Panos M. Pardalos · Victor Zamaraev
Editors

Social Networks and the Economics of Sports

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Panos M. Pardalos
Department of Industrial and Systems Engineering
University of Florida
Gainesville, FL
USA

Victor Zamaraev
Laboratory of Algorithms and Technologies
National Research University Higher School of Economics
Nizhny Novgorod
Russia

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I do not question whether I am happy or unhappy. Yet there is one thing that I keep gladly in mind—that in the great addition (their addition that I abhor) that has so many numbers, I am not one of the many units there. In the final sum I have not been calculated. And this joy suffices me.

Constantine P. Cavafy (1897)

Preface

Sports has been an integral part of human culture since ancient times and plays a key role in the economy, politics, and lifestyle of any country. Today the sports industry is complex and impacts several economic markets such as television, advertising, clothing, and manufacturing. During the 1936 Summer Olympics in Berlin, the world's first televised sporting event took place. A few years later in 1939 a college baseball game was the first televised sporting event in USA. Since then, the social media has played a complex and important role in sports. Today's Internet with Facebook, Twitter, and all types of social media sites in between make us more connected with other sports people and athletes. New marketing and economic models and tools have been developed based on these social networks and these new developments have a great influence on sports.

In addition, sports is characterized by a unique need for competitive balance. As early as 1964, the economist Walter Neale stated the 'Louis-Schmeling paradox' in that better profits could be made from a better product, which in boxing, meant two strong fighters. Louis could not have made it without a strong Schmeling. It is clear that in most businesses the ideal market position of a company is a monopoly. But in sports, it is much different. Given the paradox, a pure monopoly would be a disaster. Fans want to see a competitive balance among teams in order to keep their interest (Neale referred to this as 'inverted joint products').

Systems engineering tools can be used to study many issues in sports. For example, social network analysis deals with the interactions between individuals by considering them as nodes of a network, whereas their relations are mapped as network edges. The study of such structures lies at the intersection of different disciplines of research, including economics, sociology, and computer science. In practice, many kinds of networks have been studied, including friendship networks, scientific coauthorship networks, film collaboration networks, disease spreading networks, and urban growth networks.

This volume contains a collection of chapters, primarily based on selected talks at the international conference 'Social Networks and the Economics of Sports' that took place in Moscow, Russia, on May 27–29, 2013, enriched by several additional invited contributions from distinguished researchers around the world.

The topics covered by the chapters include:

- adaptive systems in sports;
- analysis of the Portuguese success in sport: comparing football with all the other Olympic sports;
- identification of the main trends in the senior sport tourism development in Russia;
- methods for valuation of football club;
- sports performance evaluation;
- methods of forming teams for club golf competitions;
- measuring the true ability of a team.

We take this opportunity to thank National Research University Higher School of Economics for organizing the great conference in Moscow; the authors of the papers for their dedication to this project that resulted in this excellent collection; the anonymous referees for their timely and constructive reports; and Springer management and staff for the support and technical assistance.

Gainesville, FL, USA — Panos M. Pardalos
Nizhny Novgorod, Russia — Victor Zamaraev

Contents

Contributors

Arnold Baca Department of Biomechanics, Kinesiology and Applied Computer Science, ZSU, University of Vienna, Vienna, Austria

Sergiy Butenko Department of Industrial and Systems Engineering, Texas A&M University, College Station, TX, USA

Ekaterina Buzulukova National Research University Higher School of Economics, Moscow, Russia

Taylor Corcoran The University of Arizona, Tucson, AZ, USA

Luis M. Doncel Universidad Rey Juan Carlos, Madrid, Spain

Jennifer Gossels Princeton University, Princeton, NJ, USA

Donald Hearn Department of Industrial and Systems Engineering, University of Florida, Gainesville, FL, USA

Minsoo Kang Department of Health and Human Performance, Middle Tennessee State University, Murfreesboro, TN, USA

Victor Luo Williams College, Williamstown, MA, USA

Steven J. Miller Williams College, Williamstown, MA, USA

F. Javier Otamendi Universidad Rey Juan Carlos, Madrid, Spain

Panos M. Pardalos Department of Industrial and Systems Engineering, Universidad of Florida, Gainesville, FL, USA

Jae-Hyeon Park Center for Sports and Performance Analysis, Korea National Sport University, Seoul, South Korea

Konstantin Pavlikov Department of Industrial and Systems Engineering, University of Florida, Gainesville, FL, USA

Jaclyn Porfilio Williams College, Williamstown, MA, USA

Marina Sheresheva Lomonosov Moscow State University, Moscow, Russia

Ilia Solntsev Finance University Under the Government of Russian Federation, Moscow, Russia; Research in Sports, Plekhanov Russian University of Economics, Moscow, Russia

Eduardo Tomé Universidade Europeia, Lisbon, Portugal

Stan Uryasev Department of Industrial and Systems Engineering, University of Florida, Gainesville, FL, USA

Justin Yates Department of Industrial and Systems Engineering, Texas A&M University, College Station, TX, USA

Victor Zamaraev Laboratory of Algorithms and Technologies, National Research University Higher School of Economics, Nizhny Novgorod, Russia

The Impact of Social Networks on Sports

Panos M. Pardalos and Victor Zamaraev

Abstract Social network analysis deals with the interactions between individuals by considering them as nodes of a network whereas their relations are mapped as network edges. The study of such structures lies on the intersection of different disciplines of research, including economics, sociology and graph theory. In the literature many kinds of networks have been studied, including friendship networks, scientific co-authorship networks, film collaboration networks, disease spreading networks, and urban growth networks. In this short introductory review, we focus on the social networks arising in sports. We discuss a structure of these networks as well as possible questions related to the dynamics of sports networks, which can be useful in management, economics and marketing of sports.

Keywords Social networks · Sport · NBA graph · Soccer network

1 Introduction

Many practical applications generate massive amounts of data that must be efficiently analyzed and visualized. Quite often representing such data as a large network (graph) is useful for the mentioned purposes. A graph is a set of vertices or nodes and a set of edges or links, which connect pairs of nodes. Representing a dataset as a graph means associating a certain data attributes with vertices and edges of the corresponding graph. Network analysis, applied to such a models, often provides a deeper understanding of internal structure as well as patterns of the data [1]. Examples of graph representations of real-life datasets include the Web graph that models the

P.M. Pardalos
Department of Industrial and Systems Engineering, Universidad of Florida,
Gainesville, FL, USA
e-mail: pardalos@ufl.edu; pardalos@ise.ufl.edu

V. Zamaraev (✉)
Laboratory of Algorithms and Technologies, National Research University
Higher School of Economics, Nizhny Novgorod, Russia
e-mail: vzamaraev@hse.ru

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World Wide Web [2], the call graph that represents subscriber interactions within a telecommunication network [3–5], and the market graph that reflects the structure of a stock market [6].

In social networks vertices correspond to people and edges reflect a particular kind of relationship between pairs of people [4, 7–9]. Many real-life social networks are small-world networks, that is, graphs in which most vertices are not connected to each other, but most vertices can be reached from every other vertex by a small number of steps [10]. One of the most well studied social networks is the collaboration graph of mathematicians. In this graph, mathematicians are represented by nodes, and two nodes are connected by an edge whenever the corresponding mathematicians have a joint paper. A well-known metric associated with this graph is the so-called Erdős number, which is assigned to every vertex and characterizes the distance from a given vertex to the "central" vertex corresponding to the famous graph theoretician Paul Erdős. For this reason, the collaboration graph of mathematicians is also referred to as the Erdős collaboration graph [11]. Another famous social network is the collaboration graph of movie actors, also known as the Hollywood graph, in which two film actors are linked up if they appeared in the same movie. The Bacon number is the metric related to the Hollywood graph and defined similarly to Erdős number, but the "central" vertex corresponds to the notable actor Kevin Bacon.

In this review, we survey several social networks arising in sports that reflect different relationships involving players, and discuss a structure of these networks. In Sect. 2, we provide a few basic definitions from the graph theory and network analysis, which are used in the following discussion. Section 3 provides detailed examples of sports networks, including various properties of these networks. Section 4 concludes the discussion with several questions related to the dynamics of sports networks, which can be useful in management, economics and marketing of sports.

2 Basic Notations and Terminology

Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E. The graph $G = (V, E)$ is called connected if, for every pair of distinct vertices u and v, G contains a path from u to v. The degree of a vertex is the number of edges originating from it. A clique in a graph is a set of pairwise connected vertices (see Fig. 1).

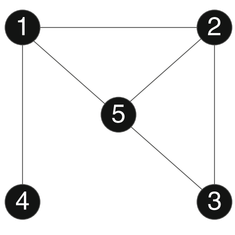

Fig. 1 Connected graph with 5 vertices and 6 edges. The degree of the fifth vertex is 3. The vertices 1, 2, 5 form a clique

Next we define several topological metrics, which are considered relevant in the network analysis [12].

1. The edge density is the ratio of the number of edges and the maximum possible number of edges. It is easy to verify that the total number of possible edges in an n-vertex graph is $n(n-1)/2$. A graph with relatively small edge density is called sparse.
2. The distance between a pair of vertices is the length of the shortest path between the vertices.
3. The diameter of the graph is the maximum distance between any pair of vertices.
4. The clustering coefficient of a network is a measure of degree to which graph nodes tend to cluster together. It is defined as the average of the clustering coefficient for each vertex. In turn the clustering coefficient for a vertex is the ratio of edges between the vertices within its neighborhood and the number of edges that could possibly exist between the vertices. Loosely speaking, clustering coefficient of a vertex reflects how close its neighbors are to being a clique. Networks with high clustering coefficient are called clustered.
5. The assortativity coefficient r $(-1 \leq r \leq 1)$ quantifies the correlation of degree between pairs of connected vertices. Networks with $r < 0$ are disassortative, which means that the vertices connect to other vertices with various degree. In assortative networks (networks with $r > 0$) the vertices are more likely to connect to vertices with similar degree [13].
6. The closeness of a vertex is the average distance to the other vertices in the network. Closeness can be regarded as a measure of how long it will take information to spread from a given vertex to other reachable vertices in the network. The closeness of the vertex is a measure of its centrality—relative importance of a vertex within the network. The vertex with the lowest closeness is called the most central node [14].

In terms of these notions, it can be stated that many social networks are sparse, clustered and have a small diameter. The last two properties are the main features of small-world networks.

3 Social Networks in Sports

Social networks arise naturally in sports, especially team sports. For example such networks may be based on the following relationships: athlete-athlete, athlete-partner, coach-athlete, athlete-clubs, etc. In addition to a single player analysis, network analysis of sports networks might be used to investigate patterns of social relations between team members, as well as to explore a behavioral dynamics of groups of teammates. This information may help coaches, players, managers and other club members in making decisions. In this section, we review several particular sports networks and discuss their topological metrics.

3.1 NBA Graph

In [15] the authors considered the NBA from the perspective of social networks. They had constructed the NBA graph and studied its structural properties. It turned out that properties of this graph are quite similar to the properties of other social networks. The vertices of the NBA graph represent all the basketball players who were playing in the NBA during the 2002/2003 season. An edge joins two given players if they ever played in the same team. The constructed NBA graph has 404 nodes and 5,492 edges between them. Thus, the edge density of this graph is rather small: 5,492/81,406 = 6.75 %.

Based on the definition, one can mention some obvious structural properties of the NBA graph. For example, the players from the same team form a clique in the graph. Also, since many players change teams, there exist links between players (vertices) from different teams (cliques). Clearly, the same structure is inherent for all collaboration networks (see Fig. 2).

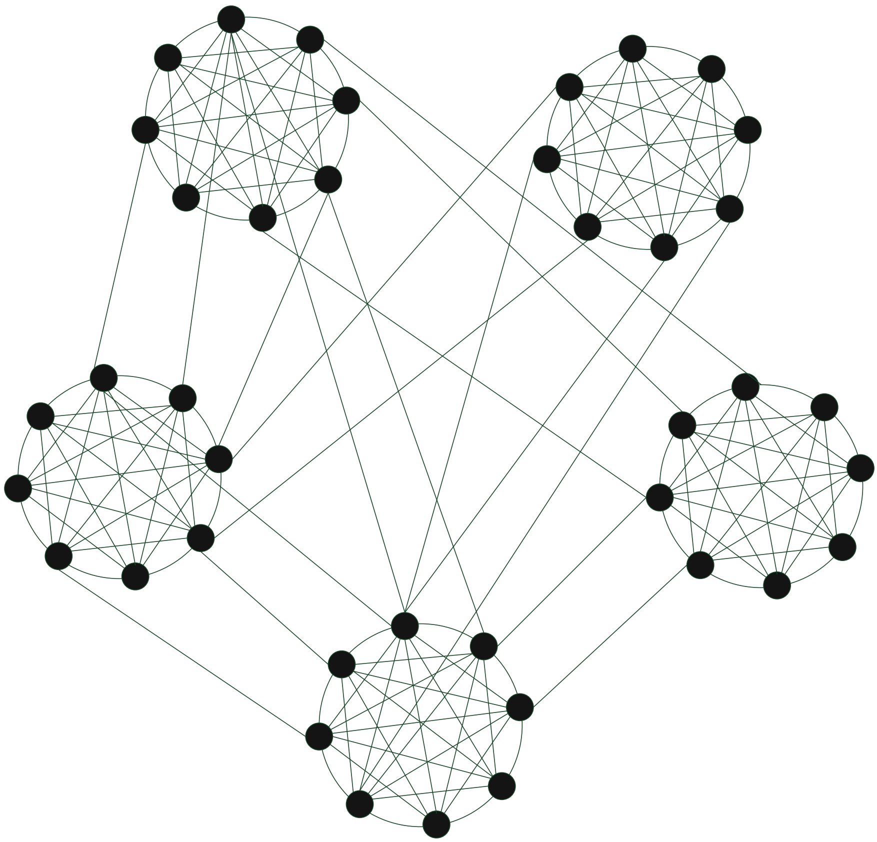

Fig. 2 A general structure of the NBA graph and other collaboration networks

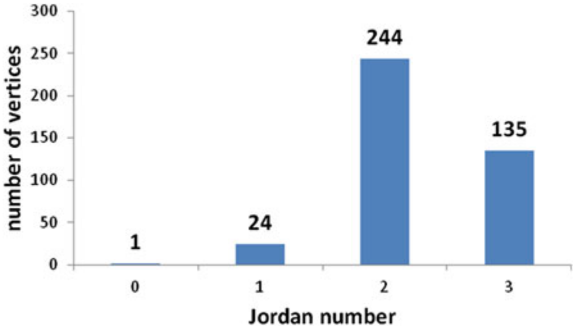

Fig. 3 Number of vertices in the NBA graph with different values of Jordan number. Average Jordan number = 2.27

The considered NBA graph is connected, due to the facts that the number of players in a basketball team is relatively small, and the players' transfers between different teams occur frequently. Furthermore, in [15] it was observed that this graph has a 'small-world' topology. In particular, the calculations showed that the maximum distance between all pairs of vertices (graph diameter) in the NBA graph is equal to 4, implying that the NBA is a very small world.

Analogously to the Erdős number for the collaboration graph of mathematicians and the Bacon numbers for the Hollywood graph, the Jordan number was introduced for the NBA graph [15]. The Jordan number of a player is defined as the distance in the NBA graph between the vertex corresponding to the player and the vertex corresponding to the outstanding basketball player Michael Jordan. Despite the fact that Michael Jordan played only for two teams through his career, and thus had relatively few "collaborators", the most players (268) in the studied NBA graph have Jordan number 1 or 2; the maximum Jordan number is only 3 (see Fig. 3). It means that all players are connected with Jordan through at most two vertices, which again confirms that NBA is a 'small world'.

3.2 Brazilian Soccer Players Network

In [16], the network analysis was applied to the Brazilian soccer championship. The authors found many interesting and unexpected results. Based on the data gathered from 32 seasons of the Brazilian soccer championship, two networks were built. The first one is the bipartite network: the first part consisting of 127 clubs (teams), and the second part is formed by 13,411 soccer players who played for those clubs between the years 1971 and 2002. A soccer player is linked up with a certain club whenever the club employed the player. It was observed that the degree distribution of vertices from the second part (i.e. the distribution of the number of clubs in which a player has ever worked) exhibits an exponential decay. For example, it is roughly 190 times more likely that an arbitrary player was a member of only two clubs than of eight clubs. Moreover, it was shown that the probability that a Brazilian soccer player has scored a certain number of goals obeys a power law.

The second considered network is the Brazilian Soccer Players (BSP) network composed of soccer players only. Two players are connected by an edge if and only if they were at the same team at the same time. The BSP network has 13,411 nodes and 315,566 links, i.e. it is very sparse network with edge density equal to 0.0035. It was also found out that the BSP network possesses the main small-world properties: high clustering (the clustering coefficient is 0.79) and low degree of separation (the average distance between players is 3.29). By investigating the time evolution of the BSP network, the authors of [16] conjectured that players' professional life is getting longer, and/or players' transfer rate between teams is increasing. It was observed that the clustering coefficient is a time decreasing function. This may reflect the exodus of the best Brazilian players to foreigner teams (which has increased, especially, in the last decades). Also the BSP network is becoming more assortative with time. This seems to indicate the existence of a growing segregationist pattern, where player transfers occur, preferentially, between teams of the same size.

3.3 Dutch Soccer Team Network

Inspired by the study of the Brazilian Soccer Players network [16], the authors of [17] studied the topological properties of the Dutch Soccer Team (DST) network. In the DST network every vertex represents a player that has played an official match for the Dutch Soccer Team. Two nodes are connected up if the corresponding players have appeared in the same match. One of the reasons to study DST network was to provide useful information for the coach of the DST, which, for instance, could be used for determining an optimal line-up in terms of defined aspects of the team. For example, a team could be assembled from as many players who have already played together as possible.

The authors considered 670 matches of the DST from the first match with Belgium (30 April 1905) up to the match with Russia (21 June 2008). The constructed network consists of 691 players and 10,450 links. It is easy to see that the edge density of the network is small—0.044. In addition, there exist a path between every pair of players in the network, i.e., the DST network is connected. Although the diameter of the network is 11, the average distance between players in the network is rather small (4.49), and the clustering coefficient is high (0.75). All this shows that the network under consideration is similar to many other social networks, in the sense that it exhibits a small world phenomenon.

The most central player (i.e. with the minimum (3.119) average distance to the other players) in the DST network is Roel Wiersma, who was active from 1954 to 1962 and played 53 games. It is quite predictable that the most central players (i.e. with the lowest values of the closeness) were active about half a century ago, because the DST has a history of about 100 years.

The evolution of the topological metrics of the DST network was investigated in [17]. The study showed that, while the number of vertices and edges increase over time, the edge density is decreasing in a nonlinear fashion and the clustering

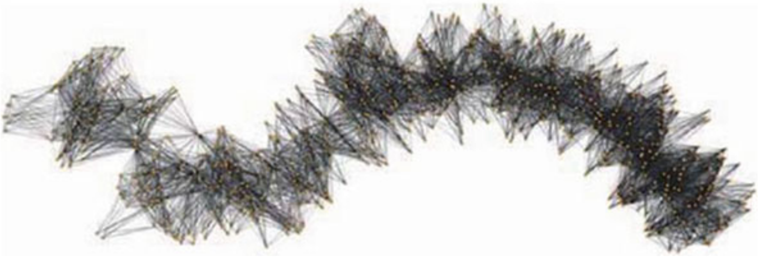

Fig. 4 Visualization of the DST network [17]

coefficient remains almost constant. Other topological metrics of the DST network, such as the average degree, the average distance and the diameter increase over time.

Finally, it is worth pointing out that authors of [17] compared the topological metrics of the DST network with those of other network models, such as the random graph of Erdős-Rényi [18], the small-world graph of Watts-Strogatz [10], and the scale-free graph of Barabási-Albert [19]. The main conclusion was that the DST network most resembles a Watts-Strogatz random graph, i.e. a graph with small-world properties.

4 Conclusions

In this review, we have surveyed several social collaboration networks in sports and discussed their structure. All such networks, which consist of players only proved to be small worlds. One might assume that networks which can be constructed in the same way for other team games or leagues have the similar structure.

There are many interesting questions related to the behavioral dynamics of sports networks. For example:

1. Using the existing patterns of connections in a sports network and a variety of graph-theoretic and statistical techniques, how can one predict a new relationships that will form in the network in the near future?
2. What are the dynamics of the information flow in a sports network? How can one extract knowledge from this information?
3. How can the future of a sports network be predicted from the current state of the network?

Answers to these questions may have implications in management, marketing, advertising strategies, ticket sales, safety in large sports events, etc.

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Application of Income Approach for Valuation of Football Club

Ilia Solntsev

Abstract The chapter treats practical problems related to the valuation of the football clubs. These problems are addressed using existing DCF model as part of income approach. The article describes in detail revenue mix and expenditure pattern of football clubs and the ways to forecast future cash flows. Besides it considers the methods of calculating the discount rate for valuation of FC and risks associated with its activities. Examples of different business models and statistics from European football market are provided.

Keywords Sport economy · Sport finance · Football economy · Valuation of football club · Discount rate for valuation of football club

1 Introduction

Football is one of the most popular sports in the World. The 2013 UEFA Champions League final was aired in more than 200 countries to an estimated global average audience of 150 million and a projected global unique reach of over 360 million viewers [3]. According to Deloitte [2], total revenue of the top football clubs in 2012/2013 reached €5.4 billion, a 8 % increase on the previous season and about 450 % on the season 1996/1997. In 1991/1992 the 22 clubs of the then Football League First Division had collective revenue of £170 m—in 2011/2012 the revenue of the 20 Premier League clubs was almost 14 times greater at over £2.3 billion, while the European football market grew to €19.4 billion [13]. Only brands of 50 football clubs are worth more than €10 billion [1].

Forbes annually evaluate top soccer teams, and in 2014 [8] "Real" Madrid, was worth $3.4 billion, more than any team in the world. "Bercelona", ranked second with a $3.2 valuation, and "Manchester United" third with $2.8. The top 20 teams were worth an average of $968 million, an increase of 26 % over 2012.

I. Solntsev (✉)
Finance University Under the Government of Russian Federation, Moscow, Russia

Research in sports, Plekhanov Russian University of Economics, Moscow, Russia
e-mail: Ilia.Solntsev@gmail.com

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Along with development of sports industry as independent economy sector, investments in its certain branches are growing: we see more businessmen investing in football clubs (FC) and consider it not just as a hobby or a social burden, but as a true business, more FC participate in M&A deals, finally, FC are returning to debt and equity capital markets, and long expected IPO of Manchester United became proof of that. Besides in the past few years there has been tremendous progress in football fans' interest for their clubs' finances. In this regard we face a real need for a modern valuation model, which will consider the specifics of the FC as a business.

2 Income Approach: General Description

If we agree, that football could be considered as business, we can use for valuation of FC three standard approaches: income, market and asset. But every economy sector has particular properties, which influence the valuation process. It means that we cannot use the same methods for valuation of oil and transport companies. As well football requires to develop a unique model and to upgrade all three approaches.

Income approach is based on the assumption that the value of the business is equal to the sum of the present values of the expected future benefits of owning this company. In accordance with Discounted Cash Flow (DCF) method, which is used within income approach, investor would not pay for business more than the present value of expected future benefits of owning that company. Similarly owner of the business would not sell his company for price which is lower than the sum of present values of future cash flows. DCF model is most appropriate for investors, who plan to buy not a package of assets (buildings, land plots, equipment, etc.), but primarily a business or stream of future cash flows, which will allow him to return his money and earn a profit.

Transferring all mentioned above to football, we understand, that we can use DCF model only for big and popular FC with positive cash flow or for those, who has a chance to join them in nearest future. For example we can find such clubs in Deloitte Football Monet League ranking (Fig. 1). But generally it is still easier to find clubs with weak business and financial performance.

If we take Russian football, even biggest FC are showing very weak results (Fig. 2). Besides it should be noted, that these 4 clubs are the only ones, who disclose their figures. They provide the statement in accordance with Russian accounting standards (RAS) and do not comment on the structure of revenue and expenses. Huge amounts are accounted as "other income" and it is unclear, what stands behind. But it is commonly known, that most all Russian FC remain afloat due to their shareholders and local authorities. The other problem is hidden transfer costs, which are not fully and properly booked. So we cannot fully rely on these statements and particularly make any conclusions or forecasts. At the same time if we find a positive dynamics in sports track record, strong management team, infrastructure base and support from fans we can try to build up a positive future cash flow. Furthermore any investor needs to see all income items, which he could earn with this particular asset. So,

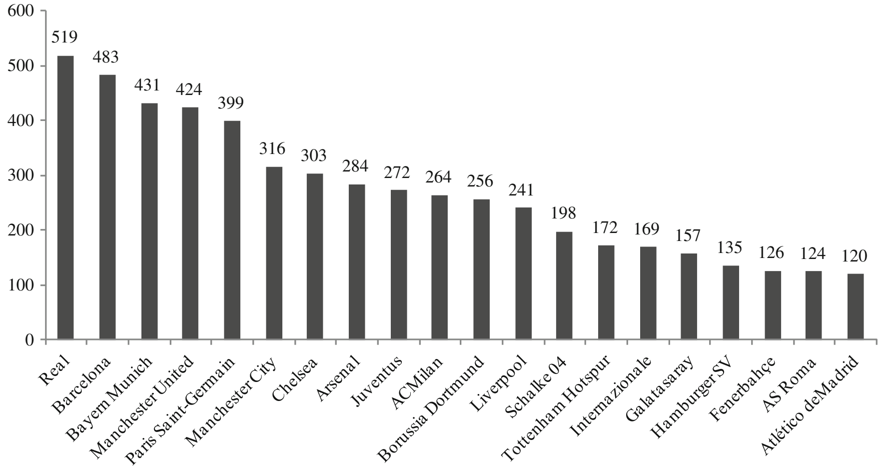
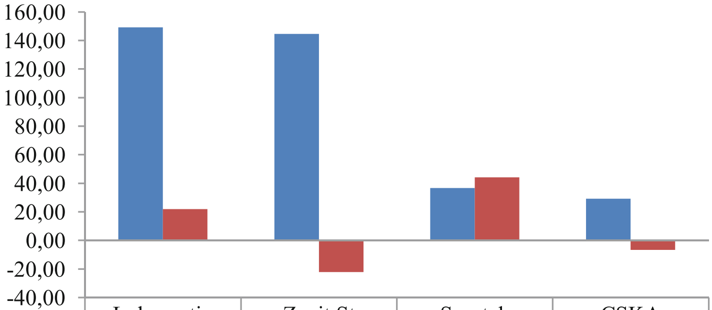


Fig. 1 Europe FC total revenues, season 2012/2013(€m). Deloitte



	Lokomotiv Moscow	Zenit St Petersburg	Spartak Moscow	CSKA Moscow
Revenue	149,22	144,65	36,63	29,18
Profit/Loss	22,00	-22,18	44,22	-6,60

Fig. 2 Financial results of major Russian FC, 2012, m$. SPARK INTERFAX (1$ = 31.6 RUR)

negative cash flow does not mean that we should not use income approach. Certainly it’s worthless to discount negative figures, but the first thing to do is to analyze the structure of revenues and expenses.

Revenue of FC consists of following items:

1. matchday revenue—selling tickets (including seasonary), food and drinks and merchandise;
2. broadcasting rights;
3. commercial revenue—sponsorship contracts and selling club's trademark products;
4. selling players;
5. participation in Champions League and League Europe;
6. stadium revenues: lease out playing field to other teams; arrange exhibitions, shows, concerts; income from commercial property on/near the stadium (offices, retail, hotel); selling naming rights;
7. media—club's TV-channel, radio station, newspaper, web site;
8. revenues from non-operating business based on FC brand. For example "Real" Madrid in March 2012 announced plans to build the resort off the coast of the United Arab Emirates, which will be located in the sea on an artificial island on the area of 430,000 m^2. It is planned that this island will become a football theme entertainment center, including a stadium for 10,000 spectators with sea view, two luxury hotels, villas, which will be available to own or rent, yacht marina and team history museum. Even more extraordinary project implements the German club "Schalke", launched the official club cemetery Beckenhausen-Sutum near the "Veltins-Arena" with 1,904 graves—similar to the founding year of the team. Cemetery will be designed like a stadium, interior elements will include "Schalke" emblem, gate and substitutes' bench. According to the newspaper The Kölner Stadt-Anzeiger, one plot on the club cemetery will cost 1,250 euros and annual service—125 euros.

Now we describe briefly all these items.

3 Revenue Structure of FC

3.1 Matchday

Much of matchday revenue comes from selling tickets, and the bulk club receives form premium seats, VIP- and sky boxes. In other words, the most affluent fans help the club to earn in foremost. However, sales of all categories of tickets are directly dependent on two factors: the popularity of the team and stadium capacity, which eventually form such an indicator as attendance.

Besides income from ticket sales will depend on sales channels: the stadium ticket office, ticket offices in the city, the club chain stores and internet sites. Another option to increase attendances—a system of discounts and loyalty programs for fans: discounts on attributes and prize draws for season ticket holders, personalized seat in the stadium, preferences for children, etc. Also one of the factors affecting the

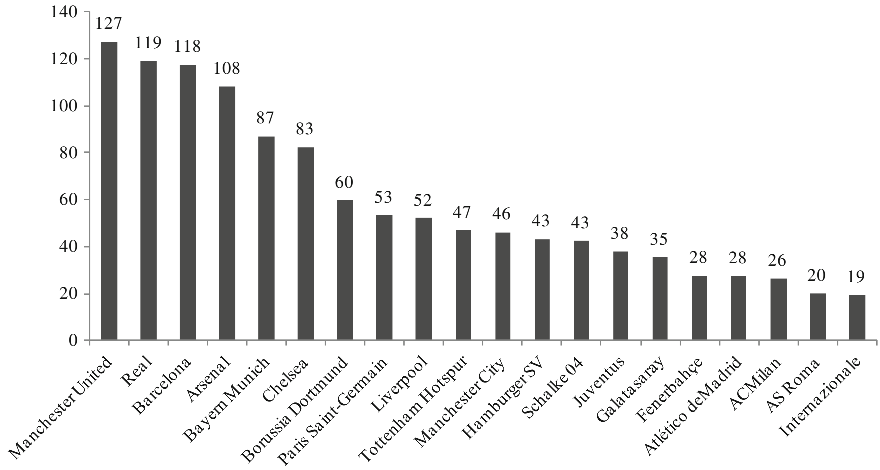


Fig. 3 Matchday revenue, season 2012/2013 (€m). Deloitte

attendance is the age of stadium—usually newly built arena filled much better than old ones.

Further the club needs to decide how many season tickets should be sold and how much—leaved for free sale. This decision will influence not only income from ticket sales. The fact is that the holders of season tickets are not very promising, as buyers of clubs' products. Typically, at the beginning of the season they get a t-shirt, and no longer make any purchases. While buyers of single tickets, in this sense, are more attractive (for example, tourists, who often buy something to remember).

People also use to buy some food and drinks, but these revenues depend on whether the team is playing on its own stadium or rents it, and on the organization of relationships with companies that provide catering services. Most of the clubs receives a percentage of the proceeds from the sale of food and beverages. As for the merchandise, the presence of fan-shop near the arena significantly helps to increase its sales, as well as several of the official points of sale in the city. For example, at the airport in Dortmund you can find even a vending machine with approximately 20 items of products from a range of "Borussia" fan-shop: T-shirts, cups and various souvenirs. Ranking of the leading clubs in Europe in terms of matchday revenue is shown in Fig. 3.

3.2 Broadcasting Revenues

In recent years revenues from the sale of television broadcasting rights have a strong upward trend that has affected even the Russian market (see Table 1). The latest contract for the right to broadcast matches of the Russian Premier League till 2015,

Table 1 Revenues from the sale of television broadcasting rights (*The Swiss Ramble*, www.sports.ru)

Country	Term, years	Total Amount (€m)[a]	Amount per year (€m)		Broadcaster
			Local	*Abroad*	
England	3 (2016)	6,499	1,333	834	SKY, BT
Italy	3 (2015)	3,000	829	171	Sky Italia, RTI
Germany	5 (2017)	2,800	628	72	Sky Deutschland
Spain		655	500	155	Sogecable, Mediapro
France	4 (2017)	2,632	610	32[b]	Al-Jazeera
Turkey	4 (2014)	1,041	260		Digiturk
Netherlands	12(2025)	1,020	85		Eredivisie Media and Marketing
					(Fox International)
Portugal			48		SportTV
Greece	4 (2015)	168.4	42.1		Nova Sports
Poland	3 (2014)	107.1	35.7		Canal+
Russia	3 (2015)	96	32		NTV+
Romania	3 (2014)	81.9	27.3		RCS-RDS, Romtelecom, Antena 1
Scotland	5 (2017)	100	20		Sky, ESPN
Bulgaria	1 (2012)		4.65		TV7, BNT1

[a] Exchange rates 1\$ = 0.8 €, 1£ = 1.25 €
[b] For 6 years

was signed with "NTV-Plus" company. The contract is amounted to $120 million. The rights for previous season were bought for $60 million, while the previous four—for $92 million, or an average of $23 million per season.

It should be noted that this source of income is largely determined by the system of selling the rights, which could be collectively or individually. The first one intends that the league, uniting all the clubs, concludes a contract with the broadcaster. The entire amount of this contract is divided between the teams in a specific pattern. For example, in England 50 % of the total amount received for the sale of domestic TV-rights equally distributed between the clubs, 25 %—depending on the number of broadcasted matches of the team and the remaining 25 %—depending on the standings at the end of the season. Money received for the sale of broadcast rights overseas, distributed equally.

When television began to cover the game in 1965, the BBC paid just £5000 for highlights, and the money was shared equally among all 92 clubs in the four divisions of English football. By 1988, after moves by the big clubs to keep more of the league's income, 50 per cent of the £44 million 4-year deal with ITV was awarded to the First Division clubs, 25 per cent to the Second Division, and 25 per cent to the Third and Fourth. The Premier League's TV deals have grown hugely: £670 million in

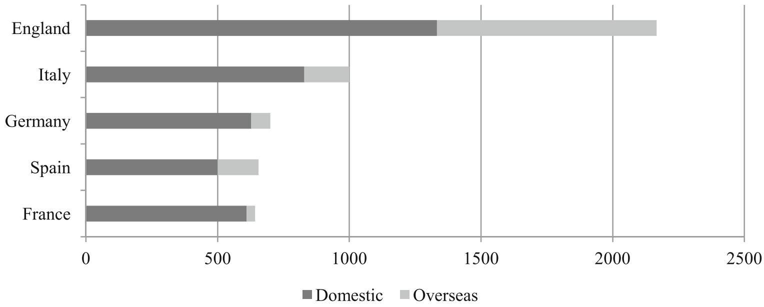


Fig. 4 Structure of broadcasting revenues in "Big Five" countries (€m) (*The Swiss Ramble*)

1997, £1.6 billion in 2001, a dip to £1.1 billion in 2004, £2.4 billion in 2007. The current deal, for 2010–2013, is £3.1 billion, including overseas rights reaping £1.4 billion. In June, in the middle of England's humdrum performance in the European Championships, the Premier League announced it had secured £3 billion from BSkyB and BT (its first entry into the field) for the right to broadcast live matches in the UK in 2013–2016 [6].

Currently the rights to broadcast English Premier League are most expensive in Europe (see Fig. 4). The lowest placed Premier League club now earns around £40 million from television alone, while clubs in the Championship earn around £2 million.

Thus, under the collective scheme all clubs receive a part from the total broadcast revenue, and given amount depends on the success in the championship. However, the most successful teams still believe that they provide primary television audience, and therefore should receive more than their less popular rivals, who, on the contrary, insist that most of the matches are played with them, and want to have just as much rights to broadcast revenue. These disputes continue permanently, but most European leagues accept collective system. And only in Spain and Portugal, this contradiction is not able to resolve, clubs sell the rights to broadcast their games on their own (for example, "Barcelona" and "Real" only for season 2011/2012 earned €140 million each).

Interesting scheme is incorporated in Italy:

1. 15 % are allocated based on the results of the team for past 5 years;
2. 10 %—based on the performances of clubs since 1946;
3. 5 %—according to the place in last championship;
4. 40 % is divided equally among all teams of Serie A;
5. 30 %—on the basis of such indicators as the population of the commune, represented by the club (5 %) and the number of fans (25 %).

It is assumed that in season 2013/2014 the league will get €17 million more, and in a year—€24 million. The money will be distributed among those teams that will take the first 10 places. Fifteen percentage—to top three teams, 10 %—to fourth and

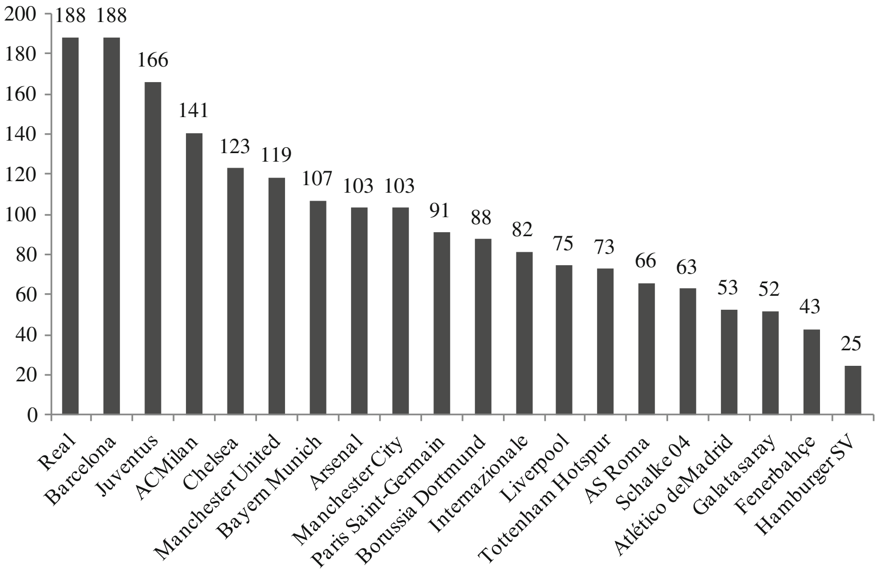


Fig. 5 Broadcast revenue, season 2012/2013 (€m) (*Deloitte*)

fifth place, the rest will get even smaller—up to 5 % to the 10th team. Three clubs that drop out the Serie A will get €30 million.

Ranking of the leading clubs in Europe in terms of revenue from the sale of television broadcasting rights is presented in Fig. 5.

3.3 Sponsorship

FC also entitled to sponsorship contracts in the following areas:

1. provider of equipment (kit supplier);
2. the main sponsor with the right to put its brand-name on T-shirts;
3. partners of the club (sometimes—with the right to put its brand-name on some part of the T-shirt (e.g., sleeves));
4. the right to put a trade name on training kit;
5. sale of stadium naming rights.

Competition for the first item takes place between the major brands of sports equipment: Adidas, Nike, Puma, Umbro. According to Sport + Markt (European Football Kit Supplier Report, 2012), in the “Big Five” countries Adidas, Puma and Nike are on the lead (Fig. 6). In season 2012/2013 of Russian Premier League Adidas was on the lead with five contracts among 16 teams.

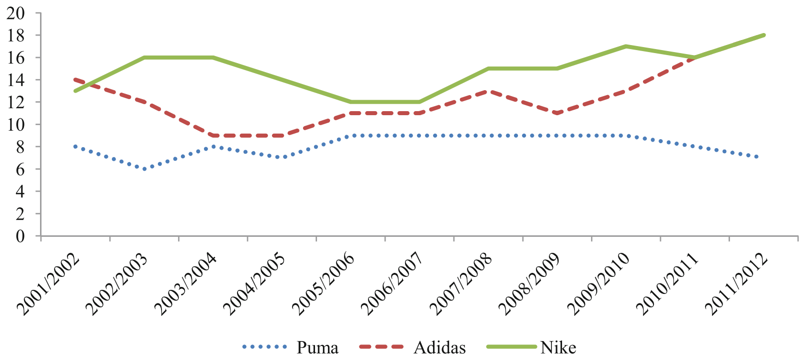


Fig. 6 Number of contracts with manufacturers of sports equipment in the "Big Five" (*Sport+Markt*)

The two significant US brands—Warrior and Under Armour have fuelled an upward trend in annual payments that suppliers are willing to be aligned with such an irresistible platform. We have seen Arsenal, Manchester City and Lazio leave long-term supplier relationships to enter more lucrative shirt deals. Warrior sports entered the market in 2012 with a deal to supply Liverpool's kits over a 6 year period. Warrior, owned by New Balance, is better known in the US market for providing lacrosse and hockey apparel. Liverpool brokered the deal over a 12 month period, speaking with all the major kits suppliers before settling on the value and exclusivity that the Warrior deal would provide. the deal was worth a reported £300 m, a record braking sum.

Under Armour is another US brand just beginning to capture UK market share, using its deal with Tottenham Hotspur as a market entry tool to tap into the football market and continue its rapid revenue rise. The company is particularly well known for its pioneering research and technological innovations and is at the forefront of the current trend for football shirts to be treated on a par with boots as serious pieces of technical apparel [1].

For 50 Football Clubs, covered by Brand Finance, this picture looks as following (Fig. 7).

It should be noted that initially the only thing clubs could expect were a few sets of kit and boots. But now the manufacturers of equipment are competing for the right to become the official supplier of the leading clubs and willing to pay for that huge money. Thus, in 2002, Nike agreed to pay £303 million and give a share of retail sales to Manchester United. Their 13-year agreement comes to an end in 2015. In May 2013 Arsenal reached an agreement with Puma for £30 m a year contract over 5 years from the end of 2013/2014. The 5-year, £170 million deal ends the Gunners' 20 year alliance with Nike. Table 2 gives an overview of recent kit sponsorship deals with European "Big 5" football federations.

Clubs can also get some serious money for the sale of the right to put a sponsor trade name on the main kit.

Table 2 Value of recent european kit sponsorship deals (*Jefferies*)

Team	Sponsor	Year	Length (years)	Annual value (£m)
France National	Nike	2011	7.5	33.5
England National	Umbro	2010	8	26.7
Spain National	Adidas	2011	7	21
Germany National	Adidas	2008	10	15.7
Italy National	Puma	2012	6	14.5

According to the World Sponsorship Monitor, in 2012 football attracted about $4.5 bn in global sponsorship—significantly more than any other sport and two-thirds more than in 2011 [5].

Back in 2012 Arsenal extended its annual shirt and stadium sponsorship deal worth £30 m annually, while Real Madrid concluded shirt sponsorship contract for €30 m per season. Manchester United, meanwhile, signed the largest shirt sponsorship deal in history—a 7 year deal worth $659 m with General Motors. In April 2013, the club also sold the rights to its training ground, training shirts and overseas tours for almost $30 m per season to Aon, the insurance group. Although the contract will come to force only in season 2014/2015, within the next two seasons "General Motors" will pay $ 18.6 million, and in season 2014/2015 the club will receive $70 million. Every year the size of sponsorship payments will grow by 2.1 %. Besides we should not forget that among other sponsors of Manchester are such companies as DHL, Chevrolet, Singha, Concha y Toro, Thomas Cook, Hublot, Turkish Airlines, Epson, Honda and Smirnoff. In July 2013 Manchester United announced a 5-year partnership with Aeroflot. The Moscow-based international airline has become the club's Official Carrier. As to DHL, there were a quite an interesting story. In 2010 the company acquired for £40 million the right to place their logo on the training kit.

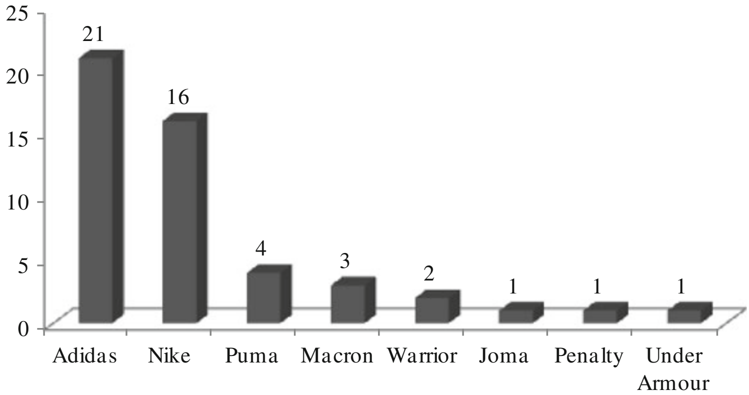


Fig. 7 Number of partnerships with clubs in the Brand Finance Football 50 (*Brand Finance*)

However, after signing an agreement with GM United has bought this contract back from DHL.

Another big contract was signed with "Barcelona"—5 year agreement with Qatar Foundation till the end of season 2015/2016 is worth €170 million. Notably this was the first deal for the Catalan club, which has no commercials on its T-shirts before. On March 2013, Barcelona has confirmed that they will be using the Qatar Airways logo on the front of their shirts, receiving €35.4 million per year.

In Russian practice we can mention a contract between "Rosseti" company and PFC "CSKA". In the period from 2013 to 2018 club will receive 4.185 billion of rubles or approximately $26 million per season. In season 2012/2013 "CSKA" had another sponsor—"Aeroflot". That contract worthed $9 million. Besides "Aeroflot" provided the team with the plane Airbus A320, and painted it red and blue colors of the club.

In 2013 FC "Spartak Moscow" has signed a contract with "Otkrytie" Bank. According to the agreement, the new stadium of the club, which will be put into operation in the September of 2014, will be called "Otkrytie Arena". In 6 years the Bank will pay approximately $40 million.

As Financial Times wrote [5], sports sponsorship kills 120 birds with one rather expensive stone. Unlike typical forms of marketing, sports sponsorship provides extra benefits, whether tickets to a game or a tour of Manchester United's training centre. The access sponsorship provides is one of main motives for the sponsors of the Champions League, which costs between £25 m and £30 m per season. For this, sponsors receive advertising surrounding the pitch and the ground as well as "break bumpers"—the short sting from a sponsor before an advert break begins. The rise of on-demand television means that live sport is one of the few times that advertisers can guarantee eyeballs. Sponsors such as Heineken also get about 1,200 tickets each to dish out to clients, staff and competition winners. About a third of the 86,000 tickets to Champion League final 2013 were dished out to sponsors and guests of UEFA, European football's governing body.

The largest transactions between football clubs, sponsors and manufacturers of equipment are shown in Fig. 8.

3.4 Merchandising

Merchandising revenue depends on the following factors:

- popularity of club and number of fans (including overseas);
- sports results;
- registered rights to identifications (brands, logos);
- sales network (including the opportunity to purchase products via the Internet);
- product range;
- pricing policy;
- title sponsor.

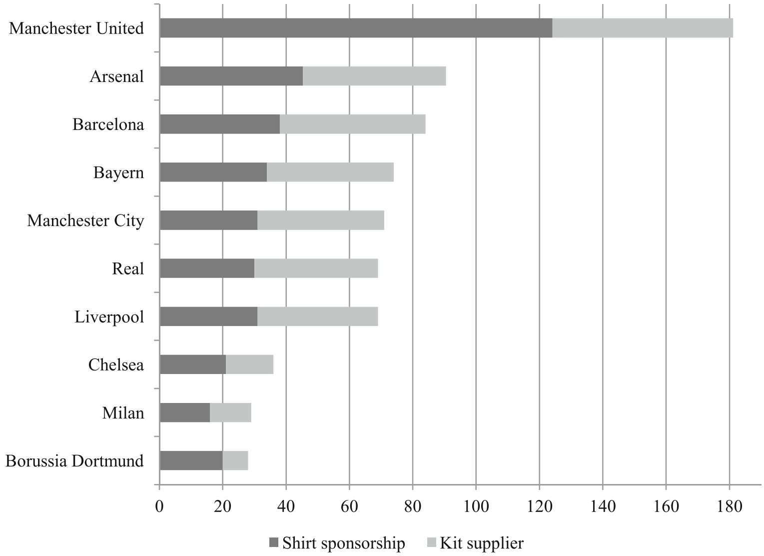


Fig. 8 The largest transactions with sponsors and manufacturers of equipment ($m) (*Brand Finance, The Swiss Ramble, Jefferies, Financial Times*)

The first two factors are closely related, but even for the popular team winning the principal match in major tournament can boost sales, and vice versa. Registered rights to branded products in the first place should help the club fight against counterfeiting, which is especially important for the Russian clubs. Product range and sales channels could be considered as accessory factors and can stimulate the growth of revenue only for those teams that have a strong base of loyal fans. Finally, the pricing depends on the region where the team plays, and income levels. However, in terms of a kit sale the price is determined by the manufacturers. For example, according to Sport + Markt (European Football Kit Supplier Report, 2012), the average price of a playing shirt replica in the "Big Five" is €65 (see Fig. 9).

Merchandising revenue could be affected by the title sponsor of the team. And sometimes this effect could be negative. For example, after signing in August 2012 sponsorship contract with "Wiesenhof" company, specializing in poultry and meat processing, sales of Bremen "Werder" shirts decreased dramatically. Team fans explained this by animal abuse, incriminated to the company. Ranking of the leading clubs in Europe in terms of commercial revenue, which include revenues from sponsors and merchandising, is presented in Fig. 10.

So, we briefly reviewed the main revenue items of Football Club. However, as in any other business, in football there are also some additional opportunities to earn money.

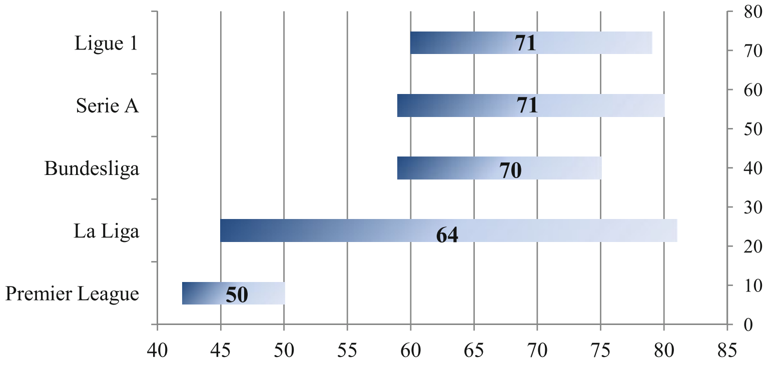


Fig. 9 The average price of a playing shirt replica in "Big Five" countries, € (*Sport+Markt*)

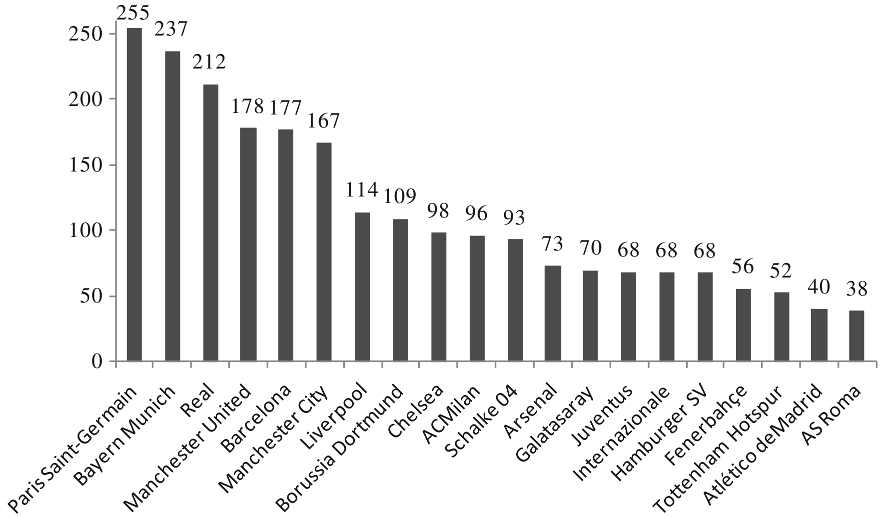


Fig. 10 Commercial revenue, season 2012/2013 (€m) (*Deloitte*)

3.5 Stadium Revenues

One of such opportunities could be afforded by the own stadium. For example, according to Sport + Markt research (International Stadia operations survey, 2011), on average following events are held on the European football stadium every year: 26 games, 2–3 concerts, 2–5 competitions in other sports, and about 14 other events (exhibitions, congresses and etc.). At the same time "sports" activity brings the stadium only 64 % of revenue. 40 % stadiums in Europe have restaurants and bars, 25 %—fitness center, 25 %—the museum of the club, which plays at the stadium, 30 % rent office space, 10 % assign some place for entertainment. It's interesting, that 12 % of stadiums have a chapel (among them Camp Nou in Barcelona).

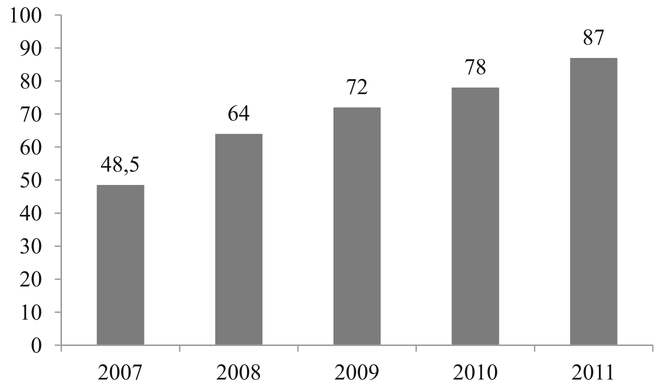


Fig. 11 Revenue of European stadiums from selling naming rights (€m) (*Sport+Markt*)

One more source of income for clubs having their own stadium—title sponsorship of arenas. According to another study of Sport + Markt (“Naming Rights Report 2011”), in 2011 European stadiums were to receive for selling naming rights approximately €87 million (see Fig. 11).

However, the opportunity to earn on selling the title of the stadium depends on the team that plays there, and more particularly—on its popularity and sport results. We also should keep in mind special attention of Union of European Football Associations: UEFA simply banned the use of trade names in its official documents. Therefore, in the Premier League, “Arsenal” plays on “Emirates Stadium”, and in the Champions League—on “Arsenal Stadium”. Nevertheless, the experience of London club should be recognized as most successful—“Arsenal” earned £100 million over 10 years, and the name “Emirates Stadium” firmly rooted in European football.

In general, contracts for selling naming rights are made at the stage of building the stadium. This is largely due to very complicated negotiations with fans, who often oppose the commercialization of stadium’s name. This primarily concerns reconstruction projects when it comes to changing the name of the arena. In this sense, the example of “Newcastle” could be very illustrative. For years the club tried to sell rights to rename home arena “St James’ Park”, each time bumping into fierce opposition to fans. In late 2011, the stadium finally becomes a name “Sports Direct Arena”. However, in autumn 2012, the new title sponsor of the “Newcastle” “Wonga” company announced recovery of old name. Therefore, the company decided to win the confidence of fans. In 2005 fans of “Borussia” Dortmund were protesting against renaming “Westfalenstadion” in favor of the insurance company “Signal Iduna” (15 years for € 20 million). Agreement was found only after one of the streets near the arena renamed in honor of the old stadium.

If the club fails to sell the name of “entire” stadium, it is possible to offer potential sponsors some alternatives. Thus, there were examples when the stadium arranged an auction to sell the right to place a name or a brand on the scoreboard of the arena and its official website for one day (FleetCenter). There have also been attempts to

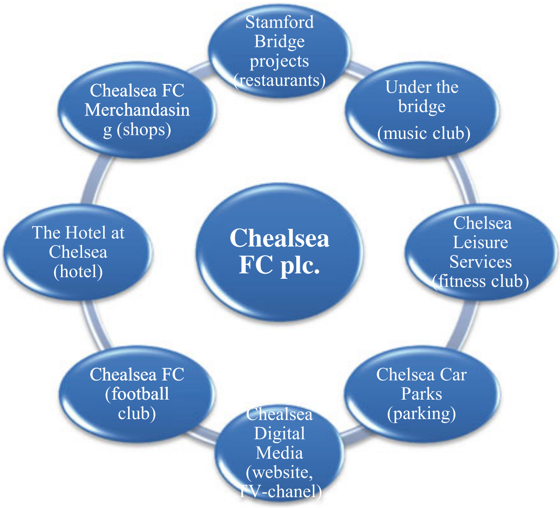


Fig. 12 Chelsea FC group of companies (*sports results*)

sell the rights to the name of the individual stands of the stadium (Giants and Jets football stadium, The Cleveland Browns stadium).

Each stadium can earn substantial share of income on advertising. Determinative in this case will be the effect, which can provide the stadium for the advertiser. In other words the football audience should be interesting for a company, which plans to place advertising on the stadium. In addition the legal restrictions to advertise individual products should be considered. The most striking example of such restrictions—a ban on beer advertising in Russian stadiums, which could have boosted stadiums revenues.

To finish with revenues that could be generated by the stadium, we must underline that the construction of the arena requires huge investments, which, in turn, decrease the total income of the club, especially if the construction was funded with borrowed money. As for these costs, we can assume that the average cost of building a one seat on modern European stadium costs approximately €3900 [9].

3.6 Additional Sources of Income

Clubs can also earn outside stadium: open branded restaurants, clubs, fitness centers etc. As an example we can hold up the structure of Chelsea FC (see Fig. 12).

Finally, club can earn serious money participating in the Euro cups. Distribution of UEFA Prize Money and figures for seasons 2012/2013 and 2011/2012 are presented in Table 3 and on Figs. 13 and 14.

In addition to these fixed sums, the clubs receive a share of the television money from the TV (market) pool, which is allocated according to a number of variables.

Table 3 Champions league bonuses in season 2011/2012 (*The Swiss Ramble*)

€ millions	Champions league		Europa league	
	2011/2012	2012/2013	2011/2012	2012/2013
Participation bonus	3.9	8.6	0.64	1.3
Match bonus:				
Each team	0.55		0.06	
Each win	0.8	1	0.14	0.2
Each draw	0.4	0.5	0.07	0.1
Group qualification				
Win group				0.4
Runners-up				0.2
Min for participation[a]	7.2	8.6	1	1.3
Max for participation[b]	12	14.6	1.84	2.5
Last 32			0.2	0.2
Last 16	3	3.5	0.3	0.35
Quarter-finalist	3.3	3.9	0.4	0.45
Semi-finalist	4.2	4.9	0.7	1
Finalist	5.6	6.5	2	2.5
Winner	9	10.5	3	5
Total for winner (min)	26.7	31.4	5.6	8.7
Total for winner (max)	31.5	37.4	6.44	9.9

[a] If the team loses all six matches of group stage
[b] If the team wins all six matches of group stage

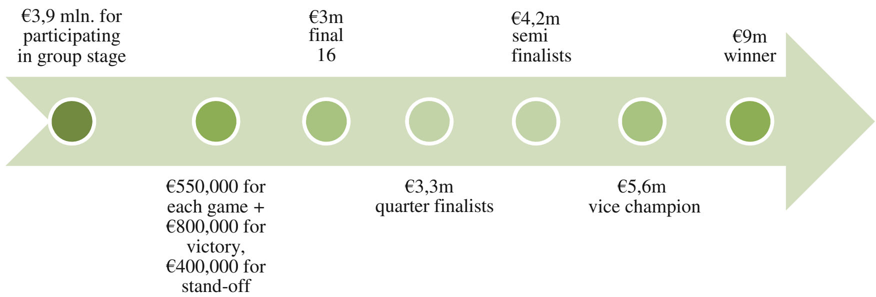
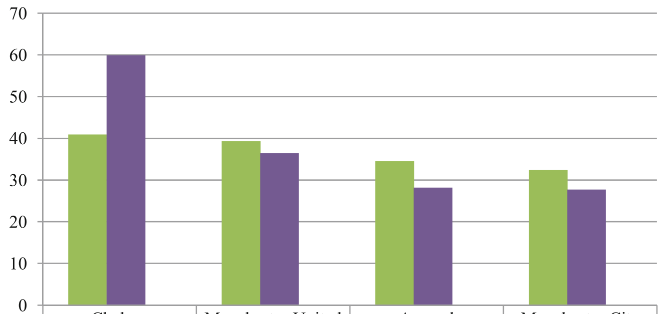


Fig. 13 Champions league bonuses in season 2011/2012 (*The Swiss Ramble*)

First, the total amount available in the pool depends on the size/value of a country's TV market, so the amount allocated to teams in England is more than that given to, say, Spain, as English television generates more revenue. Clubs can also potentially do better if fewer representatives from their country reach the group stage, as the available money is divided between fewer clubs.

In the case of the English clubs in the Champions League, the allocation works as follows:



	Chelsea	Manchester United	Arsenal	Manchester City
2012/2013	40,9	39,3	34,5	32,4
Champion League	34,4	39,3	34,5	32,4
Europa League	6,5			
2011/2012	59,9	36,4	28,2	27,7
Champion League	59,9	35,2	28,2	26,5
Europa League		1,2		1,2

Fig. 14 UEFA prize money—2012/2013 versus 2011/2012 (*The Swiss Ramble*)

1. half depends on the position that the club finished in the previous season's Premier League with the team finishing first receiving 40 %, the team finishing second 30 %, third 20 % and fourth 10 %;
2. half depends on the progress in the current season's Champions League, which is based on the number of games played, starting from the group stages [11].

3.7 *Fans, as Major Source of Income*

Ending up with the revenue items, it is worth noting that the main asset of any football club is its fans, and even money from sponsors and advertisers is largely dependent on the number of fans (prospective customers). Respectively, for the projection of future earnings analyst must know at least the approximate number of fans who will visit home games, buy t-shirts, coffee etc. But how to obtain this figure?

Any sociological research tends to seriously overestimate this numbers by controversial assumptions and extrapolations. In addition, results of studies commissioned through the request of individual clubs and, respectively, paid by them, are pretty questionable. For example, according to the survey of research firm "Nielsen", conducted in 2011, the most popular club in Russia was "Zenit" St Petersburg, which paid for the work. After interviewing 19 000 people in 38 major cities in Russia "Nielsen" found that "Zenith" is supported by 12.6 million people, Moscow "Spartak"—8.2 million, "CSKA"—6.7 million, "Rubin" Kazan—2.3 million, Moscow clubs "Dynamo" and "Lokomotiv"—1.8 million each.

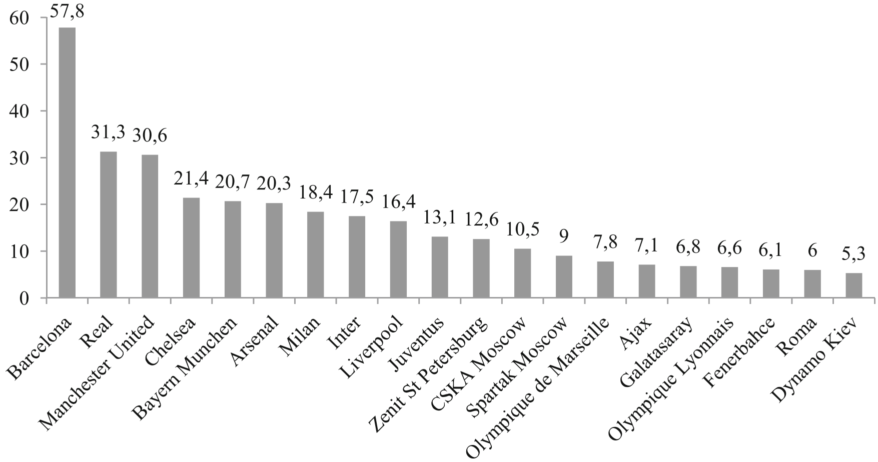


Fig. 15 Most popular football clubs in Europe, number of fans, millions (*Sport+Markt*)

In 2010 Sport + Markt surveyed 10 900 fans in age from 16 to 69 years old and published result shown in Fig. 15. But all these figures hardly could be trusted, and much less we can use it in projections of future revenue items.

The most accurate results can give a formal membership in fan club. For example, there are 226 000 people registered in a fan club of Portugal "Benfica", and in addition to Portugal, they live in the 70 countries of the world. Moreover, adult fans are paying for membership € 12 per month for themselves and € 3 for their children, which should be considered as additional income for the club. Another example of fair counting of fans could be data of "Manchester United" association of fans—MUST (Munchester United Supporters Trust). On their website we can find the counter of registered members, which showed in September 2013 more than 195 000 people. Finally we can use number of members in official groups of football club in social networks.

Now when we have considered all possible sources of revenue of football club, we can switch to its expenses.

4 Structure of Expenses

4.1 Players and Coaches

Even for people who do not consider themselves football fans, it is not a secret that the main cost item for any club is players. According to calculations of FIFA [13], the total turnover in the transfer market for the season 2011/2012 was about € 2.6 billion. Most of all, € 106 million, spent the French "Paris St. Germain", who continued to actively buy players in 2012: € 70 million for Zlatan Ibragimoviach and Thiago Silva. The owner of “Chelsea” Roman Abramovich has spent € 102 million and beat its main rival Sheikh Mansour and his “Manchester City”. Dan King from The Sun calculated that Abramovich has already spent on “Chelsea” over £2 billion.

According to CIES Football Observatory Annual Review 2013 [4], Lionel Messi would largely break the 94-million euro transfer fee record[1] (see Table 4). Estimated on the basis of an exclusive econometric model, his value is between 217 and 252 million euro. Player transfer value calculator could be found here: http://www.football-observatory.com/transfer-value. By the way, Lionel is not only the most expensive big-5 league player, but also the most decisive one for the 2012/2013 season based on the performances for five key indicators: shooting, chance creation, take on, distribution and recovery. With an estimated value between 102 and 118 million, Cristiano Ronaldo would also break his own record. At club level Barcelona holds the greatest assets from a player economic value perspective: 658 million euro. This figure is three times higher than that spent on signing the players used during 2012/2013 season. This reflects the extraordinary ability of the Catalan side to train, launch and add value to home-grown players. The second club in terms of players’ economic value is Real Madrid: 500 million euro. In 2012/2013, money spent in transfer fees to sign first team players was highly correlated to club results in all the leagues. All the champions were the biggest (Paris St. Germain and Bayern Munich), second biggest (Barcelona and Juventus) or third biggest (Manchester United) spenders in their respective league. This confirms the strong influence of money on success.

Along with the cost of players their salaries also continue to grow. Forbes [12] has been tracking the earnings of athletes since 1990 when boxer Mike Tyson ranked No. 1 with a total income of $28.6 million. Iron Mike’s haul would rank No. 25 on Forbes’ 2013 version of the world’s highest-paid athletes. Athletes are richer than ever thanks to skyrocketing television revenues and a two decade stadium building boom.

Coaches do not lag behind their players: in 2012 Jose Mourinho received € 14.8 million a year and was followed by Carlo Ancelotti with annual salary of 13.5 million; in the third place was former coach of “Barcelona” Josep Guardiola, who has earned over the year 2011 € 9.5 million.

According to Sportingintelligence’s Global Sports Salaries Survey (GSSS) for 2013, compiled in association with ESPN The Magazine [10], “Manchester City” are the best paid team in global sport. The average first-team pay at City, who have been transformed as a footballing force under the ownership of Sheikh Mansour, has been calculated at £5.2 m per year, or £100 764 per week. The most generous football clubs are presented on Fig. 16.

In terms of the cost of salaries it is interesting to calculate whether the club has enough revenue to cover the payroll. For example, “Manchester City” “salary to revenue” figure is 114 (see Fig. 17), which suggests that in fact the club does not have enough income and expenses are covered by shareholders.

[1] At the time of writing this the transfer of Gareth Bale from Tottenham Hotspur to Real Madrid in exchange for approximately € 100 million is yet to be confirmed.

Table 4 Most expensive transfers (*http://www.transfermarkt.de*)

Player	Buyer	Seller	Price, m
Cristiano Ronaldo	Real	Manchester United	94
Gareth Bale	Real	Tottenham Hotspur	91
Zinedine Zidane	Real	Juventus	73.5
Zlatan Ibrahimovic	Barcelona	Inter	69.5
Kaka	Real	Milan	65
Falcao	Monaco	Atletico Madrid	60
Luis Figo	Real	Barcelona	60
Fernando Torres	Chelsea	Liverpool	58.5
Neymar	Barcelona	Santos	57
Hulk	Zenit	Porto	55
Hernan Crespo	Lazio	Parma	55
Gianluigi Buffon	Juventus	Parma	54.2
Gaizka Mendieta	Lazio	Valencia	48
Falcao	Atletiko Madrid	Porto	47
Andrey Shewchenko	Chelsea	Milan	46

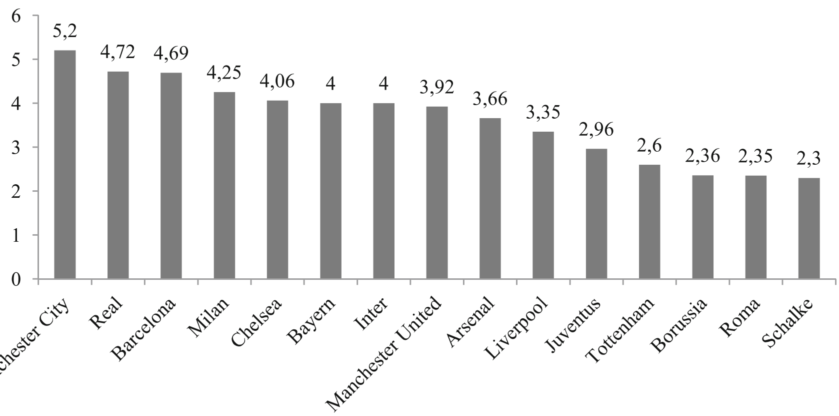


Fig. 16 Football clubs with highest average salary billions, million £ per player per year (*Sporting-intelligence's Global Sports Salaries Survey (GSSS) for 2013*)

However, we should bear in mind that expensive players can in turn increase revenue of the club in several ways. The first and most obvious—is a successful game helping win the match. This forms the economic effect. The second source of income, which is associated directly to the player is—selling T-shirts with player's name, which is especially relevant for such stars as David Beckham or Leo Mesi. Finally, the purchase of popular player can be used to enter the markets of the regions that these players represent. For example, buying Platini allowed "Juventus"

Table 5 The world's highest-paid athletes 2013 (http://www.forbes.com/athletes/list/)

Rank	Name	Sport/club	Income, m$
1	Tiger Woods	Golf	78.1
2	Roger Federer	Tennis	71.5
3	Kobe Bryant	Basketball	61.9
8	David Backham	Los Angeles Galaxy, PSG	47.2
9	Cristiano Ronaldo	Real	44
10	Lionel Messi	Barcelona	41.3
61	Wayen Rooney	Manchester United	21.1
63	Sergio Aguero	Manchester City	20.9
64	Didier Drogba	Shanghai Ahenhua, Galatasaray	20.8
67	Yaya Toure	Manchester City	20.7
68	Neymar	Santos, Barcelona	20.5
72	Fernando Torres	Chelsea	20
76	Zlatan Ibragimovich	PSG	19.7
79	Kaka	Real	19.3
85	Karlos Tevez	Manchester City	18.2
93	Steven Gerrard	Liverpool	17.2
100	Samuel Eto'o	Anji	16.4

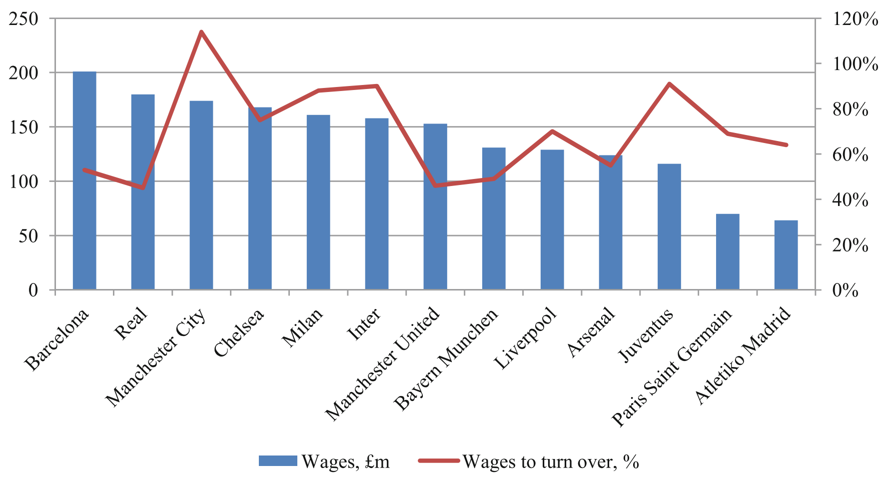


Fig. 17 Wages in leading European clubs, 2010/2011 (*The Swiss Ramble*)

to increase sales in France, Torres drew the attention of the Spanish fans to English Premier League and the Japanese Honda stimulated interest of his countrymen to the Russian championship. However, "Real" is the most successful in this regard. Stars from across the continent are playing for this club, and thus secure the support (including financial) in many countries of the world. Sometimes it's not enough to buy only one player to win new markets, and to achieve the desired results the club deliberately makes appropriate emphasis in the transfer policy. But, of course, the main purpose of each acquisition is to strengthen the team, and only secondarily—selling T-shirts with player's name.

As for access to international markets, here's another interesting example: in October 2012 "Real" in collaboration with the club "Guangzhou Evergrande" opened in China the biggest football academy in the world. All this ultimately aimed at increasing the number of fans globally and, as a consequence, the growth of income. We also need to take into account the proceeds from the immediate operation of the facilities.

4.2 Operating Expenses

Let's go back to expenses. Football clubs, like any other company, incur operating expenses, which are formed by office rent, salaries of administrative staff, payments for different consulting services (sport and finance, including insurance), support of the second team and youth academy, security costs, which also may include monetary sanctions for the illegal behavior of fans, the cost for flights and accommodation, rental of stadiums and training centers.

As for youth academy, then, for example, the Portuguese "Benfica" invested in its construction € 19.8 million and annually spends € 4 million. But if somebody signs a contract with one of its students—Nelson Oliveira, he had to pay a compensation of € 30 million. This means that only one deal could cover all the costs of the Academy for several years. Besides football school brings the club revenues, not related to the sale of players: several teams of the Academy (there are different teams for particular age) have its own sponsor. Finally Academy signed an agreement with the general title sponsor, who bought the right to give the Academy its name. Now it's officially called the Academy Saixa, named after Caixa Geral De Depositas—a major bank in Portugal. This is good example of effective and thoughtful expenses beginning sooner or later to generate income.

Another type of costs that are not relevant for all clubs, but the proportion of which could be quite substantial—service debt. For example, if we look at the cost structure of "Manchester United" for 2009–2012, interest on loans and bonds, on average, amounted to 44 %, and 33 % of funds the club allocated to pay the debt (see Fig. 18).

So, we reviewed the typical income and expenses structure of football club. Table 6 shows the revenue structure of the largest football clubs in Europe for season 2010/2011.

Few years ago, some experts using revenue structure as a base, distinguished several management models: "German" —with most part of the revenue from commercial activities, "Italian"—with an emphasis on income from broadcasting, etc. However, currently this approach is hardly applicable. This is primarily due to the fact that the revenue structure of each club is individual and can change annually. After all, football club revenue is heavily dependent on sport results—in one season the team can win the domestic championship and succeed in international tournaments, and in next one—show weaker results. At the same time the club's achievements affect all sources of income: wins give a boost to merchandise, attract new fans to the stadiums and television screens, force sponsors to be more generous. In other words, there is a kind of chain reaction, allowing the club, in one case—to increase revenue, in the other—to achieve more modest results or even incur losses. Moreover, we must remember that at the relevant rating were included only the largest representatives of national championships, and consolidated results for the league may look different. Therefore, any conclusions can be made only on the analysis of specific club or league for a few years.

The only clear trend of recent years—the growing importance of income from sale of broadcasting rights, but it does not provide any stability to the structure of income. First, the new contracts are more expensive than the previous ones. Second, the pattern of distribution of these revenues between clubs will always be controversial, which, sooner or later, will be reflected in figures.

4.3 Applying Income Approach

As we remember, Income approach is based on the assumption that the value of the business is equal to the sum of the present values of the expected future benefits of owning the company. So, the main purpose is properly identify and forecast future cash flows, generated by the FC.

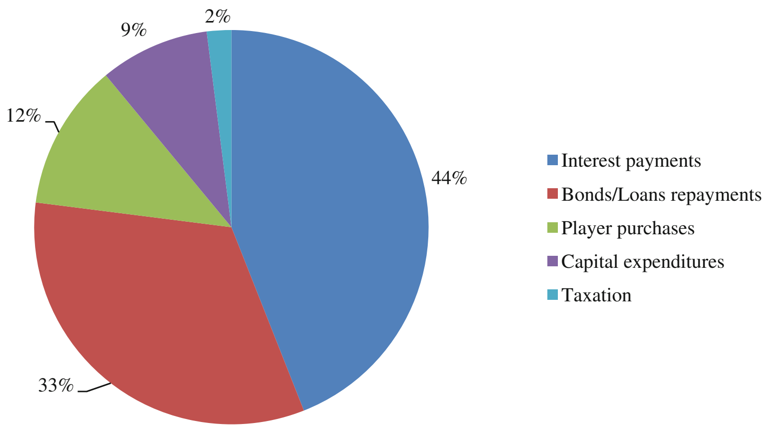


Fig. 18 Manchester United—use of funds, 2009–2012 (*The Swiss Ramble*)

Table 6 Add caption

	Rebenue	Matchday		Broadcasting		Commercial	
	€ m	€ m	%	€ m	%	€ m	%
Real	518.9	**119.0**	23	**188.3**	36	211.6	41
Barcelona	482.6	117.6	24	188.2	39	176.8	37
Bayern Munich	431.2	87.1	20	107	25	237.1	55
Manchester United	423.8	127.3	30	118.6	28	**177.9**	42
Paris Saint-Germain	398.8	53.2	13	90.9	23	254.7	64
Manchester City	316.2	46.2	15	103.1	33	166.9	53
Chelsea	303.4	82.5	27	123	41	97.9	32
Arsenal	284.3	108.3	38	103.2	36	72.8	26
Juventus	272.4	38	14	166	61	68.4	25
ACMilan	263.5	26.4	10	140.9	53	96.2	37
Borussia Dortmund	256.2	59.6	23	87.6	34	109	43
Liverpool	240.6	52.1	22	74.5	31	114	47
Schalke 04	198.2	42.5	21	62.9	32	92.8	47
Tottenham Hotspur	172.0	46.9	27	72.7	42	52.4	30
Internazionale	168.8	19.4	11	81.5	48	67.9	40
Galatasaray	157.0	35.4	23	51.9	33	69.7	44
Hamburger SV	135.4	43.2	32	24.7	18	67.5	50
Fenerbahçe	126.4	27.7	22	**43**	34	55.7	44
AS Roma	124.4	**20.1**	16	66	53	38.3	31
Atlético deMadrid United	120.0	27.5	23	52.5	44	**40**	33
Total	**5394**						
Min		19.4		24.7		38.3	
Max		127.3		188.3		254.7	
Average		59	22	97.325	37	113.38	41

However, a properly structured income and expenses are not enough. The main difficulty in applying the income approach lies in their forecasting. For these purposes we can use:

- retrospective reporting;
- business plan and strategy development;
- forecast for the industry, competitor analysis;
- interviews with the shareholders and the management (including the media).

But as we mentioned above, any forecasts are very hard to do, when we speak about football, because final results are highly dependent on sport achievements, which are difficult to predict even for the giants of world football. However, this should not be a cause for denial of the application of the income approach. Let's formulate the following principles of forecasting.

1. Projection period of 3 years.
2. Applying the retrospective performance only in a case of stable development and clear trend, confirmed by forward-looking statements by the representatives of the club.
3. Analyzing independent research to prove all assumptions, which were made (for example, reports of Deloitte; The Swiss Ramble; Soccernomics Agency; The andersred blog; Forbes; UEFA; IB, covering football industry).
4. Consider regional expansion of the club (in terms of merchandising and media rights), and plans for expansion/renovation/construction of the stadium.
5. "Pessimistic" revenue projections and "optimistic" approach to forecasting costs.
6. Considering UEFA Club Licensing System and Financial Fair Play Regulations.
7. Verification of the results from the point of view of common sense (sometimes resulting values, despite the apparent logic, do not seem realistic because of certain circumstances).

After forecasting we can move to the next step and calculate free cash flow. Usually the calculations are carried out for invested capital or free cash flow to firm (FCFF—because it's "free" to pay out to the club's investors):

$$\begin{aligned}\text{FCFF} = {} & \text{EBIT} \times (1 - \text{Tax}) + \text{Amortization \& Depreciation} \\ & - \text{Net capital investments} \mp \text{Change in working capital}\end{aligned}$$

Earnings before interest and tax (EBIT) or operating profit we can find in P&L. But football clubs are very close businesses, and independent analyst can find statement of only several FC (for example those, whose shares are available at exchanges and who are obliged to publish financial reports). If we do not have such an opportunity, we can calculate each item of income and expenditure in accordance with their basic structure discussed above. For example, we can calculate income from ticket sales taking the schedule of all games of the year (regular championship, Cup, Champions or Europe League, etc.), the average attendance at matches and ticket price ranges. Besides, as an information base, we can use interviews with shareholders and managers of the club, the opinions of experts. Of course, this method of calculation would have a high degree of uncertainty, but in the absence of reliable information such an approach could be relevant.

As for depreciation and capital expenditures, there are some specifics associated with the football players. The fact is that the club cannot buy the player himself, but only rights for him or conclude a contract for a fixed term. This contract is recognized in financial statement as intangible asset. In other words the costs associated with the acquisition of players' registrations are capitalized as intangible assets at the fair value. During the term of the contract player works for the club, being its asset, and the main goal of this asset—achieve sports results and win trophies. So the price paid for the player is amortized equally during the term of the contract, and value of a player on the balance of the club gradually declines. Costs associated with the acquisition of players' registrations include transfer fees, League levy fees, agents' fees and other directly attributable costs. These costs are amortized over the period covered by the player's contract. If player extends the contract, remaining depreciation or the value on the balance sheet is divided equally for the period of the new agreement. If the club sells the player, the profit is calculated not as money received from the buyer, but like this revenue minus the book value of a player. These rules lead to the fact that all clubs' transfer costs are accounted in non-cash expenses—as total annual depreciation. Therefore, even after large purchases, the club will not suffer a loss. We should note that the player, who joined the club from its own academy, is worth nothing on the balance sheet, and the club could earn 100 % profit. In this case, when the club publishes its balance sheet, it appears that students of the Academy as assets are not recognized at all.

Next component of Free cash flow is Net working capital. In accounting this item is calculated as difference between current assets and current liabilities. However, in business valuation the real Working Capital Requirement is taken on the basis of revenue: with income growth, grows the working capital. The amount of working capital is calculated in two steps:

1. We calculate Working Capital Requirement by comparing the working capital ratio of the subject to those of the guideline companies or by comparison to industry norms. In fact we can calculate the percentage of working capital in Revenue for our football club (for previous periods) and do the same for several public teams. As a result we will have a certain percentage (for example, 15 %) of revenue which form the working capital.
2. Knowing actual working capital we can calculate Working Capital Requirement or Excess. If actual working capital is less than required we will have Working Capital Requirement with "minus" in free cash flow, otherwise—Excess of working capital with "plus" in free cash flow.

Income approach deals with future cash flows. So after we calculate all items of FCFF, it should be discounted to the present moment of time. But firstly we need to have discount rate. The discount rate takes into account the time value of money (the idea that money available now is worth more than the same amount of money available in the future because it could be earning interest) and the risk or uncertainty of the anticipated future cash flows (which might be less than expected). In general, discount rate—is the rate of return that a buyer or investor expects to receive from an investment with a comparable level of risk. On the other hand discount rate could be considered as the alternative cost of capital.

Method of calculating the discount rate depends on the type of capital used in building up the cash flow. If we calculate the cash flow for equity (FCFE) we use Capital Assets Pricing Model—CAPM), and for all invested capital (equity + debt) or FCFF—WACC method (Weighted Average Cost of Capital). These methods are well known and their application does not cause any problems. However, the features of calculating the discount rate are largely determined by the company's industry. Sport clubs, as separate businesses, also have some special characteristics and demand a certain methodology of calculating the discount rate, which should reflect the level of risk specific to a particular team.

Table 7 β coefficients for major European public clubs

	Borussia	Juventus	Olympique	Ajax	Porto
β_L	0.678	0.157	0.268	−0.086	−0.388
β_U	0.418	0.084	0.207	−0.05	−0.224

CAPM model looks as follows:

$$E(R_i) = R_f + \beta(E(R_m) - R_f)$$

$E(R_i)$ is the expected return on the capital asset
R_f is the risk-free rate of interest such as interest arising from government bonds
$E(R_m)$ is the expected return of the market
$E(R_m) - R_f$ is known as the market premium
β (the beta) is a measure of the volatility, or systematic risk, of a security in comparison to the market as a whole. Beta is calculated using regression analysis. A beta of 1 indicates that the security's price will move with the market. A beta of less than 1, means that the security will be less volatile than the market. A beta of greater than 1 indicates that the security's price will be more volatile than the market. For example, if a stock's beta is 1.2, it's theoretically 20 % more volatile than the market. Beta coefficient is calculated as covariance of a stock's return with market returns divided by variance of market return.

So we can calculate betas for public football clubs (for example, Ajax, Roma, Juventus, Lazio, Besiktas, Fenerbahce, Galatasaray, Borussia Dortmund, Celtic, Porto, Benfica, Sporting, Millwall, Manchester United, Olympique Lyonnais). Results of such calculations are presented in Table 7. The other way is to use publicly available data from Bloomberg or Reuters. Besides we can find betas in Aswath Damodaran' research: http://aswathdamodaran.blogspot.ru/2014/06/ballmers-bid-for-clippers-investment.html

To incorporate in discount rate all possible risks we can add to the CAPM model specific risk premium for football clubs calculated using point-based system. Each risk factor is assessed by different simple question with "yes", "no" and "no data" answers. Answer "yes" corresponds to the 0 % value of risk, "No"—the maximum value of 5 %, "No Data"—the a verage value of 2.5 %. All risks are classified under the following headings: management, history, infrastructure and image, finance, sports achievements, team squad.

A significant drawback of this method is subjectivity in determining risk premiums. However, we can minimize this disadvantage by involving a larger number of risk factors. Another limitation for this method is a lack of information regarding the clubs. Moreover, with each answer "No Data" level of risk will be equal to the average amount, which does not reflect the real situation. However, despite all these limitations, this method can give much more real results than the traditional "expert" valuation without any grounds.

Table 8 Questionary for determination of specific risk premium for Manchester United

Risk factors	Answers	Risk management (%)
Specialized education and experience in football	Yes	0
Head coach has experience with top-teams	Yes	0
Club has a development plan in certain areas: transfer policy, marketing, relationship with fans, media, global development, merchandising	Yes	0
Well-defined structure of the club	Yes	0
System of rewards and penalties, and real practice of its implementation	Yes	0
Non-involvement of shareholders to the team's management	No	5
Sum:		*5*
Number of factors:		*6*
Final risk:		***0.83***
History & infrastructure & image		
The club was founded more than 20 years ago	Yes	0
The club has its own stadium	Yes	0
The club has its own training ground	Yes	0
The club has second team	Yes	0
The club has youth academy	Yes	0
The club has representative offices in other countries	Yes	0
The club sells its branded products in other countries	Yes	0
The club has more than 500 000 fans (according to several independent sociological researches, official fan-club data, social networks)	Yes	0
Additional businesses in the club's structure: TV-channel, restaurants, etc.)	Yes	0
More than 100 items of products manufactured under license agreements	Yes	0
Official fan club with more than 100 000 members	Yes	0
Fans support shareholders and management of the club	No	5
For the last three seasons the average attendance was above 80 %	Yes	0
For the last three seasons the club sells season tickets for more than 50 % seats of home stadium (or has correspond number of requests)	Yes	0
The club is regularly mentioned in media without any negative	N/A	2.50
Sum:		*7.50*
Number of factors:		*15*
Final risk:		**0.50**

(Continued)

Table 8 (Continued)

Risk factors	Answers	Risk management (%)
Finance		
Availability of ongoing financial support from shareholders	No	5
Large world companies with good reputation among sponsors	Yes	0
Debt to Equity ratio <1	No	5
Budget of the club is among the five largest in the respective league	Yes	0
The last 3 years the club finished with a net profit	No	5
Diversified income structure of the club with the share of each source not exceeding 40 %	Yes	0
The club regularly publishes annual reports, which reflect the financial position with a high degree of confidence	No	5
The club meets financial fair play requirements	Yes	0
Wage to revenue ratio $<$ 70 %	Yes	0
Sum:		*20*
Number of factors:		*9*
Final risk:		***2.22***
Sports achievements		
Over the last 5 years the club at least once won a championship and the National Cup	Yes	0
Over the last 5 years the club at least once participated in Champions League or Europe League	Yes	0
Over the last 5 years the club at least once played in final of Champions League or Europe League	Yes	0
Over the last 5 years second team at least once won a championship	Yes	0
Over the last 5 years the club won some other trophies	Yes	0
Sum:		*0*
Number of factors:		*5*
Final risk:		**0**
Team squad		
The club regularly buys and sells players at the international level	Yes	0
During the last 5 years, the number of players from club's academy in the first team does not fall below 20 % (approximately)	Yes	0
Over the past 5 years, the average age of the players if main team does not exceed 27 years	Yes	0

(Continued)

Table 8 (Continued)

Risk factors	Answers	Risk management (%)
Every season (over the past 5) the club buys at least 1 player who stable plays for the main team for several seasons	No	5
Every season (over the past 5) no more than 2 players were injured for more than 6 months	Yes	0
The presence of players that played for their National teams on last World and Euro Cups (last 8 years)	Yes	0
The presence of well-known players with high popularity and charisma	Yes	0
No Third Party Player Ownership (TPPO) agreements with players	Yes	0
Sum:		*5*
Number of factors:		*8*
Final risk:		**0.63**
Specific risk premium for MU		**4.18**

Questionary for determination of specific risk premium for Manchester United is presented in Table 8. Please note, that this questionary could be used for any football club of any league with some improvements due to the country specifics

That is it for CAPM model and free cash flow to equity. If we calculate the cash flow to invested capital the other model should be used—WACC:

$$WACC = R_e W_e + R_d W_d (1 - t)$$

W_e—market value of the club's equity;
R_e—cost of equity (calculated with CAPM model);
W_d—market value of the club's debt;
R_d—cost of debt;
t—corporate tax rate.

Exactly this model we can use to calculate the discount rate for our FCFF. The total equity value of FC may be obtained by the following formula:

$$V = \sum DCF + V_{term} - D \pm WC$$

$\sum DCF$—the sum of discounted future cash flows;
V_{term}—terminal value calculated with Gordon model;
D—long-term debt;
WC—actual working capital.

5 Conclusion

We have considered a general model of the income approach, but in practice it is not always applicable.

First, most football clubs have historically been unprofitable and were financed by private investors or municipal authorities. Knowing that DCF requires sustained and predictable profitability, it could be very hard to use DCF method for evaluation of FC. Conceptually, we cannot use income approach for valuation of companies with negative cash flow. Given that we usually calculate the future cash flow on the base of historical data, there is a risk to receive negative FCFF. In this case the value of business will be negative as well. So the main problem encountered when applying a DCF method to value football clubs is consistent profitability. Traditionally, most clubs are loss making entities. And even for those that are profitable, it is extremely difficult to forecast financial results going forward given the unpredictable nature of the game [7].

Another limitation in the application of the income approach is attributable to the heavy dependence of any football club on sports results, which makes its finance performance highly volatile and seriously difficult to predict, even in the medium term. This problem becomes especially noticeable for terminal value. After all, it is assumed that after the end of the projection period revenues will stabilize and grow with a stable rate to perpetuity. Honestly speaking this assumption is hardly applicable for any business and absolutely unreal for football.

Finally, football has always been one of the most isolated areas, not only in Russia but also in Europe. Even the clubs, whose shares are offered on the stock exchange, and regularly publish audited statements, are able to hide unwanted information and thereby distort the true picture. As a result, the analyst is forced to use the information from open sources, not confirmed by the club and make a large number of assumptions that does not add any accuracy.

Another problem comes from the calculation of discount rate: we must take into account all risks specific to the football club, and assign each of them a specific percentage. Final value of business will be largely determined by the discount rate, and every hundredth matters.

But despite all mentioned limitations any investor needs to understand, what kind of cash flow the business generates at the current moment and what he can expect in the future. Therefore we should not reject the income approach. Market and asset approaches will level these restrictions using the evidence of the market and the value of assets, respectively.

Within market approach we will use the available information (for example revenue, earnings and fan base multiplies) of other FC and data from M&A market.

Finally in the case of any company valuation, it is important to review all of the assets and liabilities of that organization. This is no exception when it comes to football club valuation. The main assets of a club (typically a stadium, training ground and player registrations) need to be weighed up versus the liabilities (normally trade creditors and debt). A club's net assets figure (total assets less total liabilities) could be considered as value of the club.

In detail these approaches will be considered in a separate article.

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The Interesting Case of Portugal in the Economy of Sport

Eduardo Tomé

Abstract This paper intends to analyze the interesting case of Portugal as sporting force. We do the study by analyzing the statistics on the Portuguese success in sport when available; specifically we end up comparing football with all the other Olympic sports. We conclude that there is a big divide, between football, and all the other Olympic sports put together. In Europe Portugal ranks in the first 6 regarding football but only in 30th (historically) or worse (taking into account the fact that some countries are very new) when regarding the Olympic Games. The reasons for that difference are related with culture, dimension and policy. In short, Portugal has a well-developed national policy and system on football, which produces results; but the country has nothing comparable in terms of the Olympics and therefore results don't happen. We therefore recommend a more balanced investment and the presence of the 4 big football clubs in the Olympic sports as a way of improving the Portuguese performance in the Olympics. That improvement would have reflections in the country health and productivity, and would be socially beneficial. The paper is original because for our knowledge it is the first time the Portuguese sports have been analyzed with such a methodology and with those results.

Keywords Economics of sport · Portugal · Football · Olympic games · Policy · Culture · Investments · Results

1 Introduction

Portugal is not a big country. But some of its sportsmen are known worldwide. Any person with a bit of knowledge in Sports heard about Eusébio, Chalana, Figo and Ronaldo (as footballers), Benfica, FCP Porto, and Sporting Lisbon (as football clubs), and Mourinho, the self denominated Special One, and more recently Lucky

E. Tomé (✉)
Universidade Europeia, Lisbon, Portugal
e-mail: eduardo.tome@clix.pt

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One (as football coach). The same people may have seen the Portuguese Selecçáo (of football) making some interesting results in the phases of the Euro and World championships since at least 2000, with some others exceptions before (World Cup 1966 in England, Euro 1984 in France). However, if we would ask a group of informed foreigners about Portuguese champions in any other sports, the answer would probably be much more difficult. Cycling fans may name Joaquim Agostinho, a legend from the seventies; athletics enthusiasts may remember Carlos Lopes, Fernando Mamede, Rosa Mota and Fernanda Ribeiro (all long distance runners from the two last decades of the 20th century) and probably that's all. Finally if we would ask the same persons to name Portuguese medalists in the Olympic Games, other than the mentioned long distance athletes, they would for sure have a very hard time providing a correct answer. And the main reason for that is that there are very few Olympic champions in Portugal: only 4 to date: Carlos Lopes (Marathon), Rosa Mota (Marathon), Fernanda Ribeiro (10,000 m) and Nelson Évpra (triple jump) all in athletics.

Therefore the idea of this paper is to try to analyze scientifically the Portuguese situation regarding the Economy of Sports. For doing that first we describe the papers' main concepts, namely European Football, Olympic Games, investment and cost benefit analysis. Secondly we expose the main theories on the Macro and Micro economic importance of Sports. Thirdly we expose the Portuguese Case. The first part of this third section is historical and statistical. In the second part we compare the Portuguese Case with countries of equal dimension. The section ends with the presentation and discussion of the results. Finally in a concluding section we present the paper's conclusions, and link them with limitations, policy implications and suggestions for further research.

2 Concepts

2.1 Football and the Olympic Games

At the time of writing (2013), football is one of the main sports in the world. European competitions are organized by the Union of European Football Associations (UEFA) [14]. UEFA organizes annually and since 1958, football club competitions, among them and most importantly the Champions League and the Europa League. Each year clubs and countries are ranked by their results in the UEFA organized competitions [7]. The rankings are available online and relate to the competitions of the previous 5 years [7]. We underline that we are only discussing men football.

The Olympic Games of the Modern Era were firstly held in Athens in 1896, by the work and inspiration of the French Pierre de Coubertin. The Games should reunite the best athletes in the world, every 4 years. The Games have embraced many changes and faced many troubles during its history. Women were allowed in 1924, professional athletes in 1984. Two World Wars impeached the Games of 1916, 1940 and 1944. Boycotts put severe problems to the 1976 (African Countries), 1980 (Western Countries) and 1984 (Eastern block countries) Games. Nowadays the Games are the biggest sporting show on Earth and every 4 years around 10,000 athletes of 200 countries have the privilege of competing in around 25 sports, all masculine and feminine, by around 300 gold, silver and bronze medals. In every Olympic Games two rankings of countries are established, one based on the medals, the other on the points obtained by the results in the first eight positions. We also note that we are not dealing with the Paralympic sports.

2.2 Cost Benefit Analysis

In Economics, investments are operations whose present costs should be compared with future benefits. They are different from consumption which is an expense from which no future return is expected. It is generically easy to define and assess the costs of any investment, at least from a point of view of accountancy. In sports, those investments are mainly tangible, done in the short run, financial, and detected via audit. These investment costs may relate to facilities, support of athletes, coaching and other team members, organizing events, administrative costs, etc. The assessment and definition of benefits is by definition much more difficult. When the investment is somehow tangible (like putting money in a bank) returns may be assessed with some simplicity and exactitude. But regarding intangible investments like knowledge, education, health, routines, brands, the definition of the returns is much more difficult. Sports are somehow an intangible investment, in the sense that even the facilities are tangible, sport is essentially an activity, which implies a performance. Indeed sports are about performance and the easiest way to assess the return of the investment is to measure the performance. Performance may be measured by number of participants, and their absolute marks or their relative classification. Also, the return of sporting activities may be defined by the financial or economic return in the medium run of the sporting activity. Sports may also relate to tourism activity: as an one off like the Games, or as permanent base like the skiing resorts in the Alps. Sports may generate income and employment. Finally it may be said that a sportive nation is less prone to disease and is more productive, and therefore sport may have a long run and lasting effect in the economy and society. So the benefits of sport are less tangible, more long run, less financial and more difficult to assess than the costs.

3 Theories

In this section we analyze the sporting scene as a multitude of sectors, the market of sports, the value in sport, the microeconomics of sports and the macroeconomics of sport. The section ends with the presentation of alternative models of sports funding, and an explanatory note on the importance of the concepts for the Economy of Sports and for the paper.

3.1 Sport as a Sector or a Multitude of Subsectors

Sports can be seen as an economic sector, or industry; even each type of sport can be seen as a small industry. In order to be competitive in the continental or world scene each country has to be able to produce, almost continuously, world beaters; those world class athletes and teams have to be breed, nurtured and accompanied throughout their careers, so that when they retire they will be the coaches and mentors and talent spotters of the younger generations. Therefore we consider that in fact the Olympic Games are the biggest show on Earth of the Sporting Industry, in which the best participants in around 25 sub sectors are shown. Very few high profile sports are not in the Games (professional football, rugby, baseball, American Football, Formula 1, golf, chess) but some are hoping to be there soon (rugby and golf). And very few events can be even comparable with the Games (the World and European Football Cups, the Rugby World Cup, the Tour de France, the Formula 1 Championship, the Chess World Championship, the Super Bowl).

3.2 The Market in Sport

Sports, or even each sport, may be considered as a market. In that market there is a demand, a supply, a price and a quantity. The demand is made by the public. The supply is made by the sporting organizations that organize the events or produce the athletes. Also, there are several economic agents involved: companies, non-profitable organizations, the public sector, international bodies, the sportsmen, and the public. Sport is an investment which at least in some part accrues the stock of sporting facilities. The funding for sports may come from the public, the sponsors, the brand buyers, or from other form of contracts.

3.3 The Value in Sport: From Tangibility and Human Capital to Intangibility and Intellectual Capital

In the 20th century the value of sport a sporting event was given by the ticket sales, and the value of a competitor by its wages, which in turn were related to the ticket sales. Skills as human capital could be important to define the wages, but there was still a high tangible presence in the market.

However things changed. In the 21st century, with the emergence of a knowledge based, intangible dominated, services economy. The value of a sporting event depends of intellectual capital related elements such as merchandizing, capacity of organization, technology used, processes of communication. Also, the value of a sportsman does not depend only of his/her skill, but it is also related to the commercial image (brand) of the sportsman, which in turn generates profitable contracts well outside the sporting arena.

3.4 A Microeconomic View of Sport

Sport may generate big earnings for sportsmen [13]. The explanation of those earnings, is, it seems, more related with social capital than with skills [13]. Sports may also generate big rewards for companies/organizations/clubs depending on the model used [1].

3.5 A Macroeconomic View of Sport

The Macroeconomic impact of Sport may be also very important [2]. Sports may generate important levels of income for nations (through big events) or regions (because of sporting related permanent revenues by using facilities like in the Alps (for skiing), or in the Portuguese Algarve (for golf, football or athletics). Therefore Sport can be responsible by a non-negligible number of jobs. However important events may have substantial effect in public budgets, if they need public funding [4].

And sports may generate exports (even indirectly by the making of swimming suits) or tourism revenues (by visitors of sites or events). Finally, sport may have an impact of ethics, and health, which may influence positively productivity.

3.6 Models of Professionals Sport Finance

With globalization, the nature of Sports changed drastically. In 20th century, many sports were managed through a model similar to the Spectators-Subsidies-Sponsor-Local (SSSL) model [1] defined for professionals sports finance. Funds were obtained by three main sources: spectators, subsidies, sponsors or local authorities. Professional sports like football were linked with big attendances rates, and thousands of spectators, and members, that keep the business going. But also, many sports and sporting collectivities were called amateur and depended on charities or subsidies given by the state or by the local authorities. Sometimes those “amateur” sports were practiced in big clubs and funded by money from the profitable and market driven (by spectators and members) professional part of the club.

However, with the 21st century, globalization and instant communication everything changed. A management model similar to the Media-Corporation-Merchandizing-Markets (MCMM) described by the same two authors for sport funding became dominant. Sports became to be funded by media contracts, managed by corporations, with global brands, aiming at a global audience; merchandizing is an essential vector of those global markets. This new business model is essentially used in football, and other professional sports, but it is also developing in almost any other sports. We may assume in fact that in the Olympic sports this model is also used.

3.7 Note on the Relevance of the Concepts and Models

The concepts we just mentioned are important to this study, or even to any broad study on the Economics of Sports because they define the theoretical background. We consider that it is decisive to understand that sport is an economic sector, and also a market. We think that sport is worth because it produces value, and it is also important to see how the creation changed in recent time. As in most correct and sound Economic analysis we should never forget the two basic sides: Micro and Macro. And finally in an era of tight funds, sport finance is an essential subject. We will explain in next section how those concepts are applied to Portuguese sports, making them relevant for the study.

4 The Portuguese Case

4.1 Political and Economic Setting

Portugal is one of the oldest countries in Europe. The Kingdom of Portugal was founded in 1143. Its frontiers with Spain date from 1297. The importance of the Portuguese in the 16th century with the Discoveries and the 18th century with the Continental Trade between Africa, Brazil and Portugal is well known. However the Napoleon Invasion left a very lasting mark of underdevelopment and poverty and the country never quite recovered its previous standing. Republic was installed in 1910 and after years of turmoil a fascist regime was installed in 1926. The democratic regime only began in 1974 and decolonization soon followed. In 1986, after 10 years of turbulent adjustment, Portugal became a member of the European Union [9].

In 1960 the country had a GDP ph of around 45 % of the EEC average. That figure rose to 55 % in 1974, but remained stable until the adhesion. The years 1986–2001 sounded like a Golden Decade, marked by the Universal Exhibition in 1998 which promoted the New Face of Portugal. And when the Euro began to circulate in Portugal in 2002, the country's GDP was of 75 % of the EU. However, in the last 10 years everything changed. The country diverged from the EU and nowadays its GDP is only of 67 % of the EU. The current rate of unemployment is of 17 % and the GDP decreased 4 % in 2012. The economic situation is in dire straits, even if since 2011 the country is having the support of the Troika (European Commission, European Central Bank and International Monetary Fund) with a loan of 78 Billion Euros [10].

4.2 History of Sport in Portugal: Main Facts

Sport has never been a very common activity in Portugal. In the Middle Ages, horse tournaments took place. And some warriors were skillful with the sword. But sport was not a feature of Portuguese education for a long time. In the last years of the Monarchy, and the beginnings of the 20th century, Sports were a characteristic of the elites [9]. Those elites founded clubs that would become popular (Benfica in 1904, Sporting Lisbon in 1906 and FC Porto in 1893). The Republic tried to symbolize the New Man using Sports; but a disaster happened. A team of six competitors was sent to the Stockholm Olympic Games, but Francisco Lázaro, the best Portuguese athlete died tragically while running the Marathon [12], even if he was celebrated as a National hero.

It was only the Estado Novo (Portuguese name for the Fascist Dictatorship between 1926 and 1974) that created a national dynamic for sports, through the "Mocidade Portuguesa". This meant that young people that attended the secondary schools had right to some sporting activities particularly on weekends. Also, the sport of the working class was promoted. A National Stadium was built near Lisbon, coupled with three University Stadiums in the three main cities—Lisbon, Porto and Coimbra. Some minor facilities were developed in secondary schools like gymnasiums and track and field. But as the drop-out rates were massive, and the educated workers were few, the "Estado Novo" sports continued to be very elitist. In accordance, the results of that investment in the international arena were very scarce: until 1960, the country won only six medals in the Olympic Games, none of them of gold and only one of them silver (see Table 4, [11]); and the only sport in which the Portuguese Team was regularly champion of Europe and the World was roller hockey, a Latin version of field hockey, in fierce competition with Spain and Italy [3].

As it happened with the economic evolution, the period 1960–1973 was marked by some sportive successes, some would say by some awakening. In the football grounds, led by Eusébio (aka the Black Panther) Benfica own two Champion Leagues (1961 and 1962) and went to two more finals (1963 and 1968), while the Selecção was only beaten the World Champions England in the 1966 tournament. Fantastic sportsmen also appeared in cycling (Joaquim Agostinho made very good showings in the cycling Tour de France from 1968 until 1983) and roller hockey (Antonio Livramento being the best player) in which the Selecao continued to win Euro and World Championships. However in the Olympic Games the lack of results continued and in 4 editions (1960, 1964, 1968 and 1972) the country only succeeded to have 1 medal, in sailing.

The revolution of 1974 tried to change things, massificating sports. But the country was in convulsion and financial resources were very scarce. Anyway, some results began to happen, mainly in athletics, by the hand of the sporting coach Mario Moniz Pereira. And in 1984, Carlos Lopes, one of his pupils, achieved the first Portuguese Gold medal in the Games. Suddenly road races became popular, on Sunday mornings. Some other good surprises had happened with the medals of Antonio Leitão (bronze in 5,000 m track in Los Angeles), Rosa Mota (also bronze in the Marathon in LA) and Armando Marques (silver in shooting in Montreal). Sports that were traditionally strong suffered the effect of the revolution but Eusebio's successors managed to be third in the Euro 1984 and some good results continued to be obtained in roller hockey. Benfica was the main team in football with some good results but no win. Tragically however Joaquim Agostinho died in 1985, while racing, and until today

Portugal has yet to find a giant cyclist like him—may be in the 2013 World Road Champion, Rui Costa.

The European adhesion and the success of its first 15 years was translated in a very big investment, a much bigger rate of participation in sports and some really improving results. In Athletics, and following Carlos Lopes, a considerable number of competitors (men and women) own European, World and Olympic medals. Rosa Mota, winner of the first Marathon in the European Championships effectively started the mood for women in sports; thankfully she had many brilliant followers, the best of all Fernanda Ribeiro, twice Olympic medalist. Lead by its controversial President Mr. Pinto da Costa, FC Porto became a force in European Football wining the Champions League in 1987. Other very good competitors appeared in Sports as diverse as sailing, and judo (see Table 4).

Finally, since the Euro became the Portuguese currency, a sense of stability has prevailed. Portugal continues to be important and very known in the world stages of football, exporting players (Figo, or Ronaldo) and coaches (Mourinho, being the best example). FC Porto accrued it importance in Europe with successes in 2004 and 2011. The athletics team earned a place in the best 12 in Europe, but Kenyan and Ethiopian super runners replaced the Portuguese at the top of the world rankings. Champions continued to exist in many disciplines, obtaining the occasional medals in the Olympic Games in sports such as athletics but also cycling, triathlon, and canoeing (see Table 4 also).

Therefore, as a general idea, Sport in Portugal was elitist until 1974, being at the service of the regime since 1926. Results were meager until then. With democracy investment happened and results appeared. But even today, particularly in Olympic sports, Portugal is a country of occasional champions and good intentions. The only sport in which the country has an established industry with stable success is professional football. Even in sports in which Benfica, FC Porto and Sporting Lisbon have teams and athletes, besides professional football, international success is a consequence of individual talent rather than societal investment or the perception of sport as an industrial activity.

4.3 History of Sport in Portugal: Main Statistics

The data that describe the results of the Portugal in football or in the Olympic Games are exposed in the following four Tables. Table 1 shows the stable success of Portuguese clubs in the UEFA competitions.

It is worth mentioning that with the exception of the post-revolution troubled years and 1971, the country was always in the top 11, and in the last 30 years Portugal has been in the top 6, 7 as a rule. That very good classification did not get worse with the drastic increase in the number of countries, following the USSR disintegration.

Table 2 highlights the highest results of Portuguese clubs in the same competitions.

In total the four major clubs participated in 17 finals and won 7 times. For such a small nation it is a remarkable achievement.

Table 1 Portuguese performance in the European football club competitions [7]

Year	Ranking	Number of clubs	Countries
1960	22	1	27
1961	7	1	27
1962	6	4	29
1963	4	5	30
1964	4	4	31
1965	4	5	32
1966	8	5	33
1967	11	5	33
1968	8	5	33
1969	9	6	33
1970	11	6	33
1971	13	5	33
1972	9	5	33
1973	9	5	33
1974	7	4	33
1975	10	4	33
1976	9	4	33
1977	11	4	33
1978	12	4	33
1979	14	4	33
1980	14	4	33
1981	15	4	33
1982	13	4	33
1983	9	4	33
1984	7	4	33
1985	7	4	33
1986	9	5	33
1987	6	5	33
1988	6	6	33
1989	7	5	33
1990	5	5	32
1991	6	5	33
1992	7	5	33
1993	7	5	39
1994	6	5	44
1995	6	5	47
1996	6	5	48
1997	6	5	48
1998	7	6	49

(Continued)

Table 1 (Continued)

Year	Ranking	Number of clubs	Countries
1999	9	6	49
2000	10	6	50
2001	10	4	51
2002	9	4	51
2003	7	4	52
2004	6	4	52
2005	6	6	52
2006	6	6	52
2007	6	6	52
2008	8	7	53
2009	10	7	53
2010	9	6	53
2011	6	5	53
2012	5	6	53
2013	5	6	53

Table 2 Portuguese finalists in the European football competitions (UEFA)

	Porto	Benfica	Sporting	Braga
Champions League				
Winner	2	2		
Runners up		5		
Europa League				
Winner	2			
Runners up		2	1	1
Cup Winners Cup				
Winner			1	
Runners up	1			

Furthermore Table 3 describes the results of Portugal in the Olympic Games.

The best position the country achieved was 23rd in 44 countries in 1924 or 26 in 59 countries in 1948. In the whole Portugal is in 63th place in 204 Olympic committees, a position which is not bad but which is much lower than the one achieved in relation with football.

Table 4 shows the medals which highlight the country's participation in the Games.

The persons included in the list deserve to be named as national heroes given the difficulty and the rarity of what they have achieved.

Table 3 Portuguese performance in the olympic games (POC)

Games	Athletes	Gold	Silver	Bronze	Total	Rank	Countries
1912 Stockholm	6	0	0	0	0		28
1920 Antwerp	13	0	0	0	0		29
1924 Paris	30	0	0	1	1	23	44
1928 Amsterdam	31	0	0	1	1	32	46
1932 Los Angeles	6	0	0	0	0		37
1936 Berlin	19	0	0	1	1	30	49
1948 London	48	0	1	1	2	26	59
1952 Helsinki	71	0	0	1	1	40	69
1956 Melbourne	11	0	0	0	0		67
1960 Rome	65	0	1	0	1	32	83
1964 Tokyo	20	0	0	0	0		93
1968 Mexico City	20	0	0	0	0		112
1972 Munich	31	0	0	0	0		121
1976 Montreal	19	0	2	0	2	30	89
1980 Moscow	11	0	0	0	0		80
1984 Los Angeles	38	1	0	2	3	23	140
1988 Seoul	65	1	0	0	1	29	159
1992 Barcelona	90	0	0	0	0		169
1996 Atlanta	107	1	0	1	2	47	197
2000 Sydney	61	0	0	2	2	69	199
2004 Athens	81	0	2	1	3	60	202
2008 Beijing	77	1	1	0	2	46	204
2012 London	77	0	1	0	1	69	204
Total		4	8	11	23	63	204

4.4 Some Comments, Comparisons and Clarifications

The Portuguese situation we just described can be clarified even further, so that the dichotomy between football and Olympic Games becomes even more evident:

(a) In relation to football Portugal has usually been placed among the countries that follow the big European countries (England, France, Germany, Italy and Spain). In fact the Portuguese clubs have managed to be in front of clubs from big countries and sporting potencies has Poland, Russia, Ukraine or Turkey. This is quite a sensational result for such a small country.

(b) Furthermore in football Portugal has had better results than the other European countries with the same size (as Austria, Belgium, Hungary or Sweden). For that class of “small countries” only the Netherlands (with 16 million habitants) has managed to obtain similar results to those of Portugal (see Table 5 column 3).

Table 4 Portuguese medalists in the olympics (Portuguese Olympic Committee)

Medal	Medalist	Olympiad	Sport	Event
Bronze	António Borges	1924 Paris	Equestrian	Team jumping
	Hélder de Souza			
	José Mouzinho			
Bronze	Frederico Paredes	1928 Amsterdam	Fencing	Team épée
	Henrique da Silveira			
	João Sassetti			
	Jorge de Paiva			
	Mário de Noronha			
	Paulo d'Eça Leal			
Bronze	Domingos de Sousa	1936 Berlin	Equestrian	Team jumping
	José Beltrão			
	Luís Mena e Silva			
Bronze	Fernando Paes	1948 London	Equestrian	Team dressage
	Francisco Valadas			
	Luís Mena e Silva			
Silver	Duarte Bello Fernando Bello	1948 London	Sailing	Swallow class
Bronze	Joaquim Fiúza	1952 Helsinki	Sailing	Star class
	Francisco de Andrade			
Silver	Mário Quina	1960 Rome	Sailing	Star class
	José Quina			
Silver	Carlos Lopes	1976 Montreal	Athletics	Men's 10,000 m
Silver	Armando Silva	1976 Montreal	Shooting	Men's trap
	Marques			
Gold	Carlos Lopes	1984 Los Angeles	Athletics	Men's marathon
Bronze	António Leitão	1984 Los Angeles	Athletics	Men's 5,000 m
Bronze	Rosa Mota	1984 Los Angeles	Athletics	Women's marathon
Gold	Rosa Mota	1988 Seoul	Athletics	Women's marathon
Gold	Fernanda Ribeiro	1996 Atlanta	Athletics	Women's 10,000 m
Bronze	Hugo Rocha	1996 Atlanta	Sailing	Men's 470 class
	Nuno Barreto			
Bronze	Fernanda Ribeiro	2000 Sydney	Athletics	Women's 10,000 m
Bronze	Nuno Delgado	2000 Sydney	Judo	Men's 81 kg
Silver	Francis Obikwelu	2004 Athens	Athletics	Men's 100 m
Silver	Sérgio Paulinho	2004 Athens	Cycling	Men's road race
Bronze	Rui Silva	2004 Athens	Athletics	Men's 1,500 m
Gold	Nelson Évora	2008 Beijing	Athletics	Men's triple jump
Silver	Vanessa Fernandes	2008 Beijing	Triathlon	Women's competition
Silver	Fernando Pimenta	2012 London	Canoeing	Men's K-2
	Emanuel Silva			

Table 5 Comparing the comparable

	Population	Football Clubs rank in the EUFA	Olympic summer games medals, rank
Austria	8.5	14	40
Azerbaijan	9	30	58[a]
Belarus	9.5	20	48[a]
Belgium	11	10	31
Bulgaria	8	28	23
Czech Republic	10.5	16	45
Greece	11	12	32
Hungary	10	27	8
Israel	8	19	85
Portugal	10.5	5	63
Serbia	7	25	84[a]
Sweden	9.7	23	10
Switzerland	8	13	25

[a] Countries participating only after 1996

(c) But, in what concerns the Olympic Games the country has much less success than the other countries of the same dimension (Austria, Belgium or Hungary) (see Table 5 column 5).

Therefore among these 11 same size European countries Portugal is by far the best positioned regarding football, but it is the worst positioned regarding the Olympic Summer Games. Hungary and Sweden have somehow the reverse position, being in the top 10 of the Olympic rankings but only in the top 30 of European football.

4.5 Qualifications and Explanations for this Situation

The data we presented generate several questions:

(a) Why is football so important in Portugal?
(b) Is Portugal so small to have anything else but football?
(c) Why is the unbalance between football and the Olympic Games so big? Is investment the clue? Or the way sports are managed? Or are the two things related?

Regarding the first question, the cultural and even religious importance of football in the country also can't be underestimated. In the country 3 daily newspapers exist (O Jogo, A Bola and O Record) printing in total around 200,000 copies each day; they usually use at least 80 % of the pages on football matters, and they are somehow affiliated to the three main clubs (FC Porto, Sporting and Benfica) respectively. Football news are regularly present time in daily news of radio and TVs and several programs discussing football matters exist in the three main TV channels. Regularly one half (4 in 8) of the matches of the Portuguese PrimeiraLiga are broadcasted by Sport TV, which also transmits UEFA Competitions, the Spanish League and the England League meaning that almost every night there is a football match that may be seen in the Portuguese TV. The importance of football is also felt in the Social Media, with every important match or occurrence of the three big clubs, plus Braga, the Selecao or some individualities like Mourinho or Ronaldo generating massive activity in the social networks. Furthermore, we could suggest that some supporters are so deeply involved in the Sport that we may say that the Dragon Stadium (from FC Porto), the Stadium of Light (from Benfica) and the Alvalade XXI Stadium (from Sporting Lisbon) are the new cathedrals of Porto and Lisbon respectively. In this context and with all the due respect, football appears almost a second, light and laicized religion, the major players being like idols or demi-gods. Anedoctically it is not for nothing that the President of FC Porto has the nickname of "The Pope", or that a cartoon caricaturizing Eusébio named in "(D)Eusébio", Deus meaning God in Portuguese. Moreover, the importance of football has historical roots and political implications. The fascist regime was said to be only preoccupied with 3 Fs—Fátima, Fado and Football [8]. But, with democracy, it seems that not everything changed for the best and some stories exist about the "dangerous connections", above all in small municipalities, between the elected party, the football clubs, and the construction and public works companies. Football is therefore sometimes in the center of a web of corruption with ramifications.

Regarding the second question: small countries tend to do much better than Portugal in other sports, but it remains to be seen if the investment in the Olympics would not divert resources from football. Anyway, we believe that things could improve in other sports and not decrease that much in football if there would be a serious sporting policy in the country. It is said that there are 10 million football coaches in the country, and that the sport most of the persons practice is seeing football on the TV in the sofa. This passive way of enjoying sport is reflected in the small figures the country has in the European Statistical Survey on Sports practice [5]. Finally if the country began to generate champions in other sports than football, the "nationalistic" part of the national enjoyment of sport would continue to exist, only its object would have been changed if not augmented.

Finally on the third question we believe there are at least two fundamental answers to the question. The first is linked with the level of investment. The second relates to the management attitude. In relation to the investment, the level of investment of Portugal in the Olympic preparation has been around 20 Million Euros (MEs), for the 2008 and 2012 Games, and will not grow for Rio 2016. Those amounts are much less than the investment of the UK in the cycling team that dominated the London Olympic and it is less than the investment of UK Sports in the most important disciplines like Cycling (40 ME), Rowing (40 ME) Athletics (30 ME), Sailing (30 ME) or Swimming (25 ME) [6].

But in the other hand, also in relation with investment, the four big Portuguese clubs are having annual budgets of at least 20 MEs (Braga) until 100 MEs (Porto). These numbers pave the way to the second part of this third question. The clubs can only be profitable because they are competitive, because they have professional managers, and because they know how to benefit from merchandizing and intangible structures of market. In a word, the four top Portuguese football clubs are profitable and winning companies. And a virtuous cycle of investment and returns (financial and sportive) exist in them. Unfortunately not the same can be said about the Portuguese Olympic Committee (POC). The POC has been managed by political appointees from the Government or retired professionals. The goals set have never been to win medals but to participate. In consequence Portugal has had traditionally larger than average teams, but less than average medal figures. And the virtuous cycle of success in football only can be mirrored by the vicious cycle of the Olympic Games.

4.6 Discussion of the Theoretical Model

Applying the concepts presented in the theoretical section, we find that the Sport sector in Portugal is dominated by the football subsector. In accordance, the market is biased to football and regarding football demand, supply and public support are high. Regarding the other Olympic sports, demand, supply and public support tend to be low. Quite interestingly, the value of football has been high, and produced using a funding model of Media-Corporation-Merchandizing-Markets (MCMM); but regarding the Olympic sports, which sometimes are still named as the "Amateurs sports", the value generated has been low, and the funding model used has been at best Spectators-Subsidies-Sponsor-Local (SSSL), when not pure amateurship. Finally, as a whole, Sports in Portugal has had a positive microeconomic impact in the persons and geographical areas that relate do professional football: footballers, managers, even businesses which relate to the winners and to football may have had benefited from it; the microeconomic effect of the Olympic sports has been much less even if occasionally some very few people and organizations may have profited from it. This microeconomic situation implies that traditionally and for long football has had a certain positive macroeconomic impact in Portugal that Olympic sports don't have; the fact that Portugal organized the Euro 2004 and never succeeded building a bid for organizing the Olympic Games speaks volumes about this specific topic.

5 Concluding Comments

5.1 Conclusions

The Portuguese football arrived at the 21st century long ago, adopting a modernized, risk assuming, competitive and professional attitude. The results, at least at club level are there for everyone to see. The four main clubs are in fact small companies with a multinational instinct, managing multinational teams aiming European glory. And they have been successful at least as a group and sometimes individually, each team having at least reached one European final since 2000. Portugal exports footballers and managers, and the discipline is well established as an economic sector. Cultural roots and elements help but they don't explain everything.

The Portuguese management of the Olympic Sports is basically still in the 20th century, and to be fair to form, sometimes it would seem using the practices described by the famous film "Chariots of Fire", describing pre-WWII practices. Amateurship may not be total but professionalism is in serious shortage, the athletes being left at their own devices to overcome with their own talent situations their rivals face in big and well organized teams. Victories, which are hailed as historical feats, may not be a miracle, but sometimes the genuinely sound like.

5.2 Recommendations

Portugal has a well-developed national policy and system on football, which produces results. But the country has nothing comparable in terms of the Olympics and therefore results don't happen. We therefore recommend a more balanced investment and the presence of the 4 big football clubs in the Olympic sports as a way of improving the Portuguese performance in the Olympics. That improvement would have reflections in the country health and productivity, and would be socially beneficial.

5.3 Suggestions for Further Studies

We would like to inspect the three following hypothesis:

1. The relation between the level of investment in football in Portugal with the level of investment in other European countries corresponds to the relation between the level of success in Portuguese football in relation with the same European countries. So, Portuguese clubs have a success above the average because they invest more than the average;
2. The relation between the level of investment in the Olympic Games in Portugal with the level the level of investment in the Olympic Games in Europe, explains the Portuguese lack of results in the Olympic Games when compared with other European countries. Thus, Portugal has less success in the Olympics than the other European countries because Portugal invests less than the average in Olympic sports;
3. The relation between the level of investment in football when compared with the level of investment in Olympic sports, is related with the level of results in football when compared with the level in Olympic sports. The lack of investment in Olympic sports explains in comparison with football explains the lack of results.

We would expect that in all the three cases, the level of investment explains much of the success in Sport.

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Senior Sport Tourism in Russia

Marina Sheresheva and Ekaterina Buzulukova

Abstract The purpose of the paper is to identify the main trends in the senior tourism development in Russia and to find out the main triggering factors which may entice local seniors to consume sport travel services. The study presented in the paper aims to examine the key determinants that should be taken into account in senior tourism marketing, as well as the specifics of consumer behavior in the consumption of senior sport tourism on the Russian emergent market. The anticipated results should fill the gap by identifying the current sport travel motivations of Russian seniors.

Keywords Senior tourism · Sport tourism · Consumer behavior · Russia

1 Introduction

Nowadays, sports and tourism management has become an important new field which has been developed by academics and practitioners. Understanding, and being able to apply, the principles of management is crucial for any organization operating in fast growing and constantly changing sport and tourism sectors [7, 10, 14, 23, 44, 48, 58].

There are lots of new opportunities in these sectors due to the influence of new technologies as well as the changing demand for different kinds of sport and tourism products. Looking at the developments in the field, one can specify inter alia fast rising of sport tourism. As sport tourism has become an ever more prominent activity, different kinds of studies in this broad and diverse topic area has also developed [8, 10, 21, 22, 25, 26, 29, 54, 69].

M. Sheresheva
Lomonosov Moscow State University, Moscow, Russia
e-mail: m.sheresheva@mail.ru

E. Buzulukova (✉)
National Research University Higher School of Economics, Moscow, Russia
e-mail: ebuzulukova@hse.ru

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Highly dynamic sport and tourism industries are quite strongly influenced by changes that take place in the contemporary global society. That's why there is a need to adjust supply to the changing demand, to find out the main requests and needs of consumers in the field, and to be ready to present sport and travel products to new emerging audiences [10, 12, 31].

One of the main trends in contemporary society is population aging [1], also called "the demographic transition" [11]. It can be defined as a process of increasing percentage of older people in proportion to the total population. In accordance with the world population statistics, increase in older adults is unprecedented in human history and is forecasted to result in prevailing of seniors in the world population by 2050 [68]. This trend has a great impact on all markets and businesses [33, 70]. Taking this into account, there is a need to unveil the importance of this audience for contemporary and future sports tourism business.

In the last two decades, an increasing amount of publication touches upon the issues of senior sport and/or tourism [19, 20, 37, 38, 43, 46, 49, 63]. The main stream of research in the field is done on the example of advanced economies, while emerging markets are still not well examined. The number of papers investigating specifics of older tourism consumers' behavior in emergent societies is relatively small, and there was no published research found on senior sports tourism in Russia. Thus, there is a clear need to fill the gap and to conduct research in the field.

The purpose of the paper is to identify the main trends in the senior tourism development in Russia and to find out the main triggering factors which may entice local seniors to consume sport travel services. The study presented in the paper aims to examine the specifics of consumer behavior in the consumption of senior sport tourism on the Russian emergent market. The anticipated results could provide understanding current sport travel motivations of Russian seniors and thus help identifying the key determinants that should be taken into account in the senior sport tourism marketing.

The paper is organized around the following topics. Firstly, we focus on the literature on the subject, including papers investigating the specifics of sport and tourism consumers, the influence of population ageing on tourism market, senior tourism and senior sport travel motivations. Secondly, we give a brief overview of the methodology and research design. Thirdly, the results are presented of the study conducted in 2013 in order to highlight attitudes of Russian seniors to the existing and potential opportunities to be engaged in sport tourism, both as funs visiting sport events and as members of different sport travel groups. In particular, the main preferences and factors preventing aged people to take part in sport tourism are discussed. The results of the study contribute to understanding older consumer behavior concerning sport tourism services and can be useful as a first step leading to further research of marketing strategy on the Russian senior tourism market.

2 Senior Consumers in Sport and Tourism: Literature Review

2.1 Sport and Tourism Consumers

Consumers are the focal point of any business activity on contemporary markets. As to sport and tourism industries, they are part of the service sector, for which a particular approach is necessary [24, 31]. It is widely accepted that services are distinct from goods because they are characterized by intangibility, heterogeneity, perishability, and inseparability [9, 41]. Services marketing "is about promises—promises made and promises kept to customers" [72, 355], and about customer experiences [53]—the sum of all experiences a customer has with a supplier of services, over the duration of his/her relationship with that supplier.

In many recent studies it is pointed out that there are the same core marketing issues and concepts which can be used in sports and tourism marketing, but in the same time there are some peculiarities which impact the implementation of the traditional marketing concepts in this fields of business [7, 14, 48, 57, 61].

A good understanding of sport consumers—be they spectators, participants, or other stakeholder (e.g. a sponsor or corporate guest)—is required to enable a more targeted sport product offering and, ultimately, a more satisfied customer [10, 32, 42, 56, 62]. The most important component of consumer satisfaction is participation in the sport consumption experience (attraction, interaction, awareness, discovery, etc). Advanced sport businesses can begin charging for the value of the "transformation" that an experience offers.

This is true also for tourism which can be seen as a way to meet a number of customer needs, including intellectual component (to undertake mental activities), social component (to make friends, build relationships, etc.), physical activities (to master new skills, challenge oneself or compete with others), and "stimulus-avoidance component" (to relax, to avoid stress and problems) [6]. In other words, there are different categories of tourism consumer motivation to be taken into account, including physical, cultural, interpersonal, status and prestige motivators [45].

There are different sport consumer typologies [64] as well as tourism consumer typologies [66]. In our case the age was used as the main criterion for distinguishing different groups of consumers, since the research question was what are the key determinants and specifics of consumer behavior in the consumption of senior sport tourism.

It was also important to choose a motivation model which could suit our research purposes.

The motivation for sport and tourism has been defined in different ways by numerous authors.

Inkson and Minnaert [31] distinguish between tourist motivation models using "four themes: tourist motivation as a result of needs; tourist motivation in relation to a 'centre'; tourist motivation as a ladder/career; and tourist motivation as a combination of push and pull factors". "Tourism in relations to needs" model was chosen which propose that tourists become involved in tourism to fulfill a need that cannot easily be fulfilled in their own environment.

2.2 Senior Consumers in Sport and Tourism

The high potential of the senior market has already captured the attention of the tourism industry. The World Tourism Organization [71] has called the senior market "an opportunity for growth for the twenty-first century". This makes necessary for tourism businesses to adapt their proposition in accordance with the changes of the average age of tourists. As Le Serre and Chevalier [37] underline, no one can deny that, from a market viewpoint, this target is highly attractive in regards to the tourism consumption. The size of this population is significant and will continue to rise; they have enough free time and financial resources required for tourism activities; on average, they are better educated, more lively and ready to travel than the previous generation; and they stay on vacation longer than the others [2, 35, 71].

In the last two decades, an increasing amount of publication touches upon the issues of senior tourism [37, 38, 43, 49, 63]. All the recent investigations propose a strong incentive for paying attention on older adults as tourism consumers and taking into account travel propensities of the elderly people [3, 36, 37, 47, 52, 67]. Age-related lifecycles are investigated [18], segmentation models of senior consumers are proposed [28, 63, 65]. The major senior travel motivations, including perceived safety and self-actualization's need, are pointed out [15–17, 27, 30, 39, 40, 51], as well as impeding factors like health, financial issues, personal preferences, and external threats like terrorism and crime, which might affect the travel decision of the elderly, are discussed [49].

Being interested in understanding senior sport tourism consumers, we aimed to find an appropriate balance between motivations of sport consumers and tourism consumers who are of rather old age and thus have some limitations (health, finance), on the one hand, and can be an attractive target audience, on the other hand.

The perception of elderly people as an important target audience for sport tourism is questioned by a very strong myth that this age group cannot be interested in sports tourism because of health problems. In addition, their overall financial situation is believed to be highly disadvantaged. Many marketers do not recognize the changes in the material condition and in the mindset of the elderly brought by "baby-boomers" and now moving into this age group and thus boosting the so-called "rise of the young-old" [50]. The size of this population is significant and will continue to grow. On average, they have enough free time and financial resources, they are better educated, more lively and ready to travel than the previous generation [2, 35, 37]. Also, marketers tend to underestimate the financial state of adult children who are often an ultimate, or at least a significant source of well-being for their parents.

Our analysis of the literature on the subject has shown that the main stream of research in the field is done on the example of advanced economies, while emerging markets are still not well examined. The number of papers investigating specifics of older tourism consumers' behavior in emergent societies is relatively small (see, for example [27]), and no papers were found to dealing with the problems of marketing sport tourism services for elderly people in emerging markets, including Russia.

This is a gap which needed to be filled, because emerging markets represent a significant part of the world economy and tend to expand their share. It is estimated that by 2035, the gross domestic product of emerging markets will permanently surpass that of all advanced markets.

This century is likely to be about marketing in the emerging markets, and the firms which have ambitions to succeed have to adapt their marketing practices [5, 13, 60]. According to Sheth [60], emerging markets should be considered as natural laboratories for marketing researchers, in which theories and assumptions can be tested, new generalizations derived, and new elements of theories are operationalized in specific settings.

2.3 Senior Tourism in Russia

Russian tourism industry has been experiencing an impressive growth since the beginning of the last decade. Currently, there are about 6.500 travel agencies (in 2002—3.300) operating in Russia; the number of tourists served by travel agencies increased from 2.8 million in 2002 to 7.7 million in 2008 [4].

Russian senior market appears to have demographic characteristics which are congruent with those for the global ageing population. In accordance with Rosstat data, contemporary Russia is home for around 50 million people 50 years old and over (that means, one-third of the entire population). Moreover, citizens aged 65+ make up about 13 % of the Russian population [55], while, by international standards, 7 % is enough to consider the population as an "old" one.

But there is still little, if any, attention to the senior segment of the market. Senior tourists in Russia are not numerous, as compared to other ages, and the last decade witnessed only a slight increase of the tourism consumption in this group [55]. It is partially due to the fact that Russian old adults are considering mostly internal travel opportunities [59], and the weak tourism infrastructure and poor marketing in the tourist sector make it difficult for tourists to learn about and visit local attractions [4].

Existing research in the field of senior tourism can be described as fragmentary. There is almost nothing to tell about peculiarities of senior tourism consumers in Russia, and there seems to be no attempts to study the issues of senior sport tourism services for elderly Russians.

At the same time, preliminary results of our pilot research uncovered the need for senior sport tourism services in Russia and therefore the need for marketers to understand Russian seniors' behavior and motivations more deeply, and to encourage them not only to consume but to co-create sport tourism products.

The study presented in the paper aims to fill the gap and to examine the key determinants that should be taken into account in the senior sport tourism marketing, as well as the consumer preferences in the segment.

3 Research Design and Methodology

The research design combines both qualitative and quantitative methodologies as part of the study.

The desk-based investigation method was used for understanding the current trends in senior tourism marketing in Russia and abroad. A wide range of both foreign and Russian literature, the results of a number of relevant studies as well as statistics, had to be examined to define the current situation on the market and to reveal the key factors that help to motivate older adults to travel. A number of in-depth interviews with open-ended questions were held with experts, from both academic and business fields of tourism marketing in Russia.

Research hypotheses were as follows:

- H1: Russian seniors are engaged in sports tourism or would be happy to be engaged
- H2: The main perceived barriers to travel of Russian seniors are lack of money and fear of health risk
- H3: Internet is growing in importance as a communication channel for senior sport consumers in Russia

The study was designed on a basis of structured questionnaire developed by authors using Laszlo, Zsuzsa [34, 37] approach. There were 22 survey questions grouped into five blocks, with the aim to identify, firstly, the purpose, nature and destinations of recent touristic trips; secondly, sport interests of respondents; thirdly, preferences and frequency of Internet use (if using at all); fourthly, the respondents' requests for sports tourism; and, finally, the socio-demographic characteristics of respondents.

Empirical data for the study were collected in the first half of 2013. Pilot survey in Moscow and Moscow region in Russia was conducted in May 2013 and has resulted in a sample of 69 respondents aged 50 years and over. After collecting the completed questionnaires, the data gathered were transferred to electronic form, processed and analyzed. Some preliminary findings based on pilot data analysis are presented and discussed below.

4 Preliminary Findings

4.1 Sample

The convenient sample consists of 69 respondents aged 50 years and over, 83 % woman and 27 % men. More than 70 % have higher education and only 19 % secondary school education. The retirement age in Russia equals 55 years for women and 60 years for men, and there were 42 respondents of retirement age in the sample. The vast majority of respondents live in big cities (93 %), 26 % can afford to spend money on travel, about the same percentage of respondents answered that they have enough money only for food and household goods. Thus, many Russians aged 50+ are able to travel in country and abroad. These data were confirmed by the fact that 40 % of the respondents indicated that they had more than 3 trips in last 5 years. It should be noted, that 65 % of the sample feel themselves younger than their actual age. According to Le Serre and Chevalier [37], psychological age is one of the most important factors that determine the level of involvement in sports. Our results also have approved this suggestion, since most of the respondents who are interested in sports activities feel themselves younger than their actual age.

4.2 Data Analysis

The research results obtained could be divided into 4 parts. Firstly, we investigated the character of travel activity of respondents and perceived barriers for travelling. Secondly, favorite sport activities and interests of the respondents, thirdly, the readiness to sport tourism and current interests in watching and visiting sport events, fourthly, the internet activities of older people and channels that could be used for promoting tourists' services.

The research results obtained can be divided into four parts.

The first part concerns the tourist activity of the respondents and the perceived barriers to travel. In the second part, the respondents' favorite sports and interests are unveiled, in the third part the respondents' preferences in watching and attending sporting events are identified. Finally, the trends in the use of the Internet by Russian seniors are determined.

In accordance with the survey results, only 22 % of the respondents did not travel during the last 5 years. Looking at the preferred travel destinations and activities of the sample, one can see that about 30 % of trips were within Russia, 16 % to the countries of the former Soviet Union, 27 % to Europe, 22 % to Asia, and 20 % to other destinations. The respondents were divided into those who prefer to travel with husband/wife or beloved person (35 %), with children and grandchildren (29 %), and with friends (25 %). More than 30 % are used to live in 3* or 4* hotels, 22 % prefer more expensive and reliable hotels, 21 % like to stay by relatives or friends, 13 % are satisfied with hostels.

For more than half of the elderly travelers in the survey, sightseeing is the best kind of tourism (54 %), beach vacations are also very popular (39 %), as well as entertaining tours (28 %), fishing and hiking (20 %). One could not expect active participation of the vast majority in sporting events, still 13 % of seniors in the pilot survey appeared to be engaged in such activity.

These data show the willingness of many Russian seniors to participate in sports. At the same time, there are barriers that prevent them from more active travelling (see Fig. 1).

The lack of money was most frequently mentioned as the main factor preventing the respondents from travelling (37 % of answers), however about two thirds of them do not consider money as main problem. Other factors, including health risk,

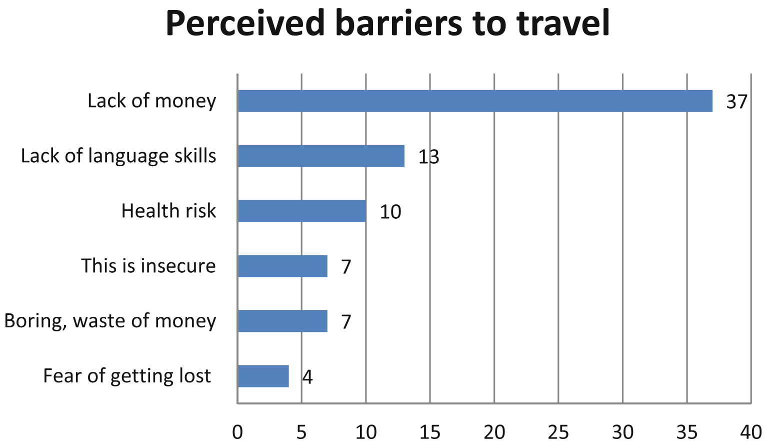

Fig. 1 Perceived barriers to travel

appeared to be much less important. Only 34 % of the respondents mentioned that they have illnesses that influence on travel activities. More than 85 % of them didn't agree with the statement that 'travelling is not for me'. Employment was one more factor that influenced the frequency of travel: 60 % of respondents, including retirees, did not have enough time to travel due to their fully or part-time employment.

The second block of questions was devoted to sports activities and interests of elderly Russians. According to the results of the pilot survey, 45 % of respondents continue to tribe sports they used to go on in their youth, weekly or more often. The most popular sport activity is walking and physical training exercises. Of course, a number of favorite non-sport activities were mentioned, such as gardening, crafts, training etc. 17 % of respondents admitted that they go on sports quite rare, all the rest are not interested in any of sports activities. The graph (Fig. 2) shows the contemporary sport interests of the respondents.

According to the data obtained, swimming, hiking, fishing, bicycle, cross-country skis and rafting are the most popular sports activities for Russian seniors. There are 40 % of respondents were involved in sports activities in youth and now have stopped do it. The most attractive destinations of sport tourism could be travelling to the seaside or riverside, and to mountains. When asked about attractive destinations of sports tourism, most respondents have chosen journey to the nature.

Further the willingness and readiness to sports tourism was investigated. It should be noted that 32 % of the respondents were not interested in sports tourism. Others were interested in hiking (32 %), sea travel (21 %), boating and rafting (16 %), fishing (15 %), horse riding (10 %), and some other sports tourism activities. The most attractive destinations are Russia (local tourism), Europe and the United States. The majority of respondents indicated that they have an active lifestyle (75 %) and that the government, from their point of view, should take care of them and give free rides to travel to improve health (62 %).

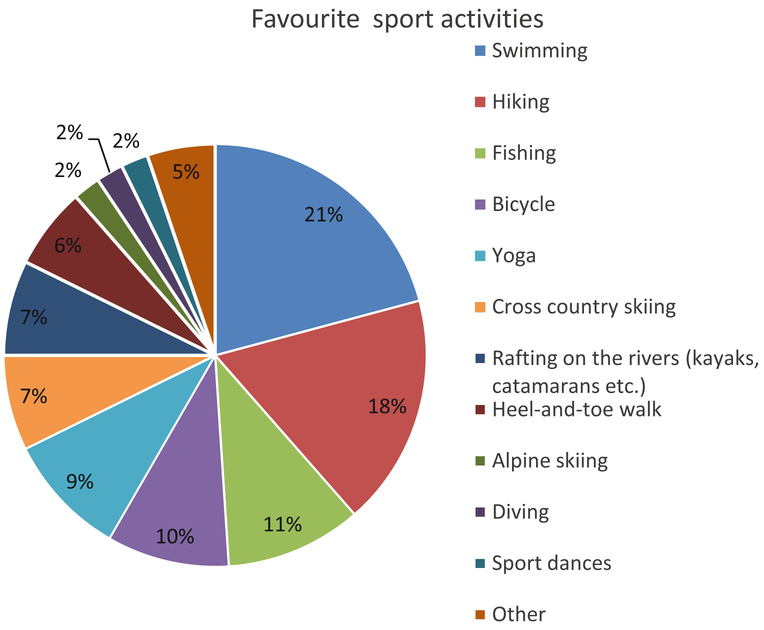

Fig. 2 Favorite sports activities

We initially hypothesized that people who are interested in sports also like to visit the sports tournaments. Respondents were asked whether they were interested in participating in sports tournaments as spectators or even participants. 36 % appeared to be fully indifferent to sporting events, 49 % of respondents were interested in attending sporting events as a spectator only, 15 % possibly as participants. At the same time, 80 % ready to go abroad, with the aim to attend such sporting events like the Olympic Games, tournaments in which their relatives are taking part, and some other international competitions. The graph shows what kinds of sporting events respondents prefer to watch, or visit (Fig. 3). Dance competitions, gymnastics, football and hockey were mentioned as the most attractive kinds of sport that attracts Russian seniors.

More than half of all seniors participated in the survey appeared to use Internet every day (52 %), and only 17 % don't use Internet at all. The web resources that are visited most actively are shown on Fig. 4.

Based on these responses, it is possible to suggest that Internet is growing in importance as one of the marketing communication channels, withal not so expensive.

Social networks appeared to be one of the most popular web resources used by respondents. Moreover, 38 % of respondents are present in more than one social network. The most popular social network is Odnoklassniki (Classmates) (37 %), Facebook is on the second place (25 %). Seniors also use Vkontakte (In Contact) (19 %) and MoyMir (My World) (14 %). Thus, there is a need to use social media marketing as a means to promote tourism services to elderly people.

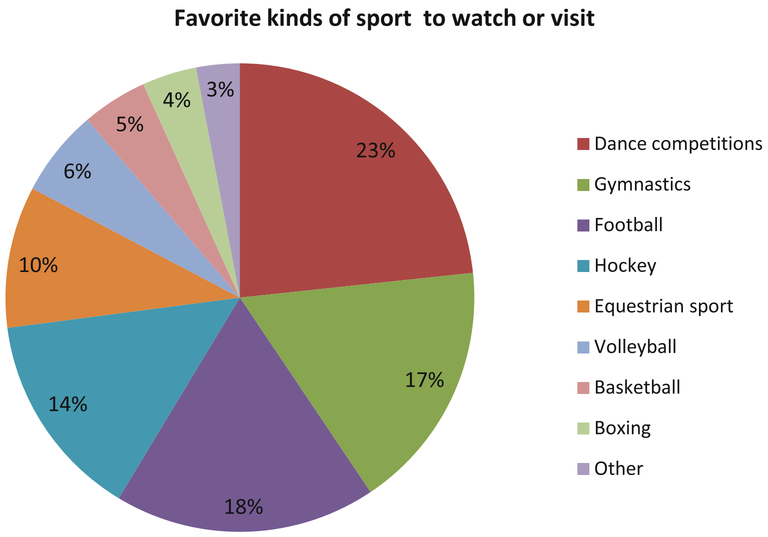

Fig. 3 Types of sports that the respondents prefer to visit

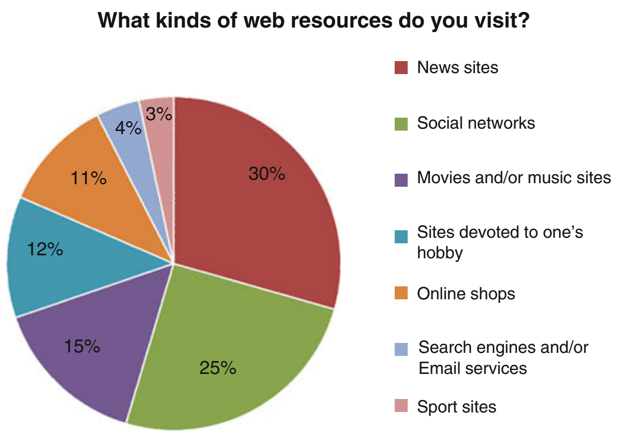

Fig. 4 Web resources most actively used by respondents

5 Discussion and Managerial Application

Preliminary findings presented are of interest for academics engaged in research in the field of population aging and sport tourism in emergent markets.

The desk research and the pilot survey conducted in 2013 have confirmed the research hypotheses listed above.

H1 was confirmed: many Russian seniors are engaged in sports tourism or would be happy to be engaged. Taking in account the number of seniors in Russia, there is a new promising audience for tourism companies, including those providing active outdoor recreation services.

H2 was confirmed for lack of money as the main perceived barrier to travel for Russian seniors, but partly rejected for fear of health risk. The majority of respondents did not express much concern associated with health risks while traveling.

H3 was confirmed, the Internet is growing in importance as a communication channel for senior sport consumers in Russia.

The preliminary results gained in our survey have direct managerial implications. Since population aging is growing dramatically and covering all regions of Russia, there are new challenges arising which are to be addressed by businesses, as well as public administration.

There are lots of opportunities in Russian regions to gain profit from organizing such tourist activities as hiking, fishing, cross-country skiing, heel-and-toe walking, etc., which usually do not require large investments and are available for people of all ages, both in terms of physical abilities, and from the point of view of monetary costs.

The literature review undertaken, as well as the empirical data obtained, has demonstrated that specific features of baby-boomers make them an attractive target audience for tourism organizations. There are millions of elderly people eager to be engaged in active outdoor recreation. It should be mentioned that they are surely not the most demanding part of Russian consumers, and many of them have enough money to spend for local sport tourism.

Thus, sport tourism activities can not only generate revenue for tourism companies, but also contribute to the local budgets and help to address a number of social problems, to increase public satisfaction and confidence in local authorities. Since sport tourism is proved to be one of the best ways to increase mental health and reduce threat of diseases, the federal and regional authorities should pay attention to the development of sports tourism routes for the elderly. Government experiments could be conducted to find balance between stimulation of seniors' sport tourism activities and reducing spending on medical treatment.

Despite the widely known benefits of Internet communications, especially those in social networks, Russian tourism organizations fail to appreciate these benefits, promoting their services basically through their own websites, and the tools are still primitive. This jeopardizes the opportunities for the development of local tourism destinations. As to seniors, they are almost fully ignored due to incorrect assumption that elderly people do not use the Internet. The results of our pilot research proved this assumption to be incorrect. Russian seniors are quite active in using Internet, and thus the channel is growing in importance and should be seriously considered as a way of communicating information to the audience.

Actually, the wide range of issues is waiting to be resolved before the Russian tourism industry can take full advantage of active outdoor recreation services, including sport tourism for all ages.

6 Conclusions and Future Research

The study presented in the paper focuses on the Russian senior sport tourism market. In the study an attempt was made to bridge the gap in understanding preferences of senior consumers and to lay the initial foundation for relevant marketing decisions of Russian organizations operating in this important segment of tourism market.

The main conclusion that can be drawn on the basis of desk research is that elderly population can certainly be considered to be a new target audience for companies operating in the tourism sector, including sports tourism.

This is also true for Russia. According to the preliminary results of our study, some features in the behavior and preferences of Russian seniors similar to those of seniors all over the world.

At the same time, there are peculiarities in main motivations of Russian seniors and in factors preventing them from traveling due to the cultural and historical context. As Russian seniors seem to show a different travel behavior and travel motivation, further research in this area is vital to be able to better anticipate the preferences and adequate tourism products for this important target group. To prove the preliminary results presented above, the sample should be extended, up to 1,000 respondents from different Russian regions. The data obtained should be further analyzed and compared with the results obtained by the researchers from other countries. There is also an obvious need for a comprehensive, empirically based segmentation model of the older consumer market in Russia.

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Evaluation in Sports Performance

Jae-Hyeon Park and Minsoo Kang

Abstract Evaluation refers to a behavior which offers qualitative meaning such as "superb" or "excellent" to information based on evidence of data. In competition or training in sports, the evaluation of an athlete's performance is generally completed by the coaches, agents and the public media, etc. The purpose of this chapter is to provide general information about the evaluation in sports performance. The evaluation purpose and process are described first, followed by the measurement properties on collected data and the evaluation methods in competition or training in sports.

1 Purpose for Sports Performance Evaluation

What role does evaluation play in sports? There are six general purposes for evaluation in sports [8], which include; placement, diagnosis, achievement confirmation, prediction, training evaluation, and motivation.

1.1 Placement

Placement-based evaluation categorizes individuals into specific groups based upon personal ability levels. A coach classifying athletes into key or non-key players according to their abilities is an example of placement-based evaluation. Another example is to determine a player's optimal playing position based on his/her ability.

J.-H. Park (✉)
Center for Sports and Performance Analysis, Korea National Sport University, Seoul, South Korea
e-mail: jhpark@knsu.ac.kr

M. Kang
Department of Health and Human Performance, Middle Tennessee State University, Murfreesboro, TN, USA
e-mail: mkang@mtsu.edu

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A potential benefit of using placement in sports could be developing personalized training protocols for groups of players based on the type of position (e.g., defensive player vs. offensive player).

1.2 Diagnosis

Diagnosis-based evaluation examines a player's or team's weaknesses and deficiencies. It is used to determine whether the problem is "the player's weak stamina, or poor understanding of team tactics." Specifically diagnosing a player's or team's problem(s) can help establish clear goals for enhancing a team's skill, which holds significant importance.

1.3 Achievement Confirmation

Achievement-based evaluation examines whether players are achieving their goals. When a player enters into a competition he/she may set a personal goal regarding his/her performance. Evaluation may be used to determine if those goals were actually met. Another example of using evaluation to measure achievement would be to examine the training levels of athletes to see if they are meeting the minimum expectations established by the coach. Achievement-based evaluation can use either criterion- or norm-referenced standards.

1.4 Prediction

Evaluation for the purpose of prediction utilizes present or past data to predict future success. Using a player's current physique and fitness to predict his/her future success as a player, or comparing the pregame power rankings of two teams to predict the outcomes of the game are examples of using evaluation for the purpose of prediction. It is impossible to build a perfect estimation model for evaluation without errors in prediction. Therefore, high quality data and well developed models are required to reduce errors.

1.5 Training Evaluation

Although in most cases evaluations are limited to specific players or teams, evaluations can be used to justify the training program itself. Information about the player's progress during training can be used to justify the training program by the coach or program managers. Also, the results from the final evaluation of the training program can be used as a method for validating the effectiveness of the selected training method with the players.

1.6 Motivation

Player motivation is also an important factor to consider when conducting evaluations in training and sports competition. Most players participating in training and competition have the desire to achieve personal goals. Evaluating the progress of these goals can stimulate and challenge players. What happens when motivation is removed from sports? What about the player who trains hard all season, but does not get a chance to evaluate themselves in a game (e.g., sitting on the bench)? This player would likely have decreased motivation to improve if there is no reward for his/her efforts. Applying appropriate evaluations during training and sports competition can help reinforce player motivation.

2 Process for Sports Performance Evaluation

The evaluation process for sports performance can be separated into designing, data gathering, analyzing, and decision-making [1]. The analytical reports from evaluation of sports performance are used to create reports for the coaching staff, scouts, or sponsors to help provide evidence and support for decision-making. Ultimately, evaluation is a tool used for decision-making [1, 10, 11]. In order to make high quality and valid evaluations, you must first have a good understanding of the entire process from design to analysis.

The first step in evaluating sports performance is the design phase. The design phase is critical to evaluation because of its direct relationship to quality. When designing an evaluation it is important to consider whom or what is the object of evaluation; what is the purpose of the evaluation; what type of data will be collected; and lastly what methods will be used to analyze the data (Table 1). The object of evaluation can be a single player, a coach, a training program or even an entire team. Other possible objects of evaluation can be facilities, environmental factors, player support system, advertisements, and/or the market [10].

After an object has been determined, the next step in the evaluation process is to define the purpose of the evaluation. Establishing the purpose of the evaluation will help provide guidance and direction for the subsequent steps. Improving sports performance or measuring achievement progress are potential purposes one might want to conduct an evaluation. How does this affect your evaluation? The purpose of your evaluation will help determine what methods you will use to collect data and how that data will be used. For example, if you want to know how a particular player's

Table 1 Designing factors on evaluation of sports performance

Objects of evaluation	*Source of data*
Players	Motions
Team	Physical fitness
Coach	Anthropometry
Program	Tactics
Environment	Competitions
Perspective of evaluation	*Property of data*
Achievement	Quantitative
Improvement	Qualitative

performance changed throughout the season you could compare his/her initial skill rankings with those at the end of the season to evaluate his/her improvement.

Once the purpose has been defined, a plan must be developed to determine what type of data should be collected. In sports performance, data can be separated into qualitative and quantitative data. Regardless of which type is used, it is important to develop a system for coding the data. In sports performance there are numerous possible sources of data. Some possible sources include video footage (to analyze player technique), player statistics (e.g., free throw percentage), physical characteristics (e.g., height, weight), injury history, etc. Other factors such as game performance records and environmental factors such as weather, wind speed/direction, and size of the audience can also be possible sources of data. The data can then be stored in a computer and analyzed (Fig. 1).

The analysis of data is guided by the purpose of the evaluation and the type of standards being used. The two types of standards are criterion- and norm-referenced standards (to be discussed later in the chapter). Following data analysis a report will be generated that can be used by the intended audience (e.g., coach, scouts, etc.) to assist with decision-making.

3 Measurement for Sports Performance Evaluation

In order to evaluate sports performance it is necessary to gather data. Data can be categorized into one of four measurement scales [7, 8]; nominal, ordinal, interval or ratio according to the characteristics of the data (i.e., does the data have order, interval characteristics, and origin?). In addition, measurements may be either observed (through direct observation), or latent [3]. Composite measures can also be created through data manipulation [5].

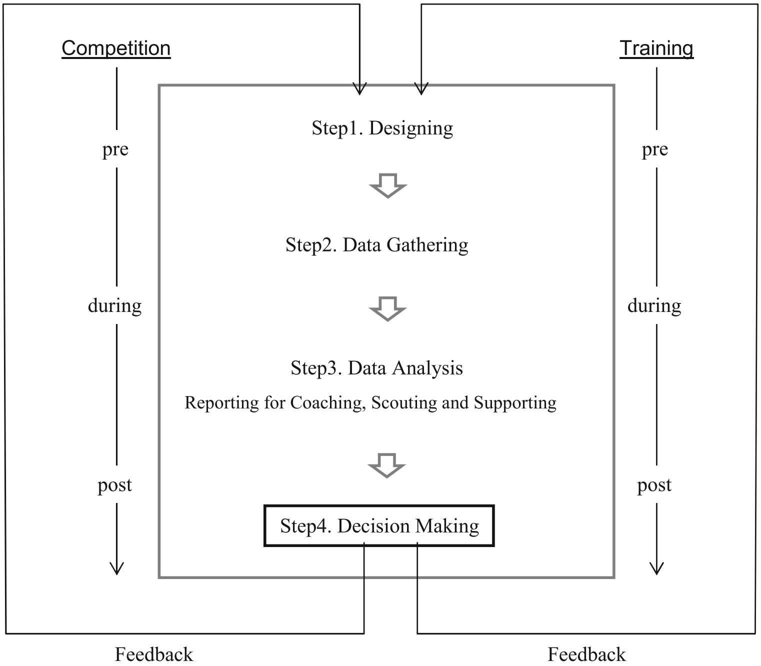

Fig. 1 Process for evaluating sports performance

3.1 Type of Measurement Scale

3.1.1 Nominal Scale

The function of a nominal scale is to classify an object. For example, the terms "male", and "female" are used to categorize different genders. Another example would be the number on the back of a player's jersey. A player with the number 30 is not twice as good as a player with the number 15. The value of this number is meaningless and only used for classification purposes. Generally when using nominal data mathematical functions are limited, and the frequency of a particular classification is calculated. For example, the number of males versus females. Nominal data can also be used to define groups such as tall versus short or baseball player versus basketball player.

3.1.2 Ordinal Scale

Ordinal scale indicates rank order among values. In sports competition, when referring to placement (i.e., 1st place, 2nd place, 3rd place etc.) an ordinal type scale is being used. Numbers measured in an ordinal scale indicate rank; however, the intervals between rankings have no meaning. An example of this is finish times in a race.

In a marathon, the difference between 1st and 2nd or 3rd and 4th is only one place difference; however, the difference in race times between rankings is not identical.

3.1.3 Interval Scale

Numbers measured on an interval scale display qualities similar to those found in ordinal scale data (i.e., the values are ordered), but unlike ordinal data the distance between values are the same. An example of interval scale data would be temperature measured in Celsius. A distinguishing characteristic of interval scale data is the absence of a "true" zero value. Using the previous example, if the temperature is 0 °C it only means that the temperature is very low. It would be inaccurate to say that because the temperature is zero we have an absence of temperature.

3.1.4 Ratio Scale

Ratio scale data include interval scale classification rankings, and interval functions with an absolute zero point. Weight and length are two good examples of ratio scale data. The number zero in ratio scale data refers to the "true" absence of a particular trait. An example of this is 0 K or the point at which all molecular motion ceases. In sports performance interval and ratio scaled data are the two most common data sources during evaluation. Many mathematical and statistical models can be applied to interval and ratio scale data making them particularly useful for evaluation purposes.

3.2 Observed or Latent Variable

3.2.1 Observed Variable

An observed variable is one that can be directly measured such as a player's physical features or their personal performance. Generally these variables are relatively easy to obtain as they can be physically observed. Cases of observed variables include acquiring various measurement values using physical scales in length, weight, volume, and time etc. Records acquired through direct observations such as number of shots and assists are also included in observed variable category. Many sports such as, track and field, swimming, and archery use direct observation as a measure of individual player's performance. The better an individual performs the higher his/her placement or ranking. Other sports may keep track of statistics that are important measures of player skills but not directly related to the outcome of the competition. For example, in a football game one might record the number of tackles or assist a particular player had, but those numbers do not directly reflect the outcome of the game. These statistics are still valuable however as they can be used for other methods of assessment such as evaluating achievement or player motivation. It is important to consider during the evaluation process the ultimate purpose or goal of the evaluation, this factor will help to determine which variables are the most important to focus on during data collection.

3.2.2 Latent Variable

Evaluation of player's performance during games and training can be classified into two categories; observed variable, which can be measured through direct observation and latent variable which cannot be measured through direct observation. One example of a latent variable is 'insecurity,' which is often experienced during competition. Insecurity is a factor which cannot be measured through direct observation because it is a latent variable derived by human imagination. To measure a latent variable such as insecurity, certain activities derived from insecurity must be observed for indirect measurement. For example, one physical characteristic that can be observed when players are insecure is abnormal heart rate. If the player's heart rate is monitored to measure insecurity indirectly, heart rate is used as an indicator to measure insecurity.

3.2.3 Composite Measures

When evaluating player's performance, there are many cases which require composite measures. Composite measures refer to a variable which is created by two or more variables mathematically combined. Batting average (batting average is computed from the ratio of hits to times at bat), frequently used in baseball, is an example of composite measurement. It is not difficult to find examples which use composite measurements for player evaluation in sports competitions. Player's scoring possession in NBA is an example of composite measurement which integrates relative values of field goals without an assist, number of assists and free throw scores.

Creating composite measurement sometimes uses simple ratio model application on two or more variables. Since sports performance results through combinations of complex factors such as player's psychology, stamina, and strategy etc., it is generally accepted that evaluation using composite measurement provides more information compared to that of a single variable. However, the issue of selecting an appropriate model for the composition of variables and giving weight values on variables must be taken under careful consideration due to its direct influence on the validity of composite measures. The National Basketball Association (NBA) is using various composite measures for team evaluation. An example of composite measures is with possession metrics in the NBA [2].

$$\begin{aligned}\text{Possession} = 0.96 \times [&(\text{Field Goal Attempts}) + (\text{Turnovers}) \\ &+ 0.44 \times (\text{Free Throw Attempts}) - (\text{Offensive Rebounds})].\end{aligned}$$

4 Methods for Sports Performance Evaluation

The content and methodology in evaluation of sports performance may change depending on which perspective of sports performance is being evaluated: achievement or improvement. The achievement approach provides information on success and achievement levels of the evaluation objects (player, team, managers etc.) when measuring their sports performance [6, 8]. Improvement approach uses the same evaluation objects but with periodical difference [9]. The evidence from performance evaluations are derived from information gathered more than twice to compare the performance of pre-and post-scores and the changed values are the basis for evaluation.

4.1 Achievement Approach

4.1.1 Grading Methods

The grading method refers to applying ranks or grades to the evaluation of objects (i.e., player, team, managers, etc.) on their sports performances. This method can be viewed as norm-referenced standard, in which a player's performance is judged relative to other peers' performance [6]. Grades can be classified in a number of different ways. One potential method is to rate objects using three tier grading system with descriptive words for each tier such as 'excellent,' 'above average,' and 'below average.' Another method is to use a traditional letter grade classification such as 'A'–'E'. In some cases it may even be possible to use seven or nine classifications. Evaluations using a grading system can sometimes apply standards that have been developed for small groups or a specific norm when targeting certain populations. The grading then creates a distribution from the comparison group and collects data to determine relative performance in a normal distribution.

The grading method incorporating standard deviation is used in various fields in wide scopes. To apply the grading method using standard deviation, the means and standard deviation from the distribution must be calculated. When this is done, it is necessary to determine the ratio of each case classified accordingly to the purpose of the grading. Table 2 provides three examples on a five category grading scale, showing the range of standard deviation and the ratio per cases.

For the first step, the range is calculated for those in grade 'C'. Next calculate the upper limit for 'B' and lower limit of 'D' grades. There is no calculation required to check the standards for 'A' and 'E' grades. If the distribution shows an average of 80, and standard deviation of 4 then the following standards (Table 2) for grades can be made.

The C range is found as follows:

$$C = \text{mean} \pm 1.0\text{sd} = 80 \pm (1.0)4 = 80 \pm 4 = 76{-}84.$$

Table 2 Percentages of case and standard deviation range examples

Grade	Standard deviation range	Percent
Example 1		
A	2.0sd or more above mean	2
B	Between +1.0sd and +2.0sd	14
C	Between +1.0sd and −1.0sd	68
D	Between −1.0sd and −2.0sd	14
E	−2.0sd or more below mean	2
Example 2		
A	1.75sd or more above mean	4
B	Between +0.75sd and +1.75sd	19
C	Between +0.75sd and −0.75sd	54
D	Between −0.75sd and −1.75sd	19
E	−1.75sd or more below mean	4
Example 3		
A	1.5sd or more above mean	7
B	Between +0.5sd and +1.5sd	24
C	Between +0.5sd and −0.5sd	38
D	Between −0.5sd and −1.5sd	24
E	−1.5sd or more below mean	7

The upper limit of the B range is found as follows:

$$B = \text{mean} + 2.0\text{sd} = 80 + (2.0)4 = 88.$$

The lower limit of the D range is found as follows:

$$D = \text{mean} - 2.0\text{sd} = 80 - (2.0)4 = 72.$$

The ranges for the grades are as follows:

A = above 88.0
B = 84.1–88.0
C = 76.0–84.0
D = 72.0–75.9
E = below 72.0

4.1.2 Criterion-Referenced Standards

The criterion-referenced standard is a standard based on a predetermined goal to evaluate the sports performance of a player or team. While the grading system using standard deviation compares a player's sports performance to other peers' performance, criterion-referenced standard uses evidence of whether the player or team has achieved the goal [4]. In general, criterion-referenced standard indicates the achievement of a goal through 'master/non-master', pass/fail, 'positive/negative.' Therefore, criterion-referenced standard uses a 2X2 contingency table made up of reliability and validity to verify the standard's stability and accuracy. However, before evaluating the quality of criterion referenced standard, first you should know how to configure the cutoff score which determines master/non-master or pass/fail.

Because the cutoff score indicates that a player or team have exceeded the predetermined goal, the relevant standards must be selected. What criterion standard should be determined if a team's striking force is to be evaluated in a soccer match? The criteria may be ball possession ratio, number of shots on the goal, or a composite measure using two or more variables. Whatever the variable is the object of evaluation, the standard relevant to the object is crucial because it influences the validity of the criterion-referenced standard.

Although it may seem unfamiliar in the field of sports analysis, various methodologies to set criterion-referenced standard have been proposed. The contrasting-group method, the borderline-group method, the Angoff method, the Bookmark method, and The body of work method are some commonly used setting standard procedures. For more detailed information, please review Gregory J. Cizek's, 'Setting performance standards.'

A 'contract method' has been rarely used setting standard method in sports performance area. A contract method refers to a predetermined agreement on the standard configuration of achievement goals by the evaluated object and the judge. An example of the contract method standards is as follows in a baseball league.

- Entering at least 15 games per season
- Achieve at least 0.320 batting average per season
- Achieve at least 0.450 on-base average per season
- Achieve more than 15 steals per season
- Achieve 7 % RBI contribution per season

Achieving three criteria from above

When examining reliability or stability of a measure often times we repeat the measure to look at the consistency of the reported values (i.e., having a participant perform the same test two days in a row to see if they get the same score). When discussing validity we are referring to the accuracy or the precision of the standard. Table 3 is the 2X2 contingency table which is distinguished according to true classification and estimated classification. It is not difficult to calculate the agreement proportion between true classification and estimated classification. In general, Kappa coefficient is calculated for reliability and Phi coefficient is calculated for validity [8].

TN: True Negative: the criterion referenced test correctly indicated the failure to reach the criterion

Table 3 2X2 contingency table from true and estimated classification

	True classification (day 2)	
Estimated classification (day 1)	TN	FN
	FP	TP

TP: True Positive: the criterion referenced test correctly indicates reaching the criterion

FN: False Negative: the criterion referenced test incorrectly indicates the failure to reach the criterion

FP: False Positive: the criterion referenced test incorrectly indicates reaching the criterion

$$\text{Proportion of agreement } (P) = \frac{TN + TP}{TN + FN + FP + TP}$$

$$\text{Kappa coefficient } (\kappa) = \frac{P_a - P_c}{1 - P_c},$$

where P_a—proportion of agreement, P_c—proportion of agreement expected by chance.

$$P_c = \frac{(TN + FN)(TN + FP) + (TP + FP)(TP + FN)}{(TN + FN + FP + TP)^2}$$

$$\text{Phi coefficient } (\phi) = \frac{(TN \cdot TP) - (FN \cdot FP)}{\sqrt{(TN + FN)(FP + TP)(TN + FP)(FN + TP)}}$$

For the proportion of agreement (P), if the value is less than 0.5, it is interpreted as no agreement. The P value must be higher than 0.5 and as the value is closer to 1, it is evaluated as higher agreement between estimated classification and true classification. The Kappa coefficient (κ) value ranges from 0 to 1, and this value also shows higher reliability as the value is closer to 1 in the evaluation. The phi coefficient (ϕ) value ranges between -1 and $+1$. If ϕ is calculated for validity purposes, a positive value is required. Negative value of the ϕ indicates high levels of classification error [8].

4.2 Improving Approach

When the same variable is measured repeatedly, we may not always obtain the same value. For example, if you were asked to shoot 10 free throws you might get 6 out of 10 one time, and 8 out of 10 the second time. Between the two trials your ability did not change so what caused the difference in scores? One factor that is always present in measurement is error. When examining the differences between pre- and post-scores we have to determine if the difference between scores are meaningful or the result of random chance.

4.2.1 Typical Error of Measurement

According to measurement theory, observed scores are always explained as the sum of true scores and error scores. Measurements without error are theoretically impossible. Usually, when improvements of sports performance on players are evaluated, the previous season records and current records are compared.

For example, in the previous season the measurement value shows 13.4 m/s in movement speeds over a 20 m distance and this season, 13.8 m/s in movement speed was measured over the same distance. In this case, did the player's speed improve? For a logical evaluation on the improvement, typical error of measure (TEM) can be applied [9]. Improvement evaluation compares the pre- and post-score, which defines the error terms as the within-subject standard deviation.

If the standard error of measurement is the error when estimating true scores with observed scores, within-subject standard deviation can be viewed as an error from measurement to measurement. At this, within-subject standard deviation is defined as the typical error of measurement (TEM). TEMs encompass both technical error and biological errors. The equation for absolute TEM is as follows

$$\text{Absolute TEM} = \sqrt{\frac{\sum d_i^2}{2n}},$$

where $\sum d_i^2$—the sum squared deviation between two measurement, n—number of cases, i—personal score.

TEM always has the same units as the observed score (e.g., centimeter, kg, minute) and TEM value interpretation is identical to that of interpreting standard deviation. The "± 1" TEM of calculated score accounts for 68 % of the range in the normal distribution and the "± 2" TEM (actually ± 1.96) includes approximately 95 % of the range. Relative interpretation is possible on TEM if the form is converted such as a coefficient of variation. The relative TEM for interpretation in comparative uses the following equation.

$$\text{Relative TEM} = \frac{TEM}{MEAN} \times 100$$

4.2.2 Evaluation of Real Change

Previously, we have known that TEM can be interpreted the same as standard deviation. Standard deviation holds 68 % area in ± 1sd range from the average in a normal distribution. In ± 2sd (actually ± 1.96), 95 % of the data distribution is encompassed.

TEM, which defines the repeated measured data and the discrepancy among measurements as the measurement error, can be applied to give meaning to value changes.

For example, let's say a certain situation evaluates the improvement of a fastball for pitcher 'L.' The observed pre speed was 145 km/h and 6 months later, the post speed was 148 km/h. The TEM value from measuring the fastball is 2.2 km/h, does the improvement in fastball speed of pitcher 'L' during the 6 months have meaning? At this question, if the improvement is evaluated on 95 % confidence interval (CI).

First, the 95 % CI requires verification on whether the changes from pre speed to post speed fall within the range of 95 %, that is, ±2 TEM. Pitcher 'L's ball speed increased 3 km/h from pre speed of 145 km/h to post speed of 148 km/h. Then the verification takes place to determine whether speed increase of 3km/hour holds meaning or not at 95 % CI. To verify the change in 95 % CI, the process is as follows:

> The change in ball speed is 3 km/h increased as post 148 km/h minus pre 145 km/h. If the pre speed is the starting point,
> 95 %CI = pre speed ± 2 TEM = 145 km/h ± 4.4 km/h 95 %CI = 140.6–149.4 km/h
> 95 %CI includes the post speed of 148 km/h from pre speed of 145 km/h.
> Therefore, the pitcher 'L's ball speed was not improved from pre speed to post speed at 95 % CI.

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Pythagoras at the Bat

Steven J. Miller, Taylor Corcoran, Jennifer Gossels, Victor Luo and Jaclyn Porfilio

Abstract The Pythagorean formula is one of the most popular ways to measure the true ability of a team. It is very easy to use, estimating a team's winning percentage from the runs they score and allow. This data is readily available on standings pages; no computationally intensive simulations are needed. Normally accurate to within a few games per season, it allows teams to determine how much a run is worth in different situations. This determination helps solve some of the most important economic decisions a team faces: How much is a player worth, which players should be pursued, and how much should they be offered. We discuss the formula and these applications in detail, and provide a theoretical justification, both for the formula as well as simpler linear estimators of a team's winning percentage. The calculations and modeling are discussed in detail, and when possible multiple proofs are given. We analyze the 2012 season in detail, and see that the data for that and other recent years support our modeling conjectures. We conclude with a discussion of work in progress to generalize the formula and increase its predictive power *without* needing expensive simulations, though at the cost of requiring play-by-play data.

1 Introduction

In the classic movie *Other People's Money*, New England Wire and Cable is a firm whose parts are worth more than the whole. Danny Devito's character, Larry the Liquidator, recognizes this and tries to take over the company, with the intent on

S.J. Miller (✉) · V. Luo · J. Porfilio
Williams College, Williamstown, MA 01267, USA
e-mail: sjm1@williams.edu; Steven.Miller.MC.96@aya.yale.edu

T. Corcoran
The University of Arizona, Tucson, AZ 85721, USA

J. Gossels
Princeton University, Princeton, NJ 08544, USA

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breaking it up and selling it piecemeal. Gregory Peck plays Jorgy, the owner of the firm, who gives an impassioned defense to the stockholders at a proxy battle about traditional values and the golden days ahead. In the climatic conclusion, Larry the Liquidator responds to Jorgy's speech which painted him a heartless predator who builds nothing and cares for no one but himself. Larry says

> Who cares? I'll tell you. Me. I'm not your best friend. I'm your only friend. I don't make anything? I'm making you money. And lest we forget, that's the only reason any of you became stockholders in the first place. You want to make money! You don't care if they manufacture wire and cable, fried chicken, or grow tangerines! You want to make money! I'm the only friend you've got. I'm making you money.

While his speech is significantly longer than this snippet, the scene in general and the lines above in particular highlight one of the most important problems in baseball, one which is easily forgotten. In the twenty-first century massive computation is possible. Data is available in greater quantities than ever before; it can be analyzed, manipulated, and analyzed again thousands of times a second. We can search for small connections between unlikely events. This is especially true in baseball, as there has been an explosion of statistics that are studied and quoted, both among the experts and practitioners as well as the everyday fan. The traditional metrics are falling out of favor, being replaced by a veritable alphabet soup of acronyms. There are so many statistics now, and so many possibilities to analyze, that good metrics are drowned out in poor ones. We need to determine which ones matter most.

In this chapter we assume a team's goal is to win as many games as possible given a specified amount of money to spend on players and related items. This is a reasonable assumption from the point of view of general managers, though it may not be the owner's goal (which could range from winning at all costs to creating the most profitable team). In this case, Devito's character has very valuable advice: The goal is to win games. We don't care if it's by winning shoot-outs 12-10 in thirteen innings, or by eeking out a win in a 1-0 pitcher's duel. We want to win games.

In this light, we see that sabermetrics is a dear friend. While there are many items we could study, we focus on the value of a run (both a run created and a run saved). We have a two-stage process. We need to determine how much each event is worth in terms of creating a run, and then we need to extract how much a run is worth. Obviously these are not constant values; a run is worth far more in a 2-1 game than in a 10-1 match. We focus below entirely on the value of a run. We thus completely ignore the first item above, namely how much each event contributes to scoring.

Our metric for determining the worth of a run is Bill James' Pythagorean Won-Loss formula: If a team scores RS runs while allowing RA, then their winning percentage is approximately $\frac{\mathrm{RS}^\gamma}{\mathrm{RS}^\gamma+\mathrm{RA}^\gamma}$. Here γ is an exponent whose value can vary from sport to sport (as well as from era to era within that sport). James initially took γ to be 2, which is the source of the name as the formula is reminiscent of the sum of squares from the Pythagorean theorem. *Note that instead of using the total runs scored and allowed we could use the average number per game, as such a change rescales the numerator and the denominator by the same amount.*

In this chapter we discuss previous work providing a theoretical justification for this formula, talk about future generalizations, and describe its implications in one of the most important economics problems confronted by a baseball team: How much is a given player worth? While much of this chapter has appeared in journals, we hope that by combining everything in one place and doing the calculations in full detail and in as elementary a way as possible that we will increase the visibility of this method, and provide support for the role of mathematical modeling in sabermetrics.

Before delving into the derivation, it's worth remarking on why such a derivation is important, and what it can teach us. In *An Enquiry Concerning Human Understanding* (1772), David Hume wrote:

> The contrary of every matter of fact is still possible, because it can never imply a contradiction, and is conceived by the mind with the same facility and distinctness, as if ever so conformable to reality. That the sun will not rise tomorrow is no less intelligible a proposition, and implies no more contradiction, than the affirmation, that it will rise. We should in vain, therefore, attempt to demonstrate its falsehood. Were it demonstratively false, it would imply a contradiction, and could never be distinctly conceived by the mind.

Hume's warning complements our earlier quote, and can be summarized by saying that just because the sun rose yesterday we cannot conclude that it will rise today. Sabermetricians frequently find quantities that appear to be well correlated with desirable outcomes; however, there is a real danger that the correlation will not persist in the future as past performance is no guarantee of future performance. (This lesson has been painfully learned by many chartists on Wall Street.) Thus we must be careful in making decisions based on regressions and other calculations. If we find a relationship, we want some *reason* to believe it will continue to hold.

We are therefore led to creating mathematical models with reasonable assumptions; thus the point of this chapter is to develop predictive mathematical models to complement inferential techniques. The advantage of this approach is that we now have a reason to believe the observed pattern will continue, as we can now point to an explanation, a reason. We will find such a model for baseball, which has the Pythagorean formula, initially a numerical observation by James that seemed to do a good job year after year, as a consequence.

The Pythagorean formula has a rich history; almost any sabermetrics book references it at some point. It is necessary to limit our discussion to just some of its aspects. As the economic consequences to a team from better predictive power are clear, we concentrate on the mathematical issues. Thus explaining how mathematical models can lead to closed-form expressions, which can solve real world problems, is our main goal. We begin in Sect. 2 with some general comments on the statistic. We describe a reasonable mathematical model in the next section, show the Pythagorean formula is a consequence, and then give a mathematical proof in Sect. 4. In Sect. 5 we examine some consequences, in particular how much a run created or saved is worth at different production levels, and in Sect. 6 we analyze data from several seasons to see how well our model and the formula do. Next we examine linear predictors for a team's winning percentage, and show how they follow from linearizing the Pythagorean formula. We end by discussing current, ongoing research into generalizing the Pythagorean formula.

2 General Comments

Before discussing why the Pythagorean formula should be true, it's worth commenting on the form it has, both in its present state and its debut back in Bill James' *1981 Baseball Abstract* [6]. Remember it says that a team's winning percentage should be $\frac{\mathrm{RS}^{\gamma}}{\mathrm{RS}^{\gamma}+\mathrm{RA}^{\gamma}}$, with γ initially taken as 2 but now typically taken to be around 1.83. One is struck by how easy the formula is to state and to use, especially in the original incarnation. All we need is to know the average number of runs scored and allowed, and the ratio can be found on a simple calculator.

Of course, back in the 1980s this wouldn't be entirely true for someone watching at home if γ were not 2, though the additional algebra is slight and not even noticeable on modern calculators, computers and even phones. One of the great values of this statistic is just how easy it is to calculate, which is one of the reasons for its popularity. You can easily approximate how much better you would do if you scored 10 more runs, or allowed 10 fewer, which we do later in Fig. 2. We can do this as we have a *simple, closed form expression* for our winning percentage in terms of just three parameters: average runs scored, average runs allowed, and an exponent γ.

This is very different than the multitude of Monte Carlo simulations which try to predict a team's record. These require detailed statistics on batters and pitchers and their interactions. Depending on how good and involved the algorithm is, we may need everything from how many pitches a batter sees per appearance to the likelihood of a runner advancing from first to third on a single hit to right field. While this data is available, it takes time to simulate thousands of games. Further, every small change in a team requires an entirely new batch of simulations. With the Pythagorean formula, we can immediately determine the impact of a player *if* we have a good measure of how many runs they will contribute or save.

Of course, as with most things in life there are trade-offs. While the closed-form nature of the Pythagorean formula allows us to readily measure the impact of players, it indicates a major defect that should be addressed. Baseball is a complicated game; it is unlikely that all the subtleties and issues can be distilled into one simple formula involving just three inputs. Admittedly, it is a major challenge to derive a good formula to predict how many runs a player will give a team, and we are ignoring this issue in this chapter; however, it is improbable that any formula as simple as this can capture everything that matters. There are thus several extensions of the Pythagorean formula; we discuss some of these in Sects. 7 and 8, as well as outline a program currently being pursued to improve its predictive power.

3 Pythagorean Formula: Model

There are many ways to model a baseball game. The more sophisticated the model, the more features can be captured, though added power comes at a cost. The cost varies from increased run-time to requiring massively more data. We give a very simple model for a baseball game, and show the Pythagorean formula is a consequence.

Of course, the simplicity of our model strongly suggests that it cannot be the full story. We return to that issue in Sect. 8, and content ourselves here with the simple case. The hope is that this simplified model of baseball is nevertheless powerful enough to capture the main features and yield a reasonably good predictive statistic. See the paper of Hammond et al. [4] for other approaches to modeling baseball games and winning percentages. Specifically, they look at James' log5 method, which also appeared in his 1981 abstract [6]. There he estimates the probability a team with winning percentage a beats a team with winning percentage b by $\frac{a(1-b)}{a(1-b)+(1-a)b}$. Interestingly, the Pythagorean formula with exponent 2 follows by taking $a = \mathrm{RS}/(\mathrm{RS}+\mathrm{RA})$ and $b = \mathrm{RA}/(\mathrm{RS}+\mathrm{RA})$, with RS the average number of runs scored and RA the average number of runs allowed.

The following model and derivation first appeared in work by the first author in [9], who introduced using a Weibull distribution to model run production. The Weibull distribution is extensively used in statistics, arising in many problems in survival analysis (see [12] for a good description of the Weibull's properties and applications). The reason a Weibull distribution is able to model well so many different data sets is that it is a three parameter distribution, with probability density function

$$f(x;\alpha,\beta,\gamma) \;=\; \frac{\gamma}{\alpha}\,((x-\beta)/\alpha)^{\gamma-1}\,e^{-((x-\beta)/\alpha)^{\gamma}} \tag{1}$$

if $x \geq \beta$ and 0 otherwise. Here α, β and γ are the three parameters of the distribution. The effect of β is to shift the entire distribution along the real line; essentially it determines the starting point. In our investigations β will always be $-1/2$, for reasons that will become clear. Next is α, which adjusts the scale of the distribution but not the shape; as α increases the distribution becomes more spread out.

The reason that α and β do not alter the shape of the distribution is that, for any distribution with finite mean and variance, we can always rescale it to have mean zero and variance 1 (or, more generally, any mean and any positive variance). Thus all α and β do are adjust these two quantities. It is γ that is the most important, as different values of γ lead to very different shapes. We illustrate this in Fig. 1. For definiteness, we may rescale and assume $\alpha = 1$ and $\beta = 0$; we see how the distribution changes as γ ranges from 1 to 2.

We are now ready to state our model. After listing our assumptions we discuss why these choices were made, and their reasonableness. *Remember, as remarked earlier, that in the Pythagorean formula it makes no difference if we use the total runs or the average per game, as rescaling changes the numerator and the denominator by the same multiplicative factor, and hence has no effect.*

Assumptions for modeling a baseball game: The average number of runs a team scores per game, denoted RS, and the average number of runs allowed per game, denoted RA, are random variables drawn independently from Weibull distributions with $\beta = -1/2$ and the same γ.

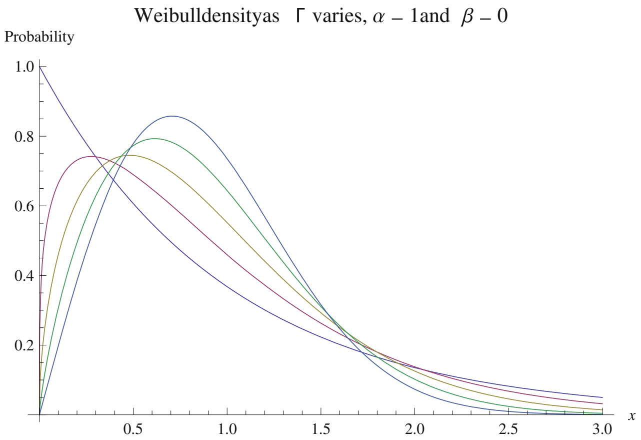

Fig. 1 The changing probabilities of a family of Weibulls with $\alpha = 1$, $\beta = 0$, and $\gamma \in \{1, 1.25, 1.5, 1.75, 2\}$; $\gamma = 1$ corresponds to the exponential distribution, and increasing γ results in the bump moving rightward

These assumptions clearly require discussion, as they cannot be right. The first issue is that we are modeling runs scored and allowed by continuous random variables and not discrete random variables. While earlier work in the field used discrete random variables (especially geometric or Poisson), the difficulty with these approaches is that it is hard to obtain tractable, closed form expressions for the probability a team scores more runs than it allows and hence wins a game. The reason is that calculus is unavailable in this case. Another way to put it is that while many people have continued in mathematics to Calculus III or IV, no one goes similarly far in classes on summation. In general, we do not have good formulas for sums, but through calculus we do have nice expressions for integrals. While the model allows for the Red Sox to beat the Yankees π to e, we must accept this if we want to be able to use calculus.

The next assumption is that these random variables are drawn from Weibull distributions. There are two reasons for this. One is that the Weibull distributions, due to their shape parameter γ, are an extremely flexible family and are capable of fitting many one-hump distributions (i.e., distributions that go up and then go down). The second, and far more important, is that calculations with the Weibull are exceptionally tractable and lead to closed form expressions. This should be compared to similar and earlier work of Hein Hundel [5], which the author learned of from the Wikipedia entry *Pythagorean expectation* [11]. In particular, the mean $\mu_{\alpha,\beta,\gamma}$ and the variance $\sigma^2_{\alpha,\beta,\gamma}$ of the Weibull are readily computed:

$$\begin{aligned}\mu_{\alpha,\beta,\gamma} &= \alpha\Gamma\left(1+\gamma^{-1}\right)+\beta \\ \sigma^2_{\alpha,\beta,\gamma} &= \alpha^2\Gamma\left(1+2\gamma^{-1}\right)-\alpha^2\Gamma\left(1+\gamma^{-1}\right)^2. \end{aligned} \tag{2}$$

Here $\Gamma(s)$ is the Gamma function, defined for the real part of s positive by

$$\Gamma(s) = \int_0^\infty e^{-u} u^{s-1} \mathrm{d}u. \tag{3}$$

The Gamma function is the continuous generalization of the factorial function, as for n a non-negative integer we have $\Gamma(n+1) = n!$.

The reason Weibulls lead to such tractable calculations is that if X is a random variable drawn from a Weibull with parameters α, β and γ, then $X^{1/\gamma}$ is exponentially distributed with parameter α^γ. Therefore a simple change of variables leads to simple integrals of exponentials, which can be done in closed form. Due to the importance of this calculation, we give full details for the computation of the mean in "Calculating the Mean of a Weibull" in Appendix (a similar calculation determines the variance). The point is that when there are several alternatives to use, certain choices are more tractable and should be incorporated. We discuss how to handle more general distributions while preserving the all-important closed form nature of the solution in Sect. 8.

The next issue is our assumption that $\beta = -1/2$. This choice is to facilitate comparisons to the discrete scoring in baseball. Using the above calculations for the mean, if β and γ are fixed we can determine α so that the mean of our Weibull matches the observed average runs scored (or allowed) per game. We can use the Method of Least Squares or the Method of Maximum Likelihood to find the best fit parameters α, β, γ to the observed data. In doing so, we need to deal with the fact that our data is discrete. By taking $\beta = -1/2$, we are breaking the data into bins $[-\frac{1}{2}, \frac{1}{2})$, $[\frac{1}{2}, 1\frac{1}{2})$, $[1\frac{1}{2}, 2\frac{1}{2})$ and so on. Notice that the *centers* of these bins are, respectively, $0, 1, 2, \ldots$. This is no accident, and in fact is the reason we chose β as we did. By taking $\beta = -1/2$ the possible integer scores are in the *middle* of each bin. If we took $\beta = 0$, as might seem more natural, then these values would lie at the endpoints of the bins, which would cause issues in determining the best fit values.

The final issue is that we are assuming runs scored and runs allowed are independent. This of course cannot be true, for the very simple reason that baseball games cannot end in a tie! Thus if we know the Orioles scored 5 runs against the Red Sox, then we know the Sox ended the game with some number other than 5. There are a plethora of other obvious issues with this assumption, ranging from if you have a large lead late in the game you might rest your better players and take a chance on a weaker pitcher, to bringing in your closer to protect the lead in a tight game. That said, an analysis of the data shows that on average these issues cancel each other out, and that subject to being different the runs scored and allowed behave as if they are statistically independent. The interesting feature here is that we cannot use a standard $r \times c$ contingency table analysis as these two values cannot be equal. This leads to an iterative procedure taking into account these *structural zeros* (values of the table that are inaccessible), which is described in "Independence Test with Structural Zeros" in Appendix.

We end this section by describing the calculation that yields the Pythagorean formula, and remarking on why we have chosen to model the runs with Weibull distributions. Let X be a random variable drawn from a Weibull with parameters α_{RS}, $\beta = -1/2$ and γ, representing the number of runs a team scores on average. Similarly, let Y be a random variable drawn from a Weibull with parameters α_{RA}, $\beta = -1/2$ and γ, representing the number of runs a team allows on average. Notice we have the same γ for X and Y, and we choose α_{RS} and α_{RA} so that the mean of X is the observed average number of runs scored per game, RS, and the mean of Y is the observed average number of runs allowed per game, RA. Thus

$$\alpha_{\mathrm{RS}} = \frac{\mathrm{RS} - \beta}{\Gamma(1 + \gamma^{-1})}, \qquad \alpha_{\mathrm{RA}} = \frac{\mathrm{RA} - \beta}{\Gamma(1 + \gamma^{-1})}. \tag{4}$$

To determine our team's winning percentage we just need to calculate the probability that X exceeds Y:

$$\mathrm{Prob}(X > Y) = \int_{x=\beta}^{\infty} \int_{y=\beta}^{x} f(x; \alpha_{\mathrm{RS}}, \beta, \gamma) f(y; \alpha_{\mathrm{RA}}, \beta, \gamma) \mathrm{d}y \, \mathrm{d}x. \tag{5}$$

For general probability densities f the above double integral is intractable (as can be seen in Hundel's work, where he used the log-normal distribution). As we'll see in the next section, the Weibull distribution leads to very simple integrals which can be evaluated in closed form. This is not an accidental, fortuitous coincidence. When first investigating this problem, Miller began by choosing f's that led to nice double integrals which could be computed in closed form; thus the choice of the Weibull came not from looking at the data but from looking at the integration! The first f Miller chose was an exponential distribution, which turns out to be a Weibull with $\gamma = 1$. Next, Miller chose a Rayleigh distribution, which is a Weibull with $\gamma = 2$. (As a number theorist working in random matrix theory, which is often used to model the energy levels of heavy nuclei, the Rayleigh distribution was one Miller encountered frequently in his research and reading, as it approximates the spacings between energy levels of heavy nuclei.) It was only after computing the answer in both these cases that Miller realized the two densities fit into a nice family, and did the calculation for general γ.

4 Pythagorean Formula: Proof

We now finally prove the Pythagorean formula, which we first state explicitly as a theorem. For completeness, we restate our assumptions.

Theorem 1 (Pythagorean Won-Loss Formula) *Let the runs scored and runs allowed per game be two independent random variables drawn from Weibull distributions with parameters* $(\alpha_{\mathrm{RS}}, \beta, \gamma)$ *and* $(\alpha_{\mathrm{RA}}, \beta, \gamma)$ *respectively, where* α_{RS} *and* α_{RA} *are chosen so that the means are* RS *and* RA*; in applications* $\beta = -1/2$. *Then*

$$\text{Won-Loss Percentage (RS, RA}, \beta, \gamma) \;=\; \frac{(\text{RS} - \beta)^\gamma}{(\text{RS} - \beta)^\gamma + (\text{RA} - \beta)^\gamma}. \tag{6}$$

Proof Let X and Y be independent random variables with Weibull distributions $(\alpha_{\text{RS}}, \beta, \gamma)$ and $(\alpha_{\text{RA}}, \beta, \gamma)$ respectively, where X is the number of runs scored and Y the number of runs allowed per game. Recall from (4) that

$$\alpha_{\text{RS}} \;=\; \frac{\text{RS} - \beta}{\Gamma(1 + \gamma^{-1})}, \qquad \alpha_{\text{RA}} \;=\; \frac{\text{RA} - \beta}{\Gamma(1 + \gamma^{-1})}. \tag{7}$$

We need only calculate the probability that X exceeds Y. Below we constantly use the integral of a probability density is 1 (for example, in moving from the second to last to the final line). We have

$$\begin{aligned}
\text{Prob}(X > Y) &= \int_{x=\beta}^{\infty} \int_{y=\beta}^{x} f(x; \alpha_{\text{RS}}, \beta, \gamma) f(y; \alpha_{\text{RA}}, \beta, \gamma) \mathrm{d}y\, \mathrm{d}x \\
&= \int_{x=\beta}^{\infty} \int_{y=\beta}^{x} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x-\beta}{\alpha_{RS}}\right)^{\gamma-1} e^{-((x-\beta)/\alpha_{\text{RS}})^\gamma} \frac{\gamma}{\alpha_{\text{RA}}} \left(\frac{y-\beta}{\alpha_{\text{RA}}}\right)^{\gamma-1} e^{-((y-\beta)/\alpha_{\text{RA}})^\gamma} \mathrm{d}y\, \mathrm{d}x \\
&= \int_{x=0}^{\infty} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x}{\alpha_{RS}}\right)^{\gamma-1} e^{-(x/\alpha_{\text{RS}})^\gamma} \left[\int_{y=0}^{x} \frac{\gamma}{\alpha_{\text{RA}}} \left(\frac{y}{\alpha_{\text{RA}}}\right)^{\gamma-1} e^{-(y/\alpha_{\text{RA}})^\gamma} \mathrm{d}y \right] \mathrm{d}x \\
&= \int_{x=0}^{\infty} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x}{\alpha_{RS}}\right)^{\gamma-1} e^{-(x/\alpha_{\text{RS}})^\gamma} \left[1 - e^{-(x/\alpha_{\text{RA}})^\gamma}\right] \mathrm{d}x \\
&= 1 - \int_{x=0}^{\infty} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x}{\alpha_{RS}}\right)^{\gamma-1} e^{-(x/\alpha)^\gamma} \mathrm{d}x,
\end{aligned} \tag{8}$$

where we have set

$$\frac{1}{\alpha^\gamma} \;=\; \frac{1}{\alpha_{\text{RS}}^\gamma} + \frac{1}{\alpha_{\text{RA}}^\gamma} \;=\; \frac{\alpha_{\text{RS}}^\gamma + \alpha_{\text{RA}}^\gamma}{\alpha_{\text{RS}}^\gamma \alpha_{\text{RA}}^\gamma}. \tag{9}$$

The above tells us that we are essentially integrating a new Weibull whose parameter α is given by the above relation; expressions like this are common (see for example center of mass calculations, or adding resistors in parallel). Therefore

$$\begin{aligned}
\text{Prob}(X > Y) \;&=\; 1 - \frac{\alpha^\gamma}{\alpha_{\text{RS}}^\gamma} \int_0^\infty \frac{\gamma}{\alpha} \left(\frac{x}{\alpha}\right)^{\gamma-1} e^{(x/\alpha)^\gamma} \mathrm{d}x \\
&=\; 1 - \frac{\alpha^\gamma}{\alpha_{\text{RS}}^\gamma} \\
&=\; 1 - \frac{1}{\alpha_{\text{RS}}^\gamma} \frac{\alpha_{\text{RS}}^\gamma \alpha_{\text{RA}}^\gamma}{\alpha_{\text{RS}}^\gamma + \alpha_{\text{RA}}^\gamma}
\end{aligned}$$

$$= \frac{\alpha_{\mathrm{RS}}^{\gamma}}{\alpha_{\mathrm{RS}}^{\gamma} + \alpha_{\mathrm{RA}}^{\gamma}}. \tag{10}$$

Substituting the relations for α_{RS} and α_{RA} of (4) into (10) yields

$$\mathrm{Prob}(X > Y) = \frac{(\mathrm{RS} - \beta)^{\gamma}}{(\mathrm{RS} - \beta)^{\gamma} + (\mathrm{RA} - \beta)^{\gamma}}, \tag{11}$$

which completes the proof of Theorem 1, the Pythagorean formula. Q.E.D.

5 The Pythagorean Formula: Applications

It is now time to apply our mathematical models and results to the central economics issue of this chapter: In each situation, how much is a run worth? We content ourselves with answering this from the point of view of the season. Thus if we score x runs and allow y, and we have a player who increases our run production by s, how much is that worth? Similarly, how much would they be worth if they prevented s runs from scoring?

We answer this question not in dollars, but in additional games won or lost. Translating the number of wins per season into dollar amounts is a fascinating and obviously important question, which the interested reader is encouraged to pursue. A good resource is Nate Silver's chapter "Is Alex Rodriguez Overpaid" in *Baseball Between the Numbers: Why Everything You Know About the Game Is Wrong* [10]. There are also numerous insightful blog posts, such as Phil Birnbaum's "Sabermetric Research: Saturday, April 24, 2010" (see [1]). In this chapter we concern ourselves with determining the number of wins gained or lost, which these and other sources can convert to monetary amounts. As not all wins are worth the same (going from 65 to 75 wins doesn't alter the fact that the season was a bust, but going from 85 wins to 95 wins almost surely punches your ticket to the playoffs), it is essential that we can determine changes from any state.

In Fig. 2 we plot the addition wins per season with $\gamma = 1.83$ and $s = 10$. We plot around a league average of 700 runs scored per season, which was essentially the average in 2012 (see Sect. 7). We let $s = 10$ as the common adage is every 10 additional runs translates to one more win per season.

Not surprisingly, the more runs we score the more valuable preventing runs is to scoring runs, and vice-versa; what is nice about the Pythagorean formula is that it quantifies exactly what this trade-off is. To make it easier to see, in Fig. 3 we plot the difference in wins gained from scoring 10 more runs to wins gained from preventing 10 more runs. The plot is positive in the upper left region, indicating that if our runs scored and allowed places us here then it is more valuable to score runs; in the lower right region the conclusion is the opposite.

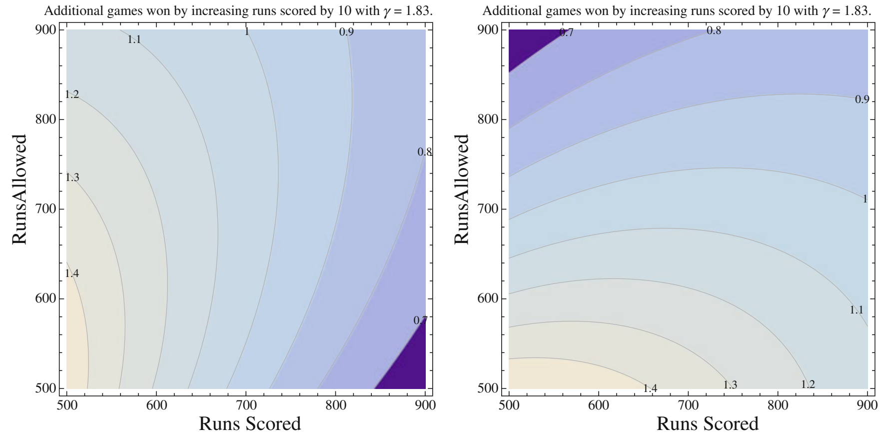

Fig. 2 The predicted number of additional wins with $\gamma = 1.83$: (*left*) scoring 10 more per season; (*right*) preventing 10 more per season. Letting $\mathscr{P}(x, y; \gamma) = x^\gamma/(x^\gamma + y^\gamma)$, the left plot is $\mathscr{P}(x + 10, y; 1.83) - \mathscr{P}(x, y; 1.83)$, while the right is $\mathscr{P}(x, y - 10; 1.83) - \mathscr{P}(x, y; 1.83)$

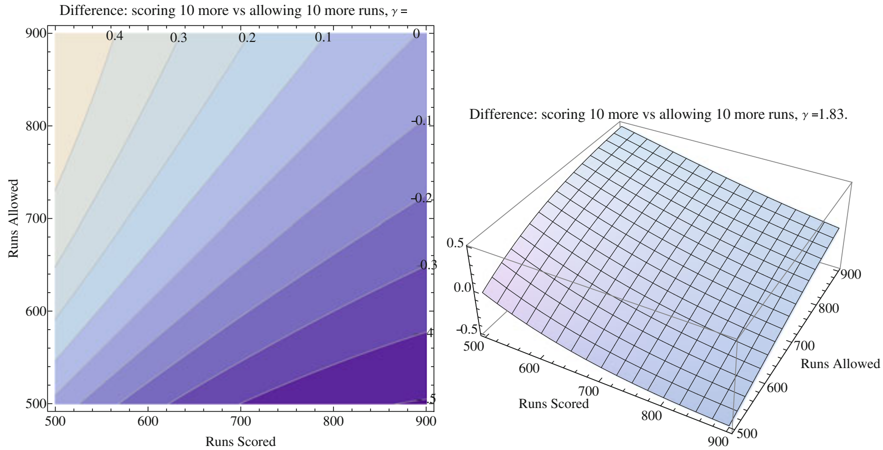

Fig. 3 The difference in the predicted number of additional wins with $\gamma = 1.83$ from scoring 10 more per season versus preventing 10 more per season. Letting $\mathscr{P}(x, y; \gamma) = x^\gamma/(x^\gamma + y^\gamma)$, the difference is $\mathscr{P}(x + 10, y; \gamma) - \mathscr{P}(x, y - 10; \gamma)$

6 The Pythagorean Formula: Verification

We have two goals in this section. First, we want to show our assumption of the runs scored and allowed being drawn from independent Weibulls is reasonable. Second, we want to find the optimal value of γ, and check the conventional wisdom that the Pythagorean formula is typically accurate to about four games a season.

There are many methods available for such analyses. Two popular ones are the Method of Least Squares, and the Method of Maximum Likelihood. As the two give similar results, we use the Method of Least Squares to attack the independence and distributional questions, and the Method of Maximum Likelihood to estimate γ and the error in the formula.

6.1 Analysis of Independence and Distributional Assumptions

We use the Method of Least Squares to analyze the 30 teams, which are ordered by the number of overall season wins and by league, from the 2012 season to see how closely our model fits the observed scoring patterns. We briefly summarize the procedure. For each team we find α_{RS}, α_{RA}, β and γ that minimize the sum of squared errors from the runs scored data plus the sum of squared errors from the runs allowed data; instead of the Method of Least Squares we could also use the Method of Maximum Likelihood (discussed in the next subsection), which would return similar values. We always take $\beta = -1/2$ and let γ vary among teams (though we could also perform the analysis with the same γ for all). We partition the runs data into the bins

$$[-0.5, 0.5),\ [0.5, 1.5],\ [1.5, 2.5],\ \ldots,\ [8.5, 9.5),\ [9.5, 11.5),\ [11.5, \infty). \tag{12}$$

Let $\mathrm{Bin}(k)$ be the kth data bin, $\mathrm{RS}_{\mathrm{obs}}(k)$ (respectively $\mathrm{RA}_{\mathrm{obs}}(k)$) be the observed number of games with runs scored (allowed) in $\mathrm{Bin}(k)$, and $A(\alpha, \beta, \gamma, k)$ be the area under the Weibull distribution with parameters (α, β, γ) in $\mathrm{Bin}(k)$. Then for each team we are searching for the values of $(\alpha_{\mathrm{RS}}, \alpha_{\mathrm{RA}}, \gamma)$ that minimize

$$\sum_{k=1}^{12} (\mathrm{RS}_{\mathrm{obs}}(k) - 162 \cdot A(\alpha_{\mathrm{RS}}, -0.5, \gamma, k))^2 + \sum_{k=1}^{12} (\mathrm{RA}_{\mathrm{obs}}(k) - 162 \cdot A(\alpha_{\mathrm{RA}}, -0.5, \gamma, k))^2 \tag{13}$$

(the 162 is because the teams play 162 games in a season; if a team has fewer games, either due to a cancelled game or because we are analyzing another sport, this number is trivially adjusted).

For each team we found the best Weibulls with parameters $(\alpha_{\mathrm{RS}}, -0.5, \gamma)$ and $(\alpha_{\mathrm{RA}}, -0.5, \gamma)$ and then compared the number of wins, losses, and won-loss percentage predicted by our model with the recorded data. The results are summarized in Table 1.

The mean of γ over the 30 teams for the 2012 season is 1.70 with a standard deviation of 0.11. This is slightly lower than the value in the literature of 1.82. The difference between the two methods is that our value of γ is a consequence of our model, whereas the 1.82 comes from assuming the Pythagorean formula is valid and finding which exponent gives the best fit to the observed winning percentages. We discuss ways to improve our model in Sect. 8.

Table 1 Results from best fit values from the Method of Least Squares, displaying the observed and predicted number of wins, winning percentage, and difference in games won and predicted for the 2012 season

Team	Obs W	Pred W	Obs %	Pred %	Diff Games	γ
Washington Nationals	98	97.5	0.605	0.602	0.5	1.76
Cincinnati Reds	97	90.7	0.599	0.560	6.3	1.80
New York Yankees	95	96.0	0.586	0.593	−1.0	1.95
Oakland Athletics	94	89.8	0.580	0.554	4.2	1.54
San Francisco Giants	94	86.1	0.580	0.531	7.9	1.72
Atlanta Braves	94	89.4	0.580	0.552	4.6	1.51
Texas Rangers	93	91.0	0.574	0.562	2.0	1.69
Baltimore Orioles	93	83.1	0.574	0.513	9.9	1.66
Tampa Bay Rays	90	90.9	0.556	0.561	−0.9	1.75
Los Angeles Angels	89	86.4	0.549	0.533	2.6	1.59
Detroit Tigers	88	94.7	0.543	0.585	−6.7	1.89
St. Louis Cardinals	88	91.0	0.543	0.562	−3.0	1.66
Los Angeles Dodgers	86	87.9	0.531	0.542	−1.9	1.65
Chicago White Sox	85	87.1	0.525	0.538	−2.1	1.66
Milwaukee Brewers	83	85.0	0.512	0.525	−2.0	1.75
Philadelphia Phillies	81	76.7	0.500	0.474	4.3	1.72
Arizona Diamondbacks	81	84.8	0.500	0.524	−3.8	1.61
Pittsburgh Pirates	79	80.3	0.488	0.496	−1.3	1.63
San Diego Padres	76	74.7	0.469	0.461	1.3	1.65
Seattle Mariners	75	74.6	0.463	0.461	0.4	1.59
New York Mets	74	75.7	0.457	0.467	−1.7	1.63
Toronto Blue Jays	73	73.7	0.451	0.455	−0.7	1.66
Kansas City Royals	72	74.8	0.444	0.462	−2.8	1.78
Boston Red Sox	69	73.6	0.426	0.455	−4.6	1.72
Miami Marlins	69	76.1	0.426	0.470	−7.1	1.74
Cleveland Indians	68	65.2	0.420	0.402	2.8	1.76
Minnesota Twins	66	65.8	0.407	0.406	0.2	1.91
Colorado Rockies	64	71.0	0.395	0.438	−7.0	1.79
Chicago Cubs	61	70.6	0.377	0.436	−9.6	1.58
Houston Astros	55	61.3	0.340	0.379	−6.3	1.61

Comparing the predicted number of wins with the observed number of wins, we see that the mean difference between these quantities is about -0.52 with a standard deviation of about 4.61. This data is misleading, though, as the mean difference is small as these are signed quantities. It is thus better to examine the absolute value of the difference between observed and predicted wins. Doing so gives an average value of about 3.65 with a standard deviation around 2.79, consistent with the empirical result that the Pythagorean formula is usually accurate to around four wins a season.

We next examine each team's z-score for the difference between the observed and predicted runs scored and runs allowed. A z-test is appropriate here because of the large number of games played by each team, a crucial difference between baseball and football. The critical value corresponding to a 95 % confidence level is 1.96, while the value for the 99 % level is 2.575. The z-score (for runs scored) for a given team is defined as follows. Let $\mathrm{RS}_{\mathrm{obs}}$ denote the observed average runs scored, $\mathrm{RS}_{\mathrm{pred}}$ the predicted average runs scored (from the best fit Weibull), σ_{obs} the standard deviation of the observed runs scored, and remember there are 162 games in a season. Then

$$z_{\mathrm{RS}} = \frac{\mathrm{RS}_{\mathrm{obs}} - \mathrm{RS}_{\mathrm{pred}}}{\sigma_{\mathrm{obs}}/\sqrt{162}}. \tag{14}$$

We see in Table 2 that both the runs scored and runs allowed z-statistics almost always fall well below 1.96 in absolute value, indicating that the parameters estimated by the Method of Least Squares predict the observed data well. We could do a Bonferroni adjustment for multiple comparisons as these are not independent comparisons, which allows us to divide the confidence levels by 30 (the number of comparisons); this is a very conservative statistic. Doing so increases the thresholds to approximately 2.92 and 3.38, to the point that all values are in excellent agreement with theory.

To further demonstrate the quality of the fit, in Fig. 4 we compare the best fit Weibulls with the Pittsburgh Pirates (who were essentially a 0.500 team and thus in the middle of the pack). The fit is excellent.

We now come to the most important part of the analysis, testing the assumptions that the runs scored and allowed are given by independent Weibulls. We do this in two stages. We first see how well the Weibulls do fitting the data, and whether or not the runs scored and allowed are statistically independent (other than the restriction that they are not equal). We describe the analysis first, and then present the results in Table 3. As the independence test is complicated by the presence of structural zeros (unattainable values), we provide a detailed description here for the benefit of the reader.

The first column in Table 3 is a χ^2 goodness of fit test to determine how closely the observed data follows a Weibull distribution with the estimated parameters, using the same bins as before. Our test statistic is

$$\sum_{k=1}^{12} \frac{(\mathrm{RS}_{\mathrm{obs}}(k) - 162 \cdot A(\alpha_{\mathrm{RS}}, -0.5, \gamma, k))^2}{162 \cdot A(\alpha_{\mathrm{RS}}, -0.5, \gamma, k)} + \sum_{k=1}^{12} \frac{(\mathrm{RA}_{\mathrm{obs}}(k) - 162 \cdot A(\alpha_{\mathrm{RA}}, -0.5, \gamma, k))^2}{162 \cdot A(\alpha_{\mathrm{RA}}, -0.5, \gamma, k)}. \tag{15}$$

This test has 20 degrees of freedom, which corresponds to critical values of 31.41 (95 % level) and 37.57 (99 % level). Of course, as we have multiple comparisons

Table 2 Method of Least Squares: z-tests for best fit runs scored and allowed

Team	Obs RS	Pred RS	z-Stat	Obs RA	Pred RA	z-Stat
Washington Nationals	4.51	4.54	−0.13	3.67	3.49	0.87
Cincinnati Reds	4.13	4.13	0.00	3.63	3.55	0.39
New York Yankees	4.96	5.02	−0.24	4.12	4.05	0.33
Oakland Athletics	4.40	4.48	−0.30	3.79	3.82	−0.15
San Francisco Giants	4.43	4.36	0.32	4.01	4.02	−0.05
Atlanta Braves	4.32	4.39	−0.27	3.70	3.76	−0.27
Texas Rangers	4.99	4.86	0.48	4.36	4.13	0.88
Baltimore Orioles	4.40	4.41	−0.09	4.35	4.26	0.35
Tampa Bay Rays	4.30	4.18	0.52	3.56	3.57	−0.04
Los Angeles Angels	4.73	4.84	−0.42	4.31	4.41	−0.38
Detroit Tigers	4.48	4.49	−0.03	4.14	3.66	2.03
St. Louis Cardinals	4.72	4.73	−0.05	4.00	4.01	−0.02
Los Angeles Dodgers	3.93	4.07	−0.67	3.69	3.63	0.29
Chicago White Sox	4.62	4.60	0.09	4.17	4.15	0.09
Milwaukee Brewers	4.79	4.89	−0.41	4.52	4.59	−0.30
Philadelphia Phillies	4.22	4.08	0.61	4.20	4.37	−0.82
Arizona Diamondbacks	4.53	4.59	−0.24	4.25	4.30	−0.26
Pittsburgh Pirates	4.02	4.12	−0.45	4.16	4.17	−0.04
San Diego Padres	4.02	4.09	−0.35	4.38	4.55	−0.76
Seattle Mariners	3.82	3.68	0.60	4.02	4.11	−0.44
New York Mets	4.01	4.06	−0.24	4.38	4.44	−0.26
Toronto Blue Jays	4.42	4.37	0.19	4.84	4.93	−0.35
Kansas City Royals	4.17	4.21	−0.17	4.60	4.63	−0.09
Boston Red Sox	4.53	4.33	0.79	4.98	4.87	0.40
Miami Marlins	3.76	3.96	−0.96	4.47	4.29	0.80
Cleveland Indians	4.12	4.06	0.22	5.22	5.21	0.00
Minnesota Twins	4.33	4.14	0.71	5.14	5.16	−0.12
Colorado Rockies	4.68	4.75	−0.29	5.49	5.53	−0.16
Chicago Cubs	3.78	3.89	−0.50	4.69	4.67	0.05
Houston Astros	3.60	3.57	0.13	4.90	5.04	−0.57

we should again perform a Bonferroni adjustment. We divide the significance levels by 30, the number of comparisons, and thus the values increase to 43.67 and 48.75. Almost all the teams are now in range, with the only major outliers being the Yankees and the Rays, the two playoff teams from the American League East.

We now turn to the final key assumption, the independence of runs scored and runs allowed, by doing a χ^2 test for independence. This test involves creating a contingency table with the requirement that each row and column has at least one non-zero entry. As the Miami Marlins had no games with 10 runs scored, we had to

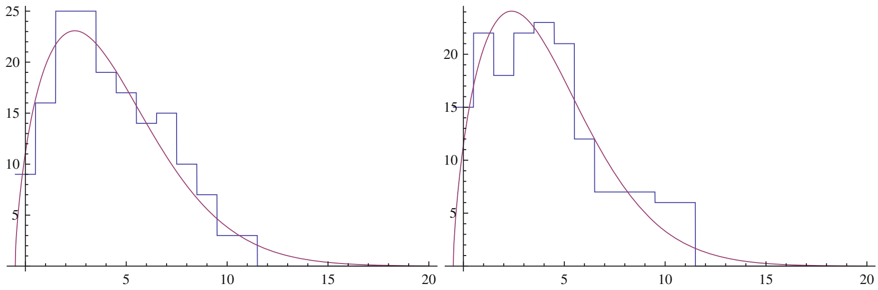

Fig. 4 Comparison of the best fit Weibulls for runs scored (*left*) and allowed (*right*) for the 2012 Pittsburgh Pirates against the observed distribution of scores

slightly modify our choice of bins to

$$[0, 1), \quad [1, 2), \quad \ldots, \quad [9, 11), \quad [11, \infty); \tag{16}$$

as we are using the observed run data from games, we can have our bins with left endpoints at the integers.

We have an 11×11 contingency table. As runs scored cannot equal runs allowed in a game (games cannot end in a tie), we are forced to have zeroes along the diagonal. The constraint on the values of runs scored and runs allowed leads to an incomplete two-dimensional contingency table with $(11-1)^2 - 11 = 89$ degrees of freedom. We briefly review the theory of such tests with structural zeros in "Independence Test with Structural Zeros" in Appendix. The critical values for a χ^2 test with 89 degrees of freedom are 113.15 (95 % level) and 124.12 (99 % level). Table 3 shows that all chi-square values for the teams in the 2012 season fall below the 99 % level, indicating that runs scored and runs allowed are behaving as if they are statistically independent. The fits are even better if we use the Bonferroni adjustments, which are 133.26 and 141.56.

6.2 Analysis of γ and Games Off

Given a dataset and a statistical model, the method of maximum likelihood is a technique that computes the parameters of the model that make the observed data most probable. Maximum likelihood estimators have the desirable property of being asymptotically minimum variance unbiased estimators. Based on the statistical model in question, one constructs the likelihood function. For our model, if we have B bins then the likelihood function is given by

Table 3 Results from best fit values from the Method of Least Squares for 2012, displaying the quality of the fit of the Weibulls to the observed scoring data, and testing the independence of runs scored and allowed

Team	RS+RA χ^2: 20 d.f.	Independence χ^2: 109 d.f
Washington Nationals	53.80	101.07
Cincinnati Reds	33.69	107.11
New York Yankees	64.02	82.82
Oakland Athletics	22.34	87.85
San Francisco Giants	14.37	89.57
Atlanta Braves	32.34	101.07
Texas Rangers	26.49	93.46
Baltimore Orioles	11.90	98.29
Tampa Bay Rays	66.35	120.25
Los Angeles Angels	28.10	105.73
Detroit Tigers	38.76	98.96
St. Louis Cardinals	36.32	117.21
Los Angeles Dodgers	31.70	123.33
Chicago White Sox	20.61	121.33
Milwaukee Brewers	49.51	98.02
Philadelphia Phillies	19.19	93.78
Arizona Diamondbacks	23.91	78.44
Pittsburgh Pirates	13.46	103.85
San Diego Padres	17.62	92.87
Seattle Mariners	9.79	113.13
New York Mets	42.88	95.66
Toronto Blue Jays	13.09	86.81
Kansas City Royals	22.51	102.39
Boston Red Sox	22.43	99.18
Miami Marlins	43.64	121.32
Cleveland Indians	26.62	83.28
Minnesota Twins	50.40	115.04
Colorado Rockies	24.30	85.79
Chicago Cubs	40.06	90.72
Houston Astros	41.16	80.48

$$L(\alpha_{\mathrm{RS}}, \alpha_{\mathrm{RA}}, -0.5, \gamma) = \binom{162}{\mathrm{RS_{obs}}(1), \dots, \mathrm{RS_{obs}}(B)} \prod_{k=1}^{B} A(\alpha_{\mathrm{RS}}, -0.5, \gamma, k)^{\mathrm{RS_{obs}}(k)} \cdot \binom{162}{\mathrm{RA_{obs}}(1), \dots, \mathrm{RA_{obs}}(B)} \prod_{k=1}^{B} A(\alpha_{\mathrm{RA}}, -0.5, \gamma, k)^{\mathrm{RA_{obs}}(k)}. \tag{17}$$

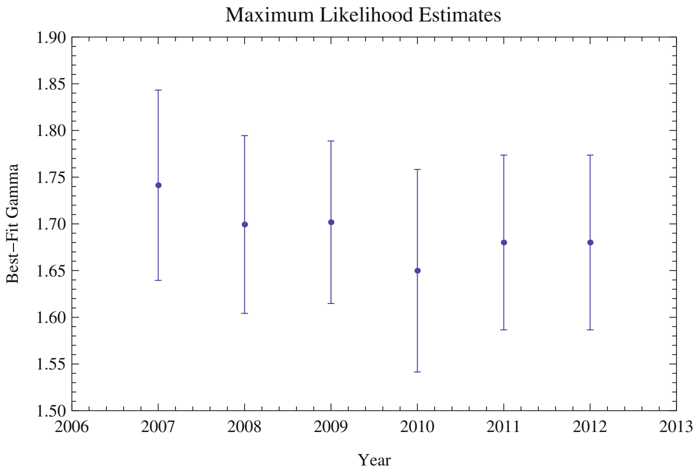

Fig. 5 Average value of γ from the Method of Maximum Likelihood

The maximum likelihood estimators are found by determining the values of the parameters α_{RS}, α_{RA} and γ that maximize the likelihood function. In practice one typically maximizes the logarithm of the likelihood because it is both equivalent to and computationally easier than maximizing the likelihood function directly.

Using our model, we calculated the maximum likelihood estimators for each team. Figure 5 displays the average values of the parameter γ for each season from 2007 to 2012, with error bars indicating the standard deviation. Note that the standard deviation of the γ values for each season are similar to each other, with 2010 having the largest deviation. The mean value of γ is about 1.69 with a standard deviation of 0.03.

Using the maximum likelihood estimators, we then calculated the predicted number of games won for each team and compared this to the observed numbers. The average absolute value of this difference is shown for each year in Fig. 6, with error bars indicating the standard deviation. The mean of the absolute value of the games off by is approximately 3.81, with a standard deviation of about 0.94; these numbers are in-line with the conventional wisdom that the Pythagorean formula is typically accurate to about 4 games per season.

7 The Pythagorean Formula: Linearization

The Pythagorean formula is not the only predictor used, though it is one of the earliest and most famous. A popular alternative is a linear statistic. For example, Michael Jones and Linda Tappin [7] state that a good estimate for a team's winning percentage

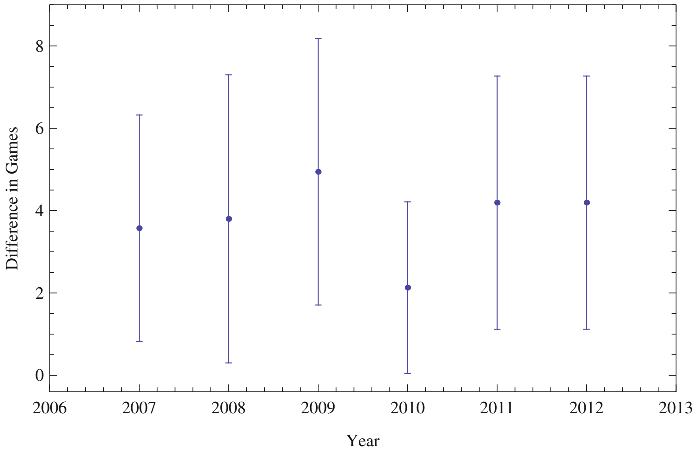

Fig. 6 Average absolute value of the difference between the observed and predicted number of wins from the Method of Maximum Likelihood

is $0.500 + \mathrm{B}(\mathrm{RS} - \mathrm{RA})$, where RS and RA are runs scored and allowed, and B is a small positive constant whose average in their studies was around 0.00065. *Note here there is a difference if we use total runs or average runs per game, as we no longer have a ratio.* We can of course use average runs per game, but that would require rescaling B; thus, for the rest of this section, we work in total runs.

While their formula is simpler to use, computers are handling all the calculations anyway and thus the savings over the Pythagorean formula is not significant. Further, by applying a Taylor series expansion to the Pythagorean formula we obtain not only this linear predictor, but also find an interpretation of B in terms of γ and the average runs scored by teams. We give a simple proof using multivariable calculus; see "Linearizing Pythagoras" in Appendix for an alternative proof that only requires one variable calculus. The multivariable argument was first given in [3] by Steven J. Miller and Kevin Dayaratna; the one-dimensional argument is from an unpublished appendix.

Given a multivariable function $f(x, y)$, if (x, y) is close to (a, b) then $f(x, y)$ is approximately the first order Taylor series about the point (a, b):

$$f(a, b) + \frac{\partial f}{\partial x}(a, b)(x - a) + \frac{\partial f}{\partial y}(a, b)(y - b). \tag{18}$$

We take

$$f(x, y) \ = \ \frac{x^\gamma}{x^\gamma + y^\gamma}, \quad (a, b) \ = \ (\mathrm{R}_{\mathrm{total}}, \mathrm{R}_{\mathrm{total}}), \tag{19}$$

where $\mathrm{R}_{\mathrm{total}}$ is the average of the total runs scored in the league. After some algebra we find

$$\frac{\partial f}{\partial x}(x, y) = \frac{\gamma x^{\gamma-1} y^{\gamma}}{(x^{\gamma} + y^{\gamma})^2}, \quad \frac{\partial f}{\partial x}(\mathrm{R_{total}}, \mathrm{R_{total}}) = \frac{\gamma}{4\mathrm{R_{total}}}, \tag{20}$$

which is also $-\frac{\partial f}{\partial y}(\mathrm{R_{total}}, \mathrm{R_{total}})$. Taking $(x, y) = (\mathrm{RS}, \mathrm{RA})$, the first order Taylor series expansion becomes

$$\begin{aligned} &f(\mathrm{R_{total}}, \mathrm{R_{total}}) + \frac{\gamma}{4\mathrm{R_{total}}}(\mathrm{RS} - \mathrm{R_{total}}) - \frac{\gamma}{4\mathrm{R_{total}}}(\mathrm{RA} - \mathrm{R_{total}}) \\ &= 0.500 + \frac{\gamma}{4\mathrm{R_{total}}}(\mathrm{RS} - \mathrm{RA}). \end{aligned} \tag{21}$$

Thus, not only do we obtain a linear estimator, but we have a theoretical prediction for the all-important slope B, namely that $\mathrm{B} = \gamma/(4\mathrm{R_{total}})$. See the paper by Dayaratna and Miller [3] for a detailed analysis of how well this ratio fits B. We content ourselves here with remarking that in 2012 the two leagues combined to score 21,017 runs (see http://www.baseball-almanac.com/hitting/hiruns4.shtml), for an average of 4.32449 runs per game per team, or an average of 700.567 runs per team. Using 1.83 for γ and 700.567 for $\mathrm{R_{total}}$, we predict B should be about 0.000653, agreeing beautifully with Jones and Tappin's findings (see http://www.sciencedaily.com/releases/2004/03/040330090259.htm).

8 The Future of the Pythagorean Formula

In the last section we saw how to use calculus to linearize the Pythagorean formula and obtain simpler estimators. Of course, linearizing the Pythagorean formula is not the only extension (and, as we are throwing away information, it is clearly not the optimal choice). In current research, the author and his students are exploring more accurate models for teams. There are two disadvantages to this approach. The first is that the resulting formula will almost surely be more complicated than the current one, and the second is that more information will be required than the aggregate scoring.

These restrictions, however, are not severe. As computers are doing all the calculations anyway, it is preferable to have a more accurate formula at the cost of additional computations that will never be noticed. The second item is more severe. The formulas under development will not be computable from the information available on common standings pages, but instead will require *inning by inning* data. Thus these statistics will not be computable by the layperson reading the sports page; however, this is true about most advanced statistics. For example, it is impossible to calculate the win probability added for a player without going through each moment of a game.

We therefore see that these additional requirements are perfectly fine for applications. Teams are concerned with making optimal decisions, and the new data required is readily available to them (and in many cases to the average fan who can write a script program to cull it from publicly available websites). The current expanded version of the Pythagorean formula, which is work in progress by the first and third authors of this paper [8], will include the following three ingredients, all of which are easily done with readily available data.

1. Write the distribution for runs scored and allowed as a linear combinations of Weibulls.
2. Adjust the value of a run scored and allowed based on the ballpark.
3. Discount runs scored and allowed from a team's statistics based on the game state.

The reason runs scored and allowed are modeled by Weibulls is that these lead to tractable, closed form integration. We can still perform the integration if instead each distribution is replaced with a linear combination of Weibulls; this is similar in spirit to the multitude of weights that occur in numerous other statistics, and will lead to a weighted sum of Pythagorean expressions for the winning percentage. An additional topic to be explored is allowing for dependencies between runs scored and allowed, but this is significantly harder and almost surely will lead to non-closed form solutions. It is highly desirable to have a closed form solution, as then we can estimate the value of a player by substituting their contributions into the formula and avoid the need for intense simulations.

The second change is trivial and easily done; certain ballparks favor pitchers while others favor hitters. The difficulty in scoring a run at Fenway Park is not the same as scoring one in Yankee Stadium, and thus ballpark effects should be used to adjust the values of the runs.

Finally, anyone who has turned on the TV during election night knows that certain states are called quickly after polls closed; the preliminary poll data is enough to predict with incredible accuracy what will happen. If a team has a large lead late in the game, they often rest their starters or use weaker pitchers, and thus the runs scored and allowed data here is not as indicative of a team's ability as earlier in the game. For example, in 2005 Mike Remlinger was traded to the Red Sox. In his first two games he allowed 5 runs to score (2 earned) while recording no outs; his ERA for the season to date was 5.45 and his win probability added was slightly negative. On August 16 the Sox and the Tigers were tied after 9 due to an Ortiz home-run in the ninth.[1] Ortiz had a three run shot the following inning, part of a 7 run offensive at the start of the tenth. With a seven run lead, this should not have been a critical situation, and Remlinger entered the game to pitch the bottom of the tenth. After retiring the first two batters, two walks and an infield single later it was bases loaded. Monroe then homered to make it 10-7, but Remlinger rallied and retired Inge. There were two reasons Papelbon was not brought in for the tenth. The first is that back then Papelbon was a starter (and in fact started that game!). More importantly, however, with a 7 run lead and just one inning to play, the leverage of the situation was low. Thus it is inappropriate to treat all runs equally. This mistake occurs in other sports; for example, when the Pythagorean formula is applied in football practitioners

[1] The data below is from http://www.baseball-reference.com/players/gl.cgi?id=remlimi01\&t=p\&year=2005 and http://scores.espn.go.com/mlb/boxscore?gameId=250816106.

frequently do not adjust for the fact that at the end of the season certain teams have already locked up their playoff seed and are resting starters.

The hope is that incorporating these and other modifications will result in a more accurate Pythagorean formula. Though it will not be as easy to use, it will still be computable with known data and not require any simulations, and almost surely provide a better evaluation of a player's worth to their team.

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Appendix

Calculating the Mean of a Weibull

Letting $\mu_{\alpha,\beta,\gamma}$ denote the mean of $f(x;\alpha,\beta,\gamma)$, we have

$$\begin{aligned}
\mu_{\alpha,\beta,\gamma} &= \int_{\beta}^{\infty} x \cdot \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1} e^{-((x-\beta)/\alpha)^{\gamma}}\,\mathrm{d}x \\
&= \int_{\beta}^{\infty} \alpha\frac{x-\beta}{\alpha} \cdot \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1} e^{-((x-\beta)/\alpha)^{\gamma}}\,\mathrm{d}x \;+\; \beta. \qquad (22)
\end{aligned}$$

We change variables by setting $u = \left(\frac{x-\beta}{\alpha}\right)^{\gamma}$. Then $\mathrm{d}u = \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1}\mathrm{d}x$ and we have

$$\begin{aligned}
\mu_{\alpha,\beta,\gamma} &= \int_{0}^{\infty} \alpha u^{\gamma^{-1}} \cdot e^{-u}\,\mathrm{d}u \;+\; \beta \\
&= \alpha\int_{0}^{\infty} e^{-u}u^{1+\gamma^{-1}}\frac{\mathrm{d}u}{u} \;+\; \beta \\
&= \alpha\Gamma(1+\gamma^{-1}) \;+\; \beta. \qquad (23)
\end{aligned}$$

Independence Test with Structural Zeros

We describe the iterative procedure needed to handle the structural zeros. A good reference is Bishop and Fienberg [2].

Let $\mathrm{Bin}(k)$ be the kth bin used in the chi-squared test for independence. For each team's incomplete contingency table, let $O_{r,c}$ be the observed number of games where the number of runs scored is in $\mathrm{Bin}(r)$ and runs allowed is in $\mathrm{Bin}(c)$. As games cannot end in a tie, we have $O_{r,r} = 0$ for all r.

We construct the expected contingency table with entries $E_{r,c}$ using an iterative process to find the maximum likelihood estimators for each entry. For $1 \leq r, c \leq 12$, let

$$E_{r,c}^{(0)} = \begin{cases} 1 & \text{if } r \neq c \\ 0 & \text{if } r = c, \end{cases} \tag{24}$$

and let

$$X_{r,+} = \sum_c O_{r,c}, \quad X_{c,+} = \sum_r O_{r,c}. \tag{25}$$

We then have that

$$E_{r,c}^{(\ell)} = \begin{cases} E_{r,c}^{(\ell-1)} X_{r,+} / \sum_c E_{r,c}^{(\ell-1)} & \text{if } \ell \text{ is odd} \\ E_{r,c}^{(\ell-1)} X_{c,+} / \sum_r E_{r,c}^{(\ell-1)} & \text{if } \ell \text{ is even.} \end{cases} \tag{26}$$

The values of $E_{r,c}$ can be found by taking the limit as $\ell \to \infty$ of $E_{r,c}^{(\ell)}$, and typically the convergence is rapid. The statistic

$$\sum_{\substack{r,c \\ r \neq c}} \frac{(E_{r,c} - O_{r,c})^2}{E_{r,c}} \tag{27}$$

follows a chi-square distribution with $(11-1)^2 - 11 = 89$ degrees of freedom.

Linearizing Pythagoras

Unlike the argument in Sect. 7, we do not assume knowledge of multivariable calculus and derive the linearization using just single variable methods. The calculations below are of interest in their own right, as they highlight good approximation techniques.

We assume there is some exponent γ such that the winning percentage, WP, is

$$\mathrm{WP} = \frac{\mathrm{RS}^\gamma}{\mathrm{RS}^\gamma + \mathrm{RA}^\gamma}, \tag{28}$$

with RS and RA the total runs scored and allowed. We multiply the right hand side by $(1/\mathrm{RS}^\gamma)/(1/\mathrm{RS}^\gamma)$ and write RA^γ as $\mathrm{RS}^\gamma - (\mathrm{RS}^\gamma - \mathrm{RA}^\gamma)$, and find

$$\begin{aligned}
\mathrm{WP} &= \frac{1}{1+\frac{\mathrm{RA}^\gamma}{\mathrm{RS}^\gamma}} = \left(1+\frac{\mathrm{RA}^\gamma}{\mathrm{RS}^\gamma}\right)^{-1} = \left(1+\frac{\mathrm{RS}^\gamma-(\mathrm{RS}^\gamma-\mathrm{RA}^\gamma)}{\mathrm{RS}^\gamma}\right)^{-1}\\
&= \left(1+1-\frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{\mathrm{RS}^\gamma}\right)^{-1}\\
&= \left(2\cdot\left(1-\frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{2\mathrm{RS}^\gamma}\right)\right)^{-1}\\
&= \frac{1}{2}\left(1-\frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{2\mathrm{RS}^\gamma}\right)^{-1}; \qquad (29)
\end{aligned}$$

notice we manipulated the algebra to pull out a 1/2, which indicates an average team; thus the remaining factor is the fluctuations about average.

We now use the geometric series formula, which says that if $|r| < 1$ then

$$\frac{1}{1-r} = 1 + r + r^2 + r^3 + \cdots. \qquad (30)$$

We let $r = (\mathrm{RS}^\gamma - \mathrm{RA}^\gamma)/2\mathrm{RS}^\gamma$; since runs scored and runs allowed should be close to each other, the difference of their γ powers divided by twice the number of runs scored should be small. Thus r in our geometric expansion should be close to zero, and we find

$$\begin{aligned}
\mathrm{WP} &= \frac{1}{2}\left(1+\frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{2\mathrm{RS}^\gamma}+\left(\frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{2\mathrm{RS}^\gamma}\right)^2+\left(\frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{2\mathrm{RS}^\gamma}\right)^3+\cdots\right)\\
&\approx 0.500 + \frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{4\mathrm{RS}^\gamma}. \qquad (31)
\end{aligned}$$

We now make some approximations. We expect $\mathrm{RS}^\gamma - \mathrm{RA}^\gamma$ to be small, and thus $\frac{\mathrm{RS}^\gamma-\mathrm{RA}^\gamma}{2\mathrm{RS}}$ should be small. This means we only need to keep the constant and linear terms in the expansion. Note that if we only kept the constant term, there would be no dependence on points scored or allowed!

We need to do a little more analysis to obtain a formula that is linear in RS − RA. Let $\mathrm{R}_{\mathrm{total}}$ denote the average number of runs scored per team in the league. We can write $\mathrm{RS} = \mathrm{R}_{\mathrm{ave}} + x_s$ and $\mathrm{RA} = \mathrm{R}_{\mathrm{ave}} + x_a$, where it is reasonable to assume x_s and x_a are small relative to $\mathrm{R}_{\mathrm{total}}$. The Mean Value Theorem from Calculus says that if $f(x) = (\mathrm{R}_{\mathrm{total}} + x)^\gamma$, then

$$f(x_s) - f(x_a) = f'(x_c)(x_s - x_a), \qquad (32)$$

where x_c is some intermediate point between x_s and x_a. As $f'(x) = \gamma(\mathrm{R}_{\mathrm{total}} + x)^{\gamma-1}$, we find

$$\mathrm{RS}^{\gamma} - \mathrm{RA}^{\gamma} \;=\; f(x_s) - f(x_a) \;=\; f'(x_c)(x_s - x_a) \;=\; \gamma(\mathrm{R}_{\mathrm{total}} + x_c)^{\gamma-1}(\mathrm{RS} - \mathrm{RA}), \tag{33}$$

as $x_s - x_a = \mathrm{RS} - \mathrm{RA}$. Substituting this into (31) gives

$$\mathrm{WP} \;\approx\; 0.500 + \frac{\gamma(\mathrm{R}_{\mathrm{total}} + x_c)^{\gamma-1}(\mathrm{RS} - \mathrm{RA})}{4\mathrm{RS}^{\gamma}} \;=\; 0.500 + \frac{\gamma(\mathrm{R}_{\mathrm{total}} + x_c)^{\gamma-1}}{4\mathrm{RS}^{\gamma}}(\mathrm{RS} - \mathrm{RA}). \tag{34}$$

We make one final approximation. We replace the factors of $\mathrm{R}_{\mathrm{total}} + x_c$ in the numerator and RS^{γ} in the denominator with $\mathrm{R}_{\mathrm{total}}^{\gamma}$, the league average, and reach

$$\mathrm{WP} \;\approx\; 0.500 + \frac{\gamma}{4\mathrm{R}_{\mathrm{total}}}(\mathrm{RS} - \mathrm{RA}). \tag{35}$$

Thus the simple linear approximation model reproduces the result from multivariable Taylor series, namely that the interesting coefficient B should be approximately $\gamma/(4\mathrm{R}_{\mathrm{total}})$.

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Adaptive Systems in Sports

Arnold Baca

Abstract Athletes voluntarily change their sportive behavior in order to improve performance or to reduce load. If this process is guided by feedback loops, characteristics of adaptive systems are met. The occurring adaptive change is relevant to achieving a goal or objective. In a similar manner, smart sports equipment may alter its properties depending on environmental conditions. In order to automatically give feedback on how to continue exercising and/or to adjust the sports equipment during the physical activity, intelligent devices are required. These devices rely on models for recognition and classification of patterns in the motion or activity currently performed. Different methods and models, such as Neural Networks, Hidden Markov models or Support Vector Machines have proven to be applicable for this purpose. Examples from recreational running, mountain-biking, exercising on weight training machines and long distance running illustrate the principle.

1 Introduction

Intelligent devices are electronic devices that can perform operations guiding its behavior with respect to its functionality and the surrounding environment to some extent autonomously. Being applicable for monitoring physical activity, thereby automatically suggesting strategies and interventions [2], they are getting increasingly important for assisting athletes during sportive activities. In doing so, miniature sensors and computing devices are attached to the athletes or integrated into the sports equipment in order to acquire and process performance or load related data. Common and easily accessible computing technologies are thus applied to implement systems, which provide athletes with feedback information on the quality of the motion just performed [1] as well as on recommendations on how to further proceed. Moreover, the acquired data are used in order to adapt the sports equipment to the current needs of the athlete.

A. Baca (✉)
Department of Biomechanics, Kinesiology and Applied Computer Science, ZSU, University of Vienna, Vienna, Austria
e-mail: arnold.baca@univie.ac.at

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Basis for almost any such system is the successful recognition of patterns characterizing the sports activity just performed and/or the prediction of certain individual parameters (e.g. heart rate), if the sports activity is continued. The respective methods are based on motion specific parameter values acquired by various sensors, which often are arranged in the form of wireless sensor networks (WSNs). Such WSNs consist of spatially distributed autonomous sensor nodes, which cooperatively monitor signals from persons (e.g. respiratory rate) or from the environment, (pre-)process these data and exchange them with other sensor nodes. Each sensor node in a sensor network typically consists of one or more sensors, a microcontroller, a communication unit for wireless data exchange and an internal power supply. The possibility of miniaturizing sensors and sensor nodes makes them perfectly suitable for acquisition of (biophysical, physiological, etc.) parameter values during regular sports activities.

The purpose of this paper is to give an overview on principles and methods underlying the development and implementation of adaptive systems in sports and to provide selected application examples.

2 Pattern Recognition and Classification

Sensors and sensor networks measuring human motion provide a large amount of data. Different methods and models have proven to be useful for identifying patterns or classes in the respective sets of parameter values acquired. Baca [5] has given a survey on most common approaches, which is summarized below.

2.1 Statistical Classification

Figure 1 illustrates the process of statistical classification.

In the first step, acquired sensor data are often pre-processed. In order to reduce the (high) dimensionality of the data set, feature extraction is accomplished next. There are different major approaches in doing this. The easiest approach is to calculate simple statistics from the (motion) data such as minimum, maximum, variance, etc. Another type of features is obtained by applying filtering techniques such as the fast Fourier transform (FFT). In many cases, a linear transformation is applied to project the data space into a new feature space with lower dimension. Principal component analysis (PCA), independent component analysis (ICA) and linear discrimination analysis (LDA) are widely used for feature extraction and dimensionality reduction [7]. In a next step, feature selection methods identify a subset of features from the original set well suited for a subsequent classification, thereby obeying various optimization criteria [11]. Efficient feature selection procedures are sequential forward selection (beginning with an empty set and repeatedly adding the feature best fulfilling the optimization criterion with the already selected) and backward selection

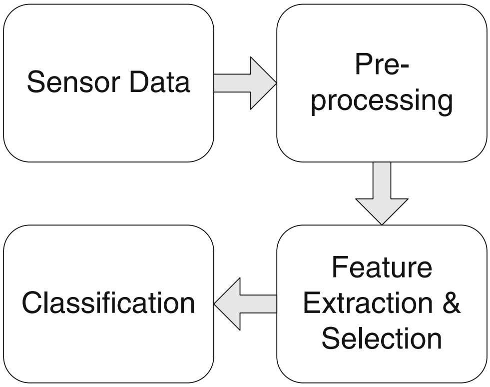

Fig. 1 Statistical classification from sensor data

(removing features repeatedly from the set) [7]. Once features have been selected, statistical classifiers are applied for identifying the corresponding class of the motion sequence performed in order to be able to select the appropriate feedback message or to initiate an adequate procedure for adapting the sports equipment. Various classification methods, such as binary classification trees, decision engines, Bayes classifier, k-Nearest Neighbour (k-NN), rule-based approaches, linear discriminant classifier (LDC), fuzzy logic techniques or Support Vector Machines (SVMs) may be utilized for this purpose.

2.2 Neural Networks

Artificial neural networks (ANNs) may also be regarded as statistical classifiers. Supervised neural networks are particularly suited for separating between different activities or performances. Among these, neural networks of type MLP (multilayer perceptron) are quite commonly applied.

Throughout the last years, the use of ANNs as a tool for analyzing human motion data has got more attention in sports and clinical biomechanics (e.g. [6, 20]). Much consideration is given to unsupervised neural nets such as self-organizing maps (SOM). Their application has proven to be helpful for identifying patterns in complex motor tasks or processes. SOMs enable to map motion processes to a two-dimensional trajectory described by a sequence of neurons. Each neuron represents a unique state of this process.

A variant of a SOM-type network as proposed by Perl (DyCon; [14, 15]) has shown to be successfully applicable for identifying classes of motions (motion types) and similarities in motion processes. Baca and Kornfeind [4], for example, trained a DyCoN of 400 neurons in order to identify types of aiming processes in biathlon shooting. After training the network, each neuron represented a feature vector representative for a set of kinematic parameters characterizing the motion of the muzzle of a biathlon gun in one of ten time intervals (of equal duration) of the total aiming motion. Similar neurons were then combined to clusters (Fig. 2).

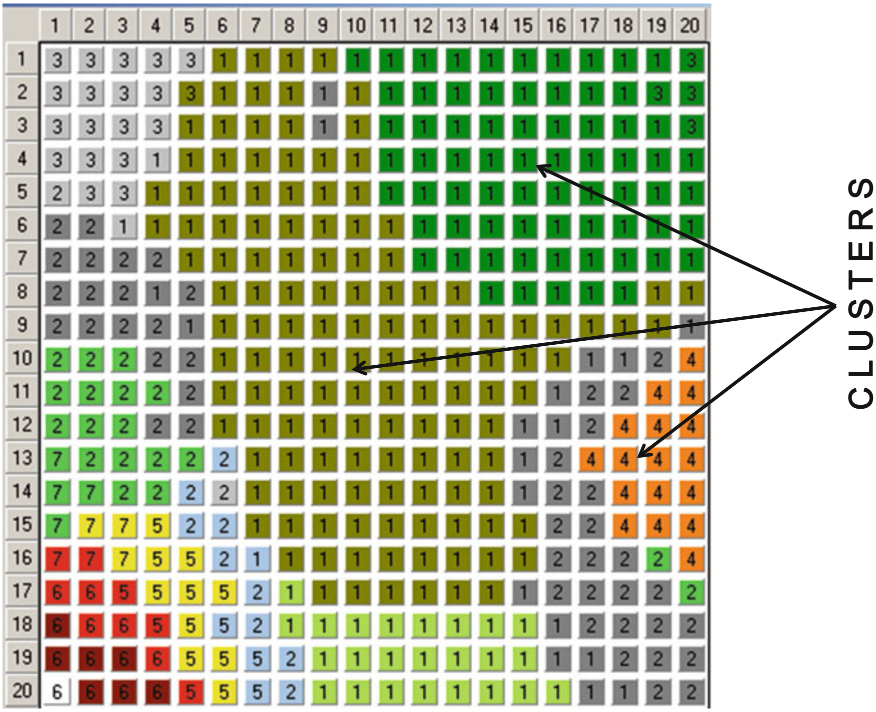

Fig. 2 DyCon. Each neuron represents a representative state of the motion process

For each analyzed shot the ten successive data sets (feature vectors) describing the aiming process were mapped to the corresponding neurons (winner neurons) of the net. The sequence of the related clusters in the respective succession was then used as 1-dimensional representation ("phase diagram") of the complex aiming motion. In a second processing step types of shots were identified taking these sequences for training and analysis of a second net.

A particular advantage of SOM-type neural networks is that process types previously not foreseen may be detected.

One probable reason for not being utilized that much as supervised neural networks for classification tasks, is their demand for a large training data set. DyCon, however, requires a substantially reduced training data set only, e.g. by adding stochastically generated data [15].

2.3 Hidden Markov Models

Hidden Markov Models (HMMs) provide another alternative in the realization of classification algorithms. They are particularly known for their applicability in temporal pattern recognition and are therefore well appropriate for being used in categorizing human motion and activity. Generally, a HMM is a statistical model for describing the characteristics of a stochastic process. There are a finite number of states, each of which is associated with a transition probability to other states. At each time one specific state is taken. The state a specific time is directly and solely influenced by the state at the previous time. After each transition from one state to the next, an output observation is generated based on an observation probability distribution associated with the current state (cf. [10]). From the observable output the underlying process may be deduced. If, for instance, different motion types shall be classified, separate HMMs are trained for each individual motion type. When evaluating a given sequence, likelihoods from each trained HMM are calculated. The sequence can then be assigned to the HMM with the largest likelihood.

2.4 Specific Models

The aim of any feedback system for athletes is to give appropriate advice on future movement execution. Recommendations can be derived from the current state (e.g. exhaustion, muscle fatigue, motion technique, etc.). Even more accurate feedback instructions can be provided, if the change of certain parameters (e.g. physiological parameters, such as heart rate) is correctly predicted. In order to do so, sophisticated models (e.g. from physiology) may be applied.

3 Adaptive Sports Equipment

In order to illustrate the principle, two examples are given:

Eskofier et al. [8] compared systems for classifying speed and track inclination groups during recreational runs. Classification was based on running speed, altitude and shoe heel compression data, which were recorded continuously while athletes ran freely outdoors. A classification system based on SVMs turned out to be best suitable for speed classification, was finally implemented and verified on the embedded microprocessor of the "adidas_1" shoe. An MLP based solution, which performed even better, was not further considered taking into account the much higher computation effort and the computing power of the microprocessor. The heel part of the "adidas_1" contained an adjustable cushioning element. The amount of vertical compression that this element allowed was regulated by a motor-driven cable system. Thus, the shoe stiffness could be adapted to the specific running speed of a particular sportsman.

Hansmann et al. [9] suggest a biomechanical-mechatronical design approach for developing an "intelligent" mountain bike. The mountain bike comprises a self sustaining adaptive mountain bike shock absorber system adapting its damping force according to the driver's behavior and environmental characteristics. A modular simulation system integrating sub-models for the human and the mechatronic system is proposed for identifying and quantifying mutual dependencies of relevant system parameters.

4 Adaptive Behavior in Exercising

Again, the principle is illustrated exemplarily:

Two papers by Novatchkov and Baca [12, 13] focus on the implementation of intelligent decision methods for the automatic evaluation of exercises in weight training. Way and cable force sensors attached to various weight machines were used, enabling the measurement of essential displacement and force determinants during exercising. The thereby determined motion parameters were applied for the development of intelligent methods adapted from conventional machine learning concepts, allowing an automatic assessment of the exercise technique and providing individuals with appropriate feedback. Whereas an MLP based solution is pursued in the 2013a-paper, the applicability of fuzzy logic techniques for the evaluation of exercises performed on weight training machines is investigated in the 2013b-study. The results showed good performance and prediction outcomes, indicating the feasibility and potency of respective methods in assessing performances on weight training equipment automatically and providing sportsmen with prompt advice. It was concluded that the implementation of such techniques can be crucial for the investigation of the quality of the execution, the assistance of athletes but also coaches, the training optimization and for prevention purposes.

A general concept for intelligent feedback systems applicable in sports training, the *Mobile Motion Advisor*, is proposed by the working group of the author [3]. The intention is to combine mobile data acquisition methods with centralized analysis routines and feedback functionality for supporting athletes. The approach is based on a feedback system providing mobile and almost real time solutions for wireless body sensor data transmission, processing and feedback provision. Characteristic parameters of the physical activity can thus be supervised continuously. In this way, athletes get individual feedback on the quality of their motion which also helps to interpret the body's reactions to physical load. Sensors, carried by the person or mounted onto the sports equipment, are used to measure different parameters like heart rate, velocity or reactive forces of an exercising person. The parameters values are sent to a smartphone application (Fig. 3) via ANT+$^{\mathrm{TM}}$ (ANT+$^{\mathrm{TM}}$ is an already well-established standard for wireless sensor data transmission used by many manufacturers of technological sports-equipment).

The measured data is then transmitted to an application server using wireless communication technologies (UMTS, HSUPA). Figure 4 shows the whole dataflow from the sensors to the Smartphone-Application (Athlete-Client) and from there to the Server (Expert-Client).

Based on the collected data (e.g. presented as Flash charts on a web interface), feedback instructions can be generated by experts and sent back to the exercising person.

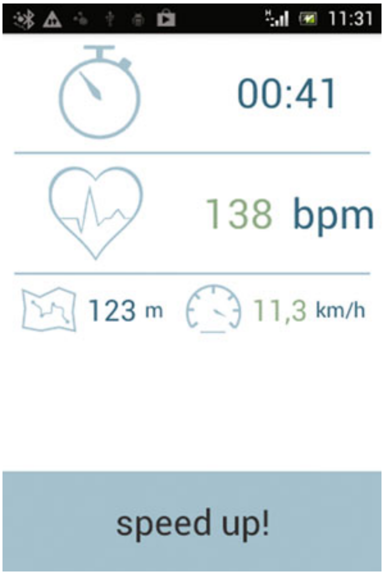

Fig. 3 Android based Smartphone Client

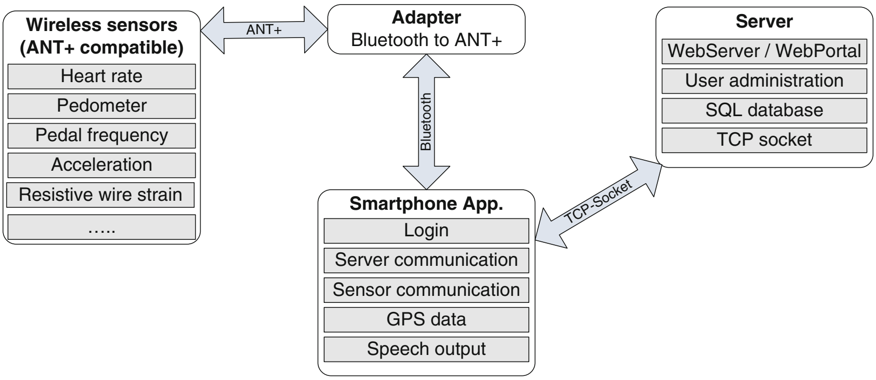

Fig. 4 Data flow from sensors to server

Moreover, sub-modules may be integrated into the server application in order to implement intelligent algorithms for processing the acquired data (Fig. 5). In this way, feedback instructions may automatically be generated or peculiarities in the data may be detected and subsequently be considered by the expert when providing feedback.

Altogether, the following goals can be achieved:

1. Assistance during sport activities
2. Improvement of training performance

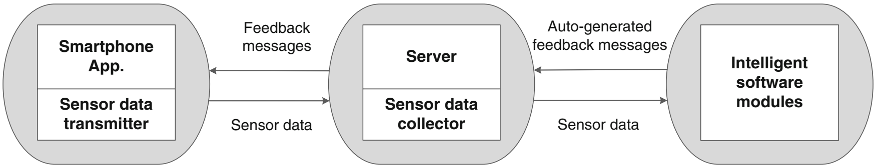

Fig. 5 Data flow between smartphone, server and intelligent software modules

3. Avoidance of overload
4. Increase of motivation by individual feedback

Coaches are able to guide a number of athletes individually and athletes get feedback of the quality of their motion which helps to interpret the body's reactions to physical load. The athletes' performances are recorded and so the bodily changes during a certain time are documented. Positive effects of physical exercise are highlighted.

A particular implementation of the proposed system for long distance running includes an intelligent feedback module for guiding further execution [5, 21]. A typical situation in long distance running is a temporary unobserved overload, which is much later followed by an unexpected break down. The reason is a delayed reduction of the reserve of the fatigue potential, which then causes a sudden overflow of strain together with a significant loss of performance. Therefore, reserve is the central aspect of predicting future performance development.

In a related prototype system, which has been developed for assisting athletes in cross country running, [22] predict performance development based on function interpolation from current environment conditions and heart rate measurements as well as from potential environment conditions of the next tracks. The main difficulty of the method lies in the correct prognosis of the heart rate. In order to overcome this problem, an antagonistic meta-model (PerPot), which has been developed by Perl [16, 17] is applied. This meta-model has proven to predict load-based performance development very precisely. Extensions of PerPot are able to determine the individual anaerobe threshold (IAT) by simulation, enabling optimization of speed and heart rate profiles in endurance sports [19].

PerPot (Fig. 6) describes physiological adaptation on an abstract level as an antagonistic process [18]: A load input flow is identically feeding a strain potential and a response potential. The response potential increases the performance potential by a positive flow, while the strain potential reduces it by a negative flow. All flows show specific delays, thus modeling the time the components of the modeled system need to react. In particular in endurance sports delays play an important role for the process of fatigue and recovering [17, 18].

In order to determine the individual performance parameters (strain potential, response potential) of an athlete required for the PerPot module, a run similar to a step-test has to be performed. Based on these parameters, the system is able to calculate the optimal goal time and speed for a given distance and track profile thereby forming the basis for the generation of feedback.

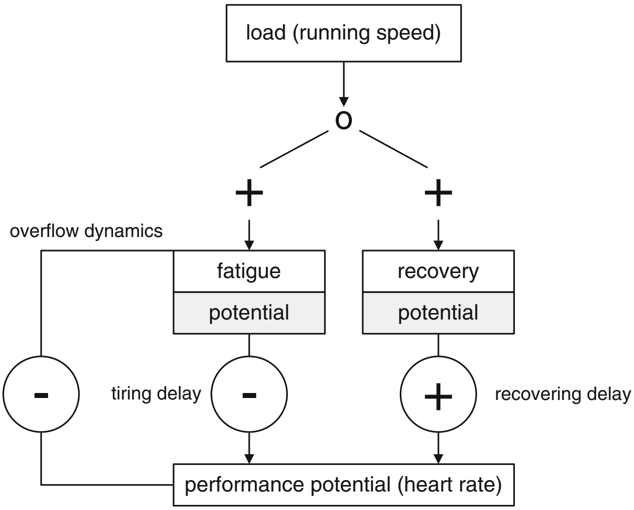

Fig. 6 Basic PerPot structure

By integrating the PerPot module into the server application of the *Mobile Motion Advisor*, it is possible to continuously update the individual performance parameters, thereby achieving a still improved fine tuning of the long distance run.

5 Conclusions

As a result of feedback loops, athletes may change their way of exercising and improve their performance or to reduce load. Intelligent devices providing individual feedback automatically in almost real time are particularly promising. Similarly, smart sports equipment may automatically adapt to the athlete's behaviour and to environmental conditions. Consequently, the implementation of such techniques is not only beneficial for the assessment of the quality of the exercise, but, moreover, for the assistance of athletes and coaches, for training optimization and for prevention purposes.

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By Sport Predictions Through Socio Economic Factors and Tradition in Summer Olympic Games: The Case of London 2012

F. Javier Otamendi and Luis M. Doncel

Abstract Socio-economic prediction of medals for London 2012 is performed "by sport" using OLS and a discretization routine. The success ratio is above 65 % for any given sports, especially for disciplines that award more than 30 medals. At the overall country level, the success raises above 85 %. The analysis of the award winning process by sports shed also new light about the critical factors that might dictate the success and that are liable to set sport policies, including the development of sound social networks and the investment on sport infrastructures to foster talent.

Keywords Olympics · Econometric modelling · Prediction · Sport policy

1 Introduction

Methods and techniques for predicting the outcomes of sport competitions are a current line of research [12, 15, 18]. The major sport competition of the modern era is the Olympic Games, both in terms of the number of sports and also in terms of audiences, and the results could also be forecasted in advance and analyzed after the Games.

Regarding prediction models, recent studies have already put under scrutiny the factors that may affect the winning of medals at the Games: economic, demographic, sociological or political. The econometric studies have been mainly performed using forecasting tools both for the Summer [3, 6, 8, 21], and the Winter Games [1, 11, 14], or for both [10]. The focus of these pioneer works has been to predict the medal share by country and not on deriving hints for future success.

Our interest is to go one step further and perform, for the first time to our knowledge, the quantitative analysis by sport and country for the Summer Olympics, after performing it for the Winter games with success [13]. Of course, the

F.J. Otamendi (✉) · L.M. Doncel
Universidad Rey Juan Carlos, Madrid, Spain
e-mail: Franciscojavier.otamendi@urjc.es

L.M. Doncel
e-mail: Luismiguel.doncel@urjc.es

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difficulty of the prediction task increases as the total medal count per sport decreases substantially in some disciplines. For example, in the 2012 London Summer Games, the sport medal count varies between a very low 6 in Handball, Basketball or Hockey to a very high 142 in Athletics, whereas the total medals awarded accounts for 962. But the extra information that can be obtained from this complex exercise is worth a try, since it can shed some light on factors that affect the "by sport" predictions.

We will take both Johnson and Ali's and Pfau's models as the benchmark to derive a new forecasting model and calculate the estimations by sports resulting in 40 different models (39 sports and the overall model). The new model includes specific socio-economic factors in order to explain the medal award winning process so as to successfully predict the outcome by sport and country. The aim is also to derive possible hints for future sport policies, since countries are willing to invest in sports in search for sport excellence and country visibility [20].

In this research, we have selected the period that starts on 1992 after the segregation of the Soviet and Balkan Republics and ends on 2012 with the 2012 London Games. They are the last Summer Games that have been held.

This article shows both the theoretical bases of the "by sport" econometric methodology in Sect. 2 and the forecasts for London in Sect. 3. The analysis of the observed deviations from the predicted outcome and the quantification of the success of the predictions are shown in Sect. 4. Section 5 concludes.

2 The "By-sport" Econometric Methodology

The first stage of this research is focused on using traditional econometric methods and tools to determine significant factors in the quest for Olympic success, but modifying the models and their predictions through the categorization of country outcomes by sport.

There is no model to our knowledge that performs an econometric analysis in general or a prediction of a medal table in particular not for the Games as a whole but by sport. What follows is the explanation of the proposed "by sport" methodology based on regression models solved using ordinary least square methods (OLS).

2.1 Step 1. Modeling the Medal Share

Our first starting point is the work of Johnson and Ali [10], JA model, with the aim of calculating the medals to be awarded to a specific country:

$$\begin{aligned} Medal_t = {} & \beta_0 + \beta_1 GDPCAP + \beta_2 GDPCAP^2 + \beta_3 POP \\ & + \beta_4 POP^2 + \beta_5 HOST + \beta_6 NEIGH + \sum_{j=7}^{10} \beta_j POL_j \\ & + \beta_{11} LFROST + \beta_{12} HFROST + \beta_{13} t + \beta_{14} MED \end{aligned}$$

As economic factors (β_1 to β_4), one might expect that countries with higher gross domestic product per capita (*GDPCAP*) or population (*POP*) should produce better athletes capable of winning Olympic medals.

Host countries (*HOST*) and neighbors (*NEIGH*) (β_5 and β_6) can also be expected to perform better on account of the increased audience support, familiarity with the sporting facilities, and other factors like investments. Being the host is a critical factor in the start line to success, motivated by the extra budget to spend on sport both on the short and on the long run, as well as the motivation of the athletes to succeed in home soil. It seems that being the host country facilitate medal winning [1]. This effect, linked with the economic benefits of hosting the Olympics [7], has increased the fights to become a host nation.

Socio-political issues (β_7 to β_{10}) have made a subset of countries performed unusually well given their circumstances (*POL* includes those countries with special types of political regimes, like Cuba). Many countries are looking for the status, prestige or symbolic capital that Olympic success provides [20]. These countries try to obtain a medal if the sport budget is appropriately spent.

As environmental variables (β_{11} to β_{12}) the model separates those countries in terms of the average number of days in which the territory is frosted (LFROST for low and HFROST for high). Finally, the number of medals to be awarded in total is included, MED (β_{14}), so as to limit the value of the output variable or medals predicted to be won.

The second starting model is that of Pfau [14], which focuses on the Winter Games, but it is the basis for the first successful set of the predictions by sport [13]. The econometric model to calculate the medal share per country (*MS*) looks as follows:

$$
\begin{aligned}
MS_t = {} & \beta_0 + \beta_1 \log(GDPCAP) + \beta_2 \log(POP) + \beta_3 HOST + \beta_4 MS_{t-1} \\
& + \beta_5 SOV + \beta_6 SCAND + \beta_7 GERM + \beta_8 ALPINE + \beta_9 NORTHAM
\end{aligned}
$$

The model restricts the effect of *GDPCAP* and *POP* and includes only the host (and not the neighbor). It also predicts in terms of shares and not total counts; in a sense, it is the same as predicting totals and including *MED* as an independent variable. The model only uses one political variable *POL* (β_5), which corresponds to soviet tradition.

The model bases its predictions in the winning tradition (β_4), including the lagged share of medals for a country, MS_{t-1}, as it is reasonable to expect some inertia in medal winning performances. Past success at the Olympics can help explain present outcomes and successes since tradition gives the countries an edge because they have already developed elite sports and built on infrastructure, social networks and expert knowledge (coaches, training methods or talent identification systems [19]).

The Pfau model finally includes, as geographical factors, groups of countries that have similar characteristics (β_6 to β_9), maybe because of the weather (the Scandinavian variable (*SCAND*) includes Norway, Sweden, and Finland), because of the mountains (the Alpine country variable (*ALPINE*) includes Switzerland, Italy, and

France), because of the location (North America variable (*NORTHAM*) includes the United States and Canada), or because of their joint socio-political history (the Germanic country variable (*GERM*) includes Germany and Austria, although they also could have been included as *ALPINE*).

We propose to combine both models to develop a new robust model to predict the medals to be won by each country and sport. Since the variable *POP* has been one of the main causes of deviations from predictions to results, with countries like India having excessive predicted values in the JA model, a new transformation of the variable has been proposed by taking the square root. The variable *NEIGH* is back in the formulation since it has been another cause of deviation when the factor is absent, especially within the Winter Games models. The variable *POL* is now unique since its impact has proven to be lower and just for soviet countries and in particular sports. Only one *FROST* variable, the one corresponding to *LFROST* is used.

Concerning the very important social context, history and tradition, the ability to invest and fund elite sport combined with the management of the resources and the development of policies leads to greater achievements in international sport [17]. Sport policies should be strategically and consciously planned in order to achieve success both in the short and in the long run [13]. On this regard, two distinct factors have been included to account for long term and short term impacts.

The first one, *MedalHistory*, is the total number of medals won in past history. In the long run, the countries might spend their budget in developing the country expertise, by building infrastructure and attracting coaches and training methods to build and acquire skill [22]. Also, investing in technological background might excel in some sports [4, 9].

The second one, $Medal_{t-1}$, corresponds to the short term success in just the previous games. There are two known possibilities that might account for short term success: "talent pipeline" [2], where the governments may grant and allow their young talents to train abroad in the best centers to improve their ability and competitiveness; and the nationalization of star athletes [16], which both increases the possibility of winning a medal and the performance of their own athletes in competition.

The proposed model is the following:

$$
\begin{aligned}
Medal_t = {} & \beta_0 + \beta_1 GDPCAP + \beta_2 GDPCAP^2 + \beta_3 \sqrt{POP} + \beta_4 HOST \\
& + \beta_5 NEIGH + \beta_6 POL_6 + \beta_7 FROST + \beta_8 MED \\
& + \beta_9 MedalHistory + \beta_{10} Medal_{t-1}
\end{aligned}
$$

2.2 Step 2. Converting to Medal Count

The outcome of any forecasting methodology is therefore the medal count (*Medal*), which is discrete in nature, but treated as a continuous variable in the models since OLS regression is used. Hence, it is necessary to directly convert the medal count into an integer number per sport and country. The models show then a rounding problem, since the sum of the medal counts per country is not likely to add up to the total count. As such, Pfau's use of the model only accounts for 249 medals in the 2006 Torino Games, whereas in fact 252 total medals were awarded. A key aspect of our methodology is therefore the conversion from medal count (continuous variable) to total medal count (finite discrete variable), to solve the rounding problem.

A mathematical routine has been developed to come up with the exact number of medals to be awarded. The continuous model output is converted into a share and initially rounded to the closest integer individually for each country and sport, and the total summed up for every country. If the total did not coincide with the total awarded, a scale factor was used to recalculate the shares and determine if the total was then exact. The routine looks for the first scaled set of shares that, after rounding, guarantees that the number of predicted and awarded medals is the same.

This rounding exercise is very much necessary whenever the purpose is to identify countries that are likely to win medals, even if they are newcomers and their total count is just 1 (newcomers or rising countries with low totals).

3 Econometric Predictions for 2012 London Olympic Games

3.1 Model Estimation

The proposed "by sport" methodology is applied to the last Summer Games that have been held and that have taken place in London, UK, in 2012, with the aim of showing the predictive power of the "by sport" methodology and of determining the factors that are statistically significant to derive policy issues. For the study period 1992–2008, thirty-nine models were then estimated to determine the statistically significant factors. The results are shown in Table 1, indicating with asterisks those beta coefficients that are significantly different than 0 at the 0.05 level[1].

The analysis of factors shows that the *POP* variable is still critical but the tradition variables clearly show a difference between recent and past history. In seventeen sports, only recent history is a significant factor, for example, in Basketball and Weightlifting, whereas just in six sports only past history is a significant factor: Artistic Gymnastics, Cycling/Mountain Bike, Equestrian/dressage, Rhythmic Gymnastics, Sailing and Triathlon. Both factors are critical in nine sports, like Archery and Athletics, whereas tradition is not significant in the other six sports (for example, Football and Trampoline).

[1] We have also done the significance analysis with the two original models to demonstrate the higher potential for analysis of the proposed model (see Appendix). In the JA model, the variables with predictive power are *GDPCAP* and *POP* as well as *MED* to account for size of the sport or number of medals to be awarded. *HOST* and *FROST* also play an important role. In the Pfau model, $MShare_{t-1}$ accounts for tradition in basically all of the sports as well as *POP*. It is very difficult for newcomer countries to win medals unless the medals are bought via nationalization [16]. The models might also be compared, in terms of full explanation, as measured by the adjusted correlation coefficient; both the Pfau model and the "by sport" summer model perform better than the JA model.

Table 1 "By-sport" models, 1992–2008

SPORT	**Constant**		*GDPCAP*		$GDPCAP^2$		$\sqrt{POP}$		*HOST*		*NEIGH*		*POL*		*FROST*		*MED*		*MedalHistory*		$Medal_{t-1}$		r^2 **(ADJUSTED)**
Archery	−0.09		0.01		0.00		0.01		0.05	*	0.05		0.07		0.12		0.00		0.14	*	0.51	*	59.7
Artistic gymnastics	−2.59		−0.03		0.00		0.18	*	−0.30		−0.30		−0.77		1.12	*	0.04		0.09	*	0.18		43.4
Athletics	3.85	*	0.00		0.00		0.05	*	−0.26		−0.26		−0.21		0.19		−0.03	*	0.11	*	0.58	*	61.9
Badminton	−4.14	*	0.00		0.00		0.03	*	−0.09		−0.09		0.32	*	0.11		0.26	*	0.15	*	0.44	*	68.5
Basketball	−0.08		0.00		0.00		0.03	*	−0.32		−0.32		−1.01	*	−0.01		0.00		−0.03		0.51	*	45.9
Beach volleyball	0.14		0.01		0.00		0.03	*	0.36	*	0.36		−0.33		−0.33	*	0.00		0.06		0.47	*	47.5
Boxing	−2.25		0.00		0.00		0.01		−0.08	*	−0.08		0.64	*	0.19		0.05		0.05		0.55	*	45.3
Canoe/kayak flatwater	−0.03		0.02		0.00		0.01		−0.34		−0.34		−0.12		0.21		0.00		0.05		0.64	*	51.9
Canoe/kayak slalom	−0.47		0.02		0.00		0.01		0.36		0.36		−0.41		0.04		0.04		0.04		0.52	*	33.6
Cycling BMX	−0.20	*	0.00		0.00		0.03	*	−0.05		−0.05		−0.97	*	0.09		0.04	*	0.00		0.00		12.2
Cycling mountain bike	−0.09		0.01	*	0.00		0.00		0.36		0.36	*	−0.08		0.04		0.01		0.13	*	0.14		15.1
Cycling road	−0.08		0.02	*	0.00	*	0.02	*	0.40		0.40		−0.19		0.02		0.00		0.03		0.17	*	18.1
Cycling track	0.12		0.07	*	0.00	*	0.03		−0.27		−0.27		−0.40		−0.10		−0.01		−0.07		0.73	*	37.0
Diving	−0.87	*	0.04		0.00	*	0.09	*	0.03		0.03		0.41		0.20		0.01		0.08		0.56	*	78.6
Equestrian/dressage	−0.27		0.02		0.00		0.01		−0.14		−0.14		−0.25		0.20		0.00		0.18	*	0.27		42.8
Equestrian/eventing	−0.45		0.02		0.00		0.02		−0.23		−0.23		−0.45		0.00		0.04		0.00		0.49	*	25.0
Equestrian/jumping	−0.10		0.01	*	0.00		0.01	*	0.17		0.17		−0.13		0.03		0.00		0.08		0.16		14.3
Fencing	0.98		0.01		0.00		0.04	*	1.25		1.25		−0.17		0.18		−0.04		0.02		0.73	*	55.2
Football	−0.02		0.00		0.00		0.02	*	−0.14		−0.14		−0.34		−0.03		0.00		0.15		0.04		9.5

(Continued)

Table 1 (Continued)

SPORT	**Constant**		*GDPCAP*		$GDPCAP^2$		$\sqrt{POP}$		*HOST*		*NEIGH*		*POL*		*FROST*		*MED*		*MedalHistory*		$Medal_{t-1}$		r^2 **(ADJUSTED)**
Handball	0.00		0.01		0.00		0.00		0.63		0.63	*	−0.15		0.24		0.00		0.00		0.09		11.0
Hockey	0.52	*	0.04		0.00		0.00		−0.37		−0.37		0.01		−0.63		0.00		0.03		0.37	*	26.2
Judo	−0.11		0.01		0.00		0.03	*	0.71		0.71		0.41	*	0.17		0.00		0.06	*	0.56	*	51.5
Modern pentathlon	−0.13		−0.01		0.00		0.00		−0.10		−0.10		−0.10		0.24	*	0.03		−0.02		0.33	*	10.1
Rhythmic gymnastics	−0.40		−0.02		0.00		0.04	*	−0.01		−0.01		−1.14	*	0.26		0.05		0.25	*	−0.01		31.9
Rowing	0.17		0.02	*	0.00	*	0.02		0.45	*	0.45		−0.17		0.20		−0.01		0.07	*	0.40	*	38.6
Sailing	0.51		0.03	*	0.00	*	0.03	*	−0.02	*	−0.02		−0.25		0.17		−0.03		0.15	*	0.01		29.8
Shooting	−0.07		0.00		0.00		0.06	*	0.22		0.22	*	0.02		0.30	*	−0.01		0.06	*	0.53	*	56.9
Swimming	2.86	*	0.03	*	0.00	*	0.06	*	0.22	*	0.22		−0.09		0.03		−0.03	*	0.09	*	0.65	*	70.6
Synchronized swimming	−0.61	*	0.02		0.00		0.02	*	−0.12		−0.12		−0.16		0.26		0.05		0.02		0.44	*	27.1
Table tennis	−1.10	*	0.00		0.00		0.04	*	0.18		0.18		0.39	*	0.15		0.07	*	0.09	*	0.53	*	70.4
Taekwondo	−0.15	*	0.00		0.00		0.02	*	−0.08	*	−0.08		0.00		0.12		0.01	*	0.12	*	0.53	*	46.8

(Contiued)

Table 1 (Continued)

SPORT	**Constant**		*GDPCAP*		*GDPCAP*^{2}		$\sqrt{POP}$		*HOST*		*NEIGH*		*POL*		*FROST*		*MED*		*MedalHistory*		$Medal_{t-1}$		r^2 **(ADJUSTED)**
Tennis	−0.63	*	0.01		0.00		0.02	*	−0.14	*	−0.14		−0.06		0.13		0.04	*	0.12	*	0.18	*	28.4
Trampoline	−0.06		0.00		0.00		0.01		0.07	*	0.07		0.25		−0.02		0.05	*	0.11		0.22		25.0
Triathlon	0.03		0.01	*	0.00	*	0.00		−0.02		−0.02		−0.08		−0.12		0.02	*	0.29	*	−0.02		15.3
Volleyball	0.27		−0.01		0.00		0.02	*	−0.01	*	−0.01		−0.26		−0.30		0.00		0.02		0.54	*	28.9
Water polo	0.45		−0.05	*	0.00	*	0.03		0.04		0.04		−1.43	*	−0.32		0.06		−0.04		0.22		19.4
Weightlifting	−0.03		−0.01	*	0.00		0.04	*	0.05		0.05		0.16		0.29	*	0.00		0.03		0.69	*	57.3
Wrestling freestyle	−0.55		0.03		0.00		0.02	*	−0.32		−0.32		−0.09		0.35	*	0.01		−0.02		0.79	*	49.0
Wrestling Greco-Roman	−0.97		0.00		0.00		0.02		−0.34		−0.34		−0.04		0.18		0.04	*	0.03		0.45	*	24.9
Country model	**−0.19**		**0.00**		**0.00**		**0.02**		**0.06**		**0.06**	*****	**−0.01**	*****	**0.16**		**0.00**		**0.10**		**0.59**		**56.1**

* significant at the 0.05 level

The results therefore show that it is obvious that whenever an athlete wins a medal, $Medal_{t-1}$, the possibility of winning it again in the following Games increases, either if the medal winner is the same athlete or any countrymen. Whenever medals are won, the chances of returning champions increase, especially in economically powerful and populated countries. Pure tradition, as demonstrated by the "MedalHistory" variable, has somewhat of a lesser impact as counted by the number of significant sports under this variable.

3.2 Model Predictions

With the proposed models, data for the period between 1992 and 2008 is used to predict the results for London 2012 before the Games. Table 2 shows the predictions for each sport, for the countries that should get at least 5 medals. The top five countries are, as expected, United Stated, China, Russia, Great Britain and Germany, the richest countries in the world.

4 Reliability of the Predictions

After the Olympic Games, a reliability study has been performed to check the prediction power of the models. First, the deviations of the predictions from the outcomes have been analysed to shed new light about the factors that influence the performance by sport and country. Second, the behavior of the sport models is assessed in terms of the number of awarded medals so as to determine the adequacy of the technique used to perform the forecasts.

4.1 Analysis of Deviations from the Predictions

We define as Absolute Deviation the difference between the outcome and the predicted values in number of medals by country and sport:

$$\text{Absolute Deviation} = \text{Outcome} - \text{Prediction}$$

If the deviation is positive, it means that any given country has improved the expectations and won more medals than forecasted. If negative, the performance of the country appears to have been poor, or at least, have not lived up to expectations.

Table 3 includes the deviations after the games for those countries whose outcome was at least five medals. The range of the absolute deviations for the country as a whole after adding the results of the individual sports (as included in column "Sum of By Sport") is between $+13$ and -7 at the country level, and between $+6$ and -5

Table 2 Predictions "by-sport" for 2012

	Archery	Artistic Gymnastics	Athletics	Badminton	Basketball	Beach Volleyball	Boxing	Canoe / Kayak Flatwater	Canoe / Kayak Slalom	Cycling BMX	Cycling Mountain Bike	Cycling Road	Cycling Track	Diving	Equestrian Dressage	Equestrian Eventing	Equestrian Jumping	Fencing	Football	Handball	Hockey	Judo	Modern Pentathlon	Rhythmin Gymnastics	Rowing	Sailing	Shooting	Swimming	Synchronized Swimming	Table Tennis	Taekwondo	Tennis	Trampoline	Triathlon	Volleyball	Water polo	Weighlifting	Wrestling Freestyle	Wrestling Greco-Roman	Country Total
United States	0	6	23	0	1	2	2	0	0	1	0	1	0	1	1	1	1	6	2	0	0	1	0	0	3	3	5	27	0	0	3	2	0	0	2	2	1	1	1	99
China	2	9	4	7	0	2	3	1	0	0	0	0	1	12	0	0	0	3	0	0	1	4	0	1	1	1	7	6	1	7	2	1	2	0	1	0	9	3	2	93
Russian Federation	1	4	16	0	1	0	2	2	1	0	1	1	1	5	0	0	0	2	0	0	0	1	1	2	1	1	4	4	2	0	1	2	1	0	1	0	6	6	3	73
Great Britain	1	1	7	0	1	0	3	1	1	0	0	1	9	0	0	2	0	0	1	0	0	1	1	0	4	4	1	7	0	0	2	2	1	0	0	1	0	0	0	52
Germany	0	2	3	0	0	0	1	7	1	1	1	1	1	2	3	2	1	2	0	0	1	2	1	0	3	1	3	4	0	1	0	1	0	1	0	0	1	0	0	47
Australia	0	0	3	0	1	0	0	2	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3	1	1	16	0	0	0	1	0	2	1	2	0	0	0	35
France	1	1	2	0	0	0	2	1	2	1	1	1	1	0	0	0	0	4	0	1	0	4	0	0	2	1	1	5	0	0	1	1	0	0	0	0	1	0	1	35
Italy	1	1	2	0	0	0	2	2	0	1	1	1	1	0	0	0	0	6	0	0	0	1	0	0	2	1	3	3	0	0	1	0	0	0	0	0	0	0	1	30
Japan	0	2	2	0	0	0	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	6	0	0	1	1	1	5	1	0	1	0	0	0	0	0	0	5	0	29
South Korea	5	1	1	3	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	2	2	0	2	4	0	0	0	0	0	3	1	1	28
Ukraine	1	2	4	0	0	0	2	2	0	0	0	0	1	1	0	0	0	1	0	0	0	1	1	1	0	1	2	1	0	0	0	0	0	0	0	0	2	3	1	27
Spain	0	1	1	0	1	0	1	3	0	0	1	1	3	0	0	0	0	0	1	1	1	1	0	0	0	2	0	1	2	0	0	2	0	0	0	0	0	1	0	24
Cuba	0	0	5	0	0	0	6	0	0	0	0	0	1	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	19
Canada	0	0	1	0	1	0	1	2	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	3	1	0	1	0	0	1	0	1	1	0	0	0	1	0	18
Belarus	0	1	5	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	3	1	1	15
Brazil	0	1	1	0	0	2	0	0	0	1	0	0	0	1	0	0	0	0	2	0	0	2	0	0	0	1	0	2	0	0	1	0	0	0	1	0	0	0	0	15

(Continued)

Table 2 (Continued)

	Archery	Artistic Gymnastics	Athletics	Badminton	Basketball	Beach Volleyball	Boxing	Canoe / Kayak Flatwater	Canoe / Kayak Slalom	Cycling BMX	Cycling Mountain Bike	Cycling Road	Cycling Track	Diving	Equestrian Dressage	Equestrian Eventing	Equestrian Jumping	Fencing	Football	Handball	Hockey	Judo	Modern Pentathlon	Rhythmin Gymnastics	Rowing	Sailing	Shooting	Swimming	Synchronized Swimming	Table Tennis	Taekwondo	Tennis	Trampoline	Triathlon	Volleyball	Water polo	Weighlifting	Wrestling Freestyle	Wrestling Greco-Roman	Country Total
Poland	0	1	2	0	0	0	0	2	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	14
Romania	0	3	2	0	0	0	1	1	0	0	0	0	0	0	0	0	0	2	0	0	0	1	0	0	2	0	0	1	0	0	0	0	0	0	0	0	0	1	0	14
India	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	2	1	0	1	0	1	0	0	0	0	1	2	0	13
Hungary	0	1	0	0	0	0	1	4	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	1	1	12
Kazakhstan	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	3	3	2	12
Netherlands	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2	1	1	0	0	0	0	1	0	0	2	1	0	3	0	0	0	0	0	0	0	0	0	0	0	12
Kenya	0	0	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11
Czech Republic	0	0	1	0	0	0	0	1	2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	0	0	0	1	0	0	0	0	0	0	0	9
Turkey	0	1	2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	1	1	9
Bulgaria	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	8
Jamaica	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8
Sweden	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	8
Denmark	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	7
Grecce	0	0	2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	1	0	7
New Zealand	0	0	1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0	0	0	1	0	0	0	0	0	7
South Africa	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	7
Uzbekistan	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2	1	7
Armenia	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	2	6

(Continued)

Table 2 (Continued)

	Archery	Artistic Gymnastics	Athletics	Badminton	Basketball	Beach Volleyball	Boxing	Canoe / Kayak Flatwater	Canoe / Kayak Slalom	Cycling BMX	Cycling Mountain Bike	Cycling Road	Cycling Track	Diving	Equestrian Dressage	Equestrian Eventing	Equestrian Jumping	Fencing	Football
Azerbaijan	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Ethiopia	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Georgia	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Indonesia	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mexico	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
North Korea	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Norway	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Switzerland	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0
Argentina	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
Croatia	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Slovakia	0	0	0	0	0	0	0	1	3	0	0	0	0	0	0	0	0	0	0
Slovenia	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Thailand	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0

	Handball	Hockey	Judo	Modern Pentathlon	Rhythmin Gymnastics	Rowing	Sailing	Shooting	Swimming	Synchronized Swimming	Table Tennis	Taekwondo	Tennis	Trampoline	Triathlon	Volleyball	Water polo	Weighlifting	Wrestling Freestyle	Wrestling Greco-Roman	Country Total
Azerbaijan	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	6
Ethiopia	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6
Georgia	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3	1	6
Indonesia	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2	0	0	6
Mexico	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	1	0	6
North Korea	0	0	2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	2	0	0	6
Norway	1	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	6
Switzerland	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	6
Argentina	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	5
Croatia	0	0	0	0	0	0	0	1	0	0	0	2	1	0	0	0	0	0	0	0	5
Slovakia	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	5
Slovenia	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	5
Thailand	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	5

Table 3 Deviations “by-sport” and country in London 2012

				Ratio																						
				85.76 %	88.05 %	Min	−7	−6	0	−1	−1	−5	−3	−1	−2	−2	−2	−1	−1	−1	−1	−3	−3	−2	−1	−1
				825	847	Max	13	0	6	2	4	6	2	1	1	4	2	1	2	1	2	5	3	3	1	1
	Predictions		Results	Successes			Absolute deviations = Results − Prediction																			
	General model	Sum of “by-sport”	London 2012	General model	Sum of “by-sport”		Total deviation	Minimum	Maximum	Archery	Artistic gymnastics	Athletics	Badminton	Basketball	Beach volleyball	Boxing	Canoe/kayak flatwater	Canoe/kayak slalom	Cycling BMX	Cycling mountain bike	Cycling road	Cycling track	Diving	Equestrian dressage	Equestrian eventing	Equestrian jumping
United States	100	99	104	100	99		5	−5	6	1	0	6	0	1	0	0	0	0	−1	1	0	2	3	−1	−1	−1
China	80	93	88	80	88		−5	−2	4	0	−1	2	1	0	−2	0	−1	0	0	0	0	2	−2	0	0	0
Russian Federation	66	73	82	66	73		9	−3	4	−1	4	2	1	0	0	4	1	−1	0	−1	1	−1	−3	0	0	0
Great Britain	38	52	65	38	52		13	−4	5	−1	3	−1	0	−1	0	2	1	1	0	0	2	0	1	3	−1	1
Germany	47	47	44	44	44		−3	−3	5	0	1	5	0	0	1	−1	−1	1	−1	0	1	2	−2	−2	1	−1
Japan	23	29	38	23	29		9	−1	6	2	1	−1	1	0	0	1	0	0	0	0	−1	−1	0	0	0	0
Australia	43	35	35	35	35		0	−6	5	0	0	0	0	0	0	0	−1	0	1	0	0	5	1	0	0	0
France	38	35	34	34	34		−1	−4	3	−1	0	0	0	1	0	−2	−1	0	−1	0	−1	2	0	0	0	0
Italy	28	30	28	28	28		−2	−2	2	0	0	−1	0	0	0	1	−2	1	−1	0	−1	−1	0	0	0	0
South Korea	29	28	28	28	28		0	−3	5	−1	0	−1	−2	0	0	0	0	0	0	0	0	−1	0	0	0	0
Ukraine	23	27	20	20	20		−7	−2	3	−1	−1	−1	0	0	0	3	1	0	0	0	0	−1	−1	0	0	0

(Continued)

Table 3 (Continued)

				Ratio																					
				85.76 %	88.05 %	Min	−7	−6	0	−1	−1	−5	−3	−1	−2	−2	−2	−1	−1	−1	−1	−3	−3	−2	−1	−1
				825	847	Max	13	0	6	2	4	6	2	1	1	4	2	1	2	1	2	5	3	3	1	1
	Predictions		Results	Successes			Absolute deviations = Results − Prediction																			
	General model	Sum of “by-sport”	London 2012	General model	Sum of “by-sport”		Total deviation	Minimum	Maximum	Archery	Artistic gymnastics	Athletics	Badminton	Basketball	Beach volleyball	Boxing	Canoe/kayak flatwater	Canoe/kayak slalom	Cycling BMX	Cycling mountain bike	Cycling road	Cycling track	Diving	Equestrian dressage	Equestrian eventing	Equestrian jumping
Netherlands	14	12	20	14	12		8	−1	2	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	−1	1
Canada	18	18	18	18	18		0	−1	2	0	0	0	0	−1	0	−1	1	0	0	0	−1	1	1	0	0	−1
Spain	18	24	17	17	17		−7	−3	3	0	−1	−1	0	0	0	−1	−1	1	0	−1	−1	−3	0	0	0	0
Brazil	14	15	17	14	15		2	−1	3	0	0	−1	0	0	0	3	0	0	−1	0	0	0	−1	0	0	0
Hungary	15	12	17	15	12		5	−1	2	0	0	1	0	0	0	−1	2	0	0	0	0	0	0	0	0	0
Cuba	23	19	14	14	14		−5	−3	1	0	0	−3	0	0	0	−2	0	0	0	0	0	−1	0	0	0	0
Kazakhstan	11	12	13	11	12		1	−1	2	0	0	1	0	0	0	2	0	0	0	0	1	0	0	0	0	0
New Zealand	9	7	13	9	7		6	−1	3	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0
Belarus	16	15	12	12	12		−3	−5	2	0	−1	−5	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Jamaica	9	8	12	9	8		4	0	4	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Spain	18	24	17	17	17		−7	−3	3	0	−1	−1	0	0	0	−1	−1	1	0	−1	−1	−3	0	0	0	0

(Continued)

Table 3 (Continued)

	Predictions		Results	Successes			Absolute deviations = Results − Prediction																			
				Ratio																						
				85.76 %	88.05 %	Min	−7	−6	0	−1	−1	−5	−3	−1	−2	−2	−2	−1	−1	−1	−1	−3	−3	−2	−1	−1
				825	847	Max	13	0	6	2	4	6	2	1	1	4	2	1	2	1	2	5	3	3	1	1
	General model	Sum of "by-sport"	London 2012	General model	Sum of "by-sport"		Total deviation	Minimum	Maximum	Archery	Artistic gymnastics	Athletics	Badminton	Basketball	Beach volleyball	Boxing	Canoe/kayak flatwater	Canoe/kayak slalom	Cycling BMX	Cycling mountain bike	Cycling road	Cycling track	Diving	Equestrian dressage	Equestrian eventing	Equestrian jumping
Iran	4	4	12	4	4		8	0	3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Kenya	11	11	11	11	11		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Poland	13	14	10	10	10		−4	−1	1	0	−1	0	0	0	0	0	−1	0	0	0	0	−1	0	0	0	0
Czech Republic	7	9	10	7	9		1	−1	1	0	0	1	0	0	0	0	0	−1	0	1	0	−1	0	0	0	0
Azerbaijan	6	6	10	6	6		4	−1	3	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Romania	13	14	9	9	9		−5	−2	2	0	0	−2	0	0	0	−1	−1	0	0	0	0	0	0	0	0	0
Denmark	8	7	9	8	7		2	−1	2	0	0	0	2	0	0	−1	−1	0	0	0	0	1	0	0	0	0
Sweden	8	8	8	8	8		0	−1	2	0	0	−1	0	0	0	−1	0	0	0	0	−1	0	0	0	1	−1
Colombia	3	2	8	3	2		6	0	2	0	0	1	0	0	0	0	0	0	2	0	1	0	0	0	0	0

(Continued)

Table 3 (Continued)

	Predictions		Results	Successes			Absolute deviations = Results − Prediction																			
				Ratio																						
				85.76 %	88.05 %	Min	−7	−6	0	−1	−1	−5	−3	−1	−2	−2	−2	−1	−1	−1	−1	−3	−3	−2	−1	−1
				825	847	Max	13	0	6	2	4	6	2	1	1	4	2	1	2	1	2	5	3	3	1	1
	General model	Sum of “by-sport”	London 2012	General model	Sum of “by-sport”		Total deviation	Minimum	Maximum	Archery	Artistic gymnastics	Athletics	Badminton	Basketball	Beach volleyball	Boxing	Canoe/kayak flatwater	Canoe/kayak slalom	Cycling BMX	Cycling mountain bike	Cycling road	Cycling track	Diving	Equestrian dressage	Equestrian eventing	Equestrian jumping
Ethiopia	6	6	7	6	6		1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Georgia	5	6	7	5	6		1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mexico	4	6	7	4	6		1	−1	2	2	−1	−1	0	0	0	0	0	0	0	0	0	0	2	0	0	0
India	4	13	6	4	6		−7	−1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
South Africa	4	7	6	4	6		−1	−4	2	0	0	−1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
North Korea	6	6	6	6	6		0	−1	2	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0
Croatia	5	5	6	5	5		1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turkey	9	9	5	5	5		−4	−1	0	0	−1	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0
Lithuania	5	4	5	5	4		1	−1	1	0	0	−1	0	0	0	1	1	0	0	0	0	0	0	0	0	0
Ireland	4	4	5	4	4		1	−1	2	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	1
Mongolia	4	3	5	4	3		2	−1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0

at the sport level. The first column includes the prediction of the country general model, that is, without performing the predictions by sport.

The largest positive difference is that the host, United Kingdom, obtained +13 medals more than the predictions. United Kingdom got +5 in Rowing, a sport where tradition and hosting plays important roles. UK got also −4 in swimming, a sport where many medals are awarded. Other than that, UK behaved consistently with the predictions.

Russia and Japan got +9 as a country. Russia has excelled in Boxing and Judo (POL variable) and Artistic Gymnastics, very technical team-oriented sport. Japan won +6 medals in Swimming, a sport with options for newcomers. The Netherlands got +8, with +2 in Hockey (traditional sport in this country) and Sailing (+2 as newcomers).

On the down side, with −7, seven less than the prediction, we find Spain, India and Ukraine. Spain has lost its edge in Tennis, with the injury of some expected winners like Rafael Nadal in a sport where the *HOST* matters (in fact, the unexpected winner was Andy Murray, who also got a medal in mixed doubles). India has been over-predicted in seven sports due to its high population *POP* and Ukraine due to the diminishing effect of the *POL* variable.

The surprises are Iran with +8 and New Zealand and Colombia with +6. Iran has specialized in Weightlifting and Wrestling (both sports with short-term tradition as a key factor). New Zealand has excelled in Rowing (sport where GDPCAP as well as tradition are critical) and Colombia in BMX (a new sport). So it looks like there is room for newcomers in specific sports.

If the analysis is further performed by sport, the largest differences are in Athletics (from +6 to −5), with United States (+6), Germany (+5) and Jamaica (+4) leading the positive outcomes. On the other side, Belarus (−5) and Cuba (−3) show the decreasing effect of the POL variable. The differences in this sport are however not too high in relative terms since the total number of medals is a huge 142.

In the other major sport, Swimming, with 102 medal, Japan (+6) and the United States (+4) have capitalized the loss of Great Britain (−4) and Australia (−6).

In Track Cycling, Australia has a +5 and Spain −3. Australia has put a strong team whereas the effect of the extraordinary recent history in Spain has provoked the over-prediction thorough the $Medal_{t-1}$ factor.

In Fencing, where $Medal_{t-1}$ and *POP* are the significant factors, South Korea has excelled (+5) whereas United States (−5) and France (−4) have lost share through short term tradition.

4.2 Success Ratio

We also use another measure to further understand the prediction power of the "by sport" models in order to give hints for improvement in the techniques that might be used for forecasting. The success ratio is defined as the number of medals correctly predicted divided by the medals awarded.

$$\text{Success Ratio} = \text{successes/medals awarded}$$

To calculate the success ratio, the following rules are used:

1. A success is counted whenever one medal was predicted and it was awarded.
2. If the number of predicted medals for a given sport and country is smaller than the number of awarded medals, the number of successes is the number of predicted medals.
3. If the number of predicted medals for a given sport and country is higher than the number of awarded medals, the number of successes is the number of awarded medals.
4. If the predicted and awarded number of medals are both 0, there is no success accounted for.

Figure 1 shows the results by sport, ordered by the number of awarded medals, with an average ratio of 63.51 %. Several clusters of sports might then be settled based on the variable *MED*:

1. The two star sports of any Olympic Games are Athletics and Swimming with high totals of medals awarded. The success ratios are high in both of them.
2. The main bulk of sports awards between 24 and 56 medals. These sports have a success ratio between 57 and 76 %, around the average of 63.5 %. The exception is on the down side Cycling Track where Australia won a very high, unpredictable number of medals, and Artistic Gymnastics, the other star sport, on the positive side, where the big traditional countries have the long term edge.
3. Team sports with 6 awarded medals are very difficult to predict and chance plays a key role. The very technical Synchronized swimming seems easier to predict due to coaching and short term tradition but the rest of sports are scattered throughout the sorted list ranging from a high 67 % in Handball and a low 33 % in Football.
4. The rest of sports award a low number of medals (less than 15) and are also difficult to predict. Cycling BMX had only been an Olympic sport once before London 2012, so the prediction was poor and even got a 0 % success ratio.

It looks like the "By sport" models are good for sports with more than 24 medals awarded. It is possible then that other techniques are used for "small sports" in order to increase the prediction power (Table 4).

The success ratio could also be used to compare the country forecasts made through the use of the single general model (without sport predictions) and the "Sum of the By Sport" aggregate model. In other words, if the prediction per individual sport is added for all the sports to come up with a predicted number for a given country, this added value could be compared to the awarded medals for the country to obtain a success ratio. The success ratio of the "Sum of the By Sport" methodology could be considered to be 88.05 % if the ratio is calculated for the country as a whole. This number is higher than the gross success ratio per country, 85.75 %, calculated after running the model just for the country as a whole without distinguishing among sports. These two calculations are shown in Table 3.

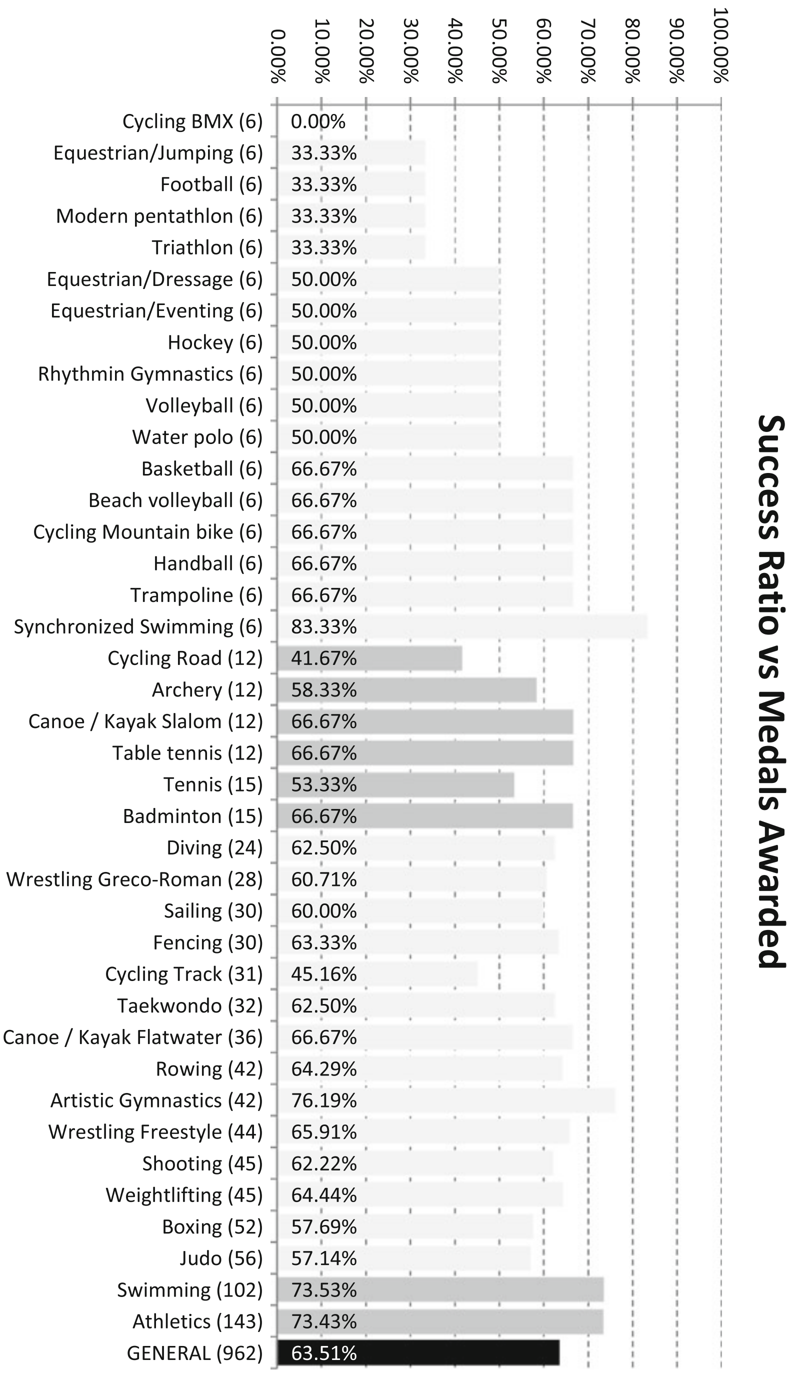

Fig. 1 Success ratio “by-sport” for 2012

Table 4 Significant factors "by-sport", 1992–2008

	Significance level = 0.05	Johnson and Ali											
	Number of significant sports	0	0	18	12	24	22	10	1	7	20	24	
	With possitive betas	0	0	17	1	23	4	10	1	5	18	15	
962	With negative betas	0	0	1	11	1	18	0	0	2	2	9	22.89 %
AWARDED MEDALS		Constant	Year	*GDPCAP*	*GDPCAP*^{2}	*POP*	*POP*^{2}	*HOST*	*NEIGH*	*POL*	*FROST*	*MED*	r^2 (ADJUSTED)
12	Archery			+	−						+	−	7.40
42	Artistic gymnastics					+	−				+	−	36.40
143	Athletics			+	−	+	−	+			+		26,10
15	Badminton						+					+	34.40
6	Basketball					+							41,40
6	Beach volleyball					+	−			−	−	+	32.50
52	Boxing					+	−	+		+	+		15.10
36	Canoe/kayak flatwater			+							+	+	3.10
12	Canoe/kayak slalom			+	−							−	1.70
6	Cycling BMX					+						−	13.50
6	Cycling mountain bike			+									5,60
12	Cycling road			+	−	+	−		+			+	19.30
31	Cycling track			+	−								12.50
24	Diving											+	67.30
6	Equestrian/dressage											−	3,30
6	Equestrian/eventing			+				+					11.40
6	Equestrian/jumping			+		+	−						14.50
30	Fencing					+	−				+	+	10.90
6	Football					+	−					+	16.80
6	Handball											+	9.80
6	Hockey										−		11.50
56	Judo			+	−	+	−			+	+	+	16.70

	Significance level = 0.05	PFAU							
	Number of significant sports	6	37	6	20	3	3	10	
	With possitive betas	0	36	5	20	3	3	7	
962	With negative betas	6	1	1	0	0	0	3	55.32 %
AWARDED MEDALS		Constant	$Mshare_{t-1}$	log *GDP*	log *POP*	*FROST*	*POL*	*HOST*	r^2 (ADJUSTED)
12	Archery		+					+	65.80
42	Artistic gymnastics	−	+		+	+			34.60
143	Athletics		+		+				77.80
15	Badminton		+				+		79.30
6	Basketball		+		+				57.90
6	Beach volleyball		+		+				42,90
52	Boxing		+				+	+	54.80
36	Canoe/kayak flatwater		+						70.70
12	Canoe/kayak slalom		+						40.60
6	Cycling BMX		−						0.70
6	Cycling mountain bike		+						13.40
12	Cycling road	−	+	+	+				19.30
31	Cycling track		+						37.70
24	Diving		+						81.70
6	Equestrian/dressage		+						53,40
6	Equestrian/eventing		+					+	40.80
6	Equestrian/jumping		+	+	+				17.30
30	Fencing	−	+		+				62.80
6	Football				+				12.30
6	Handball							−	9.10
6	Hockey		+						29.60
56	Judo		+		+		+		65.20

	Significance level = 0.05	By sport for summer											
	Number of significant sports	10	10	8	23	10	4	9	6	10	16	27	
	With possitive betas	3	8	1	23	9	4	4	5	8	16	27	
962	With negative betas	7	2	7	0	1	0	5	1	2	0	0	49.64 %
AWARDED MEDALS		Constant	*GDPCAP*	*GDPCAP*^{2}	$\sqrt{POP}$	*HOST*	*NEIGH*	*POL*	*FROST*	*MED*	*MedalHistory*	$Medal_{t-1}$	r^2 (ADJUSTED)
12	Archery					+					+	+	59.70
42	Artistic gymnastics				+				+		+		43.40
143	Athletics	+			+					−	+	+	61.90
15	Badminton	−			+			+		+	+	+	68.50
6	Basketball				+			−				+	45.90
6	Beach volleyball				+	+			−			+	47.50
52	Boxing					+		+				+	45.30
36	Canoe/kayak flatwater											+	51.90
12	Canoe/kayak slalom											+	33.60
6	Cycling BMX	−			+			−		+			12.20
6	Cycling mountain bike		+				+				+		15.10
12	Cycling road		+	−	+							+	18.10
31	Cycling track		+	−								+	37.00
24	Diving	−		−	+							+	78.60
6	Equestrian/dressage										+		42.80
6	Equestrian/eventing											+	25.00
6	Equestrian/jumping		+		+								14.30
30	Fencing				+							+	55.20
6	Football				+								9.50
6	Handball						+						11.00
6	Hockey	+										+	26.20
56	Judo				+			+			+	+	51.50

(Continued)

Table 4 (Continued)

	Significance level = 0.05	Johnson and Ali												PFAU								By sport for summer											
6	Modern pentathlon										+		4.30		+						11.50							+			+	10.10
6	Rhythmin gymnastics					+					+	+	17.70		+	−	+				26.80				+			−			+		31.90
42	Rowing			+	−	+	−	+					15.50		+	+					16.30		+	−		+					+	+	38.60
30	Sailing			+	−	+	−	+			+	+	22.40	−	+	+	+				33.50		+	−	+	+					+		29.80
45	Shooting			+		+					+		29.40	−	+		+	+			55.90				+		+		+		+	+	56.90
102	Swimming			+	−	+	−	+				+	30.10		+	+	+			+	82.10	+	+	−	+	+				−	+	+	70.60
6	Synchronized swimming			+	−	+	−				+	−	23.90		+		+				45.30	−			+							+	27.10
12	Table tennis					−	+			+	+		59.90		+						81.30	−			+			+		+	+	+	70.40
32	Taekwondo					+	−	+			+		20.20		+		+			+	40.60	−			+	+				+	+	+	46.80
15	Tennis					+	−	+			+	+	20.00	−	+		+			+	28.20	−			+	+				+	+	+	28.40
6	Trampoline						+			−		−	16.60		+						12.40					+				+			25.00
6	Triathlon			+	−							−	7.00		+						9.80		+	−						+	+		15.30
6	Volleyball					+	−						23.10		+		+			−	40.00				+	−						+	28.90
6	Water polo			−	+			+				+	16.60								4.40		−	+				−					19.40
45	Weightlifting									+	+		27.90		+		+			−	57.20		−		+				+			+	57.30
44	Wrestling freestyle					+	+			+	+	+	25.10		+		+				58.30				+				+			+	49.00
28	Wrestling Greco−Roman					+	−					−	13.80		+						32.10									+		+	24.90
962	Modelo general			+	−	+	−	+			+	−	40.20		+		+	+		+	66.00						+	−					56.10

The table includes a "+" whenever a factor is significant, that is, whenever its beta is different than 0 and positive, and a "−" whenever it is significant and its beta is less than 0, with a significant level of 0.5

5 Conclusions

The analysis of the Olympic Games under any prism is very appealing [5]. This research aims at understanding the rules of the process of awarding medals in the Summer Olympic Games and develops robust methodologies to predict the outcome of the greatest sport competition of all.

A new methodology has been introduced to develop robust models, which have been categorized "by sport" for the first time to our knowledge in order to shed new light about the medal process. Categorizing by sport has provided much more information to help to understand the outcome of the Games since the significant factors differ among sports.

Econometric models have predicted the total medal count per country with success. They also have proven satisfactory to establish several factors as statistically significant, like short term and lifelong tradition as well as being the host country. However, they have shown the necessity to use techniques other than OLS especially for sports with a low number of medals to be awarded.

In addition, this research brings sport policy issues to the forefront: the selection of the sport in which to invest to achieve short and long term success, possibly, by developing sound social networks and sport infrastructures to foster talent.

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Soccer Analytics Using Touch-by-Touch Match Data

Sergiy Butenko and Justin Yates

Abstract This paper discusses several soccer analytics directions exploiting detailed ball touch data from a soccer game. The topics discussed include visualizing team formations and quantifying territorial advantage; determining the network-based structural properties of team play, and computing the importance of individual players for the team interactions. The proposed ideas are illustrated using the data from a real-life Barclays Premier League game, which was made available by StatDNA.

1 Introduction

The availability of detailed soccer game data, such as that collected and supplied by StatDNA [1], opens up a window of opportunities for applying data mining techniques towards soccer analytics. This paper explores several methodological directions, each suitable for exploiting one of the two characteristic attributes of the available data sets, namely, the coordinate data for each player action recorded and the touch-by-touch data, which allow for representing the corresponding player interactions in the form of a social collaboration network.

The issues analyzed in this paper include visualization of team formations and quantifying territorial advantage; determining the network-based structural properties of team play, and computing the importance of individual players for the team interactions. The aim of this paper is to concisely introduce the proposed techniques at an elementary level, using illustrations based on a game selected from the available StatDNA data base. More specifically, to illustrate the ideas described, we use the data from the first half of Barclays Premier League game between Aston Villa and Arsenal that took place on November 27, 2010. A report from this game can be found online, see, e.g., [2].

S. Butenko (✉) · J. Yates
Department of Industrial and Systems Engineering,
Texas A&M University, College Station, TX 77843-3131, USA
e-mail: butenko@tamu.edu

J. Yates
e-mail: jtyates@tamu.edu

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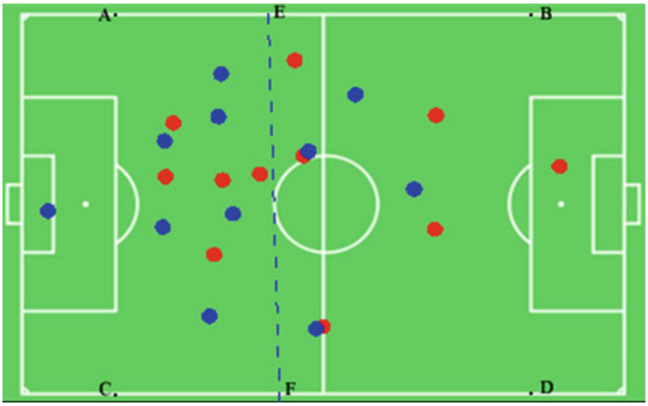

Fig. 1 The frontline for the first half of Aston Villa versus Arsenal matchup (Color figure online)

The reminder of this chapter is organized as follows. Section 2 suggests ways to quantify territorial advantage and visualize team formations based on information about the coordinates of ball touches during a game. Section 3 deals with a network representation of the touch-by-touch data and uses the concepts of a spanning tree and algebraic connectivity to characterize team's interactions. Finally, Sect. 4 ranks players according to their network centrality.

2 Quantifying Territorial Advantage and Visualizing Team Formations

While "territorial advantage" (TA) is a commonly used term in soccer circles, there is no clear definition that would allow quantifying it easily. Some television channels, such as Sky Sports, do operate with TA figures in percentage form during their broadcasts, however, no explanation can be found of how the territorial advantage is computed, leaving the fans to wonder how exactly to interpret these figures. A glance through various online forum discussions on the topic indicates that the general consensus among the fans is that the TA is directly related to the ball possession on the opponent's side of the field. However, it is also natural to assume that TA should also depend on the relative amount of possession on the team's own side versus the opponent's side of the field. Moreover, enjoying the majority of possession around the midfield is not the same as controlling the ball around the opposition's penalty area. Hence, providing a more transparent and sound definition of TA is of interest.

To address this issue, we propose to define the concept of *game frontline* at a certain time moment to be the line most closely approximating the coordinates of ball touches that took place until the time moment of interest. More specifically, the game frontline can be naturally defined as the least-squares linear fit for the touch

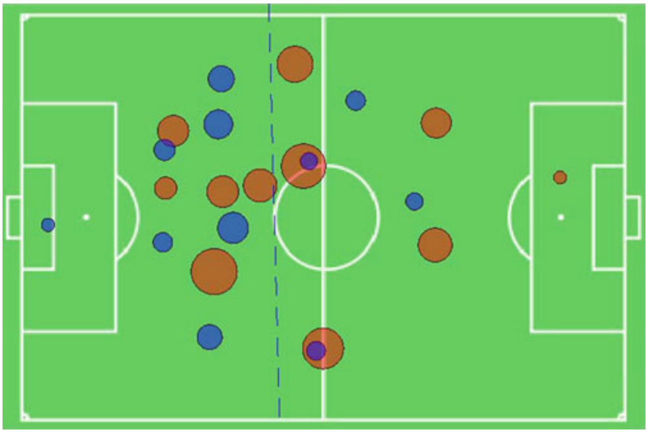

Fig. 2 The frontline for the first half of Aston Villa versus Arsenal matchup. The radius of each circle is proportional to the number of touches performed by the corresponding player

points. As an example, Fig. 1 shows the frontline for the first half of Aston Villa versus Arsenal matchup. In this figure, the blue and red dots represent the average position of touch points for each Aston Villa and Arsenal player, respectively, and the blue dashed line is the game frontline as of the end of the first half. Note that it is expected that for any interesting encounter, the game frontline will cross the field only within its part between the two penalty areas. Hence, to measure the territorial advantage it makes sense to concentrate on the rectangle created by the two sidelines and the two lines corresponding to the front sides of the penalty areas (ABDC in Fig. 2). The game frontline will split this rectangle into two polygons (most likely, trapezoids, such as AEFC and EBDF in Fig. 2), one corresponding to each side of the field. The ratio of the area of the corresponding polygon to the overall area of the rectangle can be used to express the team's territorial advantage. In the considered example, Arsenal's territorial advantage is given by

$$\mathbf{TA}(Arsenal) = \frac{S(EBDF)}{S(ABDC)} = 61.18\ \%,$$

where S is used to denote the area. This figure happens to be rather close to the percentage of the total number of touches that were made by Arsenal's players, which was equal to 60.52 % for the same data. Intuitively, the touches percentage (TP) should be more closely related to the ball possession figure rather than to the TA. To provide a more comprehensive visualization of both the TA and TP, instead of equally-sized dots we can use circles to represent players, where the radius of each circle is proportional to the number of touches performed by the corresponding player (see Fig. 2). Such a visualization leads to an immediate conclusion that Arsenal had most of the possession and dominated the midfield (two of Aston Villa midfielders were completely "eaten" by their Arsenal vis-á-vis). From this illustration, one may also observe that Arsenal's formation de facto consisted of just two defenders, three central midfielders, two wingers and three forwards, whereas Aston Villa had two echelons of defense (involving the total of six players), three midfielders and one forward. These are quite different from the standard formations presented by the media in game reports. It should be noted that the proposed visualization of the ball touch data may, in some cases, be misleading, as the average coordinates of touches do not tell much about the actual zone of activity for some players. However, overall it appears to be a useful supplementary tool in analysis of teams' performance.

One interesting parameter that is easy to measure based on the ball touch data is the *Gini coefficient* [3] which has been used to quantify the level of disparity in applications ranging from sociology and economics to comparing competitive balance in sports leagues [4]. For the values x_i, $i = 1, \dots, n$, listed in non-decreasing order, the Gini coefficient can be computed as follows:

$$G = \frac{2\sum_{i=1}^{n} i x_i}{n\sum_{i=1}^{n} x_i} - \frac{n+1}{n}.$$

When applied to touch distribution data for soccer game, the closer Gini coefficient is to zero, the more "balanced" the team play is. We propose to refer to the value of the Gini coefficient as activity disbalance. For the considered game, the activity disbalance for Aston Villa and Arsenal was equal to 0.1353 and 0.1509, respectively, indicating that the workload is slightly more evenly distributed among the Aston Villa players than Arsenal's. However, we should recall that the overall number of touches was much higher for Arsenal.

3 Network Structure and Robustness of Team Play

The availability of touch-by-touch data provides an opportunity to analyze the team interactions from the network perspective. In particular, the information on passes between pairs of players can be summarized in the form of a network, where players are represented by nodes and passing interactions between pairs of players are represented by weighted directed arcs, with the weight of the arc from player A node to player B node being the number of successful passes completed by A to B. Then social network analysis (SNA) techniques can be utilized to study the structural properties of the network representing the team play, such as connectivity, cohesiveness, and robustness. While opportunities for exploiting the SNA and, more generally, network-based data mining techniques in soccer analytics are abundant, in this preliminary study we restrict ourselves to illustrating some of the basic techniques along these lines.

The first question that we explore is, which pair-wise interactions of players are most critical for the overall connectivity of the team play? This question can be answered effectively by employing the concept of the maximum weight spanning tree in the corresponding network representing the team play. To illustrate this concept, we turn to the same example as above, the first half of Aston Villa versus Arsenal

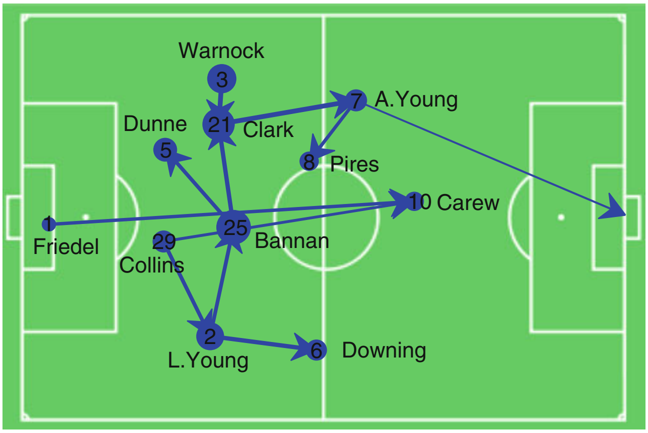

Fig. 3 The maximum spanning tree for Aston Villa

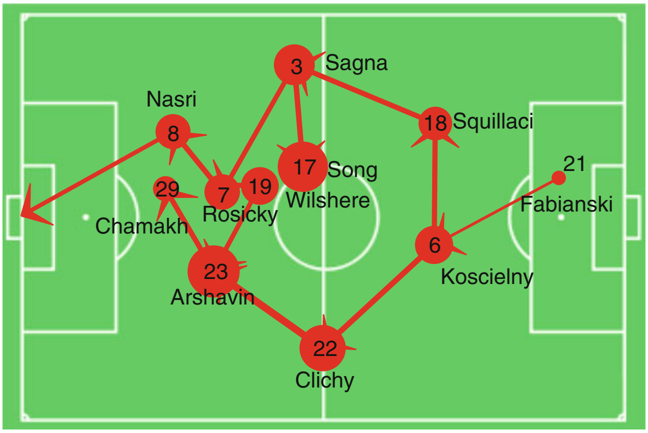

Fig. 4 The maximum spanning tree for Arsenal

game. Figures 3 and 4 show the maximum spanning trees for Aston Villa and Arsenal, respectively. Each node represents a player and is marked with the player's jersey number and name. In addition to the 11 nodes representing players, there is a node representing the opponent's goal, with an arrow in the direction of this node representing shot attempts. As before, the radius of the circle representing a node is proportional to the number of times the corresponding player touched the ball. The arcs shown in these figures represent the interactions that are most essential for the global connectivity of the team play. The thickness of the corresponding arrow is directly proportional to the number of passes between the respective pair of players. Hence, the thicker is the arrow, the more important is the role of the corresponding connection in the team play. The presence of numerous backward links in the graph

Table 1 Comparison of the computed parameters for the first half of Aston Villa versus Arsenal match

Parameter	Aston Villa	Arsenal
Ball touches (%)	39	61
Touches disparity (Gini index)	0.13	0.15
Territorial advantage (%)	42	58
Maximum spanning tree weight	53	84
Robustness	6.46	6.70

for Aston Villa indicates that the team had to defend most of the time and struggled offensively, whereas one can clearly see the two major offensive vectors exploited by Arsenal. It should be pointed out that the total weight of Arsenal's spanning tree was equal to 84 versus Aston Villa's 53.

Another important characteristic of a network is its robustness with respect to adversary actions. One of several possible ways of measuring it is by computing the algebraic connectivity of the network, which is given by the second smallest eigenvalue of the network's Laplacian [5]. In our working example, it is equal to 6.6974 for Arsenal and to 6.4624 for Aston Villa, i.e., the advantage of Arsenal in this component is not very significant.

While understanding the impact that the computed robustness value has on a team's performance is an interesting topic for future investigation, one interesting observation that we made while analyzing various games is that very high robustness value (often approaching 10) singles out Manchester United (MU) from among other English Premier League (EPL) teams. In fact, MU appeared to dominate opposition in robustness in nearly every game included in the StatDNA data set. In an intriguing exception, in a game versus Chelsea MU's robustness suddenly dropped in the second half after a dominant (with respect to robustness) first half. MU won the first half (1-0) and lost the second (0-2) despite having an advantage in touches percentage.

Table 1 summarizes the various parameters computed for the considered game.

4 Centrality Ranking of Players

Next we compute the eigenvector centrality for each node that is used to measure the importance of a node in a network in social networks as well as in other applications. One of the most well known examples illustrating the effectiveness of this measure is Google's PageRank method used to rank web pages according to their importance [6, 7]. With respect to the soccer team network introduced above, the centrality score quantifies the importance of each player in the collective team play, i.e., his role as a team player rather than his individual qualities.

Table 2 ranks the players of Aston Villa and Arsenal with respect to their centrality scores for our working example. In the table, the rank, name and centrality score of each player are given for each team, while the "In" and "Out" columns show the

Table 2 Aston Villa and Arsenal players ranked according to their centrality scores

Rank	Name	Score	In	Out
Aston Villa				
1	B. Bannan	0.1411	47	46
2	C. Clark	0.1135	34	37
3	L. Young	0.1079	32	31
4	R. Dunne	0.0915	28	26
5	R. Pires	0.0876	23	27
6	S. Warnock	0.0874	29	24
7	A. Young	0.0838	21	27
8	J. Collins	0.0779	25	17
9	S. Downing	0.0771	19	23
10	J. Carew	0.0744	16	22
11	B. Friedel	0.0578	16	10
Arsenal				
1	A. Song	0.1163	63	65
2	A. Arshavin	0.1151	64	63
3	G. Clichy	0.1072	59	56
4	L. Koscielny	0.1028	56	52
5	B. Sagna	0.1002	57	51
6	J. Wilshere	0.0914	51	46
7	T. Rosicky	0.0906	45	50
8	S. Squillaci	0.0889	50	43
9	S. Nasri	0.0790	34	46
10	M. Chamakh	0.0629	25	33
11	L. Fabianski	0.0454	15	14

number of passes received and completed, respectively, by each player. It should be noted that in this case two consecutive touches by the same player in the data set are counted as a pass from the player to himself. The centrality scores of the players are normalized so that they sum up to one. In the considered example, the centrality ranking of the players appears to be closely related to their activity level expressed by the total number of plays they were involved in. In terms of activity level, the most active Aston Villa player, Bannan was involved in as many plays (93) as Squillaci, who was only 8th most active player among the Arsenal's starting 11.

Based on the results of analysis of numerous EPL games played in 2010, we observed that, not surprisingly, central midfielders tend to rank rather highly in the centrality rankings. In a notable example, Michael Essien is ranked #1 in most Chelsea games indicating the extremely important role he plays in the team (which has been pointed out to the press by his teammates and coaches on numerous occasions).

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The Golf Director Problem: Forming Teams for Club Golf Competitions

Konstantin Pavlikov, Donald Hearn and Stan Uryasev

Abstract Club golf competitions are regular events arranged by golf directors (or professionals) for club members. Player skill levels are measured by their USGA or R&A handicaps and it is the job of the director to use the handicaps to organize teams that are, in some sense, fair. The handicap system is limited in that it does not take the variance of players' scores into account. In this paper we propose two optimization models that employ the handicap distributions from a prior study [1]. The first model directly computes team probabilities to win a single hole, and the second derives team probabilities to win from those of the players. The computational complexity of both models grows exponentially with the number of players. Using scenario optimization, with approximations, the second model is shown to give very good results for up to 40 players in reasonable computer time. Also, the solution of a real problem shows that common assumptions about the structure of fair teams are not necessarily correct.

1 Introduction

The Golf Director Problem. This paper considers the assignment of players to teams in ordinary golf club competitions, which are events arranged for members at many public and private golf clubs. Typically these competitions involve 12 or more players, all of whom have established handicaps—the lower the handicap, the better the player. The task of assigning the players to teams is that of the club golf director (or professional), who attempts to create teams that are as fair as possible, using the players' handicaps. By convention, teams are of size 4, but sometimes teams

K. Pavlikov · D. Hearn(✉) · S. Uryasev
Department of Industrial and Systems Engineering, University of Florida,
303 Weil Hall, P.O. Box 116595 Gainesville, FL 32611-6595, USA
e-mail: hearn@ufl.edu

K. Pavlikov
e-mail: kpavlikov@ufl.edu

S. Uryasev
e-mail: uryasev@ufl.edu

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of 3 or 5 are formed if the number of players is a multiple of 3 or 5. Players register in advance to play and the total is restricted to such multiples. Once the teams are formed, the golfers play an 18-hole match to determine the winning team(s). Team scores can be defined by the director in various ways. For the basic golf director problem, all players play all 18 holes and the team score is determined by counting only the lowest player score on each hole and then summing those 18 best individual scores.

In forming the teams, the golf director has only the player handicaps as a guide. Little, if any, consideration is given to the variation of scores that a player might have because the handicap system does not report measures such as the standard deviation of player performance. Instead *ad hoc* methods that aim to distribute handicaps evenly among the teams are employed. Two of these are given Sect. 4 as a part of our computational comparisons.

Optimization Models. A primary feature of the optimization models developed in this paper is that they employ the *handicap distributions* derived and validated in [1]. The distributions in Fig. 2 from that paper are given in numerical form in the appendix. Thus these models consider the full range of scores from the handicap history of each player.

We focus on models for the base case but consider only one hole of play, with the justification that the scores of different players on the 18 holes of a match can be considered independent events. Even for just one hole of play, if we assume there are four possible scores per player, then having 24 players means there are 4^{24} possible scoring scenarios, and the number of possible teams of size 4 is $O(10^{12})$. By considering only one hole of play, problems of this size are solvable with our Model 2. One might also take the view that if teams are formed on the basis of how well they do in their gross scoring, handicapping in the actual game should impact all teams in a similar way.

A common variant of this base case is to also use player handicaps in computing the team score for each hole. Each player receives a deduction from their score on certain holes, depending on their handicap, which yields their net score. For handicaps less than or equal to 18, a deduction of 1 is made for the number of holes equal to the handicap. For higher handicaps, a player receives an additional deduction of 1 for as many holes as the handicap exceeds 18. Exactly which holes are handicapped depends on the difficulty of the holes as shown on golf scorecards. A player with a handicap of 1 would get a deduction of 1 on the most difficult hole, a 2 handicap player would get the deduction on the two most difficult holes, and so on. There are many other scoring variants, such as using more than one player score per hole and/or using a special point system rather than player scores. Some of these are defined in [2]. More complex models that consider 18 holes of play and the net scoring of more than one player will be addressed in future research.

The decision variables of the models are binary variables that designate assignment of players to teams. The objective of the models is to choose these variables so that all teams have equal probability to win. This objective is a better alternative than forming teams with the same expected team score. As an example, consider two teams where team A scores 4 or 6 with equal probabilities of $\frac{1}{2}$ and team B scores

2, 3 or 10 with probabilities of $\frac{1}{3}$. Both teams have expected scores of 5, but the probability of team B to win is $\frac{2}{3}$ and that of A is $\frac{1}{3}$.

The first model deals directly with the team probabilities of winning and includes consideration of ties. It is inherently a nonlinear binary program, but, using a technique of [3] or [4], it is converted to a binary linear program by the addition of $O(mn^{\frac{n}{m}})$ variables where n is the number of players and m is the number of teams. However, as n increases this model quickly goes beyond the capabilities of optimization solvers.

The second model deals with player probabilities of winning and employs scenario generation. It is shown that all players' probabilities to win over all possible scenarios can be adequately approximated so that relatively few scenarios need be considered and solutions for $n \leq 40$ can be attained in reasonable computation time.

The computational results section provides a comparison of the scenario model with three simple heuristics for team assignments.

Related Literature. Since the golf director determines the teams, the problem studied here differs from those in the literature in which the players form the teams and then the question is how to handicap the teams for specific competitions. A good recent example is [5], which looks at handicapping teams in "scramble" competitions. This paper also provides a nice summary of academic papers relevant to golf handicapping. Another related paper that address scrambles is [6], which employs a binary integer programming model.

(Information on the United States Golf Association Handicap system is available from the website http://www.usga.org#usga. See also http://www.randa.org/#ranga for comments on handicapping by the R&A which administers the rules of golf in 128 countries around the world.)

2 Model 1: Team Probabilities to Win

For n players and m teams we introduce m vectors of {0, 1} variables:

$$x_1 = (x_1^1, \dots, x_1^n)\,, \tag{1}$$

$$\dots$$

$$x_m = (x_m^1, \dots, x_m^n)\,. \tag{2}$$

$x_t^i = 1$ means that player i is on team t. Integrality constraints are then:

$$\sum_{t=1}^{m} x_t^i = 1\,,\ i = 1, \dots, n\,, \tag{3}$$

$$\sum_{i=1}^{n} x_t^i = r \ , \ t = 1, \ldots, m \ . \tag{4}$$

where, by assumption, the team size is an integer $r = \dfrac{n}{m}$. We measure the efficiency of team t allocation by determining the quantity

$$\mathbb{P}(\text{team } t \text{ ``wins''}) \ .$$

However, there are several ways to define the probability of a team to win. It can be either as the probability of "strict" win, when the best player of a team scores strictly less that all other players, or it can be the probability that team does not lose, i.e., the probability that best player of the team scores at least as good as all other players. Clearly, these two approaches differ in dealing with ties: the first one ignores them completely and the second one incorporates the tie outcomes to the all teams with best scores. Consequently, the first definition implies that $\sum_{t=1}^{m} \mathbb{P}(\text{team } t \text{ ``wins''}) < 1$ and the second definition implies $\sum_{t=1}^{m} \mathbb{P}(\text{team } t \text{ ``wins''}) > 1$. The question is which one is correct? We believe that the second definition is more relevant. In Sect. 3 we will discuss another approach for dealing with ties.

First, we introduce the following notation which will be used throughout the paper:

$$\mathbb{P}(\text{player } i \text{ score} = j) = p_j^i \ , \qquad j = 1, \ldots, k \ , \tag{5}$$

$$\mathbb{P}(\text{player } i \text{ score} \le j) = F_j^i = \sum_{l=1}^{j} p_l^i \ , \qquad j = 1, \ldots, k \ . \tag{6}$$

In other words (5) defines the probability distribution function for every player i and (6) is the cumulative distribution function of player i. The score of a team is determined by the score of its best player. First, we find the probability that team t scores less or equal than $j, \ j < k$:

$$\mathbb{P}(\text{team } t \text{ score} \le j) = 1 - \mathbb{P}(\text{team } t \text{ score} > j) = \tag{7}$$

$$1 - \mathbb{P}\,(\text{each player's score in team } t > j) =$$

$$1 - \prod_{i \,\in\, \text{team } t} \left(1 - F_j^i\right) = 1 - \prod_{i=1}^{n} \left(1 - F_j^i x_t^i\right) \ . \tag{8}$$

Therefore, the probability of team t to score less or equal than j, $j = 1, \ldots, k$:

$$\mathbb{P}(\text{team } t \text{ score} \le j) = \begin{cases} 1 - \prod_{i=1}^{n} \left(1 - F_j^i x_t^i\right) \ , & \text{if } j < k \ , \\ 1 \ , & \text{if } j = k \ . \end{cases} \tag{9}$$

Then, we find the probability that team t scores exactly $j,\ j > 1$,

$$\mathbb{P}(\text{team } t \text{ score} = j) = \mathbb{P}(\text{team } t \text{ score} \le j) - \mathbb{P}(\text{team } t \text{ score} \le j-1)\,.$$

Therefore,

$$\mathbb{P}(\text{team } t \text{ score} = j) = \begin{cases} \prod_{i=1}^{n}\left(1 - F_{k-1}^{i} x_t^i\right), & \text{if } j = k\,, \\ \prod_{i=1}^{n}\left(1 - F_{j-1}^{i} x_t^i\right) - \prod_{i=1}^{n}\left(1 - F_{j}^{i} x_t^i\right), & \text{if } 1 < j < k\,, \\ 1 - \prod_{i=1}^{n}\left(1 - F_{1}^{i} x_t^i\right), & \text{if } j = 1\,. \end{cases} \tag{10}$$

Now, $\mathbb{P}(\text{team } t \text{ "wins"})$ can be directly obtained from (10), if we notice that

$$\mathbb{P}(\text{team } t \text{ "wins"}) = \mathbb{P}(\text{team } t \text{ score} \le \text{team } t' \text{ score})\,, \tag{11}$$

where team t' can be defined as all players not included in team t. If vector $(x_t^1, \ldots, x_t^n)$ defines team t, then apparently vector $(1 - x_t^1, \ldots, 1 - x_t^n)$ defines team t'.

$$\mathbb{P}(\text{team } t' \text{ score} \le j) = 1 - \mathbb{P}(\text{team } t' \text{ score} > j) \tag{12}$$

$$= 1 - \prod_{i \,\in\, \text{team } t'} (1 - F_j^i) = 1 - \prod_{i=1}^{n}\left(1 - F_j^i(1 - x_t^i)\right)$$

$$= 1 - \prod_{i=1}^{n}\left(1 - F_j^i + F_j^i x_t^i\right). \tag{13}$$

As before,

$$\mathbb{P}(\text{team } t' \text{ score} \le j) = \begin{cases} 1 - \prod_{i=1}^{n}\left(1 - F_j^i + F_j^i x_t^i\right), & \text{if } j < k\,, \\ 1\,, & \text{if } j = k\,. \end{cases} \tag{14}$$

Consequently,

$$\mathbb{P}(\text{team } t' \text{ score} = j) = \begin{cases} \prod_{i=1}^{n}\left(1 - F_{k-1}^{i} + F_{k-1}^{i} x_t^i\right), & \text{if } j = k\,, \\ \prod_{i=1}^{n}\left(1 - F_{j-1}^{i} + F_{j-1}^{i} x_t^i\right) - \prod_{i=1}^{n}\left(1 - F_{j}^{i} + F_{j}^{i} x_t^i\right), & \text{if } 1 < j < k\,, \\ 1 - \prod_{i=1}^{n}\left(1 - F_{1}^{i} + F_{1}^{i} x_t^i\right), & \text{if } j = 1\,. \end{cases} \tag{15}$$

Finally, we find the probability that team t "wins" versus team t':

$$\mathbb{P}(\text{team } t \text{ score} \leq \text{team } t' \text{ score}) \tag{16}$$

$$= \sum_{j=1}^{k} \mathbb{P}(\text{team } t \text{ score} \leq j \mid \text{team } t' \text{ score} = j) \cdot \mathbb{P}(\text{team } t' \text{ score} = j) =$$

$$\sum_{j=1}^{k} \mathbb{P}(\text{team } t \text{ score} \leq j) \cdot \mathbb{P}(\text{team } t' \text{ score} = j)\,. \tag{17}$$

Therefore, the "fairness" vector is

$$\big(\mathbb{P}(\text{team 1 "wins"}), \ldots, \mathbb{P}(\text{team } m \text{ "wins"})\big)\,. \tag{18}$$

To minimize the range deviation of the vector (18) components, we have the following formulation:

$$\begin{aligned}
&\text{minimize } b - a \\
&\text{subject to } b \geq \mathbb{P}(\text{team t "wins"})\,, && t = 1, \ldots, m\,, \\
&\qquad\qquad a \leq \mathbb{P}(\text{team t "wins"})\,, && t = 1, \ldots, m\,, \\
&\qquad\qquad \sum_{i=1}^{n} x_t^i = r\,, && t = 1, \ldots, m\,, \\
&\qquad\qquad \sum_{t=1}^{m} x_t^i = 1\,, && i = 1, \ldots, n\,, \\
&\qquad\qquad x_t^i \in \{0, 1\}\,, && i = 1, \ldots, n\,,\ t = 1, \ldots, m\,.
\end{aligned} \tag{19}$$

The above formulation has a drawback in that constraints depend nonlinearly on variables $x_1, \ldots, x_m$. Indeed,

$$\mathbb{P}(\text{team } t \text{ "wins"}) = \sum_{j=1}^{k} \mathbb{P}(\text{team } t \text{ score} \leq j) \cdot \mathbb{P}(\text{team } t' \text{ score} = j) = \tag{20}$$

$$\sum_{j=1}^{k} \left(1 - \prod_{i=1}^{n} \left(1 - F_j^i x_t^i\right)\right) \left(\prod_{i=1}^{n} \left(1 - F_{j-1}^i (1 - x_t^i)\right) - \prod_{i=1}^{n} \left(1 - F_j^i (1 - x_t^i)\right)\right). \tag{21}$$

In fact, $\mathbb{P}(\text{team } t \text{ "wins"})$ can be represented by some linear combination of products of decision variables,

$$x_t^{i_1} \cdot x_t^{i_2} \cdot \ldots \cdot x_t^{i_s}\,.$$

Therefore, we can linearize each of these products by introducing $m(2^n - n - 1)$ new variables defined by the following constraints, as in [3]:

$$w \geq \sum_{j=1}^{s} x_t^{i_j} - (s-1)\,, \tag{22}$$

$$w \leq x_t^{i_j}\,,\ j = 1, \ldots, s\,, \tag{23}$$

$$w \in [0, 1]\,. \tag{24}$$

Alternatively, the set of s constraints (23) can be replaced by the single constraint with binary variable w, as in [4]:

$$w \leq \frac{1}{s} \sum_{j=1}^{s} x_t^{i_j}\,, \tag{25}$$

$$w \in \{0, 1\}\,. \tag{26}$$

However, due to the integrality constraint (4), every product $x_t^{i_1} \cdot x_t^{i_2} \cdot \ldots \cdot x_t^{i_s}$ with $s > r$ equals 0, therefore the number of additional variables to introduce can be reduced to $O(mn^r)$. Still, as mentioned in the introduction, we have not pursued computational implementation of this model because the number of binary variables grows rapidly with n. Instead, we focus on the approach presented in the next section, which, in some sense, provides an alternative way to linearize the above problem.

3 Model 2: Player Probabilities to Win

In this section, we will define the quantities P_i, $i = 1, \ldots, n$, the probabilities of every player i to win the hole, based on game scenarios. To illustrate the basic idea behind this approach, consider the following game score scenario of 5 players $=$ $(5, 4, 3, 7, 9)$ that happens with some probability. Player 3 scores the lowest, therefore the scenario can be represented in terms of win/lose notation as $(a_1, \ldots, a_5) =$ $(0, 0, 1, 0, 0)$, and this scenario is fully counted towards the probability of the player i to win. Note that there are many ways for player 3 to win, i.e., the $(6, 7, 5, 8, 9)$ game score scenario leads to the same $(0, 0, 1, 0, 0)$ scenario in win/lose notation. Consider another game score example: $(3, 4, 3, 7, 9)$ that happens with probability p. Here, players 1 and 3 score the lowest. Therefore, if they are on the same team, this scenario should be fully counted towards that team. However, if they are on the different teams, then these teams tie, i.e., this particular scenario should not provide any advantage to any of the teams. We propose to split that scenario in win/lose notations $(1, 0, 1, 0, 0)$ into two artificial scenarios:

$$(a_1^1, \ldots, a_5^1) = (1, 0, 0, 0, 0) \text{ with probability } \frac{p}{2}\,, \tag{27}$$

$$(a_1^2, \ldots, a_5^2) = (0, 0, 1, 0, 0) \text{ with probability } \frac{p}{2}\,. \tag{28}$$

The motivation for this split is as follows: suppose that when a tie occurs, i.e., $(1, 0, 1, 0, 0)$, we still want to determine the winner, for instance by flipping a coin among the players with the best score. Now, let us generate S game score scenarios using the score distribution of every player. Every scenario j can be counted directly to the winning player or split into several scenarios, as described above, so that finally every scenario has only one winner. Now we can construct the matrix that has only one 1 in every row, so that one and only one player wins in every scenario. From this matrix, it is clear how to obtain the probability to win of every player, P_i:

$$P_i = \sum_{j=1}^{S} a_i^j p_j\,,$$

where S is the number of scenarios. Note that, assuming k possible scores per player, S need not be greater that k^n, the total number of score outcomes of n players.

An alternative expression for the true values of P_i is

$$P_i = \sum_{\substack{a_1=0,\ldots,a_n=0\\ 0<\sum_{l=1}^n a_l<n\\ a_i=1}}^{a_1=1,\ldots,a_n=1} \frac{1}{\sum_{l=1}^n a_l} \sum_{j=1}^{k-1} \prod_{q:\,a_q=1} p_j^q \prod_{q:\,a_q=0} \sum_{l=j+1}^{k} p_l^q + \frac{1}{n} \sum_{j=1}^{k} \prod_{l=1}^{n} p_j^l\,, \tag{29}$$

the calculation time of which also grows exponentially. In an experiment with $n = 24$, over 4 hours of computer time were required to compute the P_i values.

However, in another experiment with $n = 10$ randomly generated handicaps, we confirmed that relatively few randomly generated scenarios are required to closely approximate the true P_i values. This is shown in Table 1.

Thus we can obtain a good approximation of the vector of probabilities of players to win $P = (P_1, \ldots, P_n)$ by restricting S in the first expression above. The main

Table 1 Comparison of scenario-generated P_i versus the true values (29), 10 players

n	S	Handicaps									
		3	5	7	9	11	11	13	17	18	26
10	10, 000	0.2015	0.1701	0.1301	0.1124	0.0949	0.0897	0.0790	0.0530	0.0460	0.0233
	20, 000	0.2035	0.1622	0.1336	0.1115	0.0944	0.0942	0.0821	0.0510	0.0457	0.0218
	50, 000	0.2013	0.1656	0.1340	0.1111	0.0951	0.0950	0.0802	0.0502	0.0457	0.0218
	100, 000	0.2000	0.1646	0.1349	0.1125	0.0948	0.0955	0.0808	0.0503	0.0447	0.0219
	True P_i	0.2014	0.1643	0.1346	0.1116	0.0948	0.0948	0.0802	0.0507	0.0457	0.0222

advantage of this approach is linearity of the probability of a team to win as a function of the decision variables. Indeed, since P_i and P_j, $i \neq j$ define two events with no intersection, then the probability of a team to win can be expressed as follows:

$$\mathbb{P}(\text{team } t \text{ “wins”}) = \sum_{i=1}^{n} P_i \cdot x_t^i\,, \quad t = 1, \ldots, m\,.$$

Then, the following formulation minimizes the range of the “fairness”vector components:

$$\begin{aligned}
\text{minimize} \quad & b - a \\
\text{subject to } b \geq & \sum_{i=1}^{n} P_i \cdot x_t^i\,, && t = 1, \ldots, m\,, \\
a \leq & \sum_{i=1}^{n} P_i \cdot x_t^i\,, && t = 1, \ldots, m\,, \\
& \sum_{i=1}^{n} x_t^i = r\,, && t = 1, \ldots, m\,, \\
& \sum_{t=1}^{m} x_t^i = 1\,, && i = 1, \ldots, n\,, \\
& x_t^i \in \{0, 1\}\,, \quad i = 1, \ldots, n\,,\ t = 1, \ldots, m\,.
\end{aligned} \tag{30}$$

4 Computational Experiments

Computational results were obtained on a machine equipped with Windows 8.1 × 64 operating system, Intel Core(TM) i5-4200M CPU 2.5 GHz, 6 GB RAM, using XPress-MP software, XPRS_BAR solver [7].

To test Model 2, we used the player score distributions from [1] as shown in Table A.1 in the appendix. The website [8] provides USGA distributions of male US golfers by handicap. We employed data from this website for randomly generating players in the handicap range of 1–28, excluding very low and very high handicap players who are not normally part of club competitions. Frequencies for this handicap range are shown in Table A.2. Model 2 was then run for $S = 100{,}000$ scenarios for values of $n \leq 40$. The results were compared with a random assignment and two simple heuristics which are often employed by golf directors in making assignments manually. The manual assignments are called “ABCD” and “Zigzag”, each of which first orders the players by handicap. The “ABCD” assignment for teams of size 4 places the m players with lowest handicap in group A, the next m in group B, etc. and then randomly chooses a player from each of the four groups on each team. Each team then has an A, B, C and D player. The “Zigzag” pattern of assignment is as shown in Table 2.

Table 2 "Zigzag" heuristic illustration, allocating 15 players in 5 teams

	Handicaps														
	3	6	8	8	8	9	10	10	14	15	15	19	21	22	22
Team 1	1									1	1				
Team 2		2							2			2			
Team 3			3					3					3		
Team 4				4			4							4	
Team 5					5	5									5

The number in the column under every player (whose handicap is reported in the second row) represents the identifier of the assigned team

Table 3 Comparison of the Model 2 optimal allocation and different heuristic approaches

#	n	m	Range (30)		CPU Time (30)	"Zigzag"		"ABCD"		Random	
			min	max	(s)	min	max	min	max	min	max
1	12	3	0.3313	0.3364	0.85	0.2770	0.3925	0.3054	0.3833	0.2127	0.4777
2	16	4	0.2479	0.2509	1.26	0.2324	0.2755	0.2207	0.2838	0.1834	0.3770
3	20	5	0.1996	0.2006	2.51	0.1903	0.2175	0.1906	0.2057	0.1269	0.2744
4	24	6	0.1661	0.1673	27.22	0.1551	0.1756	0.1400	0.2099	0.1145	0.2893
5	28	7	0.1427	0.1429	89.47	0.1323	0.1550	0.1188	0.1690	0.0710	0.1978
6*	32	8	0.1229	0.1278	5.75	0.1158	0.1459	0.1009	0.1422	0.0432	0.1671
7*	36	9	0.0988	0.1276	13.80	0.0634	0.1550	0.0961	0.1460	0.0426	0.1764
8*	40	10	0.0984	0.1016	8.26	0.0920	0.1133	0.0764	0.1259	0.0643	0.1322

For every approach the minimum and maximum probabilities to win are reported. Other teams have their corresponding probabilities in the range between these reported values

The results are in Table 3. The first five cases were solved using Model 2 and show that the optimization process is best in all instances. For example, the instance 4 with $n = 24$ and $m = 6$ teams, the ideal solution would have each team with a $1/6 = 0.16666...$ probability of winning. As shown in the table the range attained, $(0.1661,\ 0.1673)$, was very close to this ideal, while the ranges for the heuristics and the random assignment were significantly larger.

For instances larger than $n = 28$, we obtained approximate solutions by a binary decomposition process. For example, for $n = 32$, Model 2 was first solved for two teams of size 16. Then the two resulting teams of that size were solved to yield four teams of size 4, resulting in 8 total teams. For $n = 40$ the first split was into two teams of size 20, then Model 2 was applied to obtain two sets of five teams of size 4. For $n = 36$ the initial split was two teams of sizes 16 and 20. Thus cases 6–8 illustrate that larger problems can be solved quickly and that the results obtained are still superior to those obtained by the heuristics.

Real data example. For further validation, a real case of 24 players being assigned to 6 teams was considered. In this problem the ordered handicaps are presented in Table 4. In Table 5 the approximate probabilities of each player to win a hole without

Table 4 Handicaps of real-life competition of 24 players

Handicaps					
4	8	11	16	17	22
5	11	13	16	17	24
5	11	14	16	17	25
6	11	15	16	21	28

Table 5 Approximate P_i for 24 player handicaps

hcp	P_i	hcp	P_i	hcp	P_i	hcp	P_i	hcp	P_i	hcp	P_i
4	0.1085190	8	0.0679458	11	0.0518596	16	0.0261438	17	0.0227471	22	0.0143381
5	0.0968831	11	0.0518465	13	0.0435318	16	0.0268632	17	0.0225537	24	0.0093904
5	0.0952413	11	0.0513759	14	0.0367221	16	0.0263349	17	0.0225386	25	0.0078451
6	0.0865406	11	0.0517914	15	0.0314653	16	0.0266498	21	0.0145629	28	0.0063104

use of handicaps are presented. These were determined over a sample of $S = 100,000$ of the 9.6×10^{15} scoring scenarios.

Using the approximate probabilities above, the Model 2 optimization problem was solved to form teams that were as close as possible in terms of the team probability to win. Ideally, each of the 6 teams would have probability of $1/6 = 0.1666667$. There are $O\left(10^{12}\right)$ team possibilities.

The solution was obtained in 31.51 s and is given below with the six teams with probability to win and, for comparison, the team average handicap:

Table 6 Optimal solution of Model 2 for the real-life set of players presented in Table 4 with the approximate vector of "win" probabilities presented in Table 5

Team	Team players	Average hcp	Probability to win
1	11, 11, 14, 16	13.00	0.166594
2	4, 16, 16, 28	16.00	0.167308
3	5, 11, 24, 25	16.25	0.165910
4	5, 16, 17, 17	13.75	0.167197
5	6, 13, 17, 22	14.50	0.167158
6	8, 11, 15, 21	13.75	0.165834

This example illustrates that certain common assumptions about the makeup of fair teams are not necessarily correct for the basic golf director problem. Specifically

- the teams' average handicaps do not have to be close,
- the low and high handicap players do not have to be on the same team, and
- players of the same handicap can be on the same team.

5 Conclusions

This paper has introduced the golf director's problem of assigning players to teams in club competitions. Two models have been investigated and compared with the conclusion that scenario optimization (Model 2) provides an effective means of determining fair teams for as many as 40 players in reasonable computer time. Larger problems can be addressed by decomposition techniques such as the one presented. Extensions of the model's basic assumptions will be investigated in future research.

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Appendix

See Tables A.1, A.2.

Table A.1 Distributions of par 4 scores by handicap

Hcp	1	2	3	4	5	6	7	8	9
0	0	0	0.2023	0.4786	0.3191	0	0	0	0
1	0	0	0.1864	0.4519	0.3597	0.002	0	0	0
2	0	0	0.164	0.4483	0.3734	0.0142	0	0	0
3	0	0	0.1437	0.4427	0.3851	0.0284	0	0	0
4	0	0	0.1254	0.4352	0.3948	0.0446	0	0	0
5	0	0	0.109	0.4256	0.4025	0.0628	0	0	0
6	0	0	0.0946	0.4141	0.4083	0.083	0	0	0
7	0	0	0.0823	0.4005	0.412	0.1052	0	0	0
8	0	0	0.0719	0.3849	0.4137	0.1295	0	0	0
9	0	0	0.0636	0.3673	0.4134	0.1557	0	0	0
10	0	0	0.0573	0.3477	0.4111	0.1839	0	0	0
11	0	0	0.053	0.3261	0.4067	0.2142	0	0	0
12	0	0	0.0508	0.3024	0.4003	0.2465	0	0	0
13	0	0	0.0435	0.2891	0.3972	0.2614	0.0089	0	0
14	0	0	0.0349	0.2796	0.3945	0.2691	0.0219	0	0
15	0	0	0.0275	0.2696	0.3904	0.2764	0.0361	0	0
16	0	0	0.0212	0.2591	0.385	0.2833	0.0514	0	0
17	0	0	0.0161	0.2482	0.3781	0.2896	0.0679	0	0
18	0	0	0.0122	0.2368	0.3699	0.2955	0.0856	0	0
19	0	0	0.0094	0.225	0.3603	0.301	0.1044	0	0
20	0	0	0.0078	0.2126	0.3492	0.3059	0.1244	0	0
21	0	0	0.0074	0.1998	0.3368	0.3104	0.1456	0	0
22	0	0	0.0082	0.1866	0.3229	0.3144	0.168	0	0
23	0	0	0.0053	0.1776	0.3148	0.3155	0.1795	0.0072	0
24	0	0	0.0015	0.1704	0.3086	0.3151	0.1867	0.0177	0
25	0	0	0	0.1604	0.3011	0.3156	0.195	0.0279	0
26	0	0	0	0.1492	0.2926	0.3161	0.2038	0.0383	0
27	0	0	0	0.1392	0.2836	0.3151	0.2122	0.0499	0
28	0	0	0	0.1305	0.2742	0.3127	0.22	0.0627	0
29	0	0	0	0.1229	0.2642	0.3087	0.2273	0.0768	0
30	0	0	0	0.1167	0.2538	0.3033	0.2341	0.0921	0
31	0	0	0	0.1117	0.2428	0.2963	0.2405	0.1087	0
32	0	0	0	0.1074	0.2319	0.2887	0.246	0.1253	0.0008
33	0	0	0	0.1002	0.2247	0.2858	0.2489	0.1325	0.008
34	0	0	0	0.0937	0.2175	0.2822	0.2511	0.1397	0.0159
35	0	0	0	0.088	0.2103	0.2778	0.2524	0.1469	0.0246
36	0	0	0	0.0831	0.2031	0.2726	0.253	0.1541	0.0341

Table A.2 Fractions of golfers with handicaps 1–28 [8]

Hcp	Probability (%)	Hcp	Probability (%)	Hcp	Probability (%)	Hcp	Probability(%)
1	1.34	8	5.07	15	5.43	22	2.45
2	1.72	9	5.46	16	4.99	23	2.11
3	2.20	10	5.78	17	4.61	24	1.79
4	2.77	11	6.02	18	4.02	25	1.52
5	3.41	12	6.09	19	3.56	26	1.27
6	4.01	13	6.06	20	3.15	27	1.06
7	4.63	14	5.82	21	2.79	28	0.87

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