MATHSKILLS

Geometry

100 Reproducible Activities

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Exploring Geometry: Points, Lines, and Angles in a Plane

Parallel Lines and Transversals • and more

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MathSkills

Geometry

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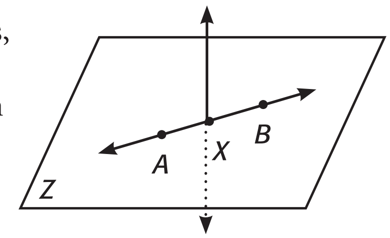
Table of Contents

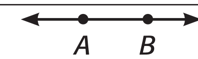
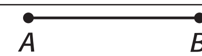
Name ______________________________ Date ____________________

Points, Segments, Rays, Lines, and Planes



In geometry, figures are created using points, lines, line segments, rays, and planes. Each item has a unique and specific definition, each a certain way to express it using symbols, and each a certain way the symbols are translated into words.

The figure to the right contains points, segments, lines and planes. Use the figure to complete the chart below.

Type of Figure	Symbol	Words	Drawing
Point	Point *A*	______________	______
Line	______	Line *AB*	
______________	$\overline{AB}$	______________	
Ray	______	Ray *AB*	______________
______________	▱*Z*	Plane *Z*	______________

Each figure also has a specific definition. Identify each type of figure. Complete each definition using the chart and figures above.

Rules for Naming Basic Figures

Point: A point has ________ size; it is shown by a ________ and named by a capital ______________.

Line: A line extends ____________________ on both sides with ________ thickness or width; a line is shown with an arrow at ________ ends.

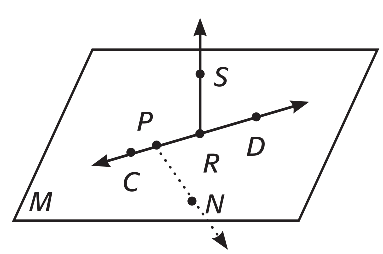
Segment: A part of a line with two points called ____________________; a segment shows the two points with ________ arrow at either end.

Ray: A ray is a part of a line that extends indefinitely in ________ direction; a ray has one ____________________.

Plane: A plane is a ________ surface that extends indefinitely in all directions and has ________ thickness.

Practice

Use the figure to the right to complete each statement.



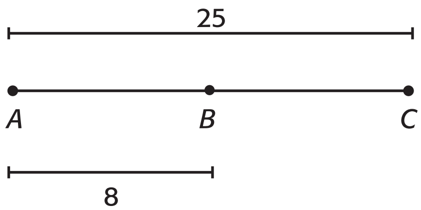
1. The plane shown in the figure is plane _____.
2. The symbol for line *CD* is _____.
3. A ray in the figure can be written using the symbol ______________.
4. $\overline{PN}$ is a symbol for ______________ *PN*.

Name ______________________________ Date ____________________

Measuring Segments

Unlike a line, a segment has a beginning point and an ending point, known as **endpoints**. You can measure the distance between the endpoints to find the measure of the segment.

Complete each statement using the figure below.



1. The measure of $\overline{AB}$ = ______.

2. The measure of ______ = 25.

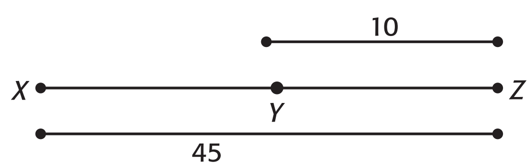
3. The measure of $\overline{BC}$ is $\overline{AC}$ – ______ or 25 – ______ = ______.

4. So, $\overline{AB} + \overline{BC}$ = ______.

Complete the rule for segment addition.

If ______ is between A and C, then $\overline{AB} + \overline{BC} = \overline{AC}$.
If $\overline{AB} + \overline{BC} = \overline{AC}$, then ______ is between A and C.

Practice



1. Find XY if Y is between X and Z, if $YZ = 10$ and $XZ = 45$.

Write an equation using what you know about segment addition. XY + ______ = XZ

Plug what you know into the equation. XY + ______ = XZ

XY + ______ = 45

Solve for the unknown segment length. XY + ______ – ______ = 45 – ______

XY = ______

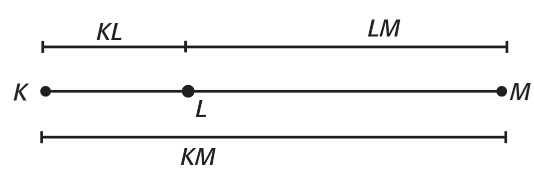
Given that *L* is between *K* and *M*, find the missing measure.

2. $KL = 10$, $LM = 17$, KM = ______

3. KL = ______, $LM = 32$, $KM = 47$.

4. $KL = 21$, LM = ______, $KM = 68$

5. $KL = 2x + 1$, $LM = 4x$, KM = ______________



Name ______________________________ Date ____________________

Using Formulas

In geometry you will use many formulas. There are formulas for finding the area, the volume or perimeter of a figure. A **formula** is a statement of a relationship between two or more quantities.

Rules for Using Formulas

1. Identify the formula to use. Determine what each variable stands for.
2. Match what you know and don't know from the problem to the variables in the formula.
3. Plug the numbers you know into the formula.
4. If necessary, use order of operations in reverse to undo operations and solve for the unknown variables.

Example

The formula for the area of a triangle is $A = \frac{1}{2}bh$. A triangle has an area of 36 cm^2 and a height of 12 cm. What is the length of the base?

Step 1 Identify the formula to use. Determine what each variable stands for.
Use the formula given, $A = \frac{1}{2}bh$.
A = area, b = base length, h = height

Step 2 Match what you know and don't know from the problem to the variables in the formula.
You know the area and the height. You need to find the base length.
$A = 36\text{ cm}^2$, b = ?, h = 12 cm

Step 3 Plug the numbers you know into the formula.
$36 = \frac{1}{2}(b)(12)$

Step 4 Solve.
$6 = b$

Practice

1. The formula for a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. What is the area of a trapezoid with base lengths of 6 and 8 and a height of 10?

   Identify the formula to use. Determine what each variable stands for.
   The formula given is $A = \frac{1}{2}h(b_1 + b_2)$

   Match what you know and don't know from the problem to the variables in the formula.
   A = ____________, b_1 = one base length
   b_2 = ____________, h = ____________
   A = _____, b_1 = _____, b_2 = _____, h = _____

   Plug the numbers you know into the formula.
   $A = \frac{1}{2}$ _____ (_____ + _____)

   Solve.
   = _____ square units

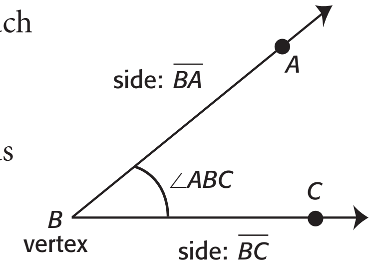
2. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. A sphere has a volume of 904 cubic units. What is the radius of the sphere? ________

3. The area of a parallelogram is 120 in.^{2}. The base measurement is 6 inches. What is the length of the height? Use the formula $A = bh$. ________

Name ______________________________ Date ________________

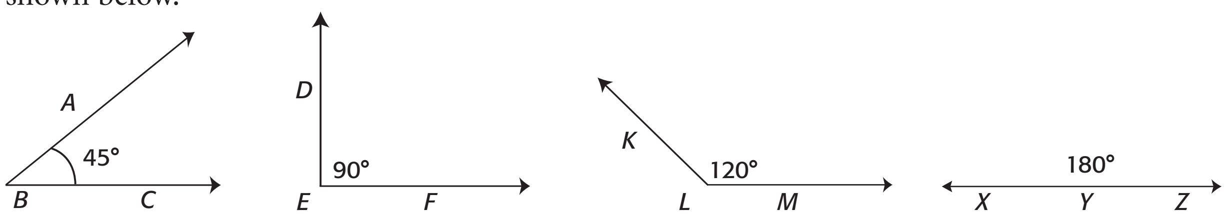
Types of Angles

An angle is made of two rays that have a common endpoint. Each ray forms a side of the angle. The common endpoint forms the vertex of the angle.



Angles are measured in degrees. An angle's measure is written as $m\angle B = 60^\circ$ or $m\angle ABC = 60^\circ$

Angles are classified by their measures. Four types of angles are shown below.



Complete the chart below.

Angle Type	Example	Measure
Acute	$\angle ABC$	________
Right	________	90˚
________________	$\angle KLM$	120˚
Straight	________	180˚

Complete the statements for the rules for classifying angles.

Rules for Classifying Angles

1. An acute angle is an angle whose measure is less than ______.

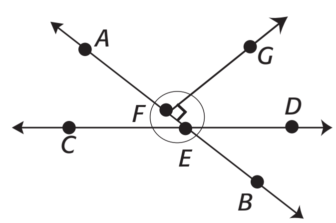
2. A ________________ is an angle whose measure is equal to 90˚.

3. An obtuse angle is an angle whose measure is ________________ 90˚.

4. A ________________ is an angle whose measure is equal to 180˚.

Practice

Refer to the figure to answer the following items.



1. $\angle AFG$ has a measure of 90˚; $\angle AFG$ is a ________________ angle.

2. $\angle AED$ appears to have a measure greater than 90˚; $\angle AED$ is an ________________ angle.

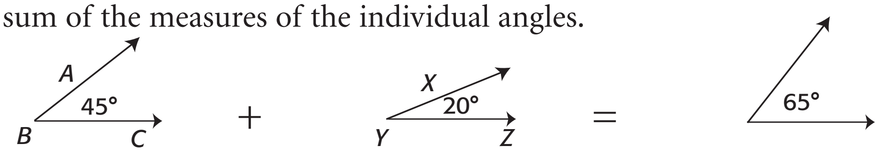
3. ____________ measures 180˚ and is a straight angle.

4. $\angle DEB$ measures ________________ 90˚; $\angle DEB$ is an acute angle.

Name ______________________ Date ______________

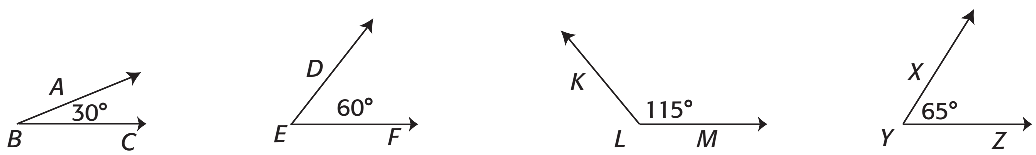
Complementary and Supplementary Angles

Like other measures, you can add angle measures. The result is an angle whose measure is the sum of the measures of the individual angles.



45° + 20° = 65°

The figures below are two set of angles. Complete the chart below.



Type	Angle Pair	Measure of One Angle	Measure of the Other Angle	Sum of the Measure
Complementary	$\angle ABC$ and $\angle DEF$	________ +	________ =	________
Supplementary	$\angle KLM$ and $\angle XYZ$	________ +	________ =	________

Complete the statement for the rules for complementary and supplementary angles.

Rules for Complementary and Supplementary Angles

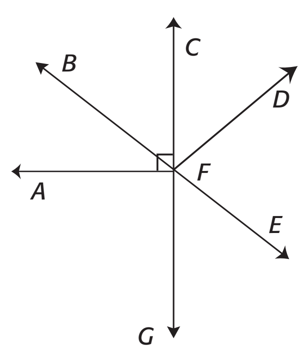
1. Two angles are ____________________ angles if the sum of their measures equals 90°.
2. Two angles are supplementary angles if the sum of their measures equals ________.

Practice

Use the figure to the right to answer the items below.

1. $\angle AFB$ is complementary to ________.
2. $m\angle CFE + m\angle EFG =$ ________
3. $\angle BFC = 35°$; $\angle BFC$ and $\angle CFE$ are supplementary.

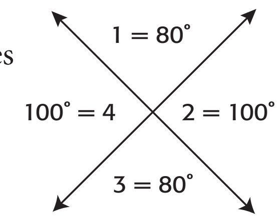
   What is the measure of $\angle CFE$? ________
4. $\angle BFC$ and ________ are complementary angles.



Name ______________________ Date ______________

Pairs of Angles

As you know when two lines intersect four angles are created, as you can see in the figure on the right. Certain relationships exist among the angles formed by intersecting lines.



Complete the chart below.

Type	Measure of One Angle	Measure of the Other Angle
Vertical Angles	m∠1 = ______	m∠3 = ______
	m∠2 = ______	m∠4 = ______
Linear Pair	m∠1 = ______	m∠2 = ______
	m∠3 = ______	m∠4 = ______

Complete the statements below.

1. ∠1 and ∠3 are ______________, _____ and _____ are also vertical angles.
2. ∠1 and ∠2 form a ______________, _____ and _____ also form a linear pair.
3. The sum of the measures of ∠1 and ∠2 is ______; the sum of the measures of _____ and _____ is 180º.
4. Another term for a linear pair is ______________ angles.

Complete the statements for rules for angle pairs.

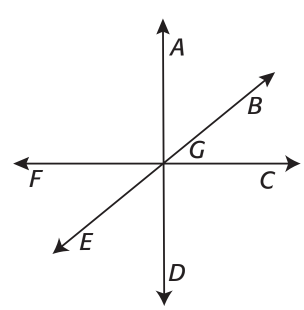
Rules for angle pairs

1. When two lines intersect, ______________ angles are created opposite one another.
2. Vertical angles have ______________ measure; they are ______________.
3. The sum of the measures of the angles in a linear pair is ______.

Practice

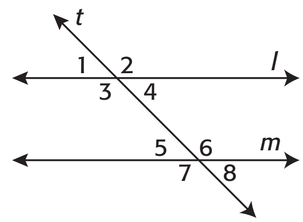
Use the figure to the right to complete the following statements.

1. ∠*AGB* and ______ are vertical angles.
2. ∠*AGB* and ∠*BGD* are ______________.
3. The measure of ∠*FGE* is 45º. The measure of ∠*EGC* is ______.
4. ∠*EGD* is supplementary to ∠______.
5. An angle congruent to ∠*DGC* is ______.



Name ______________________________ Date ______________

Parallel Lines: Types of Angles



In the figure to the right, lines *l* and *m* are parallel lines. Line *t* intersects lines *l* and *m*, line *t* is known as a transversal.

A **transversal** is a line that intersects two or more lines at different points.

As you can see, when a transversal intersects two lines, eight angles are formed. These angles are given special names.

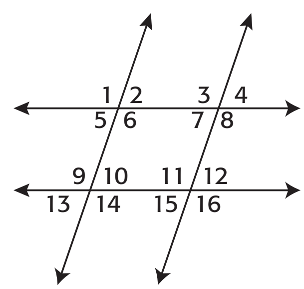
Complete the rules below by using the diagram above.

Rules for Angles Formed by Parallel Lines Intersected by a Transversal

1. Exterior angles are angles on the outside of the lines; angles 1, 2, _____, and _____ are exterior angles.
2. Interior angles are angles on the inside of the lines; angles 3, 4, _____, and _____ are interior angles.
3. Consecutive interior angles are angles that are inside the lines on the same side of the transversal; angles 3 and 5 and angles _____ and _____ are consecutive interior angles.
4. Alternate interior angles are angles that are inside the lines but on the opposite sides of the transversal; angles 3 and 6 and angles _____ and _____ are alternate interior angles.
5. Alternate exterior angles are angles that are outside the lines on opposite sides of the transversal; angles 1 and 8 and angles _____ and _____ are alternate exterior angles.
6. Corresponding angles occupy the same position on each line; angles 1 and 5 and angles 3 and 7 are corresponding angles, as are angles 2 and _____ and angles _____ and 8.

Practice

Classify each pair of angles using the figure to the right.



1. $\angle 7$ and $\angle 12$ ____________________
2. $\angle 1$ and $\angle 13$ ____________________
3. $\angle 11$ and $\angle 14$ ____________________
4. $\angle 4$ and $\angle 5$ ____________________

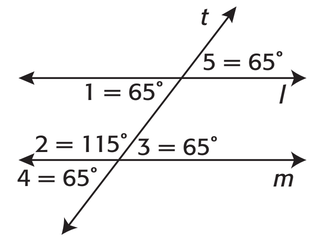
Identify the missing angle in each pair.

5. Corresponding angles: $\angle 3$ and $\angle$_____ or $\angle$_____
6. Consecutive interior angles: $\angle 6$ and $\angle$_____ or $\angle$_____
7. Interior angles: $\angle 10$ and $\angle$_____ or $\angle$_____

Name ______________________________ Date ________________

Parallel Lines: Angle Relationships

In the figure to the right lines *l* and *m* are parallel lines. Line *t* intersects line *l* and *m*, line *t* is known as a transversal.



A **transversal** is a line that intersects two or more lines at different points.

As you can see, when a transversal intersects two lines, 8 angles are formed. These angles have special relationships.

Explore the angle relationships that exist when a transversal intersects two parallel lines.

Type	Measure of Angle	Measure of Other Angle
Corresponding angle	$m\angle 1 = 65°$	____________
Alternate interior angles	____________	$m\angle 3 = 65°$
____________	$m\angle 1 = 65°$	$m\angle 2 = 115°$
Alternate exterior angles	$m\angle 5 = 65°$	____________

Use the chart to complete the statements below.

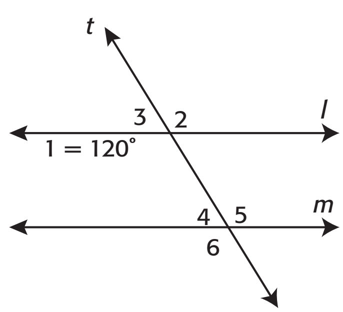
Rules for the relationships among angles formed when a transversal intersects parallel lines

1. Corresponding angles are ____________.
2. Alternate interior angles are ____________.
3. ____________ angles are supplementary.
4. Alternate exterior angles are ____________.

Practice

Use the figure to the right.

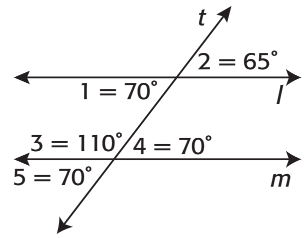
1. $m\angle 4 =$ ________
2. $m\angle 5 =$ ________
3. $m\angle 3 =$ ________
4. $m\angle 2 =$ ________
5. $m\angle 6 =$ ________



Name ______________________________ Date ________________

Proving Lines Are Parallel

In the figure to the right, lines *l* and *m* are parallel lines. When a transversal intersects two lines, 8 angles are formed. These angles have special relationships. You can use these relationships to prove lines are parallel.



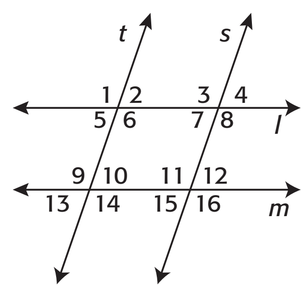
Use the figure above to help complete the following statements.

Rules for Proving Lines are Parallel

1. If two lines are intersected by a transversal and ______________ angles, such as ∠1 and ∠5, are ______________, then the lines are parallel.
2. If two lines are intersected by a transversal and ______________ angles, such as ∠1 and ∠4, are ______________, then the lines are parallel.
3. If two lines are intersected by a transversal and ______________ angles, such as ∠2 and ∠5, are ______________, then the lines are parallel.
4. If two lines are intersected by a transversal and ______________ angles, such as ∠1 and ∠3, are ______________, then the lines are parallel.

Example

State the rule that says why the lines are parallel, *m*∠5 ≅ ∠10



Step 1 State the relationship between ∠5 and ∠10.
The angles are alternate interior angles.

Step 2 State how the measures of each angle are related.
The angles are congruent.

Step 3 State why the lines *l* and *m* are parallel.
If two lines are intersected by a transversal and alternate interior angles are congruent, then the lines are parallel.

Practice

State the rule that says why the lines are parallel. Use the figure above.

1. m∠5 + m∠9 = 180°

   State the relationship between ∠5 and ∠9: The angles are ______________ angles.

   State how the measure of each angle is related: The angles are ______________.

   State why lines *l* and *m* are parallel: If two lines are intersected by a transversal and ______________ angles are ______________, the lines are parallel.

2. m∠4 ≅ m∠5 ______________
3. m∠5 ≅ m∠7 ______________
4. m∠8 ≅ m∠11 ______________
5. m∠10 + m∠11 = 180° ______________

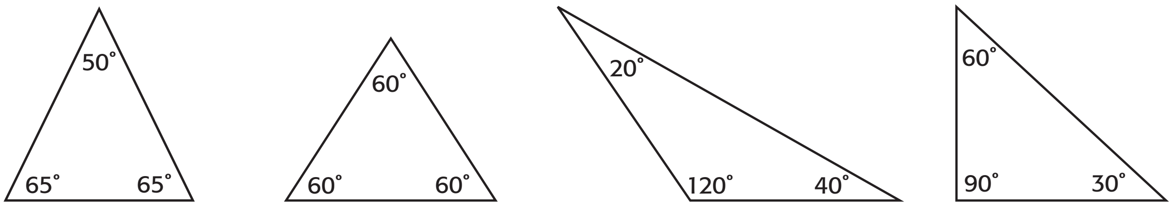
Name ______________________ Date ______________

Classifying Triangles

A triangle is a three-sided polygon. A polygon is a closed figure made up of segments, called sides, that intersect at the end points, called vertices.

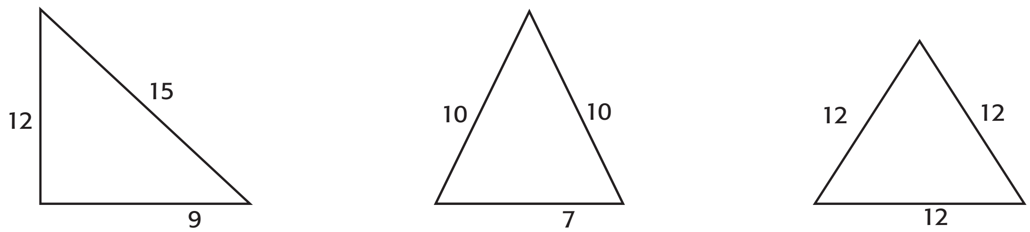
Triangles are classified by their angles and their sides.

Classifying by angle:



Acute Equiangular Obtuse Right

Classifying by side length:



Scalene Isosceles Equilateral

Use the figures above to complete the rules for classifying triangles.

Rules for classifying triangles by angle

1. An acute triangle has ______________ acute angles.

2. An equiangular triangle has three ______________ angles.

3. An obtuse triangle has one ______________ angle.

4. A right triangle has one ______________ angle.

Rules for classifying triangles by side length

1. A scalene triangle has __________ congruent sides.

2. An isosceles triangle has at least __________ congruent sides.

3. An equilateral triangle has ______________ congruent sides.

Practice

Use the figure to the right.

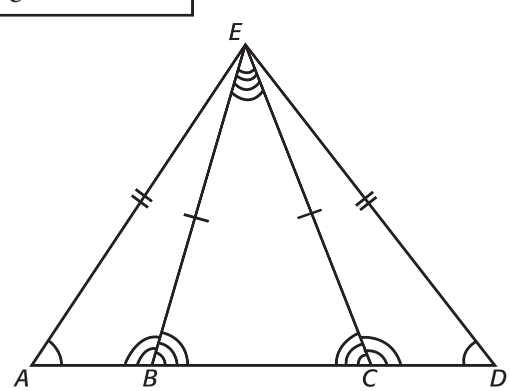
1. Name an equilateral triangle. __________

2. Name a scalene triangle. __________

3. Name an obtuse triangle. __________

4. Name an acute triangle. __________

5. Name an isosceles triangle. __________



Name ______________________________ Date ______________

Interior and Exterior Angles in Triangles

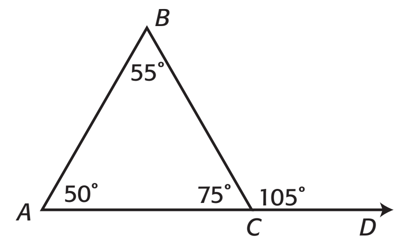
A triangle is a three-sided polygon. A triangle is made of segments, called sides, that intersect only at their endpoints, called vertices.

Sides: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

Vertices: *A*, *B*, *C*

Interior Angles: $\angle BAC$, $\angle ABC$, $\angle BCA$

Exterior Angle: $\angle BCD$



Complete each statement below.

1. The measure of $\angle BAC$ is ______, the measure of $\angle ABC$ is ______, and the measure of $\angle BCA$ is ______. If you add the measures of the interior angles, the sum is ________.
2. $\angle BCD$ is an exterior angle. The measure of $\angle BCD$ is ________. $\angle BAC$ and $\angle ABC$ are both known as remote interior angles. The measure of $\angle BAC$ is ______ and the measure of $\angle ABC$ is ______. If you add the measures of these remote interior angles, the sum is ________.
3. The measure of $\angle BCA$ is ______. The measure of $\angle BCD$ is ______. If you add the measure of $\angle BCA$ and $\angle BCD$, the sum is ______; the angles are ____________________.

Use the figure above to complete the rules for angle relationships in triangles.

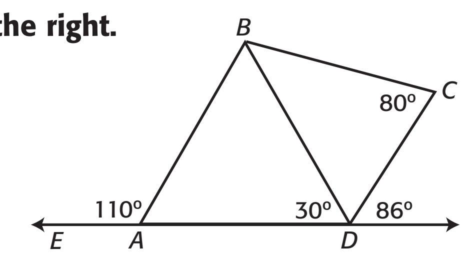
Rules for Angle Relationships in Triangles

1. The sum of the measures of the interior angles of a triangle is ________.
   $m\angle 1 + m\angle 2 + m\angle 3 =$ ________
2. The measure of an ____________________ angle of a triangle is ______________ to the sum of the measures of the two remote interior angles.

Practice

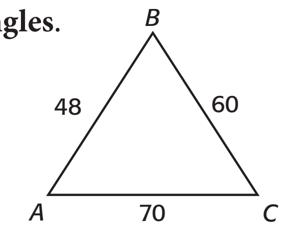
Find the measures of the angles in the figure to the right.

1. $m\angle ABD =$ ______
2. $m\angle BAD =$ ______
3. $m\angle CDB =$ ______
4. $m\angle CBD =$ ______



Name ________________________________ Date ____________________

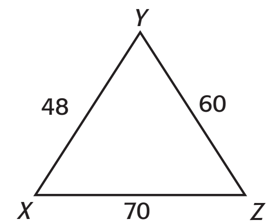
Corresponding Parts of Triangles



Triangles that are the same size and the same shape are **congruent triangles**.

As you know each triangle has six parts—three sides and three angles.

Use the figures to the right to identify corresponding parts.
Use the symbol "$\leftrightarrow$" to mean "corresponds to".



$\angle CAB \leftrightarrow \angle ZXY$ $\overline{AC} \leftrightarrow \overline{XZ}$

$\angle ABC \leftrightarrow$ ________ $\overline{AB} \leftrightarrow$ ________

________ $\leftrightarrow \angle YZX$ ________ $\leftrightarrow \overline{YZ}$

Complete the chart below.

Angle	Corresponding Angle	Relationship
$\angle CAB = 70°$	$\angle ZXY = 70°$	$\angle CAB \cong \angle ZXY$
$\angle ABC = 57°$	________	$\angle ABC \cong$ ________
________	$\angle YZX = 53°$	________ $\cong \angle YZX$

Side	Corresponding Side	Relationship
$\overline{AC}$	$\overline{XZ}$	$\overline{AC} \cong \overline{XZ}$
$\overline{AB}$	________	$\overline{AB} \cong$ ________
$\overline{BC}$	$\overline{YZ}$	________ $\cong \overline{YZ}$

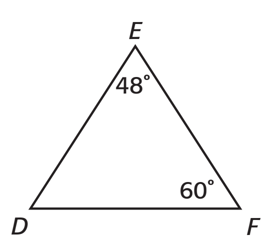
Complete the statement below for the rules for corresponding parts of congruent triangles.

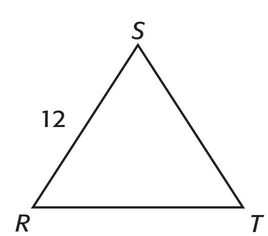
> **Rules for Corresponding Parts of Corresponding Triangles**
>
> Two triangles are congruent if and only if their ____________________ parts are ____________________.

Practice

Complete each statement. $\triangle DEF \cong \triangle RST$

1. $\overline{DE} =$ ________
2. $\angle EDF \cong$ ________
3. $m\angle RST =$ ________
4. $\overline{ST} \cong$ ________
5. $m\angle SRT =$ ________
6. $\overline{DF} =$ ________

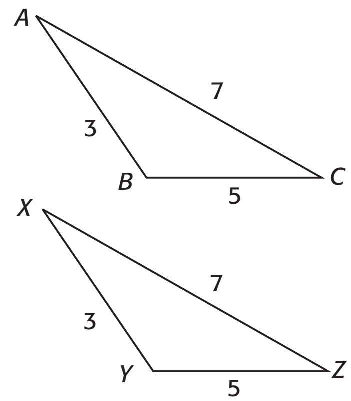




Name ________________________________ Date ____________________

Triangle Congruence: Side-Side-Side Congruence

If two triangles have three pairs of congruent corresponding angles and three pairs of congruent corresponding sides, then the triangles are congruent. However, you do not need to know that all the sides and angles are congruent in order to conclude that the triangles are congruent. You only need to know certain corresponding parts are congruent. The two triangles to the right are congruent.



Complete the chart by identifying the corresponding sides and their measures.

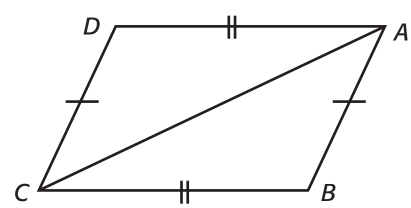
Side	Measure	Corresponding Side	Measure	Relationship Between Sides
$\overline{AB}$	3	$\overline{XY}$	3	$\overline{AB} \cong \overline{XY}$
$\overline{BC}$	5	________	________	$\overline{BC} \cong$ ________
________	________	$\overline{XZ}$	7	________ $\cong \overline{XZ}$

Complete the rule for triangle congruence.

Rule for Side-Side-Side (SSS) Postulate
If three sides of one triangle are ________________ to ________________ sides of another triangle then the two triangles are congruent.

Practice

For each figure, determine if there is enough information to prove the two triangles congruent.



1. The corresponding side to side AB: ________

Are the sides congruent? ________

The corresponding side to side AD: ________

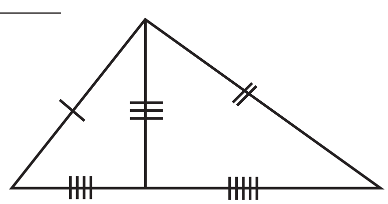
Are the sides congruent? ________

What do you notice about side AC? ________________

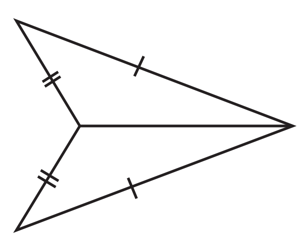
Does $\overline{AC}$ in $\triangle ABC$ correspond to $\overline{AC}$ in $\triangle ACD$? ________

Can you use SSS postulate? ________

2. ________

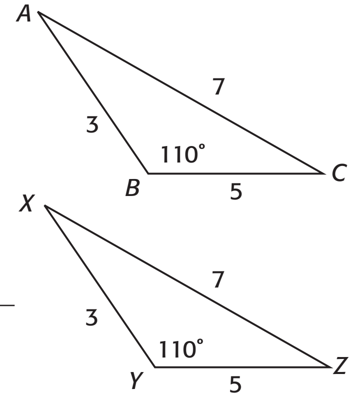


3. ________



Name ______________________________ Date ____________________

Triangle Congruence: Side-Angle-Side Congruence



You do not need to know that all the sides and angles are congruent in order to conclude that triangles are congruent. You only need to know certain corresponding parts are congruent.

The two triangles to the right are congruent. Use them to answer the following.

1. Which sides in triangle ABC form $\angle B$? $\overline{AB}$ and ____________
2. Which angle is formed from (included) $\overline{AB}$ and $\overline{AC}$? __________
3. Which two angles are made using side AB ? ______________

The two triangles above are congruent. Complete the chart by identifying the corresponding sides and angles and their measures.

Side	Measure	Corresponding Side	Measure	Relationship Between Sides
$\overline{AB}$	3	______	3	$\overline{AB} \cong \overline{XY}$
Angle	**Measure**	**Corresponding Angle**	**Measure**	**Relationship Between Angles**
______	110°	$\angle Y$	110°	______ $\cong \angle Y$
Side	**Measure**	**Corresponding Side**	**Measure**	**Relationship Between Sides**
______	5	______	______	$\overline{BC} \cong$ ______

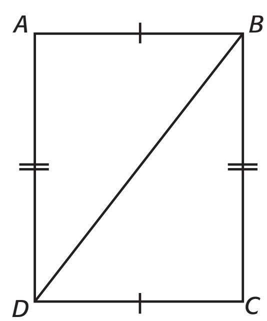
Complete the rule for triangle congruence.

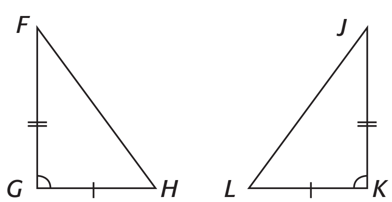
Rule for Side-Angle-Side (SAS) Postulate.

If the two sides and the included angle of one triangle are ____________________ to two ______________ and the ____________________ angles, then the triangles are congruent.

Practice

Name the included angle between each pair of sides.



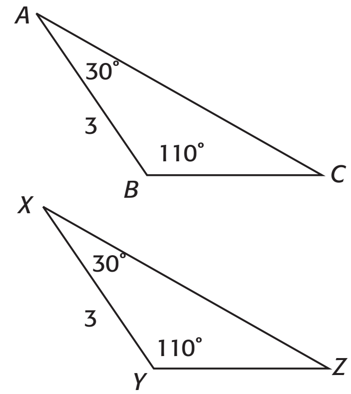
1. $\overline{AD}$ and $\overline{AB}$ ________
2. $\overline{BD}$ and $\overline{BC}$ ________
3. $\overline{BC}$ and $\overline{DC}$ ________
4. 

__________, __________

Name ______________________________ Date ______________

Triangle Congruence: Angle-Side-Angle Congruence

You do not need to know that all the sides and angles are congruent in order to conclude that triangles are congruent. You only need to know certain corresponding parts are congruent.



The two triangles to the right are congruent. Use them to answer the following questions.

1. Which side is included by $\angle A$ and $\angle C$? ______
2. Which side is included by $\angle B$ and $\angle C$? ______
3. Which side is included by $\angle A$ and $\angle B$? ______

The two triangles above are congruent. Complete the chart by identifying corresponding sides and angles and their measures.

Angle	**Measure**	**Corresponding Angle**	**Measure**	**Relationship Between Angles**
$\angle B$	110°	$\angle Y$	110°	$\angle B \cong \angle Y$
Side	**Measure**	**Corresponding Side**	**Measure**	**Relationship Between Sides**
$\overline{AB}$	3	______	______	$\overline{AB} \cong$ ______
Angle	**Measure**	**Corresponding Angle**	**Measure**	**Relationship Between Angles**
______	______	$\angle X$	30°	______ $\cong \angle X$

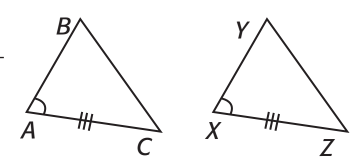
Complete the rule for triangle congruence.

Rule for Angle-Side-Angle (ASA) Postulate.

If the two angles and the included side of one triangle are ______________ to two ____________ and the ______________ side of another triangle, then the two triangles are congruent.

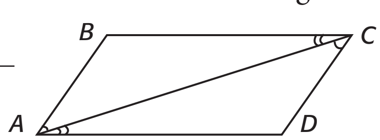
Practice

State the missing congruence that must be given to use the ASA Postulate to prove the triangles are congruent.

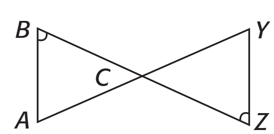


1. Which pair of corresponding angles are given? ____________
   Which set of corresponding sides are given? ____________
   Which angles are adjacent to $\overline{AC}$? ____________
   If $\triangle ABC$ and $\triangle XYZ$ are congruent by ASA, which is the other angle in $\triangle ABC$? ____________

2. ______



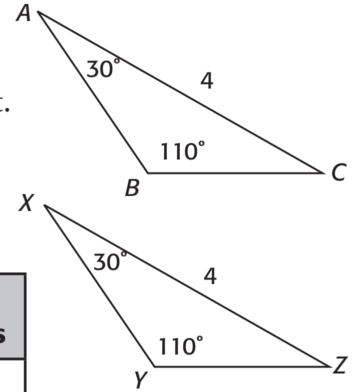
3. ______



Name ______________________________ Date ________________

Triangle Congruence: Angle-Angle-Side Congruence

You do not need to know that all the sides and angles are congruent in order to conclude that the triangles are congruent. You only need to know certain corresponding parts are congruent. These two triangles are congruent.



Complete the chart by identifying the corresponding sides and their measures.

Angle	Measure	Corresponding Angle	Angle Measure	Relationship Between Angles
$\angle A$	30°	$\angle X$	30°	$\angle A \cong \angle X$
$\angle B$	110°	______	______	$\angle B \cong$ ______
Side	**Measure**	**Corresponding Side**	**Measure**	**Relationship Between Sides**
______	______	$\overline{XZ}$	4	______ $\cong \overline{XZ}$

Complete the rule for triangle congruence.

Rule for Angle-Angle-Side (AAS) Postulate

If two angles and a non-included side of one triangle are ________________

to two angles and the ________________ non-included side of another triangle, then the triangles are congruent.

Practice

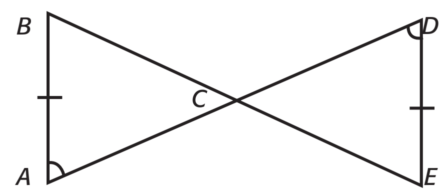
State the missing congruence that must be given to use the AAS postulate to prove the triangles are congruent.

1. Which pair of corresponding angles are given?

____

Which set of corresponding sides are given?

____



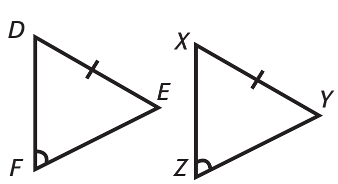
Which angles are not adjacent to the given sides? ________________

How are these angles related? ________________

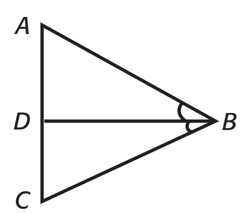
How do the measures of these angles compare? ________________

State the congruent sides and angles. ________________

2. ________________



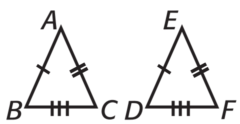
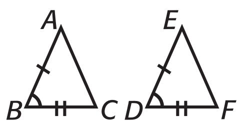
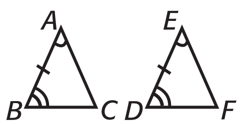
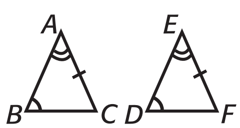
3. ________________



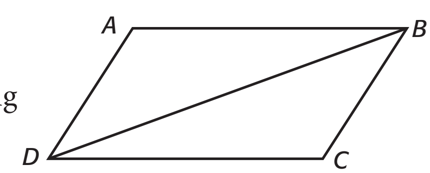
Name ______________________________ Date ____________________

Choosing the Correct Congruence Postulate

There are four postulates that show the ways in which triangle congruence is proved. By carefully looking at the two triangles and identifying corresponding parts, you can identify the postulate to use.

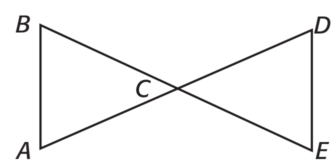
Example	What Is Given	Postulate to Use
	Three pairs of corresponding congruent sides.	Side-Side-Side Postulate
	________ pair of corresponding ________ included angles and ________ pairs of corresponding ________ sides.	Side-Angle-Side Postulate
	________ pairs of ________ corresponding angles and ________ pair of ________ corresponding included sides.	Angle-Side-Angle Postulate
	________ pairs of ________ corresponding angles and ________ pair of ________ non-included sides.	________

Triangle Congruence Hints



1. A common side can be used as one pair of corresponding sides in using SSS, ASA, or SAS postulates.

   Side $\overline{DB}$ is a side common to $\triangle ABD$ and $\triangle BCD$, for corresponding sides you can say $\overline{DB} \cong \overline{DB}$.

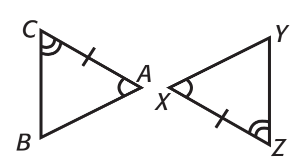


2. Remember, vertical angles are congruent. A figure with vertical angles will often not show the vertical angles are congruent.

   Angles ACB and DCE are vertical angles; you can say $\angle ACB \cong \angle DCE$.

Practice

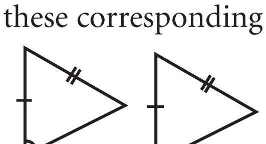
Decide if it is possible to prove the triangles are congruent. Some pairs of triangles may not include enough or the proper information.



1. What are the relationships between corresponding parts? ________________

   Which postulate uses these corresponding parts? ________________

2. ________________



Name ______________________________ Date ____________________

Isosceles Triangle Theorem

As you know, an isosceles triangle has two congruent sides. The parts of an isosceles triangle have special names.

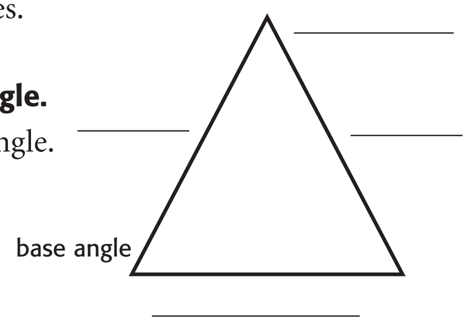
Use the definitions below to label the isosceles triangle.

Legs: The two congruent sides of an isosceles triangle.

Base: The third side of an isosceles triangle.

Base angles: The two angles next to the base.

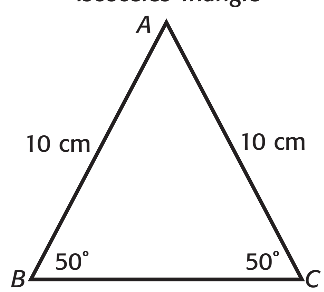
Vertex: Angle opposite the base.



Isosceles Triangle

Use the figure to the right to complete the chart.

Angle or Side	Measure
$\angle B$	________
$\angle C$	________
$\overline{AB}$	________
$\overline{AC}$	________



Use the chart above to complete the following theorems about isosceles triangles.

Isosceles Triangle Theorems

1. Base Angle Theorem

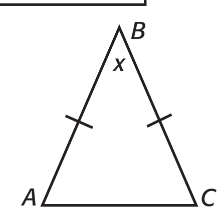
If two sides of a triangle are ________________, then the angles opposite them (base angles) are ________________.

2. Converse of the Base Angle Theorem

If two angles of a triangle are ________________, then the sides opposite the angles are ________________.

Practice

1. Find the measure of $\angle B$.



Is the triangle an isosceles triangle? Explain. — Yes $AB =$ ________

What is the measure of $\angle A$? — $\angle C \cong \angle A$, $\angle C = 55°$, $\angle A =$ ______

What is the sum of the measures of a triangle? — ________

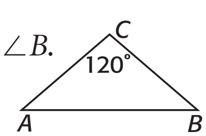
Use the two known measures and the sum of the measures of angles in a triangle to find the unknown measure.

$\angle A + \angle B + \angle C =$ ________

________ $+ \angle B + 55° = 180°$

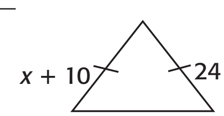
$\angle B =$ ________

2. Find the measure of $\angle A$ and $\angle B$.



____

3. Find the value of x.



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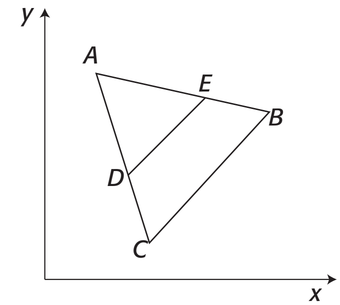
Name ______________________________ Date ______________

Triangle Mid-segment



In the figure to the right, $\overline{DE}$ is a mid-segment. A mid-segment of a triangle connects the midpoints of two sides of a triangle. As you will see, the mid-segment and sides of a triangle have special relationships. One relationship is that between the slope of the mid–segment, $\overline{DE}$, and the slope of the side opposite the mid-segment, $\overline{CB}$.

Find the slope of each segment. Slope = $\frac{y_2-y_1}{x_2-y_1}$

$\overline{CB}$: C (1, 1); B (6, 3)

Slope = $\frac{y_2-y_1}{x_2-y_1}$

= $\frac{3-1}{6-1}$ = ______

$\overline{DE}$: D (1, 3); E(3.5, 4)

Slope = $\frac{y_2-y_1}{x_2-y_1}$

Slope = $\frac{4-3}{3.5-1}$ = ____________

1. The slope of $\overline{CB}$ is _____; the slope of $\overline{DE}$ is _____. The slopes are ____________, therefore, the segments are ________________.

Another special relationship between segments is the length of each segment.

Use the Distance Formula to find the length, *d*, of each segment. Remember, $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\overline{CB}$: C (1, 1); B (6, 3)

$\overline{CB} = \sqrt{(6-1)^2 + (3-1)^2}$

$\overline{CB}$ = ____________

= _______ = _______

$\overline{DE}$: D (1, 3); E (3.5, 4)

$\overline{DE} = \sqrt{(3.5-1)^2 + (4-3)^2}$

$\overline{DE}$ = ____________

= $\sqrt{7.25}$ = _______

2. The length of $\overline{CB}$ = _______. Divide the length of $\overline{CB}$ by 2, the result is _______. So, $\frac{1}{2}$ of $\overline{CB}$ equals the length of _______.

Complete the rule below for mid–segment of a triangle by circling the correct term in each pair.

> **Mid–Segment Theorem**
>
> The segment connecting the midpoints of two sides of a triangle is ________________ to the third side; the mid–segment is ________________ as long as the third side.

Use the figure to the right to complete each statement. $\overline{DE}$, $\overline{DF}$, and $\overline{EF}$ are triangle mid-segments.

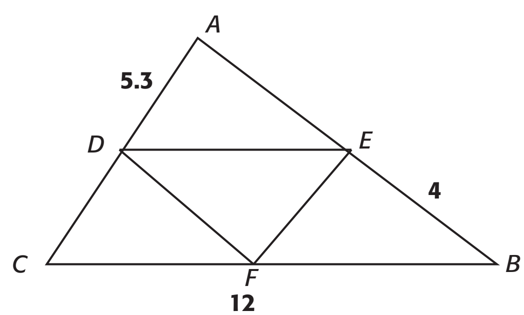
3. $\overline{DF} \parallel$ _____

4. _____ $\parallel \overline{CB}$

5. $\overline{DE}$ = _____

6. $\overline{AB}$ = _____

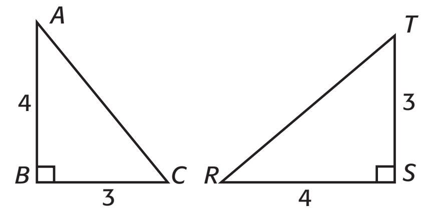
7. $\overline{EF}$ = _____



Name ______________________________ Date ____________________

Hypotenuse-Leg Theorem

As you know, you can prove two triangles are congruent using one of many postulates. Depending on what you know about the sides and angles of the two triangles, you can use postulates such as SSS, ASA, AAS, or SAS.



The triangles to the right are right triangles, but are they also congruent? There is a special theorem associated with right triangles that will allow you to prove right triangles are congruent.

Use the figures above to complete the chart and find the relationship sides of the right triangle. To find the length of each hypotenuse use the Pythagorean theorem.

$\triangle ABC$		**$\triangle RST$**		
Side	**Measure**	**Corresponding Side**	**Measure**	**Relationship Between Sides**
$\overline{AB}$	4	$\overline{RS}$	4	$\overline{AB} \cong \overline{RS}$
$\overline{BC}$	3	______	______	$\overline{BC} \cong$ ______
$(AC)^2 = 4^2 + 3^2$	______	$(\overline{RT})^2 = 4^2 + 3^2$	______	$\overline{AC}$ ______ $\overline{RT}$

When you know that three sides of one triangle are congruent to three sides of another triangle, you can use the ________ postulate. So, $\triangle ABC$ ______ $\triangle RST$.

To find if two right triangles are congruent, all you need to know is the length of one leg and the hypotenuse in each triangle.

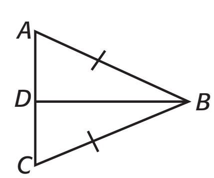
Use the results from the chart above to complete the rule for proving that right triangles are congruent.

Hypotenuse-Leg Theorem (H-L Theorem)

If the hypotenuse and leg of one right triangle are ____________________ to the ____________________ and ____________________ of another right triangle, then the triangles are congruent.

Practice

Decide whether enough information is given to use the Hypotenuse-Leg Theorem.



1. Segment $\overline{AB}$ is congruent to segment ______________.

2. Remember, a common side forms a congruent pair.

Is there another pair of congruent segments? ______________

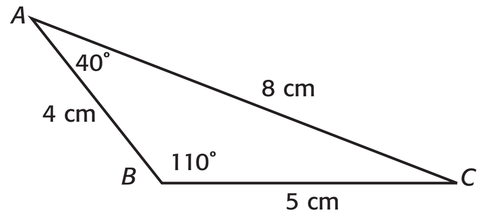
3. Can you say $\overline{AD} \cong \overline{CD}$ ______________

4. Are there any congruent angles? ______________

5. Can you prove $\triangle ABD \cong \triangle CBD$? ______________

Name ______________________________ Date ____________________

Triangle Inequalities: Inequalities for Sides and Angles



According to the Isosceles Triangle Theorem, if two sides of a triangle are congruent, then the angles opposite those sides are also congruent. But what happens when you have a triangle in which two sides are not congruent?

Use the figure to the right to complete the chart.
Then use the figure and the chart to complete the statements that follow.

Side	Measure	Angle	Measure
$\overline{AB}$	________	$\angle ABC$	________
$\overline{BC}$	________	$\angle BAC$	________
$\overline{AC}$	________	$\angle ACB$	________

1. Which is longer, $\overline{AB}$ or $\overline{BC}$? ________

2. Which angle is opposite $\overline{AB}$? ________

3. Which angle is opposite $\overline{BC}$? ________

4. Of the angles found in #2 and #3, which has the greater measure? ________

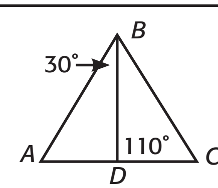
5. Is the angle with the greater measure opposite the side with the greater measure? ______________

Use the answers to the items above and the chart to complete the rule for Inequalities for Sides and Angles of Triangles.

> **Inequalities for Sides and Angles of a Triangle**
> If one side of a triangle is longer than another side, then the angle ______________ the longer side has a ______________ measure than the angle opposite the shorter side.

Practice

Use the figure to the right. Fill in each blank with < or >.



1. In $\triangle ABD$, $\overline{BD}$ ___ $\overline{BA}$.

The angle and measure of the angle opposite $\overline{BA}$ is ______________

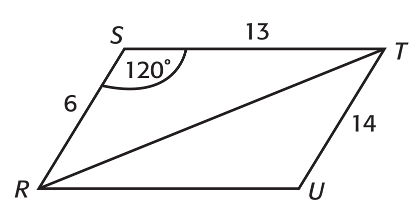
The angle and measure of the angle opposite $\overline{BD}$ in $\triangle ABD$ is ______________

Which side is opposite the angle with the greatest measure? ______________

2. In $\triangle BCD$, _____ is the longest side.

3. In $\triangle RST$, $\angle RST$ ___ $\angle SRT$.

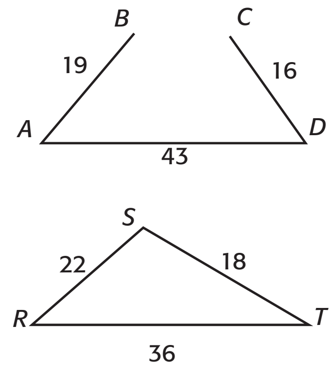
4. In $\triangle RTU$, $\angle RUT$ ___ $\angle RTU$



Name ______________________________ Date ____________________

Triangle Inequality Theorem

Look at the figures to the right. As you can see, both have three sides. But only the bottom figure is a triangle. The top figure is not a closed figure and is, therefore, **not** a triangle.



Complete the chart below. Add combinations of two sides of each triangle and compare the sum to the third side.

Figure *BADC*	Inequality Test	Is the Inequality True?
	19 + 43 > 16	Yes
	16 + 43 > ______	______
	______ + ______ > 43	______
Figure *RST*	**Inequality Test**	**Is the Inequality True?**
	22 + 18 > 36	Yes
	18 + 36 > ______	______
	______ + ______ > 18	______

Review the results in the chart and answer the questions below.

1. For Figure *BADC*, were all the inequality statements true? ____________________

2. For Figure *RST*, were all the inequality statements true? ____________________

Complete the Rule for Triangle Inequality.

Rule for Triangle Inequality

The sum of any two sides of a triangle is ____________________ than the length of the third side.

Practice

For each set of sides, determine if a triangle is formed.

1. 20, 43, 55

Is the following inequality true? 20 + 43 > 55 ______

Is the following inequality true? 43 + 55 > 20 ______

Is the following inequality true? 55 + 20 > 43 ______

Were the answers to the previous three questions "yes"? ______. Will the combination form a triangle? ______

2. 20, 33, 556 ______

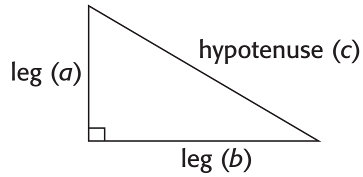
3. 15, 26, 31 ______

4. 10, 13, 18 ______

Name ______________________________ Date ____________________

The Pythagorean Theorem

A right triangle is a triangle with one 90° angle (also known as a **right angle**). In a right triangle, the sides next to the right angle are the **legs**. The side opposite the right angle is the **hypotenuse**.



In a right triangle, there is a relationship between the legs and the hypotenuse. This relationship (the Pythagorean Theorem) says that $a^2 + b^2 = c^2$

Rules for Using the Pythagorean Theorem

1. Identify the legs and the hypotenuse.
2. Plug the numbers into the Pythagorean theorem. Square the numbers.
3. If the unknown side is a leg, solve the equation for the unknown leg.
4. If the unknown side is the hypotenuse, add the squares of the two legs and then find the square root.

Example

Find the unknown length in a right triangle if $a = 5$ and $c = 13$.

Step 1	Identify the legs and the hypotenuse.	$a = 5$ is a leg; c is the hypotenuse
Step 2	Plug the numbers into the Pythagorean Theorem. Square the numbers.	$5^2 + b^2 = 13^2$
Step 3	If the unknown side is a leg, solve the equation for the unknown leg.	$25 - 25 + b^2 = 169 - 25$ $b^2 = 144$ $\sqrt{b^2} = \sqrt{144} = 12$

Practice

Find the unknown length in each right triangle.

1. $b = 15$, $a = 8$

Identify the legs and the hypotenuse.	$a = 8$ is a side. $b = 15$ is ____________
Plug the numbers into the Pythagorean Theorem. Square the numbers.	$8^2 +$ _____^{2} = _____ $64 +$ _______ = _____
If the unknown side is the hypotenuse, add the squares of the two legs and then find the square root.	$64 +$ _______ = _____ _______ = _____ _____ = _____

2. $b = 8$; $c = 10$ ____________

3. $a = 3$; $b = 4$ ____________

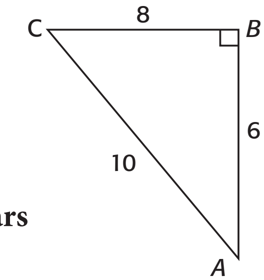
4. $a = 20$; $b = 15$ ____________

5. $a = 200$; $c = 250$ ____________

Name ______________________________ Date ____________________

Converse of the Pythagorean Theorem

You have learned that there is a special relationship between the legs of a right triangle and the triangle's hypotenuse.



The Pythagorean Theorem states that:

$$a^2 + b^2 = c^2$$

But what if you know the measure of three sides of a triangle that **appears** to be a right triangle? Suppose you are given the triangle to the right.

Can you show it is a right triangle by knowing just the length of the sides?

Use the figure to complete the chart. Assume the triangle is a right triangle.

Decide which side is the hypotenuse. The hypotenuse is the ________________ side.

Leg	**Leg**	**Hypotenuse**		
$\overline{AB}$	$\overline{BC}$	$\overline{AC}$	$AB^2 + BC^2$	AC^2
6	8	10	$6^2 + 8^2 =$ ______	$(10)^2 =$ ______

The relationship between $(\overline{AB} + \overline{BC})^2$ and $\overline{AC}^2$ is $(AB + BC)^2$ _____ AC^2

Does this satisfy the Pythagorean Theorem? _____ Is $\triangle ABC$ a right triangle? _____

Use the data in the chart and answer the questions about how to complete the Converse to the Pythagorean Theorem.

> **Converse to the Pythagorean Theorem**
>
> If the sum of the ________________ of the measure of two sides of a triangle equals the square of the measure of the ________________ side, then the triangle is a right triangle.

Practice

Use the Converse of the Pythagorean Theorem to decide if each triangle is a right triangle.

1. 10, 7, 13

Which side is the longest side, c? _____

Which sides are the shorter sides, a and b? _____ and _____

Plug the values into the Pythagorean Theorem $a^2 + b^2 = c^2$ $\_\_\_\_\_^2 + \_\_\_\_\_^2 = 13^2$

_____ + ______ = ______

______ =? ______

Is the equation true? ________________

2. 20, 21, 29 _____

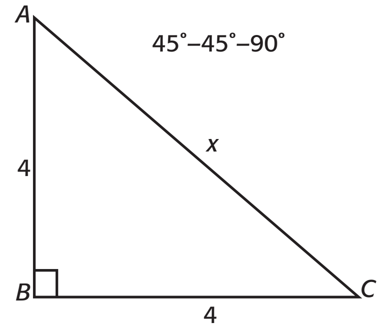
3. 7, 24, 25 _____

4. 3, 11, 12 _____

5. $\sqrt{13}$, 6, 7 _____

Name ______________________________ Date ____________________

Special Right Triangles: 45°–45°–90° Right Triangles



You have learned that there is a special relationship between the legs of a right triangle and the triangle's hypotenuse.

The Pythagorean Theorem states that:

$$a^2 + b^2 = c^2$$

Right triangles whose measures are 45º–45°–90° or 30°–60°–90° are called special right triangles.

Use the figure to the right to complete the chart.

45°–45°–90°			
Leg	**Leg**	**Hypotenuse**	
$\overline{AB}$	$\overline{BC}$	$\overline{AC}$	$AB^2 + BC^2 = AC^2$
4	4	x	$4^2 + 4^2 =$ ______ $= x^2$

Solve for *x*.

______ $= x^2$

______ $= \sqrt{x^2}$

$\sqrt{\text{\_\_\_\_\_\_}2} = \sqrt{x^2}$

______$\sqrt{2} = x$

One of the legs in the triangle is ______. Its measure is ______. The hypotenuse is ______. Its measure is ______.

> **45°–45°–90° Triangle Theorem**
>
> The relationship between a leg and the hypotenuse in a 45°–45°–90° triangle is that the ____________________ is $\sqrt{2}$ times the length of the ________.

Practice

Find the value of *x*.

1. The length of a leg in a 45°–45°–90° triangle is 6. Find the value of x, the length of the hypotenuse.

What is the relationship between a leg and the hypotenuse? ____________________ $= \sqrt{2}$ ________

Substitute the value for the hypotenuse and the leg. ______ $= \sqrt{2}$ ______

Simplify ______ $=$ ______ $\sqrt{2}$

2. In a 45°–45°–90° triangle, the length of the hypotenuse is 10. Find x, the length of a leg.

____

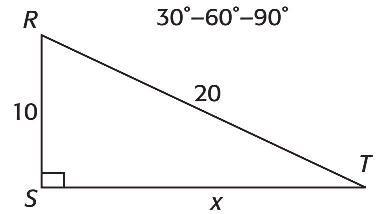
Name ______________________ Date ______________

Special Right Triangles: 30°–60°–90° Right Triangles

You have learned that there is a special relationship between the legs of a right triangle and the triangle's hypotenuse.

The Pythagorean Theorem states that: $a^2 + b^2 = c^2$

Right triangles whose measures are 45°–45°–90° or 30°–60°–90° are called special right triangles.



Use the figure to the right to complete the chart.

30°–60°–90°			
Leg	**Leg**	**Hypotenuse**	
$\overline{RS}$	$\overline{ST}$	$\overline{RT}$	$RS^2 + ST^2 = RT^2$
10	x	20	$10^2 + x^2 =$ ______

Solve for *x*.

______ $+ x^2 =$ ______

$x^2 =$ ______

$\sqrt{x^2} =$ ______

$x =$ ______

$x =$ ______ $\sqrt{3}$

Which leg is the shorter leg? ______ What is its measure? ______

Which leg is the longer leg? ______ What is its measure? ______

30°–60°–90° Triangle Theorem
The relationship between the longer leg and the shorter leg of a 30°–60°–90° triangle is that the ______________ leg is $\sqrt{3}$ times as long as the ______________ leg.

Practice

Find the value of *x*.

1. The length of a shorter leg in a 30°–60°–90° triangle is 6. Find the value of x, the length of the longer leg.

What is the relationship between a shorter leg and the longer leg? ______________ $= \sqrt{3}$ ______________

Substitute the value for the longer leg and the shorter leg. ______ $= \sqrt{3}$

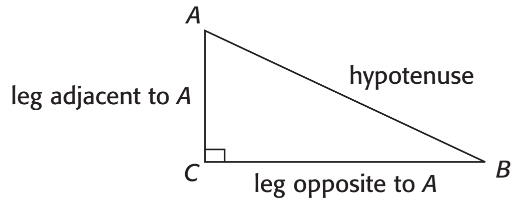
Simplify ______ $=$ ______ $\sqrt{3}$

2. In a 30°–60°–90° triangle, the length of the shorter leg is 12. Find x, the length of the hypotenuse. Hint: find the length of the longer leg, then use the Pythagorean Theorem to find the hypotenuse. ______________

Name ______________________________ Date ______________________

Trigonometric Ratios

As you know, the sides of a right triangle exhibit a special relationship known as the Pythagorean Theorem. The sides of a right triangle exhibit other special properties. The ratios of different sides of right triangles are called **trigonometric ratios**.



There are 3 basic trigonometric ratios—sine, cosine, and tangent. These ratios are based on the length of two of the sides in a right triangle.

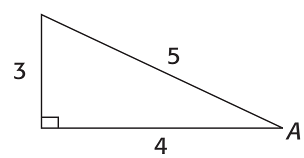
$$\text{sine of } A = \frac{\text{length of the leg opposite } A}{\text{length of the hypotenuse}} = \sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } A = \frac{\text{length of the leg adjacent } A}{\text{length of the hypotenuse}} = \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } A = \frac{\text{length of the leg opposite } A}{\text{length of the leg adjacent } A} = \tan A = \frac{\text{opposite}}{\text{adjacent}}$$

Example

Find sin *A*, cos *A*, and tan *A*.



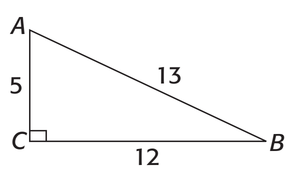
Step 1 Find the sin A $\quad \text{sine of } A = \frac{\text{length of the leg opposite } A}{\text{length of the hypotenuse}} = \sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$

Step 2 Find the cos A $\quad \text{cosine of } A = \frac{\text{length of the leg adjacent } A}{\text{length of the hypotenuse}} = \text{cosine } A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$

Step 3 Find the tan A $\quad \text{tangent of } A = \frac{\text{length of the leg opposite } A}{\text{length of the leg adjacent } A} = \tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$

Practice

1.



Find the sin A sine of A = ______________________

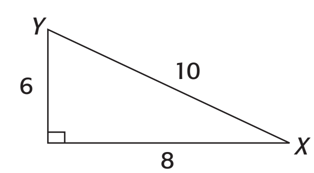
Find the cos A cosine of A = ______________________

Find the tan A tangent of A = ______________________

2. Use the triangle above to find the sin B, cos B, tan B. ______________

3. Use the triangle to the right to find the sin X, cos X, tan X.

____



4. Use the triangle to the right to find the sin Y, cos Y, and tan Y.

____

Name ______________________________ Date ____________________

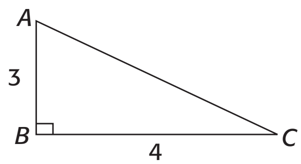
Inverse of Trigonometric Ratios

If you know the measure of two sides of a right triangle, you can find the measures of the angle of the right triangle.

> **Rules for finding the Measure of an Angle in a Right Triangle**
> **1.** Identify the relationship between the unknown angle and the sides that are given.
> **2.** Determine the trigonometric ratio to use.
> **3.** Plug the side measure into the formula. Convert the ratio to a decimal.
> **4.** Using a calculator, find the measure of the angle.

Example

Find the measure of $\angle A$



Step 1 Identify the relationship between the unknown angle and the sides that are given.
Side $\overline{AB}$ is the side adjacent to $\angle A$.
Side $\overline{BC}$ is the side opposite to $\angle A$.

Step 2 Determine the trigonometric ratio to use.
Use the tangent ratio:
$\tan A = \dfrac{\text{length of the leg opposite } A}{\text{length of the leg adjacent } A}$

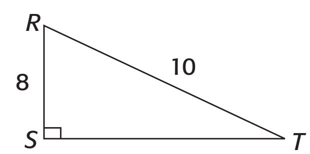
Step 3 Plug the side measure into the formula. Convert the ratio to a decimal.
$\tan A = \frac{4}{3} = 1.33$

Step 4 Using a calculator, find the measure of the angle.
$\tan A = 1.33$
$A = 53^\circ$

Practice

Find the measure of the unknown angle.

1. Find the measure of $\angle T$



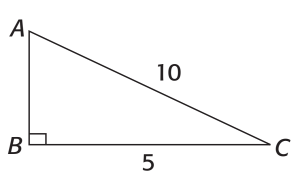
Identify the relationship between the unknown angle and the sides that are given.
Side $\overline{RS}$ is ______________ to $\angle T$.
Side $\overline{RT}$ is the ______________.

Determine the trigonometric ratio to use.
Use the ____________ ratio.
______ $T = \dfrac{\text{length of the leg opposite } T}{\text{length of the hypotenuse}}$

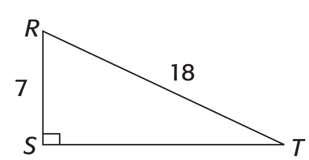
Plug the side measure into the formula. Convert the ratio to a decimal.
______ $T =$ ____________ $=$ ______ $=$ ______

Using a calculator, find the measure of the angle.
______ $T =$ ______
$T =$ ______

2. Find the measure of $\angle C$. ______



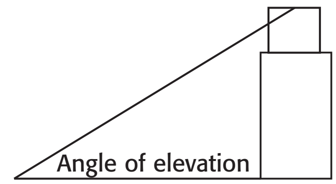
3. Find the measure of $\angle T$. ______



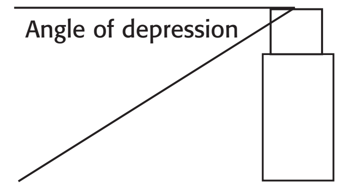
Name ______________________________ Date ____________________

Angles of Elevation and Depression

Suppose you are standing on the ground looking up at the top of a building. The angle of your line of sight is called the **angle of elevation.**



Now suppose there is a person looking down from the top of the building. The angle of the line of sight of the person at the top of the building is the **angle of depression**.



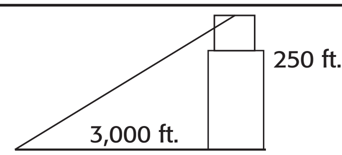
You can use what you know about trigonometric ratios to find the angle of depression or elevation. You can also use what you know to find the height of an object.

Rules for working with the Angles of Elevation and Depression

1. Identify givens and unknowns.
2. Determine the trigonometric ratio to use. Plug the values into the formula.
3. Solve for the unknown.

Example

Find the angle of elevation in the diagram to the right.



Step 1 Identify givens and unknowns.

height: 250 ft (opposite side); distance: 3,000 ft (adjacent side)

Unknown: angle of elevation.

Step 2 Determine the trigonometric ratio to use. Plug the values into the formula.

You know the opposite side and adjacent side; use the tangent ratio:

$\tan\theta = \frac{\text{length of the leg opposite } A}{\text{length of the leg adjacent } A} = \frac{250}{3{,}000}$

Step 3 Solve for the unknown.

$\tan\theta = \frac{250}{3{,}000} = 0.083 = 4.74^{\circ}$

Practice

1. Find the distance: Height is 100 ft and the angle of depression is 9°.

Identify givens and unknowns.

height: ________ (opposite side);

angle of depression: ______

Unknown: distance ____________________

Determine the trigonometric ratio to use. Plug the values into the formula.

You know the opposite side; you want to find the ____________________ side ; use the tangent ratio.

Solve for the unknown.

$\tan\theta = \frac{\text{length of the leg opposite } A}{\text{length of the leg adjacent } A}$

$= \tan$ ______ $= \frac{\quad}{x}$

$x =$ ________ $=$ ________ $=$ ________

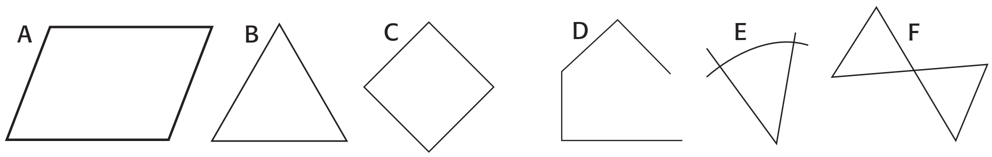
2. Distance is 200 ft. and angle of elevation is 15°. Find the height. ______________

3. Distance is 40 ft and height is 625 ft. Find the angle of elevation. ______________

Name ____________________________ Date ______________

Types of Polygons

The term **polygon** is a term that means "many–sided". Look at the figures below. Those to the left **are** polygons, while the figures to the right **are not** polygons.



Polygons

Not Polygons

Use the figures above to identify characteristics of a polygon. Circle the term in each pair that makes the statement true.

1. The figures on the left are polygons because their sides are made of ______________.

   One of the figures above (Figure E) has a side made of an ______________.

2. In the figures on the left, each segment intersects with ______________ other segments. One of the figures on the right (Figure D) has some of its sides intersecting with

   ______________ segment.

3. In the figures on the left, each segment intersects with ______________ other segments. One of the figures on the right (Figure F) has some of its sides intersecting with

   ______________ segments.

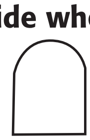
Complete the statements below defining polygons.

A polygon is a closed figure made of a certain number of ______________ lying in the same plane. In a polygon, each side intersects exactly ______________ other sides. Polygons are classified by the number of sides they possess. The chart to the right gives the names of each type of polygon.

Sides	Name	Sides	Name
3	triangle	7	heptagon
4	quadrilateral	8	octagon
5	pentagon	9	nonagon
6	hexagon	10	decagon

Practice

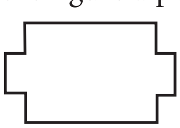
Decide whether each figure is a polygon.

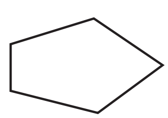
1. 

   Is the figure a closed figure? ________

   Is the figure made only of segments? ________

   Is the figure a polygon? ________

2.  ________

3.  ________

4. Name the figure C above. ______________

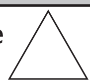
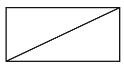
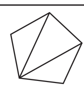
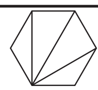
5. Name the figure in # 3. ______________

Name ______________________________ Date ____________________

Sum of Polygon Angle Measures

Each type of convex polygon has a unique value for the sum of the measures of its interior angles. For example, the sum of the measures of the angles of a triangle is 180°. If you divide a polygon into non-overlapping triangles, you can use what you know about triangles to find the sum of the measures of the interior angles of the polygon.

Complete the chart below.

Polygon	Number of Sides	Number of Triangles	Sum of Angle Measures
Triangle 	3	1	1(180°) = 180°
Quadrilateral 	______	______	____________
Pentagon 	______	______	____________
Hexagon 	______	______	____________

Complete the statements below. Use the data in the chart above.

1. A quadrilateral has ______ sides. The number of triangles formed when diagonals are formed is ______.
2. Look at the other figures. The number of triangles formed is always the number of sides (n) minus ______.
3. To find the sum of the angles, you take the number of sides (n) minus ______ and multiply by ______.

Use the data in the table and the statements above to complete the rule below.

Polygon Interior Angle Theorem

The sum of the measures of the interior angles of a convex polygon, where n is the number of sides, is ____________________.

Practice

1. Find the sum of the interior angle measures of a 12-gon.

   How many sides does the polygon have? ______

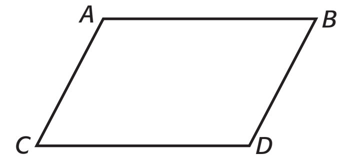
   What is the formula to use? Plug the numbers into the formula. ____________________ ____________________

   Solve. ____________________
2. Find the measure of a 15-gon. ______________
3. Find the measure of a 20-gon. ______________
4. A regular polygon has angles measuring 120°. How many sides does it have? __________

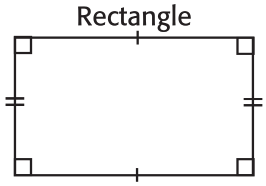
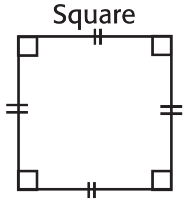
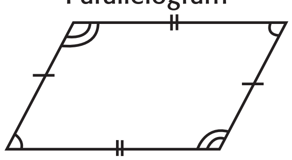
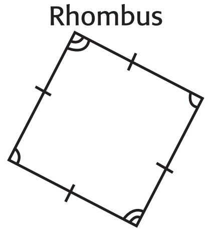
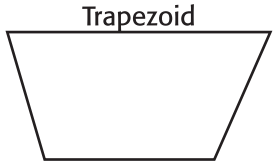
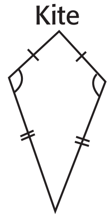
Name ______________________________ Date ________________

Types of Quadrilaterals

A **quadrilateral** is a closed plane figure with four sides. The figure to the right is a quadrilateral.

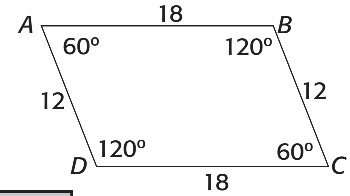


Explore the nature of quadrilaterals. Examine each quadrilateral and complete the chart summarizing the properties of each quadrilateral.

Type	Sides	Angles
Rectangle 	Opposite sides are ____________. Opposite sides are ____________.	All angles are ______.
Square 	Opposite sides are ____________. All sides are ____________.	All angles are ______.
Parallelogram 	Opposite sides are ____________. Opposite sides are ____________.	Opposite angles are ____________.
Rhombus 	Opposite sides are ____________. All sides are ____________.	Opposite angles are ____________.
Trapezoid 	Only one pair of opposite sides are ____________.	N/A
Kite 	Two pairs of adjacent sides are ____________. No pairs of sides are ____________.	One pair of opposite angles are ____________.

Name ____________________ Date ____________________

Properties of Parallelograms



The symbol for the parallelogram to the right is ▱$ABCD$.
In ▱$ABCD$, $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$.

Use the figure to the right to complete the chart.

Opposite Sides			
Side	Measure	Opposite Side	Measure
______	______	$\overline{BC}$	12
$\overline{AB}$	18	______	______

Opposite Angles			
Angle	Measure	Opposite Angle	Measure
$\angle A$	60°	______	______
______	______	$\angle B$	120°

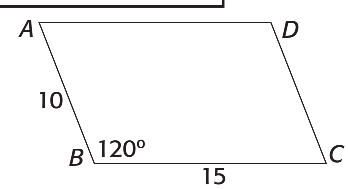
Consecutive Angles				
Angle	Measure	Consecutive Angle	Measure	Sum of Measures
$\angle A$	60°	____________	______	______
$\angle B$	120°	____________	______	______

Use the chart to complete the following statements about the properties of parallelograms.

Properties of Parallelograms

1. Opposite sides of a parallelogram are ____________. Opposite angles of a parallelogram are ____________.
2. The sum of consecutive angles in a parallelogram is 180°. Consecutive angles in a parallelogram are ____________.

Practice



1. For the parallelogram to the right, find the value of $\angle A$, $\angle D$, $\overline{AD}$, and $\overline{CD}$.

   $\angle A$ is an angle that is opposite a consecutive angle with ______, whose measure is ______.

   Consecutive angles are ____________. Therefore, m$\angle A$ = ____________.

   $\angle D$ is an angle that is opposite ______.

   Opposite angles are ____________. Therefore, m $\angle D$ = ______.

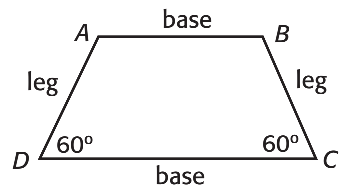
   $\overline{AD}$ is opposite ______. Opposite sides are ____________ so $\overline{AD}$ = ______.

   $\overline{CD}$ is opposite ______. Opposite sides are ____________, so $\overline{CD}$ = ______.

Name ______________________________ Date ____________________

Properties of Trapezoids

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**. The non-parallel sides are called **legs**. In a trapezoid there are two pairs of base angles, one pair for each base. The trapezoid to the right is an isosceles trapezoid, so the legs are congruent, and the angles in each pair of base angles are congruent.



Use the figure above to complete each statement. Then complete the chart.

1. One pair of base angles is $\angle A$ and ______.

2. The other pair of base angles is $\angle D$ and ______.

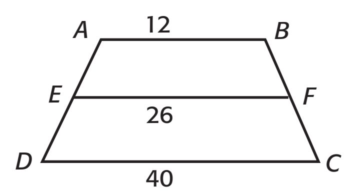
Base Angle	Measure	Base Angle Pair	Measure
$\angle A$	120˚	______	______
$\angle D$	60˚	______	______

Use the data in the table to complete the following rule.

> **Base Angles of an Isosceles Trapezoid**
>
> Both pairs of base angles in an isosceles trapezoid are ____________________.

In the figure to the below right, $\overline{EF}$ is a **median** of a trapezoid; the median is a segment that joins the midpoints of the two legs.

Use the figure to the right to complete each statement.



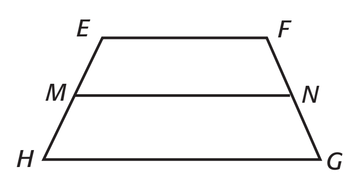
3. $\overline{AB} = 12$, $\overline{CD} = 40$; $\overline{AB} + \overline{CD} =$ ______

4. The measure of $\overline{EF}$ is ______

5. One–half of $\overline{AB} + \overline{CD} =$ ______ $\div 2 =$ ______

Practice

Use the figure to the right to answer the following questions.
EFGH **is an isosceles trapezoid.** $\overline{MN}$ **is a median.**



1. $\overline{HG} = 22$ and $\overline{EF} = 6$. Find $\overline{MN}$.

The sum of $\overline{HG}$ and $\overline{EF}$ is $\overline{HG} + \overline{EF}$; $22 + 6 =$ ______

The median is ______ the length of the sum of the two bases.

$\overline{MN} = \frac{1}{2}(\overline{HG} + \overline{EF}) = \frac{1}{2}(\underline{\qquad}) = \underline{\qquad}$

2. $\angle E = 105°$; $\angle H =$ ______

3. $\overline{EF} = 28$ and $\overline{MN} = 30$. Find $\overline{HG}$ ______

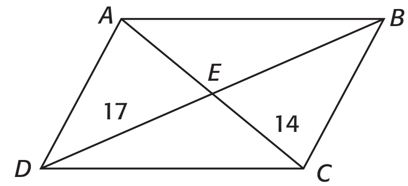
4. $\overline{EM} = 18$. Find $\overline{NG}$ ______

5. $\angle H = 70°$; $\angle G =$ ______

Name ______________________________ Date ______________________

Diagonals in Parallelograms

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. In the parallelogram to the right, *AB* is parallel to *DC*, and *AD* is parallel to *BC*. A diagonal of a polygon is a segment that joins non–connective vertices. In the parallelogram to the right, *DB* is a diagonal.



Use the figure to the above right to complete the table below. Then complete the statements that follow.

Diagonal	Measure	Segment	Measure	Segment	Measure
$\overline{AC}$	28	$\overline{AE}$	______	$\overline{CE}$	______
$\overline{DB}$	______	$\overline{DE}$	______	$\overline{BE}$	______

1. Diagonal $\overline{AC}$ is divided into two segments, $\overline{AE}$ and $\overline{CE}$. The measure of $\overline{AE}$ is ______________ the measure of $\overline{AC}$. Similarly, the measure of $\overline{DE}$ is ______________ the measure of diagonal $\overline{DB}$.
2. Another way to look at this relationship is that $\overline{AE}$ ______ to $\overline{CE}$ and $\overline{DE}$ ______ $\overline{BE}$.
3. You could also say that since one diagonal, such as $\overline{AC}$, intersects $\overline{DB}$ so that two congruent segments are formed, $\overline{DE}$ and $\overline{EB}$, the $\overline{AC}$ ______________ $\overline{DB}$.

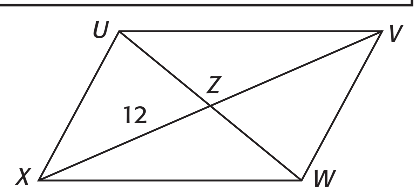
Use the data in the table to complete the rule for diagonals in a parallelogram.

Rule for Diagonals in a Parallelogram

If a quadrilateral is a parallelogram, then its diagonals ______________ each other.

Practice

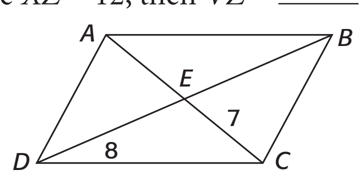
Find the unknown length or lengths.



1. Find $\overline{UZ}$ and $\overline{VZ}$. You are given that UVWX is a parallelogram and that $\overline{WU} = 28$

Finding $\overline{UZ}$:	$\overline{UZ}$ is a ____________________ of $\overline{WU}$.
What is the relationship of $\overline{UZ}$ to $\overline{WU}$?	$\overline{UZ}$ is ______________ the measure of $\overline{WU}$.
What can you conclude about $\overline{UZ}$?	$\overline{UZ}$ = ______
Finding $\overline{VZ}$:	$\overline{XV}$ is ______________ by $\overline{WU}$.
What is the relationship between $\overline{XZ}$ and $\overline{VZ}$?	$\overline{XZ}$ ______ $\overline{VZ}$
What can you conclude about $\overline{VZ}$?	Since $\overline{XZ}$ = 12, then $\overline{VZ}$ = ______

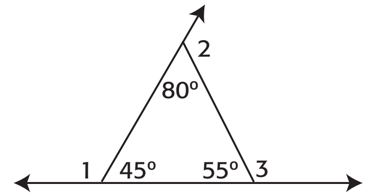
2. $\overline{BE}$ ______
3. $\overline{DB}$ ______
4. $\overline{AE}$ ______
5. $\overline{AC}$ ______

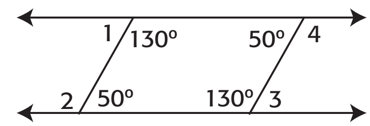


Name ______________________________ Date ____________________

Exterior Angles of a Polygon

As you know, there is a relationship between the interior angles of a convex polygon. You know that the sum of the measures of the interior angle is $(n - 2) = 180$, where n is the number of sides. There is a relationship between the exterior angles of a convex polygon. Remember, an exterior angle is an angle that forms a linear pair with the adjacent interior angle of a polygon.





Use the figures to the right to complete the chart.

Triangle				
$\angle 1$	$\angle 2$	$\angle 3$	$\angle 4$	Sum of Angles
______	______	______	N/A	______
Quadrilateral				
______	______	______	______	______

Use the data in the table above to complete the rule for exterior angles of a polygon.

Exterior Angle Sum Theorem
If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is ______.

Practice

1. Find the measure of each interior and exterior angle in a regular pentagon.

   How many sides are in a pentagon? ____

   What is the sum of the measures of the interior angles of a pentagon? The sum of the measure of a regular pentagon is ______

   What is the measure of each interior angle? ______ ÷ ____ = ______

   What is the sum of the measures of the exterior angles of a polygon? ______

   What is the measure of each exterior angle? ______ ÷ ____ = ____

2. Find the measure of each interior and exterior angle in a regular octagon. __________
3. Each exterior angle of a regular polygon measures 40º. How many sides does this polygon have? ______________
4. Each exterior angle of a regular polygon measures 60º. How many sides does this polygon have? ______________

Name ______________________ Date ______________

Proportions

A proportion is an equation that states that two ratios are equal. The following are examples of proportions.

$$\frac{5\text{ miles}}{10\text{ minutes}} = \frac{15\text{ miles}}{30\text{ minutes}} \qquad \frac{2\text{ cups}}{5\text{ gallons}} = \frac{4\text{ cups}}{10\text{ gallons}}$$

If you were to express each ratio in simplest terms, you would see they are the same. Furthermore, in a proportion, the units are the same across the top and are the same across the bottom.

Rules for Identifying Proportions

1. Place the ratios next to each other. Be sure if the numbers have units that the units are the same across the top and are the same across the bottom.
2. Write each ratio in simplest form.
3. If they are the same in simplest form, the two ratios form a proportion.

Example

Does the pair of ratios $\frac{8}{10}$ and $\frac{32}{40}$ form a proportion?

Step 1 Place the ratios next to each other. $\frac{8}{10}$ $\frac{32}{40}$

Step 2 Write each ratio in simplest form. $\frac{8}{10} = \frac{4}{5}$ $\frac{32}{40} = \frac{4}{5}$

Step 3 If they are the same in simplest form, the two ratios form a proportion. They are each in simplest form. $\frac{8}{10} = \frac{32}{40}$

Practice

Do the ratios in each pair form a proportion?

1. $\frac{10}{20} : \frac{40}{50}$

Place the ratios next to each other. $\frac{10}{20}$ $\frac{40}{50}$

Write each ratio in simplest form. $\frac{10}{20} =$ ______ $\frac{40}{50} =$ ______

If they are the same in simplest form, the two ratios form a proportion. In simplest form, the ratios ____________ equal.

They ____________ form a proportion.

2. $\frac{1}{2} : \frac{25}{30}$ ________

3. $\frac{3}{12} : \frac{9}{36}$ ________

4. $\frac{6}{15} : \frac{12}{45}$ ________

5. $\frac{3}{8} : \frac{4}{16}$ ________

6. $\frac{42}{5} : \frac{126}{15}$ ________

7. $\frac{12}{8} : \frac{16}{24}$ ________

8. $\frac{72}{27} : \frac{16}{6}$ ________

9. $\frac{28}{25} : \frac{112}{100}$ ________

Name ________________________________ Date ____________________

Solving Proportions

In some instances you will need to find a missing number in order to create a proportion. You can use the **cross products** of the two ratios to find the missing number.

Rules for Solving Proportions
1. Multiply the top of the first ratio by the bottom of the second ratio.
2. Multiply the top of the second ratio by the bottom of the first ratio.
3. Divide each side by the number in front of the missing number.

Example

Find the value that completes each proportion: $\frac{14}{35} = \frac{42}{x}$.

Step 1 Multiply the top of the first ratio by the bottom of the second ratio.

$\frac{14}{35} = \frac{42}{x}$

$14 \times x = 14x$

Step 2 Multiply the top of the second ratio by the bottom of the first ratio.

$42 \times 35 = 1470$

Set the results equal to each other.

$14x = 1470$

Step 3 Divide each side by the coefficient of the variable, x.

$14x \div 14 = 1470 \div 14$

$x = 105$

Practice

Find the value that completes each proportion.

1. $\frac{14}{4} = \frac{x}{28}$

Multiply the top of the first ratio by the bottom of the second ratio.

$\frac{14}{4} = \frac{x}{28}$

$14 \times 28 =$ ________

Multiply the top of the second ratio by the bottom of the first ratio.

$4 \times x =$ ________

Set the results equal to each other.

________ = ________

Divide each side by the coefficient of the variable, x.

________ $\div 4 =$ ________ $\div 4$

$x =$ ________

2. $\frac{x}{20} = \frac{11}{55}$ ______

3. $\frac{3}{x} = \frac{9}{18}$ ______

4. $\frac{2}{8} = \frac{x}{52}$ ______

5. $\frac{3}{10} = \frac{12}{x}$ ______

6. $\frac{8}{6} = \frac{20}{x}$ ______

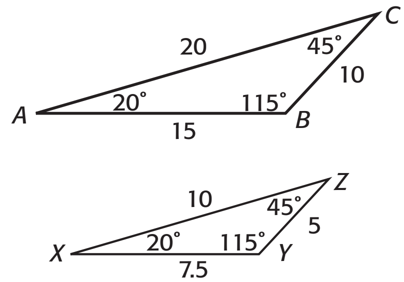
7. $\frac{80}{x} = \frac{10}{4}$ ______

8. $\frac{25}{4} = \frac{50}{x}$ ______

9. $\frac{x}{65} = \frac{4}{10}$ ______

Name ______________________ Date ______________

Similar Polygons



You know that when two polygons are congruent, the measures of their corresponding sides are congruent and the measures of their corresponding angles are congruent. In other words, the figures have the same size and shape. Figures can have the same shape, but are not the same size. When figures have the same shape but are different sizes, they are **similar figures**.

Use the two triangles to the above right to explore the nature of similar figures.

Corresponding Angles				
Angle	**Measure**	**Corresponding Angle**	**Measure**	**Relationship Between Angles**
$\angle A$	20°	$\angle X$	________	$\angle A$ ___ $\angle X$
________	________	$\angle Y$	115°	________ ___ $\angle Y$
$\angle C$	45°	________	________	$\angle C$ ___ ________

Corresponding Sides				
Side	**Measure**	**Corresponding Side**	**Measure**	**Ratio of Angle to Corresponding Angle**
$\overline{AB}$	15	$\overline{XY}$	7.5	$\frac{15}{7.5} = 2$
$\overline{AC}$	20	________	________	________
________	________	$\overline{YZ}$	5	________

Use the data in the table to complete the rule for similar polygons.

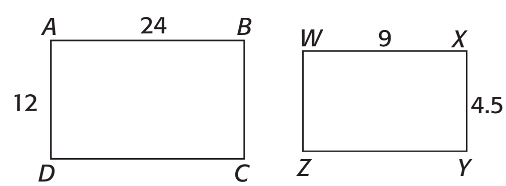
Similar Polygons

1. Corresponding angles are ________________.

2. Corresponding sides are in ________________.

Practice

Tell whether each pair of figures is similar.



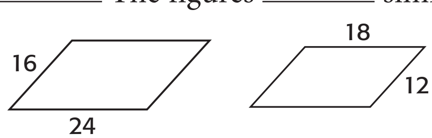
1. Are corresponding angles congruent?

$\angle A \cong \angle W$, $\angle B$ ___ $\angle X$, $\angle C$ ___ $\angle Y$, $\angle D$ ___ $\angle Z$

Are the sides proportional? $\frac{\overline{AB}}{\overline{WX}}$ = ______, ______ = ______;

________ = ________ The figures ________ similar.

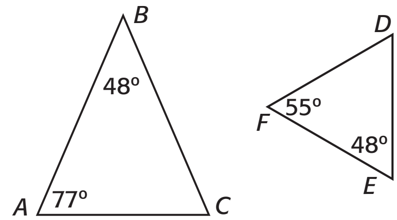
2. ________



Name ______________________________ Date ____________________

Triangle Similarity: Angle-Angle Similarity

Just like triangle congruence, there are various methods for proving that two triangles are similar. The two triangles to the right are similar.



Explore the nature of triangle similarity by completing the statements and chart below.

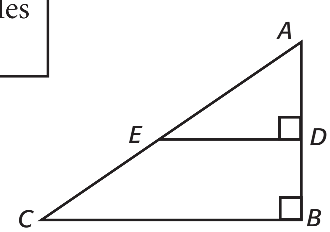
1. $\angle A$ corresponds to ______; ______ corresponds to $\angle E$; $\angle C$ corresponds to ______
2. The measure of $\angle C$ is unknown. Since the sum of the measures of the angles of a triangle is 180°, the measure of $\angle C$ is ______.
3. The measure of $\angle D$ is unknown. Since the sum of the measures of the angles of a triangle is 180°, the measure of $\angle D$ is ______.

Angle	Measure	Corresponding Angle	Measure	Relationship
$\angle A$	77°	$\angle D$	77°	$\angle A \cong \angle D$
$\angle B$	48°	________	________	$\angle B$ ___ ________
________	________	$\angle F$	55°	________ ___ $\angle F$

4. Based on the data above, all the angles of $\triangle ABC$ are ____________________ to all the corresponding angles of $\triangle DEF$.

You may have noticed that if you know the measure of two angles of a triangle, you know the measure of the third. To prove triangles similar, you need to know congruence of two pairs of angles. Use the data in the table and the completed statements to write the rule for triangle similarity.

> **Angle-Angle Similarity**
> If two angles of one triangle are ____________________ to two angles of another triangle, then the two triangles are similar.



Practice

1. Determine whether the triangles in the figure are similar.

   The two triangles are $\triangle ABC$ and ________.

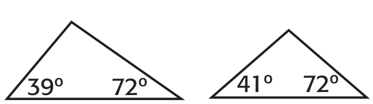
   The two triangles share ________. Since the two triangles share an angle, ________ $\cong$ ________.

   The measure of $\angle D$ is ________; the measure of $\angle B$ is ________.

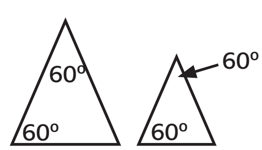
   Are there two pairs of angles that are congruent? ____________________

   Are the triangles similar? ________

2. Are the following triangles similar?



____

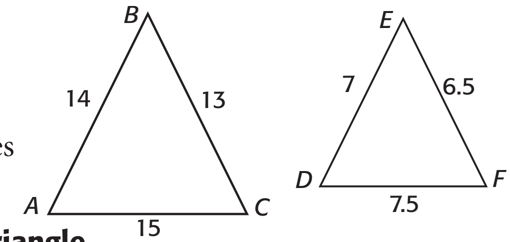


____

Name ______________________________ Date ________________

Triangle Similarity: Side-Side-Side Similarity

When you explored the nature of congruent triangles, you found different methods for proving that two triangles are congruent. Just like triangle congruence, there are various methods for proving that two triangles are similar.



The two triangles to the right are similar. Explore triangle similarity by completing the statements and chart below.

Side	Measure	Corresponding Side	Measure	Ratio of Sides
$\overline{AB}$	14	$\overline{DE}$	7	$\frac{14}{7}$ = ______
$\overline{BC}$	13	______	______	______
______	______	$\overline{DF}$	7.5	______

1. $\frac{\overline{AB}}{DE}$ = ______ and $\frac{\overline{BC}}{EF}$ = ______. Does $\frac{\overline{AB}}{DE} = \frac{\overline{BC}}{EF}$? ______
2. $\frac{\overline{AB}}{DE}$ = ______. Is the ratio of the sides in ΔABC proportional to the corresponding sides in $\triangle DEF$? ______
3. Complete the proportion: $\frac{\overline{AB}}{DE}$ = ______ = $\frac{\overline{AC}}{DF}$
4. The ratio of the sides is the **scale factor.** The scale factor for $\triangle ABC : \triangle DEF$ is ______.

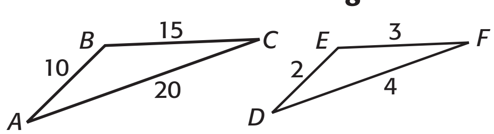
Use the data in the chart to complete the rule for triangle similarity.

> **Side-Side-Side Similarity**
> If the lengths of the corresponding sides of two triangles are ____________________, then the triangles are similar.

Practice

Determine whether the triangles are similar.

1.



$\frac{\overline{AB}}{DE}$ corresponds to ______; the ratio of the sides is ______ = 10/______ = ______

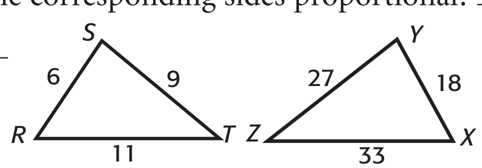
$\frac{\overline{BC}}{EF}$ corresponds to ______; the ratio of sides is ______ = 15/______ = ______

$\frac{\overline{AC}}{DF}$ corresponds to ______; the ratio of sides is ______ = 20/______ = ______

What is the ratio of corresponding sides in each case? ______ : ______

Are the corresponding sides proportional? ______ Are the triangles similar? ______

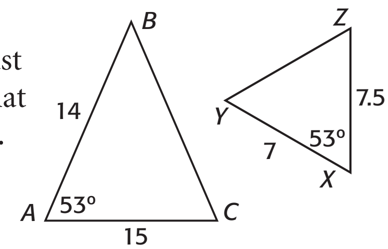
2. ______



Name ______________________________ Date ____________________

Triangle Similarity: Side-Angle-Side Similarity

When you explored the nature of congruent triangles, you found different methods for proving that two triangles are congruent. Just like triangle congruence, there are various methods for proving that two triangles are similar. The two triangles to the right are similar.



Explore the nature of triangle similarity by completing the statements and chart below.

1. $\overline{AB}$ and $\overline{AC}$ are on either side of $\angle A$. $\overline{XY}$ and $\overline{XZ}$ are on either side of $\angle X$. $\angle A$ and $\angle X$ are ____________________ angles.

Angle or Side	Measure	Corresponding Angle or Side	Measure	Relationship
$\overline{AB}$	14	$\overline{XY}$	7	$\frac{14}{7} = 2$
$\angle A$	53°	________	________	$\angle A$ ___ ________
$\overline{AC}$	15	________	________	________

2. Based on the data above, $\angle A$ is ____________________ to $\angle X$.

3. $\frac{\overline{AB}}{\overline{XY}}$ = _____ and $\frac{\overline{AC}}{\overline{XZ}}$ = _____. Does $\frac{\overline{AB}}{\overline{XY}} = \frac{\overline{AC}}{\overline{XZ}}$? ________

Are the corresponding sides of the triangles proportional? _____

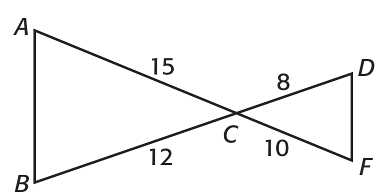
Use the data in the chart to complete the rule for triangle similarity.

Side-Angle-Side Similarity

If an angle of one triangle is ____________________ to an angle of another triangle, and the sides ____________________ these angles are in proportion, then the triangles are similar.

Practice

Determine if each pair of triangles is similar.



1. $\overline{AC}$ corresponds to _____. $\overline{BC}$ corresponds to _____.

The ratio of $\overline{AC}$ to _____ is ______________.

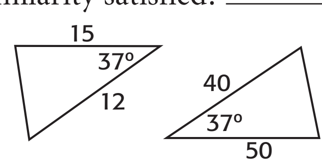
The ratio of $\overline{BC}$ to _____ is ______________.

Do the ratios form a proportion? ____________________

Name the angle between $\overline{AC}$ and $\overline{BC}$? ______________ Is this angle congruent to another angle? If so, name it. ____________________

Is SAS Similarity satisfied? ____________________

2. _____



Name ______________________________ Date ____________________

Finding Lengths in Similar Triangles

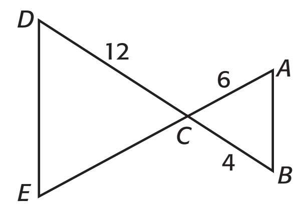
You can apply what you know about similar triangles and proportions to find the unknown length of a side in one of the triangles.

Rules for Finding Lengths in Similar Triangles

1. Create a ratio using a pair of known corresponding sides.
2. Create a ratio for the side and its measure that corresponds to the unknown side.
3. Set up a proportion using the two ratios. Use Cross Products Property.
4. Solve for the unknown.

Example

Find the length of $\overline{CE}$. $\triangle ABC$ is similar to $\triangle EDC$.



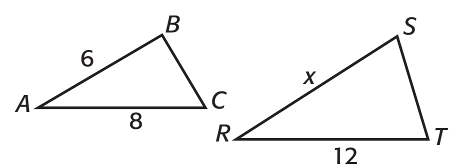
Step 1 Create a proportion using a pair of known corresponding sides.
$\overline{CB}$ and $\overline{CD}$ are corresponding sides.
$\frac{CB}{CD} = \frac{4}{12}$

Step 2 Create a ratio for the side and its measure that corresponds to the unknown side.
$\overline{AC}$ corresponds to the unknown side, $\overline{CE}$.
$\frac{AC}{CE} = \frac{6}{CE}$

Step 3 Set up a proportion using the two ratios. Use Cross Products Property.
$\frac{4}{12} = \frac{6}{CE}$

Step 4 Solve for the unknown.
$\overline{CE} = 18$

Practice



1. Find the missing length. Assume each pair of triangles are similar.

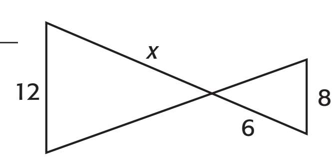
Create a proportion using a pair of known corresponding sides.
$\overline{AC}$ and $\overline{RT}$ are corresponding sides.
_____ = _____

Create a ratio for the side and its measure that corresponds to the unknown side.
$\overline{AB}$ corresponds to the unknown side, $\overline{RS}$.
_____ = _____

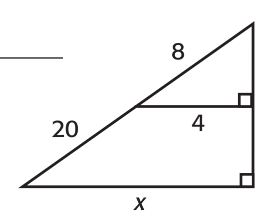
Set up a proportion using the two ratios. Use Cross Products Property.
_____ = _____

Solve for the unknown.
$\overline{RS}$ = _____

2. ______________



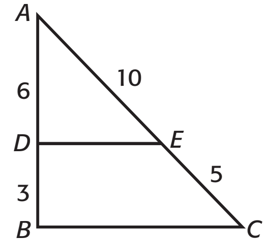
3. ______________



Name ______________________________ Date ____________________

Proportions in Triangles: Side-Splitter Theorem

Segments can be used within a triangle to make a smaller triangle. If the segment within the triangle is parallel to one of the sides, then a special relationship exists with the remaining two sides.



Use the figure to the right to explore the relationship between the sides. $\overline{DE}$ is parallel to $\overline{BC}$.

	Segment	Measure	Segment	Measure	Ratio
Side $\overline{AB}$	$\overline{AD}$	6	$\overline{DB}$	3	$\frac{\overline{AD}}{\overline{DB}}$ = ______ = ______
Side $\overline{AC}$	$\overline{AE}$	10	______	______	$\frac{\overline{AE}}{\overline{EC}}$ = ______ = ______

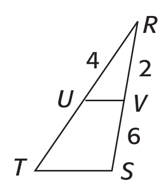
1. The ratio of the two segments formed from $\overline{AC}$ is ______ : 1.
2. The ratio of the two segments formed from $\overline{AB}$ is ______ : 1.
3. Does the ratio of each set of sides form a proportion? ________

Use the data in the table and the completed statements above to complete the Side-Splitter Theorem.

Side-Splitter Theorem
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides ____________________.

Practice

1. Solve for $\overline{UT}$. $\overline{UV}$ is parallel to $\overline{TS}$.



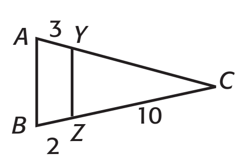
$\overline{RS}$ is divided into two segments; $\overline{RV}$ and $\overline{VS}$. Create a ratio of the segments. $\frac{\overline{RV}}{\overline{VS}}$ = ______

$\overline{RT}$ is divided into two segments; $\overline{RU}$ and $\overline{UT}$. Create a ratio of segments. $\frac{\overline{RU}}{\overline{UT}}$ = ______

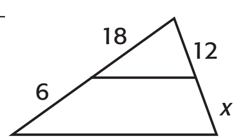
Create a proportion using the two ratios. Solve for the unknown. $\frac{2}{6}$ = ______

$\overline{UT}$ = ______

2. Find $\overline{CY}$. ______________



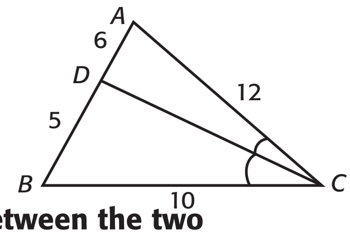
3. Solve for x. ______________



Name ______________________________ Date ____________________

Triangle-Angle Bisector Theorem

Segments can be used within a triangle to make smaller triangles. If a segment within a triangle bisects one of the angles, what type of relationship exists between the two segments that are formed and the other two sides?



Use the figure to the above right to explore the relationship between the two smaller segments. Complete each statement by filling in each blank.

1. $\overline{DC}$ ____________________ $\angle ACB$. The side opposite $\angle ACB$ is $\overline{AB}$.
2. AB is divided into two smaller segments—AD and _____.
3. The ratio of $\frac{\overline{BD}}{\overline{AB}}$ is _____.
4. The ratio of $\frac{\overline{BC}}{\overline{AC}}$ is _____ = _____.
5. The ratio from #3 ___ the ratio from #4.

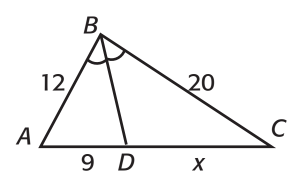
   Therefore, the ratios form a ____________________.

Use the answers to the items above to complete the Triangle-Angle Bisector Theorem.

> **Triangle-Angle Bisector Theorem**
>
> If a segment ____________________ an angle of a triangle, then it divides the opposite side into two segments that are ____________________ to the other two sides of the triangle.

Practice

Solve for the unknown length.



1. Find the length of $\overline{DC}$.

   Identify the angle that is bisected. ________

   Identify the side opposite the bisected angle. Name the segments into which the side is divided. $\overline{AC}$ is divided into $\overline{AD}$ and ________.

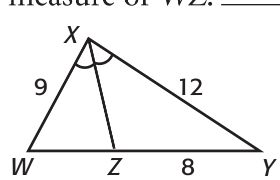
   Create a proportion of the two smaller segments to the two other sides. ________ $= \frac{\overline{AB}}{\overline{AC}}$

   ________ $= \frac{12}{20}$

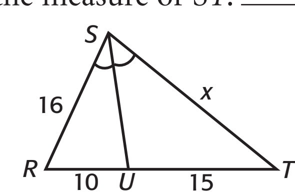
   Use Cross Product property and solve for the unknown side. (______)(20) = 12 ________

   $\overline{DC} =$ ________

2. Find the measure of $\overline{WZ}$. ____________



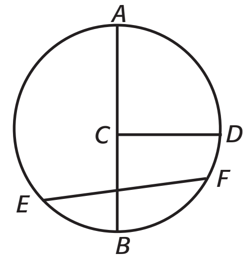
3. Find the measure of $\overline{ST}$. ____________



Name ______________________________ Date ____________________

Circles and Circumference

A **circle** is a set of points in a plane that are the same distance from a given point called the center of the circle. A circle has certain parts. These parts are shown in the figure to the right.



Examine each figure and then complete each definition below.

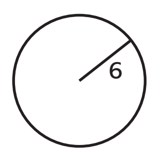
1. $\overline{CD}$ is the radius. The **radius** of a circle is a segment whose endpoints are the ____________ of the circle and a point ________ the circle.
2. C is the center of the circle. $\overline{AB}$ is the diameter. The **diameter** of a circle is a segment that passes through the ____________ of the circle with endpoints ________ the circle.
3. $\overline{EF}$ is a chord. A **chord** is a segment that has ____________ endpoints ____________ the circle. The ________________ of a circle is also a chord.
4. $\overline{AB} = 20$ and $\overline{CD} = 10$. The radius is ____________ the diameter, or the diameter is ____________ times the radius.

The distance around a circle is its **circumference**. The circumference of a circle is found using the measure of the radius or diameter.

> **Rules for Finding the Circumference**
> 1. Identify the radius or diameter of a circle. The radius is r and the diameter is d.
> 2. If the radius is given, use the formula $C = 2\pi r$; if the diameter is given, use $C = \pi d$.

Example

Find the circumference of the circle to the right. Use $\pi = 3.14$

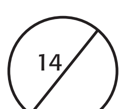


Step 1 Identify the radius or diameter of a circle. The radius is r; the diameter is d.
The radius is shown. So, $r = 6$

Step 2 If the radius is given, use the formula $C = 2\pi r$; if the diameter is given, use $C = \pi d$.
Since the radius is given, use $C = 2\pi r$.
$C = 2\pi r = 2\,(3.14)(6) = 37.68$

Practice

1. Find the circumference of the circle.



Identify the radius or diameter of a circle. The radius is r; the diameter is d.
The ________________ is shown.
So, ______ = ______.

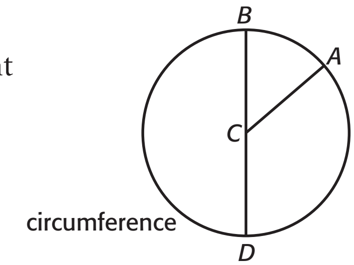
If the radius is given, use the formula $C = 2\pi r$; if the diameter is given, use $C = \pi d$.
Use the formula $C =$ ______.
$C =$ ______ = ____________ = 43.96

2. Find the circumference of a circle with a radius of 12. ____________
3. Find the radius and diameter of a circle with a circumference of 47.1. ____________
4. Find the radius and diameter of a circle with a circumference of 62.8. ____________

Name ______________________________ Date ____________________

Exploring π

A **circle** is the set of all points in a plane that are an equal distance from a given point, the center of the circle. A **diameter** is a segment that passes through the center of the circle with endpoints on the circle. A **radius** is a segment whose endpoints are the center of the circle and a point on the circle. The **circumference** is the distance around a circle.



The chart below shows the diameter and circumference of several circles. Complete the chart by finding the ratio of the circumference to the diameter. Express the ratio as a fraction and a decimal.

Circle	Circumference	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
1	37.68	12	________
2	31.4	10	________
3	25.12	8	________
4	56.52	18	________
5	47.1	15	________

Use the data in the table above to complete each statement below.

1. Is the ratio of the circumference to the diameter of each circle the same or different?

   ____

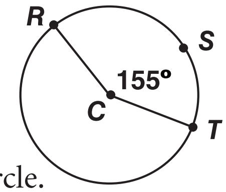
2. What is the value for the ratio of the circumference to the diameter? ________________
3. Write an equation that shows the relationship of circumference, diameter and the resulting ratio. ____________________
4. Use the relationship between the circumference and the diameter to complete the chart.

Circle	Circumference	Diameter
1	________	34
2	37.68	________
3	53.38	________
4	________	20

The ratio of the circumference of a circle to its diameter is the same for every circle. The symbol π represents this ratio.

Name ______________________________________ Date ____________________

Arc Length



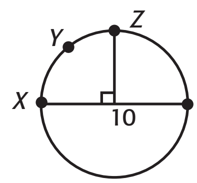
An **arc** is a part of a circle. In the circle to the right, the part of the circle from R to S to T is an arc. You write the symbol for the arc as $\overset{\frown}{RST}$. The measure of an arc is in degrees. **Arc length** is a fraction of the circle's circumference. The **measure** of $\overset{\frown}{RST}$ is equal to m$\angle RCT$ with C the center of the circle.

Rules for Finding Arc Length

1. Find the measure of the arc (in degrees).
2. Find the radius of the circle. If the diameter is given, divide the diameter by 2.
3. Plug the measure of the arc and the radius into the formula for arc length:
   Arc length $= \frac{\text{measure of arc}}{360°}(2\pi r)$

Example

Find the length of arc $\overset{\frown}{XYZ}$. Use $\pi = 3.14$



Step 1 Find the measure of the arc (in degrees). The measure of the arc is 90°.

The diameter is given; the diameter is 10.

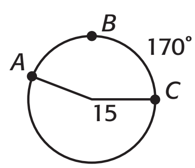
Step 2 Find the radius of the circle. If the diameter is given, divide the diameter by 2. To find the radius, divide the diameter by 2; $r = 5$.

Step 3 Plug the measure of the arc and the radius into the formula for arc length: Arc length $= \frac{90°}{360°}(2(3.14))(5) = 7.85$

Arc length $= \frac{\text{measure of arc}}{360°}(2\pi r)$

Practice

Find the length of arc $\overset{\frown}{ABC}$.



1.

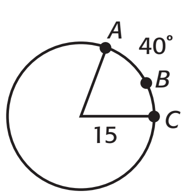
Find the measure of arc $\overset{\frown}{ABC}$ (in degrees). The measure of the arc is ________.

Find the radius of the circle. If the diameter is given, divide the diameter by 2. The radius is ________.

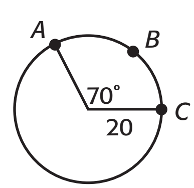
Plug the measure of the arc and the radius into the formula for arc length: Arc length $= \frac{170°}{360°}(2(3.14))(\underline{\qquad}) = 44.5$

Arc length $= \frac{\text{measure of arc}}{360°}(2\pi r)$

2. ______________



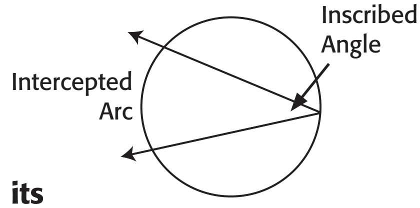
3. ______________



Name ______________________________ Date ______________

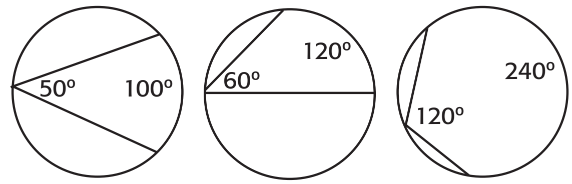
Inscribed Angles

An **inscribed angle** is an angle with its vertex on a circle and with its sides that are a chord of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.



Explore the relationship between an inscribed angle and its intercepted arc. Use the three circles below to complete the table below.

Cirlce	Inscribed Angle	Intercepted Arc
Circle 1	________	________
Circle 2	________	________
Circle 3	________	________



1. The measure of the ____________________ is ____________________ than the measure of the inscribed angle.

2. The relationship between the intercepted arc and its inscribed angle is that the intercepted arc is ______________ times the measure of the inscribed angle.

3. Another way to look at the relationship is that the measure of the inscribed angle is ______________ the measure of its intercepted arc.

Complete the relationship rule for an inscribed angle and its intercepted arc.

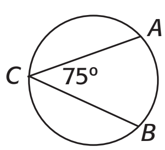
Rule for Inscribed Angles

If an angle is inscribed in a circle, then its measure is ______________ the measure of its intercepted arc: $m\angle ABC = \frac{1}{2} m\,\overset{\frown}{AC}$

Practice

Find the measure of the arc or the angle in the following circles.

1.



What is given? — The measure of the ____________________ is given.

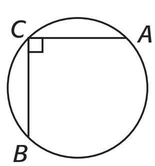
What is its measure? — Its measure is ________.

Plug the measure into the relationship between an inscribed angle and its intercepted arc. — $m\angle ACB = \frac{1}{2} m\,\overset{\frown}{AB}$

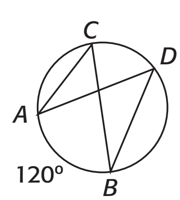
________ $= \frac{1}{2} m\,\overset{\frown}{AB}$

Solve. — ________ $= m\,\overset{\frown}{AB}$

2. ______________



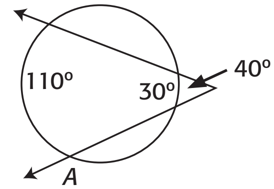
3. ______________



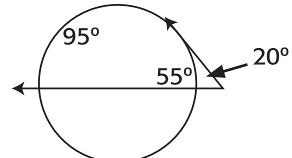
Name ____ Date ________________________

Angle Measures in Circles

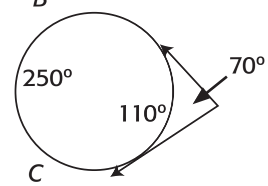
A **secant** is a line that intersects a circle at two points. A **tangent** is a line in the plane of the circle that intersects the circle at one point. Angles formed by secants and tangents intercept arcs on the circle. The three ways this can happen are shown to the right.



A



B



C

Circle	Larger Arc	Smaller Arc	Angle	Larger Arc–Smaller Arc
A	________	________	________	________
B	________	________	________	________
C	________	________	________	________

1. In Circle *A*, the difference in arc measure is ______. The measure of the angle formed by the two secants is ______. The measure of the angle is ______________ the difference in arc measure.
2. Compare the same measures in Circle *B*. The measure of the angle formed by the secant and tangent is ______________ the difference in arc measure.
3. Compare the same measures in Circle *C*. The measure of the angle formed by the two tangents is ______________ the difference in arc measure.

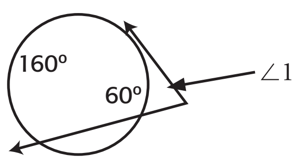
Complete the rule for the angle formed by two lines that intersect outside a circle.

> **Rule for Angles Formed by Secants and Tangents Intersecting Outside a Circle**
>
> The measure of an angle formed by two lines that intersect outside a circle is ________ the difference of the measures of the intercepted arcs: $m\angle 1 = \frac{1}{2}(x - y)$

Practice

Find the missing measure.

1.



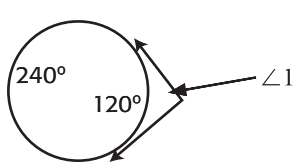
What is the missing measure? The measure of the ______________.

What is given? The measure of the two ______________.

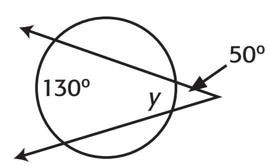
Plug the numbers into the formula. $m\angle 1 = \frac{1}{2}(x - y) =$ ______________

$= 50°$

2. ______________



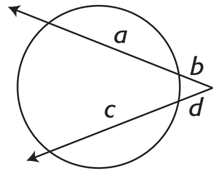
3. ______________



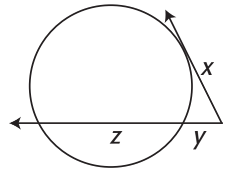
Name ______________________________ Date ____________________

Finding Segment Lengths

A **secant** is a line that intersects a circle at two points. A **tangent** is a line that intersects a circle at one point. Angles formed by secants and tangents intercept arcs on the circle. Tangents and secants can intersect outside a circle. The figures to the right show two such situations. There is a proportional relationship that exists between the segments of each line.



Secant-Secant



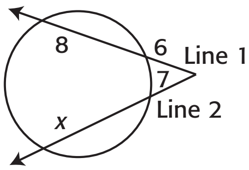
Secant-Tangent

Rules for Finding Segment Lengths

1. Find the lengths of the two segments of the secant going through the circle.
2. If the second line is a tangent, then find the length of the tangent. If the second line is another secant, then find the length of the two segments.
3. For secant/secant use: $(b + a)\ b = (d + c)\ d$; for tangent/secant use: $(y + z)\ y = x^2$
4. Solve for the unknown length.

Example

Find the missing length.



Step 1 Find the lengths of the two segments of the secant going through the circle. For Line 1, make $a = 8$ and $b = 6$.

Step 2 The second line is another secant. For Line 2, make $c = x$ and $d = 7$.

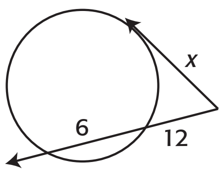
Step 3 For secant/secant use: $(b + a)b = (d + c)d$ $(6 + 8)6 = (7 + x)\ 7$

Step 4 Solve for the unknown length. $84 = 49 + 7x$

$5 = x$

Practice

1. Find the missing length.



Find the lengths of the two segments of the secant going through the circle. Make $z =$ ________ and $y =$ ________.

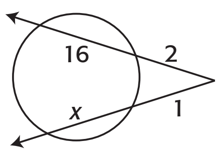
Find the length of the tangent. Tangent length = ________

For tangent/secant use: $(y + z)y = x^2$ ______________ $= x^2$

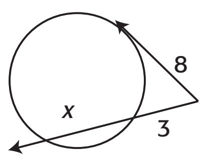
Solve for the unknown length. ________ $= x^2$

________ $= x$

2. ______________

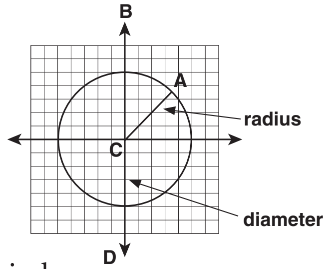


3. ______________



Name ______________________________ Date ______________________

Equation of a Circle



A **circle** is the set of all points in a plane that are an equal distance from a given point, the center of the circle. A **diameter** is a segment that passes through the center of the circle with endpoints on the circle. A **radius** is a segment whose endpoints are the center of the circle and a point on the circle.

You can write an equation of a circle in a coordinate plane. To do so, you need to know the coordinates of the center and the radius of the circle.

Rules for Finding the Equation of a Circle

1. Identify the coordinates of the center. The x-coordinate is h and the y-coordinate is k.
2. Determine the radius of the circle. Remember, the diameter is twice the radius. So, if you are given the diameter, divide the diameter by 2.
3. Plug the values into the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$

Example

Write an equation of the circle with a center at (2, 1) and a radius of 5.

Step 1	Identify the coordinates of the center. The x-coordinate is h and the y-coordinate is k.	The center is at (2, 1). $h = 2, k = 1$
Step 2	Determine the radius of the circle.	The radius is 5.
Step 3	Plug the values into the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$	$(x - 2)^2 + (y - 1)^2 = 5^2$ $(x - 2)^2 + (y - 1)^2 = 25$

Practice

Write an equation of the circle for each circle with the given center and radius or diameter.

1. Center (0, 0); radius 4.

Identify the coordinates of the center. The x-coordinate is h and the y-coordinate is k.	The center is at (0, 0). $h =$ ______, $k=$ ______.
Determine the radius of the circle.	The radius is ______.
Plug the values into the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$	$(x -$ ______$)^2 + (y -$ ______$)^2 =$ ______^{2} ______ + ______ = ______

2. Center: (5, 3); diameter: 12 ______________________________

3. Center: (0, 2); radius: 7 ______________________________

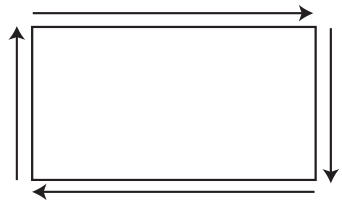
4. Center: (4, −1); diameter: 3 ______________________________

5. Center: (−2, −2); diameter: 18 ______________________________

Name ____________________________ Date ____________________

Perimeter

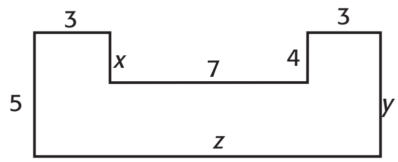
The **perimeter** of a figure is the distance around the outside of that figure. To find the perimeter of a figure, add the lengths of all of its sides. Perimeter is measured in linear units. In the case of a circle, the distance around the figure is known as the **circumference**.



Rules for Finding the Perimeter of a Figure
1. Find the length of each side.
2. Add the length of all of the sides to find the perimeter.

Example

Find the perimeter of the figure.



Step 1 Find the length of each side.

You need to find the length of some missing sides, x, y, and z.

$x = 4$, $y = 5$, $z = 3 + 7 + 3 = 13$

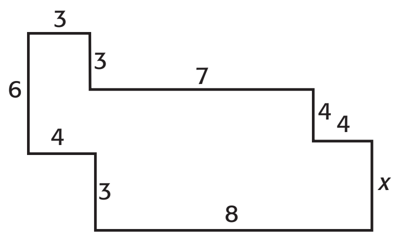
Step 2 Count the number of sides. Add the length of all the sides to find the perimeter.

There are 8 sides. The measures of all the sides are added.

$P = 5 + 3 + 4 + 7 + 4 + 3 + 5 + 13 = 44$ units

Practice

Find the perimeter of each figure.



1. Find the length of each side.

There is one length that is not known, x.

The length of x is ______.

Count the number of sides. Add the length of all the sides to find the perimeter.

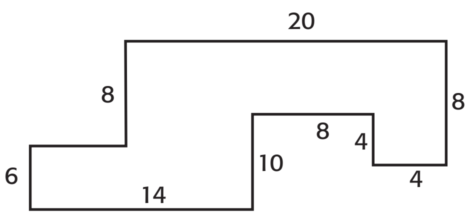
There are ______ sides to the figure.

$P =$ ______ + ______ + ______ + ______ + ______

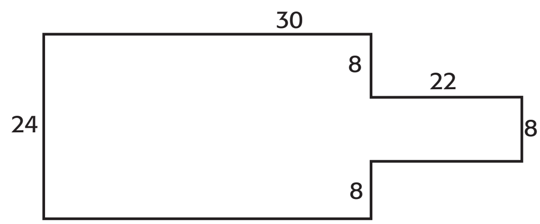
+ ______ + ______ + ______ + ______ + ______

$P =$ ______ units

2. ______________



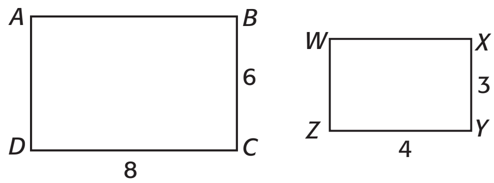
3. ______________



Name ____ Date ____________________

Perimeter and Similar Figures

Is there a relationship between the perimeters of two similar figures? As you know, in similar figures, the lengths of corresponding sides are in proportion. The **perimeter** of a geometric figure is the distance around the figure. To find the perimeter, you find the length of each side and add the lengths.



Use the figures to the above right to explore the perimeters of similar figures.

	Side	Side	Side	Side	Perimeter
ABCD	$\overline{AB}$ = ______	$\overline{BC}$ = ______	$\overline{CD}$ = ______	$\overline{DA}$ = ______	28
WXYZ	$\overline{WX}$ = ______	$\overline{XY}$ = ______	$\overline{YZ}$ = ______	$\overline{ZW}$ = ______	14

1. The ratio of sides $\overline{AB} : \overline{WX}$ is ______________ or ______________.

2. The ratio of sides $\overline{BC} : \overline{XY}$ is ______________ or ______________.

3. The ratio of the perimeters of $ABCD : WXYZ$ is ______________ or ______________.

4. The ratio of the perimeters is ______________ to the ratio of the sides.

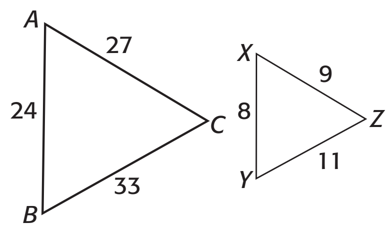
Use the data in the table and the completed statements to write the rule about perimeters of similar figures.

> **Perimeters of Similar Figures**
> If two figures are similar with the lengths of corresponding sides in the ratio $a : b$, then the ratio of their perimeters is ______________.

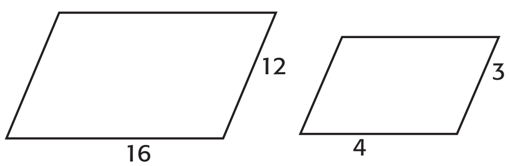
Practice

The following polygons are similar. Find the ratio of their sides and perimeters.

1. The side corresponding to $\overline{AB}$ is ________. The ratio of their measures is ________ or ________.
The perimeter of $\triangle ABC$ is ________. The perimeter of $\triangle XYZ$ is________. The ratio of the perimeter of $\triangle ABC : \triangle XYZ$ is ________ or ________.



2.



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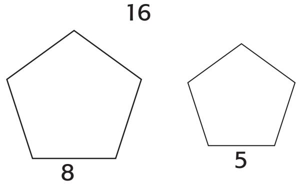
3.



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Name ______________________________ Date ______________

Area of a Triangle

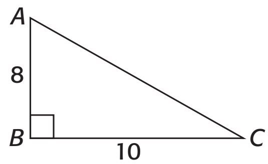
The area of a two-dimensional figure is the number of square units enclosed by the figure. A square unit is the space enclosed by a 1 unit by 1 unit square.

To find the area of a triangle you multiply half the base of the triangle by its height. In a triangle, any side is a base. The height is an altitude, from the base. Remember, an altitude is a perpendicular segment from the base to the angle opposite the base.

Rules for Finding the Area of a Triangle
1. Identify the base. Any side of the triangle can be the base.
2. Identify the height. The height can be inside, or outside the triangle.
3. The area is calculated by using the following formula: $A = \frac{1}{2}bh$.

Example

Find the area of the triangle to the right.



Step 1 Identify the base. Any side of the triangle can be the base.
The base is $\overline{BC}$; $\overline{BC} = 10$

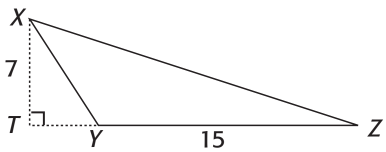
Step 2 Identify the height. The height can be inside, or outside the triangle.
In a right triangle, the height can be the other leg, so, $\overline{AB}$ is the height; $\overline{AB} = 8$

Step 3 The area is calculated by using the following formula: $A = \frac{1}{2}bh$.
$A = \frac{1}{2}bh$
$= \frac{1}{2}(10)(8) = 40$ square units.

Practice

Find the area of each triangle.

1.

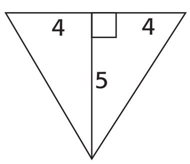


Identify the base. Any side of the triangle can be the base.
The base is $\overline{YZ}$; $\overline{YZ} =$ ______

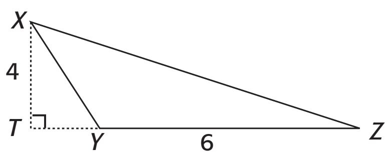
Identify the height. The height can be inside, or outside the triangle.
The height is shown by segment $\overline{XT}$ and is ____________ the triangle;
$\overline{XT} =$ ______

The area is calculated by using the following formula: $A = \frac{1}{2}bh$.
$A = \frac{1}{2}bh$
$= \frac{1}{2}$ ____________ = ______ square units

2. ____________



3. ____________



Name ______________________________ Date ______________________

Area of a Parallelogram

To find the area of a parallelogram, you simply multiply the base and the height. In a parallelogram that is a rectangle or square, any side can be the base. The height is a side adjacent to the base. In other parallelograms, such as a rhombus, the base can be any side, and the height is an altitude drawn from the base.

Rules for Finding the Area of a Parallelogram

1. Identify the base. Any side can be the base.
2. Identify the height. In a rectangle or square, the side adjacent to the base is the height. In other parallelograms, the height is an altitude from the base.
3. Multiply the length times the height: $A = bh$.

Example

Find the area of the figure to the right.

Step 1 Identify the base. Any side can be the base. — Make the base $\overline{DC}$; $\overline{DC} = 9$

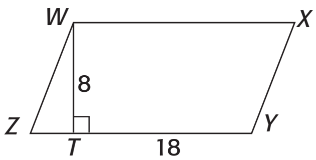
Step 2 Identify the height. In a rectangle or square, the side adjacent to the base is the height. In other parallelograms, the height is an altitude from the base. — The sides adjacent to $\overline{DC}$ are $\overline{AD}$ and $\overline{BC}$. Make the height $\overline{BC}$. The length of $\overline{BC}$ is 5.

Step 3 Multiply the length times the height: $A = bh$. — $A = bh = 9 \times 5 = 45$ square units.

Practice

Find the area of each parallelogram.

1.



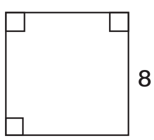
Identify the base. Any side can be the base. — Make the base side $\overline{YZ}$; the length of the base is ______.

Identify the height. In a rectangle or square, the side adjacent to the base is the height. In other parallelograms, the height is an altitude from the base. — The parallelogram is not a rectangle or a square. The height is $\overline{WT}$; the length of the height is ______.

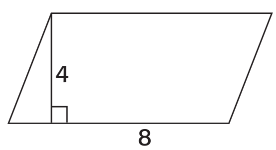
Multiply the length times the height: $A = bh$. — $A = bh$

= ______ ______= ______ square units

2. ____________________



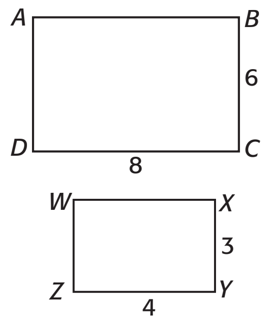
3. ____________________



Name ______________________________ Date ____________________

Area of Similar Figures

As you know, in similar figures corresponding sides are in proportion. You may also know that the ratio of the perimeters is equal to the ratio of corresponding sides, and so in proportion to the sides. What about the ratio of the areas? Is there a relationship between the areas of two similar figures?



Use the figures to the right to explore the areas of similar figures.

	Length	Width	Area ($l \times w$)
ABCD	______	______	______
WXYZ	______	______	______

1. The ratio of sides $\overline{BC}$ to $\overline{XY}$ is ________ or ________.

2. The ratio of sides $\overline{DC}$ to $\overline{ZY}$ is ________ or ________.

3. The ratio of the areas of *ABCD* to *WXYZ* is ________ or ________.

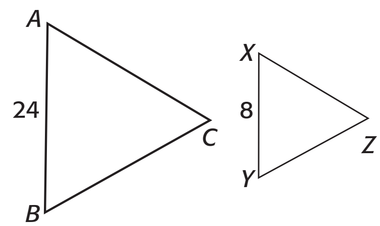
4. Compare the ratio of the sides to the ratio of the areas. The ratio of the area is the ratio of the sides ________________.

Use the data in the table and the completed statements above to write the rule about the area of similar figures.

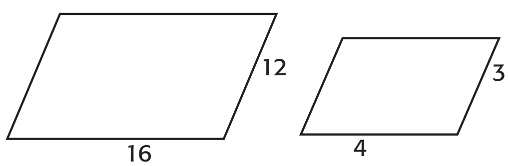
> **Areas of Similar Figures**
> If two figures are similar with the lengths of corresponding sides in the ratio of *a*:*b*, then the ratio of their areas is ____________.

Practice

1. The side corresponding to $\overline{AB}$ is ________; the ratio of their measure is ________ or ________.
Find the ratio of the areas by squaring the ratio of the sides. The ratio of the areas is ________ = ________.



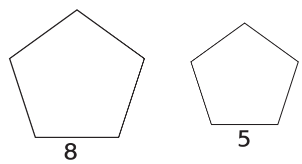
2.



Ratio of sides: ________;

ratio of areas ________

3.



Ratio of sides ________;

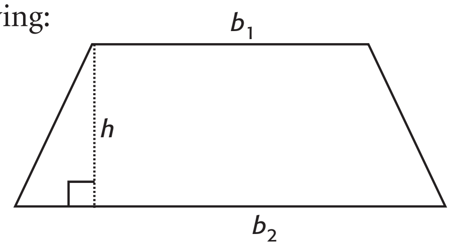
ratio of areas ________

Name ______________________________ Date ____________________

Area of a Trapezoid

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

To find the area of a trapezoid, you need to know the following:



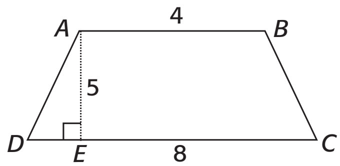
- The length of one of the bases, b_1.
- The length of the other base, b_2.
- The height of the trapezoid; the **height** is an altitude drawn from one base to the other.

Rules for Finding the Area of a Trapezoid

1. Identify each base. The bases of a trapezoid are the sides that are parallel to each other.
2. Identify the height.
3. Use the formula $A = \frac{1}{2}(b_1 + b_2)h$

Example

Find the area of the trapezoid to the right.



Step 1 Identify each base. The bases of a trapezoid are the sides that are parallel to each other.
Let $\overline{AB}$ be b_1 and $\overline{DC}$ be b_2.
$\overline{AB} = 4$; $\overline{DC} = 8$

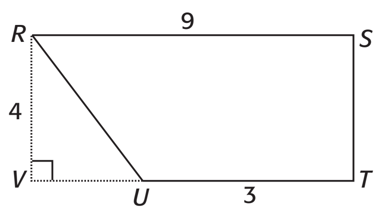
Step 2 Identify the height.
$\overline{AE}$ is the height; $\overline{AE} = 5$

Step 3 Use the formula $A = \frac{1}{2}(b_1 + b_2)h$
$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(4 + 8)(5)$
$= 30$ square units

Practice

Find the area of each trapezoid.

1.



Identify each base. The bases of a trapezoid are the sides that are parallel to each other.
Let $\overline{RS}$ be b_1, $\overline{RS} = 9$

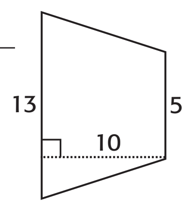
Let ______ be b_2; ______ = ______

Identify the height.
______ is the height ; ______ = ______

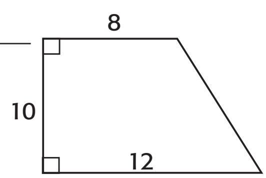
Use the formula $A = \frac{1}{2}(b_1 + b_2)h$
$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(9 +$ ______$)$ ______

= ______ square units

2. ____________________



3. ____________________



Name ______________________________ Date ____________________

Area of a Rhombus or Kite

A **rhombus** is a quadrilateral with four congruent sides. A **kite** is a quadrilateral with two pairs of consecutive congruent sides (but the opposite sides are not congruent).

To find the area of a rhombus or kite, you must draw two diagonals. Remember, a **diagonal** is a segment that joins two non-consecutive vertices. In a rhombus or a kite, the diagonals are perpendicular.

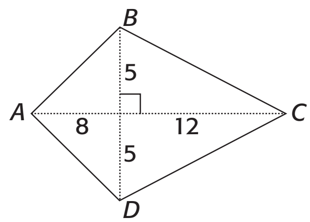
Rules for Finding the Area of a Rhombus or Kite

1. Identify two diagonals. A diagonal is a segment that joins two non-consecutive vertices. Label one diagonal d_1 and the other d_2.

2. Use the formula $A = \frac{1}{2} d_1 d_2$ to find the area.

Example

Find the area of the kite.



Step 1 Identify two diagonals. A diagonal is a segment that joins two non-consecutive vertices. Label one diagonal d_1 and the other d_2.

The diagonals are $\overline{AC}$ and $\overline{BD}$.
Let $\overline{AC}$ be d_1; $\overline{AC} = 20$
Let $\overline{BD}$ be d_2; $\overline{BD} = 10$

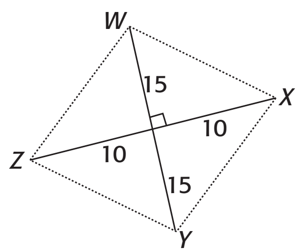
Step 2 Use the formula $A = \frac{1}{2} d_1 d_2$ to find the area.

$A = \frac{1}{2}d_1d_2 = \frac{1}{2}(20)(10)$
$= 100$ square units.

Practice

Find the area.

1.



Identify two diagonals. A diagonal is a segment that joins two non-consecutive vertices. Label one diagonal d_1 and the other d_2.

The diagonals are $\overline{WY}$ and ______.

Let $\overline{WY}$ be d_1; $\overline{WY} = 30$.

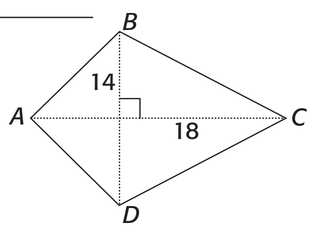
Let ______ be d_2; ______ = ______

Use the formula $A = \frac{1}{2} d_1 d_2$ to find the area.

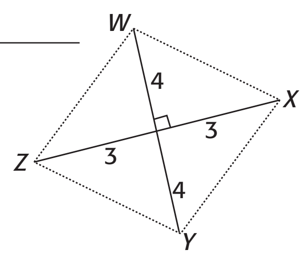
$A = \frac{1}{2} d_1 d_2 = \frac{1}{2}(30)(\_\_\_\_\_\_)$

= ______ square units

2. ____________________



3. ____________________



Name ____ Date ________________________

Area of a Circle

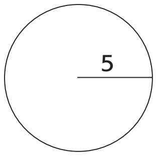
To find the area of a circle, you need to know the radius or diameter of the circle. Remember, the **radius** of a circle is a segment from the center of the circle to the edge of the circle. The **diameter** is a segment that passes through the center of the circle and whose endpoints are on the circle. The diameter is twice the radius *or*, the radius is half the diameter.

Rules for Finding the Area of a Circle

1. Identify the radius of the circle. If you are given the diameter, divide the diameter by 2.
2. Use the formula for the area of a circle: $A = \pi r^2$. Plug the radius into the formula.
3. Square the radius. Use 3.14 for π. Solve.

Example

Find the area of the circle.



Step 1 Identify the radius of the circle. If you are given the diameter, divide the diameter by 2. — The radius is 5.

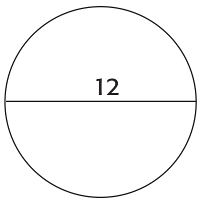
Step 2 Use the formula for the area of a circle: $A = r^2$. Plug the radius into the formula. — $A = \pi r^2$; $A = \pi(5)^2$

Step 3 Square the radius. Use 3.14 for π. Solve. — $A = 3.14(5)^2 = 78.5$ square units.

Practice

Find the area of the circle.

1.

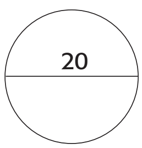


Identify the radius of the circle. If you are given the diameter, divide the diameter by 2. — In the problem, the diameter is given. The radius is ______________ the diameter, or ______.

Use the formula for the area of a circle: $A = \pi r^2$. Plug the radius into the formula. — $A = \pi r^2$; $A =$ ______________

Square the radius. Use 3.14 for π. Solve. — $A =$ (______________)(______)^2 = ______________ square units.

2. Find the area of the circle. ______________



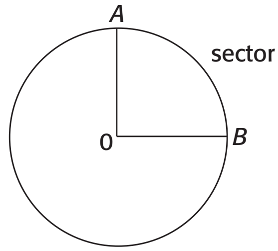
3. What is the radius of a circle with an area of 28.26 square units? ______________

4. What is the difference in area between a circle with a radius of 4 and a circle with a radius of 8? ______________

Name ____________________ Date ____________________

Area of a Sector of a Circle

A **sector** of a circle is a region in a circle bounded by an arc of the circle and two radii from the center of the arc's endpoint.

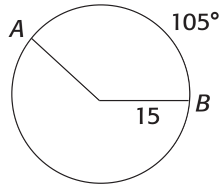


A sector is named using one arc endpoint, the center of the circle and the other endpoint. The arc in the figure to the right is $\overset{\frown}{AB}$. The area of a sector *AOB* of a circle is a sectional part of the area of the circle.

> **Rules for Finding the Area of a Sector of a Circle**
> **1.** Determine the measure of the arc. The measure of the arc is in degrees.
> **2.** Determine the radius of the circle.
> **3.** Use the formula: Area of Sector = $\frac{\text{measure of the arc}}{360} \times \pi r^2$. Use 3.14 for π.

Example

Find the area of sector *AOB*.



Step 1 Determine the measure of the arc. The measure of the arc is in degrees. — The measure of arc *AB* is 105°.

Step 2 Determine the radius of the circle. — The measure of the radius is 15.

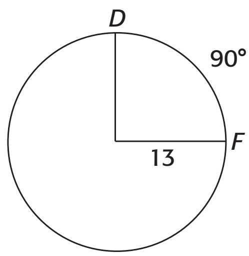
Step 3 Use the formula:
Area of Sector = $\frac{\text{measure of the arc}}{360} \times \pi r^2$
Use 3.14 for π.

Area $= \frac{\text{measure of the arc}}{360} \times \pi r^2 = \frac{105}{360} \times \pi 15^2$
$= 206.06$ square units

Practice

Find the area of the sector.

1.



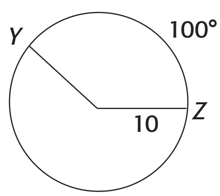
Determine the measure of the arc. The measure of the arc is in degrees. — The measure of arc ______ is ______.

Determine the radius of the circle. — The measure of the radius is ______.

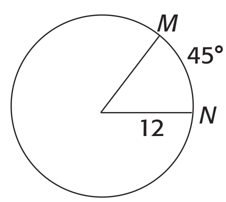
Use the formula:
Area of Sector = $\frac{\text{measure of the arc}}{360} \times \pi r^2$
Use 3.14 for π.

Area $= \frac{\text{measure of the arc}}{360} \times \pi r^2 = \frac{90}{360} \times \pi$______
$= 132.67$ square units

2. ______________



3. ______________



Name ______________________ Date ______________

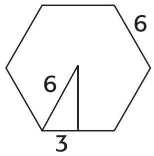
Area of Regular Polygons

To find the area of a regular polygon, you first find the center of the polygon. Then draw a segment from the center to the mid-point of any side; this segment is known as the **apothem**. Then, find the perimeter of the polygon.

Rules for Finding the Area of a Regular Polygon
1. Find the measure of the apothem.
2. Find the perimeter of the polygon.
3. Use the formula $A = \frac{1}{2}aP$ (a = the apothem; P = the perimeter).

Example

Find the area of the hexagon to the right.



Step 1 Find the measure of the apothem.

You can use the Pythagorean Theorem to find the apothem: $a^2 + b^2 = c^2$

$a^2 + 3^2 = 6^2$

$a = 5$

Step 2 Find the perimeter of the polygon.

The perimeter is the sum of the lengths of the sides of a figure.

So, $6 + 6 + 6 + 6 + 6 + 6 = 36$

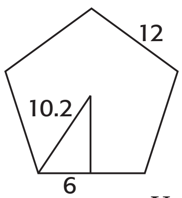
Step 3 $A = \frac{1}{2}aP$
(a = the apothem; P = the perimeter)

$A = \frac{1}{2}aP = \frac{1}{2}(5)(36) = 90$ square units.

Practice

Find the area of each figure.

1.



Find the measure of the apothem.

You can use the Pythagorean Theorem to find the apothem: $a^2 + b^2 = c^2$

$a^2 +$ ________ $=$ ________

$a =$ ________

Find the perimeter of the polygon.

The perimeter is the sum of the lengths of the sides of a figure. The figure has ______ sides.

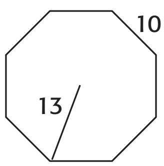
$P =$ ______________ $=$ ________

$A = \frac{1}{2}aP$
(a = the apothem; P = the perimeter)

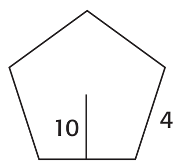
$A = \frac{1}{2}aP = \frac{1}{2}$ ______________

$=$ ______________ square units.

2. ______________



3. ______________

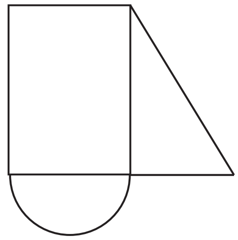


Name ______________________________ Date ____________________

Area of an Irregular Shape

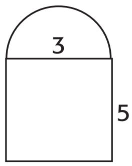
Not all figures are simple geometric shapes. The figure below right is made of a rectangle, triangle and a half circle. The area of the figure is the sum of the areas of the three figures.

Rules for Finding the Area of an Irregular Figure
1. Divide the figure into two or more simple geometric figures.
2. Identify the area formula to use for each figure.
3. Plug the appropriate dimensions into each formula. Solve.
4. Add the individual areas to find the total area.



Example

Find the area.



Step 1 Divide the figure into two or more simple geometric figures.
The figure is made of a half circle and a rectangle.

Step 2 Identify the area formula to use for each figure.
Use the formula for area of a circle and then divide the answer by 2.
Circle: $A = \pi r^2$; Rectangle: $A = lw$

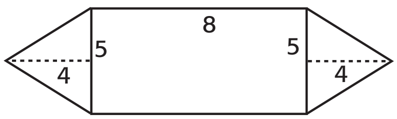
Step 3 Plug the appropriate dimensions into each formula. Solve.
Circle: $r = 3 \div 2 = 1.5$
$A = (3.14)(1.5)^2 = 7.065$ square units
$A_{\text{half circle}} = 3.5325$ square units
Rectangle: $A = lw = (5)(3) = 15$ square units

Step 4 Add the individual areas to find the total area.
$A_{\text{total}} = A_{\text{half circle}} + A_{\text{rectangle}} = 3.5325 + 15$
$= 18.5325$ sq. units

Practice

Find the area.

1.



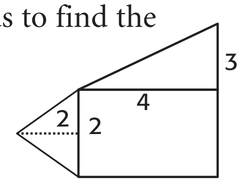
Divide the figure into two or more simple geometric figures.
The figure is divided into two triangles and one ______________.

Identify the area formula to use for each figure.
Triangle: $A =$ _______; Rectangle: $A =$ _______

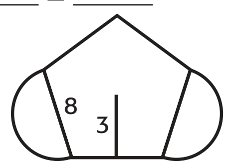
Plug the appropriate dimensions into each formula. Solve.
$A_{\text{triangle}} =$ ________ $=$ ________ square units
$A_{\text{rectangle}} =$ ________ $=$ ________ square units

Add the individual areas to find the total area.
$A_{\text{total}} =$ ______ + ______ + ______ = ______ square units

2. ______________



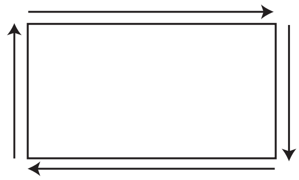
3. ______________



Name ______________________ Date ______________________

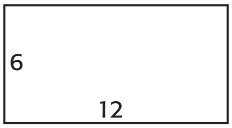
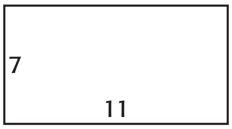
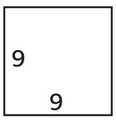
Comparing Area and Perimeter

The **perimeter** of a polygon is the sum of the length of all of its sides. The **area** of a polygon is the number of square units it encloses. For many polygons, you can use formulas to find perimeter or area.


Perimeter

Suppose you have 36 ft. of fencing. You want to make a rectangular play yard with the largest possible area.

Use the table below to explore the dimensions of each rectangle and its area.

Rectangle	Length	Width	Perimeter $2(l + w)$	Area $l \times w$
	____	____	36	____
	____	____	36	____
	____	____	36	____
	____	____	36	____

Use the data in the table to complete the following items.

1. The dimensions of the rectangle that result in the least area are ______________.
2. The dimensions of the rectangle that result in the greatest area are ______________.
3. The length and width of the rectangle with the greatest area are ______________.
4. A four-sided polygon in which the lengths and the sides are equal is a ______________.
5. For every four-sided polygon, a ______________ occupies the greatest area.
6. For rectangles with the same perimeter, as the rectangle approaches a ______________, the area ______________.

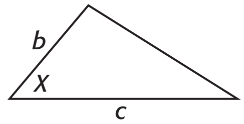
Practice

1. You want to make a play area with an area of 144 square units. Which dimensions will result in the minimum perimeter? ______________
2. You have 160 feet of fencing. You are making a rectangular dog pen. Which dimensions will give you the maximum area? What is that area? ______________.

Name ______________________________ Date ____________________

Using Trigonometry to Find the Area of a Triangle

Suppose you want to find the area of a triangle but you only know the measures of one angle and two sides, as shown in the figure to the right. You can use the formula $A = \frac{1}{2}bc(\sin X)$ to find the area of the triangle.

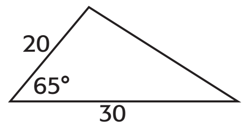


Rules for Using Trigonometry to Find the Area of a Triangle

1. Identify the known measure of an angle and the two sides that include the angle.
2. Plug the numbers into the formula $A = \frac{1}{2}bc(\sin X)$, where b and c are sides, and X is the included angle.
3. Use the sin key on your calculator to find the sine of the angle. Then solve.

Example

Find the area of the triangle to the right.



Step 1 Identify the known measure of an angle and the two sides that include the angle. $b = 20$; $c = 30$; $X = 65°$

Step 2 Plug the numbers into the formula $A = \frac{1}{2}bc(\sin X)$, where b and c are sides, and X is the angle included by sides b and c.

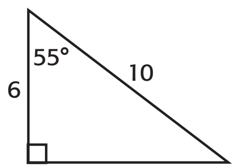
$$A = \frac{1}{2}bc(\sin X)$$
$$= \frac{1}{2}(20)(30)(\sin 65°)$$

Step 3 Find the sine of the angle. Then solve. $A = \frac{1}{2}(20)(30)(0.91) = 272$ square units

Practice

Find the area of each triangle.

1.



Identify the known measure of an angle and the two sides that include the angle. $b =$ ______; $c =$ ______; $X =$ ______

Plug the numbers into the formula $A = \frac{1}{2}bc(\sin X)$, where b and c are sides, and X is the angle included by sides b and c.

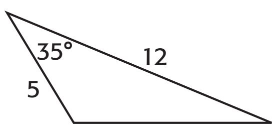
$A = \frac{1}{2}bc(\sin X)$

$= \frac{1}{2}$(______)(______)(sin ______)

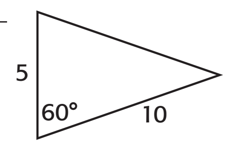
Find the sine of the angle. Then solve. $A = \frac{1}{2}$(______)(______)(______)

$= 24.6$ square units

2. ____________________



3. ____________________



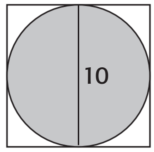
Name ______________________________________ Date ____________________

Geometric Probability

Geometric probability applies the laws of probability by comparing the values of one measure or measurement to a total measure.

Rules for Finding Geometric Probability
1. Find the measure of the favorable outcome.
2. Find the measure of all the outcomes.
3. Use the formula for probability;
   Probability of an Event = $\dfrac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$

Example



A circle is surrounded by a square as shown to the right. What is the probability that a randomly selected spot will not be inside the circle?

Step 1 Find the measure of the favorable outcome.

Subtract the area of the circle from the area of the square.

$A_{\text{square}} = l \times w = 10 \times 10 = 100$

$A_{\text{circle}} = \pi r^2 = (3.14)(5)^2 = 78.5$

$A = 100 - 78.5 = 21.5$ square units

Step 2 Find the measure of all the outcomes.

Total area is the area of the square, 100 square units.

Step 3 Use the formula for probability;

$P = \dfrac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$

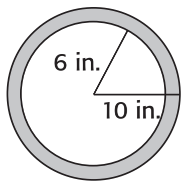
$P = \dfrac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$

$= \dfrac{21.5}{100} \times 100 = 21.5\%$

Practice

Find the probability of a randomly-selected spot being within the shaded region.

1.



Find the measure of the favorable outcome.

________________ the area of the smaller circle from the area of the larger circle.

A = ____________ – ____________

= ____________ – ____________

= ____________ square units

Find the measure of all the outcomes.

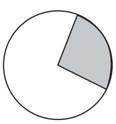
The total area is the area of the ____________ circle.

Use the formula for probability;

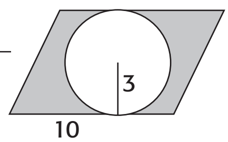
$P = \dfrac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$

P = ________ × 100 = ______%

2. ________



3. ________

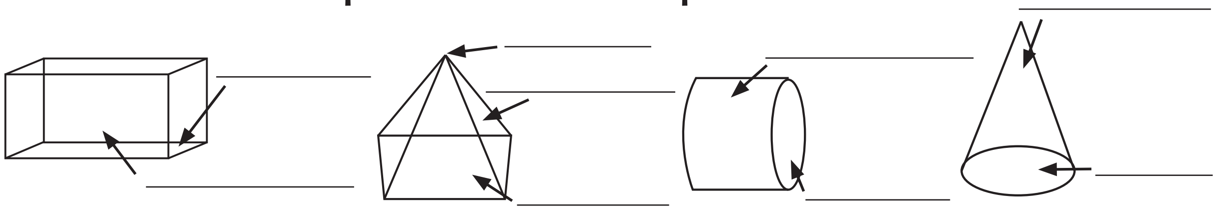


Name ______________________________ Date ________________

Types of Solids

A solid is a three dimensional figure. The parts of a solid do not lie in the same plane. Solids have length, width and height. There are four types of solids. Each is shown below.

Solids have a number of parts. Label each of the parts of a solid.



Prism Pyramid Cylinder Cone

Solids are classified by the number of bases and the nature of their surfaces. Take a closer look at each solid. Complete the chart below.

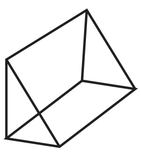
Figure	Base(s)	Lateral Face(s)
Prism	________ parallel, congruent ________	________
Pyramid	________ polygon	________
Cylinder	________ parallel, congruent ________	________ rectangle
Cone	________ circle	________ surface

Prisms are named for the shape of the base. Therefore, if a prism has a pentagon on the base, it is called a pentagonal prism.

Practice

Name each prism

1.



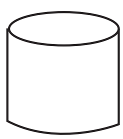
What is the shape of the base? The base is in the shape of a ________.

How many bases are there? ________

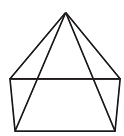
What is the shape of the lateral face? ________

What is its name? ________

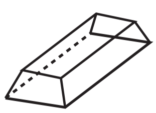
2. ________



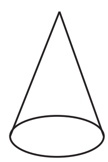
3. ________



4. ________



5. ________



Name ______________________________ Date ______________

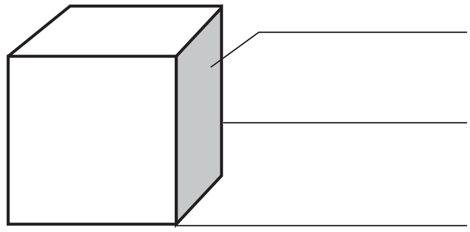
Solids and Euler's Formula

There are several parts to a solid, or polyhedron:

Face: each surface or polygon.

Edge: the segment formed by the intersection of two faces.

Vertex: a point where 3 or more edges intersect, the plural is *vertices.*



The rectangular prism to the right has _____ faces, _____ edges and _____ vertices.

The relationship between the faces, edges and vertices is known as Euler's Formula.

Rules for Using Euler's Formula

1. Identify what you are given:
faces: count the number of polygons
edges: count the number of segments
vertices: count the points where 3 or more segments meet

2. Plug the numbers into Euler's Formula: $V + F = E + 2$

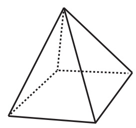
Example

How many vertices are in a pyramid with a square base?

Step 1 Identify what you are given: 1 rectangle and 4 triangles

faces: count the number of polygons — There are 5 faces.

edges: count the number of segments — There are 8 edges.



Step 2 Plug the numbers into Euler's Formula: $V + F = E + 2$

$V + 5 = 8 + 2$

$V = 5$; there are 5 vertices

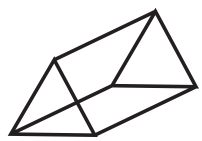
Practice

1. Find the number of edges in a triangular prism.

Identify what you are given: _____ triangles and _____ rectangles

faces: count the number of polygons — There are _____ faces.

vertices: count the points where 3 or more segments meet — There are _____ vertices.



Plug the numbers into Euler's Formula: $V + F = E + 2$

_____ + _____ $= E + 2$

_____ $= E$

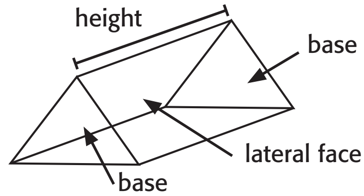
2. Find the number of vertices in a solid with 18 edges and 8 faces. _____

3. Find the number of edges in a solid with 6 faces and 6 vertices. _____

Name ______________________________ Date ____________________

Surface Area: Prisms

A prism is a three-dimensional figure with two congruent, parallel faces, known as **bases**. The height of a prism is a perpendicular segment that joins the bases. Other faces are known as **lateral faces**. The surface area of a prism is the sum of the area of the lateral faces and the area of the two bases.

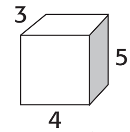


Rules for Finding the Surface Area of a Prism

1. Identify the shape of the base. Use that shape's area formula to find the area of one base. Multiply this result by 2. The final result is the area of the two bases.
2. Find the area of each lateral face. Add the areas of the lateral faces to find the lateral surface area.
3. Add the result from Rule 1 and Rule 2. The result is the surface area of the prism.

Example

Find the surface area of the rectangular prism to the right.



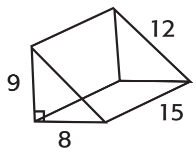
Step 1 Use the shape's area formula to find the area of one base. Multiply this result by 2.
The bases are rectangular: $A = lw$
$A = 4 \times 3 = 12$; $12 \times 2 = 24$

Step 2 Find the area of each lateral face. Add their areas to find the lateral surface area.
There are two 5×3 and two 5×4 rectangular lateral faces. Use the $A = lw$ formula.
$A = 2(5 \times 3) + 2(5 \times 4) = 70$

Step 3 Add the result from Rule 1 and Rule 2 to get the surface area of the prism.
Surface area = area of bases and area of lateral faces. Surface area = 24 + 70 = 94 square units.

Practice

Find the surface area.



1. Use the shape's area formula to find the area of one base. Multiply this result by 2.

The bases of the prism are right triangles. Use the formula $A = \frac{1}{2}bh$.

$A = \frac{1}{2}bh = \frac{1}{2}(\_\_\_\_\_\ \_\_\_\_\_) = \_\_\_\_\_$ square units

_____ × 2 = _____ square units.

Find the area of each lateral face. Add their areas to find the lateral surface area.

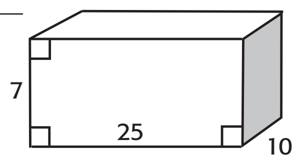
The lateral faces are rectangles. Find the area of each face using $A = lw$.

$A = 8 \times 15 + \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_$ square units

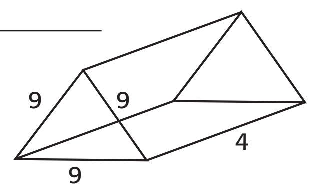
Add the result from Rule 1 and Rule 2 to get the surface area of the prism.

Surface area = _____ + _____ = _____ square units

2. ______________

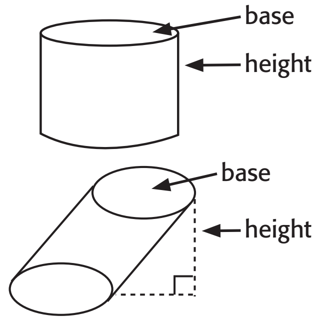


3. ______________



Name ______________________________ Date ______________

Surface Area: Cylinders



A cylinder is a three-dimensional figure with two congruent parallel bases. The **bases** of a cylinder are in the shape of a circle. To find the surface area of a cylinder you need to show an **altitude**. In a cylinder, this altitude is a perpendicular segment that joins the planes of the two bases. The **height** of the cylinder is the length of the altitude. To find the surface area of a cylinder, you need to know the **radius** of the base and the height.

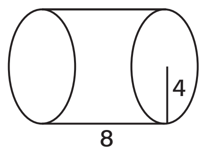
Rules for Finding the Surface Area of a Cylinder

1. Find the radius (r) of the base. Find the height (h).
2. Plug the radius and height into the formula for the surface area of a cylinder: Surface area = $2\pi rh + 2\pi r^2$

In the formula for the surface area of a cylinder, $2\pi rh$ is the lateral surface area of the cylinder, and $2\pi r^2$ is the area of the two bases.

Example

Find the surface area of the cylinder. Use π = 3.14



Step 1 Find the radius (r) of the base. Find the height (h).

radius (r) = 4; height (h) = 8.

Step 2 Plug the radius and height into the formula for the surface area of a cylinder:

Surface area = $2\pi rh \times 2\pi r^2$

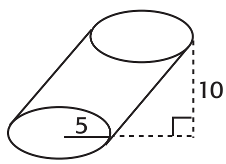
Surface area = $2(3.14)(4)(8) + 2(3.14)(4)^2$

Surface area = 200.96 + 100.48 = 301.44 sq. units

Practice

Find the surface area. Use π = 3.14

1.



Find the radius (r) of the base and the height (h).

radius (r) = _____; height (h) = _____

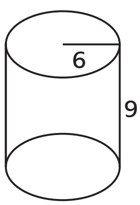
Plug the radius and height into the formula for the surface area of a cylinder:

Surface area = $2\pi rh \times 2\pi r^2$

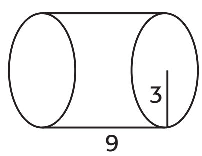
Surface area = 2(3.14)(_____)(_____) + 2(3.14)(_____)^2

Surface area = _____ + _____ = _____ sq. units

2. ____________________



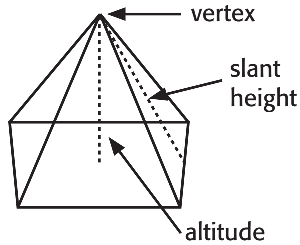
3. ____________________



Name ______________________________ Date ____________________

Surface Area: Pyramids

When you are asked to find the surface area of a pyramid, you may be given the following information:

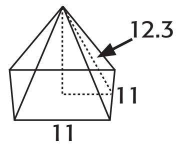


- **area** of the base
- **slant height**: the length of an altitude along the lateral face
- **altitude**: a perpendicular segment from the base to the vertex

Rules for Finding the Surface Area of a Pyramid
1. Use the area formula for the shape of the base to find the area of the base.
2. Find the slant height (l) of one of the lateral faces. Find the perimeter (p) of the base.
3. Surface area = $\frac{1}{2}pl$ + area of base

Example

Find the surface area of the rectangular pyramid.



Step 1 Use the area formula for the shape of the base to find the area of the base.

The base is a rectangle, use the formula for area of a rectangle, $A = lw$

$A = (11)(11) = 121$ square units

Step 2 Find the slant height (l) of one of the lateral faces. Find the perimeter (p) of the base.

The slant height = 12.3

The perimeter of the base is 11 + 11+ 11 + 11 = 44

Step 3 Surface area = $\frac{1}{2}pl$ + area of base

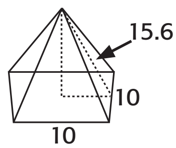
Surface area = $\frac{1}{2}(44)(12.3) + 121$

Surface area = 270.6 + 121 = 391.6 square units

Practice

Find the surface area of each figure.

1.



Use the area formula for the shape of the base to find the area of the base.

The base is a ____________; use the formula A = ______.

A = ______ = (______)(______) = ______ square units

Find the slant height (l) of one of the lateral faces. Find the perimeter (p) of the base.

The slant height (l) = ______

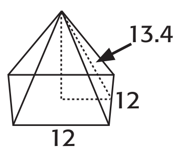
p = 10 + 10 + ______ + ______ = ______.

Surface area = $\frac{1}{2}pl$ + area of base

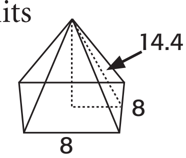
Surface area = $\frac{1}{2}$ ________ + ________

= ________ sq. units

2. ____________________



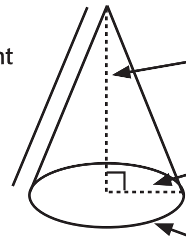
3. ____________________



Name ______________________________ Date ____________________

Surface Area: Cones

A cone is a three-dimensional figure with a circular base and a vertex. The **height** (h) of a cone is the distance between the base and the vertex. The **slant height** (l) is the distance from the vertex to a point on the edge of the base. To find the surface area of a cone, you will need to know the radius (r) of the base and the slant height (l) of the cone.

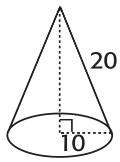


Rules for Finding the Surface Area of a Cone

1. Identify the radius (r) of the base.
2. Identify the slant height (l) of the cone.
3. Use the formula for the surface area of a cone: Surface Area = $\pi rl + \pi r^2$

Example

Find the radius of the cone to the right. Use $\pi = 3.14$



Step 1 Identify the radius (r) of the base. The radius (r) of the base is 10.

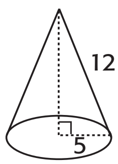
Step 2 Identify the slant height (l) of the cone. The slant height (l) is 20.

Step 3 Use the formula for the surface area of a cone: Surface Area = $\pi rl + \pi r^2$

Surface Area = $(3.14)(10)(20) + (3.14)(10)^2$
= $628 + 314 = 942$ square units

Practice

1. Find the surface area of the cone.



Identify the radius (r) of the base. The radius (r) is ______.

Identify the slant height (l) of the cone. The slant height (l) is ______.

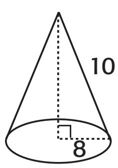
Use the formula for the surface area of a cone: Surface Area = $\pi rl + \pi r^2$

Surface area = (3.14)(______)(______) + (3.14)(______)

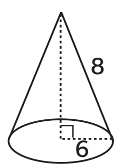
= ________ + ________

= ________ sq. units

2. ____________________

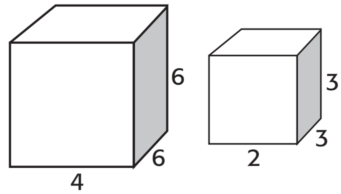


3. ____________________



Name ______________________________ Date ____________________

Surface Area of Similar Solids



Rectangular Prism A Rectangular Prism B

As you know, in similar two-dimensional figures the lengths of corresponding sides are in proportion. Are there relationships between similar solids? What about the ratio of the surface areas?

Use the solids to the right to explore the surface area relationships between similar solids. Complete the chart and the statements that follow.

	Length	Width	Height	Surface Area
Rectanglular Prism A	4	______	6	______
Rectangular Prism B	2	3	______	______

1. Select one of the dimensions from Rectangular Prism A and its corresponding dimension from rectangular Prism B. The ratio of the dimensions is ______ : ______.

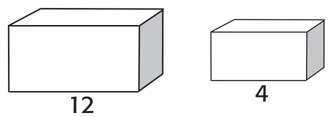
2. The ratio of the surface areas of Rectangular Prism A to Rectangular Prism B is ______ : ______ or ______ : ______.

3. Compare the ratio of corresponding dimensions to the ratio of the surface areas. The ratio of surface areas is the ratio of corresponding dimensions ____________________.

Use the data in the table and the statements to write the rule about the surface areas of similar solids.

> **Surface Area and Volume of Similar Solids**
> If the ratio of corresponding dimensions of two similar solids is $a : b$, then the ratio of their surface areas is a^2 : ______.

Practice



1. Find the surface area of the smaller rectangular prism.

The surface area of the larger solid is 468 cm^2.

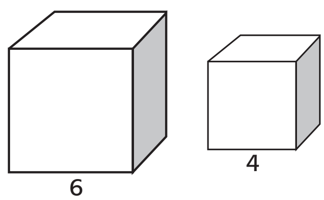
Find the ratio of the lengths. The ratio of the lengths of the larger prism to the smaller prism is 12 : ______ or 3 : ______.

Set up a proportion of the ratio of the known parts of the figures to the known surface area of one cylinder. $\frac{SA_{Large}}{SA_{Small}} = \frac{a^2}{b^2}; = \frac{\_\_\_\_\_\_}{SA_{Small}} = \_\_\_\_\_\_$

Solve using cross products. SA_{Small} ______ = 468 cm^2; SA_{Small} = ______ cm^2

2. Find the surface area of the smaller rectangular prism. The surface area of the larger solid is 216 cm^2.

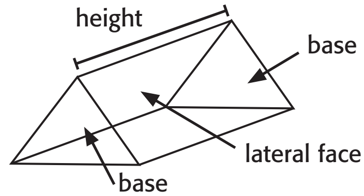
____



Name ______________________________ Date ________________

Volume: Prisms

Volume is the space that a figure occupies. Volume is measured in cubic units, such as in.^{3} (cubic inches) or m^3 (cubic meters). To find the volume of a prism, you will need to know the lengths, width and height of a prism.

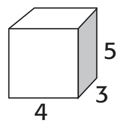


Rules for Finding the Volume of a Prism

1. Identify the shape of the base. Use the area formula for that shape to find the area of the base.
2. Identify the height of the prism.
3. Use the formula $V = Bh$ to find the volume of the prism. B is the area of the base from Rule 1 and h is the height from Rule 2.

Example

Find the volume of the prism.



Step 1 Identify the shape of the base. Use the area formula for that shape to find the area of the base.

The base is a rectangle, $A = lw$.
The length is 4 and the width is 3.
$A = 3 \times 4 = 12$

Step 2 Identify the height of the prism.

The height is 5.

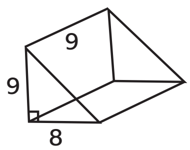
Step 3 Use the formula $V = Bh$. B is the area of the base from Rule 1 and h is the height from Rule 2.

$V = Bh = (12)(5) = 60$ cubic units

Practice

Find the volume of each prism.

1.



Identify the shape of the base. Use the area formula for that shape to find the area of the base.

The base is a right triangle. Use the formula for area of a triangle, $A = \frac{1}{2}bh$.

$A = \frac{1}{2}$______ $\times$ ______ $=$ ______ square units

Identify the height of the prism.

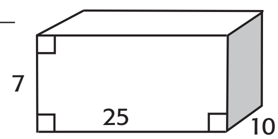
The height of the prism is ______.

Use the formula $V = Bh$. B is the area of the base from Rule 1 and h is the height from Rule 2.

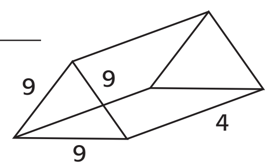
$V = Bh =$ (______)(______)

$=$ ______ cubic units

2. ____________________



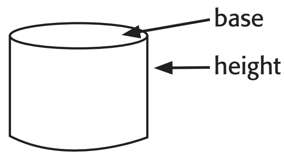
3. ____________________



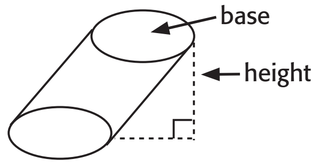
Name ______________________________ Date ____________________

Volume: Cylinders

A cylinder is a three-dimensional figure with two congruent parallel **bases**. The bases of a cylinder are in the shape of a circle. To find the surface area of a cylinder, you need to show an **altitude**. In a cylinder, this altitude is a perpendicular segment that joins the planes of the two bases. The **height** of the cylinder is the length of the altitude.



Volume is the space that a figure occupies. Volume is measured in cubic units, such as in^3 (cubic inches) or m^3 (cubic meters). To find the volume of a cylinder, you will need to know the radius of the base and the height of the cylinder.

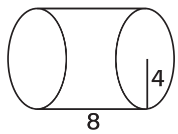


Rules for Finding the Volume of a Cylinder

1. Find the radius (r) of the base.
2. Find the height (h) of the cylinder.
3. Plug the radius and height into the formula for the volume of a cylinder: $V = \pi r^2 h$.

Example

Find the volume of the cylinder. Use $\pi = 3.14$.



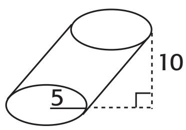
Step 1 Find the radius (r) of the base. The radius (r) of the base is 4.

Step 2 Find the height (h) of the cylinder. The height (h) of the cylinder is 8.

Step 3 Plug the radius and height into the formula for the volume of a cylinder: $V = \pi r^2 h$. $V = (3.14)(4)^2(8) = 401.92$ cubic units

Practice

1. Find the volume. Use $\pi = 3.14$.



Find the radius (r) of the base. The radius (r) is ______.

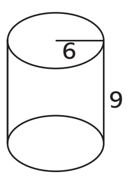
Find the height (h) of the cylinder. The height (h) is ______.

Plug the radius and height into the formula for the volume of a cylinder: $V = \pi r^2 h$.

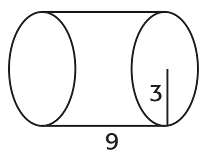
$V = (3.14)(\_\_\_\_\_)^2\ (\_\_\_\_\_)$

$= \_\_\_\_\_\_$ cubic units

2. ____________________



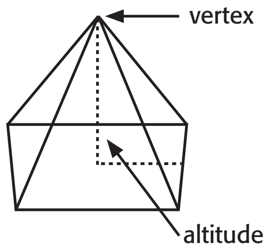
3. ____________________



Name ______________________________ Date ______________________

Volume: Pyramids

When you are asked to find the volume of a pyramid, you may be given the following information:

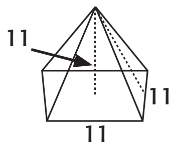


- **area** of the base
- **slant height**: the length of an altitude along the lateral face
- **altitude**: a perpendicular segment from the base to the vertex

> **Rules for Finding the Volume of a Pyramid**
> **1.** Use the area formula for the shape base to find the area of the base.
> **2.** Identify the height (or altitude) of the pyramid.
> **3.** Use the formula $V = \frac{1}{3}Bh$ to find the volume of the pyramid. B is the area of the base from Rule 1 and h is the height from Rule 2.

Example

Find the volume of the pyramid.



Step 1 Use the area formula for the shape base to find the area of the base. The base is a rectangle; $A = lw$

$A = lw = (11)(11) = 121$ square units

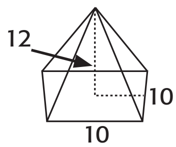
Step 2 Identify the height of the pyramid. The height is (11).

Step 3 Use the formula $V = \frac{1}{3}Bh$. $V = \frac{1}{3}(121)(11) = 443.7$ cubic units

Practice

Find the volume of the pyramid.

1.



Use the area formula for the shape base to find the area of the base. The base is a ____________;

use $A =$ ______.

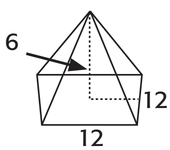
$A =$ ______ $=$ (______)(______)

$=$ ______ square units

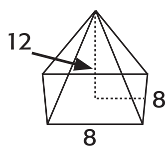
Identify the height of the pyramid. Height = ______

Use the formula $V = \frac{1}{3}Bh$. $V = \frac{1}{3}$(______)(______) = ________ cubic units

2. ____________________



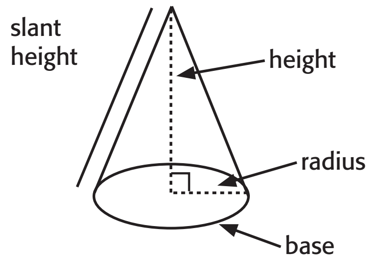
3. ____________________



Name ______________________________ Date ____________________

Volume: Cones

The **height** (h) of a cone is the distance between the base and the vertex. The **slant height** (l) is the distance from the vertex to a point on the edge of the base. Volume is the space that a figure occupies. Volume is measured in cubic units such as in.^{3} (cubic inches) or m^3 (cubic meters). To find the volume of a cone, you will need to know the radius of the base and the height of the cylinder.

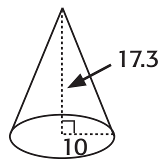


Rules for Finding the Volume of a Cone

1. Find the radius (r) of the base.
2. Find the height (h) of the cone.
3. Plug the radius and height into the formula for the volume of a cone: $V = \frac{1}{3}\pi r^2 h$.

Example

Find the volume of the cone. Use $\pi = 3.14$



Step 1 Find the radius (r) of the base. The radius (r) is 10.

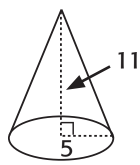
Step 2 Find the height (h) of the cone. The height (h) is 17.3.

Step 3 Plug the radius and height into the formula for the volume of a cone: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}(3.14)(10)^2\ (17.3) = 1810$ cubic units

Practice

1. Find the volume of the cone. Use $\pi = 3.14$



Find the radius (r) of the base. The radius (r) of the base is ______.

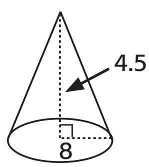
Find the height (h) of the cone. The height (h) is ______.

Plug the radius and height into the formula for the volume of a cone: $V = \frac{1}{3}\pi r^2 h$

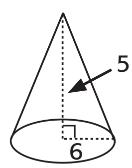
$V = \frac{1}{3}(3.14)(\_\_\_\_\_\_)^2\ (\_\_\_\_\_\_)$

$= \_\_\_\_\_\_$ cubic units

2. ____________________



3. ____________________



Name ______________________________ Date ______________________

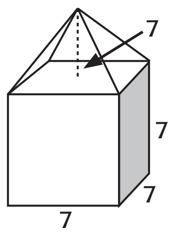
Volume of an Irregular Shape

An irregular, or complex figure is made of two or more basic solids. The total volume of an irregular solid is the sum of the volume of the basic solids.

> **Rules for Finding the Volume of an Irregular Solid**
> **1.** Divide the figure into two or more basic solids.
> **2.** Use the volume formula to use for each figure.
> **3.** Plug the appropriate dimensions into each formula.
> **4.** Add the individual volumes to find the total volume.

Example

Find the volume.



Step 1 Divide the figure into two or more basic solids.
The figure is a combination of a pyramid and a prism.

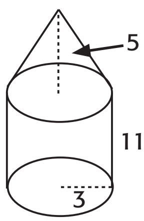
Step 2 Identify the volume formula to use for each figure.
Pyramid: $V = \frac{1}{3}Bh$; Prism: $V = Bh$

Step 3 Plug the appropriate dimensions into each formula.
$V_{pyramid} = \frac{1}{3}Bh = \frac{1}{3}(7 \times 7)(7) = 114 \text{ units}^3$
$V_{prism} = Bh = (7 \times 7)(7) = 343 \text{ units}^3$

Step 4 Add the individual volumes to find the total volume.
$V_{total} = V_{prism} + V_{pyramid} = 343 + 114$
$= 457 \text{ units}^3$

Practice

1. Find the volume.



Divide the figure into two or more basic solids.
The figure is made of a ______________ and a cylinder.

Identify the volume formula to use for each figure.
______________: $V =$ ______________

Cylinder: $V =$ ______________

Plug the appropriate dimensions into each formula.
$V_{cone} =$ ____________________

$V_{cylinder} =$ ____________________

Add the individual volumes to find the total volume.
$V_{total} = V$______________ $+ V_{cylinder}$

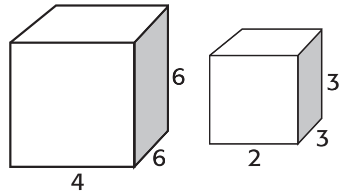
$V_{total} =$ ______________ + ______________

$=$ ______________

Name ______________________ Date ______________

Volume of Similar Solids

As you know, in similar two-dimensional figures, the length of corresponding sides are in proportion. Are there relationships between similar solids? What about the ratio between the volumes of similar figures?



Rectangular Prism A

Rectangular Prism B

Use the solids to the right to explore the surface area and volume relationships between similar solids. Complete the chart and the statements that follow.

	Length	Width	Height	Volume
Rectangular Prism A	______	______	______	______
Rectangular Prism B	______	______	______	______

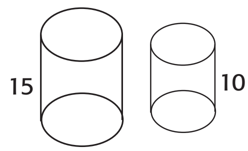
1. Select one of the dimensions from Rectangular Prism A and its corresponding dimension from rectangular Prism B. The ratio of the dimensions is ______ : ______.

2. The ratio of the volume of Rectangular Prism A to Rectangular Prism B is ______ : ______ or ______ : ______.

3. Compare the ratio of corresponding dimensions to the ratio of the volumes. The ratio of the volumes is the ratio of the corresponding dimensions ____________.

Use the data in the table and the completed statements to write the rule about the volumes of similar solids.

> **Volume of Similar Solids**
> If the ratio of corresponding dimensions of two similar solids is $a : b$, then the ratio of their volumes is ______ $: b^3$.



Practice

1. Find the missing volume in each set of similar solids. Find the volume of the larger solid. The volume of the smaller cylinder is 1130 cm^3.

Find the ratio of the heights.

The ratio of the heights of the larger cylinder to the smaller cylinder is ______ : ______ or ______ : ______.

Set up a proportion of the ratio of the known parts of the figures to the known volume of one cylinder.

$$\frac{V_{\text{Large}}}{V_{\text{Small}}} = \frac{a^3}{b^3}$$

$$\frac{V_{\text{Large}}}{\rule{2cm}{0.4pt}} = \rule{1.5cm}{0.4pt}$$

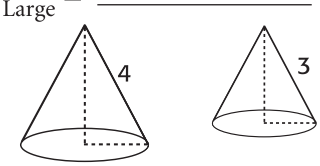
Solve using cross products.

______ $V_{\text{Large}} = 30510$ cm^3;

$V_{\text{Large}} =$ ____________

2. Find the volume of the smaller cone. The volume of the larger cone is 67 in.^{3}.

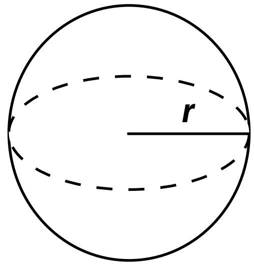
____



Name ____________________ Date ____________________

Surface Area: Spheres

A sphere is a three-dimensional figure. It is the set of all points in space that are the same distance from a point known as the **center**. Like a circle, a sphere has a **radius**—a segment with one endpoint at the center and the other endpoint on the sphere. Additionally, a sphere has a **diameter**—a segment passing through the center with endpoints on the sphere.

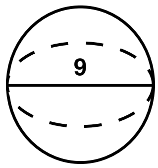


To find the surface area of a sphere, you need to know the radius (r) of the sphere. In a sphere, just like with a circle, the diameter is twice the radius, or, the radius is half the diameter of the sphere.

> **Rules for Finding the Surface Area of a Sphere**
> 1. Find the radius of the sphere. If you know the diameter, the radius is the diameter divided by 2. ($r = d \div 2$)
> 2. Use the formula for surface area of a sphere: Surface area = $4\pi r^2$. Use $\pi = 3.14$.

Example

Find the surface area of the sphere.



Step 1 Find the radius of the sphere. If you know the diameter, the radius is the diameter divided by 2. ($r = d \div 2$)

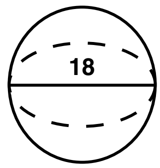
The diameter is given; $d = 9$
The radius is the diameter divided by 2. The radius (r) is 4.5.

Step 2 Use the formula for surface area of a sphere: Surface area = $4\pi r^2$. Use $\pi = 3.14$.

Surface area = $(4)(3.14)(4.5)^2$
= 254.34 square units

Practice

1. Find the surface area of each sphere. Use $\pi = 3.14$.



Find the radius of the sphere. If you know the diameter, the radius is the diameter divided by 2. ($r = d \div 2$)

The ____________________ is given, its measure is ______.

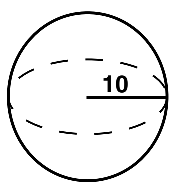
The radius (r) is ______.

Use the formula for surface area of a sphere: Surface area = $4\pi r^2$. Use $\pi = 3.14$.

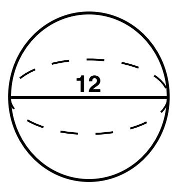
Surface area = 4(3.14)(______)^2 =

______________ square units

2. ____________________

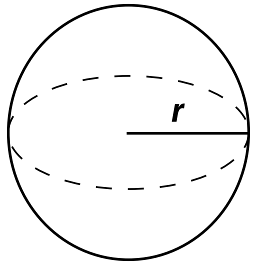


3. ____________________



Name ______________________________ Date ____________________

Volume: Spheres



A sphere is a three-dimensional figure. It is the set of all points in space that are the same distance from a point known as the **center**. Like a circle, a sphere has a **radius**—a segment with one endpoint at the center and the other endpoint on the sphere. Additionally, a sphere has a **diameter**—a segment passing through the center with endpoints on the sphere.

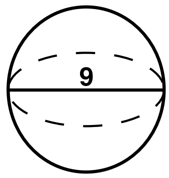
To find the volume of a sphere, you need to know the radius of the sphere. In a sphere, just like in a circle, the diameter is twice the radius. Or, the radius is half the diameter of the sphere.

Rules for Finding the Volume of a Sphere

1. Find the radius of the sphere. If you know the diameter, the radius is the diameter divided by 2. ($r = d \div 2$)
2. Use the formula for volume of a sphere: $V = \frac{4}{3}\pi r^3$. Use $\pi = 3.14$.

Example

Find the volume of the sphere. Use π = 3.14.



Step 1 Find the radius of the sphere. If you know the diameter, the radius is the diameter divided by 2. ($r = d \div 2$)

The diameter is given; $d = 9$

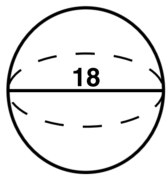
The radius is the diameter divided by 2. The radius is 4.5

Step 2 Use the formula for volume of a sphere: $V = \frac{4}{3}\pi r^3$. Use $\pi = 3.14$. The radius is r.

$V = \frac{4}{3}(3.14)(4.5)^3 = 381.51$ cubic units

Practice

1. Find the surface area of each sphere. Use $\pi = 3.14$.



Find the radius of the sphere. If you know the diameter, the radius is the diameter divided by 2. ($r = d \div 2$)

The ____________ is given.

Its measure is ______.

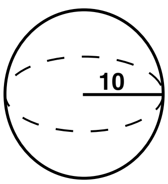
The radius, r, is ______.

Use the formula for volume of a sphere: $V = \frac{4}{3}\pi r^3$. Use $\pi = 3.14$.

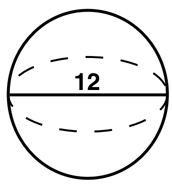
$V = \frac{4}{3}(3.14)(\_\_\_\_\_)^3$

$=$ ____________ cubic units

2. ____________



3. ____________

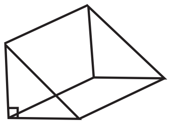
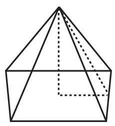
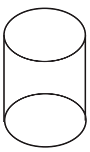
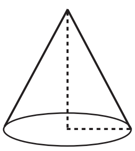
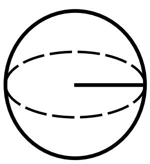


Name ____________________ Date ____________

Surface Area and Volume Formulas

The surface area of a solid is the sum of the areas of all the faces and bases of a solid. Surface area is measured in square units such as in.^{2} (square inches) or m^2 (square meters). Volume is the space that a figure occupies. Volume is measured in cubic units, such as in.^{3} (cubic inches) or m^3 (cubic meters).

The chart below will help you remember the types of solids, their properties, and their surface area and volume formulas.

Type	Description	Surface Area	Volume
 Prism	________ parallel, congruent ____________ form the bases. Lateral faces are ________________.	2(area of one ________) + (Sum of areas of ________________)	____________
 Pyramid	Has only ________ base in the shape of a polygon. Lateral faces are ________________.	________________	____________
 Cylinder	________ parallel, congruent ____________ form the bases. Lateral face is ____________ rectangle.	________________	____________
 Cone	Has only ________ base in the shape of a circle. Lateral face is ____________ surface.	________________	____________
 Sphere	It is the set of all points in space that are the ____________ distance from a point known as the ____________.	________________	____________

Name ______________________________ Date ____________________

Plotting Points on a Coordinate Plane

A point on a coordinate plane is defined by its *x*- and *y*-coordinates. The location of a point is given by an ordered pair.

Sign of Coordinate	*x*	*y*
+	right	up
–	left	down

Ordered Pair

(x, y)

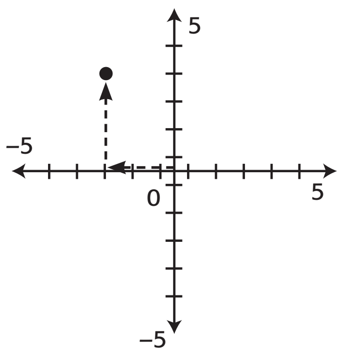
(x-coordinate, y-coordinate)

Rules for Plotting Points on a Coordinate Plane

1. Move right or left from the *y*-axis the number of units of the *x*-coordinate.
2. Move up or down from the *x*-axis the number of units of the *y*-coordinate.

Example

Graph the following point: (–3, 4).



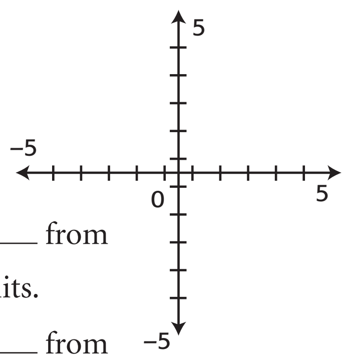
Step 1 Move right or left from the *y*-axis the number of units of the *x*-coordinate. Move left from the *y*-axis 3 units.

Step 2 Move up or down from the *x*-axis the number of units of the *y*-coordinate. Move up from the *x*-axis 4 units.

Practice

Graph the following points.

1. (2, –3)



Move right or left from the *y*-axis the number of units of the *x*-coordinate. Move ______________ from the *y*-axis ______ units.

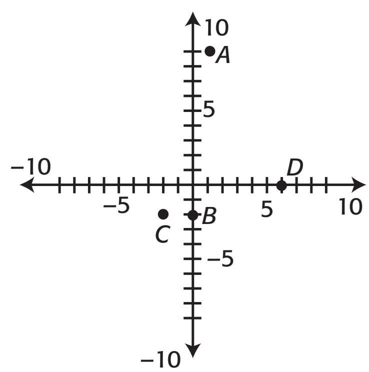
Move up or down from the *x*-axis the number of units of the *y*-coordinate. Move ______________ from the *x*-axis ______ units.

2. (–5, 0)

3. (5, 3)

4. (0, 4)

Give the coordinates for each point.



5. *A* ______________________

6. *B* ______________________

7. *C* ______________________

8. *D* ______________________

Name ______________________________ Date ______________________

Graphing a Linear Equation

When you find the solution of an equation, you are finding two values, one for *x* and one for *y*, that make the equation true. Each set of values is known as an ordered pair. You can use the ordered pairs to plot points on a coordinate plane. If the solution (ordered pairs) makes a line, then you have a **linear equation**.

> **Rules for Graphing a Linear Equation**
> 1. Create an input/output table.
> 2. Select several values for *x*.
> 3. Substitute the values for *x* into the equation. Solve for *y*.
> 4. Plot each solution on the coordinate plane. Draw a line so it goes through each point.

Example

Graph the following equation: $2x + 3 = y$.

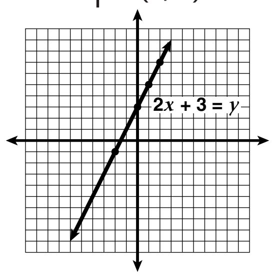
Step 1 Create an input/output table.

Step 2 Select several values for x.

Step 3 Substitute each value of *x* into the equation. Solve the equation for *y*.

Step 4 Plot each solution on a coordinate plane. Draw a line so it goes through each point.

x	$2x + 3 = y$	y	(x, y)
−2	$2(-2) + 3 = y$	−1	(−2, −1)
0	$2(0) + 3 = y$	3	(0, 3)
1	$2(1) + 3 = y$	5	(1, 5)
2	$2(2) + 3 = y$	7	(2, 7)



Practice

Graph the following equations.

1. $y = 2x + 6$

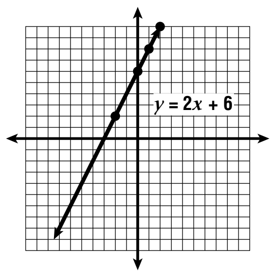
   Create an input/output table.

   Select several values for *x*.

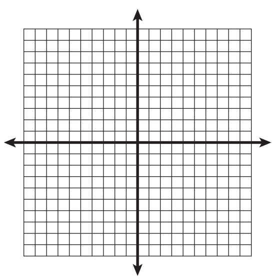
   Substitute each value of *x* into the equation. Solve the equation for *y*.

   Plot each solution on a coordinate plane. Draw a line so it goes through each point.

x	$2x + 6 = y$	y	(x, y)
2	$2(2) + 6 = y$	______	(2, 10)
0	$2(0) + 6 = y$	______	(0, 6)
______	$2(1) + 6 = y$	______	(1, 8)
______	$2(-2) + 6 = y$	______	(−2, 2)

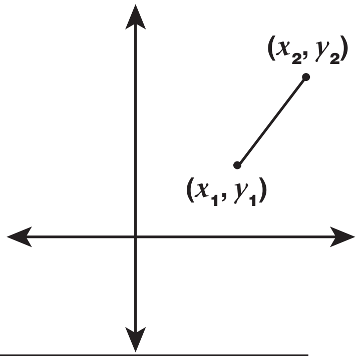


2. $y = 3x + 1$ ______________________
3. $y = 3 + 2x$ ______________________
4. $y = 5x$ ______________________
5. $2x + 2y = 6$ ______________________
6. $y = \frac{1}{2}x + 5$ ______________________



Name ______________________________ Date ________________

Distance Formula



The distance between two points in a coordinate plane can be found if the points lie on a horizontal or vertical line. If the two points are not on a horizontal or vertical line, you can use the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Rules for Finding a Distance Between Two Points Using the Distance Formula

1. Identify the coordinates of one point–make these coordinates x_1 and y_1.
2. Identify the coordinates of the other point–make these coordinates x_2 and y_2.
3. Plug the coordinates into the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example

Find the distance between *A* (4, 1) and *B* (–3 , –4).

Step 1 Identify the coordinates of one point —make these coordinates x_1 and y_1.

Make point *A* your first point.
The coordinates of *A* are (4, 1); so, $x_1 = 4$ and $y_1 = 1$.

Step 2 Identify the coordinates of the other point—make these coordinates x_2 and y_2.

Make point *B* your second point.
The coordinates of *B* are (–3, –4); so, $x_2 = -3$ and $y_2 = -4$.

Step 3 Plug the coordinates into the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(-3 - 4)^2 + (-4 - 1)^2}$
$d = \sqrt{(-7)^2 + (-5)^2} = \sqrt{74} = 8.6$

Practice

Find the distance between each pair of points.

1. *M* (–2, –1) and *N* (4, 2)

Identify the coordinates of one point —make these coordinates x_1 and y_1.

Make point *M* your first point.
The coordinates of *M* are (–2,–1);
so $x_1 =$ ______ and $y_1 =$ ______.

Identify the coordinates of the other point—make these coordinates x_2 and y_2.

Make point *N* your second point.
The coordinates of *N* are (4, 2);
so $x_2 =$ ______ and $y_2 =$ ______

Plug the coordinates into the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(\quad - \quad)^2 + (\quad - \quad)^2}$

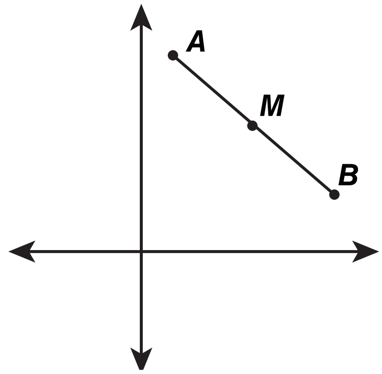
$d = \sqrt{(\_\_\_\_\_)^2 + (\_\_\_\_\_)^2} =$ ______ $=$ ______

2. *R* (9, 8) and S (–3, –6) **3.** *D* (–7, 2) and E (0, –2) **4.** *A* (3, –2) and B (5, –9)

Name ____ Date __________________________

Midpoint Formula

As you know, you can find the midpoint of a segment on a number line by finding the mean of the coordinates of the endpoints. Put another way, you add the coordinates and divide by 2. To find the midpoint of a segment on a coordinate plane, you find the average of the *x*-coordinates and the average of the *y*-coordinates.



Rules for Finding the Coordinates of the Midpoint of a Segment

1. Identify the coordinates of one of the points–make these coordinates x_1 and y_1.

2. Identify the coordinates of the other point–make these coordinates x_2 and y_2.

3. Plug the numbers into the midpoint formula: $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

Example

$\overline{AB}$ has endpoints *A* (4, 1) and *B* (8, 3). Find the midpoint.

Step 1 Identify the coordinates of one of the points–make these coordinates x_1 and y_1.

Make point A your first point. The coordinates of *A* are (4, 1); so, $x_1 = 4$ and $y_1 = 1$.

Step 2 Identify the coordinates of the other point–make these coordinates x_2 and y_2.

Make point B your second point. The coordinates of *B* are (8, 3); so $x_2 = 8$ and $y_2 = 3$.

Step 3 Plug the numbers into the midpoint formula: $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$\left(\frac{8 + 4}{2}, \frac{3 + 1}{2}\right) = \left(\frac{12}{2}, \frac{4}{2}\right) = (6,2)$

Practice

Find the midpoint of each segment with the endpoints given.

1. *M* (–2, –4) and *N* (4, 2)

Identify the coordinates of one of the points–make these coordinates x_1 and y_1.

Make point *M* your first point.
The coordinates of *M* are (–2,–4);

so, x_1 = ______ and y_1 = ______.

Identify the coordinates of the other point–make these coordinates x_2 and y_2.

Make *N* your second point.
The coordinates of *N* are (4, 2); so,

x_2 = ______ and y_2 = ______.

Plug the numbers into the midpoint formula: $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$\left(\frac{\_\_\_ + (-2)}{2}, \frac{\_\_\_ + (-4)}{2}\right) = \left(\frac{\_\_}{2}, \frac{\_\_}{2}\right) =$ ______.

2. *R* (9, 8) and *S* (–3, –6) ________________

3. *D* (–7, 2) and *E* (0, –2) ________________

4. *A* (3, –2) and *B* (5, –9) ________________

5. *Y* (4, 0) and *Z* (4, –6) ________________

Name ______________________________ Date ____________________

Slope of a Line

If you look at the graph of a linear equation, you will see it forms a straight line. You may have noticed that most lines have a "slant" to them. The slope of a line is a measure of the steepness of a line.

The slope of a line is the ratio of the vertical change (the number of units of change along the y-axis) to horizontal change (the number of units of change along the x-axis). To find the slope of a line, you pick any two points on the line. Find the difference between the y-coordinates, and then, find the difference between the x-coordinates.

Suppose a line passes through two points, for example (2, 3) and (4, 2). You make one set of coordinates (x_1, y_1), and the other set, (x_2, y_2).

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

Find the slope of a line that passes through (5, 2) and (3, 8).

Step 1 Make one set of coordinates (x_1, y_1) and the other set, (x_2, y_2). (x_1, y_1) (5, 2) (x_2, y_2) (3, 8)

Step 2 Using the equation for slope, place the numbers into the formula.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{3-5} = \frac{6}{-2} = -3$$

Step 3 Solve. The slope is −3.

Practice

Find the slope of the line passing through each set of points.

1. (1, 2) and (4, 5)

Make one set of coordinates (x_1, y_1) and the other set, (x_2, y_2) (x_1, y_1) (1, 2) (x_2, y_2) ______

Using the equation for slope, place the numbers into the formula.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\quad - 2}{\quad - 1} = \frac{\quad}{\quad} = \underline{\qquad}$$

Solve. The slope is ______.

2. (2, 3) and (4, 6) ______________

3. (−2, 2) and (0, 4) ______________

4. (4, 3) and (−1, 1) ______________

5. (−1, 5) and (4, 2) ______________

6. (3, 1) and (6, 3) ______________

7. (4, 4) and (−1, −2) ______________

Name ______________________________ Date ____________________

Slope Intercept Form

Looking at an equation can tell you certain pieces of information about the graph of that equation. An equation written with y isolated on one side of the equal sign and x on the other side of the equation is in **slope–intercept form**. An equation in slope-intercept form is written as:

y-intercept

$$y = \mathrm{m}x + \mathrm{b}$$

slope

The y-intercept is the point on the y-axis through which the line passes.

Example

Find the slope and the *y*-intercept of the line $y = -2x + 4$.

Step 1 Find the number in front of the x-term. This is the slope.

$y = \mathrm{m}x + \mathrm{b}$

$y = -2x + 4$

$\mathrm{m} = \text{slope} = -2$

Step 2 Find the term without a variable: the y-coordinate of where the line crosses the y-axis.

$y = \mathrm{m}x + \mathrm{b}$

$y = -2x + 4$

$\mathrm{b} = y\text{–intercept} = 4$

Practice

Find the slope and y-intercept for each line.

1. $y = \frac{1}{2}x + 10$

Find the number in front of the x-term. This is the slope.

$y = \mathrm{m}x + \mathrm{b}$

$y = \frac{1}{2}x + 10$

$\mathrm{m} = \text{slope} =$ ______

Be sure to include the negative if necessary.

Find the term without a variable: the y-coordinate of where the line crosses the y-axis.

$y = \mathrm{m}x + \mathrm{b}$

$y = \frac{1}{2}x + 10$

$\mathrm{b} = y\text{–intercept} =$ ______

2. $y = x + 3$ ____________________

3. $y = -\frac{3}{4}x - 6$ ____________________

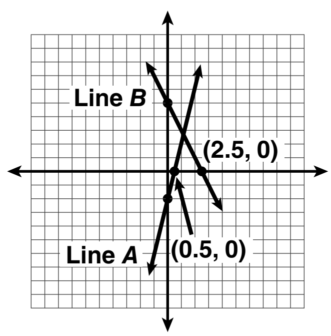
4. $2y = x + 4$ ____________________

5. $3y = -2x - 9$ ____________________

Use the graphs below to write equations in slope-intercept form.

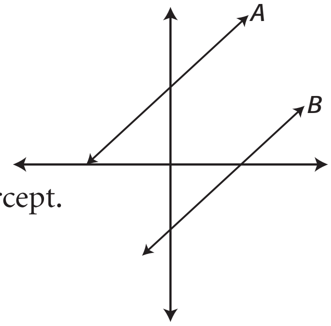
6. Line A ____________________

7. Line B ____________________



Name ______________________________ Date ____________________

Parallel Lines



Parallel lines are lines in the same plane that do not intersect.

The equation of line A is $y = 2x + 3$

The equation of line B is $y = 2x - 1$

As you can see, both lines have the same slope, but a different y–intercept.

$y = mx + b$	m = slope	b = y-intercept
$y = 2x + 3$	2	3
$y = 2x - 1$	2	-1

Rules for Parallel Lines
1. Write all equations in slope-intercept form.
2. Identify the slope of each line.
3. If the slopes are equal, the lines are parallel.

Example

Are the graphs of $y = \frac{1}{2}x + 4$ and $6y - 3x = 6$ parallel?

Step 1 Write all equations in slope-intercept form.
$y = \frac{1}{2}x + 4$ *is* in slope-intercept form.
$6y - 3x = 6 \rightarrow y = \frac{3}{6}x + 1 = \frac{1}{2}x + 1$

Step 2 Identify the slope of each line.
$y = \frac{1}{2}x + 4$; slope $= \frac{1}{2}$
$y = \frac{1}{2}x + 1$; slope $= \frac{1}{2}$

Step 3 If the slopes are equal, the lines are parallel.
The slopes are equal, so the lines are parallel.

Practice

For each set of equations, determine if graphs of the equations are parallel.

1. $y = 3x + 12$ and $6y = -3x - 6$

Write all equations in slope-intercept form.
$y = 3x + 12$ *is* in slope-intercept form.
$6y = -3x - 6$ *is not* in slope-intercept form.

$6y = -3x - 6 \rightarrow y =$ ________

Identify the slope of each line.
$y = 3x + 12$; m = ________

$6y = -3x - 6$; m = ________

If the slopes are equal, the lines are parallel.
The slopes ____________ equal.

The lines ____________ parallel.

2. $y = -\frac{1}{4}x + 5$ and $12y + 3x = 24$ ______________________________

3. $8x + 4y = 8$ and $y = -2x + 4$ ______________________________

4. $y = 2x + 6$ and $-2x + 2y = 12$ ______________________________

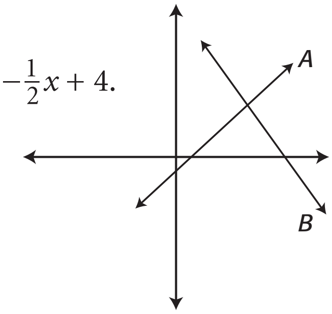
5. $y = -\frac{1}{4}x + 12$ and $8x + 6y = 9$ ______________________________

Name ______________________ Date ______________

Perpendicular Lines

Perpendicular lines are lines that intersect to form right angles.

The equation of line A is $y = 2x - 1$. The equation of line B is $y = -\frac{1}{2}x + 4$.



As you can see, the slope of one line is the opposite (negative) reciprocal of the other line.

$y = \mathrm{m}x + \mathrm{b}$	m = slope	b= y–intercept
$y = 2x - 1$	2	−1
$y = -\frac{1}{2}x + 4$	$-\frac{1}{2}$	4

Rules for Writing the Equation of a Perpendicular Line

1. Identify the slope of the known line.
2. Write the reciprocal of the slope. This is the slope of the perpendicular line.
3. Give the new slope a sign opposite to the slope of the first line.
4. Use the slope-intercept form to create the equation of a line perpendicular to the given line.

Example

Write an equation of the line that has a *y*-intercept of 2 and is perpendicular to $y = 3x + 5$.

Step 1 Identify the slope of the known line. $y = 3x + 5$; slope = m = 3

Step 2 Write the reciprocal of the slope. This is the slope of the perpendicular line. m = 3, the reciprocal is $\frac{1}{3}$.

Step 3 Give the new slope a sign opposite to the slope of the first line. The slope of the given line is positive; the perpendicular slope is negative: $-\frac{1}{3}$

Step 4 Use the slope-intercept form to create the equation of a line perpendicular to the given line. $y = \mathrm{m}x + b =$ _____ $x +$ _____

Practice

Write an equation of the line that has the given *y*-intercept and is perpendicular to the given equation.

1. $y = -\frac{1}{2}x + 2$; new y-intercept: −3

Step 1 $y = -\frac{1}{2}x + 2$; slope = m = $-\frac{1}{2}$

Step 2 m = $-\frac{1}{2}$; the reciprocal of $-\frac{1}{2}$ is _____

Step 3 The slope of the given line is ______________; the perpendicular slope is ______________ : _____

Step 4 $y = \mathrm{m}x + b =$ _____ $x +$ _____

2. $y = \frac{3}{4}x + 5$; new y-intercept: 4 ______________

3. $2y = 4x + 2$; new y-intercept: 3 ______________

4. $y = -4x + 2$; new y-intercept: −5 ______________

5. $y = x + 7$; new y-intercept: 1 ______________

Name ______________________________ Date ______________

Point-Slope Form I

There are instances in which you are given the slope and an ordered pair.
For example, you may know that the slope of a line is −2 and the graph of the equation passes through (−2, 1).

You can use the **point-slope form** of a linear equation to write an equation of the line.

slope

Point–slope form: $y - y_1 = \mathrm{m}(x - x_1)$

y-coordinate x-coordinate

Rules for Using the Point-Slope Form
1. Identify the slope, m.
2. From the ordered pair, identify the x-coordinate and the y-coordinate.
3. Use the point–slope form to write the equation: $y - y_1 = \mathrm{m}(x - x_1)$

Example

Write the equation of the line that has a slope of 3 and passes through the point (2, 5).

Step 1 Identify the slope. The slope (m) is 3.

Step 2 From the ordered pair, identify the x-coordinate and the y-coordinate. The ordered pair is (2, 5)
The x-coordinate is 2; the y-coordinate is 5.

Step 3 Use the point-slope form to write the equation.
$y - y_1 = \mathrm{m}(x - x_1)$
$y - 5 = 3\,(x - 2)$

Practice

Write the equation of the line.

1. Slope = 6; point is (−3, −1)

   Identify the slope (m). The slope is ________.

   From the ordered pair, identify the x-coordinate and the y-coordinate. The ordered pair is ________.
   The x-coordinate is ________; the y-coordinate is ________ .

   Use the point-slope form to write the equation. $y - y_1 = \mathrm{m}(x - x_1)$

   ____

2. slope = $-\frac{1}{2}$, (7, 1) ______________________________
3. slope = 2 , (−3 , −3) ______________________________
4. slope = $\frac{2}{3}$, (4, −5) ______________________________
5. slope = −3 , (−1, 3) ______________________________

Name ______________________ Date ______________

Point-Slope Form II

When you are given the slope of a line and an ordered pair identifying a point on the graph of the line, you can use the point-slope form. You can also use the point-slope form when given two ordered pairs. To use the two ordered pairs, you will need to first use the ordered pairs to find the slope.

Rules for Using Point-Slope Form Using Two Points

1. Use the formula for slope (slope = $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_2}$) to find the slope.
2. Use one set of ordered pairs for the x-coordinate and y-coordinate.
3. Use point-slope form to write the equation.

Example

Write the equation of the line that passes through (–3, –3) and (1, 5).

Step 1 Use the formula for slope ($\frac{y_2 - y_1}{x_2 - x_1}$) to find the slope.

Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{1 - (-3)} = \frac{8}{4} = 2$

Step 2 Use one set of ordered pairs for the x-coordinate and the y-coordinate.

Use the ordered pair (1, 5).
The x-coordinate is 1; the y-coordinate is 5.

Step 3 Use point-slope form to write the equation.

$y - y_1 = \text{m}(x - x_1)$
$y - 5 = 2(x - 1)$

Practice

Use the point-slope form to write an equation.

1. (−2, −2) , (0, −4)

Use the formula for slope ($\frac{y_2 - y_1}{x_2 - x_1}$) to find the slope.

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$ = ________ = — = ______

Use one set of ordered pairs for the x-coordinate and the y-coordinate.

Use the ordered pair (−2, −2).

The x-coordinate is ______;

the y-coordinate is ______.

Use point-slope form to write the equation.

$y - y_1 = \text{m}(x - x_1)$

____

2. (0, 1), (2, 2) ______________________________

3. (−6, 4), (3, −5) ______________________________

4. (2, 6), (0, 0) ______________________________

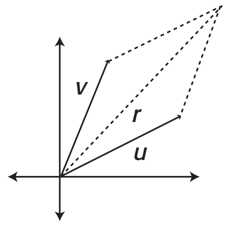
5. (−1, −4), (5, 2) ______________________________

6. (6, 0), (3, −2) ______________________________

Name ______________________________ Date ____________________

Adding Vectors

A **vector** is any quantity with magnitude <u>and</u> direction. The magnitude is the distance from the start point to the end point. The direction is the direction in which the arrow points from the start point to the end point. The diagram shows two vectors, *u* and *v*. The resultant vector *r*, is the sum of the vectors. If the vectors start at the origin, you can find the resultant vector, *r,* by adding the coordinates of their end points.

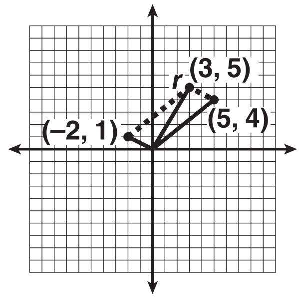


Rules for Adding Vectors

1. Find the end coordinates of one of the vectors. This is (x_1, y_1).
2. Find the coordinates of the second vector. This is (x_2, y_2).
3. Add the *x*-coordinates and add the y-coordinates: $(x_1 + x_2, y_1 + y_2)$. The resulting coordinates are the endpoint of the resultant drawn from the origin.

Example

Add vectors *a*(5, 4) and *b*(–2, 1). Write the sum of the two vectors as an ordered pair. Then draw the resultant.



Step 1 Find the end coordinates of one of the vectors. This is (x_1, y_1). — The first vector has coordinates (5, 4); $x_1 = 5, y_1 = 4$.

Step 2 Find the coordinates of the second vector. This is (x_2, y_2). — The second vector has coordinates (−2, 1); $x_2 = -2, y_2 = 1$.

Step 3 Add the *x*-coordinates and add the *y*-coordinates: $(x_1 + x_2, y_1 + y_2)$. — $(5 + (-2), 4 + 1)$

(3, 5) is the end point of the resultant vector.

Practice

1. Add vectors *a* (−3, −2) and *b* (−1, 3). Write the sum of the two vectors as an ordered pair. Then, draw the resultant.

   Find the end coordinates of one of the vectors. This is (x_1, y_1). — The first vector has coordinates ________;

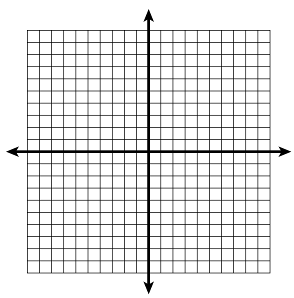
   $x_1 =$ ________, $y_1 =$ ________

   Find the coordinates of the second vector. This is (x_2, y_2). — The second vector has coordinates ________;

   $x_2 =$________, $y_2 =$ ________

   Add the *x*-coordinates and add the *y*-coordinates: $(x_1 + x_2, y_1 + y_2)$. — (______ + ______, ______ + ______)

   ________ is the end point of the resultant vector.



2. Add vectors *a* (4, 4) and *b* (−1, 2). Write the sum of the two vectors as an ordered pair. Then, draw the resultant. ____________________
3. Add vectors *a* (−6, 5) and *b* (2, −4). Write the sum of the two vectors as an ordered pair.

   Then, draw the resultant. ____________________

Name ______________________________ Date ____________________

Translations

A **translation** is often described as moving a figure from one location to another. In a translation, neither the size nor the shape of the figure changes. All the points of the figure move the same distance and in the same direction. In a translation, the points of the original figure are usually given. Each point in the new figure is followed by a **prime** (′).

> **Rules for Translation**
> 1. Identify the change in the x-coordinate and the y-coordinate of each point of the figure.
> 2. Add the change in the x-coordinate to each x-coordinate in the figure. Add the change in the y-coordinate to each y-coordinate in figure.
> 3. List the new coordinates of each point using prime notation (′).

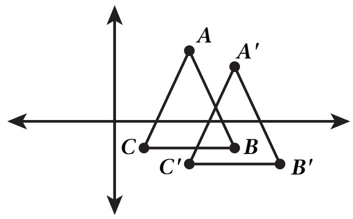
Example

A triangle has the following coordinates, *A* (3, 3), *B* (5, –1), *C* (1, –1). The triangle is translated (x, y) → (x + 2, y – 1). What are the coordinates of the translated image?

Step 1 Identify the change in the x- and y-coordinate of each point of the figure. Each x-coordinate is changed by +2; each y-coordinate is changed by −1.

Step 2 Add the change in the x-coordinate to each x-coordinate in the figure. Add the change in the y-coordinate to each y-coordinate in the figure.
$A\ (3, 3) \rightarrow (3 + 2\ , 3 - 1)$
$B\ (5\ , -1\ ) \rightarrow (5 + 2\ , -1 - 1)$
$C\ (1, -1\ ) \rightarrow (1 + 2\ , -1- 1)$

Step 3 List the new coordinates of each point using prime notation (′). A' (5, 2); B' (7, −2); C' (3, −2)



Practice

Identify the coordinates of each translated image.

1. A triangle has coordinates A (−2, −1), B (0, 2), C (1, 0). It is translated (x, y) → (x + 3 , y + 3)

   Identify the change in the x- and y-coordinate of each point of the figure. Each x-coordinate is changed by ______; each y-coordinate is changed by ______.

   Add the change in the x-coordinate to each x-coordinate in the figure. Add the change in the y-coordinate to each y-coordinate in the figure.
   A (−2, −1) → (−2 +______, −1 + ______)
   B (0, 2) → (0 ______, 2 ______)
   C (1, 0) → (1 ______, 0 ______)

   List the new coordinates of each point using prime notation (′). A' ________; B' ________; C' ________

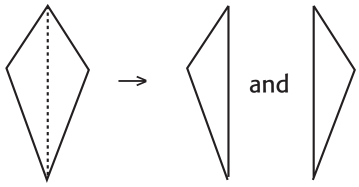
2. A parallelogram has coordinates A (−2, −3), B (−1 , −1), C (2, −1), D (1, −3). It is translated (x, y) → (x – 2, y + 1). ____

3. A trapezoid has coordinates A (0, −1), B (2, −1), C (3, −3), D (−1, −3). It is translated (x, y) → (x + 0, y + 4).

Name ______________________________ Date ____________________

Symmetry

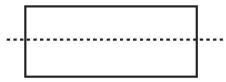
If you flip a figure over a line and the figure appears unchanged, then the figure has **line symmetry**. Another way to determine if a figure has line symmetry is to draw a line through the figure, dividing it in half. If the two halves are mirror images, then the figure has line symmetry.



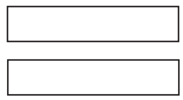
Example

For the figure to the right, find all the lines of symmetry.

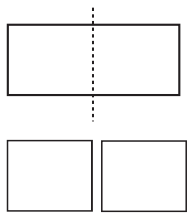
Step 1 Examine the figure and try to visualize a line that divides the image in two. Draw a dotted line for the line of symmetry.



Step 2 Draw the two halves. If the two halves are mirror images, then the line of symmetry is true.



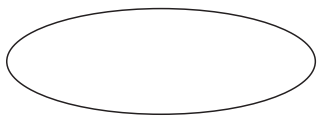
Step 3 Test other possible lines of symmetry.



Practice

Draw all lines of symmetry.

1.

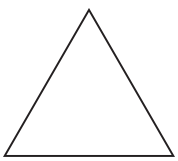


Examine the figure and try to visualize a line that divides the image in two. Draw a dotted line for the line of symmetry.

Draw the two halves. If the two halves are mirror images, then the line of symmetry is true.

Test other possible lines of symmetry.

2.

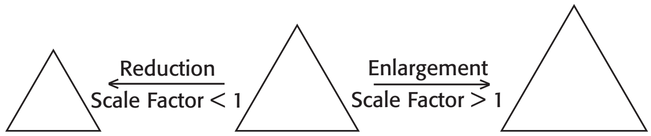


3.

Name ______________________________ Date ____________________

Dilations

A **dilation** is a transformation in which the image and the image after the dilation are similar figures. Each dilation has a **scale factor**, a description of the change in size from the original image and the resulting image. When the scale factor is greater than 1, the dilation is an enlargement. When the scale factor is less than 1, the dilation is a **reduction**.



Rules for Dilations
1. Identify the scale factor for the dilation.
2. Multiply each coordinate by the scale factor.
3. List the new coordinates using prime notation (′).

Example

A triangle has coordinates *A* (5, 2), *B* (7, –2) and *C* (3, –2). A dilation has a scale factor of $\frac{1}{3}$. What are the coordinates of the new image?

Step 1 Identify the scale factor for the dilation. Each x-coordinate is multiplied by $\frac{1}{3}$; each y-coordinate is multiplied by $\frac{1}{3}$.

Step 2 Multiply each coordinate by the scale factor.

$A\ (5, 2) \rightarrow (5 \times \frac{1}{3}, 2 \times \frac{1}{3})$

$B\ (7, -2) \rightarrow (7 \times \frac{1}{3}, -2 \times \frac{1}{3})$

$C\ (3, -2) \rightarrow (3 \times \frac{1}{3} -2 \times \frac{1}{3})$

Step 3 List the new coordinates using prime notation(′). $A'\ (\frac{5}{3}, \frac{2}{3})$, $B'\ (\frac{7}{3}, -\frac{2}{3})$, $C'\ (1, -\frac{2}{3})$

Practice

Identify the coordinates of the dilated image.

1. A triangle has coordinates A (1, 2), B (3, 5), C (4, 3). A dilation has a scale factor of 2.

   Identify the scale factor for the dilation. Each x–coordinate is multiplied by ______;
   each y-coordinate is multiplied by ______.

   Multiply each coordinate by the scale factor.
   A (1, 2) → (1 × ______, 2 × ______)
   B (3, 5) → (3 × ______, 5 × ______)
   C (4, 3) → (4 × ______, 3 × ______)

   List the new coordinates using prime notation (′). A′ ________, B′ ________, C′ ________

2. A parallelogram has coordinates A (0, –2), B (–3, 0), C (0, 0), D (–1, –2). A dilation has a scale factor of 3. ____________________

3. A trapezoid has coordinates A (0, 3), B (2, 3), C (3, 1), D (–1,–3). A dilation has a scale factor of $\frac{1}{2}$. ____________________

Name ______________________ Date ______________

If-Then Statements

You have often heard "if-then" statements, such as, "If it is Friday, then we will have pizza for lunch." An if-then statement is also known as a **conditional**. A conditional has two parts. The **hypothesis** and the **conclusion**. The hypothesis is the "If" part of the condition. The "Then" part is the conclusion. A conditional is true if every time the hypothesis is true, the conclusion is also true. A conditional is false if a counterexample is found that makes the conclusion false.

Rules for If-Then Statements

1. To write a conditional, the hypothesis is written as an "If" statement; it is followed by the conclusion, which is the "Then" statement.
2. To prove a conditional as true:
   The hypothesis must be true. The conclusion must also be true.
   If the conclusion is found to be false, then the conditional is false.

Example

Write the following statement as a conditional and show that the conditional is true or false: May is a month with 31 days.

Step 1 The hypothesis is written as an "If" statement; it is followed by the conclusion, the "Then" statement.

The hypothesis is that a month of the year can have 31 days.

Step 2 Prove the conditional as true.

The conclusion is that a month with 31 days is May.

"If a month has 31 days, then it is May."

You know by looking at a calendar that other months, such as March or July, also have 31 days. Therefore, the conditional is false.

Practice

Write the following statements as conditionals.
Show that the conditional is true or false.

1. A number divisible by 2 is an even number.

   The hypothesis is written as an "If" statement; it is followed by the conclusion, the "Then" statement.

   The hypothesis is that some ____________ are divisible by _____.

   The conclusion is that those numbers divisible by _____ are __________.

   ____

   ____

   Prove the conditional as true.

   All __________ numbers __________ divisible by 2. So, the conditional is __________.

2. Odd integers greater than 10 are not prime. ______________________

3. A right triangle has only one 90° angle. ______________________

Name ______________________________ Date ____________________

Inductive Reasoning

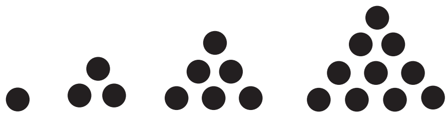
Inductive reasoning is reasoning that is based on patterns. When you use inductive reasoning, you observe a few situations and draw a conclusion based on those few instances. When you draw a conclusion, you often do so because you have observed a pattern. A conclusion you reach by using inductive reasoning is called a **conjecture**.

Rules for Identifying Patterns and Using Inductive Reasoning

1. Observe the differences between the first item in the sequence and the second item. State how the first item changed to become the second.
2. Observe the difference between the second and the third items in the sequence. State how the second item changed to become the third item. Is the way they changed the same as in Step 1?
3. Repeat the process for the next two items. If the pattern continues, then apply the pattern rule to find the next item.

Example

Find the pattern. Use the pattern to find the next two items in the sequence.



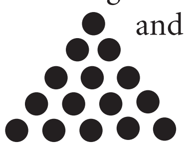
Step 1 State how the first item changed to become the second.

When you go to the next figure, you add a row with 1 more dot than the previous row.

Step 2 Observe the difference between the second and the third items in the sequence. Is the way they changed the same as in Step 1?

When you go to the next figure, you add a row with one more dot than the previous row.

Step 3 Repeat the process for the next two items. If the pattern continues, then apply the pattern rule to find the next item.

The pattern holds true in moving from figure 3 to figure 4. Therefore, the fifth figure will be  and the sixth figure will be .

Practice

Find the pattern, then use the pattern to find the next two items in the sequence.

1. 100, 50, 25, 12.5

State how the first item changed to become the second.

The second item in the sequence is __________ of the first number.

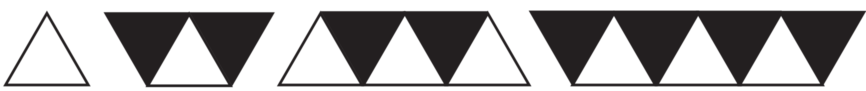
State how the second item changed to become the second. Is the way they changed the same as in Step 1?

Item 2 is 50 and item 3 is 25. The third item in the sequence is __________ of the second number.

Repeat the process for the next item. If the pattern continues, then apply the rule to find the next item.

The pattern holds true for item 4.

Therefore, the next number is ________.

2.  ______________

3. 2, −6, 18, −54, ______________

Name ______________________________ Date ____________________

Deductive Reasoning: Law of Detachment

Deductive, or logical, reasoning is the process by which a conclusion is made based upon known facts.

For example:

- You are in a room with the members of your school's swim team.
- You know that swimmers get up early for practice.
- Your friend is in the room.
- You can conclude that your friend will get up early.

As you can see, if the given statements (the first three statements) are true, deductive reasoning reaches a true conclusion (the fourth statement).

You can use two laws in deductive reasoning: the **law of detachment** and the **law of syllogism**.

Law of Detachment
If a conditional is true and its hypothesis is true, then the conclusion is true. If $p \rightarrow q$ is a true conditional and p is true, then q is true.

Example

If the measure of an angle is less than 90°, then the angle is acute. $\angle A$ has a measure of 60°. What can you conclude about $\angle A$? Use the law of detachment.

Step 1 What is given? — An angle less than 90° is acute. $\angle A$ has a measure of 60°.

Step 2 What is the relationship between $\angle A$ and the first statement? — You know the $m\angle A$ and can use that information to classify $\angle A$.

Step 3 What can you conclude? — Since $\angle A$ is 60° and an angle less than 90° is acute, $\angle A$ is acute.

Practice

Use the law of detachment to form a conclusion.

1. If an angle is obtuse, it has a measure greater than 90°. $m\angle A$ is 110°.

What is given? — An angle greater than 90° is ______________.

$\angle A$ has a measure of ______________.

What is the relationship between $\angle A$ and the first statement? — You know the $m\angle A$ and can use that information to classify ______________.

What can you conclude? — Since $\angle A$ is ______________ and an angle greater than 90° is ______________, $\angle A$ is ______________.

2. If Jamal works during the summer, he works in the library. Jamal works during the summer. ____

Name ______________________________ Date ____________________

Deductive Reasoning: Law of Syllogism

Deductive, or logical, reasoning is the process by which a conclusion is made based upon known facts.

For example:

- You are in a room with the members of your school's swim team.
- You know that swimmers get up early for practice.
- Your friend is in the room.
- You can conclude that your friend will get up early.

You can use the **law of syllogism** in deductive reasoning.

> **Law of Syllogism**
> You can state a conclusion from two true conditionals when the conclusion of one of the conditionals is the hypothesis of the other.
> If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, and p is true, then $p \rightarrow r$ is true.

Example

If Rachel is cooking, then she is making cookies. If Rachel is making cookies, then she is using raisins. What can you conclude about Rachel if she is cooking? Use the law of detachment.

Step 1 What is the first conditional? — If Rachel is cooking, then she is making cookies.

Step 2 What is the second conditional? — If Rachel is making cookies, then she is using raisins.

Step 3 Does the conclusion of the first conditional form the hypothesis of the second conditional? — Yes.

Step 4 Use the hypothesis of the first conditional and the conclusion of the second conditional to form a conclusion. — If Rachel is cooking, then she is using raisins.

Practice

Use the law of syllogism to form a conclusion.

1. If an angle is obtuse, it has a measure greater than 90°. If an angle is greater than 90°, it cannot be a complementary angle.

First conditional: ______________________

Second conditional: ______________________

Yes or no? ______________________

Conclusion: ______________________

____

2. If Pearl is reading a book, then she is reading a mystery. If she is reading a mystery, then it is a book by Stephen King. ______________________

PAGE 1 *Points, Segments, Rays, Lines, and Planes*

Type of Figure	Symbol	Words	Drawing
Point	Point *A*	Point *A*	• *A*
Line	$\overleftrightarrow{AB}$	Line *AB*	*A* *B*
Segment	$\overline{AB}$	Segment *AB*	*A* *B*
Ray	$\overrightarrow{AB}$	Ray *AB*	*A* *B*
Plane	▱*Z*	Plane *Z*	*Z*

Rules for Naming Basic Figures
Point: no, dot, letter
Line: indefinitely, no, both
Segment: endpoints, no
Ray: one, endpoint
Plane: flat, no

Practice
1. M **3.** $\overrightarrow{RS}$, $\overrightarrow{DC}$, or $\overrightarrow{CD}$
2. $\overleftrightarrow{CD}$ **4.** segment

PAGE 2 *Measuring Segments*
Complete each statement.
1. 8 **3.** $\overline{AB}$, 8, 17
2. $\overline{AC}$ **4.** $\overline{AC}$

Complete the rule.
1. *B* **2.** *B*

Practice
1. *YZ*;
YZ, 10;
$XY + 10 - 10 = 45 - 10$, 35
2. 27 **4.** 47
3. 15 **5.** $6x + 1$

PAGE 3 *Using Formulas*
Practice
1. area, the other base length; height; area, 6, 8, 10;
$A = (\frac{1}{2})(10)(6 + 8)$;
70 square units
2. $r = 6$ **3.** $h = 20$ in.

PAGE 4 *Types of Angles*

Angle Type	Example	Measure
Acute	∠*ABC*	45°
Right	∠*DEF*	90°
Obtuse	∠*KLM*	120°
Straight	∠*XYZ*	180°

Complete the statements.
1. 90° **3.** greater than
2. right angle **4.** straight angle

Practice
1. right **3.** ∠*CED* or ∠*AFB*
2. obtuse **4.** less than

PAGE 5 *Complementary and Supplementary Angles*

Type	Angle Pair	Measure of One Angle	Measure of the Other Angle	Sum of the Measure
Complementary	∠*ABC* & ∠*DEF*	30° +	60° =	90°
Supplementary	∠*KLM* & ∠*XYZ*	115° +	65° =	180°

Complete the statement.
1. complementary
2. 180°

Practice
1. ∠*BFC* **3.** 145°
2. 180° **4.** ∠*AFB*

PAGE 6 *Pairs of Angles*

Type	Measure of One Angle	Measure of the Other Angle
Vertical Angles	m∠1 = 80°	m∠3 = 80°
	m∠2 = 100°	m∠4 = 100°
Linear Pair	m∠1 = 80°	m∠2 = 100°
	m∠3 = 80°	m∠4 = 100°

Complete the statements.
1. vertical angles, ∠2, ∠4
2. linear pair, ∠3, ∠4
3. 180°, ∠3, ∠4
4. supplementary

Complete the statements for the rules.
1. vertical
2. the same, congruent
3. 180°

Practice
1. ∠*EGD* **4.** *EGA* and ∠*DGB*
2. a linear pair **5.** ∠*AGF*
3. 135°

PAGE 7 *Parallel Lines: Types of Angles*
Complete the rules.
1. 7, 8 **4.** 4, 5
2. 5, 6 **5.** 2, 7
3. 4, 6 **6.** 6, 4

Practice
1. alternate interior
2. exterior
3. alternate interior
4. alternate exterior
5. 11, 1
6. 7, 10
7. 11, 6

PAGE 8 *Parallel Lines: Angle Relationship*

Type	Measure of Angle	Measure of Other Angle
Corresponding Angle	m∠1 = 65°	m∠4 = 65°
Alternate Interior Angles	m∠1 = 65°	m∠3 = 65°
Consecutive Interior Angles	m∠1 = 65°	m∠2 = 115°
Alternate Exterior Angles	m∠5 = 65°	m∠4 = 65°

Complete the statements.
1. congruent
2. congruent
3. Consecutive interior
4. congruent

Practice
1. 60° **4.** 120°
2. 120° **5.** 120°
3. 60°

PAGE 9 *Proving Lines are Parallel*
Complete the statements.
1. corresponding, congruent
2. alternate interior, congruent
3. alternate exterior, congruent
4. consecutive interior, supplementary

Practice
1. consecutive interior; supplementary; consecutive interior, supplementary
2. alternate exterior angles
3. corresponding angles
4. alternate interior angles
5. consecutive interior angles

PAGE 10 *Classifying Triangles*
Complete the rules
Rules for Classifying Triangles by Angle
1. three **3.** obtuse
2. congruent **4.** right

Rules for Classifying Triangles by Angle
1. no **3.** three
2. two

Practice
1. △*BEC* **4.** △*BEC*
2. △*AEC* **5.** △*AED*
3. △*AEB*

PAGE 11 *Interior and Exterior Angles in Triangles*
Complete each statement.
1. 50°, 55°, 75°, 180°
2. 105°, 50°, 55°, 105°
3. 75°, 105°, 180°, supplementary

Complete the rules.
1. 180°, 180° **2.** exterior, equal

Practice
1. 80° **3.** 64°
2 70° **4.** 36°

PAGE 12 *Corresponding Parts of Triangles*
Identify the corresponding parts.
∠*CAB* ⟷ ∠*ZXY* $\overline{AC} \leftrightarrow \overline{XZ}$
∠*ABC* ⟷ ∠*XYZ* $\overline{AB} \leftrightarrow \overline{XY}$
∠*BCA* ⟷ ∠*YZX* $\overline{BC} \leftrightarrow \overline{YZ}$

Complete the chart.

Angle	Corresponding Angle	Relationship
∠*CAB* = 70°	∠*ZXY* = 70°	∠*CAB* ≅ ∠*ZXY*
∠*ABC* = 57°	∠*XYZ* = 57°	∠*ABC* ≅ ∠*XYZ*
∠*BCA* = 53°	∠*YZX* = 53°	∠*BCA* ≅ ∠*YZX*

Side	Corresponding Side	Relationship
$\overline{AC}$	$\overline{XZ}$	$\overline{AC} \cong \overline{XZ}$
$\overline{AB}$	$\overline{XY}$	$\overline{AB} \cong \overline{XY}$
$\overline{BC}$	$\overline{YZ}$	$\overline{BC} \cong \overline{YZ}$

Complete the statement.
corresponding, congruent

Practice

1. 12
2. $\angle SRT$
3. 48
4. $\overline{EF}$
5. 72
6. $\overline{RT}$

PAGE 13 *Triangle Congruence: Side-Side-Side Congruence*

Complete the chart.

Side	Measure	Corresponding Side	Measure	Relationship Between Sides
$\overline{AB}$	3	$\overline{XY}$	3	$\overline{AB} \cong \overline{XY}$
$\overline{BC}$	5	$\overline{YZ}$	5	$\overline{BC} \cong \overline{YZ}$
$\overline{AC}$	7	$\overline{XZ}$	7	$\overline{AC} \cong \overline{XZ}$

Complete the rule.

congruent, three

Practice

1. $\overline{DC}$;
   yes;
   $\overline{BC}$;
   yes;
   It is part of $\triangle ABC$ and $\triangle ACD$;
   yes;
   yes
2. no
3. yes

PAGE 14 *Triangle Congruence: Side-Angle-Side Congruence*

Answer the following.

1. $\overline{BC}$
2. $\angle A$
3. $\angle A$ and $\angle B$

Complete the chart.

Side	Measure	Corresponding Side	Measure	Relationship Between Sides
$\overline{AB}$	3	$\overline{XY}$	3	$\overline{AB} \cong \overline{XY}$
Angle	**Measure**	**Corresponding Angle**	**Measure**	**Relationship Between Angles**
$\angle B$	110°	$\angle Y$	110°	$\angle B \cong \angle Y$
Side	**Measure**	**Corresponding Side**	**Measure**	**Relationship Between Sides**
$\overline{BC}$	5	$\overline{YZ}$	5	$\overline{BC} \cong \overline{XY}$

Complete the rule.

congruent, sides, included

Practice

1. $\angle A$
2. $\angle DBC$
3. $\angle C$
4. $\angle G$, $\angle K$

PAGE 15 *Triangle Congruence: Angle-Side-Angle Congruence*

Answer the following questions.

1. $\overline{AC}$
2. $\overline{BC}$
3. $\overline{AB}$

Complete the chart.

Angle	Measure	Corresponding Angle	Measure	Relationship Between Angles
$\angle B$	110°	$\angle Y$	110°	$\angle B \cong \angle Y$
Side	**Measure**	**Corresponding Side**	**Measure**	**Relationship Between Sides**
$\overline{AB}$	3	$\overline{XY}$	3	$\overline{AB} \cong \overline{XY}$
Angle	**Measure**	**Corresponding Angle**	**Measure**	**Relationship Between Angles**
$\angle A$	30°	$\angle X$	30°	$\angle A \cong \angle X$

Complete the rule.

congruent, angles, included

Practice

1. $\angle A = \angle X$;
   $\overline{AC} = \overline{XZ}$;
   $\angle A$ and $\angle C$;
   $\angle C$
2. $\overline{AC} \cong \overline{AC}$
3. $\overline{CB}$ and $\overline{CZ}$

PAGE 16 *Triangle Congruence: Angle-Angle-Side Congruence*

Complete the chart.

Angle	Measure	Corresponding Angle	Angle Measure	Relationship Between Angles
$\angle A$	30°	$\angle X$	30°	$\angle A \cong \angle X$
$\angle B$	110°	$\angle Y$	110°	$\angle B \cong \angle Y$
Side	**Measure**	**Corresponding Side**	**Measure**	**Relationship Between Sides**
$\overline{AC}$	4	$\overline{XZ}$	4	$\overline{AC} \cong \overline{XZ}$

Complete the rule.

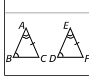
congruent, corresponding

Practice

1. $\angle A = \angle D$;
   $\overline{AB} = \overline{DE}$;
   $\angle ACB = \angle DCE$;
   vertical;
   vertical angles are congruent;
   $\angle A \cong \angle D$, $\angle C \cong \angle C$,
   $\overline{AB} \cong \overline{DE}$
2. $\angle E$ or $\angle D$ and $\angle Y$ or $\angle X$
3. $\overline{DB} \cong \overline{DB}$, $\angle A$ and $\angle C$

PAGE 17 *Choosing the Correct Congruence Postulate*

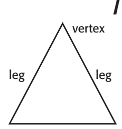
Complete the chart.

Example	What Is Given	Postulate to Use
	One, congruent, two, congruent	Side-Angle-Side Postulate
	Two, congruent, one, congruent	Angle-Side-Angle Postulate
	Two, congruent, one, congruent	Angle-Angle-Side Postulate

Practice

1. $\angle A \cong \angle X$, $\angle C \cong \angle Z$,
   $\overline{AC} \cong \overline{XZ}$;
   Angle-Side-Angle
2. cannot be proved

PAGE 18 *Isosceles Triangle Theorem*



Complete the chart.

Angle or Side	Measure
$\angle B$	50°
$\angle C$	50°
$\overline{AB}$	10 cm
$\overline{AC}$	10 cm

Complete the theorems.

1. congruent, congruent
2. congruent, congruent

Practice

1. CB;
   55°;
   180°;
   $\angle A + \angle B + \angle C = 180°$
   $55° + \angle B + 55° = 180°$
   $\angle B = 70°$
2. $\angle A = \angle B$; $m\angle A = m\angle B = 30°$
3. $x + 10 = 24$; $x = 14$

PAGE 19 *Triangle Mid-segment*

Find the slope of each segment.

$\overline{CB}$: Slope $= \frac{2}{5}$;

$\overline{DE}$: Slope $= \frac{1}{2.5} = \frac{2}{5}$

1. $\frac{2}{5}$, $\frac{2}{5}$, equal, parallel

Use the Distance Formula to find the length d of each segment.

$\overline{CB} = \sqrt{(5)^2 + (2)^2}$

$= \sqrt{29} = 5.38$

$\overline{DE} = \sqrt{(2.5)^2 + (1)^2}$

$= 2.69$

2. 5.38, 2.69, $\overline{DE}$

Complete the rule.

parallel, half

Complete each statement.

3. $\overline{AB}$
4. $\overline{DE}$
5. 6
6. 8
7. 5.3

PAGE 20 *Hypotenuse-Leg Theorem*

Complete the chart.

$\triangle ABC$		$\triangle RST$		
Side	**Meas.**	**Corresponding Side**	**Meas.**	**Relationship Between Sides**
$\overline{AB}$	4	$\overline{RS}$	4	$\overline{AB} \cong \overline{RS}$
$\overline{BC}$	3	$\overline{ST}$	3	$\overline{BC} \cong \overline{ST}$
$(\overline{AC})^2 = 4^2 + 3^2$	5	$(\overline{RT})^2 = 4^2 + 3^2$	5	$\overline{AC} \cong \overline{RT}$

SSS, $\cong$

Complete the rule.

congruent, hypotenuse, leg

Practice

1. $\overline{CB}$
2. yes, $\overline{DB} \cong \overline{DB}$
3. no
4. no
5. no

PAGE 21 *Triangle Inequalities: Inequalities for Sides and Angles*

Complete the chart.

Side	Measure	Angle	Measure
$\overline{AB}$	4 cm	$\angle ABC$	110°
$\overline{BC}$	5 cm	$\angle BAC$	40°
$\overline{AC}$	8 cm	$\angle ACB$	30°

Complete the statements.

1. $\overline{BC}$
2. $\angle BCA$
3. $\angle BAC$
4. $\angle BAC$
5. yes, it is opposite $\overline{BC}$

Complete the rule.

opposite, greater

Practice

1. $>$;
   $\angle BDA = 70°$;
   $\angle BAD = 80°$;
   $\overline{BD}$

2. $\overline{BC}$
3. >
4. >

PAGE 22 *Triangle Inequality Theorem*

Complete the chart.

Figure *BADC*	Inequality Test	Is the Inequality True?
	$19 + 43 > 16$ $16 + 43 > 19$ $16 + 19 > 43$	Yes Yes No
Figure *RST*	**Inequality Test**	**Is the Inequality True?**
	$22 + 18 > 36$ $18 + 36 > 22$ $36 + 22 > 18$	Yes Yes Yes

Answer the questions.

1. No, one statement is not true.
2. Yes

Complete the rule.

greater

Practice

1. yes;
   yes;
   yes;
   yes, yes
2. no
3. yes
4. yes

PAGE 23 *The Pythagorean Theorem*

Practice

1. a side;
   $8^2 + 15^2 = c^2$, $64 + 225 = c^2$
   $64 + 225 = c^2$,
   $289 = c^2$,
   $17 = c$
2. $a = 6$
3. $c = 5$
4. $c = 25$
5. $b = 150$

PAGE 24 *Converse of the Pythagorean Theorem*

Complete the chart.

longest

Leg	Leg	Hypotenuse		
$\overline{AB}$	$\overline{BC}$	$\overline{AC}$	$AB^2 + BC^2$	AC^2
6	8	10	$6^2 + 8^2 = 100$	$(10)^2 = 100$

=

yes, yes

Complete the rule.

squares, longest

Practice

1. 13;
   7 and 10;
   $7^2 + 10^2 = 13^2$, $49 + 100 = 169$;
   no, $149 \neq 169$
2. yes
3. yes
4. no
5. yes

PAGE 25 *Special Right Triangles: 45°-45°-90° Right Triangles*

Complete the chart.

45°–45°–90°			
Leg	Leg	Hypotenuse	
$\overline{AB}$	$\overline{BC}$	$\overline{AC}$	$AB^2 + BC^2 = AC^2$
4	4	x	$4^2 + 4^2 = 32 = x^2$

Solve for x

$\sqrt{32} = x^2$

$\sqrt{32} = \sqrt{x^2}$

$\sqrt{(16)2} = \sqrt{x^2}$

$4\sqrt{2} = x$

$\overline{AB}$, 4, $\overline{AC}$, $4\sqrt{2}$

Complete the rule.

hypotenuse, leg

Practice

1. hypotenuse $= \sqrt{2}$ leg;
   $x = \sqrt{2}(6)$;
   $x = 6\sqrt{2}$
2. $x = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

PAGE 26 *Special Right Triangles: 30°-60°-90° Right Triangles*

Complete the chart.

30°–60°–90°			
Leg	Leg	Hypotenuse	
$\overline{RS}$	$\overline{ST}$	$\overline{RT}$	$\overline{RS}^2 + \overline{ST}^2 = \overline{RT}^2$
10	x	20	$10^2 + x^2 = 20^2$

Solve for x

$100 + x^2 = 400$

$x^2 = 300$

$\sqrt{x^2} = \sqrt{300}$

$x = \sqrt{100(3)}$

$x = 10\sqrt{3}$

$\overline{RS}$, 10

$\overline{ST}$, $10\sqrt{3}$

Complete the rule.

longer, shorter

Practice

1. longer leg $= \sqrt{3}$(shorter leg);
   $x = \sqrt{3}(6)$;
   $x = 6\sqrt{3}$
2. $x = 24$

PAGE 27 *Trigonometric Ratios*

Practice

1. $\frac{\text{length of the leg opposite } A}{\text{length of the hypotenuse}} = \frac{12}{13}$;
   $\frac{\text{length of the leg adjacent } A}{\text{length of the hypotenuse}} = \frac{5}{13}$;
   $\frac{\text{length of the leg opposite } A}{\text{length of the leg adjacent } A} = \frac{12}{5}$
2. $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$
3. $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}$
4. $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$

PAGE 28 *Inverse of Trigonometric Ratios*

Practice

1. opposite, hypotenuse;
   sine, sin;
   $\sin T = \frac{\text{length of the leg opposite } T}{\text{length of the hypotenuse}}$
   $= \frac{8}{10} = 0.8$;
   $\sin T = 0.8$, $T = 53.13°$
2. $\angle C = 60°$
3. $\angle T = 22.89°$

PAGE 29 *Angles of Elevation and Depression*

Practice

1. 100 ft, 9°, adjacent side;
   adjacent;
   9°, 100
   $x = \frac{100}{\tan 9°} = \frac{100}{0.158} = 632.91$
2. 53.59 ft
3. 86.34°

PAGE 30 *Types of Polygons*

Identify characteristics of a polygon.

1. segments, arc
2. two, one
3. two, more than two

Complete the statement.

segments, two

Practice

1. yes;
   no;
   no
2. yes
3. yes
4. quadrilateral
5. pentagon

PAGE 31 *Sum of Polygon Angle Measures*

Complete the chart.

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angle Measures
Triangle	3	1	1(180°) = 180°
Quadrilateral	4	2	2(180°) = 360°
Pentagon	5	3	3(180°) = 540°
Hexagon	6	4	4(180°) = 720°

Complete the statements.

1. 4, 2
2. 2
3. 2, 180°

Complete the rule.

$(n - 2)180°$

Practice

1. 12;
   $(n - 2)180°$,
   $(12 - 2)180°$;
   $(10)(180) = 1800°$
2. 2340°
3. 3240°
4. 6 sides

PAGE 32 *Types of Quadrilaterals*

Complete the chart.

Type	Sides	Angles
Rectangle	parallel, congruent	90°
Square	parallel, congruent.	90°
Parallelogram	parallel, congruent	congruent
Rhombus	parallel, congruent.	congruent
Trapezoid	parallel	
Kite	congruent., parallel	congruent

PAGE 33 *Properties of Parallelograms*

Complete the chart.

Opposite Sides			
Side	Measure	Opposite Side	Measure
$\overline{AD}$	12	$\overline{BC}$	12
$\overline{AB}$	18	$\overline{DC}$	18
Opposite Angles			
Angle	Measure	Opposite Angle	Measure
$\angle A$	60˚	$\angle C$	60˚
$\angle D$	120˚	$\angle B$	120˚

Consecutive Angles				
Angle	Measure	Consecutive Angle	Measure	Sum of Measures
$\angle A$	60˚	$\angle D$ or $\angle B$	120˚	180˚
$\angle B$	120˚	$\angle A$ or $\angle C$	60˚	180˚

Complete the statements.

1. congruent, congruent
2. supplementary

Practice

1. $\angle B$, 120°;
supplementary,
180° – 120° = 60°;
$\angle B$;
congruent, 120°;
$\overline{BC}$, congruent, 15;
$\overline{BA}$, congruent, 10

PAGE 34 *Properties of Trapezoids*

Complete each statement.

1. $\angle B$ **2.** $\angle C$

Complete the chart.

Base Angle	Measure	Base Angle Pair	Measure
$\angle A$	120˚	$\angle B$	120˚
$\angle D$	60˚	$\angle C$	60˚

Complete the rule.

congruent

Complete each statement.

3. 52
4. 26
5. 52, 26

Practice

1. 28;
one half;
$\frac{1}{2}(28) = 14$
2. 75° **4.** 18
3. 32 **5.** 70°

PAGE 35 *Diagonals in Parallelograms*

Complete the table.

Diagonal	Measure	Seg-ment	Measure	Seg-ment	Measure
$\overline{AC}$	28	$\overline{AE}$	14	$\overline{CE}$	14
$\overline{DB}$	34	$\overline{DE}$	17	$\overline{BE}$	17

Complete the statements.

1. half, half **3.** bisects
2. ≅, ≅

Complete the rule.

bisect

Practice

1. segment, half;
14;
bisected, equal;
12
2. 8 **4.** 7
3. 16 **5.** 14

PAGE 36 *Exterior Angles of a Polygon*

Complete the chart.

Triangle				
$\angle 1$	$\angle 2$	$\angle 3$	$\angle 4$	Sum of Angles
135°	100°	125°	N/A	360°
Quadrilateral				
50°	130°	50°	130°	360°

Complete the rule.

360°

Practice

1. 5;
540°;
540° ÷ 5 = 108°;
360°;
360° ÷ 5 = 72°
2. interior angles = 135°,
exterior angles = 45°
3. 9 sides **4.** 6 sides

PAGE 37 *Proportions*

Practice

1. $\frac{1}{2}$, $\frac{4}{5}$;
are not, do not
2. no **6.** yes
3. yes **7.** no
4. no **8.** yes
5. no **9.** yes

PAGE 38 *Solving Proportions*

Practice

1. 392;
$4x$;
$4x = 392$;
$4x \div 4 = 392 \div 4$,
$x = 98$
2. 4 **6.** 15
3. 6 **7.** 32
4. 13 **8.** 8
5. 40 **9.** 26

PAGE 39 *Similar Polygons*

Explore the nature of similar figures.

Corresponding Angles				
Angle	Measure	Corresponding Angle	Measure	Relationship Between Angles
$\angle A$	20°	$\angle X$	20°	$\angle A \cong \angle X$
$\angle B$	115°	$\angle Y$	115°	$\angle B \cong \angle Y$
$\angle C$	45°	$\angle Z$	45°	$\angle C \cong \angle Z$

Corresponding Sides				
Side	Measure	Corresponding Side	Measure	Ratio of Angle to Corresponding Angle
$\overline{AB}$	15	$\overline{XY}$	7.5	$\frac{15}{7.5} = 2$
$\overline{AC}$	20	$\overline{XZ}$	10	$\frac{20}{10} = 2$
$\overline{BC}$	10	$\overline{YZ}$	5	$\frac{10}{5} = 2$

Complete the rule.

1. congruent **2.** proportion

Practice

1. ≅, ≅, ≅;
$\frac{AB}{WX} = \frac{BC}{YX}$, $\frac{24}{9} = \frac{12}{4.5}$,
2.67 = 2.67, are
2. yes

PAGE 40 *Triangle Similarity: Angle-Angle Similarity*

Complete the statements.

1. $\angle D$, $\angle B$, $\angle F$ **3.** 77°
2. 55° **4.** congruent

Complete the chart.

Angle	Measure	Corresponding Angle	Measure	Relationship
$\angle A$	77°	$\angle D$	77°	$\angle A \cong \angle D$
$\angle B$	48°	$\angle E$	48°	$\angle B \cong \angle E$
$\angle C$	55°	$\angle F$	55°	$\angle C \cong \angle F$

Complete the rule.

congruent

Practice

1. $\triangle ADE$;
$\angle A$, $\angle A \cong \angle A$;
90°, 90°;
$\angle A \cong \angle A$, $\angle D \cong \angle B$;
yes
2. no; yes

PAGE 41 *Triangle Similarity: Side-Side-Side Similarity*

Complete the chart.

Side	Measure	Corresponding Side	Measure	Ratio of Sides
$\overline{AB}$	14	$\overline{DE}$	7	$\frac{14}{7} = 2$
$\overline{BC}$	13	$\overline{EF}$	6.5	$\frac{13}{6.5} = 2$
$\overline{AC}$	15	$\overline{DF}$	7.5	$\frac{15}{7.5} = 2$

Complete the statements.

1. 2, 2, yes **3.** $\frac{\overline{BC}}{EF}$
2. 2, yes **4.** 2 : 1

Complete the rule.

proportional

Practice

1. $\overline{DE}$, $\frac{\overline{AB}}{DE}$, 2, 5;
$\overline{EF}$, $\frac{BC}{\overline{EF}}$, 3, 5;
$\overline{DF}$, $\frac{AC}{DF}$, 4, 5;
5 : 1;
yes, yes
2. yes (ratio of corresponding sides is 1 : 3)

PAGE 42 *Triangle Similarity: Side-Angle-Side Similarity*

Complete the chart.

Angle or Side	Measure	Corresponding Angle or Side	Measure	Relationship
$\overline{AB}$	14	$\overline{XY}$	7	$\frac{14}{7} = 2$
$\angle A$	53°	$\angle X$	53°	$\angle A \cong \angle X$
$\overline{AC}$	15	$\overline{XZ}$	7.5	$\frac{15}{7.5} = 2$

Complete the statements.

1. included angles
2. congruent
3. 2, 2, yes, yes

Complete the rule.

congruent, including

Practice

1. $\overline{FC}$, $\overline{CD}$;
$\overline{FC}$, $\frac{15}{10} = \frac{3}{2}$;
$\overline{CD}$, $\frac{12}{8} = \frac{3}{2}$;
yes, $\frac{3}{2} = \frac{3}{2}$;
$\angle ACB$;
yes, $\angle FCD$;
$\triangle$'s are similar
2. yes (ratio of corresponding sides is $\frac{3}{10}$)

PAGE 43 *Finding Lengths in Similar Triangles*

Practice

1. $\frac{\overline{AC}}{\overline{RT}} = \frac{8}{12}$;
$\frac{\overline{AB}}{\overline{RS}} = \frac{6}{\overline{RS}}$;
$\frac{8}{12} = \frac{6}{\overline{RS}}$;
9
2. $x = 9$
3. $x = 14$

PAGE 44 *Proportions in Triangles: Side-Splitter Theorem*

Explore the relationship between sides.

	Segment	Meas.	Segment	Meas.	Ratio
Side $\overline{AB}$	$\overline{AD}$	6	$\overline{DB}$	3	$\frac{6}{3} = 2$
Side $\overline{AC}$	$\overline{AE}$	10	$\overline{EC}$	5	$\frac{10}{5} = 2$

Complete the statements.
1. 2 : 1
2. 2 : 1
3. yes

Complete the theorem.
proportionally

Practice
1. $\frac{2}{6}$;
$\frac{4}{\overline{UT}}$;
$\frac{4}{\overline{UT}}$, 12
2. $\overline{CY} = 15$
3. $x = 4$

PAGE 45 *Triangle Angle Bisector Theorem*

Complete the statements.
1. bisects
2. $\overline{BD}$
3. $\frac{5}{6}$
4. $\frac{10}{12} = \frac{5}{6}$
5. =, proportion

Complete the theorem.
bisects, proportional

Practice
1. $\angle ABC$;
$\overline{DC}$;
$\frac{\overline{AD}}{\overline{DC}}$, $\frac{9}{\overline{DC}}$;
$(9)(20) = 12(\overline{DC})$,
$\overline{DC} = 15$
2. $\overline{WZ} = 6$
3. $\overline{ST} = 24$

PAGE 46 *Circles and Circumference*

Complete the definitions.
1. center, on
2. center, on
3. both, on, diameter
4. half, two

Practice
1. diameter, $d = 14$;
πd, $C = (3.14)(14)$
2. $C = 75.36$
3. $d = 15$, $r = 7.5$
4. $d = 20$, $r = 10$

PAGE 47 *Exploring π*

Complete the chart.

Circle	Circumference	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
1	37.68	12	$\frac{37.6}{12} = 3.14$
2	31.4	10	$\frac{31.4}{10} = 3.14$
3	25.12	8	$\frac{25.12}{8} = 3.14$
4	56.52	18	$\frac{56.52}{18} = 3.14$
5	47.1	15	$\frac{47.1}{15} = 3.14$

Complete the statements.
1. the same
2. 3.14
3. $3.14 = \frac{\text{Circumference}}{\text{Diameter}}$
4. Complete the chart.

Circle	Circumference	Diameter
1	106.76	34
2	37.68	12
3	53.38	17
4	62.80	20

PAGE 48 *Arc Length*

Practice
1. 170°;
15;
15
2. Arc length = 10.47
3. Arc length = 24.42

PAGE 49 *Inscribed Angles*

Complete the table.

Cirlce	Inscribed Angle	Intercepted Arc
Circle 1	50°	100°
Circle 2	60°	120°
Circle 3	120°	240°

Complete the statements.
1. intercepted arc, larger
2. two
3. half

Complete the rule.
half

Practice
1. inscribed angle, $\angle ACB$;
75°;
75°, 150°
2. $mAB = 180° = 6$
3. $m\angle ACB = 60°$, $m\angle ADB = 60°$

PAGE 50 *Angle Measures in Circles*

Complete the chart.

Circle	Larger Arc	Smaller Arc	Angle	Larger Arc–Smaller Arc
A	100°	30°	40°	80°
B	95°	55°	20°	40°
C	250°	110°	70°	140°

Complete the statements.
1. 80°, 40°, half
2. half
3. half

Complete the rule.
half

Practice
1. $\angle 1$;
arcs;
$\frac{1}{2}(160° - 60°)$
2. $m\angle 1 = 60°$
3. $y = 30°$

PAGE 51 *Finding Segment Lengths*

Practice
1. $z = 6$, $y = 12$;
x;
$(12 + 6)(12)$;
$216 = x^2$, $14.70 = x$
2. $x = 35$
3. $x = 18.33$

PAGE 52 *Equation of a Circle*

Practice
1. 0, 0;
4;
$(x - 0)^2 + (y - 0)^2 = 4^2$
$x^2 + y^2 = 16$
2. $(x - 5)^2 + (y - 3)^2 = 36$
3. $x^2 + (y - 2)^2 = 49$
4. $(x - 4)^2 + (y + 1)^2 = 2.25$
5. $(x + 2)^2 + (y + 2)^2 = 81$

PAGE 53 *Perimeter*

Practice
1. 2
10, $P = 6 + 3 + 3 + 7 + 4 + 4 + 2 + 8 + 3 + 4$
$P = 44$ units
2. 88 units
3. 152 units

PAGE 54 *Perimeter and Similar Figures*

Explore the perimeters of similar figures.

	Side	Side	Side	Side	Perimeter
ABCD	8	6	8	6	28
WXYZ	4	3	4	3	14

Complete the statements.
1. 8 : 4 or 2 : 1
2. 6 : 3 or 2 : 1
3. 28 : 14 or 2 : 1
4. equal

Complete the rule.
$a : b$

Practice
1. $\overline{XY}$, 24 : 8 or 3 : 1;
84, 28, 84 : 28 or 3 : 1

2. Ratio of sides is 4 : 1;
ratio of perimeters is 4 : 1
3. Ratio of sides is 8 : 5;
ratio of perimeters is 8 : 5

PAGE 55 *Area of a Triangle*

Practice

1. 15;
outside, 7;
$\frac{1}{2}(15)(7) = 52.5$ square units
2. $A = 20$ units^2 **3.** $A = 12$ units^2

PAGE 56 *Area of a Parallelogram*

Practice

1. 18;
8;
$A = (18)(8) = 144$ square units
2. $A = 64$ units^2 **3.** $A = 32$ units^2

PAGE 57 *Area of Similar Figures*

Explore the areas of similar figures.

	Length	Width	Area ($l \times w$)
ABCD	8	6	48
WXYZ	4	3	12

Complete the statements.

1. 6 : 3 or 2 : 1 **3.** 48 : 12 or 4 : 1
2. 8 : 4 or 2 : 1 **4.** squared

Complete the rule.

$a^2 : b^2$

Practice

1. $\overline{XY}$, 24 : 8 or 3 : 1;
$3^2 : 1^2 = 9 : 1$
2. Ratio of sides is 4 : 1;
ratio of areas is 16 : 1
3. Ratio of sides is 8 : 5;
ratio of areas is 64 : 25

PAGE 58 *Area of a Trapezoid*

Practice

1. Let $\overline{UT}$ be b_2; $\overline{UT} = 3$;
$\overline{RV}$; $\overline{RV} = 4$;
$A = \frac{1}{2}(9 + 3)(4) = 24$ square units
2. $A = 90$ units^2 **3.** $A = 100$ units^2

PAGE 59 *Area of a Rhombus or Kite*

Practice

1. $\overline{XZ}$, Let $\overline{XZ}$ be d_2; $\overline{XZ} = 20$;
$A = \frac{1}{2}(30)(20) = 300$ square units
2. $A = 126$ units^2 **3.** $A = 24$ units^2

PAGE 60 *Area of a Circle*

Practice

1. half, 6;
$A = \pi(6)^2$;
$A = (3.14)(6)^2 = 113.04$ square units
2. $A = 314$ units^2
3. $r = 3$
4. difference = 150.72 units^2

PAGE 61 *Area of a Sector of a Circle*

Practice

1. DF, 90°;
13;
Area $= \frac{90}{360} \times \pi \times 13^2$
$= 132.67$ units^2
2. 87.22 units^2 **3.** 56.52 units^2

PAGE 62 *Area of Regular Polygons*

Practice

1. $a^2 + 6^2 = 10.2^2$, 8.25;
5, $P = (5)(12) = 60$;
$A = \frac{1}{2}(8.25)(60)$
$= 247.5$ square units
2. $A = 480$ units^2
3. $A = 100$ units^2

PAGE 63 *Area of an Irregular Shape*

Practice

1. rectangle;
$\frac{1}{2}bh$, lw;
$A_{triangle} = \frac{1}{2}(4)(5) = 10$;
$A_{rectangle} = 8 \times 5 = 40$;
$10 + 10 + 40 + = 60$ square units
2. 16 square units
3. 110.2 square units

PAGE 64 *Comparing Area and Perimeter*

Explore the dimensions of rectangles.

Rectangle	Length	Width	Perimeter $2(l + w)$	Area $l \times w$
6 × 12	12	6	36	72
7 × 11	11	7	36	77
8 × 10	10	8	36	80
9 × 9	9	9	36	81

Complete the statements.

1. 12 × 6 **4.** square
2. 9 ×9 **5.** square
3. equal **6.** square, increases

Practice

1. 12 ×12
2. 40 × 40, 1,600 square units

PAGE 65 *Using Trigonometry to Find the Area of a Triangle*

Practice

1. $b = 6$; $c = 10$; $\angle X = 55°$;
$\frac{1}{2}(6)(10)(\sin 55°)$;
$A = \frac{1}{2}(6)(10)(0.82) = 24.6$ units^2
2. 17.21 units^2 **3.** 21.65 units^2

PAGE 66 *Geometric Probability*

Practice

1. Subtract
$A = \pi r_L^2 - \pi r_S^2$
$= (3.14)(10)^2 - (3.14)(6)^2$
$= 200.91$ square units;
larger;
$P = \frac{200.91}{314} \times 100 = 63.98\%$
2. 16.67% **3.** 52.9%

PAGE 67 *Types of Solids*

Label each of the parts of a solid.

Prism: lateral face, base
Pyramid: vertex, lateral face, base
Cylinder: lateral surface, base
Cone: lateral surface, base

Complete the chart.

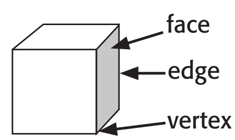
Figure	Base(s)	Lateral Face(s)
Prism	two, polygons	rectangles
Pyramid	one	triangles
Cylinder	two, circles	curved rectangle
Cone	one	curved surface (sector of a circle)

Practice

1. triangle;
2;
rectangle;
triangular prism
2. cylinder
3. rectangular pyramid
4. trapezoidal prism
5. cone

PAGE 68 *Solids and Euler's Formula*

Label and name the parts of a solid.



The rectangular prism has 6 faces, 12 edges and 18 vertices.

Practice

1. 2 triangles and 3 rectangles,
5 faces,
6 vertices;
$6 + 5 = E + 2$
$9 = E$
2. 12 vertices **3.** 10 edges

PAGE 69 *Surface Area: Prisms*

Practice

1. $A = \frac{1}{2}(8)(9) = 36$ square units
$36 \times 2 = 72$ square units;
$A = 8 \times 15 + 12 \times 15 + 9 \times 15$
$= 435$;
$72 + 435 = 507$ square units
2. $A = 990$ units^2
3. $A = 178$ units^2

PAGE 70 *Surface Area: Cylinders*

Practice

1. $r = 5$; $h = 10$;
$2(3.14)(5)(10) + 2(3.14)(5)^2$
$314 + 157 = 471$ square units
2. $A = 565.2$ units^2
3. $A = 226.08$ units^2

PAGE 71 *Surface Area: Pyramids*

Practice

1. rectangle, $A = lw$

$A = lw = (10)(10)$

$= 100$ square units;

$l = 15.6$

$p = 10 + 10 + 10 + 10 = 40;$

$\frac{1}{2}(40)(15.6) + 100 = 412$

2. $A = 465.6$ units^2

3. A=294.4 units^2

PAGE 72 *Surface Area: Cones*

Practice

1. $r = 5;$

$l = 12;$

$(3.14)(5)(12) + (3.14)(5^2)$

$= 188.4 + 78.5$

$= 266.9$ sq. units

2. $A = 452.16$ units^2

3. A=263.76 units^2

PAGE 73 *Surface Area of Similar Solids*

Complete the chart.

	Length	Width	Height	Surface Area
Rectanglular Prism A	4	6	6	168
Rectangular Prism B	2	3	3	42

Complete the statements.

1. 2 : 1 **3.** squared

2. 168 : 42 or 4 : 1

Complete the rule.

b^2

Practice

1. 12 : 4 or 3 : 1;

$\frac{468 \text{ cm}^2}{SA_{\text{Small}}} = \frac{3^2}{1};$

$SA_{\text{Small}}\ (9) = 468 \text{ cm}^2;$

$SA_{\text{Small}} = 52 \text{ cm}^2$

2. $A = 96 \text{ cm}^2$

PAGE 74 *Volume: Prisms*

Practice

1. $\frac{1}{2}(9) \times 8 = 36$ square units;

9;

$(36)(9) = 324$ cubic units

2. $V = 1{,}750$ units^3

3. V =140.30 units^3

PAGE 75 *Volume: Cylinders*

Practice

1. $r = 5;$

$h = 10;$

$V = (3.14)(5)^2(10)$

$= 785$ cubic units

2. $V = 1{,}017.36$ units^3

3. $V = 254.34$ units^3

PAGE 76 *Volume: Pyramids*

Practice

1. rectangle, $A = lw$

$A = lw = (10)(10) = 100$ square units;

12;

$V = \frac{1}{3}(100)(12) = 400$ cubic units

2. $V = 288$ units^3

3. $V = 256$ units^3

PAGE 77 *Volume: Cones*

Practice

1. 5;

11;

$V = \frac{1}{3}(3.14)(5)^2(11) = 287.83$ units^3

2. $V = 301.44$ units^3

3. $V = 188.4$ units^3

PAGE 78 *Volume of an Irregular Shape*

Practice

1. cone;

Cone: $V = \frac{1}{3}\pi r^2 h;$

Cylinder: $V = \pi r^2 h;$

$V_{\text{Cone}} = \frac{1}{3}(3.14)(3)^2(5) = 47.1$

$V_{\text{Cylinder}} = (3.14)(3)^2(11)$

$= 310.86;$

cone

$V_{\text{total}} = 47.1 + 310.86 = 358.96$

PAGE 79 *Volume of Similar Solids*

Complete the chart.

	Length	Width	Height	Volume
Rectanglular Prism A	4	6	6	144
Rectangular Prism B	2	3	3	18

Complete the statements.

1. 2 : 1 **3.** cubed

2. 144 : 18 or 8 : 1

Complete the rule.

a^3

Practice

1. 15 : 10 or 3 : 2;

$\frac{V_{\text{Large}}}{1130^3} = \frac{3^3}{2^3};$

$(8)V_{\text{Large}} = 30510 \text{ cm}^3$

$V_{\text{Large}} = 3813.75 \text{ cm}^3$

2. $V = 28.3 \text{ cm}^3$

PAGE 80 *Surface Area: Spheres*

Practice

1. diameter, 18, radius is 9;

$4(3.14)(9)^2 = 1017.36$ square units

2. 1256 units^2 **3.** 452.16 units^2

PAGE 81 *Volume: Spheres*

Practice

1. diameter, 18, radius is 9;

$V = \frac{4}{3}(3.14)(9)^3 = 3052.08$ units^3

2. 4186.67 units^3 **3.** 904.32 units^3

PAGE 82 *Surface Area and Volume: Formulas*

Complete the chart

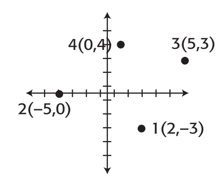
Type	Description	Surface Area	Volume
Prism	two, polygons, parallelograms	base, lateral faces	$V = Bh$
Pyramid	one, triangles	$SA = \frac{1}{2}pl +$ area of base	$V = \frac{1}{3}Bh$
Cylinder	two, circles, curved	$2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Cone	one, curved	$SA = \pi rl + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
Sphere	same, center	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

PAGE 83 *Plotting Points on a Coordinate Plane*

Practice

1. right, 2;

down, 3



5. $A(1, 9)$ **7.** $C(-2, -2)$

6. $B(0, -2)$ **8.** $D(6, 0)$

PAGE 84 *Graphing a Linear Equation*

Practice

1.

x	$2x + 6 = y$	y	
2	$2(2) + 6 = y$	10	(2, 10)
0	$2(0) + 6 = y$	6	(0, 6)
1	$2(1) + 6 = y$	8	(1, 8)
−2	$2(-2) + 6 = y$	2	(−2, 2)

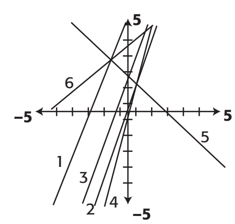
2. (−2, −5), (0, 1), (1, 4), (2, 7)

3. (−2, −1), (0, 3), (1, 5), (2, 7)

4. (−2, −10), (0, 0), (1, 5), (2, 10)

5. (−2, 5), (0, 3), (1, 2), (2, 1)

6. (−2, 4), (0, 5), (1, 5.5); (2, 6)



PAGE 85 *Distance Formula*

Practice

1. $x_1 = -2, y_1 = -1;$

$x_2 = 4, y_2 = 2;$

$d = \sqrt{(4 - (-2))^2 + (2 - (-1))^2}$

$d = \sqrt{6^2 + 3^2} = \sqrt{45} = 6.7$

2. $d = 18.4$

3. $d = 8.1$

4. $d = 7.3$

PAGE 86 *Midpoint Formula*

Practice

1. $x_1 = -2, y_1 = -4$;
$x_2 = 4, y_2 = 2$;
$(\frac{4 + (-2)}{2}, \frac{2 + (-4)}{2}) = (\frac{2}{2}, \frac{-2}{2})$
$= (1, -1)$

2. (3, 1)

3. (–3.5, 0)

4. (4, –5.5)

5. (4, –3)

PAGE 87 *Slope of a Line*

Practice

1. (x_2, y_2): (4, 5);
$\frac{5-2}{4-1} = \frac{3}{3} = 1$;
1

2. $\frac{3}{2}$

3. 1

4. $\frac{2}{5}$

5. $-\frac{3}{5}$

6. $\frac{2}{3}$

7. $\frac{6}{5}$

PAGE 88 *Slope Intercept Form*

Practice

1. $\frac{1}{2}$;
10

2. slope = 1, y-intercept = 3

3. slope = $-\frac{3}{4}$, y-intercept = –6

4. slope = $\frac{1}{2}$, y-intercept = 2

5. slope = $-\frac{2}{3}$, y-intercept = –3

6. $y = 4x - 2$

7. $y = -2x + 5$

PAGE 89 *Parallel Lines*

Practice

1. $-\frac{1}{2}x - 1$;
$3, -\frac{1}{2}$;
are not, are not

2. m = $-\frac{1}{4}$ for both equations; the graphs are parallel.

3. m = –2 for both equations; the graphs are parallel.

4. m = 2 and 1; the graphs are not parallel.

5. m = $-\frac{1}{4}$ and $-\frac{4}{3}$; the graphs are not parallel.

PAGE 90 *Perpendicular Lines*

Example

4. $y = -\frac{1}{3}x + 2$

Practice

1. –2;
negative, positive, 2;
$y = 2x + (-3)$

2. $y = -\frac{4}{3}x + 4$

3. $y = -\frac{1}{2}x + 3$

4. $y = \frac{1}{4}x - 5$

5. $y = -x + 1$

PAGE 91 *Point Slope Form I*

Practice

1. 6;
(–3, –1), –3, –1;
$y - (-1) = 6(x - (-3))$
or $y + 1 = 6(x + 3)$

2. $y - 1 = -\frac{1}{2}(x - 7)$

3. $y - (-3) = 2(x - (-3))$ or
$y + 3 = 2(x + 3)$

4. $y - (-5) = \frac{2}{3}(x - 4)$ or
$y + 5 = \frac{2}{3}(x - 4)$

5. $y - 3 = -3(x + 1)$

PAGE 92 *Point Slope Form II*

Practice

1. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-2)}{0 - (-2)} = \frac{-2}{2} = -1$;
–2, –2;
$y - (-2) = -1(x - (-2))$
or $y + 2 = -1(x + 2)$

2. $y - 2 = \frac{1}{2}(x - 2)$

3. $y - 4 = -1(x - (-6))$ or
$y - 4 = -1(x + 6)$

4. $y - 6 = 3(x - 2)$

5. $y - 2 = 1(x - 5)$

6. $y - 0 = \frac{2}{3}(x - 6)$ or $y = \frac{2}{3}(x - 6)$

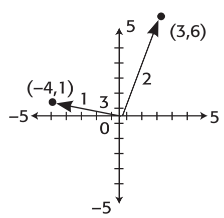
PAGE 93 *Adding Vectors*

Practice

1. (–3, –2), $x_1 = -3, y_1 = -2$;
(–1, 3), $x_2 = -1, y_2 = 3$;
(–3 + (–1), (–2 + 3)
(–4, 1)

2. resultant: (3, 6)

3. resultant: (–4, 1)



PAGE 94 *Translations*

Practice

1. +3, +3;
3, 3
+3, +3
+3, +3;
$A'(1, 2)$; $B'(3, 5)$; $C'(4, 3)$

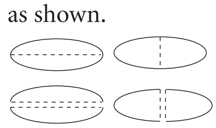
2. $A'(-4, -2)$; $B'(-3, 0)$; $C'(0, 0)$,
$D'(-1, -2)$

3. $A'(0, 3)$; $B'(2, 3)$; $C'(3, 1)$,
$D'(-1, 1)$

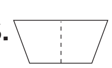
PAGE 95 *Symmetry*

Practice

1. Either a horizontal or a vertical line of symmetry can be drawn, as shown.



2. 

3. 

PAGE 96 *Dilations*

Practice

1. 2, 2;
2, 2
2, 2
2, 2;
$A'(2, 4)$; $B'(6, 10)$; $C'(8, 6)$

2. $A'(0, -6)$; $B'(-9, 0)$; $C'(0, 0)$,
$D'(-3, -6)$

3. $A'(0, \frac{3}{2})$; $B'(1, \frac{3}{2})$; $C'(\frac{3}{2}, \frac{1}{2})$,
$D'(-\frac{1}{2}, -\frac{3}{2})$

PAGE 97 *If-Then Statements*

Practice

1. numbers, 2
2, even
Sample: If a number is divisible by 2, then it is an even number;
even, are, true

2. *Sample:* If an odd number is greater than 10, then it is not a prime.
False: Sample: 11 is prime.

3. *Sample:* If a triangle is a right triangle, then it has only one 90° angle.
True: *Sample:* If a triangle does not have a 90° angle, it is not a right triangle.

PAGE 98 *Inductive Reasoning*

Practice

1. half;
half;
6.25

2.

3. 162, –486

PAGE 99 *Deductive Reasoning: Law of Detachment*

Practice

1. obtuse, 110°;
$\angle A$;
110°, obtuse, obtuse

2. *Sample:* Jamal is working in the library.

PAGE 100 *Deductive Reasoning: Law of Syllogism*

Practice

1. If an angle is obtuse, it has a measure greater than 90°;
If an angle is greater than 90°, it cannot be a complementary angle;
yes;
If an angle is obtuse, it cannot be a complementary angle.

2. *Sample:* If Pearl is reading a book, then it is a book by Stephen King.