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Quantum Theory and Local Causality

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Quantum Theory and Local Causality

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Preface

This book summarizes the results of research the authors have pursued in the past several years on the problem of implementing Bell's notion of local causality in algebraic quantum field theory and relating it to such fundamental concepts as the Common Cause Principle, Bell's inequalities, and the EPR scenario. These results have been presented at various workshops and department seminars. We wish to thank the audience and the members of the *Budapest Research Group*, the *Budapest-Kraków Research Group*, the *Center for Philosophy of Science at the University of Pittsburgh*, the *Munich Center for Mathematical Philosophy*, the *Nagoya Winter Workshop Series*, the *Sidney Edelstein Center at the Hebrew University*, the *Sigma Club at the London School of Economics*, and the *Southern California Philosophy of Physics Group* for the valuable discussions from which the present book greatly benefited.

The results contained in this book have been published by the authors in a number of papers. The authors gratefully acknowledge permissions to reuse copyrighted material. A substantial part of the main text and all figures are reproduced from these papers:

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Contents

Chapter 1
Local Causality: A Historical Introduction

Abstract In this chapter we briefly overview the history of local causality starting from the early ideas on the prohibition of the action at a distance and ending with Bell's formulation of local causality. We state the central message of the book and outline the content of the subsequent chapters.

Keywords Local causality · Action at a distance · John Bell

Local causality is the idea that causal influences propagate in spacetime continuously and with velocity smaller than the speed of light. In a 1988 interview John Stewart Bell formulated this idea as follows:

> It's the idea that what you do has consequences only nearby, and that any consequences at a distant place will be weaker and will arrive there only after the time permitted by the velocity of light. Locality is the idea that consequences propagate continuously, that they don't leap over distances. (Mann and Crease 1988)

Bell's formulation nicely discerns three components of local causality, namely the continuous propagation of physical influences, the decrease of the influence with distance, and the speed of light as the limit velocity of the propagation. The three components are logically independent. The third component has been added to the concept of local causality with the advent of Einstein's special theory of relativity. The second component is a kind of precondition of doing physics: it ensures the individuation of distant physical objects. The first component, however, has a long and intriguing prehistory in natural philosophy in the form of the prohibition of the action at a distance (McMullin 1989, 2002; Hesse 2005).

The core intuition is simple: a body can act only where it is present. Virtually every major natural philosopher from Aristotle to Descartes agreed on this principle. In Aristotle every motion that is not a self-motion of a living being is brought about by contact with another body. In the medieval Aristotelian tradition the principle became so firm that even God was bound by it. Aquinas in his *Summa Theologica* used the principle even to argue for God's omnipresence:

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> No action of an agent, however powerful it may be, acts at a distance, except through a medium. But it belongs to the great power of God that he acts immediately in all things. Hence nothing is distant from him (Aquinas, *Summa Theologica*, I, q. 8, a. 1, ad. 3)

In Cartesian physics the principle simply followed from Descartes' equating matter with extension. It appeared so self-evident that Descartes did not even make it explicit in his *Principles of Philosophy*.

However, there have always been phenomena around challenging the principle. The magnet was the prime example. From antiquity to Gilbert's *De magnete* magnetic phenomena provided a constant example for an apparently unmediated action at a distance. Another phenomenon that seemed to involve physical action across spatial distances was the ebb and flow of the tides. Tides correlating with the position of the Moon counted as a strong testimony to the palpable influence of remote celestial bodies on terrestrial affairs.

Newton's theory of gravitation provided a coherent mathematical picture for the tides, the planets orbiting around the sun, and many other celestial phenomena but gave no hint as to how exactly the attraction works. Newton's critics were agreed that the distant unmediated action of one body on another is impossible. In a letter to Bentley, Newton seems to agree with his critics:

> That gravity should be innate, inherent and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else by which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it. (Newton 1692/2004, 102)

Nonetheless, Newton was unable to fill in the gap between attracting bodies and neither were his followers. Some, including Huygens and Leibniz, argued that the notion of aether has to be called on to explain away action at a distance; others, such as Clarke, Newton's disciple, were looking for causal agencies different from mechanical contact action; still others, such as Berkeley, rejected intermediates, both mechanical and nonmechanical, and left distant correlations unexplained. Newton's mechanics dominated the thinking in natural sciences throughout the eighteenth century and action at a distance remained an ineliminable doctrine of the theory. It has been echoed even in Kant's *Metaphysical Foundations of Natural Science*: "The attraction that is essential to all matter is an unmediated action through empty space of one portion of matter on another" (Kant 1786/2004, 50; Proposition 7).

The idea that "matter cannot act where it is not" returned to physics with Faraday's investigations on electrical and magnetic phenomena. His field-theoretical paradigm provided an alternative for the action-at-a-distance paradigm of the later Newtonian tradition still prevailing on the Continent. As Maxwell writes:

> Faraday, in his mind's eye, saw lines of force traversing all space where the mathematicians saw centres of force attracting at a distance: Faraday saw a medium where they saw nothing but distance: Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids. (Maxwell 1881, p. x)

Maxwell's theory of electromagnetism provided a coherent mathematical treatment and also physical evidence for the reality of the fields transmitting the electric and magnetic attraction and repulsion. Gravitation, however, resisted any attempt to be treated within the Faraday-Maxwell field-theoretic paradigm. For that one had to wait until Einstein's theory of relativity.

It was the special theory of relativity that introduced what we called above the third requirement on local causality, namely that the speed of light puts a limit on the velocity of the propagation of physical influences. If cause precedes its effect and the time ordering between spatially related events is relative to the different inertial frames connected by the Lorentz transformation, then causal influences need to proceed within the light cone. As Einstein formulates: "According to the theory of relativity, action at a distance with the velocity of light always takes the place of instantaneous action at a distance or of action at a distance with an infinite velocity of transmission" (Einstein 1916/2006, 47).

Thus, the special theory of relativity provided a firm grasp on this third aspect of local causality that could be further deployed in the criticism of the nascent theories, such as quantum mechanics. But testing local causality in quantum mechanics was not so obvious. The reason was threefold: the new theory was probabilistic, operational, and nonrelativistic. As for the first, the probabilistic feature of quantum mechanics not only questioned whether local causality applies but it has even raised serious concerns about the applicability of the very concept of causality, as the early writings of the founding fathers attest. Second, quantum mechanics was inherently operational in the sense that its predictions referred to measurement outcomes rather than some physical values actually possessed by the system. Hence, it was difficult to implement the intuition behind local causality, namely that it is something *physical* that propagates continuously in spacetime. The third problem arose from the fact that quantum mechanics was not Lorentz covariant.

Still, the idea of local causality constantly served as an aid to the formulation and understanding of quantum mechanics. The Einstein–Podolsky–Rosen (EPR) argument was a paradigmatic example for such a clarifying role of the principle. In a nutshell, the EPR argument demonstrated that quantum mechanics is incomplete in the sense that there exist certain elements of reality which are missing from the quantum mechanical description. The argument made use of only two premises: certain perfect correlations predicted by quantum mechanics, and local causality in the form that the elements of reality in question cannot be causally influenced by remote measurement procedures.

This is the point where John Bell entered the scene. In his famous 1964 paper revisiting the EPR argument Bell showed that the assumption of local elements of reality is incompatible with the statistical predictions of quantum mechanics. His argument consisted of two parts: first, he recapitulated the EPR argument showing that the existence of the elements of reality follows from local causality (or locality, as Bell sometimes put it) and perfect (strict) correlations; second, that these elements of reality or deterministic hidden variables lead to certain inequalities violated in quantum mechanics. Interestingly, Bell's first critics often missed Bell's point and focused only on the second part of the argument (*cf.* Goldstein et al. 2011; Maudlin

2014). Thus, Bell's result has been misinterpreted as excluding only *deterministic* hidden variable models for quantum mechanics.

These early discussions on Bell's results had important consequences regarding Bell's further work. First, in his subsequent papers Bell has developed a general no-go theorem for local hidden variables that no longer relies on perfect correlations. Because perfect correlations are hard to test empirically, eliminating them from the premises lent more generality to his results. Second, these misunderstandings urged him to give a clear formulation of the assumption featured in his derivation of the inequalities, namely the assumption of local causality.

Throughout his career Bell has returned to the idea of local causality many times, providing a more and more refined formulation of the concept. To our knowledge, he addressed the question in three papers: first in his 1975, second in his 1986, and finally in his 1990 posthumous paper. In this latter paper, entitled "La nouvelle cuisine," Bell formulated local causality as follows.

> A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region V_A are unaltered by specification of values of local beables in a space-like separated region V_B, when what happens in the backward light cone of V_A is already sufficiently specified, for example by a full specification of local beables in a space-time region V_C. (Bell 1990/2004, pp. 239–240)

The figure Bell attached to his formulation of local causality is reproduced in Fig. 1.1 with the original caption. The term "beable" is Bell's innovation and is contrasted with the term "observable" used in quantum theory. "The *be*ables of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real." (Bell 1990/2004, p. 234). As Bell constantly stressed, fixing the beables of a theory is of crucial significance because local causality as a physical constraint is legitimate only if it is formulated in terms of beables, that is, "*candidates* for the description of nature," and not in terms of any constituents whatsoever of the theory.

One can rephrase Bell's local causality as follows. Local causality excludes causal processes propagating faster than the speed of light but does not exclude correlations between spatially separated events or "beables". Such correlations, namely can be

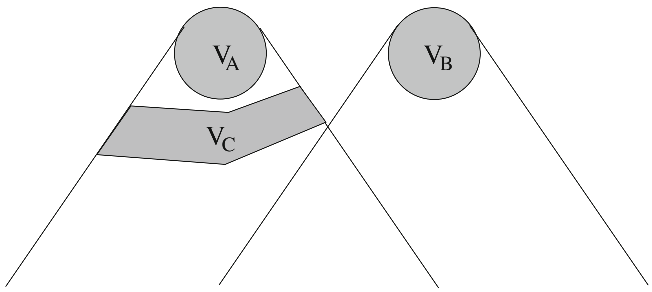

Fig. 1.1 Full specification of what happens in V_C makes events in V_B irrelevant for predictions about V_A in a locally causal theory

brought about by a common cause operating in the past of the events in question. However, fixing the past of an event at an appropriate spacetime region in a detailed enough manner, the state of this event in a locally causal theory will be fixed once and for all, and no other spatially separated event can have an impact on it.

It is not hard to see that Bell's formulation of local causality above significantly differs from his intuitive ideas expressed in his 1988 interview and quoted at the beginning of this chapter. Recall that the three components of local causality were continuous propagation of physical influences, the decrease of the influence with distance, and the speed of light as the limit velocity of propagation. Now, the third component definitely forms part of Bell's above formulation. The second component, as said in the preceding, is a general precondition of individuating distant objects. The first component, however, has been altered significantly. Continuous propagation is the idea that causal influences are mediated by intermediate physical events. A natural interpretation of this idea is to say that what happens in the distant past is *screened-off* by recent events. In an indeterministic world screening-off is taken in the probabilistic sense: in fixing the recent past the probability of present events will be fixed independently of what is going on in the distant past. By this interpretation continuous propagation turns into Markovity.

Interestingly enough, in Bell's formulation it is not the distant past that is screened-off by the "recent events" in the region V_C: the screened-off region V_B is not in the causal past of V_C. What is screened-off are events happening in spatially separated regions. The central idea of Bell's formulation of local causality is the insight that in a locally causal theory a detailed enough specification of the recent causal past of an event will fully determine the very event or at least the probability of the event. And fully determining means that no other event can contribute to fixing this probability—be it in a spatially separated region, as in Bell's formulation, or in the distant past, as in the requirement of continuous propagation.

Bell's prime merit lies in his being able to translate the philosophical intuition lying behind local causality into easily tractable mathematical terms applicable in the probabilistic formalism of quantum mechanics. More precisely, he was able to write down a mathematically precise formulation of a consequence of local causality in the context of an experiment in which measurements are performed on two subsystems that previously have interacted but are now spatially separated. His works have then set the scene for a whole research program in the foundations of quantum theory.

Bell's investigations have remained an isolated research project within theoretical physics for a long time. The mainstream was working on quantum field theory (QFT), a theory designed to unify quantum theory with the locally causal character of relativistic field theory. Interestingly enough, Bell had never formulated local causality in a QFT context. One can just speculate on why he did not implement local causality in QFT.

One reason might have been that QFT has deliberately been devised to resolve the locality issues of standard quantum mechanics. Hence, most of the physicists were convinced that QFT is relativistically local by its very construction. Another reason might have been that standard QFT is formulated in terms of field operators which do not satisfy microcausality (see below) and hence have even less chance to adopt a direct ontological interpretation than the local observable operators used in algebraic quantum field theory which at least commute if spatially separated. But for Bell it was of crucial importance to apply the notion of local causality to beables of a theory "corresponding to something real" out there in the world. He was so unsatisfied with the usual formulation of QFT that in 1984 he even came up with a suggestion on how to base QFT on beables (Bell 1984/2004). In any case, Bell never formulated local causality in a quantum field theoretical context.

So much on history. Now, let us go back to Bell's formulation of local causality. Looking at it mathematically, Bell's local causality is a probabilistic independence relation. It states that certain events become probabilistically independent from other events conditioned upon yet other events. This conditional independence relation, however, is motivated by the spatiotemporal localization of the events in question. Thus, the logical structure of Bell's local causality is a kind of *inference pattern* from spatiotemporal to probabilistic relations: if events are localized in the spacetime in a certain way, then they are to satisfy certain probabilistic independences. But in order to obtain a valid inference relation between spatiotemporal and probabilistic concepts, one needs to integrate these concepts into a *unified framework*. Without such a framework, one cannot account for the inferences from relations between spacetime regions to probabilistic independences between, say, random variables. Thus the first aim of our book is to find such a framework.

The most elaborate formalism used in quantum physics offering a general method to connect spatiotemporal and probabilistic entities is an algebraic-axiomatic form of quantum field theory, namely algebraic quantum field theory (AQFT) or local quantum physics (Haag 1992). Thus, AQFT is a mathematically transparent theory ideal for analyzing various concepts related to local causality, such as Bell's inequalities (Summers and Werner 1987a, b, 1988; Halvorson 2007), relativistic causality (Butterfield 1995, 2007; Earman 2014; Earman and Valente 2014), or the closely related (see below) Common Cause Principle (Rédei 1997; Rédei and Summers 2002; Hofer-Szabó and Vecsernyés 2012a, 2013a). For our ends, however, the full formalism of AQFT would be too much. Our intention is simply to provide a *minimal* framework that is indispensable to formulate local causality in a strict fashion both in the classical and quantum cases. We call such a framework a *local physical theory* (LPT). A LPT is based on those axioms of algebraic field theory that describe the structure of the observable algebra. Using a translation between commutative von Neumann algebras and σ-algebras it also can describe classical theories. Thus, the formal structure of a LPT integrates the two most important components of a general physical theory: a spacetime structure and an algebraic-probabilistic structure. Having the formal framework in hand, we can then accomplish the main goals of this book, namely to formulate Bell's original definition of local causality in a classical LPT, to extend it in a clear-cut way to quantum LPTs, and to relate it to other such important concepts and principles as the Common Cause Principle, Bell's inequalities, the EPR scenario, and other relativistic causality and locality concepts. The central message of the book, however, is the following somewhat heterodox statement.

Local causality in the EPR scenario can be saved by adopting noncommuting events into our ontology. In other words, one can provide a generalized noncommutative definition of Bell's local causality such that it does not imply Bell's inequalities and allows for a locally causal model for the EPR.

By extending Bell's notion of local causality to noncommutative structures we cross a line that Bell set for himself. As mentioned above, Bell always stressed that the principle of local causality should exclusively be applied to beables and not to any arbitrary ingredient of the theory. In contrast to Bell's intentions, in this book we use the notion of "beable" in a very permissive way also incorporating elements of noncommutative algebras. In this respect our move gets close to the so-called quantum logical approach to quantum mechanics (Mackey 1963; Jauch 1968; Piron 1976; Beltrametti and van Fraassen 1981). Although people in this tradition do not use the term "beable," they do use the terms "event," "conjunction," "proposition" in a similarly permissive way. Their ambition is not to give a full-fledged ontological interpretation of, say, the meeting of two noncommuting events, but rather to clarify many important logical and structural questions of the theory. Our strategy is similar. We are not able to provide a philosophically satisfactory answer to the question as to what a noncommutative ontology is. Rather, in this book we simply test how far we can get in providing a locally causal account of the EPR scenario by adopting noncommutative events.

Our book is structured as follows. In Chap. 2 we introduce the mathematical formalism of a LPT. Briefly, a LPT is an association of local operator algebras to spacetime regions regulated by three principles borrowed from AQFT: isotony, microcausality, and covariance. Depending on whether the local algebras are commutative or noncommutative, we speak about local classical theories (LCTs) or local quantum theories (LQTs). At the end of the chapter we spend some time to motivate why we use von Neumann algebras in the framework of LPTs.

Chapter 3 is devoted to the various causality and locality concepts in AQFT such as causal dynamics, local primitive causality, no-signaling, independence, and stochastic Einstein locality. There is an intensive discussion in the literature as to which of these concepts properly represents local causality in QFT. We agree with Rédei (2014) when he claims that local causality is not a single property but an intricately interconnected web of features. In this chapter we review the various concepts in the web.

At the beginning of Chap. 4 we present in turn Bell's three different formulations of local causality presented subsequently in his 1975, 1986, and 1990 papers. Then, we analyze the key concepts featured in his third, final formulation, namely "local beables," "shielder-off region," and "complete specification." We translate them into the framework of LPTs and provide a generalized definition of local causality. In the same chapter we prove Proposition 1 stating that local primitive causality makes an atomic LPT be locally causal.

In Chap. 5 we turn to Reichenbach's Common Cause Principle. The Common Cause Principle states that if there is a correlation between two events and there is no direct causal (or logical) connection between the correlated events then there exists a common cause of the correlation. We generalize this principle to the LPT framework and point out the similarities and differences between the principle and Bell's local causality.

Chapter 6 collects the most important concepts and some of the representative propositions concerning Bell's inequalities in the general C^*-algebraic setting and in the special LPT framework.

In Chap. 7 threads get intertwined: Bell's local causality, the common cause principle, and Bell's inequalities all meet in the notorious EPR scenario. In this chapter we introduce the EPR scenario and show (Proposition 2) that the violation of Bell's inequalities does *not* block the implementation of the EPR situation in a locally causal LQT because in a locally causal LQT Bell's inequalities cannot be derived.

In Chap. 8 we explicitly construct a locally causal LQT for the EPR. The model is the $1+1$ dimensional local quantum Ising model. In this chapter we show that the model is not only locally causal in Bell's sense but also able to implement four pairs of events correlating in the singlet state.

In Chap. 9 we summarize the results of the book and investigate their philosophical consequences. We argue that embracing noncommuting beables in our ontology significantly extends our explanatory sources in accounting for correlations. We also examine what price we need to pay for abandoning classicality in order to preserve local causality.

Our book fits nicely into a recent research line on a deeper conceptual and formal understanding of Bell's notion of local causality. Travis Norsen's illuminating paper on local causality (Norsen 2011) or its relation to Jarrett's completeness criterion (Norsen 2009), the paper of Seevinck and Uffink (2011) aiming at providing a "sharp and clean" formulation of local causality, or Henson's (2013b) paper on the relation between separability and Bell's inequalities all attest to this renewed interest in local causality. We comment on the points of contact with these papers. For a list of our own papers on which this book is based see the Preface.

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Chapter 2
What Is a Local Physical Theory?

Abstract In this Chapter we introduce the mathematical formalism of a local physical theory (LPT). Briefly, a LPT is an association of local operator algebras to spacetime regions subjected to the requirements of isotony, microcausality and covariance, all borrowed from algebraic quantum field theory. Depending on whether the local algebras are commutative or noncommutative, we call a LPT a local classical theory (LCT) or a local quantum theory (LQT). At the end of the chapter we motivate the application of von Neumann algebras in the LPT framework.

Keywords Local physical theory · von Neumann algebra

As stated in the Introduction, Bell's notion of local causality presupposes a framework that treats spatiotemporal and probabilistic entities in a common formalism. In this chapter we develop such a framework, called local physical theory. Instead of jumping directly to the full-fledged definition, we proceed here "inductively," unfolding step by step the various notions featuring in the definition of a local physical theory. Having listed these features we formulate the exact definition only at the end of the chapter.

The central idea of a local physical theory is the *association of local operator algebras to spacetime regions* regulated by the physically motivated requirements (Haag 1992):

1. *Isotony*. Let $\mathcal{M}$ be a globally hyperbolic spacetime[1] and let $\mathcal{K}$ be a covering collection[2] of bounded, globally hyperbolic subspacetime regions of $\mathcal{M}$ such that $(\mathcal{K}, \subseteq)$ is a directed poset under inclusion $\subseteq$. The net of local observables is given by the isotone map $\mathcal{K} \ni V \mapsto \mathcal{A}(V)$ to unital C^*-algebras; that is, $V_1 \subseteq V_2$

[1] By a spacetime we mean a connected time-oriented Lorentzian manifold. A spacetime $\mathcal{M}$ is called globally hyperbolic if $\mathcal{M}$ contains a Cauchy hypersurface, which is by definition a subset $\mathcal{S} \subset \mathcal{M}$ such that each inextendible timelike curve in $\mathcal{M}$ meets $\mathcal{S}$ at exactly one point. (See Pfäffle 2009 and references therein.)

[2] For all $x \in \mathcal{M}$ there exists $V \in \mathcal{K}$ such that $x \in V$.

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implies that $\mathcal{A}(V_1)$ is a unital C^*-subalgebra of $\mathcal{A}(V_2)$. The *quasilocal algebra* $\mathcal{A}$ is defined to be the inductive limit C^*-algebra of the net $\{\mathcal{A}(V),\ V \in \mathcal{K}\}$ of local C^*-algebras.[3]

Sometimes *additivity*, which is a stronger property than isotony, is also required for the net of observables: $\mathcal{A}(V_1) \vee \mathcal{A}(V_2) = \mathcal{A}(V_1 \cup V_2)$; $V_1, V_2, V_1 \cup V_2 \in \mathcal{K}$, where $\vee$ refers to the generated C^*-algebra in $\mathcal{A}$.

2. *Microcausality* (also called *Einstein causality*) is the requirement that local algebras belonging to spacelike (i.e., causally) separated regions commute, that is, $\mathcal{A}(V')' \cap \mathcal{A} \supseteq \mathcal{A}(V)$, $V \in \mathcal{K}$, where primes denote spacelike complement and algebra commutant, respectively.
3. $\mathcal{P}_{\mathcal{K}}$*-covariance.* Let $\mathcal{P}_{\mathcal{K}}$ be the subgroup of the group $\mathcal{P}$ of global isometries of $\mathcal{M}$ leaving the collection $\mathcal{K}$ invariant. A group homomorphism $\alpha : \mathcal{P}_{\mathcal{K}} \to \operatorname{Aut}\mathcal{A}$ is given such that the automorphisms α_g, $g \in \mathcal{P}_{\mathcal{K}}$ of $\mathcal{A}$ act covariantly on the observable net: $\alpha_g(\mathcal{A}(V)) = \mathcal{A}(g \cdot V)$, $V \in \mathcal{K}$.

Here the possible spacetimes spread from Minkowski spacetime through stationary spacetimes to generic globally hyperbolic ones where no global Killing vector field exists. Choosing the collection $\mathcal{K}$ in such a way that every $V \in \mathcal{K}$ contains only a finite number of elements of $\mathcal{K}$, one can consider theories with locally finite degrees of freedom, that is, with finite-dimensional local algebras. Otherwise the local algebras themselves are infinite-dimensional. We would like to treat classical and quantum theories on an equal footing as far as possible. The difference between the two is that the quasilocal algebra of a local classical theory is required to be commutative whereas that of a local quantum theory is required to be noncommutative.

Thus, microcausality fulfills trivially in local classical theories. On the other hand, in local quantum theories it is usually required that the quasilocal algebra is "highly noncommutative" and the local algebras are "fat enough." This is assured by algebraic Haag duality, which is a stronger requirement than microcausality:

4.Q *Algebraic Haag duality.* $\mathcal{A}(V')' \cap \mathcal{A} = \mathcal{A}(V)$, $V \in \mathcal{K}$.

Clearly, Haag duality is inherently connected to the noncommutativity of the observable algebra. In the case of commutative local algebras Haag duality would imply that $\mathcal{A}(V) = \mathcal{A}$ for any $V \in \mathcal{K}$; that is, the net structure of local algebras would be completely lost. To avoid this trivial net structure in local classical theories, one requires less than Haag duality:

4.C *Intersection property for spacelike separated regions.* The intersection property

$$\mathcal{A}(V_1) \cap \mathcal{A}(V_2) = \mathcal{A}(V_1 \cap V_2); \qquad V_1, V_2, V_1 \cap V_2 \in \mathcal{K} \tag{2.1}$$

holds for spacelike separated regions $V_1, V_2 \in \mathcal{K}$; that is, $\mathcal{A}(V_1) \cap \mathcal{A}(V_2) = \mathcal{A}(\emptyset) := \mathbb{C}\,\mathbf{1}_{\mathcal{A}}$ for them.

[3] This formulation is a special case of the general category theoretical formulation of AQFTs in curved backgrounds (Brunetti and Fredenhagen 2009). Namely, the functor from globally hyperbolic spacetimes to unital C^*-algebras is restricted to the full subcategory induced by the object $\mathcal{M}$ and the (sub)collection $\mathcal{K}$ of its subobjects.

In the case of local quantum theories this property follows from Haag duality and primitive causality (see below) if the net is additive and the quasilocal algebra is a factor; that is, its center is trivial: $\mathcal{A}' \cap \mathcal{A} = \mathbb{C}\,\mathbf{1}_{\mathcal{A}}$.[4] Note that the intersection property (2.1) is not required for disjoint pairs $V_1, V_2 \in \mathcal{K}$ if they are causally connected, $V_1 \cap (J_+(V_2) \cup J_-(V_2)) \neq \emptyset$ inasmuch as it would forbid subalgebra constituents from the past to rearrange themselves in the future. Requiring (2.1) generally would contradict primitive causality which, as we show, makes the dynamics deterministic.

Different physical realizations of a single local theory are given by unitary inequivalent representations $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ of the quasilocal C^*-algebra $\mathcal{A}$ by bounded operators $\mathcal{B}(\mathcal{H})$ on a (separable) Hilbert space $\mathcal{H}$. Inequivalent representations can be produced from essentially different states $\phi : \mathcal{A} \to \mathbb{C}$ through GNS construction. Representations are required to be locally faithful not to lose local observables. Once a particular representation is chosen, one can consider the natural von Neumann algebra extension of the local algebras by taking weak closures $\mathcal{N}(V) := \pi(\mathcal{A}(V))''$, $V \in \mathcal{K}$.

5. *Representation.* A locally faithful representation $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ is chosen where a (strongly continuous) unitary representation $U : \mathcal{P}_{\mathcal{K}} \to \mathcal{B}(\mathcal{H})$ implements $\alpha : \mathcal{P}_{\mathcal{K}} \to \operatorname{Aut}\mathcal{A}$. The local and quasilocal observables are extended as $\mathcal{N}(V) := \pi(\mathcal{A}(V))''$, $V \in \mathcal{K}$ and $\mathcal{A}_{\mathcal{H}} := \overline{\cup_{V \in \mathcal{K}} \mathcal{N}(V)} \subset \mathcal{B}(\mathcal{H})$, respectively.

It is easy to see that the net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ of local von Neumann algebras given above also obeys *isotony*, *microcausality*, in the sense that $\pi(\mathcal{A}(V'))' \cap \mathcal{B}(\mathcal{H}) \supseteq \mathcal{N}(V)$, $V \in \mathcal{K}$, and *$\mathcal{P}_{\mathcal{K}}$-covariance*. Because we concentrate on local and causal properties we do not consider further requirements on the representation π, for example, how a vacuum representation can be characterized and is chosen among the allowed representations.[5]

Here we briefly comment on the use of von Neumann algebras as local algebras in local classical theories. The crucial point is the link between von Neumann algebras and σ-algebras. Every element $S \subset \Omega$ of a σ-algebra (Ω, Σ) determines a projection χ_S in the abelian *-algebra $\mathcal{F}(\Omega, \mathbb{C})$ of complex functions on Ω, namely, χ_S is the characteristic function of the subset $S \in \Sigma$. In general, we translate local σ-algebras (Ω, Σ) to local commutative operator algebras generated by projections χ_S, $S \in \Sigma$ in the function algebra $\mathcal{F}(\Omega, \mathbb{C})$. This abundance of projections is, however, the reason why the local operator algebras cannot be represented by generic commutative C^*-algebras in a local classical theory. Namely, a commutative unital (nonunital) C^*-algebra, according to the Gelfand duality, is isomorphic to the algebra of complex-valued continuous functions (vanishing at infinity) on a (locally) compact Hausdorff

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[4]Let $V_1, V_2 \in \mathcal{K}$ be spacelike separated regions. Due to Haag duality and additivity of the net

$$\mathcal{A}(V_1) \cap \mathcal{A}(V_2) = \mathcal{A}(V_1')' \cap \mathcal{A}(V_2')' = (\mathcal{A}(V_1') \vee \mathcal{A}(V_2'))' = \mathcal{A}(V_1' \cup V_2')'. \tag{2.2}$$

Because $V_1' \cup V_2'$ always contains a Cauchy surface if V_1 and V_2 are spacelike separated bounded spacetime regions, we arrive at $\mathcal{A}(V_1' \cup V_2') = \mathcal{A}$ due to primitive causality. Therefore $\mathcal{A}(V_1) \cap \mathcal{A}(V_2) = \mathcal{A}(V_1' \cup V_2')' = \mathcal{A}' \cap \mathcal{A} =:$ Center $\mathcal{A}$.

[5]However, to stay within the quasiequivalence class of the representation π one considers only states in the folium of π (Haag 1992), that is, normal states of $\pi(\mathcal{A})''$ which lead to locally normal states, that is, normal states, by restricting them to the local von Neumann algebras $\mathcal{N}(V)$, $V \in \mathcal{K}$.

topological space. However, unless the topology is discrete, such algebras generally do not contain nontrivial projections at all. Therefore one is to consider the subclass of commutative *von Neumann algebras* in local classical theories as local operator algebras that are not only rich enough in projections, but are also generated by them.

The paradigmatic case of a commutative von Neumann algebra is the space of complex-valued essentially bounded measurable functions $L^\infty(\Omega, \Sigma, \mu)$ on the σ-finite measure space (Ω, Σ, μ). This Neumann algebra is generated by the subclass $\{\chi_S, S \in \Sigma\}$ of characteristic functions on Ω, and acts on the separable Hilbert space $L^2(\Omega, \Sigma, \mu)$ by multiplication. This subclass of characteristic functions, or equivalently, the sets of their supports form the σ-algebra (Ω, Σ) of classical events. The lattice operations and the algebra operations relate to one another as follows: $\chi_{S\wedge T} = \chi_S\chi_T$, $\chi_{S\vee T} = \chi_S + \chi_T - \chi_S\chi_T$. This σ-algebra, however, is not the most general σ-algebra one can imagine, because not every σ-algebra can be equipped by a σ-finite measure μ. Nevertheless, they give us a rich enough set of examples for classical theories. The probability measure p on the corresponding σ-algebra (Ω, Σ) can be provided by any normal state ϕ on the von Neumann algebra $L^\infty(\Omega, \Sigma, \mu)$ by $p_\phi(S) := \phi(\chi_S)$, $S \in \Sigma$.

It is a further question as to what kind of local σ-algebras can correspond to local classical theories, for example, to classical field theories on the spacetime $\mathcal{M}$ with configuration space $F^\mathcal{M} := \{\Phi\colon \mathcal{M} \to F\}$ and with field values $F = \mathbb{R}^n, \mathbb{C}^n$, for example. The maximal σ-algebra of classical events one can imagine is $(F^\mathcal{M}, \mathcal{P}(F^\mathcal{M}))$ given by the power set $\mathcal{P}(F^\mathcal{M})$ of the set of field configurations. One also needs narrower σ-algebras in tune with the net structure of the theory. This is done by taking local equivalence classes of those configurations that have the same field values on a given region $V \in \mathcal{K}$. Two field configurations $\Phi, \Psi \in F^\mathcal{M}$ are said to be *locally* V*-equivalent*, $\Phi \sim_V \Psi$, if $\Phi_{|V} = \Psi_{|V}$. The isotone net structure $\{(F^\mathcal{M}, \Sigma(V)), V \in \mathcal{K}\}$ of unital σ-subalgebras $\Sigma(V) \subset \mathcal{P}(F^\mathcal{M})$ can be given by the "cylindrical subsets" of $F^\mathcal{M}$ corresponding to the image sets of canonical projections $Z_V\colon \mathcal{P}(F^\mathcal{M}) \to \mathcal{P}(F^\mathcal{M})$, $V \in \mathcal{K}$, which map a set S of configurations onto the corresponding union of V-equivalence classes of configurations in S:

$$\mathcal{P}(F^\mathcal{M}) \ni S \mapsto Z_V(S) := \{\Phi \in F^\mathcal{M} \,|\exists\Psi \in S : \Phi_{|V} = \Psi_{|V}\} \in \Sigma(V) := Z_V(\mathcal{P}(F^\mathcal{M})). \tag{2.3}$$

Clearly, the net $\{(F^\mathcal{M}, \Sigma(V)), V \in \mathcal{K}\}$ — or $\{\Sigma(V), V \in \mathcal{K}\}$, for short — is $\mathcal{P}_\mathcal{K}$-covariant. The hard and unsolved problem is to give a probability measure on the σ-algebra $(F^\mathcal{M}, \mathcal{P}(F^\mathcal{M}))$ or on a meaningful σ-subalgebra of it. We can avoid this conundrum by choosing a locally finite covering of $\mathcal{M}$, that is, choosing a subnet $\mathcal{K}^m \subset \mathcal{K}$ in a way that every $V \in \mathcal{K}^m$ contains only a finite number of elements of $\mathcal{K}^m$, and restricting the field configurations to be piecewise constant on regions corresponding to minimal elements in $\mathcal{K}^m$. The power set of this configuration space F^{S^m}, where S^m denotes the set of minimal elements in $\mathcal{K}^m$, can also be mapped into local σ-algebras $(F^{S^m}, \Sigma_m(V))$, $V \in \mathcal{K}^m$ as before in (2.3). Although the maximal local σ-algebra $\Sigma_m(V^m)$ of a minimal region $V^m \in S^m$ is isomorphic to the power set $\mathcal{P}(F)$ of field values, one can restrict them to the Borel σ-subalgebra of $\mathcal{P}(F)$.

Then a generic local σ-algebra $\Sigma_m(V)$, $V \in \mathcal{K}^m$ is isomorphic to a finite product of the copies of corresponding Borel σ-subalgebras, because V is covered by a finite subset of S^m. We can simplify the situation further by restricting the field values F to a finite set.

Last but not least, we stress that the projections χ_S, $S \in \Sigma(V)$ in the local von Neumann algebras do not possess a direct spacetime localization: they project to subsets of $F^{\mathcal{M}}$ and not to those of $\mathcal{M}$.

Inspired by the above considerations, we define a local physical theory as

Definition 1 A *local physical theory* (LPT) is a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ of local von Neumann algebras associated with a directed poset $\mathcal{K}$ of globally hyperbolic bounded regions of a globally hyperbolic spacetime $\mathcal{M}$. The net satisfies *isotony*, *microcausality*, $\mathcal{P}_{\mathcal{K}}$*-covariance*, and the *intersection property for spacelike separated regions*. If the local von Neumann algebras are commutative, we speak about a *local classical theory* (LCT); if they are noncommutative, we speak about a *local quantum theory* (LQT).

The framework of LPTs provides the natural context in which Bell's notion of local causality can be formulated.

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Chapter 3
Locality and Causality Principles

Abstract This Chapter provides a brief overview of the interconnections between the various causality and locality concepts in algebraic quantum field theory such as causal dynamics, primitive causality, local primitive causality, no-signaling, selective and nonselective measurements, local determinism, stochastic Einstein locality.

Keywords Causal dynamics · Local primitive causality · No-signaling
Stochastic Einstein locality

With the rise of the field-theoretic paradigm in quantum physics an intensive debate has begun as to exactly how quantum field theory implements the intuitive notion of local causality (also referred to as relativistic causality or relativistic locality). Some argued that local causality is secured in quantum field theory by the very construction of the theory. Others took the position that the presence or absence of entanglement across spatially separated regions really matters when a theory is gauged against local causality (Clifton and Halvorson 2001). Still others argued that Lorentz covariance is the essential feature that implements local causality in a theory (Ruetsche 2011). Earman and Valente (2014) on the other hand regarded local primitive causality (see below) as the condition that represents local causality. Finally, Rédei (2014) took the position that local causality is not a single property but an intricately interconnected web of features.

We agree with Rédei. Our aim is to explore these interrelated concepts by starting from one specific point of the web, namely from Bell's local causality. We show how this notion can be implemented in quantum field theory, or in our terminology, into a LPT. We also see how our position is related to the above positions. Before turning to the definition of local causality, however, in the present chapter we give a brief overview of the various locality and causality principles and their relation to one another.

Let us start with *causal dynamics* or, equivalently, *causal time evolution*. The term "causal" here puts a restriction on the dynamics: the observables (classical or quantum) in a region can depend only on observables in the causal past of the

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region. The motivation for causal dynamics comes from classical field theory on a globally hyperbolic spacetime where a global time parameter can be chosen. If the field equations of the theory are symmetric hyperbolic partial differential equations (see Geroch 2010), then there exists an initial value formulation of the theory in the form: given the initial values on (a piece of) a Cauchy surface, the time evolution equation provides a unique solution in the *domain of dependence*[1] of (that piece of) the Cauchy surface. It is this restriction of the complete influences of the initial values to the domain of dependence that makes the dynamics of the theory *causal*, inasmuch as it forbids superluminal propagation (see Earman 2014).

This type of causal dynamics has three additional basic properties: It is defined within a *classical* theory. It is *Markovian* in the sense that the causal past of the domain of the initial conditions does not count in the evolution. It is *deterministic*, because fixing the (expectation) values of the observables at a certain time, the dynamics provides *unique* (expectation) values of the observables in the future (or in the past) within the domain of dependence of the initial values. We show that the properties of a LPT, classical or quantum, are not strong enough to provide us a causal dynamics. An additional property, called *primitive causality*, will ensure the dynamics in a LPT to be *deterministic* in the above sense; and only a stronger property, called *local primitive causality*, will ensure the dynamics in a LPT to be not only deterministic but also *causal*. It turns out that in the absence of local primitive causality the causality of the dynamics on the observables is meaningless. In the absence of primitive causality the notion of an initial state on the observables is already missing, because a state on the quasilocal algebra involves the prescription of the state on the proper Cauchy surface subalgebras for all time slices $t \in \mathbb{R}$. Expectation values in a generic state of such LPTs are hardly expected to show any causal properties; a causal way of state extension for Cauchy surface subalgebras is needed (*cf*; Hofer-Szabó and Vecsernyés 2015). However, Bell's local causality condition, as a simultaneous requirement for states and observables with specific support can be formulated and applied even in the absence of primitive causality or a causal state extension.

In the case of stationary spacetimes, that is when a global timelike Killing vector field exists, a natural dynamics exists in LPTs on the observables, the *covariant dynamics*: The one-parameter isometry group $T \simeq (\mathbb{R}, +)$ of $\mathcal{M}$ generated by the global timelike Killing vector field is mapped onto a one-parameter automorphism group $\{\alpha_t, t \in T\}$ of the quasilocal observable algebra $\mathcal{A}$ acting covariantly on the net (Requirement 3). In the case of a generic globally hyperbolic spacetime $\mathcal{M}$ no global timelike Killing vector field exists, therefore there is no natural dynamics on the observables in LPTs. However, a foliation $\{\mathcal{S}_t, t \in \mathbb{R}\}$ of $\mathcal{M}$ by Cauchy surfaces exists, which is indexed by a global time parameter. Such a foliation will lead to a dynamics on the observables if the observable algebra corresponding to any of the Cauchy surfaces already exhausts the quasilocal observable algebra; that is, *primitive causality* holds:

[1]The domain of dependence $D(\mathcal{S})$ of a (piece of) a Cauchy surface $\mathcal{S}$ consists of those points in $\mathcal{M}$ for which any causal curve containing them intersects $\mathcal{S}$.

6. *Primitive causality*. For any covering collection $\mathcal{K}(\mathcal{S}) \subseteq \mathcal{K}$ of any Cauchy surface $\mathcal{S}$, one has $\mathcal{A}_{\mathcal{K}(\mathcal{S})} = \mathcal{A}$.

A covering collection $\mathcal{K}(\mathcal{S}_t) \subseteq \mathcal{K}$ of the Cauchy surface $\mathcal{S}_t$ determines a subalgebra $\mathcal{A}_{\mathcal{K}(\mathcal{S}_t)}$ of $\mathcal{A}_{\mathcal{H}}$. Let us define the Cauchy surface algebra $\mathcal{A}_{\mathcal{S}_t}$ of $\mathcal{S}_t$ by the injective limit algebra of a decreasing net of subalgebras corresponding to decreasing coverings [see Brunetti and Fredenhagen (2009) for details]. Thus, in the case of primitive causality any subalgebra $\mathcal{A}_{\mathcal{K}(\mathcal{S})}$, hence any Cauchy surface subalgebra $\mathcal{A}_{\mathcal{S}}$ is equal to the whole quasilocal algebra $\mathcal{A}$. Therefore, the injective algebra morphisms corresponding to embeddings of globally hyperbolic Cauchy surface coverings into $\mathcal{M}$ become isomorphisms and one also obtains algebra isomorphisms $\iota_t : \mathcal{A}_{\mathcal{S}_t} \to \mathcal{A}, t \in \mathbb{R}$ between the Cauchy surface algebras and the quasilocal algebra. Then the isomorphism $\alpha_{t',t} := \iota_{t'}^{-1} \circ \iota_t : \mathcal{A}_{\mathcal{S}_t} \to \mathcal{A}_{\mathcal{S}_{t'}}$ provides the Cauchy time evolution isomorphism, that is, the dynamics on the observables, between the Cauchy surface algebras corresponding to time slices t and t' in the chosen foliation. In the presence of a covariant dynamics the two dynamics coincide: $\alpha_{t',t} = \alpha_{t'-t}$ if the chosen foliation of $\mathcal{M}$ by Cauchy surfaces is compatible with the action of the global time translation isometry group of $\mathcal{M}$.

But this is not the only role of primitive causality. It makes the (covariant) dynamics on the observables *deterministic*. Because a state on a single Cauchy surface algebra $\mathcal{A}_{\mathcal{S}}$, that is, a prescription of "initial (expectation) values," fixes already the state on the whole quasilocal algebra $\mathcal{A}$, the expectation values of the observables at arbitrary times can be given uniquely in terms of the (covariant) time evolution automorphisms of the observable algebra $\mathcal{A}$ and the "initial" state.

Although the dynamics $\{\alpha_{t',t}, t', t \in \mathbb{R}\}$ is deterministic, it is not necessarily *causal*. That is the deterministic dynamics per se does not ensure that

$$\alpha_{t',t}(\mathcal{A}(V_t)) \subset \mathcal{A}(V_{t'}), \quad V_{t'} := \mathcal{S}_{t'} \cap (J_+(V_t) \cup J_-(V_t)), \; V_t \subset \mathcal{S}_t; t, t' \in \mathbb{R}, \quad (3.1)$$

where $V_t := V \cap \mathcal{S}_t$ for some $V \in \mathcal{K}$ and $J_+(V_t) \cup J_-(V_t)$ is the causal cone of V_t, that is, the union of its causal future and causal past. The (deterministic) dynamics on the observables meeting the requirement (3.1) is called *causal dynamics* on the observables. It means that the "propagation" of local observable algebras under the dynamics respects the causal cone structure of the underlying spacetime. It also ensures that the state on a local algebra $\iota_{t'}(\mathcal{A}(V_{t'}))$ fixes the state on a local algebra $\iota_t(\mathcal{A}(V_t))$, if V_t is in the domain of dependence of $V_{t'}$.

The local and stronger version of primitive causality is

7. *Local primitive causality*. For any globally hyperbolic bounded subspacetime regions $V \in \mathcal{K}$, $\mathcal{A}(V'') = \mathcal{A}(V)$.[2]

[2]If $V'' \notin \mathcal{K}$ this requirement would mean that by extending $\mathcal{K}$ by the globally hyperbolic bounded subspacetime regions V'', $V \in \mathcal{K}$ and defining $\mathcal{A}(V'') := \mathcal{A}(V)$ one obtains an extended net of local algebras satisfying isotony, microcausality, and covariance.

Local primitive causality entails not only primitive causality but also the causality requirement (3.1) of the dynamics: given V_t and $V_{t'}$ as in (3.1) local primitive causality and isotony (Requirement 1) leads to $\mathcal{A} \supset \iota_{t'}(\mathcal{A}(V_{t'})) = \iota_{t'}(\mathcal{A}(V_{t'}'')) \supset \iota_t(\mathcal{A}(V_t))$.

We note that if a net satisfies Haag duality for all bounded globally hyperbolic subspacetime regions $V \in \mathcal{K}$, then it also satisfies local primitive causality for them:

$$\mathcal{A}(V) = \mathcal{A}(V')' \cap \mathcal{A} = \mathcal{A}(V''')' \cap \mathcal{A} = \mathcal{A}((V'')')' \cap \mathcal{A} = \mathcal{A}(V''), \quad V \in \mathcal{K}. \tag{3.2}$$

Conversely, requiring Haag duality only for causally complete regions (i.e., for regions $V \in \mathcal{K}$ satisfying $V'' = V$) and local primitive causality for all $V \in \mathcal{K}$ Haag duality follows for all $V \in \mathcal{K}$:

$$\mathcal{A}(V) = \mathcal{A}(V'') = \mathcal{A}((V'')')' \cap \mathcal{A} = \mathcal{A}(V''')' \cap \mathcal{A} = \mathcal{A}(V')' \cap \mathcal{A}. \tag{3.3}$$

What can we say in the absence of primitive causality? In the case of a generic globally hyperbolic spacetime there is no Cauchy dynamics $\{\alpha_{t,t'}, t, t' \in \mathbb{R}\}$ on the observables and the Cauchy surface proper subalgebras $\mathcal{A}_{\mathcal{S}_t}, t \in \mathbb{R}$ are not necessarily isomorphic. In the case of stationary spacetimes a covariant dynamics $\{\alpha_t, t \in \mathbb{R}\} \subset \mathrm{Aut}\,\mathcal{A}$ does exist but the isomorphic Cauchy surface subalgebras $\mathcal{A}_{\mathcal{S}_t}, t \in \mathbb{R}$ remain proper subalgebras of $\mathcal{A}$. Their intersection can even be trivial. Therefore there is no point in speaking about causality of the covariant dynamics because local subalgebras "propagate" into completely new local subalgebras of $\mathcal{A}$. Moreover, the covariant dynamics is not deterministic in this case; that is, the covariant dynamics and the "initial" state $\phi_s : \mathcal{A}_{\mathcal{S}_s} \to \mathbb{C}$ do not fix for $t \neq s$ the expectation values of the isomorphic but not identical proper subalgebras $\mathcal{A}_{\mathcal{S}_t}$ of $\mathcal{A}$. Hence, either one prescribes the state for the whole quasilocal algebra $\mathcal{A}$ or an extension of the initial state ϕ_s from $\mathcal{A}_{\mathcal{S}_s}$ to $\mathcal{A}$ is needed. In the first case no property forbids a generic state to reveal acausal properties. However, in the latter case properly chosen causal restrictions on the state extension procedure may lead to a subclass of states obeying causal properties (Hofer-Szabó and Vecsernyés 2015). Unfortunately, we do not know how to do such a state extension in the case of a LQT. However, in LCTs, where *conditional probabilities* of local observables have a meaning and provide local extensions of a state, a state extension procedure can be interpreted in terms of a *stochastic dynamics*, where the mentioned conditional probabilities are given by the *transition probabilities* of the underlying stochastic process. To this end there is no need for a covariant dynamics on the classical observables either. Of course, this would ensure the isomorphisms of the image σ-algebras of the random variables on the different Cauchy surfaces in the underlying stochastic process, however, a stochastic process can be defined without such isomorphisms.

In this book we do not address the problem of local causality in stochastic theories. For a specific model see Hofer-Szabó and Vecsernyés (2015). Rather, we briefly review in the rest of the chapter some further relativistic causality principles present in the literature and their relations to (local) primitive causality. These principles are formulated in a quasilocal algebra $\mathcal{A}_{\mathcal{H}}$ generated by an isotone (Requirement 1) net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ of local von Neumann algebras.

Let $\{A_k\}_{k\in K} \subset \mathcal{N}(V_A)$ be a decomposition of the unit, that is, a set of mutually orthogonal projections in the local von Neumann algebra $\mathcal{N}(V_A)$ such that $\sum_k A_k = \mathbf{1}$. The corresponding *nonselective* projective measurement is defined as a map $\mathcal{T}_{\{A_k\}} \colon \mathcal{A}_{\mathcal{H}} \to \mathcal{A}_{\mathcal{H}}$

$$\mathcal{T}_{\{A_k\}}(X) := \sum_{k\in K} A_k X A_k, \quad X \in \mathcal{A}_{\mathcal{H}}. \tag{3.4}$$

Being a unit-preserving completely positive map (even a conditional expectation) $\mathcal{T}_{\{A_k\}}$ maps states to states via

$$\phi \mapsto \phi_{\{A_k\}} := \phi \circ \mathcal{T}_{\{A_k\}}. \tag{3.5}$$

The following causality principle requires that projections (quantum events) located in spatially separated regions should be insensitive to such a change of states.

8. *No-signaling*. Let $V_A, V_B \in \mathcal{K}$ be spacelike separated. For any decomposition of the unit $\{A_k\}_{k\in K} \subset \mathcal{N}(V_A)$ and projection $B \in \mathcal{N}(V_B)$, and for any locally faithful and normal state $\phi \colon \mathcal{A}_{\mathcal{H}} \to \mathbb{C}$, we have

$$\phi_{\{A_k\}}(B) = \phi(B) \tag{3.6}$$

No-signaling follows from microcausality (Requirement 2). Schlieder (1969) showed that the converse is also true: if no-signaling holds for a decomposition of the unit $\{A_k\}_{k\in K}$ and a projection B for all normal states of a local von Neumann algebra with support containing V_A, V_B, then $[A_k, B] = 0$ for all $k \in K$. Being equivalent to microcausality no-signaling trivially fulfills in LCTs. Although it is formulated as a requirement for states, it gives a restriction for the structure of the local algebras.

Instead of non-selective projective measurements (3.4) one can also consider *selective projective measurements* using a single local projection $A \in \mathcal{N}(A)$:

$$\mathcal{T}_A(X) := AXA, \quad X \in \mathcal{A}_{\mathcal{H}}, \tag{3.7}$$

which defines a completely positive but not unit preserving map $\mathcal{T}_A \colon \mathcal{A}_{\mathcal{H}} \to \mathcal{A}_{\mathcal{H}}$. The generated state transition

$$\phi \mapsto \phi_A := \frac{\phi \circ \mathcal{T}_A}{\phi(A)} = \frac{\phi \circ \mathcal{T}_A}{(\phi \circ \mathcal{T}_A)(\mathbf{1})}, \tag{3.8}$$

sometimes called *Lüders projection* (Lüders 1950), provides another causality requirement:

9. *Independence.* For any projections $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ such that $V_A, V_B \in \mathcal{K}$ are spacelike separated regions, and for any locally faithful and

normal state ϕ, we have[3]

$$\phi_A(B) = \phi(B). \tag{3.9}$$

In the case of microcausality (Requirement 2), (3.9) implies that $\phi(AB) = \phi(A)\phi(B)$; that is, ϕ becomes a product state by restricting it to the subalgebra generated by $\mathcal{N}(V_A)$ and $\mathcal{N}(V_B)$. Hence, it is a too strong assumption, which is violated in LQTs, for example, by any entangled state. Of course, it is also violated also in the case of superluminal correlations.

In general, (completely) positive maps $\mathcal{T}\colon \mathcal{A} \to \mathcal{A}$ on a C^*-algebra $\mathcal{A}$ with the property $0 < \mathcal{T}(\mathbf{1}) \leqslant \mathbf{1}$ can be considered as generalized measurements or *operations*. They are called *inner* if $\mathcal{T}$ has the form $\mathcal{T} := \sum_i \mathrm{A}d\, K_i$ with $K_i \in \mathcal{A}$. If the K_i-s are mutually orthogonal projections one speaks about *projective* (inner) operations. Operations with $\mathcal{T}(\mathbf{1}) = \mathbf{1}$ and $\mathcal{T}(\mathbf{1}) < \mathbf{1}$ are called *nonselective* and *selective* operations, respectively. If $\mathcal{A}$ is a von Neumann algebra one usually requires T to be normal. If $\mathcal{A} = \mathcal{B}(\mathcal{H})$ this means that T is σ-weakly continuous. See, for example Werner (1987), and references therein.

A net satisfying local primitive causality (Requirement 7) also satisfies:

10. *Local determinism.* (Earman and Valente 2014) If $\phi|_{\mathcal{A}(V)} = \phi'|_{\mathcal{A}(V)}$ for any two states ϕ and ϕ' and for any globally hyperbolic spacetime region $V \in \mathcal{K}$, then $\phi|_{\mathcal{A}(V'')} = \phi'|_{\mathcal{A}(V'')}$

and consequently it also satisfies

11. *Stochastic Einstein locality.* Let $V_A, V_C \in \mathcal{K}$ such that $V_C \subset J_-(V_A)$ and $V_A \subset V_C''$. If $\phi|_{\mathcal{A}(V_C)} = \phi'|_{\mathcal{A}(V_C)}$ holds for any two states ϕ and ϕ' on $\mathcal{A}$ then $\phi(A) = \phi'(A)$ for any projection $A \in \mathcal{A}(V_A)$.

Microcausality alone does *not* entail local primitive causality. Because microcausality is equivalent to no-signaling and local primitive causality represents no-superluminal propagation (Earman and Valente 2014), it is therefore an interesting question whether there exist nets that *satisfy* local primitive causality but *violate* microcausality. Usually the translation covariant field algebra extension of the observables $\mathcal{F} \supset \mathcal{A}$, in which the localized and transportable endomorphisms—the Doplicher–Haag–Roberts morphisms—of the observables can be implemented, serve such examples: Although local field algebras are defined to be relatively local to observables

$$\mathcal{F}(V) := \mathcal{A}(V')' \cap \mathcal{F}, \quad V \in \mathcal{K}, \tag{3.10}$$

local field algebras corresponding to spacelike separated regions do not commute in general, hence microcausality fails. (For example, in the field algebra of the local quantum Ising model there are field operators with spacelike separated supports

[3] Butterfield (1995, Eqs. 3.6 and 3.7) and Earman and Valente (2014, Sect. 7.2) called (3.6) and (3.9) *parameter independence* and *outcome independence*, respectively (Shimony 1986). For the difference between parameter independence, where ϕ in (3.6) is *conditioned* on the common cause, and no-signaling, where ϕ is *unconditioned*, see Maudlin (2002) and Norsen (2011).

that anticommute.) However, local primitive causality does hold in the net of field algebras, because $V' = V'''$ and hence

$$\mathcal{F}(V) := \mathcal{A}(V')' \cap \mathcal{F} = \mathcal{A}(V''')' \cap \mathcal{F} = \mathcal{A}((V'')')' \cap \mathcal{F} =: \mathcal{F}(V''), \quad V \in \mathcal{K}. \tag{3.11}$$

Thus, for such a net of local (field) algebras no-signaling is violated whereas no-superluminal propagation holds.

In the subsequent chapters we work within the framework of a LPT. When speaking about deterministic dynamics, we also assume Requirements 6–7.

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Chapter 4
Bell's Notion of Local Causality

Abstract At the beginning of this Chapter Bell's different formulations of local causality will be reviewed. Next, we analyze the three key concepts featuring in Bell's final formulation, namely "local beables", "shielder-off region", and "complete specification." We translate them into the LPT framework and provide a generalized definition of local causality. Finally, we relate shielder-off regions to d-separating sets in a Bayesian network and prove that local primitive causality renders an atomic LPT to be locally causal.

Keywords Bell's local causality · Local beables · Shielder-off region · Complete specification · Bayesian networks · Atomicity

Local causality is one of the most important notions that Bell introduced into the foundations of quantum mechanics. To our knowledge, it crops up three times in Bell's writings and gains a more and more refined formulation. Bell addressed local causality for the first time in his "The Theory of Local Beables" delivered at the Sixth GIFT Seminar in 1975 (Bell 1975/2004, p. 54); later in a footnote added to his 1986 paper, "EPR correlations and EPW distributions," intending to clean up the first version (Bell 1986/2004, p. 200); and finally in the most elaborate form in his posthumously published, "La nouvelle cuisine" (Bell 1990/2004, pp. 239–240). It is this latter formulation that has been considered in the literature as *the* definition of local causality. In what follows we also use this third formulation but first we briefly overview the previous two.

Bell's *first formulation* of local causality reads:

> Consider a theory in which the assignment of values to some beables Λ implies, not necessarily a particular value, but a probability distribution, for another beable A. Let $p(A|\Lambda)$ denote[1] the probability of a particular value A given particular values Λ. Let A be localized in a space-time region A. Let B be a second beable localized in a second region B separated from A in a spacelike way. (Fig. 4.1.) Now my intuitive notion of local causality is that events

[1]For the sake of uniformity throughout the book we slightly changed Bell's notation and figures.

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SpringerBriefs in Philosophy, https://doi.org/10.1007/978-3-319-73933-5_4

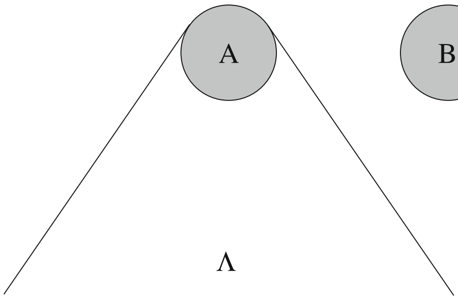

Fig. 4.1 Bell's (1975) first figure illustrating local causality

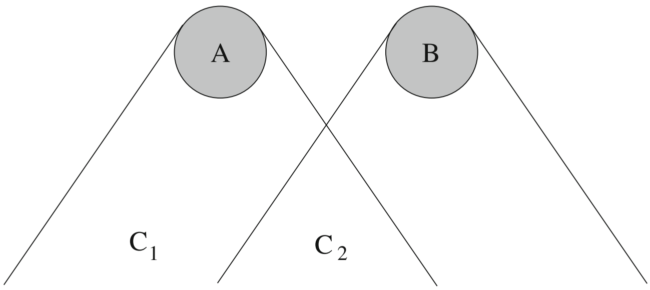

Fig. 4.2 Bell's (1975) second figure illustrating local causality

> in B should not be "causes" of events in A, and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the overlap of their backward light cones. It is perfectly intelligible then that if Λ in (4.1) does not contain a complete record of events in that overlap, it can be usefully supplemented by information from region B. So in general it is expected that
>
> $$p(A|\Lambda, B) \neq p(A|\Lambda) \tag{4.1}$$
>
> However, in the particular case that Λ contains already a *complete* specification of beables in the overlap of the light cones, supplementary information from region B could reasonably be expected to be redundant.
>
> Let C_2 denote a specification of *all* beables, of some theory, belonging to the overlap of the backward light cones of spacelike regions A and B. Let C_1 be a specification of some beables from the remainder of the backward light cone of A, and B of some beables in the region B. (See Fig. 4.2.) Then in a *locally causal theory*
>
> $$p(A|C_1, C_2, B) = p(A|C_1, C_2) \tag{4.2}$$
>
> whenever both probabilities are given by the theory. (Bell 1975/2004, p. 54)

We comment on the terminology Bell is using in his definition below in detail. Here just note that in his screening-off condition (4.2) Bell takes into account the *whole causal past* of the events in question. He does not assume some kind of Markovity rendering superfluous of the remote past regions below a certain Cauchy surface. His *second formulation* of local causality can be regarded as a step towards this Markovian direction:

> The notion of local causality presented in this reference [namely in Bell (1975/2004)] involves complete specification of the beables in an infinite space-time region. The following conception is more attractive in this respect: In a locally-causal theory, probabilities attached to values of local beables in one space-time region, when values are specified for all local beables in a second space-time region fully obstructing the backward light cone of the first, are unaltered by specification of values of local beables in a third region with spacelike separation from the first two. (Bell 1986/2004, p. 200)

Bell's second version is in a footnote; it is very succinct and contains no figure. The new element is the phrasing "space-time region fully obstructing the backward light cone of the first." This idea gets a more precise exposition in Bell's *third, final formulation* of local causality:

> A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region V_A are unaltered by specification of values of local beables in a space-like separated region V_B, when what happens in the backward light cone of V_A is already sufficiently specified, for example by a full specification of local beables in a space-time region V_C. (Bell 1990/2004, pp. 239–240)

The figure Bell attached to this formulation is reproduced in Fig. 4.3 with the original caption. Bell elaborates on his formulation as follows.

> It is important that region V_C completely shields off from V_A the overlap of the backward light cones of V_A and V_B. And it is important that events in V_C be specified completely. Otherwise the traces in region V_B of causes of events in V_A could well supplement whatever else was being used for calculating probabilities about V_A. The hypothesis is that any such information about V_B becomes redundant when V_C is specified completely. (Bell 1990/2004, p. 240)

The notions featured in Bell's formulation have become the target of intensive discussion in the philosophy of science (see Norsen 2009, 2011). Here we concentrate only on three terms, namely *local beables*, *complete specification*, and *shielding-off*.

Local beables. The notion "beable" is Bell's neologism and is contrasted with the term "observable" used in quantum theory. "The *be*ables of the theory are those

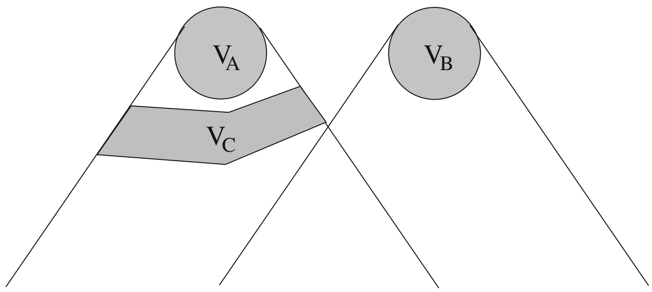

Fig. 4.3 Full specification of what happens in V_C makes events in V_B irrelevant for predictions about V_A in a locally causal theory

entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real" (Bell 1990/2004, p. 234); or elsewhere:

> The beables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on "observation." Indeed observation and observers must be made out of beables.
>
> I use the term "beable" rather than some more committed term like "being" or "beer" to recall the essentially tentative nature of any physical theory. Such a theory is at best a *candidate* for the description of nature. (Bell 1984/2004, p. 174)

The clarification of what the "beables" of a theory are, is indispensable in order to define local causality inasmuch as "there *are* things which do go faster than light. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King" (p. 236).

Beables are to be local: "*Local* beables are those which are definitely associated with particular space-time regions. The electric and magnetic fields of classical electromagnetism, $\mathbf{E}(t, x)$ and $\mathbf{B}(t, x)$ are again examples" (p. 234).

Complete specification. Local beables are to "specify completely" region V_C in order to block causal influences arriving at V_A from the common past of V_A and V_B. [For the question of *complete versus sufficient* specification see Norsen (2011), Seevinck and Uffink (2011), Hofer-Szabó (2015a).]

Shielding-off. "It is important that region V_C completely shields off from V_A the overlap of the backward light cones of V_A and V_B." Why is that so? Why is local causality not required for such regions V_C as depicted in Fig. 4.4, for example? The reason for that is the following. If V_C is localized as in Fig. 4.4, then the spacetime region *above* V_C in the common past of the correlated events may contain stochastic events (with determined probabilities by the complete specification on the region V_C) which can establish a correlation between A and B in a classical stochastic theory. The "shielding-off" condition is required just to exclude this case.

But if this is the reason, then why not also allow for regions V_C as depicted in Fig. 4.5? Allowing for shielding-off regions that *intersect* with the common past is indeed a possible interpretation of Bell's term "shielding-off." We return to this point

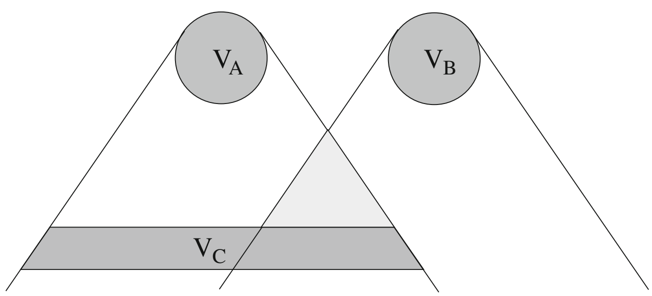

Fig. 4.4 A *not* completely shielding-off region V_C

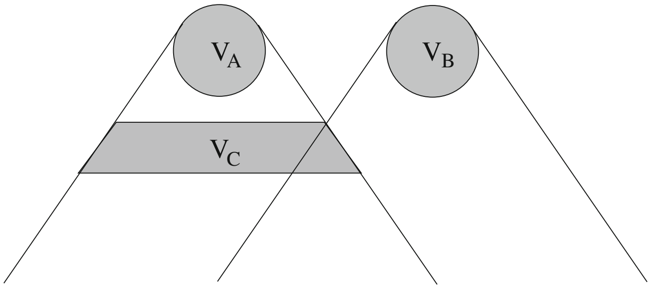

Fig. 4.5 An intersecting and completely shielding-off region V_C

in Remark 4 below. [For the relation between the localization of the region V_C and the causal Markov condition see Hofer-Szabó (2015b).]

How do Bell's three terms above "translate" into the framework of LPT? Let us see them again in turn.

Local beables. In a classical field theory beables are characterized by sets of field configurations. In our local algebraic framework local equivalence classes of field configurations, namely configurations having the same field values on a given spacetime region, generate local σ-algebras, as explained in Chap. 2. The elements of local σ-algebras capture all the beables of the theory; moreover they also provide a localization for them. Translating σ-algebras into Abelian von Neumann algebras one can use a common language for classical and quantum theories: local beables in a region $V \in \mathcal{K}$ are *elements of the local von Neumann algebra* $\mathcal{N}(V)$, which is Abelian for a classical and non-Abelian for a quantum theory.

We note here that our use of beables and hence our upcoming definition of local causality transcends Bell's ideas in an important sense. Events in a noncommutative local algebra can readily be interpreted operationally as measurement outcomes but can hardly be something ontological. It is true that at one point, namely in his *Subject and object*, Bell gives some thought to the idea whether beables can be associated with quantum mechanical observables:

> Could not one just promote *some* of the "observables" of the present quantum theory to the status of beables? The beables would then be represented by linear operators in the state space. The values which they are allowed to be would be eigenvalues of those operators. For the general state the probability of a beable *being* a particular value would be calculated just as was formerly calculated the probability of *observing* that value. (Bell 1973/2004, p. 41)

But the program envisioned here by Bell and later called the *maximal beable approach* (Halvorson and Clifton 1999) aims to identify as beables only a commutative *sub*algebra of observables that can enjoy determinate values simultaneously and not the *full* noncommutative algebra. Many interpretations of quantum mechanics can be seen as a variant of this approach. Collapse interpretations specify the maximal beable algebra for a system subject to the measurement of a non-degenerate observable. Bohmian interpretation picks a preferred observable and generates the maximal

Abelian algebra by the spectral projections of this observable. Modal interpretations (Clifton 2000; Dieks 2002) use the so-called eigenstate-eigenvalue link and identify the beables of the theory with the spectral projections of the density operator associated with the quantum state.

Contrary to these interpretations, we do not put any constraint whatsoever on local algebras that can stand for "beables." We think that at this level of generality where neither the dynamics, nor the measurement procedure, nor any other features of the system are specified, we cannot determine which elements of the local algebras should be regarded as local beables. [For some problems of the maximal beable approach in nonatomic von Neumann algebras see Ruetsche (2011), Chap. 8.] On the other hand, we do think that treating the classical and quantum case parallel manner sheds light on the general structural position of local causality in LPTs and its relations to other locality principles. We come back to this liberal understanding of "beables" in Chap. 9.

Complete specification. Complete specification of field configurations in a given spacetime region means that one specifies the field values to a prescribed value in the given spacetime region; that is, one specifies the corresponding local equivalence class (a cylindrical set) of a single configuration. In probabilistic language complete specification is translated to a probability measure having support on this local equivalence class of the single specified configuration. More precisely, complete specification is a procedure bringing about a change of the probability measure on the whole σ-algebra and resulting in the probability measure restricted to the local σ-algebra in question that has support on the local equivalence class of the single specified configuration. In the Abelian von Neumann language this corresponds to a change of the original state that results in a pure state on the local von Neumann algebra in question with value 1 on the projection corresponding to the local equivalence class of the single specified configuration. However, we would also like this change of states to be as local as possible. Therefore we translate a "complete specification of beables in a region $V \in \mathcal{K}$" as a change of state

$$\phi(X) \mapsto \phi_{\mathcal{T}}(X) := \frac{\phi \circ \mathcal{T}}{(\phi \circ \mathcal{T})(\mathbf{1})} \tag{4.3}$$

by a completely positive map $\mathcal{T}$ on the quasilocal observables obeying the properties:

$\mathbf{P}_1$: the restriction of $\phi_{\mathcal{T}}$ to the local algebra $\mathcal{N}(V)$ is pure,
$\mathbf{P}_2$: $B\mathcal{T}(\mathbf{1}) = \mathcal{T}(B) = \mathcal{T}(\mathbf{1})B$ hold for local observables B supported in V'.

Concerning property $\mathbf{P}_1$ we note that von Neumann algebras in $\mathcal{B}(\mathcal{H})$ that have a separating vector in $\mathcal{H}$, irrespective of being Abelian or non-Abelian algebras, do not possess a pure normal state (Clifton and Halvorson 2001). This is the case, for example, in AQFTs with type III local von Neumann algebras. Thus starting from a (locally) normal state ϕ on them, a normal operation $\mathcal{T}$ leads to a (locally) normal state $\phi_{\mathcal{T}}$ that cannot be pure. There are certain ways to circumvent this problem (neither of them being fully satisfactory). (1) One can use a nonnormal operation to get a pure state for the local von Neumann algebra. In this case, however, one jumps into a different quasiequivalence class of representations of observables that we just wanted to avoid by considering only (locally) normal states for the local von Neumann algebras. (2) In the case of type III (hence non-Abelian) local von Neumann algebras one can also assume the split property [see, e.g. Werner (1987) and references therein] and use the (atomic) type I intermediate von Neumann algebra to provide a pure state, hence a "full specification," for a somewhat larger local observable algebra supported in a somewhat larger local region.[2]

Concerning property $\mathbf{P}_2$ we note that *weakly localized* operations in V (Werner 1987) obey property P_2 for all elements $B \in \mathcal{N}(V)' \supseteq \mathcal{A}_{\mathcal{H}}(V')$ by definition. Moreover, if $\mathcal{T}$ is normal and $\mathcal{A}_{\mathcal{H}} = \mathcal{B}(\mathcal{H})$ then every weakly localized operation $\mathcal{T}$ with respect to $V \in \mathcal{K}$ is inner in $\mathcal{N}(V)$; that is, $\mathcal{T} = \sum_i \mathrm{A}d\ K_i$ with $K_i \in \mathcal{N}(V)$.

In a general LPT, we do not know how to characterize the operations that result in a state obeying properties P_1 and P_2, but in the case of atomic (type I) local von Neumann algebras it is almost trivial: one has to do a selective projective measurement defined in (3.7) by an atom (a minimal projection) C in the local algebra $\mathcal{N}(V)$ that induces the change of states $\phi \mapsto \phi_C$ defined in (3.8).

Shielding-off. Finally, a shielding-off region in a LQT (see Fig. 4.3) can be defined as $V_C \in \mathcal{K}$ satisfying the three localization requirements:

$\mathbf{L}_1$: $V_C \subset J_-(V_A)$
$\mathbf{L}_2$: $V_A \subset V_C''$
$\mathbf{L}_3^Q$: $V_C \subset V_B'$

that is, V_C should be in the causal past of V_A; it has to be "wide enough" such that V_A is in the domain of dependence of V_C, and it should be spacelike separated from V_B.

In a LCT a shielding-off region intersecting with the common past (see Fig. 4.5) is allowed, and requirement L_3^Q can be replaced by the weaker requirement:

$\mathbf{L}_3^C$: $J_-(V_C) \supset J_-(V_A) \cap J_-(V_B)$.

If V_C is contained in an infinitely thin Cauchy surface, requirement L_3^C coincides with requirement L_3^Q.

Given the above interpretation of the terms "local beables," "complete specification," and "shielding-off," we are now in the position to formulate Bell's notion of local causality in the framework of LPTs:

Definition 2 Let an LPT be represented by a net $\{\mathcal{N}(V),\ V \in \mathcal{K}\}$ of von Neumann algebras. Let $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ be a pair of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$. Let ϕ be a locally normal and locally faithful state on the quasilocal observables establishing a correlation $\phi(AB) \neq \phi(A)\phi(B)$ between A and B. Let $\mathcal{T}$ be an operation on the quasilocal observables obeying properties P_1 and P_2. Finally, let $V_C \in \mathcal{K}$ be a spacetime region defined by requirements L_1, L_2, and L_3^Q/L_3^C. The LPT is called *locally causal* if for any such quintuple $(A, B, \phi, \mathcal{T}, V_C)$ the following screening property holds:

[2] We thank Yuichiro Kitajima for drawing our attention to these points.

$$\phi_{\mathcal{T}}(AB) = \phi_{\mathcal{T}}(A)\phi_{\mathcal{T}}(B). \tag{4.4}$$

Remarks:

1. If the local algebras of the net are atomic,[3] the states $\phi_{\mathcal{T}}$ in Definition 2 can be replaced by the state ϕ_C given by (3.7)–(3.8), where $C \in \mathcal{A}(V_C)$ is an arbitrary atomic event, that is, a minimal projection. This converts (4.4) into the screening-off condition of the well-known form:

$$\frac{\phi(CABC)}{\phi(C)} = \frac{\phi(CAC)}{\phi(C)}\frac{\phi(CBC)}{\phi(C)}. \tag{4.5}$$

2. In the commutative case Definition 2 is just Bell's local causality. In LCTs (4.5) can be written into the standard conditional form

$$p(AB|C) = p(A|C)p(B|C), \tag{4.6}$$

or into the equivalent asymmetric form

$$p(A|BC) = p(A|C) \tag{4.7}$$

sometimes used in the literature [e.g., in Bell (1975/2004, p. 54)]. We come back to "noncommuting beables" in Chap. 8.

3. Here we briefly comment on a definition of local causality recently given by Henson (2013b). Henson's definition differs from ours in several respects. First, Henson formulates local causality in terms of *σ-algebras*. Using the recipe given in Chap. 2 to convert σ-algebras into Abelian von Neumann algebras, this difference can be easily dissolved. Second, the Henson definition applies only to *atomic* σ-algebras: his screening-off condition is equivalent to (4.5). Our more general screening condition (4.4) applies both to noncommutative and to nonatomic local algebras. Third, in Henson's definition the screener-off region V_C is not localized according to requirements L_1, L_2, and L_3^Q/L_3^C. It is an *unbounded* region, a "suitable past" of V_A and V_B.[4] In our opinion, here Henson follows Bell's *first* formulation of local causality given in Bell (1975/2004, p. 54), where the screener-off regions are identified with the *complete, unbounded causal past* of the correlated events. Our definition, on the other hand, is based on Bell's *last*, operationally more desirable definition provided in Bell (1990/2004, pp. 239–240), where the screener-off regions are only *bounded Cauchy segments* of the unbounded past regions.[5]

[3]Which is typically *not* the case in a general AQFT.

[4]Where the term "suitable past …has been left open deliberately. …It could be …the 'mutual past' …the 'joint past' or the past of one of the regions but not the other" (Henson 2013b, p. 1015). For an argument *for*, *against*, and again *for* not specifying the screener-off region see Henson (2005, 2013a), Rédei and San Pedro (2012), respectively.

[5]Cf. also Bell (1986/2004, p. 200): "The notion of local causality presented in this reference [namely in Bell (1975/2004)] involves complete specification of the beables in an infinite space-time region.

In his paper Henson shows that the lack of separability (*additivity*, in our language; see Chap. 2) does not block the derivation of Bell's inequalities. As we show, this result is in complete agreement with ours: additivity is not required in our book; hence it plays no role in the derivation of Bell's inequalities in LCTs.

4. There is a symmetric independence relation closely related to Bell's asymmetric local causality requirement. The idea comes from the theory of hyperbolic differential equations describing deterministic causal processes. As said above, in the case of hyperbolic differential equations there exists an initial value formulation of the theory: given the initial values on a segment of a Cauchy surface, the time evolution equation provides a unique solution in the domain of dependence of the segment. This also means that one can freely fix the data on two (spatially separated) segments of a Cauchy surface, V_C and V_D, and solve the initial value problem *independently* in the future domain of dependence regions $D^+(V_C)$ and $D^+(V_D)$. Thus, if $V_A \subseteq (D^+(V_C) \setminus V_C)$ and $V_B \subseteq (D^+(V_D) \setminus V_D)$, then the Cauchy data in V_C will fix the values in V_A independently of the values in V_B.
Indeterministic causal processes (see Hofer-Szabó and Vecsernyés 2015, Sect. 3) inherit this feature of deterministic causal processes. Here one can independently fix the values in regions V_C and V_D such that the stochastic evolution within $D^+(V_C)$ and $D^+(V_D)$ will be probabilistically independent. Thus, fixing the value in V_C will fix the probability of the values in $V_A \subseteq (D^+(V_C) \setminus V_C)$ irrespective of the values in V_B. And this is exactly what Bell's local causality requires.
5. Above we raised the question as to why the shielder-off regions featuring in the definition of Bell's local causality need to separate V_A from the common past of V_A and V_B, as depicted in Figs. 4.3 and 4.5, and cannot be "pushed back" in the remote past as in Fig. 4.4. Or to put it formally, why the shielder-off regions should be characterized by the criteria L_1, L_2, and L_3^Q/L_3^C. Here we claim (for the details see Hofer-Szabó 2015b, 2018) that the above localization of the shielder-off regions can be nicely translated into the theory of Bayesian networks, and it turns out that the shielder-off regions conform in a well-defined sense to the d-separating sets in Bayesian networks or m-separating sets in mixed graphs. (For causal graphs, d-separation, and m-separation see Pearl 2000; Glymour et al. 2000; Richardson and Spirtes 2002; Sadeghi and Lauritzen 2014.)
The translation goes as follows. Let $\mathcal{K}'$ be a finite subset of the covering collection $\mathcal{K}$. The collection $\mathcal{K}'$ gives rise to a mixed graph $\mathcal{G}$ as follows. Let the *vertices* of $\mathcal{G}$ be the regions V in $\mathcal{K}'$ and let there be a *directed edge* from V_1 to V_2 ($V_1, V_2 \in \mathcal{K}'$) if there is a future directed causal curve γ from a point $p_1 \in V_1$ to a point $p_2 \in V_2$ such that γ does not intersect any other region in $\mathcal{K}'$, except for V_1 and V_2. The vertices V_1 and V_2 are connected by a *bidirected edge* if there are directed edges both from V_1 to V_2 and from V_2 to V_1.
This "translation manual" uniquely assigns a causal graph $\mathcal{G}$ to a collection $\mathcal{K}'$. The graph $\mathcal{G}$ is based on the causal structure of the spacetime $\mathcal{M}$. However, being

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The following conception is more attractive in this respect." And then comes the new definition based on bounded regions.

a kind of coarse-graining, the graph also depends sensitively on the collection $\mathcal{K}'$. Choosing "small" pointlike regions, the resulting graph will comprise many edges because the regions do not causally shield one another, whereas the graph coming from collections with "fat" overlapping regions will contain fewer edges. On the two extreme ends of the scale one obtains the "densest" graph $\mathcal{G}^{max}$ and the "sparsest" graph $\mathcal{G}^{min}$. Namely, by shrinking the regions in $\mathcal{K}'$ to pointlike regions one ends up with $\mathcal{G}^{max}$ where there is an edge pointing from V_1 to V_2 whenever $V_1 \cap J_-(V_2) \neq \emptyset$. By "blowing up" the regions in $\mathcal{K}'$ such that they causally shield one another, one obtains $\mathcal{G}^{min}$ where there is no edge pointing from V_1 to V_3 if $V_1 \to V_2$ and $V_2 \to V_3$.

The type of the graph also greatly depends on the regions in $\mathcal{K}'$. For certain collections one ends up with directed acyclic graphs; for other collections one obtains mixed (not necessarily acyclic) graphs. In Chap. 8 we present a covering collection of the 1+1-dimensional Minkowski spacetime which is composed of minimal double cones of unit diameter. This covering collection leads to a directed acyclic graph. In the 2+1-dimensional Minkowski spacetime, however, one can choose a collection consisting of a finite number of double cones such that the resulting graph will be a cyclic graph composed of one single cycle: $V_1 \to V_2 \to \ldots V_n \to V_1$. Thus, not just the "density" but also the type of the causal graph sensitively depends on the collection $\mathcal{K}'$ itself.

Here we do not intend to provide a systematic treatment as to which collections lead to which type of graphs. Our aim is simply to state a proposition proven in Hofer-Szabó (2018):

Let V_A and V_B be two spacelike separated spacetime regions in $\mathcal{K}'$. Call a set $\{V_i\} \subset \mathcal{K}'$ a *shielder-off set of regions* for V_A if $\cup_i V_i$ is a shielder-off region for V_A characterized by the criteria L_1, L_2, and L_3^C. (We take the classical criterion L_3^C instead of the quantum criterion L_3^Q because Bayesian nets are classical.) Consider the graph $\mathcal{G}$ resulting from $\mathcal{K}'$ containing the vertices V_A and V_B and the shielder-off set $\{V_i\}$. Then the shielder-off set $\{V_i\}$ d-separates/m-separates V_A from V_B.

Thus, in this sense shielder-off regions are d-separating.

Coming back to Definition 2 of local causality, the main question is when is a LPT locally causal. We answer this question by the following

Proposition 1 *Let the local von Neumann algebras of a LPT be atomic. Then Bell's local causality holds if the LPT obeys local primitive causality.*

Proof If A is a projection and C is a minimal projection in an atomic von Neumann algebra then $CAC = r(C, A)C$ with $r(C, A) \in \{0, 1\}$ in the case of Abelian and $r(C, A) \in [0, 1] \subset \mathbb{R}$ in the case of non-Abelian algebras. Using the notation of Definition 2 A becomes a projection in the atomic von Neumann algebra $\mathcal{N}(V_C)$ due to local primitive causality. Thus if $C \in \mathcal{N}(V_C)$ is a minimal projection then

$$\phi_C(AB) := \frac{\phi(CABC)}{\phi(C)} = \frac{\phi(CACB)}{\phi(C)} = r(C, A)\frac{\phi(CB)}{\phi(C)} = \frac{\phi(CAC)}{\phi(C)}\frac{\phi(CBC)}{\phi(C)}$$
$$=: \phi_C(A)\phi_C(B). \tag{4.8}$$

Here we used that $CB = BC$ due to commutativity in the case of a LCT and due to the spacelike separation of V_B and V_C (ensured by requirement L_3^Q) and microcausality in a LQT. ■

By Proposition 1 our approach comes near to the position of Earman and Valente (2014) who claim that local causality should be understood as local primitive causality. Here we see that in atomic LPTs local primitive causality implies, in fact, local causality. [For a similar sufficient condition for local causality in *stochastic* LPTs see Hofer-Szabó (2015b).]

In the case of LPTs with local primitive causality but with *nonatomic* von Neumann algebras we do not know how to characterize the local manipulation on the state described in Definition 2, therefore a similar proof cannot be applied. In the case of LPTs *without primitive causality* the dynamics is not deterministic, hence an initial state on a Cauchy surface algebra does not determine the state on the whole quasilocal algebra $\mathcal{A}$. States can be forced by a properly chosen state extension procedure to show suitable causality properties. In this book we do not investigate such state extensions. [For a simple causal stochastic Ising model see Hofer-Szabó and Vecsernyés (2015)]. We stress the following: without a causal dynamics on the observables (no local primitive causality) and without the notion of initial states (no primitive causality) Bell's local causality seems to be the only surviving local causality requirement.

In the light of Proposition 1 the reader may ask how a local *quantum* theory can be locally causal if local causality implies various Bell inequalities that are known to be violated for certain sets of quantum correlations. We come back to this point in Chap. 8.

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Chapter 5
The Common Cause Principle

Abstract In this chapter we turn to Reichenbach's Common Cause Principle. The principle is generalized to the LPT framework and the status of the various Common Cause Principles in algebraic quantum field theory is investigated. Then we motivate the noncommutative generalization of the principle and compare the Common Cause Principle with Bell's local causality.

Keywords Weak · Strong and Commutative · Noncommutative Common Cause Principles

Local causality is closely related to Reichenbach's (1956) Common Cause Principle. The *Common Cause Principle* (CCP) states that if there is a correlation between two events A and B and there is no direct causal (or logical) connection between the correlated events, then there always exists a common cause C of the correlation. Reichenbach's original definition is formulated in a purely classical probabilistic setting lacking any spatiotemporal considerations; however, it can readily be generalized to the LPT framework. [For the steps of the generalization see Rédei (1997), Rédei and Summers (2002, 2007), Hofer-Szabó and Vecsernyés (2012a, b, 2013a, b) and Hofer-Szabó et al. (2013).] In this chapter we briefly overview the various CCPs and relate them to Bell's local causality.

Let $\{\mathcal{N}(V),\ V\in\mathcal{K}\}$ be a net representing a LPT. Let $A\in\mathcal{N}(V_A)$ and $B\in\mathcal{N}(V_B)$ be two events (projections) supported in spacelike separated regions $V_A, V_B\in\mathcal{K}$ that are correlated in a locally normal and faithful state ϕ. The common cause of the correlation is a set of events (projections) $\{C_k\}_{k\in K}$ that screen off the correlated events from one another, and are localized in a region $V_C\in\mathcal{K}$ in the causal past of A and B. For the precise choice of this past one has (at least) three options. One can localize the common cause either (i) in the *union*, $J_-(V_A)\cup J_-(V_B)$; or (ii) in the *intersection*, $J_-(V_A)\cap J_-(V_B)$, of the causal past of the regions V_A and V_B; or (iii) more restrictively, in the spacetime region that lies in the intersection of causal pasts of *every* point of $V_A\cup V_B$, formally $\cap_{x\in V_A\cup V_B} J_-(x)$; see (Rédei, Summers 2007).

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We refer to the above three pasts in turn as the *weak past*, *common past*, and *strong past* of A and B, respectively.

Now, we can define various CCPs in a LPT:

Definition 3 A LPT represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ is said to satisfy the *(Weak/Strong) CCP*, if for any pair $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ and for every locally faithful state ϕ establishing a correlation $\phi(AB) \neq \phi(A)\phi(B)$ between A and B, there exists a nontrivial (see below) common cause that is a set of mutually orthogonal projections $\{C_k\}_{k\in K} \subset \mathcal{N}(V_C)$, $V_C \in \mathcal{K}$ localized in the (weak/strong) common past of V_A and V_B, which decompose the unit and satisfy

$$\phi_{C_k}(AB) = \phi_{C_k}(A)\phi_{C_k}(B), \quad k \in K, \tag{5.1}$$

where the state ϕ_{C_k} is given by (3.8).

A common cause is called *trivial* if $C_k \leq X$ with $X = A, A^\perp, B$ or $B^\perp$ for all $k \in K$. If C_k commutes with both A and B for all $k \in K$, then we call it a *commuting* common cause, otherwise a *noncommuting* one, and the appropriate CCP a *Commutative/Noncommutative CCP*.

Trivial common causes provide solutions of (5.1) independently of the state ϕ. Therefore they are considered as purely "kinematic" or "algebraic" solutions that are insensitive to the actual physical environment provided by a particular state ϕ. If at least one of the algebras $\mathcal{N}(V_A)$ and $\mathcal{N}(V_B)$ is finite dimensional, then even a more trivial common cause can be given that is not sensitive to the given algebra elements A and B either. Namely, any decomposition of the unit into minimal projections of the corresponding finite dimensional algebra,[1] that is, any *maximal* (atomic) decomposition of the unit, provides a weak common cause solution of (5.1) irrespective of the chosen events in $\mathcal{N}(V_A)$ and $\mathcal{N}(V_B)$, and irrespective of the correlating state ϕ on them (Cavalcanti and Lal 2014). Therefore these trivial, maximal size solutions reflect only the structure of the underlying finite-dimensional local algebras.

What is the status of these six different notions of the Common Cause Principle in AQFT?

The question whether the Commutative Common Cause Principles are valid in a Poincaré covariant local quantum theory in the von Neumann algebraic setting was first raised by Rédei (1997). As an answer to this question, Rédei and Summers (2002, 2007) have shown that the Commutative Weak CCP is valid in algebraic quantum field theory with locally infinite degrees of freedom. Namely, in the von Neumann setting they proved that in the case of type III local von Neumann algebras for every locally normal and faithful state and for every superluminally correlated pair of projections there exists a commuting common cause of size 2 in the weak

[1] Of course the cardinality $|K|$ of these (commuting or noncommuting) common causes is uniquely determined by the finite-dimensional algebra: $|K| = \sum_r n_r$ if the finite-dimensional algebra is isomorphic to the finite direct sum of full matrix algebras, $\oplus_r M_{n_r}$.

past of the correlated projections. They have also shown (Rédei and Summers 2002, p. 352) that the localization of a common cause $C < AB$ cannot be restricted to $\big(J_-(V_A) \cup J_-(V_B)\big) \setminus J_-(V_A)$ or $\big(J_-(V_A) \cup J_-(V_B)\big) \setminus J_-(V_B)$ due to logical independence of spacelike separated algebras.

Concerning the Commutative (Strong) CCP less is known. If one also admits projections localized only in *un*bounded regions, then the Strong CCP is known to be false: von Neumann algebras pertaining to complementary wedges contain correlated projections but the strong past of such wedges is empty (see Summers and Werner 1988 and Summers 1990). In spacetimes having horizons, for example, those with the Robertson–Walker metric, the common past of spacelike separated bounded regions can be empty, yet there exist states that provide correlations among local algebras corresponding to these regions (Wald 1992).[2] Hence, CCP is not valid there. In the case of *local* algebras in Minkowski spaces there is no definite statement. We are of the opinion that one cannot decide on the validity of the (Strong) CCP without an explicit reference to the dynamics inasmuch as there is no bounded region V_C in the common past (hence neither in the strong past) for which isotony would ensure that $\mathcal{N}(V_A \cup V_B) \subset \mathcal{N}(V_C'')$. But dynamics relates the local algebras therefore $\mathcal{N}(V_A \cup V_B) \subset \mathcal{N}(V_C'' + t) = \alpha_t(\mathcal{N}(V_C''))$ can be fulfilled for certain $V_C \subseteq V_C'' \subset J_-(V_A) \cap J_-(V_B)$ and for certain time translation by t.

Coming back to the proof of Rédei and Summers, their statement is based on the crucial premise that the algebras in question are von Neumann algebras of type III. Although these algebras arise in a natural way in the context of Poincaré covariant theories, other local quantum theories apply von Neumann algebras of other types. For example, theories with locally finite degrees of freedom are based on finite-dimensional (type I) local von Neumann algebras. This raises the question of whether the commutative weak CCP is valid in these local quantum theories. To address the problem Hofer-Szabó and Vecsernyés (2012a) have chosen the local quantum Ising model (see Müller and Vecsernyés) having locally finite degrees of freedom. It turned out that the Commutative Weak CCP is *not valid* in the local quantum Ising model and it cannot be valid either in theories with locally finite degrees of freedom in general.

But why should we require commutativity between the common cause and its effects at all? Commutativity has a well-defined role in any quantum theory: observables should commute to be simultaneously measurable. In AQFT commutativity of observables with spacelike separated supports is an axiom. To put it simply, commutativity can be required for events that can happen "at the same time." But cause and effect are typically *not* these sorts of events. If one considers ordinary quantum mechanics, one easily sees that observables do not commute even with their own time translates in general. For example, the time translate $x(t) := U(t)^{-1}xU(t)$ of the position operator x of the harmonic oscillator in quantum mechanics does *not* commute with $x \equiv x(0)$ for generic t, because in the ground state vector ψ_0 we have

[2]We thank David Malament for calling our attention to this point and Wald's paper.

$$\left[x, x(t)\right] \psi_0 = \frac{-i\hbar \sin(\hbar\omega t)}{m\omega} \psi_0 \not\equiv 0. \tag{5.2}$$

Thus, if an observable A is not a conserved quantity, that is $A(t) \neq A$, then the commutator $[A, A(t)] \neq 0$ in general. Therefore why should the commutators $[A, C]$ and $[B, C]$ vanish for the events A, B and for their common cause C supported in their (weak/common/strong) past? We think that commuting common causes are only unnecessary reminiscences of their classical formulation. Due to the time delay between the correlated events and the common cause, it is also an unreasonable assumption.

Abandoning commutativity in the definition of the common cause is therefore a natural move. To our knowledge the first who raised the possibility of the noncommuting common causes were Clifton and Ruetsche (1999) criticizing Rédei (1997) who required the common cause to be commuting. They say: "[requiring commutativity] bars from candidacy to the post of common cause the vast majority of events in the common past of events problematically correlated" (p. 165). And indeed, the benefit of allowing noncommuting common causes is that the noncommutative version of the result of Rédei and Summers can be regained: as shown in (Hofer-Szabó and Vecsernyés 2013a, b), by allowing common causes that do *not* commute with the correlated events, the Weak CCP can be proven in local UHF-type quantum theories.

We close this chapter by comparing CCPs with Bell's local causality. First, note that the core mathematical requirement of both principles is the screening-off conditions (4.4) or equivalently (5.1). However, the localization of the screener-off events is different: they are localized symmetrically in the weak/common/strong past in the case of the CCPs and asymmetrically in the causal past of one of the correlating events in the case of local causality. Moreover, the subjects of these conditions are also very different: In the first case the screening-off should hold for all pairs of algebra elements supported in the spacelike regions $V_A, V_B \in \mathcal{K}$. On the contrary, different common causes are not only allowed for different triples (A, B, ϕ) but also a nontrivial dependence is expected on physical grounds. Second, in the case of local causality the screening-off condition (4.4) is required for *every* atomic event (satisfying certain localization conditions). On the contrary, in the case of the CCP the screener-offs are typically not atomic events, because finding a common cause for a correlation does not mean that we have arrived at the most detailed physical description of the situation. It simply means that at this level of description correlations can be causally accounted for. [For an opposite view see Uffink (1999) and Henson (2005).] In the case of local causality, however, it is necessary for the screener-off events to be *atomic* (provided the local algebras are atomic), because they express the Cauchy data on the shielding-off region blocking any causal information from the past. Finally, both the CCPs and Bell's local causality imply Bell's inequalities. But this is the topic for Chap. 7.

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Chapter 6
Bell's Inequalities

Abstract This Chapter collects the most important concepts and some of the representative propositions concerning Bell's inequalities in the general C^*-algebraic setting and in the special LPT framework.

Keywords Separable states · Clauser-Horne inequality

There is a notion that is even more tightly linked to Bell's name than local causality, namely Bell's inequalities. In this chapter we collect the most important concepts and some of the representative propositions concerning Bell's inequality in LPTs [see Summers (1990) and Halvorson (2007)]. We start with the general C^*-algebraic setting and then go over to the special algebraic quantum field-theoretical formulation.

In the general C^*-algebraic setting Bell's inequality is treated in the following way. Let $\mathcal{A}$ and $\mathcal{B}$ be two mutually commuting unital C^*-subalgebras of a C^*-algebra $\mathcal{C}$. A *Bell operator* R for the pair $(\mathcal{A}, \mathcal{B})$ is an element of the set:

$$\mathbb{B}(\mathcal{A}, \mathcal{B}) := \left\{ \frac{1}{2}\big(X_1(Y_1 + Y_2) + X_2(Y_1 - Y_2)\big) \,\big|\, X_i = X_i^* \in \mathcal{A};\; Y_i = Y_i^* \in \mathcal{B}; \right.$$
$$\left. -\mathbf{1} \leqslant X_i, Y_i \leqslant \mathbf{1} \right\}$$

where $\mathbf{1}$ is the unit element of $\mathcal{C}$. For any Bell operator R the following can be proven.

Theorem 1 *For any state* $\phi\colon \mathcal{C} \to \mathbb{C}$, *one has* $|\phi(R)| \leqslant \sqrt{2}$.

Theorem 2 *For separable states (i.e., for convex combinations of product states)* $|\phi(R)| \leqslant 1$.

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The *Bell correlation coefficient* of a state ϕ is defined as

$$\beta(\phi, \mathcal{A}, \mathcal{B}) := \sup \left\{ |\phi(R)| \,\middle|\, R \in \mathbb{B}(\mathcal{A}, \mathcal{B}) \right\}$$

and *Bell's inequality* is said to be *violated* if $\beta(\phi, \mathcal{A}, \mathcal{B}) > 1$, and *maximally violated* if $\beta(\phi, \mathcal{A}, \mathcal{B}) = \sqrt{2}$. An important result of Bacciagaluppi (1994) is:

Theorem 3 *If $\mathcal{A}$ and $\mathcal{B}$ are C^*-algebras, then there are some states violating Bell's inequality for $\mathcal{A} \otimes \mathcal{B}$ iff both $\mathcal{A}$ and $\mathcal{B}$ are non-abelian.*

Going over to von Neumann algebras Landau (1987) has shown that the maximal violation of Bell's inequality is generic in the following sense.

Theorem 4 *Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be von Neumann algebras, and suppose that $\mathcal{N}_1$ is abelian and $\mathcal{N}_1 \subseteq \mathcal{N}_2'$ ($\mathcal{N}'$ being the commutant of $\mathcal{N}$). Then for any state $\beta(\phi, \mathcal{N}_1, \mathcal{N}_2) \leqslant 1$. On the other hand, if both $\mathcal{N}_1$ and $\mathcal{N}_2$ are non-abelian von Neumann algebras such that $\mathcal{N}_1 \subseteq \mathcal{N}_2'$, and if $(\mathcal{N}_1, \mathcal{N}_2)$ satisfies the Schlieder property,[1] then there is a state ϕ for which $\beta(\phi, \mathcal{N}_1, \mathcal{N}_2) = \sqrt{2}$.*

Adding further constraints on the von Neumann algebras one obtains other important results such as the following two.

Theorem 5 *If $\mathcal{N}_1$ and $\mathcal{N}_2$ are properly infinite[2] von Neumann algebras on the Hilbert space $\mathcal{H}$ such that $\mathcal{N}_1 \subseteq \mathcal{N}_2'$, and $(\mathcal{N}_1, \mathcal{N}_2)$ satisfies the Schlieder property, then there is a dense set of vectors in $\mathcal{H}$ inducing states that violate Bell's inequality across $(\mathcal{N}_1, \mathcal{N}_2)$ (Clifton and Halvorson 2001).*

Theorem 6 *Let $\mathcal{H}$ be a separable Hilbert space and let $\mathcal{R}$ be a von Neumann factor of type III_1 acting on $\mathcal{H}$. Then every normal state ϕ of $\mathcal{B}(\mathcal{H})$ maximally violates Bell's inequality across $(\mathcal{R}, \mathcal{R}')$ (Summers and Werner 1988).*

Type *III* factors featuring in Theorems 5 to 6 are the typical local von Neumann algebras in AQFT with locally infinite degrees of freedom.

The Bell inequality typically used in AQFT is of the form:

$$\left|\phi\big(X_1(Y_1 + Y_2) + X_1(Y_1 - Y_2)\big)\right| \leqslant 2, \tag{6.1}$$

where $X_m \in \mathcal{N}(V_A)$ and $Y_n \in \mathcal{N}(V_B)$ are self-adjoint *contractions* (i.e., $-\mathbf{1} \leqslant X_m, Y_n \leqslant \mathbf{1}$ for $m, n = 1, 2$) supported in spatially separated spacetime regions V_A and V_B, respectively. This type of Bell's inequality is usually referred to as the *Clauser–Horne–Shimony–Holte (CHSH) inequality* (Clauser et al. 1969).

[1] The commuting pair $(\mathcal{A}, \mathcal{B})$ of C^*-subalgebras in $\mathcal{C}$ obeys the Schlieder property, if $0 \neq A \in \mathcal{A}$ and $0 \neq B \in \mathcal{B}$, then $AB \neq 0$. Because in the case of von Neumann algebras A and B can be required to be projections, the Schlieder property is the analogue of logical independence in classical logic.

[2] The center contains no finite projections.

Sometimes another Bell-type inequality is used in the EPR-Bell literature instead of (6.1): the *Clauser–Horne (CH) inequality* (Clauser and Horne 1974) defined in the following way.

$$-1 \leqslant \phi(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0, \tag{6.2}$$

where A_m and B_n are *projections* located in $\mathcal{N}(V_A)$ and $\mathcal{N}(V_B)$, respectively. It is easy to see, however, that the two inequalities are equivalent: in a given state ϕ the set $\{(A_m, B_n); m, n = 1, 2\}$ violates the CH inequality (6.2) *if and only if* the set $\{(X_m, Y_n); m, n = 1, 2\}$ of self-adjoint contractions given by

$$X_m := 2A_m - \mathbf{1} \tag{6.3}$$

$$Y_n := 2B_n - \mathbf{1} \tag{6.4}$$

violates the CHSH inequality (6.1). Therefore, the two types of Bell's inequalities can be used *ad libitum*.

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Chapter 7
The EPR Scenario

Abstract In this Chapter we outline the Einstein–Podolsky–Rosen (EPR) scenario and show that the violation of Bell's inequalities does *not* block the implementation of the EPR situation in a locally causal LQT, neither it excludes a noncommuting common causal explanation for the EPR correlations.

Keywords EPR scenario · Common causal explanation · Clauser-Horne inequality

Let us start with a succint description of the EPR scenario. Consider a pair of spin-$\frac{1}{2}$ particles prepared in the singlet state ϕ_s (see Fig. 7.1). Measure the spin of the left particle in directions $\vec{a}_m$ ($m = 1, 2$) and the spin of the right particle in directions $\vec{b}_n$ ($n = 1, 2$). Quantum mechanics yields the following probabilities for the outcomes

$$\phi_s(A_m) = \frac{1}{2} \tag{7.1}$$

$$\phi_s(B_n) = \frac{1}{2} \tag{7.2}$$

$$\phi_s(A_m B_n) = \frac{1}{2}\sin^2\left(\frac{\theta_{a_m b_n}}{2}\right) \tag{7.3}$$

where A_m and B_n denote those projections of the tensor product matrix algebra $M_2(\mathbb{C}) \otimes M_2(\mathbb{C})$ that are spectral projections with eigenvalue $+\frac{1}{2}$ of the spin operators associated with directions $\vec{a}_m$ and $\vec{b}_n$, respectively, and $\theta_{a_m b_n}$ denotes the angle between directions $\vec{a}_m$ and $\vec{b}_n$.

In Chap. 8 we show that the EPR scenario can be represented in the LPT framework where all the projections are localized in a well-defined spacetime region and there is a state on the LPT which yields the above probabilities [see Hofer-Szabó and Vecsernyés (2013)]. From this fact, however, it follows neither that this LPT will

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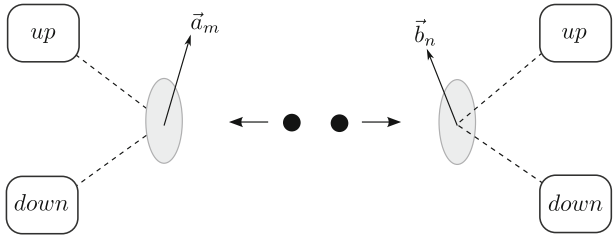

Fig. 7.1 EPR scenario for spin-$\frac{1}{2}$ particles

be locally causal nor that there will be a common causal explanation for the EPR correlations in this LPT. Hence, one can raise the following questions.

Question 1: Can the EPR scenario be implemented in a LPT that is locally causal?

Question 2: Can the EPR scenario be implemented in a LPT such that there exists a (weak/strong) common causal explanation of the EPR scenario?

The standard answer to both questions is *no*. The brief argument is this: both local causality and also the common causal explanation imply Bell's inequalities that are violated for certain measurement settings in the EPR scenario. Hence the implementability of the EPR scenario in a LPT that is locally causal or provides a common causal explanation for the correlations is impossible.

However, as we shortly show, the above reasoning crucially hinges on the assumption of *commutativity*; that is, the common cause accounting for the correlations is commuting and the LPT implementing the correlations is classical, that is, a LCT. It turns out that in the general noncommutative case Bell's inequalities cannot be derived, hence, their violation blocks neither a noncommutative common causal explanation nor the implementation of the EPR scenario in a noncommutative LPT, that is, into a LQT.

The two reasonings relating local causality and the CCPs to Bell's inequalities are parallel. Hence, we consider here only the first. The following proposition shows that a locally causal LPT does not necessarily imply Bell's inequalities. For an analogous proposition relating the common causal explanation to Bell's inequalities see (Hofer-Szabó and Vecsernyés 2013; Proposition 2).

Proposition 2 *Let $\{\mathcal{N}(V), V \in \mathcal{K}\}$ be a locally causal LPT with atomic (type I) local von Neumann algebras. Let $A_1, A_2 \in \mathcal{A}(V_A)$ and $B_1, B_2 \in \mathcal{A}(V_B)$ be four projections localized in spacelike separated spacetime regions V_A and V_B, respectively, that are correlated pairwise in the locally faithful state ϕ:*

$$\phi(A_m B_n) \neq \phi(A_m)\,\phi(B_n) \tag{7.4}$$

for any $m, n = 1, 2$. Let $V_C \in \mathcal{K}$ be a region satisfying requirements L_1, L_2, and L_3^Q / L_3^C in Definition 2 of local causality and let $\{C_k\}_{k\in K} \subset \mathcal{N}(V_C)$ be a maximal *partition of the unit containing mutually orthogonal atomic projections. Then the Clauser–Horne inequality*

$$-1 \leqslant (\phi \circ \mathcal{T}_{\{C_k\}})(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0. \quad (7.5)$$

holds for the state $\phi \circ \mathcal{T}_{\{C_k\}}$. *If* $\{C_k\}$ *commutes with* A_1, A_2, B_1, *and* B_2, *then the Clauser–Horne inequality holds for the original state* ϕ*:*

$$-1 \leqslant \phi(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0. \quad (7.6)$$

Proof It is an elementary fact of arithmetic that for any $\alpha, \alpha', \beta, \beta' \in [0, 1]$ the number

$$\alpha\beta + \alpha\beta' + \alpha'\beta - \alpha'\beta' - \alpha - \beta \quad (7.7)$$

lies in the interval $[-1, 0]$. Now let $\alpha, \alpha', \beta, \beta'$ be the conditional probabilities:

$$\alpha := \phi_{C_k}(A_m) \quad (7.8)$$
$$\alpha' := \phi_{C_k}(A_{m'}) \quad (7.9)$$
$$\beta := \phi_{C_k}(B_n) \quad (7.10)$$
$$\beta' := \phi_{C_k}(B_{n'}) \quad (7.11)$$

Plugging (7.8)–(7.11) into the (7.7) and using that the atomic partition $\{C_k\}_{k\in K}$ screens off all correlations, that is,

$$\phi_{C_k}(A_mB_n) = \phi_{C_k}(A_m)\,\phi_{C_k}(B_n) \quad (7.12)$$

we get

$$\begin{aligned} -1 \leqslant\ & \phi_{C_k}(A_mB_n) + \phi_{C_k}(A_mB_{n'}) + \phi_{C_k}(A_{m'}B_n) \\ & -\phi_{C_k}(A_{m'}B_{n'}) - \phi_{C_k}(A_m) - \phi_{C_k}(B_n) \leqslant 0. \end{aligned} \quad (7.13)$$

Multiplying the above inequality by $\phi(C_k)$, using (3.8), that is,

$$\phi_{C_k}(X) = \frac{(\phi \circ \mathcal{T}_{\{C_k\}})(XC_k)}{\phi(C_k)} \quad (7.14)$$

and summing up for the index k one obtains (7.5). If $\{C_k\}_{k\in K}$ is a *commuting* joint common cause, then $\mathcal{T}_{\{C_k\}}$ drops out from the above expression. Therefore (7.5) becomes identical to (7.6), which completes the proof. ∎

The moral of Proposition 2 is the following. Bell's inequalities can be derived in a locally causal LPT only for a *modified* state $\phi \circ \mathcal{T}_{\{C_k\}}$ in general. It can be derived for the *original* state ϕ *if* the set of atomic projections $\{C_k\}$ localized in V_C commutes with A_1, A_2, B_1, and B_2. Clearly, if the LPT is classical, the elements taken from any local algebra will commute, therefore Bell's inequalities hold for the original state ϕ in LCTs. However, going over to locally causal LQTs, commutation of $\{C_k\}$ with the correlated events is not guaranteed. If V_C is spatially separated from V_B (ensured by requirement L_3^Q but not L_3^C), then $\{C_k\}$ will commute with B_1 and B_2 due to microcausality; hence (4.4) will be satisfied, even if the B_1 and B_2 do not commute. However, in the case of local primitive causality one cannot pick a maximal partition of unit $\{C_k\}$ in $\mathcal{N}(V_C)$ (which is needed for the states ϕ_{C_k} to be pure on $\mathcal{N}(V_C)$) such that $\{C_k\}$ commutes also with projections A_1 and A_2, if $[A_1, A_2] \neq 0$. Namely, $\mathcal{N}(V_A) \subset \mathcal{N}(V_C'') = \mathcal{N}(V_C)$ due to isotony and local primitive causality, and the image $\mathcal{T}_{\{C_k\}}(\mathcal{N}(V_C))$ is a maximal *abelian* subalgebra of $\mathcal{N}(V_C)$ containing exactly those elements that commute with $\{C_k\}$. Hence, in order to commute with $\{C_k\}$, both A_1 and A_2 should be contained in $\mathcal{T}_{\{C_k\}}(\mathcal{N}(V_C))$, which cannot be the case, if $[A_1, A_2] \neq 0$.

The conclusion is that in the case of noncommuting projections A_1 and A_2 the theorem of total probability, $\sum_k \phi(C_k A_m C_k) = \phi(A_m)$, will not hold for the original state[1] ϕ at least for one of the projections A_1 and A_2. This fact blocks the derivation of Bell's inequalities for the original state ϕ. (For the details see (Hofer-Szabó and Vecsernyés 2013, p. 410.)) In short, Bell's inequalities can be derived in a locally primitive causal LQT with atomic von Neumann algebras, hence in a locally causal LQT, if the projections supported on one of the regions do commute. However, this is exactly what one expects relying on the first part of Theorem 4.

Coming back to Question 1 raised above, namely how a local *quantum* theory can be locally causal in the face of Bell's inequalities, we have arrived at the following answer. Bell's inequalities can be derived from local causality if the "beables" of the local theory are represented by *commutative* local algebras. This fact is completely analogous with the relation shown in (Hofer-Szabó and Vecsernyés 2013): Bell's inequalities can be derived from a (joint, non-conspiratorial, local) common cause if it is a *commuting* common cause. Thus, contrary to common wisdom, the answer to both Questions 1 and 2 is a qualified *yes*. The violation of Bell's inequalities does *not* block the implementation of the EPR situation in a LPT that is locally causal or provides a common causal explanation for the EPR correlations. Both common causal explanation and local causality are more general notions than what is captured by Bell's inequalities.

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G. Hofer-Szabó, P. Vecsernyés, Bell inequality and common causal explanation in algebraic quantum field theory. Stud. Hist. Phil. Mod. Phys. **44**(4), 404–416 (2013)

[1]It holds only for the state $\phi_{\{C_k\}}$ for which $\phi_{\{C_k\}}(A_m) \neq \phi(A_m)$ at least for one of the projections A_1 and A_2.

Chapter 8
A Noncommutative Locally Causal Model for the EPR Scenario

Abstract In this Chapter we explicitly construct a locally causal LQT for the EPR scenario. The model is the 1+1 dimensional local quantum Ising model (Müller and Vecsernyés). We show that the model is not only locally causal in Bell's sense but also able to implement four pairs of events correlating in the singlet state.

Keywords Discretized Minkowski spacetime · Quantum Ising model · Singlet state

In the previous chapter it was shown that the violation of Bell's inequalities does not prohibit the implementation of the EPR situation in a locally causal LPT. Here we explicitly construct such a model. Consider a discretized version of the two-dimensional Minkowski spacetime $\mathcal{M}^2$ composed of minimal double cones $\mathcal{O}^m(t, i)$ of unit diameter with their center in (t, i) for $t, i \in \mathbb{Z}$ or $t, i \in \mathbb{Z} + 1/2$. The set $\{\mathcal{O}^m_i, i \in \frac{1}{2}\mathbb{Z}\}$ of such minimal double cones with $t = 0, -1/2$ defines a "thickened" Cauchy surface in this spacetime (see Fig. 8.1). The double cone $\mathcal{O}^m_{i,j}$ stuck to this Cauchy surface is defined to be the smallest double cone containing both $\mathcal{O}^m_i$ and $\mathcal{O}^m_j$: $\mathcal{O}^m_{i,j} := \mathcal{O}^m_i \vee \mathcal{O}^m_j$. Similarly, let $\mathcal{O}^m(t, i; s, j) := \mathcal{O}^m(t, i) \vee \mathcal{O}^m(s, j)$. The directed partially ordered set (with respect to inclusion) of such double cones is denoted by $\mathcal{K}^m$. The directed subset of $\mathcal{K}^m$ whose elements are stuck to a Cauchy surface is denoted by $\mathcal{K}^m_{CS}$. Obviously, $\mathcal{K}^m_{CS}$ is invariant with respect to integer space translations and $\mathcal{K}^m$ is invariant with respect to integer space and time translations.

The net of local algebras is defined as follows. The "one-point" observable algebras associated with the minimal double cones $\mathcal{O}^m_i, i \in \frac{1}{2}\mathbb{Z}$ are defined to be $\mathcal{A}(\mathcal{O}^m_i) \simeq M_1(\mathbb{C}) \oplus M_1(\mathbb{C})$. For the unitary self-adjoint generators $U_i \in \mathcal{A}(\mathcal{O}^m_i)$ of these local algebras one demands the commutation and anticommutation relations:

$$U_i U_j = \begin{cases} -U_j U_i, & \text{if } |i - j| = \frac{1}{2}, \\ U_j U_i, & \text{otherwise.} \end{cases} \tag{8.1}$$

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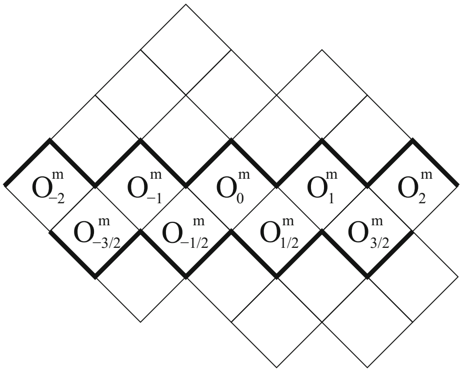

Fig. 8.1 A thickened Cauchy surface in the two dimensional Minkowski space $\mathcal{M}^2$

The local algebras $\mathcal{A}(\mathcal{O}_{i,j})$, $\mathcal{O}_{i,j} \in \mathcal{K}^m_{CS}$ are linearly spanned by the monoms

$$U_i^{k_i}\, U_{i+\frac{1}{2}}^{k_{i+\frac{1}{2}}} \;\ldots\; U_{j-\frac{1}{2}}^{k_{j-\frac{1}{2}}}\, U_j^{k_j} \tag{8.2}$$

where $k_i, k_{i+\frac{1}{2}} \ldots k_{j-\frac{1}{2}}, k_j \in \{0, 1\}$.[1]

Because the local algebras $\mathcal{A}(\mathcal{O}_{i,i-\frac{1}{2}+n})$, $i \in \frac{1}{2}\mathbb{Z}$ for $n \in \mathbb{N}$ are isomorphic to the full matrix algebra $M_{2^n}(\mathbb{C})$, the quasilocal observable algebra $\mathcal{A}$ is a uniformly hyperfinite (UHF) C^*-algebra and consequently there exists a unique (nondegenerate) normalized trace $\mathrm{Tr}\colon \mathcal{A} \to \mathbb{C}$ on it. We note that all nontrivial monoms in (8.2) have zero trace.

In order to extend the "Cauchy surface net" $\{\mathcal{A}(\mathcal{O}), \mathcal{O} \in \mathcal{K}^m_{CS}\}$ to the net $\{\mathcal{A}(\mathcal{O}), \mathcal{O} \in \mathcal{K}^m\}$ one has to classify the integer-valued time evolutions given by a group $\{\beta_t, t \in \mathbb{Z}\}$ of automorphisms of $\mathcal{A}$ (the quasilocal algebra of a Cauchy surface net) that are covariant and causal, and commute with integer space translation automorphisms of $\mathcal{A}$. The classification was given in (Müller and Vecsernyés) and it was also shown that the extended net satisfies isotony, microcausality, algebraic Haag duality, $\mathbb{Z} \times \mathbb{Z}$ covariance with respect to integer time and space translations, and primitive causality:

$$\mathcal{A}(V) = \mathcal{A}(V''), \tag{8.3}$$

where V is a finite connected piece of a thickened Cauchy surface (composed of minimal double cones from $\mathcal{K}^m_{CS}$). The double spacelike complement of V is denoted by V'', which is the smallest double cone in $\mathcal{K}^m$ containing V.

Because $\{\mathcal{A}(\mathcal{O}), \mathcal{O} \in \mathcal{K}^m\}$ is an atomic LPT obeying local primitive causality, Proposition 1 in Chap. 4 implies that the net $\{\mathcal{A}(\mathcal{O}), \mathcal{O} \in \mathcal{K}^m\}$ is locally causal.

[1]For a detailed Hopf algebraic description of the local quantum spin models see Szlachányi and Vecsernyés (1993), Nill and Szlachányi (1997), Müller and Vecsernyés.

Next we show that the EPR scenario can be implemented in our 1+1-dimensional Ising model. Consider the following group of causal time translation automorphisms of the net $\{\mathcal{A}(\mathcal{O}), \mathcal{O} \in \mathcal{K}^m_{CS}\}$ given by its generator $\beta \equiv \beta_1$.

$$\beta(U_x) = U_{x-\frac{1}{2}} U_x U_{x+\frac{1}{2}}, \quad x \in \mathbb{Z} + \frac{1}{2}. \tag{8.4}$$

(In our following example we need not even specify the particular choice for $\beta(U_x), x \in \mathbb{Z}$ from the allowed ones.) Let us consider the double cones $\mathcal{O}_A := \mathcal{O}^m(0, -1) \cup \mathcal{O}^m(\frac{1}{2}, -\frac{1}{2}), \mathcal{O}_B := \mathcal{O}^m(\frac{1}{2}, \frac{1}{2}) \cup \mathcal{O}^m(0, 1)$, and the "two-point" algebras $\mathcal{A}(\mathcal{O}_A), \mathcal{A}(\mathcal{O}_B)$ pertaining to them. (See Fig. 8.2.) A linear basis of the algebra $\mathcal{A}(\mathcal{O}_A)$ is given by the monoms

$$\mathbf{1}, \qquad U_{-1}, \qquad \beta(U_{-\frac{1}{2}}) \equiv U_{-1} U_{-\frac{1}{2}} U_0, \qquad iU_{-1}\beta(U_{-\frac{1}{2}}) \equiv iU_{-\frac{1}{2}} U_0 \tag{8.5}$$

(where i in the fourth monom is the imaginary unit). They satisfy the same commutation relations as the Pauli matrices $\sigma_0 = \mathbf{1}, \sigma_x, \sigma_y$, and σ_z in $M_2(\mathbb{C})$.

Therefore, introducing the notation

$$\mathbf{U} := (U_{-1},\ U_{-1} U_{-\frac{1}{2}} U_0,\ iU_{-\frac{1}{2}} U_0), \tag{8.6}$$

any minimal projection in $\mathcal{A}(\mathcal{O}_A)$ can be parametrized as

$$A(\mathbf{a}) := \frac{1}{2}(\mathbf{1} + \mathbf{aU}) \tag{8.7}$$

where $\mathbf{a} = (a_1, a_2, a_3)$ is a unit vector in $\mathbb{R}^3$. In the same vein, any minimal projection in $\mathcal{A}(\mathcal{O}_B)$ can be parametrized as

$$B(\mathbf{b}) := \frac{1}{2}(\mathbf{1} + \mathbf{bV}), \tag{8.8}$$

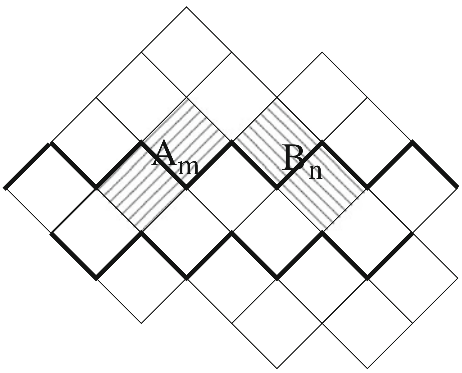

Fig. 8.2 Projections in $\mathcal{A}(\mathcal{O}_A)$ and $\mathcal{A}(\mathcal{O}_B)$

where

$$\mathbf{V} := (U_1,\ -U_0 U_{\frac{1}{2}} U_1,\ iU_0 U_{\frac{1}{2}}) \tag{8.9}$$

is the triple composed of the generators of $\mathcal{A}(\mathcal{O}_B)$ and $\mathbf{b} = (b_1, b_2, b_3)$ is a unit vector in $\mathbb{R}^3$. The projections $A(\mathbf{a})$ and $B(\mathbf{b})$ can be interpreted as the event localized in $\mathcal{A}(\mathcal{O}_A)$ and $\mathcal{A}(\mathcal{O}_B)$, respectively, pertaining to the generalized spin measurement in direction $\mathbf{a}$ and $\mathbf{b}$, respectively.

Now, consider two projections $A_m := A(\mathbf{a^m})$; $m = 1, 2$ localized in $\mathcal{O}_A$, and two other projections $B_n := B(\mathbf{b^n})$; $n = 1, 2$ localized in the spacelike separated double cone $\mathcal{O}_B$. A faithful state $\phi\colon \mathcal{A} \to \mathbb{C}$ on our system can be given in terms of an invertible density matrix $\rho \in \mathcal{A}$: $\phi(\,\cdot\,) = Tr(\rho\,\cdot\,)$. We choose a one-parameter family of such states

$$\rho(\lambda) := \mathbf{1} + \lambda\big(U_{-1}U_{-\frac{1}{2}}U_{\frac{1}{2}}U_1 - U_{-1}U_1 + U_{-\frac{1}{2}}U_{\frac{1}{2}}\big), \quad \lambda \in [0, 1). \tag{8.10}$$

For $\lambda = 1$ the state defined by (8.10) is not faithful on $\mathcal{A}$ (because the corresponding ρ is not invertible) but leads to the usual singlet state on $\mathcal{A}(\mathcal{O}_A) \vee \mathcal{A}(\mathcal{O}_B) \simeq M_2(\mathbb{C}) \otimes M_2(\mathbb{C})$. It is easy to see that the correlation between A_m and B_n in the state (8.10) will be:

$$corr(A_m, B_n) := \phi(A_m B_n) - \phi(A_m)\,\phi(B_n) = -\frac{\lambda}{4}\left\langle \mathbf{a^m}, \mathbf{b^n} \right\rangle \tag{8.11}$$

where $\langle\ ,\ \rangle$ is the scalar product in $\mathbb{R}^3$. In other words, A_m and B_n will correlate whenever $\mathbf{a^m}$ and $\mathbf{b^n}$ are not orthogonal. Now, if $\mathbf{a^m}$ and $\mathbf{b^n}$ are chosen as

$$\mathbf{a^1} = (0, 1, 0) \tag{8.12}$$
$$\mathbf{a^2} = (1, 0, 0) \tag{8.13}$$
$$\mathbf{b^1} = \frac{1}{\sqrt{2}}(1, 1, 0) \tag{8.14}$$
$$\mathbf{b^2} = \frac{1}{\sqrt{2}}(-1, 1, 0) \tag{8.15}$$

the CH inequality (6.2) will be violated at the lower bound because

$$\begin{aligned} &\phi\big(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1\big) \\ &= -\frac{1}{2} - \frac{\lambda}{4}\left(\left\langle \mathbf{a^1}, \mathbf{b^1}\right\rangle + \left\langle \mathbf{a^1}, \mathbf{b^2}\right\rangle + \left\langle \mathbf{a^2}, \mathbf{b^1}\right\rangle - \left\langle \mathbf{a^2}, \mathbf{b^2}\right\rangle\right) = -\frac{1+\lambda\sqrt{2}}{2}, \end{aligned} \tag{8.16}$$

which is smaller than -1 if $\lambda > \frac{1}{\sqrt{2}}$. Or, equivalently, the CHSH inequality (6.1) where

$$X_m := 2A_m - \mathbf{1} \tag{8.17}$$
$$Y_n := 2B_n - \mathbf{1} \tag{8.18}$$

will be violated for the above setting because

$$\begin{aligned}&\phi(X_1(Y_1 + Y_2) + X_1(Y_1 - Y_2))\\&= -\lambda \left(\left\langle \mathbf{a^1}, \mathbf{b^1} + \mathbf{b^2}\right\rangle + \left\langle \mathbf{a^2}, \mathbf{b^1} - \mathbf{b^2}\right\rangle\right) = -\lambda 2\sqrt{2}\end{aligned} \tag{8.19}$$

is smaller than -2 if $\lambda > \frac{1}{\sqrt{2}}$. Both the CH and the CHSH inequality are maximally violated for the singlet state, that is, if $\lambda = 1$.

To sum up, we have constructed a locally causal LPT in which the EPR scenario can be implemented. That is, we localized two pairs of noncommuting projections in two spatially separated regions such that the projections represented different spin outcomes in certain directions. The singlet state of the model provided just the EPR statistics of these outcomes and the CH inequalities were violated. In short, we have provided a locally causal LPT for the EPR violating Bell's inequalities.

Note again that the above model does not stand in contradiction to the common wisdom, namely that the violation of Bell's inequalities excludes a local realistic model for the EPR. The net we have provided is a LQT, a local *quantum* theory and not a LCT, a local *classical* theory. All we wanted to show that we can preserve local causality at the price of abandoning classicality. Noncommutative locally causal models are not excluded by the violation of Bell's inequalities.

Of course, it is another and highly nontrivial philosophical question as to what is the correct interpretation of a noncommutative model. Noncommuting projections are as a standard interpreted operationally: they represent the outcomes of incommensurable measurements. In order to provide a locally causal explanation of the EPR scenario in the Bellian sense one should interpret these noncommuting observables as beables representing some independently existing properties of the system. Our book leaves open the question as to whether such a noncommutative ontology can be consistently developed.

The Ising model sketched above also provides a common causal explanation for the EPR correlations. In Hofer-Szabó and Vecsernyés (2012b, Proposition 1) we have proven that the above four correlations violating the CH inequality (6.2) have a strong common cause localized in the region shown in Fig. 8.3. This result positively answers Question 2 raised in the previous chapter as to whether there is a LPT providing a common causal explanation for the EPR correlations violating Bell's inequalities.

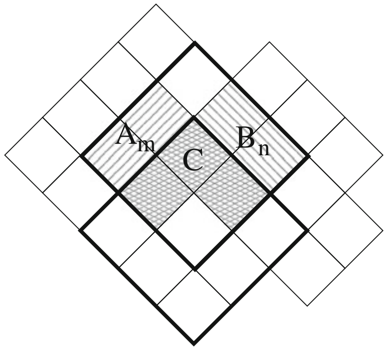

Fig. 8.3 Localization of a common cause for the correlations $\{(A_m, B_n)\}$

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Chapter 9
Summary and Outlook

Abstract In this Chapter we summarize the main results of this book and investigate their philosophical consequences. We argue that embracing noncommuting "beables" in our ontology significantly extends our explanatory sources in accounting for correlations. We also examine what price we need to pay for abandoning classicality in order to preserve local causality.

Keywords Noncommuting beables · Noncommutative ontology

In this book we aimed to implement Bell's notion of local causality in a framework, called local physical theory, that integrates probabilistic and spatiotemporal concepts in a common conceptual scheme. After giving a brief overview of the various locality and causality principles we provided a clear-cut definition of local causality. This definition transcended Bell's original intuition in the sense that it also incorporated noncommutative "beables." Having formulated local causality we gave sufficient conditions for a local physical theory to be locally causal: a theory will be locally causal if local primitive causality holds and the local von Neumann algebras are atomic (Proposition 1). Then we compared Bell's local causality with the various Common Cause Principles, overviewed the main concepts and theorems concerning Bell's inequalities in a local physical theory, and briefly outlined the EPR scenario. We found a nice parallelism here: Bell's inequalities cannot be derived either from local causality or from a common cause unless the local physical theory is classical or the common cause is commuting, respectively (Proposition 2). Finally, we explicitly constructed a simple local quantum theory implementing the EPR scenario that was locally causal and provided a common causal explanation for the EPR correlations.

All our results point in a common direction, namely towards noncommutativity. It turned out that the violation of Bell's inequalities does not exclude either a locally causal local physical theory or a local physical theory providing a common causal explanation for the correlations. Therefore Bell's inequalities in the nonclassical case do *not* play the same role as in the classical one. In the classical case there was a direct logical link between the local causality/common causal explanation and the validity

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of Bell's inequalities. Thus the violation of Bell's inequalities excludes this and only this subset of LPTs as locally causal models/possible common causal explanations. To put it differently, taking seriously the noncommutative type of description where events are represented by not necessarily commuting projections, one can provide a locally causal account/common causal explanation in a much wider range than simply sticking to commutative events.

However, it is one thing to construct a noncommutative locally causal *mathematical* model for the EPR and another thing to provide a reasonable *physical* interpretation for the model. What does it mean, for example, that a certain correlation has a noncommuting common cause? If the common cause $\{C_k\}$ is noncommuting, then the theorem of total probability $\phi(X) = \phi_{\{C_k\}}(X)$ does not hold for $X = A$, B, and AB. This fact is a straight consequence of noncommutativity: specifying a state on a maximal commutative subalgebra does not specify it on the full noncommutative algebra. The violation of the theorem of total probability means that the original probability of the correlated events cannot be reconstructed from the probabilities conditioned on the common cause in the sense of

$$\phi(X) = \sum_k \phi_{C_k}(X)\phi(C_k)$$

Thus the common causal explanation cannot be regarded as the description of the *same* physical situation at a finer level which is the traditional understanding of the common causal explanation.

Similarly, in a locally causal LQT the atomic partitions in a shielder-off region screen off the correlation, but if the partition does not commute with the correlated events then the probability of these latter cannot be reconstructed from those of the former. Thus the screener-off partitions cannot be regarded as a more detailed description of the correlations in question.

One can react to this fact in different ways. One reaction is to say that the violation of the theorem of total probability completely ruins the program of going noncommutative and so to preserve local causality and the common causal explanation [see Cavalcanti and Lal (2014)]. Another reaction would be, however, the following. Observe that the definition of the common cause does *not* contain the requirement (which our classically informed intuition would dictate) that the conditional probabilities, when added up, should give back the unconditional probabilities. To put it in a more formal way: the theorem of total probability is *not* part of the definition of the common cause. The defining property of the common cause is simply the *screening-off*. That is, common causes might not be measured without the distortion of the statistics of the original correlated events. But this fact is ubiquitous for noncommuting observables in quantum mechanics. If we tolerate this fact in general, then why not tolerate it for common causes? As we have seen, allowing noncommuting common causes can help us maintain Bell's original intuition concerning local causality.

An analogy might help here. Reichenbach's original definition of the common cause was somewhat different from the one used in this book. It contained the screener-off conditions but also contained some extra requirements. It seems that the prime reason for Reichenbach to add these extra requirements [the so-called *positive statistical relevancy* condition, see Reichenbach (1956, p. 159)] to the screener-off conditions was that these conditions together logically implied the correlation. In this sense the Reichenbachian common cause provided a Hempelian explanation for the correlation: if the common cause were present, the correlation logically followed. However, as time passed it turned out that these extra conditions are just unnecessary; they do not form part of the intuition of what a common cause is. Therefore these extra conditions have been completely dropped from the definition of the common cause in the literature. [For an attempt to define the notion of the common cause such that it preserves this deductive relation see Hofer-Szabó and Rédei (2004, 2006).]

We do not claim that abandoning commutativity in the definition of the common cause or local causality is just as unproblematic as abandoning positive statistical relevancy in Reichenbach's definition of the common cause. But we do claim that embracing noncommuting events substantially extends our explanatory sources in accounting for correlations in a locally causal manner. We are fully aware that adopting noncommuting beables in our ontology is a high price to pay for preserving local causality. This book is an invitation for philosophers to explore the prospects of such a noncommutative ontology.

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