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# Sustainable Logistics and Transportation

## Optimization Models and Algorithms

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# Sustainable Logistics and Transportation

## Optimization Models and Algorithms

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*To Murat Engin*
DIDEM CINAR

*To Marlyse Kathryn*
KONSTANTINOS GAKIS

*To Rosemary*
PANOS M. PARDALOS

# Preface

In recent years, academic studies and practical projects produce very efficient methodologies to provide sustainable logistics and transportation including environmental, economic, and social measures. The main focus of this book is to bring together the novel and recent optimization approaches for sustainable logistics and transportation operations. It provides readers the recent developments in optimization area within the context of transportation and logistics planning having the sustainability perspective.

The book provides a valuable source for university professors, researchers, as well as professionals in supply chain management. Researchers and professionals in urban and regional planning and upper-level students interested in logistics and transport systems and optimization can also utilize this book. Universities having industrial and system engineering, business administration, computer and information science, and/or mathematics departments and any organizations having transportation and logistics activities have potential readers for the book. The book can be utilized in the scope of optimization, supply chain management, transportation, and logistics courses in universities.

The book includes 10 chapters. The first chapter gives recent trends and challenges for sustainable transportation and logistics management. The rest of the book is organized into two main parts focusing on deterministic models and uncertainty. Each part includes theory and methodologies developed for various practical sustainable transportation and logistics problems as well as a deep literature review.

We would like to thank the authors of the chapters, the referees, and the publisher for their participation in producing this book. We hope that this book will be a helpful tool for researchers and professionals working in the field of logistics and transportation.

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May, 2017

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# Toward Sustainable Logistics

**Mehmet Soysal and Jacqueline M. Bloemhof-Ruwaard**

**Abstract** The fast evolution of sustainability leads to the development of a new fast-growing concept called sustainable logistics management. This research addresses recent business trends and challenges in logistics and their implications for sustainable logistics management. Additionally, we discuss policy and research developments in resource-efficient logistics and present several practice examples in sustainable logistics management from transportation business companies. The conducted research on relevant literature and sustainability reports of several leading logistics companies shows that the logistics sector is committed to sustainable development and continually looks for ways to be environmentally and socially responsible and more efficient right across the organizations.

## 1 Introduction

Logistics is the management of the flow of things between the point of origin and the point of consumption in order to meet customer requirements. Logistics covers several working areas including material handling, production, packaging, transportation, inventory management, and warehousing. Logistics contributes to an organization's success by providing the right product, at the right price, at the right store, with the right quantity, to the right customer, at the right time. The logistics sector contributes to economic growth and international competitiveness.

With increasing freight volumes due to the growing population and internationalization of markets, policy measures and strategies are developed by countries to

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increase the efficiency of freight logistics and improve supply chain sustainability. Supply chain sustainability is about consideration of environmental and social externalities of operations in supply chain management in addition to the traditional economic concerns [7]. Improving supply chain sustainability in logistics has become one of the major topics for researchers and practitioners in the last decade. On 15 August 2016, Google and Google Scholar returned about 141,000 and 2670 hits, respectively, for the English term “supply chain sustainability” that demonstrates the popularity of the term.

The fast evolution of sustainability leads to the development of a new fast-growing concept called sustainable logistics management (SLM). Therefore, for the last two decades, logistics systems have seen the transition from traditional logistics management to SLM. SLM aims to improve the supply chain sustainability in logistics systems. In this context, SLM enables the organizations to fulfill market demand by providing the right product, at the right price, at the right store, with the right quantity, to the right customer, at the right time while being as sustainable as possible.

The policy document white paper authored by the European Commission [15] states that shifting freight transport from road to rail or waterborne transport, promoting eco-innovation in freight logistics, developing multimodal freight corridors, supporting new vehicles and vessels, optimizing the performance of multimodal logistics networks, and creating frameworks for seamless information flow in the logistics chain are among the goals for achieving competitive and resource-efficient transport system in the European Union (EU). As it has been explicitly mentioned in Regulation (EU) No 1315/2013 of the European Parliament[1], to have a sustainable logistics system, member states of the EU shall pay attention on the EU goals while setting their own strategies.

In line with these policy measures and strategies, the European Union has launched several projects (e.g., SALSA[2], SCALE[3], NexTrust[4], SuperGreen[5], etc.) to increase efficiency and sustainability in logistics. The topic is accordingly on the agenda of both researchers and practitioners. Numerous qualitative and

[1]http://eur-lex.europa.eu/eli/reg/2013/1315/oj Regulation (EU) No 1315/2013 of the European Parliament and of the Council of 11 December 2013 on Union guidelines for the development of the trans-European transport network and repealing Decision No 661/2010/EU Text with EEA relevance. Online accessed: August 2016.

[2]http://www.salsaproject.eu/ The EU project SALSA aims to develop strategies to improve sustainable logistics in beef and soy supply chains from Latin America to the EU. Online accessed: August 2016.

[3]http://sfcplatform.eu/ The EU INTERREG NWE Project aims to optimize the triple bottom line of logistics for agri-food businesses. Online accessed: August 2016.

[4]http://nextrust-project.eu/ The EU project NexTrust aims to increase efficiency and sustainability in European logistics through boosting collaboration among logistics partners. Online accessed: August 2016.

[5]http://www.supergreenproject.eu/project.html The EU project SuperGreen aims to support the development of sustainable transport networks by fulfilling requirements covering environmental, technical, economic, social, and spatial planning aspects. Online accessed: August 2016.

quantitative studies have been conducted to develop innovative solutions to enhance the sustainability and efficiency of the logistics systems.

To sum up, the interest in designing and redesigning sustainable logistics networks has been increasing in the past decades. This research addresses recent business trends and challenges in logistics and their implications for sustainable logistics management. Additionally, we discuss policy and research developments in resource-efficient logistics and present several practice examples in SLM from transportation business companies. To get information about the field, relevant literature in operations research and sustainability reports of several leading logistics companies have been used. We would like to note that the choice of representative papers and company reports to illustrate the field is subjective.

The next section presents the main business trends and challenges in logistics. The subsequent section is followed by a detailed discussion on sustainable logistics management, including main motivations behind sustainable logistics and decision support models for SLM. This section is followed by presenting practical examples in SLM from transportation business companies. The last section presents conclusions drawn from the research.

## 2 Main Business Trends and Challenges in Logistics

In todays' global world, logistics is key for international trade and contributes to the prosperity and welfare of the nations. Materials, food, and products are distributed from where they are extracted, harvested, or produced to the nearby stores through logistics chains. Current logistics systems cause serious and in the long run unacceptable environmental and social damage due to, for instance, hazardous emissions, congestion, stench, noise, and the high price that has to be paid in terms of infrastructural load [46]. Below, this section briefly describes the main trends and challenges that have potential to influence the shape of the future of logistics:

- *Aging and increasing population:* World population aging is projected to accelerate in the coming years. The number of European people aged 65 or more is expected to increase from 17 to 30% of the population, by 2060 [14]. Especially in developed countries, the aged population has good socioeconomic and health conditions which motivates them to travel and see new places in the world. An aging society clearly will place more emphasis on secure and reliable transport services [14]. Note that the efficiency of logistics has a direct impact on security of the operations. We are also confronted with the fact that the world population is still growing. More people means more mobility and more transport, which might become a burden in terms of density on future transport operations.
- *Globalization:* The usage of the standard shipping container in international trade, the liberalization of international trade that reduces restrictions/barriers on the free exchange of goods between countries, the expansion and improvement

of international transport infrastructure, and the production and logistics cost differentials among countries mainly lead to the increasing of globalization since the 1970s [22]. The overall economic growth in the globalized world has led to the movement of large streams of goods (and also people) all over the world, which is expected to increase even more in the future. Accordingly, production, material handling, transportation, storage, and consumption of all these goods are important logistics-related concerns [9].

- *Technology:* Revolutionary developments in transport and communication technologies will reshape the future logistics systems. Advanced information and communication technologies [42], fuel-efficient fleet technologies and design improvements [30], and cooled (reefer) containers and data loggers for temperature history [9] can be given as some of these recent technologies that have already affected the operations in logistics.
- *Increasing e-commerce:* Rapid advance of e-commerce has a profound impact on both forward and reverse logistics [12]. Logistics systems are under pressure not only for fast delivery to customers but also for proper collection of the returned items due to several reasons such as change of mind, defective item, or late delivery. This progress has brought the term "closed-loop supply chains" which are supply chain networks that include the return processes besides the forward product movements. Numerous researchers on logistics field address this problem to have a better logistics system (see [2, 6, 20]). The literature review paper by Govindan et al. [21] on reverse logistics and closed-loop supply chain can be consulted for more information on the topic.
- *Increasing scarcity of fossil fuels:* Oil and other fossil fuels (e.g., coal, gas) are expected to become more expensive as demand increases and sources dry up [14]. According to a projection made by the European Commission, oil prices are expected to more than double in 2050 compared to the 2005 level of 59 $/barrel [16]. The increasing scarcity of fossil fuels will be reflected by the introduction of advanced methods to increase efficiency for decreasing the amount of fuel consumed per unit of output. The new methods or approaches to increase fuel usage efficiency would be (i) to use more fuel-efficient vehicles (see [29, 40]), (ii) to have optimized logistics routes using detailed fuel consumption estimations (see [4, 19]), (iii) to decrease empty returns through increased collaboration in the logistics network (see [36, 38]), or (iv) to educate drivers toward fuel-efficient driving (see [28, 45]).
- *Relationships and outsourcing:* Vertically and horizontally enlarged logistics networks will require increased collaboration and outsourcing among supply chain actors. For example, transportation and warehousing facilities can be shared by two competitors to realize fuel consumption decrease through avoiding empty vehicle runs [22]. An innovative solution in collaborative transport is referred to as "carpooling for cargo." Logistics companies are encouraged to bundle freight flows on transport modes to increase vehicle utilization, reduce empty movements, and stimulate co-modality, which allows sustainable utilization of resources [42]. Such horizontal collaboration between industry partners improves transport efficiency through increased capacity use. Another

example, many firms prefer to outsource their logistics activities to third-party logistics firms such as DHL or UPS to perform logistics operations which are not regarded as a firm's core competencies [22]. In literature, vertical and horizontal collaborations among supply chain actors have been addressed in many studies (see [23, 31, 41]).

- *Environmental and social challenges:* Another recent trend in logistics sector is to manage negative environmental and social externalities of operations. This means that environmental and social issues have been brought to the agenda of the decision-makers in logistics apart from the economic concerns. In line with that, researchers have started to focus on logistics problems by taking three main sustainability dimensions into account. For a detailed information on attempts that incorporate sustainability in logistics decision-making process, the reader is referred to the literature reviews by Seuring [35], Eskandarpour et al. [13], and Fahimnia et al. [18]. The main logistics-related environmental impacts are climate change, air pollution, noise pollution, energy use/energy efficiency, renewable energy use, biodiversity, land usage, and waste from packaging or shipping. Additionally, mobility of citizens, accessibility, employment level and conditions, and health and safety incidents can be given as foremost important logistics-related social impacts.
- *Urbanization:* The number of cities in the world with over ten million inhabitants is getting larger and larger. The logistics impacts such as congestion, noise hindrance, and air pollution are confronted in high-density urban areas. While planning delivery to urban areas, the last mile of the logistics chain is the most challenging one, since it accounts for a large proportion of shipment costs and complexity of operations [42]. Therefore, this part is often the most inefficient. The continuing urbanization, hence, requires different modes of transport and logistics systems than available today [12]. For instance, several projects (e.g., CIVITAS[6] and ELCIDIS[7]) have been undertaken in recent years to manage freight transportation in urban areas. Accordingly, several attempts on the urban distribution planning problem exist in literature such as incorporating time-dependent (e.g., [11, 26]) or stochastic (e.g., [3, 25, 34]) vehicle speed into vehicle routing problem to better manage product delivery in congested areas.

The desire for improving sustainability has mainly brought the above-listed logistical main business trends and challenges into the agenda of the logistics companies. These trends and challenges contribute to the transition toward SLM. Logistics companies have to meet the challenges that sustainability brings to their business. In this context, sustainable logistics is concerned with not only economic issues but also with environmental and social ones associated with the movement of goods through a supply chain [37].

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[6] An initiative which was launched in 2002 to redefine transport measures and policies in order to create cleaner, better transport in cities. http://www.civitas.eu/. Online accessed: August 2016.

[7] A project about electric vehicle city distribution system in Rotterdam, Netherlands. http://www.managenergy.net/resources/779. Online accessed: August 2016.

# 3 Sustainable Logistics Management

In most cases, the primary objective of traditional logistics management is to maximize profitability. Profitability calculations include only the economic costs that companies directly incur. Therefore, the wider environmental and social costs have been largely ignored in balance sheets, until recently [27]. SLM requires to manage multiple Key Performance Indicators (KPIs) from three pillars (economic, environmental, and social pillars) of sustainability. Table 1 presents an exemplary set of KPIs for sustainable logistics management.

Table 1 reveals that a company which would like to improve its sustainability performance first has to assess environmental and social externalities of its operations, assuming that the economic KPIs are already known. This assessment would allow company to identify the main environmental and social KPIs for SLM. Once the KPIs are defined, the final challenge is to achieve a more sustainable balance between economic, environmental, and social KPIs.

Here the question is why companies should bother themselves with the environmental and social objectives? Or, what are the main motivations behind being more sustainable? The coming subsection has been proposed to briefly find answers to these questions.

## *3.1 Main Motivations Behind Sustainable Logistics*

The main motivations behind the desire for paying attention on the sustainability can be summarized as follows [10]:

- *Legislative changes:* Legal restrictions on logistics exist to contribute to sustainable development, e.g., enhanced traceability and emission reduction regulations. Enhanced traceability regulations enable to preserve interest in tracking both forward product movement and reverse flows of secondary packaging and material handling equipment associated with product shipment. Enhanced emission reduction regulations enable to preserve interest in transportation energy efficiency improvement and emission reduction opportunities. For instance, there are several international standards available for airports (e.g., ISO 14001, Environmental Management Systems; ISO 50001, Energy Management Systems; or EU EMAS, EU Eco-Management and Audit Scheme) to manage their environmental performance [17]. Airports will comply with these requirements due to EU and national environmental management. These sorts of legal obligations place further pressure on companies.
- *Social attitudes*: Stakeholders' concerns on being more sustainable have been growing. They have started to become more aware of how supply chain activities cause damage to the environment and society, and they are increasingly interested in how companies are addressing sustainability challenges in their operations. The growing awareness on the topic affects both purchase decisions

**Table 1** Exemplary set of Key Performance Indicators for sustainable logistics management

| Pillars | Key Performance Indicators |
|---|---|
| Economic | Total logistics cost incurred |
| | Variance of the total logistics cost |
| | On-time delivery |
| | Late delivery |
| | Missed sales |
| | Order cycle time (lead time) |
| | Transport carriers utilized |
| | Output growth |
| | Labor productivity |
| Environmental | Product waste occurred |
| | Packaging waste occurred |
| | GHG emitted |
| | Energy used |
| | Energy from renewable sources |
| | Water used |
| | Fuel consumed |
| | Land used for production |
| | Amount of soil degradation |
| | Biodiversity affected |
| | Eco-efficiency |
| | Amount of returned product recycled |
| | Ethical transport care |
| Social | Distance between grower and distributor |
| | Profit distribution among supply chain actors |
| | Product quality |
| | Quality of life and working satisfaction |
| | Number of accrued jobs |
| | Number of accidents |
| | Contribution to traffic congestion |
| | Contribution to traffic noise |
| | Regulatory compliance |
| | Public reporting of environmental performance |

Studies of Soysal et al. [39] and Bloemhof-Ruwaard and Soysal [5] have been used while preparing the exemplary set

and consumer choices, e.g., going for the product that has a producer looking like more environmentally friendly than the competitors. Business attitude has been also changing. International companies such as Unilever or P&G are paying attention to the sustainability performances of the companies while selecting their supply chain partners. For example, Unilever sets mandatory requirements for its suppliers to establish and maintain a business relationship with Unilever. Some of these requirements are as follows: all workers are treated equally and with respect and dignity; workers' health and safety are protected at work; land rights of communities, including indigenous peoples, will be protected and promoted; business is conducted in a manner which embraces sustainability and reduces environmental impact [43].

- *Corporate commitment*: Business leaders are becoming aware of the fact that a sustainability era has already started, and they have to search for opportunities to ensure their business excels in the new era. Business leaders know that success in the sustainability era depends on strengthening the commitment to sustainability principles. Logistics company UPS has a commitment on sustainable development and leads other companies toward greater awareness and actions. To do so, company focuses on three environmental issues most related to UPS and its stakeholders: energy, emissions, and fuel supply [44].
- *Brand value*: Being more environmentally friendly or socially responsible can be regarded among the foremost important factors that have influence on customer perception and behavior. This motivates companies to take further actions on environmental and social policies. There exists a community called "Sustainable Brands" that is a learning collaboration and commerce community of over 348,000 sustainable business leaders from around the globe.[8] The community aims to inspire, engage, and equip its community to profitably innovate for sustainability.
- *Competitive advantage*: Leading organizations have seen the advantages of pursuing a sustainable business model for both the environment and society by managing externalities of the supply chain operations. Now, they would like to harness the associated benefits of being responsible organizations. Organizations are more aware of economic benefits of being sustainable; they have started to realize that win-win situations can be attained, which means that reducing externalities might provide economic benefits in the long run. For instance, reducing inputs, waste, and emissions through innovative applications contributes both to economic and environmental performance. Alignment with governmental regulations on environment and society enables to avoid paying fines or penalties and therefore can also contribute to the competitive advantage of companies. To sum up searching for ways for being more sustainable can create new source of competitive advantage.

## *3.2 Decision Support Models for SLM*

SLM has to manage a broad set of KPIs from economic, environmental, and social pillars of sustainability, as shown in Table 1. Obviously, there exist trade-offs among these indicators. Then, the challenge for logistics decision-makers is to incorporate these dimensions into the decision-making process. Focusing only on profit while

[8] http://www.sustainablebrands.com/about#null. Online accessed: August 2016.

planning logistics operations without respecting environmental and social impacts as in traditional logistics management does not guarantee long-term success for companies. There has to be a balance among economic, environmental, and social objectives when devising logistics operations strategy, and the relevant decisions are being made. These aspects increase the need for advanced decision support models for SLM that can capture current trends and challenges in logistics.

SLM has increased complexity due to the aforementioned transition from traditional logistics management, i.e., extension of indicators that have to be controlled for achieving sustainability. At this point, operations research models can be used to aid decision-making process in SLM. Especially in the last decade, researchers' tendency to address problems that have sustainability concerns has increased. A common interest is to increase the efficiency of logistics and improve supply chain sustainability by means of advanced models that incorporate the environmental and social KPIs besides the traditional ones.

Modeling approaches in SLM can be grouped into five categories: (i) mathematical programming methods such as linear programming, mixed integer linear programming, or dynamic programming models; (ii) simulation methods such as discrete event simulation, discrete time simulation, or Monte Carlo simulation; (iii) heuristic methods such as artificial intelligence techniques (approximate dynamic programming, neural networks, etc.), meta-heuristics (genetic algorithm, particle swarm optimization, etc.), or simple heuristics (nearest neighbor heuristic, greedy heuristic, etc.); (iv) hybrid models such as optimization-simulation hybrid methodologies or optimization-heuristics hybrid methodologies; and (v) analytical models such as multi-criteria decision-making methods, game theory-based approaches, or life cycle analysis-based approaches. Interested readers can be referred to the recent literature reviews by Hassini et al. [24], Seuring [35], and Brandenburg et al. [7] to get more information about the proposed decision support models for SLM.

As it has also been indicated by the literature reviews on the field, SLM requires more advanced models to capture the recent sector dynamics. Operations research models are mainly interested in finding ways to improve profitability and often do not care for operations impact on environment and society [9]. However, the field is evolving, and there exists a growing interest in how to contribute to the sustainable logistics through providing better models for logistics decision-makers. This progress is obviously beneficial for both society and industry, as improvements in decision support models will contribute to the development of sustainable logistics networks.

## 4 Sustainability on Transportation Business Companies

Sustainability is on top of the agenda of logistics companies for the coming years and requires proper strategic management. The adoption of environmentally friendly logistics networks is required for logistics companies. They should be aware of the trade-offs among three pillars of sustainability: economic, environmental, and social goals. Accordingly, companies in logistics sector have started to take actions for further improvement in sustainability. This section presents examples related to SLM from transportation business companies.

To share practice examples in SLM from transportation business companies, the Dow Jones Sustainability Indices (DJSI) have been used. For investors who believe that sustainable business practices may lead to long-term shareholder value and who wish to reflect their sustainability convictions in their investment portfolios, DJSI can be used as a family of benchmarks [32]. The DJSI family tracks the stock performance of the world's leading companies in terms of economic, environmental, and social criteria and provides benchmarks for investors to manage their sustainability investment portfolios [32]. RobecoSAM's Corporate Sustainability Assessment, which was developed in 1999, is used to analyze the companies' sustainability profiles. More information on the methodology can be obtained from [33].

To present practice examples in SLM from transportation business companies, three transportation companies have been selected from the Dow Jones Sustainability World Index, 2015. These companies are the Canadian National Railway Co (Canada), PostNL NV (the Netherlands), and Air France-KLM (France).

### *4.1 Understanding Stakeholders' Expectations*

Companies organize meetings with their stakeholders to understand their expectations. There can be many sub-motivations behind setting such meetings; the main ones can be listed as follows: (i) to identify the topics that matter most to their business, (ii) to focus their strategic priorities, (iii) to refine their reports prepared to inform stakeholders, and (iv) to inform stakeholders about the evolution of their sustainability programs.

In 2015, the Canadian National Railway Co conducted a stakeholder engagement exercise to reassess their sustainability priorities and inform the content of the sustainability report. They engaged with approximately 200 stakeholders through an electronic survey to understand the sustainability topics that most influence their decisions or perspectives of the company. The group involves employees, suppliers, governments, railway associations, customers, investors, unions, Aboriginal people, community groups, and nongovernmental organizations. While selecting stakeholders, their geographic representation and their influence and interest in companies' business are considered. Figure 1 presents the prioritization matrix that plots the most important topics.

### *4.2 Setting Sustainability Objectives*

Companies aim to reduce their environmental and social impacts by improving their operations and processes, partnering with the other companies that are willing to

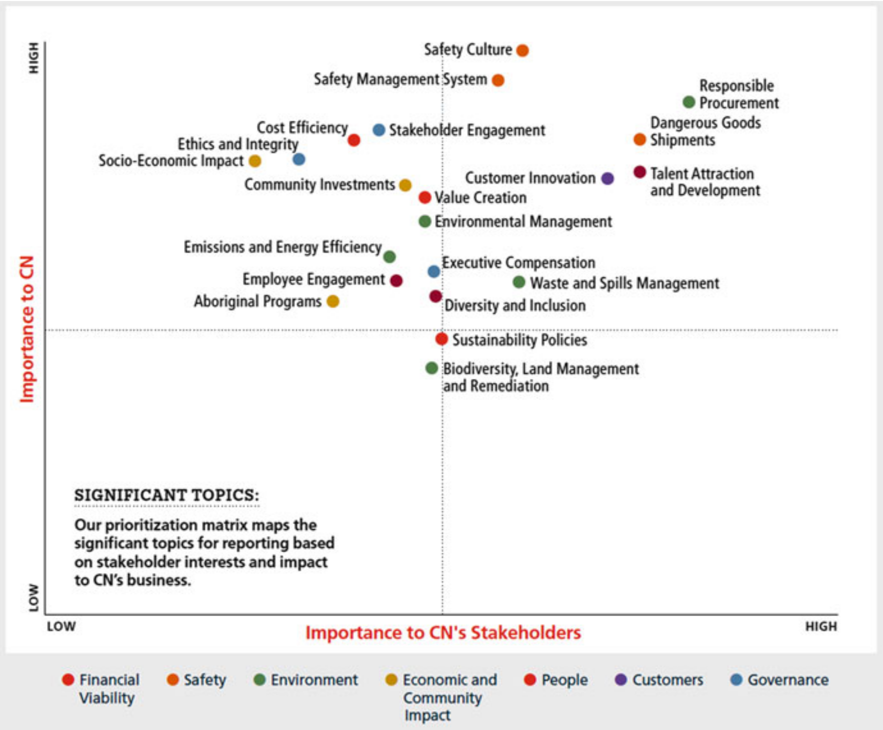


**Fig. 1** Prioritization matrix for the Canadian National Railway Co (Source: Canadian National Railway Co document [8]). In this figure, CN refers to Canadian National Railway Co

be environmentally responsible organizations and innovating in the supply chain. Accordingly, each year commitment to sustainability improvement is formalized through setting action plans with specific targets.

Air France-KLM aims to improve its sustainability performance and set the targets related to carbon footprint, biofuels, noise, local air quality, waste, energy, and biodiversity. Table 2 presents environmental and social targets for Air France-KLM. To achieve the goals set, a diverse set of measures has been implemented, focusing mainly on fleet renewal, operational efficiency, sustainable biofuels, and carbon offsetting. After defining these sorts of future goals, annual action plans are set, and progress is monitored annually by the company.

## 4.3 *Defining KPIs to Measure Sustainability Performance*

To maintain competitive focus, enhanced performance, and continuous improvement, companies define KPIs to measure their sustainability performance. Cross-functional sustainability committees in companies regularly assess the

**Table 2** Environmental and social targets for Air France-KLM

| Objectives | Schedule | Main achievements 2014 |
|---|---|---|
| *Carbon footprint* | | |
| Reduce $CO_2$ emissions by 20% compared to 2011 (tons/km) | 2020 | 6.7% reduction compared to 2011 |
| Operate regular flights powered by sustainable biofuel | 2014 | 11 flights operated |
| *Biofuels* | | |
| Create a market for sustainable biofuels | 2020 | 15 partners in KLM Corporate Biofuel Program |
| *Noise* | | |
| Expand activity while keeping noise emissions below 2005 levels | 2014 | 40.1% reduction in noise energy compared to 2005 |
| Reduce noise at Schiphol in partnership with the aviation sector | 2014 | 25.8% reduction in noise energy compared to 2000 |
| *Local air quality* | | |
| Fleet modernization | 2015 | A320s fitted with noise reduction kits by 2016 |
| Ground operations: electrically powered equipment | 2014 | Almost 50% of ground support equipment are electric |
| Implementation of fuel-efficient taxiing solutions | 2014 | TaxiBot project: towbarless towing taxiing operation for long-haul aircraft |
| *Waste* | | |
| 100% of nonhazardous waste and 60% of hazardous waste recovered | 2020 | Nonhazardous waste recycled, 90%; hazardous waste recycled, 43% |
| *Energy* | | |
| 2% annual improvement in the energy efficiency of buildings | 2016 | Plus 1.9% energy savings and 0.8% renewable energy generated by KLM's installations |
| *Biodiversity* | | |
| 100% responsible inflight catering products (from Amsterdam) | 2020 | 15 new products added to responsible inflight catering offer |

Source: Air France-KLM document [1]

**Table 3** Defined Key Performance Indicators for performance assessment in the Canadian National Railway Co

| Pillars | Key Performance Indicators |
|---|---|
| Environment | Total GHG emissions |
| | Total direct and indirect energy consumed within the organization |
| | Nitrous oxides |
| | Sulfur dioxide |
| | Particulate matter |
| | Total weight of waste generated |
| | Spend on biodiversity site assessments and remediation |
| Safety | % of workforce represented in joint union-management committees |
| | Injury frequency rate |
| | Lost time injury frequency rate |
| | Accidents |
| | Fatalities |
| People | Total number of full-time employees (end of the year) |
| | Permanent contract |
| | % of employees covered by collective bargaining agreements |
| | Total employee new hires |
| | % employee turnover number |
| | Average employee training hours |
| | Females |
| | The diversity % for Canada (visible minorities, persons with disabilities, and Aboriginals) |
| | The diversity % for the USA (minority) |
| Community and economy | Direct economic value generated (revenue) |
| | Operating costs |
| | Payments to providers of capital – dividends |
| | Payments to Canadian tax authorities |
| | Payments to US tax authorities |
| | Community investment |

Source: Canadian National Railway Co document [8]

sustainability performance through the defined KPIs and are responsible for ensuring effective implementation of companies' sustainability priorities and projects.

The sustainability performance assessment in the Canadian National Railway Co is based on a thorough analysis of its economic, environmental, and social performance, assessing issues related to environment, safety, people (employee), and community and economy. Table 3 presents defined KPIs developed for performance assessment in the Canadian National Railway Co.

Executive compensation packages in the company comprise not only issues about individual performance but also practices on supporting safe and reliable operations and ensuring environmentally and socially responsible operations. For instance, it includes instilling a strong safety culture and reducing injuries and accidents, improving fuel and emissions efficiencies, deepening employee engagement through the workforce, and ensuring solid relationships with key stakeholders.

## *4.4 Initiating New Projects to Improve Sustainability*

Environmentally friendly companies are aware of their role and impact on the environment and on the society in which they operate. They take corporate responsibility seriously. The corporate responsibility in PostNL NV is based on three pillars: being a good employer, managing sustainable operations, and a livable society.[9]

- *Being a good employer:* Companies want their employees to feel at home and are given the space to work on their own future prospects. The company achieves this through several ways. They build employee engagement by carrying out annual surveys, asking employees how they feel about PostNL as an employer. Moreover, having a multicultural diversity and encouraging more women to reach for the top make the organization stronger and more innovative and attractive.
- *Sustainable operations*: As a logistics company, PostNL NV continuously works on keeping its environmental and social impacts as small as possible through different practices. The company would like to reduce their sizeable fleet's emissions through using e-scooters, biogas, or training drivers on fuel-efficient driving. The company also looks continuously for new ways to recycle or reuse their materials. Moreover, PostNL purchases 100% carbon-neutral energy. They have initiated a new project about installing solar panels on their parcel sorting and distribution centers in the Netherlands. Finally, they have a pilot study with the Municipality of Delft on using smaller electric vehicles and more efficient routes to achieve environmentally friendly transport and to create a more livable city center. They are planning to implement this project in other municipalities in the near future as well.
- *Livable society*: The company searches opportunities to use its network for more sustainability in Dutch cities with smart city solutions. Some ideas to contribute to a more livable society are meeting elderly lonely people and keeping an eye on rubbish or graffiti in public spaces.

[9] http://www.postnl.nl/en/about-postnl/about-us/cr/. Online accessed: August 2016.

## 5 Conclusions

This research addresses developments toward sustainable logistics. Accordingly, recent business trends and challenges in logistics and their implications for sustainable logistics management have been discussed. Moreover, the research presents policy and research developments in resource-efficient logistics and presents several practice examples in SLM from transportation business companies.

The conducted research on relevant literature and sustainability reports of several leading logistics companies shows that the logistics sector is committed to sustainable development and continually looks for ways to be environmentally and socially responsible and more efficient right across the organizations. Considering environmental and social factors as well as economic benefits, therefore, becomes crucial in logistics sector. The main lessons that can be drawn from this research can be summarized as follows:

- The term "supply chain sustainability" is getting more popular in the logistics sector, and the fast evolution of sustainability leads to the development of a new fast-growing concept called sustainable logistics management.
- The trend toward ensuring sustainability has mainly brought new business trends and challenges in logistics into the agenda of the logistics companies. These trends and challenges contribute to the transition toward SLM. Logistics companies have to (i) meet the challenges that sustainability brings to their business, (ii) change the way they manage their supply chains, and (iii) find innovative ways for improving their operations to gain a competitive advantage.
- The main motivations behind the desire for paying attention on the sustainability can be listed as follows: legislative changes, social attitudes, corporate commitment, brand value, and competitive advantage.
- The research field on developing decision support models for sustainable logistics is evolving, and there exists a growing interest in how to contribute to the sustainable logistics through providing better models for logistics decision-makers.
- Practice examples in SLM from three transportation business companies existing in the Dow Jones Sustainability World Index, 2015, present (i) the importance of the sustainable logistics for the sector and (ii) main measures and actions taken by companies toward achieving sustainable logistics.

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# Part I
# Deterministic Models

# Transportation Network Regulation for Air Pollution Minimization

**Manon Raap, Maximilian Moll, and Stefan Pickl**

**Abstract** Densely populated areas are often associated with high pollution of the air. In order to decrease the air quality (health) index and thereby improve the quality of life, we aim to minimize air pollution of a transportation network especially in those highly air-polluted areas. Even though some transportation providers have invested to "green up" their operations, some can or will not sufficiently trade off their profit for reducing $CO_2$ emissions. Therefore, governmental regulation on transportation routing is necessary. This chapter is concerned with a novel problem of optimizing a governmental regulation plan, by reducing the capacity of roads. The goal is to find a regulation plan that minimizes the air pollution in dense areas on the transportation network. As a result, transportation providers must reroute their trucks in order to disburden the highest polluted areas. We propose a mixed integer linear program to solve this problem and show the applicability and low computation times of our solution in computational experiments.

## 1 Introduction

Sustainability is a rising concern in freight transportation networks because the awareness of its strain to the environment has increased. According to [7] sustainability depends on environmental, economic, and social factors. Due to the importance of finding solutions that account for these factors, there is a rapidly growing literature on this topic. An extensive literature review on the contribution of operations research to green supply chain and logistics is given in [12], in which the focus lays on planning and control of supply chain activities with respect to $CO_2$ emissions. Ramos et al. [15] aim to support tactical and operational planning decisions of reverse logistics systems while considering environmental, economic, and social objectives. Bektaş and Laporte [2] present the Pollution-

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Routing Problem, an extension of the classical vehicle routing problem, where the objective function accounts not just for the travel distance but also for the amount of greenhouse emissions, fuel, travel times, and costs. This problem is solved using a heuristical adaptive large neighborhood search by [4]. For a bi-objective extension of the Pollution-Routing Problem, Demir et al. [5] propose a combination of the adaptive large neighborhood search with a speed optimization procedure. Govindan et al. [8] aim to integrate sustainability in decision-making on distribution in a perishable food supply chain network, by introducing a two-echelon location-routing problem with time windows. Omidvar and Tavakkoli-Moghaddam [13], in particular, focus on the vehicle routing for alternative fuel vehicles (e.g., hybrid, electric, and fuel cell vehicles) minimizing the energy consumption and furthermore account for congestions at fueling stations.

A main limitation of such approaches is that transportation providers may not be willing to sufficiently trade off their profit for reducing $CO_2$ emissions. The cumulative emission of individual Pareto-optimal routings may still exceed an acceptable threshold. On the basis of the ongoing debate on environmental issues, it seems fair to suggest that governmental regulation on transportation routing is necessary. Hull [9] explores the issues of such policy integration and the according implementation mechanisms leading toward a sustainable transport system.

Employment of truck tolling schemes can be a mechanism for this purpose. Truck tolling schemes already exist in Switzerland, Austria, and Germany [11]. A postponed British system would have allowed tolls to vary by vehicle type, class of road, geographical area, and time of day. A review of worldwide truck road user tolling schemes is given in [3], in which the variables used to determine the rates for heavy trucks within different tolling schemes are examined. Capacity limitation of roads can be another mechanism to accomplish less $CO_2$ emissions locally, which is often necessary in urban regions. The mathematics behind optimal reduction of capacities on a network is widely studied in literature and is well known as the problem of network interdiction [17]. Even though most work on network interdiction aims for goals that maximally disrupt the enemy [10, 18, 16, 1], it is closely related to the problem of reducing capacity on the network to disburden highly air-polluted areas.

This chapter is concerned with the problem of optimizing a regulation plan that minimizes the air pollution in dense areas on the transportation network by reducing capacity. Since this is likely to lead to congestion on profitable roads with limited capacity, we incorporate a tolling scheme in order to avoid congestion. This results in a game in the sense that the regulator optimizes its strategy based on the optimal routing of the transportation providers *given* that strategy. Therefore, the solution approach in this chapter is inspired by the work of [14] in which a security strategy against multiple adversaries is optimized.

The remainder of this chapter is organized as follows. In Sect. 2, a formal description of the problem is given. A model and efficient reformulation hereof is described in Sect. 3, followed by the computational experiments in Sect. 4. Finally, we conclude this chapter in Sect. 5.

## 2 Problem Description

This chapter has a focus on an optimization problem[1] in order to disburden the highest air-polluted areas on a transportation network. The transportation network is described by the directed graph $(V, E)$ where $V$ is the set of nodes and $E \to V \times V$ is the set of edges. In the context of air pollution in dense areas, a spatial notion of the network is necessary as well. Therefore, we introduce an edge-dependent indication for the spatial pollution severity $\omega_{ij}$, which indicates the severity of the increase in air pollution per truck transiting along that edge. This index contains information about the density of the network around edge $(i, j)$, although other factors can be included as well.

The edges of the network are used to transport goods between nodes by means of trucks. Each truck belongs to a transportation provider $k \in K$, where $K$ is the set of transportation providers $K$. The number of trucks of transportation provider $k$ transiting along edge $(i, j) \in E$ is denoted by $x_{ij}^{(k)}$. In this chapter, the simplified assumption is made that the fleet of trucks is uniform; however, a generalization to a heterogeneous fleet is straightforward. For each transportation provider $k \in K$, source nodes for resource procurement are given by the sets $S_k \subseteq V$ and sink nodes for resource sales are given by the sets $U_k \subseteq V$. Demand is defined at each node and for each transportation provider by $d_i^{(k)}$ , where $d_i^{(k)} > 0$ if $n \in U_k$ and 0 otherwise. Demand can be served in full or in part. The price paid to transportation provider $k$ per truckload delivered at node $n$ is given by $p_n^{(k)}$, such that $p_n^{(k)} > 0$ if $n \in U_k$ and $d_n^{(k)} = 0$ otherwise. The resource is obtained by transportation provider $k$ at the source nodes in $S_k$, at no cost, and without constraint on the number of trucks. There are base transportation costs that each transportation provider faces to transport the resource along each edge $(i, j)$ that are linear in the number of trucks transiting along the edge. The cost per truck is $b_{ij} > 0$. The edge capacities are described by $w_{ij}$, which is the upper bound on the total number of trucks transiting along the edge $(i, j)$ per time unit.

The first problem is to optimize a regulation strategy $\boldsymbol{q} = \{q_{ij} \in \mathbb{N}_0 : (i, j) \in E\}$ for the case with a single transportation provider. The goal is to find a regulation strategy that minimizes the air pollution in dense areas on the transportation network, by minimizing the sum of the overall number of trucks $x_{ij}$ weighted by the pollution severity $\omega_{ij}$. As a result, the transportation provider reroutes its trucks in order to disburden the highest polluted areas. The aim here is to optimize the regulation strategy by eliminating capacity within the budget. The budget corresponds to a fraction $\beta$ of the total transportation capacity.

[1]The problem is inspired by a multi-objective optimization challenge by the MOPTA organization in coral.ie.lehigh.edu/$\sim$mopta/AIMMS_MOPTA_case_2016.pdf.

# 3 Method for Air Pollution Minimization

The goal of the method proposed in this chapter is to minimize the air pollution in dense areas on the transportation network by minimizing the sum of the overall number of trucks $x_{ij}$ weighted by the pollution severity $\omega_{ij}$. The aim here is to optimize the regulation strategy when the budget for regulation corresponds to eliminating a fraction $\beta$ of the total transportation capacity. Throughout this chapter, the case with a single transportation provider in $K$ is considered, which can be extended in a straightforward manner to the case with multiple transportation providers.

## *3.1 Min-Cost-Circulation Problem Under Regulation*

First of all, in order to minimize the air pollution in dense areas, the minimal cost circulation $\boldsymbol{x} := \{x_{ij} : (i,j) \in E\}$ for the transportation provider in $K$ given a regulation strategy $\boldsymbol{q}$ must be determined. To this end, we convert the problem to a circulation problem. For this, we extend the network $N$ by two auxiliary nodes (super source $v_0$ and super sink $v_1$) and two auxiliary sets of edges ($E_{\text{art}}$ and $E_{\text{source}}$). Edge set $E_{\text{art}} := \bigcup_{u \in U}(u, 1)$ contains edges from each sink $u \in U$ to the super sink $v_1$, and the edge set $E_{\text{source}} = \bigcup_{s \in S}(0, s)$ contains edges from super source $v_0$ to each source $s \in S$. The capacity on each edge from the super source to a source is set to infinity, i.e., $w_{0s} = \infty$, and cost to zero, i.e., $b_{0s} = 0, \forall s \in S$. Moreover, the capacity on the edge from a sink $u$ to super sink $v_1$ is set to equal the demand on sink $u$, i.e., $w_{u1} = d_u, \forall u \in U$. One final edge $(1, 0)$ with infinite capacity and zero costs is added to edge set $E_{\text{art}}$, so that the restriction that the outflow of the super sink (resp. source) must equal its inflow can easily be satisfied by sending that amount over the unrestricted edge $(1, 0)$ creating a circulation flow. The union of the artificial edges in set $E_{\text{source}}$, set $E_{\text{art}}$, and $(1, 0)$ is referred to by $E_{\text{art}}$.

The minimal cost circulation for a transportation provider on network $N$, given fixed regulation strategy $\boldsymbol{q}$, can be obtained by solving the following min-cost-circulation problem $P$.

$$\min \quad \sum_{(i,j)\in E} x_{ij} b_{ij} - \sum_{(i,j)\in E_{\text{art}}} x_{ij} p_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{j:(i,j)\in E\cup E_{\text{art}}} x_{ij} - \sum_{j:(j,i)\in E\cup E_{\text{art}}} x_{ji} = 0 \quad \forall i \in V \tag{2}$$

$$x_{ij} \leq w_{ij} - q_{ij} \quad \forall (i,j) \in E \tag{3}$$

$$x_{ij} \leq d_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{4}$$

$$x_{ij} \geq 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{5}$$

Here, the decision variable $x_{ij} \in \mathbb{R}$ represents the number of trucks routed over edge $(i, j)$ and cannot be negative. We can safely allow non-integer values for $x_{ij}$, because (by the *integrality theorem*) the relaxed problem still yields integral results as long as all input values for the capacities are integer, which is assumed to be the case here. The objective function (1) is to minimize the cost minus the reward of the circulation, which yields the maximum payoff. Constraints (2) are the typical circulation constraints that ensure the number of incoming trucks on a node to equal the number of outgoing nodes. Constraints (3) and (4) ensure that the capacity on each edge is not exceeded.

## *3.2 Optimality Conditions of Circulation Under Regulation*

The regulation strategy $\boldsymbol{q}$ must be optimized in such a way that the minimal cost circulation $\boldsymbol{x}$ for the transportation provider in $K$ given the regulation strategy $\boldsymbol{q}$ is optimal. We therefore formulate the optimality conditions for circulation $\boldsymbol{x}$ as follows.

First, associate dual variables $\boldsymbol{\lambda} = \{\lambda_i : i \in V\}$ with the constraints in (2) and dual variables $\boldsymbol{\mu} = \{\mu_{ij} : (i, j) \in E \cup E_{\text{art}}\}$ with the constraints in (3) and (4). Furthermore, add slack variable $z_{ij}^{(1)}$ to the left-hand side of each constraint in (3) and (4) to bring $P$ in standard form. The Langrangian $L_P$ of $P$ then amounts to

$$
\begin{aligned}
L_P = & \sum_{(i,j)\in E} x_{ij} b_{ij} - \sum_{(i,j)\in E_{\text{art}}} x_{ij} p_{ij} \\
& - \sum_{i \in V} \lambda_i \left( \sum_{j:(i,j)\in E\cup E_{\text{art}}} x_{ij} - \sum_{j:(j,i)\in E\cup E_{\text{art}}} x_{ji} \right) - \sum_{(i,j)\in E} \mu_{ij} \left( x_{ij} + z_{ij}^{(1)} - w_{ij} + q_{ij} \right) \\
& - \sum_{(i,j)\in E_{\text{art}}} \mu_{ij} \left( x_{ij} + z_{ij}^{(1)} - d_{ij} \right)
\end{aligned}
\tag{6}
$$

A reformulation leads to

$$
\begin{aligned}
L_P = & \sum_{(i,j)\in E} x_{ij}(b_{ij} - \lambda_i + \lambda_j - \mu_{ij}) - \sum_{(i,j)\in E_{\text{art}}} x_{ij}(p_{ij} - \lambda_i + \lambda_j - \mu_{ij}) \\
& - \sum_{(i,j)\in E\cup E_{\text{art}}} \mu_{ij} z_{ij}^{(1)} + \sum_{(i,j)\in E_{\text{art}}} \mu_{ij} d_{ij} + \sum_{(i,j)\in E} \mu_{ij}(w_{ij} - q_{ij}).
\end{aligned}
\tag{7}
$$

And we arrive at the dual $D_P$ of $P$:

$$
\max \quad \sum_{(i,j)\in E} \mu_{ij}(w_{ij} - q_{ij}) + \sum_{(i,j)\in E_{\text{art}}} \mu_{ij} d_{ij} \tag{8}
$$

$$\text{s.t.} \quad \lambda_i - \lambda_j + \mu_{ij} \leq b_{ij} \quad \forall (i,j) \in E \tag{9}$$

$$\lambda_i - \lambda_j + \mu_{ij} \leq -p_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{10}$$

$$\mu_{ij} \leq 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{11}$$

$$\lambda_i \in \mathbb{R} \quad \forall i \in V \tag{12}$$

Now slack variables $z_{ij}^{(2)}$ are added to the left-hand side of each constraint in (9) and (10) to bring the dual $D_P$ in standard form. The Langrangian $L_{D_P}$ of $D_P$ then amounts to

$$\begin{aligned} L_{D_P} = &\sum_{(i,j)\in E} \mu_{ij}(w_{ij} - q_{ij}) + \sum_{(i,j)\in E_{\text{art}}} \mu_{ij} d_{ij} \\ -&\sum_{(i,j)\in E} x_{ij}\left(\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} - b_{ij}\right) - \sum_{(i,j)\in E_{\text{art}}} x_{ij}\left(\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} + p_{ij}\right) \end{aligned} \tag{13}$$

A reformulation leads to

$$\begin{aligned} L_{D_P} = &\sum_{(i,j)\in E} \mu_{ij}(w_{ij} - q_{ij} - x_{ij}) + \sum_{(i,j)\in E_{\text{art}}} \mu_{ij}(d_{ij} - x_{ij}) \\ &- \sum_{i\in V} \lambda_i \left( \sum_{j:(i,j)\in E\cup E_{\text{art}}} x_{ij} - \sum_{j:(j,i)\in E\cup E_{\text{art}}} x_{ji} \right) - \sum_{(i,j)\in E\cup E_{\text{art}}} x_{ij} z_{ij}^{(2)} \\ &- \sum_{(i,j)\in E} x_{ij} b_{ij} + \sum_{(i,j)\in E_{\text{art}}} x_{ij} p_{ij}. \end{aligned} \tag{14}$$

Now let $\boldsymbol{x}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\mu}$ be feasible solutions for the primal $P$ and its dual $D_P$. It follows, by *complementary slackness*, that the following conditions are the optimality conditions for $\boldsymbol{x}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\mu}$.

$$\sum_{j:(i,j)\in E\cup E_{\text{art}}} x_{ij} - \sum_{j:(j,i)\in E\cup E_{\text{art}}} x_{ji} = 0 \quad \forall i \in V \tag{15}$$

$$x_{ij} + z_{ij}^{(1)} = w_{ij} - q_{ij} \quad \forall (i,j) \in E \tag{16}$$

$$x_{ij} + z_{ij}^{(1)} = d_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{17}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} = b_{ij} \quad \forall (i,j) \in E \tag{18}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} = -p_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{19}$$

$$\mu_{ij} z_{ij}^{(1)} = 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{20}$$

$$x_{ij} z_{ij}^{(2)} = 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{21}$$

$$x_{ij}, z_{ij}^{(1)}, z_{ij}^{(2)} \geq 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{22}$$

$$\mu_{ij} \leq 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{23}$$

$$\lambda_i \in \mathbb{R} \quad \forall i \in V \tag{24}$$

In summary, the feasible solution $\boldsymbol{x}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\mu}$ for an transportation provider that minimizes its cost for transportation on the network $N$ is optimal if conditions (15), (16), (17), (18), (19), (20), (21), (22), (23), and (24) hold. These optimality conditions are used in our model for optimal regulation.

### *3.3 Model for Optimal Regulation*

The model for optimal reduction of air pollution presented here is the main model that solves problem 1. It finds a regulation strategy, so that the sum of the overall number of trucks $x_{ij}$ weighted by the pollution severity $\omega_{ij}$ is minimized. Recall regulation $q_{ij} \in \mathbb{N}_0, (i,j) \in E$ to be the amount by which the capacity at edge $(i,j) \in E$ is reduced. Recall that in problem $P_1$, the regulation strategy $\boldsymbol{q}$ was fixed in order to find the optimal circulation in response to $\boldsymbol{q}$. In this final model for optimal regulation, the variables $\{q_{ij} \in \mathbb{N}_0 : (i,j) \in E\}$ become decision variables. The regulation strategy $\boldsymbol{q}$ is subject to a budget $W = \beta \sum_{(i,j)\in E} w_{ij}$, where the input variable $\beta \in [0, 1]$ is the fraction of the total transportation capacity by which the overall capacity can be eliminated. An optimal regulation strategy is obtained by solving the following mixed integer nonlinear program (MINLP):

$$\min \quad \sum_{(i,j)\in E} x_{ij}\omega_{ij} \tag{25}$$

$$\text{s.t.} \quad \sum_{j:(i,j)\in E\cup E_{\text{art}}} x_{ij} - \sum_{j:(j,i)\in E\cup E_{\text{art}}} x_{ji} = 0 \quad \forall i \in V \tag{26}$$

$$x_{ij} + z_{ij}^{(1)} = w_{ij} - q_{ij} \quad \forall (i,j) \in E \tag{27}$$

$$x_{ij} + z_{ij}^{(1)} = d_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{28}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} = b_{ij} \quad \forall (i,j) \in E \tag{29}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} = -p_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{30}$$

$$\mu_{ij} z_{ij}^{(1)} = 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{31}$$

$$x_{ij} z_{ij}^{(2)} = 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{32}$$

$$\sum_{(i,j)\in E} q_{ij} \leq \beta \sum_{(i,j)\in E} w_{ij} \tag{33}$$

$$x_{ij}, z_{ij}^{(1)}, z_{ij}^{(2)} \geq 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{34}$$

$$\mu_{ij} \leq 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{35}$$

$$\lambda_i \in \mathbb{R} \quad \forall i \in V \tag{36}$$

$$q_{ij} \in \mathbb{N}_0 \quad \forall (i,j) \in E \tag{37}$$

The objective function (25) minimizes the overall number of trucks weighted by the pollution severity. Constraints (26), (27), (28), (29), (30), (31), and (32) are the optimality conditions that ensure the circulation of the transportation provider to be the optimal response to regulation strategy $\boldsymbol{q}$. Finally, constraint (33) ensures that the total amount of capacity reduction does not exceed the given budget. This MINLP is very difficult to solve due to the bilinear constraints (31) and (32) and the non-convex nature of bilinear constraint. In the next subsection, we propose an exact linearization of this model that can be solved by algorithms that are significantly more efficient than generic algorithms for solving non-convex MINLP.

## *3.4 Efficient Reformulation of the Model for Optimal Regulation*

In general, a mixed integer nonlinear program, and especially a non-convex MINLP, is much more difficult to solve compared to a mixed integer *linear* program (MILP). A MILP is inherently convex due to the linearity of the objective function as well as of the constraints. In order to reduce the calculation time significantly, we linearize the bilinear non-convex constraints (31) and (32) in the following to obtain a computationally tractable MILP.

We start with the linear reformulation of constraint (31). For this constraint to be satisfied, the term $\mu_{ij} z_{ij}^{(1)}$ must be zero. Therefore, at least one of the decision variables $\mu_{ij}$ and $z_{ij}^{(1)}$ must be zero, i.e., $\mu_{ij} = 0$ and/or $z_{ij}^{(1)} = 0$. This can also be accomplished by means of the introduction of two binary auxiliary variables $a_{ij}, a'_{ij} \in \{0, 1\}$ and the following three types of linear constraints:

$$-\mu_{ij} \leq \bar{\mu}_{ij} a_{ij} \qquad \forall (i,j) \in E \cup E_{\text{art}} \tag{38}$$

$$z_{ij}^{(1)} \leq \bar{z}_{ij}^{(1)} a'_{ij} \qquad \forall (i,j) \in E \cup E_{\text{art}} \tag{39}$$

$$a_{ij} + a'_{ij} \leq 1 \qquad \forall (i,j) \in E \cup E_{\text{art}} \tag{40}$$

Here, in constraint (38), the term $\bar{\mu}_{ij}$ represents an upper bound of $-\mu_{ij}$. Similarly, all $\bar{\cdot}$ denote the upper bound of $\cdot$. This constraint forces auxiliary decision variable $a_{ij}$

to 1 if $\mu_{ij} < 0$. Analogously, auxiliary decision variable $a'_{ij}$ is forced to 1 if $z^{(1)}_{ij} > 0$ by constraint (39). At most one of both auxiliary decision variables $a_{ij}$ and $a'_{ij}$ is allowed the value 1 by constraint (40). Consequently, at least one of both auxiliary decision variables $a_{ij}$ and $a'_{ij}$ takes the value zero and hence $\mu_{ij} = 0$ and/or $z^{(1)}_{ij} = 0$.

For the term $x_{ij}z^{(2)}_{ij}$ in constraint (32) to be zero, we construct three types of linear constraints in a similar manner, because for this type of constraint, it holds that at least one of the decision variables $\mu_{ij}$ and $z^{(1)}_{ij}$ must be zero as well. We introduce two additional binary auxiliary variables $a''_{ij}, a'''_{ij} \in \{0, 1\}$ and the following three types of linear constraints analogously:

$$x_{ij} \leq \bar{x}_{ij}a''_{ij} \qquad \forall (i,j) \in E \cup E_{\text{art}} \tag{41}$$

$$z^{(2)}_{ij} \leq \bar{z}^{(2)}_{ij}a'''_{ij} \qquad \forall (i,j) \in E \cup E_{\text{art}} \tag{42}$$

$$a''_{ij} + a'''_{ij} \leq 1 \qquad \forall (i,j) \in E \cup E_{\text{art}} \tag{43}$$

Again, at least one of both auxiliary decision variables $a''_{ij}$ and $a'''_{ij}$ is forced to take the value zero and hence $x_{ij} = 0$ and/or $z^{(2)}_{ij} = 0$.

The upper bounds $\bar{\mu}_{ij}, \bar{z}^{(1)}_{ij}, \bar{x}_{ij}$, and $\bar{z}^{(2)}_{ij}$ can be concretized as follows:

$$\bar{\mu}_{ij} = \max_{(k,l)\in E_{\text{art}}} |p_{kl}| \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{44}$$

Here, the decision variable $\mu_{ij}$ originates from being the shadow price of the respective constraint in (3). It is well established that the shadow price corresponds to the increase of the objective value, when its constraint is relaxed by one unit. An increase of the capacity on the edge $(i, j)$ with one unit can maximally increase the flow by one unit. This one additional unit yields at most reward $\max_{(k,l)\in E_{\text{art}}} |p_{kl}|$. The derivation of an upper bound on $z^{(1)}_{ij}$ is straightforward. This variable is introduced as the slack variable for its corresponding constraint in (3) or (4), and the variables $x_{ij}$ and $q_{ij}$ are nonnegative, hence

$$\bar{z}^{(1)}_{ij} = w_{ij} \quad \forall (i,j) \in E \tag{45}$$

and

$$\bar{z}^{(1)}_{ij} = d_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{46}$$

Furthermore, the upper bound on the number of trucks $x_{ij}$ on edge $(i, j)$ is given explicitly in its constraint in (3) that ensures the number of trucks not to exceed its capacity, hence

$$\bar{x}_{ij} = w_{ij} \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{47}$$

Finally, the upper bound on variable $z_{ij}^{(2)}$ is less straightforward. Recall this decision variable to be introduced as the slack variable for its corresponding constraint in (9) or (10). Following a similar reasoning as for $\mu_{ij}$, $|\lambda_i|$ has to be bounded by the same $\max_{(k,l)\in E_{\text{art}}} |p_{kl}|$ and hence $\lambda_i - \lambda_j$ by twice that. Rearranging (9) and (10), respectively, yields the following upper bound for $z_{ij}^{(2)}$:

$$\bar{z}_{ij}^{(2)} = 3 \max_{(k,l)\in E_{\text{art}}} |p_{kl}| - p_{ij} \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{48}$$

Through the introduction of the four auxiliary binary auxiliary variables $a_{ij}$, $a'_{ij}$, $a''_{ij}$, and $a'''_{ij}$ and the six linear constraints (38), (39), (40), (41), (42), and (43), both bilinear constraints (31) and (32) can be omitted. We substitute the six linear constraints for the two bilinear constraints and obtain the following MILP $P_1$:
$(P_1)$ :

$$\min \quad \sum_{(i,j)\in E} x_{ij}\omega_{ij} \tag{49}$$

$$\text{s.t.} \quad \sum_{j:(i,j)\in E\cup E_{\text{art}}} x_{ij} - \sum_{j:(j,i)\in E\cup E_{\text{art}}} x_{ji} = 0 \quad \forall i \in V \tag{50}$$

$$x_{ij} + z_{ij}^{(1)} = w_{ij} - q_{ij} \quad \forall (i,j) \in E \tag{51}$$

$$x_{ij} + z_{ij}^{(1)} = d_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{52}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} = b_{ij} \quad \forall (i,j) \in E \tag{53}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z_{ij}^{(2)} = -p_{ij} \quad \forall (i,j) \in E_{\text{art}} \tag{54}$$

$$-\mu_{ij} - \left(\max_{i'} |p_{i'}|\right) a_{ij} \le 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{55}$$

$$z_{ij}^{(1)} - w_{ij}a'_{ij} \le 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{56}$$

$$a_{ij} + a'_{ij} \le 1 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{57}$$

$$x_{ij} - w_{ij}a''_{ij} \le 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{58}$$

$$z_{ij}^{(2)} - \left(3\max_{i'} |p_{i'}| - p_{ij}\right) a'''_{ij} \le 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{59}$$

$$a''_{ij} + a'''_{ij} \le 1 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{60}$$

$$\sum_{(i,j)\in E} q_{ij} \le \beta \sum_{(i,j)\in E} w_{ij} \tag{61}$$

$$x_{ij}, z_{ij}^{(1)}, z_{ij}^{(2)} \ge 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{62}$$

$$\mu_{ij} \le 0 \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{63}$$

$$\lambda_i \in \mathbb{R} \quad \forall i \in V \tag{64}$$

$$q_{ij} \in \mathbb{N}_0 \quad \forall (i,j) \in E \tag{65}$$

$$a_{ij}, a'_{ij}, a''_{ij}, a'''_{ij} \in \{0,1\} \quad \forall (i,j) \in E \cup E_{\text{art}} \tag{66}$$

Solving this MILP yields an optimal strategy for regulation that minimizes the pollution on the network. The objective function (49) minimizes the overall number of trucks weighted by the pollution severity. Constraints (50), (51), (52), (53), (54), (55), (56), (57), (58), (59), and (60) are the linear optimality conditions that ensure the circulation of the transportation provider to be the optimal response to regulation strategy $\boldsymbol{q}$. Finally, constraint (61) ensures that the total amount of capacity reduction does not exceed the given budget. This MILP is relatively easy to solve for instances of realistic sizes, which is shown by the results of computational experiments in Sect. 4.

## 4 Computational Experiments

In this section, we present the environment for the computational experiments and the results for problem 1: air pollution minimization. First, we show the changes in optimal behavior of the regulator and the transportation provider under varying severity of pollution on the network in Sect. 4.1. Finally, we show how much the overall pollution on the network decreases with the increase of the regulation budget in Sect. 4.2.

All computational experiments were performed on an Intel(R) Core(TM) i7-5600U CPU processor with 2.6 GHz and a usable memory of 7.7 GB. The simulation platform is written in Python, using IBM Cplex with default parameter settings to solve the instances of the proposed MILP.

### *4.1 Scenarios with Varying Severity of Pollution*

In order to show the behavior of the regulator and the transportation provider under varying severity of pollution on the network, we show the results for three scenarios for which the severity indices of the pollution vary as follows:

1. The severity of pollution is zero on the entire network (see Fig. 1);
2. There is medium severity of pollution on the city routes (see Fig. 2);
3. The severity of pollution is high on the city routes, medium on the northern routes, and low on the southern route (see Fig. 3).

### 4.1.1 Description of the Test Instances

The network $N$ represents a transportation network with 14 nodes with labels $(A) - (O)$. Most nodes $(D, E, F, G, I, J, L, M)$ are positioned inside a city. The source $(C)$ and the sink $(O)$ have their position at opposite sides of the city. A few nodes, $(H, N)$ and $(K)$, have their position outside the city. Super source $(A)$ and super sink $(B)$ are the artificial nodes that are necessarily introduced for our model formulation. The nodes are reachable through edges connecting the nodes, enabling numerous routes for the transportation provider to route its truck from its source to its sink. We refer to the routes through nodes $(H, N)$ by the *northern* routes, the route through $(K)$ by the *southern* route, and the remaining routes by the *city* routes. Each edge $(i, j) \in E$ on the network is characterized by its properties represented by the tuple $(w_{ij} - q_{ij}, b_{ij}, \omega_{ij})$, where we recall $w_{ij}$ to represent the capacity, $q_{ij}$ the regulated amount by which the capacity is reduced, $b_{ij}$ the cost, and $\omega_{ij}$ the pollution severity. The tuple $(\infty, 0, 0)$ describes the properties of artificial edges $(B, A)$ and $(A, C)$, which have infinite capacity and zero costs and zero pollution. The tuple $(d_{OB}, p_{OB}, 0)$ describes the properties of artificial edge $(O, B)$, where we recall $d_{OB}$ to represent the demand on sink $(O)$ and $p_{OB}$ to represent the reward on sink $(O)$. This artificial edge also has zero pollution. The values for $(w_{ij} - q_{ij}, b_{ij}, \omega_{ij})$ used in the three scenarios are shown in Figs. 1, 2, and 3 respectively. The artificial edge $(B, A)$ is not depicted here in favor of compactness of these figures.

Over the three scenarios, the properties of both the regulator and the transportation provider remain constant. The regulator has a budget corresponding to a fraction $\beta = 0.15$ of the total transportation capacity $\sum_{(i,j)\in E} w_{ij} = 110$. Hence, the regulator is authorized to reduce the total transportation capacity by at most 16. The demand of the transportation provider is 10 at node $(O)$. The initial edge capacities $\boldsymbol{w}$ remain constant as well as their costs $\boldsymbol{b}$ and are as indicated on the edges in Figs. 1, 2, and 3. The varying edge property is the pollution severity $\boldsymbol{\omega}$, which values are indicated on the edges in aforementioned figures as well.

### 4.1.2 Results

The results of solving the proposed MILP $P_1$ for air pollution minimization are shown in Figs. 1, 2, and 3 for the three scenarios, respectively. Recall that the goal of the regulator is to reduce the capacity on the edges such that the optimal solution of the transportation provider emits minimal air pollution. The optimal strategy to accomplish this goal is shown by dashed edges for those with reduced capacity, complemented by the resulting capacity $(w_{ij} - q_{ij})$ on the corresponding edges. Recall furthermore that the goal of the transportation provider is to minimize its costs for transportation, *given* the reduced capacity. The edges on the used routes by the transportation provider are printed thick.

The results of the first scenario on network $N_1$, in which the severity of pollution is zero on the entire network, are shown in Fig. 1. An optimal strategy for the regulator is to reduce the capacity of both edges $(C, D)$ and $(E, F)$ by 4. The initial

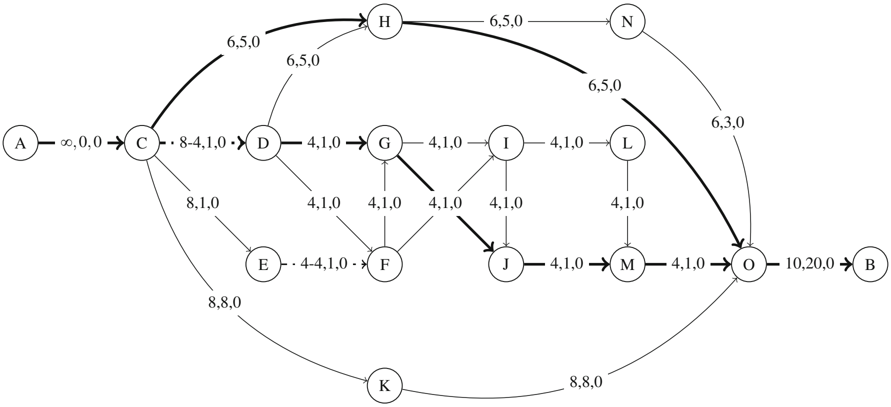


**Fig. 1** Optimal strategies of the regulator and the transportation provider on network $N_1$

capacity on edge $(C, D)$ was 8, so the remainder is 4 such that 4 trucks are routed through the city. Actually, even before the reduction of the capacity, this was already the maximum number of trucks that could be routed through the city because of the limited capacity of 4 on edge $(M, O)$. Therefore, the total reduced capacity of 8 out of allowed 16 is redundant. This can be explained by the fact that the objective is to minimize the air pollution, and, because the severity of the pollution on the entire network is zero in this scenario, any feasible regulation strategy is optimal. In order to additionally avoid redundant capacity reductions, the model can be extended with minor adaptions such as demanding a small penalty for redundant reductions. Nevertheless, besides the 4 trucks that are routed through the city, an additional 6 trucks are routed over the cheapest alternative route $(C - H - O)$, such that the demand of 10 in sink $(O)$ is satisfied. The total cost for this routing strategy is 80 and the reward is 200, resulting in a payoff of 120. The total pollution of this transport strategy is, of course, zero.

The results of the second scenario on network $N_2$, in which there is medium severity of pollution on the city routes, are shown in Fig. 2. Intuitively, the regulator aims to reduce the transport through the city, because these routes emit pollution. As we can see from the results, the regulator completely blocks the edges $(C, D)$ and $(E, F)$ with initial capacities of 8 and 4, respectively. The total reduced capacity is therefore 12, which is well within the budget. Additionally reducing capacity will apparently not result in a higher reduction of the pollution on the network. The cheapest alternative routes are the northern route $(C - H - O)$, which is used up to capacity for 6 trucks, and the southern route $(C - K - O)$, whose part of the capacity is used for the remaining 4 trucks to satisfy the demand of 10 in sink $(O)$. The total cost for this routing strategy is 124 and the reward is 200, resulting in a payoff of 96. The total pollution of this transport strategy is still zero, while the pollution of the optimal transport strategy without regulation would have been 40. Thus, this regulation strategy accomplished a pollution reduction of 40.

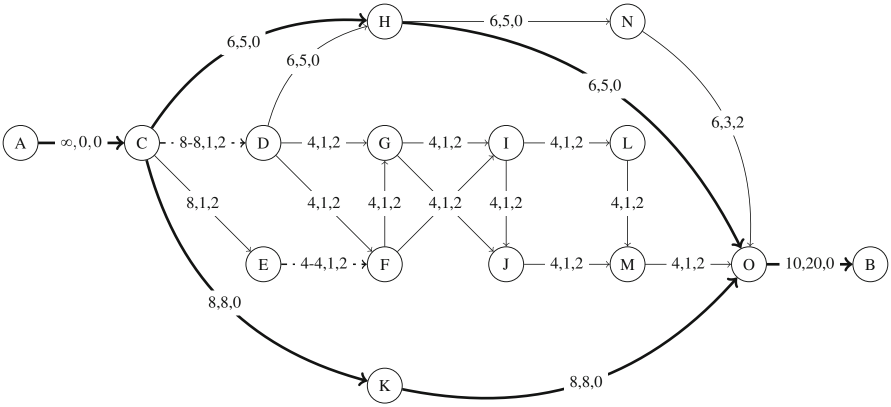


**Fig. 2** Optimal strategies of the regulator and the transportation provider on network $N_2$

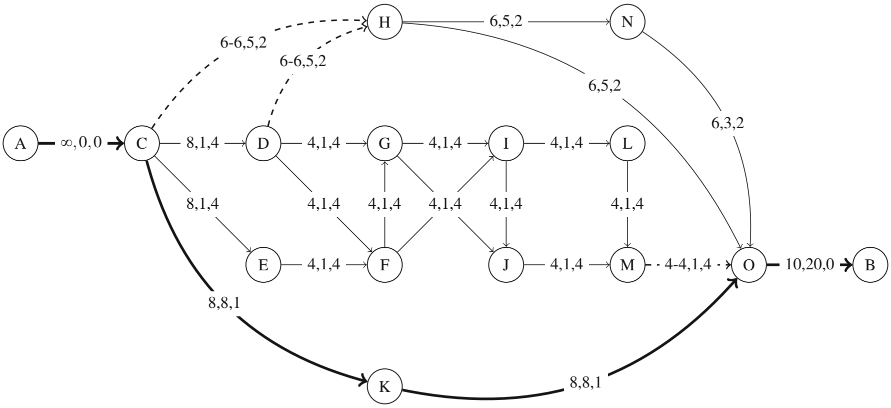


**Fig. 3** Optimal strategies of the regulator and the transportation provider on network $N_3$

The results of the third scenario on network $N_3$, in which the severity of pollution is high on the city routes, medium on the northern routes and low on the southern route, are shown in Fig. 3. Here, the intuitive solution to minimize the pollution is again to maximally reduce the transport through the city. In addition, because in this scenario the northern routes are more severely subject to pollution compared to the southern route, the regulator may aim to also reduce the capacity on these routes. As we can see from the results here, the strategy for the regulator accomplishes that goal indeed. By completely blocking the city edge $(M, O)$, no city routes lead to the source node $(O)$. Also, by completely blocking the edges $(C, H)$ and $(D, H)$ on the northern routes, these routes are also disabled. The total reduced capacity is 16 which is the maximum reduction allowed within the budget. If the budget were 1 unit higher, the regulator could additionally reduce the capacity on one of the edges on the southern route. Then, the pollution would reduce even more. Nevertheless, the optimal solution for the transportation provider, given this regulation strategy, is to use the full capacity on the southern route ($C-K-O$) for 8 trucks. Unfortunately for the transportation provider, it cannot meet the full demand of sink ($O$); however, this is not a constraint and the problem remains feasible. The total cost for this routing strategy is 128 and the reward is 160, resulting in a payoff of 32. The total pollution of this transport strategy is now 16 but corresponds to the minimum that can be achieved within the given budget. The pollution of the optimal transport strategy without regulation would have been 104. This regulation strategy accomplished a huge pollution reduction of 88.

The computation time to obtain an optimal solutions was less than 1 s on each of these instances.

## 4.2 Scenarios with Varying Budget for Regulation

The scenarios described in this section are introduced in order to show how the pollution on the network and the payoff for the transportation provider decrease with increasing budget for regulation. Moreover, the results show an increase in computation time when the budget increases.

The network used for the tests on the scenarios with varying budget is analogous to the network $N_3$ as depicted in Fig. 3. Recall that on this transportation network, the severity of pollution is high on the city routes, medium on the northern routes, and low on the southern route. We computed the optimal solutions for 10001 instances in which $\beta$ (fraction of the total capacity allowed for regulation) varies between 0 and 1 with a step size of 0.0001. All remaining parameters remain constant over the instances.

The results are shown in Fig. 4. The main and aimed result here is that when the budget for regulation increases, the pollution on the network decreases. As a side

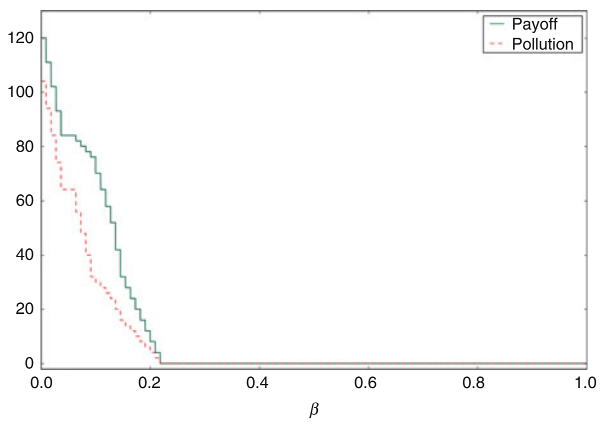


**Fig. 4** Payoff and Pollution as functions of $\beta$

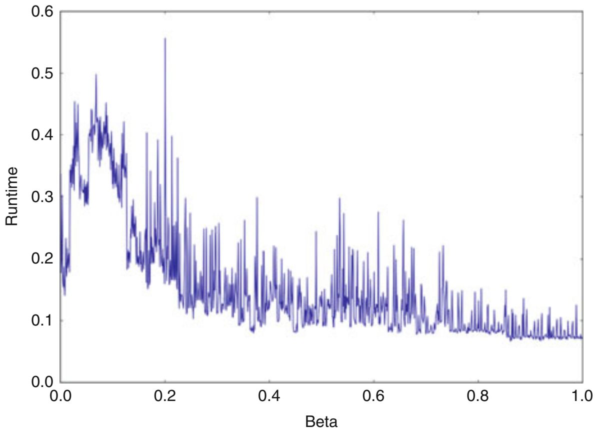


**Fig. 5** Computation times of the test instances depending on fraction $\beta$ for regulation

effect, which is not accounted for by the regulator, the payoff for the transportation provider also decreases. The results furthermore show that the pollution as well as the payoff can be completely diminished already with a relatively small budget of 24 which corresponds to $\beta = 0.22$ on a network with total capacity of 110. This effect indicates a relatively small minimum cut (see [6]) on the network. When the capacity on all edges on the minimum cut is reduced to zero, no trucks can be routed from any source to any sink. With a larger minimum cut, the necessary budget to completely disable any flow increases as well. The computation times for finding optimal solutions are shown in Fig. 5. The computational burden is very low for small $\beta$. This is intuitively explained by the fact that only few resources for regulation need to be assigned to incapacitate edges. The computation time increases then when more resources are available, so more combinations (hence solutions) are possible and the combinatorial problem becomes more complex. With higher $\beta$, the resources become so high that careful placement is not necessary to completely incapacitate the network and, hence, obtain an optimal solution of zero pollution on the transportation network.

## 5 Conclusion

With the increasing awareness of the negative environmental and health effects of air pollution, transportation providers started investing in reducing the $CO_2$ emissions of their operations. However, some transportation providers can or will not trade off their profit to obtain a satisfactory level. Governmental regulation proofs to be an option in order to accomplish this goal. The goal in this chapter was to find a regulation plan that minimizes the air pollution in dense areas, such as cities, on the transportation network by reducing capacities of the roads in those dense areas. The proposed model for air pollution minimization requires very low computation times to be solved to optimality. The computational results show a large decrease in air pollution even when a small budget is available for regulation for the test instances.

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# Cumulative VRP: A Simplified Model of Green Vehicle Routing

**Rishi Ranjan Singh and Daya Ram Gaur**

**Abstract** There has been a recent resurge of interest in vehicle routing problems, especially in the context of green vehicle routing. One popular and simplified model is that of the cumulative vehicle routing problem. In this chapter, we examine the motivation, the definition, and the mixed integer linear program for the cumulative VRP. We review some of the recent results on approximation algorithms for the cumulative VRP. A column generation-based procedure for solving the cumulative VRP is also described. We also review approximation algorithms for a stochastic version of the cumulative VRP.

## 1 Introduction

Nonrenewable energy resources are consumed directly or indirectly in today's world to run our life smoothly. Fossil fuel is one of such resource, and with its current rate of consumption, we cannot sustain it indefinitely. Only an efficient use of it will give the time to find alternative technologies and other fuel resources. Optimizing fossil fuel consumption also reduces the pollution rate and makes our planet a better place to live and breathe in. A lot of fossil fuel is burned during transportation activities. In fact, fuel cost can be as high as 60% of the transportation cost depending on the medium of the transport [59]. Therefore, by minimizing the fuel consumption, we reduce the total cost of transportation, $CO_2$ emissions, and we extend the lifespan of

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fossil fuels. The demand also affects fuel prices which in turn affects inflation [54]. Reduced utilization of fuel can also reduce the negative impact of such inflation on us.

The above considerations motived researchers to develop optimization models for minimizing fuel consumption. Fuel consumption by a vehicle is affected by several factors. Few of the important ones are the distance traveled, weight of the vehicle, vehicle speed, road inclination, traffic congestion, vehicle type, driving, aerodynamic drag, etc. [23, 25]. The readers are referred to Section 2 of the survey article due to Demir et al. [25] for more factors that affect fuel consumption in a vehicle. The inclusion of all such factors in an optimization model will make the model complex to analyze. A traditional way to tackle such complex problems is to model the problem with one or two factors in the beginning and assume the rest of the factors to be constant. Models with such assumptions do not model the original problem exactly and therefore do not provide the optimal results. Still, with such assumptions, the complexity of the general problem is reduced, and the special case of the problem is easy to model, analyze, and solve. Given the variability in the model parameters, a linear model is a good place to start the research.

One of the simplest such models was proposed by Dantzig and Ramser [22] which assumed that the fuel consumption is proportional to the distance traveled. This optimization model was termed as the vehicle routing problem (VRP). In the last decade, a simplified optimization model of fuel consumption in vehicle routing problems (VRPs) has been studied under the name of cumulative vehicle routing problems. This model generalizes the VRP model and adds another factor, the weight of the vehicle, to the fuel consumption model. It is a linear model of fuel consumption in energy minimizing vehicle routing problems. In this variant of VRPs, the objective is to minimize the *cumulative cost*, not just the distance traveled. The cumulative cost per unit distance is assumed to be proportional to the total weight of the vehicle which is the kerb weight plus the weight of the cargo on the vehicle and the distance traveled with that weight [53, 45, 68, 33].

There exist various models that consider several other factors in addition to the distance traveled by the vehicle or the cumulative weight. A review due to Demir et al. [25] describes various fuel consumption models for minimizing carbon dioxide emission in road transportation. Initially, the researchers were just focused on reducing the transportation cost. In the last two decades, several optimization models were developed in logistics and transportation with the objective of reducing the pollution and keeping our planet green. Routing of vehicles with the objective of minimizing the $CO_2$ emission is studied under the name of green vehicle routing [24, 26, 42, 52, 41, 48, 65, 66, 67]. Bektaş et al. [5], Lin et al. [50], Park and Chae [55], and Toro et al. [62] individually surveyed various models on vehicle routing problems in the context of green transportation.

Cumulative VRPs are one of the most studied variations of VRPs in the last decade and are a popular model for fuel consumption among researchers. In this chapter, we survey the literature on cumulative VRP and its variations. We define the problem mathematically and discuss a mixed integer linear programming. We summarize few of the variants of the problem considered in various studies.

We review some of the recent theoretical and experimental results on the problem. Theoretical results cover various approximation algorithms. Experimental results describe various computational results due to column generation heuristics and others. Finally, we conclude the chapter with the discussion of the possible future directions for research.

### *1.1 Cumulative VRP*

In this section, we define cumulative vehicle routing problem (Cu-VRP). These definitions are from [45, 33, 31]. We are given a complete graph $G(V, E)$ with weights on the edges satisfying the triangle inequality. All the nodes correspond to customer (clients) save for one. This special node $r$ is called the depot where a vehicle with capacity $Q$ is stationed. A demand of $d_i \leq Q$ units is at each customer node $i \in V$. The vehicle visits the nodes in some order. The depot node may be visited more than once for refilling. The vehicle picks $d_i$ units of goods at the depot and drops them at node $i$. The total quantity of the goods in the vehicle at any point is at most $Q$. The objective is to find a schedule of the vehicle that minimizes the total cumulative cost. The cumulative cost is a notion of fuel consumption and is defined below.

Cumulative cost function assumes that the rate of fuel consumption per unit distance is proportional to the overall weight of the vehicle and the kerb weight plus the weight of the cargo on the vehicle. Let $a$ be the cost of moving the empty vehicle per unit distance and $b$ be the cost of moving the unit weight of goods per unit distance. Then, the cumulative cost of moving a vehicle unit distance with the cargo of weight $w$ is $a + bw$.

A subtour is defined as an ordered sequence of nodes starting and ending with the depot node. Each subtour is a directed cycle that starts and ends at the depot node and visits at least one customer. A vehicle scheduled according to a subtour starts from the depot, traverses all the clients in the subtour in the order given by the subtour, and finally returns to the depot. A subtour is called valid or feasible if the weights of the objects delivered in a subtour from the depot are at most $Q$. A tour is defined as the collection of valid subtours. A feasible solution to Cu-VRP is a tour $T$ which visits all the clients. We can represent the breakup of a feasible solution tour $T$ into a collection of $m$ valid subtours $\{S_1, S_2, S_3, \ldots, S_m\}$. Note that each customer node is in exactly one of the subtours. Let $l_i$ denote the distance traveled by the vehicle after picking $d_i$ units of goods at the depot and offloading it at customer $i$, in some subtour $S_j$. We denote the length of the directed cycle corresponding to subtour $S_j$ by $|S_j|$. In $T$, the vehicle travels a total distance of $\sum_{j=1}^{k} |S_i|$, and each object $i$ with weight $d_i$ travels a distance of $l_i$.

The total cumulative cost $Cu(T)$ of the travel schedule given by tour $T$ is

$$Cu(T) = a \sum_{j=1}^{k} |S_j| + b \cdot \sum_{i=1}^{n} d_i l_i. \tag{1}$$

The objective is to find a tour $T^*$ such that the cumulative cost of $T^*$ is the minimum, i.e., $Cu(T^*) \leq C(T)$ for all tours $T$. Next we define a stochastic variant of cumulative VRP.

### 1.1.1 Cumulative VRP with Stochastic Demand

Cu-VRP with stochastic demands (Cu-VRPSD) is a more realistic variant of Cu-VRPs that assumes that the demands are uncertain at customer nodes. The demand at a customer node is realized only when the vehicle visits the depot. The demands are probabilistic. For a simplified analysis, the demands are assumed to fall uniformly in the interval $(0, Q]$ where $Q$ is the capacity of the given vehicle. One can possibly extend the analysis to other distributions. In Cu-VRP, the objective is to find a feasible tour with minimum cumulative cost. In Cu-VRPSDs, the objective is to find an apriori tour with the minimum expected cumulative cost.

An apriori tour is defined as an ordered sequence of the customer nodes. For a fixed configuration of the realized demands (scenario) at the customer nodes, the cumulative cost of an apriori tour can be computed as follows: the vehicle starts from the depot, visits, and meets the demands of the customer node in the order of their occurrence on the apriori tour and in between returns to the depot for refills if required. The expected cumulative cost of an apriori tour is the mean of the cumulative cost of the apriori tour for all possible configurations of demands.

Next, we take note of some of the possible variants of Cu-VRPs based on various input parameters in the definition of the problem.

## *1.2 Variants*

Below are some of the possible variants of cumulative VRPs based on various input parameters:

- *Based on capacity* ($Q$)*:* The problem is called *Capacitated*, if there is a constraint that at any point of time the vehicle can carry at most fixed constant $Q$ weights. If this constraint is relaxed, and the vehicle is assumed to have infinite capacity, the problem is called *Uncapacitated*.
- *Based on the distribution of weights:* The problem is termed as *Equal-weighted* if all objects have the same weight. The problem is termed as *Unequal-weighted* if the objects have distinct weights.
- *Based on the type of delivery:* The problem is labeled as *Split-delivery* if the delivery of the demand at a customer node can possibly occur over multiple visits. Otherwise, it is termed as *Unsplit-delivery*, because the delivery constraint requires that each customer is visited exactly once.

- *Based on the knowledge of demands:* If the demands are known at the start, the problem is called as *Deterministic*. If there is uncertainty in the demand, it is called *Probabilistic* or *Stochastic*.
- *Based on the type of graph:* If the edge costs in a given graph obeys the triangular inequality, the problem is said to be *Metric*. The problem is called a restricted version if the network is restricted to be some special graphs, tree, path, etc.
- *Based on the number of refills (offloads) allowed at the depot:* If there is exactly one vehicle at the depot which once leaves the depot loaded with objects and returns only after visiting all the customer nodes, it is called *No-refiling* variant. In this case, it can be assumed that the capacity of the vehicle is at least the sum of all the demands. If we are handling pickup demands in place of delivery demands, the variant is called *No-offloading*. Another restriction can be that the given vehicle can refill (offload) at the depot exactly $k$, at least $k$, or at most $k$ times for some constant $k$. Finally, the possibility that the vehicle can offload any number of times at the depot.
- *Based on the type of vehicles:* If there are $k$ vehicles given at the depot and all the vehicles are of the same type in terms of parameters $a$, $b$, and $Q$, then the problem is called *homogeneous*. This variant is equivalent to the case when exactly one vehicle is given at the depot but $k$ refills (offloads) are allowed. The problem is called *heterogeneous* if a fleet of vehicle is present at the depot with different parameters $a$, $b$, or $Q$.
- *Based on the number of depots:* The problem is called a single-depot problem if in the whole network, there is exactly one depot; otherwise, it is called multi-depot problem.

Gaur et al. [33] have considered the following four different versions of the deterministic metric Cu-VRP obtained by varying the capacity and the distribution of demand:

1. The vehicle has infinite capacity and all customers have equal demands.
2. The vehicle has infinite capacity and the customers have unequal demands.
3. The vehicle has capacity $Q$ and the customers have equal demands.
4. The vehicle has capacity $Q$ and the customers have unequal demands.

For each of the above variations, they gave constant factor approximation algorithms. Gaur et al. in another paper [34] considered two different versions of stochastic Cu-VRP based on the type of delivery:

1. Split Cu-VRP with stochastic demand
2. Unsplit Cu-VRP with stochastic demand

For the above two variants, Gaur et al. [34] gave constant factor approximation algorithms for metric version of the problem. The approximation ratios are improved for the case of when the network is a path or a tree.

## *1.3 Lower Bound*

Good lower bounds are integral for proving approximation ratios. Here, we mention the lower bound for different variants that are used to analyze the performance ratio of the approximation algorithms in [33, 34].

### 1.3.1 For Deterministic Variants

Gaur et al. [33] gave a lower bound for Cu-VRPs where the demands are known. This bound is a straightforward and an important extension of the lower bound due to Haimovich and Rinnooy Kan [40].

**Theorem 1 (Theorem 4, [33])** *Let $T^*$ denote an optimal TSP tour of length $|T^*|$, and let $Q$ be the capacity of the vehicle. Let $d_i$ be the demand at client i, and let $l_i$ be length of the shortest path between client i and the depot. Then, the minimum cumulative cost to meet the demands of all the clients is at least*

$$a \cdot \max\left(|T^*|, 2\frac{\sum_{i=1}^{n} d_i l_i}{Q}\right) + b\left(\sum_{i=1}^{n} d_i l_i\right).$$

This lower bound is valid for all the four variants of deterministic Cu-VRP listed above in Sect. 1.2 and considered in [33].

### 1.3.2 For Stochastic Variants

The lower bound above can be extended to handle the stochastic case of the cumulative VRP as shown below.

**Theorem 2 (Theorem 5, [34])** *Let T denote an optimal TSP tour of length $\tau$ and let $Q$ be the capacity of the vehicle. Let the stochastic demand at each client $i \in V \setminus \{r\}$ be specified by a random variable $\chi_i \in (0, Q]$, and let $l_i$ be length of the shortest path between client i and the depot r. Then, the minimum expected cumulative cost for this instance of Cu-VRPSD is at least*

$$a. \max\left\{\tau, \frac{2}{Q}\sum_{i \neq r} E[\chi_i] \cdot l_i\right\} + b. \sum_{i \neq r} E[\chi_i] \cdot l_i.$$

First term in the lower bound in Theorem 1 is on the cost of the optimal CVRP tour and is due to Haimovich and Rinnooy Kan [40]. Bertsimas [8] extended the lower bound in [40] to the stochastic version of the capacitated VRP. This is the first term in the lower bound in Theorem 2. Gupta et al. [39] used the lower bound in [8]. The lower bound in Theorem 2 is for both of the stochastic variants of metric Cu-VRP listed in Sect. 1.2.

## 2 Mathematical Formulation

Mathematical formulation first reported in [31] can be derived from [45, 44] with a modification to the objective function, to account for the cumulative cost as defined in [33]. It is a mixed integer linear programming (MILP) formulation.

Two types of variables $x_{ij}$ and $y_{ij}$ are used. Variable $x_{ij}$ is a binary variable and denotes whether the vehicle visits customer $j$ just after visiting customer $i$ or not by setting $x_{ij}$ to, respectively, 1 and 0. $y_{ij}$ is considered as an integer variable, and it denotes the total weight of the cargo moved from customer $i$ to customer $j$ by the vehicle. The MILP formulation for the cumulative vehicle routing problem is as follows:

$$\text{min}: \quad \sum_{i=0}^{n}\sum_{j=0}^{n}((a \cdot x_{ij} + b \cdot y_{ij})c_{ij}) \tag{2}$$

$$\text{s.t.}: \quad \sum_{i=0}^{n} x_{ij} = 1 \qquad (j = 1, 2, \ldots, n) \tag{3}$$

$$\sum_{i=0}^{n} x_{ip} - \sum_{j=0}^{n} x_{pj} = 0 \qquad (p = 1, 2, \ldots, n) \tag{4}$$

$$\sum_{j=0}^{n} y_{pj} - \sum_{i=0}^{n} y_{ip} = d_p \qquad (p = 1, 2, \ldots, n) \tag{5}$$

$$y_{ij} \le Q \cdot x_{ij} \qquad (i, j = 1, 2, \ldots, n) \tag{6}$$

$$x_{ij} \in \{0, 1\} \qquad (i, j = 1, 2, \ldots, n) \tag{7}$$

$$y_{ij} \ge 0 \qquad (i, j = 1, 2, \ldots, n) \tag{8}$$

Recall that $Q$ is the capacity of the vehicle and $d_p$ is the demand at the client $p$. In the above formulation, $c_{ij}$ denotes the distance from customer $i$ to customer $j$. Equation (2) is a different way of writing the objective function given in Eq. (1). Constraint in Eq. (3) ensures that each customer node has exactly one incoming edge. Constraint in Eq. (4) ensures that in-degree equals the out-degree for all the customer nodes. Equation (5) is a flow constraint which ensures that the difference between the sum of the weight carried by the vehicle on the outgoing edge minus the sum of weight carried on the incoming edge equals the demand/weight at the customer node. Equation (6) is the capacity constraint. Equation (7) is the integrality constraint on variable $x_{ij}$. Equation (8) is the nonnegativity constraints on variable $y_{ij}$.

To implement a column generation approach to solve Cu-VRPs, Gaur et al. [31] decomposed this MILP into a master problem based on the set cover formulation and a pricing problem based on the resource constrained shortest path problem. The master problem is stated in the usual way:

$$\min: \quad \sum_{j \in R} \theta_j \cdot \alpha_j \tag{9}$$

$$\text{s.t.}: \quad \sum_{j \in R} z_{ij} \cdot \alpha_j \geq 1 \qquad (i = 1, 2, \ldots, n) \tag{10}$$

$$\alpha_j \in \{0, 1\}. \tag{11}$$

where $j$ denotes a feasible subset of the customer nodes such that the sum of the demands of the clients in $j$ is at most $Q$. $z_{ij}$ is a binary variable that indicates whether node $i$ is in subset $j$ or not. The minimum cumulative cost for serving the demand of the customers in subset $j$ is denoted by variable $\theta_j$. $R$ is the set of all the feasible subsets of the customer nodes. $\theta_j$ is computed for each subset $j$. This computation is NP-hard. Variable $\alpha_j$ is a binary variable and denotes whether subset $j$ is selected in the solution to the above master problem or not by setting $\alpha_j$ to, respectively, 1 and 0.

The mixed integer linear programming (MILP) formulation for the pricing subproblem is as follows:

$$\min: \quad \sum_{i=0}^{n} \sum_{j=0}^{n} ((a \cdot x_{ij} + b \cdot y_{ij}) c_{ij}) - \sum_{i=1}^{n} \left( \pi_i \cdot \sum_{j=0}^{n} x_{ji} \right) \tag{12}$$

$$\text{s.t.}: \quad \sum_{j=1}^{n} x_{0j} = 1 \tag{13}$$

$$\sum_{j=1}^{n} x_{j0} = 1 \tag{14}$$

$$\sum_{i=0}^{n} x_{ip} - \sum_{j=0}^{n} x_{pj} = 0 \qquad (p = 1, 2, \ldots, n) \tag{15}$$

$$\sum_{j=0}^{n} y_{pj} - \sum_{i=0}^{n} y_{ip} = d_p \cdot \sum_{k=0}^{n} x_{kp} \qquad (p = 1, 2, \ldots, n) \tag{16}$$

$$y_{ij} \leq Q \cdot x_{ij} \qquad (i, j = 1, 2, \ldots, n) \tag{17}$$

$$x_{ij} \in \{0, 1\} \qquad (i, j = 1, 2, \ldots, n) \tag{18}$$

$$y_{ij} \geq 0 \qquad (i, j = 1, 2, \ldots, n) \tag{19}$$

where $\pi = \{\pi_1, \pi_2, \cdots, \pi_n\}$ are the dual prices corresponding to the current set of columns in the master problem. Recall, $y_{ij}$ is the weight of the cargo being moved from node $i$ to node $j$. Most of the constraints in the above MILP subproblem are same as given in the MILP for Cu-VRPs. Equations (13), (14), and (16) ensure that the solution is a cycle. The objective function represents the reduced cost of a cycle. The optimal solution to the above pricing subproblem is a minimum reduced cost route starting and ending at the depot.

# 3 Solution Methods

VRPs are NP-hard because of a simple reduction from TSP. Algorithmic results for general VRPs can be divided into two high-level categories. The first set of studies address the problem theoretically and guarantees to provide a solution within a constant multiplicative factor of the optimal solution. These solutions are known as approximation algorithms for VRPs. The second class of solutions contains results that are computed experimentally. The result can be exact or approximate without any worst-case guarantee. Solution techniques in the second class can be further divided into subclasses heuristic, exact, meta-heuristic, and matheuristic methods [62]. In this section, we summarize results from both classes on Cu-VRPs together with some of the well-known results on VRPS, as VRPs are a special case of Cu-VRPs.

## *3.1 Theoretical Results*

In this section, we discuss the theoretical results for Cu-VRP and its variants. These are mostly the approximation algorithms for various variants of cumulative VRPs. Recall that there are two parameters $a$ and $b$ in the objective function of Cu-VRP. If $b = 0$, Cu-VRP reduces to vehicle routing problem (VRP). If $a = 0$, Cu-VRP reduces to traveling repairmen problem (TRP) or minimum latency problem (MLP). Therefore, we start with mentioning the renowned bounds for both VRP and TRP/MLP.

Blum et al. [11] gave a constant factor approximation algorithm for MLP. They extended their algorithm further to give a constant factor approximation solution for positive-linear time-dependent TSP. This is a special case of Cu-VRP in which the vehicle has infinite capacity and the demands of all the customer nodes are met in a single trip. Therefore, the solution to this special case of Cu-VRP is a TSP tour minimizing the cumulative cost. Fakcharoenphol et al. [27] also gave constant factor approximation algorithms for MLP and TRP with $k$ repairmen ($k$-TRP). Chaudhuri et al. [14] gave better approximation factor than [11, 27] for MLP. They also improved the approximation bounds for $k$-TRP. When $a = 0$, all the proved approximation factors for TRP or capacitated MLP hold for Cu-VRP as well.

All the approximation factors proved for the deterministic and stochastic VRPs also hold for the corresponding Cu-VRPs when $b = 0$. Haimovich and Rinnooy

Kan [40] gave the first constant factor $\left(2 - \frac{1}{Q'}\right)$ approximation algorithm for the deterministic version of the capacitated VRPs with uniform demands. Here, $Q'$ represents the maximum number of objects that the vehicle can carry at any point in time. For nonuniform demands, the first constant approximation factor $\left(3 - \frac{2}{Q}\right)$ is due to Altinkemer and Gavish [1], where $Q$ is the capacity of the vehicle. These factors are under the assumption that an optimal TSP tour is known, a finding which is an NP-hard problem. In case, when an $\alpha$ factor approximate TSP tour is known, these factors increase to $1 + \alpha\left(1 - \frac{1}{Q'}\right)$ and $2 + \alpha\left(1 - \frac{2}{Q}\right)$ respectively [40, 1]. Bompadre et al. [12] improved these bounds to $1 + \alpha\left(1 - \frac{1}{Q'}\right) - \frac{1}{3Q^3}$ and $2+\alpha\left(1 - \frac{2}{Q}\right) - \frac{1}{3Q^3}$, respectively. This is an improvement in the lower order terms. Archetti et al. [2] summarized the complexity of different variants of capacitated VRPs on some special classes of networks: line, star, tree, or circle. Bertsimas [7] asked the question whether there exists any constant factor approximation algorithm for VRPs when demands are stochastic. Gupta et al. [39] settled this question by giving $1 + \alpha$ and $2 + \alpha$ factor randomized approximation algorithms for split and unsplit VRPs where the demands are stochastic in nature.

Gaur et al. [33] gave constant factor approximation algorithms for cumulative – VRP. They considered four different cases of Cu-VRP mentioned in Sect. 1.2 and gave constant factor approximation algorithm for each of the cases. Below is the result due to Gaur et al. [33] for the case when the demands are of unequal weights, and the vehicle has infinite capacity.

**Theorem 3 (Theorem 5, [33])** *Let $C$ be a traveling salesperson tour of length $|C|$. Given a metric Cu-VRP instance in which the objects are of unequal weights, and the vehicle has infinite capacity, there exists a tour with total fuel consumption at most:*

$$\left(1 + \frac{2}{\beta}\right) b \left(\sum_{i=1}^{n} w_i d_i\right) + \left(1 + \frac{\beta}{2}\right) a|C|.$$

Next theorem due to Gaur et al. [33] gives the bound for Cu-VRP in which the demands are of equal or unequal weights, and the vehicle has finite capacity $Q$.

**Theorem 4 (Theorem 6, [33])** *Let $C$ be a traveling salesperson tour of length $|C|$. Given a metric Cu-VRP instance in which the objects are of unequal weights, and the vehicle has capacity $Q$, there exists a tour with total fuel consumption at most $\left(1 + \frac{2}{\beta}\right) \cdot b \cdot \left(\sum_{i=1}^{n} w_i d_i\right) + \left(1 + \frac{\beta}{2}\right) a|C| + 4a\frac{\sum_{i=1}^{n} w_i d_i}{Q}$.*

*Further, if all vertices have unit weights, the fuel consumption can be reduced to:*

$$\left(1 + \frac{2}{\beta}\right) \cdot b \cdot \left(\sum_{i=1}^{n} d_i\right) + \left(1 + \frac{\beta}{2}\right) a|C| + 2a\frac{\sum_{i=1}^{n} d_i}{Q}.$$

By using a 1.5 factor approximate TSP tour due to Christofides [16], Gaur et al. [33] proved the following theorem:

**Theorem 5 (Theorem 7, [33])** *The approximation factors achievable in polynomial time for equal-demand infinite-capacity, unequal-demand infinite-capacity, equal-demand capacitated, and unequal-demand capacitated variants of Cu-VRPs are* 2.5*,* 2.5*,* 3.186*, and* 4*, respectively.*

Similar to the deterministic variation of Cu-VRP, if in Cu-VRPSDs we set $b = 0$, the problem reduces to VRPs with stochastic demands (VRPSDs). Therefore, the approximation factors for VRPSDs hold for the special case of Cu-VRPSDs when $b = 0$. VRPSD is an extensively studied probabilistic variant of VRPs. The readers can refer to some early survey papers on VRPSDs which are due to Bertsimas and Simchi-Levi [9], Gendreau et al. [36], Stewart and Golden [61]. We mention below some of the popular literature and results for VRPSDs.

The Ph.D. thesis of Bertsimas [7] and the subsequent paper [8] contain some of the seminal results on VRPSDs. Bertsimas [7] was the one to ask first whether there exists any constant factor approximation algorithm for VRPSD, and recently Gupta et al. [39] answer the question in affirmative by giving a randomized approximation algorithm. Gupta et al. guaranteed the existence of $1 + \alpha$ and $2 + \alpha$ factor solution for VRPSDs with split and unsplit deliveries, respectively. Gaur et al. [34] extend the methodology used by Gupta et al. [39] to give constant factor approximation algorithm for Cu-VRPs with stochastic demands (Cu-VRPSDs). Theorems below summarize the best-known results as of the writing date on Cu-VRPSDs with split and unsplit deliveries.

**Theorem 6 (Theorems 3 and 4, [34])** *Given an instance of metric Cu-VRPSD, there exists an efficiently computable apriori tour that is* $(1 + 2\alpha)$ *and* 7 *factor approximate solution for the split and unsplit variants, respectively.*

These results are for the metric case in a general graph. If the graph in the given instance is a tree or a path, the above factors are reduced as below.

**Theorem 7 (Corollary 3 and Theorem 6, [34])** *Split and unsplit Cu-VRPSDs on the tree or path can be approximated within a factor of* 3 *and* 5*, respectively.*

Gaur and Singh [32] also analyzed the upper bound on the integrality gap of the set cover formulation given in Sect. 2 for metric instances of Cu-VRP. This analysis is in the style of Bramel and Simchi-Levi [13]. The obtained bounds on the integrality gap match the existing approximation factors for capacitated Cu-VRPs due to Gaur et al. [33] and give an alternate proof of the two statements in Theorem 5. Theorem below summarizes the worst-case bounds on the integrality gap.

**Theorem 8 (Theorems 3 and 4, [32])** *The integrality gap of the set cover formulation for metric Cu-VRP with equal and unequal demands are at most* 3.18614 *and* 4*, respectively.*

Table 1 summarizes approximation ratios for various versions of Cu-VRPs when both $a$ and $b$ are non-zero.

**Table 1** Approximation ratios for Cu-VRPs. $\alpha$ is the ratio for metric TSP

| | Uncapacitated | Capacitated | | Capacitated | |
|---|---|---|---|---|---|
| | Deterministic | | | Stochastic | |
| | Graph | | | Graph | Tree |
| Equal demand | 2.5 | 3.186 | Split delivery | $(1+2\alpha)$ | 3 |
| Unequal demand | 2.5 | 4 | Unsplit delivery | 7 | 5 |
| Gaur et al. [33] | | | Gaur et al. [34] | | |

## 3.2 Experimental Results

Toro et al. [62] note four different classes of computational techniques that are used for solving VRPs: exact algorithms, heuristics, meta-heuristic, and matheuristic methods. There are relatively few experimental studies on Cu-VRPs in comparison to the number of studies on VRPs. In this section, we note some popular experimental tools and studies for VRPs as these are a special case of Cu-VRP when $b = 0$. Finally, we briefly discuss various computational methods for Cu-VRPs.

Branch and bound algorithms are one of the widely used experimental tools for solving the MILP formulations. Christofides et al. [17] gave the first branch and bound algorithm for capacitated VRPs (CVRPs). A book chapter by Toth and Vigo [63] is recommended for the branch and bound method for VRPs. Branch and cut algorithms are another tools that involve running branch and bound while simultaneously tightening the LP relaxation using cutting planes. A cutting plane corresponds to an inequality which is satisfied by all possible integral solutions of the given LP but is violated by the current solution of LP relaxation if the solution is fractional. Laporte [47] and Fisher [28] gave branch and cut algorithms for CVRP. Column generation (CG) methods are another class of algorithms that can solve an LP with a very large number of variables. It starts with a small subset of variables and iteratively generates and adds variables that have the potential to better the objective cost. In this way, CG solves a set of small-sized LP in comparison to directly solving an LP with a very large number of variables. CG algorithms for VRPs with soft time windows are due to Liberatore et al. [49] and Qureshi et al. [58]. A CG algorithm for split delivery VRPs is due to Jin et al. [43]. A popular hybridization of branch and bound with column generation is called branch and price. In branch and price method, CG is used to solve the relaxed LP at each node of the branch and bound tree. The method is called branch-cut-price if cutting planes are used to tighten the LP relaxation. Fukasawa et al. [29] and Baldacci et al. [3] gave branch-cut-price algorithms for VRPs. We move onto cumulative VRPs now.

An MILP formulation for Cu-VRP was given by Kara et al. [45, 44]. Kara et al. formulated the problem, and for two instances of capacitated VRPs, they gave a solution to the Cu-VRP. Santos et al. [60] gave a case study for cumulative VRPs based on the mathematical formulation due to Xiao et al. [68]. Gaur and

Singh [31, 32] modified the MILP in [45, 44] with the objective function based on the objective function in [33]. They use the modified MILP in their column generation matheuristic [31, 32].

Recently Lysgaard and Wohlk [51] describe a branch-cut-price algorithm for cumulative capacitated VRPs. The exact algorithms can typically solve only small-sized instances in the order of 100 clients or so. Fukasawa et al. [30] also gave a branch-cut-price algorithm for the cumulative vehicle routing problem. Xiao and Konak [65] gave an MILP formulation and algorithms to solve a similar VRP called the green vehicle routing problem. Next, we mention a few heuristic methods for VRPs and cumulative VRPs.

A two-phase heuristic for VRPs is due to Beasley [4]. A routing problem is divided into two steps when a two-phase heuristics is used. One step assigns customer nodes to a subtour, and the other one determines in which order the customer nodes in a subtour is visited. The steps can be executed in any order: cluster-first route-second or route-first cluster-second [4]. Another most widely used construction heuristic is due to Clarke and Wright [20]. Recently, Cinar et al. [19] gave a two-phase constructive heuristic for cumulative VRPs with limited time duration by adopting the heuristic due to Clarke and Wright [20].

Meta-heuristics [38, 35] are another class of computational methods that have been widely used to solve VRP and its variants. Some of the methods in this category are ant colony optimization [6], simulated annealing [15], and evolutionary algorithms [56, 57] including genetic algorithms [10, 64], Tabu Search [21], variable neighborhood search [46], etc. An extensive bibliography on the meta-heuristic approaches for VRPs is due to Gendreau et al. [37]. Based on the available literature, it is evident that several meta-heuristics exist for various variants of VRPs. For Cu-VRPs, not many meta-heuristics are known. Meta-heuristics are not as a well-explored tool for solving Cu-VRPs and remain open for examination by the future researchers. Cinar et al. [18] gave a simulated annealing-based meta-heuristic to solve cumulative VRPs with limited time duration.

Matheuristic algorithms are a newer methodology and are designed by combining heuristic or meta-heuristic techniques with exact techniques. Gaur and Singh [31, 32] gave a matheuristic for constructing solutions to Cu-VRPs and have performed a detailed computational study. They showed that a method based on rounding solutions to a linear program performs well in practice. The linear program (LP) considered in their paper is based on the set cover formulation. The solution of the LP is computed using column generation where a dynamic programming heuristic is used to solve the pricing problem. Column generation is one of the promising tools to solve large-scale linear programming relaxations. However, their approach is inexact whereas a branch-cut-price-based approach will deliver an exact answer. They [31, 32] establish the efficacy of their approach by simulations on several sets of VRP instances. The simulation results were better than the theoretical bounds on Cu-VRP due to [33].

## 4 Conclusion

Cumulative VRPs are variants of VRPs. These are one of the popular models for fuel consumption in the last decade. Cumulative VRPs also generalize minimum latency problem and the $k$-traveling repairmen problem. Following are a few open questions. NP-hardness of split cumulative VRP with stochastic demands on paths is still open. The approximation bounds of Cu-VRPs given in [33, 34] are not yet proven to be tight; a natural question is to either reduce it or prove it to be tight. The approximability of Cu-VRPs when the number of offloads is provided as an input is still an open question. The worst-case analysis on the performance ratio of the column-generation-based algorithm given in [31, 32] is still open. Extremely few heuristics and meta-heuristics are known for Cu-VRP. This provides an open direction for future research on Cu-VRPs. Cu-VRPs considered only two factors: length of the route and weight of the vehicle to model the fuel consumption. Finally, the inclusion of more factors in the objective function and performing similar studies as currently done on Cu-VRPs is another direction that is open.

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# Constructive Algorithms for the Cumulative Vehicle Routing Problem with Limited Duration

**Didem Cinar, Beyzanur Cayir Ervural, Konstantinos Gakis, and Panos M. Pardalos**

**Abstract** In this chapter, several constructive algorithms developed for the cumulative vehicle routing problem with limited duration are used as an initial solution generator algorithm for various metaheuristics. Their performance on the solution quality obtained by solution-based and population-based metaheuristics is investigated. Data sets from the literature are used for the computational tests. The computational experiments show that the performance of simulated annealing is significantly affected by the initial solution generator. Although initial solution generators do not affect the performance of genetic algorithms as much as simulated annealing, choosing the best initial solution generator is still an important issue to obtain high-quality solutions in a proper computational time.

## 1 Introduction

Vehicle routing problems (VRPs) have been an important issue in several industrial and scientific studies on environment management, sustainable logistics, and transportation systems. They mainly focus on minimizing transportation cost, which can be defined as travel time, travel distance, truck load, emission, fuel consumption, etc. while maximizing service area under the assumption that each customer is served only once.

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Although VRPs are discussed in a vast amount of studies, there are still some challenges to solve due to enhanced formulations with new constraints and objectives. Since VRP is a combinatorial optimization problem which is NP-hard in the strong sense, exact algorithms are insufficient to solve big size problems in a polynomial time. To overcome this deficiency, various heuristics, metaheuristics, and hybrid approaches have been developed to obtain reasonable solutions in an acceptable computational time. Several variants of VRPs exist in the literature such as the capacitated VRP, dynamic VRP, stochastic VRP, green VRP, VRP with time window, VRP with backhauls, the multi-depot VRP, heterogeneous or mixed fleet VRP, the split delivery VRP, and many others. Braekers et al. [15] analyzed the VRP literature extensively and classified 277 VRP articles published between 2009 and 2015 according to the problem characteristics adapted from the taxonomy proposed by Eksioglu et al. [30].

The capacitated VRP is one of the most studied types of the VRP which is firstly defined by Dantzig and Ramser [22]. The capacitated VRP takes into account vehicle capacities besides other specific VRP constraints. The cumulative VRP (CumVRP), which is defined by Kara et al. [44], is a capacitated VRP broadened by the cost function specified as a product of the distance traveled. Cost function can be illustrated as a step function which shows an increasing flow based on tour length, such as a cumulative structure. The energy minimizing VRP, the m-traveling repairman problem, and the distance minimizing school bus routing problem are addressed as different cases of the CumVRP [44]. The CumVRP can be also defined as the routing problem having the objective of minimization of sum of arrival times at customers [61].

In this study, several constructive algorithms developed in the literature for CumVRP with limited duration (CumVRP-LD) are investigated. Moreover, their performances on several metaheuristic approaches as initial solution generators are analyzed by computational experiments on data sets from the literature. Clarke and Wright algorithm (C&W), which is one of the most widely used heuristics for capacitated VRP, and a modified Clarke and Wright algorithm and a two-phase constructive algorithm developed by Cinar et al. [20] for CumVRP-LD are the constructive algorithms used as the initial solution generators in this study. A solution-based metaheuristic, simulated annealing, and a population-based metaheuristic, genetic algorithms, are used to test the performances of the constructive heuristics.

The remainder of this chapter is organized as follows. Section 2 presents the definition of CumVRP-LD. Section 3 gives a literature review on CumVRP-LD. Sections 4 and 5 give a brief information on the constructive algorithms and metaheuristics investigated in this study, respectively, and include a literature review on their application in VRP studies. Computational results are discussed in Sect. 6 and concluding remarks are given in Sect. 7.

## 2 Cumulative VRP with Limited Duration

Let $G = (V, A)$ be a directed graph where $V = \{0, 1, \ldots, N\}$ is the set of vertices and $A = \{(i,j) \mid i,j \in V, i \neq j\}$ is the set of arcs. There is a set $M = \{1, \ldots, K\}$ for vehicles which are supposed to visit all vertices in set $V$, and the demand of each vertex should be satisfied by only one vehicle. Vehicles have different capacity $Q_k (k \in M)$ and fuel consumption rates, $a_k$ and $b_k$, where $a_k$ is the fuel consumption rate of an empty vehicle $k$ per unit distance and $b_k$ is the fuel consumption rate of vehicle $k$ per load and unit distance. Vertex 0 refers to the depot wherein each vehicle starts and finishes the corresponding route and other vertices in set $V$ can be defined as customers. CumVRP-LD is a problem of finding an optimum route set and optimum assignment of these routes to the vehicles within a predetermined time limit $TL$. Fuel consumption is the objective to be minimized and computed by using distance, load, and vehicles' fuel consumption rates. Let $d_{ij}$ be the distance from vertex $i$ to vertex $j$ where $i,j = 0, 1, \ldots, N$, and $c_i$ be the demand of vertex $i$. The demand of depot (vertex 0) is zero. In each vertex, unloading time is considered as $p_i$, $i = 1, \ldots, N$. Reloading is not considered in this study. Total transportation and loading time should be within a time limit $TL$; overtime is not allowed. Each vertex should be visited once, and total demand of the vertices in the same route cannot exceed the vehicle's capacity. A feasible instance should satisfy the following inequalities:

$$(d_{0i} + d_{i0})/v \leq TL \qquad \forall i \in V \setminus \{0\} \tag{1}$$

$$c_i \leq Q_k \qquad \forall i \in V \setminus \{0\}, k \in M \tag{2}$$

where $v$ represents the average speed of the vehicles.

Ma et al. [57] and Demir et al. [27] listed several emission models from the literature to compute fuel consumption. In this study, the fuel consumption expression from Kopfer and Kopfer [51] and Kara et al. [44], which is a function of the distance, the load, and fuel consumption rates of the vehicle, is used as the objective function. Only the delivery case is considered in this study. But, the methodologies used in this study can easily be applied for collection case without loss of generality.

Let $R$ be a feasible route set and assignment where $(i, j, k) \in R$ if vehicle $k$ visits vertex $j$ immediately after vertex $i$. Total fuel consumption can be given as follows:

$$F = \sum_{(i,j,k) \in R} d_{ij}(a_k + b_k q_{ijk}) \tag{3}$$

where $F$ is the total fuel consumption of $R$ and $q_{ijk}$ is the total load transported from customer $i$ to customer $j$ by vehicle $k$. If $b_k = 0$, then the problem becomes classical VRP minimizing total distance. Since VRPs are NP-hard problems [52], CumVRP-LD is NP-hard in strong sense. Therefore, large instances may not be solved by exact optimization methods in acceptable computational times. That is why, several heuristics and metaheuristic algorithms have been developed to solve CumVRPs. A brief literature review about CumVRPs is given in the next section.

## 3 Literature Review

In recent years, green logistics have received attention from researches and policy makers to address environmental concerns with reducing energy usage, fuel consumption, and greenhouse gas emissions [11, 27, 29, 48]. Bektaş et al. [11] presented an overview of the last progresses in the green vehicle routing as a subdiscipline of the green logistics, comprising the characterization of some emission models with definitions and applications on the road transportation. Demir et al. [28] listed the vehicle, environment, traffic, driver, and operations-related factors affecting fuel consumption. Figliozzi [33] evaluated the emissions rates of vehicles considering the travel speed and distance traveled. Demir et al. [26] emphasized that emissions rates can change depending on speed limits. Soysal et al. [72] developed a MILP formulation for two-echelon capacitated VRP with environmental aspects in which fuel consumption is estimated considering vehicle types, speed, load, and distance.

In the literature, most of the studies related to CumVRP have been broadly applied in the case of energy consumption and $CO_2$ emissions minimization in order to determine an optimal routing policy, particularly in the transportation and logistics management. Recent studies on CumVRP and its variations are summarized in Table 1. Lysgaard and Wøhlk [56] used a branch-and-cut-and-price algorithm to reduce the sum of arrival times at the customers for the CumVRP. Gaur and Singh [38] considered the CumVRP as a set cover form and used column generation method to solve the problem. Dynamic programming was utilized for the pricing subproblem of the proposed model. Gaur et al. [39] examined four types of CumVRP and developed constant factor approximation algorithms.

Since CumVRPs are NP-hard problems, various heuristics and metaheuristics have been developed to obtain good solutions in a reasonable computational time for big instances. Cinar et al. [20] developed a two-phase constructive algorithm integrating Clarke and Wright algorithm and K-means clustering approach for a CumVRP with time limit constraint in order to reduce fuel consumption. Load, distance, and the features of the vehicles are taken into account for computing the amount of fuel consumption. Ke and Feng [47] developed a two-phase metaheuristic approach that combines various perturbation and local search operators in different phases of the algorithm to get better solutions. After determining customers, the algorithm is performed according to reducing the cumulative time of each route in the second phase. The proposed method presents better solutions compared to some studies in the literature. Ngueveu et al. [61] solved CumVRP using upper and lower bounding procedures. The lower bound is derived from CumVRP features, and the upper bound is analyzed by using memetic algorithms that work with a cost function in the exploration space. Ribeiro and Laporte [67] developed an adaptive

**Table 1** Recent studies on CumVRP

| Study | Year | Solution methodology | Algorithm | Problem characteristics | Objective |
|---|---|---|---|---|---|
| Rivera et al. [70] | 2016 | Exact | MILP | Multi-trip, single vehicle | Minimizing the sum of arrival times |
| Cinar et al. [20] | 2016 | Heuristic | Two-phase constructive heuristic | Limited duration | Minimizing fuel consumption |
| Alinaghian and Naderipour [5] | 2016 | Metaheuristic | Gaussian firefly algorithm | Multi-alternative graph, time-dependent routing | Minimizing fuel consumption |
| Xiao and Konak [77] | 2016 | Hybrid | MILP and iterated neighborhood search | Heterogeneous fleet, time dependent | Minimizing emissions |
| Akpinar [4] | 2016 | Hybrid | Large neighborhood search, ant colony optimization | | Minimizing total distance |
| Lima et al. [54] | 2016 | Metaheuristic | Five metaheuristic-based algorithms | Heterogeneous fleet, mixed load | Minimizing sum of fixed cost and distance cost of vehicles |
| Flores-Garza et al. [34] | 2015 | Metaheuristic | Greedy randomized adaptive search | Optional locations, limited duration | Minimizing the sum of arrival times |
| Zhang et al. [83] | 2015 | Metaheuristic | Evolutionary local search | | Minimizing fuel consumption |
| Cinar et al. [19] | 2015 | Metaheuristic | Simulated annealing | Multi-trip | Minimizing fuel consumption |
| Victoria et al. [74] | 2015 | Metaheuristic | Two-phase method based on multi-start iterated local search | Time-dependent demand | Minimizing the sum of arrival times at |
| Gaur and Singh [38] | 2015 | Exact | Column generation approach | | Minimizing the total transportation cost |
| Lysgaard and Whlk [56] | 2014 | Exact | Branch-and-cut-and-price algorithm | | Minimizing the sum of arrival times |
| Xiao et al. [79] | 2014 | Hybrid | Variable neighborhood, simulated annealing | | Minimizing the sum of arrival times |

(continued)

**Table 1** (continued)

| Study | Year | Solution methodology | Algorithm | Problem characteristics | Objective |
|---|---|---|---|---|---|
| Rivera et al. [68] | 2014 | Hybrid | Evolutionary local search algorithm | Multi-trip | Minimizing the sum of arrival times |
| Demir et al. [27] | 2014 | Metaheuristic | Adaptive large neighborhood search | | Minimizing fuel consumption and driving time |
| Ke and Feng [47] | 2013 | Metaheuristic | Two-phase metaheuristic | | Minimizing the sum of arrival times |
| Gaur et al. [39] | 2013 | Approximation | Constant factor approximation algorithms | A single vehicle with infinite capacity | Minimizing fuel consumption |
| Ozsoydan and Sipahioglu [62] | 2013 | Metaheuristic | Three metaheuristic-based algorithms | | Minimizing the sum of arrival times |
| Ribeiro and Laporte [67] | 2012 | Metaheuristic | Adaptive large neighborhood search heuristic | | Minimizing the sum of arrival times |
| Xiao et al. [78] | 2012 | Metaheuristic | Simulated annealing | | Minimizing the sum of fixed costs and fuel costs of vehicles. |
| Chen et al. [17] | 2012 | Metaheuristic | Iterated local search algorithm | | Minimizing the sum of arrival times |
| Demir et al. [26] | 2012 | Metaheuristic | Adaptive large neighborhood search | | Minimizing total fuel, emission, and driver costs |
| Ngueveu et al. [61] | 2010 | Metaheuristic | Memetic algorithm | | Minimizing the sum of arrival times |
| Figliozzi [33] | 2010 | Heuristic | Iterated route construction and improvement approach | Time dependent | Minimizing emission and fuel consumption |

large neighborhood search heuristic in order to reduce the sum of arrival times at the customers for CumVRP in emergency cases for humanity aid. Chen et al. [17] proposed an iterated local search heuristic algorithm for the CumVRP. Xiao et al. [78] considered the fuel consumption rate as a load-dependent factor in the CumVRP and used a simulated annealing algorithm to get good results.

Alinaghian and Naderipour [5] studied a time-dependent VRP model to reduce fuel consumption. While calculating fuel consumption, they have considered load, vehicle speed, road gradient, and urban traffic factors, and they developed a modified metaheuristic algorithm based on a Gaussian firefly algorithm. Xiao and Konak [77] developed a MILP model for heterogeneous green vehicle routing with time horizon to reduce $CO_2$ emissions. The problem is defined as a kind of scheduling one since it includes customer-vehicle assignment, route selection, and travel time scheduling decisions. They proposed a hybrid approach combining MILP optimization and iterative neighborhood search. Zhang et al. [83] introduced the capacitated VRP with reducing fuel consumption under three-dimensional loading constraints. The problem aimed to minimize fuel consumption considering loading plan on the route. They proposed an evolutionary local search for solving problem. Akpinar [4] developed a new hybrid algorithm combining large neighborhood search algorithm and ant colony optimization for capacitated VRP. The study focuses on improving performance of the algorithms with providing diversification in search space considering solution improvement/construction mechanisms. Flores-Garza et al. [34] developed a MILP formulation and greedy randomized adaptive search algorithm to reduce arrival times at cities in the multi-vehicle cumulative covering tour problem. Lima et al. [54] studied a capacitated rural school bus routing problem utilizing five metaheuristic-based algorithms to provide cost saving and optimal fleet size. Various local search neighborhood approaches were adapted to deal mixed loads and a heterogeneous fleet. Ozsoydan and Sipahioglu [62] compared the performances of genetic algorithms, tabu search, and a hybrid algorithm including particle swarm optimization and genetic algorithms for the CumVRP in terms of CPU times and objective values. Although hybrid algorithms outperform genetic algorithms, the best solutions were found with tabu search algorithm. Victoria et al. [74] presented the CumVRP with time-dependent demand in humanitarian logistics where the demand is dynamic and the aim is to minimize the sum of arrival times at critical nodes. They developed a MILP model and a two-phase heuristic method based on multi-start iterated local search.

Multi-trip CumVRPs, in which the vehicles are allowed to perform multiple trips to reduce the investment cost on vehicles, have been also investigated in recent years. Rivera et al. [68] considered a multi-trip cumulative capacitated VRP for disaster relief operations, which aims to reduce sum of arrival times at required nodes, and each vehicle may perform multiple trips to provide a flexibility under demand excess. The authors proposed to utilize a multi-start evolutionary local search besides an adapted split procedure and a variable neighborhood descent algorithm. Rivera et al. [69] developed a mixed integer linear programming (MILP) model, a dominance rule, and a hybrid metaheuristic for a multi-trip CumVRP. Rivera et al. [70] proposed two MILP models – flow-based model and set partitioning model – to minimize the sum of arrival times of the single vehicle, which is able to perform multiple trips for an emergency response after a disaster. Since exact algorithms are insufficient to find optimum solution for large instances, Bellman-Ford algorithm is conducted to solve CumVRP with limited capacity. The proposed method is more successful than a commercial MILP software for small instances. Cinar et al. [19] presented a MILP model for a multi-trip CumVRP with a limited duration to reduce fuel consumption that considers the distance, the load, and the features of vehicles. The authors developed a solution-based metaheuristic approach, simulated annealing, which conducts the Clarke and Wright algorithm to start with a good initial solution.

# 4 Constructive Algorithms

## *4.1 The Clarke and Wright Algorithm*

The Clarke and Wright algorithm (C&W) was developed by Clarke and Wright [21] for capacitated VRPs. Since it is easy to implement and is able to find reasonably good results in a very short time, it is one of the most widely used heuristic algorithms in the VRP literature.

Capacitated VRP is a more general version of VRP where a limited number of vehicles can perform tours to deliver the products to the customers (or collect the products from suppliers). The load of a vehicle cannot exceed the vehicle's capacity. The C&W algorithm initiates with assigning a vehicle to each vertex. If the number of vehicles is less than the number of vertices, then the algorithm creates artificial vehicles to assign a vehicle to each vertex. Then, the following saving value is calculated for each vertex pair $(i, j)$, $i < j$:

$$s_{ij} = d_{0i} + d_{0j} - d_{ij} \tag{4}$$

where $d_{ij}$ is distance between vertices $i$ and $j$ for all $i, j \in V$, $s_{ij}$ is saving value for vertex pair $(i, j)$, and $d_{0i}$ and $d_{0j}$ are distances from depot to vertices $i$ and $j$, respectively. A vertex pair $(i, j)$ having the largest savings is merged if vertices $i$ and $j$ belong to separate vehicles, the capacity constraint of the vehicle is not exceeded, and $i$ and $j$ are the first or the last customer to be merged. The algorithm proceeds until a merging is no longer possible. There are two options of the algorithm, which are sequential and parallel building of routes. The parallel version outperforms the sequential one according to performance experiments from the literature [21].

Because the C&W algorithm is easy to implement and produces fairly good solutions in a short time, it has been widely used in capacitated VRP studies. Campbell et al. [16] utilized from C&W's savings for initial trips in the proposed three-phase heuristic approach for time-limited bus routing problem. Lima et al. [54] used a modified C&W algorithm for a better initial solution of the proposed metaheuristics in a capacitated rural school bus routing problem. Li et al. [53] developed a two-stage heuristic involving C&W algorithm for VRP. Expósito-Izquierdo et al. [32] proposed a two-level solution approach to solve the clustered capacitated VRP. C&W algorithm is utilized for creating the first route for applying record-to-record travel algorithm. Junqueira and Morabito [45] defined a heuristic approach for a three-dimensional loading capacitated VRP to achieve minimum cost delivery routes. They applied the C&W algorithm or the Gillett and Miller algorithm to generate the routes. Akpinar [4] proposed a novel hybrid metaheuristic algorithm, which combines large neighborhood search algorithm and ant colony optimization, for the capacitated routing problem to improve search ability of algorithm. The C&W algorithm was utilized to compute saving values between vertices. Cinar et al. [19] applied the C&W algorithm to generate initial solution at the beginning of a simulated annealing algorithm.

To improve the performance of the C&W algorithms, various augmentations have been conducted to the standard saving formula. Gaskell [37] and Yellow [80] defined a route shape parameter, $\lambda$, to highlight the distances between customers other than the distances to depot:

$$s_{ij} = d_{0i} + d_{0j} - \lambda d_{ij} \tag{5}$$

Paessens [63] proposed a new parameter, $\mu$, which refers to asymmetry between two nodes:

$$s_{ij} = d_{0i} + d_{0j} - \lambda d_{ij} + \mu |d_{0i} - d_{0j}| \tag{6}$$

Altınel and Öncan [7] took into account demands of the vertices to compute saving:

$$s_{ij} = d_{0i} + d_{0j} - \lambda d_{ij} + \mu |d_{0i} - d_{0j}| + v \frac{c_i + c_j}{\bar{c}} \tag{7}$$

where $c_i$ and $c_j$ are the demands of vertices $i$ and $j$, respectively, $\bar{c}$ is the average demand which is used for normalization, and $v$ is the new parameter. An enhancement of C&W algorithm, called the modified C&W algorithm, proposed by Cinar et al. [20] for CumVRP is explained in the next subsection.

## *4.2 The Modified C&W Algorithm*

The modified C&W (mC&W) algorithm was developed by Cinar et al. [20], for the CumVRP-LD considering total load and fuel consumption features of vehicles, besides distances between vertices. While total distance saving is computed in C&W algorithm, total fuel consumption saving is calculated in mC&W. So, the saving depends not only on distances between vertices but also on the total load of the vehicles in mC&W algorithm.

It is assumed that two tours are numerized as 1 and 2 which represent preceding and subsequent tour, respectively. Let $f_t$ and $l_t$ be the first and last vertices in tour $t(t = 1, 2)$. The enhanced saving formulation for assigning tour 2 after tour 1 is given as follows:

$$s_{l_2 f_1} = a(d_{l_1 0} + d_{0 f_2} - d_{l_1 f_2}) + bq'_2(d_{0 f_2} - d'_1 - d_{l_1 f_2}) + b\frac{q'_1}{d'_1} \tag{8}$$

where $q'_t$ is the total demand of the vertices in tour $t$, $d'_t$ is the total distance from depot to the last customer in tour $t$, and $p'_t$ is the total service time for tour $t$. Only the savings satisfying the following inequalities are considered in each iteration:

$$q'_1 + q'_2 \leq Q \tag{9}$$

$$\frac{d'_1 + d_{l_1 f_2} + d'_2 - d_{0 f_2} + d_{l_2 0}}{v} + p'_1 + p'_2 \leq TL \tag{10}$$

where $Q$ is the capacity of a vehicle. Inequality (9) guarantees that total load on the merged tour cannot be more than vehicle's capacity. Inequality (10) ensures that total travel and service time in the merged tour cannot exceed predefined time limit. In C&W algorithm, savings are calculated and fixed at the initial iteration of the algorithm. On the other hand, savings are updated at the beginning of each iteration in mC&W which increases the computational complexity of the algorithm.

Cinar et al. [20] have validated the performance of the mC&W algorithm with data sets from the VRP literature. The mC&W algorithm provides more successful results in terms of fuel consumption compared to the C&W algorithm. On the other hand, its performance on computational time is worse than C&W. To overcome this drawback of mC&W, a two-phase constructive algorithm was developed by Cinar et al. [20] by combining clustering approach with mC&W to maintain solution quality while accelerating the computational performance.

### *4.3 A Two-Phase Constructive Algorithm*

To solve VRP problems, various heuristic-based techniques have been developed. Bowerman et al. [14] categorized the heuristic approaches to the VRP into five sections: (1) cluster-first/route-second, (2) route-first/cluster-second, (3) savings/insertion, (4) improvement/exchange, and (5) simpler mathematical programming representations through relaxing some constraints. In clustering first-route later algorithms, the nodes are initially classified into clusters, each cluster is assigned to a vehicle, and lastly a tour is determined for each vehicle. Due to its effectiveness, most of the studies utilize from clustering first-route later algorithms.

Clustering methods are one of the most remarkable data mining techniques due to the efficient managing and analyzing capabilities of huge data sets. They have a very wide application area to classify data into homogenous categories. K-means clustering is one of the well-known unsupervised learning algorithms. Different from other clustering approaches, the K-means algorithm forms a single level of clusters, not a tree structure to define the groupings. The technique uses the actual observations in the observed data set, not only their closeness (proximity) [31]. Additionally the K-means algorithm can easily come up with enormous amounts of data for clustering. The K-means algorithm separates the data into groups or clusters, and the clusters are determined by positioning centroids in sites of space. Each observation is matched with the closest one. According to an iterative way, which includes calculation of a square error function, the final positions of the centroids are identified.

A number of clustering studies have been published in VRP literature. Gao et al. [35] applied a K-means clustering algorithm to overcome the places of depots and nearby cities in each class, and then an ant colony algorithm is implemented to handle the VRP in dynamic conditions. Yücenur and Demirel [82] developed a new geometric shape-based genetic clustering algorithm to solve multi-depot VRP. Geetha et al. [40] used a K-means algorithm for clustering that reduces the multi-depot VRP to multiple VRPs. Wang et al. [76] utilized a fuzzy clustering algorithm to categorize the customers into multiple clusters. Dechampai et al. [24] handled the General Q-Delivery VRP with clustering of customer vertices method named the Multifactor-Based Evolving Self-Organizing Map. Geetha et al. [40] developed a genetic algorithm, an article swarm optimization (PSO) and a hybrid PSO to solve multi-depot VRP. The first particles in hybrid PSO are created using K-means clustering and nearest neighbor heuristic. Alvarenga Rosa et al. [23] analyzed a capacitated helicopter routing problem with creating a mathematical model and using a clustering search metaheuristic in a simulated annealing approach. New solution is composed with a simulated annealing approach and stated to the closest cluster noticing a distance metric. Expósito-Izquierdo [32] studied a clustered capacitated VRP to detect the routes of the vehicles for satisfying the demand of customers assigned into clusters. In the problem, the main point is that all the customers in the same cluster have to be fulfilled by the same vehicle.

To improve the computational performance of the mC&W algorithm, a two-phase algorithm which is based on the principle of clustering first-routing second is proposed by Cinar et al. [20]. In the first stage, using a coordinate system in two-dimensional space, vertex components are arranged according to the angle of the vertex place, and so the problem is converted to one-dimensional clustering case. The K-means algorithm is utilized to build the segment of vertices that are constructed by the angles of the vertices. In the second stage, by implementing the mC&W algorithm under time and capacity constraints, reasonable tours are obtained for each cluster which is constructed in the first stage. For a mathematical background of two-phase algorithm, the readers are referred to Cinar et al. [20].

Computational experiments, performed by using data sets from the literature, showed that two-phase constructive algorithm improves the computational time significantly with only a slight loss in total fuel consumption compared with mC&W algorithm. Although the performances of the aforementioned constructive algorithms have been analyzed in the literature for CumVRP-LD, the authors have not encountered a study that analyzes the performance of these algorithms as initial solution generators for metaheuristics. The metaheuristics used in this study to solve CumVRP-LD are explained in the next section.

## 5 Metaheuristics

Only a small number of VRP instances can be solved by exact solution methods owing to time-consuming complexities. It may not be possible to find an optimal solution in a polynomial time. Metaheuristic approaches have emerged as a response for reducing/eliminating challenges of the combinatorial optimization problems. Metaheuristic algorithms are generally simple to implement and flexible to tackle with problems with various characteristics which include continuous, discrete, or mixed objective functions.

Most of the metaheuristics are generally based on populations, which are called population-based approaches, and they simulate the collective behavior of colonies or groups in nature. Single solution-based approaches concentrate on changing and progressing a unique candidate solution, while the population-based approaches tackle with multiple candidate solutions in search space. Solution-based approaches are listed as simulated annealing, iterated local search, variable neighborhood search algorithms, etc. [13]. Evolutionary computation, genetic algorithms, and particle swarm optimization are some techniques addressed as population-based approaches.

In this study, constructive algorithms mentioned in Sect. 4 are used to generate initial solutions for simulated annealing and genetic algorithm approaches which are solution-based and population-based metaheuristics, respectively. A general discussion and literature review on simulated annealing and genetic algorithm are given in the following section.

### *5.1 Simulated Annealing*

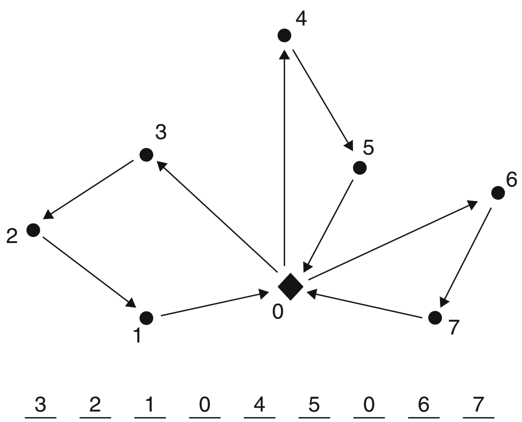
Simulated annealing (SA), firstly introduced by Kirkpatrick [49], is a probabilistic local search algorithm inspired from the physical annealing process of solid matter. It consists of heating and controlled cooling processes to reach the minimum energy configuration of its atoms. The working principle of SA is based on avoiding local optima utilizing hill-climbing procedure [12]. In order to apply the SA algorithm, the following parameters should be identified: the energy (objective) function, the candidate solution generator neighbor, the acceptance probability function, the annealing determined temperature, and the initial temperature. All these parameters can change according to the problem characteristic, and there is no rule to implement a fixed set of parameters for all problems.

In the literature, several studies employed SA approach for VRPs. Some recent studies are presented as follows. Moshref-Javadi and Lee [58] hybridized SA and variable neighborhood search to minimize total waiting time of customers in multi-commodity VRP. Yu et al. [81] proposed an SA algorithm to solve an open VRP with crossdocking. Afifi et al. [1] presented an SA-based algorithm for the VRP with time windows and synchronized visits. García-Nájera et al. [36] developed a hybrid metaheuristic algorithm based on SA to solve the VRP with stochastic demands. Afshar-Nadjafi and Afshar-Nadjafi [2] analyzed the time-dependent multi-depot VRP using SA. Mu et al. [59] considered a VRP with simultaneous pickup and delivery and developed a parallel SA algorithm using various neighborhood features. Wang et al. [76] proposed a parallel SA algorithm to reduce routing cost for VRP. Allahyari et al. [6] utilized a hybrid metaheuristic which includes GRASP (greedy randomized adaptive search procedure), iterative local search, and SA for the multi-depot capacitated VRP. Ghorbani and Jokar [41] defined a new heuristic which comprises SA and imperialist competitive algorithm to solve the multiproduct and multi-period location-routing problem. Cinar et al. [19] proposed an SA-based solution methodology for cumulative multi-trip VRP with limited duration which aims to minimize total fuel consumption.

In this study, the SA configuration used by Cinar et al. [19] is utilized to solve CumVRP-LD. A solution is represented with the order of the vertices in the tours. Tours are separated by zeros. A sample feasible solution and its representation are illustrated in Fig. 1. In this study, we performed C&W, mC&W, and two-phase constructive algorithms to generate an initial solution for SA. By this way, we aim to see the effect of the constructive algorithms on SA performance. Several runs, which start different initial solutions, will be performed to observe the performance of the constructive algorithms. In order to start with different initial solution, a random parameter is added to the saving expressions as follows:

$$s_{ij} = d_{0i} + d_{0j} - \lambda d_{ij} \tag{11}$$

**Fig. 1** A sample solution representation



$$s_{l_2f_1} = a(d_{l_10} + d_{0f_2} - d_{l_1f_2}) + bq'_2(d_{0f_2} - d'_1 - d_{l_1f_2}) + \lambda b\frac{q'_1}{d'_1} \tag{12}$$

where $\lambda$ is a random integer from the uniform distribution between 1 and 100. Equation (11) is used in C&W, while Eq. (12) is utilized in mC&W and two-phase algorithms to compute savings during initial solution generation. The operators used for the neighbor generation in SA are one-to-one exchange and reverse and delete-insert operators [50] which are illustrated in Fig. 2. One-to-one exchange operator swaps two randomly selected vertices. Reverse operator inverts the randomly selected substring. Finally, delete-insert operator removes a randomly selected vertex and inserts it in another randomly selected position.

## 5.2 Genetic Algorithms

Genetic algorithms (GAs) are a widely applied nature-inspired soft computing method. It was developed by Holland in 1975, and the applicability of GA to solving multidimensional complex problems was demonstrated by De Jong [25] and Goldberg [42]. A GA uses selection, crossover, and mutation operators, which are inspired from natural selection. It evaluates a group of candidate solutions, called population, instead of single solution. Therefore, the search mechanism works through a multiple direction simultaneously.

In GA, the population consists of individuals (solutions) encoded as chromosomes. The selection procedure begins with choosing solutions from the population for mating. A crossover operator maintains reproduction by producing new individuals (offsprings) from two selected solutions (parents) in the mating pool. Mutation operator randomly alters one or more genes in a chromosome to ensure genetic diversity. A solution is evaluated using its fitness value which is described by the objective function of the problem. Building of the initial solution at the beginning of the algorithm has a significant influence on GA performance in order to keep a broad searching ability under exploration and exploitation concepts.

Some recent studies that concentrate on GA to solve VRPs are given as follows. Pierre and Zakari [65] presented a stochastic partially optimized cyclic shift crossover for multi-objective GAs for VRP with time windows. Park et al. [64] utilized a GA for the inventory-routing problem with lost sales. The GA identified vehicle routes and replenishment times while maximizing supply chain profits. Bae and Moon [9] used a GA for the multi-depot VRP with time windows to reduce fixed costs as well as other related expenses. Karakatic and Podgorelec [46] gave a survey of GAs for solving multi-depot VRP. Ahmadizar et al. [3] proposed a GA for two-level VRP with crossdocking under transportation cost. Barkaoui et al. [10] used a hybrid GA for dynamic vehicle routing and scheduling problem. Shaabani and Kamalabadi [71] proposed a population-based SA algorithm for inventory-routing problem and compared the results with GA. Lu and Yu [55] proposed a GA for solving the pickup and delivery VRP with time windows. Vidal et al. [75] utilized

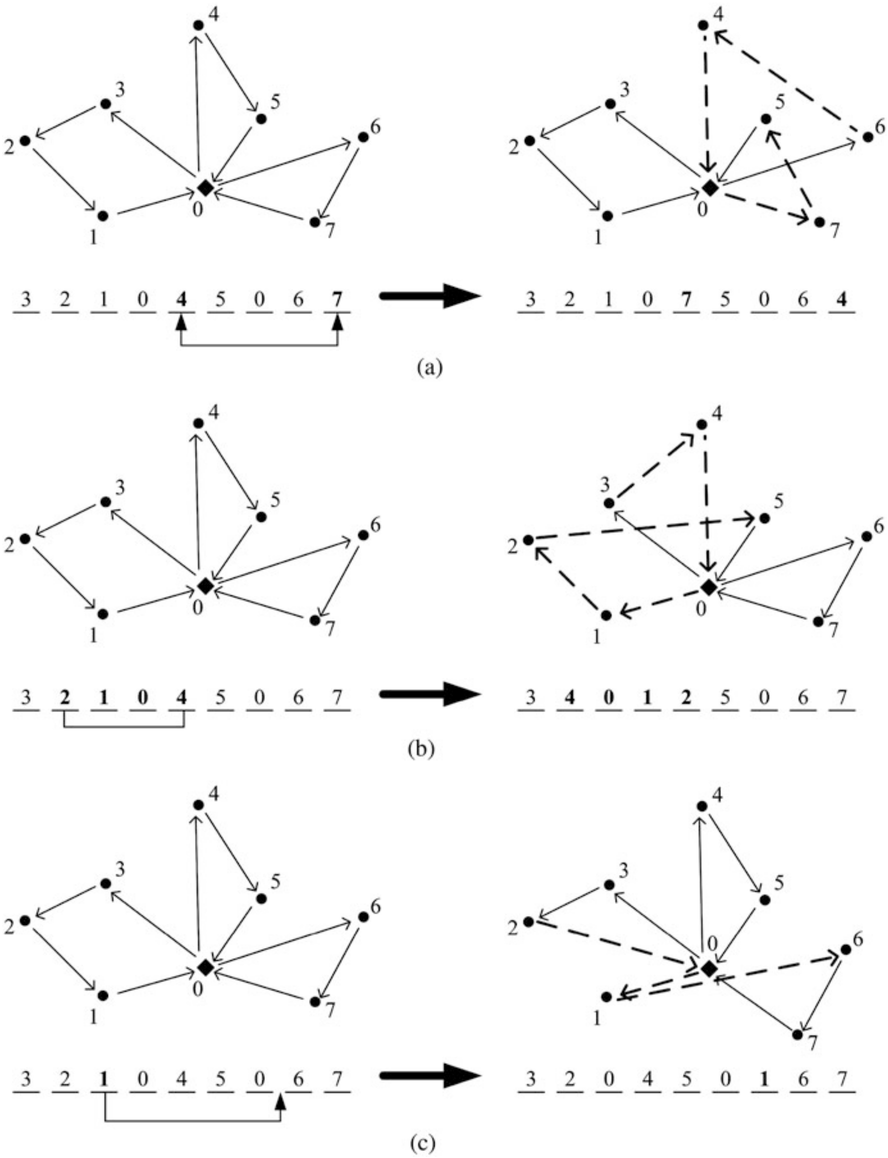



**Fig. 2** Operators used to generate neighbors [19]. (**a**) One-to-one exchange operator. (**b**) Reverse operator. (**c**) Delete-insert operator

a hybrid genetic search within each cluster for a clustered VRP. Nazif and Lee [60] used a GA for solving capacitated VRP. Pop et al. [66] presented a hybrid heuristic algorithm which combines GA and local search procedure for the generalized VRP. Anbuudayasankar et al. [8] developed an adaptive GA for bi-objective VRP with forced backhauls.

In this paper, the implementation of GA for the CumVRP-LD is given as follows. Each individual is represented as in SA (Fig. 1). Initial population is generated using constructive algorithms given in Sect. 4. Since GA is a probabilistic approach, several runs will be performed to observe the performance of the algorithm as in SA. Equations (11) and (12) are used to start with different initial populations. Elitist rule and roulette wheel selection are performed for selection. In order to satisfy monotonical improvement, a predetermined number of the best solutions in the population are reserved for the mating pool by using the elitist rule. The rest of the individuals in the mating pool are determined by roulette wheel selection, which is the most widely used selection operator. Offsprings are generated by crossover and mutation operators by using the individuals in mating pool. The operators given in Fig. 2 are used as the crossover operators in GA. The mutation operators – split-tour and merge-tour operators – are illustrated in Fig. 3. The split-tour operator divides a subtour into two separate subtours from the randomly selected position. The merge-tour operator merges two randomly selected subtours if the capacity and time limitations are satisfied after merging. The number of offsprings generated by crossover and mutation is determined according to the crossover and mutation

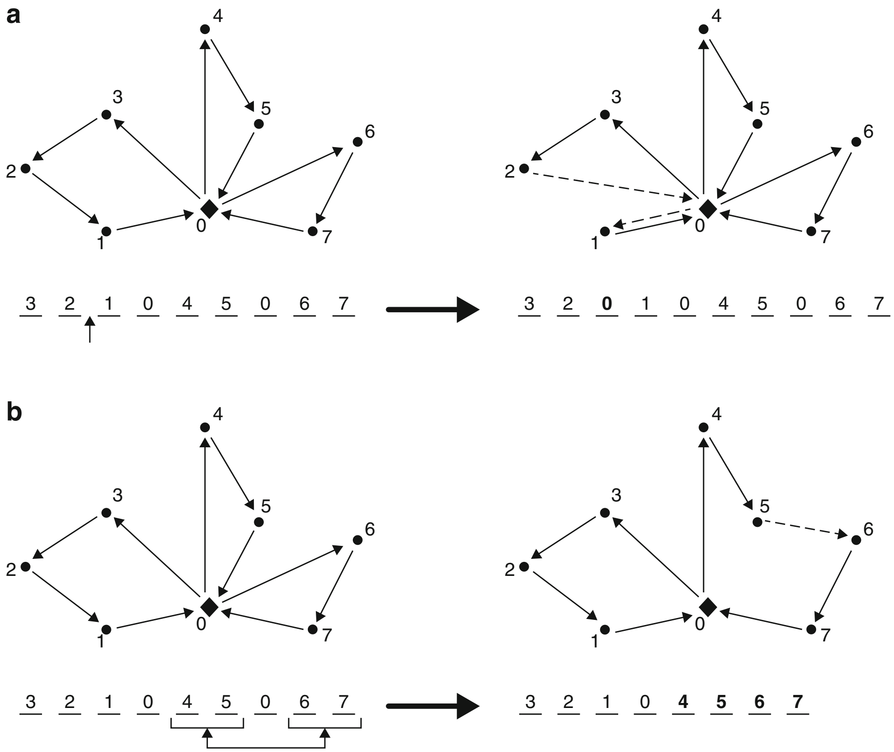



**Fig. 3** Operators used for mutation in GA. (**a**) Split-tour operator. (**b**) Merge-tour operator

probabilities, respectively. If the fitness value of the offspring is better than the parent, then it is transferred to the next generation. Iteration ends after the next generation is formed and replaced the population. The algorithm is terminated when the maximal number of generations is reached.

## 6 Experimental Analysis

Well-known capacitated VRP instances from the literature are utilized to test the performance of aforementioned constructive algorithms as initial solution generation algorithms for SA and GA. Fourteen instances in the data set of Christofides et al. [18] (C1–C14) having between 50 and 200 customers, 13 instances in the data set of Taillard [73] (T1–T13) including between 75 and 385 customers, and 20 instances in the data set of Golden et al. [43] (G1–G20) ranging from 240 to 483 are used for the computational tests. All algorithms used in this study were coded in Microsoft Visual C++ Version 10.0. The computational tests were performed on a portable work station with a 1.73 GHz Intel Core i7 processor and 4 Gb of RAM.

### *6.1 Configuration of the Experiments*

Demand, distance, and vehicles' capacity values are taken from the data sets. Other parameters are fixed as follows. Service time for each customer and the average velocity of the vehicles are fixed to 10 and 1, respectively. In order to get a tight time constraint, the time limit is fixed according to the following expression for each instance:

$$TL = \left\lceil \frac{\max_{i \in V\setminus\{0\}} d_{0i}}{10} \right\rceil \cdot 10 \cdot 2 + \left\lceil \max_{i \in V\setminus\{0\}} p_i \right\rceil \tag{13}$$

where $TL$ is time limit and $p_i$ is service time for vertex $i$. Parameter $a_k$, which refers to the fuel consumption of an empty vehicle per kilometer, is 26, and parameter $b_k$, to the fuel consumption per ton and kilometer, is 0.36 as in Kopfer and Kopfer [51]. SA and GA parameters used in the computational experiments are given in Tables 2 and 3, respectively.

**Table 2** Parameters of proposed SA

| Parameters | |
|---|---|
| Initial temperature | 20 |
| Minimum temperature | 0.1 |
| Temperature factor | 0.95 |
| Operators | One-to-one exchange (40%), reverse (30%), delete-insert (30%) |

**Table 3** Parameters of proposed GA

| Parameters | |
|---|---|
| Population size | 100 |
| Selection | Elitist rule (1%), roulette wheel (99%) |
| Crossover | One-to-one exchange (40%), reverse (30%), delete-insert (30%) |
| Mutation | Split-tour (50%), merge-tours (50%) |
| Crossover probability | 0.9 |
| Mutation probability | 0.1 |
| Number of generations | 100 |

## *6.2 Computational Results*

Both SA and GA run 25 times for each instance with each initial solution generator. To reduce the effects stemming from randomness, each run is implemented with a fixed random seed. For example, the same random seed is used to generate random numbers in the first runs of SAs and GAs with C&W, mC&W, and two-phase algorithms for each instance.

Let $A_1$, $A_2$, and $A_3$ refer to C&W, mC&W, and two-phase algorithms, respectively. $F(A_i)$ $(i = 1, 2, 3)$ represents the total fuel consumption obtained by SA with initial solution generator $A_i$. $C(A_i)$ is the total computational time (CPU time) in second achieved by SA with initial solution generator $A_i$. Pairwise comparisons of initial solution generators for SA are given in Table 4. The first column represents the name of the instances. The size of the instances is given in the second column. The number of runs where SA with $A_i$ outperforms SA with $A_j$ $(j = 1, 2, 3)$ is given by the columns between three and eight in terms of fuel consumption and computational time. For example, SA with mC&W algorithm ($A_2$) obtained better fuel consumption values in 21 runs than SA with two-phase algorithm ($A_3$) for the instance C1.

Cinar et al. [20] tested the performances of C&W, mC&W, and two-phase algorithms without hybridizing with any other algorithms. According to their experimental results, the mC&W algorithm has the best performance in terms of fuel consumption, while the C&W algorithm outperforms others with respect to CPU time. In this study, we investigate the performances of the constructive algorithms as initial solution generators while hybridizing with metaheuristics. According to Table 4, the best performance with respect to fuel consumption belongs to SA with mC&W algorithm, while the worst one is SA with C&W algorithm for most of the instances. On the other hand, the reverse is true in terms of CPU time. According to the computational time, C&W and mC&W algorithms have the best and worst performances, respectively.

The best results are obtained by different algorithms in terms of fuel consumption and CPU times. So, the question of which algorithm should be preferred to be used

**Table 4** Pairwise comparisons of initial solution generators for SA

| | | Fuel consumption | | | CPU | | |
|---|---|---|---|---|---|---|---|
| Instance | N | $F(A_1) \leq F(A_2)$ | $F(A_1) \leq F(A_3)$ | $F(A_2) \leq F(A_3)$ | $C(A_1) \leq C(A_2)$ | $C(A_1) \leq C(A_3)$ | $C(A_2) \leq C(A_3)$ |
| C1 | 50 | 0 | 0 | 21 | 24 | 25 | 12 |
| C2 | 75 | 0 | 0 | 25 | 25 | 20 | 3 |
| C3 | 100 | 0 | 0 | 25 | 25 | 16 | 0 |
| C4 | 150 | 0 | 0 | 25 | 25 | 17 | 0 |
| C5 | 199 | 0 | 0 | 25 | 25 | 25 | 1 |
| C6 | 50 | 0 | 0 | 20 | 25 | 25 | 18 |
| C7 | 75 | 0 | 0 | 25 | 25 | 19 | 1 |
| C8 | 100 | 0 | 0 | 25 | 25 | 20 | 1 |
| C9 | 150 | 0 | 0 | 25 | 25 | 18 | 1 |
| C10 | 199 | 0 | 0 | 22 | 25 | 25 | 0 |
| C11 | 120 | 0 | 0 | 25 | 25 | 23 | 2 |
| C12 | 100 | 0 | 0 | 25 | 25 | 19 | 0 |
| C13 | 120 | 0 | 0 | 25 | 25 | 21 | 0 |
| C14 | 100 | 0 | 0 | 25 | 25 | 19 | 0 |
| G1 | 240 | 0 | 0 | 24 | 25 | 23 | 14 |
| G2 | 320 | 0 | 0 | 24 | 25 | 25 | 9 |
| G3 | 400 | 0 | 0 | 25 | 25 | 25 | 3 |
| G4 | 480 | 0 | 0 | 25 | 25 | 25 | 25 |
| G5 | 200 | 0 | 0 | 18 | 25 | 25 | 17 |
| G6 | 280 | 0 | 0 | 25 | 25 | 25 | 11 |
| G7 | 360 | 0 | 0 | 25 | 25 | 25 | 17 |
| G8 | 440 | 0 | 0 | 9 | 25 | 25 | 16 |
| G9 | 255 | 0 | 0 | 17 | 25 | 25 | 5 |

(continued)

**Table 4** (continued)

| Instance | N | Fuel consumption | | | CPU | | |
|---|---|---|---|---|---|---|---|
| | | $F(A_1) \leq F(A_2)$ | $F(A_1) \leq F(A_3)$ | $F(A_2) \leq F(A_3)$ | $C(A_1) \leq C(A_2)$ | $C(A_1) \leq C(A_3)$ | $C(A_2) \leq C(A_3)$ |
| G10 | 323 | 0 | 0 | 7 | 25 | 24 | 1 |
| G11 | 399 | 0 | 0 | 15 | 25 | 25 | 1 |
| G12 | 483 | 0 | 0 | 10 | 25 | 24 | 0 |
| G13 | 252 | 0 | 0 | 7 | 23 | 25 | 18 |
| G14 | 320 | 0 | 0 | 11 | 25 | 25 | 6 |
| G15 | 396 | 0 | 0 | 10 | 25 | 24 | 9 |
| G16 | 480 | 0 | 0 | 9 | 25 | 23 | 8 |
| G17 | 240 | 0 | 0 | 16 | 25 | 22 | 3 |
| G18 | 300 | 0 | 0 | 22 | 25 | 25 | 0 |
| G19 | 360 | 0 | 0 | 25 | 25 | 25 | 0 |
| G20 | 420 | 0 | 0 | 25 | 25 | 25 | 0 |
| T1 | 385 | 0 | 0 | 25 | 25 | 24 | 0 |
| T2 | 75 | 0 | 0 | 16 | 25 | 25 | 6 |
| T3 | 75 | 0 | 0 | 7 | 25 | 25 | 8 |
| T4 | 75 | 0 | 0 | 4 | 25 | 25 | 25 |
| T5 | 75 | 0 | 0 | 24 | 25 | 25 | 1 |
| T6 | 100 | 0 | 0 | 23 | 25 | 24 | 0 |
| T7 | 100 | 0 | 0 | 25 | 25 | 21 | 1 |
| T8 | 100 | 0 | 0 | 20 | 25 | 24 | 0 |
| T9 | 100 | 0 | 0 | 25 | 25 | 23 | 0 |
| T10 | 150 | 0 | 0 | 21 | 25 | 25 | 1 |
| T11 | 150 | 0 | 0 | 17 | 25 | 25 | 1 |
| T12 | 150 | 0 | 0 | 16 | 25 | 22 | 0 |
| T13 | 150 | 0 | 0 | 21 | 25 | 24 | 1 |

as an initial solution generator for SA still stands. To decide the most appropriate initial solution generator for SA, relative deviations (RD) for C&W and mC&W are also investigated. The relative deviation is calculated as follows:

$$\mathrm{RD} = \frac{P(A_3) - P(A_i)}{P(A_3)} \cdot 100 \tag{14}$$

where $P(A_i)$ is the performance indicator for corresponding metaheuristic algorithm with initial solution generator $A_i$. The two-phase algorithm is chosen as the base algorithm to compute RD. Positive RD refers to the percentage of improvement obtained by SA with algorithm $A_i$ ($i = 1, 2$) compared with the two-phase algorithm.

Average RD values for each instance are given in Table 5. According to the RDs, SA with C&W algorithm has significantly worse results than SA with two-phase algorithm in terms of fuel consumption. However, its computational performance

**Table 5** RDs for SA algorithms with C&W and mC&W algorithms

| | | C&W | | | | mC&W | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Fuel | | CPU | | Fuel | | CPU | |
| Instance | N | av. | std dev. | av. | std dev. | av. | std dev. | av. | std dev. |
| C1 | 50 | −30.78 | 10.06 | 46.35 | 14.20 | 1.38 | 1.75 | −3.16 | 26.55 |
| C2 | 75 | −33.87 | 10.36 | 16.65 | 19.18 | 2.33 | 0.33 | −33.77 | 23.30 |
| C3 | 100 | −37.64 | 12.78 | 9.70 | 21.89 | 2.88 | 0.51 | −82.37 | 48.24 |
| C4 | 150 | −41.44 | 13.86 | 7.34 | 23.28 | 0.94 | 0.26 | −113.77 | 58.47 |
| C5 | 199 | −42.00 | 14.79 | 34.44 | 14.50 | 0.49 | 0.34 | −86.58 | 51.79 |
| C6 | 50 | −29.42 | 7.67 | 46.41 | 12.21 | 1.16 | 1.85 | 5.28 | 23.48 |
| C7 | 75 | −34.33 | 11.58 | 9.35 | 16.96 | 2.34 | 0.24 | −37.66 | 22.55 |
| C8 | 100 | −37.58 | 12.94 | 19.81 | 18.12 | 2.74 | 0.59 | −70.64 | 51.51 |
| C9 | 150 | −41.71 | 13.20 | 9.36 | 21.41 | 0.88 | 0.28 | −87.60 | 65.32 |
| C10 | 199 | −42.23 | 15.23 | 39.82 | 13.29 | 0.47 | 0.36 | −86.59 | 57.35 |
| C11 | 120 | −41.34 | 13.48 | 23.64 | 16.48 | 0.54 | 0.21 | −55.44 | 33.49 |
| C12 | 100 | −29.30 | 10.36 | 9.78 | 22.18 | 3.77 | 0.83 | −87.75 | 31.21 |
| C13 | 120 | −42.84 | 12.22 | 15.54 | 19.50 | 0.50 | 0.27 | −77.78 | 30.87 |
| C14 | 100 | −28.41 | 10.00 | 9.08 | 19.44 | 3.80 | 0.95 | −85.17 | 38.99 |
| G1 | 240 | −58.07 | 11.54 | 41.05 | 20.65 | 0.18 | 0.18 | 0.56 | 26.55 |
| G2 | 320 | −64.70 | 18.67 | 68.19 | 8.34 | 0.16 | 0.11 | −7.73 | 20.71 |
| G3 | 400 | −72.25 | 21.91 | 56.00 | 14.70 | 0.51 | 0.13 | −41.25 | 38.84 |
| G4 | 480 | −72.06 | 24.47 | 96.02 | 1.48 | 0.47 | 0.11 | 85.28 | 3.47 |
| G5 | 200 | −65.32 | 20.69 | 64.49 | 8.43 | 0.09 | 0.31 | 3.64 | 17.71 |
| G6 | 280 | −71.35 | 21.88 | 74.55 | 10.75 | 0.40 | 0.18 | −4.04 | 25.37 |
| G7 | 360 | −78.41 | 23.08 | 70.98 | 8.03 | 0.44 | 0.10 | 13.68 | 22.48 |
| G8 | 440 | −76.65 | 23.06 | 73.03 | 12.58 | −0.04 | 0.08 | 1.16 | 30.42 |
| G9 | 255 | −16.44 | 7.55 | 52.51 | 15.36 | 0.17 | 0.46 | −32.10 | 41.70 |
| G10 | 323 | −20.95 | 9.78 | 57.11 | 25.41 | −0.28 | 1.03 | −165.28 | 116.83 |

(continued)

**Table 5** (continued)

| Instance | N | C&W | | | | mC&W | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Fuel | | CPU | | Fuel | | CPU | |
| | | av. | std dev. | av. | std dev. | av. | std dev. | av. | std dev. |
| G11 | 399 | −21.15 | 9.14 | 37.06 | 22.48 | 0.24 | 0.68 | −147.87 | 121.50 |
| G12 | 483 | −22.43 | 10.56 | 43.65 | 33.45 | −0.31 | 1.16 | −137.52 | 70.58 |
| G13 | 252 | −16.16 | 5.50 | 53.75 | 15.00 | −0.93 | 1.32 | 7.50 | 37.51 |
| G14 | 320 | −19.01 | 7.52 | 60.36 | 18.33 | −0.05 | 0.91 | −26.00 | 37.65 |
| G15 | 396 | −20.21 | 8.26 | 39.38 | 23.39 | −0.03 | 0.61 | −13.12 | 28.15 |
| G16 | 480 | −19.95 | 7.85 | 45.01 | 25.25 | −0.12 | 0.55 | −18.25 | 35.46 |
| G17 | 240 | −20.62 | 7.13 | 19.51 | 24.38 | 0.77 | 1.07 | −78.94 | 47.92 |
| G18 | 300 | −37.75 | 15.48 | 40.17 | 14.26 | 1.24 | 0.95 | −117.34 | 68.66 |
| G19 | 360 | −44.00 | 18.23 | 59.83 | 11.78 | 1.65 | 0.75 | −246.08 | 164.68 |
| G20 | 420 | −43.77 | 15.36 | 55.20 | 11.24 | 1.41 | 0.42 | −146.17 | 76.43 |
| T1 | 385 | −63.26 | 22.22 | 50.20 | 17.47 | 3.59 | 0.33 | −101.38 | 43.50 |
| T2 | 75 | −12.40 | 2.00 | 38.93 | 15.24 | 0.08 | 0.27 | −15.39 | 19.13 |
| T3 | 75 | −9.17 | 2.54 | 36.17 | 10.37 | −0.22 | 0.59 | −20.29 | 25.68 |
| T4 | 75 | −15.70 | 3.01 | 59.92 | 6.72 | −0.45 | 0.47 | 27.44 | 12.02 |
| T5 | 75 | −10.28 | 2.90 | 49.39 | 8.61 | 0.53 | 0.25 | −26.37 | 17.97 |
| T6 | 100 | −14.89 | 3.62 | 29.80 | 12.72 | 0.58 | 0.45 | −64.12 | 28.35 |
| T7 | 100 | −11.87 | 2.97 | 18.90 | 17.07 | 0.66 | 0.33 | −69.04 | 46.39 |
| T8 | 100 | −11.56 | 2.76 | 22.52 | 14.34 | 0.35 | 0.49 | −25.88 | 30.85 |
| T9 | 100 | −19.70 | 3.91 | 18.81 | 14.48 | 0.68 | 0.33 | −61.07 | 28.39 |
| T10 | 150 | −12.74 | 3.82 | 28.05 | 11.78 | 0.20 | 0.36 | −40.51 | 23.60 |
| T11 | 150 | −11.28 | 2.93 | 33.11 | 9.78 | 0.12 | 0.29 | −51.37 | 27.00 |
| T12 | 150 | −11.24 | 3.19 | 21.39 | 17.90 | 0.34 | 0.86 | −70.01 | 34.74 |
| T13 | 150 | −13.57 | 3.28 | 27.71 | 18.28 | 0.33 | 0.34 | −55.04 | 31.68 |

is better than SA with two-phase algorithm. On the other hand, although the RDs for SA with mC&W are positive for most of the instances in terms of fuel consumption, the performance improvement of SA with the mC&W algorithm is significantly small compared to SA with two-phase algorithm (averages of RD are close to zero and standard deviations are very small for all instances). But SA with two-phase algorithm has quite better computational times than SA with mC&W algorithm. So, although using SA with two-phase algorithm causes slightly less quality solutions, it accelerates the computational performance of SA with mC&W algorithm significantly, and its solution quality is substantially better than C&W algorithm. These inferences overlap with the observations in Cinar et al. [20]. It can be concluded that the performance of SA is significantly affected by the initial solution generator.

Pairwise comparisons of initial solution generators for GA are given in Table 6. The algorithm having the best performance differs for each instance. Average RD values are given in Table 7. According to the RDs, GA with mC&W and GA

**Table 6** Pairwise comparisons of initial solution generators for GA

| Instance | N | Fuel consumption | | | CPU | | |
|---|---|---|---|---|---|---|---|
| | | $F(A_1) \leq F(A_2)$ | $F(A_1) \leq F(A_3)$ | $F(A_2) \leq F(A_3)$ | $C(A_1) \leq C(A_2)$ | $C(A_1) \leq C(A_3)$ | $C(A_2) \leq C(A_3)$ |
| C1 | 50 | 22 | 23 | 23 | 0 | 0 | 19 |
| C2 | 75 | 15 | 15 | 4 | 0 | 0 | 25 |
| C3 | 100 | 25 | 25 | 25 | 0 | 0 | 0 |
| C4 | 150 | 25 | 25 | 24 | 0 | 0 | 25 |
| C5 | 199 | 25 | 25 | 24 | 0 | 0 | 6 |
| C6 | 50 | 22 | 23 | 23 | 0 | 0 | 25 |
| C7 | 75 | 15 | 15 | 4 | 0 | 0 | 25 |
| C8 | 100 | 25 | 25 | 25 | 0 | 0 | 0 |
| C9 | 150 | 25 | 25 | 24 | 0 | 0 | 10 |
| C10 | 199 | 25 | 25 | 24 | 0 | 0 | 3 |
| C11 | 120 | 0 | 0 | 22 | 1 | 1 | 1 |
| C12 | 100 | 3 | 3 | 25 | 3 | 3 | 0 |
| C13 | 120 | 0 | 0 | 22 | 1 | 1 | 0 |
| C14 | 100 | 3 | 3 | 25 | 3 | 3 | 0 |
| G1 | 240 | 25 | 25 | 8 | 0 | 0 | 23 |
| G2 | 320 | 25 | 25 | 18 | 0 | 0 | 24 |
| G3 | 400 | 25 | 25 | 21 | 0 | 0 | 1 |
| G4 | 480 | 24 | 24 | 0 | 0 | 0 | 0 |
| G5 | 200 | 25 | 25 | 24 | 0 | 0 | 2 |
| G6 | 280 | 25 | 25 | 2 | 0 | 0 | 12 |
| G7 | 360 | 25 | 25 | 22 | 0 | 0 | 21 |
| G8 | 440 | 25 | 25 | 0 | 3 | 0 | 1 |
| G9 | 255 | 15 | 15 | 14 | 9 | 9 | 16 |
| G10 | 323 | 0 | 0 | 3 | 25 | 25 | 0 |

(continued)

**Table 6** (continued)

| Instance | N | Fuel consumption | | | CPU | | |
|---|---|---|---|---|---|---|---|
| | | $F(A_1) \leq F(A_2)$ | $F(A_1) \leq F(A_3)$ | $F(A_2) \leq F(A_3)$ | $C(A_1) \leq C(A_2)$ | $C(A_1) \leq C(A_3)$ | $C(A_2) \leq C(A_3)$ |
| G11 | 399 | 0 | 0 | 13 | 11 | 11 | 7 |
| G12 | 483 | 0 | 0 | 17 | 25 | 25 | 5 |
| G13 | 252 | 0 | 0 | 25 | 22 | 23 | 5 |
| G14 | 320 | 0 | 0 | 25 | 25 | 25 | 3 |
| G15 | 396 | 0 | 0 | 0 | 22 | 22 | 4 |
| G16 | 480 | 0 | 0 | 0 | 22 | 22 | 6 |
| G17 | 240 | 25 | 25 | 25 | 0 | 0 | 4 |
| G18 | 300 | 11 | 24 | 25 | 0 | 0 | 0 |
| G19 | 360 | 0 | 24 | 25 | 1 | 0 | 0 |
| G20 | 420 | 25 | 25 | 25 | 0 | 0 | 25 |
| T1 | 385 | 24 | 25 | 25 | 0 | 0 | 25 |
| T2 | 75 | 1 | 3 | 23 | 10 | 23 | 23 |
| T3 | 75 | 6 | 9 | 16 | 18 | 25 | 24 |
| T4 | 75 | 4 | 2 | 8 | 22 | 25 | 25 |
| T5 | 75 | 2 | 4 | 17 | 25 | 25 | 1 |
| T6 | 100 | 0 | 4 | 25 | 25 | 25 | 24 |
| T7 | 100 | 0 | 0 | 25 | 25 | 25 | 25 |
| T8 | 100 | 0 | 0 | 7 | 25 | 25 | 21 |
| T9 | 100 | 0 | 0 | 22 | 25 | 25 | 25 |
| T10 | 150 | 7 | 9 | 16 | 25 | 25 | 25 |
| T11 | 150 | 0 | 0 | 8 | 25 | 25 | 25 |
| T12 | 150 | 0 | 0 | 20 | 25 | 25 | 25 |
| T13 | 150 | 0 | 0 | 24 | 25 | 25 | 25 |

with two-phase algorithms have similar performances in terms of fuel consumption (averages of RD are close to zero and standard deviations are very small for all instances). This situation is similar for GA with C&W and GA with two-phase, except that for some instances (G11, G2, G3, G5, G8), GA with C&W achieves more than 10% improvement. Also, its computational performance is significantly better than GA with two-phase. Consequently, for most of the instances, initial solution generator does not affect the performance of GA as much as SA. The reason of this may be that SA generates only one solution and proceeds on this solution while the GA works on a set of solutions. Also, the most appropriate initial solution generation algorithm changes according to the instance when using GA.

**Table 7** RDs for GA algorithms with C&W and mC&W algorithms

| | | C&W | | | | mC&W | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Fuel | | CPU | | Fuel | | CPU | |
| Instance | N | av. | std dev. | av. | std dev. | av. | std dev. | av. | std dev. |
| C1 | 50 | 2.49 | 1.83 | −129.33 | 79.98 | 0.73 | 0.49 | 8.52 | 32.97 |
| C2 | 75 | 0.62 | 1.86 | −97.11 | 17.21 | −0.34 | 0.38 | 27.56 | 6.49 |
| C3 | 100 | 8.99 | 1.33 | −178.99 | 24.27 | 1.07 | 0.26 | −66.84 | 13.77 |
| C4 | 150 | 8.22 | 1.44 | −160.65 | 23.67 | 0.75 | 0.36 | 18.96 | 9.45 |
| C5 | 199 | 5.26 | 1.48 | −360.71 | 230.43 | 0.41 | 0.21 | −67.80 | 64.10 |
| C6 | 50 | 2.49 | 1.83 | −47.94 | 19.33 | 0.73 | 0.49 | 40.16 | 7.62 |
| C7 | 75 | 0.62 | 1.86 | −161.62 | 30.30 | −0.34 | 0.38 | 21.12 | 10.07 |
| C8 | 100 | 8.99 | 1.33 | −291.80 | 31.41 | 1.07 | 0.26 | −96.23 | 15.60 |
| C9 | 150 | 8.22 | 1.44 | −259.07 | 33.94 | 0.75 | 0.36 | −1.33 | 8.56 |
| C10 | 199 | 5.26 | 1.48 | −452.73 | 283.31 | 0.41 | 0.21 | −103.34 | 70.62 |
| C11 | 120 | −1.47 | 0.87 | −234.25 | 78.10 | 0.25 | 0.22 | −33.93 | 19.76 |
| C12 | 100 | −0.79 | 0.78 | −211.96 | 84.89 | 0.71 | 0.26 | −66.73 | 9.54 |
| C13 | 120 | −1.47 | 0.87 | −232.89 | 76.87 | 0.25 | 0.22 | −42.08 | 19.57 |
| C14 | 100 | −0.79 | 0.78 | −205.09 | 84.10 | 0.71 | 0.26 | −53.72 | 15.57 |
| G1 | 240 | 22.34 | 0.15 | −302.45 | 147.65 | −0.04 | 0.12 | 24.71 | 22.44 |
| G2 | 320 | 20.40 | 2.13 | −283.66 | 54.11 | 0.05 | 0.09 | 11.50 | 11.37 |
| G3 | 400 | 11.17 | 3.49 | −637.31 | 66.48 | 0.05 | 0.06 | −23.59 | 9.70 |
| G4 | 480 | 1.47 | 0.69 | −615.97 | 257.91 | −0.24 | 0.04 | −41.04 | 16.18 |
| G5 | 200 | 10.92 | 1.54 | −277.56 | 106.73 | 0.07 | 0.04 | −11.02 | 13.19 |
| G6 | 280 | 8.56 | 1.61 | −107.47 | 71.85 | −0.13 | 0.10 | −1.15 | 36.39 |
| G7 | 360 | 6.98 | 2.05 | −144.22 | 32.42 | 0.08 | 0.07 | 6.83 | 7.30 |
| G8 | 440 | 10.16 | 2.73 | −151.96 | 97.78 | −0.24 | 0.06 | −61.69 | 48.31 |
| G9 | 255 | 0.75 | 2.06 | −32.15 | 99.12 | −0.02 | 0.13 | 22.22 | 30.45 |
| G10 | 323 | −3.16 | 0.27 | 65.28 | 4.58 | −0.14 | 0.14 | −22.82 | 17.79 |
| G11 | 399 | −3.05 | 0.76 | −56.57 | 122.09 | 0.00 | 0.14 | −16.54 | 17.42 |
| G12 | 483 | −3.33 | 0.31 | 70.17 | 5.36 | 0.02 | 0.11 | −34.23 | 24.71 |
| G13 | 252 | −4.97 | 1.62 | −15.19 | 209.50 | 0.21 | 0.07 | −22.76 | 40.31 |

(continued)

**Table 7** (continued)

| Instance | N | C&W | | | | mC&W | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Fuel | | CPU | | Fuel | | CPU | |
| | | av. | std dev. | av. | std dev. | av. | std dev. | av. | std dev. |
| G14 | 320 | −4.30 | 0.26 | 58.26 | 6.66 | 0.23 | 0.13 | −23.39 | 17.80 |
| G15 | 396 | −4.07 | 1.09 | 36.67 | 89.03 | −0.25 | 0.04 | −12.43 | 20.87 |
| G16 | 480 | −2.98 | 0.88 | 44.81 | 61.85 | −0.40 | 0.07 | −15.92 | 17.08 |
| G17 | 240 | 8.37 | 0.75 | −177.15 | 123.33 | 0.42 | 0.05 | −53.88 | 49.56 |
| G18 | 300 | 1.24 | 0.59 | −244.24 | 38.43 | 1.59 | 0.14 | −56.19 | 20.24 |
| G19 | 360 | 1.45 | 0.65 | −231.44 | 49.82 | 2.06 | 0.09 | −46.84 | 42.68 |
| G20 | 420 | 3.22 | 0.65 | −141.68 | 29.76 | 0.82 | 0.19 | 35.11 | 8.08 |
| T1 | 385 | 4.53 | 1.01 | −204.64 | 98.06 | 1.84 | 0.40 | 48.77 | 16.40 |
| T2 | 75 | −0.23 | 0.29 | 12.35 | 9.72 | 0.30 | 0.19 | 13.29 | 9.41 |
| T3 | 75 | −0.48 | 1.02 | 34.95 | 15.91 | 0.16 | 0.29 | 25.74 | 11.10 |
| T4 | 75 | −1.31 | 0.99 | 42.24 | 7.52 | −0.07 | 0.15 | 23.62 | 8.76 |
| T5 | 75 | −0.40 | 0.54 | 42.70 | 6.86 | 0.19 | 0.22 | −14.81 | 7.51 |
| T6 | 100 | −0.65 | 0.59 | 40.85 | 8.39 | 0.57 | 0.12 | 12.21 | 9.23 |
| T7 | 100 | −1.21 | 0.68 | 52.30 | 5.14 | 0.38 | 0.14 | 25.89 | 3.67 |
| T8 | 100 | −2.51 | 0.63 | 50.34 | 5.60 | −0.16 | 0.30 | 6.99 | 8.92 |
| T9 | 100 | −2.07 | 1.04 | 42.38 | 8.65 | 0.26 | 0.18 | 23.34 | 5.57 |
| T10 | 150 | −0.15 | 0.66 | 39.03 | 8.42 | 0.27 | 0.39 | 25.04 | 6.59 |
| T11 | 150 | −1.23 | 0.43 | 46.44 | 4.58 | −0.03 | 0.06 | 20.53 | 3.88 |
| T12 | 150 | −1.97 | 0.47 | 43.10 | 2.32 | 0.12 | 0.15 | 12.77 | 6.55 |
| T13 | 150 | −2.71 | 0.67 | 49.01 | 5.85 | 0.17 | 0.09 | 22.07 | 6.41 |

Finally, we compared our results with the ones obtained by Cinar et al. [20]. As it is expected, using metaheuristic algorithms after finding initial solution(s) with constructive algorithms improves the solution quality, i.e., routing policies with less fuel consumption are obtained. Moreover, the computational experiments performed in this study showed that the constructive algorithms have significant effects on the performance of metaheuristics. Therefore, choosing the best initial solution generator is an important issue for metaheuristics to obtain high-quality solutions in a reasonable computational time.

## 7 Concluding Remarks

In this study, several constructive algorithms – C&W, mC&W, and two-phase algorithms – are investigated to evaluate their performances as initial solution generators of a solution-based and a population-based metaheuristic approaches developed for the CumVRP-LD. SA is used as a solution-based metaheuristic, while GA is utilized as a population-based metaheuristic. Data sets from the literature are used for the computational tests. Computational experiments show that the performance of SA is significantly affected by the initial solution generator. Although initial solution generators do not affect the performance of GA as much as SA, choosing the best initial solution generator is still an important issue to obtain high-quality solutions in a proper computational time. Since SA works on only one solution while GA evaluates a group of solutions (population), the impact of initial solution quality is higher in SA than GA.

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# Column Generation for Optimal Shipment Delivery in a Logistic Distribution Network

**George Kozanidis**

**Abstract** We consider a logistic distribution decision-making problem, in which a vehicle fleet must carry out a set of deliveries between pairs of nodes of the underlying transportation network. The goal is to maximize the number of deliveries that will be carried out, while also minimizing the number of vehicles utilized to this end. The optimization is lexicographic in the sense that the former objective exhibits higher priority than the latter one. For this problem, we develop an integer programming model formulation and an associated column generation-based solution methodology. The proposed methodology utilizes a master problem which tries to fulfill the maximum possible number of deliveries given a specific set of vehicle routes and a column generation subproblem which is used to generate cost-effective vehicle routes[1], for improving the master problem solution. We describe the steps of the proposed methodology, illustrating how it can be modified to accommodate interesting problem variations that often arise in practice. We also present extensive computational results demonstrating the computational performance of the proposed solution algorithm and illustrating how its behavior is influenced by key design parameters.

## 1 Introduction

We consider a logistic distribution network, in which a set of deliveries must be carried out by a fleet of vehicles. Each vehicle is initially positioned in one of the underlying transportation network nodes. Each delivery has a fixed departure time,

[1]We use the term *vehicle route* to denote a feasible sequence of deliveries assigned to a specific vehicle.

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as well as a fixed trip duration determined by its origin and destination network nodes. The aim is to perform as many deliveries as possible, while also minimizing the number of vehicles utilized to this end. Since the objective of maximizing the number of deliveries that will be performed exhibits higher priority than that of minimizing the number of utilized vehicles, the cost of not performing a delivery is considered significantly larger than that of utilizing an extra vehicle.

Our motivation to deal with this particular problem stems from a realistic problem setting originating in the context of airline management. Specifically, the problem arises in the daily operation of a freight transportation airline that makes scheduled deliveries between specific nodes of a logistic distribution network. One of the most important decision making problems this airline is faced with regards the optimal design of its aircraft routes, so that the number of deliveries performed is maximized, while the number of aircraft utilized to this end is minimized.

The actual problem definition dictates that the composition of each shipment is fixed; thus, its content cannot be altered. This implies that each delivery can be looked at as a scheduled flight that must be carried out independently, imposing the constraint that at most one such delivery can be carried out by a specific aircraft at any point in time. In turn, this implies that each delivery must be uninterrupted, while any two distinct deliveries carried out by the same aircraft must be non-overlapping. Due to economic efficiency reasons, aircraft deadheading is also prohibited. As a result, for any two consecutive deliveries assigned to the same aircraft, the arrival node of the preceding one must coincide with the departure node of the succeeding one.

The problem under consideration is very similar to the tail assignment problem which arises in the context of airline fleet management [12]. Motivated from theory that has been developed for addressing that problem, we develop an integer programming model formulation, and an associated column generation-based solution algorithm in what follows. The proposed methodology utilizes a *master problem* which solves the problem given a specific set of vehicle routes. This solution is gradually improved through the inclusion of additional cost-effective vehicle routes which are identified by a second optimization problem termed *column generation subproblem*. Due to the fact that excessively many feasible vehicle routes exist, out of which only very few can be part of the optimal solution, the algorithm generates only those looking promising for improving the current master problem solution. To accomplish this, the column generation subproblem utilizes the optimal dual solution of the master LP relaxation in order to identify the vehicle route with the minimum reduced cost. If this reduced cost is negative, this vehicle route is added to the master problem, causing an update of the dual information. Otherwise, the generation of cost-effective vehicle routes is terminated, since the optimal solution of the master LP problem cannot be further improved at that point.

Suitable branching on fractional vehicle route variables takes place at that point, guiding the search toward the optimal integer solution. The feasible space is partitioned by imposing integrality on a proper set of decision variables. The solution of each master problem resulting after the incorporation of the branching constraints necessitates the generation of additional vehicle routes, which become cost-effective as a consequence of the modifications caused on the feasible space due to the associated branching decisions.

The main contribution of the present work lies in the development of several innovative techniques facilitating the actual implementation of the proposed methodology, as well as in the illustration of how the challenges encountered upon its actual deployment in a realistic environment have been successfully handled. We present extensive computational results demonstrating the proposed solution algorithm's performance on realistic problems, as well as the impact of several alternative design options on its behavior. Additionally, we introduce interesting variations of the problem that often arise in practice, and we illustrate how the proposed methodology can be adapted in order to handle them.

A paper relevant to the present research is currently under publication in an international scientific journal [16]. For reasons of brevity, we refrain from repeating all the theoretical findings developed in that work in what follows. Instead, we present additional computational results illustrating the performance of the proposed solution algorithm on realistic problems of particularly large size.

The remainder of the current work is structured as follows. In the next section, we summarize the related literature, while in the one that follows after that, we present the optimization model formulation. Then, we develop the proposed solution methodology, and we investigate how its algorithmic performance is affected by alternative design parameter choices. In the following section, we elaborate on interesting problem variations, and we modify the proposed methodology in order to accommodate them. The next section presents our computational experience from the application of this methodology in a realistic environment, while the last section concludes this work.

## 2 Literature Review

In the context of optimal vehicle routing in supply chain coordination, one is often faced with linear programs having so many decision variables that it is almost impossible to explicitly consider all of them. Column generation is a very powerful methodology for addressing such problems, because it succeeds in obtaining the optimal solution of particularly large problems by explicitly considering only a very small subset of the decision variables. The robustness of column generation lies in that it can be appropriately modified to also accommodate integer programs with a vast number of decision variables. The related framework, which comprises a novel combination of column generation and branch and bound, is called *branch and price*.

An insightful technical review on column generation for integer programs using three distinct framework types is provided by Wilhelm [25]. After presenting a taxonomy of the column generation literature and proposing suitable model formulations, the author elaborates on theoretical properties, branching strategies, acceleration techniques, and alternative algorithmic designs. He also illustrates the deployment of the column generation methodology in common application areas, such as vehicle routing and assembly design.

Bard and Nananukul [1] address the problem of making production planning and vehicle routing decisions in order to minimize the total cost of a distribution network which comprises of a homogeneous vehicle fleet, a set of customers with time-varying demand, and a production facility exhibiting both production and inventory costs. The authors develop a column generation-based solution framework, which utilizes both heuristic and exact techniques. Chang et al. [5] consider a supply chain including both production and distribution decisions and develop a column generation solution framework for optimizing it.

In the related literature, there are many published works which propose branch and price solution algorithms for addressing similar vehicle deployment optimization problems in the context of passenger or freight transportation. A common example is that of the vehicle scheduling problem, which aims to find the optimal assignment of a set of scheduled duty trips to a fleet of vehicles [4]. Although that problem exhibits many similarities with the one addressed in the current work, its objective and set of constraints are typically different. The most common objective of the vehicle scheduling problem is the minimization of the transportation cost, while its model definition usually includes constraints limiting the number of utilized vehicles or prescribing the type of vehicle that can serve each trip.

Other solution methodologies that have been proposed for addressing similar vehicle deployment problems include branch and cut [18], dynamic programming [7], and Lagrangean relaxation [17]. In addition, to overcome the inherent complexity of the involved formulations, the authors often choose to compromise for near-optimal solutions by utilizing heuristic solution approaches [19].

The problem under study is very similar to the tail assignment problem, which aims to optimally assign a set of passenger flights to a fleet of commercial aircraft. Among others, Grönkvist [11, 13, 14] and Gabteni and Grönkvist [10] have developed column generation-based solution methodologies for this problem. Although the general problem context of these works is similar to that of the present work, the particular algorithmic design is different in various aspects, as demonstrated in the remainder of this work.

A slightly different version of the tail assignment problem, which aims to minimize the flight schedule disruptions, has been addressed by Borndörfer et al. [3]. After solving the master LP relaxation with column generation, the authors utilize a heuristic scheme in order to reach an integer solution. In contrast, the algorithm that we develop in what follows exhibits an exact nature, since it converges to the exact optimal solution of the problem.

# 3 Model Development

In this section, we develop the master and the column generation problem formulations. Both of them utilize the following two common sets:

***I:*** set of vehicles
***D:*** set of deliveries

The notation which is specific to each formulation is introduced in the corresponding subsection.

## *3.1 Master Problem Formulation*

For the master problem formulation, we introduce the following notation:
***Sets:***

$\boldsymbol{R_i}$**:** set of routes of vehicle $i$

***Parameters:***

$\boldsymbol{f}$**:** cost for each utilized vehicle
$\boldsymbol{h}$**:** cost for each delivery that remains unfulfilled
$\boldsymbol{a_{ird}}$**:** binary parameter that takes the value 1 if route $r$ of vehicle $i$ fulfills delivery $d$, and 0 otherwise, $i \in I$, $r \in R_i$, $d \in D$

***Decision variables:***

$\boldsymbol{x_{ir}}$**:** binary decision variable that takes the value 1 if route $r$ is assigned to vehicle $i$, and 0 otherwise, $i \in I$, $r \in R_i$
$\boldsymbol{y_d}$**:** binary decision variable that takes the value 1 if delivery $d$ remains unfulfilled, and 0 otherwise, $d \in D$

Utilizing the above mathematical notation, the master problem is formulated as follows:

$$\text{Min} \sum_{i \in I} \sum_{r \in R_i} f x_{ir} + \sum_{d \in D} h y_d \tag{1}$$

$$\text{s.t.} \sum_{r \in R_i} x_{ir} \leq 1, \forall i \in I \tag{2}$$

$$y_d + \sum_{i \in I} \sum_{r \in R_i} a_{ird} x_{ir} = 1, \forall d \in D \tag{3}$$

$$x_{ir}, y_d \text{ binary}, \forall i, r, d \tag{4}$$

The total cost, comprising of the vehicle utilization cost and the cost of unfulfilled deliveries, is minimized in the objective function (1). Cost parameter $h$ is always sufficiently larger than $f$, imposing the relative priority between the two objectives. Constraint set (2) limits the number of routes selected for each vehicle to at most 1; of course, a fixed cost equal to $f$ is imposed for each utilized vehicle. These constraints are called the *vehicle rows*. Constraints (3) ensure that each delivery is either fulfilled by a selected vehicle route or remains unfulfilled, in which case, a cost equal to $h$ is added to the objective. These constraints are called the *delivery rows*. Finally, constraint set (4) restricts the decision variables to binary values.

The above set partitioning type master problem formulation is quite typical in column generation solution methodologies. It is often utilized in numerous application contexts, such as crew assignment, vehicle scheduling, and maintenance routing [2, 21, 24]. Typically, these problems do not exhibit significant formulation requirements necessitating the development of sophisticated modeling techniques. Instead, the research focus is mostly on the development of highly effective solution approaches that will handle efficiently the excessive size of typical realistic problems. As a result, the relevant active research concentrates on the algorithmic rather than the modeling part.

### *3.2 Column Generation Subproblem Formulation*

The master problem formulation introduced above solves the optimization problem using a given set of vehicle routes. On the other hand, the column generation subproblem, introduced next, tries to identify cost-effective vehicle routes to be added to the master problem for improving its solution. This is achieved by identifying negative reduced-cost vehicle routes, with respect to the current master LP relaxation optimal solution. If the reduced cost of a vehicle route is negative, this is an indication that it has the potential to improve this solution. Therefore, it is added to the master problem, updating its dual optimal solution. On the other hand, if the reduced cost of any vehicle route is non-negative, this is an indication that the master solution cannot be further improved; therefore, the column generation procedure terminates. Complex constraints accommodating special requirements that may be present are typically incorporated into the column generation formulation. This way, the master problem formulation retains its simple structure, enabling the efficient treatment of particularly large-sized problems.

Let $dn_d$, $an_d$, $dt_d$, and $at_d$ be the departure node, the arrival node, the departure time, and the arrival time of delivery $d$, respectively. Let also $n_i$ be the node at which vehicle $i$ is initially located. For the formulation of the column generation problem, we consider a network $N = \{V, A\}$. For each vehicle $i \in I$, we denote by $F_i$ the set of deliveries which are *next-compatible* with vehicle $i$. A delivery $d$ is next-compatible with vehicle $i$ if $n_i = dn_d$. For each delivery, we define three similar sets. $B_d$ is the set of vehicles which are *previous-compatible* with delivery $d$, where vehicle $i$ is previous-compatible with delivery $d$ if delivery $d$ is next-compatible with vehicle *i*. $N_d$ is the set of deliveries which are *next-compatible* with delivery *d*, where a delivery *e* is considered next-compatible with delivery *d* if $dn_e = an_d$ and $dt_e \geq at_d$. Finally, $P_d$ is the set of deliveries which are *previous-compatible* with delivery *d*, where delivery *e* is previous-compatible with delivery *d* if delivery *d* is next-compatible with delivery *e*. The set of vertices, *V*, of network *N* includes one node for each vehicle, one node for each delivery, as well as a fictitious node, *E*, which acts as the terminal node. The set of arcs, *A*, includes edges connecting each vehicle node with its corresponding next-compatible delivery nodes, edges connecting pairs of compatible delivery nodes, and edges connecting each delivery node with the terminal node.

The column generation subproblem aims to identify the longest (minimum negative-distance, to be precise) path in the above network that begins in some vehicle node, visits at least one delivery node, and ends in the terminal node. Let *i* be the index of the vehicle associated with the initial node of a path, $\mathsf{K} \subseteq \mathsf{D}$be the set of delivery nodes this path visits, and $b_i/c_d$ be the dual value of the corresponding vehicle/delivery row in the current master LP optimal solution. The length of this path is equal to $-f + b_i + \sum_{d \in K} c_d$. Since the cost of each aircraft route is equal to *f*, finding the longest such path is equivalent to finding the minimum reduced-cost vehicle route. The terminal node can succeed any delivery node, since the trip of a vehicle practically terminates as soon as this vehicle fulfills its last delivery, without having to return to some particular depot node. On the other hand, the terminal node cannot succeed any vehicle node, since an empty vehicle route not including any delivery triggers a vehicle utilization cost without resulting in unfulfilled delivery cost savings. Since there is a one-to-one correspondence between paths of this network and actual vehicle routes, we use the terms *path* and *route* interchangeably in what follows. With these in mind, we introduce the following decision variables for the formulation of the column generation subproblem.

**Decision variables:**

$z_i$**:** binary decision variable that takes the value 1 if the route identified by the column generation subproblem pertains to vehicle *i*, and 0 otherwise, $i \in I$

$w_{id}$**:** binary decision variable that takes the value 1 if the route identified by the column generation subproblem includes a direct travel from vehicle node *i* to delivery node *d* in the associated network, and 0 otherwise, $i \in I$, $d \in F_i$

$w_{de}$**:** binary decision variable that takes the value 1 if the route identified by the column generation subproblem includes a direct travel from delivery node *d* to node *e* in the associated network, and 0 otherwise, where *e* is either a delivery node or the terminal node, $d \in D$, $e \in N_d \cup \{E\}$

$u_d$**:** binary decision variable that takes the value 1 if the route identified by the column generation subproblem fulfills delivery *d*, and 0 otherwise, $d \in D$

Utilizing the above decision variables, as well as sets $F_i$, $B_d$, $N_d$, and $P_d$, and parameters $b_i/c_d$ defined above, the column generation subproblem is formulated as follows:

$$\text{Min} \;\; f - \sum_{i \in I} b_i z_i - \sum_{d \in D} c_d u_d \tag{5}$$

$$\text{s.t.} \sum_{i \in I} z_i = 1 \tag{6}$$

$$z_i = \sum_{d \in F_i} w_{id}, \forall i \in I \tag{7}$$

$$\sum_{i \in B_d} w_{id} + \sum_{e \in P_d} w_{ed} = \sum_{g \in N_d \cup \{E\}} w_{dg}, \forall d \in D \tag{8}$$

$$u_d = \sum_{i \in B_d} w_{id} + \sum_{e \in P_d} w_{ed}, \forall d \in D \tag{9}$$

$$z_i, u_d, w_{id}, w_{ed}, w_{dg} \text{ binary}, \forall i, d, e, g \tag{10}$$

In the objective function (5), the reduced cost of the vehicle route that will be identified is minimized, which is equal to the vehicle utilization cost minus the dual value of the corresponding vehicle row, minus the sum of the duals of the corresponding delivery rows. Constraint (6) ensures that one route for exactly one vehicle will be constructed. Constraint set (7) states that the identified path must commence with a flow from a vehicle node to a delivery node. Constraint set (8) ensures flow balance at each delivery node. Incoming flow can originate either at a vehicle node or at a delivery node, while outgoing flow can be directed either to another delivery node or to the terminal node. For each delivery $d$, constraint set (9) updates the value of variable $u_d$ indicating whether this delivery is fulfilled in the associated vehicle route. Finally, constraint set (10) imposes binary values on the decision variables. Variables $u_d$ and constraint set (9) are redundant, since the fulfillment of a delivery can be determined by the flows into its corresponding node. Nevertheless, it is a good practice to include them, since they facilitate considerably the implementation of the proposed methodology, without having any significant impact on its computational requirements.

## 4 Solution Methodology

In this section, we describe the proposed solution methodology for the treatment of the problem. It is organized in a branch and price tree, that is, a branch and bound tree in which the optimal solution of each master LP tree node subproblem is obtained with column generation. We employ the best LP bound strategy for the tree node selection, which out of all active nodes selects the most promising, i.e., the one with the best LP bound on the optimal objective. If the associated solution is non-integral, the algorithm branches on a set of fractional decision variables, creating two new subproblems and their corresponding tree nodes. Each of these subproblems is then re-optimized using column generation. The procedure terminates when the optimal solution to the subproblem selected next for exploration is integer; the best LP bound node selection strategy ensures that this will be the exact optimal solution. The following subsections portray in sufficient detail the specifics of the proposed solution methodology. In order to be able to test the behavior of the algorithm extensively and compare the performance of alternative algorithmic designs, we introduce a set of realistic instances first.

## 4.1 Test Instances

For the needs of our experiments, we use as test cases six realistic problems extracted from the live database of an actual logistic distribution air carrier. The choice of an air carrier does not introduce any particular difficulties requiring special treatment. This stems from the fact that the proposed methodology does not include anything specific that would prohibit the model's applicability in different contexts, such as that of airline management. It is pretty much standard, independently of whether fleets of vehicles, trains, aircraft, or even ships are involved in the actual application.

The realistic problems drawn from the live database of the air carrier in question differ in terms of the fleet size, the number of deliveries, as well as the length of the planning horizon. The specific characteristics of these problems are presented in Table 1. All our experiments were performed on a Core 2 Duo 2.4 GHz Intel processor with 4 GB system memory, a machine with considerably inferior technical characteristics to those of machines typically encountered in practical environments. In all the computational time results that we present, we adopt the format XX:YY:ZZ, where XX is the number of hours, YY the number of minutes, and ZZ the number of seconds. We suppress the hour part when these times are less than 1 h. We also impose a predefined maximum time limit of 120 min on the

**Table 1** Data of the six realistic test instances

| No. | Fleet size | Number of deliveries | Length of planning horizon (days) | Average duration (min) |
|---|---|---|---|---|
| 1 | 31 | 247 | 1 | 179.25 |
| 2 | 43 | 352 | 1 | 173.97 |
| 3 | 57 | 449 | 2 | 177.24 |
| 4 | 69 | 561 | 2 | 173.94 |
| 5 | 80 | 654 | 3 | 172.22 |
| 6 | 93 | 769 | 3 | 168.76 |

total computational time of every computational experiment that we perform, the only exception being the experimental results reported in Table 9 which pertain to particularly large-scale problems.

### *4.2 Solving the Master LP Relaxation*

Each node of the branch and price tree is associated with a distinct master problem and its companion column generation subproblem, which originate in the two fundamental formulations (1)–(4) and (5)–(10), respectively. The optimization models of two distinct nodes differ only with respect to the extra constraints that have been added as a result of branching. The specific logic determining the addition of these constraints is explained in the following sections. The optimal solution to the master LP relaxation of each tree node is obtained with column generation, according to the logic flow depicted in Fig. 1.

The master LP formulation at the root of the branch and price tree is initialized with an empty set of routes for each vehicle and the slack variable $y_d$ of each delivery set equal to 1. It is a good practice to refrain from removing variables $y_d$ even if they drop to 0 value after some branching decision, as this could cause feasible solutions to be overlooked. This might happen when the addition of a branching constraint renders temporarily a master problem infeasible, even though this infeasibility can

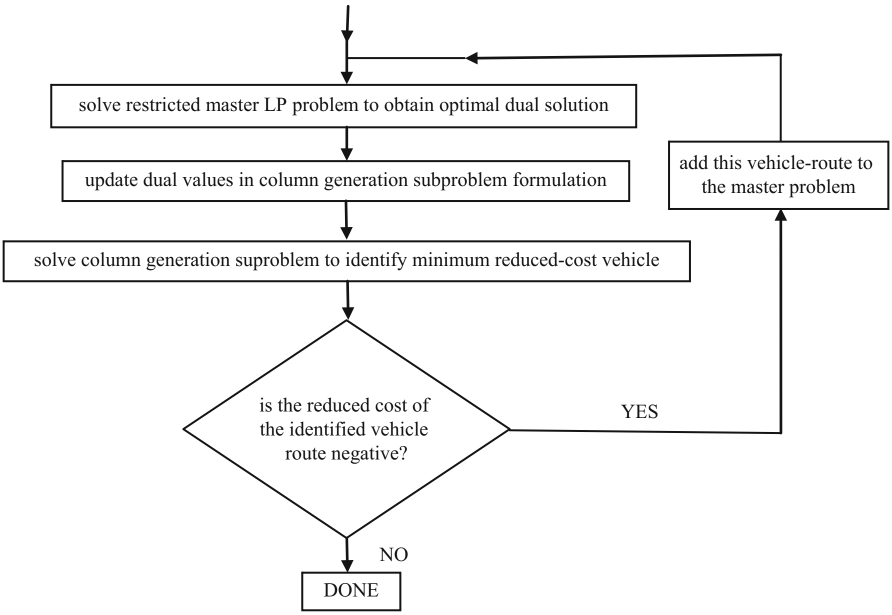


**Fig. 1** Column generation logic flow

be repaired through the generation of additional vehicle routes. In such a case, variables $y_d$ preserve the feasibility of the model, enabling the continuation of the search for the optimal integer solution in the associated subtree.

A crucial design option with significant impact on computational performance regards the algorithmic selection for the solution of the master LP relaxation at each node of the branch and price tree. Our implementation utilizes the commercial optimization software IBM ILOG CPLEX 12.5.1 [15], which has seven alternative options for the LP solver: a default one and the options to employ primal simplex, dual simplex, a network optimizer, a barrier optimizer, a sifting optimizer, and a concurrent optimizer. Table 2 presents a comparison of the three most reasonable options, i.e., primal simplex, dual simplex, and barrier optimizer without crossover. The crossover operation of the barrier algorithm transforms the possibly interior point primal optimal solution to a basic one; in our case, it is turned off as irrelevant, since our aim is to solely acquire the dual optimal solution. The particular selection of these three LP algorithms is motivated partly by our computational experience and partly by the fact that the same exact selection was considered by Grönkvist [12]. For each of the six test instances of Table 1, Table 2 presents the total computational time, the total number of generated aircraft routes, and the total number of tree nodes upon termination of the algorithm, using each of these three LP solvers for the master problem solution.

As the results of Table 2 demonstrate, there is a substantial improvement in algorithmic convergence when the barrier optimizer is employed for solving the master problem. This observation agrees with the results reported by Grönkvist [12], who attributes this behavior to two main reasons: the superior computational performance of the barrier algorithm and the fact that the dual values it provides are less extreme and therefore more suitable for column generation since they might be interior point and not basic solutions. As no significant difference in the computational performance of the three algorithms was observed in our experiments, we are leaning towards attributing this behavior to the latter of these two reasons.

A final critical remark regarding the master problem is related to its mathematical formulation. Since the feasible region remains unchanged when the upper bounds of the decision variables $x_{ir}$ and $y_d$ are set equal to infinity, it is preferable to avoid

**Table 2** Comparison of different LP solvers

| | Primal simplex | | | Dual simplex | | | Barrier optimizer | | |
|---|---|---|---|---|---|---|---|---|---|
| No. | Time | Routes | Nodes | Time | Routes | Nodes | Time | Routes | Nodes |
| 1 | 00:21 | 2087 | 53 | 00:22 | 2057 | 55 | 00:18 | 1260 | 55 |
| 2 | 02:33 | 5900 | 75 | 02:42 | 5408 | 75 | 00:46 | 3443 | 81 |
| 3 | 11:58 | 10843 | 101 | 10:25 | 9550 | 101 | 01:26 | 5643 | 109 |
| 4 | 44:00 | 17690 | 119 | 23:38 | 14541 | 115 | 02:23 | 8866 | 121 |
| 5 | >2 h | – | – | 56:10 | 19965 | 119 | 04:12 | 11940 | 137 |
| 6 | >2 h | – | – | >2 h | – | – | 06:16 | 14414 | 149 |

imposing explicit upper bound constraints ($\leq 1$) on them. This technique seems to slightly improve the computational performance of the proposed solution algorithm, since it saves the trouble of having to deal with the corresponding dual variables of these constraints.

## 4.3 Solving the Column Generation Subproblem

The column generation subproblem can be solved efficiently with a shortest path algorithm for acyclic networks (e.g., [22]), exploiting the acyclic topology of the associated network. The algorithm initializes the reduced cost of each vehicle node $i$ to $f$-$c_i$ and that of each delivery node to infinity and then scans the network arcs in topological order, identifying possible path extensions through pairs of compatible nodes and updating the corresponding reduced costs accordingly. A label denoting the reduced cost of the best path ending at each node is saved, which is updated accordingly each time an improved path is identified.

The correctness of the acyclic shortest path algorithm (ASPA) sketched above relies on the monotonicity of a path's reduced cost when this path is extended through the inclusion of additional deliveries. This key property ensures that storing the best path ending at each node of the network is sufficient for identifying the optimal solution. Since the complexity of this algorithm is linear in the total number of network arcs, its performance is considerably superior to that of integer programming solvers that can be employed alternatively for the solution of the model formulation (5)–(10). In large-scale problems, the associated time savings become substantial.

The column generation subproblem can also be modeled as a shortest path network flow problem, which possesses the total unimodularity (TUM) property; this enables the solution of the problem very fast, using efficient LP algorithms. The computational performance of such an implementation would still be substantially inferior to that of ASPA, however, since the complexity of linear programming solvers (even of those which are polynomial) is far worse than linear.

Another significant obstacle that the adoption of this implementation would raise is that it would not be applicable for the treatment of several interesting problem extensions that arise in practice. Typical such extensions are the incorporation of resource constraints and/or the incorporation of different objectives like the workload balance, for which the TUM property no longer holds, rendering the LP solvers unsuitable for the solution of the problem. For this reason, we do not pursue such an implementation.

According to Desrosiers and Lübbecke [8], a very effective enhancement for substantial computational performance improvement involves the addition of multiple vehicle routes with negative reduced cost to the master problem in each iteration. In our case, this is straightforward since ASPA identifies the optimal route of each vehicle separately. Of course, several other design variations stem out as possible, depending on the exact number of vehicles considered in each iteration, as well as on the number of routes added to the master problem for each of them.

Rather than using the same set of dual values in order to identify the optimal route of each vehicle, Grönkvist [12] has proposed a re-evaluation technique for updating these values each time a route is added to the master problem. This technique updates the dual value of a master row covered by a specific aircraft route added to the master problem as $dual^{new} = dual^{old} + \frac{rc}{\xi|M|}$, where $rc$ is the reduced cost of this route, $|M|$ is the number of master problem rows in which the corresponding decision variable appears, and $\xi$ is a smoothing parameter. The dual solution resulting after this update is not necessarily feasible, but this is not important, since our aim is to penalize the corresponding master rows in order to avoid the generation of routes exhibiting high similarity. In our implementation, we adopt the value 1 for parameter $\xi$, which provides satisfactory results.

### *4.4 Branching*

When the optimal solution to the master LP relaxation of the currently explored tree node is non-integer, the algorithm branches in order to eliminate the non-integralities. The situation is typical in a branch and bound setting. New sub-problems are created by adding constraints which eliminate fractional solutions. The most typical design chooses a fractional decision variable and partitions the solution space by setting it equal to 0 and 1. Ryan and Foster [23] have proposed an alternative branching scheme which has turned out to be very efficient for set partitioning problems. In our case, the validity of this scheme is reestablished in the following proposition.

*Proposition 1* If the optimal solution to the master LP relaxation contains one or more fractional decision variables, then there exist one vehicle row and one delivery row, such that the sum of the variables that appear in both these rows is fractional.

*Proof* When the optimal solution to the master LP relaxation contains one or more fractional decision variables, then at least one route variable is clearly fractional, too. Let $x_{ir}$ be one such variable and $d$ be a delivery that it includes (note that at least one such delivery exists since any vehicle route includes at least one delivery). Since the sum of the decision variables appearing in delivery row $d$ is equal to 1, there exists at least one other variable appearing in this constraint which is also strictly positive. If $y_d$ >0, or $x_{ir}$ is the only positive route variable of vehicle $i$ that includes delivery $d$, then the proposition holds for vehicle row $i$ and delivery row $d$. Otherwise, let $x_{ik}$ be another positive route variable of vehicle $i$ that also includes delivery $d$. Since any two route variables are always distinct (it is never cost-effective to add the same variable to the master problem twice), there exists at least one delivery with index $e \neq d$, which is covered by either $x_{ir}$ or $x_{ik}$, but not by both of them. In this case, the proposition holds for vehicle row $i$ and delivery row $e$. □

Proposition 1 enables the selection of the branching variables in an elegant way. If the master LP optimal solution is fractional, then at least one pair of rows prescribed by this proposition exists. The algorithm creates two new tree subproblems by branching simultaneously on all the variables appearing in both these rows. In the left subproblem, the sum of these variables is set equal to 0; in the right subproblem, it is set equal to 1. This feasible space partition imposes the restriction that either exactly one of the associated routes will be assigned to vehicle $i$ or none of them. We term this branching strategy *multi-branching* in order to distinguish it from the case of branching on a single variable, which we term *single-branching*.

The branching constraints are not appended to the master problem as additional constraints; they are directly incorporated into the existing formulation instead. This retains the same number of master problem constraints, saving the trouble of dealing with extra dual variables. The incorporation of the branching decision into the master problem is straightforward. In the left subproblem in which the branching variables are set equal to 0, each of them is deleted from the master problem. In the right subproblem in which the sum of the branching variables is set equal to 1, the corresponding vehicle constraint is turned into an equality, and all the remaining route variables of the same vehicle are deleted from the master problem.

As far as the column generation subproblem is concerned, the branch to 1 constraint is incorporated easily by eliminating from the associated network the node of the vehicle the branching variables belong to, as well as the nodes of those deliveries which are included in all the branching variable routes. On the other hand, in order to incorporate the 0-branch constraint, we utilize a procedure that has been proposed by Martins [20] for determining the $k$ shortest paths in a directed network. In each iteration, this procedure finds the shortest path of the current network and then modifies this network accordingly, so as to eliminate this path, without, however, eliminating any other path. Utilizing the same procedure, we are able to modify the network so that it is not possible to generate the path corresponding to the branching variable, without excluding any other path. The details of this procedure are presented in the aforementioned reference.

Alternative branching schemes can be devised through proper adjustment of several key parameters, such as the size of the branching variable set, the exact nature of the branching variable values, etc. In case that multi-branching (M) is employed, we consider two distinct choices regarding the size of the variable set, i.e., branch on the smallest (S) and branch on the first one identified (I). We do not consider the choice of branching on the largest set because our computational experience suggests that it exhibits considerably inferior computational performance. When the above options do not determine a unique branching variable set, we also consider the option of whether the selected set should be the one including the largest fractional variable or whether it should be chosen randomly. Table 3 presents the six strategies resulting from the combination of the available options (not all three options are always applicable), while Table 4 presents computational results demonstrating how the performance of the algorithm varies for the six instances when each of these strategies is adopted. In this table, the strategy index is shown in the column labeled

**Table 3** Alternative branching strategies

| Strategy | Single- (S) or multi- (M) branching | Size of branching variable set | Branch on set with largest fractional |
|---|---|---|---|
| 1 | S | – | No |
| 2 | M | S | No |
| 3 | M | I | No |
| 4 | S | – | Yes |
| 5 | M | S | Yes |
| 6 | M | I | Yes |

**Table 4** Comparison of alternative branching strategies

| | **Test problem 1** | | | **Test problem 2** | | | **Test problem 3** | | |
|---|---|---|---|---|---|---|---|---|---|
| ***s*** | ***t*** | ***r*** | ***n*** | ***t*** | ***r*** | ***n*** | ***t*** | ***r*** | ***n*** |
| 1 | 00:21 | 1634 | 45 | 00:57 | 4397 | 75 | 02:00 | 7150 | 97 |
| 2 | 00:28 | 1564 | 49 | 01:34 | 3765 | 69 | 04:22 | 6327 | 93 |
| 3 | 00:27 | 1529 | 65 | 03:07 | 4276 | 127 | 04:40 | 6727 | 155 |
| 4 | 00:18 | 1260 | 55 | 00:46 | 3443 | 81 | 01:26 | 5643 | 109 |
| 5 | 00:23 | 1379 | 51 | 01:36 | 3436 | 77 | 04:32 | 5755 | 109 |
| 6 | 00:37 | 1413 | 75 | 03:10 | 4276 | 127 | 08:44 | 6914 | 153 |
| | **Test problem 4** | | | **Test problem 5** | | | **Test problem 6** | | |
| ***s*** | ***t*** | ***r*** | ***n*** | ***t*** | ***r*** | ***n*** | ***t*** | ***r*** | ***n*** |
| 1 | 03:22 | 10128 | 109 | 05:33 | 13732 | 125 | 06:54 | 15508 | 141 |
| 2 | 09:48 | 9176 | 103 | 16:21 | 12097 | 119 | 27:20 | 15016 | 135 |
| 3 | 11:49 | 10093 | 183 | 18:37 | 12481 | 189 | 32:16 | 15828 | 221 |
| 4 | 02:23 | 8866 | 121 | 04:12 | 11940 | 137 | 06:16 | 14414 | 149 |
| 5 | 09:28 | 8813 | 115 | 16:52 | 11814 | 135 | 27:15 | 14384 | 149 |
| 6 | 23:08 | 10616 | 187 | 37:34 | 12573 | 203 | 01:06:26 | 17784 | 245 |

$s$, the computational time is shown in the column labeled $t$, the number of generated routes is shown in the column labeled $r$, and the number of tree nodes is shown in the column labeled $n$.

Table 4 reveals that branching on the single largest valued fractional variable (strategy 4) appears to exhibit the best performance. Our computational experience suggests that this is due to the fact that this strategy leads quickly to specific route selections for the vehicles. The huge feasible space leaves plenty of room to the column generation procedure for rectifying those selections which may turn out to be provisionally poor. In contrary, the multi-branching scheme eliminates multiple routes simultaneously in the 0-branch but leaves open the question of which particular one will be selected for the associated vehicle in the 1-branch.

At this point, it is crucial to emphasize the importance of the 1-branches for fast algorithmic convergence, since they make it possible to reach integer solutions fast. The 0-branches, on the other hand, stall the algorithm, since they only eliminate a few out of a huge number of alternative vehicle routes, whose inclusion into the solution has a similar effect and leads to the same objective value. Motivated by this

**Table 5** Benefit from skipping column generation on 0-branch subproblems

| No. | No. of 0-branch nodes created | Strategy adopted | | Strategy not adopted | |
|---|---|---|---|---|---|
| | | Time | Routes | Time | Routes |
| 1 | 27 | 00:18 | 1260 | 00:28 | 1465 |
| 2 | 40 | 00:46 | 3443 | 01:10 | 4042 |
| 3 | 54 | 01:26 | 5643 | 02:14 | 6237 |
| 4 | 60 | 02:23 | 8866 | 03:48 | 9560 |
| 5 | 68 | 04:12 | 11940 | 05:42 | 13133 |
| 6 | 74 | 06:16 | 14414 | 08:57 | 15916 |

observation, we adopt a "branch on 1 first" strategy, according to which the branch to 1 is always selected for exploration before its branch to 0 counterpart.

In order to further exploit the above behavior, we employ a clever strategy that skips column generation on the 0-branch nodes immediately after their creation. As the large feasible space makes backtracking rather rare, only a small percentage of these nodes actually need to be solved for reaching the optimal solution. Hence, we add each of these nodes to the tree with LP bound equal to that of its parent node, and we postpone its solution with column generation for when and if it will be subsequently selected for exploration. Table 5 presents the computational times of the algorithm and the number of vehicle routes generated when this strategy is adopted and when it is not for each of the six test instances. These results confirm our claim, since they demonstrate that the computational savings of the proposed strategy are significant, leading to computational times which are smaller by more than 35% in some cases.

## 4.5 Further Enhancements for Large-Scale Problems

Although the proposed solution methodology converges to the exact optimal solution given that it is provided with sufficient computational resources, for large-scale problems, the user must inevitably compromise for a near-optimal solution. To this end, the algorithm incorporates a suitable backtrack tolerance on the optimal objective. When the relevant option is active, the algorithm does not backtrack to tree nodes created earlier unless this tolerance is violated. Otherwise, it continues its dive in the tree, making it possible to reach an integer solution faster. Through the exact backtrack tolerance value, the user can control how close this solution will be to the exact optimum and may choose to terminate the algorithm if the quality of this solution is acceptable.

The decision for termination of the algorithm is controlled by an integer gap tolerance with respect to the master LP optimal objective. Naturally, the algorithm terminates when an integer solution satisfying this tolerance is identified. Note that, in contrary to the integer gap tolerance, the backtrack tolerance never influences the decision for algorithmic termination. Each time a new integer solution is identified, the incumbent is suitably updated, and the search for a solution that satisfies the integer gap tolerance continues.

In large-scale problems, obtaining a near-optimal solution in reasonable computational times is enabled through the premature termination of the column generation procedure, which is controlled by two threshold parameters. The first one defines a minimum improvement on the master LP objective in a given number of iterations, while the second one defines a maximum value on the reduced cost of the optimal route variable. When both these two thresholds are not met (the master LP objective improvement is not sufficiently large and the optimal reduced cost is not sufficiently negative), the column generation procedure terminates in the associated tree node. Despite the fact that the LP solution returned at that point is suboptimal, the tailing-off effect of the column generation procedure makes preferable a branching decision at that point. By careful selection of these two threshold values, the user can successfully handle much larger problems, at the cost of compromising for solutions which are not necessarily optimal.

## 5 Problem Extensions

In this section, we elaborate on some interesting problem extensions, and we explain how the proposed methodology can be modified in order to accommodate them. More specifically, we illustrate how hard/soft preferences and resource-type constraints can be incorporated, and we discuss the alternative objective of distributing equally the total workload among the vehicles.

### *5.1 Preference Incorporation*

In practical cases, a set of hard and/or soft preferences are often present, which depict the degree to which it is desirable to combine a specific delivery with a particular vehicle (V-D-type preference) or two specific deliveries consecutively on the same vehicle route (D-D-type preference). Each of these restrictions can be classified as hard (its satisfaction is absolutely necessary), or soft (its satisfaction must be fulfilled at the largest possible extent), and may express positive (must/should go with) or negative (must not/should not go with) preference.

The incorporation of a hard preference is straightforward. A hard V-D-type preference is ensured by including delivery D in every route generated for vehicle V and excluding it from any route generated for any other vehicle. Including delivery D in every route generated for vehicle V is achieved by computing a minimum distance path with origin the vehicle network node V and destination the delivery network node D and by computing a second minimum distance path with origin the delivery network node D and destination the fictitious network node. The situation is similar in the case of many V-D-type hard preferences pertaining to the same vehicle, the only difference being that the optimal path comprises of many subpaths rather than one. On the other hand, prohibiting the assignment of delivery *D* to vehicle *V* is ensured by excluding all the nodes pertaining to delivery *D* from the column generation network utilized for vehicle *V*.

Similarly, a soft preference can be either of the V-D or of the D-D type, too, and is incorporated by imposing a suitable penalty upon realization of the corresponding V-D or D-D pairing on the same route. Of course, the lower the associated preference, the higher the value of the corresponding penalty will be. Typically, a soft preference is introduced to allow the possibility of deadheading (empty trips) or to legalize a departure-arrival time incompatibility between two consecutive deliveries. The satisfaction of a type D-D soft preference depends on whether the two associated deliveries are included consecutively on the same route, independently of which vehicle this route pertains to.

The incorporation of preferences does not increase considerably the complexity of the problem, since the extension of the objective function beyond a network node still retains its monotonicity. As a consequence, the optimal solution can still be reached by saving the best partial path ending at each node of the network. In general, the inclusion of a hard preference reduces the total computational effort since it excludes certain solutions from consideration, whereas the inclusion of a soft preference has the opposite effect, since it introduces a utility-based ranking of the vehicle routes.

A special case of hard preference constraints is that of linked deliveries. Linked deliveries are pairs of deliveries which, for some reason, must necessarily be assigned to the same vehicle, independently of which specific vehicle this is. The most typical way to hard-link two deliveries $D_1$ and $D_2$ is by adding a hard must-go-with V-D preference for delivery $D_2$ and vehicle V once a partial route including delivery $D_1$ is realized. If a partial route that does not include delivery $D_1$ is realized instead, then a hard must-not-go-with V-D preference is added for vehicle V and delivery $D_2$.

Besides satisfying special technical restrictions, the incorporation of linked deliveries is a very effective technique for handling computational difficulties that inevitably arise in large-scale problems. More specifically, linking pairs of deliveries for which it is feasible and reasonable to be assigned to the same vehicle even if this is not an absolute requirement assists in overcoming such difficulties and makes it possible to reduce the involved computational effort substantially. The justification of the effectiveness of this technique lies in that it eliminates several combinations from consideration, thus reducing the size of the feasible space and, in turn, the computational effort required to find the optimal solution.

### 5.2 Incorporation of Resource Constraints

In many practical cases, the column generation procedure is subject to additional resource-type constraints. Typically, these constraints impose an upper bound either on the total number of deliveries that will be assigned to a specific vehicle or on the total travel time of that vehicle. In order to incorporate this restriction, a suitable modification of the acyclic shortest path algorithm is necessary, along the lines of the methodology that has been proposed by Desrochers and Soumis [6] for resource-constrained shortest path problems. Instead of one, this methodology stores several partial paths at each node of the network, which differ from each other in terms of their reduced cost and resource consumption. This modification is dictated by the fact that the incorporation of the resource constraint eliminates a large number of paths from the feasible space, making it necessary to monitor the associated resource consumption. Of course, a path with both worse reduced cost and higher resource consumption than another one ending at the same node can always be discarded, as it can never be part of an optimal solution. This dominance relationship limits somewhat the number of distinct paths that the algorithm needs to consider in order to reach the optimal solution; still, however, the algorithmic performance deteriorates significantly, since finding a shortest path in an acyclic network when resource constraints are present is an NP-hard problem, even if a single resource constraint is imposed and all costs and resource consumptions are positive [9].

When the problem size becomes too large, the number of distinct paths that need to be stored at each node of the network becomes excessive. As a consequence, it becomes essential to impose an upper bound on this number, discarding additional paths once this limit is reached. When this is the case, it appears more efficient to compare the associated reduced costs in order to decide which one to save and which one to discard between two alternative paths. While incorporating this limit attributes a heuristic nature to the algorithm, it gives the user the ability to control, and at the same time exploit, the compromise between solution quality and computational time. Of course, the algorithm's exact nature is restored when this limit becomes sufficiently large.

Table 6 presents results regarding the computational performance of the proposed algorithm on the six test instances when resource constraints are present. For the execution of these tests, we used four experiment sets, as described next. After computing the average number of deliveries and the average travel time, we first rounded up these two values to the next integer, imposing them as corresponding upper bounds. In the first experiment set, the maximum number of paths stored at each node was equal to 5, while in the second one, it was set equal to 10. In the next two experiment sets, the two upper bounds were imposed after rounding down the two averages to the next integer, while the values 5 and 10 were considered again for the maximum number of paths stored at each node. The results in the first line of Table 6 regard the nominal case, i.e., when no resource constraints are included. The results in the next four lines regard the four aforementioned experiment sets in the order introduced. For each of the six instances, Table 6 presents the computational times, as well as the percentage increase of the optimal objective with respect to the nominal case.

The results of Table 6 demonstrate that the incorporation of resource constraints increases substantially the algorithmic computational requirements. Specifically, the largest computational time percentage increase with respect to the nominal case is close to 400%. The optimal objective value does not appear to be influenced significantly by the incorporation of the resource constraints, with the maximum corresponding percentage increase being close to 10%. The only two exceptions to this are experiment sets 4 and 5 on test instance 6, for which the incorporation of the resource constraints increased the optimal objective value by more than 100%. Moreover, storing at most 10 instead of 5 paths at each node of the network does not appear to improve significantly the quality of the obtained solution.

## *5.3 Workload Balance*

While the problem's primary objective always seeks to maximize the number of fulfilled deliveries, sometimes the secondary objective tries to divide the workload equally among the vehicles instead of minimizing the number of utilized vehicles. Typically, this workload is defined either as the total number of deliveries or as the total travel time of all deliveries. A first modification deemed necessary as a consequence of this differentiation is that the vehicle constraints must be expressed as equalities instead of inequalities, imposing this way the selection of one route for each vehicle.

To initialize the master problem, we add an empty route (no deliveries) for each vehicle in this case; such a route may now be part of an optimal solution, in contrary to what happens when the number of utilized vehicles is minimized.

**Table 6** Computational performance of the algorithm when resource constraints are present

| | **Test instance 1** | | **Test instance 2** | | **Test instance 3** | |
|---|---|---|---|---|---|---|
| **Set** | **Time** | **% Obj incr** | **Time** | **% Obj incr** | **Time** | **% Obj incr** |
| 1 | 00:18 | – | 00:46 | – | 01:26 | – |
| 2 | 00:27 | 1.76% | 02:13 | 0.38% | 03:55 | 0.71% |
| 3 | 00:28 | 1.76% | 02:15 | 0.15% | 05:01 | 0.29% |
| 4 | 00:33 | 3.51% | 01:49 | 3.02% | 04:28 | 9.12% |
| 5 | 00:34 | 3.50% | 02:13 | 2.99% | 05:25 | 9.06% |
| | **Test instance 4** | | **Test instance 5** | | **Test instance 6** | |
| **Set** | **Time** | **% Obj incr** | **Time** | **% Obj incr** | **Time** | **% Obj incr** |
| 1 | 02:23 | – | 04:12 | – | 06:16 | – |
| 2 | 09:21 | 1.16% | 13:07 | 2.44% | 19:33 | 4.58% |
| 3 | 09:56 | 0.14% | 19:19 | 0.26% | 32:21 | 2.12% |
| 4 | 09:13 | 10.33% | 14:07 | 3.04% | 18:24 | 114.42% |
| 5 | 10:39 | 10.23% | 18:16 | 0.63% | 30:19 | 113.38% |

Another important difference is that the cost of each route is no longer fixed but varies depending on the particular workload it includes. This is incorporated by including the balance cost in the computation of the reduced cost of each path. We perform this straightforwardly by storing and updating accordingly a resource variable monitoring the workload of each network path during the column generation procedure.

The balance objective incorporation comprises yet another problem extension that necessitates the treatment of multiple paths at each network node in order to identify the optimal vehicle route. This happens because the balance cost is not a monotonic function; thus, between two distinct paths with different workloads ending at the same node, any of the two can lead to the optimal solution in the general case. In order to limit the number of paths that the algorithm considers in order to reach the optimal solution, we utilize the next proposition which establishes a suitable dominance relationship under certain special conditions.

*Proposition 2* Consider two distinct paths ending at the same node, and suppose that the reduced cost of the first one is greater or equal to that of the second one. If, in addition, the workload of the first path is greater or equal to the average workload and the deviation of the second path's workload from the average is smaller or equal to that of the first one, then the first path is dominated and can be disregarded.

*Proof* The validity of the proposition results from the fact that for any possible extension of the two paths from the current node with the same set of deliveries, the balance cost of the first one will always be greater or equal to that of the second one. Thus, the reduced cost of the route identified by extending the first path will be greater or equal to that of the second one, too. □

Apart from the fact that the total workload deviation of all vehicles from the average should be minimized, equity reasons dictate that it should also be distributed evenly among the vehicles. This can be incorporated by increasing the unit deviation penalty as the total deviation increases, for example, through the penalization of its square value; we refrain from utilizing this approach, however, in order to keep our model linear. As an alternate workaround, we define distinct deviation ranges and associated cost penalties, which penalize the workload deviations as they become larger. Our default implementation defines three parameters $r_1$, $r_2$, and $p$, such that each of the first $r_1$ deviation units from the average is penalized with cost $p$, each unit deviation between $r_1$ and $r_1$ is penalized with an additional cost equal to $p^2$, and each unit deviation above $r_2$ is penalized with an additional cost equal to $p^3$.

Table 7 presents results demonstrating the computational performance of the algorithm when the balance objective is incorporated. The first line of this table pertains to the nominal case, i.e., when the secondary objective minimizes the number of utilized vehicles. The next two lines pertain to the case in which the secondary objective tries to balance the number of deliveries, with parameters $r_1$ and $r_2$ set equal to 1 and 2, respectively. In the first case, the maximum number of paths saved at each node was set equal to 5, while in the second one it was set equal to 10. The next two lines pertain to the case in which balance is performed on the travel time of each vehicle, with parameters $r_1$ and $r_2$ set equal to 2 and 5,

**Table 7** Computational performance of the algorithm with balance used as secondary objective

| | Test instance 1 | | Test instance 2 | | Test instance 3 | |
|---|---|---|---|---|---|---|
| **Set** | **Time** | **Obj decr** | **Time** | **Obj decr** | **Time** | **Obj decr** |
| 1 | 00:18 | – | 00:46 | – | 01:26 | – |
| 2 | 00:50 | 70.32% | 03:20 | 88.17% | 05:59 | 83.33% |
| 3 | 01:01 | 70.65% | 04:00 | 89.21% | 09:31 | 84.16% |
| 4 | 00:55 | 68.87% | 03:19 | 86.96% | 06:52 | 90.06% |
| 5 | 01:08 | 71.56% | 04:19 | 87.19% | 09:42 | 90.33% |
| | **Test instance 4** | | **Test instance 5** | | **Test instance 6** | |
| **Set** | **Time** | **Obj decr** | **Time** | **Obj decr** | **Time** | **Obj decr** |
| 1 | 02:23 | – | 04:12 | – | 06:16 | – |
| 2 | 16:08 | 89.45% | 17:41 | 91.72% | 47:56 | 91.32% |
| 3 | 19:10 | 89.47% | 28:40 | 91.73% | 01:04:12 | 91.51% |
| 4 | 17:18 | 87.58% | 21:25 | 90.61% | 53:02 | 90.89% |
| 5 | 22:18 | 87.74% | 34:34 | 90.73% | 01:16:33 | 91.24% |

respectively. Again, the maximum number of paths saved at each node was set equal to 5 in the first case and equal to 10 in the second one. Parameter $p$ was always set equal to 10.

For each of the six instances, Table 7 presents the computational time, as well as the percentage by which the balance objective decreased with respect to the nominal case. To compute this, we calculated the optimal balance cost in the nominal case in which balance was not subject to optimization, and then we re-calculated it when it was optimized as the secondary objective. In all cases, the optimal value of the primary objective remained unchanged after the incorporation of the balance objective.

As the results of Table 7 demonstrate, the algorithm is capable of achieving a satisfactory level of balance, at the expense of a substantial increase in its computational requirements. Of course, the main factor attributing to the algorithmic performance deterioration is the increased number of paths that need to be stored at each node of the network. Similarly as in the case of the resource constraints, the increase on the number of paths stored at each node from 5 to 10 does not seem to improve significantly the solution quality.

## 6 Computational Implementation

In this section, we present computational results demonstrating the performance of the proposed solution methodology on a large collection of realistic instances. First, we test the algorithm on instances with similar size to that of the instances depicted in Table 1. We use the word "similar" to highlight that, being drawn from a live database of an actual air carrier, the actual number of deliveries that these instances include, for a particular fleet size, exhibits a small variation. The number

**Table 8** Computational performance of the algorithm on realistic problems of moderate size

| | Deliveries | | | Time | | | Routes | | | Nodes | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | Min | Max | Avg | Min | Max | Avg | Min | Max | Avg | Min | Max | Avg |
| 1 | 243 | 279 | 258 | 00:14 | 00:31 | 00:22 | 1203 | 2875 | 1990 | 45 | 59 | 56 |
| 2 | 348 | 396 | 364 | 00:42 | 01:09 | 00:57 | 2790 | 5367 | 4472 | 69 | 87 | 80 |
| 3 | 437 | 507 | 460 | 01:17 | 02:20 | 01:58 | 4048 | 8521 | 6976 | 59 | 113 | 105 |
| 4 | 548 | 628 | 573 | 02:01 | 04:54 | 03:58 | 6147 | 14734 | 12423 | 83 | 131 | 127 |
| 5 | 641 | 732 | 667 | 02:10 | 08:25 | 05:18 | 6543 | 22194 | 14524 | 113 | 149 | 143 |
| 6 | 769 | 886 | 796 | 06:56 | 12:31 | 08:59 | 14414 | 26059 | 19325 | 133 | 153 | 148 |

**Table 9** Computational performance of the algorithm on particularly large realistic problems

| | **Fleet** | **Flights** | | | **Time** | | |
|---|---|---|---|---|---|---|---|
| **No.** | **Size** | **Min** | **Max** | **Avg** | **Min** | **Max** | **Avg** |
| 1 | 432 | 2882 | 2917 | 2901 | 03:34:30 | 04:02:54 | 03:47:13 |
| 2 | 456 | 2958 | 2991 | 2976 | 04:01:46 | 04:35:02 | 04:11:55 |
| 3 | 481 | 3153 | 3186 | 3174 | 04:02:58 | 04:39:36 | 04:28:47 |
| 4 | 503 | 3238 | 3272 | 3257 | 04:22:33 | 04:48:51 | 04:31:26 |
| 5 | 532 | 3441 | 3483 | 3469 | 04:34:27 | 05:13:22 | 04:54:42 |
| 6 | 554 | 3511 | 3556 | 3533 | 04:45:11 | 05:32:17 | 05:08:31 |
| | **Fleet** | **Routes** | | | **Nodes** | | |
| **No.** | **Size** | **Min** | **Max** | **Avg** | **Min** | **Max** | **Avg** |
| 1 | 432 | 41011 | 43534 | 42213 | 872 | 934 | 911 |
| 2 | 456 | 42149 | 45252 | 43764 | 925 | 986 | 959 |
| 3 | 481 | 44322 | 46991 | 45235 | 1055 | 1103 | 1079 |
| 4 | 503 | 44723 | 48924 | 46543 | 1068 | 1234 | 1103 |
| 5 | 532 | 46874 | 49841 | 48250 | 1137 | 1213 | 1167 |
| 6 | 554 | 50036 | 55143 | 53224 | 1201 | 1360 | 1252 |

of random instances tested for each fleet size was always set equal to 30. Table 8 presents the minimum, average, and maximum value of the number of deliveries, the computational times, the number of routes generated, and the number of tree nodes. In all the test problems, the number of utilized vehicles was minimized as the secondary objective, while actual soft preferences were incorporated, as these had been set up by the final user.

Table 8 confirms the effectiveness of the proposed solution methodology. Since the tolerances used were negligible, the algorithm was always able to reach the exact optimum using moderate computational resources. Although considerable, the variances of the computational times, of the number of generated routes, and of the number of tree nodes cannot be characterized as over-excessive. Note that the size of the branch and price tree does not seem to increase substantially as the problem size increases. This is a very important observation, because the algorithmic computational performance is primarily affected by this factor.

In Table 9, we present results demonstrating the algorithm's computational performance on really large-scale problems. This enables the assessment of the effectiveness of the acceleration strategies presented earlier. Due to the lack of such large-sized problems in the live databases of logistic distribution air carriers, these instances have been drawn from the live database of a passenger airline. This implies that the associated flight workload concerns passenger flights instead of cargo deliveries, i.e., the underlying decision-making problem being solved is the tail assignment; this, however, makes no significant difference as far as the algorithm's application is concerned, since the problem formulation remains exactly the same.

These problems have been tested with no preference penalties, as none had been defined by the final user. In order to handle the particularly large size of these problems, the default parameter settings of the proposed algorithm were altered, as explained next. First of all, the relative integer gap tolerance was set equal to 2%, while the relative backtrack tolerance was set equal to 5%. With the cost of each uncovered delivery being equal to $10^6$ and the cost for each utilized aircraft being equal to $10^3$, two conditions were imposed for the continuation of the column generation procedure in the current tree node, a master LP objective difference at least equal to 100 on the average in two successive iterations, or an optimal reduced cost more negative than $-50$. This implies that the algorithm terminated prematurely the column generation procedure whenever both these two conditions were violated.

For six significantly large problem sizes, Table 9 presents the same computational results as those shown in Table 8. An important remark is the fact that the integer-gap tolerances do not define accurately how close to the true optimum the obtained solution is in this case. This happens because these tolerances are computed with respect to the best master LP objective, which, however, is not exact due to the premature termination of the column generation procedure. In most cases, however, these two tolerances give a pretty accurate estimate of the final solution's quality.

The results of Table 9 confirm the substantial increase in the computational requirements of the algorithm in the case of large-scale problems. Despite typically ranging in the order of minutes for the instances presented in the previous tables, the computational times now range in the order of hours. One of the main reasons for this difference is the substantial increase of the fleet sizes. The number of tree nodes and the number of generated routes have also increased drastically, but they cannot be considered over-excessive. The instances of Table 9 are considered particularly large by air transportation practitioners; therefore, these results confirm the proposed solution algorithm's capability of handling even such problems successfully.

## 7 Summary and Future Work

This work addressed the problem of fulfilling a set of deliveries in a logistic distribution network with a fleet of vehicles. For this problem, we developed an integer programming model formulation and an associated branch and price solution methodology. This methodology utilizes column generation to solve the master LP relaxation associated with each node of the branch and price tree. If the optimal solution to the currently explored subproblem is non-integral, the algorithm branches on a set of fractional decision variables, creating two new tree node subproblems. The procedure continues similarly, until the optimal solution of the ~~selected~~ subproblem is integer, which, due to the best LP node selection strategy, signifies that it is also optimal for the original integer problem.

We have used a large variety of realistic instances drawn from the live databases of actual air carriers in order to test the performance and study the behavior of the proposed solution methodology. Besides validating the correctness of this methodology and confirming its effectiveness, these tests have made it possible to fine-tune several key design parameters that have a significant impact on its performance.

Future research could be directed toward the development of clever enhancements that can accelerate the performance of the proposed methodology. We make this recommendation motivated by the fact that the OR research community unanimously perceives the column generation framework as the most suitable for addressing problems of this type. In that direction, a challenging goal for future research efforts is the evolution of the proposed methodology, so that it can become capable of handling problems with substantially larger size and considerably different characteristics and requirements.

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# Part II
# Modeling Under Uncertainty

# Sustainable Logistics Network Design Under Uncertainty

**Rozita Daghigh, Mir Saman Pishvaee, and Seyed Ali Torabi**

**Abstract** This chapter mainly discusses the mathematical programming models and methods used to design sustainable logistics networks (SLN) under epistemic uncertainty. Firstly, the relevant concepts and definitions are described and analyzed. Thereafter, a systemic review and analysis of the recent literature is provided to explore the most attractive research avenues in this area. A comprehensive description is given on environmental and social impact assessment methods in order to facilitate the quantification of environmental and social burden in the mathematical decision models. Two selected mathematical programming models for SLN design problem under uncertain data are provided and explained in detail to support quantitative decision-making in this area. Finally, a real industrial case study is described and investigated to show the applicability of the previously discussed mathematical programming methods.

## 1 Introduction

Sustainable development (SD) was defined by the World Commission on Environment and Development (WCED) as a kind of development which "meets the needs of the present generation without compromising the ability of future generation to meet their own needs" [78]. SD of a country depends on the efficient utilization of limited and irreplaceable resources. Regarding this issue and the lack of nonrenewable resources (e.g., oil, natural gas, etc.) and governmental legislation, considering various policies and corrective actions to reduce the environmental impact (EI) is necessary [14]. Therefore, enterprises rethink their strategies to ensure

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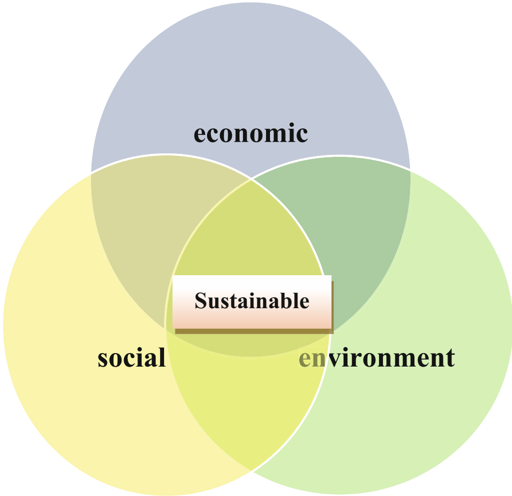


**Fig. 1** Three aspects of sustainability (Carter 2008)

the sustainability of their operations. These strategies include using raw materials compatible with the environment in production and industrial centers, reducing the use of fossil and oil energy sources, using energy-efficient technologies, green procurement, reducing packaging, employee recognition, establishing closed-loop supply chains [37], remanufacturing [42, 46], product recovery [41], reverse logistics [67, 68], and carbon emission reduction [60]. Two end-of-life methods, namely, recovery and recycling of used products, are among the most eco-efficient methods to cope with used products; however, they might increase operating cost [14]. SD must meet the following three objectives (see Fig. 1):

- Continue and support an optimum and stable level of economic development.
- Protect the environment (Env).
- Consider social (Soc) growth and meet the needs of everyone.

### *1.1 The Most Important Drivers of Sustainable Development*

According to the aspects of sustainable development, the main drivers of sustainability can be divided into three categories:

- **Economic advantages, saving and competitive advantages which include:**

  1. Saving resources
  2. Eliminating waste
  3. Productivity improvement
  4. Environmental legislation

- **Environmental concerns which include:**
  1. Global warming (rising the global temperature between 0.8 and 3.8 in this century)
  2. Reduction of the ozone layer
  3. Lack of fresh water
  4. Desertification and loss of soil quality
  5. Air pollution and acid rain
  6. Deforestation and destruction of biodiversity
- **Social concerns which include:**
  1. Lack of sanitary conditions
  2. Poverty and injustice
  3. Inequality of income
  4. Lack of equal opportunity in terms of geographical areas, gender, and age

In addition to the abovementioned drivers, awards, standards, and certifications related to environmental and social protection (e.g., SA 8000, ISO 14001, ISO 26000, GRI Guidance), consumer pressure, reputation and social image, and the Kyoto Protocol in 1997 that limits the emissions of greenhouse gases from industrialized countries are some other drivers forcing organizations to consider sustainability in all their activities.

### *1.2 Sustainable Supply Chain*

What is important is that SD must not only be deployed and implemented within the boundary of corporation ownership but also should be implemented at the whole supply chain despite its complex concept [15]. Globalization and increasing customer and government concerns about the environmental impact of activities as well as the appearance of the issue of social responsibility have led companies to employ sustainable supply chain management (SSCM). SSCM can be defined as the strategic management of information, financial and material flows, and transparent integration among the supply chain organizations in order to cover objectives at economic, environmental, and social aspects and consequently enhance the long-term performance of the whole supply chain network [13]. SSCM covers both the greenness and social responsibility of supply chain activities, simultaneously. Environmental or green supply chain management (GSCM) can be defined as the integration of environmental aspects into every supply chain management decisions, especially the strategic level decisions [56, 58, 64]. The traditional part of GSCM is dedicated to reverse logistics (RL) that includes all activities related to handling the end-of-life (EOL) products (e.g., recovery, recycling, and safe disposal). RL is useful by recapturing the value of EOL products, reducing the negative environmental impact of EOL products, and enhancing the green image of the concerned firm in the market. For example, IBM, which has profited by receiving end-of-use products, promotes secondhand items in Internet auctions and dismantles equipment as a source of spare parts [26]. Along RL process, managers can find out the necessity and importance of corporate social responsibility (CSR) in their corporate missions and strategies [16]. Indeed, CSR considers the social aspects in supply chain management to create more value for the whole society. CSR includes environment protection, workplace safety, human right, proper conditions for employees, etc. that affect different social stakeholders (i.e., consumers, staff, community activists, nongovernmental organizations, governmental legislation, and global competition) [12]. If corporations ignore social responsibility, they will be pressured by media, nongovernmental organization, professional unions, and other groups of society, and consequently they will lose a part of their market/profit. For example, popular corporations such as Shell and Walmart experienced damages on their image and profit according to media reports and campaigns of social groups [56, 58].

Sustainable development can be viewed as a source of long-term profit (i.e., the economic opportunity) against cost center (i.e., the economic threat). Saving cost by improving efficiency (e.g., reusing the value of EOL products), reducing risk (e.g., strike labor and local community complains leading to decrease profits), identifying new markets for new products (e.g., increasing demand for green products), and improving the image and reputation are examples of such long-term profits.

## *1.3 Sustainable Logistics Network Design*

Globalization of logistics networks has increased transportation distances which in turn has led to intensification of air and pollution, resource consumption, land use, and acidification which can affect the human health and ecosystem quality [47]. For instance, in the USA, from 1995 to 2006, *the total* amount of emissions by trucks increased about 7% [77]. Furthermore, governments have passed rules to support environment, such as greenhouse gas reduction regulation in European Union, Australia, and Canada. In addition, they have considered targets to decrease emission and force companies to switch over to green logistics and measure and control their carbon footprints [64].

Organizing and designing of logistics networks based on sustainable development paradigms can play fundamental role in moving toward sustainability. Network design is the initial point to start when looking for sustainable supply chain network [55]. Logistics network design (LND) is the most important strategic decision in the supply chain management. LND problem includes the determination of the number, location, capacity, and technology of the required network facilities and the quantities of aggregate flow between them to meet the demand nodes [30]. In the traditional view, the LND problem only focuses on long-term economic performance without paying attention to the environmental and social issues. However, the necessity of sustainability creates a need for considering Env and Sol objectives in the LND problem.

The remainder of this book chapter is organized as follows. In the next section, the relevant literature is reviewed. In Section 3, a comprehensive description is given on environmental and social impact assessment methods. Two selected mathematical programming models for sustainable logistics network design problem under uncertain data are presented in Section 4. The studied case and its acquired results are described in Section 5. Finally, some future research directions are presented in Section 6.

### *1.4 Uncertainty in Sustainable Logistics Network Design*

Complicated and dynamic nature of supply chain injects a high degree of uncertainty into decisions, which is an inevitable feature of any supply chain [45]. The degree of complexity in SLND is greater than traditional supply chains, since extra goals in designing sustainable supply chain networks should also be taken into account. In addition, accounting for data uncertainty especially in the strategic decisions (e.g., network design) is inevitable due to fluctuation of input parameters in a long time horizon and thus the difficulty of forecasting confident values for them.

Uncertainty is classified in different general and SCM-related categorizations. Klibi et al. [45] classify supply chain uncertainty into two groups: (1) business-as-usual uncertainty, such as usual fluctuations in demand, supply, etc., and (2) disruption uncertainty that has low frequency of happening (i.e., likelihood) but high impact. This type of uncertainty can originate from natural sources (e.g., earthquake, flood, tsunami) or man-made sources such as war, terrorist attacks, sanctions, etc. Dubois et al. [21] also classified uncertainty as (1) uncertainty in input data and (2) flexibility in constraints and goals. Uncertainty in data can be categorized into two groups [49]: (1) Randomness, which is the result of inherent randomness of the parameters, and stochastic programming methods are the most applied approaches to cope with this sort of uncertainty. (2) Epistemic uncertainty, which originates from insufficient knowledge for estimating the exact values of parameters, and possibilistic programming approaches are usually applied to cope with such uncertainty [49, 57]. Furthermore, elasticity in constraints and flexibility in goals deal with the inherent flexibility in the target values of goals and constraints for which flexible mathematical programming models are utilized to cope with such flexible target values [49]. There are different approaches to deal with uncertainty whose applications depend on the structure and context of the concerned problem, the type and the level of uncertainty in the model's parameters. Three main approaches are mostly employed to deal with uncertainty in the context of mathematical programming which include:

1. **Stochastic programming**, which can be used whenever randomness is the main source of uncertainty in input data for which random variables with known probability distributions are utilized (e.g., [10, 52]) and can be classified into two main categories: scenario-based stochastic programming with recourse and probabilistic (i.e., chance constrained) programming [62].

2. **Fuzzy programming** can handle both epistemic uncertainty in data and flexibility in goals and/or elasticity in constraints and can be classified into two main classes [40, 49, 74]: (1) possibilistic programming and (2) flexible programming. Possibilistic programming is used when there is lack of knowledge (i.e., epistemic uncertainty) about the exact value of parameters due to unavailability or insufficiency of required data. Flexible programming is used to cope with flexibility in target value of goals and/or elasticity in soft constraints.
3. **Robust optimization** provides risk-averse methods to cope with uncertainty in optimization problems. According to Pishvaee et al. [56, 58], "a solution to an optimization problem is said to be robust if it has both feasibility and optimality robustness. Feasibility robustness means that the solution should remain feasible for (almost) all possible values of uncertain parameters and optimality robustness means that the value of objective function should remain close to optimal value or have minimum (undesirable) deviation from the optimal value for (almost) all possible values of uncertain parameters." Robust programming approaches can be classified into three groups [56, 58]: (1) the hard worst-case robust programming [5, 6, 69], (2) the soft worst-case robust programming (Inuiguchi and Sakawa 1998; [7]), and (3) the realistic robust programming [50].

## 2 Literature Review

In the recent decades, interest in SSCM has increased both in academic community and among practitioners. To respond to the need of sustainability, a number of research works have been presented in the context of SCND problem. Nevertheless, the literature on sustainable SCND (SSCND) that covers all the three aspects of sustainability is very scarce. Some authors review the papers about sustainability and investigate them from different points of view. Seuring and Muller [65] reviewed 191 papers from 2002 onward and only found 21 papers that have investigated three aspects of sustainability. Carter and Rogers [13] identified papers with a conceptual framework for sustainable supply chain and prepared a literature review on sustainable supply chain (SSC). Seuring [66] reviewed papers on modeling approaches for SSCM problem. Tang and Zhou [73] reviewed 56 papers about SSC. Brandenburg et al. [9] identified and investigated 1422 papers and only found 134 papers on modeling approaches for SSC. Table 1 shows some survey articles in the field of SSC.

Despite the importance of social responsibility, the related literature is not rich and only considers the economic and environmental aspects. Generally, the relevant literature can be classified into two major groups: green SCND and SSCND.

**Table 1** Review articles in the field of SSC [9]

| Authors | Scope of supply chain | Time horizon | The number of reviewed papers | Supply chain perspective |
|---|---|---|---|---|
| Seuring and Muller [65] | Forward | 1994–2007 | 191 | General |
| Min and Kim (2012) | Forward and reverse | 1995–2010 | 519 | |
| Gold et al. (2010) | Forward | 1994–2007 | 70 | Experimental |
| Carter and Easton (2011) | Unknown | 1991–2010 | 80 | |
| Sarkis (2012) | Forward and reverse | 2000–2010 | 100 | |
| Sarkis (2011) | Forward and reverse | 1995–2010 | 150 | |
| Golicic and Smith | Unknown | 2000–2011 | 77 | |
| Hassini et al.[35] | Forward and reverse | 2000–2010 | 87 | Quantitative models |
| Seuring (2012) | Forward | 1994–2010 | 87 | |
| Tang and Zhou [73] | Forward and reverse | Unlimited | 56 | |
| Dekker et al. (2012) | Forward and reverse | Unlimited | 60 | |
| Ligin and Gupta (2010) | Forward and reverse | 1999–2010 | 540 | |
| Brandenburg et al. [9] | Forward | 2008–2012 | 134 | |

## *2.1 Green Supply Chain Network Design Literature*

Fonseca et al. [27] proposed a bi-objective model in which the entire costs and environment emission in reverse logistics were investigated. A two-stage stochastic programming model was utilized to deal with data uncertainty. Pishvaee et al. [51, 53] developed a mixed integer programming model for a multi-objective reverse logistics supply chain network design problem and proposed a simulated annealing algorithm with specific neighborhood search mechanism to solve this NP-hard model. Chaabane et al. [14] presented a mathematical programming model for designing a sustainable supply chain network and considered carbon emissions and total cost of supply chain in the aluminum industry. Pishvaee and Razmi [54] proposed a multi-objective fuzzy programming model to design an environmental sustainable supply chain under uncertainty of input parameters and used the life cycle analysis to quantify the environmental impact of the designed network. Pishvaee et al. [56, 58] presented a fuzzy programming model for designing a forward network supply chain aiming at minimizing the environment emission and total cost. Govindan et al. [31] presented a multi-objective optimization model for a sustainable two echelon location-routing problem with time windows. The purpose of the model is to determine the number and location of the facilities, the amount of aggregated flow between different echelons, and the optimal routes of the networks. In order to solve this problem, two hybrid metaheuristics based on the multi-objective particle swarm algorithm and multi-objective variable neighborhood search, were utilized. Tseng and Hung [75] suggested a strategic model according to the operating and social costs resulting from the production of greenhouse gas emissions for sustainable supply chain. In addition, they investigated the amount of $CO_2$ emission and operating cost under different scenarios in the clothing network supply chain. Govindan et al. [32] presented a bi-objective model, which integrates the sustainable supplier selection and order allocation and sustainable supply chain network design problems under stochastic demand. The model aims at minimizing the total costs and environmental effect. Hybridization of two multi-objective algorithms, namely, the adapted multi-objective electromagnetism mechanism algorithm (AMOEMA) and adapted multi-objective variable neighborhood search (AMOVNS), were used to solve the model. Talaei et al. [72] presented a novel bi-objective facility location-allocation model for closed-loop green supply chain network design problem for which the robust and fuzzy programming approaches were used to cope with the uncertainty of input data. A case study of copier industry was used to show the applicability of the proposed model.

## *2.2 Sustainable Supply Chain Network Design Literature*

Dehghanian et al. [18] developed a three-objective mathematical programming model to design a sustainable recycling network to balance all three sustainability factors. Life cycle analysis (LCA) was also used to study the environmental effects of various EOL options. A multi-objective genetic algorithm was implemented to find the Pareto-optimal solutions, and the model was implemented for rubber wastes. Pishvaee et al. [56, 58] developed a bi-objective programming model by considering social and economic aspects to optimize supply chain network under uncertainty. They proposed several robust possibilistic programming models to deal with data uncertainty. Pishvaee et al. [55] proposed a multi-objective possibilistic programming model to design a sustainable medical supply chain by considering economic, environmental, and social objectives under uncertainty. In order to solve the proposed model, a customized Benders decomposition algorithm was also implemented. Mota et al. [48] presented a multi-objective programming model to design a closed-loop supply chain in which economic, environment, and social aspects were simultaneously considered. For the first time, they implemented an environmental methodology, namely, "Recipe," in their model. Also, they investigated their model in the manufacturer and distributors of batteries in Portugal. Devika et al. [20] developed a mixed integer programming model for a multi-objective closed-loop supply chain design network to take into account all three sustainability factors simultaneously. In order to solve this complicated problem, three hybrid metaheuristics, which are based on imperialist competitive algorithm and variable neighbor search algorithm, were utilized. Finally, a glass industry case study was used to show the applicability of this approach. Ramezani et al. [59] demonstrated the fuzzy set theory application in designing a multi-period, multiple product closed-loop supply chain network. The presented model includes three-objective functions: profit maximization, delivery time minimization, and quality maximization. Using the fuzzy approach, flexible restrictions, and fuzzy coefficients, an efficient model was obtained. Mohammadi et al. [47] developed a novel sustainable hub location problem in which two new environmental-based cost function accounting for air and noise pollution of vehicles are incorporated. To cope with uncertain data, a mixed possibilistic-stochastic programming approach was proposed to construct the crisp counterpart, and also a simulated annealing and imperialist competitive algorithms are used to solve the real-sized instances. Zhang et al. [81] presented a sustainable multi-objective optimization model for sustainable supply chain network design considering multiple distribution channels. The model aims at reducing the economic cost, enlarging the customer coverage, and weakening the environmental influences. In order to solve this model, a modified multi-objective artificial bee colony algorithm is introduced. Babazadeh et al. [2] presented a multi-objective possibilistic programming model to design a second-generation biodiesel supply chain network under risk. It aims at minimizing the cost and environmental impact of all processes. To solve this multi-objective model, they used a hybrid solution approach based on the flexible lexicographic and augmented epsilon-constraint methods. Govindan et al. (2016) proposed a multi-objective model to design an optimized reverse logistics network while considering the economic, social, and environmental aspects simultaneously. To deal with data uncertainties in many parameters, the fuzzy approach was used, and for solving the model efficiently, a multi-objective genetic algorithm is devised. Daghigh et al. [17] proposed a multi-objective sustainable location-inventory model for third-party logistics providers. The model aims at minimizing the total cost and the environmental emission due to the fuel consumption of vehicles and maximizing the social responsibility subject to fair access to products, number of created job opportunities, and local community development. Input parameters of the model are tainted with epistemic uncertainty for which a possibilistic programming approach is used. To provide a systemic view on the SSCND literature, we have classified and tabulated some of the important published papers in Table 2.

## 3 Measuring Sustainability

Assessing sustainable development in macro and micro levels requires appropriate tools to quantify environmental and social impacts. In this section, a number of important environmental and social impact assessment methods are studied and described.

### *3.1 Environmental Impact Assessment (EIA)*

Each product has different EI in its life cycle stages. Life of a product or a service starts from its design phase and finishes with the end-of-life stages such as recycling, recovering, and internment. All of the activities that are done during the life cycle of a product or a service have EI due to the resources' consumption, emissions, and environmental exchanges. Focusing on the cradle-to-grave perspective of a

**Table 2** Classification of some selected papers

| Authors | year | Modeling | | Network | | | Objectives | | | Solution | | | Uncertainty | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | MINLP | MILP | forward | Reverse | Closed loop | economic | environment | social | software | exact | metaheuristic | stochastic | fuzzy | robust |
| Fonseca et al. | 2010 | | * | | * | | * | * | | | * | | * | | |
| Pishvaee et al. | 2010 | | * | | * | | * | | | | | * | | | |
| Chaabane et al. | 2012 | | * | * | | | * | * | | * | | | | | |
| Pishvaee and Razmi | 2012 | | * | | | * | * | * | | * | | | | * | |
| Pishvaee et al. | 2012 | | * | * | | | * | * | | | * | | | | * |
| Govindan et al. | 2014 | * | | * | | | * | * | | | | * | | | |
| Tseng and Hung | 2014 | * | | * | | | * | * | | * | | | | | |
| Govindan et al. | 2015 | | * | * | | | * | * | | | | * | * | | |
| Talaei et al. | 2016 | | * | | | * | * | * | | * | | | | * | * |
| Dehghanian et al. | 2009 | | * | | * | | * | * | * | | | * | | | |
| Pishvaee et al. | 2012 | | * | * | | | * | | * | | * | | | | * |
| Pishvaee et al. | 2014 | | * | | | * | * | * | * | | * | | | * | |
| Mota et al. | 2015 | | * | | | * | * | * | * | * | | | | | |
| Devika et al. | 2014 | | * | | | * | * | * | * | | | * | | | |
| Ramezani et al. | 2014 | | * | | | * | * | * | * | | * | | | * | |
| Mohammadi et al. | 2014 | * | | * | | | * | * | * | | | * | * | * | |
| Zhang et al. | 2016 | | * | * | | | * | * | * | | | * | | | |
| Babazadeh et al. | 2016 | | * | * | | | * | * | | | * | | | * | |
| Govindan et al. | 2016 | | * | | * | | * | * | * | | | * | | * | |
| Daghigh et al. | 2016 | | * | * | | | * | * | * | | * | | | * | |

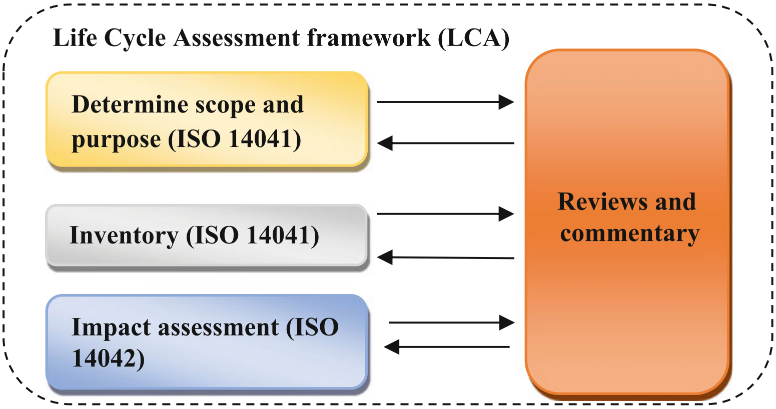


**Fig. 2** Life cycle assessment framework based on ISO 14040

product provides an appropriate framework to discover opportunities for improving the efficiency and effectiveness of the concerned system. Now, researchers and practitioners use LCA methodology to quantify and assess the EI of every product/service. The international standard organization presented ISO 14000 standard series on LCA that is the most credible structure for life cycle assessment [61]. This standard is designed in the form of a quartet structure:

- ISO 14040, which is related to the principle and framework for life cycle assessment
- ISO 14041, which is related to the purpose, scope, and life cycle inventory
- ISO 14042, which is related to the life cycle impact assessment
- ISO 14043, which is related to the interpretation of life cycle

Figure 2 shows the relationships between different parts of the life cycle assessment framework based on ISO 14040. According to this structure, at first the scope of the system and its function as well as the purpose of the application of the life cycle assessment must be described. The second stage determines the inventory (including materials, flows, and processes) of different stages of the life cycle. In the next stage, the environmental impact caused by inventory must be evaluated according to the related indicators. Reviews and commentary section is done in parallel to the aforementioned stages.

The impact assessment part consists of three elements (including the impact categories, relevant indicators, and inventory allocation to impact categories), description (including the quantitative calculation of each inventory into the relevant impact categories), and calculation of the final result through normalizing and weighting methods. Despite the advantages of LCA, this methodology requires a complex, costly, and time-consuming process, and its direct usage needs to be weighted and interpreted. Several methods have been developed as standardized and simplified versions of LCA. These methods are formed based on LCA methodology,

**Table 3** Characteristics of credible EIA methods [55]

| EIA methods | Covering midpoint Impact categories | Covering end-point impact categories | Providing normalization method | Providing weighting method | Requiring goal setting |
|---|---|---|---|---|---|
| CML2001 [34] | * | | * | | |
| Eco-indicator 99 [29] | * | * | * | * | |
| EDIP 2003 [36] | * | | * | * | * |
| EPS 2000 [70] | | * | | | |
| IMPACT 2002+ [44] | * | * | * | | |
| Ecological scarcity [8] | * | * | * | * | * |
| TRACI (Bare et al. 2003) | * | | | | |
| Recipe 2008 [28] | *[a] | * | * | * | |

[a]The method is able to assess EI based on both midpoint and end-point impacts

and most of them classify and standardize EIs in midpoint and/or end-point impact categories to select an appropriate method for EI assessment (EIA). Also, some of the methods provide normalization and weighting mechanisms to quantify the final results. The list of these methods and some of their characteristics are illustrated in Table 3.

## *3.2 Social Impact Assessment (SIA)*

World Business Council for Sustainable Development (WBCSD) defined social responsibility as "the continued organizational commitment to ethical principle and contribution to social development, while improving the quality of life of the workforce and their families as well as local communities and society in general" [79]. Generally, measuring the social responsibility is difficult due to its extensive scope and complex nature of social impacts. Also, the social responsibility subject is a multidisciplinary and multi-stakeholder issue that measuring all SIs of an activity would be impossible [56, 58]. However, ISO developed the "International Guidance standard on social responsibility-ISO 26000" to provide a comprehensive framework for SR. ISO 26000 has classified the social responsibility into seven core subjects as follows [39]:

- *Organizational governance*: it means that the organization adopts its decision by organizational governance in order to achieve its objectives. Organizational governance is the most important core subject for creating social responsibility and necessary for the fulfillment of social responsibility in six other core subjects. For example: respecting the law and responsibility.

- *Human rights*: it is related to basic rights of every human. This right is categorized into two parts: (1) political rights (e.g., the equality right against the law, right to life) and (2) economic and social rights (including the right to work and access to food and health).
- *Labor practices*: it includes all political and activities related to the work in the organization by workers (e.g., health and safety workers, hire and upgrade workers).
- *Environment*: it means the organizational activities that affect the environment. It is mentioned with respect to the high importance of environment in sustainability paradigm, Env issues are considered as a separate part from the Soc issues.
- *Fair operating practices*: it is related to ethical conduct in dealing with other organizations, suppliers, and customers.
- *Consumer issues*: it is related to that organizations are responsible to their customers and consumers and includes reducing the risk of consumer products and services and promoting the design and sustainable consumption and fair access to all segments of society.
- *Community involvement and development*: community involvement and development is the most important parts of the sustainable development. Each organization according to the type of its industry must help and affect on this process.

Methods and reporting frameworks of the social impact are weaker than those of environmental impact assessment. However, researchers and practitioners have developed some methods and guidelines based on ISO 26000 core subjects. Table 4 shows the most popular and credible methods and guidelines on SR.

## 4 Selected Fuzzy Mathematical Models

In this section, two different uncertainty mathematical models are elaborated in the context of SLND under uncertainty. For the sake of simplicity, the notations used in this section are the same as those represented in the original papers.

### *4.1 A SLND Model with Possibilistic Programming and Hybrid Solution Approach*

This problem is presented by Babazadeh et al. [2] for SLND in which possibilistic programming is used to deal with uncertainty. The simplified proposed model is

**Table 4** Characteristics of credible SIA methods [55]

| SIA methods and guidelines | Organizational governance | Human rights | Labor practices | The environment | Fair operating practices | Consumer issues | Community involvement and development |
|---|---|---|---|---|---|---|---|
| SA8000 [63] | | * | ** | | * | | * |
| GRI [33] | * | ** | ** | ** | ** | ** | ** |
| ETI [23] | | * | ** | | | | |
| FLA [25] | * | * | ** | | | | |
| GC [76] | | ** | ** | ** | * | | |
| GSLCAP [4] | | * | ** | | ** | ** | ** |

[*] partial coverage, [**] full coverage

a multi-objective, multiproduct, multimodal, and capacitated biodiesel SCND in which all involved stages from feedstock supply centers to distribution of biodiesel and by-product glycerin are integrated under uncertainty. Biodiesel is a type of biofuel whose use in the transportation sector has intensively increased due to energy crisis, environmental, and social concerns. Biofuels are divided into the first and second generations. First-generation biofuel has been commercialized worldwide, but there is a significant concern about food crisis. However, the second-generation biofuels are environmentally and socially sustainable and do not compete with food resources. *Jatropha Curcas* L. (JCL) is a nonedible feedstock which does not compete with food crops and is only used for biodiesel production. JCL seeds are harvested from farms and then shipped to JCL oil extraction. Then, JCL oil is shipped to bio-refinery centers and converted to main biodiesel product and by-product glycerin. Biodiesel is shipped to end customers through distribution centers, but glycerin is directly transported to related customers. In the considered integrated supply chain network, only the locations of final customers are known, but other locations of facilities in each layer should be selected among the candidate locations. The model has two different objectives, i.e., minimizing the environmental impact (EI) of all involved processes in the considered biodiesel SC network and minimizing the total costs subject to real-life assumptions and constraints. The model aims to determine the numbers, locations and capacities of candidate facilities, the amount of production, inventory levels, imported JCL, aggregated material flow between network nodes, and transportation modes in different periods. Feedstock supply, biodiesel and glycerin demands in constraints, cost, and environmental coefficients in the objective functions are under uncertainty. Possibilistic programming is used to deal with uncertainty due to lack of reliable historical data about uncertain parameters. In this paper, EI of all involved processes in the considered biodiesel network is assessed by Eco-indicator 99 method employing SimaPro software. The structure of the problem is depicted in Fig. 3, and the notations are described thereafter.

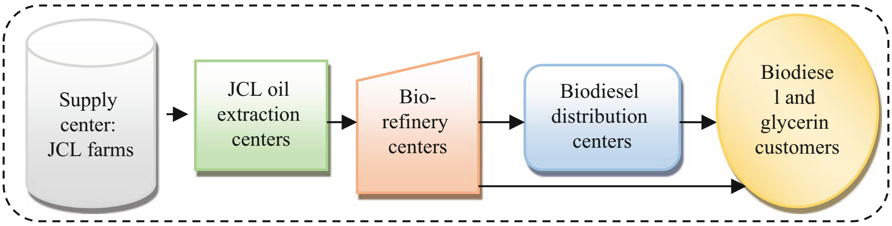


**Fig. 3** The integrated biodiesel supply network

## *Indices*

| | |
|---|---|
| $f$ | Index of candidate locations of JCL cultivation centers |
| $i$ | Index of candidate locations for collection and oil extraction centers of JCL yields |
| $j$ | Index of candidate locations for bio-refinery centers of biodiesel production |
| $k$ | Index of candidate locations for storage and distribution centers of biodiesel |
| $c$ | Index of consumer centers of biodiesel |
| $n$ | Index of consumer centers of glycerin |
| $l$ | Index of transportation mode (road and railway) |
| $t$ | Index of time period |

## *Parameters*

| | |
|---|---|
| $\widetilde{D}_{ct}$ | Demand of consumer center $c$ for biodiesel in period $t$ (ton/period) |
| $\widetilde{DE}_{nt}$ | Demand of consumer center n for glycerin in period $t$ (ton/period) |
| $\widetilde{n}_{ft}$ | Amount of JCL yields per hectare at location $f$ in period $t$ (ton/ha) |
| $\varphi$ | Conversion factor of JCL yield to JCL oil (percent) |
| $\pi$ | Conversion factor of JCL oil to biodiesel (percent) |
| $LA_f$ | Minimum land area dedicated for JCL cultivation center at location $f$ (ha) |
| $UA_f$ | Maximum land area dedicated for JCL cultivation center at location $f$ (ha) |
| $LC_i$ | Lower bound dedicated on capacity of collection and oil extraction center of JCL yields at location $i$ (ton) |
| $UC_i$ | Upper bound of capacity of collection and oil extraction center of JCL yields at location $i$ (ton) |
| $LB_j$ | Lower bound dedicated on capacity of bio-refinery center at location j (ton) |
| $UB_j$ | Upper bound dedicated on capacity of bio-refinery center at location j (ton) |
| $LS_k$ | Lower bound dedicated on capacity of storage and distribution center at location $k$ (ton) |
| $US_k$ | Upper bound of capacity of storage and distribution center at location $k$ (ton) |
| $DisJT_{fli}$ | Distance between cultivation center f and oil extraction center $i$ by mode $l$ |
| $DisOT_{ilj}$ | Distance between JCL oil extraction center $i$ and bio-refinery $j$ by mode $l$ |
| $DisBT_{jlk}$ | Distance between bio-refinery $j$ and distribution center $k$ by mode $l$ |
| $DisGT_{jln}$ | Distance between bio-refinery $j$ and consumer center $n$ by mode $l$ |
| $DisMT_{klc}$ | Distance between distribution center $k$ and consumer center $c$ by mode $l$ |

### *Cost parameters*

| | |
|---|---|
| $\widetilde{FCJ}_{ft}$ | Fixed cost of JCL cultivation at location *f* in period *t* (MIRR/period) |
| $\widetilde{FCC}_{it}$ | Fixed cost of opening oil extraction center of JCL seeds at location *i* in period *t* |
| $\widetilde{FCB}_{jt}$ | Fixed cost of opening bio-refinery center at location *j* in period *t* |
| $\widetilde{FCS}_{kt}$ | Fixed cost of opening distribution center at location *k* in period *t* |
| $\widetilde{VCJ}_{f}$ | Variable cost of JCL cultivation per hectare at location f (MIRR/ha) |
| $\widetilde{VCC}_{it}$ | Variable cost per unit capacity for oil extraction center of JCL seeds at location *i* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{VCB}_{jt}$ | Variable cost per unit capacity for bio-refinery center *j* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{VCS}_{kt}$ | Variable cost per unit capacity for distribution center *k* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{PCJ}_{ft}$ | Unit production cost of JCL seeds at location *f* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{PCB}_{jt}$ | Unit production cost of biodiesel at bio-refinery center *j* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{PCG}_{jt}$ | Unit production cost of glycerin at bio-refinery center *j* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{PCO}_{it}$ | Unit oil extraction cost from JCL seeds in oil extraction center *i* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{ICJ}_{it}$ | Unit inventory holding cost of JCL seeds at oil extraction center *i* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{ICB}_{jt}$ | Unit inventory holding cost of biodiesel at bio-refinery center *j* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{ICG}_{jt}$ | Unit inventory holding cost of glycerin at bio-refinery center *j* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{ICS}_{Kt}$ | Unit inventory holding cost of biodiesel at distribution center *k* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{JCT}_{flit}$ | Transportation cost of JCL seeds from cultivation center *f* to oil extraction center *I* by mode *l* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{OCT}_{iljt}$ | Transportation cost of JCL oil extraction center *i* to bio-refinery *j* by mode *l* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{BCT}_{jlkt}$ | Transportation cost of biodiesel from bio-refinery *j* to distribution center *k* by mode *l* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{GCT}_{jlnt}$ | Transportation cost of glycerin from bio-refinery *j* to consumer center *n* by mode *l* in period *t* (MIRR $ton^{1}$/period) |
| $\widetilde{MCT}_{klct}$ | Transportation cost of biodiesel from distribution center *k* to consumer center *c* by mode *l* in period t (MIRR $ton^{1}$/period) |
| $\widetilde{CIm}_{it}$ | Importing cost of JCL seeds in oil extraction center *i* in period *t* (MIRR $ton^{1}$/period) |

### *Environmental parameters*

| | |
|---|---|
| $\widetilde{e}x_f$ | Environmental impact of harvesting 1 t JCL seeds at location $f$ for planning horizon (pt) |
| $\widetilde{e}u_i$ | Environmental impact of establishing 1 t capacity of oil extraction center of JCL seeds at location I for planning horizon (pt) |
| $\widetilde{e}v_j$ | Environmental impact of establishing 1 t capacity of bio-refinery at location i planning horizon (pt) |
| $\widetilde{e}w_k$ | Environmental impact of establishing 1 t capacity of distribution center of biodiesel at location I for planning horizon (pt) |
| $\widetilde{EB}_j$ | Environmental impact of producing 1 t biodiesel at bio-refinery center $j$ |
| $\widetilde{EG}_j$ | Environmental impact of producing 1 t glycerin at bio-refinery center $j$ |
| $\widetilde{EO}_i$ | Environmental impact of producing 1 t JCL oil extraction center $i$ |
| $\widetilde{EIJ}_i$ | Environmental impact of inventory holding of JCL seeds at oil extraction center $i$ |
| $\widetilde{EIB}_j$ | Environmental impact of inventory holding of biodiesel at bio-refinery center $j$ |
| $\widetilde{EIG}_j$ | Environmental impact of inventory holding of glycerin bio-refinery center $j$ |
| $\widetilde{EIS}_k$ | Environmental impact of inventory holding of biodiesel at distribution center $k$ |
| $\widetilde{EJT}_{fli}$ | Environmental impact of transporting 1 t JCL seeds per km from cultivation center $f$ to oil extraction center $i$ by mode $l$ |
| $\widetilde{EOT}_{ilj}$ | Environmental impact of transporting 1 t JCL oil per km from oil extraction center $i$ to bio-refinery $j$ by mode $l$ |
| $\widetilde{EBT}_{jlk}$ | Environmental impact of transporting 1 t biodiesel per km from bio-refinery $j$ to distribution center $k$ by mode $l$ |
| $\widetilde{EGT}_{jln}$ | Environmental impact of transporting 1 t glycerin per km from bio-refinery $j$ to consumer center $n$ by mode $l$ |
| $\widetilde{EMT}_{klc}$ | Environmental impact of transporting 1 t biodiesel per km from distribution center $k$ to consumer center $c$ by mode $l$ |
| $\widetilde{EIm}_i$ | Environmental impact of importing 1 t JCL seeds imported in oil extraction center $i$ |

### *Binary decision variables*

| | |
|---|---|
| $x_f$ | 1 if location $f$ is selected for JCL cultivation; 0 otherwise |
| $u_i$ | 1 if location $i$ is selected for opening collection and oil extraction center of JCL yields; 0 otherwise |
| $v_j$ | 1 if location $j$ is selected for opening bio-refinery; 0 otherwise |
| $w_k$ | 1 if location $k$ is selected for opening storage and distribution center of biodiesel; 0 otherwise |

***Continuous decision variables***

| | |
|---|---|
| $IJ_{it}$ | Inventory level of JCL yields at collection and oil extraction center $i$ in period $t$ (ton/period) |
| $\mathrm{Im}_{it}$ | Amount of JCL yields imported at collection and oil extraction center $i$ in period $t$ |
| $IB_{jt}$ | Inventory level of biodiesel at bio-refinery $j$ in period $t$ |
| $IG_{jt}$ | Inventory level of glycerin at bio-refinery $j$ in period $t$ |
| $IS_{kt}$ | Inventory level of biodiesel at storage and distribution center $k$ in period $t$ |
| $PJ_{ft}$ | Produced amount of JCL at cultivation center $f$ in period $t$ |
| $PB_{jt}$ | Produced amount of biodiesel at bio-refinery $j$ in period $t$ |
| $PG_{jt}$ | Produced amount of glycerin at bio-refinery $j$ in period $t$ |
| $PO_{it}$ | Produced amount of JCL oil at collection and oil extraction center $i$ in period $t$ |
| $JT_{flit}$ | Transported amount of JCL yields from cultivation center $f$ to collection and oil extraction center $I$ by mode $l$ in period $t$ |
| $OT_{iljt}$ | Transported amount of JCL oil from collection and oil extraction center $i$ to bio-refinery $j$ by mode $l$ in period $t$ |
| $BT_{jlkt}$ | Transported amount of biodiesel from bio-refinery $j$ to storage and distribution center $k$ by mode $l$ in period $t$ |
| $GT_{jlnt}$ | Transported amount of glycerin from bio-refinery $j$ to consumer center $n$ by mode $l$ in period $t$ |
| $MT_{klct}$ | Transported amount of biodiesel from storage and distribution center $k$ to consumer center c by mode $l$ in period $t$ |
| $CJ_f$ | Amount of cultivated area of JCL at location $f$ (ha) |
| $CC_{it}$ | Total capacity of collection and oil extraction center $i$ in period $t$ |
| $CEC_{it}$ | Amount of capacity expansion at collection and oil extraction center $i$ in period $t$ (ton) |
| $CB_{jt}$ | Total capacity to bio-refinery $j$ in period $t$ |
| $CEB_{jt}$ | Amount of capacity expansion at bio-refinery $j$ in period $t$ |
| $CS_{kt}$ | Total capacity of storage and distribution center $k$ in period t |
| $CES_{kt}$ | Amount of capacity expansion at storage and distribution center $k$ in period $t$ |

Using aforementioned notation, the proposed mathematical model is as follows:

$$\begin{aligned}
Z1 = {} & \left(\sum_f \sum_t F\tilde{C}J_{ft}x_f + \sum_i \sum_t F\tilde{C}C_{it}u_i + \sum_j \sum_t F\tilde{C}B_{jt}v_j + \sum_k \sum_t F\tilde{C}S_{kt}w_k\right) \\
& + \left(\sum_f V\tilde{C}J_f\left(CJ_f\right)\left(CJ_f\right) + \sum_i \sum_t V\tilde{C}C_{it}CC_{it} + \sum_j \sum_t V\tilde{C}B_{jt}CB_{jt} + \sum_k \sum_t V\tilde{C}S_{kt}CS_{kt}\right) \\
& + \left(\sum_f \sum_t P\tilde{C}J_{ft}PJ_{ft} + \sum_i \sum_t P\tilde{C}O_{it}PO_{it} + \sum_j \sum_t P\tilde{C}B_{jt}PB_{jt} + \sum_j \sum_t P\tilde{C}G_{jt}PG_{jt}\right) \\
& + \left(\sum_i \sum_t I\tilde{C}J_{it}IJ_{it} + \sum_j \sum_t I\tilde{C}B_{jt}IB_{jt} \ \sum_j \sum_t I\tilde{C}G_{jt}IG_{jt} + \sum_k \sum_t I\tilde{C}S_{kt}IS_{kt}\right) \\
& + \left(\sum_f \sum_l \sum_i \sum_t J\tilde{C}T_{flit}JT_{flit} + \sum_i \sum_l \sum_j \sum_t O\tilde{C}T_{iljt}OT_{iljt} + \sum_j \sum_l \sum_k \sum_t B\tilde{C}T_{jlkt}BT_{jlkt}\right. \\
& \left. + \sum_k \sum_l \sum_c \sum_t M\tilde{C}T_{klct}MT_{klct} + \sum_j \sum_l \sum_n \sum_t G\tilde{C}T_{jlnt}GT_{jlnt}\right) + \left(\sum_i \sum_t CIm_{it}\mathrm{Im}_{it}\right) \qquad (1)
\end{aligned}$$

$$
\begin{aligned}
lZ2 = & \left( \sum_{f} \tilde{e}x_f x_f + \sum_{i} \sum_{t} \tilde{e}u_i u_i + \sum_{j} \sum_{t} \tilde{e}v_j v_j + \sum_{k} \sum_{t} \tilde{e}w_k w_k \right) \\
& + \left( \sum_{i} \sum_{t} \tilde{E}O_i PO_i + \sum_{j} \sum_{t} \tilde{E}B_j PB_{jt} + \sum_{j} \sum_{t} \tilde{E}G_j PG_{jt} \right) \\
& + \left( \sum_{i} \sum_{t} E\tilde{I}J_i IJ_{it} + \sum_{j} \sum_{t} E\tilde{I}B_j IB_{jt} + \sum_{j} \sum_{t} E\tilde{I}G_j IG_{jt} + \sum_{k} \sum_{t} E\tilde{I}S_k IS_{kt} \right) \\
& + \left( \sum_{f} \sum_{l} \sum_{i} \sum_{t} E\tilde{J}T_{fli} DisJT_{fli} JT_{flit} \right. \\
& + \sum_{i} \sum_{l} \sum_{j} \sum_{t} E\tilde{O}T_{ilj} DisOT_{ilj} OT_{iljt} \sum_{j} \sum_{l} \sum_{k} \sum_{t} E\tilde{B}T_{jlk} DisBT_{jlk} BT_{jlkt} \\
& \left. + \sum_{k} \sum_{l} \sum_{c} \sum_{t} E\tilde{M}T_{klc} DisMT_{Klc} MT_{klct} + \sum_{j} \sum_{l} \sum_{n} \sum_{t} E\tilde{G}T_{jln} DisGT_{jln} GT_{jlnt} \right) \\
& + \sum_{i} \sum_{t} E\tilde{I}M_{it} \mathrm{Im}_{it}
\end{aligned} \tag{2}
$$

$$\sum_{k} \sum_{l} MT_{klct} \geq \tilde{D}_{ct} \quad \forall c, t \tag{3}$$

$$\sum_{j} \sum_{l} GT_{jlnt} \geq \tilde{D}E_{nt} \quad \forall n, t \tag{4}$$

$$\sum_{l} \sum_{i} JT_{fliit} = PJ_{ft} \quad \forall f, t \tag{5}$$

$$PO_{it} = \varphi \sum_{f} \sum_{l} JT_{flit} \quad \forall i, t \tag{6}$$

$$PJ_{ft} = \tilde{n}_{ft} CJ_f \quad \forall f, t \tag{7}$$

$$PB_{jt} = \pi \sum_{i} \sum_{l} OT_{iljt} \quad \forall j, t \tag{8}$$

$$PG_{jt} = (1 - \pi) \sum_{i} \sum_{l} OT_{ijlt} \quad \forall j, t \tag{9}$$

$$IJ_{it} = IJ_{i,t-1} + \mathrm{Im}_{it} + \sum_{f}\sum_{l} JT_{flit} - \left(\frac{1}{\alpha}\right)\sum_{l}\sum_{j} OT_{iljt} \quad \forall i, t \tag{10}$$

$$IB_{jt} = IB_{j,t-1} + PB_{jt} - \sum_{l}\sum_{k} BT_{jlkt} \quad \forall j, t \tag{11}$$

$$IG_{jt} = IG_{j,t-1} + PG_{j,t} - \sum_{l}\sum_{n} GT_{jlnt} \forall j, t \tag{12}$$

$$IS_{kt} = IS_{k,t-1} + \sum_{j}\sum_{l} BT_{jlkt} - \sum_{l}\sum_{c} MT_{klct} \quad \forall k, t \tag{13}$$

$$\sum_{f}\sum_{l} JT_{flit} + lm_{it} \leq CC_{it} \quad \forall i, t \tag{14}$$

$$\sum_{i}\sum_{l} OT_{iljt} \leq CB_{jt} \quad \forall j, t \tag{15}$$

$$\sum_{j}\sum_{l} BT_{jlkt} \leq CS_{kt} \ \ \forall k, s \tag{16}$$

$$CC_{it} = CC_{i,t-1} + CEC_{it} \quad \forall i, t \tag{17}$$

$$v_j LB_j \leq CB_{jt} \leq v_j UB_j \quad \forall j, t \tag{18}$$

$$CB_{jt} = CB_{j,t-1} + CEB_{jt} \quad \forall j, t \tag{19}$$

$$u_i LC_i \leq CC_{it} \leq u_i UC_i \quad \forall i, t \tag{20}$$

$$CS_{kt} = CS_{k,t-1} + CES_{kt} \quad \forall k, t \tag{21}$$

$$w_k LS_k \leq CS_{kt} \leq w_k US_k \quad \forall k, t \tag{22}$$

$$x_f, y_s, u_i, v_j, w_k \geq 0 \tag{23}$$

$$\begin{aligned} &IJ_{it}, \mathrm{Im}_{it}, IB_{jt}, IG_{jt}, IS_{kt}, PJ_{ft}, PB_{jt}, PG_{jt}, PO_{it}, JT_{flit}, OT_{iljt}, BT_{jlkt}, \\ &GT_{jlnt}, MT_{klct}, CJ_f, CC_{it}, CEC_{it}, CB_{jt}, CEB_{jt}, CS_{kt}, CES_{kt} \geq 0 \end{aligned} \tag{24}$$

The objective function (1) minimizes the total expenses of biodiesel supply chain network. The first and second parts belong to fixed and variable costs of opening different facilities. The third part describes the production cost. The fourth part expresses the inventory holding cost. The fifth part states the transportation cost of various network layers. The sixth part shows the importing cost. Objective function (2) describes the sustainability environmental goal which tries to minimize the environmental impacts of all processes in the network. The first part shows the environmental impact of established facilities and related capacity installation. The second part expresses the environmental impact of production processes at different facilities. The third part states the environmental impact of inventory holding. The fourth part shows the environmental impact of transporting material between different nodes in the network, and the final part shows the environmental impact of importing JCL seeds. Constraints (3) and (4) show demands of biodiesel and glycerin at different cities for each period that should be fully satisfied. Constraint (5) ensures that all JCL seeds are collected and transported to oil extraction in each period. Constraint (6) shows the amount of JCL oil production at related facilities in each period. Constraint (7) illustrates the amount of JCL seeds produced in each cultivated area in any period. Constraints (8) and (9) state the amount of biodiesel and glycerin production at bio-refinery in each period from JCL oil. Constraints (10)–(13) are inventory balance for oil extraction center of JCL seeds, biodiesel, and glycerin at related facilities. Constraints (14)–(16) are capacity constraints at oil extraction centers of JCL seeds, bio-refineries, and distribution centers. Constrains (17)–(22) show capacity expansions and lower and upper bounds in oil extraction centers of JCL seeds, bio-refineries, and distribution centers, respectively. When a specific facility opens, its capacity must be between the lower and upper bound, and also the capacity of any open facility in each period is equal to the capacity determined in the previous period and the capacity expanded in the current period. As mentioned in the previous section, the feedstock supply and biodiesel and glycerin demands in constraints (3), (4), and (7) as well as the cost and environmental coefficients in the objective functions are tainted by epistemic uncertainty. Due to the limited historical data, a possibility distribution is estimated for each uncertain parameter using experts' subjective opinions. In this paper, a new formulation of possibilistic programming approach integrating mean and absolute deviation of the uncertain objective function (OF) is used. Possibilistic absolute deviation of uncertain OF is considered as a risk measure. The possibilistic mean-absolute deviation model based on fuzzy numbers is as follows:

$$
\begin{aligned}
&\min z = \overline{M}\left(\tilde{c}x\right) + \gamma \left|\sigma\left(\tilde{c}x\right)\right| \\
&s.t.\ \ \tilde{a}_i x \geq \tilde{b}_i \quad i = 1, \ldots, l \\
&\tilde{a}_i x = \tilde{b}_i, \quad i = l+1, \ldots, m \\
&x \geq 0
\end{aligned}
\tag{25}
$$

In fact, there is a trade-off between possibilistic mean value and absolute deviation of objective function in the above formulation. $\gamma$ is a risk coefficient that allows the decision-maker to consider risk-averse aspects in the decision-making besides considering the average condition under uncertainty. In this model, the possibilistic mean operator is used to convert the possibilistic objective functions into their crisp ones. To do so, according to Carlsson and Fuller [11], the possibilistic mean of a triangular fuzzy number $\widetilde{c}$ with three prominent points $\widetilde{\mathrm{c}} = (c^p, c^m, c^o)$ is equal to the half point of its interval-valued mean:

$$\overline{M}\left(\tilde{c}\right) = \frac{M_1\left(\tilde{C}\right) + M_2\left(\tilde{C}\right)}{2} = \frac{\left(\frac{2}{3}c^m + \frac{1}{3}c^p\right) + \left(\frac{2}{3}c^m + \frac{1}{3}c^o\right)}{2} = \left(c^p + 4c^m + c^o/6\right)$$

and the possibilistic absolute deviation for the given triangular fuzzy number $\widetilde{\mathrm{c}}$ according to Zang and Zang [80] is equal to $\sigma\left(\widetilde{c}\right) = \frac{1}{3}\left(c^o - c^p\right)$. Following these definitions and by defining the relation of fuzzy preference $N\left(\widetilde{a};\widetilde{b}\right)$ based on possibilistic mean values of fuzzy numbers, possibilistic constraints are transformed to their crisp counterparts. For any pair of fuzzy numbers $\widetilde{a}$ and $\widetilde{b}$, the degree in which $\widetilde{a}$ is bigger than $\widetilde{b}$ is defined as follows [43]:

$$\mu_N\left(\tilde{a},\tilde{b}\right) = \begin{cases} 0 & if\ M_2^a - M_1^b < 0 \\ \frac{M_2^a - M_1^b}{M_2^a - M_1^b - \left(M_1^a - M_2^b\right)} & if\ 0\epsilon\left[M_1^a - M_2^b, M_2^a - M_1^b\right] \\ 1 & if M_1^a - M_2^b > 0 \end{cases} \tag{26}$$

where $\left[M_1^a, M_2^a\right]$ and $\left[M_1^b, M_2^b\right]$ are the mean intervals of fuzzy numbers $\widetilde{a}$ and $\widetilde{b}$, respectively. When $\mu_N\left(\widetilde{a},\widetilde{b}\right) \geq \alpha$, it is said that $\widetilde{a}$ is bigger than or equal to $\widetilde{b}$, at least at degree $\alpha$ and it is shown by$\widetilde{a}{\geq_\alpha}\widetilde{b}$. It is stated that $\widetilde{a}$ is equal $\widetilde{b}$ in degree $\alpha$ if the following inequalities hold simultaneously: $\frac{\alpha}{2} \leq \mu_N\left(\widetilde{a},\widetilde{b}\right) \leq 1 - \frac{\alpha}{2}$. By considering the above definitions, the crisp counterpart of model (25) is reformulated as below:

$$\text{Min} \quad Z = \left(c^p + 4c^m + c^o/6\right) + \gamma\frac{1}{3}\left(c^o - c^p\right)$$

$$\left[(1-\alpha)\left(\frac{2}{3}a^m + \frac{1}{3}a^o\right) + \alpha\left(\frac{2}{3}a^m + \frac{1}{3}a^p\right)\right]x$$

$$\geq \alpha\left(\frac{2}{3}b^m + \frac{1}{3}b^o\right) + (1-\alpha)\left(\frac{2}{3}b^m + \frac{1}{3}b^p\right), \quad i = 1,\ldots l$$

$$\left[(1-\alpha/2)\left(\frac{2}{3}a^m+\frac{1}{3}a^o\right)+\alpha/2\left(\frac{2}{3}a^m+\frac{1}{3}a^p\right)\right]x$$

$$\geq \frac{\alpha}{2\left(\frac{2}{3}b^m+\frac{1}{3}b^o\right)}+\left(1-\frac{\alpha}{2}\right)\left(\frac{2}{3}b^m+\frac{1}{3}b^p\right), \quad i=l+1,\ldots m$$

$$\left[(\alpha/2)\left(\frac{2}{3}a^m+\frac{1}{3}a^o\right)+\left(1-\frac{\alpha}{2}\right)\left(\frac{2}{3}a^m+\frac{1}{3}a^p\right)\right]x$$

$$\geq \left(1-\frac{\alpha}{2}\right)\left(\frac{2}{3}b^m+\frac{1}{3}b^o\right)+(\alpha/2)\left(\frac{2}{3}b^m+\frac{1}{3}b^p\right), \quad i=1,\ldots l$$

$$x \geq 0 \tag{27}$$

### *4.2 Optimization of Natural Gas Supply Chain Through a Greenhouse Gas Reduction Under Uncertainty*

Natural gas is the most environmentally friendly fossil fuel because it contains less carbon. Natural gas industry is one of the most important and costly industries in which development and planning of natural gas networks as multidisciplinary projects have crucial impact on the gas-rich countries. Thus, management of environmental aspects and the best approaches to achieving high environmental performance in natural gas production, processing, and transmission must be investigated. In this part, the simplified model proposed by Azadeh et al. [1] is presented as a sample model for sustainable logistics network design in gas industry in which a possibilistic programming model is used to convert the original model into its crisp equivalent. The gas supply chain network design includes five levels: two types of supplies are (a) the gas and oil wells to provide raw materials as the first type of supply and (b) importation of the final product as the second type of supply. Producers, i.e., the refineries, are at the second level of the supply chain. In the third level, there is one type of distributor that is called compressor stations. Storage is in the fourth level acting as inventory place. Finally, in the fifth level, three consumer groups are located which are (1) power plants, (2) injection, and (3) exportation. Relations among different components of this supply chain are shown with arrows in Fig. 4. There are different properties in the route parameter like length/diameter of pipelines and hardness coefficient of the existent relations among entities. The model has two different objectives, i.e., minimization of overall environmental effects and minimization of total costs. The following notations are used for model formulation.

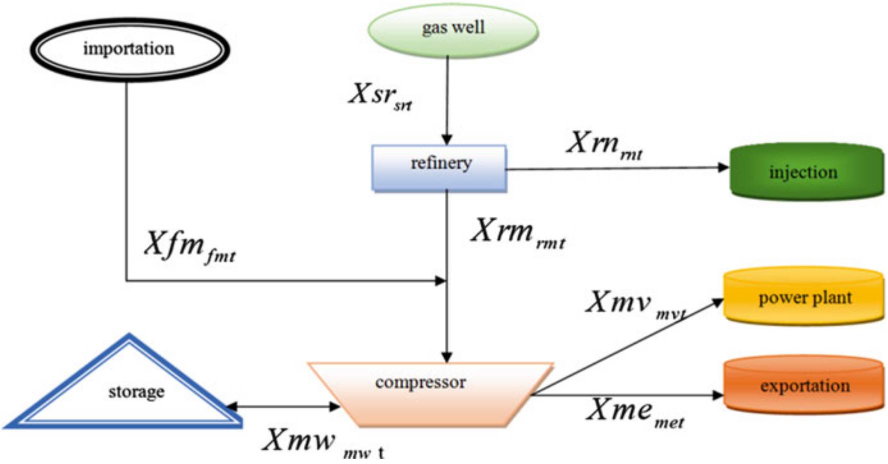


**Fig. 4** Natural gas network design

| | |
|---|---|
| $t$ | Time period $\tau - \{1, 2, \ldots \vert\tau\vert\}$ $t \epsilon \tau$ |
| $f, s$ | Suppliers set $\{s : Gas\ Well, f : importation\}$ |
| $r$ | Refineries set |
| $m$ | Compressor stations set |
| $n$ | Injection customer |
| $e$ | Exportation |
| $v$ | Power plant customer |
| $w$ | Storage tank set |
| $i$ | Starting nodes $i\epsilon\{s \cup f \cup r \cup m\}$ |
| $j$ | Finishing nodes $j\epsilon\{e \cup n \cup r \cup m \cup v\}$ |

### *Cost parameters*

| | |
|---|---|
| $\widetilde{cs}_{st}$ | Cost of supply by gas well per unit in period $t$ |
| $\widetilde{cf}_{ft}$ | Cost of supply by importation per unit in period $t$ |
| $\widetilde{cr}_{rt}$ | Cost of production by refinery per unit in period $t$ |
| $\widetilde{cm}_{mt}$ | Operation cost of compressor station per unit in period $t$ |
| $\widetilde{cw}_{wt}$ | Operation cost of storage tank per unit in period $t$ |
| $\widetilde{c}_{o}$ | Transportation cost per product unit per distance unit |
| $\widetilde{cg}$ | Social cost caused by per unit of greenhouse gas emission |
| $\widetilde{cl}_{m}$ | Fixed cost of establishing compressor |
| $\widetilde{cz}_{w}$ | Fixed cost of establishing storage |

### Capacity and demand parameters

| | |
|---|---|
| $\widetilde{sc}_{st}$ | Capacity of gas well in period t |
| $\widetilde{fc}_{ft}$ | Capacity of importation in period t |
| $\widetilde{rc}_{rt}$ | Capacity of refinery in period t |
| $\widetilde{mc}_{mt}$ | Capacity of compressor station in period t |
| $\widetilde{wc}_{w}$ | Capacity of storage tank |
| $\widetilde{nd}_{nt}, \widetilde{ed}_{et}, \widetilde{vd}_{vt}$ | Demand of each kind of customer in period t |

### Route parameters

| | |
|---|---|
| $l_{ij}$ | Length of the route between node I and node j |
| $o_{ij}$ | Hardness coefficient of the route between node I and node j |
| $\lambda_{ij}$ | $\left\{\begin{array}{ll} 1 & \textit{if there is a routebetween node i and node j} \\ 0 & \textit{otherwise} \end{array}\right\}$ |
| $Q_{ij}^{min}$ | Minimum flow rate between node I and node j |
| $Q_{ij}^{max}$ | Maximum flow rate between node I and node j |
| $\xi_r, \xi_m$ | Efficiency coefficient of refinery and compressor station, respectively |

### Greenhouse gas emissions parameters

| | |
|---|---|
| $gs$ | Average amount of greenhouse gas emissions produced by gas wells per unit |
| $gr$ | Average amount of greenhouse gas emissions produced by refineries per unit |
| $gm$ | Average amount of greenhouse gas emissions produced by compressor stations per unit |
| $gn$ | Average amount of greenhouse gas emissions produced by customers (injection) per unit |
| $gv$ | Average amount of greenhouse gas emissions produced by customers (power plant) per unit |

### Variables

| | |
|---|---|
| $xij_{ijt}$ | Amount of the gas transmitted from the *ith* level to the *jth* in period *t* for different starting and finishing nodes |
| $y_m$ | 1 if compressor station is opened at location *m* otherwise 0 |
| $L_w$ | 1if a storage tank is opened at location *w* otherwise 0 |

$$
\begin{aligned}
\text{Min } Z_1 = {} & \sum_m y_m \tilde{cl}_m + \sum_w L_w \tilde{cz}_w + \sum_s \sum_r \sum_t xsr_{srt} \left( \tilde{cs}_{st} + l_{sr}^{SR} o_{sr}^{SR} \tilde{c}_o \right) \\
& + \sum_s \sum_n \sum_t xsn_{snt} \left( \tilde{cs}_{st} + l_{sn}^{SN} o_{sn}^{SN} \tilde{c}_o \right) \\
& + \sum_r \sum_m \sum_t xrm_{rmt} \left( \tilde{cr}_{rt} + l_{rm}^{RM} o_{rm}^{RM} \tilde{c}_o \right) \\
& + \sum_f \sum_m \sum_t xfm_{fmt} \left( \tilde{cf}_{ft} + l_{fm}^{FM} o_{fm}^{FM} \tilde{c}_o \right) \\
& + \sum_r \sum_n \sum_t xrn_{rnt} \left( \tilde{cr}_{rt} + l_{rn}^{RN} o_{rn}^{RN} \tilde{c}_o \right) \\
& + \sum_m \sum_e \sum_t xme_{met} \left( \tilde{cm}_{mt} + l_{me}^{ME} o_{me}^{ME} \tilde{c}_o \right) \\
& + \sum_m \sum_v \sum_t xmv_{mvt} \left( \tilde{cm}_{mt} + l_{mv}^{MV} o_{mv}^{MV} \tilde{c}_o \right) \\
& + \sum_m \sum_w \sum_t xmw_{mwt} \left( \tilde{cm}_{mt} + l_{mw}^{MW} o_{mw}^{MW} \tilde{c}_o \right) \\
& + \sum_w \sum_m \sum_t xwm_{wmt} \left( \tilde{cm}_{mt} + l_{wm}^{WM} o_{wm}^{WM} \tilde{c}_o \right)
\end{aligned}
\tag{28}
$$

$$
\min Z_2 = \tilde{cg} \left\{
\begin{aligned}
& gs \left[ \sum_s \sum_r \sum_t xsr_{srt} \right] + gn \left[ \sum_s \sum_r \sum_t xsr_{srt} \right] \\
& + gr \left[ \sum_r \sum_m \sum_t xrm_{rmt} + \sum_r \sum_n \sum_t xrn_{rnt} \right] \\
& + m \left[ \begin{aligned} & \sum_m \sum_e \sum_t xme_{met} \\ & + \sum_m \sum_v \sum_t xmv_{mt} + \sum_m \sum_w \sum_t xmw_{mwt} \end{aligned} \right] \\
& + gv \sum_m \sum_v \sum_t xmv_{mt}
\end{aligned}
\right\}
\tag{29}
$$

$$
\sum_r xrn_{rnt} \geq \tilde{nd}_{ndt} \quad \forall n, t \tag{30}
$$

$$
\sum_m xme_{met} \geq \tilde{ed}_{dt} \quad \forall e, t \tag{31}
$$

$$
\sum_m xmv_{mvt} \geq \tilde{vd}_{vt} \quad \forall v, t \tag{32}
$$

$$\sum_{r} xsr_{srt} \leq \tilde{sc}_{st} \quad \forall s, t \tag{33}$$

$$\sum_{m} xfm_{fmt} \leq \tilde{fc}_{ft} \quad \forall f, t \tag{34}$$

$$\sum_{m} xrm_{rmt} + \sum_{n} xrn_{rnt} \leq \tilde{rc}_{rt} \quad \forall r, t \tag{35}$$

$$\sum_{w} xmw_{mwt} + \sum_{e} xme_{met} + \sum_{v} xmv_{mvt} \leq \tilde{mc}_{mt} y_m \quad \forall m, t \tag{36}$$

$$\sum_{m} \sum_{t'=1}^{t} xmw_{mwt'} - \sum_{m} \sum_{t'=1}^{t} xwm_{wmt'} \geq 0 \quad \forall w, t \tag{37}$$

$$\sum_{m} \sum_{t'=1}^{t} xmw_{mwt'} - \sum_{m} \sum_{t'=1}^{t} xwm_{wmt'} \geq \tilde{wc}_w L_w \quad \forall w, t \tag{38}$$

$$\xi_r \sum_{s} xsr_{srt} = \sum_{m} xrm_{rmt} + \sum_{n} xrn_{rnt} \quad \forall r, t \tag{39}$$

$$\xi_m \left( \sum_{m} xrm_{rmt} + \sum_{w} xwm_{wmt} + \sum_{f} xfm_{fmt} \right) = \sum_{w} xmw_{mwt} + \sum_{e} xme_{met} + \sum_{v} xmv_{mvt} \quad \forall m, t \tag{40}$$

$$xsr_{srt} \leq M\lambda_{sr}^{SR} \qquad \forall s, r, t \tag{41}$$

$$xrm_{rmt} \leq M\lambda_{rm}^{RM} y_m \forall r, m, t \tag{42}$$

$$xfm_{fmt} \leq M\lambda_{fm}^{FM} y_m \forall f, m \tag{43}$$

$$xrn_{rnt} \leq M\lambda_{rn}^{RN} \quad \forall r, n, t \tag{44}$$

$$\lambda_{sr}^{SR} Q_{sr}^{minSR} \leq xsr_{srt} \leq \lambda_{sr}^{SR} Q_{sr}^{maxSR} \quad \forall s, r \tag{45}$$

$$\lambda_{rm}^{RM} Q_{rm}^{minRM} \leq xrm_{rmt} \leq \lambda_{rm}^{RM} Q_{rm}^{maxRM} \quad \forall r, m \tag{46}$$

$$\lambda_{fm}^{FM} Q_{fm}^{minFM} \leq xfm_{fmt} \leq \lambda_{fm}^{FM} Q_{fm}^{maxFM} \quad \forall f, m \tag{47}$$

$$\lambda_{rn}^{RN} Q_{rn}^{minRN} \leq xrn_{rnt} \leq \lambda_{rn}^{RN} Q_{rn}^{maxRN} \quad \forall r, n \tag{48}$$

$$\lambda_{me}^{ME} Q_{me}^{minME} \leq xme_{met} \leq \lambda_{me}^{ME} Q_{me}^{maxME} \quad \forall m, e \tag{49}$$

$$\lambda_{mv}^{MV} Q_{mv}^{minMV} \leq xmv_{mvt} \leq \lambda_{mv}^{MV} Q_{mv}^{maxMV} \quad \forall m, v \tag{50}$$

$$\lambda_{mw}^{Mw} Q_{mw}^{minMw} \leq xmw_{mwt} \leq \lambda_{mw}^{Mw} Q_{mw}^{maxMw} \quad \forall m, w \tag{51}$$

$$\lambda_{wm}^{WM} Q_{wm}^{minWM} \leq xwm_{wmt} \leq \lambda_{wm}^{WM} Q_{wm}^{maxWM} \quad \forall w, m \tag{52}$$

$$xij_{ijt} \geq 0, y_m, L_w \in [0, 1] \tag{53}$$

The objective function (28) minimizes the total opening costs, supplying and transmission to the next level. The second objective function (Equation (29) is also related to minimizing the costs of emission of greenhouse gases across the supply chain. Constraints (30)–(32) show the demands of each customer shall be fulfilled. Constraints (33)–(38) guarantee that each supplier must have an output flow equal or less than its capacity. Constraints (39) and (40) show the balance flow in the refinery centers and compressor stations according to their efficiency coefficients. Constraints (41) and (44) explain the presence or absence of a path. If the parameter $\lambda$ accepts a value of 1, the corresponding decision variable can take a value of 1, otherwise it is zero. Constraints (45)–(52) ensure the upper and lower bounds of the gas flow which are determined according to the pipeline diameter and gas pressure.

Jimenez et al. (2007) method is used to deal with uncertain coefficients in the objective functions and constraints. This method is based on the definition of the "expected interval (EI)" and the "expected value (EV)" of fuzzy numbers. Assume that $\tilde{c}$ is a triangular fuzzy number in which $c^{mos}, c^{pes}, c^{opt}$ are the three prominent points (i.e., the most likely, the most pessimistic, and the most optimistic values), respectively. Then, the membership function EI and EV of fuzzy number $\tilde{c}$ can be defined as the following equations:

$$\mu_{\tilde{c}}(x) = \begin{cases} f_c(x) = \dfrac{x - c^{pes}}{c^{mos} - c^{pes}} & if\ c^{pes} \leq x \leq c^{mos} \\ 1 & if\ x = c^{mos} \\ g_c(x) = \dfrac{c^{opt} - x}{c^{opt} - c^{mos}} & if\ c^{mos} \leq x \leq c^{opt} \\ 0 & if\ x \leq c^{pes}\ or\ x \geq c^{opt} \end{cases} \tag{54}$$

$$\begin{aligned} EI\left(\tilde{c}\right) &= \left[E_1^c, E_2^c\right] = \left[\int_0^1 f_c^1(x)d(x), \int_0^1 g_c^1(x)d(x)\right] \\ &= \left[\frac{1}{2}\left(c^{pes} + c^{mos}\right), \frac{1}{2}\left(c^{mos} + c^{opt}\right)\right] \end{aligned} \tag{55}$$

$$EV\left(\tilde{c}\right) = \frac{E_1^c + E_2^c}{2} = \frac{c^{pes} + 2c^{mos} + c^{opt}}{4} \tag{56}$$

As it was mentioned in the previous model, when $\mu_M\left(\tilde{a}, \tilde{b}\right) \geq \alpha$ it is represented as $\tilde{a} \geq_\alpha \tilde{b}$. Now, consider the following fuzzy mathematical programming model in which all parameters are defined as triangular or trapezoidal fuzzy numbers.

$$\begin{gathered} \min z = \tilde{c}^t x \\ s.t. \\ \tilde{a}_i x \geq \tilde{b}_i \ \ i = 1, \ldots, l \\ x \geq 0 \end{gathered} \tag{57}$$

Also, a decision vector $x \in R^n$ is feasible in degree of $\alpha$ ifmin $\left\{\mu_N\left(\tilde{a}_i x, \tilde{b}_i\right) = \alpha\right\}$. According to (26), equation $\tilde{a}_i x \geq \tilde{b}_i$ is equivalent to the following equation:

$$\frac{M_2^a - M_1^b}{M_2^a - M_1^b - \left(M_1^a - M_2^b\right)} \geq \alpha \quad i = 1, \ldots, l \tag{58}$$

Finally, by the aid of the definition of expected interval and expected value of a fuzzy number, the equivalent crisp $\alpha-$parametric model of the model (58) can be written as follows:

$$\begin{gathered} \min EV\left(\tilde{c}\right) x \\ s.t. \\ \left[(1-\alpha) M_2^{a_i} + \alpha M_1^{a_i}\right] x \geq \alpha M_2^{b_i} + (1-\alpha) M_1^{b_i} \ i = 1, \ldots, l \\ x \geq 0 \end{gathered} \tag{59}$$

By considering the above mentioned definition and due to the prolongation of contents, only a part of the first objective function (Equation 28) and two constraints (Equations 30 and 33) of the defuzzied model have been presented below. Other equations are similar to these presented ones:

$$\begin{aligned} & minZ_1 \\ & = \sum_m y_m \left(\frac{cl_m^1+2cl_m^2+cl_m^3}{4}\right) \\ & + \sum_w L_w \left(\frac{cz_w^1+2cz_w^2+cz_w^3}{4}\right) \sum_s \sum_r \sum_t xsr_{srt} \left(\frac{cs_{st}^1+2cs_{st}^2+cs_{st}^3}{4}\right. \\ & \left. + l_{sr}^{SR} o_{sr}^{SR} \frac{c_o^1+2c_o^2+c_o^3}{4}\right) + \ldots \end{aligned}$$

$$\sum_r xrn_{rnt} \geq \beta\left(\frac{nd_{nt}^2+nd_{nt}^3}{2}\right)+(1-\beta)\left(\frac{nd_{nt}^1+nd_{nt}^2}{2}\right) \forall n, t$$

$$\sum_r xsr_{srt} \leq \beta\left(\frac{sc_{st}^1+sc_{st}^2}{2}\right)+(1-\beta)\left(\frac{sc_{st}^2+sc_{st}^3}{2}\right) \forall s, t \tag{60}$$

## 5 Case Study

In this section, a sustainable supply chain network design case study is reviewed which has already been presented in Pishvaee et al. [55]. The case study is related to an Iranian single-use medical needle and syringe (SMNS) manufacture named Avapezeshk (AVAP) which has itself about 70% market share (www.avapezeshk.com). SMNS as one of the key products of health has particular role in the health system. The World Health Organization reported that 16 billion injections that were performed while reusing unsterilized needles and syringes led to 8–16 million hepatitis B, 2.3–4.7 million hepatitis C, and 80,000–160,000 human immunodeficiency virus (HIV) infections around the globe. The main theme of corporate vision is sustainability, because SMNS has significant environmental effects especially in the end-of-life (EOL) phase and infected SMNS affects severely in health and social-political stability of any country. To do this, the corporate strategic committee considered three major aspects, (1) profits, (2) people, and (3) planet, which are very similar to the sustainability aspects. As a result, the company's supply chain network should be redesigned to satisfy new demands while accounting for Env and Soc issues alongside the Eco objectives. As it was mentioned before, the Env impact of other phases of the SMNS life cycle should not be neglected in the concerned problem, thus reverse supply chain should also be considered in the company's supply chain network redesign problem. There are three methods for managing the EOL phase of SMNS: (1) incineration methods at cement incinerator; (2) non-incineration methods, such as

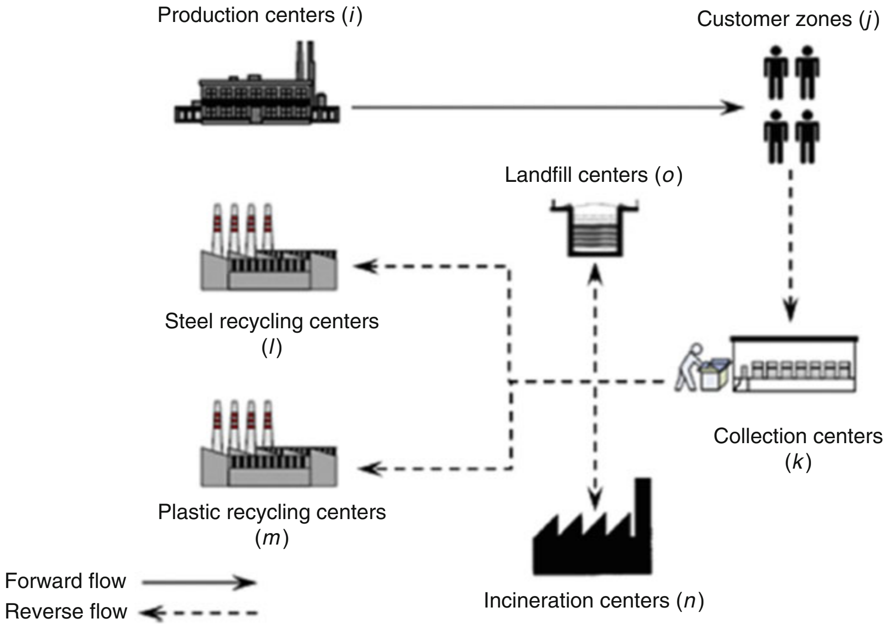


**Fig. 5** The concerned forward-reverse supply chain network [55]

safe landfill; and (3) recycling at plastic and steel recycling centers. The case study supply chain is illustrated in Fig. 5. This company has one production plant with capacity of producing about 600 million products per year in which new products are transported to the 23 customer zones including 3 foreign and 20 domestic customers in the forward network without any shortage, and after being used, the EOL SMNS (a predefined percent of customer's demand) are transported to the collection centers by the reverse flow and then shipped to incineration, landfill, or recycling centers.

For implementing the studied case, the company considered seven other candidate locations for establishing new plants in addition to one active plant. It is significant that there are four production technologies that have important effect on Soc, Env, and Eco performance. So, four types of production technologies and two capacity levels of facilities are also considered as output decisions. In the reverse flow, 11 candidate locations are available for establishing collection centers. Five joint locations also considered among the candidate ones for production centers and collection centers that may result in cost saving and also different impacts on Env and Soc performances when they are established in the same locations. Furthermore, seven steel and five plastic recycling centers and five incineration centers and eight safe landfill locations are also available for handling used products.

The model determines the number, location, and capacity of required production centers and collection centers, the best production technologies for production centers and the best EOL options as well as the aggregated material flow quantities between network facilities with the following three conflicting objectives covering the three aspects of sustainability:

- Minimizing the total cost = fixed opening costs + variable transportation and processing costs - saving from integrating facilities
- Minimizing the Env impact = damage to human health + damage to ecosystem + damage to resources
- Maximizing the SR = created job opportunities - consumer risk - damage to workers health + value of local development

The dynamic nature of supply chains leads to considerable fluctuations in input parameters, which impose a high degree of uncertainty to SCND problems in the long-term horizon. Here, the uncertain parameters due to lack of knowledge about their exact values are presented by imprecise values presented in the form of fuzzy numbers and a possibilistic programming approach with the minimum confidence level of 0.9 for possibilistic chance constraints (i.e., the credibility-based possibilistic programming approach) is used to handle these uncertain parameters in the model. GLCAP and Recipe life cycle assessment methods are incorporated in the model to estimate the relevant social and environmental impacts. In order to deal with multiple conflicting objectives of the proposed model, a posteriori fuzzy solution approach [22, 38] is used in which the range 0.8–1 is assigned to importance weight of economic objective and the range of 0–0.15 is also dedicated to other two objective functions (i.e., Env and Soc OFs).

Solving the discussed model using the aforementioned methods, one can see that the three OFs are in conflict with each other. According to such confliction between OFs, it can be concluded that when the value of protection price (i.e., the solution obtained when the model is solved only according to the cost OF) increases, more desirable value of Env and Soc protection can be achieved. Also, the cost-based objective function has a tendency toward designing a centralized network with less total cost and expensive production technologies while the environmental-based objective function offers a more decentralized network and more environmentally friendly production technologies.

Additionally, the performance of the proposed model under various minimum acceptable confidence level of chance constraints shows that when α-level value increases (in response to uncertainty with higher confidence level), it will lead to increase in values of three objective functions because more resources must be used to satisfy the chance constraints. Since the model is strongly NP-hard, an accelerated Benders decomposition algorithm utilizing three efficient acceleration methods is designed which can achieve the exact optimal solution in a reasonable time.

## 6 Future Research Direction

Given the current state-of-the-art literature in sustainable logistics network design under uncertainty, there are various avenues for further research as follows:

- Despite the fact that the corporate social responsibility is one of the main aspects of the sustainability, investigation on it is so limited in the current literature. Therefore, in order to move toward sustainable supply chain networks, it is necessary to include the social aspects beside the environmental and economic dimensions in the developed models.
- Integrating tactical and operational planning issues into the current strategic models to broaden the scope of developed models could be another interesting research direction with significant practical relevance.
- Considering the uncertainty in input parameters (i.e., operational risks) as well as uncertainty in supply chain network (i.e., disruption risks) at the same time.
- Accounting for multiple types of uncertainties in input parameters (e.g., stochastic and fuzzy data) and developing mixed uncertainty programming approaches (e.g., mixed possibilistic-stochastic programming) to cope with these uncertainties.
- Since most of real-life problems are large, and the exact methods can solve only small- to moderate-sized problem instances, devising tailored solution approaches including heuristics, metaheuristics, or matheuristics (the interoperation of metaheuristics and mathematical programming techniques) would be of particular interest.

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# Methodological Approaches to Reliable and Green Intermodal Transportation

**Emrah Demir, Martin Hrušovský, Werner Jammernegg, and Tom Van Woensel**

**Abstract** A combination of transportation modes offers environmentally friendly alternatives to transport high volumes of freight over long distances. In order to reflect the advantages of each transportation mode, it is the challenge to deal with data uncertainty during the transportation planning phase. This chapter investigates the alternative ways of modeling the uncertainty by discussing them and their characteristics in terms of solution times, the quality, and the limitations. Moreover, several real-life case studies are provided to demonstrate potential environmental benefits by considering the principles of green logistics for single-mode and intermodal transportation.

## 1 Introduction

The growing demand leads to increased transportation volumes on the limited transportation networks which leads to delays and disruptions due to unexpected events. This is particularly crucial for road transportation which has been traditionally the most preferred transportation mode and still has the major share on the modal split in Europe [43]. Moreover, road transportation is one of the main contributors to carbon dioxide equivalent ($CO_2e$) emissions from transportation that are responsible for severe impacts on climate [20]. Therefore, logistics companies are looking for

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**Table 1** Acronyms and abbreviations used in the chapter

| Notation | Description |
|---|---|
| $CO_2e$ | Carbon dioxide equivalent |
| GHGs | Greenhouse gases |
| GISND | Green intermodal service network design problem |
| SAA | Sample average approximation |
| TEU | Twenty-foot equivalent unit |
| TMS | Transport management system |
| VRP | Vehicle routing problem |

alternative transportation solutions that would minimize negative impacts of their transportation activities but still offer competitive solutions in a highly saturated market (Table 1).

One of the alternatives is intermodal freight transportation, a specialization of multimodal transportation which consecutively uses multiple transportation modes moving the goods in the same standardized loading unit (e.g., container) [17]. In addition to flexibility offered by multimodal transportation, intermodal transportation offers numerous advantages for shippers with large volumes, such as standard sizes, faster transshipments, and reduced packaging expenses [45]. However, the combination of different transportation modes requires more coordination and accurate transportation planning. Since most of the intermodal services are running according to fixed schedules, the reliability of the transportation plans is an important issue in order to avoid delays and enable on-time delivery of the goods. In this context, improved collection of real-time traffic flow information over the last decade builds the data basis for reliability of transportation plans.

While transportation literature offers extensive methods for (unscheduled) road transportation (see, e.g.,[31, 19]), these approaches are only of limited use for planning transportation activities in intermodal transportation networks, where services such as train, vessel, or flight connections follow a fixed schedule. In such cases, service network design (SND) provides promising alternatives for the reproduction of transportation flows on more than one mode. SND problems deal with the selection of available services for specific transports by offering advantages for the consolidation of transports as well as the consideration of multiple modes. Moreover, it provides methodological possibilities which enable the representation of transshipment as well as the consolidation of containers.

The research on dynamic SND problems is still in its early days, though, which leads to a lack of applications to as well as the development of new methods for service network environments. Most of the limited publications in this domain are dealing with demand uncertainty (e.g., [32, 10]), while only a minority takes travel time uncertainties into account. Input from practitioners, though, suggests that travel time uncertainty is an important source of variability to consider when trying to make accurate transportation plans.

The aim of this chapter is to give an overview about possible approaches for modeling intermodal freight transportation planning under uncertainty. For this, different approaches are at first described and then compared using a case study in order to show their advantages and weaknesses. In order to discuss different aspects of transportation activities, multiple objectives (e.g., costs, time, emissions) are considered in the models which provide managerial insights on interaction between economic and ecological objectives in transportation planning and on the benefits of using alternative transportation modes in comparison to road transportation. In this way the ecological footprint of transportation operations can be improved which contributes to achieving the objectives of the green logistics concept.

The remainder of the chapter is organized as follows. Section 2 introduces the green intermodal service network design problem and discusses important points for considering $CO_2e$ emissions in transportation planning. Section 3 describes alternative ways of uncertainty modeling. Section 4 presents case studies to highlight and compare the importance of methodological approaches to intermodal transportation. Conclusions are stated in Sect. 5.

## 2 The Green Intermodal Service Network Design Problem

The intermodal transportation chain consists of a number of transportation services served by different transportation modes that connect intermodal terminals where transshipment has to be handled. These services need to be coordinated in order to ensure smooth flow of freight in containers through the network from their origin to the destination within time windows specified by the customer. Typically, there exist various alternative routes within the network between the planned origin and destination of a container, and the aim is to find the optimal route that fulfills the criteria set by the decision maker. In this respect, the most important criteria for transportation mode choice are not only transportation costs but also safety, flexibility, and reliability, as shown in a survey performed by [48]. Since customers want to have the goods delivered on time and avoid delays which can cause additional costs or production stoppages, reliability of the system is becoming more crucial. Therefore, it is necessary to include the uncertainties in travel times caused by delays and disruptions of transportation network into the planning algorithms. In this way the created transportation plan becomes robust since it can stay feasible even if a disruption occurs, and therefore goods can be delivered on time [22].

High robustness of transportation plans is of special importance in case of intermodal transportation planning where several vehicles of different transportation modes are connected in one transportation chain. Thereby, every mode has its special characteristics that need to be considered. Whereas some services in intermodal transportation networks (e.g., rail, inland waterway) have fixed departure times according to planned schedules, other services (mostly road) are usually more flexible as they do not have fixed time slots when they can use the available infrastructure. This feature further increases the complexity of the intermodal transportation problem since the fixed departure times have to be considered when coordinating the individual services in a transportation plan. Whereas schedules can be easily incorporated into planning if only deterministic travel times under ideal conditions are considered, they might lead to disruptions of the network when delays occur and the goods are delivered to the terminal only after the next planned service has already left. In that case the goods have to wait for the next train or vessel which might result in a delay of hours or even days depending on the frequency of the connection. Alternatively, a new plan has to be found which might result in higher costs, time, or emissions. Therefore, it is necessary to include buffer times into transportation planning to avoid such situations.

Buffer times should not be too long since this would make the total transportation time longer and therefore further decrease the competitiveness of intermodal transportation in comparison to direct single-mode transportation which does not require any transshipment operations on the route. The length of the buffer times is dependent on the type and frequency of disruptions occurring on a certain route which can be derived from historical data as well as from actual real-life data about the current transportation network state and represented in form of travel time probability distribution as shown in Sect. 3. In reality, the same delay can lead to different results for different connections: e.g., a delay of 30 min can be critical for a truck operating in a just-in-time environment, whereas the same delay might not have any influence in case of an inland waterway vessel sailing 3 days between origin and destination. Besides that, the type of disruption also determines the impact of a certain disruption on the transportation: whereas a truck can usually take a detour if an accident happens on a highway, in case of a low water level on a certain river section, the vessel either has to wait for a couple of days, or goods have to be transshipped to another transportation mode [47]. If all these factors are included in the transportation planning process, the robustness of the resulting plan can be increased so that most of the disruptions are covered by the included buffer time.

A transportation planning approach which enables the inclusion of uncertainty and includes multiple objectives was first studied by [15] and followed by [24]. In their paper, the authors present a mixed-integer linear program for optimizing the transportation plan within an intermodal network considering uncertain travel times and demands. The approach chosen is called service network design (SND) in which each transportation link between two terminals is modeled as a service characterized by its origin, destination, capacity, route, departure time, planned travel time, transportation costs and emissions as well as the vehicle used for this service. This approach is also the basis for this chapter, where we investigate the transportation plan based on three different objectives which can have different weights according to the user's preferences. The objectives are transportation costs, time in form of costs for in-transit inventory and penalty costs for late deliveries at the final customer, and $CO_2e$ emissions expressed as emission costs. Since a survey by [13] has shown that this combination of objectives and especially consideration of $CO_2e$ is not usual in the current transportation management systems (TMS) responsible for planning operations, Sect. 2.1 shortly discusses requirements and possible problems for modeling emissions. After that, the mathematical model used for intermodal transportation planning is presented in Sect. 2.2.

## 2.1 Air Pollution and GHGs

The environmental impact of transportation can be measured in the form of $CO_2$e emissions which have to be calculated accurately. Using accurate calculation methods and quantifying the emissions might help to identify possibilities for their reductions which together with a proper implementation of green logistics bring more advantages than disadvantages for the logistics service providers or freight forwarders. Therefore, there is an increasing need to highlight these advantages to transportation companies.

Greenhouse gases (GHGs) are the most studied negative externality of freight transportation. These gases cause atmospheric changes and climate disruptions which are harmful to the natural and built environments and pose health risks. The primary transportation-related man-made GHGs in the Earth's atmosphere are carbon dioxide ($CO_2$), methane ($CH_4$), nitrous oxide ($N_2O$), and ozone ($O_3$). As $CO_2$ is the dominant man-made GHG, the impacts of other gases can also be calculated based on carbon dioxide equivalent ($CO_2$e) emissions [27, 6].

Despite the fact that transportation sector is one of the biggest contributors of $CO_2$e emissions, a survey performed by [13] showed that calculation of emissions is only slowly becoming part of TMS. Even when emissions are taken into account in TMS, they are only reported as an additional factor for the resulting routes, and they are not used as an optimization objective. Usually only costs are taken into account for optimization, and in case of multiple objectives, costs are combined with service, distance, time, etc. This development might be caused by multiple reasons which make the calculation of emissions challenging.

Firstly, the amount of emissions is dependent on the energy needed for moving the vehicle coming either from diesel or electricity consumption. Although the energy consumption can be easily measured after the transportation has been conducted, calculation of energy consumption before the start of the transportation is problematic as it is dependent on a number of factors which are not always known. These factors include the characteristics of the vehicle (e.g., weight, air and rolling resistance, engine), route and driving characteristics (e.g., gradient, speed, number of stops, driving behavior), and the amount of goods transported [5, 18, 12]. In order to be able to estimate the emissions, a number of different models requiring detailed inputs have been developed as shown by [12] and [14]. Besides these detailed microscopic models, emission calculators based on real-world measurements and recommended values for a typical vehicle are also available (e.g.,[9, 18, 26]). However, each of these models and calculators is based on certain assumptions which lead to discrepancies between calculated and measured emissions.

Secondly, the scope of emissions has to be determined in order to know which emissions to consider for calculation. According to the GHG protocol, emissions can be divided into three scopes: emissions from resources owned by a company, e.g., emissions from production (Scope 1), indirect emissions from purchased energy (Scope 2), and all other emissions including also other stages of supply chain, e.g., suppliers, transport, and distribution (Scope 3) [46]. Similarly, the emissions from transportation activities can either be calculated as emissions from fuel consumption directly in the vehicle (tank-to-wheel, TTW) or can also include emissions from production of the fuel (well-to-wheel, WTW). Inclusion of emissions from fuel production is especially important in cases where electric vehicles are involved since emissions from electricity consumption are equal to zero [30].

Thirdly, the monetary value of $CO_2$e emissions is unclear. Since the long-term effects of emissions on climate change and the amount of released emissions cannot be easily predicted, the estimation of emission costs is again based on a number of assumptions including different discount rates for future events and risk attitude of the decision makers. As a result, the so-called social costs of carbon emissions are estimated to be between 0 EUR and more than 700 EUR per ton of emissions depending on the model [3, 37, 23]. In the analysis of [16], there are also differences in emission costs ranging between 5 EUR and 135 EUR. Therefore the monetary value of emissions cannot be easily compared to transportation costs.

In the calculation methodology used for the model, the emissions were calculated per TEU transported by a certain service and then converted into emission costs. As the estimation of emission costs is difficult, a fixed price of 70 EUR/ton of $CO_2$e emissions was used for calculations as recommended by the German Federal Environment Agency [39]. Besides that, additional assumptions had to be made regarding the average utilization of the vehicles since the emission functions for trains and vessels are nonlinear. As a result, the utilization was assumed to be 80% for trains [39] and 90% for inland vessels [50]. Despite these additional assumptions, the results show the influence of emissions on the optimal routing decisions.

## 2.2 Mathematical Model

This part of the chapter provides a linear mixed-integer mathematical formulation of the green intermodal service network design problem (GISND). The presented model can be used to find optimal transportation plans under deterministic conditions, i.e., in situations where no uncertainty is considered. The possibilities for including uncertainty into the model are discussed in Sect. 3. The aim here is to find an optimal plan for orders $p \in \mathscr{P}$ defined by their demand $d^p$, origin $i$, and destination $j$ nodes as well as earliest release $\Gamma^p_{\text{release}}$ and due time $\Gamma^p_{\text{duetime}}$. Moreover, $\gamma^p(i,j) = \{(p \in \mathscr{P})| i \in \mathscr{N} \text{ and } j \in \mathscr{N}\}$ is a set of orders with origin $i$ and destination node $j$. The orders can be routed in a transportation network consisting of services $s \in \mathscr{S}$ (scheduled transports) and nodes $i, j \in \mathscr{N}$ (transshipment locations). Each service, since it is connected to a schedule and vehicle, is unique and connects transshipment locations $i$ and $j$. Therefore, $\delta^s(i,j,v,D^s_m) = \{(s \in \mathscr{S})| i \in \mathscr{N} \text{ and } j \in \mathscr{N} \text{ and } v \in \mathscr{V}\}$ is a set of services executed by vehicle $v$

**Table 2** Sets and parameters used in the model

| Notation | Definition |
|---|---|
| $\mathscr{N}$ | Set of all transshipment locations |
| $\mathscr{N}^+$ | Set of start terminals of transportation orders |
| $\mathscr{N}^-$ | Set of end terminals of transportation orders |
| $\mathscr{P}$ | Set of transportation orders |
| $\mathscr{S}$ | Set of transportation services |
| $\Gamma^p_{\text{release}}$ | Earliest release time of order $p$ |
| $\Gamma^p_{\text{duetime}}$ | Due time of order $p$ |
| $c_j$ | Transshipment costs per container in terminal $j$ |
| $c^s$ | Transportation costs of a service $s$ |
| $c_{\text{emi}}$ | Emissions-related costs per kg of $CO_2$e emissions |
| $c^p_{\text{pen}}$ | Penalty costs in case of late delivery of goods |
| $c^p_t$ | In-transit inventory costs per hour for order $p$ |
| $\text{cap}^s$ | Free capacities of services $s$ |
| $d^p$ | Demand (in containers) of order $p$ |
| $e_j$ | Emissions in kg per transshipment of container in terminal $j$ |
| $e^s$ | Emissions in kg per transportation of container on service $s$ |
| $L$ | Large (enough) number |
| $t_j$ | Separate loading and unloading time at terminal $j$ |
| $t^s$ | Transportation time of service $s$ |
| $T^s_{\text{min}}$ | Start of the departure time window for service s |
| $T^s_{\text{max}}$ | End of the departure time window for service s |
| $\omega_i$ | Weight for the objective $i$ |

between origin $i$ and destination node $j$ within the starting time window bounded by $T^s_{\text{min}}$ and $T^s_{\text{max}}$. In addition to that, services are characterized by their scheduled departure time $D^s$ and service travel time $t^s$ as well as service slot price $c^s$ and $CO_2$e emissions per container $e^s$. Services on the road as well as transshipment are assumed to be available when needed. We first present sets, parameters, and decision variables and then provide the mathematical formulation of the model. This model extends the model introduced by [15] by adding in-transit inventory costs to the original time-related cost component of the objective.

We now provide the sets, parameters, and decision variables used for the formulation of the mathematical model in Tables 2 and 3.

$$\text{Minimize} \quad \omega_1 \sum_{p\in\mathscr{P}} \sum_{s\in\mathscr{S}} x^{sp} c^s + \omega_1 \sum_{j\in\mathscr{N}} n_j c_j + \omega_2 \sum_{p\in\mathscr{P}} c^p_t (AD^p - \Gamma^p_{\text{release}}) +$$
$$\omega_2 \sum_{p\in\mathscr{P}} \sum_{s\in\mathscr{S}} d^p_{\text{delay}} c^p_{\text{pen}} + \omega_3 c_{\text{emi}} \sum_{p\in\mathscr{P}} \sum_{s\in\mathscr{S}} x^{sp} e^s + \omega_3 \sum_{j\in\mathscr{N}} n_j e_j \quad (1)$$

**Table 3** Decision variables used in the model

| Notation | Definition |
|---|---|
| $d^p_{\text{delay}}$ | Delay of order $p$ at destination node $j$ |
| $A^s$ | Arrival time of service $s$ at the associated destination node $j$ |
| $AD^p$ | Arrival time of order $p$ to its destination |
| $D^s$ | Departure time of service $s$ at the associated departure node $i$ |
| $\text{Delay}^{\text{qrp}}$ | Delay between preceding service $q$ and succeeding service $r$ of order $p$ |
| $l^{qr}$ | A binary variable equal to 0 if transshipment is necessary between preceding services $q$ and succeeding service $r$, 1 otherwise |
| $n_j$ | The number of containers transshipped at terminal $j$ |
| $y^s, y^{sp}$ | A binary variable equal to 1 if service $s$ is used (for order $p$) |
| $x^{sp}$ | The number of containers of order $p$ carried via service $s$ |
| $z^{\text{qrp}}$ | The number of containers of order $p$ that have to be transshipped between preceding services $q$ and succeeding service $r$ |

Subject to:

$$\sum_{s\in\delta(s\in\mathscr{S}|n=i)} x^{sp} = d^p \qquad \forall n\in\mathscr{N}|n=i, p\in\mathscr{P} \tag{2}$$

$$\sum_{s\in\delta(s\in\mathscr{S}|n=j)} x^{sp} = d^p \qquad \forall n\in\mathscr{N}|n=j, p\in\mathscr{P} \tag{3}$$

$$\sum_{s\in\delta(s\in\mathscr{S}|n=i)} x^{sp} - \sum_{s\in\delta(s\in\mathscr{S}|n=j)} x^{sp} = 0 \quad \forall n\in\mathscr{N}|(n\neq i,j), p\in\mathscr{P} \tag{4}$$

$$\sum_{p\in\gamma(p\in\mathscr{P})} x^{sp} - y^s\text{cap}^s \leq 0 \qquad \forall s\in\delta(s\in\mathscr{S}) \tag{5}$$

$$x^{sp} \leq y^{sp}L \qquad \forall s\in\delta(s\in\mathscr{S}), p\in\gamma(p\in\mathscr{P}) \tag{6}$$

$$x^{sp} \geq y^{sp} \qquad \forall s\in\delta(s\in\mathscr{S}), p\in\gamma(p\in\mathscr{P}) \tag{7}$$

$$y^s \leq \sum_{p\in\gamma(p\in\mathscr{P})} x^{sp} \qquad \forall s\in\delta(s\in\mathscr{S}) \tag{8}$$

$$\sum_{p\in\mathscr{P}} \sum_{\substack{s\in \\ \delta(s\in\mathscr{S}|i=n|j=n)}} x^{sp} - 2\sum_{p\in\mathscr{P}} \sum_{\substack{q\in \\ \delta(q\in\mathscr{S}|i=n)}} \sum_{\substack{r\in \\ \delta(r\in\mathscr{S}|j=n)}} z^{\text{qrp}} = n_n \quad \forall n\in\mathscr{N} \tag{9}$$

$$D^s + t^s - A^s \leq L(1-y^s) \qquad \forall s\in\delta(s\in\mathscr{S}) \tag{10}$$

$$A^q + t_j x^{qp} + t_j x^{rp} - 2t_j z^{\text{qrp}} - \text{delay}^{\text{qrp}} - D^r - L(1-y^{qp}) \leq L(1-y^{rp})$$
$$\forall q\in\delta(s\in\mathscr{S}|j\in\mathscr{N}), p\in\mathscr{P}, r\in\delta(s\in\mathscr{S}|i\in\mathscr{N}) \tag{11}$$

$$z^{\mathrm{qrp}} \leq L l^{qr} \qquad \forall q \in \delta(s \in \mathscr{S} | j \in \mathscr{N}), p \in \mathscr{P}, r \in \delta(s \in \mathscr{S} | i \in \mathscr{N}) \tag{12}$$

$$D^s - y^{sp} \Gamma^p_{\mathrm{release}} \geq 0 \qquad \forall p \in \mathscr{P}, s \in \delta(s \in \mathscr{S} | i \in \mathscr{N}^+) \tag{13}$$

$$A^s - a^p_{\mathrm{delay}} - \Gamma^p_{\mathrm{duetime}} \leq L(1 - y^{sp}) \qquad \forall p \in \mathscr{P}, s \in \delta(s \in \mathscr{S} | i \in \mathscr{N}^-) \tag{14}$$

$$AD^p \leq A^s - L(1 - y^{sp}) \qquad \forall p \in \mathscr{P}, s \in \delta(s \in \mathscr{S} | i \in \mathscr{N}^-) \tag{15}$$

$$T^s_{\mathrm{min}} y^s \leq D^s \leq T^s_{\mathrm{max}} y^s \qquad \forall s \in \mathscr{S} \tag{16}$$

$$\mathrm{delay}^{\mathrm{qrp}} \leq y^{qp} L \qquad \forall q \in \mathscr{S}, r \in \mathscr{S}, p \in \mathscr{P} \tag{17}$$

$$\mathrm{delay}^{\mathrm{qrp}} \leq y^{rp} L \qquad \forall q \in \mathscr{S}, r \in \mathscr{S}, p \in \mathscr{P} \tag{18}$$

$$y^s, y^{sp} = \{0, 1\} \qquad \forall s \in \mathscr{S}, q \in \mathscr{S}, r \in \mathscr{S}, p \in \mathscr{P} \tag{19}$$

$$a^p_{\mathrm{delay}}, x^{sp}, z^{\mathrm{qrp}}, \mathrm{delay}^{\mathrm{qrp}}, D^s, A^s, AD^p \geq 0 \quad \forall s \in \mathscr{S}, q \in \mathscr{S}, r \in \mathscr{S}, p \in \mathscr{P}. \tag{20}$$

The objective function (1) of the mathematical model minimizes a weighted sum of the total costs. The weights enable the reflection of individual preferences regarding direct transportation ($\omega_1$), time-related ($\omega_2$), and $CO_2$e emissions-related ($\omega_3$) costs. The direct transportation costs consist of transportation costs per container and service $c^s$, which include the fixed transportation costs per service allocated to one container as well as the direct transportation costs per container and transshipment costs per container ($c_j$). The time-related costs ($c_t^p$) are represented by in-transit inventory costs for the total time spent since the release of goods at the origin until the arrival of the order to the destination. In addition to that, charges for delayed deliveries ($c^p_{\mathrm{pen}}$) are also included in time-related costs. As the third objective, the $CO_2$e emissions-related costs per kg ($c_{\mathrm{emi}}$) for the emissions consumed per container serviced ($e^s$) and transshipped ($e_j$) are also included.

Constraints (2), (3), and (4) handle the movement of containers. While constraints (2) and (3) focus on the origin and destination nodes, constraint (4) manages the transshipment. Demand, in that regard, is positive if more containers are planned to originate from a specific node than are destined for that node. Constraint (5) ensures that capacity limits of services are adhered to. Constraints (6), (7), and (8) make sure that a service is only allowed to process any amount of containers when it is selected. While (9) tracks the transshipment necessary, constraints (10) and (11) ensure the timely sequencing of the services within the network. As seen in (10), each service has interrelated departure, service, and arrival times. In addition to the synchronization at nodes in terms of loading units (2), (3), and (4), constraint (11) takes care of the timely synchronization. It ensures the relation of sequential services at a transshipment location. This is necessary due to more or less fixed schedules of services, which permit services with earlier departure times than possible preceding services from following up on them. Constraint (12) ensures that only containers which have to change the vehicle are considered when calculating transshipment times, costs, and $CO_2$e-emissions. Constraints (13) and (14) provide the time frame for each order to plan within. The lower limit (earliest pick-up time) is fixed, while the upper limit (due date) can be bent, with penalties – if desired – allocated to late deliveries ($a^p_{\text{delay}}$). Constraint (15) defines the arrival time of the order to the destination which is dependent on the arrival of the last service which the order is carried on. Constraint (16) gives the time window within which services can depart with $T^s_{\min} = T^s_{\max}$ being valid for scheduled services. Constraints (17) and (18) ensure that the feasibility of two consecutive services is only checked if these services are designated to be used within the same routing plan. The domain of the decision variables is given in constraints (19) and (20).

## 3 Dealing with Travel Time Uncertainty

Whereas the presented mathematical model can easily calculate transportation plans in a deterministic environment, it has only limited possibilities to handle the increased complexity of the problem if stochastic factors are included. The reason for this is that considering uncertainty for different variables results in a high number of possible scenarios which cannot be handled by conventional methods, such as dynamic programming and multistage stochastic programming for realistic instance sizes. As an example, in a network with three services that can have three possible travel time realizations each, in total 27 different travel time combinations are possible, and the number of combinations is exponentially increasing with the increasing number of services and scenarios. Therefore, two possible approaches which can handle such complexity and evaluate the reliability of transportation plans under uncertainty are presented in this chapter, namely, sample average approximation and simulation-optimization. The focus here is on travel time uncertainty, but these approaches can be easily applied to other uncertain factors, such as demand or customer.

### *3.1 Sample Average Approximation*

The sample average approximation (SAA) method is used to reduce the complexity of a stochastic problem by approximating a distribution or an expected value of an uncertain variable. The approximation of a distribution is obtained by replacing the actual distribution with an empirical distribution by Monte Carlo sampling. In cases where the objective function corresponds to an expected value, it is approximated by its sample average estimate. The resulting problem is then solved by deterministic optimization methods.

The SAA method has been widely applied in the context of transportation planning and routing. Kenyon and Morton [28] use SAA to solve a stochastic vehicle routing problem (VRP) under two different objective functions: minimizing the expected completion time and maximizing the probability of completion time being below a target level. Luedtke and Ahmed [33] provide an application of SAA to a chance-constrained transportation problem with a convex feasible region where the dimension of the random vector presents a computational challenge. Wang and Meng [52] apply SAA for a schedule design problem for liner shipping services to minimize expected costs, and [53] apply it to chance-constrained liner ship fleet deployment problem. Verweij et al. [49] provide an introduction to the application of SAA to stochastic routing problems with expected value objectives.

The SND formulation presented in Sect. 2 extended by travel time uncertainty can be classified as a chance-constrained problem where a chance constraint measuring the number of successful realizations of a transportation plan under different travel time scenarios decides about the reliability of the plan. In this context, SAA method is used to approximate the true probability of the constrained event by its frequency of occurrence within the sample. In general, SAA is applied to chance-constrained stochastic problems because of two reasons: the feasible region defined by the chance constraint can be non-convex, and the probability of the constrained event may be difficult to evaluate [33, 38]. It has been shown that the optimal solution of the sampled problem converges exponentially fast to the optimal solution of the original problem as the number of scenarios increases.

The application of SAA to the GISND problem has been studied by [15]. In their approach, a number of different independent samples are created where each sample consists of $M$ scenarios representing different travel time realizations based on their probability distributions. Then, the model is solved for each of the samples which results in a number of candidate solutions. These candidate solutions are then tested on another test sample with a large number of scenarios in order to evaluate the probability that a plan is not feasible under a certain travel time combination, and therefore replanning is required. If this probability is higher than a certain value $1-\alpha$ which has to be chosen arbitrarily before the start of the process, then the plan is not considered as a feasible solution. From all feasible candidate solutions, the solution with minimal total costs is chosen at the end as an optimal solution. In order to apply the SAA method to the GISND, the mathematical model presented in Sect. 2 has to be extended by a set of travel time scenarios $M$ and the following constraints checking the reliability of the plan:

$$1-f_m^{\mathrm{qrp}} \leq \mathrm{delay}_m^{\mathrm{qrp}} \qquad \forall q \in \mathscr{S}, r \in \mathscr{S}, p \in \mathscr{P}, m \in \mathscr{M} \tag{21}$$

$$L(1-f_m^{\mathrm{qrp}}) \geq \mathrm{delay}_m^{\mathrm{qrp}} \qquad \forall q \in \mathscr{S}, r \in \mathscr{S}, p \in \mathscr{P}, m \in \mathscr{M} \tag{22}$$

$$f_m^{\mathrm{qrp}} \geq f_m^p \qquad \forall q \in \mathscr{S}, r \in \mathscr{S}, p \in \mathscr{P}, m \in \mathscr{M} \tag{23}$$

$$\sum_{m \in \mathscr{M}} f_m^p \geq M\alpha \qquad \forall p \in \mathscr{P}, \tag{24}$$

where $f_m^{\mathrm{qrp}}$ is a binary variable checking whether an order $p$ can catch the planned ensuing service after arriving to a transshipment terminal ($f_m^{\mathrm{qrp}} = 1$) or not ($f_m^{\mathrm{qrp}} = 0$), depending on the delay of the order determined by constraint (11). Based on the constraints (21), (22), and (23), the binary variable $f_m^p$ then shows whether a certain transportation plan for order $p$ is feasible ($f_m^p = 1$) or not ($f_m^p = 0$). Constraint (24) is then the chance constraint measuring the number of feasible scenarios which has to be higher than the factor $\alpha$ (e.g., 95%) in order to classify a plan as reliable.

### 3.2 Simulation-Optimization Approach

Another possibility for including uncertainty into the green intermodal transportation problem is a two-step hybrid approach combining the presented deterministic optimization model in Sect. 2 with a simulation model which is able to create and evaluate a high number of scenarios for the stochastic elements. This approach is getting more attention in the last years when it has been used for solving complex dynamic problems in supply chain management (see, e.g.,[2, 1, 35]). As an example, [40] combines simulation and optimization models in order to optimize a supply chain by combining transportation planning and production decisions including stochastic and nonlinear elements. In case of [44], a simulation-based approach is used for sustainable transportation optimization by searching for strategies that minimize the generalized costs of multimodal planning. In addition to that, [7] and [42] apply a hybrid approach combining simulation and optimization for coordinating production and distribution decisions, and [11] uses a similar method for the perishable goods industry. Whereas these contributions cover the production processes and their combination with distribution, the application of the simulation-optimization approach to the transportation planning area is very limited. Besides that, the main purpose of combining the two methods is usually the estimation of uncertain parameters by simulation which are then used for the optimization. This differs from our approach where the simulation model with stochastic travel times is used to evaluate the reliability of deterministic transportation plans created by the optimization model.

The solution procedure has been described in detail in [24] and is presented in Fig. 1. The authors considered a system which consists of an optimization model and a simulation model run in different software that are connected through a database including the relevant transportation network represented by terminals and services

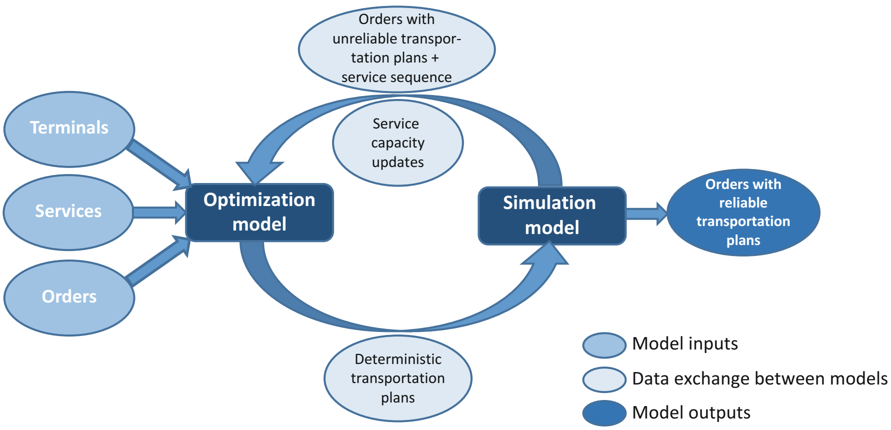


**Fig. 1** An overview of the simulation-optimization approach presented by [24]

as well as orders that need to be shipped. All of the data has to be available at the beginning of the process and is the input for the optimization model which computes the optimal transportation plan considering deterministic travel times in ideal situation where no congestion or delay occurs. In this way the optimal plan is obtained relatively quickly, and additional constraints connected to travel time uncertainty which might limit the size of the instances that can be solved to optimality can be avoided.

The transportation plans calculated by the optimization model serve as a basis for the simulation model which in the second step of the solution procedure evaluates their reliability under stochastic circumstances. In this step, the travel time is uncertain and can take different values depending on the underlying probability distribution that has to be determined in advance. During the simulation, multiple runs of the simulation model are executed in order to consider different possible travel time combinations for all services in the transportation network. Within each run, the optimal deterministic route for each order is simulated in order to see whether the plan is still feasible under the chosen travel time realizations. In this way, in addition to calculating the number of scenarios in which a plan becomes infeasible, also the problematic service or sequence of services which might lead to delay and infeasibility of the plan can be identified. The plan becomes infeasible if the containers arrive too late to an intermediate terminal, and therefore the planned subsequent service is missed. In this case the transportation process cannot be continued according to the original plan, and therefore an alternative solution has to be found. The simulation model enables to define some simple solutions in advance (e.g., using an additional truck to transport the containers directly to the destination) which can be used and simulated in case of infeasibility. In this way not only the reliability of the plan but also the additional costs in case of infeasibility can be estimated.

At the end of the simulation phase, the reliability of the plans is evaluated based on two criteria: the number of runs in which the plan was infeasible and the average additional costs of this infeasibility in comparison to the optimal deterministic solution. The thresholds for these criteria have to be set at the beginning of the optimization process, and they are decisive for classifying a transportation plan as reliable or not reliable. Transportation plans which are reliable leave the optimization process and are fixed for execution which means that the service capacity used by these plans has to be blocked and the free capacity of the services has to be updated in the database. Transportation plans which are not reliable are sent back to the optimization model together with the updated service capacities, and the whole optimization process starts from the beginning. In order to prevent the repeated choice of the unreliable plan by the optimization model, the service sequence of the plan is also used as input for the optimization model and is handled in an additional constraint so that an alternative plan has to be chosen. This process is repeated until a reliable plan is found for all orders. If there is no feasible and reliable route for an order in the considered network, a direct transportation by truck is used as a default option.

### 3.3 A Comparison of the Methods

Both presented methods can be used to get reliable transportation plans; however, the decision about which method to use and also the quality of the solution for each method might be dependent on the complexity of the problem which has to be solved. The division of the solution procedure into two steps in the simulation-optimization approach decreases the complexity for the mathematical optimization model which only has to deal with deterministic times. Therefore, larger instances can be solved than in the case of the SAA approach since the scenarios are included in the mathematical model which limits the size of the problem a solver can handle. In terms of computational time, the integration of scenarios into the model leads to faster solutions for SAA for smaller instances since the simulation model needs some time to run all scenarios and the time further increases if solutions are infeasible and further runs of the optimization and simulation model are necessary. However, whereas the computational times for SAA tend to increase exponentially with the increasing complexity of the instances, the time needed for one simulation run is rather stable. Moreover, with regard to the quality of the solution, the simulation-optimization approach evaluates the reliability based on two criteria so that some plans which are unreliable according to SAA can be accepted by simulation since the infeasibility might cause only very small cost increase that might be negligible in comparison to the higher costs of an alternative plan. Furthermore, the simulation model shows where the disruption occurs which is not reported by SAA where only a solution is chosen based on the chance constraint. The simulation model also gives possibilities to increase the number of considered scenarios and replications in order to increase the statistical significance of the solution, and the travel time can be modeled by using different probability distributions. The described differences are also illustrated by a computational comparison in Sect. 4.2.

## 4 Case Studies

The combination of different objectives and the consideration of travel time uncertainty in the optimization process often lead to trade-offs and conflicting solutions which are dependent on the priorities that the transportation planner sets before the solution process is started. In order to illustrate these trade-offs, this section consists of two case studies that show the influence of the individual objectives and the travel time uncertainty on the optimal solution. Comparing results for different objectives is not only important in the intermodal transportation, which is described in Case study B (Sect. 4.2), but can also help to improve the reliability of single-mode transportation, as it is illustrated in Case study A (Sect. 4.1).

### 4.1 Case Study A: Choosing the Optimal Route for Road Transportation

Road transportation is a very popular transportation mode with the highest share on the modal split within the EU (74.9% in 2014) [21]. The reason for this is a high flexibility of this transportation mode since most of the customers can be reached without problems via the dense road network so that a truck can be used either for the pick-up and last-mile delivery of goods within intermodal transportation chains but also for direct connections between origin and destination. However, the high volume of road freight transportation is responsible for a significant amount of emissions from transportation, and the limited infrastructure capacity in combination with dense individual passenger traffic might cause unexpected congestions or delays that might influence the reliability of this transportation mode and cause late deliveries of goods to the customer. Since the dense road network usually offers a number of alternative routes between two terminals, a comparison of the possible routes according to transportation costs, travel time, and $CO_2$e emissions can lead to transportation plans that might cause slightly higher costs but improve the environmental impact and reliability of travel times so that buffer times accounting for possible delays might be reduced. In order to be able to compare the routes according to travel time uncertainty, historical data about past trips has to be available.

The importance of travel time reliability can be illustrated on regular truck transportation of air freight between major European airports which is necessary due to consolidation of goods or due to changes in available plane capacities that are used for further transportation of the goods. As the changes in capacities are announced at very short notice, the planning process has to be fast and use very accurate data. In order to achieve this, a detailed analysis of travel times for truck transportation between the airports in Amsterdam and Frankfurt (AMS-FRA) and Amsterdam and Brussels (AMS-BRU) was conducted. The distance for AMS-FRA is approximately 450 km, whereas for AMS-BRU trucks are traveling slightly more than 200 km. The available data covered 3 weeks of transports in spring and summer 2014 in which around 300 trips were conducted for each origin-destination pair. The collected data included travel times, speed, direction, breaks, delays, departure times, and GPS coordinates of the trips [4]. As a result, three different routes could be identified for AMS-FRA and two different routes for AMS-BRU. They are displayed in Fig. 2 (routes 1–3 for AMS-FRA and 4–5 for AMS-BRU). Whereas the number of trips was almost equal for routes 4 and 5, route 1 was clearly preferred for the relation AMS-FRA.

Since the distance of the routes for each origin-destination pair is very similar, the differences in total costs and emissions between the routes are less than 2%. However, the analysis of travel times revealed important differences in travel time distributions that might have influence on the reliability of transportation. Although most of the trips were conducted during evening or night hours due to flight schedules, there were enough trips during the day and rush hours for which

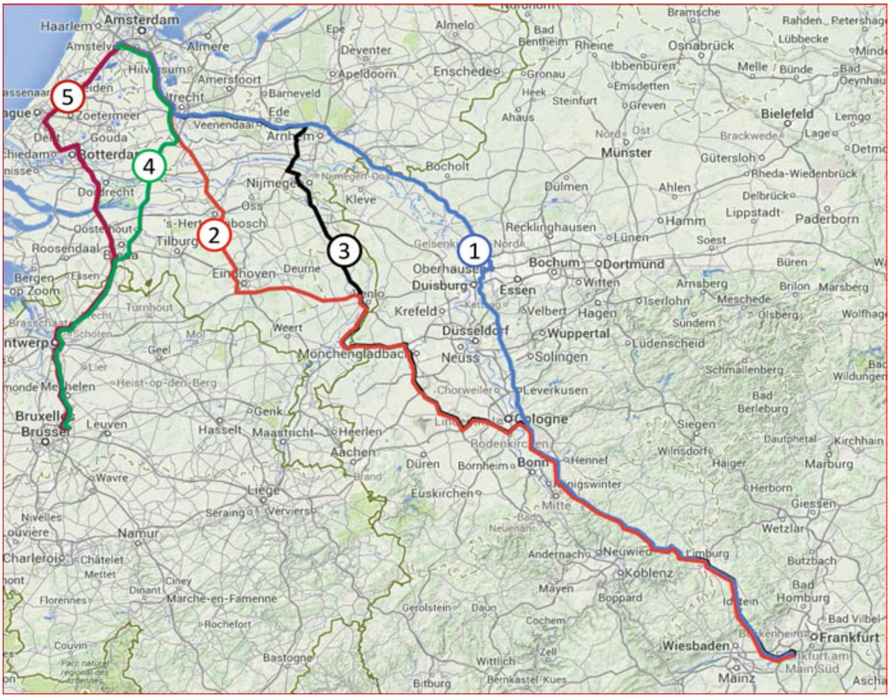


**Fig. 2** Route alternatives for AMS-FRA and AMS-BRU

typical delays could be observed. Therefore, the travel times were divided into three categories: uncongested travel time accounting for minimal travel time without any disruptions, congested travel time representing travel times with small delays caused by usual (recurring) congestion, and disrupted travel time which was observed for trips with major delays. The resulting travel times with their correspondent probability of occurrence for each route are summarized in Table 4.

As shown in Table 4, route 1, which is preferred by the truck drivers, is the route with the lowest uncongested travel time for FRA-AMS and therefore the fastest route if no disruption occurs. However, this happens only in about 75% of the cases, and the risk of delay of 1 h is about 20%, caused mainly by the fact that there are regular congestions on the route which is passing important German cities (Cologne, Düsseldorf, Dortmund). In addition to that, there is even a risk of major disruptions adding another 4 h to the transportation time. In contrast to that, route 3 has a slightly higher uncongested travel time, but the probability of congestion is lower, and also the average delay is lower which reduces the fluctuation in arrival times. Therefore, it might be more convenient to choose route 3 when trying to avoid driving on congested highways which leads to delays and, especially in a stop-and-go traffic, to higher fuel consumption and $CO_2$e emissions.

**Table 4** Travel time distribution for route alternatives

| No | Route | Uncongested travel time | | Congested travel time | | Disrupted travel time | |
|---|---|---|---|---|---|---|---|
| | | Probability [%] | Travel time [hh:mm] | Probability [%] | Travel time [hh:mm] | Probability [%] | Travel time [hh:mm] |
| 1 | AMS-Utrecht-Arnhem-Köln-FRA | 75.90 | 06:29 | 23.20 | 07:20 | 1.00 | 11:06 |
| 2 | AMS-Eindhoven-Mönchengladbach-Köln-FRA | 84.20 | 07:06 | 14.00 | 07:25 | 1.80 | 10:53 |
| 3 | AMS-Nijmengen-Mönchengladbach-Köln-FRA | 86.40 | 06:50 | 13.60 | 07:29 | 0.00 | 00:00 |
| 4 | AMS-Utrecht-Breda-Antwerp-BRU | 86.20 | 02:52 | 13.10 | 03:28 | 0.70 | 06:52 |
| 5 | AMS-Rotterdam-Antwerp-BRU | 87.20 | 02:52 | 11.00 | 03:11 | 1.70 | 10:54 |

In the case of AMS-BRU, average travel times are the same for both routes, and route 4 has a higher probability of congestion with longer average delay. However, when taking into account disrupted travel times, route 4 might be better since the probability and also average delay for this travel time category is lower.

Although the historical data cannot certainly predict the exact travel time for the next trip, the distribution of past travel times can at least help to avoid routes which might be critical when a disruption occurs. In this way the efficiency of transportation can be increased minimizing the costs, delays, and emissions. Whereas the current study was only based on a limited number of trips, recording and analyzing historical data continuously can further improve the accuracy of travel time distributions and predictions.

### *4.2 Case Study B: Intermodal Transportation Planning Under Travel Time Uncertainty*

The second case study investigates an intermodal transportation planning problem on which the differences between the methods presented in Sects. 2 and 3 and the influence of the individual optimization objectives can be presented. For this, an intermodal transportation network consisting of intermodal terminals connected by services on road, rail, and inland waterway has been created. This network is based on real-world connections where [34] and [29] were used for railway schedules, [51] was the basis for modeling inland waterway services, and road connections were designed based on [41]. In total, the network consists of 20 terminals located in Austria, Slovakia, Czech Republic, Germany, Slovenia, and Italy which are used either as ports or as collection terminals for feeder services to Western European harbors. For each terminal and service, the estimated transshipment and transportation costs, times, and emissions have been assigned. The transportation network, which was firstly defined in [15], is depicted in Fig. 3. Due to the high complexity of the network, only rail and inland waterway connections are included in the figure. These connections are extended by road services. The travel time distribution for each service was modeled as a three-point distribution with uncongested, congested, and disrupted travel time as already described in Sect. 4.1.

The analysis was conducted on an Intel(R) Core(TM) i5-5300U CPU with 2.3 GHz and 8 GB of memory. The deterministic optimization model and the SAA model were solved using CPLEX 12.6 [25], and the simulation was run in Anylogic University 7.2.0 [8]. For the inventory costs, 1 EUR/h was assumed for each order, and the penalty costs are different for each order varying between 1 and 10 EUR/h. The emission costs were estimated to be 70 EUR/t of $CO_2$e emissions as recommended by [39].The thresholds for evaluating the reliability of the plans were set to 5% for the share of infeasible scenarios and also 5% for the additional costs of unreliability. In the simulation-optimization model, an extraordinary truck which transports the goods directly to the destination is chosen as an alternative plan in

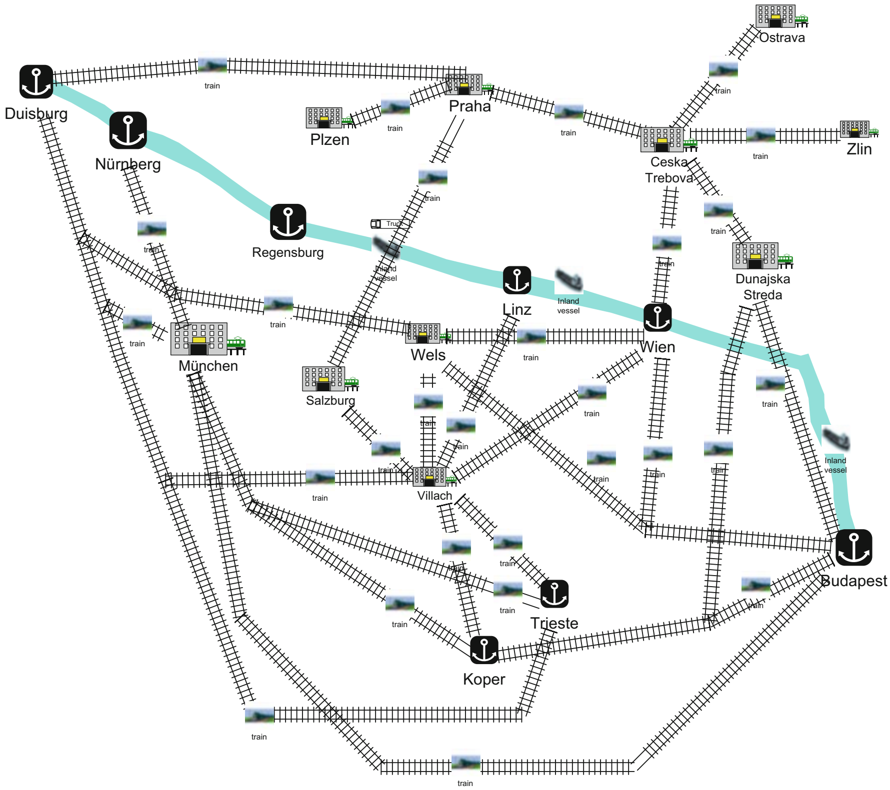


**Fig. 3** Rail and inland waterway services in the intermodal transport network

case of infeasibility. This truck is usually the fastest option if the original plan does not work; however, since it has to be organized in a short time, an increase of 25% for the transportation costs in comparison to a planned truck on the same route is assumed.

In the first step, the solutions of the deterministic optimization model, SAA, and simulation-optimization were compared with regard to computational times, quality, and limitations. For this, the number of services in the network with 20 terminals was stepwise increased from 50 to 500, and the number of orders considered varied between 1 and 20. In this setting, the deterministic model could find an optimal solution for all instances with up to 250 services; bigger instances could not be solved due to memory problems of the solver. For SAA it could be observed that the approach is limited by the number of scenarios $M$ used where more than ten scenarios for choosing candidate solutions significantly reduce the network size that can be handled by the model. Besides that, the number of orders also has a negative influence on computational time since the computational time tends to grow exponentially with the increasing number of orders. The SAA approach could find

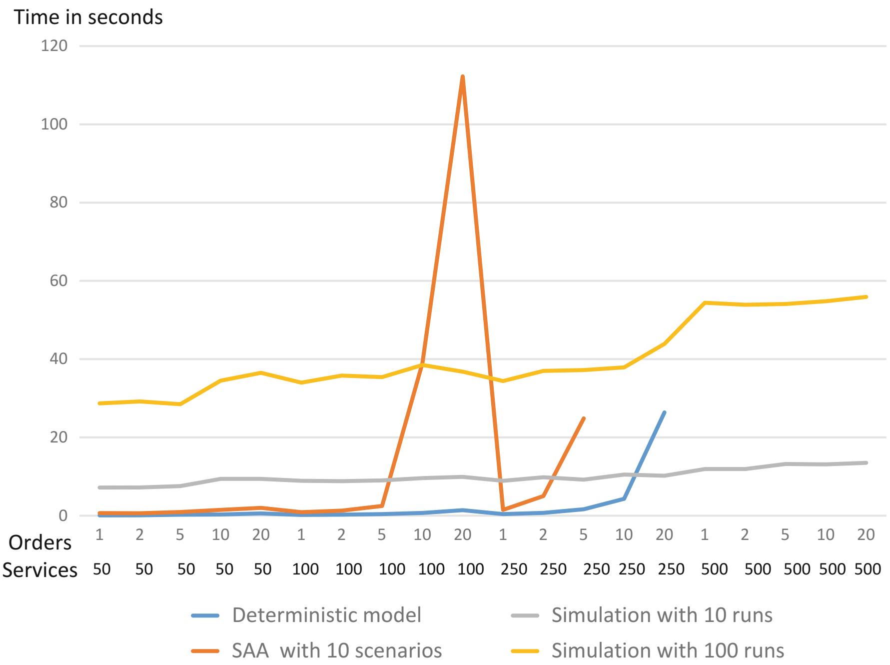


**Fig. 4** Average computational times in seconds for different methods and instances

optimal solutions for all instances with up to 250 services and 5 orders. The best results with regard to size limitations were obtained by the simulation model used in simulation-optimization where it can be seen that the model can handle all of the tested instances and the computational times are relatively stable. This means that it is more convenient to use the simulation-optimization model especially for bigger instances where the computational time using ten simulation runs is lower than in the case of SAA. In addition to that, the simulation model can also handle increased number of scenarios which improves the statistical significance of the results. This is also illustrated in Fig. 4, where computational times for deterministic model, SAA, and simulation model with 10 and 100 runs are summarized.

When looking at the quality of the results, it could be observed that both SAA and simulation-optimization are able to identify the same unreliable plans based on the number of scenarios where the plan becomes infeasible. This is mainly the case in small network with 50 services where usually very limited number of possible connections exist. However, there is a difference in the estimated total costs of the reliable solution since SAA only estimates the increase in costs due to unreliability, whereas the simulation model can also calculate the costs of the extraordinary truck. Therefore, it might happen that the estimated costs after simulation are lower than the costs from the deterministic optimization model. This might indicate that using a direct truck for a part of the route or the whole route might be a more convenient

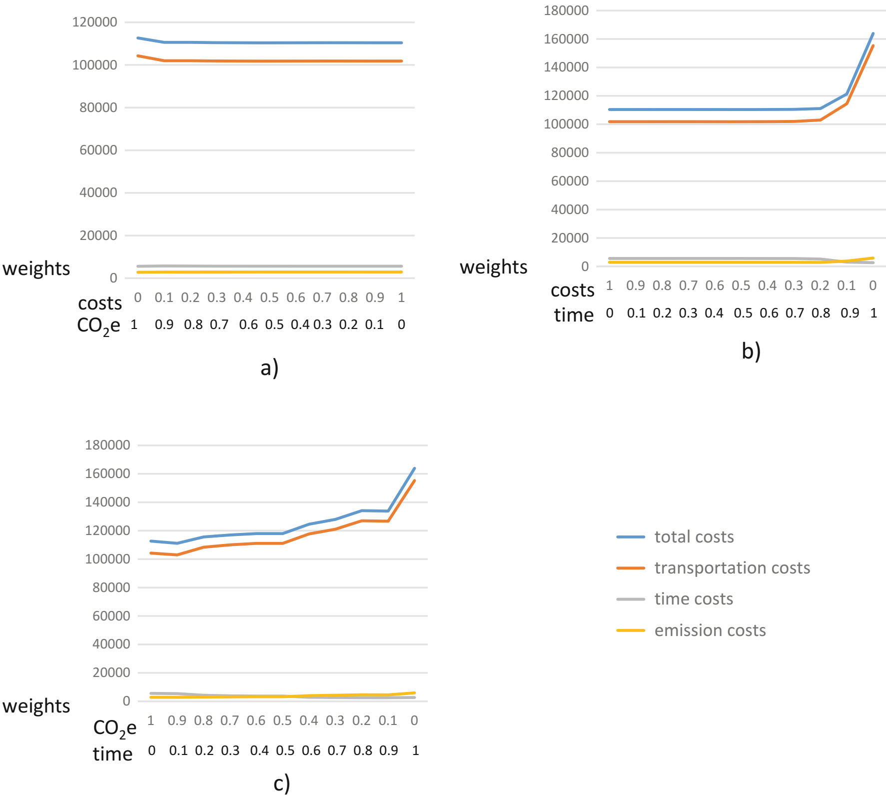


**Fig. 5** Influence of different objective weights on total costs. (**a**) Transportation costs vs. $CO_2$e emissions. (**b**) Transportation costs vs. time. (**c**) $CO_2$e emissions vs. time

option when considering the economic factors and reliability. However, this has usually a negative impact on $CO_2$e emissions.

In order to analyze the influence of different objectives on the optimal route, 50 orders were created randomly, and their optimal route was calculated for different combinations of objectives using the deterministic mathematical model in the first step. The underlying network consisted of 100 services, and the results showed that for 25 orders there was only one optimal route independent of the objective weights. For the remaining 25 orders, where a change in plans was recorded, the dominance of solutions minimizing the transportation costs is clearly visible since this cost category has the highest share on the total costs. However, there is also a visible influence of other objectives as it is shown in Fig. 5 where in each of the three graphs, a trade-off between two objectives is shown, whereas the third objective is not considered in the optimization process.

In the first graph, the trade-off between transportation costs and $CO_2$e emissions is shown where the transportation cost-minimizing solution clearly dominates the

**Table 5** Cost overview for different objectives of 25 orders

| Optimization according to | Transportation costs (EUR) | Time costs (EUR) | Emission costs (EUR) | Total costs (EUR) |
|---|---|---|---|---|
| Costs (1,0,0) | 101,847.50 | 5,638.10 | 2,919.07 | 110,404.67 |
| Time (0,1,0) | 155,304.50 | 2,655.17 | 5,917.86 | 163,877.53 |
| $CO_2$e emissions (0,0,1) | 104,265.50 | 5,594.00 | 2,802.71 | 112,662.21 |

emission minimization. A change occurs only when the emission objective has a weight of 0.8 or higher. In this case the transportation plans for some of the orders change toward more environmentally friendly transportation modes which results in about 4% saving in $CO_2$e emissions, whereas the increase in transportation costs is only 2.4%. A similar picture can be seen when the trade-off between transportation costs and time is analyzed; however, in this case the increase in total costs due to time-optimizing solutions is much higher. Due to the increased use of direct and fast truck services, the time costs can be minimized by 53%. However, this is only possible when the transportation costs are increased by 53% in comparison to the transportation cost-minimizing solution. Moreover, an increase in emissions by 103% also has to be accepted. The negative impact of time optimization on total costs is even more visible in the third graph where the trade-off between time and $CO_2$e emissions is depicted. Here the total costs are continuously increasing from the minimum when only emissions are minimized until the maximum for the time-minimizing scenario. Similar to the previous case, also here the transportation costs increase in total by 49%, whereas 53% of time costs can be saved. The increase in total emissions is with 111% even higher. The comparison of the individual cost components for optimal deterministic plans according to every single objective is displayed in Table 5.

The reliability of the calculated plans for all 50 orders was tested by the simulation model. Since the travel time uncertainty can be modeled in different ways, three different travel time distributions were used in order to compare the influence of travel times on the reliability of the plans. Besides the discrete three-point distribution, which was already used in Sect. 4.1, two continuous distributions were also applied: a shifted exponential distribution, as suggested by [36], and the uniform distribution, which is usually used if no or insufficient information about the distribution of the uncertain variable is available. The exponential distribution is shifted to the right starting at the uncongested travel time from the discrete distribution, and its shape was obtained by fitting it to three intervals (uncongested, congested, and disrupted) which were also created from the discrete distribution and have borders located in the middle between the discrete travel times for each state and probabilities corresponding to the discrete ones [24]. The borders of uniform distribution are located at the uncongested and disrupted travel time for each service. All three travel time distributions are illustrated in Fig. 6.

The output of the simulation model shows that many of the deterministic plans in the studied instance are not reliable and require replanning. This is especially

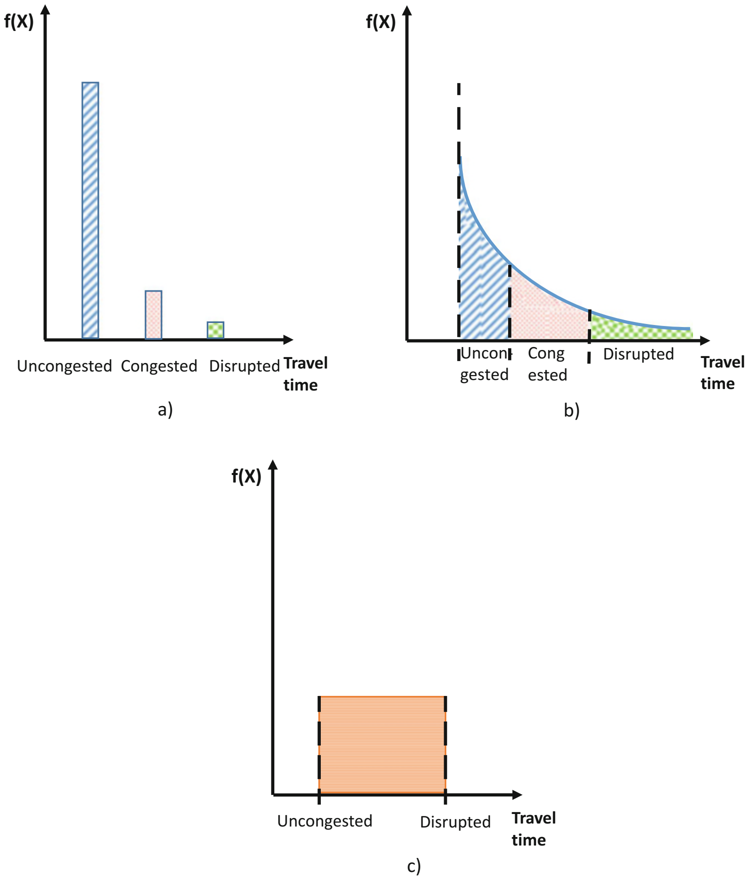


**Fig. 6** Probability distributions used for travel time uncertainty. (**a**) Discrete three-point distribution. (**b**) Continuous shifted exponential distribution. (**c**) Continuous uniform distribution

true for the time optimization where the created deterministic plans often combine various truck services which might be faster than waiting for a direct train that departs only in a couple of days. However, since the truck services are shared by many orders, the truck service has to wait until all orders are available which might result in delay for another order if the truck arrives too late to the destination. This is not such a big problem for trains and vessels where the departure time is given by the schedule and the vehicle is not waiting for a delayed order. Since the network is limited by 100 services, it is often the case that there is only one available route in the intermodal network for a certain order, and if this route is evaluated as unreliable, the only alternative is to use a direct truck, which is then suggested by the simulation model. This is especially true for orders which have to travel for very long distances and have to combine a lot of services (up to eight in the studied instance). In these cases avoiding the unreliable intermodal connection and using direct truck

**Table 6** Comparison of costs for different objectives and travel time realizations for 50 orders

| Optimization according to | Total deterministic costs | Total costs after simulation for | | |
|---|---|---|---|---|
| | | Discrete | Shifted exponential | Uniform |
| Costs | 183,949.04 | 166,493.40 | 167,771.54 | 173,396.52 |
| Time | 233,612.27 | 220,797.20 | 223,542.52 | 227,515.27 |
| $CO_2$e emissions | 186,206.59 | 174,209.05 | 174,960.51 | 180,312.37 |

might be more beneficial. In this way the model can increase the motivation of transportation planners to consider intermodal planning as an alternative since it offers them only routes which are reliable. The use of direct truck, which is not available in the original network, might be sometimes cheaper but can have negative impacts on the environment. This can be also seen in Table 6, where the costs after simulation are lower than the deterministic costs especially due to the use of direct truck connections as an alternative to the limited intermodal network.

When comparing the results for the three travel time distributions displayed in Table 6, it can be noticed that the costs in case of continuous distributions are higher than for the discrete distribution. The reason for that might be that the uncongested travel time is the most important travel time for the discrete distribution, whereas it is only the lower border for the continuous distribution, and therefore the travel times for continuous distributions are higher on average. However, since for continuous distributions any time within the specified interval can be chosen, the results give a better picture about the reliability of the plans. Especially in the case of the exponential distribution, the number of infeasible scenarios in cases where these are caused only by very small delays at the destination is decreasing, and the number of infeasible scenarios for plans where the time causing infeasibility is located between uncongested and congested time is increasing. Also it can be seen that due to the equal distribution of travel times in case of uniform distribution, some of the plans that are reliable under exponential distribution become unreliable due to higher number of scenarios with longer travel times. Due to this more conservative evaluation of the plans, the uniform distribution can be used especially in situations where the information about the real distribution of travel times is not available.

## 5 Conclusions

This chapter provided an overview of current studies aiming at intermodal transportation planning with travel time uncertainty. Even though this area is quite new and there is only limited research, very recent research has been summarized and highlighted to bring more attention on data uncertainty from both the academia and the practice.

Two possible approaches which can handle such complexity and evaluate the reliability of intermodal transportation plans under uncertainty are discussed and compared in terms of solution times, the quality, and the limitations. The methodological approaches presented in this chapter are sample average approximation and simulation-optimization, which both can easily handle travel time complexity. The focus of this chapter was on travel time uncertainty, but these approaches can be easily applied to other uncertain factors (i.e., demand and customer uncertainties).

Moreover, we investigate the transportation plan based on three different objectives which can have different weights according to the transport user's preferences. The objectives are transportation costs, time in form of inventory costs and penalty costs for late deliveries at the final customer, and $CO_2$e emissions expressed as emission costs.

Computational experiments confirm that both methods can be used to get reliable transportation plans. With regard to the quality of the solution, the simulation-optimization approach evaluates the reliability based on two criteria so that some plans which are unreliable according to SAA can be accepted by simulation since the infeasibility might cause only very small cost increase that might be negligible in comparison to the higher costs of the alternative plan. Furthermore, the simulation model shows where the disruption occurs which is not reported by SAA where only a solution is chosen based on the chance constraint. The simulation model also gives possibilities to increase the number of considered scenarios and replications in order to increase the statistical significance of the solution and the travel time can be modeled by using different probability distributions.

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# A Multiproduct Multi-vehicle Inventory Routing Problem with Uncertainty

**Secil Ercan and Didem Cinar**

**Abstract** As a result of the increase in environmental problems, green logistics has become an important subject in the supply chain literature. In this study, a multiproduct multi-vehicle inventory routing problem is modeled by considering the cost stemming from fuel consumption as an environmental objective. Demand and inventory holding costs are taken into account as uncertain parameters. A sample average approximation algorithm is used to solve the problem. The performance of the algorithm is evaluated in terms of optimality gap and computational time by using a data set from the literature. The computational experiments give promising results for further research.

## 1 Introduction

Because of increasing environmental concerns, government and business organizations have attempted to decrease carbon emissions stemming from logistics processes. Since transportation has significant impact on global warming in the world, environmentally sensitive logistics and transportation schemes have been developed to get more sustainable policies with less negative effects on the environment [3, 23].

The inventory routing problem (IRP) was first introduced by Bell et al. [5] by integrating inventory management and vehicle scheduling for the distribution of industrial gases. The traditional vehicle routing problem deals with finding the vehicle route that minimizes an economic indicator. On the other hand, IRP deals with finding optimal routing and inventory policies where no stockouts are allowable for each customer. Quantity and time of delivery to each customer and vehicle routes are the decisions that policy makers have to make simultaneously in IRP [32].

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In the literature, the most considered objective is to minimize total distribution and inventory costs during the planning horizon. Environmental concerns increase the focus on sustainability issues in transportation and logistics processes. Therefore, environmental and social factors have been taken into account in the recent IRP studies. However, although the critical parameters – such as demand of customers, distance, transportation cost, and inventory holding cost – are uncertain in real life, the majority of IRP studies have considered deterministic parameters.

In this study, a mixed-integer linear programming (MILP) model is developed for the multiproduct multi-vehicle inventory routing problem (MMIRP). The objective of the investigated MMIRP is to minimize total transportation and inventory costs. During the computation of transportation cost, fuel consumption is taken into account. Fuel consumption is computed by considering load, distance, and fuel consumption rates of the vehicles. In order to deal with uncertainties, a stochastic MILP model, which allows demand and unit inventory holding costs as uncertain parameters, is proposed. A sample average approximation algorithm is used to solve the MMIRP with uncertain demand and inventory holding cost. The main contributions of this study can be summarized as follows: (i) A MILP model is developed for MMIRP considering environmental aspects. (ii) A two-stage stochastic programming model for MMIRP taking two sources of uncertainty into account is proposed, and a sample average approximation algorithm is implemented to solve the problem.

The remainder of this chapter is organized as follows. Section 2 provides a review of recent IRP studies considering environmental factors. Section 3 presents the definition and a MILP model for MMIRP. The two-stage stochastic programming model of MMIRP is given in Sect. 4. The sample average approximation algorithm is described in Sect. 5. The computational tests and analyses are presented in Sect. 6. Finally, some conclusions and directions for future research are presented in Sect. 7.

## 2 Literature Review

Since carbon emissions have a significant role in global warming, environmental aspects have become important topics for transportation and logistics in recent years. Interested readers are referred to Lin et al. [23] for the green logistics literature including IRP studies published until 2013. In this section, a brief literature review on recent IRP studies taking into account environmental concerns is given.

Kuo et al. [22] developed a MILP and a decision-making model for the carbon footprint IRP which includes supplier selection and efficient carbon emission inventory route planning. The objective was to minimize total cost, including inventory verifier cost, service time cost, and traveling time cost. Mirzapour and Rekik [25] also employed MILP for multiproduct multi-capacitated vehicle IRP to minimize total costs including inventory holding cost and transportation cost.

A transshipment option and GHG emission restriction were considered in the model. Alkawaleet et al. [3] proposed a MILP model minimizing the sum of transportation cost, inventory cost, stock-out penalty, and carbon emission cost of a single-product IRP with a heterogeneous fleet of vehicles. Al Shamsi et al. [4] developed a mixed-integer nonlinear programming model to minimize the total transportation, inventory, and $CO_2$ emission costs for an IRP with perishable products. The vehicle load was considered to calculate the $CO_2$ emissions as well as distance. Rahimi et al. [29] proposed a bi-objective mathematical model for an IRP with perishable products. The model included both economic and social objectives. The rate of accidents and the number of expired products were taken into account as social factors. GHG emission was also considered in the model as an environmental factor. Cheng et al. [9] proposed several mixed-integer nonlinear programming models and a hybrid genetic algorithm to analyze the impacts of carbon emission regulations on a multi-period IRP problem with single-product and identical vehicles. The objective was to minimize total cost including transportation cost, inventory holding cost, and fuel consumption cost. The fuel consumption cost was calculated by using the fuel consumption rate, the vehicle's load, the distance, and the fuel price. Qiu et al. [28] proposed a MILP model for a production IRP that aims to determine the quantity of production as well as the optimum inventory and routing policy. They presented a branch-and-price heuristic to minimize operational cost and $CO_2$ emission cost. Kumar et al. [21] developed a multi-objective mathematical model that minimizes the total operational costs and total emissions for a multi-period multi-vehicle production and pollution routing problem with time window. A hybrid self-learning particle swarm optimization algorithm was implemented to solve the problem.

Some IRP studies considering fuel consumption also regard uncertainties in parameters such as demand, distances, capacity of vehicles, capacity of retail center, cost of transportation, etc. These studies generally apply chance-constrained model, fuzzy approaches and heuristics, or hybridize these methods. Soysal et al. [32] proposed a chance-contrained programming model for the multi-period IRP with perishable products. The handled IRP was a single-product problem from one supplier to many customers with uncertain demands. The total cost including the inventory cost, the waste cost, the fuel cost, and the driver cost was minimized. Distance, load, and speed were taken into account during the fuel consumption estimation. The proposed methodology was implemented to a real-life case study on the fresh tomato distribution operations of a supermarket chain. Soysal et al. [33] also used a chance-constrained model for IRP with multiproduct from many suppliers to many customers under demand uncertainties. Soysal [31] proposed a probabilistic MILP model for a multiproduct closed-loop IRP to deal with the uncertainty in demand. They used a simulation model to evaluate the solution of the proposed MILP model with respect to inventory and routing performances. Zhalechian et al. [36] designed a multi-objective mixed-integer nonlinear programming model and proposed a new two-stage approach with possibilistic programming and modified game theory. The problem was formulated to minimize total cost and environmental impacts of $CO_2$ and maximize positive social impacts for a multiproduct IRP with heterogeneous vehicles utilizing fuzzy parameters such as demand, distances, and capacities of suppliers, retailers, and vehicles. They also developed a hybrid two-stage metaheuristic algorithm and obtained lower bounds for large-size problems. Niakan and Rahimi [26] hybridized two possibilistic methods applying a fuzzy approach for a multi-objective healthcare IRP. The objectives were minimizing total cost, minimizing forecast error because of shortages and expired products, and minimizing GHG emissions. Demand, transportation cost, and shortage cost were considered as uncertain. Tavakkoli-Moghaddam and Raziei [34] presented a multi-objective possibilistic MILP model for a multi-period multiproduct location-routing-inventory problem with heterogeneous fleets in a two-echelon distribution network. The objectives were to minimize total cost, including fuel consumption cost and cost of shortages of products for customers. Uncertainty was considered both in the parameters of constraints and in the objective functions. Demand was the uncertain parameter which was represented with a fuzzy approach.

Table 1 gives a brief summary for IRP studies considering fuel consumption or $CO_2$ emission. In this study, a stochastic approach is proposed to solve a multiproduct, multi-vehicle IRP from one supplier to many customers considering demand and unit inventory holding cost as uncertain parameters. The objective is the minimizing of total cost including transportation and inventory costs. The transportation cost consists of the fuel consumption cost which is computed by considering load and vehicle features as well as distance.

**Table 1** Summary of literature on IRP considering fuel consumption of $CO_2$ emission

| Ref. | Solution approach | Product type | Vehicle type | Uncertainty | Considering load for fuel consumption? | Supplier-to-customer |
|---|---|---|---|---|---|---|
| [32] | Stochastic | Single | Identical | + | + | One-to-many |
| [25] | Deterministic | Multi | Heterogeneous | − | − | Many-to-one |
| [31] | Stochastic | Multi | Identical | + | + | One-to-many |
| [33] | Stochastic | Multi | Identical | + | + | Many-to-many |
| [9] | Deterministic | Single | Identical | − | + | One-to-many |
| [22] | Deterministic | Single | Identical | − | − | Many-to-one |
| [36] | Fuzzy | Multi | Heterogeneous | + | + | Many-to-many |
| [21] | Deterministic | Single | Identical | − | + | One-to-many |
| [29] | Deterministic | Multi | Heterogeneous | − | − | One-to-many |
| [4] | Deterministic | Single | Identical | − | + | One-to-many |
| [28] | Deterministic | Single | Identical | − | + | One-to-many |
| [26] | Fuzzy | Multi | Heterogeneous | + | − | One-to-many |
| [34] | Fuzzy | Multi | Heterogeneous | + | + | Many-to-many |
| [3] | Deterministic | Single | Heterogeneous | − | − | One-to-many |
| This study | Stochastic | Multi | Heterogeneous | + | + | One-to-many |

## 3 Multiproduct Multi-vehicle Inventory Routing Problem

Let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ be a directed graph where $\mathcal{V} = \{0, 1, \ldots, N\}$ is the set of vertices and $\mathcal{A} = \{(i,j) \mid i,j \in V, i \neq j\}$ is the set of arcs. Vertex 0 refers to the supplier, and vertices in the set $\mathcal{V}' = \mathcal{V} \setminus 0$ represent customers. There is a set of products $\mathcal{M} = \{1, \ldots, M\}$ which are distributed to the customers by the supplier. A fleet of heterogeneous vehicles, $\mathcal{K} = \{1, \ldots, K\}$, is available for delivering the products to the customers. MMIRP is the problem of finding the optimal routing and inventory policy in the planning horizon $T\,(\mathcal{T} = \{1, \ldots, T\})$ to meet the demand for each customer. The quantity of product $m \in \mathcal{M}$ that is available for delivery by the supplier is $s_m^t$ at time period $t \in \mathcal{T}$. Demand of product $m$ by each customer $i \in \mathcal{V}$ for each time period $t$ is $c_{im}^t$. It is assumed that backlogging is not allowed, so the supplier has enough products to satisfy the demand of the customers. There is no restriction on the product type for the inventory of vertices. In other words, a vertex can hold all kind of products as its inventory. The initial inventory level of vertex $i$ for product $m$ is represented by $I_{im}^0$. The unit inventory holding cost in vertex $i$ for product $m$ is $h_{im}$. The inventory holding capacity of vertex $i$ is $HC_i$. A vehicle $k \in \mathcal{K}$ has limited capacity represented by $Q_k$. Each vehicle can perform only one tour per time period and deliver the products from supplier to a subset of customers during the tour. The distance from vertex $i$ to vertex $j$ is represented by $d_{ij}$ $(i,j \in \mathcal{V})$. The notation used in this study is given in Table 2.

The objective is to minimize total transportation and inventory costs. Only the transportation cost arising from fuel consumption is taken into account. In the literature several formulations to represent fuel consumption have been proposed for routing problems [12, 24]. In this study, fuel consumption is computed by considering load, distance, and vehicle features [20, 17]. Let $x_{ijk}^t$ be a binary decision variable that is set to 1 if vehicle $k$ visits vertex $j$ immediately after vertex $i$ in time period $t$, 0 otherwise, and $q_{ijkm^t}$ be a continuous variable referring to the quantity of product $m$ carried by vehicle $k$ on arc $(i,j)$ in time period $t$. The total fuel cost ($TFC$) can be represented as follows:

$$TFC = u \sum_{(i,j)\in\mathcal{A}} d_{ij} \left( \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} a_k x_{ijk}^t + \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} b_k q_{ijkm}^t w_m \right) \tag{1}$$

where $u$ is the unit fuel cost, $a_k$ is the fuel consumption rate per kilometer for vehicle $k$ when it is empty, and $b_k$ is the fuel consumption rate per load per kilometer for vehicle $k$. Let $I_{im}^t$ be the inventory level of product $m$ in vertex $i$ at the end of period $t$. The total inventory cost ($TIC$) is computed as follows:

$$TIC = \sum_{i\in\mathcal{V}'} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} I_{im}^t h_{im} \tag{2}$$

**Table 2** Notation for MMIRP

| | |
|---|---|
| Sets | |
| $\mathcal{V}$ | Set of vertices, $\mathcal{V} = \{0, 1, \ldots, N\}$, where 0 refers to supplier |
| $\mathcal{V}' = \mathcal{V} \setminus \{0\}$ | Customers |
| $\mathcal{A}$ | Set of arcs $\mathcal{A} = \{(i,j) \mid i,j \in \mathcal{V}, i \neq j\}$ |
| $\mathcal{M}$ | Set of products $\mathcal{M} = \{1, \ldots, M\}$ |
| $\mathcal{K}$ | Set of vehicles $\mathcal{K} = \{1, \ldots, K\}$ |
| $\mathcal{T}$ | Set of time periods $\mathcal{T} = \{1, \ldots, T\}$ |
| Decision variables | |
| $x^t_{ijk}$ | 1 if vehicle $k$ visits vertex $j$ immediately after vertex $i$, 0 o.w. $(i,j \in \mathcal{V}, k \in \mathcal{K}, t \in \mathcal{T})$ |
| $q^t_{ijkm}$ | Total quantity of product $m$ transported from vertex $i$ to vertex $j$ by vehicle $k$ in time $t$ $(i,j \in \mathcal{V}, m \in \mathcal{M}, k \in \mathcal{K}, t \in \mathcal{T})$ |
| $I^t_{im}$ | Inventory level of product $m$ in vertex $i$ at the end of time period $t$ $(i \in \mathcal{V}', m \in \mathcal{M}, t \in \mathcal{T})$ |
| $R^t_{im}$ | Total amount of product $m$ received at vertex $i$ from the supplier at time $t$ $(i \in \mathcal{V}', m \in \mathcal{M}, t \in \mathcal{T})$ |
| Parameters | |
| $s^t_m$ | Quantity of product $m$ that is available for delivery by the supplier at time $t$ $(t \in \mathcal{T})$ |
| $c^t_{im}$ | Demand for product $m$ by vertex $i$ at time period $t$ $(i \in \mathcal{V}', m \in \mathcal{M}, t \in \mathcal{T})$ |
| $h_{im}$ | Unit inventory holding cost for product $m$ in vertex $i$ $(i \in \mathcal{V}', m \in \mathcal{M})$ |
| $I^0_{im}$ | Initial inventory level of vertex $i$ for product $m$ $(i \in \mathcal{V}', m \in \mathcal{M})$ |
| $HC_i$ | Inventory holding capacity of vertex $i$ $(i \in \mathcal{V}')$ |
| $d_{ij}$ | Distance from vertex $i$ to vertex $j$ $(i,j \in \mathcal{V})$ |
| $Q_k$ | Capacity of vehicle $k$ $(k \in \mathcal{K})$ |
| $a_k$ | Fuel consumption rate for empty vehicle $k$ per kilometer $(k \in \mathcal{K})$ |
| $b_k$ | Fuel consumption rate for vehicle $k$ per unit of load per kilometer $(k \in \mathcal{K})$ |
| $w_m$ | Weight of one unit of product $m$ $(m \in \mathcal{M})$ |
| $u$ | Unit fuel price |

The MILP model for MMIRP is given as follows:

$$\min\ u \sum_{(i,j)\in\mathcal{A}} d_{ij} \left( \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} a_k x^t_{ijk} + \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} b_k q^t_{ijkm} w_m \right) + \sum_{i\in\mathcal{V}'} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} I^t_{im} h_{im} \tag{3}$$

s.t.

$$\sum_{j\in\mathcal{V}, j\neq i} \sum_{k\in\mathcal{K}} x^t_{ijk} \leq 1 \qquad \forall i \in \mathcal{V}', \forall t \in \mathcal{T} \tag{4}$$

$$\sum_{j\in\mathcal{V}, j\neq i} x^t_{ijk} = \sum_{j\in\mathcal{V}, j\neq i} x^t_{jik} \qquad \forall i \in \mathcal{V}', \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5}$$

$$\sum_{k\in\mathcal{K}} x^t_{ijk} \leq 1 \qquad \forall (i,j) \in \mathcal{A}, \forall t \in \mathcal{T} \tag{6}$$

$$\sum_{j\in\mathcal{V}'} x^t_{0jk} \leq 1 \qquad \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{7}$$

$$\sum_{j\in\mathcal{V}, j\neq i} \sum_{k\in\mathcal{K}} q^t_{jikm} = R^t_{im} + \sum_{j\in\mathcal{V}, j\neq i} \sum_{k\in\mathcal{K}} q^t_{ijkm} \qquad \forall i \in \mathcal{V}', \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{8}$$

$$I^{t-1}_{im} + R^t_{im} = c^t_{im} + I^t_{im} \qquad \forall i \in \mathcal{V}', \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{9}$$

$$\sum_{m\in\mathcal{M}} q^t_{ijkm} w_m \leq Q_k x^t_{ijk} \qquad \forall (i,j) \in \mathcal{A}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{10}$$

$$\sum_{j\in\mathcal{V}'} \sum_{k\in\mathcal{K}} q^t_{0jkm} \leq s^t_m \qquad \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{11}$$

$$\sum_{m\in\mathcal{M}} I^t_{im} w_m \leq HC_i \qquad \forall i \in \mathcal{V}', \forall t \in \mathcal{T} \tag{12}$$

$$q^t_{ijkm} \geq 0 \qquad \forall i,j \in \mathcal{V}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{13}$$

$$x^t_{ijk} \in \{0,1\} \qquad \forall i,j \in \mathcal{V}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{14}$$

$$I^t_{im} \geq 0 \qquad \forall i \in \mathcal{V}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{15}$$

$$R^t_{im} \geq 0 \qquad \forall i \in \mathcal{V}', \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{16}$$

The objective function, which minimizes the total cost, is given by expression (3). The total cost is computed as the sum of the transportation cost (*TFC*) and the inventory cost (*TIC*). Constraints (4) ensure that each customer is visited at most once in each time period. Constraints (5) guarantee that if a vehicle arrives at vertex $i$ in time period $t$, then it has to leave that vertex in the same time period. According to constraints (6), at most one vehicle can use arc $(i,j)$ in each time period. Constraints (7) satisfy that a vehicle performs a tour at most once in each period. Constraints (8) ensure that the difference between incoming and outgoing amounts of product $m$ in vertex $i$ should be equal to the received amount of product $m$ by vertex $i$ in time period $t$. Constraints (9) give the inventory level of product $m$ for each vertex $i$ in time period $t$. Capacity restrictions for each vehicle are satisfied by constraints (10). Constraints (11) ensure that the total amount of product $m$ delivered by the supplier cannot exceed the available amount of product. Constraints (12) guarantee that the inventory level of a vertex cannot be more than the inventory holding capacity within each time period. Constraints (13), (14), (15), and (16) give the integrality and nonnegativity restrictions for decision variables.

## 4 Stochastic Programming Model

In this study, two-stage stochastic programming, which is the most well-known stochastic programming (SP) model [8], is used to cope with the uncertainties in the MMIRP. In two-stage SP, uncertainties are characterized by a finite set of scenarios where each scenario is a realization of the uncertain parameters with specific values. Let $\mathcal{S}$ be the set of scenarios. It is assumed that the probability of realization of scenario $s \in \mathcal{S}$, which is represented by $p_s$, is known in advance. Decision variables are divided into two groups as first-stage and second-stage variables. First-stage variables represent the decisions which are made before the realization of uncertainties. Decisions for the second-stage variables are called as recourse decisions and made after the realization of uncertainties. SP minimizes (maximizes) total cost (benefit) results from first-stage variables and expected value of costs (benefits) as given by the distribution of the finite set of scenarios. A general formulation of the scenario-based mixed-integer linear SP for a minimization problem is given as follows:

$$\min \quad \mathbf{c}^T\mathbf{x} + \sum_{s\in\mathcal{S}} p_s \mathbf{q}_s^T \mathbf{y}_s \tag{17}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \tag{18}$$

$$\mathbf{T}_s\mathbf{x} + \mathbf{W}_s\mathbf{y}_s \leq \mathbf{h}_s \qquad \forall s \in \mathcal{S} \tag{19}$$

$$\mathbf{x} \in \mathbb{R}_+^{n_1-r_1} \times \mathbb{Z}_+^{r_1} \tag{20}$$

$$\mathbf{y}_s \in \mathbb{R}_+^{n_2-r_2} \times \mathbb{Z}_+^{r_2} \qquad \forall s \in \mathcal{S} \tag{21}$$

$\mathbf{x}$ and $\mathbf{y}_s$ are the first-stage and second-stage decision variables, respectively. The objective function (17) includes the sum of costs associated with first-stage variables and expected future costs related with second-stage variables. While constraints (18) include only the first-stage decision variables, constraints (19) connect the first-stage with second-stage decision variables. Domains of first-stage and second-stage decision variables are given in constraints (20) and (21), respectively.

There are several studies on IRP dealing with uncertainties. Kleywegt et al. [19] presented a brief review of stochastic and deterministic demand in IRP and applied Markov decision processes. Juan et al. [16] also gave a detailed review of the IRP with stochastic demands. Verweij et al. [35] developed new approaches for a stochastic routing problem but not IRP. They designed a sample average approximation method for three routing problems: the shortest path problem with random travel times, the shortest path problem with arc failures, and the traveling salesman problem with random travel times. Hvattum and Løkketangen [15] implemented scenario trees and a progressive hedging algorithm for stochastic IRP. Elbek et al. [13] utilized a neighborhood search to examine the uncertainty in accretion rate for materials by minimizing the operation cost. Huang et al. [14] modified ant colony optimization to minimize the total travel length of a multiproduct IRP with uncertain demand. Bertazzi et al. [7] proposed a stochastic dynamic programming approach to minimize the total cost for IRP with stochastic demand by implementing a heuristic algorithm on an exact dynamic programming algorithm. Popovi et al. [27] also developed a stochastic variable neighborhood search algorithm for IRP in fuel delivery and compared it with different deterministic models. To handle stochastic demand in IRP, Coelho et al. [11] developed heuristic policies, Alaei and Setak [2] proposed a metaheuristic algorithm, and Juan et al. [16] hybridized a simulation model with a heuristic. Bertazzi et al. [6] presented a dynamic programming model to minimize total cost including shortage penalty cost. While many of the studies concern demand as a stochastic parameter, Agra et al. [1] considered both sailing and port times as stochastic parameters because of the weather conditions and uncertain waiting times at ports. They implemented a two-stage sample average approximation method to solve this problem.

In this study, we develop a mixed-integer linear SP model for MMIRP. The uncertainties in the problem result from the demand and inventory holding cost. The proposed mixed-integer linear SP model for MMIRP is given as follows:

$$\min\ u \sum_{(i,j)\in\mathcal{A}} d_{ij}\left(\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} a_k x^t_{ijk} + \sum_{s\in\mathcal{S}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} b_k q^t_{ijkms} w_m\right) + \sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{V}'}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} p_s I^t_{ims} h_{ims} \tag{22}$$

s.t.

$$\sum_{j\in\mathcal{V},j\neq i}\sum_{k\in\mathcal{K}} x^t_{ijk} \leq 1 \qquad \forall i\in\mathcal{V}', \forall t\in\mathcal{T} \tag{23}$$

$$\sum_{j\in\mathcal{V},j\neq i} x^t_{ijk} = \sum_{j\in\mathcal{V},j\neq i} x^t_{jik} \qquad \forall i\in\mathcal{V}', \forall k\in\mathcal{K}, \forall t\in\mathcal{T} \tag{24}$$

$$\sum_{k\in\mathcal{K}} x^t_{ijk} \leq 1 \qquad \forall (i,j)\in\mathcal{A}, \forall t\in\mathcal{T} \tag{25}$$

$$\sum_{j\in\mathcal{V}'} x^t_{0jk} \leq 1 \qquad \forall k\in\mathcal{K}, \forall t\in\mathcal{T} \tag{26}$$

$$\sum_{j\in\mathcal{V},j\neq i}\sum_{k\in\mathcal{K}} q^t_{jikms} = R^t_{ims} + \sum_{j\in\mathcal{V},j\neq i}\sum_{k\in\mathcal{K}} q^t_{ijkms} \qquad \forall i\in\mathcal{V}', \forall m\in\mathcal{M}, \forall t\in\mathcal{T}, \forall s\in\mathcal{S} \tag{27}$$

$$I^{t-1}_{ims} + R^t_{ims} = c^t_{ims} + I^t_{ims} \qquad \forall i\in\mathcal{V}', \forall m\in\mathcal{M}, \forall t\in\mathcal{T}, \forall s\in\mathcal{S} \tag{28}$$

$$\sum_{m\in\mathcal{M}} q^t_{ijkms} w_m \leq Q_k x^t_{ijk} \qquad \forall (i,j)\in\mathcal{A}, \forall k\in\mathcal{K}, \forall t\in\mathcal{T}, \forall s\in\mathcal{S} \tag{29}$$

$$\sum_{j \in \mathcal{V}'} \sum_{k \in \mathcal{K}} q^t_{0jkms} \leq s^t_m \qquad \forall m \in \mathcal{M}, \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \tag{30}$$

$$\sum_{m \in \mathcal{M}} I^t_{ims} w_m \leq HC_i \qquad \forall i \in \mathcal{V}', \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \tag{31}$$

$$q^t_{ijkms} \geq 0 \qquad \forall i, j \in \mathcal{V}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \tag{32}$$

$$x^t_{ijk} \in \{0, 1\} \qquad \forall i, j \in \mathcal{V}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{33}$$

$$I^t_{ims} \geq 0 \qquad \forall i \in \mathcal{V}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \tag{34}$$

$$R^t_{ims} \geq 0 \qquad \forall i \in \mathcal{V}', \forall m \in \mathcal{M}, \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \tag{35}$$

This SP formulation is the straightforward extension of deterministic formulation given in Sect. 3. $c^t_{ims}$ and $h_{ims}$ are scenario-dependent parameters. $c^t_{ims}$ denotes the demand of product $m$ of vertex $i$ in time period $t$ under scenario $s$, and $h_{ims}$ represents unit inventory holding cost for product $m$ in vertex $i$ under scenario $s$. In the SP model, variables are partitioned in two stages. The binary variables ($x^t_{ijk}$), which refer to the decision whether transportation activity is performed on arc $(i, j) \in A$ by vehicle $k$ in time period $t$, are the first-stage decision variables. After the transportation decisions are made, $q^t_{ijkm}$, $R^t_{ims}$, and $I^t_{ims}$ are decided as the second-stage variables, which represent the load of product $m$ transported from vertex $i$ to vertex $j$ directly by vehicle $k$, the received amount of product $m$ and the inventory level of product $m$ in vertex $i$ at the end of time period $t$ under the scenario $s$, respectively. Objective function (22) minimizes the sum of current transportation costs and expected future inventory costs. Constraints (23), (24), (25), and (26) include first-stage variables, while constraints (27), (28), (29), (30), and (31) connect the first-stage to the second-stage variables. In order to solve this problem, a sample average approximation approach is used. A brief definition of sample average approximation is given in the next section.

## 5 Methodology: Sample Average Approximation

Sample average approximation (SAA) method is a scenario-based stochastic optimization algorithm. The scenarios are mostly generated by Monte-Carlo simulation in SAA. The probability of each scenario is accepted as $1/N$ where $N$ denotes the sample size. A typical two-stage SAA method has the following steps [18]:

Step 1: Let $M$ be the number of independent samples and $N$ be the sample size. At the beginning of the algorithm, $M$ and $N$ values are determined. If $N$ is a big number, the problem complexity and computation time will increase. On the other hand, bigger $N$ values show a higher tendency to find a near optimal solution. With smaller $N$ values, the solution of the problem will not be close to the optimal solution.

Step 2: For each scenario $m = \{1, \ldots, M\}$, the following sub-steps are repeated.

Step 2.1: A sample is generated in size $N$ and the SAA problem is solved. Let $\hat{\mathbf{x}}_N^m$ and $\hat{\upsilon}_N^m$ indicate the optimal value of decision variables and objective function, respectively.

Step 2.2: In order to test the solution, a new sample is generated in size $N'$ which is bigger than $N$. The value of the first decision variables in the solution produced with size $N$ is given directly to this new problem as shown in constraint (36). Let $g_{N'}^m$ denote the optimal objective function value for each $m$, as shown in (37) where $\xi$ shows the scenario.

$$\hat{\mathbf{x}}_N^m = \hat{\mathbf{x}}_{N'}^m \qquad \forall m \in M \tag{36}$$

$$g_{N'}^m = \mathbf{c}^{\mathbf{T}}\mathbf{y} + E[Q(\hat{\mathbf{x}}, \xi)] \qquad \forall m \in M \tag{37}$$

Step 3: For a minimization problem, solving the model in sample size $N$ gives a lower bound for the optimal objective value, and sample size $N'$ gives an upper bound and vice versa. The lower bound is estimated by the mean of optimal objective function values produced in the first problem with sample size $N$, whereas the upper bound is estimated by the mean of optimal objective function values produced in the second problem with $N'$. The estimation of the lower bound is shown in (38), and the variance of the lower bound is shown in (39).

$$\overline{\upsilon}_N^M = \frac{1}{M} \sum_{m \in M} \hat{\upsilon}_N^m \tag{38}$$

$$s_{\upsilon_N^M}^2 = \frac{1}{M(M-1)} \sum_{m \in M} \left(\hat{\upsilon}_N^m - \overline{\upsilon}_N^M\right)^2 \tag{39}$$

Since $N'$ scenarios are generated in the second problem and the first stage decision variables are taken from the solution of the first problem, the second problem gives an upper bound for the optimal objective function. The estimation of the upper bound and the variance of the upper bound are shown in (40) and (41), respectively.

$$\overline{g}_{N'}^M = \frac{1}{M} \sum_{m \in M} g_{N'}^m \tag{40}$$

$$s_{g_{N'}^M}^2 = \frac{1}{M(M-1)} \sum_{m \in M} \left(g_{N'}^m - \overline{g}_{N'}^M\right)^2 \tag{41}$$

Step 4: The difference between the lower bound and the upper bound defines the optimality gap. Since the optimal solution is in this gap, a small gap is desired. If the estimation of the optimality gap and its variance are sufficiently small, there is no sufficient evidence for any significant difference between the lower and upper bounds. Then, the algorithm stops. If not, the sample size $N$ and/or $N'$ is increased, and the algorithm returns to step 2.

Step 5: The best solution is selected among the candidate solutions produced with sample size $N$ in $M$ independent samples. The minimum of $\hat{v}_N^m$ in $M$ experiments gives the best solution with decision variables $\hat{\mathbf{x}}_N^m$ and $\hat{\mathbf{y}}_N^m$.

$$v^* = \arg\min \left\{ \hat{v}_N^m : m \in \{1, \ldots, M\} \right\} \tag{42}$$

## 6 Computational Results

The data set generated by Coelho and Laporte [10] for MMIRPs is used in this study to assess the performance of the proposed methodology. The lognormal distribution is convenient to represent economic stochastic parameters because it provides nonnegativity [30]. Thus, the lognormal distribution is utilized for the inventory holding cost where the deterministic data is taken as mean and 2.5% of mean value is received as standard deviation. The demand is uniformly distributed where the parameters, minimum and maximum values, are determined by using the deterministic data points and a coefficient of 0.2. For example, if the demand of vertex $i$ is $d_i$ in the data set, then the minimum and maximum values are taken as $0.8 \times d_i$ and $1.2 \times d_i$, respectively. Other parameters, i.e., deterministic parameters, are provided from the data set.

Since Coelho and Laporte [10] computed transportation costs considering only the distances traveled by vehicles, fuel consumption rates and fuel prices do not exist in the data set. In the scope of this study, fuel rates of the vehicles ($a_k$ and $b_k$) are taken from Kopfer and Kopfer [20] and given in Table 3. For each instance, the vehicles are chosen in the order given by Table 3 according to the size of the fleet. For example, if the instance only includes one vehicle, the vehicle's fuel consumption rates are $a_1 = 26$ and $b_1 = 0.36$. In the same way, if the instance has the fleet with three vehicles, corresponding parameters for these vehicles are $a_1 = 26$, $b_1 = 0.36$, $a_2 = 20$, $b_2 = 0.76$, $a_3 = 15$, and $b_3 = 1.54$. In the data set of Coelho and Laporte [10], the number of vehicles is 5 for some instances. In this case, the fleet has 2 type 1 vehicles and 1 of each other types. Without loss of generality, the weight of each product ($w_m$) and fuel price ($u$) are fixed as 1.

**Table 3** Fuel rates for vehicles [20]

| Vehicles | $a_k$ | $b_k$ |
|---|---|---|
| 1 | 26 | 0.36 |
| 2 | 20 | 0.76 |
| 3 | 15 | 1.54 |
| 4 | 8 | 3.31 |

**Table 4** Number of samples and sample sizes for testing

| $M$ | $N$ | $N'$ |
|---|---|---|
| 5 | 20 | 100 |
| 10 | 40 | 300 |
| 15 | 60 | – |

**Table 5** Instances used in performance analysis [10]

| #customers | #products | #vehicles | Horizon |
|---|---|---|---|
| 10 | 1 | 1 | 3 |
| 10 | 1 | 1 | 5 |
| 10 | 1 | 3 | 3 |
| 10 | 1 | 3 | 5 |
| 10 | 3 | 1 | 3 |
| 10 | 3 | 1 | 5 |

The proposed SP model and SAA are coded in Microsoft Visual Studio 2013 C#. The computational tests are performed on a portable work station with a 2.20 GHz Intel Core i7-4702HQ processor and 16 Gb of RAM.

In order to determine the proper values for the parameters of SAA – the number of independent samples ($M$), sample size ($N$), and the size of test sample ($N'$) – computational experiments are performed. All values of $M$, $N$, and $N'$ tested to determine the best parameter combination are given in Table 4. The instances used to test these parameter combinations are given in Table 5. The columns refer to the number of customers (#customers), the number of products (#products), the number of vehicles (#vehicles), and the length of the planning horizon, respectively. In the data set, there are five instances generated for each problem size. The first instances of each problem size are performed with each parameter combination given in Table 4. The average percentages of optimality gap and computational time (CPU time) obtained by SAA with different parameter combinations are given in Figs. 1 and 2, respectively. The parameter combinations that result in high computational time are not shown in the figures. According to the percentages of optimality gap and computational time, the parameter combination $(M, N, N') = (10, 20, 100)$ is selected and implemented for all instances.

After selecting a proper combination for $M$, $N$, and $N'$, SAA is applied to the instances given in Table 6 to evaluate the performance of SAA. The number of decision variables and constraints is higher in the SP model than in the deterministic model. This situation results in higher computational effort in SP. Therefore, only the instances having five customers are considered in this study. There are three alternatives for the other parameters: the number of products can be 1, 3, or 5; the number of vehicles can be 1, 3, or 5; and the length of the horizon can be 3, 5, or 7. The time limit is set to 10,800 s. Computational results are placed in Table 7. Names of the instances are given in the first column. The second to fifth columns give the size of the instances. The sixth and seventh columns present the CPU time and the optimality gap attained by SAA, respectively. We exclude the experiments which take more than the time limit.

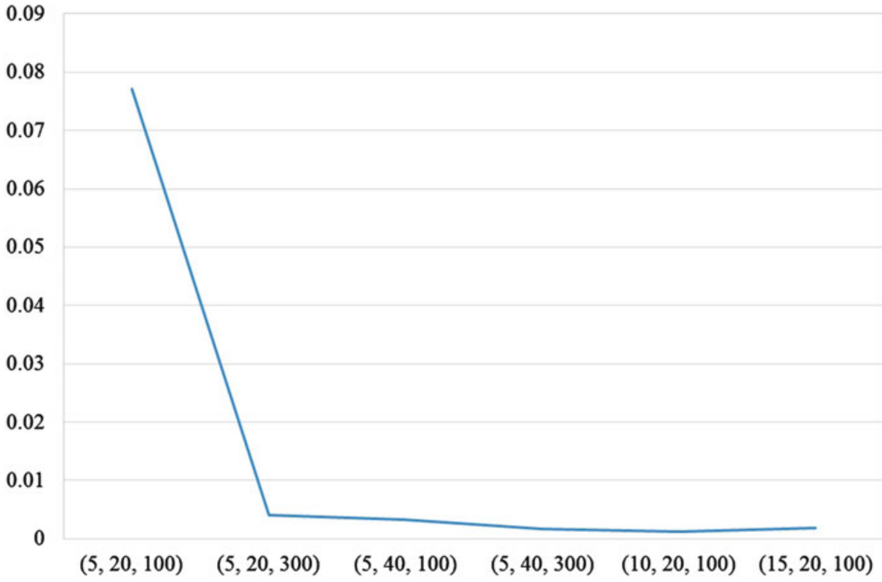


**Fig. 1** Average optimality gap for the instances used for parameter setting

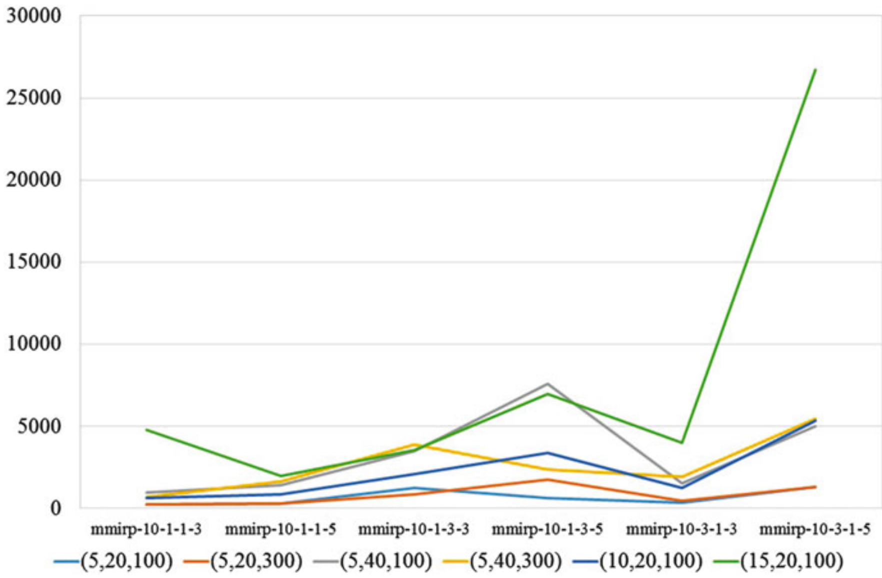


**Fig. 2** CPU time for the instances used for parameter setting

**Table 6** Instances used in performance analysis [10]

| #customers | #products | #vehicles | Horizon |
|---|---|---|---|
| 5 | 1 | 1 | 3 |
| 5 | 1 | 1 | 5 |
| 5 | 1 | 1 | 7 |
| 5 | 1 | 3 | 3 |
| 5 | 1 | 3 | 5 |
| 5 | 1 | 3 | 7 |
| 5 | 1 | 5 | 3 |
| 5 | 3 | 1 | 3 |
| 5 | 3 | 1 | 5 |
| 5 | 3 | 1 | 7 |
| 5 | 5 | 1 | 3 |
| 5 | 5 | 1 | 5 |

The optimality gap provides the closeness of the obtained results to the optimal solution. Although the optimality gap is less than 1% for all experiments, the CPU time is getting larger when the number of products and the time horizon increase. To improve the computational performance of SAA, it can be hybridized with other algorithms, or decomposition algorithms can be used. Acceleration of the algorithm in terms of computational time is left for further research.

# 7 Conclusion

In this study, a two-stage stochastic model is developed for the MMIRP and solved by the SAA. The proposed model considers uncertainty in demand and inventory holding cost. The objective of the problem is to minimize total cost including total transportation cost and total inventory cost. Total transportation cost includes also fuel consumption cost, which is an important issue for the environmental aspect. Load, distance, and vehicle features are considered during the computation of total fuel consumption. The problem is modeled as a two-stage SP problem. In the first stage, transportation routes between cities are decided. Transportation quantities and inventory levels are decision variables for the second stage.

The experiments based on data from the literature are analyzed in terms of the optimality gap and CPU time. Since the numbers of decision variables and constraints are higher in the SP model than in the deterministic model, only the instances having five customers are considered in this study. The optimality gap shows the closeness to the optimal solution. Optimality gaps that are less than 1% are obtained for all instances. However very high CPU time is observed for the big instances.

**Table 7** Computational results

| Instance | N | M | K | T | CPU time | Gap % |
|---|---|---|---|---|---|---|
| mmirp-10-1-1-3-1 | 5 | 1 | 1 | 3 | 694.26 | 0.0012 |
| mmirp-10-1-1-3-3 | 5 | 1 | 1 | 3 | 224.19 | 0.0017 |
| mmirp-10-1-1-3-4 | 5 | 1 | 1 | 3 | 140.36 | 0.0001 |
| mmirp-10-1-1-3-5 | 5 | 1 | 1 | 3 | 438.48 | 0.0008 |
| mmirp-10-1-1-5-1 | 5 | 1 | 1 | 5 | 915.23 | 0.0036 |
| mmirp-10-1-1-5-2 | 5 | 1 | 1 | 5 | 1171.72 | 0.0027 |
| mmirp-10-1-1-5-3 | 5 | 1 | 1 | 5 | 393.14 | 0.0028 |
| mmirp-10-1-1-5-4 | 5 | 1 | 1 | 5 | 811.33 | 0.0035 |
| mmirp-10-1-1-5-5 | 5 | 1 | 1 | 5 | 440.53 | 0.0053 |
| mmirp-10-1-1-7-1 | 5 | 1 | 1 | 7 | 481.92 | 0.0010 |
| mmirp-10-1-1-7-2 | 5 | 1 | 1 | 7 | 1170.57 | 0.0011 |
| mmirp-10-1-1-7-3 | 5 | 1 | 1 | 7 | 2039.82 | 0.0018 |
| mmirp-10-1-1-7-4 | 5 | 1 | 1 | 7 | 1441.51 | 0.0001 |
| mmirp-10-1-1-7-5 | 5 | 1 | 1 | 7 | 1733.68 | 0.0002 |
| mmirp-10-1-3-3-1 | 5 | 1 | 3 | 3 | 2176.60 | 0.0019 |
| mmirp-10-1-3-3-2 | 5 | 1 | 3 | 3 | 615.39 | 0.0021 |
| mmirp-10-1-3-3-3 | 5 | 1 | 3 | 3 | 731.71 | 0.0024 |
| mmirp-10-1-3-3-4 | 5 | 1 | 3 | 3 | 997.18 | 0.0021 |
| mmirp-10-1-3-3-5 | 5 | 1 | 3 | 3 | 1024.68 | 0.0028 |
| mmirp-10-1-3-5-1 | 5 | 1 | 3 | 5 | 3377.43 | 0.0005 |
| mmirp-10-1-3-5-4 | 5 | 1 | 3 | 5 | 1940.61 | 0.0016 |
| mmirp-10-1-3-5-5 | 5 | 1 | 3 | 5 | 1453.64 | 0.0026 |
| mmirp-10-1-3-7-3 | 5 | 1 | 3 | 7 | 1892.48 | 0.0017 |
| mmirp-10-1-5-3-1 | 5 | 1 | 5 | 3 | 4644.89 | 0.0055 |
| mmirp-10-1-5-3-2 | 5 | 1 | 5 | 3 | 917.25 | 0.0015 |
| mmirp-10-3-1-3-1 | 5 | 3 | 1 | 3 | 1278.72 | 0.0002 |
| mmirp-10-3-1-3-2 | 5 | 3 | 1 | 3 | 1233.30 | 0.0025 |
| mmirp-10-3-1-3-3 | 5 | 3 | 1 | 3 | 754.98 | 0.0013 |
| mmirp-10-3-1-3-4 | 5 | 3 | 1 | 3 | 2286.31 | 0.0013 |
| mmirp-10-3-1-3-5 | 5 | 3 | 1 | 3 | 3235.96 | 0.0015 |
| mmirp-10-3-1-5-1 | 5 | 3 | 1 | 5 | 5522.17 | 0.0003 |
| mmirp-10-3-1-5-2 | 5 | 3 | 1 | 5 | 4851.80 | 0.0016 |
| mmirp-10-3-1-5-3 | 5 | 3 | 1 | 5 | 10,844.44 | 0.0012 |
| mmirp-10-3-1-5-4 | 5 | 3 | 1 | 5 | 7227.30 | 0.0013 |
| mmirp-10-3-1-5-5 | 5 | 3 | 1 | 5 | 3848.28 | 0.0016 |
| mmirp-10-3-1-7-2 | 5 | 3 | 1 | 7 | 9614.08 | 0.0023 |
| mmirp-10-3-1-7-3 | 5 | 3 | 1 | 7 | 10,592.53 | 0.0014 |
| mmirp-10-3-1-7-4 | 5 | 3 | 1 | 7 | 10,459.43 | 0.0008 |
| mmirp-10-3-1-7-5 | 5 | 3 | 1 | 7 | 10,623.20 | 0.0006 |
| mmirp-10-5-1-3-1 | 5 | 5 | 1 | 3 | 6259.56 | 0.0002 |
| mmirp-10-5-1-3-2 | 5 | 5 | 1 | 3 | 5391.63 | 0.0007 |
| mmirp-10-5-1-3-3 | 5 | 5 | 1 | 3 | 3116.87 | 0.0012 |
| mmirp-10-5-1-3-4 | 5 | 5 | 1 | 3 | 3404.58 | 0.0007 |
| mmirp-10-5-1-3-5 | 5 | 5 | 1 | 3 | 3292.02 | 0.0012 |
| mmirp-10-5-1-5-2 | 5 | 5 | 1 | 5 | 9181.67 | 0.0030 |

Although the results are promising to obtain good quality solutions for MMIRP with uncertainties, the computational performance should be improved for large instances. In further studies, decomposition algorithms will be used to improve the computational performance.

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# The Impact of Bunker Risk Management on $CO_2$ Emissions in Maritime Transportation Under ECA Regulation

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**Abstract** The shipping industry carries over 90% of the world's trade, and is hence a major contributor to $CO_2$ and other airborne emissions. As a global effort to reduce air pollution from ships, the implementation of the ECA (Emission Control Areas) regulations has given rise to the wide usage of cleaner fuels. This has led to an increased emphasis on the management and risk control of maritime bunker costs for many shipping companies. In this paper, we provide a novel view on the relationship between bunker risk management and $CO_2$ emissions. In particular, we investigate how different actions taken in bunker risk management, based on different risk aversions and fuel hedging strategies, impact a shipping company's $CO_2$ emissions. We use a stochastic programming model and perform various comparison tests in a case study based on a major liner company. Our results show that a shipping company's risk attitude on bunker costs has impacts on its $CO_2$ emissions. We also demonstrate that, by properly designing its hedging strategies, a shipping company can sometimes achieve noticeable $CO_2$ reduction with little financial sacrifice.

## 1 Introduction

Maritime transport is one of the most important freight transportation modes in the world, since it is by far the most cost-effective alternative for transporting large-volume goods between continents. In 2015, more than 90% of global trade is carried by sea [13]; therefore, the shipping industry plays a vital role in the world economy.

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Due to the enormous amount of marine fuel consumed by the world fleet, the maritime sector is one of the biggest sources of $CO_2$ emissions among all transportation industries. International shipping emits approximately 2.2% of the world's anthropogenic $CO_2$ emissions. This number may further increase to 17% by 2050 if no effective control measure is applied [3].

On the other hand, fuel cost is the major cost driver in the shipping industry. It is therefore critical for a shipping company to manage its bunker purchasing and consumption properly. In practice, fuel prices are highly volatile which could bring considerable risks. Bunker risk management is then commonly applied by shipping companies in order to control the risk brought by the high volatility of the fuel cost. For example, risk measures such as CVaR (conditional value-at-risk) from the field of financial portfolio management may be used to represent a shipping company's risk aversion. Fuel hedging is also one of the popular risk control approaches in the shipping industry. As a contractual tool, it allows the shipping company to reduce its exposure to fuel risk by establishing a fixed or capped cost for its future fuel consumption.

In [11], a maritime bunker management (MBM) problem that combines tactical fuel hedging and operational ship routing and speed optimization is introduced, which aims to minimize a shipping company's expected total bunker costs based on its risk attitude. Using a case study, the authors show that the integration of the tactical and operational levels of MBM is vital for a shipping company after the implementation of Emission Control Areas (ECA) which regulate sulfur emissions. In this study, the same mathematical model and a similar case are used, but we focus on the impact of a shipping company's bunker risk management on its fleet's $CO_2$ emissions.

As individual research topics, both $CO_2$ emissions and bunker risk management have been intensively studied in the maritime transportation literature. Regarding $CO_2$ emissions, many studies focus on the relationship between speed reduction, also known as slow steaming, and emission reduction. Corbett et al. [6] evaluate whether speed reduction is a cost-effective option to mitigate $CO_2$ emissions for ships calling on US ports. Cariou [4] examines the break-even price of the maritime bunker at which the slow steaming strategy and the corresponding $CO_2$ emissions reduction are sustainable in the long run. Lindstad et al. [17] investigate the impacts of slow steaming on $CO_2$ emissions and costs in maritime transport. They show that the emissions of $CO_2$ can be decreased by 19% with a negative abatement cost and by 28% at a zero abatement cost if a proper slow steaming strategy is applied. Maloni et al. [18] show that under current conditions, extra slow steaming can achieve substantial reductions in both total cost and $CO_2$ emissions. Tai and Lin [25] compare the unit $CO_2$ emissions in the cases when daily frequency or slow steaming strategies are applied in international container shipping on Far East-Europe routes. Wong et al. [27] generalize the traditional discrete cost-based decision support model in slow steaming maritime operations into novel continuous utility-based models which balance fuel consumption, carbon emission, and service quality. Another research direction on $CO_2$ reduction in maritime transportation is green ship routing and scheduling. It extends the traditional ship routing and scheduling problems and integrates environmental concerns. Related studies can be found in, for instance, [23, 15] and [7].

As regards bunker risk management, fuel hedging is the most commonly used instrument in maritime transportation. Menachof and Dicer [19] argue that the bunker surcharges widely applied in liner shipping can be eliminated and replaced by the utilization of oil commodity futures contracts. The hedging effectiveness of futures contracts among different fuel commodities is examined and compared in [1]. Wang and Teo [26] offer a comprehensive review of all the fuel hedging instruments available on the market and integrate fuel hedging into the modeling of liner network planning. Pedrielli et al. [21] propose a game theory-based approach to optimize the fuel hedging contract so that the expected profit for the bunker supplier and the expected refueling cost for the shipping company are maximized and minimized, respectively.

To the best of our knowledge, none of the studies in the literature have explored the relationship between bunker risk management and $CO_2$ emissions in maritime transportation. Such a gap in knowledge is, to a certain degree, expected as the former had no impact on the latter in the past. This is because in most circumstances, the sailing pattern of the fleet (and hence its $CO_2$ emissions) is relatively fixed and irrelevant to the shipping company's bunker risk management, i.e., the shortest path and the slowest possible speed is usually chosen during the whole voyage, no matter what actions are taken in terms of the company's bunker risk management, such as the amounts of marine fuel hedged.

However, things have changed significantly in the shipping industry during the last decade due to the implementation of the ECA regulation. It is a regional sulfur emission control regulation that restricts the maximum sulfur content in the marine bunker burnt inside the regulated areas; see Fig. 1. The ECA regulation has forced the shipping companies, who have not invested in sulfur emission reduction technologies (scrubber system or liquefied natural gas-powered propulsion), to switch their fuels from the traditional heavy fuel oil (HFO) to the expensive marine gasoline oil (MGO) when their vessels navigate inside ECA.

One of the consequences of the ECA regulation and the substantial price difference between MGO and HFO is that the shipping companies no longer necessarily operate their fleet in the old-fashioned "shortest and slowest possible" way. They now have the motivation to change the sailing behavior of the vessels, so as to minimize the total bunker cost and simultaneously comply with the ECA regulation. Two types of potential change in sailing behavior, namely, speed differentiation and ECA evasion, are shown in [8] and [9]. We illustrate these two types of sailing behavior change in Fig. 2. First, in order to reduce the consumption of MGO, a ship may choose to use different speeds inside and outside ECA, as shown in Fig. 2a, if the voyage involves both regulated and unregulated sea areas. Second, a vessel may make a detour so that the sailing distance inside ECA, and hence its MGO consumption, can be considerably decreased; see, for example, Fig. 2b. However, to what extent the speed differentiation and ECA-evasion strategies will be applied

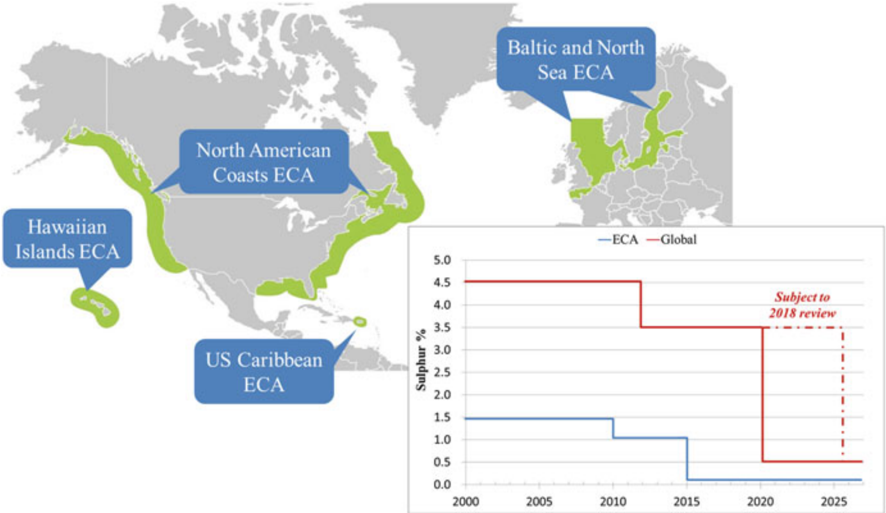



**Fig. 1** Map and requirements of the emission control areas

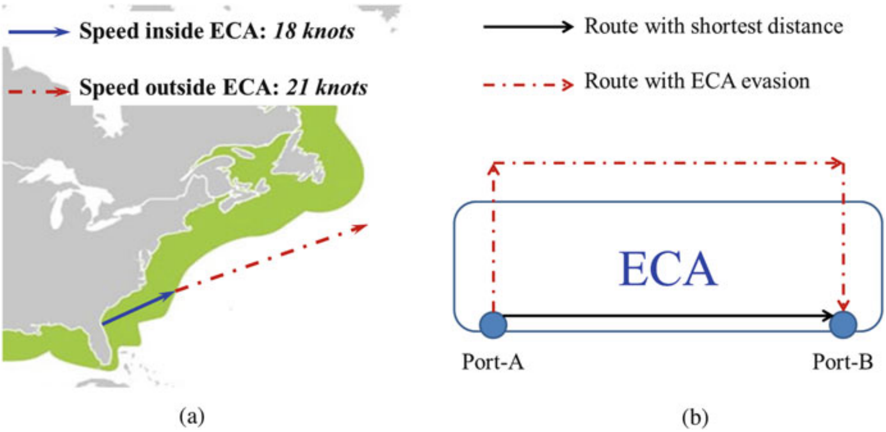



**Fig. 2** Two types of sailing behavior change after the implementation of ECA regulation. (**a**) Speed differentiation. (**b**) ECA evasion

depends on the price difference between MGO and HFO. For example, if the price difference increases, so will the incentive to reduce MGO consumption, in which case sailing a route with lower ECA involvement may be more beneficial.

A vessel's $CO_2$ emissions mainly depend on its fuel consumption. The $CO_2$ emission factors we use in this paper for MGO and HFO are 3.082 (tons/ton fuel) and 3.021 (tons/ton fuel), respectively [22]. Therefore, it is the total amount of the two fuels consumed that affects the $CO_2$ emissions the most, rather than the different combination of the two. Fuel consumption is further determined by the vessel's sailing speed and travelling distance. After the introduction of ECA, the shipping company's optimal speed and routing choices, which may include speed differentiation and ECA evasion, also depend on the prices of the two fuels and the associated hedging decisions made [11]. In this paper, we seek to investigate how different actions taken in bunker risk management, including different settings of risk aversion and fuel hedging strategies, impact the shipping company's optimal speed and routing choices and the corresponding $CO_2$ emissions.

We use the stochastic programming model introduced in Gu et al. [11] and propose various comparison tests based on different levels of risk aversion, fuel hedging strategies, and fuel prices. The tests are performed on a case based on a real liner service offered by Wallenius Wilhelmsen Logistics (WWL), one of the world's largest liner service providers for rolling equipment. We aim to provide a novel view on the relationship between bunker risk management and $CO_2$ emissions, which would hopefully contribute to the worldwide effort in reducing greenhouse gas emissions.

The rest of the paper is structured as follows. Section 2 gives the description of the maritime bunker management (MBM) problem and the mathematical model. In Sect. 3 we introduce the test case and the scenario generation process. Section 4 presents the results of our computational study. Our conclusion is given in Sect. 5.

## 2 The Problem and Mathematical Model

The problem description and the mathematical formulation are given in this section. In Sect. 2.1, we summarize the settings and assumptions of the problem. The mathematical formulation is then presented in Sect. 2.2.

### *2.1 Problem Statement*

First, we introduce four important terms, *loop*, *leg*, *leg option*, and *stretch*, which are frequently used in this paper; see Fig. 3 for illustration. A *loop* refers to a round trip calling several ports in a predetermined order, while a *leg* refers to the voyage between two consecutive ports in the loop. A *leg option* represents a possible sailing path for a leg. For different leg options of a same leg, the total sailing distance and the sailing distance inside ECA are also different. A leg option may have one or more *stretches*. When the vessel crosses the ECA border, the current stretch ends and a new one begins. We also combine the stretches of the same type (on which we assume the same speed) for every single-leg option and thus represent each leg option with only two segments: the ECA stretch and the non-ECA stretch. For example, in Fig. 3c, we combine Stretch 1 and Stretch 3 as the ECA stretch.

For simplification, the MBM problem in this paper only considers one single vessel operating on one single loop. The length of the planning period is assumed to

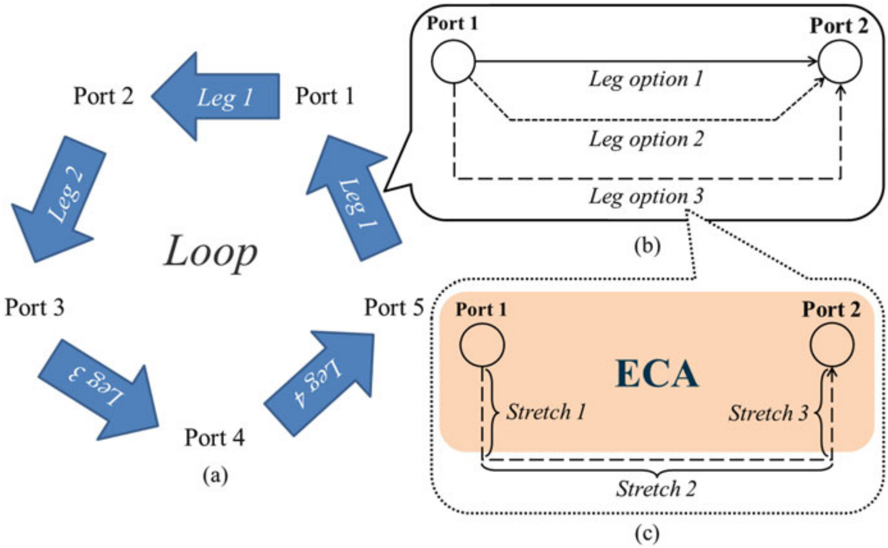



**Fig. 3** Illustration of a loop and its associated legs, leg options, and stretches

be equal to the scheduled time for the vessel to finish a round trip on the loop. The sequence of the port calls on the loop and the related leg information, including all possible leg options for every leg and the associated stretches, are also assumed to be given as input to our model.

As an essential instrument in bunker risk management, fuel hedging reduces the fuel consumers' exposure to financial risk caused by volatile fuel prices. We consider the so-called *forward-fuel contract with exit terms and physical supply* (FFC in the following) in this study. An FFC endows the shipping company with the right to buy a specified amount of a certain type of fuel with a predetermined price during an agreed time period. The forward price of a certain fuel in the FFC is normally higher than this fuel's expected price during the contract period. We assume this to be the case in our tests as well, and as a result, a risk-averse shipping company can, and normally will, use FFC for risk control purposes, while a risk-neutral one never enters the forward market due to the expected loss. However, if the shipping company realizes that the remaining fuel in the ongoing FFC is no longer needed and decides to terminate the contract, the leftovers are sold back to the fuel supplier with a penalty.

The spot prices for MGO and HFO fuels during the planning period are assumed to be stochastic, and the MBM problem can be described using a two-stage model with scenarios representing the stochastics. In the first stage, decisions with respect to the amounts of MGO and HFO to be hedged in an FFC must be made at the beginning of the planning period. The spot prices of the two fuels in different scenarios are then realized in the second stage and are assumed to remain constant during the whole planning period. Several operational decisions will be made

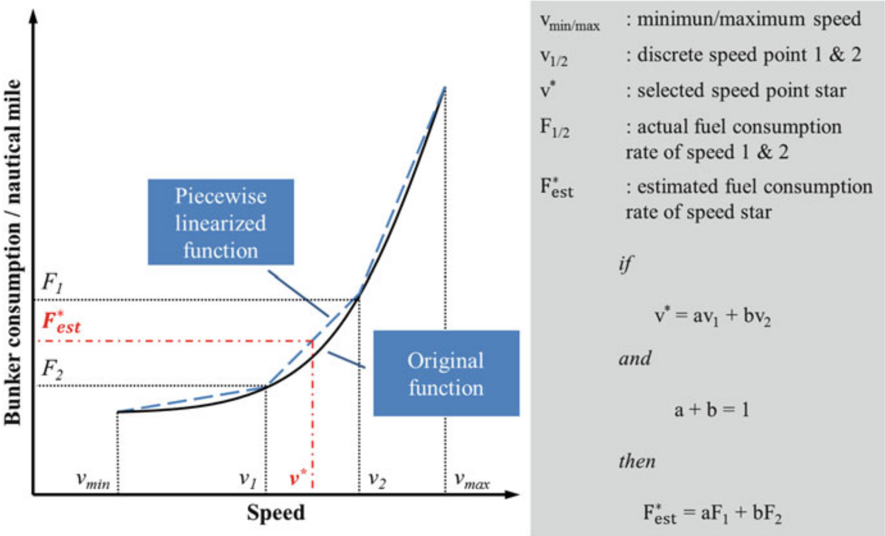



**Fig. 4** Piecewise linearization of the fuel consumption function

afterward based on the realized fuel prices and the first-stage hedging decisions. These second-stage decisions are made of two major parts. The first consists of speed and routing choices on each leg. The second part consists of fuel allocation decisions, i.e., how much spot and forward fuels should be used during operations. The objective of the MBM problem is to minimize the total bunker cost which is the sum of the first-stage purchasing costs of the forward fuels and the expected second-stage costs on spot fuels and penalties for unused forward fuels, meanwhile controlling the bunker cost risk within a desirable level.

The relationship between a vessel's sailing speed and its fuel consumption per unit distance is normally considered as a quadratic function [20]. We use a piecewise linearization approach [2] to approximate the fuel consumption rate for different sailing speeds, as shown in Fig. 4. Note that an overestimation is expected in the application of this approach (see Andersson et al. for detailed discussion), but it is normally insignificant as long as sufficient discrete speed points are used. A good estimation of the relation between speed and traveling time can also be made using this approach.

As part of the risk control measures for bunker risk management, we model the risk attitude of the shipping company using a conditional value-at-risk (CVaR) approach, which is extensively used in the field of financial risk management to evaluate various risks. The standard expression of CVaR with a discrete probability distribution can be expressed as follows:

$$\mathrm{CVaR}_\gamma(\phi) = \frac{1}{1-\gamma} \sum_{s:f(\phi,\delta_s)\geq \mathrm{VaR}_\gamma(\phi)} p_s f(\phi, \delta_s) \tag{1}$$

In (1) $\phi$ are the decision variables, $\gamma$ refers to the confidence level, $f(\phi, \delta_s)$ refers to the cost or loss function whose risk (expected value in the worst cases) needs to be controlled, $\delta$ are the random variables, while $\delta_s$ represent the realization of the random variables in scenario $s$ and $p_s$ which refers to the probability of scenario $s$. According to the definition of CVaR, only the scenarios in which the cost is larger than the VaR (value-at-risk) value ($s : f(\phi, \delta_s) \geq \mathrm{VaR}_\gamma(\phi)$) need to be accumulated for the calculation of CVaR. Since the expression of CVaR explicitly involves the VaR function, it becomes difficult to work with due to the nonlinearity. Therefore, it is common to use its equivalent auxiliary alternative [24]:

$$\mathrm{CVaR}_\gamma(\phi, \alpha) = \alpha + \frac{1}{1-\gamma} \sum_{s: f(\phi,\delta_s) \geq \alpha} p_s [f(\phi, \delta_s) - \alpha] \tag{2}$$

$$\mathrm{CVaR}_\gamma(\phi, \alpha) = \alpha + \frac{1}{1-\gamma} \sum_{s \in S} p_s [f(\phi, \delta_s) - \alpha]^+ \tag{3}$$

Note that the VaR function in (1) is replaced by the artificial variable $\alpha$ in (2). Moreover, the expression of CVaR is further simplified to (3) through the $[\ \ ]^+$ operator which produces nonnegative results.

In our model, we impose CVaR constraints on the total bunker costs to achieve the desired risk control effect. Two key parameters, a confidence level and a maximum tolerable CVaR value, are defined and used as inputs for our model. For instance, if the confidence level and the maximum CVaR value are set to 95% and 1.2 million USD, respectively, the CVaR constraints will then ensure that the expected total bunker costs in the worst 5% cases will not exceed 1.2 million USD during the planning period.

## 2.2 Mathematical Formulation

The mathematical formulation is presented as follows:

*Sets*

| | |
|---|---|
| $J$ | Set of sailing legs along the loop |
| $R_j$ | Set of leg options for Leg $j$ |
| $V$ | Set of feasible discrete speed points for the ship |
| $S$ | Set of scenarios |

*Parameters*

| | |
|---|---|
| $P^{\mathrm{MGO}-F}$ | Price per ton of MGO agreed in the forward-fuel contract |
| $P^{\mathrm{HFO}-F}$ | Price per ton of HFO agreed in the forward-fuel contract |
| $P_s^{\mathrm{MGO}-S}$ | Price per ton of MGO on spot market under scenario $s$ |
| $P_s^{\mathrm{HFO}-S}$ | Price per ton of HFO on spot market under scenario $s$ |

$P^{\text{MGO}-P}$ Penalty per ton for the unused MGO left in the forward-fuel contract
$P^{\text{HFO}-P}$ Penalty per ton for the unused HFO left in the forward-fuel contract
$\overline{W}_j$ Latest starting time for Leg $j$
$W_j^S$ Service time for Leg $j$ in the departing port
$W_{jrv}^{\text{ECA}}$ Sailing time on ECA stretches on Leg $j$ under Leg option $r$ with Speed $v$
$W_{jrv}^{N}$ Sailing time on non-ECA stretches on Leg $j$ under Leg option $r$ with Speed $v$
$D_{jr}^{\text{ECA}}$ Sailing distance on ECA stretches on Leg $j$ under Leg option $r$
$D_{jr}^{N}$ Sailing distance on non-ECA stretches on Leg $j$ under Leg option $r$
$F_v$ Fuel consumption per unit distance sailed with speed alternative $v$ (same for both HFO and MGO)
$p_s$ Probability of scenario $s$ taking place
$\gamma$ Confidence level applied in CVaR
$A_\gamma$ The maximum tolerable CVaR value under confidence level $\gamma$

*Decision variables*

$x_{jrvs}^{\text{ECA}}$ Weight of speed choice $v$ used on ECA stretches on Leg $j$ with Leg option $r$ under scenario $s$
$x_{jrvs}^{N}$ Weight of speed choice $v$ used on non-ECA stretches on Leg $j$ with Leg option $r$ under scenario $s$
$y_{jrs}$ Binary variables representing the decisions on route selection, equal to 1 if Leg option $r$ is sailed on Leg $j$ under scenario $s$ and 0 otherwise
$z_{js}^{\text{MGO}-S}$ Amount of MGO from spot market used on Leg $j$ under scenario $s$
$z_{js}^{\text{MGO}-F}$ Amount of MGO from forward contract used on Leg $j$ under scenario $s$
$z_{js}^{\text{HFO}-S}$ Amount of HFO from spot market used on Leg $j$ under scenario $s$
$z_{js}^{\text{HFO}-F}$ Amount of HFO from forward contract used on Leg $j$ under scenario $s$
$u_s^{\text{MGO}-F}$ Amount of unused forward MGO left at the end of the planning period under scenario $s$
$u_s^{\text{HFO}-F}$ Amount of unused forward HFO left at the end of the planning period under scenario $s$
$m^{\text{MGO}-F}$ Agreed amount of MGO in the forward contract
$m^{\text{HFO}-F}$ Agreed amount of HFO in the forward contract
$\alpha$ Artificial variable for CVaR constraints
$h_s$ Artificial variables for CVaR constraints under scenario $s$

The mathematical formulation of the model starts here:

$$\begin{aligned} \min \quad & P^{\text{MGO}-F} m^{\text{MGO}-F} + P^{\text{HFO}-F} m^{\text{HFO}-F} \\ & + \sum_{s \in S} p_s \Bigg\{ \sum_{j \in J} \left( P_s^{\text{MGO}-S} z_{js}^{\text{MGO}-S} + P_s^{\text{HFO}-S} z_{js}^{\text{HFO}-S} \right) \\ & - \left( P^{\text{MGO}-F} - P^{\text{MGO}-P} \right) u_s^{\text{MGO}-F} - \left( P^{\text{HFO}-F} - P^{\text{HFO}-P} \right) u_s^{\text{HFO}-F} \Bigg\} \end{aligned} \tag{4}$$

*Subject to*

$$\overline{W}_{j+1} \geq \overline{W}_j + W_j^S + \sum_{r \in R_j} \sum_{v \in V} \left( W_{jrv}^{\mathrm{ECA}} x_{jrvs}^{\mathrm{ECA}} + W_{jrv}^N x_{jrvs}^N \right) \qquad s \in S, j \in J \tag{5}$$

$$\sum_{v \in V} x_{jrvs}^{\mathrm{ECA}} = y_{jrs} \qquad s \in S, j \in J, r \in R_j \tag{6}$$

$$\sum_{v \in V} x_{jrvs}^N = y_{jrs} \qquad s \in S, j \in J, r \in R_j \tag{7}$$

$$\sum_{r \in R_j} y_{jrs} = 1 \qquad s \in S, j \in J \tag{8}$$

$$z_{js}^{\mathrm{MGO}-F} + z_{js}^{\mathrm{MGO}-S} = \sum_{r \in R_j} \sum_{v \in V} F_v D_{jr}^{\mathrm{ECA}} x_{jrvs}^{\mathrm{ECA}} \qquad s \in S, j \in J \tag{9}$$

$$z_{js}^{\mathrm{HFO}-F} + z_{js}^{\mathrm{HFO}-S} = \sum_{r \in R_j} \sum_{v \in V} F_v D_{jr}^N x_{jrvs}^N \qquad s \in S, j \in J \tag{10}$$

$$\sum_{j \in J} z_{js}^{\mathrm{MGO}-F} + u_s^{\mathrm{MGO}-F} = m^{\mathrm{MGO}-F} \qquad s \in S \tag{11}$$

$$\sum_{j \in J} z_{js}^{\mathrm{HFO}-F} + u_s^{\mathrm{HFO}-F} = m^{\mathrm{HFO}-F} \qquad s \in S \tag{12}$$

$$y_{jrs} \in \{0, 1\} \qquad s \in S, j \in J, r \in R_j \tag{13}$$

$$x_{jrvs}^{\mathrm{ECA}}, x_{jrvs}^N \geq 0 \qquad s \in S, j \in J, r \in R_j, v \in V \tag{14}$$

$$z_{js}^{\mathrm{MGO}-F}, z_{js}^{\mathrm{MGO}-S}, z_{js}^{\mathrm{HFO}-F}, z_{js}^{\mathrm{HFO}-S} \geq 0 \qquad s \in S, j \in J \tag{15}$$

$$u_s^{\mathrm{MGO}-F}, u_s^{\mathrm{HFO}-F} \geq 0 \qquad s \in S \tag{16}$$

*CVaR constraints:*

$$\alpha + \frac{1}{1-\gamma} \sum_{s \in S} p_s h_s \leq A_\gamma \tag{17}$$

$$h_s \geq 0 \qquad s \in S \tag{18}$$

$$\begin{aligned} h_s \geq\ & P^{\mathrm{MGO}-F} m^{\mathrm{MGO}-F} + P^{\mathrm{HFO}-F} m^{\mathrm{HFO}-F} \\ & - \left(P^{\mathrm{MGO}-F} - P^{\mathrm{MGO}-P}\right) u_s^{\mathrm{MGO}-F} - \left(P^{\mathrm{HFO}-F} - P^{\mathrm{HFO}-P}\right) u_s^{\mathrm{HFO}-F} \\ & + \sum_{j \in J} \left(P_s^{\mathrm{MGO}-S} z_{js}^{\mathrm{MGO}-S} + P_s^{\mathrm{HFO}-S} z_{js}^{\mathrm{HFO}-S}\right) - \alpha \qquad s \in S \end{aligned} \tag{19}$$

The objective function (4) minimizes the expected total bunker cost for the planning period. The purchasing costs for the stated amounts of both fuels in the FFC are given in the first line of (4). The second line of the objective function refers to the expected costs for the consumption of spot fuels. The last line represents the treatment of the unused forward fuels. At the end of the planning period, the leftovers in the FFC (if any) are sold back to the bunker supplier at "buyback" prices, computed as their contractual forward prices subtracted by a penalty.

Constraints (5) enforce the time constraints for all sailing legs according to the schedule. Constraints (6) and (7) connect x- and y-variables with respect to the speed-routing choices in ECA and non-ECA stretches, respectively. They ensure that the sums of the speed weights, $x_{jrvs}^{\mathrm{ECA}}$ and $x_{jrvs}^{N}$, respectively, for ECA and non-ECA stretches, are equal to 1 if Leg option $r$ is chosen for Leg $j$ in scenario $s$ and 0 otherwise. Constraints (8) ensure that only one leg option is used on any specific leg. Constraints (9) and (10) make sure that for each scenario the sum of the spot and forward fuels used on each leg equals the actual fuel consumption on that leg based on the speeds and leg options chosen. Constraints (11) and (12) ensure that the forward fuels used plus the leftovers equal the agreed amounts in the forward contract. Constraints (13), (14), (15), and (16) define the domains of the decision variables. Constraints (17), (18), and (19) are the CVaR constraints representing the risk attitude of the shipping company, restricting the risk on the total bunker costs to be within an acceptable level. Constraint (17) ensures that the actual CVaR value during the optimization will not exceed the desired risk level ($A_\gamma$). On the other hand, the $[\quad]^+$ operator in (3) is replaced by the artificial variables $h_s$ and constraints (18) and (19) for optimization purpose.

It is important to notice that the $CO_2$ emissions are not directly considered in the formulation. Instead, they can be calculated based on the optimal solutions obtained using the $CO_2$ emission factors for the two fuels (see Sect. 1), 3.082 (tons/ton fuel) and 3.021 (tons/ton fuel) for MGO and HFO, respectively. More details will be discussed in Sect. 4.1.

## 3 The Test Case and Scenario Generation

In this section, we briefly describe the test case in Sect. 3.1, while the scenario generation process is discussed in Sect. 3.2.

**Table 1** Sequence of the port calls in the case loop

| | Origin port | Destination port |
|---|---|---|
| Leg 1 | Brunswick | Galveston |
| Leg 2 | Galveston | Charleston |
| Leg 3 | Charleston | New York |
| Leg 4 | New York | Bremerhaven |
| Leg 5 | Bremerhaven | Brunswick |

## *3.1 The Test Case*

The case considered in this paper is based on a liner service offered by Wallenius Wilhelmsen Logistics (WWL). The company offers roll-on roll-off (RoRo) services for transporting cars, trucks, and other types of rolling equipment. In our case, the service loop and its corresponding schedule are adapted from one of WWL's Europe-Americas trade lanes. The sequence of the port calls in the loop is shown in Table 1. The scheduled total traveling time for a round trip on this loop is 35 days. We therefore also set the planning period to 35 days.

We further assign five leg options to each leg in the case loop. Although different leg options of a specific leg share the same origin and destination ports, they differ in terms of ECA, non-ECA, and total sailing distances. As an example, Fig. 5 illustrates all five leg options of Leg 3 (Charleston, New York), where Leg option 1 takes the shortest possible path which is completely inside ECA, and Leg option 5, on the contrary, has the least ECA sailing. The detailed information about sailing distances for each leg option of every leg is displayed in Table 2.

Additionally, the fuel consumption data we use is collected from the historical record of a real RoRo ship under normal conditions. Figure 6 shows the fuel consumption per nautical mile for seven selected discrete speed points ranging from 15 to 24 knots.

## *3.2 Scenario Generation*

As mentioned earlier, the uncertainties considered in our problem refer to the spot prices for MGO and HFO. We assume we know their marginal distributions and the correlation between them and apply a version of the scenario-generating heuristic developed by [12] in order to generate scenarios for fuel prices. However, since fuel prices are significantly dependent over time, generating fuel prices using distributions derived from historical data directly can be problematic. For example, the generated fuel prices will not be representative if the historical data during the past booming period (e.g., 2008) is directly used in the process of scenario generation, when the current market is actually in recession.

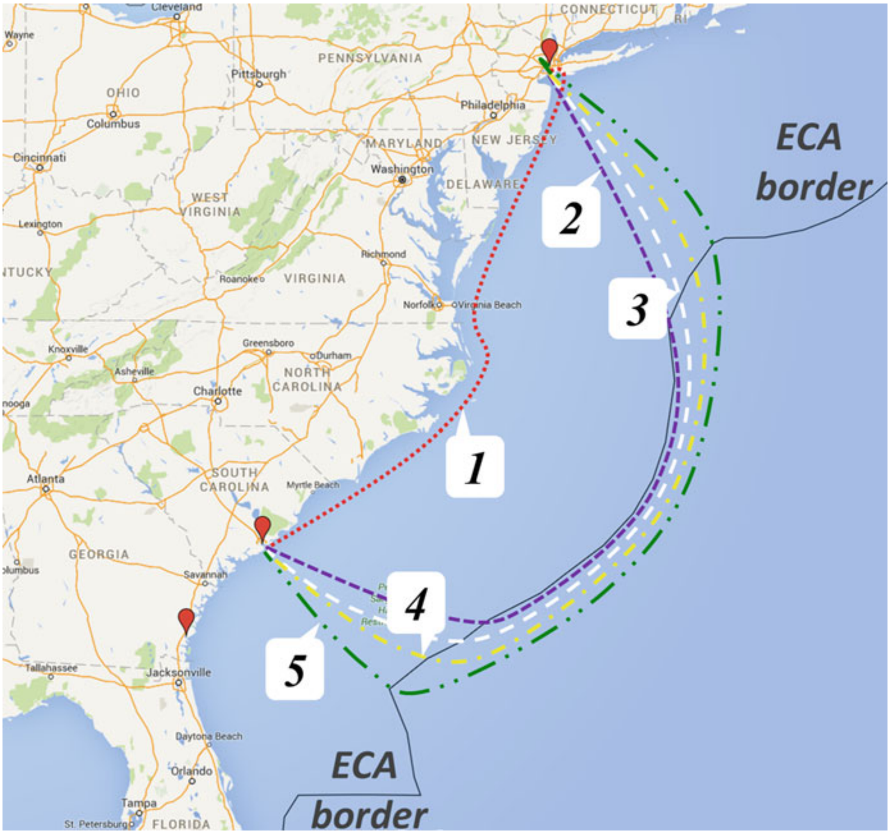



**Fig. 5** Five leg options for Leg 3 (Charleston, New York) (Google Maps, 2016)

**Table 2** Travelling distances for all ECA/non-ECA stretches

| Nautical mile | Option 1 | Option 2 | Option 3 | Option 4 | Option 5 |
|---|---|---|---|---|---|
| Leg 1 (ECA/*non-ECA*) | 1191/35 | 569/774 | 495/870 | 469/905 | 408/1062 |
| Leg 2 (ECA/*non-ECA*) | 1271/34 | 686/704 | 524/906 | 458/1083 | 397/1241 |
| Leg 3 (ECA/*non-ECA*) | 632/0 | 560/330 | 499/429 | 443/515 | 423/602 |
| Leg 4 (ECA/*non-ECA*) | 1767/1629 | 1379/2125 | 1042/2503 | 899/2652 | 752/2903 |
| Leg 5 (ECA/*non-ECA*) | 2393/1626 | 1110/2984 | 1013/3109 | 817/3337 | 751/3428 |

Therefore, we use a two-step approach to construct the scenarios for the spot-fuel prices for the next planning period. As a first step, we observe the latest fuel prices on the spot market and use them as base prices, which are also the *expected* spot prices during the planning period. Then, we generate *price increments* using the scenario generation heuristic, either positive or negative, and add them to the base prices. This approach corresponds to the special dynamic of the development of fuel price, which can be seen as a Lévy process with independent increments [16, 10].

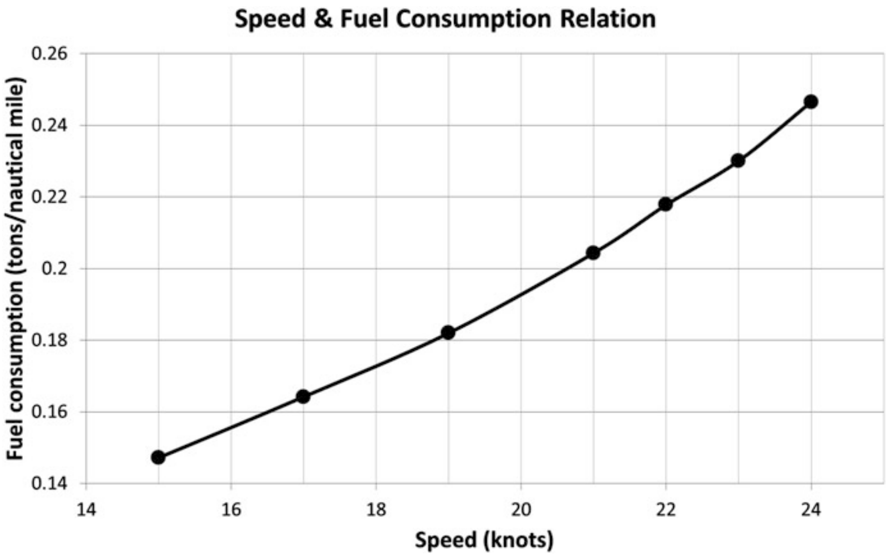



**Fig. 6** Speed and fuel consumption relation for the selected discrete speed points

We use the historical data provided by Clarkson Research Services Limited [5] to obtain an estimation of the distributions and correlation for the price increments. The data is collected from three major ports, Rotterdam, Houston, and Singapore, and consists of monthly prices of the two fuels (HFO and MGO) at these ports from January 2000 to December 2015. According to the data, the price increments of HFO and MGO are positively correlated and the correlation coefficient is estimated at 0.75. Furthermore, we assume triangular distributions for the random increments. The lower limit, mode, and upper limit that control the marginal distributions applied in the scenario-generating heuristic are set to (−40, 0, 40) and (−120, 0, 120) for HFO and MGO, respectively. The latest observations of the spot-fuel prices (used as base prices or expected spot prices) are from December 2015, and the prices of HFO and MGO are 150 USD/ton and 375 USD/ton, respectively. Also note that in our model the forward prices are always set to be marginally higher than the corresponding expected spot prices to prevent speculation.

Finally, an in-sample stability test [14] is performed to check the reliability of the scenario generation process. By comparing the results with different scenario trees generated under the same conditions, this test checks whether the optimal objective function value has a significant dependence on the specific scenario tree used. In our case, 10 scenario trees, each with 100 scenarios, are generated. The difference among the objective values solved with all 10 scenario trees is smaller than 0.02%, which shows that the scenario generation process used in this paper is stable and reliable.

## 4 Computational Study

In this section, we investigate how $CO_2$ emissions may be affected by a shipping company's risk attitude (Sect. 4.1) and its fuel hedging decisions (Sect. 4.2). The mathematical model in this paper is programmed in C++ with Microsoft Visual Studio. The commercial solver, CPLEX Optimization Studio V12.6.1, is called to solve the model in our tests. All computational tests are performed with an Intel Celeron 1.60 GHz CPU and 8 Gb RAM. The computational time does not exceed 1 min for an individual test.

### *4.1 Impact of Risk Attitude on $CO_2$ Emissions*

In our model, the shipping company's risk attitude toward its total bunker costs, i.e., its risk aversion level, can be represented by a maximum tolerable CVaR value ($A_\gamma$) and a confidence level ($\gamma$, set to a fixed value of 95% in our study). The $A_{95\%}$ value determines an upper bound of the average total cost allowed in the worst (5%) cases. A larger $A_{95\%}$ then corresponds to a higher tolerance of extreme risk and hence a lower risk aversion level. This allows us to use different $A_{95\%}$ values to represent the different levels of risk aversion, in order to study the impact of the company's risk attitude on its $CO_2$ emissions.

#### 4.1.1 Effect of Changing Risk Aversion Levels

First, we assume a "standard" risk aversion level for our case study. The corresponding $A_{95\%}$ is set to 390,000 USD which is approximately 1%[1] higher than the optimal total bunker cost in a risk-neutral case or the objective function value obtained when solving the problem without the CVaR constraints. The standard risk aversion level is then used as a benchmark for comparing the $CO_2$ emissions at different risk aversion levels.

We use in total 8 different $A_{95\%}$ values, ranging from 388,000 USD (extremely risk-averse) to 400,000 USD (least risk-averse). We also test the risk-neutral case which can be equivalently considered as having an enormously large $A_{95\%}$ value, and the CVaR constraints are thus no longer binding. We then solve the problem with each of these $A_{95\%}$ values and observe, in each case, the optimal *fuel allocation* decisions, i.e., the forward fuels $\left(z_{js}^{\text{MGO}-F}\text{ and } z_{js}^{\text{HFO}-F}\right)$ and spot fuels $\left(z_{js}^{\text{MGO}-S}\text{ and } z_{js}^{\text{HFO}-S}\right)$ consumed for every scenario $s \in S$. The amount of $CO_2$ emitted (in tonnes) for every scenario $s$ can then be calculated using the following formula:

[1]It is feasible and reasonable to restrict the risk level to such extent in this test because the forward-fuel prices are set to be only marginally higher than the expected spot-fuel prices.

**Table 3** Worst-case $CO_2$ emissions under different risk aversion levels

| Value set for $A_{95\%}$ (maximum CVaR) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| [1000 USD] | 388 | 389 | **390**[a] | 391 | 392 | 393 | 395 | 400 | Risk-neutral |
| % | −0.51 | −0.26 | **0.0** | +0.26 | +0.51 | +0.77 | +1.28 | +2.56 | – |
| Worst-case $CO_2$ emitted (average of five worst scenarios out of 100) | | | | | | | | | |
| [tonnes] | 5653 | 5695 | **5785** | 5787 | 5890 | 5930 | 6046 | 6046 | 6046 |
| % | −2.28 | −1.56 | **0.0** | +0.03 | +1.82 | +2.51 | +4.41 | +4.41 | +4.41 |

[a]Benchmark case with "standard" risk aversion

**Table 4** Example of fuel consumption for a particular scenario under different risk aversion levels

| Value set for $A_{95\%}$ (1000 USD) | 388 | 389 | 390 | 391 | 392 | 393 | $\geq$ 395[a] |
|---|---|---|---|---|---|---|---|
| MGO consumption (tonnes) | 482.2 | 473.1 | 455.4 | 454.4 | 435.5 | 426.5 | 404.8 |
| HFO consumption (tonnes) | 1379.4 | 1402.6 | 1449.8 | 1451.9 | 1504.5 | 1527.8 | 1588.5 |
| Total consumption (tonnes) | 1861.6 | 1875.7 | 1905.2 | 1906.3 | 1940.0 | 1954.2 | 1993.3 |

[a]Including the risk-neutral case

$$\mathrm{CO_2Emitted} = \sum_{j \in J} \left[ 3.082\left(z_{js}^{\mathrm{MGO}-S} + z_{js}^{\mathrm{MGO}-F}\right) + 3.021\left(z_{js}^{\mathrm{HFO}-S} + z_{js}^{\mathrm{HFO}-F}\right) \right] \quad (20)$$

Out of 100 scenarios, we can then find the 5 scenarios with the highest amounts of $CO_2$ emitted and calculate their average as the worst-case $CO_2$ emission for each given $A_{95\%}$ value.

Table 3 displays the results comparing the worst-case $CO_2$ under different risk aversion levels ($A_{95\%}$ values). We may observe that the worst-case $CO_2$ emissions increase when the company lowers its risk aversion level (or accepts a higher $A_{95\%}$ value). We may also notice that the amount of $CO_2$ emitted stops increasing and remains at 6046 tonnes when $A_{95\%}$ is above 395,000 USD.

Recall that the stochastics in our model come from the uncertain spot-fuel prices, creating risk on the total bunker costs. The introduction of CVaR constraints is therefore to contain such risk in the extreme cases. When the risk attitude is more relaxed in a shipping company's bunker risk management, i.e., with higher $A_{95\%}$, the company will have a higher willingness to take risks and rely more on the fuels from the spot market, rather than buying from the forward market. This actually allows the shipping company to operate the ship with higher flexibility in terms of more freedom to apply ECA-evasion and/or speed differentiation strategies, in order to avoid consuming the more expensive MGO. In contrast, for example, if a fair amount of MGO is already hedged, the shipping company's routing decisions may be restricted to the more traditional "shortest but more ECA involved" alternative, just to commit to the hedging contract and thus avoid paying too much penalty for unused MGO eventually. In Table 4, we show the fuel consumption for a specific scenario (where the spot prices for MGO and HFO are 413 USD/ton and 140 USD/ton, respectively) under different risk aversion levels. We can see that the total fuel consumption (bottom row in Table 4) increases when setting a higher $A_{95\%}$, and hence the $CO_2$ emissions also increase (since the emission factors for MGO and HFO are practically the same). This is due to the fact that when relying more on spot fuels (at higher $A_{95\%}$) and in the light of the significant price difference between MGO and HFO, the ship is sailing more "aggressively," such as evading ECA as much as possible and sailing as slowly as possible inside ECA (see Appendix 1 for details). The aggressive sailing has brought down the consumption of MGO but increased HFO consumption even more, which is beneficial in terms of total bunker costs but leads to an increase in total fuel consumption and eventually more $CO_2$ emitted. Once the $A_{95\%}$ exceeds 395,000 USD, nevertheless, the pattern of fuel consumption for both fuels and thus the sailing behavior remain stable in the worst scenarios, since the sailing behavior in these scenarios has already been pushed to the most aggressive level. Therefore, the average $CO_2$ emissions in the worst scenarios remain unchanged after the $A_{95\%}$ surpasses 395,000 USD, as observed in Table 3.

It is important to notice that the above results are based on studying the *worst-case* $CO_2$ emissions under different risk aversion levels, which show a clear tendency that the imposition of financial risk control measures (CVaR constraints) is also, to a certain degree, able to contain the "environmental risk" ($CO_2$ emissions in the worst scenarios). On the other hand, the relationship between *average* $CO_2$ emissions and risk aversion level is more complicated and is influenced by how much more expensive MGO is than HFO.

### 4.1.2 Influence of Price Gap Between MGO and HFO

In our model, the base prices for MGO and HFO (see Sect. 3.2), 375 USD/ton and 150 USD/ton, respectively, refer to the spot prices observed in December 2015 and are used as expected spot prices for the planning period. This has led to an Expected Spot Price Gap (ESPG in short) of 225 USD/ton. In the following test, we aim to study how *average* $CO_2$ emissions change with different ESPG. This is done by altering the base price for MGO and hence the ESPG, solving the corresponding MBM problem, and observing the average amount of $CO_2$ emitted across all scenarios (instead of the five worst scenarios). We also test for two risk settings: standard risk-averse (see Sect. 4.1.1) and risk-neutral.

Table 5 displays the average amounts of $CO_2$ emitted, for both standard risk-averse and risk-neutral settings, at various ESPG ranging from 100 USD/ton to 400 USD/ton. The corresponding expected spot MGO ranges from 250 to 550 USD/ton, while the expected spot HFO is fixed at 150 USD/ton. We can clearly see from Table 5 that for the risk-averse setting, the amount of $CO_2$ emitted becomes higher with increasing ESPG. It is also the case for the risk-neutral setting. This is in fact consistent with the conclusion shown in [9], that is, in general, $CO_2$ emissions would also increase when the price gap between MGO and HFO increases, due to a higher tendency to implement ECA-evasion and speed differentiation strategies. However, when comparing the $CO_2$ emissions between risk-averse and risk-neutral settings, i.e., the **Difference%** row of Table 5, we cannot easily tell which risk attitude is

**Table 5** Average $CO_2$ emissions at different ESPG, for both standard risk-averse and risk-neutral settings

| ESPG (USD/ton) | 100 | 150 | 200 | 250 | 260 | 270 | 300 | 350 | 400 |
|---|---|---|---|---|---|---|---|---|---|
| $CO_2$ (tonnes) Risk-averse | 5586.7 | 5618.2 | 5655.6 | 5743.3 | 5852.3 | 5988.1 | 6023.1 | 6045.2 | 6046.4 |
| $CO_2$ (tonnes) Risk-neutral | 5561.0 | 5619.7 | 5670.7 | 5821.2 | 5869.0 | 5913.0 | 5999.2 | 6043.8 | 6046.4 |
| **Difference%**[a] | −0.46 | +0.03 | +0.27 | +1.36 | +0.29 | −1.25 | −0.40 | −0.02 | 0.00 |

[a]Relative increase in $CO_2$ for the risk-neutral case compared to the risk-averse case

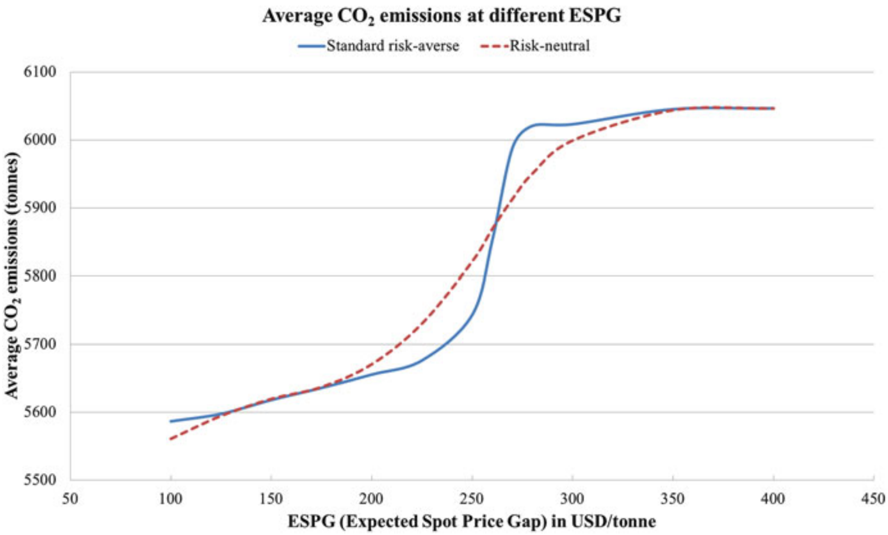



**Fig. 7** Difference of expected $CO_2$ emissions between risk-averse and risk-neutral cases under different levels of price gap

more “environmentally friendly.” We further illustrate in Fig. 7 the comparison of average $CO_2$ between risk-averse and risk-neutral settings.

In Fig. 7, the average $CO_2$ emitted at different ESPG under the standard risk-averse setting is represented by the solid line and the risk-neutral setting by the dashed line. Unlike the results shown in Sect. 4.1.1, where stronger risk aversion leads to a lower worst-case $CO_2$ emissions (which is, in fact, a general trend regardless of ESPG according to our experiments), the effect of risk aversion on average $CO_2$ emissions is undetermined and depends upon the specific ESPG that we face. For example, in Fig. 7, when the ESPG is around 250 USD/ton, the risk-averse setting has lower average $CO_2$ emissions than the risk-neutral case, whereas at around 270 USD/ton, the opposite situation is observed.

In order to explain this somewhat surprising result, we first need to explain what we call a *jump* in sailing behavior. If there is no price gap between MGO and HFO, the shipping company has no incentive to change its sailing behavior and thus sails

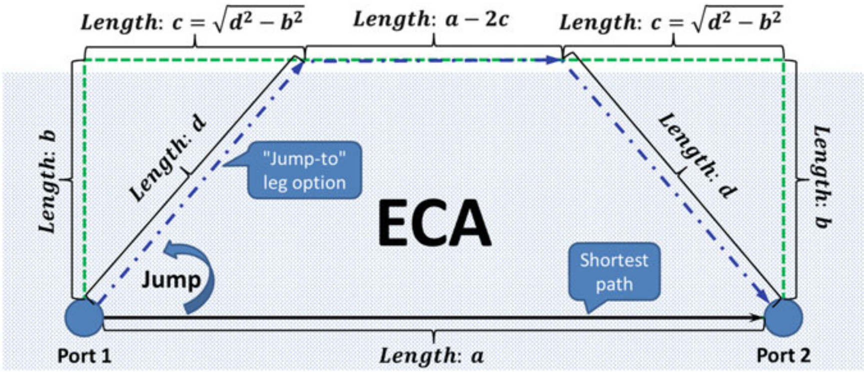



**Fig. 8** Simple example illustrating a jump in sailing behavior change

the traditional leg option (shortest path) between two ports in the same ECA. When the price gap increases, a leg option change will not immediately occur. The vessel will stick to the shortest path until the price gap between MGO and HFO reaches a certain level and then switch to another leg option, looking something like Leg Option 2 in Fig. 5; the sailing pattern makes a *jump*. Figure 8 is used to illustrate the principle of a jump. The solid line represents the traditional leg option between two ports located inside the same ECA, the dash-dot line illustrates the leg option following a jump. For simplicity of the argument, let us simply assume that one universal speed is applied both in- and outside ECA. Hence, fuel consumption is proportional to distance. The fuel cost for the traditional leg is $P^{\text{MGO}} \times a$, while the total bunker cost for the "Jump-to" leg option is $P^{\text{MGO}} \times 2d + P^{\text{HFO}} \times (a - 2c)$. Hence, until the MGO price becomes $(a - 2c)/(a - 2d)$ times as high as the HFO price, it is cheaper to sail the shortest path and a jump will not be triggered. However, once the price gap exceeds that level, the optimal leg option switches to the "Jump-to" leg option and the jump occurs. Note that jumps are a natural part of the underlying problem and not caused by the fact that we have discretized sailing patterns into possible leg options. Furthermore, if time and speed considerations are involved, the "Jump-to" leg option will need a higher average speed, thus higher fuel consumption to maintain the schedule, which leads to an even higher price gap to trigger the jump.

A risk-averse company relies mainly on the forward market while using small amounts of spot fuels as supplements. Moreover, the forward price difference approximately equals the ESPG since the prices of the forward fuels are set to be just marginally higher than the expected spot fuel prices (basis prices). Therefore, the actual price gap which decides the sailing behavior in the risk-averse setting is significantly affected by the ESPG and only slightly influenced by the Realized Spot Price Gap (RSPG in short) in each scenario. The RSPG in scenario $s$ can be expressed as:

$$\text{RSPG}_s = \text{ESPG} + \left(I_s^{\text{MGO}} - I_s^{\text{HFO}}\right) \tag{21}$$

where $I_s^{\text{MGO}}$ and $I_s^{\text{HFO}}$ represent the price increments of MGO and HFO, respectively, in scenario $s$. The risk-neutral company, however, is more willing to take market risks and thus only buys from the spot market. Hence, in the risk neutral case, the sailing pattern in each specific scenario purely depends on the RSPG in that scenario. In sum, the price gaps in most scenarios in the risk-averse setting are approximately the same as the ESPG, while the price gaps in the risk-neutral setting (RSPG) differ substantially from scenario to scenario.

Since most scenarios in the risk-averse setting have similar price gaps, the jump happens almost simultaneously in these scenarios when the ESPG increases to the level that satisfies the requirement illustrated in Fig. 8. Such a clustered change in sailing behavior (thus $CO_2$ emissions) in most scenarios brings a sudden and major increase in average $CO_2$ emissions in the risk-averse setting, as observed in Fig. 7. From (21), we see that RSPG increases together with the ESPG, but such that the scenario with the largest price increment difference will also have the largest RSPG. Hence, when ESPG increases, these scenarios with large price increment differences will first trigger a jump. Then the scenarios with moderate price increment differences (thus moderate RSPGs) follow along with the increase of ESPG, and finally the jump occurs in the scenarios which have small price increment differences (thus small RSPGs). As we can see, contrary to the clustered jump in the risk-averse setting, the jump in the risk-neutral setting happens gradually from the scenarios with larger RSPGs to the ones with smaller RSPGs. The corresponding effect on average $CO_2$ emissions is much more widely distributed along the ESPG axis, which eventually leads to a smoother increasing curve for the risk-neutral setting, as witnessed in Fig. 7.

To summarize, in this section we show that the shipping company's risk attitude has impact on its $CO_2$ emissions in various ways. On one hand, the worst-case $CO_2$ emissions will be reduced by financial risk control measures, i.e., a stronger risk aversion will lead to less $CO_2$ emitted in the worst scenarios; on the other hand, the effect of risk aversion on average $CO_2$ emissions is undetermined and is influenced by the expected price gap between MGO and HFO on the spot market.

## 4.2 *Impact of Hedging Strategies on $CO_2$ Emissions*

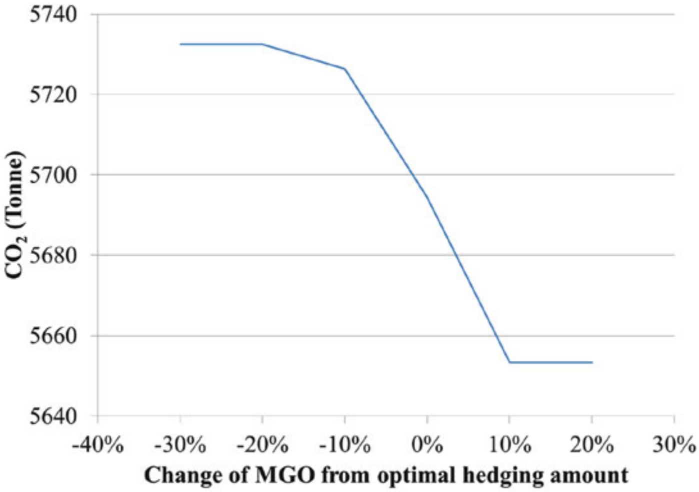
We now study how different hedging strategies affect a shipping company's expected $CO_2$ emissions. For all experiments presented in this section, we assume the company's risk attitude is always standard risk-averse (see Sect. 4.1.1), and the expected $CO_2$ emissions refer to the average amount of $CO_2$ emitted across all scenarios.

Using the input data given in Sect. 3, we can first obtain the optimal hedging amounts of both MGO and HFO by solving the stochastic MBM problem to optimality. We then fix the hedging decision for one fuel, HFO for instance, and change the hedging amount of the other (MGO) to get different combinations of hedging decisions. For each such combination, we solve the problem after fixing the hedging decisions accordingly and record the expected $CO_2$ emissions. The results are shown in Fig. 9a, where the hedged MGO varies from 70% to 120% of its optimal amount. We also show in Fig. 9b the opposite case in which we vary the hedging amount for HFO while fixing the hedged MGO at its optimal amount.

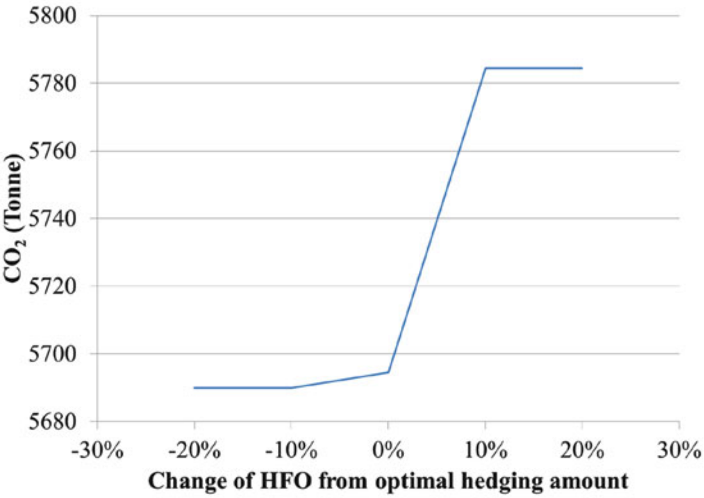
From the two charts in Fig. 9, we can see that the expected $CO_2$ emissions will (a) decrease when hedging more MGO and (b) increase when hedging more HFO. These changes in $CO_2$ emissions may be explained by the changes in the company's willingness to apply ECA-evasion and speed differentiation strategies. As mentioned earlier, when more MGO is hedged, in order to commit to the forward contract and avoid paying too much penalty for unused forward MGO, the company may be restricted to the traditional "shorter but more ECA involved" routes. In this case, the *total* fuel consumption (MGO&HFO) is usually lower because of the shorter total distance sailed, hence the $CO_2$ emissions are also lower. On the other hand, when more HFO is hedged, the company may be more likely to sail "aggressively," e.g., with as little ECA involvement as possible, in order to consume more HFO. As a result, the total sailing distance is usually longer, which eventually leads to higher total fuel consumption and more $CO_2$ emitted.

Note that the above tests only show how expected $CO_2$ emissions change when altering the hedging amount of one type of fuel alone. In addition, apart from the environmental impact, different hedging decisions may also affect the total bunker costs, which is more of a concern for most shipping companies. Therefore, in the following tests, we demonstrate the effects of simultaneously changing the hedging amounts of MGO and HFO, both environmentally (in terms of expected $CO_2$ emissions) and financially (in terms of expected total bunker costs and worst-case total bunker costs). We seek to provide an insight into the question: can we effectively reduce $CO_2$ emissions through different hedging strategies? And at what cost?

Let us look at two 3-D charts in Fig. 10. In both charts, we use the changes (%) in the hedging amounts of MGO and HFO (relative to their respective optimal amounts) as x- and y- axes, respectively. For Fig. 10a, we show two plotted surfaces representing expected $CO_2$ emissions (bottom surface) and expected total bunker costs (top surface), both are changes (%) relative to their corresponding values obtained with the optimal hedging decisions. For Fig. 10b, the surface of expected $CO_2$ emissions remains the same, and we also show the surface for the worst-case total bunker costs, computed as the average of the five worst scenarios out of 100. Let us further focus on the red areas in the two charts, where the x- (change in MGO) values and y- (change in HFO) values correspond to [+5%, +10%] and [−6%,−2%], respectively. Therefore, by hedging 5–10% more on MGO and 2–6%




(a)




(b)

**Fig. 9** Expected $CO_2$ emissions under different fuel hedging strategies. (**a**) $CO_2$ under different MGO hedging amount. (**b**) $CO_2$ under different HFO hedging amount

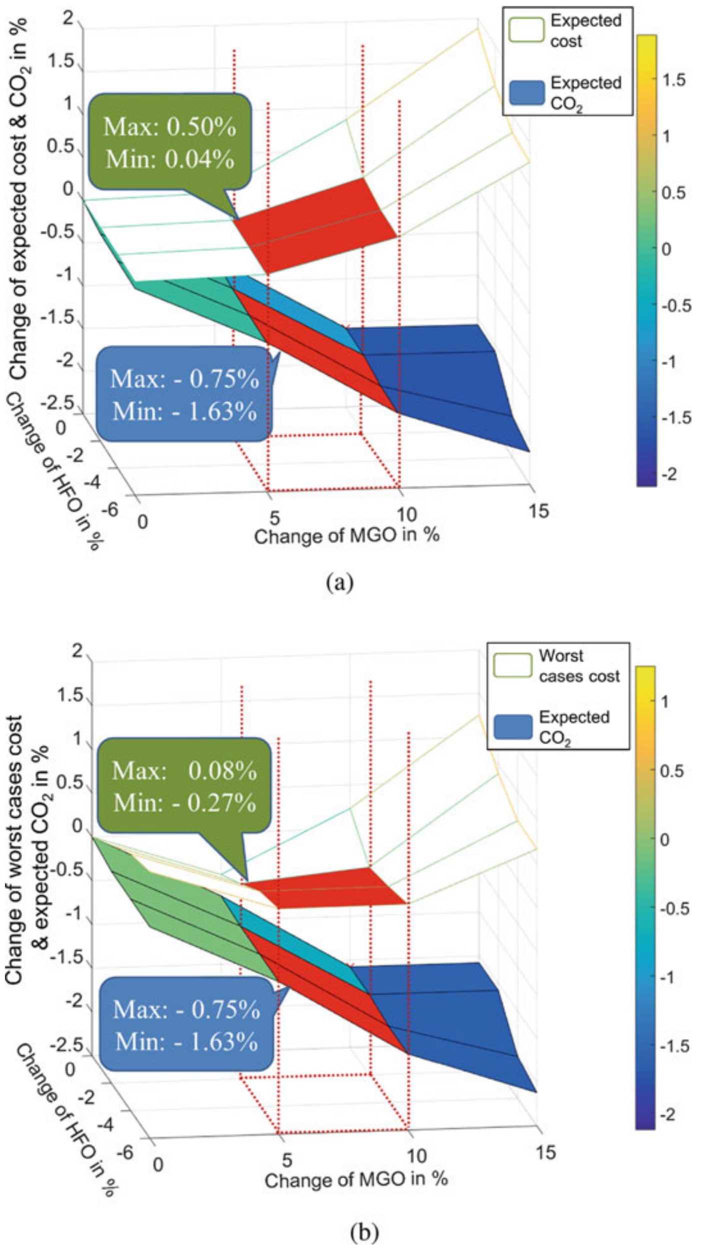



**Fig. 10** Illustration of the relation between $CO_2$ emissions and expected and worst-case total bunker costs under different hedging strategies. (**a**) Surfaces for expected total bunker costs and expected $CO_2$ emissions. (**b**) Surfaces for worst-case total bunker costs and expected $CO_2$ emissions

less on HFO, we are able to reduce expected $CO_2$ emissions by 0.75–1.63%. This is achieved at the expense of increasing the expected total bunker costs by 0.04–0.50%, which are not significant. Furthermore, we show in Fig. 10b that such reduction in $CO_2$ sometimes even coincides with an improved situation (decrease) in the worst-case bunker cost (ranging from −0.27% to 0.08%). These results are meant to provide an example that sometimes a shipping company can achieve noticeable reduction in $CO_2$ emissions with little sacrifice on its financial costs by changing the hedging strategies. For any single player in maritime transportation, such reduction may not be significant. But for the shipping industry on a global scale, this could become a sizable contribution if more companies are coming to the realization of the potential environmental benefits of proper design of hedging and other bunker risk management measures.

## 5 Conclusion

Bunker risk management is widely practiced in the shipping industry to reduce financial risk and can be vital for a shipping company to remain competitive. Nevertheless, dramatic changes have taken place after the introduction of the ECA regulation. In this paper, we use a stochastic maritime bunker management (MBM) model and a case study on a major liner shipping company to show that bunker risk management has impacts on the company's $CO_2$ emissions.

We first study the impact of the shipping company's risk attitude on its $CO_2$ emissions. The results show that stronger risk aversion can also lead to lower "environmental risk," i.e., less $CO_2$ emissions in the worst cases. Meanwhile, we also show that the effect of risk aversion on average $CO_2$ emissions is undetermined and is influenced by the expected price gap between MGO and HFO on the spot market. We then study the impact of hedging strategies on $CO_2$ emissions. We show that a shipping company can sometimes achieve noticeable reduction in $CO_2$ emissions with little sacrifice on its financial costs by changing its hedging strategies.

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## Appendix 1

The detailed speed-routing decisions in scenario no. 26 under different maximum CVaR values are shown in the following table (Table 6).

**Table 6** Sailing behaviors in scenario no. 26 under different risk aversions

| **Max CVaR (1000 USD)** | 388 | 389 | 390 | 391 | 392 | 393 | Neutral |
|---|---|---|---|---|---|---|---|
| ***Leg 1*** | | | | | | | |
| Leg option | 4 | *5* | *5* | *5* | *5* | *5* | *5* |
| ECA/non-ECA distance (nautical mile) | 496/905 | *408/1062* | *408/1062* | *408/1062* | *408/1062* | *408/1062* | *408/1062* |
| ECA/non-ECA speed (knot) | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 |
| ***Leg 2*** | | | | | | | |
| Leg option | 3 | 3 | *5* | *5* | *5* | *5* | *5* |
| ECA/non-ECA distance (nautical mile) | 524/906 | 524/906 | *397/1241* | *397/1241* | *397/1241* | *397/1241* | *397/1241* |
| ECA/non-ECA speed (knot) | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 |
| ***Leg 3*** | | | | | | | |
| Leg option | 1 | 1 | 1 | 1 | *4* | *4* | *4* |
| ECA/non-ECA distance (nautical mile) | 632/0 | 632/0 | 632/0 | 632/0 | *443/515* | *443/515* | *443/515* |
| ECA/non-ECA speed (knot) | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 | 15/15 |
| ***Leg 4*** | | | | | | | |
| Leg option | 4 | 4 | 4 | 4 | 4 | 4 | *5* |
| ECA/non-ECA distance (nautical mile) | 889/2652 | 889/2652 | 889/2652 | 889/2652 | 889/2652 | 889/2652 | *752/2903* |
| ECA/non-ECA speed (knot) | 15/20.6 | 15/20.6 | *15/20.1* | *15/20.1* | *15/20.1* | *15/20.1* | *15/20.5* |
| ***Leg 5*** | | | | | | | |
| Leg option | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| ECA/non-ECA distance (nautical mile) | 751/3428 | 751/3428 | 751/3428 | 751/3428 | 751/3428 | 751/3428 | 751/3428 |
| ECA/non-ECA speed (knot) | 15/18.1 | 15/18.1 | 15/18.1 | 15/18.1 | 15/18.1 | 15/18.1 | 15/18.1 |

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# Allied Closed-Loop Supply Chain Network Optimization with Interactive Fuzzy Programming Approach

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**Abstract** The concept of closed-loop supply chain (CLSC) has started to attract growing attention due to the consumer pressures, environmental awareness, and legislations. Managers in many companies have realized that a well-designed supply chain (SC) can improve the companies' performance in the market. Thus, a lot of companies start to focus on CLSC issues including remanufacturing, refurbishing, recycling, and disposal of end-of-life products. The body of literature on CLSC management has been overwhelmingly dominated by noncooperative studies. In order to fill up this gap in the literature, we deal with an allied SC network in cooperative environment. With the implementation of allied SCs, companies not only maximize their profit but also minimize their various costs and become more flexible and efficient in the market. Following this motivation, we develop a decentralized multilevel CLSC model for allied SCs. At the first decision level, the plants in allied SCs are considered as the upper-level DMs of the Stackelberg game. At the second level, raw material suppliers, common suppliers, assembly centers, and common collection centers are considered as the lower-level DMs of the Stackelberg game. In order to tackle each decision-maker (DM)'s unique objectives, we propose a new fuzzy analytic hierarchy process (AHP)-based interactive fuzzy programming (IFP) approach. In the IFP approach, upper-level DMs determine the minimum satisfactory level for their own objectives, and by using this value, the lower DMs evaluate their own satisfactory level. A compromise solution can be derived until termination conditions are satisfied. The primary aim of this study is

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to design a decentralized CLSC network in cooperative environment and to propose a novel IFP approach. Finally, a numerical example is implemented and analyzed in order to demonstrate the efficiency of the proposed approach.

## 1 Introduction

The traditional dynamics of business environment have changed over the last years, and supply chain management will gain a critical concern in the competitive business environment. In the future, the relationships between companies will change into the relationships between supply chains. The traditional concept of "SC," which is an important key of profitability of companies, has gained new features. Many more companies are faced to new terms in SCs: "competitive" or "alliance." In competitive SC companies conduct their actions across the other supply network entities [1]. But in alliance SC companies work together and coordinate their actions in an alliance [2]. With the use of alliance SC, companies work together, share common units, and reach the success faster than by working alone. There are some studies that consider competitive SC [3–5] in comparison with the allied SC.

The "closed-loop supply chain" becomes an increasingly growing concept with rapid technological developments, increasing consumer pressures, and government regulations. The managers of companies are responsible of their end-of-life products (EOLPs) with the increase in public awareness about environmental issues and environmental regulations [6]. Many companies such as Xerox, HP, and IBM focus on remanufacturing activities. In 2014, Fuji Xerox Australia has increased the mass of remanufactured parts for reuse by 43 percent to 280 tons and has increased mass of recycled materials up to 10 percent to 3577 tons with the remanufacturing activities [7]. In 2013 IBM, 32.200 metric tons of end-of-life products were collected for end-of-life products management. From 1995 to 2013, IBM had processed over 2 billion pounds (913,000 metric tons) of product and product waste worldwide [8]. On the other hand, we can find alliance SC examples in the automotive, aerospace, and personal computer (PC) industries, where in these examples many original equipment manufacturers (OEMs) share common suppliers [1].

Supply chain network design (SCND) includes three main decisions. Determination of capacity, location of facility, and optimal number of facilities are handled as strategic decisions [1]. Determination of produced, sent, and stored parts/products is handled as tactical decisions [12–16]. Integration of production planning and inventory control decisions such as routing and scheduling are called operational decisions [17–19]. We have encountered centralized strategies in most of the CLSC studies. But in this study, we focus on decentralized CLSC strategies [20].

In many CLSC network designs, the units may have decentralized strategies, which means all the units have different individual objectives under some parameters and constraints. In this situation, the SC units may adopt cooperative strategies by centralizing their decisions and operating jointly (e.g., they may share/mutualize some physical resources such as warehouses or vehicle fleet or may do some project jointly such as shipment consolidation, joint replenishment). In this case, each of them will get a portion of the achieved savings [21].

The Stackelberg game model is the most used modeling method in decentralized organizations. In the model there are two players; each of the players knows their strategies and determines their own strategy according to the other player's strategy. But in this model, players do not cooperate with each other. In real-world examples, top management or an executive board must think other players' strategy in overall management policy. In order to overcome this problem, Sakawa et al. [22, 23] have proposed IFP [24]. Sakawa et al. [22] have developed interactive fuzzy programming for two-level linear programming problems to overcome the problem in the above methods. Moreover, from the viewpoint of experts' imprecise or fuzzy understanding of the nature of parameters in the problem formulation process, they have extended it to interactive fuzzy programming for two-level linear programming problems with fuzzy parameters [23].

Following this motivation, in this paper we developed a decentralized multilevel CLSC model for alliance SCs. Two closed-loop supply chains with alliance in some units are taken into account. The raw material suppliers, common suppliers, assemblers, and common collection centers are handled allied units in two SCs. The proposed model is able to help how we integrate two SCs simultaneously. A novel IFP approach which is based on fuzzy AHP is proposed for a decentralized model with multiple DMs at the upper levels and multiple DMs at the lower level. In this approach, the balance of the upper-level DMs and lower-level DMs is obtained by using a ratio. With the usage of this ratio, DMs can cooperate with each other, and a compromise solution can be obtained.

The remainder of this paper is organized as follows: Section 2 presents a comprehensive literature review related to CLSC and IFP approaches. Section 3 describes problem definition and model formulation in detail; assumptions, sets, parameters, variables, objective functions, and constraints are described. Section 4 gives the novel interactive fuzzy programming approach. Section 5 presents computational results. Finally, we finish the study and give some future directions in Sect. 6.

## 2 Literature Review

A number of studies have been handled in CLSC management by many researchers. A comprehensive review of reverse logistics and CLSC is investigated by Govindan et al. [25]. A bibliometric and network analysis of green supply chain management is handled in detail by Fahimnia et al. [26].

An interactive fuzzy goal programming approach is proposed for solving a CLSC network problem by Zarandi et al. [11]. According to the numerical example, they showed that the proposed approach is adequate to solve the proposed CLSC distribution network. Pishvaee et al. [27] developed a mixed-integer linear model for CLSC network design. In order to handle uncertainty, a robust optimization model is proposed by authors. The robustness of the proposed model showed with computational results. Pishvaee and Razmi [28] presented a multi-objective fuzzy mathematical programming model for environmental SCND. An interactive fuzzy solution approach is developed based on the $\varepsilon$-constraint method and possibilistic programming approach proposed by Jiménez et al. [29]. The validity of the developed model is investigated for a real industrial case. Farahani et al. [3] provided a review of competitive environment on SCND. They categorized the related literature in SCND that considered competition in modeling SCND. After the review they proposed potential gaps and a general framework for modeling the competitive SCND problems. Rezapour et al. [4] designed a competitive SC network between two SCs. Internal and external competitions are modeled at the SCND stage of a CLSC. In order to solve the model, a modified projection method is used, and in an automotive spare parts market, the proposed model is tested. Fallah et al. [30] addressed the competition between two SCs in an uncertain environment. The developed CLSC model competes on following factors: the retail prices and incentive quantity. In order to solve the model, they proposed a possibilistic SCND model. The validity of proposed model is tested via an Iranian battery manufacturer company. In order to solve decentralized bi-level multi-objective programming problems, Toksari and Bilim [31] proposed an interactive fuzzy goal programming approach based on Jacobian matrix. In the proposed model, a single DM at the first level and multiple decision-makers at the second level are accepted. With the use of the proposed approach, DMs obtain satisfactory solutions. Subulan et al. [32] handled various recovery options including remanufacturing, recycling, and energy recovery for tire CLSC model via holistic view. The developed model was tested using an interactive fuzzy goal programming approach for the tire industry in the Aegean Region of Turkey.

Fuzzy programming approaches are the most used approach for solving multi-objective and multilevel programming models. The first approach is developed by Zimmermann [33] called max–min approach. Li et al. [34] improved the fuzzy compromise approach of Guua and Wu [35] by computing proper membership thresholds. Ahlatcioglu and Tiryaki [36] proposed two different IFP approaches for a decentralized two-level linear fractional programming. In both approaches, upper-level DM reflects the judgments with the help of analytic hierarchy process. The validity of the proposed approaches is shown with different numerical examples. Selim and Ozkarahan [37] developed a multi-objective linear programming model that selects the optimum numbers, locations, and capacity levels of plants and warehouses. A new and generic interactive fuzzy goal programming (IFGP)-based solution approach is proposed for obtaining compromise solution. Torabi and Hassini [38] proposed a multi-objective possibilistic mixed-integer linear programming model (MOPMILP). They developed a two-phase interactive fuzzy programming approach, and for the second phase, they proposed a novel interactive fuzzy approach to find an efficient compromise solution. The numerical experiments indicate that the proposed approach is better than the considered fuzzy approaches. Özceylan and Paksoy [39] used different fuzzy interactive programming approaches for solving the developed fuzzy multi-objective mixed-integer nonlinear programming model. The proposed fuzzy model is converted to the auxiliary crisp multi-objective model via two methods.

# 3 Problem Definition and Model Formulation

In this section, the developed multilevel, multiproduct, and multi-echelon CLSC model is presented. In this study, two different SCs are handled with the usage of common sources: the first SC hereafter the SC1 and the allied SC hereafter the SC2. As distinct from conventional SCs, in this study, we use common sources in the network. Not only SCs have own sources but also they can share some sources for obtaining the final product. The integration of the SC1 and SC2 is shown in Fig. 1.

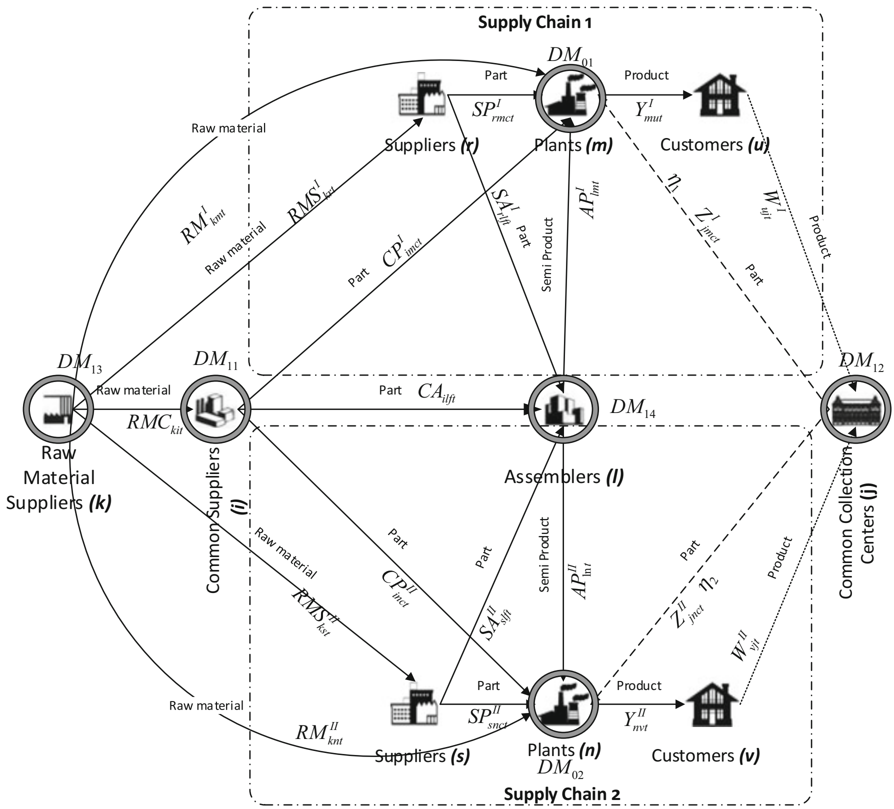


**Fig. 1** The integration of SC1 and SC2

## 3.1 Assumptions

The main characteristics and some assumptions contained in this study given as follows:

- The locations of facilities both forward and reverse SC fixed and predefined.
- The capacities of all facilities both forward and reverse are limited and fixed.
- All demands of customers must be fully satisfied at the same period.
- The cost of transportation, purchasing, inventory holding, and opening facilities is fixed and deterministic.
- The end product is assembled by different components, the semifinished product and raw material with different utilization rates by various suppliers.
- The percentage of collected product is known a priori.

## 3.2 Sets

***K*** set of raw material suppliers $k \in K$
***I*** set of common suppliers $i \in I$
***R*** set of suppliers in SC1 $r \in R$
***S*** set of suppliers in SC2 $s \in S$
***M*** set of plants in SC1 $m \in M$
***N*** set of plants in SC2 $n \in N$
***U*** set of customers in SC1 $u \in U$
***V*** set of customers in SC2 $v \in V$
***J*** set of common collection centers $j \in J$
***L*** set of assemblers $l \in L$
***C*** set of parts $c \in C$
***F*** set of parts of semifinished product $f \in F$
***T*** set of periods $t \in T$

## 3.3 Parameters

### 3.3.1 Distances and Unit Shipping Cost

$d_{km}$ distance between raw material "*k*" and plant "*m*" in SC1 (km)
$d_{kn}$ distance between raw material "*k*" and plant "*n*" in SC2 (km)
$d_{ki}$ distance between raw material "*k*" and common supplier "*i*" (km)
$d_{im}$ distance between common supplier "*i*" and plant "*m*" in SC1 (km)
$d_{in}$ distance between common supplier "*i*" and plant "*n*" in SC2 (km)

$d_{il}$ distance between common supplier "$i$" and assembler "$l$" (km)
$d_{rm}$ distance between supplier "$r$" and plant "$m$" in SC1 (km)
$d_{rl}$ distance between supplier "$r$" and assembler "$l$" in SC1 (km)
$d_{sn}$ distance between supplier "$s$" and plant "$n$" in SC2 (km)
$d_{sl}$ distance between supplier "$s$" and assembler "$l$" in SC2 (km)
$d_{lm}$ distance between assembler "$l$" and plant "$m$" in SC1 (km)
$d_{ln}$ distance between assembler "$l$" and plant "$n$" in SC2 (km)
$d_{mu}$ distance between plant "$m$" and customer "$u$" in SC1 (km)
$d_{nv}$ distance between plant "$n$" and customer "$v$" in (km)
$d_{uj}$ distance between customer "$u$" and common collection center "$j$" in SC1 (km)
$d_{vj}$ distance between customer "$v$" and common collection center "$j$" in SC2 (km)
$d_{jm}$ distance between common collection center "$j$" and plant "$m$" in SC1 (km)
$d_{jn}$ distance between common collection center "$j$" and plant "$n$" in SC2 (km)
$utc$ the shipping cost of one unit (($/ton).km)

### 3.3.2 Capacities

$as_{kt}$ the steel plate capacity of raw material supplier "k" at time period "t" (ton)
$a^{I}_{rct}$ the part "c" capacity of supplier "r" at time period "t" in SC1 (ton)
$a^{II}_{sct}$ the part "c" capacity of supplier "s" at time period "t" in SC2 (ton)
$a_{ict}$ the part "c" capacity of common supplier "i" at time period "t" (ton)
$a^{I}_{rft}$ the part "f" capacity of supplier "r" at time period "t" in SC1 (ton)
$a^{II}_{sft}$ the part "f" capacity of supplier "s" at time period "t" in SC2 (ton)
$a_{ift}$ the part "f" capacity of common supplier "i" at time period "t" (ton)
$b^{I}_{mct}$ the part "c" capacity of plant "m" at time period "t" in SC1 (ton)
$b^{II}_{nct}$ the part "c" capacity of plant "n" at time period "t" in SC2 (ton)
$aa_{lt}$ the semifinished product capacity of assembler "l" at time period "t" (ton)
$ca_{jt}$ the capacity of common collection center "j" at time period "t" (ton)

### 3.3.3 Demands

$de^{I}_{ut}$ demand of customer "u" at time period "t" in SC1 (ton)
$de^{II}_{vt}$ demand of customer "v" at time period "t" in SC2 (ton)

### 3.3.4 Fixed Costs

$\alpha^{I}_{mt}$ fixed-opening cost of plant "m" at time period "t" in SC1 ($)
$\alpha^{II}_{nt}$ fixed-opening cost of plant "n" at time period "t" in SC2 ($)
$\alpha_{jt}$ fixed-opening cost of common collection center "j" at time period "t" ($)

### 3.3.5 Purchasing Costs/Selling Prices

$p_{rc}$ purchasing cost of part “c” from supplier “r” in SC1 (\$/ton)
$p_{sc}$ purchasing cost of part “c” from supplier “s” in SC2 (\$/ton)
$p_{ic}$ purchasing cost of part “c” from common supplier “i” (original part) (\$/ton)
$p_{rf}$ purchasing cost of part “f” from supplier “r” in SC1 (\$/ton)
$p_{sf}$ purchasing cost of part “f” from supplier “s” in SC2 (\$/ton)
$p_{if}$ purchasing cost of part “f” from common supplier “i” (original part) (\$/ton)
$p_k$ purchasing cost/selling price of raw material from raw material supplier “k” (\$/ton)
$e_{jc}$ purchasing cost/selling price of part “c” from common collection center “j” (second-hand part) (\$/ton)
$e_l$ purchasing cost/selling price of semifinished product from assembler “l” (\$/ton)

### 3.3.6 Inventory Holding Costs

$hcp^{I}_{mct}$ inventory holding cost of part “c” in plant “m” at time period “t” in SC1 (\$/ton)
$hcp^{II}_{nct}$ inventory holding cost of part “c” in plant “n” at time period “t” in SC2 (\$/ton)

### 3.3.7 Ratios and Percentages

$r$ weight ratio of semifinished product in final product
$r_c$ weight ratio of part “c” in final product
$r'$ weight ratio of raw material in final product
$ro_c$ weight ratio of part “c” in one ton steel plate
$ro_f$ weight ratio of part “f” in one ton steel plate
$rf_f$ weight ratio of part “f” in semifinished product
$\eta_1$ percentage of collected amount which is resent to plants in SC1
$\eta_2$ percentage of collected amount which is resent to plants in SC2

## 3.4 Variables

### 3.4.1 Outbound Logistics of Raw Material Suppliers

$RM^{I}_{kmt}$ amount of steel plate shipped from raw material supplier “*k*” to plant “*m*” at time period “*t*” in SC1 (ton)
$RM^{II}_{knt}$ amount of steel plate shipped from raw material supplier “*k*” to plant “*n*” at time period “*t*” in SC2 (ton)

$RMS^{I}_{krt}$ amount of steel plate shipped from raw material supplier "*k*" to supplier "*r*" at time period "*t*" in SC1 (ton)
$RMS^{II}_{kst}$ amount of steel plate shipped from raw material supplier "*k*" to supplier "*s*" at time period "*t*" for SC2 (ton)
$RMC_{kit}$ amount of steel plate shipped from raw material supplier "*k*" to common supplier "*i*" at time period "*t*" (ton)

### 3.4.2 Outbound Logistics of Common Suppliers

$CP^{I}_{imct}$ amount of part "*c*" shipped from common supplier "*i*" to plant "*m*" at time period "*t*" in SC1 (ton)
$CP^{II}_{inct}$ amount of part "*c*" shipped from common supplier "*i*" to plant "*n*" at time period "*t*" in SC2 (ton)
$CA_{ilft}$ amount of part "*f*" shipped from common supplier "*i*" to assembler "*l*" at time period "*t*" (ton)

### 3.4.3 Outbound Logistics of Suppliers

$SP^{I}_{rmct}$ amount of part "*c*" shipped from supplier "*r*" to plant "*m*" at time period "*t*" in SC1 (ton)
$SP^{II}_{snct}$ amount of part "*c*" shipped from supplier "s" to plant "*n*" at time period "*t*" in SC2 (ton)
$SA^{I}_{rlft}$ amount of part "*f*" shipped from supplier "*r*" to assembler "*l*" at time period "*t*" in SC1 (ton)
$SA^{II}_{slft}$ amount of part "*f*" shipped from supplier "*s*"_ to assembler "*l*" at time period "*t*" in SC2 (ton)

### 3.4.4 Outbound Logistics of Assemblers

$AP^{I}_{lmt}$ amount of semifinished product shipped from assembler "*l*" to plant "*m*" at time period "*t*" in SC1 (ton)
$AP^{II}_{lnt}$ amount of semifinished product shipped from assembler "*l*" to plant "*n*" at time period "*t*" in SC2 (ton)

### 3.4.5 Outbound Logistics of Plants

$Y^{I}_{mut}$ amount of product shipped from plant "*m*" to customer "*u*" at time period "*t*" in SC1 (ton)
$Y^{II}_{nvt}$ amount of product shipped from plant "*n*" to customer "*v*" at time period "*t*" in SC2 (ton)

#### 3.4.6 Outbound Logistics of Customers

$W^{I}_{ujt}$ amount of used product shipped from customer "$u$" to common collection center "$j$" at time period "$t$" in SC1 (ton)
$W^{II}_{vjt}$ amount of used product shipped from customer "$v$" to common collection center "$j$" at time period "$t$" in SC2 (ton)

#### 3.4.7 Outbound Logistics of Common Collection Centers

$Z^{I}_{jmct}$ amount of part "$c$" shipped from common collection center "$j$" to plant "$m$" at time period "$t$" in SC1 (ton)
$Z^{II}_{jnct}$ amount of part "$c$" shipped from common collection center "$j$" to plant "$n$" at time period "$t$" in SC2 (ton)

#### 3.4.8 Fixed Facility Cost Variables

$Q^{I}_{mt}$ if plant "$m$" is open at time period "$t$", 1; otherwise, 0 in SC1
$Q^{II}_{nt}$ if plant "$n$" is open at time period "$t$", 1; otherwise, 0 in SC2
$QC_{jt}$ if common collection center "$j$" is open at time period "$t$", 1; otherwise, 0

#### 3.4.9 Inventory Holding Cost Variables

$Cinv^{I}_{mct}$ inventory level of part "$c$" at plant "$m$" at time period "$t$" for SC1 (ton)
$Cinv^{II}_{nct}$ inventory level of part "$c$" at plant "$n$" at time period "$t$" for SC2 (ton)

### *3.5 Objective Functions*

The objective functions of the multilevel CLSC model are given in the following equations. As mentioned in Abstract, we have six different DMs, and the objective functions are changed according to the DM.

#### 3.5.1 Objective Function of Plants in SC1 ($\mathbf{DM_{01}}$)

Objective function (1) is constituted for the plants in SC1. Also this DM is the first upper-level DM of the Stackelberg game. The objective function consists of four components, and objective function for $DM_1$ with the cost components can be formulated as follows:

$$\text{Min } Z_1 = TC_1 + PC_1 + FFC_1 + IHC_1 \tag{1}$$

The transportation cost between the facilities can be calculated by multiplying the unit transportation cost:

$$TC_1 = utc \cdot \Big[\sum_k \sum_m \sum_t RM^I_{kmt} \cdot d_{km} + \sum_r \sum_m \sum_c \sum_t SP^I_{rmct} \cdot d_{rm} + \sum_i \sum_m \sum_c \sum_t CP^I_{imct} \cdot d_{im} + \sum_m \sum_u \sum_t Y^I_{mut} \cdot d_{mu} + \sum_j \sum_m \sum_c \sum_t Z^I_{jmct} \cdot d_{jm} + \sum_l \sum_m \sum_t AP^I_{lmt} \cdot d_{lm}\Big] \tag{2}$$

The purchasing cost of raw material, original or second-hand part, and semifinished product can be calculated as follows:

$$PC_1 = \Big[\Big(\sum_r \sum_m \sum_c \sum_t SP^I_{rmct} \cdot p_{rc} + \sum_i \sum_m \sum_c \sum_t CP^I_{imct} \cdot p_{ic}\Big) + \sum_j \sum_m \sum_c \sum_t Z^I_{jmct} \cdot e_{jc} + \sum_k \sum_m \sum_t RM^I_{kmt} \cdot p_k + \sum_l \sum_m \sum_t AP^I_{lmt} \cdot e_l\Big] \tag{3}$$

The fixed-opening cost of plant "*m*" can be formulated as follows:

$$FFC_1 = \sum_m \sum_t Q^I_{mt} \cdot \alpha^I_{mt} \tag{4}$$

The inventory holding cost of part "*c*" can be formulated as follows:

$$IHC_1 = \sum_m \sum_c \sum_t Cinv^I_{mct} \cdot hcp^I_{mct} \tag{5}$$

### 3.5.2 Objective Function of Plants in SC2 ($\mathbf{DM_{02}}$)

Objective function (2) is formulated for the plants in SC2. Also this DM is the second upper-level DM of the Stackelberg game. The objective function consists of four components and similar the $DM_1$'s objective function:

$$\text{Min } Z_2 = TC_2 + PC_2 + FFC_2 + IHC_2 \tag{6}$$

$$TC_2 = utc \cdot \Big(\sum_k \sum_n \sum_t RM^{II}_{knt} \cdot d_{kn} + \sum_s \sum_n \sum_c \sum_t SP^{II}_{snct} \cdot d_{sn} + \sum_i \sum_n \sum_c \sum_t CP^{II}_{inct} \cdot d_{in} + \sum_n \sum_v \sum_t Y^{II}_{nvt} \cdot d_{nv} + \sum_j \sum_n \sum_c \sum_t Z^{II}_{jnct} \cdot d_{jn} + \sum_l \sum_n \sum_t AP^{II}_{lnt} \cdot d_{ln}\Big)$$

$$PC_2 = \Big[\Big(\sum_s \sum_n \sum_c \sum_t SP^{II}_{snct} \cdot p_{sc} + \sum_i \sum_n \sum_c \sum_t CP^{II}_{inct} \cdot p_{ic}\Big) + \sum_j \sum_n \sum_c \sum_t Z^{II}_{jnct} \cdot e_{jc} + \sum_k \sum_n \sum_t RM^{II}_{knt} \cdot p_k + \sum_l \sum_n \sum_t AP^{II}_{lnt} \cdot e_l\Big] \tag{7}$$

$$FFC_2 = \sum_n \sum_t Q^{II}_{nt} \cdot \alpha^{II}_{nt} \tag{8}$$

$$IHC_2 = \sum_n \sum_c \sum_t Cinv^{II}_{nct} \cdot hcp^{II}_{nct} \tag{9}$$

### 3.5.3 Objective Function of Common Suppliers ($DM_{11}$)

Common suppliers are the first lower-level DM. The objective function maximizes the total profit, and it composes total revenue and transportation cost. Therefore, the objective function can be written as follows:

$$\text{Max } Z_3 = TR_3 - TC_3 \tag{10}$$

The total revenue of common suppliers can be calculated as follows:

$$TR_3 = \Big(\sum_i \sum_m \sum_c \sum_t CP^{I}_{imct} \cdot p_{ic} + \sum_i \sum_n \sum_c \sum_t CP^{II}_{inct} \cdot p_{ic} + \sum_i \sum_l \sum_c \sum_t CA_{ilft} \cdot p_{if}\Big) - \sum_k \sum_i \sum_t RM_{kit} \cdot p_k \tag{11}$$

The transportation cost between common suppliers and steel plate suppliers can be determined as follows:

$$TC_3 = utc \cdot \left(\sum_k \sum_i \sum_t RM_{kit} \cdot \ d_{ki}\right) \tag{12}$$

### 3.5.4 Objective Function of Common Collection Centers (DM$_{12}$)

Common collection centers are the second lower-level DM. The objective function maximizes the total profit and consists of three components:

$$\text{Max } Z_4 = TR_4 - TC_4 - FFC_4 \tag{13}$$

The total revenue of common collection centers can be calculated as follows:

$$TR_4 = \left(\sum_j \sum_m \sum_c \sum_{t \in T} Z^I_{jmct} \cdot \ e_{jc} + \sum_j \sum_n \sum_c \sum_t Z^{II}_{jnct} \cdot \ e_{jc}\right) \tag{14}$$

The transportation cost between common collection centers and customers can be determined as follows:

$$TC_4 = utc \cdot \left(\sum_u \sum_j \sum_t W^I_{ujt} \cdot \ d_{uj} + \sum_v \sum_j \sum_t W^{II}_{vjt} \cdot \ d_{vj}\right) \tag{15}$$

The fixed-opening cost of common collection center "$j$" can be formulated as follows:

$$FFC_4 = \sum_j \sum_t QC_{jt} \cdot \ \alpha_{jt} \tag{16}$$

### 3.5.5 Objective Function of Raw Material Suppliers (DM$_{13}$)

Steel plate suppliers are the third lower-level DM, and the objective function of the DM maximizes the selling price of raw material:

$$\begin{aligned} \text{Max } Z_5 = \Big(&\sum_k \sum_r \sum_t RMS^I_{krt} \cdot \ p_k + \sum_k \sum_s \sum_t RMS^{II}_{kst} \\ &\cdot \ p_k + \sum_k \sum_i \sum_t RMC_{kit} \cdot \ p_k + \sum_k \sum_m \sum_t RM^I_{kmt} \\ &\cdot p_k + \sum_k \sum_n \sum_t RM^{II}_{knt} \cdot \ p_k\Big) \end{aligned} \tag{17}$$

### 3.5.6 Objective Function of Assemblers ($DM_{14}$)

Assemblers are the fourth lower-level DM, and the objective function of the DM maximizes the total profit:

$$\text{Max } Z_6 = PC_6 - TC_6 \tag{18}$$

$$\begin{aligned} PC_6 = {} & \left(\sum_l \sum_m \sum_t AP^I_{lmt} \cdot e_l + \sum_l \sum_n \sum_t AP^{II}_{lnt} \cdot e_l\right) \\ & - \left(\sum_r \sum_l \sum_f \sum_t SA^I_{rlft} \cdot p_{rf} + \sum_s \sum_l \sum_f \sum_t SA^{II}_{slft} \right. \\ & \left. \cdot p_{sf} + \sum_i \sum_l \sum_c \sum_t CA_{ilft} \cdot p_{if}\right) \end{aligned} \tag{19}$$

$$\begin{aligned} TC_6 = utc \cdot {} & \left(\sum_r \sum_l \sum_f \sum_t SA^I_{rlft} \cdot d_{rl} + \sum_s \sum_l \sum_f \sum_t SA^{II}_{slft} \right. \\ & \left. \cdot d_{sl} + \sum_i \sum_l \sum_c \sum_t CA_{ilft} \cdot d_{il}\right) \end{aligned} \tag{20}$$

## 3.6 Constraints

### 3.6.1 Capacity Constraints

The total quantity of raw material which is sent from steel plate suppliers to the suppliers in each SC, common suppliers, and plants in each SC should be less than or equal to the capacity of those raw material suppliers during any period:

$$\sum_m RM^I_{kmt} + \sum_n RM^{II}_{knt} + \sum_r RMS^I_{krt} + \sum_s RMS^{II}_{kst} + \sum_i RMC_{kit} \le as_{kt} \ \forall_{k,t} \tag{21}$$

The total quantity of part "*c*" which is sent from suppliers to the plants in SC1 should be less than or equal to the capacity of part "*c*" of those suppliers during any period:

$$\sum_m SP^I_{rmct} \le a^I_{rct} \forall_{r,c,t} \tag{22}$$

The total quantity of part "*f*" which is sent from suppliers to the assemblers in SC1 should be less than or equal to the capacity of part "*f*" of those suppliers during any period:

$$\sum_l SA^I_{rlft} \le a^I_{rft} \forall_{r,f,t} \tag{23}$$

The total quantity of part "*c*" which is sent from suppliers to the plants in SC2 should be less than or equal to the capacity of part "*c*" of those suppliers during any period:

$$\sum_{n} SP^{II}_{snct} \leq a^{II}_{sct} \forall_{s,f,t} \tag{24}$$

The total quantity of part "*f*" which is sent from suppliers to the assemblers in SC2 should be less than or equal to the capacity of part "*f*" of those suppliers during any period:

$$\sum_{l} SA^{II}_{slft} \leq a^{II}_{sft}.\forall_{s,f,t} \tag{25}$$

The total quantity of part "*c*" which is sent from common suppliers to the plants should be less than or equal to the capacity of part "*c*" of those common suppliers during any period:

$$\sum_{m} CP^{I}_{imct} + \sum_{n} CP^{II}_{inct} \leq a_{ict} \forall_{i,c,t} \tag{26}$$

The total quantity of part "*f*" which is sent from common suppliers to the assemblers should be less than or equal to the capacity of part "*c*" of those common suppliers during any period:

$$\sum_{l} CA_{ilft} \leq a_{ift} \forall_{i,f,t} \tag{27}$$

The total quantity of semifinished products that is sent from assemblers to the plants should be less than or equal to the capacity of those assemblers during any period:

$$\sum_{m} AP^{I}_{lmt} + \sum_{n} AP^{II}_{lnt} \leq aa_{lt} \quad \forall_{l,t} \tag{28}$$

The total quantity of products that is sent from plants to customers at SC1 should be less than or equal to the capacity of part "*c*" of those plants during any period:

$$r_c \sum_{u} Y^{I}_{mut} \leq b^{I}_{mct} Q^{I}_{mt} \forall_{m,c,t} \tag{29}$$

The total quantity of products that is sent from plants to customers at SC2 should be less than or equal to the capacity of part "*c*" of those plants during any period:

$$r_c \sum_{v \in V} Y^{II}_{nvt} \leq b^{II}_{nct} Q^{II}_{nt} \forall_{n,c,t} \tag{30}$$

The total quantity of used products collected from customers should be less than or equal to the capacity of product of common collection centers during any period:

$$\sum_{u} W^{I}_{ujt} + \sum_{v} W^{II}_{vjt} \leq ca_{jt} \cdot QC_{jt} \forall_{j,t} \tag{31}$$

### 3.6.2 Demand Constraints

The demands of all customers are fully satisfied, and the total quantity of products should be greater than the customers' demand during any period:

$$\sum_{m} Y^{I}_{mut} \geq de^{I}_{ut} \forall_{u,t} \tag{32}$$

$$\sum_{n} Y^{II}_{nvt} \geq de^{II}_{vt} \forall_{v,t} \tag{33}$$

### 3.6.3 Balance Constraints (Kirchhoff Law)

According to the principle of conservation, Kirchhoff equalities assure that the sum of flows coming into that node is equal to the sum of flows going out that node in the CLSC network.

Constraints (34) and (35) ensure the flow balance at supplier "*r*" for both part "*c*" and part "*f*":

$$\sum_{c} \sum_{k} ro_c RMS^{I}_{krt} - \sum_{m} \sum_{c} SP^{I}_{rmct} = 0 \forall_{r,t} \tag{34}$$

$$\sum_{f} \sum_{k} ro_f RMS^{I}_{krt} - \sum_{l} \sum_{f} SA^{I}_{rlft} = 0 \ \forall_{r,t} \tag{35}$$

Constraints (36) and (37) ensure the flow balance at supplier "*s*" for both part "*c*" and part "*f*":

$$\sum_{c} \sum_{k} ro_c RMS^{II}_{kst} - \sum_{n} \sum_{c} SP^{II}_{snct} = 0 \ \forall_{s,t} \tag{36}$$

$$\sum_{f} \sum_{k} ro_f RMS^{II}_{kst} - \sum_{l} \sum_{f} SA^{II}_{slft} = 0 \ \forall_{s,t} \tag{37}$$

Constraints (38) and (39) ensure the flow balance at common supplier "*i*" for both part "*c*" and part "*f*":

$$\sum_{c} \sum_{k} ro_c RMC_{kit} - \left( \sum_{m} \sum_{c} CP^{I}_{imct} + \sum_{n} \sum_{c} CP^{II}_{inct} \right) = 0 \forall_{i,t} \tag{38}$$

$$\sum_{f} \sum_{k} ro_f RMC_{kit} - \sum_{l} \sum_{f} CA_{ilft} = 0 \ \forall_{i,t} \tag{39}$$

Constraint (40) ensures the flow balance at assembler "$l$" for part "$f$":

$$\sum_r SA^I_{rlft} + \sum_i CA_{ilft} + \sum_s SA^{II}_{slft} - rf_f\left(\sum_m AP^I_{lmt} + \sum_n AP^{II}_{lnt}\right) = 0\forall_{l,f,t} \quad (40)$$

Constraints (41) and (42) ensure the flow balance of used products that are collected from customers after one period usage in each SC:

$$\sum_m Y^I_{mut} - \sum_j W^I_{uj(t+1)} = 0\forall_{u,t} \quad (41)$$

$$\sum_n Y^{II}_{nvt} - \sum_j W^{II}_{vj(t+1)} = 0\forall_{v,t} \quad (42)$$

Constraints (43) and (44) ensure the flow balance of part "$c$" at common collection center "$j$":

$$r_c\left(\eta_1\sum_u W^I_{ujt}\right) - \sum_m Z^I_{jmct} = 0\forall_{j,c,t} \quad (43)$$

$$r_c\left(\eta_2\sum_v W^{II}_{vjt}\right) - \sum_n Z^{II}_{jnct} = 0\forall_{j,c,t} \quad (44)$$

Constraints (45) and (46) ensure the flow balance of semifinished product at plants "$m$" and "$n$":

$$\sum_l AP^I_{lmt} - r\sum_u Y^I_{mut} = 0\ \forall_{m,t} \quad (45)$$

$$\sum_l AP^{II}_{lnt} - r\sum_v Y^{II}_{nvt} = 0\forall_{n,t} \quad (46)$$

Constraints (47) and (48) ensure the flow balance of raw material at plants "$m$" and "$n$":

$$\sum_k RM^I_{kmt} - r'\sum_u Y^I_{mut} = 0\ \forall_{m,t} \quad (47)$$

$$\sum_k RM^{II}_{knt} - r'\sum_v Y^{II}_{nvt} = 0\ \forall_{m,t} \quad (48)$$

### 3.6.4 Inventory Constraints

Constraints (49) and (50) calculate inventory levels of part "$c$" at plants "$m$" and "$n$":

$$Cinv^{I}_{mc(t-1)} + \sum\nolimits_{r} SP^{I}_{rmct} + \sum\nolimits_{i} CP^{I}_{imct} + \sum\nolimits_{j} Z^{I}_{jmct} - r_c \sum\nolimits_{u} Y^{I}_{mut} = Cinv^{I}_{mct} \ \forall_{m,c,t} \tag{49}$$

$$Cinv^{II}_{nc(t-1)} + \sum\nolimits_{s} SP^{II}_{snct} + \sum\nolimits_{i} CP^{II}_{inct} + \sum\nolimits_{j} Z^{II}_{jnct} - r_c \sum\nolimits_{v} Y^{II}_{nvt} = Cinv^{II}_{nct} \forall_{n,c,t} \tag{50}$$

The inventory level of part "*c*" should be less than or equal to capacity of plants "*m*" and "*n*":

$$Cinv^{I}_{mct} \leq b^{I}_{mct} \forall_{m,c,t} \tag{51}$$

$$Cinv^{II}_{nct} \leq b^{II}_{nct} \forall_{n,c,t} \tag{52}$$

### 3.6.5 Non-negativity Constraints

The following constraints show non-negativity restrictions on the decision variables:

$$X^{I}_{rmct} \geq 0 \ \forall_{r,m,c,t} \tag{53}$$

$$X^{II}_{snct} \geq 0 \forall_{s,n,c,t} \tag{54}$$

$$X^{I}_{imct} \geq 0 \ \forall_{i,m,c,t} \tag{55}$$

$$X^{II}_{inct} \geq 0 \ \forall_{i,n,c,t} \tag{56}$$

$$Y^{I}_{mut} \geq 0 \ \forall_{m,u,t} \tag{57}$$

$$Y^{II}_{nvt} \geq 0 \forall_{n,v,t} \tag{58}$$

$$W^{I}_{ujt} \geq 0 \forall_{u,j,t} \tag{59}$$

$$W^{II}_{vjt} \geq 0 \ \forall_{v,j,t} \tag{60}$$

$$Z^{I}_{jmct} \geq 0 \ \forall_{j,m,c,t} \tag{61}$$

$$Z^{II}_{jnct} \geq 0 \ \forall_{j,n,c,t} \tag{62}$$

$$Cinv^{I}_{mct} \geq 0 \forall_{m,c,t} \tag{63}$$

$$Cinv^{II}_{nct} \geq 0 \forall_{n,c,t} \tag{64}$$

### 3.6.6 Binary Variables Constraints

The following constraints show binary restrictions on the decision variables:

$$Q^{I}_{mt} = \{0, 1\} \ \forall_{m,t} \tag{65}$$

$$Q^{II}_{nt} = \{0, 1\} \ \forall_{n,t} \tag{66}$$

$$QC_{jt} = \{0, 1\} \ \forall_{j,t} \tag{67}$$

## 4 A Novel Interactive Fuzzy Programming Approach

Multilevel programming problems are the most used method in decision-making problems in which one or more upper-level DMs assess the main objectives and then the lower-level DMs assess their own [24, 40]. According to the DM's behavior, multilevel programming models can be solved by centralized or decentralized approach. Many decentralized models can be solved using the Stackelberg game at which the DM at the first level chooses a strategy and after that the other DMs determine their own strategy. In this game, there is no relationship or cooperation among DMs. Thus, Stackelberg solutions do not satisfy Pareto optimality. To overcome these difficulties, in this study, we proposed a novel IFP approach using the cooperation concept between DMs. The steps of the proposed IFP approach can be summarized as follows:

**Step 1** The upper-level DMs determine the importance weight of lower DMs by using a multi-criteria decision-making method such as AHP, ANP, etc. In this study, we used fuzzy AHP for obtaining lower-level DMs' relative weights. The pairwise comparison matrix can be shown as follows:

$$\tilde{D} = \begin{bmatrix} 1 & \cdots & \tilde{a}_{1j} & \cdots & \tilde{a}_{1n} \\ \vdots & & \vdots & & \vdots \\ \tilde{a}_{i1} & \cdots & 1 & \cdots & \tilde{a}_{in} \\ \vdots & & \vdots & & \vdots \\ \tilde{a}_{n1} & & \tilde{a}_{nj} & \cdots & 1 \end{bmatrix}$$

The weight vector of the lower-level DMs obtained by this pairwise comparison matrix, $w_j$, shows the relative weight of lower-level DM. Geometric mean is used to aggregate upper-level DMs' opinions.

## *4.1 Determining the Weights of Individual Objective Functions*

**Step 2** All DMs determine their own weights of their objectives. According to the obtained individual objective weights, all of them optimize their objectives as follows:

$$\text{Min } Z_{0i} = w_{0i1} \cdot x_1 + w_{0i2} \cdot x_2 + \cdots + w_{0il} \cdot x_m$$

$$\text{Min } Z_{1j} = w_{1j1} \cdot x_1 + w_{1j2} \cdot x_2 + \cdots + w_{1jl} \cdot x_n$$

We consider that there are $i$ DMs at the upper level $(DM_1, DM_2, \ldots, DM_m)$ $[Z_{0i}]$ and $j$ DMs $(DM_1, DM_2, \ldots, DM_n)$ $[Z_{1j}]$ at the lower level and $l$ shows the number of objectives.

$w_{0il}$: The weight of the objective $\boldsymbol{l}$ of the upper-level DM $\boldsymbol{i}$

$w_{1jl}$: The weight of the objective $\boldsymbol{l}$ of the lower-level DM $\boldsymbol{j}$

### 4.1.1 Creating the Payoff Table

**Step 3** According to the weighted sum method, the payoff table is obtained using the individual objective functions for all DMs.

### 4.1.2 Determining the Minimum Satisfactory Levels

**Step 4** The upper-level DM determines the minimum satisfactory level ($\delta_0$), and the lower DMs determine their own minimum satisfactory level based on the upper level's minimum satisfactory level.

$$\Gamma_{1j}^{L} = w_j \cdot \ \delta_0$$

### 4.1.3 Solving the Main Problem

**Step 5** The following problem is solved and the satisfactory levels are obtained for all the DMs.

$$\text{Max} \ \sum\nolimits_{i=1}^{m} \mu_{0i}\left(Z_{0i}\right) + \sum\nolimits_{j=1}^{n} w_j \cdot \ \mu_{1j}\left(Z_{1j}\right)$$

$$\mu_{0i}\left(Z_{0i}\right) \geq \delta_0$$

$$\mu_{1j}\left(Z_{1j}\right) \geq \Gamma_{1j}^{L}$$

$$Ax \leq b \tag{68}$$

### 4.1.4 Interaction between Upper- and Lower-Level DMs

**Step 6** The upper-level DMs specify the lower bound and the upper bound $[\Delta_L, \Delta_U]$ of $\Delta_j$ for the satisfactory balance between upper- and lower-level DMs. The ratio of satisfactory levels for DMs can be determined as follows:

$$\Delta_j = \frac{\mu_{1j}\left(Z_{1j}\right)}{\min\left(\mu_{0i}\left(Z_{0i}\right)\right)} \tag{69}$$

After the ratio is determined, the following conditions are examined:

(i) If $\Delta_j \in [\Delta_L, \Delta_U]$, an efficient compromise solution is obtained and algorithm is ended.

(ii) If $\Delta_j \leq \Delta_L$, the minimum satisfactory level of the lower-level DM "$j$" is determined according to the following equation:

$$\Gamma_{1j}^{L} = \frac{w_j \cdot \ \delta_0}{\Delta_U} \tag{70}$$

(iii) If $\Delta_j > \Delta_L$, the minimum satisfactory level of the lower-level DM "$j$" is determined according to the following equation:

$$\Gamma_{1j}^{L} = w_j \cdot \ \delta_0 \cdot \Delta_U \tag{71}$$

#### 4.1.5 Termination Condition

The upper-level DM updates the minimal satisfactory levels of lower-level DMs until the second or third conditions are satisfied. The IFP algorithm continues until the upper-level DMs are satisfied with the compromise solution.

## 5 Computational Experiments

In this section, we give the computational results to explore the validity of the proposed CLSC model and proposed IFP approach. We then present some managerial implications based on the computational results. The description of the data is given in following section.

### *5.1 Description of Data*

The performance of the proposed model CLSC model is investigated by randomly generated parameters. The distributions of the parameters are given in Table 1. After defining the parameters, the individual objective functions are solved.

For solving individual objective functions, three raw material suppliers, two common suppliers, two assemblers, and two common suppliers are determined in alliance SC, while four suppliers, three plants, and five customers are determined in SC1 and SC2. It is assumed that one final product includes four parts, one semifinished product, and raw material. Also, a semifinished product includes three parts. The utilization rates of the semifinished product parts and of raw material in one final product are given in Fig. 2.

The shipping cost (*utc*) was accepted as 0.75 cents per ton-km, the purchasing cost/selling price of semifinished product was accepted \$1000, and the purchasing cost/selling price of raw material was accepted \$50. It is assumed that amount of used products, defined as a percentage $\eta_1 = 0.25$ for SC1 and $\eta_2 = 0.25$ for SC2, of demand must be collected in customer zones.

The decentralized multilevel CLSC model (1)–(67) of the sample network contains 1189 variables and 666 constraints for $DM_{01}$. The mathematical model was

**Table 1** The values of random generated data

| Parameters | Corresponding random distribution |
|---|---|
| $d_{km}$, $d_{kn}$ | ∼ Uniform (100, 400) |
| $d_{ki}$ | ∼ Uniform (50, 200) |
| $d_{im}$, $d_{in}$ | ∼ Uniform (100, 300) |
| $d_{il}$, $d_{rl}$, $d_{sl}$ | ∼ Uniform (60, 240) |
| $d_{rm}$, $d_{sn}$ | ∼ Uniform (100, 300) |
| $d_{lm}$, $d_{ln}$, $d_{mu}$, $d_{nv}$ | ∼ Uniform (50, 150) |
| $d_{uj}$, $d_{vj}$ | ∼ Uniform (25, 200) |
| $d_{jm}$, $d_{jn}$ | ∼ Uniform (50, 450) |
| $as_{kt}$ | ∼ Uniform (100, 2000) |
| $a^{I}_{rct}$, $a^{II}_{sct}$ | ∼ Uniform (500, 1000) |
| $a_{ict}$ | ∼ Uniform (500, 2000) |
| $a^{I}_{rft}$, $a^{II}_{sft}$, $a_{ift}$ | ∼ Uniform (500, 1000) |
| $b^{I}_{mct}$, $b^{II}_{nct}$ | ∼ Uniform (250, 750) |
| $aa_{lt}$ | ∼ Uniform (500, 1400) |
| $ca_{jt}$ | ∼ Uniform (2000, 5000) |
| $de^{I}_{ut}$, $de^{II}_{vt}$ | ∼ Uniform (100, 400) |
| $\alpha^{I}_{mt}$, $\alpha^{II}_{nt}$ | ∼ Uniform (10,000, 100,000) |
| $\alpha_{jt}$ | ∼ Uniform (5000, 50,000) |
| $p_{rc}$, $p_{sc}$ | ∼ Uniform (50, 500) |
| $p_{ic}$ | ∼ Uniform (40, 400) |
| $p_{rf}$, $p_{sf}$ | ∼ Uniform (30, 450) |
| $p_{if}$ | ∼ Uniform (25, 400) |
| $e_{jc}$ | ∼ Uniform (800, 1200) |
| $hcp^{I}_{mct}$, $hcp^{II}_{nct}$ | ∼ Uniform (100, 500) |

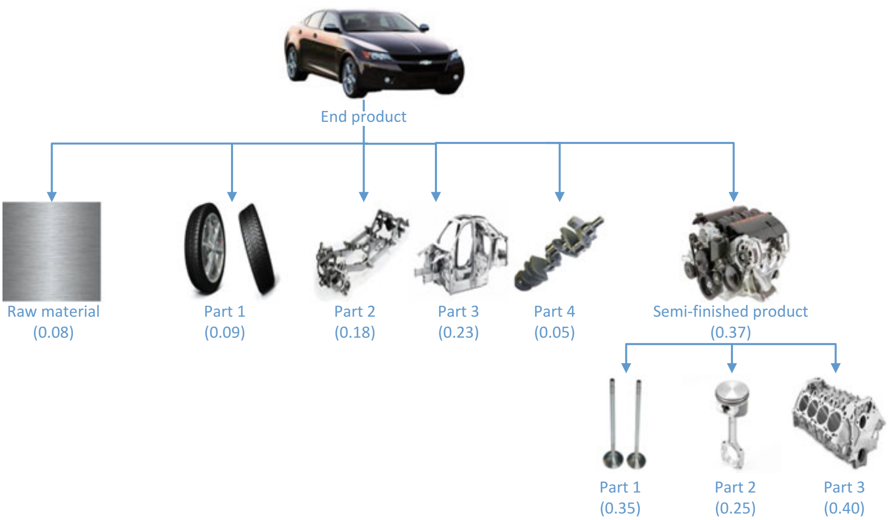


**Fig. 2** Bill of materials of the final product

solved with GAMS-CPLEX 24.0.1, on a laptop with Intel Core i5 M480 2.67 GHz and 3 GB RAM memory, and the computation time required to solve the model to optimality using the GAMS-CPLEX 24.0.1 is 0.28 CPU seconds. To give some details about the solutions, firstly we solved individual objectives for the upper-level DMs, and distribution plans of these DMs can be seen in Fig. 3 and Fig. 4.

When the model is designed, the product is considered to use at least one period before entering into recycling and so nothing returned from customers during the first period as seen in Figs. 3 and 4. According to Fig. 4, in first period, 1936.07 tons of raw materials are sent from raw material suppliers to plants, suppliers, and common suppliers. 743.33 tons of semifinished product are used for obtaining 971 tons of product in plant 3 in SC1. As it can be seen in Figs. 3 and 4, common suppliers did not open in the third period.

When the CLSC model is solved for the plants in SC1 ($DM_{01}$), the maximum cost (2,436,751.87) is obtained for purchasing cost of plants in SC2 as seen in Fig. 5. As the results show, there is no inventory cost, and purchasing cost is the highest ratio for the plants in SC1 ($DM_{01}$). According to the results, we say that the purchasing cost is the most important factor for all DMs.

Comparison of the objective function values is given in Fig. 5. When we calculated the objective function values via minimizing $Z_2$, all the objective function values were increased except for the $DM_{14}$. Note that a major cost of the production is carried out in the plants in SC2, and the plants in SC2 did not keep any inventory for production. Figure 6 shows the purchased unused parts and used parts of plants for SC1, and SC2 is given in Fig. 6.

As it is seen from Fig. 6, the amounts of purchased virgin parts stayed in the range of minimum and maximum values in the optimal solution of $Z_4$ and $Z_6$ for SC2. As expected, maximum purchased used part amounts are obtained in the solution of $Z_4$, while the minimum virgin part amounts are purchased in the solution of $Z_1$.

## 5.2 *The Solution of the CLSC Model with Novel IFP Approach*

**Step 1** In order to obtain the importance weight/relative weight of lower-level DMs, we used Chang's extent analysis method [41]. For the evaluation procedure, the linguistic terms given in Table 2 are used. In order to aggregate different expert opinions, geometric mean is used.

The assessments of upper-level DMs were presented in Table 3. It represents assessment information provided by the two upper-level DMs and aggregation of these judgments.

The weight vector of the lower-level DMs obtained by this pairwise comparison matrix was given as $(0.4504, 0.2090, 0.1416, 0.1990)$.

**Step 2** All DMs determined their own objectives and the following weight vectors are obtained (Table 4).

According to the $DM_{01}$ and $DM_{02}$, the most important factors of the objective functions are the cost of purchasing over all product parts, the cost of transportation, the fixed-opening costs, and the cost of inventory of parts, respectively. According

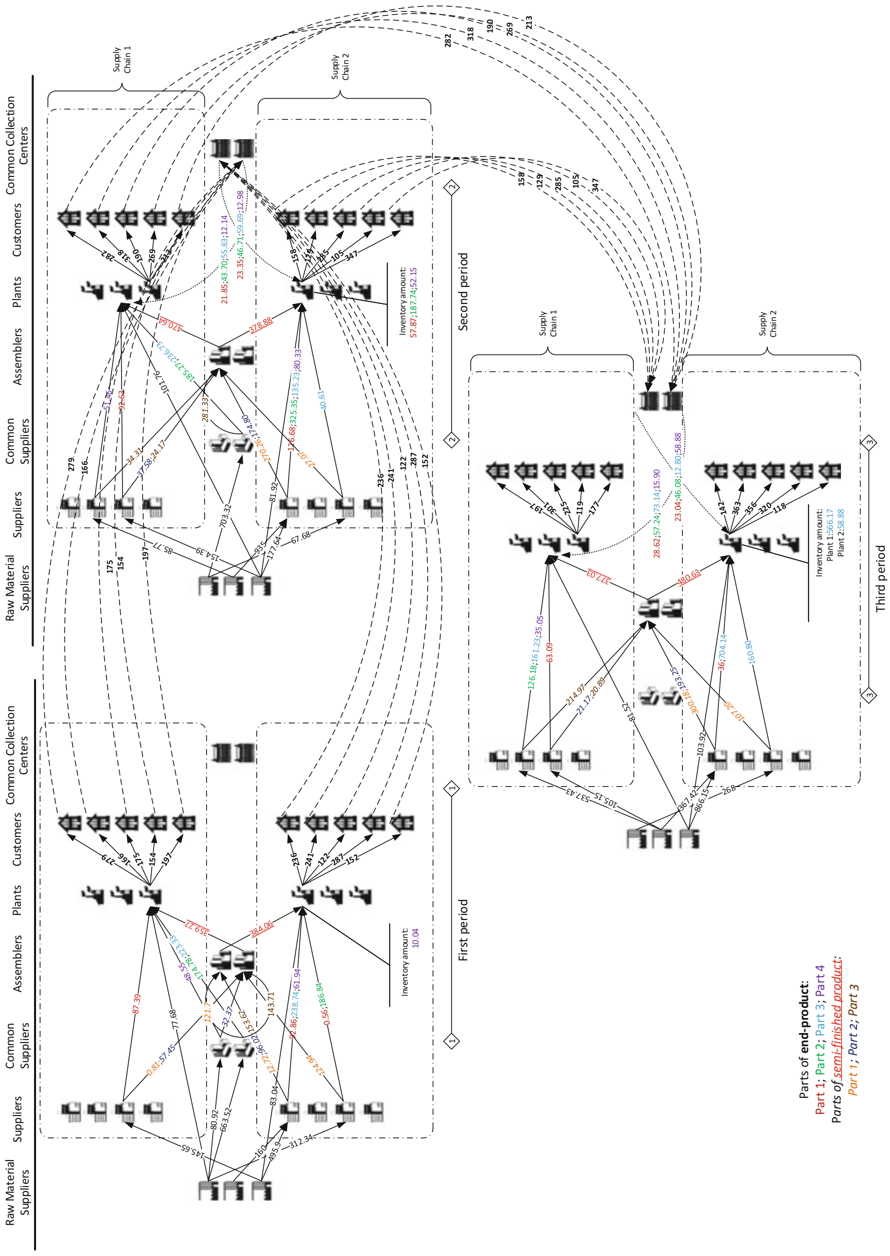


**Fig. 3** Optimal distribution plan for the plants in SC1 ($DM_{01}$)

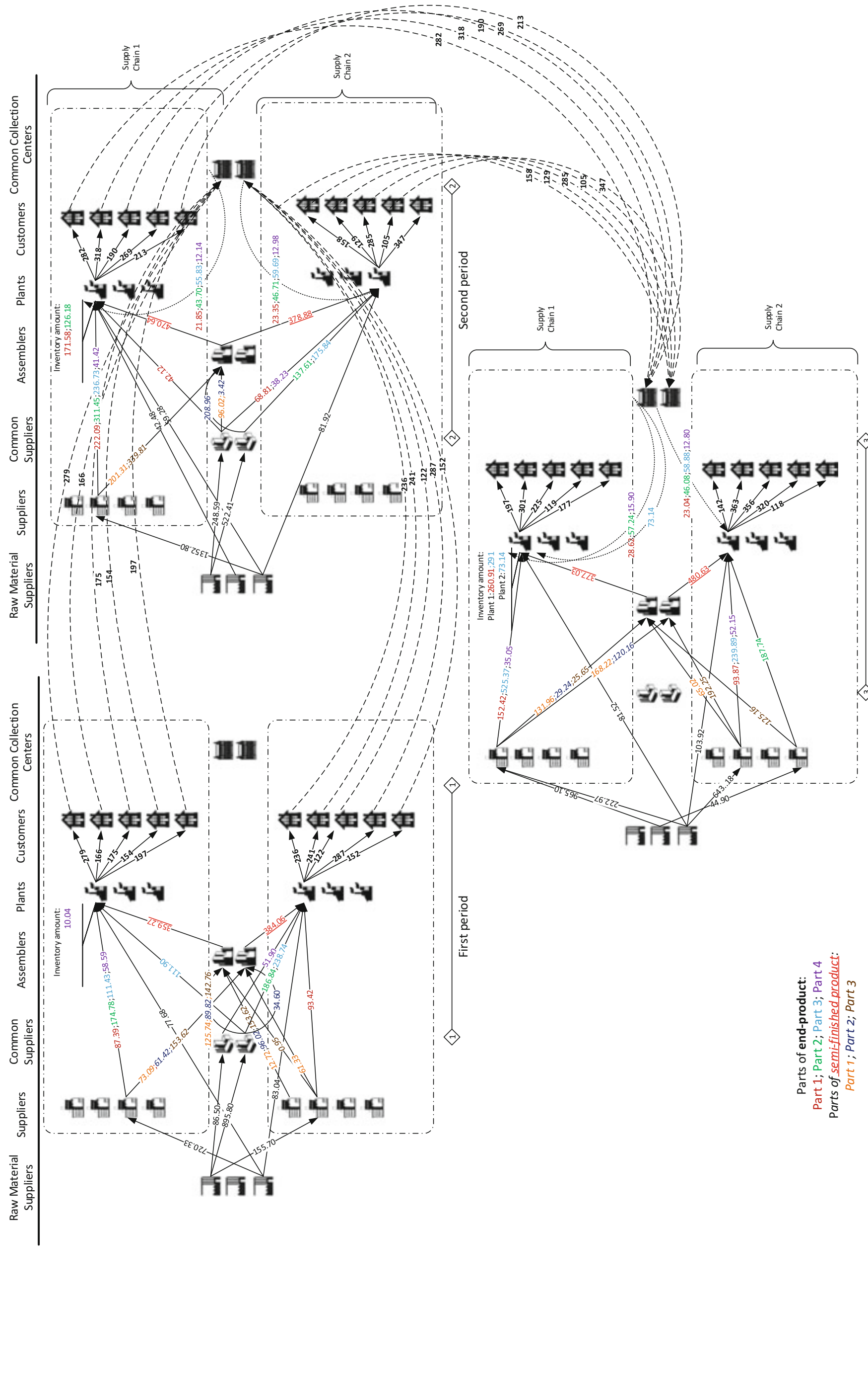


**Fig. 4** Optimal distribution plan for the plants in SC2

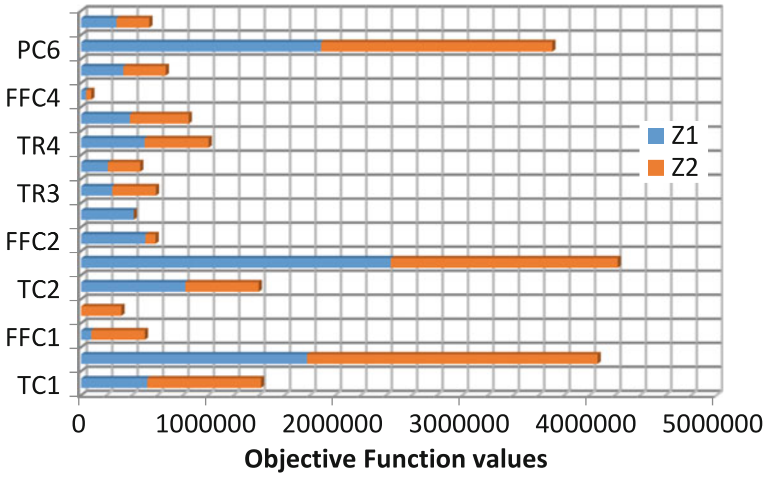


**Fig. 5** Comparison of objective function values for the plants SC1 ($DM_{01}$) and the plants SC1 ($DM_{02}$)

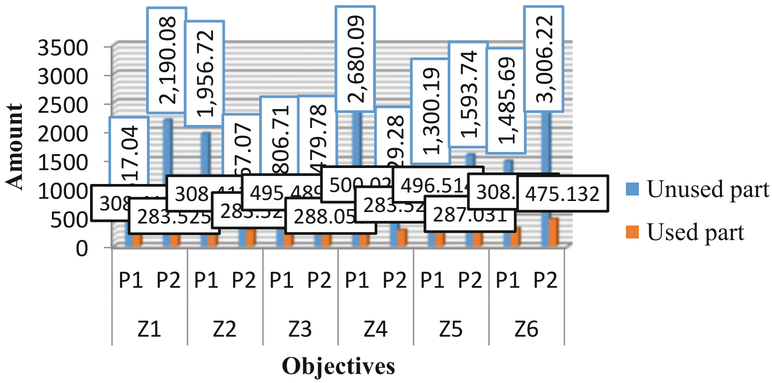


**Fig. 6** The distribution of parts according to each solution

to the $DM_{12}$, the cost of purchasing overall product parts, the fixed-opening costs, and the cost of transportation are the most important factors.

**Step 3** We obtained the following trade-off table using the weighted sum method. The membership functions belonging to objectives for DMs are obtained as the following: (Table 5)

$$\mu_{01}\left(Z_{01}\right) = \begin{cases} 1, \quad Z_{01}(x) \leq 1093235.85 \\ \frac{2110998.50 - Z_{01}(x)}{2110998.50 - 1093235.85}, \ 1093235.851 \leq Z_{01}(x) \leq 2110998.50 \\ 0, \ Z_{01}(x) \geq 2110998.50 \end{cases} \tag{72}$$

**Table 2** Linguistic scale for weight matrix

| Linguistic terms | Triangular fuzzy scale | Triangular fuzzy reciprocal scale |
|---|---|---|
| *Equal importance* | (1, 1, 1) | (1/1, 1/1, 1/1) |
| *Moderate importance* | (1, 3, 5) | (1/5, 1/3, 1/1) |
| *Essential or strong importance* | (3, 5, 7) | (1/7, 1/5, 1/3) |
| *Demonstrated importance* | (5, 7, 9) | (1/9, 1/7, 1/5) |
| *Extreme importance* | (7, 9, 9) | (1/9, 1/9, 1/7) |
| *Intermediate values* | (1, 2, 3) | (1/3, 1/2, 1) |
| | (3, 4, 5) | (1/5, 1/4, 1/3) |
| | (5, 6, 7) | (1/7, 1/6, 1/5) |
| | (7, 8, 9) | (1/9, 1/8, 1/7) |

**Table 3** Comparative judgments of the lower-level DMs and aggregated weights

| | $DM_{11}$ | $DM_{12}$ | $DM_{13}$ | $DM_{14}$ |
|---|---|---|---|---|
| $DM_{01}$ | | | | |
| $DM_{11}$ | (1, 1, 1) | (3, 5, 7) | (3, 4, 5) | (3, 4, 5) |
| $DM_{12}$ | (0.14, 0.2, 0.33) | (1, 1, 1) | (3, 5, 7) | (3, 5, 7) |
| $DM_{13}$ | (0.2, 0.25, 0.33) | (0.14, 0.2, 0.33) | (1, 1, 1) | (0.2, 0.25, 0.33) |
| $DM_{14}$ | (0.2, 0.25, 0.33) | (0.14, 0.2, 0.33) | (3, 4, 5) | (1, 1, 1) |
| $DM_{02}$ | | | | |
| $DM_{11}$ | (1, 1, 1) | (0.2, 0.33, 1) | (3, 4, 5) | (0.14, 0.2, 0.33) |
| $DM_{12}$ | (1, 3, 5) | (1, 1, 1) | (0.14, 0.2, 0.33) | (0.14, 0.2, 0.33) |
| $DM_{13}$ | (0.2, 0.25, 0.33) | (3, 5, 7) | (1, 1, 1) | (5, 6, 7) |
| $DM_{14}$ | (3, 5, 7) | (3, 5, 7) | (0.14, 0.16, 0.2) | (1, 1, 1) |
| *Aggregation of upper-level DM judgments* | | | | |
| $DM_{11}$ | (1, 1, 1) | (0.77, 1.29, 2.64) | (3, 4, 5) | (0.65, 0.89, 1.29) |
| $DM_{12}$ | (0.37, 0.77, 1.29) | (1, 1, 1) | (0.65, 1, 1.52) | (0.65, 1, 1.52) |
| $DM_{13}$ | (0.2, 0.25, 0.33) | (0.65, 1, 1.52) | (1, 1, 1) | (1, 1.22, 1.52) |
| $DM_{14}$ | (0.77, 1.11, 1.52) | (0.65, 1, 1.52) | (0.65, 0.85, 1) | (1, 1, 1) |

**Table 4** Relative weights of objectives for all DMs

| Decision-makers | Weight vector |
|---|---|
| $DM_{01}$ | (0.25, 0.55, 0.15, 0.05) |
| $DM_{02}$ | (0.25, 0.55, 0.15, 0.05) |
| $DM_{11}$ | (0.60,0.40) |
| $DM_{12}$ | (0.50, 0.40, 0.10) |
| $DM_{13}$ | (1) |
| $DM_{14}$ | (0.80,0.20) |

**Table 5** The payoff table

| | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ | $Z_5$ | $Z_6$ |
|---|---|---|---|---|---|---|
| min $Z_1$ | **1.093.235,85** | 1.580.918,18 | 43.426,38 | 287.024,06 | 332.805,75 | 1.562.666,72 |
| min $Z_2$ | 2.110.998,50 | **1.130.015,62** | 51.246,98 | 256.599,65 | 402.829,50 | 1.816.494,19 |
| maks $Z_3$ | 1.870.777,27 | 1.617.982,84 | **499.527,66** | 345.856,19 | 402.829,50 | 1.845.534,01 |
| maks $Z_4$ | 1.559.424,06 | 1.957.316,69 | 113.562,55 | **464.204,50** | 402.829,50 | 1.775.631,37 |
| maks $Z_5$ | 1.942.005,07 | 1.579.774,20 | 104.757,66 | 294.194,89 | **406.700,00** | 1.824.714,49 |
| maks $Z_6$ | 1.376.014,91 | 1.994.078,41 | 40.371,94 | 409.405,71 | 406.700,00 | **2.208.122,17** |
| The worst values | *2.110.998,50* | *1.994.078,41* | *40.371,94* | *256.599,65* | *332.805,75* | *1.562.666,72* |

$$\mu_{02}\left(Z_{02}\right) = \begin{cases} 1,\ Z_{02}(x) \leq 1130015.62 \\ \frac{1994078.41 - Z_{02}(x)}{1994078.41 - 1130015.62},\ 1130015.62 \leq Z_{02}(x) \leq 1994078.41 \\ 0,\ Z_{02}(x) \geq 1994078.41 \end{cases} \tag{73}$$

$$\mu_{11}\left(Z_{11}\right) = \begin{cases} 1,\ Z_{11}(x) \geq 499527.66 \\ \frac{Z_{11}(x) - 40371.94}{499527.66 - 40371.94},\ 40371.94 \leq Z_{11}(x) \leq 499527.66 \\ 0,\ Z_{11}(x) \leq 40371.94 \end{cases} \tag{74}$$

$$\mu_{12}\left(Z_{12}\right) = \begin{cases} 1,\ Z_{12}(x) \geq 464204.50 \\ \frac{Z_{12}(x) - 256599.65}{464204.50 - 256599.65},\ 256599.65 \leq Z_{12}(x) \leq 464204.50 \\ 0,\ Z_{12}(x) \leq 256599.65 \end{cases} \tag{75}$$

$$\mu_{13}\left(Z_{13}\right) = \begin{cases} 1,\ Z_{13}\,(x) \geq 406700.00 \\ \frac{Z_{13}(x) - 332805.75}{406700.00 - 332805.75},\ 332805.75 \leq Z_{13}\,(x) \leq 406700.00 \\ 0,\ Z_{13}\,(x) \leq 332805.75 \end{cases} \tag{76}$$

$$\mu_{14}\left(Z_{14}\right) = \begin{cases} 1,\ Z_{14}\,(x) \geq 2208122.17 \\ \frac{Z_{14}(x) - 1562666.72}{2208122.17 - 1562666.72},\ 1562666.72 \leq Z_{14}\,(x) \leq 2208122.17 \\ 0,\ Z_{14}\,(x) \leq 1562666.72 \end{cases} \tag{77}$$

**Step 4** Assume that $DM_{01}$ and $DM_{02}$ specify the minimal satisfactory level as $\delta_0 = 0.75$. Using this value, the satisfactory levels for lower-level DMs are evaluated:

$$\Gamma_{11}^{L} = w_1 \cdot \ \delta_0 = 0.4504 \cdot \ 0.75 = 0.3378$$

$$\Gamma_{12}^{L} = w_2 \cdot \ \delta_0 = 0.2090 \cdot \ 0.75 = 0.1567$$

$$\Gamma_{13}^{L} = w_3 \cdot \delta_0 = 0.1416 \cdot 0.75 = 0.1062$$

$$\Gamma_{14}^{L} = w_4 \cdot \delta_0 = 0.1990 \cdot 0.75 = 0.1492$$

**Step 5** The following problem is solved and the satisfaction levels are obtained for all the DMs.

$$\begin{aligned} \text{Max } & \left(\mu_{01}\left(Z_{01}\right) + \mu_{02}\left(Z_{02}\right)\right) + w_1 \cdot \mu_{11}\left(Z_{11}\right) + w_2 \cdot \mu_{12}\left(Z_{12}\right) \\ & + w_3 \cdot \mu_{13}\left(Z_{13}\right) + w_4 \cdot \mu_{14}\left(Z_{14}\right) \end{aligned}$$

$$\mu_{01}\left(Z_{01}\right) \geq \delta_0 = 0.75$$

$$\mu_{02}\left(Z_{02}\right) \geq \delta_0 = 0.75$$

$$\mu_{11}\left(Z_{11}\right) \geq \Gamma_{11}^{L} = 0.3378$$

$$\mu_{12}\left(Z_{12}\right) \geq \Gamma_{12}^{L} = 0.1567$$

$$\mu_{13}\left(Z_{13}\right) \geq \Gamma_{13}^{L} = 0.1062$$

$$\mu_{14}\left(Z_{14}\right) \geq \Gamma_{14}^{L} = 0.1492$$

$$\textit{Constraints } (1)\text{–}(67) \tag{78}$$

**Step 6**
**Iteration 1.** The satisfactory levels of DMs, $\mu_{01}(Z_{01}) = 0.81$, $\mu_{02}(Z_{02}) = 0.95$, $\mu_{11}(Z_{11}) = 0.53$, $\mu_{12}(Z_{12}) = 0.18$, $\mu_{13}(Z_{13}) = 0.11$, and $\mu_{14}(Z_{14}) = 0.25$, are obtained.

Assume that the upper-level DMs specify $[\Delta_L, \Delta_U] = [0.40, 0.80]$. The ratio of satisfactory levels between the upper and lower DMs is obtained using

$$\Delta_j = \frac{\mu_{1j}\left(Z_{1j}\right)}{\min\left(\mu_{0i}\left(Z_{0i}\right)\right)}$$

and then,

$$\Delta_1 = \frac{\mu_{11}(Z_{11})}{\mu_{01}(Z_{01})} = \frac{0.53}{0.81} = 0.6543$$

$$\Delta_2 = \frac{\mu_{12}(Z_{12})}{\mu_{01}(Z_{01})} = \frac{0.18}{0.81} = 0.2222$$

$$\Delta_3 = \frac{\mu_{13}(Z_{13})}{\mu_{01}(Z_{01})} = \frac{0.11}{0.81} = 0.1358$$

$$\Delta_4 = \frac{\mu_{14}(Z_{14})}{\mu_{01}(Z_{01})} = \frac{0.25}{0.81} = 0.3086$$

Since $\Delta_2, \Delta_3$, and $\Delta_4 < \Delta_L$, the DMs determine their own satisfactory levels as follows:

$$\Gamma_{12}^{L} = \frac{w_2 \cdot \delta_0}{\Delta_U} = \frac{0.1567}{0.80} = 0.1958$$

$$\Gamma_{13}^{L} = \frac{w_3 \cdot \delta_0}{\Delta_U} = \frac{0.1062}{0.80} = 0.1327$$

$$\Gamma_{14}^{L} = \frac{w_4 \cdot \delta_0}{\Delta_U} = \frac{0.1492}{0.80} = 0.1865$$

Using these values the main problem is formulated in the following equation:

$$\begin{aligned} \text{Max } & (\mu_{01}(Z_{01}) + \mu_{02}(Z_{02})) + w_1 \cdot \mu_{11}(Z_{11}) + w_2 \cdot \mu_{12}(Z_{12}) \\ & + w_3 \cdot \mu_{13}(Z_{13}) + w_4 \cdot \mu_{14}(Z_{14}) \end{aligned}$$

$$\mu_{01}(Z_{01}) \geq \delta_0 = 0.75$$

$$\mu_{02}(Z_{02}) \geq \delta_0 = 0.75$$

$$\mu_{11}(Z_{11}) \geq \Gamma_{11}^{L} = 0.3378$$

$$\mu_{12}(Z_{12}) \geq \Gamma_{12}^{L} = 0.1958$$

$$\mu_{13}(Z_{13}) \geq \Gamma_{13}^{L} = 0.1327$$

$$\mu_{14}(Z_{14}) \geq \Gamma_{14}^{L} = 0.1865$$

$$\text{Constraints}(1)\text{–}(67) \tag{79}$$

**Iteration 2.** The satisfactory degrees of DMs, $\mu_{01}(Z_{01}) = 0.81$, $\mu_{02}(Z_{02}) = 0.94$, $\mu_{11}(Z_{11}) = 0.53$, $\mu_{12}(Z_{12}) = 0.20$, $\mu_{13}(Z_{13}) = 0.13$, and $\mu_{14}(Z_{14}) = 0.26$, are obtained, and the ratios of satisfactory levels are obtained as follows:

$$\Delta_1 = \frac{\mu_{11}(Z_{11})}{\mu_{01}(Z_{01})} = \frac{0.53}{0.81} = 0.6543$$

$$\Delta_2 = \frac{\mu_{12}(Z_{12})}{\mu_{01}(Z_{01})} = \frac{0.20}{0.81} = 0.2469$$

$$\Delta_3 = \frac{\mu_{13}(Z_{13})}{\mu_{01}(Z_{01})} = \frac{0.13}{0.81} = 0.1604$$

$$\Delta_4 = \frac{\mu_{14}(Z_{14})}{\mu_{01}(Z_{01})} = \frac{0.26}{0.81} = 0.3209$$

Since $\Delta_2, \Delta_3$, and $\Delta_4 < \Delta_L$, the DMs determine their own satisfactory levels as the following:

$$\Gamma_{12}^{L} = \frac{w_2 \cdot \delta_0}{\Delta_U} = \frac{0.1958}{0.80} = 0.2447$$

$$\Gamma_{13}^{L} = \frac{w_3 \cdot \delta_0}{\Delta_U} = \frac{0.1327}{0.80} = 0.1658$$

$$\Gamma_{14}^{L} = \frac{w_4 \cdot \delta_0}{\Delta_U} = \frac{0.1865}{0.80} = 0.2331$$

Using these values the main problem is formulated in the following equation:

$$\begin{aligned}\text{Max } & (\mu_{01}(Z_{01}) + \mu_{02}(Z_{02})) + w_1 \cdot \mu_{11}(Z_{11}) + w_2 \cdot \mu_{12}(Z_{12}) \\ & + w_3 \cdot \mu_{13}(Z_{13}) + w_4 \cdot \mu_{14}(Z_{14})\end{aligned}$$

$$\mu_{01}(Z_{01}) \geq \delta_0 = 0.75$$

$$\mu_{02}\left(Z_{02}\right) \geq \delta_0 = 0.75$$

$$\mu_{11}\left(Z_{11}\right) \geq \Gamma_{11}^{L} = 0.3378$$

$$\mu_{12}\left(Z_{12}\right) \geq \Gamma_{12}^{L} = 0.2447$$

$$\mu_{13}\left(Z_{13}\right) \geq \Gamma_{13}^{L} = 0.1658$$

$$\mu_{14}\left(Z_{14}\right) \geq \Gamma_{14}^{L} = 0.2331$$

$$\text{Constraints (1)–(67)} \tag{80}$$

**Iteration 3.** The satisfactory degrees of DMs, $\mu_{01}(Z_{01}) = 0.83$, $\mu_{02}(Z_{02}) = 0.89$, $\mu_{11}(Z_{11}) = 0.53$, $\mu_{12}(Z_{12}) = 0.27$, $\mu_{13}(Z_{13}) = 0.17$, and $\mu_{14}(Z_{14}) = 0.28$, are obtained, and the ratios of satisfactory levels are obtained as follows:

$$\Delta_1 = \frac{\mu_{11}\left(Z_{11}\right)}{\mu_{01}\left(Z_{01}\right)} = \frac{0.53}{0.83} = 0.6385$$

$$\Delta_2 = \frac{\mu_{12}\left(Z_{12}\right)}{\mu_{01}\left(Z_{01}\right)} = \frac{0.27}{0.83} = 0.3253$$

$$\Delta_3 = \frac{\mu_{13}\left(Z_{13}\right)}{\mu_{01}\left(Z_{01}\right)} = \frac{0.17}{0.83} = 0.2048$$

$$\Delta_4 = \frac{\mu_{14}\left(Z_{14}\right)}{\mu_{01}\left(Z_{01}\right)} = \frac{0.28}{0.83} = 0.3373$$

Since $\Delta_2, \Delta_3$, and $\Delta_4 < \Delta_L$, the DMs determine their own satisfactory levels as follows:

$$\Gamma_{12}^{L} = \frac{w_2 \cdot \delta_0}{\Delta_U} = \frac{0.2447}{0.80} = 0.3058$$

$$\Gamma_{13}^{L} = \frac{w_3 \cdot \delta_0}{\Delta_U} = \frac{0.1658}{0.80} = 0.2072$$

$$\Gamma_{14}^{L} = \frac{w_4 \cdot \delta_0}{\Delta_U} = \frac{0.2331}{0.80} = 0.2913$$

Using these values the main problem is formulated in the following equation:

$$\begin{aligned} \text{Max } & (\mu_{01}(Z_{01}) + \mu_{02}(Z_{02})) + w_1 \cdot \mu_{11}(Z_{11}) + w_2 \cdot \mu_{12}(Z_{12}) \\ & + w_3 \cdot \mu_{13}(Z_{13}) + w_4 \cdot \mu_{14}(Z_{14}) \end{aligned}$$

$$\mu_{01}(Z_{01}) \geq \delta_0 = 0.75$$

$$\mu_{02}(Z_{02}) \geq \delta_0 = 0.75$$

$$\mu_{11}(Z_{11}) \geq \Gamma_{11}^{L} = 0.3378$$

$$\mu_{12}(Z_{12}) \geq \Gamma_{12}^{L} = 0.3058$$

$$\mu_{13}(Z_{13}) \geq \Gamma_{13}^{L} = 0.2072$$

$$\mu_{14}(Z_{14}) \geq \Gamma_{14}^{L} = 0.2913$$

$$\text{Constraints (1)–(67)} \tag{81}$$

**Iteration 4.** The satisfactory degrees of DMs, $\mu_{01}(Z_{01}) = 0.85$, $\mu_{02}(Z_{02}) = 0.84$, $\mu_{11}(Z_{11}) = 0.54$, $\mu_{12}(Z_{12}) = 0.31$, $\mu_{13}(Z_{13}) = 0.21$, and $\mu_{14}(Z_{14}) = 0.30$, are obtained, and the ratios of satisfactory levels are obtained as follows:

$$\Delta_1 = \frac{\mu_{11}(Z_{11})}{\mu_{02}(Z_{02})} = \frac{0.54}{0.84} = 0.6428$$

$$\Delta_2 = \frac{\mu_{12}(Z_{12})}{\mu_{02}(Z_{02})} = \frac{0.31}{0.84} = 0.3690$$

$$\Delta_3 = \frac{\mu_{13}(Z_{13})}{\mu_{02}(Z_{02})} = \frac{0.21}{0.84} = 0.25$$

$$\Delta_4 = \frac{\mu_{14}(Z_{14})}{\mu_{02}(Z_{02})} = \frac{0.30}{0.84} = 0.3571$$

Since $\Delta_2, \Delta_3$, and $\Delta_4 < \Delta_L$, the DMs determine their own satisfactory levels as follows:

$$\Gamma_{12}^{L} = \frac{w_2 \cdot \delta_0}{\Delta_U} = \frac{0.3058}{0.80} = 0.3822$$

$$\Gamma_{13}^{L} = \frac{w_3 \cdot \delta_0}{\Delta_U} = \frac{0.2072}{0.80} = 0.259$$

$$\Gamma_{14}^{L} = \frac{w_4 \cdot \delta_0}{\Delta_U} = \frac{0.2913}{0.80} = 0.3641$$

Using these values the main problem is formulated in the following equation:

$$\begin{aligned} \text{Max } & (\mu_{01}(Z_{01}) + \mu_{02}(Z_{02})) + w_1 \cdot \mu_{11}(Z_{11}) + w_2 \cdot \mu_{12}(Z_{12}) \\ & + w_3 \cdot \mu_{13}(Z_{13}) + w_4 \cdot \mu_{14}(Z_{14}) \end{aligned}$$

$$\mu_{01}(Z_{01}) \geq \delta_0 = 0.75$$

$$\mu_{02}(Z_{02}) \geq \delta_0 = 0.75$$

$$\mu_{11}(Z_{11}) \geq \Gamma_{11}^{L} = 0.3378$$

$$\mu_{12}(Z_{12}) \geq \Gamma_{12}^{L} = 0.3822$$

$$\mu_{13}(Z_{13}) \geq \Gamma_{13}^{L} = 0.259$$

$$\mu_{14}(Z_{14}) \geq \Gamma_{14}^{L} = 0.3641$$

$$\text{Constraints (1)–(67)} \tag{82}$$

**Iteration 5.** The satisfactory degrees of DMs, $\mu_{01}(Z_{01}) = 0.89$, $\mu_{02}(Z_{02}) = 0.79$, $\mu_{11}(Z_{11}) = 0.46$, $\mu_{12}(Z_{12}) = 0.38$, $\mu_{13}(Z_{13}) = 0.26$, and $\mu_{14}(Z_{14}) = 0.36$, are obtained, and the ratios of satisfactory levels are obtained as follows:

$$\Delta_1 = \frac{\mu_{11}(Z_{11})}{\mu_{02}(Z_{02})} = \frac{0.46}{0.79} = 0.5822$$

$$\Delta_2 = \frac{\mu_{12}(Z_{12})}{\mu_{02}(Z_{02})} = \frac{0.38}{0.79} = 0.4810$$

$$\Delta_3 = \frac{\mu_{13}(Z_{13})}{\mu_{02}(Z_{02})} = \frac{0.26}{0.79} = 0.3291$$

$$\Delta_4 = \frac{\mu_{14}(Z_{14})}{\mu_{02}(Z_{02})} = \frac{0.36}{0.79} = 0.4556$$

Since $\Delta_3 < \Delta_L$, the third DM determines its own satisfactory level:

$$\Gamma_{13}^{L} = \frac{w_3 \cdot \delta_0}{\Delta_U} = \frac{0.259}{0.80} = 0.3237$$

Using this value the main problem is formulated in the following equation:

$$\begin{aligned}\text{Max } & (\mu_{01}(Z_{01}) + \mu_{02}(Z_{02})) + w_1 \cdot \mu_{11}(Z_{11}) + w_2 \cdot \mu_{12}(Z_{12}) \\ & + w_3 \cdot \mu_{13}(Z_{13}) + w_4 \cdot \mu_{14}(Z_{14})\end{aligned}$$

$$\mu_{01}(Z_{01}) \geq \delta_0 = 0.75$$

$$\mu_{02}(Z_{02}) \geq \delta_0 = 0.75$$

$$\mu_{11}(Z_{11}) \geq \Gamma_{11}^{L} = 0.3378$$

$$\mu_{12}(Z_{12}) \geq \Gamma_{12}^{L} = 0.3822$$

$$\mu_{13}(Z_{13}) \geq \Gamma_{13}^{L} = 0.3237$$

$$\mu_{14}(Z_{14}) \geq \Gamma_{14}^{L} = 0.3641$$

$$\text{Constraints (1)–(67)} \tag{83}$$

**Iteration 6.** The satisfactory degrees of DMs, $\mu_{01}(Z_{01}) = 0.83$, $\mu_{02}(Z_{02}) = 0.79$, $\mu_{11}(Z_{11}) = 0.55$, $\mu_{12}(Z_{12}) = 0.38$, $\mu_{13}(Z_{13}) = 0.32$, and $\mu_{14}(Z_{14}) = 0.37$, are obtained, and the ratios of satisfactory levels are obtained as follows:

$$\Delta_1 = \frac{\mu_{11}\left(Z_{11}\right)}{\mu_{02}\left(Z_{02}\right)} = \frac{0.55}{0.79} = 0.6962$$

$$\Delta_2 = \frac{\mu_{12}\left(Z_{12}\right)}{\mu_{02}\left(Z_{02}\right)} = \frac{0.38}{0.79} = 0.4810$$

$$\Delta_3 = \frac{\mu_{13}\left(Z_{13}\right)}{\mu_{02}\left(Z_{02}\right)} = \frac{0.32}{0.79} = 0.4050$$

$$\Delta_4 = \frac{\mu_{14}\left(Z_{14}\right)}{\mu_{02}\left(Z_{02}\right)} = \frac{0.37}{0.79} = 0.4683$$

At the sixth iteration, the ratio between satisfactory levels is in the specified interval. Therefore, this solution satisfies all DMs and the algorithm is finished.

## 6 Implications and Discussion

As mentioned in Sect. 1, the relationships between companies have changed into relationships between SCs in order to achieve higher service level. Companies decide how they become more flexible or more efficient in their markets. They can choose to share some units that companies are become allied with usage of this choice or they can be rival in their market. From this point in this study, we developed allied SCs which the echelons in the SCs have common units in the network.

Some of the implications derived from the study are given below:

- With the use of the developed model, two SCs integrated under alliance behavior.
- In the proposed model, the end product consists of raw material, one semifinished product, and parts with different utilization rates.
- In order to obtain the balance constraints, the weight ratios are used.
- Based on the experimental results, transportation and purchasing costs have the highest ratio in the total cost when compared to inventory and fixed costs.
- The objectives of different DMs are handled simultaneously with the proposed IFP approach.
- The satisfactory balance between the upper-level and lower-level DMs guarantees a ratio.
- If the proposed IFP approach is used, the lower-level DMs can achieve higher satisfactory levels by 4%, 111%, 190%, and 48% with respect to the initial satisfactory levels.

## 7 Conclusions

As a result of globalization, management of EOLPs has gained more attention. There is a growing pressure to cooperate in SCND. In real-world examples, SCs are becoming more complex when the number of facilities increases and the integration of these facilities found out new terms in SCND. In order to fulfill customers' needs, companies are forced to compete or cooperate with each other. This paper is an early attempt to integrate two closed-loop SCs under alliance behavior.

The SC network considered in this paper consists of two allied SCs. To cope with the allied SCs, a decentralized multilevel CLSC model is developed. There are allied units in the network: raw material suppliers, suppliers, assemblers, and collection centers are common in two SCs, and these units are accepted lower-level DMs of a Stackelberg game. The upper-level DMs are plants in two SCs. Since the DMs have individual objectives, whose the upper-level DMs minimize their total cost and the lower-level DMs maximize their total revenue, an IFP approach is proposed.

The proposed IFP approach is based on the "cooperate" concept. In the IFP approach, minimum satisfactory level of upper-level DMs is guaranteed with $\delta_0$, and with the use of this value, lower-level DMs determine their own satisfactory level. Also, the minimum satisfactory level of lower-level DMs multiplies with relative weights that are determined by the fuzzy AHP method. To tackle the balance between the upper- and lower-level DMs, a ratio is constituted. According to this ratio, the lower-level DMs are updated by their minimum satisfactory level. The IFP algorithm continuous until upper-level DMs are satisfied with the compromise solution. The compromise solution guarantees the satisfactory balance between the upper-level and lower-level DMs.

The main contributions of this study are given as follows:

- A CLSC model is developed in which the alliance behavior is used for integration of two SCs.
- In the developed model, an end product consists of four parts, a semifinished product, and raw materials with different utilization rates and assembled by plants in each SC. A semifinished product is assembled with three parts by assemblers.
- The flow balance at each node is provided by weight ratios.
- In order to handle decentralized model, a novel IFP approach is proposed.
- The novel approach that allows multi-DMs at the first level and combines their judgments in an analytically structured method by using fuzzy AHP.
- In order to obtain a compromise solution for the DMs, a ratio is constituted. With the use of this value, a cooperative relation between decision-makers is provided.

However, the extension of the model and IFP approach in this paper can be also done for future researches. The uncertainty of parameters such as capacity, demand, or other relevant parameters of the problem can be handled with fuzzy, Grey theory, and stochastic modeling approaches. Heuristics algorithms such as genetic algorithms, simulated annealing, and particle swarm optimization can be used for the solution of allied CLSC models. Some of the multi-criteria decision-making methods are also combined at the novel proposed IFP approach.

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