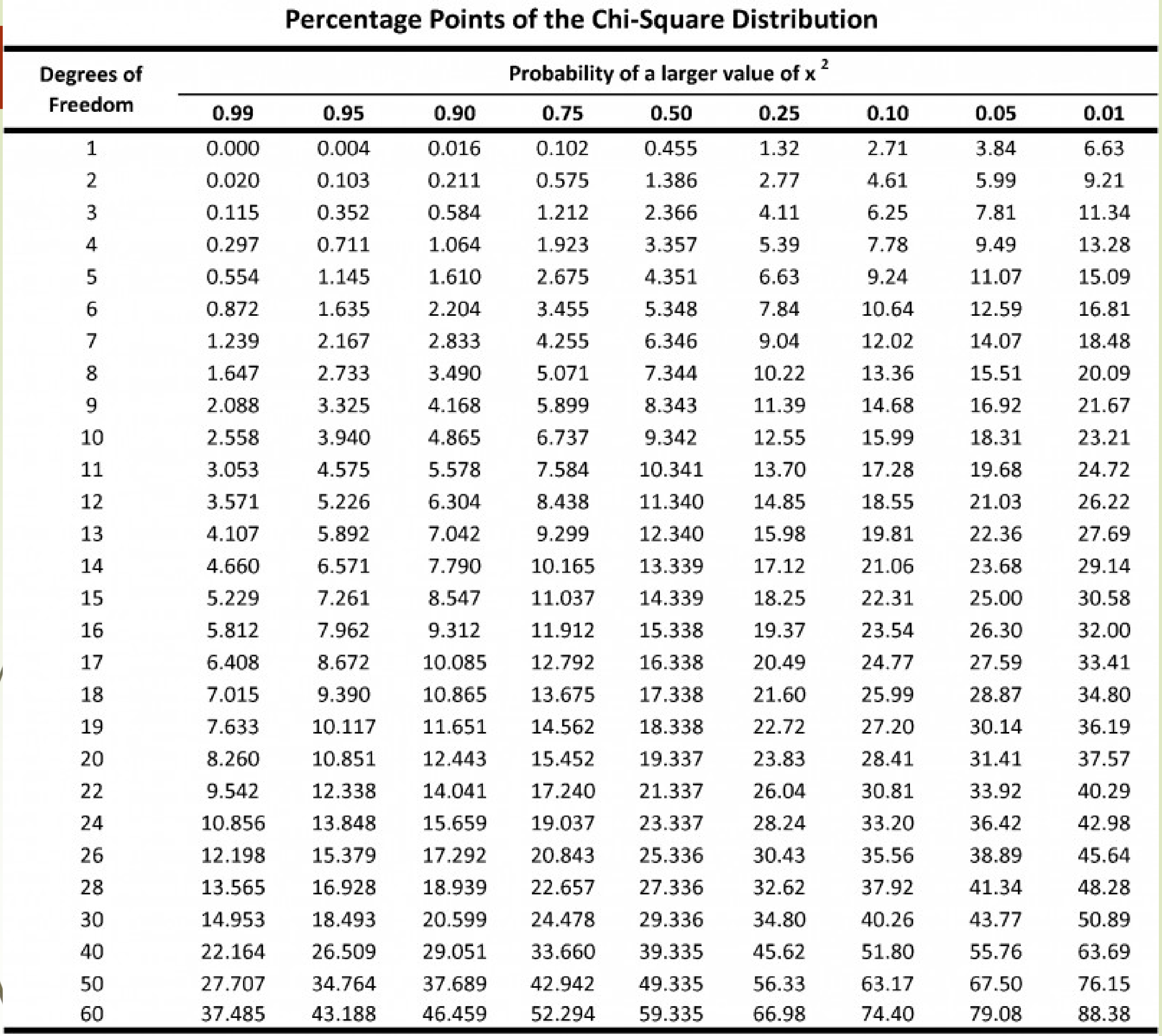
$$s_y = \sqrt{\frac{\sum_{i=1}^{n} (y - \bar{y})^2}{n-1}}$$

Sample standard deviation

$$r = \frac{s_{x,y}}{s_x \bullet s_y}$$

Sample correlation coefficient

Critical values of chi-square distribution



Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38

Example 1. Calibration and criteria testing of a dissolved oxygen model for a hypothetical stream. A waste discharge with biochemical oxygen demand (BOD) at km 0.0 causes a depletion in dissolved oxygen in a stream. Model calibration results are tabulated below (DO model) together with field measurements (DO field).

Distance (km)	Concentration (mg/L)		Distance (km)	Concentration (mg/L)	
	Simulated	Measured		Simulated	Measured
0	8.0	8.0	40	5.1	4.5
5	6.3	6.6	50	5.5	5.2
10	5.4	5.5	60	6.0	6.0
20	4.58	4.4	70	6.7	7.0
30	4.64	4.6	80	7.2	7.3

Problem: determine if the model calibration is acceptable according to the following statistical criteria: (a) Chi-square goodness-of-fit at a 0.10 significance level (a 90% confidence level); (b) Linear least-squares regression of model results (DO simulation on x-axis) versus observed data (DO measurement on y-axis) with $r^2 > 0.8$.

Solution: (a)

Distance (km)	DO Measurement (mg/L)	DO Simulation (mg/L)	Observed-Simulated	(Observed-Simulated)^2	(Observed - Simulated)^2/Simulated
0	8	8	0	0	0
5	6.6	6.3	0.3	0.09	0.014286
10	5.5	5.4	0.1	0.01	0.001852
20	4.4	4.58	-0.18	0.0324	0.007074
30	4.6	4.64	-0.04	0.0016	0.000345
40	4.5	5.1	-0.6	0.36	0.070588
50	5.2	5.5	-0.3	0.09	0.016364
60	6	6	0	0	0
80	7	6.7	0.3	0.09	0.013433
100	7.3	7.2	0.1	0.01	0.001389
Total					**0.12533**

$$\left.\begin{array}{l}\text{Degrees of freedom} = 10 - 1 = 9\\ \text{Confidence level } \alpha = 1-0.10 = 0.9\end{array}\right\} \Rightarrow \chi_0^2 = 4.168$$

χ^2

Since $\chi^2 < \chi_0^2 = 4.168$ ➔ model passes the goodness of fit test at a 0.10 significance level

Solution: (b)

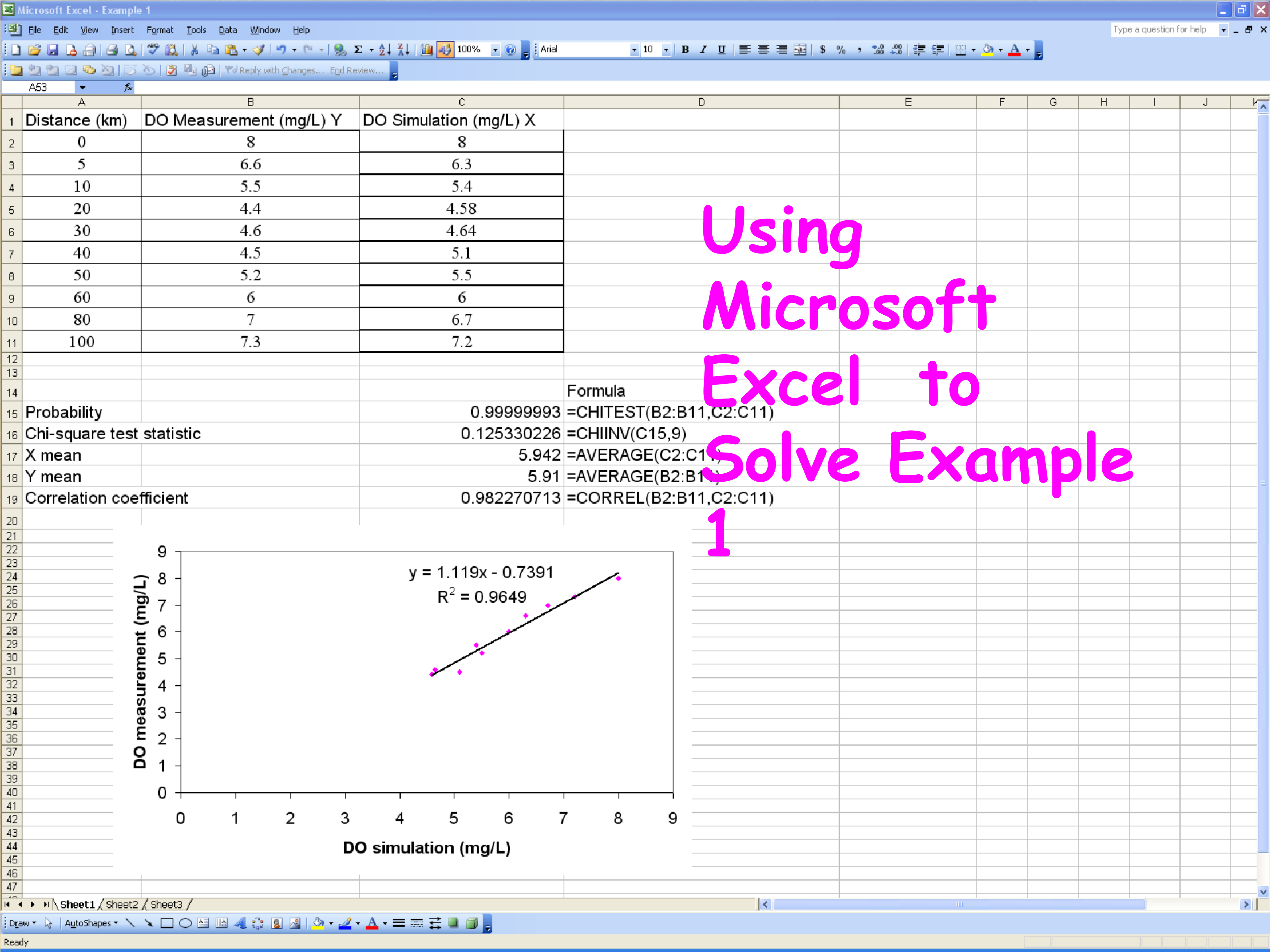
Distance (KM)	Measurement (mg/L) (Y)	Simulation (mg/L) (X)	$x_i - x$	$y_i - y$	$(x_i - x)_2$	$(y_i - y)_2$	$(x_i - x)(y_i - y)$
0	8	8	2.058	2.09	4.235364	4.3681	4.30122
5	6.6	6.3	0.358	0.69	0.128164	0.4761	0.24702
10	5.5	5.4	-0.542	-0.41	0.293764	0.1681	0.22222
20	4.4	4.58	-1.362	-1.51	1.855044	2.2801	2.05662
30	4.6	4.64	-1.302	-1.31	1.695204	1.7161	1.70562
40	4.5	5.1	-0.842	-1.41	0.708964	1.9881	1.18722
50	5.2	5.5	-0.442	-0.71	0.195364	0.5041	0.31382
60	6	6	0.058$x_i - \bar{x}$	0.09	0.003364	0.0081	0.00522
80	7	6.7	0.758	1.09	0.574564	1.1881	0.82622
100	7.3	7.2	1.258	1.39	1.582564	1.9321	1.74862
		5.942					
	5.91						
Sum					11.27236	14.629	12.6138

➔ s_x = 1.119 s_y = 1.275 $s_{x,y}$ = 1.402

$$r = \frac{s_{x,y}}{s_x \bullet s_y} = \frac{□1.402}{1.119 \times 1.275} = 0.9827$$

→ $r^2 = 0.966 > 0.80$ → Good calibration

Using Microsoft Excel to Solve Example 1



Distance (km)	DO Measurement (mg/L) Y	DO Simulation (mg/L) X
0	8	8
5	6.6	6.3
10	5.5	5.4
20	4.4	4.58
30	4.6	4.64
40	4.5	5.1
50	5.2	5.5
60	6	6
80	7	6.7
100	7.3	7.2

	Value	Formula
Probability	0.99999993	=CHITEST(B2:B11,C2:C11)
Chi-square test statistic	0.125330226	=CHIINV(C15,9)
X mean	5.942	=AVERAGE(C2:C11)
Y mean	5.91	=AVERAGE(B2:B11)
Correlation coefficient	0.982270713	=CORREL(B2:B11,C2:C11)

Mass and Concentration Relationship

- Contaminant Concentrations:
  - C_w, ppm, mg/L (mass per mass basis): concentration in water
  - C_s, ppm, mg/kg (mass per mass basis): concentration in solids
  - C_g, ppmV (volume per volume basis): concentration in gas
- Convert ppmV into mass concentration units:

$$mg/m^3 = \frac{(ppmV)\times 12.187MW}{273.15+T}$$

Where MW is the molecular weight of the compound, T is temperature (°C)

- Determination of the mass of contaminant present in a medium:
  - Mass of contaminant in liquid = (liquid volume)× (concentration in liquid) = $(V_w)(C_w)$
  - Mass of contaminant in soil = (soil mass)× (concentration in soil) = $(M_s)(C_s)$ =$(V_s\rho_b)(C_s)$
  - Mass of contaminant in air = (air volume)× (concentration in soil in mass per volume) =$(V_g)(C_g)$

Example 2: A child went into a site and played with dirt contaminated with benzene. During his stay at the site he inhaled 2 m^3 of air containing 10 ppbV of benzene and ingested a mouthful (~5 cm^3) of soil containing 3 mg/kg of benzene.

Which system (ingestion or inhalation) is exposed to more benzene? Assume the bulk density of soil is 1.8 g/cm^3, T = 20 °C, MW = 78.

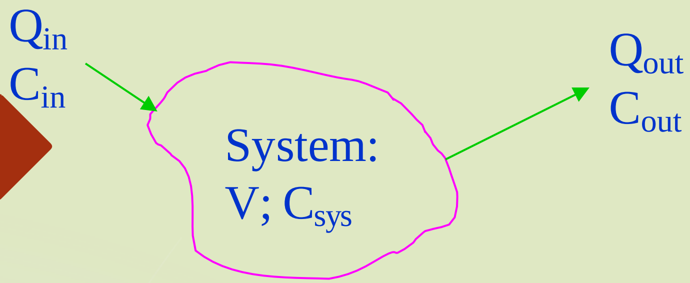
Mass Balances

- Most models are based on mass balance calculations
- statements of the principle of conservation of mass
- Two types of mass balances:
  - overall mass balance

Rate of accumulation of mass in the system	=	Rate of mass flow in of mass in the system	−	Rate of mass flow out of mass in the system	±	Reactions

- component balance

Rate of accumulation of mass of component *i* in the system	=	Rate of mass flow of component *i* into the system	-	Rate of mass flow of component *i* out of the system	+	Rate of production of component *i* by reaction	-	Rate of consumption of component *i* by reaction



Where
Q: flow rate $[L^3/T]$
C: Mass concentration $[M/L^3]$
V: System volume $[L^3]$

Mass in the system = VC_{sys}

Rate of mass accumulation = $\dfrac{\Delta(VC_{sys})}{\Delta t} = \dfrac{d\,(VC_{sys})}{dt}$

Rate of mass flow into system = $\sum_{i=1}^{N} Q_{in,i} C_{in,i}$ where N is number of mass components

Rate of mass flow out of system = $\sum_{j=1}^{N} Q_{out,j} C_{out,j}$

- For no reaction condition, the mass balance equation:

$$\frac{d\,(VC_{sys})}{dt} = \sum_{i=1}^{N} Q_{in,i} C_{in,i} - \sum_{j=1}^{N} Q_{out,j} C_{out,j}$$

- For no reaction condition, by using fluid density ➔ mass balance:

$$\frac{d(V\rho_{sys})}{dt} = \sum_{i=1}^{N} Q_{in,i}\,\rho_{in,i} - \sum_{j=1}^{N} Q_{out,j}\,\rho_{out,j} \longrightarrow V\frac{d(\rho_{sys})}{dt} + \rho_{sys}\frac{dV}{dt} = \sum_{i=1}^{N} Q_{in,i}\,\rho_{in,i} - \sum_{j=1}^{N} Q_{out,j}\,\rho_{out,j}$$

- If the density is constant (i.e. incompressible fluid)

$$\frac{dV}{dt} = \sum_{i=1}^{N} Q_{in,i} - \sum_{j=1}^{N} Q_{out,j}$$

Steady-state condition (i.e. constant volume system) ➔

$$\sum_{i=1}^{N} Q_{in,i} = \sum_{j=1}^{N} Q_{out,j}$$

Example 3: A certain pollution control device is processing a waste stream which has a flow rate of 3.0 L/s and a contaminant concentration of 102 mg/L. We want to produce a treated product flow of 2.8 L/s with a contaminant concentration of 9.0 mg/L. Determine the flow and concentration of the residual stream. Assume steady flow, constant density and constant volume system.

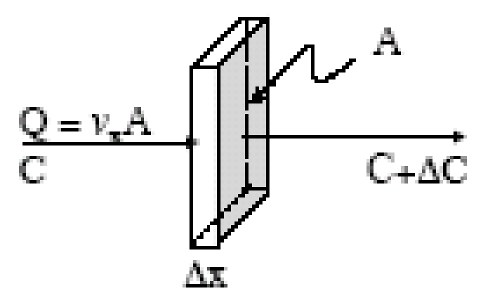
Example 4: A house has a ventilation rate of 100 m^3/h. A kerosene heater is being used in the house, which emits 1000 g/h of CO_2. What is the average steady-state CO_2 concentration in the house? Assume that the CO_2 concentration in the atmosphere is 0.568 g/m^3.

Mass Transport Process

(1) Advection

- The process by which contaminants are transported by the bulk of motion of the flowing water.
- Nonreactive contaminants are carried at an average rate equal to the average linear velocity of the water.
- Advective transport:

$J_{A,x} = Q_x C = v_x A C$



where $J_{A,x}$ = mass flux of contaminant in x direction $[MT^{-1}]$, v_x = mean water velocity in x direction $[LT^{-1}]$, C = contaminant concentration $[ML^{-3}]$, Q = volumetric flow rate $[L^3T^{-1}]$

- With **JUST** advection, mass balance equation:

Mass inflow rate = $v_x A C$

Mass outflow rate = $v_x A(C+\Delta C)$

Mass accumulation rate = $V\frac{\partial C}{\partial t} = v_x A C - v_x A(C+\Delta C) = -v_x A\,\Delta C$

due to advection

Since $V = A\Delta x$

$$\frac{\partial C}{\partial t} = -\frac{v_x A\,\Delta C}{A\Delta x} = -v_x\frac{\Delta C}{\Delta x}$$

$$\frac{\partial C}{\partial t} = -v_x\frac{\partial C}{\partial x}$$

Example 5: If the salt concentration in a river is 20 mg/L and the average river velocity is 100 cm/s, what is the average mass flux of salt in the downstream direction per unit flow area?

(2) Hydrodynamic Dispersion

- Tendency of the contaminant to spread out from the advective path
- Two processes (diffusion, dispersion)

A. Diffusion

- ➢ the movement of mass due to random water motion or mixing
- ➢ *molecular diffusion*: the mixing of dissolved chemicals due to the random walk of molecules within the fluid
- ➢ "random walk" = spreading will occur in fluid with no mean velocity

↓

Mixing caused by random molecular motion

- **Fick's First Law of diffusion** (based on Fourier's Law of heat flow)

Mass flux of solute = mass of solute passing a unit areaper unit time in a given direction ∝ a concentration gradient of solute in that direction

$$J_m \propto A \frac{dC}{dx}$$

where J_m = mass flux rate $[MT^{-1}]$, A = cross-sectional area $[L^2]$, dC/dx = concentration gradient $[ML^{-3}L^{-1}]$.

Incorporating a proportionality constant (D) into the above equation:

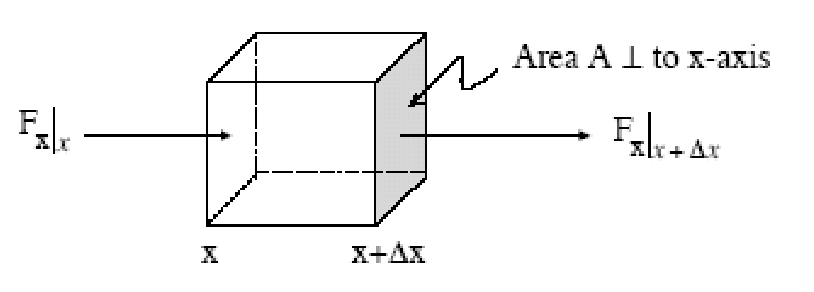
$$J_m = -DA \frac{dC}{dx}$$

$$F_x = \frac{J_m}{A} = -D \frac{dC}{dx}$$

where D = molecular diffusion coefficient $[L^2T^{-1}]$ (usually around 10^{-5} cm^2/s in water, indicating very slow movement of mass due to molecular diffusion), F_x= areal mass flux rate $[ML^{-2}T^{-1}]$

Example 6: The molecular diffusivity of a caffeine (C_9H_8O) in wateris 0.63×10^{-5} cm^{2}s^{-1}. For a 0.001 mg/cm^3 solution, calculate the mass flux in mg/s through an intestinal membrane (1000 cm^2 area) with a liquid film approximately 0.006 cm thick. How long will it take 1 mg of caffeine to move through 1000 cm^2 of intestine by assuming the above fluxrate?

➢ Fick's Second Law of Diffusion



Mass balance equation:

rate of change of mass in volume element = $V\frac{\partial C}{\partial t} = A\Delta x\frac{\partial C}{\partial t}$

Input of mass across face at x = $AF_x\big|_x$

Output of mass across face at x+Δx = $AF_x\big|_{x+\Delta x}$

due to diffusion

$$\longrightarrow \quad A\Delta x\frac{\partial C}{\partial t} = AF_x\big|_x - AF_x\big|_{x+\Delta x}$$

$$\longrightarrow \quad \frac{\partial C}{\partial t} = -\left(\frac{F_x\big|_{x+\Delta x} - F_x\big|_x}{\Delta x}\right) \longrightarrow \frac{\partial C}{\partial t} = -\frac{\partial F_x}{\partial x} = -\frac{\partial\left(-D\frac{\partial C}{\partial x}\right)}{\partial x} = D\frac{\partial^2 C}{\partial x^2}$$

- *Turbulent diffusion*: mixing caused by micro-scale turbulence or mechanical mixing ==> to minimize concentration gradients by moving mass from regions of high to low concentration

$$F_t = \frac{J_t}{A} = -\varepsilon_m \frac{dC}{dx}$$

where J_t = mass flux rate due to turbulent diffusion [MT^{-1}], ε_m = turbulent diffusion coefficient [L^2T^{-1}]

B. Dispersion

- the greater degree of mixing caused by the interaction of turbulent diffusion with velocity gradients in the water body

$$F_d = \frac{J_d}{A} = -E\frac{dC}{dx}$$

where J_d = mass flux rate due to dispersion [MT^{-1}], E = dispersion coefficient [L^2T^{-1}]

One-dimensional analytical solution of the dispersion equation

$$\frac{\partial C}{\partial t} = E\frac{\partial^2 C}{\partial x^2}$$

Instantaneous input of mass M at t = 0, x = 0
C(x) = 0 at t = 0
C (+∞) = 0 at t > 0
Assume complete mixing in y & z directions:

$$C(x,t) = \frac{M}{\sqrt{4\pi Et}}e^{-\frac{x^2}{4Et}}$$

(3) Advection-Dispersion Equation

$$\frac{\partial C}{\partial t} = -u_i\frac{\partial C}{\partial x_i} + E_i\frac{\partial^2 C}{\partial x_i^2} - R$$

One dimensional form:

$$\frac{\partial C}{\partial t} = -u_x\frac{\partial C}{\partial x} + E_x\frac{\partial^2 C}{\partial x^2} - R$$

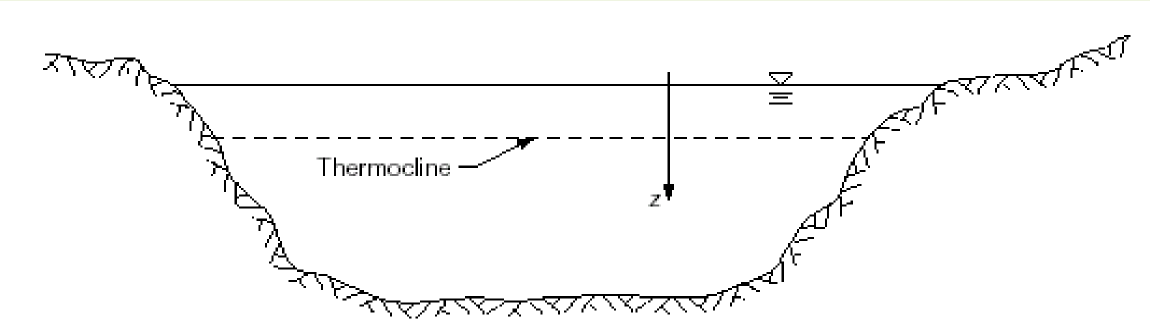
Solutions to the advection-dispersion equation (when R = kC):

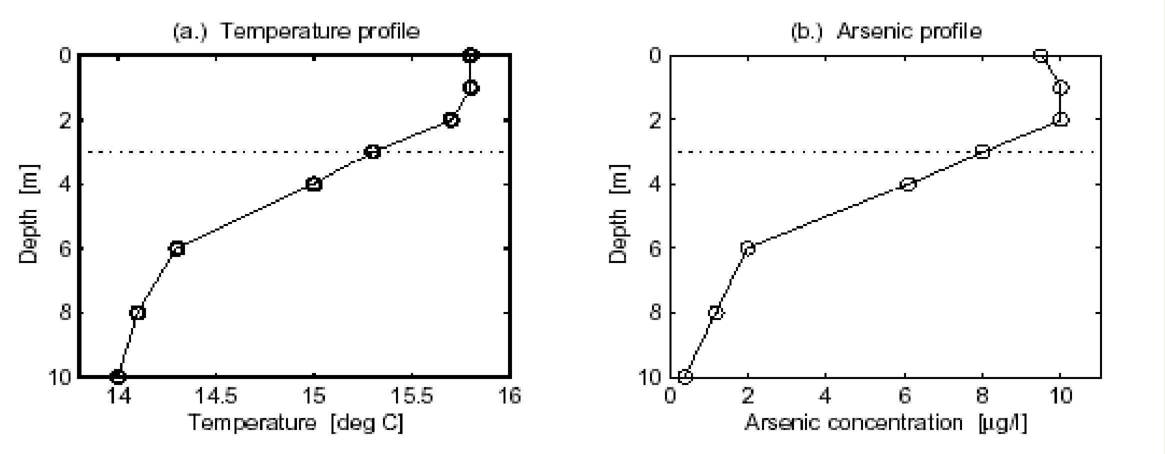
Instantaneous input of mass M at t = 0 and x = 0, C (x, 0) = 0 for x $\neq$ 0, C($\pm\infty$, t) = 0 for t > 0

$$C(x,t) = \frac{\square_a M}{\sqrt{4\pi E_x t}} \exp\{- \frac{\square_x \ (x - u\,t)^2}{4E_x t} - kt\}$$

Where M_a is the mass per unit cross-sectional area, k is first-order reaction rate constant

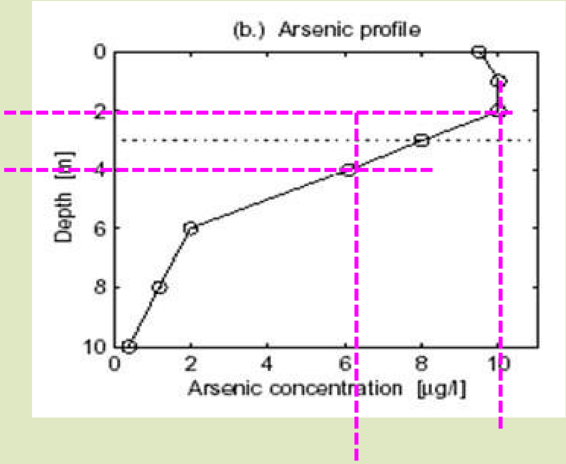
Example 7: As shown in the following figures, a small alpine lake ismildly stratified, with a thermocline (region of steepest density gradient) at 3 m depth, and is contaminated by arsenic. The molecular diffusion coefficient is D = 1 $\times 10^{-10}$ m^2/s, and a turbulent diffusion coefficient can be taken as εm = 1.5 $\times$ 10^{-6} m^2/s. After further investigation, it is determined that a freshwater spring flows into the bottom of the lake with a flow rate of 10 L/s. Determine the magnitude and direction of the diffusive and advective flux of arsenic through the thermocline (cross-sectional area at the thermocline is A = 2 $\times$ 10^4 m^2).





Observed profiles of temperature (a) and arsenic concentration (b) in an alpine lake. The dotted line at3 m indicates the location of the thermocline

(1) Molecular diffusion:



$$\frac{dC}{dz}=\frac{C_1-C_2}{Z_1-Z_2}=\frac{6.1-10}{4-2}=-1.95\,\frac{\mu g}{L\cdot m}$$

$$F_z=-D\frac{dC}{dz}=-(10^{-10}\ m^2/s)\times\left(-1.95\,\frac{\mu g}{L\cdot m}\right)\times\frac{1000\,L}{m^3}$$
$$=1.95\times10^{-7}\,\frac{\mu g}{m^2\cdot s}$$

➔ Flow direction: downward

- Mass flux due to molecular diffusion:

$$J_z=F_zA=1.95\times10^{-7}\,\frac{\mu g}{m^2\cdot s}\times(2\times10^4\ m^2)=3.9\times10^{-3}\ \mu g/s$$

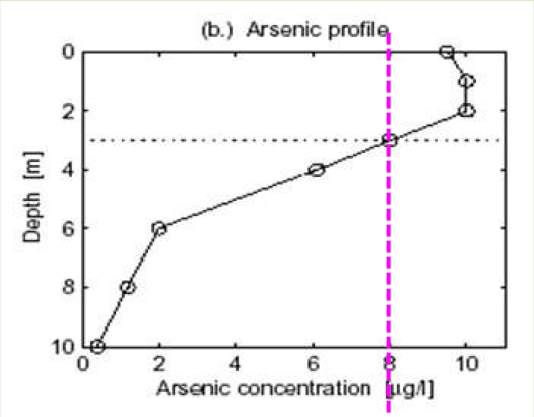
(2) Turbulent diffusion:

$$F_t=-\varepsilon_m\frac{dC}{dz}=-(1.5\times10^{-6}\ m^2/s)\times\left(-1.95\,\frac{\mu g}{L\cdot m}\right)\times\frac{1000\,L}{m^3}=2.925\times10^{-3}\,\frac{\mu g}{m^2\cdot s}$$

$$J_t=F_tA=2.925\times10^{-3}\,\frac{\mu g}{m^2\cdot s}\times(2\times10^4\ m^2)=58.5\ \mu g/s$$

(3) Advection:

$$v_A=-\frac{Q}{A}=-\frac{10\ L/s}{2\times10^4\ m^2}\times\frac{1\ m^3}{1000\ L}=-5\times10^{-7}\ m/s$$



$$F_A = v_A C = -(5\times10^{-7}\ m/s)\times(8\,\frac{\mu g}{L})\times\frac{1000\,L}{m^3}$$
$$= -4\times10^{-3}\,\frac{\mu g}{m^2\cdot s}$$

$$J_A = F_A A = -4\times10^{-3}\,\frac{\mu g}{m^2\cdot s}\times(2\times10^{4}\ m^2) = -80\ \mu g/s$$

➔ Net vertical flux

$$= J_z + J_t + J_A = 3.9\times10^{-3} + 58.5 - 80 = -21.5\ \mu g/s$$

Example 8: C_{max} was measured at time $t_1 = 5$ hr after a pulse injection of a conservative dye into a river. What is the average river velocity if the maximum concentration at t_1 occurs 1025 m downstream from the pulse injection? Estimate the dispersion coefficient for this river if σ is approximately 350 m when the chemical has traveled a distance of 1975 m to x_2.

Example (Spill Model): 10 kg of a conservative contaminant (k=0) are spilled in a stream that is 15 m wide, 3m deep (on average), and has an average velocity of 35 cm/s. If the contaminant is rapidly mixed over the cross-section of the stream: (a) derive an expression for the contaminant concentration as a function of time 500 m downstream of the spill; (b) if a peak concentration of 4 mg/L is observed 500 m downstream of the spill, estimate the longitudinal dispersion coefficient in the stream; (c) using the result in part (b), what would be the maximum contaminant concentration 1 km downstream of the spill?; and (d) if the detection limit of the contaminant is 1 μg/L, how long after release will the contaminant be detected 1 km downstream of the release point?

Solution:

(a) $A = w \cdot d = (15\text{m}) \times (3\text{m}) = 45\ \text{m}^2$

$M = 10\ \text{kg}$ ➔ $M_a = M/A = (10\ \text{kg})/(45\text{m}^2) = 0.2222\ \text{kg/m}^2$

$u_x = 35\ \text{cm/s} = 0.35\ \text{m/s}$

$x = 500\text{m},\ k = 0$

Then:

$$C(500,t) = \frac{0.2222 kg/m^2}{\sqrt{4\times 3.14 E_x t}} \exp\{-\frac{(500-0.35t)^2}{4E_x t}\}$$

$$= \frac{0.0627}{\sqrt{E_x t}} \exp\left[-\frac{(500-0.35t)^2}{4E_x t}\right]$$

where t is in seconds (s), and E_x is in m²/s

(b) $t_1 = x_1/u_x = (500m)/(0.35\ m/s) = 1429\ s$

$$C_{\max,1} = \frac{M_a}{\sqrt{4\pi E_x t_1}} = \frac{0.0627}{\sqrt{E_x t_1}} = \frac{0.0627}{\sqrt{1429 E_x}} (kg/m^3) = 4mg/L = 0.004(kg/m^3)$$

➔ solve this equation ➔ $E_x = 0.172\ m^2/s$

(c) $t_2 = x_2/u_x$ = (1000m)/(0.35m/s) = 2857 s ➔

$$C_{\max,2} = \frac{M_a}{\sqrt{4\pi E_x t_2}} = \frac{0.0627}{\sqrt{E_x t_2}} = \frac{0.0627}{\sqrt{0.172\times 2857}}(kg/m^3) = 0.00283(kg/m^3)$$

$$= 2.83 mg/L$$

(d) C(1000,t) = 1 μg/L = 10^{-6} kg/m^3

➔

$$C(1000,t) = \frac{0.0627}{\sqrt{E_x t}} \exp\left[-\frac{(1000-0.35t)^2}{4E_x t}\right]$$

$$= \frac{0.0627}{\sqrt{0.172t}} \exp\left[-\frac{(1000-0.35t)^2}{4\times 0.172t}\right] = 10^{-6}$$

Solve this equation ➔ t = 2520.4 s = 42 min.
(using Microsoft Excel "Goal Seek")

Estimating Relative Importance of Transport Processes

A. Peclet Number

$P_e = uL/E$

$P_e >> 1$ ➔ advection predominates

$P_e << 1$ ➔ dispersion predominates

B. Reaction Number

Rxn. No. = kE/u^2

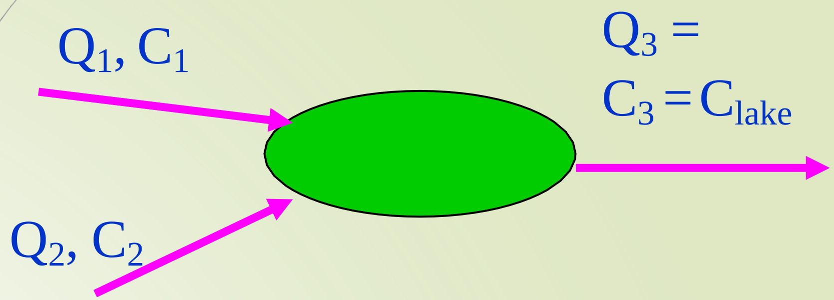
Rxn. No. < 0.1 ➔ advection predominates

Rxn. No. > 10 ➔ dispersion predominates

Compartmental (Box) Models

- a region in space (box or control volume) for which the mass balance equation is developed.
- Complete mixing assumed within the box

A. Box model for conservative pollutants and steady-state system



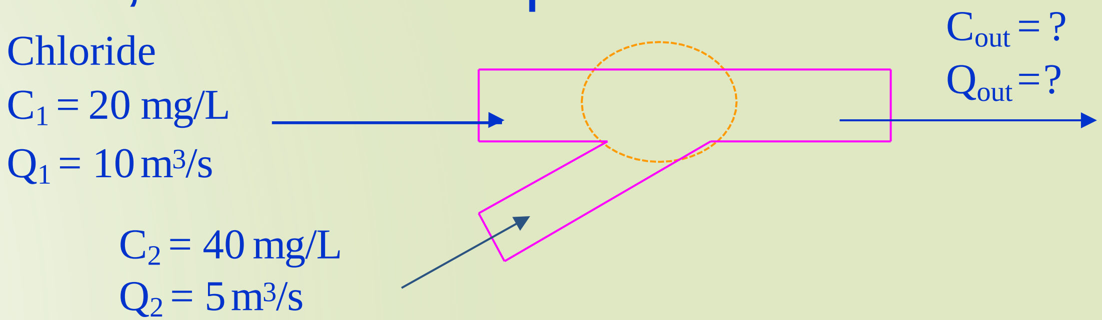
Q_1, C_1

Q_2, C_2

$Q_3 =$

$C_3 = C_{lake}$

Example 9: steady state, conservative pollutant:



Chloride

$C_1 = 20$ mg/L

$Q_1 = 10$ m³/s

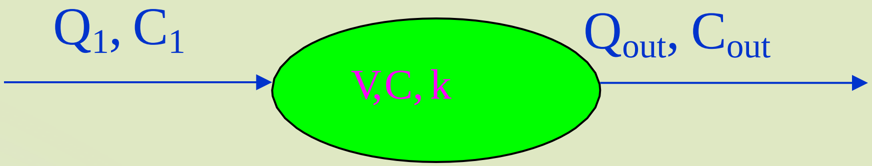
$C_2 = 40$ mg/L

$Q_2 = 5$ m³/s

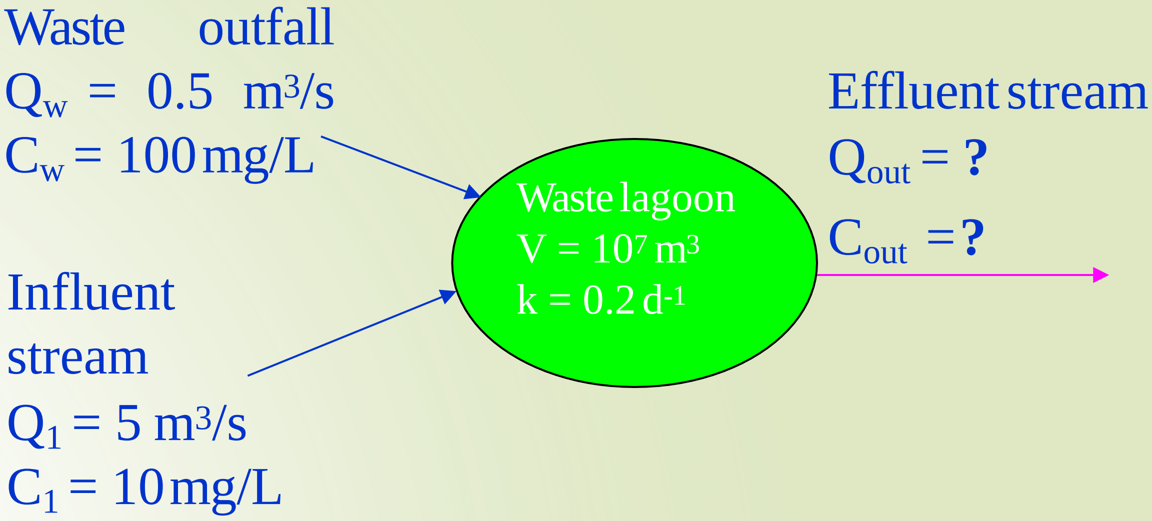
$C_{out} = ?$

$Q_{out} = ?$

B. Box model for non-conservative pollutants and steady-state system



Example 10: steady state, waste lagoon with biodegradable organic pollutant



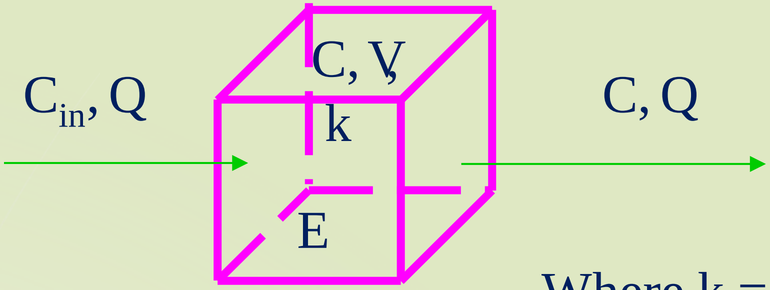
Waste outfall
$Q_w = 0.5\ m^3/s$
$C_w = 100\ mg/L$

Influent stream
$Q_1 = 5\ m^3/s$
$C_1 = 10\ mg/L$

Waste lagoon
$V = 10^7\ m^3$
$k = 0.2\ d^{-1}$

Effluent stream
$Q_{out} = ?$
$C_{out} = ?$

C. Box model for non-conservative pollutants and unsteady-state system



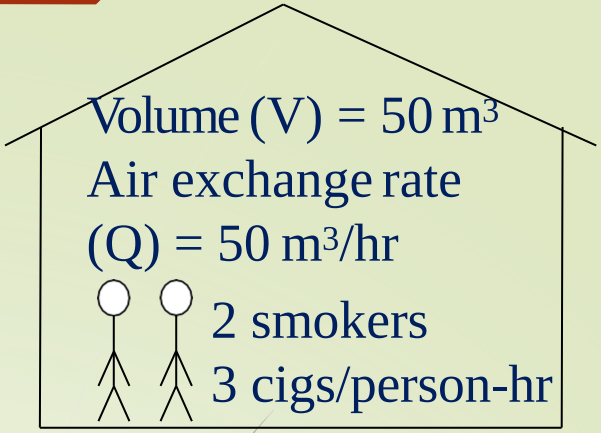
Where k = degradation constant,
E = emission rate (mass/time)

Mass Balance:

$$V\frac{dC}{dt} = E + QC_{in} - QC - kVC$$

Example 11: Formaldehyde concentration in a home with smokers

Each cigarette emits 1.4 mg



Volume (V) = 50 m^3

Air exchange rate (Q) = 50 m^3/hr

2 smokers

3 cigs/person-hr

- formaldehyde
- Formaldehyde degrades with a rate coefficient of 0.4/hr
- Threshold for eye irritation from formaldehyde is 0.06 mg/m^3
- **Question:** how long will it take to reach a formaldehyde concentration sufficient to cause eye irritation?

Questions

1. What is modeling?
2. Why we model?
3. What are things to think before modeling?
4. What is point and nonpoint sources of pollution?
5. What is mass balance equation?

Thank you

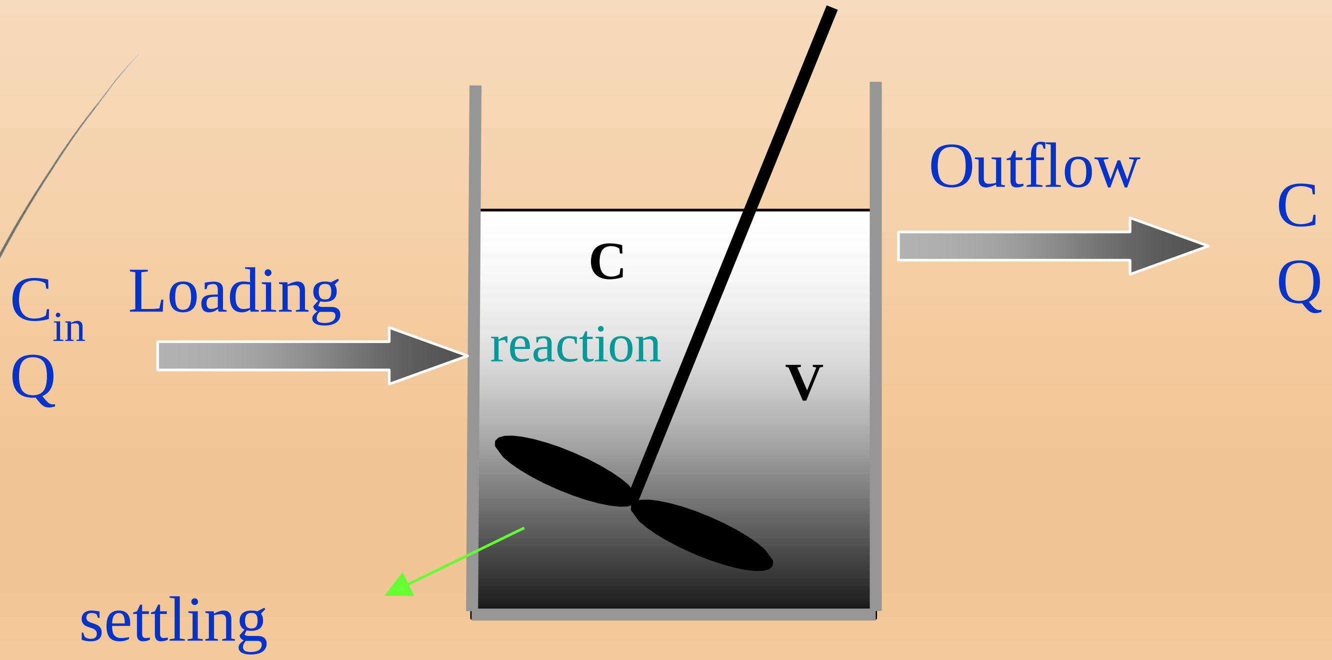
Any question ?

Chapter two

Lake water quality modelling

Mass Balance for a Well-Mixed Lake

- completely mixed system ➔ continuously stirred tank reactor (**CSTR**) ➔ outflow concentration = in-lake concentration

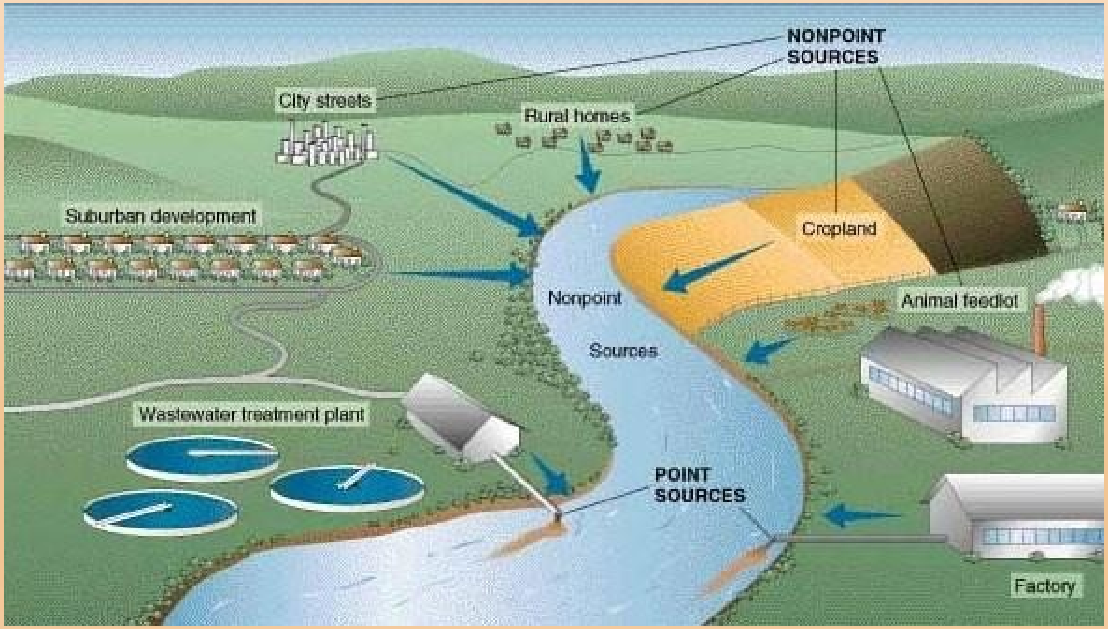


Mass loading rate

- mass m of pollutants discharged over a time period t
  ➔ $W = m/t$
- mass enters a lake from a variety of sources and in a number of ways ➔ lump all loadings into a single term:

$$W(t) = QC_{in}(t)$$

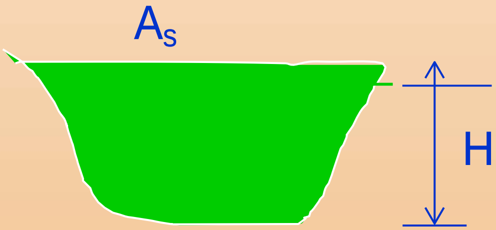
Where Q= volumetric flow rate of all water sources entering the system $[L^3T^{-1}]$, $C_{in}(t)$ = average inflow concentration of these sources $[ML^{-3}]$



Source: http://w.isaveearth.org/Waterpollutionsources.htm

Example 12: A pond having constant volume and no outlet has a surface area A_s of 10^4 m^2 and a mean depth H of 2 m. It initially has a concentration of 0.8 ppm. Two days later a measurement indicates that the concentration has risen to 1.5 ppm. What was the mass loading rate during this time? If you hypothesize that the only possible source of this pollutant was from atmosphere, estimate the flux (i.e. mass loading per unit surface area) that occurred.

Solution:



System volume $V = A_sH = (10^4\ m^2)(2m) = 2{\times}10^4\ m^3$

Mass of pollutant at t=0:

$$M_0 = VC_0 = (2{\times}10^4\ m^3)(0.8\ g/m^3) = 1.6{\times}10^4\ g$$

Mass of pollutant at t= 2 day:

$$M_1 = VC_1 = (2{\times}10^4\ m^3)(1.5\ g/m^3) = 3.0{\times}10^4\ g$$

$$\text{Mass loading } W = \frac{M}{t} = \frac{M_1 - M_0}{t} = \frac{3\times10^4\ g - 1.6\times10^4\ g}{2\ d} = 0.7\times 10^4\ g/d$$

$$\text{Mass flux } J = \frac{W}{A_s} = \frac{0.7\times10^4\ g/d}{10^4\ m^2} = 0.7 g/(m^2 \cdot d)$$

Assimilation Factor (a)

- assume a linear relationship between loading rate (W) and pollutant concentration (C)

$$a = \frac{W}{C}$$

➔ a $[L^3T^{-1}]$ ➔ represent the physics, chemistry, and biology of the receiving water

➔ C = f(W, physics, chemistry, biology)

Source: http://www.peterpatau.com/2012_06_10_archive.html

Example 13: Lake Ontario in the early 1970s had a total phosphorus loading of approximately 10,500 tons/yr and an in-lake concentration of 21 μg/L. In 1973 the state of New York and the province of Ontario ordered a reduction of detergent phosphate content. This action reduced loadings to 8000 tons/yr. (a) compute the assimilation factor for Lake Ontario; (b) what in-lake concentration would result from the detergent phosphate reduction action? (c) if the water quality objective is to bring in-lake levels down to 10 μg/L, how much additional load reduction is needed?

Solution:

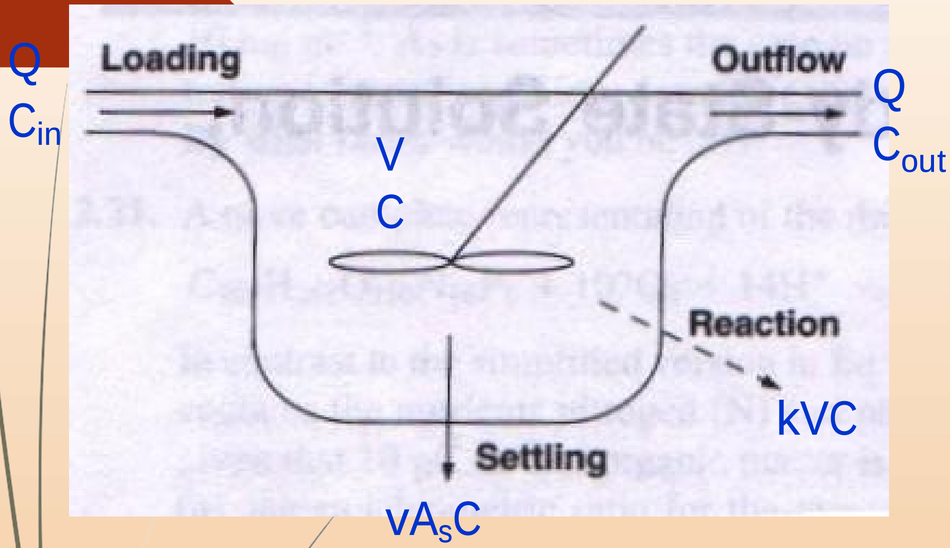
$$(1)\quad a=\frac{W}{C}=\frac{10500\ t/y}{21\ \mu g/L}=500\ (t\cdot L)/(\mu g\cdot y)$$

$$(2)\quad C_1=\frac{W_1}{a}=\frac{8000\ t/y}{500\ (t\cdot L)/(\mu g\cdot y)}=16\ \mu g/L$$

$$(3)\quad W=aC=500\ (t\cdot L)/(\mu g\cdot y)\times 10\left(\frac{\mu g}{L}\right)=5000\ t/y$$

➔ Additional load reduction = 8000 – 5000 = 3000 t/y

Mass balance for the well-mixed lake



- Loading = $W(t) = QC_{in}(t)$
- Outflow = $QC_{out} = QC$
- Reaction = $KM = KVC$
- Settling = flux of mass across the surface area of sediment-water interface = vA_sC

Where v = apparent settling velocity $[LT^{-1}]$, A_s= surface area of sediments $[L^2]$

- settling can also be estimated by using:

settling = k_sVC

where k_s = first-order settling rate constant $[T^{-1}]$

$$k_s = \frac{vA_s}{V} = \frac{vA_s}{A_sH} = \frac{v}{H}$$

where H = mean depth

- **Temperature effects of reaction:**

  - The rates of most reactions in natural waters increases with temperature

$$\frac{k(T_2)}{k(T_1)} = \theta^{T_2 - T_1}$$

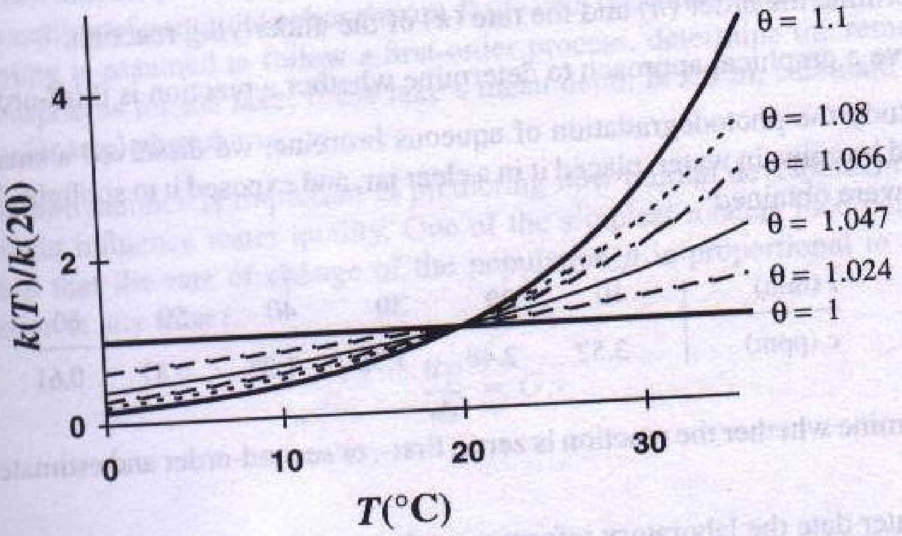
where $k(T_1)$, $k(T_2)$ = reaction rate constants at temperature T_1 and T_2 (°C), respectively, θ = constant

  - in water quality modeling, many reactions are reported at 20 °C ➔

$$k = k(20)\theta^{T-20}$$

Some typical values of θ used in water quality modeling (Chapra, 1997)

θ	Reaction
1.024	Oxygen reaeration
1.047	BOD decomposition
1.066	Phytoplankton growth
1.08	Sediment oxygen demand (SOD)



The effect of temperature on reaction rate for various values of θ.

Example 14: A laboratory provides you with the following results for a reaction: $T_1 = 4$ °C, $k_1 = 0.12$ day^{-1}; $T_2 = 16$ °C, $k_2 = 0.20$ day^{-1}. (a) Calculate θ for this reaction; (b) Determine the rate constant at 20 °C.

Solution:

(A):

$$\frac{k(T_2)}{k(T_1)}=\theta^{T_2-T_1} \rightarrow \log_{10}\frac{k(T_2)}{k(T_1)} = (T_2 - T_1)\log_{10}\theta$$

$$\rightarrow \theta = 10^{[\log_{10}k(T_2)-\log_{10}k(T_1)]/(T_2-T_1)}$$

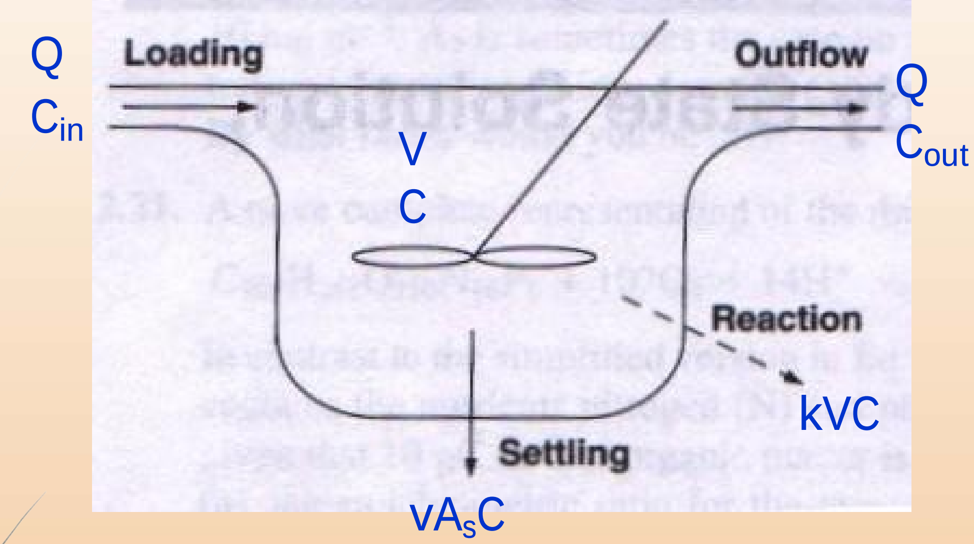
$$= 10^{[\log_{10}0.20-\log_{10}0.12]/(16-4)}=1.0435$$

(B):

$$K(16)= k(20)\theta^{16-20}$$

$$\rightarrow k(20) = \frac{k(16)}{\theta^{-4}} = \frac{0.20}{1.0435^{-4}}=0.237\ d^{-1}$$

Mass balance for the well-mixed lake



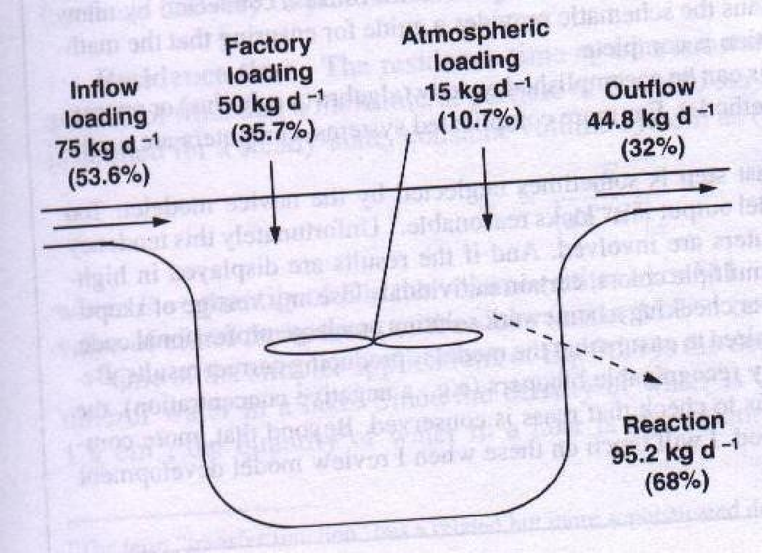
Accumulation rate = Loading rate – outflow rate – reaction rate – settling rate

➔ $$V\frac{dC}{dt} = W(t) - QC - kVC - vA_sC$$

- **Steady-State Solutions:** $$C = \frac{W}{Q + kV + vA_s}$$

$$\text{Assimilation Factor } a = \frac{W}{c} = Q + kV + vA_s$$

Example 15: A lake has the following characteristics: volume =50,000 m^3; mean depth = 2 m; inflow = outflow = 7,500 m^3/day; temperature = 25 °C. The lake receives the input of a pollutant from three sources: a factory discharge of 50 kg/day, a flux from the atmosphere of 0.6 g m^2 d^{-1}, and the inflow stream that has a concentration of 10 mg/L. If the pollutant decays at the rate of 0.25 day^{-1} at 20 °C ($\theta = 1.05$). (a) Compute the assimilation factor; (b) Determine the steady-state concentration; (c) Calculate the mass per time for each term in the mass balance equation.



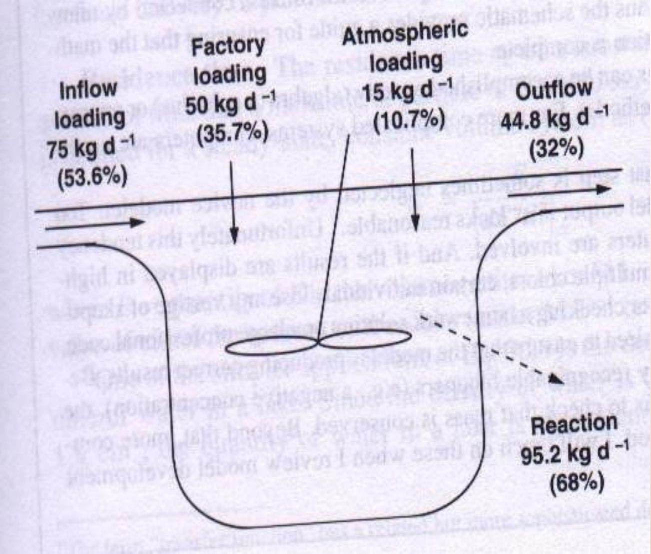
Solution:

(a):

$$k = k(20)\theta^{T-20} = 0.25 \times 1.05^{25-20} = 0.319\ d^{-1}$$

When not considering sedimentation:

$$a = \frac{W}{c} = Q + kV = 7500(m^3/d) + (0.319\ d^{-1})(50000\ m^3) = 23454\ (m^3/d)$$



(b): $A_s = \frac{V}{H} = \frac{50000\ m^3}{2\ m} = 25000\ m^2$

$$\begin{aligned} W_{atmosphere} &= JA_s \\ &= (0.6\ g\ m^{-2} d^{-1})(25000\ m^2) \\ &= 15000\ g/d \end{aligned}$$

$$\begin{aligned} W_{inflow} &= QC_{in} \\ &= (7500\ m^3/d)(10\ g/m^3) \\ &= 75000\ g/d \end{aligned}$$

➔ Total load

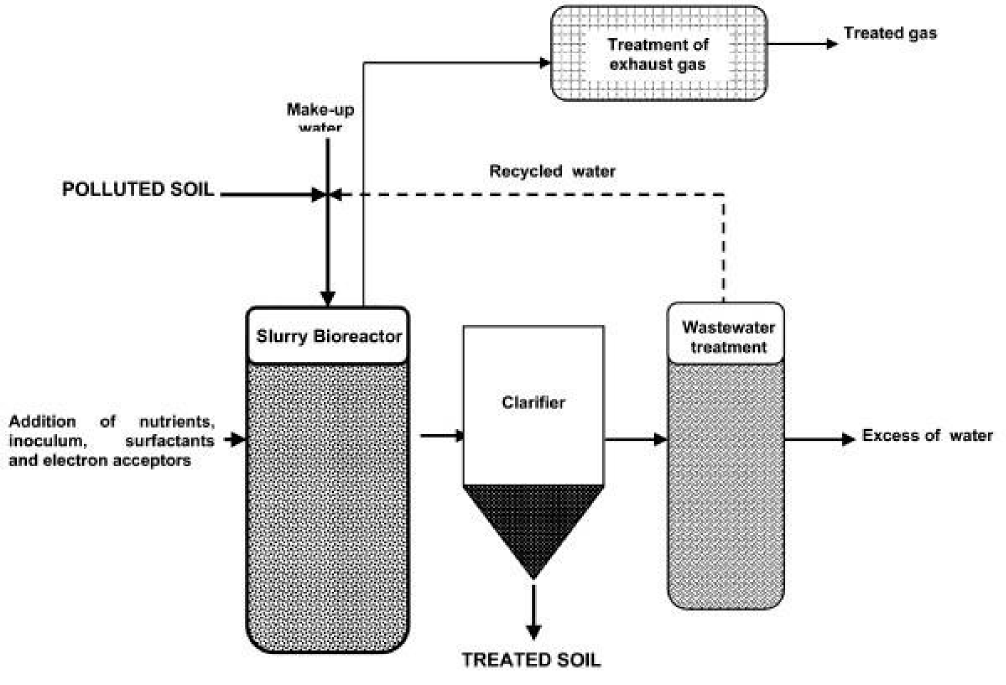
$$\begin{aligned} W &= W_{atmosphere} + W_{inflow} + W_{factory} \\ &= 15000 + 75000 + 50000 = 140000\ g/d \end{aligned}$$

➔ $C = \frac{W}{a} = \frac{140000\ g/d}{23454\ m^3/d} = 5.97\ g/m^3 = 5.97\ mg/L$

(c): Mass loss due to outflow = QC = $(7500\ m^3/d)(5.97\ g/m^3)$ $= 44769\ g/d$

Mass loss due to reaction = kVC = $(0.319\ d^{-1})(50000\ m^3)(5.97\ g/m^3) = 95231\ g/d$

Example 16: A soil reactor is used to treat soils contaminated with 1200 mg/kg of TPH. It is necessary to treat the slurry at 30 gal/min. The required final soil TPH concentration is 50 mg/kg. From a bench-scale study, the first-order reaction rate constant is 0.05/min. The contents in the reactor are fully mixed. Assume that the reactor behaves as a CSTR. Size the CSTR for this project.



Flow diagram of a typical slurry bioreactor installation. Clarifier is optional (source: Robles-Gonzalez et al., 2008. A review on slurry bioreactors for bioremediation of soils and sediments. *Microbial Cell Factories* 7:5, doi:10.1186/1475-2859-7-5

Solution:

$$C = \frac{W}{Q + kV} = \frac{QC_{in}}{Q + kV}$$

➔ $$\frac{C}{C_{in}} = \frac{Q}{Q+kV}$$

➔ $$\frac{50\ mg/kg}{1200\ mg/kg} = \frac{30\ gal/min}{30\frac{gal}{min} + (0.05\ min^{-1})V}$$

➔ V = 13800 gal

Transfer Functions and Residence Time

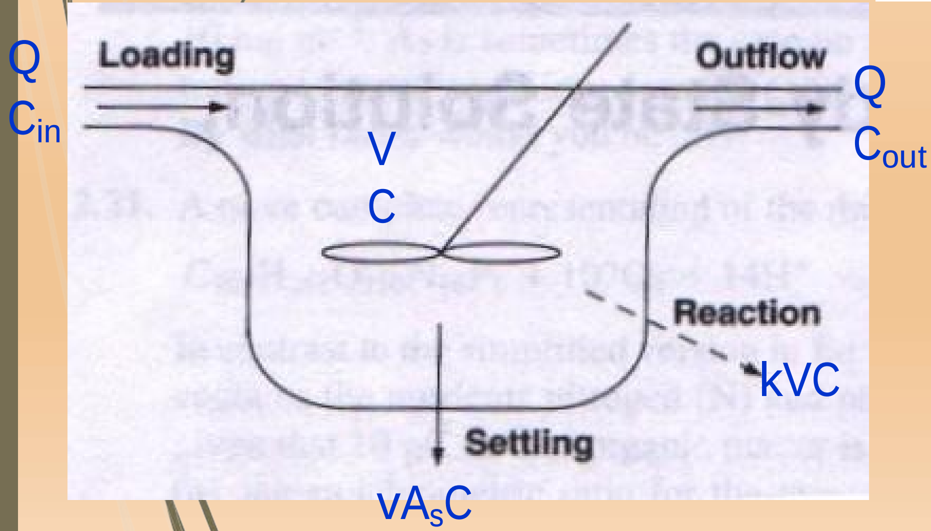
- aside from the assimilation factor, there are a variety of other ways to summarize the ability of a steady-state system to assimilate pollutants

A. Transfer Function (β) ➔ specify how the system input is transferred to an output

$$\boxed{\beta = \frac{C}{C_{in}}}$$

- ➢ For steady state:

$$C = \frac{W}{Q + kV + vA_s} = \frac{QC_{in}}{Q + kV + vA_s}$$



➔ $$\boxed{\beta = \frac{C}{C_{in}} = \frac{\square Q}{Q + kV + vA_s}}$$

- ➢ β (Dimensionless)

B. Residence Time

- the residence time (τ_E) of a substance E represents the mean amount of time that a molecule or particle of E would stay or “reside” in a system

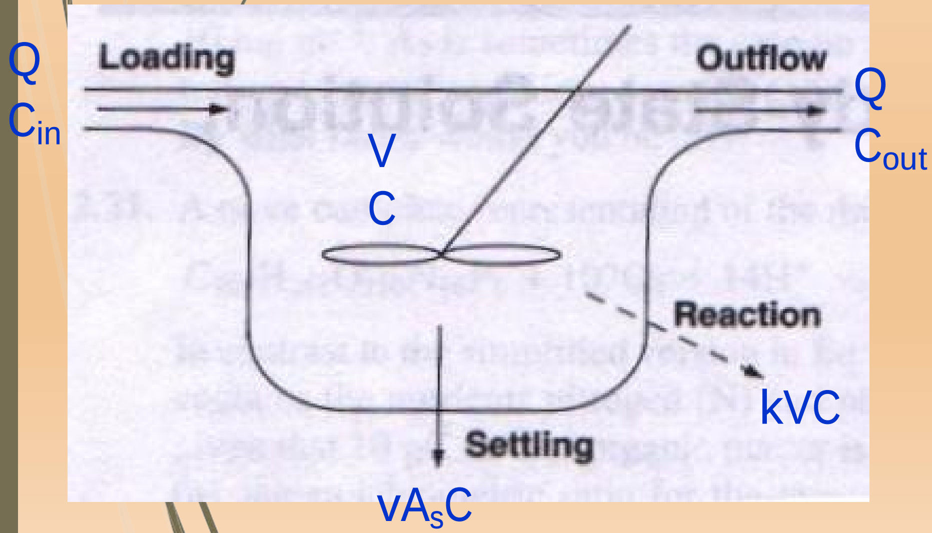
$$\tau_E = \frac{E}{\left|dE/dt\right|_{\pm}}$$

where E = quantity of E in the volume (either M or ML^{-3}), $|dE/dt|_{\pm}$ = absolute value of either the sources or the sinks (either MT^{-1} or $ML^{-3}T^{-1}$)

- ***water residence time***

$$E = \rho_w V$$

$$\textit{sink of water} = \frac{dE}{dt} = \rho_w Q$$

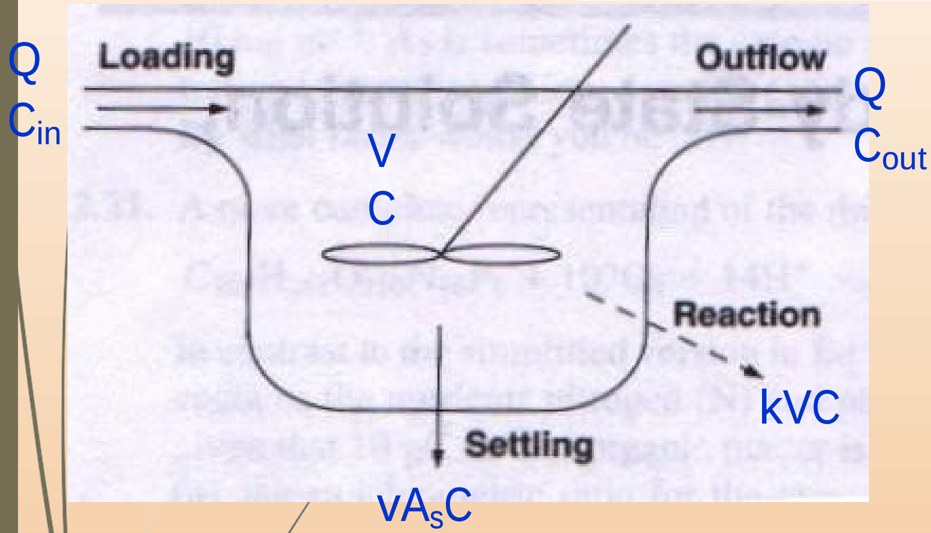


$$\rightarrow \quad \tau_w = \frac{E}{dE/dt} = \frac{\rho_w V}{\rho_w Q} = \frac{V}{Q}$$

→ the amount of time that would be required for the outflow to replace the quantity of water in the lake → measure of ***flushing rate***

Pollutant residence time

$$\text{sink of pollutant} = \frac{dM}{dt} = QC + kVC + vA_sC$$



$$\tau_C = \frac{M}{dM/dt} = \frac{VC}{QC+kVC+vA_sC}$$

$$= \frac{V}{Q+kV+vA_s}$$

Example 17: For the lake in example 15, determine (a) inflow concentration (lump all loadings together), (b) transfer function, (c) water residence time, and (d) pollutant residence time.

Solution:

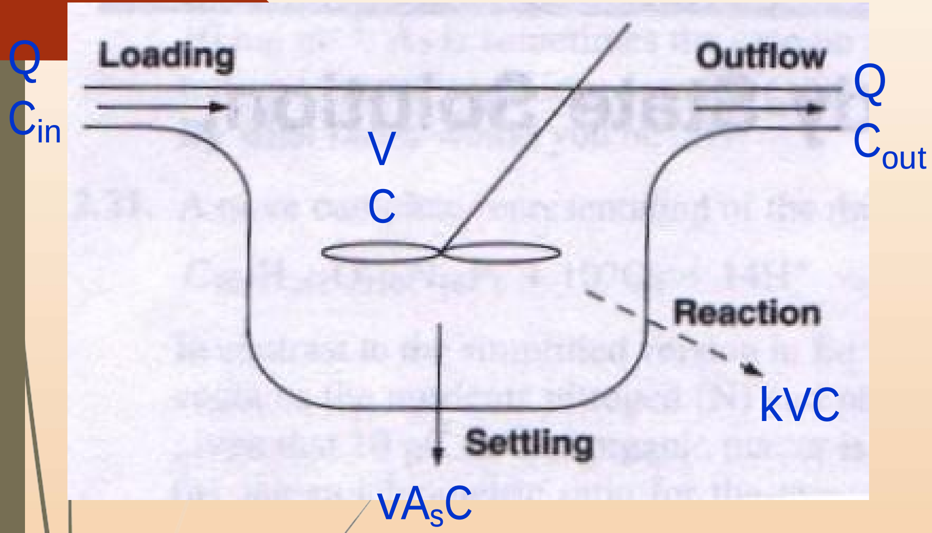
$$(a)\ \ C_{in} = \frac{W}{Q} = \frac{140000\ g/d}{7500\ m^3/d} = 18.67\ \text{g/m}^3 = 18.67\ \text{mg/L}$$

$$(b)\ \ \beta = \frac{C}{C_{in}} = \frac{Q}{Q+kV} = \frac{7500\ m^3/d}{7500\frac{m^3}{d} + (0.319\ d^{-1})(50000\ m^3)} = 0.32$$

$$(c)\ \ \tau_w = \frac{V}{Q} = \frac{50000\ m^3}{7500\ m^3/d} = 6.67\ d$$

$$(d)\ \ \tau_C = \frac{V}{Q+kV} = \frac{50000\ m^3}{7500\frac{m^3}{d} + (0.319\ d^{-1})(50000\ m^3)} = 2.13\ \text{d}$$

Mass balance for the well-mixed lake



$$V\frac{dC}{dt} = W(t) - QC - kVC - vA_sC$$

- Non-steady State Solutions:

$$\frac{dC}{dt}=\frac{W(t)}{V}-\frac{Q}{V}C-kC-\frac{vA_s}{V}C$$
$$=\frac{W(t)}{V}-\left(\frac{Q}{V}+k+\frac{v}{H}\right)C$$

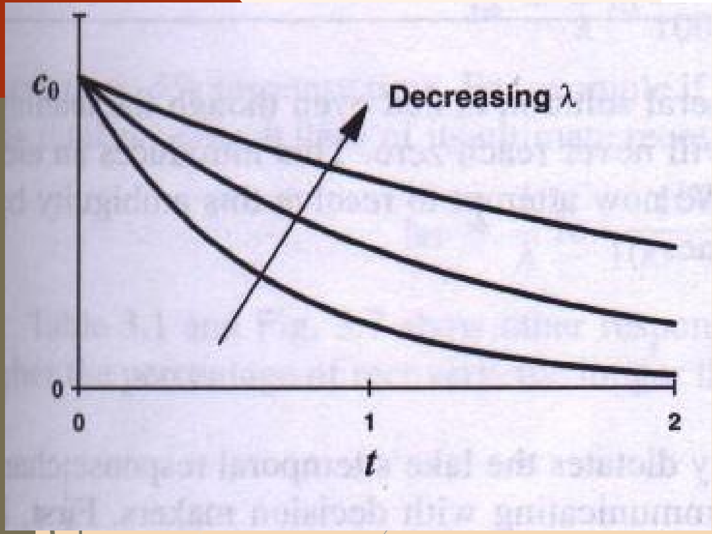
Let $\lambda = \frac{Q}{V} + k + \frac{v}{H}$ → $\frac{dC}{dt} + \lambda C = \frac{W(t)}{V}$

- where λ = eigenvalue (characteristic value)
- $C = C_g + C_p$

  where C_g = general solution for the case W(t) = 0, C_p = particular solution for specific forms of W(t)

- **General Solution** $C = C_0 e^{-\lambda t}$

Where $C = C_0$ at $t = 0$



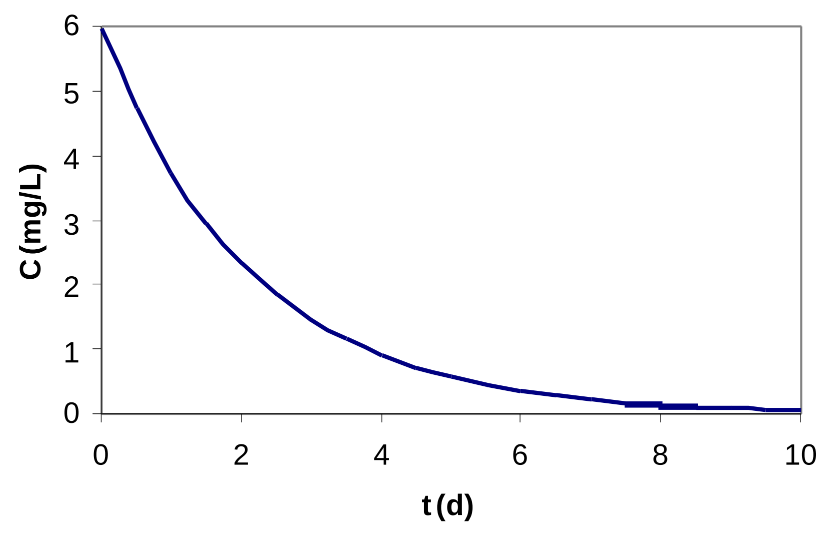
Example 18: In example 15 we determined the steady-state concentration for a lake having the following characteristics: volume = 50,000 m^3, temperature = 25 °C, mean depth = 2 m, waste loading = 140,000 g/day, inflow = outflow = 7,500 m^3/day, decay rate = 0.319 day^{-1}. If the initial concentration is equal to the steady-state level (5.97 mg/L), determine the general solution.

Solution:

$$\lambda = \frac{Q}{V} + k = \frac{7500\ m^3/d}{50000\ m^3} + 0.319\ d^{-1}$$

$$= 0.469\ d^{-1}$$

→ $C = C_0 e^{-\lambda t} = 5.97 e^{-0.469t}$



Response Time

- the time it takes for the lake to complete a fixed percentage of its recovery ➔ to decide “how much” of the recovery is judged as being “enough”

$$C = C_0 e^{-\lambda t}$$

Let $C = \frac{100-\Phi}{100} C_0$ (i.e. Φ% of recovery)

➔ $$\frac{100-\Phi}{100} C_0 = C_0 e^{-\lambda t_\Phi}$$

➔ $$t_\phi = \frac{1}{\lambda}\ln\left(\frac{100}{100-\phi}\right)$$

i.e. 50% response time (t_{50}): $$t_{50} = \frac{1}{\lambda}\ln\left(\frac{\square 100}{100-50}\right) = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Example 19: Determine the 75%, 90%, 95%, and 99% response times for the lake in example 18.

Solution:

$$(a)\ t_{75} = \frac{1}{\lambda}\ln\left(\frac{100}{100-\Phi}\right) = \frac{1}{\lambda}\ln\left(\frac{100}{100-75}\right) = \frac{ln4}{\lambda} = \frac{1.39}{\lambda}$$

$$(b)\ t_{90} = \frac{1}{\lambda}\ln\left(\frac{100}{100-\Phi}\right) = \frac{1}{\lambda}\ln\left(\frac{100}{100-90}\right) = \frac{ln10}{\lambda} = \frac{2.3}{\lambda}$$

$$(c)\ t_{95} = \frac{1}{\lambda}\ln\left(\frac{100}{100-\Phi}\right) = \frac{1}{\lambda}\ln\left(\frac{100}{100-95}\right) = \frac{ln20}{\lambda} = \frac{3.0}{\lambda}$$

$$(d)\ t_{99} = \frac{1}{\lambda}\ln\left(\frac{100}{100-\Phi}\right) = \frac{1}{\lambda}\ln\left(\frac{100}{100-99}\right) = \frac{ln100}{\lambda} = \frac{4.6}{\lambda}$$

$$(a)\ t_{75} = \frac{1.39}{\lambda} = \frac{1.39}{0.469\ d^{-1}} = 2.96\text{ d}$$

$$(c)\ t_{95} = \frac{3.0}{\lambda} = \frac{3.0}{0.469\ d^{-1}} = 6.4\text{ d}$$

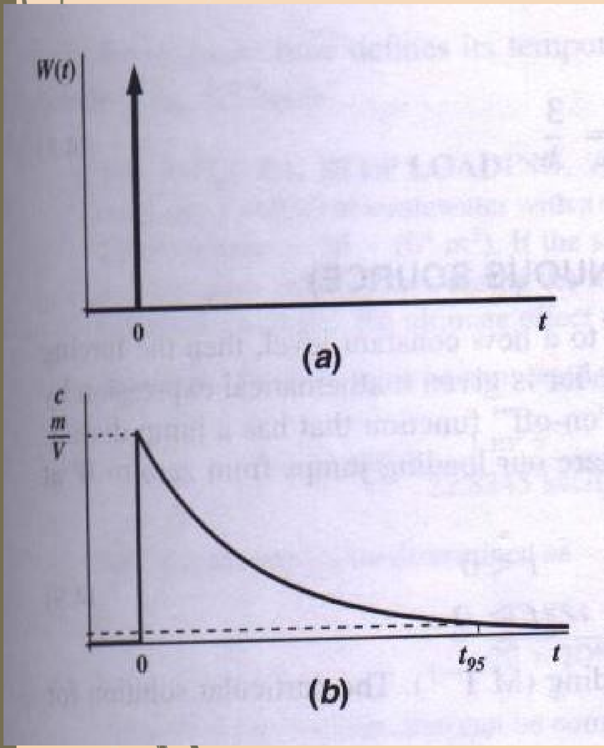
$$(b)\ t_{90} = \frac{2.3}{\lambda} = \frac{2.3}{0.469\ d^{-1}} = 4.9\text{ d}$$

$$(d)\ t_{99} = \frac{4.6}{\lambda} = \frac{4.6}{0.469\ d^{-1}} = 9.8\text{ d}$$

- ## Particular Solution for Specific Forms of Loading Functions

$$\frac{dC}{dt} + \lambda C = \frac{W(t)}{V}$$

A. Impulse Loading (Spill)



- Impulse loading ➔ represent the discharge of waste over a relatively short time period ➔ described by Dirac delta function (i.e. impulse function) δ(t)

$$\delta(t) = 0 \qquad t \neq 0$$ and $$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

➔ W(t) = mδ(t)

Where m = quantity of pollutant mass discharged during a spill [M]

$$\frac{dC}{dt} + \lambda C = \frac{m\delta(t)}{V}$$

Particular solution:

$$C = \frac{m}{V} e^{-\lambda t}$$

B. Step Loading (i.e. New Continuous Source)

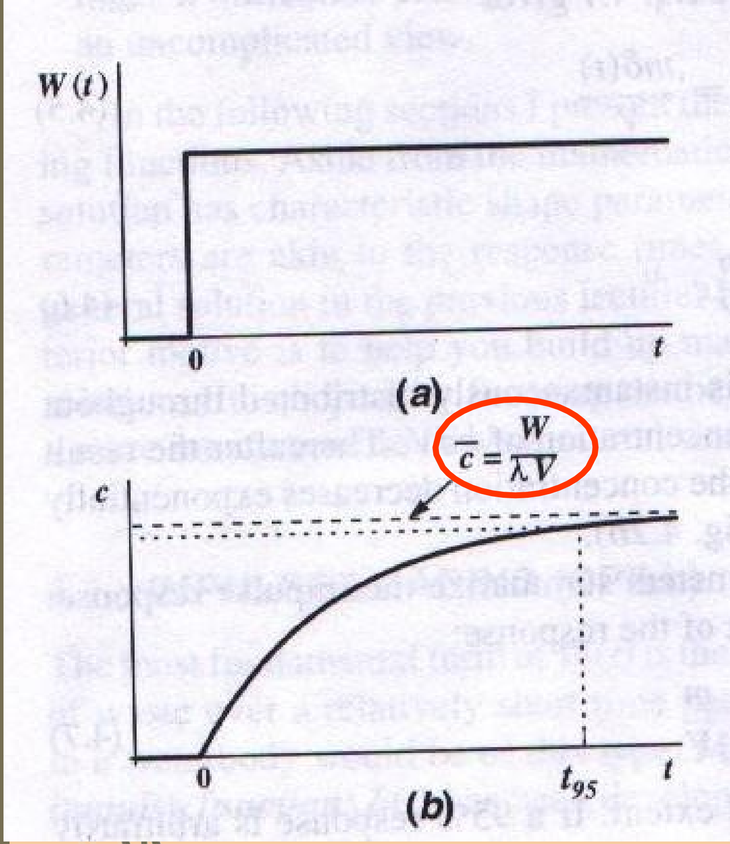
$W(t) = 0 \quad t < 0$

$W(t) = W \quad t \geq 0$ where W = the new constant level of loading $[MT^{-1}]$

$$\frac{dC}{dt} + \lambda C = \frac{W}{V}$$

Particular Solution:

$$C = \frac{W}{\lambda V}(1 - e^{-\lambda t})$$



Source: http://gizmodo.com/5844925/chicagos-...place-on-earth

Example 20: At time zero, a sewage treatment plant began to discharge 37,854 m^3/day of wastewater with a concentration of 200 mg/L to a small detention pond (volume = $20\ 10^4\ m^3$). If the sewage decays at a rate of 0.1 day^{-1}, compute the concentration in the pond during the first 2 weeks of operation (assume $C_0 = 0$ mg/L).

Solution:

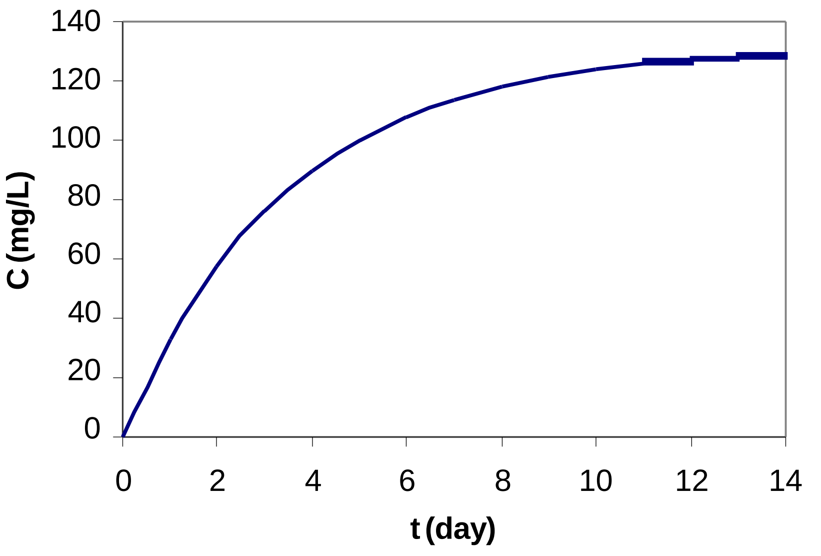
$$W = QC_{in} = (37854\ m^3/d)(200\ g/m^3) = 7.57\times10^6\ g/d$$

$$\lambda = \frac{Q}{V} + k = \frac{37854\ m^3/d}{20\times10^4\ m^3} + 0.1\ d^{-1}$$

$$C = C_g + C_p = C_0 e^{-\lambda t} + \frac{W}{\lambda V}(1 - e^{-\lambda t})$$

$$= 0 + \frac{7.57\times10^6\ g/d}{(0.2893 d^{-1})(20\times10^4\ m^3)}(1 - e^{-0.2893t})$$

$$= 131\times(1 - e^{-0.2893t})$$



$$t_{95} = \frac{3.0}{\lambda} = \frac{3.0}{0.2893\ d^{-1}} = 10.4\ d$$

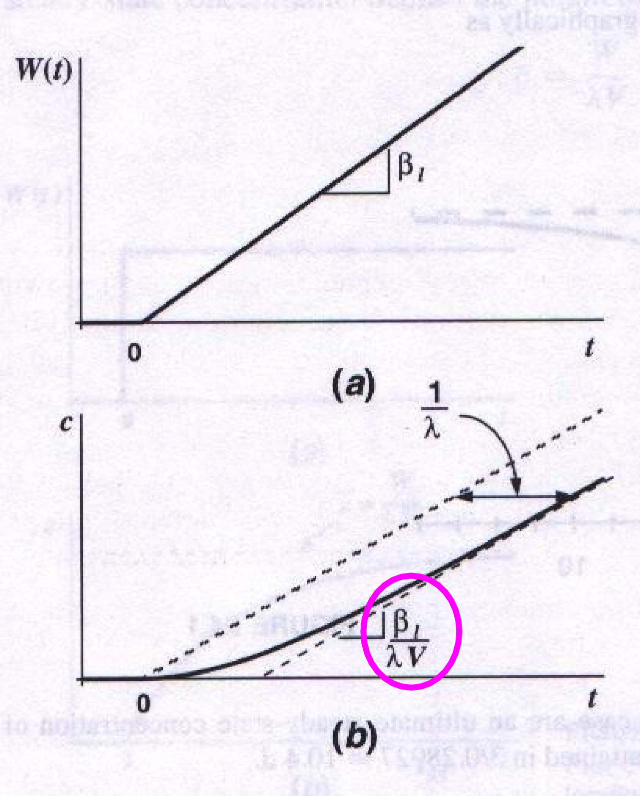
C. Linear Loading (i.e. Ramp loading)

- waste inputs represented by: **W(t) = ±β_lt**
  Where β_l = rate of change [MT^{-2}]

$$\frac{dC}{dt} + \lambda C = \frac{W(t)}{V}$$

- **Particular solution:**

$$C = \pm \frac{\beta_l}{\lambda^2 V}(\lambda t - 1 + e^{-\lambda t})$$



- for positively increasing case, after an initial start-up time (i.e. t_{95}), the particular solution becomes:

$$C = \frac{\beta_l}{\lambda V}\left(t - \frac{1}{\lambda}\right)$$

D. Exponential Loading

Waste loading represented by: $W(t) = W_e e^{\pm \beta_e t}$

where W_e = parameter denoting the value at t = 0 [MT^{-1}], β_e = rate of growth (positive) or decay (negative) of the loading [T^{-1}]

$$\frac{dC}{dt} + \lambda C = \frac{W(t)}{V}$$

-- **Particular Solution:**

$$C = \frac{W_e}{V(\lambda \pm \beta_e)}\left(e^{\pm \beta_e t} - e^{-\lambda t}\right)$$

- Decaying exponential loading is more common:

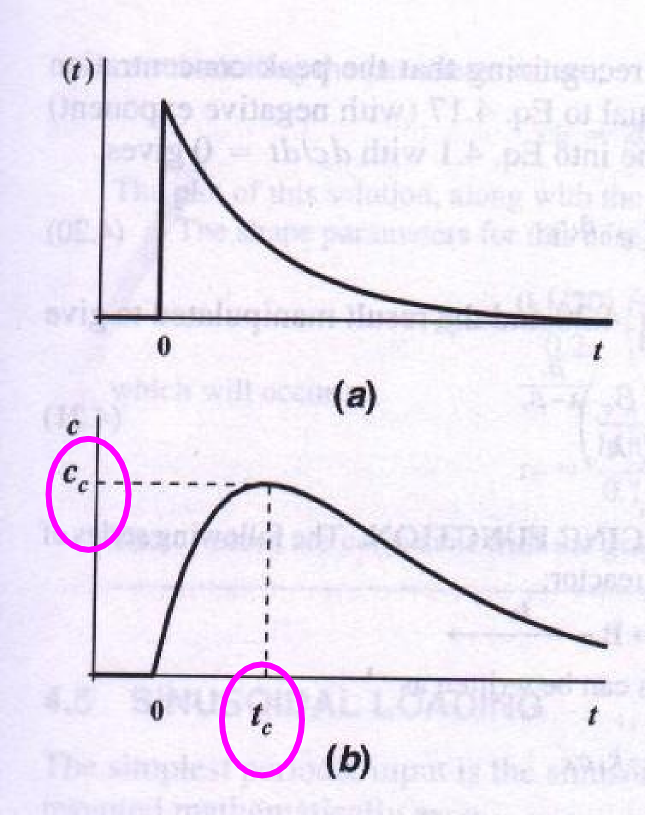
→ $$C = \frac{W_e}{V(\lambda - \beta_e)}\left(e^{-\beta_e t} - e^{-\lambda t}\right)$$

- shape parameters:

→ $$\frac{dd}{dd} = \frac{W_e}{V(?-?_e)}\left[-\beta_e e^{-\beta_e d} + \lambda e^{-\lambda d}\right] = 0$$

→ $\beta_e e^{-\beta_e d_c} = \lambda e^{-\lambda dc}$ → $\ln(\beta_e e^{-\beta_e d_c}) = \ln(\lambda e^{-\lambda dc})$

→ $$t_c = \frac{\ln(\beta_e / \lambda)}{\beta_e - \lambda}$$ and $$C_c = \frac{W_e}{\lambda V} e^{-\beta_e t_c}$$



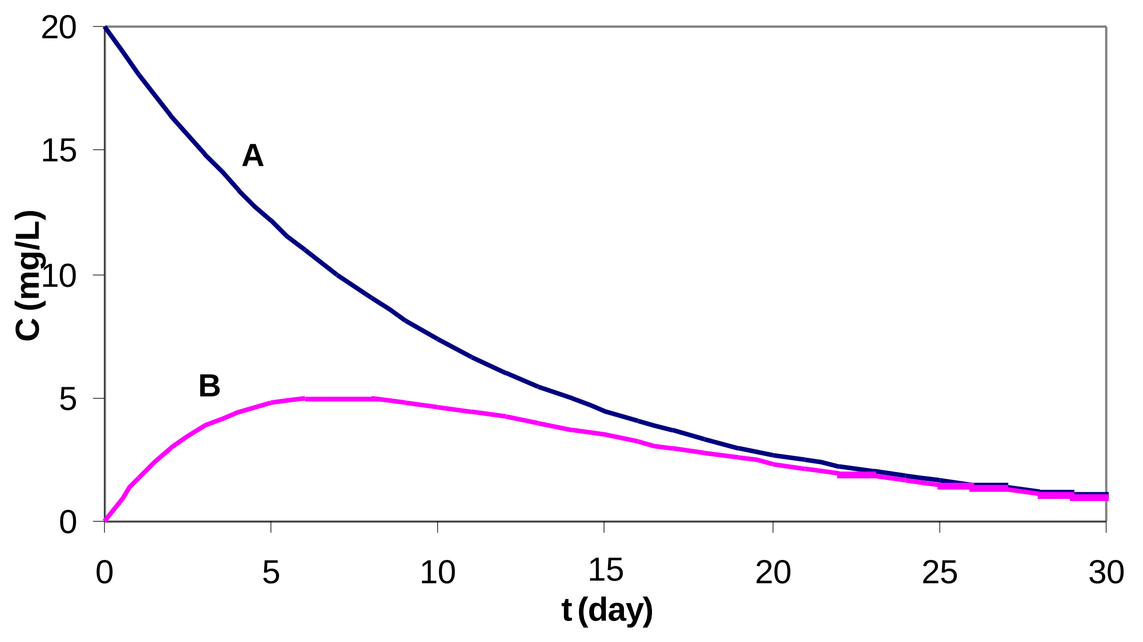
Example 21: The following series of first-order reactions takes place in a batch reactor:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

These reactions can be written as: $\frac{dC_A}{dt} = -k_1 C_A$ and $\frac{dC_B}{dt} = k_1 C_A - k_2 C_B$

suppose that an experiment is conducted where C_{A0}= 20 and C_{B0} = 0 mg/L. If k_1 = 0.1 and k_2 = 0.2 day^{-1}, compute the concentration of reactant B as a function of time. Calculate its shape parameters C_c and t_c.

Solution:



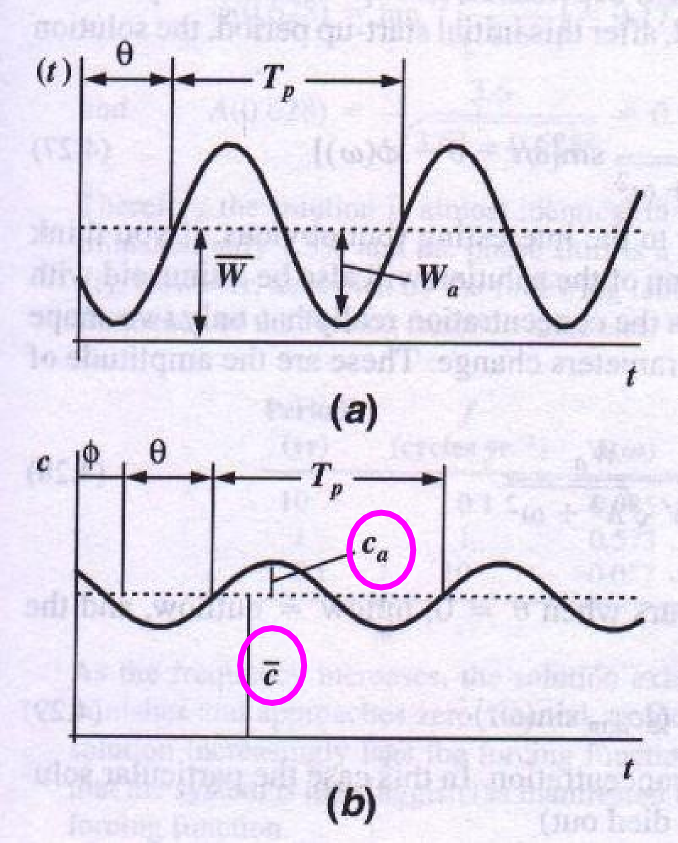
E. Sinusoidal Loading

periodic input, represented by: $W(t) = \overline{W} + W_a \sin(\omega t - \theta)$

where $\overline{W}$ = mean loading [MT^{-1}], W_a = amplitude of the loading [MT^{-1}], θ = phase shift (radians), ω = angular frequency of the oscillation (radians T^{-1}),

$\omega = \frac{2\pi}{T_p}$ where T_p = period of the oscillation [T]

$$\frac{dC}{dt} + \lambda C = \frac{W(t)}{V}$$



-- Particular Solution:

$$C = \frac{\overline{W}}{\lambda V}(1 - e^{-\lambda t}) + \frac{W_a}{V\sqrt{\lambda^2+\omega^2}}\sin[\omega t - \theta - \phi(\omega)] - \frac{W_a}{V\sqrt{\lambda^2+\omega^2}}\sin[-\theta - \phi(\omega)]e^{-\lambda t}$$

where: $\phi(\omega) = \tan^{-1}\left(\frac{\omega}{\lambda}\right)$

- after an initial start-up period:

$$C = \frac{\overline{W}}{\lambda V} + \frac{W_a}{V\sqrt{\lambda^2+\omega^2}}\sin[\omega t - \theta - \phi(\omega)]$$

$$C_a = \frac{W_a}{V\sqrt{\lambda^2+\omega^2}}$$

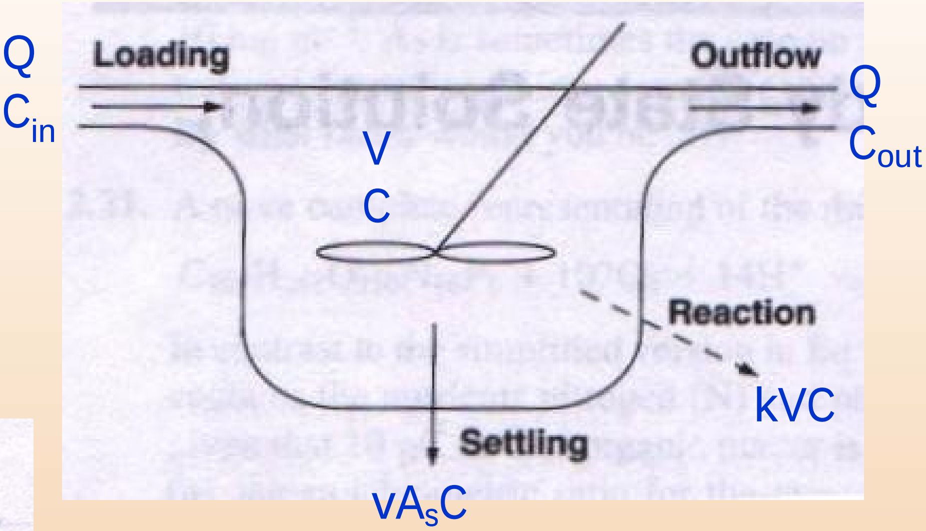
Mean

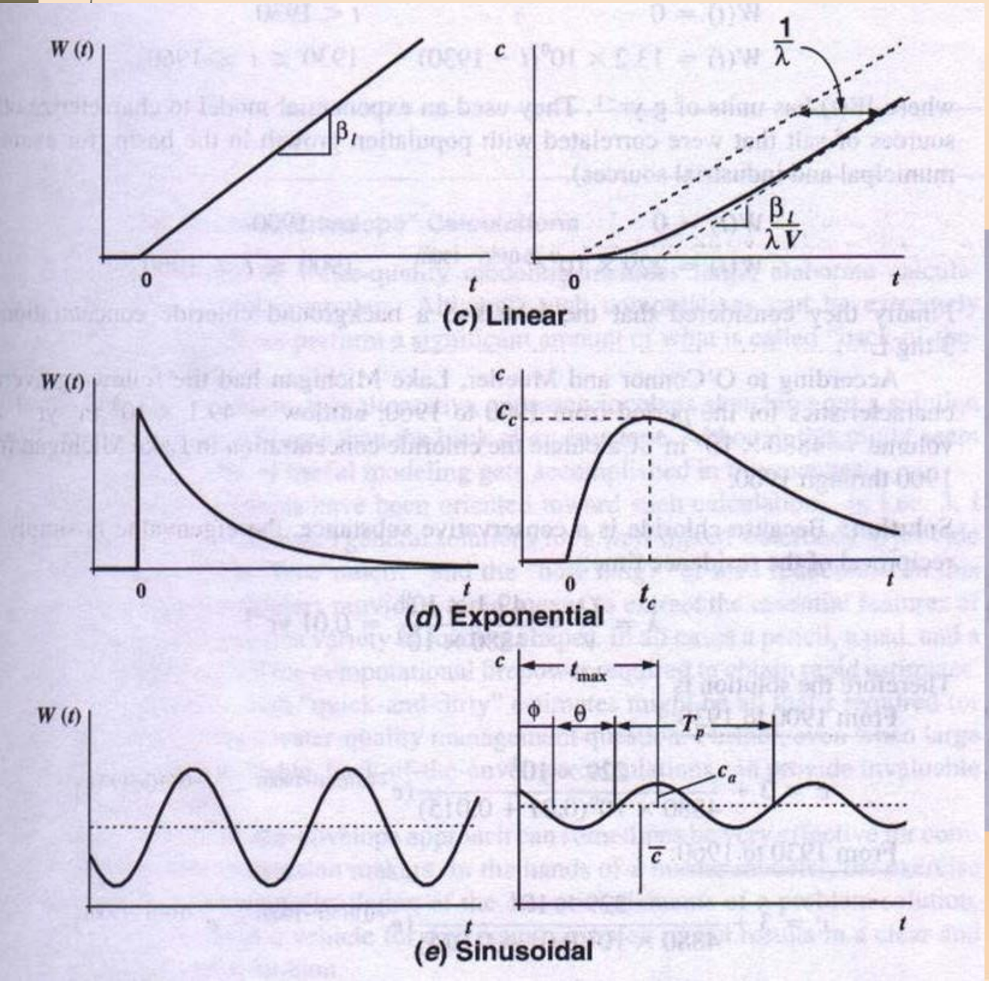
Amplitude

F. Summary of Particular Solutions

$$\frac{dC}{dt} + \lambda C = \frac{W(t)}{V}$$

- $C = C_g + C_p$

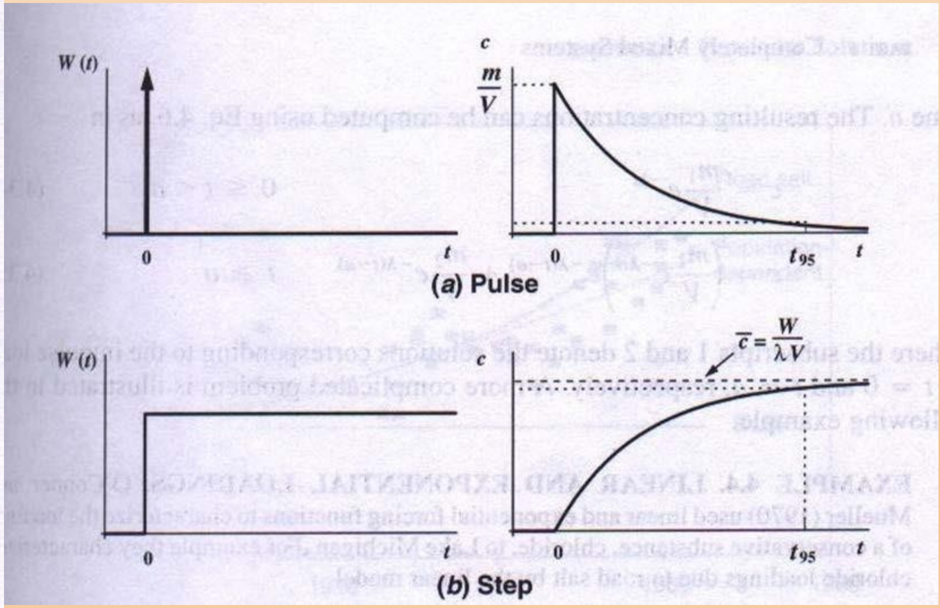




(c) Linear

(d) Exponential

(e) Sinusoidal



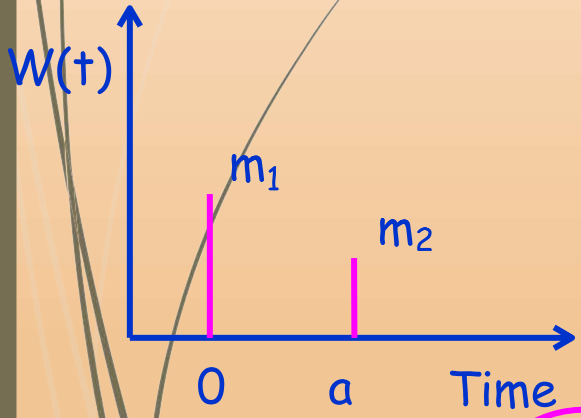
(a) Pulse

(b) Step

- **Total Solution**

$$\frac{dC}{dt} + \lambda C = \frac{W(t)}{V}$$ → $$C = C_g + \sum_{i=1}^{n} a_i C_{pi}$$

- can add loadings that are initiated at different times by merely shifting the solutions.
  - ✓ i.e. Suppose that we have two impulse loads, one that occurs at time zero and another at time a, the resulting concentrations can be calculated:



→

$$C = \frac{m_1}{V} e^{-\lambda d} \quad f f f\ 0 \leq t < a$$

$$C = \left(\frac{m_1}{V} e^{-\lambda a}\right) e^{-\lambda(d-a)} + \frac{m_2}{V} e^{-\lambda(d-a)} \quad f f f\ t \geq a$$

Example 22: O’Connor and Mueller (1970) used linear and exponential forcing functions to characterize the loadings of a conservative substance, chloride, to Lake Michigan. For example they characterized chloride loadings due to road salt by the linear model:

$$W(t) = 0 \qquad t < 1930$$

$$W(t) = 13.2 \,\square\, 10^{9}\,(t-1930) \qquad 1930 \,\square\, t \,\square\, 1960$$

Where W(t) has units of g/yr. They used an exponential model to characterize other sources of salt that were correlated with population growth in the basin (i.e. municipal and industrial sources):

$$W(t) = 0 \qquad t < 1900$$

$$W(t) = 229 \,\square\, 10^{9}\, e^{0.015(t-1900)} \qquad 1900 \,\square\, t \,\square\, 1960$$

Finally they considered that the lake had a background chloride concentration of 3 mg/L. According to O’Connor and Mueller, Lake Michigan had the following average characteristics for the period from 1900 to 1960: $Q = 49.1 \,\square\, 10^{9}\ m^{3}/yr$ and $V = 4880 \,\square\, 10^{9}\ m^{3}$. Calculate the chloride concentration in Lake Michigan from 1900 through 1960.

Solution:

$$□ = \frac{Q}{V} + k = \frac{Q}{V} + 0 = \frac{49.1\times10^9\,m^3/y}{4880\times10^9\,m^3} = 0.01\ y^{-1}$$

- From 1900 to 1930: $W(t) = 229$ □ $10^9\ e^{0.015(t-1900)}$

  □ exponential growth loading ($W_e = 229\times10^9$; □$_e = 0.015$)

□ $$C = 3 + \frac{229\times10^9}{4880\times10^9(0.01+0.015)}\left[e^{0.015(d-1900)} - e^{-0.01(d-1900)}\right]\ mm/L$$

$W(t) = 229$ □ $10^9\ e^{0.015(t-1900)}$ W

- From 1930 to 1960: $(t) = 13.2$ □ $10^9\ (t-1930)$

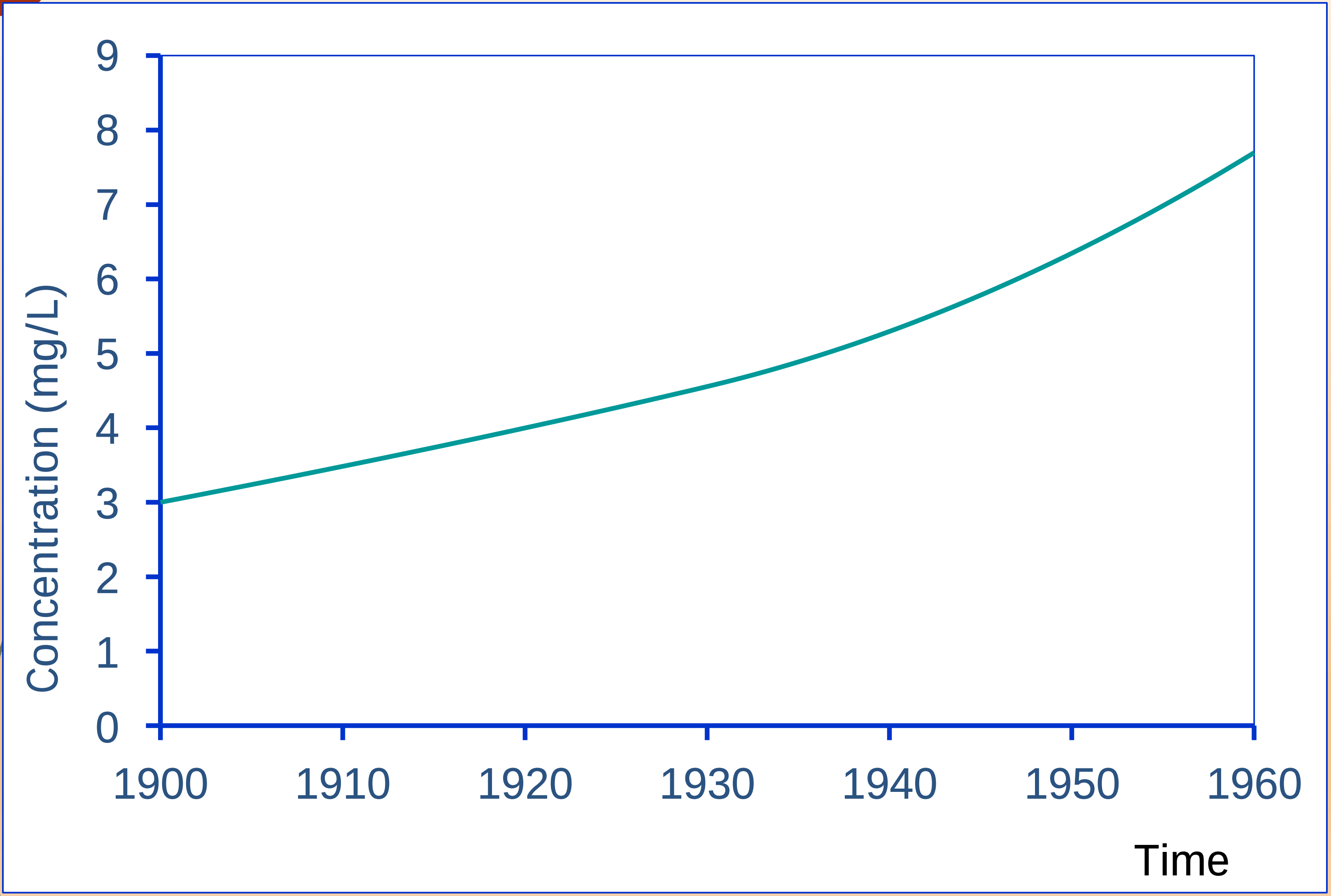
  □ exponential growth loading ($W_e = 229\times10^9$; □$_e = 0.015$)
  + linear loading (□$_L = 13.2\times10^9$)

□ $$C = 3 + \frac{229\times10^9}{4880\times10^9(0.01+0.015)}\left[e^{0.015(d-1900)} - e^{-0.01(d-1900)}\right] + \frac{13.2\times10^9}{(0.01)^2\times4880\times10^9}\left[(0.01(t-1930)-1+e^{-0.01(d-1930)}\right]$$

$$=3+1.88\left[e^{0.015(d-1900)} - e^{-0.01(d-1900)}\right]+27.05\left[0.01(t-1930)-1+e^{-0.01(d-1930)}\right]$$

Initial conditions are not included explicitly □ they are reflected in the constant background level of 3 mg/L

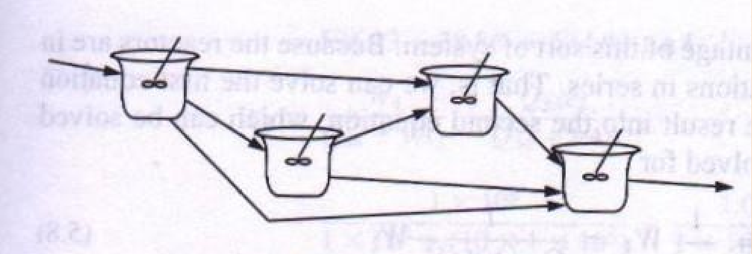
Solution:



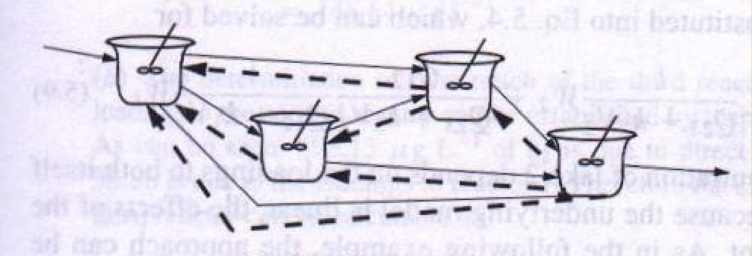
Concentration (mg/L)
0
1
2
3
4
5
6
7
8
9
1900
1910
1920
1930
1940
1950
1960
Time

Feedforward System of Reactors (e.g. a series of lakes connected by short rivers)

- model more complicated systems by combining reactors
- two general ways in which completely mixed reactors can be connected: **feedforward** and **feedback**



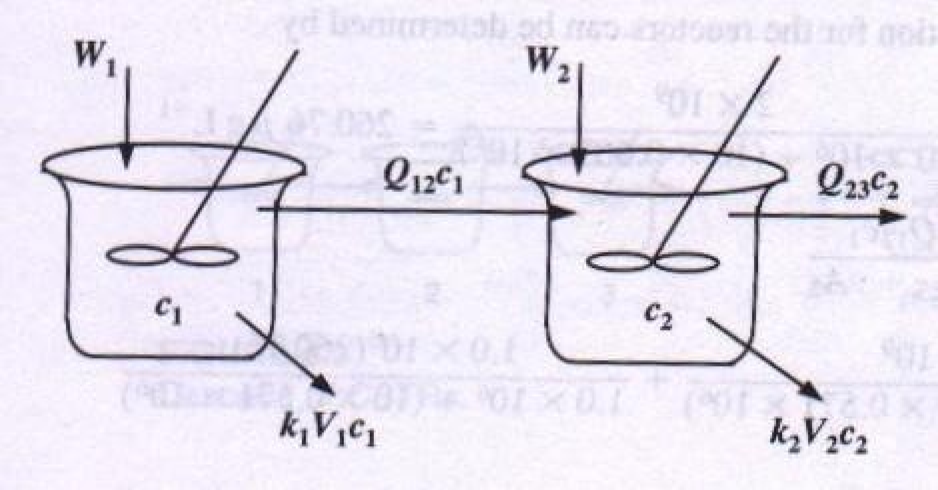
(*a*) Feedforward reactors



(*b*) Feedback reactors

Source: http://www.abettersewer.com/wp-content/uploads/2012/04/Lincoln-NE-Wastewater-Treatment-Plant.jpg

A. Two lakes in series



- Mass balance equations for reactors:

$$V_1 \frac{dd_1}{dd} = W_1 - Q_{12}C_1 - k_1V_1C_1$$

$$V_2 \frac{dd_2}{dd} = W_2 + Q_{12}C_1 - Q_{23}C_2 - k_2V_2C_2$$

➢ **Steady-state solutions:**

C_1

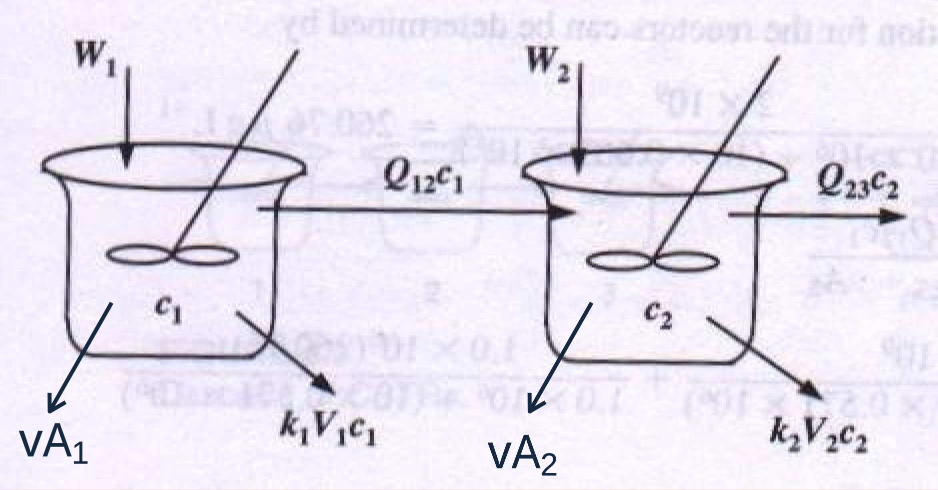
$$C_1 \square \frac{1}{Q_{12} \square k_1V_1} W_1$$

$$C_2 \quad \frac{1}{Q_{23} \square k_2V_2} W_2 \square \left(\frac{Q_{12}}{Q_{23} \square k_2V_2}\right)\left(\frac{1}{Q_{12} \square k_1V_1}\right) W_1$$

- Considering settling and reaction:

$$C_1 = \frac{\square 1}{Q_{12} + k_1V_1 + vA_1} W_1$$

$$C_2 = \frac{\square 1}{Q_{23} + k_2V_2 + vA_2} W_2 + \left(\frac{Q_{12}}{Q_{23} + k_2V_2 + vA_2}\right) C_1$$

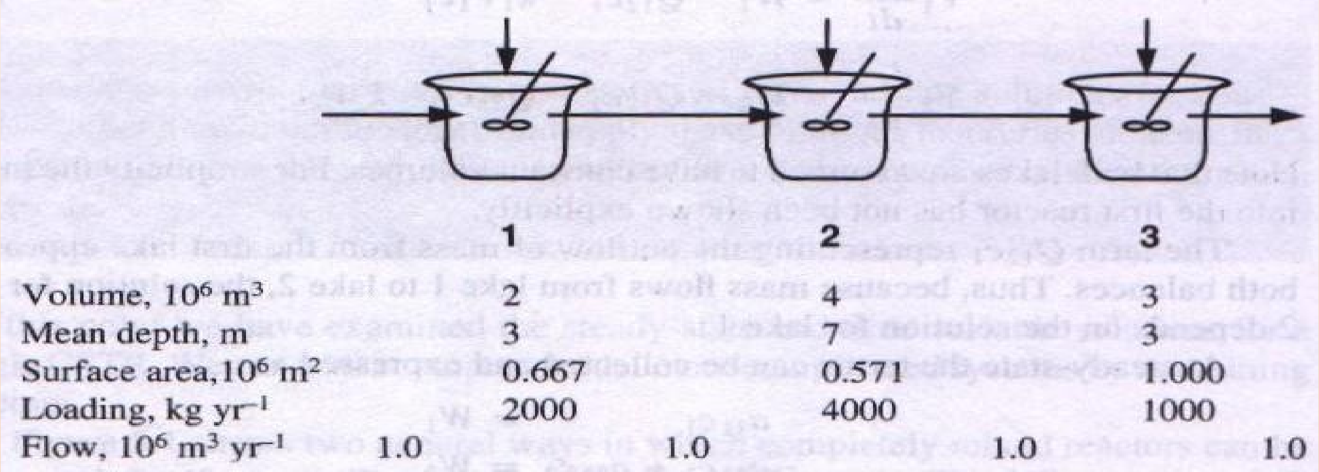


- Considering settling:

$$C_1 = \frac{\square 1}{Q_{12} + vA_1} W_1$$

$$C_2 = \frac{\square 1}{Q_{23} + vA_2} W_2 + \left(\frac{Q_{12}}{Q_{23} + vA_2}\right) C_1$$

Example 23: Suppose that three lakes connected in series have the following characteristics



		1		2		3	
Volume, 10^6 m^3		2		4		3	
Mean depth, m		3		7		3	
Surface area, 10^6 m^2		0.667		0.571		1.000	
Loading, kg yr^{-1}		2000		4000		1000	
Flow, 10^6 m^3 yr^{-1}	1.0		1.0		1.0		1.0

If the pollutant settles at a rate of 10 m/yr, (a) calculate the steady- state concentration in each of the lakes; (b) determine how much of the concentration in the third lake is due to the loading to the second lake.

Solution:

$$(a)\ C_1 = \frac{W_1}{Q_{12}+vv_1} = \frac{2\times10^6\ g/y}{1.0\times10^6\ m^3/y+(10\frac{m}{}\times0.667\times10^6\ m^2)} = 0.26\ m/m^3$$

$$C_2 = \frac{W_2}{Q_{23}+vv_2} + \frac{Q_{12}}{Q_{23}+vv_2} C_1$$

$$= \frac{4\times10^6}{1.0\times10^6+(10\times0.571\times10^6)} + \frac{1.0\times10^6}{1.0\times10^6+10\times0.571\times10^6} \times 0.26$$

$$= 0.60 + 0.03 = 0.63\ m/m^3$$

$$C_3 = \frac{W_3}{Q_{34}+vv_3} + \frac{Q_{23}}{Q_{34}+vv_3} C_2$$

$$= \frac{1\times10^6}{1.0\times10^6+(10\times1\times10^6)} + \frac{1.0\times10^6}{1.0\times10^6+10\times1\times10^6} \times 0.63 = 0.15\ m/m^3$$

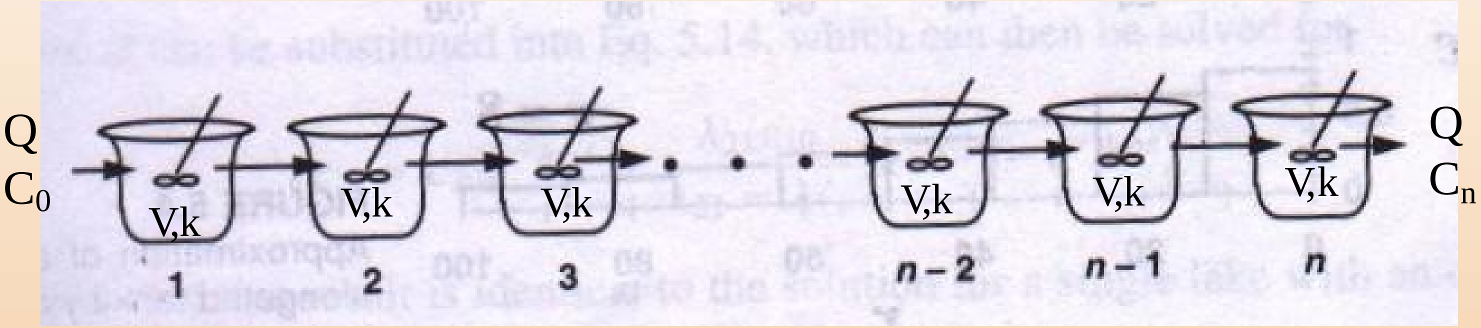
(b) 0.60 mg/L of C_2 is due to direct loading W_2

C_3 due to loading W_2 to reactor2

$$= \frac{1.0\times10^6}{1.0\times10^6+10\times1\times10^6} \times 0.60 = 0.05\ m/m^3$$

B. Cascade Model

- reactors in series have the same size and flow ⟶ **cascade** of CSTRs



- No loading:

$$C_1 = \frac{QC_0}{Q + kV}$$

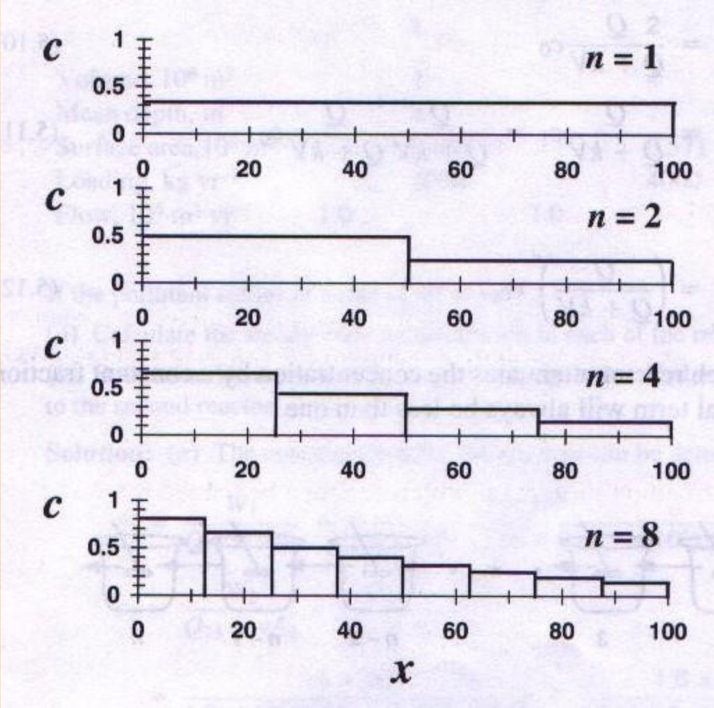
$$C_2 = \left(\frac{Q}{Q+k}\right) C_1 = \left(\frac{Q}{Q+k}\right)^2 C_0$$

$$C_n = \left(\frac{Q}{Q + kV}\right)^n C_0$$

Example 24: Use the cascade model to simulate the steady-state distribution of concentration in an elongated tank. The tank has a cross-sectional area $A_c = 10\ m^2$, length $L = 100$ m, velocity $v = 100$ m/hr, and first-order reaction rate $k = 2\ hr^{-1}$. The inflow concentration is 1 mg/L. Use n = 1, 2, 4, and 8 CSTRs to approximate the tank. Plot the results.

Solution:

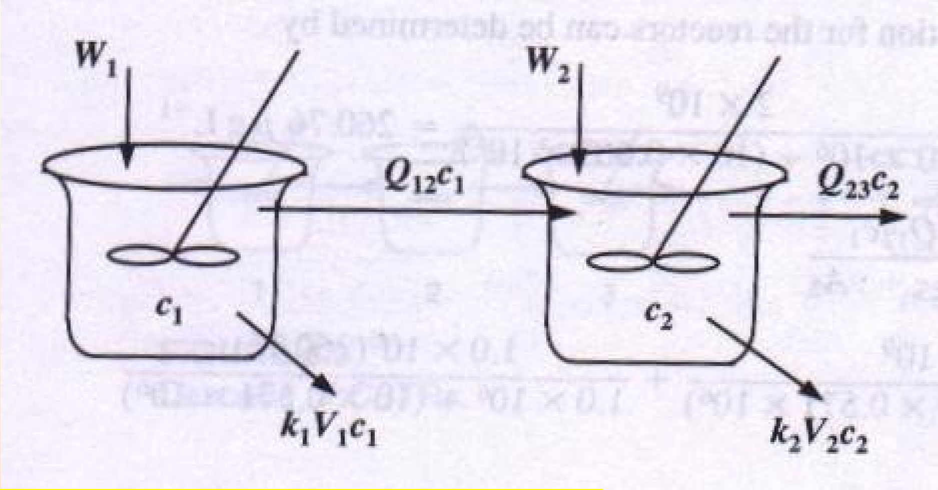
$Q = vA_c = 100\ m/h \times 10\ m^2 = 1000\ m^3/h$ Total
volume $= 100\ m \times 10\ m^2 = 1000\ m^3$



Example 25: Subsurface soil at a site is contaminated with diesel fuel at a concentration of 1800 mg/kg. Above-ground remediation, using slurry bioreactors, is proposed. The treatment system is required to handle a sluryy flow rate of 0.04 m^3/min. The required final diesel concentration in the soil is 100 mg/kg. The reaction is first-order with a rate constant of 0.1/min, as determined from a bench-scale study. Four different configurations of slurry bioreactors in the CSTR mode are considered. Determine the final effluent concentration from each of these arrangements an if it meets the cleanup requirement:

(a) One 4-m^3 reactor
(b) Two 2-m^3 reactors in series
(c) One 1-m^3 reactor followed by one 3-m^3 reactor
(d) One 3-m^3 reactor followed by one 1-m^3 reactor

C. Non-steady State Solution for Two Lakes in Series



- Mass balances:

$$V_1\frac{dC_1}{dt} = W_1 - Q_{12}C_1 - k_1V_1C_1 \qquad V_2\frac{dC_2}{dt} = W_2 + Q_{12}C_1 - Q_{23}C_2 - k_2V_2C_2$$

- ✓ Zero loadings situation ($W_1 = W_2 = 0$):

$$\frac{dC_1}{dt} = -\frac{Q_{12}}{V_1}C_1 - k_1C_1 = -\left(\boxed{\frac{Q_{12}}{V_1} + k_1}\right)C_1 \qquad \frac{dC_2}{dt} = \boxed{\frac{Q_{12}}{V_2}}C_1 - \left(\boxed{\frac{Q_{23}}{V_2} + k_2}\right)C_2$$

Let $\lambda_{11} = \frac{Q_{12}}{V_1} + k_1$ $\qquad \lambda_{21} = \frac{Q_{12}}{V_2}$ $\qquad \lambda_{22} = \frac{Q_{23}}{V_2} + k_2$

Then:

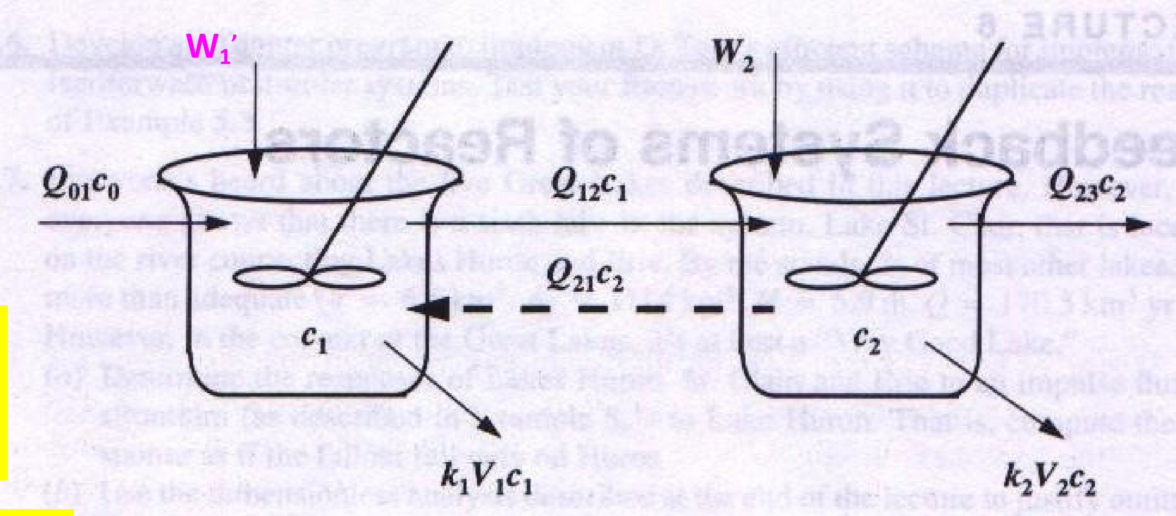
$$\frac{dC_1}{dt} = -\lambda_{11}C_1$$

$$\frac{dC_2}{dt} = \lambda_{21}C_1 - \lambda_{22}C_2$$

Initial conditions: $C_1 = C_{10}$, $C_2 = C_{20}$ at $t = 0$ ➔

$$C_1 = C_{10}e^{-\lambda_{11}t} \qquad C_2 = C_{20}e^{-\lambda_{22}t} + \frac{\lambda_{21}C_{10}}{\lambda_{22} - \lambda_{11}}\left(e^{-\lambda_{11}t} - e^{-\lambda_{22}t}\right)$$

3. Feedback System Reactors



- Mass balances:

$$V_1\frac{dC_1}{dt} = W_1' + Q_{01}C_0 + Q_{21}C_2 - Q_{12}C_1 - k_1V_1C_1$$

$$= W_1 + Q_{21}C_2 - Q_{12}C_1 - k_1V_1C_1$$

$$V_2\frac{dC_2}{dt} = W_2 + Q_{12}C_1 - Q_{23}C_2 - Q_{21}C_2 - k_2V_2C_2$$

- ✓ Steady State situation:

$$a_{11}C_1 + a_{12}C_2 = W_1$$

$$a_{21}C_1 + a_{22}C_2 = W_2$$

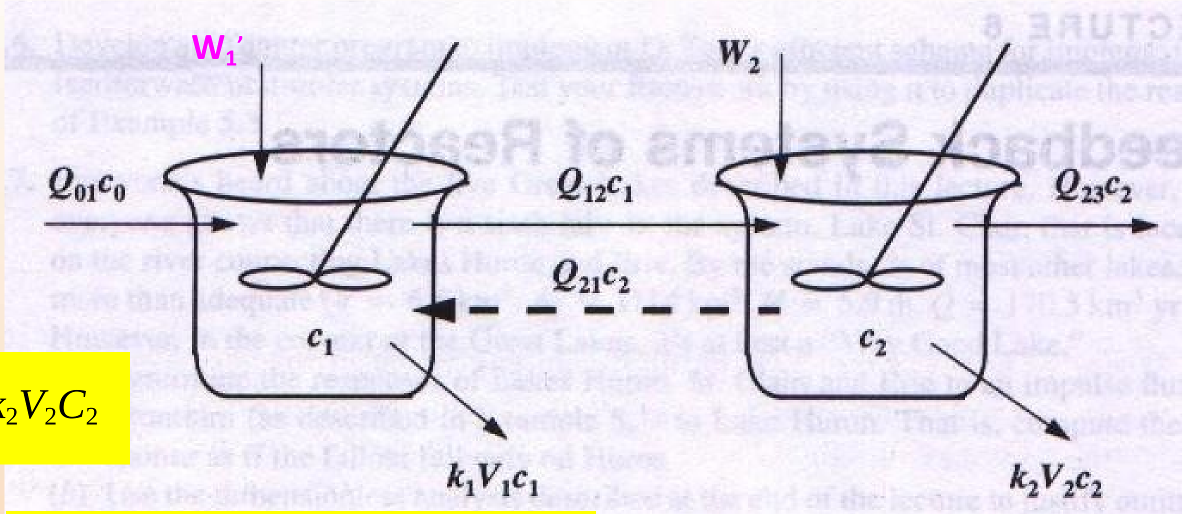
where $a_{11} = Q_{12} + k_1V_1$ $a_{12} = -Q_{21}$
$a_{21} = -Q_{12}$ $a_{22} = Q_{21} + Q_{23} + k_2V_2$

➔ Steady State Solution:

$$C_1 = \frac{1}{a_{11} - (a_{21}a_{12}/a_{22})}W_1 + \frac{1}{a_{21} - (a_{11}a_{22}/a_{12})}W_2$$

$$C_2 = \frac{1}{a_{12} - (a_{11}a_{22}/a_{21})}W_1 + \frac{1}{a_{22} - (a_{21}a_{12}/a_{11})}W_2$$

✓ Non-steady State situation, with no loading:



$$V_1\frac{dC_1}{dt} = W_1 + Q_{21}C_2 - Q_{12}C_1 - k_1V_1C_1$$

$$V_2\frac{dC_2}{dt} = W_2 + Q_{12}C_1 - Q_{23}C_2 - Q_{21}C_2 - k_2V_2C_2$$

➔ $$\frac{dC_1}{dt} = -\left(\frac{Q_{12}}{V_1} + k_1\right)C_1 + \frac{Q_{21}}{V_1}C_2$$

$$\frac{dC_2}{dt} = \frac{Q_{12}}{V_2}C_1 - \left(\frac{Q_{23}+Q_{21}}{V_2} + k_2\right)C_2$$

Let $\alpha_{11} = \frac{Q_{12}}{V_1} + k_1$ $\alpha_{12} = \frac{Q_{21}}{V_1}$ $\alpha_{21} = \frac{Q_{12}}{V_2}$ $\alpha_{22} = \frac{Q_{23}+Q_{12}}{V_2} + k_2$

Then:

$$\frac{dC_1}{dt} = -\alpha_{11}C_1 + \alpha_{12}C_2$$

$$\frac{dC_2}{dt} = \alpha_{21}C_1 - \alpha_{22}C_2$$

- initial condition: $C_1 = C_{10}$, $C_2 = C_{20}$ at $t = 0$
- **Non- steady State Solution:**

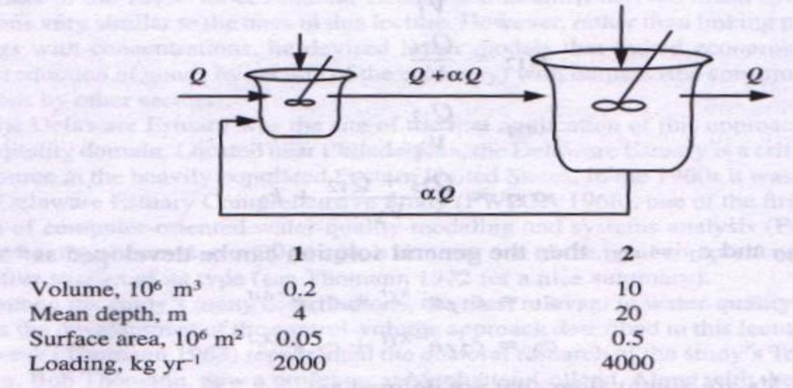
$$C_1 = C_{1f}e^{-\lambda_f t} + C_{1s}e^{-\lambda_s t} \qquad C_2 = C_{2f}e^{-\lambda_f t} + C_{2s}e^{-\lambda_s t}$$

Where:

$$\lambda_f = \frac{(\alpha_{11}+\alpha_{22}) + \sqrt{(\alpha_{11}+\alpha_{22})^2 - 4(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})}}{2} \qquad \lambda_s = \frac{(\alpha_{11}+\alpha_{22}) - \sqrt{(\alpha_{11}+\alpha_{22})^2 - 4(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})}}{2}$$

$$C_{1f} = \frac{(\lambda_f - \alpha_{22})C_{10} - \alpha_{12}C_{20}}{\lambda_f - \lambda_s} \qquad C_{1s} = \frac{\alpha_{12}C_{20} - (\lambda_s - \alpha_{22})C_{10}}{\lambda_f - \lambda_s} \qquad C_{2f} = \frac{-\alpha_{21}C_{10} + (\lambda_f - \alpha_{11})C_{20}}{\lambda_f - \lambda_s} \qquad C_{2s} = \frac{-(\lambda_s - \alpha_{11})C_{20} + \alpha_{21}C_{10}}{\lambda_f - \lambda_s}$$

Example 26: The following two lakes include feedback of a fraction of the flow (□) from the second lake back to the first lake:



	1	2
Volume, 10^6 m^3	0.2	10
Mean depth, m	4	20
Surface area, 10^6 m^2	0.05	0.5
Loading, kg yr^{-1}	2000	4000

Notice that for this case the second lake is much larger than the first. If Q = 1 □ 10^6 m^3/yr, □ = 0.5, and the pollutant settles at a rate of 10 m/yr, calculate the steady-state concentration in each of the lakes. Using the concentrations calculated in (a) as initial conditions, determine the response of each lake if their loadings are terminated at t =0.

Solution:

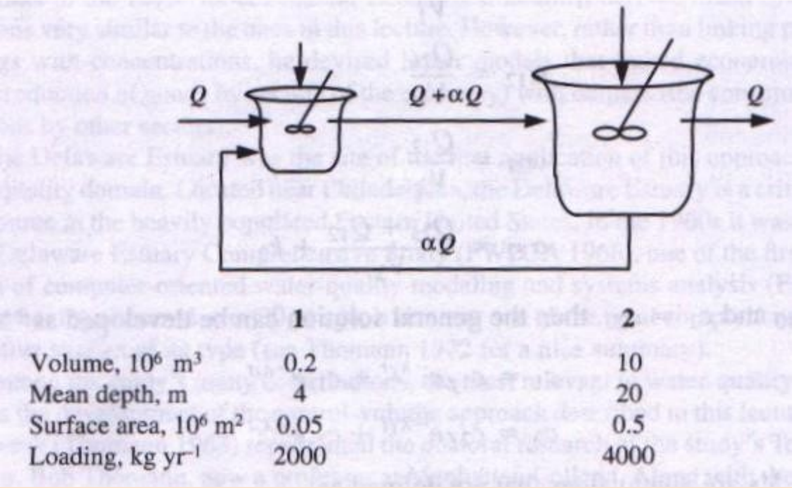
Q_{12} = (1+□)Q=(1+0.5)× 10^6 = 1.5 × $10^6 m^3/y$

Q_{21} = □Q=0.5 × 10^6 m^3/y Q_{23} = Q=1.0 × $10^6 m^3/y$

$Q_{12} = (1+\alpha)Q = (1+0.5)\times 10^6 = 1.5 \times 10^6 m^3/y$

$Q_{21} = \alpha Q = 0.5 \times 10^6 m^3/y$

$Q_{23} = Q = 1.0 \times 10^6 m^3/y$



	1	2
Volume, 10^6 m^3	0.2	10
Mean depth, m	4	20
Surface area, 10^6 m^2	0.05	0.5
Loading, kg yr^{-1}	2000	4000

$a_{11} = Q_{12} + vA_1 = 1.5 \times 10^6 + 10 \times 0.05 \times 10^6$
$= 2 \times 10^6$

$a_{12} = -Q_{21} = -0.5 \times 10^6$

$a_{21} = -Q_{12} = -1.5 \times 10^6$

$a_{22} = Q_{21} + Q_{23} + vA_2 = 0.5 \times 10^6 + 10^6$
$+ 10 \times 0.5 \times 10^6 = 6.5 \times 10^6$

$$C_1 = \frac{1}{a_{11} - (a_{21}a_{12}/a_{22})} W_1 + \frac{1}{a_{21} - (a_{11}a_{22}/a_{12})} W_2 = 1.224 \quad mg/L = 1224 \mu g/L$$

$$C_2 = \frac{1}{a_{12} - (a_{11}a_{22}/a_{21})} W_1 + \frac{1}{a_{22} - (a_{21}a_{12}/a_{11})} W_2 = 0.898 \quad mg/L = 898 \mu g/L$$

(b)

$$\alpha_{11} = \frac{Q_{12}}{V_1} + \frac{v}{H_1} = \frac{1.5\times10^6}{0.2\times10^6} + \frac{10}{4} = 10y^{-1}$$

$$\alpha_{12} = \frac{Q_{21}}{V_1} = \frac{0.5\times10^6}{0.2\times10^6} = 2.5y^{-1}$$

$$\alpha_{21} = \frac{Q_{12}}{V_2} = \frac{1.5\times10^6}{10\times10^6} = 0.15y^{-1}$$

$$\alpha_{22} = \frac{Q_{23}+Q_{12}}{V_2} + \frac{v}{H_2} = \frac{10^6+1.5\times10^6}{10\times10^6} + \frac{10}{20} = 0.75y^{-1}$$

$$\lambda_f = \frac{(\alpha_{11}+\alpha_{22}) + \sqrt{(\alpha_{11}+\alpha_{22})^2 - 4(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})}}{2} = 10.04\,y^{-1}$$

$$\lambda_s = \frac{(\alpha_{11}+\alpha_{22}) - \sqrt{(\alpha_{11}+\alpha_{22})^2 - 4(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})}}{2} = 0.71y^{-1}$$

$$C_{1f} = \frac{(\lambda_f - \alpha_{22})C_{10} - \alpha_{12}C_{20}}{\lambda_f - \lambda_s} = 978.13$$

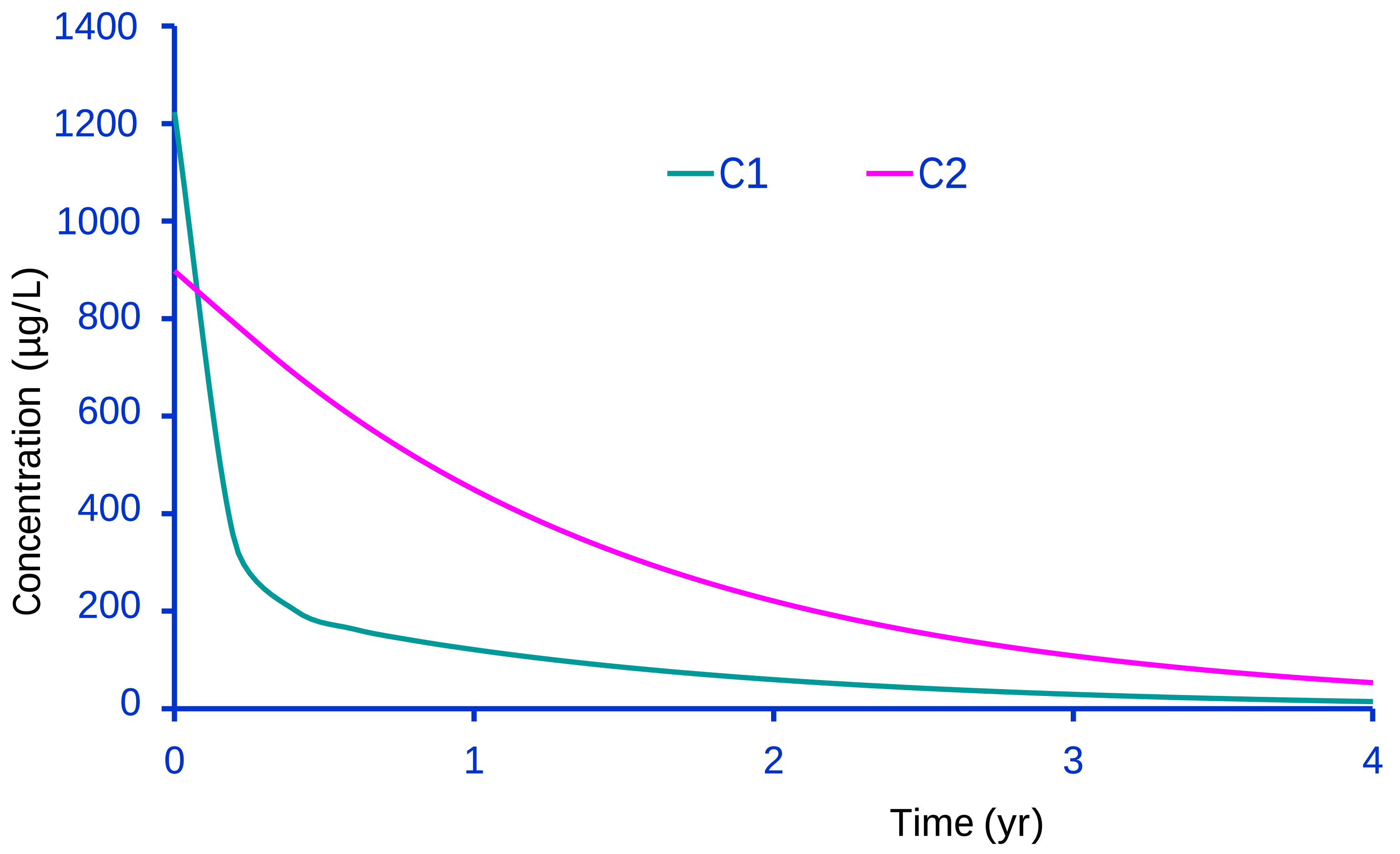
$$C_{1s} = \frac{\alpha_{12}C_{20} - (\lambda_s - \alpha_{22})C_{10}}{\lambda_f - \lambda_s} = 245.87$$

$$C_{2f} = \frac{-\alpha_{21}C_{10} + (\lambda_f - \alpha_{11})C_{20}}{\lambda_f - \lambda_s} = -15.83$$

$$C_{2s} = \frac{-(\lambda_s - \alpha_{11})C_{20} + \alpha_{21}C_{10}}{\lambda_f - \lambda_s} = 913.83$$

$$C_1 = 978.13e^{-10.04t} + 245.87e^{-0.71t}$$

$$C_2 = -15.83e^{-10.04t} + 913.83e^{-0.71t}$$

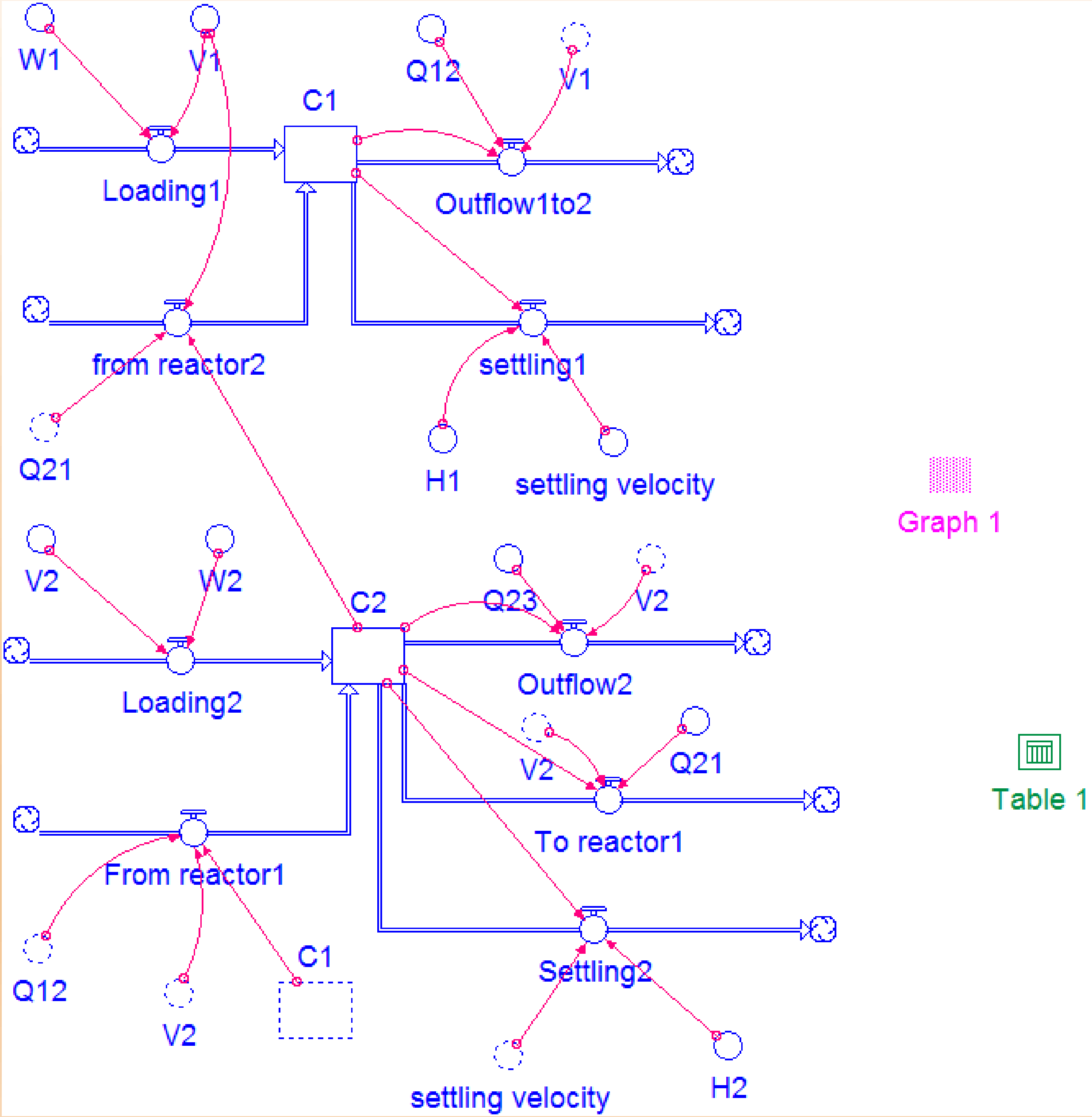


Concentration (µg/L)
1400
1200
1000
800
600
400
200
0
C1
C2
0
1
2
3
4
Time (yr)

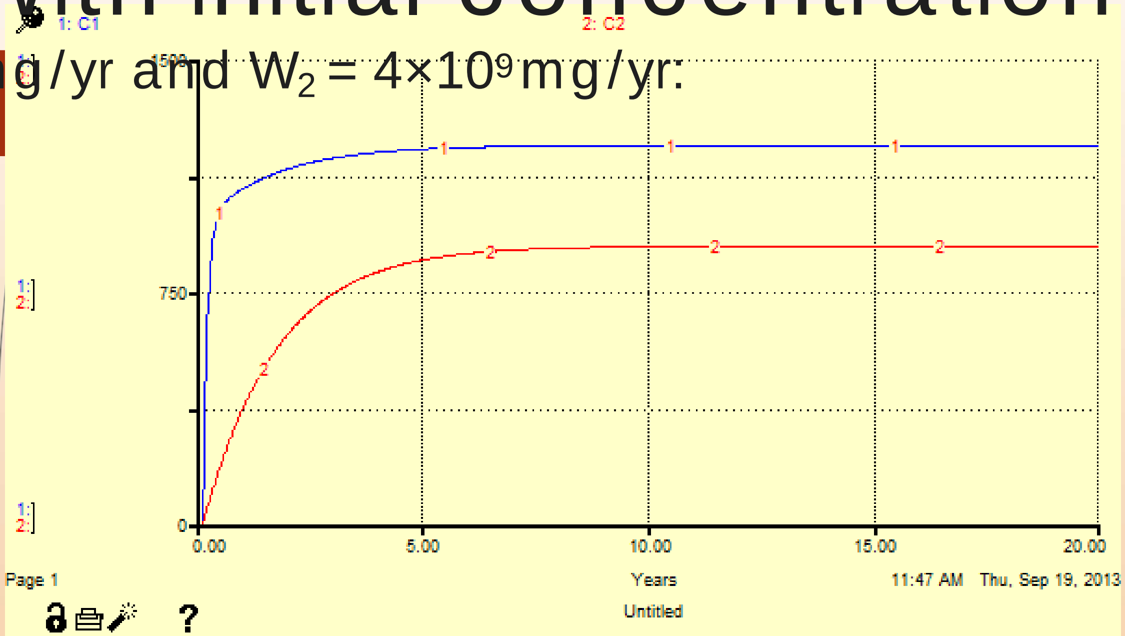
$$\frac{dC_1}{dt} = \frac{W_1}{V_1} + \frac{Q_{21}}{V_1} C_2 - \frac{Q_{12}}{V_1} C_1 - \frac{v}{H_1} C_1$$

$$\frac{dC_2}{dt} = \frac{W_2}{V_2} + \frac{Q_{12}}{V_2} C_1 - \frac{Q_{23}}{V_2} C_2 - \frac{Q_{21}}{V_2} C_2 - \frac{v}{H_2} C_2$$

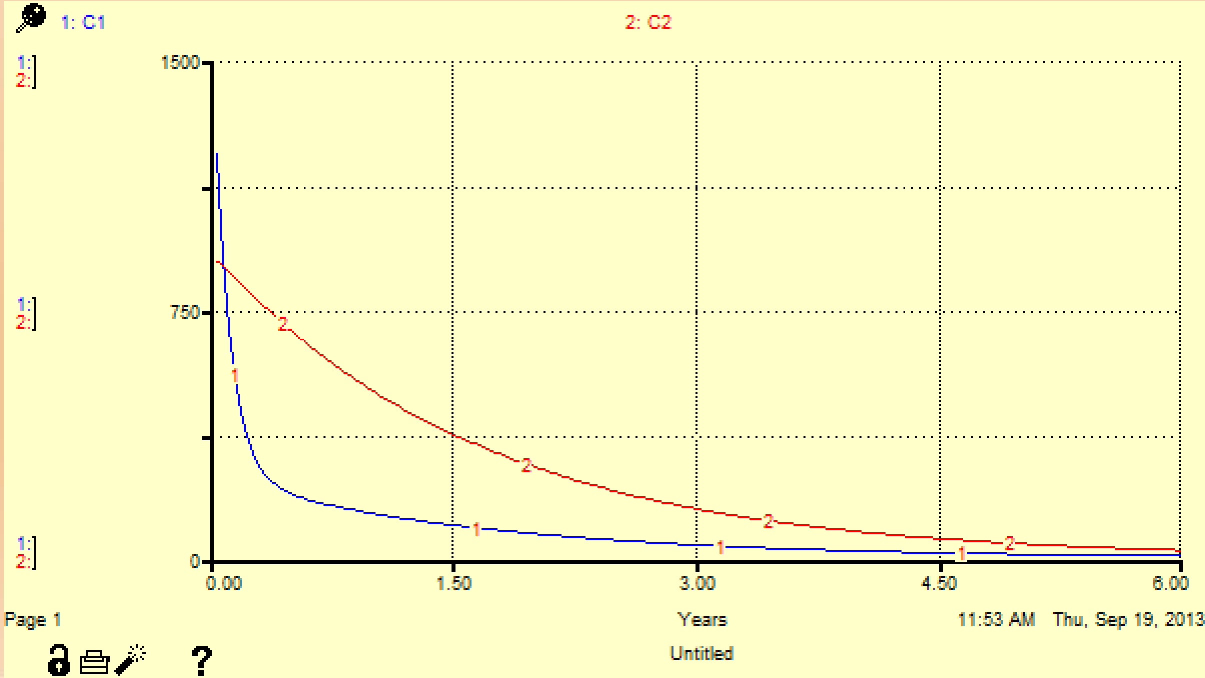
Stella Model



With initial concentration of 0, $W_1 = 2\times10^9$ mg/yr and $W_2 = 4\times10^9$ mg/yr:



With initial concentration of $C_{1,0}$=1224, $C_{2,0}$ = 898, W_1 = W_2 = 0 ➔



Thank you

Any question ?

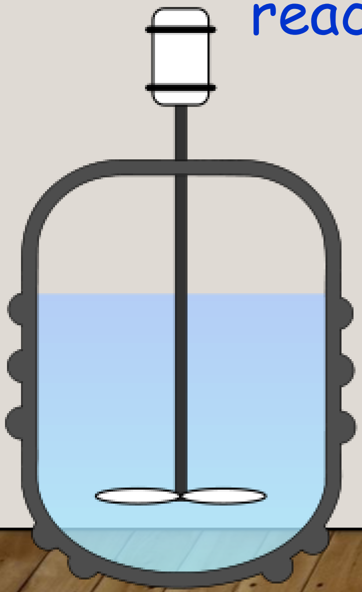
Chapter three

River water quality modelling

1. IDEAL REACTORS

- Ideal reactors are model systems for which the transport and mixing processes are exactly defined (e.g., perfect mixing in all directions or no mixing at all in the axial direction)

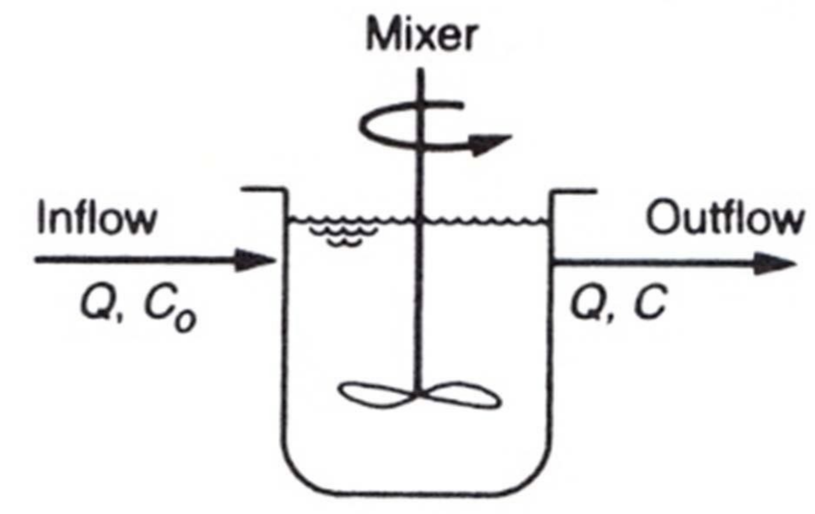
- Types of Ideal Reactors:
  - ✓ **Batch reactor:** The flow enters the reactor, is treated and then is discharged, and the cycle repeats. Once the processing starts, flow does not enter or leave the reactor. Used for small operations.



http://en.wikipedia.org/wiki/File:Batch_reactor_STR.svg

Sequencing Batch Reactor:
http://www.directindustry.com/prod/aqua-aerobic-systems-inc/sequencing-batch-reactors-sbr-89335-876857.html

- ✓ Continuously stirred tank reactor (CSTR): Complete mixing occurs uniformly throughout the reactor as fluid enters the reactor. Usually L ≅ W.




Aerobic Digester

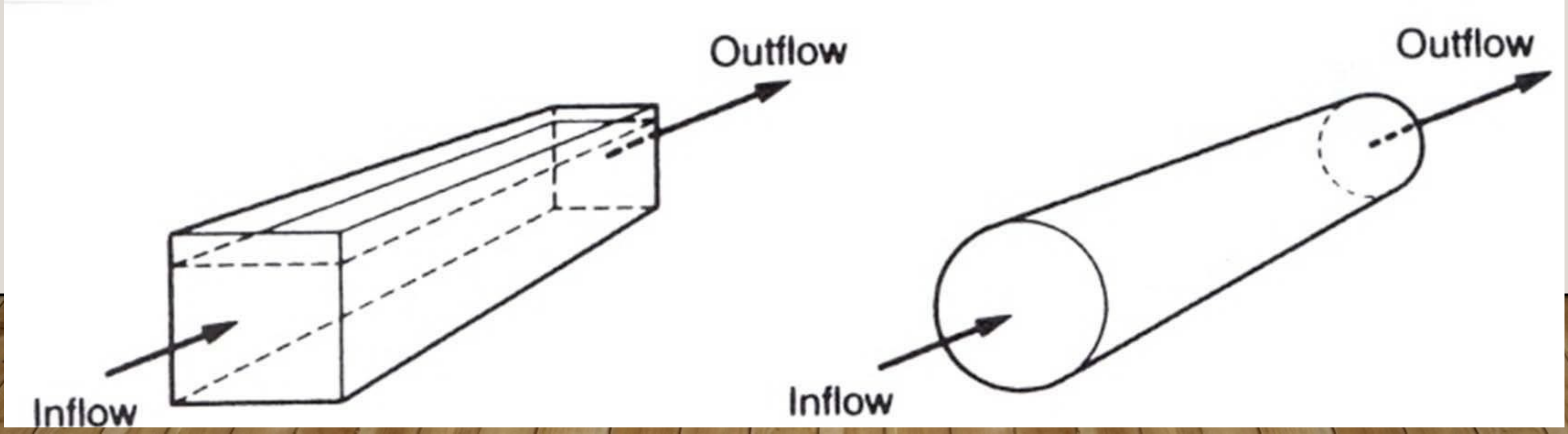
http://www.fountainsanitation.com/Treatment/AerobicDigester.htm

Aerated lagoon:

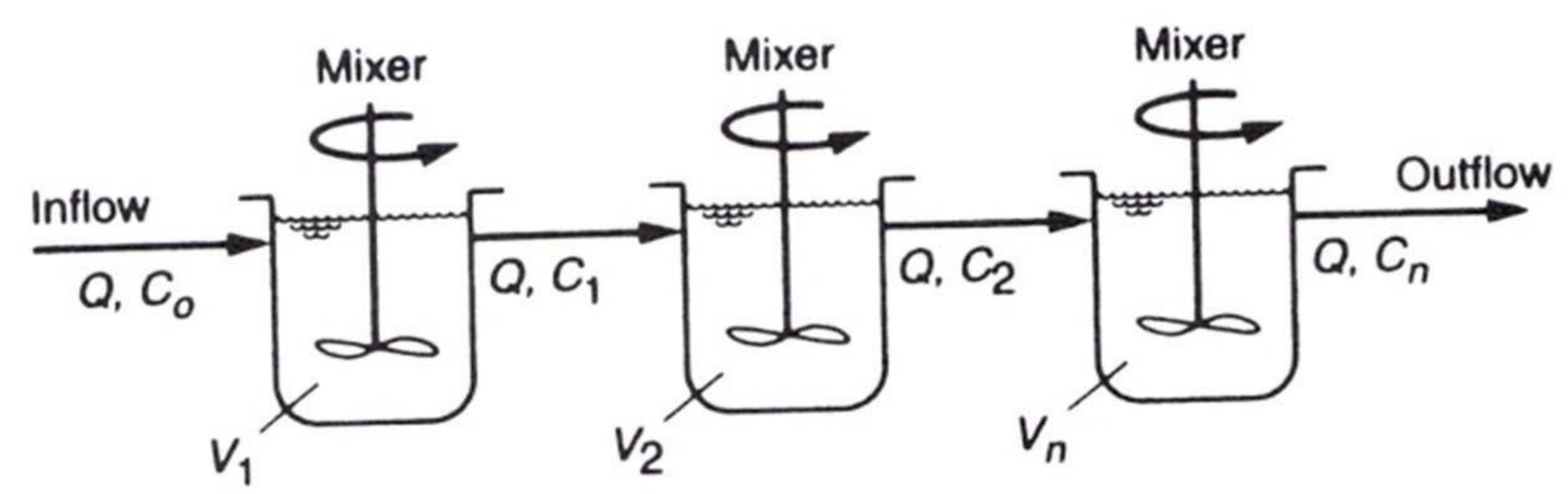
http://www.sites.mech.ubc.ca/~pougatch/lagoon.html

- ✓ Plug-flow reactor (PFR): Fluid passes through the reactor with little or no longitudinal mixing and leaves the reactor in the same sequence as they entered. Long rectangular tanks with L>>W

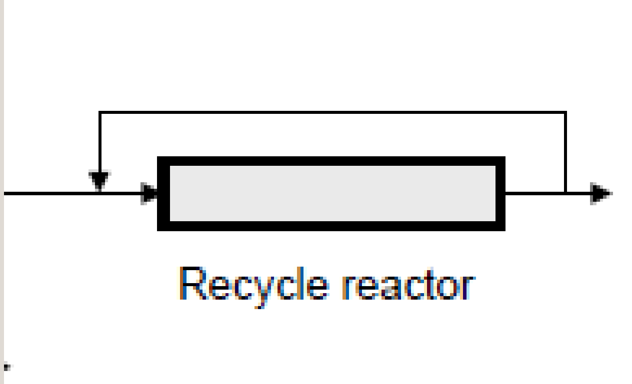
CHLORINE CONTACTTANK: HTTP://WWW.PANORAMIO.COM/PHOTO/37898431



- ✓ Stirred tank cascade

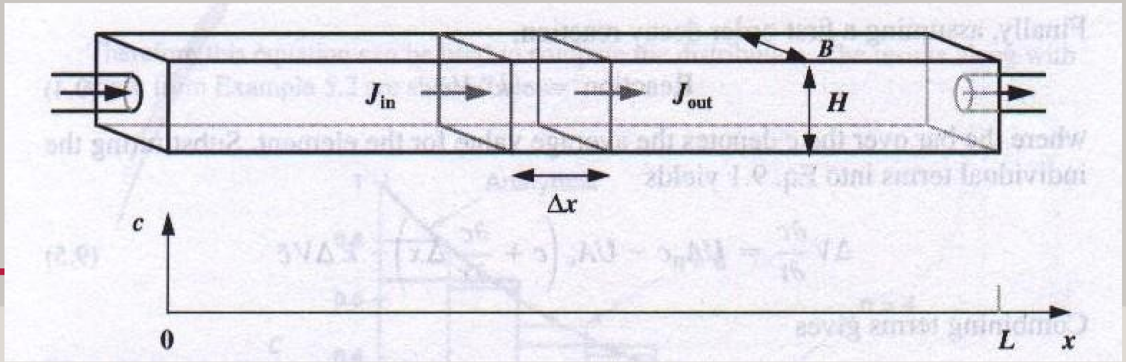


- ✓ Recycle reactor



(1)PLUG-FLOW REACTOR (PFR)

- well-mixed laterally and vertically
- **variations** in the *longitudinal* dimension (advection dominates)



- ***Mass balance equation:***

$$(\Delta V)\frac{\partial C}{\partial t} = J_{in}A_c - J_{out}A_c \pm reaction$$

where ΔV = volume of the element [L^3] = $A_c\Delta x$; A_c = cross-sectional area of the reactor [L^2] = BH, B = channel width [L], H = depth [L], J_{in} and J_{out} = mass flux in and out of the element due to transport [$ML^{-2}T^{-1}$], reaction = gain or loss of mass within the element due to reaction [MT^{-1}]

$$J_{in} = UC$$

$$J_{out} = U(C + \frac{\partial C}{\partial x}\Delta x)$$

$$reaction = K(\Delta V)C$$

→

$$(\Delta V)\frac{\partial C}{\partial t} = UCA_c - U(C + \frac{\partial C}{\partial x}\Delta x)A_c - K(\Delta V)C$$

$$= -UA_c\frac{\partial C}{\partial x}(\Delta x) - K(\Delta V)C$$

$$= -U(\Delta V)\frac{\partial C}{\partial x} - K(\Delta V)C$$

➔ $$\frac{\partial C}{\partial t} = -U\frac{\partial C}{\partial x} - kC$$

✓ Assumptions made:

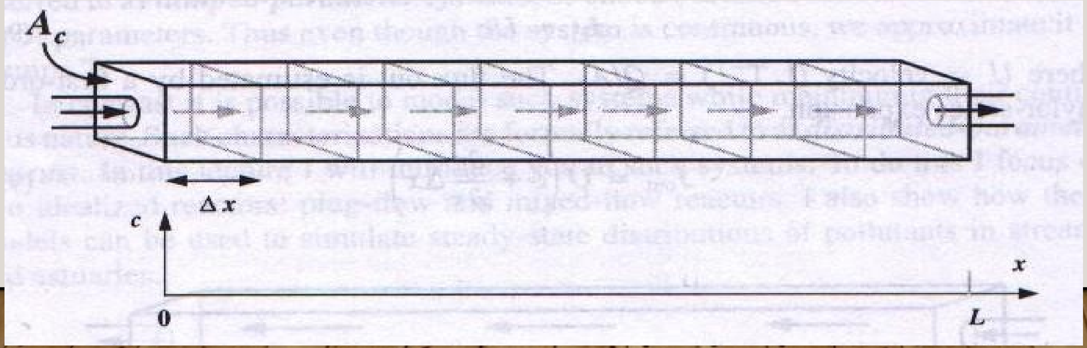
- ♦ Q constant
- ♦ A_c constant
- ♦ U constant
- ♦ k constant
- ♦ completely mixed in cross-section

➢ *Steady-State Solution:*

$$\frac{\partial C}{\partial t} = -U\frac{\partial C}{\partial x} - kC = 0 \;\rightarrow\; \frac{dC}{dx} = -\frac{k}{U}C \;\rightarrow\; C = C_0\, e^{-\frac{k}{U}x}$$

Where $C=C_0$ at $x=0$ (boundary condition)

Example 47: In Example 24 we used a cascade model to simulate the steady-state distribution of concentration in an elongated tank. The tank has a cross-sectional area $A_c = 10\ m^2$, length L = 100 m, velocity U = 100 m/hr, and first-order reaction rate $k = 2\ hr^{-1}$. Use the plug-flow model to compute the steady-state concentration distribution for the tank.

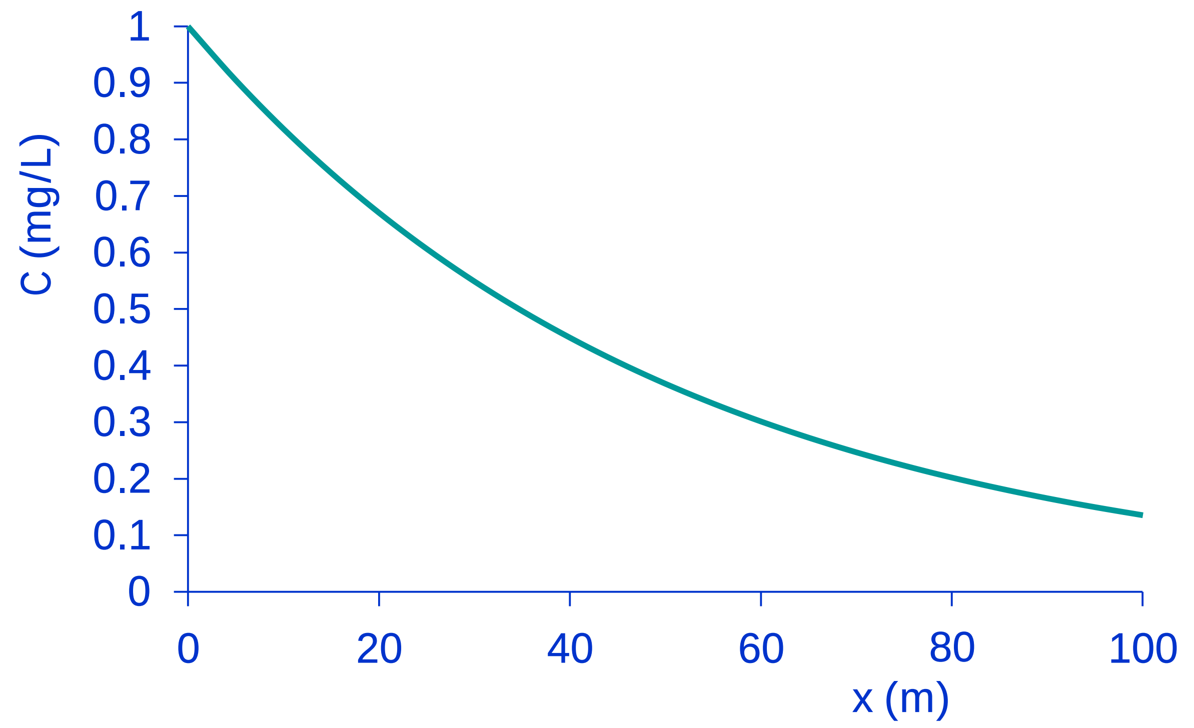


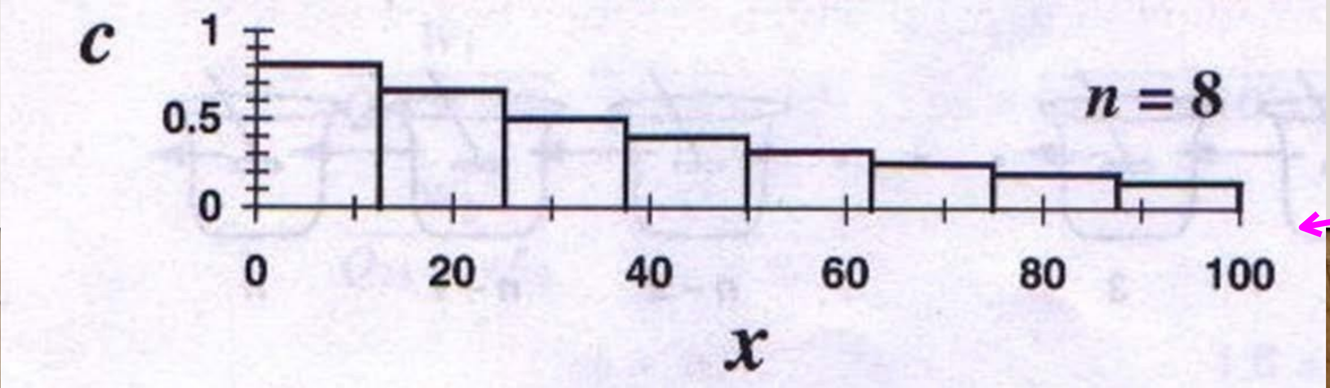
SOLUTION:

$$U = 100 m/hr, \qquad k = 2hr^{-1}, \qquad C_0 = 1 mg/L$$

➔ $C = C_0 e^{-\frac{k}{U}x} = 1\times e^{-\frac{2}{100}x} = e^{-0.02x}$ Where C is in mg/L and x is in m)

➔





From Cascade model

➢ *Estimate reaction rate constant k:*

$$C = C_0 e^{-\frac{k}{U}x} \quad ➔ \quad \frac{C}{C_0} = e^{-\frac{k}{U}x} \quad ➔ \quad \ln\left(\frac{C}{C_0}\right) = -k\left(\frac{x}{U}\right)$$

- ✓ plot ln(C/C_0) versus (x/U) ➔ Slope = -k ➔ k = -Slope

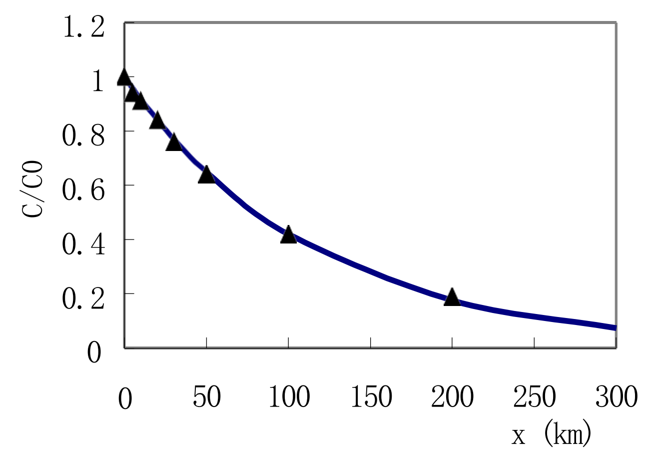
Example 48: The contaminant concentration profile in a river with mean velocity of 0.4 m/s is given below from field measurement. Estimate the degradation constant (k).

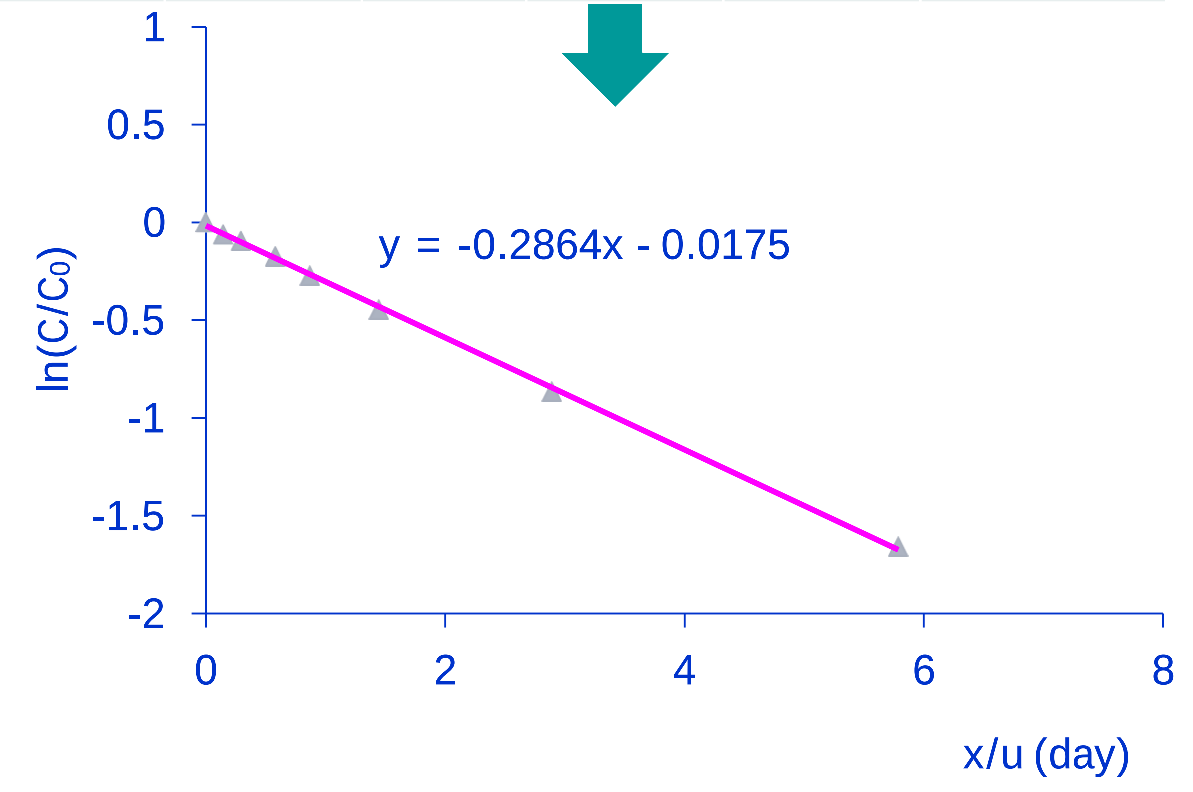
x (km)	C (mg/L)	x (km)	C (mg/L)	x (km)	C (mg/L)	x (km)	C (mg/L)
0	10.0	10	9.1	30	7.6	100	4.2
5	9.4	20	8.4	50	6.4	200	1.9

SOLUTION:

➔ k =   -Slope
0.28 = 64
day^{-1}

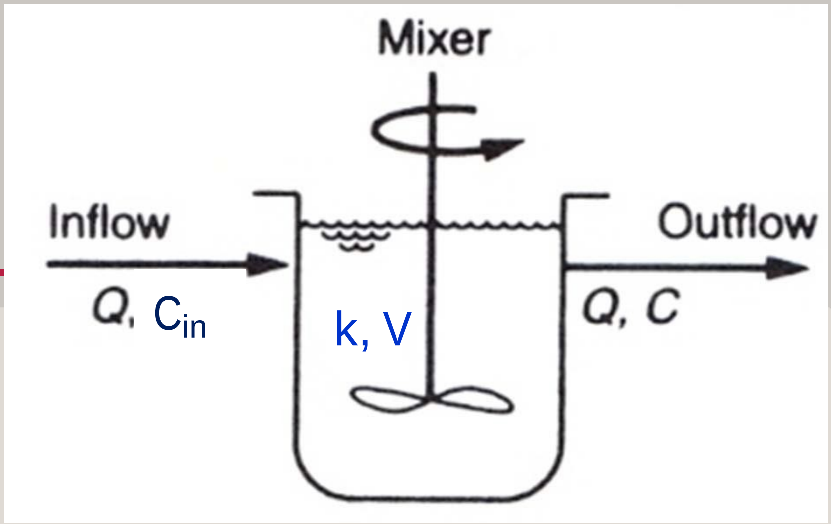
x (km)	C (mg/L)	C/C_0	$\ln(C/C_0)$	x/u (s)	x/u (day)
0	10	1	0.00	0	0.00
5	9.4	0.94	-0.06	12500	0.14
10	9.1	0.91	-0.09	25000	0.29
20	8.4	0.84	-0.17	50000	0.58
30	7.6	0.76	-0.27	75000	0.87
50	6.4	0.64	-0.45	125000	1.45
100	4.2	0.42	-0.87	250000	2.89
200	1.9	0.19	-1.66	500000	5.79





(2) COMPARISON OF CSTR AND P.F.R

(a) steady-state solution for CSTR with first-order reaction:



$$C = C_{in} \frac{Q}{Q + kV}$$

→ Performance efficiency defined by transfer function β:

→ $$\beta = \frac{C}{C_{in}} = \frac{Q}{Q + kV} = \frac{1}{1 + k\tau_w}$$

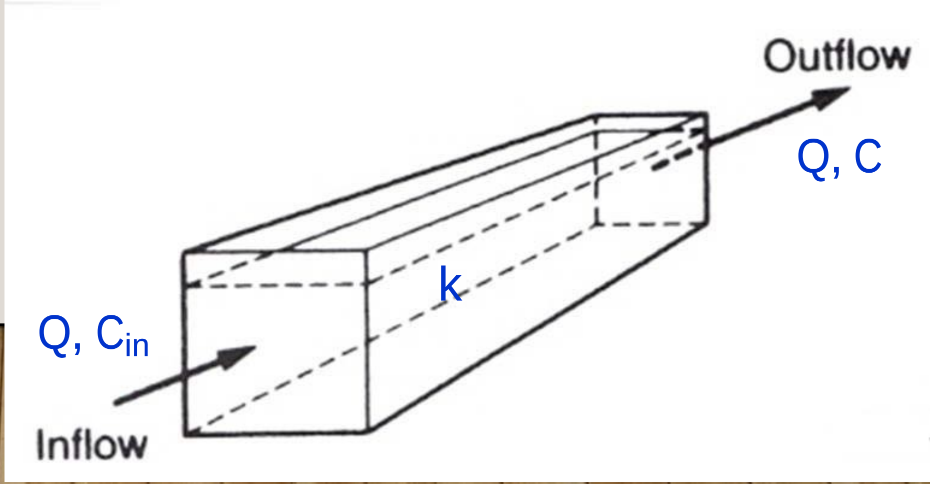
→ $$\tau_{w(CSTR)} = \left(\frac{1}{k}\right)\left(\frac{1-\beta}{\beta}\right)$$

(b) steady-state solution for PFR with first-order reaction:

$$C = C_{in} e^{-\frac{k}{U}x}$$

→ $$\beta = \frac{C}{C_{in}} = e^{-k\left(\frac{x}{U}\right)} = e^{-k\tau_{w(PFR)}}$$

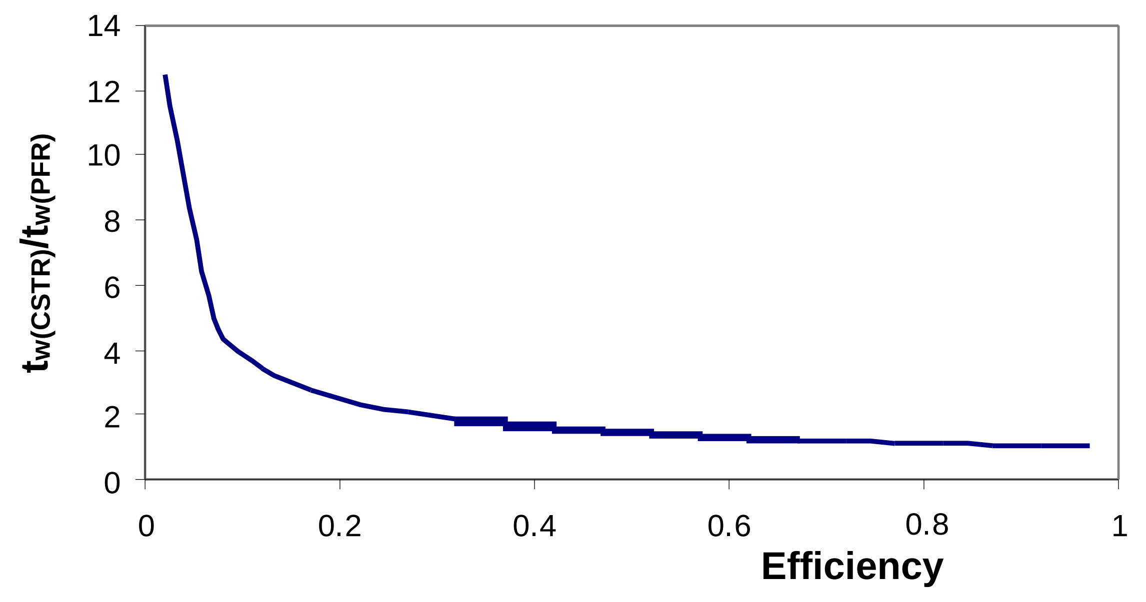
→ $$\tau_{w(PFR)} = \left(\frac{1}{k}\right)\ln\left(\frac{1}{\beta}\right)$$



(C) COMPARISON OF CSTR AND P

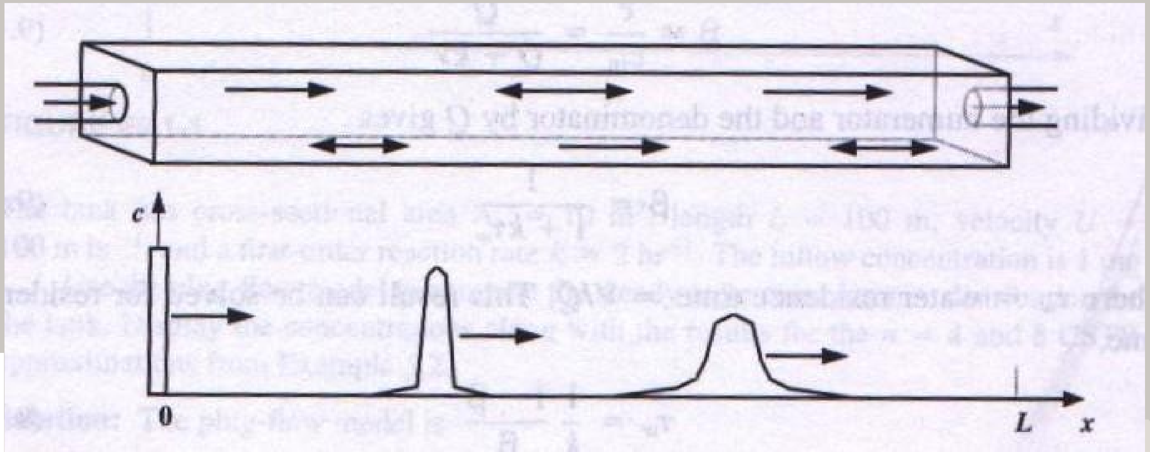
$$\tau_{w(CSTR)} = \left(\frac{1}{k}\right)\left(\frac{1-\beta}{\beta}\right)$$

$$\tau_{w(PFR)} = \left(\frac{1}{k}\right)\ln\left(\frac{1}{\beta}\right)$$



- ➢ CSTR always requires a higher residence time to achieve a desired efficiency ➔for first-order reactions the PFR is more efficient than CSTR for removing pollutants

(3) Mixed-Flow Reactor



- **Governing Equation:**

$$\frac{\partial C}{\partial t} = -U\frac{\partial C}{\partial x} + E\frac{\partial^2 C}{\partial x^2} - kC$$

- **Steady-State Solution:**

$$-U\frac{dC}{dx} + E\frac{d^2C}{dx^2} - kC = 0$$

Let $C = e^{\lambda x}$ Then: $\frac{dC}{dx} = \lambda e^{\lambda x}$ *and* $\frac{d^2C}{dx^2} = \lambda^2 e^{\lambda x}$

➔ $-U\lambda e^{\lambda x} + E\lambda^2 e^{\lambda x} - ke^{\lambda x} = 0$ ➔ $(-U\lambda + E\lambda^2 - k)e^{\lambda x} = 0$

➔ $E\lambda^2 - U\lambda - k = 0$ ➔

$$\begin{cases} \lambda_1 = \frac{U}{2E}\left(1 + \sqrt{1 + \frac{4kE}{U^2}}\right) \\ \lambda_2 = \frac{U}{2E}\left(1 - \sqrt{1 + \frac{4kE}{U^2}}\right) \end{cases}$$

✓ Boundary conditions:

$$\begin{cases} QC_{in} = QC_0 - EA_c \left.\frac{dC}{dx}\right|_{x=0} \\ \left.\frac{dC}{dx}\right|_{x=L} = 0 \end{cases}$$

✓ Steady-State Solution:

$$C = Fe^{\lambda_1 x} + Ge^{\lambda_2 x}$$

Where:

$$\lambda_1 = \frac{U}{2E}\left(1 + \sqrt{1 + \frac{4kE}{U^2}}\right) \qquad \lambda_2 = \frac{U}{2E}\left(1 - \sqrt{1 + \frac{4kE}{U^2}}\right)$$

$$F = \frac{UC_{in}\lambda_2 e^{\lambda_2 L}}{(U - E\lambda_1)\lambda_2 e^{\lambda_2 L} - (U - E\lambda_2)\lambda_1 e^{\lambda_1 L}} \qquad G = \frac{UC_{in}\lambda_1 e^{\lambda_1 L}}{(U - E\lambda_2)\lambda_1 e^{\lambda_1 L} - (U - E\lambda_1)\lambda_2 e^{\lambda_2 L}}$$

Example 49: For the same system as in Example 47, compute the concentration using the mixed-flow model with dispersion coefficients of E = 2000 and 10000 m^2/hr. Plot the results along with the PFR and CSTR models for the same tank.

SOLUTION:

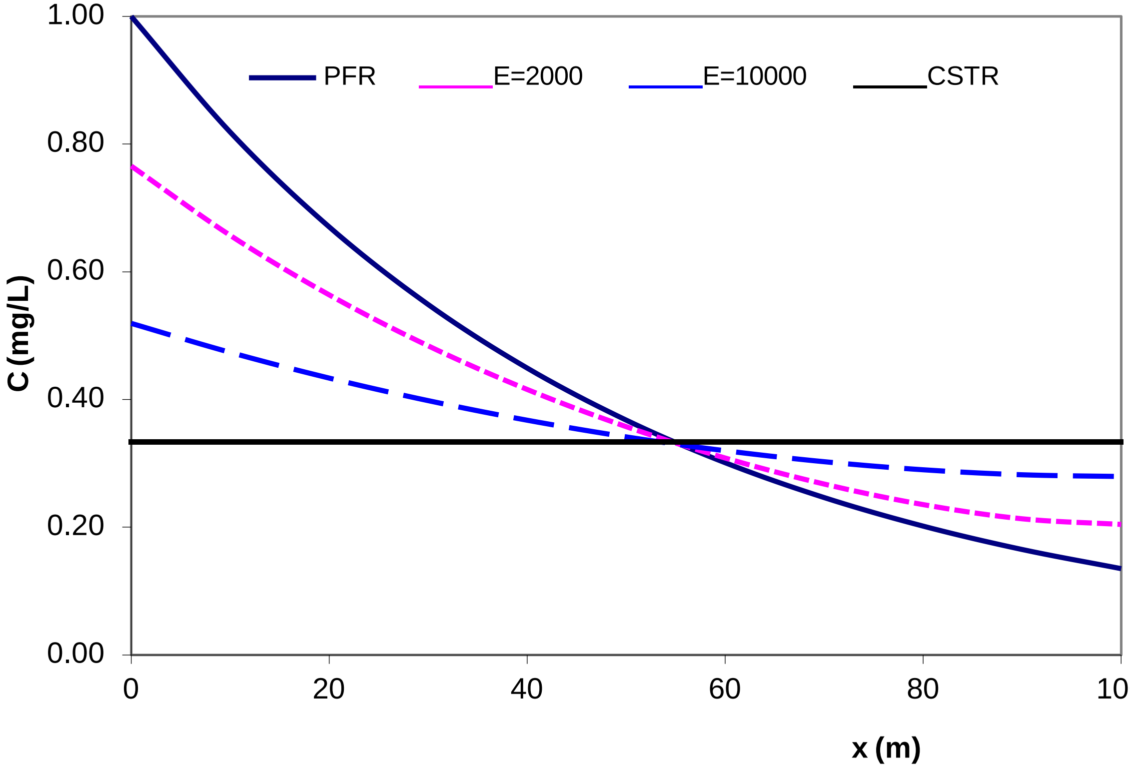
$$\lambda_1 = \frac{U}{2E}\left(1+\sqrt{1+\frac{4kE}{U^2}}\right) \qquad \lambda_2 = \frac{U}{2E}\left(1-\sqrt{1+\frac{4kE}{U^2}}\right)$$

$$F = \frac{UC_{in}\lambda_2 e^{\lambda_2 L}}{(U-E\lambda_1)\lambda_2 e^{\lambda_2 L} - (U-E\lambda_2)\lambda_1 e^{\lambda_1 L}} \qquad G = \frac{UC_{in}\lambda_1 e^{\lambda_1 L}}{(U-E\lambda_2)\lambda_1 e^{\lambda_1 L} - (U-E\lambda_1)\lambda_2 e^{\lambda_2 L}}$$

For mixed flow reactor: $C = Fe^{\lambda_1 x} + Ge^{\lambda_2 x}$

For PFR: $C = C_{in}e^{-\frac{k}{U}x}$

	E= 2000	E = 10000	PFR	CSTR
λ_1	0.065	0.020		
λ_2	-0.015	-0.010		
F	5.657E-05	0.013		
G	0.766	0.506		
x = 0	0.77	0.52	1.00	0.33
10	0.66	0.47	0.82	0.33
20	0.56	0.43	0.67	0.33
30	0.48	0.40	0.55	0.33
40	0.42	0.37	0.45	0.33
50	0.36	0.34	0.37	0.33
60	0.31	0.32	0.30	0.33
70	0.27	0.30	0.25	0.33
80	0.24	0.29	0.20	0.33
90	0.21	0.28	0.17	0.33
100	0.20	0.28	0.14	0.33

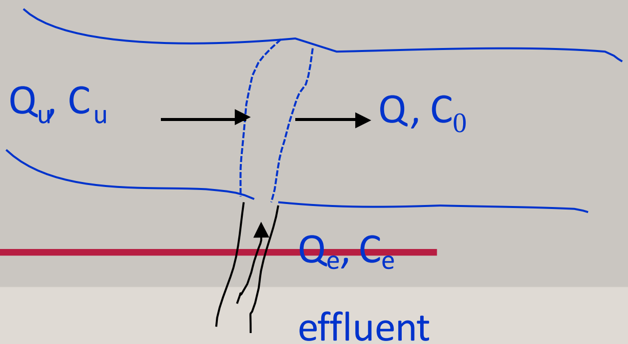


For CSTR:

$$C = \left(\frac{Q}{1+kV}\right)C_{in} = \left(\frac{UA_c}{1+kA_cL}\right)C_{in}$$

2. APPLICATION OF PFR MODEL TO STREAMS

(1) Point Source



- Mass balance equation:

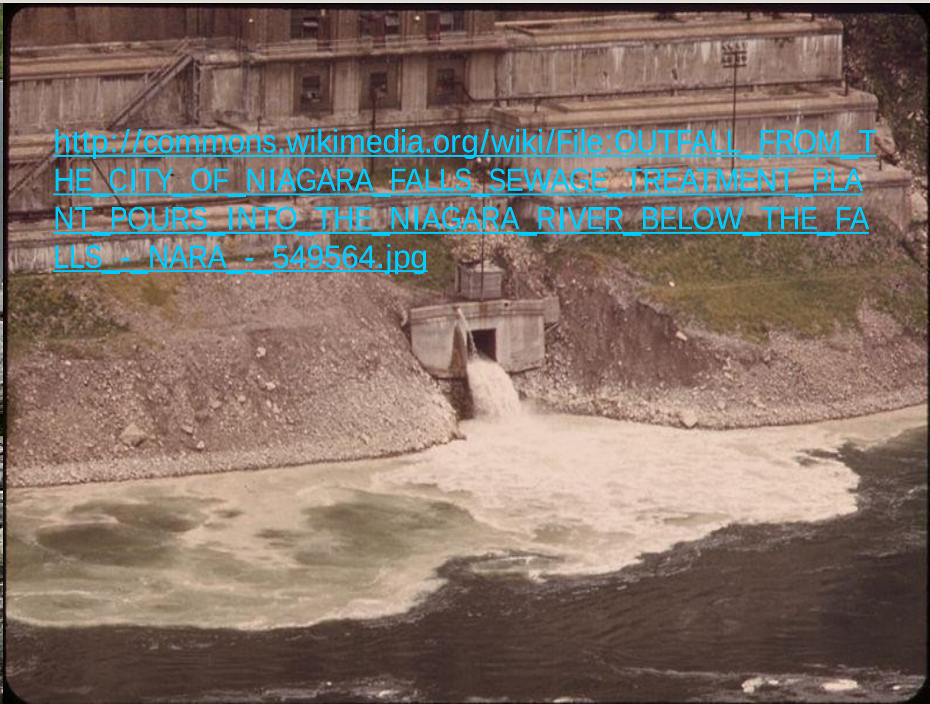
$$Q_uC_u + Q_eC_e = (Q_u + Q_e)C_0$$

$$\rightarrow \boxed{C_0 = \frac{Q_uC_u + Q_eC_e}{Q_u + Q_e}}$$

If $Q_u \gg Q_e$ and $C_e \gg C_u \rightarrow$

$$\boxed{C_0 \approx \frac{Q_eC_e}{Q_u + Q_e} = \frac{W}{Q}}$$

http://www.piquenewsmagazine.com/whistler/sewage-plant-upgrades-ahead-of-schedule/Content?oid=2159992



http://commons.wikimedia.org/wiki/File:OUTFALL_FROM_THE_CITY_OF_NIAGARA_FALLS_SEWAGE_TREATMENT_PLANT_POURS_INTO_THE_NIAGARA_RIVER_BELOW_THE_FALLS_-_NARA_-_549564.jpg

Example 50: A point source is discharged to a river having the following characteristics (see figure above) : $Q_u = 12\times10^6$ m^3/day, $C_u = 1$ mg/L, $Q_e = 0.5\times10^6$ m^3/day, and $C_e = 400$ mg/L. (a) determine the initial concentration assuming complete mixing vertically ~~and laterally; (b) calculate the concentration of~~ the pollutant for 8000 m below the discharge point. Note that the stream has a cross-sectional area of 2000 m^2 and the pollutant reacts with first-order decay (k = 0.8 day^{-1}).

SOLUTION

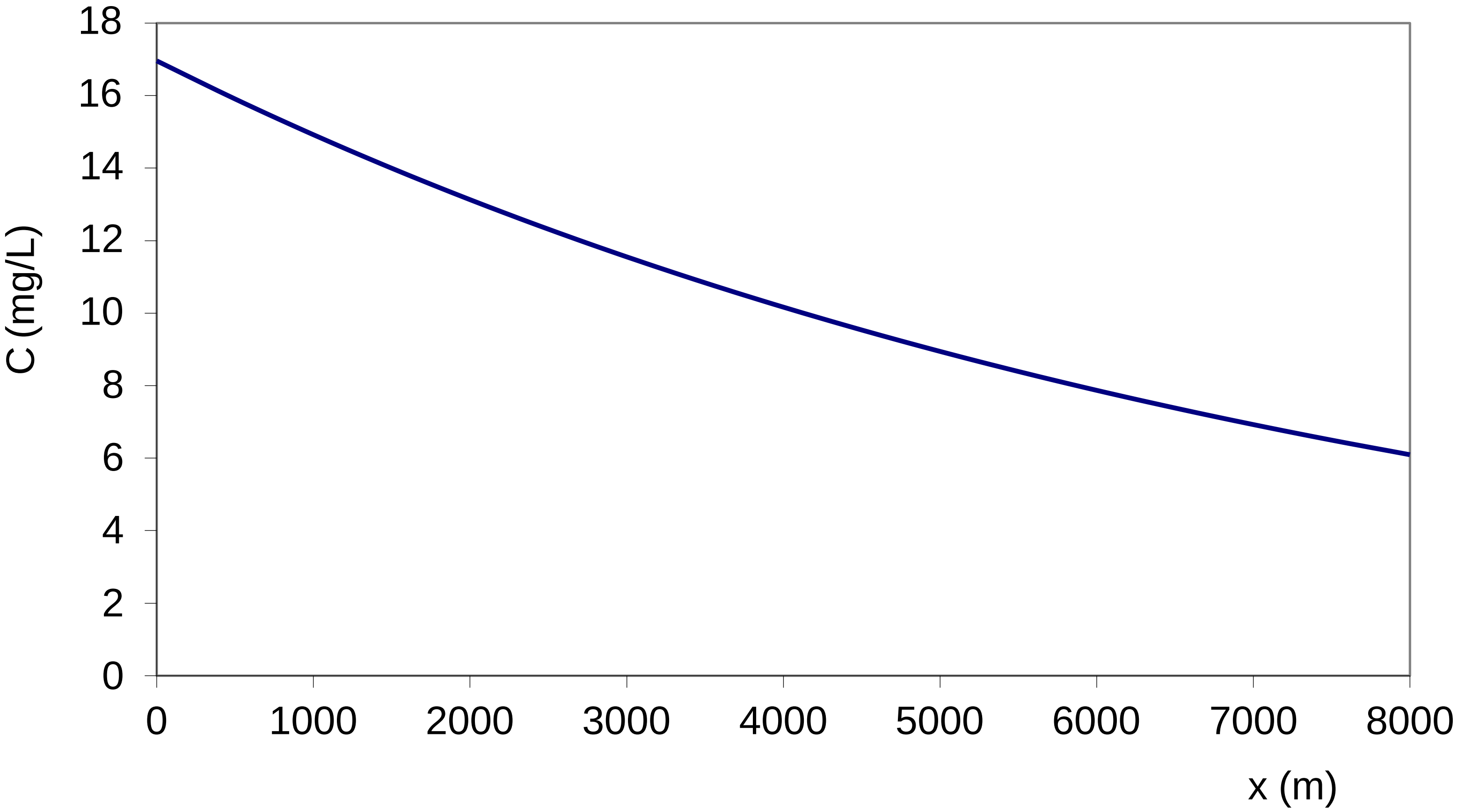
(a) $$C_0 = \frac{Q_uC_u + Q_eC_e}{Q_u + Q_e} = \frac{(12\times10^6\, m^3/d)(1mg/L) + (0.5\times10^6\, m^3/d)(400mg/L)}{12\times10^6\, m^3/d + 0.5\times10^6\, m^3/d} = 16.96mg/L$$

(b) $$U = \frac{Q}{A_c} = \frac{Q_u + Q_e}{A_c} = \frac{\left(12\times10^6 + 0.5\times10^6\right)m^3/d}{2000m^2} = 6250m/d$$

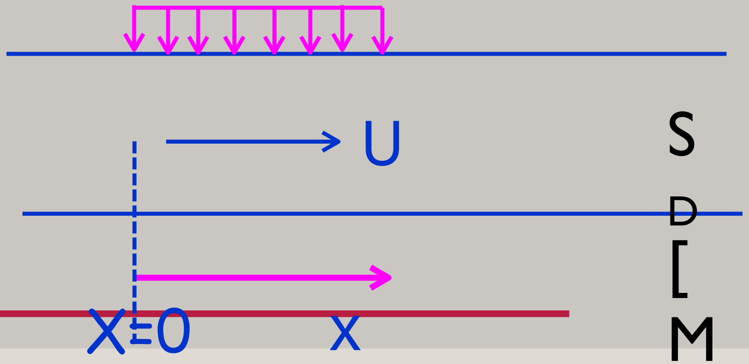
➔ $$C = C_0 e^{-\frac{k}{U}x} = 16.96e^{-\frac{0.8x}{6250}}\, mg/L$$

Where x is in m

SOLUTION:


18
16
14
12
10
8
6
4
2
0
C (mg/L)
0
1000
2000
3000
4000
5000
6000
7000
8000
x (m)

(2) Distributed Source



- For uniformly distributed source:

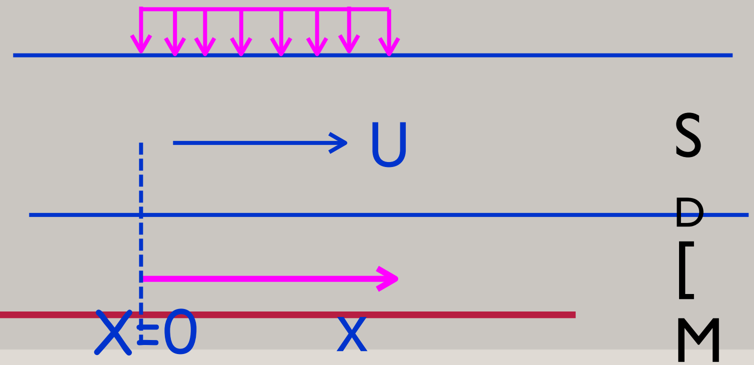
$$\frac{\partial C}{\partial t} = -U\frac{\partial C}{\partial x} - kC + S_d$$

where S_d = distributed source $[ML^{-3}T^{-1}]$

http://en.wikipedia.org/wiki/File:Runoff_of_soil_%26_fertilizer.jpg

http://oregonstate.edu/sustainability/water

Steady-State Solution:



$$-U\frac{dC}{dx} - kC + S_d = 0$$

$$\begin{cases} \dfrac{dC}{dx} + \left(\dfrac{k}{U}\right)C = \dfrac{S_d}{U} \\ C = C_0 \qquad at \qquad x = 0 \end{cases}$$

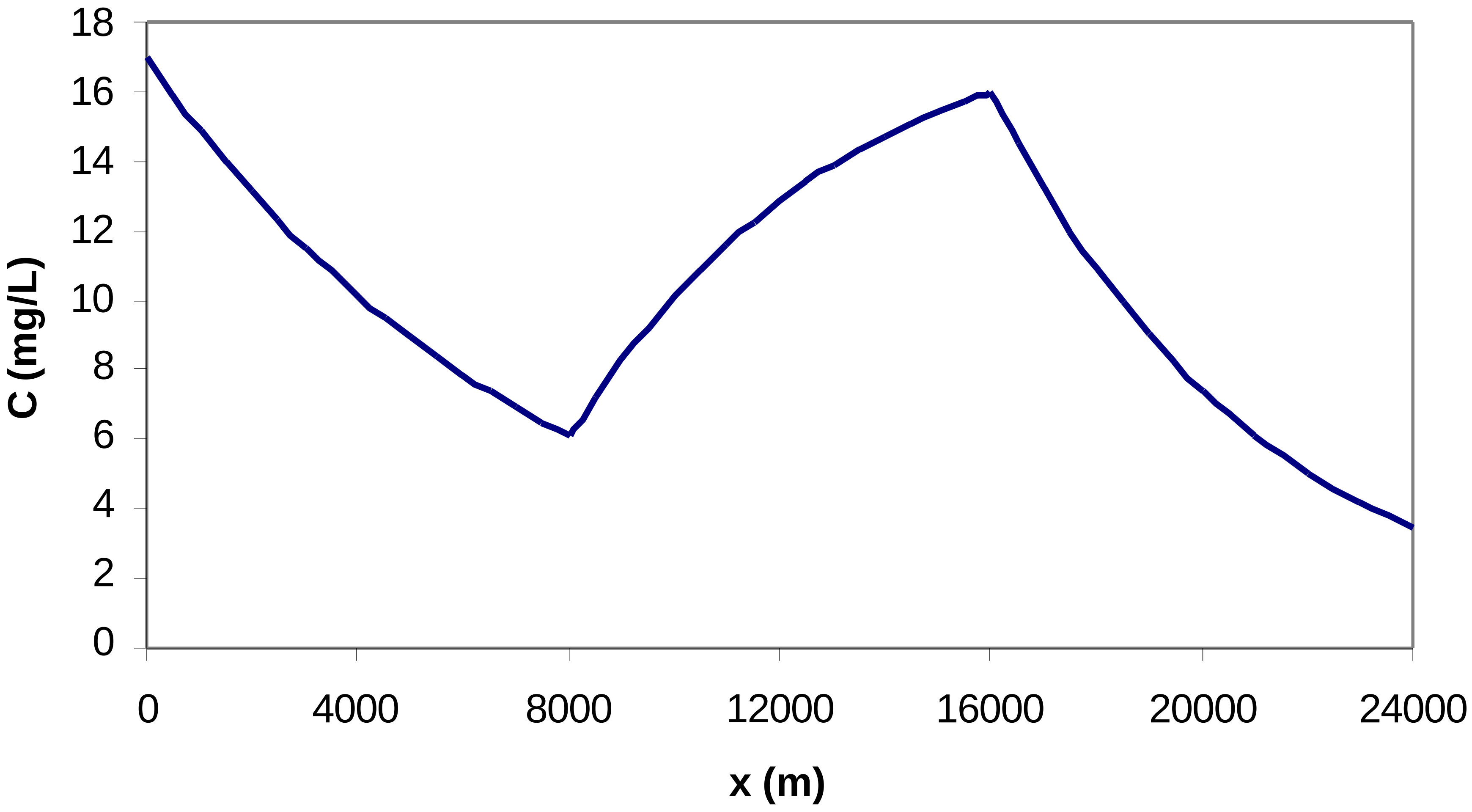
$$C = C_0 e^{-\frac{k}{U}x} + \frac{S_d}{k}\left(1 - e^{-\frac{k}{U}x}\right)$$

$S_D\,[ML^{-3}T^{-1}]$

Example 51: Suppose that for the same problem setting as in Example 50, a uniform distributed source load with no flow contribution begins 8 km downstream from the point source. The distributed source has a value of 15 g m^{-3} d^{-1} and continues for an additional 8 km. Thereafter all loadings terminate. Determine the concentration for the stretch 24 km below the point source. Note that the cross-sectional area increases to 3000 m^2 at the 8-km point.

SOLUTION:

x (m)	C (mg/L)	x (m)	C (mg/L)	x (m)	C (mg/L)
0	16.96	8000	6.09	16000	16.02
500	15.91	8500	7.25	16500	14.56
1000	14.92	9000	8.30	17000	13.23
1500	14.00	9500	9.26	17500	12.02
2000	13.13	10000	10.13	18000	10.92
2500	12.32	10500	10.92	18500	9.92
3000	11.55	11000	11.63	19000	9.01
3500	10.84	11500	12.29	19500	8.18
4000	10.16	12000	12.88	20000	7.44
4500	9.53	12500	13.41	20500	6.75
5000	8.94	13000	13.90	21000	6.14
5500	8.39	13500	14.35	21500	5.57
6000	7.87	14000	14.75	22000	5.06
6500	7.38	14500	15.12	22500	4.60
7000	6.92	15000	15.45	23000	4.18
7500	6.49	15500	15.75	23500	3.80
8000	6.09	16000	16.02	24000	3.45


C (mg/L)
0
2
4
6
8
10
12
14
16
18
0
4000
8000
12000
16000
20000
24000
x (m)

3. DISSOLVED OXYGEN IN RIVERS:

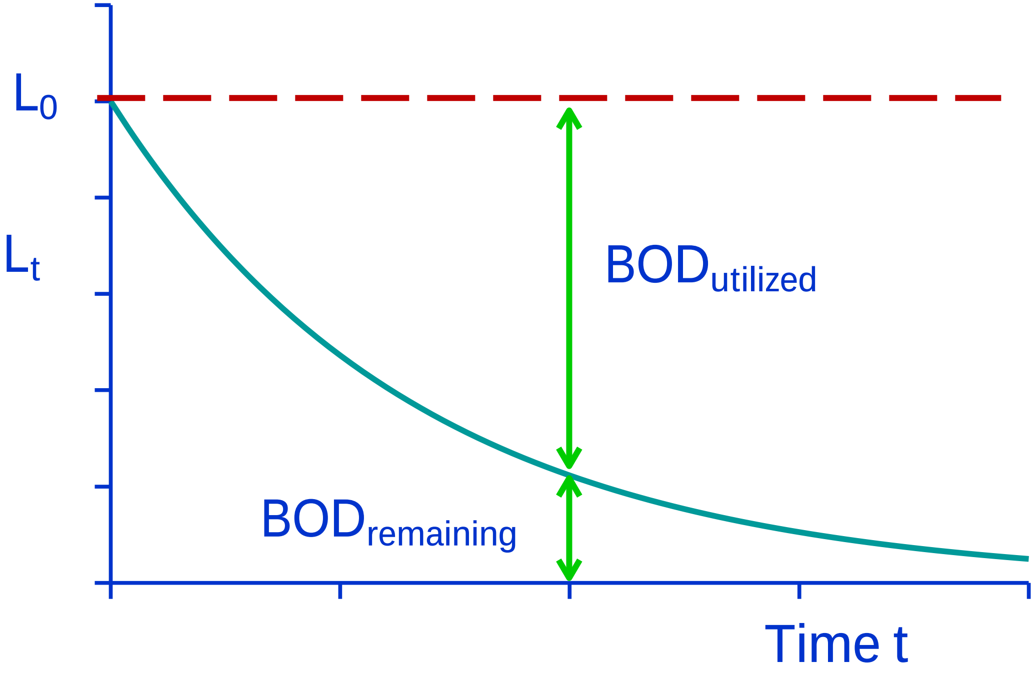
- most widely used indicators of overall ecological health of water body (e.g., fish need 4 to 5 mg/L to survive)

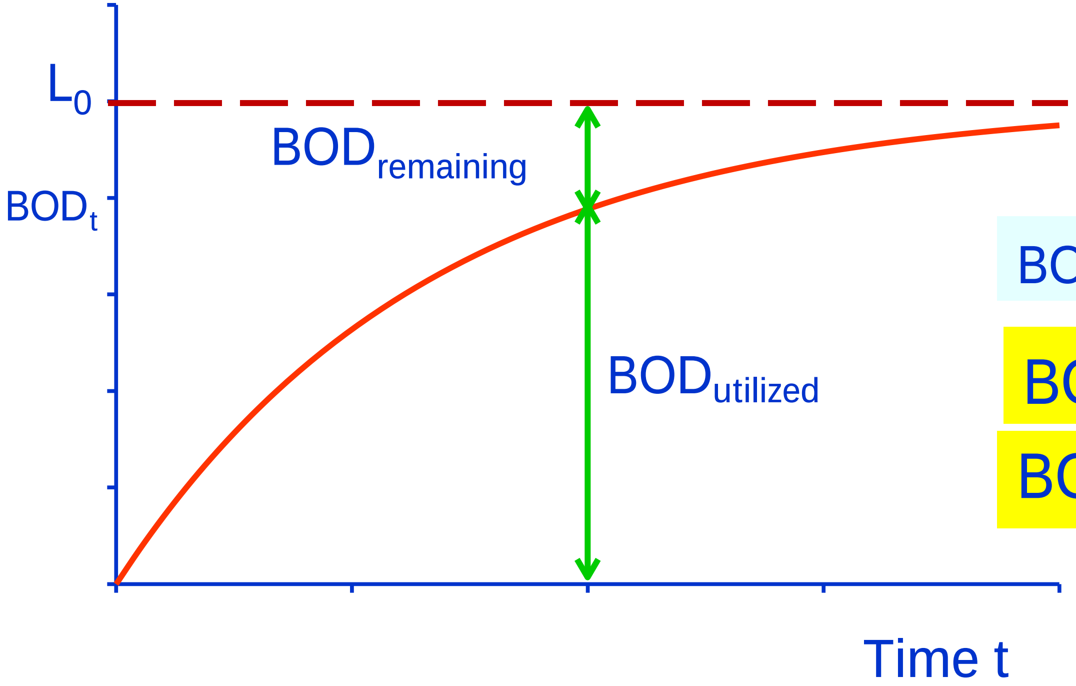
(1) Biochemical oxygen demand (BOD)

- **BOD** = the amount of oxygen required by microorganisms to oxidize organic wastes aerobically
- an indirect measure of organic content of a sample

http://winnipeg.ca/waterandwaste/sewage/NEWPCC/shutdownSummary.stm

- L_t = concentration of organic matter remaining at time t ➔ potential total O_2 demand = $BOD_{remaining}$ at time t
- L_0 = initial value of BOD ➔ the amount of total O_2 requirement to oxidize completely all of the organic material ➔ $BOD_{ultimate}$





- BOD_t = $BOD_{utilized}$ ➔ amount of O_2 consumed through time t

$$BOD_{utilized} + BOD_{remaining} = BOD_{ultimate}$$

$$BOD_{remaining} = BOD_{ultimate} \times e^{-kt}$$

$$BOD_{utilized} = BOD_{ultimate}(1 - e^{-kt})$$

$$\text{Or:} \quad BOD_t = L_0(1 - e^{-kt})$$

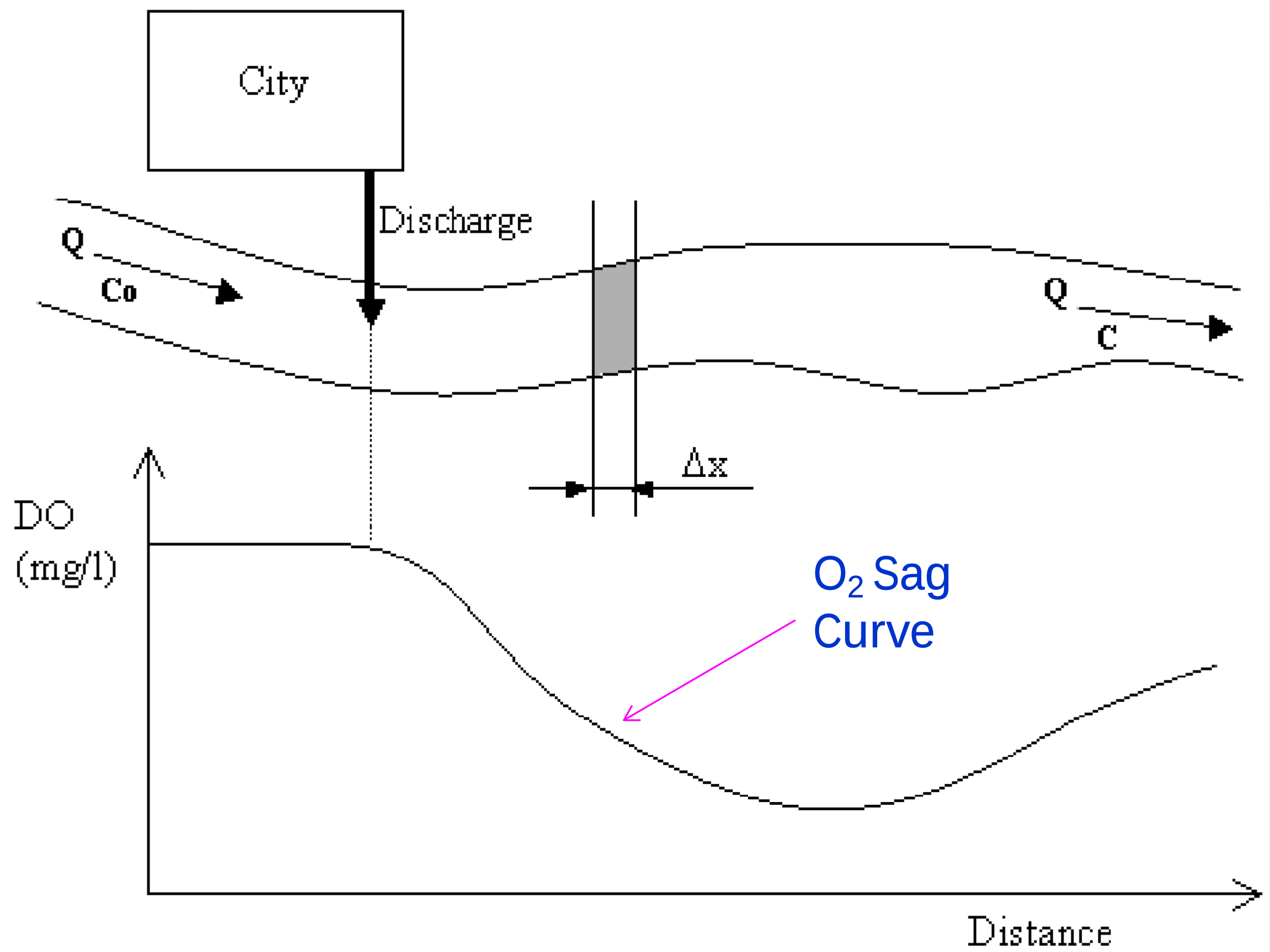
- BOD_5 ➔ standard test to measure over 5 days BOD of a waste:
  - ✓ Measure initial DO
  - ✓ Incubate sample for 5 days
  - ✓ Measure final DO
  - ✓ The BOD5 is the amount of DO used up over the 5-day period

$$BOD_5 = L_0(1 - e^{-5k})$$

➔ $$L_0 = \frac{BOD_5}{1 - e^{-5k}}$$


http://www.ysi.com/parametersdetail.php?Biochemical-Oxygen-Demand---BOD-5

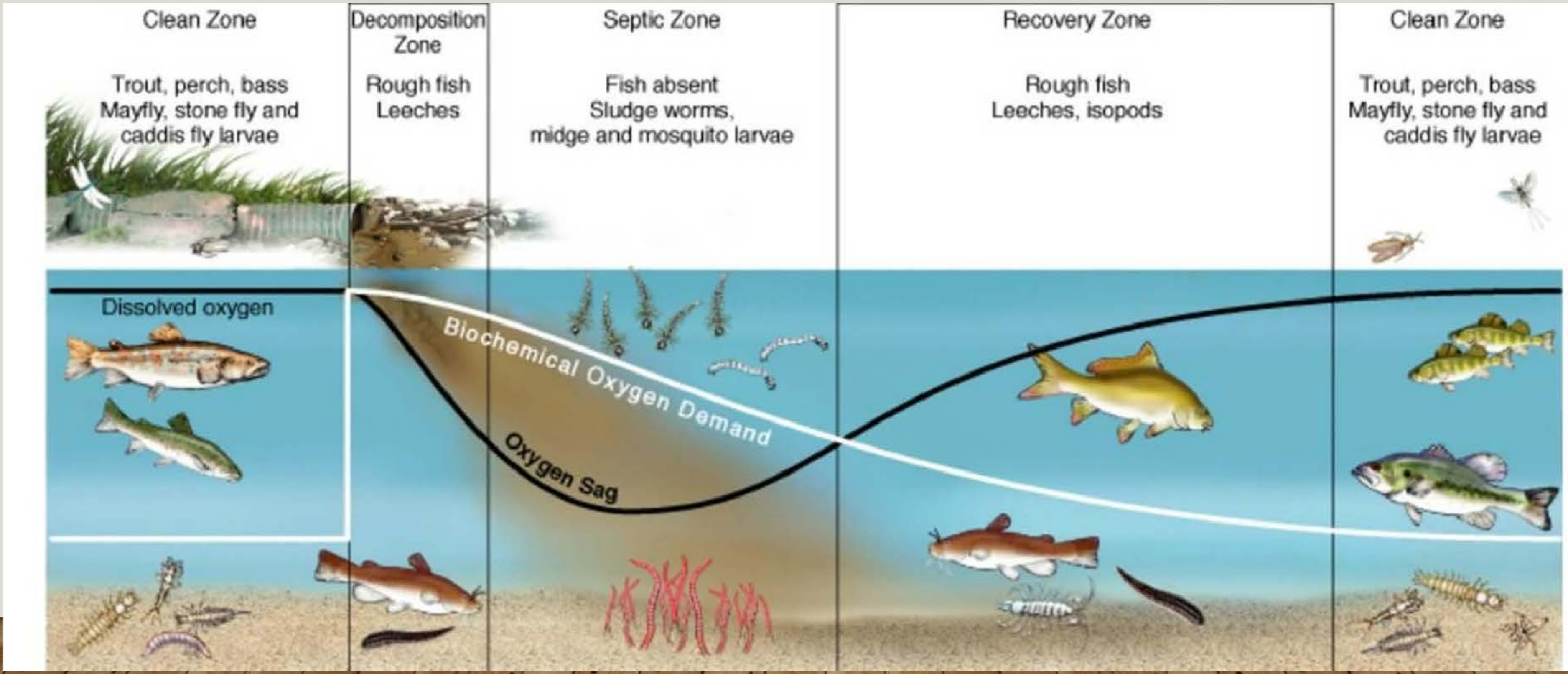
(2) Effects of O_2 demanding wastes on DO in rivers



O_2 Sag Curve

➢ Dissolved oxygen:

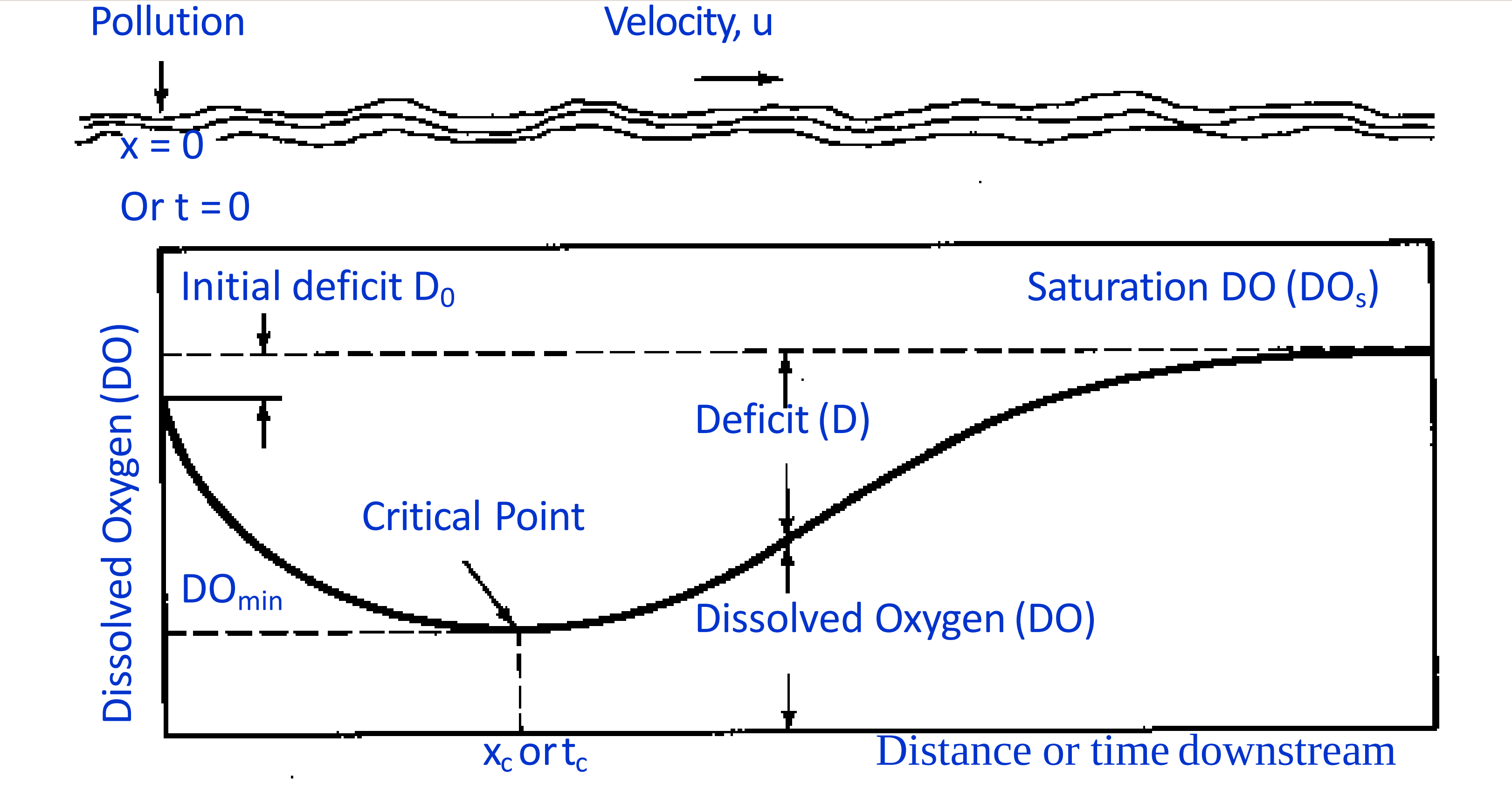
- ✓ drops in the decomposition zone (conditions are ideal for the rapid growth of plants, algae and other organisms)
- ✓ remains low and approximately constant in the septic zone (oxygen is too low to support organisms that live by aerobic respiration)
- ✓ increases in the recovery zone to its original level.



Copyright：The McGraw-Hill http://www.unc.edu/courses/2005fall/envr/051/001/05watpol05.jpg

- **Oxygen deficit D** = difference betweensaturation concentration and actual concentration of DO

$$D = DO_{sat} - DO = C_s - C$$



➢ Deoxygenation

- ✓ Rate of deoxygenation is proportional to L_t at any point ➔ the greater the BOD, the faster the deoxygenation rate

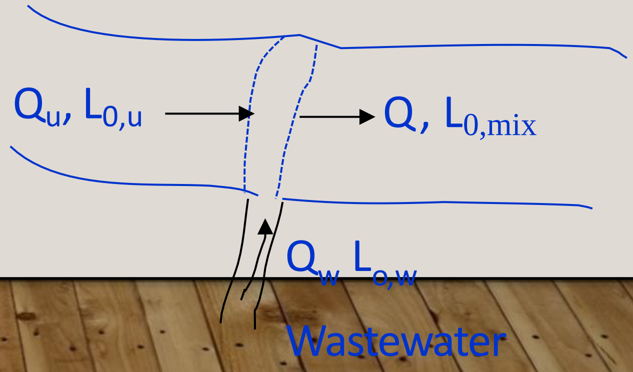
$$r_d = -\frac{d\,(DO)}{dt} = k_d L_t$$

➢ Need to know $L_{0,0}$ = $BOD_{ultimate}$ at point where river and wastewater mix

$$L_0 = \frac{BOD_5}{1 - e^{-5k}}$$

- ✓ Sample waste, measure BOD_5 ➔ calculate $L_{0,waste}$
- ✓ Sample river, measure BOD_5 ➔ calculate $L_{0,upstream}$
- ✓ Sample mixed water, measure BOD_5 ➔ calculate $L_{0,mix}$

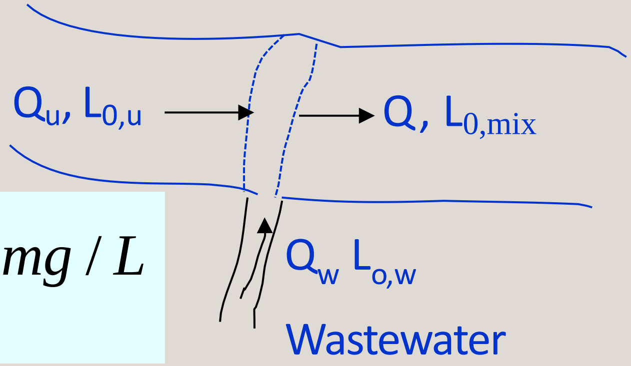
$$L_{0,mix} = \frac{Q_u L_{0,u} + Q_w L_{0,w}}{Q_u + Q_w}$$



Plug flow model:

$$L_x = L_{0,mix} e^{-\frac{k_d}{U}x}$$

Example 52: A wastewater treatment plant discharge 1.0 m^3/s of effluent with a $BOD_{ultimate} = L_0 = 40$ mg/L into a stream flowing 10 m^3/s. Upstream of the waste discharge, the stream has a $BOD_{ultimate} = 3.0$ mg/L. Assume the deoxygenation rate constant $k_d = 0.22$/day. (a) What is the $BOD_{ultimate}$ of the wastewater plus stream mixture assuming instantaneous mixing at point of discharge (i.e. L_0 at discharge of point)? (b) What is the BOD at 10000 m downstream assuming a constant cross-sectional area of 55 m^2 for the stream? (c) What is the rate of deoxygenation 10000 m downstream?



Solution:

(a)
$$L_{0,mix} = \frac{Q_u L_{0,u} + Q_w L_{0,w}}{Q_u + Q_w} = \frac{10 \times 3.0 + 1.0 \times 40}{10 + 1.0} = 6.4 mg/L$$

(b)
$$U = \frac{Q}{A_c} = \frac{Q_u + Q_w}{A_c} = \frac{11 m^3/s}{55 m^2} \times \frac{86400 s}{1d} = 1.7 \times 10^4 m/d$$

$$L_x = L_{0,mix} e^{-\frac{k_d}{U}x} = 6.4 \times e^{-(0.22 d^{-1}) \times (\frac{10000 m}{1.7 \times 10^4 m/d})} = 5.6 mg/L$$

(c)
$$r_d = k_d L_t = (0.22 d^{-1}) \times (5.6 mg/L) = 1.2 \frac{mg}{L \cdot day}$$

Reaeration

- ✓ Rate of reaeration is proportional to the oxygen deficit at any point ➔ the greater the deficit D, the faster the reaeration rate

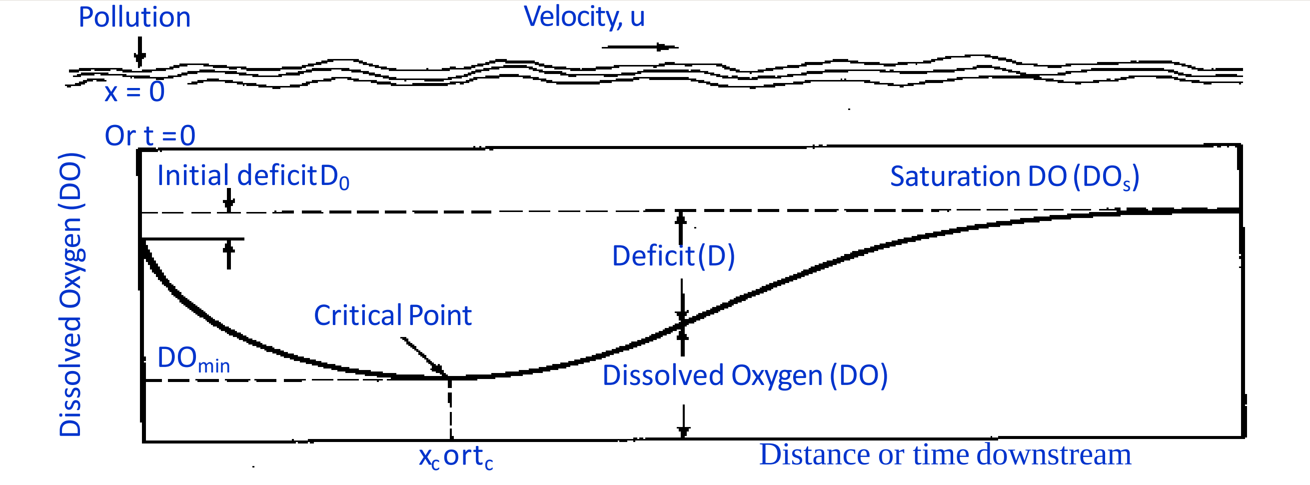
$$r_a = k_a D$$

- ✓ Reaeration rate constant depends on river velocity and depth

O'Connor-Dobbins Formula

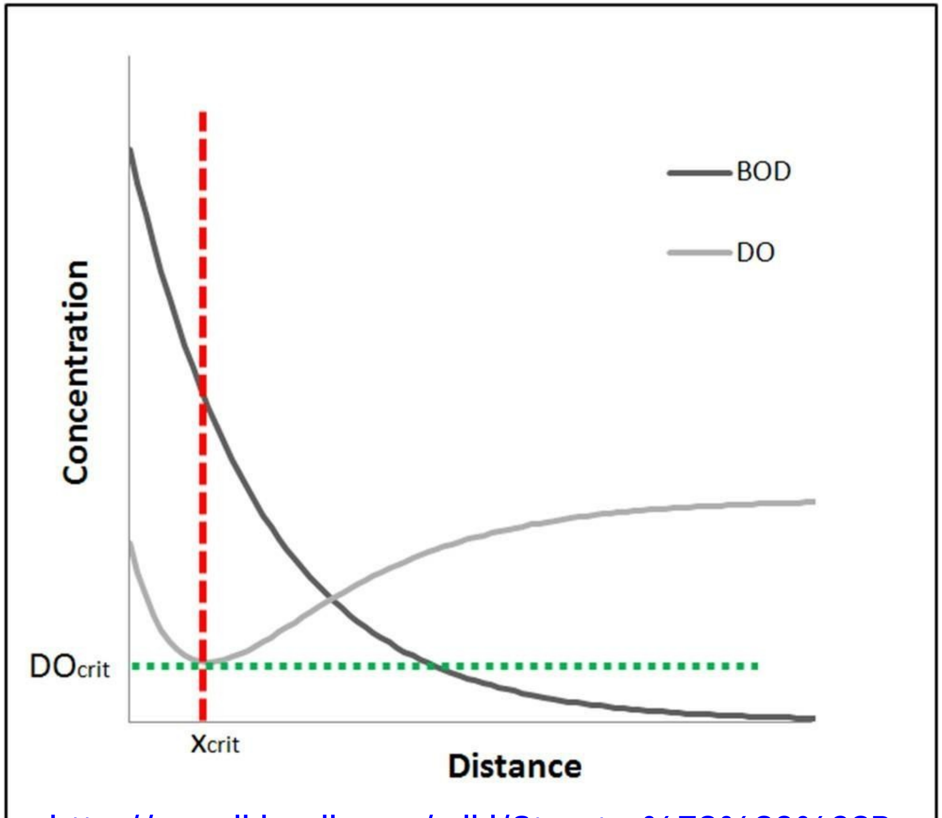
$$k_a(20^\circ C) = 3.9\sqrt{\frac{u}{H^3}}$$

where k_a (20°C) = reaeration rate constant at 20 °C (1/day); u = average stream velocity (m/s); H = average stream depth (m)

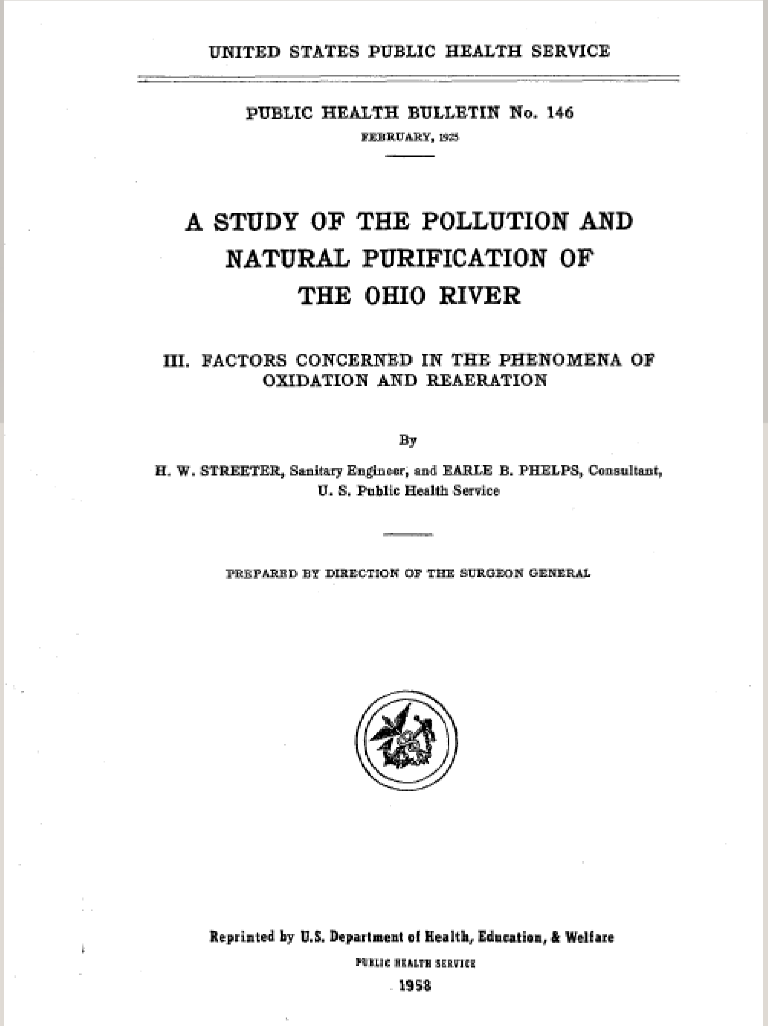


5. STREETER-PHELPS EQUATION

- The most prevalent model to describe the quality of water in streams, rivers and lakes
- Addition of wastewater (BOD) typically causes a slow decrease in O_2, followed by a gradual increase close to the dissolved oxygen (D.O.) saturation concentration
- The equation was derived by Streeter and Phelps in 1925, based on field data from the Ohio River
- Assumptions of the Model:
  - ✓ stream is an ideal plug flow reactor
  - ✓ The only reactions of interest are BOD exertion and transfer of oxygen from air to water across air-water interface



http://en.wikipedia.org/wiki/Streeter%E2%80%93Phelps_equation



UNITED STATES PUBLIC HEALTH SERVICE

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FEBRUARY, 1925

A STUDY OF THE POLLUTION AND NATURAL PURIFICATION OF THE OHIO RIVER

III. FACTORS CONCERNED IN THE PHENOMENA OF OXIDATION AND REAERATION

By

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U. S. Public Health Service

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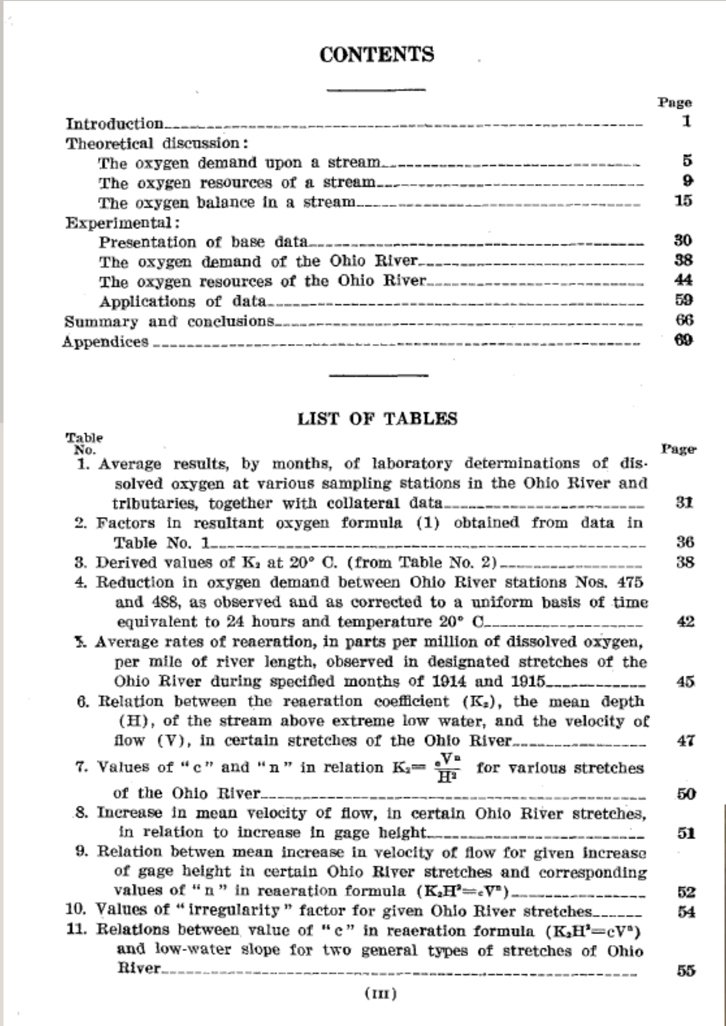
1958



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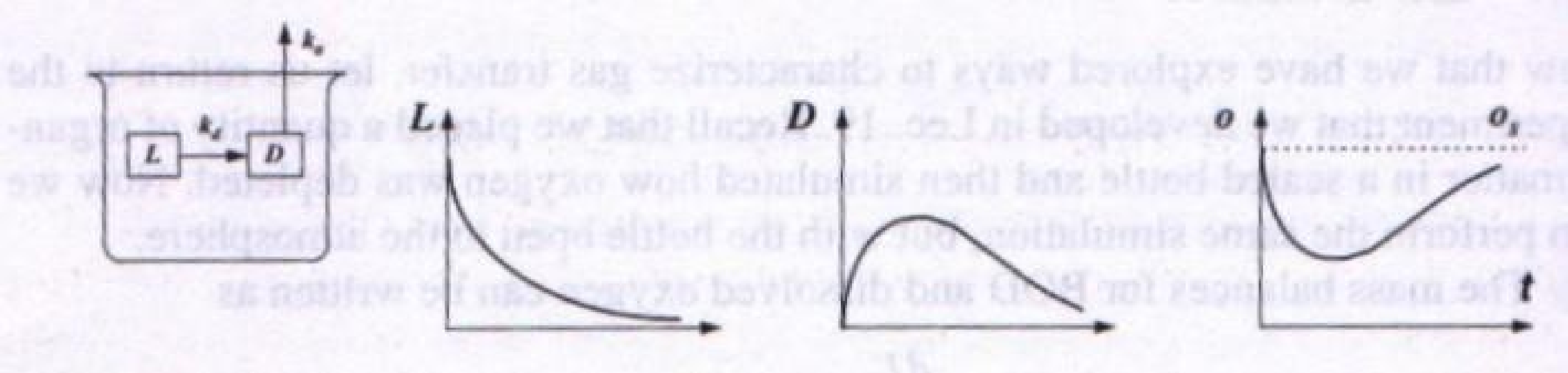
Earle Bernard Phelps (1876–1953) was a chemist, bacteriologist and sanitary expert. He is known for his contributions in sewage disinfection, water chlorination, sewage treatment, milk pasteurization, shellfish control, and for describing the "oxygen sag curve" in surface water bodies

http://www.zeably.com/Earle_B._Phelps

(1) Mass Balance for Well-Mixed System



- **for BOD:**

$$V\frac{dL}{dt} = -Vk_dL \quad \rightarrow \quad \frac{dL}{dt} = -k_dL \quad \rightarrow \quad L(t) = L_0e^{-k_dt}$$

where L_0 = initial $BOD_{ultimate}$ at t = 0, k_d = deoxygenation rate constant

- **for DO:**

$$V\frac{dO}{dt} = Vk_a(O_s - O) - Vk_dL$$

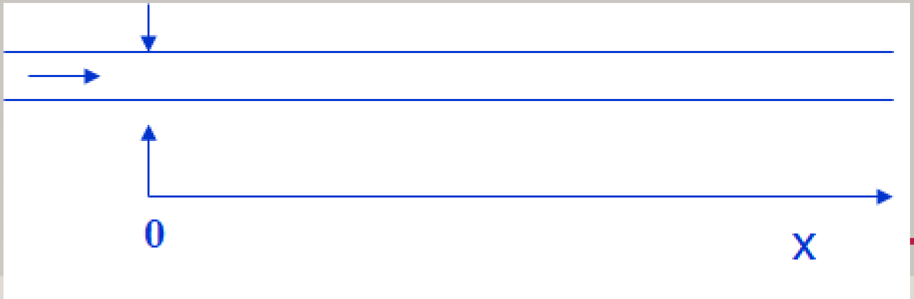
Let $D = O_s - O$ → $\frac{dD}{dt} = -\frac{dO}{dt}$

$$\rightarrow -V\frac{dD}{dt} = Vk_aD - Vk_dL \quad \rightarrow \quad \frac{dD}{dt} = k_dL - k_aD \quad \rightarrow \quad \frac{dD}{dt} + k_aD = k_dL_0e^{-k_dt}$$

$$\rightarrow D(t) = D_0e^{-k_at} + \frac{k_dL_0}{k_a - k_d}\left(e^{-k_dt} - e^{-k_at}\right)$$

Where k_a = reaeration rate constant, D_0 is DO deficit at t =0

(2) Point-Source Streeter-Phelps Equation



- the reach is steady-state and is characterized by plug flow with constant hydrology and geometry

- Governing equation:

$$\frac{\partial L}{\partial t} = -U\frac{\partial L}{\partial x} - k_d L \quad \text{and} \quad \frac{\partial D}{\partial t} = -U\frac{\partial D}{\partial x} - k_d D + k_d L$$

A. Steady-State Solution:

$$\rightarrow \quad -U\frac{dL}{dx} - k_d L = 0 \quad \rightarrow \quad \frac{dL}{dx} = -\left(\frac{k_d}{U}\right)L \quad \rightarrow \quad \boxed{L(x) = L_0 e^{-\frac{k_d}{U}x}}$$

and

$$-U\frac{dD}{dx} - k_a D + k_d L = 0 \quad \rightarrow \quad \frac{dD}{dx} + \left(\frac{k_a}{U}\right)D = k_d L = k_d L_0 e^{-\frac{k_d}{U}x}$$

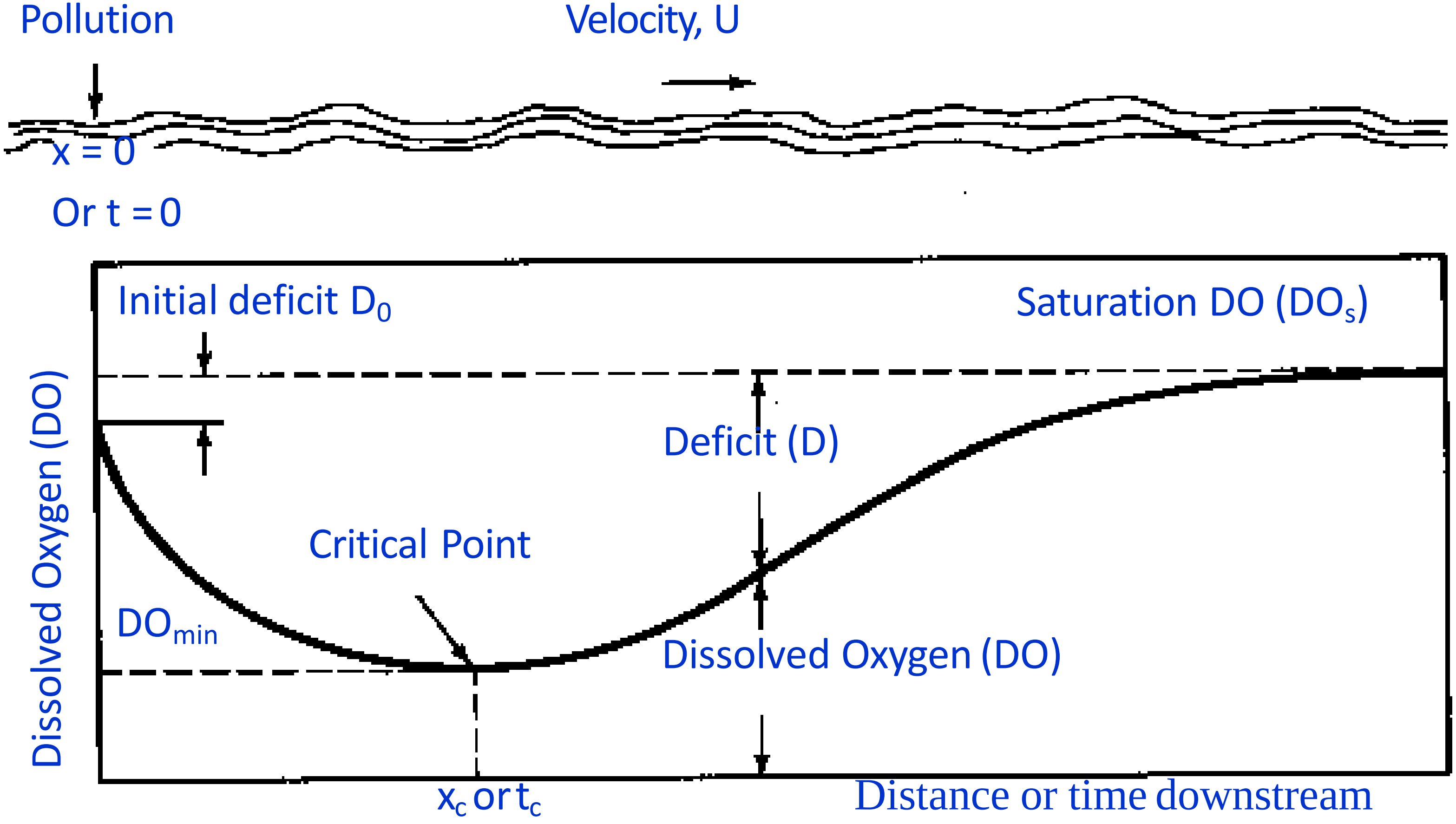
$$\rightarrow \quad \boxed{D(x) = D_0 e^{-\frac{k_a}{U}x} + \frac{k_d L_0}{k_a - k_d} \left(e^{-\frac{k_d}{U}x} - e^{-\frac{k_a}{U}x} \right)}$$

where L = L_0 and D = D_0 at x = 0, k_d = deoxygenation rate constant, k_a = reaeration rate constant

$$L(x) = L_0 e^{-\frac{k_d}{U}x}$$ and $$D(x) = D_0 e^{-\frac{k_a}{U}x} + \frac{k_d L_0}{k_a - k_d}\left(e^{-\frac{k_d}{U}x} - e^{-\frac{k_a}{U}x}\right)$$

B. Critical Point (x_c, D_c)

- Critical deficit ➔ occurs at location with maximum D



$$D = D_0 e^{-\frac{k_a}{u}x} + \frac{k_d L_0}{k_a - k_d}\left(e^{-\frac{k_d}{u}x} - e^{-\frac{k_a}{u}x}\right)$$

$\frac{dD}{dx} = 0$ occurs at $x = x_c$

$$\Rightarrow \quad 0 = -u\frac{dD}{dx} + k_d L - k_a D \quad \text{(Mass balance Equation)}$$

$$\Rightarrow \quad k_d L = k_a D_c \quad = D_c = \frac{k_d}{k_a} L = \frac{k_d}{k_a} L_0 e^{-\frac{k_d}{u}x_c}$$

Then $D_c = D_0 e^{-\frac{k_a}{u}x_c} + \frac{k_d L_0}{k_a - k_d}\left(e^{-\frac{k_d}{u}x_c} - e^{-\frac{k_a}{u}x_c}\right)$

$$\frac{k_d}{k_a} L_0 e^{-\frac{k_d}{u}x_c} = D_0 e^{-\frac{k_a}{u}x_c} + \frac{k_d L_0}{k_a - k_d}\left[e^{-\frac{k_d}{u}x_c} - e^{-\frac{k_a}{u}x_c}\right]$$

$$\left[\frac{k_d L_0}{k_a} - \frac{k_d L_0}{k_a - k_d}\right] e^{-\frac{k_d}{u}x_c} = \left[D_0 - \frac{k_d L_0}{k_a - k_d}\right] e^{-\frac{k_a}{u}x_c}$$

$$\frac{\left[k_d(k_a - k_d) - k_a k_d\right] L_0}{k_a(k_a - k_d)} e^{-\frac{k_d}{u}x_c} = \frac{D_0(k_a - k_a) - k_d L_0}{k_a - k_d} e^{-\frac{k_a}{u}x_c}$$

$$\frac{-k_d^2 L_0}{k_a \cancel{(k_a-k_d)}} e^{-\frac{k_d}{u}x_c} = \frac{[D_0(k_a-k_d)-k_d L_0]}{\cancel{k_a-k_d}} e^{-\frac{k_a}{u}x_c}$$

$$\frac{e^{-\frac{k_d}{u}x_c}}{e^{-\frac{k_a}{u}x_c}} = \frac{[D_0(k_a-k_d)-k_d L_0]k_a}{-k_d^2 L_0}$$

$$= \frac{k_a[k_d L_0 - D_0(k_a-k_d)]}{k_d^2 L_0}$$

$$\ln\left[\frac{e^{-\frac{k_d}{u}x_c}}{e^{-\frac{k_a}{u}x_c}}\right] = \ln\left\{\frac{k_a[k_d L_0 - D_0 \cancel{k_a}(k_a-k_d)]}{k_d^2 L_0}\right\}$$

$$-\frac{k_d}{u}x_c + \frac{k_a}{u}x_c = \ln\left\{\frac{k_a}{k_d}\left[1-\frac{D_0(k_a-k_d)}{k_d L_0}\right]\right\}$$

$$\Rightarrow x_c = \frac{u}{k_a-k_d}\ln\left\{\frac{k_a}{k_d}\left[1-\frac{D_0(k_a-k_d)}{k_d L_0}\right]\right\}$$

B. Critical Point (x_c, D_c)

- Critical deficit

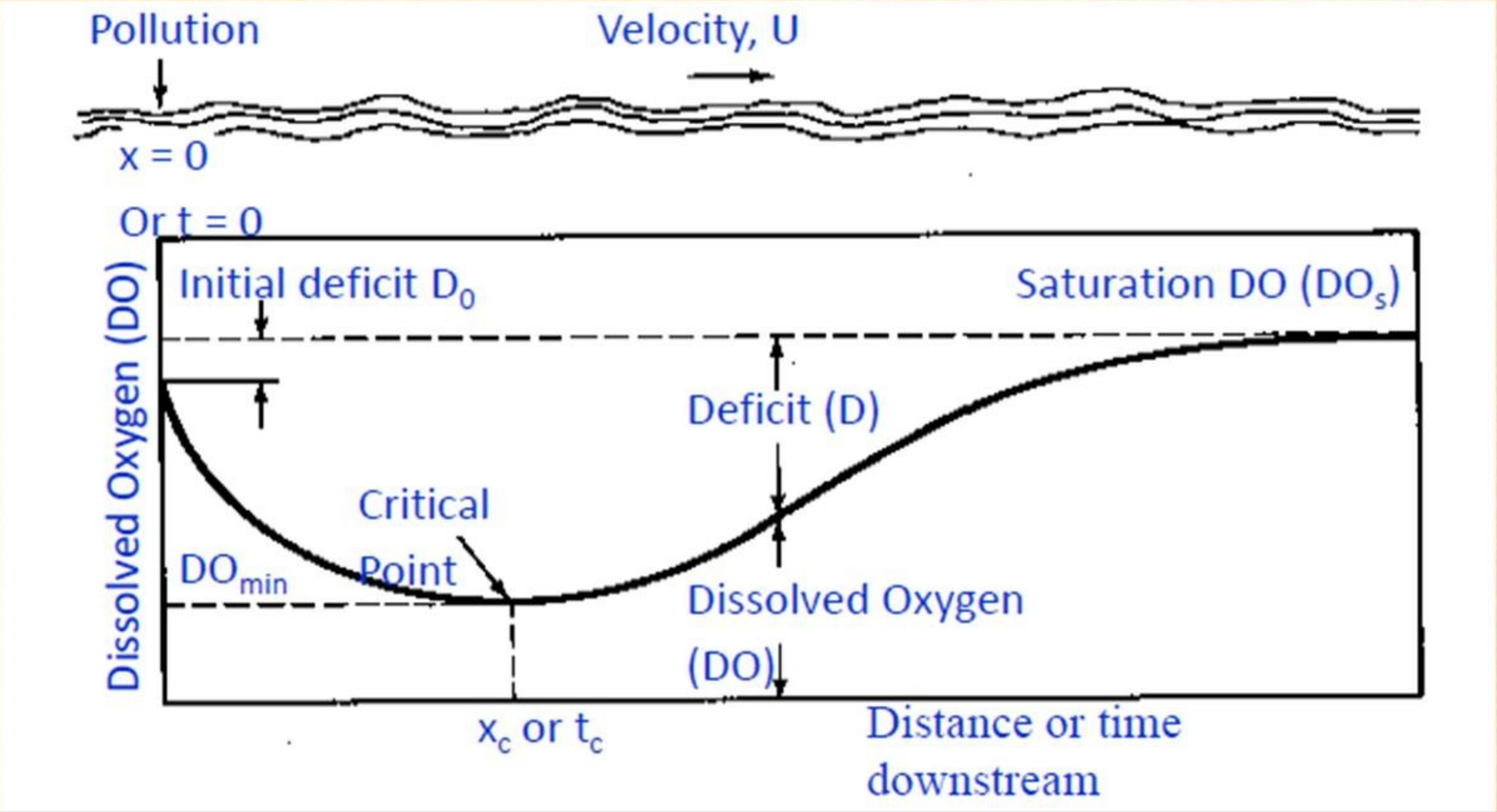
$$D_c = \frac{k_d}{k_a} L_0 e^{-\frac{k_d}{U} x_c} = \frac{k_d}{k_a} L_0 e^{-k_d t_c}$$

- Critical Distance (x_c)

$$x_c = \frac{U}{k_a - k_d} \ln\left\{\frac{k_a}{k_d}\left[1 - \frac{D_o(k_a - k_d)}{k_d L_o}\right]\right\}$$

- Critical Travel Time (t_c)

$$t_c = \frac{1}{k_a - k_d} \ln\left\{\frac{k_a}{k_d}\left[1 - \frac{D_o(k_a - k_d)}{k_d L_o}\right]\right\}$$



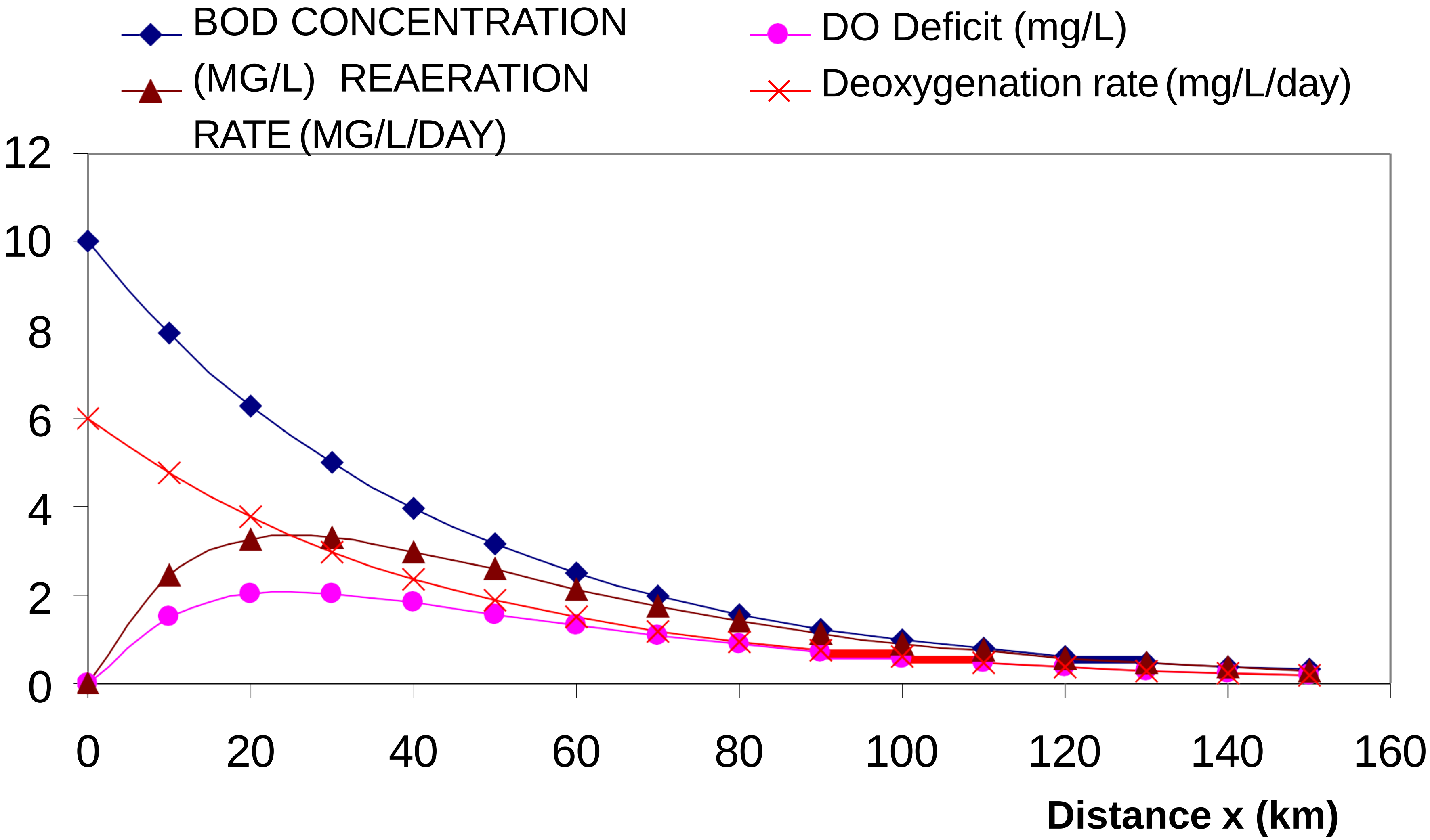
Example 53: The $BOD_{ultimate}$ of a river just below a sewage outfall is 50 mg/L. The upstream DO is saturate, with DO_{sat} = 10 mg/L. The only source of BOD in this river is this single sewage outfall. k_d = 0.30/day, k_a = 0.90/day, U = 48 km/day. (a) find critical distance downstream at which DO is minimum; (b) find minimum DO (DO at critical point); (c) what fraction of BOD must be removed from sewage to assure a minimum of 5.0 mg/L DO?

Example 54: A plug-flow stream at 20 ℃ has a velocity of 0.3 m/s,a mean depth of 1.2 m, and a deoxygenation rate constant of 0.6/day. The ultimate BOD concentration at x = 0 is 10.0 mg/L, and the DO deficit at x = 0 is 0.0 mg/L. Estimate the reaeration rate constant and plot (a) the BOD concentration versus downstream distance, (b) the DO deficit versus downstream distance, and (c) the reaeration rate (in mg/L/day) and the deoxygenation rate (in mg/L/day) versus distance. Where is the reaeration rate equal to the deoxygenation rate? What is the critical deficit and distance?

SOLUTION:

$$k_a = 3.9\sqrt{\frac{u}{H^3}} = 3.9\sqrt{\frac{0.3}{1.2^3}} = 1.63 d^{-1}$$

x (km)	L (mg/L)	k_dL (mg/L/day)	D (mg/L)	k_aD (mg/L/day)
0	10.00	6.00	0.00	0.00
10	7.93	4.76	1.52	2.47
20	6.29	3.78	2.01	3.28
30	4.99	3.00	2.03	3.30
40	3.96	2.38	1.84	2.99
50	3.14	1.89	1.58	2.58
60	2.49	1.50	1.32	2.15
70	1.98	1.19	1.08	1.76
80	1.57	0.94	0.88	1.43
90	1.25	0.75	0.71	1.15
100	0.99	0.59	0.56	0.92
110	0.78	0.47	0.45	0.73
120	0.62	0.37	0.36	0.59
130	0.49	0.30	0.29	0.47
140	0.39	0.23	0.23	0.37
150	0.31	0.19	0.18	0.29


BOD CONCENTRATION (MG/L)
REAERATION RATE (MG/L/DAY)
DO Deficit (mg/L)
Deoxygenation rate (mg/L/day)
12
10
8
6
4
2
0
0
20
40
60
80
100
120
140
160
Distance x (km)

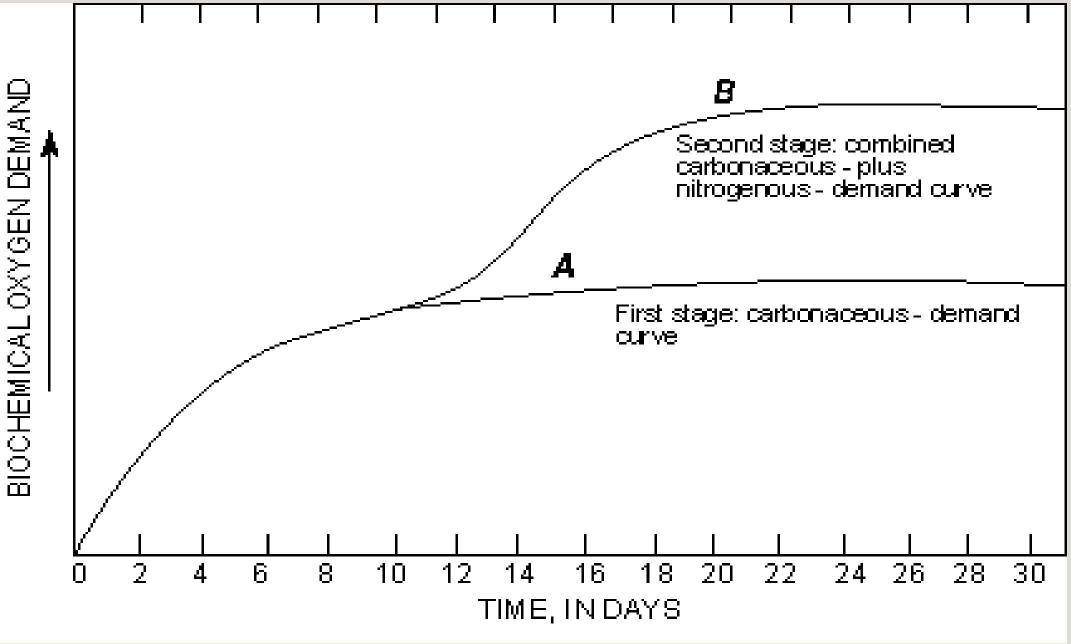
5. MODIFICATIONS TO STREETER-PHELPS EQUATION

(1) Sources and sinks of dissolved oxygen:

- Reaeration
- ~~Oxidation of carbonaceous BOD (CBOD)~~
  - ✓ represents the portion of oxygen demand involved in the conversion of organic carbon to carbon dioxide.
- Oxidation of nitrogenous BOD (NBOD)

$$NH_4^+ + 2O_2 \xrightarrow{\text{bacteria}} NO_3^- + H_2O + 2H^+$$

  - ✓ represents the portion of oxygen demand involved in the conversion of organic nitrogen, ammonia, and nitrite to nitrate.



Biochemical oxygen demand curves: (*A*) typical carbonaceous-demand curve showing the oxidation of organic matter, and (*B*) typical carbonaceous- plus nitrogeneous-demand curve showing the oxidation of ammonia and nitrite. (Modified from Sawyer and McCarty, 1978.)

(1) SOURCES AND SINKS OF DISSOLVED OXYGEN:

- Photosynthesis
- Respiration
- Sediment oxygen demand
- Background BOD (nonpoint source runoff)


http://ian.umces.edu/imagelibrary/displayimage-1032.html

(2) Background DO deficit

- Continuous input of CBOD due to runoff from agricultural land, storm water discharge, and highway runoff etc. ➔ Background BOD ➔ Background DO deficit

$$\frac{\partial D}{\partial t} = -U\frac{\partial D}{\partial x} - k_a D + k_d L$$

➔ For steady state:

$$u\frac{dD}{dx} = -k_a D + k_d L$$

- under steady-state conditions and continuous background concentration of CBOD ➔ constant background DO deficit:

$$u\frac{dD}{dx} = -k_a D_b + k_d L_b = 0$$

➔

$$D_b = \frac{k_d L_b}{k_a}$$

D_b typically 0.5 ~2 mg/L ➔ safety factor for designing waste load allocation

(3) STEADY STATE ONE-DIMENSIONAL CBOD, NBOD, AND DISSOLVED OXYGEN MASS BALANCE EQUATIONS FOR PLUG FLOW

$$\frac{\partial L}{\partial t} = -u\frac{\partial L}{\partial x} - k_r L$$

$$\frac{\partial L^N}{\partial t} = -u\frac{\partial L^N}{\partial x} - k_N L^N$$

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} - (k_d L + k_N L^N) + k_a(C_S - C) + P - R - \frac{S_B}{H}$$

Where u = river velocity [L/T]; x = distance [L]; L = CBOD concentration [M/L^3]; k_r = total CBOD loss rate constant [1/T]; L^N = NBOD concentration [M/L^3]; k_N = NBOD decay rate constant [1/T]; C = Dissolved oxygen concentration [ML^{-3}]; k_d = CBOD deoxygenation rate constant [1/T]; C_S = saturation concentration of dissolved oxygen [M/L^3]; P = Average gross photosynthetic production of dissolved oxygen [M/(L^3·T)]; R = Average respiration rate [M/(L^3·T)]; S_B = Areal sediment oxygen demand (SOD) [M/(L^2·T)]; H = Mean depth of river [L]

- Assume L = particulate CBOD + dissolved CBOD = $L_p + L_d$
  - ✓ Particulate CBOD ➔ settling rate constant k_s
  - ✓ Dissolved CBOD ➔ deoxygenation rate constant k_d
  - ✓ Loss rate constant $k_r = k_s + k_d$

Use $D = C_s - C$ ➔ $$\frac{\partial D}{\partial t} = -u\frac{\partial D}{\partial x} + (k_d L + k_N L^N) - k_a D - P + R + \frac{S_B}{H}$$

- Steady State:

$$-u\frac{dL}{dx} - k_r L = 0$$ ➔ $$L(x) = L_0 e^{-\frac{k_r}{U}x}$$ and $$-u\frac{dL^N}{dx} - k_N L^N = 0$$ ➔ $$L^N(x) = L^N{}_0 e^{-\frac{k_N}{U}x}$$

$$-u\frac{dD}{dx} + (k_d L + k_N L^N) - k_a D - P + R + \frac{S_B}{H} = 0$$

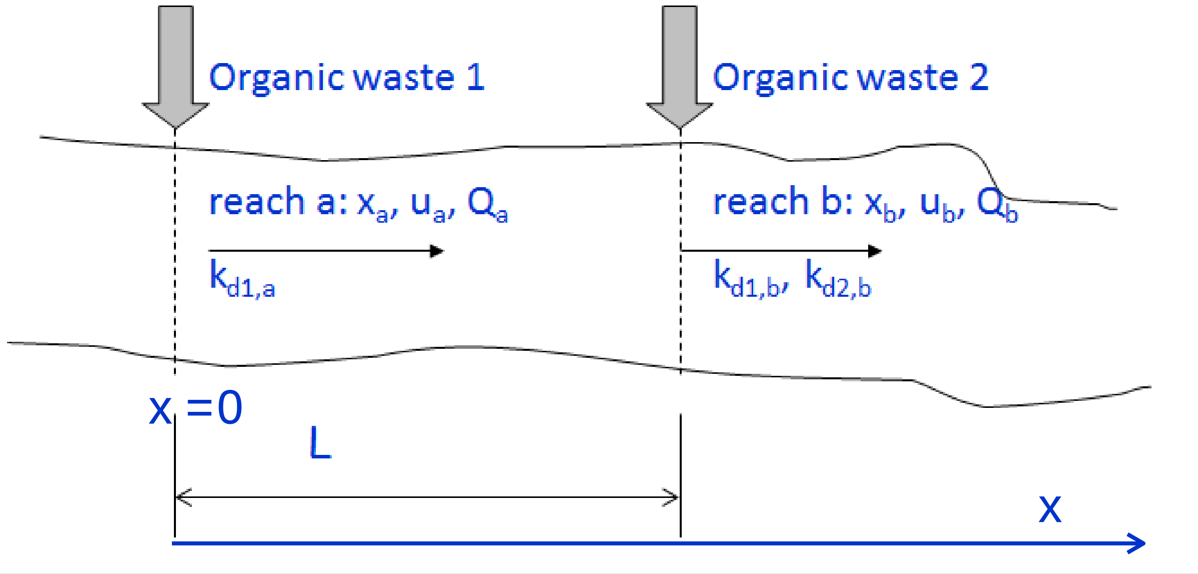
➔ $$u\frac{dD}{dx} + k_a D = k_d L + k_N L^N + (R - P) + \frac{S_B}{H} = k_d L_0 e^{-k_r\left(\frac{x}{U}\right)} + k_N L^N{}_0 e^{-k_N\left(\frac{x}{U}\right)} + (R - P) + \frac{S_B}{H}$$

(4) Principle of Superposition

- linear differential equation ➔ the additional sources and sinks of dissolved oxygen can be solved individually ➔ their solutions can be summed to yield the overall solution
- summation of the various solutions for each source and sink

$$D = D_0 e^{-k_a x/u} + \frac{k_d L_0}{k_a - k_r}\left(e^{-k_r x/u} - e^{-k_a x/u}\right) + \frac{k_N L_{N_0}}{k_a - k_N}\left(e^{-k_N x/u} - e^{-k_a x/u}\right)$$
$$+ \frac{S_B}{k_a H}\left(1 - e^{-k_a x/u}\right) + \frac{R - P}{k_a}\left(1 - e^{-k_a x/u}\right) + \frac{k_d L_b}{k_a}$$

6. Multiple Sources



u_a and u_b: flow velocity in reach a and reach b

Q_a and Q_b: flow rate in reach a and reach b

$C_{1a,0}$: BOD at x = 0 due to waste 1 in reach a

$C_{1b,0}$: BOD at x = L due to waste 1 in reach b

$C_{2b,0}$: BOD at x = L due to waste 2 in reach b

C_A AND C_B: BOD IN REACH A AND REACH B

C_{1a}: BOD due to waste 1 in reach a

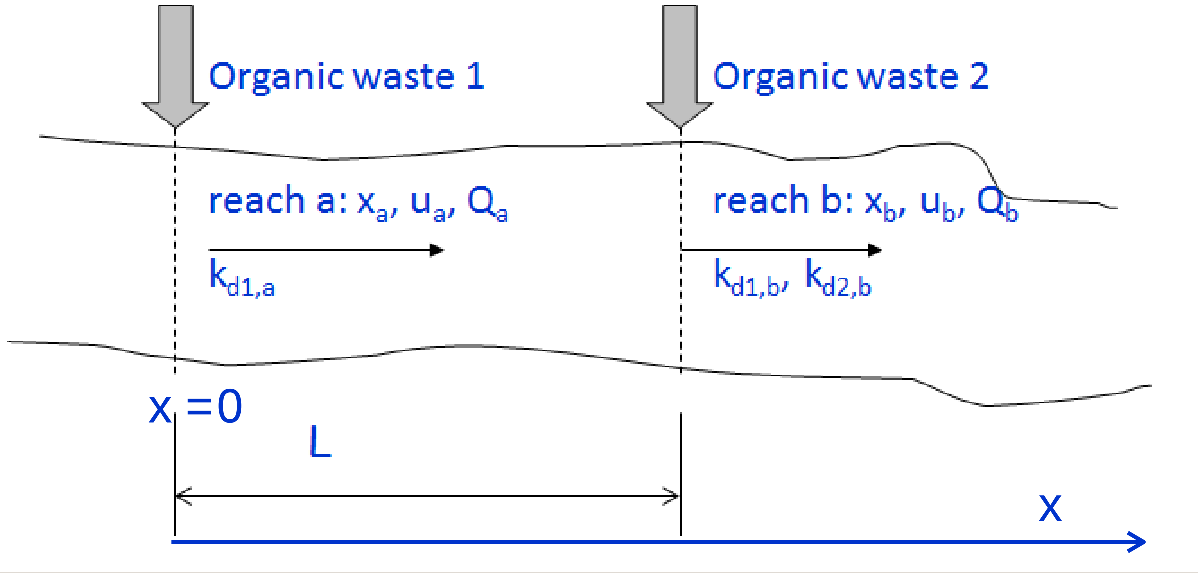
C_{1b}: BOD due to waste 1 in reach b

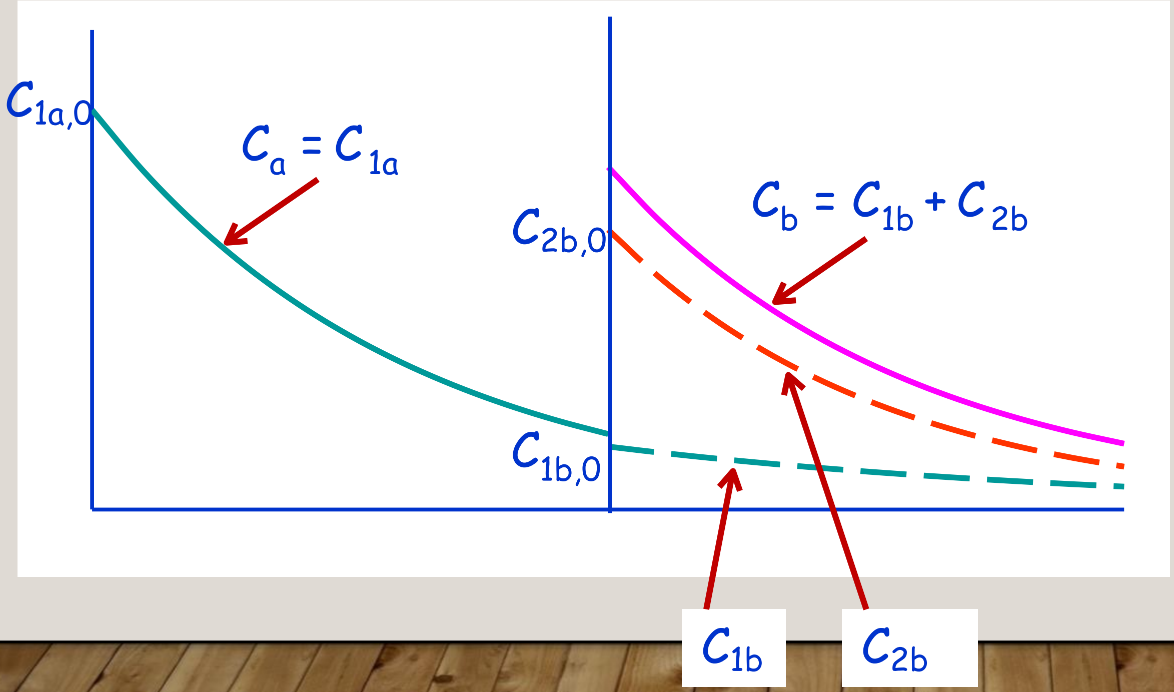
C_{2b}: BOD due to waste 2 in reach b

$k_{d1,a}$ and $k_{d1,b}$: deoxygenation rate constant of waste 1 in reach a and reach b

$k_{d2,b}$: deoxygenation rate constant of waste 2 in reach b

→ $C_a = C_{1a}$ and $C_b = C_{1b} + C_{2b}$





$$C_{1A} = C_{1A,0} E^{\frac{-k_{d\,1,a}}{u_a} x}$$

$$C_{1b,0} = \frac{\left(Q_a C_{1a,0} e^{\frac{-k_{d\,1,a}}{u_a} L} \right)}{Q_b}$$

$$C_{1b} = C_{1b,0} e^{-\frac{k_{d\,1,b}}{u_b}(x-L)}$$

$$C_{2b} = C_{2b,0} e^{-\frac{k_{d\,2,b}}{u_b}(x-L)}$$

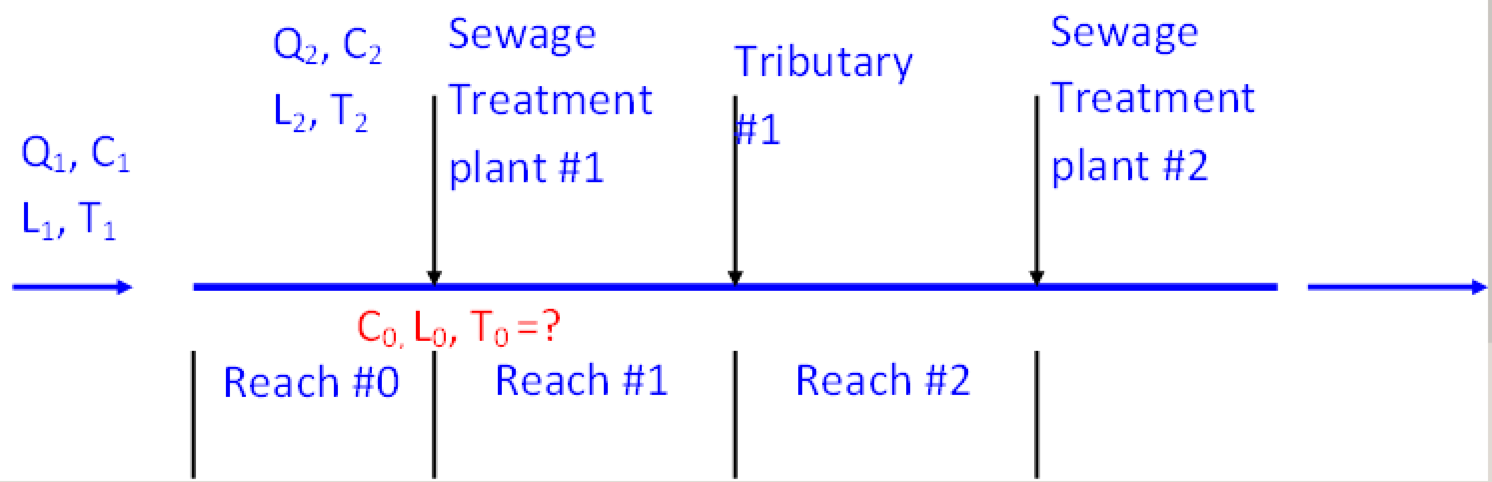
$$C_b = C_{1b} + C_{2b}$$

7. RIVER SEGMENTATION

- River segment is necessary when boundary conditions or parameters change in a dissolved oxygen model
- Boundary concentration may change:
  - ✓ Tributary input or confluence
  - ✓ Wastewater discharge
  - ✓ Dams or rapids
  - ✓ i.e. changes in the boundary concentrations D_0, L_0 and L^N_0 would require a new stream segment
- Process ➔ dividing the river into segments of constant coefficients

http://www.careers.ch2m.com/images/locations/canada/pine-creek-wastewater-treatment-plant-lg.jpg

http://www.onegeology.org/extra/kids/earthprocesses/rivers.html



➔ $$L_0 = \frac{Q_1 L_1 + Q_2 L_2}{Q_1 + Q_2}$$ and $$T_0 = \frac{Q_1 T_1 + Q_2 T_2}{Q_1 + Q_2}$$

$$C_0 = \frac{Q_1 C_1 + Q_2 C_2}{Q_1 + Q_2}$$ ➔ $$D_0 = C_{sat} - C_0$$

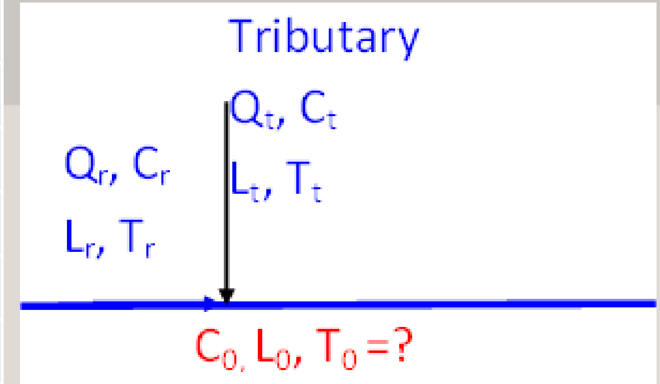
Where C_{sat} is saturated dissolved oxygen concentration at T = T_0

➔ Then for reach #1:

$$L(x) = L_0 e^{-\frac{k_d}{U}x}$$ and $$D(x) = D_0 e^{-\frac{k_a}{U}x} + \frac{k_d L_0}{k_a - k_d}\left(e^{-\frac{k_d}{U}x} - e^{-\frac{k_a}{U}x}\right)$$

Example 55: A tributary merges with a river. Calculate the concentrations of dissolved oxygen, DO deficit, CBOD, and temperature at the confluence for use in a DO model.

River	Tributary
T = 26.3 °C	T = 24.9 °C
$C_{sat,r}$ = 8.4 mg/L	$C_{sat,t}$ = 8.6 mg/L
C_r = 7.3 mg/L (DO)	C_t = 6.8 mg/L (DO)
L_r = 3.0 mg/L (CBOD)	L_t = 6.0 mg/L (CBOD)
Q_r = 2000 cfs	Q_t = 500 cfs



Solution:

$$L_0 = \frac{Q_r L_r + Q_t L_t}{Q_r + Q_t} = \frac{2000 \times 3.0 + 500 \times 6.0}{2000 + 500} = 3.6 mg/L$$

$$T_0 = \frac{Q_r T_r + Q_t T_t}{Q_r + Q_t} = \frac{2000 \times 26.3 + 500 \times 24.9}{2000 + 500} = 26°C$$

$$\rightarrow C_{sat} = 8.3 mg/L$$

$$C_0 = \frac{Q_r C_r + Q_t C_t}{Q_r + Q_t} = \frac{2000 \times 7.3 + 500 \times 6.8}{2000 + 500} = 7.2 mg/L$$

$$\rightarrow \begin{aligned} D_0 &= C_{sat} - C_0 \\ &= 8.3 - 7.2 \\ &= 1.1 mg/L \end{aligned}$$

8. ADVECTION-DISPERSION MODEL FOR LARGE RIVERS AND ESTUARIES

- Governing equation for BOD:

$$\frac{\partial L}{\partial t} = -u\frac{\partial L}{\partial x} + E\frac{\partial^2 L}{\partial x^2} - k_d L$$

- where $L(x,t)$ = BOD$_{\text{ultimate}}$ at x and t $[ML^{-3}]$, x = distance [L], t = time [T], u = mean velocity (freshwater) in the river or estuary (equal to freshwater flow rate divided by average cross-sectional area) $[LT^{-1}]$, E
- = dispersion coefficient $[L^2T^{-1}]$, k_d = deoxygenation rate constant $[T^{-1}]$.

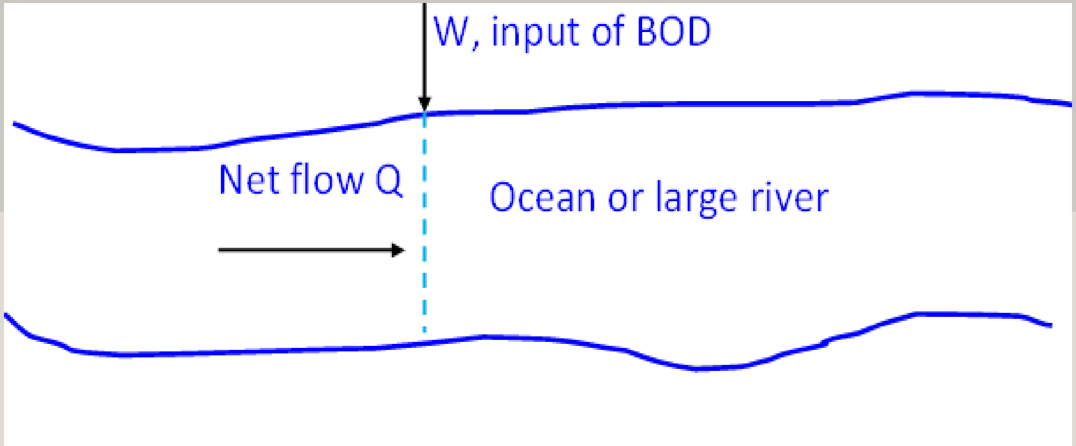
for DO:

$$\frac{\partial D}{\partial t} = -u\frac{\partial D}{\partial x} + E\frac{\partial^2 D}{\partial x^2} + k_d L - k_a D$$

where $D(x,t)$ = D.O. deficit at x and t $[ML^{-3}]$, k_a = reaeration rate constant $[T^{-1}]$.

➢ Steady-state conditions:

$$\begin{cases} -u\dfrac{dL}{dx} + E\dfrac{d^2L}{dx^2} - k_d L = 0 \\ -u\dfrac{dD}{dx} + E\dfrac{d^2D}{dx^2} + k_d L - k_a D = 0 \\ L(\pm\infty) = 0 \\ L(0) = L_0 \\ D(\pm\infty) = 0 \\ D(0) = D_0 \end{cases}$$



➢ Solutions to the steady-state equations:

$$L(x) = L_0 \exp\left[\frac{ux}{2E}(1 \pm m_1)\right]$$

$$L_0 = \frac{W}{Qm_1}$$

$$m_1 = \sqrt{1 + \frac{4k_d E}{u^2}}$$

and

$$D(x) = \frac{k_d}{(k_a - k_d)}\frac{W}{Q}\left\{\frac{1}{m_1}\exp\left[\frac{ux}{2E}(1 \pm m_1)\right] - \frac{1}{m_2}\exp\left[\frac{ux}{2E}(1 \pm m_2)\right]\right\}$$

$$m_2 = \sqrt{1 + \frac{4k_a E}{u^2}}$$

where W = wastewater $\mathrm{BOD}_{\mathrm{ultimate}}$ discharge $[MT^{-1}]$, Q = river or estuary flow rate $[L^3T^{-1}]$

Example 56: A continuous wastewater BOD discharge is released to a coastal plain estuary. Laboratory and field measurements were performed to estimate the parameters given below: E = 15 mi^2/day, k_d = 0.15/day, u = 3.57 mi/day, k_a = 0.18/day, W/Q = 18.5 mg/L. Solve for the steady-state BOD concentration and the D.O. deficit both upstream and downstream from the discharge point (x = 0).

SOLUTI

$$m_1 = \sqrt{1+\frac{4k_d E}{u^2}} \quad \sqrt{1+\frac{4\times 0.15d^{-1}\times 15mi^2/d}{(3.57mi/d)^2}} = 1.31$$

$$m_2 = \sqrt{1+\frac{4k_a E}{u^2}} = \sqrt{1+\frac{4\times 0.18\times 15}{(3.57)^2}} = 1.36$$

$$L_0 = \frac{W}{Qm_1} = \frac{18.5mg/L}{1.31} = 14.16mg/L$$

$$\frac{u}{2E} = \frac{3.57mi/d}{2\times 15mi^2/d} = 0.119mi^{-1}$$

$$\frac{u}{2E}(1+m_1) = 0.119\times(1+1.31) = 0.2749$$

$$\frac{u}{2E}(1-m_1) = 0.119\times(1-1.31) = -0.03689$$

$$\frac{u}{2E}(1+m_2) = 0.119\times(1+1.36) = 0.2808$$

$$\frac{u}{2E}(1-m_2) = 0.119\times(1-1.36) = -0.04284$$

$$\frac{k_d}{(k_a-k_d)}\frac{W}{Q} = \frac{0.15}{(0.18-0.15)}\times 18.5 = 92.5mg/L$$

➔ THEN FOR UPSTREAM (X

$$L(x) = L_0 \exp\left[\frac{ux}{2E}(1+m_1)\right] = 14.16e^{0.2749x}$$

$$D(x) = \frac{k_d}{(k_a - k_d)}\frac{W}{Q}\left\{\frac{1}{m_1}\exp\left[\frac{ux}{2E}(1+m_1)\right] - \frac{1}{m_2}\exp\left[\frac{ux}{2E}(1+m_2)\right]\right\}$$

$$= 92.5\times\left(0.763e^{0.2749x} - 0.735e^{0.2808x}\right)$$

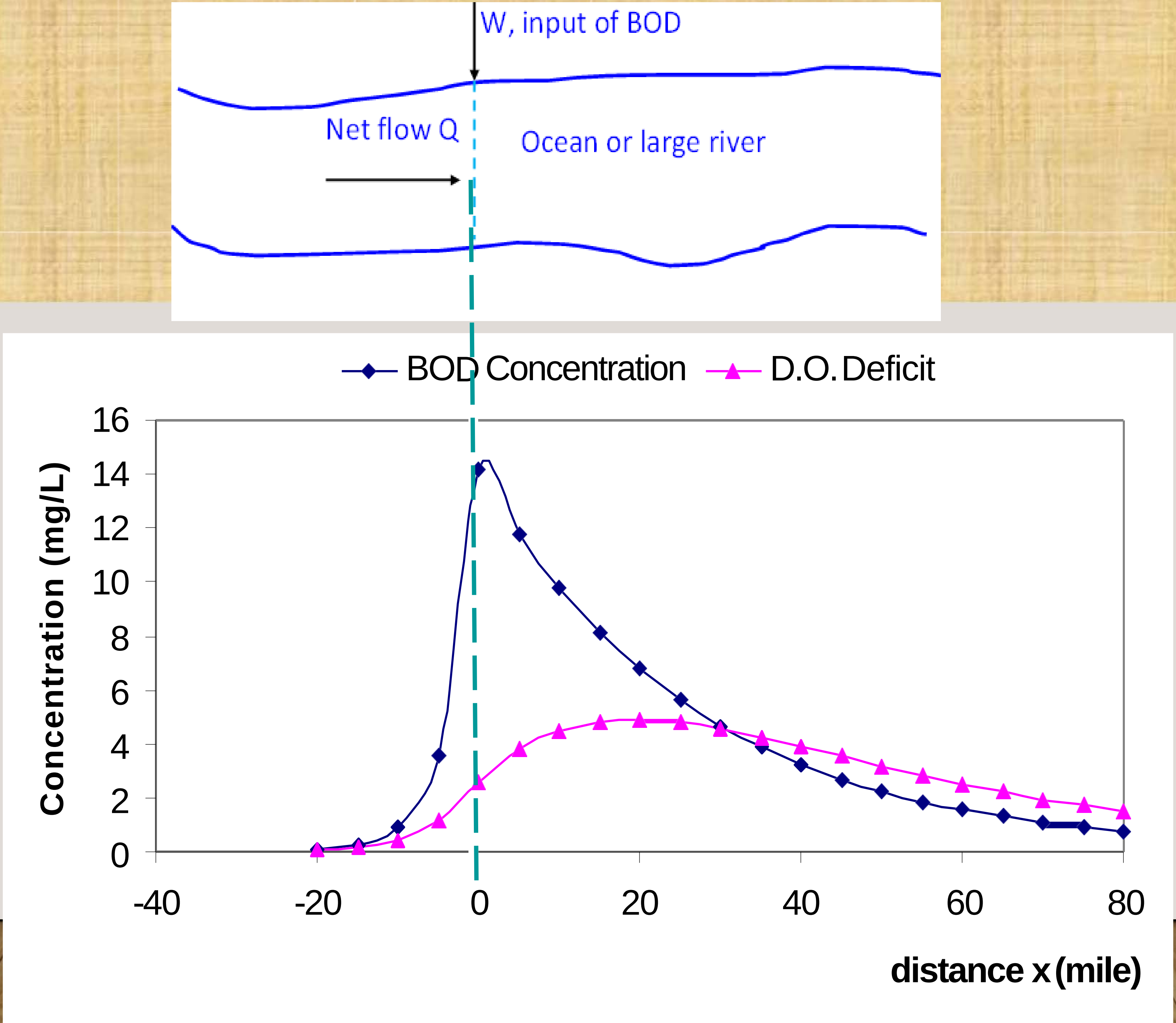
➔ Then for downstream ($x \geq 0$):

$$L(x) = L_0 \exp\left[\frac{ux}{2E}(1-m_1)\right] = 14.16e^{-0.03689x}$$

$$D(x) = \frac{k_d}{(k_a - k_d)}\frac{W}{Q}\left\{\frac{1}{m_1}\exp\left[\frac{ux}{2E}(1-m_1)\right] - \frac{1}{m_2}\exp\left[\frac{ux}{2E}(1-m_2)\right]\right\}$$

$$= 92.5\times\left(0.763e^{-0.03689x} - 0.735e^{-0.04284x}\right)$$

x (mile)	L (mg/L)	D (mg/L)
-20	0.06	0.04
-15	0.23	0.14
-10	0.91	0.42
-5	3.58	1.16
0	14.16	2.60
5	11.77	3.82
10	9.79	4.51
15	8.14	4.83
20	6.77	4.89
25	5.63	4.77
30	4.68	4.53
35	3.89	4.23
40	3.24	3.89
45	2.69	3.53
50	2.24	3.18
55	1.86	2.84
60	1.55	2.52
65	1.29	2.22
70	1.07	1.95
75	0.89	1.70
80	0.74	1.48


W, input of BOD
Net flow Q
Ocean or large river
BOD Concentration
D.O. Deficit
Concentration (mg/L)
16
14
12
10
8
6
4
2
0
-40
-20
0
20
40
60
80
distance x (mile)

Thank you
Any question ?

Chapter Four

Eutrophication and Lake Morphology

Introduction

- *Eutrophication* is an enrichment of water by nutrient salts that causes structural changes to the ecosystem such as:
  - ✓ increased production of algae and aquatic plants,
  - ✓ depletion of fish species,
  - ✓ general deterioration of water quality and other effects that reduce and preclude use
- The cultural eutrophication process consists of a continuous increase in the contribution of nutrients, mainly nitrogen and phosphorus (organic load) until it exceeds the capacity of the water body (i.e. the capacity of a lake, river or sea to purify itself) , triggering structural changes in the waters.
- These structural changes mainly depend on 3 factors:

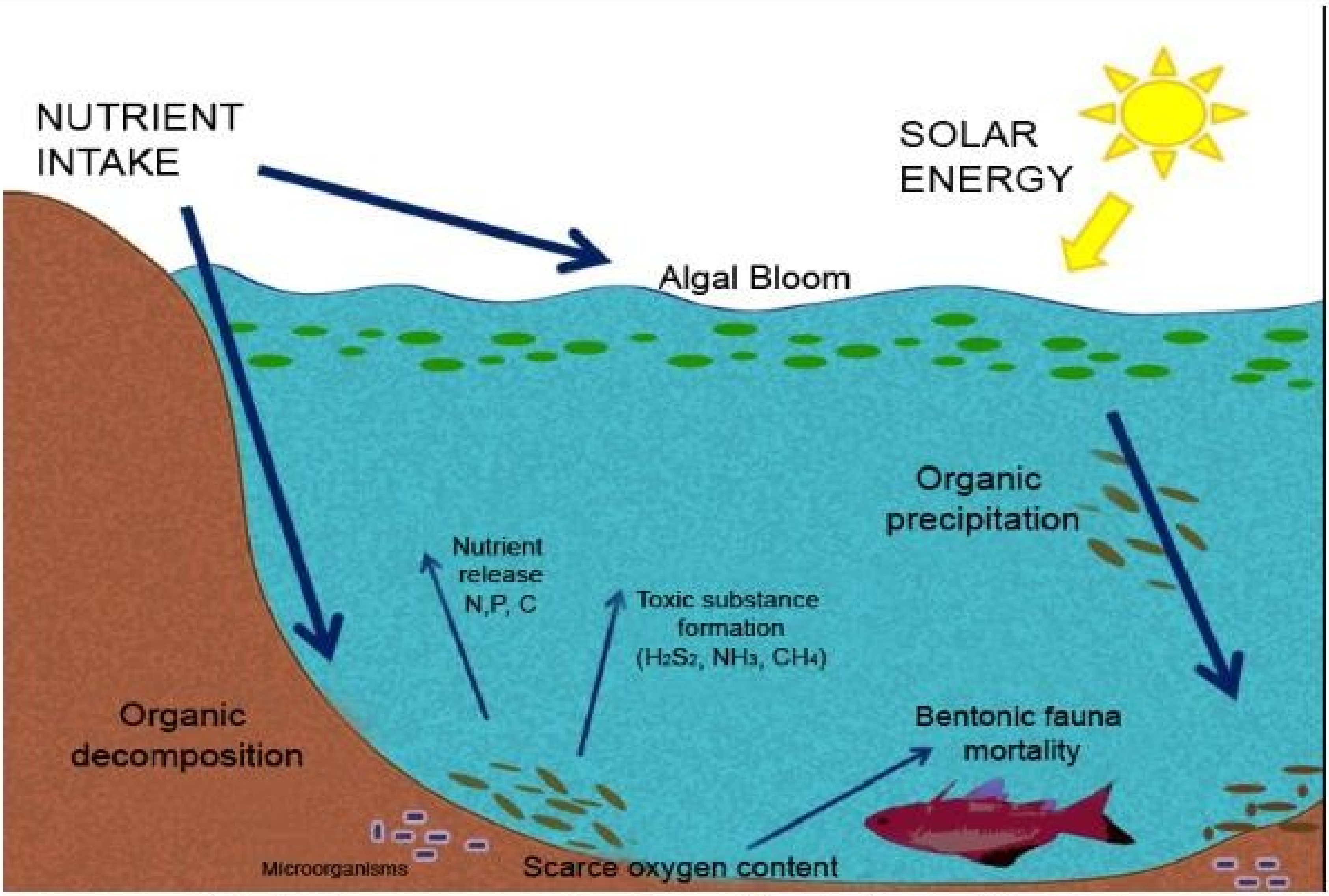
Introduction

- Use of fertilizers: Agricultural practices and the use of fertilizers in the soil contribute to the accumulation of nutrient

- Discharge of waste water into water bodies: In various parts of the world, and particularly in developing countries, waste water is discharged directly into water bodies such as rivers, lakes and seas.

- Reduction of self purification capacity: Over the years, lakes accumulate large quantities of solid material transported by the water (sediments).

Introduction

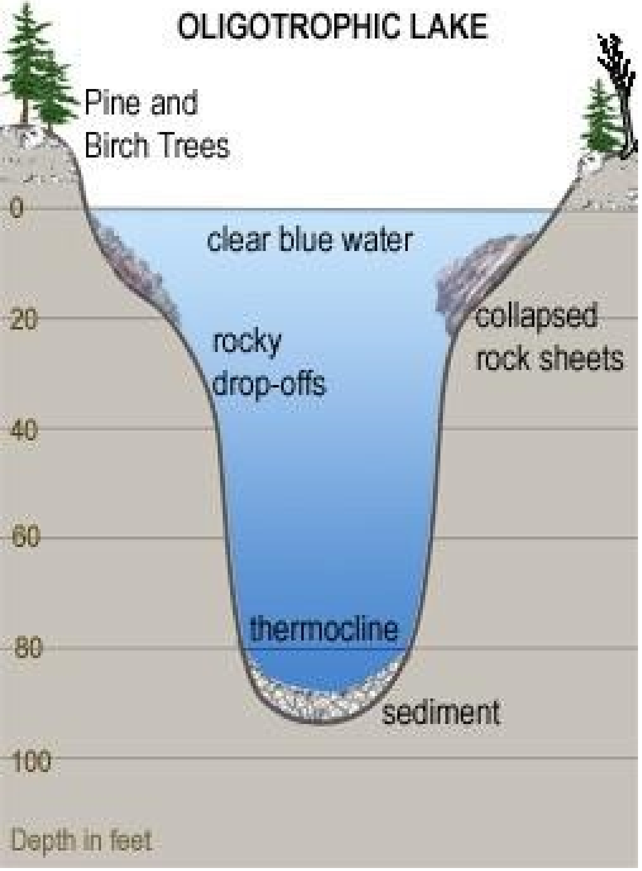


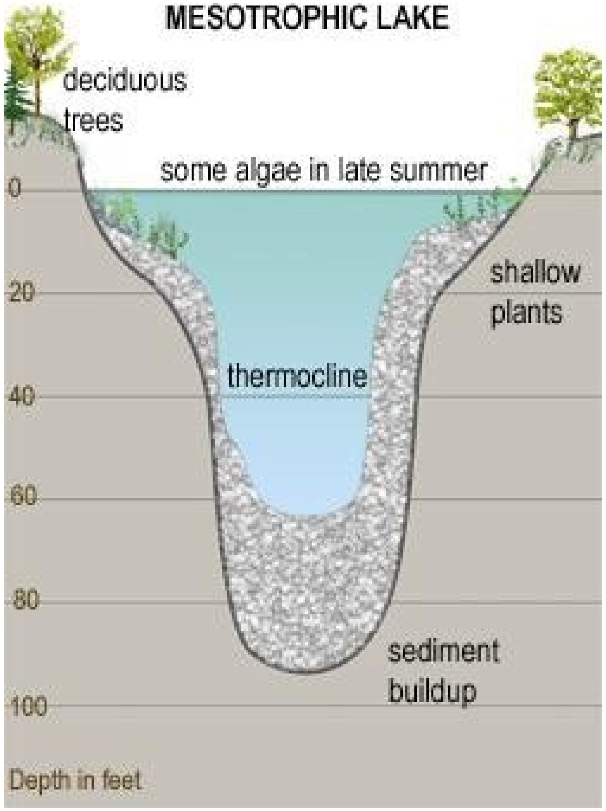
Introduction

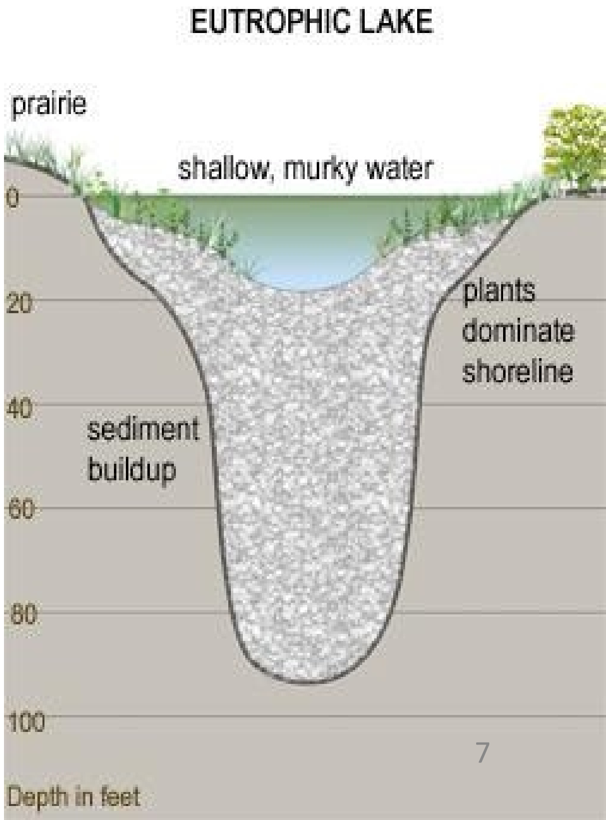
Eutrophication Problem & Nutrients

- The addition of nutrients (Phosphorus an Nitrogen) to natural water stimulates plant growth.
- When lakes, streams, and estuaries are overfertilized, the resulting excessive plant growth can become a serious water quality problem.
- "overfertilization" is referred to as "eutrophication", originally used to describe aging process whereby a lake is transformed from a lake to a marsh to a meadow (fill the lake with sediments).
- "***Cultural eutrophication***" – the lake aging process is quickened or accelerated by *excess nutrients* from ***human activities***.

Lake Classifications by Trophic Status







Measurement Techniques for eutrophication

- Counts (number/milliliter) of specific phytoplankton species;
- Cell volume of species (biomass);
- Chlorophyll-a concentration (µg/l) – a measure of the gross level of phytoplankton; or
- Secchi disk depth (m) – a surrogate (quick and easy to measurement)
- Secchi disk a small disk, divided into black and white quadrants, that is lowered into water to visually observe water clarity and estimate the depth of the euphotic zone

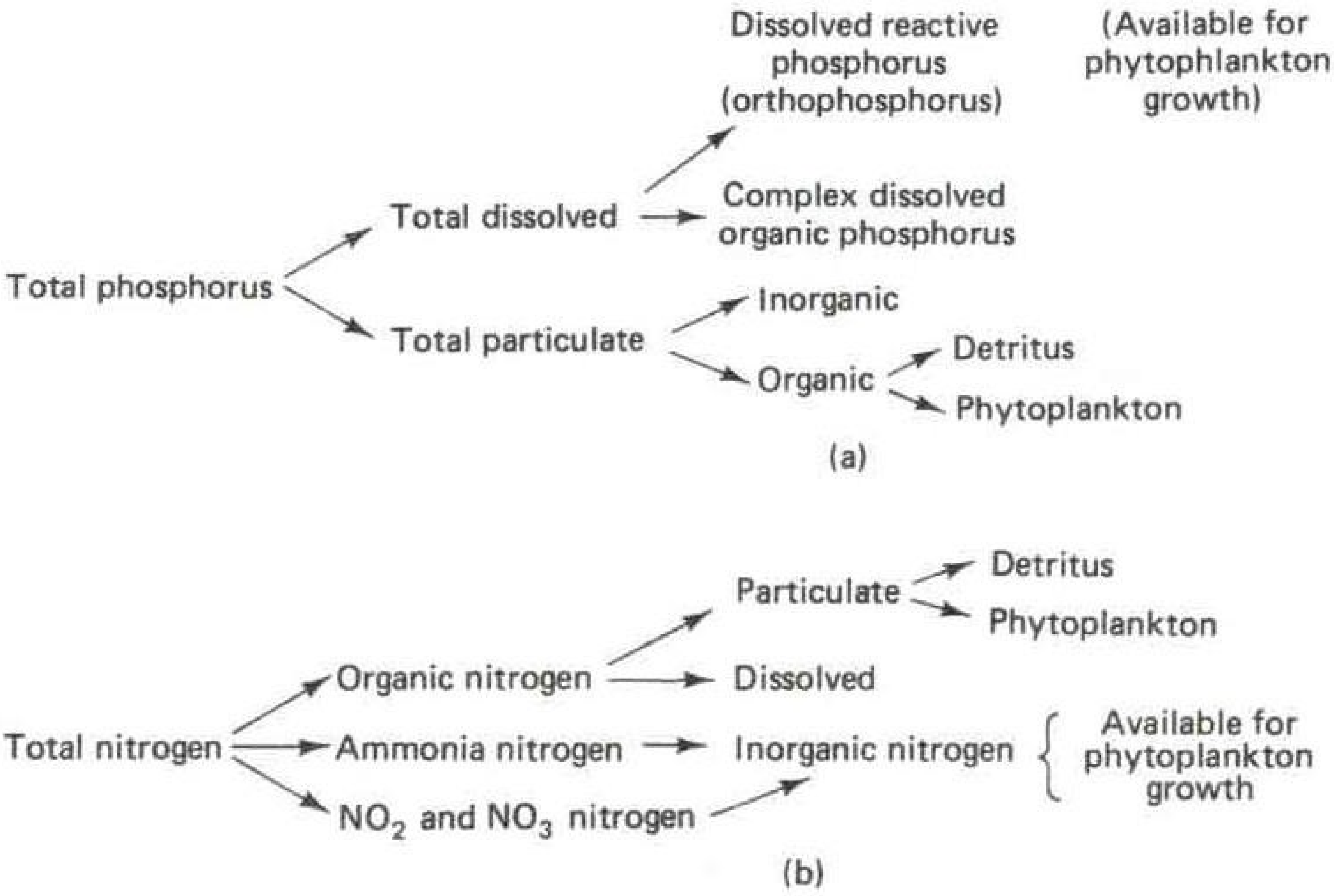
Nutrients causing eutrophication

- ❖ Phosphorus (P) and Nitrogen (N)

- ❖ Phosphorus is usually in short supply relative to the other macronutrients because
- ✓ (1) it is not abundant in the earth’s crust,
- ✓ (2) it does not exist in a gaseous form (not in atmosphere),
- ✓ (3) phosphate tends to sorb strongly to fine-grained particles, then settles to the bottom sediments.

- ❖ Many human activities results in phosphorus discharge to natural waters: human and animal waste, detergent, nonpoint sources from agricultural and urban land use (fertilizers or other phosphorus-containing chemicals)

Principal components of nutrients



Dissolved reactive phosphorus (orthophosphorus)

(Available for phytophlankton growth)

Complex dissolved organic phosphorus

Total dissolved

Total phosphorus

Total particulate

Inorganic

Organic

Detritus

Phytoplankton

(a)

Particulate

Detritus

Phytoplankton

Organic nitrogen

Dissolved

Total nitrogen

Ammonia nitrogen

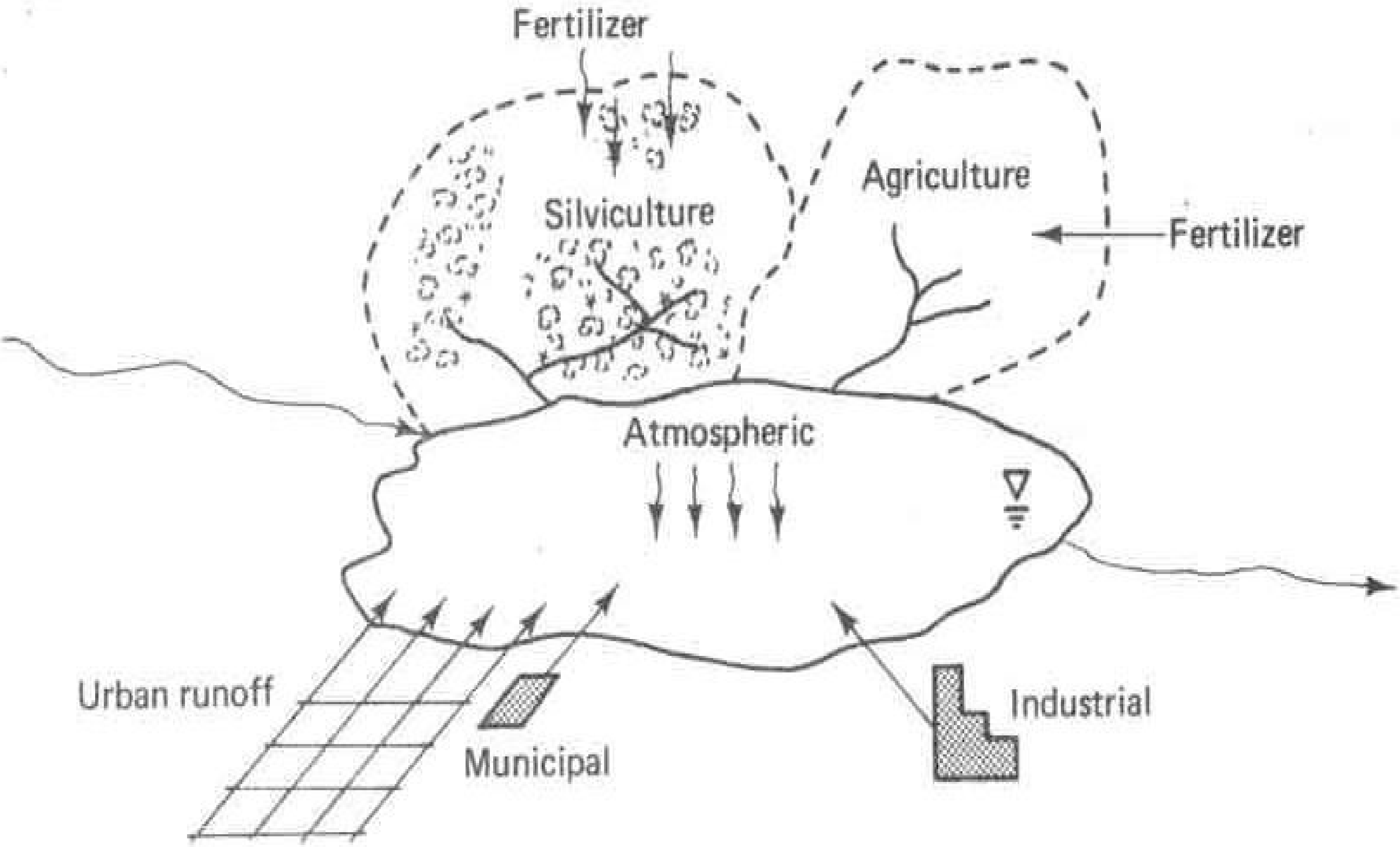
Inorganic nitrogen

Available for phytoplankton growth

NO_2 and NO_3 nitrogen

(b)

Sources of Nutrients in Eutrophication Problems



Limiting nutrient

- TN/TP >> 10: Phosphorus is the limiting nutrient
- TN/TP << 10: Nitrogen is the limiting nutrient Usually:
- Phosphorus is the limiting nutrient for inland water bodies (e.g. lakes)
- Nitrogen is the limiting nutrients for marine systems

Trophic status of lakes

Parameter	Oligotrophic	Mesotrophic	Eutrophic
Total P, mg/l	< 10	10-20	>20
Chlorophyll-a, mg/l	< 4	4-10	>10
Secchi depth, m	>4	2-4	< 2
Hypolimnion O_2 % sat	>80	10-80	< 10

Major effects of eutrophication

Effects on physical, chemical & biological parameters

- ✓ Species diversity decreases
- ✓ Plant and animal biomass increase
- ✓ Turbidity increases
- ✓ Rate of sedimentation increases
- ✓ Anoxic conditions may develop

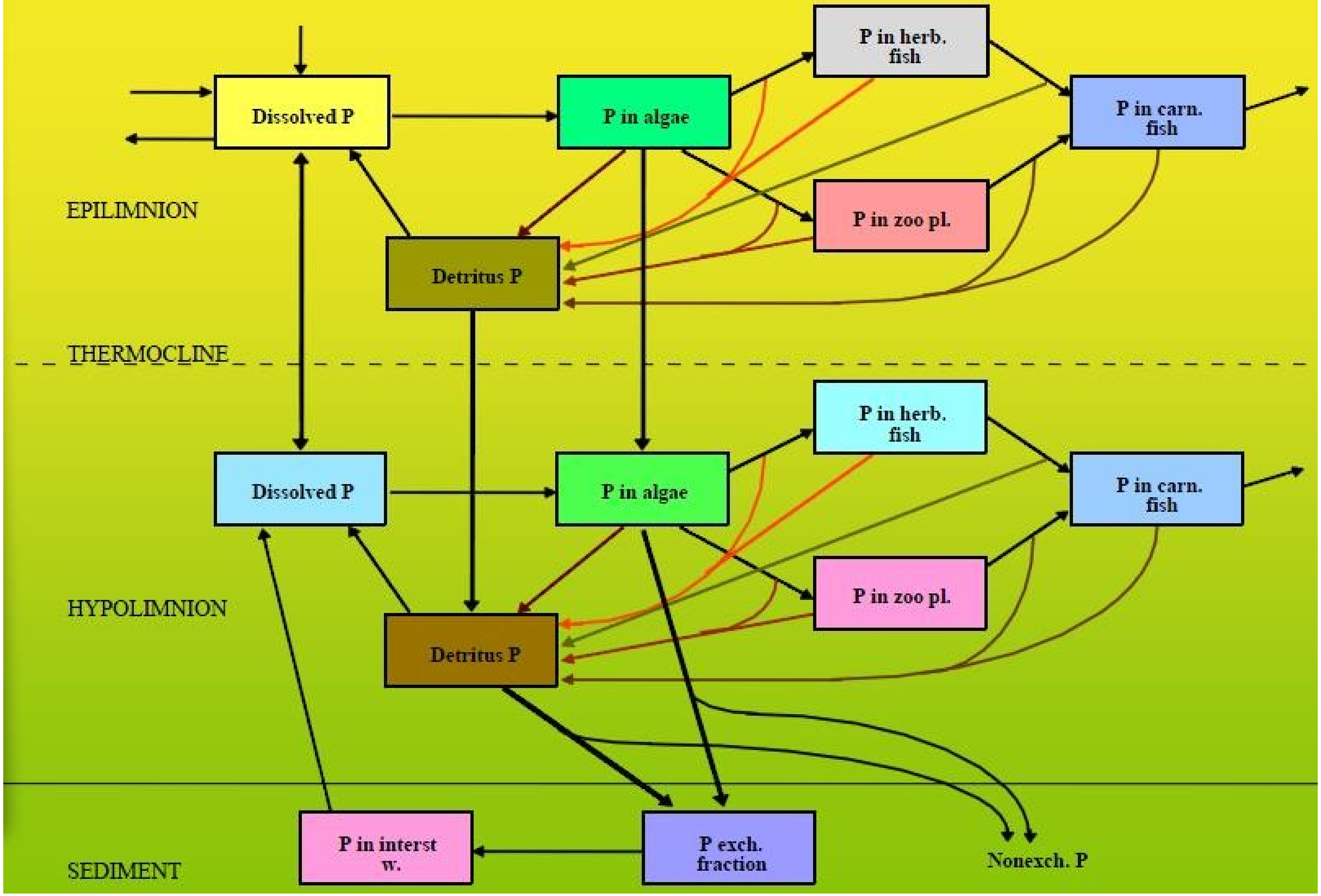
Problems

- ✓ Drinking water treatment may become difficult
- ✓ The water may have unacceptable taste or odor
- ✓ The water may be injurious to health
- ✓ The amenity value of the water may decrease
- ✓ Increased vegetation may impede water flow and navigation
- ✓ Commercially important species may disappear

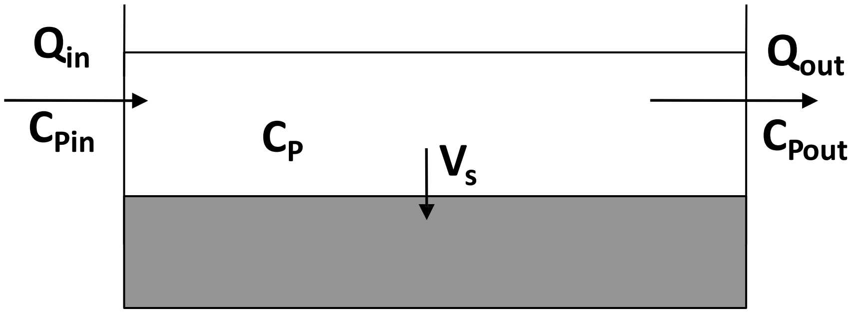
Eutrophication models

- ✓ The water quality constituents of primary interest in eutrophication modeling studies are typically nutrients, phytoplankton, and dissolved oxygen (DO).
- ✓ Have between approximately 5 and 25 constituents, or “state variables.”
- ✓ Model phytoplankton growth as a function of available nutrients, temperature, and light, and include other processes such as phytoplankton mortality, sinking, and exudation.

CONCEPTUAL DIAGRAM OF A PHOSPHORUS RECYCLE MODEL



Simple total phosphorus balance in a lake



C_P = Lake P concentration

V_s = P settling velocity

Mass balance (completely mixed, steady state & no phosphorus generation)

$$0 = Q_{in}C_{Pin} - Q_{out}C_{Pout} - k_s C_P V \qquad (Q_{in} = Q_{out} = Q;\ C_{out} = C_p)$$

$$C_P = \frac{Q\,C_{Pin}}{Q + k_s V}$$

Total phosphorus model

Or, in terms of the hydraulic detention time (t = V/Q)

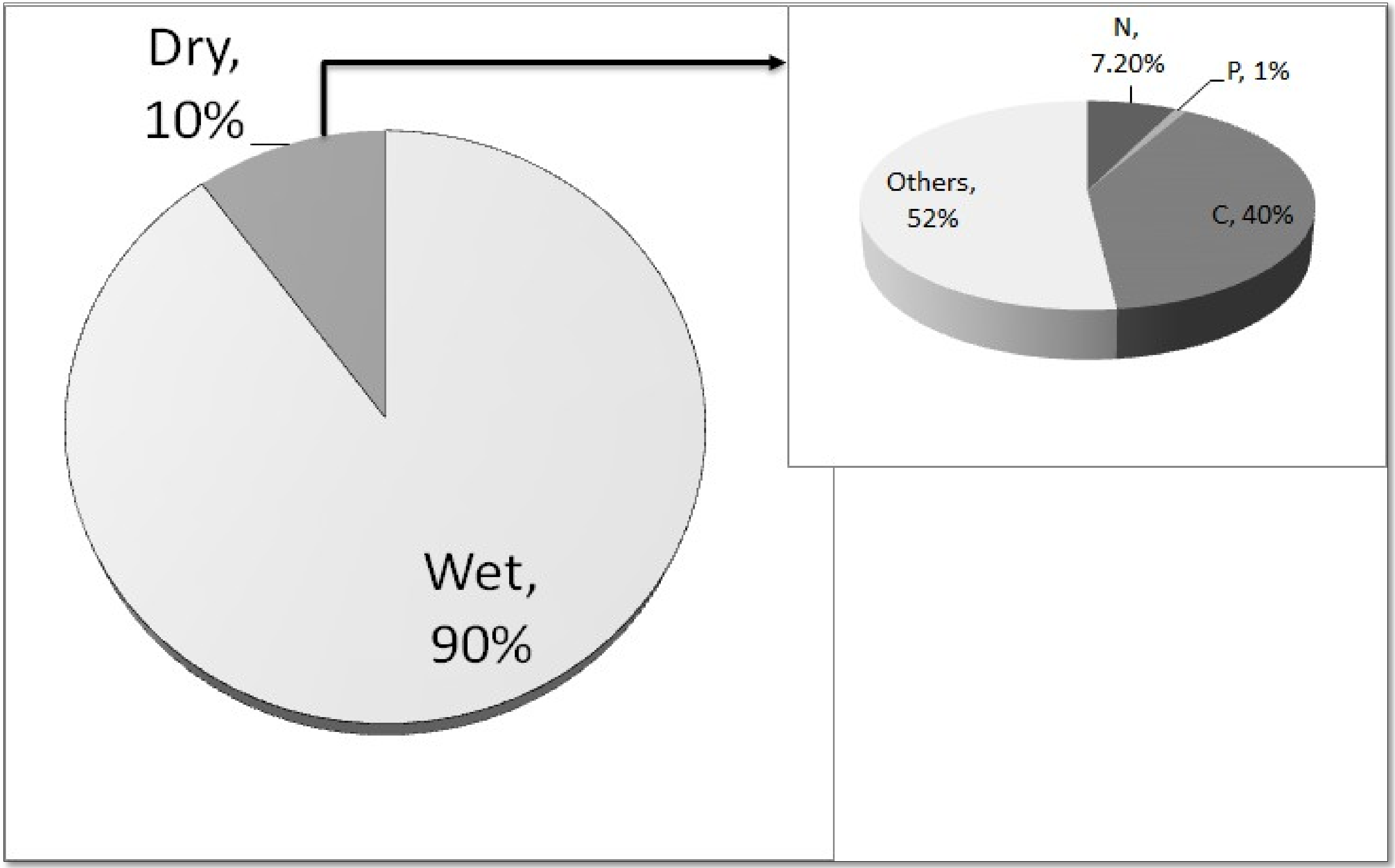
$$C_P = \frac{C_{Pin}}{k_s \tau + 1}$$

Fraction of phosphorus removed (R) = 1- C_p/C_{pin}

k_s = first-order sedimentation coefficient, T^{-1}

It is a surrogate parameter for the mean settling velocity (v_s), reciprocal mean depth (H), and an alpha factor (a) for the ratio of particulate phosphorus to total phosphorus

$$k_s = av_s/H$$


Dry,
10%
Wet,
90%
N,
7.20%
P, 1%
Others,
52%
C, 40%

LAKE MORPHOMETRY

Morphometry of lakes, ponds and streams

- Morphometry is the measurement of *external form or shape of a selected water body*.
- It is that branch of *limnology* which deals with the measurement of significant morphological features of any basin and its included water mass is known as morphometry.
- Morphometry defines a physical dimension and involves the quantification and measurement of any basin.
- These dimensions influence the water *quality and productivity* levels.
- Morphological features, age and geology of the lake basin along with the level of human interference have a direct and significant bearing on the structural and functional attributes of the *aquatic habitats*.
- Therefore, before undertaking any *limnological investigation* it is essential to prepare the maps and generate information on the morphometry and general characteristics of the area.

LAKE MORPHOMETRY

- ✓ The first step in characterizing any lake or impoundment is to determine its geometry or, as it is formally called morphometry.
- ✓ To do this(morphometry) the lake’s bathymetry must be mapped.
- ✓ Bathymetry refers to a topographic map showing depth contour lines.
- ✓ The analyst can then determine how much area is encompassed by each depth contour.
- ✓ This can be done using a mechanical device called a planimeter.
- ✓ The area can then be estimated by summing the areas of the grid cells falling within each counter.
- ✓ The resulting areas and corresponding depths represent a tabular function of the dependence of area on depth **$A(z)$** , the volume for the system can be determine by integration.

LAKE MORPHOMETRY

$$tf(H)=\int_0^H A(Z)\,dz$$

- ✓ *In addition the volume between two depths can be evaluated. For example the volume between two adjacent depths can be represented generally as*

$$tf_{i,\,i+1}=\int_{Hi}^{Hi+1} A(Z)\,dz$$

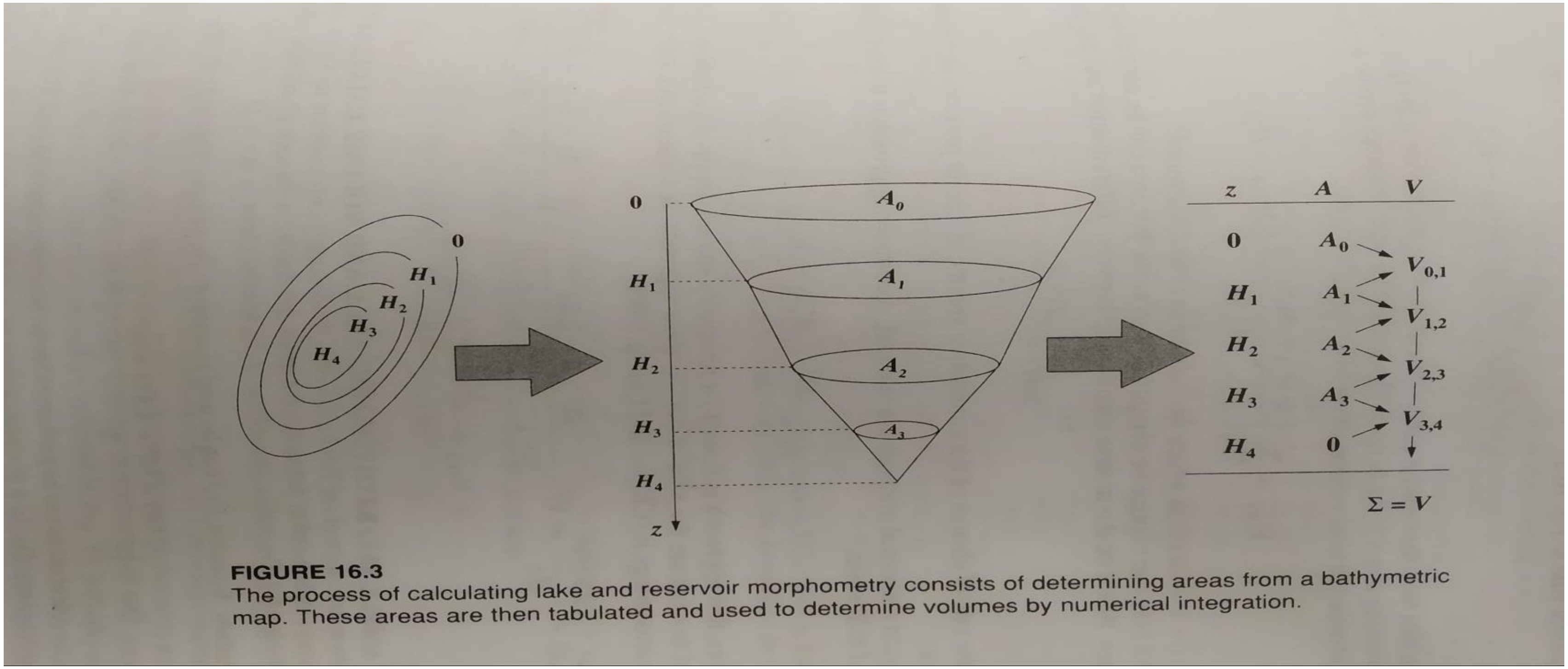
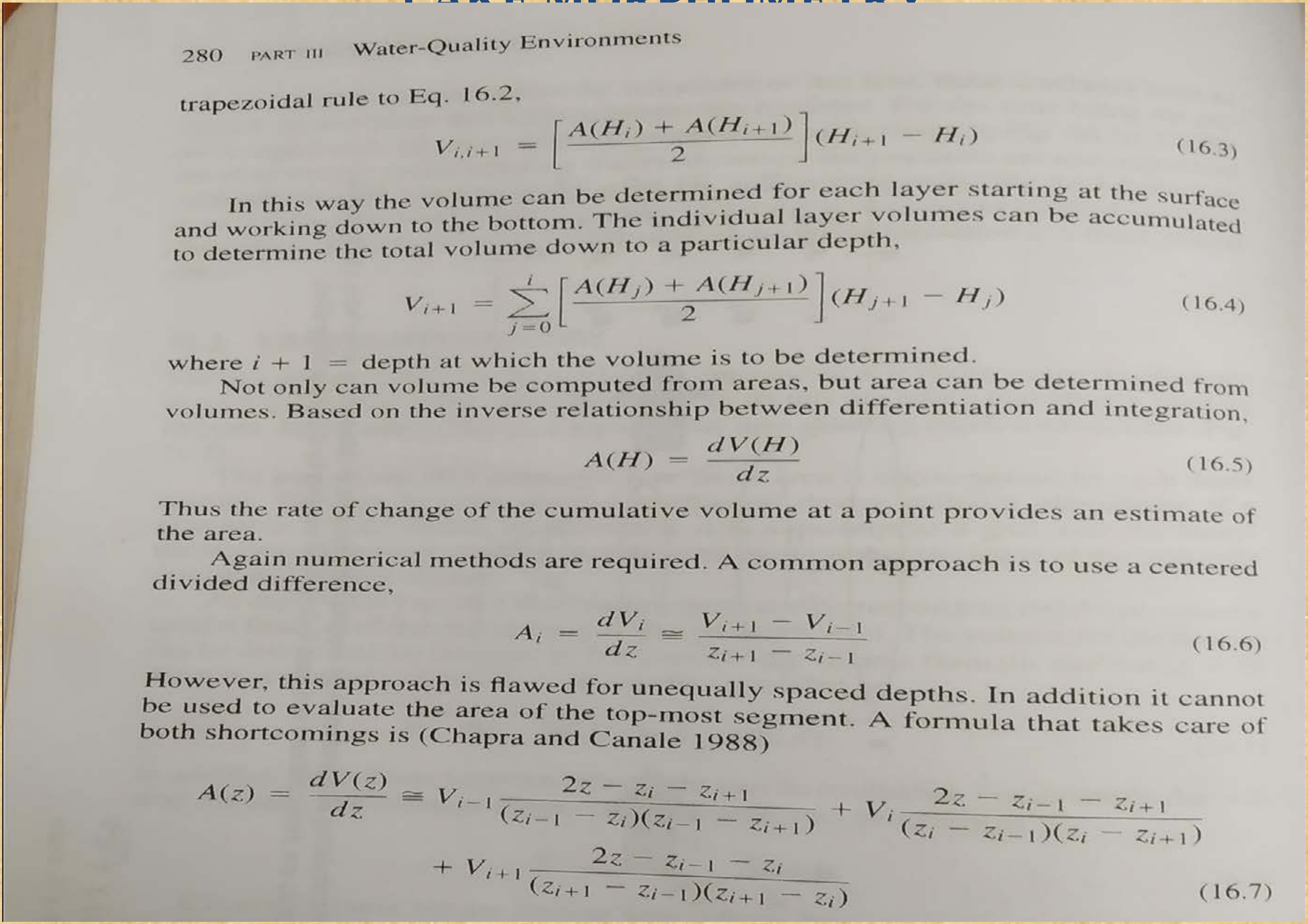


FIGURE 16.3
The process of calculating lake and reservoir morphometry consists of determining areas from a bathymetric map. These areas are then tabulated and used to determine volumes by numerical integration.

LAKE MORPHOMETRY



trapezoidal rule to Eq. 16.2,

$$V_{i,i+1} = \left[\frac{A(H_i) + A(H_{i+1})}{2}\right](H_{i+1} - H_i) \tag{16.3}$$

In this way the volume can be determined for each layer starting at the surface and working down to the bottom. The individual layer volumes can be accumulated to determine the total volume down to a particular depth,

$$V_{i+1} = \sum_{j=0}^{i}\left[\frac{A(H_j) + A(H_{j+1})}{2}\right](H_{j+1} - H_j) \tag{16.4}$$

where $i + 1 =$ depth at which the volume is to be determined.

Not only can volume be computed from areas, but area can be determined from volumes. Based on the inverse relationship between differentiation and integration,

$$A(H) = \frac{dV(H)}{dz} \tag{16.5}$$

Thus the rate of change of the cumulative volume at a point provides an estimate of the area.

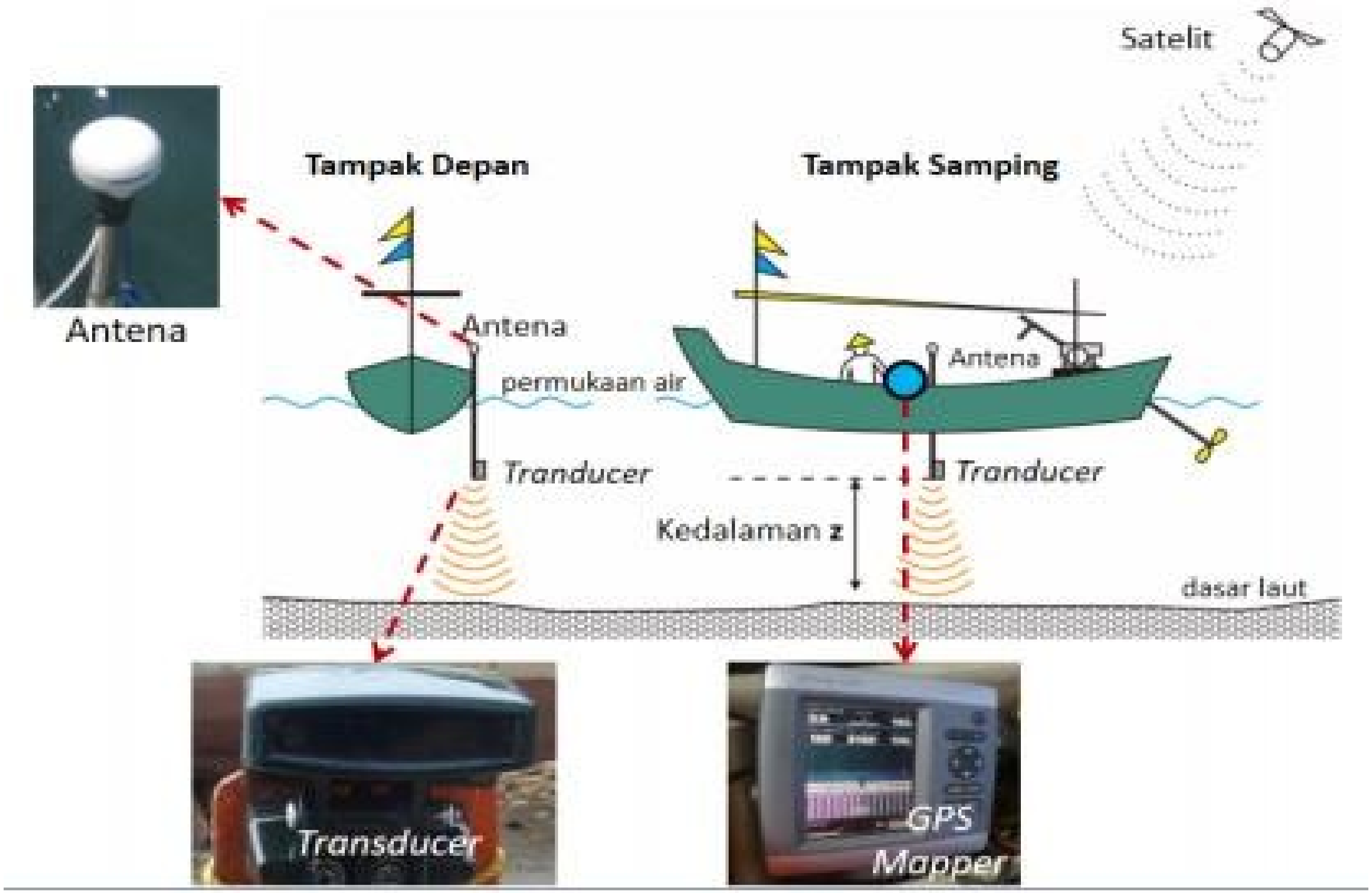
Again numerical methods are required. A common approach is to use a centered divided difference,

$$A_i = \frac{dV_i}{dz} \cong \frac{V_{i+1} - V_{i-1}}{z_{i+1} - z_{i-1}} \tag{16.6}$$

However, this approach is flawed for unequally spaced depths. In addition it cannot be used to evaluate the area of the top-most segment. A formula that takes care of both shortcomings is (Chapra and Canale 1988)

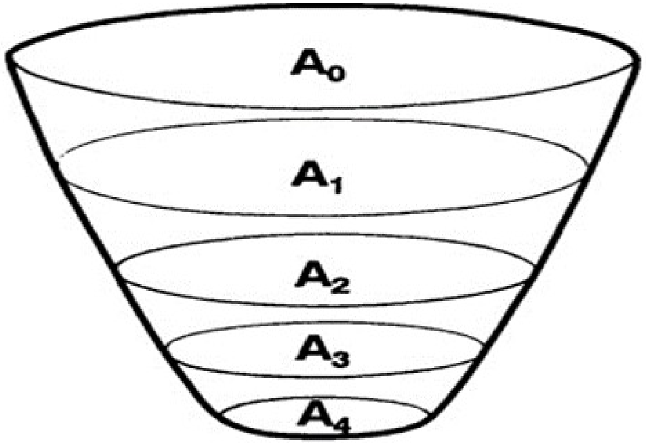
$$A(z) = \frac{dV(z)}{dz} \cong V_{i-1}\frac{2z - z_i - z_{i+1}}{(z_{i-1} - z_i)(z_{i-1} - z_{i+1})} + V_i\frac{2z - z_{i-1} - z_{i+1}}{(z_i - z_{i-1})(z_i - z_{i+1})} + V_{i+1}\frac{2z - z_{i-1} - z_i}{(z_{i+1} - z_{i-1})(z_{i+1} - z_i)} \tag{16.7}$$

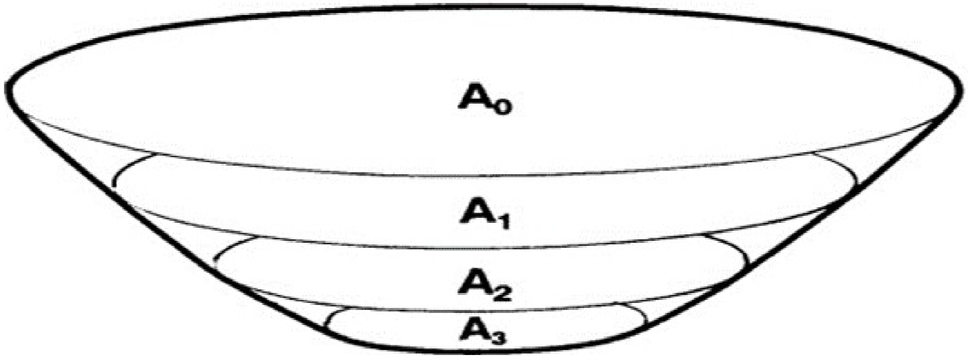
LAKE MORPHOMETRY

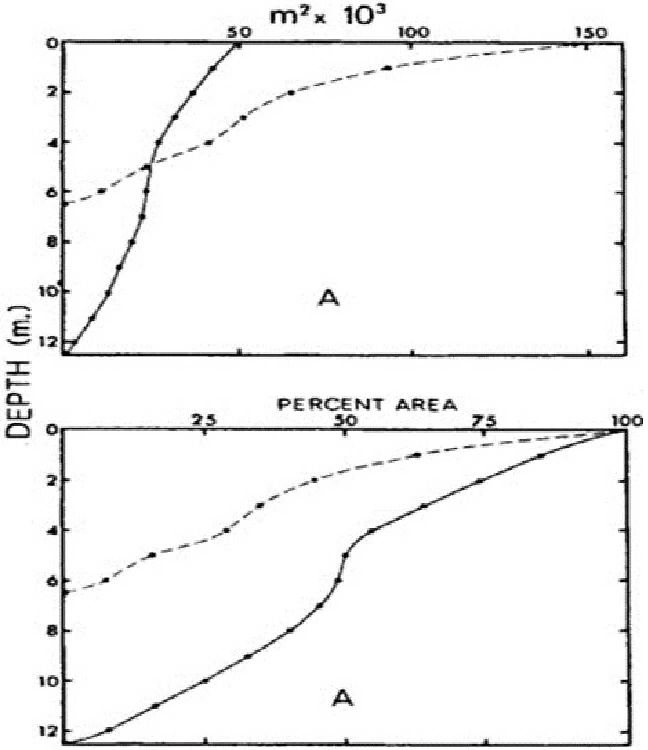


(b)

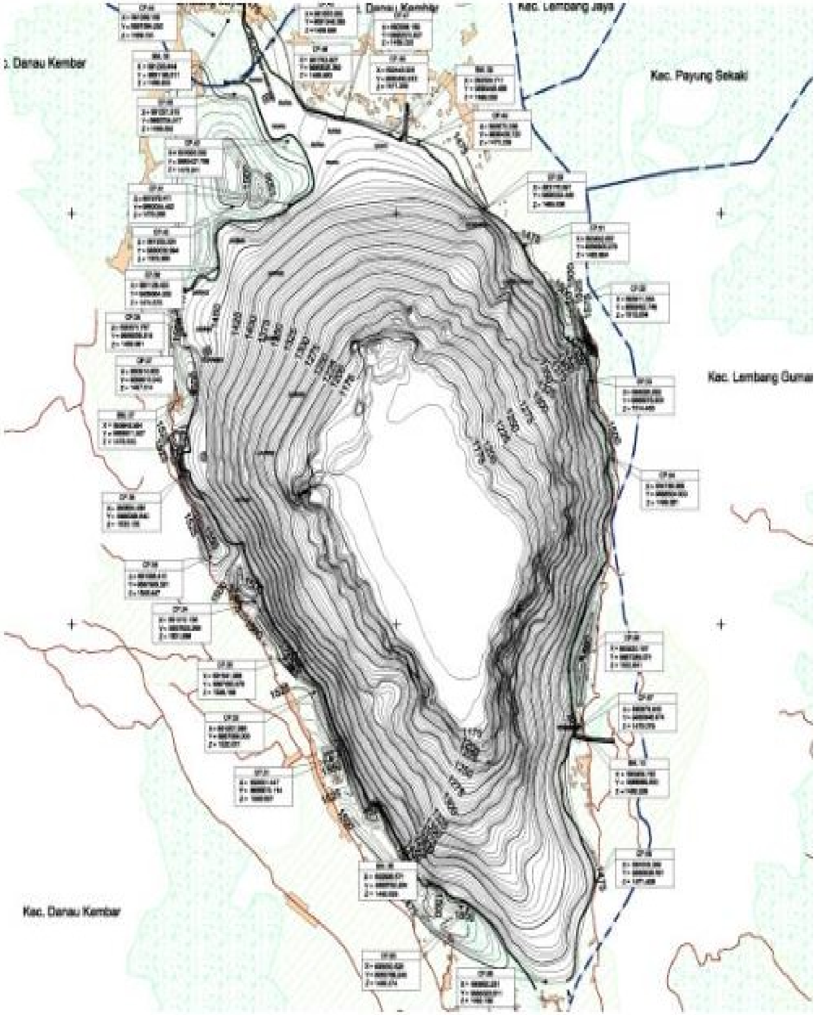
LAKE MORPHOMETRY



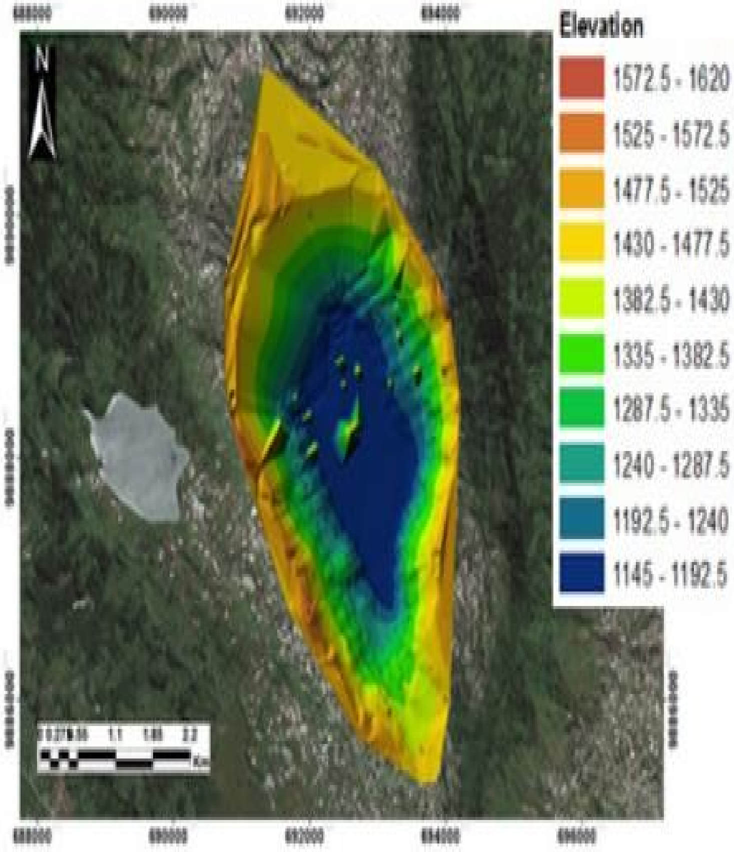




LAKE MORPHOMETRY

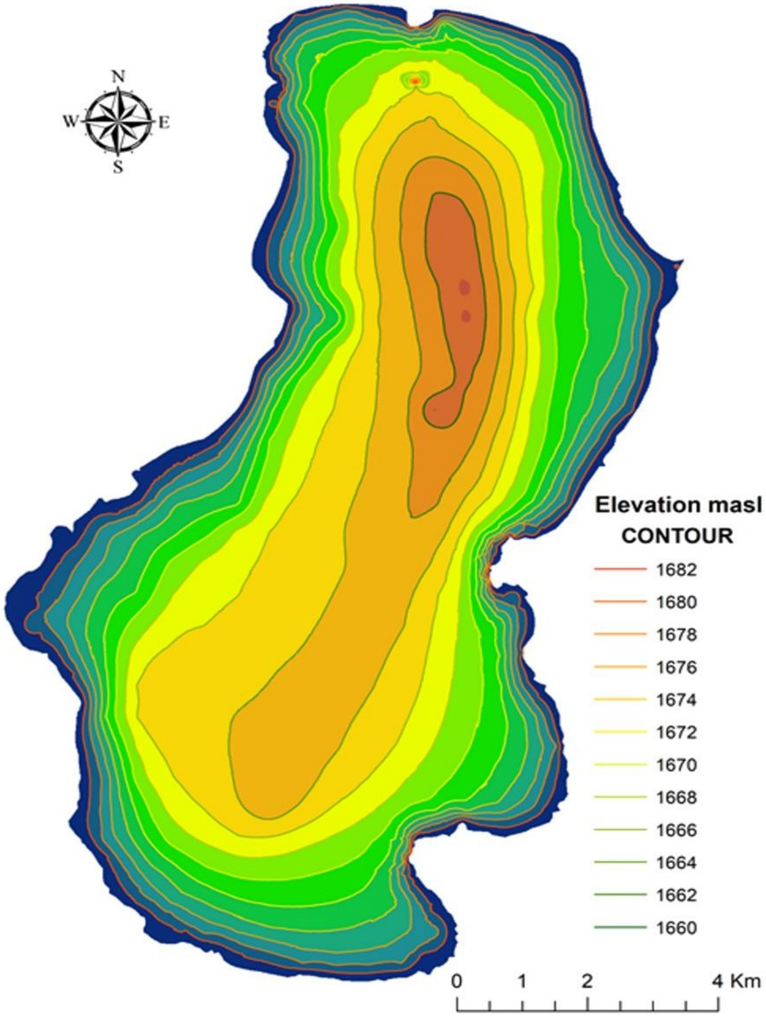


(a)

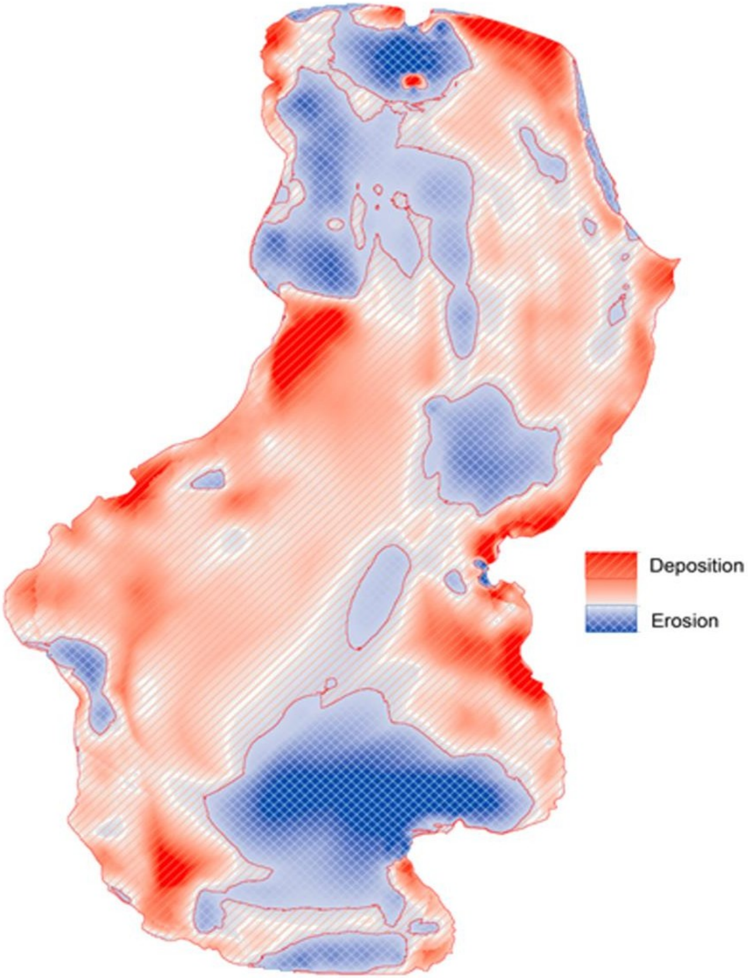


(b)

LAKE MORPHOMETRY



Lake Hawassa Bathymetry 2011



Lake Hawassa Morphometric Change 1999-2011

Chapter Five

River and Lake Water Quality Management

FACULTY OF WATER SUPPLY AND ENVIRONMNETAL ENGINEERING

G5WSEE
2020

G5WSEE
2020

Instructor: Zemed Menberu(MSc)

River and Lake Pollution Control Methods

- Pollution prevention
  - Source reduction: input changes, processes changes, product changes
  - Recycling/reuse, water conservation
  - Minimizing illicit wastewater discharge
  - Improving solid waste management, etc.
  - Regulate rate, frequency and amount of fertilizer application
- Reducing waste after generation
  - Conventional wastewater treatment
  - Alternative treatment methods
- Increasing assimilative capacity of a watercourse
  - Low flow augmentation
  - Artificial reaeration

River and Lake Pollution Control Methods

- Making better use of assimilative capacity of a watercourse
  - Regulated discharge, i.e., to temporarily store water during high flow period and to release it later when flow is low
  - To collect individual effluents to a common treatment plant and discharge it at a point where impact on quality is minimized.

- Landuse planning
  - Zoning
  - Restrictions on population density and settlement on sensitive areas
  - Erosion control
  - Reducing impervious surfaces
  - Reducing directly connected impervious surfaces
  - Low impact land development

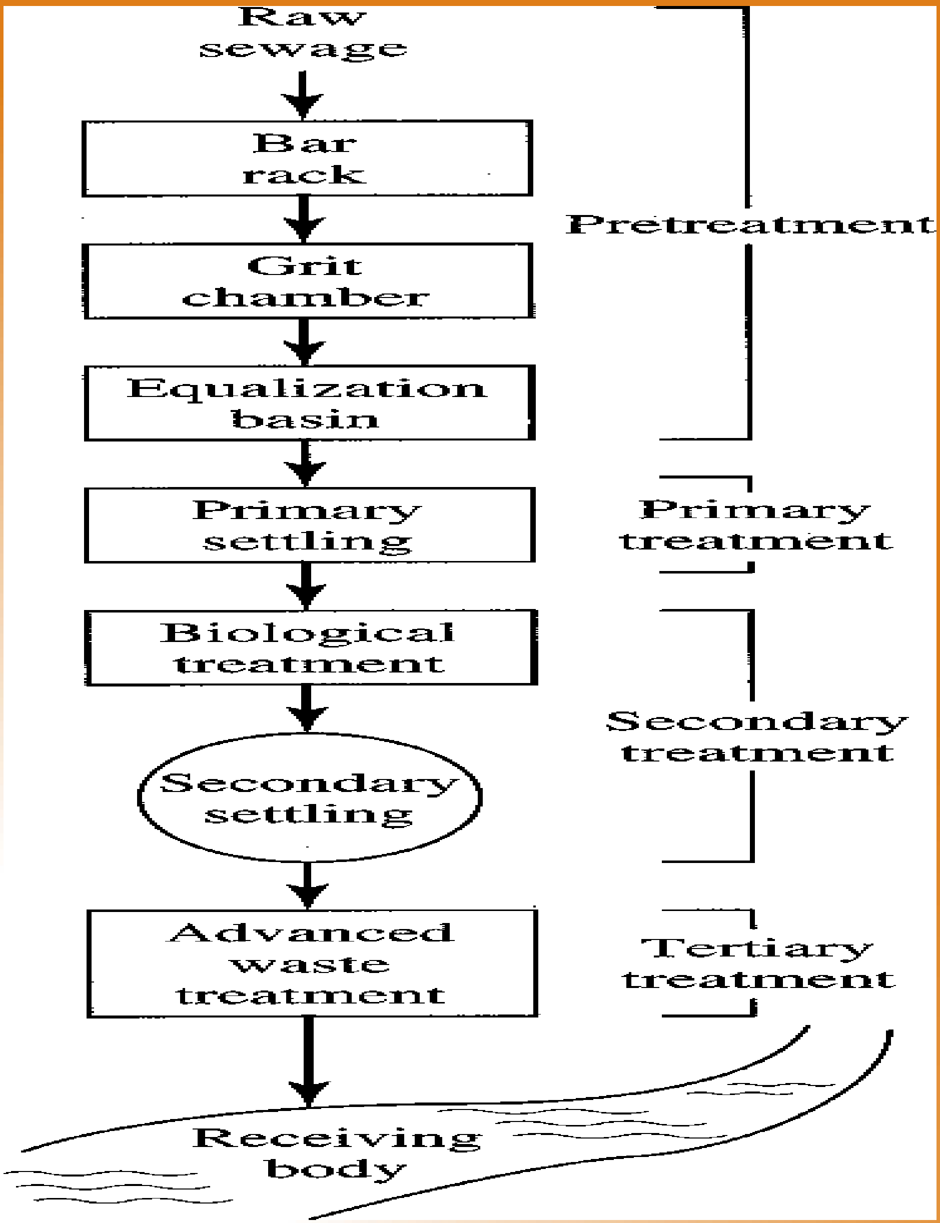
Reducing waste after generation

Treat wastewater to the required level before discharging into the receiving water system

Typical characteristics of domestic wastewater

Constituent	*Weak (mg/L)*	*Medium (mg/L)*	*Strong (mg/L)*
Alkalinity (as $CaCO_3$)	50	100	200
BOD_5	100	200	300
Suspended solids	100	200	350
TDS	200	500	1000
TKN (as N)	20	40	80
Total phosphorus (as P)	5	10	20

Conventional wastewater treatment



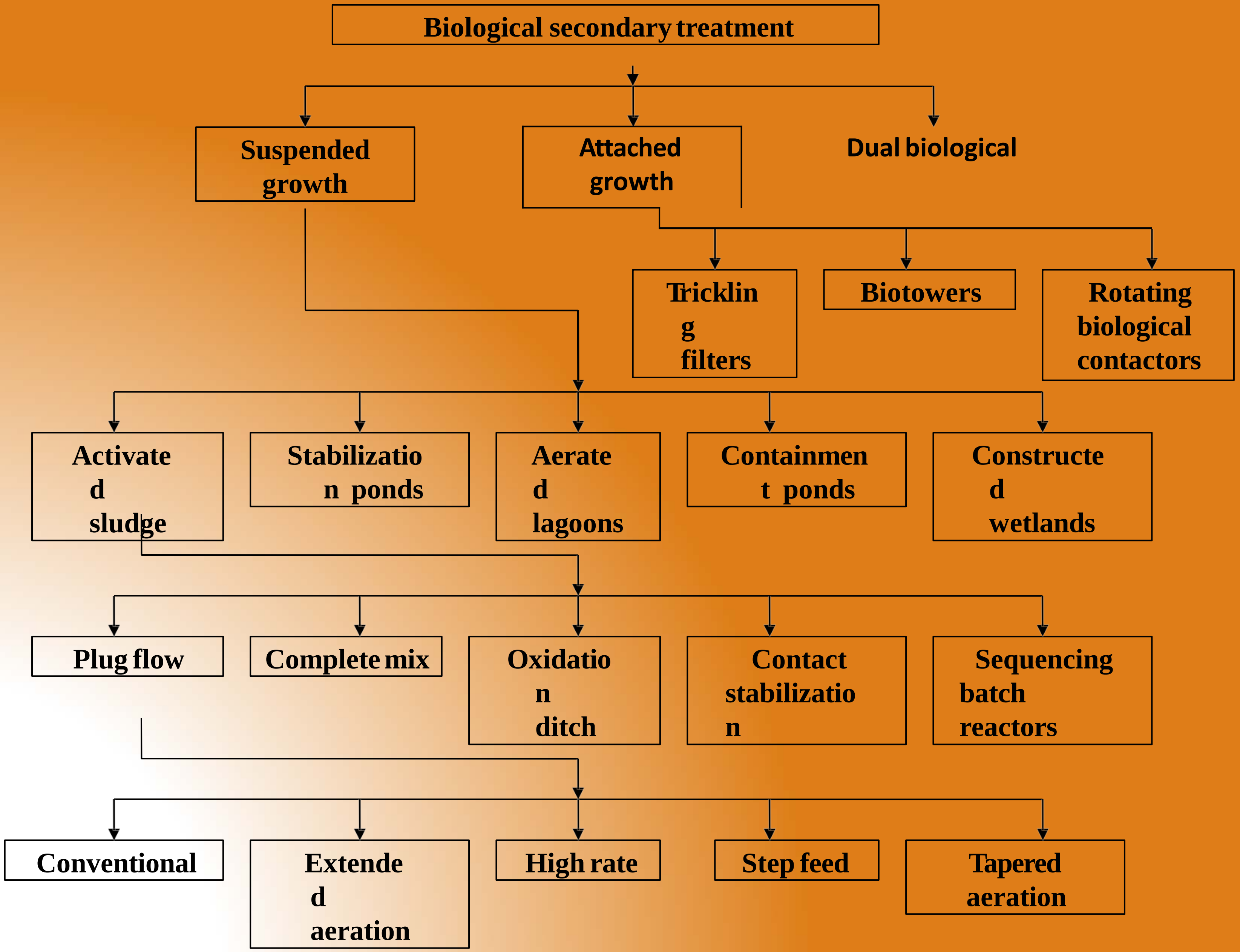
Pretreatment – removes materials that can cause operational problems, equalization optional

Primary treatment – remove ~60% of solids and ~35% of BOD

Secondary treatment – remove ~85% of BOD and solids

Advanced treatment – varies: 95+ % of BOD and solids, N, P

Biological wastewater treatment



Biological secondary treatment

Suspended growth

Attached growth

Dual biological

Trickling filters

Biotowers

Rotating biological contactors

Activated sludge

Stabilization ponds

Aerated lagoons

Containment ponds

Constructed wetlands

Plug flow

Complete mix

Oxidation ditch

Contact stabilization

Sequencing batch reactors

Conventional

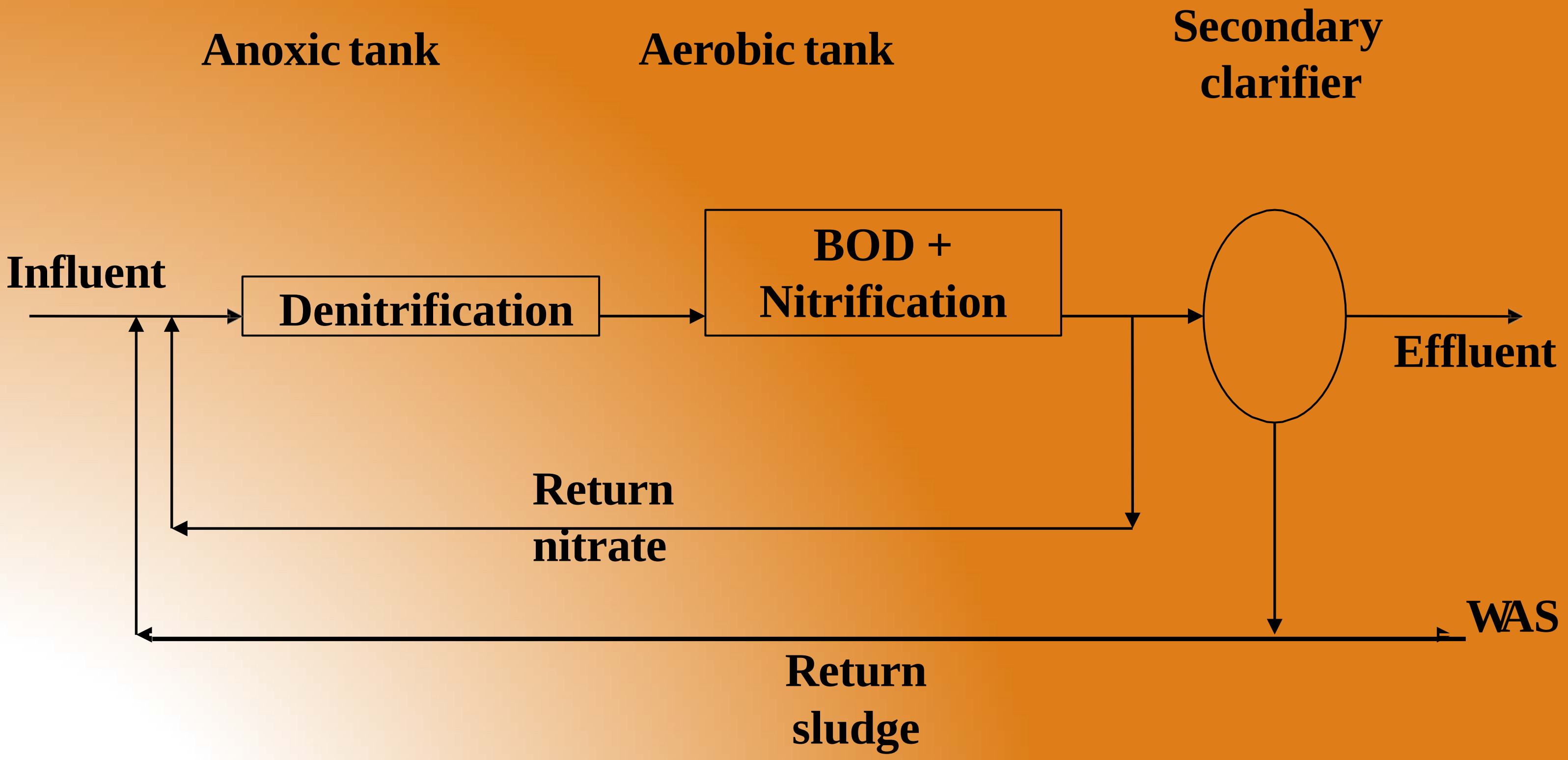
Extended aeration

High rate

Step feed

Tapered aeration

Biological Nitrogen removal



Anoxic tank

Aerobic tank

Secondary clarifier

Influent

Denitrification

BOD + Nitrification

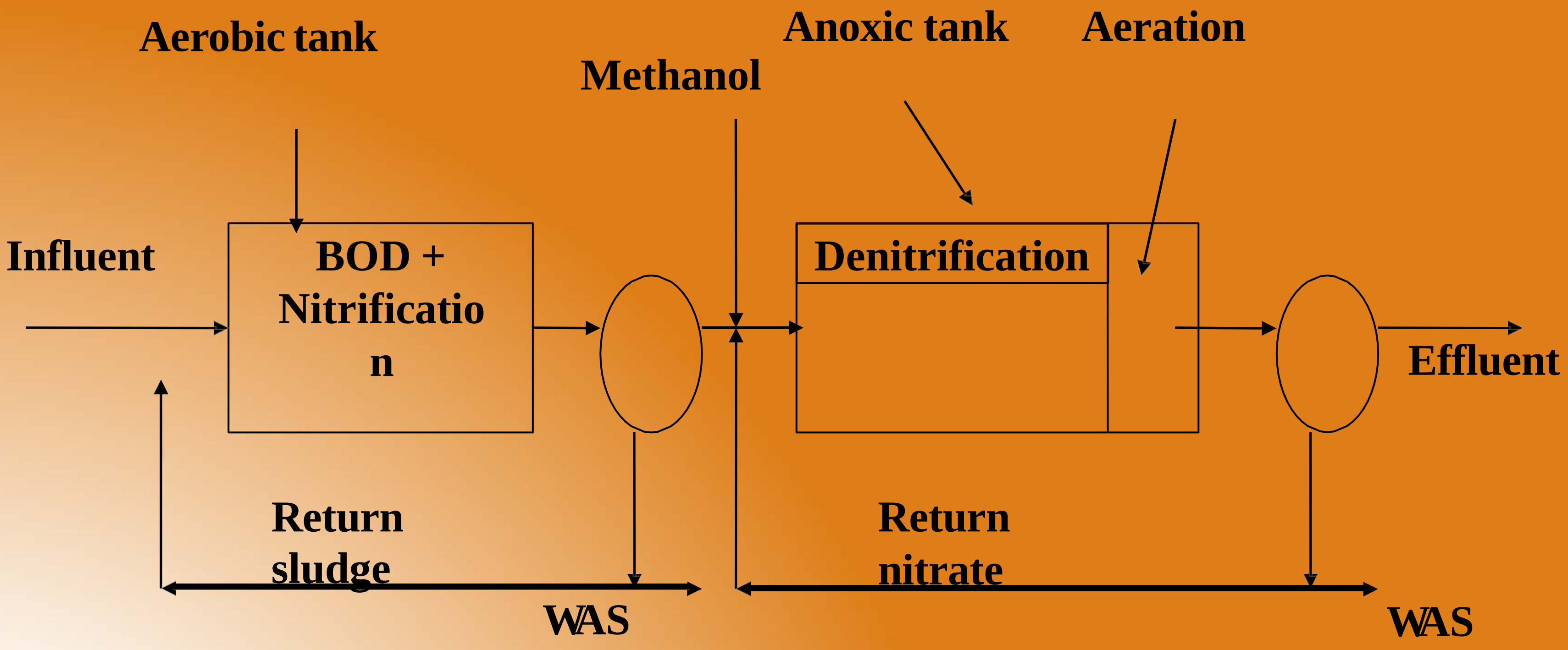
Effluent

Return nitrate

WAS

Return sludge

Biological Nitrogen removal



Aerobic tank

Methanol

Anoxic tank

Aeration

Influent

BOD + Nitrification

Denitrification

Effluent

Return sludge

WAS

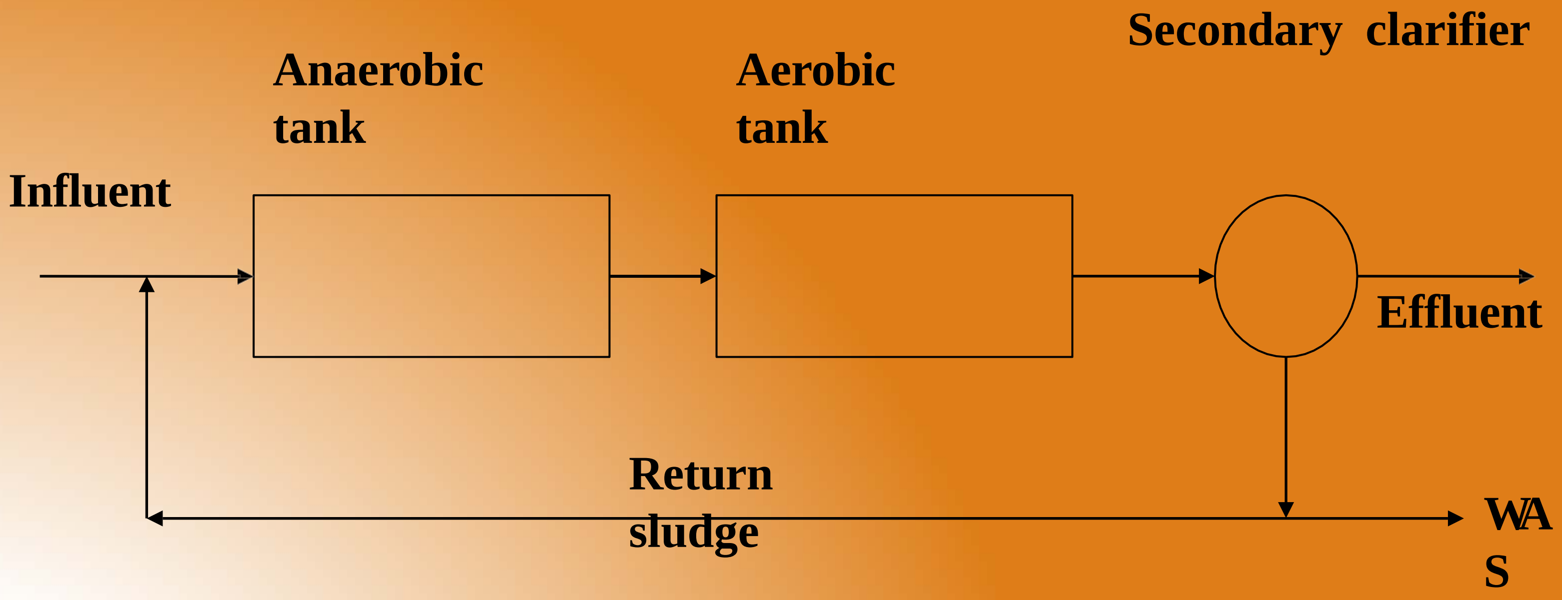
Return nitrate

WAS

- **Nitrification in aerobic environment**
  Organic matter + O_2 □ NH_3 + CO_2 + microorganisms
  NH_3 + $2O_2$ □ NO_3^- + H^+ + H_2O + microorganisms

- **Denitrification in an anoxic environment (no free oxygen)**
  Organic matter + NO_3^- □ N_2 + microorganisms

Two-stage biological P removal



Secondary clarifier

Anaerobic tank

Aerobic tank

Influent

Effluent

Return sludge

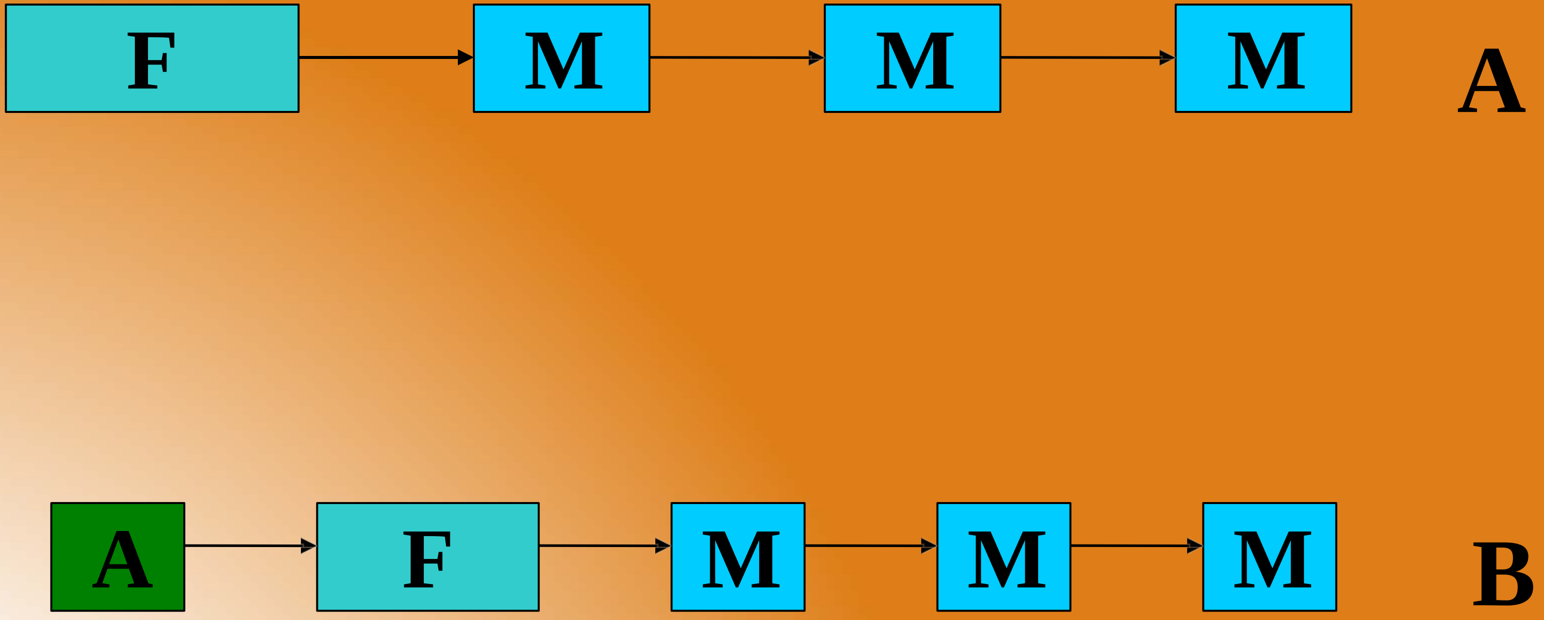
WAS

Alternative Treatments

Waste Stabilization Ponds

- Comprises series of man-made shallow ponds
- They can achieve any required degree of treatment at the lowest cost and
  - with minimum of maintenance by unskilled operators
- The removal of pathogens is considerably greater than that in other methods of sewage treatment
- They are well able to withstand both organic and hydraulic shock loads
- They can effectively treat a wide variety of industrial and agricultural wastes
- They can easily be designed so that the degree of treatment is readily altered
- The method of construction is such that, should at some future time the land be required for some other purpose, it I easily reclaimed
- The algae produced in the pond are potential sources of high-protein food which can be conveniently exploited by fish farming

Waste stabilization ponds

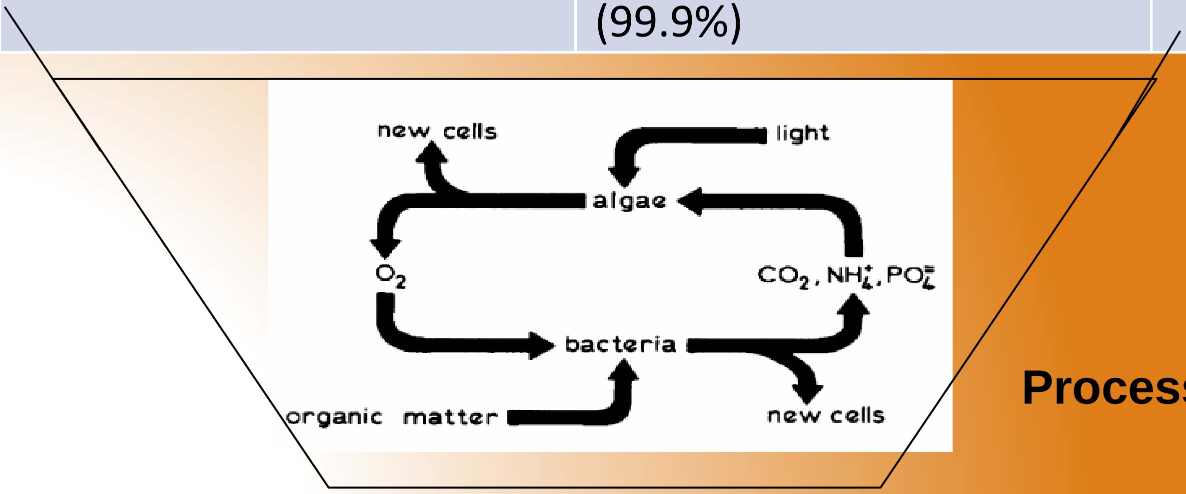


Pond layouts (a) Weak-medium strength wastes (b) Strong wastes M:

A: Anaerobic pond F: Facultative pond Maturation pond

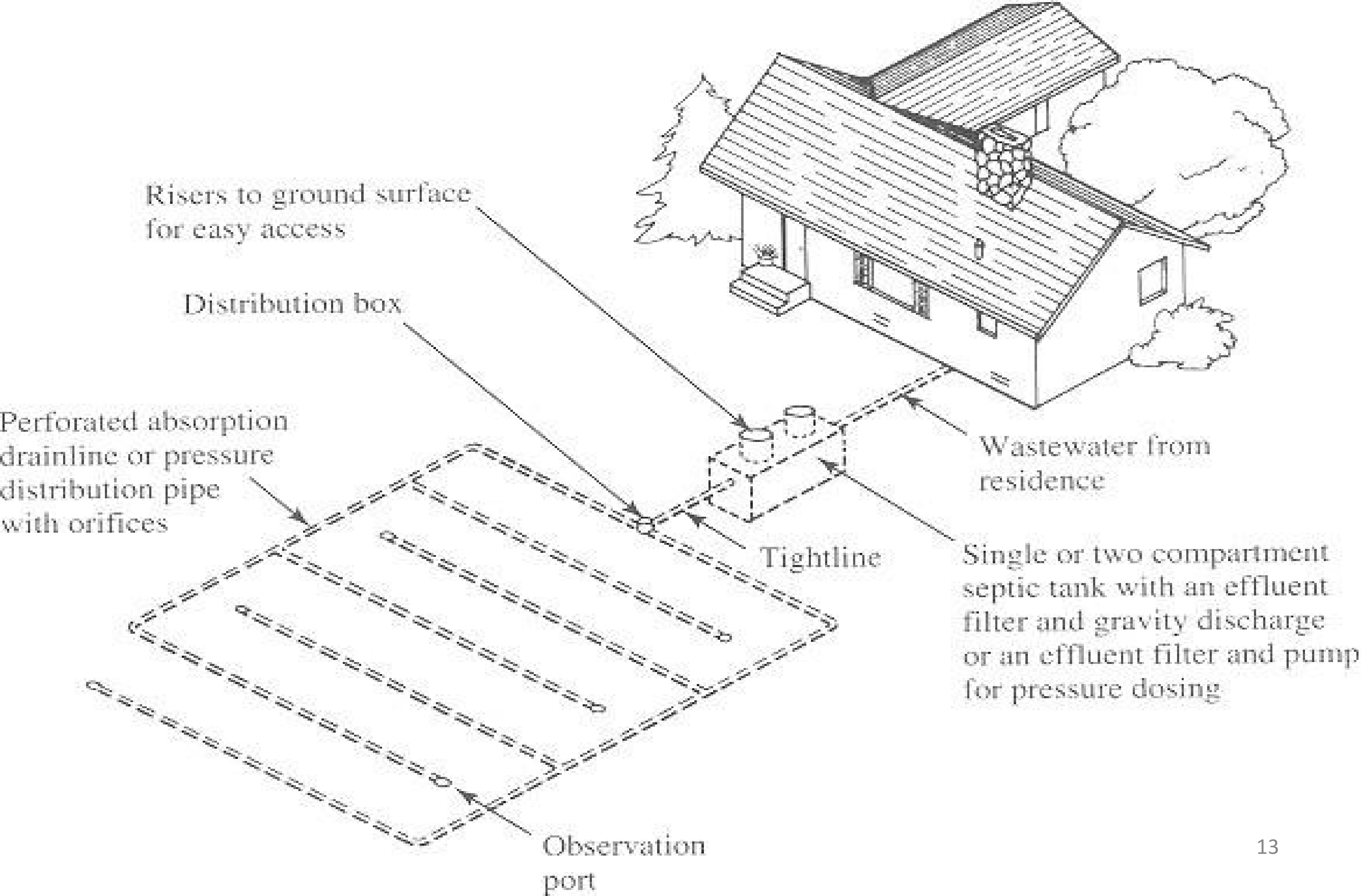
Functions and requirements of ponds

Pond	Main function	Design factors
Anaerobic	Treat strong organic waste (70% reduction in BOD5)	Depth: 2-4 m Detention time: 5 days Temp/pH: > 15°C/>6
Facultative	BOD removal	Depth: 1-1.5 m Exposed to wind Large land area Desludging: 10-15 years
Maturation	Fecal coliform removal (99.9%)	Depth: 1-1.5 m



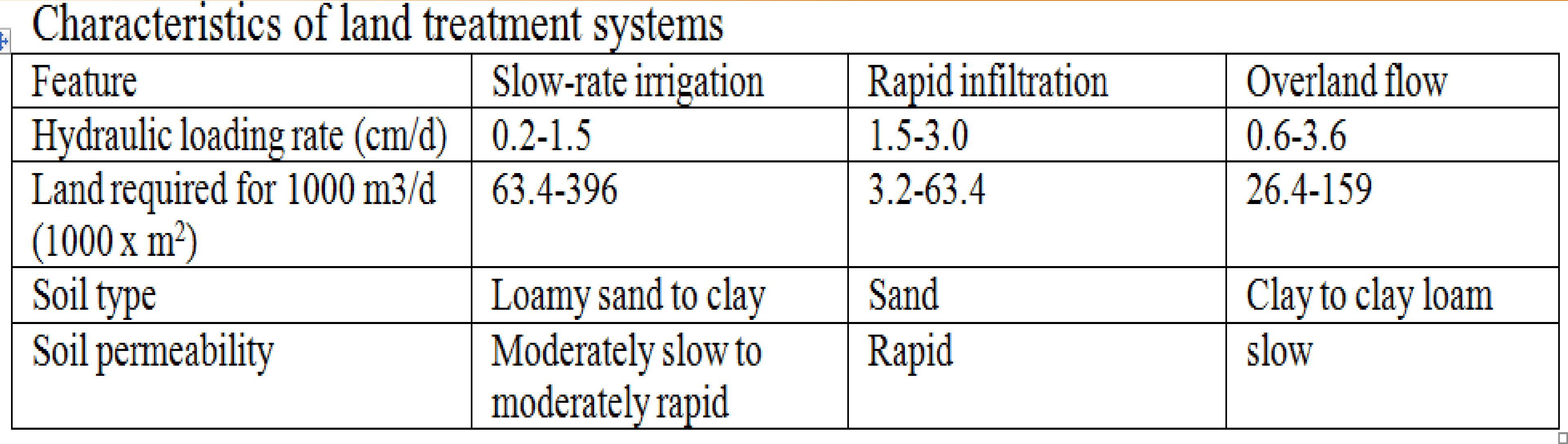
Processes in a facultative pond

Alternative treatments- Septic systems



Land treatment methods

- Application of wastewater on the land
- Plants and soil play roles in pollutant removal
- Three types of application:
    - Slow rate infiltration
    - Rapid infiltration
    - Overland flow

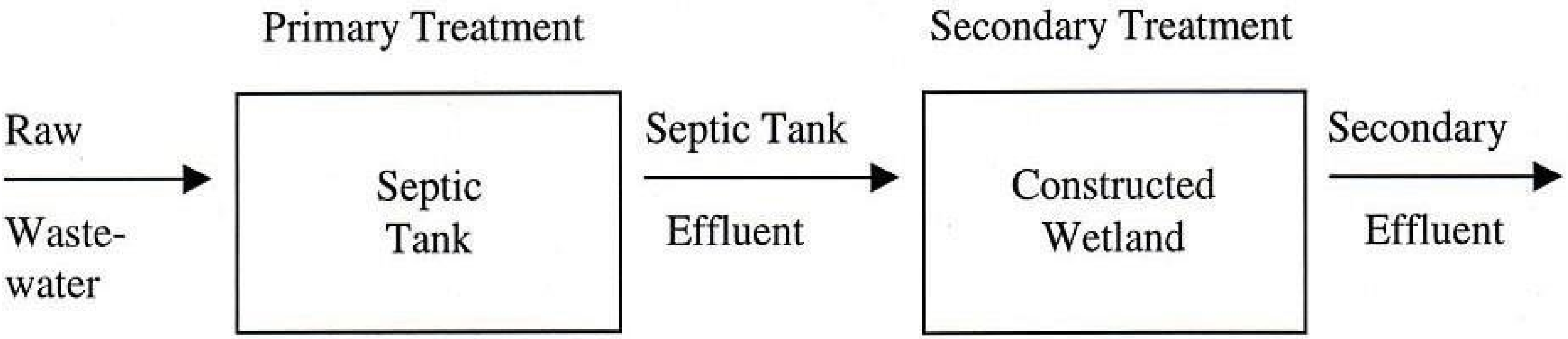


Characteristics of land treatment systems

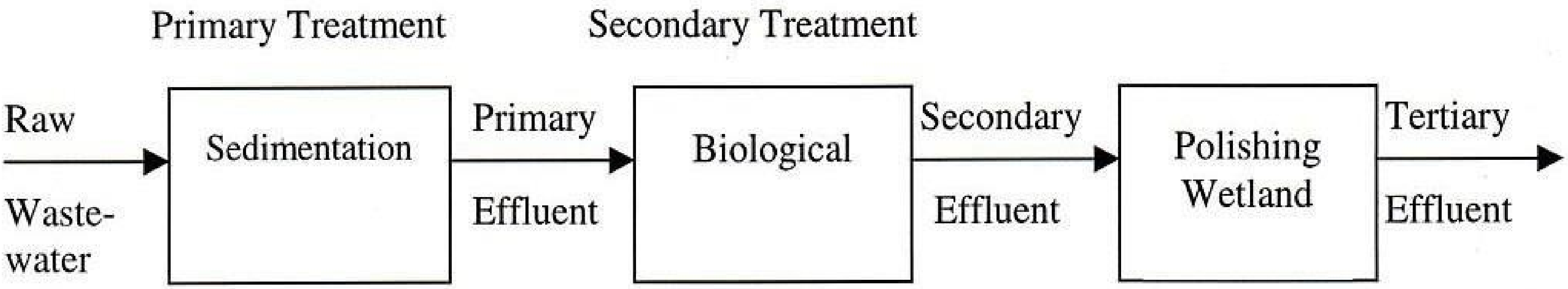
Feature	Slow-rate irrigation	Rapid infiltration	Overland flow
Hydraulic loading rate (cm/d)	0.2-1.5	1.5-3.0	0.6-3.6
Land required for 1000 m3/d ($1000 \times m^2$)	63.4-396	3.2-63.4	26.4-159
Soil type	Loamy sand to clay	Sand	Clay to clay loam
Soil permeability	Moderately slow to moderately rapid	Rapid	slow

Alternative treatments- Constructed wetlands

CONSTRUCTED WETLANDS TREATMENT TRAIN



LARGE POLISHING WETLAND TREATMENT TRAIN



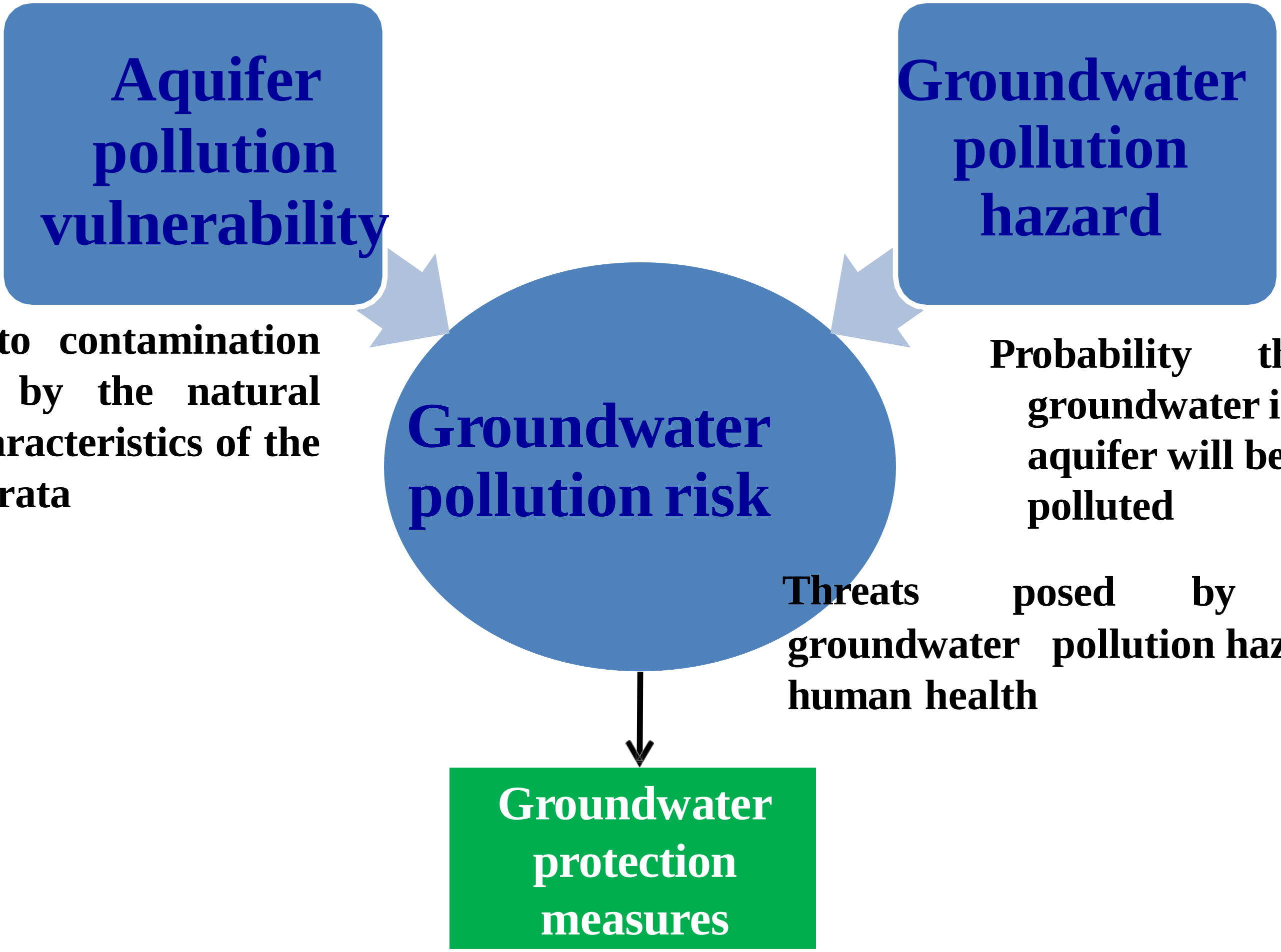
Groundwater Quality Management

- Why do groundwater supplies merit protection?
    - ➢ Vital sources of water supply in rural and urban areas
    - ➢ May take years before the impact of pollution become apparent
    - ➢ Remediation of polluted groundwater is difficult

Sources of groundwater pollution

- Municipal
  - Sewer leakage
  - Liquid wastes
  - Solid wastes
- Industrial
  - Liquid wastes
  - Tank and pipeline leakage
  - Mining activities
  - Oilfield brines
- Agriculture
  - Irrigation return flows
  - Animal wastes
  - Fertilizers
  - Pesticides
- Others
  - Spills
  - Septic tanks
  - Saline water intrusion,

Groundwater Quality Management



Aquifer pollution vulnerability

Sensitivity to contamination determined by the natural intrinsic characteristics of the geological strata

Groundwater pollution hazard

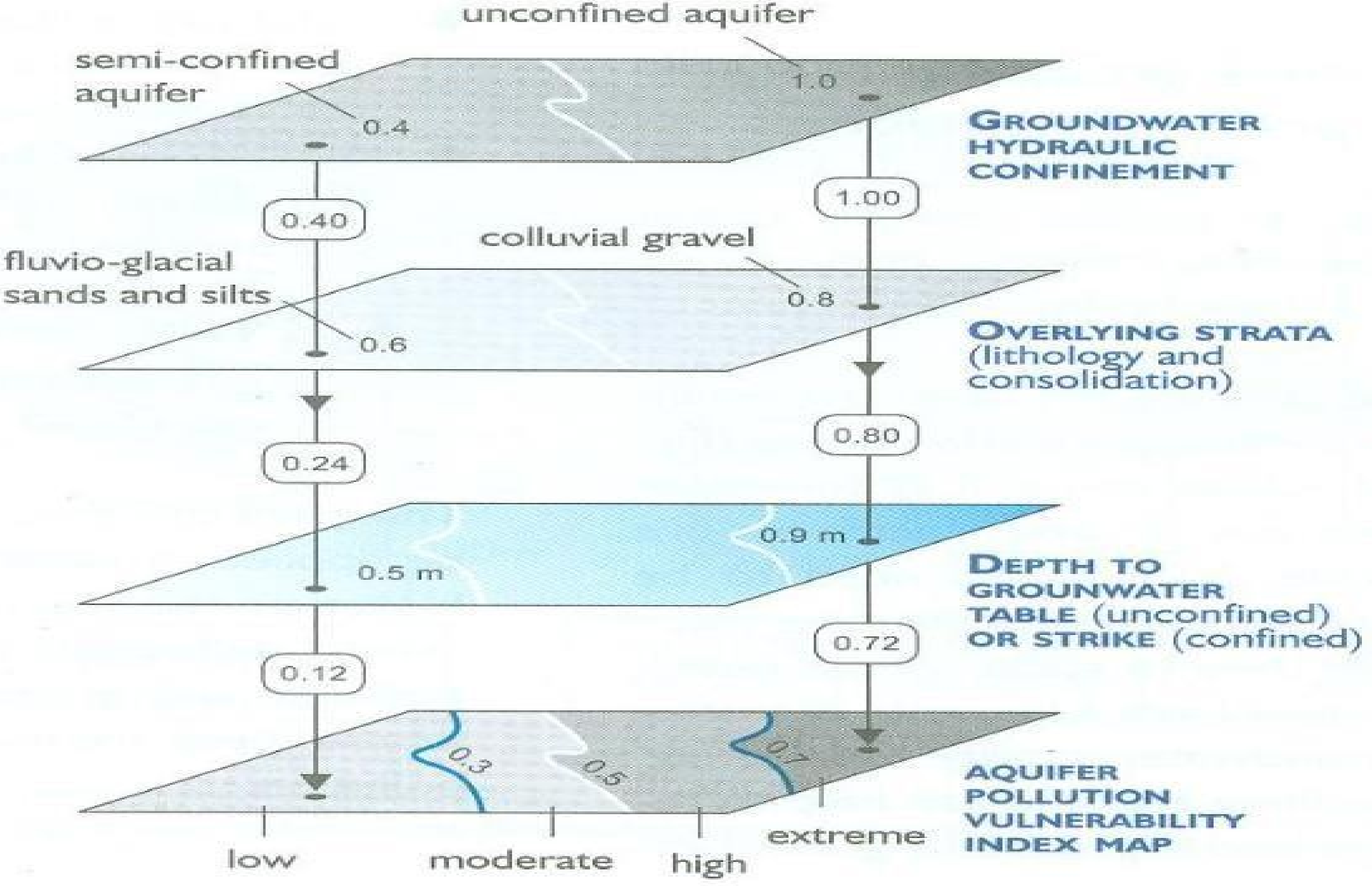
Probability that the groundwater in an aquifer will become polluted

Groundwater pollution risk

Threats posed by groundwater pollution hazard to human health

Groundwater protection measures

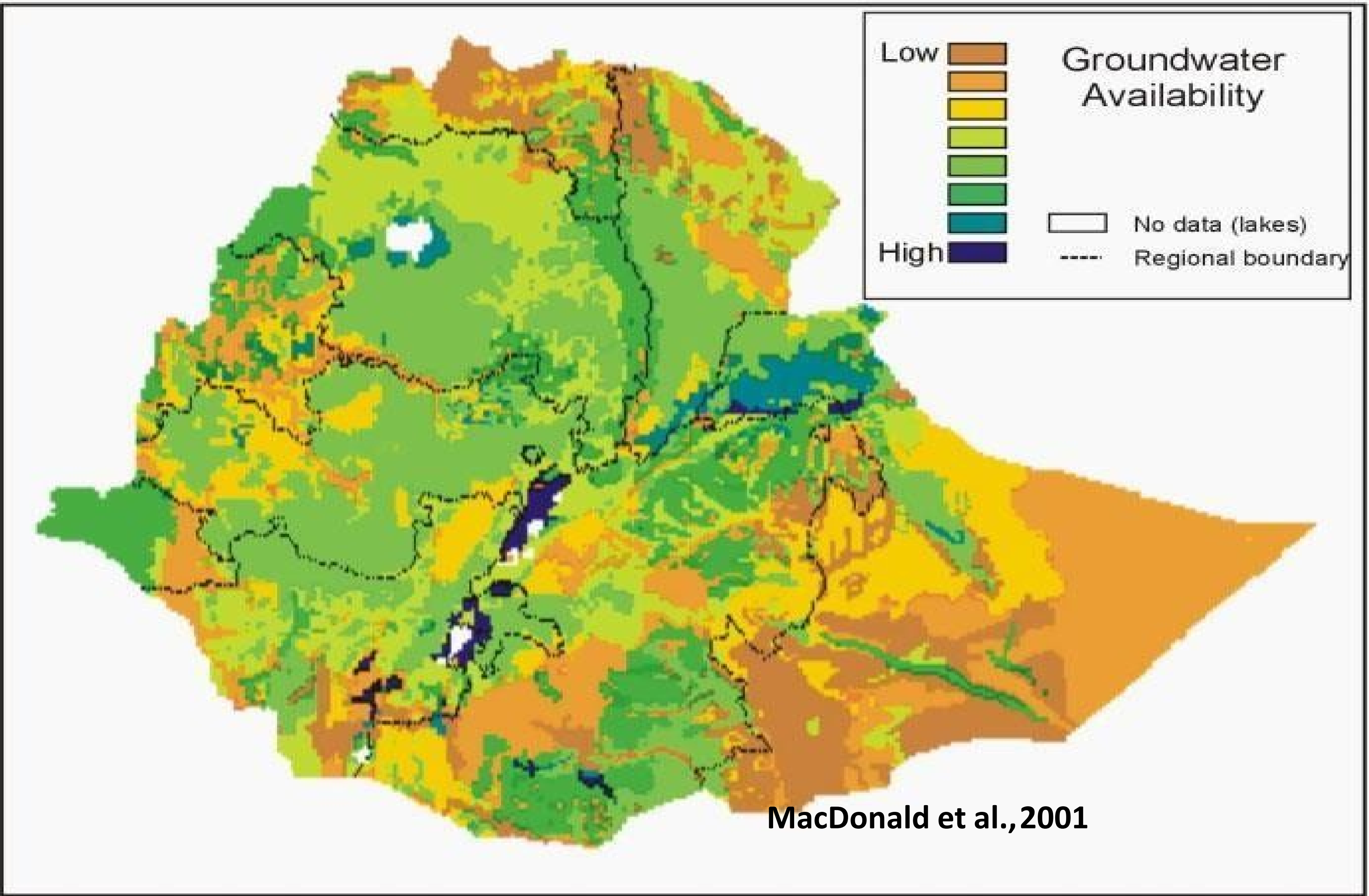
Key factors in aquifer vulnerability



Groundwater quality management practices

- Assessing actual groundwater quality status
- Prohibition of certain activities in vulnerable areas
- Prohibiting the disposal of certain level of waste except in sealed facilities
- Management of waste effluents
- Monitoring compliance with regulations/permits
- Remedial actions:
  - ➢ Pumping and treating polluted groundwater
  - ➢ Pumping/well injection to reverse the hydraulic gradient
  - ➢ Construction of physical barriers
  - ➢ Chemical or bacterial in-situ treatment of contaminated groundwater

Groundwater availability in



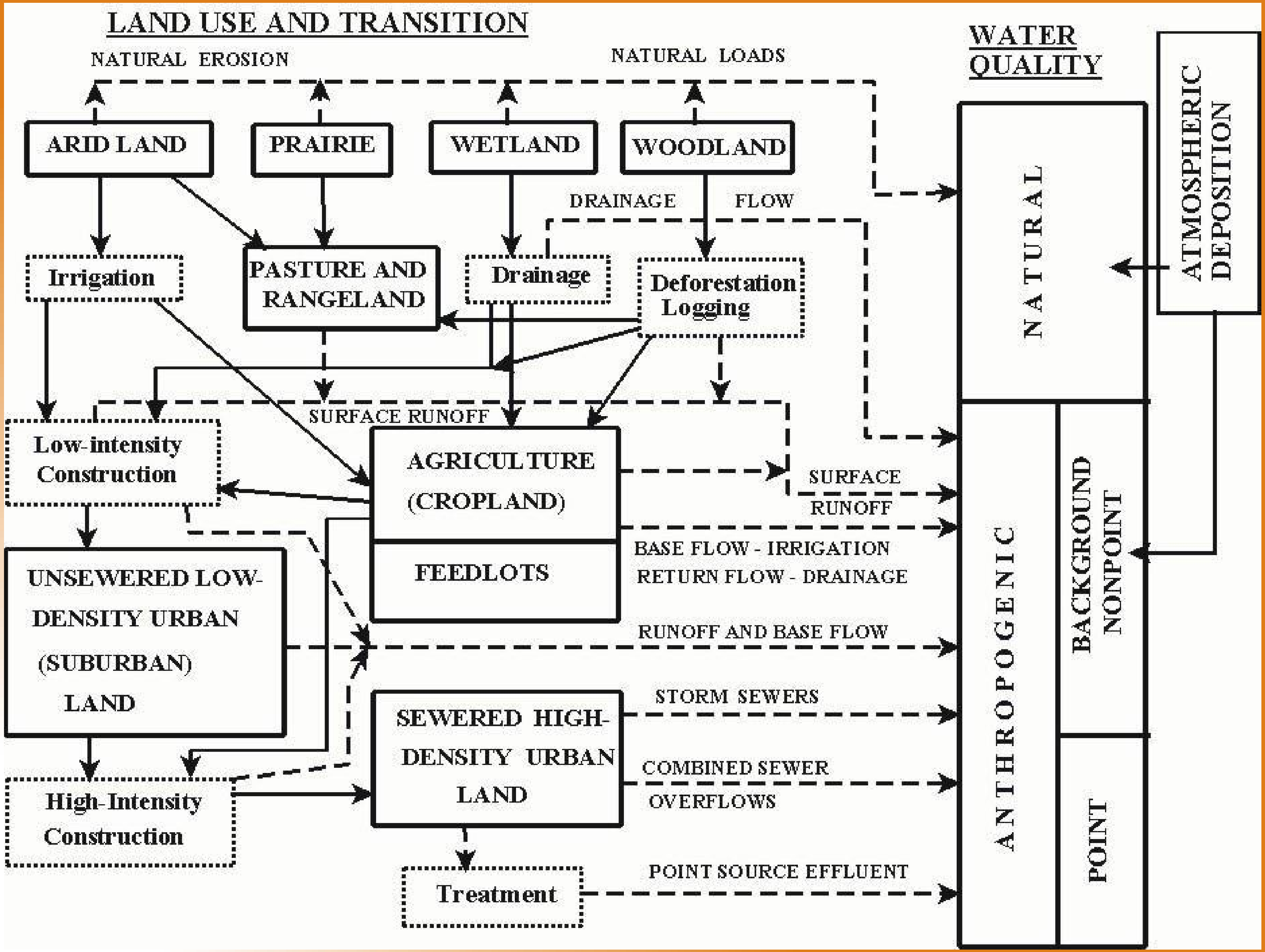
MacDonald et al.,2001

Groundwater quality in Ethiopia

- Total dissolved solids (< 500 mg/L in the highlands to > 3000 mg/L in the lowlands)
- Flouride (> 10 mg/L in Rift valley areas)
- Iron and manganese (low)
- Nitrate (higher in urban areas)
- Arsenic (expected to be low)
- No obvious problems with other trace elements

Management of Agricultural Sources of Water Pollution

Land use change and water quality



LAND USE AND TRANSITION

NATURAL EROSION

NATURAL LOADS

ARID LAND

PRAIRIE

WETLAND

WOODLAND

DRAINAGE

FLOW

Irrigation

PASTURE AND RANGELAND

Drainage

Deforestation Logging

SURFACE RUNOFF

Low-intensity Construction

AGRICULTURE (CROPLAND)

FEEDLOTS

SURFACE RUNOFF

BASE FLOW - IRRIGATION

RETURN FLOW - DRAINAGE

UNSEWERED LOW-DENSITY URBAN (SUBURBAN) LAND

RUNOFF AND BASE FLOW

SEWERED HIGH-DENSITY URBAN LAND

STORM SEWERS

COMBINED SEWER OVERFLOWS

High-Intensity Construction

Treatment

POINT SOURCE EFFLUENT

WATER QUALITY

NATURAL

ANTHROPOGENIC

BACKGROUND NONPOINT

POINT

ATMOSPHERIC DEPOSITION

Agricultural water pollution

Point sources

- Uncovered exercise or feeding areas
- Animal waste storage facilities
- Silage effluent handling facilities
- Pesticides storage and handling facilities

Nonpoint sources

- Agricultural production areas
- Transport agents
  - ✓ Excess precipitation
  - ✓ Drainage water through soil
  - ✓ Wind

Sources of Agricultural NPSP

Cultivation

Livestock production

Pollutants of concern

- Sediment
- Nutrients (Phosphorus and Nitrogen)
- Pesticides
- Pathogens
- Solid waste

Agricultural point source controls

- **Site selection**
  - As far as practicable from surface waters and down gradient from nearby (50 to 75 m) groundwater wells
  - Suitable surface and subsurface soil conditions (low organic matter, low shrink-swell potential, good compactability and adequate bearing capacity, deep water table, etc.)
- **Sizing of structural facilities**
  - Adequate capacities to hold the pollutants (and rainwater in the case of uncovered structures)
- **Design and construction**
  - Design according to accepted engineering standards and procedures
  - Use corrosion resistant construction materials
  - Follow accepted construction practices

Non-structural BMPs for controlling Agricultural NPSP

- **Education:**
  - awareness on the threats of ANPSP and importance of their control
- **Water management:**
  - reduce erosion and nutrient losses in runoff by minimizing or slowing water flow off fields.

Examples: contour tillage, buffer strips, diversions and terraces

- **Land use**
  - designating areas suitable for various types of development or conservation
- **Erosion and sediment control for cultivated crops**:
  - focuses on minimization of exposed soil and time of exposure.

Examples: conservation cover/stabilization practices, conservation tillage, cover crop, buffer zones, critical area planting, residue use, delayed seedbed preparation, indigenous weed management, mulching, strip cropping, conservation cropping;

…. BMPs for controlling Agricultural NPSP

- **Erosion and sediment controls for livestock areas**:

  Example: deferred grazing, heavy use area protection

- **Pesticide/nutrient control:**

  Good housekeeping practices, plant and soil analyses, nutrient management plan, integrated pest management, proper application of phosphorus and nitrogen, use of natural fertilizers, leguminous plants in rotation;

- **Pathogens:** focuses on livestock and manure management;

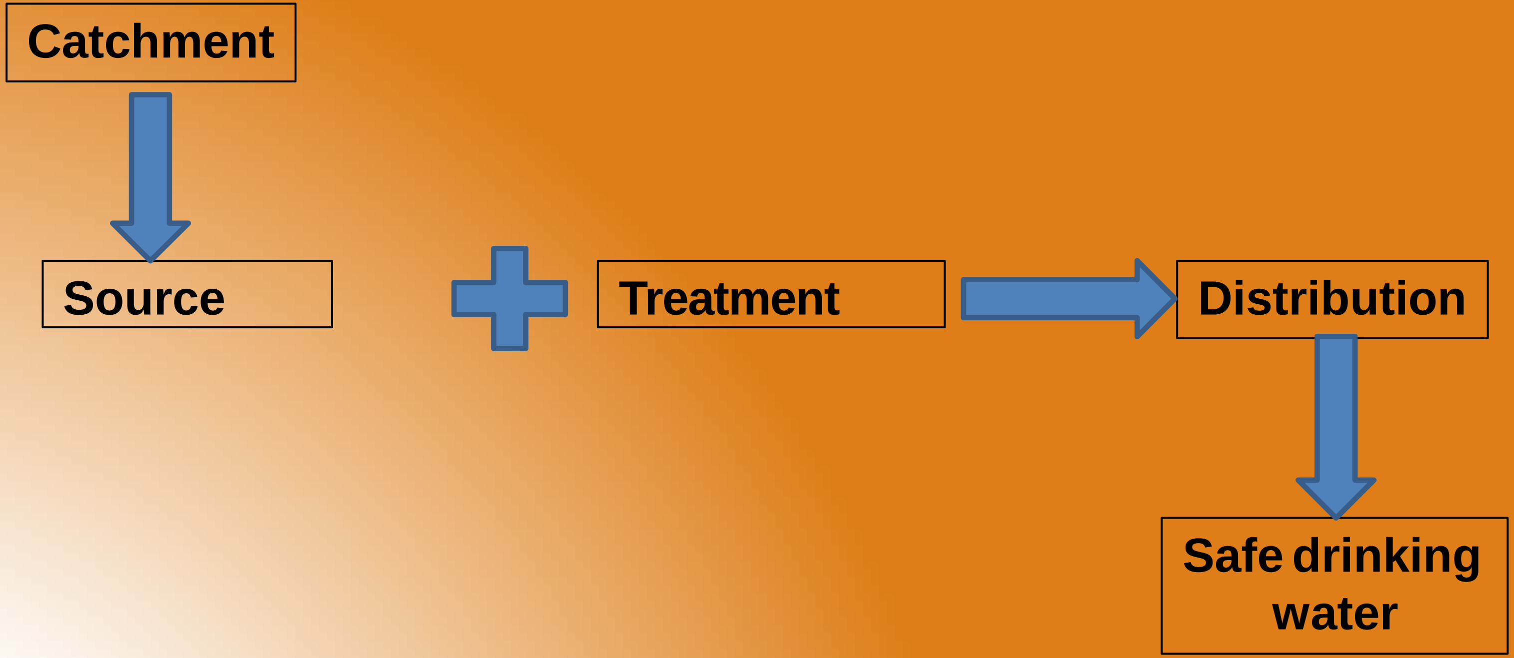
- **Proper management of solid waste**

Structural BMPs for controlling ANPSP

- **Erosion control:**
  - Contour farming, diversions, terracing, fencing, wind erosion control
- **Sediment control:**
  - Filter strips, grassed waterways, sediment basins
- **Pathogens:**
  - Waste storage ponds, stream crossing

Drinking water quality management

Safe Water Supply



Catchment

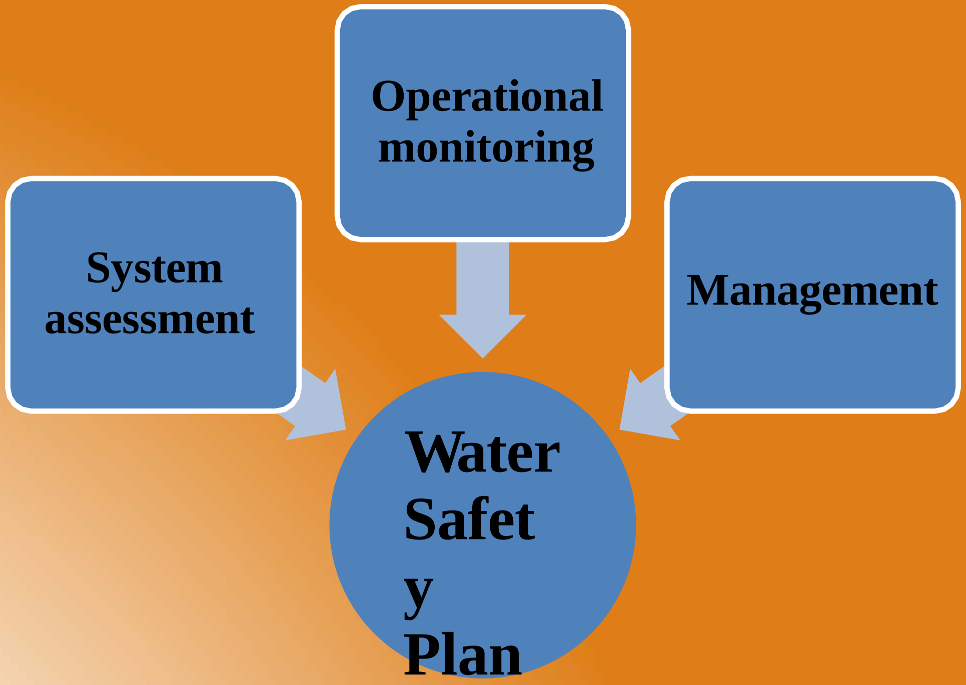
Source

Treatment

Distribution

Safe drinking water

Water Safety Plans (WSP)



Operational monitoring

System assessment

Management

Water Safety Plan

System Assessment: Determines if the water supply chain as a whole can deliver water of a sufficient quality
Monitoring: Monitoring of the control measures in the supply chain that are of particular importance in securing water safety
Management: Management plans describing the actions to be undertaken from normal conditions to extreme events

Objectives of WSP

- Minimization of contamination of source waters
- Reduction or removal of contamination through treatment processes
- Prevention of contamination during storage, distribution and handling of drinking-water.

Water Safety Plans : System Assessment

Assembling the team

- ➢ Assemble the team to prepare the water safety plan.

Documenting & describing the system

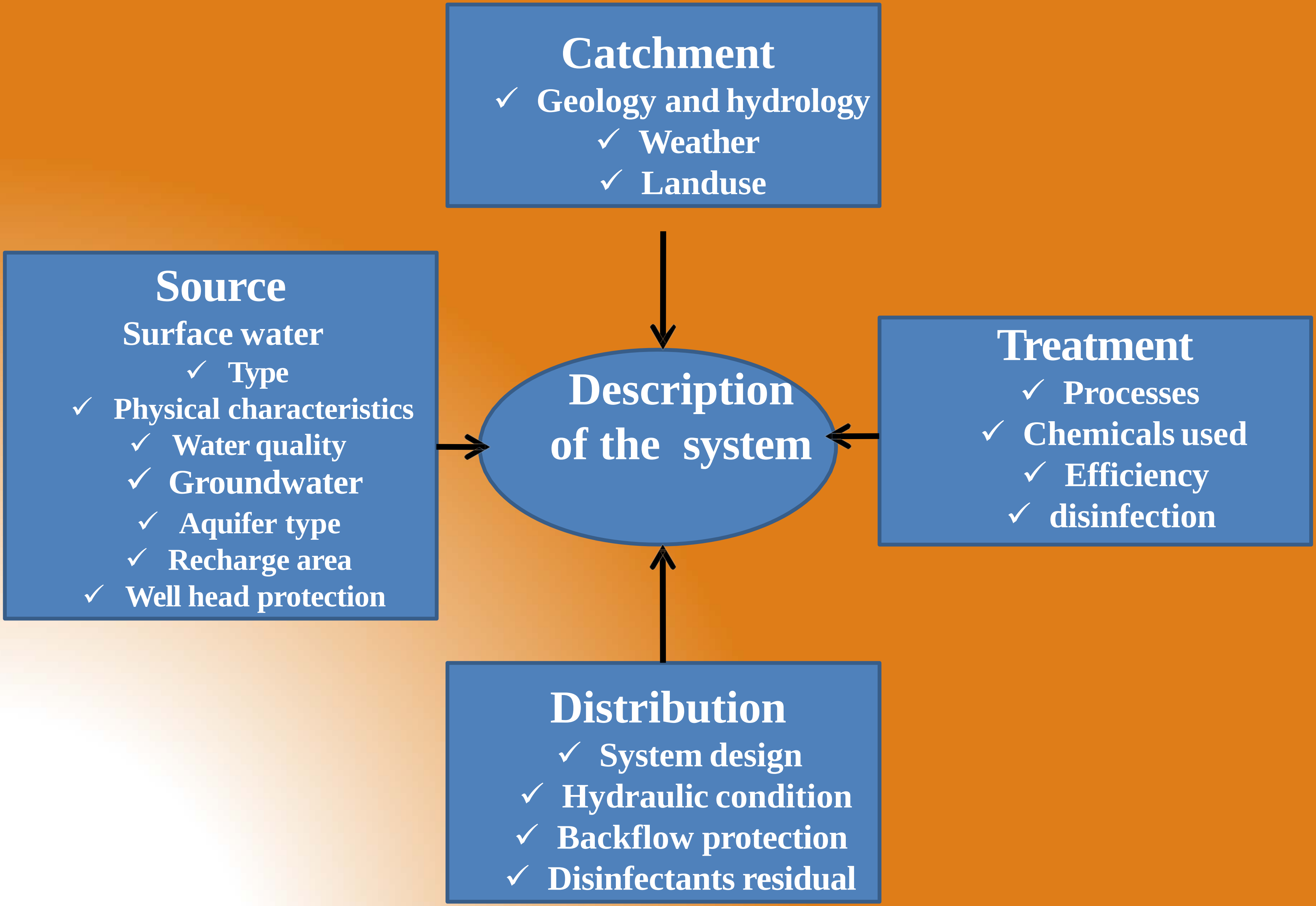
- ➢ Describe the existing water supply system from catchment to consumer.

Assessing hazards and prioritizing risks

- ➢ Undertake a hazard assessment and risk characterization to identify and understand how hazards can enter into the water supply.

Assessing the system

- ➢ Assess the existing/proposed system
  - ✓ include a description of the system and a drinking water flow diagram.



Catchment

- ✓ Geology and hydrology
- ✓ Weather
- ✓ Landuse

Source

Surface water

- ✓ Type
- ✓ Physical characteristics
- ✓ Water quality
- ✓ Groundwater
- ✓ Aquifer type
- ✓ Recharge area
- ✓ Well head protection

Description of the system

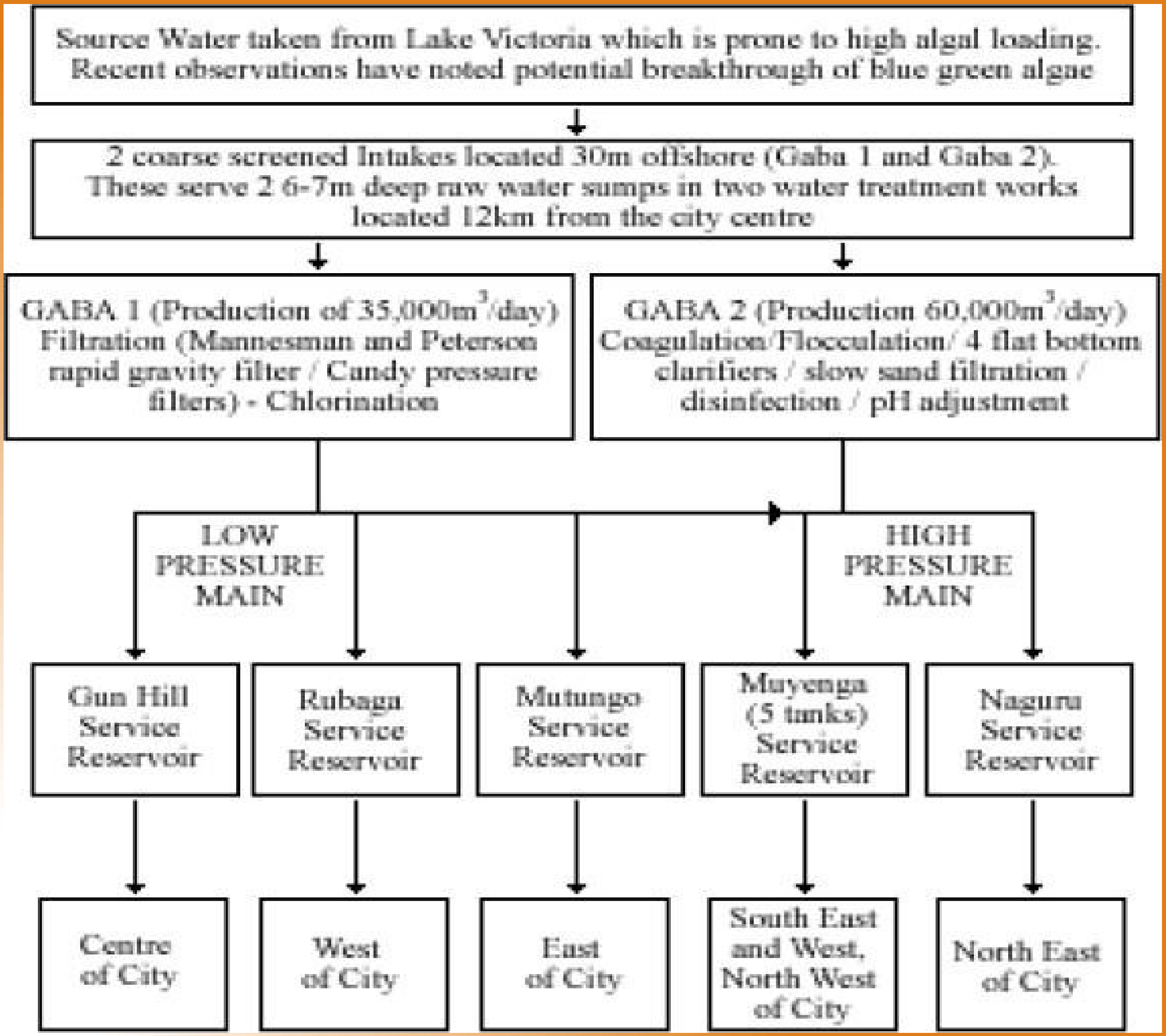
Treatment

- ✓ Processes
- ✓ Chemicals used
- ✓ Efficiency
- ✓ disinfection

Distribution

- ✓ System design
- ✓ Hydraulic condition
- ✓ Backflow protection
- ✓ Disinfectants residual

The Kampala water system



Source Water taken from Lake Victoria which is prone to high algal loading. Recent observations have noted potential breakthrough of blue green algae

2 coarse screened Intakes located 30m offshore (Gaba 1 and Gaba 2). These serve 2 6-7m deep raw water sumps in two water treatment works located 12km from the city centre

GABA 1 (Production of 35,000m^3/day) Filtration (Mannesman and Peterson rapid gravity filter / Candy pressure filters) - Chlorination

GABA 2 (Production 60,000m^3/day) Coagulation/Flocculation/ 4 flat bottom clarifiers / slow sand filtration / disinfection / pH adjustment

LOW PRESSURE MAIN

HIGH PRESSURE MAIN

Gun Hill Service Reservoir

Rubaga Service Reservoir

Mutungo Service Reservoir

Muyenga (5 tanks) Service Reservoir

Naguru Service Reservoir

Centre of City

West of City

East of City

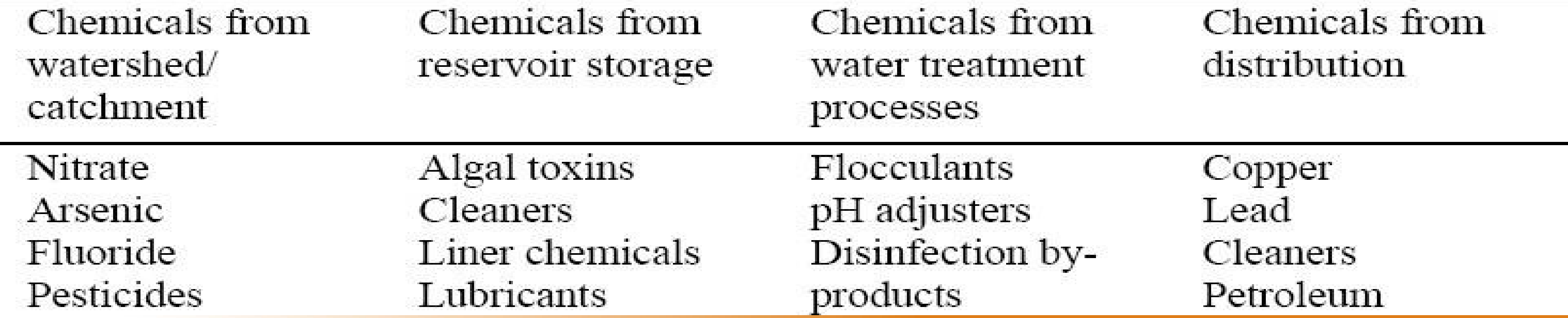
South East and West, North West of City

North East of City

Assessing hazards

Hazard → **Hazardous event** → **Risk**

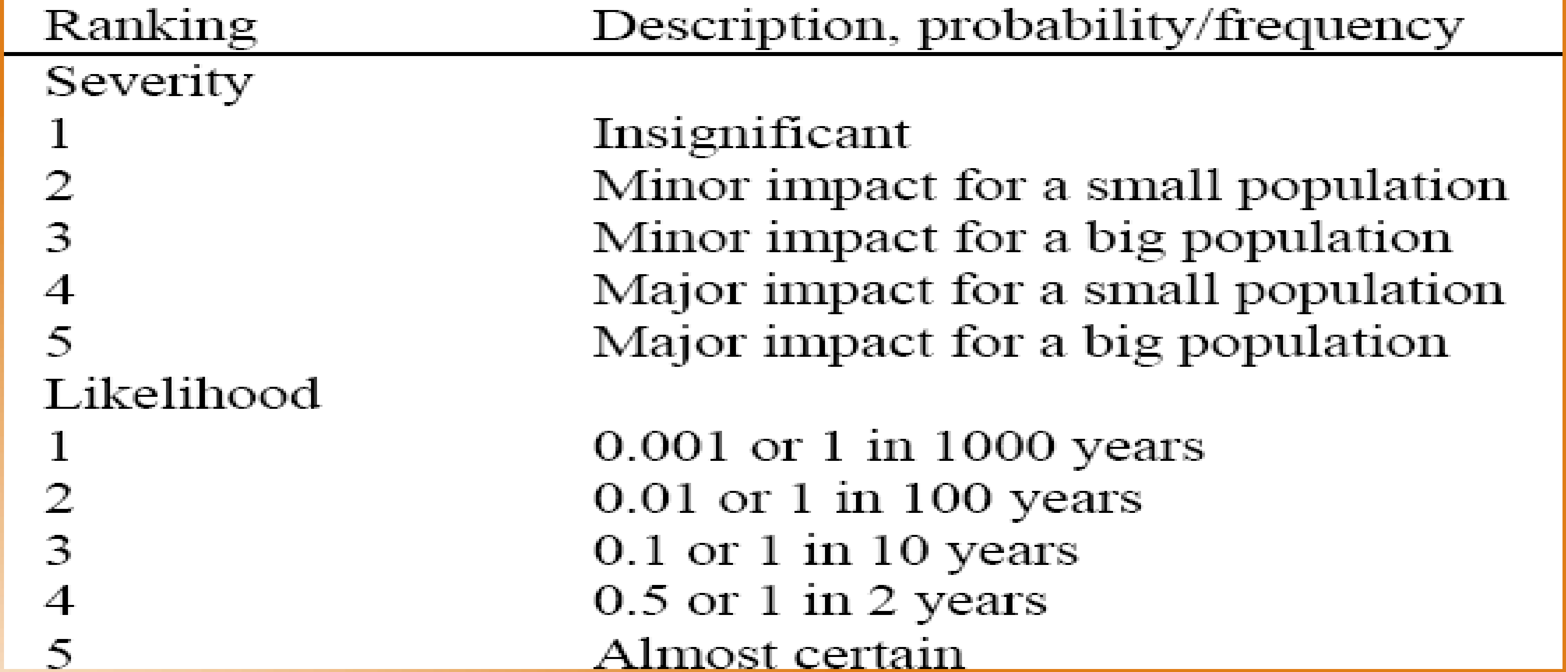
- ✓ Biological: bacteria, viruses, protozoa, etc.
- ✓ Chemical: example



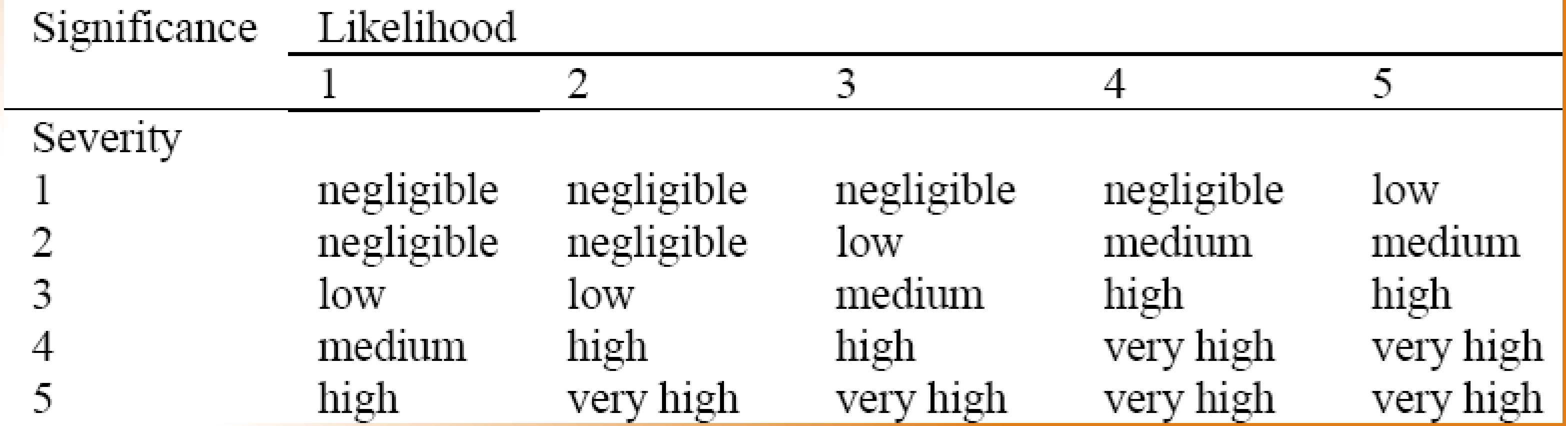
Chemicals from watershed/ catchment	Chemicals from reservoir storage	Chemicals from water treatment processes	Chemicals from distribution
Nitrate Arsenic Fluoride Pesticides	Algal toxins Cleaners Liner chemicals Lubricants	Flocculants pH adjusters Disinfection by-products	Copper Lead Cleaners Petroleum

- ✓ Physical: sediment
- ✓ Radiological

Risk analysis

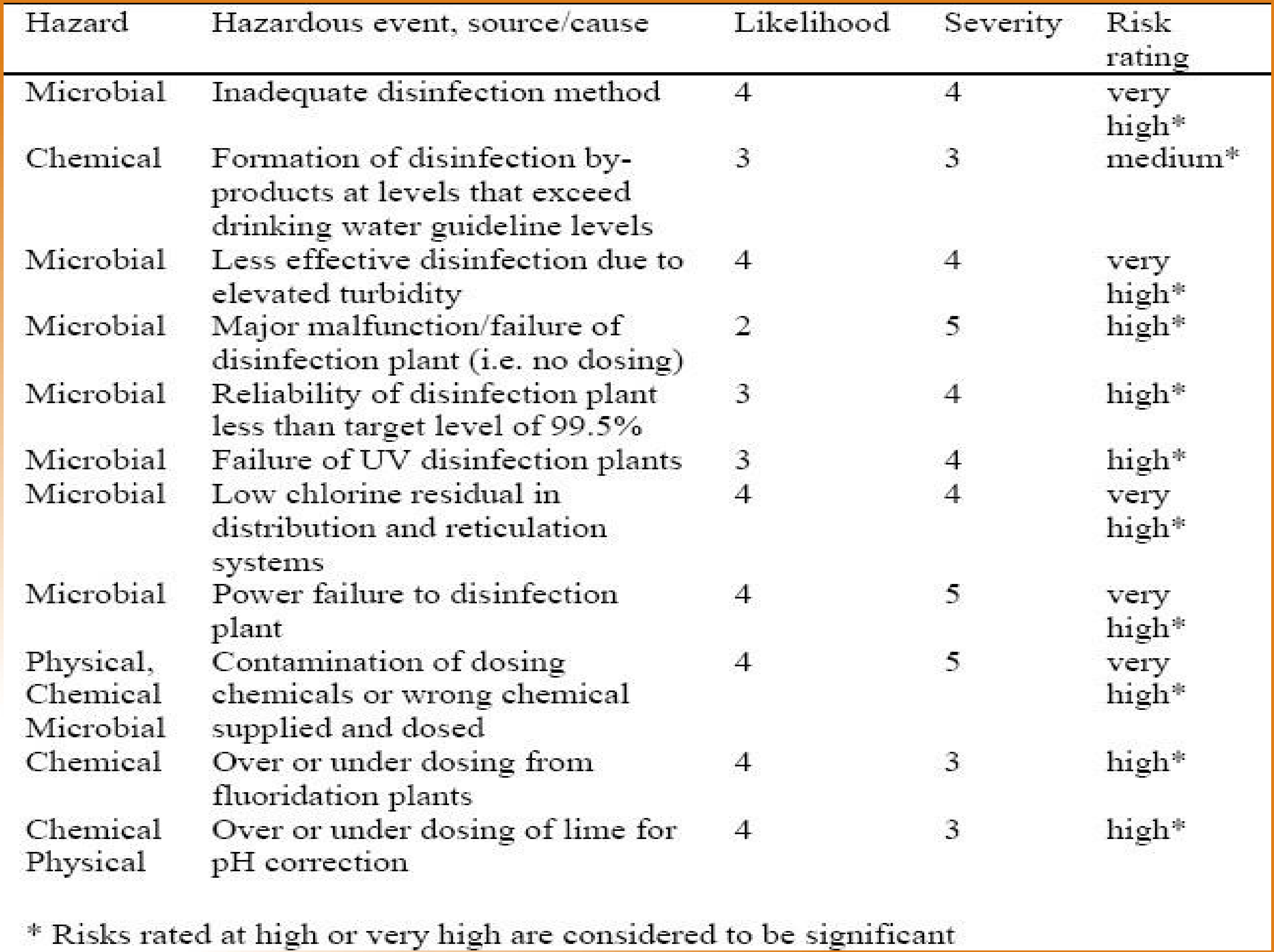


Ranking	Description, probability/frequency
Severity	
1	Insignificant
2	Minor impact for a small population
3	Minor impact for a big population
4	Major impact for a small population
5	Major impact for a big population
Likelihood	
1	0.001 or 1 in 1000 years
2	0.01 or 1 in 100 years
3	0.1 or 1 in 10 years
4	0.5 or 1 in 2 years
5	Almost certain



Significance	Likelihood				
	1	2	3	4	5
Severity					
1	negligible	negligible	negligible	negligible	low
2	negligible	negligible	low	medium	medium
3	low	low	medium	high	high
4	medium	high	high	very high	very high
5	high	very high	very high	very high	very high

Example- Hazard assessment



Hazard	Hazardous event, source/cause	Likelihood	Severity	Risk rating
Microbial	Inadequate disinfection method	4	4	very high*
Chemical	Formation of disinfection by-products at levels that exceed drinking water guideline levels	3	3	medium*
Microbial	Less effective disinfection due to elevated turbidity	4	4	very high*
Microbial	Major malfunction/failure of disinfection plant (i.e. no dosing)	2	5	high*
Microbial	Reliability of disinfection plant less than target level of 99.5%	3	4	high*
Microbial	Failure of UV disinfection plants	3	4	high*
Microbial	Low chlorine residual in distribution and reticulation systems	4	4	very high*
Microbial	Power failure to disinfection plant	4	5	very high*
Physical, Chemical Microbial	Contamination of dosing chemicals or wrong chemical supplied and dosed	4	5	very high*
Chemical	Over or under dosing from fluoridation plants	4	3	high*
Chemical Physical	Over or under dosing of lime for pH correction	4	3	high*

* Risks rated at high or very high are considered to be significant

Water Safety Plans : Monitoring

Identifying control measures

- Identify the means by which risks may be controlled.

Monitoring control measures

- Define the limits of acceptable performance and how these are monitored.

Validating effectiveness of WSP

- Establish procedures to verify that the water safety plan is working effectively and will meet the pre- determined *targets*
  – eg. health-based targets.

Operational water quality monitoring

Methods

- Sampling at selected points
- Automatic insitu sensors
- Remotes sensing (by airplane or satellite)
- Numerical model
- Customer complaints

Parameters

- pH
- Turbidity
- Color
- Coliform
- Temperature
- Residual chlorine

Water Safety Plans : Management & Communications

Developing supporting programs

- Provide a program of support for staff and infrastructure (training, upgrade and improvement, research and development, etc).

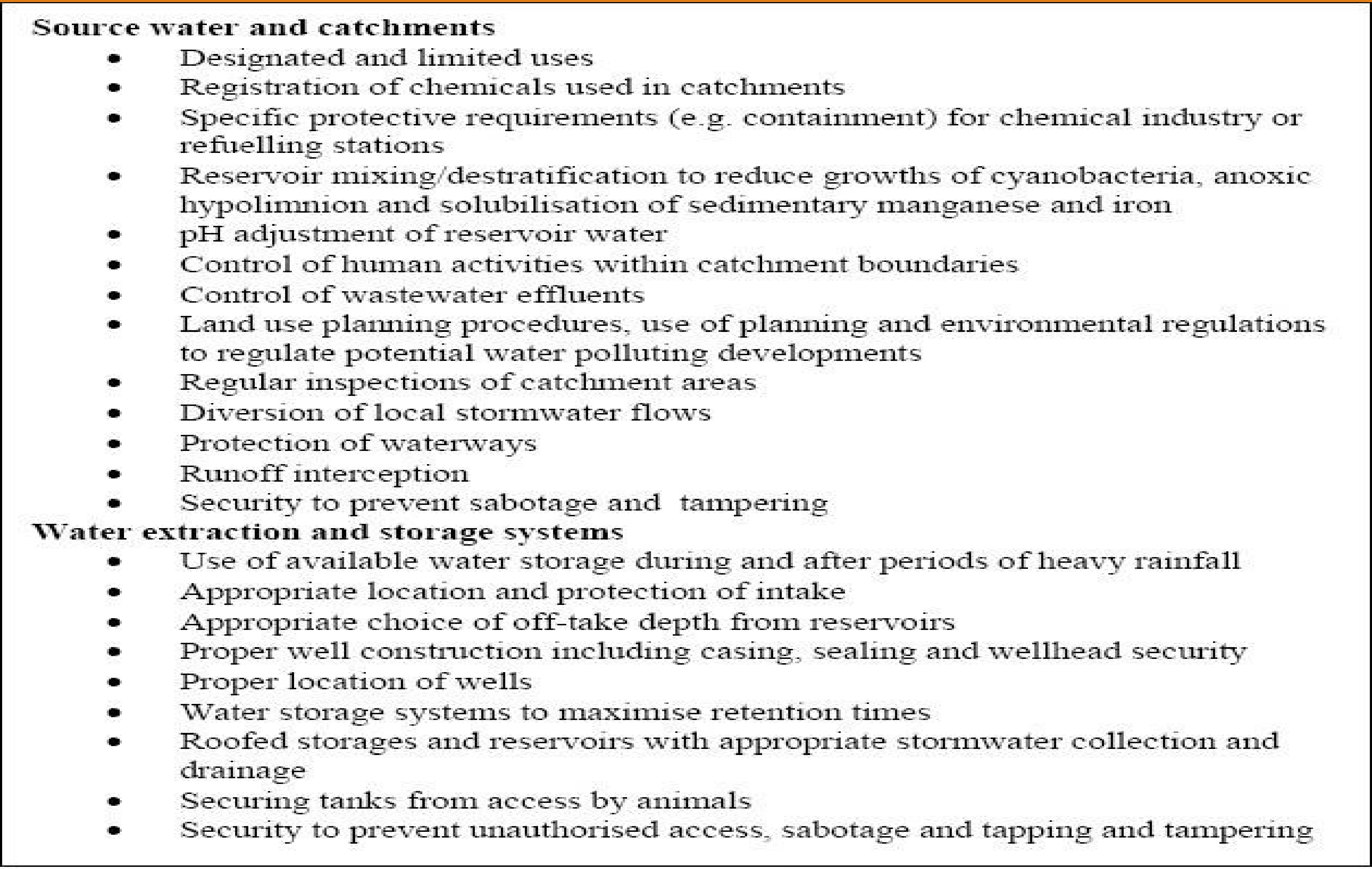
Preparing management procedures

- Prepare management procedures (including corrective actions) for normal and incident conditions.

Establishing documentation & communication procedures

- Establish documentation of the water safety plan and procedures for communicating with other parties involved in the delivery of drinking water, including the consumer.

Example- source, storage, extraction control measures



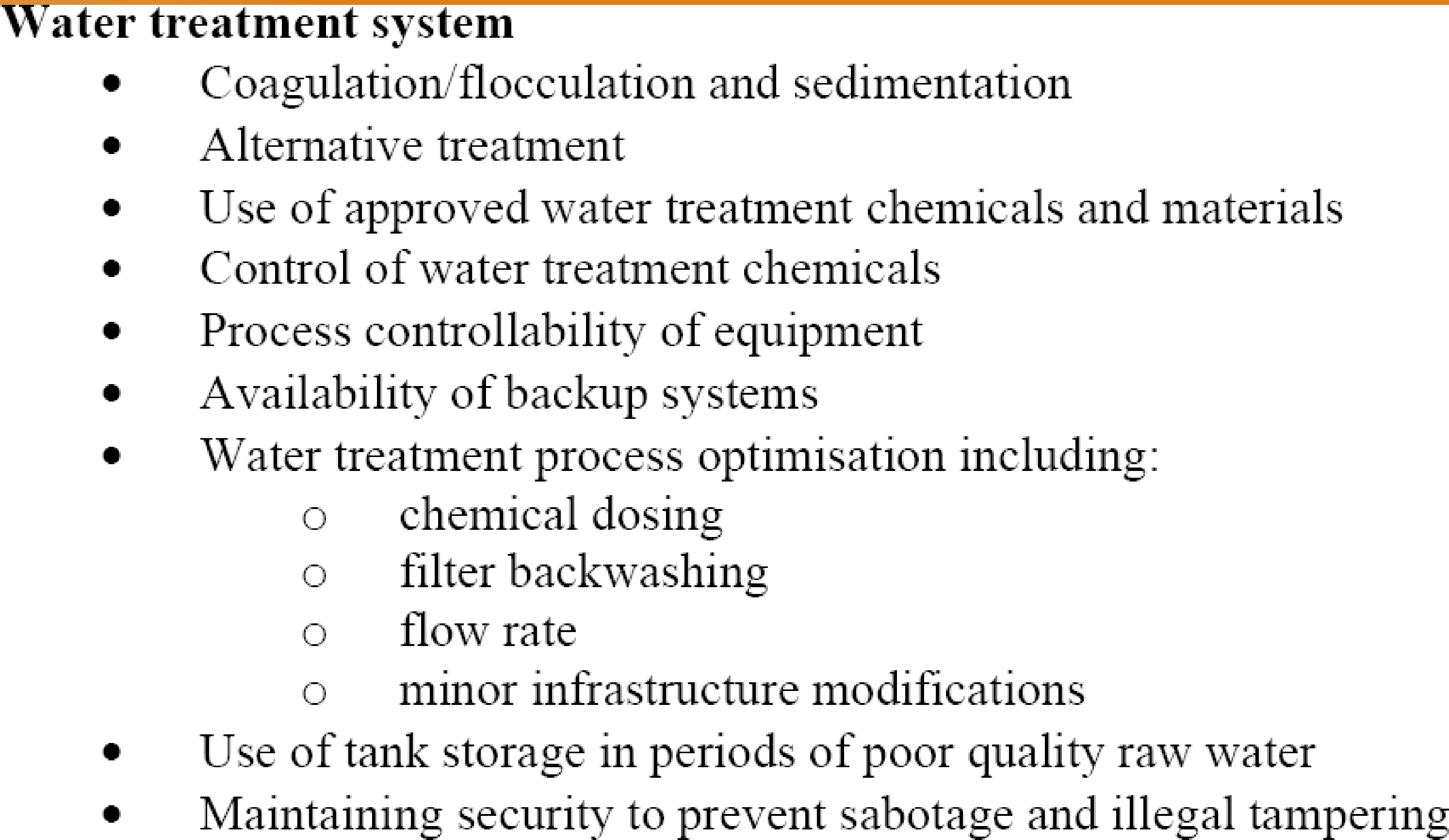
Source water and catchments

- Designated and limited uses
- Registration of chemicals used in catchments
- Specific protective requirements (e.g. containment) for chemical industry or refuelling stations
- Reservoir mixing/destratification to reduce growths of cyanobacteria, anoxic hypolimnion and solubilisation of sedimentary manganese and iron
- pH adjustment of reservoir water
- Control of human activities within catchment boundaries
- Control of wastewater effluents
- Land use planning procedures, use of planning and environmental regulations to regulate potential water polluting developments
- Regular inspections of catchment areas
- Diversion of local stormwater flows
- Protection of waterways
- Runoff interception
- Security to prevent sabotage and tampering

Water extraction and storage systems

- Use of available water storage during and after periods of heavy rainfall
- Appropriate location and protection of intake
- Appropriate choice of off-take depth from reservoirs
- Proper well construction including casing, sealing and wellhead security
- Proper location of wells
- Water storage systems to maximise retention times
- Roofed storages and reservoirs with appropriate stormwater collection and drainage
- Securing tanks from access by animals
- Security to prevent unauthorised access, sabotage and tapping and tampering

Water treatment control measures



Water treatment system

- Coagulation/flocculation and sedimentation
- Alternative treatment
- Use of approved water treatment chemicals and materials
- Control of water treatment chemicals
- Process controllability of equipment
- Availability of backup systems
- Water treatment process optimisation including:
  - chemical dosing
  - filter backwashing
  - flow rate
  - minor infrastructure modifications
- Use of tank storage in periods of poor quality raw water
- Maintaining security to prevent sabotage and illegal tampering

Causes of water quality degradation in the distribution system

- Failures at the treatment barrier
- Transformations in the bulk phase
- Corrosion and leaching of pipe material
- Biofilm formation
- Mixing between different sources of water

Effects of bacteriological growth taste-and-odor problems, discoloration, slime buildup, and economic problems, including corrosion of pipes and bio-deterioration of materials

Distribution Systems Water Quality

Water Quality deterioration factors

Structural

- Oversized pipes
- Pipe materials that can pit
- Corrosive pipes
- cross-connections
- Leaky or broken pipes

Operational

- Slow flow velocity
- Intermittent supplies
- Long storage time

Effects:
-Fading of chlorine residuals
-Allowing microbial growth
-Sediment deposits
-Degraded water quality

Water quality

- Long water detention time
- Adequate nutrient
- Warm temperature
-disinfectant's reaction with organic and inorganic matters

System operation modifications to maintain water quality

- Minimize bulk water detention time
- Maintain positive pressure
- Control the direction and velocity of the bulk water
- Maintain a disinfectant residual in the distribution system
- Prevent cross-connections and backflow

➢ Maintenance activities:

  - try to keep contaminated water out of the trench and pipe; flush the line in the vicinity of the break; apply disinfectant to potentially contaminated components; disinfect new mains; disinfect storage tanks after construction, inspection, or maintenance; and conduct bacteriological testing to confirm the absence of contaminates
  - prevent and eliminate cross-connections; cover and vent storage tanks; maintain an adequate separation from sewers; and enforce applicable building plumbing codes.

Corrosion, Biofilm and Sludge Management

Corrosion/Biofouling

- Corrosion is removal of material from the system and can lead to leaks that cause the system to be shut down while the leaks are repaired.
- It occurs when the water and pipe material create a small electrical charge that removes molecules from the metal system
- Biofouling is the undesirable accumulation of microorganisms
- Effects of internal corrosion
  - *May cause leaching of toxic metals (e.g. lead*
  - *Impart metallic taste to water*
  - *Stain plumbing fixtures*
  - *Harbours pathogens*
  - *Reduces pipe's hydraulic carrying capacity*
  - *Leads to pipe leaks and clogs*

Some Approaches to control corrosion

- Modify the water quality to make the water less corrosive, such as adjusting the pH.

- Lay down a protective lining between the water and the pipe, such as using chemical inhibitors.

- Switch to plastic pipe, which is less prone to corrosion.

Biofilm control measures

Comprehensive distribution system maintenance program (e.g. regular flushing, pipe replacement)

Reservoir maintenance (e.g. limit detention time, keep covered

Appropriate disinfection practices (e.g. increase free chlorine residual, use alternative disinfectant)

Corrosion control

Reduced nutrient level

BAT to meet total coliform rule

Personnel training

Wastewater Sludge Management

- Characteristics of wastewater sludge

  Primary clarifier sludge:

    - Contains both organic (BOD) and inorganic solids (sand, silt, etc)
    - 5% solids (95% water)

  Secondary clarifier sludge:

    - Contains dead bacteria
    - 1% solids if activated sludge unit is used
    - 5% solids if trickling filter is used
- Objectives of sludge management:
    - Removal of part or all of the water
    - Decomposition of putrescible organic matter
    - Killing of pathogens

Sludge treatment processes

- *Thickening*
  - Objective: remove water before stabilization; decrease volume
  - Processes: gravity thickening; dissolved air floatation, or centrifuge

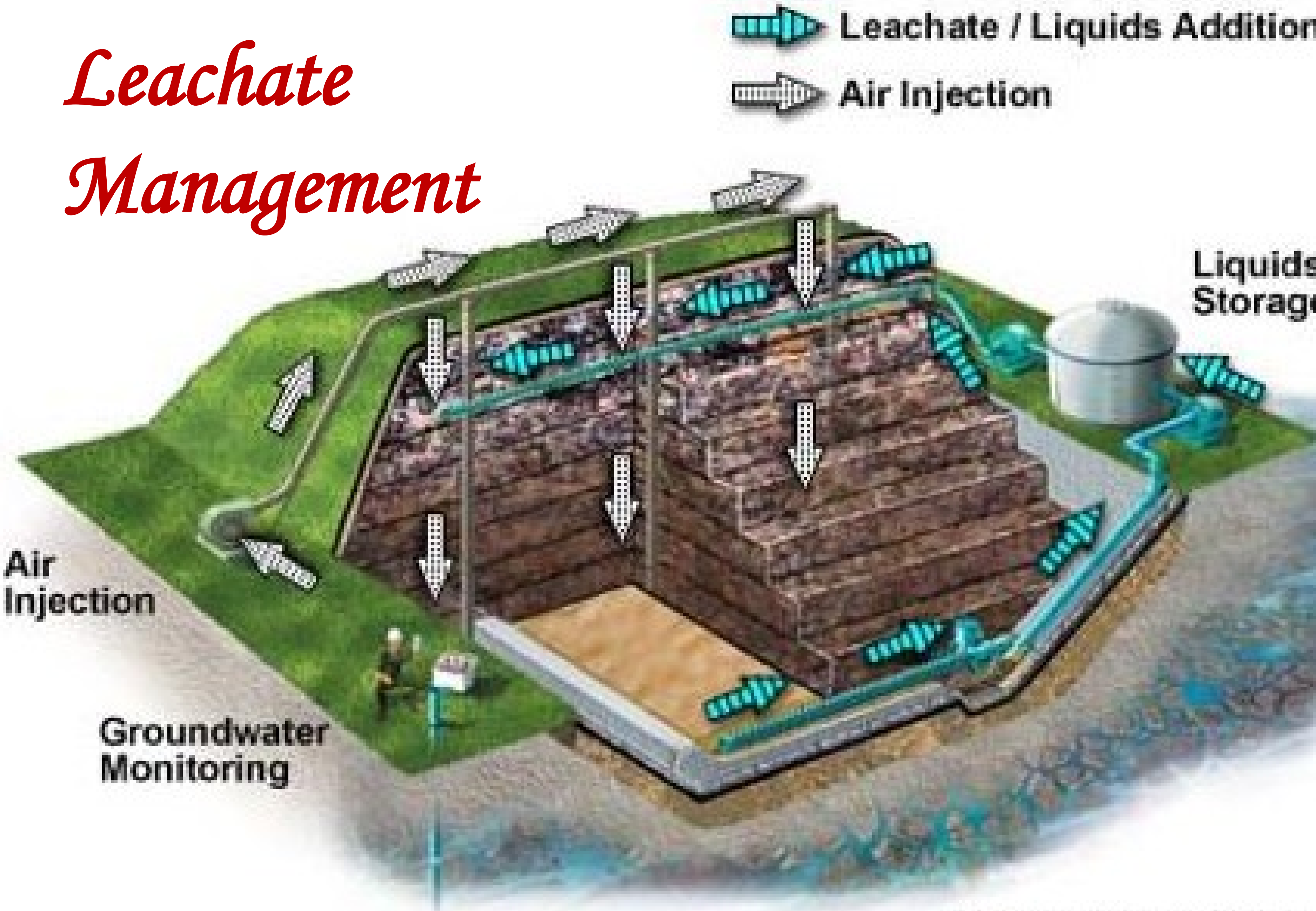
- *Stabilization:*
  - Objectives: kill pathogens; stabilize organics and eliminate odors; concentrate solids
  - Process: Anaerobic digestion

Sludge treatment processes

- ***Dewatering***
  - Objective: remove water from the stabilized sludge
  - Processes: drying beds; vacuum filter; centrifuge

- ***Final disposal***
  - Objective: dispose of the sludge in environmentally sound way
  - Processes: landfill; land application; composting

Leachate Management

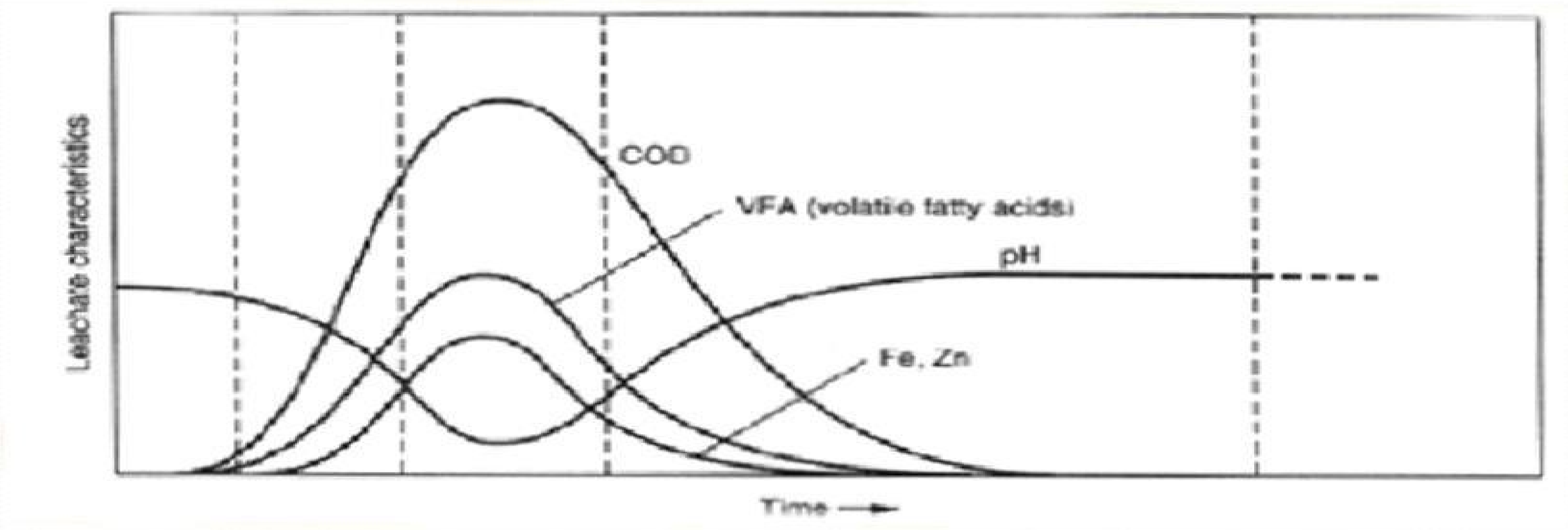


Courtesy of Waste Managem

Leachate characteristics

Leachate Properties

- Leachate is liquid that percolates through the layers of the landfill and transports organic and inorganic compounds.



Composition of typical landfill leachate

Constituent	< 2 years (mg/L)	> 10 years (mg/L)
BOD5	2000-30000	100-200
TOC	1500-20000	80-160
COD	3000-60000	100-500
TSS	200-2000	100-400
Organic nitrogen	10-800	80-120
Ammonia nitrogen	10-800	20-40
Nitrate	5-40	5-10
Total phosphorus	5-100	5-10
Ortho-phosphorus	4-80	4-8
Alkalinity as CaCO3	1000-10000	200-1000
pH	4.5-7.5	6.6-7.5
Total hardness as CaCO3	300-10000	200-500
Calcium	200-3000	100-400
Magnesium	50-1500	50-200
Potassium	200-1000	50-400
Sodium	200-2500	100-200
Chloride	200-3000	100-400
Sulfate Total	50-1000	20-50
iron	50-1200	20-200

Leachate quantity

LC = PR + SRT – SRO – EP – ST

Where

LC = leachate

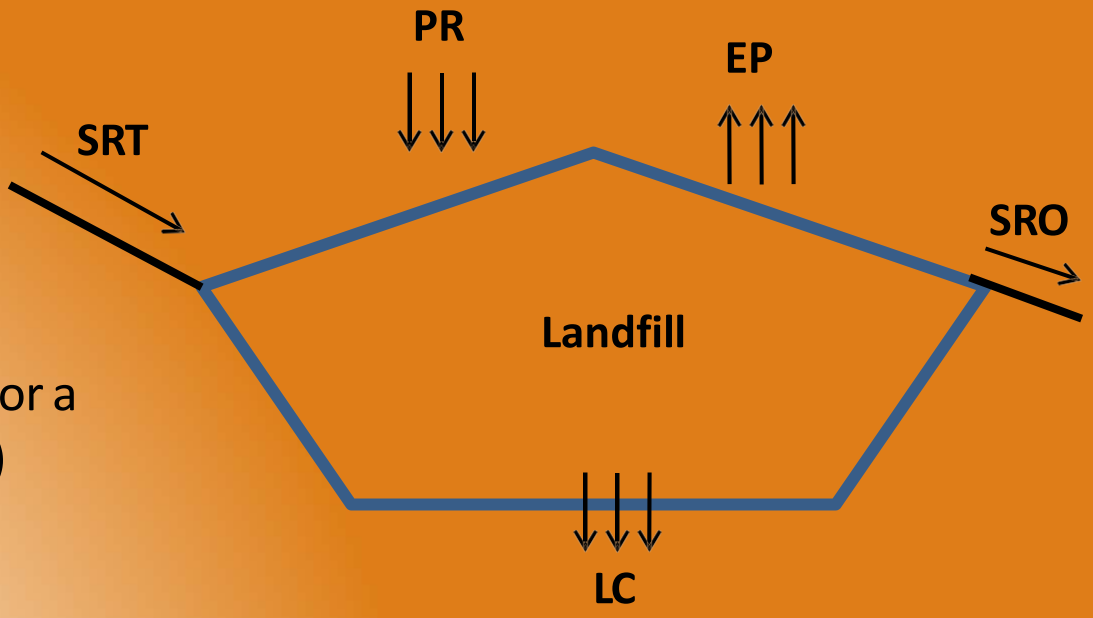
PR = precipitation

SRO = surface runoff

SRT = surface runto (SRT = 0 for a properly designed landfill)

EP = evapotranspiration

ST = change in water storage



Leachate management

- Liners
  - –Objective: To minimize infiltration of leachate into aquifer system

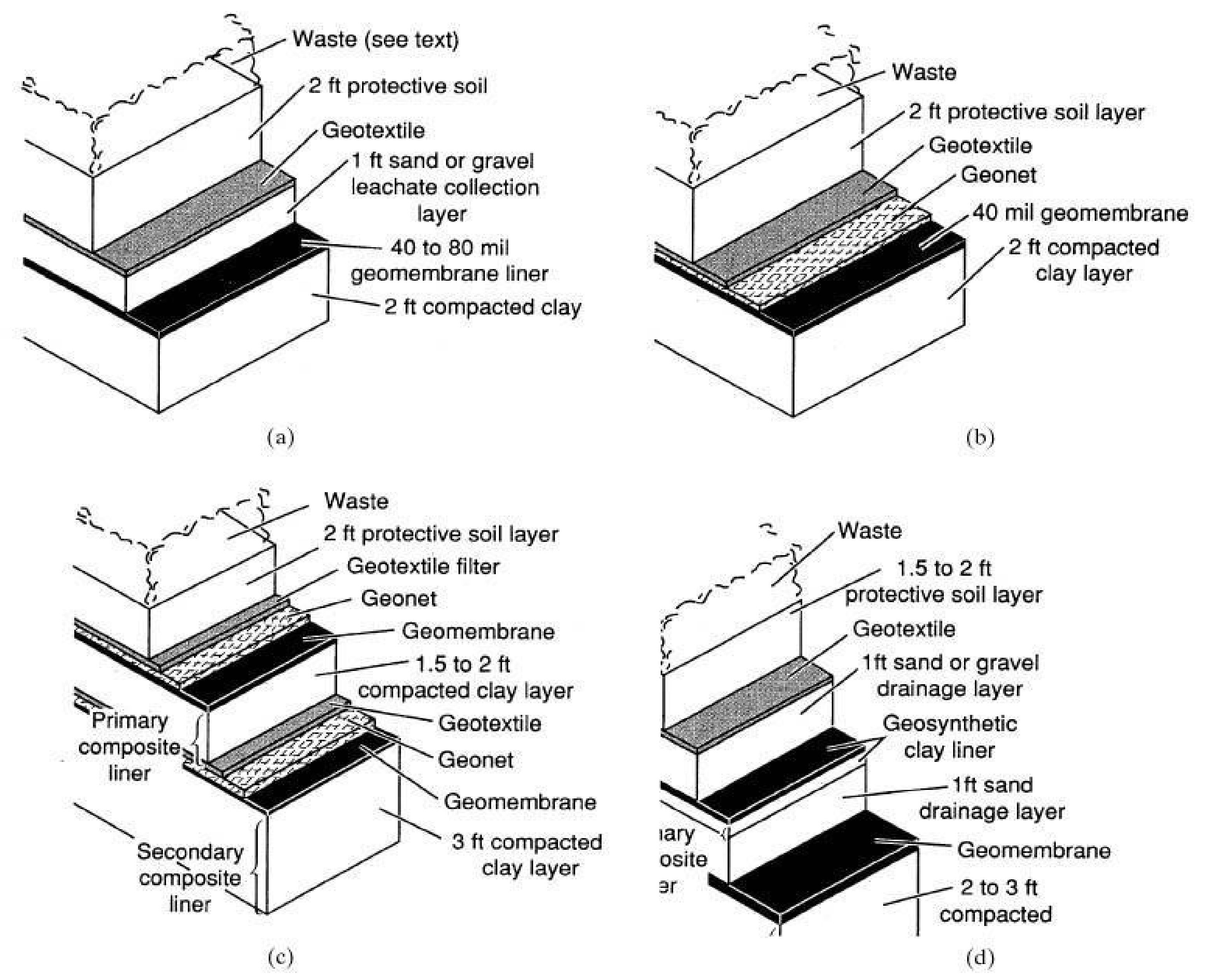
  Selection of liner: function of geology and environmental requirements

    - If there is no groundwater a single compacted clay layer could be sufficient

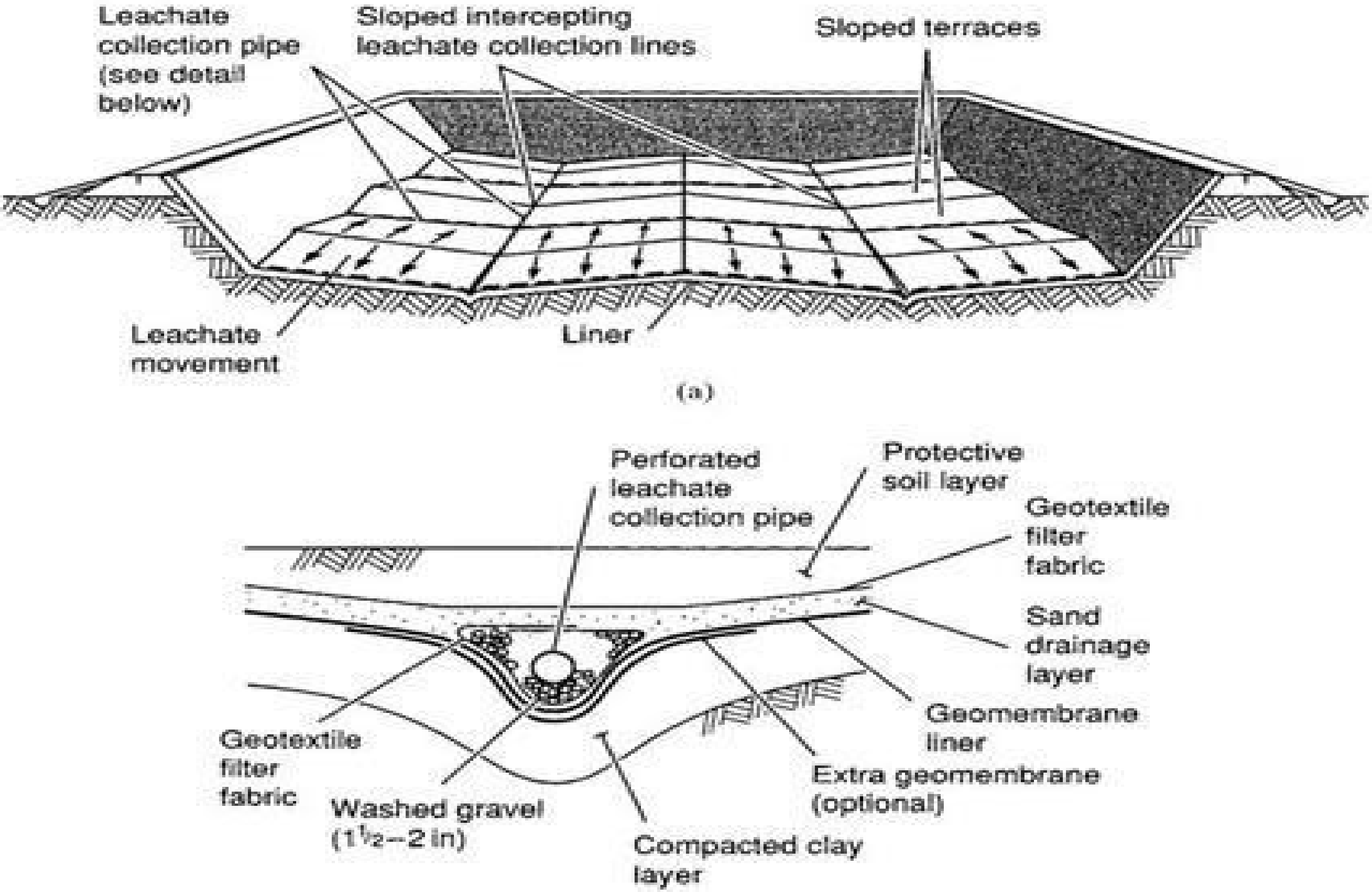
    -If groundwater pollution hazard is high use a composite liner composed of clay and geosynthetic liners

- Collection systems
  - accomplished by using a series of sloped terraces and a system of collection pipes
- Treatment
  - Recycling; evaporation; treatment + spray disposal; wetlands; treatment with domestic sewage

Leachate


Waste (see text)
2 ft protective soil
Geotextile
1 ft sand or gravel leachate collection layer
40 to 80 mil geomembrane liner
2 ft compacted clay
(a)
Waste
2 ft protective soil layer
Geotextile
Geonet
40 mil geomembrane
2 ft compacted clay layer
(b)
Waste
2 ft protective soil layer
Geotextile filter
Geonet
Geomembrane
1.5 to 2 ft compacted clay layer
Geotextile
Geonet
Geomembrane
3 ft compacted clay layer
Primary composite liner
Secondary composite liner
(c)
Waste
1.5 to 2 ft protective soil layer
Geotextile
1ft sand or gravel drainage layer
Geosynthetic clay liner
1ft sand drainage layer
Geomembrane
2 to 3 ft compacted
iary
osite
er
(d)

Typical leachate collection



Household level water improvement methods

- Education: Influence hygiene behaviors and increase awareness about the dangers of contaminated water and waterborne disease.
- Safe storage
  - Plastic or modified clay pot safe storage containers
- Disinfection
  - Boiling
  - Household chlorination
  - SODIS and UV disinfection

Household level water improvement methods

- Particle removal
  - Cloth filtration
  - Ceramic filters (candle filters, pot filters)
  - Biosand filter
  - coagulation/precipitation only
- Membrane/reverse osmosis
  - Membrane, reverse osmosis, ultrafiltration, nanofiltration
- Combined systems
  - Coagulation/precipitation + chlorine disinfection (e.g. PUR)
  - Filtration + disinfection + aesthetics (Pure-it)
- Chemical removal systems

Safe storage

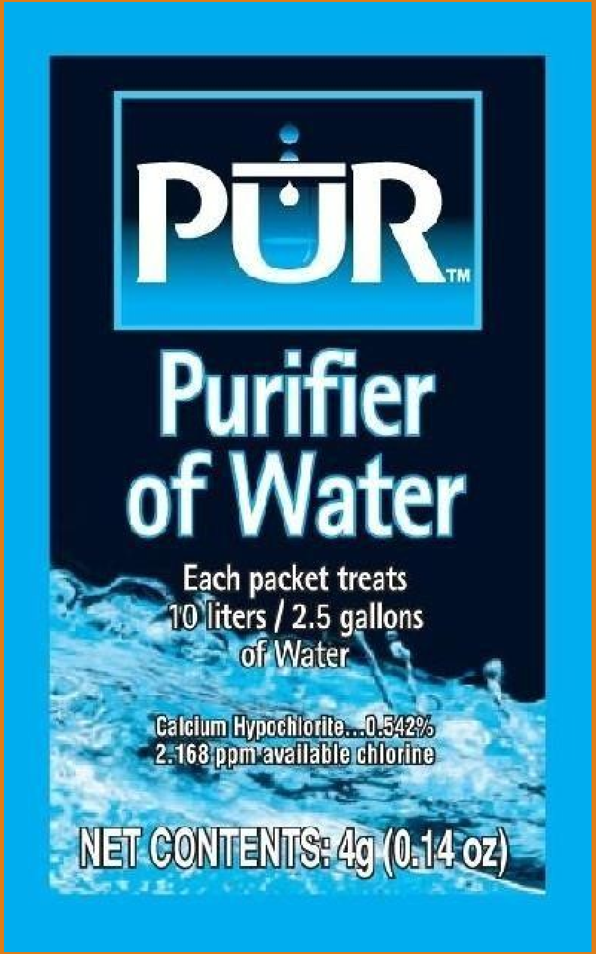
- Container: plastic or modified clay pot
- Standardized volume (10-30 liters)
- Durable, easy-to-clean material
- Inlet diameter (6-9 cm)
- Durable spout or spigot allowing a discharge of 1 liter per 15 seconds at outlet
- Instructions for use, cleaning container and disinfecting its contents

Safe Water Solution (SWS)

- Consists of dilute chlorine solution and a measuring cap
- Purifies water by inactivating microbial pathogens
- Water safe to drink after 30 min of treatment with SWS
- Reduces diarrhea by 30-50%

Purifier of Water (PuR)

- Treats through a combined process of disinfection with calcium hypochlorite and flocculation with iron sulfate
- Proven efficacy
- Especially effective in treating surface water
- Settles out particles after 5 minutes of stirring

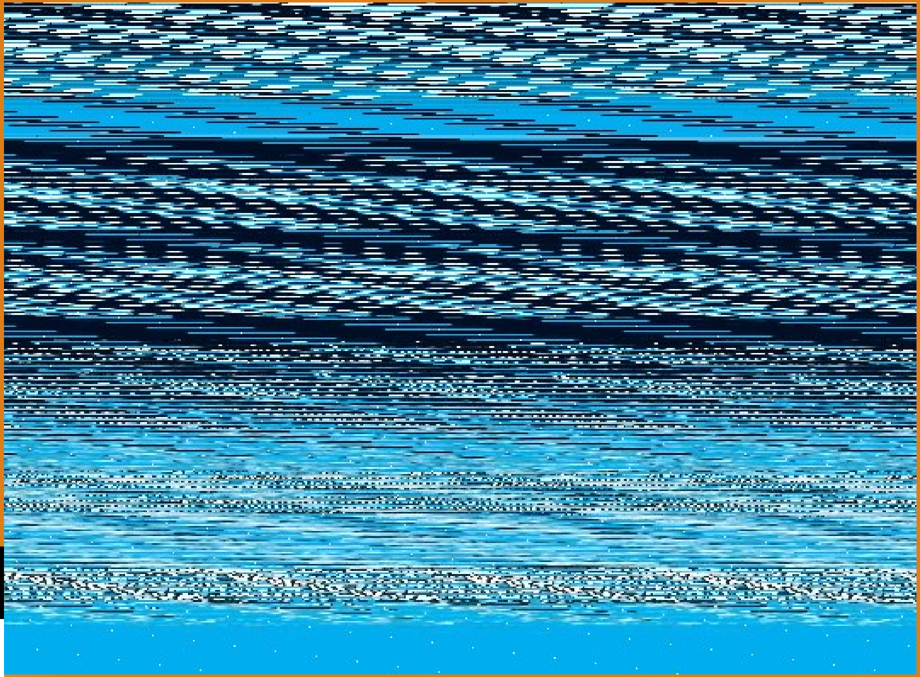


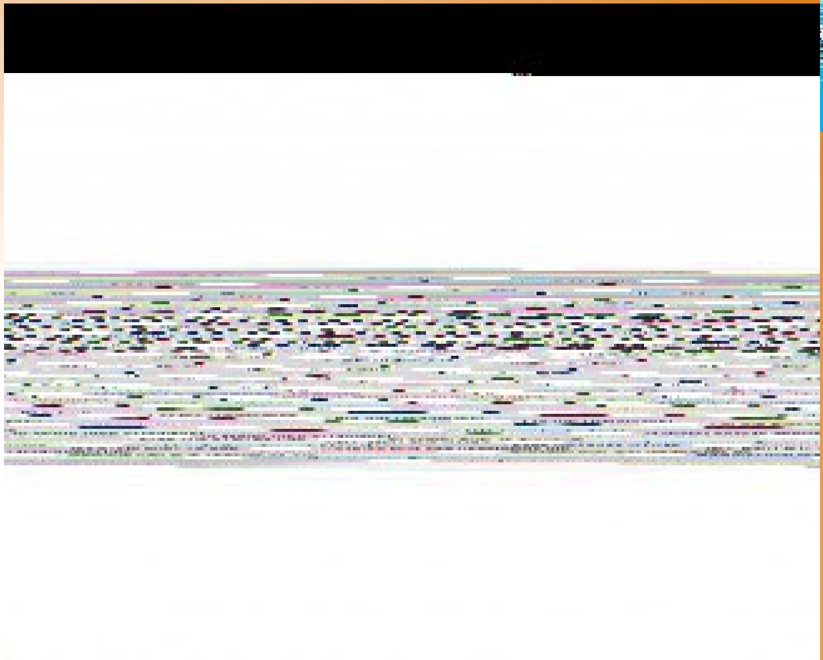
Water purification tablets (Aquatabs)

- Aquatabs are effervescent tablets that rapidly dissolve in water to kill pathogens
- Utilize active ingredient sodium dichloroisocyanurate
- Can be easily transported to rural areas

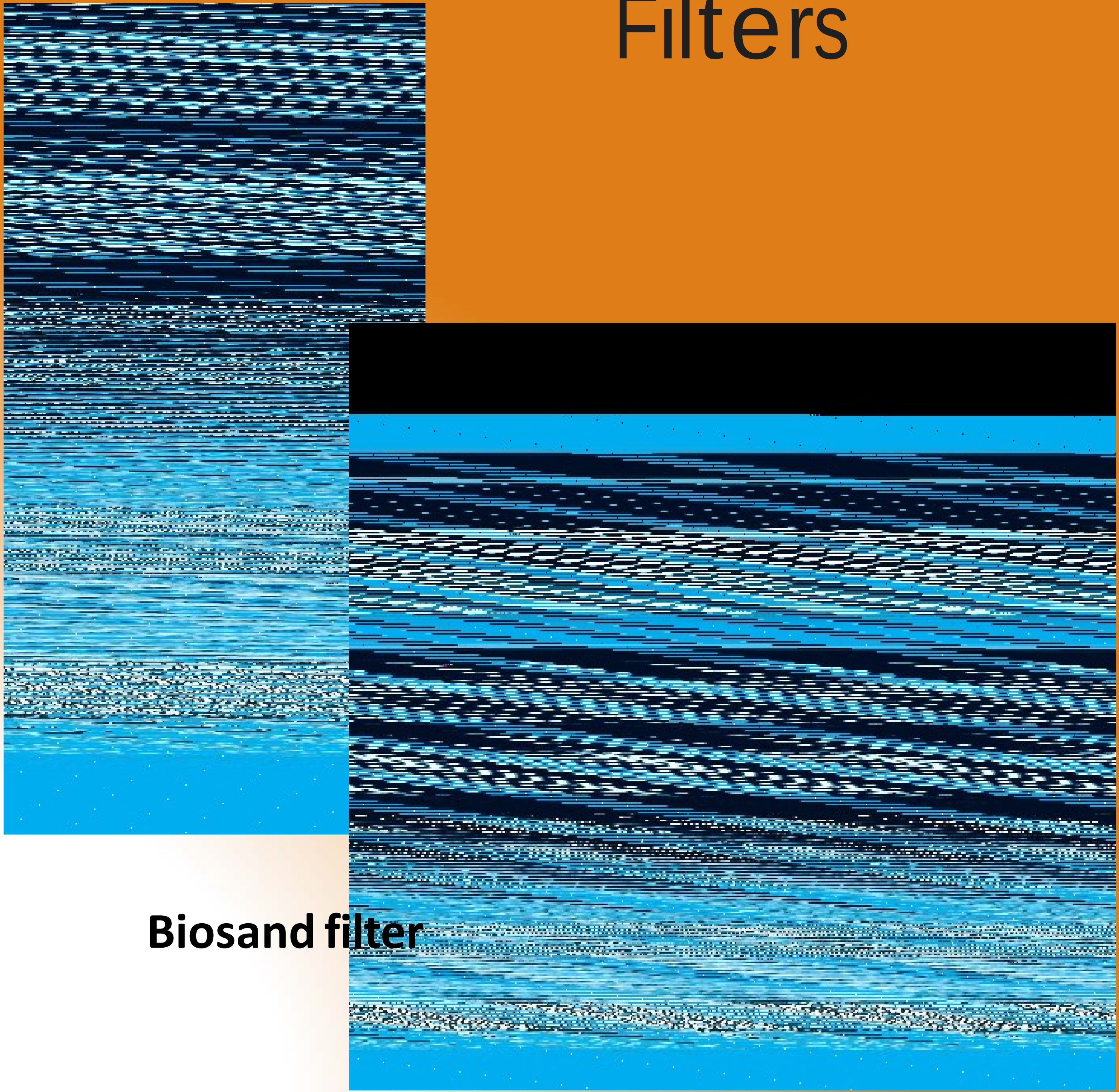
Solar Disinfection (SODIS)

- Plastic bottles exposed to solar radiation for 1-2 days
- Scientifically proven
- Highly effective against a range of microbial contaminants





Filters

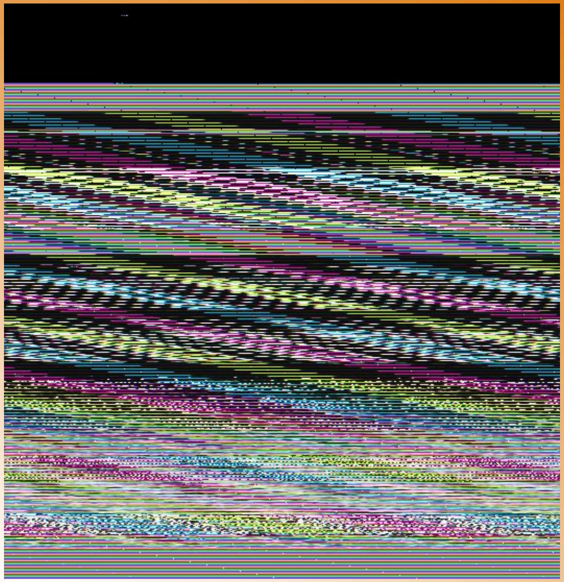


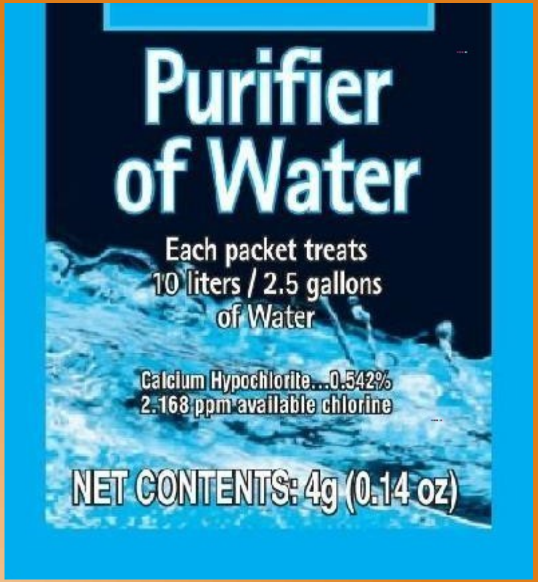
Biosand filter

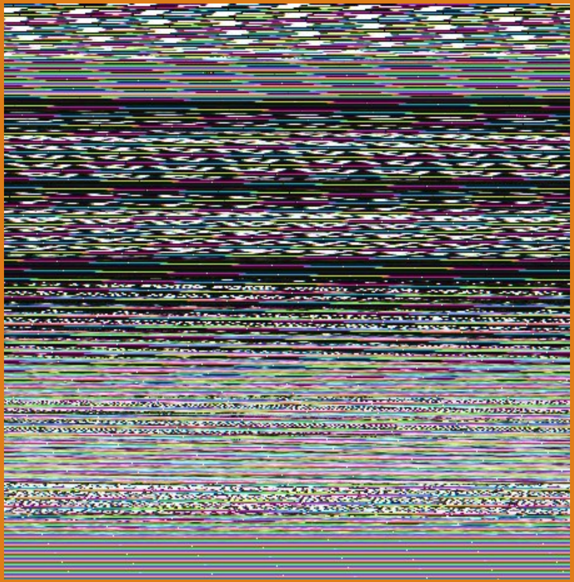
Cloth filter

Ceramic filter

Manual coagulation with Alum







Raw water

Coagulation regime

Settling time 0 min

Settling time 30 min

Thank you

Any question?