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Preface

WINE 2005, the First Workshop on Internet and Network Economics (WINE 2005), took place in Hong Kong, China, December 15-17, 2005. The symposium aims to provide a forum for researchers working in Internet and Network Economic algorithms from all over the world. The final count of electronic submissions was 372, of which 108 were accepted. It consists of the main program of 31 papers, of which the submitter email accounts are: 10 from edu (USA) accounts, 3 from hk (Hong Kong), 2 each from il (Isreal), cn (China), ch (Switzerland), de (Germany), jp (Japan), gr (Greece), 1 each from hp.com, sohu.com, pl (Poland), fr (France), ca (Canada), and in (India). In addition, 77 papers from 20 countries or regions and 6 dot.coms were selected for 16 special focus tracks in the areas of Internet and Algorithmic Economics; E-Commerce Protocols; Security; Collaboration, Reputation and Social Networks; Algorithmic Mechanism; Financial Computing; Auction Algorithms; Online Algorithms; Collective Rationality; Pricing Policies; Web Mining Strategies; Network Economics; Coalition Strategies; Internet Protocols; Price Sequence; Equilibrium. We had one best student paper nomination: "Walrasian Equilibrium: Hardness, Approximations and Tracktable Instances" by Ning Chen and Atri Rudra.

We would like to thank Andrew Yao for serving the conference as its Chair, with inspiring encouragement and far-sighted leadership. We would like to thank the International Program Committee for spending their valuable time and effort in the review process. We would like to thank the two invited speakers, Ehud Kalai and Christos Papadimitriou, for offering their insightful views of the emerging field. We would also like to thank the Organizing Committee for their services.

Finally, we owe our success to the generosity of our financial sponsors: K. C. Wong Education Foundation and Hong Kong Pei Hua Education Foundation Ltd. We also owe our success to the financial support and the clerical support of the Department of Computer Science, City University of Hong Kong.

December 2005

Xiaotie Deng
Yinyu Ye

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WINE 2005 was jointly organized by the City University of Hong Kong, Chinese University of Hong Kong, and Hong Kong Baptist University.

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Recent Developments in Equilibria Algorithms*

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Nash proved in 1951 that every game has a mixed Nash equilibrium [6]; whether such an equilibrium can be found in polynomial time has been open since that time. We review here certain recent results which shed some light to this old problem.

Even though a mixed Nash equilibrium is a continuous object, the problem is essentially combinatorial, since it suffices to identify the *support* of a mixed strategy for each player; however, no algorithm better than exponential for doing so is known. For the case of two players we have a simplex-like pivoting algorithm due to Lemke and Howson that is guaranteed to converge to a Nash equilibrium; this algorithm has an exponential worst case [9]. But even such algorithms seem unlikely for three or more players: in his original paper Nash supplied an example of a 3-player game, an abstraction of poker, with only irrational Nash equilibria.

When the number of players is large, complexity questions become moot because the input is exponential: ns^n numbers representing player utilities are needed to specify a game with n players and s strategies per player. Such games are computationally meaningful only if they can be represented succinctly. Indeed, several succinct representations of games have been proposed, chief among which are the graphical games [5]. In a graphical game players are the nodes of a graph, and the utility of a player depends only on the actions of the players that are adjacent to it on the graph. Thus, if the graph is of bounded degree, the input is polynomial in the number of players and strategies. For such games it was recently shown [8] that *correlated equilibria* (a relaxation of Nash's concept that is tractable in the normal-form representation by linear programming) can be computed in polynomial time.

In a recent paper [3] we showed that the problem of finding a Nash equilibrium in bounded-degree graphical games is equivalent to the same problem in normal-form games, and in fact normal-form games with four players are completely general in this respect. The key insight in these reductions are certain "gadgets," 4-player games simulating arithmetical operations.

Nash's existence proof relies on Brouwer's fixpoint theorem, another notoriously non-constructive result for which we do have evidence of intractability [4, 7, 1]. Is this reliance inherent? This is a restatement of the question of the complexity of the mixed Nash equilibrium problem. In a recent paper with Costas Daskalakis and David Goldberg [2] we use the arithmetical gadgets of [3], among other ideas, to provide an affirmative answer to this question: Mixed Nash equilibria in a game with 4 players (equivalently, a graphical game of degree 3) can

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simulate arbitrary Brouwer fixpoints. Hence, finding a mixed Nash equilibrium is PPAD-complete [7]— the appropriate evidence of intractability for this complexity level.

We conjecture that the 3-player case is also intractable, while the 2-player one is polynomial.

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Partially-Specified Large Games

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Abstract. The sensitivity of Nash equilibrium to strategic and informational details presents a difficulty in applying it to games which are not fully specified. Structurally-robust Nash equilibria are less sensitive to such details. Moreover, they arise naturally in important classes of games that have many semi-anonymous players. The paper describes this condition and its implications.

1 Introduction

Deciding whether to attend WINE 2005 is a participation game, where players' payoffs depend on the participation choices of others. But like many other games, it is only partially specified. It is known that players may go to a web site and click in their choice before some deadline, but it is not known in what order they will move, what information they will have, who may make agreements with whom, etc., etc.

Equilibrium analysis of a partially-specified game forces the analyst to make-up all the missing details. But since equilibrium predictions are often sensitive to such details, the analyst's predictions are often unreliable.[1]

Equilibria that are structurally robust, as described in this paper, are less sensitive to many game specifics.[2] As such, they offer a partial resolution to the difficulty above. The analyst can compute the equilibrium of a game with a minimal structure (simultaneous-one-move play), and be assured that it remains equilibrium no matter what the missing details are.

Structural robustness is a strong property that fails in the equilibria of most games. But as discussed in this paper, in games with many semi-anonymous players all the equilibria are structurally robust.[3]

[1] This high level of sensitivity is well known to researchers and users of game theory. Even small modifications, such as allowing the players to make meaningless cheap-talk announcements, may drastically alter the equilibrium of the game, see for example Crawford and Sobel (1982).

[2] Other notions of robustness and structural robustness were proposed in economics. We refer the reader to Hansen and Sargent (2001) for some examples. An earlier weaker notion of structural stability was introduced in Kalai (2004) under the name of extensive robustness.

[3] Positive results about large games were obtained earlier in the cooperative-games literature, see Aumann and Shapley (1974) for a survey.

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In addition to dealing with partially specified games, structural robustness has other important implications, even in fully specified games. For better or worse, an equilibrium with this property is more persistent, it self-purifies, has a strong ex-post Nash property, can be used to model a small-world game embedded in a larger game, and has a strong rational-expectations property in market games.

2 Definition of Structural Robustness

Starting with an n-person simultaneous-one-move game, we first describe a large number of variations on how the game may be played. Such variations allow for sequential (instead of simultaneous) moves, multiple opportunities to revise earlier choices, information transmission, commitments, delegations, etc. The eventual purpose is to identify equilibrium as being structurally robust, if it survives under all such variations.

2.1 Metagames

To describe a variation of the simultaneous move game, G, we introduce the notion of a metagame of G. The concept is natural and straight forward but the formal notations are cumbersome.[4] For this reason, it is better to explain it through a couple of simple examples.

Consider first a 2-person Match Pennies game (MP for short) with player 1 being a male who wishes to match his opponent, and player 2 being a female who wishes to mismatch her opponent.[5] The play of the game has the four possible H-P pairs, (H,H), (H,T), (T,H), (T,T), with the corresponding payoffs, (1,-1), (-1,1), (-1,1), (1,-1). A *metagame* of MP is any m-person perfect-recall extensive game $\mathcal{M}$ that has exactly one of the four H-P pairs above associated with every one of its final nodes. In other words, every play of $\mathcal{M}$ yields a play of MP.

Condition 1 ***Preservation of strategies and payoffs:*** *We restrict ourselves to metagames that satisfy three properties. First, the players of the metagame include all the original players of the underlying game MP.*

Second, any of the original MP players has metagame strategies that guarantee him, in the metagame, the same pure choices that he has in MP. For example, since player 1 can choose H in MP, he should have a metagame strategy that guarantees him (no matter what strategies are used by the opponents) to end up (with probability one) at a final node in which his label is H (the associate pair being (H,H) or (H,T)).

Third, the metagame must preserve the original payoffs in the sense that at every final node of $\mathcal{M}$, the payoffs of the original players are the same as their payoffs in MP. For example, if the metagame ends at a final node with the associate pair (H,H) then the payoffs of players 1 and 2 are (1,-1).

[4] As is the case for most formal models that involve extensive form games.

[5] Simultaneously, each one of them chooses either H or T. If the choices match, she pays him 1; it they mismatch, he pays 1 to her.

Three simple examples of metagames of MP that preserve strategies and payoffs are the following.

Example 1. **Sequential-play metagame:** player 1 moves first and chooses H or T, player 2 moves second and chooses H or T after being informed of the choice of player 1. The associated pair at a final node consists of the pair of sequential choices made on the path leading to it, and the payoffs are the MP payoffs defined at this pair.

Example 2. **Metagame with revisions:** simultaneously each of the two players chooses H or T; then, after being informed of the pair of choices, simultaneously they each choose H or T again. The associated pair at every final node of the metagame consists of the choices made in the second round, and the payoffs are the MP payoffs defined at this pair.

Example 3. **Metagame with an outsider setting the rules:** player 3, the rule setter, moves first and decides whether the game will be played simultaneously, sequentially, or simultaneously with revisions. After being informed of his choice, the original players play according to the rule chosen by him and receive the appropriate payoffs. The metagame preserves strategies and payoffs, no matter what payoffs are assigned to the outsider (the rule setter) at the final nodes.

As the reader should be able to see, one can construct a large number of interesting metagames that preserves strategies and payoffs.

2.2 Metaversions of Strategies

To require that an equilibrium "survive in all metagames," we first must clarify how a strategy of the simultaneous move game is played in a given metagame. This can be done easily due to the requirement that the metagame preserve strategies.

For example, consider the .50-.50 H-T mixed strategy of player 1 (he chooses H or T with equal probability in MP) and consider the metagame with a rule setter.

Let ψ_1 be the metagame mixed strategy in which player 1 chooses with equal probabilities one of the following two pure strategies: (1) he chooses H in every one of his information sets in the metagame, or (2) he chooses T in every one of his information sets in the metagame. This strategy guarantees that he ends up with an H (with probability .50) or guarantees that he ends up with a T (with probability .50).[6]

[6] This is stronger than simply guaranteeing a .50-.50 outcome on H or T. For example, consider a metagame that starts with nature moving left or right with equal probabilities, and players 1 and 2 follow with a simultaneous choice of H or T after observing nature's choice. To choose H after nature moves left and T after nature moves right is a strategy that guarantees player 1 .50-.50 probabilities over a final label of H or T. But this strategy cannot be decomposed into a .50-.50 lottery over one strategy that guarantees him H and one strategy that guarantees him T.

When such is the case we say that ψ_1 is a *metaversion* of the .50-.50 H-T mixed strategy. However, there are additional metaversions of the .50-.50 H-T mixed strategy. For example, player 1 may mix over the following two pure metagame strategies: (1) he chooses H in every one of his metagame information sets, or (2) he chooses T in every one of his metagame information sets except for the first round of the subgame with revisions where he chooses H.

Having defined metaversions of individual strategies, we can define metaversions of strategy profiles. For example, the strategy profile (ψ_1, ψ_2, ψ_3) in the metagame with a ruler is *a metaversion* of the strategy profile (σ_1, σ_2) in MP, if ψ_1 is a metaversion of σ_1 and ψ_2 is a metaversion of σ_2 (outsiders' strategies are not restricted).

2.3 Structural Robustness

An equilibrium σ of the simultaneous move game is *structurally robust,* if it is an equilibrium in every metagame that preserves strategies and payoffs. This means that no matter what metagame is played, in every metaversion of σ the strategies of the original players are best response.

For example, for an equilibrium $\sigma = (\sigma_1, \sigma_2)$ of MP to be structurally robust, the metaversions of σ must be optimal in every metagame of MP. If in the metagame with a ruler, for example, $\psi = (\psi_1, \psi_2, \psi_3)$ is a metaversion of σ, then ψ_1 and ψ_2 must be a best response strategies of player 1 and player 2 respectively (ψ_3 is not restricted).

We may think of σ as being a uniform Nash equilibrium in every metagame, since any metaversions of (σ_1, σ_2) are optimal no matter what the strategies of the outsiders are.[7]

It is easy to see that MP has no structurally robust equilibrium. Since the underlying game is always a metagame of itself, any structurally robust equilibrium must be equilibrium of the underlying game itself. Thus the only candidate for structural robustness in MP is the profile consisting of the pair of .50-.50 H-T mixed strategies. But this pair fails to be an equilibrium in the sequential play metagame for example. Randomly choosing H or T is no longer a best reply of the follower since she is better off mismatching player 1's observed choice.

2.4 Approximate Structural Robustness

The existence difficulty becomes less sever when the number of players is large. Consider a generalized Match Pennies game that consists of n males and n females, 2nMP. Simultaneously, they each choose H or T. A male's payoff is the proportion of females his choice matches and a female's payoffs is the proportion of males she mismatches. The .50-.50 H-T profile of mixed strategies is an equilibrium that turns out to be highly structurally robust when n is large. In the sequential-play metagame with all the males moving first, for example,

[7] Using the standard fixed point method, for every structurally robust σ and every metagame, every metaversion of σ can be completed to a metagame profile in which all the players (including the outsiders) best respond.

when the females turn to choose comes up, the distribution of male choices will be nearly one half H's and one half T's with probability close to one. Thus randomly choosing between H and T is nearly optimal for every female. (Using Chernoff bounds, for any $\varepsilon > 0$ the probability that the proportion of H-choosing males be outside the $.50 \pm \varepsilon$ range goes down to zero at an exponential rate in the number of males).

The above observation motivates an approximate definition of being (ε, ρ) structurally robust. For a given pair of such non negative numbers we require that the equilibrium of the simultaneous move game be (ε, ρ) Nash equilibrium in every metagame that preserves strategies and payoffs. Being (ε, ρ) Nash equilibrium means that the event "following a play path along which some player can improve his payoff at some information set by more than ε" has probability of at most ρ.[8]

2.5 Generalization to Bayesian Games

Due to their wide applicability, it is important to generalize the above notions to simultaneous-one-move Bayesian games.[9] The generalizations are straight forward, as illustrated by the following example of an n-person Bayesian MP game (BMP for short).

Every player i is randomly drawn to be a $male$ or a $female$ type according to commonly known individual prior probabilities. Before they make any choices, everyone is informed (only) of his own realized type. Next, simultaneously, each of the n players chooses H or T. Every player's payoff is a function of his type, his choice, and the distribution of types and choices of his opponents. For example, a $male$'s (type) payoff may be the proportion of $female$s (female types) his choice matches and a$female$'s payoff may be the proportion of $male$s she mismatches.

A metagame of BMP is again any m-person perfect recall extensive form game with an n-tuple of H-T's associated with each of its final nodes. But to be compatible with the underlying Bayesian game, it must start with a move of nature, where the types of the individual players are drawn with the same distribution as in BMP, and with every player being informed of his own realized type prior to the start of play.

An equilibrium of BMP is (ε, ρ) structurally robust if it remains an (ε, ρ) Nash equilibrium in every metagame of BMP that preserves strategies and payoffs.

3 Main Result

To keep the presentation simple, we restrict ourselves to games in which player types are restricted to be called $male$ or $female$ and player actions are restricted to be called H or T. These particular names of types and actions are not important, nor is the fact that there are only two types and two actions. It is important however, that there is a finite number of each.

[8] See Kalai (2004) for elaboration.
[9] See Harsanyi (1967/68).

Consider a family of games $\mathcal{F}$ that includes for every $n = 1, 2, ...$ many n person games (could even be uncountably many games for every n). Every n-person game $G_n \in \mathcal{F}$ is described by a collection of n pairs $G_n = (\tau_i, u_i)_{i=1}^n$ having the following interpretation. The non negative number τ_i is the probability that player i is a *male* type (male for short) and $1 - \tau_i$ is the probability that player i is a *female* type (female for short). It is assumed that types are drawn independently across players.

The payoff function of every player i, u_i, is anonymous in the sense that it may depend only on aggregate data of player i's opponents. More formally $u_i : (t_i, a_i, e_{-i}) \longmapsto r \in [0, 1]$, where $t_i = male$ or $female$, $a_i = H$ or T, and $e_{-i} = (e_{m,H}, e_{m,T}, e_{f,H}, e_{f,T})$ describes the proportions of opponents type-choice combinations. For example $e_{m,H}$ describes the proportion of player i's opponents who are *male*s who choose H.

We assume also that the collection of all payoff functions (consisting of all the functions u_i's from all the games in $\mathcal{F}$) is uniformly equicontinuous.

Theorem 1. ***Structural Robustness.*** *Given the family $\mathcal{F}$ above and an $\varepsilon > 0$, there are positive constants α and β, $\beta < 1$, such that for every n person game in the family all the equilibria are $(\varepsilon, \alpha\beta^n)$ structurally robust.*

This theorem is a generalization of a theorem in Kalai (2004). Even though the result is substantially stronger, the proof is similar and is therefore omitted (it requires that the number of player types and possible actions be uniformly bounded by finite numbers. This could possibly be replaced by assumptions of continuity and compactness).

3.1 Some Clarifications

The anonymity imposed on the individual payoff functions is far from being full anonymity, since players may have individual payoff functions and individual prior distributions over types. While this already means that the players are not anonymous individuals, it even breaks some of the anonymity in the payoff functions.

For example, we may have a formulation that includes a type whose name is *Mr.Jones*, and in a certain game only player 1 is the *Mr.Jones* type (he has prior probability one of being so and every other player has probability zero of being so). Now, while the players payoffs are anonymous in the technical sense formulated above, the payoffs of the players may depend on the action of player 1 in a non symmetric way. For example player 2 payoff may depend entirely on the distribution of actions chosen by *MrJones* types, thus making player 2 payoff be a function of player 1's action. This method of breaking anonymity is only partial, however, since we can accommodate only finitely many types and the number of players becomes infinite.

Another consequence of the individualized payoff functions and prior probabilities is that the family of games above may contain many games that are drastically different from the Match Pennies game that we started with. For example, it may contain two players, both female type with probability one, and each has a payoff function that is one if and only if she matches the opponent. This is a pure coordination game.

The Bayesian aspect of the formulation allows for interesting games. For example player 1 may be a female who wishes to match, but player 2 may have a positive prior operability of being a male who wishes to mismatch and a positive prior probability of being a female who wishes to match. Thus, player 1 may not know exactly whether he is in a coordination game or a match pennies game.

In a similar way, the family may contain prisoners' dilemma games of various sizes. When we allow n types, all n person payoff functions can be accommodated.

4 Implications

4.1 Partially Specified Games

In what sense does the above theorem help the modeling of a partially specified game? The analyst may write such a game with a minimal structure, as a simultaneous one-move game, and compute its equilibrium. This equilibrium will survive even if the structure is made richer.

For example, in a large anonymous participation game, where the choice of every player is whether or not to participate in an event, the analyst may write the game where simultaneously every player decide whether to participate or not, and compute its equilibrium. The structural robustness theorem implies that this equilibrium will be sustained no matter how the simultaneous move and informational assumptions are changed. So even if choices are made sequentially, private and public messages being transmitted according to any dynamic stochastic process, players are allowed to repeatedly revise earlier choices, players are allowed to interact and make use of outsiders, under all such possibilities the computed equilibrium is not destroyed.

However, the resolution is only partial. While all the equilibria of the simultaneous-one-move game are sustained in all extensive metagames, each metagame may have additional equilibria that are not present in the simultaneous one-move game. This puts us into another typical difficulty of game theory, namely multiplicity of equilibria. Whether one equilibrium will be sustained over others depends on focal point and related considerations.

The simultaneous-one-move equilibria have an added advantage over the others: they are the only equilibria present in the intersection of all metagames, since the simultaneous one-move game is one of the metagames of itself.

4.2 Ex-post Stability

The structural robustness property implies other game theoretic properties of the equilibrium. One important consequence is *strong ex-post stability* (also known as ex-post Nash).[10] This means that after the game is over, even with

[10] See Cremer and McLean (1985), Green and Laffont (1987) and follow up literatures, for earlier (weaker) versions of the concept.

(full or partial) hind-sight information about the types and choices of the others, no player has incentive to change (or regrets) the choice he made.

A vector of types and actions is ε ex-post Nash if for every player and for every outcome of the game, even with perfect hind-sight information about the realized types and selected actions of all his opponents, the player cannot gain an ε-improvement in his payoff by a unilateral change of his own action.

An equilibrium is (ε, ρ) *ex-post Nash* if the probability of ending up with a vector of types and actions which is ε ex-post Nash is at least $1 - \rho$.

To see that (ε, ρ) structural robustness implies (ε, ρ) ex-post Nash consider the metagame with revision discussed earlier (they play the one shot game once, observe everybody's realized types and choices, and get to revise their choices in a second round). For any equilibrium of the simultaneous move game, consider the following metaversion. Play the equilibrium in the first round, with no revisions in the second round. The structural robustness property implies that this metaversion of the equilibrium is an equilibrium of the metagame with revision. This, however, is equivalent to being ex-post Nash (also in the (ε, ρ) adjusted senses of both).

The ex-post stability is stronger than just described, because it holds even for partial information. It may be that no player can improve his payoff by more than ε when given complete information about the types and choices of his opponents, but he can improve his *expected* payoff by more than ε when he conditions on partial information about his opponents. Being strongly (ε, ρ) ex-post Nash means that this is not the case. No matter what hind-sight (perfect or imperfect) information is given to individual players, with probability at least $1 - \rho$ no one can improve his expected payoff by more than ε by a unilateral change of his selected action.

4.3 Self Purification

The ex-post Nash property may alternatively be viewed as a property of self purification. Starting with Schmeidler (1973), there is a large literature showing that large anonymous games have pure strategy Nash equilibrium. Schmeidler's paper starts by proving the existence of "mixed strategies" equilibrium, and then proving that he can purify it, i.e., replace the mixed strategies by pure ones without destroying the equilibrium property.

But when an equilibrium is (fully) ex-post Nash (in the sense defined in this paper), the profiles of pure strategies that can be generated by its play must all be Nash equilibria of the game. In other words, there is no need to purify the equilibrium since it purifies itself (through laws of large numbers).

And going beyond the purification literature of Schmeidler (which studies only simultaneous-move normal-form games), self purification holds for simultaneous-move Bayesian games. The play of any Bayesian equilibrium which is ex-post Nash must generate, for every vector of realized types, a profile of pure actions which is a Nash equilibrium of the normal-form game determined by the realized types.

There are some technical differences between the two approaches. Schmeidler and follow up papers model large games by assuming a continuum of players, while the current paper models it asymptotically by letting the number of players grow to infinity. A difficulty in Schmeidler's model is that mixed strategy profiles are hard (border line impossible) to define, since they involves a continuum of independent mixed strategies. A limitation of the current paper is that the full ex-post Nash property holds only in the limit. The reader may verify though, that the approximate notion of (ε, ρ) ex-post Nash gives rise to a natural approximate notion of self purification which becomes fully so in the limit.

4.4 Small World in a Bigger Game

Game theorists model games in isolation, ignoring strategic and informational spillovers between the game and the outside world in which the game is played. This often leads to incorrect analysis since such spillovers can easily destroy equilibria of the isolated game. It is therefore important to identify conditions under which an equilibrium persist, even when embedded in a bigger world.

Since metagames allow additional players, additional actions, back and forth communication and the like, they provide a model of a bigger world in which the underlying game may be embedded. Moreover, the condition of structural robustness is precisely the one that guarantees the survival of the equilibrium under such embedding. This makes the sufficient-conditions for structural robustness important in the discussion of the embedding issue.

Clearly, the requirement that the metagame preserve strategies and payoffs limits the scope of possible embeddings. But it is important to note that if the outside world in which the isolated game is played does not preserve its strategies and payoffs, then the isolated game should not be studied in the first place. It is simply not the game that exists in reality.

Assuming, therefore, the metagame does preserve strategies and payoffs, as a general rule it still does not preserve the equilibria of the isolated game, unless the isolated game is large as assumed in our model.

We may conclude that the equilibria of large semi-anonymous games are persevered when embedded in the outside world, provided that the embedding preserve strategies and payoffs. We should keep in mind that while the equilibria are preserved, the presence of the outside world may introduce new additional equilibria, not identified by the analysis of the isolated game.

4.5 Implementation

Mechanism designers deal with games in which the natural equilibrium may be inefficient, and aim to create new related games in which equilibria are efficient. The new game has the same set of possible types outcomes as the original game, but it is redesigned to have "good incentives." In the language of the current paper, mechanism designers often replace an underlying "bad game" by a metagame, and by doing so they *implement* a better equilibrium.

To a significant extent the implementation literature studies the possible improvements that a mechanism designer may attain under various restrictions on the possible or permissible implementing metagame. The condition that a metagame preserves strategies and payoffs is interesting in this regard. It means that the designer cannot invade the options of the participants, by disallowing them some choices or by modifying the consequences of the original choices.

Under this restriction, the structural robustness property has an interesting implication, since it limits the ability of the mechanism designer to eliminate the bad equilibrium. This means that he must resort to more subtle means of implementation. He can only hope to find a metagame in which the original bad equilibrium becomes non appealing. For example, he may be able to construct a metagame in which the original bad equilibrium involves the use of dominated strategies, or has some other non appealing aspects from focal point considerations.

The observations above may be important to real-life policy makers and system designers. Languages, measurement systems, and keyboard choices are examples illustrating that highly inefficient social equilibria are likely to persist even despite social attempts to replace them by more efficient ones.

4.6 Rational Expectations

When restricting ourselves to market games, the strong ex-post Nash property of structurally robust equilibrium implies that the equilibrium has a strong rational expectations property.[11] This may be illustrated by considering a Bayesian version of a Shapley-Shubik game, where players of random types (representing possible initial endowments, information and preferences) submit, in a one-simultaneous-move game, portions of their initial endowments to be traded in the market. Final market prices are computed as a function of the submitted quantities and net trades are executed by these final prices.[12]

In general, Nash equilibrium of such a game fails to satisfy the rational expectations property economists expect from an equilibrium. This property requires that every agent's trade be optimal at the market prices, given his individual type and the inference (bases on the observed prices) he may make about the unknown types of the others. This cannot be expected to hold for the Shapley-Shubik players since their strategies are based entirely on knowledge of their individual parameters, without knowledge of the choices made by the others (and the consequential market prices).

But an equilibrium which is strongly ex-post Nash turns out to automatically satisfy the rational expectations property. Prices, in the Shapley-Shubik game, are ex-post information. Thus, under a strong ex-post Nash condition, knowing the prices gives no player incentives to revise his choices. This means that his trade is optimal given knowledge of the prices in the same sense requited by rational expectations.

The intuition is fairly straight forward. When the game has many traders, with unknown types but known prior distributions over types, laws of large numbers allow the traders to correctly anticipate the final market prices which

[11] For examples of economic and game theoretic implications see Lucas (1972), Grossman and Stiglitz (1980), Jordan and Radner (1982) and Forge and Minelli (1998).

[12] See Shapley and Shubik (1977) and Peck (2003).

depends on aggregated data of the opponents. Therefore the ex-post Nash property and the rational expectation property both hold.[13]

The fact that the ex-post Nash property is strong implies a strong rational expectations property. The trade every player ends up with is optimal given his information, any inference he may make through the realized prices, but also any inference he may make from any other information he may acquire.

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[13] The above arguments require continuity of the payoff functions under small changes in the distribution of opponent characteristics. This is so because in the game above players are small in their strategic influence and in the information they posses (see for example McLean and Postlewaite (2002)). When individual players posses substantial inside information, continuity fails and we do not obtain the required ex-post Nash and the consequential rational expectations properties.

Exchange Market Equilibria with Leontief's Utility: Freedom of Pricing Leads to Rationality

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Abstract. This paper studies the equilibrium property and algorithmic complexity of the exchange market equilibrium problem with more general utility functions: piece-wise linear functions, which include Leontief's utility functions. We show that the Fisher model again reduces to the general analytic center problem, and the same linear programming complexity bound applies to approximating its equilibrium. However, the story for the Arrow-Debreu model with Leontief's utility becomes quite different. We show that, for the first time, that solving this class of Leontief exchange economies is equivalent to solving a known linear complementarity problem whose algorithmic complexity status remains open.

1 Introduction

This paper studies the equilibrium property and algorithmic complexity of the Arrow-Debreu competitive equilibrium problem. In this problem, players go to the market with initial endowments of commodities and utility functions. They sell and buy commodities to maximize their individual utilities under a market clearing price. Arrow and Debreu [1] have proved the existence of equilibrium prices when utility functions are concave and commodities are divisible. From then on, finding an efficient algorithm for computing a price equilibrium has became an attractive research area; see [2, 4, 6, 7, 8, 9, 10, 15, 16, 17, 21, 22, 26].

Consider a special case of the the Arrow-Debreu problem, the Fisher exchange market model, where players are divided into two sets: producer and consumer. Consumers have money to buy goods and maximize their individual utility functions; producers sell their goods for money. The price equilibrium is an assignment of prices to goods so that when every consumer buys a maximal bundle of goods then the market clears, meaning that all the money is spent and all the goods are sold. Eisenberg and Gale [12, 13] gave a convex optimization setting to formulate Fisher's problem with linear utilities. They constructed an aggregated concave objective function that is maximized at the equilibrium. Thus, finding an equilibrium became solving a convex optimization problem, and it could be obtained by using the Ellipsoid method or interior-point algorithms in polynomial time. Here, polynomial time means that one can compute an ϵ-approximate equilibrium in a number of arithmetic operations bounded by polynomial in n and $\log \frac{1}{\epsilon}$.

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It has turned out that the general Arrow-Debreu problem with linear utilities is also equivalent to a convex optimization setting (see, e.g., Nenakhov and Primak [20] and Jain [15]). The best arithmetic operation bound for solving the Arrow-Debreu problem with linear utilities is $O(n^4 \log \frac{1}{\epsilon})$; see [26]. Moreover, if the input data are rational, then an exact solution can be obtained by solving a system of linear equations and inequalities when $\epsilon < 2^{-L}$, where L is the bit length of the input data. Thus, the arithmetic operation bound becomes $O(n^4 L)$, which is in line with the best complexity bound for linear programming of the same dimension and size.

In this paper we deal more general utility functions: piece-wise linear functions, which include Leontief's utility functions. We show that the Fisher model again reduces to the general analytic center model discussed in [26]. Thus, the same linear programming complexity bound applies to approximating the Fisher equilibrium with these utilities. We also show that the solution to a (pairing) class of Arrow-Debreu problems with Leontief's utility can be decomposed to solutions of two systems of linear equalities and inequalities, and the price vector is the Perron-Frobenius eigen-vector of a scaled Leontief utility matrix. Consequently, if all input data are rational, then there always exists a rational Arrow-Debreu equilibrium, that is, the entries of the equilibrium vector are rational numbers. Additionally, the size (bit-length) of the equilibrium solution is bounded by the size of the input data. This result is interesting since rationality does not hold for Leontief's utility in general. Perhaps more importantly, it also implies, for the first time, that solving this class of Leontief's exchange market problems is equivalent to solving a known linear complementarity problem where its algorithmic complexity status remains open.

2 The Fisher Equilibrium Problem

Without loss of generality, assume that there is 1 unit good from each producer $j \in P$ with $|P| = n$. Let consumer $i \in C$ (with $|C| = m$) has an initial endowment w_i to spend and buy goods to maximize his or her individual linear substitution utility:

$$u_i(x_i) = \min_k \{u_i^k(x_i)\}, \tag{1}$$

where $u_i^k(x_i)$ is a linear function in x_{ij}—the amount of good bought from producer j by consumer i, $j = 1, ..., n$. More precisely,

$$u_i^k(x_i) = (u_i^k)^T x_i = \sum_{j \in P} u_{ij}^k x_{ij}.$$

In particular, the Leontief utility function is the one with

$$u_i^k(x_i) = \frac{x_{ik}}{a_{ik}}, \ k = j \in P$$

for a given $a_{ik} > 0$, that is, vector u_i^k is an all zero vector except for the kth entry that equals $1/a_{ik}$.

We make the following assumptions: Every consumer's initial money endowment $w_i > 0$, at least one $u^k_{ij} > 0$ for every k and $i \in C$ and at least one $u^k_{ij} > 0$ for every k and $j \in P$. This is to say that every consumer in the market has money to spend and he or she likes at least one good; and every good is valued by at least one consumer. We will see that, with these assumptions, each consumer can have a positive utility value at equilibria. If a consumer has zero budget or his or her utility has zero value for every good, then buying nothing is an optimal solution for him or her so that he or she can be removed from the market; if a good has zero value to every consumer, then it is a "free" good with zero price in a price equilibrium and can be arbitrarily distributed among the consumers so that it can be removed from the market too.

For given prices p_j on good j, consumer i's maximization problem is

$$\begin{array}{ll} \text{maximize} & u_i(x_{i1}, ..., x_{in}) \\ \text{subject to} & \sum_{j\in P} p_j x_{ij} \leq w_i, \\ & x_{ij} \geq 0, \quad \forall j. \end{array} \tag{2}$$

Let x_i^* denote a maximal solution vector of (2). Then, vector p is called a Fisher price equilibrium if there is x_i^* for each consumer such that

$$\sum_{i\in C} x_i^* = e$$

where e is the vector of all ones representing available goods on the exchange market.

Problem (2) can be rewritten as an linear program, after introducing a scalar variable u_i, as

$$\begin{array}{ll} \text{maximize} & u_i \\ \text{subject to} & \sum_{j\in P} p_j x_{ij} \leq w_i, \\ & u_i - \sum_{j\in P} u^k_{ij} x_{ij} \leq 0, \ \forall k, \\ & u_i, \ x_{ij} \geq 0, \quad \forall j. \end{array} \tag{3}$$

Besides (u_i, x_i) being feasible, the optimality conditions of (3) are

$$\begin{array}{c} \lambda_i p_j - \sum_k \pi^k_i u^k_{ij} \geq 0, \ \forall j \in P \\ \sum_k \pi^k_i = 1 \\ \lambda_i w_i = u_i. \end{array} \tag{4}$$

for some $\lambda_i, \ \pi^k_i \geq 0$.

It has been shown by Eisenberg and Gale [12, 11, 13] (independently later by Codenotti et al. [3]) that a Fisher price equilibrium is an optimal Largrange multiplier vector of an aggregated convex optimization problem:

$$\begin{array}{ll} \text{maximize} & \sum_{i\in C} w_i \log u_i \\ \text{subject to} & \sum_{i\in C} x_{ij} = 1, \quad \forall j \in P, \\ & u_i - \sum_{j\in P} u^k_{ij} x_{ij} \leq 0, \quad \forall k, i \in C, \\ & u_i, \ x_{ij} \geq 0, \quad \forall i, j. \end{array} \tag{5}$$

Conversely, an optimal Largrange multiplier vector is also a Fisher price equilibrium, which can be seen from the optimality conditions of (5):

$$\begin{array}{r} p_j - \sum_k \pi_i^k u_{ij}^k \geq 0,\ \forall i,j \\ \pi_i^k(\sum_{j\in P} u_{ij}^k x_{ij} - u_i) = 0,\ \forall i,k \\ x_{ij}(p_j - \sum_k \pi_i^k u_{ij}^k) = 0,\ \forall i,j \\ u_i \sum_k \pi_i^k = w_i,\ \forall i. \end{array} \tag{6}$$

for some p_j, the Largarange multiplier of equality constraint of $j \in P$, and some $\pi_i^k \geq 0$, the Largarange multiplier of inequality constraint of $i \in C$ and k. Summing the second constraint over k we have

$$w_i = \sum_k \pi_i^k u_i = \sum_k \pi_i^k \sum_{j\in P} u_{ij}^k x_{ij} = \sum_{j\in P}\left(x_{ij} \sum_k \pi_i^k u_{ij}^k \right),\ \forall i;$$

then summing the third constraint over j we have

$$\sum_{j\in P} p_j x_{ij} = \sum_{j\in P}\left(x_{ij} \sum_k \pi_i^k u_{ij}^k \right) = w_i.$$

This implies that x_i from the aggregate problem is feasible for (3). Moreover, note that π_i^k in (6) equals π_i^k/λ_i in (4). Thus, finding a Fisher price equilibrium is equivalent to finding an optimal Largrange multiplier of (5).

In particular, if each $u_i^k(x_i)$ has the Leontief utility form, i.e.,

$$u_i^k(x_i) = \frac{x_{ik}}{a_{ik}},\ \forall k = j \in P$$

for a given $a_{ik} > 0$. Then, upon using u_i to replace variable x_{ij}, the aggregated convex optimization problem can be simplified to

$$\begin{array}{ll} \text{maximize} & \sum_i w_i \log u_i \\ \text{subject to} & A^T u \leq e, \\ & u \geq 0; \end{array} \tag{7}$$

where the Leontief matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad \text{and variable vector} \quad u = \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_m \end{pmatrix}. \tag{8}$$

3 The Weighted Analytic Center Problem

In [26] the Eisenberg-Gale aggregated problem was related to the (linear) analytic center problem studied in interior-point algorithms

$$\begin{array}{ll} \text{maximize} & \sum_{j=1}^n w_j \log(x_j) \\ \text{subject to} & Ax = b, \\ & x \geq 0, \end{array} \tag{9}$$

where the given A is an $m \times n$-dimensional matrix with full row rank, b is an m-dimensional vector, and w_j is the nonnegative weight on the jth variable. Any x who satisfies the constraints is called a primal feasible solution, while any optimal solution to the problem is called a weighted analytic center.

If the weighted analytic center problem has an optimal solution, the optimality conditions are

$$\begin{aligned} Sx &= w, \\ Ax &= b, \;\; x \geq 0, \\ -A^T y + s &= 0, \;\; s \geq 0, \end{aligned} \tag{10}$$

where y and s are the Largrange or KKT multipliers or dual variable and slacks of the dual linear program:

$$\min \;\; b^T y \quad \text{subject to} \quad s = A^T y \geq 0,$$

and S is the diagonal matrix with slack vector s on its diagonals.

Let the feasible set of (9) be bounded and has a (relative) interior, i.e., has a strictly feasible point $x > 0$ with $Ax = b$ (clearly holds for problem (5) and (7)). Then, there is a strictly feasible dual solution $s > 0$ with $s = A^T y$ for some y. Furthermore, [26], based on the literature of interior-point algorithms (e.g., Megiddo and Kojima et al. [19, 18] and Güler [14]), has shown that

Theorem 1. *Let A, b be fixed and consider a solution $(x(w), y(w), s(w))$ of (10) as a mapping of $w \geq 0$ with $\sum_j w_j = 1$. Then,*

- *The mapping of $S^n_{++} = \{x > 0 \in R^n : \;\; e^T x = 1\}$ to $F_{++} = \{(x > 0, y, s > 0) : \;\; Ax = b, \;\; s = A^T y\}$ is one-to-one, continuously and differentiable.*
- *The mapping of $S^n_{+} = \{x \geq 0 \in R^n : \;\; e^T x = 1\}$ to $F_{+} = \{(x \geq 0, y, s \geq 0) : Ax = b, \;\; s = A^T y\}$ is upper semi-continuous.*
- *The pair $(x_j(w), s_j(w))$ is unique for any $j \in W = \{j : \;\; w_j > 0\}$, and*

$$x'_j(w) s''_j(w) = x''_j(w) s'_j(w) = 0, \;\; \forall j \notin W$$

and for any two solutions $(x'(w), y'(w), s'(w))$ and $(x''(w), y''(w), s''(w))$ of (10).

From this theorem, we see that, in the Fisher equilibrium problem (5) or (7), $u_i(w)$, the utility value of each consumer, is unique; but the price vector $p(w)$ can be non-unique.

In addition, a modified primal-dual path-following algorithm was developed in [26], for computing an ϵ-solution for any $\epsilon > 0$:

$$\begin{aligned} \|Sx - w\| &\leq \epsilon, \\ Ax &= b, \;\; x \geq 0, \\ -A^T y + s &= 0, \;\; s \geq 0. \end{aligned} \tag{11}$$

Theorem 2. *The primal-dual interior-point algorithm solves the weight analytic center problem (9) in $O(\sqrt{n} \log(n \max(w)/\epsilon))$ iterations and each iteration solves a system of linear equations in $O(nm^2 + m^3)$ arithmetic operations. If Karmarkar's rank-one update technique is used, the average arithmetic operations per iteration can be reduced to $O(n^{1.5} m)$.*

A rounding algorithm is also developed for certain types of problems possessing a rational solution, and the total iteration bound would be $O(\sqrt{n}L)$ and the average arithmetic operation bound would be $O(n^{1.5}m)$ per iteration, where L is the bit-length of the input data A, b, w. These results indicate, for the first time, that the complexity of the Fisher equilibrium problem with linear substitution utility functions is completely in line with linear programming of the same dimension and size.

4 The Arrow-Debreu Equilibrium Problem

The Arrow-Debreu exchange market equilibrium problem which was first formulated by Leon Walras in 1874 [24]. In this problem everyone in a population of m players has an initial endowment of a divisible good and a utility function for consuming all goods—their own and others. Every player sells the entire initial endowment and then uses the revenue to buy a bundle of goods such that his or her utility function is maximized. Walras asked whether prices could be set for everyone's good such that this is possible. An answer was given by Arrow and Debreu in 1954 [1] who showed that such equilibrium would exist if the utility functions were concave.

We consider a special class of Arrow-Debreu's problems, where each of the $m = n$ players have exactly one unit of a divisible good for trade (e.g., see [15, 26]), and let player i, $i = 1, ..., m$, bring good $j = i$ and have the linear substitution utility function of (1). We call this class of problems the pairing class. The main difference between Fisher's and Arrow-Debreu' models is that, in the latter, each player is both producer and consumer and the initial endowment w_i of player i is *not* given and will be the price assigned to his or her good i. Nevertheless, we can still write a (parametric) convex optimization model

$$\begin{array}{ll} \text{maximize} & \sum_i w_i \log u_i \\ \text{subject to} & \sum_i x_{ij} = 1, \quad \forall j, \\ & u_i \leq \sum_j u_{ij}^k x_{ij}, \quad \forall i, k, \\ & u_i,\ x_{ij} \geq 0, \quad \forall i, j, \end{array}$$

where we wish to select weights w_i's such that an optimal Largrange multiplier vector p equals w. It is easily seen that any optimal Largrange multiplier vector p satisfies

$$p \geq 0 \quad \text{and} \quad e^T p = e^T w.$$

For fixed u_{ij}^k, consider p be a map from w. Then, the mapping is from S_+^n to S_+^n, and the mapping is upper semi-continuous from Theorem 1. Thus, there is a $w \in S_+^n$ such that an Largrange multiplier vector $p(w) = w$ from the Kakutani fixed-point theorem (see, e.g., [22, 23, 25]). This may be seen as an alternative, restricted to the case of the linear substitution utility functions, to Arrow-Debreu's general proof of the existence of equilibria.

We now focus on the Arrow-Debreu equilibrium with the (complete) Leontief utility function:

$$u_i^k(x_i) = \frac{x_{ik}}{a_{ik}},\ \forall k = j = 1, ..., m$$

for a given $a_{ik} > 0$. Recall the parametric convex optimization model (7) where the Leontief matrix A of (8) is a $m \times m$ positive matrix. Let $p \in R^m$ be an optimal Largrange multiplier vector of the constraints. Then, we have

$$\begin{array}{ccc} u_i \sum_j a_{ij} p_j = w_i \ \forall i, & \text{and} & p_j(1 - \sum_i a_{ij} u_i) = 0 \ \forall j, \\ \sum_i a_{ij} u_i \leq 1 \ \forall j, & \text{and} & u_i, \ p_j \geq 0 \ \forall i, j. \end{array}$$

Thus, the Arrow-Debreu equilibrium $p \in R^m$, together with $u \in R^m$, satisfy

$$\begin{aligned} UAp &= p, \\ P(e - A^T u) &= 0, \\ A^T u &\leq e, \\ u, \ p &\geq 0, \end{aligned} \tag{12}$$

where U and P are diagonal matrices whose diagonal entries are u and p, respectively. The Arrow-Debreu theorem implies that nonzero p and u exist for this system of equalities and inequalities, even in general case where $A \geq 0$, that is, some $a_{ik} = 0$ in the Leontief matrix.

5 Characterization of an Arrow-Debreu Equilibrium

If $u_i > 0$ at a solution $(u, p \neq 0)$ of system (12), we must have $p_i > 0$, that is, player i's good must be priced positively in order to have a positive utility value. On the other hand, $p_i > 0$ implies that $\sum_k^m a_{ki} u_k = 1$, that is, good i must be all consumed and gone. Conversely, if $p_i > 0$, we must have $u_i > 0$, that is, player i's utility value must be positive. Thus, there is a partition of all players (or goods) such that

$$B = \{i : \ p_i > 0\} \quad \text{and} \quad N = \{i : \ p_i = 0\}$$

where the union of B and N is $\{1, 2, ..., m\}$. Then, (u, p) satisfies

$$\begin{aligned} (U_B A_{BB}) p_B &= p_B, \\ A_{BB}^T u_B &= e, \\ A_{BN}^T u_B &\leq e, \\ u_B, \ p_B &> 0. \end{aligned}$$

Here A_{BB} is the principal submatrix of A corresponding to the index set B, A_{BN} is the submatrix of A whose rows in B and columns in N. Similarly, u_B and p_B are subvectors of u and p with entries in B, respectively.

Since the scaled Leontief matrix $U_B A_{BB}$ is a (column) stochastic matrix (i.e., $e^T U_B A_{BB} = e^T$), p_B must be the (right) Perron-Frobenius eigen-vector of $U_B A_{BB}$. Moreover, A_{BB} is irreducible because $U_B A_{BB}$ is irreducible and $u_B > 0$, and $U_B A_{BB}$ is irreducible because $p_B > 0$. To summarize, we have

Theorem 3. *Let $B \subset \{1, 2, ..., n\}$, $N = \{1, 2, ..., n\} \setminus B$, A_{BB} be irreducible, and u_B satisfy the linear system*

$$A_{BB}^T u_B = e, \quad A_{BN}^T u_B \leq e, \quad \textit{and} \quad u_B > 0.$$

Then the (right) Perron-Frobenius eigen-vector p_B of $U_B A_{BB}$ together with $p_N = 0$ will be an Arrow-Debreu equilibrium. And the converse is also true. Moreover, there is always a rational Arrow-Debreu equilibrium for every such B, that is, the entries of price vector are rational numbers, if the entries of A are rational. Furthermore, the size (bit-length) of the equilibrium is bounded by the size of A.

Our theorem implies that the players in block B can trade among them self and keep others goods "free." In particular, if one player likes his or her own good more than any other good, that is, $a_{ii} \geq a_{ij}$ for all j. Then, $u_i = 1/a_{ii}$, $p_i = 1$, and $u_j = p_j = 0$ for all $j \neq i$, that is, $B = \{i\}$, makes an Arrow-Debreu equilibrium. The theorem thus establishes, for the first time, a combinatorial algorithm to compute an Arrow-Debreu equilibrium with Leontief's utility by finding a right block $B \neq \emptyset$, which is actually a non-trivial complementarity solution to a *linear complementarity problem* (LCP)

$$A^T u + v = e, \ u^T v = 0, \ 0 \neq u, v \geq 0; \tag{13}$$

see, e.g., [5] for more references on LCP. If $A > 0$, then any complementarity solution $u \neq 0$ and $B = \{j : \ u_j > 0\}$ of (13) induce an equilibrium that is the (right) Perron-Frobenius eigen-vector of $U_B A_{BB}$, and it can be computed in polynomial time by solving a linear equation. If A is not strictly positive, then any complementarity solution $u \neq 0$ and $B = \{j : \ u_j > 0\}$, as long as A_{BB} is irreducible, induces an equilibrium. The equivalence between the pairing Arrow-Debreu model and the LCP also implies that LCP (13) always has a complementarity solution $u \neq 0$ such that A_{BB} is irreducible where $B = \{j : \ u_j > 0\}$.

The pairing class of Arrow-Debreu's problems is a rather restrictive class of problems. Consider a general supply matrix $0 \leq G \in R^{m \times n}$ where row i of G represents the multiple goods brought to the market by player i, $i = 1, ..., m$. The pairing model represents the case that $G = I$, the identity matrix, or $G = P$ where P is any permutation matrix of $m \times m$.

What to do if one player brings two different goods? One solution is to copy the same player's utility function twice and treat the player as two players with an identical Leontief utility function, where each of them brings only one type of good. Then, the problem reduces to the pairing model. Thus, we have

Corollary 1. *If all goods are different from each other in the general Arrow-Debreu problem with Leontief's utility, i.e., each column of $G \in R^{m \times n}$ has exactly one positive entry, then there is always a rational equilibrium, that is, the entries of a price vector are rational numbers.*

Now what to do if two players bring the same type of good? In our present pairing class, they will be treated as two different goods, and one can set the same utility coefficients to them so that they receive an identical appreciation from all the players. Again, the problem reduces to the pairing class, which leads to rationality. The difference is that now these two "same" goods may receive two different prices; for example, one is priced higher and the other is at a discount level. I guess this could happen in the real world since two "same" goods may not be really the same and the market does have "freedom" to price them.

6 An Illustrative Example

The rationality result is interesting since the existence of a rational equilibrium is not true for Leontief's utility in Fisher's model with rational data, see the following example converted in Arrow-Debreu's setting, with three consumers each of whom has 1 unit money (the first good) and two other goods (the second and third) brought by a seller (the fourth player) who is only interested in money, adapted from Codenotti et al. [3] and Eaves [9].

$$A = \begin{pmatrix} 0 & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 \\ 0 & \frac{1}{4} & \frac{1}{5} \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad G = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

There is a unique equilibrium for this example, where the utility values of the three consumers are $u_1^* = \frac{2}{3\sqrt{3}}$, $u_2^* = \frac{1}{3} + \frac{1}{3\sqrt{3}}$, $u_3^* = \frac{10}{3} - \frac{10}{3\sqrt{3}}$, and the utility value of the seller $u_4^* = 3$. The equilibrium price for good 1 (money) is $p_1^* = 1$, and for other two goods are $p_2^* = 3(\sqrt{3} - 1)$, and $p_3^* = 3(2 - \sqrt{3})$.

However, if we treat the money from each consumer differently, that is, let

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{5} \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

Then, there are multiple rational equilibria and here are a few:

1. $B = \{1, 4\}$, with $u_1^* = u_4^* = 1$ and $p_1^* = p_4^* = 1$, and $u_2^* = u_3^* = p_2^* = p_3^* = p_5^* = 0$;
2. $B = \{2, 5\}$, with $u_2^* = u_4^* = 1$ and $p_2^* = p_5^* = 1$, and $u_1^* = u_3^* = p_1^* = p_3^* = p_4^* = 0$;
3. $B = \{3, 4\}$, with $u_3^* = 4$ and $u_4^* = 1$ and $p_3^* = p_4^* = 1$, and $u_1^* = u_2^* = p_1^* = p_2^* = p_5^* = 0$;
4. $B = \{1, 2, 3, 4, 5\}$, with an equilibrium $u_1^* = \frac{11}{30}$, $u_2^* = \frac{31}{60}$, $u_3^* = \frac{3}{2}$, $u_4^* = 1$, $p_1^* = \frac{66}{80}$, $p_2^* = \frac{93}{80}$, $p_3^* = \frac{81}{80}$ and $p_4^* = p_5^* = \frac{3}{2}$, .

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A Primal-Dual Algorithm for Computing Fisher Equilibrium in the Absence of Gross Substitutability Property

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Abstract. We provide the first strongly polynomial time exact combinatorial algorithm to compute Fisher equilibrium for the case when utility functions do not satisfy the Gross substitutability property. The motivation for this comes from the work of Kelly, Maulloo, and Tan [15] and Kelly and Vazirani [16] on rate control in communication networks. We consider a tree like network in which root is the source and all the leaf nodes are the sinks. Each sink has got a fixed amount of money which it can use to buy the capacities of the edges in the network. The edges of the network sell their capacities at certain prices. The objective of each edge is to fix a price which can fetch the maximum money for it and the objective of each sink is to buy capacities on edges in such a way that it can facilitate the sink to pull maximum flow from the source. In this problem, the edges and the sinks play precisely the role of sellers and buyers, respectively, in the Fisher's market model. The utility of a buyer (or sink) takes the form of Leontief function which is known for not satisfying Gross substitutability property. We develop an $O(m^3)$ exact combinatorial algorithm for computing equilibrium prices of the edges. The time taken by our algorithm is independent of the values of sink money and edge capacities. A corollary of our algorithm is that equilibrium prices and flows are rational numbers. Although there are algorithms to solve this problem but they are all based on convex programming techniques. To the best of our knowledge, ours is the first strongly polynomial time exact combinatorial algorithm for computing equilibrium prices of Fisher model under the case when buyers' utility functions do not satisfy Gross substitutability property.

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1 Introduction

The study of *market equilibrium* has occupied central stage within both positive (or descriptive) and normative (or prescriptive) economics. The notion of market equilibrium was first proposed by Walras [19] This equilibrium is popularly known as *competitive or Walrasian* equilibrium. Contemporary to Walras, Irving Fisher [18] also independently modeled the market equilibrium in 1891. However, Fisher's model turns out to be the special case of Walras' model. Fisher's market model assumes that there are two kinds of traders in the market: buyers and sellers who trade over a finite set of commodities. Buyers have money and utility functions for goods. Sellers have initial endowment of goods and want to earn money. The equilibrium prices are defined as assignment of prices to goods, so that when every consumer buys an optimal bundle then market clears i.e. all the money is spent and all the goods are sold. If money is also considered as a commodity then it is easy to see that Fisher model is a special case of Walras model.

The existence of market equilibrium is a deeply investigated problem. It is the seminal work of Arrow and Debreu [1] which proved the existence of competitive market equilibria under quite general setting of concave utility functions by applying Kakutani's fixed point theorem. The proof outlined by Arrow-Debreu is highly non-constructive in nature and, therefore, the natural question down the line is the existence of an efficient computation process which establishes equilibrium. In this paper, we provide the first strongly polynomial time exact combinatorial algorithm to compute Fisher equilibrium for the case when utility functions do not satisfy the Gross substitutability property. The time complexity of our algorithm is independent of the values of input data. We also show that output of our algorithm consists of only rational numbers.

2 Related Work

The recent papers by Papadimitriou [17] and Deng et al. [3] have raised and partly answered the question of efficient computability of Walrasian equilibrium. The problem of developing an efficient algorithm for computing Walrasian equilibrium has been addressed from many perspectives. Some of the algorithms are based on solving nonlinear convex programs [13, 20], following the classical approach of Eisenberg and Gale [7]. The other known algorithms are primal-dual-type, initiated by Devanur et al. [4], and auction-based introduced by Garg et al. [9].

For the special case of linear utility functions, Deng et al [3] first developed a polynomial-time algorithm for the case when the number of goods or agents is bounded. Devanur et al [4] have proposed a polynomial-time algorithm via a primal-dual type approach for Fisher's equilibrium under liner utility - a special case of Walrasian equilibrium. [5] et al have further proposed an improved approximation scheme for the same problem. Jain, Mahadian, and Saberi [12] have proposed a fully polynomial time approximation algorithm to compute Walrasian equilibrium for the case when utility functions satisfy Gross substitutability property. Garg et al. [9] have also proposed a fully polynomial time approximation algorithm for the same problem but they use auction based approach. Jain [13] gives a convex program for the linear version of Walrasian model and uses the ellipsoid algorithm and diophantine approximation to obtain a polynomial time algorithm for this case as well, thereby settling the open problem of computing the market equilibria efficiently under the scenario when utilities are linear. However, designing efficient market equilibrium algorithms for general convex utility functions is still an open problem.

The problem of computing the market equilibrium with linear utilities is comparatively simpler because they satisfy the gross substitutability, i.e., increasing the price of one good cannot decrease the demand for another. Hence for such utility functions, monotonically raising prices suffices. In contract, concave and even piecewise-linear and concave, utility functions do not satisfy gross substitutability, hence requiring the more involved process of increasing and decreasing prices. Therefore, designing market equilibrium algorithms for them remains an outstanding open problem.

3 Fisher Equilibrium and Gross Substitutability

Consider a market consisting of n buyers and m divisible goods. Buyer i has, initially, a positive amount of money m_i. The amount of good j available in the market is c_j. Let for buyer i, $X_i \subset \Re_+^m$ represents the consumption set, i.e., the set of bundles of m goods which buyer i can consume. Let $u_i : X_i \mapsto \Re$ be the utility function for buyer i. Given the prices $p_1, \ldots, p_m$, it is easy to compute the bundle $x_i \in X_i$ which will maximize buyer i's utility subject to his budget constraint. The prices $p_1, \ldots, p_m$ are said to be *Fisher or market equilibrium prices* if after each buyer is assigned such an optimal bundle, there is no surplus or deficiency of any goods. If x_{ij}^* denotes the amount of good j bought by buyer i at prices $p_1^*, \ldots, p_m^*$, then it can be verified [1] that this price vector $p_1^*, \ldots, p_m^*$ is Fisher equilibrium iff it satisfies the following conditions:

- $\left(\sum_{i=1}^n x_{ij}^* - c_j\right) \le 0 \ \forall j = 1, \ldots, m$;
- $p_j^* \left(\sum_{i=1}^n x_{ij}^* - c_j\right) = 0 \ \forall j = 1, \ldots, m$
- x_i^* maximizes $u_i(x_i)$ over the set $\left\{x_i \in X_i | \sum_{j=1}^m x_{ij} p_j^* \le m_i\right\}$
- $p_j^* \ge 0 \ \forall j = 1, \ldots, m$

The Arrow-Debreu theorem [1] says that if the utility functions $u_i(.)$ are concave then such an equilibrium price vector always exists. It is easy to see that in equilibrium, each buyer must spend his full budget.

Gross substitutability is a well-studied property [10] that has useful economic interpretation. Goods are said to be Gross substitutes for a buyer iff increasing the price of a good does not decrease the buyer's demand for other goods. Note that, whether goods are Gross substitutes or not for a given buyer i depends solely on his own utility function $u_i(x_i)$. It can be shown that not all concave utility functions satisfy this property. Computing Fisher equilibrium when the buyers utility functions do not satisfy Gross substitutability property is far more difficult problem than the case when they satisfy this property. A frequently arising utility functions that do not satisfy this property are Leontief utility function. A Leontief utility function for buyer i in Fisher's model is something like this: $u_i(x_i) = \min_j(x_{ij})$. We will be using this utility function in our problem.

4 Problem Statement

The precise problem we solve is the following: Let $T = (V, E)$ be a tree with integer capacities on edges. Let root of the tree be a source node and $T = \{t_1, \ldots, t_k\}$ be the sink nodes. Without loss of generality, we can assume that each sink t_i is the leaf node and conversely each leaf node is a sink.[1] The sinks have budgets $m_1, \ldots, m_k$, respectively. Each sink can use its budget to buy the capacities of the edges in the network. The edges of the network sell their capacities at certain prices. The objective of each edge is to fix a price which can fetch maximum money for it and, at the same time, objective of each sink is to buy capacities on edges in such a way that it can facilitate the sink to pull maximum flow from the source. In order to map this problem to Fisher's market model, we view edges as the sellers who are trying to sell their capacities and sinks as buyers who are trying to buy the capacities on the edges and whose Leontief utilities are given by $u_i(x_i) = \min_{j|j \in P_i}(x_{ij})$, where x_{ij} is amount of the capacity bought by sink t_i on edge j and P_i is the collection of edges that forms a unique path from source to sink t_i. The problem is to determine Fisher equilibrium prices for all edges and flows from the source to the sinks. It is easy to verify from previous section that edge prices $p_e \forall e \in E$ and flows f_i from source to each sink t_i, form Fisher equilibrium iff they satisfy the following conditions:

1. $f_e \leq c_e \forall e \in E$
2. For any edge $e \in E$, if $p_e > 0$, then $f_e = c_e$
3. For any edge $e \in E$, if $f_e < c_e$, then $p_e = 0$
4. $f_i = \frac{m_i}{\left(\sum_{e|e \in P_i} p_e\right)}$
5. $p_e \geq 0 \forall e \in E$, $f_i \geq 0 \forall t_i \in T$

where c_e and f_e are the capacity and total flow, respectively for the edge $e \in E$. The P_i is the set of edges which forms a unique path in the tree T from source to the sink t_i. For each edge $e \in E$, the total flow f_e is given by flow conservation equation $f_e = \sum_{t_i|e \in P_i} f_i$. Note that the first condition corresponds to capacity constraint, the second, third, and fourth conditions correspond to three equilibrium conditions, and the fifth condition is obviously a nonnegativity constraint. Also note that in view of the first condition, the second condition can be relaxed slightly and f_e=c_e can be replaced by $f_e \geq c_e$.

[1] If a sink t_i is an internal node then we can always add an additional leaf edge of infinity capacity at that particular internal node and push the sink t_i to this newly generated leaf node. Similarly, if a leaf node is not a sink then we can just remove the corresponding leaf edge from the tree.

5 Convex Programs and Equilibrium

It is interesting to see that the problem of computing market equilibrium that we sketched in previous section is captured by following Eisenberg-Gale type convex program that maximizes sum of logarithms of flows, weighted by budgets, subject to capacity constraints on the flows.

$$\begin{aligned} \text{Maximize} \quad & \sum_{t_i \in T} m_i \log f_i \qquad (1) \\ \text{subject to} \quad & \sum_{t_i | e \in P_i} f_i \le c_e \ \forall e \in E \\ & f_i \ge 0 \ \forall t_i \in T \end{aligned}$$

Let p_e's be the dual variables (also called Lagrange multipliers) corresponding to the first set of constraints; we will interpret these as prices of edges. Using KKT conditions, one can show that f_i's and p_e's form an optimal solution to primal and dual problems, respectively, iff they satisfy:

1. Primal Feasibility:
$\sum_{t_i|e\in P_i} f_i \le c_e \ \forall e \in E$
$f_i \ge 0 \ \forall t_i \in T$
2. Dual Feasibility:
$p_e \ge 0 \ \forall e \in E.$
3. Complementary Slackness:
$p_e(c_e - \sum_{t_i|e\in P_i} f_i) = 0 \ \forall e \in E$
4. Lagrange Optimality:
$f_i = \frac{m_i}{\left(\sum_{e|e\in P_i} p_e\right)} \ \forall t_i \in T$

It is easy to verify that above conditions are precisely the same as the six conditions mentioned in the previous section which must be satisfied by the output of our algorithm. Thus, we basically develop a strongly polynomial algorithm for this problem. Our primal variables are flows and dual variables are edge prices. We observe that equilibrium prices are essentially unique in the following sense: for each sink t_i, the cost of the path from source to the sink is the same in all equilibria. We call the cost of the path from source to the sink t_i as price(t_i) and it is given by $\sum_{e|e\in P_i} p_e$. A corollary of our algorithm is that equilibrium prices and flows are rational numbers. We show that this does not hold even if there are just two sources in the tree.

Eisenberg [6] generalized the Eisenberg-Gale convex program to homothetic utility functions (and thus to Leontief). An approach based on Eisenberg's result [6] was used by Codenotti et al. [2] to address the same problem that we are solving here.[2]

6 The Algorithm

Here is a high level description of our algorithm: We start with zero prices of the edges and iteratively change prices. Since prices are zero initially, all the sinks draw infinity flows and hence all the edges are over-saturated; we iteratively decrease the number of such edges. We maintain the following invariants throughout our algorithm

[2] We were not aware of these references while preparing the manuscript. Our special thanks to the referee for point out these important references.

I1. For any edge $e \in E$, if $p_e > 0$, then $f_e \geq c_e$
I2. For any edge $e \in E$, if $f_e < c_e$, then $p_e = 0$
I3. $f_i = \frac{m_i}{\left(\sum_{e|e \in P_i} p_e\right)} \forall t_i \in T$
I4. $p_e \geq 0 \forall e \in E$, $f_i \geq 0 \forall t_i \in T$
I5. $f_e = \sum_{t_i|e \in P_i} f_i \forall e \in E$

Thus, we maintain dual feasibility, complementary slackness, and Lagrange optimality conditions throughout the algorithm. The only condition that is not maintained is primal feasibility. Thus, our algorithm would terminate at the point where primal feasibility is attained. At such point, all KKT conditions would have been met and the current values of f_i and p_e would be the desired solution. At any instant during the course of the algorithm, each edge would be marked either red or green. If an edge e is marked as green then it means that primal feasibility condition $f_e \leq c_e$ is being satisfied at that edge. Initially, all the edges are red and the subroutine **make-green** converts at least one red edge into green. Later we will show that this algorithm also maintains the following invariants.

I6. The parameter price(t_i) to each sink t_i is non-decreasing throughout the algorithm, and flows f_i are non-increasing.
I7. At any instant, the price of a red edge is zero and the set of red edges forms a subtree containing the root of the original tree.

6.1 Feasible Flow

We will say that vertex u is *lower than* v if there is a path from v to u in T. Similarly edge (u, v) is *lower than* (a, b) if there is a path from b to u in T.

Consider a partition $\mathcal{P}$ of E into *red* and *green* edges. Let p_e be the current assignment of prices to the edges. Let this partition-price pair $(\mathcal{P}, p)$ satisfies the invariant **I7**. A flow f is said to be *feasible* for this partition-price pair $(\mathcal{P}, p)$ if it satisfies the five invariant conditions **I1** through **I5** mentioned earlier.

We present a subroutine below called **make-green**. Given a partition-price pair $(\mathcal{P}, p)$ which satisfies the invariant **I7**, and a feasible flow for this partition-price pair, f, **make-green** converts at least one red edge into green. It returns the new partition, new prices for edges, and a feasible flow for the new partition-price pair. The partition and the new edge prices returned by **make-green** respect the invariant **I7**. Subroutine **make-green** accomplishes this in $O(m^2)$ computations (see Section 7). **make-green** accomplishes this via an involved process that increases and decreases prices on edges. Clearly, $O(m)$ calls of **make-green** suffice. The lemmas in Section 7 lead to:

Theorem 1. *$O(m^3)$ computations suffice to find an optimal solution to convex program (1) and the corresponding equilibrium prices and flows for trees. Furthermore, such trees always admit a rational solution.*

Lemma 1. *The time taken by the algorithm does not depend on actual values of sink money and edge capacities and therefore the algorithm is strongly polynomial.*

6.2 Subroutine Make-Green

Subroutine **make-green** works iteratively. It starts by picking a topologically lowest red edge (u, v), i.e., all edges lower than (u, v) are green. If in the given feasible flow f, the edge (u, v) carries a flow less than or equal to its capacity then turn this edge into green. Otherwise, let Z be the set of vertices reachable from v by following green zero price edges. Z is called a *zero component* of the edge (u, v). Let A be the set of edges both whose end points are in Z. Let B be the set of edges incident on Z, excluding the edge (u, v). Clearly, all edges in A must be green and must have zero prices and all edges in B must be green and must have positive prices. Let $T_1 \subset T$ be the set of sinks that are sitting inside the zero component Z, $T_2 \subset T$ be the set of sinks that are sitting outside the zero component Z but are lower than vertex v, and $T_3 \subset T$ be the set of remaining sinks. Note that it is quite possible that T_1 is an empty set.

Now the idea is following. Increase the price of the red edge (u, v) and decrease the price of each edge in the set B by the same amount. This will result in price(t_i) to be undisturbed for all the sinks in the set T_2 as well as in the set T_3. However, this would decrease the price$(t_i)$for all the sinks in the set T_1. After changing these prices we would recompute the flows f_i for each sink by using the formula given by invariant **I3**. Note that f_i would remain unchanged for all the sinks in the set T_2 as well as in the set T_3. However, f_i would decrease for all the sinks in the set T_1. Thus, the flow on edge (u, v) may decrease.

The obvious question now is how much to increase the price of red edge (u, v). Note that during this process of increasing the price of edge (u, v), any one of the following two events may occur.

E1: The flow on edge (u, v) hits its capacity
E2: The price of some edge in the set B goes to zero.

If the event **E1** occurs then turn the red edge (u, v) into green and this will give a new partition with an extra green edge, the new prices for edges, and a feasible flow for the new partition-price pair. If the event **E2** occurs then we freeze the price of edge (u, v) and the edges in the set B at their current values. Now we need to reconstruct the zero component Z and the sets A, B, T_1, T_2, and T_3. We now repeat the process of increasing the price of red edge (u, v) from its current value. In order to identify which one of these two events has occurred, we maintain the following quantities:

$$m_{in} = \sum_{t_i \in T_1} m_i, \quad f_{green} = \sum_{t_i \in T_2} f_i, \quad p_{min} = \min\left(p_e | e \in B\right)$$

Now we distinguish between two cases:

- **Case 1:** $(f_{green} > c_{(u,v)})$
  It is easy to see that if this is the case then next event would be **E2**.
- **Case 2:** $(f_{green} \leq c_{(u,v)})$
  Under this case, it is east to verify that if $\frac{m_{in}}{C_{(u,v)} - f_{green}} - p_{(u,v)} \leq p_{min}$ then event **E1** will occur, otherwise event **E2** will occur.

Algorithm 1 gives the pseudo code for the subroutine **make-green**.

Algorithm 1. Subroutine make-green

Procedure $make - green(V, E, m, mark[.], f, p)$:
1: find topologically lowest edge e such that $mark[e] = red$
2: **if** $f_e \leq c_e$ **then**
3: $mark[e] \leftarrow green$
4: return
5: **else**
6: construct zero component Z, sets B, T_1, and T_2 for the edge e
7: $m_{in} \leftarrow \sum_{t_i \in T_1} m_i$
8: $f_{green} \leftarrow \sum_{t_i \in T_2} f_i$
9: $p_{min} \leftarrow \min(p_e | e \in B)$
10: **if** $f_{green} > c_e$ **then**
11: $p_{e'} \leftarrow p_{e'} - p_{min} \; \forall e' \in B$
12: $p_e \leftarrow p_e + p_{min}$
13: $f_{t_i} \leftarrow \frac{m_i}{p_e} \; \forall t_i \in T_1$
14: Go to line number 6
15: **else if** $\left(\frac{m_{in}}{C_e - f_{green}} - p_e\right) \leq p_{min}$ **then**
16: $p_{e'} \leftarrow p_{e'} - \left(\frac{m_{in}}{C_e - f_{green}} - p_e\right) \; \forall e' \in B$
17: $p_e \leftarrow \frac{m_{in}}{C_e - f_{green}}$
18: $f_{t_i} \leftarrow \frac{m_i}{p_e} \; \forall t_i \in T_1$
19: $mark[e] \leftarrow green$
20: return
21: **else**
22: Go to line number 12
23: **end if**
24: **end if**

7 Analysis

In this section we state key observations and facts pertaining to the algorithm. We have omitted the proofs due to paucity of space. However, most of the proofs are quite straightforward and follow directly from the way we have structured the algorithm.

Lemma 2. *If an edge e becomes green in an iteration then it will remain green in all the future iterations.*

Corollary 1. *The number of iterations executed by subroutine* **make-green** *is bounded by $O(m)$ and its running time is bounded by $O(m^2)$. The running time of* **make-green** *is independent of actual values of sink money and edge capacities.*

Lemma 3. *The invariant* **I7** *is maintained throughout the algorithm.*

Theorem 2. *The flow maintained by the algorithm is feasible.*

Theorem 3. *The prices p and flow f attained by the algorithm at its termination are equilibrium prices and flow.*

Lemma 4. *The invariant* **I6** *is maintained throughout the algorithm.*

7.1 Rational Solution

The results in this section highlights the fact that the solution given by our algorithm consists of only rational numbers.

Theorem 4. *As long as all the sinks in the tree draw their flow from single source, the convex program (1) has a rational solution.*

Lemma 5. *Even if there are multiple equilibria for convex program (1), the path price for each sink t_i , i.e. price(t_i), is unique.*

Lemma 6. *The path price for each sink t_i , i.e. price(t_i), is a rational number.*

7.2 Multiple Sources and Irrational Solution

If there are multiple sources present in the tree then it may give rise to irrational equilibrium prices and flows. For example, consider a tree on three nodes, $\{a, b, c\}$ and two edges $\{ab, bc\}$. Let the capacity of (a, b) be one unit and the capacity of (b, c) be two units. The source sink pairs together with their budgets are: $(a, b, 1)$, $(a, c, 1), (b, c, 1)$. Then the equilibrium price for ab is $\sqrt{3}$ and for bc it is $\frac{\sqrt{3}}{1+\sqrt{3}}$.

8 Discussion

The primal-dual schema has been very successful in obtaining exact and approximation algorithms for solving linear programs arising from combinatorial optimization problems. [4] and our paper seem to indicate that it is worthwhile applying this schema to solving specific classes of nonlinear programs. There are several interesting convex programs in the Eisenberg-Gale family itself, see [14] and the references therein. Another family of nonlinear programs deserving immediate attention is semidefinite programs. Considering the large running time required to solve such programs, it will be very nice to derive a combinatorial approximation algorithm for MAX CUT for instance, achieving the same approximation factor as [11].

Extending our algorithm to handling arbitrary directed cyclic graphs is another challenging open problem. Also interesting will be to obtain approximation algorithms for the cases where the solution is irrational. Another interesting question is to obtain an auction-based algorithm for tree (or acyclic graphs) along the lines of [8]. Such an algorithm will be more useful in practice than our current algorithm.

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Click Fraud Resistant Methods for Learning Click-Through Rates

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Abstract. In pay-per-click online advertising systems like Google, Overture, or MSN, advertisers are charged for their ads *only* when a user clicks on the ad. While these systems have many advantages over other methods of selling online ads, they suffer from one major drawback. They are highly susceptible to a particular style of fraudulent attack called *click fraud*. Click fraud happens when an advertiser or service provider generates clicks on an ad with the sole intent of increasing the payment of the advertiser. Leaders in the pay-per-click marketplace have identified click fraud as the most significant threat to their business model. We demonstrate that a particular class of learning algorithms, called *click-based algorithms*, are resistant to click fraud in some sense. We focus on a simple situation in which there is just one ad slot, and show that fraudulent clicks can not increase the expected payment per impression by more than $o(1)$ in a click-based algorithm. Conversely, we show that other common learning algorithms are vulnerable to fraudulent attacks.

1 Introduction

The Internet is probably the most important technological creation of our times. It provides many immensely useful services to the masses for free, including such essentials as web portals, web email and web search. These services are expensive to maintain and depend upon advertisement revenue to remain free. Many services such as Google, Overture, and certain components of MSN, generate advertisement revenue by selling clicks. In these *pay-per-click* systems, an advertiser is charged only when a user clicks on his ad.

A scenario of particular concern for service providers and advertisers in pay-per-click markets is *click fraud* – the practice of gaming the system by creating fraudulent clicks, usually with the intent of increasing the payment of the advertiser. As each click can cost on the order of \$1, it does not take many fraudulent clicks to generate a large bill. Just a million fraudulent clicks, generated perhaps by a simple script, can cost the advertiser \$1,000,000, easily exhausting his budget. Fraudulent behavior threatens the very existence of the pay-per-click advertising market and has consequently become a subject of great concern [5, 7, 8]. Recently, Google CFO George Reyes said, in regards to the click fraud problem, that "I think something has to be done about this really, really quickly, because I think, potentially, it threatens our business model." [8].

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A variety of proposals for reducing click fraud have surfaced. Most service providers currently approach the problem of click fraud by attempting to automatically recognize fraudulent clicks and discount them. Fraudulent clicks are recognized by machine learning algorithms, which use information regarding the navigational behavior of users to try and distinguish between human and robot-generated clicks. Such techniques require large datasets to train the learning methods, have high classification error, and are at the mercy of the "wisdom" of the scammers. Recent tricks, like using cheap labor in India to generate these fraudulent clicks [9], make it virtually impossible to use these machine learning algorithms.

Another line of proposals attempts to reduce click fraud by removing the incentives for it. Each display of an ad is called an *impression*. Goodman [2] proposed selling advertisers a particular *percentage* of all impressions rather than user clicks. Similar proposals have suggested selling impressions. For a click-through-rates of 1%, the expected price per impression in the scenario mentioned above is just one cent. Thus, to force a payment of $1,000,000 upon the advertiser, 100,000,000 fraudulent impressions must be generated versus just 1,000,000 fraudulent clicks in the pay-per-click system. When such large quantities of fraud are required to create the desired effect, it ceases to be profitable to the scammer.

Although percentage and impression based proposals effectively eliminate fraud, they suffer from three major drawbacks. First, the developed industry standard sells clicks, and any major departure from this model risks a negative backlash in the marketplace. Second, by selling clicks, the service provider subsumes some of the risk due to natural fluctuations in the marketplace (differences between day and night or week and weekend, for example). Third, by requesting a bid per click, the service provider lessons the difficulty of the strategic calculation for the advertiser. Namely, the advertiser only needs to estimate the worth of a click, an arguably easier task than estimating the worth of an impression.

In this paper, we attempt to eliminate the incentives for click fraud in a system that sells clicks. We focus on a common pay-per-click system, generally believed to be used by Google [3] among others, which has been shown empirically to have higher revenue [1, 4] than other pay-per-click systems like that of Overture [6]. This system is based on estimates of the *click-through rate* (CTR) of an ad. The CTR is defined as the likelihood, or probability, that an impression of an ad generates a click. In this system, each advertiser submits a bid which is the maximum amount the advertiser is willing to pay per click of the ad. The advertisers are then ranked based on the product of their bids and respective estimated CTRs of their ads. This product can be interpreted as an expected bid per impression. The ad space is allocated in the order induced by this ranking. Advertisers are charged *only* if they receive a click, and they are charged an amount inversely proportional to their CTR.

In pay-per-click systems, when a fraudulent click happens, an advertiser has to pay for it, resulting in a short term loss to the advertiser whose ad is being clicked fraudulently. However, in the system described above, there is a long term benefit too. Namely, a fraudulent click will be interpreted as an increased likelihood of a future click and so result in an increase in the estimate of the CTR. As the payment is inversely proportional to the CTR, this results in a reduction in the payment. If the short term loss and the long term benefit exactly cancel each other, then there will be less incentive to generate fraudulent clicks; in fact, a fraudulent click or impression will only cost the advertiser as much as a fraudulent impression in a pay-per-impression scheme. Whether this happens depends significantly on how the system estimates the CTRs. There are a variety of sensible algorithms for this task. Some options include taking the fraction of all impressions so far that generated a click, or the fraction of impressions in the last hour that generated a click, or the fraction of the last hundred impressions that generated a click, or the inverse of the number of impressions after the most recent click, and so on.

We prove that a particular class of learning algorithms, called *click-based* algorithms, have the property that the short term loss and long term benefit in fact cancel. Click-based algorithms are a class of algorithms whose estimates are based upon the number of impressions between clicks. To compute the current estimate, a click-based algorithm computes a weight for each impression based solely on the number of clicks after it and then takes the weighted average. An example of an algorithm in this class is the one which outputs an estimate equal to the reciprocal of the number of impressions before the most recent click. We show that click-based algorithms satisfying additional technical assumptions are *fraud-resistant* in the sense that a devious user can not change the expected payment of the advertiser per impression (see Section 3 for a formal definition). We provide an example that a traditional method for estimating CTR (that is, taking the average over a fixed number of recent impressions) is not fraud-resistant.

The structure of this paper is as follows. In Section 2, we describe the setting. In Section 3, we define a very general class of algorithms for learning the CTR of an ad, called CTR learning algorithms. In Section 4, we define a special class of these algorithms, called click-based algorithms, and prove that they are fraud-resistent. In Section 5, we give examples showing that other common algorithms for learning the CTR are not fraud-resistent.

2 Setting

We consider a simple setting in which a service provider wishes to sell space for a single ad on a web page. There are a number of advertisers, each of whom wishes to display their ad on the web page. The service provider sells the ad space according to the pay-per-click model and through an auction: the advertiser whose ad is displayed is charged only when a user clicks on his ad. Each advertiser i submits a bid b_i indicating the maximum amount he is willing to pay the service provider when a user clicks on his ad. The allocation and price is computed using the mechanism described below.

For each ad, the service provider estimates the probability that the ad receives a click from the user requesting the page, if it is displayed. This probability is called the *click-through-rate (CTR)* of the ad. Each bid b_i is multiplied by the estimate λ_i of the CTR of the ad. The product $\lambda_i b_i$ thus represents the expected willingness-to-pay of advertiser i per impression. The slot is awarded to the advertiser i^* with the highest value of $\lambda_i b_i$. If the user indeed clicks on the ad, then the winning advertiser is charged a price equal to the second highest $\lambda_i b_i$ divided by his (that is, the winner's) estimated CTR (that is, λ_{i^*}). Thus, if we label advertisers such that $\lambda_i b_i > \lambda_{i+1} b_{i+1}$, then the slot is awarded to advertiser 1 and, upon a click, he is charged a price $\lambda_2 b_2 / \lambda_1$. In this paper, we study the mechanism over a period of time during which the same advertiser wins the auction, and the value of $\lambda_2 b_2$ does not change. If the advertisers do not change their bids too frequently and $\lambda_1 b_1$ and $\lambda_2 b_2$ are not too close to each other, it is natural to expect this to happen most of the time. We will henceforth focus on the winner of the auction, defining $p := \lambda_2 b_2$ and $\lambda := \lambda_1$.

3 CTR Learning Algorithms

Of course, we have left unspecified the method by which the algorithm learns the CTRs. The subject of this work is to study different algorithms for computing the CTR of an advertiser. There are a variety of different algorithms one could imagine for learning the CTR of an ad. Some simple examples, described below, include averaging over time, impressions, or clicks, as well as exponential discounting.

- **Average over fixed time window:** For a parameter T, let x be the number of clicks received during the last T time units and y be the number of impressions during the last T time units. Then $\lambda = x/y$.
- **Average over fixed impression window:** For a parameter y, let x be the number of clicks received during the last y impressions. Then $\lambda = x/y$.
- **Average over fixed click window:** For a parameter x, let y be the number of impressions since the x'th last click. Then $\lambda = x/y$.
- **Exponential discounting:** For a parameter α, let $e^{-\alpha i}$ be a discounting factor used to weight the i'th most recent impression. Take a weighted average over all impressions, that is, $\sum_i x_i e^{-\alpha i} / \sum_i e^{-\alpha i}$ where x_i is an indicator variable that the i'th impression resulted in a click.

These algorithms are all part of a general class defined below. The algorithm estimates the CTR of the ad for the current impression as follows: Label the previous impressions, starting with the most recent, by $1, 2, \ldots$. Let t_i be the amount of time that elapsed between impression i and impression 1, and c_i be the number of impressions that received clicks between impression i and impression 1 (impressions 1 included). The learning algorithms we are interested in are defined by a constant γ and a function $\delta(t_i, i, c_i)$ which is decreasing in all three parameters. This function can be thought of as a discounting parameter, allowing the learning algorithm to emphasize recent history over more distant history. Let x_i be an indicator variable for the event that the i'th impression resulted in a click. The learning algorithm then computes

$$\lambda = \frac{\sum_{i=1}^{\infty} x_i \delta(t_i, i, c_i) + \gamma}{\sum_{i=1}^{\infty} \delta(t_i, i, c_i) + \gamma}.$$

The constant γ is often a small constant that is used to guarantee that the estimated click-through-rate is strictly positive and finite. Notice that in the above expression, the summation is for every i from 1 to ∞. This is ambiguious, since the advertiser has not been always present in the system. To remove this ambiguity, the algorithm assumes a *default* infinite history for every advertiser that enters the system. This default sequence could be a sequence of impressions all leading to clicks, indicating that the newly arrived advertiser is initialized with a CTR equal to one, or (as it is often the case in practice) it could be a sequence indicating a system-wide default initial CTR for new advertisers. For most common learning algorithms, the discount factor becomes zero or very small for far distant history, and hence the choice of the default sequence only affects the estimate of the CTR at the arrival of a new advertiser.

Note that all three learning methods discussed above are included in this class (for $\gamma = 0$).

- **Average over fixed time window:** The function $\delta(t_i, i, c_i)$ is 1 if $t_i \leq T$ and 0 otherwise.
- **Average over fixed impression window:** The function $\delta(t_i, i, c_i)$ is 1 if $i \leq y$ and 0 otherwise.
- **Average over fixed click window:** The function $\delta(t_i, i, c_i)$ is 1 if $c_i \leq x$ and 0 otherwise.
- **Exponential discounting:** The function $\delta(t_i, i, c_i)$ is $e^{-\alpha i}$.

4 Fraud Resistance

For each of the methods listed in the previous section, for an appropriate setting of parameters (e.g., large enough y in the second method), on a random sequence generated from a constant CTR the estimate computed by the algorithm gets arbitrarily close to the true CTR, and so it is not a priori apparent which method we might prefer. Furthermore, when the learning algorithm computes the true CTR, the expected behavior of the system is essentially equivalent to a pay-per-impression system, with substantially reduced incentives for fraud. This might lead to the conclusion that all of the above algorithms are equally resistant to click fraud. However, this conclusion is wrong, as the scammer can sometimes create fluctuations in the CTR, thereby taking advantage of the failure of the algorithm to react quickly to the change in the CTR to harm the advertiser.

In this section, we introduce a notion of fraud resistance for CTR learning algorithms, and prove that a class of algorithms are fraud-resistant. The definition of fraud resistance is motivated by the way various notions of security are defined in cryptography: we compare the expected amount the advertiser has to pay in two scenarios, one based on a random sequence generated from a constant CTR without any fraud, and the other with an adversary who can change a fraction of the outcomes (click vs. no-click) on a similar random sequence. Any scenario can be described by a time-stamped sequence of the outcomes of impressions (i.e., click or no-click). More precisely, if we denote a click by 1 and a no-click by 0, the scenario can be described by a doubly infinite sequence $\mathbf{s}$ of zeros and ones, and a doubly infinite increasing sequence $\mathbf{t}$ of real numbers indicating the time stamps (the latter sequence is irrelevant if the learning algorithm is time-independent, which will be the case for the algorithms we consider in this section). The pair $(\mathbf{s}, \mathbf{t})$ indicates a scenario where the i'th impression (i can be any integer, positive or negative) occurs at time t_i and results in a click if and only if $s_i = 1$.

Definition 1. *Let ϵ be a constant between zero and one, and $(\mathbf{s}, \mathbf{t})$ be a scenario generated at random as follows: the outcome of the ith impression, s_i, is 1 with an arbitrary fixed probability λ and 0 otherwise, and the time difference $t_i - t_{i-1}$ between two consecutive impressions is drawn from a Poisson distribution with an arbitrary fixed mean. For a value of n, let $(\mathbf{s}', \mathbf{t}')$ be a history obtained from $(\mathbf{s}, \mathbf{t})$ by letting an adversary insert at most ϵn impressions after the impression indexed 0 in $(\mathbf{s}, \mathbf{t})$. The history $(\mathbf{s}', \mathbf{t}')$ is indexed in such a way that impression 0 refers to the same impression in $(\mathbf{s}, \mathbf{t})$ and $(\mathbf{s}', \mathbf{t}')$. We say that a CTR learning algorithm is* ϵ-fraud resistant *if for every adversary, the expected average payment of the advertiser per impression during the impressions indexed $1, \ldots, n$ in scenario $(\mathbf{s}', \mathbf{t}')$ is bounded by that of scenario $(\mathbf{s}, \mathbf{t})$, plus an additional term that tends to zero as n tends to infinity (holding everything else constant). More precisely, if q_j (q'_j, respectively) denotes the payment of the advertiser for the jth impression in scenario $(\mathbf{s}, \mathbf{t})$ ($(\mathbf{s}', \mathbf{t}')$, respectively), then the algorithm is ϵ-fraud resistant if for every adversary,*

$$\mathrm{E}[\frac{1}{n}\sum_{j=1}^{n} q'_j] \leq \mathrm{E}[\frac{1}{n}\sum_{j=1}^{n} q_j] + o(1).$$

Intuitively, in a fraud-resistant algorithm, a fraudulent click or impression only costs the advertiser as much as a fraudulent impression in a pay-per-impression scheme.

Some details are intentionally left ambiguous in the above definition. In particular, we have not specified how much knowledge the adversary has. In practice, an adversary probably can gain knowledge about some statistics of the history, but not the complete history. However, our positive result in this section holds even for an all powerful adversary that knows the whole sequence (even the future) in advance. We prove that even for such an adversary, there are simple learning algorithms that are fraud-resistant. Our negative result (presented in the next section), shows that many learning algorithms are not fraud-resistant even if the adversary only knows about the learning algorithm and the frequency of impressions in the scenario. Therefore, our results are quite robust in this respect.

Another point that is worth mentioning is that the assumption that the true click-through rate λ is a constant in the above definition is merely a simplifying assumption. In fact, our results hold (with the same proof) even if the parameter

λ changes over time, as long as the value of λ at every point is at least a positive constant (i.e., does not get arbitrarily close to zero). Also, the choice of the distribution for the time stamps in the definition was arbitrary, as our positive result only concerns CTR learning algorithms that are time-independent, and our negative result in the next section can be adapted to any case where the time stamps come from an arbitrary known distribution.

In this section, we show that CTR learning algorithms for which the discounting factor, δ, depends only on the number of impressions in the history which resulted in clicks, that is the parameter c_i defined above (and not on i and t_i), are fraud-resistant. We call such algorithms *click-based* algorithms.

Definition 2. *A CTR learning algorithm is click-based if* $\delta(t_i, i, c_i) = \delta(c_i)$ *for some decreasing function* $\delta(.)$.

Of the schemes listed in the previous section, it is easy to see that only averaging over clicks is click-based. Intuitively, a click-based algorithm estimates the CTR by estimating the *Expected Click-Wait* (ECW), the number of impression it takes to receive a click.

The following theorem shows that click-based algorithms are fraud-resistant.

Theorem 1. *Consider a click-based CTR learning algorithm A given by a discounting function $\delta(.)$ and $\gamma = 0$. Assume that $\sum_{i=1}^{\infty} i\delta(i)$ is bounded. Then for every $\epsilon \leq 1$, the algorithm A is ϵ-fraud-resistant.*

Proof. The proof is based on a simple "charging" argument. We distribute the payment for each click over the impressions preceding it, and then bound the expected total charge to any single impression due to the clicks after it.

We begin by introducing some notations. For any scenario $(\mathbf{s}, \mathbf{t})$ and index i, let $S_{i,j}$ denote the set of impressions between the j'th and the $(j-1)$'st most recent click before impression i (including click j but not click $j-1$) and $n_{i,j} = |S_{i,j}|$. Then the estimated CTR at i can be written as

$$\frac{\sum_{j=1}^{\infty} \delta(j)}{\sum_{j=1}^{\infty} n_{i,j}\delta(j)}. \tag{1}$$

and hence the payment at i if impression i receives a click is

$$p \times \frac{\sum_{j=1}^{\infty} n_{i,j}\delta(j)}{\sum_{j=1}^{\infty} \delta(j)}.$$

We now introduce a scheme to charge this payment to the preceding impressions (for both with-fraud and without-fraud scenarios). Fix an impression i' for $i' < i$ and let j be the number of clicks between i' and i (including i' if it is a click). If impression i leads to a click, we charge i' an amount equal to

$$p \times \frac{\delta(j)}{\sum_{j=1}^{\infty} \delta(j)}. \tag{2}$$

for this click. Summing over all impressions preceding click i, we see that the total payment charged is equal to the payment for the click at impression i. The crux of the argument in the remainder of the proof is to show that in both scenarios, with or without fraud, the average total payment charged to an impression i in the interval $[1, n]$ by clicks occurring after i is $p \pm o(1)$. We start by proving an upper bound on the total amount charged to the impressions.

We first focus on bounding the total amount charged to impressions before impression 0. The impressions in the set $S_{0,j}$ are charged by the i'th click after impression 0 a total of

$$p \times \frac{n_{0,j}\delta(i+j)}{\sum_{k=1}^{\infty}\delta(k)}.$$

Summing over all clicks between impression 0 and impression n (inclusive), we see that the total charge to the impressions in $S_{0,j}$ is at most

$$p \times \frac{\sum_{i=1}^{\infty} n_{0,j}\delta(i+j)}{\sum_{k=1}^{\infty}\delta(k)}.$$

Summing over all sets $S_{0,j}$, we see that the total charge to impressions before 0 is at most

$$p \times \frac{\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} n_{0,j}\delta(i+j)}{\sum_{k=1}^{\infty}\delta(k)}.$$

Since the denominator $\sum_{k=1}^{\infty}\delta(k)$ is a positive constant, we only need to bound the expected value of the numerator $\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} n_{0,j}\delta(i+j)$.

Since in both with-fraud and without-fraud scenarios the probability of click for each impression before impression 0 is λ, the expected value of $n_{0,j}$ in both scenarios is $1/\lambda$. Therefore, the expectation of the total amount charged to impressions before 0 in both scenarios is at most

$$\frac{p}{\lambda} \times \frac{\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}\delta(i+j)}{\sum_{k=1}^{\infty}\delta(k)} = \frac{p}{\lambda} \times \frac{\sum_{k=1}^{\infty}k\delta(k)}{\sum_{k=1}^{\infty}\delta(k)},$$

which is bounded by a constant (i.e., independent of n) since $\sum_{k=1}^{\infty}k\delta(k)$ is finite.

We now bound payments charged to impressions after impression 0. For a fixed impression i with $0 \le i \le n$, the payment charged to i by the j'th click after i is given by Equation 2. Summing over all j, we see that the total payment charged to i is at most

$$p \times \frac{\sum_{j=1}^{\infty}\delta(j)}{\sum_{j=1}^{\infty}\delta(j)} = p. \tag{3}$$

By the above equation along with the fact that the expected total charge to impressions before impression 0 is bounded, we see that the expected total charge of all impressions is at most $np+O(1)$, and therefore the expected average payment per impression (in both with-fraud and without-fraud scenarios) is at most $p + o(1)$.

We now show that in the scenario $(\mathbf{s}, \mathbf{t})$ (i.e., without fraud), the expected average payment per impression is at least $p - o(1)$. Let k be the number of clicks in the interval $[1, n]$ and consider an impression in $S_{n,j}$. Then, by Equation 2, this impression is charged an amount equal to

$$p \times \frac{\sum_{i=1}^{j-1} \delta(i)}{\sum_{i=1}^{\infty} \delta(i)} = p - \frac{p \sum_{i=j}^{\infty} \delta(i)}{\sum_{i=1}^{\infty} \delta(i)}.$$

Therefore, the total amount charged to the impressions in the interval $[1, n]$ is at least

$$np - \frac{p \sum_{j=1}^{k+1} n_{n,j} \sum_{i=j}^{\infty} \delta(i)}{\sum_{i=1}^{\infty} \delta(i)}.$$

As in the previous case, the expected value of $n_{n,j}$ in the scenario without any fraud is precisely $1/\lambda$. Therefore, the expectation of the total charge to impressions in $[1, n]$ is at least

$$np - \frac{p \sum_{j=1}^{k+1} \sum_{i=j}^{\infty} \delta(i)}{\lambda \sum_{i=1}^{\infty} \delta(i)} \geq np - \frac{p}{\lambda} \times \frac{\sum_{i=1}^{\infty} i\delta(i)}{\sum_{i=1}^{\infty} \delta(i)}.$$

Therefore, since $\sum_{i=1}^{\infty} i\delta(i)$ is bounded, the expected average payment per impression in the scenario without fraud is at least $p - o(1)$. This shows that the difference between the expected average payment per impression in the two scenarios is at most $o(1)$, and hence the algorithm is ϵ-fraud resistant.

5 Non-click-Based Algorithms

In this section, we give an example that shows that in many simple non-click-based algorithms (such as averaging over fixed time window or impression window presented in Section 3), an adversary can use a simple strategy to increase the average payment of the advertiser per impression. We present the example for the learning algorithm that takes the average over a fixed impression window. It is easy to see that a similar example exists for averaging over a fixed time window.

Consider a history defined by setting the outcome of each impression to click with probability λ for a fixed λ. Denote this sequence by $\mathbf{s}$. We consider the algorithm that estimates the CTR by the number of click-throughs during the past l impressions plus a small constant γ divided by $l + \gamma$, for a fixed l. If l is large enough and γ is small but positive, the estimate provided by the algorithm is often very close to λ, and therefore the average payment per impression on any interval of length n is arbitrarily close to p. In the following proposition, we show that an adversary can increase the average payment by a non-negligible amount.

Proposition 1. *In the scenario defined above, there is an adversary that can increase the average payment per impression over any interval of length n, for* *any large enough n, by inserting ϵn fradulent impressions and clicking on some of them.*

Proof Sketch: Consider the following adversary: The adversary inserts ϵn fradulent impressions distributed uniformly in the interval starting at the impression indexed 1 and ending at the impression indexed $(1-\epsilon)n$ (with an outcome described below), so that in the scenario with fraud, each of the first n impressions after impression 0 is fradulent with probability ϵ. Divide the set of impressions after impression 0 into a set of intervals $I_1, I_2, \ldots$, where each interval I_j contains l impressions (real or fradulent). In other words, I_1 is the set of the first l impressions after impression 0, I_2 is the set of the next l impressions, etc. The adversary sets the outcome of all fradulent impression in I_j for j odd to click and for j even to no-click. This means that the "true" CTR during I_j is $(1-\epsilon)\lambda + \epsilon$ for odd j and $(1-\epsilon)\lambda$ for even j. The algorithm estimates the CTR for the r'th impression of the interval I_j by dividing the number of clicks during the last l impressions by l. Of these impressions, r are in I_j and $l-r$ are in I_{j-1}. Therefore, for j even, the expected number of clicks during the last l impressions is

$$r(1-\epsilon)\lambda + (l-r)\left((1-\epsilon)\lambda + \epsilon\right),$$

and therefore the estimated CTR is the above value plus γ divided by $l+\gamma$ in expectation. When l is large and γ is small, this value is almost always close to its expectation. Therefore, price of a click for this impression is close to pl divided by the above expression. Thus, since the probability of a click for this impression is $(1-\epsilon)\lambda$, the expected payment of the advertiser for this impression can be approximated using the following expression.

$$\frac{pl(1-\epsilon)\lambda}{r(1-\epsilon)\lambda + (l-r)\left((1-\epsilon)\lambda + \epsilon\right)}$$

The average of these values, for all r from 1 to l, can be approximated using the following integral.

$$\frac{1}{l}\int_0^l \frac{pl(1-\epsilon)\lambda}{r(1-\epsilon)\lambda + (l-r)\left((1-\epsilon)\lambda + \epsilon\right)} dr = \frac{p(1-\epsilon)\lambda}{\epsilon} \ln\left(1 + \frac{\epsilon}{(1-\epsilon)\lambda}\right).$$

Similarly, the total expected payment of the advertiser in the interval I_j for j odd and $j > 1$ can be approximated by the following expression.

$$\frac{p((1-\epsilon)\lambda + \epsilon)}{\epsilon} \ln\left(1 + \frac{\epsilon}{(1-\epsilon)\lambda}\right).$$

Denote $\alpha = \frac{\epsilon}{(1-\epsilon)\lambda}$. Therefore, the average payment of the advertiser per impression can be written as follows.

$$p(\frac{1}{2} + \frac{1}{\alpha}) \ln(1+\alpha)$$

Since α is a positive constant, it is easy to see that the above expression is strictly greater than p. ■

6 Discussion

In this paper, we discussed pay-per-click marketplaces and proved that a particular class of learning algorithms can reduce click fraud in a simplified setting. Our results lead to several interesting extensions and open questions.

Pay-Per-Acquisition Marketplaces. We focused on pay-per-click marketplaces. Our reasons for this were three-fold: it is a common industry model, it absorbs risk due to market fluctuations for the advertiser, and it simplifies the strategic calculations of the advertiser. The latter two of these comments can be equally well employed to argue the desirability of a pay-per-acquisition marketplace. In these marketplaces, a service provider receives payment from an advertiser only when a click resulted in a purchase. Such systems are used by Amazon, for example, to sell books on web pages: a service provider, say Expedia, can list an Amazon ad for a travel guide with the understanding that, should a user purchase the product advertised, then the service provider will receive a payment. The problem with pay-per-acquisition systems is that the service provider must trust the advertiser to truthfully report those clicks which result in acquisitions. Our results hint at a solution for this problem. We have seen that in a simple scenario with a single ad slot, click-based algorithms are fraud-resistant in the sense that the expected payment per impression of an advertiser can not be *increased* by click fraud schemes. In fact, it can also be shown that this payment can not be *decreased* either. Thus, as click-based learning algorithms reduce fraud in pay-per-click systems, *acquisition-based* learning algorithms induce truthful reporting in pay-per-acquisition systems.

Computational Issues. We have shown that click-based learning algorithms eliminate click fraud. However, in order to be practical and implementable, learning algorithms must also be easily computed with constant memory. The computability of a click-based algorithm depends significantly on the choice of the algorithm. Consider, for example, a simple click-based exponentially-weighted algorithm with $\delta(i) = e^{-\alpha i}$. Just two numbers are needed to compute this estimate: the estimate of the click-through rate for the most recent impression that lead to a click and a counter representing the number of impressions since the last click. However, other click-based algorithms have worse computational issues. Consider an algorithm in which $\delta_i \in \{0, 1\}$ with $\delta_i = 1$ if and only if $i \leq l$ for some (possibly large) l. Then at least l numbers must be recorded to compute this estimate exactly. One interesting question is how efficiently (in terms of the space) a given estimate can be computed.

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Experiments with an Economic Model of the Worldwide Web*

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Abstract. We present a simple model in which the worldwide web (www) is created by the interaction of selfish agents, namely document authors, users, and search engines. We show experimentally that power law statistics emerge very naturally in this context, and that the efficiency of the system has certain monotonicity properties.

1 Introduction

The worldwide web is an unstructured hypertextual corpus of exploding astronomical size and global availability. But perhaps the most fundamental, differentiating characteristic of the web is that is created, supported, used, and ran by a multitude of selfish, optimizing economic agents with various and dynamically varying degrees of competition and interest alignment. Web page authors want their pages to be visited because they benefit from such visits, either directly (e.g., in the case of e-shops) or otherwise (recognition, influence, etc.). End users seek in the web the most relevant and helpful sites for their information needs. Search engines want to improve their reputation (and therefore profits) by helping users to find the most relevant web pages. The selfish nature of the agents suggests immediately that economics and game theory can be used in modeling and understanding the web.

Incidentally, a very obvious and striking example of the game theoretic aspects of the web (not discussed further, however, in this paper) is the so-called *search engine spam*, whereby document authors attempt to deceive the ranking algorithm of the search engine in order to receive a high rank for their page, thus attracting visitors who would otherwise have no interest in them, while search engines devise and deploy countermeasures to such deception.

* A preliminary version of this work, without the experimental results, was presented as a poster in WWW 05 [5].

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The idea that economics can inform web search was first proposed by Hal Varian in [6]; however, the classical "economics of search" discussed there do not seem to apply directly to web information retrieval. Economic concepts were also used in the study of electronic markets [7] and for understanding the ranking of documents [8]. But these works do not develop an economic model of the web.

In this paper we introduce an economic model of the web. Our goals in proposing this model are: simplicity, modeling economy, and predictive power. We think of the web as a theater where three types of agents interact: document authors, users, and search engines. We postulate a utility *U(i,d),* unknown a priori to all agents, that a user *i* obtains if he or she obtains a document *d*; its main characteristics are randomness and clustering. We assume that search engines propose documents to the users, users choose and endorse those that have the highest utility for them, and then search engines make better recommendations based on these endorsements. This is how the web is created: it is the sum total of all these user endorsements.

There are several important questions: Does the structure of the web thus produced resemble that of the real web? How efficient (now in a concrete economic sense) are various search engine algorithms? And how is this efficiency affected by, say, the amount of randomness and clustering of the utility function?

In this paper we address these questions by performing experiments with our model; we hope that theorems will follow. We find that the web graph of our model has power-law distributed degrees, and that the efficiency of the process improves with clustering and endorsement intensity,

In section 2 we describe the model in detail. In section 3 the experimental results are presented. Finally in section 4 we provide some directions for future work.

2 The Model

We aim at a model of the www that captures some of the economic issues involved while at the same time being simple (not obscured by a multitude of extraneous details) and with some predictive power (it behaves, provably or experimentally, in ways consistent with observations about the www, without, of course, encoding such observations in its assumptions).

Our model consists of three types of actors: *documents*, *users* and a *search engines*. There are *m* users, indexed by *i*, that can be thought as simple queries asked by individuals, and *n* documents, indexed by *d*. The search engine, assumed to be unique at this stage of the model, provides users with document recommendations based on information it has about their preferences.

Our main economic assumption is this: We assume that there is a utility *U(i,d)* associated with user *i* and document *d*. This utility value represents the satisfaction user *i* will obtain if he or she is presented with document *d*. The quantitative features of the *m* x *n* matrix *U* are of central importance for our model, and will be discussed in greater detail below. Users know their utility values only for the documents they have been presented so far (since they cannot value something that is unknown to them). *The search engine initially has no knowledge of U,* but acquires such knowledge only by observing user endorsements. In the ideal situation in which the search engine knows *U*, it would work with perfect efficiency, recommending to each user the documents he or she likes the most.

Briefly, the model works as follows: The search engine recommends some documents to the users, initially at random. Every user reviews the documents seen so far and endorses those with the highest utility. To model the limited attention capacity of the user, we assume a bound on the number of documents he or she can endorse in total. The endorsement mechanism does not need to be specified, as soon as it is observable by the search engine. For example, endorsing a document may entail clicking it, or pointing a hyperlink to it. In our model we represent the endorsements as edges from the users to the respective documents. *A basic assumption of the model is that the www is created by this kind of interaction between users and documents.* The bipartite graph $S = ([m],[n],L)$ of document endorsements by users is called *the www state.* Furthermore, the search engine, by observing the www state, recommends new documents to users, who change their endorsements to new, higher utility documents. The search engine is using a *search algorithm*, which is a function mapping the www state to a set of a recommendations.

Even at this early stage, some interesting questions arise regarding the model:

1. What are the characteristics of the ultimate www state that results from this process? Do most users point to the highest-utility documents? After how many iterations does the process converge in utility achieved? Does the graph have the peculiar statistics, such as power law distribution, observed in the real web?
2. What is the efficiency or "price of anarchy" [1] of the search algorithm? In other words, which fraction of the maximum possible utility (the ideal situation where each user sees the documents of maximum utility of him or her) can be realized by a search engine?
3. What is the best search algorithm with respect to total utility? That is, which algorithm mapping www states to recommendations optimizes the price of anarchy? Note that a search engine need not be altruistic or socially conscious to strive to maximize social welfare: total user satisfaction would be a reasonable objective for a search engine in a more elaborate model in which multiple search engines compete.

In order to be able to answer to these questions we must define in more detail utility matrix U. In this paper we treat experimentally the questions 1 and 2. Question 3, although it is quite important is left open and maybe be the subject of a future work.

2.1 The Utility Matrix *U*

It turns out that the answers to the above questions depend heavily on the quantitative and the statistical characteristics of the utility matrix U. If the entries of U are completely random and uncorrelated the search engine will be confined to random sampling, naturally with quite poor results. But in reality, utilities are highly correlated. Documents have quality and value that make them more or less useful to users. Also documents and users (recall that by "users" we model queries) are clustered around topics.

To accommodate these characteristics, following [2], we model U as a *low rank matrix with added noise*. U is generated as follows: There are k topics, for some reasonable small number k. For each topic $t \leq k$ there is a document vector D_t of length n, with entries drawn independently from some distribution Q. The value 0 is very probable in Q so that about k - 1 of every k entries are 0. Also for each topic t there is a user vector R_t of length m, whose entries also follow distribution Q, with about m/k non-zero entries. In other words each entry in vectors D_t and R_t represents how relative each document and user is with respect to topic t. Finally, let N be a m by n "noise" matrix with normally and independently distributed entries with mean zero and standard deviation σ. Then the utility matrix is composed as follows:

$$U = \sum_{t=1}^{k} R_t^T \cdot D_t + N \quad (1)$$

In other words, the utility Matrix U is the sum of k rank-one matrices, plus a Gaussian noise. By modeling U like this we ensure that the resulting matrix has the desired properties. So the parameters of the model so far are k, Q, and σ.

2.2 A Search Algorithm and an Endorsement Mechanism

To specify the model in full detail we need to specify a search algorithm and an endorsement mechanism. Through the endorsement mechanism users show their preference is some documents, in a way that is observable by the search engine. In our model users link to the documents they wish to endorse. As we said earlier, there is a finite number of endorsements per user, say b. That is, each user endorses the b highest utility documents he has seen so far.

The search algorithm maps the current www state to a set of recommended documents. A very simple search algorithm is to recommend to each user, at each stage, among documents, with positive utility, the a documents of highest in-degree in the www state, where a is another integer parameter. Like many successful search algorithms, this algorithm takes into account the link structure of the graph. To capture other factors affecting search, besides link structure (such as occurrences of query terms), we have assumed that the search engine has partial knowledge of utility matrix U, by knowing whether an element of it, is zero or non-zero. Also it can be shown that the "highest in-degree" heuristic is a common specialization of the well-known Pagerank [3] and HITS [4] algorithms.

So a and b are two final parameters of our model.

3 Experiments

To validate our model, we ran several experiments for various values of the parameters. The model was implemented as an iterative process. In each iteration, the search engine observed the *www state* and proposed a documents to the users according to the highest in-degree heuristic. After that, users made their decisions, endorsing the b highest utility documents they had seen so far. This algorithm was iterated to convergence, that is, until very few changes were observed in the *www state*. The number of iterations needed for convergence was between 8 and 10. The utility matrix U was constructed according to Eq. 1 and the non-zero elements of distribution Q were chosen randomly from the uniform distribution.

In subsection 3.1 we study the degree distribution of the www state and show that for a wide range of the parameters values follows a power law distribution. In subsection 3.2 we study the efficiency or price of anarchy of the search algorithm.

In the experiments reported here we have assumed that there is no noise, that is, N in Eq. 1 is the zero matrix.

3.1 The Characteristics of the www State

It has been pointed out in several studies [9,10,11,12] that the degrees of the web graph follow power law distributions, that is, the fraction of pages with in-degree *i* is proportional to i^{-x} for some $x>1$, where the value for the parameter *x* is about 2.1 for in-degrees and 2.7 for out-degrees. In our case, only the in-degrees are significant, since out-degrees are, by definition, all equal to *b*.

We find experimentally that, for a wide range of values of the parameters *m, n, k, a, b*, the in-degree of the documents seem to be clearly power-law distributed. This is shown in Figs. 1 and 2. (We have not included in the plot the documents with zero in-degree, which are the majority. In other words, endorsements are concentrated in a very small subset of the documents.) In fact, the exponent in Figure 2 (but not of Figure 1) is quite close to the observed exponent of the in-degree power law in the real www.

More work is needed in order to define how the various parameters affect the exponent of the distribution.

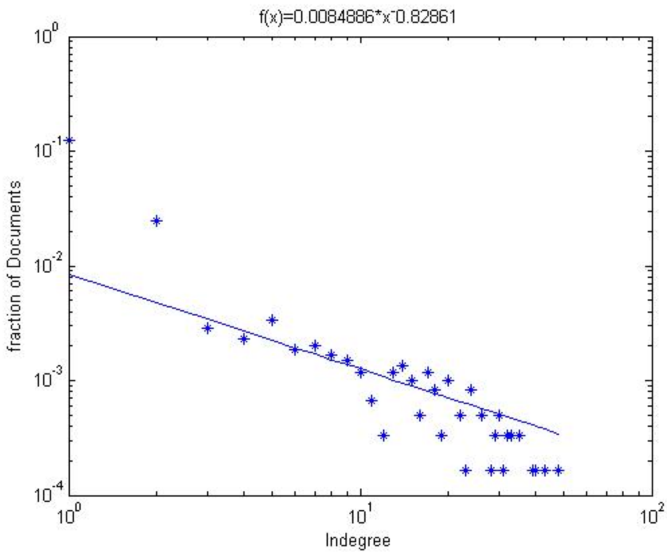

Fig. 1. Log-log plot of the in-degree distribution of an instance of the model with m=1000, n=6000, k=80, a=5, b=5

3.2 The Price of Anarchy

The second question we pose is how efficient the search algorithm can be, meaning which fraction of the maximum total utility can realize during its operation. In this situation the quantity of interest is the price of anarchy as a function of the number of iterations of the algorithm. In all experiments we made, the price of anarchy improved radically during the first 2-3 iterations and later the improvement had a slower rate. This is shown in Fig. 3.

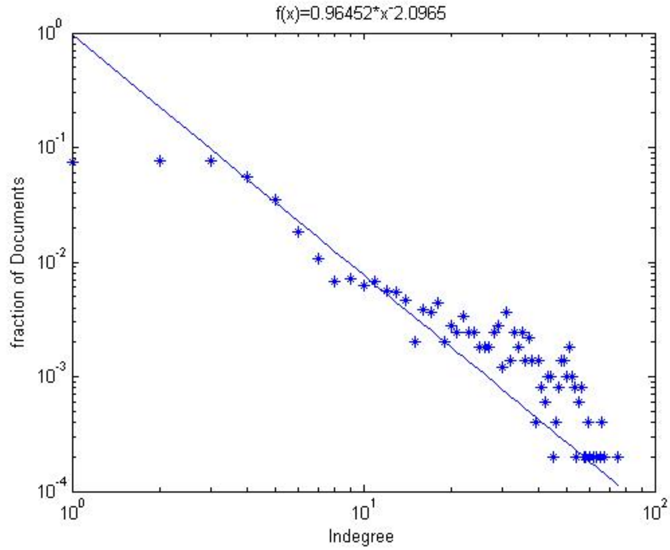

Fig. 2. Log-log plot of the in-degree distribution of an instance of the model with m=3000, n=5000, k=150, a=10, b=10

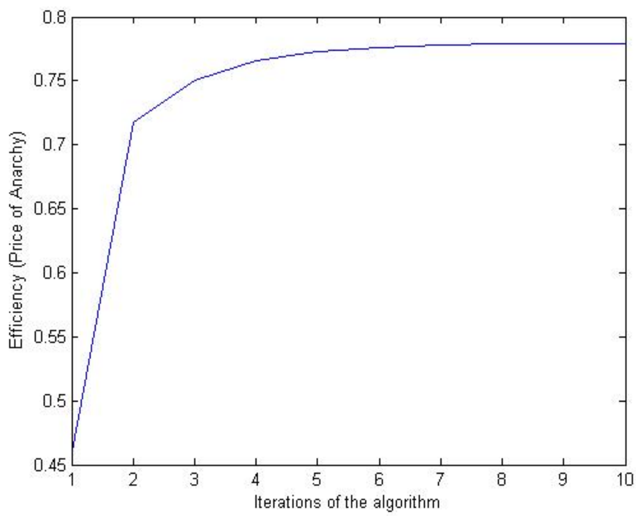

Fig. 3. Efficiency of the Search Algorithm

As we can see the algorithm performs very well since already after iteration 3 has attain more than 75% of the total attainable utility. Of course, adding noise to the model may deteriorate the efficiency.

Another interesting issue is how the efficiency of the search algorithm is affected when we vary the values of *k, a* and *b.* When the number of topics *k* increases the efficiency of the algorithm increases. This fact is clearly shown in Fig. 4.

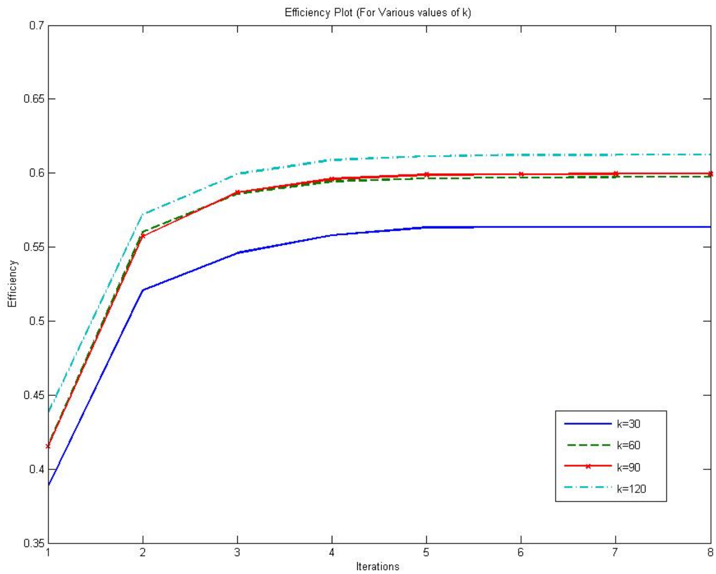

Fig. 4. Efficiency plot for various values of k (number of topics) for an instance of 1000 users, 6000 documents, *a=1, b=1*

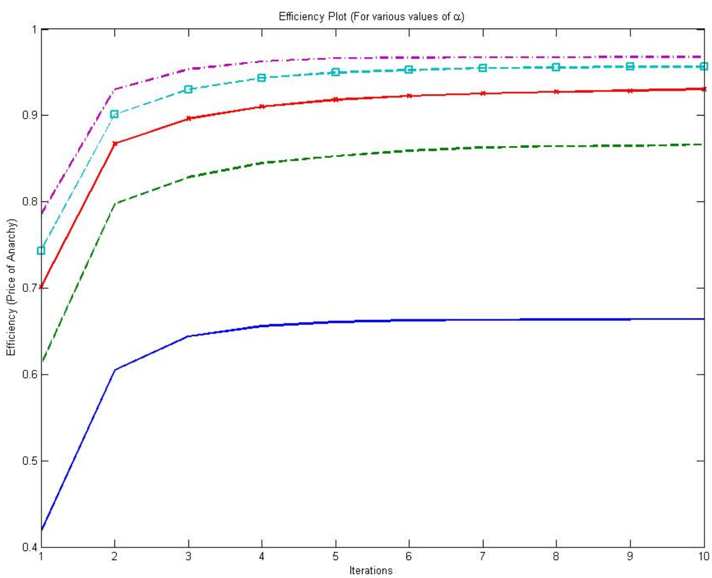

Fig. 5. Efficiency plot for various values of *a* (number of recommended documents) for an instance of 1000 users, 6000 documents, *a=2,4,6,8,10, b=2*

When a increases (the number of recommended documents by the search engine) the efficiency of the algorithm also increases. This is shown in Fig. 5 and is quite expected because the users choose from a wider collection of documents.

Increasing b (number of endorsed documents per user) causes the efficiency of the algorithm to decrease. This is quite unexpected, since more user endorsements mean more complete information and more effective operation of the search engine. But the opposite happens: more endorsements per user seem to confuse the search engine (see Fig. 6).

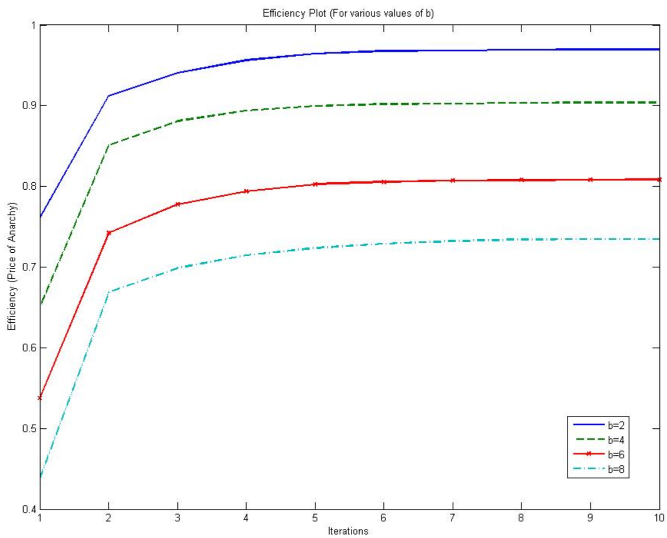

Fig. 6. Efficiency plot for various values of b (number of endorsed documents) for an instance of 1000 users, 6000 documents, $a=8$, $b=2,4,6,8$

4 Conclusion and Future Work

In this paper we propose a very simple economic model of the worldwide web and present some promising and interesting initial experimental results. Much more comprehensive experiments need to be conducted most notably adding noise to U.

Naturally, our ambition is to rigorously *prove* the power law phenomena we observed experimentally, as well as the monotonicity properties of the efficiency. Finally, we hope to use this model to approach the intriguing subject of the optimally efficient search algorithm.

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Coordination Mechanisms for Selfish Scheduling

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Abstract. In machine scheduling, a set of n jobs must be scheduled on a set of m machines. Each job i incurs a processing time of p_{ij} on machine j and the goal is to schedule jobs so as to minimize some global objective function, such as the maximum makespan of the schedule considered in this paper. Often in practice, each job is controlled by an independent selfish agent who chooses to schedule his job on machine which minimizes the (expected) completion time of his job. This scenario can be formalized as a game in which the players are job owners; the strategies are machines; and the disutility to each player in a strategy profile is the completion time of his job in the corresponding schedule (a player's objective is to minimize his disutility). The equilibria of these games may result in larger-than-optimal overall makespan. The ratio of the worst-case equilibrium makespan to the optimal makespan is called the *price of anarchy* of the game. In this paper, we design and analyze scheduling policies, or *coordination mechanisms*, for machines which aim to minimize the price of anarchy (restricted to pure Nash equilibria) of the corresponding game. We study coordination mechanisms for four classes of multiprocessor machine scheduling problems and derive upper and lower bounds for the price of anarchy of these mechanisms. For several of the proposed mechanisms, we also are able to prove that the system converges to a pure Nash equilibrium in a linear number of rounds. Finally, we note that our results are applicable to several practical problems arising in networking.

1 Introduction

With the advent of the Internet, large-scale autonomous systems have become increasingly common. These systems consist of many independent and selfish agents all competing to share a common resource like bandwidth in a network or processing power in a parallel computing environment. Practical settings range from smart routing among stub autonomous systems (AS) in the Internet [14] to selfish user association in wireless local area networks [16]. In many systems

⋆ The final parts of this paper were completed while the author was at the Theory group in Microsoft Research.

⋆⋆ The final parts of this paper were completed while the author was at the Strategic Planning and Optimization group in amazon.com.

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of this kind, it is infeasible to impose some centralized control upon the users. Rather, a centralized authority can only design protocols a priori and hope that the independent and selfish choices of the users—given the rules of the protocol—combine to create socially desirable results.

This approach, termed *mechanism design* has received considerable attention in the recent literature (see, for example, [23]). A common goal is to design system-wide rules which, given the selfish decisions of the users, maximize the total social welfare. The degree to which these rules approximate the social welfare in a worst-case equilibrium is known as the *price of anarchy* of the mechanism, and was introduced in 1999 by Koutsoupias and Papadimitriou [19] in the context of *selfish scheduling games* (see below for details). This seminal paper spawned a series of results which attempt to design mechanisms with minimum price of anarchy for a variety of games. One approach for achieving this goal [4, 7, 12] is to impose economic incentives upon users in the form of *tolls*; i.e., the disutility of a user is affected by a monetary payment to some central authority for the use of a particular strategy such as a route in a network. Another approach [3, 18, 24, 28] assumes that the central authority is able to enforce particular strategies upon some fraction of the users and thus perhaps utilize an unpopular resource. This action of the central authority is called a *Stackelberg strategy*.

A drawback of the above approaches is that many of the known algorithms assume global knowledge of the system and thus have high communication complexity. In many settings, it is important to be able to compute mechanisms locally. A third approach, which we follow here, is called *coordination mechanisms*, first introduced by Christodoulou, Koutsoupias and Nanavati [6]. A coordination mechanism is a *local* policy that assigns a cost to each strategy s, where the cost of s is a function of the agents who have chosen s. Consider, for example, the *selfish scheduling game* in which there are n jobs owned by independent agents, m machines, and a processing time p_{ij} for job i on machine j. Each agent selects a machine on which to schedule its job with the objective of minimizing its own completion time. The social objective is to minimize the maximum completion time. A *coordination mechanism* [6] for this game is a local policy, one for each machine, that determines how to schedule jobs assigned to that machine. It is important to emphasize that a machine's policy is a function only of the jobs assigned to that machine. This allows the policy to be implemented in a completely distributed fashion.

Coordination mechanisms are closely related to local search algorithms. A local search algorithm iteratively selects a solution "close" to the current solution which improves the global objective. It selects the new solution from among those within some *search neighborhood* of the current solution. Given a coordination mechanism, we can define a local search algorithm whose search neighborhood is the set of best responses for each agent. Similarly, given a local search algorithm, it is sometimes possible to define a coordination mechanism whose pure strategy equilibria are local optima with respect to the search neighborhood. The locality gap of the search neighborhood, or approximation factor of the local search algorithm, is precisely the price of anarchy of the corresponding coordination mechanism and vice versa. In particular, designing new coordination mechanisms may lead to the discovery of a new local search algorithms for a particular problem.

In this paper, we are primarily interested in the properties of *pure strategy Nash equilibria*[1] for the selfish scheduling games we define. A pure strategy Nash equilibrium is an assignment of jobs to machines such that no job has a unilateral incentive to switch to another machine. Although a non-cooperative game always has a mixed strategy equilibrium [22], it may in general not have a pure strategy equilibrium. However, by restricting ourselves to pure strategies, we are able to bound the *rate of convergence* of the mechanism. That is, if the jobs, starting from an arbitrary solution, iteratively play their best response strategies, how long does it take for the mechanism to reach a pure Nash equilibrium? Guarantees of this sort are important for bounded price-of-anarchy mechanisms to be applicable in a practical setting.

Preliminaries. In machine scheduling, there are n jobs must be processed on m machines. Job i has processing time p_{ij} on machine j. A schedule μ is a function mapping each job to a machine. The makespan of a machine j in schedule μ is $M_j = \sum_{i:j=\mu(i)} p_{ij}$. The goal is to find a schedule μ which minimizes the maximum makespan, $C_{\max} = \max_j M_j$. Different assumptions regarding the relationship between processing times yield different scheduling problems, of which we consider the following four: (i) *Identical* machine scheduling ($P||C_{\max}$) in which $p_{ij} = p_{ik} = p_i$ for each job i and machines j and k; (ii) *Uniform* or *related* machine scheduling ($Q||C_{\max}$) in which $p_{ij} = \frac{p_i}{s_j}$, where p_i is the load of job i and $s_j \leq 1$ is the speed of machine j; (iii) machine scheduling for *restricted assignment* or *bipartite* machine scheduling ($B||C_{\max}$) in which each job i can be scheduled on a subset S_i of machines, i.e., p_{ij} is equal to p_i if $j \in S_i$ and is equal to ∞ otherwise; and (iv) *unrelated* machine scheduling ($R||C_{\max}$) in which the processing times p_{ij} are arbitrary positive numbers.

In *selfish scheduling*, each job is owned by an independent agent whose goal is to minimize the completion time of his own job. To induce these selfish agents to take globally near-optimal actions, we introduce the notion of a *coordination mechanism* [6]. A coordination mechanism is a set of *scheduling policies*, one for each machine. A scheduling policy for a machine j takes as input a set $S \subseteq \{p_{1j}, \ldots, p_{nj}\}$ of jobs on machine j along with their processing times on machine j and outputs an ordering in which they will be scheduled. The policy is run locally at a machine, and so does not have access to information regarding the global state of the system (the set of all jobs, for instance, or the processing times of the jobs on other machines). A coordination mechanism defines a game in which there are n agents. An agent's strategy set is the set of possible machines $\{1, ..., m\}$. Given a strategy profile, the disutility of agent i is the (expected) completion time of job i in the schedule defined by the coordination mechanism. We study four coordination mechanisms. In the ShortestFirst and LongestFirst policies, we sequence the jobs in non-decreasing and non-increasing order of

[1] In this paper, we use the term Nash equilibria for pure Nash equilibria.

Table 1. The price of anarchy for four different policies and scheduling problems. All the upper and lower bounds hold for pure Nash equilibria. The upper bounds of the Randomized policy for $R||C_{\max}$ and $Q||C_{\max}$ are valid for the maximum of the expected load on any machine in mixed Nash equilibria. The results marked by * are proved in this paper.

	Makespan	ShortestFirst	LongestFirst	Randomized
$P\|\|C_{\max}$	$2-\frac{2}{m+1}$ [11, 29]	$2-\frac{2}{m+1}$ [11, 29]	$\frac{4}{3}-\frac{1}{3m}$ [6]	$2-\frac{2}{m}$ [11, 29]
$Q\|\|C_{\max}$	$\Theta(\frac{\log m}{\log\log m})$ [8]	$\Theta(\log m)$ ([1], *)	$\frac{4}{3}-\frac{1}{3m}\le P\le 2-\frac{2}{m+1}$ [15]	$\Theta(\frac{\log m}{\log\log m})$([8])
$B\|\|C_{\max}$	$\Theta(\frac{\log m}{\log\log m})$ [13]	$\Theta(\log m)$ ([1], *)	$\Theta(\log m)$ ([2], *)	$\Theta(\frac{\log m}{\log\log m})$ [13]
$R\|\|C_{\max}$	Unbounded [29]	[9] $\log m\le P\le m$ *	Unbounded	$\Theta(m)$ *

their processing times, respectively. In the Randomized policy, we process the jobs in a random order.[2] In the Makespan policy, we process all jobs on the same machine in parallel, and so the completion time of a job on machine j is the makespan of machine j.

We are interested in the properties of the solution imposed by the selfish strategies of the agents in a pure Nash equilibrium, in particular, the *price of anarchy*. In this setting, the price of anarchy is the worst-case ratio (over all instances) of the maximum makespan in a (pure) Nash equilibrium to the optimal makespan. For each policy and each scheduling problem, we prove upper and lower bounds on the price of anarchy. Some of these bounds are already known as the approximation factor of some local search algorithms or the price of anarchy in some selfish load balancing games. All bounds, known and new, are summarized in Table 1.

Related Work. The Makespan policy [8, 13, 19] is perhaps the best known policy among the above policies. Czumaj and Vocking [8] give tight results on the price of anarchy for the Makespan policy for mixed Nash equilibria and $Q||C_{\max}$. Gairing et al. [13] study the Makespan policy for $B||C_{\max}$ and give a polynomial-time algorithm for computing a pure Nash equilibrium with makespan at most twice the optimal makespan. They also give a tight bound for the price of anarchy of the Makespan policy for pure Nash equilibria and $B||C_{\max}$. Azar et al. [1, 2] proved that the greedy list scheduling algorithm is an $O(\log m)$-approximation algorithm for $Q||C_{\max}$ and $B||C_{\max}$. Their proof can be used to bound the price of anarchy of the LongestFirst and ShortestFirst polices for $B||C_{\max}$ and $Q||C_{\max}$.

Coordination mechanism design was introduced by Christodoulou, Koutsoupias and Nanavati [6]. In their paper, they analyzed the LongestFirst policy for $P||C_{\max}$ and also studied a selfish routing game. As we have mentioned before, the price of anarchy for coordination mechanisms is closely related to the approximation factor of local search algorithms. The speed of convergence and the approximation factor of local search algorithms for scheduling problems were studied in several papers [9, 10, 11, 17, 25, 26, 29]. Vredeveld surveyed some of the

[2] The Randomized policy is also known as the batch model [19].

results on local search algorithms for scheduling problems in his thesis [29]. The jump model in his survey [29] is similar to the Makespan policy. Moreover, the push model in his survey is related to the LongestFirst policy. It is known that the jump local search algorithm is an $\Theta(\sqrt{m})$-approximation for $Q||C_{\max}$ [25, 29]. Cho and Sahni [25] showed that the approximation factor of the shortest-first greedy algorithm is not better than $\log m$ for $Q||C_{\max}$. Furthrmore, Gonzales et al [15] proved that the longest-first greedy algotithm is a $2 - \frac{2}{m+1}$-approximation for $Q||C_{\max}$.

Ibarra and Kim [17] analyzed several greedy algorithms for $R||C_{\max}$. In particular, they proved that the shortest-first greedy algorithm is an m-approximation for $R||C_{\max}$. Davis and Jaffe [9] showed that the approximation factor of this greedy algorithm is at least $\log m$. Our lower bound example for the ShortestFirst and the LongestFirst policy is the same as the example in [9]. Davis and Jaffe [9] also gave a $\sqrt{m}$-approximation for $R||C_{\max}$. The best known approximation factor for $R||C_{\max}$ is given by a 2-approximation algorithm due to Lenstra, Shmoys and Tardos [20].

Even-Dar et al. [10] considered the convergence time to Nash equilibria for variants of the selfish scheduling problem. In particular, they studied the Makespan policy and bounded the number of required steps to reach a pure Nash equilibrium.

Our Contribution. We give almost tight bounds for the price of anarchy of pure Nash equilibria for the Randomized, the ShortestFirst, and the LongestFirst policies. We give a proof that the price of anarchy of any deterministic coordination mechanism (including ShortestFirst and LongestFirst) for $Q||C_{\max}$ and $B||C_{\max}$ is at most $O(\log m)$. This result is also implied in the results of Azar et al [1, 2] on the approximation factor of the greedy list scheduling algorithm for $Q||C_{\max}$ and $B||C_{\max}$. We also prove that any coordination mechanism based on a *universal ordering* (such as ShortestFirst and LongestFirst) has a price of anarchy $\Omega(\log m)$ for $B||C_{\max}$. In addition, we analyze the Randomized policy for $R||C_{\max}$ machine scheduling and prove a bound of $\Theta(m)$ for the price of anarchy of this policy. We further study the convergence and existence of pure Nash equilibria for the ShortestFirst and LongestFirst policies. In particular, we show that the mechanism based on the ShortestFirst policy for $R||C_{\max}$ is a potential game, and thus any sequence of best responses converges to a Nash equilibrium after at most n rounds. We also prove fast convergence of the LongestFirst policy for $Q||C_{\max}$ and $B||C_{\max}$.

2 Upper Bounds on the Price of Anarchy

2.1 The ShortestFirst Policy

We begin by bounding the price of anarchy of the ShortestFirst policy for $R||C_{\max}$. We note that there is a direct correspondence between outputs of the well-known shortest-first greedy algorithm for machine scheduling (see [17], "Algorithm D") and pure Nash equilibria of the ShortestFirst policy in $R||C_{\max}$.

Theorem 1. *The set of Nash equilibria for the* ShortestFirst *policy in* $R||C_{\max}$ *is precisely the set of solutions that can be output by the shortest-first greedy algorithm.*

The proof of this theorem is deferred to the full version of this paper. The implication is that any bound on the approximation factor of the shortest-first greedy algorithm is also a bound on the price of anarchy of the ShortestFirst policy for $R||C_{\max}$. In particular, we may use this theorem and the result of Ibarra and Kim [17] to prove that the price of anarchy of ShortestFirst for $R||C_{\max}$ is at most m.

Theorem 2. *The price of anarchy of the* ShortestFirst *policy for* $R||C_{max}$ *is at most* m.

We can further prove that the price of anarchy of the ShortestFirst policy for $Q||C_{\max}$ is $\Theta(\log m)$. This also shows that the approximation factor of the shortest-first greedy algorithm for $Q||C_{\max}$ is $\Theta(\log m)$, as was previously observed by Azar et al [1]. In fact, the bound on the price of anarchy can be derived from their result as well. Our proof, derived independently, uses ideas from the proof of the price of anarchy for the Makespan policy for $Q||C_{\max}$ by Czumaj and Vocking [8].

We prove our upper bound for any *deterministic* coordination mechanism. A coordination mechanism is deterministic if the scheduling policies do not use randomization to determine the schedules. We prove that the price of anarchy for deterministic mechanisms, and in particular for the ShortestFirst policy, for $Q||C_{\max}$ is at most $O(\log m)$. The proof is deferred to the full version of the paper.

Theorem 3. *The price of anarchy of a deterministic policy for* $Q||C_{\max}$ *is at most* $O(\log m)$. *In particular, the price of anarchy of the* ShortestFirst *policy for* $Q||C_{\max}$ *is* $O(\log m)$.

In addition, we can prove that the price of anarchy of any deterministic mechanism for $B||C_{\max}$ is $\Theta(\log m)$. This result is implied by a result of Azar et al [2] on the approximation factor of the greedy algorithm for $B||C_{\max}$, but our proof is independent of theirs and uses the ideas of the proof for the Makespan policy by Gairing et al. [13]. We defer the proof to the full version of the paper.

Theorem 4. *The price of anarchy of a deterministic policy for* $B||C_{\max}$ *is at most* $O(\log m)$. *In particular the price of anarchy of the* ShortestFirst *policy for* $B||C_{\max}$ *is* $O(\log m)$.

2.2 The LongestFirst Policy

It is easy to see that the price of anarchy of the LongestFirst policy for unrelated machine scheduling is unbounded. It is known that the price of anarchy of this policy for $P||C_{\max}$ is bounded above by $\frac{4}{3} - \frac{1}{3m}$ [6]. Theorems 4 and 3 show that the price of anarchy of the LongestFirst policy for $B||C_{\max}$ and $Q||C_{\max}$ is at most $O(\log m)$. In Section 3, we prove that this bound is tight for $B||C_{\max}$. Here, we observe that the price of anarchy of the LongestFirst policy for $Q||C_{\max}$ is at most $2 - \frac{2}{m+1}$. Note that this is the only policy for which we know that the price of anarchy for $Q||C_{\max}$ is bounded by a constant. The proof is the same as the proof of the greedy algorithm in [15] and is omitted here.

Theorem 5. *The price of anarchy of the* LongestFirst *policy for related machine scheduling ($Q||C_{\max}$) is at most $2 - \frac{2}{m+1}$.*

2.3 The Randomized Policy

In the Randomized policy, an agent's disutility is the *expected* completion time of his job. We begin by computing the condition under which an agent has an incentive to change strategies. Consider a job i on machine j and let J_j be the set of jobs assigned to machine j. Then the disutility of agent i under the Randomized policy is:

$$p_{ij} + \frac{1}{2} \sum_{i' \neq i, i' \in J_j} p_{i'j}.$$

Letting M_j be the makespan of machine j, we see that a job i on machine j has an incentive to change to machine k if and only if:

$$p_{ij} + M_j > 2p_{ik} + M_k.$$

Because of this observation, the randomized policy is the same as the Makespan policy for $P||C_{\max}$ and $B||C_{\max}$. This implies a price of anarchy of at most $2 - \frac{2}{m}$ and $O(\frac{\log m}{\log\log m})$ for these settings, respectively.

Here, we bound the price of anarchy of the Randomized policy for $Q||C_{\max}$ and $R||C_{\max}$. In fact, we prove that, in contrast to the Makespan policy the price of anarchy of the Randomized policy for $R||C_{\max}$ is not unbounded.

Theorem 6. *The price of anarchy of the* Randomized *policy for $R||C_{\max}$ is at most $2m - 1$.*

Proof. Let $\mathcal{L}$ be any pure strategy Nash equilibrium and $\mathcal{O}$ be an optimal solution. We consider two groups of jobs — those that are on different machines in $\mathcal{O}$ and $\mathcal{L}$, and those that are on the same machine. Define S_{qj} as the set of jobs on machine q in $\mathcal{L}$ that are on machine j in $\mathcal{O}$, and let $L_q = \sum_{i \in \cup_{j \neq q} S_{qj}} p_{iq}$, $O_q = \sum_{i \in \cup_{j \neq q} S_{jq}} p_{iq}$, and $R_q = \sum_{i \in S_{qq}} p_{iq}$. Thus, the makespan of a machine l in $\mathcal{L}$ is $L_l + R_l$, and the makespan of l in $\mathcal{O}$ is $O_l + R_l$. Since $\mathcal{L}$ is a Nash equilibrium, for all jobs $i \in S_{qj}$,

$$L_q + R_q + p_{iq} \leq L_j + R_j + 2p_{ij}.$$

Suppose the makespan of $\mathcal{L}$ is achieved on machine l and the makespan of $\mathcal{O}$ is achieved on machine l'. Then

$$
\begin{aligned}
|\cup_{j\neq l} S_{lj}|(L_l+R_l)+L_l &= \sum_{i\in\cup_{j\neq l}S_{lj}} (L_l+R_l+p_{il}) \\
&\leq \sum_{i\in\cup_{j\neq l}S_{lj}} (L_j+R_j+2p_{ij}) \\
&\leq |\cup_{j\neq l} S_{lj}|(L_l+R_l)+2\sum_{j\neq l}\sum_{i\in S_{lj}} p_{ij} \\
&\leq |\cup_{j\neq l} S_{lj}|(L_l+R_l)+2\sum_{j\neq l} O_j \\
&\leq |\cup_{j\neq l} S_{lj}|(L_l+R_l)+2(m-1)(O_{l'}+R_{l'}). \quad (1)
\end{aligned}
$$

Therefore, the value of the solution induced by the Nash equilibrium $\mathcal{L}$ is at most $2(m-1)(O_{l'}+R_{l'})+R_l \leq (2m-1)(O_{l'}+R_{l'})$, and so the price of anarchy is at most $2m-1$.

Unfortunately, we do not know if pure Nash equilibria exist for the Randomized policy for $R||C_{\max}$, and so the above theorem might be vacuous. However, we can extend the above proof to bound the maximum of the expected load of a machine in a mixed Nash equilibrium of the Randomized policy for $R||C_{\max}$. If M_j is the expected load of machine j in a mixed Nash equilibrium, then it is easy to show that if the probability of assigning job i to machine q is nonzero, then for any other machine j, $M_q+p_{iq} \leq M_j+2p_{ij}$. Now, we can define L_q as the expected load of jobs with positive probability on machine q that are scheduled on machines other than q in the optimum solution. Similar inequalities hold in this setting. This bounds the maximum of the expected load of any machine in a mixed Nash equilibrium. We defer the details of the proof to the full version of the paper. Note that this analysis does not hold for the expected value of the maximum load (see [8] for the difference between these two objective functions). We can further prove that this bound is tight, up to a constant factor (see Theorem 9).

Finally, we also observe a better bound for $Q||C_{\max}$. The proof is along the same lines as the proof of the Makespan policy [8] and is omitted here. This proof is also valid for the maximum expected load on any machine for mixed Nash equilibria.

Theorem 7. *The price of anarchy of the* Randomized *policy for* $Q||C_{\max}$ *is at most* $O(\frac{\log m}{\log\log m})$.

3 Lower Bounds on the Price of Anarchy

In this section, we prove lower bounds on the price of anarchy of coordination mechanisms. Our first result shows that the price of anarchy of a general class of coordination mechanisms for $B||C_{\max}$ and $R||C_{\max}$ is at least $\log m$. This is interesting in light of the fact that constant-factor LP-based approximation algorithms are known for $R||C_{\max}$ [20], and suggests that it may be hard to obtain similar approximations with local search algorithms.

We consider a class of mechanisms which are *deterministic* and use a *common tie-breaking rule*. That is, there is no randomization in the scheduling policies; and, whenever two jobs i and i' have the same processing time on a machine, one of them, say i, is *always* scheduled before the other (independent of the machine and the presence of other jobs). An example of such a mechanism is ShortestFirst with an alphabetical tie-breaking rule (i.e. if $p_{ij} = p_{i'j}$ then i is scheduled before i' if and only if $i < i'$). The example in our proof was used by Davis and Jaffe [9] to show that the approximation factor of the shortest-first greedy algorithm is at least $\log m$.

Theorem 8. *The price of anarchy of any deterministic coordination mechanism which uses a common tie-breaking rule is at least* $\log_2 m$ *for* $B||C_{\max}$ *and* $R||C_{\max}$.

Proof. Consider a deterministic coordination mechanism with a common tie-breaking rule, say alphabetically first (this assumption is without-loss-of-generality since we can always relabel jobs for the purposes of the proof such that in a tie job i is scheduled before i' whenever $i < i'$). Consider the following instance of $B||C_{\max}$: there are m jobs and m machines and the processing time of job i on machines $1, 2, \ldots, m-i+1$ is 1. Job i cannot be scheduled on machines $m-i+2, \ldots, m$. Assume that m is a power of 2 and $m = 2^k$.

Consider an assignment of jobs to machines as follows. Jobs 1 to $2^{k-1} = \frac{m}{2}$ are assigned to machines 1 to $\frac{m}{2}$ respectively. Jobs $\frac{m}{2}+1$ to $\frac{3m}{4}$ are assigned to machines 1 to $\frac{m}{4}$ respectively. Jobs $\frac{3m}{4}+1$ to $\frac{7m}{8}$ are assigned to machines 1 to $\frac{m}{8}$ respectively, and so on. It is not hard to check that this is a pure strategy Nash equilibrium of this mechanism. The makespan of this assignment is $k = \log_2 m$. In the optimal assignment job i is assigned to machine $m-i+1$. Thus the optimal makespan is 1, and so the price of anarchy is at least $\log_2 m$.

The example in the above proof can be easily changed to show that for $R||C_{\max}$, the price of anarchy of the LongestFirst and ShortestFirst policies is at least $\log_2 m$, even if there is no tie among the processing times.

Theorem 8 proves that if a coordination mechanism is deterministic and policies of different machines are the same, then we cannot hope to get a factor better than $\log_2 m$ for $R||C_{\max}$. One might hope that the Randomized policy can achieve a constant price of anarchy. However, we have the following lower bound for the Randomized policy.

Theorem 9. *The price of anarchy of the* Randomized *policy for* $R||C_{\max}$ *is at least* $m-1$.

Proof. Consider a scenario with m machines and $(m-1)^2$ jobs. Split the first $(m-1)(m-2)$ jobs into $m-1$ groups $J_1, \ldots, J_{m-1}$, each of size $m-2$ jobs. For jobs $i \in J_k$, let $p_{ik} = 1$, $p_{im} = 1/m^2$, and $p_{ij} = \infty$ for all other machines j. Form a matching between the remaining $m-1$ jobs and the first $m-1$ machines. Whenever job i is matched to machine j in this matching, set $p_{ij} = 1$ and $p_{im} = 1$.

The optimal solution has makespan 1 and assigns all jobs in $J_1, \ldots, J_{m-1}$ to the last machine and each of the remaining $m-1$ jobs to its corresponding machine in the matching. However, the solution which assigns all jobs in J_k to machine k for all $1 \leq k \leq m-1$ and all remaining $m-1$ jobs to machine m is a Nash equilibrium with makespan $m-1$. To see that this is a pure strategy Nash equilibrium, consider a job $i \in J_k$. Its disutility on machine k is $\frac{1}{2}(m-3)+1$ while its disutility if it moved to machine m would increase to $\frac{1}{2}(m-1)+1/m^2$. Therefore all jobs in $J_1, \ldots, J_{m-1}$ are playing a best response to the current set of strategies. Now consider one of the remaining $m-1$ jobs i. Say job i is matched to machine j in the matching. Then the disutility of job i on machine m is $\frac{1}{2}(m-2)+1$ while its disutility if it moved to machine j would remain $\frac{1}{2}(m-2)+1$. Since these jobs are also playing a (weakly) best response to the current set of strategies, the above scenario is a Nash equilibrium in the Randomized policy.

4 Convergence to Pure Strategy Nash Equilibria

In practice, it is undesirable if the job to machine mapping keeps changing. The system performance can be adversely affected if players keep reacting to one another's changes of strategies. A good coordination mechanism is one with a small price of anarchy and fast convergence to pure strategy Nash equilibrium. In this section we investigate the convergence of players' selfish behavior. We prove that, except for the case of the Randomized and LongestFirst policies for $R||C_{\max}$ and the Randomized policy for $Q||C_{\max}$, the selfish behavior of players converges to a pure Nash equilibrium.

We first define the notion of a state graph and a potential function. Let A_i be the set of actions of player i, $i = 1, 2, \cdots, n$. In our setting, each A_i equals the set of machines $\{1, \ldots, \}$. A state graph $G = (V, E)$ is a directed graph where V is the set of nodes $A_1 \times A_2 \cdots A_n$, and an edge labelled with i exists from state u to v if the only difference between the two states is the action of player i and i's payoff is strictly less in v. A pure strategy Nash equilibrium corresponds to a node with no outgoing edges. A potential function is a function f mapping the set of states to a totally ordered set such that $f(v)$ is strictly less than $f(u)$ for all edges $uv \in G$. In other words, whenever a player in state u changes his action and improves his payoff, the resulting state v satisfies $f(u) > f(v)$. Note that the existence of a potential function implies the state graph is acyclic and establishes the existence of pure strategy Nash equilibrium. The existence of a potential function also implies that the Nash dynamics will converge if one player takes the best response action atomically. We restrict our convergence analysis to this atomic and best-response behavior of players. A game that has a potential function is called a potential game. Many of our proofs proceed by showing that the games we have defined in this paper are in fact potential games.

We remark that the Makespan policy corresponds to a potential game. This fact has been observed in various places. In particular, Even-Dar et al. [10] give several bounds on the speed of convergence to pure Nash equilibria for this policy. The Randomized policy is the same as the Makespan policy for $B||C_{\max}$ and $P||C_{\max}$. Thus, the Randomized policy also corresponds to a potential game for $B||C_{\max}$ and $P||C_{\max}$. We do not know if the Randomized policy for $R||C_{\max}$ is a potential game or not. If it were, this would imply, among other things, that pure Nash equilibrium exist. For the rest of this section, we study the convergence for the ShortestFirst and LongestFirst policies.

4.1 Convergence for the ShortestFirst Policy

In Section 2.1, we have shown that pure strategy Nash equilibria exist for the ShortestFirst policy for $R||C_{\max}$ and can be found in polynomial time. In the following, we show that this game is a potential game and players will converge to a pure strategy Nash equilibrium. Note that this gives an alternative proof of the existence of pure strategy Nash equilibria.

Theorem 10. *The* ShortestFirst *policy for* $R||C_{\max}$ *is a potential game.*

Proof. For any state u, let $c(u)$ be the vector of job completion times sorted in increasing order. We show that as a job switches from one machine to another machine to decrease its completion time, it decreases the corresponding vector c lexicographically. Suppose the system is in state u and $c(u) = (c_1, c_2, \ldots, c_n)$. Suppose job i with completion time c_i switches machines and decreases its completion time. Call the new state v and let $c(v) = (c'_1, c'_2, \ldots, c'_n)$. Let i's completion time in v be c'_j. We know that $c'_j < c_i$. However, the change in i's action may cause an increase in the completion times of other jobs. Assume that job i switched to machine k in state v. Jobs whose completion time increases after this switch are the jobs that are scheduled on machine k and whose processing time on machine k is greater than i's processing time on machine k. Thus, the completion times of these jobs in state u (before i moves) were greater than or equal to c'_j. Thus in the resulting vector $c(v)$, we decrease an element of the vector from c_i to c'_j and we do not increase any element with value less than c'_j. Thus this switch decreases the corresponding vectors lexicographically, i.e. $c(v) < c(u)$ and so c is a potential function. This completes the proof.

Corollary 1. *Selfish behavior of players will converge to a Nash equilibrium under the* ShortestFirst *policy for* $R||C_{\max}$.

Knowing that the selfish behavior of players converges to a Nash equilibrium and the social value of a Nash equilibrium is bounded does not indicate a fast convergence to good solutions. We are interested in the speed of convergence to a Nash equilibrium. We consider the best responses of jobs and prove fast convergence to Nash equilibria for the ShortestFirst policy.

In order to prove a convergence result, we must make some assumption regarding the manner in which the ShortestFirst policy resolves ties. In particular, we will require that this tie-breaking rule is deterministic and satisfies *independence of irrelevant alternatives*, i.e., the resolution of a tie between jobs i and j is not affected by the presence or absence of job k. One such tie-breaking rule is the *alphabetically first* rule. When there is a tie between the processing times of two jobs, the alphabetically first rule always chooses the one with the smaller identifier. We note that all our upper and lower bounds hold for the ShortestFirst policy with the alphabetically first rule. For simplicity, in the proof below, we will assume our ShortestFirst policy employs the alphabetically first rule to break ties.

Theorem 11. *In $R||C_{\max}$ with the* ShortestFirst *policy, best responses of jobs converge to a Nash equilibrium after n rounds of any arbitrary ordering of jobs, when ties in the coordination mechanism are resolved using the alphabetically first rule. In other words, from any state in the state graph G, it takes at most n state traversals to end up in a state with no outgoing edges.*

Proof. In the t'th round, let i_t be the alphabetically first job which achieves the minimum possible disutility among the set of jobs $J_t \equiv J - \{i_1, \ldots, i_{t-1}\}$, fixing the strategies of jobs $J - \{i_t\}$. We prove by induction that in round t, job i_t moves to some machine and remains there in subsequent rounds.

Suppose j is any machine on which i_t achieves his minimum disutility. Then in the t'th round, a best response for i_t is to move to machine j. We show that this machine is the weakly best response of i_t for *any* set of strategies of jobs J_t. By weakly best response, we mean there is no other action that, gives the player a strictly better payoff (a smaller completion time in our setting).

First notice that the disutility of i_t on j can not increase as jobs in J_t alter their strategies. This is because any job $i' \in J_t$ has processing time at least $p_{i_t j}$ on machine j and, upon equality, is alphabetically larger or else we would have set $i_t = i'$ in the t'th round. Now consider some other machine j'. Let c_j be the completion time of job i on machine j. Then any job with completion time less than c_j on machine j' in round t must be in $\{i_1, \ldots, i_{t-1}\}$ or else we would have picked this job to be i_t in round t. Thus, the strategies of these jobs are fixed. Let i' be the job on machine j' in round t with the smallest completion time that is at least c_j. If $p_{i_t j'} < p_{i' j'}$, then the strategy of i' and all other jobs scheduled after i' on j' in round t does not affect i_t's disutility for machine j'. If $p_{i_t j'} \geq p_{i' j'}$, then even if i' leaves j' in a subsequent round, the completion time of i_t on j' is still at least i''s completion time on j' in round t, or at least c_j. Thus, it is a weakly best response for i_t to remain on machine j.

This shows that it is a weakly best response for i_t to remain on machine j in all subsequent rounds.

The next theorem proves that the bound of Theorem 11 is tight.

Theorem 12. *There are instances of $R||C_{\max}$ under the* ShortestFirst *policy for which it takes n rounds of best responses of players to converge to a Nash equilibrium.*

Proof. Suppose the processing time of job j on machine i is $1 + i\epsilon + (n - j)\frac{\epsilon}{n}$ for a sufficiently small ϵ. Starting from an empty assignment, let jobs go to their best machine in the order of $1, 2, \ldots, n$. In the first round, all jobs go to the first machine. In the second round, all jobs except the n'th job go to the second machine. In the i'th round, jobs $1, 2, n - i + 1$ will go from machine $i - 1$ to machine i. At the end, job i is scheduled on machine $n - i + 1$ and it takes n rounds to converge to this equilibrium.

4.2 Convergence for the LongestFirst Policy

In the LongestFirst policy, it is possible to prove convergence in the $Q||C_{\max}$, $B||C_{\max}$, and $P||C_{\max}$ models in a manner similar to that of Theorem 11. One just must argue that in each round the job with the longest feasible processing time (among jobs not yet considered) moves to its optimal machine and remains there in subsequent rounds. For the sake of brevity, we omit this proof.

Theorem 13. *For the* LongestFirst *policy in $Q||C_{\max}$, $B||C_{\max}$, or $P||C_{\max}$, the best responses of jobs converge to a Nash equilibrium after n rounds of any arbitrary ordering of jobs, when ties in the coordination mechanism are resolved using the alphabetically first rule.*

We note that Theorem 13 proves the existence of a Nash equilibrium in these games. We do not know how to prove that the LongestFirst policy converges in the $R||C_{\max}$model, although given that the price of anarchy would be unbounded anyway, such a proof is of somewhat questionable value.

5 Conclusion and Future Work

We have studied abstract scheduling games where the disutility of each player is its completion time. We note that our results can be applied in many practical network settings. Our results can be directly applied to the Internet setting [27] where there is a set of selfish clients, each of whom must choose a server from a set of servers. Each client tries to minimize its latency, or job completion time; the social welfare is the total system-wide latency. Similar problems arise in the wireless network setting. For example, the basic fairness and load balancing problem in wireless LANs [16] is reduced to the problem of unrelated parallel machine scheduling. Centralized algorithms have been designed for this problem in [16]. We hope to use ideas from our coordination mechanisms to design decentralized algorithms for this problem. In the third generation wireless data networks, the channel quality of a client is typically time-varying [5]; it would be interesting to study coordination mechanisms in this context given that users may exhibit selfish behavior.

Theoretically, the most interesting open problem in this paper is to find coordination mechanisms with constant price of anarchy for $B||C_{\max}$ and $R||C_{\max}$. We have shown that this cannot be achieved by any deterministic coordination mechanism which uses a common tie-breaking rule. Another problem left open in this paper is the existence of pure Nash equilibria for the Randomized policy for $R||C_{\max}$.

Through-out this paper, we assumed all information regarding job processing times was public knowledge. A new direction considered in [6] and [21] is the design of coordination mechanisms in a private information setting, i.e. where a job's processing time is a private value. In such a setting, it would be nice if a coordination mechanism incentivizes jobs to announce their true processing times. The only constant-factor price of anarchy for $Q||C_{\max}$ and the best factor for $P||C_{\max}$ in our paper are achieved using the LongestFirst policy, but this policy is not truthful. In particular, jobs can artificially inflate their length (by inserting empty cycles, perhaps) and as a result actually decrease their disutility. A truthful coordination mechanism with a constant price of anarchy in these settings would be an interesting result.

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Economic Mechanisms for Shortest Path Cooperative Games with Incomplete Information

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Abstract. In this paper we present a cooperative game theoretic interpretation of the shortest path problem. We consider a *buying* agent who has a budget to go from a specified source node s to a specified target node t in a directed acyclic network. The budget may reflect the level of utility that he associates in going from node s to node t. The edges in the network are owned by individual utility maximizing agents each of whom incurs some cost in allowing its use. We investigate the design of economic mechanisms to obtain a least cost path from s to t and to share the surplus (difference between the budget and the cost of the shortest path) generated among the participating agents in a *fair* manner. Previous work related to this problem assumes that cost and budget information is common knowledge. This assumption can be severely restrictive in many common applications. We relax this assumption and allow both budget and cost information to be private, hence known only to the respective agents. We first develop the structure of the shortest path cooperative game with incomplete information. We then show the non-emptiness of the incentive compatible core of this game and the existence of a surplus sharing mechanism that is incentive efficient and individually rational in virtual utilities, and strongly budget balanced.

1 Introduction

The shortest path problem and its many variants such as all pairs shortest paths and stochastic shortest paths occur in a wide variety of contexts and have been studied extensively. More recently, motivated by applications in grid computing, mobile ad-hoc networks, and electronic commerce, game theoretic interpretations of the shortest path problem including both the non cooperative interpretation [1, 2, 3, 4] and the cooperative interpretation [5, 6] have emerged.

In these game theoretic interpretations, economic agents control and provide access to different resources - edges and/or nodes, of the network for a price. In addition, one other agent, hereafter called the buying agent, associates a certain level of utility, in traversing the network between two specified nodes - the source s and the target t. In the remaining part of this section we first summarize the existing state-of-the-art in analyzing shortest path games and then motivate the context for this paper.

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1.1 Extant Work and Motivation

In the noncooperative game theoretic interpretation introduced in [2] it is assumed that the cost of edges in the network are only privately known. In order to find the least cost path, a Vickrey-Clarke-Groves (VCG) mechanism (see chapter 23 of [7]) is employed to elicit truthful cost information. When this approach is analyzed from the perspective of the buying agent, it has been shown that the incentives provided through this mechanism to elicit truthful cost information may be arbitrarily high [8] and cannot be avoided [3].

In the cooperative game theoretic interpretation introduced in [5, 6], the buying agent indicates his budget limitation (which may be a proxy for the utility he obtains) in going from node s to node t. The N agents owning the edges/nodes in the network then agree to cooperate among themselves to identify a shortest (least cost) path and share the surplus revenue, if any, among themselves in some *fair* way as enunciated by solutions concepts such as the core or the Shapley value. It should be noted here that this is a complete information cooperative game and no information is privately held. However, if the agents (both the buying agent and resource owners) have access to private budget and cost information, as is often the case in real world scenarios, then there is reason to expect strategic misrepresentation of either the budget and/or cost information by them. We give below two real world scenarios where such misrepresentation can be commonly expected.

- *QoS routing in communication networks*: Consider a scenario where high quality video is to be transmitted over the public Internet with a Quality of Service (QoS) guarantee. The agent requiring such a service is willing to compensate the owners of the intermediate interconnecting networks. Clearly (1) the agent is going to act strategically so that he obtains the service at the least possible cost and (2) the resource owners will bid strategically so as to maximize their share of the surplus in the transaction.
- *Supply chain procurement*: Consider a procurement scenario with an automotive assembler where a subassembly is to be procured. The way in which a subassembly maybe put together may involve many alternatives in terms of which suppliers participate in supplying components and services. All this may be captured as a network with edges representing value added by each preceding node and the network itself converging into the node representing the assembler. All nodes with no incoming edge may be grouped into the category of source nodes and the objective now is to find a shortest path from any one of the source nodes to the terminal assemblers node. Clearly the automotive assembler as well as the suppliers can be expected to behave strategically and misrepresent budget and cost information.

To model these scenarios, we need to extend the analysis of the class of shortest path cooperative games in two directions: First, by including the buying agent as a game theoretic agent and second, by treating costs and budgets as private information. These extensions prompt us to formulate the scenarios as cooperative games with incomplete information.

1.2 Contributions and Outline

Our contributions in this paper are two-fold.

1. As far as we know this is the first time that a cooperative game with *incomplete information* arising out of the shortest path problem has been addressed. We develop the structure for this game.
2. For this class of games, we investigate the design of surplus sharing mechanisms. We invoke results in cooperative game theory to show the non-emptiness of the incentive compatible core for this class of games and then prove the existence of mechanisms that are incentive efficient and individually rational in virtual utilities, and strongly budget balanced.

The structure of the paper is as follows. In Section 2, we provide the basic notation to develop the structure of the shortest path cooperative game with incomplete information (SPCG-II). In Section 3, we show the non-emptiness of the Incentive Compatible Core for this game. In Section 4, we adapt the bargaining solution based on a generalization of the Shapley Value for the SPCG-II. Finally in Section 5, we summarize the contributions of the paper.

2 A Shortest Path Cooperative Game with Incomplete Information (SPCG-II)

To begin with, in this section we set out the basic notation required and then present the structure of the SPCG-II. We consider a directed graph $N = (V, E)$ with V, the set of vertices, E the set of edges and two special nodes s (source) and t(target). We let $n = |E|$ be the number of edges. Each edge in the network is assumed to be a commodity that is owned by an agent. For expositional clarity we assume that the number of agents is equal to the number of edges in the network. However, any analysis that follows can be extended to cases where agents own multiple edges. We therefore let $I = \{1, 2, \dots, n, n+1\}$ be the set of all agents where $1, 2, \dots, n$ are edge owning agents and agent $(n+1)$ is the buying agent.

Each agent $i \in I$ has an initial endowment vector $e_i \in \Re_+^{n+1}$ where $e_{i,j} \in \{0, 1\}, \forall j \in \{1, 2, \dots, n\}$ and $e_{i,(n+1)} \in \Re$. This implies that when agent i owns the edge $j \in E$ then $e_{ij} = 1$ and is otherwise 0. Having assumed that there is a one-to-one correspondence between the edges and the agents, we have $e_{i,i} = 1$ and $e_{i,j} = 0, \forall j \neq i$. In addition the $(n+1)^{th}$ entry in the endowment vector e_i indicates the amount of money that agent i has. For any agent $i \in I$ we let T_i denote the set of possible types. The type $t_i \in T_i$ for all edge owning agents $i \in I \backslash \{(n+1)\}$ is a description of the cost that he incurs when his edge is used. The type $t_{(n+1)} \in T_{(n+1)}$ however describes the budget of the buying agent.

Let $\mathcal{C}$ denote the set of all possible coalitions or non empty subsets of I, that is, $\mathcal{C} = \{S | S \subseteq I, S \neq \emptyset\}$. For any coalition $S \in \mathcal{C}$, we let $T_S = \times_{i \in S} T_i$ so that any $t_S \in T_S$ denotes a combination of types of all agents i in S. For the grand coalition I, we let $T = T_I = \times_{i \in I} T_i$. Now, for any subset $S \in \mathcal{C}$, which includes the agent $(n+1)$, we define a set of market transactions. This follows from a shortest path computation that is carried out after the agents i in S declare their types t_i. We call this set of market transactions as the set of possible outcomes $X_S(t_S)$, such that $X_S(t_S) = \{(\tilde{e}_i)_{i \in S} | \tilde{e}_i \in \Re_+^{n+1}$ and $\sum_{i \in S} \tilde{e}_{ij} \leq \sum_{i \in S} e_{ij}, \quad \forall j \in \{1, 2, \ldots, n, n+1\}$, where $\tilde{e}_i$ is the outcome vector of agent i after the transaction is carried out. The outcome set specifies that the reallocation of resources and money is such that there is no infusion of additional resources into the system. We also define the set X_S and X as the sets that include the outcomes for all possible type declarations $t_S \in T_S$ and all possible coalitions $S \in \mathcal{C}$. So, $X_S = \bigcup_{t_S \in T_S} X_S(t_S)$ and $X = \bigcup_{S \in \mathcal{C}} X_S$.

The reallocation of resources, i.e., the edges and the money, is carried out as follows: Given the set of edges owned by the agents in S and the edge costs declared by them, a shortest path computation identifies the set of edges whose ownership is to be transferred to the buying agent. Following this, each edge agent whose edge is transferred to the buying agent is compensated according to the declared cost. The entire surplus, defined as the difference between the budget and the cost of the shortest path, that results from the transaction is then given to either the buying agent or to one of the agents who plays an active role in providing the shortest path. Note here that from the way in which we define the outcomes, there are only a finite number of outcomes, which is one greater than the number of edge agents who provide the shortest path. In reality however, we would expect the surplus to be shared among the participating agents. This sharing of the surplus is achieved by using randomized mechanisms as state contingent contracts.

Now, for any outcome $x \in X$ and any $t \in T$, we let the utility for an agent $i \in I$ be $u_i(x,t)$. For any agent i and outcome x, the final outcome vector $\tilde{e}_i$ reflects the edges that it currently owns and the money that it has after the transfers have been carried out. That is $\tilde{e}_{i,i}$ can be either 0 or 1 and $\tilde{e}_{i,(n+1)} \in \Re$. So, the payoff that the agent receives from outcome x when its type is t_i is given by $u_i(x,t) = \tilde{e}_{i,(n+1)} + (\tilde{e}_{i,i} - 1)t_i$.

We use the notation $I - i$ to denote $I \backslash \{i\}$ and we write $t = (t_{-i}, t_i)$. Similarly, (t_{-i}, s_i) denotes the vector t where the i^{th} component is changed to s_i. Now, for any $t \in T$, we let $p_i(t_{-i}|t_i)$ denote the conditional probability that t_{-i} is the combination of types for players other than i as would be assessed by player i if t_i were his type. We will assume that these probabilities are consistent as in [9]. We are now in a position to define the structure of the SPCG-II. In line with the structure for cooperative games with incomplete information in [10], the shortest path cooperative game with incomplete information can be described by the structure below.

$$\Gamma = (X, x^*, (T_i)_{i \in I}, (u_i)_{i \in I}, (p_i)_{i \in I}) \tag{1}$$

Here, X refers to the set of all outcomes for all coalitions $S \in \mathcal{C}$ that could be formed; x^* is a default outcome that results when the agents are unable to come to an agreement over the solution. In the context of the SPCG-II, the default outcome is a null transaction whose utility for all types of all agents is 0. T_i,

u_i, and p_i are as defined earlier. This structure Γ of the game is assumed to be known to all agents. In addition we assume that each agent knows his own type before the start of negotiations. Our concern now is to develop a solution to this cooperative game and interpret the results in the context of the SPCG-II and the applications introduced in Section 1.

As opposed to solution concepts for cooperative games with complete information, where the focus is on finding an allocation of the surplus value to the participating agents, in cooperative games with incomplete information the focus is on finding mechanisms or state contingent contracts that a grand coalition of all agents agree to [10, 11]. Here, in the context of the SPCG-II, we use the same conceptual apparatus as presented in [10, 12, 13, 14, 15] and find that the solution approaches for this class of games are analogous to the *Core* and the *Shapley Value*. It appears, as we shall see below, that extensions to these concepts based on important additional insights lead to the *Incentive Compatible Core* [15] and *Myerson's generalization of the Shapley Value* [10] which we adapt as solutions to SPCG-II.

3 The Incentive Compatible Core for SPCG-II

The core as a solution concept for cooperative games with complete information is based on the premise that a group of agents can cooperate and agree upon a coordinated set of actions which can then be enforced; and the resulting feasible allocations of surplus value cannot be improved upon by any other coalition. In the context of incomplete information games, however, since we are concerned with state contingent contracts rather than allocations, the meaning of the two terms - "feasible" and "improve upon" needs a precise clarification. Feasibility in this context refers to contracts that satisfy not only physical resource constraints in each information state but also the incentive constraints that arise when information is private and is inherently unverifiable. And secondly "improving upon" a state contingent contract implies that agents need to examine what information they use at the time of evaluating contracts. The evaluations may be carried out either at the ex-ante stage when none of the agents has any type information or at the interim stage when individuals know their own type information but not that of other agents. For the class of games that we are concerned with here, agents already possess private information when they enter into negotiations. So, the evaluation of contracts should be done at the interim stage and the measure of an agent's well being is based on conditionally expected utilities (conditional on private information). Our further analysis is based on this measure of evaluation. Before that we introduce some additional notation to enable the analysis that follows.

Let $\triangle X$ and $\triangle X_S$ be the sets of all possible probability distributions on X and X_S respectively. Now, we define μ and μ_S as mappings from T and T_S to $\triangle X$ and $\triangle X_S$ respectively. i.e., $\mu : T \rightarrow \triangle X; \quad \mu_S : T_S \rightarrow \triangle X_S$. Now, μ and μ_S may be viewed as direct random mechanisms. Note also that a state contingent contract can be written as function from T to $\triangle X$. So, while strictly a mechanism should be seen as a means to implement a state contingent allocation, here we interpret it as a state contingent contract. Having defined a mechanism, we can now define the conditionally expected utilities of the agents. We let $U_i(\mu, s_i|t_i)$ be the conditionally expected utility of agent i from the mechanism μ, if i's true type is t_i but he reports s_i while all other agents report their types truthfully. So we have,

$$U_i(\mu, s_i|t_i) = \sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i) \sum_{x\in X} \mu(x|t_{-i}, s_i)u_i(x,t) \tag{2}$$

$$U_i(\mu, t_i) = U_i(\mu, t_i|t_i) = \sum_{t_{-i}\in T_{-i}} p_i(t_{-i}|t_i) \sum_{x\in X} \mu(x|t)u_i(x,t) \tag{3}$$

Now, from our definition of outcomes and mechanisms, we can easily verify that the mechanisms always meet the resource feasibility constraints. Since a feasible mechanism is required to satisfy both physical resource constraints and incentive constraints, we now define Π as the set of all mechanisms which are physically feasible and Π^* as the set of all mechanisms which are also incentive compatible. So we have,

$$\Pi = \{\mu : T \rightarrow \triangle X \quad | \quad \mu(x|t) \geq 0 \quad \text{and} \quad \sum_{x\in X} \mu(x|t) = 1\} \tag{4}$$

$$\Pi^* = \{\mu \in \Pi \quad | \quad U_i(\mu, t_i) \geq U_i(\mu, s_i|t_i) \quad \forall s_i, t_i \in T_i, \forall i \in I\} \tag{5}$$

These two equations taken together imply that the set of Bayesian incentive compatible mechanisms Π^*, is a subset of Π, the set of resource feasible mechanisms that obey probability constraints. In a similar vein, we define Π_S and Π_S^* as the set of resource feasible mechanisms and incentive compatible mechanisms respectively for a coalition $S \subseteq \mathcal{C}$.

$$\Pi_S = \{\mu_S : T_S \rightarrow \triangle X_S \quad | \quad \mu(x_S|t_S) \geq 0 \quad \text{and} \quad \sum_{x_S\in X_S} \mu(x_S|t_S) = 1\} \tag{6}$$

$$\Pi_S^* = \{\mu_S \in \Pi_S \quad | \quad U_i(\mu_S, t_i) \geq U_i(\mu_S, s_i|t_i) \quad \forall s_i, t_i \in T_i, \forall i \in S\} \tag{7}$$

Now, in the spirit of the core defined for cooperative games with complete information, we can say that a mechanism or a state contingent contract is in the core of the SPCG-II if no subset of agents stands to gain by breaking away and negotiating a separate contract which gives them all a better expected utility. But in order to break away and negotiate a more beneficial contract, the agents in the breakaway coalition, say S, must be able to gain in an event that is discernible by all of them. We define an event Λ as $\Lambda = \times_{i\in I}\Lambda_i$, where $\Lambda_i \subseteq T_i$. This event Λ is discernible by a coalition S (or is common knowledge within S) if $p_i(\hat{t}_{-i}|t_i) = 0, \forall i \in S, t \in \Lambda, \text{and } \hat{t} \notin \Lambda$.

Now, coalition S has an objection to a mechanism $\mu \in \Pi^*$ if there exists a contract $\mu_S \in \Pi_S^*$ and an event Λ that is discernible by S such that the following inequality holds for all agents $i \in S$ with strict inequality holding for at least one of them.

$$U_i(\mu_S|t_i) \geq U_i(\mu|t_i), \quad \forall t_i \in \Lambda_i, \forall i \in S. \tag{8}$$

The incentive compatible core consists of all mechanisms $\mu \in \Pi^*$ to which there is no such objection. In other words if a mechanism μ has to belong to the incentive compatible core then there should not exist a coalition S, an incentive compatible mechanism μ_S and an event $\Lambda \in T$ such that:

1. Λ is discernible by S,
2. $U_i(\mu_S|t_i) \geq U_i(\mu|t_i), \quad \forall t_i \in \Lambda_i, \forall i \in S,$
3. $\sum_{i \in S} \tilde{e}_{ij} \leq \sum_{i \in S} e_{ij}, \quad \forall j \in \{1, 2, \ldots, n, n+1\}; \forall t \in T,$

The question that now arises is whether the core of such a game is non-empty. The answer to this lies in recognizing the fact that the utility functions $u_i(x, t)$ are all affine linear in t_i and from Remark 3.1 in [15], it can be deduced that the incentive compatible core is indeed non-empty. This immediately gives us the following theorem.

Theorem 1. *The shortest path cooperative game with incomplete information, $\Gamma = (X, x^*, (T_i)_{i \in I}, (u_i)_{i \in I}, (p_i)_{i \in I})$, has a non-empty incentive compatible core.*

The incentive compatible core is an important solution concept, whose non-emptiness provides strong guarantees for the stability of a coalition. Many times we are also interested in finding a single solution to the SPCG-II as opposed to a set of solutions like in the core. We address this issue next.

4 A Generalization of the Shapley Value for SPCG-II

In the case of cooperative games with incomplete information, we are concerned with finding an incentive efficient mechanism or a state contingent contract agreeable to the grand coalition I so that it is both implementable, and pareto dominates all other incentive compatible mechanisms. In addition, the mechanism chosen should fairly capture the power structure of the agents in the game and must also be an adequate compromise between the types of the agents if type information is to be protected in the bargaining process. A solution for this class of games was proposed in [10] which we adapt here to the SPCG-II.

A mechanism μ is incentive efficient iff $\mu \in \Pi^*$ and there is no other mechanism $\nu \in \Pi^*$ such that $U_i(\nu|t_i) \geq U_i(\mu|t_i), \forall i \in I, \forall t_i \in T_i$ with strict inequality holding at least for one type t_i of some agent i. Since, X and T are finite sets, the set of incentive compatible mechanisms Π^* is a closed convex polyhedron defined by incentive compatibility constraints and probability constraints. These constraints are linear and hence by the supporting hyperplane theorem if there is a set of utility transfer weights $\lambda \in \times_{i \in I} \Re^{T_i}$ then the incentive efficient mechanism μ is a solution to an appropriate linear programming problem (LP1). This is given by:

$$\max_{\mu \in \Pi^*} \sum_{i \in I} \sum_{t_i \in T_i} \lambda_i(t_i) U_i(\mu|t_i) \tag{9}$$

subject to:

$$U_i(\mu|t_i) \geq U_i(\mu, s_i|t_i), \quad \forall i \in I, \forall t_i \in T_i, \forall s_i \in T_i. \tag{10}$$

$$\mu(x|t) \geq 0, \forall x \in X; \quad \text{and} \quad \sum_{x \in X} \mu(x|t) = 1, \forall t \in T \tag{11}$$

The dual of LP1 can be constructed by letting $\alpha_i(s_i|t_i)$ be the dual variable corresponding to the incentive compatible constraint that requires agent i to not gain by claiming that his type is s_i when it is actually t_i. So, $\alpha \in \times_{i \in I} \Re^{T_i \times T_i}$ is a vector of dual variables. Now, a Lagrangian function can be written by multiplying each of the incentive compatibility constraints with its corresponding dual variable and adding it to the objective function of LP1. We then define $v_i(x, t, \lambda, \alpha)$ as the virtual utility of agent i with respect to λ and α for an outcome x when the type profile is t [10]. This is given by:

$$\begin{aligned} v_i(d, t, \lambda, \alpha) = \{\{\lambda_i(t_i) + \sum_{s_i \in T_i} \alpha_i(s_i|t_i)\} p_i(t_{-i}|t_i) u_i(x, t) \\ - \sum_{s_i \in T_i} \alpha_i(t_i|s_i) p_i(t_{-i}|s_i) u_i(x, (t_{-i}, s_i))\}/p(t) \end{aligned} \tag{12}$$

With this definition of virtual utilities, the Lagrangian of LP1 may be written in terms of the virtual utilities as:

$$\max_{\mu \in \Pi^*} \sum_{t \in T} p(t) \sum_{x \in X} \mu(x|t) \sum_{i \in I} v_i(x, t, \lambda, \alpha) \tag{13}$$

Notice that we now seek a mechanism that maximizes the sum of virtual utilities. It is also shown in [10], that when agents face binding incentive constraints, they appear to act according to the preferences of their virtual utilities and not their actual utilities. So, for cooperative games with incomplete information, the bargaining solution is based on conditional transfers of virtual utility rather than transfers of actual utility.

In the computation of the Shapley Value for cooperative games with complete information, the worth of each of the smaller coalitions serves only as a countervailing force to influence the final allocations of the surplus. Analogously, in the model of bargaining for incomplete information games, every coalition selects a threat mechanism against a complementary coalition. We note that the SPCG-II is a game with orthogonal coalitions, in the sense that the threat mechanisms only affect the payoffs of the agents in the coalition. In addition, only a coalition S which includes the buying agent $(n+1)$ can select a threat which has some positive utility for the coalition. All other coalitions can only select threats whose utility for the coalition is zero (empty threats!). We let $\Omega = \times_{S \in \mathcal{C}} \Pi_S$. That is any vector $\omega = (\mu_S)_{S \in \mathcal{C}} \in \Omega$ includes a specification of the threats μ_S that each coalition $S \in \mathcal{C}$ threatens to use in case of a breakdown in negotiations of the grand coalition. Since the significance of these threat mechanisms is only to influence the mechanism $\mu = \mu_I$ chosen by the grand coalition, we do not require them to be incentive compatible nor equitable. So, in our choice of threat mechanisms involving all coalitions S where S includes $(n+1)$ and $S \subset I$, we can restrict ourselves to those mechanisms which place the complete probability weight on the outcome which gives the maximum possible payoff to agent $(n+1)$.

We do this because one of the motivations in our application scenarios was to reduce the high payments that are seen when VCG mechanisms are used.

Now, we can define the warranted claims $W_S(\omega, t, \lambda, \alpha)$ of virtual utility of each coalition S with respect to λ and α given the type profile t and threat profile ω by considering only the parameters relevant to the coalition S and neglecting those of $I \backslash \{S\}$.

$$W_S(\omega, t, \lambda, \alpha) = \sum_{x_S \in X_S} \mu_S(x_S|t_S) \sum_{i \in S} v_i(x_S, t_s, \lambda, \alpha) \tag{14}$$

With these warranted claims of the coalitions we can build a characteristic function form game $W(\omega, t, \lambda, \alpha)$. That is, $W(\omega, t, \lambda, \alpha) = (W_S(\omega, t, \lambda, \alpha))_{S \in \mathcal{C}}$. The Shapley value of such a game is given by:

$$\phi_i(W(\omega, t, \lambda, \alpha)) = \sum_{S \in \mathcal{C}, S \supseteq \{i,(n+1)\}} \frac{(|S|-1)!((n+1)-|S|)!}{(n+1)!}(W_S(\omega, t, \lambda, \alpha)) \tag{15}$$

From the Shapley value, the expected virtual-utility payoff of agent i with type t_i is given by $\sum_{t_{-i} \in T_{-i}} P_i(t_{-i}|t_i) \Phi_i(W(\omega, t, \lambda, \alpha))$. We note here that the mechanism $\mu = \mu_I$ in the warranted claim $W_I(\omega, t, \lambda, \alpha)$ of the grand coalition I is the one that maximizes the sum of virtual utilities of all the agents in I and hence is the one which maximizes the Lagrangian of LP1 expressed in virtual utilities. That such a mechanism exists and is also individually rational is shown in [10]. And from our construction of the outcome set $X = \bigcup_{S \in \mathcal{C}} X_S$, where we ensure that there are no transfers of money into or out of the system, we can infer that the mechanism is strongly budget balanced. This discussion can be summarized as the following theorem:

Theorem 2. *The shortest path cooperative game with incomplete information, $\Gamma = (X, x^*, (T_i)_{i \in I}, (u_i)_{i \in I}, (p_i)_{i \in I})$, has a Shapley value mechanism that is incentive efficient and individually rational in virtual utilities, and strongly budget balanced.*

5 Summary

In this paper we have extended the analysis of shortest path cooperative games to scenarios with incomplete information where a buying agent is also a participant in the game. Such scenarios are routinely encountered in many real life applications such as supply chain procurement, Internet routing, etc. We have developed the structure for the shortest path cooperative game with incomplete information. We have then, using previous results in cooperative game theory, shown the following:

- the non-emptiness of the incentive compatible core of such a game.
- the existence of a Shapley Value mechanism that is incentive efficient and individually rational in virtual utilities and also strongly budget balanced.

We believe that this analysis can be extended to multi commodity network flow scenarios that capture more complex features of the motivating problems.

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Truth-Telling Reservations

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Abstract. We present a mechanism for reservations of bursty resources that is both truthful and robust. It consists of option contracts whose pricing structure induces users to reveal the true likelihoods that they will purchase a given resource. Users are also allowed to adjust their options as their likelihood changes. This scheme helps users save cost and the providers to plan ahead so as to reduce the risk of under-utilization and overbooking. The mechanism extracts revenue similar to that of a monopoly provider practicing temporal pricing discrimination with a user population whose preference distribution is known in advance.

1 Introduction

A number of compute intensive applications often suffer from bursty usage patterns [1, 3, 6, 7, 10], whereby the demand for information technology (IT) resources, such as memory and bandwidth, can at times exceed the installed capacity within the organization. This problem can be addressed by providers of IT services who satisfy this peak demand for a given price, playing a role similar to utilities such as electricity or natural gas.

The emergence of a utility form of IT provisioning creates in turn a number of problems for both providers and customers due to the uncertain nature of IT usage. On the provider side there is a need to design appropriate pricing schemes to encourage the use of such services and to gain better estimates of the usage pattern so as to enable effective statistical multiplexing. On the customer side, there needs to be a simple way of figuring out how to anticipate and hedge the need for uncertain demand as well as the costs that it will add to the overall IT operations.

Recently, it was proposed to use swing options for pricing the reservation of IT resources [3]. By purchasing a swing option the user pays an upfront premium to acquire the right, but not the obligation, to use a resource as defined in the option contract. As with the case with electricity, IT resources, such as bandwidth and CPU time, are non-storable and with volatile usage pattern. Thus, swing options provide flexibility in both the amount and the time period for which a resource is purchased, making them appealing to users whose bursty demand is hard to predict. From the point of view of the providers, if enough users purchase these options providers can offset the cost of providing peak capacity by multiplexing among many users.

Pricing a swing option for IT resources however, turns out to be difficult because of the complexity of the option contract and the lack of a good model

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of the spot market price. Moreover, there are two important problems that need resolution. First, the user needs to be able to estimate the amount of resources that need to be reserved as well as their cost; and second, the provider needs to put in place a mechanism that will induce truth revelation on the part of the user when stating the likelihood that a given reservation or option will be exercised.

As was shown in [3], the first problem can be addressed by providing the user with a simulation tool for estimating the cost of a reservation from a set of historical data, as well as a provision for entering the user's assumptions about aggressive or conservative swings. Because the prices for swings are set ahead of time and not by market forces, the forecasting tool also provides a powerful "what-if" capability to both the resource provider and the customer for estimating outright costs and risks associated with fluctuations in customer demand.

As to the provider's problem with asymmetric information, it could be argued that a user's historical usage pattern allows to predict his future demand. But in many cases, such as with new users, the data may not be available or reflect unanticipated user needs. Even worse, users may intentionally misrepresent the likelihood of their needs in order to gain a pricing advantage, with the consequent loss to the provider. In the original design of the swing option this was addressed by introducing a time dependent discount that induces early commitment to a contract. But this strategy still allows users to misrepresent their likelihoods the first time they buy an option.

This paper presents a solution to the truth revelation problem in reservations by designing option contracts with a pricing structure that induces users to reveal their true likelihoods that they will purchase a given resource. A user is allowed to adjust his option later if his likelihood changes. Truthful revelation helps the provider to plan ahead to reduce the risk of under-utilization and overbooking, and also helps the users to save cost. In addition to its truthfulness and robustness, the mechanism extracts revenue similar to that of a monopoly provider practicing temporal pricing discrimination with a user population whose preference distribution is known in advance [2, 5, 6, 8, 9, 10, 12].

A truth telling mechanism like the one we propose may be useful in many areas. For example, airline seats, hotel reservations, network bandwidth and tickets for popular shows would benefit from a properly priced reservation system, leading to both more predictable use and revenue generation.

2 The Two Period Model

Consider n users $\{1, 2, \ldots, n\}$ who live for two discrete periods. Each user may need have to consume one unit of resource in period 2, which he can buy from a resource provider either in period 1 at a discount price 1, or in period 2 at a higher price $C > 1$. In period 1, each user i knows the probability p_i that he will need the resource in the next period. It is not until period 2 that he can be certain about his need (unless $p = 0$ or 1). We also assume that the distributions of the users' needs are independent.

Suppose all the users wish to pay the least while behaving in a risk-neutral fashion. User i can either pay 1 in period 1, or wait until period 2 and pay C if it turns out he has to, an event that happens with probability p_i. Obviously, he will use the former strategy when $Cp_i > 1$ and the latter strategy when $Cp_i < 1$, while his cost is $\min(1, Cp_i)$.

This optimal paying plan can be very costly for the user. For example, when $C = 5$ and $p = 0.1$, the user always postpones the decision to buy until period 2 (because $Cp < 1$), ending up paying 5 for every unit he needs.

In what follows we describe a reservation mechanism that allows him to pay a small premium that guarantees his one unit of resource whenever he needs it in period 2, at a price not much higher than the discount price 1. In addition, the mechanism makes the user truthfully reveal his probability of using the resource to the provider, who can then accurately anticipate user demand. At a later stage, we show how this mechanism can be thought of as an option.

2.1 The Coordinator Game

To better illustrate the benefit of this mechanism, we introduce a third agent, the coordinator, who aggregates the users' probabilities and makes a profit while absorbing the users' risk. He does so in a two period game.

1. (Period 1) The coordinator asks each user to submit a probability q_i.
2. (Period 1) The coordinator reserves $\sum q_i$ units of resource from the resource provider (at price 1), ready to be consumed in period 2.
3. (Period 2) The coordinator delivers the reserved resource units to users who claim them. If the amount he reserved is not enough to satisfy the demand, he buys more resource from the provider (at the higher unit price C) to meet the demand.
4. (Period 2) User i pays

$$\begin{cases} f(q_i) & \text{if he needs one unit of resource,} \\ g(q_i) & \text{if he does not need it,} \end{cases} \tag{1}$$

where $f, g : [0, 1] \to \mathbb{R}^+$ are two functions whose forms will be specified later.

These terms are publicly announced to everyone, before step 1.

For the coordinator to profit, the following two conditions have to be satisfied:

Condition A. *The coordinator can make a profit by providing this service.*

Condition B. *Each user prefers to use the service provided by the coordinator, rather than to deal with the resource provider directly.*

The next two truth-telling conditions, although not absolutely necessary, are useful for conditions A and B to hold.

Condition T1. *(Step 1 truth-telling) Each user submits his true probability p_i in step 1, so that he expects to pay the least later in step 4.*

Condition T2. *(Step 3 truth-telling) In step 3, when a user does not need a resource in period 2, he reports it to the coordinator.*

From Condition T1, user i expects to pay $w(q_i) \equiv p_i f(q_i) + (1-p_i)g(q_i)$ in period 2. His optimal submission q_i^* is determined by the first-order condition $w'(q_i^*) = p_i f'(q_i^*) + (1-p_i)g'(q_i^*) = 0$. Truth-telling requires that $q_i^* = p_i$, or $p_i f'(p_i) + (1-p_i)g'(p_i) = 0$. Condition T2 simply requires that $f(p) \geq g(p)$ for all $p \in [0,1]$.

Now we study Condition A when all users submit their true probabilities $\{p_i\}$. Let U be the total resource usage of all users in period 2, and let W be the their total payment. Both U and W are random variables. Clearly, $\mathbb{E}\, U = \sum p_i$, and $\mathbb{E}\, W = \sum w(p_i)$.

Lemma 1. *If there exists an arbitrarily small $\epsilon > 0$ such that $w(p) \geq p + \epsilon$ for all $p \in [0,1]$, then $W - U \to \infty$ a.s. as $n \to \infty$. That is, by charging an arbitrarily small premium, the coordinator makes profit when there are many users (Condition A).*

Proof. This follows directly from the "X^4-strong law". (See e.g. [11]. The random usage of each user does not have to be identically distributed.)

The small number ϵ is merely a technical device. In what follows we will neglect it and use a weakened condition of Lemma 1 as a sufficient condition of Condition A (not rigorous): $w(p) \geq p$ for all $p \in [0,1]$.

Last, Condition B says that the user can save money by using the coordinator's service: $w(p) \leq \min(1, Cp)$ for all $p \in [0,1]$.

To summarize, the following conditions on concave f and g are sufficient for the truth-telling mechanism to work:

$$pf'(p) + (1-p)g'(p) = 0, \tag{2}$$

$$f(p) \geq g(p), \tag{3}$$

$$p \leq pf(p) + (1-p)g(p) \leq \min(1, Cp), \tag{4}$$

for all $p \in [0,1]$.

Consider the following choice[1]

$$f(p) = 1 + \frac{k}{2} - kp + \frac{kp^2}{2}, \tag{5}$$

$$g(p) = \frac{kp^2}{2}. \tag{6}$$

which satisfies (2). To check (3) and (4), we first calculate

$$w(p) = \left(1 + \frac{k}{2}\right)p - \frac{k}{2}p^2. \tag{7}$$

And then it is not hard to show

[1] This choice is not unique, but is analytically simple. For example, we could have chosen $g \propto p^a$ $(a > 1)$, which will have no essential impact on the rest of the paper.

Lemma 2. *For the choice of f and g in (5) and (6), conditions (3) and (4) are satisfied for $k \in [0, \min\{2(C-1), 2\}]$.*

Proof. Equation (3) is satisfied because

$$f(p) - g(p) = 1 + k\left(\frac{1}{2} - p\right) \geq 1 - \frac{k}{2} \geq 0. \quad (8)$$

To verify (4), we write

$$w(p) = p + \frac{k}{2}p(1-p) \geq p, \quad (9)$$

$$w(p) = 1 - (1-p)\left(1 - \frac{k}{2}p\right) \leq 1, \quad (10)$$

$$w(p) \leq Cp - \frac{k}{2}p^2 \leq Cp. \quad (11)$$

Fig. 1(a) shows the special case $C = 2$ and $k = 1.5$. As can be seen, the curve $w(p)$ lies completely in the triangular region. The difference between the upper blue curve and the red curve is the amount of money the user saves (varying with different p). The difference between the red curve and the lower blue line is the coordinator's expected payoff from one user. Note that his payoff is larger for values of p's lying in the middle of the range, and is zero for $p = 0$ and $p = 1$. This result is hardly surprising, for when there is no uncertainty the user does not need a coordinator at all. Thus, the coordinator makes a profit out of uncertainties in user behavior.

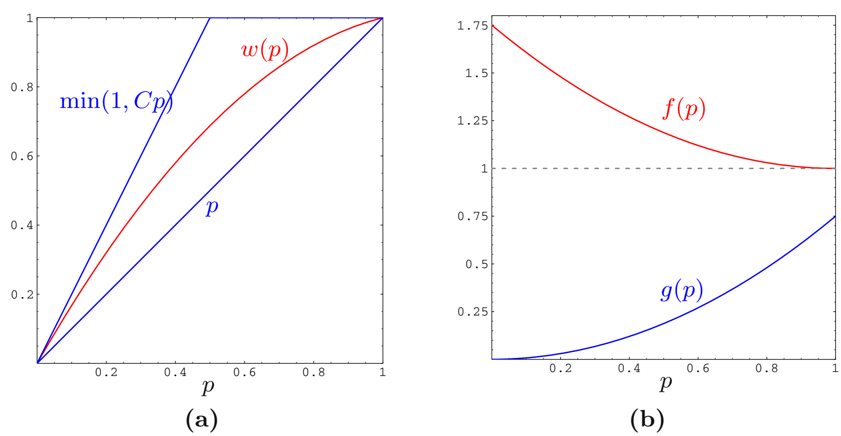

Fig. 1. Both figures are plotted under the choice $C = 2$ and $k = 1.5$. (a) Plot of $w(p)$. The curve $w(p)$ lies completely in the triangular region. (b) Payment curves of the user for $C = 2$ and $k = 1.5$. If the user needs one unit of resource in period 2, he pays according to the upper curve. Otherwise he pays according to the lower curve.

Fig. 1(b) plots the two payment curves, $f(p)$ and $g(p)$, for the same choice of parameters. After signing a contract, a user agrees to pay later either the upper curve for one unit of resource, or the lower curve for nothing. Note that $f(p)$ is strictly decreasing, a feature essential for the user to be truth-telling. A user with a high p is more likely to pay the upper curve rather than the lower curve. Knowing this, he has an incentive to submit a high probability of use and thus not to cheat.

2.2 The Reservation Contract as an Option

The contract discussed in previous sections can be equivalently regarded as an "option". Because $g(p)$ is the minimal amount the user has to pay in any event, we can ask him to pay it in period 1, and only to pay $f(p) - g(p)$ in period 2 if he needs one unit of resource at that time. Hence, by paying an amount $g(p)$, the user achieves the right but no the obligation to buy one unit of resource at price $f(p) - g(p)$ in period 2. Naturally, we may call $g(p)$ the *premium* or the *price of option*, and $f(p) - g(p)$ the *price of the resource*.

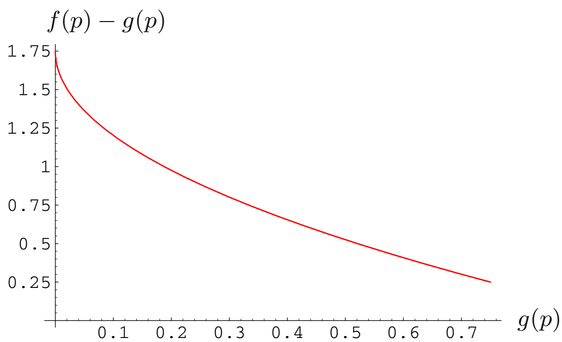

Fig. 2. Price-premium curve for the user. The horizontal axis is the premium (value of contract, value of option). The vertical axis is the price the user pays for one unit of resource in period 2.

Fig. 2 shows the parametric plot of resource price versus option price, for $p \in [0, 1]$. Instead of submitting an explicit p, the user can equivalently choose one point on this curve and pay accordingly. His probability p can then be inferred from his choice (using (5) or (6)). This alternative method may be more user-friendly because people tend to be more sensitive to monetary values rather than probabilities. We can even further simplify the curve by providing the user with a table with the values of a few discrete points along the curve.

3 A Multi Period Truth-Telling Reservation

In the previous 2-period mechanism, if a user learns more in time about the likelihood of his needing the resource, it is impossible for him to modify the original contract. To solve this issue we extend our mechanism so that the user can both submit early for a larger discount and update his probability afterwards to a more accurate one. We thus consider a dynamic extension of the problem in which the user is allowed to change his probability of future use some time after his initial submission.

3.1 The Information Structure

Assume that everyone lives for m periods. In period m the user might need to consume one unit of resource. He can reserve/buy one unit in period i at price C^{i-1}, for $i = 1, \ldots, m$.[2] The intermediate periods are introduced to exploit the user's information gaining process. We assume that at each period i, the user can always make a "best guess" of his probability of usage, given his information up to period i. Formally, his "best guess" can be described by a random process p_t adapted to an information filtration [4], satisfying the property $\mathbb{E}_t p_{t+1} = p_t$.

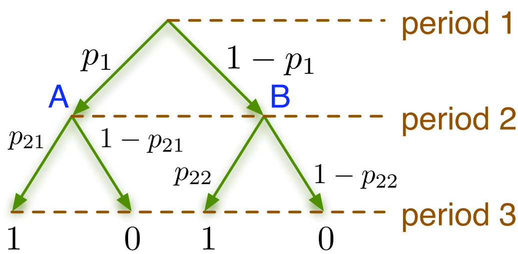

Fig. 3. The information structure for three periods

As an illustrative example, consider the three period information structure depicted in Fig. 3. The user enters state A with probability p_1 and state B with probability $1 - p_1$. If he enters state A, with probability p_{21} he will need the resource in period 3. If he enters state B, he will need the resource with probability p_{22}. Under our requirement, the user is able to make the best guess $p = p_1 p_{21} + (1 - p_1) p_{22}$ in period 1, which will change to either p_{21} or p_{22} in period 2.

3.2 The Three Period Coordinator Game

Again we describe a mechanism used by a coordinator to make profit by aggregating the user's uncertainty. A user may submit a probability in period 1, as in the 2-period setting. Additionally, when he enters period 2 he is allowed to update his probability based on his new information gained at that time. This way the user can enjoy the full discount while simultaneously utiize maximum

[2] The $(1, C, \ldots, C^{m-1})$ assumption is not essential. We could have assumed $(1, C_2, \ldots, C_m)$ instead and the main result of this section will continue to hold, just that the maths would become considerably messier.

information. His final payment in period 3 is determined by the one or two probabilities he submitted. The whole mechanism is described more rigorously as follows.

1. (Period 1) The user may submit a probability q_1.
2. (Period 1) The coordinator reserves q_1 units of resource from the resource provider (at price 1).
3. (Period 2) The user may submit a probability q_2.
4. (Period 2) The coordinator adjusts his holdings to match the new probability q_2.
5. (Period 3) If the user claims the need of one unit of resource, the coordinator delivers one reserved unit to him. If his reservation pool is not large enough, he buys more resource from the provider (at the higher unit price C^2) to meet the demand.
6. (Period 3) The user pays according to Table 1:

Table 1. The user's payment table. The columns represent his three possible submission patterns.

	q_1 not q_2	q_2 not q_1	both q_1 and q_2
uses one unit	$f_1(q_1)$	$f_2(q_2)$	$f_1(q_1) - \alpha f_2(q_1) + \alpha f_2(q_2)$
does not use	$g_1(q_1)$	$g_2(q_2)$	$g_1(q_1) - \alpha g_2(q_1) + \alpha g_2(q_2)$

In Table 1, (f_1, g_1) and (f_2, g_2) are two sets of 2-period truth-telling functions solved in Sect. 2.1.

$$f_1(p) = 1 + \frac{k_1}{2} - k_1 p + \frac{k_1 p^2}{2}, \qquad g_1(p) = \frac{k_1 p^2}{2}, \tag{12}$$

$$f_2(p) = C + \frac{k_2}{2} - k_2 p + \frac{k_2 p^2}{2}, \qquad g_2(p) = \frac{k_2 p^2}{2}, \tag{13}$$

where $k_1 \in [0, \min\{2(C^2-1), 2\}]$ and $k_2 \in [0, \min\{2(C^2-C), 2C\}]$. To make the mathematical analysis easier, we will choose $k_1 = k \in [0, \min\{2(C-1), 2\}]$ and $k_2 = Ck \in [0, \min\{2(C^2-C), 2C\}]$, so that $f_2(p) = Cf_1(p)$ and $g_2(p) = Cg_1(p)$. We require that $f_2 > f_1$ and $g_2 > g_1$, so that the user pays more when he reserves late.

Theorem 1. *Suppose $\alpha \in (0, 1/C)$. The user's optimal strategy is to submit a probability in period 1 and to adjust it in period 2. Each probability he submits is his true probability in that period. In addition, the coordinator is profitable.*

Proof. Follows from the four Lemmas in the Appendix of the full paper [13].

3.3 Three Period Options

As for the 2-period problem, there is an equivalent "option" form of the 3-period contract, which we now describe. Assume $f_2(p) = Cf_1(p)$ and $g_2(p) = Cg_1(p)$.

1. (Period 1) There are various options that the user can buy, with option price $g_1(p)$ and resource price $f_1(p) - g_1(p)$, for all $p \in [0, 1]$. The user buys one share of q_1-option at price $g_1(q_1)$.
2. (Period 2) The user can swap αC (remember $\alpha C < 1$) share of his q_1-option for a q_2-option, by paying the difference price $\alpha C(g_1(q_2) - g_1(q_1))$. Then he holds a share $(1 - \alpha C)$ of q_1-options and a share αC of q_2-options.
3. (Period 3) If the user needs one unit of resource, he executes his options. That is, he pays $(1 - \alpha C)(f_1(q_1) - g_1(q_1))$ using his q_1 option, plus $\alpha C(f_1(q_2) - g_1(q_2))$ using his q_2 option.

It is easy to verify that this option payment plan is equivalent to Table 1.

3.4 Multi Period Options

The option form of the 3-period contract can be easily extrapolated to an m-period contract ($m > 3$). Assume β is a positive number such that $\beta + \cdots + \beta^{m-2} < 1$. Such a β certainly exists. For example $0 \le \beta \le 1/2$ is enough for the condition to hold for all m. The contract now says:

1. (Period 1) There are various options that the user can buy, with option price $g_1(p)$ and resource price $f_1(p) - g_1(p)$, for all $p \in [0, 1]$. The user buys one share of q_1-option at price $g_1(q_1)$.

i. (Period i: $i = 2, \ldots, m-1$) The user can swap β^{i-1} share of his q_1-option for a q_i-option, by paying the difference price $\beta^{i-1}(g_1(q_i) - g_1(q_1))$.

m. (Period m) If the user needs one unit of resource, he executes his options. That is, he pays

$$\left(1 - \frac{\beta - \beta^{m-1}}{1 - \beta}\right)(f_1(q_1) - g_1(q_1)) + \sum_{i=2}^{m-1} \beta^{i-1}(f_1(q_i) - g_1(q_i)). \tag{14}$$

4 Mechanism Behavior

We have seen that the truth-telling reservation mechanism helps the user save money and the coordinator to make money, so they both have an incentive to use it. An interesting question to ask now is whether the resource provider himself would want to use the reservation mechanism, playing both roles of seller and coordinator. To answer this question we need to consider objective functions for both the user and the seller.

4.1 The User's Utility

In the previous sections we assumed that when it happens that the user needs one unit of resource, he has no other choice but to buy it. In reality if the on-spot price exceeds the user's financial limit, he can always choose not to buy. Because of this, the resource provider cannot set the price arbitrarily high.

Suppose the user has an expected utility in the form $u = v - c$. Here, c is the minimum expected price he has to pay for one unit of resource, estimated in period 1. v is the value of the unit to him in period 1, scaled to $v \in [0, 1]$. Equivalently, we can use period-2 value instead of period-1 value and write $u = v_2 p - c$. If the user does not buy the resource when he needs it, his utility is zero. We again assume that the user is risk-neutral.

4.2 The Seller's Problem

Direct Selling. Assume that it takes the resource provider constant cost to provide the resource, so his profit-maximization problem becomes a revenue-maximization problem (e.g., the cost of a flight is essentially independent of the number of passengers on a plane). Without using the truth-telling reservation mechanism, he can only choose a reservation price C_1 and a spot price C_2 to maximize his revenue.

To do so he must assume a prior distribution $f(v, p)$ of the users, where $f(v, p)\, dvdp$ is the fraction of users whose (v, p) lie in the small rectangle $(v, v + dv) \times (p, p + dp)$. Suppose that he has complete information about the users, i.e, he knows the real $f(v, p)$. He then faces the following maximization problem[3]:

$$\max_{0 \le C_1 \le C_2 \le 1} \iint dv\, dp\, f(v, p) I(v \ge C_1 \wedge C_2 p) C_1 \wedge C_2 p. \tag{15}$$

It is easy to calculate that for the uniform prior $f(v, p) = 1$ the maximal revenue $R_{\max} = 5/24$ is achieved at $C_1 = 1/2$ and $C_2 = 1$.

Options. Within the truth-telling reservation framework, the seller sets two prices, $f(p)$ and $g(p)$, by choosing the parameters C_1, C_2 and k. Note that C_2 does not appear explicitly in the prices, but only appears implicitly in the constraint $k \le 2(C_2 - 1)$. Thus the seller can choose a sufficiently large C_2.[4] In the many-user limit, his optimization problem becomes

$$\max_{0 \le C_1 \le 1 \le k \le 2} \iint dv\, dp\, f(v, p) I(v \ge w(C_1, k, p)) w(C_1, k, p), \tag{16}$$

where

$$w(C_1, k, p) = C_1 \left[\left(1 + \frac{k}{2}\right) p - \frac{k}{2} p^2 \right] \tag{17}$$

is the expected revenue he collects from a user whose expected value exceeds the expected cost.

For the uniform prior it can be shown that the seller's maximal revenue is again $R_{\max} = 5/24$, achieved at $C_1 = 5/8$. Hence in this case the option mechanism and direct-selling yield the same maximal revenue. While this is coincidental, as we shall see in the next section, it does suggest that the two revenues are comparable.

[3] As in standard probability texts, here $a \wedge b$ denotes the minimum of a and b, and $I(\cdot)$ is the indicator function.

[4] This may seem surprising, but remember that the user never pays the on-spot price when he buys an option! In fact, C_2 can be set greater than 1 in this case.

4.3 Other Distributions

We will now compare the two pricing schemes for other probability distributions. Again assume that v and p are independent, and v is uniform on $[0, 1]$. Assume now that p is uniformly distributed on $[a, b]$, where $0 \leq a \leq b \leq 1$.

We optimize the seller's revenue for the two schemes with multiple choices of a and b. The numerical results are shown in Table 2. It can be seen that in most cases the option mechanism performs better than the direct mechanism. In particular when the users' probabilities are concentrated at the small end (row $(0, 1/2)$, $(0, 1/3)$ and $(0, 1/5)$ in the table), the option mechanism significantly beats direct selling. This is because in the direct selling scheme, the seller has to compromise for a low C_1 for small p, therefore losing considerable profit. On the other hand, by selling options he can settle on a much higher C_1 and profit from the premium.

Table 2. The seller's revenue per person, using direct selling or options. For example, when the users' p is uniformly distributed over $(0, 1/2)$, the seller's revenue per person when using the option mechanism is 0.197.

p	$(0,1)$	$(0,1/2)$	$(1/2,1)$	$(0,1/3)$	$(1/3,2/3)$	$(2/3,1)$	$(0,1/5)$	$(2/5,3/5)$	$(4/5,1)$
direct	0.208	0.167	0.250	0.130	0.245	0.250	0.087	0.248	0.250
options	0.208	0.197	0.248	0.183	0.246	0.250	0.141	0.249	0.250

We thus conclude that the truth-telling mechanism is particularly efficient for reservations of peak demands and rare events (small p).

5 Conclusion

In this paper we presented a solution to the truth revelation problem in reservations by designing option contracts with a pricing structure that induces users to reveal their true likelihoods that they will purchase a given resource. Truthful revelation helps the provider to plan ahead to reduce the risk of under-utilization and overbooking. In addition to its truthfulness and robustness, the scheme can extract similar revenue to that of a monopoly provider who has accurate information about the population's probability distribution and uses temporal discrimination pricing.

This mechanism can be applied to any resource that exhibits bursty usage, from IT provisioning and network bandwidth, to conference rooms and airline and hotel reservations, and solves an information asymmetry problem for the provider that has traditionally led to inefficient over or under provision.

This approach can be extended in a number of ways so as to become useful in a number of realistic situations. With the addition of a simulation tool developed in the context of swing options [3], for example, users can anticipate their future needs for resources at given times and price them accordingly before committing to a reservation contract. Yet another extension would allow for the reservation of single units of a resource (airline seats or conference rooms, for example) over a time interval, as opposed to a particular date.

Given the rather inefficient way through which most bursty resources are now allocated, we believe that this mechanism will contribute to a more useful and profitable way of allocating them to those who need them, while giving the provider essential information on future demand that he can then use to rationally plan its provisioning.

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Inapproximability Results for Combinatorial Auctions with Submodular Utility Functions

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Abstract. We consider the following allocation problem arising in the setting of combinatorial auctions: a set of goods is to be allocated to a set of players so as to maximize the sum of the utilities of the players (i.e., the social welfare). In the case when the utility of each player is a monotone submodular function, we prove that there is no polynomial time approximation algorithm which approximates the maximum social welfare by a factor better than $1 - 1/e \simeq 0.632$, unless **P**= **NP**. Our result is based on a reduction from a multi-prover proof system for MAX-3-COLORING.

1 Introduction

A large volume of transactions is nowadays conducted via auctions, including auction services on the internet (e.g., eBay) as well as FCC auctions of spectrum licences. Recently, there has been a lot of interest in auctions with complex bidding and allocation possibilities that can capture various dependencies between a large number of items being sold. A very general model which can express such complex scenarios is that of combinatorial auctions.

In a combinatorial auction, a set of goods is to be allocated to a set of players. A utility function is associated with each player specifying the happiness of the player for each subset of the goods. One natural objective for the auctioneer is to maximize the economic efficiency of the auction, which is the sum of the utilities of all the players. Formally, the *allocation problem* is defined as follows: We have a set M of m indivisible goods and n players. Player i has a monotone utility function $v_i : 2^M \rightarrow \mathrm{R}$. We wish to find a partition $(S_1, \ldots, S_n)$ of the set of goods among the n players that maximizes the total utility or *social welfare*, $\sum_i v_i(S_i)$. Such an allocation is called an optimal allocation.

We are interested in the computational complexity of the allocation problem and we would like an algorithm which runs in time polynomial in n and m. However, one can see that the input representation is itself exponential in m for general utility functions. Even if the utility functions have a succinct representation (polynomial in n and m), the allocation problem may be **NP**-hard [13, 1]. In the absence of a succinct representation, it is typically assumed that the auctioneer has oracle access to the players' utilities and that he can ask queries to the players. There are 2 types of queries that have been considered. In a *value query* the auctioneer specifies a subset $S \subseteq M$ and asks player i for the value

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$v_i(S)$. In a *demand query*, the auctioneer presents a set of prices for the goods and asks a player for the set S of goods that maximizes his profit (which is his utility for S minus the sum of the prices of the goods in S). Note that if we have a succinct representation of the utility functions then we can always simulate value queries. Even with queries the problem remains hard. Hence we are interested in approximation algorithms and inapproximability results.

A natural class of utility functions that has been studied extensively in the literature is the class of submodular functions. A function v is submodular if for any 2 sets of goods $S \subseteq T$, the marginal contribution of a good $x \notin T$, is bigger when added to S than when added to T, i.e., $v(S \cup x) - v(S) \geq v(T \cup x) - v(T)$. Submodularity can be seen as the discrete analog of concavity and arises naturally in economic settings since it captures the property that marginal utilities are decreasing as we allocate more goods to a player.

1.1 Previous Work

For general utility functions, the allocation problem is **NP**-hard. Approximation algorithms have been obtained that achieve factors of $O(\frac{1}{\sqrt{m}})$ ([14, 5], using demand queries) and $O(\frac{\sqrt{\log m}}{m})$ ([12], using value queries). It has also been shown that we cannot have polynomial time algorithms with a factor better than $O(\frac{\log m}{m})$ ([5], using value queries) or better than $O(\frac{1}{m^{1/2-\epsilon}})$ ([14, 19], even for single minded bidders). Even if we allow demand queries, exponential communication is required to achieve any approximation guarantee better than $O(\frac{1}{m^{1/2-\epsilon}})$ [16]. For single-minded bidders, as well as for other classes of utility functions, approximation algorithms have been obtained, among others, in [2, 4, 14]. For more results on the allocation problem with general utilities, see [6].

For the class of submodular utility functions, the allocation problem is still **NP**-hard. The following positive results are known: In [13] it was shown that a simple greedy algorithm using value queries achieves an approximation ratio of $1/2$. An improved ratio of $1 - 1/e$ was obtained in [1] for a special case of submodular functions, the class of additive valuations with budget constraints. Very recently, approximation algorithms with ratio $1-1/e$ were obtained in [7, 8] using demand queries. As for negative results, it was shown in [16] that an exponential amount of communication is needed to achieve an approximation ratio better than $1 - O(\frac{1}{m})$. In [7] it was shown that there cannot be any polynomial time algorithm in the succinct representation or the value query model with a ratio better than $50/51$, unless **P**= **NP**.

1.2 Our Result

We show that there is no polynomial time approximation algorithm for the allocation problem with monotone submodular utility functions achieving a ratio better than $1 - 1/e$, unless **P**= **NP**. Our result is true in the succinct representation model, and hence also in the value query model. The result does not hold if the algorithm is allowed to use demand queries.

A hardness result of $1 - 1/e$ for the class XOS (which strictly contains the class of submodular functions) is obtained in [7] by a gadget reduction from the

Table 1. Approximability results for submodular utilities

	Algorithms	Hardness
Value Queries	1/2 [13]	$1 - 1/e$
Demand Queries	$1 - 1/e$ [8]	$1 - O(1/m)$ [16]

maximum k-coverage problem. For a definition of the class XOS, see [13]. Similar reductions do not seem to work for submodular functions. Instead we provide a reduction from multi-prover proof systems for MAX-3-COLORING. Our result is based on the reduction of Feige [9] for the hardness of set-cover and maximum k-coverage. The results of [9] use a reduction from a multi-prover proof system for MAX-3-SAT. This is not sufficient to give a hardness result for the allocation problem, as explained in Section 3. Instead, we use a proof system for MAX-3-COLORING. We then define an instance of the allocation problem and show that the new proof system enables all players to achieve maximum possible utility in the yes case, and ensure that in the no case, players achieve only $(1 - 1/e)$ of the maximum utility on the average. The crucial property of the new proof system is that when a graph is 3-colorable, there are in fact many different proofs (i.e., colorings) that make the verifier accept. This would not be true if we start with a proof system for MAX-3-SAT. By introducing a correspondence between colorings and players of the allocation instance, we obtain the desired result. The idea of using MAX-3-COLORING instead of MAX-3-SAT in Feige's proof system to have instances with many "disjoint" solutions is not new. The same approach is used in [10] (based on ideas of [11]) to prove a hardness result of $\log n$ for the domatic number problem.

The current state of the art for the allocation problem with submodular utilities, including our result, is summarized in Table 1. We note that we do not address the question of obtaining truthful mechanisms for the allocation problem. For some classes of functions, incentive compatible mechanisms have been obtained that also achieve reasonable approximations to the allocation problem (e.g. [14, 2, 4]). For submodular utilities, the only truthful mechanism known is obtained in [7]. This achieves an $O(\frac{1}{\sqrt{m}})$-approximation. Obtaining a truthful mechanism with a better approximation guarantee seems to be a challenging open problem.

2 Model, Definitions and Notation

We assume we have a set of players $N = \{1, ..., n\}$ and a set of goods $M = \{1, ..., m\}$ to be allocated to the players. Each player has a utility function v_i, where for a set $S \subseteq M$, $v_i(S)$ is the utility that player i derives if he obtains the set S. We make the standard assumptions that v_i is monotone and that $v_i(\emptyset) = 0$.

Definition 1. *A function $v : 2^M \to R$ is submodular if for any sets $S \subset T$ and any $x \in M \backslash T$:*

$$v(S \cup \{x\}) - v(S) \geq v(T \cup \{x\}) - v(T)$$

An equivalent definition of submodular functions is that for any sets S, T: $v(S \cup T) + v(S \cap T) \leq v(S) + v(T)$.

An allocation of M is a partition of the goods $(S_1, ..., S_n)$ such that $\bigcup_i S_i = M$ and $S_i \cap S_j = \emptyset$. The allocation problem we will consider is:

> **The allocation problem with submodular utilities:** Given a monotone, submodular utility function v_i for every player i, find an allocation of the goods $(S_1, ..., S_n)$ that maximizes $\sum_i v_i(S_i)$.

To clarify how the input is accessed, we assume that either the utility functions have a succinct representation[1], or that the auctioneer can ask value queries to the players. In a value query, the auctioneer specifies a subset S to a player i and the player responds with $v_i(S)$. In this case the auctioneer is allowed to ask at most a polynomial number of value queries.

Since the allocation problem is **NP**-hard, we are interested in polynomial time approximation algorithms or hardness of approximation results: an algorithm achieves an approximation ratio of $\alpha \leq 1$ if for every instance of the problem, the algorithm returns an allocation with social welfare at least α times the optimal social welfare.

3 The Main Result

In this Section we present our main theorem:

Theorem 1. *For any $\epsilon > 0$, there is no polynomial time $(1 - \frac{1}{e} + \epsilon)$-approximation algorithm for the allocation problem with monotone submodular utilities, unless* **P**=**NP**.

For ease of exposition, we first present a weaker hardness result of 3/4. This proof is provided here only to illustrate the main ideas of our result and to give some intuition. At the end of this Section, we explain what modifications are required to obtain a hardness of $1 - 1/e$.

The reduction for the 3/4-hardness is based on a 2-prover proof system for MAX-3-COLORING, which is analogous to the proof system of [15] for MAX-3-SAT. In the MAX-3-COLORING problem, we are given a graph G and we are asked to color the vertices of G with 3 different colors so as to maximize the number of properly colored edges, where an edge is properly colored if its vertices receive different colors. Given a graph G, let $OPT(G)$ denote the maximum fraction of edges that can be properly colored by any 3-coloring of the vertices. We will start with a *gap* version of MAX-3-COLORING: Given a constant c, we denote by GAP-MAX-3-COLORING(c) the promise problem in which the yes instances are the graphs with $OPT(G) = 1$ and the no instances are graphs with $OPT(G) \leq c$. By the PCP theorem [3], and by [17], we know:

[1] By this we mean a representation of size polynomial in n and m, such that given S and i, the auctioneer can compute $v_i(S)$ in time polynomial in the size of the representation. For example, additive valuations with budget limits [13] have a succinct representation.

Proposition 1. *There is a constant $c < 1$ such that GAP-MAX-3-COLORING(c) is* **NP***-hard, i.e., it is* **NP***-hard to distinguish whether*
YES Case: $OPT(G) = 1$, and
NO Case: $OPT(G) \leq c$.

Let G be an instance of GAP-MAX-3-COLORING(c). The first step in our proof is a reduction to the Label Cover problem. A label cover instance L consists of a graph G', a set of labels Λ and a binary relation $\pi_e \subseteq \Lambda \times \Lambda$ for every edge e. The relation π_e can be seen as a constraint on the labels of the vertices of e. An assignment of one label to each vertex is called a *labeling*. Given a labeling, we will say that the constraint of an edge $e = (u, v)$ is satisfied if $(l(u), l(v)) \in \pi_e$, where $l(u), l(v)$ are the labels of u, v respectively. The goal is to find a labeling of the vertices that satisfies the maximum fraction of edges of G', denoted by $OPT(L)$.

The instance L produced from G is the following: G' has one vertex for every edge (u, v) of G. The vertices (u_1, v_1) and (u_2, v_2) of G' are adjacent if and only if the edges (u_1, v_1) and (u_2, v_2) have one common vertex in G. Each vertex (u, v) of G' has 6 labels corresponding to the 6 different proper colorings of (u, v) using 3 colors. For an edge between (u_1, v_1) and (u_2, v_2) in G', the corresponding constraint is satisfied if the labels of (u_1, v_1) and (u_2, v_2) assign the same color to their common vertex. From Proposition 1 it follows easily that:

Proposition 2. *It is* **NP***-hard to distinguish between:*
YES Case: $OPT(L) = 1$, and
NO Case: $OPT(L) \leq c'$, for some constant $c' < 1$

We will say that 2 labelings L_1, L_2 are *disjoint* if every vertex of G' receives a different label in L_1 and L_2. Note that in the YES case, there are in fact 6 disjoint labelings that satisfy all the constraints.

The Label Cover instance L is essentially a description of a 2-prover 1-round proof system for MAX-3-COLORING with completeness parameter equal to 1 and soundness parameter equal to c' (see [9, 15] for more on these proof systems).

Given L, we will now define a new label cover instance L', the hardness of which will imply hardness of the allocation problem. Going from instance L to L' is equivalent to applying the parallel repetition theorem of Raz [18] to the 2-prover proof system for MAX-3-COLORING.

We will denote by H the graph in the new label cover instance L'. A vertex of H is now an ordered tuple of s vertices of G', i.e., it is an ordered tuple of s edges of G, where s is a constant to be determined later . We will refer to the vertices of H as nodes to distinguish them from the vertices of G. For 2 nodes of H, $u = (e_1, ..., e_s)$ and $v = (e'_1, ..., e'_s)$, there is an edge between u and v if and only if for every $i \in [s]$, the edges e_i and e'_i have exactly one common vertex (where $[s] = \{1, ..., s\}$). We will refer to these s common vertices as the *shared* vertices of u and v. The set of labels of a node $v = (e_1, ..., e_s)$ is the set of 6^s proper colorings of its edges ($\Lambda = [6^s]$). The constraints can be defined analogously to the constraints in L. In particular, for an edge $e = (u, v)$ of H, a labeling satisfies the constraint of edge e if the labels of u and v induce the same coloring of their shared vertices.

By Proposition 2 and Raz's parallel repetition theorem [18], we can show that:

Proposition 3. *It is* **NP***-hard to distinguish between:*
YES Case: $OPT(L') = 1$*, and*
NO Case: $OPT(L') \leq 2^{-\gamma s}$*, for some constant* $\gamma > 0$*.*

Remark 1. In fact, in the YES case, there are 6^s disjoint labelings that satisfy all the constraints.

This property will be used crucially in the remaining section. The known reductions from GAP-MAX-3-SAT to label cover, implicit in [9, 15], are not sufficient to guarantee that there is more than one labeling satisfying all the edges. This was our motivation for using GAP-MAX-3-COLORING instead.

The final step of the proof is to define an instance of the allocation problem from L'. For that we need the following definition:

Definition 2. *A Partition System* $P(U, r, h, t)$ *consists of a universe* U *of* r *elements, and* t *pairs of sets* $(A_1, \bar{A}_1), ...(A_t, \bar{A}_t)$*,* $(A_i \subset U)$ *with the property that any collection of* $h' \leq h$ *sets without a complementary pair* $A_i, \bar{A}_i$ *covers at most* $(1 - 1/2^{h'})r$ *elements.*

If $U = \{0, 1\}^t$, we can construct a partition system $P(U, r, h, t)$ with $r = 2^h$ and $h = t$. For $i = 1, ..., t$ the pair $(A_i, \bar{A}_i)$ will be the partition of U according to the value of each element in the i-th coordinate. In this case the sets $A_i, \bar{A}_i$ have cardinality $r/2$.

For every edge e in the label cover instance L', we construct a partition system $P^e(U^e, r, h, t = h = 3^s)$ on a separate subuniverse U^e as described above. Call the partitions $(A_1^e, \bar{A}_1^e), ..., (A_t^e, \bar{A}_t^e)$.

Recall that for every edge $e = (u, v)$, there are 3^s different colorings of the s shared vertices of u and v. Assuming some arbitrary ordering of these colorings, we will say that the pair $(A_i^e, \bar{A}_i^e)$ of P^e corresponds to the ith coloring of the shared vertices.

We define a set system on the whole universe $\bigcup U^e$. For every node v and every label i, we have a set $S_{v,i}$. For every edge e incident on v, $S_{v,i}$ contains one set from every partition system P^e, as follows. Consider an edge $e = (v, w)$. Then A_j^e contributes to all the sets $S_{v,i}$ such that label i in node v induces the jth coloring of the shared vertices between v and w. Similarly $\bar{A}_j^e$ contributes to all the $S_{w,i}$ such that label i in node w gives the jth coloring to the shared vertices (the choice of assigning A_j^e to the $S_{v,i}$'s and $\bar{A}_j^e$ to the $S_{w,i}$'s is made arbitrarily for each edge (v, w)). Thus

$$S_{v,i} = \bigcup_{(v,w) \in E} B_j^{(v,w)}$$

where E is the set of edges of H, $B_j^{(v,w)}$ is one of $A_j^{(v,w)}$ or $\bar{A}_j^{(v,w)}$, and j is the coloring that label i gives to the shared vertices of (v, w).

We are now ready to define our instance I of the allocation problem. There are $n = 6^s$ players, all having the same utility function. The goods are the sets $S_{v,i}$ for each node v and label i. If a player is allocated a collection of goods $S_{v_1,i_1}...S_{v_l,i_l}$, then his utility is

$$|\bigcup_{j=1}^{l} S_{v_j,i_j}|$$

It is easy to verify that this is a monotone and submodular utility function. Let $OPT(I)$ be the optimal solution to the instance I.

Lemma 1. *If $OPT(L') = 1$, then $OPT(I) = nr|E|$.*

Proof. From Remark 1, there are $n = 6^s$ disjoint labelings that satisfy all the constraints of L'. Consider the ith such labeling. This defines an allocation to the ith player as follows: we allocate the goods $S_{v,l(v)}$ for each node v, to player i, where $l(v)$ is the label of v in this ith labeling. Since the labeling satisfies all the edges, the corresponding sets $S_{v,l(v)}$ cover all the subuniverses. To see this, fix an edge $e = (v, w)$. The labeling satisfies e, hence the labels of v and w induce the same coloring of the shared vertices between v and w, and therefore they both correspond to the same partition of P^e, say $(A^e_j, \bar{A}^e_j)$. Thus U^e is covered by the sets $S_{v,l(v)}$ and $S_{w,l(w)}$ and the utility of player i is $r|E|$. We can find such an allocation for every player, since the labelings are disjoint.

For the No case, consider the following simple allocation: each player gets exactly one set from every node. Hence each player i defines a labeling, which cannot satisfy more than $2^{-\gamma s}$ fraction of the edges. For the rest of the edges, the 2 sets of player i come from different partitions and hence can cover at most 3/4 of the subuniverse. This shows that the total utility of this simple allocation is at most 3/4 of that in the Yes case. In fact, we will show that this is true for any allocation.

Lemma 2. *If $OPT(L') \le 2^{-\gamma s}$, then $OPT(I) \le (3/4+\epsilon)nr|E|$, for some small constant $\epsilon > 0$ that depends on s.*

Proof. Consider an allocation of goods to the players. If player i receives sets $S_1, ..., S_l$, then his utility p_i can be decomposed as $p_i = \sum_e p_{i,e}$, where

$$p_{i,e} = |(\cup_j S_j) \cap U^e|$$

Fix an edge (u, v). Let m_i be the number of goods of the type $S_{u,j}$ for some j. Let m'_i be the number of goods of the type $S_{v,j}$ for some j, and let $x_i = m_i + m'_i$. Let N be the set of players. For the edge $e = (u, v)$, let N^e_1 be the set of players whose sets cover the subuniverse U^e and $N^e_2 = N \backslash N^e_1$. Let $n^e_1 = |N^e_1|$ and $n^e_2 = |N^e_2|$. Note that for $i \in N^e_1$, the contribution of the x_i sets to $p_{i,e}$ is r. For $i \in N^e_2$, it follows that the contribution of the x_i sets to $p_{i,e}$ is at most $(1 - \frac{1}{2^{x_i}})r$ by the properties of the partition system of this edge[2]. Hence the total utility derived by the players from the subuniverse U^e is

$$\sum_i p_{i,e} \leq \sum_{i \in N_1^e} r + \sum_{i \in N_2^e} (1 - \tfrac{1}{2^{x_i}}) r$$

In the YES case, the total utility derived from the subuniverse U^e was nr. Hence the loss in the total utility derived from U^e is

$$\Delta_e \geq nr - \sum_{i \in N_1^e} r - \sum_{i \in N_2^e} (1 - \tfrac{1}{2^{x_i}}) r = r \sum_{i \in N_2^e} \frac{1}{2^{x_i}}$$

By the convexity of the function 2^{-x}, we have that

$$\Delta_e \geq r\ n_2^e\ 2^{-\frac{\sum_{i \in N_2^e} x_i}{n_2^e}}$$

But note that $\sum_{i \in N_1^e} x_i \geq 2n_1^e$, since players in N_1^e cover U^e and they need at least 2 sets to do this. Therefore $\sum_{i \in N_2^e} x_i \leq 2n_2^e$ and $\Delta_e \geq r\ n_2^e/4$. The total loss is

$$\sum_e \Delta_e \geq r/4 \sum_e n_2^e$$

Hence it suffices to prove $\sum_e n_2^e \geq (1-\epsilon) n|E|$, or that $\sum_e n_1^e \leq \epsilon n|E|$.

For an edge (u, v), let $N_1^{e, \leq s}$ be the set of players from N_1^e who have at most s goods of the type $S_{u,j}$ or $S_{v,j}$. Let $N_1^{e,>s} = N_1^e \backslash N_1^{e,\leq s}$.

$$\sum_e n_1^e = \sum_e |N_1^{e,>s}| + |N_1^{e,\leq s}| \leq \frac{2n|E|}{s} + \sum_e |N_1^{e,\leq s}|$$

where the inequality follows from the fact that for the edge e we cannot have more than $2n/s$ players receiving more than s goods from u and v.

Claim. $\sum_e |N_1^{e,\leq s}| < \delta n|E|$, where $\delta \leq c' s 2^{-\gamma s}$, for some constant c'.

Proof. Suppose that the sum is $\delta n|E|$ for some $\delta \leq 1$. Then it can be easily seen that for at least $\delta|E|/2$ edges, $|N_1^{e,\leq s}| \geq \delta n/2$. Call these edges *good* edges.

Pick a player i at random. For every node, consider the set of goods assigned to player i from this node, and pick one at random. Assign the corresponding label to this node. If player i has not been assigned any good from some node, then assign an arbitrary label to this node. This defines a labeling. We look at the expected number of satisfied edges.

For every good edge $e = (u, v)$, the probability that e is satisfied is at least $\delta/2s^2$, since e has at least $\delta n/2$ players that have covered U^e with at most

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[2] To use the property of P^e, we need to ensure that $x_i \leq 3^s$. However since $i \in N_2^e$, even if $x_i > 3^s$, the distinct sets A_j^e or $\bar{A}_j^e$ that he has received through his x_i goods are all from different partitions of U_e and hence they can be at most 3^s.

s goods. Since there are at least $\delta|E|/2$ good edges, the expected number of satisfied edges is at least $\delta^2|E|/4s^2$. This means that there exists a labeling that satisfies at least $\delta^2|E|/4s^2$ edges. But, since $OPT(L') \leq 2^{-\gamma s}$, we get $\delta \leq c' s 2^{-\gamma s}$, for some constant c'.

We finally have

$$\sum_e n_1^e \leq \frac{2n|E|}{s} + \delta n|E| \leq \epsilon n|E|$$

where ϵ is some small constant depending on s. Therefore the total loss is

$$\sum_e \Delta_e \geq \frac{1}{4}(1-\epsilon)nr|E|$$

which implies that $OPT(I) \leq (3/4+\epsilon)nr|E|$.

Given any $\epsilon > 0$, we can choose s large enough so that Lemma 2 holds. From Lemmas 1 and 2, we have:

Corollary 1. *For any $\epsilon > 0$, there is no polynomial time $(3/4+\epsilon)$-approximation algorithm for the allocation problem with monotone submodular utilities, unless* **P** = **NP**.

To strengthen the hardness to $1-1/e$, we use a different reduction from a multi-prover proof system, using the construction of Feige [9]. The new Label Cover instances that arise in this reduction are defined on a hypergraph instead of a graph. We also need to use a more general version of partition systems, as in [9]. Due to lack of space, we omit the proof for the full version of the paper.

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An Auction-Based Market Equilibrium Algorithm for a Production Model

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Abstract. We present an auction-based algorithm for the computing market equilibrium prices in a production model, in which producers have a single linear production constraint, and consumers have linear utility functions. We provide algorithms for both the Fisher and Arrow-Debreu versions of the problem.

1 Introduction

Two basic models of market equilibria, consisting of buyers and goods, were defined by Fisher [2] and Walras [20] in the 19th century. The celebrated paper of Arrow and Debreu [1] introduced a general production model and gave proofs of existence of equilibria in all these models. In the Arrow-Debreu model, each production vector (schedule) lies in a specified convex set; negative coordinates represent raw materials and positive coordinates represent finished goods. For the problem of computing the equilibrium prices, the classic work of Eisenberg and Gale [9] gave a convex program for computing equilibrium prices for Fisher's model for the case of linear utility functions. For the production model, convex programs were obtained [16, 19, 17, 18] when the model is restricted to positive production vectors only. An assumption of a set of raw goods outside the current market justifies this restricted model.

Over the last few years, there has been a surge of activity within the theoretical computer science community on the question of computing equilibria [5, 6, 13, 7, 8, 3, 15, 10, 11, 4, 14]. Perhaps the most novel aspect of this work lies in the development of combinatorial algorithms for these problems. Two techniques have mainly been successful: the primal-dual schema [6] and auctions [10]. Algorithms developed using these two approaches iteratively raise prices until equilibrium prices are found. They have been successful for models satisfying weak gross substitutability, i.e., raising the price of one good does not decrease the demand of another good.

Here we give an auction based algorithm for the following case of the production model: buyers have linear utilities for goods and producers have a single linear capacity constraint on their production schedule. As in the case of convex programs for the production model mentioned above, we assume that raw materials are external to the current market. For any given prices of goods, each

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producer chooses a feasible schedule that maximizes his profit and each buyer chooses a bundle of goods that maximizes her utility. The problem is to find prices such that the market clears, i.e., all goods produced are bought and there is no deficiency of goods.

From the point of view designing an algorithm that iteratively adjusts prices, an important difference between the consumer models of Fisher and Walras and the production models is that in the former the amount of each good available is fixed, while in the latter this changes with the prices of goods. So, while the algorithm is adjusting prices to dispose off existing goods, the amounts of goods available are also changing. However, it is easy to see that our production model (with a single constraint) satisfies weak gross substitutability in the following sense: raising the price of a good does not decrease the demand nor increase the production of another good. As a consequence, an algorithm that only increases prices can in principle arrive at the equilibrium. Indeed, our auction-based algorithm never needs to decrease prices. The actions taken by the auction-based algorithm are natural and correspond to actions performed in real markets.

2 The Production Model

We have a market with q Producers and b Consumers, and m goods. Producer s produces $z_{sj} \geq 0$ amount of good j and his production is constrained by a linear inequality of the form (5) below, where the $a_{sj} \geq 0$ and $K_s \geq 0$. The producers aim to sell the goods they produce, and they have utility for money. Consumer i has an initial endowment of e_i units of money, buys $x_{ij} \geq 0$ amount of good j, and has a linear utility function for the goods (1).

When the prices of the goods are fixed at $(p_j)_j$ consumers and producers will buy and produce so as to satisfy the following programs, termed CLP and PLP:

$$\text{Maximize}: \sum_{1\leq j\leq m} v_{ij}x_{ij} \tag{1}$$

$$\text{Subject to:} \sum_{1\leq j\leq m} x_{ij}p_j \leq e_i \tag{2}$$

$$\forall j:\ x_{ij} \geq 0 \tag{3}$$

$$\text{Maximize}: \sum_{1\leq j\leq m} p_j z_{sj} \tag{4}$$

$$\text{Subject to:} \sum_{1\leq j\leq m} z_{sj}a_{sj} \leq K_s \tag{5}$$

$$\forall j:\ z_{sj} \geq 0 \tag{6}$$

Defining α_i as the bang per buck for consumer i, and γ_s as the profit rate for producer s, and using duality theory we can write consumer and producer conditions for optimality as follows:

$$\sum_j x_{ij} p_j = e_i \tag{7}$$

$$\forall j : \alpha_i p_j \geq v_{ij} \tag{8}$$

$$\forall j : x_{ij} > 0 \Rightarrow \alpha_i p_j = v_{ij} \tag{9}$$

$$\forall i : \alpha_i \geq 0, \forall j : x_{ij} \geq 0 \tag{10}$$

$$\gamma_s > 0 \Rightarrow \forall s : \sum_j z_{sj} a_{sj} = K_s \tag{11}$$

$$\forall j : \gamma_s \geq p_j / a_{sj} \tag{12}$$

$$\forall s, j : z_{sj} > 0 \Rightarrow \gamma_s = p_j / a_{sj} \tag{13}$$

$$\forall s : \gamma_s \geq 0 \tag{14}$$

An equilibrium is a price vector $(p_j)_j$ s.t. there are productions z_{sj} and allocations x_{ij} s.t. conditions (7 to 14) hold and futhermore all produced goods are sold:

$$\forall j \ : \sum_i x_{ij} = \sum_s z_{sj} \tag{15}$$

Relaxed Equilibrium Conditions: The output of our algorithm will satisfy (8), (10), (12), (14) and (15) exactly, and will satisfy the following relaxed forms of (7), (9), (11) and (13). We will call this an ϵ-equilibrium.

(7'): $\sum_j x_{ij} p_j \leq e_i$ and $\sum_j x_{ij} p_j \geq (1-\epsilon) e_i$
(9'): $\forall j : x_{ij} > 0 \Rightarrow \alpha_i p_j \leq v_{ij}(1+\epsilon)$
(11') $\forall s : \sum_j z_{sj} a_{sj} \leq K_s$ and $\gamma_s > \epsilon \Rightarrow \sum_j z_{sj} a_{sj} = K_s$
(13'): $z_{sj} > 0 \Rightarrow \gamma_s \leq (1+\epsilon)^2 p_j / a_{sj}$

3 The Auction Algorithm

We start at very low prices $p_j = \epsilon \times a_{min}$ for all j, where $a_{min} = \min_{s,j} a_{sj}$, so that no producer produces at these prices. Buyer i has a surplus of his initial endowment e_i.

At any point during the algorithm, depending on the prices, each producer will produce a bundle of goods which satisfies (11'), (12') and (13'). We define rate$(s, j) := p_j / a_{sj}$ as the profit rate of producer s for good j. All production is completely bought at all times. Buyers buy good j at two different prices: at the current price p_j, and at $p_j/(1+\epsilon)$. We let h_{ij} be the amount of good j bought by buyer i at p_j, let y_{ij} be that at $p_j/(1+\epsilon)$ and let $x_{ij} = y_{ij} + h_{ij}$. The surplus of buyer i is $r_i = e_i - \sum_j (y_{ij} p_j/(1+\epsilon) + h_{ij} p_j)$. The total surplus is $r = \sum_i r_i$. A buyer will only buy goods which approximately maximize his bang-per-buck (9').

A buyer i with $r_i > \epsilon e_i$ tries to acquire a good j which maximizes his bang-per-buck. The auction follows the following three steps:

1. `outbid(`i`,`k`,`j`)`: If $\exists\ k : y_{kj} > 0$ then reduce y_{kj}, increase h_{ij}, return k's money and spend i's money, until either $y_{kj} = 0$ or $r_i \leq \epsilon e_i$.
2. `producers_reschedule(`i,j`)`: If after (1), $r_i > \epsilon e_i$, then the producers will react to i's request for j:
   (a) A producer s s.t. $z_{sj} > 0$ and $\sum_l z_{sl} a_{sl} < K_s$: Increase the production of j and sell it to i, until either s reaches capacity, or $r_i \leq \epsilon e_i$..
   (b) A producer s s.t. $\sum_l z_{sl} a_{sl} = K_s$, $z_{sj} > 0$, $z_{sj'} > 0$ and $p_j/a_{sj} > (1+\epsilon)p_{j'}/a_{sj'}$: Reduce the production of j' and increase that of j while maintaining the production at capacity. Buyer i is sold the extra amount of j and any buyer i' who had bought j' is now returned his money in exchange for the reduced production. This continues until either $z_{sj} = 0$ or $r_i \leq \epsilon e_i$.
3. `raise_price(`j`)`: If after (2), $r_i > \epsilon e_i$, then i will raise the price of good j by a factor of $(1+\epsilon)$. All profit rates for the producers and bang-per-bucks for the buyers are recalculated.

This process continues until each surplus r_i becomes sufficiently small.

3.1 Correctness

It is clear that during the algorithm, the producers produce within capacity and the buyers buy within budget. Conditions (8), (10), (12) and (14) hold by definition. We prove that the remaining ϵ-equilibrium conditions also hold throughout:

(9'): This holds trivially after initialization. Now buyer i is assigned a good j only if it `outbid(`i`,`k`,`j`)` or `producer_reschedule(`i`,`j`)` is performed. But these are performed only if $\alpha_i = v_{ij}/p_j$, hence (9') continues to hold. Producer_reschedule may also disallocate some good from some player, but clearly, this does not violate (9'). A call to raise_price$(\cdot, j)$ raises the price of good j and one needs to prove that (9') continues to hold. Consider two successive calls to raise_price$(\cdot, j)$. After the first call all the $h_{ij} = 0$. The second call is made only when all the $y_{ij} = 0$. But this means that a call to outbid(i, j, k) was made in between these two times. A call to outbid(i, j, k) is made only if $\alpha_i = v_{ij}/p_j$. The second raise_price only raises p_j by a factor of $(1+\epsilon)$. Hence, $\alpha_i \leq (1+\epsilon)v_{ij}/p_j$, i.e. (9') still holds.

(13'): Let rate(s, j):= p_j/a_{sj}. Take any two goods j, j', s.t. $z_{sj}, z_{sj'} > 0$. It suffices to prove that rate$(s, j') \geq$ rate(s, j) / $(1+\epsilon)^2$. Suppose not. Consider the first time this is violated. This can only happen because of a call to raise_price(j). Before this call rate$(s, j') <$ rate(s, j) / $(1+\epsilon)$. But then there should have been a producers_reschedule step in which s converts all its production of j' into j. This is a contradiction.

(15): The production of any producer is changed only in producer_reschedule and this is done keeping (15) true. Every extra amount of good produced is sold to some buyer. The buyers may also change the amount of goods they buy in an outbid operation. Even here, the invariant is maintained.

(7') and (11') do not hold throughout the algorithm, but they hold upon termination. (7') is precisely the terminating condition.

(11'): Consider the first time that γ_s became greater than ϵ. This happened because the price of some good j was raised, and j became the good with the maximum profit rate for s. But note that the price of a good is raised only if the surplus of the bidder i who raised the price of j remains significant after the procedure `producers_reschedule(`i, j`)` is run. In this procedure every producer who is producing j is made to produce more of j until he is producing to capacity. This means that s is producing at capacity when the price was raised. It is clear that after this time, the producer keeps producing at capacity, since the production level is never reduced.

Thus we have:

Lemma 1. *On termination, the algorithm finds an ϵ-equilibrium.*

3.2 Termination

It remains to prove that the algorithm indeed terminates efficiently.

Let $v_{max} = \max_{i,j} v_{ij}$ and $v_{min} = \min_{i,j} v_{ij}$

Lemma 2. *For all j, j' which are produced, $p_j/p_{j'} \leq (1+\epsilon)v_{max}/v_{min}$*

Proof. If not, there would be some amount of good j' produced, but no buyer would be willing to buy any amount of j'.

Let $a_{max} = \max_{s,j} a_{sj}$, and $K_{min} = \min_s K_s$.

Lemma 3. *For all j, $p_j < P_{max} := (1+\epsilon)em(a_{max}/K_{min})(v_{max}/v_{min})$*

Proof. If no producer is producing to capacity, then we are done, since the prices are at most ϵa_{sj}, for all s, j. Else, take any producer s who is producing to capacity K_s. Let $a_s = \max_j a_{sj}$. Then s produces at least one good upto at least $K_s/(ma_s)$ amount. Now the total amount of money in the system is e, hence the price of this good can be at most ema_s/K_s. Since the ratio of prices of any two goods which are produced is at most $(1+\epsilon)v_{max}/v_{min}$ (by Lemma 2), we get the result.

Corollary 1. *Number of calls to raise_price is at most* $\log_{1+\epsilon} P_{max}$.

In order to prove efficient convergence, we make the algorithm proceed in rounds. In a round each buyer is picked once and he reduces his surplus to 0.

Consider two successive price rises. We will first bound the number of rounds in between these price rises. Of course, in each such round, each bidder manages to reduce his surplus to 0 without raising any prices. Pick any such round R.

Lemma 4. *The total surplus reduces by a factor of* $1+\epsilon$ *in round R.*

Proof. Let r_i be the surplus of buyer i, and $r = \sum_i r_i$ be the total surplus before the round. When it is buyer i's turn in this round, he will perform a series of `outbid` and `producers_reschedule` operations.

Consider an `outbid`(i, i', j) operation. Some δ amount of good j is transferred to buyer i from some other buyer i'. But buyer i is buying at price p_j and buyer i' at price $p_j/(1+\epsilon)$. So the reduction in the surplus r_i of buyer i is $\Delta_{r_i} = \delta p_j$, and there is a net reduction in the total surplus of $\delta p_j \epsilon/(1+\epsilon) = \Delta_{r_i}\epsilon/(1+\epsilon)$.

Consider a `producers_reschedule`(i, j) operation. Some producer s converts some δ amount of production of some good j' to $(a_{sj'}/a_{sj})\delta$ amount of good j. Furthermore, s converts j' to j only if

$$p_j/a_{sj} > (1+\epsilon)p_{j'}/a_{sj'} \tag{16}$$

Buyer i buys the extra amount of good j, thus reducing his surplus by $\Delta_{r_i} = (a_{sj'}/a_{sj})\delta p_j$. The δ amount of good j' that is no longer produced causes some money to be returned to some other buyers. This is at most $\delta p_{j'}$ (it may even be less than $\delta p_{j'}$ because some buyers may have bought j' at $p_{j'}/(1+\epsilon)$). Using (16) we see that there is a net reduction in the total surplus of $\Delta_{r_i}\epsilon/(1+\epsilon)$.

As buyer i reduces his surplus from r_i to 0, we see that the total surplus goes down by $\epsilon/(1+\epsilon)r_i$. When buyer i is reducing his surplus, the surplus of another buyer can only go up. So the net reduction in the surplus during the whole round is at least $(\sum_i r_i)\epsilon/(1+\epsilon) = r\epsilon/(1+\epsilon)$. Thus the surplus at the end of the round is at most $r/(1+\epsilon)$.

Let $e = \sum_i e_i$, the total initial endowment.

Corollary 2. *There can be at most* $\log_{1+\epsilon} e$ *rounds in between two price rises.*

It remains to bound the number of outbid and producer_reschedule operations in a round which does not have a price rise. Consider player i's turn in this round during which he reduces his surplus to 0. Since he achieves this without raising any price, the number of outbids he performs is at most $(b-1)m$ and the number of producer_reschedules he causes is at most qm (recall that b is the number of buyers, q the number of producers, and m the number of goods). Thus the total number of outbids and producer_reschedules performed in a round which lies in between two price rises is at most $bm(b+q)$.

Thus we get:

Theorem 1. *The total number of operations performed by the algorithm is* $O(bm(b+q)\log_{1+\epsilon} P_{max} \log_{1+\epsilon} e)$,
where $P_{max} = (1+\epsilon)em(a_{max}/K_{min})(v_{max}/v_{min})$.

4 Arrow-Debreu Model

In this model we consider an extension of the previous model where each producer-consumer i has an initial endowment of goods denoted by $g_i = \{g_{i1}, g_{i2} \ldots g_{im}\}$. As a producer, each agent i can produce goods under constraints, and as a consumer has a linear utility function for the goods:

$$\sum_{1\le j\le m} v_{ij}x_{ij}$$

An equilibrium in this consumer-production model is a vector of prices $(p_j)_{j=1,..,m}$ so that the consumers and producers satisfy the following programs:

$$\text{Maximize}: \sum_{1\le j\le m} v_{ij}x_{ij} \tag{17}$$

$$\text{Subject to:} \sum_{1\le j\le m} x_{ij}p_j \le \sum_{1\le j\le m} g_{ij}p_j + \sum_{1\le j\le m} z_{ij}p_j \tag{18}$$

$$\forall j: \; x_{ij} \ge 0 \tag{19}$$

$$\text{Maximize}: \sum_{1\le j\le m} p_j z_{sj} \tag{20}$$

$$\text{Subject to:} \sum_{1\le j\le m} z_{sj}a_{sj} \le K_s \tag{21}$$

$$\forall j: \; z_{sj} \ge 0 \tag{22}$$

Assigning dual variables, α_i (termed bang per buck) for the consumer i, and γ_s (termed bang per buck) for producer s, and using duality theory we can write conditions for optimality as follows:

$$\sum_{1\le j\le m} x_{ij}p_j = \sum_{1\le j\le m} g_{ij}p_j + \sum_{1\le j\le m} z_{ij}p_j \tag{23}$$

$$\forall j: \alpha_i p_j \ge v_{ij} \tag{24}$$

$$\forall j: x_{ij} > 0 \Rightarrow \alpha_i p_j = v_{ij} \tag{25}$$

$$\forall i: \alpha_i \ge 0, \forall j: x_{ij} \ge 0 \tag{26}$$

$$\gamma_s > 0 \Rightarrow \forall s: \sum_j z_{sj}a_{sj} = K_s \tag{27}$$

$$\forall j: \gamma_s \ge p_j/a_{sj} \tag{28}$$

$$z_{sj} > 0 \Rightarrow \gamma_s = p_j/a_{sj} \tag{29}$$

$$\forall s: \gamma_s \ge 0 \tag{30}$$

4.1 The Algorithm

The algorithm is similar to the algorithm in the Fisher model.

Initially all items are priced at unity, unlike the Fisher case. Given these prices a production schedule is computed for each producer. The endowment of the consumers is now computed based on the amount of goods produced and available initially, i.e., $e_i = \sum_j p_{ij}(z_{ij} + g_{ij})$. The consumers bid for goods based on their utility and acquire the goods available.

At the kth iteration, which we term as a *consumption phase*, consider a consumer with surplus r_i. One of the following events can occur:

– The set of goods which maximize v_{ij}/p_j, termed the *demand* set, is computed. The buyer acquires demanded goods by an outbid procedure wherein he outbids another buyer of the good j (one who has good j at price $p_j/(1+\epsilon)$)

– If demand for good j still exists and surplus is still available but good j is not available then the producers transfer goods which are profitable in order to exhaust the surplus. The producers are considered in round robin order. Note that if the producer itself has good j as maximizing v_{sj}/p_j, i.e., in its demand set, then all its production can be allocated to j. Since the production of some goods is reduced, this generates surplus for consumers. However the surplus generated is less than the revenue gained because the goods are sold for a higher profit.
– If none of the above apply the the consumer raises the price of item j with maximum value of v_{ij}/p_j.

The algorithm terminates when the price of all items rises above 1 (i.e., all items are allocated) or when the surplus at each node is less than $a_{min}\epsilon/(1+\epsilon)$.

To prove a bound on the number of rounds, the consumers are considered in round-robin order .

4.2 Analysis

As in the case of the Fisher model, the following invariants hold during the algorithm:

Invariant 1: $x_{ij} > 0 \Rightarrow \alpha_i p_j \leq v_{ij}(1+\epsilon)$
Invariant 2: $z_{sj} > 0 \Rightarrow (p_j/a_{sj}) \geq \gamma_s/(1+\epsilon)$
Invariant 3: $\forall s\ \gamma_s = \max\{0, \max_j(p_j/a_{sj})\}$
Invariant 4: $\alpha_i p_j \geq v_{ij}$.
Invariant 5: $\sum_{1\leq j\leq m} y_{sj}a_{sj} \leq K_s$
Invariant 6: $p_j > 1$ implies that $\sum_i x_{ij} = \sum_i(g_{ij} + z_{ij})$.

These invariants can be shown valid as the algorithm proceeds. These ensure approximate optimality of the solution. To show approximate clearing, note that at termination, either all items are allocated when the prices of every item is above 1 or the surplus is very small. In the second case it can be shown, by an analysis similar to that in [10], that approximate clearing of goods occurs.

To prove termination note that the price rises are bounded by the ratio of the largest utility to the smallest, since the algorithm terminates as soon as the price of the last item rises above 1. The decrease of surplus when there is no price increase is bounded below.

For simplicity we will assume that the producers are consumers of only money, i.e., they have a valuation only on money. Money can be added as a good to achieve this. The consumption utility of producers can be modeled by a separate consumer. Consider a consumption phase between two price rises. Each such consumption phase is composed of rounds. In every round, each of the consumers exhausts his surplus at least once (else there is a price rise). However, as producers change their production schedule to goods which are in demand and more profitable to produce, the endowment also rises. Let j be a good whose price has risen and which is in demand. In a round (where every buyer is considered once) producers would be induced to change to this good. Consider a buyer

i who demands good j. Part of the surplus of buyer i goes towards acquiring j from some other buyer i' and the remaining goes to a producer who shifts his production from some good j', thus generating surplus at some buyer i''. Note that the producer also increases his endowment value when he shifts his production. The surplus at buyers i and i'' is reduced by a factor of $(1+\epsilon)$ due to the higher price of acquisition of good j. The increased value of the endowment could lead to increase in the price of money. Thus if there is no increase in price then the entire surplus reduces by a factor of $(1+\epsilon)$.

By an analysis similar to that in [10] we get a polynomial bound on the complexity which is $O(bm(b+q)\log_{1+\epsilon} P_{max} \log_{1+\epsilon}(mKP_{max}/\epsilon g_{min}))$ where $P_{max} = (1+\epsilon)(v_{max}/v_{min})$ is the maximum price achievable, $K = \max_s\{K_s\}$, g_{min} is the smallest size of the initial endowment and $v_{max}(v_{min})$ is the largest(smallest) utility.

Note: A reduction from the Fisher case of our production model (with a single production constraint) to the Fisher case without production has been recently provided in [12].

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New Algorithms for Mining the Reputation of Participants of Online Auctions

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Abstract. The assessment of credibility and reputation of contractors in online auctions is the key issue in providing reliable environment for customer-to-customer e-commerce. Confident reputation rating system is an important factor in managing risk and building customer satisfaction. Unfortunately, most online auction sites employ a very simple reputation rating scheme that utilizes user feedbacks and comments issued after committed auctions. Such schemes are easy to deceive and do not provide satisfactory protection against several types of fraud. In this paper we propose two novel measures of trustworthiness, namely, credibility and density. We draw inspiration from social network analysis and present two algorithms for reputation rating estimation. Our first algorithm computes the credibility of participants by an iterative search of inter-participant connections. Our second algorithm discovers clusters of participants who are densely connected through committed auctions. We test both measures on a large body of real-world data and we experimentally compare them with existing solutions.

1 Introduction

Electronic commerce, or e-commerce for short, is quickly becoming a noticeable market in contemporary economy. In the third quarter of 2004 e-commerce sales in the United States augmented to $17.6 billion, which made 2% of the total retail sales. Important models of e-commerce include business-to-business (B2B), business-to-customer (B2C), and customer-to-customer (C2C) commerce. The later model describes auctions, which are one of the oldest forms of economic activity known to mankind. Although known for centuries, auctions are experiencing an incredible revival due to the Internet which is providing auction participants with new and unprecedent possibilities. Cautious estimates predict that over 15% of the entire e-commerce market can be attributed to online auctions. Global auction sites, such as `www.ebay.com`, `www.onsale.com`, or `www.qxl.com`, attract millions of users every day. Auction sites differ on the frequency of auctions, closing and bumping rules, or auction types. Most popular auction protocols include English, First Price Sealed Bid, Dutch, Sealed Double Auction, and Vickrey auctions. For example, at `www.priceline.com` buyers provide bids on commodity items without knowledge of prior bids of other users, and the bids are being immediately accepted or rejected by sellers. Other auction places offer

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the possibility to bid on non-commodity items exposing the existing bids and establishing deadlines for auction termination.

Online auctions provide opportunities for malicious participants to commit fraud [11]. Fraudulent practices can occur during bidding process and after the bidding finishes. During the first stage two types of fraud are common: bid shilling and bid shielding. Bid shilling happens when a false identity is created to drive up the bidding process on behalf of a seller. On the other hand, bid shielding aims at discouraging other bidders from participating in an auction because the shielder sets the bid unusually high. At the last moment, the shielder withdraws the bid, leaving the second highest bid of shielder's partner the winner of an auction. After-bidding fraud includes sending no merchandise at all, sending merchandise that is of lower quality or inconsistent with the announcement, or never sending payment for the committed auction.

In order to successfully trade using online auction channel, users must develop trust toward their partners. Reputation ratings of participants of online auctions are both limited and unreliable. Furthermore, the anonymity and geographical dispersion encourages dishonest participants to default and deliberately commit fraud. Unfortunately, most popular auction sites employ only simplest reputation rating mechanisms which are easy to deceive and do not provide satisfactory protection against cheaters. Often, user reputation rating is a simple count of committed auctions that are described by auction parties using brief comments, usually labeled "praise", "neutral", or "complaint". This simple schema of reputation rating can be easily deceived into assigning an unfair rating to a user. Let us assume that a dishonest seller wants to increase the rating. One possibility is to create several artificial buyers who would provide extra positive comments at virtually no cost. Such unfairly high seller ratings are referred to as "ballot stuffing". Inflated seller's rating biases the system by providing the seller with an unearned reputation estimate, which allows the seller to obtain more bids or higher prices from other users [8, 10]. Still, an examination of artificial buyers' ratings would reveal their low credibility as they would not participate in any other auctions except the cheating seller's auctions. A careful cheater could disguise artificial buyers by creating a network of connections between them to form a clique. Such structure of inter-buyer connections would be hard to identify because all involved buyers would pretend to be fairly credible and their reported ratings would be high. On the other hand, a dishonest seller could create a few artificial buyers to provide unfairly low ratings to seller's competitors. Such technique is called "bad-mouthing" and reflects the negative impact of a deceitful gossip. Bad-mouthing is harder to implement because it requires an investment in winning competitor's auctions. Still, such an investment could prove beneficial if the expected gain of driving a competitor out of the market exceeds the initial cost. Sellers can also bias the system by providing discriminatory ratings, both positive and negative, to selected buyers.

In this paper we propose two novel measures of online auction participant trustworthiness. Instead of using simple participation counts, we propose to use data mining to analyze the topology of the network of seller-buyer connections to derive useful knowledge about each participant. Building upon social network analysis techniques we extract two notions that characterize auction participants: credibility and density. Our original contribution includes the definition of those novel measures, the construction of efficient algorithms to compute them, and experimental evaluation of the proposal. We use a large body of data acquired from `www.allegro.pl`, a leading Polish online auction site, to empirically prove the feasibility and benefit of our measures.

This paper is organized as follows. In Section 2 we present the related work. Section 3 introduces the concept of credibility and presents an algorithm used to compute it. In Section 4 we define the notion of density of participants. The results of the experimental evaluation of our proposal are presented in Section 5. Finally, the paper concludes with a future work agenda in Section 6.

2 Related Work

Reputation systems [9] are commonly used to build trust on the Internet, where millions of individuals perform commercial transactions without knowing each other. Web-based auction sites typically rely on trust models in which credibility of participants is assessed by counting positive and negative comments received from their trading partners after each transaction [5]. A critical analysis of this simple model can be found in [7]. The author points to the subjective nature of feedbacks, the lack of feedback context, the need to perform feedback aging. Of particular importance is the fact that positive and negative feedbacks are highly asymmetric, because users refrain from providing a negative feedback until the quality of service becomes totally unacceptable.

Several solutions have been recently proposed to address at least some limitations of current feedback-based models. In [1] the authors introduced a complaint-only trust management method. A method presented in [3] evaluates the quality of ratings assuming that a rating is credible if it is consistent with the majority of ratings for a given user. [2] proposed to introduce a trusted third party that could be used to authorize, identify, and establish the reputation of auction participants. A comparison of fraud mechanisms in online auctions and pay-per-call industry in the early 1990s can be found in [11]. The author states that the existing efforts of online auction industry self-regulation are not adequate and would not solve the problem, hence legal action must be undertaken by the government to provide consumer protection. In [12] a trust model called PeerTrust for peer-to-peer e-commerce communities was proposed. The presented model includes several trust parameters, i.e., feedback in terms of satisfaction, number of transactions, credibility of feedback, transaction context, and community context. Interesting idea of trust and distrust propagation was formulated in [4]. The authors present a method to compute trust between any two individuals based on a small amount of explicit trust/distrust statements per individual.

In our approach, rather than trying to solve all the problems with reputation assessment in Web-based auctions, we focus on just one issue that we believe is the most important, i.e. credibility of feedback. We go a step further than [12] with the goal of discovering networks of artificial auction participants created by cheating sellers and providing reciprocal positive comments. We mine the network of auction participants to derive knowledge on seller's credibility that would be independent of other users' feedbacks. To achieve this, we measure the density of each seller's neighborhood.

The problem of evaluating importance of Web pages by Web search engines can be regarded as similar to the problem of reputation assessment in online auctions. In terms of implementation details our method for credibility assessment in online auctions is similar to an algorithm proposed to evaluate the quality of Web pages, called HITS (hyperlink-induced topic search) presented in [6]. HITS divides the pages into authorities (covering a certain topic) and hubs (directory-like pages linking to authorities devoted to a certain topic). In our method, we apply a similar distinction, dividing auction participants into those that are mainly sellers and those that are mainly buyers. Our second method, i.e. the assessment of seller density, is similar to density-based clustering schemes.

3 Credibility of Participants

Given a set of buyers $B = \{b_1, b_2, \ldots, b_n\}$ and a set of sellers $S = \{s_1, s_2, \ldots, s_m\}$. Given a set of possible comments $C = \{-1, 0, 1\}$, where each value represents the "negative", "neutral", and "positive" comment, respectively. Given a set of auctions $A = \{a_1, a_2, \ldots, a_p\}$. An auction is a tuple $a_i = \langle b_j, s_k, c\rangle$ where $b_j \in B \wedge s_k \in S \wedge c \in C$. Let $S(b_j)$ represent the set of sellers who sold an item to the buyer b_j. We denote the *support* of the buyer b_j as $\mathit{support}(b_j) = |S(b_j)|$. Let $B(s_k)$ represent the set of buyers who bought an item from the seller s_k. We denote the *support* of the seller s_k as $\mathit{support}(s_k) = |B(s_k)|$. According to this formulation, the support of the participant is identical to the reputation rating measure currently employed by leading online auction providers.

Given a $m \times n$ matrix M_S. Each entry in the matrix represents the flow of support from a buyer to a seller in a committed auction. Entries in the matrix M_S are initialized as follows.

$$\forall\, i \in \langle 1, m\rangle \;\; M_S[i,j] = \frac{1}{\mathit{support}(b_j)} \;\; \mathit{if}\ \langle b_j, s_i, *\rangle \in A,\ 0\ \mathit{otherwise}$$

Given a $m \times n$ matrix M_B. Each entry in the matrix represents the flow of support from a seller to a buyer in a committed auction. Entries in the matrix M_B are initialized as follows.

$$\forall\, j \in \langle 1, n\rangle \;\; M_B[i,j] = \frac{1}{\mathit{support}(s_i)} \;\; \mathit{if}\ \langle b_j, s_i, *\rangle \in A,\ 0\ \mathit{otherwise}$$

Given a vector $S_C = [s_1, s_2, \ldots, s_m]$ of seller credibility ratings. Initially, all sellers receive the same credibility of 1. Analogously, given a vector of buyer credibility ratings $B_C = [b_1, b_2, \ldots, b_n]$. Initially, all buyers receive the same credibility of 1. Reputations of sellers and buyers are independent of each other despite the true identity of participants. In other words, the fact that a person can be a seller and a buyer at the same time is not considered. In our opinion this simplification is justified in practice and it reflects the real behavior of online auction participants who form distinct and well separated clusters of buyers and sellers. Upon the termination of the algorithm vectors S_C and B_C contain diversified credibility ratings for sellers and buyers, respectively. A reputation rating for a buyer b_j is a tuple $R(b_j) = \langle C_-, C_0, C_+ \rangle$. Each component represents the sum of credibilities of sellers participating in transactions with a given buyer and posting a negative, neutral, or positive comment, respectively. Formally, $C_- = \sum_k S_C[k]$ where $\langle b_j, s_k, -1 \rangle \in A$, $C_0 = \sum_k S_C[k]$ where $\langle b_j, s_k, 0 \rangle \in A$, and $C_+ = \sum_k S_C[k]$ where $\langle b_j, s_k, +1 \rangle \in A$. Reputation rating for a seller is defined analogously.

Our method of reputation rating estimation is based on the following recursive definition of credibility. We consider a given buyer to be highly credible if the buyer participates in many auctions involving credible sellers. Analogously, we define a given seller to be credible if the seller participates in many auctions involving credible buyers. Since there is no *a priori* estimation of credibility of participants, we assume that initially all participants have equal credibility. Then, we iteratively recompute the credibility of sellers and buyers in the following way. In each iteration we distribute the current credibility of each buyer among participating sellers. Next, we update the credibility of all sellers by aggregating credibility collected from participating buyers. After the credibility of sellers has been updated, we propagate current credibility of sellers to buyers and we refresh the appropriate ratings. We repeat this procedure several times until the credibility of sellers and buyers converge. Alternatively, the procedure can be repeated a given number of times. Our experiments suggest that in practical applications ten iterations are sufficient to estimate the credibility correctly. After assessing the credibility of all participants the credibility ratings can be used together with the database of past comments to derive the proper reputation ratings by aggregating the credibility of contractors grouped by the type of the comment issued after the transaction.

The algorithm, presented in Fig. 1, works as follows. First, all required structures are initialized as explained above. Next, the algorithm begins to iteratively recompute the credibility for sellers and buyers. The intuition behind the algorithm is that the credibility of "good" buyers quickly aggregates in "good" sellers and *vice versa*. Initial ratings consisting of simple participation counts are quickly replaced by the true credibility which reflects the importance of every participant. Casual auction participants receive significant recommendation ratings only if they trade with highly ranked sellers, otherwise their initial unitary recommendation rating dissolves among lowly rated sellers.

Presented algorithm interestingly safeguards against two popular schemes of reputation rating deception. Ballot stuffing, which is a conspiracy between a seller and a group of buyers in order to unfairly augment the reputation ranking of a seller, is prevented by quick decrease in the reputation of such buyers. This can be attributed to the fact that, accordingly to the algorithm, buyers trading

Require: $A = \{a_1, a_2, \ldots, a_p\}$, the set of committed auctions
Require: $B = \{b_1, b_2, \ldots, b_n\}$, the set of buyers
Require: $S = \{s_1, s_2, \ldots, s_m\}$, the set of sellers
Require: M_S,M_B, matrices representing the structure of the inter-participant network
Require: S_C,B_C, vectors representing the credibility of participants

Initialize matrices M_S,M_B and vectors S_C,B_C appropriately
repeat
    for all $s_k \in S$ **do**
        $S_C\left[s_k\right] = \sum_{j=1}^{n} M_S\left[j, k\right] * B_C\left[b_j\right]$
    end for
    for all $b_j \in B$ **do**
        $B_C\left[b_j\right] = \sum_{k=1}^{m} M_B\left[j, k\right] * S_C\left[s_k\right]$
    end for
until vectors S_C and B_C converge
Output S_C and B_C as credibility ratings
Compute reputation ratings $R\left(b_j\right)$, $R\left(s_k\right)$ $\forall b_j \in B$, $\forall s_k \in S$ using S_C, B_C, and A

Fig. 1. Algorithm for computing the credibility of participants

with a few sellers are generally not considered trustworthy. An attempt to create a clique of participants who try to mutually increase their reputation ratings is also prevented by the algorithm. In such case, the algorithm discovers a subset of participants with constant reputation rating. An artificial clique is indeed a closed system with no inflow or outflow of credibility. The detection of such a closed system quickly leads to the discovery of fraudulent collaboration between dishonest participants.

4 Density of Sellers

The main drawback of the credibility assessment method presented in Sec. 3 is the fact, that the credibility of a user depends on the credibility of other users directly involved in auctions with a given user. This encourages us to propose a novel measure of user credibility that indirectly employs information about participating users. We restrict our measure only to sellers, because the credibility of sellers is more important from the economical point of view (buyers risk financially more than sellers when involved in auctions with unreliable sellers).

We say that two sellers s_i and s_j are *linked* if there exist at least *min_buyers* who finalized an auction with both sellers, and the final price of each auction was at least *min_value*. The number n_{ij} of such buyers is called the *strength* of the link and is denoted as $|link\left(s_i, s_j\right)|$. We define the *neighborhood* $N(s_i)$ of a seller s_i as the set of sellers $\{s_j\}$ with whom the seller s_i is linked, given user-defined thresholds *min_buyers* and *min_value*. We call the cardinality of the neighborhood $N(s_i)$ the *density* of the neighborhood. The threshold *min_buyers* is used to select only sellers with significant volume of sales. The threshold *min_value* is used to protect against cheaters trying to impersonate credible sellers. Note that this measure does not take into account the strength of the link between any two sellers, only the number of other sellers in a given seller's neighborhood. Therefore, we introduce another measure, called *score*, and defined as

$$score\,(s_i) = \sum_{s_j \in N(s_i)} density\,(s_j) * \log_{min\_buyers} |link\,(s_i, s_j)|$$

The rationale behind the density measure is the following: a buyer who buys from two different sellers acknowledges the quality of both sellers. Experienced buyers who buy many items are used to link sellers, so we naturally discard less reliable information from unexperienced buyers. The fact that two sellers are linked indicates that either their product offer is very similar (e.g., both sellers sell used books and there are many buyers who buy books from both sellers), or that their offer is complementary (e.g., one seller sells computers and the other seller sells peripherals). Of course, the link between any two sellers may be purely coincidental, but we believe that this is the case of sellers with low density. Clusters with high density represent groups of very credible sellers. The score measure uses density of each seller in the neighborhood of the current seller and multiplies it by the strength of the link between the two. The logarithm is used to reduce the impact of very strong links between sellers.

The main benefit of the density measure is that it is very resistant to fraud. Consider a malicious seller trying to enter a cluster of reliable sellers. Linking to a single seller requires the cheater to create *min_buyers* and investing at least *min_buyers*∗*min_value* in winning auctions of a credible seller. Such investment still links the cheater to only one seller, so in order to receive higher density the cheater has to repeat this procedure several times. This feature of our measure is caused by the fact that it uses other sellers to rate a current seller, rather than using information from buyers. Several studies show that it is much easier for malicious users to create false buyer identities (e.g., to provide artificially high positive feedbacks) than to form cooperating cliques of real sellers.

5 Experimental Results

The data have been acquired from `www.allegro.pl`, the leading Polish provider of online auctions. Allegro uses a simple auction protocol: each auction has an explicit deadline and all current bids are exposed to all participants, users may use a proxy which performs stepwise bidding until the maximum bid defined by a user has been reached. The dataset contains information on 440 000 participants and 400 000 terminated auctions (with 1 400 000 bids). We have chosen 10 000 different sellers and for this group we have selected all their auctions and participants of these auctions during a period of six months. Analogously, we have selected 10 000 buyers and for this group we have collected information on all auctions and their participants during a period of six months. Therefore, we had access to full information on 20 000 users, and partial information on another 420 000 users. All experiments are conducted on Pentium IV 2.4 GHz with 480 MB of memory. Data are stored and preprocessed using Oracle 10g database.

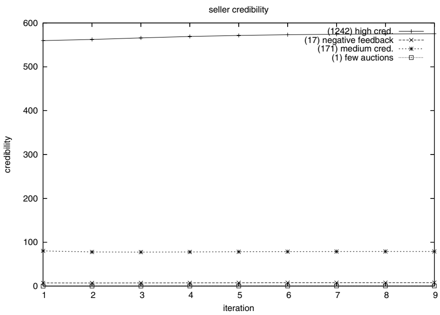

Fig. 2. Seller credibility

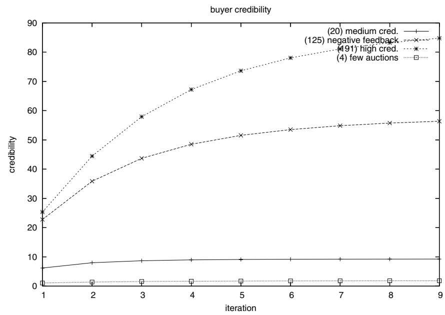

Fig. 3. Buyer credibility

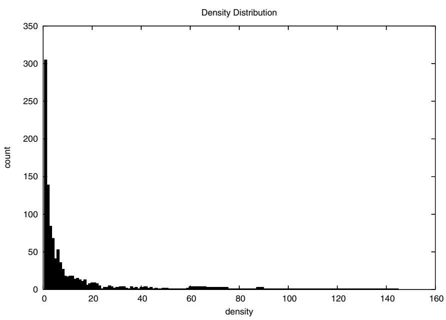

Fig. 4. Density distribution

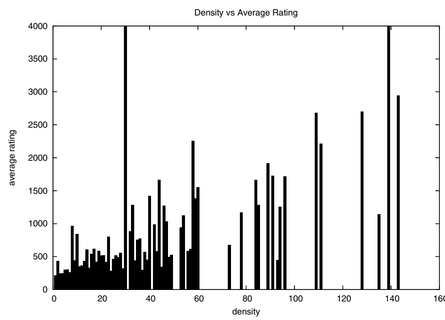

Fig. 5. Avg. rating distribution

Figure 2 depicts the convergence of credibility in subsequent iterations for selected sellers. Numbers in parentheses represent currently used reputation counts for those sellers. Observe that the relative difference in credibility for sellers (1242) and (171) is smaller than for reputation counts. Also note that credibility estimation converges after only a few iterations. The results of credibility estimation for a group buyers are depicted in Fig. 3. One can easily notice that the credibility as defined by us is not a linear function of reputation counts. Again, we observe that the computations converge after only a few iterations.

For testing of density and score measures we used the set of 10 000 sellers for whom we had data on all auctions during the period of six months. We choose threshold values $min\_buyers = 3$ and $min\_value = 50$ PLN (ca. $15). Figure 4 presents the distribution of density in the examined set. Most sellers have density lower than 6, but we also observe sellers with very high density exceeding 100. Around 10% of all examined sellers turned out to be dense (1026 out of 10 000). Figure 5 presents the distribution of average rating with respect to density. This result supports our claim that high density represents reliable and credible sellers, one can easily notice that the average rating grows quickly with the increase of the density.

Figures 6 and 7 show projections of density and score on currently used rating value. Interestingly, we find many sellers with very high rating (above 500 auc-

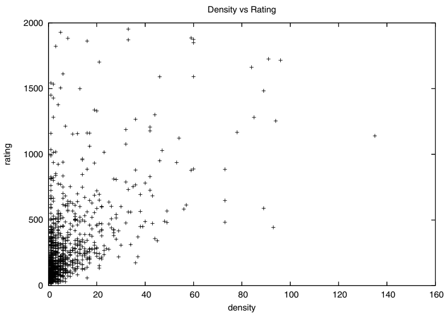

Fig. 6. Density vs Rating

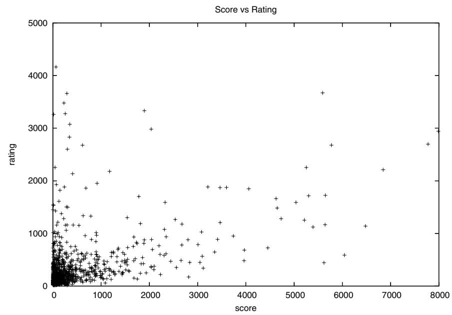

Fig. 7. Score vs Rating

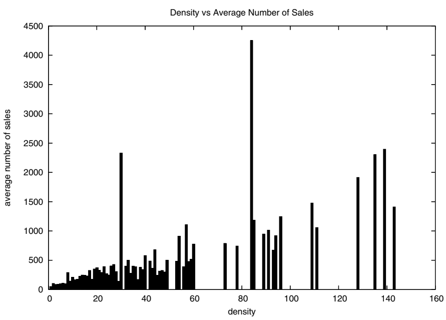

Fig. 8. Avg. number of sales distribution

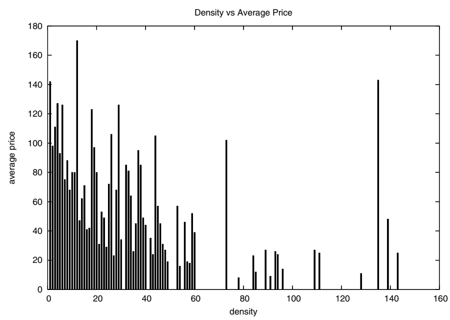

Fig. 9. Avg. price distribution

tions) with very low density. Of course, these could be users who trade low-value goods (e.g. used books, small collectibles, etc.) and they are being punished by relatively high *min_value* threshold. On the other hand, the average price of auctions in the examined set was close to 90 PLN (ca. $28), so we do not feel that our setting was too prohibitive. Figure 7 reveals a significant shift along the x-axis. This suggests that the sellers with low density and high rating have much higher average strength of the link than densely connected sellers.

Finally, Fig. 8 and 9 show the impact of density on the average number of sales and the average price of items for each seller. We discover that the density is a good predictor of the volume of sales, and sellers with high density enjoy a much higher number of auctions. On the other hand, there is no clear indication whether the density of a seller impacts the average price of offered goods.

6 Conclusions

In this paper we have presented two novel measures for reputation rating in online auction environments. Our measures, credibility and density, evaluate the reputation of auction participants by mining the topology of the network of seller-buyer relationships to retrieve useful knowledge. We believe that the patterns that we discover in the network provide additional insight into user characteristics and can be used as a predictor of user reliability. The experiments prove the practical usability and applicability of the presented solution. Our work extends previous proposals in the number of ways, namely, it considers the structure of inter-participant relationships and computes the reputation ratings iteratively by simulating the flow of credibility between auction participants. The results of conducted experiments encourage us to follow the research in the area. For the moment, we compute the reputation of a buyer and a seller disjointly, even if they are the same physical person. The next step is to combine the information about these reputation ratings for every distinct individual. An interesting, yet often disregarded, feature of online bidding is the timing of the bid. We believe that the timing carries valuable knowledge about the nature of the bid and can be successfully used to discover fraud. Finally, our experiments were conducted on a fairly small subset of the original data. Due to the immense popularity of online auctions, the volume of data to be analyzed poses a significant challenge. We plan to scale the algorithms to allow for almost real-time analysis of huge amounts of raw data.

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A Simple Characterization for Truth-Revealing Single-Item Auctions

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Abstract. We give a simple characterization of all single-item truth-revealing auctions under some mild (and natural) assumptions about the auctions. Our work opens up the possibility of using variational calculus to design auctions having desired properties.

1 Introduction

The classic work of Vickrey [9] characterizes truth-revealing auctions when the allocation is required to be efficient, i.e., maximizes global welfare. The subsequent work of Clarke [3] and Groves [5] showed that a generalization of Vickrey's mechanism leads to truth-revealing mechanisms for a much wider class of applications. Although this so-called VCG mechanism stands as a pillar of auction theory, it suffers from the drawback that it designed for the implementation of efficient allocations. If the user utilities are unrestricted, then an efficient allocation (weighted VCG) is essentially the only possible implementable solution [8], but in more specific environments the goals of the auctioneer may be very different from efficiency.

In this paper, we wish to characterize single-item auctions in which efficient allocations are not necessarily required. Such situations are quite common in emerging applications of auctions, for instance, budget constrained ad auctions, as carried out by Internet search engine companies. These auctions consist of several micro-auctions, one for each search. Search engine companies want to maximize efficiency/revenue over the entire sequence of auctions. Simply maximizing efficiency/revenue for each micro-auction would entail awarding the ad to the highest bidder, and in the presence of budget constraint of the advertisers, this will not maximize efficiency/revenue for the entire sequence of auctions. On the other hand, the search engine companies would like to make each micro-auction truth-revealing [2, 1]. Such repeated auctions are called myopic truth-revealing.

In order to arrive at a simple characterization, we need to assume that the auction satisfies some natural constraints. Once we make these very reasonable assumptions, our characterization turns out to be simple enough to be practically useful, for example, in repeated ad auctions [6].

Given bids for the single item, an *auction* determines the winner and the price charged from her. Hence, we may assume that the auction is a function from

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bids to a profit vector whose components are zero for losers and zero/positive for the winner. We assume the following conditions on the auction:

1. **Truth-revealing:** Truth-revealing should be a dominant strategy of bidders.
2. **Continuity:** The auction function, as defined above, should be continuous.
3. **Autonomy:** For any bidder i, if all other bidders bid zero and i bids high enough, then i wins.
4. **Independence:** If bidder i is the winner and some other bidder j decreases her bid, then i still wins.

We say that two auction mechanisms are *equivalent* if for identical bid vectors their profit vectors are also identical. We show that any auction mechanism satisfying the above-stated conditions is equivalent to the following simple mechanism: For each bidder i there is a strictly monotonically increasing function $f_i : \mathbf{R}_+ \rightarrow \mathbf{R}$ which we call the *rank function of bidder* i. For a bid vector b, compute the rank of each bid, i.e. $f_i(b_i)$. The winner is a bidder with maximum rank, and she pays the least amount p such that if she had bid p she would still have maximum rank.

One may ask what is the use of assigning different rank functions to different users. For instance, in the repeated ad auctions, different bidders have different budgets and they may have different ad campaigns, i.e., they want to spend their money on different sets of key words. Clearly, in this case, we need to assign different rank functions to different bidders.

Truthful auctions of digital goods have a very simple characterization: for each bidder there is a threshold function, which depends only on the remaining bids, such that this bidder wins iff she bids at least the threshold function (see [4]). Under this characterization it is essential to award multiple items, if several bidders bid strictly more than their threshold amounts. Hence this characterization does not apply to single item auctions. Our characterization leads to a set of threshold functions, one for each bidder, such that the bid of at most one bidder can be strictly bigger than her threshold. Furthermore, in the characterization of digital good auctions, the threshold functions could be arbitrarily complex functions of the bids. However, in our characterization, the threshold functions are simply the maximum of single argument functions.

Characterizations such as ours usually lead to mathematical programs for finding one particular mechanism satisfying desired properties. For instance, Moulin and Shenker's [7] characterization of group strategyproof cost sharing methods, under natural assumptions, for submodular cost functions leads to a linear program for finding one such method. Our characterization leads to a variational calculus formulation of truth-revealing auction mechanisms satisfying our natrual assumptions. This may be useful for finding rank functions that lead to an auction mechanism with desired properties.

2 Model and Definitions

We are given a single item which we want to auction to a set of n bidders. Each bidder has a private evaluation of her worth of this item. An auction takes the bids of these bidders and assign the item to one of them and decides how much to charge her. The bidder who gets the item makes a profit of her evaluation minus the price charged. All other bidders make zero profit. We will only work with deterministic auctions.

Formally, we define an auction as a function $\boldsymbol{\alpha} : \mathbf{R}_+^n \to \mathbf{R}_+^n$ which maps a bid-vector to a profit-vector. The profit vector can have at most one non-zero entry. The assumptions in the Introduction lead to the following restrictions on $\boldsymbol{\alpha}$: $\boldsymbol{\alpha}$ is continuous, whenever $\boldsymbol{\alpha}$ outputs the all-zero vector then there are at least two bidders who can increase their bids to make their own profit positive, and if the input to $\boldsymbol{\alpha}$ has only one positive bid, then the profit vector is the same as the bid vector.

We will characterize this mechanism $\boldsymbol{\alpha}$ by another mechanism $\boldsymbol{\beta}$. In $\boldsymbol{\beta}$, each bidder has a strictly monotonically increasing function $f_i : \mathbf{R}_+ \to \mathbf{R}$ which we call the *rank function of bidder* i. For a bid vector b, $\boldsymbol{\beta}$ computes the rank of each bid, i.e. $f_i(b_i)$. The winner is a bidder with maximum rank, and she pays the least amount p such that if she had bid p she would still have maximum rank. Using assumptions on $\boldsymbol{\alpha}$ we show the existence of the rank functions.

3 Useful Properties

We first modify $\boldsymbol{\alpha}$ to massage it to a more useful form and show that it satisfies some useful properties.

Fix a bidder i. When all other bidders bid zero, there is a threshold t_i such that i wins if she bids higher than t_i (by autonomy and truthfulness). From the independence property, it follows that i cannot ever win if she bids lower than t_i. Thus we normalize her bid so that a bid of t_i corresponds to a bid of zero in a new mechanism $\boldsymbol{\alpha}'$. Formally, the mechanism $\boldsymbol{\alpha}'$ adds t_i to bidder i's bid and runs $\boldsymbol{\alpha}$ on the resulting bid vector. Clearly $\boldsymbol{\alpha}(\boldsymbol{b})$ is equivalent to $\boldsymbol{\alpha}'(\boldsymbol{b} - \boldsymbol{t})$ and $\boldsymbol{\alpha}'$ inherits truthfulness, continuity and independence. Further we argue that, $\boldsymbol{\alpha}'$ satisfies what we call non-favoritism:

Non-favoritism: The winner cannot have a zero bid unless all bidders have bid zero.

Indeed, suppose that bidder 1 wins with bid zero, when bidder 2 has a non zero bid in $\boldsymbol{\alpha}'$. Thus in $\boldsymbol{\alpha}$, 1 wins with bid t_1 when 2 has bid strictly more that t_2. By independence, we can assume that bidders $3, \ldots, n$ bid zero (in $\boldsymbol{\alpha}$).

First assume that $t_1 = 0$. Then bidder one wins when the bid vector is $(0, b_2, 0, \ldots, 0)$. But this contradicts the definition of t_2 (since $b_2 > t_2$). Thus t_1 must be strictly positive.

Consider the bid vector $(t_1 - \epsilon, t_2' + \epsilon, 0, \ldots, 0)$, for $\epsilon > 0$ being arbitrarily small. We first argue that bidder one cannot be the winner for this bid vector. Indeed, if bidder one was the winner, then by independence, she would be the winner for the bid vector $(t_1 - \epsilon, 0, \ldots, 0)$ as well contradicting the definition of t_1. Thus some other player i is the winner. By independence, i is the winner under the bid vector $(0, t_2 + \epsilon, 0, \ldots, 0)$ as well. But by definition of t_2 and truthfulness, i must be two and thus the threshold for player two, under the bid vector $(t_1 - \epsilon, \cdot, 0, \ldots, 0)$ is no larger than $t_2 + \epsilon$. Clearly bidder two still wins at bid vector $(t_1 - \epsilon, b_2, 0, \ldots, 0)$ and her profit must be at least $(b_2 - (t_2 + \epsilon))$ which is bounded away from zero when ϵ approaches zero. Hence by continuity, bidder two's profit under the bid vector $(t_1, b_2, 0, \ldots, 0)$ cannot be zero. This however contradicts that assumption that bidder one was the winner. Hence we have shown that

Lemma 1. *The auction $\boldsymbol{\alpha}'$ defined above satisfies non-favoritism.*

We also note that one of the t_i's must be zero. Indeed, consider the bidder that wins $\boldsymbol{\alpha}$ when all bids are zero. Clearly, this bidder has t_i equal to zero.

We are now ready to define $\boldsymbol{\beta}$. We shall define $\boldsymbol{\beta}$ so as to be equivalent to $\boldsymbol{\alpha}'$, clearly it can be easily massaged so as to be equivalent to $\boldsymbol{\alpha}$. To define $\boldsymbol{\beta}$, we need to specify functions $f_i(b_i)$ for each i. We define $f_1(\cdot)$ as the identity function. For any $i > 1$, we define $f_i(x)$ to be the infimum of all y such that bidder one wins when the bid vector is given by $b_1 = y, b_i = x$ and $b_j = 0$ for $j \neq 1, i$. It is easy to verify that the profit vector for bids $b_1 = f_i(x), b_i = x$ and $b_j = 0$ for $j \neq 1, i$ is the zero vector. In the next section, we show that $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}'$ are indeed equivalent mechanisms.

4 Proof of Equivalence

In this section we provide the details of the proof of the main result that $\boldsymbol{\beta}$ as defined above is equivalent to the given auction $\boldsymbol{\alpha}$'.

Lemma 2. *For all $i = 1, .., n$, $f_i(0) = 0$.*

Proof. $f_1(0)$ is clearly 0. We also have $\boldsymbol{\alpha}'(\delta, 0, 0, \ldots, 0) = (+, 0, \ldots, 0),\ \forall\ \delta > 0$. By definition of f_i, we get $f_i(0) = 0$.

Lemma 3. *For all $i = 1, .., n$, f_i is strict monotonically increasing.*

Proof. f_1 is the identity function, so it is strictly increasing. We prove that f_2 is strictly increasing, the proof is the same for f_i, $i > 2$.

Suppose that there are two bid values for bidder 2, b_2 and b'_2, $b_2 < b'_2$, such that $f_2(b_2) = q_2$, $f_2(b'_2) = q'_2$, and $q_2 > q'_2$.

Then for the bid vector $(q_2 - \epsilon, b'_2, 0, \ldots, 0)$ when ϵ is small enough so that $q_2 - \epsilon > q'_2$, the definition of $f_2(b'_2)$ implies that bidder one must win. By independence then, bidder one wins for the bid vector $(q_2 - \epsilon, b_2, 0, \ldots, 0)$. This however contradicts the definition of $f_2(b_2)$.

This shows that f_2 is non-decreasing. To prove that it is strictly increasing, we use the other axioms. Assume that there are two bid values for bidder 2, $b_2 < b'_2$, such that $f_2(b_2) = f_2(b'_2) = q_2$. We have:

$$\boldsymbol{\alpha}'(q_2, b_2, 0, \ldots, 0) = (0, 0, \ldots, 0)$$

and

$$\boldsymbol{\alpha}'(q_2, b'_2, 0, \ldots, 0) = (0, 0, \ldots, 0)$$

Pick an arbitrarily small $\epsilon > 0$ and consider the bid vector $(q_2 - \epsilon, b_2, 0 \ldots, 0)$. By definition of f_2, bidder one loses and hence some bidder i must be the winner. By independence, i wins for the bid vector $(0, b_2, 0, \ldots, 0)$ as well, and by non-favoritism, i must be bidder two. Thus the threshold for bidder two, given the bids $(q_2 - \epsilon, -, 0, \ldots, 0)$ must be no larger than b_2. As a result, $\boldsymbol{\alpha}'(q_2 - \epsilon, b'_2, 0, \ldots, 0) = (0, \delta, 0, \ldots, 0)$ for some $\delta \geq b'_2 - b_2$ bounded away from zero. By continuity then, it follows that $\boldsymbol{\alpha}'(q_2, b'_2, 0, \ldots, 0)$ must be at least $\delta > 0$. This however contradicts the definition of $f_2(b'_2)$.

Theorem 1. *$\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}$ are equivalent auctions.*

Proof. It is easy to check that $\boldsymbol{\beta}$ as defined above satisfies all the axioms. We now argue that in fact $\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}$ are equivalent.

Suppose $\boldsymbol{\alpha}$' is not equivalent to $\boldsymbol{\beta}$. Then there are four different ways in which they could differ. We consider each of these four cases, and obtain a contradiction. The only direct contradiction is obtained in Case 1, while in the rest of the cases we reduce to other cases, and eventually to case 1, taking care of avoiding logical cycles.

1. Case 1: There is a bid vector $(b_1, b_2, \ldots, b_n)$ s.t. two different bidders make positive profit in $\boldsymbol{\alpha}$' and $\boldsymbol{\beta}$. Assume first that

$$\boldsymbol{\alpha}'(b_1, b_2, \ldots, b_n) = (+, 0, \ldots, 0) \quad \textit{and} \quad \boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (0, +, 0, \ldots, 0)$$

Since $\boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (0, +, 0, \ldots, 0)$, we have by definition of $\boldsymbol{\beta}$ that $\boldsymbol{\beta}(b_1, b_2, 0, \ldots, 0) = (0, +, 0, \ldots, 0)$. By definition of $f_2(b_2)$, it must be the case that $\boldsymbol{\alpha}'(b_1, b_2, 0, \ldots, 0) = (0, +, 0, \ldots, 0)$. On the other hand, by independence and the assumption that $\boldsymbol{\alpha}'(b_1, b_2, b_3, \ldots, b_n) = (+, 0, \ldots, 0)$, it follows that $\boldsymbol{\alpha}'(b_1, b_2, 0, \ldots, 0) = (+, 0, \ldots, 0)$. This however is a contradiction.

Now assume w.l.o.g that

$$\boldsymbol{\alpha}'(b_1, b_2, \ldots, b_n) = (0, +, 0, 0 \ldots, 0)$$

$$\boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (0, 0, +, 0, \ldots, 0)$$

From the definition of $\boldsymbol{\beta}$, it follows that $f_3(b_3) > f_2(b_2)$. Also, by independence, there is no loss of generality in assuming that $b_4, \ldots, b_n$ are all zero. Let b'_1 be such that $f_2(b_2) < b'_1 < f_3(b_3)$. Consider the behaviour of $\boldsymbol{\alpha}'$ under the bid vector $(b'_1, b_2, b_3, 0, \ldots, 0)$. Let j be the winner. We split cases:
(a) $j \geq 4$: This contradicts non-favoritism
(b) $j = 1$: By independence, bidder one still wins $\boldsymbol{\alpha}'$ when the bid vector is $(b'_1, 0, b_3, 0, \ldots, 0)$. This however contradicts the definition of $f_3(b_3)$.
(c) $j = 2$: By independence, bidder two still wins $\boldsymbol{\alpha}'$ when the bid vector is $(b'_1, b_2, 0, 0, \ldots, 0)$. This contradicts the definition of $f_2(b_2)$.
(d) $j = 3$: Using independence, bidder three must win $\boldsymbol{\alpha}'$ for bid vector $(0, b_2, b_3, 0, \ldots, 0)$. On the other hand,
$\boldsymbol{\alpha}'(b_1, b_2, b_3, 0, \ldots, 0) = (0, +, 0, \ldots, 0)$ along with independence implies that bidder two wins when the bid vector is $(0, b_2, b_3, 0, \ldots, 0)$. Hence we get a contradiction

Thus we have shown that when both $\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}$ have a winner with positive profit, they must agree.

2. Case 2: There is a bid vector $(b_1, b_2, \ldots, b_n)$ s.t. there is a bidder who makes positive profit in $\boldsymbol{\beta}$, but no bidder makes positive profit in $\boldsymbol{\alpha}$', i.e.,

$$\boldsymbol{\alpha}'(b_1, b_2, \ldots, b_n) = (0, 0, 0, \ldots, 0) \quad \textit{and} \quad \boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (0, +, 0, \ldots, 0)$$

where we have taken the winner in $\boldsymbol{\beta}$ to be bidder 2 w.l.o.g.

For an arbitrarily small $\epsilon > 0$, consider the mechanism $\boldsymbol{\alpha}'$ when given the bid vector $(b_1, b_2 - \epsilon, b_3, \ldots, b_n)$. Let bidder i be the winner in this case; by truthfulness, i is different from two. Since ϵ can be made arbitrarily small, for any $\delta > 0$, bidder i is the winner in $\boldsymbol{\alpha}'$ when the bid vector is given by $b'_i = b_i + \delta$, $b'_j = b_i$ for $j \neq i$. Since δ can be made smaller, $\boldsymbol{\alpha}'(\boldsymbol{b'})$ has a positive ith component. Moreover, for small enough δ, $\boldsymbol{\beta}(\boldsymbol{b'})$ has a positive second component. Thus we have reduced this to the first case.

3. Case 3: There is a bid vector $(b_1, b_2, \ldots, b_n)$ s.t. there is a bidder who makes positive profit in $\boldsymbol{\alpha}$', but no bidder makes positive profit in $\boldsymbol{\beta}$, i.e.,

$$\boldsymbol{\alpha}'(b_1, b_2, \ldots, b_n) = (0, +, 0, \ldots, 0) \quad \textit{and} \quad \boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (0, 0, 0, \ldots, 0)$$

where we have taken the winner in $\boldsymbol{\alpha}$' to be bidder 2 w.l.o.g.

Since $\boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (0, 0, 0, \ldots, 0)$, we know that there are at least two bidders with the maximum value of their rank function. If there are exactly two such bidders, then consider two subcases: bidder 2 is one of the two, or bidder 2 is not one of the two.

In the first subcase, consider the bid vector obtained by reducing the bid of bidder 2 infinitesimally to $b_2 - \delta$, for some small $\delta > 0$. By axiom 3 (Continuity), we get that $\boldsymbol{\alpha}'(b_1, b_2 - \delta, b_3, \ldots, b_n)$ remains $(0, +, 0, \ldots, 0)$, but by definition of $\boldsymbol{\beta}$ (and Lemma 3) we see that$\boldsymbol{\alpha}'(b_1, b_2 - \delta, b_3, \ldots, b_n)$ gives a positive profit to the other bidder who had highest rank. This reduces to Case 1.

In the second subcase, when bidder 2 is not one of the two highest ranked bidders in $\boldsymbol{\beta}$, consider the bid vector obtained by reducing the bid of any one of the two bidders. $\boldsymbol{\beta}$ gives a positive profit to the other of the two bidders. But by independence, we see that $\boldsymbol{\alpha}$' still gives positive profit to bidder 2. This again reduces to Case 1.

In the case that there are more than two bidders with the highest value of rank function, we can choose a bidder who is not bidder 2, and reduce his bid to 0. By independence, we see that $\boldsymbol{\alpha}$' still gives positive profit to bidder 2. Since there were at least 3 bidders with highest rank in $\boldsymbol{\beta}$ before reducing the bid, there are now at least 2 bidders with highest rank. Hence the output of $\boldsymbol{\beta}$ remains $(0, 0, \ldots, 0)$. Thus we reduce to the same case again, but this time with a smaller number of positive bids, and we can use induction on the number of positive bids. The base case is that of two highest ranked bidders, which is taken care of above.

4. Case 4: There is a bid vector $(b_1, b_2, \ldots, b_n)$ s.t. the same bidder makes a positive profit in both $\boldsymbol{\alpha}$' and $\boldsymbol{\beta}$, but makes different amounts of profit.

We assume w.l.o.g. that the winning bidder is bidder 1. In the first subcase, we have $\boldsymbol{\alpha}'(b_1, b_2, \ldots, b_n) = (p + \Delta, 0, \ldots, 0)$ and $\boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (p, 0, \ldots, 0)$, for some $p, \Delta > 0$. Suppose we reduce the bid of bidder 1 from b_1 to $b_1 - p$. We get $\boldsymbol{\alpha}'(b_1 - p, b_2, \ldots, b_n) = (\Delta, 0, \ldots, 0)$ by Axiom 1 (Truthfulness), and we get $\boldsymbol{\beta}(b_1 - p, b_2, \ldots, b_n) = (0, 0, \ldots, 0)$ by the definition of $\boldsymbol{\beta}$. Thus we have reduced to Case 3.

In the second subcase, we have $\boldsymbol{\alpha}'(b_1, b_2, \ldots, b_n) = (p, 0, \ldots, 0)$ and $\boldsymbol{\beta}(b_1, b_2, \ldots, b_n) = (p + \Delta, 0, \ldots, 0)$, for some $p, \Delta > 0$. Again, we reduce bidder 1's bid to $b_1 - p$. By Axiom 1, we get that $\boldsymbol{\alpha}'(b_1 - p, b_2, \ldots, b_n)$ is either $(0, 0, \ldots, 0)$ or $(0, \ldots 0, +, 0, \ldots 0)$, where the positive entry is in some index other than 1. By the definition of $\boldsymbol{\beta}$ we get that $\boldsymbol{\beta}(b_1 - p, b_2, \ldots, b_n) = (\Delta, 0, \ldots, 0)$. Thus we have reduced to either Case 2 or to Case 1.

Finally, note that this mechanism $\boldsymbol{\beta}$, defined by the functions $f_i : \mathbf{R}_+ \to \mathbf{R}_+$ is equivalent to the mechanism $\boldsymbol{\alpha}'$. To get a similar mechanism equivalent to $\boldsymbol{\alpha}$, we define $g_i : \mathbf{R}_+ \to \mathbf{R}$ so that $g_i(x) = f_i(x - t_i)$ for $x \geq t_i$. When i bids less than t_i, she can never win irrespective of the other bids; we can thus define $g_i(x)$ arbitrarily in this range, e.g. $g_i(x) = (x - t_i)$ for $x \leq t_i$ works and makes g_i both continuous and strictly increasing. Also note that since for a bidder i such that $t_i = 0$ (such a bidder exists, as we argued earlier), $g_i(0) = 0$. Thus the winning bidder under g_i's always has a positive value of $g_i(b_i)$ and the negative values of g_i indeed are irrelevant.

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Prediction Games
(Extended Abstract)

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Abstract. We introduce a new class of mechanism design problems called *prediction games*. There are n self interested agents. Each agent i has a *private* input x_i and a cost of accessing it. Given are a function $f(x_1, \ldots, x_n)$ which predicts whether a certain event will occur and an independent distribution on the agents' inputs. Agents can be rewarded in order to motivate them to access their inputs. The goal is to design a mechanism (protocol) which in equilibrium predicts $f(.)$ and pays in expectation as little as possible.

We investigate both, exact and approximate versions of the problem and provide several upper and lower bounds. In particular, we construct an optimal mechanism for every anonymous function and show that any function can be approximated with a relatively small expected payment.

1 Introduction

1.1 Motivation

Predictions of future events play an important role in our everyday life. Individuals want for example to know whether it will rain tomorrow, and who will win the next elections. Companies may be interested whether a marketing campaign will succeed. Governments want to predict the usefulness of public projects, etc.

A powerful method of making accurate predictions is the usage of input provided by multiple agents. Often, such predictions are more accurate than those performed by a single agent, even an expert (see e.g [2, 3, 15, 16]). Currently, multi agent predictions are mainly carried out by public opinion polls, phone surveys, and in a more limited scale by future markets. Moreover, many organizations (e.g. governments, institutions, companies) base predictions on input of external experts which do not necessarily share the interest of the organization. A major difficulty in making such predictions is that the participating agents have no incentive to spend the time and effort necessary for providing their inputs (answering long questionnaires, perform costly checks, etc.).

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Naturally, mechanisms which ask costly inputs from the agents need also to reward them. As we shall see, such rewards may need to be very high in order to ensure that the agents are motivated and developing low payment prediction mechanisms is a non trivial task. In order to demonstrate this, let us consider the following toy example.

Example (consensus). A company develops a new exciting product. It would like to predict whether the product will be ready before the holiday season. Five managers are involved in the development process. Each manager has information which indicates the chances that the company's deadline will be met. In view of the company's past experience the product is likely to be ready on time if and only if all managers predict so. This is an example of a *consensus* function. The event that the company wants to predict is whether the product will be ready on time and the inputs are the managers' indications. Assume that each manager has a positive indication with probability $1/2$ and that the time required for each manager to issue an estimation costs her $\$1,000$.

Lets examine a few intuitive methods by which the company can try to forecast its success. One option is to ask every manager to predict whether the deadline will be met and eventually reward only the managers that were correct. Consider one of the managers. The probability that another manager will have a negative indication is $15/16$. Thus, even when the manager's input is positive, the probability of meeting the deadline is only $1/16$. Therefore, all managers are better off reporting a negative estimation regardless of their actual information.

A better option is to ask each manager to report her *own* indication and reward *all* of them if and only if their **joint** prediction is correct. Consider one of the managers, say Alice. If another manager has a negative indication then Alice's input does not matter to the prediction. Thus, the probability that Alice's input matters is $1/16$. In addition, Alice can guess her input with probability $1/2$. Intuitively, the lower the chances that Alice's input is relevant, the less motivated will she be to invest the costly effort of performing her evaluation. Indeed, we shall show later that unless a manager is rewarded by at least $\frac{\$1{,}000}{1/2 \cdot 1/16} = \$32,000$ she will be better off guessing her input. In such a case the prediction of the company is arbitrary. Therefore, the intuitive mechanism which simultaneously asks the participants to compute and report their inputs must pay a sum of at least $\$160,000$, even though the actual cost of the managers is only $\$5,000$! We will show later that it is possible to design a mechanism which predicts the consensus function with an overwhelmingly smaller expected payment.

1.2 Our Contribution

We introduce a new class of mechanism design problems called *prediction games*. There are n self interested agents. Each agent i holds a *private* input bit x_i. The cost for each agent of accessing its input is c. Given is a function $f(x_1, \ldots, x_n) \rightarrow \{0, 1\}$ which predicts whether a certain event will occur. The mechanism can pay the agents in order to motivate them to access their inputs. Given an IID (Independent Identical Distribution) on the agents' inputs, the goal is to design a mechanism which in equilibrium predicts $f(.)$ and pays in expectation as little as possible.

We show that it is possible to focus on a very specific class of mechanisms. These mechanisms approach the agents serially (each agent at most once), do not reveal *any* information to the agents other than the fact that they are required to compute their input, stop when $f(.)$ is determined, and pay according to a specific scheme. In equilibrium, each agent approached computes its input and discloses it to the mechanism. We call such mechanisms *canonical*.

Perhaps the most important prediction functions are anonymous (agent symmetric) functions. These functions include majority, consensus, percentile, etc. We show that the **optimal** mechanisms for anonymous functions are canonical mechanisms which approach the agents in random order.

We show interesting connections between the expected payment of prediction mechanisms and the analysis of the influence of random variables. Using results from influence analysis, we construct upper bounds on the expected payment for both general and anonymous functions. These results also bound the sum of utilities required for computing a function in the model of [6, 7, 11].

The necessary expected payment for the exact prediction of many functions is very high. Therefore, we study approximate prediction mechanisms. These are mechanisms that allow a small probability of error. We show that **every** function can be approximated with a relatively low expected payment.

Finally, several important prediction functions are analyzed in the paper.

Our setup is basic and there is a lot of room for extensions. Yet, we believe that the insights gained in this paper apply to many real life prediction scenarios.

1.3 Related Work

Most of the vast literature on voting and group decision making assumes that the agents have free access to their inputs. Several recent papers [6, 7, 11] analyze situations of decision making with costly inputs. (A somewhat different setup was studied in [4, 5].) In these papers each agent has a utility for every possible decision and payments are not used by the mechanism. [6, 7] mainly focus on anonymous functions and [11] is dedicated to majority rules. The main concern of these papers is to characterize the functions which can be implemented in equilibrium.

Our model is similar to the model used in [6, 7, 11] but there are fundamental differences. Firstly, we allow the mechanism to *pay* the agents. Secondly, we assume that the agents are indifferent about the prediction of the mechanism. The utility of an agent is solely determined by its payment and the cost of accessing its input. Finally, correctness of the prediction can be *verified* by the mechanism.

Therefore, in our model every function can be implemented, and our main concern is how to minimize the expected payment of the mechanism.

Markets, and future markets in particular are known to be good predictors in certain situations. Recently, several artificial markets for forecasting have been established. Among the examples are the IOWA electronic markets (http://www.biz.uiowa.edu/iem/), Hollywood Stock Exchange (http://www.hsx.com/), and the Foresight Exchange site (http://www.ideosphere.com/). Such markets were studied in several papers (e.g. [2, 3]). A pioneering paper [1] provides a theoretical analysis of the power of future markets. The paper shows that many functions can be predicted (under strong assumptions such as myopic behavior of the agents) but others, such as parity, cannot.

More empirical literature exists within the forecasting community. A comparison between the game theoretic approach and other methods used in conflict forecasting can be found in articles published in the International Journal of Forecasting (e.g. [14],[15]).

Our model is based on mechanism design, a subfield of game theory and micro economics that studies the design of protocols for self interested parties. An introduction to this field can be found in many textbooks (e.g. [13–chapter 23]).

We show connections between the payment that must be made to an agent and its influence on the prediction function (the probability that its input is necessary for making the prediction). Since the seminal work of [8] influence analysis evolved significantly. Its focus is usually on bounds on the aggregate and the maximum influence. While we are usually interested in the harmonic mean, some of these results are still very helpful to us. A recent survey on influence literature can be found in [10].

Note: Due to space constraints large parts of the article were omitted from the proceedings. The full version can be found at http://iew3.technion.ac.il/ amirr/.

2 Preliminaries

2.1 The Model

In this section we introduce our model and notation. We have tried as much as possible to adopt the standard approach of mechanism design theory.

There are n self interested agents. Each agent i holds a *private* input bit x_i. The input $x = (x_1, \ldots, x_n)$ is taken from a prior distribution ϕ which is common knowledge. Each agent can access its input x_i but this access is costly. For simplicity of the presentation we assume that all agents have the same cost c. Our results hold when each agent has a different (but known) cost as well. An agent can also *guess* its input. We assume that guessing is free so the cost of an agent is either c in case it computed its input or zero. We allow the mechanism to pay the agents (for otherwise none will access its input) but not to fine them. The *utility* of each agent is the difference between its payment and cost. Each agent selfishly tries to maximize its expected utility.

We are given a boolean *prediction function* $f : \{0,1\}^n \to \{0,1\}$ which given the inputs of the agents predicts whether a certain event will occur.

A *prediction mechanism* is a communication protocol and a payment function. After communicating with the agents the mechanism announces whether the event will occur. It pays the agents *after* it is known whether the event occurred, i.e. after $f(x)$ is revealed. The mechanism is known to the participating agents.

A *strategy* for an agent is a function from its input to its behavior during the execution of the mechanism. A set of strategies $s = (s_1(x_1), \ldots, s_n(x_n))$ is a *Bayesian equilibrium* if each agent cannot improve its expected utility by unilaterally deviating from its strategy. An agent can always obtain a non negative utility by guessing its input. Therefore, any Bayesian equilibrium also guarantees an expected utility which is non negative (individual rationality).

Definition 1. (Implementation) *An* implementation *of a boolean function $g(x)$ is a pair (m, s) where m is a prediction mechanism and $s = (s_1(x_1), \ldots, s_n(x_n))$ is a*

Bayesian equilibrium in this mechanism, such that for every input x, the mechanism outputs $g(x)$. An implementation is called exact *if $g(x) = f(x)$.*

Note that the function $g(.)$ which the mechanism implements needs not necessarily be equal to $f(.)$. This will play an important role later when we study approximation mechanisms. It is not difficult to see that implementation in dominant strategies is impossible.

Consider the example of Section 1.1. The input of each manager is whether she has a positive indication that the deadline will be met. This input is uniformly distributed with $q = 1/2$. The prediction function $f(.)$ is consensus. The access cost c of each manager is $\$1,000$. A possible mechanism m would be to simultaneously ask each manager to evaluate her input and to reward it by $\$32,000$ iff the mechanism's prediction is correct. A possible equilibrium s in this mechanism is the truthful equilibrium: each agent submits a sincere estimation to the mechanism. In this equilibrium, each manager has an expected utility of $\$31,000$. (m, s) is an implementation of the consensus function.

Definition 2. (Expected payment) *Let (m, s) be an implementation. The* expected payment *$B(m)$ of the implementation is defined as the expected total payment of the mechanism when the agents follow their equilibrium strategies and the input is drawn from the underlying distribution ϕ. That is, $B(m) = \mathbf{E}[\sum_i v_i(s(x))]$ where v_i denotes the payment of each agent i.*

Similarly, the *accuracy* of the mechanism is defined as the probability that $g(x) \neq f(x)$. The analysis of influence of random variables plays an important role in our work. Most of the literature on this topic assumes that the input bits are drawn from an IID. We therefore adopt this assumption in our paper. We denote by ϕ_q the product distribution obtained when each input is independently set to 1 with probability q. We conjecture that our results also hold when there exists a $\Delta > 0$ such that each agent has its own probability $\Delta < q_i < 1 - \Delta$. However, this conjecture requires generalizations of basic results in the analysis of the influence of random variables for this setup. We assume w.l.o.g. that $0 < q < 1$, otherwise the agent's input is known to the mechanism and thus the agent is redundant. We let $q_M = \max(q, 1 - q)$ denote the maximal probability in which an agent can guess its input successfully.

Notation: Let x be an n-tuple. We denote by x_{-i} the $(n - 1)$-tuple $(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$.

2.2 Influence of Random Variables

Definition 3. (Influence) *Agent i is called* pivotal *for x_{-i} if $f(0, x_{-i}) \neq f(1, x_{-i})$. The* influence *$I_i$ of agent i is the probability that i is pivotal when x is chosen randomly according to ϕ_q.*

In other words, influence measures the extent of 'necessity' of agent i for making the prediction. We denote by $I_{max} = \max_i(I_i)$ the maximal influence of any agent.

Lemma 1. (Talagrand's Inequality) [9] *Let p denote the probability that $f(x) = 1$ when x is chosen according to ϕ_q. There exists a universal constant κ such that*

$$\sum_{i=1}^{n} I_i / \log(1/I_i) \geq \kappa p(1 - p)q(1 - q). \tag{1}$$

Lemma 2. (Maximum influence) *Let p denote the probability that $f(x) = 1$ when x is chosen according to ϕ_q. Then*

$$I_{max} = \Omega\left(p(1-p)q(1-q)\frac{\log n}{n}\right). \tag{2}$$

We often use the following well known mean inequality which states that the harmonic mean is always dominated by the arithmetic one.

Lemma 3. (Harmonic vs Arithmetic mean) *Let $a_1, \ldots, a_n$ be positive numbers. Then*

$$\frac{n}{\sum_i 1/a_i} \leq \frac{1}{n}\sum_i a_i. \tag{3}$$

3 Canonical Prediction Mechanisms

In this section we shall consider exact implementations and show that it is possible to focus on a small class of mechanisms which we call canonical. This will provide us tools for both, the construction of low payment mechanisms and for proving lower bounds on the expected payment.

We start with a standard argument in mechanism design stating that it is possible to focus on mechanisms which have a very simple communication structure.

Definition 4. (Truthful implementation) *A* truthful implementation *is an implementation with the following properties:*

- *The mechanism communicates with each agent at most once asking it to report its input. It can also pass additional information to the agent. The agent then replies with either* 0 *or* 1*. We call such a mechanism* revelation *mechanism.*
- *In equilibrium, each agent approached computes its input and reveals it to the mechanism. We call such an equilibrium* truthful.

We say that two implementations are *equivalent* if for every input vector x they announce the same predictions and hand the same payments to the agents.

Proposition 1. (revelation principle for prediction mechanisms) *For every implementation there exists an equivalent truthful implementation.*

Note that there exist many truthful mechanisms. In particular, the mechanism can choose what information to pass to an agent, the payment scheme, and the order of addressing the agents.

Consider an agent which receives some information from the mechanism. The agent can compute the likelihood of every input vector of the other agents given that the mechanism passed it this information. In other words, any information which the mechanism passes to an agent defines a probability distribution on the input of the other agents. For example, consider a mechanism for consensus in which each agent knows all the declarations of the agents that were approached before it. The first agent approached has an influence of $1/2^{n-1}$. However, if all the agents except one were already approached, this agent knows that its declaration will determine the prediction. In other words, the conditional influence of this agent is 1.

Definition 5. (Conditional influence) *We denote by $I_i(\varphi_{-i})$ the* conditional influence *of agent i. That is, the probability that agent i is pivotal given a distribution φ_{-i} on the others' input (not necessarily the original distribution).*

Remark. We assume w.l.o.g. that every approached agent has a strictly positive conditional influence. Otherwise the mechanism does not need to approach the agent.

Before proceeding, let us recall a few notations that we shall use extensively. $q = \mathbf{Pr}\big[x_i = 1\big]$ for each i. We denote by $q_M = \max(q, (1-q))$ the maximal probability by which each agent can guess its input and by ϕ_q the product distribution on all the agents. The constant c denotes the cost that each agent must incur in order to access its input. We let v_i denote the expected equilibrium payment of each agent i.

Theorem 1. (Payment characterization for exact implementation) *Let m be a revelation mechanism that implements f whenever the agents are truthful. Suppose that m passes information to agent i which defines a probability distribution φ_{-i} on the inputs of the other agents. Suppose that the other agents are truthful. Let $v_0 \geq 0$ denote the expected payment of agent i in case of a wrong prediction. Truth telling is a best response for agent i iff its expected payment in case of a true prediction is at least $v_i \geq v_0 + \frac{c}{(1-q_M)\cdot I_i(\varphi_{-i})}$.*

Proof : The only inputs that the mechanism has are the agents' declarations and the actual outcome (i.e. whether the event occurred). Agent i needs to decide whether to guess its input or compute it and reveal it to the mechanism. (Since the others are truthful, the best response strategy cannot be to compute the input and then misreport it.) We need the following claim:

Claim. Under the conditions of Theorem 1:

1. The mechanism cannot distinguish between the case where agent i guesses its input correctly and a the case where it computes it.
2. If the agent is not pivotal, the mechanism cannot distinguish between the case where agent i guesses its input according to ϕ_q and the case where it computes it.

Proof : The first point is obvious as the inputs of the mechanism in both cases are identical. In the second point we use the fact that the input of the other agents is independent of agent i. Since the agent is not pivotal, the actual outcome $f(x)$ is also independent of its input. Therefore, the distribution of the input of the mechanism is identical in both cases. □

Thus, in equilibrium, the only situation in which the expected payment of the agent is different from its payment when it is truthful, is when it guesses wrong and is pivotal. In equilibrium, this happens if and only if the **prediction** of the mechanism is wrong. Recall that the expected payment of the agent in this case is v_0 and its expected payment when the prediction is correct is v_i. Recall also that the agent can guess its correct input with probability q_M. The following matrix summarizes the utility of guessing.

	Pivotal	Not Pivotal
Correct guess	$q_M \cdot I_i(\varphi_{-i})v_i$	$q_M(1 - I_i(\varphi_{-i}))v_i$
Wrong guess	$(1 - q_M)I_i(\varphi_{-i})v_0$	$(1 - q_M)(1 - I_i(\varphi_{-i}))v_i$

Thus, the agent's strategic considerations are expressed in the following payments matrix.

Strategy/Prediction	Correct	Wrong
Compute	$v_i - c$	$v_0 - c$
Guess	$[q_M I_i(\varphi_{-i}) + (1 - I_i(\varphi_{-i}))]\, v_i$	$(1 - q_M) I_i(\varphi_{-i}) v_0$

This matrix shows the utility of the agent according to the correctness of the prediction of the mechanism. For example, if the agent chose to compute its input and the prediction was correct its utility is $v_i - c$, since it invested c in the computation and was rewarded v_i for the correct prediction. Note that in a truthful equilibrium, it is impossible that the agent computed its input but the prediction was wrong.

Thus, a necessary and sufficient condition for an agent to compute its input is that the following inequality is satisfied,

$$v_i - c \geq (1 - q_M) I_i(\varphi_{-i}) v_0 + [I_i(\varphi_{-i}) q_M + (1 - I_i(\varphi_{-i}))] v_i \tag{4}$$

We call this inequality **Incentive Elicitation Condition (IEC)**.

The bound $v_0 + \frac{c}{(1-q_M)\cdot I_i(\varphi_{-i})}$ is then obtained by a simple calculation.

This completes the proof of Theorem 1. □

Corollary 1. *Under the conditions of Theorem 1. For every exact truthful implementation m there exists another exact truthful implementation $\tilde{m}$ with an expected payment $B(\tilde{m}) \leq B(m)$ which rewards the agents an amount of $\frac{c}{(1-q_M)\cdot I_i(\varphi_{-i})}$ iff $f(.)$ is predicted correctly.*

We call the payment scheme described in Corollary 1 *canonical*. The corollary states that w.l.o.g. we can focus on such schemes. Two important issues which the mechanism designer needs to address are what additional information to transfer to each agent and when to stop the computation.

Definition 6. (Canonical mechanisms) *A* canonical mechanism *is a truthful implementation with the following properties:*

- *The mechanism approaches the agents serially and reveals no additional information to them.*
- *The mechanism never performs redundant computations.*
- *The payment scheme is canonical. The payment v_i offered to agent i in case of a correct prediction equals $\frac{c}{(1-q_M)\cdot \hat{I}_i}$ where $\hat{I}_i$ denotes the influence of agent i conditional on the fact that it is approached.*

Theorem 2. (Optimality of a canonical mechanism) *For every function $f(.)$ there exists a canonical implementation of it with an optimal expected payment among all the exact implementations of $f(.)$.*

The theorem's proof and some other results of this section can be found in the full version of this paper.

From now on we can therefore limit ourselves to canonical mechanisms and the only decision we need to take is the *order* in which the agents are approached.

4 Exact Predictions

In this section we consider exact implementations. These are implementations which in equilibrium always issue a correct prediction.

4.1 Budget Estimations

This section is omitted due to lack of space.

4.2 Optimal Mechanisms for Anonymous Functions

This subsection characterizes the **optimal** mechanism for anonymous functions. Theorem 2 implies that the only decision left when designing a mechanism is the order in which the agents are approached. We will show that for anonymous functions *random* order is best.

Definition 7. (Anonymous functions) *Anonymous functions are functions of which the value depends only on the sum of inputs.*

In other words, anonymous functions are invariant to input permutations. These functions include majority, percentile, consensus, etc.

Definition 8. (Equal opportunity mechanism for anonymous functions) *Let $f(.)$ be an anonymous function. An* equal opportunity *mechanism for $f(.)$ is a canonical mechanism which approaches the agents according to a* random *order.*

The equal opportunity mechanism can be computed in polynomial time provided that it is possible to decide in polynomial time whether $f(.)$ has already been determined.

Theorem 3. *If $f(.)$ is anonymous, the equal opportunity mechanism is an optimal implementation of it.*

From the above theorem we can get precise lower bounds on the expected payment of various anonymous functions. Our bounds imply bounds on the sum of the utilities required for computing a function in the model of [6, 7, 11]. We now analyze the optimal mechanisms for two important functions, majority and consensus.

Theorem 4. (Lower bound for majority) *An exact prediction of majority requires an expected payment of at least $\Omega(cn^{3/2})$. The payment increases exponentially with $|q - 1/2|$.*

Consensus Revisited. Let us reconsider consensus when $q = 1/2$. The expected payment (obtained according to Theorem 1) of the intuitive simultaneous mechanism is $cn2^n$ since each agent's influence is 2^{1-n} which is the probability that all the other agents declared 1. Consider the equal opportunity mechanism for consensus. The probability that an agent is approached and pivotal equals: Let k be an agent.

$$\sum_i (\mathbf{Pr}\big[\text{k's place is } i\big] \cdot \mathbf{Pr}\big[\text{k is approached in the i-th place}\big] \cdot \mathbf{Pr}\big[\text{k is pivotal}\big].$$

This equals $\frac{1}{n}\sum_i 1/2^{i-1} \cdot 1/2^{n-i} = 1/2^{n-1}$. The probability that an agent is approached is $\frac{1}{n}\sum_i 1/2^{i-1} < \frac{2}{n}$. Therefore, the agent's conditional influence is at least $n/2^n$. According to the payment characterization lemma, each agent approached receives a payment of $v_i < c2^{n+1}/n$. Thus, the expected payment of each agent is bounded by $c2^{n+2}/n^2$ and the overall expected payment is bounded by $c2^{n+2}/n$. This is an improvement by a factor of $\Theta(n^2)$ over the intuitive simultaneous mechanism. Reverting to example of section 1.1 the expected payment decreases from $\$160,000$ to less than $\$26,000$!

5 Approximate Predictions

We saw that exact predictions of functions like consensus or majority may require very large payments. An approximation mechanism for $f(.)$ is a mechanism which gets an accuracy parameter ε and in equilibrium predicts $f(.)$ with accuracy of at least $1-\varepsilon$. This section shows that **every** function can be approximated using a relatively low expected payment. The main problem that needs to be overcome is that when $f(.)$ is almost determined, agents will have very small influence. Avoiding this requires a little care. Due to space constraints we leave only the definition and the main theorem.

Definition 9. (ε-Implementation) *Let $\varepsilon \geq 0$. A (truthful) implementation is called an* ε-implementation *of $f(.)$, if $\mathbf{Pr}\big[f(x) \neq g(x)\big] < \varepsilon$ where $g(x)$ is the actual function that the mechanism implements and the probability is taken over the input distribution ϕ_q.*

Theorem 5. (Approximate predictions of general functions) *Let $\varepsilon < 1/2$. Every prediction function $f(.)$ can be ε-approximated with the following expected payment:*

$$B(f,\varepsilon) = O\left(\frac{cn^2}{\varepsilon(1-2\varepsilon)(1-q_M)\log n}\right).$$

6 Weighted Threshold Functions

The design of low payment mechanisms for non-anonymous functions is an intriguing challenge. A particularly interesting class of functions is the class of weighted threshold functions. These are functions of the form:

$$f_{w,\theta}(x) = \begin{cases} 1 \text{ if } \sum_{i=1}^{n} w_i x_i \geq \theta \\ 0 \text{ otherwise} \end{cases}$$

where w is a vector of non negative numbers and θ a positive threshold. These functions are natural to use in many prediction scenarios. For example when forming an expert committee one could set the experts' weights according to their reputation. We will show that for every two agents, and for every input vector of the other agents, the agent with the higher weight *always* has higher influence. This property indicates that characterize the optimal mechanism for weighted threshold functions may be easier than the general case.

We show that among all the mechanisms which approach the agents in a **fixed** order, the mechanism which approaches the agents by the order of their weights is the best.

Surprisingly, we demonstrate that this is not always the best deterministic mechanism. We do so by introducing a generic de-randomization technique.

Due to space constraints we leave only the definition and the main theorem.

Definition 10. (DWO mechanisms) *A* descending weight order (DWO) mechanism *is a canonical mechanism which approaches the agents serially by a fixed order determined by a descending order of their weights. (The mechanism chooses arbitrarily between agents with the same weight.)*

Theorem 6. *The DWO mechanism dominates every other fixed order mechanism.*

7 Future Research

In this paper we studied a basic class of prediction problems. Real life prediction scenarios are likely to be more complicated. Nevertheless, we believe that the insights we gained here (e.g. the usefulness of a serial approach, randomization, and zero information) are very useful for making such predictions. It would be interesting to extend our setup to several directions: in particular consideration of probabilistic prediction functions, general independent distributions, and general decision making in verifiable situations.

In this paper we assumed that the agents' inputs are independent. An important case is conditionally independent inputs. In this case, the mechanism may be able to use the fact that the agents' inputs are correlated in order to reduce its payment. Another important case is when inputs can be verified at least partially.

When $f(.)$ is polynomially computable, the greedy mechanism can be approximated in polynomial time. Devising polynomial time mechanisms with smaller payment for non anonymous functions is an intriguing challenge.

Finally, it will be interesting to study repeated prediction games where the function $f(.)$ is unknown to the mechanism but can be learnt over time.

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Walrasian Equilibrium: Hardness, Approximations and Tractable Instances

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Abstract. We study the complexity issues for Walrasian equilibrium in a special case of combinatorial auction, called single-minded auction, in which every participant is interested in only one subset of commodities. Chen et al. [5] showed that it is NP-hard to decide the existence of a Walrasian equilibrium for a single-minded auction and proposed a notion of approximate Walrasian equilibrium called relaxed Walrasian equilibrium. We show that every single-minded auction has a $\frac{2}{3}$-relaxed Walrasian equilibrium proving a conjecture posed in [5]. Motivated by practical considerations, we introduce another concept of approximate Walrasian equilibrium called weak Walrasian equilibrium. We show it is strongly NP-complete to determine the existence of δ-weak Walrasian equilibrium, for any $0 < \delta \leq 1$.

In search of positive results, we restrict our attention to the tollbooth problem [15], where every participant is interested in a single path in some underlying graph. We give a polynomial time algorithm to determine the existence of a Walrasian equilibrium and compute one (if it exists), when the graph is a tree. However, the problem is still hard for general graphs.

1 Introduction

Imagine a scenario where a movie rental store is going out of business and wants to clear out its current inventory. Further suppose that based on their rental records, the store has an estimate of which combinations (or bundles) of items would interest a member and how much they would be willing to pay for those bundles. The store then sets prices for each individual item and allocates bundles to the members (or buyers) with the following "fairness" criterion— given the prices on the items, no buyer would prefer to be allocated any bundle other than those currently allocated to her. Further, it is the last day of the sale and it is imperative that no item remain after the sale is completed. A natural way to satisfy this constraint is to give away any unallocated item for free. This paper looks at the complexity of computing such allocations and prices.

The concept of "fair" pricing and allocation in the above example is similar to the concept of market equilibrium which has been studied in economics literature for more than a hundred years starting with the work of Walras [27]. In this

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model, buyers arrive with an initial endowment of some item and are looking to buy and sell items. A market equilibrium is a vector of prices of the items such that the market clears, that is, no item remains in the market. Arrow and Debreu showed more than half a century ago that when the buyers' utility functions are concave, a market equilibrium always exists [3] though the proof was not constructive. A polynomial time algorithm to compute a market equilibrium for linear utility functions was given last year by Jain [18]. We note that in this setup the items are divisible while we will consider the case of indivisible items.

Equilibrium concepts [22] are an integral part of classical economic theory. However, only in the last decade or so these have been studied from the computational complexity perspective. This perspective is important as it sheds some light on the feasibility of an equilibrium concept. As Kamal Jain remarks in [18]— "If a Turing machine cannot efficiently compute it, then neither can a market". A small sample of the recent work which investigates the efficient computation of equilibrium concepts appears in the following papers: [8, 9, 18, 5, 24].

In this paper, we study the complexity of computing a Walrasian equilibrium [27], one of the most fundamental economic concepts in market conditions. Walrasian equilibrium formalizes the concept of "fair" pricing and allocation mentioned in the clearance sale example. In particular, given an input representing the valuations of bundles for each buyer, a Walrasian equilibrium specifies an allocation and price vector such that any unallocated item is priced at zero and all buyers are satisfied with their corresponding allocations under the given price vector. In other words, if any bundle is allocated to a buyer, the profit[1] made by the buyer from her allocated bundle is no less than the profit she would have made if she were allocated any other bundle for the same price vector.

In the clearance sale example, we assumed that every buyer is interested in some bundles of items— this is the setup for combinatorial auctions which have attracted a lot of attention in recent years due to their wide applicability [26, 7]. In the preceding discussions, we have disregarded one inherent difficulty in this auction model— the number of possible bundles are exponential and thus, even specifying the input makes the problem intractable. In this paper, we focus on a more tractable instance called *single-minded auction* [20] with the linear pricing scheme[2]. In this special case, every buyer is interested in a single bundle. Note that the inputs for these auctions can be specified quite easily. The reader is referred to [2, 1, 23, 5, 13] for more detailed discussions on single-minded auctions.

If bundles can be priced arbitrarily, a Walrasian equilibrium always exists for combinatorial auctions [6, 21]. However, if the pricing scheme is restricted to be linear, a Walrasian equilibrium may not exist. Several sufficient conditions for the existence of a Walrasian equilibrium with the linear pricing scheme have been studied. For example, if the valuations of buyers satisfy the *gross substitutes* condition [19], the *single improvement* condition [14], or the *no complementarities* condition [14] then a Walrasian equilibrium is guaranteed to exist.

[1] The profit is the difference between how much the buyer values a bundle and the price she pays for it.

[2] The price of a bundle is the sum of prices of items in the bundle.

Bikhchandani et al. [4] and Chen et al. [5] gave an exact characterization for the existence of a Walrasian equilibrium— the total value of the optimal allocation of bundles to buyers, obtained from a suitably defined integer program, is equal to the value of the corresponding relaxed linear program. These conditions are, however, not easy to verify in polynomial time even for the single-minded auction. In fact, checking the existence of a Walrasian equilibrium for single-minded auctions is NP-hard [5].

The intractability result raises a couple of natural questions: (1) Instead of trying to satisfy all the buyers, what fraction of the buyers can be satisfied ? (2) Can we identify some structure in the demands of the buyers for which a Walrasian equilibrium can be computed in polynomial time? In this paper we study these questions and have the following results:

- *Conjecture on relaxed Walrasian equilibrium.* Chen et al. [5] proposed an approximation of Walrasian equilibrium, called *relaxed Walrasian equilibrium.* This approximate Walrasian equilibrium is a natural approximation where instead of trying to satisfy all buyers, we try to satisfy as many buyers as we can. [5] gave a simple single-minded auction for which no more than 2/3 of the buyers can be satisfied. They also conjecture that this ratio is tight— that is, for any single-minded auction one can come up with an allocation and price vector such that at least 2/3 of the buyers are satisfied. In our first result we prove this conjecture. In fact we show that such an allocation and price vector can be computed in polynomial time.
- *Weak Walrasian equilibrium.* There is a problem with the notion of relaxed Walrasian equilibrium— it does not require all the winners to be satisfied. This is infeasible in practice. For instance in the clearance sale example, the store cannot sell a bundle to a buyer at a price greater than her valuation. In other words, at a bare minimum all the winners have to be satisfied. With this motivation, we define a stronger notion of approximation called *weak Walrasian equilibrium* where we try to maximize the number of satisfied buyers subject to the constraint that all winners are satisfied. In our second result we show that computing a weak Walrasian equilibrium is strongly NP-complete. We achieve this via a reduction from the Independent Set problem. Independently, Huang and Li [17] showed the NP-hardness result (but not strongly NP-hard) by a reduction from the problem of checking the existence of a Walrasian equilibrium.
- *Tollbooth problem.* With the plethora of negative results, we shift our attention to special cases of single-minded auctions where computing Walrasian equilibrium is tractable. With this goal in mind, we study the tollbooth problem introduced by Guruswami et al. [15] where the bundle every buyer is interested in is a path in some underlying graph. We show that in a general graph, it is still NP-hard to compute a Walrasian equilibrium given that one exists. We then concentrate on the special case of a tree, and show that we can determine the existence of a Walrasian equilibrium and compute one (if it exists) in polynomial time. Essentially, we prove this result by first showing that the optimal allocation can be computed by a polynomial time

Dynamic Program. In fact, the optimal allocation is equivalent to the maximum weighted edge-disjoint paths in a tree. A polynomial time divide and conquer algorithm for the latter was given by Tarjan [25]. Another Dynamic Program algorithm for computing edge-disjoint paths with maximum cardinality in a tree was shown by Garg et al. [12]. However, their algorithm crucially depends on the fact that all paths have the same valuation while our setup is the more general weighted case.

The paper is organized as follows. In Section 2, we review the basic properties of Walrasian equilibrium for single-minded auctions. In Section 3, we study two different type of approximations— relaxed Walrasian equilibrium and weak Walrasian equilibrium. In Section 4, we study the tollbooth problem— for the different structure of the underlying graphs, we give different hardness or tractability results. We conclude our work in Section 5. Due to space limitations, we omit the proofs in this paper. The proofs will appear in the full version of the paper.

2 Preliminaries

In a *single-minded auction* [20], an auctioneer sells m heterogeneous commodities $\Omega = \{\omega_1, \ldots, \omega_m\}$, with unit quantity each, to n potential buyers. Each buyer i desires a fixed subset of commodities of Ω, called *demand* and denoted by $d_i \subseteq \Omega$, with *valuation* (or *cost*) $c_i \in \mathbb{R}^+$. That is, c_i is the maximum amount of money that i is willing to pay in order to win d_i.

After receiving the submitted tuple (d_i, c_i) from each buyer i (the *input*), the auctioneer specifies the tuple (X, p) as the *output* of the auction:

- *Allocation vector* $X = (x_1, \ldots, x_n)$, where $x_i \in \{0, 1\}$ indicates if i wins d_i ($x_i = 1$) or not ($x_i = 0$). Note that we require $\sum_{i:\ \omega_j \in d_i} x_i \leq 1$ for any $\omega_j \in \Omega$. $X^* = (x_1^*, \ldots, x_n^*)$ is said to be an *optimal allocation* if for any allocation X, we have
$$\sum_{i=1}^{n} c_i \cdot x_i^* \geq \sum_{i=1}^{n} c_i \cdot x_i.$$
That is, X^* maximizes total valuations of the winning buyers.
- *Price vector* $p = (p(\omega_1), \ldots, p(\omega_m))$ (or simply, $(p_1, \ldots, p_m)$) such that $p(\omega_j) \geq 0$ for all $\omega_j \in \Omega$. In this paper, we consider linear pricing scheme, *i.e.*, $p(\Omega') = \sum_{\omega_j \in \Omega'} p(\omega_j)$, for any $\Omega' \subseteq \Omega$.

If buyer i is a winner (*i.e.*, $x_i = 1$), her (quasi-linear) *utility* is $u_i(p) = c_i - p(d_i)$; otherwise (*i.e.*, i is a loser), her *utility* is zero.

We now define the concept of Walrasian equilibrium.

Definition 1. (Walrasian equilibrium) [14] *A* Walrasian equilibrium *of a single-minded auction is a tuple (X, p), where $X = (x_1, \ldots, x_n)$ is an allocation vector and $p \geq 0$ is a price vector, such that*

1. *$p(X_0) = 0$, where $X_0 = \Omega \setminus \left(\bigcup_{i:\ x_i=1} d_i\right)$ is the set of commodities that are not allocated to any buyer.*
2. *The utility of each buyer is maximized. That is, for any winner i, $c_i \geq p(d_i)$, whereas for any loser i, $c_i \leq p(d_i)$*

Chen et al. [5] showed that checking the existence of a Walrasian equilibrium in general is hard.

Theorem 1. [5] *Determining the existence of a Walrasian equilibrium in a single-minded auction is NP-complete.*

Lehmann et al. [20] showed that in a single-minded auction, the computation of the optimal allocation is also NP-hard. Indeed, we have the following relation between Walrasian equilibrium and optimal allocation.

Theorem 2. [5] *For any single-minded auction, if there exists a Walrasian equilibrium (X, p), then X must be an optimal allocation.*

Due to the above theorem, we know that computing a Walrasian equilibrium is at least as hard as computing the optimal allocation. However, if we know the optimal allocation X^*, we can compute the price vector by the following linear program:

$$
\begin{aligned}
&\sum_{j:\ \omega_j \in d_i} p_j \leq c_i, \quad \forall\ x_i^* = 1 \\
&\sum_{j:\ \omega_j \in d_i} p_j \geq c_i, \quad \forall\ x_i^* = 0 \\
&p_j \geq 0, \quad \forall\ \omega_j \in \Omega \\
&p_j = 0, \quad \forall\ \omega_j \in \Omega \setminus \left(\bigcup_{i:\ x_i^*=1} d_i \right)
\end{aligned}
$$

Any feasible solution p of the above linear program defines a Walrasian equilibrium (X^*, p). In fact, note that if a Walrasian equilibrium exists, we can compute one which maximizes the revenue by adding the following objective function to the above linear program

$$\max \sum_{j:\ \omega_j \in \Omega} p_j$$

Thus, we have the following conclusion:

Corollary 1. *For any single-minded auction, if the optimal allocation can be computed in polynomial time, then we can determine if a Walrasian equilibrium exists or not and compute one (if it exists) which maximizes the revenue in polynomial time.*

3 Approximate Walrasian Equilibrium

Theorem 1 says that in general computing a Walrasian equilibrium is hard. In this section, we consider two notions of approximation of a Walrasian equilibrium. The first one called *relaxed Walrasian equilibrium*, due to Chen et al. [5], tries to maximize the number of buyers for which the second condition of Definition 1 is satisfied. We also introduce a stronger notion of approximation called *weak Walrasian equilibrium* that in addition to being a relaxed Walrasian equilibrium has the extra constraint that the second condition of Definition 1 is satisfied for *all* winners. We now define these notions and record some of their properties.

For any tuple (X, p), we say buyer i is *satisfied* if her utility is maximized. That is, if $x_i = 1$, then $p(d_i) \leq c_i$; if $x_i = 0$, then $p(d_i) \geq c_i$. For any single-minded auction $\mathcal{A}$, let $\delta_{\mathcal{A}}(X, p)$ denote the number of satisfied buyers under (X, p). In the following discussion, unless specified otherwise, we assume all unallocated commodities are priced at zero.

3.1 Relaxed Walrasian Equilibrium

We first define the notion of relaxed Walrasian equilibrium [5].

Definition 2. (relaxed Walrasian equilibrium) *Given any* $0 < \delta \leq 1$, *a* δ-relaxed Walrasian equilibrium *of single-minded auction* $\mathcal{A}$ *is a tuple* (X, p) *that satisfies the following conditions:*

- $p(X_0) = 0$, *where* $X_0 = \Omega \setminus \left(\bigcup_{i:\ x_i=1} d_i\right)$.
- $\frac{\delta_{\mathcal{A}}(X,p)}{n} \geq \delta$, *where* n *is the number of buyers in* $\mathcal{A}$.

Note that a Walrasian equilibrium is a 1-relaxed Walrasian equilibrium. Theorem 1 implies that it hard to maximize $0 < \delta \leq 1$ such that δ-relaxed Walrasian equilibrium exists for a single-minded auction. We now consider the following example which is due to Chen et al. [5].

Example 1. Consider the following single-minded auction $\mathcal{A}$: Three buyers bid for three commodities, where $d_1 = \{\omega_1, \omega_2\}, d_2 = \{\omega_2, \omega_3\}, d_3 = \{\omega_1, \omega_3\}$, and $c_1 = 3, c_2 = 3, c_3 = 3$. Note that there is at most one winner for any allocation, say buyer 1. Thus, under the condition of $p(\omega_3) = 0$ (since ω_3 is an unallocated commodity), at most two inequalities of $p(\omega_1\omega_2) \leq 3$, $p(\omega_2\omega_3) \geq 3$, $p(\omega_1\omega_3) \geq 3$ can hold simultaneously. Therefore at most two buyers can be satisfied.

Chen et al. conjectured that this ratio $\delta = \frac{2}{3}$ is tight [5]. We prove this conjecture in the following theorem.

Theorem 3. *Any single-minded auction has a* $\frac{2}{3}$*-relaxed Walrasian equilibrium, and thus* $\delta = \frac{2}{3}$ *is a tight bound. Further, a* $\frac{2}{3}$*-relaxed Walrasian equilibrium can be computed in polynomial time.*

	w_1	w_2	w_3	valuation
Buyer 1	+	+		3
Buyer 2		+	+	3
Buyer 3	+		+	3

Fig. 1. An example of single-minded auction where at most 2/3 buyers can be satisfied

3.2 Weak Walrasian Equilibrium

The notion of relaxed Walrasian equilibrium does not require all the winners to be satisfied. Indeed, in our proof of Theorem 3, to get a higher value of $\delta_{\mathcal{A}}(X,p)$, we may require some winner to pay a payment higher than her valuation. This is infeasible in practice: the auctioneer cannot expect a winner to pay a price higher than her valuation. Motivated by this observation, we introduce a stronger concept of approximate Walrasian equilibrium.

Definition 3. (Weak Walrasian Equilibrium) *Given any* $0 < \delta \leq 1$*, a* δ-weak Walrasian equilibrium *of single-minded auction* $\mathcal{A}$ *is a tuple* (X,p) *that satisfies the following conditions:*

- $p(X_0) = 0$*, where* $X_0 = \Omega \setminus \left(\bigcup_{i:\ x_i=1} d_i\right)$.
- *The utility of each winner is maximized. That is, for any winner* i*,* $c_i \geq p(d_i)$.
- $\frac{\delta_{\mathcal{A}}(X,p)}{n} \geq \delta$*, where* n *is the number of buyers in* $\mathcal{A}$.

Unfortunately, this extra restriction makes the problem much harder as the following theorem shows.

Theorem 4. *For any* $0 < \delta \leq 1$*, checking the existence of a* δ*-weak Walrasian equilibrium problem is strongly NP-complete.*

We use a reduction from the Independent Set problem in the proof of the above result. If we regard Independent Set and weak Walrasian equilibrium as optimization problems, then our reduction is a gap-preserving reduction [16]. Therefore we have the following conclusion:

Corollary 2. *There is an* $\epsilon > 0$ *such that approximation of weak Walrasian equilibrium problem within a factor* n^ϵ *is NP-hard, where* n *is the number of buyers.*

4 The Tollbooth Problem

As we have seen in the preceding discussions, in general the computation of a (weak) Walrasian equilibrium in a single-minded auctions is hard. In search of tractable instances, we consider a special case of single-minded auctions where the set of commodities and demands can be represented by a graph. Specifically, we consider the *tollbooth problem* [15], in which we are given a graph, the commodities are edges of the graph, and the demand of each buyer is a path in the graph. For the rest of this section, we will use the phrase tollbooth problem for a single-minded auction where the input is from a tollbooth problem.

4.1 Tollbooth Problem in General Graphs

For the general graph, as the following theorem shows, both the computation of optimal allocation and Walrasian equilibrium (if it exists) are difficult.

Theorem 5. *If a Walrasian equilibrium exists in the tollbooth problem, it is NP-hard to compute one.*

4.2 Tollbooth Problem in a Tree

In this subsection, we consider the tollbooth problem in a tree. Note that even for this simple structure, Walrasian equilibrium may not exist, as shown by Example 1. In this special case, however, we can determine whether Walrasian equilibrium exists or not and compute one (if it exists) in polynomial time. Due to Corollary 1, we only describe how to compute an optimal allocation efficiently. To this end, we give a polynomial time Dynamic Program algorithm (details are deferred to the full version). We note that computing an optimal allocation is the same as computing the maximum weighted edge-disjoint paths in a tree for which a polynomial time algorithm already exists [25]. Therefore, we have the following conclusion:

Theorem 6. *For any tollbooth problem in a tree, it is polynomial time to compute an optimal allocation, determine if a Walrasian equilibrium exists or not and compute one (if it exists).*

5 Conclusion

In the notions of approximate Walrasian equilibrium that we studied in this paper, we are concerned with relaxing the second condition of Walrasian equilibrium (Definition 1). That is, we guarantee the prices of the two approximate Walrasian equilibria clear the market (Definition 2, 3). Relaxing of the first condition is called *envy-free auction* and is well studied, *e.g.*, in [15]. Unlike the general Walrasian equilibrium, an envy-free pricing always exists, and thus, a natural non-trivial goal is to compute an envy-free pricing which maximizes the revenue [15]. Similarly, trying to compute a revenue maximizing Walrasian equilibrium is an important goal. However, even checking if a Walrasian equilibrium exists is NP-hard [5] which makes the task of finding a revenue maximizing Walrasian equilibrium an even more ambitious task. Corollary 1 shows that it is as hard as computing an optimal allocation.

Our polynomial time algorithm for determining the existence of a Walrasian equilibrium and computing one (if it exists) in a tree generalizes the result for the line case, where Walrasian equilibrium always exists [4, 5] and can be computed efficiently.

Our work leaves some open questions. For relaxed Walrasian equilibrium, we showed $\delta_{\mathcal{A}}(X, p) \geq 2/3$ for some (X, p) in any single-minded auction $\mathcal{A}$. An interesting question is how to approximate $\max_{(X,p)} \delta_{\mathcal{A}}(X, p)$ within an approximation ratio better than 2/3. In addition, we showed that for the tollbooth problem on a general graph, it is NP-hard to compute the optimal allocation, which implies that given that a Walrasian equilibrium exists, computing one is also hard. A natural question is to resolve the complexity of determining the existence of a Walrasian equilibrium (as opposed to computing one if it exists) in a general graph.

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On the Structure and Complexity of Worst-Case Equilibria*

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Abstract. We study an intensively studied resource allocation game introduced by Koutsoupias and Papadimitriou where n weighted jobs are allocated to m identical machines. It was conjectured by Gairing et al. that the fully mixed Nash equilibrium is the worst Nash equilibrium for this game w. r. t. the expected maximum load over all machines. The known algorithms for approximating the so-called "price of anarchy" rely on this conjecture. We present a counter-example to the conjecture showing that fully mixed equilibria cannot be used to approximate the price of anarchy within reasonable factors. In addition, we present an algorithm that constructs so-called *concentrated equilibria* that approximate the worst-case Nash equilibrium within constant factors.

1 Introduction

A central problem arising in the management of large-scale communication networks like the Internet is that of routing traffic through the network. Due to the large size of these networks, however, it is often impossible to employ a centralized traffic management. A natural assumption in the absence of central regulation is to assume that network users behave selfishly and aim at optimizing their own individual welfare. To understand the mechanisms in such non-cooperative network systems, it is of great importance to investigate the selfish behavior of users and their influence on the performance of the entire network.

In this paper, we investigate the price of selfish behavior under game theoretic assumptions, that is, we assume that each *agent* (i.e., user) is aware of the situation facing all other agents and aims at optimizing its own strategy. In particular, we investigate the structure of the network in a *Nash equilibrium*, i.e., a combination of mixed (randomized) strategies from which no users has an incentive to deviate. It is well known that such equilibria may be inefficient and do not always optimize the overall performance. We address the most basic case of a routing problem, a network consisting of m *identical parallel links* from an origin to a destination. There are n agents, each having an amount of traffic w_i to send from the origin to the destination. Each agent i sends the traffic using a

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possibly randomized *mixed strategy*, with p_i^j denoting the probability that agent i sends the entire traffic w_i to a link j. We assume the agents are *selfish* in the sense that each of them aims at minimizing its individual cost, i. e., the expected load on the machine it selects.

Koutsoupias and Papadimitriou [10] proposed to investigate the price of uncoordinated individual decisions in terms of the worst-case *coordination ratio*, which is the ratio between the expected social cost in the worst possible Nash equilibrium and in the social optimum. Here, we define social cost as the maximal load of a machine. In [2] and [9] it has been shown that the coordination ratio for a system of n weighted jobs and m identical machines is bounded by $\Theta\left(\frac{\ln m}{\ln \ln m}\right)$. However, for a given instance of the game, i. e., for a given vector of weights, the ratio between worst social cost of a Nash equilibrium and the optimal social cost may be significantly smaller. It is therefore an interesting question how the cost of the worst Nash equilibrium can be computed for a given instance. It has been conjectured that the fully mixed Nash equilibrium in which every agent assigns the same probability to every machine is the worst possible [6, 5, 4]. If this conjecture was true, then computing the worst Nash equilibrium would be a trivial task and its social cost could be approximated arbitrarily well using the fully polynomial randomized approximation scheme (FPRAS) presented in [3].

In this paper, we show that the Fully Mixed Nash Equilibrium Conjecture does not hold. In fact, the ratio between the social cost of the fully mixed Nash equilibrium and the worst Nash equilibrium can be almost as bad as the coordination ratio. We then present a different kind of equilibrium that concentrates the large jobs on a few machines. These *concentrated equilibria* are as bad as the worst Nash equilibrium up to constant factors. They can be computed in linear time and hence we obtain the first constant-factor approximation for worst-case equilibria on identical machines.

1.1 The Game

Koutsoupias and Papadimitriou [10] introduced a resource allocation game in which n jobs of size $w_1, \ldots, w_n \geq 0$ shall be assigned to m identical machines. Each job is managed by a selfish agent. The set of *pure strategies* for task i is $[m] := \{1, \ldots, m\}$. Let $(j_1, \ldots, j_n) \in [m]^n$ be a combination of pure strategies, one for each task. The *load* of link j is defined as

$$\lambda_j = \sum_{j_k = j} w_k .$$

The *cost* for agent i is λ_{j_i}. Every agent aims at minimizing her cost. The *social objective* is to minimize the maximum cost over all agents or, equivalently, the maximum load over all machines.

Agents may also use *mixed strategies*, i. e., probability distributions on the set of pure strategies. Let p_i^j denote the probability that agent i assigns its job to link j. Then

$$\mathbb{E}\left[\lambda_j\right] = \sum_{i \in [n]} w_i \, p_i^j .$$

The social cost of a mixed strategy profile $\mathbf{P} = (p_i^j)$ is defined as

$$SC(\mathbf{P}) \;=\; \mathbb{E}\left[\max_{j\in[m]} \lambda_j\right] \;.$$

The *expected cost of task i on link j* is

$$c_i^j \;=\; w_i + \sum_{k\neq i} w_k\, p_k^j \;=\; \mathbb{E}\left[\lambda_j\right] + (1-p_i^j)\, w_i \;.$$

A (mixed) strategy profile $\mathbf{P}$ defines a *Nash equilibrium* if and only if any task i will assign non-zero probabilities only to links that minimize c_i^j, that is, $(p_i^j) > 0$ implies $c_i^j \leq c_i^q$, for every $q \in [m]$. A Nash equilibrium is called *fully mixed* if $p_i^j > 0$ for all $i \in [n]$, $j \in [m]$. The game under consideration admits a unique fully mixed Nash equilibrium $\mathbf{F}$ in which each job is assigned with probability $\frac{1}{m}$ to each machine [11].

1.2 The Conjecture

Mavronicolas and Spirakis [11] investigate the social cost of fully mixed Nash equilibria. The motivation for their study is the hope that the techniques for the analysis of fully mixed strategies can be appropriately extended to yield upper bounds on the social cost for general equilibria. This hypothesis is formalized in the following conjecture stated in [6, 5, 4].

Conjecture 1 (FMNE conjecture). The fully mixed Nash equilibrium $\mathbf{F}$ is the worst Nash equilibrium, that is,

$$SC(\mathbf{F}) \;\geq\; SC(\mathbf{P}) \;,$$

for every Nash equilibrium $\mathbf{P}$.

Several attempts have been made to prove the conjecture. For example, it was shown that the conjecture is true for the case $m = 2$ [4] and for the case that $\mathbf{P}$ refers only to pure equilibria [6]. Furthermore, it was shown that the conjecture holds in an approximate sense if $m = n$ [1, 6]. In [3], an FPRAS for the social cost of the fully mixed Nash equilibrium is presented. Further interesting discussions can be found in [6].

The FMNE conjecture seems to be intuitive and appealing since in case of its validity it would allow for an easy identification of the worst-case mixed Nash equilibrium, whereas the worst case pure Nash equilibrium is NP-hard to compute.

1.3 Outline and Contribution

In Section 2 we give a counter-example to the FMNE conjecture that shows that mixed Nash equilibria may have a social cost that is by a factor of $\Omega\left(\frac{\ln m}{\ln\ln m}\right)$ worse than the social cost of the fully mixed Nash equilibrium. This is indeed the worst possible.

In Section 3 we present a simple algorithm that constructs a constant factor approximation of the worst Nash equilibrium in linear time.

2 The Counterexample

We present a counterexample to the FMNE conjecture. More specifically, we show that there is a family of simple instances of the game for which there exists an equilibrium $\mathbf{P}$ with

$$SC(\mathbf{P}) \;=\; \Omega\left(SC(\mathbf{F})\cdot\frac{\ln m}{\ln\ln m}\right) \; .$$

Let us remark that this is the worst possible ratio as it follows from the analyses in [2, 9] that the social cost of every Nash equilibrium can be at most $\mathcal{O}\left(\frac{\ln m}{\ln\ln m}\right)$ times the optimal social cost.

Theorem 1. *For every m, there exists an instance of the resource allocation game with m machines admitting a Nash equilibrium P with*

$$SC(\mathbf{P}) \;=\; \left(\frac{1}{4}-o(1)\right)\cdot\frac{\ln m}{\ln\ln m}\cdot SC(\mathbf{F}) \; .$$

The instance consist of $n=\mathcal{O}(f(m)\cdot m\ln m)$ jobs whose weights differ at most by a factor $\mathcal{O}(f(m)\cdot\ln m)$, where f denotes an arbitrary function in $\omega(1)$.

Proof. The counterexample uses only two different kinds of jobs: *Large jobs* of weight 1 and *small jobs* of weight $\frac{1}{k}$, $k\in\mathbb{N}$. Let $\ell\le m$ denote the number of large jobs. The number of small jobs is $k(m-\ell)$. Thus, the total weight is m and the optimal assignment has social cost 1. We show that the fully mixed equilibrium has social cost close to optimal if the parameters k and ℓ are chosen appropriately.

Lemma 1. *If $k=\Omega(f(m)\cdot\ln m)$ and $\ell=\mathcal{O}(\sqrt{n}/f(m))$ then $SC(\mathbf{F})\le 2+o(1)$.*

Proof. Recall that $\mathbf{F}$ assigns each job with probability $\frac{1}{m}$ to each of the machines.

– The assignment of the large jobs corresponds to a balls-and-bins experiment in which $\ell=\mathcal{O}(\sqrt{m}/f(m))$ balls are assigned uniformly at random to m bins. Fact 1 from the Appendix yields that for this experiment the expected number of balls in the fullest bin is $1+o(1)$. Thus, the expected maximum load due to the large jobs is $1+o(1)$, too.
– The assignment of the small jobs corresponds to a ball-and-bins experiment in which $k(m-\ell)$ balls are assigned uniformly at random to $m-\ell$ bins for $k=\Omega(f(m)\cdot\ln m)$. Fact 2 shows that for this experiment the expected number of balls in the fullest bin is $(1+o(1))\cdot k$. Since each ball corresponds to a job of weight $\frac{1}{k}$, the expected maximum load due to the small jobs is thus $1+o(1)$ as well.

Combining the upper bounds for the small and the large jobs yieds that the maximum load over all machines is at most $2+o(1)$ when taking into account all the jobs. □

Next we present a mixed Nash equilibrium whose maximum load is lower-bounded by a function in ℓ.

Lemma 2. *There exists a Nash equilibrium* $\mathbf{P}$ *with* $SC(\mathbf{P}) \geq (1 - o(1)) \cdot \frac{\ln \ell}{\ln \ln \ell}$.

Proof. We construct $\mathbf{P}$ in the following way. The small jobs are assigned using pure strategies. They are distributed evenly among the machines $1, \ldots, m - \ell$ such that each machine receives k small jobs. Hence, their load is fixed to 1. The large jobs are assigned to each of the remaining ℓ machines with probability $1/\ell$. Again, the expected load of these machines is 1. This is a Nash equilibrium since no job can improve by an unilateral move:

- For a small job i assigned to machine j_i, we have $c_i^{j_i} = 1$ and $c_i^j = 1 + 1/k$ for $j \neq j_i$.
- For a large job i, we have $c_i^j = 2 - 1/k$ if $j > m - \ell$ and $c_i^j = 2$ if $j \leq m - \ell$.

The social cost of this equilibrium equals the maximum occupancy of the balls-and-bins experiment where ℓ balls are assigned uniformly at random to ℓ bins. It is well-known that the maximum occupancy of this assignment is $(1 \pm o(1)) \cdot \frac{\ln \ell}{\ln \ln \ell}$ (see, e. g. [7]). □

The ratio between the bounds in Lemma 1 and 2 is maximized by choosing ℓ as large as possible under the constraints specified in Lemma 1. W.l.o.g., let $f(n) = \mathcal{O}(\ln n)$. We set $\ell = \Theta(\sqrt{m}/f(m))$. This way, $SC(\mathbf{P}) \geq (\frac{1}{2} - o(1)) \cdot \frac{\ln m}{\ln \ln m}$ and $SC(\mathbf{F}) \leq 2 + o(1)$. This completes the proof of Theorem 1. □

Let us remark that we can fine-tune the above example such that for $m = 14$ machines and $\ell = 3$ large jobs the expected maximum load of $\mathbf{P}$ is $17/9$ and the expected maximum load of $\mathbf{F}$ is $15/9 + 3/14 + \epsilon < 17/9$, where $\epsilon > 0$ can be made arbitrarily small by increasing the number of small jobs. Thus there is a counterexample to the FMNE conjecture with only 14 machines.

3 Approximating Worst-Case Equilibria

In this section we assume that jobs are ordered such that $w_1 \geq \cdots \geq w_n$. Also, w. l. o. g., we assume that the average load is 1, i. e. $\sum_{i=1}^n w_i = m$. Now, we define the quantities

$$M_i := \frac{e + w_i \ln(e + i)}{\ln(e + w_i \ln(e + i))} \qquad \text{and} \qquad M := \max_{i \in [n]} M_i, \tag{1}$$

where $e = 2.71\ldots$ is the Eulerian constant. We will see that the social cost of the worst Nash equilibrium of a given instance is $\Theta(M)$. In the next subsection we establish a lower bound by specifying an algorithm that outputs a Nash equilibrium of value $\Omega(M)$. Subsequently, we prove that an upper bound $\mathcal{O}(M)$ on the social cost of any equilibrium.

3.1 The Algorithm

We present an algorithm that constructs a Nash equilibrium that favors collisions between large jobs on few machines. It proceeds by partitioning the set of jobs into i large jobs and $n-i$ small jobs for a suitable index i. Then, all large jobs are assigned to machines $\{1,\ldots,k\}$ for a minimal k such that these machines are not overloaded, that is, the average load on these machines is at most 1. Additionally, small jobs are moved to machines $\{1,\ldots,k\}$ in order to guarantee that this produces a Nash equilibrium. The index i is chosen such that M_i is maximized. The pseudocode of algorithm Greedy-Nash is given below.

Algorithm 1. The Greedy-Nash algorithm

```
// find suitable threshold that separates large from small jobs
Choose i ∈ [n] such that it maximizes M_i.
// distribute largest jobs on first machines
Let W ← Σ_{j=1}^{i} w_j.
Choose k = ⌈W⌉.
p_j^l ← 1/k for j ∈ {1,...,i} and l ∈ {1,...,k}
// ensure that smaller jobs are satisfied, too
for all jobs j ∈ {i+1,...,n} in weight-decreasing order do
  if W/k ≤ (W_tot − W − w_j)/(m−k) then
    p_j^l ← 1/k for l ∈ {1,...,k}
    W ← W + w_j.
  else
    p_j^l ← 1/(m−k) for l ∈ {k+1,...,m}
  end if
end for
return ((p_i^j)_{i∈[n],j∈[m]}, M_i)
```

Note that the output of algorithm Greedy-Nash as described in the pseudocode has size $n \cdot m$. It can be represented in a compact way by specifying k and the set of jobs assigned to machines $\{1,\ldots,k\}$. This way, the algorithm has linear running time.

Intuitively, the proof of our lower bound M_i proceeds by merging the jobs such they have equal size and applying a Lemma on throwing $\Theta(k/w_i)$ balls with weight w_i into k bins.

Theorem 2. *Let M be defined as in Equation (1). A Nash equilibrium with expected maximal load $\Omega(M)$ can be constructed in time $\mathcal{O}(n)$ provided that the jobs are given in non-increasing order of weight.*

Proof. We first prove that the algorithm constructs a Nash equilibrium. We call the machines $1,\ldots,k$ *left machines* and the machines $k+1,\ldots,m$ *right machines*. Suppose that at the beginning of the for-loop all jobs $i+1,\ldots,n$ are assigned to the machines on the right with uniform probability $1/(m-k)$. Subsequently the algorithm may shift some of these jobs to the machines on the left. Before the loop starts, all jobs in $1, \ldots, i$ are satisfied because they only use the left machines and the expected load on every left machine is W/k whereas the loads on every right machine is $(W_{tot} - W)/(m-k) \geq W_i/k$ by our choice of k.

Note that it is an invariant of the loop that the total weight of jobs on left machines equals the value of the variable W. We have to show that after one pass of the loop job j is satisfied and no jobs in $\{1, \ldots, j-1\}$ become unsatisfied. Since job j goes to the group of machines on which all other jobs induce the smaller expected load, job j is obviously satisfied. If job j is assigned to the right machines the situation does not change and no other job can become unsatisfied. If job j is assigned to the left machines only other jobs on left machines can get unsatisfied. Assume that job $j' < j$ becomes unsatisfied. This job has weight $w_{j'} > w_j$ and this job being unsatisfied means that

$$\frac{W_{tot} - W - w_j}{m-k} < \frac{W + w_j - w_{j'}}{k} \leq \frac{W}{k}.$$

However, if this is the case, then job j would have been assigned to the right machines. Hence, the assignment returned by the algorithm is a Nash equilibrium.

We now show that this assignment has a social cost of at least $\Omega(M_i)$. For the time being, assume that $\sum_{j=1}^{i} w_j > 1$, that is $k \geq 2$ and the average load induced by jobs $1, \ldots, i$ on the left machines is at least $1/2$. For the purpose of the analysis we repeatedly split the jobs $1, \ldots, i-1$ into halves until their weight is in the range $[w_i, 2w_i]$. This way, the number of jobs with weight in $[w_i, 2w_i]$ is some number $i' \geq i$. In [9] it was shown that this inverse *ball fusion* does not increase the expected maximum load on the left machines. Finally, we reduce all job weights down to w_i again not increasing the expected maximum load. The average load on the left machines is still at least $1/4$. Then it follows from Fact 3 in the appendix, that now the expected maximum load is at least

$$\Omega\left(\frac{e + w_i \ln(e+k)}{\ln(e + w_i \ln(e+k))}\right)$$

This term gives a lower bound of $\Omega(M_i)$ if we assume that $w_i \geq 1/k$ as, in this case, $i' \cdot w_i \leq k$ implies $k \geq \sqrt{i'}$ which gives $\ln(e+k) = \Omega(\ln(e+i')) = \Omega(\ln(e+i))$.

We are left with two special cases in both of which we show that $M_i = \mathcal{O}(1)$ and hence is a trivial lower bound.

- $\sum_{j=1}^{i} w_j \leq 1$. In that case, our constructed Nash equilibrium has a trivial lower bound of 1. Furthermore,

  $$M_i \leq e + w_i \ln(e+i) \leq e + w_i \ln(e + 2/w_i) \leq e + 2 + w_1(e-1) = \mathcal{O}(1)$$

  since $\ln(1+\epsilon) \leq \epsilon$ for any $\epsilon > 0$.
- $w_i \leq 1/k$. Since $w_i \leq k/i$ we have $i \leq k/w_i \leq 1/w_i^2$ and hence $w_i \leq 1/\sqrt{i}$. Substituting this into M_i yields $M_i = \mathcal{O}(1)$.

Thus in all cases, the social cost is lower bounded by $\Omega(M_i)$. Since i is chosen to maximize M_i, it is also lower bounded by $\Omega(M)$. □

3.2 Upper Bound

The maximum load over all machines is equal to the maximum *height* over all jobs where the height of a job is defined as follows. We assume that jobs are thrown into the machines according to the Nash probability distribution one after another in non-increasing order of weight. The height of job i is the total weight of jobs on its machine at its insertion time. The important property of this definition is that the height of job i does not depend on the assignments of the jobs $1, \ldots, i-1$. More formally, we define indicator variables I_i^j where $I_i^j = 1$ if and only if ball $i \in [n]$ is assigned to machine $j \in [m]$. For any job $i \in [n]$, let j_i denote the machine that job i is assigned to and let

$$X_i = \sum_{k=i}^{n} I_k^{j_i} w_k$$

denote the height of this job. Obviously $H = \max_i\{X_i\}$, the maximal height over all jobs, is equivalent to the maximum load over the machines.

Theorem 3. *Let M be defined as in Equation (1). For any Nash equilibrium, it holds that $\mathbb{E}[H] \leq M$.*

Proof. Consider job i. For $\alpha \geq 1$, let $q = 2\,e\,\alpha\,M_i$. On every machine that job i assigns positive probability to, the expected total load induced by jobs $i+1, \ldots, n$ is upper bounded by 1 since we are at a Nash equilibrium, that is, $\mathbb{E}[X_i - w_i] \leq 1$. Applying a weighted Chernoff bound yields

$$\mathbb{P}[X_i - w_i \geq q] \leq \left(\frac{e}{q}\right)^{q/w_i} .$$

Observe that $q \geq e\sqrt{e + w_i \ln(e+i)}$ as $x/\ln x \geq \sqrt{x}$ for any $x \geq e$. Hence,

$$\begin{aligned}
\mathbb{P}[X_i - w_i \geq q] &\leq \left(\frac{1}{\sqrt{e + w_i \ln(e+i)}}\right)^{2e\alpha\,\ln(e+i)/\ln(e+w_i\ln(e+i))} \\
&= \left(\frac{1}{e}\right)^{e\,\alpha\,\ln(e+i)} \\
&\leq \frac{1}{i^{e\alpha}} \\
&\leq \frac{1}{i^2\,2^{\alpha}}
\end{aligned}$$

for $i \geq 2$. Applying a union bound we see that

$$\mathbb{P}[X - w_1 \geq \alpha\,M] \leq \mathbb{P}[\exists i : X_i - w_i \geq \alpha \cdot M_i] \leq 2^{-\alpha} \sum_{i=1}^{n} \frac{1}{i^2} \leq 2^{-\alpha} \cdot \frac{\pi^2}{6}$$

and hence

$$\begin{aligned}
\mathbb{E}[X] &\leq w_1 + \int_0^\infty \mathbb{P}[X - w_1 \geq t\,M]\,(M - w_1)\,dt \\
&\leq w_1 + (M - w_1)\left(\int_0^1 \mathbb{P}[X - w_1 \geq t\,M]\,dt + \int_1^\infty \mathbb{P}[X - w_1 \geq t\,M]\,dt\right) \\
&\leq w_1 + (M - w_1)\cdot\left(1 + \frac{\pi^2}{6}\int_1^\infty 2^t\,dt\right) \\
&= \mathcal{O}(M)
\end{aligned}$$

This finishes the proof of the theorem. □

4 Conclusions

We have shown that the fully mixed Nash equilibrium is not the worst-case equilibrium and does not even give a good approximation. In contrast, we have shown that concentrating large jobs on a few machines yields equilibria that approximate the worst-case within a constant factor. As these equilibria can be constructed in linear time we obtained the first constant factor approximation for the worst-case Nash equilibrium.

Our analysis is restricted to identical machines. The question whether worst-case equilibria can be approximated for the case of uniformly related machines remains open and is a challenging problem.

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Appendix

The following facts about balls and bins processes have almost surely been shown somewhere else before. Due to space limitations, we leave the proofs for the full version.

Fact 1. *Let f denote any function in $\omega(1)$. If $n \leq \sqrt{m}/f(m)$ balls are assigned independently and uniformly at random to m bins. Then the expected number of balls in the fullest bin is $1 + o(1)$.*

Fact 2. *Let f denote any function in $\omega(1)$. If $n \leq m \cdot f(m) \cdot \ln m$ balls are assigned independently and uniformly at random to m bins. Then the expected number of balls in the fullest bin is $f(m) \cdot \ln m + O(\sqrt{f(m)} \cdot \ln m) = (1 + o(1)) \cdot f(m) \cdot \ln m$.*

Fact 3. *When n balls are thrown into m bins independently, uniformly at random, the expected number of balls in the fullest bin is*

$$\Omega\left(\frac{n/m + \ln(e+m)}{\ln(e + (m/n)\ln(e+m))}\right).$$

Thus, if the balls have weight $w = m/n$, so that the average load is 1, then the maximum weight over all bins is

$$\Omega\left(\frac{1 + w\ln(e+m)}{\ln(e + w\ln(e+m))}\right).$$

Club Formation by Rational Sharing: Content, Viability and Community Structure

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Abstract. A sharing community prospers when participation and contribution are both high. We suggest the two, while being related decisions every peer makes, should be given separate rational bases. Considered as such, a basic issue is the viability of club formation, which necessitates the modelling of two major sources of heterogeneity, namely, peers and shared content. This viability perspective clearly explains why rational peers contribute (or free-ride when they don't) and how their collective action determines viability as well as the size of the club formed. It also exposes another fundamental source of limitation to club formation apart from free-riding, in the community structure in terms of the relation between peers' interest (demand) and sharing (supply).

1 Introduction

Much current research in peer-to-peer systems focuses on performance of the platform on which peers transact. Even when incentives of the peers themselves are considered, the concern is with their effects on the system load. Invariably, selfish peers are assumed always ready to participate. Incentive mechanisms are then necessary to make sure they behave nicely and do not cause excessive load. As this *performance perspective* dominates the research agenda, free-riding is often identified as a major problem, a limiting factor to be dealt with.

In reality, free-riding [1] is in fact very common in open access communities, including many successful file sharing networks on the Internet where incentive mechanisms are often scant or absent altogether [2, 3]. A problem in principle does not appear to be a problem in practice. An empirical observation offers a straightforward explanation: many peers are seemingly generous with sharing their private assets. They upload files for sharing, help one another in resource discovery, routing, caching, etc. As long as sufficiently many are contributing, free-riding may be accommodated and the community would sustain. But then why are they so generous?

Our study is in part motivated by Feldman *et al.* [4] who explains this in terms of peers' intrinsic *generosity*. Assuming that a peer's generosity is a statistical *type variable* and he would contribute as long as his share of the total cost does not exceed it, they show that some peers would choose to contribute *even as others free-ride*, as long as sufficiently many peers have high enough generosity.

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However, how the generosity is derived is left open. In contrast, [5] identifies a rational basis for peers to contribute based on a utility function predicated on the benefit of information access, which is improved as peers contribute in load sharing that eases system congestion. They demonstrate also that a sharing community may sustain in the face of free-riding without any explicit incentive mechanism.

1.1 Why Peers Participate?

In this paper, we contend that there remains another important basic question to examine, namely, why peers participate in the first place. It is of course reasonable to assume simply that peers do so for their own benefit. However, it is crucial to note that peers often benefit *differently*, even as they participate to the same extent and contribute to the same extent. In a file sharing community for instance, the benefit a peer sees would depend on whether he finds what he is interested in there, which should vary from peer to peer. In this regard, the studies cited above and many others essentially assume that peers have identical interest and sees the same benefit potential from a sharing community, which is obviously not very realistic.

1.2 Goods Type and Peer Type

As peers exhibit different interest, they would demand different things in their participation. It follows that the system serving a sharing community (*the club*) comprising such peers would have content with *a variety of goods*. The availability of the goods a peer demands from the club would determine the benefit potential he sees, and the extent of his participation. Two new type variables are implied: first, a type variable for different goods, and second, a peer type variable for different interests in the goods. For simplicity, we may replace the second by some measure of his *propensity-to-participate*, as a proxy variable conditioned on a given club with its particular content.

We believe the new type variables are essential to analyzing realistic peer-to-peer systems. We shall demonstrate this by constructing a generic model of an *information sharing club (ISC)* [6] in which peers contribute by sharing information goods. Such shared goods then make up the club's content which in turn entice peers to participate and contribute.

1.3 Viability Perspective

Unlike models taking the performance perspective, the ISC model takes a *viability perspective* and focuses instead on a more primary concern of whether the club itself has any chance to grow in size at all. In principle, this would depend on a *mutual sustenance between club membership and content*. This concern turns out to subsume free-riding, and reveals another fundamental source of limitation, in *community structure* in terms of the relation between demand and supply among the sharing peers. Generally speaking, insufficient interdependence among them for their interested goods would limit their propensity-to-participate and the size of the club they form. In the worst case, the club may get stuck in a deadlock with few members and little content, even emptiness, when there is little overlap between their interest (demand) and potential contribution (supply).

We begin our discussion in the next section by re-visiting the question of why rational peers contribute. We shall derive another peer type variable – the *propensity-to-contribute* (c.f. generosity of [4]) – from the peer's utility function of benefit and cost that arises from *both* participation and contribution. As a result, we come up with new insight regarding free-riding, in particular, when and why even generous peers may cease to contribute. Section 3 introduces the concept of goods type and club content as a distribution over goods types, and how a club may prosper on a mutual sustenance between its membership and content. Section 4 describes the ISC model and derives two viability conditions. We demonstrate how the community structure affects viability in simplistic model instances with two goods types. In the final section, we discuss the design of incentive systems in the face of two different sources of limitation, namely, free-riding and community structure.

2 ***Why*** Do Rational Peers Share?

Let peer i's contribution (or cost) to a club be C_i and the club's benefit to him is B_i. Further assume that each peer's choice of C_i directly affects its benefit (therefore, B_i is a function of C_i among other things). The peer's utility, a function of both the benefit and cost, is given by $U_i(B_i, C_i)$. It is intuitive to assume that U_i is decreasing in C_i and concave increasing in B_i. Given any particular level of contribution C_i and a corresponding level of benefit B_i, any small increment of utility is given by

$$\delta U_i(B_i, C_i) = (\partial U_i/\partial B_i)\delta B_i + (\partial U_i/\partial C_i)\delta C_i \ .$$

The (non-negative) ratio

$$-\frac{\partial U_i/\partial B_i}{\partial U_i/\partial C_i}$$

then gives us the (marginal) *exchange rate* of peer i's contribution to benefit. In other words, it is the maximum amount of contribution the peer would give in exchange for an extra unit of benefit with no net utility loss.

Although we have not yet described how to determine the club's benefit to a particular peer B_i, it suffices to say that the value of $\partial B_i/\partial C_i$ represents the *club (marginal) response* to peer i's contribution, per current levels of benefit and cost at (B_i, C_i). Of particular interest is whether this club response to peer i's *initial* contribution, viz $\partial B_i/\partial C_i|_{C_i=0}$, is enticing enough. The answer can be different for each peer. Specifically, peer i would contribute only if

$$\left(\left.\frac{\partial B_i}{\partial C_i}\right|_{C_i=0} \right)^{-1} < \ \Gamma_i \triangleq -\left.\frac{\partial U_i/\partial B_i}{\partial U_i/\partial C_i}\right|_{C_i=0} \ . \tag{1}$$

Otherwise, he prefers not to contribute and free-ride when he actually joins the club for the good benefit he sees. Note that Γ_i is a property derived from the peer's utility function, and may serve as a type variable to characterize different peers. We refer to this property as a peer's *propensity-to-contribute*.

Therefore, we have tied a peer's decision to his contribution to a club to two quantities, namely, his propensity-to-contribute Γ_i and the club response $\partial B_i/\partial C_i$ which depends on the specific club model. This treatment of peers as rational agents is similar to the formulation in [5]. It is also compatible with [4], in that each peer ends up being characterized by a type variable. The difference is that [4] chose not to further explain how its type variable of generosity, is derived. Another major difference is that Γ_i is dependent on B_i. When U_i is concave increasing in B_i due to decreasing marginal return of benefit, Γ_i would decrease as B_i increases. In other words, *improved benefit reduces a peer's propensity-to-contribute.*

Furthermore, the club response $\partial B_i/\partial C_i$ would also tend to decrease as B_i increases. A club already offering high benefit to a peer has less potential to reward further to incentivize his contribution. The club response may even reduce to naught when the maximum benefit is being offered. In this case, peer i would cease to contribute and join as a free-rider, even when his propensity-to-contribute may not be small.

In summary, as a club prospers and peers see improving benefit, the motivation to contribute would reduce on two causes: decrease in peer's propensity-to-contribute *and* decrease in club response. The latter is caused by decreasing marginal benefit of a prosperous club, which we identify as another systematic cause of free-riding (apart from peers not being generous enough).

3 *What* Do Peers Share?

Before a peer decides whether to contribute based on the marginal benefit, he first decides whether to join based on the benefit itself, namely, B_i. Research works that focus on incentive schemes often study B_i as a function of the peer's decision to contribute, namely, C_i, *only*, whence the two decisions are not differentiated and become one. Consequently, peers who contribute the same see the same benefit potential. This is the assumption we call into question here.

A salient feature of many real world peer-to-peer systems is the variety of goods being shared. The benefit that a peer receives is dependent on what he demands in the first place, and whether they are available in the current club content. Even if two peers contribute the same and demand the same, the benefit they receive would differ in general. Peers with similar interests would see similar benefit potential while peers with different interests may not. Therefore, a peer's interest, in terms of the types of goods he demands, is an important type variable. However, this would be a distribution over all goods types, which is complicated. To account for peer i's particular interest in relation to a given club, the benefit on offer, viz. B_i, would suffice. As benefit is the primary motivation for him to participate, B_i would qualify as a measure of his *propensity-to-participate*, a proxy variable for his interest conditioned on the given club.

The assumption often made, that peers contributing the same receive the same benefit, implicitly implies a single goods type. This would be a gross over-simplification that ignores the variety of goods as a principal source of peer heterogeneity. Consequently, it would overlook important structural properties of both club membership and content essential for a detailed analysis of the dynamics of club formation.

3.1 Club Formation, Membership and Content

A club would attract a peer by its range of shared goods, and to an extent which depends on the availability of the goods he demands. As a result, it would tend to attract peers who are interested in its available content. At the same time, such peers contribute to the club's content with what they share. Such is the essence of a sharing community: peers come together by virtue of the overlap between the range of goods they share (supply) and the range they are interested in (demand). With benefit (B_i) as peer's propensity-to-participate, he determines his extent of participation. With Γ_i as a threshold for the club response, as his propensity-to-contribute, he determines whether to contribute during participation.

The mutual relation between peers' demand and supply is suggestive of potentially complex coupled dynamics of club membership and content. A club would prosper on virtuous cycles of gains in membership and content, and would decline on vicious cycles of losses in both. If and when a club sustains would depend on the existence and size of any stable equilibrium. In particular, when an empty club is a stable equilibrium, it signifies a deadlock between insufficient content and insufficient membership. Otherwise, an empty club would be unstable and self-start on the slightest perturbation, growing towards some statistical equilibrium with a positive club size. In the following, we refer to such a club as being *viable*, and *viability* is synonymous to instability of an empty club.

4 A Simple Sharing Model

Here we present a simple model of an information sharing club (ISC), sharing information goods which are *non-rivalrous*[1].

The model is based on two kinds of entities: a population $\mathcal{N}$ (size N) of peers, and a set $\mathcal{S}$ of information goods. In addition, we assume the following characteristics about these peers:

1. Each peer has a supply of information goods which is available for sharing once the peer joins the ISC.
2. Each peer has a demand for information goods in the ISC.

[1] Unlike rivalrous goods such as bandwidth or storage which are congestible, non-rivalrous goods may be consumed concurrently by many users without degradation. Information goods are inherently non-rivalrous as they may be readily replicated at little or no cost.

The purpose of the model is to determine, based on the characteristics of the peer population, whether an ISC will form[2]; and if so, what is the size of this club and its content. At the heart of the model is an assumption about how a peer decides whether he joins the club. This decision process can be modelled as a function of a given peer's demand function, and a club's content. In other words, given a club with certain content (of information goods), a peer would join the club if his demand (for information goods) can be sufficiently met by the club. So we complete a cycle: given some content in the club, we can compute a peer's decision whether to join a club; given the peers' decisions we can compute the collective content of the club; from the content, we can compute if additional peers will join the club. This process can always be simulated. With suitable mathematical abstraction, this process can also be represented as a fixed point relationship that yields the club size (and content) as its solution [6].

In our mathematical abstraction, we let peer i's demand be represented by a probability distribution $h_i(s)$, and his supply be represented by another distribution $g_i(s)$, where s indexes the information goods in $\mathcal{S}$. For simplicity, we normally assume $h_i(s) = h(s)$ and $g_i(s) = g(s)$ for all i. Further, a peer's decision to join a club is based on a probability $P_i(n)$, where n is the number of peers already in the club. Let us consider what these assumptions mean (we will come back to what $P_i(n)$ is later). First, by representing a peer's supply (of information goods) using a common probability distribution, we can easily derive the distribution of the resultant content of a club of n peers. Second, given a peer's demand and a club's content, both as probability distributions, it is possible to characterize whether a peer joins a club as a Bernoulli trial (where P_i gives the probability of joining, and $1 - P_i$ gives the probability of leaving a club). Thirdly, the composition of the club (if formed) is not deterministic; rather it is given by the statistical equilibrium with peers continually joining and leaving.

As a result, the club content is given by a probability distribution composed from the supplies of n peers, where n is the expected size of the club. Each peer's joining probability, $P_i(n)$, is then given by

$$P_i(n) \triangleq E_{h_i(s)}[1 - e^{-\rho_i \Phi(n) g(s)}] \tag{2}$$

where $g(s), s \in \mathcal{S}$ is the distribution of information good shared and available in the club over a goods type domain $\mathcal{S}$, and $h_i(s)$ is a distribution representing of peer i's interest over $\mathcal{S}$. Here, $\Phi(n)$ represents the total quantity of information goods found in the club if n peers joined, which is given by

$$\Phi(n) = n\bar{k} + \phi_0 \tag{3}$$

where $\bar{k}$ is an average peer's *potential* contribution (realized when he actually joins the club) and $\phi_0 \geq 0$ represents some *seed content* of the club. The rationale for equation (2) is as follows. Given any specific s demanded by the considered

[2] Theoretically, it is also possible that more than one clubs will form, although the analysis of multiple club scenarios is outside the scope of this paper.

peer, the expected number of copies of it in the club is given by $\Phi(n)g(s)$. We assume the probability of not finding this item in the club exponentially diminishes with the quantity, which means the probability of finding that item is $1 - e^{-\rho_i \Phi(n)\, g(s)}$. Since the demand of the considered peer is actually a distribution $h_i(s)$, therefore this peer's likely satisfaction is expressed as an expectation over the information goods he may demand. Finally,

$$\rho_i = \rho(K_i) \in [0, 1] \tag{4}$$

is *search efficiency* that peer i sees, which is made dependent on his contribution K_i by some incentive system of the club to encourage contribution so that $\rho(K_i)$ is monotonically increasing in K_i.

In this model, the benefit of the club to a peer is the extent the peer chooses to join, viz $B_i = P_i$; the cost (of contribution), on the other hand, is simply $C_i = K_i$.

4.1 Peer Dynamics: Joining and Leaving

In figure (1) the club is depicted as the smaller oval, and the flux of peers continually joining and leaving the club statistically is driven by peers' propensity-to-participate (the distribution of P_i). The figure also depicts the partition of the universe into the set of potentially contributing peers (the white area), and the set of non-contributing peers (the lightly shaded area). This division is driven by the population's propensity-to-contribute (the distribution of Γ_i).

By definition, non-contributing peers have no effect on Φ or P_i. Therefore, we may refine the ISC model to ignore them and focus on the potentially contributing peers (*pc-peers*), namely, the population $\mathcal{N}$ (size N) is the population of pc-peers and n is the number of pc-peers in the club. Subsequently, $\bar{k} = 1/N \sum_{K_i>0} K_i$ and $K_i > 0$ is the positive contribution of pc-peer i.

However, the population of pc-peers is actually dependent on n, viz. $\mathcal{N}$ and N should really be $\mathcal{N}(n)$ and $N(n)$. Since the incoming rate $\bar{P}(n)$ [3] depends on n, it gives rise to a fixed point equation

$$\bar{P}(n)\; N(n) = n \tag{5}$$

for the statistical equilibrium club size indicated by n.

As pointed out in Section 2, prosperity reduces the motivation to contribute and some population in the white area would cease to be (potentially) contributing and moved to the shaded part. However, we shall assume $N(n)$ to be roughly constant here when we are studying the club's viability property which is pre-emptive to prosperity and determined by the club dynamics around $n = 0$.

[3] $\bar{P}(n) = E[P_i(n)]$ is the average joining probability. Assuming independence between the participation and contribution decisions, the average is the same when taken over either all peers or the potentially contributing peers only.

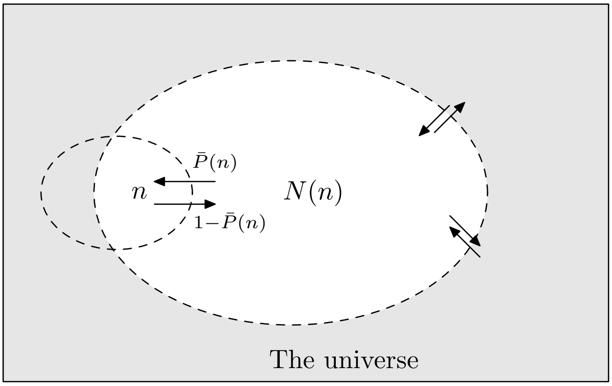

Fig. 1. Peer dynamics of the ISC model

4.2 Statistical Equilibrium of Membership and Content

The fixed point equation (5) characterizes a statistical equilibrium club size when the rates of incoming and outgoing members of the club are balanced [6]. Consequently, we have the following proposition :

Proposition 1 (Sufficient viability condition).

$$N(0)\sum_{s} g(s)h(s) > (\bar{k}\,\rho(0^+))^{-1} > 0 \qquad (6)$$

is sufficient for the empty club to be unstable where h(s) is the expected demand distribution[4]*. The club is viable with a positive equilibrium club size that satisfies equation (5).*

The membership dynamics and content dynamics are closely coupled : as pc-peers join and leave, they alter the total shared content, inducing others to revise their join/leave decisions. Pc-peer i would contribute on joining as long as his initial contribution could improve his utility, as discussed in section 2.

Whether some peers remain potentially contributing even when the club is empty is essential to viability; $N(0)$ has to be strictly positive. This leads to our second proposition:

Proposition 2 (Necessary viability condition).

$$\rho'(0)\phi_0 > 0\,. \qquad (7)$$

This means some positive incentive to entice a peer to become a contributor (from a non-contributor) and some seed content, viz. $\phi_0 > 0$, are needed for a club to be viable and not get stuck in an empty state with no contributing peers.

[4] h(s) is the demand the club sees and would be a *weighted* average of peers's demand, with the weights being their demand rates.

(However, while more seed content improves viability as well as participation (P_i), it would also tend to reduce N the same way that prosperity does. The equilibrium club size would increase only if this reduction is more than offset by the increase in P_i.)

4.3 Community Structure

Since the basis for club formation is the overlap between peers' interest and the club's content, an interesting question is whether peers tend to form small clubs due to clustering of common interest, or large clubs with diverse population of peers. Furthermore, given a club of multiple information goods, can it be decomposed and analyzed as multiple clubs of single goods? The answers to these questions would shed more light on why it is important to model an information sharing club based on content.

Suppose we have two disjoint clubs with equilibrium club sizes n_1 and n_2, formed independently based on their respective potentially contributing peer populations N_1 and N_2. When they are brought together, it is intuitive that a new club would form with at least size $n_1 + n_2$. We refer to this as *mixing* (two clubs). A key question is : will the size of the new club, n, be strictly greater than $n_1 + n_2$?

We devise two simple simulated examples below to study the effect of mixing.

First, consider a universe with only two types of information goods and two independent clubs, each with peers interested in a different single goods type only. However, when they contribute, they may share some percentage q of the other goods types. The demand and supply distributions are shown in table 1. Further we assume $\bar{k}\rho = 0.015$; $N_1 = N_2 = 100$ such that both clubs are viable (when q is reasonably small) according to proposition 1. Therefore q may be regarded as the degree of overlap between the two clubs' supply.

Table 1. Two population and two goods type scenario

peers	demand	supply
type 1	$\{1, 0\}$	$\{1 - q, q\}$
type 2	$\{0, 1\}$	$\{q, 1 - q\}$

We obtained (by simulation) the equilibrium club sizes n_1 and n_2 when the populations N_1 and N_2 are separate, and then the equilibrium club size n when they are mixed. Figure (2) shows the gain in the mixed club size, as the ratio $\frac{n}{n_1+n_2}$. With no overlap ($q = 0$), the mixed club size n is simply the sum of the two individual clubs ($n_1 + n_2$). However a larger club is formed with rapidly increasing gain as the overlap increases. A two-fold gain results with a moderate overlap of 20%.

Our second example considers the same two populations with the total size $N_1 + N_2 = 200$, except that $N_1 > N_2$ so that the second population do not make a viable club this time. Figure (3) shows what happens when these two

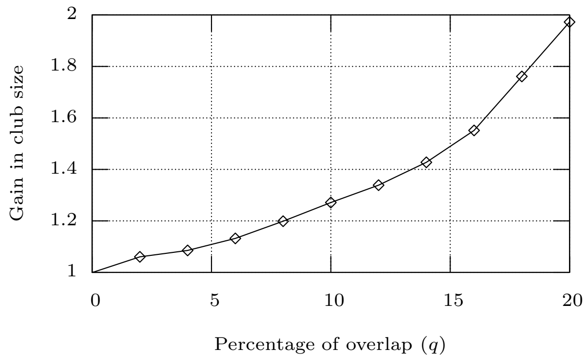

Fig. 2. Effect of mixing clubs when both are viable

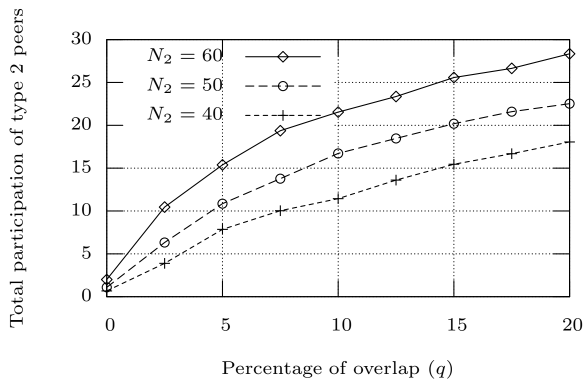

Fig. 3. Effect of mixing clubs on the non-viable club (type 2 peers)

populations are mixed with various degrees of overlap. For a good range of N_2, the non-viable club is able to form (as part of the larger mixed club) with different vigor monotonically increasing in q. These two examples demonstrate how the modeling of goods types help account for club formation: a high degree of overlap between peers' supply and demand is conducive to large club formation, and vice versa. When a large club is formed with significant mixing, it comprises gain in membership and content over any constituent specialized clubs, and is structurally different from their mere union.

4.4 Discussions

Our reasoning and analysis has so far assumed a non-rivalrous relation among peers. In practice, there always are rivalrous resources which peers may contend for sooner or later. For example, bandwidth could be scarce when delivering large files and storage space may be limited. As a club increases in size, *diseconomy of scale* due to such contention would set in and prohibit large club formation. Benefit to peers would suffer, perhaps as a consequence of a reduction in search efficiency ρ_i. In this case, the tendency to form smaller clubs with more specialized content would increase. However, economy of scale may also be at work, most notably, due to statistical effects (e.g. multiplexing gain) and/or network effects[5], which would increase the tendency to form larger clubs with a diverse population of peers.

5 Concluding Remarks: Incentivizing Sharing

The ISC example demonstrates that the overlap between peers' interest (demand) in and sharing (supply) of the variety of goods is crucial. This two goods type example, while simplistic, is suggestive of the important role played by peers who share a wider variety of goods. They may help induce virtuous cycles that improve membership and content, resulting in a larger club size. Further, they may help *niche* peer groups, otherwise not viable, to benefit from participating in a more mixed and larger community. Therefore it is conceivable that such sharing creates more "synergy" than more specialized supplies. In economics term, they create positive externality and should justify positive incentives.

The viability perspective points to the importance of maintaining a large $N(n)$ for viability is instructive to the design of incentive mechanisms. In other words, the more potentially contributing peers the better. Contribution should be encouraged especially when starting up a club, as viability depends on a large enough $N(0)$.

However, it should be emphasized that encouraging contribution may not entail discouraging free-riding. One may imagine free-riders who are discouraged by some negative incentive schemes simply demand less without becoming contributing peers, and the club does not become more viable. A positive scheme that aims to increase $N(n)$ directly is always preferred. A reasonable principle in economizing the use of incentive schemes would be to focus on those peers who are bordering on free-riding, by virtue of their propensity-to-contribute and/or the club response, to coerce them into contributing.

In fact, a club's well-being may actually be harmed when free-riding is overly discouraged. First, free-riders may behave differently and become contributors if only they stay long enough to realize more benefits in participation. Second, they may be useful audience to others, e.g. in newsgroups, BBS and forums, where wider circulation may improve utility of *all* due to network effects.

However, the ISC example has made two key assumptions, namely, constant $N(n)$ and non-rivalrous resources, so as to focus on the viability of club formation. In reality, the two limiting factors may set in at different stages. When the club is "young" and/or resourceful (abundant in all resources except those reliant on peers' sharing), viability is the critical concern. When it is "grown-up" and/or contentious (in some rivalrous resources), performance would be critical instead. The ISC example suggests n as a key parameter to watch, which

[5] Network effects are diametrically opposite to sharing costs [7] (due to consumption of rivalrous resources, say). They would help make good a non-rivalrous assumption made in the presence of the latter.

measures the club size in terms of the *total participation of potential contributing peers*. $N(n)$ would become sensitive and drop significantly when n becomes large, beyond n^{via} say. Contention would set in as system load increases with n, beyond n^{perf} say. Unless $n^{perf} \ll n^{via}$ whence the performance perspective always dominates, the viability perspective should never be overlooked.

In cases where the non-rivalrous assumption is not appropriate and sharing costs are significant [7], e.g. in processing, storage and/or network bandwidth, penalizing free-riding non-contributing peers would be more necessary to reduce their loading on the system and the contributing peers. However, as pointed out in [8], there is a trend demonstrated strongly by sharing communities on the Internet: rivalrous resources may become more like non-rivalrous as contention is fast reduced due to decreasing costs and increasing excess in resources. Because of this, it is plausible that viability would overtake performance as the central concern in many peer-to-peer systems sooner or later, if not already so.

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A Appendix

A.1 Proof of Proposition 1 (Sufficient Viability Condition)

With reference to figure 1, the average rate at which pc-peers join the club of current size n is:

$$r_{\text{join}} = (N(n) - n)\,\bar{P}(n)$$

while that of leaving is:

$$r_{\text{leave}} = n\,(1 - \bar{P}(n))$$

Hence, the net influx of pc-peers is given by:

$$r_{\text{influx}} \stackrel{\triangle}{=} r_{\text{join}} - r_{\text{leave}} = N(n)\,\bar{P}(n) - n \tag{8}$$

For an empty club, $n = 0$,

$$\begin{aligned} r_{\text{influx}} &= N(0)\,\bar{P}(0) \\ &= \sum_{i \in \mathcal{N}(0)} \sum_{s} h_i(s)(1 - e^{-\rho_i \phi_0\, g(s)}) \end{aligned} \tag{9}$$

When $\phi_0 > 0$, r_{influx} is strictly positive since the proposition implies $N(0) > 0$. The empty club is unstable and would grow in this case.

When $\phi_0 = 0$, $\bar{P}(0) = 0$ and $r_{\text{influx}} = 0$. The empty club is at equilibrium. However, its *stability* depends on the quantity

$$\begin{aligned} \left.\frac{\partial r_{\text{influx}}}{\partial n}\right|_{n=0} &= N(0)\bar{P}'(0) + \bar{P}(0)N'(0) - 1 \\ &= N(0)\bar{P}'(0) - 1 \\ &= \sum_{i \in \mathcal{N}(0)} \bar{k}\rho_i \sum_{s} g(s)h_i(s) - 1 \\ &\geq N(0)\bar{k}\rho(0^+) - 1 \\ &> 0 \end{aligned} \tag{10}$$

as implied by the proposition. The empty club is therefore also unstable and would grow at the slightest perturbation. **Q.E.D.**

A.2 Proof of Proposition 2 (Necessary Viability Condition)

According to equation (1), the contribution condition of peer i is given by:

$$\left(\left.\frac{\partial P_i}{\partial K_i}\right|_{K_i=0} \right)^{-1} < \Gamma_i \ .$$

$N(0) > 0$ only if:

$$\left.\frac{\partial P_i}{\partial K_i}\right|_{n=0,\,K_i=0} > 0$$

for some peer i. However,

$$\left.\frac{\partial P_i}{\partial K_i}\right|_{n=0,\,K_i=0} = \frac{\partial(\rho_i \Phi)}{\partial K_i} \sum_{s} g(s)h_i(s)e^{-\rho_i\,\Phi\, g(s)} \Bigg|_{n=0,\,K_i=0}$$

for which it is necessary that:

$$0 < \left.\frac{\partial(\rho_i \Phi)}{\partial K_i}\right|_{n=0,\,K_i=0} = \rho'(0)\phi_0$$

Q.E.D.

Subjective-Cost Policy Routing*

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Abstract. We study a model of interdomain routing in which autonomous systems' (ASes') routing policies are based on *subjective* cost assessments of alternative routes. The routes are constrained by the requirement that all routes to a given destination must be confluent. We show that it is NP-hard to determine whether there is a set of stable routes. We also show that it is NP-hard to find a set of confluent routes that minimizes the total subjective cost; it is hard even to approximate minimum cost closely. These hardness results hold even for very restricted classes of subjective costs.

We then consider a model in which the subjective costs are based on the relative importance ASes place on a small number of objective cost measures. We show that a small number of confluent routing trees is sufficient for each AS to have a route that nearly minimizes its subjective cost. We show that this scheme is trivially strategyproof and that it can be computed easily with a distributed algorithm that does not require major changes to the Border Gateway Protocol. Furthermore, we prove a lower bound on the number of trees required to contain a $(1+\epsilon)$-approximately optimal route for each node and show that our scheme is nearly optimal in this respect.

1 Introduction

The Internet is divided into many *Autonomous Systems* (ASes). Loosely speaking, each AS is a subnetwork that is administered by a single organization. The task of routing between different ASes in the Internet is called *interdomain routing*.

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Currently, the only widely used protocol for interdomain routing is the Border Gateway Protocol (BGP). BGP allows an AS to "advertise" routes it currently uses to neighboring ASes. An AS i with many neighbors may thus receive advertisements of many different routes to a given destination j. It must then select one of these available routes as the route it will use to send its traffic; subsequently, i can advertise this chosen route (prefixed by i itself) to all *its* neighbors. Proceeding in this manner, every AS in the Internet can eventually discover at least one route to destination j.

Thus, one of the key decisions an AS has to make is that of *route selection*: Given all the currently available routes to destination j, which one is traffic sent on? At first glance, it may seem as though ASes would always prefer the shortest route; in practice, however, AS preferences are greatly influenced by other factors, including perceived reliability and existing commercial relationships. For this reason, BGP allows ASes complete freedom to pick a route according to their own *routing policies*. The resulting routing scheme is called *policy-based routing*, or *policy routing* for short. However, BGP does place one important constraint on routing: It stipulates that an AS can only advertise a route that the advertising AS itself currently uses. This is because of the way traffic is routed in the Internet: Routers examine the destination of incoming packets and simply forward the packet to the next hop on the current route to that destination. At a given time, each AS typically has exactly one active route to the destination. Thus, the set of all ASes' routes to a given destination AS j must be *confluent*, *i.e.*, they must form a tree rooted at j.

The policy-routing aspect of interdomain routing has recently received a lot of attention from researchers. Varadhan *et al.* [1] observed that general policy routing could lead to route oscillations. Griffin, Shepherd, and Wilfong [2, 3] studied the following abstract model of general policy routing: Each AS i's policy is represented by a preference ordering over all possible routes to a given destination j. At any given time, i inspects the routes all of its neighbors are advertising to j and picks the one that is ranked highest. AS i then advertises this route (prefixed by i itself) to all its neighbors. Griffin *et al.* proved that, in such a scenario, BGP may not converge to a set of *stable paths*; the routes might keep oscillating as ASes continuously change their selection in response to their neighbors' changes. They further showed that, given a network and a set of route preferences, it is NP-complete to determine whether a set of stable paths exists. In recent work, Feamster *et al.* [4] showed that instability can arise even for restricted routing policies.

Feigenbaum *et al.* [5] extended the model of [2] by including cardinal preferences instead of preference orderings. Specifically, they assume that AS i conceptually assigns each potential route a monetary value and then ranks routes according to their value. The advantage of working with cardinal preferences is that a set of paths can be stabilized by making payments to some of the ASes: Although the ASes' *a priori* preferences may have led to oscillation (in the absence of payments), ASes preferences can be changed if they receive more money for using a less valuable route. This is the basis for the mechanism-design approach to routing, which seeks to structure incentives so as to achieve a stable, globally optimal set of routes; see [5] for further details. In the context of policy routing, the most natural global goal is to select a set of confluent routes that maximizes the *total welfare* (the sum of all ASes' values for their selected routes). However, Feigenbaum *et al.* showed that, for general valuation functions, it is NP-hard to find a welfare-maximizing set of routes; it is even NP-hard to approximate the maximum welfare to within a factor of $n^{\frac{1}{4}-\epsilon}$, where n is the number of nodes. Thus, in this model too, general routing policies lead to computationally intractable problems.

The natural approach to get around the intractability results is to restrict either the network or the routing policy. Restricting the network alone does not appear to be a very promising direction, because the hardness results hold even for fairly simple networks that cannot be excluded without excluding many "Internet-like" networks. This has led researchers to turn to restricted classes of preferences that can express a wide class of routing policies that ASes use in practice. Feigenbaum *et al.* [5] study *next-hop* preferences – preferences in which an AS i's value for a path depends only on the next AS on the path – and show that, in this case, a welfare-maximizing set of routes can be found in polynomial time. Next-hop preferences can capture the effects of i's having different commercial relationships with neighboring ASes. Similarly, in the ordinal-preference model, Gao and Rexford [6] show that, with the current hierarchical Internet structure, BGP is certain to converge to a set of stable paths as long as every AS prefers a customer route (*i.e.*, a route in which the next hop is one of its customers) over a peer or provider route; this can also be viewed as a next-hop restriction on preferences.

However, there are many useful policies that cannot be expressed in terms of next-hop preferences alone. In this paper, we study other classes of routing policies that capture realistic AS preferences. For example, an AS i might wish to avoid any route that goes through AS k, either because it perceives k to be unreliable or because k is a malicious competitor who would like to drop all of i's traffic. This leads to the *forbidden-set* class of routing policies: For each AS i, there is a set of ASes S_i such that i prefers any route that avoids S_i over any route that uses a node in S_i. We can then ask the following questions: (1) If each node uses a forbidden-set routing policy, will BGP converge to a set of stable paths?, and (2) Can we find a welfare-maximizing routing tree, *i.e.*, a set of confluent routes that maximizes the number of nodes i whose routes do *not* intersect the sets S_i? If the latter optimization problem were tractable, then this class of routing policies would be a candidate for a mechanism-design solution as in [7].

Forbidden-set policies (and many others) can be framed in terms of *subjective costs*: Each AS i assigns a cost $c_i(k)$ to every other AS k. Then, the "cost" perceived by AS k for a route P is $\sum_{k\in P} c_i(k)$; AS i prefers routes with lower subjective cost. Subjective-cost routing is a natural generalization of lowest-cost routing (in which there is a single objective measure of cost that all ASes agree upon). It is well known that lowest-cost routes can be computed easily, and hence we hope that some more general class of subjective-cost routing policies will also be tractable.

However, we find that even very restricted subsets of subjective-cost policies lead to intractable optimization problems: We show that, if all ASes rank paths based on subjective-cost assignments, it is still possible to have an instance in which there is no stable-path solution. Further, given a network and subjective costs, it is NP-complete to determine whether there is a set of stable paths. Moreover, the NP-completeness reduction only requires subjective costs in the range $\{0, 1, 2\}$ for each node. In the cardinal utility model, the outlook is not much brighter: We show that, even if all subjective costs are either 0 or 1, it is NP-hard to find a set of routes that maximizes the overall welfare; indeed, it is NP-hard even to approximate maximum welfare to within *any* factor. The forbidden-set routing policies can be formulated in terms of 0-1 subjective costs, and hence optimizing for this class is also difficult. We then turn to subjective costs with bounded ratios. We show that, if the subjective costs are restricted to lie in the range $[1, 2]$, the problem of finding a confluent tree with minimum total subjective cost is APX-hard; thus finding a solution that is within a $(1 + \epsilon)$ factor of optimal is intractable. In this case, however, an unweighted shortest-path tree provides a trivial 2-approximation to the optimization problem.

In light of all these hardness results, we consider a more restricted scenario in which the differing subjective cost assignments arise from differences in the relative importance placed on two *objective* metrics, such as latency and reliability. Thus, we suppose that every path P has two objective costs $l_1(P)$ and $l_2(P)$. We assume that AS i evaluates the cost of path P as the convex combination $\lambda_i l_1(P) + (1 - \lambda_i) l_2(P)$, where $\lambda_i \in [0, 1]$ reflects the importance i places on the first metric. Here, too, it is NP-hard to find a routing tree that closely approximates the maximum welfare. However, if we slightly relax the constraint that each AS stores only a single route to the destination, we show that it is possible to find a nearly optimal route, as follows. Given any $\epsilon > 0$, we can find a set of $O(\log n)$ trees[1] rooted at j with the following property: If each AS i chooses the route it likes best among the $O(\log n)$ alternatives, the overall welfare is within a $(1 + \epsilon)$ factor of optimal. This solution can be implemented by replacing each destination with a set of $O(\log n)$ logical destinations and then finding a lowest-cost routing tree to each of these logical destinations. The results generalize to the convex combinations of $d > 2$ objective metrics; $O(4^d \log^{d-1} n)$ trees are required in this case.This scheme is trivially strategyproof, and, further, it can be implemented with a "BGP-based" algorithm, *i.e.*, an algorithm with similar data structures and communication patterns to BGP (*cf.* [7, 5]).

The rest of this paper is structured as follows: In section 2, we introduce the subjective-cost model of routing preferences. In section 3, we study the stable-paths problem for path rankings based on subjective costs. In sections 4 and 5, we study the problem of finding a routing tree that minimizes the total subjective cost. Due to space restrictions, the proofs have been omitted from this extended abstract; they will appear in the final version of the paper.

2 Subjective-Cost Model for Policy Routing

In this section, we present the subjective-cost model of AS preferences. The model involves each AS i's assigning a cost $c_i(k)$ to every other AS k. These costs are *subjective*, because there is no requirement that $c_i(.)$ and $c_k(.)$ be consistent. We assume that each subjective cost $c_i(k)$ is non-negative. The total cost of an AS i for a route P_{ij} to destination j is

$$c_i(P_{ij}) = \sum_{k \in P_{ij}} c_i(k) \ .$$

[1] The dependence on ϵ is detailed in Section 5.

Here, the notation $k \in P_{ij}$ is used to indicate that k is a *transit node* on the the path P_{ij}; i and j are thus excluded from the summation. AS i wants to use a route P_{ij} that minimizes the cost $c_i(P_{ij})$.

The subjective-cost model can be used to express a wide range of preferences, but it does place some restrictions on AS preferences. For instance, an AS i cannot prefer a path P over a path P' whose nodes are a strict subset of P. The class of preferences that can be expressed as subjective costs includes:

– Lowest-cost routing
  If $c_i(k)$ is the actual cost of transiting AS k, minimizing the path cost is exactly lowest-cost routing.
– Routing with a forbidden set
  Let $c_i(.)$ take the following form: If $k \in S_i$, $c_i(k) = 1$, else $c_i(k) = 0$. Then any route that avoids ASes in S_i is preferred by i over any route that involves an AS in S_i.

Subjective costs can form the basis for either ordinal preferences or cardinal utilities. In section 3, we study the stable-paths problem for path rankings based on subjective costs. In sections 4 and 5, we study the problem of finding a routing tree that minimizes the total subjective cost.

3 Stable Paths with Subjective Costs

The *Stable Paths Problem (SPP)*, introduced by Griffin *et al.* [3], is defined as follows. We are given a graph with a specified destination node j. Each other node i represents an AS; there is an edge between two nodes if and only if they exchange routing information with each other. Thus, a path from i to j in the graph corresponds to a potential route from AS i to the destination. Each AS i ranks all potential routes to destination j. A *route assignment* is a specification of a path P_{ij} for each AS i such that the union of all the routes forms a tree rooted at j (*i.e.*, the confluence property is satisfied). A route assignment is called *stable* if, for every AS i, the following property holds: For every neighbor a of i, AS i does not strictly prefer the path aP_{aj} over the path P_{ij}; in other words, i would not want to change its current route to any of the other routes currently being advertised by its neighbors. The stable-paths problem is *solvable* if there is a stable route assignment.

Griffin *et al.* [2, 3] have shown that there are instances of SPP that are unsolvable, and, further, that it is NP-complete to determine whether a given SPP is solvable. Their constructions used preferences that cannot be directly expressed as subjective-cost preferences. This leads us to hope that, for subjective-cost preferences, the stable-paths problem might be tractable. Unfortunately, this is not the case. In this section, we prove that these hardness results extend to subjective-cost preferences.

Assume that the rankings assigned by ASes are based on an underlying subjective-cost assignment. Then, the stable paths problem can be viewed in terms of a strategic game, as follows: The players of this game are the ASs. Given a graph $G(V, E)$ with a specific destination j and a subjective-cost function $c : V(G) \times V(G) \to \mathcal{R}$, the *next-hop game* is defined as follows. ASes correspond to the vertices of graph G. The strategy

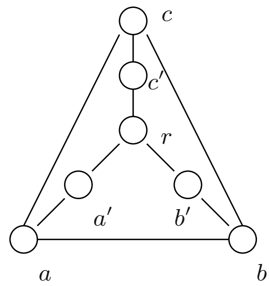

Fig. 1. A bad triangle

space for AS i is the set $N(i)$ of neighboring nodes in the graph; thus, AS i's picking the route advertised by a neighboring AS a corresponds to i's playing strategy a. Given a vector of strategies (one for each player), the cost incurred by player i is the subjective cost of its route to the destination; if there is no route from i to the destination, i's cost is ∞. A vector of strategies is a pure-strategy Nash equilibrium if, given the strategies of all the other ASes, no AS could decrease its subjective cost by changing its strategy. A pure-strategy Nash-equilibrium strategy profile must result in every AS's having some route to j, and, hence, it must correspond to a valid route assignment. Thus, proving that an SPP is solvable is equivalent to proving that the corresponding next-hop game has a pure-strategy Nash equilibrium.

Definition 1. *The* bad triangle *is defined as follows. It is a graph G with vertex set $\{a, b, c, a', b', c', r\}$ and edge set $\{aa', a'r, cc', c'r, bb', b'r, ab, bc, ca\}$. Set $c_a(c) = c_a(c') = 0$; $c_b(a) = c_b(a') = 0$; and $c_c(b) = c_c(b') = 0$. All other subjective costs are set to* 1. *A bad triangle is shown in Figure 1. (This construction is based on the* bad gadget *defined in [2].)*

In the bad triangle, AS a prefers the path (a, b, b', r) to the path (a, a', r), AS b prefers the path (b, c, c', r) to the path (b, b', r), and AS c prefers the path (c, a, a', r) to the path (c, c', r). It follows from the arguments in [2] that this network is not solvable.

We now show that, as in the case of unrestricted routing policies, it is NP-complete to determine if an SPP based on subjective-cost preferences is solvable.

Theorem 1. *Given an instance of the next-hop game, it is NP-complete to decide whether it has a pure Nash equilibrium or not.*

The proof is based on the corresponding NP-completeness proof in [2].

4 The Minimum Subjective-Cost Tree (MSCT) Problem

In this section, we assume that the subjective cost $c_i(k)$ is an actual monetary amount that is measured in the same unit across all ASes. A natural overall goal is then to minimize the sum of subjective costs, *i.e.*, to pick a set of routes $\{P_{ij}\}$ that minimizes $\sum_j \sum_i c_i(P_{ij})$. However, there is a constraint that all the routes to a single destination j must form a tree, because the packets are actually sent by forwarding. This constraint applies independently to each destination, and so we can consider the simpler problem of routing to a single destination j.

Thus, we can frame the subjective-cost minimization problem as:

Subjective-cost minimization: We are given a graph G, a set of cost functions $\{c_i(.)\}$, and a specific destination j. We want to find a set of routes $\{P_{ij}\}$ and payments p_i to each AS i such that:

1. The routes $\{P_{ij}\}$ form a tree rooted at j.
2. Among all such trees, the selected tree minimizes the sum $\sum_i \sum_{k \in P_{ij}} c_i(k)$.

We first prove that, for arbitrary cost functions, the MSCT problem is NP-hard to approximate within any multiplicative factor. Let $c_{\max} = \max_{v,u \in V(G)} c_v(u)$ and $c_{\min} = \min_{v,u \in V(G)} c_v(u)$. Then, we have the following result:

Theorem 2. *It is NP-hard to approximate the MSCT problem within a factor better than $\frac{c_{\max}}{c_{\min} n^2}$, where n is the number of vertices. In particular, it is NP-hard to approximate MSCT within any factor if $c_{\min} = 0$ and $c_{\max} > 0$.*

Note that the above theorem does not show hardness for the special cases in which $\frac{c_{\max}}{c_{\min}}$ is not large. This may be a reasonable restriction; however, we now show that this also yields an intractable optimization problem. In particular, we study the special case in which all subjective costs are either 1 or 2. We call this problem the $(1,2)$-MSCT problem. In the following, we give a hardness result for the $(1,2)$-MSCT problem.

Theorem 3. *The $(1,2)$-MSCT problem is APX-Hard.*

Theorem 3 shows that, for sufficiently small ϵ, it is hard to find a $(1+\epsilon)$-approximation for the $(1,2)$-MSCT problem. However, we note that finding a 2-approximation is easy: Simply ignore the costs, and construct an unweighted shortest-path tree with destination j. This is optimal to within a factor of 2, because the number of nodes on the shortest path from i to j is a lower bound on the subjective cost $c_i(P_{ij})$ for any path P_{ij} from i to j.

5 An Alternative Model: Subjective Choice of Metrics

In this section, we consider a more restricted preference model. We assume that there are multiple objective metrics on routes (*e.g.*, cost and latency), and ASes' preferences differ only in the relative importance they accord to different metrics. This is a non-trivial restriction only when the number of objective metrics is small; here, we first consider the case in which there are only two objective metrics on a route. The results are generalized to $d > 2$ objective metrics in Section 5.1.

Formally, suppose that any transit AS k has two associated objective "length" values $l_1(k)$ and $l_2(k)$. Both the length values can be extended to additive path metrics, *i.e.*, we can define $l_1(P_{ij}) = \sum_{k \in P_{ij}} l_1(k)$ and $l_2(P_{ij}) = \sum_{k \in P_{ij}} l_2(k)$. Note that we use the term "metric" for the ease of presentation and that we do not impose the triangle equality on the length functions l_1 and l_2.

Each AS i has a private parameter λ_i, $0 \leq \lambda_i \leq 1$. AS i's subjective cost for the route P_{ij} is given by $c_i(P_{ij}) = \lambda_i l_1(P_{ij}) + (1-\lambda_i)l_2(P_{ij})$, *i.e.*, AS i's preferences are modeled as a convex combination of the two path metrics.

It is easy to show that the APX-hardness proof for the $(1,2)$-MSCT problem (Theorem 3) can be adapted to the two-metric routing problem as well:

Theorem 4. *In the subjective-metric model, it is APX-hard to find a tree T that minimizes total subjective cost.*

We now investigate whether relaxing the confluent-tree routing constraint would lead to stronger results. If we allowed the routes to be completely arbitrary, then clearly we could have optimal routing: Each AS could simply use the route it liked the best. However, supporting these routes would either require source routing (*i.e.*, the packet header contains a full path) or a massive increase in storage at each router to record the forwarding link for each source and destination. Instead, we ask whether we can get positive results with only a small growth in routers' space requirements.

Our approach is to use a *small number* r of confluent routing trees $T_1, T_2, \cdots T_r$ to each destination j. Then, each AS i evaluates its subjective cost to j in each of the routing trees and picks a tree T_{t_i} that minimizes this subjective cost. AS i then marks each packet it sends with the header $< j, t_i >$. Each AS *en route* stores its route to j along each tree T_j; thus, it can inspect the header of each incoming packet and forward along the appropriate route.

We can prove the following result:

Theorem 5. *Suppose that, for transit AS k , $l_1(k)$ and $l_2(k)$ are integers bounded by a polynomial, i.e., $l_1(k), l_2(k) < n^c$ for some constant c. Then, for any given $\epsilon > 0$, there is a set of routing trees $T_1, T_2, \cdots T_r$ with $r = O(\frac{1}{\epsilon}[\log n + \log(\frac{1}{\epsilon})])$ such that:*

> *For each AS i, there is a tree T_{t_i} such that $c_i(T_{t_i}) \leq (1+\epsilon)c_i(P^*_{ij})$, where P^*_{ij} is the minimum-subjective-cost route from i to j.*

Further, this set of trees can be constructed in polynomial time.

We now sketch the tree construction used in the proof.

Let $\alpha = (1+\epsilon)$. Each tree T_t in our collection is the shortest-path tree for a specific convex combination of the two metrics. We name the trees after the metrics they optimize:

T_∞**:** $l_1(\cdot)$, with ties broken by minimum $l_2(\cdot)$.
$T_{-\infty}$**:** $l_2(\cdot)$, with ties broken by minimum $l_1(\cdot)$.
T_t**:** $l_t(\cdot) = \frac{\alpha^t}{1+\alpha^t} l_1(\cdot) + \frac{1}{1+\alpha^t} l_2(\cdot)$ for $t \in \{-k, -(k-1) \ldots, -1, 0, 1, \cdots k\}$, where $k = \lceil \log_\alpha(2\epsilon^{-1} n^{c+1}) \rceil$.

There are several points worth noting about this scheme: (1) It achieves a result that is slightly stronger than our initial goal – it approximately maximizes each individual node's welfare, not just the sum of all nodes' welfare. (2) The computation of the trees is oblivious to the nodes' preference information. Thus, if we assume that the objective costs are common knowledge (or verifiable), this scheme is trivially a strategyproof mechanism. (3) Each tree computation involves computing lowest-cost routes for a specific objective metric. Thus, it is easily computed within the framework of BGP itself. (In the terminology of Feigenbaum *et al.* [7, 5], there is a natural BGP-based distributed algorithm for this scheme.)

We now prove a corresponding lower bound that shows that Theorem 5 is nearly optimal.

Theorem 6. *Let $\epsilon > 0$ be given. There is a family of instances of the subjective-metric routing problem, with all weights in $[0, n^c]$ for some constant c, such that the following property holds:*

> *Any set of routing trees that contains a $(1+\epsilon)$-approximately optimal path P_{ij} for each i must have $\Omega(\log n/\epsilon)$ trees.*

(Here, n is the number of nodes of the network.)

5.1 Generalization to More Than 2 Metrics

In this section, we show that Theorems 5 and 6 generalize to the case in which there are $d > 2$ objective metrics, and an AS's subjective cost is a convex combination of these metrics.

Theorem 7. *Suppose that, for transit AS k , all lengths $l_1(k), l_2(k) \ldots, l_d(k)$ are integers bounded by a polynomial, i.e. , $l_j(k) < n^c$ for some constant c. Then, for any given $\epsilon > 0$, there is a set of routing trees $T_1, T_2, \cdots T_r$ with $r = O(4^d [\frac{(c+1)\log n + \log(\frac{2}{\epsilon})}{\log(1+\epsilon)}]^{d-1})$ such that:*

> *For each AS i, there is a tree T_{t_i} such that $c_i(T_{t_i}) \leq (1+\epsilon) c_i(P^*_{ij})$, where P^*_{ij} is the minimum-subjective-cost route from i to j.*

Further, this set of trees can be constructed in polynomial time for any constant d.

Theorem 8. *Let $\epsilon > 0$ be given and $d > 2$ be given. There is a family of instances of the subjective-metric routing problem, with all weights in $[0, n^c]$ for some constant c, such that the following property holds:*

> *Any set of routing trees that contains a $(1+\epsilon)$-approximately optimal path for each i must have $\Omega((\frac{\log n}{d \log d + d \log(1+\epsilon)})^{d-1})$ trees.*

(Here, n is the number of nodes of the network.)

6 Conclusion

In this paper, we have studied classes of ordinal and cardinal preferences based on subjective costs. The subjective-cost preference model is intuitively appealing, and it is very expressive. However, our results show that, even if the costs are restricted to a very small range, unstructured subjectivity leads to intractable problems in both models: NP-completeness of the stable paths problem for ordinal preferences and APX-hardness of the minimum subjective-cost tree problem for cardinal preferences.

The root cause of these hardness results appears to be the high dimension of the space of AS preferences. Thus, it is necessary to work with models that provide a more consistent global structure. In Section 5, we consider the case in which there are two objective cost metrics, and ASes differ in the relative importance they place on the first metric. For example, ASes may agree on the latency and packet-loss rate of each node in the network but have subjective opinions about the relative importance of latency and loss rate. Thus, in this model, the space of all AS *types* is one-dimensional. We showed that it is possible to select a small number ($O(\frac{1}{\epsilon}[\log n + \log(\frac{1}{\epsilon})])$ for a $(1+\epsilon)$-approximation) of representative types such that every ASes' preferences are closely approximated by one of the representatives; then, by picking a *set* of routing trees, each of which is optimized for a specific representative type, we can guarantee each AS a route that $(1+\epsilon)$-approximately minimizes its subjective cost. Further, this scheme is easy to implement, even in the distributed-computing context: Each destination can be replaced by a small number of logical destinations, and a lowest-cost routing algorithm (*e.g.*, the Bellman-Ford algorithm) can be used for each logical destination.

It is also possible that other models that restrict the subjectivity of the costs in some way may yield positive results. For example, the nodes' subjective costs for a given transit node k are random variables drawn from a specific distribution. Finding such models that are both realistic and tractable is an interesting avenue for future research.

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Economic Analysis of Networking Technologies for Rural Developing Regions

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Abstract. Providing network connectivity to rural regions in the developing world is an economically challenging problem especially given the low income levels and low population densities in such regions. Many existing connectivity technologies incur a high deployment cost that limits their affordability. Leveraging several emerging wireless technologies, this paper presents the case for economically viable networks in rural developing regions. We use the Akshaya Network located in Kerala, India as a specific case study. and show that a wireless network using WiFi for the backhaul, CDMA450 for the access network, and shared PCs for end user devices has the lowest deployment cost. However, if we include the expected spectrum licensing cost for CDMA450, a network with lease exempt spectrum using WiFi for the backhaul and WiMax for access is the most economically attractive option. Even with license exemption, regulatory costs comprise nearly half the total cost in the WiFi/WiMax case suggesting the possibility of significant improvement in network economics with more favorable regulatory policies. Finally, we also demonstrate the business case for a WiFi/CDMA450 network with nearly fully subsidized cellular handsets as end user devices.

1 Introduction

The lack of network connectivity in many regions around the world is as much an economic problem as a technological one. This is strikingly true for rural regions in developing countries with low income levels and low population densities. On one hand, the presence of network connectivity within these regions can have potentially several far-reaching implications including promoting literacy, improving health care [1], reducing market inefficiencies [2], increasing government transparency [3] and enabling environmental monitoring [4]. On the other hand, traditional connectivity solutions using fiber-optic networks may be ill-suited for such environments. The high infrastructure cost and the long period of time to deploy fiber-optic networks make these technologies economically less appealing and a very risky investment proposition. Additionally, high bandwidth links may not be necessary in such regions since many of the applications (e.g. messaging, voice) leveraging the network connectivity have limited bandwidth requirements. For rural markets with dispersed populations and uncertain demand, network technologies with a low cost of entry are preferable.

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Long-distance wireless technologies, especially those based on standards can enable networking in rural regions. The attractive features of these networks include the low-deployment cost, ease of deployment and the ability to cater to a wide-range of geographic terrain. On the flip side, these wireless networks have capacity constraints that limit the maximum available bandwidth and also suffer from reliability problems. The capacity that a fiber-optic network provides is orders of magnitude larger than that of a backhaul wireless network. Hence, from an economic standpoint, the *cost per bit of capacity* for a fiber-optic network is much smaller than that of a wireless-backhaul network.

In this paper, we study the economic viability of different technologies for providing network connectivity in emerging regions. Specifically, we ask the question: *What is the networking technology that can provide connectivity at the lowest cost per unit of usage while remaining profitable?* This *cost per unit of demand* is distinct from the traditional focus on *cost per unit of supply*. The cost per unit of demand can be expressed as the ratio of the *cost per user* and the *demand per user*. Sharing of devices helps reduce cost per user significantly. However sharing devices does carry a penalty: lower expected demand per user.

Any such technology requires two connectivity components: the access network and the backhaul network. The access network provides connectivity within a local region and the backhaul network provides connectivity across regions. In this paper, we consider four forms of access technologies which exhibit different coverage and capacity characteristics: WiFi [5], WiMax [6], CDMA450 [7] and WipLL [8]. We consider these access technologies in combination with three forms of backhaul technologies: Fiber(PON) [9], WiFi [5] (with directional antennas) and VIP [10]. WipLL and VIP are proprietary technologies and will be discussed later in Section 3.

Our economic study is motivated by the *Akshaya* project in India whose aim is to provide Internet connectivity within a specific district in the state of Kerala, India. Based on economic data gathered from Akshaya, we analyze and compare the cost of various communication technologies for providing connectivity. For the *Akshaya* case, we show six key results. First, WiFi with directional antennas as backhaul technology combined with WiMax as an access technology provides the most attractive economics. However, if we discount the cost of spectrum licensing, WiFi/CDMA450 has a lower cost of deployment than WiFi/WiMax. Second, we found that the largest component of the capital investment for providing connectivity is the cost of the end-user devices (either PCs or cellular handsets). Third, tower and primary and/or backup power source costs dominate the last mile and backhaul costs. These costs are not expected to decrease in the future and hence any technology that is able to provide a large coverage area per tower or lower power consumption can reduce costs. Fourth, since network equipment comprises a small portion of the overall deployment cost, reducing equipment cost per node does not lead to significant improvement in economics. In fact, a counterintuitive conclusion of this analysis suggests that increasing the network equipment cost is desirable if it leads to greater coverage area/link distance and/or lower power consumption. Our analysis shows that if the coverage area of last mile devices could be doubled with an 4x increase in price, it would yield the same rate of return. Fifth, a WiFi/CDMA450 network with cellular handsets as end user devices is economically viable even with nearly full subsidization of handsets. Limited affordability of handsets has traditionally hindered adoption of communications services in rural developing regions. Finally, costs linked to regulatory policies constitute a substantial portion of the overall network cost structure. Any reduction in such costs (e.g. lower termination rates) has a significant beneficial impact on network economics.

2 Defining the Rural Setting

The rural connectivity landscape in developing countries presents a contrast to urban telecommunications in the developed world. Three aspects are noteworthy. First, any networking technology needs to be affordable. Lowering cost at the expense of reliability (*e.g.,* no backup equipment, intermittent connectivity) and sharing of end-user devices (*e.g.,* Grameen phone [11], kiosks) may be an acceptable trade-off. Second, coverage is more important than capacity. Urban settings are capacity limited. Service providers must place multiple base stations in a small area to cater to large volume of traffic. In contrast, rural settings are coverage limited. Service providers would like a single base station to cover as large a geographical area (*i.e.,* as many users) as possible. Finally, demand is difficult to forecast. There is no real measure of demand in rural areas. However a few trends point towards a latent demand and a willingness to invest in telecommunications. For example, rural spending in China has risen threefold from 1990 to 2002 while the percentage spent on telecommunications and transportation (on an absolute level) has risen from 1% of overall spending to 6% of overall spending. A similar trend is visible in Bangladesh where rural residents devote 7% of their income to telecommunications.

Telecommunication markets can be classified according to bandwidth demand per user (peak and average demand), purchasing power per user and the population density. Areas with a high population density and high bandwidth demand per user can be serviced by fiber. On the other hands areas with low population density and high purchasing power per user (*e.g.,* Alaska, USA) can be serviced by satellite.

The rural environment in developing countries is characterized by medium to low population densities, low bandwidth demand per user and very low purchasing power per user. Hence any technology to provide connectivity to these regions must be characterized by low *cost per unit of demand.*

2.1 Economic Model for Analysis

Expenditure associated with any telecommunications deployment can be classified into two categories: *Capital Expenditure (CapEx)* and *Operating Expenditure (OpEx)*. CapEx covers the basic infrastructure for providing connectivity. In the case of a wireless network like CDMA450, CapEx will include the cost for towers, network equipment, primary or backup power sources, installation costs, one-time spectrum licensing fees etc. CapEx can either be viewed as a one-time investment or as a recurring investment where the infrastructure continues to grow over time. OpEx, on the other hand, is the cost expended by the service provider for operating and maintaining the network and supporting users. For example, OpEx includes the salaries of employees, recurring power costs, recurring spectrum licensing fees and per call termination charges.

Note that our analysis applies to the entire ecosystem for providing connectivity to rural environments irrespective of the specific allocation of economic interests between various parties in the ecosystem. Our interest is in analyzing the overall economic viability of this market and in highlighting the key technological and economic conclusions that effect the value chain in the aggregate. The impact of external subsidies may also be explored within this framework.

Economic terms: Throughout this paper, we use two standard economic measures, namely the *Net Present Value (NPV)* and *Internal Rate of Return (IRR)*, for evaluating the economic viability of different technologies. Over a specific time-period T and an appropriate discount rate r, NPV is the sum total of the future stream of revenues and expenses discounted to the present. If the NPV over time-period T is negative, the investment is deemed not profitable. Over a time-period T, IRR represents the value of the discount rate r at which NPV becomes zero. Detailed definitions of economic terms used in this paper can be found in [12].

3 Networking Technologies for the Rural Setting

To provide connectivity to a rural community, a peering/transit/exchange point provided by a standard telecommunications provider must be linked to users either using kiosks or cybercafes. This linking process is provided by two different classes of technologies: Access technologies that are used to connect individual users/centers to Points of Presence (POPs) of the network and Backhaul technologies that are used to link POPs to the worldwide network. The basic structure of such a network shown in Figure 1(a).

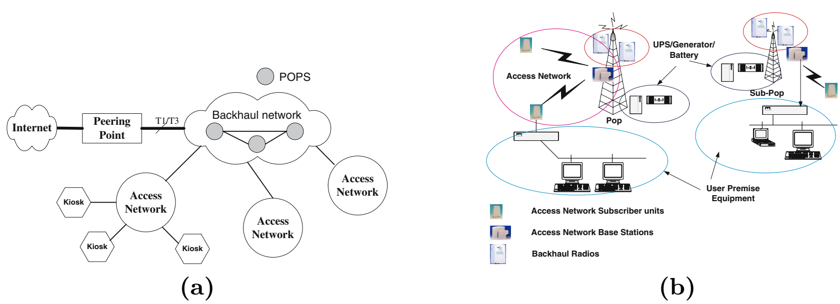


Fig. 1. (a)Basic network architecture for providing connectivity comprises a Backhaul network for connecting to the worldwide network and a set of Access networks for connecting to individual users/kiosks. (b) Corresponding elements in the *Akshaya* network.

Table 1. A comparison of capabilities and costs of various Access technologies

	WiMax	CDMA450	WiFi	WipLL
Coverage	5km	17km [7]	1-2km (omni) 5km (directional)	5km
Number of simultaneous users	16	32 on each 1.25MHz carrier	3 for 802.11b (directional) 24 for 802.11a (directional)	126 total
Spectrum Costs	unlicensed(2.4/5GHz) licensed (3.5GHz)	licensed	unlicensed	unlicensed
Spectrum availability	380MHz at 2.4GHz [13] 200MHz at 3.5GHz [13] 150MHz at 5GHz [13]	9MHz FDD	80MHz at 2.4GHz [13] 580MHz at 5GHz [13]	Same as WiFi
Peak throughput per user	4Mbps	1Mbps [7]	802.11b:5-7Mbps 802.11a:20-30Mbps	4Mbps
Base station costs current, projected (2yrs)	$1,999, $1,000	$20,000, $10,000	$500, $300	$2,100, $1,500
Subscriber unit costs current, projected (2yrs)	$1,199 $600	$250, $150	$200, $150	$500, $300

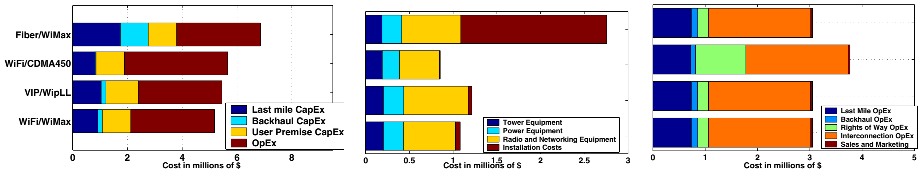
For our analysis we consider four access technologies: WiMax, CDMA450 (CDMA2000 at 450MHz), Enhanced WiFi (WiFi with high gain unidirectional antennas or directional antennas), and WipLL (see Section 4 for details). A further description of these technologies can be found in [12]. Here we reproduce relevant characteristics and costs of these four technologies (Table 1). We make two assumptions about access technologies in our analysis: Firstly, we expect the cost for subscriber units for standardized technologies (WiMax, WiFi) to be similar once deployment is widespread (see [12] for operating case assumptions). Secondly, technologies operating in the same spectrum will have similar range. Differences in average bandwidth per user may arise from differences in the Medium Access Control (MAC) schemes employed by each technology.

For backhaul Technologies we consider the three technology options: Extended range WiFi (WiFi with directional antennas), Fiber (Passive Optical Networks) and VIP (proprietary technology from Wi-LAN used in the current Akshaya network). These technologies are further explained in [12].

4 Akshaya – A Case Study

The *Akshaya* project is a large rural wireless network [14] developed as a joint project between Tulip IT [15] and the Government of Kerala, India, to provide connectivity to the Malappuram district. The aim of this project is to provide connectivity to a group of people rather than individual users. 630 *Akshaya* centers (one for every 2000 families) are located throughout the district. The estimated costs for deployment are roughly a dollar per covered population. Each *Akshaya* center (AC) is setup and maintained by local entrepreneurs who receive a subsidized loan from the Government. These sites also provide computer training to one member of each household. Each entrepreneur pays Tulip a flat fee of $20 per month for network connectivity.

The *Akshaya* network consists of wireless backhaul links based on patented VINE (Versatile Intelligent Network Environment) technology from Wi-LAN, Canada [10]. This Technology has been incorporated in the VIP radios from



(a) (b) (c)

Fig. 2. Total costs, CapEx and OpEx for building the *Akshaya* network using various technology options

Table 2. List of equipment at each node

Key terms	Explanation	Equipment
POP	A POP hosts backhaul links and an Access Base Station for connecting to *Akshaya* centers.	n Backhaul radios, 1 30m Tower 1 Access Base Station, Shelter UPS with battery, Generator/Solar
subPOP	*Akshaya* centers which also serve as POPs	n Backhaul radios, 1 9m Tower 1 Access Base Station, Shelter UPS with battery, Generator/Solar
relayPOP	Backhaul relay towers that do not host Access basestations - used to connect POPs	n Backhaul radios, 1 30m Tower Shelter, UPS with battery, Generator/Solar
AC	Village kiosk with 3 to 5 PCs	1 Access Subscriber unit, Ethernet switch 3-5 PCs, UPS with battery

Wi-LAN. Each VIP radio can serve as a basestation (using sectored antennas) or as a repeater (using directional antennas). A single radio connects to the uplink node in the backhaul network. A second radio communicates with all the downlink nodes. These two nodes are bridged using a Wireless Ethernet Bridge. Each radio has 1W transmission power and provides 11Mbps in the 2.4GHz band.

For connecting the backhaul network with the *Akshaya* Centers, (point to multi-point links) Tulip used WipLL technology from Marconi and Airspan [8]. Each WipLL basestation serves up to 127 centers using sectored or omni antenna.

An analysis of the *Akshaya* network was used to determine the key network elements required to install such a network. These network elements are depicted in Figure 1(b) and explained in Table 2. Table 2 also lists the components located as each network node.

4.1 Economic Analysis of the *Akshaya* Network

The network CapEx and OpEx was split into three main categories: Backhaul, Last Mile and User premise.

Backhaul: The backhaul costs consists of all backhaul radio equipment, POP tower costs, auxiliary power costs and associated installation and maintenance costs. Land lease costs are factored in as Operating expenses.

Last mile: The Last Mile costs consists of all access (base station and subscriber side equipment) radio equipment, subPOP tower costs, auxiliary power costs and associated installation and maintenance costs.

User Premise: The User Premise Equipment (UPE) costs consist of *Akshaya* center networking equipment and computer costs.

We calculated the total CapEx and OpEx using the following Backhaul/Last mile technology pairs: WiFi/WiMax, WIP/WipLL, Fiber/WiMax and WiFi/ CDMA450. All scenarios except WiFi/CDMA450 use 18 POPs and 16 subPOPS. CDMA450 requires 3 POPS and 3 subPOPS since a lower frequency provides a larger coverage area. Figure 2(a) shows the CapEx and OpEx (for 5 years) for each of the four technology options. The CapEx has been split into Last Mile CapEx, Backhaul CapEx and User Premise CapEx. Alternatively CapEx can be separated into tower cost, primary and/or auxiliary power equipment cost, radio and networking equipment cost and installation cost which can be seen in Figure 2(b).

Similarly, the OpEx can be classified under different categories: Last Mile OpEx (Power cost and maintenance of subPOPs), Backhaul OpEx (Power cost and maintenance of POPs), Rights of Way OpEx (Tower leasing cost, Spectrum leasing cost and miscellaneous government fees), Interconnection OpEx (Termination and leased line cost) and Sales and Marketing cost. This OpEx breakdown for different technology options can be seen in Figure 2(c). The key assumptions made in generating these figures can be found in [12]. The final NPV and IRR for all technology options can be found in Figure 3. The Time for Cash-flow Profitability was 1 year for all technologies except WiFi/CMDA450 which required 2 years due to a larger OpEx. Time to Break even was 4 years for all technologies.

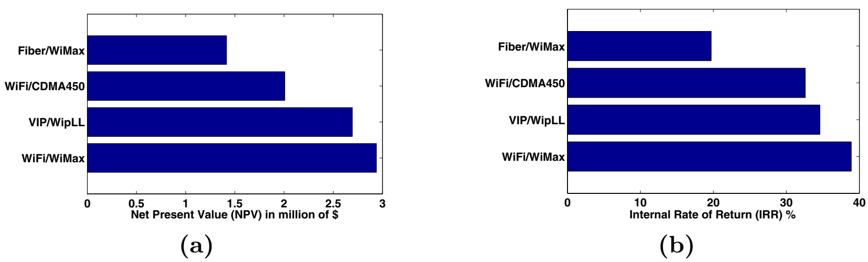


Fig. 3. (a)Net Present Value (NPV) for various technology options (b) Internal Rate of Return (IRR) for various technology options

Key items to note in the cost breakdown include:

1. For the backhaul, wireless (both WiFi and VIP) technologies have at most one-eight the backhaul CapEx as compared to the fiber scenario. While WiFi radio costs are lower than VIP, radio costs in general comprise a small portion of the overall backhaul costs and hence this difference is not significant.
2. The CDMA450 scenario exhibits the lowest CapEx since the number of POPs and subPOPS is greatly reduced. However spectrum costs[1] weigh heavily in

[1] In India spectrum licensing costs are a percentage of service revenues. In our analysis we assume the licensing costs for the 450MHz band would be similar to traditional cellular spectrum bands.

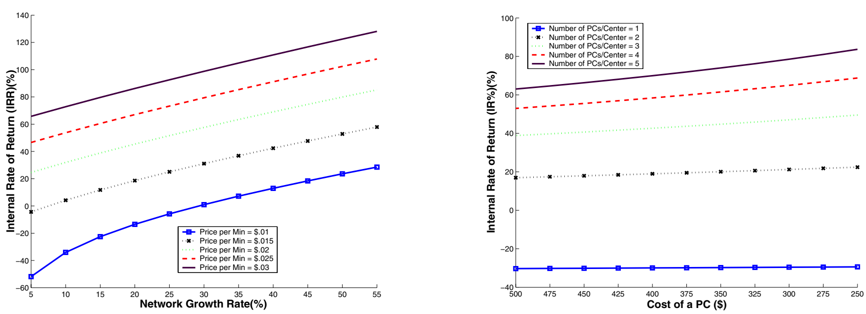
the OpEx reducing the final NPV. This can be seen in the Rights of Way OpEx for CDMA450 in Figure 2(c).

3. The Fiber scenario is based on Passive Optical Network (PON) technology using a single chassis at the peering point with two line-cards. The fiber is assumed to be strung along towers due to the hilly terrain. The bulk of the costs (three-fourths of a million dollars) is to string the fiber on towers.
4. Termination costs ($.005/minute) account for a large portion of the OpEx since we assume that a majority of the traffic is voice calls which terminate on regular telephone networks.

4.2 Sensitivity Analysis for *Akshaya*

While our model is based on data from Tulip IT, and other publicly available information, this information is a rough estimate since prices in this industry change frequently. That being the case, we decided to perform sensitivity analysis to determine the impact of changes in our assumptions on the projected financial returns from the project. This is of particular importance for the revenue drivers, given the uncertainty of demand in many rural areas. Hence we considered the effect of the price per minute and the network utilization growth rate on the net IRR for the technology choice of WiFi with WiMax. This analysis is shown in Figure 4(a). For all network utilization growth rates, the minimum price of $0.02/minute is required for profitability. Since the termination cost is $.005/minute, net value to the service provider is $0.015/minute in this case.

Similarly since User Premise Equipment comprises the majority of the CapEx in all cases except the fiber scenario, we studied the variation of the number of PCs per *Akshaya* center and the cost per PC on the IRR. It is worth noting that we assume network usage to be proportional to the number of PCs per *Akshaya* center. Again, a minimum of 2 PCs per center are required for a positive value of IRR (See Figure 4(b)). In other words, each *Akshaya* center needs sufficient



(a) **(b)**

Fig. 4. (a)Effect of network utilization growth rate and price per minute on the IRR. For all network utilization growth rates, a minimum price of $0.02 per minute is required for profitability. (b)Effect of changing cost per PC and the number of PCs per *Akshaya* center on the IRR. A minimum of 2 PCs per center are required for profitability.

demand to support at least 2 PCs to achieve a positive IRR (even with no termination cost and a free PC, a single PC would not yield positive IRR).

5 Discussion

Alternative to Sharing: Use of individual devices: In Section 2, we used the kiosk/cybercafe model to leverage the aggregate purchasing power of rural communities. While sharing of devices reduces the cost per user, it also limits the demand per user. In order to evaluate the feasibility of distributing individual devices to users, we focused on the CDMA450 case where the 6 Base stations atop the POPs and subPOPs were converted into regular cellular base stations for voice calls. Each CDMA basestation with 6 sectored antennas can serve 1000 customers. Under this model the 630 *Akshaya* centers are assumed to be cellular handset retailers. A usage of 2 min per day per subscriber was assumed. This was increased by 1 min each year to 6 minutes per day per subscriber in year 5. We neglected any upfront spectrum licensing costs for the 450MHz band. The subscribers were charged $0.02 per minute and received a subsidy of $30 on each handset.

In this scenario, recurring spectrum licensing cost represents 26% of the OpEx and 11% of the revenue. The IRR in this case is 28.5% which rises to 44.8% if recurring spectrum licensing cost is neglected. Assuming 100,000 handsets are in use (a penetration level of 1.58%), the highest handset subsidy possible to achieve break even is $38 when recurring spectrum licensing costs are taken into account. This subsidy rises to $45 if recurring spectrum licensing costs are omitted. In fact, neglecting recurring spectrum costs gives the handset scenario a better IRR than the kiosk model. With handset prices entering the sub $50 price range, this analysis suggests that handsets could be nearly fully subsidized at these usage levels and penetration rates while still achieving economic viability for the network.

Extending the *Akshaya* analysis: In most respects the *Akshaya* case represents a worst case scenario. Malappuram is a hilly region with extremely dense vegetation. Hence, Tulip has to use high towers to avoid this vegetation and get a line of sight to a POP or subPOP. If we assume a similar population density but a flat terrain, tower heights can be reduced. However, this reduction does not impact the analysis in any significant manner. If tower heights were to be reduced by half for the WiFi/WiMax scenario, the IRR only increases by 1.8%. Change in terrain/frequency which enables a reduction in number of towers has a more tangible impact on the CapEx. If we could double the coverage area and hence halve the number of POPs and subPOPs, the IRR would increase to 2.7%.

Impact of Regulation: In developing countries, telecommunication markets are usually highly regulated across several dimensions including spectrum allocation, approval for rights of way and service areas, universal service provisions, termination rules and rates, business licenses, service import tariffs. Regulation can play a crucial factor in determining the economic viability of different technologies since it effects a variety of critical business processes. For example, for the WiFi/WiMax scenario, regulatory costs in total represent 45.5% of the overall CapEx and 5 year OpEx. Eliminating regulatory costs would increase the IRR by 74.5%.

As illustrated in Figure 2(c), termination rates make up a significant portion of the OpEx. Reducing the termination rate allows service providers to lower their price which in turn is likely to increase usage due to elasticity of demand. We studied the effect of a reduction of termination charges by $.001 every year. We varied the demand elasticity from 0 to -1 (i.e. a price elasticity of $-x$ implies that a price reduction of x% would increase network utilization by x% over and above the assumed ramp in network utilization). This analysis reveals that the IRR increases from 40.8% to 55.2% as the elasticity is increased from 0 to -1.

6 Conclusions

The economic viability of communications networks in rural developing regions with severe affordability constraints, low population densities, and uncertain demand has long been a concern. This paper presents the case for economically viable networks in rural developing regions. For the *Akshaya* case study, we show that a wireless network using WiFi for the backhaul, CDMA450 for the access network, and shared PCs for end user devices has the lowest deployment cost. Once the expected spectrum licensing cost for CDMA450 is included, a network with lease exempt spectrum using WiFi for the backhaul and WiMax for access is the most economically attractive option. However, even with the WiFi/WiMax scenario, regulatory costs comprise nearly half the total cost structure of the network demonstrating the significant impact of regulatory policies on network economics. Finally, we demonstrate the business case for a WiFi/CDMA450 network with nearly fully subsidized cellular handsets as end user devices. Such a network overcomes the handset affordability constraint which has traditionally hindered adoption of communications services in rural developing regions.

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A Simple Graph-Theoretic Model for Selfish Restricted Scheduling*

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Abstract. In this work, we introduce and study a simple, graph-theoretic model for selfish *scheduling* among m non-cooperative *users* over a collection of n *machines*; however, each user is restricted to assign its unsplittable *load* to one from a pair of machines that are allowed for the user. We model these bounded interactions using an *interaction graph,* whose vertices and edges are the machines and the users, respectively. We study the impact of our modeling assumptions on the properties of Nash equilibria in this new model. The main findings of our study are outlined as follows:

- We prove, as our main result, that the *parallel links* graph is the *best-case* interaction graph – the one that minimizes expected *makespan* of the *standard fully mixed Nash equilibrium* – among all *3-regular* interaction graphs. The proof employs a graph-theoretic lemma about *orientations* in 3-regular graphs, which may be of independent interest.
- We prove a lower bound on *Coordination Ratio* [16] – a measure of the cost incurred to the system due to the selfish behavior of the users. In particular, we prove that there is an interaction graph incurring Coordination Ratio $\Omega\left(\frac{\log n}{\log\log n}\right)$. This bound is shown for pure Nash equilibria.
- We present counterexample interaction graphs to prove that a *fully mixed Nash equilibrium* may sometimes not exist at all. Moreover, we prove properties of the fully mixed Nash equilibrium for *complete bipartite* graphs and *hypercube* graphs.

1 Introduction

Motivation and Framework. Consider a group of m non-cooperative *users,* each wishing to assign its unsplittable unit *job* onto a collection of n processing (identical) *machines*. The users seek to arrive at a stable assignment of their jobs for their joint inter-

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action. As usual, such stable assignments are modeled as *Nash equilibria* [21], where no user can unilaterally improve its objective by switching to a different strategy.

We use a structured and sparse representation of the relation between the users and the machines that exploits the locality of their interaction; such locality almost always exists in complex scheduling systems. More specifically, we assume that each user has access (that is, finite cost) to only *two* machines; its cost on other machines is infinitely large, giving it no incentive to switch there. The (expected) cost of a user is the (expected) load of the machine it chooses. Interaction with just a few neighbors is a basic design principle to guarantee efficient use of resources in a distributed system. Restricting the number of interacting neighbors to just two is then a natural starting point for the theoretical study of the impact of selfish behavior in a distributed system with local interactions.

Our representation is based on the *interaction graph,* whose vertices and (undirected) edges represent the machines and the users, respectively. Multiple edges are allowed; however, for simplicity, our interaction multigraphs will be called *interaction graphs*. The model of interaction graphs is interesting because it is the simplest, non-trivial model for selfish scheduling on restricted parallel links. In this model, any assignment of users to machines naturally corresponds to an *orientation* of the interaction graph. (Each edge is directed to the machine where the user is assigned.)

We will consider *pure Nash equilibria,* where each user assigns its load to exactly one of its two allowed machines with probability one; we will also consider *mixed Nash equilibria,* where each user employs a probability distribution to choose between its two allowed machines. Of particular interest to us is the *fully mixed Nash equilibrium* [20] where every user has strictly positive probability to choose each of its two machines. In the *standard fully mixed Nash equilibrium*, all probabilities are equal to $\frac{1}{2}$. It is easy to see that the standard fully mixed Nash equilibrium exists if and only if the (multi)graph is regular.

With each (mixed) Nash equilibrium, we associate a *Social Cost* [16] which is the expected *makespan* - the expectation of the maximum, over all machines, total load on the machine. *Best-case* and *worst-case* Nash equilibria minimize and maximize Social Cost, respectively. For a given type of Nash equilibrium such as the standard fully mixed Nash equilibrium, best-case and worst-case graphs among a graph class minimize and maximize Social Cost of Nash equilibria of the given type, respectively. The assignment of users to machines that minimizes Social Cost might not necessarily be a Nash equilibrium; call *Optimum* this least possible Social Cost. We will investigate *Coordination Ratio* [16] - the worst-case ratio over all Nash equilibria, of Social Cost over Optimum. We are interested in understanding the interplay between the topology of the underlying interaction graph and the various existence, algorithmic, combinatorial, structural and optimality properties of Nash equilibria in this new model of selfish restricted scheduling with bounded interaction.

Contribution and Significance. We partition our results into three major groups:
3-regular interaction graphs (Section 3). It is easy to prove that the Social Cost of the standard fully mixed Nash equilibrium for any d-regular graph is $d - f(d, n)$, where $f(d, n)$ is a function that goes to 0 as n goes to infinity. This gives a general but rather rough estimation of Social Cost for d-regular graphs; moreover, it does not say how

the specific structure of each particular 3-regular graph affects the Social Cost of the standard fully mixed Nash equilibrium. We continue to prove much sharper estimations for the special class of 3-regular graphs. Restricting our model of interaction graphs to 3-regular graphs led us to discover some nice structural properties of orientations in 3-regular graphs, which were motivated by Nash equilibria. However, we have so far been unable to generalize these properties to regular graphs of degree higher than 3.

We pursue a thorough study of 3-regular interaction graphs; these graphs further restrict the bounded interaction by insisting that each machine is accessible to just three users. Specifically, we focus on the standard fully mixed Nash equilibrium where all probabilities of assigning users to machines are $\frac{1}{2}$. We ask which the best 3-regular interaction graph is in this case. This question brings into context the problem of comparing against each other the expected number of 2-orientations and 3-orientations - those with makespan 2 and 3, respectively. The manner in which these numbers outweigh each other brings Social Cost closer to either 2 or 3. We develop some deep graph-theoretic lemmas about 2- and 3-orientations in 3-regular graphs to prove, as our main result, that the simplest 3-regular parallel links graph is the best-case 3-regular graph in this setting. The proof decomposes any 3-regular graph down to the parallel links graph in a way that Social Cost of the standard fully mixed Nash equilibrium does not increase. The graph theoretic lemmas about 2- and 3-orientations are proved using both counting and mapping techniques; both the lemmas and their proof techniques are, we believe, of more general interest and applicability.

Bound on Coordination Ratio (Section 4). For the more general model of restricted parallel links, a tight bound of $\Theta(\frac{\log n}{\log \log n})$ on Coordination ratio restricted to pure Nash equilibria was shown in [9–Theorem 5.2] and independently in [1–Theorem 1]. This implies an upper bound of $O(\frac{\log n}{\log \log n})$ on the Coordination Ratio for pure Nash equilibria in our model as well. We construct an interaction graph incurring Coordination Ratio $\Omega(\frac{\log n}{\log \log n})$ to prove that this bound is tight for the model of interaction graphs as well. The construction extends an approach followed in [9–Lemma 5.1] that proved the same lower bound for the more general model of restricted parallel links.

The Fully Mixed Nash Equilibrium (Section 5). We pursue a thorough study of fully mixed Nash equilibria across interaction graphs. Our findings are outlined as follows:

- There exist counterexample interaction graphs for which fully mixed Nash equilibria may not exist. Among them are all trees and meshes. These counterexamples provide some insight about a possible graph-theoretic characterization of interaction graphs admitting a fully mixed Nash equilibrium. 4-cycles and 1-connectivity are factors expected to play a role in this characterization.
- We next consider the case where infinitely many fully mixed Nash equilibria may exist. In this case, the fully mixed Nash dimension is defined to be the dimension d of the smallest d-dimensional space that can contain all fully mixed Nash equilibria. For complete bipartite graphs, we prove a dichotomy theorem that characterizes unique existence. The proof employs arguments from Linear Algebra. For hypercubes, we have only been able to prove that the fully mixed Nash dimension is the hypercube dimension for hypercubes of dimension 2 or 3. We conjecture that this is true for all hypercubes, but we have only been able to observe that the hy-

percube dimension is a lower bound on the fully mixed Nash dimension (for all hypercubes).

– We are finally interested in understanding whether (or when) the fully mixed Nash equilibrium is the worst-case one in this setting. We present counterexample interaction graphs to show that the fully mixed Nash equilibrium is sometimes the worst-case Nash equilibrium, but sometimes not. For the hypercube, there is a pure Nash equilibrium that is worse (with respect to Social Cost) than the fully mixed one. On the other hand, for the 3-cycle the fully mixed Nash equilibrium has worst Social Cost.

Related Work and Comparison. Our model of interaction graphs is the special case of the model of restricted parallel links introduced and studied in [9], where each user is now further restricted to have access to only two machines. The work in [9] focused on the problem of computing pure Nash equilibria for that more general model. Awerbuch et al. [1] also considered the model of restricted parallel links, and proved a tight upper bound of $\Theta(\frac{\log n}{\log \log \log n})$ on Coordination Ratio for all (mixed) Nash equilibria. This implies a corresponding upper bound for our model of interaction graphs. It is an open problem whether this bound of $O(\frac{\log n}{\log \log \log n})$ is tight for the model of interaction graphs, or whether a better upper bound on Coordination Ratio for all (mixed) Nash equilibria can be proved.

The model of restricted parallel links is, in turn, a generalization of the so called KP-model for selfish routing [16], which has been extensively studied in the last five years; see e.g. [3, 4, 5, 7, 8, 9, 10, 11, 19, 20]. Social Cost and Coordination Ratio were originally introduced in [16]. Bounds on Coordination Ratio are proved in [3, 8, 9, 10, 20]. The fully mixed Nash equilibrium was introduced and studied in [20], where its unique existence was proved for the original KP-model. The Fully Mixed Nash Equilibrium Conjecture, stating that the fully mixed Nash equilibrium maximizes Social Cost, was first explicitly stated in [11]. It was proved to hold for special cases of the KP-model [11, 19] and for variants of this model [9, 10]. Recently the Fully Mixed Nash Equilibrium Conjecture was disproved for the original KP-model and the case that job sizes are *non-identical* [6]. This stands in sharp contrast to the model considered in this paper where job sizes are *identical*.

The model of interaction graphs is an alternative to *graphical games* [14] studied in the Artificial Intelligence community. The essential difference is that in graphical games, users and resources are modeled as vertices and edges, respectively. The problem of computing Nash equilibria for graphical games has been studied in [13, 14, 18]. Other studied variants of graphical games include the *network games* studied in [12], *multi-agent influence diagrams* [15] and *game networks* [17].

2 Framework and Preliminaries

For all integers $k \geq 1$, denote $[k] = \{1, \ldots, k\}$.

Interaction Graphs. We consider a graph $G = (V, E)$ where edges and vertices correspond to users and machines, respectively. Assume there are m users and n machines, respectively, where $m > 1$ and $n > 1$. Each user has a unit job. From here on, we shall refer to users and edges (respectively, machines and vertices) interchangeably. So, an edge connects two vertices if and only if the user can place his job onto the two machines.

Strategies and Assignments. A *pure strategy* for a user is one of the two machines it connects; so, a pure strategy represents an assignment of the user's job to a machine. A *mixed strategy* for a user is a probability distribution over its pure strategies. A *pure assignment* $\mathbf{L} = \langle \ell_1, \ldots, \ell_m \rangle$ is a collection of pure strategies, one for each user. A pure assignment induces an orientation of the graph G in the natural way. A *mixed assignment* $\mathbf{P} = (p_{ij})_{i \in [n], j \in [m]}$ is a collection of mixed strategies, one for each user. A mixed assignment $\mathbf{F}$ is *fully mixed* [20–Section 2.2] if all probabilities are strictly positive. The *standard fully mixed assignment* $\widetilde{\mathbf{F}}$ is the fully mixed one where all probabilities are equal to $\frac{1}{2}$. The *fully mixed dimension* of a graph G is the dimension d of the smallest d-dimensional space that contains all fully mixed Nash equilibria for this graph.

Cost Measures. For a pure assignment $\mathbf{L}$, the *load* of a machine $j \in [n]$ is the number of users assigned to j. The *Individual Cost* for user $i \in [m]$ is $\lambda_i = |\{k : \ell_k = \ell_i\}|$, the load of the machine it chooses. For a mixed assignment $\mathbf{P} = (p_{ij})_{i \in [m], j \in [n]}$, the *expected load* of a machine $j \in [n]$ is the expected number of users assigned to j. The *Expected Individual Cost* for user $i \in [m]$ on machine $j \in [n]$ is the expectation, according to $\mathbf{P}$, of the Individual Cost for user i on machine j, then, $\lambda_{ij} = 1 + \sum_{k \in [m], k \neq i} p_{kj}$. The *Expected Individual Cost* for user $i \in [m]$ is $\lambda_i = \sum_{j \in [n]} p_{ij} \lambda_{ij}$.

Associated with a mixed assignment $\mathbf{P}$ is the *Social Cost* defined as the expectation, according to $\mathbf{P}$, of makespan (that is, maximum load), that is, $\mathsf{SC}(G, \mathbf{P}) = \mathcal{E}_{\mathbf{P}} \left(\max_{v \in [n]} |\{k : \ell_k = v\}| \right)$, that is, Social Cost is the expectation, according to $\mathbf{P}$, of makespan (that is, maximum load). The *Optimum* $\mathsf{OPT}(G)$ is defined as the least possible, over all pure assignments $\mathbf{L} = \langle \ell_1, \ldots, \ell_n \rangle \in [n]^m$, makespan; that is, $\mathsf{OPT}(G) = \min_{\mathbf{L} \in [n]^m} \max_{v \in [n]} |\{k : \ell_k = v\}|$.

Nash Equilibria and Coordination Ratio. We are interested in a special class of (pure or) mixed assignments called *Nash equilibria* [21] that we describe here. The mixed assignment $\mathbf{P}$ is a Nash equilibrium [9, 16] if for each user $i \in [m]$, it minimizes $\lambda_i(\mathbf{P})$ over all mixed assignments that differ from $\mathbf{P}$ only with respect to the mixed strategy of user i. Thus, in a Nash equilibrium, there is no incentive for a user to unilaterally deviate from its own mixed strategy in order to decrease its Expected Individual Cost. Clearly, this implies that $\lambda_{ij} = \lambda_i$ if $p_{ij} > 0$ whereas $\lambda_{ij} \geq \lambda_i$ otherwise. We refer to these conditions as *Nash equations* and *Nash inequalities*, respectively.

The *Coordination Ratio* CR_G for a graph G is the maximum, over all Nash equilibria $\mathbf{P}$, of the ratio $\frac{\mathsf{SC}(G,\mathbf{P})}{\mathsf{OPT}(G)}$; thus, $\mathsf{CR}_G = \max_{\mathbf{P}} \frac{\mathsf{SC}(G,\mathbf{P})}{\mathsf{OPT}(G)}$. The *Coordination Ratio* CR is the maximum, over all graphs G and Nash equilibria $\mathbf{P}$, of the ratio $\frac{\mathsf{SC}(G,\mathbf{P})}{\mathsf{OPT}(G)}$; thus, $\mathsf{CR} = \max_{G,\mathbf{P}} \frac{\mathsf{SC}(G,\mathbf{P})}{\mathsf{OPT}(G)}$. Our definitions for CR_G and CR extend the original definition of Coordination Ratio by Koutsoupias and Papadimitriou [16] to encompass interaction graphs.

Graphs and Orientations. Some special classes of graphs we shall consider include the cycle C_r on r vertices; the complete bipartite graph (or biclique) $K_{r,s}$ which is a simple bipartite graph with partite sets of size r and s respectively, such that two vertices are adjacent if and only if they are in different partite sets; the hypercube H_r of dimension r whose vertices are binary words of length r connected if and only if their Hamming distance is 1. For a graph G, denote Δ_G the maximum degree of G. A graph is d-regular if all vertices have the same degree d. The graph consisting of 2 vertices and 3 parallel edges will be called *necklace*. Also, for even n, $G_{||}(n)$ will denote the *parallel links* graph, i.e., the graph consisting of $\frac{n}{2}$ necklaces.

An *orientation* of an undirected graph G results when assigning directions to its edges. The *makespan* of a vertex in an orientation α is the in-degree it has in α. The *makespan* of an orientation is the maximum vertex makespan. For any integer d, a d-orientation is an orientation with makespan d in a graph G; denote $d\text{-}\mathsf{or}(G)$ the set of d-orientations of G.

3 3-Regular Graphs

In this section, we consider the problem of determining the best-case d-regular graph among the class of all d-regular graphs with a given number of vertices (and, therefore, with the same number of edges), with respect to the Social Cost of the standard fully mixed Nash equilibrium, where all probabilities are equal to $1/2$.

A Rough Estimation. We start with a rough estimation of the Social Cost of any d-regular graph G, where $d \geq 2$. We first prove a technical lemma about the probability that such a random orientation has makespan at most $d-1$. Denote this probability $q_d(G)$.

Lemma 1. *Let I be an independent set of G. Then, $q_d(G) \leq \left(1 - \frac{1}{2^d}\right)^{|I|}$.*

We are now ready to prove:

Theorem 1. *For a d-regular graph G with n vertices, $\mathsf{SC}(\widetilde{\mathbf{F}}, G) = d - f(n,d)$, where $f(n,d) \to 0$ as $n \to \infty$.*

Cactoids and the Two-Sisters Lemma. The rest of our analysis will deal with 3-regular graphs. We will be able to significantly strengthen and improve Theorem 1 for the special case of 3-regular graphs. We define a structure that we will use in our proofs.

Definition 1 (Cactoids). *A cactoid is a pair $\widehat{G} = \langle V, \widehat{E} \rangle$, where V is a set of vertices and $\widehat{E}$ is a set consisting of undirected edges between vertices, and pointers to vertices, i.e., loose edges incident to one single vertex.*

A cactoid is called 3-regular if each vertex is incident to three elements from $\widehat{E}$. A cactoid may be considered as a standard multigraph if we add a special vertex and we replace each pointer by an edge which connects the special vertex with the vertex the pointer is incident to.

Consider now any arbitrary but fixed orientation σ of $\widehat{G}$. Call it *standard orientation*. We will now define variables $x_\alpha(e)$ for each $e \in \widehat{E}$, which take values from $\{0,1\}$ in each possible orientation α of $\widehat{G}$. The values are defined with reference to the standard orientation σ. So, take any arbitrary orientation α of $\widehat{G}$. For each $e \in \widehat{E}$, $x_\alpha(e) = 1$ if e has the same direction in α and σ, and 0 otherwise. Note that $x_\sigma(e) = 1$ for all $e \in \widehat{E}$.

We now continue with a lemma that estimates the probability that a random orientation is a 2-orientation in a 3-regular cactoid $\widehat{G}$. Consider two vertices u and v called the two sisters with incident pointers π_u and π_v. Assume that in the standard orientation σ, π_u and π_v point away from u and v, respectively. Denote $P_{\widehat{G}}(i, j)$ the probability that a random orientation α with $x_\alpha(u) = i$ and $x_\alpha(v) = j$, where $i, j \in \{0, 1\}$, is a 2-orientation. Clearly, by our assumption on the standard orientation σ, $P_{\widehat{G}}(1, 1)$ is not smaller than each of $P_{\widehat{G}}(0, 0)$, $P_{\widehat{G}}(0, 1)$ and $P_{\widehat{G}}(1, 0)$. However, we prove that $P_{\widehat{G}}(1, 1)$ is upper bounded by their sum.

Lemma 2 (Two Sisters Lemma). *For any* 3*-regular cactoid* $\widehat{G} = \langle V, \widehat{E}\rangle$ *and any two sisters* $u, v \in V$*, it holds that* $P_{\widehat{G}}(0, 0) + P_{\widehat{G}}(0, 1) + P_{\widehat{G}}(1, 0) \geq P_{\widehat{G}}(1, 1)$.

Proof. Denote b_1, b_2 and b_3, b_4 the other edges or pointers incident to the two sisters u and v, respectively. Define the standard orientation σ so that these edges or pointers point towards u or v, respectively. Denote $\widehat{G'}$ the cactoid obtained from $\widehat{G}$ by deleting the two sisters u and v and their pointers π_u and π_v. Define $P_{\widehat{G'}}(x_1, x_2, x_3, x_4)$ the probability that a random orientation α of the cactoid $\widehat{G'}$ with $x_\alpha(b_i) = x_i$ for $1 \leq i \leq 4$ is a 2-orientation. Then,

$$P_{\widehat{G}}(1,1) = \frac{1}{16} \sum_{x_1,x_2,x_3,x_4 \in \{0,1\}} P_{\widehat{G'}}(x_1, x_2, x_3, x_4) \ ,$$

$$P_{\widehat{G}}(0,0) = \frac{1}{16} \sum_{x_1 \cdot x_2 = 0, x_3 \cdot x_4 = 0} P_{\widehat{G'}}(x_1, x_2, x_3, x_4) \ ,$$

$$P_{\widehat{G}}(0,1) = \frac{1}{16} \sum_{x_1,x_2 \in \{0,1\}, x_3 \cdot x_4 = 0} P_{\widehat{G'}}(x_1, x_2, x_3, x_4) \ ,$$

$$P_{\widehat{G}}(1,0) = \frac{1}{16} \sum_{x_1 \cdot x_2 = 0, x_3, x_4 \in \{0,1\}} P_{\widehat{G'}}(x_1, x_2, x_3, x_4) \ .$$

Set now $D = 16 \cdot \big(P_{\widehat{G}}(0, 0) + P_{\widehat{G}}(0, 1) + P_{\widehat{G}}(1, 0) - P_{\widehat{G}}(1, 1)\big)$. It suffices to prove that $D \geq 0$. Clearly,

$$D = 2 \sum_{x_1 \cdot x_2 = 0, x_3 \cdot x_4 = 0} P_{\widehat{G'}}(x_1, x_2, x_3, x_4) - P(1, 1, 1, 1) \ .$$

Use now the cactoid $\widehat{G'}$ to define the probabilities $Q(i, j)$ and $R(i, j)$ where $i, j \in \{0, 1\}$ as follows: $Q(i, j)$ is the probability that a random orientation α of the cactoid $\widehat{G'}$ with $x_\alpha(b_1) = i$ and $x_\alpha(b_2) = j$ is a 2-orientation; $R(i, j)$ is the probability that a random orientation α of the cactoid $\widehat{G'}$ with $x_\alpha(b_3) = i$ and $x_\alpha(b_4) = j$ is a 2-orientation. Clearly,

$$Q_{\widehat{G'}}(i,j) = \sum_{x_3,x_4 \in \{0,1\}} P_{\widehat{G'}}(i, j, x_3, x_4) \ ,$$

$$R_{\widehat{G'}}(i,j) = \sum_{x_1,x_2 \in \{0,1\}} P_{\widehat{G'}}(x_1, x_2, i, j) \ .$$

We proceed by induction on the number of vertices of $\widehat{G}$. So, it suffices to assume the claim for the cactoid $\widehat{G'}$ and prove the claim for the cactoid $\widehat{G}$. Assume inductively that $Q_{\widehat{G'}}(0,0) + Q_{\widehat{G'}}(0,1) + Q_{\widehat{G'}}(1,0) \geq Q_{\widehat{G'}}(1,1)$ and $R_{\widehat{G'}}(0,0) + R_{\widehat{G'}}(0,1) + R_{\widehat{G'}}(1,0) \geq R_{\widehat{G'}}(1,1)$. These inductive assumptions and the definitions of $Q_{\widehat{G'}}$ and $R_{\widehat{G'}}$ imply that

$$\sum_{\substack{x_3,x_4\in\{0,1\}\\ x_1\cdot x_2=0}} P_{\widehat{G'}}(x_1,x_2,x_3,x_4) \geq \sum_{x_3,x_4\in\{0,1\}} P_{\widehat{G'}}(1,1,x_3,x_4)\ ,$$

$$\sum_{\substack{x_1,x_2\in\{0,1\}\\ x_3\cdot x_4=0}} P_{\widehat{G'}}(x_1,x_2,x_3,x_4) \geq \sum_{x_1,x_2\in\{0,1\}} P_{\widehat{G'}}(x_1,x_2,1,1)\ .$$

From the first inequality we obtain,

$$\sum_{\substack{x_3\cdot x_4=0\\ x_1\cdot x_2=0}} P_{\widehat{G'}}(x_1,x_2,x_3,x_4) \geq \sum_{x_3,x_4\in\{0,1\}} P_{\widehat{G'}}(1,1,x_3,x_4) - \sum_{x_1\cdot x_2=0} P_{\widehat{G'}}(x_1,x_2,1,1)\ .$$

From the second inequality we get,

$$\sum_{\substack{x_1\cdot x_2=0\\ x_3\cdot x_4=0}} P_{\widehat{G'}}(x_1,x_2,x_3,x_4) \geq \sum_{x_1,x_2\in\{0,1\}} P_{\widehat{G'}}(x_1,x_2 1,1) - \sum_{x_3\cdot x_4=0} P_{\widehat{G'}}(1,1,x_3,x_4)\ .$$

This yields $2\sum_{\substack{x_1\cdot x_2=0\\ x_3\cdot x_4=0}} P_{\widehat{G'}}(x_1,x_2,x_3,x_4) \geq 2P_{\widehat{G'}}(1,1,1,1)$ by adding up the last two inequalities, which implies $D \geq 0$, and the claim follows. □

Orientations and Social Cost. In this section, we prove a graph-theoretic result, namely that the regular parallel links graph minimizes the number of 3-orientations among all 3-regular graphs with the same number of vertices.

Theorem 2. *For every 3-regular graph G with n vertices it holds that $|\text{3-or}(G)| \geq |\text{3-or}(G_{||}(n))|$.*

Proof. In order to prove the claim, we start from the graph $G_0 = G = (V, E_0)$ and iteratively define graphs $G_i = (V, E_i)$, $1 \leq i \leq r$, for some $r \leq n$, in a way that G_r equals $G_{||}(n)$ and $|\text{3-or}(G_i)| \geq |\text{3-or}(G_{i+1})|$ holds for all $1 \leq i < r$.

Note that in each 3-regular graph, each connected component is either isomorphic to a necklace or it contains a path of length 3 connecting four different vertices, such that only the middle edge of this path can be a parallel edge. If in G_i all connected components are necklaces, than G_i is equal to $G_{||}(n)$, otherwise some connected component of G_i contains a path c, a, b, d with 4 different vertices a, b, c, d. In the latter case, construct a new graph $G_{i+1} = (V, E_{i+1})$ by deleting the edges $\{a, c\}, \{b, d\}$ from E_i and adding the edges $\{a, b\}, \{c, d\}$ to the graph as described in the following paragraph.

As illustrated in Figure 1, the edges incident to vertices a, b, c, d are numbered by some j, where $1 \leq j \leq 9$. In this figure, all the edges are different. This does not necessarily have to be the case. It may happen that $e_4 = e_5$ resulting in two parallel edges between a and b in G_i and three parallel edges between a and b in G_{i+1}. It may also happen that e_6 or e_7 is equal to e_8 or e_9. It is not possible that e_6 or e_7 is equal to

e_2 (or that e_8 or e_9 is equal to e_3) since we assumed that in the path c, a, b, d only the middle edge may be a parallel edge. It may be also possible that e_4 is equal to e_8 or e_9, and that e_5 is equal to e_6 or e_7. Note also that in each iteration step, the number of single edges is decreased by at least 1. So the number of iteration steps is at most n.

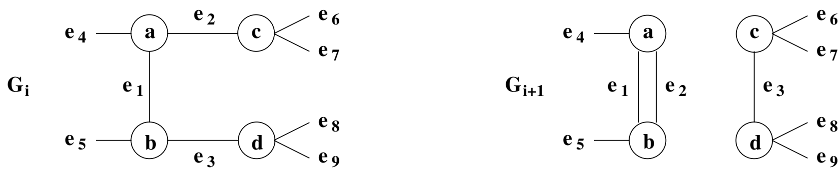

Fig. 1. Constructing the graph G_{i+1} from G_i

First, we will show that $|$3-or$(G_i)| \geq |$3-or$(G_{i+1})|$ holds if all the edges $e_1, \ldots, e_9$ are different. We will consider the more general case in which some of the e_j's are equal at the end of the proof. To make the notation simpler, we set $i = 1$, i.e., we consider the graphs G_1 and G_2. Note that there is a one-to-one correspondence between edges in G_1 and edges in G_2. This implies that any arbitrary orientation in G_1 can be interpreted as an orientation in G_2 and vice versa. Take the standard orientation of G_1 to be the one consistent with the arrows in Figure 2. The interpretation of this orientation for G_1 yields the standard orientation for G_2 (also shown in Figure 2).

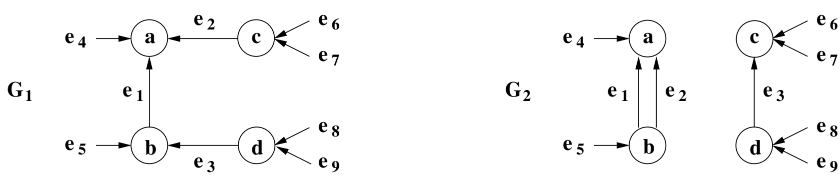

Fig. 2. The standard orientations in G_1 and G_2

We will prove our claim by defining an injective mapping F : 3-or$(G_2) \rightarrow$ 3-or(G_1). We want to use the identity mapping as far as possible. We set $C_2 = \{\alpha\ ;\ \alpha \in$ 3-or(G_2) $\alpha \notin$ 3-or$(G_1)\}$ and $C_1 = \{\alpha\ ;\ \alpha \in$ 3-or(G_1) , $\alpha \notin$ 3-or$(G_2)\}$, and we will define F such that $F(\alpha) = \alpha$ for $\alpha \in$ 3-or$(G_2) \setminus C_2$ and that $F : C_2 \rightarrow C_1$ is injective. Note that a mapping F : 3-or$(G_2) \rightarrow$ 3-or(G_1) defined this way is injective, since if $\beta \in C_1$, then $\beta \notin$ 3-or(G_2) and therefore β is not an image when using the identity function.

Let α be an arbitrary orientation. Note that all vertices $u \notin \{a, b, c, d\}$ have the same makespan in G_1 and in G_2 with respect to α. We identify first the class C_2 and consider the vertices a, b, c, d. We observe:

a has makespan 3 in G_2
$\Rightarrow x_1 = x_2 = x_4 = 1$
$\Rightarrow a$ has makespan 3 in G_1

d has makespan 3 in G_2
$\Rightarrow x_3 = 0, x_8 = x_9 = 1$
$\Rightarrow d$ has makespan 3 in G_1

b has makespan 3 in G_2
$\Rightarrow x_1 = x_2 = 0\ ,\ x_5 = 1$
$x_3 = 1 \Rightarrow b$ has makespan 3 in G_1
$x_3 = 0\ ,\ x_8 = x_9 = 1$
$\Rightarrow d$ has makespan 3 in G_1
$x_6 = x_7 = 1$
$\Rightarrow c$ has makespan 3 in G_1

c has makespan 3 in G_2
$\Rightarrow x_3 = x_6 = x_7 = 1$
$x_2 = 0 \Rightarrow c$ has makespan 3 in G_1
$x_1 = 0 \wedge x_5 = 1$
$\Rightarrow b$ has makespan 3 in G_1
$x_2 = 1 \wedge x_1 = x_4 = 1$
$\Rightarrow a$ has makespan 3 in G_1

Collecting this characterization, we construct the class C_2 as

$$C_2 = \{\alpha \notin \text{3-or}(G_1)\ ;\ x_1 = x_2 = x_3 = 0 \wedge x_5 = 1 \wedge x_6 \cdot x_7 = x_8 \cdot x_9 = 0\} \cup \{\alpha \notin \text{3-or}(G_1)\ ;\ x_2 = x_3 = x_6 = x_7 = 1 \wedge x_1 \cdot x_4 = 0\ \wedge\ (x_1 = 1 \vee x_5 = 0)\}\ .$$

In a similar way, we construct the class C_1 as

$$C_1 = \{\alpha \notin \text{3-or}(G_2)\ ;\ x_1 = 0 \wedge x_2 = x_3 = x_5 = 1 \wedge x_6 \cdot x_7 = 0\} \cup \{\alpha \notin \text{3-or}(G_2)\ ;\ x_2 = x_3 = 0 \wedge x_6 = x_7 = 1 \wedge x_8 \cdot x_9 = 0\ \wedge (x_1 = 1 \vee x_5 = 0)\}\ .$$

Now, to define F, we consider four cases about orientations $\alpha \in C_2$:

(1) Consider $\alpha \in C_2$ with
$x_2 = x_3 = x_6 = x_7 = 1\ \wedge\ x_1 \cdot x_4 = 0\ \wedge\ x_8 \cdot x_9 = 0\ \wedge\ (x_1 = 1\ \vee\ x_5 = 0)$.
Set $F(x_1, 1, 1, x_4, x_5, 1, 1, x_8, x_9, \dots) = (x_1, 0, 0, x_4, x_5, 1, 1, x_8, x_9, \dots)$.

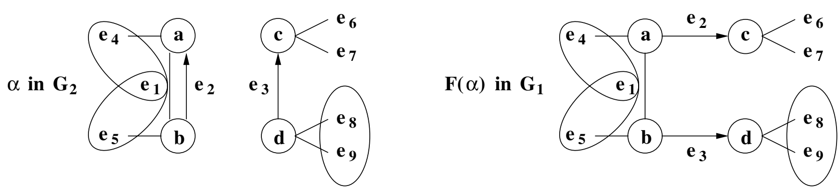

Fig. 3. The mapping F

Note that vertices from $\{a, b, c, d\}$ have the same connections to vertices outside $\{a, b, c, d\}$; therefore, $\alpha \notin \text{3-or}(G_1)$ implies that $F(\alpha) \notin \text{3-or}(G_2)$. This implies that $F(\alpha) \in C_1$.

(2) Consider $\alpha \in C_2$ with $x_1 = x_2 = x_3 = 0 \wedge x_5 = 1 \wedge x_6 \cdot x_7 = 0 \wedge x_8 \cdot x_9 = 0$.
Set $F(0, 0, 0, x_4, 1, x_6, x_7, x_8, x_9, \dots) = (0, 1, 1, x_4, 1, x_6, x_7, x_8, x_9, \dots)$.
In a way similar to case (1), we conclude that $F(\alpha) \in C_1$.

After these two cases, any orientation $\alpha \in H_2$ with

$$H_2 = \{\alpha \in C_2 \mid x_2 = x_3 = x_6 = x_7 = 1 \wedge x_1 \cdot x_4 = 0 \wedge x_8 = x_9 = 1 \wedge (x_1 = 1 \vee x_5 = 0)\}$$

has not been mapped by F, and orientations $\beta \in H_1$ with

$$H_1 = \{\beta \in C_1 \mid x_2 = x_3 = 0 \wedge x_6 = x_7 = 1 \wedge x_1 = x_4 = 1 \wedge x_8 \cdot x_9 = 0\}$$
$$\cup \{\beta \in C_1 \mid x_1 = 0\,,\ x_2 = x_3 = x_5 = 1 \wedge x_6 \cdot x_7 = 0 \wedge x_8 = x_9 = 1\}$$

are not images under F. We continue with these orientations.

(3) Set

$$H_{21} = \{\alpha \in C_2\ ;\ x_2 = x_3 = x_6 = x_7 = x_8 = x_9 = 1 \wedge x_1 = 1 \wedge x_4 = 0\}$$
$$H_{11} = \{\beta \in C_1\ ;\ x_2 = x_3 = 0 \wedge x_1 = x_4 = x_6 = x_7 = 1 \wedge x_8 \cdot x_9 = 0\}$$

We will show that $|H_{21}| \leq |H_{11}|$ holds.

Consider the cactoids T_{21} and T_{11} obtained by omitting the vertices a, b, c, d from H_{21} and H_{11}, respectively. T_{21} and T_{11} consist of edges and 6 pointers $e_j, 4 \leq j \leq 9$. Fixing the directions of the pointers in the same way as in the definitions of H_{21} and H_{11}, respectively, the number of 2-orientations of T_{21} is equal to $|H_{21}|$ and the number of 2-orientations of T_{11} is equal to $|H_{11}|$. See Figure 4 for an illustration. The pointers e_6 and e_7 have the same directions in T_{21} and T_{11} and e_5 has no specified direction in both cases. Edge e_4 has different directions in T_{21} and T_{11}. Directing edge e_4 in T_{21} towards vertex a would lead to an increased number of 2-orientations since the other vertex incident to e_4 has in this case makespan 2 with

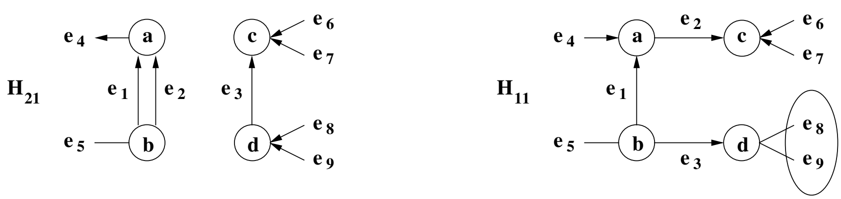

Fig. 4. Orientations from the sets H_{21} and H_{11}

a larger probability. Let $\widetilde{T}_{21}$ be the cactoid obtained from T_{21} by directing edge e_4 towards a. Then $\widetilde{T}_{21}$ and T_{11} differ only in the directions given to edges e_8 and e_9. Let $P(i, j)$ be the probability of a 2-orientation in G_2 if $x_8 = i$ and $x_9 = j$. Set $m = \frac{3}{2}n$. Then, $\frac{|H_{11}|}{2^{m-3}} = P(0, 0) + P(0, 1) + P(1, 0) \geq P(1, 1) \geq \frac{|H_{21}|}{2^{m-3}}$, because of Lemma 2. It follows that $|H_{21}| \leq |H_{11}|$.

(4) To finish the first part of the proof, set

$$H_{22} = \{\beta \in C_2; x_2 = x_3 = x_6 = x_7 = x_8 = x_9 = 1 \wedge x_1 = x_5 = 0\}$$
$$H_{12} = \{\beta \in C_1; x_1 = 0 \wedge x_2 = x_3 = x_5 = x_8 = x_9 = 1 \wedge x_6 \cdot x_7 = 0\}$$

See Figure 5 below for an illustration. In the same way as in case (3), we show that $|H_{22}| \leq |H_{12}|$.

Since $H_2 = H_{21} \cup H_{22}$ and $H_1 = H_{11} \cup H_{12}$, there exists an injective mapping F : 3-or$(G_2) \rightarrow$ 3-or(G_1) in the case that all edges $e_4, \ldots, e_9$ are different.

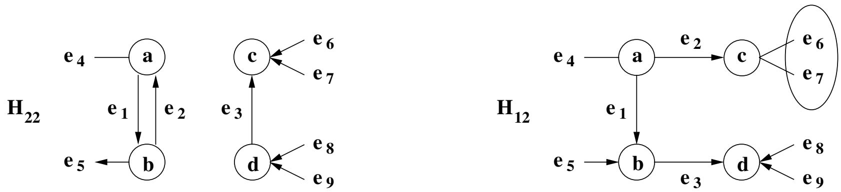

Fig. 5. Orientations from the sets H_{22} and H_{12}

Now we consider the case that some of these edges are equal. If $e_i = e_j$ then in each orientation α the variables x_i and x_j get opposite values. Recall that the construction and proof of injectivity of the mapping F, which we described above, was done in 3 steps:

(i) We defined $F(\alpha) = \alpha$ for all $\alpha \in 3\text{-or}(G_2) \setminus C_2$
(ii) In cases (1) and (2) for some well defined $\alpha = (x_1, \ldots, x_9, \ldots)$, the value $F(\alpha)$ is obtained by negating x_2 and x_3 and leaving the other directions unchanged.
(iii) $|H_2| \leq |H_1|$ is shown for the remaining cases.

Steps (i) and (ii) are not influenced by setting $x_i = \bar{x}_j$ for some $i, j \in \{4, \ldots, 9\}, i \neq j$. So it remains to consider step (iii). If $e_i = e_j$ for $i \in \{6, 7\}, j \in \{8, 9\}$, then $x_i = \bar{x}_j$ holds and this implies that $H_2 = \emptyset$, since for all $\alpha \in H_2$ it holds $x_6 = x_7 = x_8 = x_9 = 1$. Clearly, this implies $|H_2| \leq |H_1|$.

So we can assume now that $e_i \neq e_j$ for $i \in \{6, 7\}, j \in \{8, 9\}$ and we consider the case $e_4 = e_5$. We will show first that $|H_{21}| \leq |H_{11}|$ holds also in this case. Consider the cactoids T_{21} and T_{11} obtained by deleting the vertices a, b, c, d from H_{21} and H_{11}. Since edge $e_4 = e_5$ connects vertices a and b, it is also deleted when the cactoids are formed. Each of the cactoids T_{11} and T_{21} has now only the 4 pointers $e_j, 6 \leq j \leq 9$. A simple inspection of the proof given above shows that $|H_{21}| \leq |H_{11}|$ holds also in this case. Furthermore, $|H_{22}| \leq |H_{21}|$ can be shown in the same way. The cases $e_4 = e_8$ and $e_5 = e_6$ can be handled in a very similar way. This completes the proof of the claim. □

Our main result follows now as an immediate consequence of Theorem 2.

Corollary 1. *For a 3-regular graph G with* n *vertices,* $\mathsf{SC}(G, \widetilde{\mathbf{F}}) \geq \mathsf{SC}(G_{||}(n), \widetilde{\mathbf{F}}) = 3 - \left(\frac{3}{4}\right)^{n/2}$.

We can also show that equality does *not* hold in Corollary 1.

Example 1. *There is a 3-regular graph for which the Social Cost of the standard fully mixed Nash equilibrium is larger than for the corresponding parallel links graph.*

4 Coordination Ratio

In this section, we present a bound on the Coordination Ratio for pure Nash equilibria.

Theorem 3. *Restricted to pure Nash equilibria,* $\mathsf{CR} = \Theta\left(\frac{\log n}{\log \log n}\right)$.

Observation 1. *Restricted to pure Nash equilibria, for any interaction graph G,* $\mathsf{CR}_G \leq \Delta_G$*, and this bound is tight.*

5 The Fully Mixed Nash Equilibrium

In this section, we study the fully mixed Nash equilibrium. For a graph $G = (V, E)$, for each edge $jk \in E$, denote jk the user corresponding to the edge jk. Denote $\widehat{p}_{jk}$ and $\widehat{p}_{kj}$ the probabilities (according to $\mathbf{P}$) that user jk chooses machines j and k, respectively. For each machine $j \in V$, the expected load of machine j excluding a set of edges $\widetilde{E}$, denoted $\pi_{\mathbf{P}}(j) \setminus \widetilde{E}$, is the sum $\sum_{kj \in E \setminus \widetilde{E}} \widehat{p}_{kj}$. As a useful combinatorial tool for the analysis of our counterexamples, we prove:

Lemma 3 (The 4-Cycle Lemma). *Take any 4-cycle C_4 in a graph G, and any two vertices $u, v \in C_4$ that are non-adjacent in C_4. Consider a Nash equilibrium $\mathbf{P}$ for G. Then, $\pi_P(u) \setminus C_4 = \pi_P(v) \setminus C_4$.*

Non-Existence Results. We first observe:

Counterexample 1. *There is no fully mixed Nash equilibrium for trees and meshes.*

We remark that the crucial property of trees that was used in the proof of Counterexample 1 is that each tree contains at least one leaf. Thus, Counterexample 1 actually applies to the more general class of graphs with no vertex of degree 1. We continue to prove:

Counterexample 2. *For each graph in Figure 6, there is no fully mixed Nash equilibrium.*

Our six counterexample graphs suggest that the existence of 4-cycles across the "boundary" of a graph or 1-connectivity may be crucial factors that disallow the existence of fully mixed Nash equilibria. Of course, this remains yet to be proved.

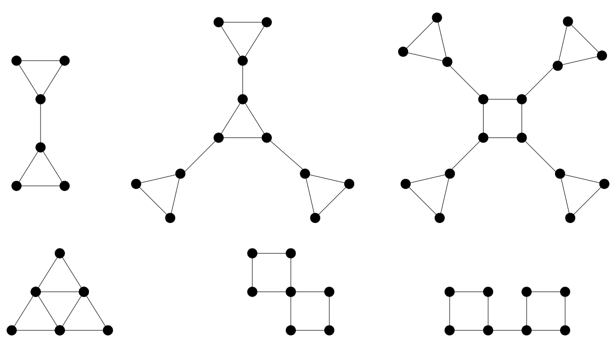

Fig. 6. Six counterexample graphs

Uniqueness and Dimension Results. For *Complete Bipartite Graphs*, we prove:

Theorem 4. *Consider the complete bipartite graph $K_{r,s}$, where $s \geq r \geq 2$ and $s \geq 3$. Then, the fully mixed Nash equilibrium $\mathbf{F}$ for $K_{r,s}$ exists uniquely if and only if $r > 2$. Moreover, in case $r = 2$, the fully mixed Nash dimension of $K_{r,s}$ is $s - 1$.*

Hypercube Graphs. Observe first that, in general, any point in $(0,1)^r$ is mapped to a fully mixed Nash equilibrium with equal Nash probabilities on all edges of the same dimension (and "pointing" to the same direction). This implies:

Observation 2. *Consider the hypercube H_r, for any $r \geq 2$. Then, the fully mixed Nash dimension of H_r is at least r.*

To show that r is also an upper bound, we need to prove that no other fully mixed Nash equilibria exist. We manage to do this only for $r \in \{2, 3\}$.

Theorem 5. *Consider the hypercube H_r, for $r \in \{2, 3\}$. Then, the fully mixed Nash dimension is r.*

Worst-Case Equilibria. We present two counterexamples to show that a fully mixed Nash equilibrium is not necessarily the worst-case Nash equilibrium, but it can be.

Counterexample 3. *There is an interaction graph for which no fully mixed Nash equilibrium has worst Social Cost.*

Counterexample 4. *There is an interaction graph for which there exists a fully mixed Nash equilibrium with worst Social Cost.*

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A Cost Mechanism for Fair Pricing of Resource Usage*

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Abstract. We propose a simple and intuitive *cost mechanism* which assigns *costs* for the competitive usage of m *resources* by n selfish *agents*. Each agent has an individual *demand*; demands are drawn according to some probability distribution. The cost paid by an agent for a resource she chooses is the total demand put on the resource divided by the number of agents who chose that same resource. So, resources charge costs in an equitable, fair way, while each resource makes no *profit* out of the agents.

We call our model the *Fair Pricing* model. Its fair cost mechanism induces a non-cooperative game among the agents. To evaluate the *Nash equilibria* of this game, we introduce the *Diffuse Price of Anarchy*, as an extension of the *Price of Anarchy* that takes into account the probability distribution on the demands. We prove:

- *Pure Nash equilibria* may not exist, unless all chosen demands are identical; in contrast, a *fully mixed Nash equilibrium* exists for all possible choices of the demands. Further on, the fully mixed Nash equilibrium is the *unique* Nash equilibrium in case there are only two agents.
- In the *worst-case* choice of demands, the *Price of Anarchy* is $\Theta(n)$; for the special case of two agents, the Price of Anarchy is less than $2 - \frac{1}{m}$.
- Assume now that demands are drawn from a *bounded, independent probability distribution*, where all demands are *identically distributed* and each is at most a (*universal* for the class) constant times its expectation. Then, the Diffuse Price of Anarchy is at most that same constant, which is just 2 when each demand is distributed symmetrically around its expectation.

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1 Introduction

Motivation, Framework and Overview. We propose and analyze a very simple and intuitive *cost mechanism* for pricing the competitive usage of a collection of m *resources* by a collection of n selfish *agents*, each coming with an individual *demand*. Demands are drawn according to some (perhaps unknown) probability distribution. This assumption is suitable for many practical situations (e.g., selfish sharing of bandwidth) where repeatedly revealing the demands allows their statistical modeling.

The key feature of our mechanism is its reliance on a very natural *fairness* principle in *cost sharing*. Namely, the cost charged to an agent for a resource she chooses is the total demand on the resource divided by the number of agents who chose that same resource; we call it *Resource Cost*.

Such a cost mechanism represents a very natural sharing scheme that is often used in real life; for example, restaurants offering an "all-you-can-eat" buffet charge a fixed price to each customer, which is calculated to alleviate all restaurant costs over customers. Moreover, Internet service providers and operators in telecommunication networks often charge a flat amount in case the demands of agents on bandwidth do not differ much – see, for example, [19, 20], or the more recent [18] and references therein. Moreover, this cost mechanism represents a *fair* pricing scheme since no resource makes any profit by overcharging, while all agents sharing the same resource are treated equally.

In reflection to the fairness principle encapsulated in its cost mechanism, our pricing model will be coined as the *Fair Pricing* model. Its fair cost mechanism induces a non-cooperative strategic game, which we call FairPricingGame, whose *players* and *strategies* are the agents and resources, respectively. We analyze the *Nash equilibria* [16] (both *pure* and *mixed*) for FairPricingGame; roughly speaking, these are stable states from which no agent has an incentive to unilaterally deviate. In particular, we consider the *fully mixed Nash equilibrium* where each agent selects each resource with non-zero probability. While offering in addition an advantage with respect to convenience in handling, the fully mixed Nash equilibrium is suitable for our economic framework under the very natural assumption that each resource offers usage to all agents without imposing any access restrictions.

We define *Optimum* as the *least possible* maximum (over resources) Resource Cost; for a Nash equilibrium, we define *Social Cost* as the expectation, over random choices of the agents, of maximum (over resources) Resource Cost. We observe that Optimum is achieved when all agents choose the same resource (Proposition 1).

The *Price of Anarchy* [9, 17] is the ratio of Social Cost over Optimum in the *worst-case* pair of chosen demands and Nash equilibrium. To evaluate the Nash Equilibria of the FairPricingGame, we use both the Price of Anarchy and the *Diffuse Price of Anarchy*, an extension of the former, which we introduce to accommodate the (arbitrary but specific) probability distribution on the demands.

The Diffuse Price of Anarchy. The main argument for using worst-case demands in the definition of the Price of Anarchy [9] is that the distribution of the demands is not known. However, the Price of Anarchy takes this argument way too far. It assumes that nothing is known about the distribution, so that any distribution on the demands is possible. The worst-case distribution prevailing in the definition of the Price of Anarchy is the one in which the worst-case demand occurs with probability one. We extend this definition to remove this assumption while avoiding to assume full knowledge about the distribution of demands. Roughly speaking, the *Diffuse Price of Anarchy* is the *worst-case*, over all allowed probability distributions, of the expectation (according to each specific probability distribution) of the ratio of Social Cost over Optimum in the worst-case Nash equilibrium.

Summary and Contribution. We prove that the FairPricingGame may not have a pure Nash equilibrium unless all chosen demands are identical (Theorem 1). The proof argues that the Resource Costs of all resources are identical in a pure Nash equilibrium. In contrast, we show that a fully mixed Nash equilibrium always exists (Theorem 2). For the case of two agents, we prove that the fully mixed Nash equilibrium is the unique Nash equilibrium (Theorem 3).

We next prove that the Price of Anarchy can be as large as $\Theta(n)$; we establish both lower and upper bounds (Theorems 4 and 6). A tighter analysis applies to the case of two agents to yield that the Price of Anarchy is then less than $2 - \frac{1}{m}$ (Theorem 5).

To mitigate the high $\Theta(n)$ bound on the Price of Anarchy, we seek structure in the probability distributions of demands. The outcome has been the identification of an interesting class of probability distributions on demands for which the Diffuse Price of Anarchy is upper bounded by a constant (Theorem 7). This is a very simple but broad class of so called *bounded, independent probability distributions* – there, roughly speaking, demands are independent and identically distributed, while each demand may not exceed a *universal* for the class constant times its expectation. Under the additional assumption that each demand is symmetrically distributed around its expectation, this universal constant is just 2. To the best of our knowledge, our work is the first to prove something nontrivial about an averaged Price of Anarchy such as the Diffuse Price of Anarchy we have introduced.

Related Work. Our Fair Pricing model is partly motivated by the KP model [9] for selfish routing; there is a vast volume of research about selfish routing with unsplittable demands (see, e.g., [2, 3, 4, 8, 9, 11]). The KP model overcharges the total demand on a resource to each and every agent choosing the resource. To the best of our knowledge, our Fair Pricing Model is the first work to explicitly formulate and evaluate, using the Price of Anarchy, a theoretical model of cost sharing with resources and selfish agents that charges the fair share of the total demand to each agent choosing the resource; of course, proportional cost sharing models such as ours have been considered before in Computer Science (e.g., in the microprocessor scheduling/sharing model).

Studied in the Economics literature are several pricing models similar to (but different than) our Fair Pricing model. These pricing models have mostly considered an economic system with a single resource and multiple agents, each choosing its individual demand as its strategy; these models have addressed the problem of identifying the most suitable cost function for the single resource according to several axiomatic criteria, such as monotonicity and envy-freeness. Two prominent examples of such pricing models are *Average Cost Pricing* [1] and *Serial Cost Sharing* [14, 15]; they will be discussed in more detail in Sect. 2. The essential differences between our Fair Pricing model and those studied in the Economics literature are that we consider multiple resources (albeit identical) and we model the strategy of an agent as some resource. As a result, associated with the two approaches are different notions of equilibria exhibiting different properties.

The Price of Anarchy was originally proposed by Koutsoupias and Papadimitriou, and further advocated by Papadimitriou [17], as a measure of performance degradation in systems with resources shared by selfish agents. The Diffuse Price of Anarchy is motivated by the *Diffuse Adversary* studied by Koutsoupias and Papadimitriou [10] as an alternative to *worst-case* adversaries usually considered for competitive analysis in online computing. So, Diffuse Competitive Ratio is to Competitive Ratio in Online Computing what Diffuse Price of Anarchy is to Price of Anarchy in Selfish Computing.

The fully mixed Nash equilibrium was originally proposed by Mavronicolas and Spirakis [13]; its various existence and uniqueness properties were subsequently studied very extensively; see, e.g., [3, 4, 5, 11, 12]. Coordination mechanisms [2] are another means for reducing the Price of Anarchy.

Hayrapetyan *et al.* [6] presented and analyzed (using the Price of Anarchy) a pricing game to capture the interaction between service providers and users over the Internet. Their pricing model addresses the competition of network managers for users via *prices* and the quality of service provided; it addresses neither fairness nor distributions on demands, which are the two main ingredients of our model.

Organization. Section 2 introduces the Fair Pricing model and summarizes some preliminary facts. Pure Nash equilibria and fully mixed Nash equilibria are treated in Sect. 3 and 4, respectively. Sections 5 and 6 present our results for the Price of Anarchy and the Diffuse Price of Anarchy, respectively. We conclude, in Sect. 7, with a discussion of our results and suggestions for further research.

2 The Fair Pricing Model

Our Fair Pricing model was originally motivated by the standard KP model for selfish routing [9]; it departs from it by encompassing some stochastic assumptions on user demands, and notions of pricing and fairness as well.

Notation. For an event E in a sample space, denote $\mathbf{Pr}\{E\}$ the probability of event E occurring. For a random variable X that follows the probability distribution D, denote $\mathcal{E}_D(X)$ the *expectation* of X (according to the probability distribution D). For any integer $m \geq 2$, denote $[m] = \{1, \ldots, m\}$.

Agents and Resources. We consider a collection $\mathcal{M} = \{1, 2, \ldots, m\}$ of identical *resources*, and a collection $\mathcal{N} = \{1, 2, \ldots, n\}$ of *agents*. Associated with an agent $i \in \mathcal{N}$ is a *demand* $w_i \in \mathbb{R}_+$. We assume that demands are chosen according to some (known) joint probability distribution D, which comes from some (known) class Δ of possible distributions. We consider D to be the steady state distribution of some ergodic stochastic process that generates demands. We will often fix a particular outcome of the experiment of choosing demands (according to D), which is a $n \times 1$ *demand vector* $\mathbf{w}$. Denote $W = \sum_{i \in \mathcal{N}} w_i$ and $\widehat{W} = \frac{W}{n}$. Note that $\frac{w_i}{\widehat{W}} \leq n$ for all agents $i \in \mathcal{N}$. We will be assuming, without loss of generality, that $w_1 \geq w_2 \geq \ldots \geq w_n$. We usually use subscripts for agents and superscripts for resources.

Strategies and Assignments. A *pure strategy* for agent $i \in \mathcal{N}$ is some specific resource; a *mixed strategy* for agent i is a probability distribution on the set of pure strategies. A *pure assignment* $\mathbf{L} \in \mathcal{M}^n$ is a collection of pure strategies, one per agent. Similarly, a *mixed assignment* $\mathbf{P}$ is a collection of mixed strategies, one per agent. A mixed assignment is represented by an $n \times m$ *probability matrix* $\mathbf{P}$ of mn probabilities p_i^j, $i \in \mathcal{N}$ and $j \in \mathcal{M}$, where p_i^j is the probability that agent i selects resource j. Clearly, for each agent $i \in \mathcal{N}$, $\sum_{j \in \mathcal{M}} p_i^j = 1$. For each agent $i \in \mathcal{N}$, the *support* of agent i in the mixed assignment $\mathbf{P}$ is the set of resources $\mathcal{S}_i = \{j \in \mathcal{M} \mid p_i^j > 0\}$; thus, the support of agent i is the set of resources which i chooses with non-zero probability. $\mathbf{P}$ is *fully mixed* [13] if for all agents $i \in \mathcal{N}$ and resources $j \in \mathcal{M}$, $p_i^j > 0$; thus, each agent selects each resource with non-zero probability.

Resource Demand, Resource Congestion, Resource Cost, Individual Cost and Resource Profit. Fix a pure assignment $\mathbf{L} = \langle l_1, l_2, \ldots, l_n \rangle$ and a resource $j \in \mathcal{M}$. Define the *Resource Demand* on resource j, denoted $W^j(\mathbf{L})$, as the total demand on resource j; that is, $W^j(\mathbf{L}) = \sum_{k \in \mathcal{N}: l_k = j} w_k$. Define the *Resource Congestion* on resource j, denoted $n^j(\mathbf{L})$, as the total number of agents on resource j; that is, $n^j(\mathbf{L}) = \sum_{k \in \mathcal{N}: l_k = j} 1 = |\{k \in \mathcal{N} \mid l_k = j\}|$. The *Resource Cost* on resource j, denoted $\mathsf{RC}^j(\mathbf{L})$, is the ratio $\frac{W^j(\mathbf{L})}{n^j(\mathbf{L})}$ if $n^j(\mathbf{L}) > 0$, and 0 otherwise. The *Individual Cost* for agent $i \in \mathcal{N}$, denoted $\mathsf{IC}_i(\mathbf{L})$, is defined to be $\mathsf{RC}^{l_i}(\mathbf{L})$; so, $\mathsf{IC}_i(\mathbf{L}) = \frac{W^{l_i}(\mathbf{L})}{n^{l_i}(\mathbf{L})}$, and the Individual Cost of agent i is the Resource Cost of the resource she chooses. Although Individual Cost is identified with Resource Cost in the specific case of the Fair Pricing model considered here, this may not be true in general. So, we chose to introduce both in order to offer convenience to future cost sharing models that will explicitly distinguish between them.

The *Resource Profit* of resource $j \in \mathcal{M}$, denoted $\mathsf{RP}^j(\mathbf{L})$, is defined as $\mathsf{RP}^j(\mathbf{L}) = \sum_{k \in \mathcal{N}: l_k = j} \mathsf{IC}_k(\mathbf{L}) - W^j(\mathbf{L})$; intuitively, the profit of a resource is the total cost it charges to agents choosing it minus the total demand it serves. This definition is very general since it applies to all possible specifications of Individual Cost. Clearly, for the specific way we defined Individual Cost in this work, all Resource Profits are zero. However, we still chose to introduce Resource

Profit as an important metric for general theoretical models of cost sharing, even though it happens to be zero in the specific case of the Fair Pricing model considered here. We believe that Resource Profit merits explicit investigation in other cost sharing models as well. We often drop the arguments of the various costs when these are clear from context.

Other Cost Sharing Models. Similar cost sharing models from the Economics literature have considered Resource Costs and Individual Costs similar to the ones we employed in our Fair Pricing model. We will comment (using the terminology adopted in this work) on the two most closely related ones, namely the *Average Cost Pricing* [1] and the *Serial Cost Sharing* [14, 15] models. Both of them use a nondecreasing *Cost Function* C^l for each resource $l \in [m]$, and take as Resource Cost the value $C^l(\sum_{k \in \mathcal{N}: l_k = l} w_k)$. This value is different than the Resource Cost adopted here, even if C^l is the identity function. The crucial difference is that the latter depends on (in particular, decreases with) the Resource Congestion for resource l, while the former ignores Resource Congestion completely. Moreover, both models seek ways to share the Resource Cost $C^l(\sum_{k \in \mathcal{N}: l_k = l} w_k)$ among the selfish agents choosing the resource l.

- In the *Average Cost Pricing* model [1], the Individual Cost of agent $i \in \mathcal{N}$ choosing resource $l \in [m]$ in the pure assignment $\mathbf{L}$ is

$$\mathsf{IC}_i(\mathbf{L}) = \frac{w_i}{\sum_{k \in \mathcal{N}: l_k = l} w_k} C^l\Big(\sum_{k \in \mathcal{N}: l_k = l} w_k\Big) \ .$$

- In the *Serial Cost Sharing* model [14, 15], there are intuitive, systematical formulas for the Individual Costs of the agents choosing a resource $l \in [m]$, which we will demonstrate for the special case of three agents with demands $w_1 \geq w_2 \geq w_3$. We refer to a pure assignment $\mathbf{L}$. The Individual Cost of agent 3 with the smallest demand is $\mathsf{IC}_3(\mathbf{L}) = \frac{C^l(3w_3)}{3}$. This is similar to (but different than) the fair share of Resource Cost employed in the Fair Pricing model, and it depends on the Resource Congestion (equal to 3) in a way identical to the one in our model. This is not true for the rest of the agents. The Individual Cost of agent 2 is

$$\mathsf{IC}_2(\mathbf{L}) = \frac{C^l(w_3 + 2w_2) - \mathsf{IC}_3(\mathbf{L})}{2} = \frac{C^l(w_3 + 2w_2)}{2} - \frac{C^l(3w_3)}{6} \ .$$

  Finally, agent 1 pays the rest of Resource Cost, and this is calculated to be

$$\mathsf{IC}_1(\mathbf{L}) = C^l(w_1 + w_2 + w_3) - \frac{C^l(w_3 + 2w_2)}{2} + \frac{C^l(3w_3)}{6} \ .$$

  All Individual Costs depend on Resource Congestion, but in a way much more involved (but probably noteworthy) than the one in our Fair Pricing model. The Serial Cost Sharing model reflects the principle that the Individual Cost of an agent does not depend on user demands that are larger than its own; this principle is violated in the Fair Pricing model.

Expectations in Mixed Assignments. For a mixed assignment $\mathbf{P}$, all resource demand, resource congestion and Resource Cost become random variables induced by the probability distribution $\mathbf{P}$. We define the *expected resource demand*, the *expected congestion demand* and the *Expected Resource Cost* as the expectations of resource demand, resource congestion and Resource Cost, respectively, according to $\mathbf{P}$. The *Conditional Expected Individual Cost* IC_i^j of agent $i \in \mathcal{N}$ on resource $j \in \mathcal{M}$ is the conditional expectation of the Individual Cost of agent i had she been assigned to resource j. The *Expected Individual Cost* IC_i of agent i is the expectation of her Conditional Expected Individual Cost on a resource; so, $\mathsf{IC}_i = \sum_{j \in \mathcal{M}} p_i^j \mathsf{IC}_i^j$.

Nash Equilibria. The definition of Expected Individual Cost completes the definition of a strategic game that models fair pricing of resource usage, which we call $\mathsf{FairPricingGame}$. We are interested in the induced (both pure and mixed) *Nash equilibria* [16] of $\mathsf{FairPricingGame}$. Formally, the pure assignment $\mathbf{L}$ is a *pure Nash equilibrium* if for each agent $i \in \mathcal{N}$, the Individual Cost $\mathsf{IC}_i(\mathbf{L})$ is minimized (given the pure strategies of the other agents); thus, no agent can unilaterally improve her own Individual Cost. The mixed assignment $\mathbf{P}$ is a *mixed Nash equilibrium* if for each agent $i \in \mathcal{N}$, the Expected Individual Cost $\mathsf{IC}_i(\mathbf{L})$ is minimized (given the mixed strategies of the other agents); thus, no agent can unilaterally improve her own Expected Individual Cost. The particular definition of Expected Individual Cost implies that for each agent $i \in \mathcal{N}$, for each resource $j \in \mathcal{M}$ such that $p_i^j > 0$, all Conditional Expected Individual Costs IC_i^j are the same and no more than any Conditional Expected Individual Cost IC_i^l with $p_i^l = 0$.

Social Cost and Optimum. We proceed to define Social Cost, Optimum and the Price of Anarchy for the specific $\mathsf{FairPricingGame}$ we consider. Associated with a mixed Nash equilibrium $\mathbf{P}$ is the *Social Cost* $\mathsf{SC}(\mathbf{w}, \mathbf{P})$, which is the expectation, over all random choices of the agents, of the maximum Resource Cost; thus, $\mathsf{SC}(\mathbf{w}, \mathbf{P}) = \mathcal{E}_{\mathbf{P}}\left(\max_{j \in \mathcal{M}} \mathsf{RC}^j\right)$. By definition of Resource Cost, we may explicitly write,

$$\mathsf{SC}\left(\mathbf{w}, \mathbf{P}\right) = \sum_{\langle l_1, l_2, \ldots, l_n \rangle \in \mathcal{M}^n} \prod_{i=1}^{n} p_i^{l_i} \max_{j \in \mathcal{M}} \left\{ \frac{\sum_{k \in \mathcal{N} : l_k = j} w_k}{|\{k \in \mathcal{N} : l_k = j\}|} \right\} .$$

On the other hand, the *Optimum* associated with a demand vector $\mathbf{w}$, denoted $\mathsf{OPT}(\mathbf{w})$, is the *least possible,* over all pure assignments, maximum Resource Cost; thus, $\mathsf{OPT}(\mathbf{w}) = \min_{\mathbf{L} \in \mathcal{M}^n} \max_{j \in \mathcal{M}} \mathsf{RC}^j(\mathbf{w}, \mathbf{L})$, and explicitly,

$$\mathsf{OPT}\left(\mathbf{w}\right) = \min_{\langle l_1, l_2, \ldots, l_n \rangle \in \mathcal{M}^n} \max_{j \in \mathcal{M}} \left\{ \frac{\sum_{k \in \mathcal{N} : l_k = j} w_k}{|\{k \in \mathcal{N} : l_k = j\}|} \right\} .$$

Proposition 1. *For any demand vector* $\mathbf{w}$, $\mathsf{OPT}(\mathbf{w}) = \frac{W}{n}$.

Proof. Fix any demand vector $\mathbf{w}$. Clearly, the pure assignment σ where all agents are assigned to the same resource achieves Social Cost $\frac{W}{n}$. Since σ is no better than the optimal assignment, it follows that $\mathsf{OPT}(\mathbf{w}) \leq \frac{W}{n}$. So, it only remains to prove that $\mathsf{OPT}(\mathbf{w}) \geq \frac{W}{n}$.

Consider any arbitrary assignment α. Let resource $l \in [m]$ be such that $\mathsf{SC}(\mathbf{w}, \alpha) = \frac{W^l(\alpha)}{n^l(\alpha)}$. By definition of Social Cost, it follows that for any resource $j \in [m]$ such that $n^j > 0$, $\frac{W^l(\alpha)}{n^l(\alpha)} \geq \frac{W^j(\alpha)}{n^j(\alpha)}$, or $\frac{n^j(\alpha)}{n^l(\alpha)} \geq \frac{W^j(\alpha)}{W^l(\alpha)}$. Summing up over all such resources $j \in [m]$ yields that $\sum_{j\in[m]:n^j>0} \frac{n^j(\alpha)}{n^l(\alpha)} \geq \sum_{j\in[m]:n^j>0} \frac{W^j(\alpha)}{W^l(\alpha)}$, or that $\frac{\sum_{j\in[m]:n^j>0} n^j(\alpha)}{n^l(\alpha)} \geq \frac{\sum_{j\in[m]:n^j>0} W^j(\alpha)}{W^l(\alpha)}$. Hence, it follows that $\frac{W^l(\alpha)}{n^l(\alpha)} \geq \frac{W}{n}$. By choice of resource l, this implies that $\mathsf{SC}(\mathbf{w}, \alpha) \geq \frac{W}{n}$. Since α was chosen arbitrarily, it follows that $\min_\alpha \mathsf{SC}(\mathbf{w}, \alpha) \geq \frac{W}{n}$ or $\mathsf{OPT}(\mathbf{w}) \geq \frac{W}{n}$, as needed to complete the proof. □

Price of Anarchy and Diffuse Price of Anarchy. The *Price of Anarchy* (also referred to as *Coordination Ratio* [9]), denoted $\mathcal{PA}$, is the maximum value, over all demand vectors $\mathbf{w}$ and (mixed) Nash equilibria $\mathbf{P}$, of the ratio $\frac{\mathsf{SC}(\mathbf{w},\mathbf{P})}{\mathsf{OPT}(\mathbf{w})}$. Proposition 1 immediately implies that

$$\mathcal{PA} = \max_{\mathbf{w},\mathbf{P}} \left(\frac{n}{W} \cdot \mathsf{SC}(\mathbf{w}, \mathbf{P}) \right) \ .$$

The *Diffuse Price of Anarchy* for the class Δ is given by

$$\mathcal{DPA}_\Delta = \max_{D\in\Delta} \left(\mathcal{E}_D \left(\max_{\mathbf{P}} \frac{\mathsf{SC}(\mathbf{w},\mathbf{P})}{\mathsf{OPT}(\mathbf{w})} \right) \right) \ .$$

Each fixed but arbitrary demand vector $\mathbf{w}$ induces a candidate value for the Price of Anarchy (corresponding to the worst-case Nash equilibrium associated with $\mathbf{w}$), which is a function of the particular demand vector. Now, each fixed but arbitrary probability distribution D induces an expectation on this value, which is a function of the particular probability distribution D. Finally, the maximum of these expectations, over all possible probability distributions, is the Diffuse Price of Anarchy. Proposition 1 immediately implies that

$$\mathcal{DPA}_\Delta = \max_{D\in\Delta} \left(\mathcal{E}_D \left(\frac{n}{W} \left(\max_{\mathbf{P}} \mathsf{SC}(\mathbf{w},\mathbf{P}) \right) \right) \right) \ .$$

3 Pure Nash Equilibria

We prove:

Theorem 1 (Inexistence of Pure Nash Equilibria). *There is a pure Nash equilibrium if and only if all demands are identical.*

Proof. Assume first that all demands are equal to w, i.e. all demands are identical. Then, in any pure Nash assignment, the Resource Cost on each resource j such that $n^j > 0$ is equal to w, which implies that all Individual Costs are also equal to w. Hence, every pure assignment is a Nash equilibrium.

Assume now that there is a pure Nash equilibrium $\mathbf{L}$. For each resource $j \in \mathcal{M}$, denote $w_1^j, \ldots, w_{n^j}^j$ the demands assigned to resource j. So, $\sum_{1 \leq k \leq n^j} w_k^j = W^j$. Fix now a resource $j \in \mathcal{M}$ with $n^j > 0$. Since $\mathbf{L}$ is a Nash equilibrium, for each agent $k \in \{1, 2, \ldots, n^j\}$ assigned to resource j, and for each resource $l \in \mathcal{M}$, $l \neq j$, it holds that $\mathsf{IC}_k^j \leq \mathsf{IC}_k^l$ or $\dfrac{W^j}{n^j} \leq \dfrac{W^l + w_k^j}{n^l + 1}$. Summing up over all such agents k yields that $W^j \leq \dfrac{n^j W^l}{n^l + 1} + \dfrac{W^j}{n^l + 1}$. Rearranging terms yields that $n^l W^j \leq n^j W^l$. This implies that for any pair of resources $j, l \in \mathcal{M}$ with $n^j, n^l > 0$, $\dfrac{W^j}{n^j} = \dfrac{W^l}{n^l}$.

Note now that for each agent $k \in \{1, 2, \ldots, n^j\}$, $\dfrac{w_k^j}{n^l + 1} \geq \dfrac{W^j}{n^j} - \dfrac{W^l}{n^l + 1}$. We consider the implications of this inequality in two possible cases.

- Assume first that $n^l = 0$ (in which case $W^l = 0$ as well). Then, $w_k^j \geq \frac{W^j}{n^j}$.
- Assume now that $n^l > 0$. In this case, recall that $\frac{W^j}{n^j} = \frac{W^l}{n^l}$. So, the inequality implies that $\frac{w_k^j}{n^l+1} \geq \frac{W^l}{n^l} - \frac{W^l}{n^l+1} = \frac{W^l}{n^l(n^l+1)}$, implying that $w_k^j \geq \frac{W^l}{n^l} = \frac{W^j}{n^j}$.

So, in all cases, $w_k^j \geq \frac{W^j}{n^j}$ for all $k \in \{1, 2, \ldots, n^j\}$. This implies that $w_1^j = w_2^j = \ldots = w_{n^j}^j = \frac{W^j}{n^j}$. Since, however, $\frac{W^j}{n^j} = \frac{W^l}{n^l}$ for any pair of resources $j, l \in \mathcal{M}$ with $n^j, n^l > 0$, it follows that all demands are identical, as needed. □

4 Fully Mixed Nash Equilibria

We prove:

Theorem 2 (Existence of Fully Mixed Nash Equilibrium). *There is always a fully mixed Nash equilibrium.*

Sketch of Proof. Fix any demand vector $\mathbf{w}$. Consider the fully mixed assignment $\mathbf{F}$, with $f_i^j = \frac{1}{m}$ for each pair of an agent $i \in \mathcal{N}$ and a resource $j \in \mathcal{M}$. We calculate the Conditional Expected Individual Cost of agent i on resource j. There are two cases.

- Assume first that no agent other than i selects resource j. This occurs with probability $(1 - \frac{1}{m})^{n-1}$, and it contributes $w_1 \, (1 - \frac{1}{m})^{n-1}$ to IC_i^j.
- Fix now any integer k, where $2 \leq k \leq n$, and assume that $k-1$ agents other than i select the resource j. This occurs with probability $(\frac{1}{m})^{k-1}(1 - \frac{1}{m})^{n-k}$. There are $\binom{n-1}{k-1}$ pure assignments where exactly $k-1$ agents (besides i) select resource j, and each agent $t \neq i$ selects resource j in exactly $\binom{n-2}{k-2}$ of these assignments. Thus, the total contribution of all such assignments to IC_i^j is $\frac{1}{k} \cdot (\frac{1}{m})^{k-1}(1 - \frac{1}{m})^{n-k} \cdot \left(\binom{n-1}{k-1} w_i + \binom{n-2}{k-2} W_{-i}\right)$, where $W_{-i} = \sum_{t \in \mathcal{N} : t \neq i} w_t$.

Hence, it follows that

$$\mathsf{IC}_i^j = w_i \left(1 - \frac{1}{m}\right)^{n-1} + \sum_{k=2}^{n} \frac{1}{k} \left(\frac{1}{m}\right)^{k-1} \left(1 - \frac{1}{m}\right)^{n-k} \left(\binom{n-1}{k-1} w_i + \binom{n-2}{k-2} W_{-i} \right) .$$

Since IC_i^j is independent of j, it follows that $\mathbf{F}$ is a fully mixed Nash equilibrium, as needed. □

Call $\mathbf{F}$, with $f_i^j = \frac{1}{m}$ for each pair of an agent $i \in \mathcal{N}$ and resource $j \in \mathcal{M}$ from the proof of Theorem 2 the *standard fully mixed Nash equilibrium*. We next present a combinatorial proof of a uniqueness property for the standard fully mixed Nash equilibrium in the case of two agents.

Theorem 3 (Uniqueness of Standard Fully Mixed Nash Equilibrium for $n = 2$). *The standard fully mixed Nash equilibrium is the unique Nash equilibrium in the case of $n = 2$ agents with nonidentical demands.*

Sketch of Proof. Fix any demand vector $\mathbf{w}$. Consider any arbitrary Nash equilibrium $\mathbf{P}$. We will prove that (necessarily) $\mathbf{P} = \mathbf{F}$, the fully mixed Nash equilibrium from Theorem 2.

We first prove a simple fact, namely that the supports $\mathcal{S}_1$ and $\mathcal{S}_2$ of agents 1 and 2, respectively, intersect. By way of contradiction, assume otherwise; that is, assume that $\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$. Without loss of generality, take that $w_1 > w_2$. Consider any resource $\ell \in \mathcal{S}_2$. Clearly,

$$\mathsf{IC}_1 = w_1 > w_1 + \frac{w_2 - w_1}{2} p_2^\ell = w_1(1 - p_2^\ell) + \frac{w_1 + w_2}{2} p_2^\ell = \mathsf{IC}_1^\ell ,$$

a contradiction to the fact that $\mathbf{P}$ is a Nash equilibrium. So, take any resource $j \in \mathcal{S}_1 \cap \mathcal{S}_2$. Clearly,

$$\mathsf{IC}_1 = \mathsf{IC}_1^j = w_1(1 - p_2^j) + \frac{w_1 + w_2}{2} p_2^j = w_1 + \frac{w_2 - w_1}{2} p_2^j , \text{ and similarly,}$$

$$\mathsf{IC}_2 = w_2 + \frac{w_1 - w_2}{2} p_1^j .$$

Note that $\mathsf{IC}_1 < w_1$ and $\mathsf{IC}_2 > w_2$. We next argue that $\mathcal{S}_1 = \mathcal{S}_2 = \mathcal{M}$.

- Assume that there exists a resource $k \in \mathcal{S}_1 \setminus \mathcal{S}_2$. Then $\mathsf{IC}_1^k = w_1 > \mathsf{IC}_1$, a contradiction.
- Assume that there exists a resource $k \in \mathcal{S}_2 \setminus \mathcal{S}_1$. Then $\mathsf{IC}_2^k = w_2 < \mathsf{IC}_2$, a contradiction.

It follows that $\mathcal{S}_1 = \mathcal{S}_2$. Assume that there exists a resource $k \notin S_1$. Then, $\mathsf{IC}_2^k = w_2 < \mathsf{IC}_2$, a contradiction. It follows that $\mathcal{S}_1 = \mathcal{S}_2 = \mathcal{M}$.

Fix now any pair of resources $j, k \in \mathcal{S}_1 = \mathcal{M}$. Since $\mathbf{P}$ is a Nash equilibrium, $\mathsf{IC}_1^j = \mathsf{IC}_1^k$, or equivalently $w_1(1 - p_2^j) + \frac{w_1 + w_2}{2} p_2^j = w_1(1 - p_2^j) + \frac{w_1 + w_2}{2} p_2^j$. or $(w_2 - w_1)p_2^j = (w_2 - w_1)p_2^k$, or $p_2^j = p_2^k$. Since $\mathcal{S}_2 = \mathcal{M}$, it follows that $p_2^j = \frac{1}{m}$ for each resource $j \in \mathcal{M}$. Similarly, we can prove that $p_1^j = \frac{1}{m}$ for each resource $j \in \mathcal{M}$. So, $\mathbf{P} = \mathbf{F}$, as needed. □

The assumption of nonidentical demands in Theorem 3 is *necessary* since every assignment is a Nash equilibrium when the two demands are identical. Moreover, it does *not* hold in general that the standard fully mixed Nash equilibrium is the unique Nash equilibrium. Consider, for example, the case where $n = 3$ and $m = 2$ with $w_1 = w_2$; it is easy to see that the mixed assignment in which agent 1 (resp., agent 2) is assigned to resource 1 (resp., resource 2), while agent 3 is assigned to each resource with probability $\frac{1}{2}$ is a Nash equilibrium (other than the standard fully mixed). We conjecture, however, that the standard fully mixed Nash equilibrium is *always* the unique fully mixed Nash equilibrium.

5 The Price of Anarchy

5.1 Lower Bound

We prove:

Theorem 4. $\mathcal{PA} \geq \frac{n}{2e}$.

Sketch of Proof. Fix any demand vector $\mathbf{w}$. Consider the fully mixed Nash equilibrium $\mathbf{F}$ from Theorem 2 where each pure assignment occurs with the same probability $(\frac{1}{m})^n$. Note that there are $m(m-1)^{n-1}$ pure assignments in which the agent with maximum demand, say w_1, is the unique agent assigned to the resource she selects. In these pure assignments, the resource chosen by agent 1 is the resource with maximum Resource Cost. Thus,

$$\mathsf{SC}(\mathbf{w}, \mathbf{F}) \geq \left(\frac{1}{m}\right)^n \left(m(m-1)^{n-1} w_1\right) = \left(\frac{m-1}{m}\right)^{n-1} w_1 \ .$$

Fix now the demand vector $\mathbf{w}$ with $w_1 = \Theta(2^n)$ and $w_i = 1$ for all agents $i \neq 1$. Then, clearly, $\frac{w_1}{W} \geq \frac{1}{2}$. Proposition 1 implies that $\mathsf{OPT}(\mathbf{w}) = \frac{W}{n}$. Hence, $\mathcal{PA} \geq \frac{n w_1}{W} \left(\frac{m-1}{m}\right)^{n-1} \geq \frac{n}{2} \left(\frac{m-1}{m}\right)^{n-1} \geq \frac{n}{2e}$ for $m = n$, as needed. □

5.2 Upper Bounds

We first prove an upper bound for the special case of 2 agents.

Theorem 5. *Assume that* $n = 2$. *Then,* $\mathcal{PA} < 2 - \frac{1}{m}$.

Sketch of Proof. Fix any demand vector $\mathbf{w}$. If $w_1 = w_2 = w$, then any (pure or mixed) assignment has Social Cost w, which is equal to Optimum. In particular, any Nash equilibrium does so, which implies that $\mathcal{PA} = 1$. So take that $w_1 > w_2$, and consider the fully mixed Nash equilibrium $\mathbf{F}$ from Theorem 2, which, by Proposition 3, is the unique Nash equilibrium in this case.

Note that each pure assignment occurs with the same probability $\left(\frac{1}{m}\right)^2$. Among all m^2 pure assignments, there are $m(m-1)$ pure assignments for which the maximum Resource Cost is w_1 (occurring when the two demands are put on different resources), and m pure assignments for which the maximum Resource Cost is $\frac{w_1+w_2}{2}$ (occurring when both demands are put on the same resource). So, $\mathsf{SC}(\mathbf{w},\mathbf{F}) = \left(\frac{1}{m}\right)^2 \left(m(m-1)w_1 + m\frac{w_1+w_2}{2}\right) = w_1\left(1-\frac{1}{m}\right) + \frac{w_1+w_2}{2}\frac{1}{m}$. Since $\mathsf{OPT}(\mathbf{w}) = \frac{w_1+w_2}{2}$, it follows that $\mathcal{PA} = \frac{2w_1\left(1-\frac{1}{m}\right)}{w_1+w_2} + \frac{1}{m} < 2 - \frac{2}{m} + \frac{1}{m} = 2 - \frac{1}{m}$, as needed. □

We now proceed to the general case of $n \geq 2$ agents. We prove:

Theorem 6. $\mathcal{PA} \leq \frac{w_1}{W} \cdot n$.

Sketch of Proof. Fix any demand vector $\mathbf{w}$. Note that for any pure assignment and any resource $j \in \mathcal{M}$ such that $n^j > 0$, $\frac{W^j}{n^j} \leq w_1$. So, for any Nash equilibrium $\mathbf{P}$, $\mathsf{SC}(\mathbf{w},\mathbf{P}) = \mathcal{E}_{\mathbf{P}}\left(\max_{j\in\mathcal{M}} \frac{W^j}{n^j}\right) \leq w_1$. So the Price of Anarchy is $\mathcal{PA} \leq \frac{w_1}{\frac{W}{n}} = \frac{w_1}{W} n$, as needed. □

Theorem 6 immediately implies:

Corollary 1. *Assume that for all agents $k \in \mathcal{N}$, $w_k \leq c \cdot \min_i w_i$, for some constant $c > 0$. Then, $\mathcal{PA} \leq c$.*

6 The Diffuse Price of Anarchy

We prove an upper bound on the Diffuse Price of Anarchy for a special case of the class Δ of probability distributions for the demands. We start by defining this special class.

Definition 1 (Bounded, Independent Probability Distributions). *The class of* bounded, independent probability distributions Δ *includes all probability distributions D for which the demands w_i, $i \in \mathcal{N}$, are independent, identically distributed random variables such that:*

- *There is some parameter $\delta_D(n) < \infty$ such that $w_i \in [0, \delta_D(n)]$ for each $i \in \mathcal{N}$.*
- *There is some (universal) constant $\ell_\Delta > 0$ such that $\dfrac{\delta_D(n)}{\mathcal{E}_D(w_i)} \leq \ell_\Delta$ for each $i \in \mathcal{N}$.*

In our proof, we will use the following general version of *Hoeffding bound* [7].

Proposition 2 (Hoeffding Bound [7]). *Let $X_1, \ldots, X_n$ be independent random variables with $a_k \leq X_k \leq b_k$, for suitable constants a_k and b_k, for each $k \geq 1$. Denote $\bar{X} = \frac{1}{n}\sum_{k=1}^n X_k$ and $\bar{\mu} = \mathcal{E}(\bar{X})$. Then, for any $t > 0$,*

$$\mathbf{Pr}\left\{\bar{X} - \bar{\mu} \leq -t\right\} \leq \exp\left(\frac{-2n^2t^2}{\sum_{k=1}^n (b_k - a_k)^2}\right) .$$

Setting $t = \varepsilon\mu$, where $0 < \varepsilon \ll 1$, and $a_k = 0$ and $b_k = \delta(n)$ for each $k \geq 1$ in Proposition 2 yields:

Corollary 2. *Let* $X_1, \ldots, X_n$ *be independent random variables with* $0 \leq X_k \leq \delta(n)$*, for some suitable* $\delta(n) > 0$*, for each* $k \geq 1$*. Denote* $\bar{X} = \frac{1}{n}\sum_{k=1}^{n} X_k$ *and* $\bar{\mu} = \mathcal{E}(\bar{X})$*. Then, for any* $t > 0$*,* $\mathbf{Pr}\left\{\bar{X} \leq (1-\varepsilon)\mu\right\} \leq \exp\left(\frac{-2n\varepsilon^2\mu^2}{\delta^2(n)}\right)$*.*

We are now ready to prove:

Theorem 7. *Consider the class* Δ *of bounded, independent probability distributions. Then:*

1. $\mathcal{DPA}_\Delta \leq \dfrac{\ell_\Delta}{1 - \ell_\Delta\sqrt{\frac{1}{2\ln n}}} + n\exp\left(-\dfrac{n}{\ln n}\right)$;
2. $\lim_{n\to\infty} \mathcal{DPA}_\Delta \leq \ell_\Delta$.

Sketch of Proof. Fix any probability distribution $D \in \Delta$. Fix any arbitrary demand vector $\mathbf{w}$ drawn according to D. By Theorem 6, $\mathcal{PA} \leq \frac{w_1}{\widehat{W}}$. We will analyze $\mathcal{E}_D\left(\frac{w_1}{\widehat{W}}\right)$. Since all demands are identically distributed, linearity of expectation implies that $\mathcal{E}_D\left(\widehat{W}\right) = \mathcal{E}_D\left(w_1\right)$. So, for any $\varepsilon > 0$,

$$\begin{aligned}
\mathcal{E}_D\left(\frac{w_1}{\widehat{W}}\right) &\leq \max_{\widehat{W} \geq (1-\varepsilon)\mathcal{E}_D(\widehat{W})} \frac{w_1}{\widehat{W}} \cdot \mathbf{Pr}\left\{\widehat{W} \geq (1-\varepsilon)\mathcal{E}_D(\widehat{W})\right\} \\
&\quad + \max_{\widehat{W} \leq (1-\varepsilon)\mathcal{E}_D(\widehat{W})} \frac{w_1}{\widehat{W}} \cdot \mathbf{Pr}\left\{\widehat{W} \leq (1-\varepsilon)\mathcal{E}_D(\widehat{W})\right\} \\
&\leq \frac{\delta_D(n)}{(1-\varepsilon)\mathcal{E}_D(\widehat{W})} \cdot 1 + n \cdot \exp\left(\frac{-2n\varepsilon^2\mathcal{E}_D^2(\widehat{W})}{\delta^2(n)}\right) \\
&\quad \text{(since } w_1 \leq \delta_D(n) \text{ and by Corollary 2)} \\
&\leq \frac{\delta_D(n)}{(1-\varepsilon)\mathcal{E}_D(w_1)} \cdot 1 + n \cdot \exp\left(\frac{-2n\varepsilon^2\mathcal{E}_D^2(w_1)}{\delta^2(n)}\right) \\
&\leq \frac{\ell_\Delta}{1-\varepsilon} \cdot 1 + n \cdot \exp\left(\frac{-2n\varepsilon^2}{\ell_\Delta^2}\right) \\
&= \frac{\ell_\Delta}{1 - \ell_\Delta\sqrt{\frac{1}{2\ln n}}} \cdot 1 + n \cdot \exp\left(-\frac{n}{\ln n}\right) \\
&\quad \left(\text{setting } \varepsilon = \ell_\Delta\sqrt{\frac{1}{\ln n}}\right) ,
\end{aligned}$$

so that $\lim_{n\to\infty} \mathcal{E}_D\left(\frac{w_1}{\widehat{W}}\right) \leq \ell_\Delta$. Since $\mathcal{DPA}_\Delta = \max_{D\in\Delta}\left(\mathcal{E}_D(\mathcal{PA})\right)$ and D was chosen arbitrarily, both claims follow. □

A special subclass of the class Δ of bounded, independent probability distributions is the class $\Delta_{sym} \subseteq \Delta$ of *bounded, independent, expectation-symmetric probability distributions*. For each distribution $D \in \Delta_{sym}$, each demand w_i is distributed symmetrically around its expectation; this happens, for example, when each demand is uniformly distributed in the interval $[0, \delta_D(n)]$. In this case, for each demand w_i, $i \in \mathcal{N}$, $\mathcal{E}_D(w_i) = \frac{\delta_D(n)}{2}$. So, in this case, $\ell_{\Delta_{sym}} = 2$, and Theorem 7(2) implies:

Corollary 3. *Consider the class Δ_{sym} of bounded, independent, expectation-symmetric probability distributions. Then,* $\lim_{n\to\infty} \mathcal{DPA}_{\Delta_{sym}} \leq 2$.

7 Discussion and Directions for Future Research

We proposed here a very intuitive and pragmatic cost mechanism for pricing the competitive usage of resources shared by selfish agents. This mechanism is both distributed and fair. We presented results for the (pure and mixed) Nash equilibria of the induced strategic game. We also presented bounds for both the Price of Anarchy (considering worst-case demands) and the Diffuse Price of Anarchy (assuming that demands of agents are drawn according to some probability distribution from some wide class).

Our Fair Pricing model provides a concrete first step toward a systematic way of treating such cost mechanisms for pricing the competitive usage of multiple resources. We are currently examining both more general pricing functions and heterogeneous cases of selfish agents. We believe that our proof techniques will be instrumental to obtaining corresponding results for related models and problems. We also believe that our proposed Diffuse Price of Anarchy is of general applicability in congestion games with players' demands drawn according to some known probability distribution. In particular, what are bounds on the Diffuse Price of Anarchy for the Average Cost Pricing model [1] and the Serial Cost Sharing model [14, 15]?

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A Delay Pricing Scheme for Real-Time Delivery in Deadline-Based Networks

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Abstract. We introduce a novel delay pricing and charging scheme in deadline-based networks to support real-time data delivery. We study the delay performance observed by individual users when different levels of deadline urgency are specified and take a market-based approach to determining a delay price. Our pricing and charging scheme can be used to prevent greedy users from gaining an advantage by specifying arbitrarily urgent deadlines, and can also aid in network load control when equipped with user budget constraints.

1 Introduction

Current and future computer networks are expected to accommodate an increasing number of real-time applications. These applications may require timely delivery of real-time data. Example real-time data include stock quote updates, bids in an online auction, state update messages in distributed multi-player interactive games, audio and video data in video conferences, and voice data in IP telephony. To ensure timely delivery of real-time data, quality of service (QoS) support at the transport network is required.

Deadline-based network resource management [12, 6] is a framework that has been developed to support real-time data delivery. In this framework, the notion of application data unit (ADU) is used. An ADU may correspond to a file or a frame in audio or video transport. Each real-time ADU is associated with a delivery deadline, which is provided by the sending application. It represents the time at which the ADU should be delivered at the receiver. The ADU deadlines are mapped to packet deadlines at the network layer, which are carried by packets and used by routers for channel scheduling. Deadline-based channel scheduling algorithms are employed inside networks; packets with more urgent deadlines are transmitted first. It has been shown that deadline-based scheduling achieves superior performance to FCFS (First-Come First-Served) with respect to the percentage of ADUs that are delivered on time [7, 6].

In deadline-based scheduling, the delay performance experienced by real-time packets is largely affected by the deadline information that they carry, which depends on the ADU deadlines provided by sending applications. If one is free to specify the ADU deadline, a sender may try to gain an advantage by using arbitrarily tight deadlines. This raises the issue of fairness as seen by network

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users. In this paper, we first study the impact of deadline urgency on the delay performance experienced by real-time data. Our experiments show that by specifying more urgent deadlines, a user can receive better service in terms of the end-to-end response time. Therefore without a control mechanism in place, a greedy user may obtain good service quality by specifying very tight deadlines. Besides deadline urgency, the delay performance in deadline-based networks is also affected by the load conditions along the ADU path. When the load is light, the delay performance is good. When the load is heavy, congestion may occur; queues at bottleneck links may grow significantly, the delay performance deteriorates.

To prevent greedy users from specifying arbitrarily urgent deadlines, and to control the level of load in order to maintain good delay performance, we develop a novel delay pricing and charging scheme that takes into account both deadline urgency and network load conditions. At each network channel, a *delay price* is periodically computed based on the traffic deadline urgency and the traffic load so that (i) the higher the level of deadline urgency, the higher the price, and (ii) the heavier the network load, the higher the price. Each passing-by packet is charged based on the delay it experiences at this channel and the current channel price: the lower the delay it experiences, the higher the charge; the higher the current channel price, the higher the charge. This charge is carried by the packet and is accumulated along the entire packet path. Depends on the size of network maximum transfer unit (MTU), an ADU may be fragmented into multiple packets for transmission. If an ADU is delivered to the receiver on-time, the ADU is charged based on the packet charges of all its packets. In determining the channel price, a market-based approach from the field of microeconomics is taken. At each channel, the demand is derived from the deadline information carried by real-time packets, and the supply reflects the amount of time that is needed to service these packets. Such a delay pricing scheme encourages users to submit deadline requirements that best match their needs and capacity. Given limited user budget for network transmissions, such a delay pricing and charging scheme may aid in the process of load control so that performance degradation due to congestion can be alleviated. We present our pricing and charging scheme, and evaluate its performance by simulation.

This paper is organized as follows. In Section 2, the delay performance in deadline-based networks when there are no pricing and charging schemes in place is studied. The delay pricing and charging scheme that we developed is presented in Section 3. Simulation results on its performance are reported in Section 4. In Section 5, we review the related literature. Finally, in Section 6, we conclude our work and suggest some topics for future research.

2 Deadline-Based Data Delivery

In this section, we study the impact of ADU deadlines on the delay performance experienced. In previous studies on deadline-based networks, only the aggregated performance of all traffic that is transmitted over the network was studied. In this work, we study the performance observed by individual ADUs and individual users. The different delay performance obtained will incur different cost to senders after we introduce our pricing and charging scheme.

We first describe our performance model. At a sender, each generated ADU is characterized by: size, source and destination addresses, deadline, and arrival time. For simplicity, only real-time ADUs are considered. The support to best-effort traffic will be discussed in Section 3.3. Segmentation of an ADU into packets is performed at the sender before the packets are admitted to the network. The maximum packet size at the network layer is 1500 bytes. Packets are routed through the network until they reach their destination node. They are then delivered to the receiver where packet re-assembly is performed. We assume that fixed shortest-path routing is used and there are no transmission errors. For simplicity, the processing times at the sender and the receiver are not included in our model, and each packet carries the deadline of the ADU to which it belongs.

The deadline-based channel scheduling algorithm implemented is the T/H-$p(m)$ algorithm [6]. T stands for the time left (or packet deadline - current time) and H is the number of remaining hops to destination. The value T/H is calculated when a packet arrives at a router, it can be viewed as the urgency of a packet; specifically, a packet with a smaller T/H means that it is more urgent. At each scheduler, there are m queues, namely Q1, Q2, ..., Qm for real-time traffic and one queue for best-effort traffic. The T/H values of packets in Q1 are the smallest among all queues, followed by Q2 which has the next smallest, then Q3, Q4, until Qm. The fraction of real-time packets that are sent to Qi is $1/m$, $i = 1..m$. Head-of-the-line priority is used to serve packets in these queues, including the best-effort queue, which has lower priority than Qm. Let T_f denote the sum of the packet transmission time and the propagation delay on the current channel. If a real-time packet is already late (T $< T_f$) upon arrival at a router, the packet is downgraded to best-effort. In our experiments, m of 4 is used.

For a real-time ADU, the delivery deadline is modeled as follows. Let x be the end-to-end latency when there is no queueing and no segmentation. Also let x_p be the end-to-end propagation delay, y the size of the ADU, and c_j the capacity of the j-th channel along the path based on shortest-path routing. Then x can be estimated by $x = x_p + \sum_j y/c_j$. The allowable delay is assumed to be proportional to x. Hence, the delivery deadline for the ADU is given by $d = arrival\ time + kx$, where k is referred to as a *deadline parameter* ($k > 1$). In general, a smaller k means that the ADU has a more urgent deadline.

A 13-node network model is used in our simulation. Its topology is depicted in Figure 1. The capacity of each channel is assumed to be 155 Mbit/sec. The value shown on each link is the distance in miles. This is used to determine the propagation delay. For each arriving ADU, the source and destination nodes are selected at random. The ADU interarrival time is assumed to be exponentially distributed, and the average ADU arrival rate is λ (in number of ADUs per second). The size of each ADU is assumed to belong to one of two ranges: [500, 1500], and [1500, 500000], in bytes. The first range reflects the sizes of small

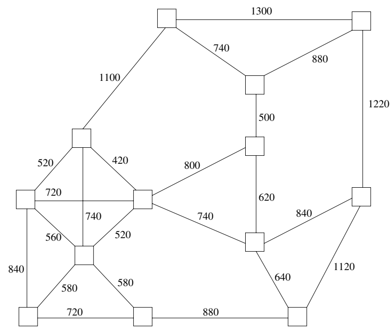

Fig. 1. Network model

ADUs, *i.e.,* one packet per ADU. The proportion of small ADUs is kept at 25%. ADU size is assumed to be uniformly distributed within each of these two ranges. At each scheduler, we assume that a finite buffer is in place. Modern routers usually have large buffer sizes that can accommodate between 100 and 200 ms' worth of data with respect to link capacity [2]. We will consider buffer sizes within this range. For simplicity, we assume that packets dropped due to buffer overflow are not re-transmitted. The performance measure of interest is the end-to-end response time.

Two experiments were carried out. The first experiment is to compare the performance of two benchmark ADUs that have the same size, arrival time, sender and receiver, but have different deadlines. The following model parameters are used for background traffic in this experiment. We choose 1200 ADUs/sec as the ADU arrival rate λ. This corresponds to 90% utilization on the bottleneck. At this level, the delay difference between two ADUs with different deadline urgency is larger, thus is more obvious, than at a lower load level. The deadline parameter k was assumed to be $1 + \tilde{e}$ where $\tilde{e}$ is exponentially distributed with mean 0.4. With this model, a variety of deadline urgency can be represented. For each outgoing channel, the total buffer size was assumed to be 3.1MByte. This corresponds to 160 ms' worth of data with respect to the link capacity. The sizes of the two benchmark ADUs are 5000 bytes. The end-to-end latency x in this case is 33 ms. Let end-to-end deadline denote the time period between when an ADU is submitted by a sending application for transmission and the ADU deadline. The end-to-end deadlines for the two ADUs are chosen to be 100 and 500 ms respectively. Thus one deadline is more urgent than the other. The response times obtained by these two ADUs are shown in Table 1. Ignore the

Table 1. Two ADUs with different deadlines

ADU	End-to-end deadline (ms)	Response time (ms)	ADU charge (cu)
1	100	39.57	1.50
2	500	120.84	0.14

Table 2. A flow when with different levels of deadline urgency

Deadline parameter	Mean response time (ms)	Mean ADU charge (cu)
$k = 1.4$	80.3	353.52
$k = 1.5$	86.4	306.30

last column for now, it can be observed that the ADU with the more urgent deadline has lower response time.

In the second experiment, we compare the performance of a benchmark flow (flow 0) when its level of deadline urgency is varied. In our experiments, a flow denotes a sequence of ADUs that are sent between a given sender and a given receiver. All background traffic is the same as in the first experiment, except that the value of $\tilde{e}$ is 0.5 for the deadline parameter k. The deadline parameter for Flow 0's ADUs is varied from 1.4 to 1.5, thus we can compare the delay performance when flow 0's deadline is more urgent than the average deadline of background traffic with the delay performance when flow 0's deadline has the same urgency as the average deadline of background traffic. Flow 0's mean response time performance is shown in Table 2. It can be observed that when with identical background traffic, flow 0 is able to achieve lower average response time when its ADU deadlines are more urgent. We conclude that in deadline-based networks, the delay performance largely depends on the deadline urgency. When competing with the same background traffic, an ADU or a flow of ADUs can raise their service priority, thus obtaining better delay performance by using more urgent deadlines. An important objective of our pricing and charging scheme is to prevent such greedy behaviours.

3 Delay Pricing in Deadline-Based Networks

In this section, we present the channel delay pricing and the packet and ADU charging scheme that we have developed. Some implementation issues are then discussed.

3.1 A Market-Based Approach to Delay Pricing

In deadline-based networks, each packet carries a deadline, which specifies the requirement on its delay performance. At each hop, the T/H value calculated indicates the delay requirement of this packet on this channel; namely, if the response time at this hop is less than or equal to T/H, and if every hop along the packet path manages to achieve so, then the packet will arrive at the receiver on-time. From an economic point of view, the finite capacity and the transmission service at each channel is the scarce resource sought by real-time packets. The packet T/H values reflect the *demand* on the resource; and the time it takes to service a set of packets signifies the capability, *i.e.,* the *supply* available at the channel. The goal at each channel is to utilize a pricing mechanism to urge the adjustment of demand so that the difference between the supply and the demand can be kept minimal.

We take a market-based approach from the field of microeconomics, and determine a market price, called *channel delay price*, at each channel based on the relation between the demand and the supply. An iterative tatonnement process [11] is used. The channel delay price is updated every *price update interval.* During each update interval, for each departing packet, the following information is recorded and accumulated: (i) the packet T/H value, and (ii) the packet response time. The packet response time is defined as the sum of the queueing delay, the packet transmission time, and the channel propagation delay. Let D^T denote the total T/H value of all departing packets, and S^T be the total packet response time of all departing packets. At the end of the update interval n, the channel delay price p for the update interval $n+1$ is defined as:

$$p_{n+1} = \{p_n + \sigma * (D - S)/S, 0\}^+ \quad (1)$$

where $D = 1/D^T$, and $S = 1/S^T$. σ is an *adjustment factor*, which can be used to trigger faster or slower responses of the channel price to the amount $D-S$. At system initialization, p_0 is set to zero. In addition, only positive channel delay prices are defined.

It should be noted that the channel price is higher (i) when the deadline urgency is higher, *i.e.,* when D^T is lower; and (ii) when the load is heavier, *i.e.,* when S^T is higher. We assume that the channel prices can be made available to network users. In response to the changes of the channel delay price, adaptive users with budget constraints may adjust their requirements in terms of the deadline urgency and the offered load. The end result is that at the channel with the heaviest load, the resource demand can be driven towards the amount of supply, and at every other channel, the demand is no greater than the supply. Under these conditions, good service quality can be achieved.

3.2 Calculation of Packet and ADU Charges

Using the channel delay pricing scheme presented above, we describe a method to calculate packet and ADU charges. Note that in this work, we focus on devising a *delay charging* scheme that aims at two objectives: (1) to provide an incentive for users to submit requests with the QoS requirement that best matches their need, and (2) to control network load so that good delay performance can be maintained. In general, network charging schemes usually contain certain charges in order to assure the return on investment; these charges may cover the cost for constructing, maintaining, and upgrading the network. In this paper, however, we do not consider these charges and focus on the delay charge only.

A per-packet per-channel charging scheme is developed in our framework. At each channel, upon each packet departure, the packet response time d_a is calculated: the queueing delay can be obtained by subtracting the packet arrival time from the current time, the transmission delay can be computed using the packet size and the channel capacity, the propagation delay is fixed and given.

Let p be the current channel delay price. The packet charge g at this channel is defined as: $g = p/d_a$. Define a new packet header field called "accumulated charge". It keeps track of the total delay charge incurred by this packet at all channels along its path. If a packet arrives at the receiver on-time, the value of this field is retrieved and is taken as the packet charge. If an ADU is delivered on-time, its ADU charge is defined as the sum of all its packets' charges. Late packets and ADUs are not charged.

3.3 Network Layer Issues

In our pricing scheme, the channel delay prices are updated periodically at constant time intervals. This can be easily implemented using either hardware or software timer interrupts. In general, the length of the update interval should not be too short, this way a good number of T/H value and response time samples can be collected to estimate the current resource demand and supply. A length that is much longer than the average packet transmission time should be used. The update interval should not be too long either, in this way the short-term traffic conditions can be accounted for.

Our pricing and charging scheme introduces some processing overhead inside routers. This includes packet response time calculation, and accumulation of the T/H values and the packet response times for all departing packets. However, because none of these operations depends on the queue size, we consider this overhead to be in-expensive in terms of implementation. The computation of delay prices only occurs once every update interval, which is much longer than the mean packet transmission time, therefore is not considered costly either. The "accumulated charge" header field can also be easily added using packet header options or similar mechanisms.

The social fairness aspect of a pricing scheme is concerned with whether some users will be prevented from accessing the network only because of their inability to pay [3]. In our discussion so far, we have assumed that there is only real-time traffic in the network. In fact, best-effort traffic can easily be accommodated in our framework. All best-effort traffic can carry a deadline of infinity. At each outgoing channel inside the network, a certain amount of bandwidth can be allocated to best-effort traffic only. This can be implemented using a fair-queueing algorithm with two classes. The deadline-based scheduling is used only within the real-time class. The best-effort class can use a low flat-rate pricing and charging scheme. Those users who can not afford the delay charges of the real-time class can use the bandwidth that is allocated to the best-effort class.

4 Performance Evaluation

In this section, we evaluate the performance of our pricing and charging scheme by simulation. We used the performance model that is described in Section 2 and added our pricing and charging scheme implementation. The following values are chosen for algorithm parameters. The adjustment factor σ in Eq.(1) is set to 0.06. The price update interval is 2 seconds' long. The simulation is run for 50 seconds. Corresponding to the two objectives of our pricing and charging scheme, we discuss two cases: differential charges based on deadline urgency, and price-based load control.

4.1 Deadline Urgency Differential Charges

In section 2, we have shown that it is possible for a user to gain higher service priority in deadline-based networks by specifying very tight deadlines. Our experiments show that this holds for both individual ADUs and for a flow of ADUs. Our solution to prevent such greedy behaviors is to introduce an ADU delay charge. In Tables 1 and 2, the last columns indicate the ADU charges in a *charge unit* (cu) using our pricing and charging scheme. In this paper, we do not associate the charge unit with any concrete monetary value, and leave this choice to network operators. It can be observed that when all traffic attributes but the deadline urgency are the same, the more urgent ADUs are charged more using our scheme. The absolute charge values depend on channel prices and packet response times along the path. We conclude that our scheme can be used to enforce differential charges based on ADU deadline urgency.

4.2 Price-Based Load Control

Our pricing and charging scheme may also aid in load control. This can be accomplished through a *delay pricing agent*. This agent is located between the users and the network. When a user submit a real-time ADU for transmission, this agent may utilize the current price information along the ADU path and the ADU deadline requirement to provide a *charge estimate* for this ADU. If the network is heavily loaded, the delay prices inside networks would be high, which may result in a high charge estimate. In this case, a user may choose not to submit the ADU for transmission until the price drops. In our simulation, we used a simple elasticity model to represent such user adaptation behavior. We assume that each sender has a fixed amount of budget for every price update interval. When an ADU is generated for transmission, the total allowable end-to-end delay is equally allocated to each hop, and the current highest channel price along the ADU path is used to compute an estimated per-hop packet charge. Cumulating the total number of hops along the path and the total number of packets in this ADU, an estimated ADU charge is obtained. If a sender has enough available budget to cover this charge, then the ADU is sent and the budget is decremented by this charge. Otherwise the ADU is refrained from entering the network.

In Figure 2, we plot the price dynamics at one bottleneck inside the network when without and with the user budget constraint. There are four graphs in this figure. The two on the left are ones when there is no budget constraint. The two on the right are ones when there is limited user budget. In our simulation, the value of budget is assumed to be 150000. The four curves in each graph are for four load levels. It can be observed that regardless of load, when with limited user budget, the price can be regulated to a fairly steady value. This is

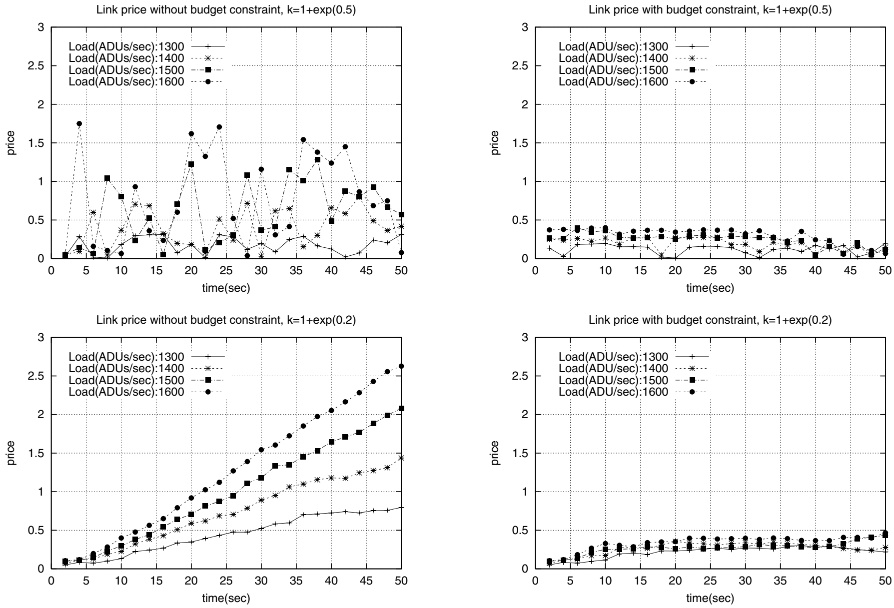

Fig. 2. Prices when without and with the budget constraint

because limited budget can limit the amount of load that enters the network. When demand and supply are approximately equal, the price becomes steady. This demonstrates the effectiveness of load control of our scheme when coupled with user budget constraints.

The difference between the top two and the bottom two graphs lies in the deadline urgency used. When deadline is less tight (see the top left graph where the mean deadline parameter is 1.5), although there is fluctuation, the demand is about equal to the supply, so the prices do not increase monotonically. When

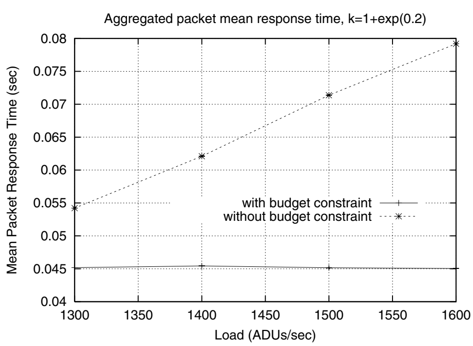

Fig. 3. Response time when with and without the budget constraint

deadline is tight (see the bottom left graph where the mean deadline parameter is 1.2), because there is no budget constraint to control the load, there is a clear mismatch between demand and supply. The ever increasing prices indicate that the supply does not keep up with the demand. Thus certain load control is needed.

Because of the effectiveness of load control of our scheme when with user budget constraints, the network delay performance can be significantly improved. In Figure 3, we plot the average response time when with and without the budget constraint. The average deadline parameter is 1.2. It can be observed that as the load increases, when without pricing and the budget constraint, the response time keeps increasing. When with pricing and the budget constraint, the response time can be kept very low regardless of the level of the offered load. We conclude that our pricing and charging scheme is effective in network load control when coupled with user budget constraints.

5 Related Work

Network pricing has been a popular subject of research. Except flat rate pricing, among dynamic pricing schemes, there are Paris Metro Pricing [9], priority-based pricing [1, 5], smart market pricing [8], competitive market pricing [4], DiffServ pricing and RNAP [11, 10]. Similar to some of above studies, in our study, we adopt a market-based approach to determining the channel price. However, differ from all above studies where pricing of bandwidth is concerned, in our work, we introduce the novel delay pricing concept. This is made available by the deadline-based framework in which each packet carries its delay requirement. The demand can easily be derived from this deadline information.

6 Conclusion

We have developed a novel delay pricing and charging scheme in deadline-based networks to support real-time data delivery. In our scheme, we make use of the concept of competitive market and determine a delay price based on delay demand and supply at each channel. Simulation results show that our scheme can incur different charges to users with different QoS requirements, and can aid in effective load control when user budget constraints are available. There are a number of interesting future work of this study, including the design of more sophisticated ADU charge estimation schemes, and the investigation of strategies to maximize the user utility.

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Game-Theoretic Analysis of Internet Switching with Selfish Users*

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Abstract. We consider the problem of Internet switching, where traffic is generated by selfish users. We study a packetized (TCP-like) traffic model, which is more realistic than the widely used fluid model. We assume that routers have First-In-First-Out (FIFO) buffers of bounded capacity managed by the drop-tail policy. The utility of each user depends on its transmission rate and the congestion level. Since selfish users try to maximize their own utility disregarding the system objectives, we study Nash equilibria that correspond to a steady state of the system. We quantify the degradation in the network performance called the price of anarchy resulting from such selfish behavior. We show that for a single bottleneck buffer, the price of anarchy is proportional to the number of users. Then we propose a simple modification of the Random Early Detection (RED) drop policy, which reduces the price of anarchy to a constant.

1 Introduction

If all Internet users voluntarily deploy a congestion-responsive transport protocol (e.g. TCP [19]), one can design this protocol so that the resulting network would achieve certain performance goals such as high utilization or low delay. However, with fast growth of the Internet users population, the assumption about cooperative behavior may not remain valid. Users are likely to behave "selfishly", that is each user makes decisions so as to optimize its own utility, without coordination with the other users. Buffer sharing and bandwidth allocation problems are prime candidates for such a selfish behavior.

If a user does not reduce its sending rate upon congestion detection, it can get a better share of the network bandwidth. On the other hand, all users suffer during congestion collapse, since the network delay and the packet loss increase drastically. Therefore, it is important to understand the nature of congestion resulting from selfish behavior. A natural framework to analyze this class of problems is that of non-cooperative games, and an appropriate solution concept is that of Nash equilibrium [24]. Strategies of the

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users are at a Nash equilibrium if no user can gain by unilaterally deviating from its current policy.

The subject of this paper is game-theoretic analysis of the Internet switching problem. We consider a bottleneck First-In-First-Out (FIFO) buffer shared by many users. The users compete for the buffer share in order to maximize their throughput, but they suffer from the created congestion. We assume that each user knows its buffer usage, the queue length and the buffer size. In reality, these parameters can be estimated only approximately (e.g. such a mechanism exists in TCP Vegas). However, in this work we rather study *lack of coordination* and not *lack of information*. The goal of a user is to maximize its utility. We assume that the utility function of a user increases when its goodput increases and decreases when the network congestion increases.

We assume that there are n users and the buffer capacity is B while $B \gg n$. In our model all packets have a constant size. Time is slotted. Every time step each user may or may not send a packet to the buffer.[1] These packets arrive in an arbitrary order and are processed by the buffer management policy one by one. Then the first packet in the FIFO order is transmitted on the output link.

The drop policy has to decide at each time step which of the packets to drop and which to accept. It can also preempt (drop) accepted packets from the buffer. Under the *drop-tail* policy, all arriving packets are accepted if the buffer is not full and dropped otherwise (when the buffer is full).

We denote by w_i^t the number of packets of user i in the buffer at the beginning of time step t. We call it the *buffer usage* of user i. We denote by $W^t = \sum_i w_i^t$ the *queue length* at time t. We also denote by W_{-i}^t the buffer usage of all users but user i, i.e., $W_{-i}^t = W^t - w_i^t$. We assume that the users are greedy, i.e., they always have data to send and the queue is never empty.

We denote by $r_i^t = w_i^t / W^t$ the *instant transmission rate* of user i at time t. We say that the system is in a *steady state* if the queue length remains constant. In a steady state, the *average transmission rate* of user i is $r_i = w_i/W$, where w_i is the average buffer usage of user i and W is the average queue length.

Now we proceed to define a utility function. Since in general the rate and the delay are incomparable values, it is natural to normalize one of these parameters. We introduce the notion of *congestion level*, that is the congestion level at time t is $L^t = \frac{W^t}{B}$. Note that the congestion level is zero when the buffer is empty and one when the buffer is full.

The utility of user i at time t is $u_i(w_i^t, W^t) = r_i^t \cdot (1 - L^t)$. We assume that user i sends a packet to the buffer at time t if its utility increases, i.e., $u_i(w_i^t + 1, W^t + 1) > u_i(w_i^t, W^t)$. In a steady state, the utility of user i is $u_i(w_i, W) = r_i \cdot (1 - L)$, where $L = W/B$. Observe that when the buffer is almost empty, the utility of each user approximately equals its transmission rate. On the other hand, when the buffer is nearly full, all users have utility close to zero. The latter situation can be viewed as "congestion collapse".

The *strategy* of each user is its buffer usage while the strategies of the other users define the background buffer backlog. Now we define a Nash equilibrium.

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[1] We note that our results can be extended to the case in which each user can send an arbitrary number of packets.

Definition 1. *The system is said to be in a* Nash equilibrium *if no user can benefit by changing its buffer usage.*

The total utility of the users in a Nash equilibrium is $\sum_{i=1}^{n} u_i(w_i, W)$. Observe that under an optimal centralized policy, all users have equal sending rates of $1/n$ and experience almost negligible delay, which results in the total utility close to 1. We define the *price of anarchy* in a Nash equilibrium to be $1/\sum_{i=1}^{n} u_i(w_i, W)$. We are also concerned with the fairness of a Nash equilibrium. A Nash equilibrium is said to be *fair* if all users have the same buffer usage.

A Nash equilibrium in a networking environment is interesting only if it can be reached efficiently (in polynomial time). We define the *convergence time* to a Nash equilibrium as the maximum number of time steps required to reach a Nash equilibrium starting from an arbitrary state of the buffer.

We demonstrate that the drop-tail buffering policy imposes a fair Nash equilibrium. However, the price of anarchy is proportional to the number of users. We also show that the system converges to a Nash equilibrium in polynomial time, namely after $O(B^2)$ time steps. Then we propose a simple modification of the Random Early Detection (RED) policy [13] called Preemptive RED (PRED) that achieves a constant price of anarchy. We note that PRED is in the spirit of CHOKe [26] (see Section 4).

Paper organization. The rest of the paper is organized as follows. In Section 2 we discuss the related work. Analysis of a single switch appears in Section 3. The PRED policy is presented in Section 4. We conclude with Section 5.

2 Related Work

Shenker [29] analyzes switch service disciplines with a $M/M/1$ model and Markovian arrival rates. The utility of each user is increasing in its rate and decreasing in the network congestion. Shenker shows that the traditional FIFO policy does not guarantee efficiency and fairness and proposes a policy called Fair Share that guarantees both of them.

Garg at al. [17] study a switching problem using a continuous fluid-flow based traffic model, which is amenable to analysis of an arbitrary network. The user's utility is an increasing function of its goodput only. Garg at al. show that selfish behavior leads to congestion collapse and propose a rate inverse scheduling service discipline under which a max-min fair rate allocation is a Nash equilibrium. Contrary to [17], we consider only a simple FIFO scheduling policy.

Unfortunately, the complexity of the policies proposed in [29, 17] is too high to implement them in the core of the Internet since they have to maintain per-flow state. Dutta et al. [9] analyze simple state-less buffer management policies under the assumption that the traffic sources are Poisson and the utility of each user is proportional to its goodput. Dutta et al. demonstrate that drop-tail and RED do not impose Nash equilibria and present a modification of RED that enforces an efficient Nash equilibrium. Differently from [9], we assume that the utility of each user depends on the network congestion as well.

In a recent paper, Gao et al. [16] propose efficient drop policy that in case of congestion drops packets of the highest-rate sender. Gao et al. show that if all sources are

Poisson, this policy results in a Nash equilibrium which is a max-min fair rates allocation. In addition, it is demonstrated that the throughput of a TCP source is a constant factor of its max-min-fairness value when competing with Poisson sources. In contrast to [16], we do not make any assumptions regarding the traffic pattern.

Unlike the works mentioned above, in this paper we study *packetized* traffic model in which sources do not control the sending rate explicitly, but rather make it on per-packet basis. This model is implicit in the TCP protocol.

Nash equilibria for network games have been extensively studied in the literature. Douligeris and Mazumdar [8] determine conditions for a Nash equilibrium for a $M/M/1$ system when the utility of a user is a function the throughput and the delay. Orda et al. [25] investigate uniqueness of Nash equilibria in communication networks with selfish users. Korilis and Lazar [21] study Nash equilibria of a non-cooperative flow control game. Congestion control schemes utilizing pricing based on explicit feedback are proposed by Kelly et al. [20]. Gibbens and Kelly [18] explore the implementation of network pricing in which users are charged for marks that routers place on packets in order to achieve congestion control. Akella et al. [1] present a game-theoretic analysis of TCP congestion control. Qiu et al. [27] consider selfish routing in intra-domain network environments.

Traditionally, in Computer Science research has been focused on finding a global optimum. With the emerging interest in computational issues in game theory, the *price of anarchy* introduced by Koutsoupias and Papadimitriou [22] has received considerable attention [6, 7, 14, 28, 11]. The price of anarchy is the ratio between the cost of the worst possible Nash equilibrium (the one with the maximum social cost) and the cost of the social optimum (an optimal solution with the minimum social cost). In some cases the price of anarchy is small, and thus good performance can be achieved even without a centralized control.

If a Nash equilibrium imposed by selfish users is not satisfiable, one can deploy resource allocation policies to improve the situation. Recently, Christodoulou at al. [5] introduced the notion of coordination mechanism, which is a resource allocation policy whose goal is to enforce a better Nash equilibrium. They show that even simple local coordination mechanisms can significantly reduce the price of anarchy.

Efficient convergence to a Nash equilibrium is especially important in the network environment, which is highly variable. The question of convergence to a Nash equilibrium has received significant attention in the game theory literature [15]. Altman at al. [2] and Boulogne at al. [4] analyze the convergence to a Nash equilibrium in the limit for a routing and a scheduling game, respectively. Even-Dar et al. [10] consider deterministic convergence time to a pure Nash equilibrium for a load balancing game (routing on parallel links).

3 Single Switch

In this section we consider a single switch. We first characterize Nash equilibria and derive the price of anarchy. Then we establish an upper bound on the convergence time. Finally, we study the Nash traffic model.

3.1 Price of Anarchy

The intuition explaining why congestion collapse does not happen is that at some point users start to suffer from the delay due to their own packets in the bottleneck buffer and would not further increase their sending rates. The next theorem shows the existence of a unique Nash equilibrium and derives the price of anarchy.

Theorem 1. *There exists a unique Nash equilibrium that has the price of anarchy of n and is fair.*

Proof. By the definition of a Nash equilibrium, no user can increase its utility by changing the buffer usage. Therefore, a Nash equilibrium is a local maximum point for the utility function of each user i. We have that the first derivative $(u_i(w_i, W))'$ must be equal to zero:

$$\frac{-w_i^2 - 2W_{-i}w_i - W_{-i}^2 + BW_{-i}}{B(w_i + W_{-i})^2} = 0,$$

and thus

$$w_i = \sqrt{BW_{-i}} - W_{-i}. \tag{1}$$

It is easy to verify that the second derivative $(u_i(w_i, W))''$ is negative: $\frac{-2W_{-i}}{B(w_i+W_{-i})^3} < 0$.

We obtain that a Nash equilibrium is fair since for each user i, $W = \sqrt{BW_{-i}}$, where W is constant. Summing Equation (1) for all users, we get: $W = B(n-1)/n$. Therefore, the unique Nash equilibrium is $w_i = B(n-1)/n^2$ for each user i. The price of anarchy in the Nash equilibrium is $1/\sum_{i=1}^{n}\left(\frac{1}{n}\cdot\frac{1}{n}\right) = n$. ∎

For simplicity, we assume that $B(n-1)/n^2$ is an integer. Otherwise, there would be an approximate Nash equilibrium in which the buffer usage of each user varies between $\lfloor B(n-1)/n^2 \rfloor$ and $\lceil B(n-1)/n^2 \rceil$ as our simulations indeed show.

3.2 Convergence

Now we analyze convergence to a Nash equilibrium. The next theorem demonstrates that the convergence time to a Nash equilibrium is proportional to the square of the buffer size.

Theorem 2. *The convergence time to a Nash equilibrium is $O(B^2)$.*

Proof. Let $w^* = B(n-1)/n^2$ be the buffer usage of each user in a Nash equilibrium. Observe that until a Nash equilibrium is reached, each time step either a user with the maximum buffer usage (greater than w^*) can benefit from decreasing its buffer usage or a user with the minimum buffer usage (smaller than w^*) can benefit from increasing its buffer usage. Note that a user can increase its buffer usage at any time if the buffer is not full, but can decrease its buffer usage only when one of its packets is transmitted out of the buffer.

Initially, we let the system run for B time steps so that the users would be able to fill the buffer. Note that at this point no user with the maximum buffer usage wishes to increase its buffer usage. In the sequel, the maximum buffer usage will only decrease and the minimum buffer usage will only increase until they become equal to w^*.

We divide time into *phases* of B time steps. Consider a phase that starts at time step t. We argue that if a Nash equilibrium is not reached by time $t + B$, then either the maximum buffer usage decreases or the minimum buffer usage increases. Otherwise, there would be a Nash equilibrium contradicting Theorem 1, which states that in a Nash equilibrium all users have the same buffer usage. Therefore, the system reaches a Nash equilibrium after B phases or in B^2 time steps. ∎

We also perform a convergence simulation. We consider different settings, where initially the buffer is filled randomly with packets of different users. The dynamics of the minimum and the maximum buffer usage for 8 and 16 users appears in Figure 1. The simulation results show that the actual convergence time is roughly proportional to the buffer size.

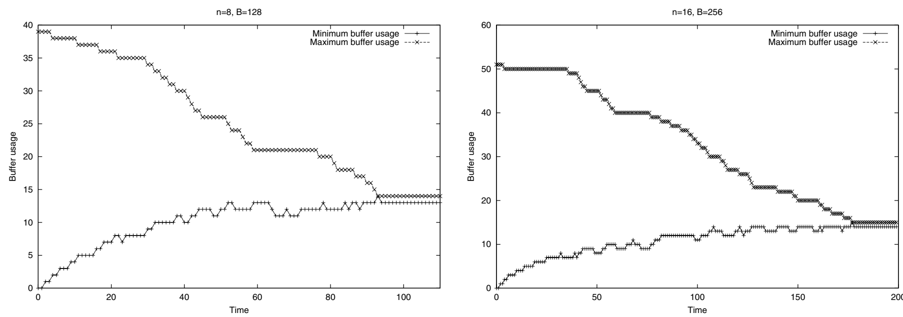

Fig. 1. Convergence to a Nash equilibrium

3.3 Nash Traffic Model

The traffic model of any QoS architecture is the regulation of the rate at which a user is allowed to inject packets into the network. There are two important policing criteria: *average rate* – limit of the long-term average rate at which packets can be injected into the network and *burst size* – the limit of the maximum number of packets that can be sent into the network over an extremely short time interval.

Definition 2. *The* (σ, ρ) leaky bucket *mechanism is an abstraction modeling a source with an average rate of* ρ *and burst size of at most* σ*. Therefore, at any time interval of length* l *there are at most* $\rho \cdot l + \sigma$ *packets entering the system.*

We show that the traffic of a user in a Nash equilibrium conforms to the leaky bucket model.

Theorem 3. *In a Nash equilibrium the traffic of each user conforms to the leaky bucket model with* $\sigma = B(n-1)/n^2$ *and* $\rho = 1/n$*.*

The theorem follows due to the fact that in order to maintain the buffer usage of $B(n-1)/n^2$, each user must send a burst of $B(n-1)/n^2$ packets and keep sending packets at the average rate of $1/n$.

4 PRED Policy

In this section we consider the problem of designing a coordination mechanism (drop policy) that will improve the price of anarchy. The main goal of a drop policy is to control the average queue length. This can be done by dropping arriving packets probabilistically when the average queue length exceeds some pre-defined threshold. Such a policy called Random Early Detection (RED) has been introduced by Floyd and Jacobson [13]. We will show that a simple modification of RED reduces the price of anarchy to a constant. Again, we assume that the users are aware of the drop policy.

The RED policy calculates the average queue length, using a low-pass filter with an exponential weighted moving average. The average queue length is compared to two thresholds, a minimum threshold and a maximum threshold. When the average queue length is less than the minimum threshold T_l, no packets are dropped. When the average queue length is greater than the maximum threshold T_h, every arriving packet is dropped. When the average queue length is between the minimum and the maximum threshold, each arriving packet is dropped with probability p, which is a function of the average queue length.

Pan et al. [26] devise a simple packet dropping scheme, called CHOKe, that discriminates against unresponsive or misbehaving flows aiming to approximate the fair queuing policy. When a packet arrives at a congested router, CHOKe picks up a packet at random from the buffer and compares it with the arriving packet. If they both belong to the same flow, then they are both dropped. Otherwise, the randomly chosen packet is left intact and the arriving packet is admitted into the buffer with the same probability as in RED.

Our goal is to ensure a small queue length. Unfortunately, CHOKe is not aggressive enough in penalizing misbehaving flows since the probability that the buffer usage of a non-responsive user decreases is inversely proportional to the queue length. We propose a modification of RED in the spirit of CHOKe, called Preemptive RED (PRED). The PRED policy is presented in Figure 2. The main feature of PRED is extra drop mechanism that drops an additional packet of the same user from the buffer when its packet is dropped by RED. Intuitively, we try to penalize users that do not respond to congestion signals. Note that if there is no penalty associated with dropped packets, users will greedily send new packets as long as they can benefit from increasing their buffer usage.

1. When a new packet arrives, apply the RED policy (**regular** drop).
2. If the packet is dropped by RED, preempt (drop) the earliest packet of the same user from the buffer, if any (**extra** drop).

Fig. 2. The Preemptive RED (PRED) drop policy

The next theorem analyzes Nash equilibria imposed by PRED.

Theorem 4. *Under the PRED policy, the price of anarchy in a Nash equilibrium is at most* $\frac{B}{B-T_l}$ *and there exists a fair Nash equilibrium.*

Proof. First we show how to select the drop probability function so that in a Nash equilibrium the queue length is bounded by T_l. Consider the expected utility of user i after sending a new packet when the queue length is $X \geq T_l$. The buffer usage of user i will increase and decrease by one with probability $1 - p$ and p, respectively. Therefore, the expected utility of user i is

$$(1 - p(X)) \cdot u_i^t(w_i + 1, X + 1) + p(X) \cdot u_i(w_i - 1, X - 1).$$

User i would refrain from sending additional packets if the expected utility is less than its current utility $u_i(w_i, X)$, and we get that $p(X)$ must be greater than

$$\frac{u_i(w_i + 1, X + 1) - u_i(w_i, X)}{u_i(w_i + 1, X + 1) - u_i(w_i - 1, X - 1)}. \qquad (2)$$

Suppose that the drop probability function satisfies inequality (2) for each user i and for each $w_i \geq T_l/n$ (e.g. $p \approx 1/2$ satisfies the above requirements).

If $T_l \geq B(n-1)/n$ then clearly there is a unique Nash equilibrium identical to that imposed by the drop-tail policy. If $T_l < B(n-1)/n$, then we argue that $w^* = T_l/n$ is a fair Nash equilibrium. We have that no user can benefit from increasing its buffer usage above T_l/n by the selection of the drop probability function and no user can benefit from decreasing its buffer usage below T_l/n since in the Nash equilibrium imposed by the drop-tail policy the buffer usage of each user is greater than T_l/n.

We also claim that in a Nash equilibrium the queue length is bounded by T_l. If it is not the case, at least one user has buffer usage greater than T_l/n. However, the selection of the drop probability function implies that its buffer usage will eventually drop to or below T_l/n, which contradicts to the stability of a Nash equilibrium. Therefore, the price of anarchy is at most $\frac{B}{B-T_l}$. ∎

Note that if the queue length exceeds T_l, new users with zero buffer usage would still keep sending new packets since they can only increase their utility. That allows to avoid Denial of Service (DoS) when the buffer is completely monopolized by old users.

We also present a simulation that illustrates the effect of PRED on the queue length. The setting is similar to that of Section 3. The dynamics of the queue length for 8 and 16 users is presented in Figure 3. It turns out that the queue length in a steady state is very close to T_l and has a small variance, as our analysis indeed shows.

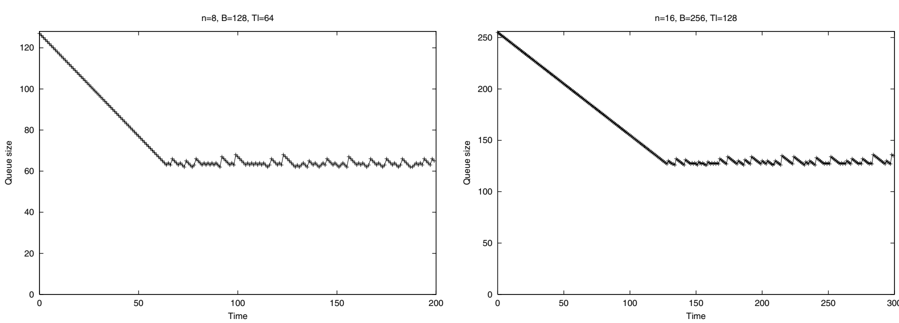

Fig. 3. The effect of PRED on the queue length

5 Concluding Remarks

The Internet is stable today mostly due to the fact that the majority of the users voluntarily use a congestion-responsive TCP protocol. However, some users can benefit from not reducing their transmission rate during congestion. Thus, if users behave selfishly, the assumption about cooperative behavior may not remain valid. Therefore, it is important to understand the nature of congestion resulting from selfish behavior.

We analyze the users' behavior by means of game theory. We consider a single bottleneck buffer under a packetized traffic model, which makes our approach more practical and applicable to real networks such as the Internet. We show that there exist efficient and fair Nash equilibria imposed by a simple FIFO buffering policy. However, the congestion created by the users is rather high. Then we propose a simple modification of RED policy, which decreases the congestion at a Nash equilibrium. Finally, we consider some natural extensions of our model.

We believe that our results can help to shed more light on the stability of the existing Internet infrastructure in presence of selfish users. Some interesting open problems include analysis of routing in general networks, alternative utility functions and interaction of greedy and non-greedy users (e.g. VBR and CBR applications).

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The Price of Anarchy of Cournot Oligopoly

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Abstract. Cournot oligopoly is typically inefficient in maximizing social welfare which is total surplus of consumer and producer. This paper quantifies the inefficiency of Cournot oligopoly with the term "price of anarchy", i.e. the worst-case ratio of the maximum possible social welfare to the social welfare at equilibrium. With a parameterization of the equilibrium market share distribution, the inefficiency bounds are dependent on equilibrium market shares as well as market demand and number of firms. Equilibrium market share parameters are practically observable and analytically manageable. As a result, the price of anarchy of Cournot oligopoly established in this paper is applicable to both practical estimation and theoretical analysis.

1 Introduction

It is well-known that Cournot oligopolistic market equilibrium generally does not maximize social welfare (also referred as social surplus, aggregate surplus), which means that Cournot oligopoly is typically inefficient. This paper quantifies the inefficiency of Cournot oligopoly by looking into the worst-case ratio of the social welfare at social optimum (SO) to the social welfare at equilibrium. The philosophy is the same as the term "price of anarchy" which was firstly introduced to congestion games and selfish routing in networks ([9], [11]).

Earlier studies on the inefficiency properties of oligopoly or monopoly focused mainly on empirical analysis (e.g. [6], [4]), while recent papers began to quantify the inefficiency. Anderson and Renault [1] parameterized the curvature of market demand, and derived bounds on the ratios of deadweight loss and consumer surplus to producer surplus. Their results require marginal costs of producers to be constant. Following a series of papers focusing on the quantification of inefficiency for various games (e.g. [2], [3], [12], [13]), Johari and Tsitsiklis [8] studied the efficiency loss in Cournot games. Their discussion on Cournot oligopoly requires the inverse demand curve to be concave, which does not hold for the widely used constant elasticity demand. This paper gives most general results regarding both cost function and market demand. For cost function, all the results of this paper allow arbitrarily convex cost function (nondecreasing marginal cost function). For market demand, this paper begins with general inverse demand function without concavity or convexity assumption, and then studies concave and convex inverse demand functions separately.

In this paper, we introduce to each firm a parameter demoting its market share at equilibrium. The price of anarchy, or inefficiency bounds of Cournot oligopoly are dependent on three terms, market demand, number of firms and equilibrium market

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shares. The parameterization of equilibrium market shares has at least three advantages. First, in contrast to cost functions typically unknown, market shares at equilibrium are observable in practice. With observed equilibrium market shares, the results of this paper can be used for practical estimation of the efficiency loss of Cournot oligopoly. Second, equilibrium market share parameters are analytically manageable, and thereby facilitate theoretical analysis. Finally, equilibrium market share parameters are effective in describing various market structures, which naturally makes the results of this paper comprehensive in terms of market structure.

This paper is organized as follows. Section 2 defines the problem under study and the necessary terms. In Section 3, we consider general inverse demand function without concavity or convexity assumption. We present several important lemmas as the fundamentals of the whole paper, and a theorem bounding the inefficiency of general Cournot oligopoly. Section 4 and Section 5 discuss concave and convex inverse demand functions, respectively. In Section 4, the concavity of the inverse demand function leads to good properties. As a result, we have refined inefficiency bounds that give rise to corollaries regarding several special cases intensively studied by previous papers. In Section 5, the convexity of the inverse demand function also leads to tightened inefficiency bounds, and the application to constant elasticity demand has meaningful analytical results. Finally, our conclusions are contained in Section 6.

2 Cournot Oligopoly: Equilibrium, Social Optimum and the Price of Anarchy

Let there be N firms, $i = 1, 2, ..., N$, which supply a homogeneous product in a noncooperative fashion. Let $p(Q)$, $Q \geq 0$ denote the inverse demand curve, where Q is the total supply in the market. Let $q_i \geq 0$ denote the ith firm's supply, then $Q = \sum_{i=1}^{N} q_i$. Let $f_i(q_i)$ denote the ith firm's total cost of supplying q_i units. A Cournot-Nash equilibrium solution is a set of nonnegative output levels q_1^*, q_2^*,…, q_N^* such that q_i^* is an optimal solution to the following problem for all $i = 1, 2, ..., N$:

$$\underset{q_i \geq 0}{\text{maximize}}\ q_i p\left(q_i + Q_i^*\right) - f_i\left(q_i\right)$$

where

$$Q_i^* = \sum_{j \neq i} q_j^*$$

It is well-known (e.g. [7], [10]) that if $f_i(\cdot)$ is convex and continuously differentiable for $i = 1, 2, ..., N$, the inverse demand function $p(\cdot)$ is strictly decreasing and continuously differentiable, and the revenue curve $Qp(Q)$ is concave, $Q \geq 0$, then (q_1^*, q_2^*, … , q_N^*) is a Cournot-Nash equilibrium solution if and only if

$$\left[p\left(Q^*\right) + q_i^* p'\left(Q^*\right) - f_i'\left(q_i^*\right)\right] q_i^* = 0 \text{ for each } i = 1, 2, ..., N \tag{1}$$

$$p\left(Q^*\right) + q_i^* p'\left(Q^*\right) - f_i'\left(q_i^*\right) \leq 0 \text{ for each } i = 1, 2, ..., N \tag{2}$$

where

$$Q^* = \sum_{i=1}^{N} q_i^*$$

Assumption 1.

(a) The cost function $f_i(\cdot)$ *is convex, strictly increasing and continuously differentiable for* $i = 1, 2, ..., N$. *In addition,* $f_i(0) = 0$.

(b) The inverse demand function $p(\cdot)$ *is strictly decreasing and continuously differentiable, and the revenue curve* $Qp(Q)$ *is concave for* $Q \geq 0$.

(c) At equilibrium, all firms are active, namely $q_i^* > 0$ *for* $i = 1, 2, ..., N$.

Note that in Assumption 1, no assumption is made on the concavity or convexity of the inverse demand function $p(\cdot)$. Part (a) and (b) of Assumption 1 ensure the existence and uniqueness of a Cournot-Nash equilibrium solution. $f_i(0) = 0$, implying that fixed cost is not considered, makes this paper consistent with previous literature ([1], [8]). Part (c) is a reasonable and weak assumption. With Part (c) of Assumption 1, the Cournot-Nash equilibrium condition (1)-(2) is simplified to be

$$p(Q^*) + q_i^* p'(Q^*) - f_i'(q_i^*) = 0 \tag{3}$$

Let s_i denote the ith firm's equilibrium market share

$$s_i = q_i^* / Q^*, \ i = 1, 2, ..., N \tag{4}$$

Without loss of generality, let the first firm have the largest equilibrium market share

$$q_1^* = \max \ \{q_i^*, i = 1, 2, ..., N\}$$

$$s_1 = \max \ \{s_i, i = 1, 2, ..., N\} \tag{5}$$

By definition, we have

$$1/N \leq s_1 \leq 1 \tag{6}$$

s_1 plays a fundamental role in both market structure description and theoretical analysis. For market structure description, we have the following observations

(a) For $N = 1$, "$s_1 = 1$" *naturally represents the case of monopoly; for* $N \geq 2$, "$s_1 = 1$" *also represents the case of monopoly, but is a limit case.*

(b) "$s_1 \to 0$ ($N \to \infty$)" *represents the case of perfect competition.*

(c) "$s_1 = 1/N$" *represents the symmetric cost case.*

Social welfare is defined to be total surplus of consumer and producer, or total benefits minus total costs, which is mathematically formulated as

$$S = \int_0^Q p(x)\,dx - \sum_{i=1}^{N} f_i(q_i) \tag{7}$$

Then the social optimization (SO) problem is given by

$$\underset{q_i \geq 0}{\text{maximize}} \int_0^Q p(x)\,dx - \sum_{i=1}^{N} f_i(q_i) \tag{8}$$

Let ($\overline{q}_1, \overline{q}_2, \ldots, \overline{q}_N$) be an optimal solution to problem (8), then the following first-order conditions hold

$$\left[p\left(\overline{Q}\right)-f_i'(\overline{q}_i)\right]\overline{q}_i=0,\ i=1,2,...,N \tag{9}$$

$$p\left(\overline{Q}\right)-f_i'(\overline{q}_i)\le 0,\ i=1,2,...,N \tag{10}$$

where

$$\overline{Q}=\sum_{i=1}^{N}\overline{q}_i$$

Let $\overline{S}$ and S^* denote the social welfare at optimum and equilibrium, respectively

$$\overline{S}=\int_0^{\overline{Q}}p(x)\,dx-\sum_{i=1}^{N}f_i(\overline{q}_i) \tag{11}$$

$$S^*=\int_0^{Q^*}p(x)\,dx-\sum_{i=1}^{N}f_i\left(q_i^*\right) \tag{12}$$

Define the following ratio

$$\rho=\overline{S}/S^* \tag{13}$$

Clearly, $\rho\ge 1$. This ratio is called the *inefficiency*, or *price of anarchy* of Cournot oligopoly. We will give upper bounds on ρ under different conditions.

With Assumption 1, $f_i(0)=0$, then social welfare defined by (7) becomes

$$S=\int_0^{Q}p(x)\,dx-\sum_{i=1}^{N}\int_0^{q_i}f_i'(x)\,dx \tag{14}$$

In some economics textbook [5], social welfare is directly defined by (14) instead of (7) regardless of the value of fixed cost, which justifies the assumption $f_i(0)=0$.

3 General Inverse Demand Function

In this section, we bound the inefficiency ratio ρ with Assumption 1 only.

Define function $\gamma(\cdot)$ as

$$\gamma(x):\ p(x)+s_1xp'(x)=p\left(\gamma(x)x\right),\ x>0 \tag{15}$$

and assume $\gamma(x)$ to be *bounded* for $x>0$. Since $p(\cdot)$ is strictly decreasing, $\gamma(\cdot)$ is generally well-defined. For example, if $p(\cdot)$ takes the form of $p(Q)=\alpha Q^{-\beta}$, $\alpha>0$, $0<\beta<1$, we have

$$\gamma(x)=\left(1-\beta s_1\right)^{-1/\beta},\ x>0 \tag{16}$$

Lemma 1. *With Assumption 1, let* $k=\overline{Q}/Q^*$ *, then it holds* $1<k\le\gamma\left(Q^*\right)$.

Proof: (a) $k>1$. If it holds $\overline{q}_i>q_i^*$ for each $i=1,2,...,N$, then $\overline{Q}>Q^*$ and $k>1$. Otherwise, without loss of generality, suppose $q_j^*\ge\overline{q}_j$, then we have

$$p\left(\overline{Q}\right)\le f_j'\left(\overline{q}_j\right) \tag{17}$$

$$\leq f_j'\left(q_j^*\right) \tag{18}$$

$$= p\left(Q^*\right) + q_j^* p'\left(Q^*\right) \tag{19}$$

$$< p\left(Q^*\right) \tag{20}$$

where (17) follows from condition (10), (18) follows from that $f_j(\cdot)$ is convex and thus $f_j'(\cdot)$ is nondecreasing, (19) follows from condition (3), and (20) follows from that $p(\cdot)$ is strictly decreasing and thus $p'\left(Q^*\right) < 0$. $p(\cdot)$ is strictly decreasing, then $p\left(\bar{Q}\right) < p\left(Q^*\right)$ leads to $\bar{Q} > Q^*$ and $k > 1$.

(b) $k \leq \gamma\left(Q^*\right)$. Since $\bar{Q} > Q^*$, thus without loss of generality, suppose $\bar{q}_j > q_j^*$, then

$$p\left(\bar{Q}\right) = f_j'\left(\bar{q}_j\right) \tag{21}$$

$$\geq f_j'\left(q_j^*\right) \tag{22}$$

$$= p\left(Q^*\right) + q_j^* p'\left(Q^*\right) \tag{23}$$

$$\geq p\left(Q^*\right) + s_1 Q^* p'\left(Q^*\right) \tag{24}$$

$$= p\left(\gamma\left(Q^*\right) Q^*\right) \tag{25}$$

where (21) follows from condition (9), (22) follows from $f_j'(\cdot)$ nondecreasing, (23) follows from condition (3), (24) follows from $p'\left(Q^*\right) < 0$, and (25) follows from (15), the definition of $\gamma(\cdot)$. $p(\cdot)$ is strictly decreasing, then $p\left(\bar{Q}\right) \geq p\left(\gamma\left(Q^*\right) Q^*\right)$ leads to $\bar{Q} \leq \gamma\left(Q^*\right) Q^*$ and $k \leq \gamma\left(Q^*\right)$, which completes the proof. ♦

Define function $\theta(\cdot)$ as

$$\theta(x) = \frac{1}{2} x^2 \left(-p'(x)\right) \Big/ \left(\int_0^x p(w)\,dw - xp(x)\right), \; x > 0 \tag{26}$$

and assume $\theta(x)$ to be *bounded* and *nonzero* for $x > 0$. $\theta(\cdot)$ is a well-defined function. For example, if $p(\cdot)$ takes the form of $p(Q) = \alpha Q^{-\beta}$, $\alpha > 0$, $0 < \beta < 1$, then

$$\theta(x) = (1-\beta)/2, \; x > 0 \tag{27}$$

If $p(\cdot)$ takes the form of $p(Q) = p_0 - \alpha Q^{\beta}$, $\alpha > 0$, $\beta > 0$, we have

$$\theta(x) = (1+\beta)/2, \; x > 0 \tag{28}$$

where $\beta = 1$ gives a linear $p(\cdot)$ with $\theta(x) = 1$ for $x > 0$.

Lemma 2. *With Assumption 1, the equilibrium social welfare* S^* *satisfies*

$$S^* \geq \left(-p'\left(Q^*\right)\right)\left[\frac{1}{2\theta\left(Q^*\right)}\left(Q^*\right)^2 + \sum_{i=1}^{N}\left(q_i^*\right)^2\right] \tag{29}$$

Proof: We have

$$\int_0^{q_i^*} f_i'(x)\,dx \le q_i^* f_i'(q_i^*) \tag{30}$$

$$= q_i^*\left(p(Q^*) + q_i^* p'(Q^*)\right) \tag{31}$$

where (30) follows from $f_j'(\cdot)$ nondecreasing, and (31) follows from condition (3). From (14), it holds

$$S^* = \int_0^{Q^*} p(x)\,dx - \sum_{i=1}^N \int_0^{q_i^*} f_i'(x)\,dx \tag{32}$$

Substitute (30)-(31) into (32), we have

$$\begin{aligned} S^* &\ge \int_0^{Q^*} p(x)\,dx - \sum_{i=1}^N q_i^*\left(p(Q^*) + q_i^* p'(Q^*)\right) \\ &= \int_0^{Q^*} p(x)\,dx - Q^* p(Q^*) + \left(-p'(Q^*)\right)\sum_{i=1}^N (q_i^*)^2 \\ &= \left(-p'(Q^*)\right)\left[\frac{1}{2\theta(Q^*)}(Q^*)^2 + \sum_{i=1}^N (q_i^*)^2\right] \end{aligned} \tag{33}$$

where (33) follows from (26), the definition of $\theta(\cdot)$. This completes the proof. ♦

Lemma 3. *With Assumption 1, the following inequality holds*

$$\rho = \frac{\bar{S}}{S^*} \le 1 + \frac{\left(-p'(Q^*)\right)\sum_{i=1}^N (\bar{q}_i - q_i^*) q_i^*}{S^*} \tag{34}$$

Proof: From (11) and (12), we have

$$\bar{S} - S^* = \int_{Q^*}^{\bar{Q}} p(x)\,dx - \sum_{i=1}^N \left(f_i(\bar{q}_i) - f_i(q_i^*)\right) \tag{35}$$

Because $p(\cdot)$ is strictly decreasing, it holds

$$\int_{Q^*}^{\bar{Q}} p(x)\,dx \le (\bar{Q} - Q^*)\, p(Q^*) \tag{36}$$

where "=" may hold only as a limit case. Because $f_i(\cdot)$ is convex, we have

$$f_i(\bar{q}_i) - f_i(q_i^*) \ge (\bar{q}_i - q_i^*) f_i'(q_i^*),\ i = 1,2,\dots,N \tag{37}$$

Substitute (36) and (37) into (35), we obtain

$$\begin{aligned} \bar{S} - S^* &\le (\bar{Q} - Q^*)\, p(Q^*) - \sum_{i=1}^N (\bar{q}_i - q_i^*) f_i'(q_i^*) \\ &= \sum_{i=1}^N (\bar{q}_i - q_i^*)\left(p(Q^*) - f_i'(q_i^*)\right) \\ &= \left(-p'(Q^*)\right)\sum_{i=1}^N (\bar{q}_i - q_i^*) q_i^* \end{aligned} \tag{38}$$

where (38) follows from condition (3). It follows immediately (34) from $\bar{S} - S^* \le \left(-p'(Q^*)\right)\sum_{i=1}^N (\bar{q}_i - q_i^*) q_i^*$, which completes the proof. ♦

Combine Lemma 2 and Lemma 3, substitute (29) into (34), we obtain

$$\rho \le \left[(Q^*)^2 + 2\theta(Q^*)\sum_{i=1}^N \bar{q}_i q_i^*\right] \Big/ \left[(Q^*)^2 + 2\theta(Q^*)\sum_{i=1}^N (q_i^*)^2\right] \tag{39}$$

With $\sum_{i=1}^{N}\bar{q}_i q_i^* \le \sum_{i=1}^{N}\bar{q}_i q_1^* = \bar{Q}q_1^* = ks_1\left(Q^*\right)^2$, $\sum_{i=1}^{N}\left(q_i^*\right)^2 = \left(Q^*\right)^2\sum_{i=1}^{N}s_i^2$, it comes

$$\rho \le \left(1+2\theta\left(Q^*\right)ks_1\right)\Big/\left(1+2\theta\left(Q^*\right)\sum_{i=1}^{N}s_i^2\right) \tag{40}$$

Define parameter γ and θ as

$$\gamma = \max_{x>0}\ \gamma(x) \tag{41}$$

$$\theta = \max_{x>0}\ \theta(x) \tag{42}$$

Theorem 1. *With Assumption 1, the inefficacy ratio* ρ *is bounded as*

$$\rho \le \left(1+2\theta\gamma s_1\right)\Big/\left(1+2\theta\sum_{i=1}^{N}s_i^2\right) \tag{43}$$

$$\le \left(1+2\theta\gamma s_1\right)/\left(1+2\theta m\right) \tag{44}$$

where

$$m = \begin{cases} 1, & N=1 \\ s_1^2 + \left(1-s_1\right)^2/\left(N-1\right), & N \ge 2 \end{cases} \tag{45}$$

Proof: The right-hand side of (40) increases with k and $\theta\left(Q^*\right)$, and we have $k \le \gamma$ from Lemma 1 and definition (41) and $\theta\left(Q^*\right) \le \theta$ by definition (42), thus we readily obtain (43) by setting $k=\gamma$ and $\theta\left(Q^*\right)=\theta$ in (40). Then (44) follows from $\sum_{i=1}^{N}s_i^2 \ge m$ ($\sum_{i=1}^{N}s_i^2 = s_1^2 + \sum_{i=2}^{N}s_i^2 \ge s_1^2 + \left(1-s_1\right)^2/\left(N-1\right) = m$, for $N \ge 2$).

This completes that proof. ♦

Theorem 1 gives the price of anarchy for general Cournot oligopoly. The two inefficiency bounds given by (43) and (44) are both determined by three terms, the market demand function (represented by θ and γ), the number of firms N and the market share distribution at equilibrium. While (43) requires the equilibrium market share of each individual firm to be known, (44) needs s_1 only because parameter m captures the worst-case market structure for any s_1, i.e. the first firm have the largest market share s_1 and each other firm have market share $\left(1-s_1\right)/\left(N-1\right)$. In general, (43) applies to practical estimation for which equilibrium market shares are observed in practice, while (44) is applicable to theoretical analysis for which it is impossible and unnecessary to know the equilibrium market shares of individual firms.

Apply (44) to the case of perfect competition, we have the following corollary which states that perfect competition, as expected, is fully efficient in terms of the maximization of social welfare.

Corollary 1. *With Assumption 1, for the case of perfect competition, namely* $s_1 \to 0$ *(* $N \to \infty$ *), it holds*

$$\rho \to 1$$

Proof: When $N \to \infty$ and $s_1 \to 0$, it follows $m \to 0$ from (45), thus the right-hand side of (44) approaches 1 given that θ and γ are bounded. Since $\rho \ge 1$, it follows immediately $\rho \to 1$, which completes that proof. ♦

4 Concave Inverse Demand Function

In this section, we study the case of concave inverse demand function, and apply our general results to the special cases studied by Johari and Tsitsiklis [8] for comparison.

Lemma 4. *With Assumption 1, if $p(\cdot)$ is concave, then it holds*

(a) $\gamma(x) \le 1+s_1$, *for* $x>0$;

(b) $$\rho = \frac{\bar{S}}{S^*} \le 1 + \frac{\left(-p'(Q^*)\right)\left[\sum_{i=1}^{N}\left(\bar{q}_i - q_i^*\right)q_i^* - \frac{1}{2}\left(\bar{Q}-Q^*\right)^2\right]}{S^*} \tag{46}$$

Proof: (a) With Assumption 1, if $p(\cdot)$ is concave, we have

$$p(x) + s_1 x p'(x) \ge p(x + s_1 x),\ x>0 \tag{47}$$

From (15), the definition of $\gamma(\cdot)$, (47) gives

$$p\left(\gamma(x)x\right) \ge p\left((1+s_1)x\right),\ x>0 \tag{48}$$

Since $p(\cdot)$ is strictly decreasing, (48) simply gives $\gamma(x) \le 1+s_1$ for $x>0$.

(b) With Assumption 1, if $p(\cdot)$ is concave, we have

$$\int_{Q^*}^{\bar{Q}} p(x)d \le \left(\bar{Q}-Q^*\right)p\left(Q^*\right) - \frac{1}{2}\left(-p'(Q^*)\right)\left(\bar{Q}-Q^*\right)^2 \tag{49}$$

Let (49) take the place of (36) in Lemma 3, then (34) of Lemma 3 simply becomes (46), which completes the proof. ♦

Part (b) of Lemma 4 in this section takes the place of Lemma 3 in last section. Combine Lemma 2 and Part (b) of Lemma 4, substitute (29) into (46), we obtain

$$\rho \le \frac{\left(Q^*\right)^2 + 2\theta\left(Q^*\right)\left[\sum_{i=1}^{N}\bar{q}q_i^* - \frac{1}{2}\left(\bar{Q}-Q^*\right)^2\right]}{\left(Q^*\right)^2 + 2\theta\left(Q^*\right)\sum_{i=1}^{N}\left(q_i^*\right)^2} \tag{50}$$

With $\sum_{i=1}^{N}\bar{q}_i q_i^* \le \sum_{i=1}^{N}\bar{q}_i q_1^* = \bar{Q}q_1^* = ks_1\left(Q^*\right)^2$, $\sum_{i=1}^{N}\left(q_i^*\right)^2 = \left(Q^*\right)^2\sum_{i=1}^{N}s_i^2$, it comes

$$\rho \le \left[1+\theta\left(Q^*\right)\left(2ks_1 - (k-1)^2\right)\right] \Big/ \left(1+2\theta\left(Q^*\right)\sum_{i=1}^{N}s_i^2\right) \tag{51}$$

Theorem 2. *With Assumption 1, if $p(\cdot)$ is concave, then it holds*

$$\rho \le \left[1+\theta\left(2+s_1\right)s_1\right] \Big/ \left(1+2\theta\sum_{i=1}^{N}s_i^2\right) \tag{52}$$

$$\le \left[1+\theta\left(2+s_1\right)s_1\right] \Big/ \left[1+2\theta m\right] \tag{53}$$

Proof: The right-hand side of (51) increases with $\theta\left(Q^*\right)$ and k, and we have $\theta\left(Q^*\right) \le \theta$ by definition (42) and $k \le 1+s_1$ from Lemma 1 and Part (a) of Lemma 4, thus we readily obtain (52) by setting $\theta\left(Q^*\right)=\theta$ and $k=1+s_1$ in (51). Then (53) follows immediately from $\sum_{i=1}^{N}s_i^2 \ge m$, which completes the proof. ♦

Theorem 2 gives the price of anarchy of Cournot oligopoly for concave inverse demand function, which is a tightened and refined counterpart of Theorem 1. Like Theorem 1, the two inefficiency bounds given by (52) and (53) are applicable to practical estimation and theoretical analysis, respectively. The bounds do not use parameter γ thanks to Part (a) of Lemma 4, a good property brought about by the concavity of $p(\cdot)$. Without γ in its formulation, (53) gives rise to the following corollaries regarding the worst-case inefficiency for several special cases.

Corollary 2. *With Assumption 1, if $p(\cdot)$ is concave, then it holds*

$$\rho \le (2+s_1)s_1/2m \tag{54}$$

where "=" may hold only if $\theta \to \infty$.

Corollary 2 gives the worst-case inefficiency for given N and s_1, and has two important applications, the case of monopoly and the symmetric cost case, both studied intensively by previous papers. For the case of monopoly, namely $N=1$ and $s_1=1$, it follows $m=1$ from (45), thus (54) gives $\rho \le 3/2$. For the symmetric cost case, namely all firms share the same cost function and thereby $s_1 = 1/N$, it follows $m = 1/N$ from (45), thus (54) gives $\rho \le 1+1/2N$. These two results are exactly the same as those of Johari and Tsitsiklis (Corollary 17 and 18 of [8]).

Corollary 3. *With Assumption 1, if $p(\cdot)$ is concave, then it holds*

$$\rho \le \left(\sqrt{4N+5}+3\right)/4 \tag{55}$$

where "=" may hold only if $\theta \to \infty$ and $s_1 = \left(\sqrt{4N+5}+1\right)/2(N+1)$.

Corollary 3 gives the worst-case inefficiency for given N. Particularly, if $N=1$, (55) gives $\rho \le 3/2$, the same as the result of Corollary 2.

Corollary 4. *With Assumption 1, if $p(\cdot)$ is concave, then it holds*

$$\rho \le \left(3+\sqrt{1+8\theta}\right)/4 \tag{56}$$

where "=" may hold only if $N \to \infty$ and $s_1 = \left(\sqrt{1+8\theta}-1\right)/4\theta$.

Corollary 4 gives the worst-case inefficiency for given θ. The most important application of Corollary 4 is the case of linear (affine) market demand function, which appears in a lot of papers. For linear market demand function, namely $\theta=1$, (56) gives $\rho \le 3/2$ (where "=" holds if and only if $N \to \infty$ and $s_1 = 1/2$). This result (including the condition for "=" to hold) is again the same as that of Johari and Tsitsiklis (Theorem 19 of [8]).

5 Convex Inverse Demand Function

In this section, we study the case of convex inverse demand function including the constant elasticity demand.

Lemma 5. *With Assumption 1, if* $p(\cdot)$ *is convex, then it holds*

$$\rho = \frac{\bar{S}}{S^*} \le 1 + \frac{\left(-p'\left(Q^*\right)\right)\sum_{i=1}^{N}\left(\bar{q}_i - q_i^*\right)q_i^* - \frac{1}{2}\left(\bar{Q}-Q^*\right)\left(p\left(Q^*\right)-p\left(\bar{Q}\right)\right)}{S^*} \tag{57}$$

Proof: With Assumption 1, if $p(\cdot)$ is convex, we have

$$\int_{Q^*}^{\bar{Q}} p(x)dx \le \left(\bar{Q}-Q^*\right)p\left(Q^*\right) - \frac{1}{2}\left(\bar{Q}-Q^*\right)\left(p\left(Q^*\right)-p\left(\bar{Q}\right)\right) \tag{58}$$

Then the proof follows the same line as the proof Lemma 3. Let (58) take the place of (36) in Lemma 3, then (34) of Lemma 3 simply becomes (57).

This completes the proof. ♦

In this section, Lemma 5, like Part (b) of Lemma 4 in last section, takes exactly the place of Lemma 3 in Section 3. Combine Lemma 2 and Lemma 5, substitute (29) into (57), we obtain

$$\rho \le \frac{\left(Q^*\right)^2 + 2\theta\left(Q^*\right)\left[\sum_{i=1}^{N}\bar{q}q_i^* - \frac{(k-1)Q^*\left(p\left(Q^*\right)-p\left(\bar{Q}\right)\right)}{2\left(-p'\left(Q^*\right)\right)}\right]}{\left(Q^*\right)^2 + 2\theta\left(Q^*\right)\sum_{i=1}^{N}\left(q_i^*\right)^2} \tag{59}$$

With $\sum_{i=1}^{N}\bar{q}_i q_i^* \le \sum_{i=1}^{N}\bar{q}_i q_1^* = \bar{Q}q_1^* = ks_1\left(Q^*\right)^2$, $\sum_{i=1}^{N}\left(q_i^*\right)^2 = \left(Q^*\right)^2\sum_{i=1}^{N}s_i^2$, it comes

$$\rho \le \left[1+\theta\left(Q^*\right)\left(2ks_1-(k-1)h\left(Q^*,k\right)\right)\right] \Big/ \left(1+2\theta\left(Q^*\right)\sum_{i=1}^{N}s_i^2\right) \tag{60}$$

where $h: E_2 \to E_1$ is a function defined as

$$h(x,k) = \left(p(x)-p(kx)\right)\big/\left(-p'(x)x\right),\ x>0,\ 1<k\le\gamma(x) \tag{61}$$

Like $\gamma(\cdot)$ and $\theta(\cdot)$, $h(x,k)$ is a well-defined function. For example, if $p(\cdot)$ takes the form of $p(Q)=\alpha Q^{-\beta}$, $\alpha>0$, $0<\beta<1$, we have

$$h(x,k) = \left(1-k^{-\beta}\right)\big/\beta,\ x>0,\ 1<k\le\gamma(x) \tag{62}$$

Furthermore, $h(x,k)$ have the following relationship with $\gamma(\cdot)$ defined by (15)

$$h\left(x,\gamma(x)\right) = s_1,\ x>0 \tag{63}$$

Assumption 2. *For any* $1/N \le s_1 \le 1$ *and* $x>0$, $\left(2ks_1-(k-1)h(x,k)\right)$ *increases with* k *for* $1<k\le\gamma(x)$.

With Assumption 2, set $k=\gamma\left(Q^*\right)$ in (60), and make use of (63), we obtain

$$\rho \le \left[1+\theta\left(Q^*\right)\left(1+\gamma\left(Q^*\right)\right)s_1\right] \Big/ \left(1+2\theta\left(Q^*\right)\sum_{i=1}^{N}s_i^2\right) \tag{64}$$

Theorem 3. *With Assumption 1 and 2, if* $p(\cdot)$ *is convex, then it holds*

$$\rho \le \left[1+\theta(1+\gamma)s_1\right] \Big/ \left(1+2\theta\sum_{i=1}^{N}s_i^2\right) \tag{65}$$

$$\leq \left[1+\theta(1+\gamma)s_1\right] \big/ (1+2\theta m) \tag{66}$$

Proof: The right-hand side of (64) increases with $\gamma(Q^*)$ and $\theta(Q^*)$, and we have $\gamma(Q^*) \leq \gamma$ and $\theta(Q^*) \leq \theta$ by definitions (41)-(42), thus we readily obtain (65) by setting $\gamma(Q^*) = \gamma$ and $\theta(Q^*) = \theta$ in (64). Then, like the proof of Theorem 1 and 2, (66) follows immediately from $\sum_{i=1}^{N} s_i^2 \geq m$, which completes the proof. ♦

Theorem 3 gives the price of anarchy of Cournot oligopoly for convex inverse demand function. Like Theorem 1 and 2, the two inefficiency bounds given by (65) and (66) are applicable to practical estimation and theoretical analysis, respectively.

Apply (66) to the constant elasticity demand, we have the following corollary.

Corollary 5. *Suppose* $p(\cdot)$ *takes the constant elasticity form,* $p(Q) = \alpha Q^{-\beta}$, $\alpha > 0$, $0 < \beta < 1$. *With Assumption 1, it holds*

$$\rho \leq \frac{1+\frac{1}{2}(1-\beta)\left[1+(1-\beta s_1)^{-1/\beta}\right]s_1}{1+(1-\beta)m} \tag{67}$$

Furthermore, monopoly $s_1 = 1$ *gives the worst-case inefficiency*

$$\rho \leq \left[3-\beta+(1-\beta)^{1-1/\beta}\right] \big/ 2(2-\beta) \tag{68}$$

and $\beta = 0.8670$ *gives the overall worst-case inefficiency* $\rho \leq 1.5427$.

Corollary 5 does not require Assumption 2 explicitly because constant elasticity demand automatically meets Assumption 2. From Corollary 5, for a constant elasticity demand, the worst-case inefficiency can be attained only by monopoly, and the overall worst-case inefficiency is that the optimal social welfare is 1.5427 times of the equilibrium social welfare.

6 Conclusion

This paper studies the price of anarchy of Cournot oligopoly, and gives most general results regarding cost function and market demand. General, concave and convex inverse demand functions are studied separately. The general inefficiency bounds given by Theorem 1-3 are determined by three terms, market demand, number of firms and equilibrium market shares. These general results are applicable to both practical estimation and theoretical analysis thanks to the practical observability and analytical manageability of the equilibrium market share parameters. Furthermore, since equilibrium market share parameters can effectively describe various special market structures, the general results can be readily applied to special cases such as monopoly, perfect competition and the symmetric cost case.

One point worth mentioning is that the general inefficiency bounds given by Theorem 1-3 are *not* independent of the cost functions of firms because the cost characteristics of firms to a large extent determine the equilibrium market shares. However, with these general results, equilibrium market share parameters are enough for practical estimation and theoretical analysis of the price of anarchy of Cournot oligopoly. Thus it is unnecessary to know any information on the cost functions (as long as the marginal costs are nondecreasing with output). In addition, inefficiency bounds independent of cost functions can simply be obtained by searching the worst-case equilibrium market share distribution, like in Corollary 3-4 where the inefficiency bounds are dependent on θ or N only.

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Incentives in Some Coalition Formation Games

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Abstract. The idea of using the core as a model for predicting the formation of coalitions and the sharing of benefits to coordinated activities has been studied extensively. Basic to the concept of the core is the idea of group rationality as embodied by the blocking condition. The predictions given by the core may run into difficulties if some individuals or coalitions may benefit from not blocking "truthfully". This paper investigates this question in games in some games that generalize assignment games. Some positive results are given, and relationships with Vickrey-Clarke-Groves mechanisms are drawn.

1 Introduction

The idea of using the core as a model for assessing the stability of arrangements made within a society has been proved quite fruitful in various contexts. In particular, as in the pioneering analysis of an assignment game by Shapley and Shubik [13], the core may predict which coalitions form and how benefits are shared within each coalition. This occurs in the situations in which diversity in individual tastes or institutional or organizational constraints do not call for the whole society to coordinate.

Underlying the concept of the core is the idea of group rationality as embodied by the blocking condition. There are however two difficulties: existence and manipulability. No stable outcome exists for a large class of games. Also, the predictions given by the core may run into difficulties if some individuals or coalitions benefit from not blocking "truthfully". It turns out that in some coalitional games, both the problems of existence and manipulability can be resolved in a manner to be made precise.

The paper is organized as follows. Section 2 introduces the model and gives some examples. Section 3 and 4 discusses the manipulability of core correspondences, and Section 5 investigates strategy-proof selections of the core.

2 The Coalitional Formation Model

Before introducing the model, it is worth recalling the basic features of the assignment game.

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The assignment game. There are two types of agents, the "buyers" and the "sellers". Each buyer is interested in buying only one item, say a house. The i-th seller values his own house to c_i while the j-th buyer values the same house at h_{ij} dollars. The important data are the "essential" coalitions, the pairs of buyer-seller, and the total value they derive by forming, here $h_{i,j} - c_i$. The possible outcomes of the market specify which pairs of buyer-seller end up making a transaction and at what price. One seeks a stable outcome, meaning that no pair consisting of a buyer and a seller can make an arrangement that is more satisfactory to both than the given one. As shown in [13], a stable outcome always exists, and each one is supported by an equilibrium price. Furthermore, there is a minimum equilibrium price vector and a maximal one. The strategic properties are the following ones:

(a) selecting the minimum equilibrium price vector gives the incentives to all buyers to reveal their true valuations. This holds true because each buyer reaches his incremental value, or Vickrey payment, at the corresponding stable arrangement ([10] and [3]; incremental values are defined later on). The same result holds for the sellers by selecting the maximal price.

(b) a buyer and a seller cannot each one achieve more than his incremental value by misrepresenting jointly their preferences (Demange [5]).

Our purpose is to explore the extent to which properties (a) and (b) generalize to the following situation.

The coalitional game. A finite set of players, the "society", $N = \{1, ..., n\}$, may organize themselves into pairwise disjoint coalitions, where as usual a coalition is a non empty subset of N. Not all coalitions may form say for organizational or institutional reasons : A collection $\mathcal{C}$ will describe the set of admissible coalitions. Throughout the paper, singletons are allowed to form, hence are members of $\mathcal{C}$.

Players only care about the coalition they join and the amount of money they receive or give. Player i's preferences are represented by a utility function u_i defined over the coalitions that include i : $u_i(S)$ gives in term of money the utility for i to be a member of coalition S.

The set of admissible i's utility functions is denoted by $\mathcal{U}_i$. I shall assume that any utility function is admissible (there is no reason for instance to assume utility functions to be increasing: a member of a coalition may dislike a newcomer or may not enjoy too large coalitions). The n-tuple $u = (u_i)_{i \in N}$ is a preference profile and $\mathcal{U} = \bigotimes_{i=1,..,n} \mathcal{U}_i$ the set of admissible profiles.

A coalition S that forms can decide to implement transfers between its members, $(t_i)_{i \in S}$. Player i receives t_i (positive or negative), hence achieves a utility level or payoff of $u_i(S) + t_i$. Feasibility requires transfers to be overall balanced: $\sum_{i \in S} t_i \leq 0$. Thus, if S forms, any payoff $(x_i)_{i \in S}$ that satisfies $\sum_{i \in S} x_i \leq \sum_{i \in S} u_i(S)$ can be achieved by S alone through adequate transfers. This leads us to define the value of S by

$$V_u(S) = \sum_{i \in S} u_i(S). \qquad (1)$$

The society may split into several self-sufficient groups owing to individuals' preferences (when players dislike large coalitions for instance) or because of the constraints on coalitions as specified by the set $\mathcal{C}$.[1] As defined in [1], a coalition structure describes how players organize themselves into admissible coalitions that are pairwise disjoint (hence membership to a coalition is exclusive) and self-sufficient (which excludes transfers across coalitions).

Definition 1. *A* $\mathcal{C}$**-(coalition) structure** *of* N *is given by* $a = (\pi, t)$ *where*
$\pi = (S_\ell)_{\ell=1,\dots,L}$ *is a partition of* N *made of elements in* $\mathcal{C}$*:* $S_\ell \in \mathcal{C}$ *and*
$t = (t_i)_{i \in N}$ *specifies transfers that are balanced within each element of* π*:* $\sum_{i \in S_\ell} t_i \leq 0$ *for each* S_ℓ *in* π.

The payoff reached by i*, denoted by* $\tilde{u}_i(a)$*, is* $\tilde{u}_i(a) = u_i(S_{\ell(i)}) + t_i$ *where* $S_\ell(i)$ *is the unique coalition of which* i *is a member.*

To analyze which structure will emerge, we rely on the standard stability notion as embodied by the blocking condition.

Definition 2. *Given a profile* u*, the coalition structure* $a = (\pi, t)$ *is said to be* **blocked** *by* T *if*

$$\sum_{i \in T} \tilde{u}_i(a) < V_u(T). \tag{2}$$

A $\mathcal{C}$**-stable structure** *is a* $\mathcal{C}$*-structure that is not blocked by any coalition in* $\mathcal{C}$*. Its payoff vector* $(\tilde{u}_i(a))_{i=1,\dots,n}$ *is called* $\mathcal{C}$**-stable payoff***. The set of* $\mathcal{C}$*-stable structures is called the* $\mathcal{C}$**-core***.*

The blocking condition (2) is justified as usual: since coalition T can achieve to its members any payoff that sums to $V_u(T)$, if (2) is met, then each individual in T could be made better off than under the structure a. Accounting for the set of admissible coalitions, the stability notion follows.

The collections that guarantee the existence of a $\mathcal{C}$-stable structure for any profile are of particular interest (as introduced by Kaneko and Wooders [8]). The guarantee imposes quite severe restrictions. For example, the collection must not contain a "Condorcet triple" that is three coalitions that intersect each other but whose overall intersection is empty (for instance, no $\mathcal{C}$-stable structure exists for a profile that gives value 1 to each of the three coalitions and zero otherwise). The absence of Condorcet triples is however not sufficient to guarantee a non empty core. A sufficient condition can be stated in terms of the balanced families. Recall that a family $\mathcal{B}$ of subsets of N is said to be *balanced* if there are nonnegative weights on the elements in the family $(\gamma_S)_{S \in \mathcal{B}}$ such that $\sum_{S, i \in S} \gamma_S = 1$ for each i. A partition is a balanced family (take weights equal to 1).

Definition 3. *A collection* $\mathcal{C}$ *satisfies the* **partition property** *if any balanced family composed with coalitions in* $\mathcal{C}$ *contains a partition.*

[1] In technical terms, the game is not super-additive. Recall that super-additivity writes as $V_u(S \cup T) \geq V_u(S) + V_u(T),$ for every $S, T \; s.t. S \cap T = \emptyset$.

Thanks to Scarf theorem [11], the partition property is sufficient for $\mathcal{C}$ to guarantee stability on any set of utility profiles. As shown in [8], the partition property is also necessary if the set $\mathcal{U}$ is rich enough, so that all super-additive transferable utility games are obtained, which is obviously the case with the whole set of possible profile.

Illustrative examples. **1.** Two-sided society (i.e. divided into two subgroups).

In the assignment game, admissible coalitions are singletons and pairs of buyer-seller and the partition property holds.

In a job market as considered by Kelso and Crawford [9], entities are firms on one side and workers on the other side (or buyers and sellers. with buyers who may be interested in buying several objects). Firms may hire several workers but an employed worker works with a single firm : Apart from singletons, a coalition is admissible if it contains a single firm. Stability is not guaranteed if there are at least two firms and three workers : $\{f_1, w_1, w_2\}$, $\{f_2, w_2, w_3\}$, $\{f_1, w_3, w_1\}$ is a Condorcet triple. Some conditions on preferences are needed to ensure the existence of a stable structure (gross substitutes condition). Also various auction mechanisms and their relationships with Vickrey-Clarke-Groves mechanism have been investigated, see for example Bikhchandani and Ostroy [2]).

2. Networks games. Individuals are linked through a network and only the coalitions that are connected can form. If the network is a tree, individuals are partially ordered, then stability is guaranteed (Demange [6]). Stability fails whenever the network contains a cycle since a Condorcet triple is easily found.

It is worth noting that dropping some coalitions from a collection $\mathcal{C}$ has two effects. On one hand less coalitions can block and on the other hand less can form, hence less structures are feasible. As a result, the cores associated with nested collections cannot be compared. According to this remark, selecting some connected sets of a tree gives a collection that satisfies the partition property and may generate interesting cores.

Incremental values. To account for the possibility of a coalition S to split up into elements of $\mathcal{C}$ let us define the superadditive function $\overline{V}_u(S)$ as follows. Denote by $\Pi_{\mathcal{C}}(S)$ the set of partitions of S made of elements in $\mathcal{C}$ (it is nonempty since it contains the partition of singletons). By choosing to partition into π, each element T in the partition can achieve to its members any payoff that sum to $V_u(T)$, which leads to a total of $V_u(\pi) = \sum_{T\in\pi} V_u(T)$. The value $\overline{V}_u(S)$ is obtained by picking out a partition of S that gives the maximal value. Formally,

$$\overline{V}_u(S) = \max_{\pi\in\Pi_{\mathcal{C}}(S)} V_u(\pi).$$

Observe that a coalition S needs to implement transfers across the distinct elements of an optimal partition so as to reach *any* share of $\overline{V}_u(S)$. Hence the super-additive characteristic function $\overline{V}_u$ does not exactly represent our coalitional game. It is nevertheless a useful tool for describing stability: if a coalition does not achieve $\overline{V}_u(T)$, then surely one of its subset in $\mathcal{C}$ can block. Two remarks follow. First, the set of $\mathcal{C}$-stable payoffs is described by the set of linear inequalities (see [8])

$$\sum_{i \in N} x_i \le \overline{V}_u(N) \text{ and } \sum_{i \in S} x_i \ge V_u(S),\ S \in \mathcal{C} \tag{3}$$

Thus, if the $\mathcal{C}$-core is non empty, which holds true under the partition property, the set described by (3) is nonempty. Furthermore, a partition at a stable structure achieves $\overline{V}_u(N)$, that is is *optimal* for N (typically such a partition is unique).

Second, defining the *incremental value* of a coalition T (to the set of all remaining players) by

$$\overline{V}_u(N) - \overline{V}_u(N - T). \tag{4}$$

yields an upper bound on the sum of the payoffs that players in T can achieve at a $\mathcal{C}$-stable payoff. To see this, note simply that if players in T get strictly more than their incremental value, then, by feasibility, $N - T$ gets strictly less than $\overline{V}_u(N - T)$, hence an admissible subset can block.

Similarly the incremental value of a player i to a coalition S possibly smaller than $N - i$, is simply defined as $\overline{V}_u(S + i) - \overline{V}_u(S)$ (to simplify notation, $\{,\}$ is dropped when there is no possible confusion. Also $S + i$ denotes the set $S \cup \{i\}$.)

3 Optimistic Manipulability and Incremental Values

The set of stable structures, if non empty, is typically multi-valued. Therefore, when a player contemplates misrepresenting his preferences he compares two subsets of coalition structures. Various notions of manipulability are possible, depending on how preferences over coalition structures are extended over subsets. We choose here a concept that answers to the following main objection. Although it seems quite surprising at first sight, it may happen that all members of a coalition prefer an alternative that they can block to *any* alternative that is stable. Why, then, should these individuals agree to block? (for a discussion see Demange [5] or [7]). As usual, given a profile u, (v_T, u_{N-T}) denote the profile with functions u_i for individuals not in T and v_i for those in T.

Definition 4. *Optimistic T can manipulate correspondence $\mathcal{S}$ at u if there is v_T and b in $\mathcal{S}(v_T, u_{N-T})$ for which*

$$\tilde{u}_i(b) > \tilde{u}_i(a), \forall\ a \in \mathcal{S}(u), \forall i \in T. \tag{5}$$

Applying the definition to the $\mathcal{C}$-core correspondence, the members of T can manipulate if by misrepresenting their preferences, a structure b that they all prefer to each C-stable structure becomes stable. When applied to a single individual, the definition amounts to assume that the individual evaluates a subset by considering the best element in the set, hence the qualification of "optimistic" manipulation.

The link between strategy-proofness and incremental values is known since the work of Vickrey [14]. The argument extends to correspondences under optimistic manipulability.

Proposition 1. *Let u be a profile. A coalition that achieves at a stable structure its incremental value cannot optimistically manipulate the core.*

Proof. Let coalition T achieve its incremental value at u. By contradiction, suppose T can optimistically manipulate. Denote by $x = \tilde{u}(b)$ the payoff vector (under the "true" preferences u) at the preferred structure b. Surely $\sum_{i \in T} x_i > \overline{V}_u(N) - \overline{V}_u(N-T)$. Also, by feasibility, $\sum_{i \in N} x_i \le \overline{V}_u(N)$ holds. These inequalities imply $\sum_{i \in N-T} x_i < \overline{V}_u(N-T)$, in contradiction with the stability of b at profile (v_T, u_{N-T}). □

It is worth noting that the core may be manipulable. Consider player 1 in the game V_u: $V_u(2,3) = V_u(2,4) = c$, $V_u(1,3,4) = d$, $V_u(1,2,3,4) = 1$, and all other values are nil. Assume $c \le 1$ and $d \le 1$ so that the game is super additive. The incremental value of player 1 is $(1-c)$. The only possible stable payoff at which it can be achieved is $(1-c, c, 0, 0)$. However, for $1 - c < d$, this payoff is not stable (it is blocked by $\{1,3,4\}$) and 1's maximum stable payoff is reached at the extreme point $(2-2c-d, 1-d, c+d-1, c+d-1)$. By lowering his utility d for $\{1,3,4\}$, player 1's maximal payoff is increased possibly up to $(1-c)$.

4 Non Manipulability Result

Proposition 2. *Let collection $\mathcal{C}$ satisfy the partition property. Then, for each coalition T in $\mathcal{C}$ there is a $\mathcal{C}$-stable structure at which that coalition reaches its incremental value. Therefore, no admissible coalition can optimistically manipulate the $\mathcal{C}$-core. This applies in particular to each singleton.*

Proof. Given profile u, let (π, t) be a $\mathcal{C}$-stable structure at which T achieves its maximal payoff, denoted by M_T. Of course, $M_T \ge \overline{V}_u(T)$. We have to show that for T admissible

$$M_T = \overline{V}_u(N) - \overline{V}_u(N-T). \tag{6}$$

If T belongs to π, T gets exactly its value at the structure: $M_T = V_u(T)$, and furthermore $V_u(\pi) = \overline{V}_u(N) = \overline{V}_u(T) + V_u(N-T)$: (6) holds. Suppose that T does not belong to π. Change u_i into v_i for each i in T by increasing $u_i(T)$ everything else equal. Denote $v = (v_T, u_{N-T})$. As long as $M_T \ge V_v(T)$, the structure (π, t) remains stable for profle v. For $M_T < V_v(T)$ (π, t) is no longer stable. Furthermore, T is a member of a partition at any stable structure for v, say (π', t'): otherwise (π', t') would also be stable at profile u and the payoffs to T (computed at u) would be strictly larger than M_T, a contradiction. Thus the value $\overline{V}_v(N)$ is given by:

$$\overline{V}_u(N) \text{ for } M_T \ge V_v(T) \text{ and by } V_v(T) + \overline{V}_u(N-T) \text{ for } M_T < V_v(T). \tag{7}$$

The continuity of the value $\overline{V}_v$ with respect to v at $M_T = V_v(T)$ yields

$$M_T + \overline{V}_u(N-T) = \overline{V}_u(N),$$

the desired result. □

The result can be proved through linear programming methods, by computing some extreme points of the set of stable payoffs (see [7]). The proof provided here is somewhat more intuitive. It makes clear that the crucial property is that the $\mathcal{C}$-core is not empty for any profile. Hence the result may extend to the case with money but without transferable utility (for an extension in an assignment game see Demange and Gale [4]).

Finally, the non manipulability result does not apply to the coalitions that are not admissible, as illustrated by the following example. There are three individuals on a line with 1 in between and $\mathcal{C}$ is the collection of all connected sets (thus only $\{2,3\}$ is not admissible) and $V_u(1,2,3) = 1$, $V_u(i) = 0$, $V_u(1,2) = c_2$ and $V_u(1,3) = c_3$. Assume c_2 and c_3 between 0 and 1 and $c_2 + c_3 > 1$. Consider players 2 and 3. Each one gets his incremental value at the same (extreme) stable payoff $(c_2 + c_3 - 1, 1 - c_3, 1 - c_2\}$. As for the non admissible coalition $\{2,3\}$, its incremental value, equal to 1, is not reached. (check that the maximal payoff is $2-(c_2+c_3) < 1$. Players 2 and 3 can be better off by falsifying their preferences as follows: 2 announces a lower utility for $\{1,2\}$, thereby lowering the value of $\{1,2\}$ hence increasing the incremental payoff of player 3, and similarly 3 makes 2 better off by lowering her utility for $\{1,3\}$.

5 Strategy-Proof Selection

We consider here a collection that satisfies the partition property. A selection of the $\mathcal{C}$-core is a function that assigns a $\mathcal{C}$-stable structure at each profile. Recall that individual i can manipulate f at u if for some v_i in $\mathcal{U}_i$

$$\tilde{u}_i(f(v_i, u_{N-i})\} > \tilde{u}_i(f(u)).$$

Function f is strategy-proof for an individual if this individual cannot manipulate f at any profile.

From the previous result, one easily derives that selecting a preferred core structure for a given player is strategy-proof for that player. Is it possible to get strategy-proofness for more than one player ? The answer is positive in an assignment game as recalled above (property (a)). This section aims at understanding under which conditions on the collection $\mathcal{C}$ a selection of the $\mathcal{C}$-core is strategy-proof for a given subset of players. As a preliminary, note that such a selection has to give to each of these players his incremental value.

Proposition 3. *Let collection $\mathcal{C}$ satisfy the partition property and consider a coalition T. A selection of the $\mathcal{C}$-core is strategy-proof for each player in T if and only if each one reaches his incremental value at any profile.*

An immediate consequence is that for a selection of the core to be strategy-proof for each player the core has to be single valued. This occurs only in the uninteresting case where no coalition apart the singletons are admissible.

A second consequence is that the incentives properties of a selection are much related to the properties of substitutes or complements as defined in [13]. We first recall the definition, restricting to two players, α and β. Denote $S + \alpha\beta$ the set $S \cup \{\alpha, \beta\}$ (and similarly $S - \alpha\beta$ for the set $S - \{\alpha, \beta\}$).

Definition 5. *Two players are* **substitutes** *at u if*

$$\overline{V}_u(S+\alpha\beta)-\overline{V}_u(S+\beta)\leq\overline{V}_u(S+\alpha)-\overline{V}_u(S)\ \textit{all}\ S,\alpha\notin S,\beta\notin S \qquad (8)$$

They are **complements** *if*

$$\overline{V}_u(S+\alpha\beta)-\overline{V}_u(S+\beta)\geq\overline{V}_u(S+\alpha)-\overline{V}_u(S)\ \textit{all}\ S,\alpha\notin S,\beta\notin S \qquad (9)$$

In other words, players α and β are substitutes (resp. complements) if the incremental value of one of the players to a coalition is not positively (resp. negatively) affected by the arrival of the other player in the coalition. To see the relationship with the incentives properties, it suffices to observe that the two players α and β simultaneously reach their incremental value at a stable structure only if

$$\overline{V}_u(N)-\overline{V}_u(N-\alpha)+\overline{V}_u(N)-\overline{V}_u(N-\beta)\leq\overline{V}_u(N)-\overline{V}_u(N-\alpha\beta) \qquad (10)$$

holds. This inequality says that the sum of the players' incremental values is less than the incremental value of $\alpha\beta$, which is an upper bound on the payoffs to $\alpha\beta$ at a stable structure. Condition (10) is surely satisfied if the players are substitutes (apply (8) to $S=N-\alpha\beta$ and rearrange). At the opposite, complements players can reach their incremental values at the same stable outcome in the very special case where (10) holds as an equality. Let us start with this case.

5.1 Complements

Recall that when $\{\alpha,\beta\}$ is admissible, the incremental value of the coalition is reached. Since also each single player gets at most his own incremental value, surely the reverse of (10) holds. This suggests that the players are complements We give here a direct proof.

Proposition 4. *Let collection $\mathcal{C}$ satisfy the partition property and $\{\alpha,\beta\}$ be in $\mathcal{C}$. Then players α and β are complements.*

Proof. For each $j=\alpha,\beta$, take an optimal $\mathcal{C}$-partition π^j of $S+j$. Add to the family $\mathcal{B}$ composed of all the elements of π^j the admissible coalition $\{\alpha,\beta\}$. $\mathcal{B}$ is composed of coalitions in $\mathcal{C}$. Furthermore it is a balanced family of $S+\alpha\beta$: Each i in $S+\alpha\beta$ belongs to 2 sets (counting twice a set that belongs to 2 partitions). Formally $\mathcal{B}$ is balanced with a weight vector γ equal to half $\delta^{\pi^\alpha}+\delta^{\pi^\beta}+1_{\{\alpha,\beta\}}$. where δ^π is the vector associated with a partition π ($\delta^\pi_C=1$ for C element of π and 0 otherwise). Consider the set of balanced weights for $\mathcal{B}$. As shown in [12], an extreme point is associated with a balanced family included in $\mathcal{B}$ and minimal (i.e. containing no balanced family). Under the partition property, a minimal balanced family is a partition. Thus, vector γ is a convex combination of some δ^π associated to partitions of $S+\alpha\beta$: there are μ_π such that

$$\mu_\pi\geq 0,\sum_\pi\mu_\pi=1 \text{ and } \delta^{\pi^\alpha}+\delta^{\pi^\beta}+1_{\{\alpha,\beta\}}=2\sum_\pi\mu_\pi\delta^\pi \qquad (11)$$

Since $\overline{V}_u(S+j) = V_u(\pi^j) = \sum_{C \in \mathcal{C}} \delta_C^{\pi^j} V_u(C)$, $j = \alpha, \beta$, one deduces

$$\overline{V}_u(S+\alpha) + \overline{V}_u(S+\beta) = 2\sum_{\pi} \mu_\pi V_u(\pi) - V_u(\{\alpha,\beta\}) \tag{12}$$

One always has $V_u(\pi) \leq \overline{V}_u(S+\alpha\beta)$ and furthermore for a partition of $S+\alpha\beta$ that contains $\{\alpha,\beta\}$, $V_u(\pi) \leq V_u(\{\alpha,\beta\}) + \overline{V}_u(S)$. Since from (11) surely $\sum_{\pi,\{\alpha,\beta\}.\in\pi} \mu_\pi = 1/2$ and $\sum_{\pi,\{\alpha,\beta\}\notin\pi} \mu_\pi = 1/2$ this gives $2\sum_\pi \mu_\pi V_u(\pi) \leq V_u(S) + \overline{V}_u(S+\alpha\beta)) + \overline{V}_u((\{\alpha,\beta\})$. This inequality together with (12) yields (9), the desired result. □

The lesson to be drawn is the following. Recall that when $\{\alpha,\beta\}$ is admissible, both players cannot be better off than at their preferred outcome by misrepresenting jointly their preferences. However by making precise how coalitions split their benefits, typically at least one of these two players will benefit from misrepresentation (since typically (10) holds as a strict inequality). There is no strategy-proof selection of the core for these two players.

5.2 Incentives for Substitutes Players

Condition (10) is necessary for a strategy-proof selection of the $\mathcal{C}$-core to exist at a given profile. Is it sufficient ? Also, what kind of restrictions on admissible coalitions ensures it is satisfied at all profile ? An answer to this question can be stated in terms of chains, which we now define.

Definition 6. *A* **chain** *between α and β is defined by two families of admissible coalitions, $(S_k,\ k = 1,..,\ell+1)$ and $(T_k,\ k = 1,..,\ell\ )$ with $\ell \geq 0$, each formed with disjoint elements, that satisfy*
α belongs to S_1 and β to $S_{\ell+1}$, no T_k contains α or β
T_k intersects S_k and S_{k+1}, $k = 1,..,\ell$.

For $\ell = 0$, a chain is simply an admissible coalition that contains both α and β. For $\ell = 1$, a chain is given by two disjoint coalitions, S_1 and S_2, one that contains α, the other β, and a third coalition T_1 that intersects both S_1 and S_2 but contains neither α nor β.

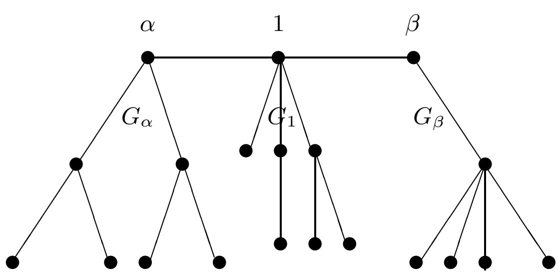

Fig. 1.

Proposition 5. *Let collection $\mathcal{C}$ satisfy the partition property. Consider two players α and β. The following properties are equivalent:*

1. *players α and β are substitutes at any profile*
2. *there is no chain between α and β*
3. *condition (10) is met for α and β at any profile.*

The absence of a chain between two players imply that no admissible coalition contains both. In particular N cannot be admissible. It is easy to check that in an assignment game there is no chain between two sellers or between two buyers. Let us consider a tree and two players α and β who are linked through player 1 as in figure 1. Letting $\mathcal{C}$ be the set of all connected coalitions except those that contain both α and β, one can show that players α and β are substitutes.

Notes and Comments. An extended version and different proofs are in Demange [7].

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Enforcing Truthful Strategies in Incentive Compatible Reputation Mechanisms

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Abstract. We commonly use the experience of others when taking decisions. Reputation mechanisms aggregate in a formal way the feedback collected from peers and compute the *reputation* of products, services, or providers. The success of reputation mechanisms is however conditioned on obtaining true feedback. Side-payments (i.e. agents get paid for submitting feedback) can make honest reporting rational (i.e. Nash equilibrium). Unfortunately, known schemes also have other Nash equilibria that imply lying. In this paper we analyze the equilibria of two incentive-compatible reputation mechanisms and investigate how undesired equilibrium points can be eliminated by using trusted reports.

1 Introduction

In a world that offers an ever increasing number of choices, we commonly use the experience of our peers when making decisions. The feedback coming from previous users can be aggregated into the *reputation* of a product, service or manufacturer, and accounts for the data that cannot be directly observed before the purchase: e.g. reliability, technical support, etc.

As reputation mechanisms become more and more popular in online markets, it is important to ensure that selfish agents have the right incentives to report honest feedback (i.e. the mechanism is incentive compatible). One way to elicit truthful information is to pay the reports according to their estimated truthfulness. Since objective verification is usually impossible, the truthfulness of a report is assessed by comparing it with other reports coming from peers. When the observations of different clients are sufficiently correlated, there exist payment rules that make truthful reporting be Nash Equilibrium (NEQ): i.e. rational agents report the truth given that all other agents report the truth. [9] and [6] describe concrete mechanisms.

Unfortunately, such mechanisms also have other NEQ points where agents lie. The existence of multiple equilibria is a serious problem when engineering real reputation mechanisms: nothing can guarantee that the desired (i.e. truthful) equilibrium strategy is selected. Moreover, the payoff generated by the truthful equilibrium is often dominated by the payoff in a non-truthful equilibrium. Thus, the selection of the truthful strategy becomes even more problematic.

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In this paper we propose a method of enforcing the selection of the truthful strategy based on trusted reports (i.e. verifiable reports coming from specialized reporters). Such reports can constitute a true reference base against which other feedback can be evaluated. When enough trusted reports are available, the incentive compatible NEQ becomes unique.

Efficiency, however, dictates that the number of trusted reports required be kept as small as possible. We therefore investigate the reputation mechanisms described in [9] and [6], and derive analytical and numerical solutions for the minimum percentage of trusted reports required to enforce the truthful strategies. Besides the results for the two specific mechanisms, the paper introduces a general methodology for eliminating undesired equilibrium points, and offers insights into the dynamics of feedback reporting mechanisms. Sections 2 and 3 briefly introduce the two reputation mechanisms. Section 4 analyzes the set of Nash equilibrium points and analytically shows how trusted reports can be used to eliminate the undesired equilibria. Numerical results are presented and interpreted in Section 5, followed by related work and a conclusion.

2 The MRZ Incentive Compatible Reputation Mechanism

In [9], Miller, Resnick and Zeckhauser (henceforth referred to as MRZ) consider that a number of clients sequentially experience the same product whose *type*[1] is drawn from a set of possible types T.[2]

The real type of the product is unknown to clients and does not change during the experiment. After every interaction, the client observes one signal s (from a set of possible signals, S, of cardinality M) about the type of the product. The observed signals are independently identically distributed conditioned on the real type t of the product. $f(s_i|t)$ denotes the probability that the signal s_i is observed when the product is of type t. $\sum_{s_i \in S} f(s_i|t) = 1$ for all $t \in T$.

After every interaction, the client is asked to submit feedback about the signal she has observed. A reputation mechanism collects the reports, and updates the probability distribution over the possible types of the product. Let p characterize the current belief of the reputation mechanism (and therefore of all agents that can access the reputation information) about the probability distribution over types. $p(t)$ is the probability that the product is of type t, and $\sum_{t \in T} p(t) = 1$. When the reputation mechanism receives a report $r \in S$, the posterior belief is updated using Bayes' Law:

$$p(t|r) = \frac{f(r|t) \cdot p(t)}{Pr[r]};$$

where $Pr[r] = \sum_{t \in T} f(r|t) \cdot p(t)$ is the probability of observing the signal r.

[1] The type of a product defines the totality of relevant characteristics of that product. e.q. quality, reliability, etc.

[2] The set of possible types is the combination of all values of the attributes that define the type. While this definition generates an infinite-size set of types, in most practical situations, approximations make the set of possible types countable. For example, the set of possible types could have only two elements: *good* and *bad*.

Every feedback is paid according to a payment rule that takes into account the current belief, the value of the report and the value of another future report submitted by some other client (called the *rater*). When this payment is defined by a *proper scoring rule*[3], MRZ show that every agent has the incentive to submit the true feedback given that the rater also reports honestly. The mechanism thus has an incentive-compatible NEQ.

One possible payment rule is:

$$R\big(p(\cdot), r, r_r\big) = \log\big(Pr[r_r|r, p(\cdot)]\big) = \log\Big(\sum_{t \in T} p(t|r) \cdot f(r_r|t)\Big);$$

where $p(\cdot)$ is the prior belief of the agent (and of the reputation mechanism), $r \in S$ is the report of the agent, $r_r \in S$ is the future report of the designated rater, and $Pr[r_r|r]$ is the posterior probability that the signal r_r will be observed by the rater given that r was observed by the current reporter. When denoting the payment received by an agent, we will frequently ignore the dependence on the belief and have $R(s_i, s_j)$ represent the payment received by an agent reporting s_i when the rater reports s_j.

3 The JF Incentive-Compatible Reputation Mechanism

Jurca and Faltings (henceforth referred to as JF) describe in [6] an incentive compatible reputation mechanism in a setting where the binary signal observed by the clients is influenced not only by the type of the service, but also by time. The probability distribution of the observed signal is thus modeled by a Markov chain of variable length.

The side-payment for reports follows a very simple rule, and does not depend on the beliefs of the agent or those of the reputation mechanism. A report is paid only if the next report submitted by some other client about the same service has the same value. The amount of the payment is dynamically scaled such that the mechanism is budget-balanced.

The Markov model for the observable signals is very appropriate for services offered by software agents, where failures are correlated. If we take the example from the previous section, and consider that the product is actually a service provided by a webservice, it is very unlikely that individual signals follow an independent distribution. A service failure due to a software or hardware problem is likely to attract other service failures in the immediate future. Likewise, a present successful invocation signals that everything works well with the infrastructure, and is probably going to be followed by other successful service invocations.

While the MRZ mechanism can also be adapted for Markov models of behavior, it requires that the model be common knowledge among the agents: i.e. all agents must agree on the length of the model and on the fact that there is a unique set of parameters characterizing that model. MRZ argue that private

[3] see [4] for an introduction to the scoring rules.

information can be accommodated by requiring the agents to first submit their private information, and than report the feedback. The computation of the payment will take into account the reported private information, and will make it rational for the agents to truthfully submit feedback afterwards.

However, reporting private information and feedback introduces additional cheating opportunities. Although no agent can obtain an expected payoff greater than the one rewarded by the truthful strategy, malicious reporters can bias the declared private information in order to make any desired report (weakly) optimal: e.g. an agent willing to bad-mouth a provider, can do so without being penalized by submitting appropriately modified private information.

Having side-payments that do not depend on the beliefs of the agents, the JF mechanism allows the agents to have private beliefs about the model of the webservice, as long as these beliefs satisfy some general constraints. Of course, the freedom of having private beliefs is paid by the constraints that limit the contexts in which incentive-compatibility is guaranteed.

4 Equilibrium Strategies

Formally, a reporting strategy of an agent a is a mapping σ from the set of signals S to the set ΔS containing all probabilistic combinations of signals from S. $\sigma(s_i) = \sum_{j \in S} \alpha_j^i s_j$ denotes that an agent a following reporting strategy σ, will report s_j with probability α_j^i given that the signal observed was s_i. $\sum_{j=1}^{M} \alpha_j^i = 1$ for all $i \in \{1, \ldots, M\}$. The set of all reporting strategies is denoted as $\mathcal{S}$.

Let σ^* be the incentive-compatible strategy, i.e. $\sigma^*(s_i) = s_i$ for all $s_i \in S$. By an abuse of notation we also use s_j to denote the "constant" reporting strategy (i.e. $s_j(s_i) = s_j$ for all $s_i \in S$) and ΔS to denote the set of all reporting strategies.

When a uses reporting strategy σ and her rater (i.e. a_r) uses the reporting strategy $\sigma' = (\beta_j^i)$, the expected payment of a when observing the signal s_i is:

$$E[\sigma, \sigma', s_i] = \sum_{j=1}^{M} \alpha_j^i \Bigg(\sum_{k=1}^{M} Pr[s_k|s_i] \cdot \Big(\sum_{l=1}^{M} \beta_l^k \cdot R(s_j, s_l) \Big) \Bigg); \qquad (1)$$

where $Pr[s_k|s_i]$ is the probability that the rater observes the signal s_k given that a has observed s_i, and the function $R(s_j, s_l)$ gives the payment made by the reputation mechanism to a when a reports the signal s_j and a_r reports s_l. For the MRZ mechanism the function $R(s_j, s_l)$ is given by one scoring rule. For the JF mechanism, the function $R(s_j, s_l)$ is 1 if $s_l = s_j$ and 0 otherwise.

Definition 1. *A reporting strategy σ is a NEQ of the reputation mechanism iff $\forall\ s_i \in S$, no agent deviates from σ, as long as her rater reports according to σ. Formally, σ satisfies: $E[\sigma, \sigma, s_i] \geq E[\sigma', \sigma, s_i], \forall s_i \in S, \sigma' \neq \sigma$.*

Definition 1 restricts possible reporting strategies to *symmetric* ones. General N-player feedback reporting games might have asymmetric Nash equilibria as well (i.e. every agent uses a different reporting strategy). However, online markets usually assume an infinite number of anonymous clients providing feedback. In this case, all reporting equilibria are symmetric. To prove that, assume an asymmetric equilibrium, and two agents, a_i and a_j using different reporting strategies: $\sigma_i \neq \sigma_j$. For both a_i and a_j the rater will be drown from the same (infinite) set of future reporters. Therefore, for both a_i and a_j, the rater's strategy will be the same strategy σ, computed as a mix of the strategies of all future reporters. Then, $E[\sigma_i, \sigma, \cdot] \geq E[\sigma_j, \sigma, \cdot]$ as σ_i is optimal for a_i and $E[\sigma_j, \sigma, \cdot] \geq E[\sigma_i, \sigma, \cdot]$ as σ_j is optimal for a_j. Consequently, $\sigma_i = \sigma_j$, and by induction all clients use the same strategy in equilibrium (i.e. the equilibrium reporting strategy is symmetric).

Both the MRZ and the JF mechanisms have many NEQ strategies. In general, finding all NEQ points of a game is a difficult problem [3]. However, for the special case of binary reputation mechanisms, Proposition 1 completely characterizes the set of equilibrium strategies.

Proposition 1. *Given a binary incentive-compatible reputation mechanism, always reporting positive feedback and always reporting negative feedback are Nash equilibria. At least one of these equilibria generates a higher payoff than the truthful equilibrium.*

Proof. Let "+" and "−" denote the positive and respectively the negative quality signals. The mechanism is incentive-compatible, so $E[\sigma^*, \sigma^*, +] \geq E[-, \sigma^*, +]$ and $E[\sigma^*, \sigma^*, -] \geq E[-, \sigma^*, -]$. Expanding $E[\cdot, \cdot, \cdot]$ and taking into account that $Pr[+|+] \geq Pr[+|-]$ (easy verifiable by applying Bayes law) we obtain: $R(+,+) \geq R(-,+)$ and $R(-,-) \geq R(+,-)$. Therefore, $E[+,+,\cdot] \geq E[-,+,\cdot]$ and $E[-,-,\cdot] \geq E[+,-,\cdot]$, and thus, according to Definition 1, the strategies $+$ and $-$ (i.e. always reporting positive, respectively negative feedback) are NEQ.

Let $\rho = \max\big(R(+,+), R(-,-)\big)$. Then,

$$\begin{aligned} E[\sigma^*, \sigma^*, +] &= Pr[+|+]R(+,+) + Pr[-|+]R(+,-) \\ &\leq Pr[+|+]R(+,+) + Pr[-|+]R(-,-) \leq \rho; \end{aligned}$$

Similarly, $E[\sigma^*, \sigma^*, +] \leq \rho$, therefore, at least one of the constant reporting NEQ strategies generates a higher expected payoff than the truthful equilibrium.

The results of Proposition 1 are valid for all binary incentive-compatible reputation mechanisms, and prove that honesty is always dominated by at least one of the constant reporting strategies. We conjecture the existence of a similar result for all IC reputation mechanisms.

4.1 The Influence of Trusted Reports

For all incentive compatible reputation mechanisms, the truthful reporting strategy σ^*, is a strict Nash equilibrium. When the report submitted by the rater is always trusted (i.e. true), the expected payment received by a, given that she has observed the signal s_i and uses the reporting strategy σ, is $E[\sigma, \sigma^*, s_i]$. As the rater's strategy is fixed, the only Nash equilibrium strategy of a is the truthful reporting strategy σ^*. Any other reporting strategy will generate a strictly lower payoff (Definition 1).

Since trusted reports are expensive, it is interesting to see if undesired Nash equilibrium points can be eliminated by using only a *probabilistic* rating against a trusted report: i.e. a report is rated with probability q against a trusted report and with probability $1-q$ against a normal report. The expected payoff to a from the equilibrium strategy σ, given that she has observed the signal s_i is then:

$$E_q[\sigma, \sigma, s_i] = q \cdot E[\sigma, \sigma^*, s_i] + (1-q) \cdot E[\sigma, \sigma, s_i]$$

The strategy σ continues to be a Nash equilibrium strategy if and only if for all other reporting strategies σ', $E_q[\sigma', \sigma, s_i] < E_q[\sigma, \sigma, s_i]$, for all signals s_i. Finding the minimum probability q such that the incentive-compatible reporting strategy remains the only Nash equilibrium strategy of the mechanism involves solving the following problem:

Problem 1. Find the minimum $q^* \in [0,1]$ such that for all q, $q^* \leq q \leq 1$, for all reporting strategies $\sigma \neq \sigma^*$, there is a signal s_i and a strategy $\sigma' \neq \sigma$ such that $E_q[\sigma, \sigma, s_i] < E_q[\sigma', \sigma, s_i]$.

Problem 1 is hard to solve in the general case, and its result are very restrictive. A relaxation would be to eliminate only those equilibrium strategies that generate a higher payoff than the incentive compatible strategy. The practical justification for this relaxation is that rational agents always choose from a set of possible equilibrium strategies the one that generates the highest payoff. Given that truthful reporting yields the highest payoff, we argue that it is not necessary from a practical perspective to eliminate all other Nash equilibrium points.

Finding the minimum probability, q^*, such that the incentive-compatible reporting strategy generates the highest payoff implies solving the following problem:

Problem 2. Find $q^* = \min(q)$, s.t. $f(q^*) = 0$, where $f(q) = \max_{\sigma, s_i} E_q[\sigma, \sigma, s_i] - E[\sigma^*, \sigma^*, s_i]$ s.t. σ is a NEQ: i.e. $E_q[\sigma, \sigma, s_k] \geq E_q[s_j, \sigma, s_k]$ for all $s_j, s_k \in S$.

Problem 2 contains two nested optimizations: (1) finding the Nash equilibrium strategy that generates the highest payoff, and (2) finding the minimum value of q (i.e. q^*) for which the highest Nash equilibrium payoff corresponds to the incentive-compatible reporting strategy. Finding the highest Nash equilibrium payoff is a NP-hard problem [3]. The function $f(q)$, on the other hand, is decreasing in q and therefore a binary search can be used to find the minimum value of q. Please note that the solutions to problem 2 also represent lower bounds for the solutions of problem 1.

The solution q^* of Problem 2 does not necessarily represent the overall percentage of trusted reports needed by the reputation mechanism. For example, the MRZ mechanism allows to reuse trusted reports. The same trusted report can be used to assess the honesty of more than one feedback. In extremis, one could imagine that the same trusted report is used to assess all other reports collected by the reputation mechanism. The actual percentage of trusted reports needed by the mechanism is hence very low, and equal to the value of q^* divided by the total number of reports that are rated against the same trusted report.

Using the same trusted report poses, however, some problems. First, all feedback has to be rated in the same time, i.e. after all reports have been submitted. This delays the side-payments to clients, and weakens the monetary incentives to report. Second, the trusted report can become outdated. In a dynamic system (service providers change their type by updating for example their infrastructure), the validity of any report is limited to a certain time window. Third, it leaves the mechanism vulnerable to mistakes of the trusted reporters.

In practice, a periodically updated set of trusted reports can be used to rate normal feedback. For any report, one trusted report can be randomly chosen from this set. Thus, a compromise can be reached between cost and stronger incentives to report the truth.

The JF mechanism, on the other hand, requires a fresh rater for every submitted report (i.e. the report of the next agent is always used to rate the present feedback). Therefore, the threshold value q^* that a report is rated against a trusted report becomes the overall percentage of trusted reports needed by the reputation mechanism.

Our analysis also offers an interesting insight for influencing the behavior of incentive compatible reputation mechanisms. From a dynamic perspective, the reputation mechanism can shift from one Nash reporting equilibrium to another. A mechanism operator will therefore be interested to know how easy it is to switch the reporting strategy from lying to truth-telling.

For example, let us take a reputation mechanism currently coordinated on an equilibrium σ. Assuming that the operator can observe σ, he can shift the reporting equilibrium to the truthful one by publicly committing to rate every report against a trusted report with probability $\bar{q} = \max_{s_i} \frac{E[\sigma,\sigma,s_i]-E[\sigma^*,\sigma^*,s_i]}{E[\sigma,\sigma,s_i]-E[\sigma,\sigma^*,s_i]}$. Note that $\bar{q} \leq q^*$, with equality holding when σ is the reporting NEQ yielding the highest payoff. Thus, in particular, the operator can drive the reputation mechanism to the truthful equilibrium in the beginning.

Once the agents have coordinated on the truthful strategy (by some external coercion or natural initiative), the operator can stop using trusted reports. It will take a significant proportion of deviators (i.e. at least $1 - q^*$) coordinated on a different payoff equilibrium in order to make it rational for the other agents to also switch to this non-truthful equilibrium. Since q^* is often quite low, this can be very useful in practice. This observation opens new research and design opportunities for practical reputation mechanisms where external intervention (e.g. trusted reports) is only periodically needed in order to (re)coordinate a sufficient proportion of agents on the desired strategy.

5 Numerical Analysis

For the JF mechanism the incentive-compatible equilibrium dominates all other equilibria only when feedback is rated against a trusted report with probability greater than $q^* = \max\left(\frac{1-Pr[-|-]}{Pr[-|-]}, \frac{1-Pr[+|+]}{Pr[+|+]}\right)$, where $Pr[+|+]$ and $Pr[-|-]$

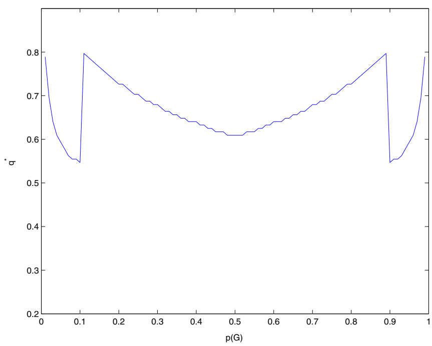

Fig. 1. Threshold value q^* for the MRZ mechanism, when $N = 2$ and $\delta = 10\%$

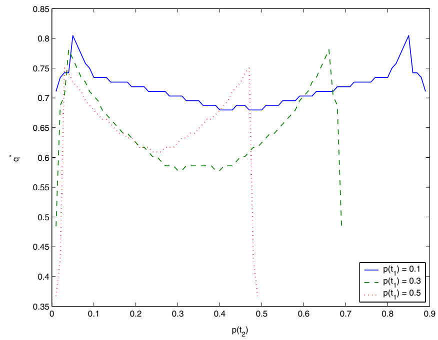

Fig. 2. Threshold value q^* for the MRZ mechanism, when $N = 3$, and $\delta = 10\%$

are the probabilities of observing a positive (respectively negative) signal in the next round given a positive (respectively negative) signal observed in the present. Both $Pr[-|-]$ and $Pr[+|+]$ vary in the interval $[0.5, 1]$ (smaller values for $Pr[-|-]$ or $Pr[+|+]$ are not allowed by the assumptions of the JF mechanism), therefore q^* can take any value between 0 and 1. For example, a webservice modeled by a Markov chain with transition probabilities $Pr[+|+] = 0.95$ and $Pr[-|-] = 0.9$[4] requires $q^* = 1/9$. The greater the correlation between the behavior of the webservice in successive transactions, the lower the threshold value, q^*. In such cases, there is little uncertainty about the signal observed by the rater, and therefore the incentive-compatible strategy of the JF mechanism yields payoffs that approach the maximum possible payoff.

For the MRZ mechanism we numerically investigate settings with N possible product types and N observable signals. Each signal characterizes one type, and uniform noise "scrambles" the observation of clients. The conditional probability distribution of signals is defined as $f(s_j|t_i) = 1 - \delta$ when $s_j = s_i$ and $f(s_j|t_i) = \frac{\delta}{N-1}$ when $s_j \neq s_i$. $\delta = 10\%$ is the "level" of noise.

For $N = 2$, Figure 1 plots the threshold value q^* against the set of possible beliefs of the clients (characterized by the prior probability assigned to the *good* type). The values of q^* range between 0.6 and 0.8. The gaps at both ends of the interval are explained by the "activation" of previously inefficient reporting strategies. When one type is very probable (e.g. the good type), the constant reporting strategy "$-$" is very inefficient. $Pr[-|-]$ is very small, therefore the payoff generated by always reporting negative feedback is lower than the payoff rewarded to the truthful strategy. As soon as the prior probability of the *bad* type becomes big enough, both constant reporting strategies are more profitable than the truthful strategy, hence more trusted reports are needed to enforce truth-telling.

[4] $Pr[+|+]$ is the probability of successful service at time $t+1$ given a successful service at time t; similarly $Pr[-|-]$ is the probability of service failure at time $t+1$ given a failure at time t.

For $N = 3$ the space of possible beliefs is two dimensional: two probabilities entirely characterizes the prior distribution over the three types. Figure 2 presents three slices through the 3-dimensional graph for three different probabilities of the type t_1: $p(t_1) = 0.1$, $p(t_1) = 0.3$ and $p(t_1) = 0.5$. The threshold value q^* varies between 0.5 and 0.8.

For higher number of types, the graphical representation of the space of beliefs becomes impossible. Moreover, solving the optimization problem for an increasing number of types (and therefore signals) becomes exponentially more difficult. However, the solution for $N = 4, 5, 6$ types and beliefs normally distributed around every type does not bring any surprises. As in the previous cases, the values of q^* range from 0.5 to 0.8, with higher values for more focused beliefs, and lower values for increased ambiguity in beliefs.

6 Related Work

In their seminal papers, Kreps, Wilson, Milgrom and Roberts [8] prove that cooperative equilibria can exist in finitely repeated games due to the reputation effect. Since then, numerous computational reputation mechanisms have been described, ranging from mechanisms based on direct interactions [2] to complex social networks [11] where agents ask and give recommendations to their peers. Centralized implementations as well as completely decentralized [1] have been investigated.

Besides the two mechanisms treated in this paper, a number of other mechanisms address the problem of eliciting honest feedback from self interested participants. For e-Bay-like auctions, the Goodwill Hunting mechanism [5] provides a way to make the sellers indifferent between lying or truthfully declaring the quality of the good offered for sale. Momentary gains or losses obtained from misrepresenting the good's quality are later compensated by the mechanism which has the power to modify the announcement of the seller.

Jurca and Faltings [7] take a different approach and achieve in equilibrium truthful reporting by comparing the two reports coming from the buyer and the seller involved in the same transaction. Using the same idea, Papaioannou and Stamoulis [10] describe a mechanism suitable for P2P environments.

This paper also relates to the vast literature concerned with computing Nash equilibrium strategies [3] and with the ongoing efforts of the networking community to design routing algorithms that have a unique Nash equilibrium point with the desired properties.

7 Conclusion

Obtaining true feedback is of vital importance to the success of online reputation mechanisms. When objective verification is not available, economic measures must exist to ensure that self interested agents truthfully report their observations. Unfortunately, existing incentive compatible schemes have multiple Nash equilibria. Moreover, lying equilibrium strategies usually yield higher payoffs than the truthful strategy.

In this paper we analyze the influence of trusted reports on the set of Nash equilibria of two existing incentive-compatible reputation mechanisms. We emphasize the existence of lying Nash equilibria, and investigate how such undesired equilibrium points can be eliminated. By having a fraction of trusted reports it is possible to have a mechanism where the truthful strategy is the only (or the most attractive) strategy to be followed.

Besides the numerical analysis of the two reputation mechanisms we also provide a general methodology for eliminating undesired equilibrium points from incentive compatible reputation mechanisms.

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Strategy/False-Name Proof Protocols for Combinatorial Multi-attribute Procurement Auction: Handling Arbitrary Utility of the Buyer

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Abstract. In this paper, we develop new protocols for a combinatorial, multi-attribute procurement auction in which each sales item (task) is defined by several attributes called quality, the buyer is the auctioneer (e.g., a government), and the sellers are the bidders. Furthermore, there exist multiple tasks, and both buyer and sellers can have arbitrary (e.g., complementary/substitutable) preferences on a bundle of tasks.

In this setting, there is a chance that a VCG protocol cannot satisfy Individual Rationality (IR) for the buyer, i.e., the buyer's utility can be negative. We show that if a surplus function is concave, then the VCG protocol satisfies IR and the protocol is also false-name-proof, i.e., using multiple identifiers provides no advantage. Furthermore, we present a modification of the VCG protocol that satisfies IR even if the concavity condition is not satisfied. The key idea of this protocol is to introduce a special type of bidder called the reference bidder. We assume that the auctioneer knows the upper-bound of the reference bidder's cost. Introducing such a reference bidder is similar to setting reservation prices in standard combinatorial auctions. Furthermore, we develop a new false-name-proof protocol that is based on the idea of the Leveled Division Set (LDS) protocol.

1 Introduction

Recently, auctions—in particular, combinatorial auctions—have attracted considerable attention from computer science/AI researchers (an extensive survey is presented in [1]). Previous studies have dealt mostly with models in which price is the unique strategic dimension. In many situations, however, conducting negotiations on multiple attributes of a deal is required. For example, when allocating tasks, the attributes of a deal may include starting time, ending deadline, accuracy level, etc. A service can be characterized by its quality, supply time, and risk of the service not being supplied. Also, a product can be characterized by several attributes such as size, weight, and supply date.

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This problem becomes more complicated when there are multiple tasks, services, or products. For example, the task of constructing a large building can be divided into many subtasks. One construction supplier might be able to handle multiple subtasks, while another company is specialized to a particular subtask. In addition, since each construction firm may contract processes under different conditions, e.g., their quality, appointed date, price and so on, the utility of the government may depend on these conditions in a complex fashion.

In this paper, we follow the model of a combinatorial, multi-attribute procurement auction presented in [2], which can handle such situations. In this model, each sales item is defined by several attributes called quality, the buyer is the auctioneer (e.g., a government), and the sellers are the bidders. Furthermore, there exist multiple items, and both buyer and sellers can have arbitrary (e.g., complementary/substitutable) preferences on a bundle of items. In this model, we assume the preference/type of the buyer is known and the research goal is to develop strategy/false-name proof protocols for sellers. This assumption is natural in such cases as the procurement by the government, etc. Aside from this assumption, this model is quite general. For example, the quality of a task can have arbitrary dimensions. Also, there is no restriction on the possible types of a seller's cost function.

In [2], we have shown that the Vickrey-Clarke-Groves (VCG) type protocol [3, 4, 5] cannot guarantee Individual Rationality (IR) for the buyer, i.e., the buyer's utility can be negative. In this paper, we first show one sufficient condition where the VCG protocol can guarantee IR for the buyer, i.e., the *concavity* of a surplus function. Furthermore, we develop a modified version of the VCG protocol that can guarantee IR even if this condition cannot be satisfied. The key idea of this protocol is the introduction of a *reference bidder*. We assume that the buyer knows the upper-bounds of the reference bidder's cost. In many situations, it is quite natural to assume that a buyer has some knowledge of the costs of some bidders.

Furthermore, the possibility of a new type of fraud called the false-name bids, which exploits the anonymity available over the Internet, is discussed in [6, 7]. False-name bids are bids submitted under fictitious names, e.g., multiple e-mail addresses. In this paper, we develop a new false-name-proof combinatorial multi-attribute procurement auction protocol based on the Leveled Division Set (LDS) protocol [7].

2 Related Works

Traditionally, very little theoretical work has been conducted on multi-attribute auctions, with the notable exception of [8]. In this work, bidders can bid on both price and quality, and bids are evaluated by a scoring rule designed by a buyer. In addition, first score and second score sealed bid auctions have been proposed. However, in this work, the quality is assumed to be one-dimensional. Also, protocols and strategies of a multi-attribute English auction were proposed in [9], and then strategy with a deadline was studied in [10]. In these studies, the processes of auctions are sequential, and they provide the automated bidder agents and their strategies. The value of the quality is extended to two dimensions.

In these studies, assigning multiple tasks is not considered. On the other hand, these works consider the incentive issues of the buyer, while we assume the buyer's type is public. Also, these works propose non-direct revelation mechanisms, which require less exposure of private information than the direct revelation protocols developed in this paper.

Bichler [11] carried out an experimental analysis of multi-attribute auctions, showing through experiments that the utility scores achieved in multi-attribute auctions were higher than those of single-attribute auctions.

Our model is based on the model introduced in [2]. we presented a VCG protocol and a false-name-proof protocol. However, the VCG protocol cannot guarantee IR in general. Also, our false-name-proof protocol can be applied only when the buyer's gross utility has an additive form.

3 Model

Here, we show the model of a combinatorial multi-attribute procurement auction presented in [2].

- There exists a single buyer 0.
- There exists a set of sellers/bidders $N = \{1, 2, \ldots, n\}$. We use the pronoun "he" to represent a seller/bidder and the pronoun "she" to represent the buyer.
- There exists a set of tasks $T = \{t_1, \ldots, t_m\}$.
- Each bidder i privately observes his type θ_i, which is drawn from a set Θ.
- For each task t_j, quality $q_j \in Q$ is defined.
- A possible allocation of tasks to bidders is represented as $\overrightarrow{B} = (B_1, \ldots, B_n)$, where $B_i \subseteq T$ and for $i \neq k$, $B_i \cap B_k = \emptyset$ holds.
- A profile of qualities is represented as $\overrightarrow{q} = (q_1, \ldots, q_m)$.
- For a quality profile $\overrightarrow{q}$ and bundle $B_i = \{t_{i,1}, t_{i,2}, \ldots\}$, we represent a projection of B_i onto $\overrightarrow{q}$ as $\overrightarrow{q}_{B_i} = (q_{t_{i,1}}, q_{t_{i,2}}, \ldots)$.
- The cost of bidder i when the allocation is B_i and the achieved quality profile is $\overrightarrow{q}_{B_i}$ is represented as $c(\theta_i, B_i, \overrightarrow{q}_{B_i})$. We assume c is normalized as $c(\theta_i, \emptyset, ()) = 0$.
- The gross utility of buyer 0, when the obtained quality profile is $\overrightarrow{q}$, is represented as $V(\overrightarrow{q})$.
- The payment from the buyer to each seller/bidder i is represented as p_i.
- We assume that each participant's utility is quasi-linear, i.e., for each seller i, his utility is represented as $p_i - c(\theta_i, B_i, \overrightarrow{q}_{B_i})$. Also, for the buyer, her (net) utility is $V(\overrightarrow{q}) - \sum_{i \in N} p_i$.
- For an unallocated task t_j, we assume that the quality of t_j is $0 \in Q$. V is normalized by $V(\overrightarrow{q}_0) = 0$ as $\overrightarrow{q}_0 = (0, 0, \ldots, 0)$.

Note that although there is only one parameter q_j for representing the quality of task t_j, this does not mean that our model can handle only one-dimensional quality, i.e., q_j can be a vector of multiple attributes.

An auction protocol is (dominant-strategy) *incentive compatible* (or *strategy-proof*) if declaring the true type/evaluation values is a dominant strategy for each bidder, i.e., the optimal strategy regardless of the actions of other bidders. Furthermore, we define that an auction protocol is *false-name-proof* if declaring the true type *by using a single identifier* is a dominant strategy for each bidder.

An auction protocol is Individually Rational (IR) if no bidder suffers any loss in a dominant-strategy equilibrium, i.e., the cost never exceeds the payment. In a private value auction, IR is indispensable; no bidder wants to participate in an auction where he might be paid less money than the amount he spent to achieve the task. Therefore, in this paper, we restrict our attention to IR protocols.

We say an auction protocol is *Pareto efficient* when the sum of all participants' utilities (including that of the auctioneer), i.e., the social surplus, is maximized in a dominant-strategy equilibrium. In our model, the obtained social surplus is represented as $V(\overrightarrow{q}) - \sum_{i \in N} c(\theta_i, B_i, \overrightarrow{q}_{B_i})$.

Example 1. There are two bidders $N = \{1, 2\}$ and two tasks $T = \{t_1, t_2\}$. In this example, we assume that for each task t_j, its quality q_j is one-dimensional and continuous. The cost function of bidder 1 can be represented as follows.

- $c(\theta_1, \{t_1\}, (q_1)) = \frac{1}{4} q_1$,
- $c(\theta_1, \{t_2\}, (q_2)) = \frac{1}{2} q_2$,
- $c(\theta_1, \{t_1, t_2\}, (q_1, q_2)) = \frac{1}{4} q_1 + \frac{1}{4} q_2$

Assume that $V(\overrightarrow{q}) = \sum_i \sqrt{q_i}$. When both t_1 and t_2 are allocated to the bidder 1 with $\overrightarrow{q} = (4, 4)$, then the cost of bidder 1 is $c(\theta_1, \{t_1, t_2\}, (4, 4)) = 2$. Social surplus is $V((4, 4)) - c(\theta_1, \{t_1, t_2\}, (4, 4)) = 2$. If we assume that the cost of bidder 2 is always greater than that of bidder 1, then this allocation is Pareto efficient.

4 VCG Protocol

4.1 Protocol Description

The VCG protocol presented in [2] can be described as follows.

Definition 1 (VCG).

- *Each bidder i declares his type $\tilde{\theta}_i$, which can be different from his true type θ_i.*
- *Based on declared types, an allocation and a quality profile is determined so that the social surplus is maximized:*

$$(\overrightarrow{B}^*, \overrightarrow{q}^*) = \arg \max_{(\overrightarrow{B}, \overrightarrow{q})} V(\overrightarrow{q}) - \sum_{j \in N} c(\tilde{\theta}_j, B_j, \overrightarrow{q}_{B_j}).$$

- *For i, payment p_i is defined as follows:*

$$p_i = [V(\overrightarrow{q}^*) - \sum_{j \neq i} c(\tilde{\theta}_j, B_j^*, \overrightarrow{q}^*_{B_j^*})] - [V(\overrightarrow{q}^{\sim i,*}) - \sum_{j \neq i} c(\tilde{\theta}_j, B_j^{\sim i,*}, \overrightarrow{q}^{\sim i,*}_{B_j^{\sim i,*}})]$$

$(\overrightarrow{B}^{\sim i,*}, \overrightarrow{q}^{\sim i,*})$ *is an allocation and quality profile that maximizes social surplus when* i *is excluded . More specifically, for an allocation that does not allocate any task to bidder* i*, i.e.,* $\overrightarrow{B}^{\sim i} = (B_1, \ldots, B_{i-1}, \emptyset, B_{i+1}, \ldots, B_n)$*,* $(\overrightarrow{B}^{\sim i,*}, \overrightarrow{q}^{\sim i,*})$ *is defined as follows:*

$$\arg \max_{(\overrightarrow{B}^{\sim i}, \overrightarrow{q}^{\sim i})} V(\overrightarrow{q}^{\sim i}) - \sum_{j \neq i} c(\tilde{\theta}_j, B_j^{\sim i}, \overrightarrow{q}_{B_j^{\sim i}}^{\sim i}).$$

4.2 Limitation of the VCG Protocol

One serious limitation of the VCG protocol is that it cannot guarantee IR for the buyer. Let us show an example.

Example 2. We assume that there are two bidders $N = \{1, 2\}$, two tasks $T = \{t_1, t_2\}$, and for each task, there are two quality levels 0 and 1, where 0 means the task is not performed at all. We assume $V((1,1)) = 10$, $V((1,0)) = V((0,1)) = V((0,0)) = 0$, i.e., these tasks are all-or-nothing for the buyer. Assume bidder 1 can execute t_1 with quality 1 at cost 1, and bidder 2 can execute t_2 with quality 1 at cost 1. In this case, the VCG protocol assigns t_1 to bidder 1 and t_2 to bidder 2. If we exclude bidder 1, there is no bidder who can perform task t_1, and thus the social surplus becomes 0. Therefore, the payment for bidder 1 in the VCG protocol is calculated as: $(10 - 1) - 0 = 9$. The payment for bidder 2 is also 9. The total payment of the buyer is 18, which is larger than her gross utility $V((1, 1)) = 10$. Thus, the VCG protocol does not satisfy IR for the buyer.

Actually, in [2] it is proved that there exists no protocol that satisfies all of the following conditions simultaneously: Pareto efficiency, strategy-proofness, and IR for both the buyer and the sellers.

Also, the VCG protocol is not false-name-proof.

5 Sufficient Condition Where VCG Is IR and False-Name-Proof

For a set of bidders X, social surplus function $U(X)$ is defined as follows.

$$U(X) = \max_{(\overrightarrow{B}, \overrightarrow{q})} V(\overrightarrow{q}) - \sum_{j \in N} c(\theta_j, B_j, \overrightarrow{q}_{B_j})$$

Definition 2. *We say* $U(\cdot)$ *is concave over bidders if for all possible sets of bidders* X, Y, Z*, where* $X \subset Y$*, the following condition holds:* $U(X \cup Z) - U(X) \geq U(Y \cup Z) - U(Y)$*.*

The intuitive meaning of this condition is as follows. The increase in the social surplus by adding a set of bidders Z becomes smaller if the original set becomes larger. In [6], it was showed that if the concavity condition is satisfied, the VCG protocol is false-name-proof in standard combinatorial auctions.

Theorem 1. *If* $U(\cdot)$ *is concave, then the VCG protocol satisfies IR for the buyer.*

Due to the space limitation, we omit the proof but we can show that the VCG protocol satisfies IR for the buyer and is also false-name-proof if the surplus function satisfies concavity by a similar proof presented in [6].

6 Modified VCG Protocol

Next, we show a modification of the VCG protocol that satisfies the buyer's IR without the concavity condition. First, we introduce a notion called *reference bidder* r, which has the following properties.

- We assume the buyer knows the upper-bound of a cost when task t_k is allocated to reference bidder at quality q_k, i.e., the buyer knows c_{r,t_k,q_k}, where $c_{r,t_k,q_k} \geq c(\theta_r, \{t_k\}, (q_k))$ holds.
- The cost of the reference bidder is additive, i.e., for arbitrary B, $\overrightarrow{q}_B = (q_1, q_2, \ldots, q_k, \ldots)$, $c(\theta_r, B, \overrightarrow{q}_B) = \sum_{t_k \in B} c(\theta_r, \{t_k\}, (q_k))$ holds. Therefore $c(\theta_r, B, \overrightarrow{q}_B) \leq \sum_{t_k \in B} c_{r,t_k,q_k}$ holds.
- For arbitrary quality profile $\overrightarrow{q} = (q_1, \ldots, q_k, \ldots)$, $V(\overrightarrow{q}) \geq \sum_{t_k \in M} c_{r,t_k,q_k}$ holds. This means that for any quality, the buyer's net utility is non-negative when all tasks are allocated to the reference bidder and the payment is equal to the sum of the upper-bounds.

It is quite natural to assume that a buyer has some knowledge on the upper-bounds of costs for some bidders. For example, assume a company is making a decision on whether to create a product in-house or to outsource the production, i.e., the company can make some profit by in-house production but the cost might be reduced by outsourcing. The company can make this decision by performing an auction. In this case, the company itself (when it does in-house production) can be considered the reference bidder.

Furthermore, a manufacturer can publish listed prices for some products. The government can use an auction to find out better deals than the listed prices by inviting other manufacturers. In such a case, the manufacturer that publishes the listed prices can be considered as the reference bidder. Also, it is a common practice in procurement auctions for Japanese governments and municipalities to set an upper-bound price, i.e., if all of the bids exceeds the upper-bound, the auction is canceled. Such an upper-bound price is calculated by estimating the cost of a typical bidder.

Now, the third assumption regarding the reference bidder might seem too restrictive. Let us assume that for a particular quality profile $\overrightarrow{q}$, this assumption does not hold, i.e., $V(\overrightarrow{q}) < \sum_{t_k \in M} c_{r,t_k,q_k}$. Even in this case, we can still use this protocol. The buyer's utility can be negative, but the worst-case loss is bounded by $V(\overrightarrow{q}) - \sum_{t_k \in M} c_{r,t_k,q_k}$. Also, if a buyer does not want to take the risk of her utility becoming negative, she can simply exclude the quality profile $\overrightarrow{q}$ if $V(\overrightarrow{q}) < \sum_{t_k \in M} c_{r,t_k,q_k}$ holds.

The modification we apply to the VCG is as follows.

- The optimal allocation and the quality profile that maximize the social surplus are calculated in the same way as for the VCG, except that for the reference bidder r, c_{r,t_k,q_k} is used instead of his true cost.
- If the quality profile is $q^* = (q_1^*, \ldots, q_k^*, \ldots)$ and a set of tasks B_r is allocated to the reference bidder, the payment to the reference bidder is equal to $\sum_{t_k \in B_r} c_{r,t_k,q_k^*}$

The following theorem holds.

Theorem 2. *The modified VCG protocol is IR for both the buyer and sellers.*

To prove Theorem 2, we use the following lemma.

Lemma 1. *For the allocation $\overrightarrow{B}^*$ and quality profile $\overrightarrow{q}^* = (q_1^*, \ldots, q_k^*, \ldots, q_m^*)$, the payment to bidder i is less than the sum of the upper-bound of the reference bidder's costs for B_i^*, i.e.,$p_i \leq \sum_{t_k \in B_i^*} c_{r,t_k,q_k^*}$ holds.*

Due to the space limitation, we omit the proof. The proof of Theorem 2 is as follows.

Proof: The only difference between this protocol and the previous VCG protocol is the existence of the reference bidder. Therefore, the protocol is IR for sellers.

The proof of the buyer's IR is as follows. By Lemma 1, for all i, $p_i \leq \sum_{t_k \in B_i^*} c_{r,t_k,q_k^*}$ holds. Then, for the net utility of the buyer, the following condition holds, since we assume the net utility of the buyer is non-negative when all tasks are performed by the reference bidder and the payment is equal to the sum of the upper-bounds.

$$V(\overrightarrow{q}^*) - \sum_{i \in N} p_i \geq V(\overrightarrow{q}^*) - \sum_{t_k \in T} c_{r,t_k,q_k^*} \geq 0$$

Therefore, the buyer's utility is non-negative. □

In general, this protocol cannot guarantee Pareto efficiency, since for the reference bidder, we use upper-bounds instead of the true costs.

7 False-Name-Proof Protocol

The VCG protocol is not false-name-proof even if we introduce the reference bidder.

In this section, we present a new false-name-proof protocol based on the Leveled Division Set (LDS) protocol [7], which was developed for standard combinatorial auctions. In this paper, for notation simplicity, we use a simplified version of the LDS protocol in which only a single division is allocated to one level.

First, we define a *division* of tasks.

- A division is a set of bundles, $D = \{B | B \subseteq T\}$, where $\forall B, B' \in D$ and $B \neq B'$, $B \cap B' = \emptyset$ holds[1].

Next, a leveled division set is defined as follows.

Definition 3. *Leveled Division Set (LDS)*

- *Levels are defined as* $1, 2, ..., max\_level$.
- *For each level, a division D_l and a quality profile $\overrightarrow{q}^l = (q_1^l, \ldots, q_k^l, \ldots, q_m^l)$ are defined.*

A leveled division set must satisfy the following conditions:

1. *$D_1 = \{T\}$: the division of level 1 consists of a bundle of all tasks.*
2. *For each level and its division, a union of multiple bundles in the division is always included in a division of a smaller level, and they have the same quality, i.e., $\forall l > 1, \forall D' \subseteq D_l, where |D'| \geq 2, B_u = \cup_{B \in D'} B$, then there exists a level $l' < l$, with a division $D_{l'}$, where $B_u \in D_{l'}$ and $\overrightarrow{q}^l_{B_u} = \overrightarrow{q}^{l'}_{B_u}$.*

Table 1. Example of LDS (# of tasks is 3)

level	division	quality (q_1, q_2, q_3)
1	$\{(t_1, t_2, t_3)\}$	(2, 2, 2)
2	$\{(t_1, t_2, t_3)\}$	(2, 2, 1)
3	$\{(t_1, t_2, t_3)\}$	(2, 1, 2)
4	$\{(t_1, t_2)\}$	(2, 2, 2)
5	$\{(t_1, t_2)\}$	(2, 1, 2)
6	$\{(t_2, t_3)\}$	(2, 2, 2)
7	$\{(t_1, t_3)\}$	(2, 2, 2)
8	$\{(t_1, t_3)\}$	(2, 1, 2)
9	$\{(t_1), (t_2), (t_3)\}$	(2, 2, 2)

Table 1 shows an example of leveled division sets, where the number of tasks is 3 and the quality q_i is 1 or 2.

For a division $D_l = \{B_{l,1}, B_{l,2}, \ldots\}$ and one feasible allocation of tasks $\overrightarrow{B} = (B_r, B_1, B_2, ..., B_n)$, we say $\overrightarrow{B}$ is allowed under D_l if the following conditions are satisfied.

1. Tasks that belong to the same bundle in a division must be allocated to the same bidder. Also, if two tasks belong to different bundles in the division, they must be allocated to different bidders, except for the case where they are allocated to the reference bidder, i.e., if $B_{l,k} \cap B_i \neq \emptyset$ for $i \neq r$, then $B_{l,k} = B_i$. Also, if $B_{l,k} \cap B_r \neq \emptyset$, then $B_{l,k} \subseteq B_r$.
2. If a task does not belong to any set in the division, it must be allocated to the reference bidder r, i.e., $\forall t_k$, if $\forall B_{l,k} \in D_l, t_k \notin B_{l,k}$ holds, then $t_k \in B_r$.

Also, we define *allowed allocations* in level l (denoted as SB_l) as a set of allocations that are allowed under D_l.

To execute the LDS protocol, the auctioneer must pre-define the leveled division set. Each bidder i declares his type $\tilde{\theta}_i$, which can be different from his true type θ_i.

To search the level in which tasks should be allocated, the auctioneer calls the procedure LDS(1). LDS(l) is a recursive procedure defined as follows.

Definition 4. *Procedure LDS(l)*

Step 1: *If there exists exactly one bidder $x \in N$ whose costs satisfy the following condition: $\exists B_x \in D_l$, where $c(\tilde{\theta}_x, B_x, \overrightarrow{q}^{\,l}_{B_x}) \leq \sum_{t_k \in B_x} c_{r,t_k,q^l_k}$,*
then compare the results obtained by the procedure VCG(l), defined below, and by LDS($l+1$) and then choose the one that gives the larger utility for bidder x.
In this case, we say bidder x is a pivotal *bidder. When choosing the result of LDS(l + 1), we don't assign any task, nor transfer money, to normal bidders other than x, although the tasks assigned to bidder x and his payment are calculated as if tasks were allocated to the other bidders. The tasks that are not assigned to bidder x are assigned to the reference bidder.*

[1] We assume that $\bigcup_{B \in D} B \subseteq T$ holds, but $\bigcup_{B \in T} B$ is not necessarily equal to T.

Step 2: *If there are at least two bidders* $x_1, x_2 \in N, x_1 \neq x_2$ *whose costs satisfy the following condition:* $\exists B_{x_1} \in D_l, \exists B_{x_2} \in D_l$, *where* $c(\tilde{\theta_{x_1}}, B_{x_1}, \overrightarrow{q}^l_{B_{x_1}}) \leq \sum_{t_k \in B_{x_1}} c_{r,t_k,q^l_k}$, $c(\tilde{\theta_{x_2}}, B_{x_2}, \overrightarrow{q}_{l,B_{x_2}}) \leq \sum_{t_k \in B_{x_2}} c_{r,t_k,q^l_k}$,
then apply the procedure VCG(l*).*

Step 3: *Otherwise: call LDS(*$l+1$*), or terminate if* $l = max\_level$.

Procedure VCG(l) is defined as follows.

Definition 5. *Procedure VCG(*l*)*

VCG(l**):** *Choose an allocation* $\overrightarrow{B}^* \in SB_l$ *such that it maximizes* $V(\overrightarrow{q}^l) - \sum_{j \in N} c(\tilde{\theta}_j, B_j, \overrightarrow{q}^l_{B_j})$. *The payment to bidder* x *(where* $x \neq r$*) is calculated as*

$$p_x = [V(\overrightarrow{q}^l) - \sum_{j \neq x} c(\tilde{\theta}_j, B^*_j, \overrightarrow{q}^l_{B_j})] - [V(\overrightarrow{q}^l) - \sum_{j \neq x} c(\tilde{\theta}_j, B^{\sim x,*}_j, \overrightarrow{q}^l_{B^{\sim x}_j})]$$

$\overrightarrow{B}^{\sim x,*}$ *is an allocation in* SB_l *that maximizes social surplus when excluding* x.

Theorem 3. *The LDS protocol satisfies IR for both the buyer and sellers.*

We omit the detailed proof due to space constraints. It is basically obvious since the LDS uses the result of VCG(l), i.e., the modified VCG protocol that satisfies IR.

The following theorem holds.

Theorem 4. *The LDS protocol is false-name-proof.*

Proof: Assume that the result of level l is used, bidder i uses two identifiers i' and i'', and each bidder is allocated $B_{i'}$ at quality $q^l_{B_{i'}}$ and $B_{i''}$ at $q^l_{B_{i''}}$, respectively. As shown by Lemma 1, the payment to i' is less than (or equal to) $\sum_{t_k \in B_{i'}} c_{r,t_k,q^l_k}$, and the payment to i'' is less than (or equal to) $\sum_{t_k \in B_{i''}} c_{r,t_k,q^l_k}$.

On the other hand, by the condition of Leveled Division Set, $B_{i'} \cup B_{i''}$ exists in the smaller level $l' < l$, and $q^l_{B_{i'}} = q^{l'}_{B_{i'}}$ and $q^l_{B_{i''}} = q^{l'}_{B_{i''}}$ hold. Therefore, if bidder i declares the cost of performing tasks $B_{i'} \cup B_{i''}$ at qualities $q^l_{B_{i'}}$ and $q^l_{B_{i''}}$ as slightly less than $\sum_{t_k \in B_{i'} \cup B_{i''}} c_{r,t_k,q^{l'}_k}$, then i becomes a pivotal bidder at level l' and the payment to i by VCG(l') is equal to $\sum_{t_k \in B_{i'} \cup B_{i''}} c_{r,t_k,q^{l'}_k}$. Since allocated tasks and qualities are the same and the payment is greater (or the same), i's utility without false-name bids is greater than (or the same as) that with false-name bids. By a similar argument, we can show that a bidder cannot increase his utility by using more than two identifiers.

Next, we show that declaring his true type is a dominant strategy assuming that each bidder uses a single identifier. If applied level l does not change, then clearly, declaring his true type is a dominant strategy since the result of the VCG(l) is used. On the other hand, applied level l can change to l' only if (1) bidder i becomes a pivotal bidder at $l' < l$ by under-declaring his costs, or (2) bidder i, who is a pivotal bidder if he declares his true type becomes a non pivotal bidder at $l' > l$ by over-declaring his costs. In case (1), bidder i cannot perform any bundle of tasks $B \in D_{l'}$ at quality $q^{l'}_B$ with a cost less than $\sum_{t_k \in B} c_{r,t_k,q_k^{l'}}$. Since his payment is equal to $\sum_{t_k \in B} c_{r,t_k,q_k^{l'}}$, his utility cannot be positive. In case (2), if bidder i prefers the result of level l', he can choose it when he declares his true type since he is a pivotal bidder. Therefore, his utility does not change.

From the above arguments, the LDS protocol is false-name-proof. □

8 Conclusions

In this paper, we developed new strategy/false-name proof protocols that can be used for combinatorial multi-attribute procurement auctions. First, we showed a single sufficient condition (concavity of a surplus function) where the VCG protocol becomes IR for the buyer and false-name-proof. Next, we presented a modification of the VCG protocol that is IR even when the concavity condition cannot be satisfied. The key idea of this protocol is the introduction of a reference bidder. By assuming that the buyer knows the upper-bound of the costs of the reference bidder, we can successfully limit the payments in the VCG protocol and guarantee the IR for the buyer. Furthermore, we developed a new false-name-proof protocol that is based on the LDS protocol.

Although there are many situations in which the existence of a reference bidder is quite reasonable, the assumption that the buyer's utility is non-negative for all possible quality profiles if the reference bidder performs all of the tasks might be too restrictive. We hope to develop protocols that can work under weaker assumptions. Also, we intend to experimentally evaluate the efficiencies of the VCG protocol and the LDS protocol.

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Practical Zero-Knowledge Arguments from Σ-Protocols

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Abstract. Zero-knowledge (ZK) plays a central role in the field of modern cryptography and is a very powerful tool for constructing various cryptographic protocols, especially cryptographic protocols in E-commerce. Unfortunately, most ZK protocols are for general $\mathcal{NP}$ languages with going through general $\mathcal{NP}$-reductions, and thus cannot be directly employed in practice. On the other hand, a large number of protocols, named Σ-protocols, are developed in industry and in the field of applied cryptography for specific number-theoretic languages (e.g. DLP and RSA), which preserves the ZK property only with respect to *honest* verifiers (i.e., they are not real ZK) but are highly practical. In this work, we show a *generic yet practical* transformation from Σ-protocols to practical (real) ZK arguments without general $\mathcal{NP}$-reductions under either the DLP or RSA assumptions.

1 Introduction

Zero-knowledge (ZK) protocols are remarkable since they allow a prover to validate theorems to a verifier without giving away any other knowledge (computational advantage). This notion was suggested in [5] and its generality was demonstrated in [4]. Since their introduction, zero-knowledge proofs have proven to be very powerful as a building block in the construction of cryptographic protocols, especially cryptographic protocols in E-commerce like identification schemes, electronic money and payment systems, and electronic voting and bidding, et al.

Unfortunately, most ZK protocols are feasible solutions for general $\mathcal{NP}$ languages with going through general $\mathcal{NP}$-reductions, and thus cannot be directly employed in practice, although ZK protocols for general languages (i.e., $\mathcal{NP}$) show important plausibility since many important statements are in $\mathcal{NP}$. On the other hand, a large number of protocols, named Σ-protocols, are developed in industry and in the field of applied cryptography for specific number-theoretic languages (e.g. DLP and RSA), which preserves the ZK property only with respect to *honest* verifiers (i.e., they are not real ZK) but are highly practical. Thus, it's naturally desirable to bridge the gap between theory and practice in the field of ZK by developing a *generic yet practical* transformation from Σ-protocols to (real) ZK protocols *without general* $\mathcal{NP}$*-reductions*.

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In this work, we show how to transform any Σ-protocols into zero-knowledge arguments *without going through general* $\mathcal{NP}$*-reductions* under either the DLP or RSA assumptions. The transformation also can be easily extended to be applicable to any public-coin HVZK protocols. Our transformation only incurs additional constant rounds (specifically, 3 rounds). In particular, if the starting Σ-protocols are practical then the transformed protocols are also *practical.* To this end, we investigate and employ, in a novel way, the DLP and RSA based trapdoor commitment schemes [1, 9], with the OR-technique and the Naor-Yung key pair trick that are originally introduced in the public-key encryption PKE setting [8].

1.1 Related Works

Recently, Micciancio and Petrank presented a transformation for achieving ZK *proofs* from HVZK protocols without general $\mathcal{NP}$-reductions *under the decisional Diffie-Hellman (DDH) assumption* [7]. In comparison, our transformation is for ZK *arguments* and *under either DLP (that is weaker than DDH) or RSA assumptions.* We note that the additional round-complexity incurred by both the Micciancio-Petrank transformation and our transformation is the same (i.e. 3 rounds).

2 Preliminaries

In this section, we quickly recall basic definitions and major tools used in this work.

We use standard notations and conventions below for writing probabilistic algorithms, experiments and interactive protocols. If A is a probabilistic algorithm, then $A(x_1, x_2, \cdots; r)$ is the result of running A on inputs $x_1, x_2, \cdots$ and coins r. We let $y \leftarrow A(x_1, x_2, \cdots)$ denote the experiment of picking r at random and letting y be $A(x_1, x_2, \cdots; r)$. If S is a finite set then $x \leftarrow S$ is the operation of picking an element uniformly from S. If α is neither an algorithm nor a set then $x \leftarrow \alpha$ is a simple assignment statement. By $[R_1; \cdots; R_n : v]$ we denote the set of values of v that a random variable can assume, due to the distribution determined by the sequence of random processes $R_1, R_2, \cdots, R_n$. By $\Pr[R_1; \cdots; R_n : E]$ we denote the probability of event E, after the ordered execution of random processes $R_1, \cdots, R_n$.

Definition 1 (interactive argument system). *A pair of probabilistic polynomial-time interactive machines, $\langle P, V\rangle$, is called an interactive argument system for a language L if the following conditions hold:*

- *Completeness. For every $x \in L$, there exists a string w such that for every string z,* $\Pr[\langle P(w), V(z)\rangle(x) = 1] = 1$.
- *Computational soundness. For every polynomial-time interactive machine P^*, and for all sufficiently large n's and every $x \notin L$ of length n and every w and z,* $\Pr[\langle P^*(w), V(z)\rangle(x) = 1]$ *is negligible in n. An interactive protocol*

is called a proof *for L, if the soundness condition holds against any (even power-unbounded)* P^* *(rather than only PPT* P^**).*

An interactive system is called a public-coin system if at each round the prescribed verifier can only toss coins and send their outcome (random challenge) to the prover.

Definition 2 ((black-box) zero-knowledge). *Let* $\langle P, V\rangle$ *be an interactive system for a language* $L \in \mathcal{NP}$*, and let* R_L *be the fixed* $\mathcal{NP}$ *witness relation for L. That is,* $x \in L$ *if there exists a* w *such that* $(x, w) \in R_L$*. We denote by* $view_{V^*}^{P(w)}(x)$ *a random variable describing the transcript of all messages exchanged between a (possibly malicious) PPT verifier* V^* *and the honest prover* P *in an execution of the protocol on common input* x*, when* P *has auxiliary input* w*. Then we say that* $\langle P, V\rangle$ *is (black-box) zero-knowledge if there exists a probabilistic (expected) polynomial-time oracle machine* S*, such that for every probabilistic polynomial-time interactive machine* V^* *and for every* $x \in L$*, the following two probability distributions are computationally indistinguishable:* $\{view_{V^*}^{P(w)}(x)\}_{x\in L}$ *and* $\{S^{V^*}(x)\}_{x\in L}$*. Machine* S *is called a black-box simulator for* $< P, V >$*.*

2.1 Σ-Protocols and Σ_{OR}-Protocols

Σ-protocols have been proved to be a very powerful cryptographic tool and are widely used in numerous important cryptographic applications including digital signatures, efficient electronic payment systems, electronic voting systems, et al. We remark that a very large number of Σ-protocols have been developed in the literature, mainly in the field of applied cryptography and in industry.

Definition 3 (Σ-protocol [2]). *A 3-round public-coin protocol* $\langle P, V\rangle$ *is said to be a* Σ*-protocol for a relation* R *if the following hold:*

- *Completeness. If* P*,* V *follow the protocol, the verifier always accepts.*
- *Special soundness. From any common input* x *of length* n *and any pair of accepting conversations on input* x*,* (a, e, z) *and* (a, e', z') *where* $e \neq e'$*, one can efficiently computes* w *such that* $(x, w) \in R$*. Here* a*,* e*,* z *stand for the first, the second and the third message respectively, and* e *is assumed to be a random string (i.e. the random challenge from* V*) of length* k *(that is polynomially related to* n*) selected uniformly at random in* $\{0, 1\}^k$*.*
- *Perfect SHVZK (Special honest verifier zero-knowledge). There exists a probabilistic polynomial-time (PPT) simulator* S*, which on input* x *(where there exists a* w *such that* $(x, w) \in R$*) and a random challenge string* $\hat{e}$*, outputs an accepting conversation of the form* $(\hat{a}, \hat{e}, \hat{z})$*, with the same probability distribution as the real conversation* (a, e, z) *between the honest* $P(w)$*,* V *on input* x*.*

Σ-Protocol for DLP [10]. The following is a Σ-protocol $< P, V >$ proposed by Schnorr [10] for proving the knowledge of discrete logarithm, w, for a common input of the form (p, q, g, h) such that $h = g^w \ mod \ p$, where on a security parameter n, p is a uniformly selected n-bit prime such that $q = (p-1)/2$ is also a prime, g is an element in $\mathbf{Z}_p^*$ of order q. It is also actually the first efficient Σ-protocol proposed in the literature.

- P chooses r at random in $\mathbf{Z}_q$ and sends $a = g^r \ mod \ p$ to V.
- V chooses a challenge e at random in $\mathbf{Z}_{2^k}$ and sends it to P. Here, k is fixed such that $2^k < q$.
- P sends $z = r + ew \ mod \ q$ to V, who checks that $g^z = ah^e \ mod \ p$, that p, q are prime and that g, h have order q, and accepts iff this is the case.

Σ-Protocol for RSA [6]. Let n be an RSA modulus and q be a prime. Assume we are given some element $y \in Z_n^*$, and P knows an element w such that $w^q = y$ mod n. The following protocol is a Σ-protocol for proving the knowledge of q-th roots modulo n.

- P chooses r at random in Z_n^* and sends $a = r^q \mod n$ to V.
- V chooses a challenge e at random in Z_{2^k} and sends it to P. Here, k is fixed such that $2^k < q$.
- P sends $z = rw^e \mod n$ to V, who checks that $z^q = ay^e \mod n$, that q is a prime, that $gcd(a, n) = gcd(y, n) = 1$, and accepts iff this is the case.

The OR-proof of Σ-protocols [3]. One basic construction with Σ-protocols allows a prover to show that given two inputs x_0, x_1, it knows a w such that either $(x_0, w) \in R_0$ or $(x_1, w) \in R_1$, but without revealing which is the case. Specifically, given two Σ-protocols $\langle P_b, V_b \rangle$ for R_b, $b \in \{0, 1\}$, with random challenges of, without loss of generality, the same length k, consider the following protocol $\langle P, V \rangle$, which we call Σ_{OR}. The common input of $\langle P, V \rangle$ is (x_0, x_1) and P has a private input w such that $(x_b, w) \in R_b$.

- P computes the first message a_b in $\langle P_b, V_b \rangle$, using x_b, w as private inputs. P chooses e_{1-b} at random, runs the SHVZK simulator of $\langle P_{1-b}, V_{1-b} \rangle$ on input (x_{1-b}, e_{1-b}), and let $(a_{1-b}, e_{1-b}, z_{1-b})$ be the output. P finally sends a_0, a_1 to V.
- V chooses a random k-bit string s and sends it to P.
- P sets $e_b = s \oplus e_{1-b}$ and computes the answer z_b to challenge e_b using (x_b, a_b, e_b, w) as input. He sends (e_0, z_0, e_1, z_1) to V.
- V checks that $s = e_0 \oplus e_1$ and that conversations (a_0, e_0, z_o), (a_1, e_1, z_1) are accepting conversations with respect to inputs x_0, x_1, respectively.

Theorem 1. [3] *The protocol Σ_{OR} above is a Σ-protocol for R_{OR}, where $R_{OR} = \{((x_0, x_1), w) | (x_0, w) \in R_0 \text{ or } (x_1, w) \in R_1\}$. Moreover, for any malicious verifier V^*, the probability distribution of conversations between P and V^*, where w is such that $(x_b, w) \in R_b$, is independent of b. That is, Σ_{OR} is perfectly witness indistinguishable.*

The perfect SHVZK simulator of Σ_{OR} [3]. For a Σ_{OR}-protocol of above form, denote by S_{OR} the perfect SHVZK simulator of it and denote by S_b the perfect SHVZK simulator of the protocol $\langle P_b, V_b \rangle$ for $b \in \{0,1\}$. Then on common input (x_0, x_1) and a random string $\hat{e}$ of length k, $S_{OR}((x_0, x_1), \hat{e})$ works as follows: It firstly chooses a random k-bit string $\hat{e}_0$, computes $\hat{e}_1 = \hat{e} \oplus \hat{e}_0$, then S_{OR} runs $S_b(x_b, \hat{e}_b)$ to get a simulated transcript $(\hat{a}_b, \hat{e}_b, \hat{z}_b)$ for $b \in \{0,1\}$, finally S_{OR} outputs $((\hat{a}_0, \hat{a}_1), \hat{e}, (\hat{e}_0, \hat{z}_0, \hat{e}_1, \hat{z}_1))$.

2.2 Σ-Provable Trapdoor Commitments

We recall some perfectly-hiding trapdoor commitment schemes and clarify some additional properties about them which are critical for our purpose.

Definition 4 (perfectly-hiding trapdoor (string) commitments TC). *A (normal) trapdoor commitment scheme (TC) is a quintuple of probabilistic polynomial-time (PPT) algorithms TCGen, TCCom, TCVer, TCKeyVer and TCFake, such that*

- *Completeness.* $\forall n, \forall v$ *of length* k *(where* $k = k(n)$ *for some polynomial* $k(\cdot)$*),* $\Pr[(TCPK, TCSK) \xleftarrow{R} \text{TCGen}(1^n); (c, d) \xleftarrow{R} \text{TCCom}(1^n, 1^k, TCPK, v) : \text{TCKeyVer}(1^n, TCPK) = \text{TCVer}(1^n, 1^k, TCPK, c, v, d) = 1] = 1.$
- *Computational Binding. For all sufficiently large* n*'s and for any PPT adversary* A*, the following probability is negligible in* n *(where* $k = k(n)$ *for some polynomial* $k(\cdot)$*):* $\Pr[(TCPK, TCSK) \xleftarrow{R} \text{TCGen}(1^n); (c, v_1, v_2, d_1, d_2) \xleftarrow{R} \text{A}(1^n, 1^k, TCPK) : \text{TCVer}(1^n, 1^k, TCPK, c, v_1, d_1) = \text{TCVer}(1^n, 1^k, TCPK, c, v_2, d_2) = 1 \bigwedge |v_1| = |v_2| = k \bigwedge v_1 \neq v_2].$
- *Perfect Hiding.* $\forall$ $TCPK$ *such that* $\text{TCKeyVer}(TCPK, 1^n) = 1$ *and* $\forall$ v_1, v_2 *of equal length* k*, the following two probability distributions are identical:* $[(c_1, d_1) \xleftarrow{R} \text{TCCom}(1^n, 1^k, TCPK, v_1) : c_1]$ *and* $[(c_2, d_2) \xleftarrow{R} \text{TCCom}(1^n, 1^k, TCPK, v_2) : c_2].$
- *Perfect Trapdoorness.* $\forall$ $(TCPK, TCSK) \in \{\text{TCGen}(1^n)\}$, $\exists v_1$, $\forall v_2$ *such that* v_1 *and* v_2 *are of equal length* k*, the following two probability distributions are identical:* $[(c_1, d_1) \xleftarrow{R} \text{TCCom}(1^n, 1^k, TCPK, v_1);$ $d_2' \xleftarrow{R} \text{TCFake}(1^n, 1^k, TCPK, TCSK, c_1, v_1, d_1, v_2) : (c_1, d_2')]$ *and* $[(c_2, d_2) \xleftarrow{R} \text{TCCom}(1^n, 1^k, TCPK, v_2) : (c_2, d_2)].$
  Furthermore, from any (c, v_1, d_1, v_2, d_2)*, where* $|v_1| = |v_2| = k$*, such that* $\text{TCVer}(1^n, 1^k, TCPK, c, v_1, d_1) = \text{TCVer}(1^n, 1^k, TCPK, c, v_2, d_2) = 1$*, the* $TCSK$ *can be efficiently extracted.*

As shown in Definition 4, known trapdoor commitment schemes work in two rounds as follows: In the first round, the commitment receiver generates and sends the $TCPK$ to the commitment sender. In the second round, on $TCPK$ and the value v (of length k) to be committed, the sender computes $(c, d) \leftarrow TCCom(TCPK, v)$ and sends c as the commitment, while keeping the value v and the decommitment information d in private. The trapdoor is the corresponding $TCSK$ with which one can equivocate the commitment at its wish. But, for our purpose, we need TC schemes that satisfy the following additional requirements:

1. Public-key Σ-provability. On common input $TCPK$ and private input $TCSK$, one can prove, by Σ-protocols, the knowledge of $TCSK$.
2. Commitment Σ-provability. On common inputs $(1^n, 1^k, TCPK, c, v)$ and private input d, where $(c, d) \stackrel{R}{\leftarrow} \text{TCCom}(1^n, 1^k, TCPK, v)$, one can prove, by Σ-protocols, the knowledge of d such that $\text{TCVer}(1^n, 1^k, TCPK, c, v, d) = 1$.

We call a trapdoor commitment scheme satisfying the above two additional properties a *Σ-provable trapdoor commitment* scheme. We note both the DLP-based [1] and the RSA-based [9] trapdoor (string) commitment schemes are Σ-provable trapdoor commitment schemes with perfect hiding and trapdoorness properties.

Consider the DLP-based perfectly-hiding trapdoor commitment scheme [1]: On a security parameter n, the receiver selects uniformly an n-bit prime p so that $q = (p-1)/2$ is a prime, an element g of order q in $\mathbf{Z}_p^*$. Then the receiver uniformly selects w in $\mathbf{Z}_q$ and sets $h = g^w \ mod \ p$. The receiver sets $TCPK = (p, q, g, h)$ and keeps w as $TCSK$ in secret. To commit a value v in $\mathbf{Z}_q$, the sender first checks that: p, q are primes, $p = 2q + 1$ and g, h are elements of order q in $\mathbf{Z}_p^*$ (this in particular guarantees the existence of w). If the above checking is failed, then the sender halts announcing that the receiver is cheating. Otherwise (i.e., the above testing is successful), then the sender uniformly selects $d \in \mathbf{Z}_q$ (the decommitment information), and sends $c = g^d h^v \mod p$ as its commitment. The public-key Σ-provability is direct from the above Σ-protocol for DLP [10]. Commitment Σ-provability follows from the fact that given (c, v), proving the knowledge of the decommitment information d is equivalent to proving the knowledge of the discrete-logarithm of c/h^v with g as the base.

Now, consider the RSA-based perfectly-hiding trapdoor commitment scheme [9]: Let n be a composite number and $q > n$ be a prime number, the receiver randomly chooses w from Z_n^* and computes $y = w^q \mod n$. The $TCPK$ is set to be (n, q, y) and $TCSK = w$. To commit a value $v \in Z_q$, the sender firstly checks that: n is a composite number, $q > n$ is a prime number and y is in Z_n^* (this in particular guarantees the existence of w). If the above checking is successful, then the sender randomly chooses $d \in Z_n^*$ and computes $c = y^v d^q$ as its commitment. The public-key Σ-provability is direct from the above Σ-protocol for RSA [6]. Commitment Σ-provability also follows from the Σ-protocol for RSA [6], by observing that given (c, v) proving the knowledge of the decommitment information d is equivalent to proving the knowledge of the q-th root of c/y^v modulo n.

3 The Transformation from Σ-Protocols to Practical ZK Arguments

For any language L that admits a Σ-protocol, we present for the same language a 6-round black-box zero-knowledge argument. Let $\langle P_L, V_L \rangle$ be a Σ-protocol for a language L and denote by (a_L, e_L, z_L) the messages exchanged between honest P_L and honest V_L on a common input $x \in L \cap \{0,1\}^n$, where e_L is a k-bit string chosen uniformly at random from $\{0,1\}^k$. Below, we deliver some high-level overview about the transformation.

In the transformation, on the top of running the starting Σ-protocol $\langle P_L, V_L \rangle$, each verifier generates and sends *a pair of* public-keys, $(TCPK_0, TCPK_1)$, for a Σ-provable trapdoor commitment scheme. Then, the verifier V proves to the prover P that it knows one secret-key of either $TCPK_0$ or $TCPK_1$ by executing the Σ_{OR}-protocol (guaranteed by the public-key Σ-provability of the underlying Σ-provable trapdoor commitment scheme). After that, the main-body of the transformed protocol is the Σ-protocol $\langle P_L, V_L \rangle$, but with a coin-tossing mechanism that determines the random challenge in $\langle P_L, V_L \rangle$. The underlying coin-tossing mechanism has the following properties: if one knows one secret-key of either $TCPK_0$ or $TCPK_1$, then it can set the outcomes (i.e. the random challenge) at its wish (this is to facilitate ZK simulation); On the contrary, if one does not know any secret-key w.r.t. $(TCPK_0, TCPK_1)$, then it cannot convince the honest verifier a false statement (this assures the soundness property). The transformed protocol $\langle P, V \rangle$ is depicted in Figure 1 (page 295).

The transformation depicted in Figure-1 works starting from any Σ-protocols, but we remark that it can be easily extended to be applicable to any public-coin HVZK protocols. For the additional round-complexity incurred by the transformation in comparison with the original protocol $\langle P_L, V_L \rangle$, it's easy to check that the number of the incurred additional rounds is that of the first three stages that can be combined into 3. Specifically, Stage-2 and Stage-3 can be combined into Stage-1.

Theorem 2. *Assuming (perfectly-hiding) Σ-provable trapdoor commitment schemes, the transformed protocol $\langle P, V \rangle$ is a black-box ZK argument for L.*

Proof (sketch). Black-box zero-knowledge. For any PPT adversary V^*, on input $x \in L$ the ZK simulator S works as follows by running V^* as a subroutine and playing the role of the honest prover.

For the key-pair $(TCPK_0, TCPK_1)$ sent by V^* at the beginning of Stage-1, on which the Σ_{OR}-protocol of Stage-1 is executed. If V^* cannot successfully finish the Σ_{OR}-protocol in Stage-1, then S aborts (just as the honest prover does in real interactions) and outputs the transcript up to now.

In case V^* successfully finishes the Σ_{OR}-protocol on $(TCPK_0, TCPK_1)$ in Stage-1, we denote by a_{OR}, e_{OR}, z_{OR}, the first-round message, the second-round message, and the third round message of the Σ_{OR}-protocol of Stage-1 respectively, where e_{OR} is the random challenge (of length k) sent by S (emulating the action of the honest prover in the second-round of the Σ_{OR}-protocol of Stage-1). Then, S keeps doing the following trials until extracting the secret-key of either $TCPK_0$ or $TCPK_1$: S rewinds V^* to the state that it just sent the a_{OR}, runs V^* further from this point by sending it a new random challenge e'_{OR}, runs V^* further and look forward to receiving back again a valid third-round message z'_{OR}. Note that according to the special soundness of Σ-protocols (recall that

Common input. An element $x \in L \cap \{0,1\}^n$.
P private input. An $\mathcal{NP}$-witness w for $x \in L$.

Stage-1. On a security parameter n, the verifier V generates $(TCPK_0, TCSK_0) \stackrel{\text{R}}{\leftarrow}$ TCGen$(1^n, r_0)$, $(TCPK_1, TCSK_1) \stackrel{\text{R}}{\leftarrow}$ TCGen$(1^n, r_1)$ for a *Σ-provable trapdoor commitment* scheme, where r_0 and r_1 are two independent random strings used by TCGen. The verifier V sends $(TCPK_0, TCPK_1)$ and proves to the prover P that: it knows either $TCSK_0$ or $TCSK_1$ by running the Σ_{OR}-protocol guaranteed by the public-key Σ-provability property of the underlying Σ-provable trapdoor commitment scheme. The witness used by V in this stage is $TCSK_b$ for a randomly chosen bit b from $\{0,1\}$.

Stage-2. The prover P selects uniformly at random two independent random strings $r_P^{(0)} \stackrel{\text{R}}{\leftarrow} \{0,1\}^n$ and $r_P^{(1)} \stackrel{\text{R}}{\leftarrow} \{0,1\}^n$, computes $\alpha_0 = \text{TCCom}(TCPK_0, r_P^{(0)})$ and $\alpha_1 = \text{TCCom}(TCPK_1, r_P^{(1)})$ using the underlying Σ-provable trapdoor commitment scheme. Finally, the prover sends (α_0, α_1) to the verifier.

Stage-3. The verifier V uniformly selects $r_V \stackrel{\text{R}}{\leftarrow} \{0,1\}^n$ and sends r_V to P.

Stage 4. Stage 4 includes the following three steps:

Stage 4.1. Running the SHVZK simulator S_{OR} (of the Σ_{OR} protocol guaranteed by the commitment Σ-provability property of the underlying Σ-provable trapdoor commitment) on common input $(\alpha_0, \alpha_1, r_V)$ and a random challenge $\hat{e}$ of length k, the prover obtains a transcript $(\hat{a}, \hat{e}, \hat{z})$. (Informally, here the prover uses the simulator S_{OR} to "pretend" that one of (α_0, α_1) commits to r_V). Then P runs $P_L(x, w)$ to compute a_L, and sends $(\hat{a}, a_L)$ to V.

Stage 4.2. The verifier sends back P a random value e'_L of length k.

Stage 4.3. The prover computes $e_L = \hat{e} \oplus e'_L$ and $z_L = P_L(x, w, a_L, e_L)$. P then sends $(\hat{e}, \hat{z}, z_L)$ to the verifier.

Verifier's decision The verifier accepts if and only if $(\hat{a}, \hat{e}, \hat{z})$ is an accepting conversation on $(\alpha_0, \alpha_1, r_V)$ for showing that one of (α_0, α_1) commits to r_{V^*} and $(a_L, e_L = \hat{e} \oplus e'_L, z_L)$ is an accepting conversation for showing $x \in L$.

Fig. 1. The transformed protocol $\langle P, V \rangle$ from $\langle P_L, V_L \rangle$

Σ_{OR} is itself a Σ-protocol), from two successful conversations with respect to the same first-round message (i.e. a_{OR}) but for two different challenges (i.e. e_{OR} and e'_{OR}), S can successfully extract the secret-key of either $TCPK_0$ or $TCPK_1$. Also, note that S works in this (secret-key extraction) stage within time inversely proportional to the probability that V^* successfully finishes Phase-1, thus this (secret-key extraction) stage in simulation will be done in expected polynomial-time.

After successfully extracting the secret-key of either $TCPK_0$ or $TCPK_1$, denoted by $TCSK_b$ for $b = 0$ or 1, S goes into next stages as follows:

In Stage-2, S works just as the honest prover does and we denote by (α_0, α_1) the message sent by S in Stage 2 (that commit to two independent random values, $r_P^{(0)}$ and $r_P^{(1)}$, respectively); In Stage-3, V^* sends a string, denoted by r_{V^*}, to S; In Stage 4, S firstly runs the SHVZK simulator, S_L, of the Σ-protocol $\langle P_L, V_L \rangle$ to get a simulated transcript $(\hat{a}_L, \hat{e}_L, \hat{z}_L)$ (for showing $x \in L$). Then, on common input $(\alpha_0, \alpha_1, r_{V^*})$ and with $TCSK_b$ as the witness, S computes out the first-round message, denoted by a, of the Σ_{OR}-protocol (guaranteed by the commitment Σ-provability property of the underlying Σ-provable trapdoor commitment scheme). Then, S sends $(a, \hat{a}_L)$ to V^* as the Stage-4.1 message. After receiving back a random value e'_L from V^* in Stage 4.2, S sets $e = \hat{e}_L \oplus e'_L$, computes z on $(\alpha_0, \alpha_1, r_{V^*}, a, e)$ by using $TCSK_b$ as the trapdoor, and finally sends $(e, z, \hat{z}_L)$ to V^* in Stage 4.3.

Below, we show that the output of S is indistinguishable from the view of V^* in real interactions. The differences between simulated transcript and the real interaction transcript lie in that of Stage-4.

- In real interactions, the transcript of Stage-4 is: $\{(\hat{a}, a_L), e'_L, (\hat{e}, \hat{z}, z_L)\}$, where $(\hat{a}, \hat{e}, \hat{z})$ is a *simulated* conversation on $(\alpha_0, \alpha_1, r_{V^*})$ for showing that one of (α_0, α_1) commits to r_{V^*}, and $(a_L, e_L = \hat{e} \oplus e'_L, z_L)$ is a *real* conversation for showing $x \in L$.
- In the ZK simulation, the transcript of Stage-4 is: $\{(a, \hat{a}_L), e'_L, (e, z, \hat{z}_L)\}$, where $(a, e = \hat{e}_L \oplus e'_L, z)$ is a *real* conversation on $(\alpha_0, \alpha_1, r_{V^*})$ for showing that one of (α_0, α_1) commits to r_{V^*}, and $(\hat{a}_L, \hat{e}_L = e \oplus e'_L, \hat{z}_L)$ is a *simulated* conversation for showing $x \in L$.

Note that in real interaction transcript, $\hat{e}$ is a truly random string, and $\hat{a}$ perfectly hides $\hat{e}$ that is guaranteed by the *perfect* SHVZK property of Σ-protocols (specifically, simulated transcript $(\hat{a}, \hat{e}, \hat{z})$ of a Σ-protocol is identical to real interaction transcript (a, e, z), but in real interactions e is independent of a). Thus V^* cannot choose e'_L dependently on $\hat{e}$, which means that $e_L = \hat{e} \oplus e'_L$ is a truly random string. Similarly, in the simulated transcript, $\hat{e}_L$ is a truly random string and $\hat{a}_L$ perfectly hides $\hat{e}_L$, which means that $e = \hat{e}_L \oplus e'_L$ is also a truly random string. Then, the indistinguishability between real interaction transcript and simulated transcript outputted by S is directly from the *perfect* SHVZK property of Σ-protocols (note that Σ_{OR} is itself a Σ-protocol), which says that the distribution of real interaction transcript of a Σ-protocol is identical to that of simulated transcript outputted by the SHVZK simulator.

Computational soundness. We first note that a computational power unbounded prover can easily convince the verifier of a false statement since it can extract the secret-keys if its computational power is unbounded. Hence the protocol $\langle P, V \rangle$ depicted in Figure 1 constitutes an argument system rather than a proof system.

We now proceed to prove that the protocol $\langle P, V \rangle$ satisfies computational soundness. Specifically, suppose a PPT adversary P^* can convince the honest verifier V of a false statement $x \notin L$ with non-negligible probability p, then we will construct an algorithm E that takes $TCPK$ as input and outputs the corresponding $TCSK$ with probability $\frac{p^4}{4}$ in polynomial-time, which breaks the one-wayness of $TCPK$ (i.e. violating the computational-binding property of the underlying Σ-provable trapdoor commitment scheme).

On common input $TCPK$, the algorithm E firstly runs TCGen (of the underlying Σ-provable trapdoor commitment) to get: $(TCPK', TCSK') \xleftarrow{R} TCGen(1^n)$. Then, E randomly selects a bit $b \xleftarrow{R} \{0, 1\}$, sets $TCPK_b$ be $TCPK'$, $TCSK_b$ be $TCSK'$ and $TCPK_{1-b}$ be $TCPK$. E sends $(TCPK_0$,

$TCPK_1$) to P^* at the beginning of Stage-1, and proves to P^* that it knows the secret-key of either $TCPK_0$ or $TCPK_1$, by executing the Σ_{OR}-protocol on $(TCPK_0, TCPK_1)$ with $TCSK_b = TCSK'$ as its witness. After receiving the Stage-2 message, denoted by (α_0, α_1), from P^*, E does the following:

- $k := 1$.
- While $k \leq 2$ do:
  - Uniformly selects $r_V \xleftarrow{R} \{0,1\}^n$ and sends r_V to P^* as the Stage-3 message.
  - Acts accordingly further until receiving the last Stage 4.3 message from P^*. We denote by $(\hat{a}, a_L)$ the Stage 4.1 message from P^*.
  - After receiving the Stage 4.3 message from P^*, E rewinds P^* to the point that P^* just sent Stage 4.1 message and sends back a new random Stage 4.2 message to P^*. Such a rewinding is called the *knowledge rewinding.*
  - Runs P^* further from the above knowledge rewinding point until receiving back again a Stage-4.3 message.
  - $k := k + 1$. This means that E will rewind P^* to the point that P^* just sent the Stage-2 message (α_0, α_1) and send back a new random Stage-3 message, denoted by r'_V. Such a rewinding is called the *major rewinding.*

According to the above description of E, conditioned on P^* always succeeds in completing its interactions with E, then for the *knowledge-rewindings* both in the interactions prior to the major rewinding and in the interactions posterior to the major rewinding, there are two cases to be considered:

1. E gets two different accepting conversations of the Σ-protocol $\langle P_L, V_L \rangle$ on input x with respect to the same Stage 4.1 message (specifically, the same a_L message). According to the special soundness property of Σ-protocols, this means a witness for $x_i \in L$ can be efficiently extracted.
2. E gets two different accepting conversations of the Σ_{OR} on input $(\alpha_0, \alpha_1, r_V)$ with respect to the same Stage 4.1 message (specifically, the same message $\hat{a}$). According to the special soundness property of Σ-protocol, this means that a decommitment information d can be efficiently extracted such that $\mathrm{TCVer}(TCPK_\sigma, \alpha_\sigma, r_V, d) = 1$ for $\sigma = 0$ or 1.

Firstly, we note that since we assume $x \notin L$ the first case above will not appear. For the second case above, since b is chosen randomly (by E) in $\{0,1\}$, and the Σ_{OR}-protocol executed in Stage-1 is perfectly witness indistinguishable, conditioned on $x \notin L$ and P^* always succeeds in completing its interactions with E, the probability of $\sigma = 1 - b$ in both the interactions prior to the major rewinding and the interactions posterior to the major rewinding is $\frac{1}{4}$. Note that $TCPK_{1-b} = TCPK$, and from two valid different decommitments (i.e. r_V and r'_V) of the same Stage-2 message (specifically, the same $\alpha_\sigma = \alpha_{1-b}$) the corresponding secret-key $TCSK$ can be efficiently extracted.

Since P^* cannot distinguish whether it is interacting with the honest verifier or E, then with the same non-negligible probability p P^* convinces E of a false statement $x \notin L$. We conclude that with probability $\frac{p^4}{4}$, E can extract the secret-key of $TCPK = TCPK_{1-b}$, i.e. $TCSK_{1-b}$, in polynomial-time. This violates the one-wayness of $TCPK$ (i.e. the computational binding property of the underlying Σ-provable trapdoor commitment scheme).

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Bayesian Communication Leading to a Nash Equilibrium in Belief*

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Abstract. A Bayesian communication in the p-belief system is presented which leads to a Nash equilibrium of a strategic form game through messages as a Bayesian updating process. In the communication process each player predicts the other players' actions under his/her private information with probability at least his/her belief. The players communicate privately their conjectures through message according to the communication graph, where each player receiving the message learns and revises his/her conjecture. The emphasis is on that both any topological assumptions on the communication graph and any common-knowledge assumptions on the structure of communication are not required.

Keywords: p-Belief system, Nash equilibrium, Bayesian communication, Protocol, Conjecture, Non-corporative game.

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1 Introduction

This article relates equilibria and distributed knowledge. In game theoretical situations among a group of players, the concept of mixed strategy Nash equilibrium has become central. Yet little is known the process by which players learn if they do. This article will give a protocol run by the mutual learning of their beliefs of players' actions, and it highlights an epistemic aspect of Bayesian updating process leading to a mixed strategy Nash equilibrium for a strategic form game.

As for as J.F. Nash [7]'s fundamental notion of strategic equilibrium is concerned, R.J. Aumann and A. Brandenburger [1] gives epistemic conditions for

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mixed strategy Nash equilibrium: They show that the common-knowledge of the predictions of the players having the partition information (that is, equivalently, the **S5**-knowledge model) yields a Nash equilibrium of a game. However it is not clear just what learning process leads to the equilibrium. The present article aims to fill this gap from epistemic point of view.

Our real concern is with what Bayesian learning process leads to a mixed strategy Nash equilibrium of a finite strategic form game with emphasis on the epistemic point of view. We focus on the Bayesian belief revision through communication among group of players. We show that

Main theorem. *Suppose that the players in a strategic form game have the p-belief system with a common prior distribution. In a communication process of the game according to a protocol with revisions of their beliefs about the other players' actions, the profile of their future predictions induces a mixed strategy Nash equilibrium of the game in the long run.*

Let us consider the following protocol: The players start with the same prior distribution on a state-space. In addition they have private information given by a partition of the state space. Beliefs of players are posterior probabilities: A player *p-believes* (simply, *believes*) an event with $0 < p \leq 1$ if the posterior probability of the event given his/her information is at least p. Each player predicts the other players' actions as his/her belief of the actions. He/she communicates privately their beliefs about the other players' actions through messages, and the receivers update their belief according to the messages. Precisely, the players are assumed to be rational and maximizing their expected utility according their beliefs at every stage. Each player communicates privately his/her belief about the others' actions as messages according to a protocol,[1] and the receivers update their private information and revise their belief.

The main theorem says that the players' predictions regarding the future beliefs converge in the long run, which lead to a mixed strategy Nash equilibrium of a game. The emphasis is on the two points: First that each player's prediction is not required to be common-knowledge among all players, and secondly that each player send to the another player not the exact information about his/her belief about the actions for the other players but the approximate information about the the other players' actions with probability at lest his/her belief of the others' actions.

This paper organized as follows: In section 2 we give the formal model of the Bayesian communication on a game. Section 3 states explicitly our theorem and gives a sketch of the proof. In final section 4 we conclude some remarks. We are planning to present a small example to illustrate the theorem in our lecture presentation in the conference WINE 2005.

2 The Model

Let Ω be a non-empty *finite* set called a *state-space*, N a set of finitely many *players* $\{1, 2, \ldots n\}$ at least two ($n \geq 2$), and let 2^Ω be the family of all subsets

[1] When a player communicates with another, the other players are not informed about the contents of the message.

of Ω. Each member of 2^{Ω} is called an *event* and each element of Ω called a *state*. Let μ be a probability measure on Ω which is common for all players. For simplicity it is assumed that (Ω, μ) is a *finite* probability space with μ *full support.*[2]

2.1 *p*-Belief System[3]

Let p be a real number with $0 < p \leq 1$. The *p-belief system* associated with the partition information structure $(\Pi_i)_{i\in N}$ is the tuple $\langle N, \Omega, \mu, (\Pi_i)_{i\in N}, (B_i(*, p))_{i\in N}\rangle$ consisting of the following structures and interpretations: (Ω, μ) is a finite probability space, and i's *p-belief operator* $B_i(*; p)$ is the operator on 2^{Ω} such that $B_i(E, p)$ is the set of states of Ω in which i *p-believes* that E has occurred with probability at least p ; that is, $B_i(E; p) := \{\omega \in \Omega \mid \mu(E \mid \Pi_i(\omega)) \ \geq \ p \ \}$.

Remark 1. When $p = 1$ the 1-belief operator $B_i(*; 1)$ becomes knowledge operator.

2.2 Game on *p*-Belief System[4]

By a *game G* we mean a *finite* strategic form game $\langle N, (A_i)_{i\in N}, (g_i)_{i\in N}\rangle$ with the following structure and interpretations: N is a finite set of players $\{1, 2, \dots, i, \dots n\}$ with $n \geq 2$, A_i is a finite set of i's *actions* (or i's pure strategies) and g_i is an i's *payoff function* of A into $\mathbb{R}$, where A denotes the product $A_1 \times A_2 \times \cdots \times A_n$, A_{-i} the product $A_1 \times A_2 \times \cdots \times A_{i-1} \times A_{i+1} \times \cdots \times A_n$. We denote by g the n-tuple $(g_1, g_2, \dots g_n)$ and by a_{-i} the $(n-1)$-tuple $(a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$ for a of A. Furthermore we denote $a_{-I} = (a_i)_{i\in N\setminus I}$ for each $I \subset N$.

A probability distribution ϕ_i on A_{-i} is said to be i's *overall conjecture* (or simply i's *conjecture*). For each player j other than i, this induces the marginal distribution on j's actions; we call it i's *individual conjecture* about j (or simply i's conjecture *about* j.) Functions on Ω are viewed like random variables in the probability space (Ω, μ). If $\mathbf{x}$ is a such function and x is a value of it, we denote by $[\mathbf{x} = x]$ (or simply by $[x]$) the set $\{\omega \in \Omega | \ \mathbf{x}(\omega) = x\}$.

The information structure (Π_i) with a common prior μ yields the distribution on $A \times \Omega$ defined by $\mathbf{q}_i(a, \omega) = \mu([\mathbf{a} = a]|\Pi_i(\omega))$; and the i's overall conjecture defined by the marginal distribution $\mathbf{q}_i(a_{-i}, \omega) = \mu([\mathbf{a}_{-i} = a_{-i}]|\Pi_i(\omega))$ which is viewed as a random variable of ϕ_i. We denote by $[\mathbf{q}_i = \phi_i]$ the intersection $\bigcap_{a_{-i}\in A_{-i}}[\mathbf{q}_i(a_{-i}) = \phi_i(a_{-i})]$ and denote by $[\phi]$ the intersection $\bigcap_{i\in N}[\mathbf{q}_i = \phi_i]$. Let $\mathbf{g}_i$ be a random variable of i's payoff function g_i and $\mathbf{a}_i$ a random variable of an i's action a_i. Where we assume that $\Pi_i(\omega) \subseteq [a_i] := [\mathbf{a}_i = a_i]$ for all $\omega \in [a_i]$ and for every a_i of A_i. i's action a_i is said to be *actual* at a state ω if $\omega \in [\mathbf{a}_i = a_i]$; and the profile a_I is said to be actually played at ω if $\omega \in [\mathbf{a}_I = a_I] := \bigcap_{i\in I}[\mathbf{a}_i = a_i]$ for $I \subset N$. The pay off functions $g = (g_1, g_2, \dots, g_n)$ is said to be *actually*

[2] That is; $\mu(\omega) \neq 0$ for every $\omega \in \Omega$.

[3] Monderer and Samet [6].

[4] Aumann and Brandenburger [1].

played at a state ω if $\omega \in [\mathbf{g} = g] := \bigcap_{i \in N}[\mathbf{g}_i = g_i]$. Let **Exp** denote the expectation defined by $\mathbf{Exp}(g_i(b_i, \mathbf{a}_{-i}); \omega) := \sum_{a_{-i} \in A_{-i}} g_i(b_i, a_{-i})\ \mathbf{q}_i(a_{-i}, \omega)$.

A player i is said to be *rational* at ω if each i's actual action a_i maximizes the expectation of his actually played pay off function g_i at ω when the other players actions are distributed according to his conjecture $\mathbf{q}_i(\cdot\,; \omega)$. Formally, letting $g_i = \mathbf{g}_i(\omega)$ and $a_i = \mathbf{a}_i(\omega)$, $\mathbf{Exp}(g_i(a_i, \mathbf{a}_{-i}); \omega) \geq \mathbf{Exp}(g_i(b_i, \mathbf{a}_{-i}); \omega)$ for every b_i in A_i. Let R_i denote the set of all of the states at which i is rational.

2.3 Protocol [5]

We assume that the players communicate by sending *messages*. Let T be the time horizontal line $\{0, 1, 2, \cdots t, \cdots\}$. A *protocol* is a mapping $\mathrm{Pr} : T \to N \times N, t \mapsto (s(t), r(t))$ such that $s(t) \neq r(t)$. Here t stands for *time* and $s(t)$ and $r(t)$ are, respectively, the *sender* and the *receiver* of the communication which takes place at time t. We consider the protocol as the directed graph whose vertices are the set of all players N and such that there is an edge (or an arc) from i to j if and only if there are infinitely many t such that $s(t) = i$ and $r(t) = j$.

A protocol is said to be *fair* if the graph is strongly-connected; in words, every player in this protocol communicates directly or indirectly with every other player infinitely often. It is said to contain a *cycle* if there are players $i_1, i_2, \ldots, i_k$ with $k \geq 3$ such that for all $m < k$, i_m communicates directly with i_{m+1}, and such that i_k communicates directly with i_1. The communications is assumed to proceed in *rounds*[6]

2.4 Communication on *p*-Belief System

A *Bayesian belief communication process* $\pi(G)$ with revisions of players' conjectures $(\phi_i^t)_{(i,t) \in N \times T}$ according to a protocol for a game G is a tuple

$$\pi(G) = \langle \mathrm{Pr}, (\Pi_i^t)_{i \in N}, (B_i^t)_{i \in N}, (\phi_i^t)_{(i,t) \in N \times T} \rangle$$

with the following structures: the players have a common prior μ on Ω, the protocol Pr among N, $\mathrm{Pr}(t) = (s(t), r(t))$, is fair and it satisfies the conditions that $r(t) = s(t+1)$ for every t and that the communications proceed in rounds. The revised information structure Π_i^t at time t is the mapping of Ω into 2^Ω for player i. If $i = s(t)$ is a sender at t, the message sent by i to $j = r(t)$ is M_i^t. An n-tuple $(\phi_i^t)_{i \in N}$ is a revision process of individual conjectures. These structures are inductively defined as follows:

- Set $\Pi_i^0(\omega) = \Pi_i(\omega)$.
- Assume that Π_i^t is defined. It yields the distribution $\mathbf{q}_i^t(a, \omega) = \mu([\mathbf{a} = a] | \Pi_i^t(\omega))$. Whence

[5] C.f.: Parikh and Krasucki [8].

[6] There exists a time m such that for all t, $\mathrm{Pr}(t) = \mathrm{Pr}(t+m)$. The *period* of the protocol is the minimal number of all m such that for every t, $\mathrm{Pr}(t+m) = \mathrm{Pr}(t)$.

- R_i^t denotes the set of all the state ω at which i is *rational* according to his conjecture $\mathbf{q}_i^t(\cdot\,;\omega)$; that is, each i's actual action a_i maximizes the expectation of his pay off function g_i being actually played at ω when the other players actions are distributed according to his conjecture $\mathbf{q}_i^t(\cdot\,;\omega)$ at time t.[7]
- The message $M_i^t : \Omega \to 2^\Omega$ sent by the sender i at time t is defined by

$$M_i^t(\omega) = \bigcap_{a_{-i}\in A_{-i}} B_i^t([a_{-i}]; \mathbf{q}_i^t(a_{-i}, \omega)),$$

where $B_i^t : 2^\Omega \to 2^\Omega$ is defined by

$$B_i^t(E; p) = \{\omega \in \Omega \mid \mu(E \,|\, \Pi_i^t(\omega)) \geq p\ \}.$$

Then:

– The revised partition Π_i^{t+1} at time $t+1$ is defined as follows:
  - $\Pi_i^{t+1}(\omega) = \Pi_i^t(\omega) \cap M_{s(t)}^t(\omega)$ if $i = r(t)$;
  - $\Pi_i^{t+1}(\omega) = \Pi_i^t(\omega)$ otherwise,
– The revision process $(\phi_i^t)_{(i,t)\in N\times T}$ of conjectures is inductively defined as follows:
  - Let $\omega_0 \in \Omega$, and set $\phi_{s(0)}^0(a_{-s(0)}) := \mathbf{q}_{s(0)}^0(a_{-s(0)}, \omega_0)$
  - Take $\omega_1 \in M_{s(0)}^0(\omega_0) \cap B_{r(0)}([g_{s(0)}] \cap R_{s(0)}^0; p)$,[8] and set $\phi_{s(1)}^1(a_{-s(1)}) := \mathbf{q}_{s(1)}^1(a_{-s(1)}, \omega_1)$
  - Take $\omega_{t+1} \in M_{s(t)}^t(\omega_t) \cap B_{r(t)}([g_{s(t)}] \cap R_{s(t)}^t; p)$, and set $\phi_{s(t+1)}^{t+1}(a_{-s(t+1)}) := \mathbf{q}_i^{t+1}(a_{-s(t+1)}, \omega_{t+1})$.

The specification is that a sender $s(t)$ at time t informs the receiver $r(t)$ his/her individual conjecture about the other players' actions with a probability greater than his/her belief. The receiver revises her/his information structure under the information. She/he predicts the other players action at the state where the player p-believes that the sender $s(t)$ is rational, and she/he informs her/his the predictions to the other player $r(t+1)$.

We denote by ∞ a sufficient large τ such that for all $\omega \in \Omega$, $\mathbf{q}_i^\tau(\cdot\,;\omega) = \mathbf{q}_i^{\tau+1}(\cdot\,;\omega) = \mathbf{q}_i^{\tau+2}(\cdot\,;\omega) = \cdots$. Hence we can write $\mathbf{q}_i^\tau$ by $\mathbf{q}_i^\infty$ and ϕ_i^τ by ϕ_i^∞.

Remark 2. The Bayesian belief communication is a modification of the communication model introduced by Ishikawa [3].

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[7] Formally, letting $g_i = \mathbf{g}_i(\omega)$, $a_i = \mathbf{a}_i(\omega)$, the expectation at time t, $\mathbf{Exp}^t$, is defined by $\mathbf{Exp}^t(g_i(a_i, \mathbf{a}_{-i}); \omega) := \sum_{a_{-i}\in A_{-i}} g_i(a_i, a_{-i})\ \mathbf{q}_i^t(a_{-i}, \omega)$. An player i is said to be rational according to his conjecture $\mathbf{q}_i^t(\cdot, \omega)$ at ω if for all b_i in A_i, $\mathbf{Exp}^t(g_i(a_i, \mathbf{a}_{-i}); \omega) \geq \mathbf{Exp}^t(g_i(b_i, \mathbf{a}_{-i}); \omega)$.

[8] We denote $[g_i] := [\mathbf{g}_i = g_i]$.

3 The Result

We can now state the main theorem:

Theorem 1. *Suppose that the players in a strategic form game G have the p-belief system with μ a common prior. In the Bayesian belief communication process $\pi(G)$ according to a protocol among all players in the game with revisions of their conjectures $(\phi_i^t)_{(i,t)\in N\times T}$ there exists a time ∞ such that for each $t \geq \infty$, the n-tuple $(\phi_i^t)_{i\in N}$ induces a mixed strategy Nash equilibrium of the game.*

The proof is based on the below proposition:

Proposition 1. *Suppose that the players in a strategic form game have the p-belief system with μ a common prior. In the Bayesian belief communication process $\pi(G)$ in a game G with revisions of their conjectures, if the protocol has no cycle then both the conjectures $\mathbf{q}_i^\infty$ and $\mathbf{q}_j^\infty$ on $A \times \Omega$ must coincide; that is, $\mathbf{q}_i^\infty(a;\omega_\infty) = \mathbf{q}_j^\infty(a;\omega_{\infty+t})$ for $(i,j) = (s(\infty), s(\infty+t))$ and for any $t = 1, 2, 3, \cdots$.*

Proof. Let us first consider the case that $(i,j) = (s(\infty), r(\infty))$. In view of the construction of $\{\Pi_i^t\}_{t\in T}$ we can observe that

$$\Pi_j^\infty(\xi) \subseteq W_i^\infty(\omega) \quad \text{for all } \xi \in W_i^\infty(\omega). \tag{1}$$

It immediately follows that $W_i^\infty(\omega)$ is decomposed into a disjoint union of components $\Pi_j^\infty(\xi)$ for $\xi \in \Pi_i^\infty(\omega)$;

$$W_i^\infty(\omega) = \bigcup_{k=1,2,\ldots,m} \Pi_j^\infty(\xi_k) \quad \text{where} \quad \xi_k \in W_i^\infty(\omega). \tag{2}$$

It can be observed that

$$\mu([\mathbf{a} = a]|\ W_i^\infty(\omega)) = \sum_{k=1}^{m} \lambda_k \mu([\mathbf{a} = a]|\ \Pi_j^\infty(\xi_k)) \tag{3}$$

for some $\lambda_k > 0$ with $\sum_{k=1}^m \lambda_k = 1$.[9] Since $\Pi_i(\omega) \subseteq [a_i]$ for all $\omega \in [a_i]$, we can observe that $\mathbf{q}_i^\infty(a_{-i};\omega) = \mathbf{q}_i^\infty(a;\omega)$. On noting that $W_i^\infty(\omega)$ is decomposed into a disjoint union of components $\Pi_i^\infty(\xi)$ for $\xi \in \Pi_i^\infty(\omega)$, we can obtain $\mathbf{q}_i^\infty(a;\omega) = \mu([\mathbf{a} = a]|\ W_i^\infty(\omega)) = \mu([\mathbf{a} = a]|\ \Pi_i^\infty(\xi_k))$ for any $\xi_k \in W_i^\infty(\omega)$. It follows by (3) that, for each $\omega \in \Omega$ there exists a state $\xi_\omega \in \Pi_i^\infty(\omega)$ such that $\mathbf{q}_i^\infty(a;\omega) \leq \mathbf{q}_j^\infty(a;\xi_\omega)$ for $(i,j) = (s(\infty), t(\infty))$.

On continuing this process according to the *fair* protocol, the below facts can be plainly verified: For each $\omega \in \Omega$ and for sufficient large $\tau \geq 1$,

1. For any $t \geq 1$, $\mathbf{q}_{s(\infty)}^\infty(a;\omega) \leq \mathbf{q}_{s(\infty+t)}^\infty(a;\xi_t)$ for some $\xi_t \in \Omega$; and
2. $\mathbf{q}_i^\infty(a;\omega) \leq \mathbf{q}_i^\infty(a;\xi) \leq \mathbf{q}_i^\infty(a;\zeta) \leq \cdots$ for some $\xi, \zeta, \cdots \in \Omega$.

Since Ω is finite it can be obtained that $\mathbf{q}_i^\infty(a;\omega_\infty) = \mathbf{q}_j^\infty(a;\omega_{\infty+t})$ for $(i,j) = (s(\infty), s(\infty+t))$ for every a, in completing the proof.

[9] This property is called the *convexity* for the conditional probability $\mu(X|*)$ in Parikh and Krasucki [8].

Proof of Theorem 1: We denote by $\Gamma(i)$ the set of all the players who directly receive the message from i on N; i.e., $\Gamma(i) = \{ j \in N \mid (i, j) = \Pr(t)$ for some $t \in T\}$. Let F_i denote $[\phi_i^\infty] := \bigcap_{a_{-i} \in A_i} [\mathbf{q}_i^\infty(a_{-i}; *) = \phi_i^\infty(a_{-i})]$. It is noted that $F_i \cap F_j \neq \emptyset$ for each $i \in N$, $j \in \Gamma(i)$.

We observe the first point that for each $i \in N$, $j \in \Gamma(i)$ and for every $a \in A$, $\mu([\mathbf{a}_{-j} = a_{-j}] \mid F_i \cap F_j) = \phi_j^\infty(a_{-j})$. Then summing over a_{-i}, we can observe that $\mu([\mathbf{a}_i = a_i] \mid F_i \cap F_j) = \phi_j^\infty(a_i)$ for any $a \in A$. In view of Proposition 1 it can be observed that $\phi_j^\infty(a_i) = \phi_k^\infty(a_i)$ for each $j, k, \neq i$; i.e., $\phi_j^\infty(a_i)$ is independent of the choices of every $j \in N$ other than i. We set the probability distribution σ_i on A_i by $\sigma_i(a_i) := \phi_j^\infty(a_i)$, and set the profile $\sigma = (\sigma_i)$.

We observe the second point that for every $a \in \prod_{i \in N} \operatorname{Supp}(\sigma_i)$, $\phi_i^\infty(a_{-i}) = \sigma_1(a_1) \cdots \sigma_{i-1}(a_{i-1}) \sigma_{i+1}(a_{i+1}) \cdots \sigma_n(a_n)$: In fact, viewing the definition of σ_i we shall show that $\phi_i^\infty(a_{-i}) = \prod_{k \in N \setminus \{i\}} \phi_i^\infty(a_k)$. To verify this it suffices to show that for every $k = 1, 2, \cdots, n$, $\phi_i^\infty(a_{-i}) = \phi_i^\infty(a_{-I_k}) \prod_{k \in I_k \setminus \{i\}} \phi_i^\infty(a_k)$: We prove it by induction on k. For $k = 1$ the result is immediate. Suppose it is true for $k \geq 1$. On noting the protocol is fair, we can take the sequence of sets of players $\{I_k\}_{1 \leq k \leq n}$ with the following properties:

(a) $I_1 = \{i\} \subset I_2 \subset \cdots \subset I_k \subset I_{k+1} \subset \cdots \subset I_m = N$:

(b) For every $k \in N$ there is a player $i_{k+1} \in \bigcup_{j \in I_k} \Gamma(j)$ with $I_{k+1} \setminus I_k = \{i_{k+1}\}$.

We let take $j \in I_k$ such that $i_{k+1} \in \Gamma(j)$. Set $H_{i_{k+1}} := [\mathbf{a}_{i_{k+1}} = a_{i_{k+1}}] \cap F_j \cap F_{i_{k+1}}$. It can be verified that $\mu([\mathbf{a}_{-j-i_{k+1}} = a_{-j-i_{k+1}}] \mid H_{i_{k+1}}) = \phi_{-j-i_{k+1}}^\infty(a_{-j})$. Dividing $\mu(F_j \cap F_{i_{k+1}})$ yields that

$$\mu([\mathbf{a}_{-j} = a_{-j}] \mid F_j \cap F_{i_{k+1}}) = \phi_{i_{k+1}}^\infty(a_{-j}) \mu([\mathbf{a}_{i_{k+1}} = a_{i_{k+1}}] \mid F_j \cap F_{i_{k+1}}).$$

Thus $\phi_j^\infty(a_{-j}) = \phi_{i_{k+1}}^\infty(a_{-j-i_{k+1}}) \phi_j^t(a_{i_{k+1}})$; then summing over a_{I_k} we obtain $\phi_j^\infty(a_{-I_k}) = \phi_{i_{k+1}}^\infty(a_{-I_k-i_{k+1}}) \phi_j^\infty(a_{i_{k+1}})$. It immediately follows from Proposition 1 that $\phi_i^\infty(a_{-I_k}) = \phi_i^\infty(a_{-I_k-i_{k+1}}) \phi_i^\infty(a_{i_{k+1}})$, as required.

Furthermore we can observe that all the other players i than j agree on the same conjecture $\sigma_j(a_j) = \phi_i^\infty(a_j)$ about j. We conclude that each action a_i appearing with positive probability in σ_i maximizes g_i against the product of the distributions σ_l with $l \neq i$. This implies that the profile $\sigma = (\sigma_i)_{i \in N}$ is a mixed strategy Nash equilibrium of G, in completing the proof. □

4 Concluding Remarks

We have observed that in a communication process with revisions of players' beliefs about the other actions, their predictions induces a mixed strategy Nash equilibrium of the game in the long run. Matsuhisa [4] established the same assertion in the **S4**-knowledge model. Furthermore Matsuhisa [5] showed a similar result for ε-mixed strategy Nash equilibrium of a strategic form game in the **S4**-knowledge model, which gives an epistemic aspect in Theorem of E. Kalai and E. Lehrer [2]. This article highlights the Bayesian belief communication with missing some information, and shows that the convergence to an exact Nash equilibrium is guaranteed even in such the communication on approximate information.

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The Bahncard Problem with Interest Rate and Risk*

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Abstract. This paper investigated a new framework for the competitive analysis of the Bahncard problem. In contrast to the earlier approach we introduce the interest rate i and the risk tolerance t into the model, in which the traveller can develop the optimal trading strategies based on his risk preference. Set $\alpha = \frac{1}{1+i}$. We prove that the Bahncard problem with the interest rate is $1 + (1-\beta)\alpha^{m^*+1}$-competitive, where m^* is the critical point. Then we further design a t-tolerance strategy and present a surprisingly flexible competitive ratio of $1 + \frac{(1-\beta)\alpha^{m^*}}{tr^* - (1-\beta\alpha^{m^*})}$, where r^* is the optimal competitive ratio for the Bahncard problem with the interest rate and β is the percentage of discount.

1 Introduction

An extensive study of the online problems began in the 1980s in the seminal work of Sleator and Tarjan [9] on list accessing and paging algorithms. Within the theoretical computer science community, the competitive ratio has become a standard approach for the analysis of the online problems. Nevertheless, one argument against the use of this approach is that the online players are inherently risk-averse as they are optimized with respect to the worst-case event sequences. A number of approaches have been developed in an attempt to remedy this situation. Raghavan [3] proposed a competitive strategy against the statistical adversary whose request sequence was required to satisfy certain distributional requirements. Al-Binali [1] analyzed a financial game using the competitive analysis framework to include a flexible risk management mechanism. Our risk-reward competitive analysis allows the online players to benefit from their own capability in correctly forecasting the coming request sequences and to control their risk of performing to select a set of near optimal algorithms. Such risk algorithm can be favorable for the online players who may prefer somewhat inferior but guaranteed performance to better average performance.

In this paper we study the Bahncard problem originally proposed by Fleischer [2]. Let $BP(C, \beta, T)$ denote this problem in which a Bahncard costs C,

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reduces any regular ticket price p to βp, and is valid for time T. For example, $BP(240\,DM, \frac{1}{2}, 1\,year)$ means that if the traveller spends $240\,DM$ for a Bahncard, he is entitled to a discount of 50% price reduction on nearly all train tickets for one year. Fleischer presented a $(2-\beta)$-competitive solution using the traditional deterministic online algorithm for $BP(C, \beta, T)$. However, the key factors such as the net present value and the risk are not considered in [2]. In fact, accounting for the market interest rate i and controlling risk tolerance t are the essential features of any financial decision-making. We introduce the nominal interest rate in the market into the model and improve the traditional competitive analysis which is risk averse by allowing a traveller to provide and benefit from a forecast with respect to the optimal offline algorithm should his forecast be correct. Although these are only one step toward a more realistic solution of the problem, the introduction of these parameters considerably complicates the analysis.

This paper is organized as follows. Section 2 outlines the traditional competitive analysis of the Bahncard problem with the interest rate denoted by $BP(C, \beta, T, i)$. One interesting feature is that the introduction of the interest rate diminishes the uncertainty involved in decision making. For all such sensible value of $i > 0$, we prove that optimal deterministic competitive ratio is $1+(1-\beta)\alpha^{m^*+1}$ (m^* is the critical point), which is strictly improved with the interest rate. In section 3 we present such a framework that generalizes competitive analysis and allows for flexible risk management. The framework extends traditional competitive analysis by introducing the risk and the capability. The online traveller often owns such capability that can make a correct forecast about the future requirements. Therefore, for $BP(C, \beta, T, i)$, the traveller can estimate his own investing capability to choose a maximum acceptable risk level t and a set of forecasts F, and then develop a t-tolerance strategy that can maximize the reward should his forecast be correct. Then the t-tolerance strategy is $1+\frac{(1-\beta)\alpha^{m^*}}{tr^*-(1-\beta\alpha^{m^*})}$-comepetitive. Substituting $\alpha = 1$ and $t = 1$ into the above formula, we obtain the $(2-\beta)-$competitive ratio which is the best attainable result presented by Fleischer. Finally, conclusions and open questions are reported in section 4.

2 The Bahncard Problem with Interest Rate

When considering alternative financial decisions an agent must consider their net present value. Thus the market interest rate is an essential feature of any reasonable financial model. In this section while accounting for the interest rate, we determine the optimal strategies of the online Bahncard problem, using the following notation:

i The nominal interest rate in the market.

$BP(C, \beta, T, i)$. The Bahncard problem with the interest rate where a Bahncard costs $C > 0$, reduces any ticket price p to βp, and is valid for time T.

$\delta = \delta_1, \delta_2, \ldots, \delta_n$. A sequence of travel requests for n travelling periods.

$\delta_j(t_j, p_j)$ The travel request where t_j is the travel time and p_j is the regular ticket price.
$\alpha = 1/(1+i)$. The present discount value to unit of money.
t_k The time of purchasing a Bahncard.
ε The current ticket price at the time of purchasing a Bahncard.
$C^* = C/(1-\beta)$. The break-even value whether to buy a Bahncard.
$p^I(\delta) = \Sigma_{j:t_j \in I} p_j \alpha^j$. The present cost of all travel requests in the time interval I. If $p^I(\delta) \leq C^*$, the time interval I is called the cheap interval, otherwise it is called the expensive interval.

2.1 The Lower Bound

In this section we show a lower bound on the deterministic competitive ratio for $BP(C, \beta, T, i)$. It is assumed that $p_1 < (C \cdot i)/(1-\beta)$. Because the regular ticket price p_1 at the beginning time t_1 must be less than the present discount value of the critical cost. Otherwise, the online traveller can attain a competitive ratio of 1 by simply purchasing a Bahncard at the first request.

Our argument here follows from the lower bound proof for $BP(C, \beta, T, i)$. Regardless of what strategy the online player is followed, the adversary can construct a sequence of the travel requests consisting of the rules:

1. The travel requests are arbitrarily dense, so that all requests are in intervals of length T.
2. If the online traveller does not buy a Bahncard, the adversary continues the game.
3. Once the online traveller buys a Bahncard, the adversary will stop showing requests.

The Critical Point for $\boldsymbol{BP(C, \beta, T, i)}$. Set $\alpha^j = 1/(1+i)^j$, where $j = 1, \ldots, n$. For $m \leq n$ the present discount value of the critical cost is

$$\sum_{j=1}^{m} p_j \alpha^j - \beta \sum_{j=1}^{m} p_j \alpha^j = C\,.$$

In other words, the offline traveller has known the future travel sequence, so he can achieve the optimal break-even point m^* in advance(For simplicity we assume that m^* is an integer.), which is the root of the above equation. Then we can achieve the present value of the critical cost $\sum\limits_{j=1}^{m^*} p_j \alpha^j = \frac{C}{1-\beta}$. Therefore for any n travel periods the optimal offline cost $C_{OPT}(\delta)$ is presented by

- for $n \leq m^*$, let $\sum\limits_{j=1}^{n-1} p_j \alpha^j + \varepsilon \alpha^n$ be the optimal cost.
- for $n > m^*$, let $C + \beta \sum\limits_{j=1}^{n-1} p_j \alpha^j + \beta \varepsilon \alpha^n$ be the optimal cost.

Theorem 1. *Any deterministic algorithm for $BP(C, \beta, T, i)$ is at least $1 + (1-\beta)\alpha^{m^*+1}$-competitive.*

Proof. For $n \le m^*$ consider the online algorithm A. Let $C_A(\delta)$ be the total cost incurred by the algorithm A that buys ticket at the regular price for the $n-1$ periods and then purchases a Bahncard. Thus A pays

$$C_A(\delta) = \sum_{j=1}^{n-1} p_j \alpha^j + (C + \beta\varepsilon)\alpha^n.$$

It is clear that optimal choice of the offline travller against A would not buy a Bahncard. Then the competitive ratio is

$$r_A^1 = \frac{\sum\limits_{j=1}^{n-1} p_j \alpha^j + (C+\beta\varepsilon)\alpha^n}{\sum\limits_{j=1}^{n-1} p_j \alpha^j + \varepsilon\alpha^n}$$

$$= \frac{(\sum\limits_{j=1}^{n-1} p_j \alpha^j)/\alpha^n + (C+\beta\varepsilon)}{(\sum\limits_{j=1}^{n-1} p_j \alpha^j)/\alpha^n + \varepsilon}.$$

Note that r_A^1 has the monotonous decreasing character. Therefore, the online player will take the maximum possible at $\sum\limits_{j=1}^{n-1} p_j \alpha^j = C^* - \alpha^n \varepsilon$. Hence, the competitive ratio is

$$r_A^1 \ge 1 + (1-\beta)\alpha^{m^*+1} - \frac{\varepsilon(1-\beta)^2 \alpha^{m^*+1}}{C}.$$

Let $\varepsilon = 0$, and then we obtain the lower bound

$$r_A^1 \ge 1 + (1-\beta)\alpha^{m^*+1}.$$

Next consider the online algorithm A with $n > m^*$. Then for this case we obtain

$$r_A^2 = \frac{\sum\limits_{j=1}^{n-1} p_j \alpha^j + (C+\beta\varepsilon)\alpha^n}{C + \beta \sum\limits_{j=1}^{n-1} p_j \alpha^j + \beta\varepsilon\alpha^n}.$$

Note that r_A^2 has the monotonous increasing character. When $\sum\limits_{j=1}^{n-1} p_j \alpha^j = C^* - \alpha^n \varepsilon$, we also can achieve the same lower bound $r_A^2 \ge 1 + (1-\beta)\alpha^{m^*+1}$. □

2.2 The Upper Bound

We are interested in the interval of length T. Let $S = p^{(t-T,t]}(\delta)$ be the sum of all regular requests in $(t-T,t]$. In this section we do not want the current request at time t to be included in the summation when computing S. Then we speak of the S^-*cost*. Therefore the SUM strategy is pessimistic about future in the sense that it always buys at the latest possible time when $S^- \geq C^*$.

Theorem 2. *SUM is* $1 + (1-\beta)\alpha^{m^*+1} -$ *competitive for* $BP(C, \beta, T, i)$.

Proof. During the cheap interval I, never buying a Bahncard is the optimal strategy. If I is an expensive time interval of length at most T then OPT has at least one cheap request in I. Let C_{SUM} and C_{OPT} denote the online traveller's and the offline one's cost during I, respectively. We divide I into three subphases I_1, I_2, I_3(some of which can be empty); in I_1 and I_3 the online traveller has a valid Bahncard, whereas it must pay regular prices in I_2. For $j \in 1, 2, 3$, let $s_j = p^I(\delta)$. Then

$$r_{SUM} = \frac{C\alpha^{I_1+I_2+1} + s_2 + \beta(s_1 + s_3)}{C + \beta(s_1 + s_2 + s_3)}$$

If $s_1 = s_3 = 0$, then we can achieve the following inequality

$$r_{SUM} \leq \frac{C\alpha^{I_1+I_2+1} + \sum_{j=I_1}^{I_1+I_2} p_j\alpha^j}{C + \beta \sum_{j=I_1}^{I_1+I_2} p_j\alpha^j} .$$

If the above quotient takes its minimum value at $s_2 = C^*$(not including the current request), we can attain a better competitive ratio

$$r_{SUM} \leq 1 + (1-\beta)\alpha^{m^*+1} .$$

□

Observing the function of $1 + \frac{1-\beta}{(1+i)^{m^*+1}}$, we have the following property: for $i > 0$ and $0 < \beta < 1$, the optimal competitive ratio is strictly decreasing with i and β. If $\alpha = 1$ (neglect the net present value), then the ratio is $2 - \beta$ presented by Fleischer.

3 The $BP(C, \beta, T, i)$ In a Risk-Reward Framework

3.1 A Risk-Reward Framework

The basic definition of the risk-reward framwork is described as follows: when an online traveller is risk-averse, he will use the traditional online algorithm A and achieve the optimal competitive ratio. If the online traveller is a risk-seeker, our risk-reward framework allows him to provide and benefit from such capability that can correctly forecast the future, and then beat the optimal competitive ratio obtained by the classical competitive analysis.

First, the definition of the competitive ratio is presented as follows. For a request sequence $\delta\langle\delta_1, \delta_2, \ldots, \delta_n\rangle \in R^n$, the total cost of the online algorithm A

is denoted by $C_A(\delta)$, and the optimal offline cost on δ is defined as $C_{OPT}(\delta) = min(cost_A(\delta))$. An algorithm is $r-$ competitive if there exists a constant α such that

$$C_A(\delta) \leq r_A C_{OPT}(\delta) + \alpha .$$

The optimal competitive ratio for the same problem is

$$r^* = inf(r_A) = inf \frac{C_A(\delta)}{C_{OPT}(\delta)} .$$

Second, we define the risk of an online algorithm to be the ratio between the different algorithms. That is, the online traveller could prefer to expose the risk, with respect to the maximum opportunity cost that the algorithm A may incur over the optimal online algorithm. Therefore we define the risk of the algorithm A to be r_A/r^* . Let t be the risk tolerance. The set of all algorithms that respect the traveller's risk tolerance can be denoted by

$$I_t = \{ A \mid r_A \leq tr^* \quad t \geq 1 \} .$$

Then we describe the reward function R on the basis of the risk preference over the optimal online algorithm. Facing with the coming request δ, the online traveller with the capability could always make a correct forecast $\bar{\delta}$, denoted by $F(\bar{\delta}) \in \delta$ ($F(\bar{\delta}) \notin \delta$ implies the false forecast). Hence, if there is $F(\bar{\delta}) \in \delta$, the online traveller could achieve the better competitive ratio $\hat{r}^*$ by a risk algorithm A:

$$\hat{r}^* = inf_{F(\bar{\delta})\in\delta}(r_A) = inf_{F(\bar{\delta})\in\delta} \frac{C_A(\bar{\delta})}{C_{OPT}(\bar{\delta})} .$$

Comparing $\hat{r}^*$ with r^*, we use the improvement of this risk algorithm A over the optimal online algorithm to measure the reward function R:

$$R = sup_{A\in I_t}\{r^*/\hat{r}^*\} .$$

Generally speaking, the bound of R can be presented as $R \in [1, r^*]$. For the upper bound when $t = 1$, $\exists\, A$, such that $\hat{r}^* \geq 1 \Longrightarrow \exists\, A$, such that $R \leq r^*$. For the lower bound, note that for all online algorithms the restricted ratio will always be $r \leq r^*$, and then $\hat{r}^* \leq r^* \Longrightarrow R \geq 1$.

3.2 The T-Tolerance Strategy

We analyze $BP(C, \beta, T, i)$ by the t-tolerance strategy based on the two possible forecasts of $n \leq m^*$ and $n > m^*$.

For the case of $n \leq m^*$, $BP(C, \beta, T, i)$ has the optimal competitive ratio such that $\hat{r}^* = 1$. It is because that both the offline and online travellers will never purchase any Bahncards in this situation.

The t-tolerance strategy for $n > m^*$ in two stages. In first stage the algorithm is under the threat that forecast is incorrect, and chooses a threshold to ensure a competitive ratio of $r_A \leq tr^*$. The second stage begins when the forecast comes true, the algorithm chooses a threshold as small as possible subject to the stage 1.

Theorem 3. *For $BP(C, \beta, T, i)$, the t-tolerance strategy is* $1 + \frac{(1-\beta)\alpha^{m^*}}{tr^* - (1-\beta\alpha^{m^*})}$-*competitive.*

Proof. For $n > m^*$, there are two cases.

Case 1: $\sum_{1 \le j \le n-1} p_j \alpha^j \le \frac{C}{(1-\beta)} - \varepsilon \alpha^n$.

According to the preceding risk definition $r_A \le tr^*$, we obtain

$$r_A = \frac{\sum_{j=1}^{n-1} p_j \alpha^j + (C + \beta\varepsilon)\alpha^n}{\sum_{j=1}^{n-1} p_j \alpha^j + \varepsilon\alpha^n}$$

$$\le [1 + (1-\beta)\alpha^{m^*+1}]t.$$

The function of r_A has the monotonous decreasing character with the total cost of ticket price. Then, the total cost of ticket price Σp_i satisfies

$$\sum_{1 \le j \le n-1} p_j \ge \frac{C\alpha^n + \beta\varepsilon\alpha^n - \varepsilon tr^* \alpha^n}{tr^* - 1}.$$

Case 2: $\sum_{1 \le j \le n-1} p_j \alpha^j > \frac{C}{(1-\beta)} - \varepsilon \alpha^n$.

According to the preceding risk definition $r_A \le tr^*$, we obtain

$$r_A = \frac{\sum_{j=1}^{n-1} p_j \alpha^j + (C + \beta\varepsilon)\alpha^n}{C + \beta \sum_{j=1}^{n-1} p_j \alpha^j + \beta\varepsilon\alpha^n}$$

$$\le [1 + (1-\beta)\alpha^{m^*+1}]t.$$

Set $\varepsilon = 0$. If $r^* < (1/t\beta)$, then we have the following inequation:

$$\sum_{1 \le j \le n-1} p_j \le \frac{tCr^* - C\alpha^n}{1 - t\beta r^*}$$

where $r^* = 1 + (1-\beta)\alpha^{m^*+1}$.

For the online traveller, should the forecast $n > m^*$ be correct, he wants to minimize r_A subject to $r_A \le tr^*$. Since we know the function of r_A in case 2 is monotonically increasing in $\sum p_i$. r_A is minimized when $\sum p_i$ is the smallest. Therefore substituting $\sum p_i = \frac{C\alpha^n + \beta\varepsilon\alpha^n - \varepsilon tr^* \alpha^n}{tr^* - 1}$ into the equation r_A of case 2, we obtain the restricted optimal competitive ratio

$$\hat{r}^* = 1 + \frac{(1-\beta)\alpha^{m^*}}{tr^* - (1-\beta\alpha^{m^*})}$$

□

Corollary 1. *If the forecast $n > m^*$ is correct, the reward of the t-tolerance strategy for $BP(C, \beta, T, i)$ is $R \longrightarrow 2 - \beta$.*

Proof. $R = \frac{r^*}{\hat{r}^*} \approx 2 - \beta$ as $t \longrightarrow \infty$ and $i \longrightarrow 0$. □

4 Conclusion

In this paper, the interest rate and the risk tolerance are introduced into the online Bahncard problem to make our model can be adopted to reality even further. Compared with the results of Fleischer's in [2], a more flexible competitive ratio can be achieved. However, there are some interesting problems as follows.

⋆ In this paper, we assume that β is a constant. But the percentage discount β often takes on the different values at some stage which make the competitive analysis more complex.
⋆ For the t-tolerance strategy, the risk tolerance t can be in the form of interval number, better simulating the behavior preference, such as $t \in [a, b]$.

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Optimal Starting Price in Online Auctions*

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Abstract. Reserve price auctions are one of hot research topics in the traditional auction theory. Here we study the starting price in an online auction, counterpart of the public reserve price in a traditional auction. By considering three features of online auctions, the stochastic entry of bidders (subject to Poisson process), the insertion fee proportional to the starting price, and time discount, we have analyzed the properties of extremum points of the starting price for maximizing seller's expected revenue, and found that, under certain conditions, the optimal starting price should be at the lowest allowable level, which is contrary to results from the classic auction theory and finds its optimality in reality. We have also developed a general extended model of multistage auction and carried out analysis on its properties. At last, some directions for further research are also put forward.

1 Introduction

It's well known that in an endogenous entry auction, the optimal auction should be zero reserve combined with optimal entry fee or optimal reserve combined with zero entry fee. That is they are alternative. In online auctions such as eBay auctions, seller can't charge entry fee from potential bidders, so they have to set optimal reserve to maximize their expected revenue.

In an online auction, seller can set some parameters to feature her auction. She may activate the Buy-It-Now option, choose among different durations and payment methods, initiate the starting price at high level or low level and set hidden reserve price and so on. The most important consideration of a seller in online auctions is how to set the starting price and hidden reserve price[1]. Because the hidden reserve price is seldom used by sellers in online auction[7][12], the starting price plays an important role on affecting the result of an auction. Lower starting price can attract more bidders while having risk in a the higher possibility of receiving lower final price thus lower expected revenue. On the other hand, if the starting price is too high, the item will not be auctioned off at all.

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How to trade-off? Is there any optimal starting price for maximizing the seller's ex ante expected revenue? Do sellers in online auction have some rationalities in setting their starting price optimally? The starting price, an alternative of public reserve price in traditional auction theory, is thus our main attention in this paper.

As it is well known, the online auction market is different from that of the traditional auction. Some features have changed the seller's expected revenue. The first one is the stochastic entry of bidders, which results in uncertainty in the number of participants in an online auction. While in the traditional auction theory, the number of bidders is critical to determine the seller's expected revenue. The second one is that in online auctions such as eBay auctions, a seller needs to pay the insertion fee (roughly proportionally to the starting price) to initiate her auction. This will affect the seller's decision making on how high the starting price should be. The last one is about the depreciation of item's valuation due to the duration of an online auction. In the case of products with a rapid depreciation like consumer electronics, computer equipment and fragile or perishable goods like flower and food, long auctions can reduce the value of the product and reduce the seller's revenue [10]. That is, the time discount factor is also needed to be considered when formulating the seller's expected revenue.

We intend to formulate the seller's expected revenue by considering above more realistic conditions, so as to analyze the optimal starting price policy the seller should adopt with the objective of maximizing her expected revenue. In addition, noting that in practice, online auction markets give sellers opportunities of re-auctioning their items with very cheap costs, we'll extend our model to a multistage auction and call for further research. It should be noted that all the discussions in this paper is under the IPV (Independent Private Value) assumption.

This paper is organized as follows. We give out the basic model and discuss several basic properties of extremum points of starting price for maximizing the seller's expected revenue in section 2. Subsequently in section 3, we analyze the optimality of the starting price in online auctions. The extended model for a multistage auction is suggested in section 4, and section 5 concludes the paper and puts forward several directions for further research.

2 Basic Model

We consider a risk-neutral seller who chooses to auction one item on an eBay-like online auction site. She doesn't set any hidden reserve price, and initiates her auction by paying an insertion fees proportional to the starting price r, which is prescribed by her. we assume this insertion fee rate is c_0, the intrinsic value of the item to the seller is v_s with $v_s \geq 0$, and the number of bidders is fixed at n. Suppose that bidders have independent private values (assessed at the end of the auction) to the item being auctioned, and the common cumulative distribution function (c.d.f.) of a bidder's valuation is $F(v), v \in [\underline{v}, \overline{v}]$, which is twice differentiable. The probability density function (p.d.f.) is $f(v)$.

Because online auctions, such as eBay auctions, are essentially second-price auctions in which the bidders' equilibrium bidding behaviors are the same with or without ambiguity about the number of bidders [6], we can use the similar method of [11] to construct the seller's expected revenue providing there are fixed n bidders participate:

$$E(\pi_s, n) = v_s F^n(r) + n\int_r^{\bar{v}} (vF'(v) + F(v) - 1)F^{n-1}(v)dv - c_0 r. \qquad (1)$$

The expressions for $E(\pi_s, n)$ is composed of three parts. The first is that when no bid surpasses the starting price, i.e. the highest valuation of bidders is less than the starting price, in which case the item doesn't sell at the auction. The auction revenue increasing from this outcome is v_s and occurs with probability $F^n(r)$. The second is that the highest bid is at least equal to the starting price, so the item sells in the auction. This outcome's contribution to $E(\pi_s, n)$ is

$$n\int_r^{\bar{v}} (vF'(v) + F(v) - 1)F^{n-1}(v)dv$$

(from Prop.1 of [11]). At last, the seller pays the insertion fee $p_0 = c_0 r$ definitely to initiate her auction.

In online auctions, bidders can place their bids at any point in the duration. We assume that the bid or bidder arrivals follow a Poisson process, i.e. the probability of n bidders participating in the auction is:

$$p_n = \frac{\lambda e^{-\lambda}}{n!}, n = 0, 1, 2, \cdots \qquad (2)$$

where $\lambda > 0$ is the average number of bidders during the duration of auction.

As discussed above, we consider the effect of duration on item's valuation and denote the time discount factor as $\delta \in [0, 1)$. Therefore, the seller can only receive $\frac{v_s}{1+\delta}$ by retaining the item in case of the failed auction.

By further considering the above two assumptions resulting from the characteristics of online auctions, the seller's expected revenue should be:

$$E(\pi_s) = \sum_{n=0}^{\infty} p_n[n\int_r^{\bar{v}} (vF'(v) + F(v) - 1)F^{n-1}(v)dv + \frac{v_s}{1+\delta}F^n(r)] - c_0 r. \quad (3)$$

To get the extremum point of r for the seller, consider the first-order necessary optimality condition for $E(\pi_s)$, i.e.:

$$\frac{\partial E(\pi_s)}{\partial r} = \sum_{n=0}^{\infty} p_n[n(1 - rf(r) - F(r))F^{n-1}(r) + \frac{nv_s}{1+\delta}F^{n-1}(r)f(r)]|_{r=r^*} - c_0 = 0.$$

Combined with equation (2), solving and simplifying it gives:

$$r^* = \frac{1 - F(r^*) - \frac{c_0}{\lambda}e^{\lambda(1-F(r^*))}}{f(r^*)} + \frac{v_s}{1+\delta}. \qquad (4)$$

Following We'll discuss the properties of the extremum point r^*.

Lemma 1. *Denote*

$$H(v) = f^2(v)(2 - c_0 e^{\lambda(1-F(v))}) + f^{'}(v)(1 - F(v) - \frac{c_0}{\lambda} e^{\lambda(1-F(v))}),$$

and

$$M(v) = v - \frac{1 - F(v) - \frac{c_0}{\lambda} e^{\lambda(1-F(v))}}{f(v)}, v \in (\underline{v}, \overline{v}),$$

if $H(v) \geq 0$*, then* $M(v)$ *increases in* v*.*

Proof. Taking the derivative of $M(v)$ w.r.t. v gives

$$\frac{dM(v)}{dv} = 1 - \frac{f^2(v)[c_0 e^{\lambda(1-F(v))} - 1] - f^{'}(v)[1 - F(v) - \frac{c_0}{\lambda} e^{\lambda(1-F(v))}]}{f^2(v)} = \frac{H(v)}{f^2(v)},$$

that is $\frac{dM(v)}{dv} \geq 0 \Leftrightarrow H(v) \geq 0$. □

Proposition 1. *The extremum point* r^* *for* $E(\pi_s)$ *may be unique, non-unique or non-existent at all.*

Proof. By definition, $H(v)$ can be negative or positive. From Lemma 1, $\frac{dM(v)}{dv}$ has the same signal as that of $H(v)$. So $M(v)$ may be non-monotone in $[\underline{v}, \overline{v}]$. On the other hand, from Equation (4), the extremum point r^* should satisfy: $M(r^*) = \frac{v_s}{1+\delta}$. So, r^* may be unique, non-unique or non-existent at all. □

Consider the boundary points of the interval of r. Because at $r = \overline{v}$, no bidders will participate, the expected revenue can't be maximized at $r = \overline{v}$.

Proposition 2. *Fixing* λ*,* δ*,* v_s *and* c_0*, the expected revenue* $E(\pi_s)$ *can achieve its global maximum at either* r^* *or* $r = \underline{v}$*, where* r^* *meets the Equation (4).*

Theorem 1. *If* $H(v) \geq 0$*,* $v \in [\underline{v}, \overline{v}]$*, then the expected revenue* $E(\pi_s)$ *can achieve its global maximum at* r^**, which is determined in Equation (4).*

Proof. From equation (4), we have $M(r^*) = \frac{v_s}{1+\delta}$. Because $H(v) \geq 0$, from Lemma 1, we know $M(v) = v - \frac{1-F(v)-\frac{c_0}{\lambda}e^{\lambda(1-F(v))}}{f(v)}$ is increasing in v. Thus for $\underline{v} < r < r^*$, $M(r) \leq \frac{v_s}{1+\delta}$, i.e. $1 - rf(r) - F(r) \geq -\frac{v_s}{1+\delta} f(r) + \frac{c_0}{\lambda} e^{\lambda(1-F(r))}$. Thus, $\frac{\partial E(\pi_s)}{\partial r} \geq 0$, for $\underline{v} < r < r^*$. Similarly, $\frac{\partial E(\pi_s)}{\partial r} \leq 0$, for $r^* < r < \overline{v}$. Therefore, $E(\pi_s)$ can achieve its maximum at r^*. □

Remark 1. Theorem 1 has indicated that if the parameter combinations of c_0, λ and $F(\cdot)$ make $H(v)$ satisfy the strong condition $H(v) \geq 0$ within the whole interval $[\underline{v}, \overline{v}]$, then the extremum point r^* is unique. Furthermore, the seller's expected revenue can achieve global maximum at it.

Because the insertion fee rate c_0 is fixed at each auction site, following we'll only discuss the relations between r^* and λ, δ. Denote

$$K(r^*, \lambda, \delta) = r^* - \frac{1 - F(r^*) - \frac{c_0}{\lambda} e^{\lambda(1-F(r^*))}}{f(r^*)} - \frac{v_s}{1+\delta} = 0.$$

Then

$$\frac{dr^*}{d\lambda} = -\frac{\frac{\partial K}{\partial r^*}}{\frac{\partial K}{\partial \lambda}} = -\frac{\frac{dM}{dr^*}}{\frac{c_0 e^{\lambda(1-F(r^*))}}{f(r^*)\lambda^2}[\lambda(1-F(r^*))-1]}.$$

Similarly,

$$\frac{dr^*}{d\delta} = -\frac{\frac{\partial K}{\partial r^*}}{\frac{\partial K}{\partial \delta}} = -\frac{\frac{dM}{dr^*}}{\frac{v_s}{(1+\delta)^2}}.$$

Intuitively, the higher the discount factor δ is, the lower the starting price should be, i.e. $\frac{dr^*}{d\delta}$ should be less than zero, which requires $\frac{dM}{dr^*} \geq 0$. On the other hand, because r^* can't be too high generally, $\lambda(1-F(r^*))-1 \geq 0$ is easy to be held. Thus, generally, $\frac{dr^*}{d\lambda} \leq 0$. That is when the average number of bidders λ reach a certain level (satisfying $\lambda(1-F(r^*))-1 \geq 0$), the larger expected number of bidders on average, the lower the optimal starting price should be. This is consistent with the intuition. The more the bidders, the higher probability the bidders with higher valuation will arrive, resulting in more furious competition among higher valuator, lower starting price can play an important role in attracting these high level bidders.

3 Optimal Starting Price

In the traditional auction theory (e.g., [5][9][11]), the optimal starting price r^* maximizing the seller's expected revenue should satisfy: $r^* = v_s + \frac{1-F(r^*)}{f(r^*)}$, which implies $r^* > v_s$, so there is a positive probability the seller refuses the entry of bidders with valuations exceeding the valuation to the seller. However, in online auction market, there are nontrivial percentage of sellers set the starting price at the lowest allowable level. An obvious discrepancy between theory and practice motivates us to explore the possibility of setting the starting price equal to the lowest allowable level other than above the true value to the seller. We open up our study by an example as follows.

Example 1. Suppose the valuations of bidders are distributed uniformly on unit interval, i.e. $F(v) = v$, where $v \in [0,1]$, the discount factor $\delta = 0.05$, insertion fee rate $c_0 = 0.03$, and true value to the seller $v_s = 0.2$. We derive the condition under which should seller start the auction from the lowest price, zero. Given the above assumption, and from Equation (4), the extremum points should satisfy:

$$0.691 - r^* = \frac{0.015}{\lambda} e^{\lambda(1-r^*)}. \tag{5}$$

It's easy to verify that the right hand side of Equation (5) is increasing in λ and the left hand side is less than 0.691. So there exists λ^*, when $\lambda \geq \lambda^*$, no r^* satisfies Equation (5). That is, the maximum of the expected revenue is achieved at the starting price $r = 0$. For this example, the optimal starting price should

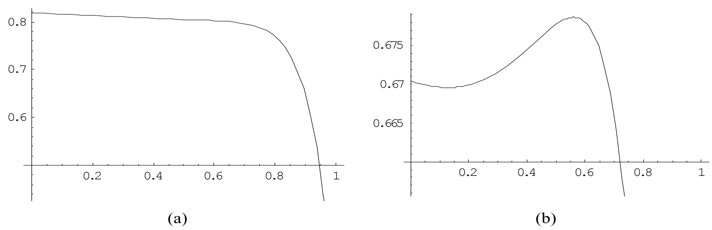

Fig. 1. The expected revenue varied with starting price, (a)$\lambda = 11$, (b)$\lambda = 6$

be zero for all $\lambda \geq 11$. Figure 1(a) gives the expected revenue varied with the starting price from 0 to 1.

While, if the average number of bidders λ is less than 11, say at 6, then there exist two extremum points, one is the global minimum $r_1^* = 0.1284$ and the other is the global maximum $r_2^* = 0.5603$, which are plotted in Figure 1(b).

The implication behind example 1 is very intuitive. In fact, it originates from the following theorem.

Theorem 2. *For fixed c_0, $F(v), v \in [\underline{v}, \overline{v}]$, δ and v_s, there exists such threshold λ^* that:*

(1)for $\lambda < \lambda^*$, there exist one or two extremum points for $E(\pi_s)$;
(2)for $\lambda \geq \lambda^*$, the maximum of $E(\pi_s)$ is achieved by letting starting price equal to the lowest allowable level $\underline{v}$.

Proof. Define $T(v,\lambda) = -\frac{c_0}{\lambda}e^{\lambda(1-F(v))} + [1 - F(v) - f(v)(v - \frac{v_s}{1+\delta})]$, from Equation (4) we know that the extremum point of $E(\pi_s)$ must be the root of $T(v,\lambda) = 0$, for fixed λ. On the other hand, according to L'Hospital's Rule,

$$\lim_{\lambda\to\infty} -\frac{c_0}{\lambda}e^{\lambda(1-F(v))} = \lim_{\lambda\to\infty} -c_0(1-F(v))e^{\lambda(1-F(v))} = -\infty;$$

on the other hand, for fixed c_0, $F(v), v \in [\underline{v}, \overline{v}]$, δ and v_s, $1 - F(v) - f(v)(v - \frac{v_s}{1+\delta})$ must be bounded. So there must exist a threshold λ^*, for all $\lambda > \lambda^*$, $T(v,\lambda) < 0$, which induces that $T(v,\lambda) = 0$ has no root. That is no available extremum point determined by equation (4). Because we have excluded the possibility of maximum point at $r = \overline{v}$, it is clear immediately that the expected revenue for seller must achieve its maximum at the lowest starting price $\underline{v}$. This has proved the second claim.

Similarly, we can easily verify that

$$\lim_{\lambda\to 0} -\frac{c_0}{\lambda}e^{\lambda(1-F(v))} = -\infty$$

and note $\frac{\partial T(v,\lambda)}{\partial \lambda} = -\frac{c_0}{\lambda^2}e^{\lambda(1-F(v))}[\lambda(1-F(v)) - 1]$. So $T(v,\lambda)$ is increasing with λ when $\lambda(1-F(v)) \leq 1$ and decreasing with λ when $\lambda(1-F(v)) > 1$. Combined with the fact that

$$\lim_{\lambda\to 0} T(v,\lambda) < 0, \lim_{\lambda\to\infty} T(v,\lambda) < 0,$$

if $T(v,\lambda)$ crosses the axis from below once, it must cross the axis again from above; if it just reaches the axis and decreases subsequently, unique root is found.

At last, we'll indicate that, for $\lambda < \lambda^*$, there always exist at least one solution for extremum condition (4). We rearrange it and find that r^* must satisfy:

$$e^{\lambda(1-F(v))} = \frac{\lambda}{c_0}[1 - F(v) - f(v)(v - \frac{v_s}{1+\delta})]. \tag{6}$$

For fixed v, denote the left hand side of (6) is $A(\lambda)$ and the right hand side is $B(\lambda)$. For certain λ^*, when $\lambda < \lambda^*$, there exists $v \in [\underline{v}, \overline{v}]$ that brings $A(\lambda)$ crosses $B(\lambda)$ one or two times. That is the first claim of Theorem 2 comes into exist. □

Remark 2. The implication behind Theorem 2 is that the average number of bidders λ is the critical determinant of optimal starting price. There exists a threshold λ^*, when $\lambda > \lambda^*$, the optimal starting price should be at the lowest allowable level. This enlightens the seller to pay more attention on attracting the participants into her auction, so as to achieve the maximal expected revenue at the lowest starting price. However, for those less popular items or short auctions, the optimal starting price should be other than the lowest one.

4 Model Extension

Now we consider the possibility of re-auction. In reality, some sellers prefer higher final price to higher probability of auctioning off the item. Then they usually choose very high starting price regardless of possibility of attracting no bidders at all. Were the item not auctioned off, the seller will re-auction it just after the end of current auction with same or lower starting price, by paying another insertion fee. This process will continue until the item is auctioned off or the seller decides to never auction it again. We describe this auction process as a multistage auction. Suppose the seller's belief of the longest stage is m, the c.d.f. of valuations in each auction is $F_i(v), v \in [\underline{v}_i, \overline{v}_i], i = 1, 2, \cdots, m$, then the seller's problem should be:

$$\max_{r_i, i\in\{1,2,\cdots m\}} E(\pi_1) \tag{7}$$

s.t.

$$\begin{cases} E(\pi_1) = \sum_{n=0}^{\infty} p_n[n\int_{r_1}^{\overline{v}_1}(vF_1'(v) + F_1(v) - 1)F_1^{n-1}(v)dv + \frac{E(\pi_2)}{1+\delta}F_1^n(r_1)] - c_0 r_1 \\ E(\pi_2) = \sum_{n=0}^{\infty} p_n[n\int_{r_2}^{\overline{v}_2}(vF_2'(v) + F_2(v) - 1)F_2^{n-1}(v)dv + \frac{E(\pi_3)}{1+\delta}F_2^n(r_2)] - c_0 r_2 \\ \cdots\cdots \\ E(\pi_m) = \sum_{n=0}^{\infty} p_n[n\int_{r_m}^{\overline{v}_m}(vF_m'(v) + F_m(v) - 1)F_m^{n-1}(v)dv + \frac{v_s}{1+\delta}F_m^n(r_m)] - c_0 r_m. \end{cases}$$

That is the seller should choose a series of optimal starting price $r_i^*, i \in \{1, 2, \cdots m\}$ for each stage of auction so as to maximize the total expected revenue $E(\pi_1)$.

How to solve problem (7) is still a challenge. While, from some simple analyses and numerical study can we get some properties of this extended model.

Proposition 3. *The optimal starting price of current stage, 1) isn't affected by that of any previous stage, and 2) affects the previous optimal starting price.*

Remark 3. This property is very straightforward: the effect of the optimal starting price of current stage on that of previous stage is through its effect on current optimal expected revenue because of the expected revenues' recursion in problem (7). This proposition enlightens the seller to determine the optimal starting price series r_i^* backward.

Theorem 3. *If $H(v)$ (defined in Lemma 1) is larger than zero and assume $F_i(v) = F_j(v) = F(v)$, $v \in [\underline{v}, \overline{v}]$, then the optimal starting price series r_i^* will be non-increasing in $i, i \in \{1, 2, \cdots m\}$.*

Proof. According to previous analysis, the extremum point r_i^* in each stage should satisfy:

$$M(r_i^*) = \begin{cases} \frac{v_s}{1+\delta}, & i = m, \\ \frac{E(\pi_{i+1})}{1+\delta}, & \text{otherwise.} \end{cases}$$

Lemma 1 has shown that if $H(v)$ is lager than zero, then $M(v)$ is increasing in v. Obviously, $E(\pi_m) \geq v_s$, which results to $r_{m-1}^* \geq r_m^*$. Noting further that the model also needs $E(\pi_1) \geq E(\pi_2) \geq \cdots \geq E(\pi_m)$, for otherwise, the maximum of $E(\pi_1)$ can't be achieved (the auction may be continued infinitely). So we get $r_1^* \geq r_2^* \geq \cdots \geq r_m^*$. □

Remark 4. The result in Theorem 3 may be intuitive. When the seller anticipates multi-stage of auction, she may set a very high starting price at the first stage to look for the higher valuator. If she successes, she can get a higher revenue; otherwise, she can set a lower starting price in the next stage of auction so as to improve the probability of auctioning off the item. Once she decides the last opportunity of re-auction (i.e. the stage m), she can set the optimal starting price according to the discussion in section 3.

Example 2. Recall Example 1 in section 3, and assume $m = 4$. Considering different level of λ, the optimal starting price and corresponding expected revenue for each stage are shown in table 1.

Table 1. Optimal starting price and resulted expected revenue for each stage

Stage	$\lambda = 4$		$\lambda = 6$		$\lambda = 11$	
i	r_i^*	$E(\pi_i^*)$	r_i^*	$E(\pi_i^*)$	r_i^*	$E(\pi_i^*)$
4	0.574685	0.581486	0.560260	0.678683	0.000000	0.818205
3	0.767389	0.681668	0.815625	0.757464	0.884778	0.845557
2	0.816800	0.723277	0.854720	0.785441	0.898481	0.853474
1	0.837226	0.743121	0.868517	0.797060	0.902428	0.855996
Increasing rate	45.68%	27.80%	55.02%	17.44%	–	4.62%

From table 1 we can find that:

(1) Both the optimal starting price and corresponding expected revenue are decreasing in the stage of auction regardless of average number of arrived bidders. That is, when there is anticipation of re-auction, to maximize her expected revenue, the seller should set decreasing optimal starting prices, from higher to lower. In practical online auction market, there is a kind of seller characterized by following behavior: setting very high starting price when she auctioned her item first time. Fortunately enough, she can get excessive profit; if the item is not auctioned off in the first stage, she can re-auction it with a little lower starting price just after the end of the first stage. This process continues until she sets a suddenly lower starting price to maximize the expected revenue of last stage.
(2) The expected revenue is also increasing in the average number of bidders under each stage. This is consistent with the intuition and previous study[8]. This finding implies that attracting bidders as many as possible to participate the auction is the main source of revenue for the seller.
(3) The multi-stage auction can provide the seller more expected revenue than just one-shot auction. The difference is just $E(\pi_1) - E(\pi_m)$. As in table 1, all these difference is positive. However, the effect may be weakened as the average number of arrived bidders λ increases. As in this example, the increasing rate of expected revenue is the highest 27.80% at $\lambda = 4$, then decreases to 17.44% at $\lambda = 6$, finally to the lowest level of only 4.62% at $\lambda = 11$.
(4) The increasing rate of optimal starting price is increasing in λ.

If we consider the "damaged good effect" of re-auction: the bidders generally regard the re-auctioned items to be damaged and lower their offers[2], then they will value the item decreasingly as the stages go on. Consequently, $F_i(v)$ should first order stochastic dominate $F_j(v)$, for every $i < j$. i.e. $F_i(v) < F_j(v)$, for every $i < j$. Under this circumstance, it is further difficult to extract the analytical solution to problem (7).

5 Conclusion Remark

How to set appropriate starting price may be the most important problem for sellers in online auctions. In this paper, we make some efforts on this topic. By considering three features of online auctions—the stochastic entry of bidders (subject to Poisson process), the insertion fee proportional to the starting price, and time discount, we have investigated the properties of extremum points of the starting price for maximizing seller's expected revenue. Through some examples and analyses, we found that under certain conditions, the optimal starting price should be at the lowest allowable level, which is contrary to results from the classic auction theory and finds its optimality in reality. We have also developed a general extended model of multistage auction, given out analysis on its properties, and call for further research on its analytical solution.

It should be noted that setting the starting price at the lowest allowable level may be served as a kind of fraud. Kauffman and Wood[4] illustrated the prevalence of reserve price shilling, which is motivated by the avoidance of auction site's insertion fees. By setting a low starting price, and then secretly bidding that amount up by pretending to be a bidder, sellers can save money when selling items in online auction. This behavior should be investigated in further research.

The objectives of sellers in online auctions may not always be maximizing their expected revenue. For firm who is trying to dump its excess inventory, seller who clear her attic full of used tools, furniture, toys etc., the goal of using online auction is just to sell out the items as soon as possible. Many firms have found that online auctions are also a tool for managing inventory and marketing new products [3][10]. Under all above circumstances, the sellers are more inclined to let bidders compete with each other and determine the final price themselves without imposing any reserve. That is starting the auction from the lowest level finds its optimality in reality. So empirical study should proceed to investigate whether items with above characteristics are more likely be auctioned with the lowest allowable starting price in online auction sites.

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Time Interval-Based Prepaid Charging of QoS-Enabled IP Services

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Abstract. Driven by the analysis of existing prepaid charging solutions with their limitations, a novel approach of prepaid charging for Internet Protocol (IP) services based on time intervals has been developed. This approach combines key advantages of hot billing with a tariff flexibility and the guarantee of minimal fraud for providers. The technical effort of real-time charging is reduced to a minimum and ensures economic efficiency as well as scalability in large-scale environments for providers who achieve a better customer binding in prepaid cases.

1 Introduction

As soon as commercial services are offered in an Internet Protocol (IP)-based network, the service delivery has to be charged for. After the tariff scheme for the specific service has been determined, two charging options—pre- or postpaid—can be applied. The postpaid option, which makes the customer pay after his service usage, is very widespread throughout the whole telecommunication market. But especially in the mobile telecommunication market, the prepaid model has proven to be successful. However, as the use of high-value IP services, *e.g.*, Voice-over-IP (VoIP) or streaming services, does show a significant increase in use, a prepaid option for such services is required, thus, ensuring an optimal economic and technical efficiency.

During many years, the role of charging for telecommunication services has been studied thoroughly [8, 23], where charging is defined as a process that is applied by service providers and network operators on the usage of services. In contrast to telecommunication systems and their standards, *charging* in IP networks is very often referred to as *pricing* and the two terms are used interchangeably. An overview of pricing in IP networks is given in [14, 15]. It is a noticeable development that the two worlds of telecommunication systems and IP networks will merge together into a so-called *all-IP* network [10, 22]. From a charging perspective, this merging means to combine the most sophisticated features and functionalities from both systems.

To distinguish charging from other processes connected with, [17] defines a layered model, where charging is positioned on top of metering and accounting, but below billing. Charging maps accounting data into monetary units by applying a tariff, i.e. evaluating a tariff function. Billing defines the collection of charging records and the deliv-

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ery of a bill to the user. Orthogonal to this layered model different charging options, namely prepaid and postpaid, have to be distinguished. Within the prepaid approach, the customer is buying a certain amount of credits—typically represented in monetary units—prior the service usage. These credits are kept at the provider's side and are referred to as the customer's balance [20]. With the prepaid charging option, the customer's balance needs to be checked periodically in real-time during service consumption. To reduce the real-time credit-checking effort, this paper introduces a novel concept for prepaid charging. Instead of monitoring a set of different variables, it defines a time interval, wherein the customer can use prepaid services without being able to spend more than given limit of monetary units. The technical overhead imposed due to the real-time charging and the risk of fraud are both minimized.

The remainder of the paper is organized as follows. Sect. 2 shows existing solutions and Sect. 3 lists the requirements. In Sect. 4 service and tariff modelling builds the basis for Sect. 5, which develops the mathematical background of time intervals. In Sect. 6 the approach is evaluated and finally, Sect. 7 draws conclusions.

2 Existing Solutions

In mobile telecommunication networks prepaid systems are well established and they can be categorized into four groups as shown in [20]: (1) Service Node, (2) Intelligent Network (IN), (3) Handset-Based (SIM card-based), and (4) the concept of Hot Billing. These prepaid systems present in today's 3G networks—UMTS [5] and CDMA2000 [1, 2]—are well suited to handle classical circuit-switched services and basic packet-switched services. The evolution of today's 3G networks is towards an all-IP architecture [3, 4]. In IP-networks prepaid charging is addressed by the two IETF working groups, AAA [6] and RADEXT [7]. The former extends the diameter base protocol [9] with the so-called *Credit-Control Application* (CCA) [16]. The extensions made by the latter [21] focus on prepaid charging only, while CCA can also be used for postpaid charging. Apart from the differences in the protocol messages, the two extensions handle prepaid charging quite similarly and interoperability between them is provided, too.

Besides the differences in network architecture and technology, the basic operation of prepaid charging is as follows. First, the user's balance is checked whether sufficient credits are left when he is requesting a prepaid service. Then, an initial amount of credits is reserved prior to service usage and if they are not used up entirely, restored to the user's balance after the service usage. Depending on the amount of reserved credits, the customer can use the whole or only a part of the service, which makes it necessary to re-check and re-request credits during service consumption in real-time to prevent overuse of credits. From an operator's point of view, this real-time processing of credit (re-)checks is a considerable cost factor, because it consumes many resources in its network. To reduce the direct costs of prepaid services, the number of credit checks is minimized according to a statistical model for the customer's service usage. However, if the statistical model fails, the customer's balance can become negative due to overuse of credits, i.e. the operator loses revenue. Eqn. (1) shows the total costs for a service C_{tot} as the sum of expected costs for credit checks $E[C_c]$ and the expected revenue loss

$E[R_l]$. The dilemma is that minimizing *$E[C_c]$* increases *$E[R_l]$*, as the analysis for the voice call service in service node prepaid systems shows, [12].

$$C_{tot} = E[C_c] + E[R_l] \tag{1}$$

Another concept to reduce the costs of real-time charging, is a hybrid approach—i.e. between pre- and postpaid—the so-called *hot billing* [20]. In hot billing, charging data records (CDR) are only processed after service usage and not during service consumption. But there are still signalling costs that occur when processing CDR's, i.e. there is a similar term of expected costs as *$E[C_c]$* in Eqn. (1). The number of CDR's generated can be decreased by increasing the number of completed calls being accumulated in one CDR. However, reducing the number of CDR's increases the expected revenue loss. And again, a trade off has to be found between saving costs and a raised risk of revenue loss, as an analysis for the voice call service shows in [11].

Some operators stated to have revenue loss up to 20%, because finding an optimal solution to Eqn. (1) is not a trivial task. Besides this, existing solutions solve Eqn. (1) only for single services with simple tariff functions. But one of the major requirement imposed on future prepaid systems, is that they are able to cope with a multi-service environment. And in such a competitive environment it's also essential that providers can make use of more flexible tariffs. This means that more complex non-linear tariffs functions depending on several parameters will become important, *e.g.*, a combined abc-tariff as introduced by [18]. Thus, prepaid charging in the multi-service environment of an all-IP network architecture requires further studies.

The newly developed prepaid charging approach based on time intervals and presented in this paper represents an elegant solution to Eqn. (1) combined with very low technical overhead: First, it minimizes *both* the costs for credit checks and the risk of potential revenue loss. Second, the introduced tariff modelling allows for nearly arbitrary complex tariff functions. And third, to be prepared for multi-service environments, the concept of service bundles is introduced, thus allowing for a step away from a simple single service solution.

3 Assumptions and Requirements

The core assumption is that an all-IP network architecture supporting QoS is in place, such as the upcoming releases of 3G networks, [3, 4] or as being developed within the Daidalos project [13]. The concept of these all-IP architectures is similar, i.e. separate IP multimedia domains are interconnected and operate independently on IP. Fig. 1 shows the Daidalos all-IP architecture from provider A's perspective, where domain A is the home and domain B the foreign domain for its customers. Therefore, the A4C systems (Authorization, Authentication, Accounting, Auditing, and Charging) are named accordingly, i.e. A4C.h and A4C.f. A4C systems are based on the IETF AAA Architecture, [6] with extensions for auditing and charging. Between provider A and B a business-to-business relation builds the basis for roaming of customers between them.

Prepaid charging is implemented with the Diameter Credit Control Application, whereas the Accounting Gateway (AG) is acting as the client and the A4C as the server.

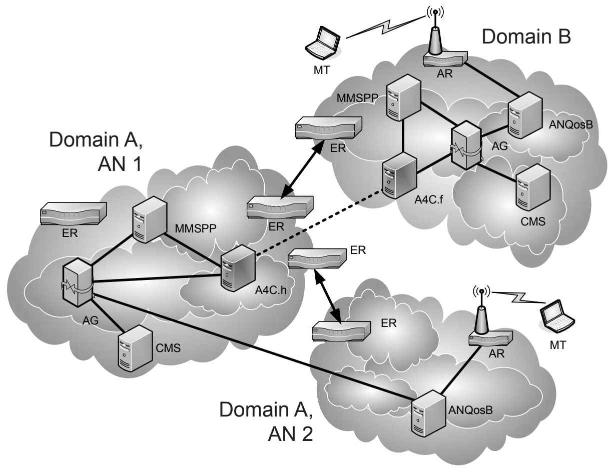

Fig. 1. Daidalos all-IP architecture with two providers with their domains and access networks

During any service consumption, the Central Monitoring System (CMS) is constantly collecting metering information and forwards them to the AG. The process of charging, i.e. the mapping of accounting data into monetary units, is done within the A4C. All services offered in the Daidalos architecture are based on IP with QoS, where best-effort traffic is treated as a service with low QoS. To be able to support prepaid charging based on time intervals, the accounting system in this architecture has to meet the following requirements: (1) on the transport level, accounting must provide for detailed information on all QoS parameters of the traffic class used by the service, (2) on the application layer, high level service usage data must be accounted for, *e.g.*, the number of messages sent, (3) the ability to support inter- and intra-domain mobility is needed, and (4) a correlation of accounting data/events per session/customer has to be performed.

4 Services and Tariff Model

Within the all-IP architecture any kind of QoS-enabled services based on IP are supported, reaching from network up to application services. The former consist of IP with QoS and are used as the transport services for the latter. Application services are categorized into session- and event-based groups, like sending messages. Session-based services have session setup-, communication-, and session release-phases. To identify a service unambiguously and to associate accounting data with it, each service is associated with a unique service ID, denoted as s_i.

4.1 Service Bundeling

The multi-service environment of an all-IP architecture introduces a new problem for prepaid charging, because the credits in monetary units represent a singular value, which has to be shared among all the services. Using *n* services in parallel means that up to *n* distributed processes are accessing and modifying the credits independently. If a providers has a big customer basis, then the load imposed on the centralized A4C component increases dramatically. To overcome this scalability problem for such a multi-service environment, so-called *service bundles* are introduced. A service bundle consists of *I* services $s_1, s_2, \ldots, s_I$ with prepaid charging option, which can all be used in parallel. Instead of performing credit checking of the services independently, it will be done on a per bundle basis. Therefore, each bundle is associated with a time interval, which is the only parameter to be monitored per bundle. This service bundle concept is comparable with the credit-and resource-pools from diameter CCA and the radius extensions for prepaid respectively. However, the basis for these approaches are simple linear tariff functions depending on one variable, cf. [19] for a thorough analysis.

4.2 Tariff Modelling

With every service s_i a tariff function $f_i(.)$ depending on one or more variables is associated. If *m* denotes the total number of different variables per bundle for all services, then the $f_i(.)$ are defined in the following way,

$$f_i(\underline{x}) \qquad \underline{x} \in \mathbb{R}^m \qquad f_i : \mathbb{R}^m \curvearrowright \mathbb{R} \tag{2}$$

The input to (2) is accounting data representing the amount of resources that have been consumed by using service s_i and the output is the charge to be paid. Vector $\underline{x}$ is partitioned in such way that every tariff function $f_i(.)$ uses a different subset of $\underline{x}$ as its input. For the ongoing sections, $f_i(.)$ must meet the following mathematical properties: (1) $f_i(.)$ is represented by a smooth or piecewise smooth functions and (2) $f_i(.)$ is monotonically increasing. With the definition of (2) and its properties, providers have all the flexibility they need to define user incentive tariffs while maximising their revenue.

5 Time Intervals

So far, the *I* tariff functions $f_i(.)$ from a bundle depend on *m* different variables as shown in (2). Existing prepaid solutions would now use an independent real-time monitoring for those *m* variables. However, the approach in this paper defines consecutive time intervals, wherein the customer can use prepaid services without being able to spend more than given limit of monetary units. Therefore, instead of monitoring *m* variables, only the length of the time intervals has to be observed. To define and apply time intervals, the tariff functions have first to be transformed into time-based functions.

5.1 Transformation into Time-Based Tariff Functions

For given specific service s_i from the bundle, a subset of all *m* variables is used as an input while the rest of the variables belongs to other services. Since those other variables are not needed by s_i, they can be set to zero. Now, whenever the absolute value of

$\underline{x}$ increases, it can only be due to resource consumption by this service s_i. Instead of looking at $f_i(\underline{x})$ in an (up to) m-dimensional space, one can study the behavior of $f_i(\underline{x})$ in the time domain: The time is divided into discrete time intervals, wherein the behavior of (2) is studied, cf. (3).

$$[t_j, t_{j+1}] \qquad \forall j \in \mathbb{N}: \quad t_j < t_{j+1} \tag{3}$$

With the borders of the time intervals, values of $\underline{x}$ are assigned as shown in (4). At the beginning of the interval $\underline{x}_j$ represents the amount of resources that have been consumed so far—i.e. up to the current time interval—by service s_i. If $\underline{x}_j > 0$, then $f_i(\underline{x}_j)$ is the charge, which has already been subtracted from the customer's credit.

$$t_j \rightsquigarrow \underline{x}_j, \quad t_{j+1} \rightsquigarrow \underline{x}_{j+1} \qquad \underline{x}_j, \underline{x}_{j+1} \in \mathbb{R}^m \tag{4}$$

If service s_i is not used within a time interval, then the charge to be paid at t_{j+1} is zero. Due to the mathematical properties of $f_i(.)$, any usage of service s_i within the interval (3) must lead to resource consumption and hence the charge to be paid at t_{j+1} is the $\underline{\Delta}$ between $f_i(\underline{x}_j)$ and $f_i(\underline{x}_{j+1})$. The upper bound of $\underline{\Delta}$ is represented by $\underline{\Delta}_{max}$ which is the maximum of resources that could be consumed by service s_i in the given interval. If the resource consumptions are maximal, then also the charge is maximal, cf. (5).

$$f_i(\underline{x}_j + \underline{\Delta}_{max}) > f_i(\underline{x}_j + \underline{\Delta}) \qquad \forall \underline{\Delta} \in \left]\underline{0}, \underline{\Delta}_{max}\right[\tag{5}$$

The charge to be paid at t_{j+1} is given by (6). Although $\underline{x}_j$ and $f_i(\underline{x}_j)$ are known from the last interval, $c_i(.)$ cannot be evaluated before t_{j+1}, since $\underline{x}_{j+1}$ is still unknown.

$$c_i(\underline{x}_j, \underline{x}_{j+1}) = f_i(\underline{x}_{j+1}) - f_i(\underline{x}_j) \quad c_i : \mathbb{R}^m \curvearrowright \mathbb{R} \tag{6}$$

The maximum charge to be paid for resource consumptions of service s_i in the time interval (3) is defined as $cmx_i(.)$, cf.(7).

$$cmx_i(\underline{\Delta}_{max}) = f_i\left(\underline{x}_j + \underline{\Delta}_{max}\right) - f_i(\underline{x}_j) \geq c_i(\underline{x}_j, \underline{x}_{j+1}) \quad cmx_i : \mathbb{R}^m \curvearrowright \mathbb{R} \tag{7}$$

To be able to calculate $cmx_i(.)$ before t_{j+1}, $\underline{\Delta}_{max}$ has to be represented by functions depending on time only. To achieve this, every variable x_k of $\underline{\Delta}_{max}$ is being substituted by functions depending on time only. This yields to $\underline{\Delta}_{max}(t)$, where $t \in [0, t_{j+1} - t_j]$. Therefore, for a given length of the time interval, the maximal consumable resources by s_i can now be calculated and hence the maximum charge to be paid, cf. (8).

$$\begin{aligned} cmx_i\left(\underline{\Delta}_{max}\right) \mathrel{\widehat{=}} cm_i(t) &= f_i\left(\underline{x}_j + \underline{\Delta}_{max}(t)\right) - f_i(\underline{x}_j) \quad cm_i : \mathbb{R} \curvearrowright \mathbb{R} \\ &\geq c_i(\underline{x}_j, \underline{x}_{j+1}) \end{aligned} \tag{8}$$

The maximum charge to be paid in the time interval (3) is given by $cmx_i(\underline{\Delta}_{max})$ corresponding to $cm_i(t)$, which depends on t only. The process of substitutions is repeated for all services s_i in the bundle, yielding to $CM(t)$, cf. (9).

$$CM(t) = \sum_{i=1}^{I} cm_i(t) \geq \sum_{i=1}^{I} c_i(\underline{x}_j, \underline{x}_{j+1}) \quad CM : \mathbb{R} \curvearrowright \mathbb{R} \tag{9}$$

For a given amount of c credits from customers' balance, the charging system will calculate a time interval t, such that (9) is exactly equal to c. I.e., the left hand side of (9) is given and the length of the time interval will be calculated. This time interval t is the shortest possible interval, wherein the customer can use up c credits.

5.2 Applying Time Intervals

The formal definition of time intervals per service bundle from (9) constitutes the core part of the Time Interval Calculation Algorithm (TICA). Its major principle is outlined here, while the full details may be obtained from [19]. TICA takes the resource consumption from a current interval—$\underline{x}_u$—as an input and calculates based on remaining credits the upcoming length of the time interval for this service bundle. However, before TICA can be applied, two important constants have to be defined.

The maximal time it takes to perform the credit (re-)check is t_c and t_{min} is the minimal length of a time interval that the AG is able to support. If the time interval calculated by TICA is too short (no credits left), the AG is the first part of a Policy Enforcement Point (PEP). If a "not enough credits" message arrives at the AG from the charging system, some or all services are removed from the list of services allowed for this customer. Depending on the type of service, this updated list is communicated from the AG to the ANQoS-Brokers or to the MMSPP, cf. Fig. 1. These two systems constitute the second part of the PEP, i.e. they are responsible for disconnecting the customer.

In Fig. 2, TICA is applied to the calculation of three consecutive time intervals—labelled as t_1, t_2, and t_3. The message sequence chart starts when a user requests a prepaid service. This request is forwarded to the AG, which contacts the A4C.h to check user's credits, cf. step (1) in Fig. 2. On the basis of remaining credits in monetary units (hatched rectangle), the charging system calculates t_{1*} by applying TICA, cf. step (2). The area of the stretched hatched rectangle is the same as the one shown at step (1)—i.e. within t_{1*} at most these credits can be used up and no overuse is possible. After t_{1*} has been calculated, t_c is subtracted and the result is checked against t_{min}, cf. step (2). In the example, t_1 is greater then t_{min} and, hence, the calculated interval is valid and is communicated back to the AG. At this point, the AG starts to monitor t_1 and the customer can start using services. Within the time of t_1, accounting data is accumulated in the AG, (shaded rectangle). After t_1 is reached, the AG forwards accounting data $\underline{x}_u$ to the charging system, combined with a request for a new time interval (user continues using services).

During the time needed to perform credit checking in the charging system t_c, service consumption can continue and, hence, more accounting data is accumulated at the AG. This accounting data is forwarded to the A4C.h at the end of the next interval, i.e. t_2. At step (3), the charging system will rate for the actual resource consumption within t_1, i.e. the used resources $\underline{x}_u$. The resulting charge is subtracted from initial credits (rectangle with a light-gray background). Remaining credits are used for the calculation of the sec-

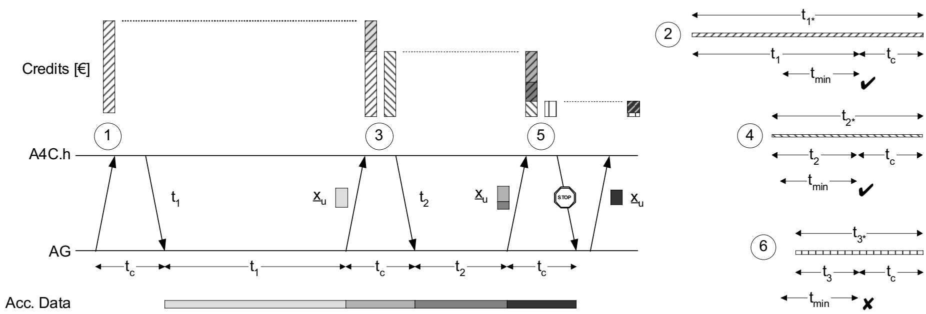

Fig. 2. Example application of TICA with message flows shown between AG and A4C.h

ond valid time interval, t_2 in step (4). After t_2 is reached, the AG will again ask for a renewal of the time interval, step (5). At step (6), TICA returns an invalid—i.e. too short—interval. Therefore, A4C.h sends a "stop" message to the AG, which will lead to a termination of all running services of this user. Finally, the accumulated accounting data from the last t_c is sent to A4C.h.

6 Evaluation

To evaluate TICA, a real-world scenario for a provider *A* offering multimedia IP services on the Daidalos architecture has been defined: (s_1) Live-streaming TV, (s_2) Video Call, (s_3) VoIP, and (s_4) Messaging. Each service uses its own level of QoS and it is assumed that sufficient network resources (QoS) are available always to use any combination of s_1 to s_4. Provider *A* offers all these services with the prepaid charging option and user-incentive tariff functions in order to compete in a multi-provider environment. Therefore, tariffs are rather complex functions depending on several variables—totalling in 7 different variables here. Tariff functions are represented as (6), are transformed into (8), and finally into (9), cf. [19] for detailed formulas addressing s_1 to s_4.

For a different amount of initial credits, consecutive time intervals have been calculated until all credits were exhausted. For typical prepaid card values between 20 € to 50 €, Fig. 3 illustrates that the first time interval shows a length between approximately 10 and 20 minutes. Smaller initial amount of credits yield in shorter time intervals and, therefore, generate credit re-checks at a higher rate. During any running time interval, only its length has to be monitored by provider *A*. However, the scenario showed that for services s_2 and s_4, two additional variables are needed to be observed, too. In the current implementation, only the expiry of the time interval triggered a credit re-check. Therefore, only a small additional load was imposed on the AG.

The following simulations formed the proof of those theoretical mathematical properties of (9), i.e. it's impossible for provider *A* to lose revenue due to overuse of credits. This is in contrast to existing service node and hot billing prepaid solutions, where the credit checking frequency is reduced at the expense of an increased risk for revenue loss. Thus, TICA is advantageous and ensures a providers economic efficiency. Even if

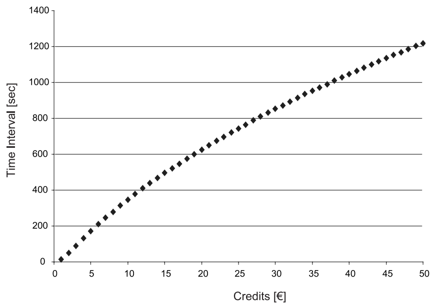

Fig. 3. Length of the first time interval as a function of the initially available credits

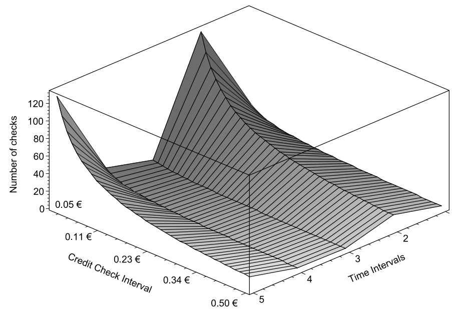

Fig. 4. Number of credit check for the service node prepaid approach

only the number of credit checks is compared, TICA performs much better than existing approaches. Due to the different concept of prepaid in hot billing, a direct comparison was only possible with the service node prepaid approach based on [12], cf. Fig. 4. For those simulations, one set consisting of five consecutive time intervals was selected. The set contains time intervals with low (3), medium (1, 4), and high (2, 5) service consumptions. In the service node approach, user's credits are checked every *I* €, here reaching from 0.50 € to 0.05 €. Note that TICA uses always exactly *one* credit check per interval, independent of the actual service consumption. Therefore, this leads to a plane with height $z = 1$ (not shown). Fig. 4 shows that for time intervals with low service consumption, the service node approach performs good (3) and reasonable for medium service usage (1, 4). For time intervals with a high service consumption (2, 5) the number of credit checks starts to increase around 0.20 €. For different scenarios of [12], the optimal credit checking frequency for the service node approach, was between 0.16 € and 0.05 €, thus, the time interval approach is more efficient in technical terms.

7 Conclusions and Further Work

An analysis of existing prepaid systems for IP-based networks has shown that no suitable solution exists to solve efficiency matters in an integrated manner. Therefore, the concept of time interval-based prepaid charging for service bundles has been introduced. Orthogonal to this simplification of the charging, the tariff modelling introduced allows for the definition of flexible and user-incentive tariff schemes. To calculate time intervals for each bundle, tariff functions have to be transformed into time-based functions by applying the TICA algorithm. Finally, the TICA algorithm has been evaluated in different scenarios for the Daidalos all-IP architecture. The approach derived within this paper is economically and technically reliable, flexible, and scalable, since the overall development considered those characteristics. Reliability is achieved through formal properties, where it was shown that is impossible to overuse reserved credits. Flexibility is given by supporting any kind of services based on IP and by allowing arbitrary tariff functions. Scalability is improved compared to existing alarm-based prepaid systems by reducing the overhead of real-time monitoring. Further work will extend the service bundle concept to support a dynamic adding and removing of services to and from the bundle. Calculating the upper bound of resource consumptions will be investigated further by incorporating sophisticated traffic modeling.

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Mining Stock Market Tendency Using GA-Based Support Vector Machines

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Abstract. In this study, a hybrid intelligent data mining methodology, genetic algorithm based support vector machine (GASVM) model, is proposed to explore stock market tendency. In this hybrid data mining approach, GA is used for variable selection in order to reduce the model complexity of SVM and improve the speed of SVM, and then the SVM is used to identify stock market movement direction based on the historical data. To evaluate the forecasting ability of GASVM, we compare its performance with that of conventional methods (e.g., statistical models and time series models) and neural network models. The empirical results reveal that GASVM outperforms other forecasting models, implying that the proposed approach is a promising alternative to stock market tendency exploration.

1 Introduction

Mining stock market tendency is regarded as a challenging task due to its high volatility and noisy environment. There have many studies using artificial neural networks (ANNs) in this area. The early days of these studies focused on application of ANNs to stock market prediction, such as [1-3]. Recent research tends to hybridize several artificial intelligence (AI) techniques [4]. Some researchers tend to include novel factors in the learning process. Kohara et al. [5] incorporated prior knowledge to improve the performance of stock market prediction. Tsaih et al. [6] integrated the rule-based technique and ANN to predict the direction of the S&P 500 stock index futures on daily basis. Similarly, Quah and Srinivasan [7] proposed an ANN stock selection system to select stocks that are top performers from the market and to avoid selecting under performers. They concluded that the portfolio of the proposed model outperformed the portfolios of the benchmark model in terms of compounded actual returns overtime. Kim and Han [8] proposed a genetic algorithms approach to feature discretization and the determination of connection weights for ANN to predict the stock price index. They suggested that their approach reduced the dimensionality of the feature space and enhanced the prediction performance.

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Although a large number of successful applications have shown that ANN can be a very useful tool for stock market modeling and forecasting [9], some of these studies, however, showed that ANN had some limitations in learning the patterns because stock market data has high volatility and noise. ANN often exhibits inconsistent results on noisy data [10]. Furthermore, ANN also suffers from difficulty in trapping into local minima, overfitting and selecting relevant input variables [11].

In order to overcome the above main limitations of ANN, a novel intelligent learning algorithm, genetic algorithm-based support vector machine (GASVM) approach, is proposed in this study. First of all, the SVM, a novel intelligent algorithm developed by Vapnik and his colleagues [12] is used to avoid local minima and overfitting of traditional neural network models. Actually, many traditional neural network models had implemented the empirical risk minimization (ERM) principle; SVM implements the structural risk minimization (SRM) principle. The former seeks to minimize the mis-classification error or deviation from correct solution of the training data but the latter searches to minimize an upper bound of generalization error. In addition, the solution of SVM may be global optimum while other neural network models may tend to fall into a local optimal solution. Thus, overfitting is unlikely to occur with SVM [12]. Subsequently, to select relevant variable and reduce the complexity of SVM, a genetic algorithm is used. Therefore the proposed GASVM approach has two distinct advantages. One is that the computations of GASVM are reduced by the decrease of model inputs and running speed will be accelerated. Another is that GASVM can avoid some defects of neural network models, such as local minima and overfitting.

Although the proposed GASVM has the above advantages, there are few studies for the application of SVM in mining stock market tendency. Kim [13] applied SVM to predict the direction of changes of Korea composite stock price index. Recently, Huang et al. [14] examined the predictability of stock index with SVM. They showed that SVM outperformed the BP networks on the criteria of hit ratios. However, in the existing literature, no studies mentioned the related input variables selection. Our approach fills up the gap in the literature.

The main motivation of this study is to propose a new data mining approach for exploring stock market tendency and to test the predictability of the proposed GASVM model by comparing it with conventional models and neural network models. The rest of the study is organized as follows. The next section will describe the proposed GASVM model building process in detail. In Section 3, we give an experiment scheme and Empirical results and analysis are reported in this section. The concluding remarks are given in Section 4.

2 Model Building Process

In this section, the GASVM model building process is presented in detail. First of all, a basic theory of the SVM in classification is described. Then the genetic algorithm for variable selection will be proposed to reduce the model complexity of SVM. Based on the genetic algorithm and SVM, a GASVM model is built finally.

2.1 The Basic Theory of SVM

The SVM used here is the support vector classification (SVC) proposed by Vapnik [12]. The basic idea of SVM is to use linear model to implement nonlinear class boundaries through some nonlinear mapping the input vector into the high-dimensional feature space. A linear model constructed in the new space can represent a nonlinear decision boundary in the original space. In the new space, an optimal separating hyperplane is constructed. Thus SVM is known as the algorithm that finds a special kind of linear model, the maximum margin hyperplane. The maximum margin hyperplane gives the maximum separation between the decision classes. The training examples that are closest to the maximum margin hyperplane are called support vectors. All other training examples are irrelevant for defining the binary class boundaries.

For the linearly separable case, a hyperplane separating the binary decision classes in the three-attribute case can be represented as the following equation:

$$y = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 \tag{1}$$

Where y is the outcome, x_i are the attribute values, and there are four weights w_i to be learned by the learning algorithm. In Equation (1), the weights w_i are parameters that determine the hyperplane. The maximum margin hyperplane can be represented as the following equation in terms of the support vectors:

$$y = b + \sum \alpha_i y_i \mathbf{x}(i) \cdot \mathbf{x} \tag{2}$$

Where y_i is the class value of training example $\mathbf{x}(i)$, $\cdot$represents the dot products. The vector $\mathbf{x}$ represents a test example and the vectors $\mathbf{x}(i)$ are the support vectors. In this equation, b and α_i are parameters that determine the hyperplane. From the implementation point of view, finding the support vectors and determining the parameters b and α_i are equivalent to solving a linearly constrained quadratic programming.

As mentioned above, SVM constructs linear model to implement nonlinear class boundaries through the transforming the inputs into the high-dimensional feature space. For the nonlinearly separable case, a high-dimensional version of Equation (2) is represented as follows:

$$y = b + \sum \alpha_i y_i K(\mathbf{x}(i) \cdot \mathbf{x}) \tag{3}$$

The function $K(\mathbf{x}(i) \cdot \mathbf{x})$ is defined as the kernel function. There are some different kernels for generating the inner products to construct machines with different types of nonlinear decision surfaces in the input space. Choosing among different kernels the model that minimizes the estimate, one chooses the best model. Com Common examples of the kernel function are the polynomial kernel $K(x, y) = (xy + 1)^d$ and the Gaussian radial basis function $K(x, y) = \exp\left(-(x - y)^2 / 2\sigma^2\right)$ where d is the degree of the polynomial kernel and σ is the bandwidth of the Gaussian radial basis function kernel [13]. The construction and selection of kernel function is important to SVM, but in practice the kernel function is often given directly.

For the separable case, there is a lower bound 0 on the coefficient α_i in Equation (3). For the non-separable case, SVM can be generalized by placing an upper bound C on the coefficient α_i in addition to the lower bound [4].

2.2 Feature Vector Selection with Genetic Algorithm for SVM Modeling

In this study, we use genetic algorithm (GA) to extract feature vector of model inputs for SVM modeling. To date, GA has become a popular optimization method as they often succeed in finding the best optimum in contrast to most common optimization algorithms. Genetic algorithm imitates the natural selection process in biological evolution with selection, mating reproduction and mutation, and the sequence of the different operations of a genetic algorithm is shown in the left part of Fig. 1. The parameters to be optimized are represented by a chromosome whereby each parameter is encoded in a binary string called gene. Thus, a chromosome consists of as many genes as parameters to be optimized. Interested readers can be referred to [15-16] for more details. In the following GA for feature variable selection is discussed.

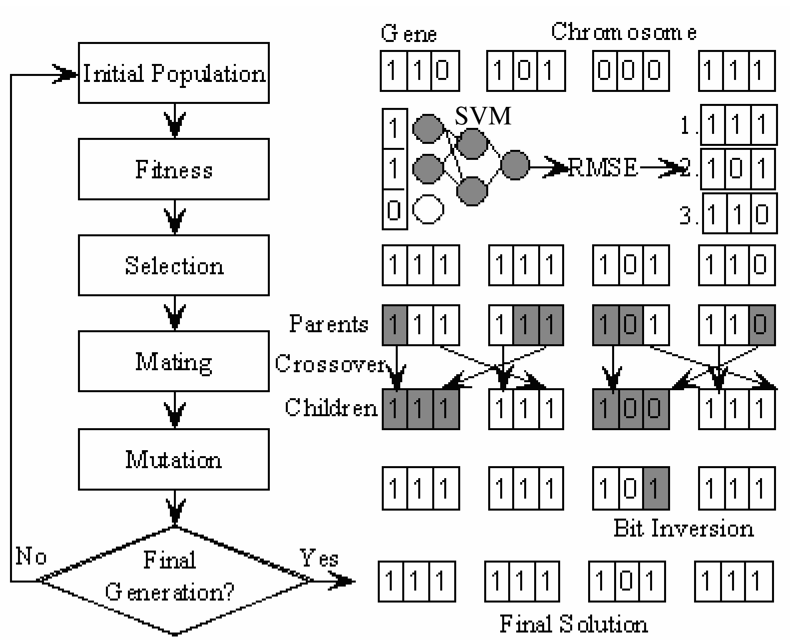

Fig. 1. The variable selection with the genetic algorithm for SVM

First of all, a population, which consists of a given number of chromosomes, is initially created by randomly assigning "1" and "0" to all genes. In the case of variable selection, a gene contains only a single bit string for the presence and absence of a variable. The top right part of Fig. 1 shows a population of four chromosomes for a three-variable selection problem. In this study, the initial population of the GA is randomly generated except of one chromosome, which was set to use all variables.

The binary string of the chromosomes has the same size as variables to select from whereby the presence of a variable is coded as “1” and the absence of a variable as “0”. Consequently, the binary string of a gene consists of only one single bit. The subsequent work is to evaluate the chromosomes generated by previous operation by a so-called fitness function, while the design of the fitness function is a crucial point in using GA, which determines what a GA should optimize. Here the goal is to find a small subset of variables from many candidate variables. In this study, the SVM is used for modeling the relationship between the input variables and the responses. Thus, the evaluation of the fitness starts with the encoding of the chromosomes into SVM model whereby “1” indicates that a specific variable is used and “0” that a variable is not used by the SVM model. Then the SVM models are trained with a training data set and after that, a testing data set is predicted. Finally, the fitness is calculated by a so-called fitness function f. For a prediction/classification problem, for example, our fitness function for the GA variable selections can use the following form:

$$f = 0.3 RMSE_{training} + 0.7 RMSE_{testing} - \alpha (1 - n_v / n_{tot}) \tag{4}$$

where n_v is the number of variables used by the SVM models, n_{tot} is the total number of variables and *RMSE* is the root mean square error, which is defined in Equation (5) with N as total number of samples predicted, y_t as the actual value and $\hat{y}_t$ as the predicted value:

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (\hat{y}_t - y_t)^2} \tag{5}$$

From Equation (4), we find that the fitness function can be broken up into three parts. The first two parts correspond to the accuracy of the SVM models. Thereby $RMSE_{training}$ is based on the prediction of the training data used to build the SVM models, whereas $RMSE_{testing}$ is based on the prediction of separate testing data not used for training the SVM models. It was demonstrated in [17] that using the same data for the variable selection and for the model calibration introduces a bias. Thus, variables are selected based on data poorly representing the true relationship. On the other hand, it was also shown that a variable selection based on a small data set is unlikely to find an optimal subset of variables [17]. Therefore, a ratio of 3:7 between the influence of training and testing data was chosen. Although being partly arbitrary this ratio should give as little influence to the training data as to bias the feature selection yet taking the samples of the larger training set partly into account. The third part of the fitness function rewards small models using only few variables by an amount proportional to the parameter *a*. The choice of *a* will influence the number of variables used by the evolved SVM. A high value of results in only few variables selected for each GA whereas a small value of a results in more variables being selected. In sum, the advantage of this fitness function is that it takes into account not only the testing error of test data but also partially the training error and primarily the number of variables used to build the corresponding SVM models.

After evolving the fitness of the population, the best chromosomes with the highest fitness value are selected by means of the roulette wheel. Thereby, the chromosomes are allocated space on a roulette wheel proportional to their fitness and thus the fittest chromosomes are more likely selected. In the following mating step, offspring chromosomes are created by a crossover technique. A so-called one-point crossover technique is employed, which randomly selects a crossover point within the chromosome. Then two parent chromosomes are interchanged at this point to produce two new offspring. After that, the chromosomes are mutated with a probability of 0.005 per gene by randomly changing genes from "0" to "1" and vice versa. The mutation prevents the GA from converging too quickly in a small area of the search space. Finally, the final generation will be judged. If yes, then the optimized subsets are selected. If no, then the evaluation and reproduction steps are repeated until a certain number of generations, until a defined fitness or until a convergence criterion of the population are reached. In the ideal case, all chromosomes of the last generation have the same genes representing the optimal solution.

2.3 GA-Based SVM Model in Data Mining

Generally, SVM cannot reduce the input information. When the input space dimension is rather large, the process of solving SVM problem will require too much time. It is therefore necessary for SVM to preprocess the input feature vectors. In this study, the genetic algorithm is used to preprocess the input feature vectors. Then the processed feature vectors are sent to SVM model for learning and training. Thus, a novel forecasting approach, GA-based SVM (GASVM) model integrating GA and SVM, is formulated for data mining, as illustrated in Fig. 2.

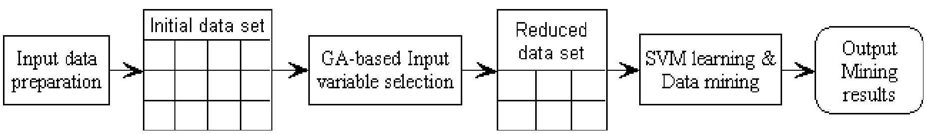

Fig. 2. The main process of GASVM for data mining

As can be seen from Fig.2, the GASVM data mining model comprises three phases: data preparation, input variable selection, and SVM learning and mining. Generally, for a specific problem, the first step is to collect and prepare some input data (initial data set) in terms of factor analysis. The second step is to select some typical feature variables using GA method mentioned previously, and a reduced data set can be obtained. The final step is to train SVM model for mining purpose and to output the mining results accordingly.

3 Empirical Study

3.1 Research Data

The research data used in this study is technical indicators and the direction of change in the daily S&P500 stock price index. Since we attempt to mine the stock index movement tendency, technical indicators are used as input variables. This study selects 18 technical indicators to make up the initial attributes, as determined by the review of domain experts and prior research [13]. The descriptions of initially selected attributes are presented in Table 1.

Table 1. Initially selected feature indicators and their formulas

Feature indicators	Formulas
Price (P)	x_t, (t = 1, 2, …, n)
Stochastic oscillator (SO)	$((x_t - x_l(m))/(x_h(m) - x_l(m)))$
Moving stochastic oscillator (MSO)	$\frac{1}{m}\sum_{i=t-m+1}^{t}(SO_{t-i})$
Slow stochastic oscillator (SSO)	$\frac{1}{m}\sum_{i=t-m+1}^{t}(MSO_{t-i})$
Rate of change (ROC)	x_t/x_{t-m}
Momentum (M)	x_t-x_{t-m}
Moving average (MA)	$\frac{1}{m}\sum_{i=t-m+1}^{t}x_i$
Moving variance (MV)	$\frac{1}{m}\sum_{i=t-m+1}^{t}(x_i - \overline{x}_t)^2$
Moving variance ratio (MVR)	MV_t^2 / MV_{t-m}^2
Exponential moving average (EMA)	$a \times x_t + (1 - a) \times x_{t-m}$
Moving average convergence & divergence (MACD)	$\sum_{i=t-m+1}^{t}EMA_{20}(i) - \sum_{i=t-m+1}^{t}EMA_{40}(i)$
Accumulation/ distribution oscillator (ADO)	$((x_l(m) - x_t)/(x_h(m) - x_l(m)))$
Disparity5 (D5)	x_t / MA_5
Disparity10 (D10)	x_t / MA_{10}
Price oscillator (OSCP)	$(MA_5 - MA_{10})/MA_5$
Commodity channel index (CCI)	$(M_t - SM_t)/0.015D_t$ where $M_t = x_h(t) + x(t) + x_l(t)$, $SM_t = \sum_{i=t-m+i}^{t}M_i / m$, $D_t = \sum_{i=t-m+i}^{t}\lvert M_i - SM_t\rvert / m$.
Relative strength index (RSI)	$100 - \frac{100}{1 + RS}$ where $RS = \frac{\sum_{i=t-m+1}^{t}(x(i) - x(i-1))^{+}}{\sum_{i=t-m+1}^{t}(x(i) - x(i-1))^{1}}$
Linear regression line (LRL)	$\frac{m \times \sum_{i=t-m+1}^{t} i \times x(i) - \sum_{i=t-m+1}^{t} i \times \sum_{i=t-m+1}^{t} x(i)}{m \times \sum_{i=t-m+1}^{t} i^2 - \left(\sum_{i=t-m+1}^{t} i\right)^2}$

In order to evaluate the forecasting ability of the proposed GASVM model, we compare its performance with those of conventional methods, such as statistical and time series model, and neural network model, as well as individual SVM model without GA preprocessing. Typically, we select random walk (RW) model, autoregressive integrated moving average (ARIMA) model, individual back-propagation neural network (BPNN) model and individual SVM model as the benchmarks.

For RW and ARIMA models, only the foreign exchange series P (i.e., price) is used. Each of the RW and ARIMA models is estimated and validated by in-sample data. The model estimation selection process is then followed by using an empirical evaluation, e.g., RMSE, which is based on the out-of-sample data.

For individual BPNN model, the BPNN has 18 input nodes because 18 input variables are employed. By trial and error, we select 36 hidden nodes. Training epochs are 5000. The learning rate is 0.25 and the momentum term is 0.30. The hidden nodes use sigmoid transfer function and the output node uses the linear transfer function.

Similarly, individual SVM model has also 18 input variables, the Gaussian radial basis function are used as the kernel function of SVM. In SVM model, there are two parameters, i.e., upper bound C and kernel parameter σ, to tune. By trial and error, the kernel parameter σ is 10 and the upper bound C is 70.

For GASVM model, the GA is firstly used as preprocessor for input variable selection. Then the reduced variables are sent to the SVM model for learning and forecasting. In this study, eleven variables, i.e., price (P), stochastic oscillator (SO), rate of change (ROC), moving average (MA), moving variance ratio (MVR), moving average convergence & divergence (MACD), disparity5 (D5), price oscillator (OSCP), commodity channel index (CCI), relative strength index (RSI) and linear regression line (LRL), are retained by GA-based variable selection. Accordingly, some other parameters settings are similar to the individual SVM model.

This study is to mine and explore the tendency of stock price index. They are categorized as “0” and “1” in the research data. “0” means that the next day’s index is lower than today’s index, and “1” means that the next day’s index is higher than today’s index. The entire data set covers the period from January 1 2000 to December 31 2004. The data sets are divided into two periods: the first period covers from January 1 2000 to December 31 2003 while the second period is from January 1 2004 to December 31 2004. The first period, which is assigned to in-sample estimation, is used to network learning and training. The second period is reserved for out-of-sample evaluation.

3.2 Experiment Results

Each of the models described in the last section is estimated and validated by in-sample data. The model estimation selection process is then followed by an empirical evaluation based on the out-of-sample data. At this stage, the relative performance of the models is measured by hit ratio. Table 2 reports the experimental results.

From Table 2, the differences between the different models are very significant. For example, for the GBP test case, the D_{stat} for the RW model is only 51.06%, for the ARIMA model it is 56.13%, and for the BPNN model D_{stat} is only 69.78%; while for the proposed GASVM forecasting model, D_{stat} reaches 84.57%, which is higher than the individual SVM, implying that the GA-based variable selection has a significant impact on SVM forecasting.

In addition, in the experiments, we also find that the GASVM computing is faster then BPNN and SVM. The main reason is that the GA-based variable selection procedure reduces the model input space and thus saves the training time and speeds the SVM learning. Therefore, the proposed GASVM model can have some comparative advantages relative to individual SVM and BPNN. First of all, there is less parameter to tune for GASVM than for BPNN. Second, the GASVM can overcome some shortcomings of BPNN, such as overfitting and local minima. Third, the input space of the GASVM is smaller, and the learning speed of GASVM is faster than the individual SVM model.

Table 2. The prediction performance comparison of various models

Mining models	Hit ratios (%)
RW	51.06
ARIMA	56.13
BPNN	69.78
SVM	78.65
GASVM	84.57

4 Conclusions

This study proposes using a GA-based SVM data mining model that combines the genetic algorithm and support vector machine to predict stock market tendency. In terms of the empirical results, we find that across different forecasting models for the test cases of S&P 500 on the basis of same criteria, the proposed GASVM model performs the best. In the proposed GASVM model test cases, the D_{stat} is the highest, indicating that the nonlinear ensemble forecasting model can be used as a viable alternative solution for mining stock market tendency.

Acknowledgements

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Model-Based Analysis of Money Accountability in Electronic Purses*

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Abstract. The Common Electronic Purse Specifications (CEPS) define requirements for all components needed by an organization to implement a globally interoperable electronic purse program. In this paper we describe how we model purchase transaction protcol in CEPS using formal specification language. We define and verify the money accountability property of the CEPS, and we address its violation scenario in the presence of communication network failures. Using model checking technique we find that transaction record stored in the trusted-third party plays a essential role in satisfying the accountability property.

Keywords: Formal specification and verification, security, e-commerce protocol, CEPS, model checking, money accountability, Casper, FDR.

1 Introduction

The use of smart cards as electronic purses and smart credit card/debit cards is increasing the market potentiality of electronic commerce. The technology for secure and stable electronic payment cards is being driven by big companies. Visa, Proton and a number of European financial institutions have collaborated to create the Common Electronic Purse Specifications (CEPS)[1], with the purpose of having some degree of international interoperability in all national purse schemes.

One of the most important requirements in electronic commerce protocols is to ensure the accountability on electronic transactions. *Money accountability* means that money is neither created nor destroyed in the process of an electronic transaction between customer and merchant. For example, a customer loads an e-money on smart card from the bank, and attempts to use the coin to pay a merchant. Unfortunately, communication networks fails in the transit of e-money from the customer to the merchant. In this situation, the consumer cannot be convinced that the e-money has been spent correctly. This is a critical

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issues for money accountability, because the customer and the merchant can be susceptible to dispute between transacting parties. In other words, without adequate accountability assurance in CEPS, there would be no means to settle legal disputes against fraudulent individuals.

In this paper we address the accountability problem in the CEPS, and we verify it with model checking technique. First, we model accountability property and CEPS process in CSP (Communicating Sequential Processes) process algebra language, respectively. Second, the property model is expressed as *SPEC*, and the CEPS is modelled as *SYSTEM*. Lastly, we use the FDR (Failure Divergence Refinement) model checking tool. If *SYSTEM* is a refinement of *SPEC*, the set of behaviors generated by *SYSTEM* is a subset of those generated by *SPEC*. It means that the CEPS satisfies the accountability property.

The remainder of this paper is organized as follows. Section 2 describes some related work. Section 3 gives a brief overview of the CEPS. Section 4 briefly describes model checking technique, and describes how the CEPS and the accountability property can be modelled and verified using CSP and FDR. Finally, section 5 concludes this paper.

2 Related Work

There has been many research in using model checking to verify the security aspects of protocols. However, relatively little research has been carried out on formal analysis of accountability in e-commerce protocols.

Il-Gon Kim et al.[5] used the FDR model checker to verify the secrecy and authentication properties of a m-commerce protocol.

Heintze et al.[3] researched on non-security properties such as money atomicity and goods atomicity of two e-commerce protocols(NetBill[2] and Digicash[4]).

Indrakshi Ray et al.[7] proposed mechanisms that desirable properties addressed in [3] are still preserved in a fair-exchange protocol despite network failures.

Jan Jürjens et al.[6] investigated the security aspect of the CEPS using AUTOFOCUS tool. The author modelled CEPS with several diagrams which supported by the AUTOFOCUS tool. Then he added intruder model in order to intercept messages and learn secrets in the messages. Finally, the author used the AUTOFOCUS connection to the model checker SMV.

In this paper we specify the behaviour of the CEPS purchase transaction protocol in presence of network failures, using model checking technique proposed in [3][7]. We focus on verifying the accountability property in the CEPS, not in the viewpoint of security properties such as confidentiality and authentication. So far, there is no research to specify and verify the accountability of the CEPS using model checking. In this regard our work is different from the work of Jan Jürjens et al.

3 CEPS

In this section we give a brief overview of the Common Electronic Purse Specification and describe its purchase transaction protocol.

The Common Electronic Purse Specification (CEPS) was proposed by Visa, Proton and a number of European financial institutions, in order to have a common standard for all national purse schemes. Currently, stored value smart cards(called "electronic purses") have been proposed to allow an electronic purse application which conforms to CEPS. It is designed as a standard for 72 different e-purse systems worldwide to work together.

Herein we address the central function of CEPS, the purchase transaction, which allows the cardholder to use e-money on a smart card to pay for goods. In this paper we assume that the communicating participants in the purchase transaction protocol should consist of three part; customer's smart card, PSAM in the merchant's POS (Point Of Sale) device, and bank. The POS device embeds Purchase Security Application Module (PSAM), which is used to store and process data transferred from cardholders. The bank plays a role of checking the validity of customer's card and account.

3.1 Purchase Transaction Protocol

The PSAM starts the protocol (see Fig.1) after the smart card(CARD) is inserted into the POS device, by sending a debit request to the card. The debit request message contains an e-money amount for a deal. In this paper we focus on addressing the accountability property, not security properties. For the detailed notation and meaning of each data and key used in messages, the reader can see [1]. At step 2, the CARD responds with the message containing e-money singed by the customer. The message of purchase response includes the e-money amount signed by the customer. Then the PSAM forwards the customer's electronic payment order (EPO) to the BANK. At step 4, the BANK checks the validity of this EPO and sends to the PSAM a receipt of the fund transfer. Finally, if the electronic transaction between the CARD and the PSAM has completed successfully, then the PSAM sends 'transaction completed' message to the CARD.

1. PSAM → CARD : debit request
2. CARD → PSAM : purchase response
3. PSAM → BANK : endorsed signed EPO
4. BANK → PSAM : signed receipt
5. PSAM → CARD : transaction completed

Fig. 1. Purchase transaction protocol of the CEPS

4 CSP Specification and FDR Model Checking of the Purchase Transaction Protocol

In this section we describe how we model and verify the purchase transaction protocol and the accountability property using CSP/FDR.

We use CSP process algebra language to encode communicating channel and each participant. A CSP process denotes a set of sequences of events, where an event represents a finite state transition. A CSP process may be composed of component processes which require synchronization on some events. In this paper we encode communicating channel, the customer, the merchant, and the bank processes in CSP.

In FDR model checking tool, the refinement model is described as a process, say SYSTEM, and the property model is specified as another process, say SPEC. If SYSTEM is a refinement of SPEC, it means that the set of the possible behaviors of SYSTEM is a subset of the set of possible behaviors of SPEC. This relationship is written SPEC $\sqsubseteq$ SYSTEM in FDR. If SYSTEM is not a refinement of SPEC, the FDR tool generates a counter-example that describes under what scenario the property is violated. In the following subsections, SYSTEM represents the purchase transaction protocol and SPEC means the accountability property.

4.1 Communication Environment Process

We model asynchronous communication channel which proposed in [3][7], instead of using synchronous channel, because the asynchronous communication is useful to consider a network failure in the CSP model.

Fig.2 shows logical communication channel in the purchase transaction protocol. For example, the customer uses two communication processes(*COMMcp* and *COMMcb*) in order to communicate with the merchant and the bank, respectively. For example, the process *COMMcp* reads a data from channel *coutp*(which is connected to the channel *coutp* in the CARD) and writes the data to channel *pinc* and the process *COMMpc* reads a data from channel *poutc* and writes it to channel *cinm*. In a failure network, the transaction data between the customer and the merchant may be lost. To reflect on this unreliable communication channels, we model that *COMMcp* and *COMMpc* could lose sending or receiving data non-deterministically.

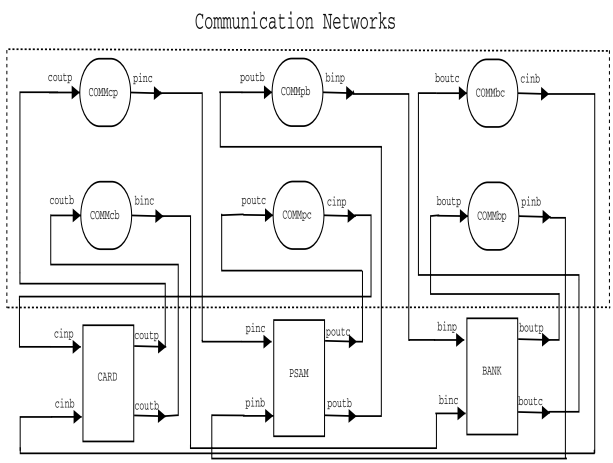

Fig. 2. Logical communication channel in purchase communication protocol

4.2 The CARD Process

We model the CARD process could load e-money from the BANK process if the customer's account holds enough balance. Then the CARD waits for the debit request(*DebitA* or *DebitB*) from the PSAM. If the CARD receives *DebitA* or *DebitB* correctly, it sends electronic payment order(*epo_TokenA* or*epo_TokenB*) to the PSAM. In the model we allow the CARD to send two different electronic payment orders for one purchase transaction, because this will be considered evidence to detect fradulent double deposit of the merchant. Because the CEPS provides on-line and off-line communication between the CARD and the PSAM, the following scenarios could be possible. As the customer's response to the debit request is in transit, the communication network fails. Thus the customer is left in an uncertain situation whether the e-money has been spent correctly or not. This is a critical issue for money accountability, because the merchant could be accused of double deposit. To confirm the accountability, we allow the customer to go to the bank and wait for the arbitration decision based on the transaction record. To reflect on this scenario, we model that the card process sends second

```
CARD = STOP |~| LOAD_CARD
LOAD_CARD = coutb!loadtoken ->
            cinb?x ->
            if x==token then loaded_token -> (USE_TOKEN [] KEEP_TOKEN)
            else STOP

USE_TOKEN = cinp?x -> DEBIT_CARD(x)
KEEP_TOKEN = cKeepsToken -> STOP

DEBIT_CARD(x) = if (x==DebitA) then
                (coutp!epo_TokenA -> EPO_TOKEN_SENT)
                else if (x==DebitB) then
                (coutp!epo_TokenB -> EPO_TOKEN_SENT)
                else RETURN_TOKEN

RETURN_TOKEN = coutb!paymentAlready ->
             cinb?x ->
             (if (x==refundSlip) then REFUND_RECEIVED
             else if (x==depositSlip) then epo_tokenSpent -> ARBITRATION
             else ERROR_DEBIT)

REFUND_RECEIVED = STOP
EPO_TOKEN_SENT = (cinp?y ->
                 (if (y==ok) then epo_tokenSpent -> (END |~| USE_TOKEN)
                 else if (y==no) then STOP
                 else RETURN_TOKEN))
                [] USE_TOKEN
                [] (timeoutEvent -> RETURN_TOKEN)
```

Fig. 3. The CARD process

EPO(*epo_TokenB*) again, if the response from the merchant is not received in a specified time after sending first EPO(*epo_TokenA*) to the merchant.

The CARD process's main behaviours could be summarized as follows:

– use token(*USE_TOKEN*) : the customer may use a token for purchasing a good
– load token(*LOAD_CARD*) : the customer may load a token from the bank
– token holding(*KEEP_TOKEN*) : the customer may keep a token for future use, not spending it to purchase goods
– token return(*RETURN_TOKEN*) : the consumer may return token to the bank for refund, in case of incorrect transaction.

4.3 The PSAM Process

The PSAM process starts with sending the debit request message(*DebitA* or *DebitB*) to the CARD; the *SEND_DEBIT_A* and *SEDN_DEBIT_B* states represent this step. Then the PSAM waits for the response(*epo_TokenA* or *epo_TokenB*)

```
PSAM = STOP |~| REPEATED_DEBIT_REQUEST(none)

REPEATED_DEBIT_REQUEST(previousDebitRequest) =
if (previousDebitRequest == none) then (SEND_DEBIT_A [] SEND_DEBIT_B)
else if (previousDebitRequest == DebitA) then SEND_DEBIT_B
else SEND_DEBIT_A

SEND_DEBIT_A =  (poutc!DebitA -> WAIT_FOR_RESPONSE(epo_TokenA))
SEND_DEBIT_B =  (poutc!DebitB -> WAIT_FOR_RESPONSE(epo_TokenB))

WAIT_FOR_RESPONSE(epo_Token) = pinc?x ->
if (x==epo_Token) then (mGets_epoToken->FORWARD_EPO_TO_BANK(epo_Token))
else poutc!bad_epoToken -> NO_TRANSACTION

NO_TRANSACTION = STOP
FORWARD_EPO_TO_BANK(epo_Token)=poutb!epo_Token -> WAIT_FOR_BANK(epo_Token)

WAIT_FOR_BANK(epo_Token) =
(pinb?x -> (if x==depositSlip then M_MAY_BE_FRAUD(epo_Token)
           else if x==refundSlip then (mGetsRefundSlip -> STOP)
           else if x==alreadyDeposited then FRAUD_DISCOVERED
           else if x==badBalance then poutc!no -> STOP
           else STOP))

FRAUD_DISCOVERED = STOP
M_MAY_BE_FRAUD(epo_Token) = END |~| FORWARD_EPO_TO_BANK(epo_Token)|~|
                            REPEATED_DEBIT_REQUEST(epo_Token)   |~|
                            poutc!ok -> STOP
```

Fig. 4. The PSAM process

from the CARD. After receiving the EPO (Electronic Payment Order) token, the PSAM sends it to the BANK in order to check the validity and the balance of the cardholder. If the payment token from the customer has been settled successfully, the merchant will get a deposit slip; denoted by *depositSlip*. When a merchant attempts to deposit a coin twice, the PSAM process receives *alreadyDeposited* data from the bank.

Herein the *M_MAY_BE_FRAUD* state is used to trace the fraud when a merchant attempts to deposit a coin twice for one purchase transaction.

4.4 The Bank Process

The most important function in the BANK process plays a role in recording logs about an electronic transaction in order to guarantee the accountability. If the BANK decides that an EPO token of the customer is valid, then it debits the

```
 BANK =  binc?x -> (if (x==loadtoken) then
                    (debitC -> boutc!token -> RECORD_LOG(0,0,0))
                    else STOP)

 RECORD_LOG(Flag, A, B) =
  binc?x -> (if (x==paymentAlready) then
            (if (Flag==0) then
               (creditC -> boutc!refundSlip -> RECORD_LOG(1,0,0))
             else if (A==1 or B==1) then
              (arbitration -> boutc!depositSlip -> RECORD_LOG(Flag, A, B))
             else RECORD_LOG(Flag, A, B))
            else RECORD_LOG(Flag, A, B))
[] binp?x ->(if (x==epo_TokenA) then
            (if (Flag==0) then
               (creditM -> boutp!depositSlip -> RECORD_LOG(1,1, B))
             else if (B==0 and A==1) then
               (boutp!alreadyDeposited -> mFraud -> RECORD_LOG(Flag, A, B))
             else if (B==1 and A==1) then
               (creditM -> boutp!depositSlip -> RECORD_LOG(Flag, A, B))
             else if (A==1 and B==1) then STOP
             else (arbitration -> boutp!refundSlip -> RECORD_LOG(Flag, A, B)))
           else if (x==epo_TokenB) then
             if (Flag==0) then
               (creditM -> boutp!depositSlip -> RECORD_LOG(1,A,1))
             else if (A==0 and B==1) then
               (boutp!alreadyDeposited -> mFraud -> RECORD_LOG(Flag, A, B))
             else if (A==1 and B==0) then
               (creditM -> boutp!depositSlip -> RECORD_LOG(Flag, A, B))
             else if (A==1 and B==1) then STOP
             else arbitration -> boutp!refundSlip -> RECORD_LOG(Flag, A, B)
          else RECORD_LOG(Flag, A, B))
```

Fig. 5. The BANK process

balance of the card and credits the account of the merchant(denoted by events *debitC* and *creditM*). In addition, the BANK process can settle out arbitration state caused by fraud of a customer or a merchant. For example, after a merchant gets a deposit slip by finishing a successful transaction with a customer, he/she may try to deposit an e-money twice. In this situation, the BANK process warns that the merchant has already deposited and it may be fraud by the merchant(shown by *boutp!alreadyDeposited* and *mFraud* events).

4.5 Failure Analysis of Money Accountability Property

Accountability is one of the most critical requirements to electronic commerce protocols and it can be divided into two categories; money accountability and goods accountability.

Money accountability property means that money is neither created nor destroyed in the steps of an electronic commerce transaction[3]. *Goods accountability* property represents that a merchant receives payment if and only if the customer receives the goods[3].

In this paper, we do not deal with failure analysis of goods accountability property because goods delivery is not included in the CEPS. Money accountability in the CEPS could be considered in the viewpoint of a customer and a merchant. Customer's money accountability property may be defined as the following trace specification.

```
SPECc = STOP |~| (loaded_token ->
                                ((epo_tokenSpent -> STOP) |~|
                                (cKeepsToken -> STOP) |~|
                                (creditC -> STOP)))
```

Customer's money accountability written in CSP means that once e-money is loaded into a smart card, the customer may choose keep it for future use, spend it for purchasing goods, or return it for refund. After using FDR tool, we found that customer's money accountability property is satisfied in the spite of unreliable communication networks and merchant's fraudulent behaviour.

```
SPECm = STOP |~| (mGets_epoToken ->
                                ((creditM -> STOP) |~|
                                (mGetsRefundSlip -> STOP)
```

Merchant's money accountability described in CSP represents that once e-money is transferred into the PSAM, the merchant's account balance may be incremented and the merchant may get a refund slip for incorrect transaction. When we run FDR tool, it shows the following counterexample which represents that merchant's money accountability property(*SPECm* process) may be violated due to a merchant fraud.

```
coutb.loadtoken, binc.loadtoken, debitC, boutc.token, cinb.token,
loaded_token, poutc.DebitB, cinp.DebitB, coutp.epo_TokenB,
pinc.epo_TokenB, mGets_epoToken, poutb.epo_TokenB, binp.epo_TokenB,
creditM, boutp.depositSlip, pinb.depositSlip, poutc.DebitA,
cinp.DebitA, coutp.epo_TokenA, pinc.epo_TokenA, mGets_epoToken,
boutp.alreadyDeposited, mFraud
```

This sequence of CSP events may show the scenario where a merchant attempts to deposit an e-money twice for a deal with a customer. After the customer sends an e-money for a good price to the merchant(shown by *coutp.epo_TokenB* event), communication network fails between the customer and the merchant. The merchant finishes a transaction with a bank and the account balance of the merchant is incremented(*poutb.epo_TokenB*, *creditM*, and *pinb.depositSlip*). Then the customer sends another e-money again (*coutp.epo_TokenA*) to the merchant because the customer can't confirm the transaction result due to the network failure. At this moment a malicious merchant attempts to use the customer's e-money again by sending it to the bank.

However, the merchant's fraud behaviour can be detected by the transaction record in the bank. Therefore, above counterexample doesn't mean a violation of the merchant's money accountability property. When we modify the *SPECm* process to contain at least one fraud event *mFraud*, we confirm that the CEPS model satisfies the merchant accountability property. This result means that the merchant fraud may happen in the e-commerce transaction based on the CEPS, by tampering of a POS device and man-in-the-middle attack. However the risk of the merchant accountability violation against the merchant fraud could be solved through comparing the signed transaction log information stored in the card and the bank.

5 Conclusion

In this paper we analyzed the money accountability properties of the Common Electronic Purse Specification (CEPS) using model checking approach. We have also shown how model checking using CSP and FDR can be used to describe e-commerce transaction in the CEPS including communication failure networks and detect the violation scenario of the accountability properties.

In our modelling of the CEPS we have abstracted away the purchase transaction of the CEPS by focusing on the non-security aspect. We have found that the risk of customer security against the merchant may bring about the cause to violate the money accountability. In addition, we have also identified that the transaction record in the trusted-third party such as the bank could provide the most effective solution to guarantee the accountability in the e-commerce protocol.

In the future we plan to analyze accountability properties of the Load Security Application Module (LSAM) which plays a role in loading e-money into the card.

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Applying Modified Fuzzy Neural Network to Customer Classification of E-Business

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Abstract. With the increasing interest and emphasis on customer demands in e-commerce, customer classification is in a crucial position for the development of e-commerce in response to the growing complexity in Internet commerce logistical markets. As such, it is highly desired to have a systematic system for extracting customer features effectively, and subsequently, analyzing customer orientations quantitatively. This paper presents a new approach that employs a modified fuzzy neural network based on adaptive resonance theory to group users dynamically based on their Web access patterns. Such a customer clustering method should be performed prior to Internet bookstores as the basis to provide personalized service. The experimental results of this clustering technique show the promise of our system.

1 Introduction

In the recent years, the on-going advance of Internet and Web technologies has promoted the development of electronic commerce. Enterprises have been developing new business portals and providing large amount of product information in order to expand their markets and create more business opportunities. In addition to providing a new channel, e-commerce (EC) has the potential of serving customer better if it can take good advantage of information technology and develop EC-specific marketing strategy. Ec, not just the purchase of goods and services over the Internet, is a broad term. It encompasses all electronically conducted business activities, operations, and transaction processing. With the development of the Inter and EC, companies have changed the way they connect to and deal with their customers and partners. Businesses hence could overcome the space and time barriers and are now capable of serving customers electronically and intelligently. New enterprises should manage a particular customer's Web experiences by customer personalization and retain the communication or interaction with the customer. Such understanding of customers can be applied to transform customer information into quality services or products.

In the approach, we presents a framework that dynamically groups users according to their Web access and transactional data, which consist of the customers' behavior on web site, for instance, the purchase records, the purchase date, amount paid, etc. The proposed system is developed on the basis of a modified fuzzy ART neural network, and involves two sequential modules including: (1) trace customers' behavior on web site and generate customer profiles, (2) classify customers according customer profile using neural network. In second module, we employ a modified fuzzy

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ART, that is a kind of adaptive resonance theory neural network, for the following reasons. Similar to other neural network strategies, it can plastically adapt to such complex (often uncertain or inconsistent) and correlated (non-linear and not isolated) situations in market analysis rather than those linear functions such as K-means clustering model. And some of the methods like self-organizing map algorithm is suitable for detailed classification, rather than preliminary clustering, such as customer analysis. And some self-organizing map algorithms need to specify the expected number of clusters in advance, which may affect the clustering results due to subjective parameter setting.

The remainder of the paper is organized as follows. Section 2 presents the relate work. In Section 3, the framework to automatically extract user preference and recommend personalized information is expatiated in detail. Section 4 presents three classifier used in our experiments briefly. Implementation issues and the results of empirical studies are presented in Section 5. Finally, the conclusion can be found in Section 6.

2 A Customer Cluster Framework

In this section, an on-line customer cluster framework is presented, which is performed prior to an Internet bookstore in our experiment. The main idea is that customer preference could be extracted by observing customer behavior, including the transaction records, the transaction time and the products pages customer browsed. Then the result of first module is organized in a hierachical structure and utilized to generate customer profile respectively. Finally customer profile could be grouped into different teams using modified fuzzy ART neural network. The framework includes three modules: customer behavior recording, customer profile generating and customer grouping.

2.1 Customer Behavior Recording

Most personalization systems gather customer preference through asking visitors a series of questions or needing visitors rating those browsed web pages. Although relevance feedback obtained directly from customers may make sense, it is troublesome to customers and seldom done. And since customer interests often change likely, it is important to adjust the user preference profile incrementally. Although relevance feedback is effective, customers are overloaded. In the paper, we present a customer behavior recording module to collect the training data without user intervention through tracking the customers behavior on a e-commerce web site. In the paper, the customer behavior is divided two types: transaction record and customer operation. The transaction record includes the type and number of products customer purchased. The customer operation on product pages or images includes the browsing time, the view frequency, saving, booking, clicking hyperlinks, scrolling and so on.

According to some relate works, visiting duration of a product pages or images is a good candidate to measure the preference. Hence, in our work, each product page or image, whose visiting time is longer than a preset threshold (e.g. 30 seconds), is analyzed and rated. Periodically (e.g. every day), the module analyzes the activities of the previous period, whose algorithm is shown as follows:

```
BEGIN
For each product category Pi a customer browsed
{ if (Pi doesn't exist in customer log file)
       {favorite(Pi)=0;}
  for each (transaction record on Pi)
       {favorite(Pi)= favorite(Pi)+transaction-number*0.03;}
  While (browsing time of product page or image belonging to
Pi) > threshold
       { switch (happened operation)
            {case (saving, booking operation happened):
                 favorite(Pi)= favorite(Pi)+0.02;break;
             default: favorite(Pi)= favorite(Pi)+0.01;break;}
       }
}
updating favorite(Pi) in Customer Log file;
END
```

where the function *favorite(P_i)* measures the favorite degree of a certain product category in e-commerce web site, and the record in customer log file is shown as follows: *product-id, category, favorite*. The *category* element is the category path of a resource, what is a path from the root to the assigned category according the hierarchical structure of Internet bookstore. For example, in a Internet bookstore, "JavaBean" category is a subclass of "Java" category, "Java" category is a subclass of "Programming" category, and "Programming" category is a subclass of "Computer & Internet" category, then the category path of the product pages or images belonging to "JavaBean" is "/JavaBean/Java/Programming/Computer&Internet".

2.2 Customer Profile Generating

In this approach, we employ a tree-structured scheme to represent customer profile, with which customers specify their preference. Generator could organize customer preference in a hierarchical structure according the result of Recorder and adjust the structure to the changes of customer interests. Customer profile is a category hierarchy where each category represents the knowledge of a domain of user interests, which could easily and precisely express customer's preference. The profile enhances the semantic of user interests and is much closer to a human conception. The logical structure of the preference tree is shown as follows:

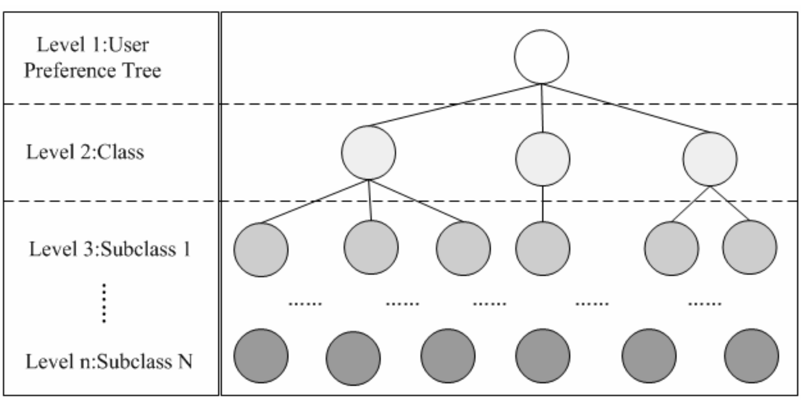

Fig. 1. The Logical Structure of Customer profile

Customer profile is established according the hierarchical structure of a certain e-commerce web site. It means the number of levels and the categories in the profile are similar with the web site. Each node in the tree, representing a category might be interested in, is described by an energy value E_i what indicates the favorableness of a product category. E_i controls the life cycle of a category in a profile. The energy increases when customers show interest in the product category, and it decreases for a constant value for a period of time. Relatively, categories that receive few interest will be abstracted gradually and finally die out. Based on the energy values of categories, the structure of customer profile can be modulated as customers interests change. The algorithm is shown as follows:

```
BEGIN
for each (product Pi in customer log file f)
{ inserting(Pi);
  if ( Energy Ei of Pi) >1 { Ei =1; }
}
if (the days from the last updating) > threshold {updat-
ing(f)}
END
```

To construct customer profile, we employ two Functions: *inserting* and *updating*. The inserting operation is utilized to insert new categories into a profile and adjust the energy values of existing categories. The updating operation is utilized to remove those categories customers don't interest anymore. The two operation just like planting a new kind flower and pruning for a plant in a garden. And we must keep the energy value from 0 to 1, what is expected by the modified fuzzy ART neural network.

2.2.1 Inserting

The Customer Log File mentioned in section 3.1 is considered as the basis of inserting operation to construct the customer profile. For each product in a log file, we first check if the category of the product exists in the preference tree. If the category exists, the *energy* value of the category should be refreshed. If the category does not exist, we will create the category in customer profile, whose *energy* is the value of *favorite*. Then the *energy* value of the new node should be calculated. The following method is used to calculate the new energy value of each category:

$$E_i = \frac{\sum_{p \in P_i^{new}} W_{p,i}}{\left| P_i^{new} \right|} + \lambda \times E_i \qquad (1)$$

where E_i is the energy value of product category C_i, P_i^{new} is the set of the products assigned to the category C_i in customer log file , the absolute value $\left|P_i^{new}\right|$ is the number of products in P_i^{new} , and $W_{p,i}$ is the *favorite* of the product *p*. The parameter λ, called *decaying factor*, is set from 0 to 1, hence the older records have less effects to the representation of category. In our experiment, λ is assigned to 0.98.

2.2.2 Updating

Since customer interests often change, it is important to adjust the customer profile incrementally, in order to represent customer interests accurately. In discussion of the changes of customer interests, it is found that there are two types of the user interests. One is the long-term interest and the other is the short-term interest. The long-term interest often reflects a real user interest. Relatively, the short-term interest is usually caused by a hot products event and vanishes quickly. The updating operation is designed to adjust the part reflecting customer short-term interests.

In contrast to the inserting operation that adds the new interesting categories into customer profile, the updating operation is the mechanism to remove the out-of-favor categories. Categories with a continual attention can continuously live, otherwise, they will become weak and finally die out. In customer preference, every category's energy value should be reduced a predefined value ψ periodically (e.g. 15 days). The parameter ψ, called *aging factor*, is used to control the reduction rate.

When no or few products browsed in a category, its energy value will decline gradually. If a category's energy value is less than (or equal to) a pre-defined threshold, we remove the category from user preference tree. To keep a personal view on part to the trend of customer interest, categories with low energy value are removed.

2.3 Customer Cluster

Customer cluster could group customers into different teams according their profiles using adaptive neural network. Nowadays, there are various approaches to cluster analysis, including multivariate statistical method, artificial neural network, and other algorithms. However, some of the methods like self-organizing map algorithm implies some constraints: the need to choose the number of clusters a priori, heavier computational complexity and merging the groups representing the same cluster, because the SOM, by approximating the distribution patterns, finds more than one group representing the same cluster. Moreover, successive SOM results depend on the training phase and this implies the choice of representative training examples. For this reason, we employ a modified fuzzy ART, one of the clustering methods using neural network, for cluster analysis.

The Fuzzy ART [9] network is an unsupervised neural network with ART architecture for performing both continuous-valued vectors and binary-valued vectors. It is a pure winner-takes-all architecture able to instance output nodes whenever necessary and to handle both binary and analog patterns. Using a 'Vigilance parameter' as a threshold of similarity, Fuzzy ART can determine when to form a new cluster. This algorithm uses an unsupervised learning and feedback network. It accepts and input vector and classifies it into one of a number of clusters depending upon which it best resembles. The single recognition layer that fires indicates its classification decision. It the input vector does not match any stored pattern, it creates a pattern that is like the input vector as a new category. Once a stored pattern is found that matches the input vector within a specified threshold (the vigilance $\rho \in [0,1]$), that pattern is adjusted to make it accommodate the new input vector. The adjective fuzzy derives from the functions it uses, although it is not actually fuzzy. To perform data clustering, fuzzy ART instances the first cluster coinciding with the first input and allocating new groups when necessary (in particular, each output node represents a cluster from a group). In the paper, we employ a modified Fuzzy ART proposed by Cinque al. [10] to solve some problems of traditional Fuzzy ART [10,11]. The algorithm is shown as follows:

```
BEGIN
For each (input vector V_i)
{ for each (exist cluster C_i) {C*=argmax(choice(C_i,V_i));}
  if match(C*,V_i)≥ρ {adaptation(C*,V_i);}
  else { Instance a new cluster; }
}
END
```

Function *choice* used in the algorithm is the following:

$$choice\ (C_j, V_i) = \frac{(|C_j \wedge V_i|)^2}{|C_j| \cdot |V_i|} = \frac{(\sum_{r=1}^{n} z_r)^2}{\sum_{r=1}^{n} c_r \cdot \sum_{r=1}^{n} v_r} \tag{2}$$

It computes the compatibility between a cluster and an input to find a cluster with greatest compatibility. The input pattern V_i is an n-elements vector transposed, C_j is the weight vector of cluster J (both are n-dimensional vectors). "$\wedge$" is fuzzy set intersection operator, which is defined by:

$$\begin{aligned} x \wedge y &= \min\{x, y\} \\ X \wedge \mathrm{Y} &= (\mathrm{x}_1 \wedge y_1, \cdots, \mathrm{x}_n \wedge y_n) = (\mathrm{z}_1, \mathrm{z}_2, \cdots, \mathrm{z}_n) \end{aligned} \tag{3}$$

Function *match* is the following:

$$match\ (C^*, V_i) = \frac{|C^* \wedge V_i|}{|C^*|} = \frac{\sum_{r=1}^{n} z_r}{\sum_{r=1}^{n} c_r^*} \tag{4}$$

This computes the similarity between the input and the selected cluster. The *match* process is passed if this value is greater than, or equal to, the vigilance parameter $\rho \in [0,1]$. Intuitively, ρ indicates how similar the input has to be to the selected cluster to allow it to be associated with the customer group the cluster represents. As a consequence, a greater value of ρ implies smaller clusters, a lower value means wider clusters.

Function *adaptation* is the selected cluster adjusting function, which algorithm is shown as following:

$$adaptation\ (C^*, V_i) = C^*_{new} = \beta(C^*_{old} \wedge V_i) + (1-\beta)C^*_{old} \tag{5}$$

Where the learning parameter $\beta \in [0,1]$, weights the new and old knowledge respectively. It is worth observing that this function is not increasing, that is $C^*_{new} < C^*_{old}$.

In the study, the energy values of all leaf nodes in a customer profile consist an *n-elements* vector representing a customer pattern. Each element of the vector represents a product category. If a certain product category doesn't include in customer profile, the corresponding element in the vector is assigned to 0. Pre-processing is required to ensure the pattern values in the space [0,1], as expected by the fuzzy ART.

3 Other Classifiers Used in Our Experiments

To verify our proposed system, we built traditional fuzzy ART and SOM classifier. In this section, these classifiers are briefly described.

3.1 Traditional Fuzzy ART

Adaptive resonance theory (ART) describes a family of self-organizing neural networks, capable of clustering arbitrary sequences of input patterns into stable recognition codes. Many different types of ART networks have been developed to improve clustering capabilities, including ART1, ART2, ART2A, and fuzzy ART etc. The modified fuzzy ART presented in the paper is similar with traditional fuzzy ART, but employs different *choice* function. The choice function utilized in traditional fuzzy ART is as following:

$$choice\ \ (C_j, V_i) = \frac{|C_j \wedge V_i|}{\alpha + |V_i|} = \frac{(\sum_{r=1}^{n} z_r)}{\alpha + \sum_{r=1}^{n} v_r} \tag{6}$$

Where α is choice parameter providing a floating point overflow. Simulations in this paper are performed with a value of $\alpha \approx 0$.

3.2 Self-organizing Maps

The self-organizing maps or Kohonen's feature maps are feedforward, competitive ANN that employ a layer of input neurons and a single computational layer. Let us denote by *y* the set of vector-valued observations, $y = [y_1, y_2, \ldots, y_m]^T$, the weight vector of the neuron *j* in SOM is $w_j = [w_{j1}, w_{j2}, \ldots, w_{jm}]^T$. Due to its competitive nature, the SOM algorithm identifies the best-matching, winning reference vector w_i (or winner for short), to a specific feature vector *y* with respect to a certain distance metric. The index *i* of the winning reference vector is given by:

$$i(y) = \arg\min_j \{\ \| y - w_j \| \}\ ,\ j = 1, 2, \ldots, n \tag{8}$$

where *n* is the number of neurons in the SOM, $\|\cdot\|$ denotes the Euclidean distance. The reference vector of the winner as well as the reference vectors of the neurons in its neighborhood are modified using:

$$w_i(t+1) = w_i(n) + \Lambda_{i,j}(t)[x(t) - w_i(t)]\ ,\ t = 1,2,3,\ldots \tag{9}$$

Where $\Lambda_{i,j}(t)$ is neighourhood function, and *t* denotes discrete time constant. The neighbourhood function $\Lambda_{i,j}$ used in equation (9), is a time decreasing function which determines to which extent the neighbours of the winner will be updated. The extent of the neighbourhood is the radius and learning rate contribution, which should both decrease monotonically with time to allow convergence. The radius is simply the maximum distance at which the nodes from the winner are affected. A typical smooth Gaussian neighbourhood kernel is given bellow in equation (10).

$$\Lambda_{i,j}(t) = \alpha(t) \cdot \exp\left(-\frac{\| r_i - r_j \|^2}{2\sigma(t)}\right) \tag{10}$$

where $\alpha(t)$ is the learning rate function, $\sigma(t)$ is the kernel width function, $\| r_i - r_j \|^2$ is the distance of BMU i unit to current unit j. There are various functions used as the learning rate $\alpha(t)$ and the kernel width functions $\sigma(t)$. For further details about the SOM please refer to [12] and [13].

4 Experiment

In the experiment, we construct an experimental web site and the proposed framework utilizing Java servlet and Java bean. The trial simulated 15 customers behavior on an experiment Internet bookstore over a 30-day period, and they were pre-grouped 4 teams. The experimental web site is organized in a 4-level hierarchy that consists of 4 classes and 50 subclasses, including 5847 book pages and images obtained from www. Amazon.com. As performance measures, we employed the standard information retrieval measures of recall (r), precision (p), and F1(*F1=2rp/(r+p)*).

The trial results were compared with the clustering algorithm performed by SOM and traditional fuzzy ART. All the experiments were conducted by limiting human interaction to adjust parameter with intuitive effects. Comparisons were made in the following context. Traditional fuzzy ART was simulated by an original implementation. It was used in the fast learning asset (with β =1) with α set to zero. Values for the vigilance parameter ρ were found by trials. In the simulation of *k*-means, parameter *K* representing the number of clusters is assigned to 7 by trials. In particular, we used a rectangular map with two training stages: the first was made in 750 steps, with 0.93 as a learning parameter and a half map as a neighborhood, and the second in 400 steps, with 0.018 as a learning parameter and three units as a neighborhood. Map size was chosen by experiments. In the proposed system, decaying factor λ is assigned to 0.95, aging factor ψ is set to 0.03, β is set to 1, and vigilance parameter ρ is assigned to 0.90 by trials. With the growth of vigilance parameter, the amount of clusters is increased too. Figure 2 shows the increase in the number of clusters with increased vigilance parameter values ranging from 0.85 to 0.97.

Figure 3 illustrates the comparisons of three algorithms mentioned before, including precision, recall and F1. The average for precision, recall and F1 measures using the SOM classifier are 81.7%, 78.3%, 79.9%, respectively. The average for precision, recall and F1 measures using the traditional fuzzy ART classifier are 87.3%, 84.8%, 86%, respectively. And the average for precision, recall and F1 measures using the k-means classifier are 81.6%, 76.9%, 79.2%, respectively. In comparison with the proposed system, the precision, recall, and F1 measures are 92.3%, 88.1%, 90.15%, respectively. This indicates that if the parameters are selected carefully, the proposed framework could group users pattern accurately.

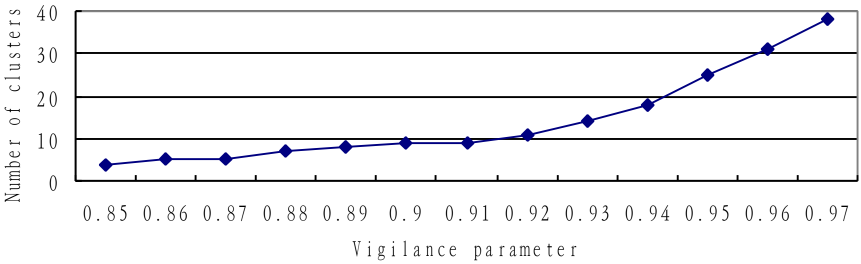

Fig. 2. The vigilance parameter increase with the clusters increasing

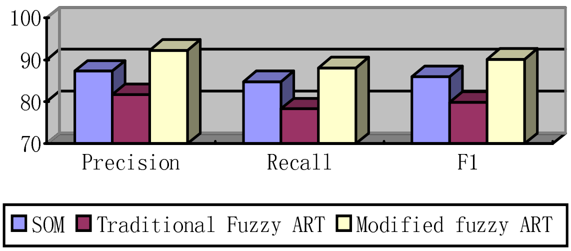

Fig. 3. The comparison of SOM, tranditional ART and modified fuzzy ART algorithm

5 Conclusions

In this paper, we have presented a new framework to automatically track customer access patterns on an Internet commerce web site and group customers using an adaptive neural network. Our approach, essentially based on neural network computation, i.e., learning capacity, satisfies some of its main requirements: fast results, fault and noise tolerance. A pattern grouping module totally independent of the application was also proposed. The cluster system made up of the modified fuzzy ART and the customer pattern track module, was very simple to use. As such system does not use specific knowledge, by adopting the most proper operators, it becomes possible to customize it to different scenarios.

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Fuzzy Comprehensive Evaluation of E-Commerce and Process Improvement

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Abstract. This paper presents an evaluation-factor system of electronic commerce (e-commerce) and provides a fuzzy comprehensive evaluation method to analyze and estimate the development of enterprise e-commerce. Besides, an optimized fuzzy evaluation model utilizing the genetic algorithm is applied to improve the process of the evaluation. Based on a recent survey on e-commerce of some enterprises in Guangzhou, the evaluation results show that the evaluation-factor system is reasonable; the fuzzy comprehensive evaluation model is effective and the optimized fuzzy comprehensive evaluation method makes the evaluation more accurate.

1 Introduction

Electronic commerce[1, 2] (e-commerce), which involves carrying out commerce on the web, breaks through the limitation of the traditional commerce and enhances the management of information flow, currency flow, etc. The implementation of e-commerce promotes the efficiency of the commerce, reduces the time of business transactions and enlarges the profits of the enterprises.

With the rapid growth and wide use of e-commerce, the comparison analysis of e-commerce gradually becomes crucial. Through the comparison analysis we are able to objectively and comprehensively learn the situation of enterprise e-commerce, find out the advantages of it and diagnose the existing problems, which would make a great influence on the further development of the e-commerce in the enterprises.

On the basis of a resent research[1] on the development of enterprise e-commerce in Guangzhou, this paper is organized as follows. In Section 2, an evaluation-factor system of e-commerce is introduced. Section 3 describes the fuzzy evaluation method with its implementation results. Section 4 presents the improvement of the evaluation process. Concluding remarks are in Section 5.

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2 Evaluation-Factor System of E-Commerce

Enterprise e-commerce is influenced by many factors. Considering the information flow, the currency flow, the physical distribution and other aspects, we propose an evaluation-factor system, which consists of 11 first-level factors and 37 second-level factors as shown in Section 3.

All the second-level factors are divided into 5 grades, according to the development of e-commerce in China. Grade1 refers to the best condition, while Grade5 refers to the worst condition. For instance, the grade-division of a second-level factor is shown in Table1.

Table 1. Grade-division of Access Quantity

Second-level factor	Grade1	Grade 2	Grade 3	Grade 4	Grade 5
Access quantity	Above 3000 times/day	1000-3000 times/day	300-1000 times/day	100-300 times/day	Below 100 times/day

3 Fuzzy Comprehensive Evaluation for Enterprise E-Commerce

We design several questionnaires such as factor-importance questionnaire, e-commerce enterprise questionnaire, customer questionnaire and so on for the research on the development of enterprise e-commerce in Guangzhou. During the research, we received 16 completed factor-importance questionnaires from the e-commerce experts in the district, 8 finished e-commerce enterprise questionnaires from 8 e-commerce sample enterprises with the questionnaires from their customers. The 8 sample enterprises are Guangzhou Hongtian E-Commerce Ltd. (S1), Guangdong Guoxun Communication Ltd. (S2), Guangdong World Information Technology Ltd. (S3), Guangzhou Highsun Ltd. (S4), Guangdong Guangsheng Hotel Group (S5), Guangzhou Wanjia Ltd. (S6), Guangzhou Wanfujing Department Store Ltd. (S7) and Guangdong Haihong Medicine E-commerce Ltd. (S8).

To evaluate enterprise e-commerce, a two-level fuzzy comprehensive evaluation[3,4] method is used. For the first-level fuzzy comprehensive evaluation, we combine the second-level factors belonging to the same first-level factor to compose an evaluation factors set, therefore there are 11 evaluation factors sets. The remarks set for the first-level evaluation is defined as $V_1 = \{\text{Grade1, Grade2, Grade3, Grade4, Grade5}\}$. All the first-level factors in the evaluation-factor system compose an evaluation factors set for the second-level fuzzy comprehensive evaluation. And the remarks set for the second-level evaluation is defined as $V_2 = \{\text{Excellent, Good, Normal, Bad, Worse}\}$.

Based on the factor-importance questionnaires we determine the weight of the factors in the evaluation-factor system by comparing the factor with the other factors in the same evaluation set. According to the grade-division criterion, we change the data derived from 8 sample enterprises through the e-commerce enterprise questionnaires combining with the observed results into the corresponding grades. The weights and the grades of the factors are shown in Table2.

Table 2. Weights & Grades of the Second-level Factors in 8 Sample Enterprises

First-level factors (weight)	Second-level factors (weight)	S1	S2	S3	S4	S5	S6	S7	S8
Customer resources (0.1051)	Access quantity (0.2218)	1	1	1	2	5	1	4	2
	Number of registered users (0.2484)	2	1	3	3	5	4	4	5
	Education attainments distribution (0.1109)	3	3	3	3	3	3	3	3
	Age distribution (0.0955)	3	3	3	3	3	3	3	3
	Monthly income distribution (0.1071)	3	3	3	3	3	3	3	3
	Types of business distribution of enterprise users (0.1129)	1	3	2	3	5	3	5	1
	Size distribution of enterprise users (0.1035)	1	3	2	3	5	3	5	1
Customer service (0.0807)	Transaction speed (0.5741)	1	2	3	2	1	2	3	1
	Quality of sales and after sales service (0.4259)	2	2	3	2	2	4	2	2
Delivery system (0.0754)	Delivery speed (0.3411)	2	3	3	2	2	2	4	2
	Delivery scope (0.2802)	2	4	3	2	2	4	4	3
	Satisfaction of the delivery service (0.2307)	1	2	3	2	2	2	3	2
	Size of delivery crew (0.1480)	2	4	3	5	2	3	3	5
Online transaction system (0.0807)	Flexibility of payment (0.2486)	1	3	2	2	3	3	3	2
	Number of electronic identifications (0.1567)	1	4	4	4	4	4	3	4
	Number of electronic signatures (0.1502)	4	4	4	4	4	4	4	4
	Times of security complaints (0.2120)	1	2	1	1	1	1	1	1
	Number of security attacks (0.2325)	5	3	2	2	1	1	3	3
Construction of software & hardware (0.1388)	Software & hardware investment (1)	1	1	1	1	1	1	1	1
Combination of inner and outer information management (0.0754)	Combination of e-commerce with sales, financial, inventory and other management systems (1)	2	3	5	4	5	3	3	1
Network technique & service (0.1285)	Rate of successfully logging in (0.2912)	1	1	1	2	2	1	1	1
	Access charge (0.1242)	4	4	1	4	5	5	4	4
	Stabilization of the website (0.1689)	3	2	2	3	2	2	3	3
	Linking of the web pages (0.1030)	1	2	2	3	2	2	2	1
	Design of the website (0.1535)	2	2	2	3	2	3	2	2
	Update frequency (0.1593)	1	1	1	1	1	1	2	1

Marketing (0.0723)	Advertisement (0.2289)	1	2	1	1	2	5	5	2
	Number of search engines & rank (0.2177)	1	1	1	1	1	1	1	1
	Number of linkages with other websites (0.1653)	4	2	1	2	5	5	5	3
	Adaptability of the commodity/service online (0.3881)	1	2	2	3	1	1	2	1
Effect of implementation of e-commerce (0.1119)	Sales turnover ratio (0.5405)	1	3	2	3	3	1	3	1
	Online payment ratio (0.4595)	1	3	4	4	5	5	5	1
Training (0.0680)	Information training ratio (0.4361)	2	4	1	2	1	2	5	5
	E-commerce training ratio (0.5639)	3	3	1	2	1	3	3	3
Financial position (0.0631)	Sales profit rate (0.3842)	1	3	4	4	3	2	1	2
	Profit growth rate (0.3202)	1	5	1	1	3	5	1	1
	Inventory turnover ratio (0.2956)	1	3	2	1	1	4	3	1

The value of reliability vector A_i (i=1, 2, …, 5) and the relation matrix R for the two-level fuzzy comprehensive evaluation are derived from empirical knowledge

$$A_1=\{0.5333, 0.2667, 0.1333, 0.0667, 0\} \tag{1}$$

$$A_2=\{0.2105, 0.4211, 0.2105, 0.1053, 0.0526\} \tag{2}$$

$$A_3=\{0.1, 0.2, 0.4, 0.2, 0.1\} \tag{3}$$

$$A_4=\{0.0526, 0.1053, 0.2105, 0.4211, 0.2105\} \tag{4}$$

$$A_5=\{0, 0.0677,0.1333, 0.2667, 0.5333\} \tag{5}$$

and

$$R=\begin{bmatrix} 0.5333 & 0.2667 & 0.1333 & 0.0667 & 0 \\ 0.2105 & 0.4211 & 0.2105 & 0.1053 & 0.0526 \\ 0.1 & 0.2 & 0.4 & 0.2 & 0.1 \\ 0.0526 & 0.1053 & 0.2105 & 0.4211 & 0.2105 \\ 0 & 0.0667 & 0.1333 & 0.2667 & 0.5333 \end{bmatrix} \tag{6}$$

3.1 First-Level Fuzzy Comprehensive Evaluation

If u_i (i=1, 2, …, n) refers to a second-level factor of an evaluation factors set and the value of u_i is Grade j (j=1, 2, …, 5), let

$$B=(B_i)_{n\times 1}=(b_{ik})_{n\times 5} \tag{7}$$

then

$$B_i = A_j \circ R = (b_{i1}, b_{i2}, ..., b_{i5}) \tag{8}$$

where

$$b_{ik} = \sum_{p=1}^{5} a_{jp} r_{pk} \quad (k = 1, 2, ..., 5) \tag{9}$$

Considering the effect that the second-level factors make on the first level factor, we denote the weight vector of the factors (as shown in Table2) by W and denote the possibilities of the first-level factor belongs to each grade under certain conditions of the second-level factors by C

$$C = W \circ B = (c_1, c_2, ..., c_5) \tag{10}$$

where

$$c_k = \sum_{i=1}^{n} w_i b_{ik} \quad (k = 1, 2, ..., 5) \tag{11}$$

If $c_k = \max\{c_i \mid i = 1, 2, \ldots, 5\}$, then define the value of the first-level factor as Grade k .

3.2 Second-Level Fuzzy Comprehensive Evaluation

Similar to the first-level fuzzy comprehensive evaluation, the fuzzy comprehensive evaluation method is applied to the fuzzy evaluation set composed by 11 first-level factors. The result of the second-level fuzzy comprehensive evaluation is the ultimate result of enterprise e-commerce. The two-level fuzzy comprehensive evaluation results of the 8 sample e-commerce enterprises are shown in Table3.

Table 3. The Ultimate Results of Two-level Fuzzy Comprehensive Evaluation

Enterprises	S1	S2	S3	S4	S5	S6	S7	S8
Evaluation Results	Excellent	Good	Good	Good	Good	Good	Normal	Excellent

4 Improvement of Fuzzy Comprehensive Evaluation

Since the reliability vector, the weight vector and the fuzzy relation matrix are empirical data derived from some e-commerce experts, it is inevitable that these data would be influenced by the subjective point of view to some extent, which plays a direct role on the results of the evaluation. When errors arise, we need to adjust the parameters such as A_i, R and W . In other words, if the results directly evaluated by the experts (we regard them as the actual results) are identical with the comprehensive evaluation results (the evaluation results), then the evaluation model seems to be reasonable; else the evaluation model need modifying.

Concretely, compared the fuzzy comprehensive evaluation results with the actual results of the 8 sample enterprises in Guangzhou as shown in Table4, the accuracy of the comprehensive evaluation method is 75%. In order to improve the accuracy, genetic algorithm[5] is used to optimize the fuzzy relation matrix R and the elements of the weight vector W.

Table 4. Fuzzy Comprehensive Evaluation Results & Actual Results

Enterprises	S1	S2	S3	S4	S5	S6	S7	S8
Evaluation results	Excellent	Good	Good	Good	Good	Good	Normal	Excellent
Actual results	Excellent	Good	Excellent	Good	Good	Good	Good	Excellent

4.1 Population Initialization

Combine the elements of empirical data R and the weight vector W of the 11 first-level factors as an initial individual and create the other individuals randomly on the basis of the initial one to form a population. Let

$$R^{(0)} = (r_{ij}{}^{(0)})_{5\times5} = R \tag{12}$$

and

$$W^{(0)} = (w_1{}^{(0)}, w_2{}^{(0)}, ..., w_{11}{}^{(0)}) = W \tag{13}$$

then the initial individual $X^{(0)}$ can be described as follows

$$X^{(0)} = (r_{11}{}^{(0)}, r_{12}{}^{(0)}, ..., r_{55}{}^{(0)}, w_1{}^{(0)}, w_2{}^{(0)}, ..., w_{11}{}^{(0)}) \tag{14}$$

Denote the error radius of $r_{ij}{}^{(0)}$ (i =1, 2, …, 5, j =1, 2, …, 5) by ε_R (ε_R >0), and the error radius of $w_k{}^{(0)}$ (k =1, 2, …, 11) by ε_W (ε_W >0). The corresponding elements of each random individual is in the open interval $(r_{ij}{}^{(0)} - \varepsilon_R, r_{ij}{}^{(0)} + \varepsilon_R)$ or $(w_k{}^{(0)} - \varepsilon_W, w_k{}^{(0)} + \varepsilon_W)$. If the number of the population is q □then the initial population

$$Z_0 = \{X^{(0)}, X^{(1)}, ..., X^{(q-1)}\} \tag{15}$$

The best individual in population Z_0 not only contains the empirical information but also is not worse than $X^{(0)}$.

4.2 Fitness Function

Each individual in the population represents a possible solution to the problem. A fitness value $f(X^{(t)})$ is assigned to member $X^{(t)}$ ($t = 0, 1, \ldots, q-1$). Individuals that represent better solutions are awarded higher fitness values.

Since there are 8 sample enterprises in this paper, let T_i （i =1,2, …, 8） be the evaluation results of the samples and O_i be the actual results of the samples, then to each individual $X^{(t)}$ the error between the evaluation results and the actual results is

$$E(X^{(t)}) = \sum_{i=1}^{8} \left| T_i - O_i \right| \tag{16}$$

The fitness function for individual evaluation is defined as

$$f(X^{(t)}) = \frac{1}{E(X^{(t)}) + \sum_{i=1}^{5}\sum_{j=1}^{5}(r_{ij}^{(t)} - r_{ij}^{(0)})^2 + \sum_{k=1}^{11}(w_k^{(t)} - w_k^{(0)}) + a} \tag{17}$$

where a is a constant real number.

4.3 Optimization Results

A standard genetic algorithm[6] is applied to optimize the parameters of the fuzzy evaluation method. Each element of the relation matrix and the weight vector, which composes the individual regards as a gene. Real number encoding is used to realize the optimization.

The standard genetic algorithm is designed as follows:

1. Create a population, the size of which is 50. And the corresponding elements of each individual is in the interval $({r_{ij}}^{(0)} - \varepsilon_R, {r_{ij}}^{(0)} + \varepsilon_R)$ or $({w_k}^{(0)} - \varepsilon_W, {w_k}^{(0)} + \varepsilon_W)$, where ε_R =0.03 and ε_W =0.03
2. Evaluate the fitness of each individual, where $a = 0$
3. Utilize fitness-proportionate selection, two-point crossover and mutation to create a new generation. Generally, the possibility of crossover p_c = 0.60~1.00[6] and the possibility of mutation p_m = 0.005~0.01[6]
4. Go back to the second step and repeat the process until the generation evolutionary time reaches 200 or the fitness value of one individual of the present generation is above 150

The initial relation matrix $R^{(0)}$ and the weight vector $W^{(0)}$ are shown below

$$R^{(0)} = \begin{bmatrix} 0.5333 & 0.2667 & 0.1333 & 0.0667 & 0 \\ 0.2105 & 0.4211 & 0.2105 & 0.1053 & 0.0526 \\ 0.1 & 0.2 & 0.4 & 0.2 & 0.1 \\ 0.0526 & 0.1053 & 0.2105 & 0.4211 & 0.2105 \\ 0 & 0.0667 & 0.1333 & 0.2667 & 0.5333 \end{bmatrix} \tag{18}$$

and

$$W^{(0)} = (0.1051\ \ 0.0807\ \ 0.0754\ \ 0.0807\ \ 0.1388\ \ 0.0754\ \ 0.1285\ \ 0.0723\ \ 0.1119\ \ 0.0680\ \ 0.0631) \tag{19}$$

Design a program using Delphi 6 to optimize the parameters in the fuzzy comprehensive evaluation. When defining $p_c = 0.65$ and $p_m = 0.005$, we obtain the best individual in the 181 generation. The optimized relation matrix and the weight vector are shown below

$$R^{(181)} = \begin{bmatrix} 0.5396 & 0.2492 & 0.1280 & 0.0657 & 0.0174 \\ 0.2376 & 0.4241 & 0.1942 & 0.1063 & 0.0378 \\ 0.1241 & 0.1990 & 0.3896 & 0.1837 & 0.1036 \\ 0.0452 & 0.1197 & 0.2050 & 0.4018 & 0.2284 \\ 0.0031 & 0.0876 & 0.1493 & 0.2417 & 0.5183 \end{bmatrix} \tag{20}$$

and

$$W^{(181)} = (0.1001\ \ 0.0878\ \ 0.0617\ \ 0.0902\ \ 0.1295\ \ 0.0765\ \ 0.1269\ \ 0.0741\ \ 0.1220\ \ 0.0658\ \ 0.0654) \tag{21}$$

Use the optimized parameters $R^{(181)}$ and $W^{(181)}$ to evaluate the e-commerce situation of the 8 sample enterprises in Guangzhou. The evaluation results are identical with the actual results. The accuracy of the fuzzy comprehensive evaluation method rises from 75% to 100%, which proves the effectiveness of the optimization.

5 Conclusions

According to the characteristics and the regulations of the development of enterprise e-commerce, it is necessary to construct an objective and systematic evaluation criterion and a comprehensive evaluation method to evaluate and analyze enterprise e-commerce. This paper outlines an evaluation-factor system including the factors of information flow, currency flow, physical distribution, etc. A two-level fuzzy comprehensive evaluation method and an optimized method are proposed. Based on the data derived from the experts, enterprises and customers, the implementation of the approach to evaluate e-commerce of the enterprises in Guangzhou proves that both the evaluation-factor system and the approach are effective. Furthermore, the use of the evaluation-factor system and the evaluation method can be easily extended.

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Application of Integrated Web Services-Based E-Business and Web Services-Based Business Process Monitoring

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Abstract. In this paper, a solution that aims to help develop web services-based e-business and web services-based business process monitoring is provided. Some issues, such as security models and access control policy, asynchronous transaction support, and reliability of transactions in the process of developing e-business, are addressed in this solution. Moreover, we provide a mechanism to define the web services provided by supplier organizations and to record their business objectives to enable performance measurement.

1 Introduction

Web services are self-contained and modular applications that can be described, published, located, and invoked over the web. They are essentially founded upon three major technologies: Web Services Description Language (WSDL) [9], Universal Description, Discovery and Integration (UDDI) [8], and the Simple Object Access Protocol (SOAP) [10].

How to efficiently and reliably develop e-business solutions through the integration of existing applications and systems has been one of the major topics in the e-commerce community. Most of the development approaches are characterized by their significant complexity and high usage of development resources [3], [6]. Some research into web services [2], [4], [5], [7] has focused on the framework needed to develop web services, but there has been little if any research in the area of analyzing the performance of the web service. There are still some other issues in using web services for developing e-business solutions that need to be resolved, such as the unreliability of communication, security models and access control policy, asynchronous transaction support, and the unreliability of transactions.

In this paper, we provide a solution that aims to help develop web services-based e-business and web services-based business process monitoring. Some of the problems mentioned in the above paragraph are addressed. Moreover, we provide a mechanism to define the web services provided by supplier organizations and to record their

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business objectives, thus enabling performance measurement. A key benefit of this mechanism is that the data is stored only once but provides information both to the organization acting as the customer and the organization acting as the supplier.

2 Main Architecture

Lambros et al. [6] explain the interface between the service registry, service requestor and service provider in the context of the establishment and commencement of a web service relationship. We extend this web services model to incorporate the Services Manager Service (SMS). Figure 1 shows the web services provided by the SMS and how these services are utilized by different organizations. Also, in Figure 1 we provide a solution for organizations in developing web services-based e-business, named WSES.

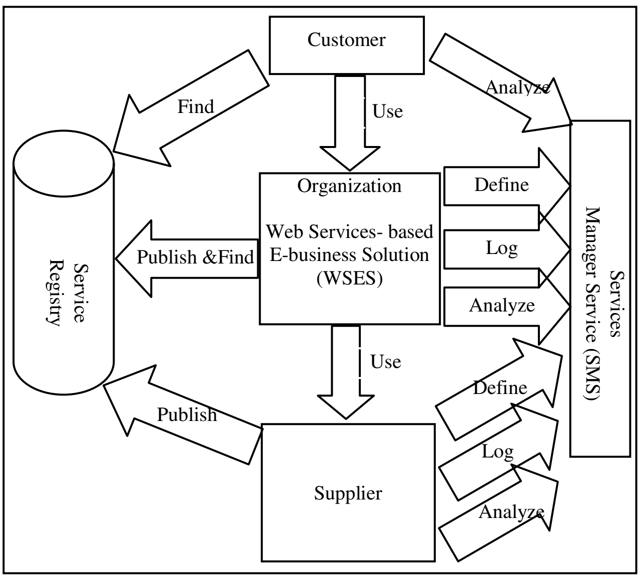

Fig. 1. Services manager-based web services (SMWS)

Figure 1 detailed this relationship in the context of an organization that acts as both a supplier of services to its customers and a user of the services of its suppliers. An organization wishing to supply a new web service must first publish that web service within the Service Registry. In publishing the web service, the organization provides a service description that contains the details of the interface and implementation of the service. This includes its data types, operations, binding information and network location. This definition is constructed using the WSDL. We have named the model shown in Figure 1 SMWS.

The major components of the SMWS are as follows:

(1) SMS: provides a mechanism for organizations to define, log and analyze the web services that they participate in.
(2) WSES: provides a solution for organizations in developing a web services-based e-business. Each of the two components is now discussed in more detail in the following sections.

3 Framework of WSES

The WSES architecture (shown in Figure 2) consists of a number of common IT artifacts that deal with B2B program interactions, web interface handing, security and resource access policies, business control, system exception handling and recovery, interfaces to the data sources and host systems/applications, and data control flows. We call this perspective of e-business solutions the system perspective. Also, an e-business solution should address a set of business problems by implementing relevant business logic and presenting critical and up-to-date business information to relevant decision makers. We call this perspective of e-business solutions the application perspective.

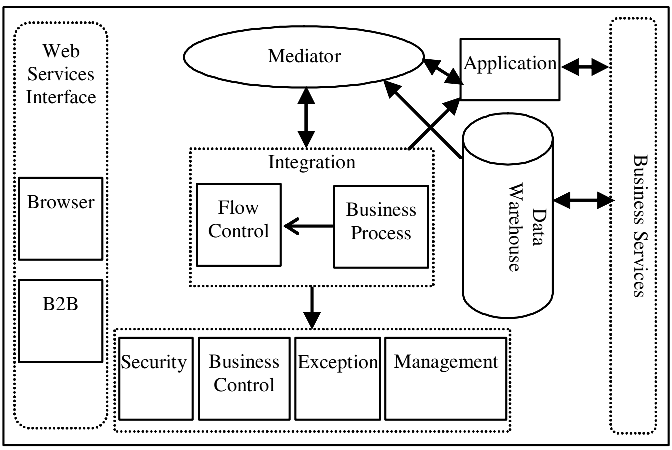

Fig. 2. WSES architecture

The framework depicted in Figure 2 has the following key features:

(1) All the key system perspective artifacts and application perspective components are defined as web services that have standards-based service and interface description.
(2) The system perspective artifacts consist of various interface services (B2B, Browser, Application and Data), a set of common services (Security, Business Control, Exception, etc.), and the integration services (Flow Control and Business Process).
(3) The Integration service may contain a special process brokering service, the so-called Adaptive Document (ADoc) [4], [5] can contain a state machine whose

state transitions are driven by either external events or the completion of the activities in the associated flows.

(4) The Mediator plays a central role in pulling together all different IT artifacts in forming a complete e-business solution.

One of the key aspects of the WSES is an unbundling of typical e-business solutions into modular service components with a process-based composition model. This is important not only for application development (as discussed above), but also for more manageable application deployments and operations. Conventional applications that are used by many e-businesses tend to lump business logic, process definition and data objects all together. This is certainly undesirable if these applications are to be re-used as a part of other solutions or applications, which is quite common in the e-business environment. The WSES will provide a process-based flexible composition of services to form e-business solutions with the support of all the necessary system services.

3.1 Security Models and Access Control Policy

The Mediator defines an object (service) that encapsulates how a set of objects (services) interacts. The Mediator promotes loose coupling by keeping objects (services) from referring to each other explicitly, and it allows their interaction to be varied independently. This is a crucial aspect of the WSES since it takes advantage of the Web services' standardized interfaces so that the Mediator is capable of interacting with all the services in a standards based fashion. This, in combination with the process/flow-based services composition, will permit very flexible composition and re-composition on an as-needed basis whenever the requirements change.

The WSES proposed in this paper consists of a security service that provides both the necessary authentication and authorization needs of an e-business enterprise. Going through the Mediator, all service requests and responses can be properly controlled to make them available only to the authenticated and authorized role players.

3.2 Asynchronous Transaction Support

The WSES will utilize the ADoc model to support the asynchronous transactions. As it is known, the web services requests-responses do not maintain a correlation between different calls. However, if we model each transaction as an ADoc that has an embedded state machine, different web services' calls can be correlated through a unique correlation ID assigned by a service offered to the Mediator in the WSES. Then, relevant asynchronous responses to the original transaction can be tied back to the ADoc corresponding to the transaction.

3.3 Reliability of Transactions

HTTPR [1] is a protocol on top of the popular HTTP. HTTPR supports the application interfaces for both SOAP and Message Queue (MQ). Therefore, a service requestor may receive a transaction message through either SOAP or MQ based interfaces. The message will be assigned a unique correlation ID and persist at the MQ Manager. The

MQ at the service requestor can then create an HTTPR data stream and use HTTP POST to send the message over to the service provider who receives the message, stores it in the local Queue Manager, and includes the correlation ID of the message as part of the HTTP POST response. Then, the MQ at the service requestor will be assured of the reliable delivery of transaction messages to the service provider.

4 Architecture of SMS

In [6], the authors provided a web service model, but they provided no mechanism to monitor the web service model for performance measurement by any of the participating organizations. That is, they did not provide a mechanism to measure, for example, volume throughput or service level attainment.

The business problem we address is that of extracting this knowledge through the establishment and subsequent analysis of web service audit logs. Our approach in solving this problem is to design a generic SMS framework that exists outside the individual organization.

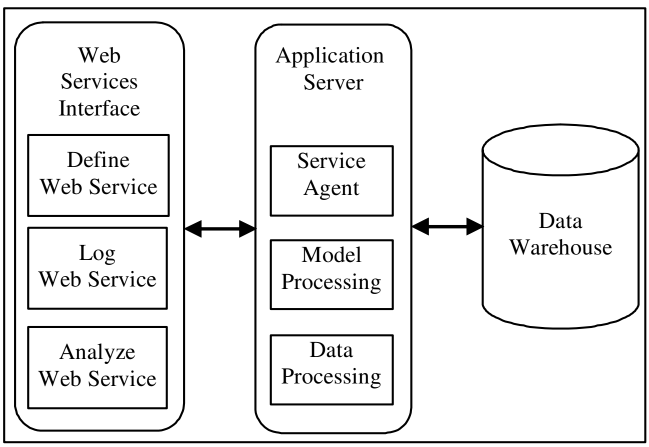

Fig. 3. SMS architecture

Figure 3 details the architecture for the SMS. The major components of the SWS are as follows:

(1) The Web Services Interface of SMS provides a mechanism for organizations to define, log and analyze the web services that they participate in, as well as letting customers analyze web services.
(2) The Application Server implements the business logic of all the services. It includes the service agent, model processing, data processing, etc.
(3) The Data Warehouse is the data repository of our solution. It holds the summary data, model data, and the web service definition data received from the *Define web service* and the web service logging data received from the *Log web service*.

4.1 SMS Interface

The interface provides the place for customers and organizations to interact with the system. Those organizations providing web services require the ability to define the web service. For each instance where a customer requests a web service, the organizations providing the web service require the ability to log details relating to the progress of that web service instance. These same organizations require the ability to analyze the usage and overall performance of this web service in relation to predefined service levels and other goals defined during the definition of the web service. So, the SMS provides the interface for three main services: *Define web service*, *Log web service*, and *Analyze web service*.

The *Define web service* provides services to identify a web service, its owner and performance measurement information such as service level cycle times and costs. In addition, the organization may wish to establish volume targets and service prices relating to individual customer relationships that would also be established using one of these services.

The *Log web service* provides services to log the activity states of a web service such as request, commencement and completion for a specific web service instance. When a supplier receives a request to access a web service, they would log the receipt of this request using the *Log web service*. The supplier similarly would request to log the commencement, completion and any other chosen states of the web service as these milestones are reached.

The *Analyze web service* provides a mechanism where by information relating to web services that the organization participates in can be requested. This information can be supplied whether the user participates as a supplier or a customer of the service. As such, two organizations have access to the same information, through different web services, one as the supplier and the other as the customer.

4.2 SMS Data Warehouse

There are several categories of data stored within the SMS, such as the data for the business process, definition data, log data, summary data, and data for the model process.

Data from heterogeneous data sources for business processes is stored in the data warehouse. Data received as a result of the enactment of the *Define web service* is stored within the Definition Data tables. Data received as a result of the *Log web service* is stored in Log Data table and, together with the definition data, is used by Data Processing to create Summary Data tables. Model data is constructed, improved and destroyed by Model Processing.

4.3 SMS Application Server

As shown in Figure 3, the Service Agent provides the mechanism to access data stored within the SMS Data Warehouse and transmit the results to the web service interface.

The data received as part of enactment of the *Log web service* is forwarded through to the Log Data table. Data Processing uses this information together with information contained in the Definition Data tables to construct summary data that is stored in the Summary Data tables. Model Processing uses the information contained in both the

Definition Data tables and the Summary Data tables to construct model data for both active customers and active suppliers.

5 Case Study: Use of Solutions

In the following subsections, an example to illustrate how the SMWS (see Figure 1) can be used in developing real-world web services-based e-business applications and in developing web service-based business process monitoring for inter-organizational business processes. This case study represents an organization that has implemented a Financial Portfolio Selection using web services. Our goal is to demonstrate the concepts discussed so far using a concrete example.

5.1 Web Services Definition

A high level sequence diagram for the registration and definition of web services is shown in Figure 4.

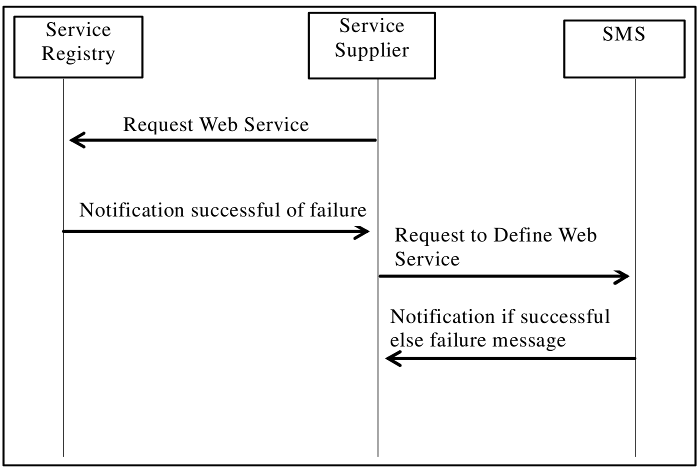

Fig. 4. Web service registration and definition

This organization (shown as the Service Supplier) has established a web service to enable its customers to place orders and has published that web service within the service registry. In publishing the web service, the organization provided a service description containing the details of its data types, operations, binding information and network location. This definition was constructed using WSDL.

5.2 Web Services Logging

Once the definition of the web service is completed, logging can commence (shown in Figure 5). Upon receipt of a request from a customer to commence a web service, a request to the *Log web service* is triggered by the supplier to log the request of their web service with the SMS.

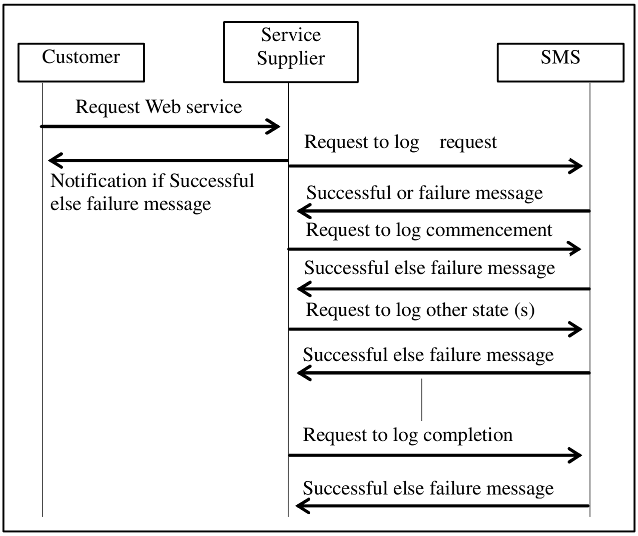

Fig. 5. Web service logging

5.3 Web Services Analysis

To provide the ability for organizations to analyze the enactment of their web services, and the enactment of web services they use that are supplied by other organizations, a series of the *Analyze web services* are provided (shown in Figure 6).

5.4 Use of WSES

Our solution enables feedback on the organization's performance measures through the analysis of web services. Furthermore, this solution demonstrates advantages over more traditional developing of web service-based e-business solutions. Here, we discuss the development of a Participant Enablement solution for a Portfolio Selection Exchange (PSE).

There is a problem to facilitate the trading partners on boarding process. The customization of business rule processes and workflows for each participant during initial sign-on stage are similarly problematic. This is quite common for the PSE. The participants to this service can run into very significant numbers. Without an efficient and automated solution, the costs for setting up new customers and maintaining existing customers could become prohibitive.

Our solution to this problem is based on the WSES and concentrates more specific on participant registration and service provisioning. This is mainly to capture the participant's security profile and its requirements for various services. This will involve initial participant profile establishment, profile validation, service selection, and possibly payment scheme setup.

Most of these tasks can be automated through a properly defined flow and an ADoc for the state management. The participants of the solution include various system artifacts in the framework such as a Browser GUI interface, a Participant Profile, the

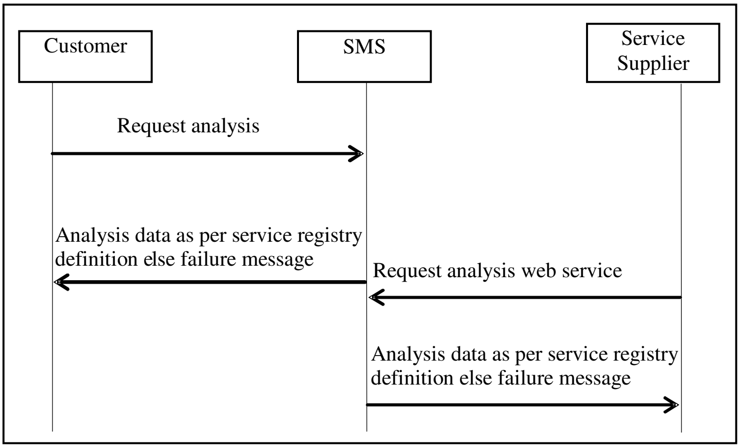

Fig. 6. Web service analyses

Integration Flow, the Security System Service and the Intelligent Controller. Both the registration and service provisioning processes can be long-running, so it is important to be aware of the states since the responses to participants' requests may vary depending on the current process states. Therefore, the ADoc is also likely to be used in the solution.

6 Conclusions

This paper has described the Services Manager-based Web Services (SMWS) solution that provides web services to define, log and analyze other web services. Also, this solution has addressed the key question of how to take advantage of web services' flexible and standards-based capabilities to improve the adaptability and cost-efficiency of e-business solutions. The solution presented in this paper has provided feasible approaches to deal with these issues.

The work described in this paper can be extended in several ways for future research. Firstly, as previously indicated, work has commenced on a proposed standard for the Web Services Description Language (WSDL) used by the Services Manager Service (SMS) providers. Secondly, in this paper, we deal mainly with a security model of the web services where most of these services are within an enterprise. In a highly distributed environment, many different security models of web services should be allowed and supported. Even though there are standards being set up, a true challenge still exists in terms of how to support web services in a highly distributed environment with a very high security requirement.

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A Note on the Cramer-Damgård Identification Scheme

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Abstract. In light of the recent work of Micali and Reyzin on showing the subtleties and complexities of the soundness notions of zero-knowledge (ZK) protocols when the verifier has his public-key, we reinvestigate the Cramer-Damgård intended-verifier identification scheme and show two man-in-the-middle attacks in some reasonable settings: one simple replaying attack and one ingenious interleaving attack. Our attacks are independent of the underlying hardness assumptions assumed.

Keywords: Cryptography, identification scheme, Σ_{OR}, man-in-the-middle attacks.

1 Introduction

Identification protocol is one of the major cryptographic applications, especially in E-commerce over the Internet. Feige, Fiat and Shamir introduced a paradigm for identification (ID) schemes based on the notion of zero-knowledge (ZK) proof of knowledge [6, 5]. In essence, a prover identifies himself by convincing the verifier of knowing a given secret. Almost all subsequent ID schemes followed this paradigm. But, all previous Fiat-Shamir-like ZK-based ID schemes suffer from a weakness, as observed by Bengio et al [1]. Specifically, a malicious verifier may simply act as a moderator between the prover and yet another verifier, thus enabling the malicious verifier to pass as the prover. In [2] Cramer and Damgård presented a simple yet efficient ZK-based (specifically, Σ_{OR}-based) solution for preventing aforementioned man-in-the-middle attacks. Essentially, beyond the novel use of Σ_{OR} in the identification setting, in the Cramer-Damgård ID scheme not only the identification prover but also the identification verifier are required to have public-keys. In other words, the Cramer-Damgård scheme is an *intended-verifier* ID scheme. Though the intended-verifier property is necessary to prevent aforementioned man-in-the-middle attacks, it brings other security issues, as we shall observe in this paper, in light of the recent work of Micali and Reyzin [8] on showing the subtleties and complexities of the soundness notions of zero-knowledge (ZK) protocols when the verifier has his public-key.

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2 Description of the Cramer-Damgård Intended-Verifier ID Scheme

In this section, we first present the basic tools used in the Cramer-Damgård ID scheme and then give the protocol description of the Cramer-Damgård ID scheme.

We assume the following form of *3-round protocol* is considered, which is known as Σ-protocols. Suppose P and V are probabilistic polynomial-time (PPT) machines, on common input x to P and V, and a w such that $(x, w) \in R$ is the only advantage of P over V that he knows w. The *conversation* of a 3-round protocol $\langle P, V \rangle$ is defined as a 3-tuple, say (a, e, z), where a is the first message sent from P to V, e is a random string sent from V to P, and z is replied by P to V. After this 3-round conversation, V would decide to accept or reject based on the conversation.

2.1 Σ-Protocol and Σ_{OR}-Protocol

Definition 1 (Σ-protocol). *A 3-round protocol $\langle P, V \rangle$ is said to be a Σ-protocol for a relation R if the following holds:*

- **Completeness.** *If prover P and verifier V follow the protocol, the verifier always accepts.*
- **Special soundness.** *From any common input x of length n and any pair of accepting conversations on input x, (a, e, z) and (a, e', z') where $e \neq e'$, one can efficiently compute w such that $(x, w) \in R$. Here a, e, z stand for the first, the second and the third message respectively, and e is assumed to be a string of length k (that is polynomially related to n) selected uniformly at random from $\{0, 1\}^k$.*
- **Perfect Special honest verifier zero-knowledge (SHVZK).** *There exists a probabilistic polynomial-time (PPT) simulator S, which on input x (where there exists a w such that $(x, w) \in R$) and a random challenge string $\hat{e}$, outputs an accepting conversation of the form $(\hat{a}, \hat{e}, \hat{z})$, with the same probability distribution as that of the real conversation (a, e, z) between the honest $P(w)$, V on input x.*

Σ-protocols have been proved to be a very powerful cryptographic tool and are widely used in numerous important cryptographic applications including digital signatures, efficient electronic payment systems, electronic voting systems, et al. We remark that a very large number of Σ-protocols have been developed in the literature, mainly in the field of applied cryptography and in industry. Below, we give Σ-protocol examples for DLP and RSA.

Σ-Protocol for DLP [9]. The following is a Σ-protocol $< P, V >$ proposed by Schnorr [9] for proving the knowledge of discrete logarithm, w, for a common input of the form (p, q, g, h) such that $h = g^w \ mod \ p$, where on a security parameter n, p is a uniformly selected n-bit prime such that $q = (p-1)/2$ is also a prime, g is an element in $\mathbf{Z}_p^*$ of order q. It is also actually the first efficient Σ-protocol proposed in the literature.

- P chooses r at random in $\mathbf{Z}_q$ and sends $a = g^r \ mod\ p$ to V.
- V chooses a challenge e at random in $\mathbf{Z}_{2^k}$ and sends it to P. Here, k is fixed such that $2^k < q$.
- P sends $z = r + ew \ mod\ q$ to V, who checks that $g^z = ah^e \ mod\ p$, that p, q are prime and that g, h have order q, and accepts iff this is the case.

Σ-Protocol for RSA [7]. Let n be an RSA modulus and q be a prime. Assume we are given some element $y \in Z_n^*$, and P knows an element w such that $w^q = y$ mod n. The following protocol is a Σ-protocol for proving the knowledge of q-th roots modulo n.

- P chooses r at random in Z_n^* and sends $a = r^q \mod n$ to V.
- V chooses a challenge e at random in Z_{2^k} and sends it to P. Here, k is fixed such that $2^k < q$.
- P sends $z = rw^e \mod n$ to V, who checks that $z^q = ay^e \mod n$, that q is a prime, that $gcd(a, n) = gcd(y, n) = 1$, and accepts iff this is the case.

The OR-proof of Σ-protocols [3]. One basic construction with Σ-protocols allows a prover to show that given two inputs x_0, x_1, it knows a w such that either $(x_0, w) \in R_0$ or $(x_1, w) \in R_1$, but without revealing which is the case. Specifically, given two Σ-protocols $\langle P_b, V_b \rangle$ for R_b, $b \in \{0, 1\}$, with random challenges of, without loss of generality, the same length k, consider the following protocol $\langle P, V \rangle$, which we call Σ_{OR}. The common input of $\langle P, V \rangle$ is (x_0, x_1) and P has a private input w such that $(x_b, w) \in R_b$.

- P computes the first message a_b in $\langle P_b, V_b \rangle$, using x_b, w as private inputs. P chooses e_{1-b} at random, runs the SHVZK simulator of $\langle P_{1-b}, V_{1-b} \rangle$ on input (x_{1-b}, e_{1-b}), and let $(a_{1-b}, e_{1-b}, z_{1-b})$ be the simulated conversation. P now sends a_0, a_1 to V.
- V chooses a random k-bit string e and sends it to P.
- P sets $e_b = e \oplus e_{1-b}$ and computes the answer z_b to challenge e_b using (x_b, a_b, e_b, w) as input. He sends (e_0, z_0, e_1, z_1) to V.
- V checks that $e = e_0 \oplus e_1$ and that both (a_0, e_0, z_o) and (a_1, e_1, z_1) are accepting conversations with respect to (x_0, R_0) and (x_1, R_1), respectively.

Theorem 1. [4] *The above protocol Σ_{OR} is a Σ-protocol for R_{OR}, where $R_{OR} = \{((x_0, x_1), w) | (x_0, w) \in R_0 \text{ or } (x_1, w) \in R_1\}$. Moreover, for any malicious verifier V^*, the probability distribution of conversations between P and V^*, where w satisfies $(x_b, w) \in R_b$, is independent of b. That is, Σ_{OR} is perfectly witness indistinguishable.*

2.2 Description of Protocol

Let X and Y be two parties, and let f_X and f_Y be two one-way functions that admit Σ-protocols. The following description of protocol is taken from [4, 2], in which X plays the role of identification prover and Y plays the role of identification verifier.

Key Generation. On a security parameter n, randomly select x_X and x_Y of length n each in the domains of f_X and f_Y respectively, compute $pk_X = f_X(x_X)$ and $pk_Y = f_Y(x_Y)$. pk_X and pk_Y are the public-keys of X and Y respectively and x_X and x_Y are their corresponding secret-keys.

The ID Protocol. In order to identify himself to the *intended* verifier Y with public-key pk_Y, X proves to Y that he knows either the preimage of pk_X (i.e. x_X) or the preimage of pk_Y (i.e. x_Y), by executing the Σ_{OR}-protocol on common input (pk_X, pk_Y). We denote by a_{XY}, e_{XY}, z_{XY} the first, the second and the third message of the Σ_{OR}-protocol respectively.

3 Two Man-in-the-Middle Attacks

In this section, we show two attacks on the Cramer-Damgård ID scheme in some reasonable settings: one replaying attack and one interleaving attack.

3.1 The Replaying Attack

As shown in [2, 4], the intended-verifier property of the Cramer-Damgård ID scheme prevents a malicious verifier to pass as the prover to another *different* verifier. But, we observe that a simple replaying attack enables an adversary (the man-in-the-middle) to identify himself as the (honest) verifier to the (honest) prover. In other words, the Cramer-Damgård ID scheme suffers from the man-in-the-middle attack when it is used for mutual identification purpose between two players X and Y, in which both X and Y identify themselves to each other concurrently with reversed playing role in the two concurrent protocol executions.

Now, suppose X (with public-key pk_X) is identifying himself to Y (with public-key pk_Y) and an adversary A (i.e. the man-in-the-middle) controls the communication channel between X and Y and wants to identify himself as Y to X. The following is the message schedule of the adversary:

Move-1: After receiving a_{XY} from X, A sets $a_{YX} = a_{XY}$ and sends a_{YX} back to X.

Move-2: After receiving the random challenge e_{YX} from X, A sets $e_{XY} = e_{YX}$ and sends back e_{XY} as the random challenge to X.

Move-3: After receiving z_{XY} from X, A sets $z_{YX} = z_{XY}$ and sends z_{YX} back to X.

Clearly, if X can successfully identify himself to Y (which means (a_{XY}, e_{XY}, z_{XY}) is an accepting conversation on (pk_X, pk_Y) with X playing the role of identification prover and Y playing the role of identification verifier), then (a_{YX}, e_{YX}, z_{YX}) is also an accepting conversation on (pk_Y, pk_X) with X playing the role of identification verifier and the adversary A playing the role of identification prover (which means that A has successfully impersonated himself as Y to X).

3.2 The Interleaving Attack

We consider a scenario in which two parties X (with public-key pk_X) and Y (with public-key pk_Y) identify each other internally, but they *externally* identify themselves as a group with public-key (pk_X, pk_Y) to outside parties (say, a third party T with public-key pk_T). That is, when X (or Y) identifies himself to an outsider party T, X (or Y) just convinces T that he is either X or Y without revealing exactly who he is. Specifically, X (or Y) convinces T that he knows the preimage of either pk_X or pk_Y or pk_T, by executing the Σ_{OR} on (pk_X, pk_Y, pk_T) with pk_X (or pk_Y respectively) as his private witness. We remark that this scenario is meaningful in certain applications. Now, suppose the honest player X is identifying himself to the honest player Y, then we show an interleaving attack that enables an adversary A (i.e. the man-in-the-middle who controls the communication channel between X and Y) to convince T that he is one member of the player group $\{X, Y\}$ (i.e. he is either X or Y). The following is the specification of the interleaving message schedule of A who is the man-in-the-middle between X and Y. We remark the interleaving attack is ingenious in comparison with the above simple replaying attack.

Move-1: After receiving a_{XY} from X, A first generates a simulated conversation that he knows the preimage of pk_T (by running the SHVZK simulator as shown in the description of Σ_{OR}). Denote by $(\hat{a}_T, \hat{e}_T, \hat{z}_T)$ the simulated transcript, where $\hat{e}_T$ is a random string. Then, A sends $(a_{XY}, \hat{a}_T)$ to T.

Move-2: After receiving the random challenge e_T from T, A sets $e_{XY} = e_T \oplus \hat{e}_T$, and sends e_{XY} to X as the random challenge in the protocol execution between X and Y.

Move-3: After receiving z_{XY} from X, A sends $(z_{XY}, \hat{z}_T)$ to T.

Note that from the point view of T: $(\hat{a}_T, \hat{e}_T, \hat{z}_T)$ is an accepting conversation on pk_T, (a_{XY}, e_{XY}, z_{XY}) is an accepting conversation on (pk_X, pk_Y) for proving the knowledge of the preimage of either pk_X or pk_Y, and furthermore $e_{XY} \oplus \hat{e}_T = e_T$. This means A has successfully identified himself to T as one member of the player group $\{X, Y\}$.

4 Concluding Remarks

Identification protocol is one of the major cryptographic applications, especially in E-commerce over the Internet, and the Cramer-Damgård intended-verifier ID scheme is a famous one (due to its conceptual simpleness and highly practical efficiency) that may have been employed in practice. Though the intended-verifier property is necessary to prevent man-in-the-middle attacks of certain types, but as shown in this work, the intended-verifier property (i.e. letting the verifier also have his public-key) brings other security issues. Note that the two attacks shown in this work are all related to the intended-verifier property. In particular, if the identification verifier (e.g. Y) has no public-key (say, pk_Y), but, rather *freshly* generates and sends the "public-key message" (i.e. pk_Y) to the identification prover in each invocation, then our attacks will not work. But a verifier with a public-key suffers from other security vulnerabilities, as we mentioned in Section 1. We note that the security vulnerabilities we reported in this paper are not an incidental phenomenon. Actually, the underlying reason behind the above two attacks is just the subtleties and complexities of soundness notions of ZK protocols in public-key models when the verifier has his public-key. Specifically, Micali and Reyzin showed in [8] that for ZK protocols although an adversary cannot get more advantages by concurrent interactions than by sequential interactions in the standard model, but, the soundness notion in the public-key model (when the verifier has his public-key) turns out to be much subtler and more complex than that in the standard model [8]. In particular, they showed that in the public-key setting concurrent interactions are strictly more powerful to an adversary than only sequential interactions.

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Formal Analysis and Improvement of the State Transition Model for Intrusion Tolerant System

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Abstract. Intrusion tolerance is an emerging network security technique, which enables the victim server systems to continue offering services (or degraded services) after being attacked. A state transition model has been presented to describe the dynamic behaviors of intrusion tolerant systems. In this paper, we build an attack finite state system based on the recent network attacks, and use SMV, a model checking tool, to analyze the intrusion tolerant systems by the interaction of the system model and the attack model. The analysis results demonstrate that not all types of attacks can be mapped to the system model. We improve this state transition model, whose correctness is proved by SMV. In addition, we give two attack instances mapped to our improved model.

1 Introduction

Intrusion tolerance [1] is an emerging network security technique that aims to enable systems to continue offering normal services or degraded services after being attacked. As modern computer systems become more complex, it is inevitable that there exist some security vulnerabilities in large-scale systems. Moreover, the attack methods are various and unpredictable. So it is, in many cases, more important to determine whether a victim server can continue offering services than to determine which type of attack has occurred. Intrusion tolerant systems are measures we take to keep the compromised system functioning properly. There are several types of attacks, such as compromise of confidentiality, compromise of data integrity and compromise of user/client authentication, etc. An attack intrusion tolerant system is inherently tied to functions and services that require protection.

Systematic research on intrusion tolerance has emerged since 1999, and some advancement has been achieved recently [2]. However, little attention has been paid to the formal verification of intrusion tolerant systems, so we cannot ensure that the system implement or the system model indeed satisfies certain specifications.

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Model checking [3, 4, 5] is a formal verification technique, which verifies whether a model M, often deriving from a hardware or software design, satisfies a logical specification S, i.e. $M \models S$. Model checking has been applied successfully in several areas, particularly in the verification of digital circuits and communication protocols. In the early stage, while checking finite-state system, states were represented explicitly and the model-checking algorithm explored the whole state space. However, as the system becomes large, state explosion problem may occur. Symbolic model checking is developed to solve this problem to some extent, and is so called because sets of states are represented by Boolean functions. Moreover, model-checking algorithms are improved to reduce the size of state space. SMV (Symbolic Model Verifier), which was developed by K. McMillan [6], is one of the symbolic model checking tools. It is practical even for large systems.

This paper presents the SMV analysis and improvement of the intrusion tolerant systems and discusses how two of the known network security attacks fit into the model. The rest of the paper is organized as follows. Section 2 introduces symbolic model checking, SMV and CTL. Section 3 presents the intrusion tolerant system and attack state transition models in detail. Section 4 uses SMV to verify the model by interacting of the two models. Section 5 improves the original intrusion tolerant system model and also uses SMV to verify its properties. Section 6 describes two known network attacks, and show how the target system is mapped to the state transition model. We summarize the paper in Section 7.

2 Symbolic Model Checking, SMV and CTL

In symbolic model checking, Boolean functions are represented as OBDDs [7], and the set of states satisfying temporal logic specifications are calculated according to the fixed-point theory. Using a symbolic model checker to verify whether a system satisfies a property includes three steps:

1. Model the system using the description language of a model checker, arriving at a model M.
2. Code the property using the specification language of the model checker, resulting in a temporal logic formula ϕ.
3. Run the symbolic model checker with inputs M and ϕ. The model checker outputs 'true' if M satisfies ϕ ($M \models \phi$) and 'no' otherwise; in the latter case, most model checkers also output a trace of system behavior, which causes this failure.

SMV (figure 1) is a well-known symbolic model checker originally developed by Ken McMillan. Its main idea is to utilize symbolic model checking algorithm to verify whether a system model satisfies a specification expressed by a Computation Tree Logic (CTL) formula. SMV is applied widely in the verification of complex digital circuits and communication protocols.

SMV is a model checker for the temporal logic Computation Tree Logic (CTL), and that the systems to be verified are finite state transition systems described

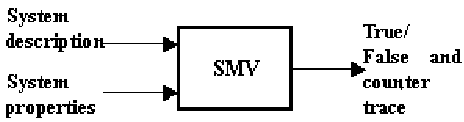


Fig. 1. SMV

in SMV language. CTL is a temporal logic, having connectives that allow us to refer to future. It is also a branching-time logic, which means that its model of time is a tree-like structure in which the future is not determined; there are different paths in the future, and one of which might be the 'actual' path that is realized. The syntax of CTL is defined as follows:

1. Atomic formulas are CTL formulas;
2. If f and g are CTL formulas, $\neg f$, $(f \vee g)$, $(f \wedge g)$, AXf, EXf, AFf, EFf, AGf, EGf, $A(fUg)$, $E(fUg)$ are CTL formulas.

The symbols A (along all paths) and E (along at least one path) are path operators. The symbols X (next state), F (some future state), G (all future states) and U (until) are state operators. Path operators and state operators must appear in pair.

3 State Transition Model for Intrusion Tolerant System (STMITS) and Attack State Transition Model (ASTM)

3.1 Overview of STMITS

Figure 2 [8] depicts the intrusion tolerant system state transition model. This model represents the system configuration that depends on the actual security requirements. The system is in vulnerable state V if it enables a user to use the resource without authorization, or there is a flaw or a hole (of the software or/and of the hardware) in the system. If the system management detects the penetration and exploration phases of an attack, it can take some strategies to bring the system from the state V back to the good state G. Otherwise the system will enter the active attack state A. If the system can mask the attack's impact, the system will enter masked compromised state MC. If the intrusion tolerance strategies fail to recognize the active attack state and limit the damage, the system will enter undetected compromised state UC. When an active attack in exploitation phase is detected, the system will enter the triage state TR. According to protect the functions and services, the system can recover to state G, or enter the graceful degradation state GD. If the system fails to offer degraded service, it will enter fail-secure state FS. If all of the above strategies fail, the system will enter failed state F. Recovering the full services after an attack and returning to the good state by manual intervention is represented by transitions denoted with dashed lines.

The system enables multiple intrusion tolerance strategies to exist and supports different levels of security requirements. The system focuses on impacts

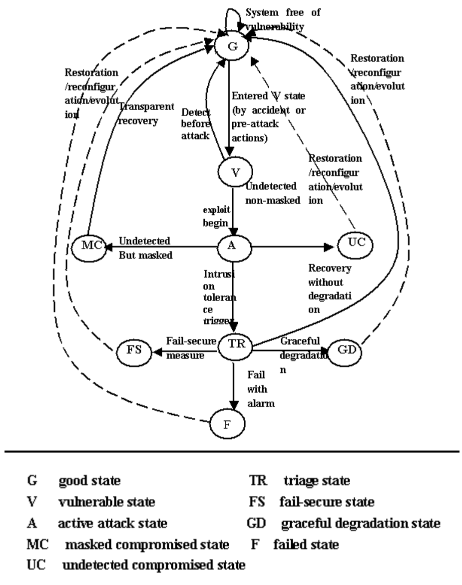


Fig. 2. State transition model for intrusion tolerant system

rather than specific attack procedures, so if some unknown attacks' effect can be mapped to the model, we can use the model to deal with such unknown attacks.

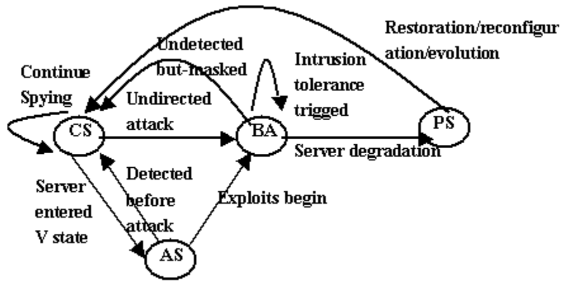
3.2 Attack State Transition Model

When an attack occurs, the attacker's actions should be simultaneous with the behaviors of the system being attacked. The state of the system is the result of the interaction of the system behaviors and the attack behaviors. According to the network attack instances and the state transition diagram of intrusion tolerant system, we construct the attack state transition model (figure 3).

Initially, the attacker continues spying the server system to seek occasions to attack the server, so the initial state of the model is CS (continually spying the server).

With the development of mutual firewall technique and the intrusion detection systems (IDS), it becomes more difficult for attackers to intrude the server before a new fault (in software or in hardware) is found. Nowadays, attackers usually attack the server indirectly by exploiting the vulnerability of the network infrastructure. If such attack occurs, although the server doesn't have any faults, the attack state will transfer from continually spying state CS to attack beginning state BA.

Recently, much research has been focused on Distributed Denial of Service (DDoS) attack [9]. Some researchers have found over 12,000 DDoS attacks in three weeks, and the actual number may be much higher [10]. Most researchers believe that DDoS attacks will constitute an increasing threat to the Internet



CS	**continually spying the server**	**AS**	**server in V state**
BA	**attack beginning**	**PS**	**attack partly success**

Fig. 3. Attack state transition model

in the future [11]. DDoS attacks are attacks on systems designed to cause a loss of service to users. Such attacks are not designed to gain access to the systems; their sole purpose is to simply deny authorized access to systems. Such attacks rely on neither any specific network protocol nor any faults in the systems. When the attackers disrupt the system in this way, the attack state will transit from state CS to state BA.

When there are faults in the server systems, the attacker will enter state AS. When the server system is updated before the attackers begin an attack, the attack state will transit back to state CS. If the attackers attack the system successfully by exploiting these system faults, the attack will be brought back to state BA.

In the state BA, if the server system doesn't detect the attack but masks the attack's impact, the attacker will enter state CS. If the system detects the attack, it will take some tolerant intrusion strategies to eliminate the impacts produced by the attack. During the tolerance stage the attackers are still attacking the system, so the attack will remain in state BA. If the intrusion tolerant strategies succeed, the attack fails and the attack state will transit back to state CS. If the system provides degradation service, the attack will enter state PS.

4 SMV Analysis of the State Transition Models

When we use SMV to verify a concurrent system, the states of the system must be finite. Since the states of STMITS and the states of ASTM are both finite, we can use SMV to verify them.

4.1 Description for the System Model

The intrusion tolerant process includes two parts: $\{server, attack\}$. In our SMV program, we use parallel modules to describe the server and the attack.

In the SMV modules, the next state is determined by the message input buffer and the state transition variables. If there are legal messages or the state transition variable is true, it can use operand '*init*()' or '*next*()'. The operand '*init*()' stands for the initial state or the initial value of message sent to the other model. The operand '*next*()' stands for the value of state variable in the next state or the value of the message sent to the other module in the next state.

1. According to the state transition model for the server (figure 2):

   In the server module, the server state is denoted by the variable *state*. So the set of the possible values of *state* is
   {`good, vulnerable`, `active_attack`, `mc, uc`,`triage`,
   `fs, degradation, failed`},
   where '`good`' denotes the state G, '`vulnerable`' denotes the state V, etc.

   The set of message `buff_s_to_a` denotes the message buffer, which the server may send to attack, is
   {`nones, entered_v`,`detected_before_attack`,`undetected_but_masked`,
   `undetected_non_masked`, `intrusion_tolerance_triggered`,
   `recovery_without_degradation`,`fail_secure_measure`,
   `graceful_degradation`, `fail_with_alarm`,
   `restoration_reconfiguration_evolution`}, where '`nones`' means no messages.

   The server module has three state transition variables, `v_begin`, `att_begin` and `tri_begin`. `v_begin` is a Boolean variable. If there is a fault in the server, `v_begin` is true. In this case the initial state of the server is '`vulnerable`' and the initial output message is '`entered_v`'. Otherwise, $init(state) :=$ `good`, and $init($`buff_s_to_a`$) :=$ `nones`.

   When the server is in '`active_attack`' state, its state transition variable is `att_begin`. The value set of `att_begin` is $\{0, 1, 2\}$. When the server is in '`triage`' state, its state transition variable is `tri_begin`, and its value set is $\{0, 1, 2, 3\}$. The transition label in figure 1 illustrates what such integer values stand for. When the server is in '`good`' state, the attacker is not able to find any holes in the server, and the server's next state is '`good`', next output message is '`nones`'. In other cases, next value of *state* and `buff_s_to_a` can be obtained according to figure 2 in the similar way, and we will not show all the details due to limited space.

   The SMV module for server model is simply described as follows (the full module is omitted for limited space):

```
Module server_prc(v_begin, att_begin, tri_begin, buff_in)

{ input v_begin : boolean;
  input att_begin : {0,1,2};
  input tri_begin :{0,1,2,3};
  input buff_in : {nonea, undirected_a,exploits_b};
  buff_s_to_a : {...};
```

```
  state : {...};
    if(v_begin) {
      init(state):=vulnerable;
      init(buff_s_to_a) := enter_v;}
    else {
      init(state) := good;
      init(buff_s_to_a) := nones;}
    switch(state){ good: {next(state) := good;
                          next(buff_s_to_a) :=nones;}
                          vulnerable : . . .
                          .
                          .
                          .}
}
```

2. According to the attack state transition model (figure 3):

   In the attack module, the attack state is also denoted by the variable *state*, and the value set of *state* is {`spying`,`attacking`, `server_in_v`, `partly_success`}. The set of message `buff_a_to_s` denotes the message buffer, which the attack may send to the server, is {`nonea`, `undirected_attack`,`exploits_begin`}, where ’`nonea`’ means no messages. When the attack is in the initial state, the attacker spies the server, and if there is a fault in the server, the attacker will attack the server. So the attack's initial state is ’`server_in_v`’, and meanwhile sends the message ’`exploit_b`’ to the server model. If there are no faults in the server, the attacker may attack the server indirectly, and the attack's initial state is ’`spying`’, the initial message sent to the server model, is ’`undirected_a`’. In other cases, next value of *state* and `buff_a_to_s` can be obtained according to figure 3 in similar way.

   The SMV module for attack model is simply described as follows (the full module is omitted for limited space):

```
Module attack_prc(buff_in){

    input buff_in : {...};
    buff_a_to_s : {...};
    state : {...};
    if(buff_in = in_v)
    { init(state):= server_in_v;
      init(buff_s_to_a):= exploits_b;}
    else
       {init(state) := spying;
        init(buff_a_to_s) := undirected_a;}
    switch(state)
         { spying : ...
           partly_success :...
```

```
                .
                .
                . }
}
```

3. The main module
   In the main module, server and attack module run parallel. Module server's variable `buff_s_to_a` is module attack's input `buff_in`, and module attack's variable `buff_a_to_s` is module server's input `buff_in`. The main module is as follows:

```
Module main(){

v : boolean;
tri : {0,1,2,3};
att : boolean;
attack :attack_prc(server.buff_s_to_a);
server : server_prc(v, att, tri, attack.buff_a_to_s);
assert G((server.state=active_attack)-> (attack.state=attacking));
assert G((attack.state=attacking)-> (server.state=active_attack));
assert G((server.state=active_attack)-> F(server.state=good));

assert G((attack.state=attacking)->F((attack.state=spying)
|(attack.state=partly_success)));

assert G((attack.state=partly_success)-> F(attack.state=spying));
}
```

4.2 System Specifications

Whenever the server is being attacked, the attacker must be attacking it at the same time. Expressed as a CTL formula, such specification is

$$AG((server.state = \texttt{active\_attack}) \rightarrow (attack.state = attacking)) \qquad (1)$$

Another important specification to verify is that the intrusion tolerant system can manifest all types of attack, that is if the attacker is in attack state, the server must be in being attacked state. Expressed as a CTL formula, such specification is

$$AG((attack.state = attacking) \rightarrow (server.state = \texttt{active\_attack})) \qquad (2)$$

The whole CTL formula (2) means that in all paths of the intrusion tolerant system, as long as the attacker enters '`attacking`' state, the server will enter '`active_attack`' state.

For the whole intrusion tolerant system, if the server has been attacked, it will be recovered in the future. Expressed as a CTL formula, such specification is

$$AG((server.state = \texttt{active\_attack}) \rightarrow AF(server.state = good)) \qquad (3)$$

Mapped to ASTM, when the attacker attacks the server, the server can take some intrusion tolerant strategies, and then the server will continually offer service or degraded service. In another words, all the successful-attacking state will return to the state PS or state CS. Expressed as a CTL formula, such specification is

$$AG((attack.state = attacking) \rightarrow AF((attack.state = spying)|(attack.state = \texttt{partly\_success}))) \quad (4)$$

All providing-degraded-service states of the server will transmit to initial state G. Mapped to ASTM, the partly attack success state will return to state CS. Expressed as a CTL formula, such specification is

$$AG((attack.state = \texttt{partly\_success}) \rightarrow AF(attack.state = spying)). \quad (5)$$

4.3 Verification Results

We run SMV with above modules, and the verification results show that specification (1) is true. Specification (2) is false, and SMV gives the counter path: when the attack model transmits from state CS to state BA, the server is still in state G, rather than enter state A. The indirect attacks cannot be mapped to the intrusion tolerant system. Specification (3) is true. Specification (4) is false, and SMV gives the concurred path, where the attacker enters state BA, but the server model is still in state G. The concurred path shows that the server model cannot tolerate the indirect attacks. Specification (5) is true. So we can conclude that the state transition model for intrusion tolerant system is not complete, that is, not all types of attack can be mapped to such model. More details are omitted here due to the limited space.

5 Improved-STMITS and Its SMV Analysis

5.1 Improved Model

The indirect attack and DDoS attack cannot be mapped to the original intrusion system model. The two types of attacks may occur even though there is no vulnerability in the system, and during such attack process, the server enters state A from state G.

Although there are not effective methods to avoid DDoS attacks and it is very difficult to trace the DDoS attacks' organizers, some intrusion tolerant strategies have been put forward, such as Source Path Isolation Engine (SPIE)[13], Internet firewall, and North America's sink hole [12].

In order to map all types attacks to the intrusion tolerant system model, we change the original model by adding the state transition from state G to state A, that is, the system can enter state A directly without passing state V. This transition stands for the system being attacked indirectly or suffering from DDoS attack. Figure 4 depicts the improved-model.

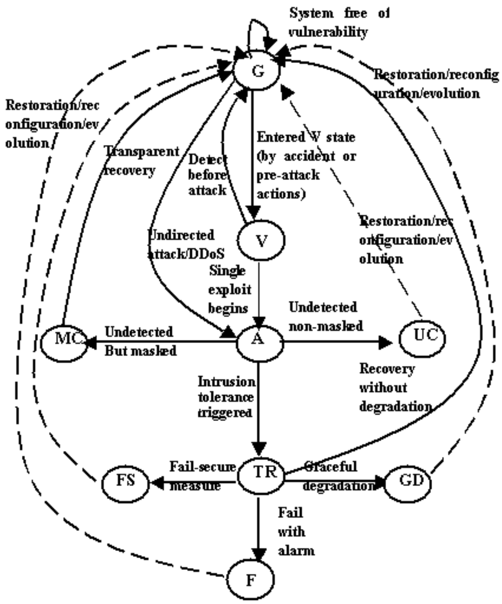


Fig. 4. Improved model

5.2 SMV Analysis of the Improved Model

There is no change in the attack module, so we only need to modify the server module in the original SMV program. The main change is that when the server is in state G, if the message sent by the attack is 'undirected_a', the server will enter state A. The SMV program for the improved model is simply described as follows:

```
Module server_prc(v_begin, att_begin, tri_begin, buff_in)
 { . . .
   if(v_begin)
   {init(state) :=vulnerable; init(buff_s_to_a) :=  enter_v;}
   else {init(state) := good; init(buff_s_to_a) := nones;}
   switch(state){
     good: {
          if(buf_in=undirected_a)
             {next(state) := active_attack;
              if (att_begin=0)
                 {next(buff_s_to_a):=but_masked;}
              else if(att_begin=1)
               {next(buff_s_to_a) := n_m;}
              else
             {next(buff_s_to_a) := tolerance;}
             }
             else {next(state):= good; next(buff_s_to_a):= nones;}
```

```
                vulnerable : . . .
              }
     }
}
```

We verify the improved model using the above SMV program, and the verification result shows that the five specifications in section 4.2 are all true.

6 Case Studies of the Improved-Model

6.1 Indirect Attacks

Cisco Internet Operating System (IOS) is Cisco's proprietary routing software and the core operating system running on Cisco routers. CSCdp58462 (Cisco Bug ID) is a Cisco IOS buffer overflow vulnerability, which attackers can exploit to run arbitrary instructions, or carry out DoS attacks. The detailed information of this vulnerability can be found in [13]. When the server's ISP uses the network infrastructures with Cisco IOS, the attackers can directly attack the network infrastructures, which provide service for the server, to compromise the usability of the server indirectly.

Figure 5 depicts the whole attack and the intrusion tolerant procedure mapped to the improved model. The server is in state G initially, and the attackers begin to attack the router, bringing the server to state AS. Once ISP detects the attack, the server enters state TR. When ISP updates IOS, the server can go back to state G. In state A, if ISP is not aware of the fault of IOS or what type of attack has occurred, the server will enter state UC. If the server changes another ISP, it will return to state G, and continue offering the service without degradation.

6.2 TCP SYN Flooding Attack

Figure 6 illustrates the process of TCP SYN flooding attack. Such attack may exhaust the resource of the server by making use of TCP protocol's three-way

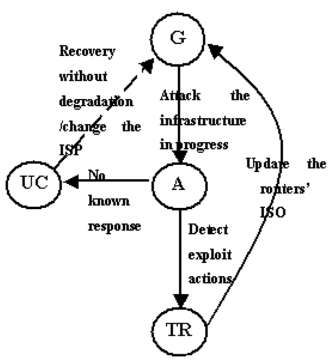


Fig. 5. State transition diagram for indirect attack

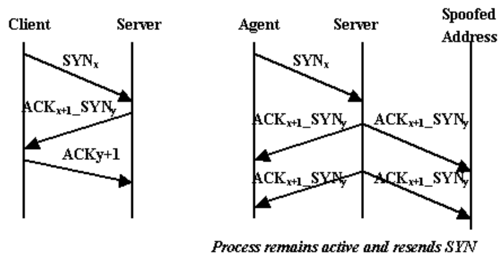


Fig. 6. TCP SYN attack

handshake mechanism. When a client attempts to establish a TCP connection to a server, the client first sends an SYN message to the server. The server then acknowledges the SYN by sending an `SYN_ACK` back to the client, and waits for the ACK message from the client. If the ACK message of the client cannot arrive at the server, the server will send the `SYN_ACK` message to the client again, and after SYN Timeout (about 30s-2min), throws away the connection. If a malicious attacker sends mass TCP SYN packet to the server's TCP port and makes a lot of half-open connections, the server will consume very much resource to maintain a large half-open connection table. If the server's TCP/IP protocol stack is lack of robustness, it will cause memory error. Even if the server is robust enough, the server is busy in dealing with the attacker's forged TCP connection request, and unable to deal with normal requests.

There are many schemes to defend against DDoS attacks, but none of them can solve the problem thoroughly. At present, the main techniques to defend against DDoS attacks are attack detection and filters.

When the server is being attacked, the server is in state A. The system administrator can detect the attack very easily by using intrusion detection system or with the connected routers. The detection standards include the abnormal high data flow and the abnormal reduction of the service performance. If data flow at one of the server's TCP ports is too high and most of the packets are TCP SYN

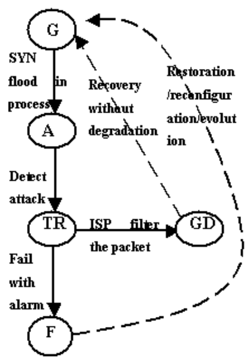


Fig. 7. State transition diagram for DDoS attack

packets, it can be ascertained that the server is being DDoS attacked and the server enters state TR. In this state, the server uses TcpDump to check the SYN packet, and records the features of the packet. If the address of sending the SYN packets is RFC1918's private address, it can configure the access control list on the subway router to filter the private address packets. The upstream routers of the network providing service to the server limit the largest network flow to the server being attacked. Meanwhile, contact the upper ISP, tell them the features of the packets, and let them filter the packets in their network. There are, as yet, no effective measures to defend against DDoS attacks, so in this case, the server's performance has to be degraded and the server enters state GD. If all above strategies fail, the server enters state F. After the attack ceases, the server returns to state G. Figure 7 shows the whole intrusion tolerant process.

7 Conclusions

In this paper, we proposed a formal method for verification of intrusion tolerant systems. According to general network attacks, we build an attack transition state model. By interacting of the server model and the attack model, we used SMV to carry out the verification automatically.

With the development of network technique, there will be more means to attack server systems. Intrusion tolerant system model should be modified to deal with such new attacks. The server is in the environment, which includes network users, ISP' network, and the server systems. Therefore, not only the server system's fault but also other factors can cause the server to be attacked.

According to the verification results, we improved the original model, verified it with SMV and at last gave the case studies of the improved model.

Our future work includes building a multi-agent system model for intrusion tolerant systems in a complex network environment, and verifying its temporal properties with the emerging approaches for model checking multi-agents [14].

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Secure Fingerprint-Based Remote User Authentication Scheme Using Smartcards

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Abstract. Biometrics and smartcards have the potential to be a very useful combination of technology. Among the various biometrics technological advances in use today, fingerprint recognition seems to be particularly suitable for smartcard systems. In 2002, Lee-Ryu-Yoo proposed a fingerprint-based remote user authentication scheme using smartcards. This scheme, however, was found to be potentially vulnerable to some forgery attacks and is not easily repairable. The current paper presents an improved scheme that is more secure and efficient.

Keyword: User authentication, Smartcard, Fingerprint verification.

1 Introduction

User authentication is an important part of security, along with confidentiality and integrity, for systems that allow remote access over untrustworthy networks, like the Internet. Traditional remote user authentication methods mainly employ the possession of a token (magnetic card, etc.) and/or the knowledge of a secret (password, etc.) in order to establish the identity of an individual. A token, however, can be lost, stolen, misplaced, or willingly given to an unauthorized user, and a secret can be forgotten, guessed, or unwillingly-or willingly-disclosed to an unauthorized user. The science of biometrics has emerged as a powerful tool for remote user authentication systems. Since it is based on physiological and behavioral characteristics of the individual, biometrics does not suffer from the disadvantages of traditional methods [1]. Also, biometrics and smartcards have the potential to be a very useful combination of technology. First, the security and convenience of biometrics allow for the implementation of high-security applications regarding smartcards. Second, smartcards represent a secure and portable way of storing biometric templates, which would otherwise need to be stored in a central database. Among the various biometric technological tools in use today, fingerprint recognition seems to be particularly suitable for smartcard systems [1].

In 1981, Lamport [2] proposed a remote password authentication scheme using a password table to achieve user authentication. In 2000, Hwang and Li [3] pointed out that Lamport's scheme suffers from the risk of a modified password table. Also, there is the cost of protecting and maintaining the password

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table. Therefore, they proposed a new user authentication scheme using smart cards based on the ElGamal public key cryptosystem [4] to eliminate risks and costs. Hwang and Li's scheme can withstand replay attacks and it can also authenticate users without maintaining a password table. In 2002, Lee-Ryu-Yoo [5] proposed an improved version of Hwang and Li's scheme by using a fingerprint verification mechanism in the user's smartcard and two secret keys for the servers. Their scheme strengthened system security by verifying the smartcard owner's fingerprints. Their fingerprint verification method is based on minutiae extraction and matching [6, 7, 8]. Lee-Ryu-Yoo scheme, however, was found to be potentially vulnerable to some forgery attacks [9, 10]. In 2005, Ku-Chang-Chiang also pointed out that Lee-Ryu-Yoo is still vulnerable to forgery attacks and is not easily repairable [11].

Accordingly, the current paper presents an improved scheme with better security and efficiency than Lee-Ryu-Yoo's scheme. In addition, the proposed scheme achieves the same advantages as Lee-Ryu-Yoo's scheme that employs a fingerprint verification mechanism to strengthen its security, plus it has the following merits: (1) The proposed scheme can resist forgery attacks and is repairable, (2) users can freely choose and securely change their password, and (3) the computational costs are less than Lee-Ryu-Yoo's scheme.

This paper is organized as follows: In Section 2, Lee-Ryu-Yoo scheme is briefly reviewed and then, Ku-Chang-Chiang's cryptanalysis on the Lee-Ryu-Yoo scheme in presented. In Section 3, an improved Lee-Ryu-Yoo's fingerprint-based remote user authentication scheme using smartcards is propsed. In Section 4 and 5, the security and efficiency of the proposed scheme is analyzed. Finally, the conclusion is offered in Section 6.

2 Related Research

In this section, Lee-Ryu-Yoo scheme is briefly reviewed and then Ku-Chang-Chiang's cryptanalysis on the scheme is offered. The notations used to describe Lee-Ryu-Yoo scheme and the proposed scheme are described as follows:

- U_i represents a legal user.
- ID_i and PW_i denote the identity and password of U_i, respectively.
- S represents the remote server (system).
- SK_1 and SK_2 denote two secret keys of S.
- $f(\cdot)$ denotes a public one-way hash function.
- p denotes a public large prime.
- $\oplus$ represents the bitwise XOR operator.
- T represents the current timestamp of the input device.
- T' represents the receiving timestamp of the system.

2.1 Review of Lee-Ryu-Yoo Scheme

There are three phases in Lee-Ryu-Yoo scheme including a registration phase, a login phase and an authentication phase. These three phases are reviewed in the following:

Registration Phase

R1. U_i submits ID_i to S for registration.

R2. S computes $ID_i' = (ID_i)^{SK_1} \bmod p$ and $PW_i = (ID_i')^{SK_2} \bmod p$.

R3. S delivers PW_i and a smartcard containing $(f(\cdot), p)$ to U_i through a secure channel.

Login Phase

L1. U_i inserts his smartcard into the card reader, keys in ID_i and PW_i, and imprints his fingerprint on the fingerprint input device. If the fingerprint of U_i is not successfully verified, the login process is terminated.

L2. U_i's smartcard generates a random number r using coordinates of the minutia of an input fingerprint, and then computes
$C_1 = (ID_i)^r \bmod p$,
$t = f(T \oplus PW_i) \bmod (p-1)$,
$M = (ID_i)^t \bmod p$,
$C_2 = M(PW_i)^r \bmod p$, where T is the current timestamp.

L3. U_i's smartcard outputs (C_1, C_2, T), and then U_i sends his login request message $C = (ID_i, C_1, C_2, T)$ to S.

Authentication Phase

A1. Upon receiving U_i's login request message C at time T', S checks the validity of ID_i. If ID_i is not a valid identity, S rejects U_i's login request.

A2. If $(T' - T) \geq \Delta T$, where ΔT denotes the expected valid time interval for a transmission delay, S rejects U_i's login request.

A3. If the equation $C_2(C_1^{SK_2})^{-1} \bmod P = (ID_i^{SK_1})^{f(T \oplus PW_i)} \bmod p$ holds, S accepts U_i's login request. Otherwise, S rejects U_i's login request.

2.2 Ku-Chang-Chiang's Cryptanalysis

Ku-Chang-Chiang presented an effective forgery attack on Lee-Ryu-Yoo scheme and determined that the scheme is not easily repairable. The attacks are summarized as follows:

Forgery Attack

(1) Suppose that the impersonation target of the adversary is U_i, a specific privileged user of S. The adversary can try to find ID_a with a valid format such that $ID_a = ID_i^k \bmod p$, where k is an integer within the interval $[2, p-2]$.

(2) If the adversary succeeds in finding such an ID_a, he submits ID_a to S for registration. Then, S will compute $ID_a' = (ID_a)^{SK_1} \bmod p$ and $PW_a = (ID_a)^{SK_2} \bmod p$, and then will deliver PW_a and a smartcard containing $(f(\cdot), p)$ to the adversary.

(3) Next, the adversary can compute

$$\begin{aligned} PW_a^{-k} \bmod p &= (ID_a)^{SK_2 \times (-k)} \bmod p \\ &= (ID_a^{-k})^{SK_1 \times (SK_2)} \bmod p \\ &= (ID_i)^{SK_1 \times (SK_2)} \bmod p, \end{aligned}$$

which yields PW_i.

(4) From now on, the adversary can use ID_i and PW_i to impersonate U_i in order to log-in S. As U_i is a privileged user, such an impersonation attack may result in serious security problems. Note that U_i's smartcard is not involved in the above attack.

Repairable Problem

Suppose that the adversary has obtained U_i's password PW_i by performing the above forgery attack. Such an attack cannot be prohibited even if U_i has detected that PW_i has been compromised. As $PW_i = (ID_i')^{SK_2} \bmod p = (ID_i^{SK_1})^{SK_2} \bmod p$, the value of PW_i is determined only by ID_i, SK_1 and SK_2. Since SK_1 and SK_2, however, are commonly used for authenticating all users rather than what is specifically used for only U_i, it is unreasonable that S should change SK_1 and SK_2 to recover the security of U_i only. In addition, it is also impractical to change ID_i, which should be uniquely tied to U_i.

3 Proposed Scheme

This section proposes an improvements on Lee-Ryu-Yoo's fingerprint-based remote user authentication scheme using smartcards. There are three phases in the proposed scheme including registration, login and authentication. Before accessing a remote system, a new user has to personally imprint his fingerprint on the input device and offer his identity to the system in the registration center. The proposed scheme is illustrated in Figure 1.

Registration Phase

R.1 U_i selects ID_i, PW_i and a random number N freely. Then, U_i personally imprints his fingerprint on the input device. Then, U_i computes $f(PW_i, N)$ and sends ID_i and $f(PW_i, N)$ to S.

R.2 If it is U_i's initial registration, S creates an entry for U_i in the account database and stores $n = 0$ in this entry. Otherwise, S sets $n = n + 1$ in the existing entry for U_i. Next, S computes $K = f(SK_1, ID_i, n)$ and $R = K \oplus f(PW_i, N)$, and then writes $\{R, f(\cdot)\}$ into U_i's smartcard and releases it to U_i through a secure channel.

R.3 U_i enters N into his smartcard.

Login Phase

L1. U_i inserts his smartcard into the card reader, keys in ID_i and PW_i, and imprints his fingerprint on the fingerprint input device. If the fingerprint of U_i is not successfully verified, the login process is terminated.

L2. U_i's smartcard computes $f(PW_i, N)$ by using stored N and extracts $K = R \oplus f(PW_i, N)$.

L3. U_i's smartcard generates a random number r using coordinates of the minutia of the input fingerprint, and then computes $C_1 = f(K, r, T)$, where T is the current timestamp.

L4. U_i's smartcard outputs (r, C_1, T), and then U_i sends his login request message $C = (ID_i, r, C_1, T)$ to S.

Authentication Phase

A1. Upon receiving U_i's login request message C at time T', S checks the validity of ID_i. If ID_i is not a valid identity, S rejects U_i's login request.

A2. If $(T' - T) \geq \Delta T$, where ΔT denotes the expected valid time interval for a transmission delay, S rejects U_i's login request.

A3. S retrieves n from the account database of U_i and then computes $K' = f(SK_1, ID_i, n)$. If the equation $C_1 = f(K', r, T)$ holds, S accepts U_i's login request. Otherwise, S rejects U_i's login request.

Password Change

C1. Whenever U_i decides to change the old password to a new password, U_i imprints his fingerprint on the fingerprint input device.

C2. If the fingerprint of U_i is not successfully verified, the password change process is terminated. Otherwise, U_i inputs the old password PW_i and the new password PW_i'.

C3. U_i's smartcard computes a new $R' = R \oplus f(PW_i, N) \oplus f(PW_i', N)$, and then replaces the old R with the new R' on the smartcard.

Shared Information: $f(\cdot)$
Information held by User U_i: ID_i,PW_i
Information held by Remote System S: SK_1,ID_i,n

User U_i		**Remote System** S
n-th **Registration Phase:**		
Select ID_i,PW_i,N	$(ID_i, f(PW_i, N))$ $\longrightarrow$	$n \leftarrow n + 1$
		Store ID_i and n in U_i's DB
		$K \leftarrow f(SK_1, ID_i, n)$
		$R \leftarrow K \oplus f(PW_i, N)$
Enter N into Smartcard	(Smartcard) $\longleftarrow$	Store $(R, f(\cdot))$ in Smartcard
Login Phase:		
Input ID_i,PW_i,fingerprint		
Verify fingerprint		
Generate r, T		
$K \leftarrow R \oplus f(PW_i, N)$		
$C_1 \leftarrow f(K, r, T)$	$C = (ID_i, r, C_1, T)$ $\longrightarrow$	
Authentication Phase:		Verify ID_i and T
		Retrieve n from U_i's DB
		$K' \leftarrow f(SK_1, ID_i, n)$
		Verify $C_1 \stackrel{?}{=} f(K', r, T)$

Fig. 1. Proposed Fingerprint-Based Remote User Authentication Scheme

4 Security Analysis

This section analyzes the security of the proposed fingerprint-based remote user authentication scheme. First, we define the security terms [12] needed for the analysis of the proposed scheme.

Definition 1. *A weak secret key (password) is a value of low entropy* $W(k)$*, which can be guessed in polynomial time.*

Definition 2. *A strong secret key is a value of high entropy* $H(k)$*, which cannot be guessed in polynomial time.*

Definition 3. *A secure one-way hash function is a function* f*, such that for each* x *in the domain of* f*, it is easy to compute* $f(x)$*; but for essentially all* y *in the range of* f*, it is computationally infeasible to find any* x *such that* $y = f(x)$*.*

Under the above definitions, the following theorems are used to analyze the security properties in the proposed scheme.

Theorem 1. *The proposed scheme can resist a guessing attack.*

Proof. The proposed scheme is based on a secure one-way hash function and fingerprint verification. Due to the fact that a one-way hash function is computationally difficult to invert, it is extremely hard for any adversary to derive the system secret key SK_1 from K. Even if the smartcard of U_i is picked up by an adversary, it is still difficult for the adversary to derive SK_1. It's also difficult for the adversary to obtain U_i's password PW_i from $f(PW_i, N)$, since the adversary cannot know the random value N.

Theorem 2. *The proposed scheme can resist a forgery attack.*

Proof. If an adversary tries to forge a valid parameter C_1, they must have the system secret information SK_1 or the secret value K of U_i, because C_1 must be derived from K, r or T. This is infeasible, however, as K has to be obtained from the system's secret information SK_1 because it is a one-way property of a secure one-way hash function. Therefore, the proposed scheme can resist Ku-Chang-Chiang's forgery attack unlike Lee-Ryu-Yoo's scheme.

Theorem 3. *The proposed scheme can resist a replay attack.*

Proof. A replay attack can be prevented by checking the time stamp at Step 2 in the authentication phase. An adversary may try to modify the time stamp to achieve the replay attack. It does not work unless $C_1 = f(K, r, T)$ is modified to a correct value. It is difficult to modify C_1 correctly, however, without knowing K. Note that the random number r is generated by using the coordinate of the imprint fingerprint minutiae. This method can generate a one-time random number r because the picture of the matched minutiae is always different [5].

Theorem 4. *The proposed scheme can resist a stolen smartcard attack.*

Proof. In case the adversary can wangle a legal U_i's smartcard and password PW_i, he still cannot pass fingerprint verification in the login phase. In comparing an adversary's with the minutiae template stored on the smartcard, the illegal smartcard, the 'illegal access' will be rejected.

Theorem 5. *The proposed scheme is reparable.*

Proof. If U_i finds or suspects that his K has been compromised, he can select a new random number N' and a new password PW_i', and then, $f(PW_i, N')$ can be computed. Next, U_i can re-register to S by using $f(PW_i, N')$. Upon receiving U_i's re-registration request, S will set $n' = n + 1$ and compute $K^* = f(SK_1, ID_i, n')$. Next, S stores $R^* = K^* \oplus f(PW_i, N')$ in U_i's new smartcard. After receiving the new smartcard from S through a secure channel, U_i enters N' into it. From now on, U_i can securely login S by using his new smartcard and new password PW_i'. In the meanwhile, the compromised K has been revoked automatically, i.e. the login request of the adversary who had obtained K will be rejected.

Theorem 6. *The proposed scheme can resist an insider attack.*

Proof. Since the user U_i registers to S by presenting $(ID_i, f(PW_i, N))$ instead of (ID_i, PW_i), an insider of S cannot directly obtain PW_i without knowledge of the random number N.

Theorem 7. *The proposed password change is secure.*

Proof. Since the smartcard could verify the fingerprint of U_i in Step 2 of the password change when the smartcard was stolen, unauthorized users cannot change the new password of the smartcard.

5 Performance Comparison

The computation costs of Lee-Ryu-Yoo's scheme and the proposed scheme are summarized in Table 1. In the registration, login and authentication phases, Lee-Ryu-Yoo's scheme requires a total of six exponentiation operations, two hash operations, four multiplication operations and two bitwise XOR operations, The proposed scheme, on the other hand, requires a total of six hash operations and two bitwise XOR operations. It is obvious that our scheme is more efficient than that of Lee-Ryu-Yoo's scheme.

Table 1. A comparison of computation costs

	Lee-Ryu-Yoo's scheme		Proposed scheme	
	User	System	User	System
Registration phase	No	1Exp+1Mul	1Hash	1Hash+1Xor
Login and Authentication phase	3Exp+1Hash +1Mul+1Xor	2Exp+1Hash +2Mul+1Xor	2Hash+1Xor	2Hash
Password change	Not supported		2Hash+1Xor	

Exp: Exponentiation operations; Mul: Multiplication operations;
Hash: One-way Hash operations; Xor: Bitwise XOR($\oplus$) operations.

6 Conclusion

In the current paper, improvements to Lee-Ryu-Yoo's fingerprint-based remote user authentication scheme were proposed. While the proposed scheme achieves the same advantages as Lee-Ryu-Yoo's scheme by employing a fingerprint verification mechanism to strengthen its security, it also can boast of the and has the following: (1) The proposed scheme can resist forgery attacks and is repairable, (2) users can freely choose and securely change their password, and (3) the computational costs are less than Lee-Ryu-Yoo's scheme. Therefore, the improved scheme is designed to repair the security weaknesses of Lee-Ryu-Yoo's scheme and to be more efficient.

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New Authentication Protocol Providing User Anonymity in Open Network*

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Abstract. User authentication is an operation whereby one user is aware of the identity of an another user involved in a protocol. In 2004, Park presented an authentication protocol providing user anonymity based on the secret-key certificate and error-correcting codes called PA protocol. In this paper, it will be argued that PA protocol is vulnerable to the man-in-the-middle attack and does not provide a sufficient level of security. Then, an improved protocol to fix this problem is proposed.

Keywords: User authentication, Key exchange, User anonymity, Mobile network, Error-correcting codes.

1 Introduction

In an open wireless mobile telecommunication network, each entity should be able to secure communication with another entity over an insecure channel. Usually, secure communication can be provided by building upon a secret key [1]. With a shared secret key, two or more entities of an exchange must share the same secret key, and the shared key must be protected from others. An authenticated key exchange protocol provides secure communication over an insecure network with a shared secret key. Since secure communication is difficult to achieve in open network, many communication protocols neglect an individual user's privacy. In such circumstances where the user identity should remain anonymous, the identity of the mobile user is not revealed to the public [2-3].

For an authenticated key exchange protocol, Wilson and Menezes identified two fundamental security goals: Implicit key authentication and explicit key authentication [4]. In the following, A and B denote two honest entities. A key

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agreement protocol is said to provide *implicit key authentication* if entity A is assured that no other entity aside from a specifically identified second entity B can learn the value of a particular session key. A key agreement protocol is said to provide *explicit key authentication* if entity A is assured that the second entity B has actually computed the agreed session key. A key agreement protocol which provides implicit key authentication to both participating entities is called an *authenticated key agreement* (AK) protocol, while one providing explicit key authentication to both participating entities is called an *authenticated key agreement with key confirmation* (AKC) protocol.

In 2004, Park [5] proposed an authentication protocol providing user anonymity and un-traceability using the secret-key certificate and error-correcting codes called the PA protocol. His idea is valuable for conveying authentication tokens and a secret-key certificate using error-correcting codes and an artificial error vector. The PA protocol, however, is vulnerable to the man-in-the-middle attack [6-9]. Also, the PA protocol does not provide implicit or explicit key authentication, which can cause security flaws within an entire systems.

In this paper, we show that the PA protocol is vulnerable to the man-in-the middle attack and does not provide implicit or explicit key authentication. In addition, we propose an improved protocol to overcome these weaknesses.

In Section 2, the PA protocol will be reviewed. Section 3 will outline the problems of their protocol. In Section 4, an authentication protocol providing user anonymity will be presented. In Section 5, the security of the protocol will be analyzed. In Section 6, the two protocols will be compared. Finally, Section 7 will offer concluding remarks.

2 Review of the PA Protocol

This section reviews the main ideas of the PA protocol. The notations used this paper are the same as those used in the PA protocol. An identity of each mobile subscriber (MS) is denoted by id, and $f()$ is a symmetric-key encryption function. The MS and the AS share a secret key of the AS (k_{AS}) beforehand. Let h be a pseudo-random generator with an input length of n bits and an output length of $2n$ bits. Let $h_0(x)$ and $h_1(x)$ be the left and right halves of the output of h, respectively. The PA protocol is divided into three stages:(1) subscription stage, (2) certificate issue stage and (3) session key generation stage. An illustration of the PA protocol is shown in Fig 1.

The subscription performed in the subscription stage for each subscriber is described as follows:

(1) During the subscription stage, the mobile subscriber MS computes a chain of authentication tokens $x_{i-1} = h_0(x_i)$ and session keys $k_i = h_1(x_i)$ for $i = s, s-1, ..., 1$, s is the network accessing time described in the secret-key certificate used as the expiration period.
(2) The MS sends the root authentication token x_0 to the authentication server (AS) in a secure fashion using error-correcting codes.

MS		**AS**
computes x_0 $x_{i-1} = h_0(x_i)$	$\xrightarrow{x_0\ (secure\ channel)}$	
	$\xleftarrow{f(k_{AS}, [id \parallel x_0])}$	issues $f(k_{AS}, [id \parallel x_0])$
sends transformed messages	$\xrightarrow{[i \parallel x_i],\ f(k_{AS}, [id \parallel x_0])}$	
		compares i and compute $h_0(x_i),\ h_0(x_{i-1}), ...,\ h_0(x_1)$ checks $x_0 \stackrel{?}{=} h_0(x_1)$
computes $k_i = h_1(x_i)$		computes $k_i = h_1(x_i)$

Fig. 1. Illustration of the PA protocol

The certificate issuing performed in the certificate issue stage for each subscriber MS is described as follows:

(1) In the certificate issuing stage, the AS issues to each mobile subscriber the secret-key certificate $f(k_{AS}, [id \parallel x_0])$, which is constructed by symmetrically encrypting both id and x_0 with the secret key k_{AS} of the AS, where " $\parallel$ " is a concatenation operation, id is the real identity of the MS, and x_0 is the value used for authenticating the MS and session key generation.

The key generation performed in the session key generation stage for each subscriber MS is described as follows:

(1) In the session key generation stage, MS passes $[i \parallel x_i]$ to the AS with his secret-key certificate to access the mobile network at session number i.
(2) Then, the AS compares session number i with the one stored in its database.
(3) The AS checks if $h_0(x_1)$ is the same as value x_0, which was retrieved from the secret-key certificate after computing a series of authentication tokens.

$$h_0(x_i), h_0(x_{i-1}), ..., h_0(x_1)$$

(4) If the authentication is successful, both parties update the session number.

In order to convey the secret-key certificate and provide user un-traceability, however, Park used the algebraic structure of the error-correcting codes. The secret-key certificate $f(k_{AS}, [id \parallel x_0])$ is encoded using a linear error-correcting code, then, an artificial error vector $e = [i \parallel x_i]$ is added to an encoded secret-key certificate in the session key generation stage [5][10][11].

A linear error-correcting code of length N, dimension K and minimum distance D is denoted by (N, K, D). A binary K tuple m can be encoded to an

N-bit codeword $c = (c_1 c_1 ... c_{N-1} c_N) = m \cdot G$, where G is a $K \times N$ generator matrix. After generating a certain binary (N, K, D) linear block code with a decoding algorithm, the AS produces an associated $K \times N$ generator matrix G together with a parity-check matrix H, which is only known to the AS.

An error vector e, which is added to the codeword c results in vector $r = c + e$. If the Hamming weight of e is less than or equal to $t = \lfloor (D-1)/2 \rfloor$, r can be decoded into c based on the syndrome vector $s = r \cdot H^T$, where H is an $(N-K) \times N$ parity-check matrix, such that $G \cdot H^T = 0$ is a null matrix. The error vector can be used to carry $[i \parallel x_i]$ with an order-preserving mapping algorithm and an inverse mapping algorithm [5][12].

3 Weakness of the PA Protocol

In this section, we show that the PA protocol is vulnerable to the man-in-the-middle attack and does not provide implicit or explicit key authentication. When such an attack is launched, the attacker can compute the i_{th} session key k_i.

In the PA protocol, a secret-key certificate $f(k_{AS}, [id \parallel x_0])$ is securely transacted between the MS and AS because only the AS knows G, H and k_{AS}. The $[i \parallel x_i] = e^{(i)} = z_i$, however, is open to the network. The illustration of the man-in-the-middle attack scenario is described as shown in Fig. 2.

MS		**Attacker**
transform $z_i = [i \parallel x_i]$ to error vector $e^{(i)}$ transform $f(k_{AS}, [id \parallel x_0])$ to $c = m \cdot G$ with mapping algorithm		
sends	$\xrightarrow{c + e^{(i)} \textit{ to AS}}$	capture $c + e^{(i)}$ ignore c compute $z_i = [i \parallel x_i]$ from $e^{(i)}$ with inverse mapping algorithm
computes $k_i = h_1(x_i)$		computes i_{th} session key $k_i = h_1(x_i)$

Fig. 2. Illustration of the man-in-the-middle attack scenario in the session key generation stage

The man-in-the-middle attack, performed in the session key generation stage, is described as follows:

(1) Suppose that the attacker intercepts the communication between the MS and AS. In the PA protocol, the attacker receives message $c + e^{(i)}$ from the network during a legal MS and sends it to the AS in the session key generation stage.

(2) After capturing $c+e^{(i)}$, the attacker can divide $e^{(i)}$ from it and can compute an integer $z_i = [i \parallel x_i]$ by using an inverse mapping algorithm from a binary error vector $e^{(i)} = (e_1^i, e_2^i, ..., e_N^i)$ of weight t.

Algorithm 1 shows how to transform an integer z_i into a binary error vector $e^{(i)} = (e_1^i, e_2^i, ..., e_N^i)$ of weight t.

Algorithm 1

$for j = 1, 2, ..., N\{$
$\quad if\ z_i \geq \binom{N-j}{t}\ then\ \{e_j^{(i)} \leftarrow 1;$
$\quad\quad z_i \leftarrow (z_i - \binom{N-j}{t});\ t \leftarrow (t-1);\}$
$\quad else\ e_j^i \leftarrow 0;$
$\}$

Algorithm 2 shows how to transform a binary error vector $e^{(i)} = (e_1^i, e_2^i, ..., e_N^i)$ of weight t to an integer z_i.

Algorithm 2

$z_i \leftarrow 0;$
$for j = 1, 2, ..., N\{$
$\quad if\ e_j^{(i)} = 1\ then$
$\quad\quad \{z_i \leftarrow z_i + \binom{N-j}{t};\ t \leftarrow (t-1);\}$
$\}$

Note that the parameters t, N, D and mapping algorithms for a specific error-correcting code are known to the public. After performing Algorithm 2, the attacker can easily compute the i_{th} session key $k_i = h_1(x_i)$ from $[i \parallel x_i]$. Now, the attacker can decrypt all encrypted messages at session i with the revealed session key k_i. Hence, the PA protocol is vulnerable to the man-in-the-middle attack.

Furthermore, an implicit or explicit session key authentication is not provided in the PA protocol. The MS and AS do not have a way to confirm that they share a session key. After receiving the $[i \parallel x_i]$, $f(k_{AS}, [id \parallel x_0])$ from the MS in the session key generation stage, the AS only checks $x_0 \stackrel{?}{=} h_0(x_1)$ with a series of authentication tokens, except for their shared session key $k_i = h_1(x_i)$ implicitly or explicitly. Hence, the PA protocol can not achieve fundamental security requirements of the AK or AKC protocol.

4 The Proposed Protocol

The proposed protocol can be divided into three stages as in the PA protocol. The subscription stage and the certificate issue stage are the same as those in the PA protocol. In the following, the session key generation stage of the proposed protocol will be described. The illustration of the proposed protocol is shown in Fig. 3.

MS		**AS**
computes x_0 $x_{i-1} = h_0(x_i)$		
	$\xrightarrow{x_0\ (secure\ channel)}$	
		issues $f(k_{AS}, [id \parallel x_0])$
	$\xleftarrow{f(k_{AS}, [id \parallel x_0])}$	
computes $k_0 = h_1(x_0)$	$\xrightarrow{f(k_0, [i \parallel x_i]),\ f(k_{AS}, [id \parallel x_0])}$	
		compute $k_0 = h_1(x_0)$ decrypt $f(k_0, [i \parallel x_i])$ compares i and computes $h_0(x_i),\ h_0(x_{i-1}), ...,\ h_0(x_1)$ checks $x_0 \stackrel{?}{=} h_0(x_1)$
computes $k_i = h_1(x_i)$		computes $k_i = h_1(x_i)$

Fig. 3. Illustration of the proposed protocol

The session key generation stage for each subscriber (MS) is described as follows:

(1) In the session key generation stage, the MS passes $f(k_0, [i \parallel x_i])$ to the AS with his secret-key certificate in order to access the mobile network at session i.
(2) The MS can compute the root session key $k_0 = h_1(x_0)$ with the root authentication token x_0. Note that the root authentication token x_0 is distributed securely in a pre-defined way.
(3) Then, the AS decrypts $f(k_0, [i \parallel x_i])$ with the root session key $k_0 = h_1(x_0)$.
(4) The AS checks if $h_0(x_1)$ is the same as the value x_0, which was retrieved from the secret-key certificate after computing a series of authentication tokens.

$$h_0(x_i),\ h_0(x_{i-1}), ...,\ h_0(x_1)$$

(5) If authentication is successful, both parties update the session number. In order to encrypt data at session i, the AS can compute the session key $k_i = h_1(x_i)$, which is shared with the MS from x_i.

In order to avoid the man-in-the-middle attack, we apply one additional symmetric-key encryption function. In the session key generation stage, the MS computes $f(k_0, [i \parallel x_i])$ with the root session key $k_0 = h_1(x_0)$. Furthermore, with this additional symmetric-key encryption function, the proposed protocol can achieve the property of the AK protocol. The AS assures that the MS knows the value of the root authentication token x_0 by decrypting $f(k_0, [i \parallel x_i])$ from the MS.

5 Security Analysis

In this section, we analyze the proposed protocol under the security of the symmetric encryption function and the intractability of the pseudo-random generator h. In the following, some possible attacks against the proposed protocol are presented under the above assumptions. The proposed scheme should be able to satisfy the following theorems.

Theorem 1. *The proposed protocol resists the man-in-the-middle attack to reveal session key k_i.*

Proof: An adversary tries to reveal the i_{th} session key k_i from transmitted messages $f(k_0, [i \parallel x_i])$, $f(k_{AS}, [id \parallel x_0])$. If the adversary knows the root authentication token x_0, he/she can obtain the root session key $k_0 = h_1(x_0)$. In order to obtain x_0 from the transmitted messages, however, this is the equivalent to solving the underlying symmetric encryption function $f()$. With the use of a cryptographic encryption function, it may be difficult to reveal session key k_i.

Theorem 2. *The proposed protocol provides user anonymity.*

Proof: First, assume that an adversary might be able to collect the transmitted messages in the protocol run, then he/she tries to find the identity of the attending participants. If the adversary knows the AS's secret key k_{AS}, he/she can obtain the participant's identity from the transmitted secret-key certificate. In computing k_{AS} from the public value of the proposed protocol is the equivalent to solving the AES algorithm.

Theorem 3. *The proposed protocol resists the MS impersonation.*

Proof: An adversary tries to impersonate MS by forging the $f(k_{AS}, [id \parallel x_0])$, $f(k_0, [i \parallel x_i])$. It, however, is impossible to compute $f(k_0, [i \parallel x_i])$ without the legal MS's root secret token x_0. The AS can verify $f(k_0, [i \parallel x_i])$ value with $x_0 \stackrel{?}{=} h_0(x_1)$ equation in the session key generation stage. The root authentication token x_0 is protected by the intractability of the pseudo-random generator h.

Theorem 4. *The proposed protocol resists the AS impersonation.*

Proof: An adversary tries to impersonate AS by replaying previously-captured messages or forging messages. It, however, is impossible to compute session key $k_i = h_1(x_i)$ without the legal AS's secret key k_{AS}. This is because only the legal AS can generate k_i after computing a series of authentication tokens $h_0(x_i),\ h_0(x_{i-1}), ...,\ h_0(x_1)$, in the session key generation stage. Without k_i, the adversary can not learn any information from the encrypted messages.

Theorem 5. *The proposed protocol resists the session key compromise.*

Proof: An adversary wishes to derive the session key from the transmitted messages of a session key generation stage. The adversary can obtain $f(k_{AS}, [id \parallel$

$x_0]), f(k_0, [i \parallel x_i])$. To compute session key $k_i = h_1(x_i)$, the adversary can obtain AS's secret key k_{AS} or root session key k_0 to recover x_0. The process of obtaining k_{AS} and k_0 from the transmitted messages is under the security of the symmetric encryption function and the intractability of the pseudo-random generator h.

Theorem 6. *The proposed protocol provides user anonymity in the case of a session key compromise.*

Proof: When a previously generated session key k_i is somehow revealed, the adversary tries to identify the MS who makes a request for the service from AS. To verify the identity of the MS, the adversary should verify transmitted messages $f(k_{AS}, [id \parallel x_0])$, $f(k_0, [i \parallel x_i])$ with a revealed session key in the released protocol run. It, however, is impossible to verify the attending MS. This is because no messages can be used in the as a user identifying equation.

6 Comparisons

In this section, we compare the proposed protocol and PA protocol while preserving user anonymity in terms of several features. Table 1 summarizes the main features of the PA protocol and the proposed protocol.

Table 1. The comparison of the proposed protocol and the PA protocol

Security property	The PA protocol	The proposed protocol
Man-in-the-middle attack	×	○
Session key compromise	×	○
MS impersonation	×	○
Implicit key authentication	×	○

As in Table 1, the PA protocol is vulnerable to the man-in-the-middle attack. In the proposed protocol, in order to avoid this weakness, the MS sends an encrypted message $f(k_0, [i \parallel x_i])$ to the AS with a root session key k_0. With this change, the proposed protocol can also resist a session key compromise and MS impersonation. As long as the generation of the $f(k_0, [i \parallel x_i])$ depends upon a secret root authentication token x_0 and a root session key k_0, it can be guaranteed that this value is generated by a legal MS.

In addition, the proposed protocol is the AK protocol with the root session key k_0 because all session keys are generated by the root authentication token x_0. The AS and MS implicitly assure that their session key is shared with $k_0 = h_1(x_0)$.

The contrast between the PA protocol and the proposed protocol is that although the proposed one can provide implicit key authentication and it is able to resist the man-in-the-middle attack, this protocol can provide user anonymity the same as that of the PA protocol but with only one additional symmetric encryption function.

Table 2. The comparison of the computational efficiency

Computational property	The PA protocol	The proposed protocol
Error-correcting code	1	0
Symmetric encryption	1	2
Number of message flow	3	3
Number of message	4	4

Table 2 summarizes the computational efficiency of the PA protocol and the proposed protocol.

As in Table 2, the proposed protocol provides user anonymity with one extra symmetric encryption. The use of an error-correcting code, however, involves high computational costs to the system. With the same number of messages and flows, the proposed protocol provides adequate user anonymity in resource-limited mobile communication environments.

7 Conclusions

In this paper, we have demonstrated the weaknesses of the PA protocol. Their protocol is vulnerable to the man-in-the-middle attack. Anyone can easily compute a shared session key with public parameters of specific error-correcting codes and an error vector. To overcome this weakness, we have proposed a new authentication protocol that provides user anonymity. Our protocol makes use of the additional symmetric encryption function by using a pre-distributed secret value x_0, which is called root authentication token, without an error-correcting code. The proposed protocol not only provides the security property of the AK protocol but it also guarantees user anonymity.

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Effective Filtering for Collaborative Publishing

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Abstract. In little over the last decade the World Wide Web has established itself as a medium of interaction, communication, content delivery, and collaboration, opening doors of opportunity never before available to humanity, and on a scale unprecedented in human history. At the same time, *information overload*, due to democratization of content creation and delivery, remains a major problem. In this paper, we postulate that the problems of democracy are solved by democracy itself: harnessing the people power of the world wide web through *collaborative filtering* of content is the natural solution to the information overload problem; and we present approaches to promote such collaboration.

We show that the standard PageRank Algorithm, inspired by the effectiveness of citation-structure analysis ("all links are good, and the more the better") to estimate the relative importance of articles in scientific literature, is becoming less effective in this increasingly democratized world of online content. As long as uniformly edited content produced by media companies and other corporate entities dominated online content, the topological similarity of the web to the world of scientific literature was maintained sufficiently well. The explosion of unedited blogs, discussion fora, and wikis, with their "messier" hyperlink structure, is rapidly reducing this similarity, and also the effectiveness of standard PageRank-based filtering methods.

We assume a slightly modified Web infrastructure in which links have positive and negative *weights*, and show that this enables radically different and more effective approaches to page ranking and collaborative content filtering, leading to a vastly improved environment to incentivize content creation and co-operation on the World Wide Web, helping realize, in essence, a vastly more efficient information economy in today's online global village.

1 Introduction

In little over a decade, the World Wide Web has transformed itself into the major medium of interaction, communication, content delivery, and collaboration for the global village of Humanity. Not only has this medium enabled worldwide collaboration on a scale unprecedented in human history, but is also itself showing a dramatic rate of change as it continually redefines itself. At the same time, the Web remains ill-equipped and handicapped on numerous fronts to reach its full potential because of design decisions that were made decades ago without its

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current scale and scope of use in mind. The problem of *information overload* is one such issue that is rapidly becoming more and more acute with the ever-faster democratization of the online content creation and delivery industry. In this paper, we postulate that the problems of democracy are solved by democracy itself: harnessing the people power of the world wide web through *collaborative filtering* of content is the natural solution to the information overload problem; and we present approaches to promote such collaboration. While publication on the web used to be largely done by media companies and other corporate entities to disseminate information and advertise till a short while ago, it has now seen a dramatic explosion in content creation and publishing (in the form of online photo albums, music, short films, “blogs”, wikis, etc) by a large and ever more rapidly increasing number of individuals.[1] The ongoing explosion in profusion of content is rapidly reducing the effectiveness of the web by overwhelming content consumers with an uncategorized and undifferentiated slew of material, in which finding what they want is not much easier than looking for a needle in a haystack.

This is certainly not a new problem. Indeed, this very same issue has been the focus of quite a lot of research for quite some time. Keyword-based search engines have been around for almost as long as the Web has been in existence, and their effectiveness has been steadily going up. The earliest search engines like AltaVista used a simple keyword occurrence-frequency based approach to estimate the relevance of a webpage to a set of keywords given by an user in her query. This was very susceptible to keyword spamming. Subsequently, the HITS algorithm of Kleinberg [11], augmented by the approach of Bharat and Henzinger [1] and that of Chakrabarti et. al. [5] rely on local analysis of the structure of the Web around each result webpage to estimate the relative importance of webpages in a given context. These and related approaches [3, 7] are especially susceptible to link spamming because they do not look at the global structure of the internet. With the continued march of Moore’s Law and steady exponential increase in computing power available at a given price-point, the PageRank algorithm of Brin and Page [4, 13], that substantially minimizes some of the above difficulties by looking at the entire structure of the World Wide Web, has become economically and commercially feasible, and led to the immensely successful internet giant Google[2]. Improvements in page ranking technology have continued [16, 12], with the study into topic-sensitive ranking and the use of multiple rankings with various biases [6, 9, 14, 10]. Approaches have been proposed to take into account structural differences in web topology across different subject areas, to make sure that subject areas that involve pages with higher rates of citations (in-links) do not drown out “more relevant” pages that come from

[1] The blog tracker Technorati reported in August 2005 that it sees about 900,000 new blog posts created every day, or about 10.4 new posts created per second, and about 80,000 new blogs created per day, or about one new blog created per second. Technorati currently tracks about 14 million blogs and 1.3 billion links; the number of blogs is continuing to double every 5.5 months; and 55% of all blogs continue to remain active.

[2] `http://www.google.com`

subject areas with a less heavy use of citations. Bharat and Mihaila propose an approach to improve results for most common queries [2]. Other approaches to scaling PageRanks appropriately to more accurately reflect desirable rankings by attempting to better discover relevance of a given webpage to a particular context, have been proposed [8, 15]. A far more ambitious goal, the "Semantic Web", has been the subject of active research for quite some time now.

We observe in this paper that all these approaches to *mine reputations* from the topological structure of the web are seriously handicapped in that they make an implicit assumption that *all citations are good*, and *the more, the better.* This assumption is largely true of scientific literature, and that is because of the careful peer-reviewed process in which scientific literature is selected for publication, that is largely successful in preventing substandard content from getting published. An overwhelming amount of content published on the world wide web has, for the most part so far, been produced by large media companies, and other corporate entities. Content published through these websites have always been carefully edited for uniformly high quality. Companies like Geocities[3] which allow anybody to publish content in the form of personal homepages, did make it possible for unedited content of good or bad quality to be published, but such content was small enough in amount, and followed the link-structural rules of the rest of the internet ("all citations are good") to a large extent (the vast majority of people did not criticize or post negative information about other people, products, or services on their personal homepages, and almost all links could be interpreted as positive recommendations), that they did not constitute a fundamental anomaly. Link spam based attacks (of which Googlebombing is a celebrated example) and a constant arms race between "search engine optimization" companies and search vendors had continued, but the impact of such attacks on the quality of mainstream search results had remained relatively small.

However, in the past few years, there has been a new wave of explosion of interest in the world wide web. Collaborative content creation through *wikis* (like Wikipedia[4]) and discussion fora has exploded, and the community of weblogs[5] or "blogs" (the community popularly known as the "blogosphere") is growing exponentially; blogs are no longer (often boring) personal journals, but are fast emerging as a formidable alternative to the mainstream news and entertainment industry. Indeed, most mainstream news industry majors (like CNBC, BBC, etc) are attempting to fight the war against the private army of bloggers by establishing "official" blogs of their own, maintained by their correspondents and newscasters. Indeed, the significance of the emergence of blogs can be qualitatively noted from the fact that blogs and their concomitant "comment spam" had deleterious effects on the accuracy of ranking algorithms used by major search engines; effects significant enough to cause rivals Google, MSN Search, and Yahoo! Search to work with major blog-hosting sites (like Blogger, Flickr, LiveJournal, MSN Spaces, etc) to agree on a modification of HTML introducing

[3] `http://www.geocities.com`
[4] `http://www.wikipedia.org`
[5] e.g. `http://www.blogger.com`

the "`rel=nofollow`" attribute on hyperlinks (to indicate links that should be ignored by the search engine) in January 2005[6].

Our results indicate that significant parts of this world of online content do not topologically resemble citation graphs of scientific literature.[7] In contrast to the latter world, where highly-cited publications are almost always of high quality, bloggers often post links to other sites (like advertisements, or other bloggers) with strong criticisms, and a lot of highly-criticized websites have a high in-degree. Interpreting the *syntactic* presence of those links to carry the *semantics* of *positive recommendation* leads to vastly different results from a more semantic approach to analyzing link-structural topology that takes into account the *meaning* of each link.

In Section 2 we present our modified page ranking algorithm, called the *Semantic* PageRank Algorithm, and compare it with the original PageRank Algorithm on a set of about 2000 webpages collected from popular discussion fora and weblogs on a variety of topics, and a topologically similar structure of 50,000 nodes generated automatically using a randomized model of the appropriate topological structure obtained by manual inspection of the original pages. The reason we do not choose any of the performance-enhanced or otherwise augmented versions of PageRank (like [9, 8]) is because our core argument is against the applicability of blind interpretation of all links as uniformly positive recommendations, and all improvements of the original PageRank algorithm suffer from making this same implicit assumption. In Section 3 we evaluate the effectiveness of our Semantic PageRank Algorithm in enabling collaborative filtering of content on the world wide web. In Section 4 we comment on directions of future research, and conclude.

2 The Semantic PageRank Framework

2.1 PageRank Revisited

The PageRank Algorithm [13] has proved very successful in mining information about the relative importance of different webpages from the syntactic topological structure of the world wide web. Syntactic topology refers to the structure of a set of documents, as obtained purely from the presence or absence of references (or citations or links) on one to the other. On the other hand, the *semantic topology* of a set of documents refers to the structure obtained from the presence of *weighted references* on webpages to other webpages, with the weights drawn

[6] `http://googleblog.blogspot.com/2005/01/preventing-comment-spam.html`

[7] Differences between the web and well-controlled collections (like scientific literature) have been noted before [4], but the focus has been on the massive heterogeneity of kinds of documents, and document quality, on the web. We focus instead on the difference in the inter-relationship between the citation topology and document quality between the web and well-controlled collections, and its implications for methods that use the citation topology to infer document quality on the web, like the PageRank Algorithm.

from the set of integers representing the strength of recommendation carried by the reference. Syntactic topological analysis can thus be thought of as a special case of semantic topological analysis, where the weights are restricted to the set $\{0, 1\}$, with 0 and 1 denoting respectively the absence or presence of a reference.

The central idea of the PageRank Algorithm can be summarized as follows:

$$\forall w \in W. R^{i+1}(w) = \epsilon \cdot \Sigma_{w' \in P_w} \frac{R^i(w')}{O_{w'}} + \frac{(1-\epsilon)}{|W|}$$

where W is the set of all webpages, $R^j(w)$ is the PageRank of webpage w computed after j iterations, P_w is the set of *predecessors* of webpage w, i.e., pages that link to w, and O_w is the *out-degree* or the number of links going out of webpage w, and $0 < \epsilon < 1$ (usually chosen to be 0.85 or 0.90) is a *damping factor*. More details can be found in [4, 13].

The PageRank Algorithm is thus *syntax-driven* and *semantics-agnostic* in interpreting link structure: if a webpage A contains a link to a webpage B, the algorithm concludes that webpage A *recommends* B (ignoring the possibility that A might, in fact, *negatively recommend* B). In other words, the algorithm implicitly assumes that the fraction of links representing negative recommendations on the world wide web is small enough that the semantics of a link (whether the link makes a positive or negative recommendation) can be safely ignored. The approach is closely related to the citation-based approaches used in evaluating relative importance of scientific literature, where this assumption is largely true because the checks and balances involved in peer-reviewed publication are largely successful in preventing publication of substandard work. However, on the world wide web, where no such restriction exists on publication of webpages, this assumption is not guaranteed to hold. Since the algorithm was introduced in 1998-99, this assumption has been largely true about the world wide web so far, because of the nature of most of the content on the web so far, largely produced by media companies and other corporate entities had followed a certain structural pattern similar to that seen in scientific literature.

However, in the rapidly changing environment on the web, the semantics-agnosticity assumption is quickly becoming untenable. Indeed, as the experiment described in Section 2.3 shows, for certain classes of webpages (like those from discussion forums, weblogs, etc), this assumption can lead to results widely divergent from what one would expect based purely on one's understanding of the quality and importance of these webpages. In such cases our semantics-driven page ranking approach is found to find a much more accurate estimation of the relative importance of these webpages.

2.2 Semantic PageRank

The modified version of PageRank we use to get semantic page ranking is summarized as follows:

$$\forall w \in W. R^{i+1}(w) = \epsilon \cdot [\Sigma_{w' \in PP_w} \frac{NR^i(w') \cdot l_{w'}(w)}{TP_{w'}} - \Sigma_{w' \in PN_w} \frac{NR^i(w') \cdot l_{w'}(w)}{TN_{w'}}] + \frac{(1-\epsilon)}{|W|}$$

where W is the set of all webpages, $R^j(w)$ is the PageRank of webpage w computed after j iterations, $NR^j(w)$ is the *normalized* PageRank for a recommender website, defined as $NR^i(w) = \max(0, R^i(w))$, PP_w (PN_w) is the set of *positively (negatively) recommending predecessors* of webpage w, i.e., pages that link to w with a hyperlink of positive (negative) weight, and $l_{w'}(w)$ is the weight of the link from w' to w, and TP_w (TN_w) is the *total positive (negative) out-degree* or the sum of the weights of all links of positive (negative) weight going out of webpage w, and $0 < \epsilon < 1$ is a *damping factor*. Note the asymmetry in the definition of NR; it makes a webpage judged to be substandard by others to become progressively less important in determining the relative importance of other webpages. The ideal case for our experiments involves a hypothetical modification of HTML. The current version of HTML uses a syntax to represent hyperlinks as shown by the following example: `<A HREF="http://www.cs.berkeley.edu">CS Division</A>`. Ideally, we envision HTML to support a *relevance* or *recommendation level* attribute in the `A` tag, in addition to the hyperlink URL itself. Thus, we envision a syntax to represent hyperlinks in a future version of HTML to be as shown in the following example: `<A HREF="http://www.cs.berkeley.edu" REL=Score10>CS Division</A>`. This modification of HTML would be no more severe than that involved in the recent introduction of the "`rel=nofollow`" attribute by major search engine vendors.

2.3 Experiment 1

We experimentally validated our results as follows. Preliminary analyses were performed on the semantic topological structure of webpages downloaded from a set of discussion forums and blogs on a variety of topics. This validated our intuition that a lot of links should be interpreted as negative recommendations in order to obtain the best approximation to the relative ranking of pages, with respect to the relative rankings corresponding to the user-assigned ratings for the forums' pages.

Using a randomized model created by human inspection of the link structure of these pages, we simulated blog and discussion fora content of the order of 50,000 nodes. Each node represented a page and was given an *intrinsic* score representing the user feedback for that page. Pages were linked to each other with weighted edges representing individual publishers' opinions of pages produced by other publishers. A small fraction (around 5% of all publishers) of misbehaving publishers were introduced into the model to represent the behavior of link spammers.

For syntactic and semantic PageRank computation, all webpages were assigned an initial PageRank of 5. The syntactic and semantic page ranking algorithms were run till either a fixed point was reached in the PageRank values computed, or 100 iterations had been performed, whichever happened earlier.

The standard syntactic PageRank Algorithm as described in Section 2.1 was run on the accumulated data, assuming all links to be a recommendation of weight 1, and the results were normalized linearly to a scale from -10 and 10. The semantic PageRank Algorithm was run on the data set, and the results were normalized linearly to yield for each webpage a score between -10 and 10. The implementation involved about 2000 lines of OCaml code, and completed execution in less than 4 hours on an IBM T40 laptop with a 1.6GHz processor

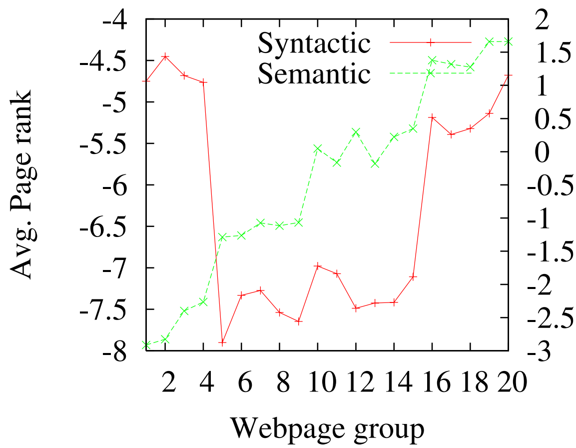


Fig. 1. Comparison between syntactic and semantic page ranking

and 768 MB RAM. The results were compared with the intrinsic scores of each webpage. The results are shown in Figure 1. The websites are grouped into 20 groups with increasing levels of actual importance (as measured by intrinsic scores). Some of the worst (according to intrinsic scores) pages were heavily criticized and thus highly in-linked, pushing up their syntactic page rank, while as the figure shows, the semantic page rank (almost) uniformly increases from left to right, showing the correct ordering of page quality.

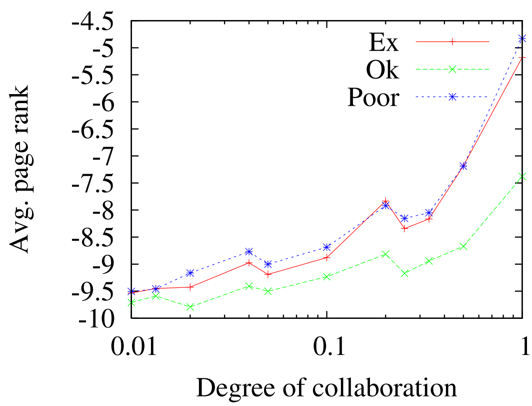
3 Collaborative Content Filtering

Using the semantic page ranking algorithm we have described in Section 2, we now evaluate how amenable our ranking scheme is to facilitate collaborative filtering of content on the web, to enable users to help each other in sifting through the immense amount of information available to find the bits and pieces that are most valuable.

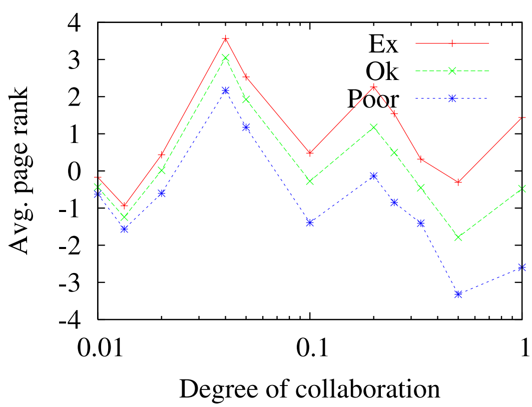
We simulate the existence of *content aggregator* websites whose contents can be modified by anybody. Readers who like or dislike content created by content publishers can post positive or negative links respectively to such content on these aggregator websites. In this paper we assume that a large fraction (around 95%) of all users modifying the aggregator sites act in good faith. In this framework, we ran the following experiment to find out how effectively newly created content is categorized by the content aggregation sites, when syntactic and semantic page ranking is used respectively.

3.1 Experiment 2

We started the simulation with about 10,000 webpages. To this collection we added, uniformly at random, 1000 new links to fresh pages each classified as "Excellent", "OK", and "Poor", to represent new content being added into the existing web of pages, in order to to analyze the effectiveness of content discovery and filtering; so there was no need for us to actually generate any content for these 1000 new webpages. Now we simulated visits by users through this web, and posts by them positively or negatively recommending the newly added pages on a subset of the original set of 10,000 webpages that act as the content aggregator sites. A small (about 5%) of users are dishonest and engage in link spamming (for example, positively recommending a page with "Poor" content), but the rest of the users share a consistent opinion of the quality of content. For simplicity, our model is not suited for cases like where the content itself may be controversial, and substantial segments of the user population may hold significantly different views on its quality depending on their own biases. We ignored such cases in this paper because our goal was to use a simple but not unrealistic model of user behavior that is applicable to the problem of filtering non-controversial content (like photo albums, music, fiction, etc) of which



(a) Syntactic page ranking



(b) Semantic page ranking

Fig. 2. Results of content filtering with the two page ranking schemes

there is an abundance, and thus a need for collaborative filtering. The results of our simulation are shown in Figure 2. The "degree of collaboration" in the figure represents the ubiquity of content aggregator sites. The figure shows the average page rank of the three groups ("Excellent", "OK", "Poor") of new webpages added, after filtering has been done by the visitors, through positive or negative recommendations posted on the content aggregator webpages. As the figure shows, semantic page ranking is able to make the "Excellent" pages rise up higher in importance with respect to the "Poor" ones, while syntactic page ranking is unable to distinguish between good and bad comments, and hence causes both "Excellent" and "Poor" webpages to be ranked higher. Thus, the semantic approach is able to take advantage of user collaboration to filter content with a better discrimination for quality.

4 Conclusions and Future Work

We have shown that the standard PageRank Algorithm which works well on citation graphs of scientific literature under the assumption "all links are good", does not perform so well on some classes of webpages with a preponderance of negative recommendations. We assume a slightly modified Web infrastructure in which links have positive and negative weights, and show that this framework enables radically different and more effective approaches to page ranking, and collaborative content filtering.

Much work remains to be done. The convergence properties of the semantic page ranking algorithm need to be carefully studied. Research into reputation systems and a framework of incentives in the form of micropayments is necessary to study possibilities of enhanced collaboration on content aggregator sites. While our current results show that the system is robust against small amounts of link spam, further work is needed to find approaches to minimize its impact when faced with relatively larger populations of spam pages. Automated techniques to derive positive or negative intentions of citations from the hyperlink anchor text or surrounding text would allow getting the benefits of this approach without making more options available to link spammers. Finally, there is a huge amount of work already done on syntactic page ranking [16, 12, 6, 9, 14, 10, 8, 15], and the relationships with those improvements on the PageRank Algorithms with our semantic page ranking scheme need to be studied carefully.

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Experimental Evaluation of an eBay-Style Self-reporting Reputation Mechanism

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Abstract. We experimentally studied the effects of a eBay-style self-reporting reputation mechanism in an double-sided exchange economy in which participants have the option of not fulfilling their contracts. We found that submitted reports quite accurately reflected their transactions and this mechanism maintaining a high contract fulfillment rate. The inaccurate reports, which were about 5% of the total, were heavily biased towards bad ratings when the transaction is successful. This is strong evidence that the inaccurate reports were not results of random errors, but derived from an underlying behavior effect. Our experimental design allowed identifying the effect of reputation mechanism on endogenous market behavior.

1 Introduction

Reputation has been an important aspect of commerce since the emergence of exchange economies [1]. Reputations can ensure promised actions are taken without the expense of external enforcement mechanisms or third party monitoring such as credit card companies. The Internet and subsequent development of e-commerce allow an increasing number of small players to engage in buying and selling. eBay is a prime example of how small businesses, particularly those serving niche markets, can overcome the previously forbidding marketing costs and reach customers with relatively low information costs. This trend leads further to transactions that take place entirely via the Internet when the product or service itself can be delivered on-line in addition to using the Internet to identify, negotiate and pay for the transaction.

However, this environment increases the importance of establishing trust in a market where everyone can choose to be anonymous, people may only participate in a few transactions, each transaction may be of relatively low value, and transactions readily cross jurisdictional boundaries raising the difficulty of legal contract enforcement. eBay approached this issue, with some success, with their feedback mechanism in which participants rate the performance of the other party in their transactions.

Establishing trust through repeated interactions has been studied in several contexts, particularly with the iterated Prisoner's Dilemma [2]. These give rise to strategies, such as tit-for-tat, to ensure cooperation. Another example is the experimental study of the "lemon" market in which reputation substantially affects behavior [3]. Unlike the Prisoner's Dilemma scenario, people in a market

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can choose not to do business with those deemed untrustworthy, or offer different terms based on perceived level of trust. Large companies can spread risk among many transactions (e.g., insurance) so have predictability arising from averaging over many individuals. On the other hand, small-scale transactions on the Internet lack this feature, perhaps leading risk-adverse people to avoid transactions that could benefit both parties. Such avoided transactions reduce market efficiency and hence decrease the potential economic gains from Internet's reduction in information and transaction costs. Thus an important question is to what extent reputation mechanisms can aid such markets. Analysis of eBay-like markets suggests that although a feedback mechanism has desirable features in theory [4], such a market may not be efficient, fair and stable in practice.

In this paper, we experimentally examine a self-reporting mechanism similar to the feedback used by eBay. Effective experimental study of reputation mechanisms requires experiments long enough for behavior to stabilize. We found laboratory experiments taking place within a few hours can provide enough transaction history to distinguish "good" from "bad" behaviors and allow identifying aggregate effects on the market [5]. Our experiments include noise, which models, for example, the situation in a single transaction where the intention to pay on time cannot be distinguished, if delayed by the mail, from the intention to pay late.

We found that the self-reporting mechanism successfully maintained high fulfillment rates and we provide an analysis of individual behavior in the use of the mechanism. In particular, the reports show a high level of accuracy (95%). Furthermore, most of the inaccurate reports were instances where a player gave a bad rating for someone who *successfully* completed the transaction. This points to more subtle underlying behavior that warrants additional scrutiny. In particular, we speculate that a small percentage of the reports (5%) were used strategically to punish another player or to deliberately lower their reputation.

After describing prior related experimental studies of reputation, Sec. 3 presents our experimental setup. We then discuss the experimental results, both in terms of the choices individuals made on whether to fulfill their contracts and their effect on overall market efficiency.

2 Reputation Mechanism Experiments

A number of experimental studies have addressed the performance of various reputation mechanisms. In one approach [6], participants face an abstracted version of the transaction, namely the "trust game" where one player can choose to send money to a second, this amount is then substantially increased and the second player can choose to share some of that gain with the first player. By removing many of the complexities involved in market transactions, this game provides a simple context to study the effect of different information policies about revealing past behaviors. Addressing market efficiency requires more complex experimental scenarios [7, 3].

In contrast to this work, our experiments provide a broader set of endogenous choices for the players. First, the players can explicitly decide who they wish to do business. Although not studied in this paper, this feature allows examining whether people choose to use reputation information to ostracize those with low reputations or give them poor prices based on their higher perceived risk. Second, both buyers and sellers make delivery choices and so face a moral hazard for which reputations are relevant. In the context of a reputation mechanism based on self-reported information, this setup for reputation on both sides of a market allows players to misreport their experience as possible punishment for a poor report on their own reputation. More generally, our setup allows for the formation of clusters of mutually high-reputation trading arrangements. Third, our experiments include a full market so prices and trading volumes are determined endogenously, providing a broader view of the macroeconomic consequences of different information policies than is possible in more restricted scenarios.

3 Experimental Design

Reputation mechanisms could have complicated effects on markets. Our experiments were designed to evaluate both aggregate and individual behaviors. In particular, the experiments provide information on how people use self-reporting mechanisms and respond to unfulfilled contracts. Our experiments had two essential components: an exchange economy and an information policy for revealing past behaviors to participants. In the remainder of this section, we describe each of these in turn. All subjects received web-based instructions[1]. Each participant had to qualify by successfully passing a web-based quiz before participating in the experiment.

3.1 Exchange Economy

The first component of our experiment was an exchange economy of a single homogenous good. We used standard experimental techniques to create the market [8]. Supply and demand were generated by methods of induced value and induced cost. That is, each unit of good a buyer purchased was redeemed for a pre-determined amount, specified in an experimental currency with an announced rate at which it would be exchanged into dollars at the end of the experiment. Similarly, each unit of good a seller sold cost a pre-determined amount. Thus a buyer could profit by purchasing a unit below its redemption value, and a seller could profit from a sale above the unit's cost.

An experiment consisted of a number of periods. In each period, buyers and sellers received tables listing their redemption values and costs, respectively. The aggregate supply and demand was kept constant across periods. This fact was publicly announced at the beginning of each experiment. However, each redemption value on the demand curve and each cost on the supply curve was assigned to a random individual in each period. Thus, although the aggregate supply

[1] Available at http://www.hpl.hp.com/econexperiment/marketinfo-base/instructions.htm

and demand did not change, an individual's supply and demand did change. The primary reason for this design feature is to prevent subjects learning each other's supply and demand and using this information to augment reputation information. For example, if I know that seller A always has only 3 units to sell at a cost below the specified price, I can deduce his intention to not fulfill if he offers 4 units for sale. We would like the subjects to make that determination solely based on the information provided by a controlled reputation mechanism.

We used a discrete form of double auction as the market institution as opposed to the more common continuous time version which allows a subject to submit an offer or accept an offer at any time as long as the market is open [8]. Each period consisted of a fixed number of rounds. Buyers and sellers took turns making offers (setting a price and a quantity) and accepting offers made by others. We allowed players to have only one offer at a time, although they could offer to buy or sell multiple units. There are two reasons for this form of market. First, a discrete time, round-based, design gives subjects more time between decisions to study and use information relevant to reputation compared to a continuous time version in which they may only have seconds to make a decision. Second, subjects needed to be able to choose who they would do business with. This choice was more natural in a double auction setting than a call market or other institution with a central clearing mechanism. To this end, we allowed the subjects to add a filter to their offer limiting who was permitted to accept it. Each participant could accept as many offers as were available to them. Subjects were able to see all offers, including those they were not permitted to accept. We provided this information to speed up the price formation process. When an offer was accepted, it became a contract – an agreement for the seller to produce and send the goods and for the buyer to send payment.

The key feature of our experiment was that contracts were not binding. After the last round of exchanging offers in a period, both buyers and sellers were given a list of the contracts they had signed for that period. They then decided whether or not to fulfill each contract. That is, buyers chose whether or not to send payment, and sellers chose whether or not to send the goods promised. This created an environment similar to online transactions between anonymous parties when there was no contract enforcement mechanism. Participants who chose not to fulfill their contracts avoided the associated cost of fulfillment (i.e., the payment in the case of a buyer, and the production cost in the case of a seller).

The experiment included noise: a fixed probability that either payment or goods would be lost "in transit". This probability was announced to the participants in advance. When this probabilistic loss occurred, the sending party was notified that their part of the exchange was not delivered to the recipient. However, the recipient received no such notification. Thus, for instance, a seller not receiving the contracted payment from a buyer would not know whether the buyer chose not to pay or whether the payment was lost.

3.2 Information Policies

The second component of our experiment was the information policy. This controlled the information available to subjects when they made trading and contract decisions. The focus of this series of experiment was the effect of past transaction information.

The past transaction information available to subjects varied with the experiment. Five treatments were conducted with different combinations of information policies and noise, as listed in Table 1. In each treatment, all information was displayed by period, with one row on a spreadsheet representing a period. Totals were given on a separate row. Market price (the average price of accepted contracts, weighted by volume), market volume, and personal payoffs were given in all treatments. The information policies were:

– Low information: Agents were given historical information about only their own transactions. Buyers were given the total value (the sum of price times quantity) of all contracts they signed with each seller, and the value-weighted percentage of contracts that were fulfilled by the buyer and by each seller. Sellers were given analogous information. In this case, the display merely summarized information already available from that player's transactions in previous periods.
– High information: Agents were given historical information about all transactions that took place between any buyer and any seller. All agents were given the total value of contracts that each agent signed, and the value-weighted percentage that he or she fulfilled.
– Self-reported ratings: An additional stage was added after contract fulfillment in which agents rated other players for each contract signed. After receiving information about whether they received payment or goods for each contract, they were asked to give the appropriate agent a positive (+) or negative (-) rating. After all players submitted their ratings, the ratings were made public: players saw value-weighted percentages of contracts signed by a given player for which he or she received a positive rating (in addition to the total value of contracts). If all players gave positive or negative ratings if and only if their contracts were fulfilled or not, respectively, then the information available with this policy would be similar to that for the high information case.

The treatment was announced on the day of the experiment, and subjects were given complete and accurate information about the rules and nature of the game, including the probabilistic loss of payment and goods (i.e., the amount of noise).

4 Results

In this section, we describe the experiments we performed and the resulting behaviors.

4.1 Overview

We conducted a total of 8 experimental sessions, summarized in Table 1. The first one was a pilot experiment with 8 subjects. The rest had at least 12 subjects. The first 3 experiments had no noise. The rest of the experiments used a noise probability of 10%.

Table 1. Overview of the experiments, showing for each one the number of subjects, number of periods, whether noise was added and the information policy. In the pilot experiment (number 1), the supply and demand was different from the rest.

experiment	subjects	periods	noise?	information policy
1	8	12	no	high
2	12	13	no	high
3	14	16	no	low
4	16	14	yes	low
5	16	14	yes	high
6	16	14	yes	self
7	16	16	yes	low
8	14	16	yes	self

In all experiments, market prices converged reasonably well to equilibrium within 3 periods, as expected from prior studies [8]. Thus we were able to study the effect of information policy choices in the context of a rapidly equilibrating underlying market. Notice we use the term "equilibrium" loosely here.

As expected, all experiments exhibited strong end-game effects. Subjects were told when the game would end two periods ahead of time. Furthermore, they had an expectation of finishing by 5pm on the day of the experiment. Contract fulfillment decreased sharply around 4 periods before the end of an experiment. We use all of the data, including those close to the end-game, to compare information policies. We found about 10 periods in each experiment minimally affected by the end-game, providing an indication of the effects and dynamics of reputation likely to arise in the context of a long series of repeated transactions.

4.2 Fulfillment Rates

We measure aggregate contract fulfillment by the *period fulfillment rate*. To define this value, we viewed each of the contracts signed during a period as two separate transactions, the payment sent by the buyer and the goods sent by the seller, each of which could be fulfilled or not. Each contract involves a price per unit and number of units to exchange, and its value is the product of this price and number of units. The period fulfillment rate for the buyers is the ratio of the number of contracts they fulfill to the total number of contracts. We use a similar definition for the sellers. The overall fulfillment rate is the average of those for the buyers and sellers in that period. Fulfillment is not equivalent to actual delivery: as described above, even when a person decides to fulfill a contract, the payment

Table 2. Observed fulfillment in the experiments. Values show the average and standard deviation of the fulfillment over periods 3 to 11, inclusive. The most significant comparison is among experiments 4 through 8, since the others did not have noise or used a different supply and demand function.

experiment	information policy	average	standard deviation
1	high	57%	20%
2	high	90%	9%
3	low	85%	7%
4	low	47%	11%
5	high	83%	9%
6	self	77%	15%
7	low	64%	16%
8	self	86%	12%

or goods could be lost in transit due to noise and thus not delivered to the other party. Table 2 gives the observed values for our experiments. As one can see, the two self-reporting experiments resulted in fulfillment rates (77% and 86%) substantially higher the low information experiments (47% and 64%) and close to or as good as the high information experiment (86%). This is evidence that this particular reputation mechanism sustains a high fulfillment rate as well as if full transaction information were available.

4.3 Behavior of the Self-reporting Mechanism

Table 3 shows the accuracy of the reports by the individuals in our two experiments involving self-reporting. We see the accuracy is quite high, and consistently for all participants. As a further detail on the use of the reports, Table 4 show how the ratings compare with the actual behavior. In most of the cases, the rating (good or bad) correctly corresponds to actual experience with the contract (the good or payment received or not, respectively). In the remaining cases, we see people are much more likely to give a bad report in spite of receiving the contracted good or payment, than they are to give a good rating in spite of not receiving it. This bias toward bad reporting could reflect a desire to punish an individual for poor past behavior (including negative ratings) or a desire to lower the reputation of potential competition. In either case, more scrutiny to

Table 3. Accuracy of self-reporting. For each participant in each of the two experiments, the value is the fraction of reports that match the actual behavior of the other party to the contract(s) entered into by that individual. The average accuracy in each experiment is 0.94.

expt.																
6	0.93	0.97	1.00	0.94	0.94	0.98	0.96	0.96	0.92	0.94	0.96	0.91	0.93	0.91	0.94	0.89
8	0.97	0.89	0.95	0.89	0.98	0.95	0.98	0.94	0.98	0.85	0.98	0.95	0.93	0.98		

Table 4. Comparison of ratings and actual behaviors for all periods and participants in the experiments using self-reporting

experiment 6		
rating	received	not received
good	0.544	0.029
bad	0.083	0.344
experiment 8		
rating	received	not received
good	0.612	0.003
bad	0.123	0.262

the data may shed some light on how the self-reporting mechanism was used strategically.

Furthermore, due to the noise introduced in the experiments, not receiving the contracted value does not mean the person intentionally did not fulfill: the item could have been lost due to noise.

4.4 Individual Behavior

We are interested in how individuals respond to fulfillment. Do they screen out those perceived as unlikely to fulfill contracts? Or do they enter into contracts with such people but at less favorable prices? Do they use a tit-for-tat strategy to encourage fulfillment?

We found subjects tend not to fulfill contracts with the people who didn't fulfill prior contracts. There is strong evidence that people are engaging in tit-for-tat strategies. Specifically, we analyze each individual in the following way. For each individual, we compute the total value of of his or her contracts that were not fulfilled by the other party. This serves as a naive value of how much that player distrusts a potential trader. We regress this value on the percentage of contracts that this person chooses to fulfill to the same trader. In all 78 subjects (of experiment 4 through 8), this regression yields a negative coefficient. That is, a subject tends not to fulfill his contract with another trader if this trader has not fulfilled a prior contract with him or her before. Furthermore, in 64 out of the 78 subjects (82%), this coefficient is significant.

5 Conclusion and Future Work

We described a series of experiments studying the effect of reputation mechanisms. Three different mechanisms were tested: low information policy when subjects observed records of their own transactions, high information when aggregate statistics of all transactions were common knowledge and a self-reporting mechanism in which subjects scored their partners after each transaction. Results show that an ebay-style self-reporting mechanism was largely effective in preventing "cheating" and the reports were highly accurate (95%). However, we also observe a heavy bias in the inaccurate reports towards giving "bad" ratings to players who successfully fulfilled their contracts. It is likely that these inaccurate reports are intentional and were used strategically to either punish past transgression or to lower the reputation of the competition. This will be one interesting issue to examine in the future.

This experimental design can be easily expanded to examine a wide range of issues. For instance, with respect to self-reporting we could also compare centralized (e.g., eBay) to distributed (e.g., word of mouth recommendations) reputation mechanisms. This issue will probably depend on whether preferences are homogenous or not. For example, every seller prefers the buyer to pay promptly. Thus reputation based on paying behavior can have a common measurement. However, reputation about recommending movies will vary with the preferences of the person.

There is a direct equivalence between fulfillment as used in our experiments and that on eBay: namely not sending goods or payment. However, "fulfillment" can be interpreted to have continuous values on eBay as opposed to the way it was set up in the experiment. For example, a seller can send sub-standard goods to the buyer while advertising for perfection, or buyers could delay, but still ultimately deliver, payment. Our experimental design could be extended to treat this question by allowing sellers to choose the quality of good to produce (with different costs and benefits to the buyer) and thereby have the option of sending a lower quality good than specified in the contract with the buyer. Identification policy is another interesting issue our experimental setup could study. This would broaden the experiment to examine issues of anonymity, the ability to change identity at will and after markets of buying, selling and/or leasing identity in similar set ups. This last option, involving markets for reputation, has been studied theoretically [9] and could be readily added to our experiment design by allowing players to trade identities, and their associated transaction histories.

Privacy is another issue closely linked with reputation mechanism design. Disclosure of personal information may facilitate the establishment of one's reputation. For example, eBay requires an email address. Obviously if an address is required and there are ways to track down a trading partner, it is easier to establish trust. However, people may prefer to keep this information private, even if they incur some cost due to less trust on the part of others. Such concerns could be addressed without need for trusted third parties through the use of cryptographic protocols [10].

By combining markets with various information policies, we were able to study the effect of reputation mechanisms under controlled laboratory conditions. Even within the limited time available for such experiments, our design allowed us to observe differences in behavior due to the amount of past transaction information revealed to participants. Moreover, our experiment design readily extends to address a variety of interesting questions beyond those described here, such as changing or trading identities. These experiments complement larger, but less controlled, field studies of reputation in practice, such as used by eBay, and theoretical studies relying on simplifying assumptions of rational behavior or limited to deal with analytically tractable games.

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An Architecture for Evolutionary Adaptive Web Systems

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Abstract. This paper present an architecture based on evolutionary genetic algorithms for generating online adaptive services. Online adaptive systems provide flexible services to a mass of clients/users for maximising some system goals, they dynamically adapt the form and the content of the issued services while the population of clients evolve over time. The idea of online genetic algorithms (online GAs) is to use the online clients response behaviour as a fitness function in order to produce the next generation of services. The principle implemented in online GAs, "the application environment is the fitness", allow to model highly evolutionary domains where both services providers and clients change and evolve over time. The flexibility and the adaptive behaviour of this approach seems to be very relevant and promising for applications characterised by highly dynamical features such as in the web domain (online newspapers, e-markets, websites and advertising engines). Nevertheless the proposed technique has a more general aim for application environments characterised by a massive number of anonymous clients/users which require personalised services, such as in the case of many new IT applications.

1 Introduction

The research on the topic of adaptive systems has mainly focused on architectures based on knowledge representation and reasoning [1], fuzzy reasoning [2][3] and probabilistic models [4]. These approaches are often able to give an adequate account of uncertainty and dynamical aspects of the domain, but they also require a great effort in building a detailed model of the problem. Despite of the good qualitative response, they often reflect too rigidly the domain constraints at modeling time. When the environment, i.e. the constraint of the domain, evolves, the system performance tends to decrease until the model needs to be modified or redesigned.

The increasing diffusion of mass services based on new information technologies (ITs) poses new requirements and goals on adaptive systems which are seemingly contradictory, such as the problem of providing adaptive personalized services to a mass of anonymous users [4]. Sometimes models of user behavior [1] for the new services does not even exist, and, in addition, services and technologies appear and disappear very quickly thus vanishing the effort of building accurate models. The growing interest in self adaptive and self modeling systems is partially motivated by these reasons.

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The two leading approaches to self adaptation, i.e. genetic algorithms [5][6] and neural networks [7][8] are characterized by somewhat symmetrical features which are worth to be pointed out: *neural networks (NNs) tends to be online systems while GAs operate offline*. GA usually operates offline in the sense that they can be seen as building a simulated application environment in which they evolve and select the best solution among all the generations, under the well known Darwinian principle of "survival of the fittest".

Some works [9][10][11] have introduced "real world" issues into the GA loop, in the interactive GAs approach [12] the user is inserted in the algorithm with the role of providing the fitness functions by interacting with the GA, in other works still following the offline approach [13][14] about machine learning by GA, historical real data are used as fitness function.

Despite of their offline nature GA are able of a highly dynamical behavior. The main reason is that the knowledge about "reasoning" structure of GA is embedded in the population chromosomes: when the population evolves the structure evolves as well. GA concepts such as *cross over* and *mutation* have no counterpart in NNs approach, but they are a powerful tools which can allow a GA to make fast hill climbing of local minimum and plateau in optimization problems [6].

The idea of bringing these adaptive features in the *online system* scenario is made more challenging from the facts that the population of clients asking for services is evolving over time, then their response to services changes.

In this paper we propose a new approach, *online genetic algorithms* (online GAs) which tries to combine timely responses with the adaptive behavior of GAs. The basic idea of online GAs is to evolve populations by using the application world as a fitness function, under the principle "the real world is the fitness".

The goal of systems based on online GAs is to give a timely response to a massive set of clients requesting services, and to be able to adapt services to clients, both changing over time in unknown and unpredictable way.

As noted in the beginning, it is not realistic to rely on the hypothesis of detailed user models [1][15]. The increasing consciousness of privacy issues, legal limitations on personal info [16] and the growth of mobile and pervasive interfaces accessible from casual users, often make the user model impossible to collect. The anonymity of users/clients is then a structural constraint in mass adaptive services.

In the next paragraphs we will motivate the online GAs approach by analyzing the features of a sample dynamical scenario regarding an online newspaper management system.

The principles and the architectural scheme of the online genetic algorithms approach will be presented, an example application and experimental results will be discussed.

2 The Online Adaptive Scenario: A Web Newspaper

Let us consider as a typical scenario for online genetic algorithms: the problem of managing the generation of an online newspaper with the goals of maximizing customers, i.e. readers, contacts.

The problem, well known to journal editors, is to build a newspaper in order to publish news according to the newspaper politics and mission, and selling it at its best. Selling news in this contexts means the goal of capturing readers attention for reading the articles, and for, possibly, satisfying the newspaper advertisers. Online readers browse time by time the newspaper web site and read the news which interest them. It is assumed that a good journal will collect a great number of contacts and many users will spend time in reading it. Managing editors of online newspapers have a great advantage with respect to their hard paper colleagues: while a conventional paper journal is limited and bounded to a single daily edition (except the cases of extraordinary events), an online editors, instead, can make timely adaptation of the newspaper to the latest news, thus maximizing the impact of the newspaper on the readers.

Online media have the likely feature that can be produced and delivered instantaneously such that, in principle, each user can read his own single, personalized and different copy of the journal. The main issues and source of difficulties, in the newspaper scenario are the lack of information about the users and the unpredictable dynamical evolution of all the elements which characterized it, in particular:

- anonymity of clients
- dynamical evolution of potential services
- dynamical evolution of clients
- dynamical evolution of client goals

These evolutionary features are shared by a wide class of online problems.

2.1 Anonymity of Clients

Anonymity of clients means that no hypotheses can be made about profiles of the users of online services. As discussed in the introduction the typical assumption for online newspaper is that the information available to the system comes from anonymous user sessions, where users cannot be identified, nor recognized from previous sessions [16].

2.2 Dynamical Evolution of Potential Services

The purpose of online systems is to provide the best of their currently available services for maximizing the client impact [17], the situation is made more complex since *the services that are issued by the providers can vary over time in unpredictable way*.

News, seen as services, are characterized by a lifetime cycle (i.e. they appear, disappear and are archived), and the news flow is by its nature unpredictable. Thus the news editor task is to select according to the editorial line, which news best interest and impress their readers, among the available ones.

2.3 Dynamical Evolution of Clients

The set clients connected with the online system evolves over time in unpredictable way. The set of connected clients are not always the same, since new clients come and previous sessions disconnect.

In the case of online newspaper there can be made some general assumption about the target users. Users are assumed to have somewhat homogeneous features like in the case of readers of newspaper specialized in economics, politics, sports etc. Nevertheless the instantaneous audience profile of online newspaper can vary over time. For instance students can connect mainly in the afternoon, while corporate workers can connect in different time range. In addition, external factors and unpredictable events, such as holidays or exceptional events, can make different classes of readers to connect in unexpected time/dates.

Even assuming that we have a way of determining the ideal journal for the current audience given the currently available news, the newspaper edition will be no more adequate after some time, since the audience will change unpredictably.

2.4 Dynamical Evolution of Client Goals and Attitudes

Goals and attitudes of the single clients can vary and depend on time. As we as pointed out before, external events of general interest can make the journal audience vary, but can also make the interests of the audience to vary. Economical or political events can induce a shift in the typical interest of the readers. Moreover even assuming to have a fixed audience, with fixed goals, is not possible to produce a fixed "ideal newspapers", since people expect that newspaper vary: it would be unlikely to read every day the same identical news; typical users of online newspapers connect to the system many times a day, expecting to read more news on topics of their interest.

2.5 Model of Service Impact Factor

A model of the impact factor of service cannot be easily defined and require classification effort. The goal of the newspaper editor is to catch the attention of most of its readers by selecting the appropriate news and preparing a suitable edition according to the newspaper editorial line, i.e. mission, policy and cultural goals.

The typical tools available to an editor to maximize the impact of the service he provides (i.e. the news) are: *selections* of the news among the continuous flow (deciding which news are currently published and which news go to archive); *location* of the news in the grid of the newspaper layout (the position of the news usually reflect is evidence or priority in editor's intention); *presentation form* of the news, which regards aspects such are selecting a *title* for the news, and or selecting a possible *picture* accompanying it, and sometime also long or short versions of the article. These tasks are usually regarded to as an "art" which the newspaper editor performs by the help of his/her experience.

It is worth noticing that some factors, such as the news position in the layout, are not necessarily determining the reader's priority. A well prepared journal, for example, usually offers a mix of different news (i.e. not many news on the same topic). The visibility strictly depend not only in the position but also in the context in which news are presented. Sometimes hot emerging topics require breaking this rules and, when it happens most part of the journal news are devoted to a single topic.

The next paragraph will describe a framework based on genetic algorithms for providing adaptive services in highly evolutionary environment to a massive audience of anonymous users, such as in the newspapers scenario.

3 Online GA Scheme

GAs have been classically proposed for use in an *offline schema*. In the offline approach populations of solutions are evolved offline for a given number of generations in order to produce a best evolved solution (usually determined in the last generation) which given as system output. For instance in classical optimization problems [6] GA are used for exploring a search space of solutions and the best minimum/maximum value found over all generation is produced. In GA applied to learning problems, such as discovering stock market rules [14], real data about stock market are used to evolve the population, but again, the best solution is computed offline, and it is used in the *real market* afterwards. A different approach is that of Interactive Genetic Algorithms [9] [12] [18] where the real world is included into the GAs loop under the principle that “the user is the fitness”, i.e. the user participates to a cooperative optimization process. In some interactive GAs applications to robot learning [13], the real world is used to evolve the solution, but GAs uses real world in an offline phase of training.

In the online GAs approach we propose to literally implement the evolutionary metaphor which originally motivates GAs. In our proposal the basic elements and concepts of GAs such as population, chromosomes, crossover, mutation and selection mechanism still exists but, they are extended with the innovative but simple assumption that “*the real world is the fitness*”, i.e. the application world, representing the environment, is used to selects the best surviving fittest solutions, moreover all generated solutions are output to the system and no interactive cooperation is required to users/clients.

The basic scheme of an online GAs wrt offline GAs is depicted in figure 1. In *online GAs* a population of solutions is evolved with usual genetic mechanisms, with the difference that the solutions in the population are actually "executed", and the client/users behaviors/responses upon solutions are used as a fitness to evolve the next generation of online solutions. In other words the fitness function resides in the real world and it is expected to give timely response to the evolution of clients' population and domain modification.

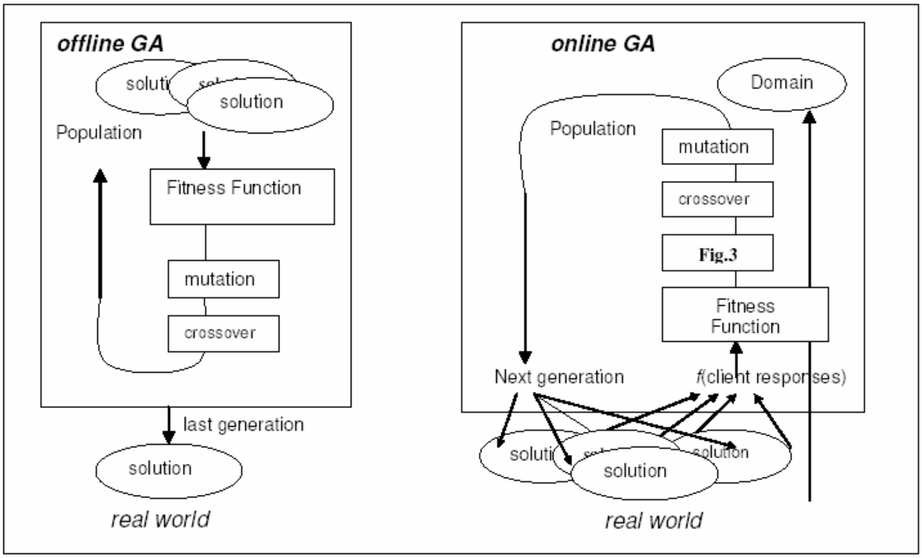
3.1 Updating Phase

Since in online GAs the problem domain also evolves unpredictably (for instance the provided services changes, i.e. the flow of incoming/outcoming news), then there is the need for a novel phase of *updating*, not present in classical GA . The purpose of the *updating* phase is to establish and implementing a policy about how to adapt the current population of solutions to domain changes (such as how to replace an article which is disappeared from the journal, because expired or delete after explicit editor's decision).

Online GAs can be used when it is required to dynamically adapt to evolution and changes in the problem domain, moreover application domains best suitable for the adoption of online GAs are characterized by:

- a *solution space* with "many" valid solutions to explore, i.e. the solution space with not unique or few valid solutions
- a *set of clients* which require solutions to be used immediately

- an *optimization function* which measures the efficacy of a solution given in output, which can be "sensed" by the system in the external world as a response/result produced by clients.



It is worth noticing that online genetic algorithms would not be a real possibility without the new ITs. A massive diffusion of the internet, mobile phones services, on demand phone services has made possible application servers where a huge number of anonymous clients (with no distinction among final users or software agents) are concurrently requiring services in an automated framework which directly connects consumers to service providers. The services providers are usually optimizing very simple functions which are completely inside their “sensing” scope such as *time-spent*, *services bought*, *money charged*, *advertising clicked*.

4 An Online GA

The pervasive dynamical and unpredictable evolution of all the key elements in the newspaper scenario represents a difficult challenge for adaptive systems which should provide adequate services in answers to clients' requests.

News to be offered in the newspaper are continuously flowing in from news agency and journalists. Different classes of anonymous individual readers continuously connect and disconnect in order to read interesting news The goals and interests of the individuals vary in an unpredictable way (people get bored of old news). The impact of news upon users depends of the form, the position and the context in which the news are given, and it is hard to be deterministically modeled [9][19]. Finally the editor policy represents a pervasive constraint to be respected throughout the journal editing.

In the following we present the architecture of a sample online GAs applied to the newspaper evolutionary scenario.

4.1 Domain and Constraints

A newspaper has a typical layout and structure in term of sections of topics, which are part of the recognizable corporate image. No editors are available to modify it, moreover the editor usually want to have control over the proposed news in order to implement the editorial policy.

In order to reflect these constraints the structure and layout of the journal do not evolve, and the editor decides which news include/exclude in/from the newspaper and how to assign (or remove) them to sections, let the sections, for instance: *TopStories, National, International, Sports, Health and Technology*. A limited set of headlines (for example 4 headlines) is reported in the front page for each section; the sections occupy fixed layout positions; the section headlines are chosen among the articles available inside the sections.

For each single article we will assume that the newspaper editor provide a set of possible alternative formats for each article, i.e. alternative titles, texts and pictures to be used for presentation. The task of the editor is to decide how to update the set of news and formats, while the online GA actually build the newspapers deciding which articles will be inserted into the sections headlines and which alternative formats will be used in the articles presentation.

4.2 Population and Individuals

The individuals which compose the population of the current generation consist of the different versions of the newspaper which have been issued to the currently requesting online readers.

4.3 Time Intervals

In order to make the online GA having a sufficient number of individuals in the population, and a sufficient time to evaluate user response, i.e. fitness of the individual, it is needed to fix a time interval value, i.e. the duration of the minimal interval of time from one generation to the next one. If, for instance, a newspaper has 6000 contacts per hour, a time granularity of 1 minute guarantee, guarantee an average population of 100 individuals, but doe not allows to evaluate responses whose duration is greater than one minute.

4.4 Fitness Function

The fitness function measures the adequacy of the solution in term of client response.

According to the anonymity hypothesis the system is able to "sense" user sessions, but not to recognize user from previously started session. Sensing data are easily collected from the web server log files. In the newspaper problem the fitness of a given solution k (i.e. the individual version of the newspaper) is defined as

$$F(k) = w_s t_s + w_{ca}\, n_{ca} + w_{cn}\, n_{cn} + w_{int}\, (\Sigma_{(i=1..}n_{cn)}\, t_{si}/t_r) \;/\; n_{cn} - w_{nohl}\, c_{nohl}$$

The listed parameters reflect the general criteria that reward as positive, in particular: t_s is the *total time spent* on the newspaper (measured as the time between the first and the last browser request); n_{ca} the *number of clicks on* newspaper *advertisings*; n_{cn} *number of clicks on news* (i.e. how many news have been red; the Σ term computes the *average interest of news*, where the interest is measured as the time spent t_{si} on a single news with respect to the time t_r needed to read the news (skipping rapidly a news means little interest wrt carefully reading it); the minus terms c_{nohl} in $F(v)$ penalizes the situations in which the readers find *no interest in headlines* and go straight to sections to read particular news, i.e. in other words it penalize at a certain extent the journal versions in which the content is interesting while presentation is not. Weights w_{as} , w_{ca} , w_{cn} , w_{int} and w_{nohl} are used to tune the contributions of the respective terms to the global fitness.

4.5 Chromosomes

The individuals, i.e., the single newspaper versions, are encoded by a set of *sections vectors* each one encoding a section of the newspaper.

Each element in the journal chromosome specifies a single news in term of its position in the section headlines (0 means not in headlines), and its presentation i.e. values indicating which title, text, and picture, the newspaper edition will contain for the given article among the different available versions.

4.6 Selection

A standard *proportional to fitness* selection method is used in order to determine the intermediate population used for crossover and mutation. The more the fitness is high more chances are given to individuals to survive. On the intermediate population thus determined crossover and mutation are applied.

4.7 Crossover

The purpose of crossover is to generate a new journal version from two individual chromosomes. The two offspring replace the parents. Again a proportional to fitness reproduction criteria is used.

The crossover is operated *section by section* on the whole chromosome. For each section a linear crossover point is determined (see dashed line in the figure below) for splitting the section subvector. The respective subsections of the two parents are then combined. Restoring valid solutions can be necessary after crossover reproduction. Suppose that a given section is allowed h headlines; the split point position can divide the section segments such that one offspring segment contains more than h headlines while the other has less than h, i.e. the solution is not valid. In this case in order to restore a valid solution we move headlines from the longest to the shortest one selecting them randomly. Another case of invalid solution is when two headlines in a section points to the same position (another headline position must be empty), in this case the tie is broken randomly. Note the criteria guarantee that all headlines in the parents will be again headlines in the offspring.

4.8 Mutation

Mutations (see figure 4) are operated at different levels with different priorities.

-headline mutation, is the operation which moves a news from sections into headlines and vice versa, since an headline mutation is a dramatic change in a newspaper version, the probability P_h of headline mutation is kept relatively low, on the other hand and additional factor Pnew is considered, Pnew, is giving more probability to become section headline to new articles wrt old ones;

-format mutation, this mutation tends to adapt the form in which the single news are given, i.e. order, titles, alternative texts and accompanying pictures, the probability P_f of this mutation is slightly high than the previous one.

A *format mutation* is realized by choosing randomly a format component (*order*, *title*, *text* and *picture*) and a feasible random value in the domain of the format component (e.g. one title over three available candidates). Format mutation of *ordering* is only applied to headline news and consists in swapping an article with another one randomly selected among the headlines.

Headline mutation is realized by randomly selecting the incoming article (taking into account of P_{new} to give priority to new articles), selecting the outcoming article and swapping them among headline and section.

4.9 Update Adaptation Phase

The adaptation phase concerns the problem of adapting the population of solution which were made invalid by external modifications. For example when the news editor decides that an old article has to be archived and/or a new one has to be inserted into the journal, some individual in the current population could be no more valid. In restoring the validity of the solution we use the following criteria:

- incoming news are added to the respective section with maximum P_{new}
- outcoming news not appearing in section hot headlines are simply deleted from the section
- outcoming news which are on the section headlines are replaced by shifting up the section headlines, and operating a format mutation on the last position where simple insertion replace swapping

The array representation is updated accordingly.

5 Conclusions

Online GAs represent a new approach to systems which provide adaptive services to a large number of anonymous clients/users which evolves over time in unpredictable way.

The basic idea of online genetic algorithms is that "the world is the fitness", i.e. the fitness function resides in the application environment and it can be evaluated by sensing the environment i.e. by evaluating clients/users response to the current solutions. A phase of adaptation is added to usual GA schema for restoring validity to solutions made invalid by evolution in the problem domain.

Online GAs are related with interactive GAs methods [12][9][10], in which the real world appear in GA in the form of the user cooperation to the selection process, or in the form of environment guided training [13]. The main difference between online GAs and interactive GAs is that in interactive GAs, GAs are used in a sort of offline simulation in order to select a *final* optimized solution or behavior, used by the application. Instead online GAs based applications made immediate use of the solutions population. The main issues which motivates the adoption of online GAs have been discussed in the framework of the newspaper scenario. Online GAs represents an answer in all those situation in which adaptation is required, while few o no data are available about users' profiles and attitudes [17]. The increasing diffusion of massive distributed services based on the new ITs , the increasing consciousness and laws about the privacy issues, motivates the apparently contradictory request of providing adaptive services to unknown users in dynamical domains.

Preliminary experimental results on a simplified version of the newspaper application confirm the validity of the online GAs approach. Open theoretical and practical issues need to be further investigated in the framework of online GAs such as the problem of time granularity with respect to the time needed for fitness evaluation; defining effective methods for tuning GA parameters and weights, and discussing typical GA issues such as co-evolution [17] in the context of online GAs. Moreover the integration between online GA and other non evolutionary techniques such as fuzzy and probabilistic analysis are worth to be investigated.

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A Collaborative E-Learning System Based on Multi-agent

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Abstract. Based on the analysis of the process of the collaborative e-learning system and the characteristics of Multi-agent technology, this paper brings forward a collaborative e-learning system framework founded on Multi-agent. Meanwhile, it provides with the key technology to realize e-learning and the arithmetic to search for the collaborator, carries out further analysis of the system's operating mechanism, multi-agent's logic structure, Agent capacity and time complexity. According to this framework, it is easy to realize the Web-based e-learning among several individuals and the knowledge's intertransferring, which is good to improve the e-learning efficiency and lift the knowledge level of the whole internet organization.

1 Introduction

In recent years, e-learning has attracted broad attention in academic and business communities. In fact, e-learning is the use of network technology to design, deliver, select, administer, and extend learning [1]. It is obvious that e-learning is the result of combining the traditional learning styles with variety of modern net technology. E-learning initiatives are expanding in academic and corporate setting driving interest in lifelong learning [3], [4], [5]. At present many international organizations and scholars have attached much importance on the research on e-learning. Thereinto, the educational technology standardization movement has also grown to become a significant force, including such organizations as IMS Global Learning Consortium, IEEE, Dublin Core, ISO, ADL, which are standardizing important base technologies for e-learning applications [6]. Digital technologies have opened new directions for experimentation in the field of learning [7]. At the same time, it has become increasingly evident that the collaborative learning will not be accepting Web technology for agent very quickly, although the potential benefits are many. Additionally, many e-learning applications are highly monolithic and seriously lacking in flexibility [8]. The kind of intelligent computer support enabled by Semantic Web descriptions, such as software agents and self-describing systems, is

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not taken into account in the design [6]. The use of information retrieval techniques in e-learning domains has been limited too, with some work done on using information retrieval techniques to structure the learning material [9].

This paper brings forwards a collaborative e-learning framework based on multi-agent which possesses the characteristics of intelligent mutual cooperation and intelligent search. Through this framework, the learners can use the existing computer network in the organization to obtain the needed knowledge from other individual learners through the UI to lift their own knowledge level.

The rest of this paper is organized as follows. Section 2 describes the process of the collaborative e-learning. Section 3 explores the Multi-agent and its characteristics. Section 4 proposes collaborative e-learning system based on Multi-agent. Conclusions and future work can be found in the last section.

2 Process of the Collaborative E-Learning

Collaborative e-learning is a new type of teaching pattern. It mainly embodies the application of new technologies such as network into collaborative learning. It uses the collaborative environment supported by the computer network [10] to carry out the collaborative learning, in the form of group work, between teachers and students, students and students based on their discussion, cooperation and communication to achieve the learning targets. The collaborative e-learning usually includes:

- composing the learning groups;
- conducting the real-time, synchronous communication, including synchronous BBS or the video conferencing etc;
- collecting, organizing and sharing knowledge, which involves the group members' obtaining relevant knowledge and then offering to the group;
- sharing the existed files resources online and discussing them among the cooperators through e-mail, video conferencing, in addition, editing the sharing files collectively by use of the online note-taking system;
- building an interactive platform for e-learning to let the learners interact and cooperate.

The interactive modes in the process of the collaborative e-learning includes: the interaction between the learners and the contents, the interaction between the learners and the mentors, the interaction between learners [11]. Thereinto, the mentors can be a teacher or a software system. The interaction between learners can be the one between two individuals; also can be the synchronous interaction, such as the interactive discussion among several group members under the collaborative learning, or the asynchronous interaction including BBS, e-mail. The interaction between associates can help learners consolidate their knowledge, check up their hypothesis, prove their choices and attitudes, and send the knowledge which is outdated to some learners but useful to other learners.

The aim of the e-learning is to realize the sharing and efficient using of knowledge. Therefore, the position of the mentor and learner is relative in the learning groups. Suppose learner *A* has done much research on a field and has mastered much knowledge in this field. Accordingly, some knowledge will lose the previous reference value to learner *A*. At the same time, learner *B* is just stepping into this field. And such knowledge is possibly important to him. Thus, this learning group is made up of learner *A* and learner *B*, and *A* acts as the mentor.

In addition, the collaborative learner should have mutual learning target and each learner should obey this target. During the cooperation, each member not only should strictly execute the mutual agreement, but also can understand such mutual target to achieve common understanding and convert to internal demand and self-conscious action. Each cooperation member must make sure of his/her own role in the cooperating and target-realization, and undertake his/her responsibility. In the process of collaborative learning, there is no individual success; there is only the realization of the common target. The process of collaborative learning is also a process for the group members to modulate, improve and advance each other to reach the mutual success. The formation process of the grouping of the collaborative e-learning is shown as figure 1.

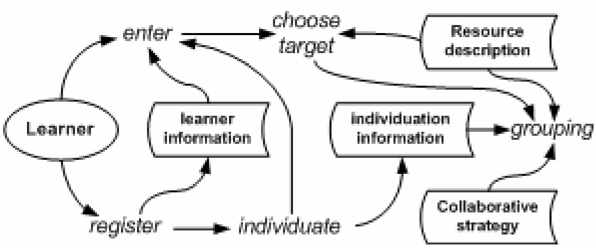

Fig. 1. The formation process of the grouping of the collaborative e-learning

3 Multi-agent and Its Characteristics

There are many network technologies which can support the e-learning, including XML, XML Schema, P2P, and other Web technologies from the W3C and elsewhere. Agent technology, as a kind of intelligentized network, has been applied to e-learning system [12], [13], [14]. Agent technology comes from artificial intelligence: Generally, Agent is considered as an autonomic entity. It can apperceive the environment and give certain judgment and reasoning to outside information to control its decision and action [14], [15], and accomplish some certain tasks.

Agent usually possesses the following features:

- Autonomy: Agent's action and behavior are controlled by its own knowledge, internal state and its apperceiving for external environment; its operation is not interfered by people or other agents.
- Responsibility: Agent can perceive the changes in environment and react to them.
- Social ability: Agent can interact and communicate with other agents or people in some agent collaboration language. In multi-agent system, Agent should have collaborative and negotiatory ability.
- Target-conducting ability: Agent can layout the action steps to realize certain target.

- Intelligence: Agent can reason according to the facts and rules in knowledge base, and the agent operating in the complex environment should also have learning or self-adaptive ability.
- Movability: Agent, as an active entity, can extend across the platform to roam on the internet to help the users collect information, and its state and behavior have continuity.

The system based on multi-agent is one in which multi agents can communicate and collaborate with each other to finish the task together. This system not only has the characteristics of resources sharing, expandability, strong reliability and flexibility that the common distributing system possesses, but also can solve big-scale, complex problems through the negotiation between every agent. Thus, the system possesses strong reliability and self-organization ability. Due to the above-mentioned characteristics, agent technology is so fit to be applied to the e-learning system.

4 Collaborative E-Learning System Based on Multi-agent

The following is a collaborative e-learning system framework based on multi-agent as figure 2 shows. Because the core of e-learning is the collaboration between the learners, the main problem this paper needs to solve is the cooperation between the learners, that is, when *learner agent* representing the learners doesn't clearly show the destination, it only needs to bring forward the demanding knowledge; then the *controlling agent* searches for the collaborative agent which has the same demanding *learner agent*, and *teacher agent* to build the collaborative group; the learners can obtain organizational knowledge or the other collaborative learners' knowledge through UI, and can realize the e-learning through direct learning or communication with others (such as e-mail, discussion online or video communication and so on). The following are the general hypothesis of framework:

- Only considering the main agents: *learner agent*, *teacher agent* and *controlling agent*. The function of *teacher agent* is to tell the *controlling agent* what knowledge can be offered, to provide *learner agent* with knowledge, and to answer questions from *learner agent* in real time. The function of the *learner agent* is to request *controlling agent* some knowledge, to form a group with the *learner agent* who has the same target and to build connection with the corresponding *teacher agent* to go on communication. The *controlling agent* is divided into *attempering agent*, *local agent* and *classifying agent*. The function of *attempering agent* is to manage the linear list (In this paper, a linear list is a finite sequence of agents.) and offer the search for the overall knowledge; *local agent* provides the cross-type search of different knowledge for all agents in its district; *classifying agent* is in charge of the communicating between *learner agents* in the same group and the communicating between *learner agents* and *teacher agents*. Thus, the function of the *controlling agent* is search *teacher agent* for *learner agents* and build collaborative group, which embodies the agent's autonomy.
- Any *teacher agent* only offers limited types of related knowledge.

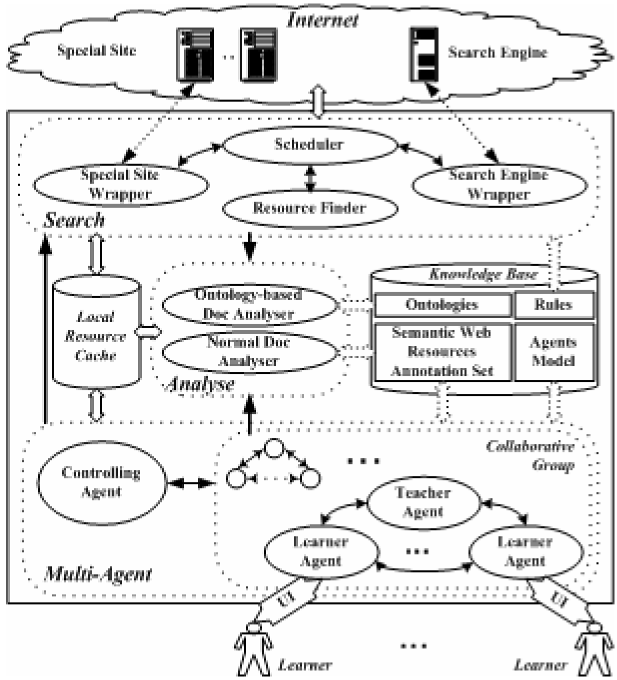

Fig. 2. The collaborative e-learning system framework based on multi-agent

4.1 Operating Mechanism of the Collaborative E-Learning System

The operating mechanism of the collaborative e-learning system is: the learner will be one user after registering and logging up. The system will send the learner's obvious learning request and connotative interests to *learner agent* through UI. *Controlling agent* makes decisions according to the information in existing Agents Model, the state of *teacher agent* and other *learner agents* and the information store in the information resources base of the semantic network; moreover, it will also send the decision to *learner agent*. Under necessary conditions, the analysis module and the information collecting module of the net resource will be activated. The collecting module will obtain the original information from the search engine according to the scheduled requests, analyze the information resources of internet using special arithmetic according to the rule definition of knowledge base, and then collect the potentially useful knowledge. The intelligent analysis module uses ontology module and related arithmetic in knowledge base to filter, analyze, take out and understand the collected information, and then put the processed information into knowledge base. Meanwhile, analysis module brings up some evolvement suggestions for ontology according to the analysis results. The knowledge base management module evolves ontology, renews the resources on time, carries out the basic check upon the resources and renews and maintains Agents Model through the communication with the outside. Through *controlling agent*'s coordinating, *learner agent* achieves the connection between *learner agents* and *teacher agents*, builds collaborative learning group, and finally realizes collaborative e-learning.

4.2 Logic Structure of Multi-agent

The multi-agent structure, which combines the linear list and hierarchy, is adopted according to the commonness of knowledge structure of the collaborative learning and the characteristics of the knowledge classification, as figure 3 shows. The linear lists are classified according to the districts. A district is corresponding to a one-dimension linear list and each linear list contains some *local agent*s. However, *local agent* cannot connect with *teacher agent* and *learner agent*. The system sets *attempering agent* to build collaborative learning group for *learner agents*. Furthermore, when the system searches *teacher agents*, the *attempering agent* is asked to point out the current search linear list. According to the knowledge contents, the hierarchy classifies *teacher agents* grade by grade that is expressed as tree, and whose node is *classifying agent*. The lower the grade, the more detailed the knowledge classification. *Teacher agent* is leave node, and it doesn't have classifying property. However, it can connect with the corresponding *classifying agent* according to the provided information. *Learner agent* adopts federation structure, but it must connect with one *classifying agent* which is fit for its learning target. At the same time, as leave node, it often connects with *classifying agent* by the means of handy combination. Several *learner agents* who have the same learning target make up of the collaborative learning group.

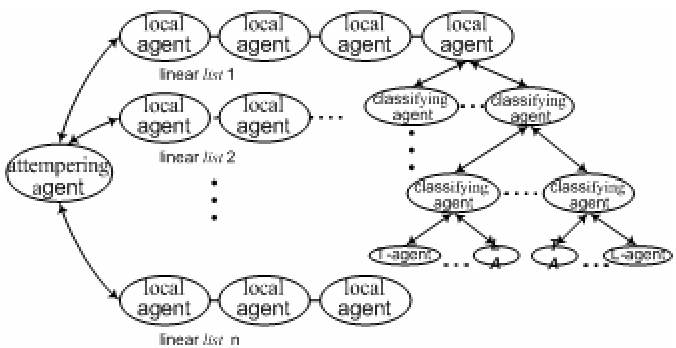

Fig. 3. The hierarchy structure of multi-agent

4.3 Capacity of Agent

The capacity of agent refers to the service one agent can offer to the other agents. The capacity knowledge is adopted to describe the capacity of agent. It comes into being with the emerging of the agent. Capacity knowledge is the base for the multi-agent's connection in the collaborative e-learning system and contains four expandable properties: linear list mark, agent type mark, knowledge classifying mark and agent number. Linear list mark can be expanded to linear list name and the *local agent* name it belongs to. Agent type mark refers to the agent type the agent belongs to. Classifying mark illustrates the type of target knowledge, and implies the middle node from *local agent* to *teacher agent*. Agent number is used to distinguish the different *learner agent* in the same knowledge classification or target. In the collaborative e-learning system, the Agents which have the same knowledge are not the exclusive, but their capacity knowledge should be the only. This exclusiveness makes it possible for *learner agent* to join in the group according to learning target and choose the suitable *teacher agent*.

The whole learning process begins with *learner agent*. And *controlling agent* connects it to one *classifying agent* according to its capacity knowledge. *Classifying agent* picks up its capacity knowledge, analyzes its capacity and judges whether the corresponding *teacher agent* is local connection. If it is not, *classifying agent* will upload this *learner agent*'s request to the upper *classifying agent*. The upper *classifying agent*s will do the similar things in turn as the lower Agent until some upper *classifying agent* can fix a *teacher agent* and send the learning request. Then the *teacher agent* reacts to the request to send knowledge. There is at least one *teacher agent* in each learning group. The other *learner agents* who have the same target can repeat the above process to join the group at any time.

4.4 Search Arithmetic and the Analysis of Time Complexity

In the above system, the time that the learners need to join in one collaborative learning group will affect learners' interest in online learning. Therefore, there is a need of analysis of the arithmetic to the search for *teacher agent* and other agents and the time complexity of arithmetic in order to affirm the rationality of the arithmetic. During the analysis, the search strategy should be first fixed. When *controlling Agent* receives a joining request, it should first analyze the classifying information of this request, and then go on the search for the collaborative learning. There are some conditions for such search:

- If it relates the learning group, then the *classifying agent* can join in the group;
- If it doesn't, then sends the search request to the upper *classifying agent* grade by grade;
- If the search in the district fails, then sends request to the *attempering agent*. *Attempering agent* points out the next *local agent* to search according to the priority rule;
- If the search fails in the same linear list district, then it is up for *attempering agent* to point out the next linear list to search until all the linear lists are searched.

During the searching, any search succeeds, and the followings can be canceled. The result of searching is the *learner agent* joining a group which has the same target and starting collaborative e-learning.

To analyze search arithmetic and its time complexity, further rational predigested is needed. Suppose there is a group of *teacher agent* under a *classifying agent* and replaced by a leave node. Then, when the search comes to the *classifying agent* who directly connects to the leave node, the search result to the branch is clearly shown. The arithmetic for searching the leave node can be described as following:

```
while (Is there any unsearched linear lists?) {
Search in the next linear list;
while (Is there any unsearched node in the linear list?) {
Search the leave node of the tree;
if (succeeds in searching?) then jumps out of all circles;
}
}
```

While analyzing the time complexity of the arithmetic, the process of uploading grade by grade can be omitted; and the postponed time of sending messages between the agents needn't be considered. Therefore, *the time complexity of searching* = *the time for searching linear list* × *searching time for tree*.

For *attempering agent*, several linear lists are equal to a linear list, which can be combined by all the linear lists. Suppose the linear list *attempering agent* uses is l, and there are n nodes, x is any a given node showing the *local agent* the needing *teacher agent* locates. Classifying according to the positions which x appears in l, then the condition of n+1 may emerge. Suppose the probability of x in l is q, and suppose the probability of x in every position of l is the same, the average time complexity of linear list is

$$A(n) = q \times (n+1)/2 + (1-q) \times n \,. \tag{1}$$

When $q = 0$, the complexity under the worst condition is:

$$W(n) = n \,. \tag{2}$$

The tree here is an irregular tree, showing that the knowledge classification in the existing network is limited, and the search for the tree can be finished within an upper time limit. So the main factors that can affect searching time should be the number of the nodes in the linear list, and the number of the nodes in the linear list is the only possible part that increases in big scope. Thus, it can be believed that the time complexity of searching is the same as the linear list and both are ***O(n)*** in the practical application. Furthermore, in the certain domains, it is impossible for the increasing speed and scale of the overall node to inflate exceedingly quickly. In this sense, this system possesses strong practicability.

The particular situation of this system which the above-mentioned framework cannot be said more because of the limited paper space.

5 Conclusions and Future Work

This paper applies multi-agent technology to the e-learning, and proposes a collaborative e-learning framework based on multi-agent. However, the research work in this paper mainly focuses on technical side. It doesn't deep explore the motivation of users who participate in collaborative e-learning. For example, how to stay and keep users active when they always provide more knowledge than they can get? Furthermore, since e-learning is a completely new research field with the development of internet, it is still at the beginning stage, whether in idea or in the technology. Therefore, the proposed system in this paper still needs the learners to lift their own self-restriction ability to avoid getting lost in information and information's overloading and strengthen their distinguishing ability to systematize their learning.

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A Class of Possibilistic Portfolio Selection Models and Algorithms

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Abstract. In this paper, a crisp possibilistic variance and a crisp possibilistic covariance of fuzzy numbers are defined, which is different from the ones introduced by Carlsson and Fullér. The possibilistic portfolio selection model is presented on the basis of the possibilistic mean and variance under the assumption that the returns of assets are fuzzy numbers. Especially, Markowitz's probabilistic mean-variance model is replaced a linear programming model when the returns of assets are symmetric fuzzy numbers. The possibilistic efficient frontier can be derived explicitly when short sales are not allowed on all risky assets and a risk-free asset.

1 Introduction

The mean-variance methodology for the portfolio selection problem, posed originally by Markowitz [1], has played an important role in the development of modern portfolio selection theory. It combines probability and optimization techniques to model the behavior investment under uncertainty. The return is measured by mean, and the risky is measured by variance, of a portfolio of assets. In Markowitz's mean-variance model for portfolio selection, it is necessary to estimate the probability distribution, strictly speaking, a mean vector and a covariance matrix. It means that all mean returns, variances, covariances of risky assets can be accurately estimated by an investor. The basic assumption for using Markowitz's mean-variance model is that the situation of asset markets in future can be correctly reflected by asset data in the past, that is, the mean and covariance of assets in future is similar to the past one. It is hard to ensure this kind of assumption for real ever-changing asset markets. It would be very difficult to obtain the explicit solution of efficient portfolio under general constraints such as non-negativity constraints on correlated assets (precluding short sales)(see Markowitz [1], Perold [2], Pang [3], VÖRÖS [4] and Best [5], etc..).

It is well-known that the returns of risky assets are in a fuzzy uncertain economic environment and vary from time to time, so the future states of returns

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and risks of risky assets cannot be predicted accurately. Recently, a few of authors such as Watada [6], Tanaka and Guo [7,8], Wang and Zhu [11] etc., studied the fuzzy portfolio selection problem. Watada [6] presented portfolio selection models using fuzzy decision theory. Tanaka and Guo [7,8] proposed the portfolio selection models based on fuzzy probabilities and possibilistic distributions. Zhang [12] introduced the admissible efficient portfolio model under the assumption that the expected return and risk of asset have admissible errors. Zhang and Wang [13] discussed the general weighted possibilistic portfolio selection problem.

In this paper, we consider the portfolio selection problem based on a new crisp possibilistic variance and a new crisp possibilistic covariance of fuzzy numbers. We present a new possibilistic mean-variance model for portfolio selection. Especially, we obtain a linear programming model when the returns of assets are symmetric fuzzy numbers. The possibilistic efficient portfolios can be easily obtained when short sales are not allowed on all risky assets and there exists a risk-free investment.

2 Possibilistic Mean and Possibilistic Variance

A fuzzy number A is a fuzzy set of the real line $\mathcal{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by $\mathcal{F}$.

Let A be a fuzzy number with $\gamma-$ level set $[A]^\gamma = [a(\gamma), b(\gamma)](\gamma > 0)$.

Carlsson and Fullér [10] defined the lower and upper possibilistic mean values of A as

$$M^L(A) = \int_0^1 2\gamma a(\gamma)d\gamma = \frac{\int_0^1 a(\gamma)Pos[A \le a(\gamma)]d\gamma}{\int_0^1 Pos[A \le a(\gamma)]d\gamma},$$

$$M^U(A) = \int_0^1 2\gamma b(\gamma)d\gamma = \frac{\int_0^1 b(\gamma)Pos[A \ge b(\gamma)]d\gamma}{\int_0^1 Pos[A \ge b(\gamma)]d\gamma},$$

where

$$Pos[A \le a(\gamma)] = \Pi((-\infty, a(\gamma)]) = \sup_{u \le a(\gamma)} A(u) = \gamma,$$
$$Pos[A \ge b(\gamma)] = \Pi([b(\gamma), +\infty]) = \sup_{u \ge b(\gamma)} A(u) = \gamma.$$

Moreover, Carlsson and Fullér [10] also defined the crisp possibilistic mean value of A as

$$\overline{M}(A) = \int_0^1 \gamma[a(\gamma) + b(\gamma)]d\gamma = \frac{M^L(A) + M^U(A)}{2},$$

According to Carlsson and Fullér [10], we easily get the following conclusion.

Theorem 2.1. *Let $A_1, \dots, A_n$ be n fuzzy numbers, and let $\lambda_0, \lambda_1, \dots, \lambda_n$ be $n+1$ real numbers. Then*

$$\overline{M}(\sum_{i=1}^n \lambda_i A_i + \lambda_0) = \lambda_0 + \sum_{i=1}^n \lambda_i \overline{M}(A_i),$$

where the addition of fuzzy numbers and the multiplication by a scalar of fuzzy number are defined by the sup-min extension principle [9].

Carlsson and Fullér [10] defined the crisp possibilistic variance and covariance of fuzzy numbers as

$$Var(A) = \frac{1}{2}\int_0^1 \gamma[b_1(\gamma) - a_1(\gamma)]^2 d\gamma$$

and

$$Cov(A,B) = \frac{1}{2}\int_0^1 \gamma[b_1(\gamma) - a_1(\gamma)][b_2(\gamma) - a_2(\gamma)]d\gamma,$$

where $[A]^\gamma = [a_1(\gamma), b_1(\gamma)]$ and $[B]^\gamma = [a_2(\gamma), b_2(\gamma)]$.

Remark 2.1. Clearly, for any fuzzy numbers $A, B \in \mathcal{F}$ and real numbers $\lambda, \mu \in \mathcal{F}$ it follows that

$$Cov(A,B) \geq 0$$

and

$$Var(\lambda A + \mu B) = \lambda^2 Var(A) + \mu^2 Var(B) + 2|\lambda\mu| Cov(A,B)d\gamma$$

$$\geq \lambda^2 Var(A) + \mu^2 Var(B),$$

which is not consistent with probability theory.

We introduce a new crisp possibilistic variance and a new crisp possibilistic covariance as follows.

Definition 2.1. *The possibilistic variance of A with $[A]^\gamma = [a(\gamma), b(\gamma)]$ is defined as*

$$\overline{Var}(A) = \int_0^1 \gamma([M^L(A) - a(\gamma)]^2 + [M^U(A) - b(\gamma)]^2)d\gamma.$$

Definition 2.2. *The possibilistic covariance of $A, B \in \mathcal{F}$ is defined as* $\overline{Cov}(A,B)$

$$= \int_0^1 \gamma[(M^L(A) - a_1(\gamma))(M^L(B) - a_2(\gamma)) + (M^U(A) - b_1(\gamma))(M^U(B) - b_2(\gamma))]d\gamma$$

where $[A]^\gamma = [a_1(\gamma), b_1(\gamma)]$ and $[B]^\gamma = [a_2(\gamma), b_2(\gamma)]$.

The following theorems show properties of the possibilistic variance.

Theorem 2.2. *Let A and B be two fuzzy numbers and let λ, μ and $\lambda_0 \in \mathcal{R}$. Then*

$$\overline{Var}(\lambda A + \mu B + \lambda_0) = \lambda^2 \overline{Var}(A) + \mu^2 \overline{Var}(B) + 2|\lambda\mu| \overline{Cov}(\phi(\lambda)A, \phi(\mu)B),$$

where $\phi(x)$ is a sign function of $x \in \mathcal{R}$.

Proof. Let $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ and $[B]^\gamma = [b_1(\gamma), b_2(\gamma)], \gamma \in [0,1]$. Suppose $\lambda < 0$ and $\mu < 0$. Then

$$[\lambda A + \mu B + \lambda_0]^\gamma = [\lambda a_2(\gamma) + \mu b_2(\gamma) + \lambda_0, \lambda a_1(\gamma) + \mu b_1(\gamma) + \lambda_0].$$

We obtain

$$\begin{aligned}\overline{Var}(\lambda A+\mu B+\lambda_0) &= \int_0^1 \gamma(M^L(\lambda A+\mu B+\lambda_0)-\lambda a_2(\gamma)-\mu b_2(\gamma)-\lambda_0)^2 d\gamma+\\ &\quad \int_0^1 \gamma(M^U(\lambda A+\mu B+\lambda_0)-\lambda a_1(\gamma)-\mu b_1(\gamma)-\lambda_0)^2 d\gamma\\ &= \int_0^1 \gamma(\lambda M^U(A)+\mu M^U(B)-\lambda a_2(\gamma)-\mu b_2(\gamma))^2 d\gamma+\\ &\quad \int_0^1 \gamma(\lambda M^L(A)+\mu M^L(B)-\lambda a_1(\gamma)-\mu b_1(\gamma))^2 d\gamma\\ &= \lambda^2\overline{Var}(A)+\mu^2\overline{Var}(B)+2\lambda\mu\overline{Cov}(-A,-B),\end{aligned}$$

that is,

$$\overline{Var}(\lambda A+\mu B+\lambda_0)=\lambda^2\overline{Var}(A)+\mu^2\overline{Var}(B)+2|\lambda\mu|\overline{Cov}(\phi(\lambda)A,\phi(\mu)B),$$

where

$$\phi(x)=\begin{cases}1 & \text{if } x>0,\\ 0 & \text{if } x=0,\\ -1 & \text{if } x<0.\end{cases}$$

Similar reasoning holds for the case $\lambda\geq 0$ and $\mu\geq 0$.

Suppose now that $\lambda>0$ and $\mu<0$. Then

$$[\lambda A+\mu B+\lambda_0]^\gamma=[\lambda a_1(\gamma)+\mu b_2(\gamma)+\lambda_0,\lambda a_2(\gamma)+\mu b_1(\gamma)+\lambda_0].$$

We get

$$\begin{aligned}\overline{Var}(\lambda A+\mu B+\lambda_0) &= \int_0^1 \gamma(M^L(\lambda A+\mu B+\lambda_0)-\lambda a_1(\gamma)-\mu b_2(\gamma)-\lambda_0)^2 d\gamma+\\ &\quad \int_0^1 \gamma(M^U(\lambda A+\mu B+\lambda_0)-\lambda a_2(\gamma)-\mu b_1(\gamma)-\lambda_0)^2 d\gamma\\ &= \int_0^1 \gamma(\lambda M^L(A)+\mu M^U(B)-\lambda a_2(\gamma)-\mu b_2(\gamma))^2 d\gamma+\\ &\quad \int_0^1 \gamma(\lambda M^U(A)+\mu M^L(B)-\lambda a_1(\gamma)-\mu b_1(\gamma))^2 d\gamma\\ &= \lambda^2\overline{Var}(A)+\mu^2\overline{Var}(B)+2|\lambda\mu|\overline{Cov}(\phi(\lambda)A,\phi(\mu)B).\end{aligned}$$

Similar reasoning holds for the case $\lambda<0$ and $\mu\geq 0$,
which ends the proof.

Theorem 2.3. *Let $A_1,\ldots,A_n$ be n fuzzy numbers, and let $\lambda_0,\lambda_1,\ldots,\lambda_n$ be $n+1$ real numbers. Then*

$$\overline{Var}(\sum_{i=1}^n \lambda_i A_i+\lambda_0)=\sum_{i=1}^n \lambda_i^2\overline{Var}(A_i)+2\sum_{i>j=1}^n|\lambda_i\lambda_j|\overline{Cov}(\phi(\lambda_i)A_i,\phi(\lambda_j)A_j),$$

where $\phi(x)$ is a sign function of $x\in\mathcal{R}$.

Proof. From the proof of Theorem 2.2, the conclusion holds obviously.

Let $A_1, \ldots, A_n$ be n fuzzy numbers and let $c_{ij} = \overline{Cov}(A_i, A_j), i, j = 1, \ldots, n$.

Then the matrix

$$\overline{\mathbf{Cov}} = (c_{ij})_{n \times n}$$

is called as the possibilistic covariance matrix of fuzzy vector $(A_1, A_2, \ldots, A_n)$.

We can prove that the possibilistic covariance matrix has the same properties as the covariance matrix in probability theory.

Theorem 2.4. $\overline{\mathbf{Cov}}$ *is a nonnegative definite matrix.*

Proof. From the definitions of possibilistic covariance, it follows that

$$\overline{Cov}(A_i, A_j) = \overline{Cov}(A_j, A_i), i, j = 1, \ldots, n.$$

Therefore, $\overline{\mathbf{Cov}}$ is a real symmetric matrix.

Especially,

$$c_{ii} = \overline{Cov}(A_i, A_i) = \overline{Var}(A_i), i = 1, 2, \ldots, n.$$

Let $[A_i]^\gamma = [a_i(\gamma), b_i(\gamma)], i = 1, \ldots, n.$

For any $t_i \in \mathcal{R}(i = 1, \ldots, n)$,

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} t_i t_j$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \int_0^1 \gamma([M^L(A_i) - a_i][M^L(A_j) - a_j] + [M^U(A_i) - b_i][M^U(A_j) - b_j]) t_i t_j d\gamma$$

$$= \int_0^1 \gamma[\sum_{i=1}^{n} t_i (M^L(A_i) - a_i(\gamma))]^2 d\gamma + \int_0^1 \gamma[\sum_{i=1}^{n} t_i (M^U(A_i) - b_i(\gamma))]^2 d\gamma \geq 0.$$

Hence, $\overline{\mathbf{Cov}}$ is the nonnegative definite matrix.

This concludes the proof of the theorem.

3 Possibilistic Portfolio Selection Model

We consider a portfolio selection problem with n risky assets and a risk-free asset in this paper. Here, the return rate r_j for risky asset j is considered a fuzzy number, $j = 1, 2, \ldots, n$. Let r_0 be the return rate of risk-free asset and x_j be the return rate of the risky asset j. Then the return associated with a portfolio $(x_1, x_2, \ldots, x_n)$ is

$$r = \sum_{i=1}^{n} x_i r_i + r_0(1 - \sum_{i=1}^{n} x_i).$$

From Theorems 2.1, the possibilistic mean value of r is given by

$$\overline{M}(r) = \sum_{i=1}^{n} \overline{M}(x_i r_i + r_0(1 - \sum_{i=1}^{n} x_i)) = \sum_{i=1}^{n} x_i \overline{M}(r_i) + r_0(1 - \sum_{i=1}^{n} x_i).$$

From Theorems 2.2, the possibilistic variance of r is given by

$$\overline{Var}(r) = \sum_{i=1}^{n} x_i^2 \overline{Var}(r_i) + 2 \sum_{i>j=1}^{n} x_i x_j \overline{Cov}(r_i, r_j).$$

In order to describe conveniently, we introduce the following notations:

$$\begin{array}{l} \mathbf{x} = (x_1, x_2, \ldots, x_n)', \\ \mathbf{r} = (r_1, r_2, \ldots, r_n)', \\ \mathbf{F} = (1, 1, \ldots, 1)', \\ \mathbf{M} = (\overline{M}(r_1), \overline{M}(r_2), \ldots, \overline{M}(r_n))', \\ \mathbf{C} = (\overline{Cov}(r_i, r_j))_{n \times n}. \end{array}$$

$\mathbf{M}$ is the possibilistic mean vector, $\mathbf{C}$ is the possibilistic covariance matrix.

Thus, the possibilistic mean value of r is rewritten as

$$\overline{M}(r) = \mathbf{M}'\mathbf{x} + r_0(1 - \mathbf{F}'\mathbf{x}). \tag{1}$$

The possibilistic variance of r is rewritten as

$$\overline{Var}(r) = \mathbf{x}'\mathbf{C}\mathbf{x}. \tag{2}$$

Analogous to Markowitz's mean-variance methodology for the portfolio selection problem, the possibilistic mean value correspond to the return, while the possibilistic variance correspond to the risk. From this point of view, the possibilistic mean-variance model of portfolio selection can be formulated as

$$\begin{array}{ll} \min & \mathbf{x}'\mathbf{C}\mathbf{x} \\ s.t. & \mathbf{M}'\mathbf{x} + r_0(1 - \mathbf{F}'\mathbf{x}) \geq \mu, \\ & \mathbf{F}'\mathbf{x} = 1, \quad \mathbf{x} \geq \mathbf{0}. \end{array} \tag{3}$$

From [13], the optimal solution of (3), $\mathbf{x}^*$, is called as the possibilistic efficient portfolio. The possibilistic efficient frontier can be obtained by solving (3) for all possible μ.

Let $r_i = (a_i, \alpha_i, \beta_i), i = 1, \ldots, n$ be triangular fuzzy numbers with center a_i, left-width $\alpha_i > 0$ and right-width $\beta_i > 0$.

Then a $\gamma-$level of r_i is computed by

$$[r_i]^\gamma = [a_i - (1-\gamma)\alpha_i, a_i + (1-\gamma)\beta_i],$$

for all $\gamma \in [0, 1], i = 1, \ldots, n$,
and therefore,

$$M^L(r_i) = a_i - \frac{\alpha_i}{3}, M^U(r_i) = a_i + \frac{\beta_i}{3},$$

$$\overline{M}(r_i) = a_i - \frac{\alpha_i}{6} + \frac{\beta_i}{6},$$

$$\overline{Var}(r_i) = \int_0^1 \gamma[(\frac{2}{3}\alpha_i - \gamma\alpha_i)^2 + (-\frac{2}{3}\beta_i + \gamma\beta_i)^2] d\gamma = \frac{\alpha_i^2 + \beta_i^2}{36},$$

$$\overline{Cov}(r_i, r_j) = \frac{\alpha_i\alpha_j + \beta_i\beta_j}{36}.$$

Then the possibilistic mean value of the return associated with the portfolio **x** is given by

$$\overline{M}(r) = \sum_{i=1}^{n}(a_i + \frac{\beta_i - \alpha_i}{6})x_i + r_0(1 - \sum_{i=1}^{n} x_i)$$

The possibilistic variance of the return associated with the portfolio **x** is given by

$$\overline{Var}(r) = \frac{1}{36}[\sum_{i=1}^{n}(\alpha_i^2 + \beta_i^2)x_i^2 + \sum_{i\neq j=1}^{n}(\alpha_i\alpha_j + \beta_i\beta_j)x_i x_j].$$

Thus, the possibilistic mean-variance model of portfolio selection problem may be described by

$$\begin{aligned} \min \quad & \overline{Var}(r) = \sum_{i=1}^{n}(\alpha_i^2 + \beta_i^2)x_i^2 + \sum_{i\neq j=1}^{n}(\alpha_i\alpha_j + \beta_i\beta_j)x_i x_j \\ s.t. \quad & \sum_{i=1}^{n}(a_i + \tfrac{\beta_i - \alpha_i}{6} - r_0)x_i \geq \mu - r_0, \\ & \sum_{i=1}^{n} x_i = 1, \\ & x_i \geq 0, i = 1, \ldots, n. \end{aligned} \tag{4}$$

The model (4) contains $3n + 1$ unknown parameters, but the conventional probabilistic mean-variance model contains $n^2 + 3n + 2/2$ unknown parameters. Clearly, the unknown parameters are greatly decreased to compare the model (4) with conventional probabilistic mean-variance methodology.

Using the algorithms in [13] to (4) for all possible μ, the possibilistic efficient frontier is derived explicitly.

Especially, if $r_i = (a_i, \alpha_i), i = 1, \ldots, n$ are symmetric triangular fuzzy numbers, that is $\alpha_i = \beta_i$, then

$$\overline{M}(r) = \sum_{i=1}^{n} a_i x_i + r_0(1 - \sum_{i=1}^{n} x_i),$$

$$\begin{aligned} \overline{Var}(r) &= [\sum_{i=1}^{n} \alpha_i^2 x_i^2 + \sum_{i\neq j=1}^{n} \alpha_i\alpha_j x_i x_j]/18 \\ &= [\sum_{i=1}^{n} \alpha_i x_i]^2/18. \end{aligned}$$

The possibilistic mean-variance model (4) is equal to the following linear programming:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \alpha_i x_i \\ s.t. \quad & \sum_{i=1}^{n} (a_i - r_0) x_i \geq \mu - r_0, \\ & \sum_{i=1}^{n} x_i = 1, \\ & x_i \geq 0, i = 1, \ldots, n. \end{aligned} \tag{5}$$

The possibilistic mean-variance model of portfolio selection problems (5) is simple linear programming model, so the possibilistic efficient portfolios are easily obtained by some related algorithms for solving linear programming problem.

4 Conclusions

Fuzzy number is a powerful tool used to describe an uncertain environment with vagueness and ambiguity. In this paper, we have defined a new crisp possibilistic variance and a new crisp possibilistic covariance of fuzzy numbers. We have discussed the portfolio selection problem which returns of assets are fuzzy numbers. We have obtained the linear programming model replaced Markowitz's mean-variance model when returns of assets are triangular fuzzy numbers. The possibilistic efficient portfolios can be easily obtained by some related algorithms for solving linear programming problem.

Acknowledgements

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An Empirical Study of Volatility Predictions: Stock Market Analysis Using Neural Networks

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Abstract. Volatility is one of the major factor that causes uncertainty in short term stock market movement. Empirical studies based on stock market data analysis were conducted to forecast the volatility for the implementation and evaluation of statistical models with neural network analysis. The model for prediction of Stock Exchange short term analysis uses neural networks for digital signal processing of filter bank computation. Our study shows that in the set of four stocks monitored, the model based on moving average analysis provides reasonably accurate volatility forecasts for a range of fifteen to twenty trading days.

1 Introduction

Variations in stock market return and influence from economic data can cause notable fluctuation in short term stock performance. Prediction of volatility in the stock market is therefore an important issue in maximizing the probability of accurately modeling the movement of stocks in a short term basis. Such model must take into consideration any random activities that may affect the accuracy of prediction [1]. An experimental model based on [2] that evaluates the credibility has been developed for the forecast of volatility. The model focuses on temporal sequence processing with minimum mean-squared error (MMSE) adaptive filtering algorithm is developed for stock market prediction using probabilistic neural network.[3], [4], [5]. The temporal dependencies are modeled with econometric algorithm in [6].

The main objective of this model is to study the volatility of stock market performance within less than one month of trading, by the evaluation of four stocks listed in the New Zealand Stock Exchange (NZSX). This model compares the stocks to identify consistence and difference in volatility of the same period of time.

This paper is organized as follows: Section 2 outlines the neural network model and the algorithms associated with the model. Section 3 discusses the training sequence of the neural network followed by result analysis in Section 4. Finally, we conclude the experimental results in Section 5.

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2 Neural Networks

In general terms, neural networks differentiate subtle pattern of changes as they analyze captured data and to learn as new patterns emerge. While neural networks have been widely used in financial analysis for many years [7], [8], a number of design options are available offering different degrees of performance under different conditions. The model that we propose is based on a linear optimal model as described below.

2.1 Linear Optimal Model

The neural network is modeled as a linear time invariant (LTI) system as:

$$x_{n+1} = A.x_n + .B.y_n \tag{1}$$

where x and y can be expressed as an n-dimensional vector such that:

$$x_n = (.x_1 , x_2 , \ldots , x_n)^T \tag{2}$$

The superscript (T) denotes the transpose of an (n x n) matrix that determines the constants A and B. The neural network used is a multi-layered neural network based model that maps the observations into inputs with the mappings realized by the network that has been trained to obtain the necessary response [9]. The MMSE filter bank proposed in [10] is used for providing synaptic connections with the weight factor as described as x in eq. (2). The weight x, which is obtained by using the filter shown in Fig. 1, represents the vector of signals conveyed between the neurons. The neuron of each layer is illustrated in Fig. 2, where the operation of each layer's node is expressed as:

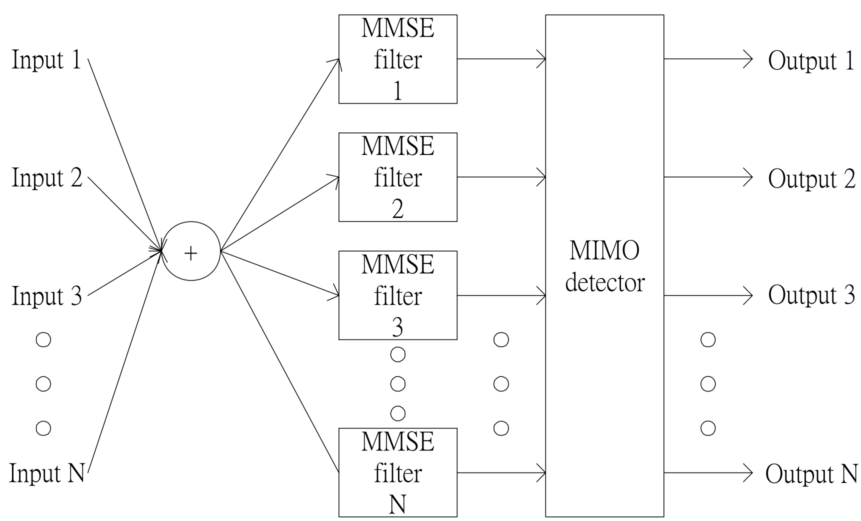

Fig. 1. Filter bank for weight calculations

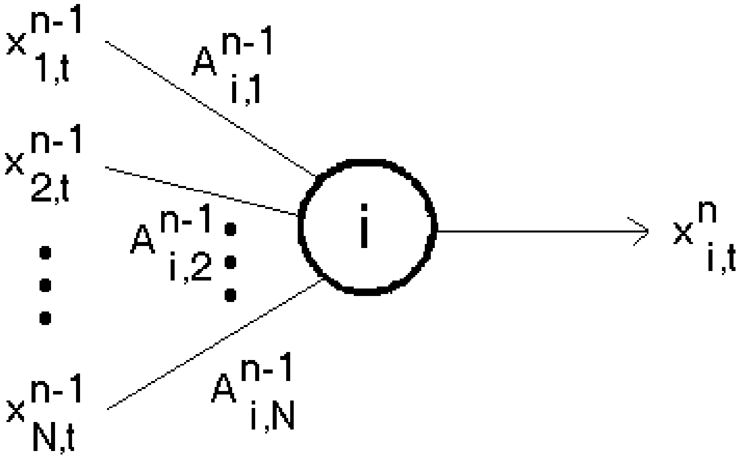

Fig. 2. Neuron

$$z_{i,t}^{n} = \sum A_{i,j}^{n-1}.x_{j,t}^{n-1} \tag{3}$$

$$A_{i,t}^{n} = \theta_{i}^{n+1}, x_{N,t}^{n} = 1\{n : 1 \le n \le N-1\} \tag{4}$$

2.2 Process

The neural network is aimed at performing four processes in a sequential for modeling the volatility analysis by providing a logical description of temporal dependencies of any changes in statistical regularity detected during the desired time frame.

Error Estimation. An error function E(x), based on the training sequence of the neural networks, is deduced with estimation e so that:

$$E_{i}^{n}(x) = \frac{1}{2}(x_i - e_i)^T.(x_i - e_{i)} \tag{5}$$

Data Range Selection. To define the time frame of interest. It is observed, from results presented in Section 4, that the model gives most accurate estimation for between 15-20 days.

Input Definition. Model inputs are selected once the range is known. External parameters that may influence the market are selected as indicators. These are deduced by exponential smoothing for forecast F at m period in advance:

$$F_m = (S_t + m.b_t).I_{t+m-L} \tag{6}$$

where S is the smoothed observation with respect to the index t denoting a given time period. b is the trend factor and I is the seasonal factor and L is the period of season. The trend factor b as a function of observation y is defined by:

$$b = \frac{1}{L}\sum_{i=1}^{L}\frac{y_{L+i} - y_i}{L}$$

Rules Selection. Model rules are defined by evaluating the model forecasts from the indicators and observations. This is then applied to historical data to evaluate the following parameters: correlation, mean absolute and relative error, and mean squared error.

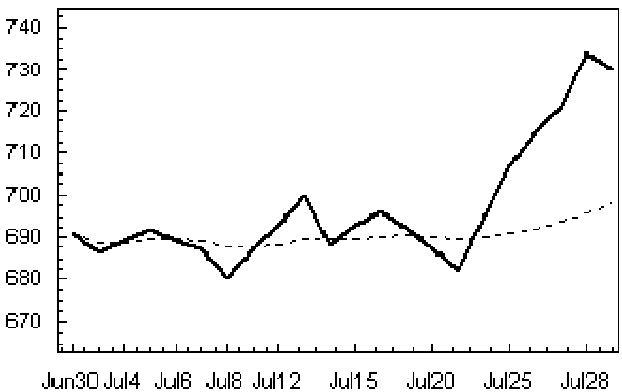

Fig. 3. Moving average: one month

2.3 Moving Average

Moving average charts plotted with closing price within a specific time frame are widely used for financial analysis. Their primary objective is to smooth a set data to facilitate identifying trends, which is found to be particularly effective when analyzing volatile movements.

Exponential moving average (EMA) is used to reduce the lag in moving average since these moving averages are lagging indicators as they are obtained based on previous closings of data. Here, weights are applied to data acquired from later days over a specific period-based calculation given by:

$$EMA = W.(Pc - EMAp) + EMAp \quad (7)$$

where $W = \frac{2}{1+N}$

The current EMA is obtained from closing price of previous trading day Pc and previous EMA (EMAp), with the weight W applied such that W is a function determined by the number of period N. W is calculated by using the network shown in Fig. 1 with an N-weight FIR filter. Data collected within the specified period is

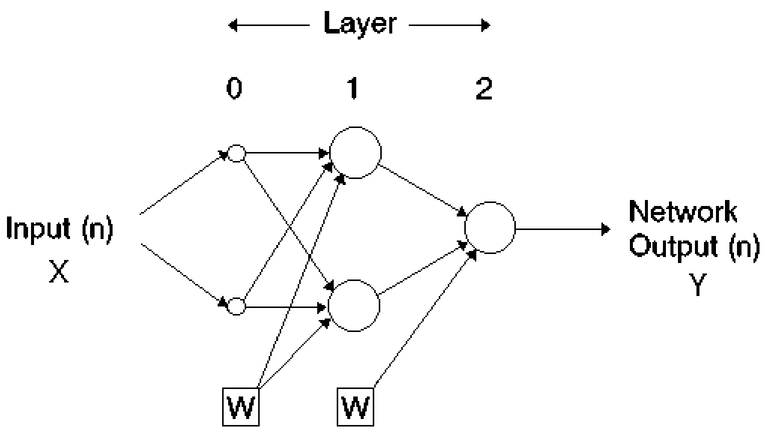

Fig. 4. Network architecture

filtered and sampled. Each set of data is multiplied by the complex weight followed by a summation to obtain the current EMA as shown in Fig. 3 with a 20-day period as:

$$EMA = \frac{1}{N}\sum_{p=1}^{N} | EMA_p |. \tag{8}$$

3 Training Sequence

A feed-forward neural network is used for training [11]. Constructed with the node illustrated in Fig. 2, the network is derived as shown in Fig. 4. In this architecture, layers 0, 1, and 2 represent the input, pattern, and summation, respectively. The weight W is applied to all layers except the output. The feed-forward network is trained using a set of patterns derived the training set based on EMAp with supervised learning. The pattern has N number of elements which forms the number of input nodes. The training algorithm with temporal back propagation derived from the principle proposed by [12] reduces the total network errors by iteratively adjusting the weight W from eq. (7). The pattern node output Y is obtained by training pattern P of input X as:

$$Y = e^{-\frac{(P-X)^T.(P-X)}{2o^2}} \tag{9}$$

Where σ is the smoothing function calculated by the procedure described in [13].

4 Evaluation of Forecast

The measure of volatility is evaluated by summing the conditional mean of EMAp and the error resulted from an idiosyncratic zero-mean, constant-variance smoothing function σ [14]. Forecasting evaluation has been performed with stocks listed in the NZX on data sets acquired in multiples of 20-day periods. Computed correlation dimension for stocks have been tested as an evaluation of commotion [15]. The neural network forecasting results are summarized in Table 1.

The neural network is trained for direct prediction of trends. Error that causes false predictions is minimized to less than 10% with training of less than 50 passes. The performance is primarily controlled by the number of training iterations. The network will be failed to extract important features and subtle details from the training set if insufficient training iterations is committed. Conversely, over training will imply the network learns details of the training set to the detriment of its ability to abstract general features if too many iterations are committed. Also, this would significantly increase the processing requirements.. The number of run it has gone through the full training process, epochs, is evaluated as a performance of the total sum-squared error. This is shown in Fig. 5. Finally, the mean error forecast performance is plotted in Fig. 6 showing a moderate improvement in forecasts compared to simple moving average computations.

Table 1. Neural network test results between July 2004 and June 2005

Data Set	Profit opportunity count	Missed opportunity count	Error count	Network Prediction
A	51	33	1	36
B	48	21	1	39
C	73	35	2	41
D	38	6	0	48
E	41	10	2	37
F	62	19	1	51

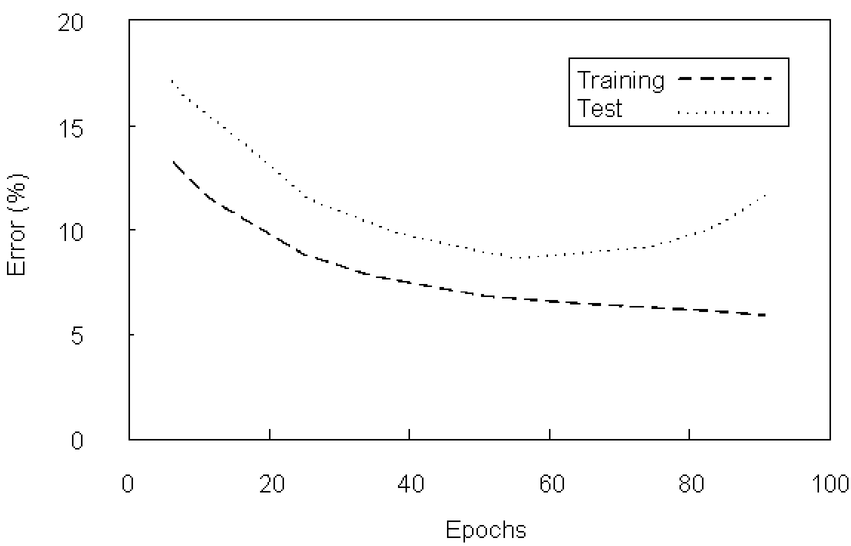

Fig. 5. Error performance

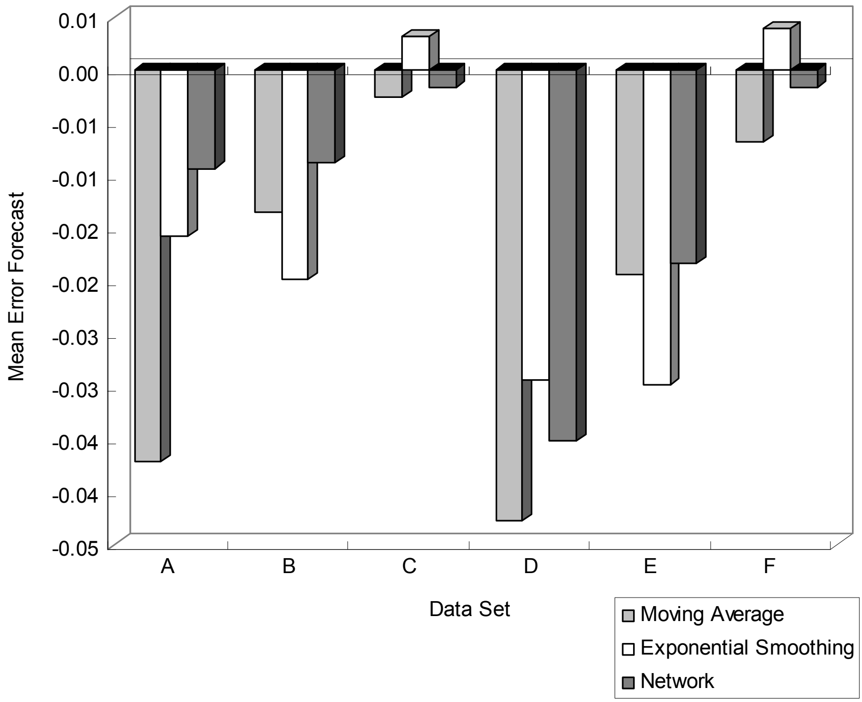

Fig. 6. Mean error forecast comparison

5 Conclusions

This paper evaluates the performance of a 3 layer neural network that is constructed to learn predicting volatility of selected stocks based on exponential moving averaging with data collected in blocks of 20-day closing prices. The network models temporal dependence of movement based on statistically volatility and moving averages which provides reasonably accuracy of forecast measures to better than 10% by using data collected for 6 stocks listed in the New Zealand Exchange. Minimization of complexity has enabled training to be performed reasonably quickly without running into over training that may lead to decrease in the ability to observe general features. Further, this network does not require extensive memory for a large amount of data.

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A New Algorithm Based on Copulas for Financial Risk Calculation with Applications to Chinese Stock Markets

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Abstract. This paper concerns the application of copula functions in calculating financial market risk. The copula function is used to model the dependence structure of multivariate financial assets. After introducing the traditional Monte Carlo simulation method and the pure copula method we present a new algorithm named mixture method based on copula's properties and the dependence measure, Spearman's rho. This new method is used to simulate daily returns of two stock market indices in China, Shanghai Stock Composite Index and Shenzhen Stock Composite Index and then calculate six risk measures including VaR and conditional VaR. The results are compared with that derived from the traditional Monte Carlo method and the pure copula method. From the comparison we show that for lower confidence level, the traditional Monte Carlo and pure copula method perform better than mixture method, while for higher confidence level, the mixture method is a better choice.

1 Introduction

The problem of modelling asset returns is one of the most important issues in finance. People generally use Gaussian process because of their tractable properties of easy computation. However, it is well-known that asset returns are fat-tailed. For multivariate case, the joint normal distribution and more generally the elliptical distribution restricts the type of association between margins to be linear, but other dependence structures such as rank correlation and tail dependence should also be considered by risk managers. These two difficulties, Gaussian assumption and dependence structure, can be effectively solved by copulas.

As we know, linear correlation has the serious deficiency that it is not invariant under non-linear strictly increasing transformation, while the dependence measures derived from copulas can overcome this shortcoming and have more

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broad applications (Nelson 1997, Wei et al. 2002, Vandenhende 2003). Furthermore, copulas can be used to describe more complex multivariate dependence structures, such as non-linear and tail dependence (Hürlimann 2003). With the development of computer software and information technology, the theory of copulas experienced rapid development since the end of 1900's (Nelsen et al. 2001, Genest 2001, Rodríguez-Lallena 2003).

But copulas have not been used in finance until 1999. After that, copulas are often cited in financial literatures. Mendes and Leal (2002) and Hurlimann (2004) investigated the problem of fitting multivariate distributions to financial data. Mendes and Moretti (2002), Mendes and Rafael (2004), and Cossette et al. (2002) studied the problem of calculating financial risk using copulas. In this paper, we use copulas to model the dependence structure of multivariate financial assets and design a new algorithm to calculate the financial risk in Chinese Stock Market.

This paper is structured as follows: Section 2 gives the definition of copulas and several important dependence measures. In Section 3 we introduce the six financial risk measures which are always used by risk managers. Our focus is on the calculation of VaR. In Section 4 we first introduce briefly the traditional Monte Carlo method and the pure copula method for VaR calculation, then present our algorithm based on copulas. Section 5 devotes to the calculation of financial risk in Chinese Stock Market using the three simulation methods given in Section 4. The comparison of the results derived from the three methods are also made in this section. Section 5 concludes the paper.

2 Copulas and Dependence Measures

In what follows we give the definition of copula functions and some related dependence measures. Readers interested in more details can refer to Nelson (1999). Here we consider the bivariate case; nevertheless, all the results carry over to the general multivariate setting.

Definition 1. *A two-dimensional copula is a real function defined on* $I^2 = [0,1] \times [0,1]$, *with range* $I = [0,1]$, *such that*

1) $\forall\ (u,v) \in I^2,\ C(u,0) = 0 = C(0,v),\ C(u,1) = u, C(1,v) = v;$

2) $\forall\ [u_1,u_2] \times [v_1,v_2] \in I^2$ *with* $u_1 \le u_2, v_1 \le v_2$,

$$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \ge 0.$$

As such, copulas can represent the joint distribution function of two standard uniform random variables U_1, U_2:

$$C(u_1,u_2) = P(U_1 \le u_1, U_2 \le u_2).$$

We can use this feature to rewrite via copulas the joint distribution function of two (even non-uniform) random variables. The most interesting fact about copulas in this sense is the following Sklar's Theorem.

Theorem 1. *(Sklar, 1959) Let $F(x,y)$ be a joint distribution with continuous margins $F_1(x)$ and $F_2(y)$. Then there exists a unique copula C such that*

$$F(x,y) = C(F_1(x), F_2(y)). \tag{1}$$

We can see from the theorem that any copula C pertaining to function F can be expressed as

$$C(u,v) = F(F_1^{-1}(u), F_2^{-1}(v)) \tag{2}$$

Sklar's theorem is very important because it provides a way to analyze the dependence structure of joint distribution without studying the marginal distributions.

An often-used copula function is the following Gaussian copula

$$C_\rho(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left\{\frac{-(s^2 - 2\rho st + t^2)}{2(1-\rho^2)}\right\} \tag{3}$$

The copula of any distribution F captures different types of dependence between variables. Here we present two dependence concepts which will be used in this paper.

(i) Pearson correlation. Pearson correlation coefficient $\rho(X,Y)$ for random variables X and Y is a measure of linear dependence. If X and Y are independent, then $\rho(X,Y) = 0$; if they are perfect linear dependent, $\rho(X,Y) = \pm 1$. Linear correlation is a natural dependence measure for multivariate normally, or more generally, elliptically distributed random variables, but it is not invariant under non-linear strictly increasing transformations.

(ii) Spearman's rho. Let X and Y be two random variables with marginal distribution functions F_1 and F_2 and joint distribution function F. *Spearman's rho* is defined by

$$\rho_s(X,Y) = \rho(F_1(X), F_2(Y)),$$

where ρ is the linear correlation coefficient.

Spearman's rho is a kind of rank correlation and can be considered as a measure of the degree of monotonic dependence between X and Y, whereas linear correlation only measures the degree of linear dependence. The linear correlation coefficient depends on margins and joint distribution (or equivalently copulas), being affected by nonlinear transformations, while Spearman's rho is not affected by nonlinear transformations and depends only on copulas. Using copulas Spearman's rho can be expressed as:

$$\rho_s(X,Y) = 12 \int_0^1 \int_0^1 (C(x,y) - xy)dxdy, \tag{4}$$

3 Risk Measures

In practice, many risk managers employ VaR (value-at-risk) as a tool of measuring risk. Briefly speaking, VaR is the maximal potential loss of a position or a portfolio in some investment horizon under a given confidential level. Precisely, VaR can be defined from the following

$$Pr(X > -VaR_\alpha) = \alpha,$$

where X is a variable representing the portfolio's return in investment horizon, α is the confidence level.

While VaR is a very powerful tool for risk management, it is not a coherent risk measure, that is, it is not sub-additive. For this reason, in recent years a modified risk measure based on VaR— conditional VaR ($CVaR$ in brief)— is brought out to overcome this problem. Shortly speaking, $CVaR$ gives the mean loss that exceeds VaR, that is,

$$CVaR_\alpha = E[X|X > VaR_\alpha(X)] \tag{5}$$

In this paper we will also calculate the following four risk measures. The expected shortfall $e_X(\alpha)$ and the median shortfall $e^*_X(\alpha)$ are defined as

$$e_X(\alpha) = E[X - VaR_\alpha(X)|X > VaR_\alpha(X)] \tag{6}$$

$$e^*_X(\alpha) = Median[X - VaR_\alpha(X)|X > VaR_\alpha(X)] \tag{7}$$

These two measures represent the expected (median) excess of loss beyond $VaR_\alpha(X)$. The following two risk measures

$$m_X(\alpha) = \frac{VaR_\alpha(X) + e_X(\alpha)}{VaR_\alpha(X)}, \quad m^*_X(\alpha) = \frac{VaR_\alpha(X) + e^*_X(\alpha)}{VaR_\alpha(X)} \tag{8}$$

represent the expected (median) total loss of a portfolio standardized by its VaR.

From the definitions of the above six risk measures we can see that the key point for the calculation of them is the calculation of VaR, which is the focus of our algorithm. In next section we will give three methods for the calculation of VaR.

4 Methods for VaR Calculation

In this section we first briefly introduce the traditional Monte Carlo method for VaR calculation, then present two algorithms based on copulas and Monte Carlo method.

4.1 Traditional Monte Carlo Method

In Monte Carlo method for calculating VaR, one first get the possible distribution from assets' historical data, then generate variables according with this distribution and construct portfolio's possible payoff, from which one can obtain the estimate of VaR for a given confidence level. In traditional Monte Carlo method one always assume that the marginal and joint distributions for asset returns are normal distribution.

The following algorithm generates variables r_1 and r_2 with normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ and linear correlation ρ from two $[0,1]$-uniform variables u_1 and u_2:

1) Put $s_1 = \sqrt{-2\ln u_1}\sin(2\pi u_2),\ s_2 = \sqrt{-2\ln u_1}\cos(2\pi u_2)$;

2) Put $r_1 = \sigma_1 s_1 + \mu_1,\ r_2 = \sigma_2(\rho s_1 + \sqrt{1-\rho^2}s_2) + \mu_2$.

4.2 Pure Copula Method

From above algorithm we can see that the traditional Monte Carlo method restricts the joint distribution for asset returns to be normal, but it is well-known that in practice asset returns are not normal. To overcome this problem one can use a copula function to give the joint distribution properly characterizing the dependence structure of asset returns.

The following algorithm generates variables with a given copula function C being the joint distribution:

1) Generate random variables U, W with $[0,1]$-uniform distribution;

2) For a given copula function C, calculate $C_U^{-1}(W)$, and put $V = C_U^{-1}(W)$;

3) Put $r_1 = F_1^{-1}(U), r_2 = F_2^{-1}(V)$, where F_1 and F_2 are the given marginal distributions for assets' returns.

For the aim of comparison we call this algorithm a pure copula method. In this method one can choose any copula function that meet his demand.

4.3 Mixture Method

In the following we will present our method, which we call a mixture method, since in this method a mixture distribution is used to describe the dependence structure between assets returns. Here the mixture distribution means the linear combination of two distributions, with Spearman's rho ρ_s being the combination coefficient. Before giving this algorithm, we first give the following lemma (Embrechts et al., 2002) which will help to get the main result proved.

Lemma 1. *Let X and Y be two random variables with distributions F_1 and F_2, ρ_{min} and ρ_{max} are the corresponding minimum and maximum linear correlations, thus $\rho \in [\rho_{min}, \rho_{max}]$. Put*

$$F_L(x,y) = \max(F_1(x) + F_2(y) - 1, 0),\ \ F_U(x,y) = \min(F_1(x), F_2(y)),$$

$$\lambda = \frac{\rho_{max} - \rho}{\rho_{max} - \rho_{min}},\quad F(x,y) = \lambda F_L(x,y) + (1-\lambda)F_U(x,y),$$

then the bivariate mixture distribution given by $F(x,y)$ has margins F_1 and F_2 and linear correlation ρ.

To go further we put

$$\lambda_s = \frac{\rho_s^{max} - \rho_s}{\rho_s^{max} - \rho_s^{min}},$$

$$F^{'}(x,y) = \lambda_s F_L(x,y) + (1-\lambda_s)F_U(x,y),$$

where ρ_s is the Spearman's rho between variables X and Y, that is, $\rho_s(X,Y) = \rho(F_1(X), F_2(Y))$. Then we get the following theorem from above Lemma.

Theorem 2. *The random vector generated by the following algorithm has the joint distribution $F'(X,Y)$ with margins F_1, F_2 and Spearman's rho ρ_s:*
1. Simulate U_1 and U_2 independently from standard uniform distribution;
2. If $U_1 \le \lambda_s$, take $(X,Y)^T = (F_1^{-1}(U_2), F_2^{-1}(1-U_2))^T$;
3. If $U_1 > \lambda_s$, take $(X,Y)^T = (F_1^{-1}(U_2), F_2^{-1}(U_2))^T$, where superscript T means the transpose of a vector..

Proof: From Sklar's theorem, the random vector with joint distribution $F^{'}(X,Y)$ has copula

$$C(F_1(x), F_2(y)) = \lambda_s C_L(F_1(x), F_2(y)) + (1-\lambda_s)C_U(F_1(x), F_2(y)) \quad (9)$$

where C_L and C_U are copulas corresponding to joint distributions $F_L(x,y)$ and $F_U(x,y)$, respectively.

Let $u = F_1(x), v = F_2(y)$, and substitute into (9) we have

$$C(u,v) = \lambda_s W(u,v) + (1-\lambda_s)M(u,v)$$

where $W(u,v)$ and $M(u,v)$ are the upper and lower Fréchet-Hoeffding bounds of copula $C(u,v)$:

$$W(u,v) = \max(u+v-1, 0), \quad M(u,v) = \min(u,v).$$

Since the Spearman's rho ρ_s between X and Y can be thought of as the linear correlation between standard uniform random variables $U = F_1(x)$ and $V = F_2(y)$, we conclude the proof from Lemma 1.

5 Application to Chinese Stock Markets

In this section, we will use the above three methods to calculate the financial risk in Chinese Stock Markets. We choose Shanghai Stock Composite Index and Shenzhen Stock Composite Index to form an equally weighted portfolio and compute the six risk measures defined in Section 3 for this portfolio.

The data for the two market indexes consist of 1200 daily prices from January 4, 2001 to December 31, 2004. Simple statistics of the two indexes and the portfolio are given in table 1.

To calculate the six risk measures, we first model the margins for the asset returns, then adding an appropriate dependence structure. Using copulas we can take into account the leptokurtic property of asset returns shown in Table 1. In order to choose a suitable distribution we check the fitness effect of the margins of asset returns. The following Fig. 1 displays the fitness effect tested by constructing p-p plots using Laplace and normal distributions.

Table 1. Statistics of the two indexes and the portfolio

	R_{SH}	R_{SZ}	R_P
N (Valid)	1200	1200	1200
N (Missing)	23	23	23
mean	.0000287	-.0001005	-.0000359
Std. Deviation	.0136276	.0142559	.0138640
Skewness	.896	.685	.795
Kurtosis	7.143	6.182	6.732

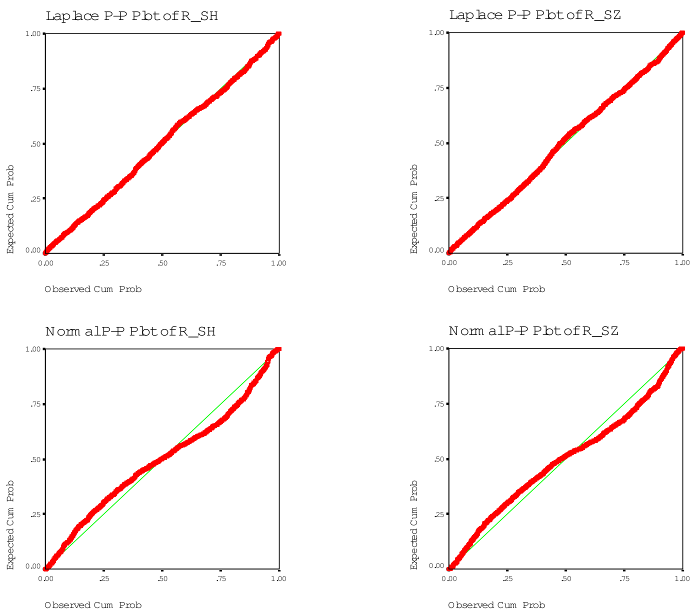

Fig. 1. P-P plots of Laplace and normal distribution fitted to Shanghai and Shenzhen index data

In figure 1, the above two plots are the fitness effect test for Laplace distribution fitted to Shanghai and Shenzhen Index data, respectively, while the nether two plots are for normal distribution. From the figure we can see that normal distribution which are usually fat-tailed is not so good a description for asset returns as Laplace distribution which represents the leptokurtic property of asset returns in a better way. For this reason we will use Laplace distribution to fit the margins.

In calculating the risk measures, we make two simplification: first, it is assumed no time dependence between the daily index returns; second, only the empirical estimates of the risk measures are computed, which works well for small α.

In Monte Carlo simulation, the classical bivariate normal distribution assumption is used to calculate the risk measures. In the pure copula method, Laplace distribution is used for the asset return margins and Gaussian copula is used to describe the dependence structure. From the introduction of the methods in Section 4 it is easy to see that the traditional Monte Carlo method is equivalent to the pure copula method if Gaussian copula is used to model the bivariate distribution for asset returns. Different copulas may be employed in pure copula method to describe the dependence structure between assets returns to investigate the effect of different copulas on risk measures. But here we only use Gaussian copula because our focus is to compare the three simulation methods and show the usefulness of copulas in measuring financial risk rather than to investigate the effect of different copulas on risk measures. In the mixture method, we use Laplace distribution for the margins and the mixture distribution for the dependence structure between asset returns.

The empirical estimates of VaR_α, $CVaR_\alpha$, $e_X(\alpha)$, $e^*_X(\alpha)$, $m_X(\alpha)$ and $m^*_X(\alpha)$ for the equally weighted portfolio and for confidence level $\alpha = 0.05$, 0.01 and 0.005 are given in table 2.

Table 2. Results for six risk measures under three methods

	Traditional Monte Carlo			Pure Copula Method			Mixture Method		
α	.05	.01	.005	.05	.01	.005	.05	.01	.005
VaR	-17.48	-23.45	-23.86	-17.28	-29.24	-35.22	-10.72	-19.24	-22.7
$CVaR$	1.24	0.38	-0.26	1.59	0.65	0.49	0.87	0.28	0.18
e_X	18.73	23.83	24.12	18.87	29.89	35.71	11.59	19.52	22.88
e^*_X	18.43	23.93	24.28	17.75	29.48	35.45	10.83	19.17	22.59
m_X	0.071	0.016	0.011	-0.092	-0.022	-0.014	-0.081	-0.015	-0.008
m^*_X	0.054	0.020	0.017	-0.028	-0.008	-0.007	-0.011	0.003	0.005

In order to check the reliability of the computed results, one can observe the portfolio's real value at a certain moment, and compare the computed results with the real change of portfolio's value to do backtest. For this, we calculate VaR=-14.46 using the real data from the security markets. Comparing the results derived from the three methods shown in Table 2, we come to the two conclusions: first, for lower confidence level, the traditional Monte Carlo and pure copula method perform better than mixture method, while for higher confidence level, the mixture method is a better choice; second, the dependence structure between asset returns plays an important role in calculating risk measures, while the form of marginal distributions for asset returns puts relatively little impact on risk measures.

6 Conclusions

In this paper, we design a new algorithm using copulas to calculate financial risk. From this paper we can see that copula is a very powerful tool for financial risk measurement in that it fulfills one of its main goals: modelling the dependence structure between individual risks.

Our work deals with the simulation problem in calculating portfolio's VaR and other risk measures. Based on copula theory and the dependence measure, Spearman's rho, a new simulation algorithm is presented and is used to calculate the financial risk in Chinese Stock Markets. The portfolio is composed of Shanghai Stock Composite Index and Shenzhen Stock Composite Index with equal weight. Six risk measures derived from this new method are compared with those obtained from the pure copula method and traditional Monte Carlo simulation. The comparison result show that this new method has obvious advantage over the other two methods in two respects: first, it is free of choosing the best suitable margin to model asset returns, rather than constrained by the distribution assumption as in the traditional Monte Carlo method; second, in terms of technology, this method is easier to understand and more practical than the pure copula method which involves intractable computations.

In the original version of our paper we have another part of work. In that part we use extreme copula to describe the dependence structure between two extreme events and study the extreme risk. Stress test is made to describe the extreme events of the portfolio and tail dependence is calculated to measure the correlation of the extreme risks. From our point of view that part is also important and useful. But because of the limitation of the length of the paper we have to cut this part off and form another paper.

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Design of Anonymity-Preserving User Authentication and Key Agreement Protocol for Ubiquitous Computing Environments*

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Abstract. The spread of mobile devices, PDAs and sensors enabled the construction of ubiquitous computing environments, transforming regular physical spaces into "Smart space" augmented with intelligence and enhanced with services. However, the deployment of this computing paradigm in real-life is disturbed by poor security, particularly, the lack or proper authentication, authorization and key management techniques. Also, it is very important not only to find security measures but also to preserve user privacy in ubiquitous computing environment. In this paper, we propose efficient user authentication and key agreement protocol with anonymity for the privacy-preserving for ubiquitous computing environments. Our scheme is suitable for distributed environments with the computational constrained devices by using MAC-based security association token.

1 Introduction

Ubiquitous computing environments or smart spaces promote the spread of embedded devices, smart gadgets, appliance and sensors. We envision a Smart space to contain hundreds, or even thousands, of devices and sensors that will be everywhere, performing regular tasks. Providing new functionality, bridging the virtual and physical worlds, and allowing people to communicate more effectively and interact seamlessly with available computing resources and the surrounding physical environment. However, the real-life deployment of smart spaces is disturbed by poor and inadequate security measures, particularly, authentication, authorization and key management techniques. These mechanisms are inadequate for the increased flexibility required by distributed environments. Moreover, traditional authentication methods using symmetric key techniques either do not well in massively distributed environments, with hundreds or thousands of embedded devices like smart space. Authentication methods using public key techniques would clearly be a better solution compared to authentication methods using symmetric key techniques. But it may inappropriate to use public key techniques in smart space with the computational constrained devices.

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In this paper, we propose efficient user authentication and key agreement protocol with anonymity for the privacy-preserving for ubiquitous computing environments. Our protocol is suitable for distributed environments with the computational constrained devices by using MAC-based security association token.

We organize the paper as follows. In section 2, we describe Kerberos system, secure device association mechanism and authenticated key agreement protocol. In section 3, we propose user authentication and key management mechanism using MAC-based security association token. Security consideration and performance evaluation will be stated in section 4, and finally the conclusions are given in section 5.

2 Related Work

Kindberg and Zhang[2] have described protocols for validating secure associations setup spontaneously between devices. The secure device association method employs lasers, which provide relatively precise physically constrained channels. The initiator device is equipped with a laser whose output can be rapidly switched to provide a data stream. The target device is equipped with a securely attached light sensor that can read the data emitted by the initiator device. The initiator device creates new session key K_S and sends it to target device using laser. Kindberg and Zhang scheme has merit to enable nomadic users to securely associate their devices without communication with trusted third parties. Because K_S is shared between initiator and target device without authentication, the scheme also has potential security vulnerabilities, particularly, denial of service attacks.

Leighton-Micali scheme[4] focuses on the efficient, authenticated key agreement protocol based symmetric techniques. Key management using public key techniques would clearly be a better solution compared to key management using symmetric techniques from the standpoint of both ease of management and scalability. However, implementation of public key management techniques requires wide-spread deployment of public key infrastructure. Also, it may inappropriate to use public key techniques in smart space with the computational constrained devices. Leighton-Micali scheme may be a good solution in distribution environments with the computational constrained devices.

3 Proposed Scheme

3.1 Protocol Architecture

This paper proposes efficient user authentication and key agreement protocol with anonymity for the privacy-preserving for distributed environments with the computational constrained devices by using MAC-based security association token. The security association token is similar to Kerberos's ticket and employs concept of Leighton-Micali scheme[4]'s authenticated key agreement mechanism. Figure 1 presents user authentication and key agreement protocol proposed in this study.

Our protocol components have a authentication server, service association server, target devices and user authentication portal.

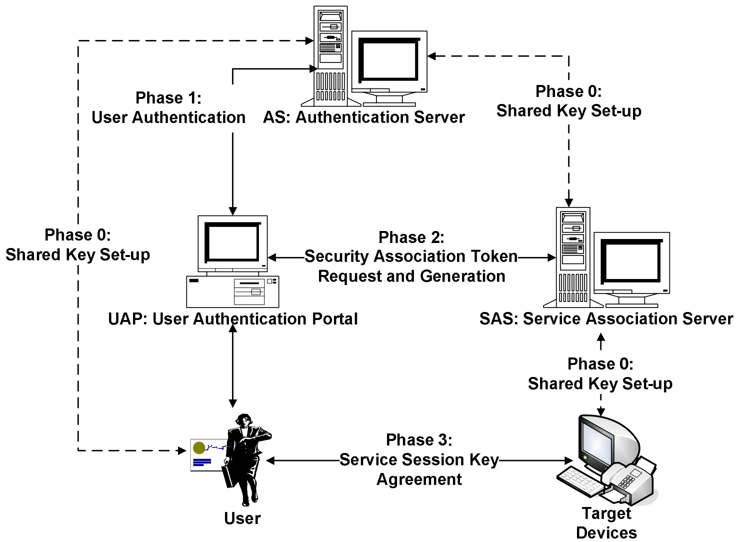

Fig. 1. User Authentication and Key Agreement Protocol Architecture

- AS(Authentication Server) performs user authentication and generates anonymous certificate for user to preserve user privacy.
- SAS(Service Association Server) generate security association token for users after validating user's anonymous certificate.
- UAP(User Authentication Portal) is not only a mediator between user and AS but also is a mediator between user and SAS. UAP requests user's anonymous certificate(or security association token) for AS(or SAS) and receive and transfer it to user.
- Target devices provide users with ubiquitous services in smart space.

The user authentication and key agreement protocol proposed in this study consist of three phases, which comprised of user authentication, security association generation, and service session key agreement. In addition, there is shared key set up as a preprocessing.

3.1.1 Symbols

$\|\|$	A symbol for the concatenation A and B for A $\|\|$ B
$\|A\|$	A symbol for the length of A
$\oplus$	A symbol for the XOR computation A and B for A $\oplus$ B
CMAC(A, B)	A CBC-MAC computation for message B using key A
K_U	A shared master key between user U and AS
K_{SAS}	A shared master key between SAS and AS
K_{Ti}	A shared master key between target device T_i and SAS
$PK_{U,SAS}$	A pair-wise public key between user U and SAS
BK_U	A smart space base key between user U and SAS
$PK_{U,Ti}$	A pair-wise public key between user U and target device T_i

$SK(U,T_i)$	A service session key between user U and target device T_i
R_{AS}	A random number generated by AS
R_{SAS}	A random number generated by SAS
R_U	A random number generated by user U

3.1.2 Phase 0: Shared Key Setup

- K_U: K_U is to be set up between AS and user U. The K_U is used to issue and validate user's anonymous certificate.
- K_{SAS}: K_{SAS} is to be set up between AS and SAS.
- K_{Ti}: K_{Ti} is to be set up between SAS and target device T_i.

The AS can issue user's anonymous certificate with K_U and K_{SAS}. User U and SAS can verify pair-wise digital signature of anonymous certificate with K_U and K_{SAS}, respectively. K_{Ti} is used to issue security association token and to verify pair-wise digital signature security association token.

3.2 Phase 1: User Authentication

We assume that user can access available UAP by using their own authentication devices, such as PDA, smart badge, mobile phone, password, etc. Figure 2 shows that UAP requests user's anonymous certificate for AS and receive it. AS validate $AUTH_U$ value with K_U. If the $AUTH_U$ is valid, AS consider as legal user and issue anonymous certificate for user and transfer to UAP. Then the UAP transfers the anonymous certificate to a user. The profile of anonymous certificate is presented in table 1.

A user can validate the anonymous certificate using K_U. The user first computes $CMAC(K_U,$ toBeSignedACInfo$)$ and check that it is equal to the $AUTH_U$ value.

Table. 1. Anonymous Certificate Profile

Field		Descriptions
toBeSignedACInfo	Anonymous_Id	User's anonymous identifier generated by AS
	SAS_Id	SAS identifier generated by AS
	Validity	Validity period of anonymous certificate
	Nonce_AS(R_{AS})	Nonce value generated by AS
	$PK_{U,SAS}$	A pair-wise public key between user U and SAS (It can be compute $PK_{U,SAS} = KM_U \oplus KM_{SAS}$ after computing $KM_U = CMAC(K_U, R_{AS})$, $KM_{SAS} = CMAC(K_{SAS}, R_{AS})$)
$AUTH_U$		A pair-wise digital signature, which is generated by AS by computing $CMAC(K_U,$ toBeSignedACInfo$)$, validates anonymous certificate with K_U.
$AUTH_{SAS}$		A pair-wise digital signature, which is generated by AS by computing $CMAC(K_{SAS},$ toBeSignedACInfo$)$, validates anonymous certificate with K_{SAS}.

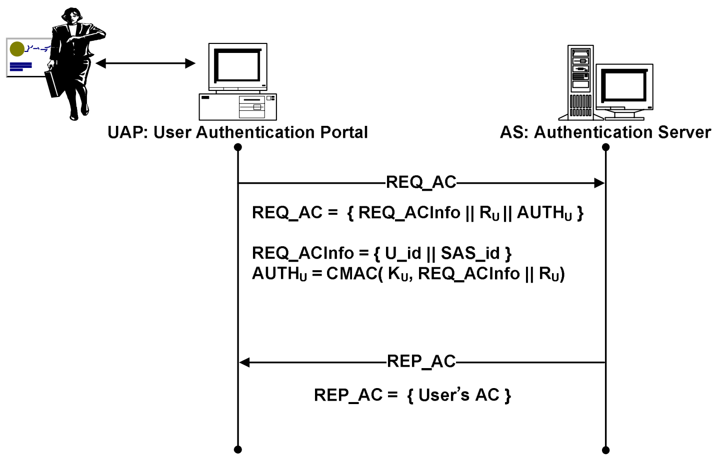

Fig. 2. Phase 1: User Authentication

The user can compute a smart space base key BK_U between a SAS and a user from the $PK_{U,SAS}$ of the anonymous certificate. If the length of $PK_{U,SAS}$ is 16 bytes(i.e., the block size of a cryptographic algorithm), the BK_U will be computed by concatenating the upper 8 bytes of KM_U and KM_{SAS} after calculating $KM_U = CMAC(K_U, R_{AS})$, and $KM_{SAS} = PK_{U,SAS} \oplus KM_U$. If the size of $PK_{U,SAS}$ is 8 bytes, the BK_U will be computed by concatenating KM_U and KM_{SAS}.

3.3 Phase 2: Security Association Token Generation

Phase 2 issues a security association token after verifying user's anonymous certificate. The security association token issuing procedures are as follows. Table 2 presents the profile of a security association token.

1. The UAP requests security association token by transferring user's anonymous certificate to SAS.
2. SAS validates user's anonymous certificate with K_{SAS}. The validation process can be performed by comparing it with the $AUTH_{SAS}$ field after computing $CMAC(K_{SAS}, toBeSignedACInfo)$.
3. If user's anonymous certificate is valid, SAS can compute smart space base key BK_U from $PK_{U,SAS}$ of user's anonymous certificate. The computation of BK_U is almost the same as computation of user's smart space base key. If the length of $PK_{U,SAS}$ is 16 bytes(i.e., the block size of a cryptographic algorithm), the BK_U will be computed by concatenating the upper 8 bytes of KM_U and KM_{SAS} after calculating $KM_{SAS} = CMAC(K_{SAS}, R_{AS})$, and $KM_U = PK_{U,SAS} \oplus KM_{SAS}$. If the size of $PK_{U,SAS}$ is 8 bytes, the BK_U will be computed by concatenating KM_U and KM_{SAS}.
4. SAS computes $AUTH\_Holder = CMAC(BK_U, toBeSignedSATInfo)$ and $AUTH\_Target = CMAC(K_{Ti}, toBeSignedSATInfo)$ using smart space base key BK_U and K_{Ti} respectively.

Table. 2. Security Association Token Profile

Field		Descriptions
toBeSignedSATInfo	SAT_No	Serial number of security association token
	Issuer_Id	Issuer identifier of security association token(i.e., SAS ID)
	Holder_Id	Holder identifier of security association token(i.e., Anonymous identifier in user's anonymous certificate)
	Validity	Validity period of security association token
	Nonce_SAS(R_{SAS})	Nonce value generated by SAS
	Target_Id	Target device identifier
	$PK_{U,Ti}$	A pair-wise public key between user U and T_i (It can be computed $PK_{U,Ti} = KM_{Ti} \oplus BKM_{U,Ti}$ after computing $KM_{Ti} = CMAC(K_{Ti}, R_{SAS})$, $BKM_{U,Ti} = CMAC(BK_U, R_{SAS})$)
AUTH_Holder		A pair-wise digital signature, which is generated by SAS using a smart space base key BK_U.
AUTH_Target		A pair-wise digital signature, which is generated by SAS using a shared key K_{Ti} between T_i and SAS

3.4 Phase 3: Service Session Key Agreement

This stage generate service session key $SK(U, T_i)$ between user U and target device T_i using user's security association token, which is issued from SAS. The service session key agreement procedures are as follows.

1. A user can generate service session key $SK(U, T_i)$ using pair-wise public key $PK_{U,Ti}$ in user's security association token and smart space base key BK_U. If the size of $PK_{U,Ti}$ is 16 bytes, the $SK(U, T_i)$ will be generated by concatenating the upper 8 bytes of $BKM_{U,Ti}$ and KM_{Ti} after computing $BKM_{U,Ti} = CMAC(BK_U, R_{SAS})$, and $KM_{Ti} = PK_{U,Ti} \oplus BKM_{U,Ti}$. If the size of $PK_{U,Ti}$ is 8 bytes, the $SK(U,T_i)$ can be generated by concatenating $BKM_{U,Ti}$ and KM_{Ti}.
2. On receiving the security association token, target device T_i first computes $CMAC(K_{Ti}, toBeSignedSATInfo)$ and check that it is equal to the AUTH_Target value in user's security association token. If the check succeeds, target device T_i can compute service session key $SK(U, T_i)$. The computation of $SK(U, T_i)$ is almost the same as computation of user's service session key. If the size of $PK_{U,Ti}$ is 16 bytes, the $SK(U, T_i)$ will be generated by concatenating the upper 8 bytes of $BKM_{U,Ti}$ and KM_{Ti} after computing $KM_{Ti} = CMAC(K_{Ti}, R_{SAS})$, and $BKM_{U,Ti} = PK_{U,Ti} \oplus KM_{Ti}$. If the size of $PK_{U,Ti}$ is 8 bytes, the $SK(U,T_i)$ can be generated by concatenating $BKM_{U,Ti}$ and KM_{Ti}.

4 Security Consideration and Performance Evaluation

4.1 Security Consideration

For the user authentication and key agreement protocol proposed in this paper, we assume that AS(Authentication Server) is trusted as a TTP(Trust Third Party). User's anonymous certificate is only generated by AS, because it can be generated using K_{SAS} and K_U. In addition, the validation of user's anonymous certificate is secure because the $AUTH_{SAS}$ can only be verified by SAS and the $AUTH_U$ can only be verified by user U. Moreover, because the computation of KM_U and KM_{SAS} from the pair-wise public key $PK_{U,SAS}$ can only be computed by user U and SAS as described in Leighton-Micali scheme[4], the smart space base key BK_U is also secure. User's security association token is only generated by SAS, because it can be generated using smart space base key BK_U and shared key K_{Ti} between SAS and target device T_i. Also, the validation of user's security association token is secure because the AUTH_Holder can only be verified by user U and AUTH_Target can only be verified by target device T_i. Moreover, because the computation of $BKM_{U,Ti}$ and KM_{Ti} from the pair-wise public key $PK_{U,Ti}$ can only be compute by user U and target device T_i as described in Leighton-Micali scheme, the service session key $SK(U, T_i)$ is secure.

4.2 Performance Evaluation

For performance evaluation, Kerberos system which has a similar structure to our scheme is used as a comparative system. Table 3 presents the summary of Kerberos message exchange protocol.

Table. 3. Summary of Kerberos Message Exchanges

(a) Authentication Service Exchange
(1) $C \rightarrow AS: ID_c \parallel ID_{tgs} \parallel TS_1$ (2) $AS \rightarrow C: E_{Kc}[K_{c,tgs} \parallel ID_{tgs} \parallel TS_2 \parallel Lifetime_2 \parallel Ticket_{tgs}]$ $Ticket_{tgs} = E_{Ktgs}[K_{c,tgs} \parallel ID_c \parallel AD_c \parallel ID_{tgs} \parallel TS_2 \parallel Lifetime_2]$
(b) Ticket-Granting Service Exchange
(3) $C \rightarrow TGS: ID_v \parallel Ticket_{tgs} \parallel Authenticator_c$ (4) $TGS \rightarrow C: K_{c,tgs}[K_{c,v} \parallel ID_v \parallel TS_4 \parallel Ticket_v]$ $Ticket_{tgs} = E_{Ktgs}[K_{c,tgs} \parallel ID_c \parallel AD_c \parallel ID_{tgs} \parallel TS_2 \parallel Lifetime_2]$ $Ticket_v = E_{Kv}[K_{c,v} \parallel ID_c \parallel AD_c \parallel ID_v \parallel TS_4 \parallel Lifetime_4]$ $Authenticator_c = E_{Kc,tgs}[ID_c \parallel AD_c \parallel TS_3]$
(c) Client / Server Authentication Exchange
(5) $C \rightarrow V : Ticket_v \parallel Authenticator_c$ (6) $V \rightarrow C: E_{Kc,v}[TS_5 + 1]$ $Ticket_v = E_{Kv}[K_{c,v} \parallel ID_c \parallel AD_c \parallel ID_v \parallel TS_4 \parallel Lifetime_4]$ $Authenticator_c = E_{Kc,v}[ID_c \parallel AD_c \parallel TS_5]$

For performance evaluation between Kerberos system and our scheme, we use AES algorithm. Table 4 and 5 respectively shows the cryptographic algorithm performance and experimental results between Kerberos system and our scheme. In Kerberos system, the size of ID_c, ID_{tgs}, AD_c, ID_v, TS_1, TS_2, TS_3, TS_4, and TS_5 were assumed as 4 bytes. And $Lifetime_2$ and $Lifetime_4$ were assumed as 8 bytes, respectively. $Ticket_{tgs}$ and $Ticket_v$ were assumed as 48 bytes and $Authenticator_c$ was assumed as 16 bytes. In addition $K_{c,tgs}$ and $K_{c,v}$ were also assumed as 16 bytes, respectively.

Table. 4. Performance of cryptographic algorithm

<table>
<tr><td rowspan="2">AES-CBC</td><td>Encryption (Decryption)</td><td colspan="2">Round key generation for encryption</td><td>Round key generation for decryption</td></tr>
<tr><td>570(Mbps)</td><td colspan="2">0.306(micro-sec)</td><td>0.572(micro-sec)</td></tr>
<tr><td rowspan="2">AES-CBC-MAC</td><td colspan="2">CBC-MAC</td><td colspan="2">Round key generation for CBC-MAC</td></tr>
<tr><td colspan="2">606(Mbps)</td><td colspan="2">0.306(micro-sec)</td></tr>
<tr><td colspan="5">* Pentium IV 3.2 Ghz, Windows XP SP2, Microsoft Visual C++ 6.0</td></tr>
</table>

Table. 5. Experimental result between Kerberos system and our scheme

	Kerberos system (AES-CBC)	Our scheme (AES-CBC-MAC)
Client / User	5.206(micro-sec)	2.816(micro-sec)
AS	2.354(micro-sec)	2.106(micro-sec)
TGS / SAS	4.721(micro-sec)	3.269(micro-sec)
Server / Target Device	2.869(micro-sec)	1.681(micro-sec)

* Pentium IV 3.2 Ghz, Windows XP SP2, Microsoft Visual C++ 6.0

As we saw with table 5, our scheme is better than Kerberos system in sense of cryptographic computation processing.

5 Conclusion

In this paper, we propose efficient user authentication and key agreement protocol with anonymity for the privacy-preserving for ubiquitous computing environments. Our scheme is suitable for distributed environments with the computational constrained devices by using MAC-based anonymous certificate and security association token instead of using public key encryption technique. And our proposed protocol is better than Kerberos system in sense of cryptographic computation processing.

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An Efficient Identity-Based Key Exchange Protocol with KGS Forward Secrecy for Low-Power Devices

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Abstract. For an ID-based key exchange (KE) protocol, KGS forward secrecy is about the protection of previously established session keys after the master secret key of the Key Generation Server (KGS) is compromised. This is the strongest notion of forward secrecy that one can provide for an ID-based KE protocol. Among all the comparable protocols, there are only a few of them providing this level of forward secrecy and all of these protocols require expensive bilinear pairing operations and map-to-point hash operations that may not be suitable for implementation on low-power devices such as sensors. In this paper, we propose a new ID-based KE protocol which does not need any pairing or map-to-point hash operation. It also supports the strongest KGS forward secrecy. On its performance, we show that it is faster than previously proposed protocols in this category. Our protocol is signature-based in which the signature scheme is a variant of a scheme proposed by Bellare et al. in Eurocrypt 2004. We show that the variant we proposed is secure and also requires either less storage space or runtime computation than the original scheme.

1 Introduction

Since the first set of identity-based (ID-based) Key Exchange (KE) protocols were proposed [16, 14, 13, 12] in late '80s and early '90s, there has been a revival of interest in ID-based KE protocols recently [17, 18, 11, 15] due to the discovery of several new applications of pairings on elliptic curves [6].

On security, most of them [16, 13, 12, 17, 18, 15] only support *partial* or *perfect* forward secrecy but not the **KGS forward secrecy** [14, 11], the strongest notion of forward secrecy in the context of ID-based KE protocols. By partial forward secrecy, previously established session keys will remain secure after the secret key of one communicating party is compromised. By perfect forward secrecy, previously established session keys will remain secure after the secret

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keys of both communicating parties are compromised. By KGS (Key Generation Server) forward secrecy, previously established session keys will still remain secure even after the long-term secret key of the KGS is compromised. Note that compromising the KGS secret key implies compromising secret keys of all parties in an ID-based cryptosystem. Hence the KGS forward secrecy is the strongest notion among these three.

On performance, according to the state-of-the-art results in [1, 2], one bilinear pairing operation requires at least 10 times more multiplications in the underlying finite field than an elliptic curve point scalar multiplication does in the same finite field. For low-power devices such as sensors, cellphones and low-end PDAs, which are usually characterized by limited battery lifetime and low computational power, applications using bilinear pairings can be too expensive to implement. In addition, most of the ID-based cryptosystems require a special hash function called map-to-point hash function [6, 7] for converting a user's identifying information to a point on the underlying elliptic curve. This operation is also time consuming and cannot be treated as a conventional hash operation which is commonly ignored in performance evaluation. A map-to-point hash function, on the other hand, is usually implemented as a probabilistic algorithm and is more expensive than a point scalar multiplication in terms of computation time.

Our Contributions. We propose a new ID-based KE protocol which does not require any pairing or map-to-point hash. It also supports the strongest KGS forward secrecy. The protocol is also shown secure under the model defined by Canetti and Krawzcyk [10]. Our protocol is signature-based in which the signature scheme is ID-based and is a variant of BNN-IBS proposed by Bellare et al. in [4]. We show that our scheme is secure and also requires either less storage space or runtime computation than the BNN-IBS. On the performance of our ID-based KE protocol, we show that it is faster than all comparable protocols.

Paper Organization. In Sec. 2, we give a definition for ID-based signature schemes. This is followed by the description of our ID-based signature scheme and its security analysis in Sec. 3. In Sec. 4, we propose an ID-based KE protocol and analyze its security using a modular approach proposed by Bellare et al. [3]. Performance evaluation of the protocol is given in Sec. 5. We conclude the paper in Sec. 6. Due to the page limitation, we skip the section of related work. For details, please refer to our full paper [19].

2 IBS: Security Model

An *ID-based signature (IBS) scheme* is a tuple $(MKGen, UKGen, Sig, Ver)$ of polynomial-time algorithms. The first three may be randomized but the last one is not.

- A Key Generation Server (KGS) runs the master-key generation algorithm $MKGen$ on input 1^k, where $k \in \mathbb{N}$ is a security parameter, to obtain a master public/secret key pair (mpk, msk).

– The KGS can then run the user-key generation algorithm $UKGen$ on msk and identity $ID \in \{0,1\}^*$ to generate a secret key usk for a user identified by ID. It is assumed that usk is securely transported to that user.
– On input usk and message $m \in \{0,1\}^*$, the signing algorithm Sig returns a signature σ.
– On input mpk, ID, m and σ, the verification algorithm Ver returns an accept/reject decision to indicate whether signature σ is valid for identity ID and message m.

For correctness, we require that for all $k \in \mathbb{N}$, $m \in \{0,1\}^*$, $ID \in \{0,1\}^*$, if $(mpk, msg) \leftarrow MKGen(1^k)$, $usk \leftarrow UKGen(msk, ID)$ and $\sigma \leftarrow Sig(usk, m)$, then $Ver(mpk, ID, m, \sigma) = 1$. We now provide the formal definition of a secure IBS scheme in terms of existential unforegability against chosen message and ID attacks (in short euf-cma-ida).

Definition 1. *Let $(MKGen, UKGen, Sig, Ver)$ be an IBS scheme, $\mathcal{A}$ an adversary, and $k \in \mathbb{N}$ a security parameter. Consider the game below which is run by a simulator/challenger $\mathcal{S}$.*

– *$\mathcal{S}$ executes $MKGen(1^k)$ to get the master public/secret key pair (mpk, msk).*
– *$\mathcal{S}$ runs $\mathcal{A}$ on 1^k and mpk. During the simulation, $\mathcal{A}$ can make queries onto the following oracles.*
  - CreateUser(ID)*: If ID is not yet* created*, $\mathcal{S}$ executes $UKGen(msk, ID)$ to get the user's secret key usk_{ID}. ID is said to be* created *from now on and 1 is returned. Otherwise, (that is, if ID has already been* created*) 0 is returned.*
  - Corrupt(ID)*: If ID has been* created*, usk_{ID} is returned and ID is said to be* corrupted*; otherwise, $\perp$ is returned for failure.*
  - Sign(ID, m)*: If ID has not been* created*, return $\perp$ for failure. Otherwise, $\mathcal{S}$ executes $Sig(usk_{ID}, m)$ to get a signature σ and returns σ. Then, m is said to be* signed by *ID.*
– *$\mathcal{A}$ is to output a triple (ID^*, m^*, σ^*).*

$\mathcal{A}$ wins if $Ver(mpk, ID^*, m^*, \sigma^*) = 1$, ID^* is created *but* not *corrupted and m^* is* not *signed by ID^*. The IBS scheme is* euf-cma-ida secure *if for all probabilistic polynomial-time (PPT) algorithm $\mathcal{A}$, it is negligible for $\mathcal{A}$ to win the game.*

In the next section, we describe a new IBS scheme which is euf-cma-ida secure as defined in this section.

3 An IBS Scheme Based on EC-DLP

Let $k \in \mathbb{N}$ be a security parameter, $ID \in \{0,1\}^*$ an identity, and $m \in \{0,1\}^*$ a message. Let $\mathbf{F}$ be a finite field, $\mathscr{C}$ an elliptic curve defined over $\mathbf{F}$, and P an element of large prime order p in $\mathscr{C}$. Let G be a cyclic subgroup of $\mathscr{C}$ generated by the 'base' point P, such that the elliptic curve discrete log problem (EC-DLP) is intractable. We assume that $(\mathscr{C}, \mathbf{F}, P, p)$ is a sequence of system-wide parameters[1]. Let $H_1 : \{0,1\}^* \rightarrow Z_p$ and $H_2 : \{0,1\}^* \rightarrow Z_p$ be two hash functions. For security analysis, we view them as random oracles [5].

Master-Key Generation
$(mpk, msk) \leftarrow MKGen(1^k)$: The trusted KGS (Key Generation Server) randomly picks $x \in_R \mathbb{Z}_p$ and computes $P_{pub} = xP$. The master public key mpk is set to P_{pub} and the master secret key msk is set to (x, P_{pub}).

User-Key Generation
$usk \leftarrow UKGen(msk, ID)$: If ID is not created, the KGS sets the user's secret key usk to (c, s, ID, P_{pub}) where $c = H_1(P_{pub}, ID, cP_{pub} + sP)$ and $s \in \mathbb{Z}_p$. This secret key is generated as follows.

1. Randomly pick $r \in_R \mathbb{Z}_p$, compute $R = rP$ and $c \leftarrow H_1(P_{pub}, ID, R)$.
2. Compute $s = r - cx \mod p$.

Signature Generation
$\sigma \leftarrow Sig(usk, m)$: Given the user's secret key $usk = (c, s, ID, P_{pub})$, a signature $\sigma = (c, T, \pi)$ on message m is generated as follows.

1. Randomly pick $t \in_R \mathbb{Z}_p$, compute $T = tP$.
2. Compute $e \leftarrow H_2(P_{pub}, ID, m, T, c)$ and $\pi = t - es \mod p$.

Signature Verification
$1/0 \leftarrow Ver(mpk, ID, m, \sigma)$: To verify the user's signature $\sigma = (c, T, \pi)$ on message m, the verifier computes $e \leftarrow H_2(P_{pub}, ID, m, T, c)$ and checks if

$$c \stackrel{?}{=} H_1(P_{pub}, ID, cP_{pub} + e^{-1}(T - \pi P)).$$

If the equation holds with equality, return 1; otherwise, return 0.

3.1 Discussions

Note that T should be a nonce, that is, each value of T should only be used once. For each new signature generation, a new T should be used. In addition, the discrete logarithm of T should only be known to the signer. Otherwise, the user's secret key usk would be compromised. Hence in practice, the value of t should be destroyed once π is computed.

A Variant of BNN-IBS. We consider our IBS scheme described above as a variant of an IBS scheme called BNN-IBS which was proposed by Bellare et al. in [4–Sec. 7.3 of the full paper]. In the following, we explain the differences between them and show that our IBS scheme is more 'friendly' to low-power devices as it requires less storage or computation resources.

[1] For formality, one can include this set of parameters into the master public/private key pair (mpk, msk) and have this set of parameters be generated by some prime order elliptic curve cyclic subgroup generator.

In BNN-IBS scheme, the component c in the user's secret key usk is replaced with R and the computation of s is changed to $r + cx \bmod p$. In other words, component c is computed as $H_1(P_{pub}, ID, sP - cP_{pub})$ in BNN-IBS scheme, and the user's secret key usk now becomes (R, s, ID, P_{pub}) where $R = sP - cP_{pub}$. To sign a message m, the following steps are carried out and a signature $\sigma = (R, S, T, \pi)$ is generated.

1. Compute $S = sP$, randomly pick $t \in_R \mathbb{Z}_p$ and compute $T = tP$.
2. Compute $e \leftarrow H_2(P_{pub}, ID, m, R, S, T)$ and $\pi = t + es \bmod p$.

To verify the signature, $c = H_1(P_{pub}, ID, R)$ and $e = H_2(P_{pub}, ID, m, R, S, T)$ are first computed, and the following equations are then checked for equality.

$$\pi P \stackrel{?}{=} T + eS \tag{1}$$

$$S \stackrel{?}{=} R + cP_{pub} \tag{2}$$

First of all, comparing with our scheme, the signature size of BNN-IBS scheme is larger. Secondly, the BNN-IBS scheme requires one more scalar multiplication for computing S in signature generation than ours. Although this additional operation can be saved by precomputing S and then caching it at the signer side, it will then require the signer to have more memory space for caching this precomputed value. It turns out that the BNN-IBS scheme either requires one more scalar multiplication during the runtime of signature generation or needs more storage space than our scheme.

The BNN-IBS scheme has been shown to be euf-cma-ida secure (Def. 1) in the random oracle model [5] under the assumption that the discrete logarithm problem is hard (in [4–Sec. 7.3 of the full paper]). In the following, we show that our IBS scheme is also euf-cma-ida secure.

Theorem 1. *If there exists a PPT adversary $\mathcal{A}$ which wins the game of Def. 1 for the IBS scheme proposed above with probability at least ϵ, then there exists a PPT adversary $\mathcal{B}$ which wins the game of Def. 1 for the BNN-IBS scheme with probability at least ϵ.*

Proof. We describe how to construct $\mathcal{B}$ when $\mathcal{A}$ is given. As defined in Def. 1, a challenger $\mathcal{S}$ simulates a game which captures the notions of adaptive chosen message attacks and ID attacks. At the end of the game, the adversary in the game is to output a triple (ID^*, m^*, σ^*) such that ID^* is not corrupted, m is not signed by ID^* and σ^* is a valid signature of ID^* on message m^*.

Given an adversary $\mathcal{A}$ which breaks the euf-cma-ida security of the IBS scheme proposed above, we construct an adversary $\mathcal{B}$ which will break the euf-cma-ida security of the BNN-IBS scheme by running $\mathcal{A}$ and answering $\mathcal{A}$'s queries as follows.

- CreateUser: $\mathcal{B}$ relays such a query directly to $\mathcal{S}$ and relays back the answer from $\mathcal{S}$ to $\mathcal{A}$.
- Corrupt: $\mathcal{B}$ relays such a query to $\mathcal{S}$. Suppose the user's secret key usk returned by $\mathcal{S}$ is (R, s, ID, P_{pub}), $\mathcal{B}$ then queries $\mathcal{S}$ for $H_1(P_{pub}, ID, R)$. Suppose the answer of $\mathcal{S}$ is $\bar{c}$. $\mathcal{B}$ then sets c to $-\bar{c} \bmod p$ (will be explained

shortly) and sends (c, s, ID, P_{pub}) to $\mathcal{A}$ as the simulated answer to $\mathcal{A}$'s Corrupt query.

- Sign: $\mathcal{B}$ relays such a query to $\mathcal{S}$. Suppose $\mathcal{S}$'s answer to query $\mathsf{Sign}(ID, m)$ is $\sigma = (R, S, T, \pi)$, $\mathcal{B}$ then queries $\mathcal{S}$ for $H_1(P_{pub}, ID, R)$. Suppose the answer of $\mathcal{S}$ is $\bar{c}$, then it must be the case that $R = S - \bar{c}P_{pub}$ for having σ be valid according to Eq. (2) on page 504. $\mathcal{B}$ sets c to $-\bar{c} \bmod p$ and sends $\sigma' = (c, T, \pi)$ to $\mathcal{A}$ as the simulated answer to $\mathcal{A}$'s $\mathsf{Sign}(ID, m)$ query.
- H_1: For any query of H_1 from $\mathcal{A}$, $\mathcal{B}$ relays it to $\mathcal{S}$. Suppose the answer of $\mathcal{S}$ is $\bar{c}$, $\mathcal{B}$ sets the answer for $\mathcal{A}$ to $-\bar{c} \bmod p$.
- H_2: For any query of H_2 from $\mathcal{A}$, $\mathcal{B}$ handles it in the following two cases depending on the query input.

  Case 1: If the query is on $(P_{pub}, \tilde{ID}, \tilde{m}, \tilde{T}, \tilde{c})$ where $\tilde{T}$ is some point and $-\tilde{c} \bmod p$ is the answer of $\mathcal{S}$ on query $H_1(P_{pub}, \tilde{ID}, \tilde{R})$ for some point $\tilde{R}$, then $\mathcal{B}$ queries $\mathcal{S}$ for $H_2(P_{pub}, \tilde{ID}, \tilde{m}, \tilde{R}, \tilde{S}, \tilde{T})$ where $\tilde{S} = \tilde{R} - \tilde{c}P_{pub}$. Suppose the answer of $\mathcal{S}$ is $\bar{e}$. $\mathcal{B}$ sets the answer for $\mathcal{A}$ to $-\bar{e} \bmod p$.

  Case 2: Otherwise (that is, at least one component of the input does not satisfy the form shown in Case 1), $\mathcal{B}$ randomly picks a value in $\mathbb{Z}_p$ as the answer to $\mathcal{A}$. Consistency (for replying the same value for the same queries) is maintained by having a table of queries values and answers maintained by $\mathcal{B}$.

When $\mathcal{A}$ outputs a valid signature (c^*, T^*, π^*) with message m^* and identity ID^*, due to the random oracle assumption, $\mathcal{A}$ must have queried for $c^* = H_1(P_{pub}, ID^*, R^*)$ where R^* is some point in order to pass all the steps of signature verification described in Sec. 3 (Note that the return of $\mathcal{S}$ for that H_1 query is $-c^* \bmod p$.). $\mathcal{B}$ sets $\sigma^* = (R^*, S^*, T^*, \pi^*)$ where $S^* = R^* - c^*P_{pub}$ and outputs the triple (ID^*, m^*, σ^*).

Analysis. To check the correctness of the simulation, first note that the $usk^{New} = (c, s, ID, P_{pub})$ of our protocol satisfies

$$c = H_1^{New}(P_{pub}, ID, sP + cP_{pub})$$

while the $usk^{BNN-IBS} = (\bar{c}, s, ID, P_{pub})$ of BNN-IBS satisfies

$$\bar{c} = H_1^{BNN-IBS}(P_{pub}, ID, sP - \bar{c}P_{pub}).$$

By setting $c = -\bar{c} \bmod p$, we can see that the output of H_1^{New} is the complement of that of $H_1^{BNN-IBS}$. Hence in the simulation above, we set the answer to $\mathcal{A}$ for queries of H_1 to the complement of the answer made by $\mathcal{S}$ for obtaining the reduction. Due to the similar reason, we also need to set the answer to $\mathcal{A}$ for queries of H_2 to the complement of the corresponding answer made by $\mathcal{S}$.

Obviously, the running time of $\mathcal{B}$ is in polynomial of that of $\mathcal{A}$. In addition, from $\mathcal{A}$'s point of view, all queries are simulated or relayed correctly. $\mathcal{A}$ cannot distinguish a simulated environment and a real game. Hence, if $\mathcal{A}$ makes a forgery of the IBS scheme proposed above, the reduction above correctly transforms the forgery to a forgery of the BNN-IBS scheme. □

4 An ID-Based Key Exchange Protocol

We now propose an ID-based key exchange (KE) protocol. The KE protocol is built using the IBS scheme described above. In the following, we first describe our scheme, then analyze its security under the Canetti-Krawczyk model [10] (in short, CK-model), and finally in the next section, we show that the scheme is much faster than previously proposed protocols.

Let $k \in \mathbb{N}$ be a security parameter. Let A and B be the initiator and responder, respectively. They are identified by $ID_A, ID_B \in \{0,1\}^*$, respectively. Let the secret key usk_A of A be $(c_A, s_A, ID_A, P_{pub})$ and the secret key usk_B of B be $(c_B, s_B, ID_B, P_{pub})$, which are generated according to the User-key Generation algorithm described in Sec. 3. Suppose A and B have a unique session-id ψ shared already. Below is the description of the ID-based KE protocol which is illustrated in Fig. 1.

Step 1. A picks $t_\alpha \in_R \mathbb{Z}_p$, computes $\alpha = t_\alpha P$, and sends (ID_A, ψ, α) to B.

Step 2. Upon receipt of (ID_A, ψ, α), B picks $t_\beta \in_R \mathbb{Z}_p$, computes $\beta = t_\beta P$ and sends $(ID_B, \psi, \beta, c_B, T_B, \pi_B)$ to A, where (c_B, T_B, π_B) is B's signature on message $m_B = (\psi, \beta, \alpha, ID_A)$. B also computes the session key $\gamma = t_\beta \alpha$ and erases t_β.

Step 3. Upon receipt of $(ID_B, \psi, \beta, c_B, T_B, \pi_B)$, A checks the correctness of each component in the incoming message and checks if the signature verification algorithm $Ver(P_{pub}, ID_B, m_B, (c_B, T_B, \pi_B))$ returns 1, where $m_B = (\psi, \beta, \alpha, ID_A)$. If the verification succeeds, A sends $(ID_A, \psi, c_A, T_A, \pi_A)$ to B where (c_A, T_A, π_A) is A's signature on $m_A = (\psi, \alpha, \beta, ID_B)$. A then computes $\gamma' = t_\alpha \beta$, erases t_α, and outputs the session key γ' under session-id ψ.

Step 4. Upon receipt of $(ID_A, \psi, c_A, T_A, \pi_A)$, B checks the correctness of each component in the incoming message and determines if the signature verification $Ver(P_{pub}, ID_A, m_A, (c_A, T_A, \pi_A))$ returns 1, where $m_A = (\psi, \alpha, \beta, ID_B)$. If the verification succeeds, B outputs the session key γ under session-id ψ.

$$
\begin{aligned}
A \rightarrow B &: ID_A,\ \psi,\ \alpha \\
A \leftarrow B &: ID_B,\ \psi,\ \beta,\ c_B, T_B, \pi_B \\
A \rightarrow B &: ID_A,\ \psi,\ c_A, T_A, \pi_A
\end{aligned}
$$

Fig. 1. An ID-based KE protocol

4.1 Security Analysis

The protocol can be viewed as a Diffie-Hellman key exchange followed by a signature-based mutual authentication. T_A and T_B correspond to the key contributions of A and B, respectively, and signatures (c_A, T_A, π_A) and (c_B, T_B, π_B) of the IBS scheme proposed in the previous section correspond to the mutual authentication. In the full paper [19], we show that the protocol can be constructed using the modular approach introduced in [3, 10]. In the following, we skip the details of the model defined by Canetti and Krawzcyk [10] (in short, CK-model) and apply the CK-model directly to show the security of our protocol. In [10], the following protocol called Protocol SIG-DH has been shown to be secure under the CK-model.

$$\begin{aligned} A \to B &: ID_A,\ \psi,\ \alpha \\ A \leftarrow B &: ID_B,\ \psi,\ \beta,\ SIG_B(m_B) \\ A \to B &: ID_A,\ \psi,\ SIG_A(m_A) \end{aligned}$$

In the protocol, $m_B = (ID_B, \psi, \beta, \alpha, ID_A)$ and $m_A = (ID_A, \psi, \alpha, \beta, ID_B)$. To instantiate the signature scheme using the IBS scheme described in Sec. 3, we have SIG_A become (c_A, T_A, π_A) and SIG_B become (c_B, T_B, π_B). In addition, from the computation of e in the signature generation phase, we can see that some components in m_A and m_B are also redundant. By further eliminating those redundant components, the final optimized protocol will become the one shown in Fig. 1.

In the following, we further evaluate our ID-based KE protocol by considering some additional features or attacks that are not captured by the CK-model.

KGS Forward Secrecy. Our ID-based KE protocol constructed above using the modular approach of [3, 10] is essentially the Protocol SIG-DH of [10] which is secure under the CK-model. This also implies that the protocol satisfies perfect forward secrecy. Although the CK-model does not capture the KGS forward secrecy, we can still see that our protocol supports the KGS forward secrecy as session keys are solely derived from contributions α and β.

Key Compromise Impersonation Resilience (KCIR). As defined in [8], a protocol provides resistance to key compromise impersonation if compromise of the long-term secret of a party A does not allow the adversary to masquerade *to* A as a different party. To see that compromising A's secret usk_A does not allow the adversary to masquerade to A as B, we notice that the adversary has to provide a signature of B in the second message of the protocol before making A accept. As long as α is a nonce and B's signature is existentially unforgeable, the adversary cannot provide a correct signature. Similar reasons can be applied to explain the KCIR of B as well.

5 Performance Analysis

In Table 1, we summarize the number of different operations of some well-known ID-based KE protocols and our protocol proposed above. We ignore the time taken by conventional hash operations and point addition operations as they are much more efficient when compared with pairings, scalar multiplications, and map-to-point hash operations.

According to the state-of-the-art results in [1, 2], one pairing operation requires at least 10 times more multiplications in the underlying finite field than a point scalar multiplication does in the same finite field. Hence those pairing-based KE protocols are much slower than the one proposed in this paper. When comparing with old, non-pairing based protocols such as [16, 13, 12], our protocol is also

Table 1. Performance Comparison

	Smart [17]	Yi [18]	Chen-Kudla [11]	Our Protocol
Pairing	2	1	1	0
Scalar Multiplication	2	3	4	6
Map-to-point Hash	1	1	1	0

much faster because each communicating party of these old, non-pairing based protocols needs to do expensive modular exponentiation operations. For protocols proposed in [14], they can be implemented under an elliptic curve group and one of the protocols in [14] is also believed to support the KGS forward secrecy. On its performance, the protocol requires each communicating party to carry out seven scalar multiplications. Hence it is slightly less efficient than our protocol. In addition, the protocol is not known to be provably secure. In [9], Burmester showed that the protocol is vulnerable to an attack called triangle attack. Since the triangle attack is captured in the CK-model and our protocol is proven secure in this model, therefore, our protocol is not vulnerable to this attack.

6 Conclusion

In this paper, we proposed an ID-based signature scheme and showed that it is a variant of the BNN-IBS scheme proposed by Bellare et al. [4]. Our scheme is more efficient than BNN-IBS as it requires less storage space or less runtime computation. Using our ID-based signature scheme, we proposed a new ID-based KE protocol. The protocol does not require any pairing operation or map-to-point hash operation. It also supports the strongest KGS forward secrecy. For security analysis, we show that it can be constructed using the modular approach of [3]. On its efficiency, we showed that it is faster than all comparable ID-based KE protocols.

For the full version of this paper, please check [19].

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To Build a Blocklist Based on the Cost of Spam

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Abstract. We studied the objective of spam based on a financial profit truth - the cost for sending spam against anti-spam techniques should be little than the return from the negligible response from the recipients. Spammers have to leave some real contact information in the spam for the recipients to touch them easily, no matter what methods they use to fight against anti-spam techniques. In this paper, we present a method to automatically identify such contact information entities in spam, and build an online blocklist for the spam filters to classify spam, especial unsolicited commercial email (UCE).

1 Introduction

With rapid advance of Internet, email has become a very important communication medium to a great many people. Within two or three years, the spam problem has grown astronomically. According to the spam statistical report of Symantec, there are over 60 percent of worldwide Internet email identified as spam, about 80 percent of spam are involved in commercial solicitations (UCE) [1]. And beyond e-mail, instant-messaging spam and SMS spam on mobile phones have also become problems.

Nowadays, people have been developing different techniques in different ways to fight with spam. We are involving in a long battle between those who want to send spam (spammers) and those who want to stop it. Spammers often change their email boxes frequently, modify the style, format or content of message to fight against anti-spam techniques. However, when we investigate the objective of spammers, we find there also exists a financial profit truth in spam, especially in UCE – the cost used to succeed against anti-spam techniques and send spam should be little than the return from the negligible response from recipients. The profit makes it worth the expense of sending spam. Therefore, spammers must leave some real contact information in the spam so that a recipient can find out them easily if he interests in the content of a spam. Since current email service can not support electronic financial transactions independently.

In this paper, we describe a method to automatic identify such real contact information entities in the spam, and build an online blocklist containing them. We studied the economic cost for spammers to change different type of contact information, and developed an efficient searching algorithm for the blocklist according to the economic cost. In practical, this blocklist can be used to set up a personalized email filter for an

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email client or an email server in a company. It also can be used as the features to enhance the effectiveness of content-based spam filters based on machine learning methodologies [2] [3] [4]. The same approach also can be used in mobile telecom Short Message Service, or SMS spam filtering. In a collaborative filtering system [5] [6], when a user classifies an email as spam, signature is computed on the email and added into the collective knowledge database. A signature is computed on every new email received and compared to the database of known spam. If the signature matches one in database, it is deemed spam. The algorithm used to compute signatures here is the key. The same idea of the blocklist can be involved in computing the signature and makes the algorithm to be more robust (or fuzzy).

2 Identify Contact Information Entities in Spam

Email has provided a fast and efficient electronic communication medium to people, but it still lacks enough security techniques to support electronic commerce deals or transactions independently nowadays. A UCE spammer has to provide some real contact information in spam, otherwise a recipient can not know how to touch him even if he is interesting in the spam. To identify these contact information entities contained in spam, we analyzed three public spam corpora collected by SpamAssassin [7] and a personal spam corpus collected from author's email boxes. Our experimental results show that, we can identify 92.7% emails in these corpora to be containing at least one pieces of contact information in their message body (excluding email header). These contact information entities can be divided into four types: email box, Uniform Resource Locator, or URL link, telephone number (includes fax number) and mail address (including contact person or company name and the address).

When we want to use the contact information to build a blocklist for email filtering or classification, we shall consider what method spammers can use to attack the blocklist. Certainly spammers can avoid the detection of the blocklist by changing the contact information in spam frequently. Therefore we should design and implement our blocklist searching strategy based on the economic cost with respect to changing each type of contact information.

To our knowledge, we find the cost for changing each kind of contact information is quite different to a spammer who wants to change it frequently. We discuss the cost of four types of contact information as the following.

Email Box. Today a spammer can easily register a new email box from many email service providers without any fee, e.g. hotmail, yahoo. As the cost of delivery, storage, and processing is borne directly by the recipient, spam could be seen as the electronic equivalent of "postage-due" junk mail. A spammer often seek out and make use of vulnerable third-party systems such as open email relays and open proxy servers, which do not properly check who is using the email server and pass all emails to the destination address. Once an email box has been indicted as an inbox of a spammer, the spammer can replace it with a new email box in the new spam, and still keeps the box without losing his old customers. Since the concept of spam is personalized to individual. It is not easy for the email service provider to collect legal proof to terminate the email account if the spammer didn't send spam by using the box directly.

URL Link. A URL link in a spam usually redirects the recipient to visit a website set up by the spammer, or provides content information data for a spam composed with html format. It needs more technical skills for people to compose web pages. Establishing a website often costs much more, spammer has to pay registration fee of DNS domain name, monthly rent of service of the ISP. It will cost a spammer much more time and money if he wants to change the domain name, or move from this ISP to anther. Although there are some ISPs provide some free-charge web host service today, the resources what they provide are rather limited (for example, few ISPs provide virtual web host service for an independent domain). A website hosting there often looks lacking public reputation to attract people.

Telephone Number. The fee for renting a telephone or fax line is a bit expensive. The cost of changing a telephone number frequently is much more expensive and also not allowed by most telecommunication providers. On the other hand, a spammer will lose his old customers when he changes his phone or fax numbers.

Mail address. The cost for changing the address of company or home will be largest among four types of contact information, if the address is real one.

Concluding above discussion, if we use W to indicate the economic cost of changing one of contact information respectively, the amount of the cost can be indicated as below,

$$W_{email} < W_{URL} \leq W_{tel} < W_{address} \tag{1}$$

Since all email boxes and URL links are created obeying the standard of SMTP protocol, HTTP protocol and RFCs related to them. Telephone numbers are strings consisting of a sequence of 0-9 digital number. We can use a regular expression pattern matching algorithm to identify and extract these three types of contact information entities with high accuracy. For mail address, we have tried to use some machine learning methods to identify them in the emails. Unfortunately, the precision of the result is still not good enough to be present in this paper. However, we also discover that there are only 1.4% of emails manually identified as containing mail address in our experimental corpora. Furthermore, there are only 0.02% of emails containing only mail address without any other contact information entity inside. In this paper, we just use three types of contact information (email box, URL link and phone number) to present our idea.

3 Build a Personalized Online Blocklist

Different from common sense of term "blocklist" in anti-spam methodologies, our blocklist is rather similar to a blacklist of spammers. Since the blocklist contains contact information entities of spammers. We used the blocklist to set up a simple email filter. The email filter works as, when an email arrives, the email filter splits the email into sequence string lines firstly. Then it uses pattern matching technique to identify and extract three types of contact information in the lines of message body. After that, these contact information will be processed and compared with the patterns in the blocklist. If the matching result is satisfied the defined conditions, the email will be classified as a spam, otherwise the email will be passed through as a ham (good email).

3.1 Build the Blocklist with User Feedback

The blocklist works as the main part of an online email filter. At the initial state, the blocklist contains nothing. When an email is manually identified as a spam by the user, the email filter will automatically identify and extract the contact information entity in the email. Then the entities are converted into regular expression string patterns and added into the blocklist. If a new email contains same contact information arrives, the email filter will classify it as spam. On the other hand, if the email filter classifies a ham as a spam, or a spam as a ham, the error can be corrected by the user manually. Corresponding patterns in the blocklist will be updated to void same error. Such procedure is called as the aggregative training of the blocklist, as we know.

3.2 Data Structure of Blocklist

To handle different types of contact information, the blocklist consists of three pattern lists, which store corresponding type of contact information respectively (as shown in Fig.1.). A phone number is notated with a sequence of several digital numbers (0-9). We can directly save it into the list as a string. In this section, we will mainly describe how to process and store other two types of contact information entities, email box and URL link.

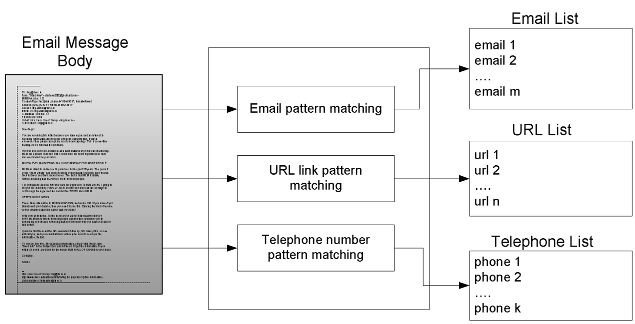

Fig. 1. Different contact information are processed and store in to three lists independently

A regular email box can be divided into two parts: an account name (or alias) and a DNS domain name of the mail server, as well as a URL link can be divided into two parts: a DNS name or IP address of the web server host, and a pathname of the webpage file. According to the rules of DNS domain name, we design and develop a suffix tree to construct Email List and URL List. Here we present an example to illustrate the suffix tree structure.

When we consider an email box *ilug@linux.ie*, we can split it into substrings, *ie, linux, @ilug*. For a URL link *http://www.linux.ie/mailman/listinfo/ilug*, it can be split into *ie, linux, www, mailman,listinfo, ilug*. Then *ie* becomes the root of the tree, substrings for DNS domain name become the left leaves, and substrings for user account or pathname become the right leaves respectively. Fig. 2 demonstrates how to use this suffix tree to notate an email box and a URL link.

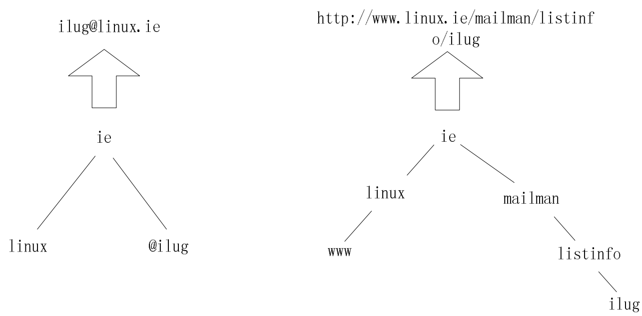

Fig. 2. A sample: using suffix tree to notate an email box and a URL link

The major drawback of using blocklist as email filter is that, with increasing amount of the elements in the blocklist, the performance of the filter will drop down. To handle huge amount of spam today, how to keep the good performance of the blocklist becomes a crucial problem we have to overcome. We consider two ways to optimize the query operation of the blocklist and reduce the drawback. First, we developed an optimization algorithm to keep the blocklist compact. This compacting algorithm has been used in optimizing Email list and URL list. Second, we apply dynamic programming algorithm to the suffix tree to implement an efficient approximate string matching query in the Email and URL lists.

3.3 Compacting the Blocklist

The basic idea of the algorithm is similar to candidate-elimination learning algorithm [8]. Each element in the Email or URL list contains several substrings to consist a regular expression pattern. We can think that these substrings compose a hypothesis. Here we present an example to describe this algorithm. If there is an element "*ie, linux, www, mailman, listinfo, ilug*" in URL list, then it can compose a hypothesis as below,

$$H_1 =< ie,\ linux,\ www,\ mailman,\ listinfo,\ ilug > \qquad (2)$$

In practical, this hypothesis tells us that a spammer can use the directory */mailman/listinfo* in the host *www.linux.ie* for spam. When a new element "*ie, linux, www, mailman, images*" is inserted into the list, another directory /mailman/images in the same host has been used for spam too.

$$H_2 =< ie,\ linux,\ www,\ mailman,\ listinfo,\ images > \quad (3)$$

Empirically we can assume that spammers possibly have right to use directory */mailman* for spam too. Then we get a new hypothesis as below,

$$H_3 = H_1 \vee H_2 =< ie,\ linux,\ www,\ mailman,\ ? > \quad (4)$$

When we apply this algorithm to the suffix tree of Email list and URL list, we get a result as shown in Fig. 3**.**

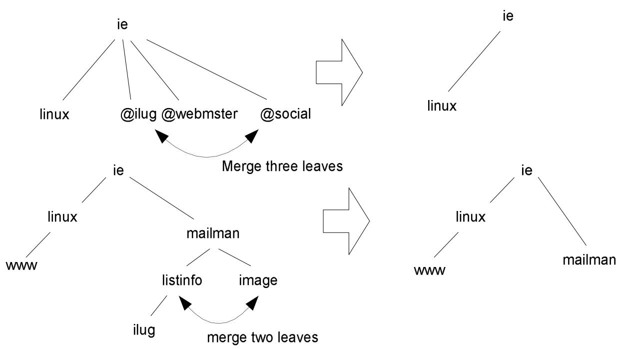

Fig. 3. Merge different leaves at same level in the suffix tree

To avoid blocking some common email service providers (e.g. hotmail.com, yahoo.com, gmail.com), we also set up a white list to keep them. The compacting algorithm will skip the compacting the patterns containing these domain names in the Email list.

3.4 Query the Blocklist

When an email arrives, the email filter identifies and extracts contact information from the message body respectively. Then each contact information entity will be compared with the patterns in the corresponding list. Actually this comparison is a search problem in the suffix tree. The search starts at the root, traverses the tree in left to right order, comparing corresponding nodes in the two trees one by one. The search will stop if it finds out an error (a pair of nodes doesn't match exactly). Otherwise it will continue to traverses whole nodes in the tree. Table 1 illustrates the details of the searching algorithm.

In order to improve performance of querying the blocklist, we also add two parameters for each pattern in the lists in our implementation. One is a counter to record the number of the pattern being matched, another records latest updated date of the pattern. We use two parameters in eliminating the patterns which are not counted any more as the spammers have changed their contact information along time.

Table 1. Dynamic programming algorithm for suffix tree matching

```
function searchList(string line, string array pattern)
 begin
     float similarity = 0;
     for each element in array pattern
        if(similarCal(line, pattern[]) >= similarity)
            similarity = similarCal(line, pattern[]);
            if(similarity = 1)
                  return similarity = 1;
            endif
        endif;
     endfor;
     return similarity;
 end
function similarCal(string pattern, string contact)
 begin
    integer matchdtrings = 0;
    string array host_pattern = split (pattern);
    string array path_pattern = split (pattern);
    string array host_contact = split (contact);
    string array path_contact = split (contact);
    for each element in host_pattern
       if(host_pattern[].equal.host_contact[])
           matchstring = matchstring + 1;
       else break;
       endif
    endfor
    if(matchstring = length(host_pattern))
       for each element in path_pattern
         if(path_pattern[].equal.path_contact[])
             matchstring = matchstring + 1;
         else break;
         endif
       endfor
    endif
    if(matchstring >=2)
         return matchstring/length(pattern);
    else
         return 0;
    endif
 end.
```

3.5 Searching Priority

We assign the priority order of searching three lists with respect to the economic cost of them. As the discussion in Section 2, the searching order for three lists shall be arranged as: Firstly, searching algorithm searches Telephone list to do exact string matching, then it searches URL list by using approximate string matching, finally it will search Email list. The search will be stopped if a full pattern matching result is returned.

Another search strategy we also used in our experiments is that. If an email contains at least one URL link or telephone number entity, then searching algorithm just searches corresponding lists and skips the steps for searching Email list.

4 Experiments

To test the performance of our blocklist, we use three public spam corpus suits (named public corpus 1, public corpus 2 and public corpus 3 in this paper) and a public hard ham corpus collected by SpamAssassin, a personal spam corpus and a personal ham corpus collected from author's inbox in practical. Public corpus 1 and 2 contain 500 emails respectively, and public corpus 3 contains 1,391 emails. There are 250 emails in public hard ham corpus, 1,126 emails in personal spam corpus and 350 emails in personal ham corpus. During processing of our blocklist filter, there are total 16,897 email box, 18,376 URL link, and 2,010 telephone number entities can be identified and extracted in the four spam corpuses. There are total 252 emails can not be identified, the average percentage is 7.13%. Fig. 5 depicts the number of different entities distribution of each corpus.

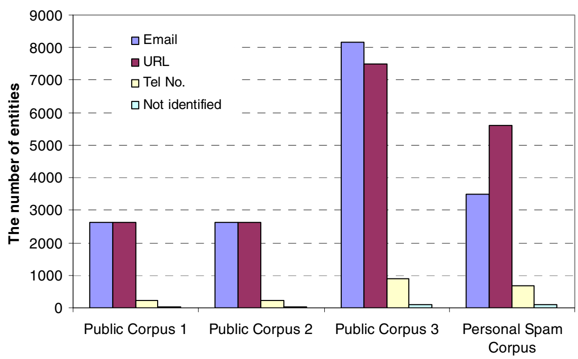

Fig. 4. The number of contact information entities in the experimental corpuses

To test the efficiency of the compacting algorithm, we write another program which builds a URL List to contain all different URL patterns without any pattern merging. We use this program to measure the amount of different URL links in each corpus, and compare them with the results of compacted URL list. Fig. 6 shows the results. The average of compression ratio can go up to 10:1.68 in the experiment.

We also concern false positives error problem in the blocklist filter: in some case, a ham might be classified as a spam because it contains a contact information entity has been added into the blocklist. We processed the public hard ham corpus and compared the result with public corpus 2. In the result of public hard ham corpus, there are 393 email boxe, 1085 URL link, and 67 Telephone number entities. The overlap of URL links are 17 (1.57%), email boxes are 5 (1.2%), and there is no overlap of telephone number in two corpuses. When we compare the results of personal ham and spam corpuses, there is no overlap entity between the two corpora.

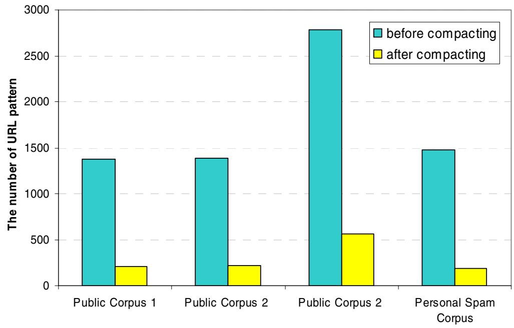

Fig. 5. The comparison result for the efficiency of compacting algorithm

5 Related Work and Our Conclusions

Fighting with spam is a constant battle. We can not expect that we can develop a new technology to solve the problem at all. There is no "right way" or "best way". The best way to reduce spam is to use as many ways as possible in a coordinated and co-operative manner [9]. The heart of an anti-spam system is classifying messages, and filtering them based on the user or organizational preferences. Setting up a blacklist might be the first method for spam reduction. In 1975, Postel had proposed the destination host/IMP (mail server) would keep a list of sources to refuse [10]. Nowadays blacklist is commonly used in many ISPs and public Web email service providers, e.g. hotmail. Bayesian filtering has become a popular spam-filtering technique today. In a Bayesian filter, a message is tokenized into words to count their occurrence probabilities. If a message contains many words which are only used in spam, and few which are never used in spam, it is likely to be spam. Well-tuned Bayesian filters can make significant performance to individual user or organization [2] [3] [5]. But their performance is lowered drastically when they run at ISP level [11].

Beside these work, we are also interesting in information extraction and segmentation by using learning and linguistic methodologies. W. Cohen recent paper presents a learning approach for extracting signature and reply lines from email. They compared several learning algorithms, including recently developed sequential learning algorithms (Conditional Random Fields, or CRFs, Conditional Markov Models, or CMMs and Voted Perceptrons), and no-sequential learning algorithms (booted decision tree, Naïve Bayes and SVM). The experimental extracting accuracy has achieved higher than 98-99% in a 617-messages dataset [12]. Recalling the mail address extraction problem we mentioned in this paper. How to identify and extract such piece of string is still a barrier we want to overcome in the future work.

In this paper, we present an implementation of automatically identifying and extracting three types of contact information entities in spam. These contact information entities are converted into string matching patterns to build a blocklist for spam filtering. To overcome the main drawback of the blocklist – the query time increases with the number of patterns inside the blocklist, we developed a compacting algorithm to optimize the patterns in the list aiming at the construction of email address and URL link. The experimental result shows the compacting algorithm is high efficient. By studying and comparing the economic cost of different type of contact information, we concluded a priority order for enhancing the searching performance of the blocklist at the searching strategy level. For further work, we are planning to use the same idea of blocklist to implement a collaborative filtering system. We believe there are still more room to improve our work presented in this paper.

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Total Dominating Set Games*

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Abstract. In this paper, we consider cooperative games arising from total domination problem on graphs. We introduce two games, rigid total dominating set game and relaxed total dominating set game, and focus on their cores. First, a common necessary and sufficient condition for the balancedness of the two total dominating set games is obtained. Next, the computational issues on cores are discussed. We prove that in general the problems of testing the balancedness and testing the membership of the core are all $\mathcal{NP}$-hard for both total dominating set games.

Keywords: Rigid total dominating set game, relaxed total dominating set game, core, balancedness, $\mathcal{NP}$-hard.

1 Introduction

A cooperative cost game $\Gamma = (N, \nu)$ consists of a player set $N = \{1, 2, \cdots, n\}$ and a characteristic function $\nu : 2^N \rightarrow R$, where for each subset S of N (called a coalition), $\nu(S)$ represents the cost incurred by the coalition of players in S without participation of other players. The central problem in cooperative game is how to distribute the total cost $\nu(N)$ among the individual players in a 'fair' way. Different requirements for fairness and rationality lead to different distributions of cost which are generally referred to solution concepts of cooperative games. Among many of these solutions concepts, the core has attracted much attention from researchers. A distribution vector $x = \{x_1, x_2, \cdots, x_n\}$ is called an *imputation* of the game $\Gamma = (N, \nu)$ if $\sum_{i \in N} x_i = \nu(N)$ and $\forall i \in N : x_i \leq \nu(\{i\})$ (individual rationality). The *core* of the game $\Gamma = (N, \nu)$ is defined as:

$$\text{Core}(\Gamma) = \{x \in R^n : \ x(N) = \nu(N) \text{ and } x(S) \leq \nu(S), \ \forall S \subseteq N\},$$

where $x(S) = \sum_{i \in S} x_i$ for $S \subseteq N$. The set of constraints imposed on Core(Γ) is called subgroup rationality which ensures that no coalition has an incentive

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to split from the grand coalition N and does better on its own. If the core of a game is non-empty, then the game is called *balanced*.

In this paper, we discuss cooperative cost games that arise from total domination problem on graphs. Given a graph $G = (V, E)$ with vertex weight function $\omega : V \rightarrow R_+$ that assigns a fixed cost to each vertex in V, a total dominating set of G is a subset $T \subseteq V$ such that every vertex in V is adjacent to at least a vertex in T. The total domination problem is to find a minimum weighted total dominating set of G, i.e. a total dominating set that minimizes the total cost of its vertices. Total dominating set were introduced by Cockayne, Dawes and Hedetniemi [1], and total domination problems have been extensively studied. An illustration of total domination problem is the following example. Consider a number of regions in which certain rescue facilities are going to be placed. There is a fixed cost for the placement of a facility in a certain region. The problem is to select the regions in which to place rescue facilities at minimum placement cost in total, such that each region should be rescued and helped by a rescue facility placed in its neighboring region when necessary. This problem can be viewed as a total domination problem on the graph of the connections among the regions.

A nature question arising from the above example is how to allocate the total cost of placing the rescue facilities among the participating regions. In this paper, we use cooperative game theory to study this problem. We introduce two closely related cooperative cost games, the rigid total dominating set game and the relaxed total dominating set game, to model the cost allocation problem.

Let $G = (V, E; \omega)$ be a graph where vertices in V correspond to the considered regions, edges in E represent pairs of regions which are neighbors and $\omega : V \rightarrow R_+$ describes the fixed cost of placement in each region. The two total dominating set games are both defined on G with the player set being V. They have the same cost value of the grand coalition which equals the minimum weight of a total dominating set of G, while may take different cost value of other coalitions. In the rigid total dominating set game, coalitions are only allowed to place rescue facilities in their own regions, i.e. each coalition will find a total dominating set with minimum weight only in the induced subgraph corresponding to itself. By dropping this requirement, we establish the relaxed total dominating set game. In spite of the differences between the two games, we present a close relationship between their cores, and prove a common necessary and sufficient condition for the balancedness of the two games.

The main technique we use in this work is linear program duality characterization of cores. Linear integer programming and the duality theory have proven itself a very powerful tool in the study of cores. Shapley and Shubik [13] formulated a two-sided market as the assignment game, and showed that the core is exactly the set of optimal solution of a linear programming dual to the optimal assignment problem. This approach is further exploited in the study of linear production game [12], partition game [6], packing and covering games [3], etc. Also the linear and integer programming techniques find its application in some games derived from facility location problems, such as, facility location game [9] and dominating set games [14]. For the facility location game defined on a tree, Curiel [2] proved that the core is always non-empty and an element of the core can be obtained via the corresponding dual programming. Goemans and Skutella [9] studied facility location games in general cases. They establish a special *canonical* relaxation of integer program formulation for a facility location problem and showed that the core of a facility location game is non-empty if and only if there is no integrality gap for this relaxation problem. Velzen [14] studied three kinds of cooperative games that arise from the weighted domination problem on graphs. It was shown that the core of each game is non-empty if and only if the corresponding linear programming relaxation of the weighted domination problem has an integer optimal solution, and in this case, an element in the core can be found in polynomial time. Our work in this paper is in much spirits as that of Velzen's work.

This paper is organized as follows. In section 2, we give definitions of total dominating set games and present a relationship between the cores. Section 3 is dedicated to the balancedness of both games. We first present characterizations of the cores, and then derive a common necessary and sufficient condition for the balancedness of the two games. In section 4, we consider the computational complexity issues concerning the core. It is proved that for general graphs, the problems of testing balancedness and testing whether a given cost distribution belongs to the core are both $\mathcal{NP}$-hard for both total dominating set games.

2 Definitions of Total Dominating Set Games

Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E. Two distinct vertices $u, v \in V$ are called *adjacent* if $(u, v) \in E$. For any non-empty set $V' \subseteq V$, the induced subgraph by V', denoted by $G[V']$, is a subgraph of G whose vertex set is V' and whose edge set is the set of edges having both endpoints in V'. The *open neighborhood* $N(v)$ of vertex $v \in V$ consists of the vertices adjacent to v, i.e., $N(v) = \{u \in V : (u, v) \in E\}$. For any subset $S \subseteq V$, we denote $N(S) = \bigcup_{v \in S} N(v)$.

A *total dominating set* of graph G is a set of vertices $T \subseteq V$ such that $N(v) \cap T \neq \emptyset$ for each $v \in V$. That is, every vertex in V is adjacent to at least one vertex in T. Let $\omega : V \to R_+$ be a cost function on the vertices. The *total domination problem* is to find a so-called minimum weighted total dominating set of G, which minimizes the total cost of its vertices. The cost of a minimum weighted total dominating set is called the *weighted total domination number*, denoted by $\gamma_t(G, \omega)$. Throughout the paper, we assume that graph G has no isolated vertex to ensure the existence of total dominating set of G.

Let $G = (V, E; \omega)$ be a graph with vertex weight function $\omega : V \to R_+$. The *rigid total dominating set game (rigid TDS game)* $\Gamma = (V, c)$ corresponding to G is defined as:

1) The player set is $V = \{1, 2, \cdots, n\}$;
2) For each coalition $S \subseteq V$,

$$c(S) = \begin{cases} \gamma_t(G[S], \omega) & \text{if } G[S] \text{ has total dominating set} \\ +\infty & \text{otherwise} \end{cases}$$

In the rigid TDS game, each coalition can not place rescue facilities in vertices not belonging to itself. We define another related game, *the relaxed total dominating set game (relaxed TDS game)*, by dropping the requirement that coalitions are only allowed to use vertices corresponding to members of the coalition. Formally, the *relaxed TDS game* $\widetilde{\Gamma} = (V, \tilde{c})$ corresponding to G is defined as:

1. The player set is $V = \{1, 2, \cdots, n\}$;
2. For each coalition $S \subseteq V$,

$$\tilde{c}(S) = \text{mim}\{\gamma_t(G[T], \omega) : T \supseteq S \text{ and } G[T] \text{ has total dominating set}\}.$$

Since coalitions have more choice of placing the facilities in the relaxed TDS game than in the rigid TDS game, for all $S \subset V$, it holds that $c(S) \geq \tilde{c}(S)$. For the grand coalition V, $c(V) = \tilde{c}(V) = \gamma_t(G, \omega)$. It follows that

$$\text{Core}(\widetilde{\Gamma}) \subseteq \text{Core}(\Gamma). \tag{2.1}$$

Moreover, we show that $\text{Core}(\widetilde{\Gamma})$ coincides with the nonnegative part of $\text{Core}(\Gamma)$.

Theorem 2.1. *Let $\Gamma = (V, c)$ and $\widetilde{\Gamma} = (V, \tilde{c})$ be the rigid and relaxed TDS games corresponding to graph $G = (V, E; \omega)$, respectively. Then we have*

$$\text{Core}(\widetilde{\Gamma}) = \text{Core}(\Gamma) \cap R^n_+. \tag{2.2}$$

Proof. It is easy to see that the relaxed TDS game $\widetilde{\Gamma} = (V, \tilde{c})$ is monotonic, i.e., $\tilde{c}(S) \leq \tilde{c}(T)$ for every $S \subseteq T$. For each $x \in \text{Core}(\widetilde{\Gamma})$, we have that $x_i = \tilde{c}(V) - \sum_{j \in V \setminus \{i\}} x_j \geq \tilde{c}(V) - \tilde{c}(V \setminus \{i\}) \geq 0$ for every $i \in V$. Also followed from (2.1), it holds that $\text{Core}(\widetilde{\Gamma}) \subseteq \text{Core}(\Gamma) \cap R^n_+$.

On the other hand, we shall show that $\text{Core}(\Gamma) \cap R^n_+ \subseteq \text{Core}(\widetilde{\Gamma})$. Let $x \in \text{Core}(\Gamma) \cap R^n_+$. Clearly, $x(V) = c(V) = \tilde{c}(V)$. Let $T \subset V$ be an arbitrary subset such that $c(T) > \tilde{c}(T)$, and let $T_0 \subseteq V$ be a subset such that $T \subseteq N(T_0) = \overline{T}$ and $\omega(T_0) = \tilde{c}(T)$. It follows that $c(\overline{T}) = \tilde{c}(\overline{T}) = \tilde{c}(T)$. Hence we have

$$x(T) \leq x(\overline{T}) \leq c(\overline{T}) = \tilde{c}(T),$$

where the first inequality holds because $x \geq 0$ and the second inequality holds because $x \in \text{Core}(\Gamma)$. Therefore, $x \in \text{Core}(\widetilde{\Gamma})$. ■

3 Balancedness of Total Dominating Set Games

In this section, we first provide descriptions of the cores for the rigid and relaxed TDS games. Based on these descriptions, we prove a common necessary and sufficient condition for the balancedness of the two games.

For a graph $G = (V, E)$, the adjacent matrix of G, denoted by $A(G) = [a_{ij}]$, is a $|V| \times |V|$-matrix with rows and columns indexed by the vertices in V respectively, where $a_{ij} = 1$ if vertex i and j are adjacent, and $a_{ij} = 0$ otherwise. Then the total domination problem can be formulated as the following 0-1 program:

$$\text{IP}(G): \quad \gamma_t(G, \omega) = \min\{\omega x : A(G)x \geq 1, x \in \{0, 1\}^{|V|}\}. \tag{3.1}$$

The relaxed TDS game $\widetilde{\Gamma} = (V, \tilde{c})$ belongs to the class of covering games introduced in Deng et al. [3]. The following theorem provides a characterization of the core and a necessary and sufficient condition for the balancedness of the relaxed TDS game $\widetilde{\Gamma}$. For the proof of this theorem we refer to Deng et al. [3]. Consider the LP relaxation LP(G) and its dual DLP(G) of IP(G):

$$\begin{array}{ll} \text{LP}(G): & \min\{\omega x : A(G)x \geq 1, x \geq 0\}; \\ \text{DLP}(G): & \max\{y1 : yA(G) \leq \omega, y \geq 0\}. \end{array} \tag{3.2}$$

Theorem 3.1. *Let $\widetilde{\Gamma} = (V, \tilde{c})$ be the relaxed TDS game corresponding to graph $G = (V, E; \omega)$. Then Core$(\widetilde{\Gamma}) \neq \emptyset$ if and only if there is no integrality gap for the relaxation LP(G) of the total domination problem on graph G. In such case, a vector $z = (z_1, z_2, \cdots, z_n)$ is in the core if and only if it is an optimal solution to the dual program DLP(G).*

In the rest of this section, we will show that the balancedness of the rigid TDS game is equivalent to that of the relaxed TDS game. That is, both TDS games are balancedness if and only if the LP relaxation LP(G) has an integer optimal solution. For this purpose, we first introduce a kind of vertex subset, called *basic T-set*, which plays an important role in the description of the core elements for the rigid TDS game.

Definition. Let $G = (V, E)$ be a graph. A subset $B \subseteq V$ is called a *basic T-set* of G if it satisfies one of the following conditions:

(1) $G[B] = K_3$ (the complete graph with 3 vertices);
(2) $|B| \geq 2$, there exists a vertex $v \in B$ such that $G[B]$ is a v-star, i.e., $B \subseteq \{v\} \cup N(v)$ and any two vertices in $B \setminus \{v\}$ are not adjacent ($G[B] = K_2$ is included in this case).

The set of all basic T-sets of G is denoted by $\mathcal{B}$.

Let T be a total dominating set of graph $G = (V, E)$. It is easy to see that T can be partitioned into several basic T-sets $B_1, B_2, \cdots, B_k \in \mathcal{B}$, (i.e. $B_i \cap B_j = \emptyset$ and $\bigcup_{i=1}^{k} B_i = T$), and correspondingly, the vertex set V can be partitioned into k disjoint subset $V_1, V_2, \cdots, V_k$ such that $B_i \subseteq V_i \subset N(B_i)$ $(i = 1, 2, \cdots, k)$ and $\bigcup_{i=1}^{k} V_i = V$. In the next lemmas, we first provide a description of the core elements of the rigid TDS game, and then make use of this description to show that if the rigid TDS game is balanced, then there exists a nonnegative core element. (In the rest of this paper, for simplicity, we let $\omega(S) = \sum_{i \in S} \omega_i$ for each subset $S \subseteq V$.)

Lemma 3.2 *Let $\Gamma = (V, c)$ be the rigid TDS game corresponding to graph $G = (V, E; \omega)$. It holds that $x \in$ Core(Γ) if and only if*

(1) $x(V) = c(V) = \gamma_t(G, \omega)$;

(2) For each basic T-set $B \in \mathcal{B}$ and each subset $S : B \subseteq S \subseteq N(B)$, $x(S) \leq \omega(B)$.

Proof. Suppose that $x \in \text{Core}(\Gamma)$. Then we have $x(V) = c(V)$. For each $B \in \mathcal{B}$, and each subset $S : B \subseteq S \subseteq N(B)$, B is a total dominating set of S. It implies that S is a coalition with cost at most $\omega(B)$, that is, $x(S) \leq c(S) \leq \omega(B)$.
Now we prove its sufficiency. Let S be an arbitrary coalition. If there is no total dominating set in $G[S]$, then $x(S) < c(S) = +\infty$. Otherwise, let $T \subseteq S$ be a minimum weight total dominating set of $G[S]$, i.e. $S \subseteq N(T)$ and $\omega(T) = c(S)$. Let $\{B_1, B_2, \cdots, B_t\}$ be a basic T-set partition of T, then there exists a corresponding partition $\{S_1, S_2, \cdots, S_t\}$ of S such that $B_i \subseteq S_i \subseteq N(B_i)$ $(i = 1, 2, \cdots, t)$. Hence, we have

$$x(S) = \sum_{i=1}^{t} x(S_i) \leq \sum_{i=1}^{t} \omega(B_i) = \omega(T) = c(S),$$

where the inequality follows from our assumption 2). Therefore, $x \in \text{Core}(\Gamma)$. ■

Lemma 3.3. *Let $\Gamma = (V, c)$ be the rigid TDS game corresponding to graph $G = (V, E; \omega)$. If Core$(\Gamma) \neq \emptyset$, then there exists an $x \in$ Core(Γ) such that $x \geq 0$.*

Proof. Suppose $x \not\geq 0$ for all $x \in \text{Core}(\Gamma)$. Let $y = (y_1, y_2, \cdots, y_n)$ be an element of Core(Γ) such that the number of zero component is as large as possible. Since $y \not\geq 0$, we assume that $y_{i_0} < 0$ and $j_0 = \text{argmin}\{d_G(i_0, j) : y_j > 0\}$, that is, j_0 is the vertex nearest to vertex i_0 with $y_{j_0} > 0$. Then we define another cost allocation $\tilde{y}$ as follows:

$$\tilde{y}_k = \begin{cases} y_{i_0} + \varepsilon & k = i_0 \\ y_{j_0} - \varepsilon & k = j_0 \\ y_k & i \in N \setminus \{i_0, j_0\} \end{cases}$$

where $\varepsilon = \min\{y_{j_0}, -y_{i_0}\}$. It is obvious that $\tilde{y}(V) = y(V)$, and $\tilde{y}_{i_0} \leq 0$, $\tilde{y}_{j_0} \geq 0$ and at least one of $\tilde{y}_{i_0}$ and $\tilde{y}_{j_0}$ is zero. We shall show that $\tilde{y}$ is also an element of Core(Γ).

According to Lemma 3.2, showing that $\tilde{y} \in \text{Core}(\Gamma)$ boils down to showing that for all $B \in \mathcal{B}$ and $S : B \subseteq S \subseteq N(B)$, it holds that $\tilde{y}(S) \leq \omega(B)$. Since $y \in \text{Core}(\Gamma)$, we only need to consider coalitions containing i_0 and not containing j_0, which has the allocated cost at $\tilde{y}$ is larger than that at y. Let $B \in \mathcal{B}$ and let $S : B \subseteq S \subseteq N(B)$ be a subset such that $i_0 \in S$ and $j_0 \notin S$. We distinguish between three cases.

Case 1. $j_0 \in N(B)$.

Then $S' = S \cup \{j_0\} \subseteq N(B)$ and

$$\tilde{y}(S) \leq \tilde{y}(S) + \tilde{y}_{j_0} = \tilde{y}(S') = y(S') \leq \omega(B).$$

The last inequality holds because $y \in \text{Core}(\Gamma)$ and $B \subseteq S' \subseteq N(B)$.

Case 2. $j_0 \notin N(B)$ and $i_0 \in S \setminus B$.

Then $S' = S \setminus \{i_0\}$ satisfies $B \subseteq S' \subseteq N(B)$, and hence

$$\tilde{y}(S) = \tilde{y}(S') + \tilde{y}_{i_0} = y(S') + \tilde{y}_{i_0} \leq \omega(B).$$

The second equality holds because $\tilde{y}_i = y_i$ for all $i \in S \setminus \{i_0\}$, and the last inequality holds because $y \in \text{Core}(\Gamma)$ and $\tilde{y}_{i_0} \leq 0$.

Case 3. $j_0 \notin N(B)$ and $i_0 \in B$.

First we suppose $B' = B \setminus \{i_0\} \in \mathcal{B}$. Let $U = \{i_0\} \cup [(N(i_0) \cap S) \setminus B]$. Then $S \setminus U$ satisfies that $B' \subseteq S \setminus U \subseteq N(B')$ and for all $k \in U$, $d_G(i_0, k) \leq d_G(i_0, j_0)$, which implies that $y_k \leq 0$. Hence we have

$$\tilde{y}(S) = \tilde{y}(S \setminus U) + \tilde{y}(U) \leq y(S \setminus U) \leq \omega(B') = \omega(B) - \omega(i_0) \leq \omega(B).$$

Secondly, we suppose $B' = B \setminus \{i_0\} \notin \mathcal{B}$. Then i_0 and B must be the case either $G[B] = K_2$ or $G[B]$ ($|B| > 2$) is an i_0-star, which we depict in figure 1. It implies that $d_G(i_0, k) < d_G(i_0, j_0)$ for all $k \in S$ and $y_k \leq 0$. Hence $\tilde{y}(S) \leq 0 \leq \omega(B)$.

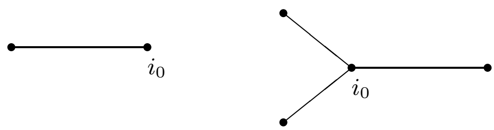

Fig. 1. The position of vertex i_0 in B

Therefore, we conclude that $\tilde{y} \in \text{Core}(\Gamma)$ and $\tilde{y}$ has the number of zero component less than that in y by 1 or 2, which contradicts to our assumption of y. Hence there must be an element $x \geq 0$ in Core(Γ). ■

In the the following theorem, a common necessary and sufficient condition for the balancedness of the two TDS games is given.

Theorem 3.4. *Let $\Gamma = (N, c)$ and $\widetilde{\Gamma} = (N, \tilde{c})$ be the rigid and relaxed TDS games corresponding to graph $G = (V, E; \omega)$, respectively. The following statements are equivalent:*

1) $\Gamma = (V, c)$ is balanced;
2) $\widetilde{\Gamma} = (V, \tilde{c})$ is balanced;
3) LP relaxation LP(G) has integer optimum solution.

In such case, an optimal solution to the dual program of LP(G) is in both Core(Γ) and Core$(\widetilde{\Gamma})$.

Proof. The equivalence of 2) and 3) follows from Theorem 3.1. "2) → 1)" follows from the observation (2.1) that Core$(\Gamma) \subseteq$ Core$(\widetilde{\Gamma})$. And "1) → 2)" can be proved by Lemma 3.3 and the result Core$(\widetilde{\Gamma})$ = Core$(\Gamma) \cap R^n_+$ given in Theorem 2.1. ■

In general, the corresponding TDS games may have empty core. However, it was proved that for some kinds of special graphs, such as, trees and interval graphs, LP(G) always has integer optimal solutions [10, 11], it follows that the corresponding TDS games are always balanced and a core element can be computed by solving the dual program of LP(G) in polynomial time.

4 Computational Complexity on Cores

The computational complexity as a rational measure for game theoretical concepts has attached more and more attention recently. Deng and Papadimitriou [4] found a game for which the core is nonempty if and only if a certain imputation (Shapley value in this case) is in the core. For the minimum cost spanning tree game, the core is always non-empty and a core element can be found in polynomial time [8], however, Faigle, et al. [5] showed that the membership testing problem is *co-NP*-complete. Deng, et al. [3] discussed the complexity concerning the core for a class of combinatorial optimization games. Goemans and Skutella [9] recently showed that, for a facility location game, if the core is nonempty, a core element can be found in polynomial time, and membership testing problem can also be solved in polynomial time. However, it is $\mathcal{NP}$-complete to decide whether the core is non-empty.

The computational complexity issues concerning the cores of the TDS games will be the focus of this section. We first prove that for general graphs the problem of testing balancedness for both TDS games is $\mathcal{NP}$-complete. As a corollary, we show that the problem of testing whether a given distribution belongs to the core is also $\mathcal{NP}$-hard. However, when the core is non-empty, finding a core element can be carried out in polynomial time for both games by Theorem 3.4.

Theorem 4.1. *Let $\Gamma = (N, c)$ and $\widetilde{\Gamma} = (N, \tilde{c})$ be the rigid and relaxed TDS game corresponding to graph $G = (V, E; \omega)$, respectively. Then it is $\mathcal{NP}$-complete to decide whether the core of $\Gamma = (N, c)$ or $\widetilde{\Gamma} = (N, \tilde{c})$ is non-empty.*

Proof. By Theorem 3.4, it suffices to show that it is $\mathcal{NP}$-complete to decide whether or not the LP relaxation LP(G) has an integer optimal solution. The problem of deciding whether LP(G) has an integer optimal solution is obviously in $\mathcal{NP}$. To prove the $\mathcal{NP}$-hardness, we construct a polynomial time transformation from a well known $\mathcal{NP}$-complete problem, 3-SATISFIABILITY [7]. Consider an arbitrary instance of the 3-SATISFIABILITY:

Instance: Collection $\mathcal{C} = \{C_1, C_2, \cdots, C_m\}$ of clauses on a finite set $U = \{x_1, x_2, \cdots, x_n\}$ of variables such that $|C_i| = 3$, $i = 1, 2, \cdots, m$.

Question: Is there a set of values for these Boolean variables (called a truth assignment) in U such that all the clauses in $\mathcal{C}$ are true?

Construct a graph $G = (V, E)$ as follows:

a) Corresponding to each variable x_i in U, there are three vertices $\{v_i, a_i, \overline{v}_i\}$. The two vertices v_i and $\overline{v}_i$ represent literals x_i and $\overline{x}_i$, respectively. The vertex a_i is connected to both x_i and $\overline{x}_i$.

b) Corresponding to each clause $C_j = (\tilde{x}_{j1}, \tilde{x}_{j2}, \tilde{x}_{j3})$ in $\mathcal{C}$ is a vertex b_j. Denote $W = \{b_1, b_2, \cdots, b_m\}$.

c) The vertex b_j corresponding to $C_j = (\tilde{x}_{j1}, \tilde{x}_{j2}, \tilde{x}_{j3})$ is connected to three vertices representing the corresponding literals in clause C_j respectively. For example, the vertex b_1 corresponding to clause $C_1 = (x_1, \overline{x}_2, x_3)$ is connected to three vertices v_1, $\overline{v}_2$ and v_3.

d) The vertex set of the graph G is $V = \bigcup_{i=1}^{n}\{v_i, a_i, \overline{v}_i\} \cup W$. The vertex weight function is defined as:

$$\omega(u) = \begin{cases} 4 & u \in W \\ 1 & u \in V \setminus W \end{cases}$$

The graph $G = (V, E; \omega)$ is depicted in Figure 2.

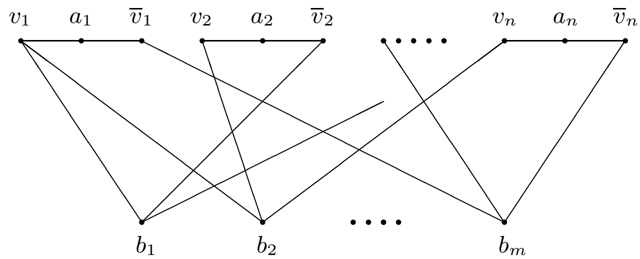

Fig. 2. Graph $G = (V, E)$

Now we claim that the 3-SATISFIABILITY problem is satisfiable if and only if the LP relaxation LP(G) for the total domination problem on the constructed graph G has an integer optimal solution.

It is easy to verify that the solution z given in the following is an optimal solution to LP(G), and the optimum value of LP(G) is $2n$:

$$z(v) = \begin{cases} 1/2 & v = v_i, \overline{v}_i \; i = 1, 2, \cdots, n \\ 1 & v = a_i \quad\; i = 1, 2, \cdots, n \\ 0 & v = b_j \quad\; j = 1, 2, \cdots, m \end{cases} \tag{4.1}$$

We first assume that the 3-SATISFIABILITY problem is satisfiable. Let t be a truth assignment satisfying all the clauses in $\mathcal{C}$. We define a 0-1 function $y : V \rightarrow \{0, 1\}$ as follows:

For $t(x_i) =$ "true", let $y(v_i) = y(a_i) = 1$ and $y(\overline{v}_i) = 0$, $\forall i \in \{1, 2, \cdots, n\}$;
For $t(x_i) =$ "false", let $y(\overline{v}_i) = y(a_i) = 1$ and $y(v_i) = 0$, $\forall i \in \{1, 2, \cdots, n\}$;
For each b_j, let $y(b_j) = 0$, $\forall j \in \{1, 2, \cdots, m\}$

Clearly, y is an feasible solution to linear program LP(G) with objective value $2n$, so y is an optimal solution to LP(G).

Conversely, assume that LP(G) has an integer optimal solution y. By the optimization of y (objective function value is $2n$) and the vertex weight function defined on graph G, we have that

1) $y(b_j) = 0, \quad \forall j \in \{1, 2, \cdots, m\}$;
2) $y(a_i) = 1, \quad \forall i \in \{1, 2, \cdots, n\}$;
2) For each $i \in \{1, 2, \cdots, n\}$, there must be only one vertex in $\{v_i, \overline{v}_i\}$ with the corresponding variable value being 1.

Thus we define a truth assignment t as follows:
$t(x_i) =$ "true", if $y(v_i) = 1$, $\forall i \in \{1, 2, \cdots, n\}$;
$t(x_i) =$ "false", if $y(\overline{v}_i) = 1$, $\forall i \in \{1, 2, \cdots, n\}$.

It is easy to show that such a truth assignment can guarantee that each clause is satisfied. In fact, for any clause C_j, if all the three literals in it is not "true", then the constraint corresponding to the clause vertex b_j in LP(G) is not satisfied by y. This contradicts to that y is a feasible solution to LP(G). Therefore, the 3-SATISFIABILITY problem is satisfiable. ■

Corollary 4.2. *Let $\Gamma = (N, c)$ and $\widetilde{\Gamma} = (N, \tilde{c})$ be the rigid and relaxed TDS game corresponding to graph $G = (V, E; \omega)$, respectively. Given a cost distribution vector $z \in R^n$, it is $\mathcal{NP}$-hard to decide whether z is in Core(Γ) or Core($\widetilde{\Gamma}$).*

Corollary 4.3. *Let $\widetilde{\Gamma} = (N, \tilde{c})$ be the relaxed TDS game corresponding to graph $G = (V, E; \omega)$. If Core($\widetilde{\Gamma}$) $\neq \emptyset$, then both computing a core element and checking whether a given cost distribution belongs to the core can be carried out in polynomial time for $\widetilde{\Gamma}$.*

Corollary 4.4. *Let $\Gamma = (N, c)$ be the rigid TDS game corresponding to graph $G = (V, E; \omega)$. If Core(Γ) $\neq \emptyset$, then computing a core element can be carried out in polynomial time for Γ.*

Although the balancedness condition of the two TDS games is the same, we are not aware of a polynomial time algorithm for checking whether a given cost distribution belongs to the core for the rigid TDS game, when it is balanced. We guess this problem is $\mathcal{NP}$-hard.

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Local Flow Betweenness Centrality for Clustering Community Graphs

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Abstract. The problem of information flow is studied to identify de facto communities of practice from tacit knowledge sources that reflect the underlying community structure, using a collection of instant message logs. We characterize and model the community detection problem using a combination of graph theory and ideas of centrality from social network analysis. We propose, validate, and develop a novel algorithm to detect communities based on computation of the Local Flow Betweenness Centrality. Using LFBC, we model the weights on the edges in the graph so we can extract communities. We also present how to compute efficiently LFBC on relevant edges without having to recalculate the measure for each edge in the graph during the process. We validate our algorithms on a corpus of instant messages that we call MLog. Our results demonstrate that MLogs are a useful source for community detection that can augment the study of collaborative behavior.

1 Introduction

Organizations are increasingly investing in knowledge management solutions to manage and leverage both implicit and explicit knowledge assets. Community detection and expertise location have become important aspects of knowledge management systems. The problem of finding the right person to help with a particular task or knowledge area persists in both academic and corporate spheres. Time and effort are spent searching for relevant information when another person in the community could easily provide assistance. Expertise can be surprisingly difficult to find, even in institutions that invest money to attract and retain world-class experts, given the mergers, growth, globalization, and employee turnover that diminish the effectiveness of informal social networks.

Frequently, there is a performance gap because community detection and expertise location solutions focus explicitly on assimilated digital information and miss a vast amount of the information in the general office which transpires through other

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modalities such as paper, telephone, and meetings. Until a few years ago, instant messaging was a multitasking activity that appealed only to adolescents eager to chat with as many friends as possible – often with one hand on the keyboard, and the other on the telephone. In recent times, corporate users have discovered that instant messaging facilitates the exchange of small but often critical details back and forth efficiently, providing yet another shortcut in the already light-speed process of electronic messaging. A typical e-mail thread may cycle three to five days towards completion. However, when empowered with instant messaging, the same participants in that e-mail discussion can make decisions in real time, and perhaps shorten that cycle to five minutes. Therefore, corporate instant message logs are an example of a naturally occurring modality in the general office environment from which organizational, collaborative and knowledge assets can be extracted.

In this context, our goal is to transform a corporate instant-message corpus into a knowledge asset that can be used for community detection, organizational change management, project management, etc. In today's enterprises people frequently collaborate with others who are not members of their “official” team, but organize spontaneously in “virtual teams”. We refer to such virtual teams as communities. Keeping track of such teams is made difficult by their very flexibility and volatility. One way is to ask people to list the members of their virtual teams, but this is slow, costly and difficult to keep up to date. Another way is to reorganize the company around the virtual teams, but this is practical only when the virtual teams are stable enough and long lasting, which is rare in modern organizations.

In this paper, we exploit the observation that members of virtual teams tend to communicate with all the other members in their team through a variety of modalities, of which instant messaging is often a significant one. Figure 1shows the type of community we are interested in detecting: sets of nodes densely connected internally, but with lower density of external links [7].

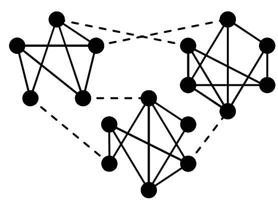

Fig. 1. Graph with community structure

If we represent an instant message communication log as a graph where the nodes represent people and the edges represent an instant message between two persons, a community will appear as a set of highly interconnected nodes with few connections to nodes outside the community. We present an efficient model and algorithm for finding such highly interconnected subsets of people. The rest of this paper is organized as follows: section 2 describes related work; section 3 introduces our basic framework for community detection; and section 4 discusses our validation methodology and experimental results.

2 Related Work

Social Network Analysis (SNA) is the mapping and measuring of relationships and flows between people, groups, organizations, computers or other information/knowledge processing entities [26]. This research approach has developed rapidly over the past twenty years, principally in sociology [5, 10], science studies [3, 4, 16], and communication science [19, 20, 12]. A social network is a set of nodes (people, organizations or other social entities) connected by a set of relationships, such as friendship, affiliation or information exchange [25]. The nodes in the network are the people and groups while the links show relationships or flows between the nodes. SNA has been used to identify structures in social systems based on the relations among the nodes rather than the attributes of individual cases [20, 19]. One of the methods used to understand networks and their participants is to evaluate the location of the participants in the network. Measuring the network location is finding the "centrality" of a node. These measures help determine the importance, or prominence, of a node in the network. Three of the most popular centrality measures are: degrees, betweenness, and closeness [15]. Girvan and Newman [6, 7] propose algorithms that involve iterative removal of edges from a network to split it into communities, the edges removed being identified using one of a number of possible centrality measures, and subsequently, these measures are, crucially, recalculated after each removal. However, there are limitations to this since it is not scalable – even the best algorithms to compute centrality indices run in $O(nm)$ and $O(nm+n^2\log n)$ time on unweighted and weighted graphs, respectively, where m is the number of links and n the number of nodes [2].

With this introduction to key SNA measures, let us switch to the hyperlinked environment on the Web. Hyperlinks let individuals or organizations running websites on the Internet expand their social or communication relations by making possible easy and direct contact among people or groups. Using hyperlinks, people can practice bilateral communication and coordination that crosses and/or strengthens off-line boundaries within and between organizations. Jackson [9] suggested that the methods of SNA are applicable to the study of hyperlinks among websites. This is in contrast to hyperlink research which did not adopt SNA methods [8, 11, 23]. Research employing SNA hyperlink is referred to as hyperlink network analysis (HNA) [21].

It seems appropriate to relate SNA to data-mining research from structured and unstructured data. The goal of data mining is to discover or derive new information from data, finding patterns across datasets, and/or separating signal from noise [13]. Text data mining has led to the research field of corpus-based computational linguistics, where statistics over large text collections are computed in order to discover useful patterns. These patterns are used for various subproblems within natural language processing, such as part-of-speech tagging, word sense disambiguation, and bilingual dictionary creation [1]. Another way to view text data mining is as a process of exploratory data analysis [24, 14] that leads to the discovery of previously unknown information, or to answer as yet unanswered questions.

Finally, the area of collaborative filtering, particularly on the Web, has relevance to this area of research. "Collaborative filtering" or "social filtering" of information [17, 18] attempts to automate the process of "word-of-mouth" by which people recommend products or services to one another. The generalized approach to collaborative filtering systems is as follows:

- preferences of a large group of people are registered;
- by means of a similarity metric, a subgroup of people is selected whose preferences are similar to those of the person who is seeking advice;
- a (possibly weighted) average of the preferences of that subgroup is calculated;
- a resulting preference function is used to recommend options on which the advice-seeker has expressed no personal opinion as yet.

As an example, in Ungar et al. [22] the symmetric relationship of people and movies is used to group movies based on who watched them and use movie groups to group people. The authors define a formal statistical model: people and movies belong to classes that are derived by the estimation-model process. The model randomly assigns people and movies to classes and for each class assigns a link with probability $p_{k,m}$ that a person in class k is linked to a movie in class m.

A natural extension of the idea of community detection would lead to the study of information propagation. A major body of research on information flow has been on the basis of the analogy between the spread of disease and the spread of information. Much epidemiology literature, including Girvan et al. [6], is based on homogeneous networks in which a node's contact points are chosen randomly from the sample space. This is different from the current problem at hand in which there is an underlying network of communication and information flow. The study of disease propagation models would enable us to understand how communities dynamically change as nodes are added and removed from a network.

The focus of this paper, however, is community detection on the basis of information flow from a given snapshot of the underlying communication network. We characterize and model the community detection problem using a combination of graph theory and ideas of node centrality from social network analysis. We propose, validate and develop a new algorithm to detect communities based on computing the Local Flow Betweenness Centrality (LFBC). Using LFBC, we model the weights on the edges of the graph so we can extract communities effectively. We also present an approach for efficient computation of the LFBC on relevant edges without the need to recalculate the measure for each edge in the graph during each pass of the iterative process.

3 Information Flow

We concentrate on defining the information flow measure that best captures the communities implicit in an MLog. Social network researchers measure network activity for a node by using the concept of degrees – the number of direct connections a node has. This leads to the concept of "connector" or "hub" in a network. One might hypothesize that more connections would be better. In practice, however, having connections only to an immediate cluster does not help make an important connection. What matters more is where the connections lead. The betweenness-centrality measure captures the degree of influence a node has over the information that flows in a network. Betweenness centrality captures a powerful role in the network, but can also be a single point of failure. If the pattern of direct and indirect connections between two nodes allows them to access all the nodes in the network more quickly than anyone else, they have a high degree of "closeness". They have the shortest paths to all others – they are close to everyone else. They are in an excellent position to monitor the information flow in the network – they have the best view of what is happening in the network.

We begin with a baseline algorithm for community detection based on betweenness centrality.

3.1 Betweenness Centrality

Betweenness Centrality (BC) has been established as an important quantity to characterize how influential a node is in communications between each pair of nodes. The communication paths between a pair of nodes (i, j) are the shortest pathways, and the number of such pathways is denoted by $c(i, j)$. Among them, the number of the shortest pathways running through a node k is denoted by $c_k(i, j)$ and the fraction $c_k(i, j)/c(i, j)$ by $g_k(i, j)$. Then the BC of the node k is defined as the accumulated amount of $g_k(i, j)$ for all pairs (i, j) for i distinct from j and k [25]. Without considering the existence of multiple shortest pathways between node i and node j the BC of a node k can be seen as the number of shortest pathways passing through k during the activity of the whole network. Likewise, BC of an edge can be computed as well.

Betweenness centrality is, in some sense, a measure of the influence that a node has over the flow of information through the network. Conceptually, high betweenness nodes lie on a large number of non-redundant shortest paths between other nodes; they can thus be thought of as "bridges" or "boundary spanners." For our purpose of detecting communities, we exploit this definition such that a high BC measure indicates a node that is bridging two communities. In general, BC has been shown to increase exponentially with connectivity as a power law, and, therefore, is a computationally involved network measure. Current algorithms computing betweenness can deal with networks of up to several hundred nodes efficiently. However, with the increasing interest in Web magnitude datasets, there is an increasing demand for efficient computation of centrality indices on networks with millions of nodes.

3.2 BC for Community Detection

Given the definition of Betweenness Centrality of a node, we define a baseline algorithm that uses BC to detect communities in the graph implicitly defined by an MLog. An MLog is represented as a graph where the nodes represent people and each edge represents an instant message exchanged between two persons. The baseline algorithm is as follows:

1. Detect connected components in graph.
2. Compute BC for each node in the subgraph.
3. Remove the node with the highest BC since it is a "boundary spanner".
4. Create two new connected components and repeat from 2 until the subgraph has the desired number of nodes in the community.

In Figure 2, nodes a and b represent two nodes (people) in a network. The BCs of nodes a and b are denoted as BC_a and BC_b respectively. Now the network containing only nodes a and b is extended with the addition of a new community (that may comprise several interconnected nodes) to which node b is connected. The point of connection of node b to the new community is node c. By definition of BC, the introduction of this new community to the network will increase the BC of node b.

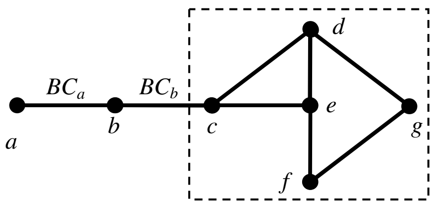

Fig. 2. The BC_j changes when a new community is added

This means that node b is likely to be considered a "bridge node" or "boundary spanner" and will be eliminated from the graph due to its high BC. This appears counter intuitive to us in terms of the community comprising of nodes a and b weakening due to node b's connection to a few nodes in a new community. If we model the number of instant messages exchanged between two persons as the weight on the edge connecting them, it is not clear how the weights may be used to compute the BC of an edge. To avoid the separation of two nodes resulting from the addition of a new community far from them and to make use of the information embedded in the weights we introduce the *Local Flow Betweenness Centrality*.

3.2.1 Local Flow Betweenness Centrality in Information Flow

The aim is to remove edges iteratively in the graph G implied by an MLog, and to detect communities arising from this process. Consider the edge (a, b) in G and all the edges (k_i, a) and (h_j, b) such that k_i is connected to a but not with b, and h_j is connected to b but not with a as in Figure 3.

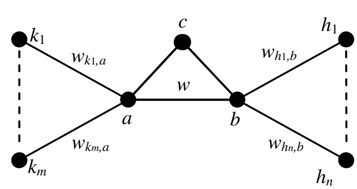

Fig. 3. The LFBC of (a, b) is computed as in (1)

Let $w_{x,y}$ represent the weight on the edge between nodes x and y. Metaphorically any source nodes h_j is trying to communicate a certain amount of information $w_{hj,b}$ to the destination a, and to do so it "stresses" the edge (b, a) with "capacity" $w_{a,b}$, by passing through the mediator b. At this stage we are not considering the presence of multiple mediators. The LFBC is computed as in (1) and provides a way to measure local information flow.

$$LFBC_{a,b} = \left(\sum_{i=1}^{m} w_{k_i,a} + \sum_{j=1}^{n} w_{h_j,b} \right) - w_{a,b} \tag{1}$$

It is intuitive that the LFBC will be higher on inter-community than intra-community edges, because the number of "missing" edges is lower inside than outside a community.

The final stress on each edge is calculated as the difference between the total local flow passing through such an edge discounted by the capacity of that edge, where the number of conversations between two persons is a rough estimate of the capacity of an edge. To illustrate LFBC, let's consider the example in Figure 4.

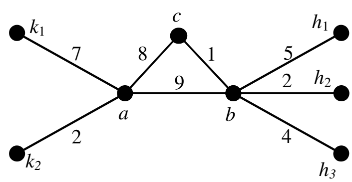

Fig. 4. The LFBC of (a, b) is 11

We should notice that the weights on edges (a, c) and (b, c) do not participate in the computation of $\text{LFBC}_{a,b}$ because c is connected to a and with b. All k and h nodes, in order to communicate with node b and node a, respectively, stress the edge (a, b).

We reason as follows: within a community almost all the nodes are connected to all the others, and therefore there is no "stress" due to a source trying to communicate with a destination. In contrast, the edges that separate two communities are stressed by the weight of the information flow that must occur between the two adjacent communities for information to flow.

If it is "in between" or "separating" two communities, so that many communications have to pass through it, an edge has a high stress factor. Using BC computed over the nodes in Figure 3, the baseline algorithm would remove node b after the first iteration. However, b is an important node in the community comprising a and b. Using LFBC computed over edges, instead of BC over the nodes we modify the baseline algorithm as follows:

1. Detect connected components in graph.
2. Compute LFBC for each edge in the subgraph.
3. Remove the edge with the highest LFBC (highest stress) since it is an edge that inhibits information flow.
4. Create two new connected components and repeat from 2 until the subgraph has the desired number of nodes in the community.

3.2.2 Local Information Flow

As shown before the LFBC is computed by hypothesizing a local flow of information between two nodes that are not directly connected in order to determine whether they are part of the same community. The use of the LFBC instead of the BC relies on the assumption that an edge which is "globally" in between communities, at the level of the whole network, should also be "locally" in between, i.e., with respect to its immediately adjacent links. Recalculating the BCs after the removal of an edge is computationally expensive because it involves virtually all edges in the graph. Using the LFBC instead presents the advantage that when an edge is removed to recalculate the LFBCs it is possible to subtract the weight of such an edge from all the edges previously "stressed" by it. Based on the definition of LFBC we should note that the edges involved are only some of the ones adjacent to it. Consider the network shown in Figure 5.

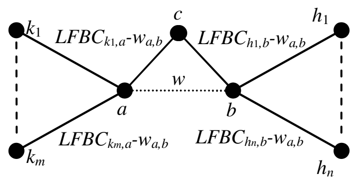

Fig. 5. Local computation of LFBCs when (a, b) is removed

Edges are labeled with the current values of LFBC. The elimination of the edge (a, b) effects only the recalculation of the EFS for edges (a, k_i) and (a, h_j) where k_i is connected to a but not to b, and h_j is connected to b but not to a. Therefore, the notion of Local Information Flow dramatically reduces the computational cost.

3.3 Algorithm to Detect Communities

We now arrive at the final algorithm to detect communities from an MLog corpus in the following manner: we transform the log into an undirected graph G, where each person participating in the conversation is represented by a node. Two nodes are connected by an edge if they are engaged in an instant message. Each edge carries a weight corresponding to the number of messages exchanged between the two persons. Note that directionality of the edge is ignored at this stage because we do not consider any directionality of the communication exchanged through an instant message.

The MLog is modeled as follows: let m be the total number of individuals present in the log, and A an $m \times m$ square matrix. Each element of this matrix, denoted as $a_{i,j}$, is an integer that represents the number of messages exchanged between the person i and the person j. The matrix A is a square symmetrical matrix that corresponds to the connectivity matrix of an undirected weighted graph. Communities are represented by subgraphs. The aim of the algorithm is to identify subgraphs where each node is connected to many nodes in the same subgraph and to few nodes outside the subgraph. In order to measure the cohesion of a subgraph we use these definitions:

- ***k*-core:** a subgraph Q with n vertices is a k-core if any internal node is adjacent to at least k nodes in Q.
- ***k*-core-factor:** the k-core-factor of a subgraph Q is the maximum k for which Q is a k-core.
- **community:** A subgraph Q with n vertices and a certain k-core-factor is accepted as a community if k is greater or equal to $\alpha \exists (n\text{-}1)$ with $\alpha \in [0,1]$.

The final algorithm is as follows:

1. Detect connected components in graph.
2. For each connected component, verify if the connected component matches our definition of community, for a given value of α.
3. Compute LFBC for each edge in subgraph.
4. Cut the edge with the highest stress factor since it is most locally in between, and repeat from 2.

4 Community Detection

Our experimental corpus consists of two datasets: the first is a test dataset of 350 corporate instant messages contributed by 100 users. The participants in this test dataset were a global corporate team whose members spanned groups in research, development, marketing and sales. Subject matter experts who understood both the user community that contributed the data as well as the content compiled the ground truth which comprised of 6 classes/user-groups and 14 users. For this test corpus, we had access to both, the link information in the MLog such as user *a* sent an instant message to user *b* at certain time, as well as the actual transcript of the message that was exchanged.

The second dataset was a larger, unknown corpus consisting of a snapshot of IBM's global, intranet instant message logs. The logs were collected over a 2-hour span, at peak time maximizing global usage by about 320,000 employees. The size of the test corpus was ~220,000 instant message logs with about 100,000 unique employees/nodes. For this corpus, due to privacy and legal concerns we had access to only the link information in the Mlog, such as the fact that user *a* messaged user *b* at time *t*.

4.1 Validation Methodology

We consider validation of communities detected from MLogs similar for validation of clustering algorithms. There are several ways to measure the precision of clustering algorithms based on either some external or internal measures. An external validity measure matches some prior knowledge about the data that is usually a set of classes and labels for that data. In the internal measure case, instead, the cluster validity is based on the data features; this is equivalent to using the measure as the clustering objective function itself. When there is no prior knowledge about the results, one way to evaluate clustering is to ask humans to judge the quality of the results. An alternative to this is to ask humans to cluster the dataset in classes that will be treated as ideal clusters to compare with the algorithm results for the evaluation.

We use this approach for our test dataset – subject matter experts classified the 100 users from the test corpus into 5 distinct communities. For the larger intranet, global dataset we used a data-warehousing/business intelligence approach to validation: i.e., we generated communities of aggregated nodes – Country, Division and Work Location. We then asked the appropriate executive subject-matter experts to judge the quality of our findings. While this is a subjective measure with no quantitative number to assign to the quality of our results, our findings were remarkably accurate and reflected the business collaborations very effectively.

4.2 Experimental Results

Figure 6 shows the users' connectivity graph for the smaller, test dataset – this encodes the ground truth against which we wish to verify our community detection results. From Figure 6, we see that we have four distinct components that do not interact with each other, and 5 communities. The shaded nodes in each ellipse are the nodes belonging to the 5 communities as identified by the subject-matter expert. The node highlighted with a box around it is very well connected in the graph – i.e. there are edges from this node to many other nodes in the graph. As a result, when the BC measure is used to detect communities [7], the algorithm ends up removing the edge between two nodes in the first community. This substantiates our intuition explained in Figure 2 where the addition of a new community results in weakening the cohesiveness between nodes in the first community. In contrast, the LFBC measure models this correctly by employing the concept of Local Flow and by using the extra information available as the weights on the edges. The LFBC based community identification detected the communities in the ground truth reported in Figure 6 with an accuracy of 100%.

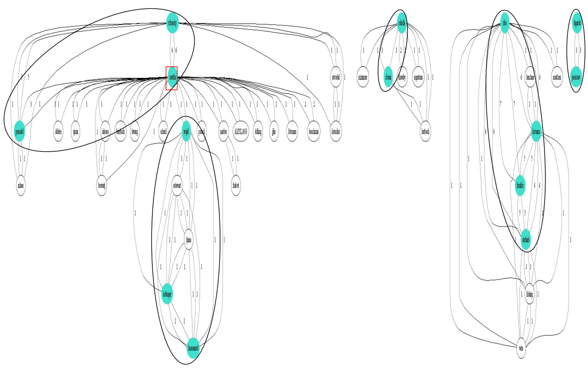

Fig. 6. Ground truth communities in the test dataset

Figures 7, 8, 9 and 10 show the results of the algorithm on the larger corporate MLog. For this corpus, we did not survey the participants to identify ground truth about user communities. Instead, we aggregated the user nodes by country, division and work location by querying the corporate LDAP directory services, and detected communities among these aggregated nodes and validated them through visual inspection by the subject matter experts. Figures 7, 8 and 9 reflect the communities whose detection was based on the aggregations of nodes by country. All users belonging to a specific country were aggregated by representing the communications between two countries as the weight on the edge. The shadow over the world map indicates the part of the world where it was nightfall during the period of IM capture. Figure 7 is a representation of the network traffic with the aggregated nodes by country. The red links show communications involving at least one country where it is night.

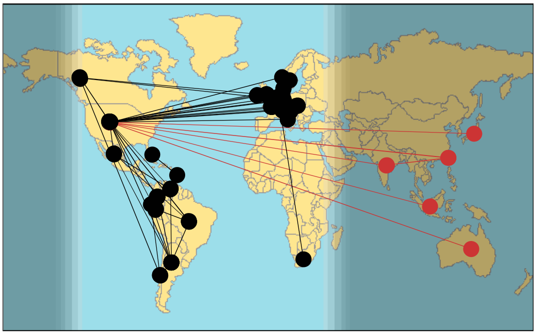

Fig. 7. IM communications aggregated by country

Figure 8 shows the 10 strongest links that comprise the USA, Canada, Brazil, Mexico, the UK, Germany, France, Ireland and India.

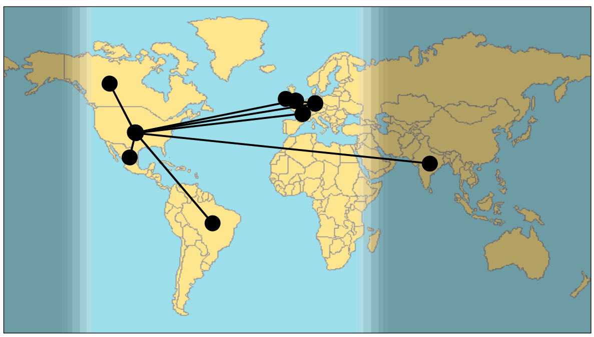

Fig. 8. 10 heaviest connections

Figure 9 shows by means of different colors different communities identified through the algorithm based on the EFS without considering the USA.

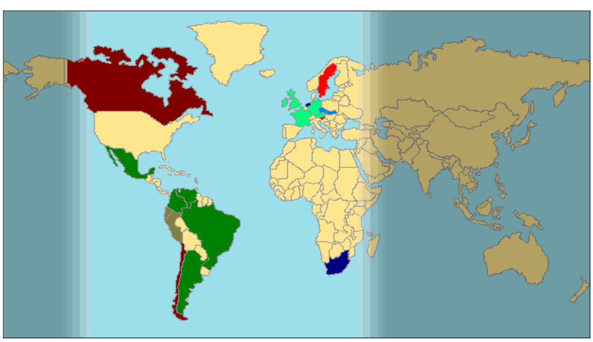

Fig. 9. Communities without US with α=0.5

Figure 10 shows a different aggregation – along the divisions within the IBM corporation.

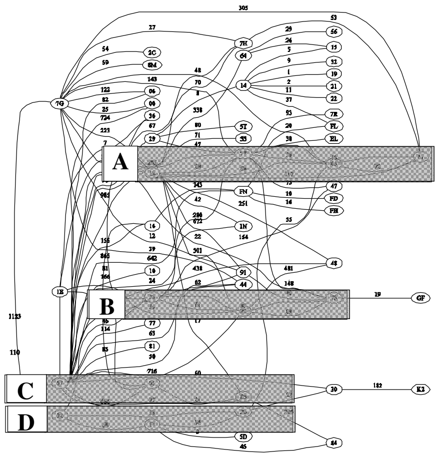

Fig. 10. Communities without US with α=0.5

The communities labeled A, B, C, and D, which correctly represent Sales, Integrated Supply Chain, Server Brand Management, and Storage Technology, represent a high degree of collaboration among the nodes in that specific community. The validation methodology for these communities consisted of surveys and feedback from the appropriate IBM business executives that could relate the collaborations to meaningful business tasks.

Finally, we also posed the following findings and questions along the lines of business intelligence to the appropriate business executives within IBM. These questions were deemed to be extremely relevant to feeling the "pulse" of the business.

- Why is the link between Canada and US the strongest?
- Why are Spain and Portugal isolated?
- Why is Argentina so central in South America?
- Why does India generate so much IMs in the night?
- Is South American expertise lost to the European market?
- Why is the tie between Canada and Chile so strong?
- Why is MLog traffic so heavy within Brazil – Rio, Manaus and Sao Paulo?
- Why do the divisions Integrated Supply Chain and Maintenance Services collaborate so much?
- Should the S&D division be talking to other divisions more?

The relevance and accuracy of these findings were validated by executives in the appropriate lines of business within IBM. Various lines of business substantiated the reasons for these sets of collaborations. The results were validated in two ways: first, some set of collaborations confirmed working businesses and projects while others indicated problems in terms of how a particular project was being run due to lack of the right collaborations. For the sake of confidentiality the specific project details cannot be discussed in this paper. While this is not a quantitative measure such as those reflected by clustering accuracy measures or precision/recall measures, the combination of the ground-truth validation of the small experimental corpus, together with the validation of such "business intelligence" on the large corpus leads us to believe in the promise of the LFBC- based approach.

5 Conclusion

We have presented a model for characterizing communities using a combination of graph theory and ideas of node centrality from social network analysis. We have introduced a new algorithm to detect communities based on *Local Flow Betweenness Centrality* (LFBC). Using the LFBC we present a framework for modeling the weights on the graph edges corresponding to community characteristics. We also present an approach for efficient computation of the LFBC on relevant edges without the need to calculate the measure for every edge in the graph at each iteration. We have validated our algorithms on a corpus of instant messages that we call MLog. In terms of future work, we would like to formalize the performance gains due to the "local" nature of the LFBC computation, and generalize the algorithm to deal with directed and undirected graphs. Finally, we would like to define robust evaluation measures that can be used on different experimental corpora to measure the accuracy of community detection algorithms.

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Computerized Collaborative Support for Enhancing Human's Creativity for Networked Community*

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Abstract. The rapid development of computer and Internet enables people who have common interests to build loosely coupled networked communities. Through stimulating individual's creativity to augment community's communicative and collaborative abilities for acquiring new ideas and knowledge is becoming highlight research and achieving more and more experts' attentions. In this paper, we focus on exploring effective computerized collaborative support for enhancing human's creativity for networked community. Versatile aids are explored, such as visualization of expert opinion structure, clustering of contributed opinions for concept formation and idea/knowledge detecting and growing, etc. all integrated into a group argumentation environment (GAE) with a simple example.

Keywords: Creativity, Networked Community, Dual Scaling Method, Centroid.

1 Introduction

In daily life, many interesting phenomena can be described and explained through network. For instance, scale free network [1], such as World Wide Web, actor connectivity and science coauthorship, can aid to solve practical problems better. Also, social network has become one of highlights in academic research. Social network analysis (SNA) pays more attention to the relationships between people, and their roles played in the network [2]. Above all, computers and computer networks as advanced information techniques has been becoming an integral part of our life. It promotes and facilitates the research on network and its phenomena, such as powerful development and advancement on electric business, networked economy and knowledge networking, etc. Via the network, people who gather with common interests, form common ground and consensus. That builds the funentmental organization as a vitual team, networked group or community, even society. Global networking provides the convenient way to facilitate communication and interaction free from the limitation of time and location.

In general, community as a group allows free discussion, speech, brainstorming, dynamic data/information/knowledge sharing and transferring [3]. Debate, negotiation,

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argumentation and consensus building are common communicative and collaborative activities for community. Collaboration and technical support collaborative tools are key elements for productivity of improvement in both physical and networked settings. Nunamaker divided collaboration into three levels toward problem solving: the first one is collective level, in which individuals are independent and uncoordinated. Community starts from individuals, while an individual is a potentially unique community. The second one is coordinative level. All community's individuals are limited to share their information in this level. The highest level is concerted level in which collaborative communities are established [4]. In our opinions, accordingly, some technical applications can be easily found to match the three levels. Personal Web homepages, advertisements, etc. can work as collective level; BBS, forum, messager, etc, for information sharing can represent the coordinative level; The highest level tools may include group support system (GSS), computer supported cooperative work (CSCW), etc. Similar to Nunamaker's view, Mase et, al also provided three modes from the group thinking perspective. Individual thinking mode as the foundation for group thinking is the thinking of community's each constituent member. There is no interaction and individual's deep thinking in isolation. The cooperative thinking mode is also referred as the communication mode. Individuals as pre-community work together cooperatively to understand each other through their interaction. Furthermore, community can not only share their information and ideas, but create new things all together in collaborative thinking mode [5]. New creation and favorable collaborative tools are the important factors to build and maintain a positive and active networked community.

Based on the different type of communities, new creation can be new design, new products and new theories, etc. Expert community through distributed argumentation, pays more attention to the created and emerged new ideas, new knowledge, even their wisdoms. For that, how to effectively and efficiently exploit individuals' implicit knowledge, externalize their mental models, stimulate his/her intuition, insight and creativity, and augment their communicative and interactive abilities together with computerized support is a major concern. The content or platform for group collaboration for the networked community also can be called '*ba*', a Japanese word, where idea/knowledge is created, shared and exploited for different domains' experts for creative problem solving [6].

In this paper, we concentrate on computerized collaborative support for enhancing human's creativity for networked community during argumentation process. Versatile computerized aids have been developed, such as visualization of expert opinion structure, clustering of contributed opinions for concept formation and idea/knowledge detecting and growing, etc. all integrated into a group argumentation environment (GAE), to support the emergence of a *ba* for knowledge creation.

2 Computerized Support for Enhancing Information Sharing and Knowledge Creation for Networked Community

To facilitate group argumentation for enhancing information sharing and knowledge creation for networked community, heavy endeavors have been engaged in computerized support with tremendous advances of information and network technologies. Absorbing some ideas from AIDE [7], AA1 [8], the architecture of we developed versatile aids for community is given.

2.1 Architecture of Computerized for Group Argumentation

Fig. 1 shows the four layers of the architecture of the integrated group argumentation environment (GAE) which is based on client/server framework and mainly includes an online electronic brainstorming argumentation room (BAR).

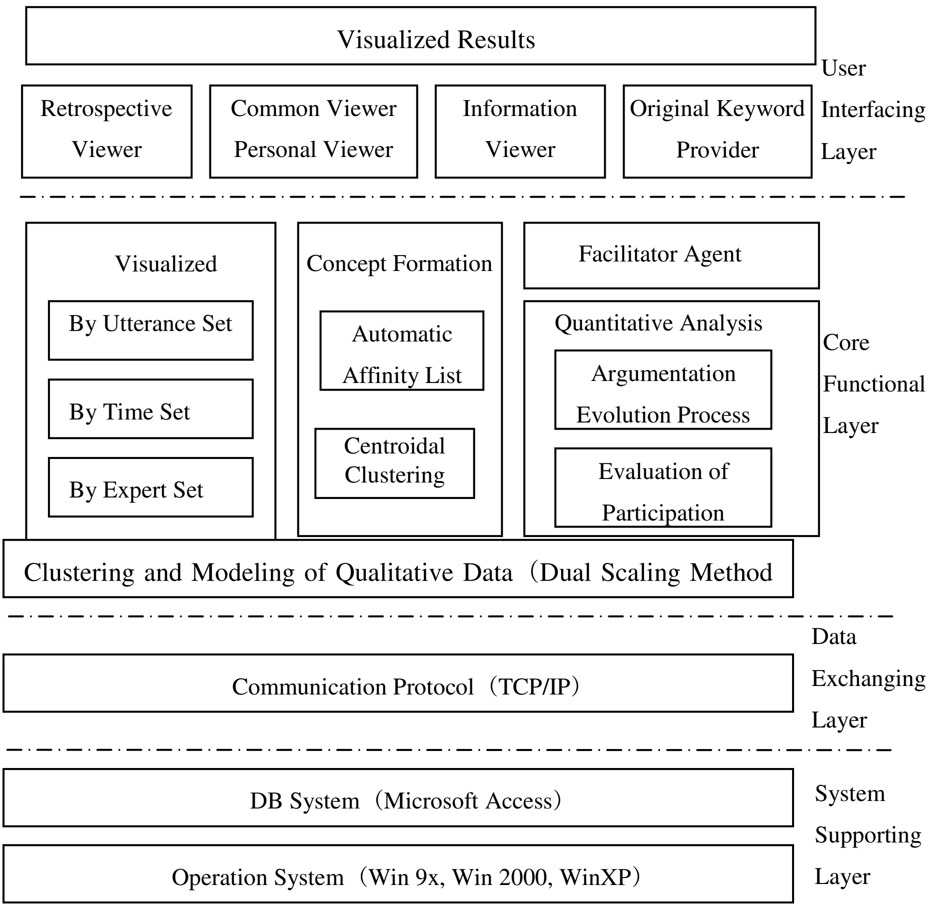

Fig. 1. Architecture of Group Argumentation Environment

Both user interfacing layer and core functional layer show what can be achieved at client window of BAR, though some services provided by server. Firstly, GAE can be regarded as a '*ba*' for networked community. Furthermore, by providing visualized thinking structure during the group working process together with a variety of analytical mechanisms about the process and participants, it aims to support emergence of creativity, even wisdom. Follows introduce some salient functions.

2.2 Visualized Shared Memory for Group Argumentation

As shown in Fig. 2(a), the main client window for visualized shared memory is consisted of event record area, dialoguing area and visualizing area.

Visualized analysis transforms qualitative knowledge into a 2-dimenstional map, which helps the participants to understand community's others' opinions easier, find common interest, stimulate further thinking, acquire intuition and insight, facilitate knowledge sharing and new ideas generation. Following are two visualized viewers:

Common viewer, a discussion space as a joint thought space for all participants of community. Via the 2-dimensional space, the idea association process to stimulate participants' thinking, idea generation, tacit knowledge surfacing and even wisdom emergence is exhibited based on the utterances and keywords from participants. The global structure and relationships between participants and their utterances are shared by all participants in the session. It helps users to acquire a general impression about each participant's contributions toward the discussing topic, and understand the relationships of each thinking structure about the topic between participants.

Personal viewer, a personalized space where records individual thinking process during discussion. It provides a personalized idea-gathering space where the relationships between utterances and keywords are visualized. Individual creativity may be stimulated through personalized work via this personalized space. It helps the user to understand how one piece of information (utterance) affects the group thinking process and understand the relationships between each participant's mental process.

Fig. 2(b) shows retrospective analysis which applies same mechanism as both viewers and provides participants to "drill down" the discussing process for visualized partial perspectives. Further analysis of pieces of discussion such as selected intervals of discussion or combination of any selected participants may be helpful to detect the existence or formulating process of a micro community and acquire further understanding about participants' thinking structure.

From the visualized structure about the discussion in the course, the standpoints of participants could be estimated based on distances between participants. All those are based on a $n \times m$ frequency matrix constructed by n utterance-objects and m keyword-objects (see the Table 1). In general, the matrix is a sparse matrix since keywords are only mentioned by some utterances. This frequency matrix is changing dynamically. As more utterances submitted, more rows and columns are appended. By the terms of graph theory, frequency matrix describes the relations between vertex (participants or utterances) and edge (sharing keywords).

In Table 1, where w_{ij} =0, when keyword *j* is not mentioned in utterance *i* otherwise, $w_{ij} = W_{ij}$. The weighting policy is as follows: keywords appearing frequently throughout an entire argumentation process are very general words, which are not important for the utterance-object and are lightly weighted. On the other hand, keywords frequently used in a certain utterance-object or referred again after a long interval are important for the utterance-object [9].

Table 1. Utterance sets and keyword sets

Y \ X	keyword$_1$ x_1	keyword$_2$ x_2	$\cdots$	keyword$_m$ x_m	
utterance$_1$ y_1	w_{11}	w_{12}	$\cdots$	w_{1m}	$y_1 = \sum_{i=1}^{m} w_{1i} x_i$
utterance$_2$ y_2	w_{21}	w_{22}	$\cdots$	w_{2m}	$y_2 = \sum_{i=1}^{m} w_{2i} x_i$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
utterance$_n$ y_n	w_{n1}	w_{n2}	$\cdots$	w_{nm}	$y_n = \sum_{i=1}^{m} w_{ni} x_i$

We applied an exploratory, descriptive multi-variant statistical method—dual scaling, to analyze and process the matrix [10,11]. Dual scaling has some of the characteristics of correspondence analysis and exploratory factor analysis. The math underlying dual scaling is based on calculations of eigenvectors and eigenvalues of a frequency matrix. As a result, a pair of utterances with more common keywords may locate closer in the 2-dimention space. In the common viewer, utterance object is the participants; participants who share more keywords may be within a cluster. Here share keywords may mean participants hold similar concerns toward those keywords.

Different from the topological graph, the above algorithm formed graph is an interpretable graph, which reflects the data's nature in the database. But the topological structures have been designed and the forms are structured. In our research, we want to cluster the utterances and keywords of the experts in networked community, and the aim is to externalize the mental process of the human thinking. Here, we think, the interpretable diagram is more suitable to embody the thinking activities than the topological graph.

2.3 Facilitator Agent

If fewer ideas are contributed by participants, the chairman can launch facilitator agent. Once every two minutes in default, the agent extracts the most infrequently posted keyword and submits it with the userID of "Conversation" if no more keywords are provided. It not only takes a more fervor environment, also stimulates participants' further thinking and interaction. As far as the most infrequent keyword is concerned, it effectively extends ideation of participants because they have to keep silence if no more new ideas can be produced after focusing heavily on one thesis for a long time. In a word, "Conversation" can help create new great ideas as a virtual participant. Appling facilitator agent participant in argumentation process also embodies the man-machine interaction.

2.4 Record of Original Keyword Provider

Boden distinguishes creativity into two senses: psychological creativity (P-creativity), and historical one (H-creativity). A valuable idea is P-creative if the person in whose mind it arises could not have had it before, no matter how many people may have had the same idea already. By contrast, a valuable idea is H-creative if it is P-creative and no one else has ever had it before with respect to the whole of human history [12]. We agree to Boden's claim that P-creativity is more critical than H-creativity. In group argumentation, if you are the original keyword provider, the keywords which represent your ideas are your P-creativity results, as shown in Fig. 2(c). The function of record of original keyword provider in GAE system is to assist the users in finding what they had not noticed so far (P-creativity) that could lead them to really creative work at last.

2.5 Concept Formation

Concept formation means automatic summarizing and clustering of experts' utterances and detecting the typical keywords as meaningful groups or sub-communities of ideas based on visualized maps. The following are two methods to support concept formation:

1) Automatic affinity diagram (AAD): sometimes called the KJ diagram after its creator, Kawkita Jiro. AAD is to map the 2-dimension personal structure into 16×16 grids. As Fig. 2(e) showed, those utterances which fall into same cell are regarded as one cluster.

2) Centroidal clustering algorithm: centroid is the center of each set produced with cluster and given by $C_m = \frac{1}{n}\sum_{i=1}^{n} t_{mi}$. Combining K-means clustering method [13], which equation is $m_i = \frac{1}{m}\sum_{j=1}^{m} t_{ij}$, we use it to get k centroids, where k is an assumed number of clusters. The closest keyword to the centroid could be regarded as cluster label.

2.6 Idea/Knowledge Detecting and Growing During Argumentation Process

During group argumentation process, participants contribute and share their opinions (utterances, keywords) continuously. Locating and detecting current focuses and some representative ideas from the mass information with some quantitative methods may help stimulate experts' further thinking. For that, clustering algorithm of centroid is used to extract those typical keywords as ideas/knowledge based on the two-dimensional maps produced in common viewer of GAE. The concrete algorithm is shown in 2.5. Through recording series of extracted keywords at different given time, the process of idea/knowledge growing and evolution in group argumentation is explained well, as shown in Fig. 2(f).

The detailed introduction of other functions of GAE, such as evaluation of participation by calculation of eigenvectors about agreement matrix and dissimilarity matrix for further testing of some assumptions about individual impacts towards group behaviors, and information support for customized search, abstract and summarization, can see the reference [14].

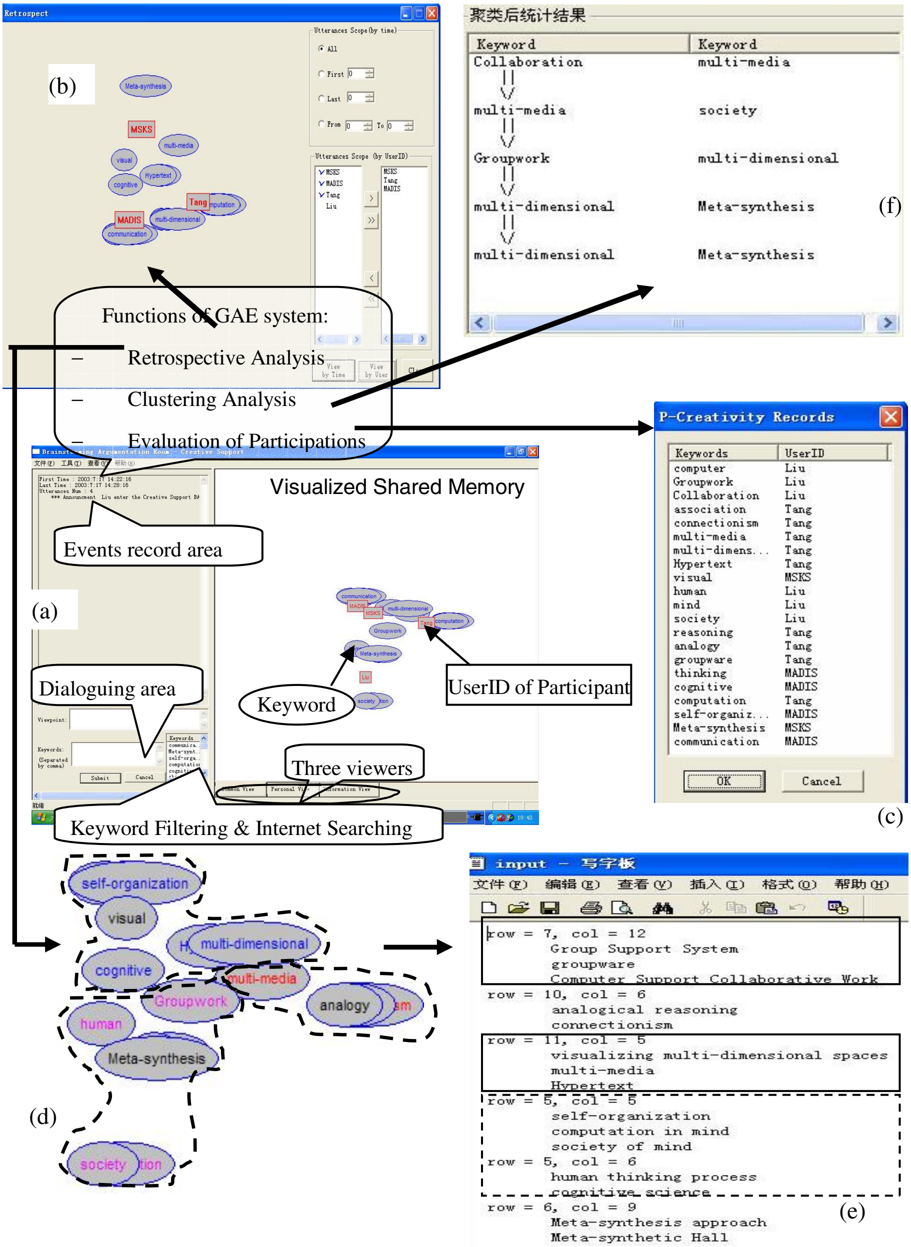

Fig. 2. Client Window of BAR (a) Main client window, b(b) Retrospective viewer, (c) Original keyword provider, (d) Clustering analysis (K=3), (e) KJ Editor (16×16), (f) Argumentation Evolution Process

3 An Example of GAE

In this example, the topic for discussion is about group support systems. Four persons whose registered IDs are Tang, Liu, MSKS and MADIS respectively participated the discussion and formed a networked community. Fig. 2 shows basic analysis taken in this test. Fig. 2(a) is a whole perspective of all concerned participants' contributions. It shows participants who share more common keywords locate closer in the 2-dimension space. Fig. 2(b) is the opinion structures of Users MADIS, Tang and MSKS as a subset community formed in retrospective viewer. Fig. 2(d) shows 3 clusters by K-means clustering method, where keywords 'visual', 'analogy' and 'Meta-synthesis' are acquired as the label (centroid) of each cluster.

Fig. 2(e) shows the affinity list based on personal viewer, which divides the whole utterance set into 6 cells according to their space relationship. It could be seen the utterances in one cell are related to each other. For example, all 3 utterances within Cell [row 7, col=12] are about GSS or similar tool systems, then that cell could be titled as group support system. On the other hand, all 3 utterances within Cell [row=11, col=5] exhibit concerns on man-machine interaction. Automatic affinity list could be regarded as a rough classification about participants' opinions during the brainstorming session. Further processing could be taken to acquire a more reasonable classification.

Dynamic visualized structures of the concerned topic may reinforce the stimulation and facilitate further thinking during community interactive process. The evolving diagrams may also help to find some hidden structures to aid communication and collaboration for community. Such a work is oriented to maintain an interactive *ba* and facilitate for idea emergence during group divergent thinking process.

4 Concluding Remarks

In this paper, we focus on computerized collaborative support for enhancing human's creativity for networked community. Research on creativity and knowledge creation together with computerized supports provides basis for our research [12, 15-17]. What we are exploring is not only a computerized support tool for communities' communication and interaction, but also expecting to support the emergence *ba* for creative problem solving. Our developed group argumentation environment exhibits our ideas, which acts as a virtual *ba* promoting members exchange ideas, stimulating their creativity and enhancing argumentation effects.

Our current work is still at very initial stage from both research and practice [14, 18-20]. From the research perspective, currently we mainly concentrate on cognitive modes and mental models for individual of community, and group communication and collaboration behaviors and responses. The aim of GAE is to support dynamic emergence of a knowledge creation environment (*ba*). Lots of further work are under exploration, such as better human-machine interaction, opinion synthesis in consideration of expert's background, and evolving process of keyword network to detect the pathway of knowledge creation, etc. More experiments, that is, building multi-communities, will also be undertaken for verification and validation of GAE in practice.

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New Results on Online Replacement Problem

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Abstract. The replacement problems are extensively studied from a number of disciplines including economics, finance, operations research, and decision theory. Much of previous theoretical work is "Bayesian". In this paper, we restudy the on-line replacement problem by using the competitive analysis. The goal is to minimize the total cost of cumulative payment flow plus changeover costs. Firstly, a refusal strategy is proposed and the competitive ratio for $k = 1$ is obtained. Furthermore, a new time-independent strategy S_{new} is presented and we prove that it is r-competitive when $M \in [c, d]$. Finally, weights are introduced to the original model and some results are achieved.

1 Introduction

R.Bellman [1] studied the replacement problem via dynamic programming. This problem is described as follows: The flow rate $f(t)$ is chosen by an adversary from the real interval $[m, M]$ where $0 < m \leq M$. For a start, suppose that both m and M are known to the online player. At each time $0 < t < T$ the online player can changeover and continue paying money at the rate $f(t)$. The changeover cost is 1 unit. The player chooses any number k of changeover times. The total cost consists of payment flow and changeover costs. The objective function is

$$y = k + \sum_{i=0}^{k} (t_{i+1} - t_i) f(t_i)$$

The previous research work [5, 7] of the replacement problems is mainly based on the conventional "average case analysis" . R.E.Yaniv et.al. [2, 4, 8] first initiated the worst-case competitive analysis of the *on-line replacement* problem. Namely, the online player knows nothing about the future but decides to select a replacement strategy based on past and present information. The off-line player knows the strategy which the on-line player adopt and chooses a flow function to maximize the competitive ratio [3, 9]. In the paper [2], some results were obtained. For example, the general lower bound on the competitive ratio of any deterministic strategy were presented, which is $\Theta(\frac{lnM}{lnlnM})$ for a fixed m.

In the paper [6], Azar et.al. studied the discrete time replacement problem variant with multiple, permanent replacement options and achieved some results.

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For the convex variant a simple 7-competitive algorithm was obtained and the competitive ratio $O(min\{log(cr_{max}), loglog(cf_{max}), log(cn_{max})\})$ was achieved to study the non-convex variant.

2 Our Contributions

In this paper, we study the on-line replacement problem P_0 and variant problem P_1. P_1 is the weighted version of P_0. Based on the time-independent strategy $S_\rho^{***}(m, M)$ [2], we present a refusal strategy which the refusal times sequence is no-increasing. Namely, the online player makes the ith changeover at time t if $f(t) \leq M_i(t)$. Otherwise, it refuses to changeover. In this case, when the changeover save costs more than the penalty costs($\frac{1}{\rho^{k-1}} times$) the online player will choose to changeover. we obtain the competitive ratio $\sqrt{\frac{M+1}{m+1}}$ for $\sqrt{\frac{M+1}{m+1}} < 2$.

Next, we propose a new time-independent strategy S_{new} in which the online player changes over for the ith time when the flow rate decreases to the level of M_i. We get the following results: The changeover sequence M_i designed strictly decreases below m within $k+1$ steps; S_{new} is r-competitive for $M \in [c, d]$ where $c = m + \sqrt{r}$ and $d = \frac{(\sqrt{r(\lfloor r \rfloor+1)} - \sqrt{\frac{\lfloor r \rfloor}{r}})m + r - \lfloor r \rfloor}{1 - \sqrt{\frac{\lfloor r \rfloor}{r}} + \sqrt{r(\lfloor r \rfloor+1)} - r}$.

Finally, we present the weighted replacement problem. The weighted value $\eta \in (0, 1)$ weights the relative importance of payment flow and changeover. It has very realistic meaning to consider this factor. For example, the replacement costs of cars are clearly different from those of computers and this will affect the choice of the replacement value in different phase. The introduction of this parameter does lead our mathematic model further to approach the real life. In this section, we propose a characterization theorem that gives necessary and sufficient conditions. This characterization theorem provides an efficient tool for determining the competitive ratio. In the rest of this section, we construct a refusal strategy $WS(m, M)$ and obtain the following results: The changeover time is $k = \lceil (\beta m + 1)(r - 1) \rceil$. If $\sqrt{\frac{\beta M}{\beta m+1}} \leq \frac{\beta m+2}{\beta m+1}$, then $r = \sqrt{\frac{\beta M}{\beta m+1}}$ and $k = 1$.

3 Replacement Problem P_0

Let $\{M_i\}_{i=1}^k$ be the changeover thresholds sequence which is strictly decreasing within the open interval (M, m) and $\{b_i\}_{i=1}^k$ be the refusal times sequence which is non-increasing with the time horizon $[0, 1]$.

3.1 A Refusal Strategy Based on $S_\rho^{***}(m, M)$

In the paper [3], a time-independent policy $S_\rho^{***}(m, M)$ with the changeover thresholds sequence $\{M_i\}_{i=1}^k$, has been presented, which shows that the sequence defined by this strategy decreases below m within $k+1$ steps for sufficiently large ρ. We quote as follows:

The sequence of changeover thresholds $\{M_i\}$ is

$$\begin{cases} M_0 &= M \\ M_{i+1} &= \frac{M_i+k}{\rho} - 1, \qquad \text{integer } i \geq 1 \end{cases} \tag{1}$$

where each $\rho > 1$, and $k = \lfloor \rho \rfloor$.

This is an approximately optimal policy for $m > 0$ which has a stronger upper bound for the general case. Therefore, we propose a new refusal policy based on above sequence of changeover thresholds. In this section, we consider the refusal strategy S_r, which is the time-independent strategy $S_\rho^{***}(m, M)$ with refusal time.

3.1.1 Competitive Analysis of S_r When $k = 1$

It pays the penalty ($\frac{1}{\rho^{k-1}} times$) the online player should changeover. We present a refusal times sequence $\{b_i\}$ as follows.

$$b_i = \begin{cases} 1 - \frac{1}{\rho^{k-1}(M_{i-1}-M_i)} & 0 \leq i \leq k \\ 0 & i = k+1 \end{cases} \tag{2}$$

Lemma 1. *For all* $1 \leq i \leq k$, $\{b_i\}$ *is non-increasing.*

Proof. From (1), we can get $M_j = \frac{M}{\rho^j} + \frac{\rho-k}{(\rho-1)\rho^j} + \frac{k-\rho}{\rho-1}$ by induction on j. Therefore, we obtain

$$\begin{aligned} M_{i-1} - M_i &= \frac{M}{\rho^{i-1}} + \frac{\rho-k}{(\rho-1)\rho^{i-1}} + \frac{k-\rho}{\rho-1} - \frac{M}{\rho^i} - \frac{\rho-k}{(\rho-1)\rho^i} - \frac{k-\rho}{\rho-1} \\ &= (M + \frac{\rho-k}{\rho-1})(\frac{1}{\rho^{i-1}} - \frac{1}{\rho^i}) \\ &= \frac{M(\rho-1)+\rho-k}{\rho^i} \end{aligned} \tag{3}$$

For every $\rho > 1$, $M_{i-1} - M_i$ is strictly decreasing with i. Hence, substitute equality (2) with (3), we get the following result:

$$b_i = 1 - \frac{\rho^{i-k+1}}{M(\rho-1)+\rho-k}$$

It is not hard to see that $\{b_i\}$ is non-increasing with i. □

Lemma 2. *For all* $1 \leq i \leq k$, $b_i \in [0, 1]$.

Proof. From (3), we obtain

$$\rho^{k-1}(M_{i-1} - M_i) = \rho^{k-1-i}(M(\rho-1)+\rho-k) \tag{4}$$

We can observe that

$$\rho^{k-i-1} \geq \rho^{-1} \tag{5}$$

for $k \geq i$.

From the assumption $M > \frac{\lfloor \rho \rfloor}{\rho - 1}$, it implies that

$$M(\rho - 1) + \rho - k > \rho \tag{6}$$

Hence, substitute (4) with (5) and (6), we obtain

$$\rho^{k-1}(M_{i-1} - M_i) > 1 \tag{7}$$

It clearly follows that $b_i \in [0, 1]$ from (2) and (7). □

3.1.2 Competitive Analysis of S_r When $k = 1$

Theorem 1. *The competitive ratio of S_r is $\sqrt{\frac{M+1}{m+1}}$ for $\sqrt{\frac{M+1}{m+1}} < 2$.*

Proof. When $k = 1$, from (1) and (2), we can write S_r as follows:

$$M_1 = \frac{M+1}{r} - 1 \tag{8}$$

$$\begin{aligned} b_1 &= 1 - \frac{1}{\rho^0 (M - M_1)} \\ &= 1 - \frac{1}{M + 1 - \sqrt{m+1} M + 1} \end{aligned} \tag{9}$$

$$M_2 = \frac{M_1 + 1}{r} - 1 = \frac{M+1}{r^2} - 1 \leq m \tag{10}$$

Solving the inequality (10), we obtain $r \geq \sqrt{\frac{M+1}{m+1}}$. We know that $r < 2$ for the assumption of $k = \lfloor r \rfloor$, hence $\sqrt{\frac{M+1}{m+1}} < 2$.

In the rest of this section, we will claim that the refusal strategy S_r will achieve the competitive ratio r. Like the paper [3], for $b_2 = 0$, the condition C_1 can be rewrite as follows:

$$M \leq r \cdot min\{M, M_1 + 1, Mb_1 + m(1 - b_1) + 1, M_1 b_1 + m(1 - b_1) + 2\} \tag{11}$$
$$M_1 + 1 \leq r \cdot min\{M, M_1 + 1, m + 1, m + 2\} \tag{12}$$
$$M_1 + 1 \leq r \cdot min\{M, m + 1, m + 2\} \tag{13}$$

We know that $M > m + 1$ and $M_1 > m$. Clearly, inequalities (12) and (13) hold for $m + 1$ is the minimum of the righthand side.

Next, we will check inequalities (11). The following four cases should be considered.

Case 1. $M \leq rM$, it's trivially holds for $r \geq 1$.

Case 2. $M \leq r(M_1 + 1) = M + 1$ holds.

Case 3. $M \le r(Mb_1 + m(1-b_1)+1)$. Substituting r with $\sqrt{\frac{M+1}{m+1}}$ and b_1 with (10), we obtain following result.

$$\begin{aligned} r(Mb_1 + m(1-b_1)+1) &= r(M(1-\frac{1}{M-M_1}) + m\frac{1}{M-M_1} + 1) \\ &\ge r(M+1+\frac{\frac{M+1}{r^2}-1-M}{M-\frac{M+1}{r}+1}) \\ &= rM-1 \end{aligned} \tag{14}$$

We note that the assumption $M > \frac{k}{r-1}$($k = 1$ in this case), which implies that $rM - 1 > M$. Hence, this inequality holds.

Case 4. $M \le r \cdot (M_1 b_1 + m(1-b_1)+2)$. We can see that

$$\begin{aligned} r \cdot (M_1 b_1 + m(1-b_1)+2) &= r(M_1(1-\frac{1}{M-M_1}) + (\frac{M+1}{r^2}-1)\frac{1}{M-M_1} + 2) \\ &= r(M_1+2)-1 \\ &= r(M+1) \end{aligned} \tag{15}$$

Therefore this inequality holds for $r \ge 1$.

Finally, For $b_1 = 1 - \frac{1}{M-M_1}$ we can prove that $Mb_1 + M_1(1-b_1)+1 = M$. It is easy to see that condition C_2 is identical to *Case* 4, and condition C_2 holds. □

3.2 A New Optimal Strategy

In this section, we propose another time-independent strategy S_{new} which can obtain the competitive ratio r when M is an element of the real interval.

Let $\{M_i\}_{i=1}^{k}$ be the changeover thresholds sequence. Then we define

$$\begin{cases} M_0 = M \\ M_i = M - \sqrt{\frac{i}{r}}(M-m) \qquad \text{integer } i \ge 1 \end{cases} \tag{16}$$

where $r > 1$ and $k = \lfloor r \rfloor$

Lemma 3. *In this case, the sequence $\{M_i\}$ strictly decreases below m within $k+1$ steps.*

Proof. For $k = \lfloor r \rfloor$, we obtain that $r < k+1 \le r+1$. Set $i = k+1$, and we get the following result from (16).

$$\begin{aligned} M_{k+1} &= M - \sqrt{\frac{k+1}{r}}(M-m) \\ &\le m \end{aligned} \tag{17}$$

□

Theorem 2. *For $M \in [c, d]$, S_{new} is r-competitive.*

Proof. Set $b_{k+1} = 0$ and $b_i = 1, i = 1, 2, \cdots, k$. The two sufficient and necessary conditions of the paper [2] reduce to the following condition.
C_1 *for all* $0 \le i \le j \le k$,

$$M_i + j \le r \cdot min\{M, M_{i+1} + 1\} \tag{18}$$

Set $c = m + \sqrt{r}$, and $d = \frac{(\sqrt{r(\lfloor r \rfloor+1)} - \sqrt{\frac{\lfloor r \rfloor}{r}})m + r - \lfloor r \rfloor}{1 - \sqrt{\frac{\lfloor r \rfloor}{r}} + \sqrt{r(\lfloor r \rfloor+1)} - r}$.

For $M \ge c$, we can get

$$\begin{aligned} M &\ge m + \sqrt{r} \\ &= M - \sqrt{\frac{1}{r}}(M - m) + 1 \\ &= M_1 + 1 \end{aligned} \tag{19}$$

Hence we only confirm that $M_i + j \le r \cdot (M_{i+1} + 1)$ holds.

For $M \le d$, and substituting $\lfloor r \rfloor$ with k, we can obtain the following result.

$$M - \sqrt{\frac{k}{r}}(M - m) + k \le rM - \sqrt{r(k+1)}(M - m) + r \tag{20}$$

For $i \in [1, k]$, inequality (20) can be expressed as follows.

$$M - \sqrt{\frac{i}{r}}(M - m) + k \le rM - \sqrt{r(i+1)}(M - m) + r$$

Namely, $M_i + k \le r(M_{i+1} + 1)$. Hence, condition C_1 holds and S_{new} is r-competitive. □

4 Weighted Replacement Problem P_1

In this section, we consider a class of the weighted replacement problem. As discussed in the introduction, to model this problem, we select an objective function to trade off payment flow and changeover costs. The factor $\eta \in (0, 1)$ weights the relative importance of payment flows and changeover. Compared with original problem P_0, the objective function of P_1 is defined as follows:

$$y' = \eta k + (1 - \eta) \sum_{i=0}^{k} (t_{i+1} - t_i) f(t_i) \tag{21}$$

where k is the changeover times, $\eta \in (0, 1)$ is the factor that weights the cost of payment flow versus the cost of changeover.

Without loss of generation, let $\beta = \frac{1-\eta}{\eta}$ and the equation (21) can be expressed:

$$y = k + \beta \sum_{i=0}^{k} (t_{i+1} - t_i) f(t_i) \tag{22}$$

We present a characterization theorem which can help to establish a refusal strategy and achieve a competitive ratio r.

Lemma 4. *S is r-competitive if and only if the following two conditions hold:* C_1 *for* $0 \le i \le j \le k$,

$$\beta M_i b_{j+1} + \beta M_j (1 - b_{j+1}) + j \le r \cdot Min \begin{bmatrix} \beta M_0 \\ \beta M_{i+1} + 1 \\ \beta M_0 b_{j+1} + \beta m (1 - b_{j+1}) + 1 \\ \beta M_{i+1} b_{j+1} + \beta m (1 - b_{j+1}) + 2 \end{bmatrix}$$

C_2 *for* $0 \le i < j \le k$

$$\beta M_i b_j + \beta M_j (1 - b_j) + j \le r \cdot Min \begin{bmatrix} \beta M_0 \\ \beta M_{i+1} + 1 \\ \beta M_0 b_j + \beta m (1 - b_j) + 1 \\ \beta M_{i+1} b_j + \beta m (1 - b_j) + 2 \end{bmatrix}$$

Proof. The proof is similar to the proof of the paper [3] presented. Hence we omit it here. □

4.1 The Refusal Strategy $WS(m, M)$

In this section, we construct a refusal strategy $WS(m, M)$ and analyze its competitive ratio. The sequence of changeover thresholds $\{M_i\}$ is

$$\begin{cases} M_0 &= M \\ M_{i+1} &= \frac{\beta M_i + i}{\beta r} - \frac{1}{\beta}, \qquad 0 \le i < k \end{cases} \tag{23}$$

where each $r > 1$.

And the refusal times sequence $\{b_i\}$ is

$$b_i = \begin{cases} 1 - \frac{1}{\beta(M_{i-1} - M_i)}, & 0 \le i \le k \\ 0, & i = k + 1 \end{cases} \tag{24}$$

In the rest of this section, we will claim $b_i \in (0, 1)$. Firstly, we obtain M_i by induction on j.

$$M_i = (M_0 + \frac{r^2}{\beta(r-1)^2}) r^{-i} - \frac{r^2}{\beta(r-1)^2} + \frac{i}{\beta(r-1)}$$

Therefore, set $\Delta = M_0 + \frac{r^2}{\beta(r-1)^2}$, and we can get following result:

$$\begin{aligned} M_{i-1} - M_i &= \frac{\Delta}{r^{i-1}} - \frac{r^2}{\beta(r-1)^2} + \frac{i-1}{\beta(r-1)} - \frac{\Delta}{r^i} + \frac{r^2}{\beta(r-1)^2} - \frac{i}{\beta(r-1)} \\ &= \frac{\beta M_0 (r-1)^2 + r^2 - r^i}{\beta(r-1) r^i} \end{aligned} \tag{25}$$

which implies that the difference $M_{i-1} - M_i$ is decreasing with i.

Using the equations $M_{k-1} = \frac{r(\beta M_k+1)-k+1}{\beta}$ and $M_k = \frac{r(\beta m+1)-k}{\beta}$, we get

$$\begin{aligned} M_{k-1} - M_k &= \frac{r(\beta M_k + 1) - k + 1}{\beta} - \frac{\beta M_k}{\beta} \\ &= (r-1)M_k + \frac{r-k+1}{\beta} \\ &= \frac{(r-1)(r(m\beta+1)-k)}{\beta} + \frac{r-k+1}{\beta} \end{aligned} \tag{26}$$

It is easy to see that

$$k < r(\beta m{+}1) - \beta m \tag{27}$$

for $M_{k+1} = m$ and $M_k > m$.

Substituting inequation (25) into equation (24), we achieve

$$\begin{aligned} M_{k-1} - M_k &= \frac{(r-1)(r(m\beta+1)-k)}{\beta} + \frac{r-k+1}{\beta} \\ &> \frac{1}{\beta} \end{aligned} \tag{28}$$

Therefore, we can get following Lemma from (22) and (26).

Lemma 5. *For* $0 \le i \le k$, $b_i \in (0,1)$.

We know that M_{k+1} is the minimal value, so $M_{k+1} = m \le M_{k+2} = \frac{\beta m+k+1}{\beta r} - \frac{1}{\beta}$. We can obtain

$$k \ge r(\beta m+1) - \beta m - 1 \tag{29}$$

From (27) and (29), we achieve the following result.

Lemma 6. *The changeover time is* $k = \lceil(\beta m+1)(r-1)\rceil$.

Theorem 3. *If* $\sqrt{\frac{\beta M}{\beta m+1}} \le \frac{\beta m+2}{\beta m+1}$*, then* $r = \sqrt{\frac{\beta M}{\beta m+1}}$ *and* $k = 1$.

Proof. If $\sqrt{\frac{\beta M}{\beta m+1}} \le \frac{\beta m+2}{\beta m+1}$, then we obtain $k \le 1$ from Lemma 6. Namely, WS strategy consists of one changeover threshold. Set $r = \sqrt{\frac{\beta M}{\beta m+1}}$, we can get following result from (23).

$$\begin{aligned} M_1 &= \frac{\beta M}{\beta r} - \frac{1}{\beta} \\ &= \frac{\sqrt{\beta M}\sqrt{\beta m+1} - 1}{\beta} \end{aligned} \tag{30}$$

And

$$\begin{aligned} b_1 &= 1 - \frac{1}{\beta M + 1 - \sqrt{\beta M}\sqrt{\beta m+1}} \\ &= \frac{\beta M - \sqrt{\beta M(\beta m+1)}}{\beta M + 1 - \sqrt{\beta M(\beta m+1)}} \end{aligned} \tag{31}$$

We know that b_2 is set to zero. Therefore condition C_1 reduces to the following equalities from Lemma (5).

$$\beta M \leq r \cdot min\{\beta M, \beta M_1{+}1, \beta M b_1 + \beta m(1-b_1){+}1, \beta M_1 b_1 + \beta m(1-b_1) + 2\} \quad (32)$$

$$\beta M_1 + 1 \leq r \cdot min\{\beta M, \beta M_1 + 1, \beta m + 1, \beta m + 2\} \quad (33)$$

$$\beta M_1 + 1 \leq r \cdot min\{\beta M, \beta m + 1, \beta m + 2\} \quad (34)$$

It is easy to see that $min\{\beta M, \beta M_1 + 1, \beta m + 1, \beta m + 2\} = \beta m + 1$ for $\beta M > \beta m + 1$. Otherwise, the on-line and the off-line player never changeover and the competitive ratio is 1. We only verify $\beta M_1 + 1 \leq r(\beta m + 1)$ for inequality (33) and (34). It holds for $M_2 = m$.

Next we consider the inequality (32) which has four cases discussed.

Case 1 and *Case* 2 clearly hold. By setting $i = 0$ the result can be obtained from (23).

Considering *Case* 3, and substitute b_1 with (31) and r with $\sqrt{\frac{\beta M}{\beta m+1}}$ we will confirm the following inequality holds.

$$\beta(M - M_1)^2 \geq M - m \quad (35)$$

For $M_1 = \frac{\sqrt{\beta M}\sqrt{\beta m+1}-1}{\beta}$ and $\sqrt{\frac{\beta M}{\beta m+1}} \leq \frac{\beta m+2}{\beta m+1}$, so we need to prove

$$\begin{aligned} \beta(M - M_1)^2 &\geq \frac{(\beta(M-m)-1)^2}{\beta} \\ &\geq M - m \end{aligned} \quad (36)$$

We know that $\beta M \geq \beta m + 1$, hence inequality (36) holds.

Case 4, we need to check that the following inequality holds.

$$\beta M \leq r(\beta M_1 b_1 + \beta m(1 - b_1) + 2) \quad (37)$$

Similar to *Case* 3, we substitute b_1 and r again, and to verify the inequality (37) holds we need to prove $M - M_1 \geq M_1 - m$. This inequality obviously holds.

Finally, for $0 \leq i < j \leq 1$, condition C_2 is $\beta M b_1 + \beta M_1(1 - b_1) + 1 \leq r \cdot min\{\beta M, \beta M_1 + 1, \beta M b_1 + \beta m(1 - b_1) + 1, \beta M_1 b_1 + \beta m(1 - b_1) + 2\}$. It is easy to see that $\beta M b_1 + \beta M_1(1 - b_1) + 1 = \beta M$ for substituting b_1 with $1 - \frac{1}{\beta(M-M_1)}$. Therefore condition C_2 holds. □

5 Concluding Remarks

In this paper, we restudy the online replacement problem and introduce the weight in the original model. Although this is only one step toward a more realistic solution of the problem, the introduction of this parameter considerably complicates the analysis and achieves some different results. There are some possible extensions that will lead towards more realistic models. For example, because the replacement problem is an important area of the financial decision, the methods of measurement used must be based on the concept of "time value of money". These include the methods of discount rate of return(DRR), net present value(NPV) and so on. Another research direction is how to introduce the risk-reward model [10] to this problem.

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Online Bin Packing of Fragile Objects with Application in Cellular Networks

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Abstract. We study a specific bin packing problem which arises from the channel assignment problems in cellular networks. In cellular communications, frequency channels are some limited resource which may need to share by various users. However, in order to avoid signal interference among users, a user needs to specify to share the channel with at most how many other users, depending on the user's application. Under this setting, the problem of minimizing the total channels used to support all users can be modeled as a specific bin packing problem as follows: Given a set of items, each with two attributes, weight and fragility. We need to pack the items into bins such that, for each bin, the sum of weight in the bin must be at most the smallest fragility of all the items packed into the bin. The goal is to minimize the total number of bins (i.e., the channels in the cellular network) used. We consider the on-line version of this problem, where items arrive one by one. The next item arrives only after the current item has been packed, and the decision cannot be changed. We show that the asymptotic competitive ratio is at least 2. We also consider the case where the ratio of maximum fragility and minimum fragility is bounded by a constant. In this case, we present a class of online algorithms with asymptotic competitive ratio at most of $1.7r$, for any $r > 1$.

Keywords: Bin packing; Channel assignment; On-line algorithm.

1 Introduction

In a typical cellular network, various users communicate with the base station using various frequency channels. While in a CDMA (code division multiple access) system, since each channel has a capacity much larger than the bandwidth requirement of a single user, it is possible to allow many users to share the same channel. Therefore to maximize the bandwidth utility in such system, one

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straightforward approach is to assign as many users to a single channel. However such assignment may result in interference between the users on the same channel, which affects the quality of communication. There exists a trade-off between the bandwidth utilization and communication quality. Roughly speaking, the fewer the users to share a common channel, the better the communication quality of each user. The requirements on communication quality may differ from users depending on the applications the users are running. To quantify the quality of communication of a user on a channel, we use a measure called the signal to noise ratio (SNR) [7, 9, 10]. Suppose there are M users communicating with a central base station at the same time with the same channel. If user i transmits with power p_i, then the signal received by the base station is $s_i = p_i g_i$, where g_i is the gain on the channel for user i. The SNR of user i is given by

$$\frac{s_i}{N_0 + \sum_{1 \le j \le M, j \ne i} s_j}, \tag{1}$$

where N_0 is the background noise power, e.g., the receiver thermal noise, which is assumed a constant. Generally speaking, the larger the value of SNR the better the communication quality. Suppose that each user specifies a minimum SNR that the user can tolerate. The objective of the channel assignment problem (CAP) is to assign the users to channels so that the SNR of each user of each channel is better than the minimum SNR the user specifies.

To tackle the CAP, Bansal, Liu and Zankar [1] defined the problem of bin packing with fragile objects (BPFO). The traditional bin packing problem [3, 2] is defined as follows. A sequence of (ordinary) items is given where each item has weight in $(0, 1]$. The goal is to pack all items into a minimum number of bins so that the total item weight in each bin is no more than 1. For BPFO, besides weight w_i, each (fragile) item is associated a *fragility* f_i. In each bin, the total weight of the items in the bin must be no more than the minimum fragility among the items in the bin. Precisely, suppose k items $a_{i_1}, a_{i_2}, \ldots, a_{i_k}$ are packed into the same bin. The packing is *feasible* if

$$w_{i_1} + w_{i_2} + \ldots + w_{i_k} \le \min\{f_{i_1}, f_{i_2}, \ldots, f_{i_k}\}.$$

The goal of BPFO is to find a feasible packing of all items with the minimum number of bins.

It can be shown that BPFO can model CAP. The frequency channels can be regarded as bins and the users as the items. For each user i, we let the amount of power received at the base station s_i be the weight w_i of item i. Suppose user i requires a minimum SNR of value β_i. From the SNR expression 1, we can see that user i can tolerate at most $s_i\beta_i - N_0$ power from other users on the same channel and still maintain the communication quality. We let $f_i = s_i + s_i\beta_i - N_0$. Then, CAP can be seen as a special case of BPFO. In the rest of the paper, we will focus on solving the BPFO.

Suppose an item is denoted as (w, f) where w is the weight and f is the fragility of the item. Fig. 1 gives an example of two feasible ways of packing the items $(1, 4), (2, 6), (2, 6), (2, 6)$ and $(3, 6)$. One (non-optimal) solution is to pack the first two items into one bin. The sum of weights is 3, which is less than the minimum fragility 4. Then the next two items are packed into one bin and the final item into another bin (see Fig. 1(a)). In the optimal solution, one can pack the first item and the last item into one bin and the other items into another bin (see Fig. 1(b)).

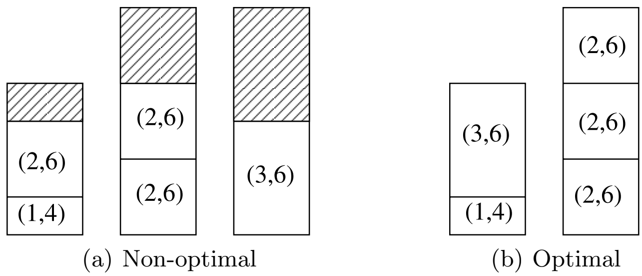

Fig. 1. Two feasible ways of packing the items $(1,4),(2,6),(2,6),(2,6),(3,6)$

Bansal, Liu and Zankar [1] studied the offline version BPFO, i.e., the set of items is given in the beginning. In this paper, we consider the on-line version of BPFO where the items arrive one by one, and the weight and fragility of an item are not known until the item arrives. This on-line version of BPFO also models the on-line version of CAP where the users arrive one by one and the power and minimum SNR of user are not known until the user arrives. For the on-line BPFO, when an item arrives, it must be assigned to one of the bins before the next item becomes known. Our goal is still to find a feasible packing of all items which minimize the number of bins.

The classical bin packing problems have been studied extensively. It is well known that the problem is NP-hard [6]. For other results in bin packing, the readers are referred to the survey papers [3, 2, 4]. For the offline BPFO, Bansal, Liu and Zankar [1] gave a 2-approximation algorithm, i.e., the algorithm can always pack the items with at most two times the number of bins used in the optimal solution. They also showed that no algorithm can achieve an approximation ratio of less than 3/2 unless $P = NP$. Then they gave an algorithm that uses at most the same number of bins as the optimum solution but the sum of item weight in each bin can be at most twice the minimum fragility of that bin.

Competitive analysis: To evaluate the on-line algorithms presented in this paper, we adopt the conventional measure of *competitive ratio*. For any input sequence I, let $A(I)$ and $OPT(I)$ denote the number of bins used by the on-line algorithm A and an optimal offline algorithm, respectively. The on-line algorithm A has a competitive ratio c if there exists a constant h such that

$$A(I) \leq c \cdot OPT(I) + h$$

for any input sequence I. In such case, we also say that A is a *c-competitive* on-line algorithm. The *asymptotic competitive ratio* R_A^∞ of A is defined by

$$R_A^\infty = \limsup_{m\to\infty} R_A^m \quad \text{where} \quad R_A^m = \max\{A(I)/OPT(I) \mid OPT(I) = m\}.$$

For simplicity we may use just A and OPT instead of $A(L)$ and $OPT(L)$ if the context is clear.

Our results: Let k be the *fragility ratio*, which is the ratio between maximum fragility and minimum fragility. We show that no on-line algorithm can have asymptotic competitive ratio less than 2 when k can be arbitrary large. We also show that the asymptotic competitive ratio of an any-fit algorithm, which include the classical first-fit and best-fit, is at least k. For the case that k is bounded by a constant, we develop a class of on-line algorithms that achieves the asymptotic competitive ratio $1.7r$ for any $r > 1$.

The remainder of this paper is organized as follows. In Section 2 we show that when the fragility ratio can be arbitrary large there is no on-line algorithm that is better than 2-competitive. In Section 3 we consider that the fragility ratio is bounded by a constant k. Section 3.1 shows that the asymptotic competitive ratio of an any-fit algorithm cannot be better than k. Section 3.2 gives a new class of algorithms and shows that its asymptotic competitive ratio $1.7r$ for any $r > 1$.

2 Unbounded Fragility Ratio

In this section we assume that the fragility ratio k can be arbitrary large. We show that the asymptotic competitive ratio is at least 2 for any on-line algorithm.

Theorem 1. *For the problem of packing fragile objects, no on-line algorithm has a asymptotic competitive ratio less than 2.*

Proof. First, we give an adversary of at most 6 items. We show that for any on-line algorithm the competitive ratio of packing these items is at least 2. Then we argue that the adversary can be extended to a long sequence that the optimal solution needs an arbitrary large number of bins.

The adversary begins with two items a_1 and a_2, where $a_1 = (\varepsilon, 1/4 + \varepsilon)$ and $a_2 = (1/4, 1)$, for $0 < \varepsilon < 1/12$. In the following, we exhaust all the ways how an on-line algorithm A can pack these two items and the subsequent items and show that the competitive ratio is at least 2.

Case 1. If the items a_1 and a_2 are packed into two different bins, the adversary stops. Clearly, $A = 2$ and $OPT = 1$.

Case 2. Consider the items a_1 and a_2 are packed into the same bin, namely B_1. Since the smaller fragility among these two items is $1/4 + \varepsilon$, which is the same as the total weight. B_1 is considered full in this case. The adversary gives another two items a_3 and a_4, where $a_3 = (\varepsilon, 1/4 + \varepsilon)$ and $a_4 = (1/4, 1)$, respectively.

Case 2.1. Consider the items a_3 and a_4 are packed into the same bin. Then adversary has the final two items $a_5 = (1/4 - \varepsilon, 1/4 + \varepsilon)$ and $a_6 = (1/4, 1)$ arrive, which obviously cannot be packed into one bin. So the algorithm needs to use 4 bins in total, i.e., $A = 4$. On the other hand, the optimal offline algorithm can pack a_1, a_3 and a_5 into one bin, and the remaining items into another bin, thus $OPT = 2$.

Case 2.2. Consider the items a_3 and a_4 are packed into two different bins, namely B_2 and B_3, respectively. In this case, the adversary gives an item $a_5 = (2\varepsilon, 1/2 + 2\varepsilon)$.

Case 2.2.1. If the item a_5 is packed together with item a_3, i.e., into bin B_2. The adversary gives the last item $a_6 = (1/4 - \varepsilon, 1/4 + \varepsilon)$. Since a_6 cannot fit into the existing three bins, B_1, B_2 and B_3, it has to be packed into a new bin. Therefore, $A = 4$. On the other hand, the optimal offline algorithm can pack a_1, a_3 and a_6 into one bin, and a_2, a_4 and a_5 into another bin. Then $OPT = 2$.

Case 2.2.2. If the item a_5 is packed together with item a_4, i.e., into bin B_3. The adversary gives the last item $a_6 = (1/4 + \varepsilon, 1)$. Since a_6 cannot fit into the existing three bins, B_1, B_2 and B_3, it has to be packed into a new bin. Therefore, $A = 4$. On the other hand, the optimal offline algorithm can pack items a_1, a_3, a_5 into one bin and items a_2, a_4, a_6 into another bin. Then $OPT = 2$.

Case 2.2.3. If the item a_5 is packed into a new bin, then the adversary stops. We have $A = 4$ but the optimal offline algorithm can use two bins, i.e., $OPT = 2$.

From all above cases, we show that $R_A \geq 2$. To bound the asymptotic competitive ratio, we can give an item sequence of length arbitrary large, so that the number of bins required by the optimal solution is arbitrary large. Let I_1 denote the sequence of items given by the adversary above (with at most 6 items). We define an arbitrary long sequence, composed by subsequence, $I_1, I_2, \ldots, I_m$, where I_i is obtained from I_{i-1} as follows. Since the fragility can be arbitrary large, we can make a copy of the sequence I_{i-1} and consider it as I_i but with the weight and the fragility of the items to be scaled up by the a ratio so that the minimum weight of items in I_i is larger than the maximum fragility of the items in I_{i-1}. In that case, any item in I_i cannot be packed into the bins containing items of I_{i-1} and the optimal solution needs to use as many bins as the length of the sequence, which is arbitrary large. Together with the fact that for every subsequence I_i the on-line algorithm uses at least two times the number of bins of the optimal solution for I_i, we have the asymptotic competitive ratio for any on-line algorithm at least 2. □

3 Bounded Fragility Ratio

In this section we assume that the fragility ratio k is a given constant. We first analyze the any-fit algorithm and then develop some new algorithms.

3.1 Any-Fit Algorithms

We consider a common type of algorithms, called the any-fit algorithms. An any-fit algorithm packs an item into a non-empty bin if the bin can accommodate the item, otherwise the item is packed into an empty bin. There are special cases of the any-fit algorithm, e.g., first-fit and best-fit which differ in the way of choosing which non-empty bin to pack the item. In the following theorem, we show that an any-fit algorithm has an asymptotic competitive ratio at least k.

Theorem 2. *The asymptotic competitive ratio of an any-fit algorithm cannot be better than k.*

Proof. The adversary sequence consists of $2kn$ items: $a, b, a, b, \ldots$, where $a = (1 - 1/kn, k)$ and $b = (1/kn, 1)$. An any-fit algorithm packs each pair items a and b into one bin. Thus, the number of bins used by the any-fit algorithm is kn. For the optimal solution, one can pack k items a together into one bin, and all the kn items a_2 in one bin. Then $OPT = n + 1$. The theorem follows as n can be arbitrary large. □

3.2 Items Divided by Classes

We define a general method to pack the items by first dividing the items into *classes*. A predefined parameter $r > 1$ is used. An item with fragility f belongs to the class s for an integer s if $f \in [r^s, r^{s+1})$. As the fragility ratio is at most k, the number of classes is at most $\lceil \log_r k \rceil$. We only pack items of the same class into the same bin. The bin which stores an item of class s is denoted as a class s bin. The general method is described as follows. When a new item with fragility f arrives, we find an integer s such $r^s \leq f < r^{s+1}$. If there is a class s non-empty bin that the item can fit in, then pack the item into one of those bins. Otherwise, pack the item into an empty bin.

We give two specific implementations of the general method, namely the ANF_r and AFF_r. They differ in the way of packing items of the same class. In short, ANF_r makes use of next-fit and AFF_r makes use of first-fit. Precisely, for ANF_r, it maintains at most one *active* non-empty bin only for each class. If the item cannot fit in the active bin, the bin is closed and the item is packed into an empty bin which becomes the only active bin in the class. It is clear that next-fit takes $O(1)$ time to pack each item and $O(n)$ time to pack a sequence of n item. For AFF_r, according to the order in which the bins are used, it finds the first non-empty bin in the class where the new item can fit it and packs the item into that bin. However, in the worst case it takes $\Theta(n \log n)$ time to pack a sequence of n items [8].

In the following we give a tight analysis of the performance of ANF_r.

Lemma 1. *The asymptotic competitive ratio of algorithm ANF_r cannot be better than $2r$.*

Proof. We give an adversary as follows. There are $2rn$ pairs of items $(1/2, r)$ and $(\varepsilon, 1)$ arriving, where $\varepsilon > 0$ is a sufficiently small number. After that, a sequence of items with small weights and increasing fragilities arrives. Precisely, the whole item sequence is

$$\underbrace{(1/2, r), (\varepsilon, 1), (1/2, r), (\varepsilon, 1), \ldots, (1/2, r), (\varepsilon, 1)}_{4rn \text{ items}}, (\varepsilon, r^2), (\varepsilon, r^3), \ldots, (\varepsilon, k).$$

It can be seen that $ANF_r = 2rn + T - 1$ where $T \leq \log_r k$ is the number of classes. For an optimal solution, it can pack $2r$ items $(1/2, r)$ into one bin and all the items with weight ε into another bin. Hence, $OPT = n + 1$. The asymptotic competitive ratio of algorithm ANF_r is $(2rn+T-1)/(n+1)$, which is arbitrary close to $2r$ as n and hence OPT are arbitrary large. □

Before we analyze the upper bound on the asymptotic competitive ratio of ANF_r, we give a lower bound on the number of bins to pack a set of items in the following lemma.

Lemma 2. *Given a set of n items (w_i, f_i) for $1 \leq i \leq n$, the n items cannot be packed with less than $\sum_{i=1}^{n} w_i/f_i$ bins.*

Proof. For any packing of the n items, consider one of the bins. Suppose that t items $(w_{j_1}, f_{j_1}), \ldots, (w_{j_t}, f_{j_t})$ are packed into the bin. We have $\sum_{i=1}^{t} w_{j_i} \leq \min_{i=1}^{t}(f_{j_i})$ and thus $\sum_{i=1}^{t} (w_{j_i}/f_{j_i}) \leq \sum_{i=1}^{t} w_{j_i}/\min_{i=1}^{t}(f_{j_i}) \leq 1$. Counting all the bins in the packing, we can see that the number of bins used is at least $\sum_{i=1}^{n} w_i/f_i$.

Theorem 3. *The asymptotic competitive ratio of algorithm ANF_r is no more than $2r$.*

Proof. Consider an item sequence I. We create another item sequence I' based on I. For each item (w, f) in I, there is a corresponding item (w, f') in I', with the same weight but different fragility. Precisely, if $r^s \leq f < r^{s+1}$ for some integer s, i.e., the item of I in class s, then we have $f' = r^{s+1}$ and the corresponding item of I' is in class $(s+1)$. It is easy to see that the number of bins required to pack the items in I' is no more than that for items in I, i.e., $OPT(I') \leq OPT(I)$, because each item has a larger fragility.

Let S and S' be the sets of integers that denote the classes required to divide the items in I and I', respectively. By the construction of I', we can see that for each $i \in S$, we have $i+1 \in S'$. Let q_i and q'_i be the total weight of items in class i for I and I', respectively. We have $q_i = q'_{i+1}$ according to the construction of I'. By Lemma 2, we have $OPT(I') \geq \sum_{i \in S'} q'_i/r^i = \sum_{i \in S} q_i/r^{i+1}$.

Consider I and the packing of items in class i by ANF_r. Let b_i be the number of bins used by ANF_r for the items in class i. We have $ANF_r(I) = \sum_{i \in S} b_i$. We claim that $b_i \leq 2rq_i/r^{i+1} + 1$ for any $i \in S$. The claim is proved as follows. We say that two bins of the same class are "adjacent" if in the execution of ANF_r one of the bins is closed and immediately the other one is made active. For any two adjacent bins of the same class i, we can see that their total weight is at least r^i, otherwise all items in the two bins can be packed into one. This implies that on average, except the last one if b_i is odd, each bin stores items of total weight at least $r^i/2$. Therefore, except the last bin if b_i is odd, if we consider each bin has weight just $r^i/2$, the total weight of all bins is at most the total weight of items in class i. Precisely, we have $(b_i - 1)r^i/2 \leq q_i$, i.e., $b_i \leq 2rq_i/r^{i+1} + 1$.

As a result, we have

$$B_{ANF_r}(I) = \sum_{i \in S} b_i \leq \sum_{i \in S} (2rq_i/r^{i+1} + 1) \leq 2rB_{\text{OPT}}(I') + |S| \leq 2rB_{\text{OPT}}(I) + |S|$$

where $|S| \leq \log_r k$ is the number of classes. □

In the following we analyze the performance of AFF_r and give an upper bound on its asymptotic competitive ratio. Our analysis makes use of the function W defined in [5] and some lemma and theorem related to the classical bin packing [5]. The function $W : [0,1] \to [0, 8/5]$ is defined as follows:

$$W(\alpha) = \begin{cases} \frac{6}{5}\alpha & \text{for } 0 \leq \alpha \leq \frac{1}{6}, \\ \frac{9}{5}\alpha - \frac{1}{10} & \text{for } \frac{1}{6} < \alpha \leq \frac{1}{3}, \\ \frac{6}{5}\alpha + \frac{1}{10} & \text{for } \frac{1}{3} < \alpha \leq \frac{1}{2}, \\ \frac{6}{5}\alpha + \frac{4}{10} & \text{for } \frac{1}{2} < \alpha \leq 1. \end{cases}$$

Lemma 3 (Garey et al. [5]). *Given a set of numbers $S = \{w_1, w_2, \ldots\}$ with total at most 1, i.e., $\sum_{w_i \in S} w_i \leq 1$, we have $\sum_{w_i \in S} W(w_i) \leq 1.7$.*

Theorem 4 (Garey et al. [5]). *In the classical bin packing problem, given any sequence I of n items with weights $w_1, w_2, \ldots, w_n$, the number of bins used by first-fit, $FF(I)$, satisfies the inequality: $FF(I) \leq \sum_{1 \leq i \leq n} W(w_i) + 1$.*

From Lemma 3, we have a lower bound on the number of bins used in the classical bin packing.

Corollary 1. *In the classical bin packing problem, given any sequence I of n items with weights $w_1, w_2, \ldots, w_n$, the number of bins required to pack all items of I is at least $\sum_{1 \leq i \leq n} W(w_i)/1.7$.*

Theorem 5. *$AFF_r(I) \leq 1.7r \cdot OPT(I) + \log_r k$ for any item sequence I.*

Proof. Consider an item sequence I. We create a new item sequence I' so that for each item (w, f) in I, there is a corresponding item (w, f') in I' with $f' = r^s$ where $r^s \leq f < r^{s+1}$ for some integer s. It can be proved that $AFF_r(I) \leq AFF_r(I')$.

Consider a packing by AFF_r for items of I'. Let S and S' be the sets of integers that denote the classes required to divide the items in I and I', respectively. Note that $S = S'$ because for any item of I the corresponding item of I' falls into the same class as the item of I. Let b'_i be the number of bins used for the items in class i and let $T^i = \{w^i_1, w^i_2, \ldots\}$ be the weights of items in class i. As $S = S'$, the definition of the set T^i is the same for both I and I'. Since all items in class i have the same fragility r^i, we can apply Theorem 4 and have an upper bound on b'_i, i.e., $b'_i \leq \sum_{w^i_j \in T^i} W(w^i_j/r^i) + 1$. Summing up for all classes, we have

$$\begin{aligned} AFF_r(I') = \sum_{i \in S'} b'_i &\leq \sum_{i \in S'} \sum_{w^i_j \in T^i} W(w^i_j / r^i) + |S'| \\ &\leq r \cdot \sum_{i \in S'} \sum_{w^i_j \in T^i} W(w^i_j / r^{i+1}) + |S'| \\ &\leq r \cdot \sum_{(w,f) \in I} W(w/f) + |S| \end{aligned} \tag{2}$$

because $S = S'$ and for each class i item $(w, f) \in I$ we have $f < r^{i+1}$ and the function W is monotonic increasing.

We define a classic bin packing problem instance I^c by the input sequence I. For each item (w, f) in I, we create an item in I^c with weight w/f. The number of bins used by the optimal packing for I^c, $OPT^c(I^c)$, is at most the number of bins used by the optimal packing for I, i.e., $OPT^c(I^c) \leq OPT(I)$, because we can always transform the latter solution to a former one. By Corollary 1, we have $1.7 \cdot OPT^c(I^c) \geq \sum_{(w,f) \in I} W(w/f)$. Together with Inequality 2, we have

$$AFF_r(I) \leq AFF_r(I') \leq 1.7r \cdot OPT^c(I^c) + |S| \leq 1.7r \cdot OPT(I) + |S|$$

where $|S| \leq \log_r k$ is the number of classes. □

Remarks: Suppose that the items of the input sequence consist of only d distinct fragilities. We can modify our algorithm AFF to group only the items of the same fragility to the same class and then apply first-fit to pack the items. If $d < k$, then AFF in this case has a slightly better performance in terms of the additive constant, i.e., $AFF \leq 1.7r \cdot OPT + d$.

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Online Algorithms for the Vehicle Scheduling Problem with Time Objective

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Abstract. Falling across the problem of the unforeseen congested events in the logistic transportation, we build the online vehicle scheduling model with the recoverable congested vertices in this paper. Our optimization objective minimizes the total time of online scheduling. Considering the dynamic occurrence characteristics of the congested vertices one by one, we first introduce three online scheduling strategies, i. e., Greedy Strategy, Reposition Strategy, and Waiting Strategy, and analyze the advantages and disadvantages of competitive performances of the three strategies. Then we propose the Simple Choice Strategy. By competitive performance analysis, we think that the Simple Choice Strategy could be an optimal scheduling scheme for online vehicle transportation.

1 Introduction

In practice, the carrier often transports goods by the vehicle from the origin to the destination. The crossings and roads of traffic networks are viewed as the vertices and edges of the weighted and undirected graph $G = (V, E)$, where V is the set of all vertices with $|V| = n$, and where E is the set of all weighted edges. Let O be the origin and D the destination respectively, and O_i $(1 \leq i \leq n-2)$ be the other vertex. Let $w(x, y)$ be the time needed for the vehicle to move from the vertex x to the vertex y in graph G. In fact, the carrier always hopes that the vehicle can move along the optimal route so as to minimize the time, while the optimal route is not creditable, because some vertices may be blocked so as not to pass through. Let us first consider the following problems:

Q1. If the carrier knows the congested sequence $R_k = (r_1, r_2, \cdots, r_k)$ and the relatively recoverable time sequence $T(R_k) = (t(r_1), t(r_2), \cdots, t(r_k))$ in advance, then how the vehicle is scheduled to minimize the total time?

Q2. If the information of the congested sequence R_k and the relatively recoverable time sequence T_k is dynamically available, i.e., when the carriers can only know the past and present information, but can not know the future information, how the vehicle is scheduled to minimize the total time?

Since Q1 is an offline problem, the carriers can obtain the optimal route by using the classical optimization theory. While Q2 is an online problem, with the dynamic recurrence of the congested vertices and the recoverable time of the

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relatively congested vertex one by one, it is difficult to deal with Q2. However, in recent years, there occurs an interesting research focusing in the field of algorithms, which is the online algorithm. The method of competitive analysis can be used to solve online problem in some sense [1–9].

2 Basic Assumptions

For the convenience of discussion, basic assumptions of online vehicle scheduling problem are stated as follows.

(1) Suppose the vehicle always meet k ($k > 0$) number vertices, while going from O to D by whatever route, and suppose $R_k = (r_1, r_2, \cdots, r_k)$ be a congested sequence, and $R_i = (r_1, r_2, \cdots, r_i)$ ($0 \leq i \leq k$) be a subsequence of R_k. For $i = 0$, R_0 means that the vehicle doesn't meet the congested vertex.

(2) Blockages happen at the crossings. Let $t(r_i)$ ($i = 1, 2, \cdots, k$) be the recoverable time of the congested vertex r_i, $T(R_k) = (t(r_1), t(r_2), \cdots, t(r_k))$ be the total recoverable time sequence of R_k, i.e., $T(R_k) = \sum_{i=1}^{k} t(r_i)$. Once the congested vertex is recoverable, it cannot be blocked again.

(3) Graph G' deriving from the removal of the congested vertices in Graph G is also connected.

(4) Let $T_{opt}(OD|R_i)$ ($0 \leq i \leq k$) be the minimum required time from O to D when knowing the occurrence of the congested sequence R_i in advance. For $i = 0$, let $T_{opt}(OD|R_i) = T_{opt}(OD)$.

(5) Suppose that the vehicle meet the congested vertex r_i at the adjacent vertex O_i ($1 \leq i \leq k$). Let $T_{opt}(O_iD|R_i)$ be the minimum time for the vehicle to move along the optimal route from O_i to D when the congested sequence R_i is known in advance.

(6) Let $TT_{ALG}(OD|R_i)$ ($1 \leq i \leq k$) be the total time that online algorithm ALG spent from O to D in the face of the congested sequence R_i one by one.

(7) Suppose that $T_{opt}(OD) + T(R_k) > T_{opt}(OD|R_k)$. Otherwise, the vehicle can reach the destination within the shortest time after each recovery of the roads when meeting each congestion.

Definition 1. *For any graph G, $T_{opt}(OD|R_k)$ and $TT_{ALG}(OD|R_k)$ is called off-line time and on-line time separately.*

Definition 2. *For any graph G, if there is only constant α, related to the number k of sequence of the congested sequence R_k, and any constant β meets this condition*

$$TT_{ALG}(OD|R_k) \leq \alpha T_{opt}(OD|R_k) + \beta,$$

then ALG is called as α-competitive for the on-line problem [1–5].

In fact, the following lemma can be directly obtained from the graph theory.

Lemma 1. *It holds that*

$$T_{opt}(OD) \leq T_{opt}(OD|R_1) \leq T_{opt}(OD|R_2) \leq \cdots \leq T_{opt}(OD|R_k).$$

3 Analysis on the Three Basic Strategies

Literature [10] has the minimum cost needed for vehicles in transportation as its optimized objective, putting forward the competitive algorithms of greedy strategy and reposition strategy. Here we will draw out these two strategies and analyze their competitive ratio and competitive performance, taking time as the optimized objective.

3.1 Greedy Strategy

Greedy Strategy: When the carrier reaches O_i and finds that the next vertex r_i is congested, he can choose the shortest path from the current vertex O_i to D after excluding the congested vertex sequence R_i, and then moves along this path.

Denote the Greedy Strategy as A, and we obtain the following theorem.

Theorem 1. *For online vehicle scheduling problem with the congested sequence R_k, the competitive ratio of A is $2^{k+1} - 1$.*

Proof. The vehicle starts from O and takes the route according to Greedy Strategy when meeting congested vertex and gets to the destination D, then the route that the vehicle takes is $OO_1O_2 \cdots O_kD$. The total time it takes is $TT_A(OD|R)$, which satisfies

$$TT_A(OD|R_k) \leq T_{opt}(OD) + T_{opt}(O_1D|R_1) + T_{opt}(O_2D|R_2) + \cdots + T_{opt}(O_kD|R_k).$$

Note that $T_{opt}(OO_1), T_{opt}(O_1O_2|R_1), T_{opt}(O_2O_3|R_2), \cdots, T_{opt}(O_{k-1}O_k|R_{k-1})$, are parts of $T_{opt}(OD), T_{opt}(O_1D|R_1), T_{opt}(O_2D|R_2), \cdots, T_{opt}(O_{k-1}D|R_{k-1})$, respectively. Using lemma 1, it follows that

$$\begin{aligned}
T_{opt}(O_1D|R_1) &\leq T_{opt}(O_1O) + T_{opt}(OD|R_1) \leq 2T_{opt}(OD|R_1), \\
T_{opt}(O_2D|R_2) &\leq T_{opt}(O_2O_1|R_1) + T_{opt}(O_1O) + T_{opt}(OD|R_2) \\
&\leq 2^2 T_{opt}(OD|R_2), \\
&\cdots, \\
T_{opt}(O_{k-1}D|R_{k-1}) &\leq T_{opt}(O_{k-1}O_{k-2}|R_{k-2}) \\
&\quad + \cdots + T_{opt}(O_1O) + T_{opt}(OD|R_{k-1}) \\
&\leq 2^{k-1} T_{opt}(OD|R_{k-1}), \\
T_{opt}(O_kD|R_k) &\leq T_{opt}(O_kO_{k-1}|R_{k-1}) + T_{opt}(O_{k-1}O_{k-2}|R_{k-2}) \\
&\quad + \cdots + T_{opt}(O_1O) + T_{opt}(OD|R_k) \\
&\leq 2^k T_{opt}(OD|R_k).
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
TT_A(OD|R_k) &\leq T_{opt}(OD) + 2T_{opt}(OD|R_1) + 2^2 T_{opt}(OD|R_2) \\
&\quad + \cdots + 2^k T_{opt}(OD|R_k) \\
&\leq (2^{k+1} - 1) T_{opt}(OD|R_k).
\end{aligned}$$

This ends the proof.

Remark 1. Note that, regarding to Greedy Strategy, the online time that the vehicle takes to move goods from O to D can be $2^{k+1}-1$ times as much as it takes for the off-line problem in the worst case. So this strategy is not feasible in such case. But if the time taken to go along the optimized route every time when meeting with the congested vertex is "relatively shorter", it's quite economic to adopt this strategy. In normal situation, as for the congested vertex, the time it takes to go along the roads can be either short (i.e., "good road") or long (i.e., "bad road"). Going along the "good road" is of course more economic and going along the "bad road" takes longer time. So it comes to the situation that it's a pity to give it up but risky to adopt. The competitive performance of this strategy is positive, but risky.

3.2 Reposition Strategy

Reposition Strategy: When the carrier reaches O_i and finds that the next vertex r_i is congested, he moves back to the origin O along the current route and then chooses the optimal path in G exclusive of the set of the known congested vertices.

Denote Reposition Strategy as B, and we have the following theorem.

Theorem 2. *For online vehicle scheduling problem with the congested sequence R_k, the competitive ratio of B is $2k+1$.*

Proof. The vehicle departs from the origin O. According to Reposition Strategy, the total online time taken to go from O to D for the vehicle when meeting the congested vertex is $TT_B(OD|R)$. It satisfies

$$\begin{aligned} TT_B(OD|R_k) &\le 2T_{opt}(OD) + 2T_{opt}(OD|R_1) + 2T_{opt}(OD|R_2) \\ &\quad + \cdots + 2T_{opt}(OD|R_{k-1}) + T_{opt}(OD|R_k) \\ &\le 2T_{opt}(OD|R_k) + 2T_{opt}(OD|R_k) + 2T_{opt}(OD|R_k) \\ &\le \ + \cdots + 2T_{opt}(OD|R_k) + T_{opt}(OD|R_k) \\ &= (2k+1)T_{opt}(OD|R_k). \end{aligned}$$

This completes the proof.

Remark 2. Note that, regarding to Reposition Strategy, the time that the vehicle takes to move goods from the origin to the destination can be $2k+1$ times as much as it takes for the off-line problem in the worst case. Compared with Greedy Strategy, Reposition Strategy is avoiding the risk from taking the "bad road" but missing the opportunity of taking the "good road". Taking Reposition Strategy is not risky but conservative. So we say, the competitive performance of Reposition Strategy is safe but conservative.

3.3 Waiting Strategy

Waiting Strategy: The carrier stays waiting every time when meeting a congested vertex until it is recoverable, and then he continues moving along the original route.

Denote Waiting Strategy as C. We have that the online time is $TT_C(OD|R_k)$= $T_{opt}(OD) + T(R_k)$, and the offline time is $T_{opt}(OD|R_k)$. Thus, it follows that

$$\frac{TT_C(R_k)}{T_{opt}(OD|R_k)} = \frac{T_{opt}(OD) + T(R_k)}{T_{opt}(OD|R_k)}.$$

Remark 3. The competitive ratio of Waiting Strategy is not only connected with the number of the congested vertex but the waiting time $T(R_k)$. When $T(R_k)$ is small, the competitive ratio is close to 1; but when $T(R_k)$ is large, the competitive ratio is large, too. Therefore, the competitive ratio of Waiting Strategy is increasing with $T(R_k)$, to an endless amount. So the competitive performance of Waiting Strategy is both economic and risky.

Here, a very practical question is lying before the carriers: which strategy are they to choose to arrange their vehicles? Let's first look at a simple example.

3.4 A Simple Example

Suppose in Figure 1, the vehicle, going from O to D, will in turn meet three congested vertices $R = \{r_1, r_2, r_3\}$ at the the adjacent vertices O_i $(i = 1, 2, \cdots, 18)$ while travelling by whatever route. And we have $t(r_1) = 2$, $t(r_2) = 1$, $t(r_3) = 0.5$. To distinguish two routes between O_6 and D, we mark a node O_0. Suppose that no information about three congested vertices or three adjacent vertices be known before hand, nor be the information about the removing time known.

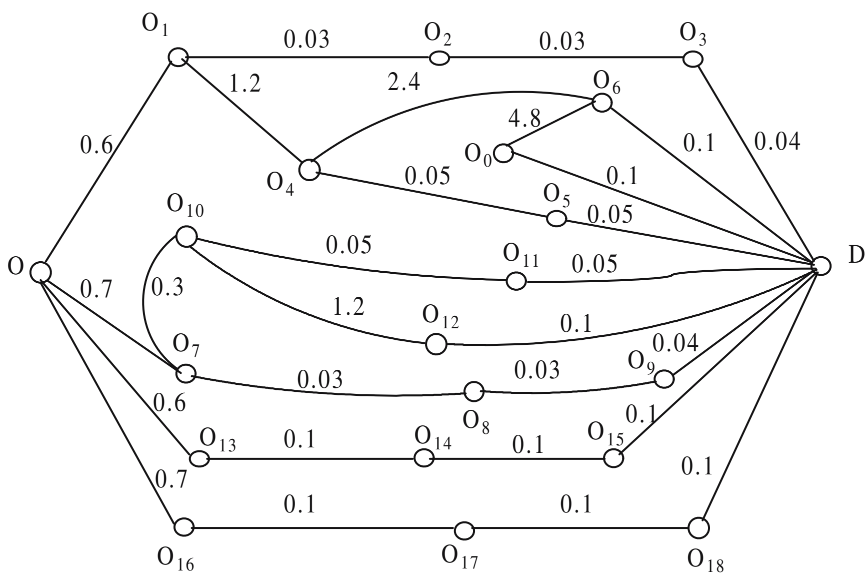

Fig. 1. The Diagram of the Competitive Analysis

The weight of edges shows the time for the vehicle to go through the distance. What is to be considered now is which road to take so that the time is relatively less.

(1) If the vehicle takes the route according to Greedy Strategy, going along the optimized route $OO_1O_2O_3D$, it meets the congested vertex r_1 when coming to O_1; if r_1 is known, the optimized route $O_1O_4O_5D$ from O_1 to D can be figured out. Then the vehicle goes along this route. When it reaches vertex O_4 and meets the congested vertex r_2, the optimized route O_4O_6D from O_4 to D can be figured out if r_1, r_2 is known. Then the vehicle goes on along this route. When it comes to vertex O_6 and meets the congested vertex r_3, if r_1, r_2, r_3 is already known, the optimized route O_6O_0D from O_6 to D can be figured out., then the vehicle goes on along this route to the destination D. Therefore, the route that the vehicle takes is $OO_1O_4O_6O_0D$. It's easy to figure out the time how long it takes the vehicle to go through this route, which is 9.1.

(2) If the vehicle takes the route according to Reposition Strategy, it goes along the optimized route $OO_1O_2O_3D$ and reaches the vertex O_1. Then when it meets the congested vertex r_1, it returns along O_1O. If r_1 is already given, going along the optimized route $OO_7O_8O_9D$, it will return along O_7O at the vertex O_7 when meeting congested vertex r_2; if r_1, r_2 is given, going along the optimized route $OO_{13}O_{14}O_{15}D$, it will return along $O_{13}O$ at the vertex O_{13} when meeting congested vertex r_3; if r_1, r_2, r_3 is given, going along the optimized route $OO_{16}O_{17}O_{18}D$, it will reach the destination D. So the route that the vehicle takes is $OO_1OO_7OO_{13}OO_{16}O_{17}O_{18}D$. It's easy to figure out the time the vehicle takes is 4.8.

(3) If the vehicle takes the route according to Waiting Strategy, it will stop to wait at the vertex O_1 when meeting congested vertex r_1 while going along the optimized route $OO_1O_2O_3D$. In a time period of two after the congested vertex r_1 is recoverable, the vehicle continues its trip along the original route. Then it reaches vertex O_2 and meets congested vertex r_2. After waiting for a time-period of one, the congested vertex r_2 is recoverable and the vehicle continues its trip along the original route and reaches the vertex O_3. At the vertex 3 it meets congested vertex r_3 then wait for a time period of 0.5. After congested vertex r_3 is removed, this vehicle goes along the same route and reaches the destination D. It's easy to figure out the time that the vehicle takes in the whole trip is 4.2.

(4) Now let's consider another strategy. For the convenience of discussion, we assume that the minimum time be the optimal route, because $w(\cdot)$ shows the time of the optimal route, i.e., the minimum time. The vehicle starts at O along the optimized route $OO_1O_2O_3D$ and meets congested vertex r_1 at the vertex O_1. If r_1 is given, the optimized route from O_1 to D can be figured out, which is $w(O_1O_4O_5D) = 1.3$; and the optimized route from O to D, which is $w(OO_7O_8O_9D) = 0.8$. Meanwhile, both the waiting time at vertex O_1, which is $t(r_1) = 2$, and the sum of it with the distance $w(O_1O_2O_3D) = 0.1$ to go after the congested vertex r_1 is removed, can be figured out, which is 2.1. It can be seen that $w(OO_7O_8O_9D) = 0.8$ is the shortest among the three alternatives. Then the decision is to go back from O_1 to O, and go on along

$OO_7O_8O_9D$. It meets congested vertex r_2 when coming to the vertex O_7. If r_1, r_2 is given, the following items can all be figured out: the optimized route from O_7 to D is $w(O_7O_{10}O_{11}D) = 0.4$ and the optimized route from O to D is $w(OO_{13}O_{14}O_{15}D) = 0.9$; the waiting time at the vertex O_7 is $t(r_2) = 1$, plus the distance $w(O_7O_8O_9D) = 0.1$ to go after r_2 is recoverable, the sum is 1.1. The smallest among the three is $w(O_7O_{10}O_{11}D) = 0.4$, so the vehicle chooses to go along this route. It comes to the vertex O_{10} and meets congested vertex r_3. Knowing r_1, r_2, r_3, the optimized route $w(O_{10}O_{12}D) = 1.3$ from O_{10} to D; and the optimized route $w(OO_{13}O_{14}O_{15}D) = 0.9$ from O to D can be figured out. And the waiting time at the vertex O_{10} is $t(r_3) = 0.5$; the sum of it with the distance $w(O_{10}O_{11}D) = 0.1$ to go after the congestion at the vertex r_3 is recoverable, is 0.6. And the smallest of the three is $t(r_3) = 0.6$, so the decision is to wait at the vertex O_{10} for a time period of 0.5 until the congestion at the vertex r_3 is recoverable. The vehicle goes on along the route $O_{10}O_{11}D$ and reaches the destination D. It's easy to figure out the time for the vehicle to finish the whole trip is 2.8.

Remark 4. It can be seen from this example that, just as what has already been said, the performance of Greedy Strategy is active but risky; Reposition Strategy, conservative but safe; and the performance of Waiting Strategy is directly related to $T(R_k)$. In Case (4), Reposition Strategy is adopted to avoid the risk of going along the "bad road". It also choose Greedy Strategy to grasp the chance of going the "good road". Meanwhile, it chooses to wait for some time to go on the optimized route. As a result, it is better than other strategies in practical use. Being illuminated by Case (4), we hope to find a good strategy in way of combining the three strategies together, so that the best choice may be made among the three strategies: Greedy Strategy, Reposition Strategy and Waiting Strategy, every time when the vehicle meets congestion. Thus, we will present the following Simple Choice Strategy that is more economic and feasible.

4 Competitive Analysis of Simple Choice Strategy

Simple Choice Strategy: When the vehicle meets a congested vertex r_i with the number i $(i = 1, 2, \cdots, k)$, the carrier first figures out

$$T_i = \min\{T_{opt}(O_iD|R_i), T_{opt}(OD|R_i), t(r_i) + T_{opt}(O_iD|R_{i-1})\},$$

and then he chooses which strategy to go the later route according to the following condition.

(1) If $T_i = T_{opt}(O_iD|R_i)$, the vehicle will choose the route according to Greedy Strategy.

(2) If $T_i = T_{opt}(OD|R_i)$, the vehicle will go according to Reposition Strategy.

(3) If $T_i = t(r_i) + T_{opt}(O_iD|R_{i-1})$, the vehicle will go according to Waiting Strategy .

Note that, to figure out T_i, knowing $T_{opt}(O_iD|R_i), T_{opt}(OD|R_i)$ and $t(r_i) + T_{opt}(O_iD|R_{i-1})$. If three of them are equal, or at least two of them are equal, the carrier takes $T_i = t(r_i) + T_{opt}(O_iD|R_{i-1})$ first, then $T_i = T_{opt}(O_iD|R_i)$.

Denote Comparison Strategy as D, and then it comes the following theorem.

Theorem 3. *For online vehicle scheduling problem with the congested sequence R_k and the relatively recoverable time sequence $T(R_k)$, the competitive ratio of D is $2k+1$.*

Proof. (1) For all $i \in \{1, 2, \cdots, k\}$, if there always comes $T_i = T_{opt}(O_i D|R_i)$, then the vehicle can choose the route according to Greedy Strategy all the way. The total time is $TT_D(OD|R_k)$, which satisfies

$$\begin{aligned} TT_D(OD|R_k) &\leq w(OO_1)+T_{opt}(O_1D|R_1)+T_{opt}(O_2D|R_2)+\cdots+T_{opt}(O_kD|R_k) \\ &\leq T_{opt}(OD)+T_{opt}(OD|R_1)+T_{opt}(OD|R_2)+\cdots+T_{opt}(OD|R_k). \end{aligned}$$

Using lemma 1, it follows that

$$TT_D(OD|R_k) \leq (k+1)T_{opt}(OD|R_k).$$

(2) In the worst case, for all $i \in \{1, 2, \cdots, k\}$, if there comes $T_i = T_{opt}(OD|R_i)$, then the vehicle will go along the route according to Reposition Strategy all the way. According to theorem 2, the total time is $TT_D(OD|R_k)$, which satisfies

$$TT_D(OD|R_k) \leq (2k+1)T_{opt}(OD|R_k).$$

(3) For all $i \in \{1, 2, \cdots, k\}$, if there always comes $T_i = t(r_i)+T_{opt}(O_iD|R_{i-1})$, then the vehicle will go on the route according to Waiting Strategy. The total time is

$$\begin{aligned} TT_D(OD|R_k) &= T_{opt}(OD) + T(R_k) \\ &\leq T_{opt}(OD) + \sum_{i=1}^{k} T_{opt}(OD|R_i) \\ &\leq (k+1)T_{opt}(OD|R_k). \end{aligned}$$

(4) There exist j vertices $r_{i_1}, r_{i_2}, \cdots, r_{i_j}$ $(1 \leq i_1 < i_2 < \cdots < i_j \leq k, 1 \leq j < k)$, so that

$$\begin{aligned} T_{i_1} &= T_{opt}(OD|r_1, r_2, \cdots, r_{i_1}), \\ T_{i_2} &= T_{opt}(OD|r_1, r_2, \cdots, r_{i_2}), \\ &\cdots, \\ T_{i_j} &= T_{opt}(OD|r_1, r_2, \cdots, r_{i_j}). \end{aligned}$$

(i) The vehicle starts from O. Before the vehicle reaches the congested vertex r_{i_1}, it is either going along the road of Greedy Strategy or along the road of Waiting Strategy. According to (1) and (3), the time it takes w_{11} satisfies

$$w_{11} \leq TT_D(OD|R_{i_1-1}) \leq i_1 T_{opt}(OD|R_k).$$

In this case, because $T_{i_1} = T_{opt}(OD|R_{i_1})$, the vehicle needs to return to the origin O by the original road according to the Simple Choice Strategy. The vehicle takes $w_{12} \leq w_{11}$ to get back, as it doesn't need to wait at the original congested vertex.

So far, the time that the vehicle needs to go back and forth w_1 satisfies

$$w_1 = w_{11} + w_{12} \leq 2i_1 T_{opt}(OD|R_k).$$

(ii) The vehicle starts from O in the second time. Before it reaches the congested vertex r_{i_2}, it is either going along the road of Greedy Strategy or Waiting Strategy. According to (1) and (3), the time it takes w_{21} satisfies

$$w_{21} \leq TT_D(OD|r_1, r_2, \cdots, r_{i_1}, r_{i_1+1}, r_{i_1+2}, \cdots, r_{i_2-1}) \leq (i_2 - i_1)T_{opt}(OD|R_k).$$

In this case, similar to (i), it takes the vehicle w_2 of time to finish its trip back and forth, which satisfies

$$w_2 \leq 2w_{21} \leq 2(i_2 - i_1)T_{opt}(OD|R_k).$$

The rest steps may be deduced by analogy.

(iii) For the time of j, the vehicle starts from O, going either along the road of Greedy Strategy or Waiting Strategy. Before reaching the congested vertex r_{i_j}, the vehicle goes back by the original road to O, similar to (ii). This time, it takes the vehicle w_j of time to finish its trip back and forth, which satisfies

$$w_j \leq 2(i_j - i_{j-1})T_{opt}(OD|R_k).$$

(iv) For the time of $j+1$, the vehicle starts from O, going along the route either of Greedy Strategy or of Waiting Strategy, until it reaches the destination D. According to (1) and (3), the time that the vehicle takes to cover this distance is w_{j+1}, which satisfies

$$w_{j+1} \leq (k - i_j + 1)C_{opt}(OD|R).$$

Therefore, from the origin O to the destination D, the total time is

$$\begin{aligned} TT_D(OD|R_k) \leq \sum_{l=1}^{l=j+1} w_l &\leq 2i_1 T_{opt}(OD|R_k) + 2(i_2 - i_1)T_{opt}(OD|R_k) \\ &+ \cdots + 2(i_j - i_{j-1})T_{opt}(OD|R_k) + (k - i_j + 1)T_{opt}(OD|R_k) \\ &= (k + i_j + 1)T_{opt}(OD|R_k). \end{aligned}$$

Taking (1), (2), and (3) together, according to the definition of competitive ratio, we can come to the conclusion that the competitive ratio of the Simple Choice Strategy D is $2k+1$ for online vehicle scheduling problem with the congested sequence R_k. This completes the proof.

Remark 5. From the discussion above, it can be seen that Simple Choice Strategy is ultimately promoted in competitive performance, although it has the same competitive ratio that is $2k+1$ as Reposition Strategy. It fully takes the dynamic features of the congested vertices into account and the advantages of the three strategies: Greedy Strategy, Waiting Strategy and Reposition Strategy. Different strategies are adopted in different situations, so that the best choice is made among the three strategies every time when the vehicle meets congestion. Greedy Strategy is used to fully grasp the chance of going the good road; in compulsive situation, Reposition Strategy is used to avoid the risk of going the bad road or waiting for too long; choosing to wait for a short time is to go on along the optimized route and avoid the risk of going the bad road. Furthermore, it can avoid the vehicle to make unnecessary reposition, which is a conservative way. All in all, the Simple Choice Strategy is a better strategy to integrate the advantages of all these strategies.

5 Conclusion

The congested vertex problem, which is not foreseeable in choosing what route to take, is a difficulty in dispatching goods. It is worthwhile to fully discuss how to put the theoretical results into practice. This paper is giving a tentative solution for online vehicle scheduling problem in transportation, which of course needs further discussion, for example, how to make the choice of optimized route, which can save time and cost, whether the congested vertex can recover as time goes on, etc.

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On Solving Coverage Problems in a Wireless Sensor Network Using Voronoi Diagrams

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Abstract. Owing to numerous potential applications, wireless sensor networks have been the focus of a lot of research efforts lately. In this note we study one fundamental issue in such networks, namely the coverage problem, in which we would like to determine whether a region of interest is sufficiently covered by a given set of sensors. This problem is motivated by monitoring applications in sensor networks, as well as robustness concerns and protocol requirements. We show that the coverage problem and some of its variants can be treated in a unified manner using suitable generalizations of the Voronoi diagram. As a result, we are able to give algorithms that have better runtimes than those proposed in previous works (see, e.g., [5, 6]). Our approach also yields efficient algorithms for coverage problems where the sensing region of a sensor is an ellipse or an L_p–ball, where $p \geq 1$.

1 Introduction

1.1 Background

Due to recent advances in wireless communication and embedded micro–sensing MEMS technologies, inexpensive wireless nodes capable of sensing, storing, processing, and communicating data are becoming increasingly common and readily available. Such devices provide the means for building large–scale wireless sensor networks for various applications, such as distributed data storage [11], target tracking [12], and habitat monitoring [13], just to name a few. In these applications, sensors collect information about their surroundings, and the data so obtained are then aggregated to give a complete picture of the region of interest. Hence, a fundamental issue in the design of wireless sensor networks is coverage, i.e. how well are the sensors covering the region of interest. This issue has been tackled by various researchers in the sensor networks community. Typically, a sensor s is modelled as a point p_s in space, and its sensing region is modelled as an Euclidean ball centered around p_s. From this simple model, many different coverage problems have been proposed. For example, the authors of [10] considered the problems of finding the *maximal breach path* and the *maximal support path* in a network, which informally can be viewed as the paths that are least and

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best monitored in the network, respectively. The maximal support path problem is further investigated by the authors of [9] in which energy consumptions of the sensors are also being taken into account. The authors of [14, 15], also motivated by energy consumption considerations, studied the problem of finding a subset of sensors $\mathscr{C}$ so that the region of interest S is covered by those sensors (i.e. each point in S is within the sensing range of some sensor in $\mathscr{C}$). This problem is later generalized by the authors of [16], in which they considered finding a subset of sensors $\mathscr{C}$ of *minimal cardinality* such that (i) the communication graph formed by the sensors in $\mathscr{C}$ is connected, and (ii) the region of interest S is k–covered by those sensors (i.e. each point in S is within the sensing ranges of k distinct sensors in $\mathscr{C}$). They proposed an algorithm that yields a subset whose cardinality is at most $O(\log n)$ times larger than the optimal and has the required properties.

In this note, we consider the following two coverage problems: (i) given a set of sensors $\mathscr{C}$ with their sensing regions and a region of interest S, determine whether every point in S is k–covered by the sensors in $\mathscr{C}$, where $k \geq 1$ is a given constant; (ii) given a set of sensors $\mathscr{C}$ with their sensing regions and a region of interest S, determine the largest k such that S is k–covered by the sensors in $\mathscr{C}$. These two problems are motivated by robustness concerns as well as protocol requirements. For example, triangulation–based localization protocols require at least three sensors to localize an object, and hence we would like every point in S to be at least 3–covered. Regarding the first problem, Huang and co–authors [5, 6] presented an $O(nm \log m)$ (resp. $O(nm^2 \log m)$) algorithm for determining whether a region in $\mathbb{R}^2$ (resp. $\mathbb{R}^3$) is k–covered by the given n sensors, where m is the maximum number of sensing regions that can intersect the sensing region of a particular sensor. Since m can be of order $\Theta(n)$ (see Figure 1), their algorithms have worst–case runtimes of $O(n^2 \log n)$ (for the 2–d case) and $O(n^3 \log n)$ (for the 3–d case), respectively. Also, it is curious that the runtimes of both algorithms are independent of k. Thus, it is natural to ask whether there exists an algorithm that can exploit the tradeoff between runtime and the parameter k. Regarding the second problem, a naïve approach would be to run the above algorithm $k(\leq n)$ times, resulting in a worst–case $O(n^3 \log n)$ (resp. $O(n^4 \log n)$) algorithm for the 2–d case (resp. 3–d case). However, to the best of our knowledge, this problem has not been addressed directly in the sensor networks community.

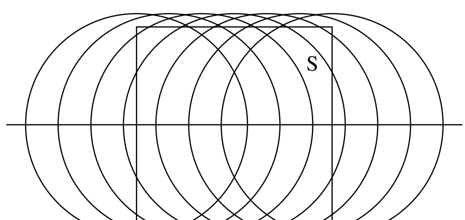

Fig. 1. A scenario where $m = \Theta(n)$: the $n+1$ circles are centered at $(1 - 2i/n, 0)$, where $i = 0, 1, \ldots, n$, and each of them has radius $1 + \epsilon$ for some $\epsilon > 0$

1.2 Our Contribution

We propose to treat the aforementioned coverage problems in a unified manner using suitable generalizations of the Voronoi diagram. This provides a rigorous mathematical framework for tackling those problems and enables us to design algorithms that have much more favorable worst–case runtimes than those proposed in previous works. Specifically, we show that for the 2–d case, if the sensors have identical sensing ranges, then problem (i) can be solved in $O(n \log n + nk^2)$ time by computing the so–called *k–th order Voronoi diagram* [4, 8]. Thus, our algorithm is independent of the configuration of the sensors and depends only on the number of sensors, n, and the parameter k. Moreover, for $k = o(\sqrt{n \log n})$, our algorithm has a better runtime than that proposed in [5]. In the case where the sensing ranges are different, we are able to obtain an $O(n \log n)$ algorithm to determine whether a region $S \subset \mathbb{R}^2$ is 1–covered. Our framework also allows us to study coverage problems in which the sensing region is not a ball. In particular, we give efficient algorithms for determining whether a region $S \subset \mathbb{R}^2$ is 1–covered when the sensing regions are modelled as ellipses or L_p–balls, where $p \geq 1$. Both of these algorithms are faster than the straightforward adaptation of the techniques presented in [5]. For problem (ii), we obtain an $O(n^3)$ (resp. $O(n^4)$) algorithm for the 2–d case (resp. for the 3–d case) by using a well–known relationship between power diagrams and arrangements [2]. This is again faster than a direct application of the algorithms described in [5, 6].

2 Problem Statement

We formulate the aforementioned coverage problems as follows. For simplicity of discussion, let the region of interest S be the cube $\{x \in \mathbb{R}^d : \|x\|_\infty \leq 1\}$. Let $a_i \in \mathbb{R}^d$ be the position of the i–th sensor, and let $B_i \equiv B(a_i, r_i) = \{x \in \mathbb{R}^d : \|x - a_i\|^2 \leq r_i^2\}$ be the sensing region of sensor i, where $1 \leq i \leq n$. In other words, the sensing region of sensor i is a ball centered at a_i with radius r_i. In the sequel, we shall mainly concern ourselves with the cases $d = 2$ and 3. For any given integer $k \geq 1$, we say that $x \in S$ is *k–covered* if there exist indices $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ such that $x \in \cap_{j=1}^k B_{i_j}$. We would then like to answer the following queries in an efficient manner:

Query 1: (k–COVERAGE; k–COV) Is every point in S k–covered by the balls $B_1, \ldots, B_n$? In other words, is it true that for every $x \in S$, there exist indices $1 \leq i_1 < \cdots < i_k \leq n$ such that $x \in \cap_{j=1}^k B_{i_j}$?

Query 2: (MAX–k–COVERAGE; MAX–k–COV) Determine the largest k such that every point $x \in S$ is k–covered by the balls $B_1, \ldots, B_n$.

3 The Proposed Solutions

In the above problems, we are required to certify that every point in S is k–covered by the given balls. Thus, in order to obtain efficient algorithms for both problems, we could attempt to find a finite and small set of points in S such that the queries can be answered by examining only these points. Indeed, our techniques are motivated by this idea, and they depend on the notion of *power* defined as follows. Given the ball $B(a,r)$, we define the *power* of a point $x \in \mathbb{R}^d$ with respect to $B(a,r)$ by $\mathrm{pow}(x, B(a,r)) = \|x-a\|^2 - r^2$. Note that:

$$\mathrm{pow}(x, B(a,r)) \begin{cases} < 0 \text{ if } x \in \mathrm{int}(B(a,r)) \\ = 0 \text{ if } x \in \partial B(a,r) \\ > 0 \text{ if } x \in \mathbb{R}^d \backslash B(a,r) \end{cases}$$

As we shall see, the notion of power allows us to treat the aforementioned problems in a unified manner.

3.1 The k–Cov Problem in $\mathbb{R}^2$ with Identical Disks

To begin, let us consider a version of the k–Cov problem where $d = 2$ and $r_i \equiv r$ for all $i = 1, 2, \ldots, n$. In other words, we are interested in the two–dimensional case of the k–Cov problem, with the sensing ranges of all sensors being identical. Let $B_1 \equiv B(a_1, r), \ldots, B_n \equiv B(a_n, r)$ be the given balls, and let $\mathscr{C} = \{a_1, \ldots, a_n\}$ be the set of centers. For any subset $U \subset \{1, 2, \ldots, n\}$ with $|U| = k$, define:

$$\begin{aligned} \mathrm{cell}(U) &= \left\{ x \in \mathbb{R}^2 : \mathrm{pow}(x, B_u) < \mathrm{pow}(x, B_v) \ \forall u \in U,\, v \in \{1,2,\ldots,n\}\backslash U \right\} \\ &= \left\{ x \in \mathbb{R}^2 : \|x - a_u\|^2 < \|x - a_v\|^2 \ \forall u \in U,\, v \in \{1,2,\ldots,n\}\backslash U \right\} \end{aligned}$$

The set $\mathrm{cell}(U)$ is the so–called *Voronoi region* of U. Intuitively, $\mathrm{cell}(U)$ is the set of points in $\mathbb{R}^2$ closer to all points in U than to any point in $\mathscr{C}\backslash U$. It is well–known (see, e.g., [4, 8]) that the Voronoi regions of all subsets of cardinality k induce a subdivision of $\mathbb{R}^2$, called the *k–th order Voronoi diagram* of $\mathscr{C}$ and denoted by $V_k(\mathscr{C})$. Moreover, such a subdivision is polyhedral. Now, observe that for $x \in \mathrm{cell}(U) \cap S$, we have x being k–covered iff $\mathrm{pow}(x, B_u) < 0$ for all $u \in U$. This suggests the following algorithm for determining whether S is k–covered or not. First, we compute the k–th order Voronoi diagram $V_k(\mathscr{C})$ of the point set $\mathscr{C}$. Then, as we will demonstrate below, it suffices to check whether the following points are k–covered: (i) the vertices of $V_k(\mathscr{C})$ in S, (ii) the intersections between the sides of S and $V_k(\mathscr{C})$, and (iii) the corners of S. We say that $x \in S$ is a *critical point* if it belongs to one of the above three categories. Thus, in order to correctly answer the query, it suffices to check for the critical points. We summarize our algorithm below (see Algorithm 1).

Theorem 1. *Algorithm 1 correctly answers the query.*

Proof. For each $x \in S$, let $a(x)$ be its k–th closest neighbor in $\mathscr{C}$. Clearly, we have $\mathrm{pow}(x, a(x)) > 0$ if x is not k–covered. Let $\hat{x} \in S$ be such that $\mathrm{pow}(\hat{x}, a(\hat{x}))$ is maximized. Note that such an $\hat{x}$ exists since S is compact and the power distance to the k–th closest point in $\mathscr{C}$ is a continuous function. We now consider two cases:

Algorithm 1 Testing k–Coverage by Identical Disks in $\mathbb{R}^2$

Given: A set of n disks $B_1 \equiv B(a_1, r), \ldots, B_n \equiv B(a_n, r)$ in $\mathbb{R}^2$, and an integer $k \geq 1$.

Compute the k–th order Voronoi diagram $V_k(\mathscr{C})$ for the set of points $\mathscr{C} = \{a_1, \ldots, a_n\}$.
for each vertex v of $V_k(\mathscr{C})$ that lies within S **do**
  Let $\{a_{i_1}(v), \ldots, a_{i_k}(v)\}$ be the k closest neighbors of v in $\mathscr{C}$.
  if $\exists j$ such that $\text{pow}(v, a_{i_j}(v)) > 0$ **then**
    Return No
  end if
end for
for each side l of the square S **do**
  Let $\{p_j\}_{j=1}^M$ be the union of the two endpoints of l and the intersections between l and $V_k(\mathscr{C})$.
  for each $j = 1, 2, \ldots, M$ **do**
    Let $a_{i_1}(p_j), \ldots, a_{i_k}(p_j)$ be the k closest neighbors of p_j in $\mathscr{C}$.
    if $\exists t$ such that $\text{pow}(p_j, a_{i_t}(p_j)) > 0$ **then**
      Return No
    end if
  end for
end for
Return Yes

Case 1: $\hat{x} \in \text{int}(S)$
We claim that $\hat{x}$ is a vertex of $V_k(\mathscr{C})$. Suppose that this is not the case. Then, we have the following possibilities:

Case 1.1: $\hat{x} \in \text{cell}(U)$ for some $U \subset \{1, 2, \ldots, n\}$ such that $|U| = k$. Then, the gradient of $\text{pow}(\cdot, a(\hat{x}))$ at $\hat{x}$ is $d(\hat{x}) = 2(\hat{x} - a(\hat{x}))$. In particular, an ascent direction is given by $d(\hat{x})$ (if $\hat{x} = a(\hat{x})$, then all directions are ascent directions). Now, let $x' = \hat{x} + \alpha d(\hat{x})$, where $\alpha > 0$ is sufficiently small so that $x' \in \text{cell}(U)$. Then, we have $\|x' - a(\hat{x})\|^2 > \|\hat{x} - a(\hat{x})\|^2$. Moreover, since every $x \in \text{cell}(U)$ has the same k closest neighbors in $\mathscr{C}$, we have $\|x' - a(x')\|^2 \geq \|x' - a(\hat{x})\|^2$. This shows that $\hat{x}$ is not a maximizer, which is a contradiction.

Case 1.2: $\hat{x}$ lies on the interior of an edge e in $V_k(\mathscr{C})$ defined by the intersection of the closure of two cells, say $e \equiv \text{int}\left(\overline{\text{cell}(U)} \cap \overline{\text{cell}(V)}\right)$, where $|U| = |V| = k$. Let a_i, a_j be two distinct k–th closest neighbors of $\hat{x}$ in $\mathscr{C}$ (cf. [8]). Note that the equation of the line L_e containing e is given by $\|x - a_i\|^2 = \|x - a_j\|^2$, or equivalently,

$$L_e : 2(a_j - a_i)^T x = \|a_j\|^2 - \|a_i\|^2$$

Define $u = (a_j - a_i)/\|a_j - a_i\|$, and let $v \neq 0$ be such that $u^T v = 0$. Note that v is a vector along the direction of L_e. Now, consider the point $x' = \hat{x} + \alpha v$ for some $\alpha \in \mathbb{R}$ to be determined shortly. Note that for sufficiently small $|\alpha| > 0$, we have $x' \in e$. Moreover, we compute:

$$\|x' - a_j\|^2 = \|\hat{x} - a_j\|^2 + \alpha^2 \|v\|^2 + 2\alpha v^T (\hat{x} - a_j) \tag{1}$$

Thus, if $\alpha \neq 0$ is chosen with a suitable sign with $|\alpha|$ small enough, then we have $2\alpha v^T(\hat{x} - a_j) \geq 0$ and $x' \in e$. This implies that $\|x' - a_j\|^2 > \|\hat{x} - a_j\|^2$. Moreover, x' has the same k closest neighbors as $\hat{x}$ in $\mathscr{C}$ (since $x' \in e$), and for all points q on the edge e, we have $\|q - a_i\|^2 = \|q - a_j\|^2 \leq \|q - a_k\|^2$ for all $k \notin U \cup V$. This contradicts the definition of $\hat{x}$.

Case 2: $\hat{x} \in \text{bd}(S)$
Let l be the side of S such that $\hat{x} \in l$. Suppose that $\hat{x} \in \text{int}(l) \cap \text{cell}(U)$ for some U. We claim that $\hat{x}$ is not a maximizer. To see this, let $v \neq 0$ be a vector along the direction of l. Now, consider the point $x' = \hat{x} + \alpha v$ for some $\alpha \in \mathbb{R}$ to be determined shortly. As before, we have $x' \in \text{int}(l) \cap \text{cell}(U)$ if $|\alpha| > 0$ is chosen to be sufficiently small. Moreover, using (1), we see that α can be chosen such that $\|x' - a(\hat{x})\|^2 > \|\hat{x} - a(\hat{x})\|^2$ (recall that $a(\hat{x})$ is the k–th closest neighbor of $\hat{x}$ in $\mathscr{C}$). Since every $x \in \text{cell}(U)$ has the same k closest neighbors in $\mathscr{C}$, we have $\|x' - a(x')\|^2 \geq \|x' - a(\hat{x})\|^2$. This again contradicts the definition of $\hat{x}$.

Theorem 2. *Algorithm 1 runs in time $O(n \log n + nk^2)$.*

Proof. The k–th order Voronoi diagram $V_k(\mathscr{C})$ of the n–point set $\mathscr{C}$ can be computed in $O(n \log n + nk^2)$ time using the algorithm described in [1]. As is shown in [8], the total number of vertices, edges and cells in $V_k(\mathscr{C})$ is bounded by $O(k(n-k))$. Thus, the remaining steps can be done in $O(k^2(n-k))$ time. This completes the proof.

The algorithm above possesses some interesting features that deserve further discussion. First, observe that we have reduced the coverage problem to an optimization problem, in which we are trying to find an $x \in S$ such that the distance to its k–th closest neighbor in $\mathscr{C}$ is maximized. Such a viewpoint enables us to answer the coverage query by examining only a set of critical points, and the efficiency of our algorithm comes from the facts that (i) the k–th order Voronoi diagram can be computed efficiently, and (ii) there exists a small set of critical points. As we shall see in subsequent sections, these ideas can be used to obtain efficient algorithms for other coverage problems. Secondly, note that the same analysis would go through if the region S is, say, a convex polygon with a constant number of sides. All we need is that the number of candidate maxima on the boundary of S is small. Thirdly, we observe that the k–th order Voronoi diagram depends only on the locations of the centers $a_1, \ldots, a_n$ but not on the common sensing range r. Thus, it is not necessary to re–compute $V_k(\mathscr{C})$ every time we change the value of r. It suffices to check the critical points described in the algorithm, and this requires only $O(nk^2)$ time. This opens up the possibility of a binary search strategy to determine the smallest r such that S is k–covered.

3.2 The 1–Cov Problem with Various Sensing Regions

In most previous works, the sensing region of a sensor is modelled as an Euclidean ball. In this section, we show how our techniques can be extended to yield efficient algorithms for the 1–Cov problem with various sensing regions. As before, we shall assume that S is the cube $\{x \in \mathbb{R}^d : \|x\|_\infty \leq 1\}$.

Covering with Non-uniform Disks. Suppose that we are given a set of balls $B_1 \equiv B(a_1, r_1), \ldots, B_n \equiv B(a_n, r_n)$, and we would like to determine whether S is covered by these balls. To answer this query, we first compute the *power diagram* of the collection $\mathscr{B} = \{B_1, \ldots, B_n\}$ [2], which is a collection of cells of the form:

$$\text{cell}(i) = \left\{x \in \mathbb{R}^2 : \text{pow}(x, B_i) < \text{pow}(x, B_j)\ \forall j \neq i\right\}$$

where $\text{pow}(x, B_i) = \|x - a_i\|^2 - r_i^2$ for $x \in \mathbb{R}^2$ and $i = 1, 2, \ldots, n$. Clearly, a point $x \in S$ is not covered iff $\min_{1 \le i \le n} \text{pow}(x, B_i) > 0$. Thus, we see that S is not covered iff the optimal value of the optimization problem

$$\begin{array}{lll} \text{maximize} & \min\limits_{1 \le i \le n} \text{pow}(x, B_i) & \\ \text{subject to} & x_j^2 \le 1 & j = 1, 2 \end{array} \tag{2}$$

is positive. Using the arguments in the proof of Theorem 1, it can be shown that the candidate maxima of (2) are precisely the critical points introduced before, i.e. (i) the vertices of the power diagram $P(\mathscr{B})$, (ii) the intersections between the sides of S and $P(\mathscr{B})$, and (iii) the corners of S. Thus, to determine the optimal value of (2), we can proceed as in Algorithm 1. Since the power diagram $P(\mathscr{B})$ of the collection $\mathscr{B}$ can be computed in $O(n \log n)$ time and the total number of vertices, edges and cells in $P(\mathscr{B})$ is bounded by $O(n)$ [2], we obtain the following theorem:

Theorem 3. *The* 1–Cov *problem in* $\mathbb{R}^2$ *can be solved in* $O(n \log n)$ *time.*

We remark that the above approach works in the three–dimensional case as well. Specifically, by following the arguments in the proof of Theorem 1, one can show that it is enough to check whether the following points are covered: (i) the vertices of the power diagram $P(\mathscr{B})$ that lie in S, (ii) the 0–flats that arise from the intersections between the sides of S and $P(\mathscr{B})$, and (iii) the corners of S. Since the power diagram $P(\mathscr{B})$ in $\mathbb{R}^3$ can be computed in $O(n^2)$ time and the total number of features (i.e. vertices, edges, ...) in $P(\mathscr{B})$ is bounded by $O(n^2)$ [2], we have the following theorem:

Theorem 4. *The* 1–Cov *problem in* $\mathbb{R}^3$ *can be solved in* $O(n^2)$ *time.*

Note that in both cases, our algorithms have much better worst–case runtimes than those proposed in [5, 6].

Covering with Ellipses. Suppose that we are given a collection of ellipses $E_1, \ldots, E_n$, where E_i is the ellipse defined by $(x - a_i)^T Q_i (x - a_i) \le r_i^2$, with Q_i symmetric positive definite, and we would like to answer the query with the B_i's replaced by the E_i's. As before, we may define $\text{pow}(x, E_i) = (x - a_i)^T Q_i (x - a_i) - r_i^2$ for $x \in \mathbb{R}^2$. To answer the query, we first compute the "power diagram" of the collection $\{E_1, \ldots, E_n\}$. Although this "power diagram" is no longer polyhedral, we can still show that the optimum must occur at a critical point as defined before. First, by using a similar argument as before, it is easy to see that $\hat{x}$

cannot lie in the interior of a cell. Now, the intersection σ of the closure of two cells is a portion of a quadratic curve $\mathcal{C}_\sigma$, whose equation is given by:

$$\mathcal{C}_\sigma : (x - a_i)^T Q_i (x - a_i) - r_i^2 = (x - a_j)^T Q_j (x - a_j) - r_j^2$$

Let $\hat{x} \in \text{int}(\sigma)$, and consider the two vectors $v_1 = Q_i(\hat{x} - a_i)/\|Q_i(\hat{x} - a_i)\|$ and $v_2 = Q_j(\hat{x} - a_j)/\|Q_j(\hat{x} - a_j)\|$. Note that v_i is the gradient vector of the function $\text{pow}(\cdot, E_i)$ at $\hat{x}$, where $i = 1, 2$. Thus, if the angle spanned by v_1 and v_2 is strictly less than π, then we can increase the objective value by moving along the curve $\mathcal{C}_\sigma$ in the direction that lies within the cone spanned by v_1 and v_2, which is a contradiction. On the other hand, if the angle spanned by v_1 and v_2 is exactly π, then we can find an $v \neq 0$ such that $v^T v_1 = v^T v_2 = 0$. Now, for any $\alpha \neq 0$, we have:

$$\begin{aligned} &(\hat{x} + \alpha v - a_i)^T Q_i (\hat{x} + \alpha v - a_i) - r_i^2 \\ &= (\hat{x} - a_i)^T Q_i (\hat{x} - a_i) - r_i^2 + 2\alpha v^T Q_i (\hat{x} - a_i) + \alpha^2 v^T Q_i v \\ &= (\hat{x} - a_i)^T Q_i (\hat{x} - a_i) - r_i^2 + \alpha^2 v^T Q_i v \end{aligned}$$

and similarly,

$$(\hat{x} + \alpha v - a_j)^T Q_j (\hat{x} + \alpha v - a_j) - r_j^2 = (\hat{x} - a_j)^T Q_j (\hat{x} - a_j) - r_j^2 + \alpha^2 v^T Q_j v$$

Since Q_i and Q_j are positive definite, we see that $(\hat{x}+\alpha v - a_i)^T Q_i (\hat{x}+\alpha v - a_i) > (\hat{x} - a_i)^T Q_i (\hat{x} - a_i)$ and $(\hat{x}+\alpha v - a_j)^T Q_j (\hat{x}+\alpha v - a_j) > (\hat{x} - a_j)^T Q_j (\hat{x} - a_j)$ for all $\alpha \neq 0$. It follows that by moving along $\mathcal{C}_\sigma$, we can increase the objective value, which again contradicts the definition of $\hat{x}$. Finally, the case where $\hat{x} \in \text{bd}(S)$ can be handled as before.

The "power diagram" for ellipses can have complexity $\Theta(n^{2+\epsilon})$ for any $\epsilon > 0$, and it can be computed in $O(n^{2+\epsilon})$ time as well [3]. Thus, we have the following theorem:

Theorem 5. *The* 1–Cov *problem in* $\mathbb{R}^2$ *with ellipsoidal sensing regions can be solved in* $O(n^{2+\epsilon})$ *time, for any* $\epsilon > 0$.

Covering with L_p–Balls. Let $p \geq 1$ and $r > 0$ be fixed, and suppose now that we are given a collection of L_p–balls $B_1, \ldots, B_n$, where B_i is defined by $\|x - a_i\|_p^p \leq r^p$ (i.e. all the B_i's have the same radius). As before, we would like to answer the query of whether S is covered by the B_i's. For this purpose, we define $\text{pow}(x, B_i) = \|x - a_i\|_p^p - r^p$ for $x \in \mathbb{R}^2$ and formulate an optimization problem similar to (2). The optimal value and the optimal solution can then be determined by first computing the L_p–Voronoi diagram of the point set $\{a_1, \ldots, a_n\}$ (see [7]) and then checking the objective values at the corresponding critical points. We omit the details and summarize the results as follows:

Theorem 6. *The* 1–Cov *problem in* $\mathbb{R}^2$ *with identical* L_p *sensing regions can be solved in* $O(n \log n)$ *time.*

3.3 The Max–k–Cov Problem

We now turn our attention to the Max–k–Cov problem, which is to determine the largest k such that S is k–covered by the given balls $B_1, \ldots, B_n$ (here, we consider the Euclidean metric). To do this, we first use the algorithm in [2] to compute all the k–th order power diagrams (where $1 \leq k \leq n-1$). This takes $O(n^3)$ time for the 2–d case and $O(n^4)$ time for the 3–d case. Then, we can check the critical points in each of these diagrams as before and determine the largest k such that S is k–covered by the balls $B_1, \ldots, B_n$. Since the total number of features in all the k–th order power diagrams is bounded by $O(n^3)$ for the 2–d case and by $O(n^4)$ for the 3–d case [2], we obtain the following theorem:

Theorem 7. *The* Max–k–Cov *problem in $\mathbb{R}^2$ (resp. $\mathbb{R}^3$) can be solved in $O(n^3)$ (resp. $O(n^4)$) time.*

Notice that this is more efficient than a direct application of the algorithms in [5, 6].

4 Conclusion

In this note we have proposed to use suitable generalizations of the Voronoi diagram to treat various coverage problems that arise from the design of wireless sensor networks. We have shown that in many cases, the runtimes of our algorithms are better than those proposed in earlier works. Moreover, our approach allows us to handle sensing regions whose shapes cannot be conveniently modelled as Euclidean balls. We remark, however, that the proposed algorithms are not distributed in nature. Since it is desirable to have decentralized computations in a wireless sensor network, an interesting future direction would be to find efficient distributed algorithms for the coverage problems discussed in this note.

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A Computational Model of Mortgage Prepayment Options

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Abstract. This paper develops a computational model of residential mortgage prepayment over discrete time. The prepayment options at every payment date are embedded in the model by using real options approach and according to Chinese residential mortgage loan payment schedule. The computational approach of the model is shown in the paper. A forecasting for cash flow of the bank and a penalty strategy designed against prepayment behavior under the prepayment option are given in the paper as the applications of the model.

1 Introduction

Now, it has been well understood by academic and practitioner researchers that the mortgage prepayment is a kind of risk [1] ,[2], [3], [4]. And developing a good model is a central task in the evaluation of mortgage and mortgage-backed securities. In China, since the house commercial process began at 1998, the residential mortgage loan is the main choice by Chinese households when they take their house-buying decision. So, Chinese single-family residential mortgage loan market is an emerging market and has an enormous size. The most Chinese households have no ability to pay the house-buying charge when they enter the mortgage loan contracts. For lack the experiences of domestic residential mortgage loan market operation, it is difficult for commercial banks to forecast the borrower's prepayment behaviors and therefore, the mortgage loan contracts is not initially designed against the prepayment behaviors. As mortgage prepayment phenomena increasingly occurs, it has been received extensive attention in Chinese residential mortgage loan market [5]. The household's decision to prepay is based, of cause on a variety of factors, some directly relate to interest rates of the mortgage loan, others mainly due to the prepayment ability of household. The changing of Chinese household's payment abilities usually comes from following sides. First, considering the level of household's income, the most borrowers prefer the mortgage loan contracts with longer expiration. However, within the longer period, the increasable savings may improve the borrower's payment ability. Second, a part of borrowers with enough payment ability enter the mortgage loan contracts in order to keep their investing ability to other markets. For them, when the mortgage loan interest rate floats, they may change their investing portfolio and choose prepayment. Moreover,

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some borrower's prepayment ability currently comes from the contracted–house trade. Finally, developing of Chinese economic conditions enhance the prepayment ability of Chinese household's payment and let a number of households have necessary conditions to hold prepayment options. Subsequently, the analysis and forecasting of mortgage prepayment cash flow have become increasingly crucial to Chinese commercial banks and regulatory bodies. For their practical important, the attention they have received in literatures. Some studies are undertaken to investigate the value of prepayment options [6], [7]. Some approaches for modeling the prepayment of a mortgage pool have been introduced [8], [9]. Some models dealing with prepayment phenomenon took into account default and refinancing cost of the mortgage, for e.g. [10]. However, the existing models on the prepayment options normally accompany a fixed–rate mortgage. Of cause the papers mentioned above are important for understanding the borrower's prepayment behavior on theoretic perspective. But the models more closely associated with Chinese mortgage loan market's operation may have more advantages in practice. This paper develops a model of prepayment over discrete period. The prepayment options at every payment date are embedded in the model by using real options theory and according to Chinese residential mortgage loan payment contracts. The interest rates of the mortgage loan involved in the model are variables. A computational approach for valuing the prepayment options is shown in the paper. Forecasting cash flow of the bank under the prepayment option and a penalty strategy designed against prepayment behavior are given in the paper as some applications of the model.

The remained sections of this paper are organized as following: section 2 is used to develop the model to describe the prepayment option values. The approach of valuing the prepayment option is given in section 3, the approach mainly used to show the computational feature of the model. Section 4 is served to show some applications of the model, which are used to illustrate how to use the model to forecast the cash flow for the bank that issues mortgage loan contracts, and how to design the penalty if the banks want to have no loss when prepayment behavior occurs.

2 Model and Analysis

First let us consider a payment schedule corresponding the residential mortgage loan contracts used in China: The household and loan bank sign a contract of residential mortgage loan, in which, the amount borrowed by a household is A, and the household will pay the constant amount of principal every month in T years. in other words, the borrower has $12T$ monthly payment dates, the scheduled payment is the monthly principal, Π and interest R at each time. In more details, let n be the current payment date, $n = 1, 2, \ldots, 12T$ the principal Π and the interests should be paid at the nth payment day, says R_n are shown as following respectively.

$$\Pi = \frac{A}{12T}, \quad R_n = \frac{12T + 1 - n}{12T} A \frac{1}{12} \gamma_n$$

Where, γ_n is the float interest rate of mortgage loan in the nth month, therefore the interest R_n is the interest of capital amount occupied by borrower in the nth month. So the current monthly scheduled payments are the sum of principal and interest, $\Pi + R_n$. The float interest rate of mortgage loan γ_n is an uncertainty effect factor in borrower's payment. However, in every payment date, the borrower may choose the prepayment strategy. The residential mortgage contracts usually do not contain relevant terms against the prepayment behavior in China. Therefore, one can believe that a mortgage contract contains a series of options permitting the borrowers not to make the scheduled payment. The borrower's ability to prepay represents a call options. The contract length of these options is one–month and exercisable on the payment date. Here, the one of the exercising way occurring in China is considered: the borrower select to prepay the loan entirely. Since prepayment has been viewed as a kind of default behavior, assume that the borrower has to pay penalty $Q(n)$ for his prepayment in nth month. The cost to change the contract, says C, for his prepayment behavior is assumed to be charged by loan bank. Formally, if the borrower exercises the prepayment option at the period n, he has to pay the following mount to terminate the mortgage loan contract.

$$\sum_{i=n}^{12T} \Pi + Q_n + C$$

Of cause, this call option is not necessarily exercised optimally. Whether the borrower decides to prepay is based on the comparison of value of current payment to exercise option and the discounted presented value of his all-future scheduled payment. Let us denote by $G(n)$ the difference of borrower presented value of scheduled payment and the current value of prepayment.

$$G(n) = \sum_{i=n}^{N} \frac{\Pi + R_i}{\rho_{in}} - \sum_{i=n}^{N} (\Pi + C + Q_n) \tag{1}$$

The first term in the $G(n)$ presented the value of his all future scheduled payment which is discounted by future capital market interest rate $\rho_{i,n}$. The subscripts i, n are used to denote that the rate $\rho_{i,n}$ discounts the ith month capital to the nth month. $G(n)$ is random from the perspective of period n, so we must take its expected value. The expected value of $G(n)$ can be viewed as benefit from the borrower's prepayment decision. From the viewpoint of NPV, in the period n, if the expected value of $G(n)$ conditional on the information of period n take the positive value, $E_n[G(n)] > 0$, the borrower will have incentive to prepay the mortgage loan and get the expected benefit $E_n[G(n)]$. But it may not correct from the viewpoint of real options theory. During the life of loan contract, the borrower holds prepayment options at every payment date with total number $12T$. Let $F(n)$ be the value of prepayment option at period n. If the borrower prepays at payment date n, he gets an immediate profit flow $E[G(n)]$, but gives up the opportunity or option to prepay in the next payment date, which is valued at $F(n+1)$. The value $F(n+1)$ must be accounted in the cost of the opportunity to exercise the current prepayment option. by using Bellman's principle of optimality, the Bellman equation satisfied by $F(n)$ is shown as following:

$$F(n) = \max\{E_n[G(n)] - \frac{F(n+1)}{1+\rho_{n+1,n}}, 0\} \tag{2}$$

If $F(n)$ is positive and at same time the borrower has the ability to prepay, the prepayment behavior of borrower will occur. In the last payment date, for borrower has no chance to prepay, we have $F(12T) = 0$. From the equation 2, it can be derived that whenever $F(n)$ is positive, then $E_n[G(n)]$ is positive, but the converse is not true. In other words, when the borrower decides to prepay by using real options rule, the same decision will be leaded by using NPV rule. Conversely, a prepayment decision resulted by NPV approach may not true for it ignores the flexibility of exercising prepayment option. This model focuses on the prepayment behavior, the other default factors are not involved in the model. So we assume that borrowers who may make prepayment decisions have the ability to pay the scheduled payment of the loan contracts.

3 Valuing the Prepayment Options

In the section 2, the recursive structure about the value of option to prepay has been given. The aim of this section is to discuss the problem how to find the value $F(n)$ and which parameters are accessible in the practice.

3.1 Evaluate the Expected Value of $G(n)$

First let us calculate the expected value $E_n[G(n)]$. From the formula 1, $G(n)$ is the random variable for it contains interest rate γ_n and the discount factor $\rho_{i,n}$. The penalty $Q(n)$,and the cost of changing the contract, C are given by the bank. Usually the penalty $Q(n)$ is designed as the function of the parameters of mortgage loan contract A, T, prepayment date n and contingents to the state of interest rate of mortgage loan, γ_n. In the section 4, a penalty aimed to against prepayment is designed, and C is assumed to be constant. Also in the practice, the float interest rate of mortgage loan and interest rate of capital market can be model as stochastic processes. So we have

$$E_n[G(n)] = \sum_{i=n}^{N} E\left(\frac{\Pi + R_i}{\rho_{i,n}}\right) - \sum_{i=n}^{N} (\Pi + C + E_n[Q_n]) \tag{3}$$

If the processes γ_n, $\rho_{i,n}$ are given, then the all parameters needed in calculation of $G(n)$ are accessible.

3.2 Evaluate the Value of Prepayment Option $F(n)$

The value of prepayment option $F(n)$ can be determined from recursive structure 2. We can start at time T and work our way backward to find $F(n)$. For

$F(12T) = 0$, we can easily get the $F(12T-1) = max\{E_{12T-1}[G(n)], 0\}$, by using the same process, the sequence of options value : $F(12T)$, $F(12T-1)$, $\ldots$, $F(2)$, $F(1)$ can be obtained. The situations that we are interested in are those components taking positive value in the sequence, that is $F(n_i) > 0$, $i = 1, 2, \ldots, m$. In the corresponding payment month n_i, the prepayment behaviors may occur. Whether the borrower decides to prepay or not is based, of cause on the variety of factors, such as interest rate variations, income variations and so on. Now, whether the borrower chooses to prepay at once or not depends on whether the scheduled payment level is above the prepayment level. Of cause, $F(n) > 0$ is not sufficient condition to make prepayment decision. In China, the loan amount A is usually larger than the household's initial payment ability; one of the factors affecting prepayment behavior is borrower's income shock. If the borrower receives a good shock in his income or some other factors that affect borrower's payment ability take change, the actual prepayment will occur.

Beyond the prepayment ability of borrower, the model shows that the values of prepayment options are underlying interest rate process and sensitive to changes in interest rates and also determined by the term structure of interest rate process.

4 The Applications of the Model

4.1 The Cash Flow Forecasted by the Prepayment Model

In the theoretic research and practice, the risks of prepayment make the timing of bank's cash flow difficult to predict. However, estimating the cash flow associated residential mortgage loan under prepayment uncertainty is every important for a commercial bank to prevent from its liquidity risks and is one of the most important problems to mortgage-backed securities. This section suggests a way of estimating cash flow of the mortgage loans as one of the applications of the mortgage prepayment model developed in the previous section.

From the model, those borrowers who have the payment abilities in the n_ith month will prepay since $F(n_i) > 0$, where $0 \leq i \leq m$. Suppose M residential mortgage loan contracts with the same expiration T are signed at the same time. Let p_i be the expected proportion of the borrowers who have the prepayment abilities in the n_ith month, then there will be p_iM borrowers exercise their prepayment options; therefore there will be p_iM loan contracts are terminated in the n_ith month. When $n = n_i$, the nth month cash flow is consisting of two parts, one of them is associated with the prepayments and the other part is associated with the scheduled payments. When n is between the two prepayment months, that is the situation of $n_i < n < n_{i+1}$, there is no borrower will prepay, the cash flow in the n_ith month will only be the part associated with the scheduled payments. So the expected cash flows of the M mortgage loan contracts can be estimated as following:

$$\begin{array}{ll} ME_n[\Pi + R_n] & 1 < n < n_i \\ ME_n\left[p_n[(12T+1-n)\Pi + C + Q_n] + (1-\sum_{k=1}^{i} p_{n_k})(\Pi + R_n)\right] & n = n_i \\ M(1-\sum_{k=1}^{i} p_{n_k})E_n[\Pi + R_n] & n_i < n < n_{i+1} \\ M(1-\sum_{k=1}^{m} p_{n_i})E_n[\Pi + R_n] & n = N \end{array} \tag{4}$$

The result shows that using this model, the problem about the uncertainty of cash flow in the mortgage-backed securities may be solved. But the key parameter used in the estimation of cash flow is prepayment proportion at period n_i, noted by p_i. The proportion p_i is defined as the ratio of "the borrowers who have prepayment ability" to "the borrowers who do not end the mortgage contract" in the $n_i th$ month. The estimation for the borrower's prepayment ability which is affected by some factors mentioned in the section 1. The changing process of the borrower's payment ability is usually the complex process. Here, a feasible process is suggested to estimate the proportion by using the history dates provided. The process is described as the following steps:

1. Identify the N historic mortgage contracts which have the same expiration into K subsets. All the M_k contracts in kth subset are signed at the same time.
2. In the subset k, $(k = 1, 2, \ldots, K)$, find out the jth month in which the prepayment behavior have occurred, and denote this month by $n_{k,j}$. Where $n_{k,j}$ is served to denote in the kth subset, the jth month in which prepayment will occur. And $q_{k,j}$ is used to denote the propotion of the borrowers who have chosen to prepay.
3. In the subset k, the proportion $q_{k,j}$ of borrowers must get their repayment abilities between the $n_{k,j-1}th$ month and $n_{k,j}th$ month, including the $n_{k,j}th$ month. So the proportion of the borrowers who have the prepayment abilities just in the nth month , $n_{k,j-1} < n \leq n_{k,j}$, is shown as the following if we assume that the distribution is uniform over the payment date.
$$\frac{q_{k,j}}{n_{k,j} - n_{k,j-1}}$$
4. Evaluate the weighted sum:
$$q_n = \frac{\sum_i^K \frac{q_{k,j}}{n_{k,j-1} - n_{k,j}}}{N}$$
Take the value as the proportion of all the borrowers who have the prepayment abilities just in the nth month.
5. Evaluate the proportion p_i, of all the borrowers who will prepay at the month n_i,
$$p_i = \sum_{n_{i-1}+1}^{n_i} q_n$$
From the $n_{i-1}th$ month to the $n_i th$ month, the borrowers who have the prepayment abilities have to prepay in $n_i th$ month, since the $n_i th$ month is the latest prepayment time after the $n_{i-1}th$ month.

4.2 A Penalty Strategy Designed Against the Prepayment Behavior

Though there are many ways to design the penalty, in this section, a special penalty model aimed at against the prepayment behaviors of the borrowers is built. From the section 3, the necessary condition of prepayment is $F(n) > 0$, so if $Q(n)$ is large enough the prepayment behavior will never occur. Denote $Q^*(n)$ as the smallest penalty that satisfies $F(n) = 0$. From recursive structure 2, $Q^*(n)$ can be designed as following:

$$Q^*(n) = E_n\left(\sum_{i=n}^{12T} \frac{\frac{A}{12T} + \frac{12T+1-n}{12T} A\gamma_n}{1 + \rho_{i,n}}\right) - \left(\sum_{i=n}^{12T} \frac{A}{12T} + C\right) \quad (5)$$

So , in order to against the prepayment behavior, any penalty $Q(n)$ satisfies $Q(n) \geq Q^*(n)$ is available .

5 Conclusions

In Chinese residential mortgage loan market, the prepayment options can be valued using the model developed in this paper. Employing the value of the prepayment option derived in the paper, it is easy for us to identify the prepayment dates from the payment dates. The prepayment ability of the borrower will result in actual prepayment at the these payment dates. The results show that the values of prepayment options are underlying the mortgage loan interest rate and the interest rate of capital market and sensitive to changes in the interest rates. The model can be used to predict the cash flow of bank under the prepayment behavior and also can be used to design the penalty model aimed to against prepayment behavior.

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Efficient Algorithms for the Electric Power Transaction Problem

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Abstract. We present two efficient algorithms for solving the electric power transaction problem. The electric power transaction problem appears when maximizing the social benefit on electric power transactions among some private companies. The problem is a special case of the minimum cost flow problem defined on a network with many leaves, where each leaf corresponds to a (private) company who wants to sell or buy electric power.

Our first algorithm is based on the minimum mean cycle canceling algorithm and the second algorithm uses a linear time median finding algorithm. The first algorithm finds an optimal solution in $O(n \log n\ k^5 \log(kC))$ time where n is the number of leaves, k is the number of non-leaf vertices and C is the highest electric power price per unit that companies may offer. The time complexity of the second algorithm is bounded by $O((n + k^3)2^k k!)$ time, which is linear in n. In many practical instances, k is small and n is very large, hence these algorithms solve the problem more efficiently than the ordinary network flow algorithms.

Keywords: Electric power transaction, minimum cost flow, median finding.

1 Introduction

In Japan, the law about the electric power industry was amended so that private companies are now able to transact electric power with each other. The electric power market consists of some areas. Each area corresponds to exactly one electric power company and every private company attending the market belongs to exactly one area. All private companies can exchange their electric power via facilities of the electric power companies in their areas. The areas are connected by high-capacity electric wires, and private companies in different areas can exchange the electric power via the high-capacity wires. The transaction fee between any private companies is zero, even if they belong to different areas.

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To determine a transaction pattern, auctions are held as follows. First, each private company bids the price and quantity of electric power he wants to sell or buy. A transaction has to be made between a pair of private companies such that one wants to sell and the other wants to buy the electric power, and the selling price per unit is less than or equal to the buying price per unit. Then, the auctioneer will find a transaction pattern which maximizes the *social benefit* defined by the sum total of benefits, ((selling price) − (buying price)) × (the amount of transaction), over all transactions between a pair of private companies. The optimization problem to find a transaction pattern which maximizes the social benefit is called *electric power transaction problem.* We can formulate the electric power transaction problem as a minimum cost flow problem on a directed graph defined on a set of vertices corresponding to the areas (electric power companies) and private companies.

We can solve the electric power transaction problem in strongly polynomial time by algorithms for minimum cost flow problems, e.g., see Goldberg and Tarjan [4, 5] and Orlin [7]. Known bounds of the computational complexities of the algorithms above are greater than $O(v^2)$ where v denotes the number of vertices. When we formulate the electric power transaction problem as a minimum cost flow problem, the corresponding network has a special structure; since every private company belongs to exactly one area, the corresponding vertex becomes a leaf. Additionally, the number of areas are usually small and the number of private companies is very large. In this paper, we discuss an algorithm whose time complexity have smaller degree with respect to the number of private companies.

We propose two algorithms for the electric power transaction problem. First, we refine the minimum mean cycle canceling algorithm by Goldberg and Tarjan [5]. Our second algorithm enumerates all (possibly infeasible) flows between areas satisfying dual feasibility and complementarity conditions, and checks primal feasibility by a median finding algorithm and a maximum flow algorithm. The first algorithm solves the problem in $O(n \log n\ k^5 \log(kC))$ time, where n denotes the number of private companies, k the number of areas, and C the highest electric power price per unit. The time complexity of the second algorithm is bounded by $O((n + k^3)2^k k!)$.

The organization of this paper is as follows. In Section 2, we describe the electric power transaction problem and formulate it as a minimum cost flow problem. In Section 3, we propose our first algorithm based on the minimum mean cycle canceling algorithm. Section 4 gives our second algorithm based on a linear time median finding algorithm.

2 Minimum Cost Flow Formulation

In this paper, $\mathbb{R}_+$ and $\mathbb{Z}_+$ denotes the set of non-negative real numbers and non-negative integers, respectively. Let $\Gamma = \{1, 2, \ldots, k\}$ be the set of areas. We consider the complete directed graph with vertex set Γ. We denote the directed edge set of the complete graph by F. A high-capacity electric wires from area

i to i' is defined by a capacity $u_{ii'} \in \mathbb{R}_+$ of the directed edge $(i, i') \in F$. When there does not exists a wire from i to i', we set the capacity $u_{ii'} = 0$. For each area $i \in \Gamma$, S_i and B_i denotes the set of private companies who want to sell or buy their electric power, respectively. We define $n = \sum_{i \in \Gamma} |S_i| + \sum_{i \in \Gamma} |B_i|$. Throughout this paper, we assume that $n \geq k$. We denote the jth company in S_i by s_{ij} and the jth company in B_i by b_{ij}. We represent the bids of private companies in S_i by a pair $(\underline{\boldsymbol{p}}_i, \underline{\boldsymbol{u}}_i)$ where $\underline{\boldsymbol{p}}_i \in \mathbb{Z}_+^{|S_i|}$ is the "prices per unit" vector, and $\underline{\boldsymbol{u}}_i \in \mathbb{R}_+^{|S_i|}$ is the vector of upper bounds of the power supplies. Similarly, the bids of private companies in B_i is denoted by a pair $(\overline{\boldsymbol{p}}_i, \overline{\boldsymbol{u}}_i) \in \mathbb{Z}_+^{|B_i|} \times \mathbb{R}_+^{|B_i|}$, where $\overline{\boldsymbol{u}}_i$ is the vector of upper bounds of the power demands. We assume that each element of $\underline{\boldsymbol{u}}_i$ and $\overline{\boldsymbol{u}}_i$ is positive.

For each $i \in \Gamma$, we introduce variable vectors $\underline{\boldsymbol{x}}_i \in \mathbb{R}_+^{|S_i|}$ which represent the amounts of electric power that private companies in S_i sell. Similarly, we introduce variable vectors $\overline{\boldsymbol{x}}_i \in \mathbb{R}_+^{|B_i|}$ $(i \in \Gamma)$. We denote the volume of electric power flow on the wire $(i, i') \in F$ by a variable $x_{ii'}$. Then we can write down the electric power transaction problem as follows,

$$
\begin{array}{lll}
\text{(P)} & \text{min. } \sum_{i \in \Gamma} (\underline{\boldsymbol{p}}_i^\top \underline{\boldsymbol{x}}_i - \overline{\boldsymbol{p}}_i^\top \overline{\boldsymbol{x}}_i), & \\
& \text{s. t. } \underline{\boldsymbol{x}}_i^\top \mathbf{1} - \overline{\boldsymbol{x}}_i^\top \mathbf{1} - \sum_{i' : (i,i') \in F} x_{ii'} + \sum_{i' : (i',i) \in F} x_{i'i} = 0 & (\forall i \in \Gamma), \\
& \quad \mathbf{0} \leq \underline{\boldsymbol{x}}_i \leq \underline{\boldsymbol{u}}_i & (\forall i \in \Gamma), \\
& \quad \mathbf{0} \leq \overline{\boldsymbol{x}}_i \leq \overline{\boldsymbol{u}}_i & (\forall i \in \Gamma), \\
& \quad 0 \leq x_{ii'} \leq u_{ii'} & (\forall (i, i') \in F),
\end{array}
$$

where $\mathbf{1}$ denotes the all-one vector (with a comfortable dimension). Here we note that the maximum of social benefit is equal to the value $-\sum_{i \in \Gamma} (\underline{\boldsymbol{p}}_i^\top \underline{\boldsymbol{x}}_i - \overline{\boldsymbol{p}}_i^\top \overline{\boldsymbol{x}}_i)$ with respect to an optimal solution. It is easy to see that the problem above is a minimum cost flow problem. We can obtain an optimal transaction pattern among private companies by decomposing an optimal flow of the problem above to a set of flows connecting pairs of private companies.

In the rest of this section, we transform the problem above to a minimum cost circulation flow problem, which helps the discussions in the next section. First, we introduce a complete directed graph (Γ, F) with vertex set Γ and edge set F. Each element in F is a cost-free directed edge whose capacity is equal to the capacity of the corresponding electric wire. Next we introduce a set of n vertices corresponding to private companies, and one artificial vertex r (see Fig. 1). We also introduce sets of edges $\cup_{i' \in \Gamma} (S_{i'} \times \{i'\})$ and $\cup_{i' \in \Gamma} (\{i'\} \times B_{i'})$. The cost of the edge $(s_{ij}, i) \in \cup_{i' \in \Gamma} (S_{i'} \times \{i'\})$ is $(\underline{\boldsymbol{p}}_i)_j$ (the jth element of vector $\underline{\boldsymbol{p}}_i$), and its capacity is $(\underline{\boldsymbol{u}}_i)_j$. Similarly, we set the cost and the capacity of the edge $(i, b_{ij}) \in \cup_{i' \in \Gamma} (\{i'\} \times B_{i'})$ to $-(\overline{\boldsymbol{p}}_i)_j$ and $(\overline{\boldsymbol{u}}_i)_j$, respectively. Finally, we introduce a cost-free directed edges $\cup_{i \in \Gamma} (B_i \times \{r\})$ and $\cup_{i \in \Gamma} (\{r\} \times S_i)$ whose capacities are infinity. Clearly, the minimum cost circulation flow problem on the network defined above is equivalent to the electric power transaction problem. The network has $k + n + 1$ vertices and $k(k-1) + 2n$ edges.

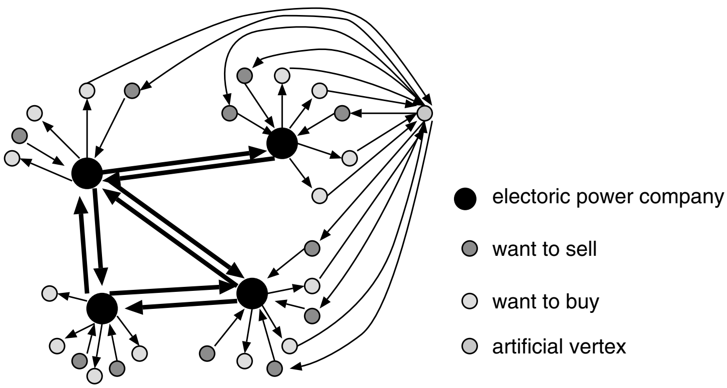

Fig. 1. network of the electric power transaction problem

3 Cycle Canceling Algorithm

In this section, we refine the well-known minimum mean cycle canceling algorithm [5] for minimum cost circulation flow problem. The minimum mean cycle canceling algorithm sets the initial flow values on all edges to zero. Then, it iteratively augments the flow along a cycle whose average cost over its edges is the smallest on the *residual network* defined below. Let us consider a network $G = (V, E)$ with edge cost $\boldsymbol{c} \in \mathbb{R}^{|E|}$ and lower (upper) edge capacity $\boldsymbol{l} \in \mathbb{R}^{|E|}$ ($\boldsymbol{u} \in \mathbb{R}^{|E|}$), respectively. We first make copies of edges in E and reverse the directions of the copies. We call the copy of edge e as $\overline{e}$ and we call the set of $\overline{e}$'s as $\overline{E}$. Given a flow $\boldsymbol{f} \in \mathbb{R}^{|E|}$ on G satisfying $\boldsymbol{l} \leq \boldsymbol{f} \leq \boldsymbol{u}$, the residual network with respect to $\boldsymbol{f}$ is a network $\widetilde{G} = (V, \widetilde{E})$ with an edge set

$$\widetilde{E} = \{e \in E \mid f_e < u_e\} \cup \{\overline{e} \in \overline{E} \mid f_e > l_e\}.$$

The cost of an edge $e \in \widetilde{E}$ is $c(e)$ if $e \in E$, and the cost of an edge $\overline{e} \in \widetilde{E}$ is $-c(e)$ if $\overline{e} \in \overline{E}$. The lower (upper) capacity of an edge $e \in \widetilde{E}$ is 0 and $u_e - f_e$ if $e \in E$, and if $\overline{e} \in \overline{E}$ the lower and upper capacity of $\overline{e}$ are 0 and f_e. In [5], Goldberg and Tarjan showed that the algorithm terminates in at most s iterations, where s is the smallest integer satisfying

$$\left\lfloor \frac{s-1}{\widetilde{m}} \right\rfloor \leq -\frac{\log(lC)}{\log(1-1/l)},$$

$\widetilde{m}$ is the number of edges, l is the length (the number of edges) of a longest simple cycle in a residual network, and C is the maximum of the edge costs.

In every residual network with respect to a flow on the electric power transaction network, the lengths of simple cycles are bounded by $\mathrm{O}(k)$, since every simple cycle contains at most two private companies. The number of edges of a residual network is bounded by $\mathrm{O}(n + k^2)$. Thus, the number of iterations required in the minimum mean cycle canceling algorithm is bounded by $\mathrm{O}(k(n + k^2)\log(kC))$.

If a minimum mean cycle in a residual network contains an edge from a private company to a corresponding area $i \in \Gamma$, the cost of the edge attains the minimum over edges from private companies to the area i in the residual network. Thus, when we find a minimum mean cycle in a residual network, for each area i, we only need to consider an edge from a private company to the area i. Similarly, for each area $i \in \Gamma$, we only need to consider an edge from the area i to a private company whose edge cost is the minimum among the costs of edges emanating from the area i to private companies in the residual network. Hence, for finding a minimum mean cycle in the algorithm, it is sufficient to find a minimum mean cycle on a network with $O(k)$ vertices, which requires $O(k^3)$ time. In the following, we show that we can construct the network with $O(k)$ vertices in $O(k^2 + \log n)$ time at each iteration. For each area, we maintain the set of edges emanating from private companies to the area by an ordinary heap structure with keys corresponding to the edge costs (in the residual network). Similarly, we also maintain the set of edges from an area to private companies with a heap structure. Then we can construct a network with $O(k)$ vertices in $O(k^2)$ time for finding a minimum mean cycle efficiently. The update procedure of the heap structures requires $O(\log n)$ time, since every simple cycle (in a residual network) contains at most two private companies. From the above, each iteration of the minimum mean cycle canceling algorithm requires $O(k^3 + \log n)$ time.

The total time complexity of the (modified) minimum mean cycle canceling algorithm is bounded by $O(k(n+k^2)\log(kC)(k^3+\log n)) = O(n \log n\, k^5 \log(kC))$.

4 Linear Time Algorithm

4.1 Optimality Condition

Here, we describe an optimality condition of minimum cost flow problem P. The duality theory on linear programming implies that a given feasible flow $(\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}, \boldsymbol{x})$ (a feasible solution of problem P) is optimal if and only if there exists a potential vector $\boldsymbol{q} \in \mathbb{R}^{|\Gamma|}$ satisfying that

$$\begin{aligned}
&\text{(C1)}\ \forall i \in \Gamma,\ \forall s_{ij} \in S_i, \begin{cases} (\underline{\boldsymbol{p}}_i)_j > q_i \ \Rightarrow\ (\underline{\boldsymbol{x}}_i)_j = 0, \\ (\underline{\boldsymbol{p}}_i)_j < q_i \ \Rightarrow\ (\underline{\boldsymbol{x}}_i)_j = (\underline{\boldsymbol{u}}_i)_j, \end{cases} \\
&\text{(C2)}\ \forall i \in \Gamma,\ \forall b_{ij} \in B_i, \begin{cases} q_i > (\overline{\boldsymbol{p}}_i)_j \ \Rightarrow\ (\overline{\boldsymbol{x}}_i)_j = 0, \\ q_i < (\overline{\boldsymbol{p}}_i)_j \ \Rightarrow\ (\overline{\boldsymbol{x}}_i)_j = (\overline{\boldsymbol{u}}_i)_j, \end{cases} \\
&\text{(C3)}\ \forall (i, i') \in F, \begin{cases} q_i < q_{i'} \ \Rightarrow\ x_{ii'} = u_{ii'}, \\ q_i > q_{i'} \ \Rightarrow\ x_{ii'} = 0. \end{cases}
\end{aligned}$$

We say that a potential vector $\boldsymbol{q}$ is *optimal*, if there exists a feasible flow $(\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}, \boldsymbol{x})$ satisfying the optimality condition above. It is well-known that we can check the optimality of a given potential by using an ordinary max flow algorithm and it requires $O(n + k^3)$ time [3]. The potential vector $\boldsymbol{q}$ is called a shadow price vector.

4.2 Single Area Case

In this subsection, we consider a single area case, i. e., $k = 1$. Given a potential q of a unique area, we partition the set of private companies as follows;

$$(S_*, S, S^*) = (\{j \in S_1 \mid (\underline{\boldsymbol{p}}_1)_j < q\}, \{j \in S_1 \mid (\underline{\boldsymbol{p}}_1)_j = q\}, \{j \in S_1 \mid (\underline{\boldsymbol{p}}_1)_j > q\}),$$
$$(B_*, B, B^*) = (\{j \in B_1 \mid (\overline{\boldsymbol{p}}_1)_j < q\}, \{j \in B_1 \mid (\overline{\boldsymbol{p}}_1)_j = q\}, \{j \in B_1 \mid (\overline{\boldsymbol{p}}_1)_j > q\}).$$

It is easy to see that a potential q is optimal if and only if a pair of intervals $[\sum_{j\in S_*}(\underline{\boldsymbol{u}}_1)_j, \sum_{j\in S_*\cup S}(\underline{\boldsymbol{u}}_1)_j]$ and $[\sum_{j\in B^*}(\overline{\boldsymbol{u}}_1)_j, \sum_{j\in B^*\cup B}(\overline{\boldsymbol{u}}_1)_j]$ has an intersection. When the intervals above intersect, we can easily construct a feasible solution of P satisfying conditions (C1) and (C2) (note that, in a single area case, we do not need condition (C3)). When the intervals above are disjoint, either

$$\textstyle\sum_{j\in B^*\cup B}(\overline{\boldsymbol{u}}_1)_j < \sum_{j\in S_*}(\underline{\boldsymbol{u}}_1)_j \quad \text{or} \quad \sum_{j\in S_*\cup S}(\underline{\boldsymbol{u}}_1)_j < \sum_{j\in B^*}(\overline{\boldsymbol{u}}_1)_j$$

holds. In the former case, it is easy to see that every optimal potential is strictly less than q. In the latter case, every optimal potential is strictly greater than q. Fig. 2 shows an algorithm based on an ordinary prune and search technique. For any subset $S' \subseteq S_1$, we denote the multi-set consisting of prices in the private companies in S' by $p(S')$. Similarly, we define $p(B')$ for a subset $B' \subseteq B_1$.

Put $(S_*, S, S^*) := (\emptyset, S_1, \emptyset)$, $(B_*, B, B^*) := (\emptyset, B_1, \emptyset)$, $\underline{f} := 0$ and $\overline{f} := 0$.
while $(|S| + |B| > 0)$
  Let q be a median of a larger set of $p(S)$ and $p(B)$.
  Let $S' := \{j \in S \mid (\underline{\boldsymbol{p}}_1)_j < q\}$ and $S'' := \{j \in S \mid (\underline{\boldsymbol{p}}_1)_j \le q\}$.
  Let $B' := \{j \in B \mid (\overline{\boldsymbol{p}}_1)_j > q\}$ and $B'' := \{j \in B \mid (\overline{\boldsymbol{p}}_1)_j \ge q\}$.
  if $(\overline{f} + \sum_{j\in B''}(\overline{\boldsymbol{u}}_1)_j < \underline{f} + \sum_{j\in S'}(\underline{\boldsymbol{u}}_1)_j)$
    /* every optimal potential is strictly less than q */
    Let $(S_*, S, S^*) := (S_*, S', S^* \cup S \setminus S')$.
    Let $(B_*, B, B^*) := (B_*, B \setminus B'', B^* \cup B'')$ and $\overline{f} := \overline{f} + \sum_{j\in B''}(\overline{\boldsymbol{u}}_1)_j$.
  else if $(\underline{f} + \sum_{j\in S''}(\underline{\boldsymbol{u}}_1)_j < \overline{f} + \sum_{j\in B'}(\overline{\boldsymbol{u}}_1)_j)$
    /* every optimal potential is strictly greater than q */
    Let $(B_*, B, B^*) := (B_* \cup B \setminus B', B', B^*)$.
    Let $(S_*, S, S^*) := (S_* \cup S'', S \setminus S'', S^*)$, and $\underline{f} := \underline{f} + \sum_{j\in S''}(\underline{\boldsymbol{u}}_1)_j$.
  else
    Let $(S_*, S, S^*) := (S_* \cup S', S'' \setminus S', S^* \cup S \setminus S'')$,
    Let $(B_*, B, B^*) := (B_* \cup B \setminus B'', B'' \setminus B', B^* \cup B')$.
    return (S_*, S, S^*), (B_*, B, B^*) and q.
end while

Fig. 2. an algorithm to find an optimal market price

We can show the correctness of the algorithm in Fig. 2 easily, since the partitions (S_*, S, S^*), (B_*, B, B^*) of private companies satisfy that

$[\forall q_1 \in p(S_*),\ \forall q_2 \in p(S),\ \forall q_3 \in p(S^*),\ q_1 < q_2 \text{ and } q_2 < q_3]$, and
$[\forall q_1 \in p(B_*),\ \forall q_2 \in p(B),\ \forall q_3 \in p(B^*),\ q_1 < q_2 \text{ and } q_2 < q_3]$,
during iterations.

We discuss the time complexity of the algorithm. We can find a median of n elements in $\mathrm{O}(n)$ time [1]. Thus, each iteration requires $\mathrm{O}(|S \cup B|)$ time. At every iteration, the cardinality of $S \cup B$ decreases at most a factor of $\frac{3}{4}$, because the size of the larger set among S and B decreases at most a factor of half. Thus, for some constant C, the total computational time T of the algorithm satisfies

$$T \leq Cn + (3/4)Cn + (3/4)^2 Cn + \cdots \leq 4Cn = \mathrm{O}(n).$$

When we have an optimal potential q^*, we can find the minimum and the maximum value of the optimal potentials, denoted by $q^{\min}$ and $q^{\max}$, as follows. Let q' be the maximum of the set of prices (elements of $\underline{\boldsymbol{p}}$ and $\overline{\boldsymbol{p}}$) which are strictly less than q^*. If q' is an optimal potential, we set $q^{\min} = q'$; else, we set $q^{\min} = q^*$. Let q'' be the minimum of prices which are strictly greater q^*. If q'' is an optimal potential, we set $q^{\max} = q''$; else, we set $q^{\max} = q^*$. This procedure correctly finds $q^{\min}$ and $q^{\max}$, since $\underline{\boldsymbol{u}}_1$ and $\overline{\boldsymbol{u}}_1$ are positive vectors. The time complexity of the procedure is bounded by $\mathrm{O}(n)$. Clearly, every potential between $q^{\min}$ and $q^{\max}$ is also optimal. We need this procedure in the next subsection.

4.3 General Case

If we know that an optimal potential vector satisfies that the potential of area i_1 is strictly less than that of area i_2, we can conclude that every optimal flow satisfies that the flow value from i_1 to i_2 is $u_{i_1 i_2}$, and the flow value from i_2 to i_1 is zero. Thus, given a permutation π of area-set Γ, we can check the existence of an optimal potential vector satisfying $q_{\pi(1)} < q_{\pi(2)} < \cdots < q_{\pi(k)}$ as follows. First, we fix flow values between areas using the optimality condition (C3) under the assumption that there exists an optimal potential vector satisfying $q_{\pi(1)} < q_{\pi(2)} < \cdots < q_{\pi(k)}$. More precisely, we set $x_{ii'} = u_{ii'}$, if $\pi(i) < \pi(i')$; $x_{ii'} = 0$, if $\pi(i) > \pi(i')$. Since the variables $x_{ii'}$ $((i,i') \in F)$ are fixed, we can decompose problem P into k subproblems and solve each subproblem defined on an area, independently. The constraint

$$\underline{\boldsymbol{x}}_i^\top \mathbf{1} - \overline{\boldsymbol{x}}_i^\top \mathbf{1} - \textstyle\sum_{i':(i,i') \in F} x_{ii'} + \sum_{i':(i',i) \in F} x_{i'i} = 0 \quad (\forall i \in \Gamma)$$

appearing in a subproblem means that the value $\underline{\boldsymbol{x}}_i^\top \mathbf{1} - \overline{\boldsymbol{x}}_i^\top \mathbf{1}$ is equivalent to an obtained constant $h_i = \sum_{i':(i,i') \in F} x_{ii'} - \sum_{i':(i',i) \in F} x_{i'i}$. We can check the feasibility of all the subproblems in $\mathrm{O}(n)$ time. If all the subproblems are feasible, we solve subproblems by a modified version of the algorithm in Fig. 2 as follows. When h_i is non-negative, we set the initial value of $\overline{f}$ to h_i in the algorithm appearing in Fig. 2. If h_i is negative, we set the initial value of $\underline{f}$ to $-h_i$. It is easy to see that the modified version of the algorithm correctly solves all the subproblems in $\mathrm{O}(n)$ time. We also obtain the minimum and the maximum of the optimal potentials, denoted by $q_i^{\min}$ and $q_i^{\max}$, for each subproblem indexed by area $i \in \Gamma$ in $\mathrm{O}(n)$ time. Finally, we check the existence of a vector $\boldsymbol{q} \in \mathbb{R}^{|\Gamma|}$

satisfying $q_{\pi(1)} < q_{\pi(2)} < \cdots < q_{\pi(k)}$ and $q_i^{\min} \leq q_i \leq q_i^{\max}$ ($\forall i \in \Gamma$). More precisely, if there exists a vector $\boldsymbol{q^*} \in \mathbb{R}^{|\Gamma|}$ satisfying $q^*_{\pi(1)} \leq q^*_{\pi(2)} \leq \cdots \leq q^*_{\pi(k)}$ and $q_i^{\min} \leq q_i^* \leq q_i^{\max}$ ($\forall i \in \Gamma$), we can conclude that the potential vector $\boldsymbol{q^*}$ is optimal and an optimal solution of P is obtained by merging the fixed flow among areas and optimal solutions of subproblems. We can check the existence of vector $\boldsymbol{q^*}$ in O(k) time. Thus, the total time complexity of the procedure above is bounded by O($n+k$) = O(n) time. We describe our algorithm in Fig. 3 briefly. If problem P has an optimal potential vector with mutually distinct elements, we can solve problem P in O($nk!$) time.

·For each pair $(i, i') \in F$, we set $x_{ii'} := \begin{cases} u_{ii'} & (\pi(i) < \pi(i')), \\ 0 & (\pi(i) > \pi(i')). \end{cases}$
·Decompose problem P into k subproblems
·**if** (a subproblem is infeasible) **then return** 'none exist'.
·Solve all the subproblems and obtain the minimum and the maximum, denoted by $q_i^{\min}$ and $q_i^{\max}$, of the optimal potentials for each $i \in \Gamma$.
· **if** $(\exists \boldsymbol{q^*} \in \mathbb{R}^{|\Gamma|}, q^*_{\pi(1)} \leq q^*_{\pi(2)} \leq \cdots \leq q^*_{\pi(k)},\ q_i^{\min} \leq q_i^* \leq q_i^{\max}\ (\forall i \in \Gamma))$,
  return $\boldsymbol{q^*}$.
·**else return** 'none exist'.

Fig. 3. the algorithm to find an optimal feasible flow

In the rest of this section, we discuss the general case that optimal potential values of some areas are the same. An *ordered partition* $\phi = (\Gamma_1, \Gamma_2, \ldots, \Gamma_{k'})$ of Γ is a sequence of mutually disjoint non-empty subsets of Γ satisfying $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \cdots \cup \Gamma_{k'}$. Now we describe our algorithm. For each ordered partition $\phi = (\Gamma_1, \Gamma_2, \ldots, \Gamma_{k'})$, we execute the following procedure. First, we construct a smaller sized electric power transaction problem as follows. We identify all the areas in Γ_ℓ for each Γ_ℓ in ϕ and denote the obtained pseudo-areas by $\gamma_1, \gamma_2, \ldots, \gamma_{k'}$. For each pseudo-area γ_ℓ, we define sets of private companies who wants to sell or buy by $S(\gamma_\ell) = \cup_{i \in \Gamma_\ell} S_i$ and $B(\gamma_\ell) = \cup_{i \in \Gamma_\ell} B_i$, respectively. We define the capacity $c(\gamma_\ell, \gamma_{\ell'})$ of the wire from γ_ℓ to $\gamma_{\ell'}$ by $c(\gamma_\ell, \gamma_{\ell'}) = \sum_{(i,i') \in \Gamma_\ell \times \Gamma_{\ell'}} u_{ii'}$. Next, we apply the algorithm in Fig. 3 to the above problem defined on pseudo-areas $\{\gamma_1, \gamma_2, \ldots, \gamma_{k'}\}$ with respect to the (trivial) permutation $(\pi(\gamma_1), \pi(\gamma_2), \ldots, \pi(\gamma_{k'})) = (\gamma_1, \gamma_2, \ldots, \gamma_{k'})$. It requires O($n$) time. If the algorithm returns a potential vector $(q^*(\gamma_1), q^*(\gamma_2), \ldots, q^*(\gamma_{k'}))$, we set a potential vector $\boldsymbol{q^*} \in \mathbb{R}^{|\Gamma|}$ of the original problem by $q^*(i) = q^*(\gamma_\ell)$ ($\forall i \in \Gamma_\ell$). Finally, we check the optimality of the obtained potential vector $q^* \in \mathbb{R}^{|\Gamma|}$ by employing a maximum flow algorithm, which requires O($n + k^3$) time [3]. If $q^* \in \mathbb{R}^{|\Gamma|}$ is optimal, return the pair of potential vector $\boldsymbol{q^*}$ and a flow (a feasible solution of problem P) obtained by a maximum flow algorithm, which satisfy the optimality conditions (C1) (C2) and (C3).

Since the number of ordered partitions of Γ is bounded by $2^k k!$, the total time complexity of the algorithm above is bounded by O($(n+k^3)2^k k!$). It is clear that if the algorithm above returns a flow vector, the obtained solution is optimal for problem P. Thus we need to show that the algorithm above returns at least one solution. We say that an ordered partition $\phi = (\Gamma_1, \Gamma_2, \ldots, \Gamma_{k'})$ is *consistent with* a potential vector $\boldsymbol{q} \in \mathbb{R}^{|\Gamma|}$ if the pair satisfies that $[\forall \ell,\ \forall i, i' \in \Gamma_\ell,\ q(i) = q(i')]$ and $[\ell < \ell' \Rightarrow [\forall i \in \Gamma_\ell, \forall i' \in \Gamma_{\ell'}, q(i) < q(i')]]$.

We show that the algorithm above returns an optimal solution at an iteration when an ordered partition consistent with an optimal potential vector is examined. Assume that $\phi = (\Gamma_1, \Gamma_2, \ldots, \Gamma_{k'})$ is consistent with an optimal potential vector $\boldsymbol{q'} \in \mathbb{R}^{|\Gamma|}$. When we apply the procedure above to ϕ, the algorithm in Fig. 3 returns a potential vector, denoted by $(q^*(\gamma_1), q^*(\gamma_2), \ldots, q^*(\gamma_{k'}))$, since there exists an optimal potential vector consistent with ϕ. The procedure constructs a potential vector $\boldsymbol{q^*} \in \mathbb{R}^{|\Gamma|}$ of the original problem satisfying $q^*(i) = q^*(\gamma_\ell)$ $(\forall i \in \Gamma_\ell)$. Let $(\underline{\boldsymbol{x}}^*, \overline{\boldsymbol{x}}^*, \boldsymbol{x}^*)$ be an optimal solution of problem P. In the rest of this section, we show that the pair $\boldsymbol{q^*}$ and $(\underline{\boldsymbol{x}}^*, \overline{\boldsymbol{x}}^*, \boldsymbol{x}^*)$ satisfies the optimality conditions (C1) (C2) and (C3), and thus $\boldsymbol{q^*}$ is optimal. The definition of ϕ and inequalities $q^*(\gamma_1) \le q^*(\gamma_2) \le \cdots \le q^*(\gamma_{k'})$ directly implies condition (C3). We only need to show that the pair $\boldsymbol{q^*}$ and $(\underline{\boldsymbol{x}}^*, \overline{\boldsymbol{x}}^*, \boldsymbol{x}^*)$ satisfies the conditions

$$\text{(C1-}\ell\text{)} \quad \forall i \in \Gamma_\ell,\ \forall s_{ij} \in S_i,\ \begin{cases} (\underline{\boldsymbol{p}}_i)_j > q_i \ \Rightarrow\ (\underline{\boldsymbol{x}}_i)_j = 0, \\ (\underline{\boldsymbol{p}}_i)_j < q_i \ \Rightarrow\ (\underline{\boldsymbol{x}}_i)_j = (\underline{\boldsymbol{u}}_i)_j, \end{cases}$$

$$\text{(C2-}\ell\text{)} \quad \forall i \in \Gamma_\ell,\ \forall b_{ij} \in B_i,\ \begin{cases} q_i > (\overline{\boldsymbol{p}}_i)_j \ \Rightarrow\ (\overline{\boldsymbol{x}}_i)_j = 0, \\ q_i < (\overline{\boldsymbol{p}}_i)_j \ \Rightarrow\ (\overline{\boldsymbol{x}}_i)_j = (\overline{\boldsymbol{u}}_i)_j, \end{cases}$$

for each Γ_ℓ in ϕ'.

We define a linear programming problem P_ℓ by;

$$\begin{array}{lll}
(\mathrm{P}_\ell) & \text{min.} & \sum_{i \in \Gamma_\ell} (\underline{\boldsymbol{p}}_i^\top \underline{\boldsymbol{x}}_i - \overline{\boldsymbol{p}}_i^\top \overline{\boldsymbol{x}}_i), \\
& \text{s. t.} & \underline{\boldsymbol{x}}_i^\top \mathbf{1} - \overline{\boldsymbol{x}}_i^\top \mathbf{1} - \sum_{i':(i,i') \in F} x_{ii'} + \sum_{i':(i',i) \in F} x_{i'i} = 0 \quad (\forall i \in \Gamma), \\
& & \mathbf{0} \le \underline{\boldsymbol{x}}_i \le \underline{\boldsymbol{u}}_i \ (\forall i \in \Gamma_\ell), \qquad \underline{\boldsymbol{x}}_i = \underline{\boldsymbol{x}}_i^* \quad (\forall i \notin \Gamma_\ell), \\
& & \mathbf{0} \le \overline{\boldsymbol{x}}_i \le \overline{\boldsymbol{u}}_i \ (\forall i \in \Gamma_\ell), \qquad \overline{\boldsymbol{x}}_i = \overline{\boldsymbol{x}}_i^* \quad (\forall i \notin \Gamma_\ell), \\
& & x_{ii'} \text{ is free} \quad (\forall (i,i') \in F \cap \Gamma_\ell^2), \quad x_{ii'} = x_{ii'}^* \ (\forall (i,i') \in F \setminus \Gamma_\ell^2).
\end{array}$$

We introduce dual variables $\boldsymbol{q} \in \mathbb{R}^{|\Gamma_\ell|}$, and $(\sigma_i, \beta_i) \in \mathbb{R}^{|S_i|} \times \mathbb{R}^{|B_i|}$ for each $i \in \Gamma_\ell$. Then the constraints of the dual problem D_ℓ of P_ℓ is defined by;

$$\begin{array}{ll}
q(i) - q(i') = 0 & (\forall (i,i') \in F \cap \Gamma_\ell^2), \\
q(i) - (\underline{\boldsymbol{p}}_i)_j \le (\sigma_i)_j & (\forall i \in \Gamma_\ell,\ \forall s_{ij} \in S_i), \\
(\overline{\boldsymbol{p}}_i)_j - q(i) \le (\beta_i)_j & (\forall i \in \Gamma_\ell,\ \forall b_{ij} \in S_i), \\
\sigma_i \ge \mathbf{0} \ (\forall i \in \Gamma_\ell), & \beta_i \ge \mathbf{0} \ (\forall i \in \Gamma_\ell).
\end{array}$$

Here we omit the objective function of D_ℓ.

It is clear that problem P_ℓ is essentially equivalent to a (single pseudo-area) subproblem defined on pseudo-area γ_ℓ. Since $q^*(\gamma_\ell)$ is an optimal potential of the subproblem, it is easy to show that a feasible solution of D_ℓ defined by $\boldsymbol{q^*} = q^*(\gamma_\ell)\mathbf{1}$, $(\sigma_i^*)_j = \max\{q^*(i) - (\underline{\boldsymbol{p}}_i)_j, 0\}$ $(\forall i \in \Gamma_\ell, \forall s_{ij} \in S_i)$, and $(\beta_i^*)_j = \max\{(\overline{\boldsymbol{p}}_i)_j - q^*(i), 0\}$ $(\forall i \in \Gamma_\ell,\ \forall b_{ij} \in B_i)$, is optimal for D_ℓ.

The duality theory shows that if a pair of feasible solution of P_ℓ and a vector $\boldsymbol{q} \in \mathbb{R}^{|\Gamma_\ell|}$ satisfy the constraints $q(i) - q(i') = 0$ $(\forall (i,i') \in F \cap \Gamma_\ell^2)$ and conditions (C1-ℓ) and (C2-ℓ), then the feasible solution is optimal for P_ℓ. Since $\boldsymbol{q'}$ is an optimal potential vector and $(\underline{\boldsymbol{x}}^*, \overline{\boldsymbol{x}}^*, \boldsymbol{x}^*)$ is optimal for problem P, the pair satisfies conditions (C1), (C2) and (C3), and thus satisfies (C1-ℓ) and (C2-ℓ). The definition of Γ_ℓ implies that $\boldsymbol{q'}$ satisfies constraints $q'(i) - q'(i') = 0$ $(\forall(i, i') \in F \cap \Gamma_\ell^2)$. It implies the optimality of a subvector of $(\underline{\boldsymbol{x}}^*, \overline{\boldsymbol{x}}^*, \boldsymbol{x}^*)$ to problem P_ℓ, where the subvector is obtained by restricting to the index sets $S_i(i \in \Gamma_\ell)$, $B_i(i \in \Gamma_\ell)$ and $F \cap \Gamma_\ell^2$.

The duality theory also implies that any pair of optimal solutions of P_ℓ and D_ℓ satisfy conditions (C1-ℓ) and (C2-ℓ). Thus the pair $(\underline{\boldsymbol{x}}^*, \overline{\boldsymbol{x}}^*, \boldsymbol{x}^*)$ and $\boldsymbol{q}^*$ satisfies the conditions (C1-ℓ) and (C2-ℓ).

5 Conclusion

We introduced the electric power transaction problem which appears in the auction of the electric power, and formulated it as a minimum cost flow problem. We presented two fast algorithms for the problem. One is based on the minimum mean cycle canceling algorithm by Goldberg and Tarjan. The other enumerates all flows on the core network which are candidates of optimal flows, and for each flow it checks whether the flow is really optimal or not with a median finding algorithm. Assume the number of private companies which attend the auction is n and the number of bidding areas is k, the first algorithm solves the problem in $\mathrm{O}(n \log n \; k^5 \log(kC))$ time, while the other solves the problem in $\mathrm{O}((n + k^3)2^k k!)$ time, where C is the largest value of the prices the private companies offer. We can assume that k is very small in practice. When k is a constant, the time complexity of the first algorithm is $\mathrm{O}(n \log n \log C)$ and that of second algorithm is $\mathrm{O}(n)$.

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Looking for Arbitrage or Term Structures in Frictional Markets

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Abstract. In this paper we consider a frictional market with finitely many securities and finite and discrete future times. The frictions under consideration include fixed and proportional transaction costs, bid-ask spreads, and taxes. In such a market, we find that whether there exists an arbitrage opportunity does not dependent on the fixed transaction costs. Under a reasonable assumption, the no-arbitrage is equivalent to the condition that the optimal value of some linear programming problem is zero, and to the existence of a so-called consistent term structure. These results permit us to identify and to find arbitrage and consistent term structures in polynomial time. Two linear programming problems are proposed, each of which can identify and find the arbitrage opportunity or the consistent term structure if either exists.

1 Introduction

No-arbitrage is a generally accepted condition in finance. For frictionless financial markets, it is very well understood. The pioneer work of Ross (1976), for example, characterized arbitrage with the existence of positive valuation or pricing operators in discrete time. This approach has been widely adopted in various models, for instance, by Harrison and Kreps (1979), Green and Srivastava (1985), Spremann (1986), and Flåm (2000). In reality, however, financial markets are never short of friction. Investors are required to pay transaction costs, commissions and taxes. Selling and buying prices are differentiated with ask-bid spread. A security is available at a price only for up to a maximum amount. One may buy or sell a stock at an integer number of shares (or an integer number of hundreds of shares). Friction is a de facto matter in financial markets.

Study of arbitrage in frictional markets has attracted more and more attention in recent years and a body of literature has emerged. Garman and Ohlson (1981) extended the work of Ross (1976) to markets with proportional transaction costs and showed that equilibrium prices in markets with proportional transaction costs are equal prices in the corresponding markets with no friction plus a "certain factor". Later, Prisman (1986) studied the valuation of risky

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assets in arbitrage-free economies with taxation. Ross (1987) extended the martingale analysis of no-arbitrage pricing to worlds with taxation in a one-period setting and showed that the absence of arbitrage implies the existence of different shadow prices for income streams that are subject to differing tax treatment. Recently, Dermody and Rockafellar (1991) investigated no-arbitrage pricing of government bonds in the presence of transaction costs and taxes. Dermody and Prisman (1993) extended the results of Garman and Ohlson (1981) to markets with increasing marginal transaction costs and showed the precise relation of the "certain factor" to the structure of transaction costs. Jouini and Kallal (1995) investigated, by means of martingale method, the no-arbitrage problem under transaction costs and short sale constraints respectively. Jaschke (1998) presented arbitrage bounds for the term structure of interests in presence of proportional transaction costs. Ardalan (1999) showed that, in financial markets with transaction costs and heterogeneous information, the no-arbitrage imposes a constraint on the bid-ask spread. Deng, Li and Wang (2000) presented a necessary and sufficient condition for no-arbitrage in a finite-asset and finite-state financial market with proportional transaction costs. This result allows ones to use polynomial time algorithms to look for arbitrage opportunities by applying linear programming techniques. This necessary and sufficient was generalized to the case of multiperiod by Zhang, Xu and Deng (2002).

Although the literature on models with friction is rapidly growing, there are only a few papers dealing with fixed transaction costs and, to the best of our knowledge, rare work on algorithmic study of arbitrage under realistic frictions although it is important, interesting and challenging. In this paper we study the computation issues of arbitrage and term structures with fixed and proportional transaction costs, bid-ask spreads, and taxes.

2 The Market and Preliminaries

Consider a market of n fixed income securities (or bonds) $i = 1, 2, \ldots, n$. Let $0 = t_0 < t_1 < t_2 < \ldots < t_m$ be all the payment dates (or the times to maturities) that can occur, which need not be equidistant. A cash stream is a vector $w = (w_1, w_2, \ldots, w_m)^T$, where ^{T} denotes the transposition of vector or matrix, and w_j is the income received at time t_j and may be positive, zero or negative. Assume that bond i pays the before-tax cash stream $A_i = (a_{1i}, a_{2i}, \ldots, a_{mi})^T$. So we have the $m \times n$ payoff matrix $A = (A_1, A_2, \ldots, A_n)$.

Bond i can be purchased at a current price p_i^a, the so-called ask price. There is also a bid price p_i^b at which bond i can be sold. The difference between these two prices, the so-called bid-ask spread, reflects a type of friction. This friction exists in most economic markets. We form the ask price vector $p^a = (p_1^a, p_2^a, \ldots, p_n^a)^T$ and the bid price vector $p^b = (p_1^b, p_2^b, \ldots, p_n^b)^T$.

The second type of friction considered in this paper is transaction costs including fixed and proportional. We assume that the fixed transaction cost is c_i if bond i is traded and that no fixed transaction cost occurs if no trading of bond i. The c_i is a positive constant regardless of the amount of bond i traded.

Denote $c = (c_1, c_2, \dots, c_n)^T$ the fixed transaction cost vector. Besides the fixed transaction cost, there is additional transaction cost that is proportional to the amount of the bond traded. Let λ_i^a and λ_i^b be such fees if one dollar of bond i is bought and sold respectively. Here $0 \le \lambda_i^a, \lambda_i^b < 1, i = 1, 2, \dots, n$. Denote $\lambda^a = (\lambda_1^a, \lambda_2^a, \dots, \lambda_n^a)^T$ and $\lambda^b = (\lambda_1^b, \lambda_2^b, \dots, \lambda_n^b)^T$.

The third type of friction incorporated into our model is taxes. Here we concentrate only on a single investor as a member of just one tax class among many. For all investors in this class, the tax amount at time t_j for holding one unit of bond i in long position is assumed to be t_{ji}^a, and the after-tax income at that time is then $a_{ji} - t_{ji}^a$; whereas the tax amount for holding one unit of bond i in short position is t_{ji}^b as a credit against the obligation to pay a_{ji} at time t_j, and the net after-tax payment to be made is then $a_{ji} - t_{ji}^b$. Let T^a be the $m \times n$ matrix whose entries are t_{ji}^a, and T^b the $m \times n$ matrix whose entries are t_{ji}^b.

Now, the bond market considered in this paper can be described by the 8-tuple $\mathcal{M} = \{p^a, p^b, \lambda^a, \lambda^b, c, A, T^a, T^b\}$.

Every investor in the fixed tax class under consideration will modify his or her position. Let the modification be $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, called also a portfolio, where x_i is the number of units of bond i modified by the investor.

For any portfolio $x = (x_1, x_2, \dots, x_n)^T$, we let $x_i^+ = \max\{x_i, 0\}$ be the long position taken in bond i (in number of units) and $x_i^- = -\min\{x_i, 0\}$ the short position in bond i, and let $x^+ = (x_1^+, x_2^+, \dots, x_n^+)^T$ be the vector of buy orders and $x^- = (x_1^-, x_2^-, \dots, x_n^-)^T$ the vector of sell orders. Then $x_i = x_i^+ - x_i^-$ and $x_i^+ x_i^- = 0$ for $i = 1, 2, \dots, n$. The complementary constraints $x_i^+ x_i^- = 0$, $i = 1, 2, \dots, n$ mean that each bond is in either long position or short position.

Define the function $\delta : \mathbb{R} \to \mathbb{R}$ by $\delta(x) = 1$ if $x \neq 0$ or 0 if $x = 0$.

If trading a portfolio $x = (x_1, x_2, \dots, x_n)^T$, then the investor pays the cost $f(x) := \sum_{i=1}^n (1+\lambda_i^a) p_i^a x_i^+ - \sum_{i=1}^n (1-\lambda_i^b) p_i^b x_i^- + \sum_{i=1}^n c_i \delta(x_i^+ - x_i^-)$ in the present and receive the after-tax gain $g_j(x) := \sum_{i=1}^n (a_{ji} - t_{ji}^a) x_i^+ - \sum_{i=1}^n (a_{ji} - t_{ji}^b) x_i^-$ at future date t_j for $j = 1, 2, \dots, m$. The after-tax cash stream of gains generated by the portfolio x is then the vector $G(x) := (g_1(x), g_2(x), \dots, g_m(x))^T$.

When the gains at some dates are positive, no liabilities will be claimed and there exist surplus gains at those dates. These surplus gains can be transformed for use at the succedent dates. Hence we can enlarge the market $\mathcal{M}$ by introducing a dummy bond, indexed by $i = 0$, that costs one dollar at date 0 and immediately pays back the one dollar. This means that $p^a, p^b, \lambda^a, \lambda^b, c, x, A, T^a$, T^b, from now on, replaced by

$$\begin{pmatrix} 1 \\ p^a \end{pmatrix}, \begin{pmatrix} 1 \\ p^b \end{pmatrix}, \begin{pmatrix} 0 \\ \lambda^a \end{pmatrix}, \begin{pmatrix} 0 \\ \lambda^b \end{pmatrix}, \begin{pmatrix} 0 \\ c \end{pmatrix}, \begin{pmatrix} x_0 \\ x \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & A \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & T^a \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & T^b \end{pmatrix}$$

respectively, where x_0 can be interpreted as the amount we have to deduce from the current income $-f(x)$ to help cover the future obligations $G(x)$. Thus, $f(x) := x_0^+ - x_0^- + \sum_{i=1}^n (1 + \lambda_i^a) p_i^a x_i^+ - \sum_{i=1}^n (1 - \lambda_i^b) p_i^b x_i^- + \sum_{i=1}^n c_i \delta(x_i^+ - x_i^-)$, $g_0(x) := x_0^+ - x_0^-$, $g_j(x) := \sum_{i=1}^n (a_{ji} - t_{ji}^a) x_i^+ - \sum_{i=1}^n (a_{ji} - t_{ji}^b) x_i^-$, $j = 1, 2, \dots, m$, $G(x) := (g_0(x), g_1(x), \dots, g_m(x))^T$. Further let $p^+ = (1, (1 + \lambda_1^a) p_1^a, \dots, (1 +$

$\lambda_n^a)p_n^a)$ and $p^- = (1, (1-\lambda_1^b)p_1^b, \ldots, (1-\lambda_n^b)p_n^b)$. Then, $f(x) = p^+x^+ - p^-x^- + \sum_{i=1}^n c_i\delta(x_i^+ - x_i^-)$ and $G(x) = (A - T^a)x^+ - (A - T^b)x^-$.

For convenience, we use the vector notation $x \geqq y$ to indicate that $x_i \geq y_i$ for all i, and denote by B the lower-triangular $(m+1) \times (m+1)$-matrix whose diagonal and lower-triangular elements all are ones.

Definition 1. *A portfolio x is said to be a strong arbitrage if it has a negative date-0 cost (i.e., $f(x) < 0$) and a non-negative cumulative after-tax cash stream (i.e., $BG(x) \geqq 0$). A portfolio x is said to be a weak arbitrage if it satisfies $f(x) \leq 0$ and $BG(x) \geqq 0$ with at least one strict inequality.*

Definition 2. *A term structure is a discount factor vector u such that $u \in K := \{u \in \mathbb{R}^{m+1} : \ 1 = u_0 \geq u_1 \geq u_2 \geq \ldots \geq u_m \geq 0\}$.*

Lemma 1. *A vector $u = (1, u_1, u_2, \ldots, u_n)^T$ is a term structure if and only if $u^T B^{-1} \geqq 0^T$.*

3 Looking for Arbitrage and Term Structures

Theorem 1. *Whether there exists a strong (weak) arbitrage in the market $\mathcal{M}$ is independent of the fixed transaction costs c. In particular, the market $\mathcal{M}$ excludes strong (weak) arbitrage if and only if the market $\mathcal{M}$ without fixed transaction costs excludes strong (weak) arbitrage.*

Proof. We only prove the case of strong arbitrage. Let c and c' be two different fixed transaction cost vectors and x a strong arbitrage under c. Then $BG(x) \geqq 0$ and $p^+x^+ - p^-x^- + \sum_{i=1}^n c_i\delta(x_i^+ - x_i^-) < 0$. Noting that $\delta(\lambda x) = \delta(x)$ and $G(\lambda x) = G(x)$ for any $x \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$, we can take a positive number λ large enough so as to $p^+(\lambda x^+) - p^-(\lambda x^-) + \sum_{i=1}^n c_i'\delta(\lambda x_i^+ - \lambda x_i^-) < 0$ and $BG(\lambda x) \geqq 0$. This immediately means that λx is a strong arbitrage under c'. □

Assumption 1. $p_i^a \geq p_i^b$ *and* $t_{ji}^a \geq t_{ji}^b$ *for all* $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$.

Assumption 1 demonstrates that, for each bond, the bid price is not higher than the ask price and the tax amount for buying one unit bond is not lower than the tax amount for selling one unit bond. The former condition is usually satisfied. The latter condition may cover a more general case (in many cases the two amounts could well be the same).

Consider the following linear programming problem:

$$(LP) \qquad \begin{cases} \text{minimize} & p^+x^+ - p^-x^- \\ \text{subject to} & B(A - T^a)x^+ - B(A - T^b)x^- \geqq 0, \ x^+, \ x^- \geqq 0. \end{cases}$$

Theorem 2. *Under Assumption 1, the market $\mathcal{M}$ excludes*

(1) strong arbitrage if and only if the optimal value of (LP) is zero;

(2) weak arbitrage if and only if the optimal value of (LP) is zero and its every optimal solution satisfies the equality constraint $B(A - T^a)x^+ - B(A - T^b)x^- = 0$.

Proof. We need only to consider the market $\mathcal{M}$ without fixed transaction costs by Theorem 1 and show only assertion (2). Let x be a portfolio such that $f(x) \leq 0$ and $BG(x) \geqq 0$. Then the corresponding (x^+, x^-) satisfies $p^+x^+ - p^-x^- \leq 0$ and is a feasible solution to problem (LP). Since the optimal value of (LP) is zero, it follows that $p^+x^+ - p^-x^- = 0$ and hence (x^+, x^-) is an optimal solution to (LP). Since every optimal solution of (LP) satisfies the equality constraint $B(A - T^a)x^+ - B(A - T^b)x^- = 0$, we have $f(x) = 0$ and $BG(x) = 0$. Thus, by the definition, the market excludes weak arbitrage. Conversely, assume that the market excludes weak arbitrage. Let (x^+, x^-) be a feasible solution of (LP). For $i = 0, 1, \ldots, n$, let $y_i^\pm = x_i^\pm - \min\{x_i^+, x_i^-\}$. Then $y_i^\pm \geqq 0$, $y_i^+ y_i^- = 0$, and, by Assumption 1, it can be checked that

$$p^+y^+ - p^-y^- \leq p^+x^+ - p^-x^-,$$

$$B(A - T^a)y^+ - B(A - T^b)y^- \geqq B(A - T^a)x^+ - B(A - T^b)x^- \geqq 0.$$

Hence, it must holds that $p^+x^+ - p^-x^- \geq 0$ for otherwise $y = y^+ - y^-$ would be a weak arbitrage. This means that the objective function of (LP) is nonnegative at any feasible solution. On the other hand, it is clear that $(x^+, x^-) = (0, 0)$ is feasible to (LP) and at which the objective function vanishes. Hence, the optimal value of (LP) is zero. Furthermore, if $(\hat{x}^+, \hat{x}^-)$ is an optimal solution to (LP), then $p^+\hat{x}^+ - p^-\hat{x}^- = 0$ and $B(A - T^a)\hat{x}^+ - B(A - T^b)\hat{x}^- \geqq 0$. Hence, $B(A - T^a)\hat{x}^+ - B(A - T^b)\hat{x}^- = 0$ because, otherwise, the market would have a weak arbitrage $\hat{y} = \hat{y}^+ - \hat{y}^-$ where $\hat{y}_i^\pm = \hat{x}_i^\pm - \min\{\hat{x}_i^+, \hat{x}_i^-\}$. □

The dual linear programming problem of (LP) is given by

$$(DP) \quad \begin{cases} \text{maximize} & y^T 0 \\ \text{subject to} & y^T B(A - T^a) \leqq p^+, \ -y^T B(A - T^b) \leqq -p^-, \ y \geqq 0. \end{cases}$$

Let $u = B^T y$. Then $u \in K$ by Lemma 1 and (DP) can be written as

$$(DP)' \quad \begin{cases} \text{maximize} & u^T 0 \\ \text{subject to} & u^T (A - T^a) \leqq p^+, \ u^T (A - T^b) \geqq p^-, \ u \in K. \end{cases}$$

Theorem 3. *Under Assumption 1, the market $\mathcal{M}$ excludes*

(1) strong arbitrage iff there exists a term structure u that satisfies

$$u^T (A - T^a) \leqq p^+ \quad \text{and} \quad u^T (A - T^b) \geqq p^-; \tag{1}$$

(2) weak arbitrage iff there exists a term structure u that satisfies (1) and

$$1 = u_0 > u_1 > u_2 > \ldots > u_m > 0. \tag{2}$$

Proof. Conclusion (1) immediately follows from Theorem 2 and the duality theory of linear programming. Now we show assertion (2) and consider only the market $\mathcal{M}$ without fixed transaction costs by Theorem 1.

Sufficiency. Assume that there exists a term structure u that satisfies (1) and (2). By (1), the market excludes strong arbitrage. Suppose to the contrary that there is a weak arbitrage, which is a portfolio x with zero cost $f(x) = p^+x^+ - p^-x^-$ and non-negative and non-zero cash stream $BG(x) = B(A - T^a)x^+ - B(A - T^b)x^-$. Let $y = (B^{-1})^T u$. Then (2) implies that y is strictly positive and (1) leads to $0 = f(x) = p^+x^+ - p^-x^- \geq u^T(A - T^a)x^+ - u^T(A - T^b)x^- = y^T B(A - T^a)x^+ - y^T B(A - T^b)x^- = y^T(BG(x)) > 0$, a contradiction.

Necessity. Assume that the market excludes weak arbitrage and hence strong arbitrage. By (1), there exists a term structure that satisfies (1). Suppose to the contrary that this term structure does not satisfy (2). Then any solution of the dual problem (DP) has at least one component equal to zero. Since the arithmetical mean of finite many solutions of (DP) is also a solution, all the solutions of (DP) have at least one common component that is equal to zero. That is, there exists an index j corresponding to a date to maturity such that $y_j = 0$ for all solutions y of (DP). Consequently, the linear programming problem

$$(LP)_j \quad \begin{cases} \text{maximize} & y^T e_j \\ \text{subject to} & y^T B(A - T^a) \leqq p^+, \ -y^T B(A - T^b) \leqq -p^-, \ y \geqq 0 \end{cases}$$

has the optimal value of zero, where e_j is the vector with all zero components except for the j-th equal to one. Hence, $(LP)_j$'s dual problem

$$(DP)_j \quad \begin{cases} \text{minimize} & p^+x^+ - p^-x^- \\ \text{subject to} & B(A - T^a)x^+ - B(A - T^b)x^- \geqq e_j, \ x^+, x^- \geqq 0 \end{cases}$$

has the optimal value of zero. An optimal solution of $(DP)_j$ is in fact a portfolio with a zero cost and a non-negative cumulative cash stream whose component at date j is at least one, and hence a weak arbitrage, a contradiction. □

Since linear programming is known to be solvable in polynomial time, We conclude that: *Under Assumption 1, the followings are polynomially solvable: 1) to identify whether there exists a strong (weak) arbitrage, 2) to identify whether there exists a consistent term structure (that satisfies (2)), 3) to find a strong (weak) arbitrage if it does exist, and 4) to find a consistent term structure (that satisfies (2)) if it does exist.*

Computationally this justifies the general belief that any arbitrage opportunity will be short-lived since it will be identified very quickly, be taken advantage of, and subsequently bring economic states to a no-arbitrage one.

We already knew from Theorem 3 that when u is a term structure, each positive component of

$$\varepsilon_i^a = \sum_{j=1}^{m} (a_{ji} - t_{ji}^a)u_j - (1 + \lambda_i^a)p_i^a, \quad i = 1, 2, \ldots, n \tag{3}$$

$$\varepsilon_i^b = (1 - \lambda_i^b)p_i^b - \sum_{j=1}^{m} (a_{ji} - t_{ji}^b)u_j, \quad i = 1, 2, \ldots, n \tag{4}$$

points to a bond i which, at best, remains "wrongly" priced. Furthermore, when $\varepsilon_i^a > 0$, bond i is worthy of being bought because it exhibits positive net return (taking the fixed transaction cost as zero; see Theorem 1 for the reason); when $\varepsilon_i^b > 0$, bond i should be sold.

Expressions (3) and (4) also hold when the summation $\sum_{j=1}^m$ is replaced by $\sum_{j=0}^m$ and hold for $i = 0$ with $\varepsilon_0^a = \varepsilon_0^b = 0$. Thus, the vectors $\varepsilon^a = (\varepsilon_0^a, \varepsilon_1^a, \ldots, \varepsilon_n^a)$ and $\varepsilon^b = (\varepsilon_0^b, \varepsilon_1^b, \ldots, \varepsilon_n^b)$ represent a direction in which existing portfolios should move. This insight inspires us to consider the *minimal pricing error*, i.e. the optimal value of the linear programming problem

$$(LP1) \quad \begin{cases} \text{minimize} & \varepsilon^a \mathbf{1} + \varepsilon^b \mathbf{1} \\ \text{subject to} & u^T(A - T^a) \leqq p^+ + \varepsilon^a, \ u^T(A - T^b) \geqq p^- - \varepsilon^b \\ & \varepsilon^a \geqq 0, \varepsilon^b \geqq 0, u \in K \end{cases}$$

where $\mathbf{1}$ is the vector whose components are all ones. Clearly, $\varepsilon_0^a = \varepsilon_0^b = 0$ in any optimal solution of $(LP1)$.

Replacing u with $z = u^T B^{-1}$ in $(LP1)$, the dual problem of $(LP1)$ is

$$(DP1) \quad \begin{cases} \text{maximize} & -p^+ x^+ + p^- x^- \\ \text{subject to} & B(A - T^a)x^+ - B(A - T^b)x^- \geqq 0 \\ & x^+ \leqq \mathbf{1}, \ x^- \leqq \mathbf{1}, \ x^+, x^- \geqq 0. \end{cases}$$

The optimal value of $(DP1)$ can be interpreted as the *maximal arbitrage profit* per unit transaction volume in each bond.

Since $(DP1)$ has a bounded feasible solution set which is nonempty (indeed $x^+ = x^- = 0$ is a feasible solution), it must have an optimal solution. By the duality theory of linear programming, $(LP1)$ and $(DP1)$ both have optimal solutions and their optimal values equal. In other words, *the minimal pricing error equals the maximal arbitrage profit.*

This fact implies that arbitrage possibilities are kept small by traders who try to exploit them. The way traders construct their intended arbitrage transactions determines in which sense the pricing error is kept small.

Theorem 4. *Let $(\overline{x}^+, \overline{x}^-; \overline{u}, \overline{\varepsilon}^a, \overline{\epsilon}^b)$ be a primal-dual optimal solution to $(DP1)$ and r^* the optimal value. Under Assumption 1, if $r^* = 0$, then $\overline{u}$ is a consistent term structure and the market $\mathcal{M}$ excludes strong arbitrage (and weak arbitrage if further $\overline{u}$ satisfies (2)); if $r^* \neq 0$, then $(\overline{x}^+, \overline{x}^-)$ provides a strong arbitrage and the associated arbitrage profit is r^*.*

Proof. Let $r^* = 0$. Then the minimal pricing error, i.e. the optimal value of $(LP1)$ is zero. Since $(\overline{u}, \overline{\varepsilon}^a, \overline{\epsilon}^b)$ is an optimal solution of $(LP1)$, we have $\overline{\varepsilon}^a = \overline{\epsilon}^b = 0$. This implies that $\overline{u}$ is a consistent term structure. Now let $r^* \neq 0$. Then $r^* > 0$ because $(x^+, x^-) = (0, 0)$ is a feasible solution of the maximization problem $(DP1)$ with objective value 0. Hence, $(\overline{x}^+, \overline{x}^-)$ is a strong arbitrage with arbitrage profit r^*. □

Since $(DP1)$ always has an optimal solution and (LP) does not, solving $(DP1)$ is more convenient than solving (LP).

When applying Theorem 4 and finding that $r^* = 0$ and $\overline{u}$ does not satisfy (2), to identify the existence of weak arbitrage and to find a one if it exists in an analogous manner we can use the result stated below.

Consider the linear programming problem $(DP)_j$ and its dual problem $(LP)_j$ in which we let $u = B^T y$ (i.e., the problem $(DP)'$ with a replacement of the objective function by $u^T B^{-1} e_j$).

Theorem 5. *Assume that the market $\mathcal{M}$ excludes strong arbitrage and that Assumption 1 holds. Let $(\overline{x}^{j+}, \overline{x}^{j-}; \overline{u}^j)$ be a primal-dual optimal solution of $(DP)_j$ and r^{j*} its optimal value. If $r^{j*} = 0$ for some $j \in \{0, 1, \cdots, m\}$, then $(\overline{x}^{j+}, \overline{x}^{j-})$ is a weak arbitrage; If $r^{j*} \neq 0$ for all $j \in \{0, 1, \cdots, m\}$, then $\sum_{j=0}^{m} \overline{u}^j/(m+1)$ is a consistent term structure that satisfies (2) and the market $\mathcal{M}$ excludes weak arbitrage.*

Proof. It directly follows from the proof of Theorem 3 (2). □

4 Applications

As a byproduct, the linear programming problem $(DP)_j$ can be used to valuate the Arrow security (or state price) at date (or state) j in the market $\mathcal{M}$ without fixed transaction costs. The optimal value of $(DP)_j$ is the minimal price of a portfolio whose cumulative cash stream is at least e_j. When the market is complete, the minimal cost is just the price of the Arrow security. When the market is incomplete, the cumulative cash stream of the minimal cost portfolio is in fact strictly greater than e_j for some j and hence the minimal cost is the supremum of the price of the Arrow security for state j.

As an application of Theorem 3, we can check the market efficiency, i.e. determine the maximal range of oscillation of bid and ask prices that exclude strong arbitrage. The minimal ask price for the i-th bond can be obtained by finding term structures that satisfy (1):

$$\begin{cases} \text{minimize} & u^T(A - T^a)e_i \\ \text{subject to} & u^T(A - T^a) \leqq p^+,\ u^T(A - T^b) \geqq p^-,\ u \in K \end{cases}$$

Analogously, the maximal bid price for the i-th bond can be obtained by solving the linear programming problem

$$\begin{cases} \text{maximize} & u^T(A - T^b)e_i \\ \text{subject to} & u^T(A - T^a) \leqq p^+,\ u^T(A - T^b) \geqq p^-,\ u \in K. \end{cases}$$

Of cours, if we further want to maintain the absence of weak arbitrage, to get the minimal ask price and the maximal bid price for the i-th bond we need only to substitue the last constraint $u \in K$ with (2).

Finally, as a numerical example we apply the method developed in this paper to a simple economy where $p^+ = (1, 1, 5/2)$, $p^- = (1, 1/2, 2)$, and

$$A - T^a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & -1 \end{pmatrix}, \quad A - T^b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & 3 \\ 0 & 1 & 0 \end{pmatrix}.$$

Solving problem $(DP1)$ yields that a primal-dual optimal solution of $(DP1)$ is $(\overline{x}^+, \overline{x}^-; \overline{u}, \overline{\varepsilon}^a, \overline{\epsilon}^b) = (1, 0, 0; 1, 0, 0; 1, 1, 1/3, 1/6; 0, 0, 0; 0, 0, 0)$ and its optimal value is $r^* = 0$. By Theorem 4, there exists no strong arbitrage and $\overline{u} = (1, 1, 1/3, 1/6)$ is a consistent term structure. However, the consistent term structure $\overline{u}$ does not satisfy (2). To investigate the existence of weak arbitrages, problem $(DP)_j$ has to be solved for all indexes $j = 0, 1, 2, 3$. It turns out that a primal-dual optimal solution $(\overline{x}^{j+}, \overline{x}^{j-}; \overline{u}^j)$ to $(DP)_j$ and its optimal value r^{j*} are as below:

$$\begin{aligned}
&j = 0 : \overline{x}^{0+} = (1, 0, 0), \quad \overline{x}^{0-} = (0, 0, 1/4), \overline{u}^0 = (1, 1/2, 1/2, 0), \quad r^{0*} = 1/2;\\
&j = 1 : \overline{x}^{1+} = (4/3, 0, 0), \overline{x}^{1-} = (0, 0, 1/3), \overline{u}^1 = (1, 1, 1/3, 1/4), \quad r^{1*} = 2/3;\\
&j = 2 : \overline{x}^{2+} = (0, 0, 1/4), \overline{x}^{2-} = (0, 0, 0), \quad \overline{u}^2 = (1, 5/8, 5/8, 0), \quad r^{2*} = 5/8;\\
&j = 3 : \overline{x}^{3+} = (0, 0, 1/3), \overline{x}^{3-} = (0, 0, 0), \quad \overline{u}^3 = (1, 5/6, 5/6, 5/6), r^{3*} = 5/6.
\end{aligned}$$

Because none of these problems has the optimal value of zero, Theorem 5 implies that there exists no weak arbitrage and $\sum_{j=0}^{3} \overline{u}^j/4 = (1, 71/96, 55/96, 13/48)$ is a consistent term structure that satisfies (2).

5 Conclusion

In this paper, we discussed strong and weak arbitrages and consistent term structures in fractional markets with fixed and proportional transaction costs, bid-ask spreads, and taxes. We concluded that the existence of strong (weak) arbitrages is independent of the fixed transaction costs and that no strong (weak) arbitrage is equivalent to the fact that the optimal value of some linear programming problem is zero (and its very optimal solution makes the inequality constraints becoming equality constraints) and to the existence of consistent term structures (that satisfies (2)). These characterizations extend some known results in discrete time security markets. Further, two linear programming problems are constructed and used to identify and find a strong (weak) arbitrage and a consistent term structure (that satisfies (2)). The computation of the method can be completed in polynomial time by using linear programming techniques.

The results demonstrated in this work are not limited to the model in this paper. For example, the described multi-period setting with one single outcome state per period may be interpreted as a one-period investment problem with n assets and m different outcome states. The methods dealt with them are much the same. It is also interesting to consider computational issues in a more general setting of friction or/and time (period). Such extensions require more sophisticated tools and are worthy of investigation further in future.

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A Framework on Compound Knowledge Push System Oriented to Organizational Employees

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Abstract. Organizational employees have different knowledge demands and the knowledge is compound. So how to push the right compound knowledge to the right organizational employees becomes important. This paper attempts to propose a framework for compound knowledge push system for organizational employees to solve the problem. Firstly, the compound push mechanism is given out based on the analysis of the knowledge needs of organizational employees. Secondly, after introducing the key IT, the framework is presented with the illumination of every body's function of the system. Finally, an application case is studied to illustrate the compound knowledge push system's operation mechanism based on the framework. Applying the system can give the employees all-around knowledge backing, and it will enhance the knowledge management level of organizations.

1 Introduction

In the Knowledge Economic Age, knowledge management has become a necessary method for an organization to develop and improve competence ability. With the comprehensive application of IT and the popularization of Internet, it is not only much easier for an organization to obtain knowledge from external, but also more convenient to share and create knowledge within the organization, which makes knowledge store increase sharply. Simultaneously, emergence of knowledge overloading and knowledge getting lost reflects the fact that knowledge in organization lacks effective management. In organizations, although knowledge store and knowledge source are abundant, employee's ability to obtain knowledge is seriously deficient. So how to build a knowledge push system for organizational employees to solve the above problem is urgent.

Recent years, researches on knowledge push have drawn more attention from academic circle to enterprises. Tso, S.K., Lau, H.C.W., Ip, R.W.L.[1] presented a fuzzy push delivery scheme which can 'observe' the movements of the user within the Web pages and then evaluates the personal interests and preferences of the user. With the information available from the fuzzy scheme, Web site information favoured by

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visitors can be 'pushed' to them without even being requested. Liu, L., Pu, C., Tang, W.[2] described a design of a distributed event-driven continual query system – OpenCQ. In OpenCQ, users may specify to the system the information they would like to monitor. Whenever the information of interest becomes available, the system immediately delivers it to the relevant users. Celik, A., Datta, A., Narasimhan, S.[3] proposed a new protocol - single encoding multiple decoding (SEMD) to handle secure access and subscription services for broadcast. Acharya, S., Franklin, M., Zdonik, S.[4] studied how to augment the push-only model with a 'pull-based' approach of using a backchannel to allow clients to send explicit requests for data to the server. Cai, J., Tan, K.L[5] refered an intergrated distribution-based information system(DBIS), its fist model is a data push model which delivers data stored in it to clients.

These literatures make the concept of knowledge push clear and give out the detailed explains and advice from the aspects of push forms, contents and achieving ways, but all these studies focus on knowledge push from the point of users' interests and hobby. To organizational employees, their demand of knowledge not only comes from their interest and hobby, but also comes from requires of posts and workflows. With regard to knowledge demand characteristics of organizational employees , relevant researches on knowledge push is still blank at present. So this paper attempts to propose a compound knowledge push framework for organizational employees to meet organizational employees' compound knowledge demand.

This paper is organized as following: In section 2, the knowledge push mechanism for employees is analyzed. In section 3, the key IT is introduced. In section 4, a compound knowledge push framework for organizational employees is presented and every body's function of the framework is described. In section 5, an application is given out to illustrate operation mechanism of the system based on the framework. In section 6, the conclusions are drawn.

2 Analysis on Compound Knowledge Push Mechanism

The attribute of knowledge needed by organizational employees has two sides: organizational attribute and natural attribute. Organizational attribute points that a employee who is an organizational member must master the knowledge needed by his/her post and workflow, to complete work and improve the decision level. Natural attribute points that a employee who is a natural man has the demands of obtaining knowledge from personal interest and hobby, these knowledge consists of work skills, the use of new tools and new methods, obtaining which will improve work efficiency. Simultaneously, we also can construct communities of practice on the base of personal preference, which is help for organizational culture. Because of knowledge's complexity, personal interest diversity and the concrete requirement of work surrounding, it is necessary to analyze the compound knowledge push mechanism.

2.1 Knowledge Push Based on Employee's Real-Time Interest

The existing research mainly focuses on how to scout the knowledge to meet users' needs in the vast and verified knowledge/information field, such as knowledge push via e-mail, web or channel which are booked manually by users. The above research suppose that external environment is dynamically varying and user's interest and hobby is relatively static. However, fact is not so. User's interest may change at any time and these changes reflect on the user's actions of using computer, such as browsing web, starting up some applications. So it is not feasible to constantly revise the user's interest setup by use of traditional push technology[6]. Now we need a intelligent system to push knowledge according to employee's real-time interest which can be obtained by monitoring the real-time data flow. For example, monitor HTTP flow to get the WWW website and then further get knowledge character in this website; monitor WINDOWS system information to get the open application and obtained focus; monitoring keyboard input to get the input text. All these information constitutes the employee's real-time interest, by which system pushes related knowledge. For instance, when system acquire the information that an employee is using Microsoft Office, it will push the related information about upgrade information and use skill, which benefits to enhance work efficiency.

2.2 Knowledge Push Based on Employee's Fixed Interest

This push mechanism is similar to traditional one, by which knowledge is pushed according to the contents booked by employee in personal preference base. Firstly, an employee sets his/her preference by hands and saves it into preference base. Additionally, according to the evaluation of the pushed knowledge (which is stored in knowledge training set, including the knowledge based on real-time interest and based on fixed interest) by employee, continuously expand and revise personal preference. Finally, the system pushes knowledge based on the preference base that exactly reflects the employee's fixed interest. Methods to mine personal preference can use clustering, artificial nerve network, fuzzy logic, rough set and other technologies and methods to achieve. We also can construct communities of practice on the base of personal preference to enhance organizational culture.

2.3 Knowledge Push Based on Post

Organizational structure decides the post setup and employees in the different posts have different knowledge demands. For example, warehouse administrators need the knowledge about logistics, accountants needs knowledge about finance. So it needs to build a post knowledge demand base, contents in which point out the post knowledge characteristics, that is, which post should master what kind of knowledge and what degree of knowledge. When a employee logs in, firstly, system checks the employee's identity and then reads the employee's post information from basic information table. Secondly, determines knowledge characteristics according to the information from post knowledge demand base. Finally, scouts the right knowledge from knowledge space and push it to the employee.

2.4 Knowledge Push Based on Workflow

Organizations complete routine and perform tasks often by corresponding workflow. Workflow orients to subject and can be seen as the integration of different post based on operation logic. So the knowledge demand of post involves in workflow composes knowledge demand of workflow. Workflow application often be divided into several phases and the knowledge demand in every phase is continuous, namely, the work in this phase depends on knowledge which is accumulated and created in last phase. System can push right knowledge to employees according to the characteristics of knowledge needed by workflow itself and its phase information. So it needs to build a workflow knowledge demand base which points out the workflow's knowledge characteristics and the address where the knowledge of different phases stores. That is to say, what kind of knowledge and what degree knowledge must have to achieve in this workflow, and locate the knowledge accumulated and created in every phase. Firstly, system interacts with workflow engine to get the workflow identity and its phase information. It finds the post involved in workflow, and then gets knowledge characteristics of the workflow from post knowledge demand base and knowledge store address in last phase from the workflow knowledge demand base. Finally, it scouts in knowledge space, which can build a knowledge buffer base to store the knowledge accumulated and created in each phase for each active workflow, and push the obtained knowledge to employees.

3 Analysis on Key Information Technology

3.1 Software Agent

Agent is derived from artificial intelligence that is generally defined as following: It is an autonomic entity which could apperceive environment and make certain judgment and reasoning of external information to conduct decision-making and activity by itself, in order to finish some tasks[7]. According to the difference of performing entities, agents can be classified into human agent, hardware agent and software agent. Among them, software agent is a software that can perform given tasks for users. It has some intelligence degree to allow to perform part of the tasks and interacts with environment in an appropriate way. It provides a new approach to solve the knowledge management problems under an open, distributed and complex web environment at present[8, 9].

The compound knowledge push system proposed in this paper is a complex and intelligent one. Identifying when employees login system, tracking employees' interest in real-time, scouting and pushing knowledge, all these need intelligent entities to complete automatically on background, and the system runs normally depending on the continuous interaction among intelligent entities as well. Software agent technology is the suitable choice for the system's realization. Based on the research on agent application and the analysis of system operation mechanism[10-12], soft agent is a better choice to achieve the system. Software agent owns some characteristics: autonomisation, reactivity, social ability, goal-guiding, intelligence, and these characteristics can fully satisfy the system's running requirements. Achieving the system needs stall a personal agent into every employee's operation platform, and stall general push agent and special agent in servers.

3.2 Knowledge Grid

Knowledge grid is put forward by Fran Berman in 2001 for the first time[13], the relatively whole definition of it is: Knowledge grid is an intelligent interconnect environment, it can make the users or virtual roles gain, issue, share and manage knowledge resource effectively. It provides the knowledge service to users or other services and aids achieving knowledge creation, coordinating work, problem solving and decision supporting[14]. In this paper, we definite knowledge space as the store space owning the ability to deal with knowledge/information intelligently by using knowledge grid technology, in which, main store entities include knowledge base, post knowledge demand base, workflow knowledge demand base, and personal preference base. The knowledge range covered by knowledge space is extensive. It not only includes the employee private knowledge and organization private knowledge, but also includes the knowledge obtained from external. When system scouts the knowledge, the sequence of scouting is local scouting by personal agent, internal scouting by organizational agent (general push agent and special push agent) and external scouting by organizational agent. Knowledge stored in knowledge base is defined on categories set by an organization and it can be divided into concept, axiom, rule and method from simplicity to difficulty, and also can be sorted into person private knowledge, organization private knowledge and public knowledge[15]. The relation about category, degree and privileges of knowledge is shown in fig. 1.

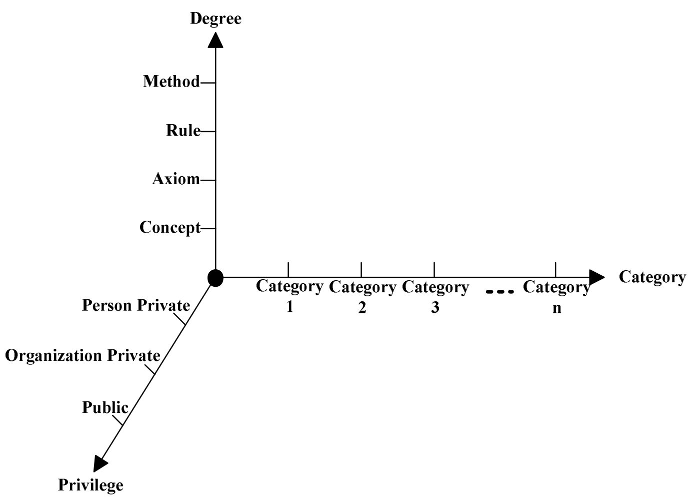

Fig. 1. Three-dimension relation about category, degree and privilege of knowledge

The knowledge category shown in Fig.1. can be sorted into more specific subclass. We can construct the inferior subclass with the same structure.

4 Structure of a Compound Knowledge Push Framework

According to the above analysis on compound knowledge push mechanism and key IT, we present a framework for compound knowledge push system. It is shown in fig.2.

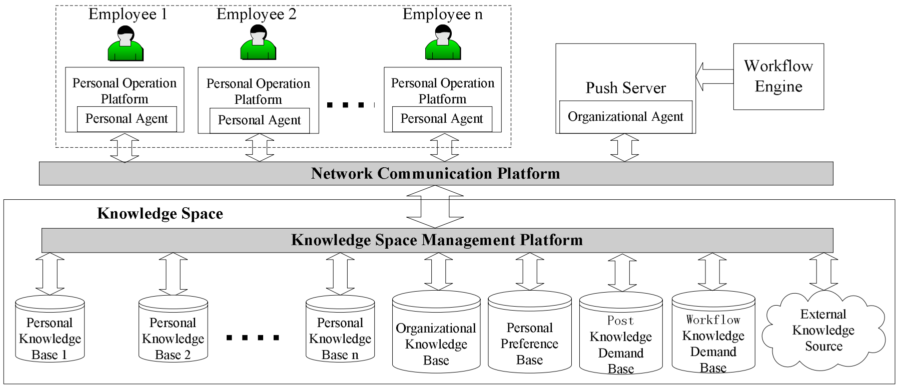

Fig. 2. A compound knowledge push framework for organizational employees

The analysis on main bodies' function of the framework is described as the following:

Knowledge Space. The knowledge/information stored in knowledge space consists of person private knowledge, organization private knowledge and knowledge obtained from external environment. The storage of knowledge is distributed. knowledge can be stored in local personal knowledge base or in organizational server(organizational knowledge base, personal preference base, post knowledge demand base, workflow knowledge demand base), even external knowledge source such as Internet. This knowledge is managed by knowledge space management platform which uses knowledge grid as its core technology. To make the system framework clear, we denote this knowledge and management platform as knowledge space. When software agent puts forward knowledge request, knowledge space management platform charges of searching knowledge and feeding back scouted results.

Personal Agent. Personal agent is installed in employee's operation platform. It has these function:

– Complete identifying(user's name, password and other secure mechanisms) and connects employees with information of post and preference.
– Track the employee's interest changing at any time, scout local knowledge and push it to employee. Send knowledge request to general agent if employee is not satisfied.
– Receive and show the scouted results of knowledge.
– Appraise the results to expand and revise personal preference.
– Use suitable mining tools and strategy to complete personal preference mining.

Organizational Agent. Organizational agent is installed in push server, and it runs on four patterns:

– Knowledge push pattern based on employee's real-time interest. Organizational agent receives knowledge request from personal agent, scouts knowledge in internal and external environment and the search is charge of knowledge space management platform, feeds scouted results back to personal agent, expands and revises the personal preference according to the evaluation from employee and mines personal preference by using suitable mining tools and strategy.

- Knowledge push pattern based on employee's fixed interest. Organizational agent scouts external environment at regular time or at the time when external knowledge related to employee's interest and hobby varies, pushes the late information and knowledge to the employee.
- Post-based knowledge push pattern. When an employee logins system and passes the identifying by personal agent, organizational agent obtains post information from employee's basic information table, reads corresponding knowledge characteristic from post knowledge demand base, scouts knowledge in knowledge space and pushes the scouted results to the employee.
- Workflow-based knowledge push pattern. Organizational agent interacts with workflow engine to read workflow ID and current phase information, reads involved post information from workflow knowledge demand base to make sure the knowledge characteristic, simultaneously, locates the address where the knowledge stores in last phase, and pushes the related knowledge which is scouted from knowledge space to employee.

5 Case Study

We have developed a prototype of compound knowledge push system called KEMS(Knowledge Express Mail Service) based on the framework and it is used in some software development company. Figure 3 gives out the running interface of KEMS on the personal operation platform of a designer in the company. The running result of KEMS shows that all-around knowledge supporting via KEMS in software development can shorten the development cycle and improve the knowledge management level of the company.

The running of system is sustained by the interaction of personal agent with organization agent in background. The personal agent installed in employee's operation platform is client software. It interacts and coordinates with organization agent installed in server to complete the work of knowledge scouting, pushing, showing and feeding within the knowledge space, and make sure the system run normally. The operation mechanism of KEMS is shown in fig. 4.

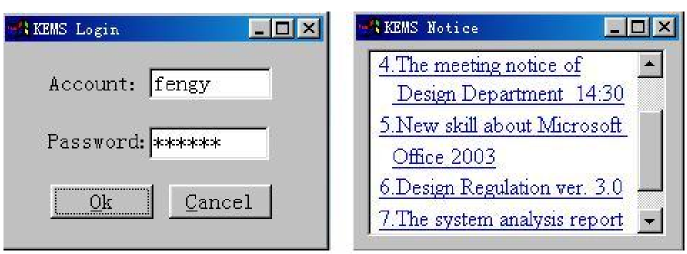

Fig. 3. Screen shot of KEMS's running interface

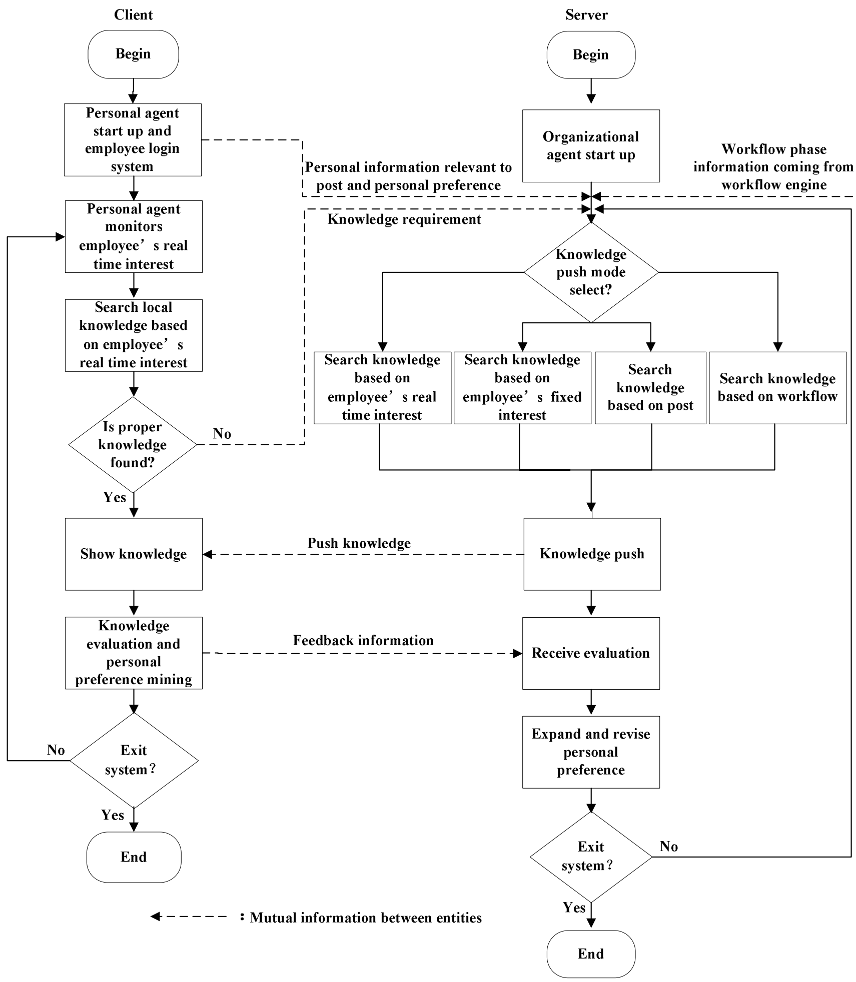

Fig. 4. Operation mechanism of KEMS

For the pushed knowledge, personal agent can deal it in many ways. The most direct way is to make it shown in the pop-up window, or saved in local disks with reminding employees to look up by notice form when employees are online. The knowledge also can be mailed to employees with the relevant notice being sent to employees' mobile phone when employees are offline.

6 Conclusions

This paper analyzes the push mechanism for organizational employee's knowledge demand, introduces the key IT, proposes a framework for compound knowledge push system, and gives out a case of enterprise to illustrate that this framework can satisfy the employee's knowledge demands and improve work efficiency. Because of the complexity of knowledge management, research on knowledge push is in the first step regardless of theory study or practice study. Therefore, the research in this paper aims to analyze the framework for compound knowledge push system, and does some practice about compound knowledge push system based on the enterprise's knowledge demands in reality.

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Effective Decision Making by Self-evaluation in the Multi-agent Environment

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Abstract. Generally, in multi-agent systems, there are close relations between behavior of each individual agent and the group of agents as a whole, so a certain information about the relative state of each agent in the group may be hided in each agent behavior. If this information can be extracted, each agent has the possibility to improve its state by seeing only its own behavior without seeing other agents' behaviors. In this paper, we focus on "power-law" which is interesting character seen in the behavior of each node of various kinds of networks as one of such information. Up to now, we have already found that power-law can be seen in the efficiently behaving agents in Minority Game which is the competitive multi-agent simulation environment. So, in this paper we have verified whether it is possible for each agent in the game to improve its state by seeing only its own behavior, and confirmed that the performance gain was actually possible.

Keyword: Minority game, Indirect coordination, Power law.

1 Introduction

Recently, it has been understood that various networks from the metabolic and ecosystem networks to the social-community networks and the technologically formed networks have same network structures "sale-free network", when the behavior of each node is efficient [1]. We can see whether a network has the characteristic of scale-free network or not by verifying whether a flow type of information exchanged between nodes of the network has "power-law" or not. The fact that various kinds of above networks are scale-free network means it is essentially desirable for each network to have the characteristic of scale-free network. At this point, power-law is the most important characteristics of forming scale-free network. For example, in the Internet it is known that power-law can be seen in the flow of packets when the number of packets of flowing is the maximum in the traffic [2]. This means if we can correct the flow of traffic to follow of power-law when the flow of traffic becomes an inefficiency situation due to some troubles, the flow of traffic may be corrected

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to an efficient situation. To make this, it is necessary to understand the forming mechanism of power-law in the traffic of the Internet, then the control of several parameters those relate closely to form power-law becomes possible.

Multi-agent system is a network where one agent is one node, and the behavior of each agent (node) forms the characteristic of the system (network). Generally, in multi-agent systems each agent decides its behavior by considering the state of other agents, and each agent's behavior influences the group of whole agents and then, the behaviors of the group influence each agent. At this point, it can be thought that the behavior of each agent includes certain information of its relative state in the group of whole agents. So, if each agent can extract this information, it can know its relative state in the group by seeing only its behavior, and becomes possible to improve its behavior efficiently. In scale-free network, power-law is seen in the behavior of an individual node, and in this paper, we focus on "power-law" which is interesting character seen in the behavior of each node of various kinds of networks as one of such information. Up to now, we have been studying the Minority-Game (MG), which is a competitive simulation environment to analyze social economic model [3][4]. In MG, a lot of agents who have local view play a simple game, and a social community network is formed in which one agent is one node of the network. And we have already discovered the fact that power-law was seen in the behavior of agents having high winning-ratio [5]. So, in this paper, first, we have analyzed why power-law was formed in agent behavior and second, verified whether agent

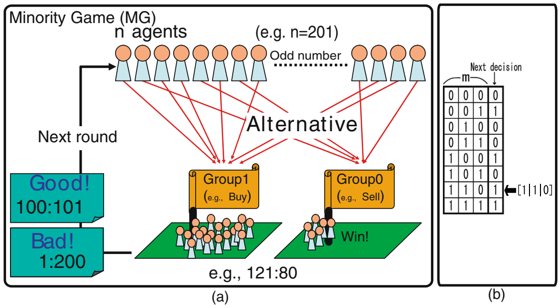

Fig. 1. Rules and strategy table

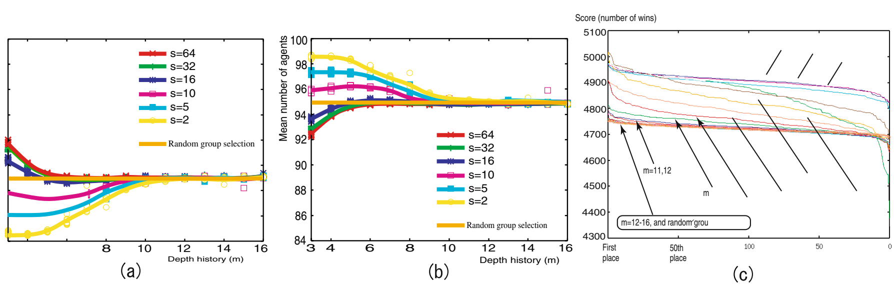

Fig. 2. (a) Standard deviations of the number of winning agents. (b) Mean numbers of winning agents. (c) Rankings of agents for each m.

behavior can be improved or not by checking whether power-law is seen in its behavior or not. The reason to choose Minority Game is that agent's rule is so concise that the simulation can be easily executed and it can be thought that the analysis of the mechanism by which power-law is formed may be easy compared with the real-networks like the Internet .

First, we will describe about Minority Game and the interesting behaviors of agents, then show that power-law can be seen in agent's behavior. Next, we will analyze the mechanism by which power-law is generated, and discuss important parameters that generate power-law. And, the experiment and its result of whether agent's efficiency can be improved by controlling these parameters are described.

2 Minority Game

Firstly, we review the rules of Minority Game (see Fig. 1 (a)). We have n agents, each of which is an autonomous agent that independently chooses between two alternatives (group 0 or group 1) according to its own behavioral rules. In each round of the game, all of the agents choose one alternative or the other, and the agents that then finish in the minority group are considered to be winners. Each winner is awarded one point, and the total numbers of points awarded to all agents is the profit in this round of the game. Therefore, the smaller the difference between the numbers of agents in the majority and minority groups, the better the result.

Each agent makes its selection based on one of multiple strategy tables that it holds. The entities in the table contain all combinations of m past winning-group choices along with next decisions that corresponds to each of the combinations (see Fig. 1 (b)). At the beginning of the game, each agent prepares s strategy tables, and the next decision entries (0 or 1) of each strategy table is stored randomly.

In the first round of the game a m-past-winning-group-history is randomly set, for example to [1|1|0], and each agent randomly selects one of its s strategy tables and sees the next decision entry corresponding to [1|1|0] (see Fig. 1(b)). And if the next decision entry is group 1, the agent selects group 1. Then if the agent wins, one point is assigned as profit to the selected strategy table. If the agent loses, one point is deducted. After the scoring update of strategy tables of all the agents, the m-past-winning-group-history is updated from [1|1|0] to [1|0|1] because group-1 becomes winning-group in this game.

In the second and subsequent rounds of the game, the strategy table that has the highest number of points is always selected. This cycle is repeated a predetermined number of times, and the final result of the game is the total number of points acquired by winning agents across all rounds.

2.1 Emerged Behavior of Agents

Then, the following overall order is formed through such the simple rules [3][4]. We executed the game by 201 agents. Firstly, the standard deviation of the number of winning agents is shown in Fig. 2(a). The game was played for the number of rounds described below with the agents possessing various numbers of strategy tables, $s = \{2, 5, 10, 16, 32, 64\}$ and the strategy tables having various history depths, $m = 3$ to 16. One trial for each parameter pair, (s, m), is 10,000 rounds of the game, and ten trials were conducted for each pair. Fig. 2(b) shows the mean numbers of winning agents. The horizontal lines in Fig. 2 (a) and (b) represent the standard deviation and mean value when all of the agents made random choices. These graphs show that, for the lower values of s, the standard deviation became lowest and the mean number of winning agents became highest when m was from three to six. Fig. 2 (c) shows the rankings of the 201 agents by average score. In the case where every agents randomly selected "group 0" or "group 1", they could get approximately 4750 points. On the other hand, the mean score was high when the standard deviation was small (m=3 to 5) and, although some differences between agents can be seen in the scores, all or almost all of the agents were able to achieve stable high scores.

This means that some kind of emerged behavior among the agents was driving the winning-group ratio closer to 100:101 in these cases. Especially, the most interesting characteristic is that, although we would expect behavior based on longer histories to be more efficient, m larger than 10 produce results that are the same as those of random behavior when we ran the game by 201 agents. At this point, as for n and m, the following relation has already been known that there exists the constant relation between m and $\sigma^2/2N$ (σ is the standard deviation of Fig. 2 (a)), in this paper, we executed the game by 101, 201, and 301 agents.

3 Agent Behavior and Power-Law

In [5], we have investigated how the strategy tables are used by each agent to analyze each agents behavior in detail. Fig. 3 (a) and (b) show the transitions of the points for each strategy table held by the 25th-place agent of m=3, and the

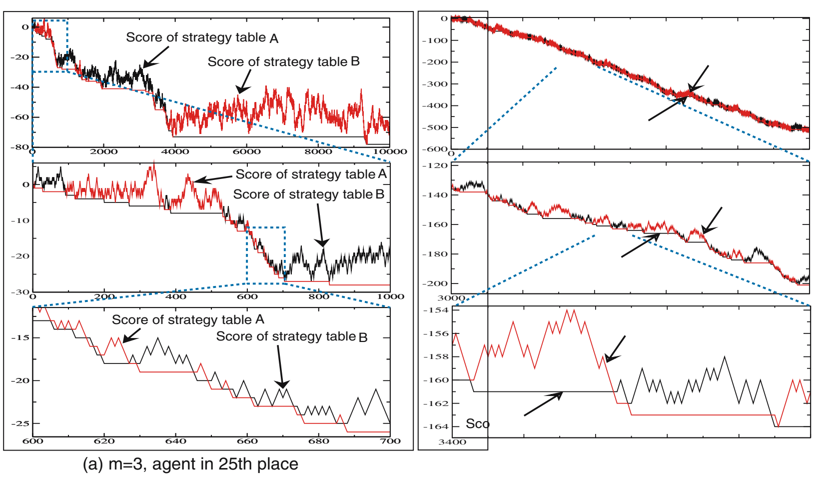

Fig. 3. Which strategy table was used? Scores for (a) m=3, 25th place agent and (b) m=14, 200th place agent in (upper) 10000, (middle) 1000, and (lower) 100 games

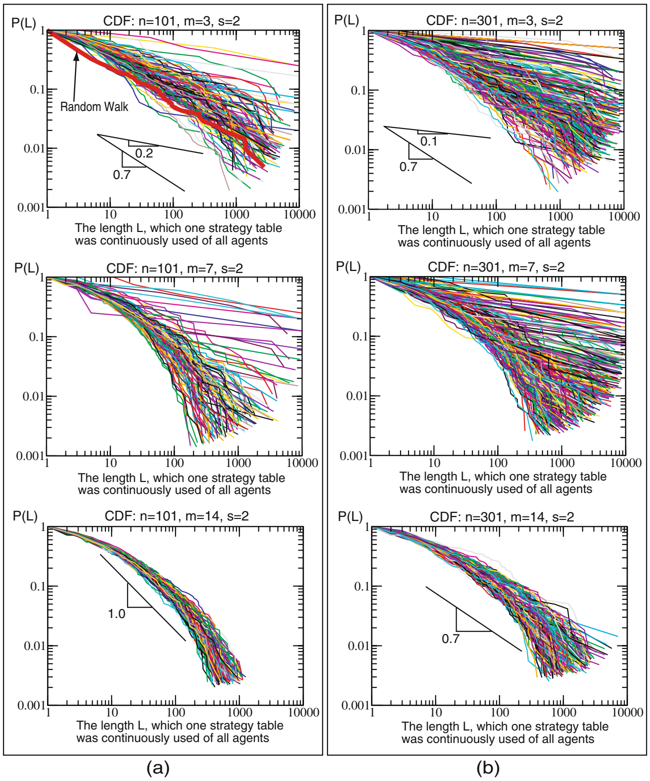

Fig. 4. No. of steps L over which either strategy table was in continuous use for (a) n=101 and (b) n=301 with (upper) For m=3 (all 101 agents and 301 agents), (middle) For m=7 (all 101 agents and 301 agents) (lower) For m=14

200th-place agent of m=14 when the game was played by 201 agents and each agent has 2 strategy tables (s=2).

Then we have discovered the following curious fact. While the 25th-place agent in Fig. 3 (a) used both strategy tables, there was no fixed period for the continuous use of one strategy table. In other words, a fractal characteristic is visible in this agent behavior; that is, the usage of strategy tables shows self-similarity. Fig. 4 shows, on log-log scales, per-agent histograms of the periods over which either of the strategy tables was continuously used. Results for m=3, 7, and 14 are given for (a) $n = 101$, and (b) $n = 301$. Power-law can be seen in the cases of m=3 of (a) and (b) (graphs were nearly straight lines). On the other hand, returning to Fig. 3 (b), no fractal characteristic is visible in the results for the 200th-place agent. There was certain fix period for continuous use of one strategy table. As Fig. 4 shows, power-law cannot be seen in m=14 of n=101 and n=301 (graph is not straight line). And the performance of agents with m=14 becomes as same as random selection has already verified in Fig. 2. So, these results show that power-law is not seen in agent behavior of large m. Interestingly, the histograms for agents with m=7 show an interesting mix of the two types of results; some are similar to the graphs for m=3, while others are similar to those for m=14. It can be thought that power-law can be seen only for high winning-ratio agents.

Fig. 5 shows the slopes[1] of the histograms in Fig. 4 for all agents, from that in first place to that in last place ((a) n=101 and (b) n=301). As you can see, the consistent relation can be seen that is the slope of power-law in high winning-ratio agents are near to 0 (more horizontal) than that in low winning-ratio agents.

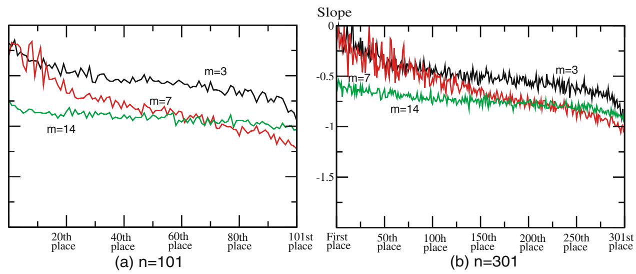

Fig. 5. Slopes of the histograms in Fig. 5

4 Mechanism of Forming Power-Law

First, we consider about (1) why winning-ratio of agent having small m becomes higher than agent of having large m and (2) why difference between the number of minority-group agent and majority-group becomes a little in small m.

4.1 Mechanism of Forming Power-Law (1)

To begin with, since the total of "0" and "1" becomes stochastically equal in each strategy table because "0" and "1" of each strategy table is stored at random, so the total of "0" and "1" that all agents select corresponding to each combination of m-past-winning-group-history must become stochastically equal too. Therefore, in each game, difference between the total of agent in minority-group and majority-group always must become a little, and after the game is repeated many times, almost all agent's winning-ratio will become approximately equal.

At this point, most biggest difference between the strategy table of small m and big m is the number of entries each strategy table has. For example, since the strategy table of m=3 has only 8 entries, the strategy table of m=14 has 16,384 entries. Next, it can be thought that even if total of "0" and "1" becomes equal stochastically, there exist various kinds of distribution of "0" and "1" in each strategy table. Here, an important point is that, for each m-past-winning-group-history one combination of distribution of "0" and "1" that each agent selects by seeing its strategy table's one of entry corresponding to the m-past-winning-group-history is fixed. And, since the expected value of occurrence of

[1] The slope is calculated by the least-squares method.

the combination like total of “0” and “1” becomes quite unequal is low, total of “0” and “1” may become equal in almost all combinations in m=3, because number of entry is only 8 in m=3. But in m=14, number of entry becomes quite big “16,384”, so the possibility of occurrence of the combination like total of “0” and “1” becomes quite unequal may become high. For MG, the combination of total of “0” and “1” of quite unequal decreases the performance of agents, so when m becomes large the average of winning-ratio of all agents must become low, because only small number of agents can be winner.

Table 1. Small m is more superior than large m

m	3	5	9	13
Average	46.6	46.5	43.2	33.7
σ	2.8	2.9	3.0	4.1

To verify this hypothesis, we calculated the average and standard-deviation of the total of “0” of all agent for each m-past-winning-group history. For one m, we executed the game under 100 kinds of strategy tables and calculated average.

Then, the following result was confirmed (see Table 1); when m became large the average of total of “0” became lower than that of small m. This means there were big difference between total of 0 and 1 in large m, and the standard deviation became also large, this means the average was not steady in each game. As a result, it can be understood that winning-ratio becomes nearly 50% steady when m is small.

4.2 Mechanism of Forming Power-Law (2)

At this point, as for the rule of scoring for strategy table, that is when an agent becomes winner the selected strategy table's score is increased 1 point and in case of loser 1 point is decreased and if probability of winning and losing is nearly equal, transition of strategy table's point under these situation can be consider as same as random-walk. It has already been known that when a certain value changes like the random-walk, the probability density of histogram of the period whose point is 0 or more follows power-law. Therefore, it can be thought that power-law has been generated since the score change of the strategy table is similar to the random-walk when m is small[2].

Then, we played MG by the following three kinds of strategy table's scoring rules. (Rule-1) Agents select strategy tables sequentially. The interval of exchange is randomly set. (Rule-2) One point is added to the score for a selected strategy table that wins, but two points are subtracted if it loses. (Rule-3) If the agent loses a single game, the strategy table is exchanged for the next-scoring table, even when the currently strategy table still has the most points. As shown in Fig. 6, extreme winner agents and loser agents were generated in all the three

[2] In Fig. 4 (a), we ran random-walk 10000 steps, and calculated the CDF of histograms of the number of steps where a positive value or a negative value was consecutive.

scoring rules, and power-law like the random walk was not generated certainly. This result means the scoring rules of 1 point increasing and 1 point decreasing are important.

5 Improvement of Winning-Ratio Using Power-Law

In Minority Game, as described above, when agents can behave efficiently power-law characteristic can be seen in their behavior, and as shown in Fig. 6, there are consistent relation between the slope of power-law and the score in each agent. And, since the parameters that generate power-law are m and the strategy table's scoring rule, if we want an agent to change its behavior without changing these parameters it only has to renew its strategy table.

Since 0 and 1 of the strategy table is stored at random, even if an agent renews its strategy table a dramatic effect might not be able to be hoped basically. However, when the strategy table of the low winning-ratio agent is renewed some effect may be expected.

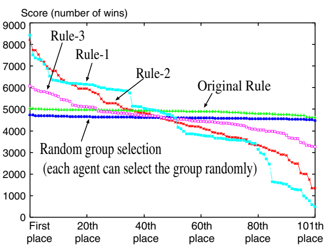

Fig. 6. Scores

Then the following experiment by which the agent who has to renew its strategy table is selected by the following three kinds of evaluation methods was done. As for the experiment, we ran the game by 101 agents and the number of strategy tables that each agent has is set to 2 (s=2). After the game is repeated 10,000 times (10,000 times is one turn), each agent's efficiency is evaluated by the following three kinds of methodologies, then both strategy tables of the selected agent are renewed and their score are initialized to 0, then next 10,000 times is started. In the experiment we repeated the renewing process 10 times that is we ran the game 100,000 times.

1. Since the agent with its power-law slope being nearly 0 has high winning-ratio, we randomly select u pieces of agents whose power-law slope become lower than the previous turn, and selected u pieces of agents renew their strategy tables. In this evaluation, since the calculation of each agent's power-law is computable only from its behavior, each agent need not know the total of agents in the game. However since this evaluation uses only the change of slope even the high winning-ratio agent may renew its strategy tables.
2. We randomly select u pieces of agents whose power-law slope become lower than the certain value (0.75), and selected agents renew their strategy tables. In this evaluation, we use the following knowledge that is, there are consistent relation between the power-law slope and the winning-ration in each agent and this consistent relation does not depend on the total of participating agents in the game. In this evaluation, each agent can know its approximate

position without knowing the total of participating agents, and agents only having low winning-ratio can renew their strategy tables.

3. We select u pieces of agents having low score, and selected agents renew their strategy tables. In this evaluation, it is necessary for each agent to know the following global information, each agent's score and the number of total of participating agent. This evaluation can be thought as the best way to select the agent to renew its strategy table. We prepared this evaluation for the comparison to above evaluation-1 and -2.

Fig. 7 shows the score of 101 agents of nine kinds of evaluation settings. We ran the three kinds of games (u=5, u=10, and u=20) for each evaluation. And as mentioned above, in each situation, strategy table renewing process was repeated 10 times and score of agents after the each renewing process was plotted to the graph of each evaluation setting. As a result, in all settings it can be confirmed that the winning-ratio has improved in all almost agents.

First of all, u = 5 was the best in all three kinds of evaluation. The reason can be thought as follows; if we select too many agents, many renewed strategy tables may destroy the current good strategy table's combination of high winning-ratio agents and thus the effect of renewing may be lost. As for the evaluation-1 and -2, evaluation-2 was better than evaluation-1 as we expected because in evaluation-2, only the agents having low winning-ratio are able to renew their strategy tables.

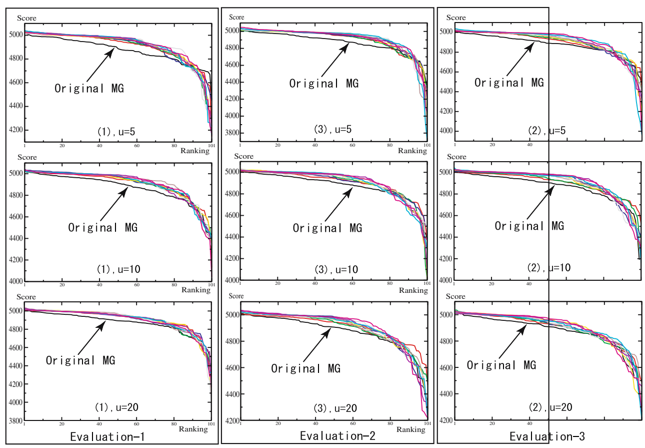

Fig. 7. Score of 9 kinds of evaluation settings

Consequentially, evaluation-2 was the most effective though it had been expected that evaluation-3 was at first most effective. While the evaluation-3 uses the several global information, it pays attention only to the score transition of agent. On the other hand, the evaluation-1 and -2 pay attention to the agent behavior. So, in the following two situations, "the score is good by chance even if the evaluation of behavior is bad" and "the score is bad by chance even if the evaluation of behavior is good", agent should not renew its strategy tables in the former situation, and should renew them in the latter situation, thus this experiment result shows paying attention to agent behavior is effective as the evaluation methodology.

6 Conclusion

Multi-agent system is a network where one agent is one node, and there are close relations between behavior of each agent and the group of agents as a whole, so a certain information about the state of each agent may be hided in each agent behavior. In Minority Game, such information to recognize agent state was power-law. Power-law is the important feature to characterize the behavior of the node and it is interesting that power-law is seen in the behavior of the node on various kinds of networks. In this paper, we verified whether the efficiency of each agent behavior was able to be improved only by seeing its behavior by checking whether power-law is seen in its behavior or not in Minority Game, and confirmed that the performance gain was actually possible. It can be expected that power-law is the important factor to characterize the state of a system, especially in the following systems; multi-agent system for the artificial economic market, collective intelligence, swarm intelligence, and ubiquitous information communication systems, etc. In the future work, we plan to apply this analytical technique to an actual system like the ubiquitous information communication system and to verify whether an efficient performance improvement is possible.

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Overlay Based Mapping Egress Service Path Between MPLS Domains

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Abstract. It is a critical issue for interdomain service routing across MPLS domains to map egress service path. In spite of being a de facto interdomain routing standard, BGP can not effectively fulfil the interdomain traffic engineering. To solve the problem, we set up an overlay model, and formulate the problem as a Markov Decision Problem. According to the model, we get related network resource state through service nodes measurement on a common measurement platform. Using service state, we design related cost function. To simplify the problem, we decompose the Markov Decision Problem, and design a heuristic proportional mapping algorithm.

1 Introduction

Service routing is an overlay network routing, which provides corresponding service capability and setup according to traffic engineering requirements. For example, we often need to setup VPN services across different MPLS network domains. Next Generation Networks (NGN) is expected to support services across different networks with diverse requirements. It brings great challenges to the MPLS networks, which will be the core networks in NGN.

To set up service path between MPLS domains, we must find the egress path. In this context, destinations for interdomain traffic may be reachable through multiple egress routers within a domain. According to IEEE RFC [1], traffic engineering (TE) deals with the issue of performance evaluation and performance optimization of operational IP networks and encompasses the measurement, characterization, modeling and control of the Internet traffic. So the objective of egress service path mapping for service routing between MPLS domains is twofolded: to find an egress for feasible service path; to optimize the usage of the network resource by balancing the load according to traffic engineering.

Since the Border Gateway Protocol (BGP) [6] is a de facto standard for interdomain routing, which determines routes for packets that cross multiple autonomous systems. It can be also used to find the egress service path. As a path vector routing protocol, BGP is similar to any other distance vector routing protocol that doesn't take into account service metrics. Its criteria for selecting the

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best path are based on the length of AS path. Due to the complexity of service routing, BGP hardly support different service routing.

Because MPLS domains are owned by different network providers, they have different service providing methods and policies. At the same time, different MPLS domains lack common languages to understand each other. Therefore, the core of the problem is how to map service capacity between two MPLS domains which don't know much about each other.

To address the problem, we must decompose function from routers. we introduce an innovative idea—service semantic P2P overlay network to open the underlay network capability. It organize the resources of service routing across different network domain based on ontology. Based on the overlay architecture, we design related formalization model and algorithms.

The rest of the paper is organized as follows. In section 2, we introduce the related works. In section 3, we set up the overlay model and formulate the problem. In section 4, we analysis the method to get interdomain network information based on overlay model. In section 5, we set up cost function according to service state. In section 6, we decompose the Markov Decision Problem, and design a distributed heuristic proportional mapping methods. Finally, a conclusion is reached.

2 Related Works

Many methods to setup interdomain routing are based on BGP. RFC 2547 [5] setup MPLS VPN based on BGP. A. Farrel, J. P. Vasseur, andY. Ayyangar [14] propose a framework for interdomain MPLS Traffic Engineering, and specify signaling (RSVP-TE) and path computation mechanisms for setting up inter-AS service routing. Bonaventure [2] focuses on how to distribute flexible QoS information by BGP in different network scenarios. Cristallo and Jacqenet [12] propose a new attribute, the QoS_NLRI (Network Level Reach ability Information), for BGP UPDATE message to carry QoS information. Bressoud [11] determines an optimal selection of outgoing links and associated border routers, where the selection optimizes the ISP's network resource utilization. However, these solutions can only select an egress router based on prefix reach ability and the egress link capacity information, without knowing whether the selected egress can satisfy the traffic flow's end-to-end traffic engineering requirement and the neighbor domains accept the service.

BGP can not resolve all the problem related to traffic engineering. Recently, overlay network is proposed to decompose the functions from routers [7] [8]. Xiaohui Gu of University of Illinois gave a framework to composite the QoS-aware service for large-scale peer-to-peer systems [9]. These solutions try to achieve interdomain TE by allowing explicit routing on the overlay. However, they do not possess the status of the underlying network, and have no control to routing on the underlay. Due to these reasons, we must open the underlay network capability to setup service routing.

To open network capability, Parlay/OSA [4] has been proposed. Through Parlay API, MPLS traffic engineering capability can be mapped up. All kinds of traffic engineering requirements can be computed in different Parlay service capability servers. To provide a distributed environment, service convergence and virtualization is the key for setting routing across heterogeneous MPLS domains. Parlay can't provide such kind of capability. OGSA (Open Grid Services Architecture) [3] is such service convergence and virtualization platform. Based on concepts and technologies from the Grid and Web Services communities, OGSA defines a uniform exposed service semantic (the Grid service); defines standard mechanisms for creating, naming, and discovering transient Grid service instances; provides location transparency and multiple protocol bindings for service instances; and supports integration with underlay native platform facilities. OGSA also defines interfaces, associated conventions and mechanisms.

Different MPLS domains need a common language to understand each other. To solve the semantic problem between different MPLS domains, Semantic Overlay Networks (SONs) [10] is proposed to be a way to make that MPLS domains can understand each other by grouping peers sharing the same schema information.

Studying these technologies to realize service routing across different MPLS domains is worth. We can combine these technologies to solve the MPLS interdomain egress path problem.

3 System Model

3.1 Overlay Model

To decompose functions from routers, we design an overlay model (fig. 1). At first, we use Parlay API to open the MPLS network capability, which include connectivity management and policy management. To get network resource status between different MPLS domains, we use a common network measurement platform. In the model, we decompose the function of each MPLS border node to grid service node. For each MPLS border node, relative service nodes are setup. These service nodes decompose the functions from both MPLS border nodes and Parlay Gateway, and provide a distributed computation. These capabilities opened are wrapped as Grid Services, which can effectively manage MPLS network resources and their states. On the foundation, we setup a single semantic image—ontology. Ontology [13] is a formal, explicit specification of a shared conceptualization. In each MPLS domain, we map egress service path based on semantic through computing cost and relative egress resource. Based on ontology, we can get, describe, express and compose services to setup service routing across different MPLS domains and other networks. At the same time, different service relationships can be setup through semantic. In semantic grid level, different MPLS domains and other access networks can understand each other through ontology, and become a transparent homogeneity network. In such a homogeneity network, we can setup P2P service routing.

3.2 Problem Formulation

For MPLS domain AS under consideration, we are given a set of neighbors $A_1, \ldots, A_q$ and a set of Grid service pipe $L_1, \ldots, L_n$ between ingress-egress

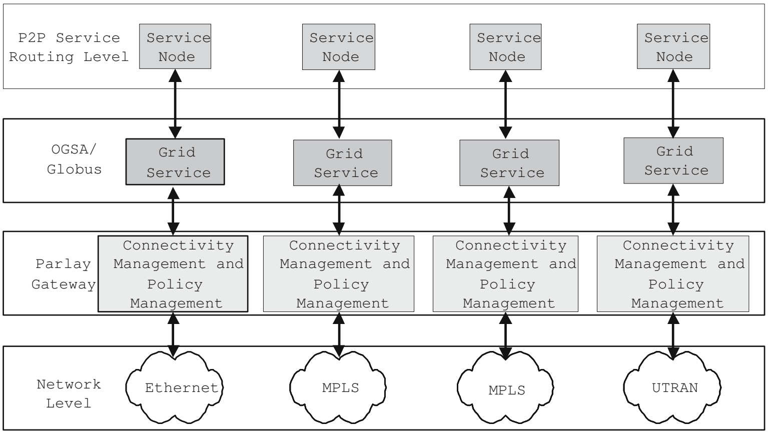

Fig. 1. Overlay Model

node pairs. Grid service pipes are instances of grid services which wrap the traffic trunks in MPLS domains between ingress nodes and egress nodes. There may be more than one path between two neighbors. Service traffic is carried between the AS under consideration and its neighbors through these ingress-egress node pairs.

The external BGP peering session at the border routers receives advertisements for network prefixes across the border nodes. Let $P_1, \ldots, P_m$ denote the set of prefix advertisements received across all border nodes. Traffic from neighbor A_h ingressing through border node b_i may be measured or estimated through a service network resource measurement platform. Let $t(h, i, k)$ denote such kind of traffic. The egress node set $\{j\}$ is determined by measurement function $\delta(h,i,k,s)$, which is measured according to the neighbor A_h, ingress node i, the destination P_k of egress outing data traffic, and service type s.

We formulate our problem as a partially observed Markov Decision Process (MDP). Given the call of service type s, let $x_s^{ijl}(t)$ be the number of the calls of service type s using Grid service pipe l between service node i and j at time t. Formula (1) denotes the vector containing the number allocated to the different grid service pipes taken by calls of service type s at time t.

$$x_s(t) = \{x_s^{ijl}(t)\}^T \tag{1}$$

Formula (2) contains the vector of different call types and respective grid service pipes at time t.

$$x(t) = [x_1^T(t), x_2^T(t), ..., x_m^T(t)] \tag{2}$$

We define the one-step cost function at state $Y(t)$ and control action $\vec{u}(t)$ by

$$Cost_s^{ijl}(Y(t), \vec{u}(t), Y(t + \Delta t)) \tag{3}$$

The optimization objective function is formula (4), which is subject to the network constraints and traffic engineering constraints. We assume $E[x_s^{ijl}(t)]$ is the expected value of $x_s^{ijl}(t)$ determined by the traffic model of the service type s.

$$\min \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{s=1}^{S} Cost_s^{ijl}(Y(t), \vec{u}(t), Y(t + \Delta t)) * E[x_s^{ijl}(t)] \tag{4}$$

The formula (5) shows for every egress that the intra domains capability can not be exceeded. C_{ij}^{intra} is the capability of between the service nodes i and j within the MPLS domains.

$$\sum_{s=1}^{S} \sum_{l=1}^{L} x_s^{ijl}(t) * t(h, i, k) \leq C_{ij}^{intra} \tag{5}$$

The formula (6) shows for every egress that the inter domain links capability can not be exceeded. C_j^{inter} is the interdomain links capability in the neighbor domains of egress node j.

$$\sum_{i=1}^{I} \sum_{s=1}^{S} \sum_{l=1}^{L} x_s^{ijl}(t) * t(h, i, k) \leq C_j^{inter} \tag{6}$$

The formula (7) shows for every egress that the related service capability of the neighbor domains can not be exceeded. $X(s)_j^{inter}$ is the related service capability of service ontology type s in the neighbor domains for the egress.

$$\sum_{i=1}^{I} \sum_{l=1}^{L} x_s^{ijl}(t) * t(h, i, k) \leq X(s)_j^{inter} \tag{7}$$

At the same time, it also includes other traffic engineering constraints.

4 Inter Domain Information Acquirement

To get the necessary information to determine the egress service node for the service path is very important. Measurement is a direct way to get such kinds of information. Our overlay model does not require each node to perform traffic measurement. Measurement is performed on a common measurement platform. Only the ingress and the egress service nodes are required to participate in the measurement process. The measurements include network measurement and service measurement.

The network measurement is provided by ingress grid service nodes to measure service path. The measure path is from the ingress service nodes to egress service node, and then to the neighbor domains' ingress service nodes. At first, we need use BGP to get the reachability information. AS (Autonomous System) interconnect via dedicated links and public network access points, and exchange routing reachability information through external BGP peering sessions. BGP allows import and export policies to modify the routing decision from the shortest-path default. The unit of reachability provided by BGP is the network prefix, which is an aggregation of IP addresses in a continuous block. A route advertisement indicates the reachability of a network. On the basis of the BGP routing table, the BGP decision process will select the best route toward each known network.

For the purpose of balancing the loads among grid service pipes, the available bandwidth appears to be a desirable metric to measure. However, it is not easily measured. Packet delay is a metric that can be reliably measured. The delay of a packet on a grid service pipe can be obtained by transmitting a probe packet from the ingress service node to the egress service node, and then to the domain which we want to measure. Packet loss probability is another metric that can be estimated by a group of probe packets. Packet loss probability can be estimated by encoding a sequence number in the probe packet to notify the egress node how many probe packets have been transmitted by the ingress node, and another field in the probe packet to indicate how many probe packets have been received by the egress node.

Service measurement is also an import issue. In each MPLS domain, all opened service capability, are registered in grid register server based on semantic. According to the semantic, each MPLS domain can sent service measurement requirement to neighbor domains. When a neighbor MPLS domain receive such kinds of requirement, it will response according to its policy.

5 Cost Computations

To allocate resource for service routing between MPLS domains, the key is the dynamical cost computation for different service states. We formulate our dynamic policy by considering a request for a new grid service pipe setup with network status and priority set P.

When the resource is enough, we only need consider the setuping priority to determine the order in which path selection is done for traffic trunks at the connection establishment and under fault scenarios. Through network status provided by measurement, we can compute the setuping cost with following formula:

$$SC_l(S,t) = \sigma_1 \cdot P^l_{loss} + \sigma_2 \cdot P^l_{delay} + \sigma_3 \cdot SP(S) \tag{8}$$

P^l_{loss} is the probability of packets dropped on grid service pipe l during the (t, t+1) period. It reflects the switch capacity of the LSR in two border nodes. P^l_{Delay} is the delay of Grid service pipe l during the (t, t+1) period. The cost will increase with the increase of P^l_{Delay}. *SP(S)* is the setuping priority of the service ontology type. The priority is determined by the services' SLA between service providers and customers. σ_1, σ_2, σ_3 is the relative weighting coefficients of P^l_{loss}, P^l_{Delay} and $SP(S)$. Their values reflect the importance of these factors.

If the resource is not enough, we must consider holding and preempting priority. Only when the preempting priority of the service is higher than the holding priority of the preempted service, the resource can be allocated. When a set of grid service pipes is chosen to be preempted, these grid service pipes will be tore down and could be rerouted through extra signaling and routing decisions.

In order to avoid or minimize rerouting, we propose to reduce the number of preempted grid service pipes by selecting a few low-priority grid service pipes. After a grid service pipe is selected for rate reduction, there will be signaling to inform the originating service node of the rate reduction. In the future, whenever it exist available bandwidth in the network, the lowered-rate grid service pipe would fairly increase their rate to the original reserved bandwidth.

After setting up the service routing, the holding priority is higher than the setuping priority. Then, we use a simple linear relation to define the holding cost, where h is less than 1.

$$HC_l(S,t) = h * (\sigma_1 \cdot P^l_{loss} + \sigma_2 \cdot P^l_{delay} + \sigma_3 \cdot SP(S)) \tag{9}$$

Preempting cost is related to the preempting priority of the service and network status. To have the widest choice on the overall objective that each service provider needs to achieve, we define the following objective function, which is chosen as a weighted sum based on the above-mentioned criteria:

$$PC(S,t) = \alpha_1 \cdot n(l) + \alpha_2 \sum_{i=0}^{k} b(S,t,i)/B(S,t,i) + \alpha_3/Pp(S) \tag{10}$$

$n(l)$ represents the number of preempted grid service pipes. If the number of preempted grid service pipes is larger, the affected service path will also become larger. $b(S,t,i)$ represents the bandwidth of pipe i which will be preempted. $B(S,t,i)$ is the original bandwidth of pipe i. We also wish that the preempted resource is little. $\sum_{i=0}^{k} b(S,t,i)/B(S,t,i)$ is the sum of preempted resource ratio. When the value is larger, the cost will also become larger. $Pp(S)$ represents the preemption priority of preempted grid service pipes. Coefficient α_1, α_2, and α_3 are the values that can be configured in order to stress the importance of each component.

6 Algorithms for Egress Selection

6.1 Decomposition of MDP

To simplify the problem, we decompose the Markov Decision Problem. We assume the service arrivals are determined by their traffic models. Their state distributions are statistically independent. These assumptions are commonly made in network performance analysis. In particular, they imply that a call for a path consisting of grid service pipes is decomposed into independent grid service pipe calls characterized by the same mean holding time as the original call. Then the Markov process for a given policy π can be described separately for each grid service pipe, which is in terms of the grid service pipe state $x = \{x_j\}$ and the transition rates defined by service arrival rates $\lambda^s_j(x,\pi)$ and departure μ_j. These parameters are estimated based on some sample statistics measured using above common measurement platform.

For a given routing policy, each grid service pipe penalty process can be described independently by the set $\{\lambda_j^s(x,\pi),\mu_j\}$. Then we can compute cost according to these parameters.

6.2 Distributed Heuristic Proportional Mapping

According to the decomposition of MDP, we design a distributed method to map service path between MPLS domains in algorithm 1. At first, we confirm the service constraint of formula (5), (6), (7) and other traffic engineering. We assumed that all paths between a source and a destination are disjoint and their capabilities are known through measurement. In practice, paths between a source and a destination have shared links. These paths may also share links with path between other source-destination pairs.

To make a decision whether a service path from its domain to destination neighbor domains, we must consider all service which are carried to the egresses of the neighbor domains. To simplify the problem, we use heuristic proportional method to determine. Then we design a decision function to determine select which egress in formula (11).

Algorithm 1. Distributed Heuristic Proportional Mapping

DHPM(a, b, s)
{
  If a service routing request access
  {
    $min = 0$
    get service ontology kind
    get the egress set $\{(i,j)\}$ for the service s, where $j \in P_k$
    for each egress (i,j)
    {
      (1) It must satisfy $\sum_{s=1}^{S}\sum_{l=1}^{L} x_s^{ijl}(t) * t(h,i,k) \le C_{ij}^{intra}$
      (2) It must satisfy $\sum_{i=1}^{I}\sum_{s=1}^{S}\sum_{l=1}^{L} x_s^{ijl}(t) * t(h,i,k) \le C_j^{inter}$
      (3) It must satisfy $\sum_{i=1}^{I}\sum_{l=1}^{L} x_s^{ijl}(t) * t(h,i,k) \le X(s)_j^{inter}$
      (4) It satisfies other traffic engineering constraints.
      (5) find minimization $Predictedcost(i,j,l,s)$
      (6) if $Predictedcost(i,j,l,s)$ < min
          $min = Predictedcost(i,j,l,s)$
    }
    If $min > 0$
      Accept the service routing mapping
    else
      Reject the service routing mapping
  }
}

$$C_s^{ijl}(t) = \frac{Cost_s^{ijl}(Y(t), \vec{u}(t), Y(t+\Delta t)) * E[x_s^{ijl}(t)]}{\sum_{s=1}^{M} Cost_s^{ijl}(Y(t), \vec{u}(t), Y(t+\Delta t)) * E[x_s^{ijl}(t)]} * x_s^{ijl}(t) \quad (11)$$

To avoid fluctuating too much, we use an exponentially weighted moving average (EWMA) method. This is, $Predictedcost(i, j, l, s) = \beta * C_s^{ijl}(t) + (1\text{–}\beta)$. When a service pipe appears for the first time, we will directly use its cost in the current interval to predict in the next interval (since it does not have any other history yet).

According to the predicted cost, we can determine the proportion of resource assignment between different service nodes, accept and reject different service routing in service pipe model. The assignment will support the service ontology mapping to accept service routing based on service ontology in pipe model.

7 Conclusions

The service routing across different MPLS domains is difficult. The most important thing is to open the underlay network capability. We propose a new model – P2P Semantic Overlay, which decompose the function of router into the service node through grid service. The model use Parlay gateway to mapping MPLS service capability. The service status is gotten through a common measurement platform. The model also including egress path computing. At the same time, we formulate the problem, and propose a MDP model. At last, we decompose the problem and propose a new algorithm. The algorithm is a distributed method which will greatly reduce the computation complexity. It will be a new method to solve the service routing problem based on overlay model by combining semantic grid. Using traffic model to assign network resource in internet is also a trend.

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An Adaptive Group-Based Reputation System in Peer-to-Peer Networks*

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Abstract. As more and more P2P applications being popular in Internet, one of important problem to be solved is inspiring users to cooperate each other actively and honestly, the reputation mechanism which is a hot spot for P2P research has been proposed to conquer it. Because of the characters of virtuality and anonymous in the network, it is very easy for users with bad reputations to reenter the system with new identities to regain new reputations in the reputation systems. In order to get rid of the impact of whitewashers and improve the system performance and efficiency, we propose a new probability-based adaptive initial reputation mechanism. In this new mechanism, newcomers will be trusted based on system's trust-probability which can be adjusted according to the actions of the newcomers. To avoid the system fluctuating for actions of a few whitewashers, we realize the new reputation mechanism in system with group-based architecture, which can localize the impact of whitewashers in their own groups. Both performance analysis and simulation show that this new adaptive reputation mechanism is more effective.

1 Introduction

Oram [16] gives a simple definition of peer-to-peer (P2P) networks as: "P2P is a class of applications that take advantage of resources storage, cycles, content, human presence available at the edges of the Internet". As large number of P2P applications, such as Gnutella [1], BitTorrent [2], Skype [3], etc., being popular in Internet, they attract a lot of users and system designers.

According to the research of the running P2P applications [4][5][6], a lot of drawbacks of the real P2P systems have been disclosed that performance of the most P2P systems can't reach or even be proximal to the expectation of users and system designers. The major reason is lacking of the effective cooperation mechanism inherently in the P2P systems, so not all participators can be encouraged to take part in the systems actively and friendly. When a P2P user

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tries to download a file from another user in the same application, he may worry about the virus or attack embedded in that file; the user shares resources with others but who do not; and so on. All of these risks destroy the trust among the system users, so that the users will remind themselves more careful when they take actions in the system full of hazards, which holds back the users' footstep to cooperate with others.

It is an effective solution to construct the reputation system in the P2P networks to build up the trust among the users. But the anonymousness and inherent virtuality of the Internet hamper the implementation of the practical reputation system. In a glimpse of security technologies, it is still difficult to map the user identities of Internet into their social identities uniquely, which means that users can easily get on the Internet with cheap pseudonyms. So the users can reenter the network system by changing their network identities to get new reputation values to avoid the penalty imposed on them, which can't be identified from the fresh users to the network. We call the former "whitewashers" and "newcomers" for the latter [8]. There is an important problem to be solved that how to limit the whitewashers' evil behaviors and encourage the newcomers' beneficial actions through the reputation system, which can make the P2P networks more cooperative and effective. The users' initial reputation values in the reputation system have impacted their behaviors. To make the P2P networks more cooperative, some researchers [7] have proposed radically to trust all the users initially when they enter the network, so everyone can trade with others rapidly and extensively. But this solution nourishes the whitewashers, which can get new good reputation and do harm to the performance of P2P networks. So others [7] advise conservatively trusting nobody initially at the beginning of users' participating to the network, which intents to eliminate the whitewashers by imposing the serious penalty on them. But in reality, the useful behaviors of newcomers have also been unfairly restricted in the case of this strategy. It will take newcomers very long time to cumulate enough reputation values to take part in the cooperation in the network, which decreases the network efficiency seriously in the case of P2P users joining and departing the network with high frequency.

A new group-based probability initial reputation system has been proposed in this paper to enhance the network cooperative efficiency and block the whitewashers' destruct behaviors adaptively. When joining the network, every user is assigned a new reputation value based on a probability indicated by a system argument, which is called Initial Trust Probability (abbr. ITP) adjusted according to the behaviors of all the new users. In addition, to avoid the shake of whole network impacted by a few evil users in a certain local, it doesn't use a globe argument of ITP for new users' reputation values, but set the one in each local where a group is built up to maintain all the users in coverage of this local. In this group-based reputation system, the impact of whitewasher can be effectively limited in the area of group he belongs to; furthermore, the huge overhead used to maintain the globe argument can be decreased largely and dispersed into every group in network.

The remainder of this paper is organized as following: Section 2 describes the proposed adaptive group-based reputation system in detail, which is organized into two subsections named group-based reputation architecture and adaptive initial reputation mechanism respectively. Theoretical analysis and simulation results to the performance of the new reputation system are given in section 3 and 4, respectively. Finally, section 5 concludes the paper.

2 An Adaptive Group-Based Reputation System

It is very difficult to distinguish the whitewasher from newcomer in the P2P network, and the impact of whitewasher can not be eliminated completely. If we trust all new users including whitewashers and newcomers without doubt, the efficiency of the network can be improved in the case of fewer whitewashers, but the system runs high risk of attacks with more whitewashers; otherwise, if we don't trust all the new users completely, the risk can be cut down, but the system cooperative efficiency may be decreased serious especially with fewer whitewashers, so the best solution should be designed based on tradeoff between restricting the whitewasher and encouraging newcomer.

Based on the above analysis, we propose an adaptive initial reputation mechanism under the group-based reputation architecture, which attempts to reach that tradeoff. In this method, the new users will be assigned good reputation values with high probability (i.e. Initial Trust Probability, ITP) when there are fewer whitewashers in the P2P networks; as the number of whitewashers has been detected to be increasing, the ITP is decreased corresponding to alleviate the serious impacts of the whitewashers. In order to decrease the overhead used to maintain the globe argument in the whole network and confine the impact of whitewasher within small area, we design a group-based reputation architecture to realize the adaptive initial reputation mechanism, in which every user is organized into an unique group in a certain rule, such as in the rule of physical topology [10], interest-based locality [11][14], etc.. Furthermore, the relationship among the users can also be organized into a hierarchy [12]. In the paper, we design this architecture in the complete distributed style, and each group maintains an individual ITP to assign the reputation to every covered new user.

2.1 Group-Based Reputation Architecture

As showing in the Fig.1, in the group-based reputation architecture, all the users are organized into groups. In simplicity, we assume that one user belong to only one group (in the case of the user belonging to more than one group, the user can be looked as joining the different group with different identity). Each group is assigned a unique identity called group id (GId), and each user in each group is also assigned an unique identity locally called member id (MId), so every user can be identified uniquely by combining his MId with GId. Furthermore, an individual ITP must be maintained consistently by the members in each group.

In each group, all the members contribute a part of their storage to cooperatively store and maintain the reputation information set which consists of

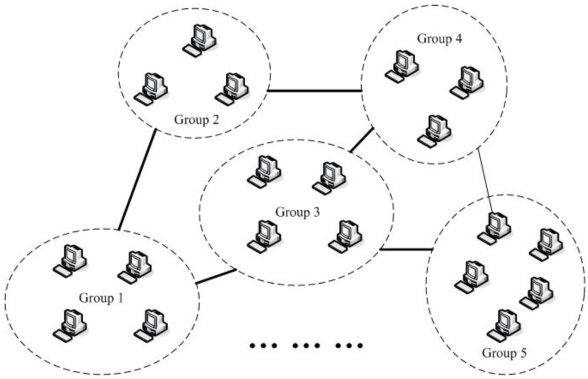

Fig. 1. The group-based reputation architecture in the P2P networks can be organized by taking into account the physical topology, interest locality, and so on

member id (MId), user id (UId), number of action and reputation value for each member. The UId is registered by user to indicate his physical identity such as IP address, MAC address, which can be realized by underlying technologies and used to encourage the user to reenter the network with same UId to get his reserved reputation value. The number of action is used to determine whether the user is new user or old user. The reputation value is the indication of the user's reliability. This reputation information can be located in the common storage through the distributed hash table (DHT).

When the user wants to cooperate with others, he firstly checks the reputation value of cooperator through the group-based reputation system, and then determines whether to do. For example, we consider the cooperation between user A and user B . If two users are in same group, user A can get the reputation information of B through DHT; otherwise if two users are in different groups, user A must search for B's group and retrieves his reputation information from it. User B does the same as A. At the end of cooperation, user $B(A)$ reports the behavior of $A(B)$ to $A(B)$'s group members to let them calculate the reputation for $A(B)$ according to it.

2.2 Adaptive Initial Reputation Mechanism

Based on the group-based reputation architecture described above, we farther propose a probability-based adaptive initial reputation mechanism attempting to reach the point of tradeoff between restricting the whitewasher and encouraging newcomer.

We set one initial trust probability (ITP) argument which is maintained by all the members in each group, and the group assigns the new user with new reputation value based on the ITP which should be adjusted according to the new user's behavior.

We consider that the whole P2P networks is organized into N groups numbered by $G_i = 1, 2, 3 \ldots N$, and there is an ITP denoted by P_i in each group G_i, further r denotes the user's reputation value which consists of two parts of value field denoted by $[0, R_u]$ and $[R_u, R_t]$, the user is in the state of trust.

When a new user A joins a group G_i, the three members with highest reputation value in this group respectively, which means they may work in correct way with highest probability, calculate A's reputation value T_j^A based on P_i, then the final reputation value r^A is deduced by the member with highest reputation integrating the three calculations. This mechanism can effectively avoid the moral risk of single referee which may judge selfishly based on its own bias. But too many referees may induce the more complexly distributed computing. The method is detailed as following:

1. The three members (referees) in group G_i calculate the reputations for the new user A respectively.
   - $p_j = random(0, 1), j = 1, 2, 3$
   - if $(p_j <= P_i)$, then $T_j^A = 1$, i.e. the new user is trusted by referee j; if $(p_j > P_i)$, then $T_j^A = 0$, i.e. the new user isn't trusted by referee j.
2. The referee with highest reputation value calculates the final reputation for the new user A by integrating the above three calculations.
   - $T^A = (T_1^A \bigcup T_2^A) \bigcap (T_2^A \bigcup T_3^A) \bigcap (T_1^A \bigcup T_3^A)$, i.e. the new user A will be trusted in the case of at least two referees trusting him.
   - if $(T^A = 1)$, then $r^A = random(R_u, R_t)$; if $T^A = 0$, then $r^A = random(0, R_u)$.

The user B reports the behavior of user A to A's group, then the user with highest reputation adjusts the ITP P_i according to it. The principle to adjust is as following:

1. P_i should be increased as the probability of normal action of the new user;
2. When the new user behaves correctly, P_i should be increased with small span in order to keep the whitewasher within limits;
3. When the new user behaves incorrectly, P_i should be decreased in large span which is scaled as increasing the number of uncooperative behaviors to protect the system from attacks;

Furthermore, the ITPs of neighbor groups may be useful for the group, so the group G_i periodically collects its neighbors' ITPs $P_j, j = 1, 2 \ldots$, and then uses them to calculate its ITP by some weights, for instance, the calculative formula is $P_i = w_i \cdot P_i + \sum_j (w_j \cdot P_j)$, s.t. $w_i + \sum_j w_j = 1, w_i, w_j > 0$.

In simplicity, all w_j can be same and w_i can be set larger in order to make the impacts of the neighbors weaker.

3 Performance Evaluation

Before analyzing the performance of this two architectures, we firstly define the mathematical symbols which would be used in the following analysis:

n the total number of users;
p_c the average probability of normal behavior of each new user;
p_{tr} the real trusted probability of each user;
P_i the initial reputation probability of user i's group;
μ_n the Average System Utility Gain (ASUG) contributed by one normal action, $\mu_n > 0$;
$-\mu_d$ the ASUG contributed by one abnormal action, $\mu_d > 0$;

The average whole system utility gain is evaluated as following in three different mechanisms.

1. **The mechanism of distrusting all the new users**
All the new users will be distrusted, so it's difficult for them to cooperate with others, which contributes mostly nothing to the system, so the system gain is $U_1 = 0$.
2. **The mechanism of trusting all the new users**
All the new users are trusted without doubt in this mechanism, so they can fully cooperate with others, which contribute to the performance of system. We calculate the average system utility gain by taking whitewasher and newcomer into account. The average contribution is:

$$\begin{aligned} U_2 &= n \cdot \int_0^1 [p_c \cdot \mu_n + (1 - p_c) \cdot (-\mu_d)] \cdot d(p_c) \\ &= n \cdot \int_0^1 [p_c \cdot (\mu_n + \mu_d) - \mu_d] \cdot d(p_c) = \frac{1}{2}(\mu_n - \mu_d) \cdot n \end{aligned} \quad (1)$$

As showing in the Equation.1, if $\mu_n > \mu_d$, i.e. the gain made by a user's normal action is bigger than absolute value of the one by his abnormal action, then the ASUG is positive; in the contrary, the ASUG is negative if $\mu_n < \mu_d$, which disrupt the system performance.
3. **Adaptive initial reputation mechanism**
In this mechanism, p_{tr} of user i is calculated by the mechanism of three referees: $p_{tr} = 3 \cdot (1 - P_i) \cdot P_i^2 + P_i^3$. Furthermore, P_i should be increased with the p_c, denoted as $P_i = f(p_c)$. To be simple, we assume $f(p_c) = p_c$, so the average contribution made by one new user i is:

$$\begin{aligned} U_3 &= n \cdot \int_0^1 p_{tr} \cdot [p_c \cdot \mu_n + (1 - p_c) \cdot (-\mu_d)] \cdot d(p_c) \\ &= \frac{7}{20}(\mu_n - \frac{3}{7} \cdot \mu_d) \cdot n \end{aligned} \quad (2)$$

If $\mu_d < \frac{7}{3} \cdot \mu_n$, then the ASUG is positive; otherwise it is negative if $\mu_d > \frac{7}{3} \cdot \mu_n$.

Comparing the Equation.1 with the Equation.2, the third mechanism can reach the better ASUG when , which condition can be satisfied in general because destruct impact of the abnormal action always is experientially larger than the constructive impact of the normal action.

4 Simulation Results

In this simulation, we only concentrate on the effect of adaptive initial reputation mechanism for all new users who will be deleted after its acting for k times during system running. The ITP P_i in each group is adjusted in the following rules:

1. $P_i = min\{P_i + \frac{P_i}{m_i \cdot k}, \overline{P}\}$, when the new user acts correctly. m_i is the number of users in group G_i, and $\overline{P}$ is the upper bound of P_i. P_i should be increased in slow step to avoid system collapse easily due to evil attack.
2. $P_i = max\{\underline{P}, P_i - a \cdot (1 - P_i)\}, 0 < a < 1$, when the new user acts incorrectly. a is a coefficient, and $\underline{P}$ is the lower bound of P_i. When the abnormal action is detected in first time, it is may be occasional, so we can decrease the P_i in small span to keep the degree of the cooperation; and then the decreasing span is increased as the times of the abnormal action, which can limit the abnormal user and protect the system from being attacked.

The reputation value of each user is adjusted in the following rules:

1. $r_i = min\{r_i + b, \overline{R}\}$, when a new user acts correctly. b is a constant and $\overline{R}$ is the upper bound of the reputation value;
2. $r_i = min\{r_i/2, \underline{R}\}$, when a new user acts abnormally. $\underline{R}$ is the lower bound of the reputation value.

Table 1. The major parameters and their values in the simulation

Parameter	Value
Number of groups N	50
Average number of member in each group m_i	50
Probability of joining of a new user	0.5
Probability of abnormal action of a new user p_c	$0.1 \cdots 0.9$
Upper bound of reputation value $\overline{R}$	100
Lower bound of reputation value $\underline{R}$	5
Upper bound of ITP $\overline{P}$	0.95
Lower bound of ITP $\underline{P}$	0.05
a	0.5
Simulating time	60 s

The major parameters of the simulation are listed in Table.1. And the simulation result is showed in the Fig.2.

We can compare the three mechanism in the simulation result as follow:

1. In the case of low probability p_c of abnormal action, the total ASUG of "Trust all" has hit the highest point because it incentives all the users to

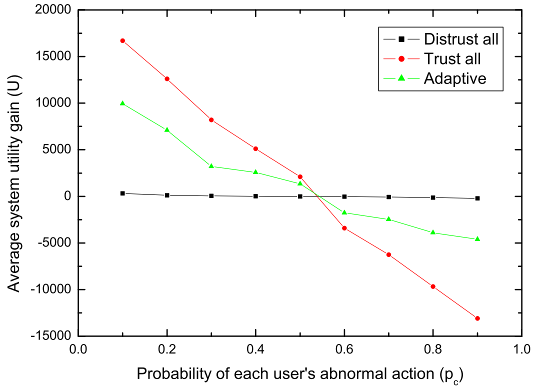

Fig. 2. The average system utility gains in the three different initial reputation mechanisms, the "Adaptive" mechanism's performance is between the other two ($\mu_d = \mu_n = 10$)

cooperate completely with each other. But as p_c increasing, the ASUG of "Trust all" is decreased sharply, and reaches the lowest point because "Trust all" also give the complete freedom to the abnormal user to attack the system and not stop them.

2. Regardless of p_c, "Distrust all" has not almost contributed any ASUG because no user can be trusted which prevents the cooperation, so "Distrust all" leads to worst efficiency.
3. In the case of low p_c, the performance of "Adaptive" mechanism is significantly better than the "Distrust all", but worse than the "Trust all" because the "Adaptive" doubt all the new users with some degree, which cannot make all them cooperate fully in this condition; however, in the case of high p_c, the "Adaptive" is worse than the "Distrust all", but largely better than "Trust all" because it restricted a large number of abnormal actions. In totally, the average whole gain of the "Adaptive" is 16% larger than the one of the "Trust all".

Totally, the performance of "Adaptive" is between "Trust all" and "Distrust all". Ideally, the ASUG of "Adaptive" should be close to that of "Trust all" when p_c is low and "Distrust all" when p_c is high, because "Adaptive" need to adjust ITP according to users' action at expense of system utility. But, it can adapt to any value of p_c.

5 Conclusions

In this paper, we propose a group-based adaptive reputation system to limit the whitewasher's behaviors and improve the performance of the P2P networks effectively. The globe fluctuation of the P2P networks can be avoided under the group-based reputation architecture where the impact of whitewasher can be limited locally in his group. Through the evaluation and simulation, it is demonstrated that the average system utility gain can be improved by at least 16 percent comparing with other two mechanisms. Through the proposed mechanism is proved to be effective, the some parts of it can be further researched in the future, for example, is it reasonable for the method of adjusting the initial reputation probability of group? Is it fair for the rules of calculating the user's reputation? All these problems have important impacts on the system performance and should be solved more rationally and effectively.

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A Comparative Study on Marketing Mix Models for Digital Products*

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Abstract. The rise of the Internet and electronic commerce provide a huge marketspace and unique transaction process for digital products. It is significant to discuss whether established marketing models can be revised for digital products. First, the unique features of digital products are systematically reviewed, and then three typical digital products, including e-books, anti-virus, and online translation services, are analyzed and compared utilizing three established marketing models, including 4P, 4C, and 4S. We find that these marketing mix models have different suitability for three typical digital products. The intention of this paper is to provide a reference for enterprises in selecting marketing mix model according to product's categories and to provide a marketing strategy tool kit.

1 Introduction

The integration of technology will lead to an overall shift in the behavior and competition strategy of market participants [1]. It is interesting to discuss whether established marketing models can serve for digital products while great changes happen in market environment and in product's characteristics. By nature, digital products can be transacted and delivered through the Internet [2]. Information, payment, and delivery related with digital product's transaction are integrated into the Internet channel. For the transactions of physical products, it must be accompanied by off-line or physical logistics [3].

With unique characteristics in economic and physical property, digital products have many differences from physical products in marketing strategy. For example, all digital products can be transacted without the need for a medium to carry them, since the cost of distribution and production is near zero. The problem is that margin-cost pricing is not available. So the suitability of 4P (product, price, place, promotion) marketing model in an E-environment has been questioned [4]. Mahajan and Venkatesh [5] share the same opinion, opportunity and challenge are brought to E-Business with the rapid development of the Internet, still existing models may have objective conflicts, long-

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term profit maximization is appropriate for E-Business firms as well, yet, increasing customer share and realizing profits and/or cash flow may be important in the short term, so existing models may not fit well in E-Business situations.

About the validity of 4P marketing mix model in digital space, a typical argument is "... marketers should focus on playing an active role in the construction of new organic paradigms for facilitating commerce in the emerging electronic society underlying the web, rather than infiltrating the existing primitive mechanical structures... [6]". Following this issue, many new models are put forward to replace the 4P model in digital marketspace[2][7][8][9][10][11][12][13], such as 4C, 4S, 5P, 7P, ICDT and three "flow" models. Among these alternatives, the 4C and 4S model have wildspread influence. According to Lauterborn, the 4P model can not fit well in E-market, which should be replaced by the 4C model (consumer wants and needs, cost to satisfy, convenience to buy, and communication)[7]. The 4S web marketing model (scope, site, synergy and system) is put forward by Constantinides [10], which is designed for depicting enterprise strategy in E-Business.

Therefore, the 4P marketing mix model may not valid on the Internet age, the 4C and the 4S model are some alternatives. The intention of this research is to study the suitability of those three marketing mix models for digital products in digital space. It is hoped to find which marketing mix model fit well for different types of digital products. Therefore, a tool kit of marketing strategies is developed for professionals selling digital products.

2 Characteristics and Categories of Digital Products

Different marketing strategies may be applied to different types of digital products due to their unique characteristics [14]. It is very important to sum up the characteristics of digital products for investigating the suitability of the established marketing mix model. Most digital products share the following economic characteristics.

Production. The production of digital products is always associated with a huge fixed cost and negligible margin cost [15].

Public goods. Digital products have some consumption characteristics of public goods, such as non-exclusiveness and non-rivalry [16].

Network externality. Contrary to the basic principle of traditional economics, digital products with positive feedback abide by the principle of "more abundant, more precious" [17].

It is possible that non-digital products share some characteristics above. But digital products also possess some unique physical characteristics at the same time.

Attrition-free. Once produced, they will exist forever with the same quality. Furthermore, the competition must spread between new digital products and second-hand digital products.

Changeability. The content of digital products can be changed or customized easily [18]. The integrity can not be controlled by manufacturer after downloaded by users.

Replication. It is most meritorious that digital products can be shared, replicated, stocked and transferred easily. After the first copy of a digital product is created, it can be manufactured with a very low marginal cost. Digital products are composed of text, graph, and voice etc. They are heterogeneous because all the components can be reconstructed quickly and easily. Therefore, it is important to classify various digital products.

In this paper, we adopt the framework put forward by Hui & Chau [19] which classified digital products into three categories according to three dimensions including trialability, granularity and downloadability (Fig. 1). Those are utilities and tools, content-based digital products and online services.

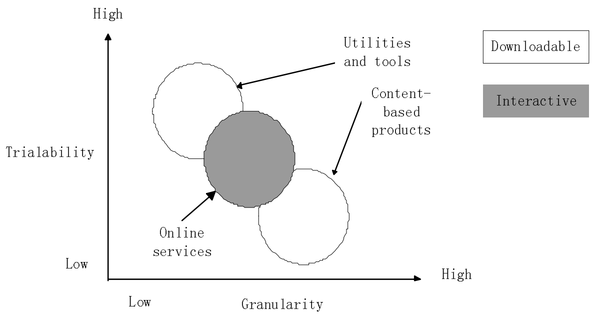

Fig. 1. classifying digital products [19]

These three categories of digital products exhibit differences in trialability, granularity and downloadability(See Table 1). We argue that different categories of digital products require different marketing mix models.

Table 1. Characteristic comparison of different categories of digital products [19]

Category	Trialability	Delivery mode	Granularity	Example
Content-based digital products	Low	By download	High	e-book
Utilities and tools	High	By download	Low	Anti-virus software
Online services	Medium	Interactive	Medium	Online translation

3 Marketing Strategy Analysis of Different Digital Products

The classic 4P marketing model, the 4C marketing model and the 4S marketing model are summarized from different business backgrounds. Constantinides suggest that the background of 4P is industrialization which is characterized by enterprise-centric[10]. With the shift of power from enterprise to consumer, the 4C model is put forward, which is characterized by customer-centric. The 4S marketing model was created for E-Business environment [10]. For different business backgrounds of three marketing mix models, We argue that the 4P, 4C and 4S marketing mix models have different suitability for three categories digital products.

In this section, we will analyze the suitability of three models for three different categories of digital products.

3.1 4P Marketing Mix

The 4P model delimit four distinct, well-defined and independent management processes. Despite the consistent effort by many physical businesses to deal with the 4P in an integrated manner, the drafting but mainly the implementation of the P policies remains largely the task of various departments and persons within the organization. Even more significant thought is the fact that the customer is typically experiencing the individual effects of each of the 4Ps in diverse occasions, times and places, even in case that companies take great pains to fully integrate their marketing activities internally[10]. Enterprise can promote products through mixing four “P” strategies. A clear marketing strategy tool kit for enterprises selling digital products is provided through analyzing different categories of digital products with 4P as shown in Table 2. We find that promotional tools of content-based digital products are more than others and combining trialability is different in three categories of digital products shown in Table 1. We argue that the less trialability a digital product is, the more promotional tools it needs.

3.2 4C Marketing Mix

With market competition shifting from product-oriented into customer-oriented, some defects of 4P emerge. Under this condition, 4C marketing mix model is put forward by Lauterbom who suggest the marketing strategies that involved product, price, place and promotion are passe. Consumer wants and needs, cost to satisfy, convenience to buy and communication are the catechism for our times[7]. This model considers a marketing problem from the consumer perspective[10]. The content includes four points. First, what the customer want should be sold rather than what you can manufacture. Second, enterprise should take every efforts to decrease the cost of fulfilling the customer’s demand. Third, enterprise should take every efforts to give convenience to customer for purchasing. Finally, communication with customer is more important than promotion. Analysis based on the 4C model of three categories digital products is shown in Table 3.

Now that we have a marketing strategy tool kit which are from customer’s perspective for enterprises selling digital products. At the same time, we find that the less trialability a digital product is, the more communication tools it needs, which is the same finding as the 4P model.

Table 2. 4P marketing strategy of different digital products

	Content-based digital products (e-book)	Utilities and tools (anti-virus software)	Online services (online translation services)
Product	Products individual (the ssreader divided e-book into single chapter to sell) Lock in (caj browser used by China National Knowledge Infrastructure)	Quickly update version (kv3000 anti-virus software put new version each year) Products individual (if you purchase UFIDA U8 soft, you can choose the module you need) Binding (WPS Office) Versioning (all version of WPS Office)	Lock in (QQ client) Products individual (stock trade online of West Stock)
Price	Nonlinear pricing (ssreader's reading card: RMB35/quarter，RMB100/year，RMB180/two years) Individual price (products sold in second hand market) Value transfer (e-journal provided in cfan web site, sell quantity of journal is one of the largest.) Two-part price (fix monthly fee of telephone and payment each time)	Group price (user of kv3000 net version/RMB10/month) Name your price(priceline.com.tw) Two-part price(after purchase U8 soft of UFIDA, you should pay for using every year) Value transfer(newhua.com is free , revenue come from advertisement for huge visiting stream)	Individual price (lesson online service of new oriental in second hand market) Two-part price (there are initial ISP service payment then pay for using each year)
Place	Web store (dangdang.com)	Web site constructed by manufacturer (jiangmin.com) web store (newhua.com)	Web site constructed by manufacturer (russky.com)
Promotion	Recommendation(top 10 of ssreader.com) Personalization(When Mary returns to the dangdang.com, it responds "Hello Mary") Comparison shopping (pconline.com) Customization (Yahoo China) User comment list (ssreader.com) Individual recommendation ("we know you buy some E-business books last time, there are some related books for you") Rules-based system("the exercises book is a good complement to this book") Ordering tools(shopping baskets) Advertisement is involved in product(book of ssreader.com) Requirement register (ssreader.com) Digital watermark (photosl.net)	Rules-based system (before downloading update package, please select version) Ordering tools (shopping baskets) Advertisement is involved in product (flashget) Trial (WPS Office provide limited trial) Authorization limitation (KV3000 net version can check virus, but can not update) FAQs (jiangmin.com)	Affiliates(russky.com) FAQs (russky.com) Instant Message (QQ) BBS (bbs.russky.com)

3.3 4S Marketing Mix

The 4S model (web-marketing model, WMM) was put forward by Constantinides [10]. It describes web marketing strategy with four elements begin with "S" in-

Table 3. Marketing strategy of different categories digital products based on 4C

	Content-based digital products (e-book)	Utilities and tools (anti-virus)	Online services (online translation services)
Consumer wants and needs	Products individual (the ssreader divided e-book into single chapter to sell) Binding(browser is bind into e-book) E-coupons(sozhao.com/tools/yhq/dangdang.asp) User comment list (ssreader.com) Requirement register (ssreader.com) Classified advertisement (8848.com)	Quickly update version (kv3000 anti-virus software put new version each year) Products individual (if you purchase UFIDA U8 soft, you can choose the module you need) Binding (WPS Office) Versioning (all version of WPS Office) Advertisement of different category (newhua.com)	Products individual (stock trade online of West Stock) Version (online translation of netat.net.cn can restrict different subject) Advertisement of different category(multi-language translation of netat.net.cn)
Cost of satisfy	Nonlinear pricing (ssreader's reading card: RMB35/quarter, RMB100/year, RMB180/2 years) Individual price (products sold in second hand market) Two-part price (fix monthly fee of telephone and pay each time) Value transfer (e-journal provided in cfan web site, sell quantity of journal is one of the largest.)	Group price(user of kv3000 net version/RMB 10/month) Name you price (priceline.com.tw) Two-part price (after purchase U8 soft of UFIDA, you should pay for using every year) Value transfer (newhua.com is free , revenue come from advertisement for huge visiting stream)	Individual price (lesson online service of new oriental in second hand market) Two-part price (there are initial ISP service payment then pay for using each year)
Convenience to buy	Web store (dangdang.com) Ordering tools (shopping baskets) Security policy (see the privacy and security notice on ssreader.com) Comparison shopping (pconline.com) Virtual reality (e360.cn)	Web site constructed by manufacturer (jiangmin.com) Ordering tools (shopping baskets) Security policy (see the privacy and security notice on ssreader.com) Trial (WPS Office provide limited trial)	Web site constructed by manufacturer (russky.com) Security policy (see the privacy and security notice on ssreader.com)
Communication	Recommendation (the suggestion on ssreader.com) Personalization (When Mary returns to the dangdang.com, it responds "Hello Mary") Customization (Yahoo China) Individual recommendation ("we know you buy some E-business books last time, there are some related books for you") Rules-based system ("the exercises book is a good complement to this book") Trial (free 17 pages in ssreader.com) Online auction (ebay.com.cn) FAQs (ssreader.com) Digital community (Sun digital community)	Rules-based system (before downloading update package, please select version) Advertisement is involved in product (flashget) Trial (WPS Office provide limited trial) Authorization limitation (KV net version can check virus, but can not update) FAQs (jiangmin.com) Auction (ebay.com.cn)	Affiliates (russky.com) FAQs (russky.com) Instant message(QQ) BBS (bbs.russky.com) Digital community (Sun digital community)

Table 4. Marketing strategy of different category digital products based on 4S

	Content-based digital products (e-book)	Utilities and tools(anti-virus)	Online services(online translation services)
Scope (strategy and objective)	Market segmentation (demographic variables, geographic variables, psychographic variables and behavioral variables) Potential customers (profiles, motivation, behavior and needs) Internal analysis(internal resources, value process, and the web sustaining technology) Strategic role of the web activities (information platform, educational, promotional and transactional)	Same as left	Same as left
Web site (online experience)	Link exchange (Coolgo e-book, ssreader) Advertisements Factors of web site (domain, content, design, layout, atmosphere etc)	Optimize of speed of web site Factors of web site (domain, content, design, layout, atmosphere etc)	Optimize of speed of web site Factors of web site (domain, content, design, layout, atmosphere etc)
Synergy (integration)	Web store (dangdang.com) The back office (physical book store)	Web site construct by manufacturer (jiangmin.com) The front office (updating product installed in client)	Web site construct by manufacturer (russky.net/trans/) Third parties (outsource of logistic)
System (technology requirement)	Technology requirement of web site (stabilization, security, software, hardware, protocol, system service etc) Preliminary payment system (ssreader)	Technology requirement of web site (stabilization, security, software, hardware, protocol, system service etc) Instant payment system (kv3000) Post-payment system (net version RMB10/month)	Technology requirement of web site (stabilization, security, software, hardware, protocol, system service etc) Instant payment system(in web site (russky.net/trans/) payment can be finished by SMS(short message system) and network)

cluding scope, site, synergy and system. The goal of this model is to design and develop marketing mix for BtoC online projects through controlling four "S" elements. In 4S model, the scope element is of primarily strategic character and outlines the decisions to be made on four areas: (a) the strategic and operational objectives of the online venture; (b) the market definition including measuring the market potential and the identification/classification of the potential competitors, visitors and customers of the site; (c) the degree of readiness of the organization for E-Commerce; (d) the strategic role of E-Commerce for the organization. The web site is therefore the functional platform of communication, interaction and transaction with the web customer. The prime mission of the web Site is to attract traffic, establish contact with the online target markets and brand the online organization. The synergy factor embraces a wide range of issues divided into three categories: the front office, the back office and the third parties. The front office refers to conventional corporate communication and distribution strategies; The back office synergy includes three issues: (a) the integration of E-Commerce physical support into existing organizational processes; (b) the legacy integration; (c) integration of the online operation into the company's value system. The success in virtual marketplace often requires cooperation with Internet partners outside the organization and its value system.

Finally, the system factor identifies the technological issues as well as the site servicing issues to be addressed by the E-Commerce management.

The 4S model brings out how to prepare a web marketing strategy on two issues. First, on strategic layout, the main strategic problem is planned which is insurance of creating successful web marketing strategy. The 4S model emphasizes that web marketing strategy should consistent with enterprise strategy. Web marketing strategies should integrate with other operation strategy and take full advantage of competition advantage. Second, on operation layout, a methodology for making web marketing strategy is provided.

4 Discussion

There is an interesting finding in Table 2 and Table 3 that the lower trialability of digital products is, the more promotional tools it needs. A probable explanation is that digital products are a kind of experience products, consumers can understand the product after purchasing it [3]. So the best promotion tool is trial, through trial, consumer will persuade himself to purchase. Refering Table 1, utilities and tools is the best in trialability while content-based product is the worst in trialability. So promotion of content-based product should be achieved by multi-marketing tools except trial. The other two categories of products can be promoted by trial. That is to say, three categories of products have notable difference in promotion tools. We can sum up this point as:

Trial is the best promotion tool for digital products. With high trialability, the digital products needs less promotion tools. At the same time, with low trialability, the digital products needs more promotion tools.

Delivery mode in Table 1 can partly explain different place strategies in Table 2. Value of content-based products can transfer through download mode, if adopting a web store, the distributions cost is low. Value transfer of online services need to interact with the customer, web site construction by manufacturer is only a feasible way. For frequent updates (e.g., update of the package) of utilities and tools, enterprises selling utilities and tools often constructed web site by itself. Web stores is a feasible tool on condition that updates are not required.

Comparing 4S model with the 4P and 4C models, we find strategic elements involved in 4S model that distinguish from others. It is combined of strategy marketing with tactics marketing. For products needing manufactures construction web site by itself as a distribution channel, it is necessary to select a target market from the market segmentation according to position. Those are important to design web site. The web site of online service is often constructed by manufacturers and there are some strategic elements, including position and target market selection, affect web site's business status except promotion instrument including advertisement etc. That is to say, the 4S model is a feasible marketing strategic tool kit for enterprises selling online service. There are parts of enterprises of selling utilities and tools constructing a web site by themselves, the 4S model is suitable for them. Enterprises selling content-based products seldom construct web site by itself, the suitability of 4S model is low. The 4C model considers a marketing problem from the consumer perspective. Customer's demand can be easy fulfill while the difference ability of products is high. Therefore, 4C model is suitable for products with high difference ability. Content-based products have the highest difference ability, which can reconstruct with different segments to distinguish from others and with no additional cost. Online services have a medium difference ability which decided by its high trialability, but some cost accompany trialing. Utilities and tools have low difference ability which can not be reconstructed randomly. In summary, the 4C model has high suitability of content-based products and medium suitability of online service and low suitability of utilities and tools. The 4P model has more controllable ability of four elements than others, so the 4P model is suitable for utilities and tools well. The 4S model is suitable for utilities and tools on condition that manufacturer constructing web site himself. Under this condition, it is best to combine the 4P model and the 4S model. The other two products have medium suitability for the 4P model. Analyzing of the above can summarized into three findings:

For content-based products, the 4P model is much more suitable than the 4C model, while the 4C model is more suitable than the 4S model.

For utilities and tools, the 4P and the 4S models are more suitable than the 4C model.

For online service, the 4S model is more suitable than the 4P and 4C models.

5 Conclusion

There are three contributions of this study. First,with unique characteristics in economic and physical property, the classic marketing models including 4P, 4C and 4S have different conditions in applying to the selling of digital product. Second, for each catagroities of digital products, the priority of suitability of established marketing mix models is provided. Third, there is an interesting finding that the less trialability a digital product is, the more promotional tools it needs.

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Optimal Pricing for Web Search Engines

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Abstract. The research and developments on web search engines have been raised to be one of the hottest topics since the popular usage of the internet. Accordingly, how to price the software becomes an important problem and is still unsolved with satisfactions. Using the Principle-Agent method in economics, the pricing model for web search engines based on the theory of Brown Motion with drifts is established. The stopping time is defined for the model, and the expected benefit of the web-search-engine provider over the rental horizon is derived, with a special case for the outright sale. By maximizing the benefit, the optimal price for the outright sale, the optimal monthly rental and the optimal selling price after renting the search engine for a period of time are discussed. Finally, given sets of parameters, simulations are implemented, and the results show that the optimal prices are reasonable.

Keywords: Search engine; Software pricing; Principal-Agent model; Stochastic process.

1 Introduction

With the development of internet and eCommerce, the demand of web search engines becomes larger and larger. More and more people find the information on the webpages. Some famous software providers have joined in and divide the market of web search engines. For example, the Microsoft announced its project plan in the area of web search engines in 2004.

In addition to the generic web search engines like google.com, many companies provide search features to allow customers to browse and search within their websites. Since the search engine technology is independent of the web-building technology, most of the companies lease or purchase the search engine and simply incorporate it into their web sites, as opposed to coding it themselves. Those company-ranged search engines help their customers to browse and are able to find most of the contents they need within the company's websites. As a result, fewer customers call the customer service department for helps, and hence reducing the cost of customer service.

In this paper, we study the best rental price that the search-engine provider should offer, also the best sale price. On one hand, the search-engine provider

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is willing to increase its income. Clearly, increasing the prices directly help to realize this goal. On the other hand, it also wants to keep certain amount of clients. As a result, reducing the rental and sale prices is a good choice. The conflict is solved by optimizing the income with regard to the prices it offers.

The search engine is a sort of software that was born with the web and applied to the web. Pricing of the search engine is involved in many factors and is a novel topic in the eCommerce economics.

There are several approaches for software pricing, the monopoly-based approach, the value-based approach, the utility-based approach, the supply-demand-based approach. In practice, the software providers generally use a combination of the above approaches. For example, conditioning on the existence of a surplus of the supply of web service, the queue theory was used in [1] to price the software. Also, for the similar software, the providers with big names could offer lower prices than the average [1]. In some cases, it is a good choice to lease software monthly, based on the number of users, or the usage, or a combination of the two. Sometimes, the companies are willing to rent the software first with an option to purchase it later. [2] considered the issue from the consumer side and found that it is less expensive to rent for the first 2 years and then purchase it. [6] employed the linear programming in pricing software and discovered the providing of monthly rental choices could increase the market size, and hence enhancing the benefit of the software providers. [1, 3, 4, 5] gave a convey.

In this paper, we apply the Principal-Agent model [7, 8] in pricing the web search engine. The provider of the search engine is the principal, and the purchaser of the search engine is the agent. Our target is to maximize the benefit of the provider under certain conditions for the purchaser.

This paper is organized as follows. Section 2 derives the conditions for a lessee to start renting and to keep the rental, respectively. Section 3 optimizes the rental price and the sale price on the provider side. Solutions are discussed in section 4. Section 5 concludes the paper.

2 Conditions for Lessee to Start Renting and to Keep the Rental

Let C_0 be the average cost before using the search engine, C_k the cost in the kth month after using the search engine, k=1, 2, 3, ..., then $X_k = C_0 - C_k$ is the saving in the k^{th} month.

We assume that C_0 is a normal random variable with the mean μ_0 and the variation σ_0^2, $C_0 \sim N(\mu_0,\, \sigma_0^2), C_k$, is an i.i.d. normal random variable with the mean $\mu_k (< \mu_0)$ and the variation σ_k^2, $C_k \sim N(\mu_k, \sigma_k^2)$, then $X_k = C_0 - C_k, \quad k = 1, 2, 3, \cdots$, is the normal random variable with the mean $\mu = \mu_0 - \mu_k > 0$, and the variation $\sigma^2 = \sigma_0^2 + \sigma_k^2 > 0, X_k \sim\ N(\mu, \sigma^2)$. Suppose

w = the monthly rental, paid on the 1st day of each month,
D = the lock-in period, i.e., the least months of rental, if the lessee is to rent,
R= the reservation value of the lessee,

H = the longevity of the search engine
(after a period of time, the software is out of time).

First of all, we derive the condition for the lessee to start renting.

Clearly, the month rental should be less than the amount of saving each month, $w < X_k, k = 1, 2, 3, \cdots$, or $0 < X_k - w$. However, generally this is still not enough to encourage the lessee to start renting. If at the end of the k^{th} month, the saving amount $X_k - w$ is larger than the reservation value of the lessee,

$$R < X_k - w$$

then the lessee is willing to start renting. We define

$$R = \theta X_k$$

where, θ =the coefficient of the reservation value, in general, $\theta \in (0.5, 1)$. Hence, for $k = 1, 2, 3, \cdots$, $\theta X_k < X_k - w$, so

$$(w <) \quad \frac{w}{1-\theta} < X_k.$$

Clearly, if the above equation is satisfied, the client starts to lease the search engine. Taking the expectation on both sides we obtain

$$\frac{w}{1-\theta} < \mu, \quad \text{or,} \quad E\left\{\frac{w}{1-\theta}\right\} < EX_k, k = 1, 2, 3, \cdots$$

To attract the client, the expected monthly saving should be larger than $\frac{w}{1-\theta}(> w)$.

Assume that the client starts to lease the search engine in month 0. We next find out the conditions under which the client keeps leasing the search engine.

Let $S_K = \sum_{k=1}^{K} X_k$ be the total saving from month 1 to month K. Clearly, S_K is the normal distribution $N(\mu K, \sigma^2 K)$, and $\{S_K\}$ is an independent incremental Gaussian process.

We further assume that after the client starts to lease the search engine, as long as the average monthly saving $\frac{S_K}{K}$ from month 1 to month K satisfies,

$$\frac{w}{1-\theta} < \frac{S_K}{K}, \quad D \leq K \leq H,$$

the client keeps the lease. Equivalently, it can be rewritten as

$$\frac{w}{1-\theta}K < S_K.$$

If $\frac{w}{1-\theta}K \geq S_K$, the client stops using the search engine. Taking the expectation on both sides, we have

$$E\left\{\frac{w}{1-\theta}K\right\} < ES_K,$$

or

$$\frac{w}{1-\theta}K < \mu K.$$

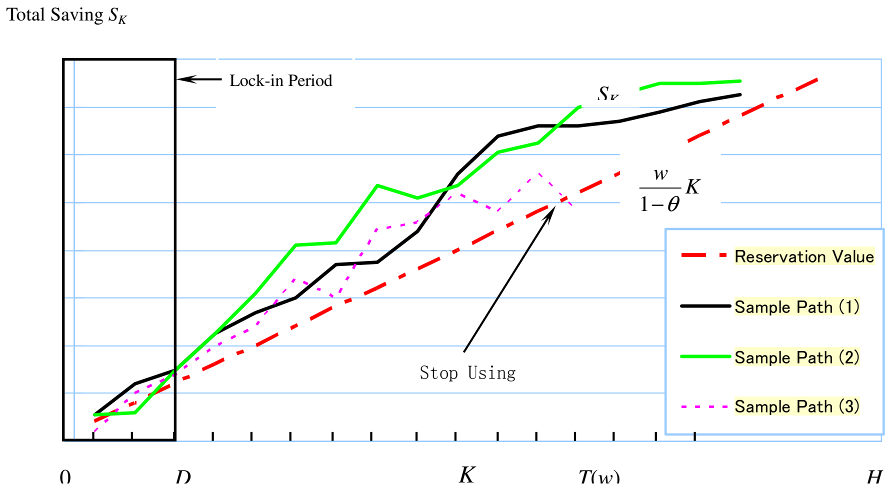

Fig. 1. The sample paths of stochastic process S_K. In the lock-in period, even if S_K is below the slope, the client must keep the lease. After the lock-in period, to keep the client, $\frac{w}{1-\theta}K < S_K$ must hold. Clearly, $\frac{w}{1-\theta}$ is the slope. If S_K is always above the slope, the client shall renew the lease. If $S_K \leq \frac{w}{1-\theta}K$, or below the slope, the client stops leasing the search engine.

Namely, if a client is to be attracted, the total saving should be larger than $\frac{w}{1-\theta}K(> wK)$.

The sample paths of stochastic process S_K are exemplified in Fig.1.

It is not difficult to find that in the first several months, the lesser is easy to lose the clients. This is the reason that a lock-in period is designed. In the beginning, the trajectory of the stochastic process is very close to the slope. As the time goes on, the fact that the stochastic process has a positive drift $(0 < w < \mu)$, then the stochastic process leaves the slope farther and farther and the chance of hitting the slope is less and less.

3 The Best Rental Price and the Best Sale Price

Define the stopping time [9, 10] of the stochastic process with regard to the area under the slope is

$$T(w) = \inf\{D \leq K \leq H : \quad S_K \leq awK\}$$

where $a = \frac{1}{1-\theta}$. If $S_K > awK, D \leq K \leq H$, namely, the stochastic process of the search engine is never under the slope. In this case, let $T(w) = H$. As a result, $T(w)$ is either the first time that the stochastic process S_K hits the slope K, or H, the longevity of the search engine. $T(w)$ is therefore the time that the lessee stops leasing the search engine.

Suppose after the lessee has rented the search engine for K months, $K \;\; 0$ or $D \leq K \leq H$, she decides to purchase the search engine. Then what is the fair purchasing price? We find it in the following.

Noting that $K < T(w)$ or $K \leq T(w)-1$, we know that after the search engine has been for K months, the expected present value of the purchasing price of the search engine is

$$Q_K(w) = E\left[\sum_{k=K}^{T(w)-1} d^k w\right]$$

$$= w\,E\left[\sum_{k=K}^{T(w)-1} d^k\right]$$

where d is the monthly discount factor. At the end of the Kth month, the purchasing price of the search engine is $\frac{Q_K(w)}{d^K}$.

In particular, if $K = 0$, the purchasing price without any leasing process is

$$Q_0(w) = w\,E\left[\sum_{k=0}^{T(w)-1} d^k\right]$$

Clearly, according to the model, no matter the lessee rents and then buys, or outright buys without any leasing stage, the benefits for the lesser are the same. Namely, there is no arbitrage opportunity.

For calculating the best purchasing price, the lesser should optimize the following equation with regard to w,

$$w^* = \arg\max_w Q_K(w) = \arg\max_w wE\left[\sum_{k=K}^{T(w)-1} d^k\right]$$

Solving the above equation leads to the best monthly rental w^*.

Next, we derive the above model [11]. From

$$\begin{aligned} Q_K(w) &= w\,E\left[\sum_{k=K}^{T(w)-1} d^k\right] \\ &= w\left[\sum_{k=K}^{H-1} d^k E\left\{ I_{\{\, k<T(w)\,\}}\right\}\right] \end{aligned}$$

where the indicator function is

$$I_{\{s<t\}} = \begin{cases} 1, & s < t \\ 0, & s \geq t \end{cases}$$

hence

$$\begin{aligned} Q_K(w) &= w\left[\sum_{k=K}^{H-1} d^k \left[1\,P\{\,k < T(w)\,\} + 0P\{\,k \geq T(w)\,\}\right]\right] \\ &= w\left[\sum_{k=K}^{H-1} d^k\, P\{\,k < T(w)\,\}\right] \\ &= w\left[\sum_{k=K}^{H-1} d^k\, \left[1 - P\{\,T(w) \leq k\,\}\right]\right] \end{aligned}$$

We need to first know the distribution of the stopping time $P\{T(w) < k^*\}$. $T(w)$ is the first time that the stochastic process hits the slope after the D^{th} month. From the properties of the independent increment Markov process, given the state of the D^{th} month S_D, $T(w)$ and the stochastic process is irrelevant to the states before the D^{th} month. We divided $P\{T(w) < k^*\}$ into two parts by conditioning on S_D

$$P(T(w) \le k^*) = E[I_{\{T(w)\le k^*\}}] = E\left[E(I_{\{T(w)\le k^*\}}|S_D)\right]$$

First, we calculate the conditional expectation. Clearly, it is a function of S_D. If $k^* > D$, we have

$$E\left[I_{\{T(w)\le k^*\}}\middle| S_D\right] = \begin{cases} 0, & S_D \le awD \\ E\left[I_{\left\{\inf\left\{D\le k:\ \sum\limits_{i=D+1}^{k} X_i \le -S_D + awD + aw(k-D)\right\}\le k^*\right\}}\middle| S_D\right], & S_D \ge awD \end{cases}$$

Let $\{Y_i\}_{i=1,2,\cdots}$ be a series of random variables with the same distribution of $\{X_i\}_{i=1,2,\ldots}$. Since S_D and $\sum_{i=D+1}^{k} X_i$ are independent, the non-zero term of the step function is equal to

$$E\left(I_{\left\{\inf\left\{0\le k:\ -\sum\limits_{i=1}^{k} Y_i + awk \ge S_D - awD\right\}\le k^*-D\right\}}\middle| S_D\right), \quad S_D - awD > 0$$

Suppose $S_k^y = -\sum_{i=1}^{k} Y_i + awk$, $S_D^1 = S_D - awD$, then we have $S_k^y \sim N(k(-\mu + aw), k\sigma^2)$, and $S_D^1 \sim N(D(\mu - aw), D\sigma^2)$. As a result, the above equation can be rewritten as

$$E\left(I_{\left\{\inf\left\{0\le k:\ S_k^y \ge S_D^1\right\}\le k^*-D\right\}}\middle| S_D^1\right), \quad S_D^1 > 0$$

hence

$$\begin{aligned} P(T(w) \le k^*) &= E\left[E\left[I_{\{T(w)\le k^*\}}\middle| S_D^1\right]\right] \\ &= \int_0^\infty E\left[I_{\left\{\inf\{0\le k:\ S_k^y \ge x\}\le k^*-D\right\}}\right] f_{S_D^1}(x)dx \\ &= \int_0^\infty P\left\{\left\{\inf\left\{0 \le k:\ S_k^y \ge x\right\} \le k^* - D\right\}\right\} f_{S_D^1}(x)dx \end{aligned}$$

Let $W_t \sim N\left(t(-\mu + aw), t\sigma^2\right)$ be the Brownian motion with the drift. Then S_k^y is the stochastic process generated by W_t on the integer time point. Define the stopping time as follows,

$$\tau_x = \inf\left\{0 < t:\ W_t \ge x\right\}$$

hence

$$P\left\{\inf\left\{0 \le k:\ S_k^y \ge x\right\} \le k^* - D\right\} = P\left\{\tau_x \le k^* - D\right\}$$

or

$$P(T(w) \leq k^*) = \int_0^\infty P\left(\tau_x \leq k^* - D\right) f_{S_D^1}(x)dx$$

For calculating the stopping time of τ_x, we introduce the following lemma [12, 13].

Lemma 1. *If $B(t)$ is a standard Brownian motion, then the stopping time of $W_t = \mu^* t + \sigma B(t)$, $\tau_x = \inf(0 < t : \ W_t \geq x)$ is associated with the following probability density*

$$f_{T_a}(t) = \frac{a}{\sigma\sqrt{2\pi t^3}} \exp\left(-\frac{(a-\mu^* t)^2}{2\sigma^2 t}\right), \quad t > 0.$$

Consequently,

$$\begin{aligned} P(T\ (w) \leq k^*) &= \int_0^\infty \int_0^{k^*-D} \tfrac{x}{\sigma\sqrt{2\pi t^3}} \exp\left(-\tfrac{(x-\mu^* t)^2}{2\sigma^2 t}\right) f_{S_D^1}(x)dtdx \\ &= \int_0^\infty \int_0^{k^*-D} \tfrac{x}{2\pi\sigma^2\sqrt{Dt^3}} \exp\left(-\tfrac{(x^2+(\mu-aw)^2 tD)(t+D)}{2\sigma^2 tD}\right) dtdx \end{aligned}$$

Exchanging the order of integrations leads to

$$\begin{aligned} &P\ (T(w) \leq k^*) \\ &= \int_0^{k^*-D}\int_0^\infty \tfrac{1}{2\pi\sigma^2\sqrt{Dt^3}} \left(-\tfrac{\sigma^2 tD}{t+D}\right) \exp\left(-\tfrac{(\mu-aw)^2(t+D)}{2\sigma^2}\right) \left(-\tfrac{x(t+D)}{\sigma^2 tD} \exp\left(-\tfrac{x^2(t+D)}{2\sigma^2 tD}\right)\right) dxdt \\ &= \int_0^{k^*-D} \tfrac{\sqrt{D}}{2\pi\sqrt{t}(t+D)} \exp\left(-\tfrac{(\mu-aw)^2(t+D)}{2\sigma^2}\right) dt \\ &\overset{t=y^2}{=} \int_0^{\sqrt{k^*-D}} \tfrac{\sqrt{D}}{\pi(y^2+D)} \exp\left(-\tfrac{(\mu-aw)^2}{2\sigma^2}(y^2+D)\right) dy \end{aligned}$$

Hence

$$\begin{aligned} Q_K(w) &= w\ \left[\sum_{k=K}^{H-1} d^k\ [1 - P\{\,T(w) \leq k\,\}]\right] \\ &= \sum_{k=K}^{H-1} d^k w \left\{1 - \int_0^{\sqrt{k-D}} \tfrac{\sqrt{D}}{\pi(y^2+D)} \exp\left(-\tfrac{(\mu-aw)^2}{2\sigma^2}(y^2+D)\right) dy\right\} \\ &= \tfrac{w\left(d^K - d^H\right)}{1-d} - \sum_{k=K}^{H-1} d^k \int_0^{\sqrt{k-D}} \tfrac{w\sqrt{D}}{\pi(y^2+D)} \exp\left(-\tfrac{(\mu-aw)^2}{2\sigma^2}(y^2+D)\right) dy \end{aligned}$$

Taking the derivatives of the above equation with respect to w on both sides, we have

$$\begin{aligned} &\tfrac{\partial Q_K(w)}{\partial w} \\ &= \tfrac{\left(d^K - d^H\right)}{1-d} - \sum_{k=K}^{H-1} d^k \int_0^{\sqrt{k-D}} \left[\tfrac{\sqrt{D}}{\pi\left(y^2+D\right)} + \tfrac{wa\sqrt{D}(\mu-aw)}{\pi\sigma^2}\right] \exp\left(-\tfrac{(\mu-aw)^2}{2\sigma^2}(y^2+D)\right) dy \\ &= \tfrac{Q_K(w)}{w} - \tfrac{\sqrt{2D}aw}{\sqrt{\pi}\sigma} \exp\left(-\tfrac{D(\mu-aw)^2}{2\sigma^2}\right) \sum_{k=K}^{H-1} \left\{d^k \left[\varPhi\left(\tfrac{\sqrt{k-D}(\mu-aw)}{\sigma}\right) - \tfrac{1}{2}\right]\right\} \end{aligned}$$

where $\Phi(x)$ is the standard normal distribution function, $\Phi(x)=\int\limits_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2}\right)dt$. If $K\leq D$, then from $T(w)\geq D$, $P(T(w)<K)=0$, we obtain

$$Q_K(w)=\frac{w\left(d^K-d^H\right)}{1-d}-\sum_{k=D+1}^{H-1}d^k\int_0^{\sqrt{k-D}}\frac{w\sqrt{D}}{\pi(y^2+D)}\exp\left(-\frac{(\mu-aw)^2}{2\sigma^2}(y^2+D)\right)dy$$

$$\frac{\partial Q_K(w)}{\partial w}=\frac{Q_K(w)}{w}-\frac{\sqrt{2D}aw}{\sqrt{\pi}\sigma}\exp\left(-\frac{D(\mu-aw)^2}{2\sigma^2}\right)\sum_{k=D+1}^{H-1}\left\{d^k\left[\Phi\left(\frac{\sqrt{k-D}(\mu-aw)}{\sigma}\right)-\frac{1}{2}\right]\right\}$$

where in the above two equations, since the integral for D is 0, k starts from $D+1$.

Since $w<\frac{\mu}{a}$, we only need to optimize $Q_K(w)$ with regard to w^* within the interval $[0,\frac{\mu}{a}]$. Since $\frac{\partial Q_K(w)}{\partial w}$ is a decreasing function in the interval $[0,\frac{\mu}{2a}]$, $\left.\frac{\partial Q_K(w)}{\partial w}\right|_{w=0}>0$, and $\left.\frac{\partial Q_K(w)}{\partial w}\right|_{w=\frac{\mu}{a}}>0$, we know that $Q_K(w)$ is an increasing function when it approaches 0 and $\frac{\mu}{a}$. If $\frac{\partial Q_K(w)}{\partial w}<0$ in $[0,\frac{\mu}{a}]$, then there exists a maximum point in $[0,\frac{\mu}{a}]$.

Since $Q_K(w)$ and its derivative are too complicated to find an analytical solution for the maximal value of $Q_K(w)$ and w^*, in the following we experiment the different parameters for $Q_K(w^*)$and w^*.

4 Discussions

Assume that the interest rate is p. Then the discounter factor is $d=\frac{1}{1+\frac{p}{12}}=\frac{12}{12+p}$. If $p=5\%$, then $d=0.9959$. Let $\mu=\$20000$, $\sigma=\$10000$, $\theta=\frac{2}{3}$, $H=36$ months, $D=3$ months, $K=5$. Namely, the longevity of the software is 3 years, and the lock-in period is 3 months. Substituting into $Q_K(w)$, we obtain $Q_K(w)$ and $\frac{\partial Q(w)}{\partial w}$ (see Fig.2).

It can be seen that starting from time 0, $Q(w)$ increases almost linearly, and then increases with the reduced values. Afterwards, it decreases and increases again. It is interesting to notice that $Q(w)$ decreases as w increases if $w>w^*$.

Table 1 gives $Q*_K(w)$ in the interval $[0,\frac{\mu}{a}]=[0,6667]$ for different H and K.

It can be seen from Table 1 that in most of cases, $Q(w)$ is located in the local maximum of interval $[0,\frac{\mu}{a}]$. This maximum is the best monthly rental, maximizing the income for the search-engine provider. However, in some cases, for example, $H=24$, $K=7$, $Q(w)$ achieves the maximum on the upper bound $\frac{\mu}{a}$.

It can also be seen from Fig. 2 that $Q(w)$ always increases in the total time period. However, if $w>w_1$, the increment rate decreases. As a result, the marginal income is small as w increases. For example, $\frac{\Delta Q(w)}{\Delta w}\approx 3$, comparing to $\frac{\Delta Q(w)}{\Delta w}\approx 13$ if $w<w_1$. It is therefore that the pricing by the sale department of the search engine provider is allowable to be flexible in the real world in the interval $(w_1\ ,\ w^*]$ for attracting more clients, based on the "flat" increase of the marginal benefit.

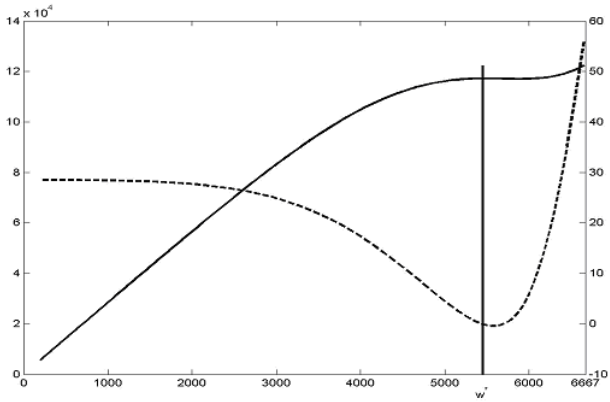

Fig. 2. $Q_(w)$(solid line, left axis) and $\frac{\partial Q(w)}{\partial w}$(dotted line, right axis). The horizontal axis is for the monthly rental w, the left axis is for $Q(w)$, and the right axis is for $\frac{\partial Q(w)}{\partial w}$. $Q(w)$ reaches the local minimum at w^*, shown by the vertical line on the right part of the figure.

Table 1. The time points to start leasing and the best monthly rental w^* and $Q(w^*)$

	$H = 24$		$H = 36$		$H = 48$		$H = 60$	
	w^*	$Q(w^*)$	w^*	$Q(w^*)$	w^*	$Q(w^*)$	w^*	$Q(w^*)$
$K = 0$	6667	111879	6667	154624	6667	194142	5537	222756
$K = 3$	6667	91962	6667	134707	5462	167846	5366	206482
$K = 5$	6667	79914	5450	117248	5343	157898	5290	196650
$K = 7$	6667	69931	5300	108542	5250	149291	5250	188091
$K = 9$	5350	57449	5250	100301	5250	141095	5200	179891
$K = 11$	5300	49404	5250	92307	5200	133096	5200	171916
$K = 13$	5250	41527	5200	84444	5200	125258	5198	164078
$K = 15$	5200	33774	5200	76714	5200	117527	5188	156349
$K = 17$	5200	26135	5200	69074	5192	109888	5181	148712
$K = 19$	5200	18577	5200	61516	5185	102331	5176	141156
$K = 21$	5200	11094	5195	54033	5179	94849	5172	133675
$K = 22$	5200	7379	5191	50318	5177	91135	5170	129960
$K = 23$	5200	3681	5188	46621	5174	87438	5168	126263

5 Conclusion

The pricing issue for the search engine incorporated in the web servers is discussed in this paper. The model is formulated by analyzing the Brownian motion and the stopping time. The optimal rental price and the optimal purchasing price are found experimentally based on the derivative of the expected present value of the purchasing price.

With the development of the eCommerce, more and more websites need the search engines. The model in this paper provides a tool for pricing them.

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Pricing Strategy of Mixed Traditional and Online Distribution Channels Based on Stackelberg Game

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Abstract. Strategies of pricing based on Stackelberg competition between manufacturer and distributor are discussed under the condition that both traditional and online channels exist. A system consisting of a manufacturer, a distributor and customers is considered, in which the manufacturer can sell products to the distributor, who, in turn, sells the products to customers through traditional channel, or the manufacturer can transact directly with the customers in an electronic manner. The manufacturer and the distributor establish models with the respective objection of maximizing expected profit. The manufacturer regards the price of products sold to distributor through traditional and online channels as decision variables, while the distributor's decision variable is the price of products sold to customers, and the distributor's decision variable is the function of the manufacturer's decision variable. At last, the numerical examples are used to show the application of pricing models.

1 Introduction

The rapid growth and adoption of the Internet has already impacted on all aspects of business heavily, including customer acquisition, marketing, human resource management, finance, information systems, and operations. Electronic commerce can be defined informally as a business process that uses the Internet or other electronic medium as a channel to complete business transactions. The changes in researches on supply chain management under electronic commerce can be divided into three aspects [1].

First, some issues related to supply chain management have not been changed in principle, although electronic commerce may have had an impact on some of their parameters. For example, a firm still need to take into account the interplay between fixed and variable costs, while making decisions related to procurement or setting up additional capacity. With the prevalence of the Internet, the firm might be able to obtain a lower procurement price or salvage excess capacity through market mechanisms more easily. Next, some existing supply chain issues have become important as a result of electronic commerce. For example, mass customization has gained a lot of momentum under electronic commerce, because firms can allow customers to specify customizations of their offerings interactively. Finally, some issues new to supply chain management have emerged. One example is how a firm coordinates Internet

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and traditional distribution channels in terms of prices as well as information and product flows.

The potential impact of the Internet on supply chain management makes the importance of relevant study under electronic commerce to be on the increase, and many researches on pricing under electronic commerce are done [2, 3]. The development of electronic commerce states that the relationship between online and traditional markets is not substitutive but supplementary. From a supply chain standpoint, the integration of traditional and online markets is more attractive because it promises profit gains, inventory reduction, and increased customer services [1]. Under the condition that both traditional and online channels exist, the strategy of product pricing is different from the strategy in traditional channel, and even a single firm needs to decide how to price products differentially over the traditional and the online channels. So it is necessary to study the pricing strategy of mixed traditional and online distribution channels[4 ~ 6]. Models of pricing of mixed traditional and online distribution channels based on the coordination and Bertrand competition are discussed in this paper, and a sketchy way to determine the optimal price is given.

2 Description of Question

A system consisting of a manufacturer, a distributor and customers is considered, in which the manufacturer can sell products to the distributor, who, in turn, sells the products to customers through traditional channel, or the manufacturer can transact directly with the customers in an electronic manner (i.e., sell via online channel) which is allowed by the development of electronic commerce and is heretofore not possible. The following is the definition of some symbols:

c_M, manufacturer's unit production cost;

p_{MD}, price of products sold by manufacturer via traditional channel (i.e., sell products to distributor);

p_{MR}, unit price of products sold by manufacturer via online channel (i.e., sell directly products to customers in an electronic manner);

p_{DR}, unit price which distributor sells products to customers;

θ, uncertain factors which can't be controlled by manufacture and distributor;

$q_{MD}(p_{MD}, p_{DR}, \theta)$, demand via traditional channel (the quantity of products bought by distributor from manufacturer), for convenience of description, the demand via traditional and online channels depends on the price in the two channels and the uncertain factors θ;

$q_{MR}(p_{MR}, p_{DR}, \theta)$, demand via online channel (the quantity of products bought by customers from manufacturer);

$q_{DR}(p_{MR}, p_{DR}, \theta)$, demand via traditional channel (the quantity of products bought by customers from distributor);

$c_{MD}(q_{MD})$, transaction cost in traditional channel (i.e., between manufacturer and distributor), it depends on the volume of transaction between manufacturer and distributor;

$c_{MR}(q_{MR})$, transaction cost in online channel (i.e., between manufacturer and customers), it depends on the volume of transaction between manufacturer and customers.

Let π_M and π_D denote the manufacturer's profit and the distributor's profit, respectively, and then the function of profit can be expressed as follows:

$$\begin{aligned}\pi_M = p_{MD} \cdot q_{MD}(p_{MR}, p_{DR}, \theta) + p_{MR} \cdot q_{MR}(p_{MR}, p_{DR}, \theta) \\ - c_M \cdot [\, q_{MD}(p_{MR}, p_{DR}, \theta) + q_{MR}(p_{MR}, p_{DR}, \theta)] \\ - c_{MD}\,(q_{MD}(p_{MR}, p_{DR}, \theta)) - c_{MR}\,(q_{MR}(p_{MR}, p_{DR}, \theta))\end{aligned} \tag{1}$$

$$\pi_D = p_{DR} \cdot q_{DR}(p_{MR}, p_{DR}, \theta) - p_{MD} \cdot q_{DR}(p_{MR}, p_{DR}, \theta) \tag{2}$$

For convenience of description, the distributor's holding cost isn't considered, and then equation (2) can be written as

$$\pi_D = (p_{DR} - p_{MD}) \cdot q_{MD}(p_{MR}, p_{DR}, \theta)\,. \tag{3}$$

3 Models of Pricing Based on Stackelberg Game

In the Stackelberg game setting, there is price competition between the manufacturer and the distributor. The manufacturer as a leader determines the prices of products sold through traditional and online channels, and then the distributor as a follower selects the optimal price after knowing the manufacturer's decision.

3.1 Pricing Model of Distributor

The distributor determines the optimal price according to the following model after knowing the manufacturer's prices of products sold through traditional and online channels.

$$\max_{p_{DR}} E\pi_D(p_{DR}) = E[(p_{DR} - p^*_{MD}) \cdot q_{MD}(p^*_{MR}, p_{DR}, \theta)] \tag{4}$$

$$\text{s. t.} \qquad p_{DR} \ge p^*_{MD} \ge c_M \tag{5}$$

We take the form similar to that in paper [7] to deal with constraint condition (5), and then the objection function can be turn into a general form:

$$E\pi_{DG}(p_{DR}) = E[(p_{DR} - p^*_{MD}) \cdot q_{MD}(p^*_{MR}, p_{DR}, \theta)] \tag{6}$$

$$-\frac{1}{2}a_1(p_{MD}^* - c_M)^2 - \frac{1}{2}a_2(p_{DR} - p_{MD}^*)^2$$

where a_1 and a_2 are all constraint parameters, and a_1 , a_2 >0。

The fist-order condition of $E\pi_{DG}$ is as follows:

$$E[q_{MD}(p_{MR}^*, p_{DR}, \theta) + (p_{DR} - p_{MD}^*) \cdot q_{MDD}(p_{MR}^*, p_{DR}, \theta)] \\ - a_2(p_{DR} - p_{MD}^*) = 0 \tag{7}$$

Where q_{MDD} denotes the derivative of q_{MD} with respect to p_{DR} .

From equation (7), we can get,

$$p_{DR}^* = p_{DR}^*(p_{MD}^*, p_{MR}^*) \tag{8}$$

Equation (8) shows that the distributor's optimal price is the function of the manufacturer's optimal prices of products sold through traditional and online channels.

3.2 Pricing Model of Manufacturer

In the Stackelberg game setting, the manufacturer determines the optimal prices of products sold through traditional and online channels, p_{MD}^* and p_{MR}^* , according to the following model.

$$\begin{aligned} &\max_{p_{MD}, p_{MR}} E\pi_M(p_{MD}, p_{MR}) \\ &= E\{p_{MD} \cdot q_{MD}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta) \\ &\quad + p_{MR} \cdot q_{MR}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta) \\ &\quad - c_M[\, q_{MD}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta) \\ &\quad + q_{MR}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta)] \\ &\quad - c_{MD}\,[q_{MD}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta)] \\ &\quad - c_{MR}\,[q_{MR}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta)]\} \end{aligned} \tag{9}$$

$$\text{s. t.} \quad p_{MD} \ge c_M \tag{10}$$

$$p_{MR} \ge c_M \tag{11}$$

Similarly we can get the general objection function,

$$\begin{aligned} &E\pi_{MG}(p_{MD}, p_{MR}) \\ &= E\{p_{MD} \cdot q_{MD}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta) \\ &\quad + p_{MR} \cdot q_{MR}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta) \\ &\quad - c_M[\, q_{MD}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta) \\ &\quad + q_{MR}(p_{MR}, p_{DR}(p_{MD}, p_{MR}), \theta)] \end{aligned} \tag{12}$$

$$-c_{MD}\,[q_{MD}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]$$
$$-c_{MR}\,[q_{MR}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]\}$$
$$-\frac{1}{2}a_3(p_{MD}-c_M)^2-\frac{1}{2}a_4(p_{MR}-c_M)^2$$

where a_3 and a_4 are all constraint parameters, and $a_3, a_4 > 0$.

The fist-order condition of $E\pi_{MG}$ is as follows:

$$\begin{aligned}
&E\{q_{MD}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad+p_{MD}\cdot q_{MDT}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad+p_{MR}\cdot q_{MRT}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad-c_M[\,q_{MDT}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad+q_{MRT}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]\\
&\quad-c_{MDT}\,[q_{MD}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]\\
&\quad-c_{MRT}\,[q_{MR}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]\}\\
&\quad-a_3(p_{MD}-c_M)=0
\end{aligned} \tag{13}$$

$$\begin{aligned}
&E\{p_{MD}\cdot q_{MDM}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad+q_{MR}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad+p_{MR}\cdot q_{MRM}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad-c_M[\,q_{MDM}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)\\
&\quad+q_{MRM}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]\\
&\quad-c_{MDM}\,[q_{MDM}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]\\
&\quad-c_{MRM}\,[q_{MR}(p_{MR},p_{DR}(p_{MD},p_{MR}),\theta)]\}\\
&\quad-a_4(p_{MR}-c_M)=0
\end{aligned} \tag{14}$$

Where

q_{MDT} and q_{MRT} denote the derivatives of q_{MD} and q_{MR} with respect to p_{MD}, respectively;

q_{MDM} and q_{MRM} denote the derivatives of q_{MD} and q_{MR} with respect to p_{MR}, respectively;

c_{MDT} and c_{MRT} denote the derivatives of c_{MD} and c_{MR} with respect to p_{MD}, respectively;

c_{MDM} and c_{MRM} denote the derivatives of c_{MD} and c_{MR} with respect to p_{MR}, respectively.

From equation（13）,(14)and（8）, we can get the manufacturer's optimal prices which he sells the products through traditional and online channels, p^*_{MD} and p^*_{MR}, and the distributor's optimal price which he sells the products to customers, p^*_{DR}, in the Stackelberg game setting.

4 Numerical Example

Assume that the relationship between the demand via traditional channel (the quantity of products bought by the distributor from the manufacturer) and the influencing factors are as follows:

$$q_{MD}(p_{MR}, p_{DR}, \theta) = (k_{MD} - h_{MD} \cdot p_{DR}) + \lambda(p_{MR} - p_{DR}) + \theta \tag{15}$$

The relationship between the demand via online channel (the quantity of products bought by customers from the manufacturer) and the influencing factors are as follows:

$$q_{MR}(p_{MR}, p_{DR}, \theta) = (k_{MR} - h_{MR} \cdot p_{MR}) + \lambda(p_{DR} - p_{MR}) + \theta \tag{16}$$

Where

k_{MD} and k_{MR} denote the market bases through traditional and online channels, respectively;
h_{MD} and h_{MR} denote the marginal channel demand per respective channel through traditional and online channels, respectively;
λ denotes the shift between the two channels with regards to the price;
θ denotes uncertain factors which can't be controlled by manufacture and distributor, and it is a random variable which is Normal with mean 0 and variance 1;

The relationship between the transaction cost and the volume of transaction in online channel (i.e., between the manufacturer and customers), and the relationship between the transaction cost and the volume of transaction in traditional channel (i.e., between the manufacturer and the distributor) are as follows, respectively:

$$c_{MD}(q_{MD}) = b_{MD} \cdot q_{MD}^2 + d_{MD} \cdot q_{MD} \tag{17}$$

$$c_{MR}(q_{MR}) = b_{MR} \cdot q_{MR}^2 + d_{MR} \cdot q_{MR} \tag{18}$$

Where b_{MD}, d_{MD}, b_{MR} and d_{MR} are all parameters, and b_{MD}, d_{MD}, b_{MR}, $d_{MR} > 0$.

Let $C_M = 100$; $k_{MD} = 2000$; $k_{MR} = 2000$; $h_{MD} = 0.1$; $h_{MR} = 0.2$; $\lambda = 1$; $\mu = 0$; $\sigma^2 = 1$; $b_{MD} = 0.5$; $d_{MD} = 3$; $b_{MR} = 0.5$; $d_{MR} = 4$; $a_1 = a_2 = a_3 = a_4 = 97.8$.

And then the general expected profit functions of the manufacturer and the distributor are as follows, respectively:

$$E\pi_{DG}(p_{DR}) = (p_{DR} - p_{MD}) \cdot (2000 - 1.1p_{DR} + p_{MR}) \\ - 48.9(p_{MD} - 100)^2 - 48.9(p_{DR} - p_{MD})^2 \tag{19}$$

$$E\pi_{MG}(p_{MD}, p_{MR}) = (2000 + p_{MR} - 1.1p_{DR}) \\ \times (p_{MD} - 0.5p_{MR} + 0.55p_{DR} - 997) \\ + (2000 + p_{DR} - 1.2p_{MR}) \cdot (1.6p_{MR} - 0.5p_{DR} - 996) \\ + 20p_{MR} + 10p_{DR} - 400001 - 48.9(p_{MD} - 100)^2 \\ + 20p_{MR} + 10p_{DR} - 400001 - 48.9(p_{MD} - 100)^2 \\ - 48.9(p_{MR} - 100)^2 \tag{20}$$

Solving the model in the Stackelberg game setting, the optimal pricing strategy of the manufacturer as a leader is: the optimal price through traditional channel (i.e., sell the products to distributor) $p^*_{MD} \approx 111$; the optimal price through online channel (i.e., sell directly the products to customers) $p^*_{MR} \approx 124$.

The optimal pricing strategy of the distributor as a follower is: the optimal price of products sold by distributor to customers $p^*_{DR} \approx 131$.

5 Conclusion

Pricing strategies based on the Stackelberg game under the condition that both traditional and online channels exist are discussed in this paper. In the Stackelberg game setting, there is price competition between the manufacturer and the distributor, and the manufacturer as a leader determines the prices of products sold through traditional and online channels, and then the distributor as a follower selects the optimal price after knowing the manufacturer's decision. When the manufacturer solves the pricing model, he must consider the influence of his optimal pricing strategy on the distributor's decision, that is, the distributor's decision variable is the function of the manufacturer's decision variables. In this paper, we do an initial research on pricing strategies based on the Stackelberg game, and the comparison of pricing strategies based on the Stackelberg game to other settings is not done. In future research, we'll compare the pricing strategies under different settings so as to direct practice well.

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Packing Trees in Communication Networks*

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Abstract. Given an undirected edge-capacitated graph and a collection of subsets of vertices, we consider the problem of selecting a maximum (weighted) set of Steiner trees, each spanning a given subset of vertices without violating the capacity constraints. We give an integer linear programming (ILP) formulation, and observe that its linear programming (LP-) relaxation is a fractional packing problem with exponentially many variables and with a block (sub-)problem that cannot be solved in polynomial time. To this end, we take an r-approximate block solver to develop a $(1-\varepsilon)/r$ approximation algorithm for the LP-relaxation. The algorithm has a polynomial coordination complexity for any $\varepsilon \in (0,1)$. To the best of our knowledge, this is the first approximation result for fractional packing problems with only approximate block solvers and a coordination complexity that is polynomial in the input size and ε^{-1}. This leads to an approximation algorithm for the underlying tree packing problem. Finally, we extend our results to an important multicast routing and wavelength assignment problem in optical networks, where each Steiner tree is also to be assigned one of a limited set of given wavelengths, so that trees crossing the same fiber are assigned different wavelengths.

1 Introduction

Multicast is an efficient approach to deliver data from a source to multiple destinations over a communication network. This approach is motivated by emerging telecommunication applications, e.g., video-conferencing, streaming video and distributed computing. In particular, a multicast session is established by finding a Steiner tree in the network that connects the multicast source with all the multicast destinations.

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In this paper we address the following *Steiner tree packing* problem, that is fundamental in multicast communications. We are given a communication network represented by an undirected graph, a capacity associated with every edge in the graph, and a set of multicast *requests* (each defined by a subset of vertices to be connected, called *terminals*). A feasible solution to this problem is a set of Steiner trees, each spanning a multicast request, such that the number of Steiner trees crossing the same edge is bounded by the capacity of that edge. The goal is to maximize the total *profit/throughput* (the weighted sum of the successfully routed requests). It is worth noting that some requests may not be successfully routed due to the edge capacity. This problem arises in the communication network that provides multicast communication service to multiple groups of users in order to realize the routing that attains the maximum global profit for the whole network with limited bandwidth resources.

A special case in which the same request is spanned by the maximum number of edge-disjoint Steiner trees was studied in [10]. The authors presented a $4/|S|$-asymptotic approximation algorithm, where S is the terminal set to be connected. This problem was further studied in [14] and an algorithm was proposed to find $\lfloor \lambda_S(G)/26 \rfloor$ edge-disjoint Steiner trees, where $\lambda_S(G)$ is the size of a minimum S-cut in G. Another generalization is to find the maximum collection of Steiner forests spanning different requests [15]. There are also many applications in this category (see [14]). A related problem of realizing all given multicast requests as to minimize the maximum edge congestion was studied from theoretical and experimental aspects in [2, 5, 6, 12, 16, 23]. This is essentially equivalent to a routing problem in VLSI design [19]. Another related problem is that of realizing all given multicast requests at the minimum cost. This problem was studied in [4] and [13] for the special case of all Steiner trees connecting the same set of vertices, and in [22] for the general case where for each Steiner tree a different set of vertices is given.

We show that the relaxation of the Steiner packing problem is a *fractional packing problem* in Section 2, which has attracted considerable attention in the literature [7, 17, 25]. In general, a block solver is called to play a similar role to the separation oracle in the ellipsoid methods in [9]. The approximation algorithm in [7] is only for the case that the block problem is polynomial time solvable. In addition, the approximation algorithms in [17, 25] have coordination complexity depending on the input data, and are thus not polynomial in the input size. A problem related to fractional packing is the convex min-max resource-sharing problem, which is studied in [8, 24]. If the block problem is $\mathcal{NP}$-hard, an approximation algorithm is designed in [11] with polynomial coordination complexity.

To date, we are not aware of any approximation results for the Steiner tree packing problem in its full generality (where for each Steiner tree is required to connect a set of vertices). Furthermore, we are not aware of any approximation algorithm for fractional packing problems with coordination complexity polynomial in the input size while the block problem is $\mathcal{NP}$-hard.

The contribution of this paper can be summarized as follows. We formulate the Steiner tree packing problem in its full generality as an ILP, and observe that its

LP-relaxation is a fractional packing problem with exponentially many variables and an $\mathcal{NP}$-hard block problem. We thus develop a $(1-\varepsilon)/r$-approximation algorithm for fractional packing problems with polynomial coordination complexity, each iteration calling an r-approximate block solver, for $r \geq 1$ and any given $\varepsilon \in (0,1)$. This is the first result for fractional packing problems with only approximate block solvers and a coordination complexity strictly polynomial in input size and ε^{-1}. In fact, the coordination complexity of our algorithm is exactly the same as in [7] where the block problem is required to be polynomial time solvable. Then we present an algorithm for the Steiner tree packing problem and also apply our approximation algorithm for integer packing problems to establish a method to directly find a feasible solution. We extend our results to an important multicast routing and wavelength assignment problem in optical networks, where each Steiner tree is also to be assigned one of a limited set of given wavelengths, so that trees crossing the same fiber are assigned different wavelengths.

The remainder of this paper is organized as follows. In Section 2 we give an ILP formulation of the Steiner tree packing problem. Then we present and analyze the approximation algorithm for fractional packing problems in Section 3 and use it to develop an approximation algorithm for the integer Steiner tree packing problem in Section 4. The approach to directly find integer approximate solutions is discussed in Section 5. The multicast routing and wavelength assignment problem in optical networks is studied in Section 6. Finally, Section 7 concludes the paper. Due to the limit of space we do not give all proofs of our results in this version. We refer the readers to the full version of our paper [21] for details.

2 Mathematical Programming Formulation

We are given an undirected graph $G = (V, E)$ representing the input communication network, and a set of multicast requests $S_1, \ldots, S_K \subseteq V$ to be routed by Steiner trees. Each edge $e_i \in E$ is associated with a capacity c_i indicating the bandwidth of the corresponding cable. Denote by $\mathcal{T}_k$ the set of all Steiner trees spanning S_k, $k \in \{1, \ldots, K\}$. The number of trees $|\mathcal{T}_k|$ may be exponentially large. Furthermore, we define an indicator variable $x_k(T)$ for each tree as follows: $x_k(T) = 1$ if $T \in \mathcal{T}_k$ is selected for routing S_k; Otherwise $x_k(T) = 0$. In addition, each request S_k is associated with a weight w_k to measure its importance in the given multicast communication network. Therefore, the Steiner tree packing problem can be cast as the following ILP:

$$\begin{array}{lll} \max \sum_{k=1}^{K} w_k \sum_{T\in\mathcal{T}_k} x_k(T) & & \\ \text{s.t.} \sum_{k=1}^{K} \sum_{T\in\mathcal{T}_k \& e_i\in T} x_k(T) \leq c_i, & \forall e_i \in E; & \\ \sum_{T\in\mathcal{T}_k} x_k(T) \leq 1, & k = 1, \ldots, K; & \\ x_k(T) \in \{0,1\}, & \forall T \,\&\, k = 1, \ldots, K. & \end{array} \quad (1)$$

The first set of constraints in (1) means that the congestion of each edge is bounded by the edge capacity. The second set of constraints shows that at most one tree is selected to realize the routing for each request. It is possible that in a feasible solution, no tree is chosen for some requests, i.e., some requests may not be realized, due to the edge capacity constraints.

The special cases of the Steiner tree packing problem studied in this work have been shown $\mathcal{APX}$-hard [10, 14, 15], so is our underlying problem. There may be exponentially many variables in (1). Thus many exact algorithms such as standard interior point methods can not be applied to solve its LP-relaxation. The LP-relaxation of (1) may be solved by the volumetric-center [1] or the ellipsoid methods with separation oracle [9]. However, those approaches will lead to a large amount of running time.

As usual, we first solve the LP-relaxation of (1), and then apply rounding techniques to obtain a feasible solution. We call the linear relaxation of the Steiner tree packing problem as the *fractional Steiner tree packing problem*, and its solution as the *fractional solution* to the Steiner tree packing problem. The LP-relaxations of (1) is in fact a fractional packing problem [7, 17, 25]. However, the approximation algorithm in [7] is only for the case that the block problem is polynomial time solvable. Unfortunately, it is not the case for the Steiner tree packing problem as its block problem is the *minimum Steiner tree problem*. In addition, the approximation algorithms in [17, 25] both lead to complexity bounds that depend on the input data, and only result in pseudo polynomial time approximation algorithms. Thus, we need to study approximation algorithms for fractional packing problems with approximate block solvers and input data independent complexity.

3 Approximation Algorithm for Fractional Packing Problems

In this section, we develop an approximation algorithm for fractional packing problems based on the approach in [7]. Our algorithm allows that the block problem can only be approximately solved. Our complexity is still strictly polynomial in the input size and ε^{-1}, which is superior to the methods in [17, 25].

We consider the following fractional packing problem:

$$\max\{c^T x | Ax \le b, x \ge 0\}. \tag{2}$$

Here A is a $m \times n$ positive matrix, and $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$ are positive vectors. In addition, we assume that the (i,j)-th entry $A_{i,j} \le b_i$ for all i and j. The corresponding dual program is:

$$\min\{b^T y | A^T y \ge c, y \ge 0\}. \tag{3}$$

Similar to the strategies in [7, 8, 11, 17, 24, 25], an (approximate) block solver is needed, which is similar to the separation oracle for the ellipsoid methods. For a given $y \in \mathbb{R}^m$, the block problem is to find a column index q that $(A_q)^T y/c_q = \min_j (A_j)^T y/c_j$. In our algorithm, we assume that we are given the following

Table 1. Approximation algorithm for fractional packing problems

$\delta = 1 - \sqrt{1-\varepsilon}$, $u = (1+\delta)((1+\delta)m)^{-1/\delta}$, $k = 0$, $x^k = 0$, $f^k = 0$, $y_i^k = u/b_i$, $D^k = um$;
while $D^k < 1$ **do** {*iteration*}
  $k = k + 1$;
  call $ABS(y^{k-1})$ to find a column index q;
  $p = \arg\min_i b_i/A_{i,q}$;
  $x_q^k = x_q^{k-1} + b_q/A_{p,q}$;
  $f^k = f^{k-1} + c_q b_p/A_{p,q}$;
  $y_i^k = y_i^{k-1}[1 + \delta(b_p/A_{p,q})/(b_i/A_{i,q})]$;
  $D^k = b^T y^k$;
end do

approximate block solver $ABS(y)$ that finds a column index q that $(A_q)^T y/c_q \leq r \min_j (A_j)^T y/c_j$, where $r \geq 1$ is the approximation ratio of the block solver.

Our algorithm is an iterative method. We first maintain a pair of a feasible primal solution x to the fractional packing problem (2) and an infeasible dual solution y. At each iteration, based on the current dual solution y, the algorithm calls the approximate block solver once. Then the algorithm increases the component of the primal solution x corresponding to the returned column index by a certain amount and multiples the dual solution y by a factor larger than 1. This iterative procedure does not stop until the dual objective value is more than 1 (though the dual solution may be still infeasible). The algorithm is shown in Table 1. In the algorithm, D^k is in fact the dual objective value for the dual vector y_k at the k-th iteration, though it can be infeasible. Let $\mathcal{OPT}$ denote the optimum dual value (also the optimum objective value of the primal program according to the duality relation). In addition, we assume that the algorithm stops at the t-th iteration. We have the following bound:

Lemma 1. *When the algorithm stops,* $\mathcal{OPT}/f^t \leq r\delta/\ln(um)^{-1}$.

The solution x_t delivered by the algorithm could be infeasible and some packing constraints may be violated. Thus we need to scale the solution by an appropriate amount to obtain a feasible solution.

Lemma 2. *The scaled solution* $x_S = x^t/\log_{1+\delta}((1+\delta)/u)$ *is feasible for (2) and the corresponding objective value is* $f^t/\log_{1+\delta}((1+\delta)/u)$.

Now we are ready to show the performance bound of the solution:

Theorem 1. *When the algorithm stops, the scaled solution* x_S *is a* $(1-\varepsilon)/r$-*approximate solution to the fractional packing problem (2).*

Proof. According to the duality relation, the optimum dual value $\mathcal{OPT}$ is also the optimum objective value of the primal problem (2). Thus we need to examine the objective value corresponding to the feasible solution x_S. According to the definition $u = (1+\delta)((1+\delta)m)^{-1/\delta}$, we have $\ln(um)^{-1} = (1-\delta)\ln[m(1+\delta)]/\delta$

and $\ln((1+\delta)/u) = \ln[m(1+\delta)]/\delta$. Denote by $\mathcal{ALG}$ the objective value of the solution delivered by our algorithm. From the above relations and Lemma 1, the following bound holds:

$$\frac{\mathcal{ALG}}{\mathcal{OPT}} = \frac{f^t}{\mathcal{OPT}\log_{1+\delta}((1+\delta)/u)} \geq \frac{\ln(um)^{-1}}{r\delta}\frac{\ln(1+\delta)}{\ln((1+\delta)/u)} = \frac{(1-\delta)\ln(1+\delta)}{r\delta}$$

According to the elementary inequality $\ln(1+z) \geq z - z^2/2$ for any $0 \leq z \leq 1$, we have $r_{ALG} = \inf(\mathcal{ALG}/\mathcal{OPT}) \geq (1-\delta)(\delta - \delta^2/2)/(r\delta) \geq (1-\delta)^2/r = (1-\varepsilon)/r$.

Theorem 2. *There exists a $(1-\varepsilon)/r$-approximation algorithm for the fractional packing problem (2) that performs $O(m\varepsilon^{-2}\ln m)$ iterations, calling an r-approximate block solver once per iteration, for any $\varepsilon \in (0,1]$.*

Thus we have developed the first algorithm that find a $(1-\varepsilon)/r$-approximate solution to fractional packing problems (2) with a complexity polynomial in the input sizes and ε^{-1}, provided an approximate block solver. It is a generalization of the approximation algorithm in [7].

4 Approximation Algorithm for Steiner Tree Packing

We first study the LP-relaxation of (1).

Theorem 3. *There is a $(1-\varepsilon)/r$-approximation algorithm for the fractional Steiner tree packing problem with complexity $O((m+K)K\varepsilon^{-2}\beta\ln(m+K))$, where r and β are the approximation ratio and the complexity of the minimum Steiner tree solver called as an oracle, respectively.*

Proof. To use our approximation algorithm for fractional packing problems, the only problems are to identify the block problem and to find an (approximate) solver. Notice that the dual vector $y = (y_1, \ldots, y_m, y_{m+1}, \ldots, y_{m+K})^T$ consists of two types of components. The first $m = |E|$ components $y_1, \ldots, y_m$ corresponds to the edges $e_1, \ldots, e_m$. The remaining K components $y_{m+1}, \ldots, y_{m+K}$ in y corresponds to the second set of constraints in (1). It is easy to verify that the block problem is as follows: to find a tree T that $\min_k \min_{T\in\mathcal{T}_k}(\sum_{e_i\in T} y_i + y_{m+k}\delta_{k,T})/w_k$. Here the indicator $\delta_{k,T} = 1$ if $T \in \mathcal{T}_k$, and otherwise $\delta_{k,T} = 0$. To solve the block problem, one can search for K trees corresponding to the K requests separately, such that each tree routes a request with the minimum of $\sum_{e_i\in T} y_i$. Afterwards, for each of these K trees, the additional term y_{m+k} is added, and the sums are divided by w_k respectively. Thus the tree with the minimum value of $(\sum_{e_i\in T} y_i + y_{m+k}\delta_{k,T})/w_k$ over all K trees is selected, which is the optimum solution to the block problem. Since the value y_{m+k} is fixed for a fixed request k at each iteration, the block problem is in fact equivalent to finding a tree spanning the request S_k that $\min_{T\in\mathcal{T}_k}\sum_{e_i\in T} y_i$, for $k = 1, \ldots, K$. Regarding y_i the length of edge e_i for $i = 1, \ldots, m$, the block problem is in fact the minimum Steiner tree problem in graphs. Thus, we can use the approximation algorithm developed in Section 3 with an approximate minimum Steiner tree solver to obtain a feasible solution to the LP-relaxation of (1), and the theorem follows.

Unfortunately, the minimum Steiner tree problem is $\mathcal{APX}$-hard [3]. Thus, the approximation algorithm in [7] is not applicable in this case. We apply randomized rounding [18, 19] to find a feasible (integer) solution. As indicated in [18, 19], to guarantee non-zero probability that no constraint is violated, a scaling technique is necessary to be employed. Denote by c the minimum edge capacity. Suppose that there exists a scalar v satisfying $(ve^{1-v})^c < 1/(m+1)$. From [18], we can immediately obtain the following bound:

Theorem 4. *There is an approximation algorithms for the Steiner tree packing problem such that the objective value delivered is at least*

$$\begin{cases} (1-\varepsilon)vOPT/r - (\exp(1)-1)(1-\varepsilon)v\sqrt{OPT\ln(m+1)/r}, & \text{if } OPT > r\ln(m+1); \\ (1-\varepsilon)vOPT/r - \dfrac{\exp(1)(1-\varepsilon)v\ln(m+1)}{1+\ln\left(r\ln(m+1)/OPT\right)}, & \text{otherwise,} \end{cases}$$

where OPT is the optimal objective value of (1).

In (1) there are exponential number of variables. However, by applying our approximation algorithm for fractional packing problems in Section 3, we just need to generate K approximate minimum Steiner trees for the K requests at each iteration corresponding to the current dual vector. Thus there are only $O((m+K)K\varepsilon^{-2}\ln(m+K))$ Steiner trees generated in total. This is similar to the column generation technique for LPs, and the hardness due to exponential number of variables in (1) is overcome.

5 Integrality

A solution to the fractional packing problem (2) has *integrality* w if each components in the solution is a non-negative integer multiple of w. In this case we modify the approximation algorithm in Table 1 as follows: At the k-th iteration, after calling the approximate block solver, the increments of x and y are $x_k(q) = x_{k-1}(q) + w$ and $y_k(i) = y_{k-1}(i)\{1 + \delta[w]/[b(i)/A(i,q)]\}$, and the following result follows:

Theorem 5. *If $w \leq \min_{i,j} b_i/A_{i,j}$ in the fractional packing problem (2), then there exists an algorithm that finds a $(1-\varepsilon)/r$-approximate solution to (2) with integrality $w\delta/(1+\log_{1+\delta} m)$ within $O(m\varepsilon^{-2}\rho\ln m)$ iterations, where $\rho = \max_{i,j} b_i/A_{i,j}$.*

Corollary 1. *If $b_i/A_{i,j} \geq (1+\log_{1+\delta} m)/\delta$ for all i and j, then there exists an algorithm that finds a $(1-\varepsilon)/r$-approximate solution to integer packing problems within $O(m\varepsilon^{-2}\rho\ln m)$ iterations.*

Corollary 2. *If all edge capacities are at least $(1+\log_{1+\delta}(m+K))/\delta$, then there exists an algorithm that finds a $(1-\varepsilon)/r$-approximate integer solution to the Steiner tree packing problem (1) within $O((m+K)K\varepsilon^{-2}c_{\max}\beta\ln(m+K))$ time, where r and β are the approximation ratio and the complexity of the minimum Steiner tree solver called as the oracle, and $c_{\max}$ is the maximum edge capacity.*

We have presented a pseudo polynomial time approximation algorithm for integer packing problems. However, this approach is still useful, as it can directly lead to an integer solution and can avoid the rounding stage. We believe that there exist instances in practice that our approximation algorithm for integer packing problem works efficiently.

6 Multicast Routing and Wavelength Assignment in Optical Networks

In the multicast routing and wavelength assignment problem in optical networks, we are given an undirected graph $G = (V, E)$, a set of multicast requests $S_1, \ldots, S_K \subseteq V$, and a set $\mathcal{L} = \{1, \ldots, L\}$ of wavelengths. It is assumed that every edge represents a bundle containing multiple fibers in parallel. In particular, we let $c_{i,l}$ denote the number of fibers of edge $e_i \in E$ that have wavelength $l \in \mathcal{L}$. Note that wavelengths that are not available in a fiber are assumed to be pre-occupied by existing connections in the network. The goal is to find routing trees with the maximum total profit/throughput, such that every selected request is realized by a Steiner tree and assigned one of the given wavelengths, and that trees crossing the same fiber are assigned different wavelengths.

Denote by $\mathcal{T}_k$ the set of all trees spanning the request S_k, for all $k = 1, \ldots, K$. Here $|\mathcal{T}_k|$ could be exponentially large. Then we define an indicator variable $x_k(T, l)$ as follows: $x_k(T, l) = 1$ if $T \in \mathcal{T}_k$ is selected for routing S_k and is assigned wavelength l; Otherwise $x_k(T, l) = 0$. In addition, each request S_K is associated with a weight w_k indicating its importance in the given multicast optical network. Thus the ILP of the problem is as follows:

$$\begin{array}{lll} \max \sum_{k=1}^{K} w_k \sum_{l=1}^{L} \sum_{T \in \mathcal{T}_k} x_k(T, l) & & \\ \text{s.t. } \sum_{k=1}^{K} \sum_{T \in \mathcal{T}_k \& e_i \in T} x_k(T, l) \le c_{i,l}, & \forall e_i \in E \,\&\, l \in \mathcal{L}; & \\ \sum_{l=1}^{L} \sum_{T \in \mathcal{T}_k} x_k(T, l) \le 1, & k = 1, \ldots, K; & \\ x_k(T, l) \in \{0, 1\}, & \forall T,\, l \in \mathcal{L} \,\&\, k = 1, \ldots, K. & \end{array} \tag{4}$$

The first set of constraints ensures that each of these trees can be routed through a separate fiber. The second set of constraints indicate that the we just need to route each request by at most one tree and assign at most one wavelength to it.

First, for the fractional multicast routing and wavelength assignment problem, we have the following result (see [21] for proof):

Theorem 6. *There is a $(1 - \varepsilon)/r$-approximation algorithm for the fractional multicast routing and wavelength assignment problem in optical networks with complexity $O((mL + K)KL\varepsilon^{-2}\beta \ln(mL + K))$, where r and β are the approximation ratio and the complexity of the minimum Steiner tree solver called as the oracle, respectively.*

Similar to Section 4, for any real number v satisfying $(ve^{1-v})^c < 1/(m + 1)$, where $c = \min_{i,l} c_{i,l}$ is the minimal capacity, we can obtain a bound for the integer solution by randomized rounding [18, 19]:

Theorem 7. *There is an approximation algorithms for the multicast routing and wavelength assignment problem in optical networks such that the objective value delivered has the same bound as in Theorem 4, where OPT is the optimal objective value of (4).*

Furthermore, we can apply our approximation algorithm for integer packing problems described in Section 5 to (4) and directly obtain an integer solution:

Theorem 8. *If all edge capacities are at least* $(1 + \log_{1+\delta}(mL + K))/\delta$*, then there exists an algorithm that finds a* $(1 - \varepsilon)/r$*-approximate integer solution to the multicast routing and wavelength assignment problem in optical networks (4) within* $O((mL + K)LK\varepsilon^{-2}c_{\max}\beta \ln(mL + K))$ *time, where* r *and* β *are the approximation ratio and the complexity of the minimum Steiner tree solver called as the oracle, and* $c_{\max}$ *is the maximum capacity.*

7 Conclusions and Future Research

in this paper, we have addressed the problem of maximizing a Steiner tree packing, such that each tree connects a subset of required vertices without violating the edge capacity constraints. We have developed a $(1 - \varepsilon)/r$ approximation algorithm to solve the LP-relaxation provided an r-approximate block solver. This is the first approximation result for fractional packing problems with only approximate block solvers and a coordination complexity that is polynomial in the input size and ε^{-1}. This generalizes the well-known result in [7] while the complexity is the same, and it is superior to many other approximation algorithms for fractional packing problems, e.g., [17, 25]. In this way we have designed approximation algorithms for the Steiner tree packing problem. Finally, we have studied an important multicast routing and wavelength assignment problem in optical networks. We are further interested in both theoretical and practical extensions of this work. An interesting problem is to develop approximation algorithms for fractional/integer packing problems with other properties. From the practical point of view, we aim to develop more realistic models for routing problems arising in communication networks and design strategies to (approximately) solve them efficiently. We have applied our approximation algorithm for fractional packing problems to the global routing problem in VLSI design [20]. In addition, we are working on implementation of our approximation algorithms with challenging benchmarks to explore their power in computational practice.

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Truthful Algorithms for Scheduling Selfish Tasks on Parallel Machines

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Abstract. We consider the problem of designing truthful mechanisms for scheduling *selfish tasks (or agents)*–whose objective is the minimization of their completion times– on parallel identical machines in order to minimize the *makespan*. A truthful mechanism can be easily obtained in this context (if we, of course, assume that the tasks cannot shrink their lengths) by scheduling the tasks following the increasing order of their lengths. The quality of a mechanism is measured by its approximation factor (price of anarchy, in a distributed system) w.r.t. the social optimum. The previous mechanism, known as SPT, produces a $(2-1/m)$-approximate schedule, where m is the number of machines. The central question in this paper is the following: *"Are there other truthful mechanisms with better approximation guarantee (price of anarchy) for the considered scheduling problem?"* This question has been raised by Christodoulou et al [1] in the context of coordination mechanisms, but it is also relevant in centrally controlled systems. We present (randomized) truthful mechanisms for both the centralized and the distributed settings that improve the (expected) approximation guarantee (price of anarchy) of the SPT mechanism. Our centralized mechanism holds for any number of machines and arbitrary schedule lengths, while the coordination mechanism holds only for two machines and schedule lengths that are powers of a certain constant.

1 Introduction

The Internet is a complex distributed system where many entities wish to maximize their own profits. Protocols organize this network, and their aim is to maximize the social welfare. The underlying assumption is that the agents on which the protocols are applied are trustworthy. This assumption is unrealistic in some settings as the agents might try to manipulate the protocol by reporting false information in order to get some advantages. With false information, even the most efficient protocol may lead to unreasonable solutions if it is not designed to cope with the selfish behavior of the single entities.

In this paper, we deal with the problem of scheduling tasks on parallel identical machines in order to minimize the *makespan*, (also known as $P||C_{max}$). There are m identical machines and n tasks of arbitrary lengths, where each task is owned by an *agent*. The lengths of the tasks are known to their owner only.

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In the first part of the paper, we focus on the following process: at first the agents declare their lengths, then given these bids the system allocates the tasks to the machines. The objective of the system is to minimize the makespan, i.e. the date at which the last task finishes its execution. The aim of each agent is to minimize its completion time and thus an agent may lie if by doing so, she can improve its completion time.

The field of Mechanism Design can be useful to deal with the selfishness of the agents. Its main idea is to pay the agents to convince them to perform strategies that help the system to optimize a global objective function. The most famous technique for designing truthful mechanisms is perhaps the Vickrey-Clarke-Groves (VCG) mechanism [8, 9, 10]. However, when applied to combinatorial optimization problems, this mechanism guarantee the truthfulness under the hypothesis that the objective function is *utilitarian* (i.e. the objective function is equal to the sum of the agents' valuation) and that the mechanism is able to compute the optimum (for instance, it works for the shortest path problem [3]). Archer and Tardos introduce in [4] a method which allows to design truthful mechanisms for several combinatorial optimization problems to which the VCG mechanism does not apply. However, neither approach can be applied to our problem and thus we design a new ad-hoc mechanism that is able to retain truthfulness.

In the second part of the paper, we change our setting and we are interested to the development of a truthful *coordination mechanism* [1] for the same scheduling problem. The notion of coordination mechanism has been introduced in order to improve the performance of a system with independent selfish and non-colluding agents. In a *coordination mechanism*, we assume that the system designer can select the scheduling policies of each machine (e.g. each machine schedules its tasks in order of decreasing lengths), but the designer must design the system one and for all (i.e. it should not depend on the values bidded by the tasks). Another important and natural condition is the decentralized nature of the problem: the scheduling on a machine should depend only on the lengths of the tasks assigned to it and should be independent of the tasks' lengths assigned to the other machines. Knowing the coordination mechanism and the values bidded by the other tasks, each task chooses on which machine she will be scheduled, and she is then scheduled on it, according to the policy of this machine.

A truthful mechanism can be easily obtained (if we, of course, assume that the tasks cannot shrink their lengths) by scheduling the tasks following the increasing order of their lengths. This mechanism can also be adapted to a truthful coordination mechanism. This mechanism, known as SPT, produces a $(2-1/m)$-approximate schedule. The central question in this paper is the following: *"Are there other truthful mechanisms with better approximation guarantee (price of anarchy) for the considered scheduling problem?"*

1.1 Results in This Paper

Any algorithm with the property that increasing task length implies non-decreasing completion time will always be truthful, and this property is necessary for truthtelling to be a dominant strategy. Since there is no dominant strategy with an approximation ratio better than the one of SPT, we focus, like in [4], on randomized truthful mechanisms. Thus, we assume that each agent aims to maximize her *expected* profit. A mechanism is then called *truthful* if, for each agent, bidding her true schedule length maximizes her expected profit regardless of what the other agents bid.

In Section 3, we consider the selfish task allocation model and we give a centralized algorithm which is truthful even if the values of the lengths are not restricted, and has an expected approximation ratio of $2 - \frac{1}{m+1}\left(\frac{5}{3} + \frac{1}{3m}\right)$, which is smaller than the one of an SPT schedule (e.g. if $m = 2$ its approximation ratio is smaller than 1.39 whereas it is 1.5 for an SPT schedule).

In Section 4, we consider the two-machines case. We first study a *coordination mechanism* in which the first machine always schedules its tasks in order of increasing lengths, and the second machine schedules its tasks with a probability $p > \frac{2}{3}$ in order of increasing lengths and with probability $(1-p)$ in order of decreasing lengths. The expected approximation ratio of this (randomized) coordination mechanism, that we prove to be $\frac{4}{3} + \frac{p}{6}$, is better than the one of SPT ($\frac{3}{2}$). We show that this coordination mechanism is truthful if the tasks are powers of a constant larger than or equal to $\frac{4-3p}{2-p}$, but not if the values of the task lengths are not restricted. We also show that if $p < \frac{1}{2}$ then this coordination mechanism is not truthful even if the tasks are powers of any integer larger than 1. In Section 4.3, we consider the other randomized coordination mechanisms that combine deterministic coordination mechanisms in which the tasks are scheduled in order of increasing or decreasing lengths (and thus which have expected approximation ratios better than the one of SPT), and give negative results on their truthfulness.

1.2 Related Works

Scheduling with selfish agents have been intensively studied these last years started with the seminal work of Nisan and Ronen [3] and followed by a series of papers [4], [6], [5], [7]. However, all these works differ from our paper since in their case, the selfish agents were the machines while here we consider that the agents are the tasks.

The most closely related work is the one by Christodoulou et al [1] who considered the same model but only in the distributed context of coordination mechanisms. They proposed different coordination mechanisms with a price of anarchy better than the one of the SPT mechanism. Nevertheless, these mechanisms are not truthful.

2 Preliminaries

We are given m machines (or processors) and n tasks $T_1, \ldots, T_n$. Let l_i denote the execution time (or length) of task T_i. We will say that a task T_i is larger than a task T_j if and only if $l_i > l_j$ or ($l_i = l_j$ and $i > j$). The machines have the same speed, and the length of each task is known by an agent, its owner. Each agent declares a value b greater than or equal to the real length of the task (we make the assumption, like in [1] that the agents cannot shrink their lengths). The aim of each agent is to minimize its completion time, and an agent may lie if by doing so she can improve its completion time.

We consider two different models of execution:

- in the first one, used in Section 3, if T_i bids a value $b > l_i$, then its execution time remains l_i,
- in the second one, used in Section 4, we assume that if T_i bids a value $b > l_i$, then its execution time is b, i.e. T_i (or its owner) will not get the result of its execution before b time units after the beginning of the execution of T_i. This model of execution is called the *weak model of execution* in what follows.

We adopt the following definition of *randomized mechanism*: A randomized mechanism can be seen as a probability distribution over deterministic mechanisms, for instance given two deterministic mechanisms $M1$ and $M2$, with a probability p the mechanism will be $M1$ and with probability $(1 - p)$ it will be $M2$.

In the *centralized setting* (Section 3), the schedule will be obtained as follows: given the randomized mechanism, the agents will declare their lengths and the system will assign them to the machines following the deterministic mechanism $M1$ with probability p or $M2$ with probability $(1 - p)$.

In the *distributed setting* (Section 4), given the randomized mechanism, each task bids a value which represents its length, and then the selected deterministic coordination mechanism is announced to the tasks (it is $M1$ with probability p and $M2$ with probability $(1 - p)$). Each task chooses on which processor it will be scheduled, according to the policies of the processors: it goes on the processor on which it will minimize its expected completion time.

We say that a (randomized) mechanism is truthful if for every task the expected completion time when it declares its true length is smaller than or equal to its expected completion time in the case where it declares a larger value. More formally, we say that a mechanism M is *truthful* if $E_i(l_i) \leq E_i(b_i)$, for every i and $b_i \geq l_i$, where $E_i(b_i)$ is the expected completion time of task T_i if it declares b_i. In order to evaluate the quality of a randomized mechanism, we use the notion of expected approximation ratio (price of anarchy).

3 Truthful Centralized Mechanism

We give in this section a randomized mechanism for the centralized setting. The idea is to propose a new deterministic mechanism which when combined with a mechanism scheduling the tasks in the decreasing order of their lengths provides a truthful randomized mechanism.

3.1 Algorithm: LS_δ

Let us consider the following algorithm, denoted by SPT_δ in the sequel:

Let $\{T_1, T_2, \ldots, T_n\}$ be n tasks to be scheduled on $m \geq 2$ identical processors, $\{P_1, P_2, \ldots, P_m\}$. Let us suppose that $l_1 \leq l_2 \leq \cdots \leq l_n$.

Tasks are scheduled alternately on $P_1, P_2, \ldots, P_m$, in order of increasing length, and T_{i+1} starts to be executed when exactly $\frac{1}{m}$ of task T_i has been executed. Thus T_1 starts to be scheduled on P_1 at time 0, T_2 is scheduled on P_2 at time $\frac{l_1}{m}$, T_3 is scheduled on P_3 (on P_1 if $m = 2$) when $\frac{1}{m}$ of T_2 has been executed, i.e. at time $\frac{l_1}{m} + \frac{l_2}{m}$, and so forth...

The schedule returned by SPT$_\delta$ will be called a SPT$_\delta$ schedule in the sequel. Figure 1 shows an SPT$_\delta$ schedule, where $m = 3$.

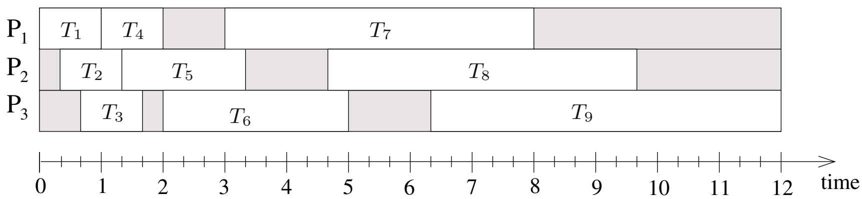

Fig. 1. SPT$_\delta$ schedule

Theorem 1. *SPT$_\delta$ is $2 - \frac{1}{m}$-approximate: the makespan of an SPT$_\delta$ schedule is smaller than or equal to $(2 - \frac{1}{m})OPT$, where OPT is the makespan of an optimal schedule for the same tasks.*

Proof: We have n tasks $T_1, \ldots, T_n$, such that $l_1 \leq \cdots \leq l_n$, to schedule on m processors. Each task T_i starts to be executed exactly when $\frac{1}{m}$ of T_{i-1} has been executed. So, if $n \leq m$, then the makespan of the SPT$_\delta$ schedule is: $\frac{1}{m}(l_1 + \cdots + l_{n-1}) + l_n \leq \frac{1}{m}(n-1)\, l_n + l_n \leq \frac{(2m-1)\, l_n}{m} \leq (2 - \frac{1}{m})\, l_n \leq (2 - \frac{1}{m})\, OPT$, since $l_n \leq OPT$.

Let us now consider the case where $n > m$. Let $i \in \{m+1, \ldots, n\}$. Task T_i starts to be executed when $\frac{1}{m}$ of T_{i-1} is executed, and T_{i-1} started to be executed when $\frac{1}{m}$ of T_{i-2} was executed, etc., $T_{(i-m)+1}$ started to be executed when $\frac{1}{m}$ of T_{i-m} was executed. So the idle time between T_i and T_{i-m} is $idle(i) = \frac{1}{m}(l_{i-m} + l_{i-m+1} + \cdots + l_{i-1}) - l_{i-m}$.

Let $i \in \{2, \ldots, m\}$. The idle time before T_i is equal to $idle(i) = \frac{1}{m}(l_1 + \cdots + l_{i-1})$, and there is no idle time before T_1, which starts to be executed at time 0. Thus, the sum of the idle times between tasks is $\sum_{i=2}^{n} idle(i) = \frac{1}{m}((m-1)l_{n-m+1} + (m-2)l_{n-m+2} + \ldots + l_{n-1})$.

Let $j \in \{n-m+1, \ldots, n-1\}$. Let $end(j)$ be the idle time in the schedule after the end of task T_j and before the end of T_n: $end(j) = l_{j+1} - \frac{m-1}{m}\, l_j + end(j+1)$, where $end(n) = 0$. So the sum of the idle times after the last tasks and before the end of the schedule is $\sum_{j=n-m+1}^{n-1} end(j) = (m-1)\left(l_n - \frac{m-1}{m}\, l_{n-1}\right) + (m-2)\left(l_{n-1} - \frac{m-1}{m}\, l_{n-2}\right) + \cdots + \left(l_{n-m+2} - \frac{m-1}{m}\, l_{n-m+1}\right)$.

The sum of the idle times on the processors, from the beginning of the schedule until the makespan, is the sum of the idle times between tasks (and before the first tasks), plus the sum of the idle times after the end of the last task of a processor and before the makespan. It is equal to $\sum_{i=2}^{n} idle(i) + \sum_{j=n-m+1}^{n-1} end(j) = \frac{1}{m}((m-1)l_{n-m+1} + (m-2)l_{n-m+2} + \cdots + l_{n-1}) + (m-1)\left(l_n - \frac{m-1}{m}\, l_{n-1}\right) + (m-2)\left(l_{n-1} - \frac{m-1}{m}\, l_{n-2}\right) + \cdots + \left(l_{n-m+2} - \frac{m-1}{m}\, l_{n-m+1}\right) = (m-1)\, l_n$.

Let ξ be the makespan of an SPT$_\delta$ schedule. ξ is the sum of the tasks plus the sum of the idle times, divided by m: $\xi = \frac{\left(\sum_{i=1}^{n} l_i\right)+(m-1)\, l_n}{m} = \frac{\sum_{i=1}^{n} l_i}{m} + \frac{(m-1)\, l_n}{m}$. Since $\frac{\sum_{i=1}^{n} l_i}{m} \leq OPT$ and $l_n \leq OPT$, we have: $\xi \leq (2 - \frac{1}{m})\, OPT$. □

Let us consider the following algorithm, denoted by LS_δ in the sequel: Let m be the number of processors. With a probability of $\frac{m}{m+1}$, the output schedule is an SPT$_\delta$ schedule; and with a probability $\frac{1}{m+1}$, the output schedule is an LPT schedule.

Theorem 2. *The expected approximation ratio of LS_δ is* $2 - \frac{1}{m+1}\left(\frac{5}{3} + \frac{1}{3\,m}\right)$.

Proof: The approximation ratio of an SPT$_\delta$ schedule is $2 - \frac{1}{m}$ (see Theorem 1), and the approximation ratio of an LPT schedule is $\frac{4}{3} - \frac{1}{3\,m}$ (see [2]). Thus the expected approximation ratio of LS_δ is $\frac{m}{m+1}\left(2 - \frac{1}{m}\right) + \frac{1}{m+1}\left(\frac{4}{3} - \frac{1}{3\,m}\right) = \frac{1}{m+1}\left(2m - 1 + \frac{4}{3} - \frac{1}{3m}\right) = \frac{1}{m+1}\left(2(m+1) - \frac{5}{3} - \frac{1}{3m}\right) = 2 - \frac{1}{m+1}\left(\frac{5}{3} + \frac{1}{3\,m}\right)$. □

3.2 Truthfulness

Theorem 3. *LS_δ is truthful.*

Proof: Let us suppose that we have n tasks $T_1, \ldots, T_n$, ordered by increasing lengths, to schedule on m processors. Let us show that any task T_i does not have incentive to bid a length higher than its true length. Let us suppose that task T_i bids $b > l_i$, and that, by bidding b, T_i is now larger than all the tasks $T_1, \ldots, T_x$, and smaller than T_{x+1}. In the LPT schedule, the tasks T_{x+1} to T_n are scheduled in the same way, whatever T_i bids (l_i or b). By bidding b, T_i can, at best, start $(T_{i+1} + \cdots + T_x)$ time units before than if it had bidded l_i. Thus the expected completion time of T_i in LS_δ decreases by at most $\frac{1}{m+1}\,(T_{i+1} + \cdots + T_x)$ time units when T_i bids b instead of l_i.

On the other hand, by bidding b instead of l_i, T_i will end later in the SPT$_\delta$ schedule: in this schedule, tasks from T_{i+1} to T_x will be started before T_i. Since a task T_j starts to be scheduled when $\frac{1}{m}$ of its predecessor T_{j-1} is executed, by bidding b, T_i starts $\frac{1}{m}\,(T_{i+1} + \cdots + T_x)$ time units later than if it had bidded l_i. Thus, the expected completion time of T_i in LS_δ is increased by $\frac{m}{m+1}(\frac{1}{m}$ $(T_{i+1} + \cdots + T_x)) = \frac{1}{m+1}\,(T_{i+1} + \cdots + T_x)$.

Thus, as a whole, the expected completion time of T_i cannot decrease when T_i bids a higher value than l_i, and we can deduce that LS_δ is truthful. □

Note that in the case where $m = 2$, the expected approximation ratio of LS_δ is $\frac{25}{18} < 1.39$. This algorithm is truthful, even in the case where the tasks can take any value, and it has a better approximation ratio than $SSL(p)$ introduced in Section 4 (but LS_δ is not a coordination mechanism because a processor has to know the tasks scheduled on the other processors).

We can also note that, since the approximation ratio of an SPT$_\delta$ schedule is $2 - \frac{1}{m}$ (like SPT) and the approximation ratio of an LPT schedule is $\frac{4}{3} - \frac{1}{3m}$, the schedule returned by LS_δ is, in the worst case, $2 - \frac{1}{m}$-approximate, which is not worse than the approximation ratio of an SPT schedule.

4 Truthful Coordination Mechanisms

4.1 Coordination Mechanism: $SSL(p)$

Let us first consider the following algorithm, denoted by $SSL(p)$ in the sequel:

Let $p \in \mathbb{R}$ such that $0 \leq p \leq 1$. With a probability of p, the output schedule is an SPT schedule: the tasks are greedily scheduled in order of increasing length. With a probability $(1-p)$, the output schedule is an SPT-LPT schedule: an SPT-LPT schedule is a schedule in which a processor, denoted by P_{SPT}, schedules the tasks in order of increasing lengths, and the other processor, denoted by P_{LPT}, schedules the tasks in order of decreasing lengths. A task T_i is scheduled on P_{SPT} if the total length of the tasks smaller than T_i is smaller than or equal to the total length of the tasks larger than T_i; otherwise it is scheduled on P_{LPT}.

We can easily transform the centralized algorithm $SSL(p)$ into a (randomized) coordination mechanism. Indeed, we can obtain, as showed in [1], an SPT-LPT schedule by having a processor, P_{SPT}, which schedules its tasks in order of increasing sizes and the other processor, P_{LPT}, which schedules its tasks in order of decreasing sizes. Thus, each task T_i will go on P_{SPT} if the total length of the tasks smaller than T_i is smaller than or equal to the total length of the tasks larger than T_i; otherwise T_i will have incentive to go on P_{LPT}. Likewise, we can obtain an SPT schedule by having two processors P_1 and P_2 which schedule tasks in order of increasing sizes, and P_2 which adds a little idle time ε (which we know to be smaller than the length of any task) before its first task, at the very beginning of the schedule. In this way, the smallest task will go on P_1, the second smallest on P_2, and so forth, and we will get the only possible Nash equilibrium, which is an SPT schedule. Hence, the coordination mechanism corresponding to $SSL(p)$ is the following one:

Let $p \in \mathbb{R}$ such that $0 \leq p \leq 1$. Let ε be a small number smaller than the length of every task. The first processor P_1 schedules, starting at time 0, its tasks in order of increasing sizes. The second processor P_2 schedules with a probability p its tasks in order of increasing sizes, starting its first task at time ε; and P_2 schedules, with a probability $(1-p)$, its tasks in order of decreasing sizes, starting its first task at time 0.

Theorem 4. *The expected approximation ratio of $SSL(p)$ is $\frac{4}{3} + \frac{p}{6}$.*

Proof: The approximation ratio of an SPT schedule is $\frac{3}{2}$ (see [2]), and the approximation ratio of an SPT-LPT schedule is $\frac{4}{3}$ (see [1]). Thus the expected approximation ratio of $SSL(p)$ is $p\,\frac{3}{2} + (1-p)\,\frac{4}{3}$, i.e. $p(\frac{3}{2} - \frac{4}{3}) + \frac{4}{3} = \frac{4}{3} + \frac{p}{6}$. $\square$

4.2 Truthfulness

In this section, we will use the weak model of execution, as explained in the Preliminaries. When we assume that all the tasks are powers of a constant C, then we assume that a task can only bid a value which is a power of C. If it was not the case (i.e. if a task bids a value which is not a power of C), we could round the value of this task to the nearest higher power of C.

Theorem 5. *Let $p \in \mathbb{R}$ and such that $\frac{2}{3} < p \leq 1$. Algorithm $SSL(p)$ is truthful if the tasks are powers of any constant $C \geq \frac{4-3p}{2-p}$.*

Proof: Let us suppose that we know that the tasks are powers of C, and thus they have to bid a value which is a power of C. Let us suppose that a task T_i, of length l_i, bids l_k ($l_k > l_i$). Let us show that the expected completion time of T_i is smaller when T_i bids l_i rather than l_k. Let $\Gamma = \{T_1, \dots, T_i, \dots, T_k, \dots, T_{n+1}\}$ be $n+1$ tasks (n tasks, plus a task T_k which represents the task T_i which bids l_k instead of l_i), and let us suppose that $l_1 \leq \dots \leq l_i \leq \dots \leq l_k \leq \dots \leq l_{n+1}$. If T_i bids l_i then the tasks we have to schedule are the tasks $\Gamma \setminus T_k$; if T_i bids l_k, then the tasks to be scheduled are $\Gamma \setminus T_i$ (thus T_k represents T_i in this case). $SSL(p)$ is truthful if, for every i, the expected completion time of T_i is smaller if it bids l_i than if it bids any other value $l_k > l_i$.

Thus, it is truthful if the *worst* expected completion time of T_i when it bids l_i is always smaller than the *best* completion time of T_i when it bids $l_k > l_i$. The worst expected completion time of T_i which bids l_i in an SPT schedule is $\frac{\sum_{j=1}^{i-1} l_j}{2} + l_i$: this is the case when T_i starts to be executed when all the smaller tasks have already been completed. The best expected completion time of T_i which bids l_k in an SPT schedule is $\frac{(\sum_{j=1}^{k} l_j) - l_i}{2}$: this is the case when T_k is completed at the same time as T_{k-1}.

There are two cases for T_i in the SPT-LPT schedule: it is either scheduled on P_{SPT} after the tasks which are smaller than l_i, and ends at time $\sum_{j=1}^{i} l_j$ (case 1), or it is scheduled on P_{LPT} after the tasks which are larger than l_i, and then ends at time $(\sum_{j=i}^{n+1} l_j) - l_k$ (case 2). It is the same thing in the case where T_i bids l_k: T_k is either scheduled on P_{SPT} and then ends at time $(\sum_{j=1}^{k} l_j) - l_i$ (case A), or it is scheduled on P_{LPT} and then ends at time $\sum_{j=k}^{n+1} l_j$ (case B). In the SPT-LPT schedule, T_i (resp. T_k) chooses between the cases 1 and 2 (resp. the cases A and B) the one that minimizes its completion time.

$SSL(p)$ is truthful if the worst completion time of T_i which bids l_i in an SPT schedule, times p, plus the completion time of T_i which bids l_i in an SPT-LPT schedule, times $(1-p)$, is smaller than the best completion time of T_i which bids l_k (T_i is then identified by T_k) in an SPT schedule, times p, plus the completion time of T_k in an SPT-LPT schedule, times $(1-p)$. Thus, $SSL(p)$ is truthful if:

$$p\left(\frac{\sum_{j=1}^{i-1} l_j}{2} + l_i\right) + (1-p)\left(\min\begin{cases}\sum_{j=1}^{i} l_j \\ (\sum_{j=i}^{n+1} l_j) - l_k\end{cases}\right)$$
$$\leq p\left(\frac{(\sum_{j=1}^{k} l_j) - l_i}{2}\right) + (1-p)\left(\min\begin{cases}(\sum_{j=1}^{k} l_j) - l_i \\ \sum_{j=k}^{n+1} l_j\end{cases}\right)$$
$$\Leftrightarrow (1-p)\left(\min\begin{cases}\sum_{j=1}^{i} l_j \\ \sum_{j=i}^{n+1} l_j - l_k\end{cases}\right) \leq p\,\frac{(\sum_{j=i}^{k} l_j) - 3\,l_i}{2} + (1-p)\left(\min\begin{cases}\sum_{j=1}^{k} l_j - l_i \\ \sum_{j=k}^{n+1} l_j\end{cases}\right)$$

There are now four cases to consider (the four combinations of the two choices of T_i and the two choices of T_k). Due to space limitations, we consider these four cases in the extended version of the paper. □

Figure 2 *Left* gives an illustration of Theorem 5: if we know that the tasks are powers of a constant larger than or equal to $C(p)$, then $SSL(p)$ is truthful. Figure 2 *Right* illustrates Theorem 4 and shows the expected approximation ratio of $SSL(p)$.

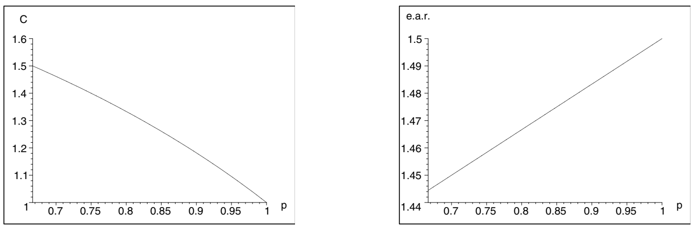

Fig. 2. *Left:* If the tasks are powers of a constant larger than or equal to $C(p)$ then $SSL(p)$ is truthful. *Right:* Expectation of the approximation ratio (e.a.r) of $SSL(p)$.

We saw that $SSL(p)$ is truthful if the tasks are powers of $C = \frac{4-3p}{2-p}$. In fact, the only sufficient condition we have for this algorithm to be truthful is that, for every i, $l_{i+1} = l_i$ or $l_{i+1} \geq C \times l_i$. Thus, if we know that the lengths of the tasks belong to a set $S = \{x_1, x_2, \ldots, x_k\}$ such that for each j, $x_{j+1} \geq C \times x_j$, then $SSL(p)$ is truthful. However, $SSL(p)$ is not truthful if the possible values of the tasks are not restricted, and it is not truthful if $p < \frac{1}{2}$, even if the tasks are powers of any integer $B > 1$ (the proofs of Theorems 6 and 7 can be found in the extended version of the paper).

Theorem 6. *Let* $p \in \mathbb{R}$ *be any number such that* $0 \leq p < 1$. *Algorithm* $SSL(p)$ *is not truthful if the tasks can take any value.*

Theorem 7. *Let* $p \in \mathbb{R}$ *be any number such that* $0 \leq p < \frac{1}{2}$. *Algorithm* $SSL(p)$ *is not truthful, even if the tasks are powers of an integer* B *(*$B > 1$*), whatever the value of* B *is.*

4.3 Other Coordination Mechanisms: Negative Results

$SL(p)$ is the algorithm where we have with a probability p an SPT schedule, and with a probability $(1-p)$ an LPT schedule. $LSL(p)$ is the algorithm where we have with a probability p an LPT schedule, and with a probability $(1-p)$ an SPT-LPT schedule. We saw in Section 4.1 that there exist coordination mechanisms which return an SPT or an SPT-LPT schedule. Likewise, by adding small delays on the processors - which both schedule the tasks in order of decreasing lengths -, the authors showed in [1] a coordination mechanism which returns an LPT schedule (the delays are here negligible since we can fix them as small as we want). Let us now give negative results on the truthfulness of this mechanisms. The proofs of Theorems 8, 9 and 10 can be found in the extended version of the paper.

Theorem 8. *Let $p \in \mathbb{R}$ be any number such that $0 \leq p < 1$. Algorithm $SL(p)$ is not truthful if the tasks can take any value.*

Theorem 9. *Let $p \in \mathbb{R}$ be any number such that $0 \leq p < \frac{1}{2}$. Algorithm $SL(p)$ is not truthful, even if the tasks are powers of a constant B ($B > 1$), whatever the value of B is.*

Theorem 10. *Let $p \in \mathbb{R}$ be any number such that $0 \leq p < 1$. Algorithm $LSL(p)$ is not truthful, even if the tasks are powers of a constant B ($B > 1$), whatever the value of B is.*

In the negative results of this section, we used the weak model of execution: we assume that if T_i bids a value $b > l_i$, then its new execution time is b. Of course, these results also hold for the second execution model, in which if T_i bids a value $b > l_i$, then its new execution time will still be l_i (T_i does not have to wait b time units after its start to get its result).

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Incentive Compatible Multiagent Constraint Optimization

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Abstract. We present in this paper an incentive-compatible distributed optimization method applied to social choice problems. The method works by computing and collecting VCG taxes in a distributed fashion. This introduces a certain resilience to manipulation from the problem solving agents. An extension of this method sacrifices Pareto-optimality in favor of budget-balance: the solutions chosen are not optimal anymore, but the advantage is that the self interested agents pay the taxes between themselves, thus producing no tax surplus. This eliminates unwanted incentives for the problem solving agents, ensuring their faithfulness.

1 Introduction

In this paper we concentrate on social choice problems, which are ubiquitous in our society. Typically, such problems include a set of public entities, that take some decisions based on the preferences expressed by a set of (private) agents. The goal is to adopt the set of decisions that best match the preferences expressed by the private agents, possibly subject to a set of feasibility constraints.

In systems with self interested agents, it is often a problem to guarantee optimal outcomes because each agent may have the incentive to manipulate the system in a way that is profitable to itself. Such manipulations steer the final outcome away from a global optimum which is otherwise achievable.

The VCG tax mechanism is a way to ensure that the agents in the system are always better off by declaring their true preferences, thus allowing for the optimal outcome to be chosen. The mechanism works by fining the participating agents with taxes which are proportional to the damage that they cause to others. Thus, the agents do not have an incentive to understate their valuations because the outcome chosen would not be the best for them, and do not overstate because that would induce a high amount of tax they would have to pay for hurting the others.

Traditionally, such mechanisms have been studied and applied in centralized systems. Feigenbaum and Shenker started in [5] a new line of research in *Distributed Algorithmic Mechanism Design (DAMD)*. DAMD is a fusion of the more traditional algorithmic-oriented AI and the recent interest in distributed computing and incentive compatibility in multiagent environments. They focus on multicast cost sharing and interdomain routing problems. This work is similar in spirit with theirs, but our focus is on constraint optimization applied to public decision problems.

Parkes and Shneidman present in [10] an approach for incentive compatible distributed computation. The goal in their approach is to distribute the computation to

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the self interested agents themselves, and take the computational burden off the center. They use VCG taxes to make *faithfulness* (see also [12]) an ex-post Nash equilibrium: the agents have the incentive to execute the correct algorithm, without manipulation. Their approach requires the presence of a center that selects an outcome, enforces it, and collects taxes. Trusted channels between the center and the agents are required such that the agents report their types to the center.

In our approach both the optimization itself, and the computation of VCG taxes are done in a distributed fashion by the agents controlling the public decision variables. We present also a budget balanced extension of the algorithm that eliminates all interest of the problem solving agents (public entities) in the problem, therefore ensuring faithfulness in the sense of [12].

In the following, we present in Section 2 some definitions and notation, in Section 3 the basic optimization algorithm that will be used in this paper, and in Section 4 a VCG-based extension of the basic optimization method. Section 5 presents a randomized algorithm that is budget balanced. Section 6 presents experimental results on meeting scheduling problems, and Section 7 concludes.

2 Definitions and Notation

Definition 1. *A discrete* multiagent constraint optimization problem *(MCOP) is a tuple* $< \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mathcal{R} >$ *such that:*

$\mathcal{A} = \{A_1, ..., A_n\}$ *is a set of selfish agents interested in the optimization problem;*
$\mathcal{X} = \{X_1, ..., X_m\}$ *is the set of public decision variables/solving agents;*
$\mathcal{D} = \{d_1, ..., d_m\}$ *is a set of finite domains of the variables* $\mathcal{X}$.
$\mathcal{C} = \{c_1, ..., c_q\}$ *is a set of constraints, where a constraint* c_i *is a function* $c_i : d_{i1} \times .. \times d_{ik} \rightarrow \{-\infty, 0\}$ *that returns 0 for all allowed combinations of values of the involved variables, and* $-\infty$ *for disallowed ones.*
$\mathcal{R} = \{r_1, ..., r_p\}$ *is a set of relations, where a relation* r_i^j *is a function* $d_{i1} \times .. \times d_{ik} \rightarrow \mathbb{R}$ *specified by agent* A_j *which denotes how much utility* A_j *assigns to each possible combination of values of the involved variables (negative values can be thought of as costs).* R_j *is the set of relations specified by agent* A_j.

This framework allowes us to model social choice-like problems, where a set of "public" agents $\mathcal{X}$ jointly choose an overall optimal outcome out of a set of possible solutions. The feasibility of a solution is decided by the constraints in $\mathcal{C}$, which are domain dependent, and are imposed by the agents $\mathcal{X}$. The choice between several feasible solutions is made according to the preferences of the "private" agents $\mathcal{A}$, stated through the relations $\mathcal{R}$. Formally, the optimal solution to such an optimization problem is a complete instantiation X^* of all variables in $\mathcal{X}$, s.t. $X^* = argmax_X(\sum_{r_i \in \mathcal{R}} r_i(X) + \sum_{c_i \in \mathcal{C}} c_i(X))$, where the values of r_i and c_i are their corresponding values for the particular instantiation X. Notice that the second sum is either $-\infty$ if X is an infeasible assignment, or 0 if it is feasible. We restrict our attention to problems that have feasible solutions. Notice that simply maximizing utility is sufficient to find the optimal, feasible solution, since if any of the constraints in $\mathcal{C}$ are unsatisfied, the resulting overall utility is $-\infty$.

In this paper we deal with unary and binary relations, being well known that higher arity relations can also be expressed in these terms with little modifications.

This framework is similar to a weighted CSP framework where we allow both positive and negative costs and we do utility maximization as opposed to cost minimization.

3 DPOP: A Dynamic Programming Algorithm for MCOP

Each agent $A_i \in \mathcal{A}$ has a set of preferences on the outcome of the optimization problem, expressed by the set of relations $R_i \subset \mathcal{R}$. The agents $\mathcal{A}$ declare their relations to the agents in $\mathcal{X}$ concerned by those relations. Afterwards, the agents X_i execute a distributed optimization procedure yielding an assignment $\mathcal{X}^*$ of the variables X_i that maximizes the overall utility for the agents $\mathcal{A}^*$.

The optimization algorithm of choice is *DPOP* ([11]). *DPOP* is an instance of the general bucket elimination scheme from [2], which is adapted for the distributed case, and uses a DFS traversal of the problem graph as an ordering.

For now, we assume the agents $\mathcal{A}$ declare their relations *truthfully*, therefore *DPOP* produces the correct optimal solution. In Section 4 we adapt the VCG mechanism to *DPOP* to ensure truthfulness.

DPOP has 3 phases. In the first phase (see Section 3.1), the pseudotree structure is established. One node is chosen among the nodes from $\mathcal{X}$, and a custom distributed DFS algorithm is initiated from that node. The second phase (see section 3.2) is a bottom-up utility propagation, and the third phase (see section 3.3) is a top-down value assignment propagation. A formal description (pseudocode) can be found at the end of section 3.3.

It has been proved in [11] that *DPOP* produces a linear number of messages. Its complexity lies in the size of the *UTIL* messages (the *VALUE* messages have linear size). The largest *UTIL* message produced by Algorithm 1 is space-exponential in the width of the pseudotree induced by the DFS ordering used.

3.1 Pseudotrees

Definition 2. *A pseudo-tree arrangement of a graph G is a rooted tree with the same nodes as G and the property that adjacent nodes from the original graph fall in the same branch of the tree (e.g. X_0 and X_{12} in Figure 1).*

As it is already known, a DFS (depth-first search) tree is also a pseudotree, although the inverse does not always hold. We thus use as pseudotree a DFS tree, generated through a distributed DFS algorithm. Due to lack of space, we can only sketch this algorithm here. The process is started from the root, and the nodes pass messages to their neighbors, adding themselves in the context of these messages. Whenever a node receives a message from a neighbor, with itself in the context, then pseudo parent/pseudo child relationships are established, otherwise parent/child relationships. The result of this algorithm is that all nodes consistently label each other as parent/child or pseudoparent/pseudochild.

Figure 1 shows an example of a pseudotree that we shall refer to in the rest of this paper. It consists of *tree edges*, shown as solid lines, and *back edges*, shown as dashed lines, that are not part of the DFS tree. We call a path in the graph that is entirely

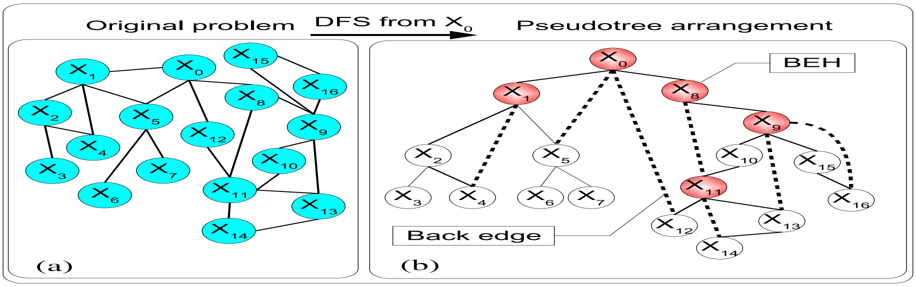

Fig. 1. A problem graph and one of its possible rooted DFS trees

made of tree edges, a *tree-path*. A *tree-path associated with a back-edge* is the tree-path connecting the two nodes involved in the back-edge (such a tree path is always unique, and included in a branch of the tree). For each back-edge, the higher node involved in that back-edge is called the *back-edge handler* - BEH (e.g. 0, 1, 8).

We define the following elements (refer to Figure 1):

Definition 3. ***P(*** X***)*** *- the parent of a node* X*: the single node on a higher level of the pseudotree that is connected to the node* X *directly through a tree edge (e.g.* $P(X_4) = X_2$*).* ***C(*** X***)*** *- the children of a node* X*: the set of nodes lower in the pseudotree that are connected to the node* X *directly through tree edges (e.g.* $C(X_1) = \{X_2, X_5\}$*).* ***PP(*** X***)*** *- the pseudo-parents of a node* X*: the set of nodes higher in the pseudotree that are connected to the node* X *directly through back-edges (*$PP(X_4) = \{X_1\}$*).* ***PC(*** X***)*** *- the pseudo-children of a node* X*: the set of nodes lower in the pseudotree that are connected to the node* X *directly through back-edges (e.g.* $PC(X_0) = \{X_5, X_{12}\}$*).*

3.2 Bottom-Up *UTIL* Propagation

Definition 4. $UTIL_i^j$ *- the UTIL message sent by agent* X_i *to agent* X_j*; this is a multidimensional matrix, with one dimension for each variable present in the context.* $dim(UTIL_i^j)$ *- the whole set of dimensions (variables) of the message (*$X_j \in dim(UTIL_i^j)$ *always). The semantics of such a message is similar to an n-ary relation having as scope the variables in the context of this message (its dimensions). The size of such a message is the product of the domain sizes of the variables from the context.*

Definition 5. *The* $\oplus$ *operator (join):* $UTIL_i^j \oplus UTIL_k^j$ *is the join of two UTIL matrices. This is also a matrix with* $dim(UTIL_i^j) \cup dim(UTIL_k^j)$ *as dimensions. The value of each cell in the join is the sum of the corresponding cells in the two source matrices.*

The semantics of this operation is the creation of a new relation between the union of the variables, equivalent to the two relations.

Definition 6. *The* $\perp$ *operator (projection): if* $X_k \in dim(UTIL_i^j)$*,* $UTIL_i^j \perp_{X_k}$ *is the projection through optimization of the* $UTIL_i^j$ *matrix along the* X_k *axis: for each tuple of variables in* $\{dim(UTIL_i^j) \setminus X_k\}$*, all the corresponding values from* $UTIL_i^j$ *(one for each value of* X_k*) are tried, and the best one is chosen. The result is a matrix with one less dimension (*X_k*).*

This projection has the semantics of a precomputation of the optimal utility achieved with the optimal values of X_k, for each instantiation of the other variables. It can also be seen as eliminating variable X_k and producing a new relation on the rest of the variables.

The *UTIL* propagation starts bottom-up from the leaves and propagates up to the root only through tree edges. The leaf nodes initiate this process, and then each node X_i relays these messages only to its parent:

- Wait for *UTIL* messages from all children. Perform join, project self out of the join and send the result to the parent.
- If root node, X_i receives all its *UTIL* messages as vectors with a single dimension, itself. It can then compute the optimal overall utility corresponding to each one of its values (by joining all the incoming *UTIL* messages) and pick the optimal value for itself (project itself out).

Top Down *UTIL* Propagation. After the bottom-up propagation, the root has global information, but all other nodes have accurate *UTIL* information only about their subtrees. We extend the *UTIL* propagation by making it *uniform*: now it also goes top-down, from each node to its children. A *UTIL* message from a parent to its child summarizes the utility information from all the problem except the subtree of that child.

If a node joins all the messages received from all its tree neighbors (parent and children), then that node obtains a global view of the system, thus becoming logically equivalent to the root. Projecting everything out (itself included) of this join gives the optimal value in the overall optimal solution.

The process is initiated by the root. Each X_i (root included) computes for each of its children X_j a $UTIL_i^j$ message. The computation is similar to the bottom-up one: *UTIL* messages from all neighbors except the respective child are joined, projections are applied, and the message is send to the child.

3.3 *VALUE* Propagation

The *VALUE* phase is a top-down propagation phase, initiated by the root after receiving all *UTIL* messages. Based on these *UTIL* messages, the root assigns itself the optimal value that maximizes the sum of utility of all its subtrees (overall utility). Then it announces its decision to its children and pseudochildren by sending them a *VALUE*($X_i \leftarrow v_i^*$) message.

Upon receipt of the *VALUE* message from its parent, each node is able to pick the optimal value for itself in a similar fashion, and then in its turn, send its *VALUE* messages. When the *VALUE* propagation reaches the leaves, all variables in the problem are instantiated to their optimal values, and the algorithm terminates.

4 VCG-Based Incentive Compatible Optimization Protocol

We now consider that the agents $A_i \in \mathcal{A}$ are self interested, and thus will try to adjust their declarations such that they obtain beneficial manipulations of the optimization process.

Algorithm 1: *DPOP - distributed pseudotree optimization procedure*

DPOP($\mathcal{X}, \mathcal{D}, \mathcal{C}, \mathcal{R}$): each agent X_i does:

Construct DFS tree; after completion, X_i knows $P(i), PP(i), C(i), PC(i)$

Bottom-up *UTIL* propagation protocol

wait for *UTIL* messages ($X_k, UTIL_k^i$) from all children $X_k \in C(i)$

$JOIN_i^{P(i)} = \left(\left(\bigoplus_{c\in C(i)} UTIL_c^i\right) \oplus \left(\bigoplus_{c\in\{P(i)\cup PP(i)\}} R_i^c\right)\right)$

if X_i *is root* **then** start *VALUE* propagation, and top-down *UTIL* propagation

else compute $UTIL_{X_i}^{P(i)} = JOIN_i^{P(i)} \perp_{X_i}$ and send it to P(i)

Top-down *UTIL* propagation protocol

foreach $X_k \in C(X_i)$ **do** compute $UTIL_i^k$ and send it to X_k

***VALUE* propagation protocol**

get and store in $agent\_view$ all *VALUE* messages ($X_k \leftarrow v_k^*$)

$v_i^* \leftarrow argmax_{X_i}\left(JOIN_{X_i}^{P(i)}[v(P(i)), v(PP(i))]\right)$

Send *VALUE*($X_i \leftarrow v_i^*$) to all $C(i)$ and $PC(i)$

It was shown in [6] that the only possible incentive compatible mechanism for optimization is of the form of a VCG mechanism ([13, 1, 7]). Ephrati and Rosenschein show in [3] for the first time how Clarke taxes can be used in multiagent systems for coordination problems in a way that induces incentive compatibility. We show in the following how to compute the Clarke taxes in a distributed fashion, and present a modified version of the *DPOP* algorithm that induces incentive compatibility.

Notice that since *DPOP* is a complete algorithm, it does not suffer from the non-truthfulness problem of approximate methods, as shown by Nisan and Ronen in [9]

To be able to compute the Clarke taxes, we need a mechanism that systematically leaves out an agent from the optimization process throughout the whole problem. We achieve this by simply including into each UTIL message that travels through the system, a corresponding part for each of the agents in $\mathcal{A}$. Namely, a $UTIL_i^j$ message sent by agent X_i to X_j is the union of n $UTIL_i^j(A_l)^*$ messages and n $UTIL_i^j(-A_l)$ messages (one for each agent A_l). The complexity of this scheme is thus $2\times n\times O(DPOP)$, where $n = |\mathcal{A}|$. A message $UTIL_i^j(A_l)^*$ is equivalent to a normal $UTIL_i^j$ message, but is computed by aggregating only the utility agent A_l obtains in the optimal solution. A message $UTIL_i^j(-A_l)$ is equivalent to the utility of all agents but A_l, and is computed by aggregating the utility of all agents except A_l, in the solution obtained by systematically ignoring A_l's relations.

When all propagations are completed, all agents X_i are able to compute the VCG taxes for all agents A_l. They do this as detailed in Algorithm 2, lines 8-13. In words, the tax that A_l has to pay equals the difference between the utility of the other agents in the solution when A_l is not present, and their utility when A_l is present.

We imagine that the agents X_i can split up between themselves the amount of tax that they collect from the agents A_l, in order to cover their costs for running the optimization process. Because the taxes are computed in a distributed fashion, and all agents $\mathcal{X}$ compute and receive the same $TAX(A_i)/m$ from the agents A_i it is more difficult for an agent X_j to simply claim exaggerated taxes from the agents A_i. Alternatively, the

Algorithm 2: *Truthful Distributed VCG-based optimization procedure*

ICDPOP($\mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mathcal{R}$)- changes from DPOP:

UTIL Propagation()

wait for *UTIL* messages $(X_k, UTIL_k^i)$ from all children $X_k \in C(i)$

foreach $A_l \in \mathcal{A}$ **do**

$JOIN_i^{P_i}(A_l)^* = \left(\bigoplus_{c \in C_i} UTIL_c^i(A_l)^*\right) \oplus \left(\bigoplus_{c \in \{P_i \cup PP_i\}} R_i^c(A_l) \oplus C_i^c\right)$

$JOIN_i^{P_i}(-A_l) = \left(\bigoplus_{c \in C_i} UTIL_c^i(-A_l)\right) \oplus \left(\bigoplus_{c \in \{P_i, PP_i\}} R_i^c(\mathcal{A} \setminus A_l) \oplus C_i^c\right)$

$UTIL_i^{P(i)}(A_l)^* = JOIN_i^{P(i)}(A_l)^* \perp_{X_i}$

$UTIL_i^{P(i)}(-A_l) = JOIN_i^{P(i)}(-A_l) \perp_{X_i}$

end

send $UTIL$ message to parent, $UTIL_i^j = \bigcup_{A_l} \{UTIL_i^{P(i)}(A_l)^*, UTIL_i^{P(i)}(-A_l)\}$

when last *UTIL* message arrives (from $P(X_i)$), execute Compute_taxes()

Compute_taxes()

foreach $A_l \in \mathcal{A}$ **do**

$JOIN(A_l)^* = \oplus_{X_j \in TreeNeighbors(X_i)} UTIL_j^i(A_l)^*$ (see section 3.2)

$JOIN(-A_m) = \oplus_{X_j \in TreeNeighbors(X_i)} UTIL_j^i(A_l)^{-A_m}$ (see section 3.2)

$UTIL(A_l)^* = JOIN(A_l)^* \perp_{\mathcal{X}}$

$UTIL(-A_m) = JOIN(-A_m) \perp_{\mathcal{X}}$

$TAX(A_l) = UTIL(-A_l) - \sum_{A_m \neq A_l} UTIL(A_m)^*$

cash in $\frac{TAX(A_l)}{m}$ from A_l

end

tax can be wasted ([3] shows that it must not return to the agents $\mathcal{A}$, otherwise incentive compatibility is broken).

The resulting algorithm is described in Algorithm 2. The *VALUE* phase is the same (as in DPOP, the optimal solution is chosen).

5 Budget Balanced VCG-Based Distributed Optimization

As it was already shown in game theory ([6, 8]), all mechanisms applied to general social choice problems that generate optimal outcomes must use a VCG-like tax, and cannot be budget balanced.

This poses sometimes a problem, since the collected tax can create undesired incentives for the entity collecting it (an auctioneer will introduce false bids to drive up the prices, a power plant operator will create artificial shortages, etc.) In our case, the agents $\mathcal{X}$ have the incentive to manipulate the optimization such that bad solutions are obtained when all the agents $\mathcal{A}$ are present, and good solutions are obtained when individual agents A_i are left out. The differences in utility translate into VCG taxes that they may collect afterwards. This problem can be solved either by throwing away the tax (utility is wasted), or by designing a budget balanced scheme that generates no tax surplus. Either one of these alternatives ensures *algorithm faithfulness* as defined by Shneidman and Parkes in [12], because the agents $\mathcal{X}$ no longer have any interest to cheat.

It has been shown in [4] that if one renounces Pareto optimality (not necessarily optimal solutions are generated), then it is possible to have a budget-balanced, incentive compatible protocol that generally generates good solutions. The basic idea is to randomly leave one agent out of the optimization process, and make the others pay their taxes to the one which was left out. The mechanism is obviously budget balanced, since the taxes are paid between the agents, but it is no longer Pareto optimal, because the solution obtained in the end is not the optimal one (the relations of the excluded agent were left out of the optimization). It was shown in [4] that the mechanism is also incentive compatible and individually rational. The solutions found are good overall, since only a single agent is excluded from the optimization. Also, the excluded agent gets the tax surplus from the other agents as a compensation for the possible loss that it incurred by not having its relations included in the optimization.

We adapt this idea to our case by having the nodes $\mathcal{X}$ select randomly an agent A_i who is going to be left out of the optimization process. This can be done at the same time as choosing the root of the DFS tree, using a similar mechanism. Alternatively, the agents $\mathcal{A}$ themselves can select one of them to be excluded. Subsequently, all the *UTIL* propagations are performed ignoring A_i's relations.

The process is similar to the previous one applied to a problem that does not include A_i. The differences from *ICDPOP* are listed in Algorithm 3. The solution obtained now is the optimum for $\mathcal{A} \setminus A_i$. Also, the taxes computed by the new algorithm are not collected by $\mathcal{X}$ anymore, but by A_i. Thus, the agents $\mathcal{X}$ do not have any interest to manipulate the process anymore. This holds unless collusion with a subset of agents $\mathcal{A}_i$ is possible. Collusion is a well-known problem of the VCG tax, so here we assume it is prevented by an external mechanism.

Algorithm 3: *Budget balanced distributed incentive compatible optimization*

BBICDPOP($\mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mathcal{R}$)- changes from ICDPOP, when agent A_k is excluded:

UTIL Propagation()

...

1 compute $UTIL$ msgs: $UTIL = \bigcup_{A_l \neq A_k} \{UTIL_i^{P(i)}(A_l)^*, UTIL_i^{P(i)}(-A_l, A_k)\}$

Compute_taxes()

foreach $A_l \in \{\mathcal{A} \setminus A_k\}$ **do**

...

2 $TAX(A_l) = UTIL(-A_l, A_k) - \sum_{A_m \neq A_l, A_k} UTIL(A_m)^*(-A_k)$

3 instruct A_l to pay $\frac{TAX(A_l)}{m}$ to A_k

end

6 Experimental Evaluation

We experimented with distributed meeting scheduling problems. These problems can be thought of as social choice problems if we have on one hand a set of agents $\mathcal{A}$ who want to schedule meetings, and on the other hand a set of agents $\mathcal{X}$ who will host these meetings. The COP model of such problems consists of a variable for each meeting,

Table 1. Evaluation on distributed meeting scheduling problems

Agents ($\lvert\mathcal{A}\rvert$)	Meetings ($\lvert\mathcal{X}\rvert$)	Width	Messages	Max size(DPOP)	Max size(ICDPOP)
10	4	2	24	64	1280
20	5	3	33	512	20480
30	14	3	95	512	30720
40	15	4	109	4096	320K
56	27	5	201	32768	3.5M
70	34	5	267	32768	4.375M
80	41	6	324	262144	40M
100	50	6	373	262144	50M

denoting its start time. There are inequality constraints between the meetings that share a participant (an agent A_i cannot participate in 2 meetings simultaneously). The agents $\mathcal{A}$ have preferences about the starting time of each meeting they participate in, stated through unary constraints on the respective variables.

Random meetings are generated, each with a certain utility for each agent. The agents $\mathcal{X}$ try to find the schedule that maximizes the overall utility for the agents $\mathcal{A}$.

Table 1 shows how our algorithm scales up with the size of the problems. The columns denote (in order): $|\mathcal{A}|$ is the number of self interested agents, $|\mathcal{X}|$ is the number of public decision variables, then the width of the resulting problems, the total number of messages sent during the algorithm, the maximal message size for simple *DPOP*, and the maximal message size for the VCG based *DPOP*.

As expected, we notice that the complexity increases across two dimensions: first the complexity of the underlying optimization problem (given by the induced width), and second, the number of self interested agents. The dependence of the complexity on the induced width produces very good results for loose problems, where the interests of the agents $\mathcal{A}$ are relatively decoupled. In these cases, where not all agents $\mathcal{A}$ are interested in all public variables $\mathcal{X}$, the resulting problems are loose, and easy to solve by the agents $\mathcal{X}$. The second complexity dimension can be observed by comparing consecutive rows that have the same induced width, but different numbers of self interested agents, e.g: rows 20-30, 56-70 and 80-100.

In any case, the fact that the algorithm produces a linear number of messages (even if they are big) is a great advantage in a distributed system, where a large number of small messages produce important overheads. For example, a backtracking based algorithm like a distributed branch and bound or ADOPT explore sequentially a large number of states, and produce an exponential number of small messages. This is why we think that a dynamic programming approach like *DPOP* is better suited for optimization tasks in distributed environments.

7 Conclusions and Future Work

We presented an incentive-compatible distributed optimization method, that computes and collects VCG taxes in a distributed fashion. We also present a budget-balanced extension of this method, that sacrifices Pareto-optimality. This eliminates unwanted incentives for the problem solving agents. We believe that this dynamic programming approach is a very good choice for multiagent systems, especially when the underlying problems are loosly connected.

As future work, we consider using approximate versions of *DPOP* to deal with difficult optimization problems, and computational complexity to counter the loss of incentive compatibility.

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Design of Incentive Compatible Mechanisms for Stackelberg Problems

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Abstract. This paper takes the first steps towards designing incentive compatible mechanisms for hierarchical decision making problems involving selfish agents. We call these Stackelberg problems. These are problems where the decisions or actions in successive layers of the hierarchy are taken in a sequential way while decisions or actions within each layer are taken in a simultaneous manner. There are many immediate applications of these problems in distributed computing, grid computing, network routing, ad hoc networks, electronic commerce, and distributed artificial intelligence. We consider a special class of Stackelberg problems called SLRF (Single Leader Rest Followers) problems and investigate the design of incentive compatible mechanisms for these problems. In developing our approach, we are guided by the classical theory of mechanism design. To illustrate the design of incentive compatible mechanisms for Stackelberg problems, we consider first-price and second-price electronic procurement auctions with reserve prices. Using the proposed framework, we derive some interesting results regarding incentive compatibility of these two mechanisms.

1 Mechanism Design and Stackelberg Problems

The *Theory of Mechanism Design* is an important discipline in the area of Welfare Economics. The area of Welfare Economics is concerned with settings where a policy maker faces the problem of aggregating the individual preferences into a collective (or social) decision and the individuals' actual preferences are not publicly known. The theory of mechanism design aims at studying how this privately held information can be elicited [2, 4, 7]. The state-of-the-art literature on mechanism design theory deals with situations where individuals are symmetric, that is to say, no single individual dominates the decision process. However, there are situations arising in Welfare Economics, Sociology, Engineering, Operations

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Research, Control Theory, and Computer Science where individuals take decisions in a hierarchical manner. A simple example is that of a situation wherein one of the individuals (or a group of individuals), called *leader(s)*, has the ability to enforce his preference on the other individual(s), called *follower(s)*. In such problems, the policy maker first invites the leaders to reveal their privately held information in a simultaneous manner. After receiving this information the policy maker broadcasts it among the followers and the followers respond to this by revealing their preferences in a simultaneous manner. After receiving the preferences from all the individuals, the policy maker aggregates the information into a social decision. The problem faced by the individuals in such a situation can be naturally modeled as a *Stackelberg game* following the seminal work of Stackelberg [15]. Following are some interesting examples where one can see these problems arising naturally.

- Task allocation in parallel/distributed systems
- Scheduling in grids
- Internet routing
- Admission, routing, and scheduling in telecom networks
- Flow control, routing, and sequencing in manufacturing systems
- Auctions in electronic commerce

In a recent work, Roughgarden [14] considered the problem of job shop scheduling, where fraction of the jobs are scheduled by a centralized authority (leader) and the remaining jobs are scheduled by selfish users (followers). He modeled this scheduling problem as Stackelberg game and showed that it is NP-hard to compute Stackelberg strategies. The underlying Stackelberg game in this problem is complete information game and there is no privately held information by the players which we require to elicit truthfully. Thus, the problem considered in this paper is about computing the optimal strategies of the leader and the followers and not about designing a mechanism for Stackelberg problem.

1.1 Contributions and Outline of the Paper

The major contributions of this paper are as follows.

- We investigate the mechanism design problem for SLRF games [1]. In this new framework, we define the notion of *Bayesian Stackelberg Incentive Compatible (BaSIC)* social choice functions. To the best of our knowledge, this is the first time mechanisms are being investigated in the context of Stackelberg problems.
- To illustrate our approach, we investigate the Bayesian Stackelberg incentive compatibility of *first-price and second-price procurement auctions with reserve prices*. We obtain two key results in this regard. The first result shows that in the first-price auction with reserve prices, the social choice function is BaSIC for the buyer but not for the sellers. The second result shows that in the second-price auction with reserve prices, the social choice function is BaSIC for the sellers but not for the buyer.

[1] We keep using the phrases SLRF games and SLRF problems interchangeably.

The organization of the paper is as follows. Section 2 presents a crisp review of relevant concepts in Stackelberg games. A more detailed treatment can be found in [1]. In Section 3, we motivate the Stackelberg mechanism design problems by means of two examples - *first-price procurement auction with reserve prices (F-PAR) and second-price procurement auction with reserve prices (S-PAR).* We then describe the problem of designing incentive compatible mechanisms for SLRF problems in Section 4. In Section 5, we state two important results concerning the Bayesian Stackelberg incentive compatibility of the two mechanisms F-PAR and S-PAR. Due to paucity of space, we are unable to provide the proofs for these results. Interested readers are urged to look into our recent technical report [3].

2 Stackelberg Games

2.1 Stackelberg Games with Incomplete Information

To begin with, we consider the following *noncooperative finite game with incomplete information* in its strategic form (also called a Bayesian game [11]):

$$\Gamma^b = (N, (C_i)_{i\in N}, (\Theta_i)_{i\in N}, (\phi_i)_{i\in N}, (u_i)_{i\in N})$$

where $N = \{1, 2, \ldots, n\}$ is a nonempty set of players, and, for each i in N, C_i is a nonempty set of actions available to player i; Θ_i is a nonempty set of possible types of player i; $\phi_i : \Theta_i \mapsto \Delta\Theta_{-i}$ is a belief function which gives the subjective probability of player i about the types of the other players for a given type of his own; and $u_i : C \times \Theta \mapsto \Re$ is the utility function of player i, where $C = \times_{j\in N} C_j$, $\Theta = \times_{j\in N}\Theta_j$, and $\Theta_{-i} = \times_{j\in N-\{i\}}\Theta_j$. Note that ΔS for any set S is the set of all probability distributions over S. A pure strategy s_i for player i in the Bayesian game Γ^b is defined to be a function from Θ_i to C_i.

In the above description of the game Γ^b, it is an implicit assumption that all the players choose their actions simultaneously. However, it is possible to impose an additional structure of hierarchical decision making on this game where agents choose their actions in a sequential manner as suggested by the hierarchy. The hierarchy is defined as a sequence of nonempty pairwise disjoint subsets of players, $H = H_1, H_2, \ldots, H_h$, where h represents the total number of levels in the hierarchy. In this setup, after learning their types, first, all the players in hierarchy level 1, i.e. H_1, choose their actions simultaneously. The actions (but not the types) chosen by all these players are announced publicly to the rest of the players. Next, all the players in hierarchy level 2, i.e. H_2, choose their actions simultaneously and again the chosen actions by all these players are announced publicly to the rest of the players. This process continues until all the players announce their actions.

A Bayesian game Γ^b together with hierarchical decision making can be called a *Bayesian Stackelberg (BS) game* and would have the following representation.

$$\Gamma^b_s = ((H_j)_{j=1,\ldots,k}, (C_i)_{i\in N}, (\Theta_i)_{i\in N}, (\phi_i)_{i\in N}, (u_i)_{i\in N})$$

If ($h = 2$), that is, the players are divided into two levels of hierarchy - H_1 and H_2, the games are referred to as *Leader-Follower Games*. The players in H_1 are called *leaders* and the players in H_2 are called *followers*. The followers, having observed the actions taken by the leaders, choose their actions in a simultaneous manner. Within the class of leader-follower games, there is an interesting subclass of games where H_1 is a singleton set. Such games are called *single leader rest followers (SLRF) games*. In an *SLRF Bayesian Stackelberg game*, one player is declared as the leader and after learning her type, she first takes her action. The action taken by the leader becomes common knowledge among the followers but her type remains unknown to the followers. Followed by the action of the leader, all the followers, *who have already learned their types*, take their actions simultaneously.

2.2 Pure Strategy Bayesian Stackelberg Equilibrium for SLRF Games

In this section, we would like to characterize the solution of the SLRF Bayesian Stackelberg games. The natural choice for the solution of such games is a combination of Stackelberg equilibrium and Bayesian Nash equilibrium. We call such a solution as Bayesian Stackelberg (BS) equilibrium. In what follows is a characterization of BS equilibrium.

1. **The Set of Followers' Optimal Response Strategy Profiles.** Let us assume that after learning her type $\theta_l \in \Theta_l$, the leader takes an action $c_l \in C_l$. For any such action $c_l \in C_l$, the set $R(c_l)$ below, which is a set of pure strategy profiles of the followers, is called as the set of *followers' optimal response (or rational reaction) strategy profiles.*

$$R(c_l) = \{s_{-l} \in \times_{j \in N-\{l\}} \times_{\Theta_j} C_j | v_{\theta_j}(c_l, c_j, s_{-l,j}) \leq v_{\theta_j}(c_l, s_{-l}) \\ \forall c_j \in C_j \; \forall \theta_j \in \Theta_j \; \forall j \in N - \{l\}\} \quad (1)$$

   where (c_l, s_{-l}) is an action-strategy profile of the players in which the leader takes an action c_l and the followers take actions as suggested by the corresponding pure strategy for them in the profile (s_{-l}). Similarly, $(c_l, c_j, s_{-l,j})$ is an action-strategy profile in which the leader takes an action c_l, follower $j \in N$ takes an action c_j and rest of the followers take actions as suggested by the corresponding pure strategy for them in the profile (s_{-l}). The quantities $v_{\theta_j}(c_l, s_{-l})$ and $v_{\theta_j}(c_l, c_j, s_{-l,j})$ are the expected payoffs to the player j when his type is θ_j, and the players follow the action-strategy profile (c_l, s_{-l}) and $(c_l, c_j, s_{-l,j})$, respectively. Note that risk averse and risk neutral followers will always play an optimal response against any action taken by the leader.
2. **Secure Strategy Set of Leader.** Assuming that $R(c_l)$ is nonempty for each $c_l \in C_l$, we call a strategy $s_l^* \in S_l$ of the leader l to be a secure strategy if it satisfies the following security constraint for each $\theta_l \in \Theta_l$:

$$s_l^*(\theta_l) = \arg \max_{c_l \in C_l} \min_{s_{-l} \in R(c_l)} \sum_{\theta_{-l} \in \Theta_{-l}} \phi_l(\theta_{-l}|\theta_l) u_l(c_l, s_{-l}(\theta_{-l}), (\theta_l, \theta_{-l})) \quad (2)$$

Note that a risk averse and risk neutral leader will always play a secure strategy. An implicit assumption behind the above relation is that the game Γ_s^b is a finite game. However, if we allow the sets Θ_i to be infinite, then the relation (2) gets modified in the following manner:

$$s_l^*(\theta_l) = \arg \max_{c_l \in C_l} \min_{s_{-l} \in R(c_l)} E_{\theta_{-l}}[u_l(c_l, s_{-l}(\theta_{-l}), (\theta_l, \theta_{-l}))|\theta_l] \qquad (3)$$

Further if we allow the sets $(C_i)_{i \in N}$ to be infinite then in the above relation, we need to replace max and min by sup and inf respectively.

3. **Bayesian Stackelberg Equilibrium.** A strategy profile $s^* = \left(s_l^*, t_{-l}^*\right)$ is said to be a Bayesian Stackelberg equilibrium if s_l^* is a secure strategy for the leader and $t_{-l}^* : C_l \mapsto \cup_{c_l \in C_l} R(c_l)$ is a rational reaction strategy of followers against s_l^*, that is $t_{-l}^*(s_l^*(\theta_l)) \in R(s_l^*(\theta_l)) \ \forall \theta_l \in \Theta_l$. One can also define the Bayesian Stackelberg equilibrium in mixed strategies for SLRF Bayesian Stackelberg games. However, we omit this because it is not required for this paper.

3 Stackelberg Mechanism Design Problems: Motivating Example

Consider an electronic procurement marketplace where a buyer b registers himself and wishes to procure a single indivisible object. There are n potential sellers, indexed by $i = 1, 2, \ldots, n$, who also register themselves with the marketplace. We make the following assumptions, which are quite standard in the existing literature on auction theory. A comprehensive discussion about these assumptions can be found in [9, 8, 17].

A1: Risk Neutral Bidders: The buyer and all the n sellers are risk neutral.

A2: Independent Private Value (IPV) Model: Each seller i, and buyer b draw their valuations θ_i, and θ_b, respectively (which can be viewed as their types) from distribution $F_i(.)$ and $F_b(.)$, respectively. $F_i(.), i = 1, 2, \ldots, n$ and $F_b(.)$ are mutually independent. Let $\Theta_i, i = 1, 2, \ldots, n$ and Θ_b denote the set of all possible types of the sellers and buyer, respectively. This implies that $F_i(.), i = 1, 2, \ldots, n$ and $F_b(.)$ are probability distribution functions of the random variables $\Theta_i, , i = 1, 2, \ldots, n$ and Θ_b, respectively.

A3: Symmetry among Sellers: The sellers are symmetric in following sense:
$\Theta_1 = \Theta_2 = \ldots = \Theta_n = \Theta \,; F_1(.) = F_2(.) = \ldots = F_n(.) = F(.)$

A4: Properties of $F(.)$ and Θ: We assume that $F(.)$ and Θ satisfy:
$\Theta = [\,\underline{\theta}\,, \overline{\theta}\,], \underline{\theta} > 0$
$F(.)$ is twice continuously differentiable
$f(\theta) = F^{'}(\theta) > 0; \forall \ \underline{\theta} \ \leq \theta \leq \overline{\theta}$

The marketplace first invites the buyer to report his type. Based on his actual type θ_b, the buyer first reports his type, say, $\hat{\theta_b} \in \Theta_b$ to the marketplace. The declared type of the buyer, that is $\hat{\theta_b}$, is treated as the price above which the buyer is not willing to buy the object. This price is known as reserve price. The marketplace publicly announces this reserve price among all the sellers. Now, the sellers are invited to submit their bids (or types) to the marketplace. Based on actual type θ_i, each seller i bids (or reports) his type say $\hat{\theta}_i \in \Theta_i$. After receiving the bids from all the sellers, the marketplace determines the winning seller, the amount that will be paid to him, and the amount that will be paid by the buyer. These are called as *winner determination* and *payment rules.* In E-commerce, such a trading institution is known as *procurement auction with reserve prices (PAR).* It is easy to see that the above problem is a Stackelberg problem and designing the winner determination and payment rules for this problem in a way that there is no incentive for buyer and sellers to reveal untruthful valuation is essentially a problem of designing an incentive compatible mechanism for Stackelberg problem. Depending on what winner determination and payment rules are employed by the marketplace, it may affect the incentive compatibility property. Following are two well known and existing mechanisms for PAR.

1. **First-Price Procurement Auction with Reserve Prices (F-PAR):** In this setting, the marketplace first discards all the received bids that fall above the reserve price announced by the buyer. Next, the seller whose bid is the lowest among the remaining bids is declared as the winner. The winner transfers the object to the buyer and the buyer pays to the winning seller an amount equal to his bid, that is $\hat{\theta}_i$. If there is no bid below the reserve price then no deal is struck. On the other hand, if there is a tie among the winning bids then the winner is chosen randomly, where each of the lowest valued bids has an equal chance of winning.
2. **Second-Price Procurement Auction with Reserve Prices (S-PAR):** In this setting, the winner determination rule is the same as F-PAR but the payment rules are slightly different. The winning seller transfers the object to the buyer and the buyer pays to him an amount equal to second lowest valued bid, if such a bid exists, otherwise an amount equal to the reserve price. Further, if there is no bid below the reserve price then no deal is struck. If there is a tie among winning bids, the winner is chosen randomly, where each of the lowest bids has an equal chance of winning.

It is easy to see that if the buyer announces a reserve price of $\hat{\theta}_b = \overline{\theta}$, then the procurement auction with reserve price will simply become the classical version of procurement auction with no reserve price.

4 Mechanism Design for SLRF Problems

A mechanism can be viewed as an institution, which a social planner deploys, to elicit the information from the agents about their types and then aggregate this information into a social outcome. Formally, a mechanism for an SLRF problem is a collection of action sets $(C_1, \ldots, C_n)$ and an outcome function $g : \times_{i \in N} C_i \mapsto X$, that is $\mathcal{M}_{\text{SLRF}} = ((C_i)_{i \in N}, g(.))$. A mechanism $\mathcal{M}_{\text{SLRF}}$ combined with possible types of the agents $(\Theta_1, \ldots, \Theta_n)$, probability density $\phi(.)$, Bernoulli utility functions $(u_1(.), \ldots, u_n(.))$, and description of leader agent l defines a Bayesian Stackelberg game Γ_s^b which is induced among the agents when the social planner invokes this mechanism as a means to solve the SLRF problem. The induced Bayesian Stackelberg game Γ_s^b is given by:

$$\Gamma_s^b = (\{l\}, N - \{l\}, (C_i)_{i \in N}, (\Theta_i)_{i \in N}, \phi(.), (\overline{u_i})_{i \in N})$$

where $\overline{u_i} : C \times \Theta \mapsto \Re$ is the utility function of agent i and is defined in the following manner: $\overline{u_i}(c, \theta) = u_i(g(c), \theta_i)$, where, we recall that $C = \times_{i \in N} C_i$, and $\Theta = \times_{i \in N} \Theta_i$.

4.1 Social Choice Function

A social choice function is a function $f : \times_{i \in N} \Theta_i \mapsto X$, which a social planner or policy maker uses to assign a collective choice $f(\theta_1, \ldots, \theta_n)$ to each possible profile of the agents' type $\theta = (\theta_1, \ldots, \theta_n)$. The set X is known as collective choice set or outcome set. For example, in the context of PAR, an outcome may be represented by a vector $x = (y_b, y_1, \ldots, y_n, t_b, t_1, \ldots, t_n)$, where $y_b = 1$ if the buyer receives the object, $y_b = 0$ otherwise, and t_b is the monetary transfer received by the buyer. Similarly, $y_i = -1$ if the seller i is the winner, $y_i = 0$ otherwise, and t_i is the monetary transfer received by the seller i. The set of feasible alternatives is then

$$X = \{(y_b, y_1, \ldots, y_n, t_b, t_1, \ldots, t_n) | y_b + \sum_{i=1}^{n} y_i = 0, t_b + \sum_{i=1}^{n} t_i \leq 0\}$$

In view of the above description, the general structure of the social choice function for PAR is

$$f(\theta_b, \theta) = (y_b(\theta_b, \theta), y_1(\theta_b, \theta), \ldots, y_n(\theta_b, \theta), t_b(\theta_b, \theta), t_1(\theta_b, \theta), \ldots, t_n(\theta_b, \theta)) \quad (4)$$

where $\theta = (\theta_1, \ldots, \theta_n)$. Note that $y_b(.)$, and $y_i(.)$ depend on the winner determination rule whereas $t_b(.)$ and $t_i(.)$ depend on the payment rule.

4.2 Implementing a Social Choice Function in Bayesian Stackelberg Equilibrium

We say that the mechanism $\mathcal{M}_{\text{SLRF}} = ((C_i)_{i \in N}, g(.))$ implements the social choice function $f : \times_{i \in N} \Theta_i \mapsto X$ in Bayesian Stackelberg equilibrium if there is a pure strategy Bayesian Stackelberg equilibrium $s^* = \left(s_l^*, t_{-l}^*\right)$ of the game Γ_s^b induced by $\mathcal{M}_{\text{SLRF}}$ such that $g\left(s_l^*(\theta_l), (t_{-l}^*(s_l^*(\theta_l)))(\theta_{-l})\right) = f\left(\theta_l, \theta_{-l}\right) \; \forall \left(\theta_l, \theta_{-l}\right) \in \times_{i \in N} \Theta_i$.

By making use of the definition of Bayesian Stackelberg equilibrium, we can say that $s^* = \left(s_l^*, t_{-l}^*\right)$ is a pure strategy Bayesian Stackelberg equilibrium of the game Γ_s^b induced by the mechanism $\mathcal{M}_{\text{SLRF}}$ iff leader Plays a secure strategy and followers play an optimal response.

4.3 Bayesian Stackelberg Incentive Compatibility

1. **Bayesian Stackelberg Incentive Compatibility for the Leader.** An SCF $f(.)$ is said to be Bayesian Stackelberg incentive compatible (BaSIC) for the leader (or truthfully implementable in BS equilibrium for the leader) if the direct revelation mechanism $\mathcal{D}_{\text{SLRF}} = ((\Theta_i)_{i \in N}, f(.))$ has a *BS equilibrium* $s^* = (s^*_l, t^*_{-l})$ in which $s^*_l(\theta_l) = \theta_l, \forall \theta_l \in \Theta_l$. That is, truth telling is a BS equilibrium strategy for the leader in the game induced by $\mathcal{D}_{\text{SLRF}}$.
2. **Bayesian Stackelberg Incentive Compatibility for the Followers.** An SCF $f(.)$ is said to be Bayesian Stackelberg incentive compatible (BaSIC) for the followers (or truthfully implementable in BS equilibrium for the followers) if the direct revelation mechanism $\mathcal{D}_{\text{SLRF}} = ((\Theta_i)_{i \in N}, f(.))$ has a *BS equilibrium* $s^* = (s^*_l, t^*_{-l})$ in which $t^*_{-l}(\theta_l) = ((s^*_j)_{j \in N-l}) \ \forall \theta_l \in \Theta_l$, where $s^*_j(\theta_j) = \theta_j \ \forall \theta_j \in \Theta_j \ \forall j \in N - \{l\}$. That is, truth telling is a BS equilibrium strategy for the followers in the game induced by $\mathcal{D}_{\text{SLRF}}$.
3. **Bayesian Stackelberg Incentive Compatibility.** An SCF $f(.)$ is said to be Bayesian Stackelberg incentive compatible (BaSIC) if it is BaSIC for both the leader and the followers.

5 Incentive Compatibility of Reserve Price Procurement Auctions

In this section, we state two key results pertaining to the Bayesian Stackelberg incentive compatibility of the social choice functions for first-price and second-price procurement auctions with reserve prices that were defined earlier in Section 3. Due to paucity of space, we are unable to include the proofs for these results. We urge the interested reader to refer to our recent technical report [3].

Theorem 1. Under the assumptions **A1 - A4**, the social choice function for the first-price procurement auction with reserve prices is BaSIC for the buyer but is not BaSIC for the sellers. The BS equilibrium of the BS game induced by this function among the sellers and the buyer is given by $s^* = (s^*_b, t^*_{-b})$ where

$$s^*_b(\theta_b) = \theta_b \ \forall \theta_b \in \Theta_b = [\, \underline{\theta} \,, \overline{\theta} \,]$$
$$t^*_{-b}(\hat{\theta}_b) = (s^*(.), \ldots, s^*(.)) \ \forall \hat{\theta}_b \in \Theta_b = [\, \underline{\theta} \,, \overline{\theta} \,]$$

That is, in the BS equilibrium, the buyer announces his true valuation itself as the reserve price, and for any announced reserve price $\hat{\theta}_b$, the sellers bid as suggested by the (symmetric) BN equilibrium strategy profile $(s^*(.), \ldots, s^*(.))$, where

$$s^*(\theta_i) = \begin{cases} \theta_i & : \quad \theta_i \in [\hat{\theta}_b, \ \overline{\theta} \,] \\ \theta_i + \frac{1}{[1-F(\theta_i)]^{n-1}} \int_{\theta_i}^{\hat{\theta}_b} [1 - F(x)]^{n-1} dx & : \quad \theta_i \in [\, \underline{\theta} \,, \hat{\theta}_b \,] \end{cases}$$

Proof: The proof of this result is based on a systematic analysis of different possible scenarios for bidding by the sellers after the reserve price is made known to the sellers. The analysis leads to quite interesting insights. For details of the proof, refer to [3].

Corollary. If $\hat{\theta}_b = \overline{\theta}$, then F-PAR will be the same as the traditional first-price procurement auction with no reserve price.

Theorem 2. Under the assumptions **A1 - A4**, the social choice function for the second-price procurement auction with reserve prices is BaSIC for the followers but is not BaSIC for the leader. The BS equilibrium of the BS game induced by this function among the sellers and the buyer is given by $s^* = (s_b^*, t_{-b}^*)$ where

1. $s_b^*(.)$ is the solution of the differential equation $F(s_b^*(\theta_b)) = (\theta_b - s_b^*(\theta_b)) f(s_b^*(\theta_b))$ with boundary condition $s_b^*(\underline{\theta}) = \underline{\theta}$
2. $t_{-b}^*(\hat{\theta}_b) = (s^*(.), \ldots, s^*(.))$ $\forall \hat{\theta}_b \in \Theta_b = [\ \underline{\theta}\ , \overline{\theta}\ ]$ where $s^*(\theta_i) = \theta_i \quad \forall \theta_i \in \Theta_i = [\ \underline{\theta}\ , \overline{\theta}\ ]$

That is, the buyer always announces $s_b^*(\theta_b)$ as the reserve price if his true valuation is θ_b. For any reserve price $\hat{\theta}_b$ announced by the buyer, the sellers always bid their true valuation.

Proof: The proof of this result is also based on a systematic analysis of different possible scenarios for bidding by the sellers after the reserve price is made known to the sellers. The analysis leads to quite interesting insights. For details of the proof, refer to [3].

Corollaries and Insights

1. The optimal reserve price strategy of the buyer in S-PAR when all the sellers draw their types independently from uniform distribution over the set $[0, 1]$ is given by $s_b^*(\theta_b) = \theta_b/2$.
2. Announcing $s_b^*(\theta_b)$ as the reserve price is a better strategy for the buyer in S-PAR than announcing true type as the reserve price.
3. Announcing the true type as the reserve price is a better strategy for the buyer than always fixing $\overline{\theta}$ as the reserve price. Another interpretation of this result is that buyer will be better off in S-PAR if he fixes his true type as reserve price as compared to having no reserve price.
4. If the buyer announces his true type as the reserve price then for any given type θ_b, his expected payoff and the expected revenue paid by him is the same as in F-PAR. The classical Revenue Equivalence theorem [16, 12, 6, 5, 10, 13] can be derived as a special case of the above result.

6 Summary

In this paper we have taken the first steps in extending the classical mechanism design theory to Stackelberg problems in general and SLRF problems in particular. These problems are natural in areas such as distributed computing, grid computing, network routing, ad hoc networks, electronic commerce, and distributed artificial intelligence. To illustrate the approach, we have taken two examples from the domain of electronic commerce - first-price and second-price procurement auctions with reserve prices and investigated the Bayesian Stackelberg incentive compatibility of these two mechanisms. To the best of our knowledge, this is the first attempt in the direction of designing incentive compatible mechanisms for Stackelberg problems. This opens up an avenue for solving many important problems, for example:

– designing fair pricing schemes for ad hoc and wireless networks
– developing fair routing algorithms in wireless ad hoc networks
– designing efficient scheduling policies in the grid computing environment.

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Proportional QoS in Differentiated Services Networks: Capacity Management, Equilibrium Analysis and Elastic Demands

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Abstract. The Differentiated Services (Diffserv) architecture is a scalable solution for providing Quality of Service (QoS) over packet switched networks. By its very definition, Diffserv is not intended to provide strict performance guarantees to its subscribers. We purpose in this paper a particular form of *relative* performance guarantees. Specifically, the network manager's goal is to maintain pre-defined ratios between common congestion measures over the different service classes. We assume that each service class is advertised with a constant price. Thus, in order to induce its goal, the manager dynamically allocates available capacity between the service classes. This scheme is studied within a network flow model, with self-optimizing users, where each user can choose the amount of flow to ship on each service class according to its service utility and QoS requirements. We pose the entire problem as a *non-cooperative game*. Concentrating on a simplified single-link model with multiple service classes, we establish the existence and uniqueness of the Nash equilibrium where the relative performance goal is obtained. Accordingly, we show how to compute and sustain the required capacity assignment. The extension to a general network topology is briefly outlined.

1 Introduction

Background and Motivation. The need for providing service differentiation over the Internet has been an ongoing concern in the networking community. The Differentiated Services (Diffserv) architecture [4] has been proposed by the IETF as a scalable solution for QoS provisioning. Instead of reserving resources per session (e.g., as in the Integrated Services (IntServ) model [22]), packets are marked to create a smaller number of packet classes, which offer different service qualities. The Diffserv proposal suggests to combine simple priority mechanisms at the network core with admission control mechanisms at the network edges only, in order to create diverse end-to-end services.

The two principal Diffserv classes that have been formalized are the Expedited Forwarding (EF) [7] and the Assured Forwarding (AF) [10] services. The premise of the EF is to provide no-loss and delay reduction to its subscribers. AF is intended for users who need reliable forwarding even in times of network congestion. Ongoing IETF work concentrates on defining the engineering

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and architectural aspects of Diffserv-enabled routers (e.g., [2]). However, current technical specifications deliberately do not quantify the actual *service characteristics*, which users will obtain by using the above mentioned classes. Apparently, service characteristics would have to be defined and publicly declared in order to make the distinction between the service classes meaningful to the user (and possibly worth paying for).

The Diffserv network cannot offer strict quality guarantees, as resources are allocated to the service classes based on some average network conditions [6]. Instead, the provider may declare upper-bounds on QoS measures, or alternatively provide looser guarantees, such as probabilistic or time-dependent guarantees. Another option is to offer *relative* quality guarantees. This option can be easily quantified and advertised, as illustrated by the following exemplifying rule: "service class Q will offer an end-to-end average delay, which is at least two times less than any other class, independent of the level of congestion". When a user buys class Q it is aware of what it gets, and expects the provider to uphold the agreement conditions. In this paper, we focus on the *proportional* QoS model, whereby QoS ratios between the classes are announced. Specifically, we concentrate on *delay* ratios, although analogous definitions may be suggested when considering other QoS measures. The proportional QoS model benefits from implementation-related pros. Ratios are easier to maintain in comparison with absolute end-to-end guarantees, primarily because they may hold for different levels of congestion, and secondly because keeping the ratios locally (on a node basis) leads to fulfilling this objective on the network level.

We shall examine capacity allocation as the main network management tool for achieving the proportional QoS design objective. The goal of capacity allocation is to keep the announced delay ratios, irrespectively of complementary means for network traffic control, such as pricing and admission control. Our focus in this paper is on a simplified single link network, where the network manager owns a fixed amount of capacity to be divided among the link's offered service classes in order to maintain the QoS ratios objective. Generally, it is easy to calculate the appropriate capacity allocation when the traffic in each service class is fixed. In this paper, however, we consider the interaction of the user behavior and network conditions. Within a standard flow model (see [3], Sec. 5.4) we represent the user population as a finite set of self-optimizing decision makers. The users are heterogenous with respect to their cost functions, which reflect their *price-quantity-quality* tradeoffs. Furthermore, users may modify their flow quantities (i.e., *elastic* users [11]), and may also shift their traffic from one service class to the other in response to current congestion conditions. We pose the overall problem as a non-cooperative game between the manager and the (selfish) users, and explore the associated capacity management policies and equilibrium conditions.

In a recent paper [14], we have considered a similar problem with static (fixed) user demand in the general network context. The present paper extends this work (for a single-link network) to the case of elastic demand, as well as adding a price term to the overall user's cost.

Related Literature. *Service differentiation approaches:* Several research papers have addressed the model of finitely many classes, in which no strict performance guarantees are given. The simplest approach for providing differentiated services is Odlyzko's Paris Metro Pricing (PMP) proposal [17]. The idea of PMP is to create service differentiation by giving a different price to each service class. Other papers [9, 13, 16] explicitly consider elements such as the user model, the scheduling mechanism and the network objective (e.g., a social or an economic objective). The major concern is usually in calculating the prices that would lead to the network objective. An additional common ground is that the network does not declare any kind of QoS guarantees. Users are assumed to acquire the best deal there is with respect to their quality-price tradeoff. We deviate from the last assumption, by considering the upholding of the service characteristic as a primary management priority.

Selfish routing: Since our model assumes that users are allowed to split traffic between the service classes, our work is related to selfish routing models. Game-theoretic analysis is widely used to study the working conditions of these models. The involved issues include the existence and uniqueness of an equilibrium point, its calculation, and its properties (such as the degradation of performance due to selfish behavior, known as the "price of anarchy" [21]). A common routing model, originated from the field of transportation networks, has considered networks shared by infinitesimal users (see [1] for a survey). The case of finitely many users, each carrying substantial flow has been introduced to the networking literature more recently (see [12, 18, 20]). In [12], the equilibrium properties are applied for network design (namely, link capacity assignment), where the objective is to obtain the socially optimal working point. We use a similar routing model to represent the user's choice of service class.

Proportional QoS: Dovrolis et al. [8] proposed a class of schedulers, based on the Proportional Delay Differentiation (PDD) model. This model aims at providing predetermined delay ratios. The schedulers are implemented by observing the history of the encountered delays (or alternatively, by measuring the delay of the packet at the head of each service class), and serving the class which most deviates from its nominal delay ratio. In the present work we do not rely on PDD schedulers, but rather use a capacitated links model that may be considered a proxy to existing scheduling schemes such as GPS [19].

Contribution and Organization. This paper proposes schemes for inducing proportional QoS through capacity allocation, when users can react to the allocation decisions. The precise definitions of the network and user models are given in Section 2. The analysis of this model is presented in Section 3, which establishes the existence and uniqueness of the equilibrium point for the network-users game, and presents an algorithm for its computation. An explicit formula is obtained for the best response map of the network, namely the capacity assignment that ensures the QoS-ratio objective for *fixed* network flows, which may be used as a basis for an adaptive capacity assignment scheme. Due to length constraints, the proofs of some claims are omitted, the reader is referred to [15] for full details.

2 The Single Link Model

Our basic model considers a single link, which supports several service classes. As a stand-alone model, it may be viewed as an approximation of a single path in a network, where the variations in traffic due to other (intersecting) network paths are neglected. Let $\mathcal{I} = \{1, 2, \ldots, I\}$ be a finite set of users, which share a link that offers a set of service classes $\mathcal{A} = \{1, 2, \ldots, A\}$. Since each service class is characterized by its own price and performance measure (to be described in the sequel), it would be convenient to consider the link with its respective service classes as a two terminal (source-destination) network, which is connected by a set of parallel arcs. Each arc represents a different service class. Thus, the set of arcs is also denoted by $\mathcal{A}$, and the terms service class and arc are used interchangeably. We denote by f_a^i the flow which user i ships on arc a. User i is free to choose any assignment of $f_a^i \geq 0$. The total demand of each user will be denoted by $f^i \triangleq \sum_{a\in\mathcal{A}} f_a^i$. Turning our attention to an arc $a \in \mathcal{A}$, let f_a be the total flow on that arc, i.e., $f_a = \sum_{i\in\mathcal{I}} f_a^i$. Also, denote by $\mathbf{f}_a$ the vector of all user flows on arc a, i.e., $\mathbf{f}_a = (f_a^1, \ldots, f_a^I)$. The user flow configuration $\mathbf{f}^i$ is the vector $\mathbf{f}^i = (f_1^i, \ldots, f_A^i)$. The flow configuration $\mathbf{f}$ is the vector of all user flow configurations, $\mathbf{f} = (\mathbf{f}^1, \ldots, \mathbf{f}^I)$. A user flow configuration is *feasible* if its components obey the nonnegativity constraints, as described above. We denote by $\mathbf{F}^i$ the set of all feasible user flow configurations $\mathbf{f}^i$, and by $\mathbf{F}$ the set of all feasible flow configurations $\mathbf{f}$.

The network manager has a constant capacity C, to be divided between the service classes. This capacity cannot be statically assigned, since the manager cannot predict in advance the number of customers and their preferences. Practically, the network manager would modify the current capacity assignment at slower time scales than the user routing decisions, after periodically measuring class performance. We denote by c_a the allocated capacity at arc a. The capacity allocation of the manager is the vector $\mathbf{c} = (c_1, \ldots, c_A)$. An allocation $\mathbf{c}$ is feasible if its components obey the nonnegativity and total capacity constraint, namely (i) $c_a \geq 0$, $a \in \mathcal{A}$ and (ii) $\sum_{a\in\mathcal{A}} c_a = C$. The set of all feasible capacity allocations $\mathbf{c}$ is denoted by Γ. Finally, a system configuration is feasible if it is composed of a feasible flow configuration and a feasible capacity allocation.

Pricing. Each service class $a \in \mathcal{A}$ has a *constant* price p_a per unit traffic. Thus, the network usage price of user i is $\sum_{a\in\mathcal{A}} f_a^i p_a$. Prices may be viewed as an indirect mean for admission control [6], and (among other things) prevent flooding of the better service classes. In this paper, however, we concentrate on capacity assignment as the management tool, assuming that prices are static (or change on a slower time scale). The issue of price setting in our context is left for future work.

The *performance measures* of both the users and the manager are specified through their respective cost functions, which they wish to minimize. We denote by $J^i(\mathbf{f}, \mathbf{c})$, $i \in \mathcal{I}$ the cost function of user i, and by $J^M(\mathbf{f}, \mathbf{c})$ the cost function of the manager. The costs of the users and the manager are related to the *congestion* level at each of the service classes. We shall use the well known *M/M/1 latency function*

$$D_a(f_a, c_a) = \begin{cases} \frac{1}{c_a - f_a} & f_a < c_a \\ \infty & \text{otherwise} \end{cases} \quad (1)$$

as a congestion measure of each service class. Here c_a is the transmission capacity measured in the same units as the flow f_a. This latency function is often used to model delay costs in communication networks [3], and provides a clear sense of link capacity.

User cost functions. Users are distinguished by their utility function $U^i(f^i)$, which quantifies their utility for shipping a total flow f^i. We make the following assumption on U^i.

Assumption 1. *For every user $i \in \mathcal{I}$, the utility function $U^i : [0, C] \to \Re$ is increasing, bounded, concave and continuously differentiable.*

We note that utility functions with the above characteristics are commonly used within the networking pricing literature [6, 11]. We define U^i in the set $[0, C]$ only, since a total flow of $f^i > C$ cannot be accommodated by the network. We note that a user may split its flow among different service classes in order to minimize the total cost. The total cost J^i of each user i is comprised from its delay cost, its network usage price, minus its utility, namely

$$J^i(\mathbf{f}, \mathbf{c}) = \beta^i \sum_{a=1}^{A} f_a^i D_a(f_a, c_a) + \sum_{a=1}^{A} f_a^i p_a - U^i(f^i), \quad (2)$$

where $\beta^i > 0$ represents the delay sensitivity of user i, and D_a is defined in (1).

Manager cost function. The objective of the manager is to impose certain predetermined ratios between the average delays of the service classes. Taking the delay of class 1 as a reference, the ratios are described by a vector $\rho = (\rho_2, \ldots, \rho_A)$, $0 < \rho_a < \infty$, where the manager's objective is to have the delays $D_1, \ldots, D_A$ obey

$$D_a(f_a, c_a) = \rho_a D_1(f_1, c_1), \quad (3)$$

where $\rho_1 \stackrel{\triangle}{=} 1$. We refer to that relation as the *fixed ratio objective.* For concreteness, we may assume that $\rho_1 \leq \rho_2 \leq \cdots \leq \rho_A$, so that service classes are ordered from best to worst. Similarly, prices are expected to satisfy $p_1 \geq p_2 \geq \cdots \geq p_A$, although this is not essential for our results. In functional terms, the manager's cost function may thus be defined as

$$J^M(\mathbf{f}, \mathbf{c}) = \begin{cases} 0 & \text{if (3) holds,} \\ \infty & \text{otherwise.} \end{cases} \quad (4)$$

An alternative objective of the manager that will be considered, is to minimize a weighted sum of the delay functions, that is

$$\bar{J}^M(\mathbf{f}, \mathbf{c}) = \sum_{a \in \mathcal{A}} w_a D_a(f_a, c_a), \quad (5)$$

where $w_a > 0,\ a \in \mathcal{A}$. As we shall see, there is a close relation between the fixed ratios objective and the weighted sum objective (5).

The interaction between the manager and the users will be referred to as the *users-manager game*. A Nash Equilibrium Point (NEP) of that game is a feasible system configuration $(\tilde{\mathbf{f}}, \tilde{\mathbf{c}})$ such that

$$J^M(\tilde{\mathbf{f}}, \tilde{\mathbf{c}}) = \min_{\mathbf{c} \in \Gamma} J^M(\tilde{\mathbf{f}}, \mathbf{c}), \tag{6}$$

$$J^i(\tilde{\mathbf{f}}^i, \tilde{\mathbf{f}}^{-i}, \tilde{\mathbf{c}}) = \min_{\mathbf{f}^i \in \mathbf{F}^i} J^i(\mathbf{f}^i, \tilde{\mathbf{f}}^{-i}, \tilde{\mathbf{c}}) \quad \forall i \in \mathcal{I},$$

where $\tilde{\mathbf{f}}^{-i}$ stands for the flow configurations of all users, but the ith user. Namely, the NEP is a network working point, where no user, nor the manager, finds it beneficial to change its flow or capacity allocation. Our users-manager game is characterized by the finiteness of the NEP costs, since users can always ship a flow of zero to encounter a finite cost. We shall formally prove this attribute in the next section.

3 Capacity Assignment and Equilibrium Analysis

In this section we analyze the equilibrium point and suggest capacity assignment schemes that induce the ratios objective. First, we show that the manager has a unique best response with respect to the ratio objective (3). This response is a simple solution to a set of linear equations. Then we prove the existence and uniqueness of an equilibrium point, in which the desired ratios are met. Accordingly, we show the equivalence between the best response with respect to (3) and the best response with respect to the modified manager objective function (5) and discuss its implications.

Theorem 1 considers the best response capacity assignment of the manager, given any (fixed) flow configuration.

Theorem 1. *Consider a fixed flow configuration* $(f_1, \ldots, f_A)$ *and a desired ratio vector* ρ. *If* $\sum_{a \in \mathcal{A}} f_a < C$, *there exists a unique capacity allocation* $\mathbf{c} \in \mathbf{\Gamma}$ *such that (3) is met. This allocation is explicitly given by*

$$c_a - f_a = (C - \sum_{\alpha \in \mathcal{A}} f_\alpha) \frac{\rho_a^{-1}}{\sum_{\alpha \in \mathcal{A}} \rho_\alpha^{-1}}. \tag{7}$$

Proof. The allocation (7) is derived from solving the following set of *linear* equations: $\rho_a(c_a - f_a) = (c_1 - f_1)$, $a = 2, \ldots, A$; and $\sum_{a=1}^{A} c_a = C$. □

The above result allows the manager to explicitly calculate its *best response* assignment, namely the capacity assignment that will satisfy the fixed ratio objective given the *current* network flows. This calculation requires just the total flows in each service class, which are easy to measure. The next theorem establishes the existence and uniqueness of the equilibrium point.

Theorem 2. *For every delay ratios vector* ρ, *there exists a unique Nash equilibrium point. This NEP has finite costs for the manager and for the users. In particular, the ratios objective of the manager is satisfied.*

Proof. The proof follows from the next four lemmas.

Lemma 1. *For every delay ratio vector ρ, there exists a NEP. Furthermore, in every NEP the costs of the users and the manager are finite and the ratio objective is met.*

Proof. See [15].

Lemma 2. *Let $D_1, \ldots, D_A$ be the class delays at some NEP. Then the following equations are met at the equilibrium for every $i \in \mathcal{I}$ and every $a \in \mathcal{A}$*

$$\begin{aligned} \beta^i \left(D_a + f_a^i D_a^2\right) + p_a &= U^{i'}(f^i) \quad \text{if } f_a^i > 0, \\ \beta^i D_a + p_a &\geq U^{i'}(f^i) \quad \text{if } f_a^i = 0. \end{aligned} \tag{8}$$

Proof. Observe that

$$\begin{aligned} \frac{dJ^i(\mathbf{f},\mathbf{c})}{df_a^i} &= \beta^i \left(\frac{1}{(c_a - f_a)} + \frac{f_a^i}{(c_a - f_a)^2} \right) + p_a - U^{i'}(f^i) \\ &= \beta^i \left(D_a(f_a, c_a) + f_a^i D_a^2(f_a, c_a) \right) + p_a - U^{i'}(f^i). \end{aligned} \tag{9}$$

Then (8) may be readily seen to be the KKT optimality conditions [5] for minimizing the cost function (2) of user i subject to the flow constraint $f_a^i \geq 0$. □

Lemma 3. *Consider a NEP with given class delays $D_1, \ldots, D_A$. Then the respective equilibrium flows f_a^i are uniquely determined.*

Proof. Consider the following optimization problem in $(f_1^i, \ldots, f_A^i)$:

$$\begin{cases} \min \sum_{a=1}^{A} \frac{1}{2} \beta^i D_a^2 {f_a^i}^2 + f_a^i \left(\beta^i D_a + p_a\right) - U^i(\sum_a f_a^i) \\ \text{s.t} \qquad f_a^i \geq 0 \end{cases} , \tag{10}$$

where we assume that the delays D_a are fixed. Note that (10) is a strictly convex optimization problem, since the objective function is the sum of a diagonal quadratic term (with $\beta^i D_a^2 > 0$ for every a) and the negation of U^i, where U^i is concave by Assumption 1. Thus, this problem has a unique minimum, which is characterized by the KKT optimality conditions. It is now readily seen that the KKT conditions for (10) coincide with the conditions in (8). Thus, by Lemma 2, any set of equilibrium flows $(f_a^i)_{a \in \mathcal{A}}$ is a solution of (10). But since this solution is unique, the claim is established. □

Lemma 4. *Consider two Nash equilibrium points $(\mathbf{f}, \mathbf{c})$ and $(\tilde{\mathbf{f}}, \tilde{\mathbf{c}})$. Then $D_a(f_a) = D_a(\tilde{f}_a)$ for every $a \in \mathcal{A}$.*

Proof. Define $D_a \triangleq D_a(f_a)$ and $\tilde{D}_a \triangleq D_a(\tilde{f}_a)$. Assume that

$$\tilde{D}_\alpha > D_\alpha \text{ for some } \alpha \in \mathcal{A}. \tag{11}$$

Then $\tilde{D}_a > D_a$ for every $a \in \mathcal{A}$ since the ratios are met in both equilibria (Lemma 1). Noting (4) and (7) it follows from (11) that $\sum_{a \in \mathcal{A}} \tilde{f}_a > \sum_{a \in \mathcal{A}} f_a$, which implies that there exists some user j for which

$$\tilde{f}^j = \sum_{a \in \mathcal{A}} \tilde{f}_a^j > \sum_{a \in \mathcal{A}} f_a^j = f^j. \tag{12}$$

We next contradict (12) by invoking the next two implications:

$$f_a^j = 0 \Rightarrow \tilde{f}_a^j = 0 \tag{13}$$

$$f_a^j > 0 \Rightarrow f_a^j > \tilde{f}_a^j. \tag{14}$$

Their proof is based on the KKT conditions (8). Since the utility U^j is concave, then by (12) we have $\lambda^j \triangleq U^{j'}(f^j) \geq U^{j'}(\tilde{f}^j) \triangleq \tilde{\lambda}^j$. If $f_a^j = 0$, then $\beta^j D_a + p_a \geq \lambda^j \geq \tilde{\lambda}^j$. Since $\tilde{D}_a > D_a$, then $\beta^j \tilde{D}_a + p_a > \tilde{\lambda}^j$, hence $\tilde{f}_a^j = 0$. To prove (14) note first that it holds trivially if $\tilde{f}_a^j = 0$. Next assume $\tilde{f}_a^j > 0$. Then by (8)

$$\beta^j D_a + \beta^j D_a^2 f_a^j + p_a = \lambda^j \geq \tilde{\lambda}^j = \beta^j \tilde{D}_a + \beta^j \tilde{D}_a^2 \tilde{f}_a^j + p_a. \tag{15}$$

Since $\tilde{D}_a > D_a$ (hence $\tilde{D}_a^2 > D_a^2$), and β^j are positive, we must have $f_a^j > \tilde{f}_a^j$ in order for (15) to hold, which establishes (14). Summing user j's flows according to (13)-(14) yields $\sum_{a \in \mathcal{A}} \tilde{f}_a^j \leq \sum_{a \in \mathcal{A}} f_a^j$, which contradicts (12). Thus $\tilde{D}_\alpha \leq D_\alpha$. Symmetrical arguments will lead to $\tilde{D}_\alpha \geq D_\alpha$, i.e., $\tilde{D}_\alpha = D_\alpha$, hence $\tilde{D}_a = D_a$ for every $a \in \mathcal{A}$. □

The last two lemmas imply that the user flows and the class delays in equilibrium are unique. The capacities in the equilibrium must also be unique since $c_a = f_a + \frac{1}{D_a}$, where $f_a = \sum_{i \in I} f_a^i$. This establishes the uniqueness of the NEP, and completes the proof of Theorem 2. □

A possible criticism of the fixed ratio objective, as defined in (3), is that it does not account at all for absolute congestion measures, namely the delays themselves rather then their ratios. However, the next result shows that by achieving the ratio objective, the manager in fact minimizes the cost function $\bar{J}^M$ (defined in (5)), which is just an appropriately weighted sum of the delays over the different service classes.

Theorem 3. *Consider the users-manager game, whose NEP is defined in (6), and an additional game, which is similar except that J^M is replaced by $\bar{J}^M$. If the parameters are such that $w_a = \frac{1}{\rho_a^2}$ for every $a \in \mathcal{A}$, then the two games are equivalent in the sense that their (unique) equilibrium points coincide.*

Proof. See [15].

Note that the weights w_a are inversely proportional to ρ_a^2, which assigns higher weight to better (lower relative delay) service classes, as might be expected. From an algorithmic point of view, it should be noted that $\bar{J}^M$ is a convex and continuous function, and therefore may provide a sound basis for an iterative (e.g., gradient-based) algorithm that may be used by the manager to minimize this cost, with the goal of eventually reaching the desired fixed-ratio equilibrium. Yet, a specific consideration of such an algorithm is beyond the scope of this work.

We conclude this section by considering the computation of the NEP. Recall that the uniqueness of the user flows was established in Lemma 3 via the definition of strictly convex optimization problems. Hence, by solving the same optimization problems, the NEP can be efficiently calculated. The only unknown variable which is required for the computation is D_1. In our case, an iterative search for D_1 may be easily performed, by comparing the total flow $\sum f_a$ (for a given D_1), which is obtained from both the best response formula (7) and also from the solutions to (10). More details on the search method are given in [15].

Remark 1. General Networks. It is obviously of interest to extend the results of this section to a general network topology. In a recent paper [14], we have made such an extension for a model with fixed (plastic) user demands. Since the same extension can be similarly used here, we briefly outline its key features. The following setup is applied: (i) each user has a unique fixed path from its source to its destination; (ii) the QoS ratio objective is maintained on a link basis, i.e., management is performed via a *distributed* approach, where the capacity adaptation is performed locally, at the link level. Observe that if the above two features are maintained, then the QoS ratios are met *end-to-end* for every user. The game framework now includes network *managers*, one for every link. Due to the locality of the capacity management, the formula for the best response map (Theorem 1) and the use of $\bar{J}^M$ instead of J^M (Theorem 3) are trivially extended to the general network case; so is the proof for the existence of the equilibrium. The issue of uniqueness of the equilibrium (as in Lemma 4) is however more complicated in the general network case and is currently an open problem.

4 Conclusion

The proposed approach to QoS provisioning in Diffserv networks focuses on maintaining relative congestion measures in the different service classes. Our analysis demonstrates the feasibility of this approach, and in particular establishes the existence and uniqueness of a working point, which satisfies this QoS objective in an elastic, reactive and heterogenous user environment. Our results provide an efficient algorithm for computing the Nash equilibrium. However, from the manager's viewpoint, this computation requires complete knowledge of the users' preferences, which may not be available. An alternative scheme is to use adaptive capacity assignment, for example by utilizing the best-response map (7), which requires only the total flows in each service class, which are easy to measure. The analysis of such a scheme is an important issue for future research. Additional research topics include the price setting issue, proportional QoS with other cost functions and QoS measures, and the equilibrium dynamics of capacity allocation algorithms.

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Can "Bill-and-Keep" Peering Be Mutually Beneficial?

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Abstract. We analyze "Bill-and-Keep" peering between two providers, where no money exchanges hands. We assume that each provider incurs costs from its traffic traversing its as well as the peer's links, and compute the traffic levels in Nash equilibrium. We show that Nash strategies are not blind, i.e., they are neither pure hot-potato nor pure cold-potato strategies. Rather, the Nash strategies involve strategically splitting traffic between a provider's own links and its peer's. We derive necessary and sufficient conditions for both the providers to be better (or worse) off in Nash equilibrium compared to the blind strategies.[1] We also analyze society's performance as a whole and derive necessary and sufficient conditions for the society to be better (or worse) off. In particular we establish that, under Bill-and-Keep peering, while it is not possible for two asymmetric providers to be both worse off, it is certainly possible for both to be better off.

1 Introduction

Today's Internet is composed of many distinct networks, operated by independent network providers, also referred to as Internet Service Providers (ISPs). Each provider is interested in maximizing its own utility and the objectives of the providers are not necessarily aligned with any global performance objective. Most relationships between providers may be classified under one of two categories [7] : *transit* and *peer*. In a transit relationship, a traffic-originating provider pays a transit provider to carry traffic destined to nodes outside the originator's local network. On the other hand, in a peering relationship the providers agree to accept and carry traffic from each other.

In this paper, we focus primarily on peering relationships. In a peering arrangement, a pair of providers agree to install bi-directional links at multiple peering points to accept traffic from each other. In today's Internet, peering relationships are mostly "Bill-and-Keep" [1]. In this arrangement, the providers don't charge each other for the traffic accepted on the peering links. This arrangement is also referred to as "Zero-Dollar" peering or "Sender-Keep-All" (SKA) peering [3]. Under the peering relationship, since the ISPs are interested in minimizing their own costs, they predominantly use the nearest-exit or *hot-potato* routing [6], where outgoing traffic exits a provider's network as quickly as possible. In some cases, where the receiver is a bigger player and is able to exert its market power, the routing is farthest-exit or *cold-potato* [7].

Various aspects of ISP peering have been analyzed by [5], [1], [4], [9], [8]. [5] was the first paper to analyze ISP peering in depth from an economic perspective. It analyzed the impact of access charge on strategies of the providers and showed that, in a

[1] By better off we mean weakly better off, i.e., the cost in Nash equilibrium is less than or equal to the cost under blind strategies.

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broad range of environments, operators set prices for their customers as if their customers' traffic were entirely off-net. [8] extended the models in [5] to include the fact that the ISPs are geographically separated. It thus analyzed the local ISP interaction separately from the local and transit ISP interaction. It also analyzed the economics of private exchange points and showed that they could become far more wide spread. Both [5] and [8] used linear pricing schemes assuming fixed marginal costs. In addition, they assumed hot-potato routing. [9] extended the models in [5] to include customer delay costs, finding that they have a substantial effect on market structure. [4] used a different model of ISP peering. It assumed that customers are bound to ISPs, subject to general, non-linear marginal costs. It then looked at how ISPs could charge each other in response to the externality caused by their traffic.

All these models ignored one important aspect of peering: they did not consider the case when the provider incurs costs even when its traffic flows on its peers links. This would happen, for example, if the providers care about the end-to-end quality of service (QOS) for traffic originating within their networks. In this paper we focus on this situation, and show that this cost structure has a substantial effect on how ISPs route traffic. Moreover, we look at the case where peering happens in the absence of pricing, with the costs incurred on the peer's links serving as a proxy for a transfer price. Our results imply that Bill-and-Keep peering, currently used mainly due to ease of implementation, can be beneficial if combined with non-myopic routing.

We analyze peering decisions using non-cooperative game theory [2] in a simple two-provider model. Analyzing Nash equilibria, under mild assumptions on the cost structures, we show that it is not in the provider's interest to route traffic in a hot-potato or a cold-potato fashion in equilibrium. That is, the *blind* strategies are not Nash. Rather, the Nash strategy involves strategically splitting traffic among peering points. We also show that, in Nash equilibria, it is not possible for both ISPs to be worse off with respect to the blind strategies. We then show that it is possible to have the two remaining scenarios in equilibrium, where either one or both ISPs are better off. We derive necessary and sufficient conditions on the cost functions for each of the two cases to occur. In addition, under a specific pricing scheme, we show that peers who would peer and be both better off under Bill-and-Keep peering, will choose to use cold-potato peering and effectively not peer at all.

We are also interested in the performance of the society as a whole. Therefore, we compare society's performance in Nash equilibrium versus the blind strategies. We derive necessary and sufficient conditions for the society to be better or worse off. In this process, we show that society is always better off under Bill-and-Keep peering if the costs incurred are linear.

2 The Model and the Nash Strategies

We look at a two ISP peering model. A realistic model would include the actual topologies of the two ISPs. However, most of the insights can be gained from the following simple model (figure 1); analysis of a generalized model is work in progress. We have two ISPs, S and R, with two peering points, $P1$ and $P2$. The ISPs have nodes S_i and R_i, respectively, located right next to the peering point $P_i, i \in \{1, 2\}$. We assume that

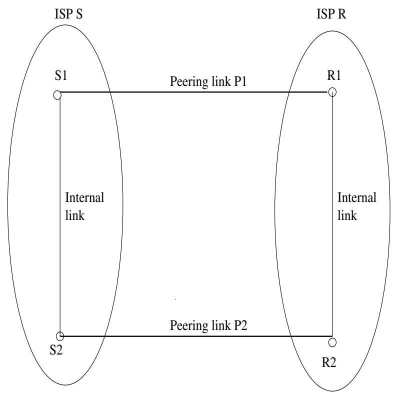

Fig. 1. The Peering Model

ISP S sends one unit of traffic from node S_1 to node R_2, and similarly, ISP R sends one unit of traffic from node R_1 to S_2. We also assume that the cost to send traffic across the peering links is zero.

The ISPs have the choice to split these flows between the two peering points. We look at the two components for ISP S first. The hot potato component, f_S^R, goes over to ISP R at the closest peering point P_1, and then travels to R_2 on the internal link R_1R_2. The cold potato component, f_S^S, first travels on the internal link S_1S_2, and then crosses over to ISP R at the farthest peering point P_2. The hot potato and cold potato components for ISP R, i.e., f_R^S and f_R^R, can be described in a similar way. Overall, ISP S carries flows f_S^S and and f_R^S on its internal link, whereas ISP R carries flows f_S^R and f_R^R on its internal link. The cost incurred per unit of traffic on ISP S's links is given by $C_S(f_S^S + f_R^S)$ and that on ISP R's link is given by $C_R(f_S^R + f_R^R)$.

We assume that these per unit cost functions are strictly increasing, convex and twice differentiable. We also assume that the per unit cost of one ISP carrying all the traffic is more than the per unit cost of the other ISP carrying zero traffic, i.e.,

$$C_S(2) \geq C_R(0) \tag{1a}$$

$$C_R(2) \geq C_S(0) \tag{1b}$$

In Nash equilibrium, given f_R^S, ISP S solves (by choosing $0 \leq f_S^R \leq 1$)

$$\begin{aligned}&\text{minimize } J_S(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_S^S + C_R(f_S^R + f_R^R)f_S^R\\&\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1,\end{aligned}$$

and ISP R solves (by choosing $0 \leq f_R^S \leq 1$)

$$\begin{aligned}&\text{minimize } J_R(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_R^S + C_R(f_S^R + f_R^R)f_R^R\\&\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1,\end{aligned}$$

given f_S^R.

The first order conditions are given by

$$C'_S(f_S^S + f_R^S)f_S^S + C_S(f_S^S + f_R^S) = C'_R(f_S^R + f_R^R)f_S^R + C_R(f_S^R + f_R^R) \quad (2a)$$
$$C'_S(f_S^S + f_R^S)f_R^S + C_S(f_S^S + f_R^S) = C'_R(f_S^R + f_R^R)f_R^R + C_R(f_S^R + f_R^R). \quad (2b)$$

Note that the second order conditions will be satisfied automatically since the per unit cost functions are strictly increasing and convex, especially at interior point solutions.

We next show that the Nash strategies don’t include blind strategies.

Proposition 1. *In Nash equilibrium, no ISP uses a blind strategy.*

Proof. We prove this in two parts. We first prove that, in Nash equilibrium, no ISP routes pure cold potato. We then prove that no ISP routes pure hot potato. The proofs are very similar and we have omitted the second part.

We now show that it is not possible for ISP R to do pure cold potato routing (a similar argument can be applied to ISP S). This is done in three steps. First, it is not possible for both ISPs to do pure cold potato routing. This would require $f_S^R = f_R^S = 0$, and[2]

$$\frac{\partial J_S}{\partial f_S^R}(0,0) = -C'_S(1) - C_S(1) + C_R(1) \geq 0 \quad (3a)$$
$$\frac{\partial J_R}{\partial f_R^S}(0,0) = -C'_R(1) - C_R(1) + C_S(1) \geq 0. \quad (3b)$$

This requires

$$C_S(1) + C'_S(1) \leq C_R(1) \leq C_S(1) - C'_R(1), \quad (4)$$

which is a contradiction since $C_S(x)$ and $C_R(x)$ are strictly increasing.

Second, it is not possible for ISP S and R to do hold potato and cold potato routing, respectively. This would require $f_S^R = 1$, $f_R^S = 0$, and

$$\frac{\partial J_S}{\partial f_S^R}(1,0) = -C_S(0) + C'_R(2) + C_R(2) \leq 0 \quad (5a)$$
$$\frac{\partial J_R}{\partial f_R^S}(1,0) = C_S(0) - C'_R(2) - C_R(2) \geq 0. \quad (5b)$$

This requires

$$C_S(0) \geq C_R(2) + C'_R(2), \quad (6)$$

which is a contradiction from (1) and the fact that $C_R(x)$ is strictly increasing.

Finally, it is not possible for ISP R to do cold potato routing and ISP S to send some nonzero amount, but not all, off its traffic to ISP R. This would require $0 < f_S^R < 1$, $f_R^S = 0$, and

$$\frac{\partial J_S}{\partial f_S^R}(f_S^R,0) = -C_S(f_S^S) - C'_S(f_S^S)f_S^S + C'_R(f_S^R+1)f_S^R + C_R(f_S^R+1) = 0 \quad (7a)$$
$$\frac{\partial J_R}{\partial f_R^S}(f_S^R,0) = C_S(f_S^S) - C'_R(f_S^R+1) - C_R(f_S^R+1) \geq 0. \quad (7b)$$

[2] The partial derivatives are evaluated at (f_S^R, f_R^S).

This requires

$$C'_S(f_S^S)f_S^S \leq C'_R(f_S^R+1)(f_S^R-1) \leq 0, \tag{8}$$

which is a contradiction since $0 < f_S^R < 1$ and $C_S(x)$ and $C_R(x)$ are strictly increasing. □

3 Individual Performance

In this section we provide necessary and sufficient conditions for the individual players to be better or worse off in Nash equilibrium compared to the blind strategies. We also provide examples showing that both ISPs can be better off in Nash equilibrium. In addition, we illustrate potential problems with introduction of pricing.

3.1 Preliminaries

We start by writing (2) in an alternate form. Using $f_S^S + f_S^R = 1$, $f_R^S + f_R^R = 1$, and denoting $f_S^R - f_R^S = f_d$, $f_S^R + f_R^S = f_a$, we get

$$C'_S(1-f_d)(1-f_d) + 2C_S(1-f_d) = C'_R(1+f_d)(1+f_d) + 2C_R(1+f_d) \tag{9a}$$

$$f_a = 1. \tag{9b}$$

In Nash equilibrium, we denote the solution to (9) as f_d^{Nash}. We also denote the Nash equilibrium costs of ISP S and R as J_S^{Nash} and J_R^{Nash}, respectively. We then get the following lemma.

Lemma 1. *The ISPs costs in Nash equilibrium are equal.*

Proof. From (9), in Nash equilibrium, we have $f_S^S = f_R^S = \frac{(1-f_d^{Nash})}{2}$ as well as $f_S^S = f_R^S = \frac{(1-f_d^{Nash})}{2}$. This ensures that the costs at equilibrium are equal, i.e.,

$$J_S^{Nash} = J_R^{Nash} = J_{common}^{Nash} = \frac{J_{total}(f_d^{Nash})}{2}, \tag{10}$$

where we define the sum of costs as

$$J_{total}(f_d) = [C_S(1-f_d)(1-f_d) + C_R(1+f_d)(1+f_d)]. \tag{11}$$

□

We next define the following three differences

$$\Delta_1(f_d) = C_S(1-f_d) - C_R(1+f_d) \tag{12a}$$

$$\Delta_2(f_d) = C_S(1-f_d) + C'_S(1-f_d)(1-f_d) - C_R(1+f_d) - C'_R(1+f_d)(1+f_d) \tag{12b}$$

$$\Delta_3(f_d) = \Delta_1(f_d) + \Delta_2(f_d). \tag{12c}$$

Then, in Nash equilibrium, from (9), we get

$$\Delta_3(f_d^{Nash}) = 0 \tag{13a}$$

$$\Delta_1(f_d^{Nash}) = -\Delta_2(f_d^{Nash}) \tag{13b}$$

as well as

Lemma 2. *In Nash equilibrium, $\Delta_3(0) f_d^{Nash} \geq 0$.*

Proof. First consider $f_d^{Nash} \geq 0$. Since, from (13), $\Delta_3(f_d^{Nash}) = 0$, and $\Delta_3(f_d)$ is non-increasing in f_d, the result follows. The argument for $f_d^{Nash} < 0$ is similar □

Finally, we derive some useful inequalities, as follows. Since $C_S(x)$ and $C_R(x)$ are twice differentiable, strictly increasing, and convex, $C_S(x)x$ and $C_R(x)x$ are twice differentiable, strictly increasing, and strictly convex. Using Jensen's inequality, we get

$$C_S(1-f_d)(1-f_d) \geq C_S(1) - [C'_S(1) + C_S(1)]f_d \tag{14a}$$

$$C_R(1+f_d)(1+f_d) \geq C_R(1) + [C'_R(1) + C_R(1)]f_d, \tag{14b}$$

and

$$C_S(1) \geq C_S(1-f_d)(1-f_d) + [C'_S(1-f_d)(1-f_d) + C_S(1-f_d)]f_d \tag{15a}$$

$$C_R(1) \geq C_R(1+f_d)(1+f_d) - [C'_R(1+f_d)(1+f_d) + C_R(1+f_d)]f_d. \tag{15b}$$

Now, using (11), and (12) we get

$$J_{total}(f_d) \geq J_{total}(0) - \Delta_2(0) f_d \tag{16}$$

as well as

$$J_{total}(0) \geq J_{total}(f_d) + \Delta_2(f_d) f_d. \tag{17}$$

3.2 Necessary and Sufficient Conditions

We first show that, under our assumptions, it is not possible for both the ISPs to be worse off.

Lemma 3. *In Nash equilibrium, both ISPs are worse off only if $\Delta_1(0)\Delta_3(0) \geq 0$.*

Proof. For both to be worse off, from lemma 1, this requires

$$J_{common}^{Nash} = \frac{J_{total}(f_d^{Nash})}{2} \geq max(J_S^{blind}, J_R^{blind}), \tag{18}$$

where $J_S^{blind} = C_S(1)$ and $J_R^{blind} = C_R(1)$ when both ISPs are doing pure cold potato routing.[3] Using (11), (17) and (13), we get

$$\begin{aligned} \frac{J_{total}(f_d^{Nash})}{2} &\geq max(C_S(1), C_R(1)) \geq \frac{J_{total}(0)}{2} \\ &\geq \frac{J_{total}(f_d^{Nash})}{2} - \frac{\Delta_1(f_d^{Nash}) f_d^{Nash}}{2}. \end{aligned} \tag{19}$$

This requires

$$\Delta_1(f_d^{Nash}) f_d^{Nash} \geq 0, \tag{20}$$

which necessitates $\Delta_1(0) f_d^{Nash} \geq 0$. This, from lemma 2, is the same as $\Delta_1(0)$ $\Delta_3(0) \geq 0$. □

[3] Similarly, $J_S^{blind} = C_R(1)$ and $J_R^{blind} = C_S(1)$ when both ISPs are doing hot potato routing. In both cases, the min, max and avg operations give the same results.

Proposition 2. *In Nash equilibrium, both ISPs cannot be worse off.*

Proof. The case $f_d^{Nash} = 0$ is straightforward. From (11) and (10), we get $J_{common}^{Nash} = \frac{C_S(1)+C_R(1)}{2}$. Since $max(C_S(1), C_R(1)) \geq \frac{C_S(1)+C_R(1)}{2} \geq min(C_S(1), C_R(1))$, we must have one ISP better off and the other worse off. Thus, both ISPs can't be worse off.

Next, we look at $f_d^{Nash} \neq 0$. We first consider $f_d^{Nash} > 0$. Both ISPs worse off requires

$$J_{common}^{Nash} \geq C_S(1). \tag{21}$$

Since C_S is convex, using $C_S(1) \geq C_S(1 - f_d^{Nash}) + C_S'(1 - f_d^{Nash})f_d^{Nash}$, we get

$$J_{common}^{Nash} \geq C_S(1 - f_d^{Nash}) + C_S'(1 - f_d^{Nash})f_d^{Nash}, \tag{22}$$

which, using (10) and (11), simplifies to

$$(1 + f_d^{Nash})[C_R(1 + f_d^{Nash}) - C_S(1 - f_d^{Nash})] \geq 2C_S'(1 - f_d^{Nash})f_d^{Nash}. \tag{23}$$

Now, since $C_S(x)$ is strictly increasing, using (12) we get, $\Delta_1(f_d^{Nash}) < 0$ as well as $\Delta_1(f_d^{Nash})f_d^{Nash} < 0$.

The argument for $f_d^{Nash} < 0$ is similar. Thus, in both cases, both ISPs worse off necessitates

$$\Delta_1(f_d^{Nash})f_d^{Nash} < 0. \tag{24}$$

This contradicts (20) of lemma 3. □

Now, we look for necessary conditions for both ISPs to be better off.

Proposition 3. *In Nash equilibrium, both ISPs are better off only if $\Delta_2(0)\Delta_3(0) \geq 0$.*

Proof. We first consider $f_d^{Nash} \leq 0$. Both ISPs better off requires

$$J_{common}^{Nash} \leq C_S(1). \tag{25}$$

Since C_S is convex, using $C_S(1) \leq C_S(1 - f_d^{Nash}) + C_S'(1)f_d^{Nash}$, we get

$$J_{common}^{Nash} \leq C_S(1 - f_d^{Nash}) + C_S'(1)f_d^{Nash}, \tag{26}$$

which, using (10) and (11), simplifies to

$$(1 + f_d^{Nash})[C_R(1 + f_d^{Nash}) - C_S(1 - f_d^{Nash})] \leq 2C_S'(1)f_d^{Nash}. \tag{27}$$

Now, since $C_S(x)$ is strictly increasing, using (12) we get, $\Delta_1(f_d^{Nash}) \geq 0$ as well as $\Delta_1(f_d^{Nash})f_d^{Nash} \leq 0$. From (13), this requires

$$\Delta_2(f_d^{Nash})f_d^{Nash} \geq 0, \tag{28}$$

which necessiates $\Delta_2(0)f_d^{Nash} \geq 0$. This, from lemma 2, is the same as $\Delta_2(0)\Delta_3(0) \geq 0$.

The argument for $f_d^{Nash} \geq 0$ is similar. □

Finally, we provide a sufficient condition for both ISPs to be better off.

Proposition 4. *If the costs under blind strategies are equal then both the ISPs are better off in Nash equilibrium.*

Proof. We start by noting that, when $C_S(1) = C_R(1)$, we have $J_S^{blind} = J_R^{blind}$. Also, from (10), we have $J_S^{Nash} = J_R^{Nash}$. This rules out the possibility that one ISP is strictly better off and the other one is strictly worse off. In addition, propostion 2 rules out the possibility that both are strictly worse off. Thus, the only remaining possibility is that they are both better off. □

3.3 Examples

We provide two examples that show that both ISPs can be better off. We look for functions $C_S(x)$ and $C_R(x)$ satisfying the following properties. First, they satisfy $C_S(1) = C_R(1)$ which, from (10), gives $J_S^{blind} = J_R^{blind} = J_{common}^{blind} = \frac{J_{total}(0)}{2}$, where J_{common}^{blind} is defined to be the common cost under blind strategies. Second, they satisfy $J_{common}^{Nash} \leq J_{common}^{blind}$, which is the same as $J_{total}(f_d^{Nash}) \leq J_{total}(0)$ - this basically requires one of the internal link costs, i.e., $C_S(x)x$ and $C_R(x)x$, to be more convex at $x = 1$. Now, both ISPs benefit by choosing an $f_d^{Nash} \approx 0$ such that the link with the more convex cost function carries less traffic.

Example 1. When $C_S(x) = x$ and $C_R(x) = x^2$, we get $J_{common}^{Nash} = 0.97 \leq 1.0 = J_{common}^{blind}$.

Example 2. This example uses more realistic per unit cost functions, given by $C_S(x) = \frac{\theta_S^N}{(\theta_S^D - x)}$ and $C_R(x) = \frac{\theta_R^N}{(\theta_R^D - x)}$ (in an $M/G/1$ queue, θ^N would be proportional to the variance of service times, whereas θ^D would be the capacity of the link). Using $\theta_S^N = 1.00, \theta_S^D = 1.10, \theta_R^N = 2.00, \theta_R^D = 1.20$, we get $J_{common}^{Nash} = 9.75 \leq 10.0 = J_{common}^{blind}$.

3.4 Can Pricing Be Bad?

In this section we illustrate potential problems with moving away from Bill-and-Keep peering toward a situation where ISP S charges ISP R an amount $p_S f_R^S$ and ISP R charges ISP S an amount $p_R f_S^R$, for some prices p_S and p_R. We consider the sequential game where the ISPs first pick prices, having committed to optimally choosing traffic splits thereafter. That is, ISP S solves (by choosing $0 \leq f_S^R \leq 1$)

$$\text{minimize } J_S(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_S^S + C_R(f_S^R + f_R^R)f_S^R$$
$$\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1,$$

and ISP R solves (by choosing $0 \leq f_R^S \leq 1$)

$$\text{minimize } J_R(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_R^S + C_R(f_S^R + f_R^R)f_R^R$$
$$\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1$$

for fixed p_S and p_R, and this is used, in turn, to calculate the optimal p_S and p_R.

Using $C_S(x) = x$ and $C_R(x) = x^2$ (the cost functions from section 3.3), and solving in the above manner, we get $f_S^R = f_R^S = 0$. This is the same as the situation when the ISPs are routing blindly. In this case both the ISPs are worse off compared to peering without pricing.

4 Society's Performance

The necessary conditions for the society to be better (or worse) off turn out to be the same as the ones stated in propostion 3 and lemma 3. Intuitively, this makes sense since the society must be better (or worse) off when both the ISPs are better (or worse) off.

Now we look at sufficient conditions for the society to be better off or worse off. The following propositions summarize our results.

Proposition 5. *If* $\Delta_1(0)\Delta_3(0) \leq 0$ *then the society is better off in Nash equilibrium.*

Proof. This implies, from lemma 3, that society can't be strictly worse off. Thus, it must be better off. □

Proposition 6. *If* $\Delta_2(0)\Delta_3(0) \leq 0$ *then the society is worse off in Nash equilibrium. In addition, one ISP is better off and the other worse off.*

Proof. This implies, from proposition 3, that society can't be strictly better off. Thus, it must be worse off. Also, since proposition 2 rules out he possibility that both are worse off, it must be that one ISP is worse off and the other one is better off. □

We finish the paper with the following specific result

Proposition 7. *When the per unit cost functions are linear, society is better off in Nash equilibrium.*

Proof. To do so, we use the linear cost functions $C_S(x) = \theta_S x$ and $C_R(x) = \theta_R x$, where $\theta_S > 0$ and $\theta_R > 0$. In Nash equilibrium, we get

$$f_S^S = f_R^S = \frac{\theta_R}{(\theta_S + \theta_R)} \tag{29a}$$

$$f_S^R = f_R^R = \frac{\theta_S}{(\theta_S + \theta_R)}. \tag{29b}$$

Next, we assume that the society is strictly worse off. This gives

$$J_{total}^{Nash} = \frac{4\theta_S\theta_R}{(\theta_S + \theta_R)} > (\theta_S + \theta_R) = J_{total}^{blind}, \tag{30}$$

which reduces to

$$0 > (\theta_S - \theta_R)^2, \tag{31}$$

which is always false. Thus, we have a contradiction. □

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Design of P2P Grid Networking Architecture Using *k*-Redundancy Scheme Based Group Peer Concept

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Abstract. Currently, there are many system requirements to guarantee the scalability, fault-tolerance, and high performance in the rapidly evolving research fields such as Grid and P2P technologies. Due to the dynamic nature of the Grid computing and P2P in networks, the behavior of system might be very unpredictable in respect of reliability. In this paper, we propose the Group Peer based P2P Grid system architecture with *k*-Redundancy Scheme[1] that can satisfy these requirements. With respect to load balancing, this P2P Grid system clearly shows that redundancy scheme of group peer guarantees the reliability for resource or service discovery. Through the theoretical analysis and simulation, we discuss the performance of the proposed scheme.

1 Introduction

The many meaningful trials for the development of Grid computing [1, 3] and Peer-to-Peer (P2P) [4, 5] networks as the prominent distributed computing paradigm, enable us more closer to access and use some network based computing services from any where at any time freely. Grid computing has emerged as a significant research area; distinguished from the conventional distributed computing style by concentrating on large scale resource sharing, innovative applications, and high performance orientation. On the other hand, P2P in the past years has got an immense popularity by supporting two main classes of applications such as file sharing and highly parallel computing. These two technologies appear to have the same ultimate objectives: the pooling and coordinated use of large number of distributed resources.

However, the current architecture tends to focus on different requirements in a same aspect. First, with regard to resource management Grid computing can provide more robust and powerful set of resources because of an idea of virtual organization (VO) [1]. And in the case of target applications, one significant point of differentiation between implemented Grid and P2P system is that the former is apt to have more data-centric characteristic. In contrast, P2P is far larger than Grid since P2P supports

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the strong functionality in terms of connectivity even though P2P exists on the unstructured based network architecture. Yet, Grid computing still stays at the conception of organization based model. Finally when it comes to fault, just simple check pointing and restarting are deployed to Grid information system. For greater reliability, Grid developers should consider fault tolerance at the application level. For that reason, we need to us decentralized P2P algorithms, which avoid critical failure points. However a few of noteworthy researches have made efforts currently how to combine Grid to P2P with a view of high reliability.

In this paper, we propose the Group Peer based P2P Grid Architecture with *k*-Redundancy Scheme for improving availability in data-centric P2P Grid applications. In the section 3, we discuss about the consideration of system design with P2P point of view. In section 4, we show the numeric evaluation with *k*-redundancy algorithm for reducing a rate of system error depending on the number of group size and the degree of incoming and outgoing links. Finally, we discuss about the performance of proposed architecture through the analysis of results from the simulations.

2 Related Works

Nevertheless some researchers point out the differences between P2P and Grid as mentioned above, requirements of Grid and P2P overlap in many regards. In the article [4], authors emphasize that Grid can benefit from the more flexible connectivity models used in P2P networks. And they explain possibility of the commonalities and synergies between two technologies in terms of the connectivity, access services, resource discovery, and fault tolerance.

Some authors in [2] talk about super peer networks, and it assumes that the degree of redundancy of super peers is 2, since if system allows the increment of *k*-redundant super peers, then the number of open connection among super peers increases exponentially as k^2. This assumption can be feasible for reducing the experimental complexity however it is not enough to reflect more interactive distributed networks e.g., Grid. We believe that P2P Grid would provide more powerful resource sharing technique for all distributed applications in P2P Grid networks. Two different projects [7, 8] provide useful measurement results through the experiment using the Grid information service over 20 hours. The authors find each usage rate of Grid operations and rank them according to a principle of decision by majority. Hence we analyze these performance trace data and classify operations to each atomic action as the unit of Gird operation. Furthermore, the power-law based network topology [7, 9], each node can be ranked by the degree of the links amongst all nodes.

3 Consideration of System Design

In this section we consider the consideration for the design of group peer (GP) networking architecture concerning enhancement for Grid service reliability provided by *k*-redundancy scheme.

3.1 Group Peer Based P2P Grid Networking Architecture

In order to provide functionalities oriented to P2P networks such as the efficiency of search, autonomy, load balancing, and robustness into Grid computing technology, we need to employ the suitable model of P2P networks e.g., super peer networks [2].

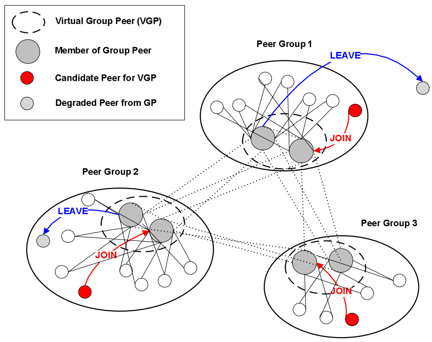


Fig. 1. Group peer networking architecture with *2*-redundancy

As depicted in figure 1, basically this architecture consists of three groups, which include individual peers. Each peer can freely join to or leave from a group under the observation of a virtual group peer (VGP) and it can be classified into two types of peer such as a group peer and a client peer (CP). The former has many responsibilities to manage a group and peers and to communicate with other group peers. In addition group peer has a lot of functionalities as follows: control message processing, resource discovery (e.g., services, data, computing elements), and store the metadata of client peers with index. Incidentally, it as aspect of Grid middleware has to take active roles in the quality of services (QoS) negotiation, job execution with monitoring, and management of result sets. However the latter, client peer, can be considered as a resource consumer and a resource provider at the same time in the proposed system based on P2P Grid computing environments. As described in figure 1, there are two group peers in a group relatively. This means that only two group peers have to handle all connections from client peers in a group, in spite of large growth of CP. Shortly without any alternative ways, it is obvious that group peer will experience a problem regarding overload. Therefore, we discuss a feasible algorithm that improves performance in the manners of system load and reliability.

3.2 *k*-Redundancy Scheme for P2P Grid

Fundamentally, we consider a data-centric application such as file sharing, data streaming, and storage service. Existing Grid system (e.g., Globus Toolkit [12]) is based on high-speed networks to transfer large amounts of data, yet it would not give any progressive solutions how to protect some failures of single-point in order to guarantee integrity and validation of large volumes of traffic data. Hence to avoid this kind of defeat, we suggest the *k*-redundancy algorithm to overcome the instability especially due to heavy burden of loads on group peers. It offers criteria to generate new group peers for the reduction of additional loads on VGP caused by the large growth of request for example resource or service discovery.

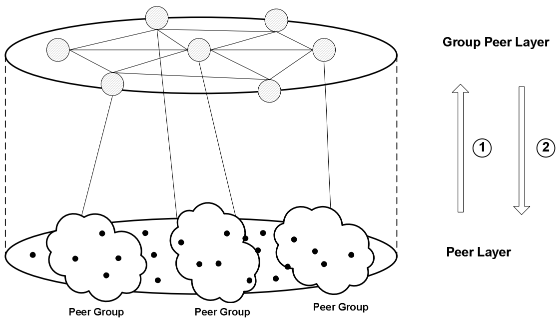


Fig. 2. This figure shows the election and degradation of group peers. The arrow with number 1 is to represent an election flow for choosing new group peer to decrease system load or to repair some unexpected failures. And 2nd arrow means a flow to degrade group peer into client peer owing to lack of capability to perform a P2P Grid jobs or services.

With overlay networks point of view, the upper plane is a group peer layer as the management domain composing of group peers. In this plane, every group peer takes the form of full mesh networks. Shortly, group peers are formalized to pure P2P. The lower plane includes network elements, which can be considered to P2P Grid resources. Some peers out of cloud are to mean client peers that have a possibility to take part in P2P Grid networks. In this overlay architecture, centralized searching based GP can archive the decentralized resource discovery providing global view through querying among GPs. This way can provide effects of reduction in terms of complex query such as flooding based searching protocol in the Gnutella [6].

3.3 Analytical Model

By using the analytical model shown in figure 3, we calculate the system load from incoming, outgoing traffic and processing power on group peers for resource or service discovery. Figure 3 shows the basic symbols to represent values of each factor. Especially, Deg_{IN} and Deg_{OUT} mean the degree of incoming links and outgoing links respectively. With the Deg_{IN} and Deg_{OUT}, we determine the unit of cost originated from incoming and outgoing links. Therefore by using these symbols we can compute the relativity of each action's occurrence respectively which can be expressed to a percentage of arbitrary number. In a paradigm of the distributed computing, every operation can be divided into several atomic actions. Especially in this paper, we determine that atomic actions such as add, modify, delete, and search are smallest units of execution for resource discovery. To show clarity of this model, we present the detailed notations in next section.

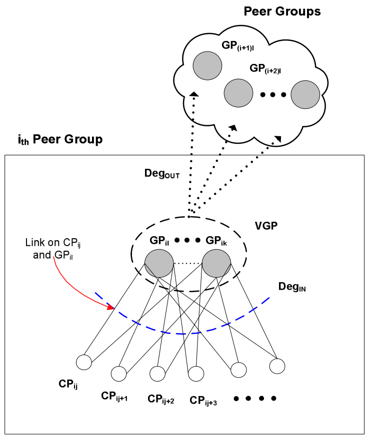


Fig. 3. Analytical model for GP based P2P Grid architecture

4 Numerical Evaluation of the GP Based P2P Grid

4.1 Network Load on a Group Peer

The Degree of Incoming and Outgoing Link. Frequently, client peers get or lose links to a group peer or member in VGP. So, Deg_{IN} can be dynamically changed according to the number of client peers. This condition can be applied to Deg_{OUT} because it might be affected by Deg_{IN}. Let define average value of Deg_{IN} as follows:

$$Avg.Deg_{IN} = \frac{\sum_{i=1}^{N/Group_{SIZE}} \sum_{l=1}^{k} \left(\sum_{j=1}^{\#CP_i - k} deg_{IN_i}^{jl} \right)}{N} \cdot Group_{SIZE} \tag{1}$$

Where, $\#CP_i$ is the number of client peers in the i^{th} group, N denotes the total number of peers in entire networks, and $Group_{SIZE}$ means the $\#CP$ in each group. We assume that all groups are composed of the same size of group. Especially, $Deg_{OUT}^{GP_{il}}$ is the degree of outgoing links from the l^{th} group peer in the i^{th} group. In same manner of previous derivation, we find the average value of Deg_{OUT} of a particular GP.

The Load of Network on a Group Peer. We can derive the expected network load with respect to the incoming links considering the Deg_{IN}.

$$\begin{aligned} E\left[GP_{i,IN-LOAD}^{k} \mid J_I\right] = \sum_{GP_l \in Networks,\, l=1}^{k} \sum_{CP_l \in Networks,\, j=1}^{\#CP_i} \Bigg\{ & E\left[\frac{\alpha \cdot deg_{IN_i}^{jl}}{k} \mid J_I\right] \cdot E\left[f_{ADD_i}^{jl}\right] + E\left[\frac{\beta \cdot deg_{IN_i}^{jl}}{k} \mid J_I\right] \cdot E\left[f_{DEL_i}^{jl}\right] \\ & + E\left[\frac{\chi \cdot deg_{IN_i}^{jl}}{k} \mid J_I\right] \cdot E\left[f_{MOD_i}^{jl}\right] + E\left[\frac{\delta \cdot deg_{IN_i}^{jl}}{k} \mid J_I\right] \cdot E\left[f_{SER_i}^{jl}\right] \Bigg\} \end{aligned} \tag{2}$$

Where, $E\left[GP_{i,IN-LOAD}^{k} \mid J_I\right]$ means the expected load of incoming links under the condition of k-redundant group peer in the i^{th} group. And parameters such as $\alpha, \beta, \chi, and\ \delta$ are set to constant values, which correspond to the proportion of the number of occurrence of particular atomic action within the Grid operations at job instance J_l. These parameters affect the degree in incoming link as weighted values. Next, frequency parameters such as $f^{jl}_{ADD_i}, f^{jl}_{DEL_i}, f^{jl}_{MOD_i}, and\ f^{jl}_{SER_i}$ should be derived from the rate how often peers try to send query or control message made of individual atomic action. Since the expected load of outgoing links to exchange data and message among VGPs can be modeled by similar manner, we estimate the expected value of outgoing network load. Moreover, we use the average degree of outgoing link, $\lfloor Avg.Deg_{OUT} \rfloor$ to reduce the unexpected fractions through the flooring. From the Eq. (1) and (2), finally we obtain a result to determine the average network load on a single VGP.

$$E\left[GP^k_{i,NET-LOAD} \mid J_l\right] = E\left[GP^k_{i,IN-LOAD} \mid J_l\right] + E\left[GP^k_{i,OUT-LOAD} \mid J_l\right]. \tag{3}$$

4.2 Processing Load on a Group Peer

In this part we discuss the system load by processing messages in a group peer. Thus, we need to consider how to estimate a performance of processing a message in a group peer. As a feasible way to calculate processing load, we take a proportion of elapsed time to the total execution time for computing message including four different requests divided into atomic actions. Each message for a particular atomic action has fixed byte size. Thus, if we get the proportion of each atomic action in a packet, we expect the average time to process each message. Additionally, we should consider that processing load dynamically could be changed by the influences of a volume and frequency of incoming and outgoing messages. Consequently, we define a formula to express the expected processing load on a group peer as follows:

$$\begin{aligned} E\left[GP^k_{i,PRO-LOAD} \mid J_l\right] &= \sum_{l=1}^{k} E\left[C_{T_{PRO}} \mid J_l\right] \cdot E[F_T] \\ &= \sum_{l=1}^{k} \left\{ E[\frac{t^l_{ADD} \cdot (deg^l_{IN,OUT})}{k \cdot T^l_{PRO}} \mid J_l] \cdot E[f^l_{ADD}] + E[\frac{t^l_{MOD} \cdot (deg^l_{IN,OUT})}{k \cdot T^l_{PRO}} \mid J_l] \cdot E[f^l_{MOD}] \right. \\ &\quad \left. + E[\frac{t^l_{DEL} \cdot (deg^l_{IN,OUT})}{k \cdot T^l_{PRO}} \mid J_l] \cdot E[f^l_{DEL}] + E[\frac{t^l_{SER} \cdot (deg^l_{IN,OUT})}{k \cdot T^l_{PRO}} \mid J_l] \cdot E[f^l_{SER}] \right\} \end{aligned} \tag{4}$$

From Eq. (4), t is the unit time to execute a message with processing options, such as add, delete, modify, and search. And T means the total processing time for all messages requested by client peers in a group. Since it is necessary to process all requests via a group peer or VGP, the $\deg^l_{IN,OUT}$ is defined to the degree of network link including incoming and outgoing traffic on a group peer.

4.3 Overall Load and System Availability

In the previous section 4.2, we described the network and processing load respectively on a single group peer. In one group, we cannot account for correct patterns of load made by all connections among VGPs, and unexpected conditions causing additional overhead to the other groups. For these reasons, we have to show the system load in the whole groups in P2P Grid networks in order to reflect of more realistic system architecture. Furthermore, since the local group peer frequently shows instability like failures of single point, we derive an expectation function with respect to system load at the job instant J_I.

$$E\left[GP^k_{LOAD} \mid J_I\right] = \sum_{i=1}^{N/Group_{SIZE}} E\left[GP^k_{i,LOAD} \mid J_I\right] = \sum_{i=1}^{N/Group_{SIZE}} \left\{E\left[GP^k_{i,NET-LOAD} \mid J_I\right] + E\left[GP^k_{i,PRO-LOAD} \mid J_I\right]\right\} \tag{5}$$

As the understandable solution, we utilize the probabilistic estimation approach to evaluate our analytical model with respect to system availability. We simplify the system availability model below:

$$A_{GP_i}(q, \#GP_i) = \sum_{r=0}^{\#GP_i - q} C(q, \#GP_i) \cdot A_{GP_{il}}^{\#GP_i - r} \cdot (1 - A_{GP_{il}})^r \tag{6}$$

Where, $\#GP_i$ is defined as the number of group peers in the i^{th} P2P group and q means the number of failures. $A_{GP_{il}}$ is an availability of the l^{th} group peer belonging to the i^{th} group when failure occurs q times with the $\#GP_i$. $C(q, \#GP_i)$ is the cost within the same condition as mentioned above. Therefore, by extending a model of system availability we derive real load to be treated by the Group peer based P2P Grid system.

$$Avl.E\left[GP^k_{LOAD} \mid J_I\right] = \sum_{i=1}^{N/Group_{SIZE}} \left\{E\left[GP^k_{i,LOAD} \mid J_I\right] \cdot A_{GP_i}(q, \#GP_i)\right\} \tag{7}$$

Equation (5) and (7) provides different point of view about system load. The equation (5) is the overall load to be managed by the entire system. On the other hands, equation (7) means the system availability. In short, there obviously exist differences between the load can be handled and be requested to perform by the system. After all, we use the quantitative analysis to get a real error rate due to a probabilistic instability of the system. Until now, we discuss the numerical evaluation of the proposed system with respect to the load performance and system availability based on the *k*-redundancy scheme.

5 Simulation and Discussion

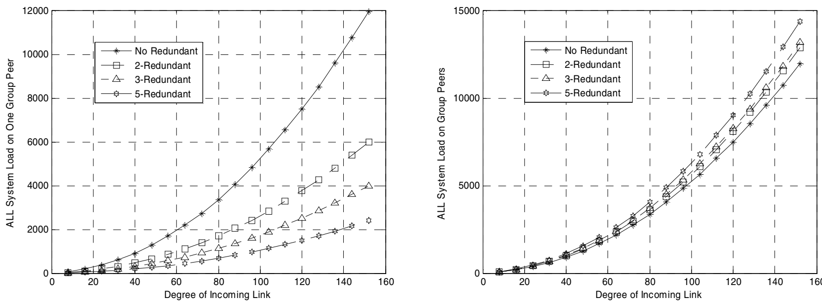
To evaluate the effective of the proposed P2P Grid system architecture, in table 1 we set the initial values of parameters that denote a status of real networks [6, 7, 8]. Basically, we assume that the networks cover 10,000 (*N*) peers for a consideration of scalability. Then each group is constructed with 10 ~ 50 peers including the group peers.

Table 1. Configuration of parameters

Parameters	Default range of values
α, β, χ, δ	$for\ \alpha + \beta + \chi + \delta \le 1$
$\#CP_i$	$\sim Normal(\mu_{CP}, \sigma^2_{CP}),\ \sigma^2_{CP} = \omega\mu_{CP}\ for\ 0 \le \omega < 1\ (eg., \omega = 0.15)$
$A_{GP_i}(q, \#GP_i)$	$\sim Uniform(0.5,\ 0.95)$

5.1 Overall Load on GP vs the Degree of Incoming Links

As for evaluation of the *k*-redundancy scheme in Group Peer based P2P Grid system, firstly we compare the overall load occurred from single group peer in a group using different Deg_{IN} range of 10 through 150. According to analytical model shown in the figure 3, we simulate the proposed architecture with various degree of redundancy from 1 to 5. The simulation results in figure 4 show the effectiveness of load balancing. As increasing *k*, each group peer would have a lower system load. It means that the proposed system could be a solution against failures of single-point owing to overhead.



(a) Load on a single GP (b) Load on all GPs

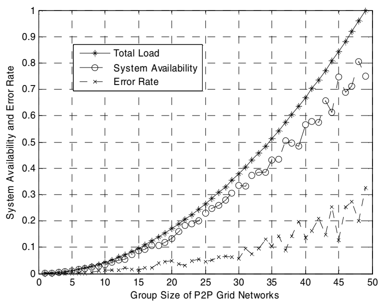
Fig. 4. (a) Overall load on a group peer vs the degree of incoming links. (b) Overall load on group peers vs the degree of incoming links. And $Group_{SIZE}$ is fixed to 20 for both of them.

Therefore, *k*-redundancy scheme is an acceptable scheme for sharing loads, which might be converged to a single group peer, if *k* equals to 1. Under identical conditions for evaluation of overall loads, we show overall loads on all group peers (or VGP) according to the various numbers of incoming links. In the case of the small number of incoming links, however there is no difference of the load as increase of the amount of loads in comparison of *k*=1 and *k*=5 respectively. We see much larger gap of load in figure 4. On the other hand, as for large *#CP*, all group peers in VGP experience the increment of system loads, since group peers should deal with more frequent requests such as join, leave, and resource discovery which make additional overhead in VGP. Therefore we have to determine the degree of redundancy proportional to the overall loads.

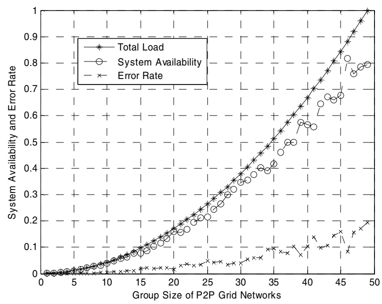
5.2 System Availability, Error Rate vs Group Size

In this section, we show system availability, which means the actual criterion of system performance. And we describe error rate versus the different size of group. We use a probabilistic estimation approach to calculate availability in the proposed system architecture.

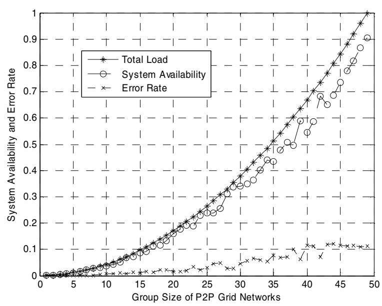
Figure 5 compares the system availability and error rate versus the size of group under different degree conditions. Figure 5-(a) shows large error rates over the range of group size. At maximum value of group size, especially there is a big fraction over than 30% approximately. It means that without any consideration for protecting system from the unexpected instabilities, system could easily fail to execute jobs. Thus we should decide how large *k* could be used to enhance the performance. And from the figure 5-(b) and (c), we can see that *k*-redundant group peers overcome the problem encountered in figure 5-(a). Namely, system can obtain more powerful capability to process jobs, so that Group peer based P2P Grid Architecture decreases a rate of failures as much. In an evaluation step of the 5-redundant group peer, the system availability is much closer to the total amount of load handled by all group peers. As a result, *k*-redundancy algorithm can support the load balancing and guarantee the high availability of system with probabilistic model-based approach.



(a) *No – Redundancy* (k =1)



(b) *2 – Redundancy* (k =2)



(c) *5 – Redundancy* (k =5)

Fig. 5. System availability and error rate vs size of p2p group

6 Conclusion

In this paper, we introduced the Group Peer based P2P Grid Architecture using the *k*-redundancy concept for improving the capabilities to process system loads and the probabilistic availabilities to avoid failures of single-point. As the first phase to archive these benefits, we design the Grid architecture based on P2P networks. Especially we propose the Group Peer concept as the P2P Grid middleware. Secondly, we describe the detailed procedures how the proposed system outperforms the conventional scheme in terms of scalability and flexibility. Through performance evaluations, we have shown that effectiveness of load balancing in a group and overall system availability could be determined by the degree of redundancy for group peers. The proposed redundancy scheme decreases the error rate of the system as well.

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An Integrated Classification Method: Combination of LP and LDA*

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Abstract. Behavior analysis of credit cardholders is one of the main research topics in credit card portfolio management. Usually, the cardholder's behavior, especially bankruptcy, is measured by a score of aggregate attributes that describe cardholder's spending history. In the real-life practice, statistics and neural networks are the major players to calculate such a score system for prediction. Recently, various multiple linear programming based classification methods have been promoted for analyzing credit cardholders' behaviors. As a continuation of this research direction, this paper proposes an integrated classification method by using the fuzzy linear programming (FLP) with moving boundary and Fisher Linear Discriminant analysis(LDA). This method can improve the accurate rate in theory. In the end, a real-life credit database from a major US bank is used for explaining the idea as an example.

1 Introduction

Data Mining, an intersection area of human intervention, machine learning mathematical modeling and databases, is a powerful information technology (IT) tool in today's competitive business world. Classification is one of the important functions of data mining. There are two steps in classification. In the first step, a model is built describing a predetermined set of data classes or concepts, which is also known as supervised learning. In the second step, the model is used for classification. Classification has numerous applications in different disciplines, including medicine [14], credit card portfolio and credit ranking [9,10], strategic management [15].

In credit cardholder behavior analysis, the issue of predicting bankruptcy in advance and avoiding huge charge-off losses is critical for credit card issuers (or banks). Generally, the goal of using the classification models for credit cardholder behavior analysis is to find the common characters and patterns of personal bankruptcy. Although the commercial software (e.g.IBM Intelligence Miner and SAS Enterprise Miner) can be adopted to predict patterns of credit card bankruptcy, the prediction accuracy in specific aspects is not satisfactory. In fact,

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both practitioners and researchers have tried a number of quantitative methods to conduct the credit card portfolio management to meet the specific needs, namely, statistics [7], neural networks [8], and multiple criteria linear programming [6, 10]. A common characteristic of these models is that they divided the behaviors of the cardholders into two predefined classes: bankrupted accounts and non-bankrupted ones according to their historical records. Then by those methods they calculate the Kolmogorov-Smirnov (KS) value which measures the largest separation of the two cumulative distributions of bankrupted accounts and non-bankrupted ones in a training set [2]. Thus, these methods can be generally regarded as two-class classification models in credit cardholder behavior analysis.

In the linear programming discriminant models, the misclassification of data separation can be described by two kinds of objectives in a linear system. First, the minimal distances of observations from the critical value are maximized (MMD). Second, separate the observations by minimizing the sum of the deviations (MSD) of the observations. This deviation is also called "overlapping." The compromise solution in multiple criteria linear programming locates the best trade-offs between MMD and MSD for all possible choices.

Fisher linear discriminant analysis (LDA) (Fisher, 1936) is based on the minimum incorrect rate ECM. It regards that the different groups have different means and variances. Later it has been improved to QDA[13], which is better than LDA in some aspect. Fisher linear discriminant analysis has been successfully applied to face recognition area in the past few years.

According to different characters of data set, different classification methods will show their advantages. But different method has its shortcoming too, thus an integration method would be a good idea to integrate their advantage together while reducing their shortcomings. In this paper, we analyzed the pattern of misclassification of LP by the FLP model with moving boundary, and proposed an integration method of LP and LDA, which can improve the accuracy of LP. Moreover we explained in theory and by example.

This paper will proceed as follows. In section 2 we review the basic concepts of linear programming discriminant models and the MCLP compromise solution to data analyses. In Section 3 we review the ideas of Fisher linear discriminant model. In Section 4 we elaborate the model of FLP model with moving boundary and get the algorithm of mixing linear programming discriminant method and Fisher linear discriminant method together for credit cardholder behavior analysis. In Section 5 we use a real-life credit database from a major U.S. bank for empirical study to show how our method works. The further research problems will be outlined in Section 6.

2 LP, FLP and MCLP Models for Classification

Research of linear programming (LP) approaches to classification problems was initiated by Freed and Glover [3]. A basic framework of two-class problems can be presented as:

Given a set of r variables (attributes) about a cardholder $a = (a_1, \cdots, a_r)$, let $A = (A_{i1}, \cdots, A_{ir})$be the development sample of data for the variables, where $i = 1, \cdots, n$ and n is the sample size. We want to determine the best coefficients of the variables, denoted by $X = (x1, \cdots, xr)$, and a boundary value b (a scalar) to separate two classes: G(Good for non-bankrupt accounts) and B(Bad for bankrupt accounts); that is,

$A_iX \leq b$, $A \in B$ (Bad) and $A_iX \geq b$, $A \in G$(Good).

To measure the separation of Good and Bad, we define:

$\alpha_i =$ the overlapping of two-class boundary for case α_i (external measurement);
$\alpha=$ the maximum overlapping of two-class boundary for all cases $A_i(\alpha_i < \alpha)$;
$\beta_i =$ the distance of case Ai from its adjusted boundary (internal measurement).
$\beta=$ the minimum distance for all cases Ai from its adjusted boundary $(\beta_i > \beta)$.

A simple version of Freed and Glover's [3] model which seeks MSD can be written as:

$$\begin{aligned} \textit{Minimize} \ & \sum \alpha_i \\ \textit{Subject to:} \ & A_iX \leq b + \alpha_i, A_i \in B, \\ & A_iX \geq b - \alpha_i, A_i \in G, \end{aligned} \tag{1}$$

Where A_i are given, X and b are unrestricted, and $\alpha_i \geq 0$.

The alternative of the above model is to find MMD:

$$\begin{aligned} \textit{Maximize} \ & \sum \beta_i \\ \textit{Subject to:} \ & A_iX \geq b - \beta_i, A_i \in B, \\ & A_iX \leq b + \beta_i, A_i \in G, \end{aligned} \tag{2}$$

Where A_i are given, X and b are unrestricted, and $\beta_i \geq 0$.

A hybrid model [4] that combines models (M1) and (M2) can be:

$$\begin{aligned} \textit{Minimize} \ & \sum \alpha_i - \sum \beta_i \\ \textit{Subject to:} \ & A_iX = b + \alpha_i - \beta_i, A_i \in B, \\ & A_iX = b - \alpha_i + \beta_i, A_i \in G, \end{aligned} \tag{3}$$

Where A_i are given, X and b are unrestricted, and α_i,$\beta_i \geq 0$ respectively.

The advantage of this conversion is to easily utilize all techniques of LP for separation, while the disadvantage is that it may miss the scenario of trade-offs between these two separated criteria.

Shi et al [10] applied the compromise solution of multiple criteria linear programming (MCLP) to miimize the sum of α_i and maximize the sum of β_i simultaneously. A two-criteria linear programming model is stated as:

$$\begin{aligned} \textit{Minimize} \ & \sum \alpha_i \ \ \textit{and} \ \ \textit{Maximize} \sum \beta_i \\ \textit{Subject to:} \ & A_iX = b + \alpha_i - \beta_i, A_i \in B, \\ & A_iX = b - \alpha_i + \beta_i, A_i \in G, \end{aligned} \tag{4}$$

Where A_i are given, X and b are unrestricted, and α_i,$\beta_i \geq 0$ respectively.

Define the membership functions can be expressed respectively by:

$$\mu_{F_1}(x) = \begin{cases} 1, & \text{if } \sum \alpha_i \geq y_{1U}; \\ \frac{\sum \alpha_i - y_{1L}}{y_{1U} - y_{1L}}, & \text{if } y_{1L} < \sum \alpha_i < y_{1U} \\ 0, & \text{if } \sum \alpha_i \leq y_{1L} \end{cases}$$

$$\mu_{F_1}(x) = \begin{cases} 1, & \text{if } \sum \beta_i \geq y_{2U}; \\ \frac{\sum \beta_i - y_{2L}}{y_{2U} - y_{2L}}, & \text{if } y_{2L} < \sum \beta_i < y_{2U} \\ 0, & \text{if } \sum \beta_i \leq y_{2L} \end{cases}$$

So FLP classification Model (5) can be obtained by He and Shi etc [1], which mean we relax the objective function.

$$\begin{aligned} \textit{Minimize} \;\; & \xi \\ \textit{Subject to:} \;\; & \xi \leq \frac{\sum \alpha_i - y_{1L}}{y_{1U} - y_{1L}}, \\ & A_i X = b + \alpha_i - \beta_i, A_i \in B, \\ & A_i X = b - \alpha_i + \beta_i, A_i \in G, \end{aligned} \tag{5}$$

Where A_i, y_{1L}, y_{1U} and y_{2L} are known, X and b are unrestricted, and α_i,β_i,$\xi \geq 0$, respectively.

In compromise solution approach [9,11], the best trade-off between $-\sum \alpha_i$ and $\sum \beta_i$ is identified for an optimal solution.To explain this, assume the "ideal value" of $-\sum \alpha_i$ be $\alpha^* > 0$ and the "ideal value" of $-\sum \beta_i$ be $\beta^* > 0$.Then, if $-\sum \alpha_i > \alpha^*$, the regret measure is defined as $-{d_\alpha}^+ = \sum \alpha_i + \alpha^*$, otherwise, it is defined as 0. If $-\sum \alpha_i < \alpha^*$, the regret measure is defined as ${d_\alpha}^- = \sum \alpha_i + \alpha^*$; otherwise, it is 0. Thus an MCLP model for two-class separation is presented as:

$$\begin{aligned} \textit{Minimize} \;\; & {d_\alpha}^- + {d_\alpha}^+ + {d_\beta}^- + {d_\beta}^+ \\ \textit{Subject to:} \;\; & \alpha^* + \sum \alpha_i = {d_\alpha}^- - {d_\alpha}^+, \\ & \beta^* - \sum \beta_i = {d_\beta}^- - {d_\beta}^+, \\ & A_i X = b + \alpha_i - \beta_i, A_i \in B, \\ & A_i X = b - \alpha_i + \beta_i, A_i \in G, \end{aligned} \tag{6}$$

Where A_i,α_i,and β_i are given, X and b are unrestricted, and α_i,β_i, ${d_\alpha}^-$, ${d_\alpha}^+$, ${d_\beta}^-$, ${d_\beta}^+ \geq 0$ respectively.

The MCLP compromise approach is to determine the classification of Good and Bad according to the minimization of the "distance" function ${d_\alpha}^- + {d_\alpha}^+ + {d_\beta}^- + {d_\beta}^+$ (which offers the best trade-off of MMD and MSD).

Note that for the purpose of classification, a better classifier must have the higher accuracy rate. The basic idea is to given a threshold of correct classification as a simple criterion; the better classifier can be found through the training process whenever the accuracy rate of the model exceeds the threshold [6]. Through adjusting the value of b, we can get the criteria we need.

3 Fisher LDA Models for Classification

For two-group classification, the objective is to maximize the projection points of the two group centers, which is estimated by the mean value:

$$Maximize f(\hat{a}) = \frac{(\hat{a}'\bar{X}_1 - \hat{a}'\bar{X}_2)^2}{\hat{a}' S_P \hat{a}} \tag{7}$$

Here, the sample expectation of each group $\bar{X}_1, \bar{X}_2$ and the whole sample covariance S_P are estimated respectively as following:

$$(\bar{X}_1)_{P*1} = \frac{1}{n_1}\sum_{j=1}^{n_1} X_{1j}, \quad (S_1)_{P*P} = \frac{1}{n_1 - 1}\sum_{j=1}^{n_1}(X_{1j} - \bar{X}_1)(X_{1j} - \bar{X}_1)'$$

$$(\bar{X}_2)_{P*1} = \frac{1}{n_2}\sum_{j=1}^{n_2} X_{2j}, \quad (S_2)_{P*P} = \frac{1}{n_2 - 1}\sum_{j=1}^{n_2}(X_{2j} - \bar{X}_2)(X_{2j} - \bar{X}_2)'$$

By *Cauchy-Schowarz* inequality, the optimal solution for M7 can be gotten as

$$\hat{a} = S_P{}^{-1}(\bar{X}_1 - \bar{X}_2)$$

Thus the two theories describe this classification method [13,16].

Theorem 1. $\hat{y} = a'X = (\bar{X}_1 - \bar{X}_2)' S_P^{-1} X$ *is the optimal solution to the projection function* $f(\hat{a}) = \frac{(\hat{a}'\bar{X}_1 - \hat{a}'\bar{X}_2)^2}{\hat{a}' S_P \hat{a}}$

Theorem 2. *Fisher classification rule: if* $\hat{y_0} = a'X_0 = (\bar{X}_1 - \bar{X}_2)' S_P^{-1} X_0 \geq \hat{m}$, *then the sample will be put in Group one, otherwise the sample will be in Group two. Here,*

$$\hat{m} = \frac{1}{2}(\bar{X}_1 - \bar{X}_2)' S_P{}^{-1}(\bar{X}_1 + \bar{X}_2)$$

This method is especially good if the distribution of the datum is normal. Fisher linear discriminant analysis has been successfully applied to face recognition area in the past few years. In fact there are two kinds of mistakes in two-classification problem; risks of putting one element belonging to a group into another. The ideal classification method is to make the two kinds of mistake as less as possible, i.e. to improve the accurate rate at the same time. Another trade-off method from the Fisher LDA, We can control one mistake rate in a level and improve the accurate rate at the same time.

4 The Integration Idea of FLP and Fisher LDA

Define the membership functions can be expressed respectively by:

$$\mu_{F_1}(x) = \begin{cases} 1, & \text{if } \sum \alpha_i \geq y_{1U}; \\ \frac{\sum \alpha_i - y_{1L}}{y_{1U} - y_{1L}}, & \text{if } y_{1L} < \sum \alpha_i < y_{1U} \\ 0, & \text{if } \sum \alpha_i \leq y_{1L} \end{cases}$$

$$\mu_{D_1}(x)=\begin{cases}1, & \text{if } A_iX\le b+\alpha_i,\quad i\in B\\ 1-\frac{A_iX-(b+\alpha_i)}{d_0}, & \text{if } b+\alpha_i<A_iX<b+\alpha_i+d_0,\quad i\in B\\ 0, & \text{if } A_iX\ge b+\alpha_i+d_0,\quad i\in B\end{cases}$$

$$\mu_{D_2}(x)=\begin{cases}1, & \text{if } A_iX\ge b-\alpha_i,\quad i\in G\\ 1+\frac{A_iX-(b-\alpha_i)}{d_0}, & \text{if } b-\alpha_i-d_0<A_iX<b-\alpha_i,\quad i\in G\\ 0, & \text{if } A_iX\le b-\alpha_i-d_0,\quad i\in G\end{cases}$$

Here d_0 is a flexible coefficient. So we get the next model based on (M1)

$$\begin{aligned}&\textit{Maximize }\ \xi\\ &\textit{Subject to }\ \xi\le\frac{\sum\alpha_i-y_{1L}}{y_{1U}-y_{lL}}\\ &\qquad\qquad \xi\le\frac{\sum\beta_i-y_{2L}}{y_{2U}-y_{2L}}\\ &\qquad\qquad \xi\le 1-\frac{A_iX-(b+\alpha_i)}{d_0},\quad A_i\in B\\ &\qquad\qquad \xi\le 1+\frac{A_iX-(b-\alpha_i)}{d_0},\quad A_i\in G\end{aligned}\tag{8}$$

Where A_i, y_{1L} , y_{1U} , y_{2L} and y_{2U} are known, X and b are unrestricted, d_0 is a flexible coefficient, and α_i, β_i, $\xi\ge 0$.

From (M8) we can think about this problem: how about the result if we move b when we compare the score with boundary?

An algorithm of FLP with moving b can be outlined as:

Step 1. Build a data mart for a task data mining project.

Step 2. Generate a set of relevant attributes or dimensions from a data mart. Transform the scales of the data mart into the same numerical measurement and determine predefined classes, classification threshold τ , training set and verifying set.

Step 3. Give a class boundary value b^*. Use model (M8) to learn and compute the overall score A_iX of the relevant attributes or dimensions over all observations repeatedly.

Note: After get the overall score A_iX^*, not classify A_iX^* with boundary b^*,but with b^*.

Step 4. Adjust ε to find the super or feasible line $b^*\pm\varepsilon d_0$

Step 5. If the accuracy rate of classification exceeds the threshold , go to step 6. Otherwise, go back to Step 3 and choose another boundary value b^{**}.

Step 6. Apply the final learned score X^* and final boundary $b^*\pm\varepsilon d_0$, to predict the unknown data in the verifying set.

From the experimental result based on the algorithm of FLP with moving b, we find that most of the score pointed listed near the b^*, so when $b^*\pm\varepsilon d_0$ is regarded the new boundary, the classification accurate changes a lot, which means most of the data misclassified are near the boundary. Table 2 in section 5 shows us some result about this. A new idea to improve the accurate rate of LP, FLP and MCLP comes with above result. That is the idea of integration of LP and Fisher LDA. There are three steps in the process.

A new idea to improve the accurate rate of LP, FLP and MCLP comes with above result. That is the idea of integration of LP and Fisher LDA. There are three steps in the process.

Step 1. Training, built the LP classification model and LDA model based on the training data in the data warehouse.

Step 2. Testing; compute A_iX to get the score by LP model, if A_iX near the boundary b, which means $A_iX \in [b - \varepsilon_1, b + \varepsilon_1]$, then go to step 3, otherwise predict it with the LP model.

Step 3. Use Fisher LDA model to predict it.

The process above shows us how the integration idea of LP and LDA works. Section 5 lists an example to show the idea based on the algorithm of FLP with moving boundary.

5 Example

In this section, a real-life data mart with 65 derived attributes and 1000 records of a major US bank credit database is first used to train the FLP with moving boundary classifier. Then, the training solution is employed to predict the spending behaviors of another 5000 customers from different states. Finally, we use the distribution chart of the training score A_iX to show the reason we integrate the LP method and Fisher LDA.

There are two kinds of accuracy rates involved in this section. The first one is the absolute accuracy rate for Bad (or Good) which is the number of actual Bad (or Good) identified divided by the total number of Bad (or Good). The second is called catch rate, which is defined as the actual number of caught Bad and Good divided by the total number of Bad and Good.

The following computation is based on (M1), FLP of (M1) and (M8) (FLP of M1 with moving boundary) because we want only to show the reason of this integration idea, although the classification result of model 1 is not the best one among them.

Table 1 indicates that FLP model is a little better for LP for this credit card portfolio classification. And the following table shows the training result and testing result of (M8), an FLP model with moving boundary. It also shows that the little change of the boundary will influence the change of accurate rate.

The distribution of score A_iX with the fixed boundary the boundary 0.5 for group “Good” and “Bad” based on (M8) shows that many scores are near the boundary respectively. That is the reason why the accurate rate changes a lot when the boundary is moved a little. What is more, the distribution of data misclassified shows most of their scores are near the boundary 0.5 for 191

Table 1. Training result of unbalanced 1,000 records

	Absolute Accuracy Rate (M1)			Absolute Accuracy Rate (FLP of M1)		
Different b value	Good(%)	Bad(%)	Catch Rate by (M1)	Good(%)	Bad(%)	Catch Rate) by (FLP of M1)
b=2	0.880233	0.371429	0.809	0.881395	0.3714285	0.81
b=1.1	0.87907	0.378571	0.809	0.87907	0.3785714	0.809
b=0.5	0.87907	0.378571	0.809	0.880233	0.3785714	0.81
b=-0.5	0.682558	0.242857	0.621	0.712791	0.2642857	0.65
b=-1.1	0.682558	0.242857	0.621	0.712791	0.2642857	0.65
b=-2	0.682558	0.242857	0.621	0.712791	0.2642857	0.65

Table 2. Training result of 1000 records and Testing results of 5000 records

		Training :Absolute Accuracy Rate (M8)			Testing: Absolute Accuracy Rate (M8)		
Different b value	b moved	Good(%)	Bad(%)	Catch Rate by (M8)	Good(%)	Bad(%)	Catch Rate) by (M8)
b=2	1.8	1	0.007143	0.861	0.998327	0.004908	0.8364
d0=0.1b	2	0.880233	0.378571	0.81	0.861888	0.393865	0.7856
	2.04	0.655814	0.9	0.69	0.663321	0.790184	0.684
	2.05	0.622093	0.928571	0.665	0.627001	0.819632	0.6584
	2.1	0.49186	0.957143	0.557	0.513501	0.892025	0.5752
	2.2	0.327907	0.992857	0.421	0.346237	0.959509	0.4462
b=1.1	1.09	0.973256	0.128571	0.855	0.940741	0.158282	0.8132
d0=0.1b	1	0.87907	0.378571	0.809	0.861649	0.393865	0.7854
	1.11	0.77093	0.778571	0.772	0.766069	0.644172	0.7462
b=0.5	0.49	0.989535	0.042857	0.857	0.972043	0.068712	0.8248
d0=0.1b	0.5	0.87907	0.378571	0.809	0.861888	0.392638	0.7854
	0.51	0.654651	0.9	0.689	0.662605	0.790184	0.6834

Table 3. Training and Testing result of LP and LDA

	Training			Testing		
	T-g	T-b	Total	T-g	T-b	Total
LP						
Good	756	104	860	3607	578	4185
Bad 87	53	140	495	320	815	
Total	843	157	1000	4102	898	5000
LDA						
Good	725	135	860	3293	892	4185
Bad	39	101	140	176	639	815
Total	764	236	1000	3469	1531	5000

misclassified records. That demonstrates the problem existing in LP classification in this Credit Classification.(The distribution charts are omitted due to the limited space.)

Table 4. Misclassified record number of LP and LDA

LP	Share	LDA
104	63	135
87	51	39
191	114	174

The next question we should consider is that how about result of Fisher LDA for this data set. If LDA and LP misclassified all of the same records, the integration idea is not useful. From Section 2 and Section 3 we know the two methods are based on different idea in theory. Considering the misclassified records in Table 4, we find that LP and LDA method don't share all the records, which shows the feasibility of the integration method.

6 Concluding Remarks

In this paper, we have proposed an integrated method of LP and LDA, which can improve the accuracy of LP. Moreover we explained in theory and by example. According to different characters of data set, different classification methods will show their advantages. For example, neural network and SVM method shows its high accurate rate in classification. QDA and LDA show their power dealing with Gauss distribution data set. LP and MCLP are good for knowledge explanation and easy to understand. But different method has its shortcoming too, so an integration method may be a good idea to integrate their advantage together while reducing their shortcomings. Here the integration idea of LP and LDA is proposed. To explain this method we just give a simple example based on the real data from a bank. However there are several detail problems to be researched in future, for example, how ε_1 is valued and how it affects the accurate rate will be studied. In theory the integration idea of LP and LDA can be used to FLP-LDA, MCLP-LDA. More testing will be done on this method and the result of these ongoing projects will be reported in the near future.

Acknowledgement

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Cooperation Evolution of Indirect Reciprocity by Discrimination

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Abstract Cooperation is an important speciality of human behaviour. Unlike most creatures, people frequently cooperate with genetically unrelated strangers, often in large groups, with people they will never meet again. These patterns of cooperation can be explained by the theory of indirect reciprocity. That is to say, cooperation appears because it confers the image of a valuable community member. In a sense, to discriminate a valuable member is prerequisite for cooperation. By analytic model and computer simulations in this paper, we show the essence of cooperation mechanism consists of discriminators and punishment. In particular, we show that discriminators of different grades have dissimilar effects.

1 Introduction

Throughout human evolutional history, crucial human activities like hunting big game, sharing meat and warfare constituted public goods. Every group member benefits from the "goods", even though he did not pay any costs of providing the goods. It raises the question of why such cooperative activities[1] as warfare and big game hunting appear [6, 18].

Explanation of human cooperation is often based on genetic relatedness (kin selection [8, 9]) and repeated game (e.g., reciprocity [2,3,20]) . Kin selection is an important explanation for human cooperation as well as for other animals. Kin selection has become a template of explanation and is gradually extended to cooperation among non-kin [16]. However it seems to be implausible to explain human cooperation among large group of unrelated individuals in this way. As for the repeated game, though capable of explaining why self-regarding individuals would cooperate, it is too demanding to meet with in the real world. Indirect reciprocity predominates in real

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[1] In this paper cooperation means an individual behavior that incurs personal costs in order to engage in a joint activity that confers benefits to other members of one's group(b>c),namely altruism. This narrow definition excludes mutually beneficial interactions (mutualisms).

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world, i. e. one does not expect a return from the recipient, but from someone else. Cooperation is channeled towards the valuable members of the community. This has been called the “I won't scratch your back if you won't scratch their backs” [4]-principle.

Indirect reciprocity involves reputation and status, and results in every group member being continually assessed and re-assessed [1]. Some authors [15,17,21,22,23] explained cooperation and beneficial activities by signal. For example, Smith and Bliege [17] explore by means of costly signaling theory the altruistic behavior, which exists in turtle hunting and public feasting in Meriam of Torres Strait, Australia.

Boyd and Richerson [5] were the first to develop a mathematical model of cooperation based on indirect reciprocity. In their model, individuals are arranged in loop. Boyd and Richerson investigated two strategies: up stream TFT(Tit For Tat) and downstream TFT. Their analysis of the indirect reciprocity indicated that the conditions necessary for the evolution of indirect reciprocity become restrictive as group size increases. In a sense, this indirect reciprocity is similar to direct reciprocity. For the diversity of society structure, human agents and their interactions tend to form embedded graphs which are not rings [7].

Cooperation flourishes if altruistic punishment is possible, and breaks down if it is ruled out [6]. In other words, punishment is a kind of public goods. Discrimination and punishment mechanism are hence prerequisite.

Nowak and Sigmund [13] developed a model of indirect reciprocity in virtue of image scores and modeled the discriminators and defectors. In their model, each individual has an image score and a strategy. Each payoff is determined by image score and strategy in random match pairs. Simultaneously image score of the donor is adjusted. They concluded that cooperation mechanism could be established by image score in case of sufficient interactions. They further developed dynamic replication equation of cooperators, defectors and discriminators based on indirect reciprocity [14].

In the real world, each person discriminates for his unique cognitive and physical capacity. It is not enough to comprehend the cooperation mechanism with single discriminator.

In view of the complexity of human society, in this paper, we introduce two kinds of discriminators which are sorted into low-grade and high-grade. Low-grade discriminators are more tolerant, that is, they will cooperate except that the co-player chose defection twice in the last two activities. We label it tolerant TFT. High-grade discriminators are more stricter, that is, they will cooperate if only the co-player chose cooperation twice in the last two activities. We label it Tit for two Tat.[2] In this paper, we explore the relationship among the cooperation mechanism, different discriminators and non-discriminators with mathematical model and computer simulation. We show that cooperators are easily invaded by defectors, and discriminators can protect cooperators from invading by defectors. The essence of cooperation mechanism consists of discriminators and punishment. And tolerant TFT is more inclined to

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[2] In fact, these two discriminator strategies in this case coincide with a variant of Tit For Tat or Tit for two Tat which based on observation not their own experience in his last two interactions.

establish cooperation mechanism and Tit for two Tat is in reverse. In summary, Tit for two Tat is a kopis with opposite effects under different conditions.

The paper is organized as follows. In Section 2, we develop a basic model showing the process of cooperation evolution of indirect reciprocity by discrimination and analyzing the payoff of each kind of individuals. In Section 3, we show that under plausible conditions the equilibrium is dynamically stable, and in Section 4, we simulate different initial conditions by computer. Section 5 draws some conclusions and implications for further research.

2 A Basic Model

Assume in a group that there are two kinds of individuals: non-discriminators and discriminators. The former always give help, or never. We shall denote the frequency of the cooperators by x_1, and that of the defectors by x_2. The discriminators consist of low and high. We denote the frequency of them x_3 and x_4 respectively. These discriminators assess each member in the group and keep track of their image score. Low-grade discriminators are more tolerant, that is they will cooperate unless the co-player chose defect in the last two activities. High-grade discriminators are more stricter, that is they will cooperate if only the co-player chose cooperation in the last two activities.

Now we shall assume that each generation experiences a certain number of rounds of interactions. In each round, every player is both in the position of a donor and a recipient. In each of these roles, the player interacts with a randomly chosen co-player. If only few rounds occur, then the likelihood of meeting the same co-player twice is very small. In this paper we take no account of this possibility. Simultaneity, we assume all can be observed in the group, namely complete information. We also assume there is no mutation and social learning in and between each generation. In this case, the standard model is replication dynamics. The rate of one strategy's increase in the group is assumed to be linear function of payoff relative to average payoff. Therefore, payoff is the rate of successful reproduction.

From the last two rounds, discriminators can distinguish among those who have helped twice and thereby acquired score G(good)[3], those who have helped once and thereby acquired score M(medium), and those who have withheld assistance twice, and acquired score B(bad). Low-grade discriminators help G-players and M-players. High-grade discriminators only help G-players. In the n-th round, g_n, m_n, b_n denote the frequency of that score is G, M and B respectively.

We can obtain $g_1 = x_1 + x_3$, $m_1 = 0$, $b_1 = x_2 + x_4$. Then,

$$\begin{cases} g_2 = x_1 + (x_1 + x_3)x_3 \\ m_2 = (x_2 + x_4)x_3 + (x_1 + x_3)x_4 \\ b_2 = x_2 + (x_2 + x_4)x_4 \end{cases} \tag{1}$$

[3] There is a subjective probability in Nowak and Sigmund 'paper. In this paper we assume p=1, that is low-grade discriminator deem the co-player is good at the beginning of each generation.

Clearly,

$$\begin{cases} g_n = x_1 + g_{n-1}(x_3 + x_4) + m_{n-1}x_3 \\ b_n = x_2 + b_{n-1}(x_3 + x_4) + m_{n-1}x_4 \\ \quad m_n = 1 - g_n - b_n \end{cases} \tag{2}$$

In the first round, the payoff[4] for a cooperator is $-c + b(x_1 + x_3)$. The payoff for a defector is similarly $b(x_1 + x_3)$. The payoff for a low-grade discriminator is $-c + b(x_1 + x_3)$ and that for a high-grade discriminator is $b(x_1 + x_3)$.

In the second round, the payoff for a cooperator is $-c + b(x_1 + x_3 + x_4)$. The payoff for a defector is bx_1. The payoff for a low-grade discriminator is $(b-c)(g_2 + m_2)$ and that for a high-grade discriminator is $(b-c)g_2$.

In the n-th round(n>3), the payoff for a cooperator is $-c + b(x_1 + x_3 + x_4)$. The payoff for a defectors is bx_1. For a low-grade discriminator, the proportion of G and B is g_{n-1} and b_{n-1}. Adding up, the payoff is $(b-c)g_n$. The payoff for a high-grade discriminator is similarly $(b-c)g_n$.

The structure of a game has invariability if the same value is subtracted from all payoff functions [10]. We then obtain as normalized payoff functions

$$P_i = \tilde{P}_i - \tilde{P}_2 (i = 1,2,3,4) \tag{3}$$

For n=2, normalized payoff functions

$$\begin{cases} P_1 = b(x_3 + x_4) - 2c \\ P_2 = 0 \\ P_3 = b[x_3 + (x_1 + x_3)x_4] - c[1 + x_1 + x_3 + (x_1 + x_3)x_4] \\ P_4 = b(x_1 + x_3)x_3 - c[x_1 + (x_1 + x_3)x_3] \end{cases} . \tag{4}$$

3 Discussion

We can use replication dynamics equation [12,19]

$$\dot{x}_i = x_i (P_i - \overline{P}), \overline{P} = \Sigma x_i P_i \tag{5}$$

to investigate the evolution of the frequencies of the four types of players under the influence of selection in the simplex

$$S_4 = \{x = (x_1, x_2, x_3, x_4) \in R^4 : x_i \geq 0, \Sigma x_i = 1 (i = 1,2,3,4)\}. \tag{6}$$

For simplicity, we discuss the evolution of the frequencies of the four types of players in the case of n = 2 mostly.

For b>c, if each x_i is not equal to 0, there is no real number solution in the simultaneous equations(4-5). From this, we have the proposition 1.

Proposition 1. Simplex S4 has no fixed point in its interior.

According to proposition 1, the system would converge to the convex surface after much iteration. As a result, four kinds of individuals can not coexist.

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[4] In this paper, the payoff is expected payoff for the random match pair.

Here we discuss in the four situations as follow.

Type 1: $x_4 = 0$

It follows immediately that the replication equation admits no interior fixed point in $S123 = \{x = (x_1, x_2, x_3) \in R^3 : x_i \geq 0, \Sigma x_i = 1(i = 1,2,3)\}$.

When $b > 2c$, in the absence of defectors, the edge $x_1 x_3$ consists of fixed points: both types do equally well. In the absence of low-grade discriminators, defectors win, the flow points from x_1 to x_2. In the absence of cooperators, there is a fixed point F_{23} on the edge $x_2 x_3$, namely $x_2 = (b - 2c)/(b - c)$. The system on the edge $x_2 x_3$ is bi-stable. See Figure 1.

When $b = 2c$, the two points of F_{23} and x_3 superpose, line AF_{23} moves up. When $2c \geq b > c$, the system has only one fixed point x_2 and ends up with a regime of defection.

Type 2: $x_3 = 0$

It follows immediately that the replication equation admits no interior fixed point in $S124 = \{x = (x_1, x_2, x_4) \in R^3 : x_i \geq 0, \Sigma x_i = 1(i = 1,2,4)\}$.

When $b > 2c$, in the absence of cooperators, the edge $x_2 x_4$ consists of fixed points: both types do equally well. In the absence of high-grade discriminators, defectors win, the flow points from x_1 to x_2. In the absence of cooperators, there is a fixed point F_{14} on the edge $x_1 x_4$, namely $x_1 = (b - 2c)/(b - c)$. Except edge $x_1 x_4$, the system is in defection. See Figure 2.

When $b = 2c$, the two points of F_{14} and x_4 superpose. When $2c \geq b > c$, the system converges to $x_2 x_4$ and ends up with a regime of defection.

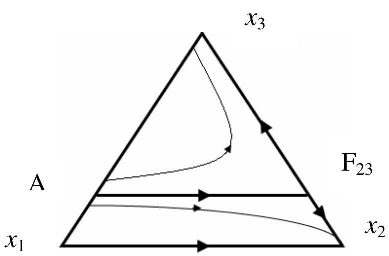

Fig. 1. Phase portrait of the model of S123

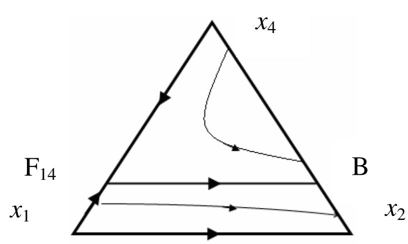

Fig. 2. Phase portrait of the model of S124

Type 3: $x_2 = 0$

There is no interior fixed point in $S134 = \{x = (x_1, x_3, x_4) \in R^3 : x_i \geq 0, \Sigma x_i = 1(i = 1,3,4)\}$.

When $b > 3c$, in the absence of high-grade discriminators, the edge $x_1 x_3$ consists of fixed points: both types do equally well. In the absence of cooperators, there are two fixed points $F_{34}\left(x_3 = \frac{1}{2}\left(1 + \sqrt{\frac{b-3c}{b-c}}\right)\right)$ and $F'_{34}\left(x_3 = \frac{1}{2}\left(1 - \sqrt{\frac{b-3c}{b-c}}\right)\right)$. The system on the edge

x_3x_4 is bi-stable. In the absence of low-grade discriminators, there is a fixed point F_{14} on the edge x_1x_4, namely $x_1 = (b-2c)/(b-c)$. The system is in cooperation. See Figure 3.

When $3c \geq b > 2c$, in the edge x_3x_4, the flow points from x_3 to x_4. At this time, the system converges to F_{14} or x_1x_3. When $2c \geq b > c$, the system converges to x_4.

Type 4: $x_1 = 0$

The system converges to x_2x_4 and ends up with a regime of defection. See Figure 4.

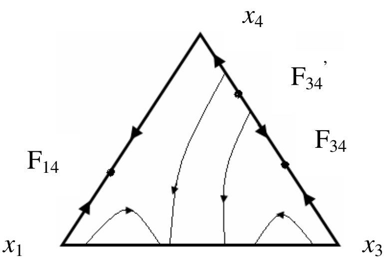

Fig. 3. Phase portrait of the model of S134

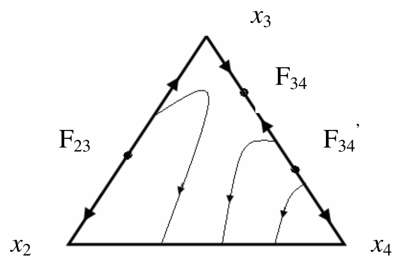

Fig. 4. Phase portrait of the model of S234

By the quotient rule for the replication dynamics, let $x_1/x_2 = U$, $x_D = x_3 + x_4$, then

$$\dot{U} = U[b(n-1)(x_3 + x_4) - 2c] = U[b(n-1)x_D - 2c] \quad (7)$$

From (10), we have the proposition 2.

Proposition 2. Discrimination is the key factor of cooperation mechanism. Cooperation mechanism establishes when $x_D > 2c/b(n-1)$; defection mechanism establishes when $x_D < 2c/b(n-1)$.

Proposition 2 shows the essence of cooperation mechanism. According to proposition 2, only one kind of the non-discriminators vanishes at first(e.g., in Figure 5-6, defectors vanish first). Then discriminators and the left non-discriminator converge to equilibrium.

Cooperators would be invaded by defectors when n=1. With the interaction growing, that is, n increases, cooperation mechanism would be established with more facility.

Figure1-4 show low-grade discriminators promote the establishment of cooperation. On the contrary, high-grade discriminators promote the establishment of defection.

Let us consider discriminators, $x_3/x_4 = V$,

$$\dot{V} = V\{(b-c)[(x_2 + x_4)x_3 + (x_1 + x_3)x_4] - c\} \quad (8)$$

From (8), we have the proposition 3.

Proposition 3. The ascending and descending of frequency of different grade discriminators is independent of interaction degree, and it is related with original distribution and b/c.

4 Simulation

The group consists of 4000 individuals. The image score ranges from −1 to +1, the strategy (k) from −1 to +2. The strategy, k = −1, represents unconditional cooperators, while the strategy, k = +2, represents defectors. In each round of the game, two individuals are chosen at random; one as donor, the other as recipient. The donor cooperates if the image score of the recipient is greater than or equal to the donor's k. Cooperation means the donor pays a cost, c, and the recipient obtains a benefit, b. The image score of donor increase 1 unit. There is no payoff in the absence of cooperation. Simultaneously, the image score of donor decrease 1 unit.

In the beginning of each generation all players have image score 0. In each generation m = 16000 pairs are chosen; each player has, on average, 4 interactions. It means n=2. The probability of direct reciprocity is negligibly small. At the end of each generation, players produce offspring proportional to their payoff. Parameter values: b = 1, c = 0.1. In Figure 5-6, t = 0 denote the different frequency of strategies at the begin. The second and the third t value show the moment of defectors' disappearance and the moment of high-grade discriminators' disappearance.

The results of Figure 5-6 indicate that four kinds of individuals can not coexist. System would converge to the convex surface at equilibrium or near equilibrium. In the

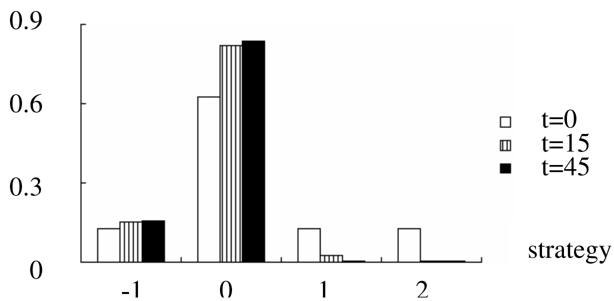

Fig. 5. Simulation results in the case of more low-grade discriminators

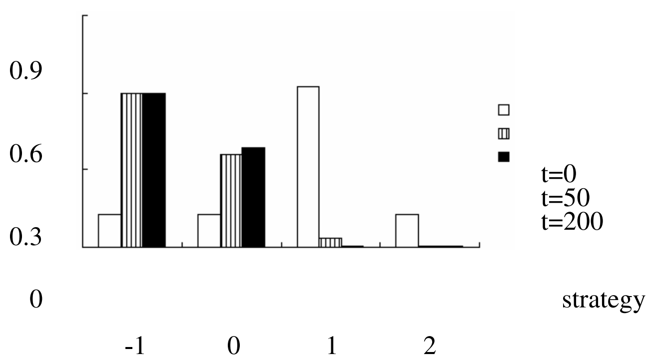

Fig. 6. Simulation results in the case of more high-grade discriminators

cases mentioned at Figure 5-6, $2c/b(n-1)=0.2$ and $x_D > 2c/b(n-1)$, cooperation mechanism establishes. It is consistent with Proposition 1 and 2.

5 Conclusion

In the cooperation evolution of indirect reciprocity, players make tradeoff between avoiding cost in the short term and increasing chance of being helped by co-player in the long term. That is, in the short term, avoiding the cost yields, of course, the higher payoff. In the long term, however, performing the altruistic act increases the image score of the donor and may therefore increase the chance of obtaining a benefit in a future encounter as a recipient. In this paper discriminators are sorted into low and high. We explore the relationship among the cooperation mechanism, different discriminators and non-discriminators with mathematical model and computer simulation. We show that cooperators are easily invaded by defectors, and discriminators can protect cooperators from invading by defectors. Simultaneously, in discriminators, low-grade is more inclined to establish cooperation mechanism and high-grade is in reverse. In summary, the high-grade discriminator is a kopis, having opposite effect under different condition.

Our evidence has a certain extent implications for the evolutionary study of human behaviour. In the past, human cooperation has mainly been explained in terms of kin selection, reciprocity. These theories focus attention on mechanisms other than discrimination. We show that the essence of cooperation mechanism consists in discriminator and punishment. Like in human society, each person discriminates for his unique cognitive and physical capacity. It is all important to comprehend the indispensable influence of discrimination to establish cooperation mechanism.

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Fixed-Point Model and Schedule Reliability of Morning Commuting in Stochastic and Time-Dependent Transport Networks

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Abstract. This paper presents a fixed-point model for studying the morning commuting behavior in stochastic and time-dependent transport networks. The model considers the network uncertainty caused by supply and demand variations as well as the commuters' perception errors on travel disutility. A new performance index called schedule reliability, i.e., the probability that commuters can arrive at destination on time or within a given time threshold early or late, is introduced to evaluate the network service level. The solution algorithm which combines the Monte Carlo simulation approach with the method of successive average is proposed together with a numerical example for demonstration.

1 Introduction

Service quality provided by an urban transport system is very important for all network users, particularly for morning commuters who seem to value more highly a reduction in variability than in the mean travel time for their journeys [1]. The variability is a result of fluctuations in travel demands and road supplies associated such stochastic events as traffic accidents, vehicle breakdowns, signal failures and adverse weather [8]. This variability may let commuters be unable to arrive at workplaces on time, consequently a vast amount of schedule delay costs occur. Hence, it is meaningful to investigate the reliability of morning commuting in congested and uncertain transport networks.

Substantial attention has recently been paid to this topic, termed transport network reliability in literature. The existing studies can be classified into such categories as connectivity reliability [2], travel time reliability [3, 11, 12], capacity reliability [6, 7, 8], behavioral reliability [16, 21], and potential reliability [4, 5]. However, all these studies mainly focus on time-stationary (static) paradigm, little on time-dependent or dynamic paradigm. Static models and associated reliability measures are unable to reveal the complex temporal and spatial interaction between road supply and traffic demand, and thus cannot be used to evaluate the time-varying service level of a transport network over the time of a day.

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This paper aims at developing a framework of analyzing the schedule reliability of morning commuting in stochastic and time-dependent networks. The schedule reliability is defined as a probability that commuters can arrive at destinations on time or within a given time threshold early or late. This index can be used to evaluate the service level of a transport network during morning rush hour. We propose a fixed-point model which can simultaneously determine the commuters' departure time and route choices on the basis of taking the commuters' responses to network uncertainty into full account. For solving this model, we design a heuristic algorithm that embeds the Monte Carlo simulation approach into the method of successive average.

The paper is organized as follows. Section 2 gives some basic considerations for setting our study. Section 3 formulates the fixed-point model. The contents about heuristic algorithm and schedule reliability are presented in Section 4. Numerical results on an application of the model and algorithm to an example network are provided in Section 5. Section 6 concludes the paper.

2 Basic Considerations

2.1 Actual Travel Disutility

Consider a transportation network $G = (N, A)$, where N is the set of nodes and A is the set of links. Let R denote the set of origins r, $r \in R \subset N$, S the set of destinations s, $s \in S \subset N$. Let P_{rs} be the set of all routes p connecting origin r and destination s, $p \in P_{rs}$. The whole study period $[0, T]$ is discretized into equal time intervals that are sequentially numbered by $t \in T = \{0, 1, \ldots, \bar{T}\}$. Let δ be the length of an interval, then $\bar{T}\delta = T$. The value of T is sufficiently large so that all commuters can complete their journeys within the study period $[0, T]$.

Let $T_p^{rs}(t)$ be the actual route travel time of commuters departing from origin r at interval t to destination s via route p, $c_a(t)$ the actual link travel time of commuters on link a during interval t. The route time $T_p^{rs}(t)$ is the sum of all link travel times along this route [10], i.e..

$$T_p^{rs}(t) = \sum_a \sum_{k \geq t} c_a(k) \delta_{apt}^{rs}(k), \forall p, r, s, t \tag{1}$$

where the indicator variable, $\delta_{apt}^{rs}(k)$, equals 1 if the flow on route p departing from origin r at interval t to destination s arrives at link a at interval k, and 0 otherwise.

Link travel time often fluctuates over the time of a day because of various stochastic events like vehicle breakdowns, signal failures, roadwork and adverse weather [1]. In this paper, the actual link travel time $c_a(t)$ is assumed to follow independently the normal distribution, i.e., $c_a(t) \sim N\left(\tau_a(t), \left(\sigma_a(t)\right)^2\right)$, where $\tau_a(t)$ is the expected link travel time of commuters on link a during interval t and $\sigma_a(t)$ is the standard deviation. Previous studies have shown that the standard deviation is a function of the associated mean [20]. For simplicity, we set $\sigma_a(t) = \rho_a \tau_a(t)$, where ρ_a is a link-related constant.

Let $\varphi_p^{rs}(t)$ denote the actual route travel disutility of commuters departing from origin r at interval t to destination s via route p. Following [18, 19], we attribute the actual route travel disutility to the in-vehicle travel time, the penalty of arriving early or late at destination and a discrete lateness penalty term, i.e.,

$$\varphi_p^{rs}(t) = \alpha T_p^{rs}(t) + \Theta_s(t) + \theta D_L^{rs}(t), \forall p, r, s, t \tag{2}$$

where $\Theta_s(t)$ is the schedule delay cost of early or late arrival at destination s, $D_L^{rs}(t)$ is a dummy variable which is equal to 1 if commuters departing at interval t arrive late and 0 otherwise, α is the value of time, and θ is an additional discrete lateness penalty. Equation (2) states that the disutility jumps up by the value θ once commuters arrive late [18, 19].

The schedule delay cost of early or late arrival can be formulated below [13]

$$\Theta_s(t) = \begin{cases} \beta\big(t_s^* - \Delta_s - t - T_p^{rs}(t)\big), & \text{if} \quad t + T_p^{rs}(t) < t_s^* - \Delta_s \\ \gamma\big(t + T_p^{rs}(t) - t_s^* - \Delta_s\big), & \text{if} \quad t + T_p^{rs}(t) > t_s^* + \Delta_s \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where $[t_s^* - \Delta_s, t_s^* + \Delta_s]$ is the desired arrival time window at destination s without any schedule delay penalty, t_s^* is the middle point of the time window, and $\beta(\gamma)$ is the unit cost of early (late) arrival.

2.2 Expected Travel Disutility

The expected link travel time experienced by commuters who enter link a during interval k, can be expressed as a function of all inflows entering that link by interval k [10], i.e.,

$$\tau_a(k) = \varphi\big(u_a(1), u_a(2), \ldots, u_a(k)\big), \quad \forall a, k \tag{4}$$

where $u_a(k)$ is the inflows that enter link a during interval k, and

$$u_a(k) = \sum_{rs} \sum_{p} \sum_{t \le k} u_{apt}^{rs}(k), \quad \forall a, k \tag{5}$$

where $u_{apt}^{rs}(k)$ is the inflow of link a during interval k for these commuters who depart from origin r to destination s via route p during interval t, and

$$u_{apt}^{rs}(k) = f_p^{rs}(t)\delta_{apt}^{rs}(k), \quad \forall a, p, r, s, t, k \tag{6}$$

where $f_p^{rs}(t)$ is the departure flow rate of route p between OD pair (r, s) during interval t, $\delta_{apt}^{rs}(k)$ is defined in Equation (1).

Referring to Equation (2), the expected route travel disutility, $\Omega_p^{rs}(t)$ can be formulated as

$$\Omega_p^{rs}(t) = \alpha E[T_p^{rs}(t)] + E[\Theta_s(t)] + \theta E[D_L^{rs}(t)], \quad \forall p, r, s, t \tag{7}$$

Note that all link travel times, $c_a(t)$, $a \in A$, are assumed to be random variables with normal distribution, the route travel time $T_p^{rs}(t)$ can then be regarded as a random variable with normal distribution, i.e.,

$T_p^{rs}(t) \sim N\left(E[T_p^{rs}(t)], \left(\sigma\left(T_p^{rs}(t)\right)\right)^2\right)$, where $E[T_p^{rs}(t)] = \sum_a \sum_{k\geq t} \tau_a(k)\delta_{apt}^{rs}(k)$ and $\sigma\left(T_p^{rs}(t)\right) = \sqrt{\left(\sum_a \sum_{k\geq t} \left(\sigma_a(k)\right)^2 \delta_{apt}^{rs}(k)\right)}$.

The probability that commuters departing from origin r at interval t arrive late at destination s, can be formulated as

$$P\{t + T_p^{rs}(t) > t_s^* + \Delta_s\} = \int_{t_s^*+\Delta_s-t}^{\infty} N(x)\mathrm{d}x = 1 - \Phi\left(\frac{t_s^* + \Delta_s - t - E[T_p^{rs}(t)]}{\sigma\left(T_p^{rs}(t)\right)}\right) \qquad (8)$$

where $N(\cdot)$ denotes the density function of the normal distribution variable $T_p^{rs}(t)$ and $\Phi(\cdot)$ the distribution function of a standard normal distribution. It is easy to understand that the expected value of the dummy variable $D_L^{rs}(t)$ in fact represents the possibility of arrival late, i.e., $E[D_L^{rs}(t)] = P\{t+T_p^{rs}(t) > t_s^*+\Delta_s\}$.

The remainder is to compute the expected value of the schedule delay cost $\Theta_s(t)$ given by Equation (3). When $t + T_p^{rs}(t) < t_s^* - \Delta_s$, the expected schedule delay cost of early arrival is given by

$$\int_0^{t_s^*-\Delta_s-t} \beta(t_s^* - \Delta_s - t - \xi)N(\xi)\mathrm{d}\xi = \beta\left(\frac{\sigma\left(T_p^{rs}(t)\right)}{\sqrt{2\pi}}\right) \times\left(\exp\left(-\frac{1}{2}\left(\frac{t_s^* - \Delta_s - t - E[T_p^{rs}(t)]}{\sigma\left(T_p^{rs}(t)\right)}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{E[T_p^{rs}(t)]}{\sigma\left(T_p^{rs}(t)\right)}\right)^2\right)\right) +\beta\left(t_s^*-\Delta_s-t-E[T_p^{rs}(t)]\right)\left(\Phi\left(\frac{t_s^* - \Delta_s - t - E[T_p^{rs}(t)]}{\sigma\left(T_p^{rs}(t)\right)}\right)-\Phi\left(-\frac{E[T_p^{rs}(t)]}{\sigma\left(T_p^{rs}(t)\right)}\right)\right) \qquad (9)$$

Similarly, when $t + T_p^{rs}(t) > t_s^* + \Delta_s$, the expected schedule delay cost of late arrival is given by

$$\int_{t_s^*+\Delta_s-t}^{\infty} \gamma(t - t_s^* - \Delta_s + \xi)N(\xi)\mathrm{d}\xi = \gamma\left(\frac{\sigma\left(T_p^{rs}(t)\right)}{\sqrt{2\pi}}\right)\exp\left(-\frac{1}{2}\left(\frac{t_s^* + \Delta_s - t - E[T_p^{rs}(t)]}{\sigma\left(T_p^{rs}(t)\right)}\right)^2\right) +\gamma\left(t_s^* + \Delta_s - t + E[T_p^{rs}(t)]\right)\left(1 - \Phi\left(\frac{t_s^* + \Delta_s - t - E[T_p^{rs}(t)]}{\sigma\left(T_p^{rs}(t)\right)}\right)\right). \qquad (10)$$

3 Fixed-Point Model

Denote $\hat{\Omega}_p^{rs}(t)$ as the perceived route travel disutility by commuters who depart from r to s via route p at interval t. The perceived disutility consists of systematic and random components, i.e.,

$$\hat{\Omega}_p^{rs}(t) = \Omega_p^{rs}(t) + \xi_p^{rs}(t), \quad \forall p, r, s, t \qquad (11)$$

where the systematic component $\Omega_p^{rs}(t)$ is given by Equation (7) and $\xi_p^{rs}(t)$ represents the random error of perceiving the expected route travel disutility. Note that up to now, we have introduced two types of random errors, one caused by the network's physical condition and another by the commuters' perception. The route perception error is the sum of all link perception errors, i.e.,

$$\xi_p^{rs}(t) = \sum_a \sum_{k \geq t} \xi_a(k) \delta_{apt}^{rs}(k), \quad \forall p, r, s, t \tag{12}$$

Suppose that commuters make their choices on departure time and route in terms of the minimal perceived travel disutility, the probability of choosing interval t for departure and route p for travel is then given by

$$P_p^{rs}(t) = P\{\hat{\Omega}_p^{rs}(t) < \hat{\Omega}_q^{rs}(k) \mid \forall k, t \in T, p, q \in P_{rs}, k \neq t, q \neq p\}, \forall p, r, s, t \tag{13}$$

Clearly, $P_p^{rs}(t)$ is dependent on the probability distribution of random variable $\xi_a(t)$. In this paper, it is assumed that the random variable $\xi_a(t)$ follows a normal distribution with zero mean, i.e., $\xi_a(t) \sim N\Big(0, \big(\lambda_a \tau_a(t)\big)^2\Big)$, where λ_a is a link-related constant.

Let Q_{rs} be the total demand of commuters between OD pair (r,s) within the whole study period $[0,\ T]$, which is assumed to be a random variable too. Let q_{rs} and σ_{rs} denote the mean value and standard deviation of the total demand, respectively. We then have

$$Q_{rs} = q_{rs} + \omega \sigma_{rs}, \quad \forall r, s \tag{14}$$

where ω is a random variable generated from standard normal distribution $N(0, 1)$. Hence, the route inflow can be formulated as

$$f_p^{rs}(t) = Q_{rs} P_p^{rs}(t), \quad \forall p, r, s, t \tag{15}$$

According to Equations (11)-(15), it can be inferred that the route inflow $\mathbf{f}$ is the function of systematic component $\mathbf{\Omega}$, which is the function of route travel time $\mathbf{T}$, which is in turn the function of route inflow $\mathbf{f}$ in terms of Equations (1)-(6). Therefore, this supply-demand circular dependence gives the following fixed-point equilibrium

$$\mathbf{f}^* = \mathbf{Q} \cdot \mathbf{P}\Big(\mathbf{\Omega}\Big(\mathbf{T}\big(\tau(\mathbf{f}^*)\big)\Big)\Big) \tag{16}$$

where the bold symbols represent the vectors of corresponding variables.

Note that all functions defined above are continuous subject to route inflows. According to the Brouwer's fixed-point theory, there exists at least one solution to the fixed-point problem (16). However, the indicator variable in Equation (1) depends on the link travel times, which in turn depends on link inflows. Consequently, the route travel times are essentially non-linear and non-convex [13]. Thus, the fixed-point model (16) is non-convex, which implies multiple local solutions may exist.

4 Solution Method and Schedule Reliability

4.1 Solution Method

In this section, a heuristic algorithm that combines the Monte Carlo simulation method with the method of successive average (MSA) is proposed to solve the fixed-point model (16). The step-by-step procedure of the solution method is given below.

Step 1 (Initialization). Choose an initial route inflow $\mathbf{f}^{(1)}$ and set $n = 1$.
Step 2 (Outer loop operation). Set $\kappa = 1$.
Step 3 (Inner loop operation, i.e., stochastic network loading).
Step 3.1. Calculate the link inflow $\mathbf{u}^{(\kappa)}$ and the expected link travel time $\tau^{(\kappa)}$.
Step 3.2. Generate random link travel times and perception errors by Monte Carlo simulation, and compute perceived route travel disutility $\hat{\Omega}_p^{rs(\kappa)}(t)$.
Step 3.3. Generate OD demands by Monte Carlo simulation, assign OD demands onto the network by all-or-nothing loading, and yield auxiliary route inflow $\tilde{\mathbf{g}}^{(\kappa)}$.
Step 3.4. Update the route inflow, $\mathbf{g}^{(\kappa)} = \left((\kappa - 1)\mathbf{g}^{(\kappa-1)} + \tilde{\mathbf{g}}^{(\kappa)}\right)/\kappa$.
Step 3.5. If the sample number κ is less than a pre-specified sample size, then let $\kappa = \kappa + 1$ and go to Step 3.1; Otherwise, let $\mathbf{g}^{(n)} = \mathbf{g}^{(\kappa)}$ and go to Step 4.
Step 4 (Update). Update the route inflow, $\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} + (\mathbf{g}^{(n)} - \mathbf{f}^{(n)})/n$.
Step 5 (Convergence check). If the relative gap $G = \|\mathbf{f}^{(n+1)} - \mathbf{f}^{(n)}\|/\|\mathbf{f}^{(n)}\|$ is less than a pre-specified precision, ε, then stop; otherwise, let $n = n + 1$ and go to Step 2.

In Step 1, the initial route inflow can be set to be zero. In Steps 3, every stochastic network loading generates a new minimum-disutility tree. So, the total number of minimum-disutility trees required by the algorithm equals the number of stochastic network loading carried out in the inner loop multiplied by the number of iterations in the MSA outer loop. In Step 5, the relative gap G measures how closely the outputs generated at iteration n approach the equilibrium condition (16).

4.2 Schedule Reliability

The schedule reliability in this study is defined as the probability that commuters between a given OD pair departing at a certain time interval arrive at destination on time or within a time threshold early or late. We introduce two indexes for measuring the schedule reliability, namely the route based one and the OD based one. The OD based schedule reliability is the weighted average of all route based schedule reliabilities for a given OD pair.

Let $P_p^{rs}(k_1, k_2|t)$ denote the schedule reliability of early arriving within a specified time threshold $[k_1, k_2]$ for commuters who depart from r at interval t to s via route p. It can mathematically be written as

$$P_p^{rs}(k_1,k_2|t) = P\{k_1 < t_s^* - \Delta_s - t - T_p^{rs}(t) < k_2\}$$
$$= \Phi\left(\frac{t_s^* - \Delta_s - k_1 - t - E[T_p^{rs}(t)]}{\sigma\big(T_p^{rs}(t)\big)}\right) - \Phi\left(\frac{t_s^* - \Delta_s - k_2 - t - E[T_p^{rs}(t)]}{\sigma\big(T_p^{rs}(t)\big)}\right) \quad (17)$$

Similarly, the schedule reliability of late arriving within a specified time threshold $[k_1, k_2]$ is

$$P_p^{rs}(k_1,k_2|t) = P\{k_1 < t - t_s^* - \Delta_s + T_p^{rs}(t) < k_2\}$$
$$= \Phi\left(\frac{t_s^* + \Delta_s + k_2 - t - E[T_p^{rs}(t)]}{\sigma\big(T_p^{rs}(t)\big)}\right) - \Phi\left(\frac{t_s^* + \Delta_s + k_1 - t - E[T_p^{rs}(t)]}{\sigma\big(T_p^{rs}(t)\big)}\right) \quad (18)$$

In addition, the probability of punctually arriving within the time window $[t_s^* - \Delta_s, t_s^* + \Delta_s]$ is

$$P_p^{rs}(t_s^* - \Delta_s, t_s^* + \Delta_s|t) =$$
$$= \Phi\left(\frac{(t_s^* + \Delta_s - t) - E[T_p^{rs}(t)]}{\sigma\big(T_p^{rs}(t)\big)}\right) - \Phi\left(\frac{(t_s^* - \Delta_s - t) - E[T_p^{rs}(t)]}{\sigma\big(T_p^{rs}(t)\big)}\right) \quad (19)$$

Then, the OD based schedule reliability is given by

$$P_{rs}(k_1,k_2|t) = \sum_{p\in P_{rs}} P_p^{rs}(k_1,k_2|t) f_p^{rs}(t) \Big/ \sum_{p\in P_{rs}} f_p^{rs}(t) \quad (20)$$

It should be pointed out that with different time thresholds, the schedule reliability introduced above can reflect the differences of the network's service levels during different time periods of a day. This cannot be achieved in the static paradigm that only studies the issues of one period.

5 Numerical Results

In this section, we present the numerical results of applying the proposed research framework to an example network. The Nguyen and Dupuis network [17], shown in Fig. 1, consists of 13 nodes, 19 links and two OD pairs (1,3) and (2,4). The study horizon is from 6:00AM to 10:00AM and is divided into 16 time intervals with 15 minutes each. For each time interval, the following link travel time function is adopted

$$\tau_a(t) = (L_a/S_a)\Big(1.0 + 0.15 \times \big(u_a(t)/C_a\big)^4\Big), \quad \forall a, t \quad (21)$$

where L_a and S_a are the link length and the free-flow speed, respectively, C_a is the capacity of link a. The network data are: $q_{13} = 20,000$, $q_{24} = 10,000$, $\sigma_{13} = 2,000$,$\sigma_{24} = 1,000$, L_a=24km for link 18, 16km for links 4 and 13, and 8km for all other links,S_a=40(km/hr) for all links, C_a=3000(veh/hr) for all links. Other input data are: α=6.4($/h), β=3.9($/h), γ=15.21($/h), θ=0.58($), t_s^*=9.0h for all s, Δ_s=0.25h for all s, ρ_a=0.2 for all links, λ_a=0.15 for all links, ε=0.001. The sample size in Monte Carlo simulation is κ=2000.

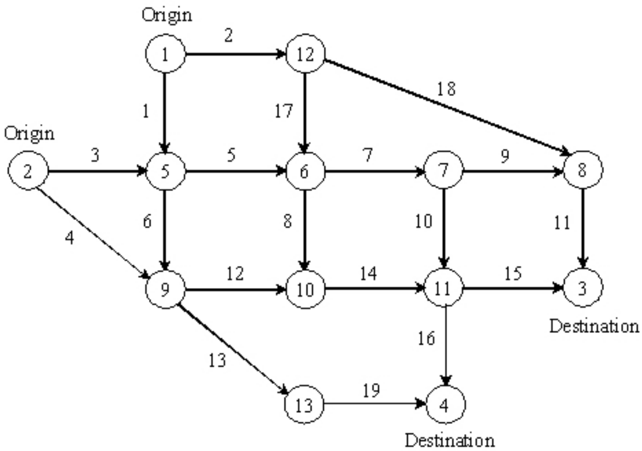

Fig. 1. The Nguyen and Dupuis network with node and link numbers

Fig. 2 shows the departure flows of morning commuting between OD pairs (1,3) and (2,4). It can be seen that most commuters leave origins during 7:30-8:30 for possible arrivals at workplaces on time. Interval 8:00-8:15 is the peak for departure.

Figs. 3 and 4 show the schedule reliabilities for commuters between OD pairs (1,3) and (2,4), respectively, all against four different departure intervals from 7:30 to 8:30. On the x-axis of these two figures, $[-a, -b]$ represents the time threshold for arrival early from b to a minutes, $[a, b]$ is the time threshold for arrival late from a to b minutes, and 0 denotes the time window for non-penalty (or on-time) arrival. Both figures show that all commuters departing within 7:30-7:45 arrive at workplaces early; for OD pair (1,3), 41% of the commuters arrives early 15 to 30 minute and 58% arrives early less than 15minutes; for OD pair (2,4), these two percentages are 44% and 55% respectively. In Fig. 3 for OD pair (1,3), if commuters leave origin 1 within 7:45-8:00, they can arrive at destination

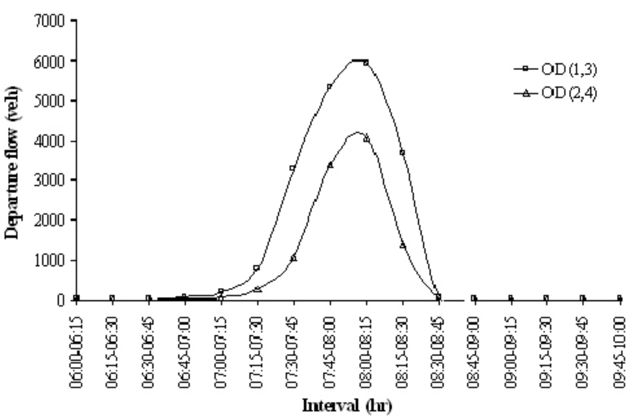

Fig. 2. Departure flows between two OD pairs

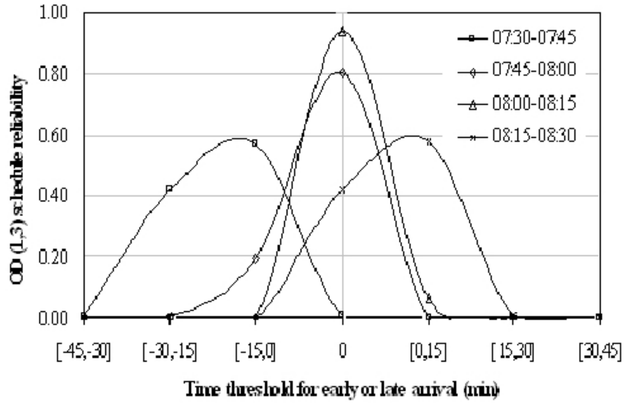

Fig. 3. Schedule reliabilities for commuters between OD pair (1,3)

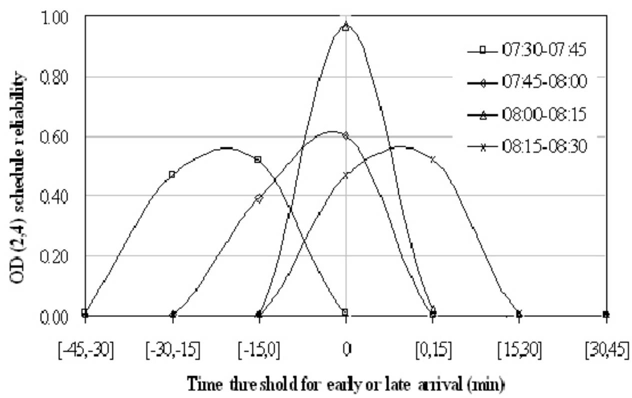

Fig. 4. Schedule reliabilities for commuters between OD pair (2,4)

3 punctually with probability 80% and arrive early less than 15 minutes with probability 20%; if commuters leave within 8:00-8:15, they can arrive on time with probability 94% and arrive late less than 15 minutes with probability 6%; if commuters leave after 8:15, about 58% of them arrive late less than 15 minutes and others arrive on time. In Fig. 4 for OD pair (2,4), similar numerical analyses can be carried out. A distinct difference from Fig. 3 is that when commuters leave origin 2 within 7:45-8:00, the probability for on-time arrival at destination 4 is only 60% and that for arrival early increases to 40%. This is caused by the different traffic conditions on the paths connecting these two OD pairs.

6 Conclusions

In this paper, a fixed-point problem for modeling the morning commuting behavior in stochastic and time-dependent networks is formulated. A heuristic algorithm that combines the Monte Carlo simulation with the method of successive average is proposed for solving the fixed-point problem and demonstrated by numerical results in an example network. The proposed approach can be used to study the complex temporal and spatial interactions between road supply and traffic demand, and thereby evaluate the time-varying service level of road networks through computing the schedule reliabilities of arriving destinations. Future research work may focus on the extension of the proposed model by incorporating activity and parking behavior into trip chains [14, 15] and considering the risk-taking behavior of travel [9].

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Computing Equilibria in a Fisher Market with Linear Single-Constraint Production Units

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Abstract. We study the problem of computing equilibrium prices in a Fisher market with linear utilities and linear single-constraint production units. This setting naturally appears in ad pricing where the sum of the lengths of the displayed ads is constrained not to exceed the available ad space. There are three approaches to solve market equilibrium problems: convex programming, auction-based algorithms, and primal-dual. Jain, Vazirani, and Ye recently proposed a solution using convex programming for the problem with an arbitrary number of production constraints. A recent paper by Kapoor, Mehta, and Vazirani proposes an auction-based solution. No primal-dual algorithm is proposed for this problem.

In this paper we propose a simple reduction from this problem to the classical Fisher setting with linear utilities and without any production units. Our reduction not only imports the primal-dual algorithm of Devanur et al. to the single-constraint production setting, but also: i) imports other simple algorithms, like the auction-based algorithm of Garg and Kapoor, thereby providing a simple insight behind the recent sophisticated algorithm of Kapoor, Mehta, and Vazirani, and ii) imports all the nice properties of the Fisher setting, for example, the existence of an equilibrium in rational numbers, and the uniqueness of the utilities of the agents at the equilibrium.

1 Introduction

The Model. Consider a market with m buyers and n sellers, each seller offering one unit of a good for sale. Each buyer i has a budget B_i, and a utility function u_i that specifies the utility of the buyer for each bundle of goods. Throughout this paper, we assume that u_i's are linear, i.e., the utility of i can be written as $\sum_j u_{ij}x_{ij}$, where u_{ij} is a non-negative number indicating the utility of buyer i for good j (the good sold by the j'th seller), and x_{ij} is the amount of good j bought by i. A market equilibrium is a price vector $\mathbf{p} \in \mathbb{R}^n$ and an allocation $\mathbf{x}$ of goods to buyers such that:

(i) The allocation maximizes the utility of each buyer at the current prices subject to his/her budget. More precisely, for every buyer i, the vector $\mathbf{x}$ is a solution of the maximization program

$$\text{maximize} \quad \sum_j u_{ij}x_{ij}$$

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$$\text{subject to } \sum_j p_j x_{ij} \leq B_i.$$

(ii) The demand and the supply for each good are equal, i.e., for every good j, $\sum_i x_{ij} = 1$. Note that as a corollary, we get that the budget of every buyer is also completely spent in buying the allocated bundle of goods.

The setting defined above is a special case of a more general setting formulated by Arrow and Debreu [1]. One of the features of the model by Arrow and Debreu is that it allows for *production units*. In this paper, we consider a very simple form of production units, defined as follows: in the Fisher model defined above, assume instead of the sellers, we have l production units. The k'th production unit has the capability of producing any bundle of goods $\mathbf{y}_k \in \mathbb{R}^n$ that satisfies a single linear constraint of the form $\mathbf{a}_k.\mathbf{y}_k \leq 1$. Clearly, this model is a generalization of the Fisher model defined above, since each seller in the Fisher setting can be viewed as a production unit producing a single good. Linear single-constraint production units appear in settings like advertisement pricing [5], where each good corresponds to a placement of an advertisement, and each producer corresponds to a page that can "produce" any combination of ad placements whose hieghts sum to any value less than or equal to the height of the ad space.

A market equilibrium in the Fisher model with production units is given by a price vector $\mathbf{p} \in \mathbb{R}^n$ (p_j is the price of the j'th good) and an allocation $\mathbf{x} \in \mathbb{R}^{m \times n \times l}$ (x_{ijk} is the amount of the j'th good purchased from the k'th seller by the i'th buyer) satisfying the following conditions:

– At these prices, each buyer is allocated with a bundle maximizing his/her utility subject to his/her budget.
– At these prices, each producer is asked to produce a bundle maximizing his/her revenue subject to his/her production constraint.
– Total quantity of each good produced is the same as the total quantity of each good bought by the buyers (this condition is covered by the previous two conditions because of our choice of notation, for clarity we still write it as a separate condition).

The model of linear single-constraint production units that we consider in this paper is very restrictive, and might be considered too simplistic to be practical. However, even in this simple model, there are relatively complicated combinatorial algorithms proposed to compute the market equilibrium prices [7]. The point of this paper is to show that such algorithms can be obtained through a simple and intuitive reduction to the setting without production units, and then using any of the numerous algorithms proposed for that setting [3, 2, 4].

2 The Reduction

The idea of the reduction is simple. We can view each production unit k as a unit that originally owns one unit of "raw material", and can transform this raw material to any bundle of goods satisfying $\mathbf{a}^k.\mathbf{y}^k \leq 1$. In order to reduce the model with production units to the simple buyer-seller setting, we construct a market in which each seller is selling the raw materials, and buyers directly buy the raw materials and convert them to their desired goods. We then give a simple transformation that converts the prices for raw materials to prices for the goods.

An alternative way to see the intuition behind our reduction is through the Eisenberg-Gale convex program [3]. This convex program which was originally formulated to solve the Fisher problem without production and later extended [6] to solve the Fisher problem with production. These convex programs intuitively say that among all feasible productions and allocations of the goods to the buyers, an equilibrium choses the production and allocation which maximizes the budget-weighted geometric mean of the buyers' utilities. Note that the convex program does not specify who does the production. In the setting of this paper, sellers produce finished goods from the raw material. Our reduction is that buyers buy the raw material and produce those finished goods which are most beneficial to them. Clearly, the set of feasible productions and allocations remains the same. Hence, the convex program of [6] does not really change. Therefore, the solution to both instances remain the same. To convert it into the classical Fisher setting with linear utilities and without production, we further show that the buyers can dissolve their production constraint into their utility functions - hence no production remains.

We now give a precise formulation of our reduction. Given an instance M of the market equilibrium problem with production units with m buyers, n goods, and l producers described by budgets $\mathbf{B} \in \mathbb{R}^m$, utilities $\mathbf{u} \in \mathbb{R}^{m\times n}$, and production constrains $\mathbf{a} \in \mathbb{R}^{l\times n}$, we construct another instance M' as follows: in M' we have m buyers (each corresponding to a buyer in M), and l sellers (each corresponding to a production unit in M) each offering one unit of a different good. We call the good sold by the k'th seller *the k'th raw material.* The budget of buyer i is B_i, and her utility for one unit of the k'th raw material is $u'_{ik} := \max_j \frac{u_{ij}}{a_{kj}}$.

Let $\mathbf{p}'$ denote the equilibrium price vector for the market M', and $\mathbf{x}'$ be a corresponding allocation. Such an equilibrium is guaranteed to exist and can be computed using any of the algorithms proposed for the linear Fisher model [3, 2, 4]. We define a price vector $\mathbf{p}$ for the goods in M by letting $p_j := \min_k p'_k.a_{kj}$. We now show that this price vector induces an equilibrium in M. To do this, we construct an allocation $\mathbf{x}$.

For every buyer i and seller k in M', take a good j that maximizes u_{ij}/a_{kj}, and define $x_{ijk} := x'_{ik}/a_{kj}$. For every other j, define $x_{ijk} := 0$. The following two lemmas show that the price vector $\mathbf{p}$ with the allocation $\mathbf{x}$ form a market equilibrium for M.

Lemma 1. *For every buyer i^*, the bundle given by the allocation $\mathbf{x}$ to i^* is an optimal feasible bundle for i^* at prices $\mathbf{p}$.*

Proof. First, we verify that i^* can afford the bundle given to her by $\mathbf{x}$. For every production unit k, by the definition of $\mathbf{x}$, $\sum_j p_j x_{i^*jk} = p_{j^*} x'_{i^*k}/a_{kj^*}$, where j^* is a good maximizing u_{i^*j}/a_{kj} over all j. Therefore, by definition of $\mathbf{p}$, $\sum_j p_j x_{i^*jk} = p'_k.a_{kj^*}.(x'_{i^*k}/a_{kj^*}) = p'_k x'_{i^*k}$. Hence, $\sum_{k,j} p_j x_{i^*jk} = \sum_k p'_k x'_{i^*k} \leq B_{i^*}$.

To show the optimality of this bundle, we need to show that for every j^*, k^*, if $x_{i^*j^*k^*} > 0$, then the good j^* maximizes the ratio u_{i^*j}/p_j over all j. By the definition of $\mathbf{x}$, $x_{i^*j^*k^*} > 0$ implies that $j^* \in \text{argmax}_j\{u_{i^*j}/a_{k^*j}\}$ and $x'_{i^*k^*} > 0$. Since $(\mathbf{p}', \mathbf{x}')$ is a market equilibrium in M', the latter inequality implies that $k^* \in \text{argmax}_k\{u'_{i^*k}/p'_k\}$. By combining these two statements and the definition of $\mathbf{u}'$, we can conclude that $(j^*, k^*) \in \text{argmax}_{(j,k)}\{\frac{u_{i^*j}}{a_{kj}p'_k}\}$, or equivalently, that j^* maximizes $\max_k\{\frac{u_{i^*j}}{a_{kj}p'_k}\} = \frac{u_{i^*j}}{\min_k(a_{kj}p'_k)} = u_{i^*j}/p_j$ over all j.

Lemma 2. *For every production unit k^*, the bundle given by $\mathbf{x}$ is the optimal feasible bundle that k^* can produce.*

Proof. We start by showing that the bundle given by $\mathbf{x}$ can be produced under k^*'s production constraint. For every buyer i, by the definition of $\mathbf{x}$, $\sum_j a_{k^*,j}x_{ijk^*} = x'_{ik^*}$. Therefore, $\sum_{i,j} a_{k^*,j}x_{ijk^*} = \sum_i x'_{ik^*} \leq 1$, since there is exactly one unit of the k^*'th raw material in M'.

To show the optimality of the bundle, we need to show that for every i^*, j^*, if $x_{i^*j^*k} > 0$, then the good j^* maximizes the ratio p_j/a_{k^*j} over all j. By the argument in the proof of the previous lemma, $x_{i^*j^*k} > 0$ implies that $k^* \in \text{argmax}_k\{\frac{u_{i^*j^*}}{a_{kj^*}p'_k}\}$. Therefore, $k^* \in \text{argmin}_k\{a_{kj^*}p'_k\}$. By the definition of $\mathbf{p}$, this means that $p_{j^*} = a_{k^*j^*}p'_{k^*}$. Therefore, for every good j,

$$\frac{p_j}{a_{k^*j}} = \frac{\min_k\{p'_k a_{kj}\}}{a_{k^*j}} \leq \frac{p'_{k^*}a_{k^*j}}{a_{k^*j}} = \frac{p'_{k^*}a_{k^*j^*}}{a_{k^*j^*}} = \frac{p_{j^*}}{a_{k^*j^*}},$$

as desired.

The above lemmas show that $(\mathbf{p}, \mathbf{x})$ is an equilibrium for M. Therefore, we have the following theorem.

Theorem 1. *For every instance M of the market equilibrium problem with linear single-constraint production units, there is an instance M' of the market equilibrium problem without production units such that*

- *Given M, M' can be constructed in polynomial time.*
- *Given an equilibrium of M', an equilibrium of M can be constructed in polynomial time.*

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Majority Equilibrium of Distribution Centers Allocation in Supply Chain Management

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Abstract. In this paper, we consider the distribution center allocation problem decided through an optimal utility value under the majority rule in supply chain management. A location of the distribution center is a majority rule winner with optimal utility value if no other location in the network where more than half of the retailers would have, is with better utility value than the winner. We define a weight function and established the network model for the cases with one or even more than one distribution centers to be located. We show that there exists a modified weak quasi-Condorcet winner if the distribution center allocation graph model is a tree. Based on above discussion we proposed an practical majority equilibrium method for general distribution center allocation problems.

1 Introduction

In many management problems, majority vote is often the ultimate decision making tool. The concept of majority equilibrium captures such a democratic spirit in requiring that no other solutions would please more than half of the participants (or more than half of the total voting weight for participants with weighted voting powers). Therefore, a majority equilibrium solution is a stable point solution under a democratic (sometimes weighted) decision making mechanism, which is employed not only in public management but also in business management decision making processes. Such a perfectly defined solution concept, however, may not always exist. As in the famous Condorcet paradox, three agents have three different orders of preferences, $A > B > C$, $B > C > A$, $C > A > B$ among three alternatives A, B and C, would not yield a majority equilibrium solution. In reality, the paradox phenomena would have to be dealt with and a solution should be settled.

The distribution center location problem in supply chain management is a case which would fit into one such decision problem. In this model, a group of collaborating retailing agents would have to decide on locations to set up distribution centers that would benefit the majority of the agents. A closely related setting is considered by Demange for continuous and discrete spatial models of collective choice, aiming at characterization of the location problem of public services as a result of public voting process [4]. To facilitate a rigorous study of the related problem, Demange proposed four types of majority equilibrium

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solutions (call Condorcet Winners) and discussed corresponding results concerning conditions for their existences.

A weighted version of the discrete model of Demange for distribution center allocation problem is of particular interests to us: The environment is represented by a network $G = ((V, w), (E, l))$ that link retailers together. In the model, for each $i \in V$, $w(i)$ represents the voting power of retailers reside at i, which can be decided by the voting system or by the decision power at a vertex. For each $e \in E$, $l(e)$ represents the distance between two retailers i and j. We will consider a special type of utility function: the sum distance between the location of the distribution center to all retailers, on which each retailer want to minimize. While each desires to have the distribution center to be close to itself, the decision has to be agreed upon by a majority of the votes.

Following Demange [4], a location $x \in V$ is a strong (resp. weak) Condorcet winner if, for any $y \in V$, the total weight of vertices that is closer to x than to y is more (resp. no less) than the total weight of vertices that is closer to y than to x. Similarly, it is a quasi-Condorcet winner if we change "closer to x than" to "closer to x than y or of the same distance to x as y". Of the four types of majority winner, strong Condorcet winner is the most restrictive of all, and weak quasi-Condorcet winner is the lest restrictive one and the other two are between them. For discrete models considered by Romero [13], Hansen and Thisse [10], it was known that, the order induced by strict majority relation (the weak Condorcet order) in a tree is transitive. Therefore, a weak Condorcet winner in any tree always exists. In addition, Demange extended the existence condition of a weak Condorcet winner to all single peaked orders on trees [5].

In the paper, motivated by the above results, we structured the weighted function based on the majority rule and proposed an optimal utility function for the distribution center allocation problem. Our study distinguishes from previous work in our focus in weighted function and optimal utility value issues with the imperfect information from majority voting. The weighted function will depend on the majority voting process and the utility function is defined as the sum value of the distance between the location of the distribution center to retailers. In Section 2, we establish our majority voting process, introduce some denotations, define formal formulation of the single distribution center location problem and modify the definition issue of Condorcet winners. in Section 3. We present a linear algorithm for finding a modified weak quasi-Condorcet winners of a tree with the proposed vertex-weight function and edge-length functions; and prove that in the case, the modified weak quasi-Condorcet points are the points which minimize the total weight-distance to the individuals' locations. Based on above discussion we will propose a practical majority equilibrium method for general cases in Section 4. Section 5 is dedicated to conclude the paper.

2 Denotations and Definitions

In [4], Demange has surveyed and discussed some spatial models of collective choice, some results concerning the transitivity of the majority rule and the existence of a majority winner. Let $S = \{1, 2, \cdots, n\}$ be a society representing a set of n individuals, and X be a set of alternatives (or choice space). Each individual $i \in S$ has a preference order, denoted $\geq_i$, on X. The n-tuple $(\geq_i)_{i\in S}$ is called the *profile* of the society. Given a profile $(\geq_i)_{i\in S}$ of the society X, an alternative $x \in X$ is called:

(a) *Weak quasi-Condorcet winner* if for every $y \in X$ distinct of x,

$$|\{i \in S : y >_i x\}| \leq \frac{n}{2}; \text{ i.e. } |\{i \in S : x \geq_i y\}| \geq \frac{n}{2}.$$

(b) *Strong quasi-Condorcet winner* if for every $y \in X$ distinct of x,

$$|\{i \in S : y >_i x\}| < \frac{n}{2}; \text{ i.e. } |\{i \in S : x \geq_i y\}| > \frac{n}{2}.$$

(c) *Weak Condorcet winner* if for every $y \in X$ distinct of x,

$$|\{i \in S : x >_i y\}| \geq |\{i \in S : y >_i x\}|.$$

(d) *Strong Condorcet winner* if for every $y \in X$ distinct of x,

$$|\{i \in S : x >_i y\}| > |\{i \in S : y >_i x\}|.$$

In [3], motivated by Demange's results and based on the proposed formal formulation of the public facility location problem with a single facility in a network, L. Chen et al are interested in classify the types of networks for which a Condorcet winner can be found in linear time. As a warm-up example, they present the solution for trees and a linear algorithm for finding the weak quasi-Condorcet winners of a tree with vertex-weight and edge-length functions, and prove that in the case, the weak quasi-Condorcet points are the points which minimize the total weight-distance to the individuals' locations. Furthermore, they give a sufficient and necessary condition for a point to be a weak quasi-Condorcet point for cycles in the case the edge-length function is a constant, and present a much more interesting linear time algorithm.

Howerer, in Demange's definitions of Condorcet winner, only the number of the society's members are countered, but the decision power difference among candidate locations or the voting asymmetric information are ignored. And in [3], although the definition of vertex-weight is introduced it is needed to establish the meaningful and exact vertex-weight function. In this paper, we try to establish weighted function and utility function to modify Demange's definitions and give the exact definition of vertex-weight in [3]. First of all, we will give some denotations and definition needed afterward.

In this paper we consider the following majority voting process:

The definition of majority voting process

Step 1. Design the vote such that it includes the following three options: the name of the retailer, all of the distribution center location's candidates, and the distance of the retailer's location to the distribution center location voted.

Step 2.Hand out the votes and every retailer can choose one or more location listed in the vote. Let k denote the number of locations voted by a retailer. The retailer's decision power for the voted locations is defined as just $\frac{1}{k}$.

Step 3. Collect the votes and make decision.

We first consider a single distribution center location problem.

Denote $V = v_1, v_2, \cdots, v_n$ the set of n distribution center candidate locations. In every location v_i,$i = 1, 2, \cdots, n$, there are u_i retailers and denote them as $(u_1^i, u_2^i, \cdots, u_{u_i}^i)$.

Denote $d(u_k^i, v_j)$ the distance from k-th retailer in location i to location j, where $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, n$ and $k = 1, 2, \cdots, u_i$.

Denote by $d_G(v, v')$ the length of a shortest chain joining two locations v and v' in G, and call it *distance between the two locations v and v' in G*.

Define the weight function at vertex v_i as $\omega(v_i) = \sum_{j=1}^{u_i} f_i(u_j^i)$ where $f_i(u_j^i)$ is defined as follows:$f_i(u_j^i) = \frac{1}{k_{ij}}$, if the retailer u_j^i marked k_{ij} candidates including the vertex v_i in V and $0 < k_{ij} \leq n$, where $i = 1, 2, \cdots, n$; otherwise, $f_i(u_j^i) = 0$, with the retailer u_j^i does not vote the vertex v_i. We call the value $f_i(u_j^i)$ is the decision power of retailer u_j^i at location v_i and the weight function $\omega(v_i)$ is the decision power of vertex v_i by the weighted cumulating votes.

For any $R \subseteq V$, we set $\omega(R) = \sum_{i \ \| v_i \in R} \sum_{j=1}^{u_i} f_i(u_j^i)$. In particular, if $R = V$, we write $\omega(G)$ instead of $\omega(V)$, i.e. $\omega(G) = \sum_{i=1}^{n} \sum_{j=1}^{u_i} f_i(u_j^i)$. A vertex v of G is said to be *pendant* if v has exact one neighbor in G.

We can model our distribution center allocation problem as follows:

Graph model of distribution center allocation (GMPFA)
We consider the undirected graph model $G = (V, E)$ of order n with the weight function ω that assigns to each vertex v of G a non-negative weight $\omega(v)$ defined above, and a length function l that assigns to each edge e of G the distance between the two end locations of the edge e. If P is a chain of G, then we denote by $l(P)$ the sum of lengths of all edges of P.

Modified definitions of Condorcet winner in the model (GMPFA)
Given a graph $G = (V, E)$ with $V = \{v_1, v_2, \cdots, v_n\}$, each $v_i \in V$ has a preference order $\geq_i$ on V induced by the distance on G. That is, we have $x \geq_i y$ for any two vertices x and y of G if and only if $d_G(v_i, x) \leq d_G(v_i, y)$. The following definition is an extension of that given in [4].

Definition. Given a graph $G = (V, E)$ and profile $(\geq_i)_{v_i \in V}$ on V, denote $\Phi = \{i \ \| v_i \in V, \ u >_i v_0\}$ and $\Psi = \{i \ \| v_i \in V : v_0 \geq_i u\}$. A vertex v_0 in V is called:

(1) *Modified weak quasi-Condorcet winner*, if for every $u \in V$ distinct of v_0,

$$\omega(\{v_i \in V : u >_i v_0\}) \leq \frac{\omega(G)}{2}; \quad \text{i.e.} \quad \omega(\{v_i \in V : v_0 \geq_i u\}) \geq \frac{\omega(G)}{2}.$$

or in other words,

$$\sum_{i \in \Phi} \sum_{j=1}^{u_i} f_i(u_j^i) \leq \frac{\sum_{i=1}^{n} \sum_{j=1}^{u_i} f_i(u_j^i)}{2};$$

i.e.

$$\sum_{i \in \Psi} \sum_{j=1}^{u_i} f_i(u_j^i) \geq \frac{\sum_{i=1}^{n} \sum_{j=1}^{u_i} f_i(u_j^i)}{2}. \tag{1}$$

(2) *Modified strong quasi-Condorcet winner,* if for every $u \in V$ distinct of v_0,

$$\omega(\{v_i \in V : u >_i v_0\}) < \frac{\omega(G)}{2}; \quad \text{i.e.} \quad \omega(\{v_i \in V : v_0 \geq_i u\}) > \frac{\omega(G)}{2}.$$

or or in other words,

$$\sum_{i \in \Phi} \sum_{j=1}^{u_i} f_i(u_j^i) < \frac{\sum_{i=1}^{n} \sum_{j=1}^{u_i} f_i(u_j^i)}{2};$$

i.e.

$$\sum_{i \in \Psi} \sum_{j=1}^{u_i} f_i(u_j^i) > \frac{\sum_{i=1}^{n} \sum_{j=1}^{u_i} f_i(u_j^i)}{2}. \tag{2}$$

(3) *Modified weak Condorcet winner,* if for every $u \in V$ distinct of v_0,

$$\omega(\{v_i \in V : v_0 >_i u\}) \geq \omega(\{v_i \in V : u >_i v_0\}).$$

or in other words,

$$\sum_{i \in \Psi} \sum_{j=1}^{u_i} f_i(u_j^i) \geq \sum_{i \in \Phi} \sum_{j=1}^{u_i} f_i(u_j^i). \tag{3}$$

(4) *Modified strong Condorcet winner,* if for every $u \in V$ distinct of v_0,

$$\omega(\{v_i \in V : v_0 >_i u\}) > \omega(\{v_i \in V : u >_i v_0\}).$$

or in other words,

$$\sum_{i \in \Psi} \sum_{j=1}^{u_i} f_i(u_j^i) > \sum_{i \in \Phi} \sum_{j=1}^{u_i} f_i(u_j^i). \tag{4}$$

Example. Denote by K_2 and K_3 the complete graphs of orders 2 and 3, respectively. Suppose that the length functions on edge set and the weight functions on vertex set in K_2 and K_3 are constant which means that the decision power at any location is same. Then K_2 has modified weak Condorcet winners, and hence has also modified weak quasi-Condorcet winners, but has no modified strong Condorcet winners, and hence has no modified strong quasi-Condorcet winners; K_3 has modified strong quasi-Condorcet winner, modified weak Condorcet winner and modified weak quasi-Condorcet winner, but has no modified strong Condorcet winner.

In this paper, we will only consider the algorithm for finding modified weak quasi-Condorcet winner of a tree. The properties and algorithms for other three types of Condorcet winners can be discussed by the similar way.

3 Weak Quasi-Condorcet Winner of a Tree

Romero, Hansen and Thisse pointed out that the family of orders induced by a distance on a tree guarantees the existence of a weak Condorcet winner. Furthermore, the weak Condorcet points are the points which minimize the total distance to the individuals' locations [4]. In this section we propose a linear algorithm for finding the modified weak quasi-Condorcet winners on a tree with vertex-weight and edge-length functions; and prove that in this case, the modified weak quasi-Condorcet winners are the same as points which minimize the total weight-distance to the individuals' locations. In fact, the same conclusions hold for modified weak Condorcet winners.

Given two vertices $v, x \in V$, the set of *quasi-friend* vertices of v relative to x is defined as

$$F_G(v,x) = \{u : \ d_G(u,v) \leq \ d_G(u,x)\};$$

and the set of *hostile* vertices of v relative to x is defined as

$$H_G(v,x) = \{u : \ d_G(u,v) > \ d_G(u,x)\}.$$

By the definition of modified weak quasi-Condorcet winner, a vertex $v_0 \in V$ is a modified weak quasi-Condorcet winner of G, if for any vertex $x \neq v_0$,

$$\omega(F_G(v_0,x)) \geq \ \frac{1}{2}\omega(G), \quad \text{or equivalently,} \ \ \omega(F_G(v_0,x)) \geq \ \omega(H_G(v_0,x)),$$

or

$$\sum_{i \in \Phi} \sum_{j=1}^{u_i} f_i(u_j^i) \leq \frac{\sum_{i=1}^{n} \sum_{j=1}^{u_i} f_i(u_j^i)}{2};$$

i.e.

$$\sum_{i \in \Psi} \sum_{j=1}^{u_i} f_i(u_j^i) \geq \frac{\sum_{i=1}^{n} \sum_{j=1}^{u_i} f_i(u_j^i)}{2}. \tag{5}$$

Similar to the proofs in [3], we can show that

Theorem 1. *Every tree has one modified weak quasi-Condorcet winner , or two adjacent modified weak quasi-Condorcet winners. We can find it or them in linear time.*

Theorem 2. *Let T be a tree. Then v_0 is a modified weak quasi-Condorcet winner of T if and only if v_0 is a barycenter of T.*

Theorem 3. *Let $T = (V,E)$ be a tree, $N = \{1,2,\cdots,n\}$ be the set of retailers with positive weight $\omega : N \to R^+$. The majority rule $\pi : V^n \to V$ of choosing modified weak quasi-Condorcet winners satisfies the property of (group)strategy-proofness.*

4 A Practical Algorithm to Get Modified Weak Quasi-Condorcet Winner for Connected Distribution Centers Allocation

We consider the undirected graph model $G = (V, E)$ of order n with a weight function ω that assigns to each vertex v_i of G with $\omega(v_i) = \sum_{j=1}^{u_i} f_i(u_j^i)$ where $i = 1, \cdots, n$ and $f_i(u_j^i)$ is defined as above, and a length function l that assigns to each edge e of G a positive length $l(e)$.Notice that for any $R \subseteq V$, we set $\omega(R) = \sum_{i,\|v_i \in R} \sum_{j=1}^{u_i} f_i(u_j^i)$. In particular, $\omega(V) = \omega(G) = \sum_{i,\|v_i \in V} \sum_{j=1}^{u_i} f_i(u_j^i)$ and notice that a vertex v of G is said to be *pendant* if v has exact one neighbor in G. In section 3, we have shown the existence of the majority equilibrium when the connected undirected graph model $G = (V, E)$ of order n of distribution center allocation problem is a tree. From the proofs in section 3 and with the minimum spanning tree technique we can establish a practicable algorithm to get the modified weak Quasi-Condorcet winner for the connected graph cases.

A Practical Algorithm for the connected graph cases
Step 1. Find the minimum spanning tree of the graph G, and denote it as $T = (V, l'(e)$ where $l'(e) \subseteq l(e)$;

Step 2. Take the pendant vertex v of T such that $w(v) < \frac{1}{2}w(T)$;

Step 3. $T - v \Rightarrow T, w(v) + w(u) \Rightarrow w(u)$, where u is the (unique) neighbor of v;

Step 4. $n - 1 \Rightarrow n$, If $n = 1$, or, $n = 2$ and the two vertices have the same weights, then stop; otherwise go to Step 2.

According to the proof of Section 4 in [3], we can also claim that there exists a modified weak quasi-Condorcet winner for the proposed Practical Algorithm. The proof is omitted here.

5 Conclusions

In this work, we consider the distribution center location problem decided via a voting process under the majority rule. Our study follows the network model that has been applied to the study of similar problems in economics [3], [13], [10], [14]. Our mathematical results depend on understanding of combinatorial structures of underlying networks.

Many problems open up from our study. The complexity study for other rules for distribution center location is very interesting and deserves further study. And it would be interesting to extend our study to other areas and problems of public decision making process.

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Traversal Pattern Mining in Web Environment

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Abstract. There have been researches about analyzing the information retrieval patterns of log file to obtain users' information search patterns in web environment. Algorithms that find the frequently traversed path pattern from search path inputs are suggested mainly. But one of the existing works' problems is to provide inadequate solution for complex, that is, general topological patterns. This paper suggests an efficient algorithm for deriving the maximal frequent traversal pattern from general paths.

1 Introduction

Nowadays there has been increasingly important about the extent of efficiency and accuracy that users acquire their wanted information in web environment. Also user's access pattern to web pages is very useful information to web system designer and market analyzer, etc. It may be very useful to improve industry benefits.

Current several researches in mining user patterns in a web environment focus mainly on two directions: 1) computing rule-based patterns [1], 2) computing topology-based patterns [2, 3, 4, 5, 6, 7, 8]. In a rule-based approach, we view the input, user access records, as a table in a relational DB, and output the discovered association rules. In a topological oriented approach, we view a web environment, the linked documents, as a directed graph: and view a user access to the web as a walk in the directed graph along edges. Then frequent user walk patterns, restricted to a specific topology, are to be computed. In this paper, we are interested in the latter approach. The existing researches in a topological oriented approach have two aspects of limitations, one is to consider only simple web search patterns, that is forward traversal path, the other is to apply complicated algorithm by using additional backward traversal path pattern mining algorithm, even though the former problem has been solved. In this paper we suggest an efficient MFTP(Maximal Frequent Traversal Pattern) algorithm to solve the problems.

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2 Preliminary

Fig. 1 shows a general web page traversal network in which each node (A,B,C,D,E) means a web page(document, site), and each bi-directional edge a traversal arc. The network is modeled as a directed graph with a root node. Therefore the graph with n nodes has ${}_nC_2/2$ undirected edges in case of undirected graph, and ${}_nC_2$ directed edges in case of directed graph.

The existing researches [2, 3] do not count backward traversal, and so the general traversal path is not considered. For example, "ABCAE" traversal path with backward traversal sub-path CA is not included as the output of the algorithm. The fact is not proper. Although the problem have been solved in another researches [4, 5, 6], the algorithm also has the problem that it consists of complicated procedures because of considering an additional processing procedure dealing with computing backward traversal.

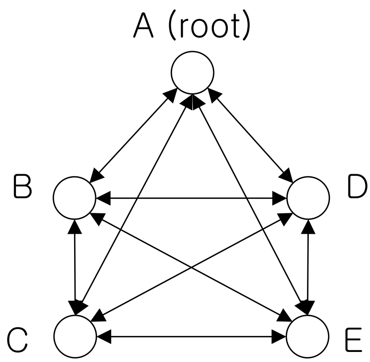

Fig. 1. Web Page Traversal Network

In this paper we suggest a new efficient algorithm finding the maximal frequent web traversal path(s) which is based on Apriori algorithm [7] and Graph Theory [8]. The actually differentiated idea from the existing researches is that this paper preserves the sequential order of traversal path in applying the algorithm, but in previous ones, not the sequence but the itemsets are counted.

Table 1. Traversal Path examples

Case	*Traversal Path Sequence*
1	A
2	ABCBA
3	ABCA
4	ABCDE
5	ABCADE

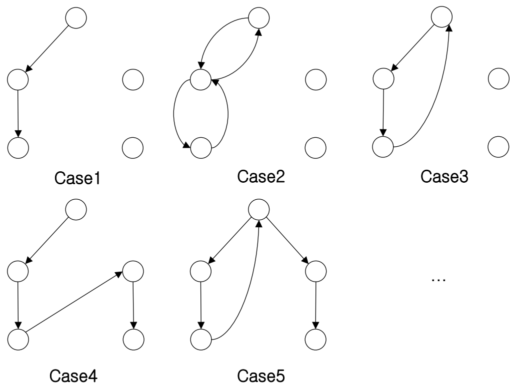

Fig. 2. Each Traversal Path depicted on Graph

Table 1 shows the sample of paths extractable from fig. 1. In fig. 2, each path in table 1 is depicted on the directed graph of fig. 1. These cases are mostly general paths including backward traversal sub-path. From now on we try to explain our proposed algorithm with the examples.

3 MFTP Algorithm

Case 2, 3, and 5 in fig. 2 include backward traversals. In this section we will explain the operation steps in MFTP algorithm.

3.1 Steps of Algorithm

Fig. 3 explains the steps in applying MFTP algorithm easily in graphical format. In D(database), the web page traversal paths obtained from log file are stored as input of the algorithm. They are accessed every time as the frequency time of C_k is to be computed. C_k is the set of possible sequential sub-paths with k length and also is the superset of L_k. L_k is the set of candidates of maximal frequent traversal path in k-th step, and is extracted by selecting the paths with above the minimal frequency time or user defined frequency time. The final output is L_k at that time the algorithm will terminate. It is noted in the algorithm that sub-paths must be ordered, that is "AB" is not equal to "BA" and "ABBC" is equal to "ABC" actually.

Algorithm MFTP
-Input: $V = \{ v_1, ..., v_i \}$ here, V is a node set
-Output: L_k
-Steps
  Step 1: Initialization step $C_1 = \{ v_1, ..., v_i \}$, k=1
  Step 2: Repeat until finding MFTP
    Compute the frequency time each element of C_k appears in D
    Compute L_k set by selecting the elements of C_k with above

D

Path	Sequence
1	ABC
2	ABCBA
3	ABCA
4	ABCDE
5	ABCADE

=> (D search)

C1

subpath	Freq.
A	8
B	6
C	5
D	2
E	2

=>

L1

subpath	Freq.
A	8
B	6
C	5

AB
AC
BC
BA
CA
CB

=> (D search)

C2

subpath	Freq.
AB	5
AC	-
BC	5
BA	1
CA	2
CB	1

=>

L2

subpath	Freq.
AB	5
BC	5
CA	2

ABBC
ABCA
BCAB
BCCA
CAAB
CABC

=> (D search)

C3

subpath	Freq.
ABC	5
ABCA	2
BCAB	-
BCA	2
CAB	-
CABC	-

=>

L3

subpath	Freq.
ABC	5

Fig. 3. Steps of MFTP Algorithm

the minimal frequency time or user defined frequency time
C_{k+1}=L_k X L_k (Execute cartesian product to preserve the order)
Convert the duplicated nodes into one node
k=k+1
Go to Step 2

3.2 Comparison with the Existing Algorithms

The algorithm suggested in this paper is superior to the existing ones in two aspects, generality because of considering backward traversal and simplicity

Table 2. Comparison Results

Comparison Factors	MFTP	[2-3]	[4-7]
Generality	-support cyclic graph -considers both forward and backward path	-supports only acyclic pattern -neglects backward path (ex; "ABCBA")	-supports cyclic graph -considers both forward and backward path
Simplicity	-simple algorithm by using sequential sub-path as in fig. 3	-simple algorithm -but deficient of generality	-complicated algorithm by using not sequential sub-path but itemsets[7] or using separate algorithm after computing backward traversal[4-6]

in consideration of not itemset but sequential sub-path. Table 2 shows the differences.

4 Conclusion

We have suggested the web page traversal pattern mining algorithm called MFTP and explained the advantages over the existing ones. Its best features are to satisfy simplicity and generality. However further studies have to be done for providing its quantitative superiority in addition to qualitative analysis given in this work.

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An Analysis of Search Engine Switching Behavior Using Click Streams

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Abstract. In this paper, we propose a simple framework to characterize the switching behavior between search engines based on user click stream data. We cluster users into a number of categories based on their search engine usage pattern during two adjacent time periods and construct the transition probability matrix across these usage categories. The principal eigenvector of the transposed transition probability matrix represents the limiting probabilities, which are proportions of users in each usage category at steady state. We experiment with this framework using real click stream data focusing on two search engines: one with a large market share and another with a small market share. The results offer interesting insights into search engine switching. The limiting probabilities provide empirical evidence that small engines can still retain its fair share of users over time.

1 Introduction

Web search has become a very competitive field in recent years. With virtually zero switching cost and large revenue from the sponsored listing business, big portals such as Yahoo!, MSN and AOL and specialized web search engines such as Google and AskJeeves are improving the search experience to expand their market share. In previous studies, Mukhopadhyay *et al* [2] concluded that lower quality search engines can survive web search competition because there is no trade-off between price and quality. Telang *et al* [1] studied the choice of search engines with survey and click streams and concluded that both loyalty—determined by the difficulty transferring learning to other engines—and the quality of search results impact the users' choice of search engines.

Our research takes a pure metrics view of switching behavior. It is motivated by a desire to characterize the search engine competition and forecast future trends. In this paper, we propose a framework for estimating *user engagement*, *user preferences* and the *trends* of the web search competition using click streams. We focus on *interaction* metrics: the main statement is about the probability of users switching from one engine to another over a specific time period. We also try to paint a picture of the ultimate market share of search engines when the competition reaches equilibrium assuming the current switching trend continues.

In a nutshell, the input to the framework is user click streams converted into sequences of search sessions over a specific period of time. We cluster all these se-

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quences into a number of usage categories. We look at usage categories of sequences belonging to two (adjacent) time periods and construct a transition probability matrix that depicts the probability of users migrating from one usage category to another. The limiting probabilities of the transition probability matrix represent the ultimate market share for each engine, that is, the fraction of users who will remain in each usage category when the system reaches the steady state. The number of transitions necessary for convergence forecasts the time at which the competition will reach equilibrium. The set of metrics produced by our framework portrays the flow of market share between search engines and reveal the level of user engagement for each search engine.

We use clustering to abstract user sequences and to segment users. The clusters or usage categories can be obtained by various clustering techniques such as K-means, hierarchical clustering methods, Gaussian mixtures and etc. We experimented with these methods and found out that they lead to the same conclusions. In this paper, we report our results using K-means.

We test our framework by analyzing the switching behavior between a "big" search engine—one with high market share—and a small engine—one with a low market share—using click stream data from an ISP. The results show that the big search engine has higher user engagement and is currently taking market share from the small engine. However, there are still users who are switching from the big engine to the small engine and those who are quite loyal to the small engine, which leads to a non-trivial market share (about 10% below its current share) at steady state assuming the current trend holds. It indicates that users do have different preferences and some users elect to primarily use the small engine even though the big engine is perceived to have higher quality and to provide a better user experience. The results provide some empirical evidence that such small engines can survive the competition as long as they maintain their current quality level with respect to the big players.

The main contributions of the paper are

- a simple yet insightful framework to characterize search engine switching behavior
- a set of key metrics—user share, user engagement, user preference and trends—for competitive analysis of user switching behavior
- empirical measurements of steady-state market shares of two com peting engines

The rest of paper is organized as follows. Section 2 details the framework. Section 3 describes the format of the input data. Section 4 describes the detailed results of applying a big search engine and a small engine into our framework. Section 5 concludes our paper with the implications of our research in web search competition.

2 Framework

First we partition the user click stream into *user sequences*. The click stream will be divided into individual sessions, each session being assigned a representative timestamp. Each session will then be characterized according to its usage across search engines and be assigned a label[1]. After labeling, we specify two adjacent time periods t and $t+1$. We then construct two sequences of labeled sessions (S_t, S_{t+1}) for each user according to the session timestamps where S_t represents the sequence during time period t; S_{t+1} represents the sequence during time period $t+1$. These *user sequences* (S_t, S_{t+1}) are the input to our framework.

We estimate the transition probabilities P_{ij} from usage class i to usage class j from time period t to time period $t+1$. To define the usage classes, we apply clustering procedure to the user sequences during time periods t and $t+1$ and find K clusters. The resulting clusters are interpretable and the clusters representing loyalists for individual search engines and switchers that frequently switch between search engines inter- and intra- search sessions are identified.

Each user will be assigned two cluster memberships $C_t = f(S_t)$ and $C_{t+1} = f(S_{t+1})$ where f is the model generated from the clustering procedure and C_t and $C_{t+1} \in \{1, 2, 3, \ldots, K\}$. We construct the frequency table of the number of users who transition from class i to class j from t to $t+1$. Let $\boldsymbol{F_{ij}}$ denote the number of users who transition from class i to j from t to $t+1$. Let $\mathbf{P}$ denote the transition probability matrix and each element $\boldsymbol{P_{ij}}$ denote the conditional probability that a user will be in class j during time period $t+1$ given that she is in class i during time period t. That is,

$$P_{ij} \equiv \Pr(j \ at \ t+1 \| i \ at \ t)$$

$\boldsymbol{P_{ij}}$ can be estimated as follows

$$\hat{P}_{ij} \equiv \frac{F_{ij}}{\sum_{j=1}^{K} F_{ij}}$$

and

$$\sum_j \hat{P}_{ij} = 1$$

$\mathbf{P}$ describes the search engine switching behavior, or *trend*, of the underlying population from time period t to time period $t+1$. From $\mathbf{P}$, we can make inferences about how loyal the users are with respect to individual search engines. We can also infer if a particular engine is losing users to another search engine.

We can also forecast the transition probabilities from time t to $t+s$ as $\mathbf{P}^s$. When s approaches infinity and assuming $\mathbf{P}$ is aperiodic, $\mathbf{P}^s$ will converge to $\mathbf{P}^*$ with all the rows equal to the vector of limiting probabilities. Let $\boldsymbol{\Pi}^T = (\boldsymbol{\pi_1}, \boldsymbol{\pi_2}, .. \boldsymbol{\pi_K})$ denote the vector of convergent probabilities where $\sum_i \pi_i = 1$. $\boldsymbol{\Pi}$ is the principal eigenvector of $\mathbf{P}^T$ since $\mathbf{P}^T\boldsymbol{\Pi} = \boldsymbol{\Pi}$ with eigenvalue 1.

We can thus compute the limiting probabilities ($\boldsymbol{\Pi}$) efficiently by just computing the principal eigenvector of $\mathbf{P}$ transpose, which is actually equivalent to the PageRank computation Assuming that the current trend ($\mathbf{P}$) does not change, the interpretation of the limiting probabilities is that ultimately the search engine competition will reach equilibrium at which the share of users in each cluster will remain constant. However,

[1] For example, given two search engines X and Y, we can assign the following labels: 'X' for a session using X only, 'Y' for a session using Y only, and 'XY' for both. This can be extended to multiple engines.

trends do change. The limiting probabilities therefore cannot be interpreted literally. Limiting probabilities, nonetheless, do offer a distilled view of the transition probability matrix: instead of looking at the K x K transition probability matrix, we can first compare the K limiting probabilities and the current probabilities to see the trend and then go back to the transition probability matrix for the details.

The introduction of the limiting probabilities brings up another interesting question: how long or how many transitions will it take to reach the equilibrium? This question can be easily answered mathematically by devising a simple distance measure between iterations and starting from the initial probabilities y, return the number of iterations to convergence s* when the distance is smaller than some predefined error threshold ε.

Finally, this framework can be leveraged to discover anecdotal examples to better understand reasons behind switching. For example, we look at the queries, click patterns, query reformulations and etc of users who suddenly go from being a loyal user of engine A to a loyal user of engine B. It provides search engines with valuable information about how they lose and gain users from other search engines and can help the search engines refine their user experience.

3 Data

The data used here is the complete user click streams from an ISP that provides dial-up service to North America. There are 12 weeks of click stream data, collected from July 25, 2004 to October 16, 2004. The click stream is in the following triplet format: *(USER ID , TIMESTAMP, URL VISITED)*

We sessionize the data using the 30-minute idleness criteria according to userid and timestamp. We then characterize each session's search engine identity. Here we are only interested in the usage of the small search engine A and the major search engine B. We keep only the search sessions with nonzero usage of either A or B and ignore sessions without any search engine usage. Each search session is given a label as follows:

A: *if only engine A is searched in this session*
B: *if only engine B is searched in this session*
C: *if both engine A and B are searched in this session*

Each sequence will then be broken into two groups according to the timestamps. The first group is the first six weeks of data from July 25, 2004 to September 4, 2004; the second group is the second 6 weeks of data from September 5, 2004 to October 16, 2004. Finally, we select out the users who have at least 5 search sessions in the first six weeks and at least 5 search sessions in the second six weeks.

There are around 2.5 million users appeared in the click streams. Around 1.2 million (46.89%) of them search at least once on either A or B during this 12 week period. There are 214,597 users or 18% of searchers who have at least 5 search sessions during both the 1st and 2nd 6 week periods. These are the users selected in our study and we will refer them as *heavy searchers* in the remaining sections.

4 Results

4.1 Summary Statistics

Of these 214,597 heavy searchers, a majority of them search only the big search engine B during the 12-week period. Using strict binary classification, Table 1 summarizes the percentage of heavy searchers who searched only A, only B or at least once on both A and B.

Table 1. Distribution of Heavy Searchers

	1st 6 weeks	2nd 6 weeks	12 week period
Engine A only	2,639 (1.23%)	2,434 (1.13%)	1,436 (0.67%)
Engine B only	176,446(82.22%)	173,053(80.85%)	158,142(73.69%)
Switchers	35,512 (16.55%)	39,110 (18.22%)	55,019 (25.64%)

Note that the distributions vary significantly across different time periods and usage classification. For example, the percentage of users using engine B only vary from 82% to 74% when using 6 and 12-week period respectively. Also, the distribution of switchers varies from 16% to 25%.

The heavy searchers consume 38.15 sessions on average during the 12-week period. Of all the sessions, 2.83% of them are A sessions while 96.07% and 1.1% of them are B and C sessions. 50.14% of the sessions are consumed in the 1st 6-week period while 49.86% are consumed in the 2nd half so it's roughly about 50/50. The breakdown of A, B and C sessions of the 1st and 2nd 6-week periods are 2.84%, 96.16%, 0.99% and 2.81%, 95.98%, 1.21% respectively. There is an increase of switching sessions from 1st to 2nd 6-week period. This trend will be better captured in Section 4.4.

4.2 Clustering

The 1st and 2nd 6-week period of data is concatenated and plugged into the K-means clustering algorithm. The algorithm returns 10 clusters with R-squared = 0.977. The features used in the K-means algorithm are the percentages of A, B and C sessions consumed by each user. The clustering results of K-means are shown in Table 2. The 1st column is the cluster ID. The 2nd column is the limiting probabilities that we will discuss in detail in Section 4.4. The 3rd column is the percentage of population falling into the 1st 6-week period while the 4th column is the percentage in the 2nd 6-week period. The 2nd to 4th columns will add up to 100% respectively since they represent the percentage of heavy searchers in each cluster. The 5th to 7th columns are the centers of the clusters. Note that for each row, the 5th to 7th column will also add up to 100 because the centers are based on the percentage of sessions using the small engine ("%A), big engine ("%B"), and both ("%C"). The last column is our interpretation of each cluster by comparing the cluster centers and examining the members in each cluster. The rows are sorted by "%B". Table 3 illustrates some examples to the clusters.

Since B has a much larger market share, 6 out of 10 clusters (cluster ids 5 to 10) represent the heavy searchers who search B most of the time. We can further group the 10 clusters into 3 main categories: *prime-A* (cluster 1 and 2), *prime-B* (cluster 5 to 10) and *switchers* (cluster 3 and 4). Table 4 summarizes the population breakdown after grouping. One interesting finding from the clustering and grouping is although 16.55% and 18.22% of heavy searchers used both engines during the 1st and 2nd 6-week periods respectively as shown in Table 1, the *"real" switchers*, who switch between engine A and B often are of very low percentage (1.45% and 1.52% respectively for 1st and 2nd 6-week periods).

Table 2. Cluster Memberships

ID	Limiting Probabilities	1st 6-week population	2nd 6-week population	% A	% B	% C	Cluster Interpretation
1	1.89%	2.14%	2.03%	95.00	2.71	2.29	A Loyalists
2	1.11%	1.08%	1.08%	67.86	23.21	8.93	A Primary
3	0.39%	0.27%	0.33%	30.45	29.00	40.55	Switcher I
4	1.31%	1.18%	1.19%	40.89	53.37	5.74	Switcher II
5	1.05%	0.67%	0.90%	12.04	62.17	25.79	B Primary
6	1.92%	1.72%	1.75%	20.00	76.68	3.32	B Principal
7	2.25%	1.63%	2.04%	2.36	81.52	16.13	B Principal using A as Backup
8	2.88%	2.73%	2.73%	9.13	89.72	1.15	B Loyalists checking out A
9	4.98%	3.92%	4.73%	0.45	92.38	7.18	B Loyalists using A as backup occasionally
10	82.22%	84.66%	83.21%	0.03	99.92	0.05	B Purists

Table 3. Examples for Each Cluster

Cluster ID	Examples
1	AAAAAAACBAAAAAAAAAAA AAAAA AAAAAAAAAAAAAAAAAAAAACAAACAA
2	AACCCBBAAAAAAC AABCABAACAAAAA AACCAA
3	CBBACC ACBCAAAC AABACABACCCCAACAA
4	AAACABBBAABBA BACAB ABBBBA
5	BBBBBCBACBBABBBBCBCAB BBCCCBBCCBBBB BBBBCACAB
6	ABABBBBBBB BBBBBBBBBBCAAABA BACBBBBB
7	CBBBBBBBBBBBACBBCBBBABBBB BCBBBBBBCBBBBBCBBBBBBBC BBCBB
8	BBBBBBBBBABBBB BBBBBBABBBBCBBABBBBBBCBB BBBBBBBBBBBABBB
9	BBBBBBCBBBABBBBBBBBBBBBBBBBBBBBBBBBB BBBBBBBBBBBBBCBBBBB BBBBBBBBBBBBBBBBBBBBBBCCBBBBBBBBBB
10	BBBBBBBBBBBBBBBBBBBBBB BBBBBBBBBBBBBBB BBBBB

We believe Table 4 is a more accurate portrait of distribution of heavy searchers because Table 1 is using a strict binary classification rule that even if a person consumes 99 sessions in engine B and 1 session in engine A, it will still be classified as a switcher rather than a *prime-A* user. On the other hand, Table 4 used the naturally found clusters to segment the users and a smaller swing of percentage of *switchers* is observed. Moreover, the percentage of *switchers* during the 12-week period according to the binary classification rule is 25.6%, which is a big jump from the 6-week time period (which is 16.55% and 18.22%). The clustering estimate, however, is much more stable (from 1.45% and 1.52% to 1.38%) with varying time windows. We therefore conclude the clustering and regrouping procedure is a better way to classify users than the commonly-used binary classification rule.

Table 4. Cluster Memberships after Grouping

	Limiting probabilities	1st 6 weeks	2nd 6 weeks	12 week Period
Prime-A (1,2)	3.00%	3.22%	3.11%	3.04%
Prime-B (5-10)	95.30%	95.33%	95.37%	95.58%
Switchers(3,4)	1.70%	1.45%	1.52%	1.38%

4.3 Transition Probabilities

We construct the transition probability matrix to describe user migration patterns from the 1st to the 2nd 6-week periods. Table 5 is the transition probability matrix where each cell describes the estimated probability of going from a *from-cluster* to a *to-cluster*. Each row will therefore add up to 1. We apply the same grouping mentioned in Section 4.2 and summarized it in Table 6.

Table 5. Transition probability matrix (%)

from \ to	1	2	3	4	5	6	7	8	9	10
1	72.81	16.85	1.68	4.81	0.81	1.46	0.35	0.33	0.15	0.76
2	29.41	27.52	5.38	16.24	4.13	7.36	2.24	2.37	1.21	4.13
3	8.05	15.92	18.66	14.90	13.36	9.08	7.36	3.42	1.54	7.71
4	6.93	14.56	3.96	21.37	6.89	14.80	5.30	9.14	2.85	14.21
5	1.26	4.20	5.10	9.58	15.38	11.12	13.36	7.83	10.14	22.03
6	1.22	4.20	1.79	10.21	5.55	14.03	7.64	11.98	8.15	35.22
7	0.23	1.11	1.23	3.31	6.36	6.78	13.11	7.16	15.25	45.47
8	0.20	1.07	0.41	3.48	2.69	7.95	5.81	11.54	12.02	54.82
9	0.04	0.20	0.17	0.69	2.01	2.62	6.78	6.35	15.84	65.30
10	0.02	0.06	0.05	0.24	0.32	0.83	1.26	1.94	3.87	91.44

From Table 6, we can clearly see that non-switchers tend to stay in the same group between the two time periods. Engine B has a much more cohesive user base than engine A: around 99% of *prime-B* searchers will continue to be *prime-B* searchers as opposed to 79% for *prime-A* searches. The *switchers* are more likely to switch to other groups and become either *prime-A* or *prime-B* user. This suggests that *switchers* are probably evaluating both engines and about a quarter of them will still be evaluating or decide to use both of them in the next period while half of them decide to go for B and about less than a quarter of them decide to go for A. The ratio of probabilities of switchers-to-prime-B over switchers-to-prime-A is an interesting metric to measure user preferences: it is two times (51.17 / 21.95 = 2.33) more likely for a switcher to become a *prime-B* user than a *prime-A* user.

Table 6. Transition probability matrix after grouping

From \ To	Prime-A (1,2)	Prime-B (5-10)	Switchers(3,4)
Prime-A (1,2)	78.67%	9.76%	11.57%
Prime-B (5-10)	0.27%	98.93%	0.80%
Switchers(3,4)	21.95%	51.17%	26.87%

4.4 Limiting Probabilities and Time to Convergence

If the trend depicted by Tables 5 and 6 holds, what will the final population distribution be? Table 2 and 4 list out the limiting probabilities for each cluster and each major category respectively. It turns out that although engine A is losing users to B from 1st to 2nd 6-week periods, at steady-state about 3% of the searchers will still use engine A as the primary engine and about 1.7% of the searchers will still use engine A along with engine B (*switchers*). This is empirical evidence that the small engine A can still survive as long as they maintain their market position at the current level with respect to other big players.

This non-zero market share is attributed to the fact there are searchers who will **prefer** engine A even though they were originally *prime-B* users. The transition probability matrix thus represents an irreducible Markov chain—after many transitions, the population distribution will reach equilibrium resulting in a non-zero market share for A.

Finally, to estimate time to convergence, we used the maximum percentage difference as the distance function and set the threshold ε to 0.1%. The returned s^* is 8, which means after about 8 transitions (or 6*8 = 48 weeks), the equilibrium will be reached. It implies the time to equilibrium between engine A and B is less than one year away. It is interesting that the convergence time is just less than one year, which suggests the web search competition is close to equilibrium.

4.5 The Usage Profiles

In Section 4.2, we used the percentages of sessions using A, B and C (both) as the clustering features. This doesn't account for the actual number of sessions consumed by each user. Table 7 summarizes the mean and median number of sessions consumed during the 12-week period by each cluster. Each transition cell (mean or media number of sessions consumed by people who transition from cluster i to j from 1st to 2nd

6-week period). We observe that the mean and median for each cluster barely change from the 1st to 2nd 6-week periods. Another interesting fact is that *prime-B* users on average consume much more sessions (38.75 for mean, 28 for median) than *prime-B* users and *switchers.* It provides another piece of evidence that users of engine B are much more engaged. The off-diagonal cells suggest that people who transition from one group to the other consume less sessions. Especially for people who switch from *prime-B* to other groups, the difference is dramatic (from 38.92 to 21.98 and 23.9). It suggests that *prime-B* users, if they don't consume a lot of sessions in the absolute term, the chance of losing them to engine A would be higher. In other words, if a search engine can make users search more, the chance of losing users to another search engine will be lower.

Table 7. Mean/Median Number of Sessions

From \ To	Prime-A (1,2)	Prime-B (5-10)	Switchers (3,4)	Row Mean
Prime-A (1,2)	26.66/21	22.32/18	24.53/20	25.99/20
Prime-B (5-10)	21.98/18	38.92/28	23.90/20	38.75/28
Switchers (3,4)	24.26/20	25.03/20	28.22/21	25.72/20
Column Mean	26.03/20	38.75/28	25.16/20	

4.6 Key Metrics

We define the following metrics[2] with respect to 2 engines A and B:

User Share. We define user share of A during time *t* as the share of *prime-A* users during time *t.* From Table 4, the user share of A is 3.22% during the 1st 6-week period whereas the user share of B is 95.33%.

User Engagement. We define user engagement of A as the probability that users will remain in *prime-A* during the second period. From Table 6, the engagement of A is 78.67% whereas the engagement of B is 98.93%.

User Preference. We define user preference of B to A as the ratio of probabilities of switchers-to-prime-B over switchers-to-*prime-A*. We construe preferences as a choice made after an evaluation process. We consider the fact that the *switchers* in the first period as users who are in the process of evaluation. This doesn't take into account users who make abrupt changes from *prime-B* to *prime-A* or vice versa. From Table 6, the preference of A to B is 2.33, which means *switchers* are over twice more likely to prefer B to A.

Trends. We define trend of A as the pair of time to convergence and the percentage difference of the current share of *prime-A* users and the share of *prime-A* users at equilibrium[3]. From Table 4, the trend for A is (3-3.22)/3.22 = -6.8% and time to convergence is 48 weeks.

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2 Extending these definitions to multiple engines is not difficult.

3 More detailed metrics can be obtained by extrapolating P and starting probabilities y, that is, to compute yP^s to forecast the market share after s transitions.

5 Conclusions

This paper presents a simple framework to characterize the switching behavior between search engines based on user click stream data. The framework focuses on obtaining aggregate usage metrics, switching probabilities, and usage forecasting. We employed clustering methods to abstract the user behavioral patterns and used simple probability models to describe the switching behavior.

Our findings indicate that such a simple framework can provide insightful metrics. For example, the transition probabilities can be computed regularly across multiple time periods and across multiple engines to get a better sense of the competitive landscape. Also, the metrics about user preference and engagement can be derived from the transition probability matrix. The limiting probabilities offer a distilled view of the trend and the time to convergence indicates how far we are from the equilibrium.

Users who sample different search engines (so called switchers) of dramatic quality differences are of very low percentage and are more likely to become loyal to an engine than to remain as switchers: our data shows that only about a quarter of these switchers will remains switchers in the next time period. Engines with small market share can retain its fair share of users provided that they maintain good quality of service and differentiate themselves from large engines. If a search engine can make users search more, the chances of losing users to another search engine will be lower. Also, user engagement, user preferences, market share and number of search sessions consumed are all positively correlated with one another. It provides empirical evidence that to increase market share, search engines should work to improve user engagement and preference as defined in this paper.

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Study on Improving Efficiency of Knowledge Sharing in Knowledge-Intensive Organization

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Abstract. Knowledge sharing is an important part in knowledge management. The value of knowledge only can be seen in its transfer, sharing and utilizing. The transfer of individual tacit knowledge to organizational capacity can improve the competitiveness of a organization. However, there are a lot of barriers in knowledge sharing, especially in those knowledge-intensive organizations, where knowledge is very important for individuals to keep advantage, so they are usually unwilling to share it with others, or 'contribute' their personal knowledge to the organization. At present, most studies are emphasizing on how to create a good physical environment and platform to improve it. In this paper, we'll analyze it using game theory. We think that to improve the efficiency of knowledge sharing efficiency, motivation mechanism and physical platform must be improved simultaneously, especially in knowledge-intensive organization, motivation mechanism is more important. Then, to problem of 'prisoner' dilemma' between individuals knowledge sharing, this paper will show how to design motivation method to improve efficiency of it.

Keywords: Efficiency of Knowledge Sharing, Game theory, Knowledge-Intensive Organization.

1 Introduction

The postindustrial society is centred on the use of intangible intellectual capital and knowledge resource (Bell, 1973; Castells, 1996; Kallinikos,1996). Especially the

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Knowledge-intensive organizations, they are based on their capability of making use of intangible, intellectual resources and assets. Knowledge sharing is an important component of knowledge management. Because of synergistic effect of knowledge sharing, both employees and organizations are beneficiaries, so the sharing of knowledge within organization has become an important method to get competitive advantage for knowledge-intensive organizations. Knowledge sharing within organization has four benefits: first, can realize the knowledge share from individual to group or team, therefore increase and enrich the structure of intellectual capital of a organization; second, can change employees' individual advantage to organizational advantage, lessen organization's dependency on individuals, and reduce possible loss of job-hopping; third, change organizational advantage into individual advantage, individuals can get more concentrated knowledge from organization, and therefore increase personal competitive ability; forth, the cost of accumulating knowledge within organization will be much less than those of purchasing from outside market.

Teece has proposed the concept of knowledge share for a long time. After this, knowledge sharing has gradually become a hotspot in knowledge management. In Kought and Zander (1992) 's opinion, the capacity of knowledge share is an important element for a organization to survive (Kought B. and U. Zander),. Currently, the study for knowledge sharing includes several parts: classification of knowledge sharing, sharing methods, barriers of knowledge sharing, organizational study, trust and evaluation, etc. Dixon (2000) has studied the face to face knowledge communication and sharing, Lynne(2001) suggested to use a electronic knowledge-base to realize synchronous and asynchronous knowledge communication and sharing. After studying the possible barriers of knowledge sharing in individual, technology and organization, Richard and Gillian (2000) thought that the main barrier of knowledge sharing is individual. Currently, studies for the efficiency of knowledge sharing are mainly on the characteristic of knowledge itself and the influence of knowledge sharing, which emphasizes that the tacit character of knowledge or the fuzzy causality of knowledge is the main barrier of knowledge sharing (Zander and Kought,1992), and believes that the development of organization knowledge depends on the specific situation of a organization.

Using game theory, we'll analyze the main reason that holds up knowledge sharing. Supposing all employees are reasonable man, individuals in an organization will have a motivation to keep his knowledge secret, thus forming a 'prisoner' dilemma'. We believe, to improve the efficiency of knowledge sharing, must solve two problems: physical environment (platform or place) and motivation mechanism. Especially in knowledge-intensive organizations, motivation mechanism is more important. For the problem of 'prisoner' dilemma' in knowledge sharing, this paper will show how to design an effective motivation mechanism to improve it.

2 Characteristic and Model of Knowledge Sharing in Knowledge-Intensive Organizations

2.1 Characteristic of Knowledge Sharing in Knowledge-Intensive Organizations

Knowledge sharing involves two parties: knowledge resource and knowledge recipient. Davenport and Prusak (1998) have proposed a formula of knowledge sharing: share (or sharing)=transmit+absorb. This says that knowledge must be shared from knowledge resource to knowledge recipient (Jeffrey C., 2003). The aim of knowledge sharing is to transfer knowledge from resource to recipient. Therefore, in certain extent, knowledge share and knowledge sharing is a same concept.

In knowledge economic society, intangible intellectual and knowledge resources are key resources of an organization, naturally, knowledge sharing becomes the main approach to utilize and spread knowledge in knowledge-intensive organizations:

First, knowledge sharing becomes the main approach that employees acquire knowledge and make creation, so it has a unique function in knowledge-intensive organizations; Second, knowledge is the hard-core of an employee's ability, so the factitious barrier will be quite great in knowledge sharing; Third, the objective of knowledge sharing is to achieve a corporate task, because of the characteristic of knowledge-intensive organization, this task is always a new knowledge product (or material object); Fourth, knowledge sharing is a mutual process based on knowledge creation, members always play dual roles: knowledge resource and knowledge recipient; Fifth, frequency and depth of knowledge sharing is greater than other non-knowledge-intensive organizations, so the synergetic effect of knowledge sharing will be greater than others; Sixth, by knowledge sharing, except realizing organization mission, employees' knowledge and creative ability will also be improved greatly.

2.2 Barriers of Knowledge Sharing in Knowledge-Intensive Organizations

In knowledge-intensive organizations, there are more barriers in knowledge sharing than others that's why the approach of knowledge sharing will be different from those non-knowledge-intensive organizations. Davenport and Prusak (1998) call these confinement factors as "attrition" or "noise" in knowledge sharing. In knowledge-intensive organizations, these barriers can be classified as:

First, nature of knowledge itself. Because in knowledge-intensive organizations, knowledge is more complex and tacit, so it's more difficult to clarify and explain in words, which makes the biggest barrier in knowledge sharing.

Second, transmitter of knowledge. The cost of knowledge sharing is very low (almost is 0)(Arrow K J.,1962; Romer, Paul M. 1986), so once knowledge is shared, one will lose its exclusive right with it, this exclusive right is generally regarded as employee's value in an organization or his career guarantee, especially in those knowledge-intensive organizations. So, for those employees that have similar knowledge, it is very difficult to share knowledge with others. Additionally, since knowledge is secretly belonging to different individuals, it is impossible to gather it by a simple centralization plan.

Third, lack of physical environment and motivation mechanism. Knowledge sharing need a face-to-face communication, organization should provide appropriate places, platform and motivation methods to make employees willing to share. This is key and a challenge for knowledge management.

2.3 A Model of Improving Efficiency of Knowledge Sharing in Knowledge-Intensive Organizations

Knowledge is boundary-less, dynamic and if not used at a specific time or place, is of no value. Use of knowledge requires the concentration of knowledge resources at a certain space or time.

By using knowledge theory, Foss (1996) explains the performance and achievement difference between different organizations. In his opinion, different situation determines that organizations have different knowledge, which will influence the performance and achievement of a organization. Leonard-Barton (1992) using four quadrants to construct a knowledge system, among which, the core quadrant is organization's evaluation to knowledge scene and structure, other three quadrants respectively are technical system of organization, management system of control and creation, individual skill and knowledge. In analyzing the knowledge flow of transnational organizations, Gupta and Govindarajan (1991) use task environment, structure character and behavior demand to define primary scene, study the relations between these three scene and knowledge flow.

Nonaka and Takeuchi(1995) proposed an organizational knowledge creating model based on Polanyi's distinction tacit knowledge and explicit knowledge. Nonaka also emphasized a "Ba" during enabling process. The concept of ba has recently been explored as a supportive platform for knowledge creation (Nonaka and Konno, 2001). Ba can be thought as a shared mental space for emerging relationships. This space can be physical (e.g. office, dispersed business space), virtual (e.g. e-mail, teleconference) or mental (e.g. shared experiences, ideas, ideals).

In these models, we can see it mainly emphasizes on the function of "environment", "place" or "platform" in improving the efficiency of knowledge sharing. That is, how to create a good environment, tool or platform to make players convenient and willing to share their knowledge. In knowledge sharing, scene is important, but in knowledge-intensive organizations, situation is more complex and difficult, create some physical environment (place or platform) only can make employees share a little knowledge, once personal benefit involved, for reasonable consideration, employees will "keep secret" of their knowledge. So, to share knowledge well, motivation must be considered. Current knowledge sharing theories are seldom regard motivation as an important element, or just regard it as an element of scene creation. But when self-motivation is absent, or use management control methods and organizational background to replace it.

So, to share knowledge effectively, organizations must solve two problems simultaneously, physical environment (platform, place) and motivation mechanism. Figure 1 shows the above-mentioned relations. The efficiency and effectiveness of knowledge sharing in organizations is determined by effectiveness of physical environment and motivation mechanism.

In quadrant I, physical environment is good and motivation mechanism is weak. In this situation, organizations can provide good physical environment and place for employees to share knowledge, such as meeting room, good organization culture, good organization structure and skill, etc., where employees feel happy and like to share knowledge with others, but once personal benefit is involved, they'll consider it rationally and keep core knowledge secret. To improve efficiency and effectiveness of knowledge sharing, design of motivation mechanism is a must.

In quadrant II, physical environment is bad and motivation mechanism is weak, which is a common phenomenon in some organizations, they don't think it's necessary to share knowledge among employees. In this situation, effect of knowledge sharing is worst, especially those tacit knowledge. To improve the knowledge sharing

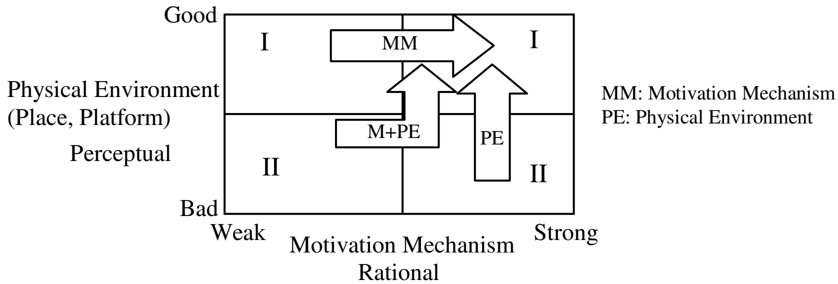

Fig. 1. Knowledge Sharing Matrix

in these organizations, one should design motivation mechanism and create appropriate physical environment to make employees willing to share knowledge.

In quadrant III, the leader of organization has realized the importance of knowledge and designed some motivation mechanism, but not realize the importance of organization's physical environment, so fail to create a physical place or platform for knowledge sharing, restrict employees' passion. For these organizations, they should take more consideration in physical environment.

In quadrant IV, the effect of knowledge share is the best. In these organizations, good motivation mechanism and physical environment encourage employees to share knowledge perceptually and rationally. The combination of physical environment and motivation mechanism can cause a high efficient knowledge sharing, both in quality and quantity.

In knowledge sharing, organization should know which quadrant it belongs to, and then take specific method accordingly. For the role of physical environment in knowledge, many researchers have had a detailed paper. So in this paper, we'll emphasize on the role of motivation mechanism in knowledge sharing within organization.

3 Game Analysis of Knowledge Sharing Behavior Among Individuals in Knowledge-Intensive Organizations

If without motivation mechanism, individuals of organization are unwilling to share knowledge with others because they are afraid of losing their advantages. A simple complete information static gaming model can help to analyze this process. Suppose A and B are individuals of an organization, they are rational, the knowledge they'll share and sharing is useful for organization.

3.1 The Process of Knowledge Sharing

According to the situation, knowledge can be sorted into two parts: a, nontransferable knowledge, mainly tacit knowledge that cannot be transfered or imitated in a short term, or cost too much to share. b, transferable knowledge, mainly explicit and part tacit knowledge, Which is part of core technical knowledge.

In the process of knowledge sharing, except directly absorbing resource knowledge of others, individuals will also obtain new creative value because of synergetic and leverage effect of knowledge. These new creative value includes synergetic value and multiplication value. The more dependence among organization members are, the more the synergetic value is.

Suppose in an organization, knowledge sharing between player A and B.

U_{A1} 、 U_{A2}、 U_{B1} 、 U_{B2} ：Player A and B's nontransferable knowledge value and shareable knowledge value

α_A ($0 < \alpha_A < 1$) and α_B ($0 < \alpha_B < 1$): Knowledge absorb coefficient.

U_{A3} 、 U_{B3}: Synergetic value after knowledge sharing because of synergetic effect of knowledge

U_{A4}、 U_{B4}: Multiplication value because of leverage of knowledge

U_{A5} 、 U_{B5} Recipient using obtained knowledge and causes value loss of knowledge provider, such as knowledge loss because of losing monopoly.

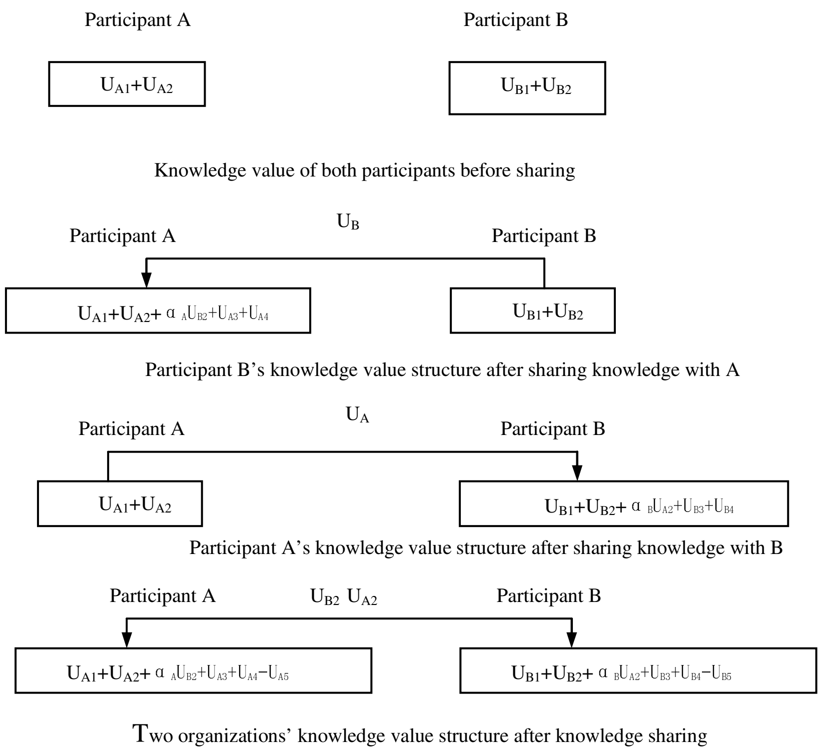

Fig. 2. Knowledge Sharing Process

In the process of knowledge sharing in knowledge-intensive organizations, the changing process of knowledge value of players can be explained as figure 2. In practice, players A and B will communicate and share knowledge simultaneously for several rounds, the flow of knowledge is mutual and coinstantaneous. For analyzing convenient, we divided it into two processes: A to B process and B to A process. In this way, we'll analyze each organization's knowledge gain after a simple round.

a. A shares knowledge to B. B absorbs shareable knowledge value $(\alpha_B U_{A2})$ of A, at the same time, because of synergetic effect and leverage combined with knowledge of A, causes new value (include synergetic value U_{B3} and multiplication value U_{B4}).

b. shares knowledge to A. A absorbs shareable knowledge value $(\alpha_A V_{B2})$ of B, at the same time, because of synergetic effect and leverage combined with knowledge of B, causes new value U_{A3} 、 U_{A4} .

c. After knowledge sharing of them, the knowledge value structure of each is shown as figure 2. Obviously, both values of A and B are increased than before.

3.2 Game Analysis in Knowledge- Intensive Enterprise

In the process of knowledge sharing, player A's strategy will influence player B's. Contrarily, action of player A is also restricted by player B. It is a game process.

Suppose all the players are rational. The aim of sharing knowledge is to maximize their payoffs. Simply, we just consider the situation of only two players: A and B. Each player selects one strategy: sharing knowledge or not. So there are four combinations. Its payoff matrix is as table 1.

Table 1. Payoff Matrix of the Game

A's Strategy	B's Strategy	
	Sharing	Not sharing
Sharing	$\alpha_A U_{B2}+U_{A3}+U_{A4}-U_{A5}$, $\alpha_B U_{A2}+U_{B3}+U_{B4}-U_{B5}$	$-U_{A5}$, $\alpha_B U_{A2}+U_{B4}$
Not sharing	$\alpha_A U_{B2}+U_{A4}$, $-U_{B5}$	0, 0

M.levy has demonstrated that there is no relevancy between low synergetic value and high multiplication value By comparing synergetic value with value loss while increased value is bigger than value loss, We can see that the game has two Equilibrium outcomes, as table 2, there are two equilibrium outcomes:

a. Prisoners' dilemma. When any player's synergetic value is less than value loss, (not-Sharing, not-sharing) is the only equilibrium outcome. That is to say, to each player the not- sharing payoff is better than sharing strategy absolutely, so they both select not-sharing strategy. It is dominant strategy equilibrium. But we can see that if both of them select not sharing, their payoffs are less than the payoffs ones when they all select sharing ones. It is the conflict between collectivity and individual rationality. That is to say, though cooperation and sharing knowledge are favorable to each player, but in the one time game, the two players would also select not sharing and get into prisoners' dilemma. The reason is that when synergetic value is less than value loss, the synergetic value from knowledge sharing can't make up risk from cooperation. So players both select not sharing.

In knowledge intensive organizations, players are risk-adverse. So how to design a mechanism to let them select strategy of sharing knowledge is a critical problem.

b. Assurance Game. Only when the two players' synergetic value are all bigger than their value loss (U_{A3} >U_{A5} and U_{B3} >U_{B5}), the equilibrium outcome are (not-Sharing, not-sharing) and (Sharing, Sharing). That is to say, players will get their maximal benefits while they make the same choose. There is First-Mover advantage. When synergetic value is bigger than value loss, players will wait and see. If A player selects sharing strategy, B's best selection is sharing, if A selects not-sharing, player B's best strategy is not-sharing.

Table 2. Equilibrium Outcome of the Game

	Equilibrium Outcome
$U_{A3}<U_{A5}$, $U_{B3}<U_{B5}$,	(not-sharing, not-sharing)
$U_{A3}<U_{A5}$, $U_{B3}>U_{B5}$,	(not-sharing, not-sharing)
$U_{A3}>U_{A5}$, $U_{B3}<U_{B5}$,	(not-sharing, not-sharing)
$U_{A3}>U_{A5}$, $U_{B3}>U_{B5}$,	(not-sharing, not-sharing) (sharing, sharing)

4 Improve Efficiency of Knowledge Sharing in Knowledge-Intensive Organization

4.1 How to Deal with Prisoner' Dilemma

Infinitely repeated game can break prisoner' dilemma. In a repeated game, players (employees) will think how his present action will influence the other player's future strategy. He is not only get his present outcome but also his future outcome. So in a repeated game, cooperation (Knowledge sharing) is possible. If employees did not know when they would leave the organization exactly, the knowledge sharing game can be looked as infinitely repeated game. So as long as the player is patient enough, the equilibrium outcome can appear. Equilibrium outcome and its stability of the infinitely repeated game are concerned with player's strategy, as follows:

a. Grim Strategy: At the very beginning, players select sharing knowledge, then if one selects sharing knowledge the other would selects it as well. But if the other player select not sharing one time, he will select not sharing forever.

Let $R_A=\alpha_A U_{B2}+U_{A3}+U_{A4}-U_{A5}$ $T_A=\alpha_A U_{B2}+U_{A4}$ $S_A=-U_{A5}$ $P_A=0$

In this condition, if the player selects sharing knowledge strategy, his expected utility is: $U_{AC}=\frac{R_A}{1-\delta}$

If he select not sharing strategy, his expected utility is: $U_{AD}=T_A+\frac{\delta P_A}{1-\delta}=T_A$

Here δ is discount factor, it reflects player's attitude to the future income. The Bigger of δ represents the more important the future income to the player. δ also reflects the feasibility of players meet each other in the future. Bigger δ, more feasibility.

When $U_{AC} > U_{AD}$, players will select sharing knowledge.

$$\delta > \frac{T_A - R_A}{T_A - P_A} = 1 - \frac{R_A}{T_A}$$

So, as long as δ is big enough, effective knowledge sharing equilibrium outcome can be realized by repeated game. It means that bigger the long term expected utility and longer the time to work together, then easier for knowledge sharing.

b. Tit-for-Tat Strategy. In infinitely repeated game, Tit-for-Tat is the best strategy. But the strategy is not fit for the knowledge sharing game. First, It is difficult to have both "retaliative" and "charitable". Once break faith will result in permanent estrangement hard to cooperate again. Second, because of the difficulties of expression and receiving information, it is also difficult to measure how much knowledge is shared during the game, it is impossible to select alternatives in sharing and not sharing continually. The third, Tit-for-Tat Strategy need players know the other player's strategy in short time, it is impossible in reality.

4.2 How to Improve Efficiency of Knowledge Sharing in Knowledge-Intensive Organization

We can see that, suppose players are rational, they will select not-sharing which is the best strategy to them. So, to improve efficiency of knowledge sharing in knowledge-intensive organization, except for physical environment, the organization must emphasis on motivation mechanism. It has been proved that, as long as the discount factor is big enough, the effective knowledge sharing can be realized. Factors influence discount are: expected income, possibility of long term relationship, return for cooperate, punish to betrayer.

So, the knowledge intensive organization can improve its efficiency of Knowledge sharing by:

1. Keeping employee team in a relative stability, to improve their long term expected outcome and cooperate opportunities which will improve discount factor and realize effective equilibrium outcome.
2. Establishing motivation mechanism, encouraging employee to share knowledge, improve expected outcome for long term cooperation and decrease the excepted outcome of betrayer and temptation of the not-sharing
3. Creating an effective and cultural environment for knowledge sharing.

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Traffic Models for Community-Based Ranking and Navigation

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Abstract. We investigate multinomial mixture traffic models for community based ranking and navigation. A "highway" model of source-destination traffic is formulated which aggregates the traffic through an underlying network of highways, onramps and offramps. The model extracts community structure from source-destination traffic information, but in addition captures the aggregate "highway" traffic between the communities. This important distinction extends the highway traffic analysis beyond clustering. The analysis discovers communities, rankings of the communities, rankings of destinations within each community, and transition probabilities between the communities. The model can be used for community-based collaborative filtering when applied to similarity graphs for music or movies.

1 Introduction

Traffic engineering and network design have been extensively studied in the engineering communities. The investigations cover how best to route traffic based on an existing connectivity graph, and optimizing connectivity paths to best fit the traffic. Source-destination traffic matrix estimation has been addressed from a statistical perspective in e.g.[1][2]. Here we present a probabilistic "highway" traffic model of source-destination random walk traffic on an underlying sparse graph. Our goal in the analysis is to model both the underlying community structure and the aggregate traffic between communities. Viewing the source-destination traffic matrix as a weighted "traffic graph", we seek to discover both tightly connected regions in the graph, and an underlying "highway" backbone structure in the graph. The traffic graph analysis has internet applications to community-based link analysis and collaborative filtering.

Our traffic models extends latent variable models which have appeared under the names Latent Class Models [3], Aggregate Markov Models [4]-[6], Non-negative Matrix Factorization (NMF)[7], and probabilistic LSA (pLSA)[8]. Many of the recent applications of these models have been in the fields of natural language processing and information retrieval. These latent variable models when applied to source-destination traffic data translate into a "hub" traffic model with only onramp and offramp traffic to latent hubs. The highway traffic latent variable model contains both highway entrance and exit hubs, and highway traffic between them. This allows the model to find both tightly interconnected

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communities, and the traffic flow between them. In addition to the analysis of source-destination traffic data, the highway traffic model is applicable to the analysis of random walk traffic on a source-destination connectivity graph. In related work, spectral clustering based on finding communities which minimize transitions between different communities has received considerable attention in image segmentation[9][10].

An outline of this paper is as follows. First we describe the highway traffic model and it's relation to a hub traffic model. Numerical experiments are presented in Section 3 for traffic analysis of a synthetic and an autonomous systems connectivity graph, and for topic community based ranking and navigation on a computer skills graph. Section 4 describes some properties of the highway traffic model, and the paper concludes with a discussion section.

2 Highway and Hub Traffic Models

Consider traffic flow data consisting of n_{ij} counts of traffic from source $X = i$ to destination $X' = j$. We assume that all sources are destinations, and destinations sources. Discrete latent variables H and H' are introduced which characterize the underlying entrance hubs and exit hubs on the highway. We assume that all entrances are exits, and vice versa. Our model of traffic flow consists of onramp traffic from sources to highway entrances, highway traffic from entrances to exits, and offramp traffic from highway exits to destinations. The model assigns a probability of going from source i to destination j of:

$$p(i,j) = \sum_{k,l} \alpha_{ik}\beta_{kl}\gamma_{jl},$$

where $\alpha_{ik} = P(X = i|H = k)$, $\beta_{kl} = P(H = k, H' = l)$, and $\gamma_{jl} = P(X' = j|H' = l)$. In words, α_{ik} is the fraction of traffic at entrance k from source i, β_{kl} is the probability of going from entrance k to exit l on the highway, and γ_{jl} is the fraction of traffic at exit l that proceed to destination j. The double sum in the expression is over all highway entrances and exits. Note that the traffic model is probabilistic, and in general allows for more than one highway route from source to destination. We further impose a constraint equating the onramp and offramp traffic distributions:

$$\gamma_{jl} = \alpha_{jl}.$$

Thus the fraction of traffic at exit l which continue to destination j is equal to the fraction of traffic at entrance l which originate from j. The model parameters are specified by $\alpha(x|h) = P(x|h)$ and $\beta(h,h') = P(h,h')$, which specify respectively the onramp/offramp traffic distribution, and highway traffic between the entrances and exits. The analysis provided by the model parameters in the context of community-based ranking and navigation are as follows. Each state of the latent variables H and H' correspond to a community. The onramp/offramp traffic distribution α provides the ranking within each community, whereas the highway traffic β describes the navigation transition probabilities between communities. The marginal distribution of H (or H') can be used to rank the communities.

Let the total amount of observed traffic be $N = \sum_{i,j} n_{ij}$, and let $\tilde{p}_{ij} = n_{ij}/N$ be the observed empirical joint distribution $\tilde{p}(x = i, x' = j)$. The log-likelihood function is given by

$$\mathcal{L} = N \sum_{x,x'} \tilde{p}(x, x') \log[\sum_{h,h'} \alpha(x|h)\beta(h, h')\alpha(x'|h')].$$

Maximizing the likelihood of the observed source-destination traffic counts is equivalent to minimizing the following Kullback-Leibler divergence:

$$\mathcal{D}(\tilde{p}(x, x') \parallel \sum_{h,h'} \alpha(x|h)\beta(h, h')\alpha(x'|h')).$$

The EM algorithm gives the following update equations

E-step

$$q(h, h'|x, x') = \frac{p(x, x', h, h')}{\sum_{hh'} p(x, x', h, h')}$$

where $p(x, x', h, h') = \alpha(x|h)\beta(h, h')\alpha(x'|h')$.

M-step

$$\alpha(x|h) = \frac{\tilde{p}(X = x, H = h) + \tilde{p}(X' = x, H' = h)}{\tilde{p}(H = h) + \tilde{p}(H' = h)}.$$

$$\beta(h, h') = \tilde{p}_{hh'},$$

where $\tilde{p}_{xh}$, $\tilde{p}_{x'h'}$, $\tilde{p}_h$, $\tilde{p}_{h'}$, and $\tilde{p}_{hh'}$ are the corresponding marginals of $\tilde{p}_{xx'} q(hh'|xx')$.

Representing the model parameters α and β as matrices, the highway traffic model seeks an approximation of the empirical traffic distribution $\tilde{p}$ by minimizing

$$\mathcal{D}(\tilde{p} \parallel \alpha\beta\alpha^t).$$

In comparison, a traffic model with the same structure as pLSA/NMF [4][7][8] seeks to minimize

$$\mathcal{D}(\tilde{p} \parallel AB).$$

The traffic interpretation of this model, which will be referred to as the "hub" traffic model, consists of an onramp distribution to the hubs from the sources, the hub distributions, and the offramp distributions from hubs to destinations. The highway model assumes more structure in the traffic data, and is a constrained version of the hub model. In particular, a highway model can always be represented as a hub model by equating corresponding terms in $(\alpha\beta)(\alpha^t) = (A)(B)$, effectively folding in the highway traffic between entrances and exits into the onramp traffic distribution specified by A. This comes at the cost of reduced sparseness of the onramp traffic distribution, and an increase in complexity of the hub model. Without equating onramp to offramp traffic in the highway model, the highway traffic has extra degrees of freedom since we can always write $\alpha\beta\gamma = (\alpha\beta)(I)(\gamma)$. Here the onramp traffic incorporates the highway traffic, and now there is no cross-traffic between entrances and exits. By equating onramp to offramp traffic, these degrees of freedom are removed in the highway traffic term β. The highway and hub traffic models differ in complexity, sparseness and structure.

3 Numerical Experiments

3.1 Synthetic Graph Analysis

We start with a simple example analysis which elucidates the highway traffic model's ability to find communities and their interrelations. A simple synthetic graph consisting traffic between 12 nodes is depicted on the left in Figure 1. Directed edges correspond to one traffic count in the given direction, whereas undirected edges represent a traffic count in both directions. The empirical joint source-destination distribution for the graph has exact decompositions according to both the highway and hub traffic models, with zero KL-divergences. Thus, the comparison here is in terms of the structure each model extracts from the data. For the highway model, the EM algorithm described above is run for 100 iterations starting from random initializations for $\alpha(x|h)$ and $\beta(h, h')$. The exact decomposition of the graph consists of $k = 4$ fully connected communities consisting of 3 nodes each, given by $\alpha(x|h = i)\beta(h = i, h' = i)\alpha(x'|h' = i)$. These are depicted in the top four subgraphs on the right in Figure 1. In addition, the relations between the communities, as given by $\alpha(x|h = i)\beta(h = i, h' = j)\alpha(x'|h' = j)$, is depicted in the bottom four subgraphs.

The highway model correctly discovers the underlying communities in the graph, as well as the onramp/offramp community-based ranking parameters. In Figure 2, the onramp/offramp distribution parameter α, and the highway traffic parameter β are displayed. In addition, a binary representation of the graph's highway backbone structure is visualized by thresholding β. For comparison, we fit a hub traffic model to the data. The corresponding graph decomposition if shown in Figure 3. The hub model all traffic within communities together with all outbound traffic from that community. The hub model essentially incorporated the highway traffic distribution into the offramp distribution.

The highway model's analysis of this simple traffic graph successfully captures the tightly knit communities and their interrelations. In addition, the highway traffic matrix $\beta(h, h')$ describes the highway backbone traffic structure in the data.

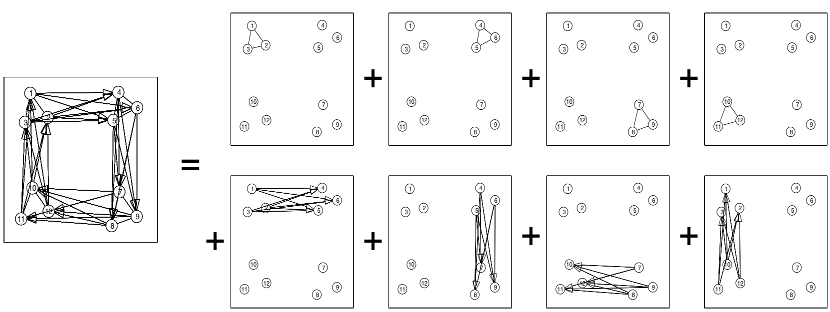

Fig. 1. Synthetic graph decomposition based on the highway traffic model. The decomposition consist of four subgraphs of tightly knit communities and four subgraphs of relations between the communities.

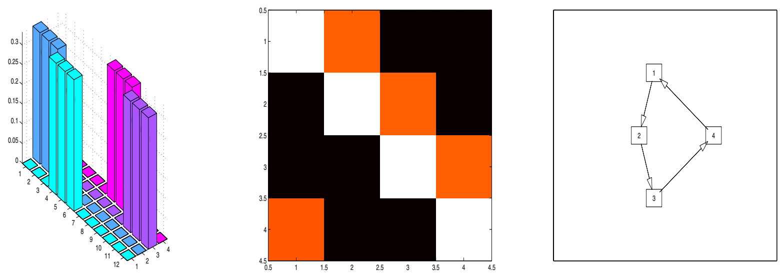

Fig. 2. The highway model's onramp/offramp distribution (left), highway traffic β (center), and highway traffic visualization (right)

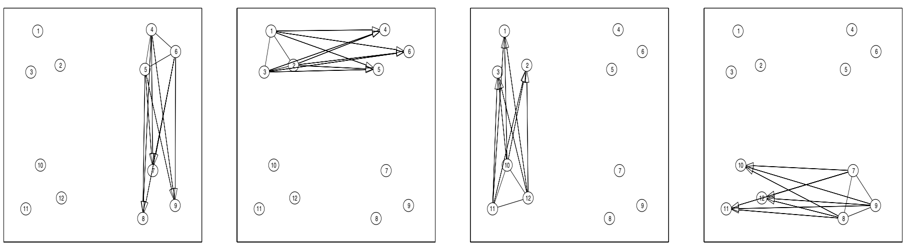

Fig. 3. Synthetic graph decomposition based on the hub traffic model

3.2 Autonomous System Connectivity Graph

We analyzed simulated internet traffic data based on an undirected connectivity graph between Autonomous Systems (AS). The connectivity graph consists of AS paths in BGP routing tables collected by the server *route-views.oregon-ix.net*. This data is the basis of the power-law analysis in [11], and is publicly available at *http://topology.eecs.umich.edu/data.html*. After trimming out nodes with no edges, we are left with an undirected binary AS connectivity graph with 13233 interconnected AS nodes.

We compared the highway traffic model to the hub traffic model normalizing for the complexity differences of the two models. With k latent hub states in the hub model, and n sources/destinations, the hub model has $[2k(n-1)+k-1]$ parameters. In contrast, the highway model with the same number k or entrances/exits contains only $[k(n-1)+k^2-1]$ parameters. We compared the two models using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and predictive test-set log-likelihoods. The simulated traffic data was constructed as follows. For the training set, we performed 100000 *single* random walk steps on the connectivity graph. The 100000 source nodes were sampled in accordance with the stationary distribution of the random walk on the connectivity graph. Traffic from source nodes are assumed to follow each of the edge paths with equal probability. Since multinomial mixture models can be prone to over-fitting problems, we added a single pseudo-count traffic for each

Table 1. AIC and BIC scores for the highway and hub models

values $\times 10^6$	k=26	k=51	k=100	k=197
Highway AIC	**4.90**	**5.41**	**6.59**	**9.07**
Hub AIC	5.56	6.72	9.16	14.15
Highway BIC	**8.34**	**12.2**	**19.9**	**35.0**
Hub BIC	12.4	20.2	35.5	66.1

edge in the connectivity graph. For the test set, 20000 single random walk steps were simulated.

In Table 1 the AIC and BIC scores for the highway and hub models are tabulated for a number of different k values. For each model and each k, 10 EM runs with random parameter initializations are performed. Scores for the best respective runs are reported in the table. The highway traffic model has significantly better (lower) AIC and BIC scores than the hub traffic model.

3.3 Random Walk Traffic on Computer Skills Graph

We analyzed a smaller, easily interpretable computer skills graph to demonstrate the traffic model's ability to extract communities, their relationships, and community-based rankings. A computer jobs description data set, provided courtesy of Prof. Richard Martin and the IT consulting firm Comrise was analyzed. The raw data consists of a collection of computer job descriptions, each of which contain a subset of 159 computer skills the hiring manager considered important for the job. The most frequently occurring skills keywords in the job descriptions are "unix", "pc(ibm)", "windows95", "windowsnt", "c" and "oracle". Entries along the diagonal of the co-occurrence matrix contain the number of times each skill occurred over all the job descriptions. The elements of this matrix is interpreted as a the amount of (co-occurrent) traffic between pairs of job skills. This interpretation is equivalent to the normalization used in the random walk view of segmentation [9]. From the co-occurrent traffic information on the computer skills graph, we seek to extract out underlying computer skill topic communities, and the underlying backbone connectivity structure between the topic communities.

A visualization of the computer skills traffic graph is shown in Figure 4(a) using the *GraphViz* [12] spring model graph layout program from *AT&T Labs-Research*. Only edge connections with average transition probability greater than .085 are shown. Even though the graph is not very large with 159 nodes, the visualization is not easily readable, and only provides vague clues to relationships between various skills.

From the job skills co-occurrence table the observed empirical joint distribution $\tilde{p}_{xx'}$ is constructed. The EM algorithm is used to find the maximum likelihood estimators for the conditional $\alpha(x|h)$ and the joint $\beta(h, h')$.

Since the onramp and offramp traffic distributions are equal in the highway model, we will simply refer to the offramp traffic. The offramp traffic distribution from a few exits (topic communities) are tabulated in Table 1. This specifies the

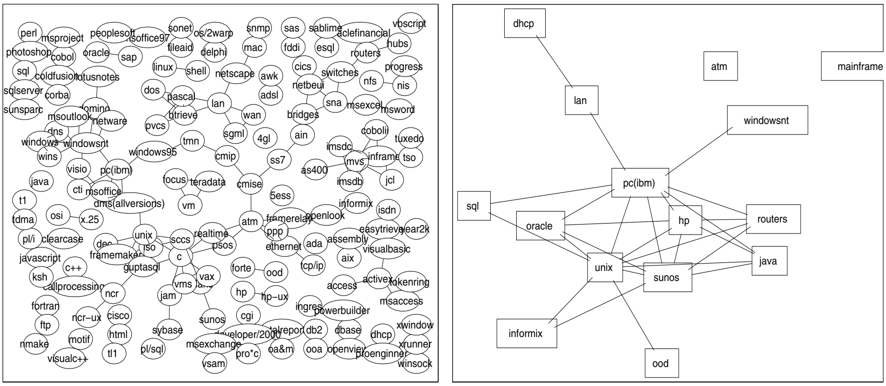

Fig. 4. (a) Graph layout of the computer skills traffic graph using *GraphViz's* spring model layout algorithm. **(b)** Highway traffic visualization - each node in this graph is a highway entrance or exit hub, and corresponds to a computer skills community. Onramp and offramp distributions to sources and destinations are tabulated in Table 1.

fraction of traffic at the specified exit which flow to each destination node. The destination computer skill with the largest traffic fraction is used as the label for the exit. The five top skills, ranked in descending order of their conditional probabilities are shown for each exit (community). These are the top 5 ranked skills for each topic community. This describes the traffic model's community-based ranking system.

From Table 1, we see a **UNIX** skills community containing *Unix*, *C* and *C++*, and a **SUNOS** operating systems community containing *SunOS*, *solaris* and *sunsparc*, a **HP** cluster with *HP*, *HP-UX*, and a **JAVA** community containing in descending community ranking *Java, html, javascript, perl* and *cgi.* The model also identified skills groups affiliated with Microsoft, containing skills *PC(IBM), Windows95, MSoffice, MSproject* and *dos.*

In addition to the communities of related computer skills, the model also extracts out the relationships between the communities. This can be used for community-based navigation of the computer skills graph which is analogous to

Table 2. Onramp/offramp traffic distribution for highway traffic model. The skills with highest traffic fraction to/from the latent community states are listed in the first row, and used to label the clusters in Figure 4(b). Each column represents an entrance/exit hub. Fractions of traffic to/from each skill is listed next to the skill name.

UNIX	.156	SUNOS	.174	HP	.181	PC(IBM)	.214	JAVA	.154
c	.154	solaris	.169	hp-ux	.146	win95	.184	html	.142
c++	.123	tuxedo	.016	tcp/ip	.076	msoffice	.146	jscript	.075
sybase	.050	sunsparc	.009	nis	.008	msproject	.050	perl	.046
jam	.004	oa&m	.007	nfs	.007	dos	.027	cgi	.031

highway navigation. In Figure 4(b), we used *GraphViz* [12] to visualize the underlying highway traffic between entrance and exit hubs as defined by $\beta(h, h')$. This backbone traffic structure in the source-destination traffic data is visualized by thresholding $\beta(h, h')$ into a binary adjacency matrix. We emphasize that this is only for visualization purposes; the model contains more information than is visualized. From the highway traffic graph, we see tightly coupled highway traffic between the **Unix, SunOS, HP** communities, as well as the **Java** and **SunOS** communities. The highway traffic model successfully finds computer skills communities as well as the relationships between the communities.

4 Highway Traffic Model Properties

The highway traffic model can in general describe non-symmetric traffic data. By equating the onramp with the offramp traffic distributions in the model, we obtain the following conditional distribution within each community:

$$p(x, x'|h = h') = \alpha(x|h)\alpha(x'|h).$$

This is the highway model's predicted probability of transiting from source x to highway entrance h, and immediately exiting to destination x'. This conditional distribution matrix has rank 1 and satisfies the detailed balance condition. Thus, within each community, the traffic is assumed symmetric, and one can show that $\pi_h(x) = \alpha(x|k)$ is simply the stationary distribution of the random walk within each community. Even though the random walk within each community is reversible, the highway model can describe non-reversible traffic depending on the highway traffic distribution $\beta(h, h')$. If the highway traffic $\beta(h, h')$ between communities is symmetric with respect to source community (entrance) and destination community (exit), thereby satisfying the detailed balance condition, then the highway model describes symmetric source-destination traffic. One can verify that if the empirical traffic distribution is symmetric, and the highway traffic distribution $\beta(h, h')$ is initialized symmetric, then it will remain symmetric under all subsequent updates under the EM algorithm. Thus reversible source-destination traffic will be modeled with a reversible highway traffic model.

The traffic model approximates the empirical traffic flow in a maximum likelihood or minimum KL-divergence sense. For example, an approximation of the source traffic distribution can be obtained as follows. Let the source distribution of the highway traffic model be $\pi(x) = \sum_{h,h'} \alpha(x|h)\beta(h, h')$. Using Pinsker's inequality [13] we can bound the total variation distance between the empirical source distribution $\tilde{p}(x)$ and the source distribution of the highway model $\pi(x)$

$$\begin{aligned} &\sum_{x,x'} |\tilde{p}(x) - \pi(x)| \\ &\leq \sqrt{2\mathcal{D}(\tilde{p}(x) \parallel \pi(x))} \end{aligned}$$

$$\leq \sqrt{2\mathcal{D}(\tilde{p}(x,x') \parallel \sum_{h,h'} \alpha(x|h)\beta(h,h')\alpha(x'|h'))}.$$

Similarly, the highway model can approximate any empirical traffic flow from a source set of nodes to a destination set, with the KL-divergence providing a bound on the approximation error.

5 Discussion

The highway and hub traffic models are constructed with various structures and associated complexities in the multinomial mixture. This is analogous to controlling the covariance structure in Gaussian distributions, from spherical Gaussian, to Graphical Gaussian models and Factor Analysis, to the full Gaussian with arbitrary covariance structure. Hub models with the same probabilistic structure as pLSA/NMF have been applied in the information retrieval setting to decompose document-word matrices [7][8] and document-citation matrices [15]. In those settings, pLSA does not provide a probabilistic generative model, and is not able to cleanly assign predictive probabilities to new documents. Latent Dirichlet Allocation [16] improves on pLSA by providing a proper generative model. In the source-destination traffic setting we consider, the sources/destinations constitute a fixed set, and the traffic models properly defines probabilities for new traffic between the sources and destinations. The traffic models are properly defined probabilistic models of source destination traffic.

In summary, the highway model extracts out communities and relational information in the form of highway traffic between the communities. The model finds communities, community-based rankings, navigation probabilities between communities, and rankings of the communities themselves. It is related to spectral clustering algorithms where the interest is in finding communities of nodes with minimal traffic between the communities [9][10]. The highway traffic model extends the framework of minimizing traffic flow between communities and provides a low rank highway based approximation to the empirical source-destination traffic. In the relational data research field, symmetric graphs with binary edge weights have been investigated in the context of link detection [17] and relational modeling [18]. In contrast, the traffic models are used for analyzing more general weighted traffic graphs. We are pursuing extensions of the highway traffic model to address the selection of the number of highway entrances/exits, as well as traffic models with highways and freeways. The highway model can also be extended from an unsupervised to a semi-supervised setting with some observations of highway and onramp/offramp traffic counts.

Acknowledgments

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On the Efficacy of Detecting and Punishing Selfish Peers

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Abstract. We study the phenomenon of free-riding in peer-to-peer (P2P) systems via an abstract model of a public good provision game with incomplete information. Each user in the model has an intrinsic contribution cost and decides whether to contribute or free-ride based on the expected benefit derived from the system. We first consider the impact of positive network externalities—common in P2P settings—on the equilibrium level of free riding and show that such network effects reduce free riding slightly but are insufficient to prevent it. We then consider the use of an incentive mechanism based on the detection and punishment of selfish peers, explicitly modelling key design tradeoffs inherent in any realistic detection mechanism. We show that detection and punishment can reduce free riding, but that the risk of falsely accusing cooperative peers can diminish their effectiveness. Finally, we consider what level of detection would maximize the social welfare of the network. We find that a surprisingly low level of detection can be optimal and that the residual level of free riding at optimum depends critically on the overhead of detecting selfishness and the probability of falsely identifying cooperative peers.

1 Introduction

Peer-to-peer (P2P) networks can be thought of as public goods in that they rely on voluntary contribution of resources from individual users (the peers) and, like classic public goods, they exhibit some aspects of non-rivalry and non-excludability (see, e.g. [1, 2, 3, 4]). Under the purely voluntary provision of public goods, strategic behavior typically results in the free-riding problem: individuals contribute less than the socially optimal amount in anticipation that others will contribute. Empirical studies have verified that free-riding is a wide-spread phenomenon in P2P networks [1, 5, 6]

While economists have traditionally understood the free riding phenomenon as a stable equilibrium outcome, there have been significant recent studies of various incentive mechanisms to mitigate or eliminate free-riding. Proposed incentive mechanisms include reputation-based service differentiation [7], micro-payments for usage [8], penalty mechanisms [9], and fixed entry fee schemes [10].

In this work, we construct a model of a P2P network based on a voluntary public good provision game with incomplete information, for which free-riding is established

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as an equilibrium outcome. We use this model to study the impact of positive network externalities commonly found in P2P networks upon the level of free-riding at equilibrium and to evaluate the effectiveness of incentive schemes based on the detection and punishment of selfish peers. Our findings can be summarized as follows:

- In general, positive network effects reduce the motivation for free-riding. From traditional public good models, it is known that free-riding increases as the network becomes large. Positive network externalities, therefore, work in opposition to the free-riding effect. The impact of such network effects, however, is modest and incentive mechanisms are needed to significantly improve levels of contribution.
- We study the impact of error-prone detection of selfish behavior, allowing both failures to detect some selfish actors as well as the false accusation of cooperative peers. Both types of error limit the effectiveness of punishment as incentive mechanism. Since false accusations become more likely as selfishness is detected more aggressively, it is necessary to seek an optimal level of detection precision.
- Considering the tradeoff between true and false detection as well as the cost of detecting selfish behavior, we seek a detection rate to optimize the social welfare of the system. We find that the optimal level of detection depends on various parameters of the system and can be surprisingly low, even in systems with low detection costs and low false alarm rates.

The remainder of this paper is organized as follows: Following a review of related work in this area, Section 3 presents the basic model with its equilibrium notion. Section 4 analyzes the model in the absence of network effects case as a baseline for comparison, and discusses the impact of network externalities. Section 5 extends the model to incorporate features of an imperfect detection mechanism and investigates the system equilibrium under a detection-based penalty mechanism. In Section 6, we study the determination of the optimal detection rate, which maximizes the sum of all the payoffs of all peers in the network. Concluding remarks close the article with a brief discussion of further research topics.

2 Related Work

Our work is related to that of Asvanund, et al. [1], which also examines network effects in P2P architectures. This work sought to empirically measure negative and positive network externalities in music-sharing P2P network. Our work is distinguished by our use of a game-theoretic model to explain how positive network externalities influence on peers' provision decision in equilibrium and to quantify the effects of penalty mechanism.

Our work also complements that of Feldman, et al. [9], which also studied the use of penalty mechanisms to mitigate free-riding problem. Feldman pays particular attention to the social cost of free identities—so-called *cheap pseudonyms*—which can limit the effectiveness of reputation-based incentives. In contrast, our work mainly deals with the question of whether realistic detection mechanisms can work, even if pseudonyms are difficult or expensive to create.

Krishnan, et al. [2] make use of a game-theoretic analytic framework to evaluate free-riding in P2P networks. These authors assume content-sharing cost incurs a lump-sum cost, which is common knowledge and is the same for each user. In contrast, we consider a more realistic Bayesian framework where each individual has private information about the cost associated with his contribution, with the costs of other peers independently drawn from an identical, commonly known distribution. That is, Krishnan's analysis is for a symmetric contribution game with complete information, for which the solution concept is a pure strategy Nash equilibrium; the model in our work is a game of incomplete information and its solution concept is a Bayesian Nash equilibrium.

Finally, we note that the possibility of user exclusion, studied by Antoniadis, et al [10], is not considered in our model, which concerns ex post non-cooperative behavior. That is, we consider the moral hazard problem arising after the decisions about who participates have already been made. By assumption, in our model, each peer would be better to stay in the network rather than to opt staying out since the utility from the participation in the network is sufficiently high.

3 The Basic Model

We now present an abstract model of a P2P system as a continuous public good provided through voluntary contributions. Our model is an extension of a general framework that has been widely explored in the public good literature [11, 12, 13, 14].

We consider a group of $n \geq 2$ peers contemplating their contributions to a P2P network. Contribution to the network has a cost $c_i \in [\underline{c}, \bar{c}]$ for peer i. These costs are assumed to be private information; that is, an individual i knows with certainty his own cost, but has incomplete information about the costs of other peers. From the point of view of peer i, the costs of other peers are assumed to be independent random variables drawn from some distribution over the finite interval $[\underline{c}, \bar{c}]$. Let us denote the cumulative distribution function and its density as $Z(c_i)$ and $z(c_i)$, respectively.

Peer to peer systems are often characterized by positive network externalities. In a file-sharing network, for example, having more peers increases the probability of successfully locating a desired file. Similarly, in an ad hoc wireless network, having more peers may increase the connectivity, resilience, or spatial reach of the network. In general, such network effects can be modelled by allowing the benefit that a peer derives from the network to depend on how many peers contribute resources. As the number of cooperative peers gets larger, the greater is the payoff for each user of the network. We therefore assume the utility of each peer is an increasing function $f(k)$, where k denotes the number of peers choosing to contribute. As is common in the literature regarding network externalities, we assume that the utility function is (weakly) increasing in k, but with diminishing marginal increments as k becomes large. Possible candidate functions for $f(k)$ include $1 + \log(k)$, or the harmonic number $(\sum_{i=1}^{k} \frac{1}{i})$. Under this general set-up, the absence of network effects can be expressed as a special case: if $f(k)$ is constant for any $1 \leq k \leq n$ and $f(k) = 0$ for $k = 0$, the payoff remains at a constant level as long as at least one participant contributes.

In the symmetric Bayesian-Nash equilibrium, a peer contributes if and only if his cost is less than or equal to his expected benefit from contributing. This benefit, in turn, depends on how many other users contribute and on the presence of network effects. Note that the peer must evaluate its benefit in expectation due to the incomplete information about the private costs of others. A peer contributes if and only if his cost is less than or equal to a *cutoff value* c^*, which is given by a solution to the following equation[1]:

$$\sum_{i=0}^{n-1} \{f(n-i) - c^*)\} \binom{n-1}{i} Z(c^*)^{n-1-i} (1 - Z(c^*))^i =$$
$$\sum_{i=0}^{n-1} f(n-1-i) \binom{n-1}{i} Z(c^*)^{n-1-i} (1 - Z(c^*))^i \quad (1)$$

The left-hand side of equation (1) is peer j's expected payoff when choosing to contribute given other players' equilibrium strategies; the right-hand side is the expected payoff from free riding. In each term of the summation, i represents the number of peers who choose not to contribute. Note that the choice to contribute requires the peer to pay his privately known contribution cost c_j. The payoff $f(n-i)$ is obtained when i among the n peers choose to free-ride. The probability that i out of the $n-1$ peers other than peer j choose to free ride is given by the binomial distribution, assuming that peer contribution costs are i.i.d. A symmetric Bayesian Nash equilibrium is constituted by the profile of this threshold strategy played by all peers simultaneously.

We must acknowledge that a peer's own contribution is arguably treated unrealistically in our model in the following sense: If only one peer contributes to the system, all peers receive a payoff of $f(1)$ *including the contributor, himself.* With most P2P networks, it is common to assume that a peer derives utility only from the resources of others. One can show, however, that deriving utility from one's own contribution is a necessary condition for the formation of networks in our model. As an informal justification of this aspect, we interpret the self-induced utility as a so-called "warm glow" effect that models the altruism that is often required to bootstrap real-world networks.

4 Network Effects

In this section, we analyze the basic model and study how the network effect is related to the issue of free-riding. Rearranging equation (1) yields the following equation:

$$\sum_{i=0}^{n-1} \{f(n-i) - f(n-1-i)\} \binom{n-1}{i} Z(c^*)^{n-1-i} (1 - Z(c^*))^i - c^* = 0, \quad (2)$$

which affords a more intuitive interpretation. The first term of (2) represents the expected marginal benefit from the choice of contribution, while the second term is the threshold contribution cost. The marginality principle—the equality of marginal benefit and marginal cost—holds in equilibrium.

As a baseline, we consider a special case in which the benefit from the network is constant regardless of the number of contributors. Let us denote the cut-off value in the absence of network effects as c^b. The following Proposition summarizes our findings:

[1] Note that we assume a peer would choose to contribute if the marginal benefit is exactly equal to his contribution cost, which is innocuous because of its continuous distribution.

Proposition 1. *(a) The equilibrium cut-off value in the presence of the network effect is greater than or equal to the cut-off value in its absence, i.e.,* $c^b \leq c^*$*. That is, positive network externalities make a peer more likely to contribute to the network given a fixed number of users.*
(b) In the absence of network effects, the equilibrium cut-off value decreases with the network size, i.e., $\frac{dc^b}{dn} < 0$.

Proof. (a) Without the loss of generality, we can normalize the payoffs of a sole contributor to be one, i.e., $f(1) = 1$*. Also, if no one makes a contribution, the network provides zero utility, i.e.,* $f(0) = 0$*. We keep this assumption for the remainder of this paper.*

At equilibrium, the level of an individual cost must be equal to the expected benefit from contribution. From the equilibrium condition (2), we know that H(n,c) = 0, where

$$
\begin{aligned}
H(n,c) &\equiv \sum_{i=0}^{n-1} \left(f(n-i) - f(n-1-i)\right) \binom{n-1}{i} Z(c^*)^{n-1-i} \left(1 - Z(c^*)\right)^i - c^* \\
&= \left[(1 - Z(c^*))^{n-1} - c^*\right] + \\
&\quad \sum_{i=0}^{n-2} \{f(n-i) - f(n-1-i)\} \binom{n-1}{i} Z(c^*)^{n-1-i} \left(1 - Z(c^*)\right)^i
\end{aligned}
$$

For the comparison of the two thresholds, c^b *and* c^**, let us evaluate the given* $H(n,c)$ *at* c^b*. In this case, the first bracketed term becomes zero because this term is identical to the equilibrium condition in the absence of network effects. The second term turns out to be non-negative in the presence of network effects. In addition, we can see that* $H(n,\overline{c}) = -\overline{c} < 0$*, which implies there exists at least one* c^* *in the range such that* $c^b \leq c^*$.

(b) The first bracketed term is identical to the equilibrium condition in the absence of network effects. In this case, the second term is cancelled out because $f(n-i) - f(n-1-i) = 0$ *for* $0 \leq i \leq n-2$*. Thus, our benchmark is a special case of this more general formulation. In the absence of network effects, the equilibrium condition is*

$$\left(1 - Z(c^b)\right)^{n-1} - c^b \equiv G(n,c) = 0.$$

Applying the Implicit Function Theorem, then $\frac{dc^b}{dn} = -\frac{G_n}{G_c} < 0$ *because (i)* $G(n+1) - G(n) = \left(1 - Z(c^b)\right)^{n-1} - \left(1 - Z(c^b)\right)^n < 0$ *since* $0 < \left(1 - Z(c^b)\right) < 1$ *and (ii)* $G_c = -z(c^b)(n-1)\left(1 - Z(c^b)\right)^{n-1} < 0$.[2] □

Part (a) of Proposition 1 states that free-riding can be limited by the presence of network effects. The underlying intuition for this result is straightforward: positive network effects strengthen the incentive to contribute by increasing the rewards from doing so,

[2] In this proof, G_n and G_c denote partial derivatives. In view of the discrete nature of n, we have adopted the heuristic of evaluating G_n as the first-order difference $G(n+1) - G(n)$, although technically, the Implicit Function Theorem does not strictly apply. We note that using the "complementary" heuristic of allowing n to be a continuous value also proves the same result.

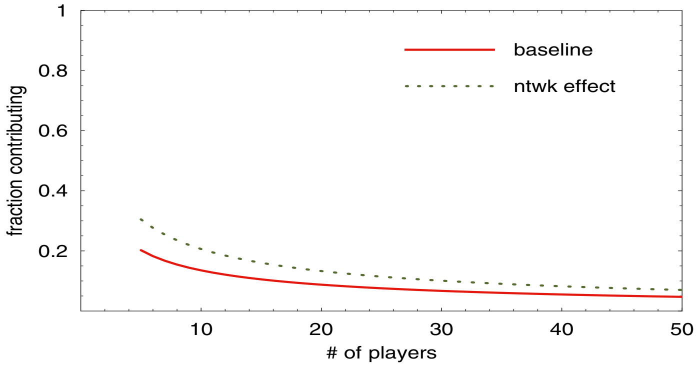

Fig. 1. Fraction of contributors as a function of the number of participants for an instance of the model with $f(n) = \log n$. Cutoff values are shown for both the baseline case (c^b) and the network effect case (c^*).

thereby inducing contribution from some peers who would otherwise have chosen to free-ride.[3] Part (b) of the Proposition states that a peer will be less likely to make a contribution in a larger network, which is a restatement of the classical free-riding problem in the provision of public goods.

Figure 1 illustrates Proposition 1 graphically for peer costs uniformly distribution of over [0, 2] and a logarithmic benefit function $f(n) = 1 + \log n$. We observe in the figure that the fraction of contributing peers—which is proportional to the cutoff value under a uniform cost distribution—decreases (i.e. free riding increases) even in the presence of positive network effects, although this fact is not guaranteed by Proposition 1.[4] In general, it is not the case that that dc^*/dn decreases monotonically. We conjecture, however, that it is asymptotically decreasing for realistic payoff functions. We leave a proof of this conjecture for future work.

5 Penalty Mechanism

We next turn our attention to the use of a penalty-based incentive mechanism to mitigate free-riding whereby selfish users are detected by honest peers and some punishment exacted. Although the details of a detection mechanism are typically application specific, we model detection abstractly as follows: We assume that a peer who fails to contribute can be detected with some probability p, which can be chosen by the system designer. Adopting terminology from signal detection theory, we refer to p as the *detection rate*

[3] It is worth noting that, by the same argument, negative network externalities—so-called congestion effects—could lead to a network with increased free-riders. Well-designed P2P networks, however, usually exhibit good scaling and load-balancing properties, making such a scenario unlikely.

[4] Recall that Proposition 1 merely states that voluntary contribution becomes less likely in a larger network *in the absence of* the network effects.

of the incentive scheme. We assume further that the system is able to punish a detected free-rider by removing any and all payoff derived from using the system.

With this penalty mechanism, the equilibrium condition becomes

$$\sum_{i=0}^{n-1} \{f(n-i) - c^p\} \binom{n-1}{i} Z(c^p)^{n-1-i} (1 - Z(c^p))^i =$$
$$\sum_{i=0}^{n-1} \{(1-p) \cdot f(n-1-i)\} \binom{n-1}{i} Z(c^p)^{n-1-i} (1 - Z(c^p))^i, \quad (3)$$

where c^p is the equilibrium cutoff value. As before, a contributing peer pays his contribution cost, while a free-riding peer does not. However, because the penalty scheme reduces the expected payoff from free-riding, the trade-off between free-riding and contribution is significantly altered. In equilibrium, as above, every peer follows the symmetric strategy of contributing if his cost is less than or equal to the cutoff value.

Proposition 2. *The equilibrium cut-off value in the presence of a penalty scheme is greater than or equal to the cut-off in the absence of a penalty scheme, i.e.,* $c^* \leq c^p$.

Proof. Rearranging terms in the equation (2) and define the equilibrium condition $J(n,c) = 0$ *as follows:*

$$J(n,c) \equiv \sum_{i=0}^{n-1} \{f(n-i) - (1-p)f(n-1-i) - c^p\} \binom{n-1}{i} Z(c^p)^{n-1-i} (1 - Z(c^p))^i$$
$$= \left[\sum_{i=0}^{n-2} \{f(n-i) - f(n-1-i)\} \binom{n-1}{i} Z(c^p)^{n-1-i} (1 - Z(c^p))^i\right] +$$
$$[(1 - Z(c^p)) - c^p] +$$
$$\sum_{i=0}^{n-2} \{p\, f(n-1-i)\} \binom{n-1}{i} Z(c^p)^{n-1-i} (1 - Z(c^p))^i$$

The first two bracketed terms are equivalent to the equilibrium condition previously considered (i.e. the network effect is present but no penalty mechanism is used). These terms become zero evaluated at c^*. *Thus,* $J(n, c^*) > 0$, *which implies that there exists at least one* c^p *such that* $c^p \geq c^*$ *since* $J(n, \overline{c}) < 0$. □

5.1 The Impact of False Positives

Our model allows for the imperfect detection of selfish behavior by assuming selfish peers go undetected with probability $1 - p$. In addition to such detection failures, *false positive* errors can also occur whereby cooperative peers are wrongly identified as selfish. The risk of undeserved punishment due to false positives would naturally reduce the expected payoff for cooperative peers. Intuition therefore suggests that false positives diminish the effectiveness of an incentive mechanism.

In any realistic detection algorithm, the false positive rate and detection rate are not dependent quantities and must be traded against each other [15]. Algorithms which detect misbehavior very aggressively tend to suffer from increased false positive rates in varying degrees depending on the type of algorithm used[5] and on the observable differences between selfish and cooperative behavior in a particular application context.

Analysis of a simple model of classification in the presence of noise (omitted here for reasons of space) suggests that false positive rate is well modelled as an exponential function of detection rate $\phi(p) = p^\gamma$, where the parameter $\gamma \geq 1$ can be chosen to determine the strength of the dependence. Under this class of functions, a perfect detection rate $p = 1$ inevitably leads to a false positive rate $\phi(1) = 1$. However, for high values of γ, detection rate can be increased to a relatively high intermediate value with little effect on the false positive rate. In the limit $\gamma = \infty$, $f(p) = 0$ as long as $p < 1$. In this case, the detection rate can become arbitrarily close to one with no false positives.

Denoting the cutoff value in the presence of false positives as c^f, we can state the following result.

Proposition 3. *For low values of γ, the the equilibrium cutoff value c^f is less than c^p. As $\gamma \to \infty$, false positives become less severe and $c^f \to c^p$,*

Proof. The equilibrium condition demands equality between a contributor's a free-rider's expected payoffs. That is, in equilibrium the following equation must hold:

$$\sum_{i=0}^{n-1} \left\{(1-p^\gamma) f(n-i) - c^f\right\} \binom{n-1}{i} Z(c^f)^{n-1-i} \left(1 - Z(c^f)\right)^i$$
$$= \sum_{i=0}^{n-1} \{(1-p) \cdot f(n-1-i)\} \binom{n-1}{i} Z(c^f)^{n-1-i} \left(1 - Z(c^f)\right)^i$$

Rearranging terms, we define the equilibrium condition $L(n, c) = 0$ as follows:

$$L(n, c) \equiv \sum_{i=0}^{n-1} \left\{(1-p^\gamma) f(n-i) - (1-p) f(n-1-i) - c^f\right\} \times \binom{n-1}{i} Z(c^f)^{n-1-i} \left(1 - Z(c^f)\right)^i$$

$$L(n, c) = \left[\sum_{i=0}^{n-2} \{f(n-i) - f(n-1-i)\} \binom{n-1}{i} Z(c^f)^{n-1-i} \left(1 - Z(c^f)\right)^i\right] + \left[\left(1 - Z(c^f)\right)^{n-1} - c^f\right] + \sum_{i=0}^{n-1} \{p\, f(n-1-i) - p^\gamma\, f(n-i)\} \binom{n-1}{i} Z(c^f)^{n-1-i} \left(1 - Z(c^f)\right)^i$$

Hence, $L(n, c^p; \gamma = \infty) = J(n, c^p) = 0$ but $L(n, c^p) > J(n, c^p) = 0$ as long as γ is not infinity. As γ decreases, these two conditions diverge so that c^f becomes smaller than c^p. □

[5] To take a trivial example, a detection algorithm that classifies all peers as selfish will have perfect detection but also a false positive rate of one.

Proposition 3 states that the risk of false positives distorts the equilibrium outcome of a penalty mechanism. If there is a possibility that cooperative peers will be misclassified as free-riders and punished, the penalty scheme does not achieve its full effectiveness as some peers who would otherwise have chosen to contribute may choose to become free-riders.

6 Optimal Detection Rate and the Social Planner's Problem

We found in the previous section that the network augmented with a penalty scheme may induce peers to take a cooperative action, provided that cooperative peers are not falsely punished with a high probability. However, implementing a detection mechanism is not without cost to the system as a whole. Detecting selfish behavior in a distributed system may require peers to log information, gossip about reputations, or play the role of third-party in other peers' transactions, all of which require system resources that could otherwise be devoted to service. Thus an additional consequence of detecting selfishness is to diminish the service capacity of the network. In some systems, high detection rates may only be achieved at an extremely high cost.

Up to this point, our model has overlooked this capacity reduction effect because it is felt by selfish and cooperative peers alike and thus has no effect on decision making or on the equilibrium operating point. To the system designer, however, there is a tradeoff to be made: By operating at a high detection rate, cooperation can be increased at the cost of reduced service capacity. Alternatively, if a higher level of free riding can be tolerated, some overhead can be recovered by using a less effective penalty mechanism.

To understand this tradeoff, we cast the designer in the role of a social planner who seeks to maximize the total welfare of users in the system. We model the impact of capacity reduction as a multiplicative penalty $\chi(p) = 1 - p^{\alpha}$ that reduces the payoffs of both selfish and cooperative nodes. In this formulation, the payoff when k participants contribute is

$$\chi(p)\, f(k)$$

We define the social welfare to be the sum of expected payoffs of all network users—contributors as well as free riders—net of the overhead.

Definition 1 (Social Welfare).

$$W = \sum_{i=0}^{n} \{(1 - p^{\gamma}) \left(\chi(p)\, f(n-i) - E[\, c/, |\, c < c^f]\right) \cdot (n-i) + \\ (1-p)\, \chi(p)\, f(n-i) \cdot i\}\, Z(c^f)^{n-i} \left(1 - Z(c^f)\right)^i$$

Observe that although each contributor pays his private cost, this is equivalent, in expectation, to all contributors paying the same cost $E[c|c < c^f]$. Note further that when i peers choose to free ride, free riders and contributors alike receive the same payoff $\chi(p)\, f(n-i)$.

The system designer must solve the social planner's problem of choosing a detection rate p that maximizes social welfare

$$p^* \in \arg\max W, \quad \text{where}\, 0 \le p^* < 1$$

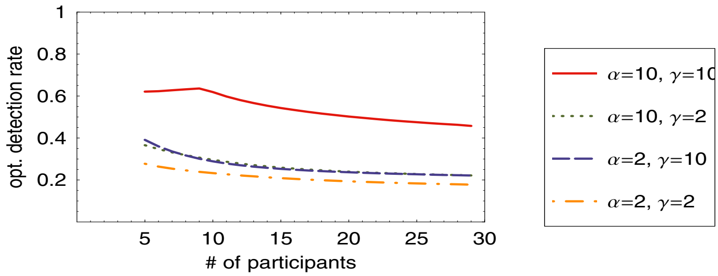

Fig. 2. Optimal detection rate as a function of number of participants for four scenarios differing in the penalty overhead function (parameter α) and the false positive growth function (parameter γ)

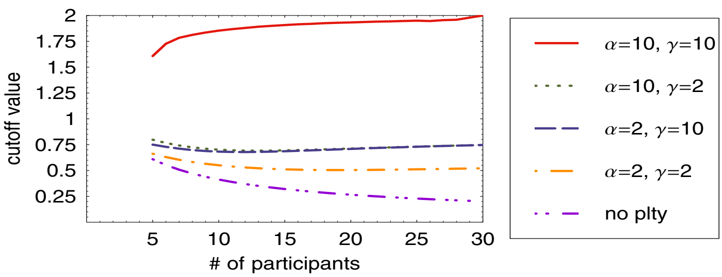

Fig. 3. Fraction of contributing participants at the optimal detection rate as a function of number of participants for four scenarios differing in the penalty overhead function (parameter α) and the false positive growth function (parameter γ). For comparison, the fraction of contributors is also shown when a network effect is present but no penalty is imposed.

Solving this optimization problem analytically by evaluating the first-order conditions with respect to the detection rate for specific functions f, Z and χ is generally intractable, so we rely on numeric techniques.

Figure 2 illustrates the optimal detection rate as a function of n in case where $f(k) = 1 + \log(k)$, $Z(c)$ is the uniform distribution over [0, 2], and $\chi(p) = 1 - p^\alpha$ for several combinations of parameters α and γ. Of the parameter combinations shown, the case most favorable to detection is when detection overhead is modest ($\alpha = 10$) and false positive rates grow slowly ($\gamma = 10$). The optimal detection rate in this case converges to a value of around 0.5, which is significantly higher than for other parameter combinations. Figure 3 plots the resulting level of cooperation at the optimal detection rate. We observe that when conditions are favorable for detection (i.e. $\alpha = 10, \gamma = 10$) the optimal detection rate, although well below perfect detection, is sufficient to guarantee near full cooperation. For the remaining three parameter settings, high overheads ($\alpha = 2$) or severe false positives ($\gamma = 2$) suppress the optimal detection rate value. Figure 3 shows that while cooperation is modestly increased by the penalty mechanism in these cases, a significant amount of free riding persists at optimal detection rates.

Perhaps the most striking feature of Figure 3 is that only in the case most favorable to detection ($\alpha = 10$, $\gamma = 10$) do we observe an increasing trend toward full cooperation as the network grows in size. In all other cases the trend is decreasing and full cooperation appears to be unattainable. Indeed, it appears that *either* high overhead or a high false positive rate alone is sufficient to prevent the incentive mechanism from inducing full cooperation. This result suggests the existence of a boundary in the space of parameters α and γ that separates two classes of system: those for which near-full cooperation is achieved at the social optimum from those for which some (possibly substantial) residual free riding is optimal.

7 Concluding Remarks

We have presented a general model of P2P systems as public good provision game of incomplete information which we have used to study the properties of incentive mechanism, incorporating abstract features commonly found in real-world P2P networks—specifically, positive network effects and a tradeoff between detection rate and false positive rate. The resulting model is tractable yet confirmed some rather intuitive results, namely,

- Positive network effects reduce free riding, but only modestly.
- Detection and punishment reduce free riding but the need to avoid false positives limits the aggressiveness with which selfish peers can be detected.

In addition to these intuitive results, we presented evidence that the socially optimal level of free riding depends critically on fundamental system parameters such as the cost of detection in terms of system overhead and the intrinsic difficulty of classifying selfish and cooperative peers. We conjecture that for many P2P systems overheads are likely to be high and the signature of selfish behavior is likely to be noisy. In such cases, our model suggests, the cost of achieving full cooperation is too high and some level of free riding must be tolerated.

While our focus has been on P2P networks, some of our results may have broader applicability in any public goods context where free riding cannot be reliably detected.

There are several additional interesting topics that can be treated under our modelling framework. For example, it would be interesting to explore the special case of a bimodal uniform cost distribution. We believe that such a distribution would be an appropriate model for realistic P2P networks which contain what we call an *altruistic core*—a subset of users with effectively zero cost to contribute. We would also like to relax the independence assumption and explore the effect of correlated costs on the system equilibrium.

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Semantic Web Recommender System Based Personalization Service for User XQuery Pattern

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Abstract. Semantic Web Recommender Systems is more complex than traditional Recommender System in that it raises many new issues such as user profiling, navigation pattern. Semantic Web based Recommender Service aims at combining the two fast-developing research areas Semantic Web and User XQuery. Nevertheless, as the number of web pages increases rapidity, the problem of the information overload becomes increasingly severe when browsing and searching the World Wide Web. To solve this problem, personalization becomes a popular solution to customize the World Wide Web environment towards a user's preference. The idea is to improve by analyze of user query pattern for recommender service in the Web and to make use for building up the Semantic Web. In this paper, we present a user XQuery method for personalization Service using Semantic Web.

1 Introduction

Over the last few years, this has lead to a growing interest in personalization. Personalization can be defined as the use of technology and user information to tailor e-commerce and m-commerce interactions between a business and each individual user. The purpose of this information technology combined with marketing practices unique to the WWW, can be described as follows: i) Better serve the user by anticipating needs, ii) Make the interaction efficient and satisfying, iii) Build a relationship that encourages the user to return for subsequent purchases. Also, Automated recommender systems intend to provide user with recommendations of products they might appreciate, taking into account their past ratings profile and history of purchase or interest.[1] Most successful systems apply social filtering techniques, dubbed collaborative filtering.[2],[3] These systems identify most similar users and make recommendations based upon products user utterly fancy. In order to create efficient personalization service, an approach for improving user query performance is to discover frequent Web based Commerce World Model. However, Commerce World, which is, E-Commerce and M-Commerce providing such kinds of services are still limited in terms of effectiveness. This paper proposes a solution to overcome these limitations. After presentation of structural approaches regarding user profiles, we introduce personalized XML Query technology using user queries. This paper is organized as

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follows: This paper is organized as follows: Section 2 describes content-based filtering systems, manual decision rule systems and collaborative filtering systems as related work. Section 3 describes Semantic Web Recommender Systems as Web based Commerce World as in our proposed system. Section 4 presents the interactions of user profiles and XML Query in the Semantic Web in terms of implementation and evaluation. In Section 5, we propose our conclusion.

2 Related Work

In general, recommendation systems manage information overload by helping a user choose among an overwhelming number of possibilities. Also, Recommender Systems have begun attracting major research interest during the early nineties.[4] These systems broadly fall into three classes based on the techniques they use to narrow the range of likely choices. Most web-based recommended systems can be divided into three classifications; content-based on filtering systems, manual decision rule systems, collaborative filtering systems, and link based systems.

2.1 Content-Based Filtering System (CFS)

Content-Based Filtering Systems. Content-based filtering systems utilize machine learning techniques such as naïve bayes to analyze WWW pages, Usenet News, E-mail, and other types of electronic content amenable to automatic textual analysis.[5] Also, in content-based filtering systems, the user profile and the document profile are represented as a weighted vector in terms of keywords. In recommendation systems, in which content-based filtering is applied, recommendations are made for a user based solely on a profile built from analyzing the content of items that the user has rated in the past. InfoFinder and WebWatcher are examples of such systems. Content-based filtering systems have several defects.[6],[7] The first problem is that only a very shallow analysis of specific content can be applied. As well as characteristics of the content itself, there are many other aspects of the content, such as public quality and time consumed in loading content. The system ignores such factors. The second problem is over-specialization. When the system can only recommend items scoring highly against a user's profile, the user is restricted to seeing items similar to those already rated.

2.2 Manual Decision Rule System (MDRS)

Manual Decision Rule Systems. Manual decision rule systems describe rules described by the web site operator, taking into account static profile information, and a session user history throughout a user's session. A representative example of this system is Broad vision. Broad vision provides a range of business solutions including Content Management Solutions(CMS), personalization solutions, commerce solutions, and enterprise portal systems. These systems can be used to solve difficult problems, using the power of personalization and robust content management to create leading edge applications and enterprise portals.[8],[9] In this way, rules are influenced in contents that are offered to particular users.

2.3 Collaborative Filtering System (CFS)

Collaborative Filtering Systems. Collaborative filtering systems select items for a particular user when they are also relevant to other similar users. In addition, collaborative filtering systems provide predicted information applicable with user preferences through a correlation engine based on clarified information and estimating the degree of user preference. A recommendation system, utilizes a collaborative filtering system, but does not analyze an item at all. The system recommends items for a user solely based on similarities to other users. GroupLens and Ringo are examples of such systems.[10],[11] A collaborative recommendation system solves problems relating to content-based recommendations. Using other user's ratings allows us to deal with any kind or content and receive items with dissimilar content to those seen in the past. Since other user's feedback influences recommendations, there is the potential to maintain effective performance even when given fewer ratings from any individual user. Nevertheless, this system has some problems. The first problem is that the coverage of ratings could be too sparse, resulting in insufficient recommendations. Recommendations may not be accurate in some cases, for example, in the case of new items being inserted into the database, or that the number of users is too small relative to the number of the items in the system. The second problem is that there will not be many like-minded users for a user whose tastes are unusual, compared with the rest of the users.[12],[13]

3 Proposed System

3.1 Semantic Web Recommender System of Category Module

The method considers the creation of a Semantic Web Recommender Systems as Web based Commerce World according to four aspects:

Management of the Product Information Contents Module (PIC Module) provides recommendable information contents as using user domain knowledge.

Definition of the Navigational Structure by user query Module (NS Module) deal with the navigation structure of the Semantic Web based Recommender System informational content by user query. This navigation method utilize by Resource Description Framework and XML.

Definition of the Personalized User Interface Module (PUI Module) show the formatting of contents associated to the navigational structure by user query.

Identification and description of Potential Users Module (PU Module) provide information about the potential users such as their background, knowledge, preference and history in order to define user categories and usage needs of these potential users.

It would be noticed that identifying the four aspects help mastering the complexity of the Semantic Web Recommender System.

3.2 SWRS of Category for Personalization Service Module

We assert that design a Semantic Web Recommender System is a four process module step, in which is created at each step, based on the previous one, and the last step is the actual system. This sub section provides a major description of the different processes, how these relate to each other and how these contribute to Semantic Web Recommender System design. Fig.1 shows the process steps, the precedence among them and their major process as each schema.

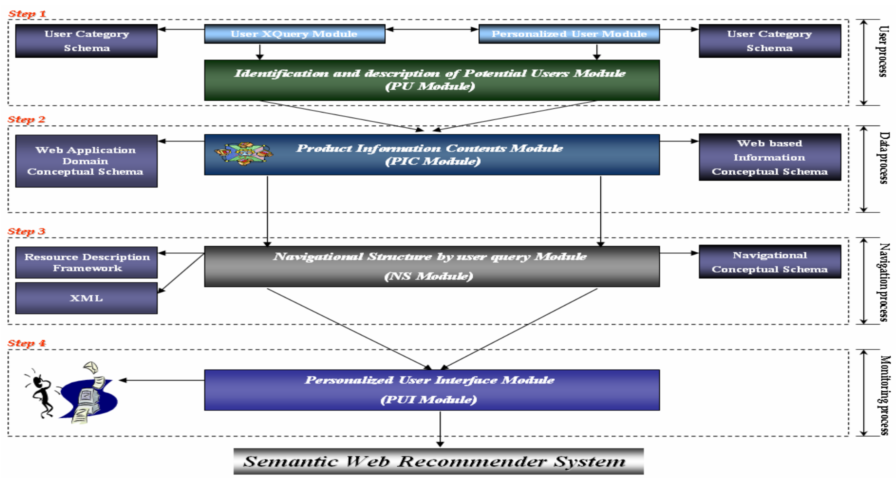

Fig. 1. Semantic Web Recommender System for Personalization Service Architecture

3.3 User XQuery Module in Semantic Web

In this step, Semantic Web data is often explained as "schema" or "self-describing", terms that indicate a separate description of the type or structure of data. Typically, when we store or program with a piece of data, we first describe the structure(type, schema) of that data and then create instances of that type schema . In Semantic Web data, we directly describe the data using special syntax. We represent recordlike or tuplelike structures: {name: "JHKim", E-mail: "ziro@yahoo.com", Cell No: "2907211"}. This is a set of pairs such as name: "JHKim" consisting of a label and a value. The values may themselves be other structures as in {name: {first: "Kim"}, {last: "JH"}, E-mail: "ziro@yahoo.com", Cell No: "2907114"}. Also, we represent structural categories in e-commerce and m-commerce.: {ShoppingMall Name: "Nate-Shop", Great Classification: "Computer", Bisection Kind: "Notebook", Subdivision Kind: "Samsung"}. In this way, the syntax makes it easy to describe sets of tuples in the Semantic Web. XML queries can be modeled as query pattern trees by XPath. In addition to element tag names, a query pattern tree may also consist of wildcards "*" and relative paths "//" by an Equivalence Classes Tree(ECTree). The wildcard "*" indicates the ANY label, while the relative paths "//" indicates zero or more labels. We assume the query pattern tree doesn't contain sibling repetitions, that is, the siblings in a query pattern tree have distinct labels. Formally we define this as follows, XML Query Pattern Tree: A query pattern tree is a rooted tree, XQPT=<V,E> is denoted by root(XQPT). For each edge e=(V1, V2), node V1 is the parent of node V2. Each vertex V has a label, denoted by v.label, whose value is in {"//", "*"} U tagSet, where the tagSet is the set of all element and attribute names in the schema.

3.3.1 User XQuery

XQuery is a powerful way to search XML documents for specific information. It is based on XPath and has the Schema statements. Also, XPath supports complex queries and complex result constructions using a nested clause. We now introduce a new set of syntax rules called the XML Query Pattern Tree, which is a generalization of XTree based on XPath. It has a tree structure like the structure of XML Schema and RDF. In the querying part of an XQuery(User Query), this is based on user profile information for personalization and Product Data(Transaction Data, Action Data) related to the ontology in the XML Database. We define the XML Query Pattern Tree and Mining XQuery Stream.[14]

3.4 User Module for Personalization Service

In this step, PU Module classify by user category schema of the SWRS. This process is mainly based on the concept of user inclination and user profile which will be described in more details in the next step. Also, This Module classify user category in existing recommender system as follows: i) Nominative Data(ND), ii) Anonymous Demographics(AD), iii) Application Data(AD). First, generate structures using XML in order to express correlations between XSchema and RDF in the Semantic Web. Second, by using the Schema, we can offer descriptions and constraints of documents using XML syntax, support for data types, description of content models and the reuse of elements via inheritance, extensibility, ability to write dynamic schemas, and self documenting capabilities when a stylesheet is applied. Third, using a XML document and XSchema, we are intended for situations in which this information needs to be processed by applications. In this way, personalization is achieved using XML Schema and RDF for the Semantic Web. This method is capable of operating not only on multiple documents, but also on document fragments.

3.5 Web Based Information Domain Module

In this step, the information domain schema of the SWRS is created. It's concern that main process during this step is to capture the SWRS domain semantics with very little attention for the users, tasks or needs. The object and relationships content the basis of the SWRS is used by information conceptual schema. As creating by information domain, PIC Module provide web application domain, if the application is a web interface to an existing database application.

3.6 Navigation Structure Module

In this step, the navigation structure module is created by navigation conceptual schema as using of the navigation documents over the information domain, that take into account the profiles of the intended users. The navigation document is composed Resource Description Framework and XML. This document represents the best navigation method for users having certain profiles to achieve needs.

4 Implementation and Evaluation

4.1 User Module for Personalization Service

User Module aims at capturing a series of relevant information which characterize a user, such as his experience of the application domain, background, preferences and history. According to the user Module for Personalization Service Description will have an important role to play in this paper, so we have designed a domain specific by a user xquery and schema. We use the XML and RDF class as a common superclass for concept Age, Sex, Job and Hobby, so they can be expressed as:

$$\textit{Personalization Service} \subseteq T$$
$$\textit{Age} \subseteq \textit{Personalization Service}$$
$$\textit{Sex} \subseteq \textit{Personalization Service}$$
$$\textit{Job} \subseteq \textit{Personalization Service}$$
$$\textit{Hobby} \subseteq \textit{Personalization Service}$$

We include the each of the four sets as follows: Age={10, 20, 30, …, 60}, Sex={Male, Female}, Job={Student, Researcher, Programmer,…., Professor}, and Hobby={Tennis, Game, Swimming,…}. By doing so, we can be expressed Resource Description Framework as:

Personalization Service 1: Class User Module for PS with Age

```
Typeof: Typeof(rdf:SchemaLocation="Age.xsd")
          document("rdf_PSAge.xml")
               //rdf:Description[@rdf:about="rdf:SchemaLocation"]
             /rdf:type/@rdf:resource
SuperClassof: SuperClassof(rdf:SchemaLocation)
             document("classHierrarchy.xml")
Domain:Domain(URI="Age.xsd")
                document("propertyHierarchy.xml")
```

Personalization Service 2: Class User Module with Sex

```
Typeof: Typeof(rdf:SchemaLocatioin="Sex")
          document("rdf_sex.xml")
               //rdf:Description[@rdf:about="rdf:SchemaLocation"
              ]/rdf:type/@rdf:resource
Domain:Domain(URI="sex.xsd")
          document("propertyHierarchysex.xml")//
InstanceOf:document("rdf_Category.xml")
          //rdf:Description[@resource="(SchemaLocation="sex.xsd")
           "]...............................................
```

In order to quantify the user parameters values during the runtime, the method uses fuzzy set theory and fuzzy logic. This Fig.2 shows an example with the Age parameter by using RDF. Thus, with respect to fuzzy set theory, the proposed method provides a user module approach which is true reality as it's allows a recommender system to classify the users in fuzzy categories.

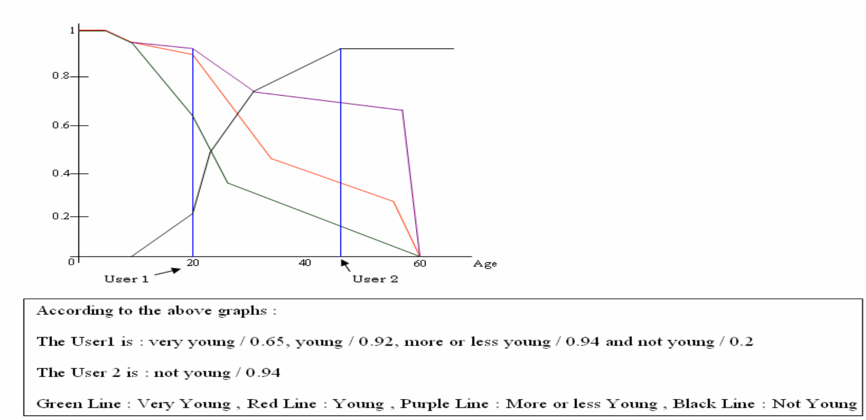

Fig. 2. Age parameter Graph using Fuzzy Theory

4.2 Navigation Structure by User Query Module

The aim of the navigation structure module is to operationalize the user query in terms. Fig.3 shows an overview together with the relationship. As shown in Fig.3, Navigation Structure Module is a set of defined navigation methods by user query. User query is a powerful way to search XML documents for specific information. It is based on XPath and has the Schema statements. Also, XPath supports complex queries and complex result constructions using a nested clause.

Fig. 3. Navigation Structure for Recommender System Model

We now introduce a new set of syntax rules called the XML Query Pattern Tree, which is a generalization of XTree based on XPath. It has a tree structure like the structure of XML Schema and RDF. In the querying part of an XQuery (User Query), this is based on user profile information for personalization and Product Data(Transaction Data, Action Data) related to the ontology in the XML Database. We define the XML Query Pattern Tree and Mining XQuery Stream. Table1 gives XPath expressions, according to the XML document in recommender system based on XDB within ontology.

Table 1. XQuery by Xpath Expression

XQuery by Xpath Expression	Description
/Cat/SMName/@GC	Get Attribute "GC" of each SM Name
/CG/SMName/BK	Get Element "BK" of each SM Name
//BK	Get all Element named "BK" regardless of their absolute path
/CG/SMName/*	Get all subelement of each SM Name
/CG/SMName/@*	Get all attributes of each SM Name
Cat:Category, SM:Shopping Mall, GC:Great Category, BK:Bisection Kinds	

4.3 The XQuery Execution

In this Fig.4, In order to demonstrate the XQuery management of the cokas provider implementation, a cokas local network was set up with provider peer configured with the RDF knowledge base depicted in Fig.4. As before, "co:" is an abbreviation for the cokas based URI localhost://cokas.skku.edu/rdf/. The XQuery states, we find all resources that have a type and a domain.

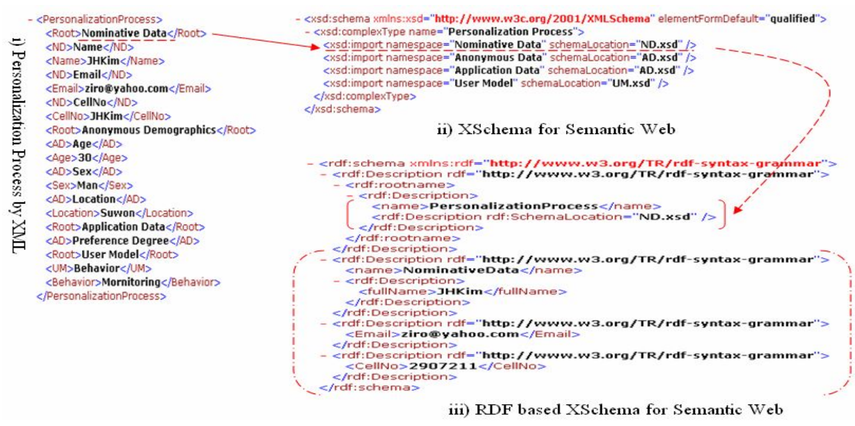

Fig. 4. XQuery Execution Procedure

4.4 Evaluation

Our experiments have intended to show the following. First, is to show the possibility of using XML syntax and dynamic schemas. Second, is to show the possibility of personalized recommendations for the world of commerce and ubiquitous environments. We proposed each of the modules existing on the PU Module and used XML Schema and RDF as a foundation. This paper provides personalization evaluation policy in ubiquitous frameworks, having advantages as follows. i) User Adaptation. This is an integrated end-user support method for system building, composing, and user feedback. This involves a process of gathering user-information during interaction with the user, which is then used to deliver appropriate content and services, tailor-made to the user's needs. ii) Context Awareness in commerce world. Context awareness drives adaptability of pervasive computing systems in the world of commerce. Discovering, extracting, interpreting and validating context will make a significant contribution to increasing efficiency, flexibility and feasibility of pervasive computing systems. iii) Efficiency of Recommendation. We can improve personalization recommendation in terms of efficiency. That is, the PU Module is a personalized document category for user queries used in the design of a web based user experience history.

5 Conclusion

In this paper, we presented a semantic web recommender system for personalization service using a user xquery method. The user XQuery provides an important recommendation service that assists e-commerce and m-commerce by creating personalized document retrieval and recommender systems. In order to provide these services effectively, the user XQuery should be a "user orientated query pattern". Also, as described in this paper, this is achieved through the utilization of a XML based e-commerce and m-commerce using the Semantic Web, including XML Query patterns, mining XQuery streams, personalization, user profiling and other XML based technologies.

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Multi-unit Combinatorial Reverse Auctions with Transformability Relationships Among Goods

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Abstract. In this paper we extend the notion of multi-unit combinatorial reverse auction by adding a new dimension to the goods at auction. In such a new type of combinatorial auction a buyer can express transformability relationships among goods: some goods can be transformed into others at a transformation cost. Transformability relationships allow a buyer to introduce his information as to whether it is more convenient to buy some goods or others. We introduce such information in the winner determination problem (WDP) so that not only does the auction help allocate the optimal set of offers—taking into account transformability relationships—, but also assesses the transformability relationships that apply. In this way, the buyer finds out what goods to buy, to whom, and what transformations to apply to the acquired goods in order to obtain the required ones.

1 Introduction

Since many reverse (or direct) auctions involve the buying (or selling) of a variety of different assets, combinatorial auctions [3, 7] (CA) have recently deserved much attention in the literature. In particular, a significant amount of work has been devoted to the problem of selecting the winning set of bids [12, 2]. Nonetheless, to the best of our knowledge, while the literature has considered the possibility to express relationships among goods on the bidder side—such as complementarity and transformability (e.g. [4],[13])—, the impact of the eventual relationships among the different assets to sell/buy on the bid-taker side has not been conveniently addressed so far.

Consider that a company devoted to the assembly and repairing of personal computers (PCs) requires to assembly new PCs in order to fulfil his demand. Figure 1 graphically represents the way a PC is assembled. Our graphical description largely borrows from the representation of Place/Transition Nets (PTN) [6], a particular type of Petri Net. Each circle (corresponding to a PTN *place*) represents a good. Horizontal bars connecting goods represent assembly/disassembly operations, likewise *transitions* in a PTN. Assembly and disassembly operations are labelled with an indexed *t*, and shall be referred to as *transformability relationships*. In particular t_1 and t_2 represent disassembly operations whereas t_3 and t_4 stand for assembly operations. An arc connecting

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a good to a transformation indicates that the good is an *input* to the transformation, whereas an arc connecting a transformation to a good indicates that the good is an *output* from the transformation. In our example, a motherboard is an *input good* to t_2, whereas CPU, RAM, USB and empty motherboard are *output goods* of t_2. Thus, t_2 represents the way a motherboard is taken into pieces (disassembled). The labels on the arcs connecting *input goods* to transitions, and the labels on the arcs connecting *output goods* to transitions indicate the units required of each *input good* to perform a transformation and the units generated per *output good* respectively. In figure 1, the labels on the arcs connected to t_3 indicate that 1 motherboard is assembled from 1 CPU, 4 RAM units, 3 USBs and 1 empty motherboard at a cost of 8 EUR. Each transformation has an associated cost every time it is carried out. In our example, assembling a motherboard via t_3 costs 8 EUR, while taking a motherboard into pieces via t_2 costs 7 EUR.

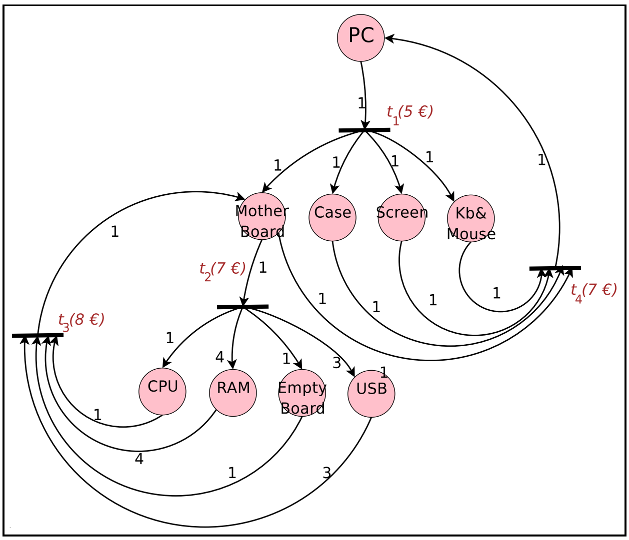

Fig. 1. Graphical representation of an RFQ with t-relationships

Say that the company's warehouse contains most of the components composing each PC. However, there are no components to assemble motherboards. Therefore, the company would have to start a sourcing [5] process to acquire such components. For this purpose, it may opt for running a combinatorial reverse auction [13] with qualified providers. But before that, a professional buyer may realise that he faces a decision problem: shall he buy the required components to assemble them in house into motherboards, or buy already-assembled motherboards, or opt for a *mixed purchase* and buy some components to assemble them and some already-assembled motherboards? This concern is reasonable since the cost of components plus transformation (assembly) costs may eventually be higher than the cost of already-assembled motherboards. Hence, the buyer requires a combinatorial reverse auction mechanism that provides: (a) a language to express required goods along with the relationships that hold among them; and (b) a winner determination solver that not only assesses what goods to buy and to whom, but also the transformations to apply to such goods in order to obtain the initially required ones. In this paper we try to provide solutions to both issues.

Firstly, since commercial e-sourcing tools [11] only allow buyers to express fixed number of units per required good as part of the so-called *Request for Quotation* (RFQ), we have extended this notion to allow for the introduction of transformation relationships (*t-relationships* henceforth) among goods. Thus, we introduce a formal definition of a *Transformability Network Structure* (TNS) that largely borrows from Place/Transition Nets [6], where transitions stand for t-relationships and places stand for goods.

Secondly, we extend the formalisation of multi-unit combinatorial reverse auction (MUCRA), departing from the model in [12], to introduce transformability by applying the expressiveness power of multi-set theory. Additionally, we provide a mapping of our formal model to integer programming that takes into account t-relationships to assess the winning set of bids along with the transformations to apply in order to obtain the buyer's initial requirements.

Finally, we empirically analyse how the introduction of t-relationships affects scalability with respect to a classical multi-unit combinatorial reverse auction.

The paper is organised as follows. In section 2 we provide some background knowledge on place/transition nets and multi-sets. In section 3 we present a formal model of multi-unit combinatorial reverse auctions with t-relationships among goods, along with its winner determination problem and its mapping to integer programming. Section 4 is devoted to illustrate some preliminary, experimental results. Finally, section 5 draws some conclusions and outlines directions for future research.

2 Background

In this section we introduce some background knowledge on multi-sets and place/transition nets.

A *multi-set* is an extension to the notion of set, considering the possibility of *multiple appearances* of the same element. A *multi-set* $\mathcal{M}_X$ over a set X is a function $\mathcal{M}_X : X \rightarrow \mathbb{N}$ mapping X to the cardinal numbers. For any $x \in X$, $\mathcal{M}_X(x) \in \mathbb{N}$ is called the *multiplicity* of x. An element $x \in X$ *belongs* to the multi-set $\mathcal{M}_X$ if $\mathcal{M}_X(x) \neq 0$ and we write $x \in \mathcal{M}_X$. We denote the set of multi-sets over X by X_{MS}. Given the multi-sets $\mathcal{M}_S, \mathcal{M'}_S \in S_{MS}$, their union is defined as: $\mathcal{M}_S \cup \mathcal{M'}_S(x) = \mathcal{M}_S(x) + \mathcal{M'}_S(x)$.

Following [6], a *Place/Transition Net Structure* (PTNS) is a tuple $N = (G, T, A, E)$ such that: (1) G is a set of *places*; (2) T is a finite set of *transitions* such that $P \cap T = \emptyset$; (3) $A \subseteq (G \times T) \cup (T \times G)$ is a set of *arcs*; (4) $E : A \rightarrow \mathbb{N}^+$ is an *arc expression* function. A *marking* of a PTNS is a multi-set over G. A PTNS with a given initial marking $\mathcal{M}_0 \in G_{MS}$ is denoted by $PTN = (N, \mathcal{M}_0)$ and it is called a *Place/Transition Net* (PTN). The graphical representation of a PTNS is composed of the following graphical elements: places are represented as circles, transitions are represented as bars, arcs connect places to transitions or transitions to places, and E labels arcs with values (see figure 1).

A *step* is a non-empty and finite multi-set over T. A step $\mathcal{S} \in T_{MS}$ is *enabled* in a marking $\mathcal{M} \in G_{MS}$ if the following property is satisfied: $\forall g \in G \sum_{t \in \mathcal{S}} E(g,t)\mathcal{S}(t) \leq \mathcal{M}(g)$.

Let step $\mathcal{S}$ be enabled in a marking $\mathcal{M}_1$. Then, $\mathcal{S}$ may *occur*, changing the $\mathcal{M}_1$ marking to another $\mathcal{M}_2 \in G_{MS}$ marking. Setting $Z(g,t) = E(t,g) - E(g,t)$ $\mathcal{M}_2$ is expressed as: $\forall g \in G$ $\mathcal{M}_2(g) = \mathcal{M}_1(g) + \sum_{t \in \mathcal{S}} Z(g,t)\mathcal{S}(t)$. Moreover, we say that marking $\mathcal{M}_2$ is *directly reachable* from marking $\mathcal{M}_1$ by the occurrence of step $\mathcal{S}$, and we denote it by $\mathcal{M}_1[\mathcal{S} > \mathcal{M}_2$.

A *finite occurrence sequence* is a finite sequence of steps and markings:$\mathcal{M}_1[\mathcal{S}_1 > \mathcal{M}_2 \ldots \mathcal{M}_n[\mathcal{S}_n > \mathcal{M}_{n+1}$ such that $n \in \mathbb{N}$ and $\mathcal{M}_i[\mathcal{S}_i > \mathcal{M}_{i+1}$ $\forall i \in \{1, \ldots, n\}$. $\mathcal{M}_1$ is called the *start marking*, while $\mathcal{M}_{n+1}$ is called the *end marking*. The *firing count multi-set* $\mathcal{K} \in T_{MS}$ associated to a finite occurrence sequence is the union of all its steps: $\mathcal{K} = \bigcup_{i \in \{1,2,\ldots,n\}} \mathcal{S}_i$.

A marking $\mathcal{M}''$ is *reachable* from a marking $\mathcal{M}'$ iff there exists a finite occurrence sequence having $\mathcal{M}'$ as start marking and $\mathcal{M}''$ as end marking. We denote it as $\mathcal{M}'[\mathcal{S}_1 \ \ldots \mathcal{S}_n > \mathcal{M}''$, where $n \in \mathbb{N}$. Furthermore the start and end markings are related by the following equation:

$$\forall g \in G \qquad \mathcal{M}''(g) = \mathcal{M}'(g) + \sum_{t \in \mathcal{K}} Z(g,t)\mathcal{K}(t). \tag{1}$$

The set of all possible markings reachable from a marking $\mathcal{M}'$ is called its *reachability set*, and is denoted as $R(N, \mathcal{M}')$.

In [10], Murata shows that in an *acyclic* Petri Net a marking $\mathcal{M}''$ is *reachable* from a marking $\mathcal{M}'$ iff there exists a multi-set $\mathcal{K} \in T_{MS}$ such that expression 1 holds (which is equivalent to say that the state equation associated to a PTN admits an integer solution). As a consequence, when a Petri Net is acyclic, the reachability set $R(N, \mathcal{M}')$ is represented by

$$R(N, \mathcal{M}') = \{\mathcal{M}'' \mid \exists \mathcal{K} \in T_{MS} : \forall g \in G\, \mathcal{M}''(g) = \mathcal{M}'(g) + \sum_{t \in \mathcal{K}} Z(g,t)\mathcal{K}(t)\}. \tag{2}$$

3 MUCRA with T-Relationships

3.1 Transformability Network Structures

A Transformability Network Structure describes the different ways in which our business is allowed to transform goods and at which cost. More formally, a *transformability network structure* (TNS) is a pair $S = (N, C_T)$, where $N = (G, T, A, E)$ is a Place-Transition Net Structure and $C_T : T \to \mathbb{R}^+$ is a cost function. The cost function associates a *transformation cost* to each *t-relationship*. In this context we associate: (1) the *places* in G to a set of goods to negotiate upon; (2) the *transitions* in T to a set of *t-relationships* among goods; (3) the *directed arcs* in A along with their weights E to the specification of the number of units of each good that are either consumed or produced by a transformation.

The values of C and the values of E label respectively transitions (between parenthesis) and arcs in figure 1.

In the following example, we formally specify the Transformability Network Structure $S = (N, C_T)$, graphically represented in figure 1: (1) $G = \{$PC, Motherboard, Case, Screen, Kb&Mouse, CPU, RAM, Empty Board, USB}; (2) $T = \{t_1, t_2, t_3, t_4\}$; (3) $A = \{(PC, t_1), (t_1, motherboard), (t_1, case), (t_1, screen), (t_1, kb\&mouse), (motherboard, t_2), (t_2, CPU), (t_2, RAM), (t_2, EmptyBoard), (t_2, USB), \ldots\}$; (4) $E(PC, t_1) = 1, E(t_1, motherboard) = 1, E(t_1, case) = 1, E(t_1, screen) = 1, E(t_1, kb\&mouse) = 1, E(motherboard, t_2) = 1, E(t_2, CPU) = 1, E(t_2, RAM) = 4, E(t_2, EmptyBoard) = 1, E(t_2, USB) = 3, \ldots$; (5) $C_T(t_1) = 5$ EUR, $C_T(t_2) = 7$ EUR, $C_T(t_3) = 8$ EUR, $C_T(t_4) = 7$ EUR.

Given a Place/Transition net $PTN = (N, \mathcal{M}_0)$, if we consider $\mathcal{M}_0$ as a good configuration, PTN defines the space of good configurations *reachable* by applying tranformations to $\mathcal{M}_0$. The application of tranformations is obtained by firing transitions on PTN. Hereafter, we consider the concepts of *transformation step*, *enabling of a transformation step*, *occurrence of a transformation step* and *transformation sequence* as the counterparts to, respectively, *step*, *enabling of a step*, *occurrence of a step*, and *finite occurrence sequence* on a PTN.

We also need to define the concept of transformation cost, taking into account the cost of transforming good configuration $\mathcal{M}_0$ into another good configuration $\mathcal{M}_1 \in R(N, \mathcal{M}_0)$ by means of some transformation sequence $J = (\mathcal{S}_1, \ldots, \mathcal{S}_n)$. The $\mathcal{K}$ firing count multi-set associated to J accounts for the number of times a transition in the sequence is fired. Thus, the cost of transforming good configuration $\mathcal{M}_0$ into good configuration $\mathcal{M}_1$ amounts to adding the transformation cost of each transition in the firing count multi-set $\mathcal{K}$ associated to J. We assess the transformation cost associated to J as $C_{TS}(J) = \sum_{\mathcal{S} \in J} \sum_{t \in \mathcal{S}} C_T(t)\mathcal{S}(t) = \sum_{t \in \mathcal{K}} C_T(t)\mathcal{K}(t)$. Notice that the transformation cost of a transformation sequence only depends on its firing count multi-set.

3.2 Winner Determination Problem (WDP) for MUCRA with T-Relationships

In a classic MUCRA scenario, an RFQ can be expressed as a multi-set $\mathcal{U} \in G_{MS}$ whose multiplicity indicates the number of units required per good. In the example of figure 1, if $\mathcal{U}(motherboard) = 1, \mathcal{U}(CPU) = 1, \mathcal{U}(RAM) = 4, \mathcal{U}(EmptyBoard) = 1, \mathcal{U}(USB) = 3$, $\mathcal{U}$ would be representing a buyer's need for 1 motherboard (M), 1 CPU (C), 1 empty board (E), 4 RAM units (R), and 3 USB (U) connectors. Nonetheless, since t-relationships hold among goods, the buyer may have different alternatives depending on the bids he receives. If we represent each bid as a multi-set $\mathcal{B} \in G_{MS}$, whose multiplicity indicates the number of units offered per good, the buyer might, for example, have the following alternatives:

1. $\mathcal{M}_0 = \{M, C, R, R, R, R, E, U, U, U\}$. Buy all items as requested.
2. $\mathcal{M}_1 = \{M, M\}$. Buy 2 motherboards, and then disassemble 1 motherboard into 1 CPU, 4 RAM units, 1 Empty Board, and 3 USB connectors at cost $C_T(t_2) =$ 7EUR. The overall cost of the purchase results from the cost of the acquired units plus the additional transformation cost.

Notice that both alternatives allow the buyer to obtain his initial requirement, though each one at a different cost. The goal of the WDP is to assess what alternative to select.

We begin by defining the set of possible auction outcomes. Given a set of bids B, a possible auction outcome is a pair (W, J), where $W \subseteq B$, and $J = (\mathcal{S}_1, \ldots, \mathcal{S}_n)$ is a transformation sequence, such that the application of J to $PTN = (N, \cup_{\mathcal{B} \in W} \mathcal{B})$

allows a buyer to obtain a good configuration that fulfils his requirements in $\mathcal{U}$. More formally, the set of possible auction outcomes is defined as[1]:

$$\Omega = \{(W, J), W \subseteq B \mid \exists \mathcal{X} \in G_{MS} \ (\bigcup_{\mathcal{B} \in W} \mathcal{B})[J > \mathcal{X}, \mathcal{X} \supseteq \mathcal{U}\}. \tag{3}$$

To each auction outcome (W, J) we associate an *auction outcome cost* as follows:

$$C_O(W, J) = \sum_{\mathcal{B} \in W} C_B(\mathcal{B}) + C_{TS}(J) \tag{4}$$

where $C_B : B \rightarrow \mathbb{R}^+$ stands for the bid cost function.

Definition 1 (Winner Determination Problem). *Given a set of bids B, an RFQ $\mathcal{U} \in G_{MS}$, and a transformability network structure $S = (N, C_T)$, the winner determination problem for a MUCRA with t-relationships amounts to assessing the auction outcome $(W^{opt}, J^{opt}) \in \Omega$ that minimises the auction outcome cost function C_O. Formally,*

$$(W^{opt}, J^{opt}) = \arg \min_{(W,J) \in \Omega} C_O(W, J). \tag{5}$$

3.3 Mapping to Integer Programming

In section 2, we defined the reachibility set according to equation 2 for the case of acyclic Petri nets. Thus, if we restrict to the case of acyclic TNS, a finite occurrence sequence J is completely specified by its firing count vector $\mathcal{K}$. Then, we can rewrite expressions 3 and 4 respectively as follows:

$$\Omega = \{(W, \mathcal{K}), W \subseteq B, \mathcal{K} \in T_{MS} \mid \exists \mathcal{X} \in G_{MS} \ (\bigcup_{\mathcal{B} \in W} \mathcal{B})[\mathcal{K} > \mathcal{X}, \mathcal{X} \supseteq \mathcal{U}\}. \tag{6}$$

$$C_O(W, \mathcal{K}) = \sum_{\mathcal{B} \in W} C_B(\mathcal{B}) + C_{TS}(\mathcal{K}) \tag{7}$$

where $C_{TS}(\mathcal{K}) = \sum_{t \in \mathcal{K}} C_T(t)\mathcal{K}(t)$. Hence, the WDP when considering acyclic TNSs can be restated, from equation 5, to assess:

$$(W^{opt}, \mathcal{K}^{opt}) = \arg \min_{(W,\mathcal{K}) \in \Omega} C_O(W, \mathcal{K}) \tag{8}$$

We can model the problem of assessing $(W^{opt}, \mathcal{K}^{opt})$ as an Integer Programming problem. For this purpose, we need to associate integer variables to the elements in: (1) a generic subset of bids ($W \subseteq B$); and (2) a generic firing count multi-set ($\mathcal{K}$).

In order to represent W we assign a binary decision variable $x_{\mathcal{B}}$ to each bid $\mathcal{B} \in B$, standing for whether $\mathcal{B}$ is selected ($x_{\mathcal{B}} = 1$) or not ($x_{\mathcal{B}} = 0$) in W. A multi-set is uniquely determined by its mapping function $\mathcal{K} : T \rightarrow \mathbb{N}$. Hence, we represent a multi-set $\mathcal{K} \in T_{MS}$ by considering an integer decision variable q_t for each $t \in T$. Each q_t

[1] Assuming free disposal.

represents the multiplicity of element t in the $\mathcal{K}$ multi-set. Thus, the translation into integer programming of expression (8) becomes:

$$min[\sum_{\mathcal{B}\in B} x_B p(\mathcal{B}) + \sum_{t\in T} q_t c(t)] \tag{9}$$

subject to $x_{\mathcal{B}} \in \{0,1\}$. Notice that leaving the $q_t (t \in T)$ decision variables unbounded is utterly unrealistic because it is equivalent to say that the buyer has got the capability of applying as many transformations as required to fulfil $\mathcal{U}$. In practice, a buyer's production capacities are constrained, and therefore it is realistic to assume that the number of in-house transformations that he can apply are constrained. Hence, we add the following constraints to equation 9: $\forall t \in T\ q_t \in \{0, 1, \ldots, max_t\}$, where $max_t \in \mathbb{N}$.

Besides, we capture the side constraints enforcing that the selected bids, along with the transformations applied to them, fulfil $\mathcal{U}$ by translating expression 6 into linear programming. We consider a set of PTNs such that $PTN = (N, \mathcal{L})$, where $\mathcal{L} = \cup_{\mathcal{B}\in W} \mathcal{B}$. Moreover, we consider all the finite occurrence sequences of $PTN = (N, \mathcal{L})$ that transform $\mathcal{L}$ into a configuration that at least fulfils $\mathcal{U}$. Under the hypothesis of N being acyclic we can express the reachability set of $\mathcal{L}$ as follows:

$$\forall g \in G\ \ \mathcal{M}(g) = \mathcal{L}(g) + \sum_{t\in\mathcal{K}} Z(g,t)\mathcal{K}(t). \tag{10}$$

Next, we select the elements in the reachability set $[\mathcal{L} >$ that at least fulfil $\mathcal{U}$:

$$\forall g \in G\ \ \mathcal{L}(g) + \sum_{t\in\mathcal{K}} Z(g,t)\mathcal{K}(t) \geq \mathcal{U}(g) \tag{11}$$

Hence, substituting marking $\mathcal{L}$ by $\sum_{\mathcal{B}\in B} x_{\mathcal{B}}\mathcal{B}(g)$ we finally obtain the following side constraints:

$$\forall g \in G\ \ \sum_{\mathcal{B}\in B} x_{\mathcal{B}}\mathcal{B}(g) + \sum_{t\in T} Z(g,t) q_t \geq \mathcal{U}(g).$$

4 Experiments

The main purpose of our preliminary experiments is to empirically evaluate the benefits and drawbacks of introducing transformability relationships. With this aim we compared the scalability of the MUCRATR solver with respect to a traditional MUCRA solver on large instances.

The solvers for the MUCRATR WDP and MUCRA WDP have been developed with the aid of ILOG's[1] CPLEX 9.0. The benchmark has been generated with the aid of MATLAB 7.0 [9]. The solver for MUCRA's WDP uses a state-of-the-art Integer Programming formulation, that exploits the analogy of a multi-unit combinatorial auction WDP with a well known optimisation problem: the Multi Dimensional Knapsack Problem (MDKP). For a complete explanation refer to [3].

A problem instance for a MUCRA is composed of a a multi-unit RFQ, a set of multi-unit multi-item bids, whereas a MUCRATR additionally needs a TNS. Thus, firstly we built some problem instances for the MUCRATR and we solved them with the

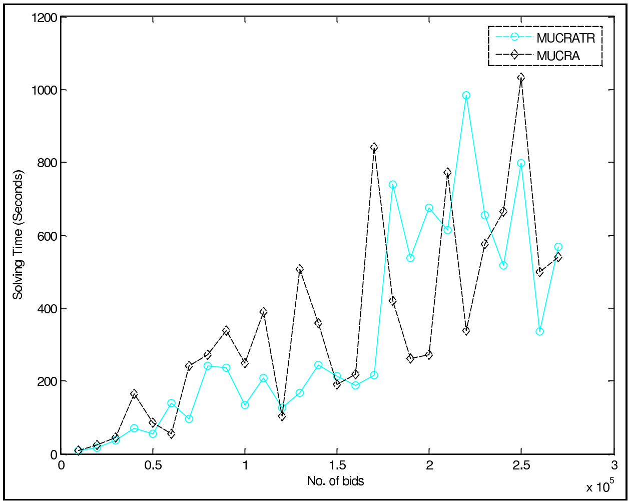

Fig. 2. Comparation of MUCRATR and MUCRA solvers for a normal distribution

MUCRATR solver. Next, we solved the very same instances with the MUCRA solver considering only bids and RFQ.

In [8], Leyton-Brown specifies an algorithm to create MUCA instances whose purpose is to test WDP algorithms. We have adapted his algorithm to generate MUCRA instances. It is well known from [13] that a MUCRA is the dual case to a MUCA.

The existence of a TNS has led us to change some aspects of Leyton-Brown's algorithm. Firstly, instead of assigning an independent average price to each good, we have to take into account the t-relationships connecting goods. We assign goods' prices so that the sum of input good costs plus the transformation cost equals the sum of the output good costs (*adapted price distribution*). Consider, for instance, the example depicted in figure 1: the default price distribution can generate problem instances in which a PC price is lower than its USB's prices, whereas our pricing policy creates a sort of equilibrium among prices. Next, we consider more realistic to weight the average price of each bid via a normal probability distribution instead of a uniform one (concretely we used a normal distribution with mean 1 and variance 0.1).

In the following we describe the parameter settings of our experiments. We performed a single experiment in which the only parameter varying was the number of bids generated, ranging from 1000 to 270000.

The number of negotiated items was set to 20, the maximum number of units of a single item that a buyer can ask for was set to 15. The maximum number of units a bidder can offer for a single item was set to 20. The decaying probabilities employed to generate the number of goods per bid and the number of units offered per bit per item were both set to 0.8. The number of t-relationships imposed among the goods was set to 8.

Figure 2 depicts the results of this preliminary scalability test. Notice that we obtained very similar results to a state-of-the-art solver that does not take into account t-relationships. Thus, we can conclude that the introduction of t-relationships does not suppose a significant time overload with respect to a traditional combinatorial auction.

5 Conclusions and Future Work

In this paper we have presented a formalisation and an integer programming solution to the winner determination problem of a new type of multi-unit combinatorial reverse auction that allows for expressing t-relationships on the buyer side. Several advantages derive from such a new type of auction. On the one hand, it allows a buyer to incorporate his uncertainty as to whether it is better to buy a required bundle of goods, or alternatively buy some goods to transform them into the former ones, or even opt for a mixed purchase and buy some goods as required and some others to be transformed. This is achieved by introducing t-relationships among goods into the winner determination problem. Therefore, not only does the winner determination solver assess what goods to buy and to whom, but also the transformations to apply to such goods in order to obtain the initially required ones. To the best of our knowledge, this is the first type of auction aimed at also handling buyers' uncertainty. As a side effect, the introduction of t-relationships is expected to increase competitiveness among bidders, and thus obtain better deals since bidders that otherwise would not be competing are put together to compete. Finally, our integer programming solution can be readily implemented with the aid of linear programming libraries.

We also performed some preliminary experiments comparing our solver for the WDP for MUCRATR with a state-of-the-art MUCRA solver. We compared the differences in terms of solving time and auction outcome cost. The results showed two main issues: (1) there is no significant, computational overload when solving a MUCRATR WDP with respect to solving a MUCRA WDP; and (2) there are always savings in terms of costs when running a MUCRATR, being outstanding for small-medium auction scenarios (less than 100 bids). Nonetheless, notice that the preliminary experiments we have run deserve further elaboration in order to thoroughly validate our early hypothesis.

As future work, it is our aim to further elaborate along several directions. Firstly, we aim at theoretically analysing the benefits in terms of savings that our mechanism provides with respect to multi-unit combinatorial reverse auctions. Secondly, we believe that it is important to research on the theoretical properties of our mechanism from a mechanism design point of view. And finally, the complexity of bidding in MUCRATRs along with decision support mechanisms for bidders shall be studied.

Acknowledgements

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Winner Determination in Discount Auctions

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Abstract. Discount auctions is a market mechanism for buying heterogeneous items in a single auction. The bidders are suppliers and a bid consists of individual cost for each of the items and a non-decreasing discount function defined over the number of items. The winner determination problem faced by the buyer is to determine the winning suppliers and their corresponding winning items. We show that this problem is $\mathcal{NP}$-hard upon reduction from the set covering problem. The problem has an embedded network structure, which is exploited to develop heuristics and an exact branch and bound algorithm. Computational experiments were performed to evaluate the proposed algorithms.

1 Introduction

With the advent of the electronic commerce, many innovative auctions mechanisms have been designed to cater the needs of various trading environments. The design of Internet auctions [1] is inherently a multidisciplinary activity requiring expertise from disciplines like economics, game theory, operations research, and computer science. One of the key issues in auction design is the *winner determination problem* (WDP) or the *bid evaluation problem*. The WDP is an optimization problem faced by the auctioneer to select a set of winning bidders and their corresponding winning quantity of items, such that the total profit (cost) is maximized (minimized). The WDP need to be solved once or several times in an auction depending on the auction dynamics. The solution time of the WDP play a significant role in the auction implementation.

Trading of multiple items in a single auction mechanism is one of the trading scenarios that has received much attention, partly due to the US Federal Communications Commission's auction for spectrum licenses. One of the famous auction formats for trading multiple items is *combinatorial auctions* (CA) [2]. In CA, a set of heterogeneous items are auctioned and bidders can bid for a combination (subset) of items (also called as a *package* or *bundle*) by quoting a single price. This is applicable in scenarios where the cost of a combination of items can be more or less than the sum of individual costs of items. By quoting a single price for a combination of items, a combinatorial bid can express complementarity or substitutability among the items in the package. The WDP in CA is to select

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a set of winning bidders and their winning package of items, which is $\mathcal{NP}$-hard [3]. The buying version of CA is a reverse auction in which the auctioneer is a buyer who wants to procure a set of items and the suppliers are bidders who bid on package of items. Similar to the selling version, algorithmic issues for buying version are well studied [4]. An alternate auction mechanism called as *discount auctions* (DA) for procurement of multiple goods was proposed in [5]. It is useful in common procurement scenarios where the seller has positive costs for each of the items and there are no synergies in selling a subset or package of items. However, to promote sales, the seller offers discount which is a function of number of the items bought. Thus, DA is applicable in scenarios where the supplier is concerned about the number of items sold rather than the combination of items sold (as in CA). A discount bid consists of two parts: individual costs for each of the items and a discount function over the number of items. The size of the bid is linear, whereas in CA, the size in worst case is exponential as the bidder has to submit different bids for different subsets. Despite its simplicity, we show in this paper that the WDP in DA is $\mathcal{NP}$-hard and propose heuristics and an exact branch and bound algorithm to solve the WDP.

The rest of the paper is organized as follows. In Section 2, we define the DA. The complexity of the WDP is discussed in Section 3. Heuristics are developed in Section 4 and an exact branch and bound algorithm in Section 5. We conclude the paper in Section 6.

2 Discount Auctions

The buyer is interested in procuring M different items. Each of the item is indivisible, *i.e.* it can be supplied by only one supplier. A item is denoted by index m and a supplier by index j. Each supplier can submit only one discount bid and hence the index j denotes both the supplier and his bid. The *discount bid* j consists of two parts: (1) cost Q_j^m for each item m and (2) discount θ_j^i for i $(= 1, \dots, M)$ items. The bid can be compactly expressed as an ordered pair of M-tuples: $((Q_j^1, \dots, Q_j^m, \dots, Q_j^M), (\theta_j^1, \dots, \theta_j^i, \dots, \theta_j^M))$. Note that m denotes a particular item and i denotes any i number of items. If the buyer procures items $2, 4$, and 7 from bid j, then the cost of procurement is $(1-\theta_j^3)(Q_j^2+Q_j^4+Q_j^7)$. This is different from *volume discount* auctions [6], which are used in procuring multiple units of the same item. All the Q_j^m are positive (possibly infinite for an unavailable item) and the θ_j^i are non-decreasing over i (the discount cannot decrease with the number of items bought). One of the main issues in auction design is bid preparation and communication. In CAs, the number of possible combinatorial bids from a supplier is 2^{M-1} (one bid for each subset). Thus both the bid preparation and communication (to the buyer) is costly. In DAs, only one discount bid is submitted from a supplier and its length is linear in the number of items $(2M)$.

3 Complexity of the Winner Determination Problem

In CA, the bidder provides a single price for a subset of items, and he can submit different bids for different subsets. A discount bid can be converted into combinatorial bids for various subsets by determining the cost for each of the subsets from the discount bid. Thus DA can be considered as a special case of CA where the cost of individual elements are known, but they vary depending on the number of items in the subset. Despite its simplicity when compared with CA, we show that the WDP of the DA is $\mathcal{NP}$-hard.

Theorem 1. *The WDP of the discount auctions is* $\mathcal{NP}$*-hard.*

Proof. We prove the hardness by showing that the decision version of the WDP is $\mathcal{NP}$-complete upon reduction from the minimum set cover. The decision versions of the WDP of DA and the minimum set cover are defined below respectively.

Definition 1 ([DAuc]).
INSTANCE: *Set of goods* $G = \{1, \dots, M\}$*, set of discount bids* $J = \{1, \dots, N\}$*, where a discount bid* $j \equiv ((Q_j^1, \dots, Q_j^M), (\theta_j^1, \dots, \theta_j^M))$ *with* $Q_j^m \geq 0 \ \forall m \in G$ *and* $0 \leq \theta_j^i \leq \theta_j^{i+1} \leq 1$*,* $1 \leq i < M$*, and a goal* $K \geq 0$*.*
QUESTION: *Does there exist a winning set* $J' \subseteq J$*, which defines a partition* $P = \{B_j : B_j \subseteq G, j \in J'\}$ *of* G*, such that the total cost of procurement* $\sum_{j \in J'} (1 - \theta_j^{|B_j|}) \sum_{m \in B_j} Q_j^m \leq K$*?*

Definition 2 ([SCov]).
INSTANCE: *Collection* C *of subsets of finite set* S*, positive weight* $w_R \ \forall R \in C$*, and a goal* $H \geq 0$*.*
QUESTION: *Does there exist cover* $C' \subseteq C$ *for* S *such that* $\sum_{R \in C'} w_R \leq H$*?*

The minimum set cover [SCov] is $\mathcal{NP}$-complete [7]. First we note that [DAuc] is in $\mathcal{NP}$: given a winning set $J' \subseteq J$, one can verify whether it defines a partition and the procurement cost is less than K, in polynomial time. Let an instance of [SCov] be given. We construct an instance of [DAuc] in the following way:

- $|M| = |S|$, $|J| = |C|$
- Create a bid j for each of the subset $R \in C$ as follows:

$$Q_j^m = \left\{ \begin{matrix} w_R \text{ if } m \in R \\ \infty \ \ \text{otherwise} \end{matrix} \right\}, \ \forall m$$

$$\theta_j^i = \frac{i-1}{i}, \ 1 \leq i \leq M$$

- $K = H$

The above reduction can be clearly done in polynomial time. We now show that the reduction is valid by showing that an instance of [SCov] is a *yes* iff if its reduction [DAuc] is a *yes* instance.

($\Leftarrow$) Let there exist a yes instance of [DAuc] with $J' \subseteq J$ defining a partition of G with procurement cost $\leq K$. A cover C' for [SCov] can be constructed as follows. For every $j \in J'$, include the corresponding subset R in C'. Note that $B_j \subseteq R$ as $m \notin R$ implies $Q_j^m = \infty$. The cost of procurement from winning bid j is given by

$$
\begin{aligned}
&(1-\theta_j^{|B_j|})\sum_{m\in B_j} Q_j^m \\
&= \left(\frac{1}{|B_j|}\right)|B_j|w_R \\
&= w_R
\end{aligned}
$$

Thus the cost of procurement from each bid is equal to the weight of the corresponding subset in C'. Since the winning bids partition G, the collection C' covers S with cost $\leq K = H$.

($\Rightarrow$) Let there exist a yes instance for [SCov] with cover C'. The solution to [DAuc] can be constructed as follows. For every subset $R \in C'$, include its corresponding bid j in J'. Since C' covers S, J' also covers G. If an item is supplied by more than one supplier then it can be removed from its respective suppliers except one. This will still satisfy the goal and is a partition of G.

Proposition 1. *Following special cases are solvable in polynomial time:*

1. *Same Cost:* $Q_j^m = Q_j^* \geq 0$, $\forall j, m$
2. *Same Discount:* $\theta_j^i = \theta_j^*$, $\forall j, i$
3. $|M| \leq 2$

Proof. (1) This special case is a multi-unit auction of a single good. The requirement of the buyer is M units of a single good and the suppliers submit a bid with unit cost Q_j^* and a discount function. The WDP can be formulated as the following integer programming problem:

$$
\text{(P)}: \qquad \min \sum_j \sum_i (1-\theta_j^i) i Q_j^i v_j^i \tag{1}
$$

subject to

$$
\sum_i v_j^i \leq 1, \ \forall j \tag{2}
$$

$$
\sum_j \sum_i i v_j^i = M \tag{3}
$$

$$
v_j^i \in \{0,1\}, \ \forall j, i \tag{4}
$$

The above formulation is a *multiple choice knapsack problem* [8] with (2) as the multiple choice constraints and (3) as the knapsack constraint. Though the generic multiple choice knapsack problem is $\mathcal{NP}$-hard, the above problem is solvable in linear time due to its cost structure. We show this using the duality theory. The linear programming dual of the above problem is:

$$
\text{(D)}: \qquad \max \pi M + \sum_i \beta_j \tag{5}
$$

subject to

$$
i\pi + \beta_j \leq (1-\theta_j^i) i Q_j^i, \ \forall j, i \tag{6}
$$

$$
\beta_j \geq 0, \ \forall j \tag{7}
$$

Let $j' = \arg\min_j\{(1-\theta_j^M)MQ_j\}$. Assign $\beta_{j'} = (1-\theta_{j'}^M)MQ_{j'}$ and $\beta_j = 0$, $\forall j \neq j'$ and $\pi = 0$. This is a feasible dual solution with objective value $\beta_{j'}$. A feasible solution to the primal problem can be constructed from this dual solution. Assign $V_{j'}^M = 1$, $V_{j'}^i = 0$, $\forall i \neq M$, and $V_j^i = 0$, $\forall j \neq j'$, $\forall i$. This is a feasible primal solution with the same objective value as of the dual and hence by duality theory it is an optimal solution to the linear relaxation. This optimal solution to the linear relaxation is integral and hence it is also optimal to the original integer programming problem. Thus the optimal solution to this WDP is $\min_j(1-\theta_j^M)MQ_j$, which can be solved in linear time.

(2) If the discounts are the same for each bidder, then it is equivalent to no discount, which can be solved in polynomial time by choosing the bidder with minimum cost for each item.

(3) If $M = 1$, then WDP can be solved in linear time by choosing the bid with minimum cost. For $M = 2$, there are only two possibilities: buy at most one item from each buyer and buy both the items from a single buyer. The first problem is an assignment problem and the second is choosing the minimum cost for both items from N bidders. The optimal solution is the one with the minimum cost among the above two solutions.

4 Heuristics to Find Feasible Solutions

The WDP has an embedded network structure [5] which can be exploited to obtain feasible solutions. Each bid is supply node with supply quantity in the interval $[0, M]$ and each item is a demand node with unit demand. The cost of flow c_j^m from bid j to item m depends on the total number of items supplied from j. This is different from the conventional nonlinear network flow problems where the cost of flow varies nonlinearly with the total flow on the link. Let p_j^{im} be the *effective price* of m, when j supplies i items. By assigning $c_j^m = p_j^{Mm}$, $\forall j, m$, we obtain a relaxation of the WDP. This relaxed problem is a *transportation problem* which can be solved in polynomial time. We develop heuristics in this section and an exact algorithm in the next section, exploiting this network structure.

Though the WDP is hard, one can easily obtain feasible solutions to the WDP. Moreover, every feasible solution is within a known approximation ratio, which depends on the instance of the problem. Let $\overline{Y}^m$ be the maximum effective cost of item m over all the bidders and similarly let $\underline{Y}^m$ be the minimum effective cost. Let $\epsilon = \max_m \frac{\overline{Y}^m}{\underline{Y}^m}$. If Z^* and Z' denote the objective value of the optimal solution and a feasible solution to the WDP, respectively. Then,

$$\begin{aligned} &\textstyle\sum_m \underline{Y}^m \leq Z^* \leq Z' \leq \sum_m \overline{Y}^m \\ \Rightarrow &\textstyle\sum_m \underline{Y}^m \leq Z^* \leq Z' \leq \epsilon \sum_m \underline{Y}^m \\ \Rightarrow &\qquad\quad Z^* \leq Z' \leq \epsilon Z^* \end{aligned}$$

The obvious values for $\overline{Y}^m$ and $\underline{Y}^m$ are $\max_j p_j^{1m}$ and $\min_j p_j^{Mm}$. Let $\overline{S}^*$ and $\underline{S}^*$ be the maximum and minimum number of items supplied among the winning bids in the optimal solution. If a feasible solution satisfy the above optimal supply bounds, then its objective value Z' will be bounded as follows:

$$Z^* \leq Z' \leq \max_m \frac{\max_j p_j^{\underline{S}^* m}}{\min_j p_j^{\overline{S}^* m}} Z^*$$

Thus by using the optimal bounds one can obtain a *good* feasible solution. The optimal bounds are not known in advance and one possible way is to try out all possible lower and upper bounds combinations, which is $O(M^2)$. Let WDP$(\underline{S}, \overline{S})$ denote the WDP with a constraint that each of the winning bids should supply items in range $[\underline{S}, \overline{S}]$. This is a *supply constrained* auction, which is commonly used in procurement [9]. The restriction on the number of winning items for any supplier is a business constraint imposed by the buyer. The upper bound reduces the risk of over exposure due to few number of winning suppliers and the lower bound reduces the cost of handling large number of winning suppliers. It can be easily seen that the WDP$(\underline{S}, \overline{S})$ is $\mathcal{NP}$-hard as one of its instance WDP$(0, M)$, which is the original problem, is $\mathcal{NP}$-hard. Hence we consider only the problems with $\underline{S} = 0$ and use its relaxation to develop the following two heuristics.

Heu-1

- *Relaxed Problems*: For each of $\overline{S} = 1, \ldots, M$ and $\underline{S} = 0$, create an associated transportation problem with supply for bid j as $[0, \overline{S}]$ and cost $c_j^m = p_j^{\overline{S}m}$ for link (j, m).
- *Feasible Solution*: Solve the relaxed problems and the optimal solution of each of the relaxed problems provide a feasible trade to the original WDP. Evaluate the cost of the feasible trades consistent with the discounts and choose the one with the least cost.

Heu-2

- *Relaxed problems*: For each of the discounts $\theta' \in \{\theta_j^i : \forall j, i\}$, create a transportation problem with $\underline{S}_j = 0$, $\overline{S_j} = \max\{i : \theta_j^i \leq \theta'\}$ for each j and cost $c_j^m = p_j^{\overline{S}_j m}$ for link (j, m).
- *Feasible Solution*: Same as in Heu-1.

Computational experiments were conducted to evaluate the performance of the heuristics. An instance of DA was randomly generated as follows. For each bid j, discount for one item was chosen as 0 and the discount for M items was chosen randomly from 10% to 40%. The discounts for the $1 < i < M$ was chosen linearly in between 0 and the maximum discount. The individual costs of the items were randomly chosen from a predetermined range. Figure 1 shows the average optimality gap for problems taken over 50 instances. The performance of Heu-2 was better as expected, as it solves nearly NM transportation problems, whereas Heu-1 solves only M transportation problems. The performance of Heu-2 is also poor, as we shall show in the next section that the branch and bound finds the optimal solution with less number of evaluations than that of Heu-2.

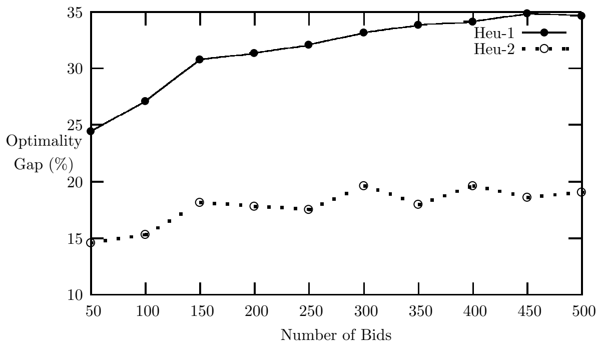

Fig. 1. Average optimality gap of the heuristics for 5 items

5 An Exact Branch and Bound Algorithm

B&B is an exact intelligent enumerative technique that attempts to avoid enumerating a large portion of the feasible integer solutions [10]. It is a widely used approach for solving discrete optimization, combinatorial optimization, and integer programming problems in general. The B&B approach first partitions the overall set of feasible solutions into two or more sets and as the algorithm proceeds the set is partitioned into many simpler and smaller sets, which are explored for the optimal solution. Each such set is represented algebraically by a *candidate problem* (CP). A typical iteration of B&B consists of:

- **Selection/Removal** of a CP from the list of CPs
- Determining the **lower bound** of the selected CP
- **Fathoming** or **pruning**, if possible, the selected CP
- Determining and updating the **incumbent** solution, if possible
- **Branching strategy**: If the CP is not fathomed, branching creates subproblems which are added to the list of CPs

The algorithm first starts with the original problem as the only CP in the list, considering the entire feasible set of solutions. The above steps are repeated until the list of CPs is empty. The popular way of implementing B&B is to use the integer or MILP formulation of the problem and use its linear relaxation to obtain the lower bounds. A MILP formulation for the WDP was presented in [5]. However, we use an alternate approach by utilizing the network substructure of the problem. The details of each of the above steps for the WDP are explained below.

Candidate Problem. The CP is the algebraic representation of a set of feasible solutions to the WDP. We use the following problem as the CP: each bid j has only a supply in range $[\underline{S}_j, \overline{S}_j]$. The CP has feasible solutions to WDP iff $\underline{S}_j \leq \overline{S}_j$, $\forall j$ and $\sum_j \underline{S}_j \leq M \leq \sum_j \overline{S}$. The supply constrained DAs discussed above are different from a CP as it need not satisfy the inequality $\sum_j \underline{S}_j \leq M$. The first CP has $\underline{S}_j = 0$ and $\overline{S}_j = M$, $\forall j$, which contains all the feasible solutions.

Lower Bounds and Incumbent Solutions. The lower bound on the objective value of CP provides the lower bound on the objective values of the feasible solutions contained in that CP. We determine the lower bound to the CP by solving it as an *interval transportation problem* with supply for j bounded in the interval $[\underline{S}_j, \overline{S}_j]$ and with cost on the link (j, m) as $p_j^{\overline{S}_j m}$. The optimal objective value of the interval transportation problem is the lower bound to the associated CP. The optimal solution is a feasible trade to the WDP. This is an incumbent solution and its cost is an upper bound to the optimal cost of the WDP. The best incumbent solution obtained so far in the algorithm is stored.

Fathoming and Pruning. If the lower bounding technique provides an optimal solution to the CP, then the CP is said to be fathomed, that is, no further probe into the CP is required. If the lower bound of a CP is greater than the objective value of the known feasible solution, then the CP is pruned from further analysis as it cannot guarantee a better solution than what is already obtained.

Branching Strategy. If the CP is not fathomed or pruned or infeasible, then it is partitioned into two CPs and added to the list of CPs. We create two child CPs by partitioning the feasible solutions of the parent CP into two sets, each represented by a child CP. Let δ_j be the number of items supplied by bid j in the optimal solution of the interval transportation problem corresponding to the parent CP. Obviously $\delta_j \in [\underline{S}_j, \overline{S}_j]$. We choose the branching bid as follows:

$$j' = \arg\min_j \{(\overline{S}_j - \delta_j) : \ \delta_j < \overline{S}_j\} \tag{8}$$

The first child CP is created by modifying the $\overline{S}_{j'}$ of the parent CP to $\delta_{j'}$ and the second child CP is created by modifying the $\underline{S}_{j'}$ of the parent CP to $\delta_{j'} + 1$. If the child CPs are feasible then their lower bounds are evaluated. If they are not fathomed or pruned, then they are added to the list of CPs.

Search Strategy. In each iterative step of B&B, a CP is selected and removed from the list of CPs for further analysis. We used *best first search* (BFS) as the search strategy, which explores the best CP from the current list of CPS. This reduces the search space, but it has to store all the unexplored CPs in memory. BFS is implemented by creating a *binary heap* that holds the list of CPs. At every iteration, the root of the heap, which is the CP with the least lower bound, is deleted from the queue and explored. If it is not fathomed or pruned, then two new CPs are generated and added to the queue. It is worth noting that the algorithm can be easily adapted to supply constrained DAs which are commonly encountered in practice.

We performed computational experiments to evaluate the performance of the proposed branch and bound algorithm based on the solution time and the number of lower bound evaluations of CP. The experiments were carried out on a Linux based PC equipped with a 3GHz Intel Xeon processor with 4GB RAM and the algorithms were coded in Java. Figure 2 shows the average solution time taken over 50 random instances. The solution time varies more with M than with

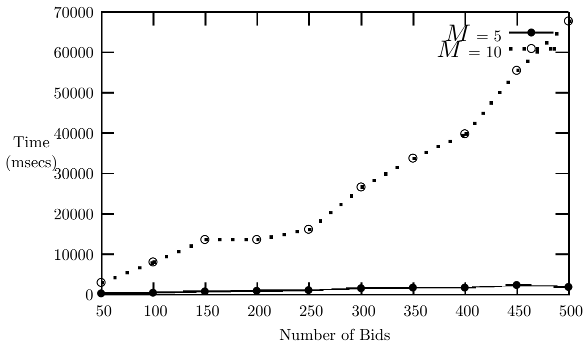

Fig. 2. Average solution time for branch and bound

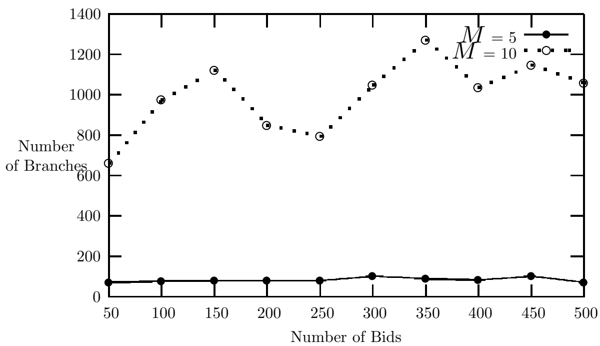

Fig. 3. Average number of lower bound evaluations

the number of bids N. The average number of lower bound evaluations (interval transportation problems) is shown in Figure 3. Currently we are experimenting with other possible data generation techniques to study the sensitivity of the algorithm with respect to the problem instance.

6 Conclusions

Discount auctions is a market mechanism for buying multiple items in a single auction. It is applicable in scenarios where the sellers have positive cost for each of the items and are interested in increasing the sales by providing discount to the buyer. We showed that the winner determination for this auction is $\mathcal{NP}$-hard and proposed heuristics and an exact branch and bound algorithm. The problem has an interesting combinatorial structure and it is worth investigating other possible exact and approximation schemes. From the auction perspective, an important research problem is to design a progressive mechanism with certain preferable economic properties. The multi-unit version of this auction is another interesting problem, which demands multiple units of each item. The interval transportation structure used in the branch and bound algorithm is not preserved in this problem and hence it needs a different solution approach.

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On the Competitive Ratio of the Random Sampling Auction

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Abstract. We give a simple analysis of the competitive ratio of the random sampling auction from [10]. The random sampling auction was first shown to be worst-case competitive in [9] (with a bound of 7600 on its competitive ratio); our analysis improves the bound to 15. In support of the conjecture that random sampling auction is in fact 4-competitive, we show that on the equal revenue input, where any sale price gives the same revenue, random sampling is exactly a factor of four from optimal.

1 Introduction

Random sampling is the most prevalent technique for designing auctions to maximize the auctioneer's profit when the bidders' valuations are a priori unknown [2, 3, 4, 7, 8, 10, 11]. The first and simplest application of random sampling to auctions is in the context of auctioning a *digital good*.[1] For this problem, the *random sampling optimal price* auction (RSOP) from [10] works by selecting a bipartition of the bidders uniformly at random and offering the optimal sale price for each part to the other part.

It is well known that, on many classes of interesting inputs, RSOP performs very close to optimally [2, 12]. Further, it was shown in [9] that RSOP is always within a constant factor of a natural benchmark for optimality even on worst-case inputs. Their analysis is not tight; they obtain an upper bound of 7600 on the competitive ratio of RSOP, well shy of the current conjectured ratio of four.

There are a number of compelling reasons for trying to prove the conjecture that RSOP is 4-competitive. First, it is one of the most natural profit-maximizing auctions, and having a tight analysis of its performance is interesting in itself. Second, an auction that is near optimal on many natural inputs and never very bad in the worst case has practical value. Finally, because of its simplicity, RSOP,

[1] Or any good where there are more units for sale than there are bidders.

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is easily adapted to other more general settings which benefit from improved analysis of RSOP for digital goods (E.g., double auctions [3], online limited supply auctions [11], and combinatorial auctions [2, 8]).

In this paper we give a new analysis of the random sampling optimal price auction for digital goods that tightens the upper bound on the competitive ratio to 15. Specifically, we show that the expected profit of RSOP is at least a factor of 15 of $\mathcal{F}^{(2)}$, the benchmark profit of the *optimal single price sale of at least two items*. We also show that on the pathological input where any single sale price gives the same revenue, i.e., the *equal revenue* input, the expected profit of the random sampling auction is at least $\mathcal{F}^{(2)}/4$. We refer the reader to [9] for motivation and discussion of this analysis framework and the choice of profit benchmark.

2 Preliminaries

We are considering auctioning a digital good to n bidders. Since the random sampling auction is *incentive compatible* we assume that each bidder bids their true valuation for the good. Let $\mathbf{b} = (b_1, \ldots, b_n)$ be the vector of bids sorted in decreasing order. Consider the following definitions from the literature [7, 10].

Definition 1 (RSOP). *The* random sampling optimal price auction *uniformly partitions the bidders into two parts, computes the optimal sale price for each part, and offers this sale price to the bidders in the opposite part.*

Definition 2 ($\mathcal{F}^{(2)}$). *The profit from the* optimal single price sale of at least two items *is:*

$$\mathcal{F}^{(2)}(\mathbf{b}) = \max_{i \geq 2} i b_i.$$

Definition 3 (Competitive Ratio). *The* competitive ratio *of an auction is the minimum value β for which the expected profit of the auction on any input is at least $\mathcal{F}^{(2)}/\beta$. An auction is β-*competitive *if its competitive ratio is at most β.*

Definition 4 (Equal-revenue Input). *The* equal revenue input *is the bid vector $\mathbf{b} = (b_1, \ldots, b_n)$ with $b_i = 1/i$.*

We will be employing this competitive framework to analyze the performance of RSOP. Our main result is the proof of the following theorem.

Theorem 1. *RSOP is 15-competitive.*

In proving Theorem 1 and analyzing RSOP on the equal revenue input, we will use an analytical tool which gives an exact computation of the probability of an event, $\mathcal{E}_\alpha$, defined as follows. Consider a discrete random walk on a line such that in each time step, the walk takes one step forward or stays put, independently with probability 1/2. If we start at the origin at time $i = 1$, then $\mathcal{E}_\alpha$ is the event that at no time $i \geq 1$ is the random walk further than αi from the origin. We prove the following lemma.

Lemma 1.

$$\Pr[\mathcal{E}_{\frac{3}{4}}] = 1 - \tfrac{1}{81}\left(\left(17+3\sqrt{33}\right)^{1/3} - 1 - 2\left(17+3\sqrt{33}\right)^{-1/3}\right)^4$$
$$= 0.912622 \pm 4 \times 10^{-6}.$$

It seems likely that this lemma was known prior to the present work, though we are not aware of any proof which has previously appeared elsewhere. Our proof technique provides a closed-form value of $\Pr[\mathcal{E}_\alpha]$ for a limited range of α values. For α of the form $\frac{k-1}{k}$ we find the value of $\Pr[\mathcal{E}_\alpha]$ implicitly as the unique root of a k-th degree polynomial on the interval $(0,1)$. For arbitrary values of α we describe a computer aided proof that $\Pr[\mathcal{E}_\alpha]$ lies in an interval of width ϵ (with proof length proportional to $\log(1/\epsilon)$).

3 Proof of Lemma 1

Let S_i be the variable for the position of a random walk on a line that starts at the origin at time $i=1$ and proceeds in each round to stay put or move forward each with probability 1/2. The lemma calls for the calculation of the probability, $\Pr[\mathcal{E}_\alpha]$, that for all i, $S_i/i \leq \alpha$, in particular for $\alpha = \frac{3}{4}$.

For any α that takes the form $\alpha = \frac{k-1}{k}$ where k is an integer, we can rewrite $\mathcal{E}_\alpha$ as the event that for all i, $(k-1)(i-S_i) - S_i \geq 0$. By setting $Z_i = (k-1)(i-S_i) - S_i$, we have that $(Z_1, Z_2, Z_3, \dots)$ is a random walk which increases by $k-1$ with probability 1/2 and otherwise decreases by 1 (where, because $S_1 = 0$, the walk starts with $Z_1 = k-1$). We also note that $\Pr[\bar{\mathcal{E}}_\alpha]$ is equal to the probability of ruin for this asymmetric random walk, which we denote by p_k (for a general study of the probability of ruin, see for example [6–Chapter XIV]). In general, let

$$p_j = \Pr\left[\text{exists } i,\ Z_i = Z_1 - j\right].$$

Because the random walk is memoryless and never decreases by more than 1, $p_j = (p_1)^j$ for $j > 0$. By expanding the probability conditionally on the value of S_2, we have

$$p_1 = \tfrac{1}{2} + \tfrac{1}{2}p_k$$
$$= \tfrac{1}{2}\left(1 + p_1^k\right).$$

The polynomial $f(x) = x^k - 2x + 1$ has $f(0) = 1$ and $f(1) = 0$. Since $f'(x) = kx^{k-1} - 2$ has one root on the interval $(0,1)$ and $f''(x) = k(k-1)x^{k-2}$ is positive on this interval, there is a unique root of f on the interval $(0,1)$; call this root r. Since $0 \leq p_1 \leq 1$ and $f(p_1) = 0$, we must have either $p_1 = 1$ or $p_1 = r$. We will prove that $p_1 < 1$, thus establishing that $p_1 = r$. Let $(Y_1, Y_2, \dots)$ be the absorbing random walk given by $Y_i = Z_i - k + 2$ until it drops to $Y_i = 0$, where it stays. More formally,

$$Y_i = \begin{cases} 0, & \text{if } Y_j = 0 \text{ for some } j < i; \\ Z_i - Z_1 + 1, & \text{otherwise.} \end{cases}$$

Let W_i be the random variable given by $W_i = r^{Y_i}$. We claim $(W_1, W_2, \ldots)$ satisfies the martingale property that $\mathrm{E}\left[W_{i+1} \mid W_i\right] = W_i$. This holds trivially when $Y_i = 0$. For $Y_i > 0$, the equation $\mathrm{E}\left[W_{i+1} \mid W_i\right] = W_i$ follows from the calculation

$$\mathrm{E}\left[r^{Z_{i+1}} \mid Z_i\right] = \tfrac{1}{2}r^{Z_i - 1} + \tfrac{1}{2}r^{Z_i + k - 1} = \tfrac{1}{2}\left(r^{-1} + r^{k-1}\right) r^{Z_i} = r^{Z_i}.$$

(The final equality follows from $f(r) = r^k - 2r + 1 = 0$.) Since $(W_1, W_2, \ldots)$ is a martingale we have $\mathrm{E}[W_t] = W_1 = r$ for all $t \geq 1$.

Now let

$$p_{1,t} = \Pr[\text{exists } i \leq t, Z_i = Z_1 - 1] = \Pr[W_t = 1].$$

Defining $\mathcal{A}_t$ to be the event that $Z_t = Z_1 - 1$ and $Z_s > Z_t$ for all $s < t$, we see that the events $\mathcal{A}_t$ are disjoint and measurable, and that $p_{1,t} = \sum_{i=1}^{t} \Pr(\mathcal{A}_i)$ and $p_1 = \sum_{i=1}^{\infty} \Pr(\mathcal{A}_i)$. This establishes that $p_1 = \lim_{t\to\infty} p_{1,t}$ while also confirming that $\mathcal{E}_\alpha$ is a measurable event since it is the union of the events $\mathcal{A}_i$ $(1 \leq i < \infty)$.

Since W_t is a non-negative random variable, we have $p_{1,t} = \Pr[W_t = 1] \leq \mathrm{E}[W_t] = r$. Recalling that $p_1 = \lim_{t\to\infty} p_{1,t}$, we conclude that $p_1 \leq r < 1$, as claimed. This completes the proof that $p_1 = r$ and that $p_j = r^j$ for all $j \geq 1$.

Recall that to calculate $\Pr[\mathcal{E}_{\frac{3}{4}}]$ we are intersted in the case that $k = 4$. When $k = 4$ the polynomial f is a quartic equation, and the root r is given exactly by Ferrari's formula [5]. So

$$p_1 = \tfrac{1}{3}\left[\left(17 + 3\sqrt{33}\right)^{1/3} - 1 - 2\left(17 + 3\sqrt{33}\right)^{-1/3}\right],$$

and to complete the proof we have

$$\begin{aligned}\Pr\left[\mathcal{E}_{\frac{3}{4}}\right] &= 1 - p_4 = 1 - p_1^4 \\ &= 1 - \tfrac{1}{81}\left[\left(17 + 3\sqrt{33}\right)^{1/3} - 1 - 2\left(17 + 3\sqrt{33}\right)^{-1/3}\right]^4.\end{aligned}$$

4 Proof of Theorem 1

In our random partition, we call the side of the bipartition with b_1 on it the *bad* side, and we call the other side the *good* side. We may assume that b_1 is larger than $\mathcal{F}^{(2)}$, since this can only increase the gap between the expected profit of RSOP and $\mathcal{F}^{(2)}$. (Increasing b_1 can not change $\mathcal{F}^{(2)}$, nor can it change the revenue obtained by RSOP from bidders on the bad side of the bipartition; moreover, if b_1 is sufficiently large then RSOP will obtain zero revenue from bidders on the good side of the bipartition.) Let $X_i \in \{0, 1\}$ be an indicator random variable for the event that bidder i is on the good side. By definition $X_1 = 0$. Let $S_i = \sum_{j=1}^{i} X_j$ be the number of bidders with bid at least b_i on the good side. Note that S_i is a random variable that behaves like the random walk under consideration in Lemma 1. Let $i^\star$ denote the index of the bidder whose value $b_{i^\star}$ maximizes revenue on the good side (meaning $S_{i^\star} b_{i^\star} \geq S_j b_j$ for all j, with ties broken arbitrarily).

Then the profit of the random sampling auction is

$$RS = (i^\star - S_{i^\star})\, b_{i^\star}.$$

We now show that $\mathbf{E}[RS] \geq \mathcal{F}^{(2)}/15$.

Let i' denote the index of the bidder whose value $b_{i'}$ is the optimal sale price for the full set of bids (meaning $i'b_{i'} \geq jb_j$ for all $j \geq 2$, with ties broken arbitrarily). We shall first provide a bound for the case when i' is even, and later explain how the same bound (or in fact, a slightly better one) can be obtained when i' is odd.

Consider the event $\mathcal{B} = \{S_{i'} \geq i'/2\}$. Using the fact that i' is even, it follows that $\Pr[\mathcal{B}] = 1/2$ (because the event holds when the majority of i' highest bidders other than the largest bid are on the good side). In this case, the optimal single price profit from the good side is

$$S_{i^\star} b_{i^\star} \geq S_{i'} b_{i'} \geq \mathcal{F}^{(2)}/2.$$

To avoid unnecessary subscripts, we set $\mathcal{E} = \mathcal{E}_{\frac{3}{4}}$.[2] If event $\mathcal{E}$ occurs, then, for all i, we have

$$(i - S_i)b_i \geq \tfrac{1}{4} i b_i \geq \tfrac{1}{3} S_i b_i.$$

If both $\mathcal{E}$ and $\mathcal{B}$ occur then

$$RS = (i^\star - S_{i^\star})b_{i^\star} \geq \frac{1}{3} S_{i^\star} b_{i^\star} \geq \mathcal{F}^{(2)}/6.$$

Thus, the expected profit of RSOP is at least

$$\mathrm{E}[RS] = \mathrm{E}\left[(i^\star - S_{i^\star})b_{i^\star}\right] \geq \Pr[\mathcal{E} \cap \mathcal{B}]\mathcal{F}^{(2)}/6.$$

By Lemma 1, $\Pr[\mathcal{E}] \geq 0.9$, so

$$\Pr[\mathcal{E} \cap \mathcal{B}] = 1 - \Pr[\bar{\mathcal{E}} \cup \bar{\mathcal{B}}] \geq 1 - \Pr[\bar{\mathcal{E}}] - \Pr[\bar{\mathcal{B}}] = \Pr[\mathcal{E}] - \tfrac{1}{2} \geq 0.4.$$

Therefore $\mathrm{E}[RS] \geq \mathcal{F}^{(2)}/15$.

We now address the case when i' is odd. In this case we consider the event $\mathcal{C} = \{S_{i'} \geq (i'-1)/2\}$. It is not hard to see that $\Pr[\mathcal{C}] = 1/2 + 2^{-i'}\binom{i'-1}{(i'-1)/2}$. It can be verified that for every odd $i' \geq 3$, a straightforward modification of the proof that was given for the case that i' is even gives a bound that is at least $1/15$. This completes the proof of the theorem.

5 Random Sampling and the Equal Revenue Input

We now discuss the performance of RSOP on the equal revenue input (See Definition 4). This input is of particular interest because the intuition motivating

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[2] It is possible to perform this calculation with the event $\mathcal{E}_\alpha$ for $\alpha \neq \frac{3}{4}$. This produces an upper bound on the competitive ratio of RSOP of $\left[\left(\Pr[\mathcal{E}_\alpha] - \frac{1}{2}\right)\left(\frac{1-\alpha}{\alpha}\right)\right]^{-1}$. Computer calculations following the same flavor as those made in Section 5 suggest this is minimized when $\alpha = \frac{3}{4}$.

random sampling does not apply to it. A price that looks good for one part, because disproportionately many bids are above it, is a bad price for the other part because disproportionately few bids are above it. In this section we discuss a computer-aided calculation that shows that the RSOP on the equal revenue input has expected profit at least $\mathcal{F}^{(2)}/4$ (and this is tight when there are only $n = 2$ bidders). Note that on the equal revenue input, n bidders with $b_i = 1/i$, the optimal single price sale obtains profit $\mathcal{F}^{(2)}(\mathbf{b}) = 1$.

Define the event $\mathcal{E}_\alpha^n = \left\{\text{for all } i \leq n, \frac{S_i}{i} \leq \alpha\right\}$ with S_i as defined in previous sections. Fix a positive integer N and let $\alpha_i = i/N$. Then let $\mathcal{A}_i^n = \mathcal{E}_{\alpha_i}^n \cap \bar{\mathcal{E}}_{\alpha_{i-1}}^n$ be the event that some S_i/i exceeds α_{i-1} but none exceed α_i. Thus, $\Pr[\mathcal{A}_i^n] = \Pr[\mathcal{E}_{\alpha_i}^n] - \Pr[\mathcal{E}_{\alpha_{i-1}}^n]$. The events $\mathcal{A}_i^n$ are disjoint and if $\mathcal{A}_i^n$ holds then $(1-\alpha_i) \leq RS \leq (1-\alpha_{i-1})$. Therefore, the profit of RSOP on the n bid equal revenue input satisfies

$$\sum_{i=1}^{N-1} \Pr[\mathcal{A}_i^n](1-\alpha_i) \leq \mathrm{E}[RS] \leq \sum_{i=1}^{N-1} \Pr[\mathcal{A}_i^n](1-\alpha_{i-1}). \tag{1}$$

What remains is to show how we can calculate $\Pr[\mathcal{E}_\alpha^n]$ and extend the above discussion to get a bound $\mathrm{E}[RS]$ for the equal revenue input with any n.

Calculating $\Pr[\mathcal{E}_\alpha^n]$ for General α

The proof in Section 3 can be easily adapted to give an implicit value of $\Pr[\mathcal{E}_\alpha]$ whenever $\alpha = \frac{k-1}{k}$ for some integer k. When $k \leq 5$, it is possible to turn this into a closed-form solution. We now describe an alternative method for calculating $\Pr[\mathcal{E}_\alpha^n]$ which leads to bounds on $\Pr[\mathcal{E}_\alpha]$ that do not require α to be of any special form,

Fix a value of α. To bound the value of $\Pr[\mathcal{E}_\alpha^n]$, we define

$$p(i,j) = \Pr\left[S_i = j \cap S_{i'} \leq \alpha i' \text{ for all } i' \leq i\right]$$

and note that $\Pr[\mathcal{E}_\alpha^n] = \sum_{j=0}^n p(n,j)$. We now can use a standard computer algebra package, like Mathematica, to evaluate $\Pr[\mathcal{E}_\alpha^n]$ using the following recurrence. The initial conditions are derived from the fact our random walk starts at time $i = 1$ (i.e., $S_1 = 0$).

$$p(i,j) = \begin{cases} \frac{1}{2}p(i-1,j-1) + \frac{1}{2}p(i-1,j), & \text{if } 0 \leq j \leq \alpha i; \\ 0, & \text{otherwise.} \end{cases}$$

$$p(1,j) = \begin{cases} 1, & j = 0; \\ 0, & \text{otherwise.} \end{cases}$$

For reasonable values of n, it is possible to evaluate $\Pr[\mathcal{E}_\alpha^n]$. For example, for $\alpha = \frac{3}{4}$, $\Pr[\mathcal{E}_{\frac{3}{4}}^{200}]$ equals

$$\frac{2291448392245272775271057660365355171921931581972190277749\,9}{2510840694154672305534315769283066566440942177785613805158\,4}.$$

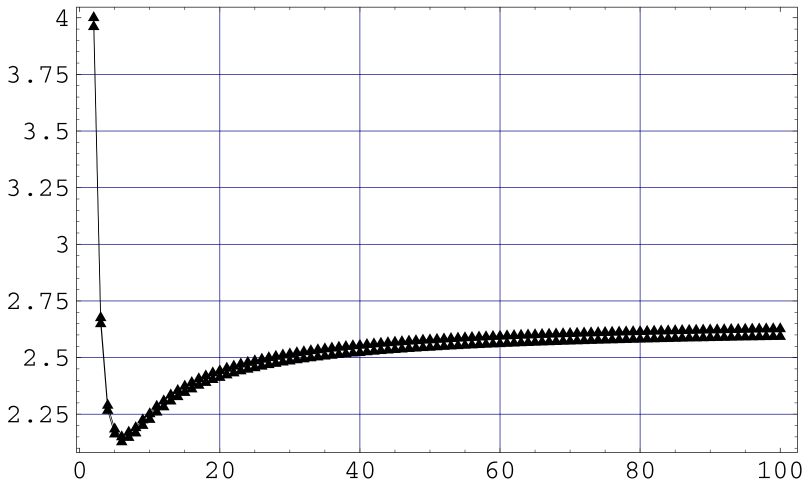

Fig. 1. Upper and lower bounds on $\mathcal{F}^{(2)}/\,\mathrm{E}[RS]$ when $N = 200$ for equal revenue input with $n = 2, \ldots, 100$

Bounding E[*RS*] for all *n*

Using the recurrence relationship for $\Pr[\mathcal{E}_\alpha^n]$ with different values of α and equation (1), we can calculate $\Pr[\mathcal{A}_i^n]$ and $\mathrm{E}[RS]$ for any given n. This calculation shows that as n increases, the ratio $\mathcal{F}^{(2)}/\,\mathrm{E}[RS]$ is not monotonic. A plot of the upper and lower bounds on $\mathcal{F}^{(2)}/\,\mathrm{E}[RS]$ obtained by taking $N = 200$ appears in Figure 1. For $N = 200$ and $n = 100$ this calculation shows that $2.59 \leq \mathcal{F}^{(2)}/\,\mathrm{E}[RS] \leq 2.63$ and growing. To extend this bound to all values of n, we must still prove that for large n the ratio does not eventually grow bigger than four.

To do so, we get bounds on $\Pr[\mathcal{E}_\alpha]$ in terms of $\Pr[\mathcal{E}_\alpha^n]$. $\Pr[\mathcal{E}_\alpha^n]$ is an upper bound of $\Pr[\mathcal{E}_\alpha]$. We can get a lower bound by applying the union bound as follows:

$$\begin{aligned} 1 - \Pr[\mathcal{E}_\alpha^n] \leq \Pr[\bar{\mathcal{E}}_\alpha] &\leq 1 - \Pr[\mathcal{E}_\alpha^n] + \sum_{i \geq n} \Pr\left[S_i \geq \alpha i\right] \\ &\leq 1 - \Pr[\mathcal{E}_\alpha^n] + \sum_{i \geq n} e^{-(\alpha - 1/2)^2 i/3} \\ &= 1 - \Pr[\mathcal{E}_\alpha^n] + \frac{e^{-(\alpha-1/2)^2 n/3}}{1 - e^{-(\alpha-1/2)^2/3}}. \end{aligned}$$

As an aside, an alternate proof of Lemma 1 can be obtained from the above bound with $n = 800$ to show that

$$\Pr[\mathcal{E}_{\frac{3}{4}}] = 0.912622 \pm 4 \times 10^{-6}.$$

We now use these upper and lower bounds on $\Pr[\mathcal{E}_\alpha]$ to get a lower bound on $\Pr[\mathcal{A}_\alpha]$. For any $n \geq n_0$,

$$\Pr[\mathcal{A}_i^n] \geq \Pr[\mathcal{E}_{\alpha_i}] - \Pr[\mathcal{E}_{\alpha_{i-1}}^{n_0}] \geq \Pr[\mathcal{E}_{\alpha_i}^{n_0}] - \frac{e^{-(\alpha_i - 1/2)^2 n_0/3}}{1 - e^{-(\alpha_i - 1/2)^2/3}} - \Pr[\mathcal{E}_{a_{i-1}}^{n_0}].$$

This bound is only useful when $\alpha_i > 1/2$ and n_0 is sufficiently large. Fortunately, to get a good lower bound on $\mathrm{E}[RS]$ for large n it suffices to consider only the contribution to the sum in equation (1) from the terms with high α. For sufficiently large i_0 and n_0 we are left with the bound:

$$\mathrm{E}[RS] \geq (1-\alpha_{i_0})\Pr[\mathcal{E}_{\alpha_{i_0}}] + \sum_{i=i_0+1}^{N-1} \Pr[\mathcal{A}_i^{n_0}](1-\alpha_i).$$

Taking $n_0 = 500$, $N = 100$, and $i_0 = 70$ (so $\alpha_{i_0} = 0.7$) and using the computer to prove bounds on the terms in this sum shows that for all $n \geq 500$, $\mathrm{E}[RS] \geq \mathcal{F}^{(2)}/3.6$. This, combined with the computer proof outlined previously for $n \leq n_0$, completes the proof showing that RSOP is 4-competitive on the equal revenue input.

6 Conclusions

As we mentioned in the introduction, the random sampling technique is widely applicable to the design of profit maximizing mechanisms. The basic RSOP auction has been generalized and applied to the problem of designing double auctions [3], online limited supply auctions [11], multi-unit auctions for bidders with budget constraints [4], combinatorial auctions [2, 8], and knapsack auctions [1]. With exception of the work of Hajiaghayi et al. [11] on online limited supply auctions, all of these generalized applications of RSOP are given with analyses that obtain *promise* style bounds. A typical promise style bound would state that if n, the number of bidders, is large enough then the random sampling auction's profit is near optimal. No bound is given if the promise is not met. There are several reasons for using such a promise bound. First, it allows for $(1-\epsilon)$-approximations, with ϵ a parameter that improves with the restrictiveness of the promise. Such bounds are of interest as $(1-\epsilon)$-approximations are not possible in the worst case competitive framework of this and preceding papers [7]. Second, the analysis of RSOP in [9] is complicated and gives such a loose bound that generalizing it to other contexts seems to be of marginal worth. In contrast, our improved analysis opens up the possibility of doing a worst case analysis of the random sampling auction for some of these more general applications.

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The Pricing Strategies for Agents in Real E-Commerce

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Abstract. The paper presents the pricing strategies for automatic agent, which is design for e-commerce in the reality. Given the characters of e-commerce in reality, the paper firstly model the bargaining process; Secondly classify the bargaining scenarios and present the optimal bargaining strategy under sub-incomplete information for corresponding bargaining scenarios; then extend them to the optimal pricing strategies under incomplete information condition; Thirdly, discuss the conditions for convergence of optimal strategy; Finally, the analysis shows that the bargaining strategic profiles form the sequential equilibrium and the agreement is unique under certain conditions.

1 Introduction

Apply automatic agent to e-commerce is mainly build the bargaining strategy model to make the agent represent the user bargaining automatically [1, 2, 3]. The agents' preference of the agreement is different because of the different utility distribution among the agents according to the agreements. So the agent adopts pricing strategy to reach the agreement that maximizes its utility. So, many researchers focus on the pricing strategy in e-commerce. However the bargaining in reality is time limited and make-decision from the incomplete information, the Refs ignored the characters, which lead to the model is not available for the applications. From this point, the paper presents the pricing strategy for agents in real e-commerce environment. It models the bargaining process firstly; then presents the optimal pricing strategy according to the incomplete information, which is based on the discussion in the sub-incomplete information; finally, discuss the characters of the optimal strategy under some condition. The optimal strategy is applicable for the e-commerce bargaining in reality while there are still have some work being done in the future.

2 The Bargaining Model

The bargaining model has four factors [4]: protocol; bargaining strategies; information state; bargaining agreement. In the section, model the bargaining process to present optimal pricing strategy.

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2.1 The Bargaining Protocol

The paper adopts an alternating offers protocol [4]. Here, let b denote the buyer, s the seller and $[IP^a, RP^a]$ denote the range of price that are acceptable to agent a, where $a \in \{b, s\}$, the price between the interval $[RP^s, RP^b]$ is agreement zone. The initial prices IP^b, IP^s lie outside of the agreement zone. The agents alternately propose offers at times in $\Gamma = \{0,1,2...\}$. Each agent has a deadline. T^b denotes agent a's deadline where $a=b$. $p^t_{b\to s}$ denotes the price offered by b at time t. Thus the response action of s is as:

$$A^s(t, p^t_{b\to s}) = \begin{cases} \text{Quit} & \text{if } t > T^s \\ \text{Accept} & \text{if } U^s(p^t_{b\to s}) \geq U^s(p^{t+1}_{s\to b}) \\ \text{Offer } p^{t+1}_{s\to b} & \text{otherwise .} \end{cases} \tag{1}$$

Agents' utilities are defined with the following Von Neumam-Morgenstern utility functions that incorporate the effect of time discounting: [5].

$$U^a(p,t) = U^a_p(p) U^a_t(t) \tag{2}$$

$U^a_p(p)$ and $U^a_t(t)$ are undimensional utility function. Here, given the other attribute. $U^a_p(p)$ is defined as :

$$U^a_p(p) = \begin{cases} RP^b - p & \text{for the buyer} \\ p - RP^s & \text{for the seller .} \end{cases} \tag{3}$$

Define $U^a_t(t) = (\delta^a)^t$, where δ^a （$0 \leq \delta^a \leq 1$） is the discounting factor, if δ^a is lower while a is more impatient, and vice versa. In the bargaining, each agent wants to reach the agreement for avoiding the conflicting. Consequently, it is optimal for a to offer RP^a at its deadline, if it has not done so earlier.

2.2 Counter-Offer Generation

The agent generates the counter-offer according to its bargaining strategy except choosing the initial offer randomly. The offer made by a to its opponent $\hat{a}$ at time t （$0 \leq t \leq T^a$） is modeled as follows:

$$p^t_{a\to\hat{a}} = \begin{cases} IP^a + \phi^a(t)(RP^a - IP^a) & \text{for } a = s \\ RP^a + (1 - \phi^a(t))(IP^a - RP^a) & \text{for } a = b \text{ .} \end{cases} \tag{4}$$

Where, $\phi^a(t)$ is ensure that: ① $\phi^a(t)$ is time-dependent; ② $0 \leq \phi^a(t) \leq 1$. Define as follows:

$$\phi^a(t) = k^a + (1 - k^a)\left(\frac{t}{T^a}\right)^{\frac{1}{\psi}} \text{ .} \tag{5}$$

Where, k^a is the constant and lies in [0, 1]. If $k^a \in (0,1)$, the initial offer varies between (IP^a, RP^a); Furthermore, according to the value of ψ, three extreme sets: Boulware (B), Conceder (C), Linear (L) show clearly different patterns of behavior (see the details in the Refs 6,7).

The counter-offer $p^t_{a\to\hat{a}}$ depends on IP^a, FP^a, t and ψ. These four variables form an agent's strategy.

Definition 1. An agent's strategy S^a is defined as a quadruple whose elements are the initial price IP^a, the final price FP^a beyond which the agent does not concede, time t^a at which the final price is offered and ψ^a. Thus: $S^a = \langle IP^a, FP^a, t^a, \psi^a \rangle$.

Definition 2. The bargaining outcome *(O)* is $\langle (p,t), \hat{C} \rangle$. where $p \in [RP^s, RP^b]$ and $t \in [0, \min(T^b, T^s)]$. $\hat{C}$ denotes the conflicting outcome.

The optimal bargaining strategy is determined by the four elements in Def 1 that will give each agent maximum possible utility. An agent's optimal strategy depends on the information it has. Therefore, define the information state for each agent and then show how the optimal strategies are determined.

2.3 Agents' Information States

The information state of *a*, I^a is that the information it has about the bargaining parameters. Define as: $I^a = \langle RP^a, T^a, U^a, S^a, L_p^{\hat{a}}, L_t^{\hat{a}} \rangle$ Where, RP^a, T^a, U^a and S^a are the information about *a*'s own and $L_p^{\hat{a}}$, $L_t^{\hat{a}}$ are its beliefs about its opponent $\hat{a}$'s deadline and reservation price. $L_t^{\hat{a}} = \langle T_i^{\hat{a}}, \alpha_i^{\hat{a}} \rangle$ $(1 \le i \le n, T_i^{\hat{a}} \in \Gamma)$ denotes the probability $\alpha_i^{\hat{a}}$ with which the $\hat{a}$'s deadline is $T_i^{\hat{a}}$; $L_p^{\hat{a}} = \langle RP_j^{\hat{a}}, \beta_j^{\hat{a}} \rangle$ $(1 \le j \le m)$ denotes the probability $\beta_j^{\hat{a}}$ with which $\hat{a}$'s reservation price is $RP_j^{\hat{a}}$. In the paper, suppose I^a being static.

Each agent's information state is its private information that is not known to its opponent, which is different from "common knowledge" in game theory.

2.4 Bargaining Scenarios

The column 1 and 2 in Table.1 show the relationships between deadline and corresponding bargaining scenario. An agent bargains in one of the three scenarios; Column 3 is the possible bargaining scenarios for its opponent $\hat{a}$:

Table 1. Bargaining scenario and interacting scenarios

Relationship for deadlines	*a*'s bargaining scenario	Bargaining scenarios for $\hat{a}$
$T_n^s < T^b$	B_1	B_2, B_3
$T_k^s < T^b \le T_{k+1}^s$ $(k+1<n)$	B_2	B_1, B_2, B_3
$T_n^s > T^b$	B_3	B_1, B_2

2.5 Optimal Strategies

The paper describes how optimal strategies are obtained for players that expected utility maximizes. The discussion is from the perspective of the buyer (although the same analysis can be taken from the seller). In order to simplify the discussion, the paper first assumes that the seller's reservation price is known and obtains the optimal strategy, then extend the analysis to the more general case.

Each agent's optimal strategy is determined on the basis of its own information state. The paper then determines if this mutual strategic behavior of agent results in equilibrium.

2.5.1. Optimal Strategies for the Buyer When RP Is Known. The strategies should ensure agreement by the earlier deadline. Otherwise the bargaining ends in conflict. For $0 < \delta^b < 1$, b wants to reach the agreement as quickly as possible. The buyer's action function in all the scenarios is same and defines as follow:

$$A^b(t, p^t_{s\to b}) = \begin{cases} \text{Quit} & \text{if } t > T^b \\ \text{Accept} & \text{if } p^t_{s\to b} \le S^b_o(t) \\ \text{Offer } S^b_o(t')\ (t' = t+1) & \text{otherwise} . \end{cases} \quad (6)$$

List the buyer's optimal strategies for all the three scenarios as column 2, 3 in Table 2. Here $T' = t_0 + 1$ (t_0 is the initial time of bargaining).

Table 2. Compare the optimal strategies for b when RP^s is known with the ones when neither RP^s nor T^s is known

Bargaining scenario	Time t during bargaining	Optimal strategy	Time t during bargaining	Optimal strategy
B_1	$t \le T'$	$< IP^b, RP^s, T', C >$	$t \le T'$	$< IP^b, RP_I{}^s, T', C >$
	$t > T'$	$< RP^s, RP^s, T^s_n, L >$	$t > T'$	$< RP_I{}^s, RP_I{}^s, T^s_n, L >$
B_2	$t \le T'$	$< IP^b, RP^s, T', C >$	$t \le T'$	$< IP^b, RP_I{}^s, T', C >$
	$T' < t \le T^s_k$	$< RP^s, RP^s, T^s_k, L >$	$T' < t \le T^s_k$	$< RP_I{}^s, RP_I{}^s, T^s_k, L >$
	$t > T^s_k$	$< RP^s, RP^b, T^b, B >$	$t > T^s_k$	$< RP_I{}^s, RP^b, T^b, B >$
B_3	$t \le T'$	$< IP^b, RP^b, T', C >$	$t \le T$	$< IP^b, RP^b, T', C >$
	$t > T'$	$< RP^b, RP^b, T^b, L >$	$t > T'$	$< RP^b, RP^b, T^b, L >$

2.5.2. Optimal Strategy for the E-commerce in Reality. During the bargaining, the opponent's deadline and reservation price is neither known. Here, discuss the optimal strategies of b when T^s and RP^s is neither known. If the optimal strategies do not include RP^s in table 2, i.e. the optimal strategies is not related to the RP^s. In the section, it's enough to discuss the b in B1 and B2. b's action function is same as formula (6). According t I^b, the number of possible strategies is $m \times n$.

In B1, b's optimal strategy is to offer RP^s from the beginning to the end. When RP^s is uncertain, b's EU from the strategy S^b_i is

$$EU^b_i = \sum_{x=1}^{i-1}\sum_{y=1}^{n}\gamma^s_{x,y}U^b(\hat{C}) + \sum_{x=1}^{n}\gamma^s_{i,x}U^b(RP^s_i, t_1) + \sum_{x=i+1}^{m}\sum_{y=1}^{n}\gamma^s_{x,y}U^b(p_1, t_2) \quad . \quad (7)$$

Where, $RP^s_x \le p_1 \le RP^s_i$, $T' \le t_1 \le T^s_x$ and $T' \le t_2 \le T^s_y$.

If s is in B_3, then $t_1 = t_2 = T'$; if s is in B_2, then $t_1 = T_x^s$ and $t_2 = T_y^s$. If s is in B_3 and makes offer at T', then $p_1 = RP_x^s$, while $p_1 = RP_i^s$ if b makes offer at T' ; On the other hand , if s is in B_2 and makes offer at the earlier deadline, then $p_1 = RP_x^s$, else $p_1 = RP_i^s$. Let eu_1^b denote the value of Eq.(7) if s is in B_3 ; eu_2^b denote the value of Eq.(7) if s is in B_2. EU_i^b becomes:

$$EU_i^b = \frac{1}{2} eu_1^b + \frac{1}{2} eu_2^b \ . \tag{8}$$

The values of i, j that give the buyer the maximum EU are denoted as I and J . b 's optimal strategy for B_1, in terms of I and J , is listed in Table 2.

In B_2, b 's expected utility from s_i^b is:

$$\begin{aligned} EU_i^b = & \sum_{x=1}^{i-1} \Big(\sum_{y=1}^{k} \gamma_{x,y}^s U^b(\hat{C}) + \sum_{y=k+1}^{n} \gamma_{x,y}^s U^b(p_1, T^b) \Big) + \sum_{x=1}^{k} \gamma_{i,x}^s U^b(RP_i^s, t_1) \\ & + \sum_{x=k+1}^{n} \gamma_{i,x}^s U^b(p_2, t_2) + \sum_{x=i+1}^{m} \Big(\sum_{y=1}^{k} \gamma_{x,y}^s U^b(p_3, t_3) + \sum_{y=k+1}^{n} \gamma_{x,y}^s U^b(p_4, t_4) \Big) \end{aligned} \ . \tag{9}$$

Where, $T' \le t_1 \le T_x^s$ and $T' \le t_2 \le T^b$ and $T' \le t_3 \le T_y^s$ and $T' \le t_4 \le T^b$ and $RP_x^s \le p_1 \le RP^b$ and $RP_i^s \le p_2 \le RP^b$ and $RP_x^s \le p_3 \le RP_i^s$ and $RP_i^s \le p_4 \le RP^b$.

If s is in B_3, then $t_1 = T'$, otherwise $t_1 = T_x^s$; if s is in B_3, then $t_3 = T'$ and $t_3 = T_y^s$; For all possible s's scenarios, $t_2 = t_4 = T^b$. $p_1 = p_2 = p_4 = RP^b$ if buyer makes offer at earlier deadline, else $p_1 = p_2 = p_4 = RP_I^b$; Finally, $p_3 = RP_x^s$ if s is in B_3 and makes offer at T'; But $p_3 = RP_i^s$ if s is in B_3 and b makes offer at T' . For the remaining scenarios, $p_3 = RP_x^s$ if s makes offer at the earlier deadline, else $p_3 = RP_i^s$. Since RP_I^b is not known to b, it can only take RP^b as the value of p_1, p_2, p_4 . Let eu_1^b denote the value of Eq.(9) if s is in B_3 ; eu_2^b denote the value of Eq.(9) if s is in B_1, B_2. EU_i^b becomes:

$$EU_i^b = \frac{1}{3} eu_1^b + \frac{2}{3} eu_2^b \ . \tag{10}$$

The values of i, j that give the buyer the maximum EU are denoted I and J. The b 's optimal strategy for B_2, in terms of I and J, is listed in column 4,5 of Table 2.

2.6 Discussion: Condition for Convergence of Optimal Strategy

To ensure the convergence of optimal strategy, the conditions listed in Table4. must be satisfied.

Table 3. Conditions for convergence of optimal strategies

Bargaining scenario	Condition for convergence	
	Buyer's strategy	Seller's strategy
B_1	$RP_I^s \ge RP^s$	$RP_I^b \le RP^b$
B_2	$RP_I^s \ge RP^s$	$RP_I^b \le RP^b$
B_3	None	None

If agents satisfy the conditions listed above, then the final bargaining outcome is shown as follow:

Table 4. The final outcome of bargaining

Bargaining scenarios		Bargaining outcome (price, time)
Buyer	Seller	
B_1	B_2	$<RP_I^s, T^s>$ or $<RP^s, T^s>$
B_1	B_3	$<RP_I^s, T'>$ or $<RP^s, T'>$
B_2	B_1	$<RP_I^b, T^b>$ or $<RP^b, T^b>$
B_2	B_2	$<RP_I^s, T^s>$ or $<RP^s, T^s>$, if $T^s < T^b$
		$<RP_I^b, T^b>$ or $<RP^b, T^b>$, if $T^s > T^b$
		$<RP^s, T^b>$ or $<RP^b, T^b>$, if $T^s = T^b$
B_2	B_3	$<RP_I^s, T'>$ or $<RP^s, T'>$
B_3	B_1	$<RP_I^b, T'>$ or $<RP^b, T'>$
B_3	B_2	$<RP_I^b, T'>$ or $<RP^b, T'>$

The outcomes listed above are available if this mutual strategic behavior of agents leads to equilibrium. The following discusses the strategic profile is sequential equilibrium in game.

3 Discussion: Equilibrium Agreements

Describe the bargaining process based on dynamic game of incomplete information. The extensive game G, $G =< N, H, P, J^b, J^s >$ [4]: Under the incomplete information, an agent always doesn't know the actual decision node it is at. This is because although an agent knows the price offer by opponent, it does not know opponent's pricing strategy. The element of information set is the subset of its decision nodes. In the paper, an agent's information set is decided by its information state which describes opponent's possible $m \times n$ strategy. The agent makes offer under certain information set which is equivalent to it under corresponding information state.

To discuss the strategies profiles, use the solution concept of sequential equilibrium for the game. There are three key notions related to sequential equilibrium: assessment, sequential rationality and consistency. An assessment is a pair (σ, μ), where σ is a strategy profile and μ is a function that assign to every information set a probability measure on the set of histories in the information set: μ is referred as the belief system. $\mu(\{S_{11}^a, S_{12}^a, \ldots, S_{mn}^a\})(S_{i,j}^a) = \gamma_{i,j}^a$ denotes that $\hat{a}$ believes that a play strategy $S_{i,j}^a$ with probability $\gamma_{i,j}^a$. Recall that $\gamma_{i,j}^a$ is obtained from $\hat{a}$'s lotteries, L_p^a and L_t^a. The bargaining strategies' profiles satisfy the Theorems following:

Theorem1. The assessment $(\sigma, \mu)_{x,y}$ forms a sequential equilibrium of the game for $1 \le x \le 3$ and $1 \le y \le 3$, where $\sigma_a = S_O^a$ for scenario x, $\sigma_{\hat{a}} = S_O^{\hat{a}}$ for scenario y, and $\mu(\{S_{11}^a, S_{12}^a, \ldots, S_{mn}^a\})(S_{i,j}^a) = \gamma_{i,j}^a$ for $1 \le i \le m, 1 \le j \le n$.

Proof. An assessment is a sequential equilibrium of game if it is sequentially rational and consistent. An assessment is sequentially rational if a's strategy of is best response to the other player's strategies, given a 's beliefs at that information set. An assessment is consistent if there is a sequence $((\sigma^n, \mu^n))_{n=1}^{\infty}$ of assessments that converges to (σ, μ) and has the properties that each strategy profile σ^n is completely mixed and that each belief system μ^n is derived from σ^n using Bayes' rule.

Recall that an agent always plays the strategy that offers its own reservation price at its deadline. Thus agent a believes that $\gamma_{i,j}^{\hat{a}}$ is the probability with which the opponent will play the strategy $S_{i,j}^{\hat{a}}$ that offers $RP_i^{\hat{a}}$ at time $T_j^{\hat{a}}$. The different strategies that agent a can play and the expressions for computing a's utility for different strategies are given in last section. Agent a gets maximum EU from strategy S_o^a defined in terms of $RP_I^{\hat{a}}$ and $T_J^{\hat{a}}$. Thus strategy S_o^a is agent a's optimal strategy.

At the turn for opponent $\hat{a}$ offering, $\hat{a}$ forms its information set since it does not know the strategy used by agent a. However, $\hat{a}$ believes that agent a will play strategy $S_{i,j}^a$ with probability $\gamma_{i,j}^a$ where offers the final price RP_i^a at time T_j^a. If it plays strategy $S_{p,q}^{\hat{a}}$ for $1 \le p \le m$ and $1 \le q \le n$, $\hat{a}$'s EU depends on a's strategy and is given by the expression:

$$EU_{p,q}^a = \sum_{i=1}^{m}\sum_{j=1}^{n} \gamma_{i,j}^{\hat{a}} EU^a(S_{p,q}^a, S_{i,j}^{\hat{a}}) \quad . \tag{11}$$

The values of p and q that give agent $\hat{a}$ the maximum EU form its optimal strategy. From last section that agent $\hat{a}$'s optimal strategy is $S_o^{\hat{a}}$ for p = I and q = J. No matter which node in the information set agent $\hat{a}$ is at, strategy $S_o^{\hat{a}}$ is better than the other strategies. The strategy $S_o^{\hat{a}}$ is agent $\hat{a}$'s optimal strategy which agent $\hat{a}$ uses. The assessment $(\sigma, \mu)_{x,y}$ is therefore sequentially rational ($1 \le x \le 3, 1 \le y \le 3$).

The following proves the consistency of the strategy profile and the beliefs. The assessment $(\sigma, \mu)_{x,y}$ in which $\sigma_a = S_o^a$ for scenario x, $\sigma^{\hat{a}} = S_o^{\hat{a}}$ for scenario y, and $\mu(\{S_{11}^a, S_{12}^a, ..., S_{mn}^a\})(S_{i,j}^a) = \gamma_{i,j}^a$ for $1 \le i \le m, 1 \le j \le n$ is consistent since it is the limit as $\varepsilon \to 0$ of assessments $(\sigma^\varepsilon, \mu^\varepsilon)$ where:

i. $$\sigma_a^\varepsilon = (\varepsilon\gamma_{11}^a, \varepsilon\gamma_{12}^a, ..., (1-\varepsilon)\gamma_{I,J}^a, ..., \varepsilon\gamma_{mn}^a) \quad . \tag{12}$$

ii. $$\sigma_{\hat{a}}^\varepsilon = (\varepsilon, \varepsilon, ..., (1-\varepsilon), ..., \varepsilon) \quad . \tag{13}$$

iii. $$\mu^a(\{S_{11}^a, S_{12}^a, ..., S_{mn}^a\})(S_{i,j}^a) = \gamma_{i,j}^a \quad . \tag{14}$$

For $1 \le i \le m, 1 \le j \le n$ and every ε, the entry $(1-\varepsilon)$ in $\sigma_{\hat{a}}$ is for agent $\hat{a}$'s optimal strategy.

The assessment $(\sigma, \mu)_{x,y}$ in which $\sigma_a = S_O^a$ for scenario x, $\sigma_{\hat{a}} = S_O^{\hat{a}}$ for scenario y, and $\mu(\{S_{11}^a, S_{12}^a, ..., S_{mn}^a\})(S_{i,j}^a) = \gamma_{i,j}^a$, for $1 \le i \le m, 1 \le j \le n$ is therefore a sequential equilibrium of the game G, for $1 \le x \le 3$ and $1 \le y \le 3$.

Theorem 2. If the conditions for convergence of optimal strategies are true, the time of agreement is unique for each possible scenario combination. The price of equilibrium agreement is unique if the agents have different deadlines, and $RP_I^a = RP^a$ for $T^a < T^{\hat{a}}$.

Proof. It is straightforward to verify the uniqueness of the time of equilibrium agreement from Table 4. In Table 4, the price of agreement is either RP_I^s or RP^s for $T^s < T^b$, i.e., in rows 1, 2, 4, and 7. On the other hand the price of agreement is either RP_I^b or RP^b for $T^s > T^b$, i.e., rows 3, 5, 8 and 9. Thus for each scenario combination, there are two possible values for the price of agreement. When the agents have different deadlines, the price of agreement is either RP^a or RP_I^a for $T^a < T^{\hat{a}}$. The price of agreement for $T^s = T^b$ is either RP^s or RP^b. This means that the equilibrium solution cannot be unique when $T^s = T^b$. But when the agents have different deadlines, the equilibrium solution is unique if $RP^a = RP_I^a$.

4 Summary and Future Work

The paper presents a novel model for the optimal pricing strategy for bargaining in real e-commence, which is under time limit and incomplete information. The strategy profile forms sequential equilibrium and unique under some conditions. In practice, there is a wide range of environment in which bargaining can take place. In the future, we will extend the optimal bargaining strategy to muti-agent (beyond three); secondly, build the bargaining simulation platform to prove the strategy effective and efficient; finally, emphasize on the optimal strategies for barraging on multi-issue.

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Why Do Information Gatekeepers Charge Zero Subscription Fees?

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Abstract. In the Internet market, the revenue of an information gatekeeper comes in two forms: advertising fees paid by firms who post their prices at the gatekeeper's site, and subscription fees paid by consumers who obtain the list of prices charged by different firms from the gatekeeper's site. By extending Varian's (1980) Model of Sales, this paper establishes conditions for an information gatekeeper to offer totally free subscriptions to consumers.

1 Introduction

In the Internet market, the revenue of an information gatekeeper comes in two forms: advertising fees paid by firms who post their prices at the gatekeeper's site, and subscription fees paid by consumers who obtain the list of prices charged by different firms from the gatekeeper's site. It is interesting to observe that most gatekeepers charge zero subscription fees to consumers who acquire information from their sites. Why do gatekeepers provide consumers with information free of charge? This paper aims to answer this question.

Michael Baye and John Morgan (2001) set up a model that is based on Varian's (1980) Model of Sales. They establish an equilibrium characterized by the following features: The optimizing gatekeeper sets positive subscription fees low enough to induce all consumers to subscribe, and sets relatively high advertising fees to induce only partial participation by firms; Advertised prices are lower than unadvertised monopoly prices; All consumers purchase from the firm that offers the lowest price at the gatekeeper's site.

Michael Baye and John Morgan (2001) fail to explain why a gatekeeper charges zero subscription fee to consumers who subscribe at the gatekeeper's site. Zero subscription fees make sense since different charges may result in quite different market structures. Low as it is, a positive subscription fee still has important effects on trade behavior. Moreover, only partial participation by consumers is observed on the Internet, which conflicts the equilibrium established in Michael Baye and John Morgan (2001).

This paper aims to formulate a model to justify the following conclusion: The optimizing gatekeeper provides consumers with totally free subscriptions in equilibrium.

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2 The Basic Model

In the market for a homogenous product, there are $n \geq 2$ firms, $l \geq 2$ consumers and a monopolistic gatekeeper. Let $N = \{1, 2, \cdots, n\}$ be the set of firms and $L = \{1, 2, \cdots, l\}$ be the set of consumers. Firms, consumers and the gatekeeper are assumed to be risk neutral. Consumers are evenly divided among n local firms. Local markets are completely segmented: Consumers in local market i only have access to firm i; Each local market is served by a single local firm.

The Internet offers the potential for a gatekeeper to eliminate geographic boundaries through the creation of the Internet market. The optimizing gatekeeper charges fees to firms and consumers who post and access information from her site. The fee paid by firms is called an advertising fee and denoted $\tau \geq 0$. The fee paid by consumers is called a subscription fee and denoted $s \geq 0$. There is no information leakage between subscribers and nonsubscribers. If a firm advertises, he has to charge the same price to the consumers in his local market that he posts at the gatekeeper's site. For simplicity, the cost of delivering goods to consumers is zero.

The above assumptions are based on Michael Baye and John Morgan (2001). However, two modifications are made in this paper.

The Internet market is only accessible to a fraction q of consumers, where $q \in (0, 1)$ is exogenously given. This assumption reflects the fact that only those consumers who acquire adequate computer skills and adopt the right search strategy can locate the gatekeeper's site. Of these ql consumers, those who subscribe are permitted to buy from any of the firms who post their prices at the gatekeeper's site.

All consumers have identical income $I > 0$, but they differ in their preference for the product. Consumer preference for the product is indicated by a parameter $t > 0$. The preference parameter of a consumer is private information to herself. The preference parameter t is drawn independently of each other from an interval $[\underline{t}, 1]$ according to the continuously differentiable distribution function $H(t)$, where $\underline{t} \in (0, 1)$ is given and $H(t)$ is common knowledge. A consumer of type t has the following indirect utility function

$$u(p, y) = y - tv(p) \tag{1}$$

where $p > 0$ is the price the consumer pays for the product, $y = I - s$ if the consumer is a subscriber and $y = I$ if the consumer is a nonsubscriber, function $v(p)$ is continuously differentiable with $v'(p) > 0$. There exists a unique $p_m \in (0, \infty)$ such that

$$p_m = \arg \underset{p \geq 0}{Max}\, v'(p)(p - c) \tag{2}$$

where $c \geq 0$ is constant marginal production cost.

Assume that, as long as the price listing services on the Internet are setup, the operation cost of the gatekeeper is negligible. The gatekeeper's expected profit is

$$\phi(s, \tau) = bls + an\tau - K \tag{3}$$

where $K > 0$ is the fixed setup cost of establishing the price listing services.

The timing of the above pricing game is as follows: The gatekeeper announces advertising (τ) and subscription (s) fees; Given advertising and subscription fees, consumers who are accessible to the Internet market decide whether or not to subscribe to the gatekeeper's site; Given the fraction $b = q\left[1 - H\left(t^*\right)\right]$ of consumers who subscribe, firms price their products and decide whether or not to post their prices on the gatekeeper's site. This paper will engage in backward induction to derive the gatekeeper's fee-setting strategy.

Mixed-Strategy Equilibrium: Suppose $t^* \in [\underline{t}, 1]$, $a \in [0,1]$ and $F(p)$ is a distribution with support $\left[\underline{p}, \overline{p}\right] \subset [0, \infty)$. Given the distribution $H(t)$ of consumer type, advertising fee $\tau \geq 0$ and subscription fee $s \geq 0$, $\{t^*, [a, F(p)]\}$ is a mixed-strategy equilibrium of the above pricing game, if t^*, a and $F(p)$ satisfy the following conditions:

1. Provided that each firm advertises his price with probability a and that the advertised prices have the distribution $F(p)$. For any of the ql consumers who are accessible to the Internet market, paying the subscription fee $s \geq 0$ is optimal if the consumer type $t > t^*$, and not paying the subscription fee $s \geq 0$ is optimal if the consumer type $t \leq t^*$;

2. Provided that a fraction $b = q\left[1 - H\left(t^*\right)\right]$ of consumers pay the subscription fee $s \geq 0$, it is optimal for each firm to post his price with probability a, and the advertised prices have the distribution $F(p)$.

3 Firm Pricing and Advertising Strategies

Lemma 1: Given the price $p \geq 0$ of the product, the individual demand function of a consumer of type $t \in [\underline{t}, 1]$ is $x(p) = tv'(p)$.

Lemma 1 implies that there is no income effect in the individual demand. Other conditions being equal, the quantity a consumer purchases is irrelevant whether or not the consumer pays subscription fee $s \geq 0$. Given the same price paid for the product, the quantity a consumer purchases depends on the preference parameter t. The proof of lemma 1 is simply an application of the Roy's Identity:

$$x(p, y) = -\frac{\partial u(p, y)}{\partial p} \Bigg/ \frac{\partial u(p, y)}{\partial y} = tv'(p) \tag{4}$$

Proposition 1: Given the fraction $b \in (0,1)$ of consumers who subscribe. In equilibrium, the probability that a firm posts his price at the gatekeeper's site is

$$a(\tau, b) = 1 - \left[\frac{n\tau}{(n-1)\, lb\pi(p_m)}\right]^{\frac{1}{n-1}} \tag{5}$$

where

$$\pi(p) \equiv (p - c)\, v'(p) \int_{\underline{t}}^{1} t dH(t), \text{ for each } p \in \left[\underline{p}, \overline{p}\right] = \left[\underline{p}, p_m\right] \tag{6}$$

The advertised prices have the following distribution

$$F(p) = \frac{1}{a}\left(1 - \left\{\frac{\pi(p_m)\, l\left[(1-b) + b(1-a)^{n-1}\right] - \pi(p)\, l(1-b) + n\tau}{nbl\pi(p)}\right\}^{\frac{1}{n-1}}\right) \tag{7}$$

for every $p \in \left[\underline{p}, p_m\right)$, where

$$\underline{p} = \pi^{-1}\left\{\frac{l(n-1)(1-b)\pi(p_m) + n^2\tau}{l(n-1)[1+(n-1)b]}\right\} \tag{8}$$

Firms who do not advertise charge monopoly price $p = p_m$ in their local markets.
Proof of proposition 1: Consider the representative firm $i \in N$ and the representative consumer $j \in L$. Firm i charges price $p \in \left[\underline{p}, \overline{p}\right]$ while each of the other $n-1$ firms advertises his price with probability $a > 0$, and each advertising firm engages in nondegenerate mixed-strategy $F(\cdot)$ with support $\left[\underline{p}, \overline{p}\right]$.
If firm i advertises, then the probability that consumer j buys from firm i is

$$\begin{aligned} \delta_A &= (1-b)\tfrac{1}{n} + b\sum_{k=0}^{n-1} C_{n-1}^k a^k (1-a)^{n-1-k}[1-F(p)]^k \\ &= (1-b)\tfrac{1}{n} + b[1-aF(p)]^{n-1} \end{aligned} \tag{9}$$

where the second equality follows by the Binomial Theorem. There are a total of $l \geq 2$ consumers who buys from firm i with probability δ_A. The number of consumers firm i attracts has a Binomial distribution with parameters δ_A and $l \geq 2$. Thus, the expected profit for the advertising firm who charges price p is given by

$$\begin{aligned} \pi(A,p) &= \pi(p)\sum_{k=0}^{l} kC_l^k \delta_A^k (1-\delta_A)^{l-k} - \tau \\ &= l\pi(p)\,\delta_A - \tau \\ &= l\pi(p)\left\{(1-b)\tfrac{1}{n} + b[1-aF(p)]^{n-1}\right\} - \tau \end{aligned} \tag{10}$$

If firm i does not advertise, then the probability that consumer j buys from firm i is

$$\delta_N = (1-b)\frac{1}{n} + b(1-a)^{n-1}\frac{1}{n} = \frac{1}{n}\left[(1-b) + b(1-a)^{n-1}\right] \tag{11}$$

The expected profit to firm i who charges price p in his local market is given by

$$\begin{aligned} \pi(N,p) &= \pi(p)\sum_{k=0}^{l} kC_l^k \delta_N^k (1-\delta_N)^{l-k} = l\pi(p)\,\delta_N \\ &= \tfrac{l}{n}\pi(p)\left[(1-b) + b(1-a)^{n-1}\right] \end{aligned} \tag{12}$$

By equation (2), the above expected profit is maximized at price $p = p_m$. Firms who do not advertise their prices simply charge the monopoly price $p = \overline{p} = p_m$ to his local customers.

In equilibrium, the representative firm is indifferent whether advertise his price or not. That is, the expected profit function of (10) valued at any $p \in \left[\underline{p}, \overline{p}\right] = \left[\underline{p}, p_m\right]$ equals that of (12) valued at $p = \overline{p} = p_m$. Hence

$$\begin{aligned} \pi(A,p) &= l\pi(p)\left\{(1-b)\frac{1}{n} + b\left[1 - aF(p)\right]^{n-1}\right\} - \tau \\ &= \frac{l}{n}\pi(\overline{p})\left[(1-b) + b(1-a)^{n-1}\right] = \pi(N,\overline{p}) \end{aligned} \tag{13}$$

Solving for $F(p)$ in equation (13) gives (7). Letting $F(p_m) = 1$ in equation (13) and solving for a yields (5). Substituting (5) in (13), setting $F\left(\underline{p}\right) = 0$ and solving for $\underline{p}$ yields (8). This completes the proof of proposition 1.

4 Consumer Shopping Strategies

To find consumer subscription decisions, one has to compare utilities conditional on subscribing and nonsubscribing. The utility derived from subscription depends on the price paid by subscribers. Lemma 2 gives the distribution of the price paid by subscribers.

Lemma 2: Suppose that advertised prices have a nondegenerate distribution $F(p)$ with support $\left[\underline{p}, p_m\right]$,and that there are $k \geq 1$ firms who advertise at the gatekeeper's site. The distribution of the price paid by subscribers is given by

$$F_{\min}^{k}(p) = 1 - \left[1 - F(p)\right]^{k}, \text{ for each } p \in \left[\underline{p}, p_m\right] \tag{14}$$

Proof of lemma 2: Let $\{p_i\}_{i=1}^{k}$ denote the prices advertised by k firms, and let $p_{\min}^{k} = \min\{p_i\}_{i=1}^{k}$ denote the lowest price. It follows from the definition of accumulative distribution that

$$\begin{aligned} 1 - F_{\min}^{k}(p) &= prob\left\{p_{\min}^{k} \geq p\right\} = prob\left\{p_i \geq p : i = 1, 2, \cdots, k\right\} \\ &= \left[1 - F(p)\right]^{k} \end{aligned} \tag{15}$$

Rearranging terms of the above equation yields (14).

Proposition 2: Given the probability $a \in (0,1)$ that each firm advertises, the distribution $F(p)$ of advertised prices with support $\left[\underline{p}, p_m\right]$, and the subscription fee $s \geq 0$ charged by the gatekeeper. If s is sufficiently small, there exists a unique

$$t^*(a,s) = s\left[\int_{\underline{p}}^{p_m} v'(p)\left\{\left[1 - aF(p)\right] - \left[1 - aF(p)\right]^{n}\right\}dp\right]^{-1} \in \left[\underline{t}, 1\right] \tag{16}$$

such that consumers of type $t \leq t^*$ do not subscribe and consumers of type $t > t^*$ subscribe. The fraction of subscribers is given by

$$b(a,s) = q\left[1 - H(t^*)\right] \tag{17}$$

Proof of proposition 2: It follows from proposition 1 and lemma 2 that the expected utility of a subscriber can be expressed as follows

$$\begin{aligned} u(S,t) &= (1-a)^n \left[I - tv(p_m)\right] - s \\ &\quad + \sum_{k=1}^{n} C_n^k a^k (1-a)^{n-k} \int_{\underline{p}}^{p_m} \left[I - tv(p)\right] d\left\{1 - \left[1 - F(p)\right]^k\right\} \\ &= I - tv(p_m) - s + t \sum_{k=1}^{n} C_n^k a^k (1-a)^{n-k} \int_{\underline{p}}^{p_m} v'(p) \left\{1 - \left[1 - F(p)\right]^k\right\} dp \end{aligned} \tag{18}$$

The expected utility of a nonsubscriber is given by

$$\begin{aligned} u(N,t) &= \left[I - tv(p_m)\right](1-a) + a \int_{\underline{p}}^{p_m} \left[I - tv(p)\right] dF(p) \\ &= I - tv(p_m) + at \int_{\underline{p}}^{p_m} v'(p) F(p)\, dp \end{aligned} \tag{19}$$

Subtracting (19) from (18) yields the difference between expected utilities conditional on subscribing and nonsubscribing

$$\begin{aligned} &\Delta u(t) \equiv u(S,t) - u(N,t) \\ &= t\left(\sum_{k=1}^{n} C_n^k a^k (1-a)^{n-k} \int_{\underline{p}}^{p_m} v'(p) \left\{1 - \left[1 - F(p)\right]^k\right\} dp - a \int_{\underline{p}}^{p_m} v'(p) F(p)\, dp\right) - s \\ &= t\left(\sum_{k=0}^{n} C_n^k a^k (1-a)^{n-k} \int_{\underline{p}}^{p_m} v'(p) \left\{1 - \left[1 - F(p)\right]^k\right\} dp - a \int_{\underline{p}}^{p_m} v'(p) F(p)\, dp\right) - s \\ &= t \int_{\underline{p}}^{p_m} v'(p) \left\{\left[1 - aF(p)\right] - \left[1 - aF(p)\right]^n\right\} dp - s \end{aligned} \tag{20}$$

The derivative of the above function satisfies

$$\frac{d}{dt}\left[\Delta u(t)\right] = \int_{\underline{p}}^{p_m} v'(p) \left\{\left[1 - aF(p)\right] - \left[1 - aF(p)\right]^n\right\} dp > 0 \tag{21}$$

If subscription fee $s > 0$ is sufficiently small, then there exists a unique $t^* \in [\underline{t}, 1]$ such that $u(S, t^*) = u(N, t^*)$. Setting (20) equal zero and solving for t yield (16). Since the difference $\Delta u(t)$ of expected utilities is a strictly increasing function of consumer type t, it is in the consumer's interests not to subscribe if consumer type $t \le t^*$; it pays to subscribe if consumer type $t > t^*$. This completes the proof of proposition 2.

Given a and $F(p)$, equation (17) suggests that the fraction b of consumers who subscribe depends on t^*; equation (16) suggests that the value of t^* depends on the subscription fee s. By proposition 2, the rate of change of b as s changes can be measured by the derivative $\partial b/\partial s = -qH'(t^*)\,\partial t^*/\partial s$, where $\partial t^*/\partial s$ is a constant and $H'(t^*)$ is the only variable. In the eyes of the gatekeeper, $H'(t^*)$ decides the change of b caused by an increase of the subscription fee. Hence $H'(t^*)$ can be interpreted as the reaction by consumers to a small change of s.

5 Gatekeeper Fee-Setting Strategies

By equation (3), propositions 1 and 2, the gatekeeper who charges subscription (s) and advertising (τ) fees yields the following expected profit

$$\phi(s,\tau) = b(a,s)\,ls + a(b,\tau)\,n\tau - K \tag{22}$$

where a and b are given by (5) and (17) respectively. Proposition 3 states that, if a is sufficiently small, then the gatekeeper may find it optimal to charge zero subscription fee.

Proposition 3: Suppose $t^* \in [\underline{t}, 1]$. If $1 - t^* H'(t^*) < 0$, then there exists $\varepsilon_a > 0$ such that $\partial\phi(s,\tau)/\partial s < 0$ for any $a \le \varepsilon_a$, $s > 0$ and $\tau > 0$.

Proof of proposition 3: By the Chain Rule, differentiating (22) with respect to s gives

$$\begin{aligned} \frac{\partial\phi(s,\tau)}{\partial s} &= b(a,s)\,l + ls\left[\frac{\partial b(a,s)}{\partial s} + \frac{\partial b(a,s)}{\partial a}\frac{\partial a(b,\tau)}{\partial b}\frac{\partial b(a,s)}{\partial s}\right] + n\tau\frac{\partial a(b,\tau)}{\partial b}\frac{\partial b(a,s)}{\partial s} \\ &= b(a,s)\,l + \frac{\partial b(a,s)}{\partial s}\left\{ls + \frac{\partial a(b,\tau)}{\partial b}\left[ls\frac{\partial b(a,s)}{\partial a} + n\tau\right]\right\} \end{aligned} \tag{23}$$

Differentiating (5) with respect to b yields

$$\frac{\partial a(b,\tau)}{\partial b} = \frac{\partial}{\partial b}\left\{1 - \left[\frac{n\tau}{(n-1)lb\pi(p_m)}\right]^{\frac{1}{n-1}}\right\} = \frac{1}{n-1}\left(\frac{1-a}{b}\right) > 0 \tag{24}$$

for any $a, b \in (0,1)$. Denote

$$\begin{aligned} A(a) &= \int_{\underline{p}(a)}^{p_m} v'(p)\left\{[1 - aF(p)] - [1 - aF(p)]^n\right\} dp > 0 \\ B(a,p) &= [1 - aF(p)]^{n-1} \\ C(a,p) &= [1 - aF(p)] - [1 - aF(p)]^n = B^{\frac{1}{n-1}}(a,p) - B^{\frac{n}{n-1}}(a,p) \end{aligned} \tag{25}$$

It follows from equations (17) and (16) that

$$\begin{aligned} \frac{\partial b(a,s)}{\partial s} &= -\frac{q}{A(a)}H'\left[\frac{s}{A(a)}\right] \\ \frac{\partial b(a,s)}{\partial a} &= sqH'\left[\frac{s}{A(a)}\right]\frac{A'(a)}{A^2(a)} \end{aligned} \tag{26}$$

It follows by Liebnitz's Rule that

$$\begin{aligned} A'(a) &= \int_{\underline{p}(a)}^{p_m} v'(p)\frac{\partial C(a,p)}{\partial a}dp - v'\left[\underline{p}(a)\right]C\left[a, \underline{p}(a)\right] \\ &= \int_{\underline{p}(a)}^{p_m} v'(p)\frac{\partial C(a,p)}{\partial a}dp - v'\left[\underline{p}(a)\right]\left(\left\{1 - aF\left[\underline{p}(a)\right]\right\} - \left\{1 - aF\left[\underline{p}(a)\right]\right\}^n\right) \\ &= \int_{\underline{p}(a)}^{p_m} v'(p)\frac{\partial C(a,p)}{\partial a}dp \\ &= \frac{1}{n-1}\int_{\underline{p}(a)}^{p_m} v'(p)\,B^{\frac{2-n}{n-1}}(a,p)\left[1 - nB(a,p)\right]\frac{\partial B(a,p)}{\partial a}dp \end{aligned} \tag{27}$$

Substituting (5) in (7) and rearranging terms yields

$$B\left(a,p\right)=\left[1-aF\left(p\right)\right]^{n-1}=\frac{\pi(p_m)l\left[(1-b)+b(1-a)^{n-1}\right]-\pi(p)l(1-b)+n\tau}{bnl\pi(p)} \tag{28}$$

which implies that

$$\begin{array}{l} B\left(a,p\right)=\left[1-aF\left(p\right)\right]^{n-1}\geq\left(1-a\right)^{n-1}>0 \\ \frac{\partial B(a,p)}{\partial a}=-\frac{(n-1)(1-a)^{n-2}\pi(p_m)}{n\pi(p)}<0 \end{array}, \text{ for every } p\in\left[\underline{p},p_m\right] \tag{29}$$

Since $aF\left(p\right)\leq a$ for any $a\in(0,1)$ and $p\in\left[\underline{p},p_m\right]$, it follows that

$$1-nB\left(a,p\right)=1-n\left[1-aF\left(p\right)\right]^{n-1}\leq 1-n\left(1-a\right)^{n-1} \tag{30}$$

Let

$$\varepsilon_a=1-\left(\frac{1}{n}\right)^{\frac{1}{n-1}} \tag{31}$$

then $\varepsilon_a>0$. If $a\leq\varepsilon_a$, then

$$1-nB\left(a,p\right)\leq 1-n\left(1-a\right)^{n-1}\leq 0, \text{ for every } p\in\left[\underline{p},p_m\right] \tag{32}$$

It follows from equations (24), (25), (26), (29) and (32) that

$$\frac{\partial b}{\partial s}<0,\ \frac{\partial b}{\partial a}>0 \text{ and } \frac{\partial a}{\partial b}>0\text{ , for any } a\leq\varepsilon_a,\ s>0 \text{ and } \tau>0 \tag{33}$$

If $1-t^*H'\left(t^*\right)<0$, then

$$\begin{aligned} \frac{\partial\phi(s,\tau)}{\partial s}&=b\left(a,s\right)l+\frac{\partial b(a,s)}{\partial s}\left\{ls+\frac{\partial a(b,\tau)}{\partial b}\left[ls\frac{\partial b(a,s)}{\partial a}+n\tau\right]\right\} \\ &<l\left[b+s\frac{\partial b(a,s)}{\partial s}\right]=l\left\{q\left[1-H\left(t^*\right)\right]-\frac{sq}{A(a)}H'\left[\frac{s}{A(a)}\right]\right\} \\ &=l\left\{q\left[1-H\left(t^*\right)\right]-qt^*H'\left(t^*\right)\right\}=ql\left[1-H\left(t^*\right)-t^*H'\left(t^*\right)\right]<0 \end{aligned} \tag{34}$$

where the first inequality follows from (33), the second equality follows from (17) and (26), and the third equality follows from (16). This completes the proof of proposition 3.

Proposition 4 establishes that, even if zero subscription fee strategy is optimal, there still exists a positive advertising fee strategy that maximizes the expected profit of the gatekeeper.

Proposition 4: If $1-\underline{t}H'\left(\underline{t}\right)<0$, then $s^*=0$ is the optimal subscription fee strategy; there exists $\tau^*>0$ such that τ^* maximizes the expected profit of the gatekeeper.

Proof of proposition 4: By (16), if $s=0$, then $t^*=\underline{t}$; by (17), if $t^*=\underline{t}$, then $b=q\left[1-H\left(\underline{t}\right)\right]=q$. Thus, equation (3) can be rewritten as follows

$$\phi\left(0,\tau\right)=a\left(b,\tau\right)n\tau-K=a\left(q,\tau\right)n\tau-K \tag{35}$$

Differentiating (5) with respect to τ yields

$$\frac{\partial a\left(b,\tau\right)}{\partial \tau}=\frac{\partial}{\partial \tau}\left\{1-\left[\frac{n\tau}{\left(n-1\right)lb\pi\left(p_m\right)}\right]^{\frac{1}{n-1}}\right\}=-\frac{1}{n-1}\left(\frac{1-a}{\tau}\right) \tag{36}$$

Differentiating (35) with respect to τ gives

$$\frac{\partial \phi\left(0,\tau\right)}{\partial \tau}=n\left[a\left(q,\tau\right)+\tau\frac{\partial a\left(q,\tau\right)}{\partial \tau}\right]=\left(\frac{n}{n-1}\right)\left[na\left(q,\tau\right)-1\right] \tag{37}$$

where the second equality follows from (36). Setting (37) equal zero yields the first order condition for the gatekeeper's expected profit maximization problem

$$\frac{1}{n}=a\left(q,\tau^*\right)=1-\left[\frac{n\tau^*}{\left(n-1\right)lq\pi\left(p_m\right)}\right]^{\frac{1}{n-1}} \tag{38}$$

where the second equality follows from equation (5). Rearranging terms of (38) yields the gatekeeper's optimal advertising fee strategy

$$\tau^*=ql\left(\frac{n-1}{n}\right)^n\pi\left(p_m\right) \tag{39}$$

Differentiating (37) with respect to τ yields

$$\frac{\partial^2 \phi\left(0,\tau^*\right)}{\partial \tau^2}=\left(\frac{n^2}{n-1}\right)\frac{\partial a\left(q,\tau^*\right)}{\partial \tau^*}=-\left(\frac{n}{n-1}\right)^2\left[\frac{1-a\left(q,\tau^*\right)}{\tau^*}\right]<0 \tag{40}$$

where the second equality follows from (36). Equation (40) implies that (39) is also sufficient for the gatekeeper's expected profit maximization problem.
The above discussion only proves that (39) is optimal conditional on $s=0$. It remains to prove that, if $1-\underline{t}H'\left(\underline{t}\right)<0$, then $s^*=0$ is the optimal subscription fee strategy for the gatekeeper. To this end, one only needs to prove

$$\frac{\partial \phi\left(0,\tau^*\right)}{\partial s}<0 \tag{41}$$

By (38), if the gatekeeper charges advertising ($\tau=\tau^*$) and subscription ($s^*=0$) fees to firms and consumers who advertise and obtain information at the gatekeeper's site, the optimal advertising strategy for firms is $a\left(q,\tau^*\right)=1/n$. In order to make use of proposition 3, one only needs to prove

$$\frac{1}{n}=a\left(q,\tau^*\right)\leq\varepsilon_a=1-\left(\frac{1}{n}\right)^{\frac{1}{n-1}},\text{ for every } n\geq 2 \tag{42}$$

which is equivalent to

$$n^{n-2}\leq\left(n-1\right)^{n-1},\text{ for every } n\geq 2 \tag{43}$$

which is equivalent to

$$\sum_{k=0}^{n-2} C_{n-2}^{k} (n-1)^{k} = [(n-1)+1]^{n-2} \le (n-1)(n-1)^{n-2} = \sum_{k=0}^{n-2} (n-1)^{n-2} \tag{44}$$

for every $n \ge 2$, where the first equality follows from the Binomial Theorem. To see (44) holds, one only needs to prove

$$C_{n-2}^{k} (n-1)^{k} \le (n-1)^{n-2}, \text{ for any } n \ge 2 \text{ and } k = 0, 1, \cdots, n-2 \tag{45}$$

which is equivalent to

$$C_{n-2}^{k} \le (n-1)^{n-2-k}, \text{ for any } n \ge 2 \text{ and } k = 0, 1, \cdots, n-2 \tag{46}$$

Since

$$C_{n-2}^{k} = \tfrac{(n-2)!}{k![(n-2)-k]!} \le \tfrac{(n-2)!}{k!} = \prod_{j=1}^{n-2-k} (n-1-j) = (n-1)^{n-2-k} \tag{47}$$

for any $n \ge 2$ and $k = 0, 1, \cdots, n-2$, it follows that

$$a(q, \tau^{*}) = \frac{1}{n} \le 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}} = \varepsilon_{a}, \text{ for every } n \ge 2 \tag{48}$$

By proposition 3, if $1 - \underline{t} H'(\underline{t}) < 0$ and (48) holds, then (46) holds. This completes the proof of proposition 4.

6 Conclusions

This paper has shown how the gatekeeper finds it in her interest to charge zero subscription fee to consumers who acquire information at the gatekeeper's site. This paper has also solved explicitly the optimal subscription and pricing strategies of consumers and firms.

Proposition 4 can be interpreted as follows: If the size $q \in (0, 1)$ of the Internet market is sufficiently small, and the reaction $H'(\underline{t})$ by subscribers is sufficiently large, then it is optimal for the gatekeeper to offer free subscription services.

This paper only considers the market with a monopoly gatekeeper. The analysis concerning multiple gatekeepers with this framework is analogous with that of Michael R. Baye and John Morgan (2001). If there are multiple gatekeepers, one might expect competition to result in even lower subscription and advertising fees. This may partially explain why gatekeepers offer other services free of charge to consumers, such as free e-mail boxes besides totally free subscriptions. As a result of lower advertising fees, the probability that firms advertise goes up, while the number of consumers a gatekeeper attracts is driven smaller by competition. This explains why one actually observes more advertising firms than this paper predicts.

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Cost-Driven Web Service Selection Using Genetic Algorithm*

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Abstract. Web services composition has been one of the hottest research topics. But with the ever increasing number of functional similar web services being made available on the Internet, there is a need to be able to distinguish them using a set of well-defined Quality of Service (QoS) criteria. The cost is the primary concern of many business processes. In this paper, we propose a new solution using Genetic Algorithm (GA) in cost-driven web service selection. GA is utilized to optimize business process composed of many service agents (SAg). Each SAg corresponds to a collection of available web services provided by multiple service providers to perform a specific function. Service selection is an optimization process with taking into account the relationships among the services. Better performance has been gotten using GA in the paper than using local service selection strategy. The global optimal solution might also be achieved with proper GA parameters.

1 Introduction

Web services are self-describing software applications that can be advertised, located, and used across the Internet using a set of standards such as SOAP, WSDL, and UDDI [1]. A single web service is most likely inadequate to serve the customers' business needs; it takes a selection of various web services composed together to form a business process. Web services composition [2] has been one of the hottest research topics in this new field. However, with the ever increasing number of functional similar web services being made available on the Internet, there is a need to be able to distinguish them using a set of well-defined Quality of Service (QoS) criteria. QoS is a broad concept that encompasses a number of non-functional properties such as cost, response time, availability, reliability, and reputation [3]. These properties apply both to standalone web services and to web services composed of other web services (i.e., composite web services). Typically, cost and time are two primary factors that customers are concerned about.

The challenge is to select the services that not only satisfy the individual requirements but also best fit the overall composed business process. Therefore, the entire business process needs to be optimized prior to execution. The philosophy of Genetic Algorithm (GA) [4] mimics the evolution process of "the survival of the fittest" in

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nature. GA is a parallel global optimization algorithm with high robustness, and it is not restricted by natures of the optimization problem, such as continuity and differentiability. Only the objective function and the corresponding fitness level influence the direction of search, so it is very suited to those complicated optimization problems that cannot be handled efficiently by traditional optimization algorithms (methods of calculus or techniques of exhaustive search).

In this paper, we take the cost as the primary concern of many business processes. Using an integer string to represent a web services composition, the best one is the string that leads to the lowest cost. A service selection model using GA is proposed to optimize business process composed of many service agents (SAg). Individual SAg corresponds to a collection of available web services provided by multiple service providers to perform a specific function. Service selection using GA is an optimization process taking into account the relationships among the services.

The remainder of the paper is organized as follows. Section 2 discusses related work. Section 3 describes GA utilized in our web service selection model. In Section 4, a service selection case is presented. Finally, Section 5 concludes the paper.

2 Related Work

In [2], authors present a framework SELF-SERV for declarative web services composition using state-charts. Their service selection method uses a local selection strategy. The service selection is determined independently to other tasks of the composite services. It is only locally optimal.

In [5], authors make research on the end-to-end QoS issues of composite service by utilizing a QoS broker that is responsible for coordinating the individual service component to meet the quality constraint. Authors design the service selection algorithms used by QoS brokers to meet the end-to-end QoS constraints. The service selection problem is modeled as the Multiple Choice Knapsack Problem (MCKP). Authors give three algorithms and recommend the Pisinger's algorithm as the best one. But their solution is usually too complex for run-time decisions.

In [6], authors present the Web Services Outsourcing Manager (WSOM) framework via a mathematical model for dynamic business processes configuration using existing web services to meet customers' requirements, and propose a novel mechanism to map a service selection space {0,1} to utilize global optimization algorithms - Genetic Algorithms. The binary encoding is adopted in the algorithm, but it is not human-readable. When the business process is complicated, the chromosome will be too lengthy. Moreover, GA utilization in the service selection is not given in detail, especially about how the relationships among the services will affect the global optimization.

3 GA Utilized in Web Service Selection Model

3.1 Web Service Selection Model Architecture

The web service selection model is based on a multi-agent platform, which is composed of multiple components (See Fig. 1).

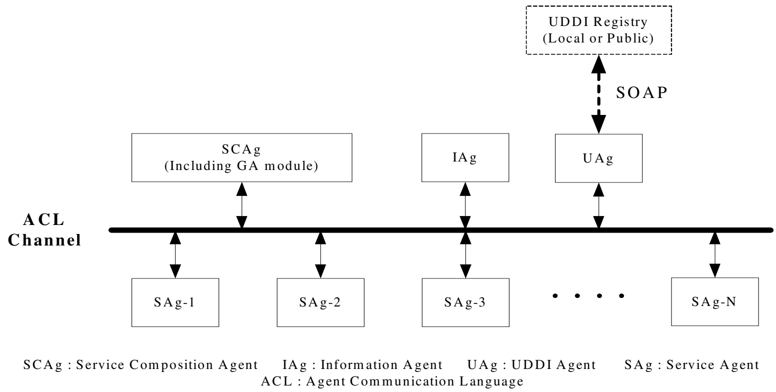

Fig. 1. Model architecture

1. **Service Agent (SAg)** corresponds a list of web services that have the specific function. Its capability depends on the specific function. SAg possesses all useful information about those web services, such as cost, response time, and so on. In order to facilitate dynamic selection, up to date information concerning parameters that affect the decision of web service activation must be gathered.
2. **UDDI agent (UAg)** is the broker of inner SAg when asking for outer UDDI registry. SAg might get information about required web services via UAg.
3. **Information agent (IAg)** is the information center of the model. All SAg must register themselves with it.
4. **Service composition agent (SCAg)** is responsible for administering the composite business process as a services flow (SF). Using the modeling tool, users can model the SF in advance. The model is composed of many predefined or customized SAg. GA module is the core of SCAg. We will depict it in the remainder of the paper. Before the composite business process is instantiated, SCAg will activate GA module to optimize the service selection to get the final executable SF.
5. **ACL channel** [7] is the communication bus among those agents mentioned above.

3.2 Problem Objective Function

When the cost becomes the sole primary factor that customers are concerned about, the service selection is equivalent to a single-objective optimization problem. There are mainly two kinds of service flow.

3.2.1 Pipeline Service Flow. For a pipeline services flow that has N steps (N SAg in an execution path) $(S_1, S_2, ..., S_N)$, it only has one type structure: sequence one. The control flow will come through all its SAg, and only one proper web service in each SAg will be chosen to bind to. The overall cost of the SF is equal to the summation cost of all its components.

$$\begin{aligned} &\cos t(SF) = \sum_{i=1}^{N} \cos t(S_{ik}),\ 1 \le k \le size(S_i) \\ &size(S_i) = |S_i|,\ i = 1, ..., N \end{aligned} \tag{1}$$

The objective function is: $Min\ \cos t(SF)$.

3.2.2 Hybrid Service Flow. For a hybrid services flow that has N steps (No more than N SAg in an execution path) $(S_1, S_2, ..., S_N)$, we assume it has three type structures: sequence one, parallel one and conditional branching one. The overall cost of the hybrid SF is described as follows:

$$\begin{aligned} &\cos t(SF) = \sum_{i=1}^{N} \cos t(S_{ik}) * CF_i,\ 1 \le k \le size(S_i) \\ &size(S_i) = | S_i |,\ i = 1,...,N \\ &CF_i \in \{0,1\},\ i = 1,...,N \end{aligned}$$

For some SAg linked by conditional branching structure, only one of them can be passed by the control flow at one execution. Thus the hybrid SF has some different execution path. The overall cost of each execution path can always be represented by the summation cost of its subset components.

$$\begin{aligned} &\cos t(SF) = \sum_{i=1}^{M} \cos t(S_{ik}),\ 1 \le k \le size(S_i) \\ &size(S_i) = | S_i |,\ i = 1,...,M \\ &M \le N \end{aligned} \tag{2}$$

The objective function is also: $Min\ \cos t(SF)$.

3.3 GA Module

The philosophy of GA mimics the evolution process of “the survival of the fittest” in nature. A fitness function is firstly defined to evaluate the quality of an individual, called a chromosome, which represents a solution to the optimization problem. GA starts with a randomly initialized population. The population then evolves through a repeated routine, called a generation, in which GA employs operators such as selection, crossover, and mutation borrowed from natural genetics to improve the fitness of individuals in the population. In each generation, chromosomes are evaluated by the fitness function. After a number of generations, highly fit individuals, which are analogous to good solutions to a given problem, will emerge.

3.3.1 Solution Representation. The representation scheme determines how the problem is structured in GA and also influences the genetic operators that are used. Which kind of solution representation is used in GA depends on characteristics of the optimization problem. Traditional GA encodes each problem solution into a binary string, called chromosome, which facilities studying GA by schema theorem. However, the binary encoding is not human-readable, which makes it difficult to develop efficient genetic operators that can make good use of the specialized knowledge for the service selection problem.

The integer encoding provides a convenient and natural way of expressing the mapping from representation to solution domain. With integer encoding, the interpretation of the solution representation for service selection is straightforward, so the integer encoding is used in this paper. The solution to service selection is encoded into a vector of integers, where the i^{th} element of an individual is k if the k^{th} web service in SAg i is selected. For example, the service flow has four SAg

$\{S_1, S_2, S_3, S_4\}$, each of which has four candidate services, and one possible individual is $[1\,4\,2\,3]$. Then the corresponding composite service is $\{S_{11}, S_{24}, S_{32}, S_{43}\}$, which means the 1^{th} service of S_1, the 4^{th} service of S_2, the 2^{th} service of S_3 and the 3^{th} of S_4 are selected concurrently.

3.3.2 Fitness Function. The fitness function is responsible for evaluating a given individual and returning a value that represents its worth as a candidate solution. The returned value is typically used by the selection operator to determine which individual instances will be allowed to move on to the next round of evolution, and which will instead be eliminated. Highly fit individuals, relative to the whole population, have a higher probability of being selected for mating, whereas less fit individuals have a correspondingly low probability of being selected.

To facilitate the selection operation in GA, the global minimization problem is usually changed into a global maximization problem. Through transforming Eq. (1) or Eq. (1'), the proper fitness function for the service selection problem can be obtained:

$$F = \begin{cases} U - \sum_{i=1}^{N} \cos t(S_{ik}) & \cos t(SF) < U,\ 1 \le k \le size(S_i) \\ 0 & \cos t(SF) \ge U. \end{cases} \tag{3}$$

Where U should select an appropriate positive number to ensure the fitness of all good individuals are positive in the feasible solution space. On the other hand, U can also be utilized to adjust the selection pressure of GA. When U is increased, the relative fitness of good individuals are reduced, so the selection pressure is decreased, which can prevent the evolution process from premature convergence to get trapped into local minimums. But a too large U will slow down the evolution process, therefore the computing time will increase.

3.3.3 Population Initialization. The population initialization creates the first population and determines the starting point for the GA search routine. An individual is generated by randomly selected web service for each SAg of the services flow, and the newly generated individual is immediately checked to see whether the corresponding solution satisfies the constraints. If any of the constraints is violated, then the generated individual is regarded as invalid and discarded. The process is repeated until enough individuals are generated. The population size $pop_{size} \in [20, 100]$ is generally recommended.

3.3.4 Genetic Operators. GA uses *selection operator* to select the superior and eliminate the inferior. The individuals are selected according to their fitness - the more suitable the more chances they have to reproduce. For the service selection problem, the popular roulette wheel method is used to select individuals to breed. This method selects individuals to reproduce based on their relative fitness, and the expected number of offspring is approximately proportional to that individual's performance, i.e. if one individual is twice as fit as another, then it is twice as likely to be selected and so on. At the same time, it is hoped the fittest individual can be retained in the offspring, so an elitism reservation strategy is applied, which always passes the fittest individual to the next generation, i.e. the individual with the highest fitness does not take part in crossover and mutation operation, instead it is used to replace the offspring with the lowest fitness generated by the crossover and mutation operation.

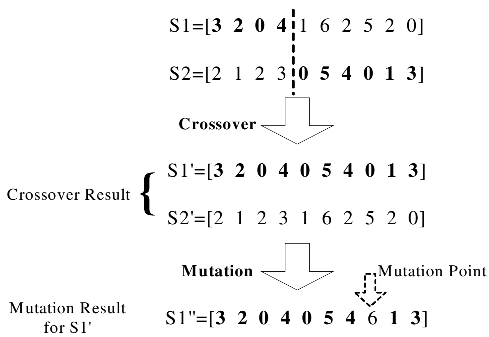

Fig. 2. Crossover operator and Mutation operator

GA uses ***crossover operator*** to breed new individuals by exchanging partial chromosome of the mated parents in stochastic mode. The crossover probability $p_c \in [0.5, 1.0]$ is generally recommended. The purpose of crossover is to maintain the qualities of the solution set, while exploring a new region of the feasible solution space. For the service selection problem, the single-point crossover operator is used: single crossover point is generated in the mated parents at random, then the two parents exchange the tail portion after the crossover point to create two new offspring (See Fig. 2). After each crossover operation, the offspring are immediately checked to see whether the corresponding solutions are valid. If any of constraints is violated, then both offspring are discarded and the crossover operation for the mated parents is retried. If the valid offspring still cannot be obtained after a certain number of retries, the crossover operation for these two parents is given up to avoid a possible infinite loop.

GA uses ***mutation operator*** to add diversity to the solution set by replacing an allele of a gene by another in some chromosomes in stochastic mode. The mutation probability $p_m \in [0, 0.05]$ is generally recommended. The purpose of mutation operation is to avoid losing useful information in the evolution process. From another point of view, the mutation operation can improve the local search performance of GA. Together with crossover operation they complete the local and global search of the solution space. For the service selection problem, the stochastically selected SAg is randomly bound to a web service different from the original one (See Fig. 2). After an offspring is mutated, it is also immediately checked to see whether the corresponding solution is valid. If any of constraints is violated, then the mutated offspring is discarded and the mutation operation for the offspring is retried. Because the mutation operation is carried out at a very small probability, the probability of generating invalid solution is also very small. Even if the invalid solution is generated, a valid solution can be easily obtained through a number of retries.

4 A Case Study and the Implementation

We assume that there are twenty tasks in a pipeline SF and each of them can be accomplished by a service agent (SAg) (See Fig. 3). The SF may be a practical "Travel arrangement" composite service. Each kind of services might have multiple providers, and a service provider could provide several kinds of services. Those business entities may be competitors and do not work with each other. In other way, business entities would prefer to work with those within the alliance, and the alliance claims that there should be some degree discount if customers select some of them. Due to those business rules, there are relationships such as partnerships, alliances, competitors among the services. We must take into account those relationships when select the final services as part of the optimization process.

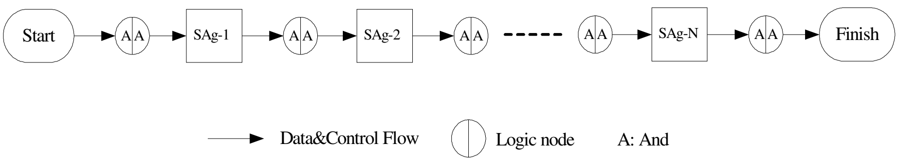

Fig. 3. A pipeline services flow

4.1 Initial Data

From Table 1, we can see each SAg might have various candidate numbers from 5 to 15 predefined at random. Different choices of web services in each SAg might have different cost. We have marked each SAg's maximum cost with red color and minimum cost with blue color in Table 1.

Table 1. Services cost

	WS1	WS2	WS3	WS4	WS5	WS6	WS7	WS8	WS9	WS10	WS11	WS12	WS13	WS14	WS15
SAg1	642	683	642	691	616	663	612								
SAg2	1025	1093	1063	1083	1091	1015	1078	1011							
SAg3	435	472	474	454	407	410	480	424							
SAg4	1314	1393	1308	1367	1390	1349	1385	1350	1356	1327					
SAg5	1763	1799	1732	1752	1714	1746	1768	1741	1732	1719	1749	1707			
SAg6	2077	2075	2057	2089	2016	2061	2046	2092	2073	2051	2083	2090	2079	2098	2007
SAg7	521	539	512	549	502	580	579	525							
SAg8	1451	1447	1410	1449	1452	1457	1427	1430	1443	1474	1408	1464	1427	1493	1432
SAg9	603	645	661	605	604										
SAg10	808	829	814	822	885	868	811	814							
SAg11	822	839	839	802	801	841	870	831							
SAg12	1786	1795	1796	1769	1720	1787	1714	1756	1747	1782	1703	1765			
SAg13	986	947	953	943	935	975	912	927	926	909	924	960	991	920	991
SAg14	1483	1465	1408	1404	1483	1492	1465								
SAg15	781	755	759	742	790	751	758	775	763	778	772	767	732	773	
SAg16	1588	1553	1595	1528	1585	1545	1591	1580	1591						
SAg17	875	870	819	868	895	884	851	877	858	852	848	832	882		
SAg18	1536	1531	1525	1591	1551	1555	1575	1518	1502	1504	1532	1576			
SAg19	1640	1644	1649	1692	1635	1619	1685	1697	1660	1671	1603	1692	1697		
SAg20	1437	1427	1439	1418	1403	1442	1495	1408	1442	1457	1478	1484	1410	1408	1452

For Eq. (2), we take the summation of all SAg's maximum cost as the constant *U*, and get *U* =**$24385**. The individual fitness can be regarded as how much can be saved after the SF execution if the travel agency has been given the money *U* to arrange the traveler's trip. If we use the local selection strategy, we might get a final services sequence as follows:

{ **SAg1**:*WS7*, **SAg2**:*WS8*, **SAg3**:*WS5*, **SAg4**:*WS3*, **SAg5**:*WS12*, **SAg6**:*WS15*, **SAg7**:*WS5*, **SAg8**:*WS11*, **SAg9**:*WS1*, **SAg10**:*WS1*, **SAg11**:*WS5*, **SAg12**:*WS11*, **SAg13**:*WS10*, **SAg14**:*WS4*, **SAg15**:*WS13*, **SAg16**:*WS4*, **SAg17**:*WS3*, **SAg18**:*WS9*, **SAg19**:*WS11*, **SAg20**:*WS5* }

And its overall cost is the summation of all SAg's minimum cost, so it equals to **$22777**. In this case, the individual fitness is **$1608.**

According to some given business rules, we define constraints and corresponding actions (See Table 2). For the special individual, its fitness should be modified with the discount given. And for the invalid individual, it will be discarded. The expression *{B(SAgi) = m, B(SAgj) = n}* means that SAg i binds to the m^{th} service and SAg j binds to the n^{th} service concurrently.

Table 2. Constraint library

	CONSTRAINT	TYPE	ACTION
1	*{ B(SAg1)=1, B(SAg3)=2 }*	Special	Modify (Discount=0.1)
2	*{ B(SAg12)=2, B(SAg15)=2 }*	Special	Modify (Discount=0.2)
3	*{ B(SAg2)=2, B(SAg4)=3, B(SAg5)=4 }*	Special	Modify (Discount=0.2)
4	*{ B(SAg9)=2, B(SAg18)=5 }*	Special	Modify (Discount=0.1)
5	*{ B(SAg8)=2, B(SAg10)=5 }*	Special	Modify (Discount=0.2)
6	*{ B(SAg3)=6, B(SAg5)=4, B(SAg7)=3, B(SAg11)=5 }*	Invalid	Discard

In the paper, some GA parameters are given as follows:

- Solution representation: Integer encoding (serial number of services in SAg starting from zero)
- Population size: $pop_{size} = 40$
- Crossover probability: $p_c = 0.8$
- Mutation probability: $p_m = 0.05$
- Maximum generations: $Gen_{\max} = 2000$.

4.2 Evolution Procedure

The initial population is randomly generated. After $Gen_{\max}$ generations, we have gotten the final optimal solution:

$$[1\ \ 2\ \ 2\ \ 3\ \ 4\ \ 15\ \ 5\ \ 2\ \ 2\ \ 5\ \ 5\ \ 2\ \ 10\ \ 4\ \ 2\ \ 4\ \ 3\ \ 5\ \ 11\ \ 5]$$

It is also represented as

{ **SAg1**:WS1, **SAg2**:WS2, **SAg3**:WS2, **SAg4**:WS3, **SAg5**:WS4, **SAg6**:WS15, **SAg7**:WS5, **SAg8**:WS2, **SAg9**:WS2, **SAg10**:WS5, **SAg11**:WS5, **SAg12**:WS2, **SAg13**:WS10, **SAg14**:WS4, **SAg15**:WS2, **SAg16**:WS4, **SAg17**:WS3, **SAg18**:WS5, **SAg19**:WS11, **SAg20**:WS5 }.

The solution fitness is **$3193**.

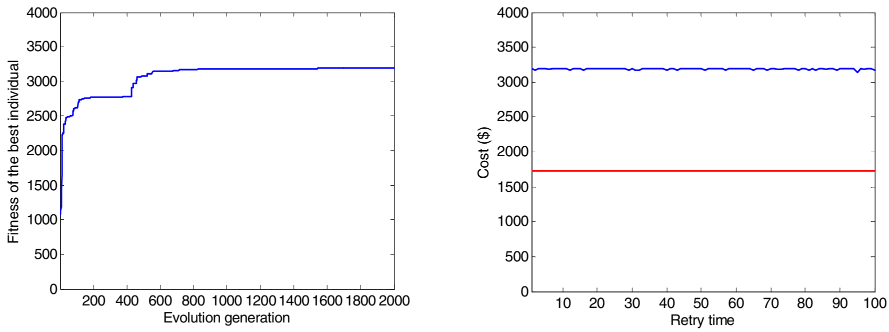

Fig. 4. (a) GA evolution process (b) Multi-time GA optimization

We can see the GA evolution process from Fig. 4(a). The best fitness of the population has a rapid increase at the beginning of the evolution process then convergences slowly. It also means the overall cost of the SF is generally decreasing with the evolution process. For better solution, the whole optimization process can be repeated for a number of times (100 times in this paper, and different initial population in each time), and the best one in all final solutions is selected as the ultimate solution to the service selection problem. From Fig. 4(b), we can see the result. The red line represents the solution using local selection strategy. The blue curve represents all best solutions at 100 GA tests. Apparently, all best solutions using GA have better fitness than those using local selection strategy. In addition, the fact about their similar fitness proves that our GA could find the global optimal solution to the service selection problem.

5 Conclusion and Future Work

A single web service is most likely inadequate to serve the customers' business needs; it takes a selection of various web services composed together to form a business process. The cost is the primary concern of many business processes. In the paper, we propose a new solution using Genetic Algorithm (GA) in cost-driven web service selection. Using an integer string to represent web services composition, the best one is the string that leads to the lowest cost. Service selection is an optimization process with taking into account the relationships among the services. Seen from the experiment result, better performance can be gotten than that using local service selection strategy. The global optimal solution can be achieved with GA within short time. In future, we will consider more QoS factors besides cost as the objectives of service selection and implement multi-objective global optimization using GA.

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On Modeling Internet QoS Provisioning from Economic Models

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Abstract. The modeling of Internet quality of service (QoS) provisioning is a multidisciplinary research subject. From the viewpoint of game theory, we propose a model that combines QoS index with price factor. We use the MultiNomial Logit (MNL) to model the choice behaviors of users. Each service class is considered an independent competing entity, which aims at maximizing its own utility. Based on noncooperative game, we demonstrate the existence and uniqueness of equilibria between QoS levels and prices among various service classes, and demonstrate the properties of equilibria. We also verified these results via numerical analysis.

1 Introduction

With the increasing development of Internet applications, there exists a demand for multiple QoS levels on the Internet. Over the years, network engineers have developed a number of QoS architectures and mechanisms. A core issue that concerns network users, service providers, and network engineers is the constantly changing traffic behavior. Such behavior depends on the aggregated traffic load, which is the result of many users' individual decisions on how to make use of the network. These decisions are affected by the incentives that users face. Thus it is important to bring price into network design and to integrate the QoS levels with price factor when considering Internet QoS provisioning.

Recently, many researchers studied the provisioning of Internet QoS based on economic mechanisms. The M3I (Market-Managed Multi-services Internet) Project funded by the European Union is aimed at studying Internet resource management system. More specifically, it was proposed that different service classes are charged differently, allowing network users to select from multiple prices and QoS levels [1]. In [2], the pricing issue when providing multiple service classes in telecommunications networks was considered. Each service class has different QoS requirements, and is characterized by a (service type, source-destination pair) tuple, where the service type

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can be voice, video, or data, etc. The optimal pricing problem is formulated as a nonlinear integer expected revenue optimization problem and is solved for prices and resource allocations that are necessary to provide connections with guaranteed QoS.

Another body of research has considered prices as an endogenous variable, which is determined as a function of the degree of saturation inside the network. Typically in this approach, the price is called a shadow price; it can be considered the Lagrange multiplier of inequality constraints such as capacity restrictions; it is used to achieve the equilibrium between link bandwidth demand and supply [3][4]. However, little work has been reported on systematically modeling the equilibria between QoS levels and prices on the Internet.

Recently, network game has undergone significant development [5]. In [6], the equilibria between QoS levels and prices of several service network providers have been investigated based on linear demand function. The assumption of linear demand function makes the problem easy to solve, but does not have scientific foundation. In this paper, we explicitly model customer choice behavior using a MultiNomial Logit (MNL) model, which is a form of random utility model. MNL is both a theoretically sound and an empirically well-tested model of customer choice behavior; it has been widely used to forecast traffic demand in airlines [7]. It constitutes a natural candidate for a choice-based optimization model. Base on the MNL model, we investigated the equilibria between QoS levels and prices; explicitly considered the effect of demand substitute, that is, the demand for certain traffic class is described as a function of prices and QoS levels of all traffic classes supported by the network. Our method was inspired by [8], in which the authors investigated the equilibria between all retailers' prices and service levels. We applied their idea to the problem of Internet QoS provisioning. In our model, we do not take into account network topology, but rather model each service class as a single entity. In other words, the price and QoS provided by each service class do not depend on the source, destination, or distance, etc. that underlay each user request. On the other hand, we use proper queuing model to describe the amount of resource requested by service classes.

This paper is organized as follows: the service model and general assumptions are described in section 2, which are used to prove several theorems in later sections. In section 3, we systematically characterize the equilibrium behaviors of an Internet QoS provisioning model under two possible scenarios. (1) Price-competition only: in this case, we assume that the QoS levels of all traffic class are exogenously chosen and characterize how the price equilibria vary with the chosen QoS levels. (2) Simultaneous price and QoS level competition: in this case, each of the traffic classes simultaneously chooses a QoS level and a price. We provide three theorems about existence and uniqueness of equilibria, and demonstrate the properties of equilibria. A numerical application of these theorems is given in section 4. Finally, we briefly conclude our paper in section 5.

2 Service Model and General Assumptions

We first present the notations that are used in this paper:

$\boldsymbol{p}=(p_1,\ldots,p_N)$: the vector of prices of service classes, p_i denotes price of service class *i;* We denote a vector using bold face, e.g. $\boldsymbol{p}$ and denote its *i*-th component using p_i;

$\boldsymbol{f}=(f_1,\ldots,f_N)$: the vector of QoS levels of service classes, f_i denotes the QoS level of service class i;

$d_i(\boldsymbol{p},\boldsymbol{f}): R_+^{N+N} \rightarrow R_+$.:demand for service class i, which depends on the entire price vector and the entire QoS vector;

$\mu_i(f_i,d_i)$: the amount of resource required by the service class i in order to provide the QoS level f_i for demand d_i;

$R_i(f_i,d_i)$: the cost of service class i to provide the QoS level f_i for demand d_i;
$U_i(\boldsymbol{p},\boldsymbol{f})$: the net revenue of service class i (the utility function of service class i);

Let us consider a network in which there are N service classes, the set of which is denoted by $I=\{1,2,\ldots,N\}$. Each service class has two parameters with respect to the service it offers: $(\boldsymbol{p},\boldsymbol{f})\in R_+^{N+N}$ .Each service class i experiences a demand for its service: $d_i : R_+^{N+N} \rightarrow R_+$,which is a function of a price vector and a QoS vector.

In this paper, we explicitly consider the effect of demand substitute, that is, the demand function of service class i is dependent not only on its own parameters, p_i and f_i, but also on the prices and QoS levels offered by its competitor classes. We make the following assumptions regarding the shape of demand function:

(I) For $i=1,\ldots,N$, $\dfrac{\partial d_i(\boldsymbol{p},\boldsymbol{f})}{\partial p_i} \le 0$, $\dfrac{\partial d_i(\boldsymbol{p},\boldsymbol{f})}{\partial f_i} \ge 0$;

(II) For all $j \ne i$, $\dfrac{\partial d_i(\boldsymbol{p},\boldsymbol{f})}{\partial p_j} \ge 0$, $\dfrac{\partial d_i(\boldsymbol{p},\boldsymbol{f})}{\partial f_j} \le 0$;

(III) For all $i=1,\ldots,N$, $\sum_{j=1}^{N} \dfrac{\partial d_i}{\partial p_j} < 0$ (or $\sum_{j=1}^{N} \dfrac{\partial d_j}{\partial p_i} < 0$).

These assumptions are intuitive: assumption (I) denotes that the demand for a service class decreases with its own price, and increases with its own QoS level; assumption (II) represents the effect of demand substitute among all service classes, that is, if one service class increases its own price (or QoS level), this will result in an increase (or decrease) in the demand of his competitors; assumption (III) denotes no service class's sales are expected to increase under a uniform price increase (similarly, aggregate sales usually decrease if one of the service classes increases its price.)

The service model that we study in this paper is shown in Fig. 1: service class i purchases resource $\mu_i(f_i,d_i)$ from the network and offers service to demand $d_i(\boldsymbol{p},\boldsymbol{f})$. The utility function of service class i is represented as the profit of service class i, which equals the revenue obtained from providing service to customers minus the cost of purchasing required resources.

Assume the network owner charges each service class a cost per unit of resource requested, v, and the amount of resource μ_i requested by service class i is dependent on the demand it experiences and on the QoS it wishes to offer (the higher the

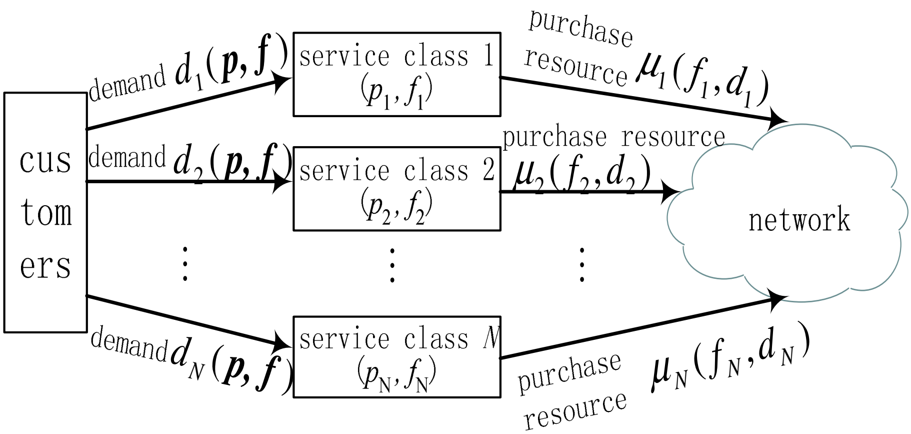

Fig. 1. Illustration of mutli-service model

demand and the better the QoS, the higher μ_i will be). For example, if let QoS level f_i denote the probability of successful transmission, for M/M/1 queuing model, $f_i=1-d_i/\mu_i$, then $\mu_i=d_i/(1-f_i)$Thus, in order to provide the QoS level f_i to demand d_i, the fee paid by service class i can be written as: $R_i(f_i,d_i)=v\mu_i(f_i,d_i)$.

Assume that the resource capacity μ_i required by service class i has the following form: $\mu_i(f_i,d_i)=d_ig_i(f_i)$, where g_i is a convex, two-differentiable and increasing function. This assumption is intuitive: the operational costs increase convexly with the service level.

The net revenue of service class i can be represented as: $p_id_i(\boldsymbol{p},\boldsymbol{f})-R_i(f_i,d_i)$, which is defined as the utility function of service class i. We have:

$$U_i(\boldsymbol{p},\boldsymbol{f})=(p_i-vg_i(f_i))d_i(\boldsymbol{p},\boldsymbol{f}) \tag{1}$$

The strategy space, $\boldsymbol{R}_i$, of service class i, is given as follows: $R_i=\{(p_i,f_i):0\le p_i^{\min}=\hat{g}_i(f_i)\le p_i\le p_i^{\max};0\le f_i^{\min}\le f_i\le f_i^{\max}\}$ Where $\hat{g}_i(f_i)=vg_i(f_i)$.

Assumption A: $p_i^{\max}=\max\{p_i^0,1+v\alpha_ig_i(f_i)\}$, where p_i^0 is the price that can make demand for service class i negligible, and α_i denotes certain constant value.

In theory, the demand functions themselves can take on a number of forms, and each has its own consequences upon the resulting equilibria. In this paper, we use MNL to model customer choice behavior, which will be defined further in the following section. Many marketing models characterize the market shares obtained by competing service classes via a vector of attraction value $\boldsymbol{a}=(a_1,\ldots,a_N)$. The market share achieved by a given service class i is given by its attraction value divided by the industry's total value, i.e., $m_i=a_i\Big/\sum_{j=0}^{N}a_j$, in which a_0 is the value of no-purchase option (in this paper, we let a_0=0). Attraction models are among the most commonly used market share models, both in empirical studies and in theoretical models. Assume that the attraction value of service class i is given by a general, twice differentiable function of its price and service level, i.e., $a_i=a_i(p_i,f_i)$, with $\frac{\partial a_i}{\partial p_i} \leq 0, \frac{\partial a_i}{\partial f_i} \geq 0$.

Most attraction models assume a specific structure. We explicitly model consumer choice behavior using a MultiNomial Logit (MNL) model. Generally, In MNL, a_i is represented as the linear function of various attributes (those attributes represent the attraction of service class i in various aspects). Below we pay special attention to the following generalization equation: $a_i(p_i,f_i)=\exp\{b_i(f_i)-\alpha_i p_i\}$, where $\alpha_i>0$.

Assumption B: $b_i(f_i)$ is twice differentiable, increasing and concave. This permits us to represent settings where the marginal increase in a service attraction value due to an increase in its QoS level, is non-negative but decreasing in QoS level. Alternatively, if a service class wants to maintain a given attraction value, it needs to compensate for a price increase with ever larger increases in its service level.

3 Modeling QoS Provisioning Based on Game Theory

The goal of this paper is to investigate the equilibria of prices and QoS levels of N service classes, and to provide mathematical foundation to Internet QoS provisioning. We systematically characterize the equilibrium behavior under two possible scenarios. (1) The equilibrium of price-competition only: in this case, we assume that service levels of classes are exogenously chosen, and characterize how the price equilibrium and price strategy vary with the chosen service levels. (2) The equilibrium of simultaneous price and service level competition: in this case, service classes simultaneously choose service levels and price strategies.

In the rest of the paper, we let $\tilde{x} = \log x$.

3.1 Price-Based Nash Equilibrium

Let $U_i(\boldsymbol{p},\boldsymbol{f})$ denote the utility function of service class i, $\boldsymbol{p}$ denote the price vector of service classes, and the vector of QoS levels, $\boldsymbol{f}$, be fixed at some predetermined value, $\hat{f}$. The price-based Nash equilibrium in $\boldsymbol{p}$ at $\hat{f}$ is the vector $\boldsymbol{p}^*$ that solves the following system for all $i \in I$. $U_i(p^*,\hat{f}) = \max U_i(p_1^*,\ldots,p_{i-1}^*,p_i,p_{i+1}^*,\ldots,p_N^*,\hat{f})$.

This equilibrium corresponds to price equilibrium for a fixed QoS vector $\hat{f}$.

Theorem 1: Adopting generalized MNL model, under the conditions of assumptions (III) and (A), the game of price has a unique equilibrium $\boldsymbol{p}^*(\boldsymbol{f})$. The price value in equilibrium is the solution to the following equation:

$$\frac{\partial \tilde{U}_i}{\partial p_i} = \frac{1}{p_i - vg_i(f_i)} - \alpha_i(1-\frac{d_i}{M}) = 0 \tag{2}$$

For paper space, we omit the detailed proof, which can be found in [10].

3.2 Price and QoS-Based Equilibrium

Let $U_i(\boldsymbol{p},\boldsymbol{f})$ be the utility function of service class i, when the pair of parameter vectors that are set simultaneously by all service classes is given by $(\boldsymbol{p},\boldsymbol{f})$, the price and Qos-based Nash equilibrium is the vector $(\boldsymbol{p}^*,\boldsymbol{f}^*)$ that solves the following system for all $i \in I$.

$$U_i(\boldsymbol{p}^*,\hat{\boldsymbol{f}}) = \max U_i(p_1^*,\ldots,p_{i-1}^*,p_i,p_{i+1}^*,\ldots,p_N^*,f_1^*,\ldots,f_{i-1}^*,f_i,f_{i+1}^*,\ldots,f_N^*)$$

Assumption C: $\lim_{f_i \to f_i^{\max}} g_i'(f_i) = +\infty$ This assumption holds for a large number of queue systems [6].

Theorem 2: The simultaneous price and QoS-based game has a Nash equilibrium $(\boldsymbol{p}^*,\boldsymbol{f}^*)$ and f_i^* is the solution to the following equations:

$$vg_i'(f_i) = b_i'(f_i)/\alpha_i \text{ , if } \frac{g_i'(f_i^{\min})}{b_i'(f_i^{\min})} \le \frac{1}{v\alpha_i}; \tag{3}$$

$$f_i^* = f_i^{\min}, \qquad \text{if} \frac{g_i'(f_i^{\min})}{b_i'(f_i^{\min})} > \frac{1}{v\alpha_i}. \tag{4}$$

p_i^* is the solution to (2) under the result of f_i^*.

Interesting readers can find the detailed proof in [10].

3.3 Properties of Equilibria

In the previous subsection, we demonstrate that there exist equilibria of the QoS and price game. In this subsection, we investigate the properties of these equilibria. We explicitly consider the effect of demand substitute, so a change in one of the service classes' QoS level will result in an increase or decrease in each of the equilibrium prices. Let $\delta_i(f_i) = vg_i'(f_i) - b_i'(f_i)/\alpha_i$ and f_i^0 denotes the QoS levels obtained from (3) and (4).

Theorem 3: In the price-based game of Internet QoS provisioning, the equilibrium price of service class i, p_i^* is strictly increasing in f_i; the equilibrium price of service class j ($j \neq i$), p_j^* is strictly decreasing in f_i for $f_i < f_i^0$, and strictly increasing in f_i for $f_i > f_i^0$.

For paper space, we omit the detailed proof. Interesting readers can find the detailed proof in [10]. In the subsection, a numerical example is provided to graphically demonstrate the obtained theorems above.

4 Numerical Analysis

We suppose in this section that the measure defining the QoS, f_i, corresponds to some function of the loss probability, $f_i = (1 - P_{loss}^i)^{1/s}$, where P_{loss}^i is the loss probability of service class i, which can be denoted as $P_{loss}^i = G(\rho_i)$, where $\rho_i = d_i / \mu_i$ is the traffic intensity, and $s \geq 1$ is a scaling coefficient that adjusts the relative importance of the QoS parameter with respect to price. For s>1, the QoS increases as a concave function of its parameter, that is, we allow for decreasing rates of return on the quality of service provide. We consider the scenario when there are two service classes and use M/M/1/2 queuing system to model the loss probability. This model can be considered a good approximation for the loss probability when these service classes are used at access network, and the bottleneck (in terms of loss) is in the shared input buffering area to the network.

Let s=2, v=2, a_1=exp($20f_1$−$2p_1$), a_2=exp($12f_2$−$2p_2$), at equilibrium, we have f_1=0.82, f_2=0.74, p_1=4.98, p_2=2.02. The relation between the QoS levels and prices of the two service classes are showed in following figures.

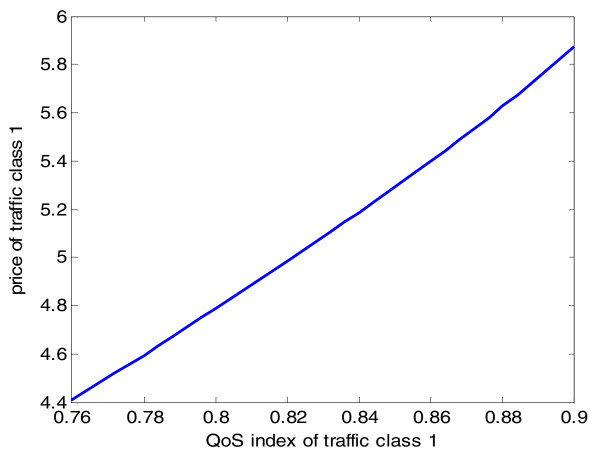

Fig. 2. The relation between QoS index of traffic class 1 and price of traffic class 1

It can be observed from Fig. 2 (Fig. 3) that, when the QoS level of traffic class 2 is fixed in equilibrium value (i.e. f_2=0.74), the relation between QoS level of traffic class 1 and price of traffic class 1 (traffic class 2). From Fig. 2, we found that the price of traffic class 1 increases with its own QoS level. While for f_1<0.82, price of traffic class 2 decreases with the QoS level of traffic class 1; for f_1>0.82, price of traffic class 2 increases with QoS level of traffic class 1. the reason for this phenomenon is that, when the QoS level of traffic class 1 exceeds the equilibrium value determined by theorem 1, the effect of traffic class 1's price increase (making the

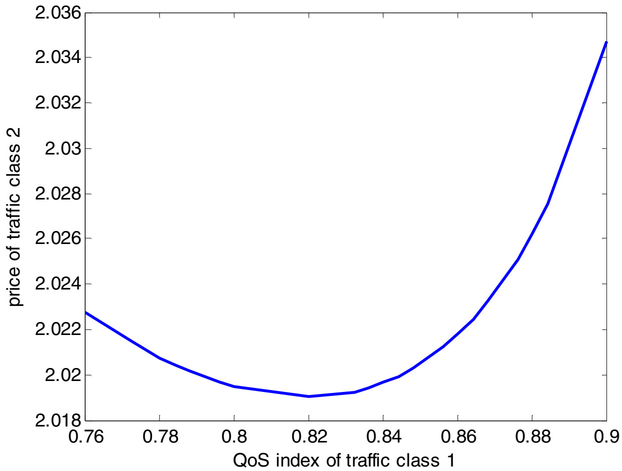

Fig. 3. The relation between QoS index of traffic class 1 and price of traffic class 2

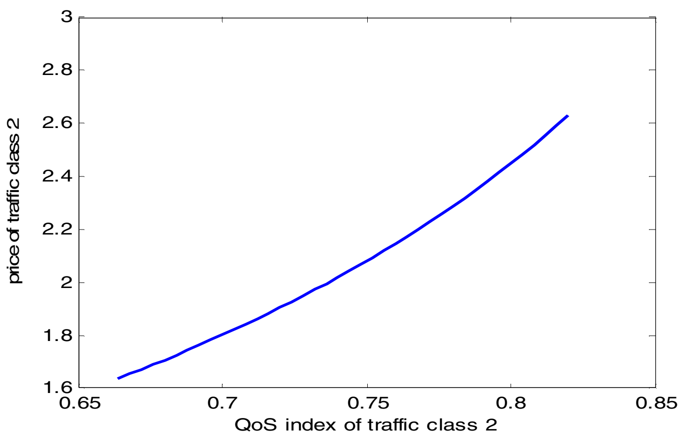

Fig. 4. The relation between QoS index of traffic class 2 and price of traffic class 2

demand for traffic class 1 decrease) surpasses the effect of traffic class 1's QoS level increase (making the demand for traffic class 1 increase), this results in the relatively increased demand for traffic class 2, thus the price of traffic class 2 increases. Those results illustrated in Fig. 2 and Fig. 3 are consistent with the conclusion drawn in theorem 3.

From Fig. 4 and Fig. 5, we observe the similar results with Fig. 2 and Fig. 3. We briefly describe them as follows: when QoS level of traffic class 1 is fixed in the equilibrium value determined by theorem 1 (i.e. f_1=0.82), the price of traffic class 2 increases with its own QoS level. For f_2<0.74, price of traffic 1 decreases with the QoS

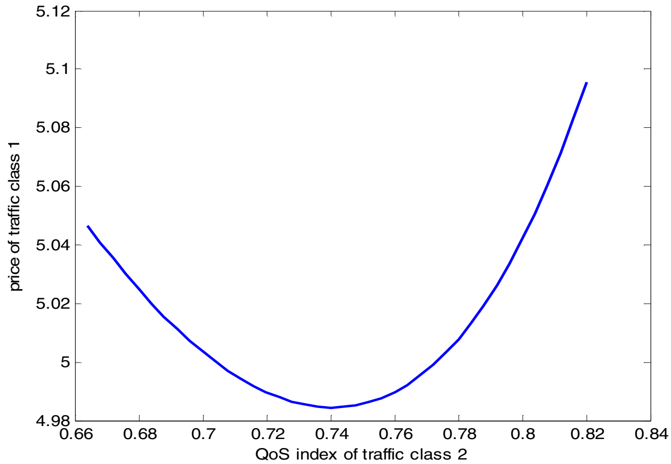

Fig. 5. The relation between QoS index of traffic class 2 and price of traffic class 1

level of traffic class 2; for f_2>0.74, price of traffic class 1 increases with the QoS level of traffic class 2. These results also verify the theorems obtained in section 3.

5 Conclusion

In this paper, we study the modeling of Internet QoS provisioning from the viewpoint of game theory, and obtain the equilibria between prices and QoS levels when there are multiple traffic classes. The motivation of this research is that price factors can provide proper incentive for customers to use network resources rationally, so we can introduce the price factor into the field of network engineering as certain control signal, and simultaneously consider QoS level and price factor in Internet QoS provisioning. We adopt MNL to model the customers' choice behavior. MNL is both a theoretically sound and empirically well-tested model of consumer choice behavior; it has been widely used to forecast traffic demand. It is a natural candidate for a choice-based optimization model. We assume that the service classes are independent, competitive entities which try to maximize their own utilities (In this paper, the utility function of service class is defined as the revenue obtained from providing the service to the customers minus the cost paid to network owner for purchasing network resources). Based on noncooperative game theory, we prove the existence and uniqueness of equilibria between prices and QoS levels among multiple service classes, and demonstrate the properties of equilibria. In our analysis, the attraction functions of service classes are important; they determine the equilibrium values between prices and QoS levels. In practice, user consumption survey and the Maximum Likelihood Estimate (MLE) can be used to obtain the MNL model. Those methods have been proved to be robust in practice. In conclusion, we have made an initial investigation on modeling Internet QoS provisioning through integrating QoS levels with the price factor. We believe that our work is useful to future research into this area.

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Metadata and Information Asset for Infomediary Business Model on Primary Product Market

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Abstract. On a marketplace, classic intermediaries based on products become less profitable, but infomediary, an intermediary based on information, plays a new important role. To intermediate meaningful and timely information among participants, it has to maintain a data warehouse with an information asset based on a new attribute, which is metadata of physical transaction, as an alternative of conventional attribute on relational data model. The metadata aggregates the raw transactions more pertinently than current attribute, and the corresponding information asset serves substantially for decision making. In this paper, we describe the constitution of metadata on primary product market and the value of multidimensional information asset for market participants. The application shows that each participant can increase its profit by using the information asset, and so can the infomediary by vending it.

1 Introduction

As the Internet businesses and technologies advance, a new market calls for a new role of intermediary, which is based on the information about the marketplace. We call the intermediary "infomediary", which is a compound word of information and intermediary, and its role becomes more important whereas a traditional intermediary becomes less profitable. On the early days of infomediary, the issues were focused on deriving the revenue from capturing customer information [1], but the interests gradually moved from the profiling of customer's behavior to all kinds of Internet-related businesses [2]. As a standpoint of third party among customers and vendors, much research about electronic intermediary's role has been progressed [3] [4], but research rarely exists on how an infomediary can make an information asset that the market participants are willing to pay for. We propose how to deal with raw transaction records and convert them into meaningful information asset by abstracting them. To satisfy each user's need, a new attribute on relational data model, which is a metadata of physical transaction, will be proposed because the CA(conventional attribute), i.e. the field of relational database table, has many problems. Being relieved

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from describing the structure of data in 1970s [5], the notion of metadata is used for expressing data about data [6], implying the knowledge derived from market data. In the remainder of this paper, "metadata" will be used as the same meaning as "NA(new attribute)" but "metadata" emphasizes the implication of data abstraction, whereas "NA" does the criterion for data aggregation on data warehouse.

2 Infomediary and Data Abstraction

The advent of infomediary imposes a fundamental change on the market. It means the de-construction of the traditional vertical value chain [7] and expanded chance to make a profit by information instead of goods or services on electronic commerce environment. Meanwhile, the infomediary can still play its role on future market structures. Wise & Morrison showed how the B2B commerce will evolve in the future, and anticipated that the information about a product is more valuable than the product itself [8]. If distinct and valuable information facilitates business models of next generation, they will be competitive and successful in ever-challenging e-business marketplace. However, these researches mentioned conceptual models on a strategic level, and they didn't provide clear explanation about how to embody the models by building information goods for an infomediary to sell, although an infomediary has to preserve well-defined data repository ready for the information users so the construction of elaborate metadata and information asset is a necessary condition for the competitiveness.

As a viewpoint of business model, let us see why the metadata, i.e. abstracted data, is necessary for an infomediary. In Fig. 1, any business model without infomediary, which is marked as "physical business model", physically intermediates the goods or services so it contains detailed data about each transaction. On the other hand, infomediary model doesn't intermediate physical flow. Instead, it gathers raw data from many sources and participants, and provides metadata, which is knowledge about physical transaction, for the market participants concerned. Such a metadata expressed in a form of information or knowledge is

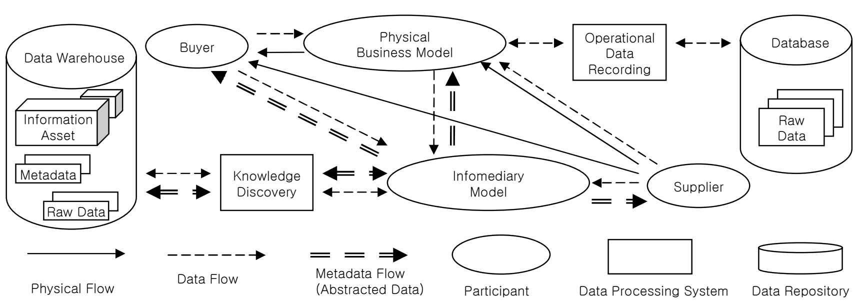

Fig. 1. Data flow on marketplace and infomediary

a result of integration and analysis of various raw data, and an infomediary always has to keep the metadata up-to-date for a short time-to-market. Because of this characteristic of abstraction and simultaneity, an infomediary is required to have DW, where metadata and information assets exist, whereas other business models maintain raw data only in database.

3 Metadata for Infomediation

In this paper, information asset is embodied as a data cube, and the content of the asset is the transactions aggregated by metadata, which is derived by knowledge discovery. In other words, the information asset supplies multiple data views by NAs as well as CAs. The relationship among them are described in Fig. 2. Attribute is the basis for aggregating data, and it is also an axis of multidimensional information asset. The concept of NA is introduced because many problems sometimes occur when we aggregate the raw transactions by CA. The trouble is that the transactions of different features are mixed just because current attribute has the same value, and the transactions of a similar feature can be partitioned because of different values. Moreover, CA cannot provide the axis that the users need, who want to aggregate and observe data by a necessary criterion. For an example on an auction market for primary product, refer to Fig. 3, where two agents exist for selling products instead of farmers. The bargained price achieved by agent A scatters in a narrow range and that of the other does in a wide range. Provided that the prices are for the same product on the same market and period, they suggest a significant information to us:

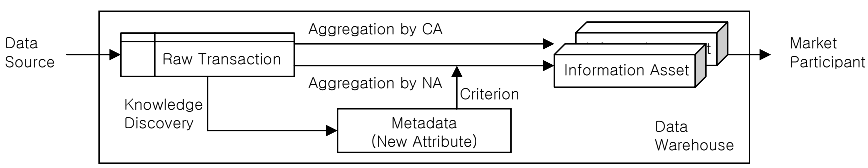

Fig. 2. Raw transaction, metadata, and information asset

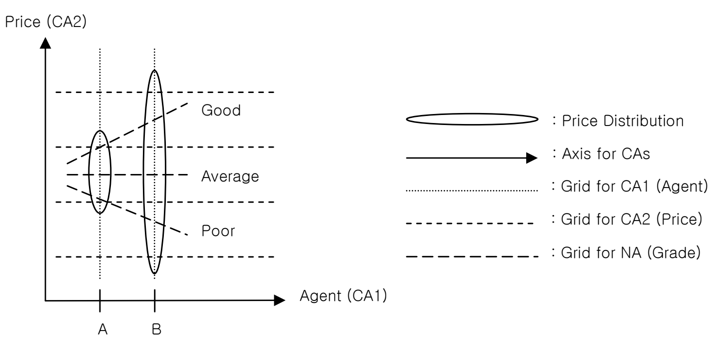

Fig. 3. Finding new attribute from conventional attribute

agent B has an ability to sell the product of good grade for a higher price. But once the transactions are aggregated by agent(CA), the information disappears. But if we lay down a specific grade for each transaction, the grade can be an alternative dimension for data view and aggregation.

A metadata can be found by grasping the characteristics of a transaction distinguished from others. Because it is hard to determine the characteristics by watching one transaction alone, we have to observe transactions of a certain range at the same time and give a new attribute to the transactions of the same characteristics. In Fig. 3, the grade can be guessed by observing the prices of the transactions for each agent, and better grade will be given for the product of higher price. After accumulating the relationship among grade, agent, and price, we can discover a knowledge, which has agent and price as input, and grade as output. The use of new attribute has some advantages. First, it provides significant information to users rather than enumerating meaningless information. Second, it allows an infomediary to provide timely information asset by storing information that user wants on DW beforehand. For the last, we can forecast the future because the metadata reflects the pattern of the product, price and quantity while traditional database table with CA still stays in past. Ultimately, if an infomediary makes NAs and creates information asset along them, they can serve market participants well.

A concrete procedure for detecting new attributes and creating information asset is shown in Fig. 4. After the recognition of the need for NA, useful data model for determining NA, such as data clustering, is selected on the consideration of data availability. For attribute extension, necessary auxiliary attributes(AAs) which can be simply computed by combining some CAs are appended, e.g., total traded amount can be derived by multiplying price by quantity. Necessary external factors are also appended and irrelevant CAs are elim-

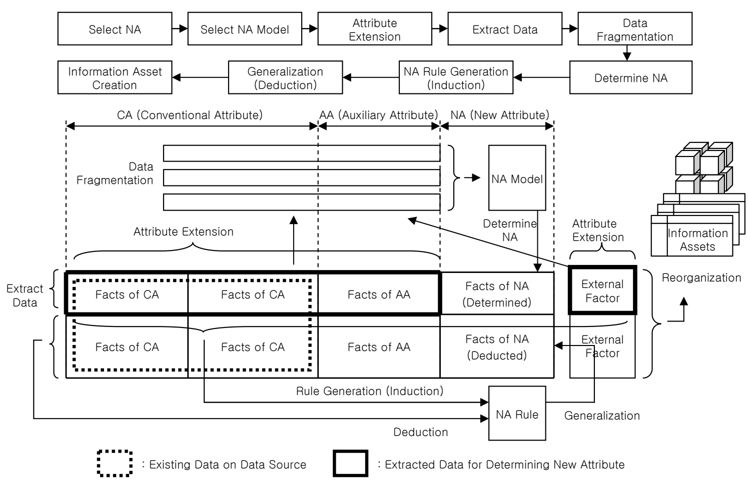

Fig. 4. Detecting new attribute and building information asset

inated. After extracting necessary transactions giving criteria to the attributes, they must be fragmented by every case of CAs that affect the determination of NA. For our example of Fig. 3, the transactions must be fragmented by each agent. After determination of NA, the next two steps are optional: induction and deduction. If a sampling was done in data extraction, all the data don't have the values for NA. So a rule from CAs to NA must be induced and NA must be obtained for other data by applying the rule. The NA can be evaluated as a standpoint of how more it divided the transactions effectively in comparison with the separation of CA only. In other words, the variations of a measure(price or quantity) with NA must be much greater than the variation without NA. So a meaningful NA must satisfy the following condition, which is known as information gain:

$$\mathbf{F}_{n=1}^{N}\left(\sqrt{\sum_{c=1}^{C-1}(M_{n,(c+1)} - M_{n,c})^2}\right) \gg \sqrt{\sum_{c=1}^{C-1}\left(\mathbf{F}_{n=1}^{N}(M_{n,(c+1)}) - \mathbf{F}_{n=1}^{N}(M_{n,c})\right)^2}, \tag{1}$$

where N is the number of instances of the NA and C is the number of instances of a CA. $M_{n,c}$ means the aggregated measure by NA n and CA c, and $\mathbf{F}$ is an aggregation function such as sum or average for the measure.

4 Application to Primary Product Marketplace

The application of NA is mainly based on the transaction data of apple on auction markets in Korea, where farmers entrust auction brokers with the sale, and wholesalers bid a price. So we can say the participants of the marketplace are farmer, wholesaler, and broker. There are several brokers on one market, and they subtract some percents as a commission from the total amount traded. So the farmers and brokers try hard to sell the apples at a higher price but the wholesalers try to buy at a lower price. Meanwhile, the market participants have to make a complex decision, and each decision variable is connected to one another. For example, a farmer can harvest his apples early. The immature apples last long in a storage; that means the farmer has a wide range of selection for time attribute. So the opportunities to make more profits are extended, but the farmer don't have any choice but to accept a fall in price because immature apples don't taste as good as mature ones. To support such

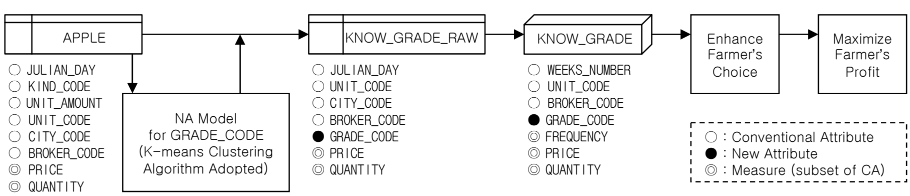

Fig. 5. Data flow and conversion for information asset

```
grade.m - 메모장
파일(F) 편집(E) 서식(O) 보기(V) 도움말(H)
% Quality Grade Detection by K-means Clustering
% Prepare Traditional Dimensions
dim_str='APPLE.JULIAN_DAY, WEEKS_NUMBER, UNIT_CODE, CITY_CODE, BROKER_CODE, PRICE, QUANTITY';
Weeks_number    =   [42;43;44;45];
Unit_code       =   [50;60;70];
Broker_code     =   ['A1'; 'A3'; 'A5'];
Grade_code      =   ['A';'B';'C'];
GRADE_REC={};
GRADE_RAW_REC={};
% Criteria for Extracting Data
from_str=' from APPLE, DIM_TIME where ';
Weeks_number_str='(Weeks_number > 41 and Weeks_number < 46)';
Product_str='(KIND_CODE=''1'' and UNIT_AMOUNT=15)';
Unit_code_str='(UNIT_CODE=50 or UNIT_CODE=60 or UNIT_CODE=70)';
Broker_code_str='(BROKER_CODE=''A1'' or BROKER_CODE=''A3'' or BROKER_CODE=''A5'')';
join_str=' and APPLE.JULIAN_DAY = DIM_TIME.JULIAN_DAY';
% Connect to Data Warehouse and Extract Data
conn = database('APPLE', '', '');
curs = exec(conn, ['select ',dim_str,from_str,Weeks_number_str,' and ',Product_str,' and ',Unit_code_str,' and ',Broker_code_str,join_str]);
curs=fetch(curs);
for w_n = 1:size(Weeks_number,1) % Data Fragmentation by WEEKS_NUMBER
  for u_c=1:size(Unit_code,1) % Data Fragmentation by UNIT_CODE
    for b_c = 1:size(Broker_code,1) % Data Fragmentation by BROKER_CODE
      % Determine Quality Grade
      index=find((curs.data{:,2}==Weeks_number(w_n))&(curs.data{:,3}==Unit_code(u_c))&(strcmp(curs.data(:,5),Broker_code(b_c,:))==1));
      [idx,c]=kmeans(curs.data{index,6},3,'emptyaction','drop','replicates',60); % K-means Clustering into 3 Clusters
      [Y,I]=sort(c); % First Row, Last Grade
      for g = 1:size(Grade_code,1) % Start from Grade A to C
        idx_grade=index(find(idx==I(size(Grade_code,1)+1-g))); % Find data of Target Grade
        % Add New Raw Record with GRADE_CODE
        GRADE_RAW_REC=[GRADE_RAW_REC;curs.data(idx_grade,1),curs.data(idx_grade,3:5),repmat({Grade_code(g,:)},size(idx_grade)),curs.data
(idx_grade,6:7)];
        avg_price=double(sum(curs.data{idx_grade,6}.*curs.data{idx_grade,7})./sum(curs.data{idx_grade,7})); % Compute Weighted AVG Price
        tot_qty=double(sum(curs.data{idx_grade,7})/3); % Compute Total Quantity for 3 years and divide by 3
        frequency=double(size(idx_grade,1)/3); % Compute the Number of Transactions for 3 years and divide by 3, i.e., frequency
        GRADE_REC=[GRADE_REC;{Weeks_number(w_n),Unit_code(u_c),Broker_code(b_c,:),Grade_code(g,:),frequency,avg_price,tot_qty}]; % New Rec.
      end
    end
  end
end
% insert Highly-Summarized Data into Data Warehouse
colnames={'WEEKS_NUMBER';'UNIT_CODE';'BROKER_CODE';'GRADE_CODE';'FREQUENCY';'PRICE';'QUANTITY'};
colnames_raw={'JULIAN_DAY';'UNIT_CODE';'CITY_CODE';'BROKER_CODE';'GRADE_CODE';'PRICE';'QUANTITY'};
insert(conn,'KNOW_GRADE',colnames,GRADE_REC);
insert(conn,'KNOW_GRADE_RAW',colnames_raw,GRADE_RAW_REC);
close(curs);
close(conn);
```

Fig. 6. Source code for detecting NA(quality grade) and storing on DW

a participant's decision, the related raw data is converted into information asset as described in Fig. 5. The source table of our concern is named APPLE, which has eight CAs. JULIAN_DAY tells when the transaction was occurred. It is continuous count of days since January 1, 1990. UNIT_AMOUNT is the weight of a box, and UNIT_CODE is the number of apples in one box. Generally, a box of less UNIT_CODE(big apples) is traded at higher price than one of more UNIT_CODE even though they have the same UNIT_AMOUNT. Like the example of previous section, a new attribute that stands for the quality grade is developed and designated as "GRADE".

On the market, it is observed that the same apples are sold at different prices on the same day. It is mainly due to the quality of apple, but market or broker

Table 1. Variations by attributes and information gain

Attribute		WEEK	UNIT	BROKER
Price Variation	Without GRADE	1874	4548	2229
	With GRADE	1992	4551	2258
	Info. Gain	118	3	29
Quantity Variation	Without GRADE	5925	19633	14047
	With GRADE	9322	19798	15500
	Info. Gain	3397	165	1453

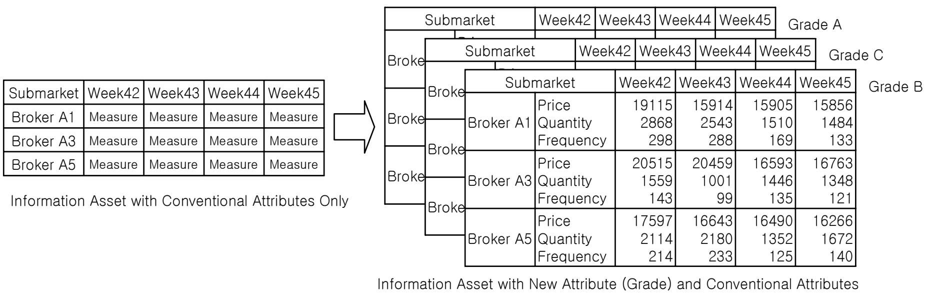

Fig. 7. Information asset by week, broker, and grade

also affects the price. So the NA is developed to see how much price each grade is sold at. As a metadata on marketplace, it reflects the relationship among product, time, price/quantity, and broker. Fig. 6 shows the procedure described in Fig. 4. Raw data having time record of harvesting season is selected to help farmers, who are troubled with when to harvest and where to sell. To detect grade, the raw data were divided into each set of same week, size, and broker. The fragmented data is the input of our grade detection model, which adopts K-means clustering algorithm, which minimizes the sum of squared Euclidean distance from each data to the centroid of the corresponding cluster. After that, the raw data is aggregated by week, unit, broker, and grade. To see how much the new attribute discriminates the price and quantity, the variation of price and quantity with NA is calculated as shown in Equation 1 of Sect. 3. The variation without grade is also derived and shown in Table 1. In the table, the differences between two variations are expressed as information gains, which are distinctively high in week attribute for both price and quantity. That means the grade information must be associated with week attribute for being a useful information asset. Fig. 7 shows an exemplary graphic presentation of the information asset, where week, broker, and grade attributes are combined.

5 Value Assessment for Information Asset

Each market participant selects some submarkets partitioned by products, times, and counterparts, and an information asset helps each participant reallocate the resource on the submarkets. In other words, owing to the characteristics of metadata that tells the relationship among submarkets associated with price and quantity, it points out where to move in a feasible submarket hyperspace to maximize the profit. The value of an information asset is measured by subtracting the profit without the asset from that with the asset. Since the value of information asset depends on how much each user reallocates the resources, an infomediary has to develop an information asset that divides the phase of transaction (e.g. price and quantity) considerably so that the information users can reshuffle the resources.

Table 2 shows the resource of the farmers in one region(CITY_CODE) by grade, and the table was not available without the metadata. Meanwhile, the

Table 2. Resource of a farmer

Grade	A	B	C	Average/Total
Price	19700	15134	13000	15885
Quantity	20	67	10	97

Table 3. The farmer's existing allocation for Grade B and enhanced submarket choice

Submarket		Week 42	Week 43	Week 44	Week 45
Broker A1	Price		15166	14857	15239
	Quantity		3	14	48
	Move to		42nd, A3	43rd, A3	45th, A3
	ΔPrice		4601	4554	907
Broker A3	No record				
Broker A5	Price		14500		
	Quantity		2		
	Move to		42nd, A3		
	ΔPrice		3872		

information asset shows the desirable submarket to move toward for selling their products for higher prices. Table 3 shows the farmer's current transaction of grade B and where to move toward to maximize the profit. Although the apples of same grade code are same products, those can be spoiled during storage or they may not be available in earlier weeks. So a movement throughout the brokers is for free, but a shift along week is restricted to the previous and next week only. ΔPrice is the difference of prices of two different submarkets on Fig. 7. We assume that the farmer's profit will increase by the ΔPrice, regardless of the price of the farmer's past transaction. If the price of target submarket on information asset is less than that of the farmer's current submarket, the movement will be cancelled. A ΔPrice would be cut down if the moving quantity were too much in comparison with the total quantity of target submarket. It would also be cut down when to move to the submarket of small frequency. In this way, the increased profit is calculated at 128839 ($\sum \Delta$Price$\cdot$Quantity) for grade B. Like

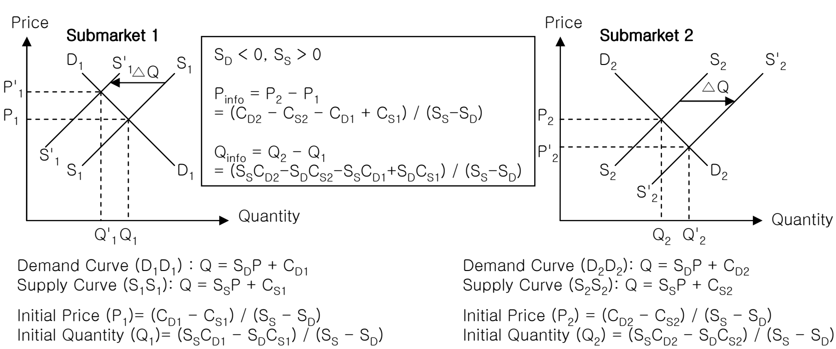

Fig. 8. Supplier's reallocation between two submarkets

this, the value of the information asset for the farmer reaches 169895, including 31926 for grade A and 9130 for grade C.

If all suppliers use an information asset, each supplier cannot enjoy the full advantage that he or she achieved when only a small number of user received the asset. They all move toward the same submarket, and the desired price and quantity are not realized as predicted. That means the infomediary cannot price the information asset as much as before any more. To see how much the participants maximize their profits under a given information asset, let us assume two submarkets as shown in Fig. 8. The price and quantity are discriminated in two submarkets by P_{info} and Q_{info}, and suppliers reallocate their quantity to maximize the whole revenue. Assuming the supply decreases in submarket 1 by ΔQ, the optimal quantity and the maximized increment of total revenue (ΔTR) are as follows:

$$\Delta Q = \frac{S_S - S_D}{4S_D}(S_D P_{info} + Q_{info}) \quad \text{and} \quad \Delta TR = -\frac{(S_D P_{info} + Q_{info})^2}{8S_D},$$

which is obtained by finding the solution of $\partial \Delta TR / \partial \Delta Q = 0$ [9].

This result has various implications on information market. First, ΔTR is never negative; increase in total revenue is always guaranteed by the advent of infomediary. Second, the bigger P_{info} and Q_{info} are given, the more ΔTR will be enjoyed. So an infomediary has to develop NAs which discriminate price and quantity in a great deal. Needless to say, a quantity-discriminating information asset is meaningless when released to few participants, because the transfer from one submarket to another is not followed by a price change. Third, as $|S_D|$ becomes bigger, i.e., the customers react sensitively to price change, ΔTR by price discrimination increases more. So, on those markets, an infomediary must develop NAs that discriminate price much. On the other hand, ΔTR by quantity discrimination becomes less as $|S_D|$ has greater value. Last, S_S is irrelevant with suppliers' decision making. For the information customers' decision making, we can find ΔQ that maximizes the decreased total cost (ΔTC) in the same way as for the suppliers'. Assuming they decrease their quantity by ΔQ in market 1 and increase the same amount in market 2, the results are as follows:

$$\Delta Q = -\frac{S_S - S_D}{4S_S}(S_S P_{info} + Q_{info}) \quad \text{and} \quad \Delta TC = \frac{(S_S P_{info} + Q_{info})^2}{8S_S}.$$

6 Conclusions and Future Works

In this paper, data warehousing and information asset creation methodology were suggested as a standpoint of infomediary business model, especially for a primary product market. The effect of information asset to the marketplace was also explained. A new viewpoint to attributes was suggested and exemplary metadata was made. Information asset was created by using the NAs, and market participants can benefit from the asset. The contribution of this paper is summarized as a concrete model of data warehousing for information asset generation and the pricing for the asset. Further researches needs to be carried out such as automated NA creation without the intervention of knowledge engineer or proactive customized information asset pushing model.

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On Protection of Threatened Unstructured Overlays: An Economic Defense Model and Its Applications

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Abstract. The presence of power-law connectivity distributions and small-world characteristics in current unstructured overlay networks, so useful to speed up the communication process, ironically, also exposes some fatal topological weaknesses, e.g., being extremely vulnerable under intentional targeted intrusions, which seriously reduces their intrusion survivability. As a remedy, we in this paper propose a novel generalized and practical analytical formulation called *Economic Defense Model*, to characterize the intrusion spreading in these networks and provide guidelines for controlling the epidemic outbreaks. Based on (but much different from) currently existing methods, our model focuses on two key concepts of efficiency and cost, by giving deep insight into the role of topological properties, like the scale free behaviors, the small-world-like phenomena, the statistical significance of both nodes and links during dynamic topology evolution over time. Moreover, we propose a novel economic defense strategy and then perform a case study to examine how efficiency and economy principles combine up to shape the epidemics and immunization in these overlays.

1 Introduction

Many social, biological, and communication systems can be properly described as complex networks with nodes representing individuals or organizations and edges mimicking the interactions among them [1, 2]. The word "complex" means that large numbers of nonlinear interactions exist between the elements, and that any approach that cut the system into parts would fail. Recently disruptive unstructured overlay networks, represented by Gnutella [3], demonstrate strong complex network characteristics like power-law degree distributions and "small-world" phenomena [4]. These emergent topological properties, so useful to speed up the information exchange, ironically, also favor at the same time the spreading of computer viruses and intrusions, e.g., being extremely vulnerable under intentional targeted attacks [5], which seriously reduces their intrusion survivability. Great efforts, and our paper also belongs to this, are being made to examine the impact of structural properties on the protection of these threatened overlays [6], following two key directions below:

1. Determine optimal topology designs against real-world attack scenarios.
2. Determine optimal defense strategies against deliberately-designed intrusions.

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In fact, the structure of the network can be as important as the nonlinear interactions between the elements. An accurate description of the coupling architecture and a characterization of the structural properties of the network can be of fundamental importance also to understand the dynamics of the system. This line of research has gain exciting achievements, the representative ones of which are (a) the susceptible-infected-susceptible (SIS) model [7, 8] and the susceptible-infected-removed (SIR) model [9, 10, 11] valid for epidemic spread in homogeneous random topologies, (b) the Barabási-Albert (BA) network model special for epidemic spreading in scale-free topologies [5, 12, 13], (c) the targeted immunization strategy [14] and the acquaintance immunization strategy [15] for halting this epidemics.

Though the above achievements has shown to have a lot of appeal to characterize and remedy the topological weaknesses of the threatened overlays we just mentioned, some big holes still remain to be filled. In particular in this paper we show that the study of a more practical defense model that leads to the concept of economic attack-tolerant overlays poses new challenges, which can in fact be overcome by using a dynamic immunization paradigm and a generalization of the ideas presented by [14] and [15]. The attack-tolerant overlay network can be defined in a general and more economic way by focusing on how efficiently and economically the network is immune from various deliberately-designed intrusions, and by giving deeper insight into the role of intrinsic topological properties, like the scale free power-law distributions, the small-world-like structure, the statistical significance of both nodes and links during dynamic evolution over time. The defense model we propose is valid both for unweighted and weighted networks and brings the application of the immunization in realistic threatened unstructured overlay networks to a more practical phase.

The rest of this paper is organized as follows. In Section 2 we examine currently existing models and illuminate their limitations. In Section 3 we present our economic defense model based on two leading concepts of (a) efficiency and (b) cost. Efficiency measures how well the immunization information propagates over the network, and cost measures how expensive it is to propagate this immunization information. We define an economic attack-tolerant overlay network as a low-cost system that halts the epidemic spreading of various intrusions efficiently both on a global and on a local scale. To apply our model to real-world networks, we perform a case study on Gnutella-like overlays in Section 4, where we devise a novel economic defense strategy based on our proposed model, and then conduct extensive simulations to justify its performance gains by making comprehensive comparisons with the other two well-known models. Finally, we conclude this paper in the last section and highlight some directions of our future work.

2 Existing Models and Their Limitations

Epidemic and immunization models are heavily affected by the connectivity patterns characterizing the population in which the infective agent spreads. So we start by introducing the following three families of topology models (see Fig.1) that are usually used to represent real-world complex networks:

1. *Homogeneous exponential network*: This kind of network has a connectivity distributed peaked at an average connectivity $<k>$, and decaying exponentially fast for $k < \langle k \rangle$ and $k < \langle k \rangle$. A typical example of this kind of network is the

Erdös-Rényi network [16], shown in Fig.1(a). In these networks connectivity has only very small fluctuations ($\langle k^2 \rangle \sim \langle k \rangle$). This is equivalent to a homogeneity assumption of the system's connectivity.

2. *Small-world network*: This particular class of networks, named *small worlds* in analogy with the concept of the small-world phenomenon observed more than 30 years ago in social systems [17], are in fact highly clustered like regular lattices, yet having small characteristic path lengths like random graphs. The network model (see Fig.1(b)) proposed by Watts and Strogatz in [18] has triggered a large interest on the study of the properties of small worlds.
3. *Heterogeneous scale-free network*: This kind of networks represent a very interesting case since they exhibit a scale-free power-law connectivity distribution $P(k) \sim k^{-\gamma}$ for the probability $P(k)$ that a node of the network has k connections to other nodes (see Fig.1(c)). For connectivity exponents in the range $2<\gamma<3$ (e.g., $\gamma \approx 2.3$ in the case of Gnutella [19]) this fact implies that each node has a statistically significant probability of having a very large number of connections compared to the average connectivity $<k>$ of the network. In mathematical terms, the implicit divergence of $<k^2>$ is signaling the extreme heterogeneity of the connectivity pattern, and it is easy to foresee that this property is going to change drastically the behavior of epidemic outbreaks in scale-free networks.

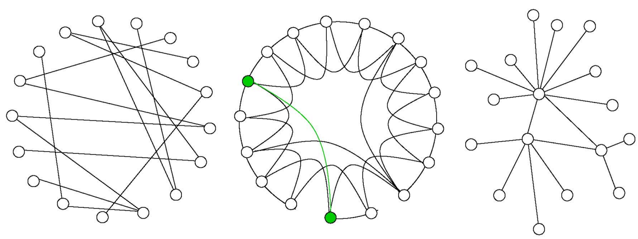

Fig. 1. Three families of analytical network topology models. (a) Erdös-Rényi (Exponential tail) network, (b) Watts-Strogatz network, and (c) Scale-free (Power law tail) network.

Next, we reexamine currently well-known epidemic and immunization models that are essentially deduced from these network models, and then outline their limitations.

SIR and SIS: As the basic homogeneous epidemic spreading models, the susceptible-infected-removed (SIR) model and susceptible-infected-susceptible (SIS) mode have been extensively studied in the past years [7-11]. In SIR model, the individuals are classified in three classes according to their states: susceptible (will not infect others but may be infected), infected (have infectivity), removed (recover from the illness and have immunity thus will not take part in the epidemic process). Assume that a susceptible individual will be infected by a certain infected one during one time step with probability β, and the recovering rate of infected ones is γ. Then, in SIR model, the epidemic process can be described by the following equations:

$$\frac{ds}{dt} = -\beta is, \quad \frac{di}{dt} = \beta is - \gamma i, \quad \frac{dr}{dt} = \gamma i \tag{1}$$

where s, i and r denote the ratio of susceptible, infected, and removed individuals to the whole population, respectively. The SIR model is not suitable when the individuals cannot acquire immunity after recovering from the infection. The SIS model is often used for this case, which is very similar to SIR model. The only difference between them is that in SIS model, the infected individuals will return to the susceptible state after recovering (see Fig.2), while in SIR model, they will be removed. Hence in SIS model, corresponds to formula (1), we have

$$\frac{ds}{dt} = -\beta is + \gamma i, \quad \frac{di}{dt} = \beta si - \gamma i \tag{2}$$

Obviously, both the SIR and SIS model are entirely deduced from homogeneous networks, and assume the same (constant) infection rate or immunization rate on all links and nodes, which means that all the nodes and links are treated as equivalent, irrespective of their corresponding connections or weights. These limitations result in their incompetence for modeling the inhomogeneous unstructured overlays.

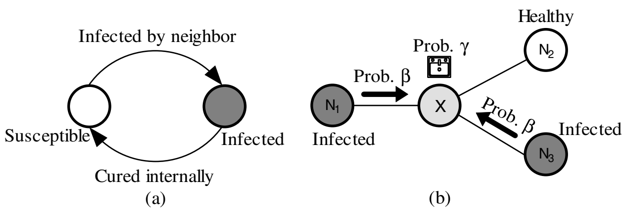

Fig. 2. The epidemics and immunization process in the SIS model. (a) Cured nodes immediately become susceptible; (b) Homogeneous infection rate β on all links between infected and susceptible nodes, homogeneous immunization rate γ for all infected nodes.

Epidemics and Immunization Model for Small-World Networks. Moore and Newman investigate the epidemics and immunization in Newman-Watts networks [20, 21], which are the small-world networks similar to Watts-Strogatz networks addressed above. The difference is that in Newman-Watts networks, only the shortcuts are added and no edges are rewired. They consider the SIR process as a site percolation. Denote φ the ratio of the number of shortcuts to Nk, where $2k$ is the average degree of the original one-dimensional lattice and N is the number of nodes; they yield the expression of the site percolation (epidemics) threshold p_c:

$$\varphi_{site} = \frac{(1-p_c)^k}{2kp_c[2-(1-p_c)^k]} \tag{3}$$

Further more, they investigate the bond percolation in Newman-Watts networks, and prove that when k=1, the thresholds of site and bond percolation are the same. By using the method of generating functions, they yield the threshold for bond percolation in the case k=2:

$$\varphi_{bond} = \frac{(1-p_c)^3(1-p_c+p_c^{\ 2})}{4p_c(1+3p_c^{\ 2}-3p_c^{\ 3}-2p_c^{\ 4}+5p_c^{\ 5}-2p_c^{\ 6})} \tag{4}$$

In general, these results are substantially based on the analytical studies of SIS/SIR models on the above NW or WS networks and thus can not go beyond the limitations we just point out.

Epidemics and Immunization Model for Scale-Free Networks. In order to fully take into account connectivity fluctuations in a analytical description of the SIS model, Pastor-Satorras and Vespignani obtained an approximate solution for network by using the mean-field theory. Denote p_k the probability that a randomly picked node is of degree k, λ the probability that a susceptible individual will be infected by an infected one during one time step, and $\Theta(\lambda)$ the probability that any given edge points to an infected node. They obtained the famous equation below:

$$\lambda\left(\sum_k kp_k\right)^{-1}\sum_k \frac{k^2 p_k}{1+k\lambda\Theta(\lambda)}=1 \tag{5}$$

The inhomogeneous connectivity distribution of many real scale-free networks is first reproduced by Barabási-Albert (BA) network model, which incorporates two ingredients common to real networks: *growth* and *preferential attachment*. The model starts with m_0 nodes. At every time step t a new node is introduced, which is connected to m of the already existing nodes. The probability Π_i that the new node is connected to node i depends on the connectivity k_i of node i such that $\Pi_i=k_i/\sum_j k_j$. For large t the connectivity distribution is a power law following $P_k=2m^2/k^3$. An analytic solution for the BA network can be got by substituting the obtained P_k expression to formula (5), showing the prevalence density ρ of infections below:

$$\rho = 2e^{-1/m\lambda} \tag{6}$$

This result shows the absence of any epidemic threshold or critical point in the model. This indicates that infections can proliferate on these scale-free networks whatever spreading rates they may have. In view of this weakness, it becomes a major task to find optimal immunization strategies oriented to minimize the risk of epidemic outbreaks in these networks. Although this model emphasizes the role of topology in epidemic modeling and provides an unexpected result that radically changes many standard conclusions on epidemic spreading, it is limited to (a) BA network based predictions, (b) only allowing power laws of γ=3, and (c) unweighted graphs with low clustering coefficient, which is clearly not suitable for unstructured overlays.

Three classes of Immunization strategies. Focused on defense of scale-free networks against spreading of viruses or attacks, three classes of immunization strategies below are recently proposed. Comparisons of their principles are shown in Fig.3.

1. *Random Immunization*: The simplest immunization strategy proposed in [22], consists in the random introduction of immune nodes in the networks, in order to get a uniform immunization density that has been verified a success in homogeneous networks. However, this strategy gives the same weight to very connected nodes (with the largest infection potential) and to nodes with a very small connection (which are relatively safe), and thus is totally ineffective for scale-free networks due to their extremely large connection fluctuations.
2. *Targeted Immunization*: The authors in [6] propose the so-called "Targeted Immunization" schemes, which mean to immune the nodes with larger degree

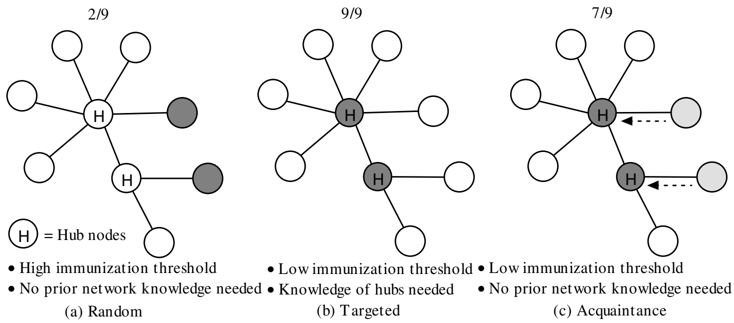

Fig. 3. An example: Immune 2 of 9 nodes using the strategies of (a) Random Immunization, (b) Targeted Immunization, and (c) Acquaintance Immunization

firstly. They prove that, the whole population must get vaccinated if one wants to effectively control the epidemic propagation by using random immunization strategy. While if the targeted immunization schemes are used, the critical immunization threshold will decay to: $g_c=e^{-1/m\lambda}$.

3. *Acquaintance Immunization*: Cohen et al in [15] design an efficient immunization strategy named "acquaintance immunization", which can avoid accounting the nodes' degrees. The proposed strategy contains two processes, first some nodes are randomly chosen, and then their random acquaintances are vaccinated. Since the nodes with larger degree have greater chance to be chosen than those small-degree ones, this strategy performs much better than random immunization especially in scale-free networks.

As is shown in Fig.3, all these immunization strategies, despite their many differences in efficiency and applicability, are focusing on local behaviors by immunization of nodes (with the same or different importance) and are static in nature (do not interact with infection process), without considering the role of traffic flow and familiarity between nodes, not to mention the cost to reach a desired immunization level.

3 Economic Defense Model

In this section we present our *Economic Defense Model* to extend the above analysis of epidemics and immunization from topological to weighted networks, by focusing on the concepts of efficiency and cost, and by taking the following factors into account without being limited by them: (a) Immunization strategies must concentrate on nodes that are statistically significant, (b) Statistically significant nodes are not necessarily limited to ones that are highly connected (We will build mathematical models to identify the most significant nodes in power-law models), and (c) Other system parameters may also matter.

The topological properties of a graph are fully encoded in its adjacency matrix A, whose elements a_{ij} are 1 if a link connects node i to node j, and 0 otherwise. The indices i, j run from 1 to N where N is the size of the network and we use the convention a_{ii}=0. Similarly, a *weighted network* is entirely described by introducing another matrix W whose entry w_{ij} ($0 \le w_{ij} \le 1$) gives the weight on the edge connecting the vertices i and j (and $w_{ij}=0$ if the nodes i and j are not connected). Throughout this paper we consider undirected graphs, with only the case of symmetric weights $w_{ij}=w_{ji}$. The degree of a node i is defined as the number k_i of neighbors: $k_i=\Sigma_j(a_{ij})$.

To account for the importance of links in our model, the weight w_{ij} of a link can be identified with (a) the familiarity or acquaintance to mimic the evolution and reinforcements of interactions in the case of Acquaintance Immunization of [15], or (b) the network flow and traffic load to determine the betweenness links of a graph in the case of the model in [23]. To characterize the significance of nodes more precisely, we introduce the concept of *vertex strength* s_i, defined as [24]:

$$s_i = \sum_{j \in V(i)} w_{ij} \tag{7}$$

where the sum runs over the set $V(i)$ of neighbors of i. The strength of a node integrates the information about its connectivity and the weights of its links, and can be considered as the natural generalization of the connectivity. In the case of Gnutella-like overlay networks, s_i provides the actual traffic going through the vertex i and is an obvious measure of the processing capacity and importance of each node.

To propagate immunization information to *right* nodes for curing, our solution is to make the selection probability $P_i(k_i, s_i, \alpha)$ be dependant on the parameters of k_i, s_i, and α, following:

$$P(i) \propto \left(k_i \cdot s_i\right)^{\alpha} \tag{8}$$

where k_i and s_i are the degree and vertex strength of node i respectively, $0<\alpha<1$ is a tunable parameter as an enhancement on the selection of nodes with high connectivity and importance. By the formula (8), we couple the statistical significance of both nodes and links between them into our defense model, which is expected to bring the immunization of inhomogeneous real-world networks to a more practical phase. According to the results in [5, 26], we can deduce the epidemic prevalence threshold ρ_c, which depends on the fraction f_v of vaccinated nodes following:

$$\rho_c(f_v) = \frac{2e^{-1/\alpha m^{\alpha}(\bar{s})^{1-\alpha}(1-f_v)}}{1-e^{-1/\alpha m^{\alpha}(\bar{s})^{1-\alpha}(1-f_v)}} \tag{9}$$

To examine whether a defense model or strategy is efficient and economical in propagating immunization information over the network, below we propose two leading concepts of immunization *efficiency* and *cost*, according to [27].

Immunization Efficiency: This variable, denoted by E, measures how efficiently immunization information (e.g., the defense messages issued by the Intrusion Responding Systems when activated by the Intrusion Detecting Systems, or the updating messages of virus definitions in the anti-virus software) can be proliferated over the network.

$$E(G) = \frac{\varepsilon(G)}{\varepsilon(G^{ideal})} = \frac{\frac{1}{N(N-1)}\sum_{i\neq j\in G} a_{ij}w_{ij}}{\frac{1}{N(N-1)}\sum_{i\neq j\in G} a_{ij}} = \frac{\sum_{i\neq j\in G} a_{ij}w_{ij}}{\sum_{i\neq j\in G} a_{ij}} \tag{10}$$

where the immunization efficiency $E(G)$ is normalized to be in the interval [0, 1] due to $0 \le w_{ij} \le 1$. The higher $E(G)$ is, the more efficient is the defense strategy operating on the graph G. Moreover, if we apply this formula to the sub-graph of i's neighbors, it will be a metric of local clustering coefficient of i, and can thus tell us how much the network is fault tolerant.

Immunization Cost: No defense measures can improve the efficiency without resulting in some amount of immunization *cost*, which has been paid little attention to in the aforementioned studies. It is therefore crucial to consider weighted networks and to define a proper variable to quantity the immunization cost. In order to do so, we define the immunization cost (denoted by C) of the graph G as:

$$C(G) = \frac{\sum_{i \ne j \in G} a_{ij} \gamma(\varepsilon_{ij})}{\sum_{i \ne j \in G} \gamma(\varepsilon_{ij})} \tag{11}$$

where γ is the so-called *cost evaluator* function, which calculates the cost needed for a defense strategy to conduct an immunization task with the given efficiency. Different cost evaluators should be used for application- and structure-specific networks.

With our defense model based on weighted graphs and focused on two leading design principles of the immunization efficiency and cost, we can then define the key notion of *Economic Defense Model*, which is an inhomogeneous-networks-oriented dynamic defense model having high immunization efficiency and low cost.

4 Applications to Gnutella-Like Unstructured Overlays

To justify the performance of our proposed model, in this section we apply it to the immunization of Gnutella-like overlays subject to deliberately intrusions. First, we develop a novel defense policy based on our defense model. The policy operates as follows: when intrusions are detected by IDS (Intrusion Detection System), the immunization mechanisms are activated, taking measures to inject the immunization messages into the infected nodes and to progressively spread them across the network hop by hop, with a selection probability calculated locally according to the Economic Defense Model. Beforehand, the intrusions are generated and diffused over the network in the following fashion: Given a threatened network, an intrusion procedure is executed to identify critical nodes/links, which will then be removed in order to cause maximum network damage. Methods of identification can be based on (a) optimal path/hubs, or (b) traffic load of links.

We investigate our Model (denoted by "Economic" in simulation results) by referring to two well-known models of *Targeted Immunization* (Targeted) and *Acquaintance Immunization* (Acquaintance), and by focusing on the following two aspects: (1) performance of immunization model, and (2) its applicability to real-world networks.

In Fig.4 we make extensive performance comparisons of these models and find that, our model results in higher efficiency (see Fig.4(a)) and lower cost (Fig.4(b)) compared with the other both models when fix at the same local clustering level. Besides, the approximately linear relation between the efficiency and the local clustering coefficient means that the former can be used as a metric of the latter, which

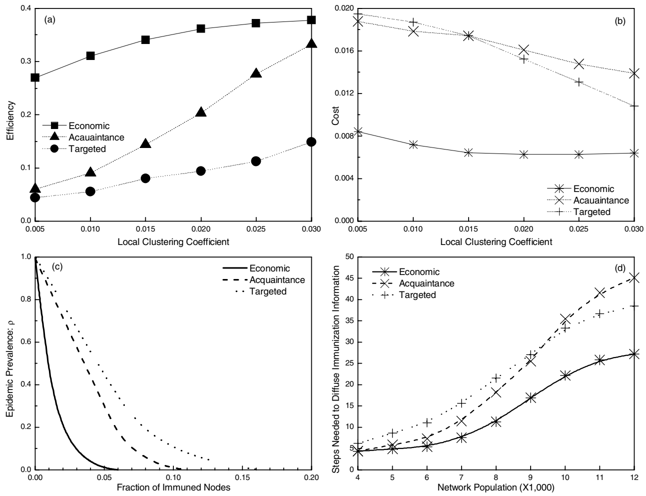

Fig. 4. Comparisons in performance aspects of three immunization strategies by simulations on a Gnutella overlay network. (a) Immunization efficiency; (b) Immunization cost; (c) Epidemic prevalence level; and (d) Steps needed to diffuse immunization information over the network.

verify the judgment in Section 3. From Fig.4 (c) and (d) we can see that, our model performs much better than the Targeted and Acquaintance models, with fewer nodes (Fig.4(c)) and steps (Fig.4(d)) needed to control the intrusions' epidemic prevalence.

Fig.5 (a) plots the immunization threshold as a function of power-law exponent of the Gnutella network, showing that our defense model is obviously more effective and thus is more suitable to the immunization applications of this kind of unstructured overlays. To examine whether our model meet the condition of power-law distributions, we plot probability distribution *P*(s) as a function of vertex strength *s* in Fig.5 (b). The data fitting between *P*(s) and *s* demonstrates strong power-law relations, in all the variable levels of traffic factor δ.

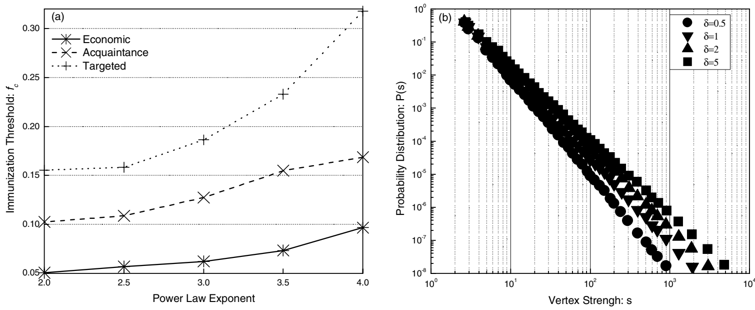

Fig. 5. (a) Immunization threshold f_c as a function of power-law exponent; (b) Probability distribution $P(s)$ as a function of vertex strength s, in cases of different traffic factors δ

5 Conclusions and Future Work

In this paper, we propose a novel immunization model, called *Economic Defense Model*, as a remedy of intrusion weaknesses in threatened unstructured overlays, in a topological perspective. Our model is built on existing well-known models, but extends them to a more general and practical case by taking into account weight-driven reinforcement mechanisms and statistical dynamics of nodes/links to characterize the immunization process of real-world networks, and by focusing on two leading concepts of efficiency and cost. In particular, the model yields significant performance gains and small-world-like behavior that has been observed in several real-world systems. These merits have also been verified by extensive simulations on a realistic Gnutella network using a new economic defense strategy. However, so far, the fundamental problems have not been solved, such as, what is the ultimate factor impacting the immunization process of unstructured overlays? This will be our future work.

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Outsourcing Internet Security: Economic Analysis of Incentives for Managed Security Service Providers

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Abstract. Firms hesitate to outsource their network security to outside security providers (called Managed Security Service Providers or MSSPs) because an MSSP may shirk secretly to increase profits. In economics this secret shirking behavior is commonly referred to as the Moral Hazard problem. There is a counter argument that this moral hazard problem is not as significant for the Internet security outsourcing market because MSSPs work hard to build and maintain their reputations which are crucial to surviving competition. Both arguments make sense and should be considered to write a successful contract. This paper studies the characteristics of optimal contracts (payment to MSSPs) for security outsourcing market by setting up an economic framework that combines both effects. It is shown that an optimal contract should be performance-based. The degree of performance dependence decreases if the reputation effect becomes more significant. We also show that if serving a large group of customers helps the provider to improve service quality significantly (which is observed in the internet security outsourcing market), an optimal contract should always be performance-based even if a strong reputation effect exists.

1 Introduction

It is predicted that the security outsourcing market, where firms contract with outside information security vendors to meet their organizational demands, will grow at a compound rate of double digits from $4.1 billion in 2001 to $9.0 billion in 2006. Experts predict that this growth rate will continue through 2008[10]. Two major reasons explain the quick expansion of security outsourcing. First, it offers production cost advantages. For example, for security device (firewalls, IDSs) management, a security engineer may cost $8,000 to $16,000 per month. In order to provide 24*7 support, this figure needs to at least be tripled. For the same functions, MSSPs charge between $600 and $4,000 per month. Counterpane, one of the most successful MSSPs, charges only 4% and 10% of cost a firm incurs to monitor network security. [19].

Second, security service providers have richer experiences, updated technology and better trained expertise by specializing in this area and serving diverse

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clients. A large client base also contributes to the improvement of the service quality because a service provider that monitors more networks is more likely to correlate attacks, identify new attacking patterns, and warn customers of events beyond their perimeters. In this sense, the security service provider also serves as an efficient information-sharing mechanism on security issues if he has a big customer base, and customers enjoy positive information spilled over from other customers.

Despite these benefits, a big doubt about the quality of security services still remains because of the well known 'moral hazard' problem. That is, the service providers can shirk secretly to increase their profits. A survey by Jeffrey Kaplan published in Business Communication Review (2003) [15] reports that 40.6% of the firms surveyed have such concerns. A counter argument is that the moral hazard problem is not as significant for security outsourcing market because MSSPs have to work hard to build and keep a good reputation which is crucial to survive the competition. Buyers will not contract network security to outside providers that cannot offer high quality service consistently. Both arguments are reasonable and should be considered to write a successful contract.

The moral hazard problem has been studied extensively in economics. Holmstrom(1979) [14] studies this problem in a one-period formulation and proposed that optimal contract under moral hazard is performance-based. Lambert(1983) [18] studies a moral hazard problem in a finitely-repeated model and shows that optimal payment depends on performance history. Spear and Srivastava(1987) [22] present similar results in analyzing an infinitely repeated model.

One main result in research on reputation is that reputation effect can mitigate the moral hazard problem. More complicated effects have been observed for different businesses. Banerjee and Duflo(2000) [7] study Indian software development and propose that reputation may explain why a larger proportion of software firms which have been established more years get payback from cost overruns, where the cost overrun may have been caused by software firms' moral hazard behavior. Gomes(2000) [11] shows that in the stock market, reputation effect is significant whenever the moral hazard problem is significant. It also shows that if a firm can finance through ways other than the stock market, its ability to build reputation is unrelated with growth opportunities. Dejong et al.(1985) [9] discover that in a laboratory market, reputation helps alleviate the moral hazard problem, but at the same time, there is evidence that reputable agents use opportunities to falsely advertise attempting to deceive the principals.

In the IT outsourcing literature, moral hazard has long been recognized as a major risk of an outsourcing project and incentive-compatible contracts are proposed to solve this problem [8], [6]. Aubert et al. [1] identify contractual difficulties [16] and diminished quality of services [2] as a major undesirable consequences of IT outsourcing. Aubert also suggests that diminished quality of services may be correlated with size of the provider.

This paper studies characteristics of optimal contracts (in form of payments to MSSPs) for security outsourcing market by setting up an economic framework that combines moral hazard and reputation effects, where reputation of a provider is reflected by how many customers it has. It is shown that an optimal contract should be performance-based. The degree of performance dependence decreases if the reputation effect becomes more significant. We also show that if serving a large group of customers helps the provider to improve service quality significantly (which is observed in the Internet security outsourcing market), an optimal contract should be performance-based even if a strong reputation effect exists.

The rest of this paper is organized as follows: In Sections 2, we survey and define managed security services. Section 3 contains a baseline moral hazard model. In Section 4 we present our model by adding reputation effect to moral hazard model for security outsourcing. We end with conclusions in Section 5. The Appendix contains summary tables for managed security services and service providers.

2 A Survey of Managed Security Services

With the increasing frequency and impact of cyber attacks as well as new government regulations (HIPAA[13], Sarbanes-Oxley[21], Gram-Leach-Bliley[12], California Disclosure Laws[3]), more firms are seeking to outsource the function of Internet security protection to expert providers. Thus MSSPs have emerged, characterized by technical capabilities and responsiveness to meet buyers diversified needs.

Managed security services can be categorized into 5 groups: (1) assessment, (2) monitoring, (3) threat and incidence control, (4) identity management and (5) consulting. The assessment sub-category includes services that evaluate a firm's security processes by penetration test on a regular basis. Security monitoring contains services such as managed firewall services, managed intrusion detection systems (IDS), managed intrusion prevention systems (IPS) and data analysis. Threat intelligence addresses how to measure and manage threats before they cause harm. Threats are detected by finding correlation among the network behavior across the Internet that the provider monitors. Early warning can be given even before attacks have reached firms' security perimeter. Once an incidence is discovered, incidence control service will be in action following pre-specified procedures to minimize impacts on firms. Figure 2 in the Appendix provides a list of security services.

The list of managed security services is expanding as demand for new security technology emerges. MSSPs may concentrate on just one or any combination of these services based on their technology strength and background. Different packages of services have been developed to meet diversified needs from global enterprizes to small businesses.

Based on our survey, managed security services possess the following attributes: tailored security solution design, expert security consulting support, 24×7 security monitoring, real-time security analysis, and real-time security incident response.

Figure 3 in the Appendix is a summary of major managed security services providers with the corresponding services they offer.

3 Baseline Model

It has been well documented that for the principal-agent problem, where a principal hires an agent but cannot observe agent's effort level, principal's payment to the agent should depend on realized performance. In other words, a performance-based contract is recommended. This contingent plan is expected to give agent hard working incentives by dumping some risks on him.

Following Spear and Srivastava(1987) [22], the standard model dealing with agent's moral hazard problem over infinite horizon, can be written in a recursive form as the following,

$$\begin{aligned} K(v) = & \max_{p(y),w(y),a} \int \{y - p(y) + \rho K(w(y))\} f(y|a) dy \\ \text{st} \quad & \int \{u(p(y)) + \rho w(y)\} f(y|a) dy - \phi(a) \geq v \qquad (PK) \\ & a \in \arg\max \int \{u(p(y)) + \rho w(y)\} f(y|a) dy - \phi(a) \qquad (IC) \end{aligned} \tag{1}$$

where the variables are defined as:

- y, output of current period
- $p(y)$, payment to the agent
- v, the agent's revenue stream discounted to current period
- $w(y)$, the agent's revenue from next period on
- $u(\cdot)$, the agent's utility function
- a, the agent's effort level
- $\phi(a)$, cost that the agent incurs by working at effort level a
- $f(y|a)$, probability distribution of output given the agent's effort level is a

By using the recursive formulation, this model utilizes the idea of dynamic programming: optimize one period at a time assuming optimal behavior in following periods. Thus, the objective function consists of two parts: $y - p(y)$ is the principal's payoff in the *current* period and $K(w(y))$ represents the principal's best payoff from *next* period on. With the discounting factor ρ, $y - p(y) + \rho K(w(y))$ represents principal's discounted profit. Since output y is random, expected payoff is calculated by taking the integration w.r.t y.

The first constraint is usually called the promise keeping(PK) constraint. It restricts principal's choice set of $p(y)$ and $w(y)$ to those that provide decent payoff to the agent. The second constraint is called the incentive compatibility(IC) constraint. This constraint incorporates moral hazard behavior of the agent into this model. That is, for any given a contract $p(y)$ and $w(y)$, the agent always chooses an effort level a that works best for its own good. Altogether, this model shows how a principal maximizes his profit by choosing a current period payment $p(y)$ and a future payoff $w(y)$ to the agent in presence of agents' moral hazard behavior. An optimal contract derived from this model is expected to provide suppliers incentives to work hard.

The following assumption (MLRP: Monotone Likelihood Ratio Property) is made in [22]. Assume that the distribution function of output y satisfies $\forall y_2 > y_1$, the likelihood ratio $\frac{f(y_2|a)}{f(y_1|a)}$ is monotone increasing in effort level a. This assumption is equivalent to $\frac{d}{dy}[\frac{f_a(y,a)}{f(y,a)}] \geq 0$.

The MLRP assumption means $\forall a_2 > a_1, y_2 > y_1$, we have $\frac{f(y_2|a_2)}{f(y_1|a_2)} \geq \frac{f(y_2|a_1)}{f(y_1|a_1)}$. In other words, at a higher effort level a_2, a higher output y_2 is more likely to be realized than a lower output y_1. Spear and Srivastava(1987) [22] shows that when the MLRP condition holds, optimal solutions $p(y)$ and $w(y)$ are performance-based. Good performance is rewarded from both high payment this period and increased higher payoff in the following periods.

4 Moral Hazard Model for Internet Security Outsourcing

In the context of security outsourcing, the security buyer is the principal and the security service provider is the agent. The baseline model can be applied to study the moral hazard problem in security outsourcing scenario with one modification: a larger customer base helps improve service quality. From the survey of managed security services in section 2, we see that most managed services have this feature. In our model, this effect is introduced through $f(y|a,N)$. That is, besides depending on the effort level a, the distribution function of service performance y also depends on number of customers of the service provider, N. The effect of reputation is added to the baseline model through $N' = G(y)$, which means the provider's number of customers next period is dependent on current performance y. Then the model for internet security outsourcing is written as:

$$
\begin{aligned}
K(v,N) = & \max_{p(y),w(y),a} \int \{y - p(y) + \rho K(w(y),N')\} f(y|a,N)dy \\
\text{st} \quad & N \cdot \int \{(u(p(y)) + \rho w(y))\} f(y|a,N)dy - \phi(a,N) \\
& \qquad + (N' - N) \cdot \int \rho w(y) f(y|a,N)dy \geq v \qquad (PK) \\
& a \in \arg\max\{N \cdot \int \{u(p(y)) + \rho w(y)\} f(y|a,N)dy - \phi(a,N) \\
& \qquad + (N' - N) \cdot \int \rho w(y) f(y|a,N)\}dy \qquad (IC) \\
& N' = G(y)
\end{aligned} \tag{2}
$$

In the baseline model, it is assumed that the number of customers does not change from period to period. So it suffices to calculate provider's benefit from just one buyer. When number of customers is different for each period, we have to calculate total benefit the provider gets from all its customers. Therefore, the first term in (PK) and that in (IC) are multiplied by customer size N. The second terms in conditions (PK) and (IC) represent service provider's change in revenue due to change in number of customers. If $N' > N$, the provider gets additional benefit due to increase in demand.

The effect of N on service quality is added through $f(y|a, N)$. We assume a larger customer base help the provider to increase expected service quality, i.e,

Assumption 1. *At same effort level a, expected output is monotone in N, i.e, for $N_1 < N_2$, $\int yf(y|a, N_1) < \int yf(y|a, N_2)$.*

$\phi(a, N)$ denotes the cost of serving one out of N customers by working at effort a. It captures the provider's economy of scale, i.e., $\phi(a, N_2) < \phi(a, N_1)$ for $N_1 < N_2$. Change in number of customers due to performance is introduced through function $G(\cdot)$ which is assumed to be continuous and differentiable w.r.t y.

We can simplify the optimization problem (2) by cancelling terms out and dividing all terms in the (PK) and (IC) equation by N:

$$\begin{aligned} K(v, N) = & \max_{p(y), w(y), a} \int \{y - p(y) + \rho K(w(y), N')\} f(y|a, N) dy \\ \text{st} \quad & \int \{u(p(y)) + \frac{N'}{N} \rho w(y)\} f(y|a, N) dy - \phi(a, N) \geq v \quad (PK) \\ & a \in \arg\max \int \{u(p(y)) + \frac{N'}{N} \rho w(y)\} f(y|a, N) dy - \phi(a, N) \qquad (IC) \\ & N' = G(y) \end{aligned} \tag{3}$$

Rogerson(1985) [20] proved that under regularity conditions, it suffices to use first order condition of (IC) instead of (IC) itself. Therefore, we substitute (IC) by its first order condition . Also, we substitute N' in the objective function by $G(y)$. Then the maximization problem is written as:

$$\begin{aligned} K(v, N) = & \max_{p(y), w(y), a} \int \{y - p(y) + \rho K(w(y), G(y))\} f(y|a, N) dy \\ \text{st} \quad & \int \{u(p(y)) + \frac{G(y)}{N} \rho w(y)\} f(y|a, N) dy - \phi(a, N) \geq v \qquad (PK) \\ & \int [u(p(y)) + \frac{G(y)}{N} \rho w(y)] f_a(y|a, N) dy - \phi_a(a, N) = 0 \qquad (FOC) \end{aligned} \tag{4}$$

We put λ as the Lagrangian multiplier on the (PK) constraint and μ as the Lagrangian multiplier on the first order condition of (IC) constraint. Then conditions for the optimal solution are derived by taking first order derivative w.r.t the choice variables $p(y)$ and $w(y)$:

$$\{p(y)\} \qquad -1 + \lambda u'(p(y)) + \mu u'(p(y)) \frac{f_a(y|a, N)}{f(y|a, N)} = 0 \tag{5}$$

$$\{w(y)\} \qquad \rho K'(w(y), G(y)) + \frac{G(y)}{N} \rho \lambda + \frac{G(y)}{N} \rho \mu \frac{f_a(y|a, N)}{f(y|a, N)} = 0 \tag{6}$$

First order conditions (5) and (6) implies:

$$\frac{1}{u'(p(y))} = \lambda + \mu \frac{f_a(y|a, N)}{f(y|a, N)} \tag{7}$$

$$-K_w(w(y), G(y)) = \frac{G(y)}{N}(\lambda + \mu \frac{f_a(y|a,N)}{f(y|a,N)}) \tag{8}$$

Assumption 2. *MLRP-N. For $y_2 > y_1$ the likelihood ratio $\frac{f(y_2|a,N)}{f(y_1|a,N)}$ is monotone increasing in effort level a for all firm sizes N, i.e., $\frac{d}{dy}[\frac{f_a(y|a,N)}{f(y|a,N)}] \geq 0$.*

This assumption is a natural extension of Spear and Srivasava(1987) [22] to the scenario where customer size is included in the model explicitly.

Lemma 1. *Under MLRP-N, $p(y)$ is monotonically increasing in performance, i.e, $p'(y) > 0$*

Proof. Because the provider's utility function $u(\cdot)$ is concave in $p(y)$, Lemma 1 follows directly from first order derivative of equation (7) w.r.t y.

This result shows that format of optimal current payment $p(y)$ under the security outsourcing scenario conforms with standard results in moral hazard literature Holmstrom(1997) [14], Lambert(1983) [18] and Spear and Srivastava(1987) [22]. In the next step we show that format of optimal continuation payment $w(y)$ is a lot more complicated. We need to define two opposite conditions. First of all, we assume that the functional form of the provider's benefit function $K(\cdot)$ does not change over time. In particular, partial derivatives of $K(\cdot)$ w.r.t its arguments do not change, i.e., $K_{vN}(v, N) = K_{wG}(w(y), G(y))$. Also, let subscripts denotes partial derivatives.

Definition 1. ***Non-decreasing Marginal Impact Condition(NDMIC).*** *Reduction in buyer's benefit due to increasing in $w(y)$ is smaller if the provider has more customers. That is, $K_{vN}(v, N) = K_{wG}(w(y), G(y)) \geq 0$.*

Since v is the total payment to the provider, it can be decomposed into current payment $p(y)$ and future payment $w(y)$. In the maximization problem (4), if v increases, it implies $p(y)$ and $w(y)$ also increase so that the (PK) condition is not violated. Therefore buyer's benefit $K(v, N) = \max \int \{y - p(y) + \rho K(w(y), N)\} f(y|a, N)$ will decrease. Otherwise, the buyer can increase $p(y)$ and $w(y)$ and both the buyer and the provider get better off. NDMIC says that the impact on buyer's benefit $K(w(y), G(y))$ by increasing $w(y)$ is smaller if the provider's number of customers increases.

Similarly, we have a definition for Decreasing Marginal Impact Condition, which exactly opposite of NDMIC.

Definition 2. ***Decreasing Marginal Impact Condition(DMIC).*** *Reduction in buyer's benefit due to increasing in $w(y)$ is bigger if the provider has more customers. That is, $K_{vN}(v, N) = K_{wG}(w(y), G(y)) \leq 0$*

In this paragraph, we give some intuition how the distribution function $f(y|a, N)$ determines which of these two conditions holds. It suffices to analyze how $K_v(v, N)$ changes with N. How much $K(\cdot)$ decreases w.r.t v depends on two factors. Factor (1) is the $\frac{1}{N}$ coefficient in the (PK) condition of the maximization problem. Factor (2) is the distribution function $f(y|a, N)$ in the maximization problem. For effect of (1), if $\frac{1}{N_1} < \frac{1}{N_2}$, to satisfy the (PK) condition, larger increase in either $p(y)$ or $w(y)$ or both is required if the provider has N_1 customers. Then decrement in buyer's payoff $K(\cdot)$ due to increment in v is bigger, i.e., $K_v(v, N_1) < K_v(v, N_2) < 0$. In other words, being a small fraction of a big service provider, it is harder(more costly) to exert influence on the provider. For effect of (2), by Assumption 1, a provider with more customers N_1 will get larger expected output($\int (u(p(y)) - \rho w(y))f(y|a, N_1)dy$). Therefore, for same increase in total payment v, less increase $p(y)$ and $w(y)$ is required to keep the (PK) condition because of effect of $f(y|a, N)$. Therefore, $K(\cdot)$ deceases less if the provider has more customers. In other words, the buyer benefits from technology of scale of a provider with many customers. Combining effects from factor (1) and factor (2), how $K_v(\cdot)$ changes with N is ambiguous, i.e., the second derivative $K_{vN}(\cdot)$ can be either positive or negative. If factor (1) is dominant, DMIC holds. If factor (2) is dominant, NDMIC holds.

Graphically, NDMIC and DMIC are shown in Figure 1.

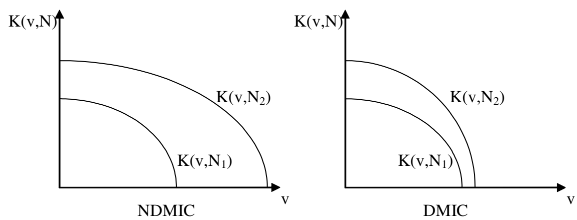

Fig. 1. NDMIC and DMIC for $N_2 > N_1$

Lemma 2. *When NDMIC is satisfied, $w(y)$ is strictly monotone increasing in y, i.e. $w'(y) > 0$.*

Proof. Take derivative of equation (8) w.r.t y,

$$\begin{aligned} LHS:\ & -K_{ww}(w(y), G(y))w'(y) - K_{wG}(w(y), G(y)) \cdot G'(y) \\ RHS:\ & \frac{d}{dy}\{\frac{G(y)}{N}(\lambda + \mu\frac{f_a(y|a, N)}{f(y|a, N)})\} > 0 \end{aligned} \tag{9}$$

Assumption 2 together with $G'(y) > 0$ implies RHS > 0. $K(\cdot)$ function is strictly concave in $w(y)$, which means $K_{ww} < 0$ [22]. Rearranging terms and applying NDMIC yields the result.

Lemma 2 shows that when NDMIC is satisfied, even there is significant reputation effect, continuation payment to the provider $w(y)$ should be performance-based. This is because if NDMIC is satisfied, the $K(\cdot)$ function is less concave in v. Therefore, cost to add more variation in v (thus $p(y)$ and $w(y)$) is lower. This more performance-based payment urges the service provider to work harder, which will in turn increase next period's number of customers $G(y)$. This result shows that reputation effect does not eliminate necessity of performance-based contracts.

Lemma 3. *When DMIC is satisfied, the larger the reputation effect is, the smaller the variance of $w(y)$ will be.*

Proof. Since $K(w(y), G(y))$ is strictly concave in $w(y)$, by Jensen's inequality, a payment schedule with same mean as $w(y)$ but smaller variance would increase $K(\cdot)$ strictly. In addition, DMIC means $K(w(y), G(y))$ is more concave in $w(y)$ when $G(\cdot)$ increases. Therefore, the buyer have additional incentive to smooth $w(y)$ when $G(\cdot)$ increases. On the other hand, a bigger variation in $w(y)$ gives the provider larger incentive to work hard. Then the buyer's problem is to find a payment schedule $w(y)$ with smallest variance yet has enough variation so that the service provider has to work hard. This job is easier in markets with reputation effect where additional variation in provider's payoff is added through the $G(y)$ function. Let $\hat{y}$ denote the borderline performance so that $G(\hat{y}) = N$. Then if $y_1 > (\hat{y})$ and $y_2 < (\hat{y})$, we have $G(y_1) > N$ and $G(y_2) < N$. From the maximization problem (4), since the provider's next period's payoff equals to $\frac{G(y)}{N} w(y)$, we have $\frac{G(y_2)}{N} w(y_2) < w(y_2) < w(y_1) < \frac{G(y_1)}{N} w(y_1)$, which means more variation is added to payment to the provider. Also, the bigger the reputation effect is, the more the variation added. Let $w_o(y)$ denotes payment to the provider if there is no reputation effect. Then the buyer can propose $w(y) = \frac{N}{G(y)} w_o(y)$, which has smaller variance compared with $w_o(y)$. Under the payment $w(y)$, the buyer gets higher $K(\cdot)$ since $w(y)$ has smaller variance compared with $w_o(y)$. Also, provider's benefit $\frac{G(y)}{N} w(y)$ does not change therefore the provider will work as hard and get same expected payment as if there is no reputation effect. Therefore, when the provider faces reputation effect, optimal solution $w(y)$ should have smaller variance compared with $w_o(y)$. Intuitively, this Lemma means that reputation helps to make the contract incentive compatible so that less variation in the payment is needed.

5 Conclusions

By modelling reputation effect under the moral hazard context, we show that the optimal contract should be performance-based. When effect of provider's customer base is not significant, the optimal contract changes less with performance when the reputation effect is more important for the provider. This conclusion conforms with previous research that reputation effect mitigates moral hazard problem.

We also show that if serving a large group of customers improves service quality significantly, the optimal contract should be performance-based even when a strong reputation effect exists.

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Appendix

Services	Description
Application security/code review	Scan web application code for vulnerabilities and insecure coding techniques
Security policy compliance	Perform regularly scheduled audits to ensure continued compliance and identify nonconformance with a company's established information security policy and government or industry-specific regulations(e.g. Sarbanes-Oxley and HIPAA)
Vuln. assessment and management	Perform penetration test on systems for known vulnerabilities
Certificates	Assessing firms' compliance with government, industry, partner and customer requirements, and issue proof of compliance
Risk management	Help customers to make decisions to accept exposure or to reduce vulnerabilities by either mitigating the risks or applying cost effective controls.
Managed firewall services	24*7 monitoring of all traffic through firewall, for service outage
Managed VPN services	Similar as managed firewall services, usually is a firewall add-on
Email anti-spam/antivirus	Scan content(email messages and attachments, SMTP, HTTP, FTP, file transfers for potential malicious code or junk mails
Managed IDS	24*7 monitoring of all network traffic, detect and analyze anomalies for true attacks
Managed IPS	24*7monitoring, proactively blocks threats rather than detect them after the fact
Security monitoring	Similar as IDS, can draw data from a wider variety of sources, and provide more in-depth analysis
Threat intelligence	Based on provider's research on real world events, offers a series of features including early warning of emerging threats, threat severity measurement, immediate notification and consultation
Incidence response and forensics	Provide responses to security bleaches base on five cornerstones of effective incidence management and response: detection, assessment, forensics, containment, and recovery
Authentication	Verifies and confirms identity of individuals who are accessing sensitive information, or conducting high value B2B transactions on an extranet
Identity management	Administer user authentication, access rights, access restrictions, account profiles, passwords and other similar attributes
Consulting	The practice of helping firms to improve security level through professional analysis

Fig. 2. List of security services

Company name/URL(Country)	App. security/code review	Vuln. assessment & mgmt	Certificates	Security policy compliance	Risk management	Managed firewall	Managed VPN	Email antivirus/antispam	IDS	IPS	Security monitoring	Incidence resp. & forensics	Threat intelligence	Authentication	Identity management	Consulting
AT&T/www.att.com(USA)	v	v				v	v	v	v							
Aspect/www.aspectsecurity.com/home.html(USA)	v	v														v
Avaya/www.avaya.com(USA)		v				v		v	v		v	v				
Aventail/www.aventail.com(USA)		v									v					
Cisco Systems/www.cisco.com(USA)		v			v	v	v		v	v				v		v
Computer Associates/www.ca.com(USA)		v											v		v	
Computer Science Corporation/www.csc.com(USA)	v					v			v		v	v		v		
Counterpane/www.counterpane.com(USA)		v				v		v	v	v	v					v
Cybertrust/www.cybertrust.com(USA)		v	v	v		v		v	v		v		v		v	
Dreaming Tree Tech. Inc//www.firewalls.com(USA)						v	v	v		v						
EDS/www.eds.com(USA)		v									v					v
Entrust/www.entrust.com(USA)			v	v				v							v	
Farm9/farm9.com(USA)	v	v				v		v	v							
FiberLink/www.fiberlink.com(USA)		v						v						v		
FrontBridge/www.frontbridge.com(USA)				v				v								
GeoTrust/www.geotrust.com(USA)			v												v	
Guardian Digital/www.guardiandigital.com(USA)											v				v	
IBM/www.ibm.com(USA)	v	v				v		v	v		v	v				
Internet Security Services/www.iss.net(USA)	v	v		v		v			v	v		v				
LURHQ/www.lurhq.com(USA)		v		v		v			v	v	v		v			
McAfee/http://www.mcafee.com/us(USA)		v				v		v						v		v
MCI/www.mci.com(USA)		v				v		v	v	v				v		
MessageLabs/www.massagelabs.com(USA)								v								
Netsec/www.netsec.net(USA)	v	v		v	v	v					v	v	v			
Netifice/www.netifice.com/default.html(USA)			v			v		v	v	v						
NT objectives/www.ntobjectives.com(USA)	v															v
NUVO/www.nuvo.com(Canada)				v				v	v		v					
Positive Networks/www.positivenetworks.com(USA)						v	v	v						v		
Postini/www.postini.com(USA)				v				v								
Qualys/www.qualys.com(USA)		v		v												
RedSiren/www.redsiren.com(USA)	v	v				v	v		v	v	v	v				
RSA security/www.rsasecurity.com(USA)	v													v		
Solutionary/www.solutionary.com(USA)	v					v	v		v		v	v				
Sonicwall/www.sonicwall.com(USA)						v	v	v		v						
SurfControl/www.surfcontrol.com(USA)								v			v					
Symantec/www.symantec.com(USA)	v	v		v		v		v	v	v	v	v				
Tata Group/www.tata.com(Inida)		v									v					v
TriGeo/www.trigeo.com(USA)								v								
TruSecure/www.trusecure.com(USA)	v	v	v			v		v	v		v	v				
Tenable network security/www.tenablesecurity.com(USA)		v				v			v							
Ubizen/www.ubizen.com(USA)		v		v		v	v	v	v		v	v				
Vericept/www.vericept.com(USA)																v
Verisign/www.verisign.com(USA)		v	v	v		v	v	v	v	v	v		v	v		v
VigilantMinds/www.vigilantminds.com(USA)	v	v		v		v	v	v	v	v	v		v			

1. MCI offers management only services, good for firms with existing devices
2. Internet Security Services offers money back satisfaction guarrantee
3. Dreaming Tree Technology Inc offers 30 days satisfaction or money back guarantee
4. symantec bought Riptech in 2002 and Brightmail in 2004
5. verisign acquired Guardent
6. Aventail sold its managed SSL to Netifice
7. Cybertrust is the result of a merger of Betrusted and TruSecure, , and is the majority owner of Ubizen

Fig. 3. Managed security services providers summary

Secure Construction of Virtual Organizations in Grid Computing Systems

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Abstract. Virtual organization (VO) is an important abstraction for designing large-scale distributed applications involving extensive resource-sharing. Existing works on VO mostly assumes that the VO already exists or is created by mechanisms outside of their system model. The VO construction is challenging and critical due to its dynamic and distributed nature. This paper presents a VO Construction Model and an implementation algorithm which is based on a threshold approach and is secure and robust in that events such as member admission, member revocation, VO splitting and merging etc. can be handled without centralized administration. Also authentication and communications among VO members are efficient and without tedious key exchanges and management usually needed in VO built upon the Grid Security Infrastructure (GSI).

Keywords: Virtual organization, grid computing system, system security, threshold approach.

1 Introduction

In recent years, the Grid Computing System (GCS) has emerged as a special form of distributed computing and is distinguished from conventional distributed computing systems by its focus on dynamic and larger-scale resource sharing over a wide geographical distribution [1]. GCS may be viewed as a sophisticated distributed computing system with high-level abstractions and services to facilitate large-scale sharing of resources in a dynamic manner. Resource sharing in GCS is built on the Virtual Organization (VO) model [1] with objectives to facilitate resource sharing and problem solving dynamically across organizations.

The concept of VO was first introduced in [2]. In general, VO is set up to respond quickly to the increasing complexities of the business environment and the associated pressures that place on traditional enterprises. From a system's point of view, VO is an architectural tool for designing distributed systems that aims to provide a flexible abstraction for implementing distributed systems with complex inter-process relationships [3, 4, 5] as well as the provision of flexibility and scalability with complex inter-process relationship [6]. On the other hand, the notion of VO has enabled GCS to support a wide variety of distributed commercial applications. For example, e-commerce applications in the healthcare

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industry typically require large-scale sharing of resources (medical records, laboratory test results and expensive equipments) by participating organizations. Being task-driven is a key characteristic of VO which is frequently restructured, sustained to run efficiently and responsively, and dissolved again to give way for the creation of the next collaborative opportunity. In practical situations, VO is a temporary cooperation of independent members to share resources. Numerous external and internal relationships must be handled across time and space in the management of VO.

Due to the dynamic nature of VOs and the complex relationships among VO participants, coordination and control mechanisms in traditional distributed computing are ineffective in managing VOs. The operation and functioning of a VO involves, firstly, the enrolment of participants and their resources and, secondly, the establishment of relationship among participants and resources. Thus the implementation and management of VO require that these two operations be carefully controlled and in accordance with the security policies of the VO. However, the tasks of specifying and enforcing the policies are challenging in that each participating organization may nominate a large number of users to access the shared resources and, for accountability and liability reasons typical in business applications, a VO construction model that is able to handle the admission and revocation of members, the merging, splitting and termination of VOs in a dynamic and secure manner is required.

At present, the Open Grid Services Architecture (OGSA) [7] provides a variety of VO service structures by integrating a collection of resources with heterogeneous and often dynamic characteristics. Since VO is usually deployed over open networks, security services and mechanisms are obviously very important. However, the bulk of its prior works have been in the context of authentication and communication. The proposed Grid Security Infrastructure (GSI) [8] mainly emphasizes identity authentication (based on public key) and encrypted messaging (based on SSL).

While security services provided in OGSA are certainly important, it lacks a secure VO construction model that is designed to manage dynamic VO events such as admitting and revoking members, merging and splitting of VOs, etc. Besides, there is no efficient authentication mechanism that is able to determine whether a party is an authorized member of a VO. The existing authentication mechanism in GSI is based on a simple public key infrastructure which is more suitable for point-to-point collaboration, but may not be efficient in meeting the needs of secure VO in which the collaboration may be one-to-many, many-to-many or many-to-one. Without an efficient and secure scheme for VO construction, VO can only either operate in a completely open manner (i.e. have no admission control) or admit member on some ad hoc basis. For example, existing literatures on GCS tend to assume that the VO already existed or are to be established by mechanisms outside of the VO model. It is therefore an important area to devise security mechanisms that address the concerns and requirements of VO and cater for the flexibility and control on sharing relationships needed to establish VO. The GSI has proposed a framework for the secure operations of

VO which emphasizes in identity authentication and secure messaging. It does not provide a secure mechanism for the control of events such as VO creation, admitting new members, revoking existing members, splitting, merging and VO termination which will happen throughout the life cycle of the VO.

This paper analyzes the characteristics of VO events and the requirements of VO construction. It introduces a new method to construct a secure and robust VO that aims to meet these requirements. In this paper, we adopt the Grid Computing System (GCS) as a basic model for organizing VO. The rest of the paper is organized as follows. In Section 2, the proposed system model for secure VO construction will be discussed. Section 3 describes the construction of VO based on a threshold approach. Section 4 concludes the discussion of this paper.

2 A System Model for VO Construction

Unlike prior work on VO which almost invariably assumed that the VO already existed and mainly focus on its operation and functioning, we aim to devise a practical and efficient mechanism to facilitate the construction and evolution of VOs. In this connection, we first define a system model which makes realistic assumption about VOs in a typical operating environment. Specifically, our model assumes that:

1. All members in a VO are true peers with a flat structure.
2. All members have identical rights and responsibilities without centralized administration privilege.
3. Members may or may not have prior working relationship and may not trust each other.
4. The cooperation is driven by events.

Our model also assumes that VO is established dynamically in response to the occurrence of events. This is something different from the existing VO model where resource management is centralized. This decentralized VO model will enable the resulting VO to have much flexibility in terms of resources management and security control. However, practical consideration will impose more challenges to the construction mechanism of VO.

In this model, the VO members themselves are responsible for admitting new members or revoking an existing one. However, it would not be efficient if every member in the VO has to participate in every event especially when the number of members in the VO is large. In this case, construction is typically based on some form of limited consensus among current members which is equivalent to attaining a threshold (or minimum number) of current members who agree to admit a prospective new member or revoke an existing member. Based on this assumption, the VO model will possesses the following characteristics:

1. The resulting VO is a temporary collaborative community in response to a specific business need or urgent event and will be dissolved when the need or event ceases to exist. It will go through a life cycle with four distinct phases, namely, creation, operation, evolution and termination [4].

2. VO members can join and leave at any time. A single VO can split into multiple VOs based on members' interest. On the other hand, multiple VOs can merge into a single VO if common tasks appear. Finally, a VO will terminate once the specific mission is over.
3. VOs are distributed with no single VO memebr in charge of the VO construction events. The IT infrastructure is built upon insecure distributed environment.
4. Members in a VO will remain autonomous and independent. Each member will maintain its own share of responsibility and control of the the VO.

With the aforementioned characteristics in mind, we will impose the following requirements in the VO construction model:

1. Each VO will be assigned a unique identifier which will be updated dynamically according to events in the VO life cycle. Each member of the VO can compute the identifier independently according to some pre-defined criteria. Membership to a VO is determined by the possession of a valid identifier.
2. Admission of new VO member is authorized by a group of existing members based on a limited consensus principle. New member is not able to access information of the VO prior to its admission.
3. Any member may be revoked from the VO either voluntarily or by a group of members in the VO based on a limited consensus principle. Revoked member will not be able to access information in the VO after revocation.
4. A single VO can be split into multiple VOs. Each new VO will create its own secure environment and unique identifier.
5. Multiple VOs can be merged into a new single VO with a new VO identifier.
6. VO members are able to authenticate and communicate with each other in an efficient and secure manner.

In order to satisfy all the requirements as stated for the construction of VO, we propose a VO construction model that is based on a threshold approach in order to handle all the events in the VO life cycle. In the threshold approach, there are two choices of the threshold, namely fixed threshold and dynamic threshold. A fixed threshold is essentially a t-out-of-n model where the threshold t is fixed and the number of members n may varies throughout the VO life cycle. For dynamic threshold, t shrinks or grows according to the value n. Since threshold updating is an expensive operation, it would not be practical to update the threshold for every construction event. Instead, within the life cycle, a VO will maintain a value called lim. Let n_{old} be the VO size before the occurrence of an event and n_{cur} be the VO size when the event happen. A new threshold update is triggered only when $|n_{\text{cur}} - n_{\text{old}}| > lim$.

Now, we can define the VO construction model using the tuple $T = \langle V, L, C \rangle$ where

V represents the set of members $\{m_1, \ldots, m_n\}$

L represents the states of the VO life cycle $\{\text{creation, operation, evolution, termination}\}$

C represents the construction events $\{\text{VO creation, member admission, member revocation, VO split, VO merge, VO termination}\}$.

3 VO Construction Algorithm

Our VO construction algorithm is based on the secure group communication framework proposed by [9] which makes use of a threshold approach to perform event operations without a trusted central administrator. The Group Key Distribution Scheme (GKDS) proposed possesses a characteristic in that the secret sharing and membership management in the system is dynamic in nature compare with traditional key distribution scheme where the secret sharing and storage is basically static. We propose to modify the GKDS algorithm such that it is more suitable for the implementation of a VO construction model. The modifications are summarized as follow:

1. The layered membership relationship in GKDS is simplified such that every member in the proposed VO model possesses the same amount of secret information and become true peer to each other.
2. There are only three events in the GKDS, namely group initialization, member admission and member revocation. The proposed algorithm will include also VO splitting and merging.
3. In GKDS, there is a security vulnerability which demands the number of member revoked each time does not exceed the threshold t. This is to prevent the shared secret being revealed by revoked members if the number exceeds t. However, the algorithm does not take care of the case when the cumulative number of revoked members across several revocations exceed t. In this case, the cumulative revoked members are able to reveal the shared secret if they are able to communicate with each other. To avoid this to happen, the proposed algorithm will first check the number of cumulative revoked members since the first revocation. If the number exceed t, the shared secret will be updated by the current remaining members to avoid the secret being revealed by revoked members.
4. The GKDS functions are extended to implement a secure authentication and communication environment for the VO.

The following subsections describe the proposed algorithm for the operations of the various events in the VO life cycle.

3.1 VO Creation

Let $V = \{m_1, \ldots, m_n\}$ be the initial set of members who would like to form a VO. These n members will authenticate each other using authentication techniques such as PKI-based certification and there is a secure communication channel between each pair of users in V before the VO is created. An initial threshold value $t \leq n$ is chosen and agreed by all members. Also, two large prime p and q is generated such that $q|(p-1)$ and a generator g of the multiplicative group $GF(p)$ is chosen. Members in V jointly compute a secret symmetric polynomial through the following steps:

1. For each $1 \leq r \leq n$, m_r construct a random symmetric polynomial of degree at most t

$$G_r(x,y) = \sum_{j=0}^{t}\sum_{k=0}^{t} a_{j,k}^{(r)} x^j y^k \bmod q$$

where $a_{j,k}^{(r)} \in GF(q), 0 \le j \le t, 0 \le k \le t$ are random and $a_{j,k}^{(r)} = a_{k,j}^{(r)}$ for all j,k.

2. m_r sends $G_r(x,s)$ to m_s for all $1 \le s \le n$ through the pair-wise communication channels.
3. For each $1 \le r \le n$, m_r, after receiving the secret information from other members in Step 1, computes,

$$F_r(x) = \sum_{i=1}^{n} G_i(x,r).$$

Note that $F_r(x) = F(x,r)$, where $F(x,y) = \sum_{i=1}^{n} G_i(x,y)$ is a symmetric polynomial of degree at most t.

3.2 VO Identifier Computation

After the VO creation process, every member possesses ite own secret polynomial. For the VO identifier computation process, assume member m_1 is nominated as the initiator. Let $M = \{i|m_i \in V\}$. m_1 performs the follow steps:

1. m_1 randomly generates an integer $r \in GF(q)$ and compute $G = g^r \bmod p$.
2. m_1 randomly chooses a t-subset I of nonzero elements from $GF(q)$ such that and $I \cap M = \emptyset$.
3. m_1 computes $G_j = G^{F_1(j)} \bmod p$ for all $j \in I$, and broadcasts the message $S = \{G, G_j||j|j \in I\}$ to all members in the VO.
4. Each member $m_i \in V$, including m_1, uses its own secret polynomial $F_i(x)$ and S to compute the VO identifier K independently where

$$K = (G^{F_i(1)})^{L(I\cup i,j)} \times \prod G_j^{L(I\cup i,j)} = G^{F_1(0)} = g^{rF_1(0)} \bmod p$$

where $L(V,u) = \prod_{v\in V, v\neq u} \frac{v}{v-u} \bmod q$.

Whenever there is an event (such as new member admission, etc.), the VO identifier K and the member's secret polynomial need to be updated. To perform an update, each member compute a new secret polynomial given by $F_i^*(x) = K + F_i(x)$. With this new polynomial, each member in the VO is able to compute the new K according to step 4.

3.3 Member Admission

Although in our proposed VO construction model, members can be admitted or revoked at any time. However, for the sake of efficient performance, requests for admission and revocation within a certain time interval are aggregated and processed in one batch. New VO member admission is authorized by a group of at least $t+1$ existing VO members. Suppose at a certain time interval, a set R of $t+1$ existing members agree to admit a set of new members N. Let $R_{id} = \{i|m_i \in R\}$. The admission is performed by the following steps:

1. Each existing member update its secret polynomial and the identifier K.
2. Each new member in N is assigned a unique identifier j randomly chosen from $GF(q)$ such that $N_{id} \cap M = \oslash$, where $N_{id} = \{j|m_j \in N\}$.
3. Each member $m_i \in R$ computes $F_{m_i}(j)$ for all $j \in N_{id}$ and distributes $\{F_{m_i}(j)||i\}$ to new member m_j over a secure channel.
4. Each new member $m_j \in N$ computes $F_i(j)$ the $t+1$ values received from the $t+1$ existing members in R based on the Lagrange interpolation

$$F_j(x) = F(x, j) = \sum_{i \in R_{id}} F_i(j) \prod_{k \in R_{id}, k \neq i} \frac{x-k}{i-k} \bmod q.$$

Every member (including the new members) is now possessing its own secret polynomial and is able to update the VO identifier accordingly.

3.4 Member Revocation

The model allows any member to leave the VO voluntarily or be revoked by a group of members. Whenever there is member revocation, an existing member m_R (not in the revocation list) will be nominated as the executing officer. Let O denotes the set of members who will leave the VO in the present time interval and let O_T denotes the set of members who have left the VO starting from the moment the first member left the VO. Members who have been revoked are, in essence, do not possess the VO identifier anymore and are not able to participate in any VO collaboration activity. Hence, the VO identifier should be updated as soon as any member being revoked from the VO. The following steps describe how members are revoked from the VO:

1. Update $O_T = O \bigcup O_T$.
2. If $|O_T| \geq t+1$, let $O_T = O$ and ask all remaining members to update their secret polynomial and K.
3. If $|O_T| < t+1$, set $O = O_T$.
4. A member m_R is nominated as the executing officer. Let $M = \{i|m_i \in V\}, O_{id} = \{i|m_i \in O\}$.
5. m_R randomly generates an integer $r \in GF(q)$ and compute $G = g^r \bmod p$.
6. m_R randomly chooses a subset I of nonzero elements from $GF(q)$ where $|I| = t - |O|$, and and $I \bigcap M = \emptyset$.
7. m_R computes $G_j = G^{F_{m_R}(j)} \bmod p$ for all $j \in I \bigcup O_{id}$.
8. m_R broadcasts the message $S = \{G, R, G_j||j|j \in I \bigcup O_{id}\}$.
9. Each member $m_i \in V/O$, including m_R, uses its own secret polynomial $F_i(x)$ and message S to update K given by

$$K = (G^{F_i(R)})^{L(O_{id} \cup I \cup i, j)} \times \prod_{j \in O_{id} \cup I} G_j^{L(O_{id} \cup I \cup i, j)} = G^{F_R(0)} = g^{rF_R(0)} \bmod p.$$

It is noted that even though members who have been revoked will still received the broadcasted message S. In order to prevent the revoked members to obtain the updated identifier K, S must consist at most t pieces of secret information from the revoked members. Hence, at any one time interval, the maximum number of members to be revoked cannot exceed t.

3.5 VO Splitting and Merging

There are occasions when a VO is required to split into several new VOs. It may due to conflicts of interest among members or because of technical problems such as network faults or congestion. The splitting of a VO into new VOs can be treated as the revocation of groups of members from the existing VO and these revoked groups of members create new VOs themselves. Hence, the splitting of the VO can be handled in the following steps:

1. Groups of members who would like to form new VOs are revoked from the existing VO according to the member revocation procedures.
2. Remaining members will update the VO identifier accordingly.
3. Groups of members revoked from the existing VO will create new VOs following the VO creation steps.

Similarly, VOs can be merged based on needs such as common interest or network restructuring, etc. In the merging process, a particular VO in the group of VOs to be merged is chosen as a base VO. Then members from other VOs can be admitted into the base VO according to the VO member admission procedures. After all members in the other VOs have been admitted into the base VO, the merging process is completed and a new VO is formed.

3.6 VO Termination

Termination is the last event in a VO life cycle. It happens when all the collaborative tasks are accomplished and there is no need to further maintain the relationships. Termination of VO is done by revoking all the VO members (probably in batches across several time intervals).

3.7 Secure Communication Environment in VO

Collaborations between VO members require a secure communication environment. Communication mode within the VO is not necessarily one-to-one. It may be one-to-many or many-to-many depending on needs. The GSI provides authentication and secure communication services based on PKI, X.509 certificates and SSL. It is viable if only one-to-one communications are required. For communications that are in the one-to-many or many-to-many modes, the maintenance of certificates and public keys is tedious and may not even possible if the VO membership changes dynamically. The VO construction model proposed also provides an infrastructure for secure communication among VO members. It is built upon the symmetric characteristic of the polynomial $F(x, y)$ and the common VO identifier and does not require exchange of certificates and public keys between members.

Since each VO member share the same identified K and it is updated whenever there is membership changes in the VO, hence it is possible to make use of K as the encryption and decryption key for secure communication between VO members. For authentication and secure communication between any two particular members, the polynomial $F(x, y)$ is used. Suppose member m_x and m_y would like to authenticate and communicate with one another. Since $F(x, y) = F(y, x)$ which is a piece of secret shared between m_x and m_y only, it can be treated as a private key for authentication and communication. If m_x uses $F(x, y)$ to encrypt message and send it to m_y, only m_y is able to decrypt the message using $F(y, x)$.

4 Conclusion

With the advances in grid computing systems (GCS), resources sharing and task-driven collaboration between different parties and organizations is gaining popularity and has become an important trend in today's e-business environment [10, 11, 12]. The notion of VO has enabled GCS to support a wide variety of applications especially suitable for organizing and implementing distributed commercial applications. At present, most VOs construction adopt the Open Grid Services Architecture (OGSA) which provides a set of services for the administration of the VO activities. Since most VOs are built upon open networks such as the Internet, security has become a critical issue. The Grid Security Infrastructure (GSI), which is widely adopted by most grid applications, provides a number of security services based on public key infrastructure, X.509 certificates, and the Secure Socket Layer (SSL) communication protocol. With the increase in complexity in today's e-business applications, the complexity of the VOs formed to serve these e-business applications are also becoming more and more complex with members join and leave in a highly dynamic manner. In this respect, VOs that rely on GSI for the provision of security services may encounter difficulties in the maintenance of public keys and certificates.

In this paper, we have proposed a VO construction model which aims to remedy the problems faced by VOs built upon PKI-based security framework. In the model, events in the VO life cycle such as member admission and revocation, etc. are handled by a decentralized mechanism making use of a threshold approach. Through the computation of symmetric polynomials, each member in the VO will share a common secret, known as the VO identifier and a secret of its own, known as the secret polynomial which allow the VO to operate events such as member admission and revocation without a centralized administration. Also, authentication and secure communications can be carried out among VO members without the hassle of certificate and key exchanges. The updating of these "secrets" can be done through a series of polynomial computations in a highly efficient manner compare with certificate and key maintenance in the PKI setting.

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A Graph-Theoretic Network Security Game*

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Abstract. Consider a network vulnerable to viral infection. The system security software can guarantee safety only to a limited part of the network. We model this practical network scenario as a non-cooperative multi-player game on a graph, with two kinds of players, a set of *attackers* and a *protector* player, representing the viruses and the system security software, respectively. Each attacker player chooses a node of the graph (or a set of them, via a probability distribution) to infect. The protector player chooses independently, in a basic case of the problem, a simple path or an edge of the graph (or a set of them, via a probability distribution) and cleans this part of the network from attackers. Each attacker wishes to maximize the probability of escaping its cleaning by the protector. In contrast, the protector aims at maximizing the expected number of cleaned attackers. We call the two games obtained from the two basic cases considered, as the *Path* and the *Edge* model, respectively.

We are interested in the associated *Nash equilibria* on them, where no network entity can unilaterally improve its local objective. We obtain the following results:

- The problem of existence of a pure Nash equilibrium is $\mathcal{NP}$-complete for the Path model. This opposed to that, no instance of the Edge model possesses a pure Nash equilibrium, proved in [4].
- We compute, in polynomial time, mixed Nash equilibria on corresponding graph instances. These graph families include, regular graphs, graphs that can be decomposed, in polynomially time, into vertex disjoint r-regular subgraphs, graphs with perfect matchings and trees.
- We utilize the notion of *social cost* [3] for measuring system performance on such scenario; here is defined to be the utility of the protector. We prove that the corresponding *Price of Anarchy* in any mixed Nash equilibria of the game is upper and lower bounded by a linear function of the number of vertices of the graph.

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1 Introduction

Motivation. This work considers a problem of *Network Security*, related to the protection of a system from harmful procedures (e.g. viruses, worms). Consider an information network where the nodes of the network are insecure and vulnerable to infection such as, viruses, Trojan horses, the *attackers*. A *protector*, i.e. system security software, is available in the system but it can guarantees security only to a limited part of the network, such as a simple path or a single link of it, chosen via a probability distribution. Each harmful entity targets a location (i.e. a node) of the network via a probability distribution; the node is damaged unless it is cleaned by the system security software. Apparently, the harmful entities and the system security software have conflicting objectives. The security software seeks to protect the network as much as possible, while the harmful entities wish to avoid being caught by the software so that they be able to damage the network. Thus, the system security software seeks to maximize the expected number of viruses it catches, while each harmful entity seeks to maximize the probability it escapes from the security software.

Naturally, we model this scenario as a non-cooperative multi-player strategic game played on a graph with two kinds of players: the *vertex players* representing the harmful entities, and the *edge* or the *path player* representing each one of the above two cases for the system security software considered; where it chooses a simple path or a single edge, respectively. The corresponding games are called the *Path* and the *Edge* model, respectively. In both cases, the Individual Cost of each player is the quantity to be maximized by the corresponding entity. We are interested in the *Nash equilibria* [7, 8] associated with these games, where no player can unilaterally improve its Individual Cost by switching to a more advantageous probability distribution.

Summary of Results. Our results are summarized as follows:

- We prove that the problem of existence of pure Nash equilibria in the Path model is $\mathcal{NP}$-complete (Theorem 1). This opposes to that, the simpler case of this model, i.e. that the Edge model posses no pure Nash equilibrium [4].
- [4] provides a graph-theoretic characterization of mixed Nash Equilibria for the Edge model. Unfortunately, this characterization only implies an exponential time algorithm for the general case. Here, we utilize the characterization in order to compute, in polynomial time, mixed Nash equilibria for specific graph instances of the game. In particular, we combine the characterization with a suitable exploration of some graph-theoretic properties of each graph family considered to obtain polynomial time mixed Nash equilibria. These graph families include, regular graphs, graphs that can be partitioned into vertex disjoint regular subgraphs, graphs with perfect matchings and trees (Theorem 3, Proposition 2, Theorems 4 and 5, respectively).
- We measure the system performance with respect to the problem considered utilizing the notion of the *social cost* [3]. Here, it is defined to be the number of attackers catch by the protector. We compute upper and lower bounds

of the social cost in any mixed Nash equilibria of the Edge model. Using these bounds, we show that the corresponding Price of Anarchy is upper and lower bounded by a linear function of the number of vertices of the graph (Theorem 6).

Due to space limits, some proofs are omitted; we include them in the full version of the paper [5].

Related Work and Significance. This work is a step further in the development of the new born area of *Algorithmic Game Theory.* It is also one of the only few works to model *network security problems* as a strategic game. Such a research line is that of *Interdependent Security* games, e.g. [2]. However, we remark that *none* of these works, with an exception of [2], study Nash equilibria on the games considered. This work is also one of the only few works that study games exploiting heavily *Graph-Theoretic* tools. In [2], the authors study a security problem and establish connections with variants of the Graph Partition problem. In [1], the authors study a two-players game on a graph, establish connections with the k-server problem. In a recent work of ours [4], we consider the simpler of the two games considered here, the Edge model. We provide a non-existence result for pure Nash equilibria of the model and a polynomial time algorithm for mixed Nash equilibria for bipartite graphs. Finally, our results contribute toward answering the general question of Papadimitriou [10] about the complexity of Nash equilibria for our special game.

2 Framework

Throughout, we consider an undirected graph $G(V, E)$, with $|V(G)| = n$ and $|E(G)| = m$. Given a set of vertices $X \subseteq V$, the graph $G \backslash X$ is obtained by removing from G all vertices of X and their incident edges. For any vertex $v \in V(G)$, denote $\Delta(v)$ the degree of vertex v in G. Denote $\Delta(G)$ the maximum degree of the graph G. A *simple* path, P, of G is a path of G with no repeated vertices, i.e. $P = \{v_1, \cdots, v_i \cdots v_k\}$, where $1 \leq i \leq k \leq n$, $v_i \in V$, $(v_i, v_{i+1}) \in E(G)$ and each $v_i \in V$ appears at most once in P. Denote $\mathcal{P}(G)$ the set of all possible simple paths in G. For a tree graph T denote $root \in V$, the root of the tree and $leaves(T)$ the leaves of the tree T. For any $v \in V(T)$, denote $parent(v)$, the parent of v in the tree and $children(v)$ its children in the tree T. For any $A \subseteq V$, let $parents(A) := \{u \in V : u = father(v),\ v \in A\}$.

2.1 Protector-Attacker Models

Definition 1. *An information network is represented as an undirected graph $G(V, E)$. The vertices represent the network hosts and the edges represent the communication links. For $\mathsf{M} = \{\mathsf{P}, \mathsf{E}\}$, we define a non-cooperative game $\Pi_{\mathsf{M}} = \langle \mathcal{N}, \{S_i\}_{i \in \mathcal{N}}, \{\mathsf{IC}\}_{i \in \mathcal{N}} \rangle$ as follows:*

- *The set of players is $\mathcal{N} = \mathcal{N}_{vp} \cup \mathcal{N}_p$, where $\mathcal{N}_{vp}$ is a finite set of* vertex *players vp_i, $i \geq 1$, $p = \{pp, ep\}$ and $\mathcal{N}_p$ is a singleton set of a player p which* *is either (i) a* path *player and* $p = pp$ *or (ii) an* edge *player and* $p = ep$*, in the case where* $\mathsf{M} = \mathsf{P}$ *or* $\mathsf{M} = \mathsf{E}$*, respectively.*

– *The strategy set* S_i *of each player* vp_i*,* $i \in \mathcal{N}_{vp}$*, is* V*; the strategy set* S_p *of the player* p *is either (i) the set of paths of* G*,* $\mathcal{P}(G)$ *or (i)* E*, when* $\mathsf{M} = \mathsf{P}$ *or* $\mathsf{M} = \mathsf{E}$*, respectively. Thus, the strategy set* $\mathcal{S}$ *of the game is* $\left(\times_{i \in N_{vp}} S_i\right) \times S_p$ *and equals to* $|V|^{|\mathcal{N}_{vp}|} \times |\mathcal{P}(G)|$ *or* $|V|^{|\mathcal{N}_{vp}|} \times |E|$*, when* $\mathsf{M} = \mathsf{P}$ *or* $\mathsf{M} = \mathsf{E}$*, respectively.*

– *Take any* strategy profile $\mathbf{s} = \langle s_1, \ldots, s_{|\mathcal{N}_{vp}|}, s_p \rangle \in \mathcal{S}$*, called a* configuration.
  - *The* Individual Cost *of vertex player* vp_i *is a function* $\mathsf{IC}_i : \mathcal{S} \rightarrow \{0, 1\}$ *such that* $\mathsf{IC}_i(\mathbf{s}) = \begin{cases} 0, & s_i \in s_p \\ 1, & s_i \notin s_p \end{cases}$ *; intuitively,* vp_i *receives* 1 *if it is not caught by the player* p*, and* 0 *otherwise.*
  - *The* Individual Cost *of the player* p *is a function* $\mathsf{IC}_p : \mathcal{S} \rightarrow \mathbb{N}$ *such that* $\mathsf{IC}_p(\mathbf{s}) = |\{s_i : s_i \in s_p\}|$.

We call the games obtained as the Path *or the* Edge *model, for the case where* $\mathsf{M} = \mathsf{P}$ *or* $\mathsf{M} = \mathsf{E}$*, respectively.*

The configuration $\mathbf{s}$ is a *pure Nash equilibrium* [7, 8] (abbreviated as *pure NE*) if for each player $i \in \mathcal{N}$, it minimizes IC_i over all configurations $\mathbf{t}$ that differ from $\mathbf{s}$ only with respect to the strategy of player i. We consider *mixed strategies* for the Edge model. In the rest of the paper, unless explicitly mentioned, when referring to mixed strategies, these apply on the Edge model. A *mixed strategy* for a vertex player (resp., edge player) is a probability distribution over vertices (resp., over edges) of G. A *mixed strategy profile* $\mathbf{s}$ is a collection of mixed strategies, one for each player. Denote $P_{\mathbf{s}}(ep, e)$ the probability that edge player ep chooses edge $e \in E(G)$ in $\mathbf{s}$; denote $P_{\mathbf{s}}(vp_i, v)$ the probability that player vp_i chooses vertex $v \in V$ in $\mathbf{s}$. Denote $P_{\mathbf{s}}(vp, v) = \sum_{i \in \mathcal{N}_{vp}} P_{\mathbf{s}}(vp_i, v)$ the probability that vertex v is chosen by some vertex player in $\mathbf{s}$. The *support* of a player $i \in \mathcal{N}$ in the configuration $\mathbf{s}$, denoted $D_{\mathbf{s}}(i)$, is the set of pure strategies in its strategy set to which i assigns strictly positive probability in $\mathbf{s}$. Denote $D_{\mathbf{s}}(vp) = \bigcup_{i \in \mathcal{N}_{vp}} D_{\mathbf{s}}(i)$. Let also $ENeigh_{\mathbf{s}}(v) = \{(u, v) \in E : \ (u, v) \in D_{\mathbf{s}}(ep)\}$. Given a mixed strategy profile $\mathbf{s}$, we denote $(\mathbf{s}_{-x}, [y])$ a configuration obtained by $\mathbf{s}$, where all but player x play as in $\mathbf{s}$ and player x plays the pure strategy y.

A mixed strategic profile $\mathbf{s}$ induces an *Expected Individual Cost* IC_i for each player $i \in \mathcal{N}$, which is the expectation, according to $\mathbf{s}$, of its corresponding Individual Cost (defined previously for pure strategy profiles). The mixed strategy profile, denoted as $\mathbf{s}^*$, is a *mixed Nash equilibrium* [7, 8] (abbreviated as mixed NE) if for each player $i \in \mathcal{N}$, it maximizes IC_i over all configurations $\mathbf{t}$ that differ from $\mathbf{s}$ only with respect to the mixed strategy of player i. Denote $BR_{\mathbf{s}}(x)$ the set of *best response (pure) strategies* of player x in a mixed strategy profile $\mathbf{s}$, that is, $\mathsf{IC}_x(\mathbf{s}_{-x}, y) \geq \mathsf{IC}_x(\mathbf{s}_{-x}, y')$, $\forall\ y \in BR_{\mathbf{s}}(x)$ and $y' \notin BR_{\mathbf{s}}(x)$, $y' \in S_x$, where S_x is the strategy set of player x (see also [9–chapter 3]). A *fully* mixed strategy profile is one in which each player plays with positive probability all strategies of its strategy set.

For the rest of this section, fix a mixed strategy profile $\mathbf{s}$. For each vertex $v \in V$, denote $Hit(v)$ the event that the edge player hits vertex v. So, $P_{\mathbf{s}}(Hit(v)) = \sum_{e \in ENeigh(v)} P_{\mathbf{s}}(ep, e)$. Define the minimum hitting probability $P_{\mathbf{s}}$ as $\min_v P_{\mathbf{s}}(Hit(v))$. For each vertex $v \in V$, denote $m_{\mathbf{s}}(v)$ the expected number of vertex players choosing v (according to $\mathbf{s}$). For each edge $e = (u, v) \in E$, denote $m_{\mathbf{s}}(e)$ the expected number of vertex players choosing either u or v; so, $m_{\mathbf{s}}(e) = m_{\mathbf{s}}(u) + m_{\mathbf{s}}(v)$. It is easy to see that for each vertex $v \in V$, $m_{\mathbf{s}}(v) = \sum_{i \in \mathcal{N}_{vp}} P_{\mathbf{s}}(vp_i, v)$. Define the maximum expected number of vertex players choosing e in $\mathbf{s}$ as $\max_e m_{\mathbf{s}}(e)$. We proceed to calculate the Expected Individual Costs for any vertex player $vp_i \in \mathcal{N}_{vp}$ and for the edge player.

$$\mathsf{IC}_i(\mathbf{s}) = \sum_{v \in V(G)} P_{\mathbf{s}}(vp_i, v) \cdot (1 - P_{\mathbf{s}}(Hit(v)) \quad (1)$$

$$\mathsf{IC}_{ep}(\mathbf{s}) = \sum_{e=(u,v) \in E(G)} P_{\mathbf{s}}(ep, e) \cdot m_{\mathbf{s}}(e) = \sum_{e=(u,v) \in E(G)} P_{\mathbf{s}}(ep, e) \cdot (\sum_{i \in N_{vp}} P_{\mathbf{s}}(vp_i, u) + P_{\mathbf{s}}(v_i, v)) \quad (2)$$

Social Cost and Price of Anarchy. We utilize the notion of *social cost* [3] for evaluating the system performance.

Definition 2. *For model* M, $\mathsf{M} = \{\mathsf{P}, \mathsf{E}\}$, *we define the* social cost *of configuration* $\mathbf{s}$ *on instance* $\Pi_{\mathsf{M}}(G)$, $\mathsf{SC}(\Pi_{\mathsf{M}}, \mathbf{s})$, *to be the sum of vertex players of* Π_{M} *arrested in* $\mathbf{s}$. *That is,* $\mathsf{SC}(\Pi_{\mathsf{M}}, \mathbf{s}) = \mathsf{IC}_p(\mathbf{s})$ *(*$p = \{pp, vp\}$*, when* $\mathsf{M} = \mathsf{P}$ *and* $\mathsf{M} = \mathsf{E}$*, respectively). The system wishes to maximize the social cost.*

Definition 3. *For model* M, $\mathsf{M} = \{\mathsf{P}, \mathsf{E}\}$, *the* price of anarchy, $\mathsf{r}(\mathsf{M})$ *is*

$$\mathsf{r}(\mathsf{M}) = \max_{\Pi_{\mathsf{M}}(G), \mathbf{s}^*} \frac{\max_{\mathbf{s} \in \mathcal{S}} \mathsf{SC}(\Pi_{\mathsf{M}}(G), \mathbf{s})}{\mathsf{SC}(\Pi_{\mathsf{M}}(G), \mathbf{s}^*)}.$$

2.2 Background from Graph Theory

Throughout this work, we consider the (undirected) graph $G = G(V, E)$.

Definition 4. *A graph G is* polynomially computable r-factor *graph if its vertices can be partitioned, in polynomial time, into a sequence $G_{r_1} \cdots G_{r_k}$ of k r-regular disjoint subgraphs, for an integer k, $1 \leq k \leq n$. That is, $\bigcup_{1 \leq i \leq k} V(G_{r_i}) = V(G)$, $V(G_{r_i}) \bigcap V(G_{r_j}) = \emptyset$ and $\Delta_{G_{r_i}}(v) = r$, $\forall \ \leq i, j \leq k \leq n$, $\forall v \in V$. Denote $G'_r = \{G_{r_1} \bigcup \cdots \bigcup G_{r_k}\}$ the graph obtained by the sequence.*

A graph G is *r-regular* if $\Delta(v) = r$, $\forall v \in V$. A *hamiltonian* path of a graph G is a simple path containing all vertices of G. A set $M \subseteq E$ is a *matching* of G if no two edges in M share a vertex. A *vertex cover* of G is a set $V' \subseteq V$ such that for every edge $(u, v) \in E$ either $u \in V'$ or $v \in V'$. An *edge cover* of G is a set $E' \subseteq E$ such that for every vertex $v \in V$, there is an edge $(v, u) \in E'$. A matching M of G that is also an edge cover of the graph is called *perfect* matching. Say that an edge $(u, v) \in E$ (resp., a vertex $v \in V$) is *covered* by the vertex cover V' (resp., the edge cover E') if either $u \in V'$ or $v \in V'$ (resp., if there is an edge $(u, v) \in E'$). A set $IS \subseteq V$ is an *independent set* of G if for all vertices $u, v \in IS$, $(u, v) \notin E$.

A *two-factor* graph is a *polynomially computable r-factor* graph with $r = 2$. It can be easily seen that there exist exponential many such graph instances. Moreover, these graphs can be recognized in polynomial time and decomposed into a sequence $C_1, \ldots, C_k$, $k \leq n$, in polynomial time via Tutte's reduction [11]. Thus, the class of *polynomially computable r-factor* graphs contains an exponential number of graph instances. The problem of finding a maximum matching of any graph can be solved in polynomial time [6].

3 Nash Equilibria in the Path Model

We characterize pure Nash Equilibria of the Path model.

Theorem 1. *For any graph G, $\Pi_{\mathsf{P}}(G)$ has a pure NE if and only if G contains a hamiltonian path.*

Proof. Assume that G contains a hamiltonian path. Then, consider any configuration $\mathbf{s}$ of $\Pi_{\mathsf{P}}(G)$ in which the path player *pp* selects such a path. Observe that path's player selection includes all vertices of G, that the player is satisfied in $\mathbf{s}$. Moreover, any player vp_i, $i \in \mathcal{N}_{vp}$ cannot increase its individual cost since, for all $v \in V(G)$, v is caught by *pp* and, consequently, $\mathsf{IC}_i(\mathbf{s}_{-i}, [v]) = 0$. Thus, $\mathbf{s}$ is a pure NE for $\Pi_{\mathsf{P}}(G)$.

For the contrary, assume that $\Pi_{\mathsf{P}}(G)$, contains a pure NE, $\mathbf{s}^*$, but the graph G does not contain a hamiltonian path. Then, the strategy of the path player, $\mathbf{s}^*_{pp}$, is not a hamiltonian path of G. Thus, there must exist a set of vertices $U \subseteq V$ such that, for any $u \in U$, $u \notin \mathbf{s}^*_{pp}$. Since $\mathbf{s}^*$ is a NE, for all players vp_i, $i \in \mathcal{N}_{vp}$, it must be that $\mathbf{s}^*_i \in U$. Therefore, there is no vertex player located on path $\mathbf{s}^*_{pp}$ which implies that *pp* is not satisfied in $\mathbf{s}^*$; it could increase its individual cost by selecting any path containing at least one vertex player. Thus $\mathbf{s}^*$ is not a NE, which gives a contradiction. □

Corollary 1. *The problem of deciding whether there exists a pure NE for any $\Pi_{\mathsf{P}}(G)$ is $\mathcal{NP}$-complete.*

4 Nash Equilibria in the Edge Model

We proceed to study Nash equilibria in the Edge model. In [4–Theorem 1] it was proved that if G contains more than one edges, then $\Pi_{\mathsf{E}}(G)$ has no pure Nash Equilibrium. For mixed NE, it was proved that:

Theorem 2 (Characterization of Mixed NE). *[4] A mixed strategy profile* $\mathbf{s}$ *is a Nash equilibrium for any $\Pi(G)$ if and only if:*

1. $D_{\mathbf{s}}(ep)$ *is an* edge cover *of* G *and* $D_{\mathbf{s}}(vp)$ *is a* vertex cover *of the graph obtained by* $D_{\mathbf{s}}(ep)$.
2. *(a)* $P_{\mathbf{s}}(Hit(v)) = P_{\mathbf{s}}(Hit(u)) = \min_v P_{\mathbf{s}}(Hit(v)),\ \forall\ u,v \in D_{\mathbf{s}}(vp)$ *and (b)* $\sum_{e \in D_{\mathbf{s}}(ep)} P_{\mathbf{s}}(ep,e) = 1$.
3. *(a)* $m_{\mathbf{s}}(e_1) = m_{\mathbf{s}}(e_2) = \max_e m_{\mathbf{s}}(e),\ \ \forall\ e_1 = (u_1,v_1), e_2 = (u_2,v_2) \in D_{\mathbf{s}}(ep)$ *and (b)* $\sum_{v \in V(D_{\mathbf{s}}(ep))} m_{\mathbf{s}}(v) = \nu$.

Here, we provide a estimation on the payoffs of the vertex players in any Nash equilibrium.

Lemma 1. *For any* $\Pi_{\mathsf{E}}(G)$*, a mixed NE,* $\mathbf{s}^*$*, satisfies* $\mathsf{IC}_i(\mathbf{s}^*) = \mathsf{IC}_j(\mathbf{s}^*)$ *and* $1 - \frac{2}{|D_{\mathbf{s}^*}(vp)|} \leq \mathsf{IC}_i(\mathbf{s}^*) \leq 1 - \frac{1}{|D_{\mathbf{s}^*}(vp)|},\ \forall i,j \in \mathcal{N}_{vp}$.

4.1 Mixed Nash Equilibria in Various Graphs

Regular, Polynomially Computable *r*-Factor and Two-Factor Graphs

Theorem 3. *For any* $\Pi_{\mathsf{E}}(G)$ *for which* G *is an* r*-regular graph, a mixed NE can be computed in constant time* $O(1)$.

Proof. Construct the following configuration $\mathbf{s}^r$ on $\Pi_{\mathsf{E}}(G)$:

$$\text{For any } i \in \mathcal{N}_{vp},\ P_{\mathbf{s}^r}(vp_i,v) := \tfrac{1}{n},\ \forall v \in V(G) \text{ and then set, } \mathbf{s}_j^r := \mathbf{s}_i^r,\ \forall j \neq i, j \in \mathcal{N}_{vp}\ .\ \text{Set } P_{\mathbf{s}^r}(ep,e) := \tfrac{1}{m},\ \forall e \in E. \tag{3}$$

Obviously, $\mathbf{s}^r$ is a valid (*fully*) mixed strategy profile of $\Pi_{\mathsf{E}}(G)$. We prove that $\mathbf{s}^r$ is a mixed NE for $\Pi_{\mathsf{E}}(G)$. Recall that in any r-regular graph, $m = r \cdot n/2$. By eq. (1) and the construction of $\mathbf{s}^r$, we get, for any $v,u \in V (= D_{\mathbf{s}^r}(vp_i))$, $i \in \mathcal{N}_{VP}$

$$\begin{aligned} IC_i(\mathbf{s}_{-i}^r,[v]) &= 1 - P_{\mathbf{s}}(Hit(v)) = 1 - \frac{|ENeigh(v)|}{m} = 1 - \frac{|ENeigh(u)|}{m} \\ &= IC_i(\mathbf{s}_{-i}^r,[u]) = 1 - \frac{r}{m} = 1 - \frac{2}{n}. \end{aligned} \tag{4}$$

The above result combined with the fact that $D_{\mathbf{s}^r}(vp_i) = V = S_i$, concludes that any vp_i is satisfied in $\mathbf{s}^r$. Now consider the edge player; for any $e = (u,v),\ e' = (u',v') \in E$, by eq. (2) and the construction of $\mathbf{s}^r$, we get

$$\begin{aligned} IC_{ep}(\mathbf{s}_{-ep}^r,[e]) &= \sum_{i \in \mathcal{N}_{VP}} (P_{\mathbf{s}^r}(vp_i,v) + P_{\mathbf{s}^r}(vp_i,u)) = \sum_{i \in \mathcal{N}_{VP}} (P_{\mathbf{s}^r}(vp_i,v') + P_{\mathbf{s}^r}(vp_i,u')) \\ &= IC_{ep}(\mathbf{s}_{-ep}^r,[e']) = \sum_{i \in \mathcal{N}_{VP}} 2 \cdot \frac{1}{n} = \frac{2\nu}{n} \end{aligned} \tag{5}$$

The above result combined with the fact that $D_{\mathbf{s}^r}(ep) = E = S_{ep}$, concludes that ep is also satisfied in $\mathbf{s}^r$ and henceforth $\mathbf{s}^r$ is a mixed NE of $\Pi_{\mathsf{E}}(G)$. It can be easily seen that the time complexity of the assignment $O(1)$. □

Corollary 2. *For any $\Pi_{\mathsf{E}}(G)$ for which G is contains an r-regular factor subgraph, a mixed NE can be computed in polynomial time $O(T(G))$, where $O(T(G))$ is the time needed for the computation of G_r from G.*

Proof. Compute an r-regular factor of G, G_r in polynomial time, denoted as $O(T(G))$. Then apply the mixed strategy profile $\mathbf{s}^r$ described in Theorem 3 on the graph G_r. See [5] for a full proof. □

Proposition 1. *For any $\Pi_{\mathsf{E}}(G)$ for which G is a* two-factor *graph, a mixed NE can be computed in polynomial time, $O(T(G))$, where $O(T(G))$ is the (polynomial) time needed for the decomposition of G into vertex disjoint cycles.*

Perfect Graphs

Theorem 4. *For any $\Pi_{\mathsf{E}}(G)$ for which G has a perfect matching, a mixed NE can be computed in polynomial time, $O(\sqrt{n} \cdot m)$.*

Proof. Compute a perfect matching M of G using a known such algorithm (e.g. [6] and requiring time $O(\sqrt{n} \cdot m)$). Construct the following configuration $\mathbf{s}^p$ on $\Pi_{\mathsf{E}}(G)$:

$$\begin{array}{l} \text{For } any\ i \in \mathcal{N}_{vp},\ P_{\mathbf{s}^p}(vp_i, v) := \frac{1}{n},\ \forall v \in V(G) \text{ and set } \mathbf{s}^p_j := \mathbf{s}^p_i, \\ \forall j \neq i, j \in \mathcal{N}_{vp}\ .\ \text{Set } P_{\mathbf{s}^p}(ep, e) := \frac{1}{|M|},\ \forall e \in E. \end{array} \tag{6}$$

Obviously, $\mathbf{s}^p$ is a valid mixed strategy profile of Π_{E}. Note that $|M| = n/2$. We first prove that any $i \in \mathcal{N}_{vp}$ is satisfied in $\mathbf{s}^p$. Note that each vertex of G is hit by exactly one edge of $D_{\mathbf{s}^p}(ep)$. Thus, by eq. (1), for any $i \in \mathcal{N}_{vp}$, $v, u \in V$,

$$\begin{aligned} IC_i(\mathbf{s}^p_{-i}, [v]) &= 1 - P_{\mathbf{s}^p}(Hit(v)) = 1 - P_{\mathbf{s}^p}(Hit(u)) = IC_i(\mathbf{s}^p_{-i}, [u]) \\ &= 1 - \frac{1}{|M|} = 1 - \frac{2}{|n|} \end{aligned}$$

The above result combined with the fact that $D_{\mathbf{s}^p}(vp_i) = V = S_i$ concludes that any vp_i is satisfied in $\mathbf{s}^p$. Now, as it concerns the edge player, note that $\mathsf{IC}_{ep}(\mathbf{s}^p_{-ep}, [e])$ depends only on the strategies of the vertex players in $\mathbf{s}^p$. Furthermore, these strategies are the same as the strategies of the vertex players on configuration $\mathbf{s}^r$ of Theorem 3. Henceforth, using the same arguments as in the theorem we conclude that the edge player is satisfied in $\mathbf{s}^p$. Since both kinds of players are satisfied in $\mathbf{s}^p$, the profile is a mixed NE for Π_{E}. For the time complexity of the assignment, see [5]. □

Trees. In Figure 1 we present in pseudocode an algorithm, called $\mathsf{Trees}(\Pi_{\mathsf{E}}(T))$, for computing mixed NE for trees graph instances. Note that in [4], a polynomial time algorithm for finding NE in bipartite graphs is presented. Thus, the same algorithm can apply for trees, since trees are bipartite graphs. However, that algorithm computes a NE of $\Pi_{\mathsf{E}}(T)$ in time $O(n^{2.5}/\sqrt{\log n})$, while the algorithm presented here computes a NE in linear time $O(n)$.

Algorithm $\mathsf{Trees}(\Pi_{\mathsf{E}}(T))$

1. Initialization: $VC := \emptyset$, $EC := \emptyset$, $r := 1$, $T_r := T$.
2. Repeat until $T_r == \emptyset$
   (a) Find the leaves of the tree T_r, $leaves(T_r)$.
   (b) Set $VC := VC \cup leaves(T_r)$.
   (c) For each $v \in leaves(T_r)$ do:
   If $parent_{T_r}(v) \neq \emptyset$, then $EC := EC \cup \{(v, parent_{T_r}(v)))\}$,
   else $EC := EC \cup \{(v, u)\}$, for any $u \in children_T(v)$.
   (d) Update tree: $T_{r+1} := T_r \backslash leaves(T_r) \backslash parents(leaves(T_r))$. Set $r := r + 1$.
3. Define a configuration $\mathbf{s}^t$ with the following support:
   For any $i \in \mathcal{N}_{VP}$, set $D_{\mathbf{s}^t}(vp_i) := VC$ and $D_{\mathbf{s}^t}(ep) := EC$. Then set $D_{\mathbf{s}^t}(vp_j) := D_{\mathbf{s}^t}(vp_i)$, $\forall\, j \neq i$, $j \in \mathcal{N}_{VP}$.
4. Determine the probabilities distributions of players in $\mathbf{s}^t$ as follows:
   ep : $\forall\, e \in D_{\mathbf{s}^t}(ep)$, set $P_{\mathbf{s}^t}(ep, e) := 1/|EC|$. Also, $\forall\; e' \in E(T)$, $e' \notin D_{\mathbf{s}^t}(ep)$, set $P_{\mathbf{s}^t}(ep, e') := 0$.
   For any vp_i, $i \in \mathcal{N}_{VP} : \forall\, v \in D_{\mathbf{s}^t}(vp_i)$, set $P_{\mathbf{s}^t}(vp_i, v) := \frac{1}{|VC|}$. Also, $\forall\, u \notin D_{\mathbf{s}^t}(vp_i)$, set $P_{\mathbf{s}^t}(vp_i, u) := 0$. Then set $\mathbf{s}_j^t = \mathbf{s}_i^t$, $\forall\, j \neq i$, $j \in \mathcal{N}_{VP}$.

Fig. 1. Algorithm $\mathsf{Trees}(\Pi_{\mathsf{E}}(T))$

Lemma 2. *Set VC, computed by Algorithm $\mathsf{Trees}(\Pi_{\mathsf{E}}(T))$, is an independent set of T.*

Lemma 3. *Set EC is an edge cover of T and VC is a vertex cover of the graph obtained by EC.*

Lemma 4. *For all $v \in D_{\mathbf{s}^t}(vp)$, $m_{\mathbf{s}^t}(v) = \frac{\nu}{|D_{\mathbf{s}^t}(vp)|}$. Also, for all $v' \notin D_{\mathbf{s}^t}(vp)$, $m_{\mathbf{s}^t}(v') = 0$.*

Lemma 5. *Each vertex of IS is incident to exactly one edge of EC.*

Proof. By Lemma 3, for each $v \in IS$ there exists at least one edge $e \in EC$ such that $e = (v, u)$. Assume by contradiction that there exists another edge, $(v, u') \in EC$. But since by step **2** of the algorithm for each vertex added in IS we add only one edge incident to it in EC, we get that it must be that $u' \in IS$. However, this contradicts to that IS is an independent set, proved in Lemma 2. □

By Lemmas 3(EC is an edge cover of G) and 5, we can show that:

Lemma 6. *For all $v \in D_{\mathbf{s}^t}(vp)$, $P_{\mathbf{s}}(Hit(v)) = \frac{1}{|D_{\mathbf{s}^t}(ep)|}$. Also, for all $v' \notin D_{\mathbf{s}^t}(vp)$, $P_{\mathbf{s}}(Hit(v')) \geq \frac{1}{|D_{\mathbf{s}^t}(ep)|}$.*

Theorem 5. *For any $\Pi_{\mathsf{E}}(T)$, where T is a tree graph, algorithm $\mathsf{Trees}(\Pi_{\mathsf{E}}(T))$ computes a mixed NE in polynomial time $O(n)$.*

Proof. **Correctness:** We prove the computed profile $\mathbf{s}^t$ satisfies all conditions of Theorem 2, thus it is a mixed NE. **1.**: By Lemma 3 . **2.**: By Lemma 6. **3.(a)**: Note that, $D_{\mathbf{s}^t}(vp)$ is an independent set of G and also a vertex cover of $D_{\mathbf{s}^t}(vp)$, by Lemmas 2, 3, respectively. Thus, by Lemma 4, for any $e = (u, v) \in D_{\mathbf{s}^t}(ep)$, we have $m_{\mathbf{s}^t}(e) = m_{\mathbf{s}^t}(v) + m_{\mathbf{s}^t}(u) = \frac{\nu}{|D_{\mathbf{s}^t}(vp)|} + 0$. **3.(b)**: Since VC is an independent set of G, for any $e = (u, v) \in E$, $e \notin D_{\mathbf{s}^t}(ep)$, $m_{\mathbf{s}^t}(e) = m_{\mathbf{s}^t}(v) + m_{\mathbf{s}^t}(u) \leq \frac{\nu}{|D_{\mathbf{s}^t}(vp)|} = m_{\mathbf{s}^t}(e')$, where $e' \in EC$.

Time Complexity: See [5]. □

4.2 The Price of Anarchy

Lemma 7. *For any $\Pi_{\mathsf{E}}(G)$ and an associated mixed NE $\mathbf{s}^*$, the social cost $\mathsf{SC}(\Pi_{\mathsf{E}}(G), \mathbf{s}^*)$ is upper and lower bounded as follows:*

$$\max\left\{\frac{\nu}{|D_{\mathbf{s}^*}(ep)|}, \frac{\nu}{|V(D_{\mathbf{s}^*}(vp))|}\right\} \leq \mathsf{SC}(\Pi_{\mathsf{E}}(G), \mathbf{s}^*) \leq \frac{\Delta(D_{\mathbf{s}^*}(ep)) \cdot \nu}{|D_{\mathbf{s}^*}(ep)|} \tag{7}$$

These bounds are tight.

Theorem 6. *The Price of Anarchy for the Edge model is $\frac{n}{2} \leq \mathsf{r}(\mathsf{E}) \leq n$.*

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Nash Equilibria and Dominant Strategies in Routing

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Abstract. Nash equilibria and dominant strategies are two of the major approaches to deal with selfishness in an automated system (AS), where each agent is a selfish entity.

In this paper, we consider the scenario when the receiver(s) and the relay links are both selfish, which generalizes the previous scenario in which either the relay links are selfish or the receivers are selfish. This also advances all previous studying in routing by taking into account the budget balance ratio. We prove that no mechanism can achieve budget balance ratio greater than $\frac{1}{n}$ when truthful revealing is a dominant strategy for each of the relay links and receivers. Here, n is the number of vertices in the network. In the meanwhile, we also present a mechanism that achieves the budget balance ratio $\frac{1}{n}$ and is truthful for both the receivers and relay links, which closes the bounds. When we relax the truthful revealing requirement to Nash Equilibrium for relay links, we present a mechanism that achieves an asymptotically optimal budget balance ratio.

1 Introduction

More and more research effort has been done to study the non-cooperative games recently. Among various forms of games, the unicast/multicast routing game [11, 4] and multicast cost sharing game [6, 2] have received a considerable amount of attentions over the past few years due to its applications in the Internet. However, both unicast/multicast routing game and multicast cost sharing game are one folded: the unicast/multicast routing game does not treat the receivers as selfish while the multicast cost sharing game does not treat the links as selfish. In this paper, we study the scenario, which we called *multicast system*, in which both the links and the receivers could be selfish.

In the first part, we study the α*-stable* multicast system that satisfies the following main properties: (1) strategyproofness for both the links and receivers; and (2) α-budget-balance. To illustrate our approaches, we first focus on the

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unicast system which is a special case of multicast system. We prove that if we use the least cost path for unicast routing, then there does not exist an α-stable unicast system such that $\alpha > \frac{1}{n}$, where n is the number of the nodes in the graph. On the other side, we present an $\frac{1}{n}$-stable unicast system, and further extend this idea to construct an $\frac{1}{r \cdot n}$-stable multicast system where r is the number of receivers in a multicast game.

In the second part, we relax the dominant strategy requirement to Nash Equilibrium for the links and study the performance of the Nash Equilibria for a multicast system. Again, we first study the unicast scenario and propose a unicast system that achieves $\frac{1}{2}$-budget-balance factor under any Nash Equilibrium. We then extend this to the multicast game which results in a multicast system with budget balance factor $\frac{1}{2r}$ under *any* Nash Equilibrium for the links.

2 Technical Preliminaries

2.1 Mechanism Design

A standard model for mechanism design is as follows. There are n agents $1, \ldots, n$. Each agent i has some private information t_i, called its *type*, only known to itself. The agent' types define the *type vector* $t = (t_1, t_2, \ldots, t_n)$. Each agent i has a set of strategies A_i from which it can choose. For each strategy vector $\mathbf{a} = (a_1, \ldots, a_n)$ where agent i plays strategy $a_i \in A_i$, the *mechanism* $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ computes an *output* $o = \mathcal{O}(\mathbf{a})$ and a *payment* vector $\mathcal{P}(\mathbf{a}) = (\mathcal{P}_1(\mathbf{a}), \ldots, \mathcal{P}_n(\mathbf{a}))$. A *valuation* function $v(t, o)$ assigns a monetary amount to agent i for each possible output o and t. Let $u_i(t, o)$ denote the *utility* of agent i at the output o and type vector t. Here, following a common assumption in the literature, we assume the utility for agent i is quasi-linear, i.e., $u_i(t, o) = v(t, o) + \mathcal{P}_i(\mathbf{a})$. Let $\mathbf{a}|^i b$ denote that every agent j, except i, plays strategy a_j, and agent i plays the strategy b. Let $\mathbf{a}_{-i}$ denote the strategies played by all agents other than i.

A strategy vector $a^\star$ is a *Nash Equilibrium* if it maximizes the utility of each agent i when the strategies of all the other agents are fixed as $a^\star_{-i}$, i.e., $u_i(t, \mathcal{O}(a^\star)) \geq u_i(t, \mathcal{O}(a^\star_{-i}|^i a'_i))$ for all i and all $a'_i \neq a^\star_i$. A strategy a_i is called a *dominant* strategy for agent i if it maximizes agent i's utility for all possible strategies of the other agents. If $\mathbf{a}$ is a dominant strategy vector for agents, then $\mathbf{a}$ is also a Nash Equilibrium.

A *direct-revelation* mechanism is a mechanism in which the only actions available to each agent are to report its private type. A direct-revelation mechanism is *incentive compatible* (IC) if reporting valuation truthfully is a dominant strategy. A direct-revelation mechanism satisfies *individual rationality* (IR) if the agent's utility of participating in the output of the mechanism is at least its utility if it did not participate the game at all. A direct-revelation mechanism is *truthful* or *strategyproof* if it satisfies both IC and IR properties.

A *binary demand game* is a game $\mathcal{G}$ such that (1) the range of the output method $\mathcal{O}$ is $\{0, 1\}^n$; (2) the valuation of the agents are not *correlated*. Binary demand game has been studied extensively [5, 1, 7, 4] and the type t_i of agent i could be expressed as the cost c_i in many applications. Here, if agent i provides a certain service, then its cost $c_i \geq 0$; if agent i requires a certain service, then its cost $c_i \leq 0$. It is generally known that [5, 1, 7, 4] if a mechanism $\mathcal{M}$ is strategyproof, then $\mathcal{O}$ should satisfy a certain *monotonicity* property: for every agent i, if it is selected when it has a cost c_i, then it is still selected when it has a cost $c_i' < c_i$. If $\mathcal{O}_i(\mathbf{c}) = 0$, we require that $\mathcal{P}_i(\mathbf{c}) = 0$, which is known as *normalization*. If $\mathcal{O}$ is monotonic and the payment scheme is normalized, then the *only* strategyproof mechanism based on $\mathcal{O}$ is to pay $\kappa_i(\mathbf{c})$ to agent i if it is selected and 0 otherwise, where $\kappa_i(\mathbf{c})$ is the threshold cost of i being selected.

2.2 Multicast Payment Sharing Mechanism

In this paper, we model a network by a link weighted graph $G = (V, E, c)$, where V is the set of all nodes and c is the cost vector of the set E of links. For a multicast session, let Q denote the set of all receivers. In game theoretical networking literatures, usually there are two models for the multicast cost/payment sharing.

Axiom Model (AM). All receivers must receive the service, or equivalently, each receiver has an infinity valuation [3]. In this model, we are interested in a sharing method ξ that computes how much each receiver should pay when the receiver set is R and cost vector is $\mathbf{c}$.

Valuation Model (VM). There is a set $Q = \{q_1, q_2, \cdots, q_r\}$ of r possible receivers. Each receiver $q_i \in Q$ has a valuation η_i for receiving the service. Let $\eta = (\eta_1, \eta_2, \ldots, \eta_r)$ be the valuation vector and η_R be the valuation vector of a subset $R \subseteq Q$ of receivers. In this model, we are interested in a sharing mechanism $\mathcal{S}$ consisting of a *selection scheme* $\sigma(\eta, \mathbf{c})$ and a *sharing method* $\xi(\eta, \mathbf{c})$. Here $\sigma_i(\eta, \mathbf{c}) = 1$ (or 0) denotes that receiver i receives (or does not receive) the service, and $\xi_i(\eta, \mathbf{c})$ computes how much the receiver q_i should pay for the multicast service. Let $\mathbb{P}(\eta, \mathbf{c})$ be the total payment for providing the service to the receiver set. For the notational consistency, we denote the sharing method and total payment under AM as $\xi(\eta_R^{=\infty}, \mathbf{c})$ and $\mathbb{P}(\eta_R^{=\infty}, \mathbf{c})$, where $\eta_R^{=\infty}$ denotes a valuation vector where each individual valuation is infinity. The utility of a receiver i is denoted as $u_i(\eta, \mathbf{c})$.

In the valuation model, a receiver who is willing to receive the service is not guaranteed to receive the service. For notational simplicity, we abuse the notations by letting $\sigma(\eta, \mathbf{c})$ be the set of actual receivers decided by the selection method σ. Under the Valuation Model, we need to find a sharing mechanism that is *fair* according to the following criteria.

1. **Budget Balance** (BB): For the receiver set $R = \sigma(\eta, \mathbf{c})$, $\mathbb{P}(\eta, \mathbf{c}) = \sum_{q_i \in Q} \xi_i(\eta, \mathbf{c})$. If $\alpha \cdot \mathbb{P}(\eta, \mathbf{c}) \leq \sum_{i \in R} \xi_i(\eta, \mathbf{c}) \leq \mathbb{P}(\eta, \mathbf{c})$, for some given parameter $0 < \alpha \leq 1$, then $\mathcal{S} = (\sigma, \xi)$ is called α-budget-balance. If budget balance is not achievable, then a sharing scheme $\mathcal{S}$ may need to be α-budget-balance.
2. **No Positive Transfer** (NPT): Any receiver q_i's sharing should not be negative. In other words, we don't pay the receiver to receive.
3. **Free Leaving** (FR): The potential receivers who do not receive the service should not pay anything, *i.e.*, if $\sigma_i(\eta, \mathbf{c}) = 0$, then $\xi_i(\eta, \mathbf{c}) = 0$.

4. **Consumer Sovereignty** (CS): For any receiver q_i, if η_i is sufficiently large, then q_i is guaranteed to be an actual receiver. In other words, fix any η_{-i}, there must exist a valuation x for q_i such that $\forall y \geq x$, $\sigma_i((y, \eta_{-i}), \mathbf{c}) = 1$.
5. **Group-Strategyproof** (GS): Assume that η is the valuation vector and $\eta' \neq \eta$. If $u_i(\eta', \mathbf{c}) \geq u_i(\eta, \mathbf{c})$ for each $q_i \in \eta$, then $u_i(\eta', \mathbf{c}) = u_i(\eta, \mathbf{c})$.

A sharing mechanism $\mathcal{S}$ that is α-budget-balance and satisfies the remaining criteria (*i.e.*, NPT, FR, CS, GS) is α*-fair*. It is generally known that if a sharing method ξ satisfies cross-monotonicity and NPT under Axiom Model, one can explicitly construct a fair sharing mechanism $\widetilde{\mathcal{S}} = (\widetilde{\sigma}, \widetilde{\xi})$ as shown in [6, 2].

2.3 Problem Statement

Assume there is a graph $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$. Let $\mathbf{c}$ be the cost vector of the links, *i.e.*, $c(e_i) = c_i$ and $\mathbf{c} = (c_1, c_2, \ldots, c_m)$. Given a source s and a set of receiver Q, multicast first chooses a receiver set $R \subseteq Q$, and then constructs a tree rooted at s that spans the receivers set R. In this paper, we focus on the *least cost path tree* (LCPT), which is the union of the least cost paths from the source to each receiver, due to the reason that LCPT is most widely used in practice. Let $\mathsf{LCPT}(R, \mathbf{c})$ be the LCPT when the cost vector is $\mathbf{c}$ and the actual receiver set is R. We are interested in a *multicast system* $\Psi = (\mathcal{M}, \mathcal{S})$ consisting of a mechanism $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ and a sharing scheme $\mathcal{S} = (\sigma, \xi)$. A multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ is α*-stable* if it satisfies that (1) $\mathcal{M}$ is strategyproof (2) $\mathcal{S} = (\sigma, \xi)$ is α-fair for some α. For any Nash Equilibrium (NE) $\widetilde{\mathbf{b}}$ for links, if $\alpha \cdot \mathbb{P}(\eta, \widetilde{\mathbf{b}}) \leq \sum_{q_i \in Q} \xi_i(\eta, \widetilde{\mathbf{b}}) \leq \mathbb{P}(\eta, \widetilde{\mathbf{b}})$, for some given parameter $\alpha \leq 1$, then $\mathcal{S} = (\sigma, \xi)$ is α-NE-budget-balance. Comparing with definition of α-budget-balance, we replace the actual cost vector $\mathbf{c}$ with any NE $\widetilde{\mathbf{b}}$ for the links. Similarly, we have the definition Nash Equilibrium Consumer Sovereignty (NE-CS).

A sharing scheme $\mathcal{S}$ is *NE-strategyproof* if $\eta_i - \xi_i(\eta, \widetilde{\mathbf{b}}) \geq \eta'_i - \xi_i(\eta|^i\eta'_i, \widetilde{\mathbf{b}}')$ for any receiver i, any valuation $\eta'_i = \eta_i$, and any NE $\widetilde{\mathbf{b}}$ for the links under η and any NE $\widetilde{\mathbf{b}}'$ for the links under $\eta|^i\eta'_i$. In other words, receiver q_i can not increase its utility by falsely declaring its valuation to affect the NE of the links under any circumstance. $\mathcal{S}$ is α-NE-fair if it is NE-strategyproof and satisfies NE-CS, NPT and FR. A multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ is α-NE-stable if it satisfies that (1) there exists a NE for the links; (2) $\mathcal{S}$ is α-NE-fair under any NE $\widetilde{\mathbf{b}}$. If there is only one receiver, which we assume to be q_1, then it is a *unicast system*, which is a special case of multicast system.

Following we present some notations that are used in this paper.

Notations: The path with the lowest cost between two odes s and t is denoted as $\mathsf{LCP}(s, t, \mathbf{c})$, and its cost is dented as $|\mathsf{LCP}(s, t, \mathbf{c})|$. Given a simple path P in the graph G with cost vector $\mathbf{c}$, the sum of the cost of links on path P is denoted as $|\mathsf{P}(\mathbf{c})|$. For a simple path $P = v_i \rightsquigarrow v_j$, if $\mathsf{LCP}(s, t, \mathbf{c}) \bigcap P = \{v_i, v_j\}$, then P is called a *bridge* over $\mathsf{LCP}(s, t, \mathbf{c})$. This bridge P *covers* link e_k if $e_k \in \mathsf{LCP}(v_i, v_j, \mathbf{c})$. Given a link $e_i \in \mathsf{LCP}(s, t, \mathbf{c})$, the path with the minimum cost that covers e_i is denoted as $\mathrm{B}_{\min}(e_i, \mathbf{c})$. We call the bridge $\mathrm{B}_{mm}(s,t,\mathbf{c}) = \max_{e_i \in \mathsf{LCP}(s,t,\mathbf{c})} \mathrm{B}_{\min}(e_i, \mathbf{c})$ the *max-min cover* of the path $\mathsf{LCP}(s,t,\mathbf{c})$.

A bridge set $\mathscr{B}$ is a *bridge cover* for $\mathsf{LCP}(s,t,\mathbf{c})$, if for every link $e_i \in \mathsf{LCP}(s,t,\mathbf{c})$, there exists a bridge $\mathrm{B} \in \mathscr{B}$ such that $e_i \in \mathsf{LCP}(v_{s(\mathrm{B})}, v_{t(\mathrm{B})}, \mathbf{c})$. The *weight* of a bridge cover $\mathscr{B}(s,t,\mathbf{c})$ is defined as $|\mathscr{B}(s,t,\mathbf{c})| = \sum_{B \in \mathscr{B}(s,t,\mathbf{c})} \sum_{e_i \in B} c_i$. Notice that a link may be counted multiple times here. A bridge cover $\mathscr{B}$ is a *minimal bridge cover* (MBC), if for each bridge $B \in \mathscr{B}$, $\mathscr{B} - B$ is not a bridge cover. A bridge cover is a *least bridge cover* (LB), denoted by $\mathbb{LB}(s,t,\mathbf{c})$, if it has the smallest weight among all bridge covers that cover $\mathsf{LCP}(s,t,\mathbf{c})$.

3 Dominant Strategies and Multicast Systems

In this section, we study how to design a multicast system that is α-stable with large α. We present some results on both the negative and positive sides.

3.1 α-Stable Unicast System

Unicast routing [9] may be one of the introductory problems that bring algorithm mechanism design to the attention of the computer scientists. Fortunately, the unicast routing problem is solved by using the celebrated VCG mechanism in the seminal paper [9] by Nisan and Ronen. However, one important question that has not been addressed in any previous literatures is who is going to pay the payments to the agents. By assuming that the unicast routing is receiver-driving, the very naive way is that the receiver should pay the payment.

We use $\mathcal{P}^{\mathsf{UVCG}}$ to denote the VCG payment for unicast under AM. The payment to a link $e_k \in \mathsf{LCP}(s, q_1, \mathbf{c})$ according to VCG mechanism is $\mathcal{P}_k^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}) = |\mathsf{LCP}(s, q_1, \mathbf{c}|^k\infty)| - |\mathsf{LCP}(s, q_1, \mathbf{c}|^k 0)|$. The payment to link that is not on the LCP is 0. Simply applying the sharing scheme $\widetilde{\mathcal{S}}$ obtains the unicast system Ψ^{VCG} as follows: first computing $\mathbb{P}^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{d})$, q_1 is charged $\mathbb{P}^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{d})$ and receives the service if $\eta_1 \geq \mathbb{P}^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{d})$; q_1 is charged 0 and does not receive the service otherwise. Each link receives its VCG payment if q_1 receives the service and 0 otherwise. Regarding the unicast system Ψ^{VCG}, we have:

Theorem 1. *For unicast system $\Psi^{\mathsf{VCG}} = (\mathcal{M}^{\mathsf{VCG}}, \mathcal{S}^{\mathsf{VCG}})$, $\mathcal{S}^{\mathsf{VCG}}$ is fair. However, $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof.*

Proof. Obviously, $\mathcal{S}^{\mathsf{VCG}}$ is fair. We then show that $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof by giving a counter example in Figure 1. Consider the graph in Figure 1 in which $c_i = 1$ for $1 \leq i \leq k$, $c_{k+1} = a \cdot k$ and the valuation for receiver q_1 is $a \cdot k + 1$. The VCG payment to link e_i is $a \cdot k - k + 1$ for $1 \leq i \leq k$ and the total payment to all links is $k(a \cdot k - k + 1)$. Thus, if every link e_i reveals its true cost, the receiver will reject the service. Consequently, every link receives payment 0 and has a utility 0. Consider the scenario when link e_1 reports its cost as $a \cdot k + 1 - \epsilon$ for a small positive ϵ. The total payment to all links is $a \cdot k + 1$ when $\epsilon = \frac{1}{k-1}$. Then, receiver q_1 accepts the service and pay $a \cdot k + 1$. Consequently, link e_1 receives a payment $k + 1$, and its utility is k. This violates the IC property. Thus, $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof.

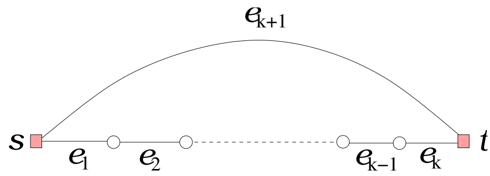

Fig. 1. $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof: link e_1 can lie to increase its utility

If we take both the links and the receiver q_1 into account as agents, then the unicast system is still a binary demand game. Thus, we have the following lemma which is a simple application of the binary demand game [5, 1, 7, 4].

Lemma 1. *If* $\Psi = (\mathcal{M}, \mathcal{S})$ *is a unicast system that is* α*-stable, then*

1. *There exists a function* $\zeta(\mathbf{c})$ *such that (a)* $\sigma_1(\eta_1, \mathbf{c}) = 1$ *if and only if* $\eta_1 \geq \zeta(\mathbf{c})$*, (b)* $\xi_1(\eta_1, \mathbf{c}) = \zeta(\mathbf{c})$ *when* $\sigma_1(\eta_1, \mathbf{c}) = 1$*,*
2. *If* $c_j < c'_j < \mathcal{P}_j^{\mathsf{UVCG}}(q_1, c)$*, then* $\zeta(c) \leq \zeta(c|^j c'_j)$*.*

The proof of this lemma is omitted due to space limit. Based on Lemma 1, the following theorem reveals a negative result of α-stable unicast system.

Theorem 2. *If* $\Psi = (\mathcal{M}, \mathcal{S})$ *is an* α*-stable unicast system, then* $\alpha \leq \frac{1}{n}$*.*

Proof. We prove it by presenting an example graph in Figure 1. Consider the cost vector $c^{(1)} = c|^1(ak - k + 1 - \epsilon)$. From Lemma 1, the sharing by q_1 is $\xi_1(\eta_1, \mathbf{c}) = \zeta(\mathbf{c})$ and

$$\zeta(c) \leq \zeta(c^{(1)}) \tag{1}$$

Recall that for any valuation $\eta_1 \geq \zeta(c^{(1)})$, $\sigma_1(\eta_1, \mathbf{c}^{(1)}) = 1$, *i.e.*, the LCP between $\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)})$ is selected. Since mechanism $\mathcal{M}$ is strategyproof, the payment to link e_i is a threshold value [4] $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)})$ which does not depend on c_i. Now we prove by contradiction that $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) \leq \mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, c^{(1)})$. For the sake of contradiction, assume that $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) > \mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}^{(1)})$. Recall that $\mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}^{(1)}) = |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i\infty)| - |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i 0)|$. However, when we set the cost of e_i as $\hat{\mathbf{c}}_i = |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i\infty)| - |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i 0)| + \delta$ for a sufficiently small positive value $\delta < \kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) - \mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}^{(1)})$, e_i is still on path $\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)})$, which is a contradiction. Thus, for graph shown in Figure 1, $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) \leq 1 + \epsilon$ for $2 \leq i \leq k$ and $\kappa_1(\eta_1, \mathbf{c}_{-1}^{(1)}) = a \cdot k - k + 1$. Since Ψ is a binary demand game, $\mathcal{P}_i(\eta_1, c^{(1)}) = \kappa_i(\eta_1, c_{-i}^{(1)})$. Thus, the total payment to all links is $\mathbb{P}(\eta_1, \mathbf{c}^{(1)}) = \sum_{e_i} \mathcal{P}_i(\eta_1, \mathbf{c}^{(1)}) \leq a \cdot k - k + 1 + (k-1) \cdot (1 + \epsilon) = a \cdot k + (k-1) \cdot \epsilon$. Recall that $\mathcal{S}$ is α-budget-balance, then $\zeta(\mathbf{c}^{(1)}) \leq \mathbb{P}(\eta_1, \mathbf{c}^{(1)}) \leq a \cdot k + (k-1) \cdot \epsilon$. By combining Inequality (1) and the above inequality, we have $\zeta(c) \leq \zeta(c^{(1)}) \leq a \cdot k + (k-1) \cdot \epsilon$. Similarly, let cost vector $\mathbf{c}^{(i)}$ be $\mathbf{c}|^i(a \cdot k + 1 - \epsilon)$ for $1 \leq i \leq k$. Let χ be a large positive number such that $\chi \geq \max_{1 \leq i \leq k} \xi(\mathbf{c}^{(i)})$. Consider the cost vector c and receiver valuation χ, we argue that $\kappa_i(\chi, \mathbf{c}_{-i}) \geq ak - k + 1 - \epsilon$ for any $1 \leq i \leq k$. Considering any link e_i, $1 \leq i \leq k$, if it reports its cost as $ak - k + 1 - \epsilon$, then the cost vector is $\mathbf{c}^{(i)}$. From the way we choose the valuation χ, the receiver q_1 will receive the service. Thus, e_i is also selected. From IR, $\kappa_i(\chi, \mathbf{c}_{-i}) = \mathcal{P}_j(\chi, \mathbf{c}) = \mathcal{P}_j(\chi, \mathbf{c}^{(i)}) \geq ak - k + 1 - \epsilon$. Therefore, $\mathbb{P}(\chi, \mathbf{c}) = \sum_{e_i} \mathcal{P}_i(\chi, \mathbf{c}) \geq k \cdot (ak - k + 1 - \epsilon)$. This obtains that

$$\alpha \leq \frac{\zeta(c)}{\mathbb{P}(\chi, \mathbf{c})} \leq \frac{ak + (k-1) \cdot \epsilon}{k \cdot (ak - k + 1 - \epsilon)}.$$

Let $\epsilon \to 0$, $a \to \infty$ and $k = n$, then $\alpha \leq \frac{1}{n}$. This finishes our proof.

Theorem 2 gives an upper bound for α for any α-stable unicast system Ψ. It is not difficult to observe that even the receiver q_1 is cooperative, Theorem 2 still holds. Following we present an $\frac{1}{n}$-stable unicast system that is based on the max-min cover of the LCP.

Algorithm 1. An $\frac{1}{n}$-stable unicast system $\Psi^{\mathsf{DU}} = (\mathcal{M}^{\mathsf{DU}}, \mathcal{S}^{\mathsf{DU}})$

Compute $\mathsf{LCP}(s, q_1, \mathbf{d})$, and set $\phi = \omega(\mathrm{B}_{mm}(s, t, \mathbf{d}), \mathbf{d})$.
if $\eta_1 \geq \phi$ **then**
    Each link $e_k \in \mathsf{LCP}(s, q_1, \mathbf{d})$ is selected and receives a payment $\mathcal{P}_k^{\mathsf{UVCG}}(\eta_1^{=\infty}, c)$; all other links are not selected and get a payment 0.
    Receiver q_1 is granted the service and charged ϕ.
else
    All links are not selected and each link receives a payment 0.
    Receiver is not granted the service and is charged 0.

Theorem 3. *Unicast system* $\Psi^{DU} = (\mathcal{M}^{DU}, \mathcal{S}^{DU})$ *is* $\frac{1}{n}$*-stable.*

The proof is omitted here due to space limit. Theorem 3 closes the gap between the upper and lower bound by presenting a tight bound $\frac{1}{n}$ for the budget balance factor α for unicast.

3.2 Multicast System

In Section 3.1, we consider how to construct a unicast system $\Psi = (\mathcal{M}, \mathcal{S})$ such that $\mathcal{M}$ is strategyproof and $\mathcal{S}$ is α-budget-balance with a large budge balance factor α. In this section, we discuss how to construct a multicast system. Under Axiom Model, Wang *et al.* [11] gave a strategyproof multicast mechanism $\mathcal{M}^{\mathsf{LCPT}} = (\mathcal{O}^{\mathsf{LCPT}}, \mathcal{P}^{\mathsf{LCPT}})$. For a link $e_k \in \mathsf{LCPT}(R, \mathbf{c})$, they compute an intermediate payment $\mathcal{P}_k^{\mathsf{UVCG}}(\eta_i^{=\infty}, \mathbf{c}) = |\mathsf{LCP}(s, q_i, \mathbf{c}|^k\infty)| - |\mathsf{LCP}(s, q_i, \mathbf{c}|^k 0)|$ to link e_k based on each downstream receiver q_i of e_k on the LCPT tree. The final payment to a link $e_k \in \mathsf{LCPT}(R, \mathbf{c})$ is $\mathcal{P}_i^{\mathsf{LCPT}}(\eta_R^{=\infty}, \mathbf{c}) = \max_{q_j \in R} \mathcal{P}_i^{\mathsf{UVCG}}(\eta_j^{=\infty}, \mathbf{c})$, where $\eta_R^{=\infty}$ is the valuation vector such that $\eta_i = \infty$ if $q_i \in R$ and 0 otherwise. They also present a payment sharing scheme based on $\mathcal{M}^{\mathsf{LCPT}}$ that is reasonable [10]. By generalizing the unicast system Ψ^{DU}, we present a multicast system Ψ^{DM} (illustrated in Algorithm 2) based on the tree LCPT. Here, DM stands for the multicast system with dominant strategy requirement for the links.

Algorithm 2. Multicast system $\Psi^{\mathsf{DM}} = (\mathcal{M}^{\mathsf{DM}}, \mathcal{S}^{\mathsf{DM}})$ based on tree LCPT

Compute path $\mathsf{LCP}(s, q_j, \mathbf{d})$ and set $\phi_j = \frac{\omega(\mathrm{B}_{mm}(s,q_j,\mathbf{d}),\mathbf{d})}{r}$ for every $q_j \in Q$.
Set $\mathcal{O}_i^{\mathsf{DM}}(\eta, \mathbf{d}) = 0$ and $\mathcal{P}_i^{\mathsf{DM}}(\eta, \mathbf{d}) = 0$ for each link $e_i \notin \mathsf{LCP}(s, q_j, \mathbf{d})$.
for each receiver q_j **do**
  if $\eta_j \geq \phi_j$ **then**
    Receiver q_j is granted the service and charged $\xi_j^{\mathsf{DM}}(\eta, \mathbf{d})$, set $R = R \cup q_j$.
  else
    Receiver q_j is not granted the service and is charged 0.
Set $\mathcal{O}_i^{\mathsf{DM}}(\eta, \mathbf{d}) = 1$ and $\mathcal{P}_i^{\mathsf{DM}}(\eta, \mathbf{d}) = \mathcal{P}_i^{\mathsf{LCPT}}(\eta_{\bar{R}}^{\infty}, \mathbf{d})$ for each link $e_i \in \mathsf{LCPT}(R, \mathbf{d})$.

Theorem 4. *The multicast system $\Psi^{DM} = (\mathcal{M}^{DM}, \mathcal{S}^{DM})$ is $\frac{1}{r \cdot n}$-stable, where r is the number of receivers.*

The proof of this theorem is omitted here. Recall that the unicast system is a special case of multicast system. Thus, for any multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ that is α-stable, the budget balance factor is at most $\frac{1}{n}$. In this section, we propose a multicast system Ψ^{DM} that achieves a budge balance factor $\frac{1}{r \cdot n}$. It is of interests to find some multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ that achieves a larger budget balance factor while $\mathcal{M}$ is strategyproof and $\mathcal{S}$ is α-fair. Our conjecture is that the upper bound of α is also $\Theta(\frac{1}{rn})$.

4 Nash Equilibrium and Multicast Systems

In light of the inefficiency of the multicast/unicast mechanism that is strategyproof for both links and receivers, it is natural to relax the dominant strategy to a weaker requirement – Nash Equilibrium. In this section, we study how to design multicast/unicast system that is α-NE-stable with a small additive ϵ.

4.1 Unicast Auction in Axiom Model

In this section, we disregard the receiver valuation and show how to design a mechanism that can induce some Nash Equilibria for links that can pay comparably smaller than the strategyproof mechanism does. Notice that, in [8], Immorlica *et al.* showed that if we simply pay whatever the link reports, which is known as the *first price auction*, there does not exist Nash Equilibrium. Due to the non-existence of the Nash Equilibrium, they propose a modified first price auction that can achieve ϵ-Nash Equilibrium with a small additive value. With further modification of the auction rules, we obtain a unicast auction that induces efficient Nash Equilibria. The high level idea of our unicast auction is as follows. We require the agents to bid two bids instead of one: the first bid vector $\mathbf{b}$ is used to find the LCP, the second bid vector $\mathbf{b}'$ is used to determine the payment. In the meanwhile, we also give a small "bonus" to all links such that each link e_i gets the maximum bonus when it reports its true cost, *i.e.*, $b_i = c_i$.

Algorithm 3 sends the data along $\mathsf{LCP}(s, q_1, \mathbf{b})$ and broadcasts something to the network with a probability ρ. Following theorem (its proof is omitted here) reveals the existence and several properties of Nash Equilibria.

Algorithm 3. FPA Mechanism $\mathcal{M}^{\mathsf{AUC}} = (\mathcal{O}^{\mathsf{AUC}}, \mathcal{P}^{\mathsf{AUC}})$

1: **for** each link $e_i \in G$ **do**
2: Set $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = f_i(s, q_1, \mathbf{b})$, where $f_i(s, q_1, \mathbf{b}) = \tau \cdot \left[b_u \cdot (n \cdot b_u - \sum_{e_j \in G - e_i} b_j) - \frac{h_i^2}{2}\right]$. Here, b_u is the maximum cost any link can declare.
3: Every link sends a unit size dummy packet and $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}})$ for every link $e_i \in G$ $\rho = \tau \cdot (n \cdot b_u - \sum\limits_{e_i \in G} b_i)$.
4: Compute the unique path $\mathsf{LCP}(s, q_1, \mathbf{b}')$ by applying certain fixed tie-breaking rule consistently.
5: **for** each link e_i **do**
6: **if** $e_i \in \mathsf{LCP}(s, q_1, \mathbf{b}')$ **then**
7: Set $\mathcal{O}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = 1$ and $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = b_i'$.
8: **else**
9: Set $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = {}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) - \gamma \cdot (b_i - b_i')^2$.

Theorem 5. *There exists NE for FPA mechanism* $\mathcal{M}^{AUC}$ *and for any NE* $\widetilde{\mathbf{b}} = \langle \mathbf{b}, \mathbf{b}' \rangle$ *we have (1)* $\mathbf{b} = \mathbf{c}$*; (2)* $LCP(s, q_1, \mathbf{c}) = LCP(s, q_1, \mathbf{b}')$*; (3) For any* $e_i \in LCP(s, q_1, \mathbf{c})$, $|LCP(s, q_1, \mathbf{b}')| = |LCP(s, q_1, \mathbf{b}'|^i\infty)|$.

Theorem 6. *Assume that* $\widetilde{\mathbf{b}} = \langle \mathbf{b}, \mathbf{b}' \rangle$ *is a NE of* $\mathcal{M}^{AUC}$ *and* ϵ *is a fixed positive value, then by properly setting the parameter* τ, $|\mathbb{LB}(s, q_1, \mathbf{c})| + \epsilon > \mathbb{P}^{\mathrm{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) \geq \frac{|\mathbb{LB}(s,q_1,\mathbf{c})|}{2}$. *Moreover, the inequalities are tight.*

4.2 Unicast and Multicast System in Valuation Model

Based on the Auction Mechanism Ψ^{AUC}, we design a unicast system that is $\frac{1}{2}$-NE-stable with small additive ϵ as follows: (1) Execute Line $1-2$ in Algorithm 3; (2) Compute $\mathbb{LB}(s, q_1, \mathbf{b})$, and set $\phi = \frac{|\mathbb{LB}(s,q_1,\mathbf{b})|}{2}$; (3) If $\phi \leq \eta_1$ then set $\sigma_1^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) = 1$ and $\xi_1^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) = \phi$. Every relay link on LCP is selected and receives an extra payment b_i'. (4) Set $\mathcal{P}_i^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) = \mathcal{P}_i^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) - \gamma \cdot (b_i' - b_i)^2$ for each link $e_i \notin \mathsf{LCP}(s, q_1, \mathbf{b}')$.

Theorem 7. *The unicast system* $\Psi^{AU} = (\mathcal{M}^{AU}, \mathcal{S}^{AU})$ *has Nash Equilibria for links, and* Ψ^{AU} *is* $\frac{1}{2}$*-NE-stable with* ϵ *additive, for any given* ϵ.

With the unicast system Ψ^{AUC}, we can simply extend the unicast system to a multicast system by treating each receiver as a separate receiver and applying the similar process as in the unicast system Ψ^{AU}. Notice that the bid vector is $\widetilde{\mathbf{b}} = (\mathbf{b}, \mathbf{b}^{(1)\prime}, \mathbf{b}^{(2)\prime}, \ldots, \mathbf{b}^{(r)\prime})$. The details are omitted here due to the space limit. For more details, please refer [12].

5 Conclusion

In this paper, we study the multicast system in networks consisting of selfish, non-cooperative relay links and receivers. We first prove that no unicast system can achieve α-stable when $\alpha > \frac{1}{n}$. We then present a unicast system that is $\frac{1}{n}$-stable, which closes the bounds of the budget balance factor. We extend this idea to a multicast system that is $\frac{1}{rn}$-stable where r is the number of the receivers. We also consider how to relax the strategyproofness requirement for the links and propose the FPA mechanism in Axiom Model that provably reduces the inevitable overpayment by achieving Nash equilibrium for the relay links. Based on the FPA mechanism, we propose a unicast system and a multicast system that are $\frac{1}{2}$-NE-stable with ϵ additive and $\frac{1}{2r}$-NE-stable with ϵ respectively.

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Atomic Selfish Routing in Networks: A Survey*

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Abstract. In this survey we present some recent advances in the literature of atomic (mainly network) congestion games. The algorithmic questions that we are interested in have to do with the *existence* of pure Nash equilibria, the *efficiency* of their construction when they exist, as well as the *gap* of the best/worst (mixed in general) Nash equilibria from the social optima in such games, typically called the *Price of Anarchy* and the *Price of Stability* respectively.

1 Introduction

Consider a model where selfish individuals (henceforth called **players**) in a communication network having varying service demands compete for some shared resources. The quality of service provided by a resource decreases with its *congestion*, ie, the amount of demands of the players willing to be served by it. Each player may reveal his actual, unique choice of a subset of resources that satisfies his service demand, or he may reveal a probability distribution for choosing (independently of other players' choices) one of the possible (satisfactory for him) subsets of resources. The players determine their actual behavior based on other players' behaviors, but they do not cooperate. We are interested in situations where the players have reached some kind of an equilibrium state. The most popular notion of equilibrium in non-cooperative game theory is the *Nash equilibrium*: a "stable point" among the players, from which no player is willing to deviate unilaterally. In [18], the notion of the *coordination ratio* or *price of anarchy* was introduced as a means for measuring the performance degradation due to lack of players' coordination when sharing common goods. A more recent measure of performance is the *price of stability* [1], capturing the gap between the best possible Nash equilibrium and the globally optimal solution. This measure is crucial for the network designer's perspective, who would like to propose (rather than let the players end up in) a Nash equilibrium (from which no player would like to defect unilaterally) that is as close to the optimum as possible.

A realistic scenario for the above model is when *unsplittable* traffic demands are routed selfishly in general networks with load-dependent edge delays. When

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the underlying network consists of two nodes and parallel links between them, there has been an extensive study on the existence and computability of equilibria, as well as on the price of anarchy. In this survey we study the recent advances in the more general case of arbitrary *congestion games*. When the players have identical traffic demands, the congestion game is indeed isomorphic to an *exact potential game* ([24], see also Theorem 1 of this survey) and thus always possesses a *pure Nash equilibrium*, ie, an equilibrium where each player adopts a pure strategy. We shall see that varying demands of the players crucially affect the nature of these games, which are no longer isomorphic to exact potential games. We also present some results in a variant of congestion games, where the players' payoffs are not resource-dependent (as is typically the case in congestion games) but *player–specific*.

Roadmap. In Section 2 we formally define the congestion games and their variants considered in this survey. We also give some game-theoretic definitions. In Section 3 we present most of the related work in the literature, before presenting in detail some of the most significant advances in the area. In Section 4 we present some of the most important results concerning unweighted congestion games and their connection to the *potential games*. In Section 5 we study some complexity issues of unweighted congestion games. In Section 6 we present Milchtaich's extension of congestion games to allow *player–specific* payoffs, whereas in Section 5.2 we study some existence and computability issues of PNE in weighted congestion games. Finally, in Section 7 we study the price of anarchy of weighted congestion games. We close this survey with some concluding remarks and unresolved questions.

2 The Model

Consider having a set of resources E in a system. For each $e \in E$, let $d_e(\cdot)$ be the **delay** per player that requests his service, as a function of the total usage (ie, the *congestion*) of this resource by all the players. Each such function is considered to be *non-decreasing* in the total usage of the corresponding resource. Each resource may be represented by a pair of points: an entry point to the resource and an exit point from it. So, we represent each resource by an arc from its entry point to its exit point and we associate with this arc the **charging cost** (eg, the delay as a function of the load of this resource) that each player has to pay if he is served by this resource. The entry/exit points of the resources need not be unique; they may coincide in order to express the possibility of offering a *joint service* to players, that consists of a sequence of resources. We denote by V the set of all entry/exit points of the resources in the system. Any nonempty collection of resources corresponding to a directed path in $G \equiv (V, E)$ comprises an **action** in the system.

Let $N \equiv [n]$[1] be the set of players, each willing to adopt some action in the system. $\forall i \in N$, let w_i denote player i's **traffic demand** (eg, the flow rate from

[1] $\forall k \in \mathbb{N},\ [k] \equiv \{1, 2, \ldots, k\}$.

a source node to a destination node), while $\mathcal{P}^i \equiv \{a^i_1, \ldots, a^i_{m_i}\} \subseteq 2^E \setminus \emptyset$ (for some $m_i \geqslant 2$) is the collection of actions, any of which would satisfy player i (eg, alternative routes from a source to a destination node, if G represents a communication network). The collection $\mathcal{P}^i$ is called the *action set* of player i and each of its elements contains at least one resource. Any $n-$tuple $\varpi \in \mathcal{P} \equiv \times_{i=1}^n \mathcal{P}^i$ is a **pure strategies profile**, or a **configuration** of the players. Any real vector $\mathbf{p} = (\mathbf{p^1}, \mathbf{p^2}, \ldots, \mathbf{p^n})$ s.t. $\forall i \in N,\ \mathbf{p^i} \in \Delta(\mathcal{P}^i) \equiv \{\mathbf{z} \in [0,1]^{m_i} : \sum_{k=1}^{m_i} z_k = 1\}$ is a probability distribution over the set of allowable actions for player i, is called a **mixed strategies profile** for the n players.

A **congestion model** $\big((\mathcal{P}^i)_{i\in N}, (d_e)_{e\in E}\big)$ typically deals with players of identical demands, and thus the resource delay functions depend only on the *number* of players adopting each action ([8, 24, 25]). In the more general case, ie, a **weighted congestion model** is the tuple $((w_i)_{i\in N}, (\mathcal{P}^i)_{i\in N}, (d_e)_{e\in E})$. That is, we allow the players to have different (but fixed) demands for service (denoted by their weights) from the whole system, and thus affect the resource delay functions in a different way, depending on their own weights. We denote by $W_{\text{tot}} \equiv \sum_{i\in N} w_i$ and $w_{\max} \equiv \max_{i\in N}\{w_i\}$.

The **weighted congestion game** (WCG in short) $\Gamma \equiv \big(N, E, (w_i)_{i\in N}, (\mathcal{P}^i)_{i\in N}, (d_e)_{e\in E}\big)$ associated with this model, is the game in strategic form with the set of players N and players' demands $(w_i)_{i\in N}$, the set of shared resources E, the action sets $(\mathcal{P}^i)_{i\in N}$ and players' cost functions $(\lambda^i_{\varpi^i})_{i\in N, \varpi^i\in\mathcal{P}^i}$ defined as follows: For any configuration $\varpi \in \mathcal{P}$ and $\forall e \in E$, let $\Lambda_e(\varpi) = \{i \in N : e \in \varpi^i\}$ be the set of players wishing to exploit resource e according to ϖ (called the **view** of resource e wrt configuration ϖ). We also denote by $x_e(\varpi) \equiv |\Lambda_e(\varpi)|$ the *number* of players using resource e wrt ϖ, whereas $\theta_e(\varpi) \equiv \sum_{i\in\Lambda_e(\varpi)} w_i$ is the **load** of e wrt to ϖ. The **cost** $\lambda^i(\varpi)$ **of player** i **for adopting strategy** $\varpi^i \in \mathcal{P}^i$ in a given configuration ϖ is equal to the cumulative delay $\lambda_{\varpi^i}(\varpi)$ of all the resources comprising this action:

$$\lambda^i(\varpi) = \lambda_{\varpi^i}(\varpi) = \sum_{e\in\varpi^i} d_e(\theta_e(\varpi)) \,. \tag{1}$$

On the other hand, for a mixed strategies profile $\mathbf{p}$, the **(expected) cost of player** i **for adopting strategy** $\varpi^i \in \mathcal{P}^i$ wrt $\mathbf{p}$ is

$$\lambda^i_{\varpi^i}(\mathbf{p}) = \sum_{\varpi^{-i}\in\mathcal{P}^{-i}} P(\mathbf{p}^{-i}, \varpi^{-i}) \cdot \sum_{e\in\varpi^i} d_e\left(\theta_e(\varpi^{-i} \oplus \varpi^i)\right) \tag{2}$$

where, $\varpi^{-i} \in \mathcal{P}^{-i} \equiv \times_{j\neq i}\mathcal{P}^j$ is a configuration of all the players except for i, $\mathbf{p}^{-i} \in \times_{j\neq i}\Delta(\mathcal{P}^j)$ is the mixed strategies profile of all players except for i, $\varpi^{-i} \oplus a$ is the new configuration with i definitely choosing the action $a \in \mathcal{P}^i$, and $P(\mathbf{p}^{-i}, \varpi^{-i}) \equiv \prod_{j\neq i} p^j_{\varpi^j}$ is the occurrence probability of ϖ^{-i} according to $\mathbf{p^{-i}}$.

A WCG in which all players have identical traffic demands is called **unweighted congestion game** (UCG). A WCG in which all players are indistinguishable (ie, they have the traffic demands and the same action set), is called **symmetric**. When each player's action set $\mathcal{P}^i$ consists of sets of resources that comprise (simple) paths between a unique origin-destination pair of nodes (s_i, t_i) in (V, E), we refer to a **(multi–commodity) network congestion game** (NCG). If additionally all origin-destination pairs of the players coincide with a unique pair (s, t) we have a **single–commodity NCG** and then all players share exactly the same action set. Observe that in general a single–commodity NCG is not necessarily symmetric because the players may have different demands and thus their cost functions will also differ.

Dealing with Selfish behavior. Fix an arbitrary (mixed in general) strategies profile $\mathbf{p}$ for a WCG $\left(N, E, (w_i)_{i\in N}, (\mathcal{P}^i)_{i\in N}, (d_e)_{e\in E}\right)$. We say that $\mathbf{p}$ is a **Nash Equilibrium (NE)** if and only if $\forall i \in N,\ \forall \alpha, \beta \in \mathcal{P}^i,\ \ p^i_\alpha > 0 \Rightarrow \lambda^i_\alpha(\mathbf{p}) \leqslant \lambda^i_\beta(\mathbf{p})$. A configuration $\varpi \in \mathcal{P}$ is a **Pure Nash Equilibrium (PNE)** if and only if $\forall i \in N, \forall \alpha \in \mathcal{P}^i,\ \ \lambda^i(\varpi) = \lambda_{\varpi^i}(\varpi) \leqslant \lambda_\alpha(\varpi^{-i} \oplus \alpha) = \lambda^i(\varpi^{-i} \oplus \alpha)$. The **social cost** $\mathrm{SC}(\mathbf{p})$ in this WCG is $\mathrm{SC}(\mathbf{p}) = \sum_{\varpi\in\mathcal{P}} P(\mathbf{p}, \varpi) \cdot \max_{i\in N}\{\lambda_{\varpi^i}(\varpi)\}$ where $P(\mathbf{p}, \varpi) \equiv \prod_{i=1}^n p^i_{\varpi^i}$ is the probability of configuration ϖ occurring, wrt the mixed strategies profile $\mathbf{p}$. The **social optimum** of this game is defined as $\mathrm{OPT} = \min_{\varpi\in\mathcal{P}} \{\max_{i\in N}[\lambda_{\varpi^i}(\varpi)]\}$. The **price of anarchy** (PoA in short) for this game is then defined as $\mathcal{R} = \max_{\mathbf{p}\text{ is a NE}} \left\{\frac{\mathrm{SC}(\mathbf{p})}{\mathrm{OPT}}\right\}$.

Potential Games. Fix an arbitrary game in strategic form $\Gamma = \left(N, (\mathcal{P}^i)_{i\in N}, (U^i)_{i\in N}\right)$ and some vector $\mathbf{b} \in \mathbb{R}^n_{>0}$. A function $\Phi : \mathcal{P} \to \mathbb{R}$ is called an **ordinal potential** for Γ, if $\forall \varpi \in \mathcal{P},\ \forall i \in N, \forall \alpha \in \mathcal{P}^i,\ \ \mathrm{sign}\left[\lambda^i(\varpi) - \lambda^i(\varpi^{-i} \oplus \alpha)\right] = \mathrm{sign}\left[\Phi(\varpi) - \Phi(\varpi^{-i} \oplus \alpha)\right]$. It is a **b-potential** for Γ, if $\forall \varpi \in \mathcal{P},\ \forall i \in N, \forall \alpha \in \mathcal{P}^i,\ \lambda^i(\varpi) - \lambda^i(\varpi^{-i} \oplus \alpha) = b_i \cdot [\Phi(\varpi) - \Phi(\varpi^{-i} \oplus \alpha)]$. It is an **exact potential** for Γ, if it is a $\mathbf{1}$−potential.

Configuration Paths and Discrete Dynamics Graph. For a WCG $\Gamma = \left(N, E, (w_i)_{i\in N}, (\mathcal{P}^i)_{i\in N}, (d_e)_{e\in E}\right)$, a **path** in $\mathcal{P}$ is a sequence of configurations $\gamma = (\varpi(0), \varpi(1), \ldots, \varpi(k))$ s.t. $\forall j \in [k],\ \varpi(j) = (\varpi(j-1))^{-i} \oplus \pi_i$, for some $i \in N$ and $\pi_i \in \mathcal{P}^i$. γ is a **closed path** if $\varpi(0) = \varpi(k)$. It is a **simple path** if no configuration is contained in it more than once. γ is an **improvement path** wrt Γ, if $\forall j \in [k],\ \lambda^{i_j}(\varpi(j)) < \lambda^{i_j}(\varpi(j-1))$ where i_j is the unique player differing in his strategy between $\varpi(j)$ and $\varpi(j-1)$. That is, the unique defector of the j^{th} move in γ is actually willing to make this move because it improves his own cost. The **Nash Dynamics Graph** of Γ is a directed graph whose vertices are configurations and there is an arc from a configuration ϖ to a configuration $\varpi^{-i} \oplus \alpha$ for some $\alpha \in \mathcal{P}^i$ if and only if $\lambda^i(\varpi) > \lambda^i(\varpi^{-i} \oplus \alpha)$. The set of best replies of a player i against a configuration $\varpi^{-i} \in \mathcal{P}^{-i}$ is defined as $BR_i(\varpi^{-i}) = \arg\max_{\alpha\in\mathcal{P}^i}\{\lambda^i(\varpi^{-i} \oplus \alpha)\}$. Similarly, the set of best replies against a mixed profile $\mathbf{p^{-i}}$ is $BR_i(\mathbf{p^{-i}}) = \arg\max_{\alpha\in\mathcal{P}^i}\{\lambda^i_\alpha(\mathbf{p^{-i}} \oplus \alpha)\}$. A path γ is a **best–reply improvement path** if each defector jumps to a best–reply pure strategy. The **Best Response Dynamics Graph** is a directed graph whose vertices are configurations and there is an arc from a configuration ϖ to a configuration $\varpi^{-i} \oplus \alpha$ for some $\alpha \in \mathcal{P}^i \setminus \{\varpi^i\}$ if and only if $\alpha \in BR_i(\varpi^{-i})$ and $\varpi^i \notin BR_i(\varpi^{-i})$. A (finite) strategic game Γ possesses the **Finite Improvement**

Property (FIP) if any improvement path of Γ has finite length. Γ possesses the **Finite Best Reply Property (FBRP)** if every best–reply improvement path is of finite length.

Isomorphism of Strategic Games. Two strategic games $\Gamma = \big(N, (\mathcal{P}^i)_{i\in N}, (U^i)_{i\in N}\big)$ and $\tilde{\Gamma} = \Big(N, (\tilde{\mathcal{P}}^i)_{i\in N}, (\tilde{U}^i)_{i\in N}\Big)$ are **isomorphic** if there exist bijections $g : \mathcal{P} \mapsto \tilde{\mathcal{P}}$ and $\tilde{g} : \tilde{\mathcal{P}} \mapsto \mathcal{P}$ such that $\forall \varpi \in \mathcal{P}$, $\forall i \in N$, $U^i(\varpi) = \tilde{U}^i(g(\varpi))$ and $\forall \tilde{\varpi} \in \tilde{\mathcal{P}}, \forall i \in N,\ \tilde{U}^i(\tilde{\varpi}) = U^i(\tilde{g}(\tilde{\varpi}))$.

Layered Networks. We consider a special family of networks whose behavior wrt the PoA, as we shall see, is asymptotically equivalent to that of the parallel links model of [18] (which is actually a 1-layered network): Let $\ell \geqslant 1$ be an integer. A directed network $G = (V, E)$ with a distinguished source–destination pair (s, t), $s, t \in V$, is an **ℓ-layered network** if every (simple) directed $s - t$ path has length exactly ℓ and each node lies on a directed $s - t$ path. In a layered network there are no directed cycles and all directed paths are simple. In the following, we always use $m = |E|$ to denote the number of edges in an ℓ-layered network $G = (V, E)$.

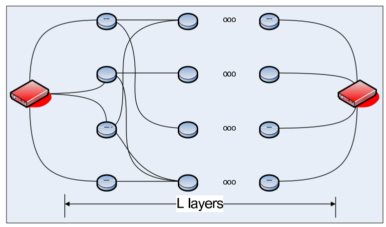

Fig. 1. A layered network

3 Related Work

Existence and Tractability of PNE. It is already known that any UCG(where the players have the same demands and thus, the same affection on the resource delay functions) is guaranteed to have at least one PNE: actually, Rosenthal ([25]) proved that any potential game has at least one PNE and it is easy to write any UCG as an exact potential game using Rosenthal's potential function[2] (eg, [8–Theorem 1]). In [8] it is proved that a PNE for any unweighted single–commodity NCG[3] (no matter what resource delay functions are considered, so long as they are non-decreasing with loads) can be constructed in polynomial time, by computing the optimum of Rosenthal's potential function, through a nice reduction to min-cost flow. On the other hand, it is shown that even for a symmetric congestion game or a multi–commodity UCG, it is PLS-complete to find a PNE (though it certainly exists).

The special case of single–commodity, parallel-edges NCG where the resources are considered to behave as parallel machines, has been extensively studied in recent literature. In [10] it was shown that for the case of players with varying demands and uniformly related parallel machines, there is always a PNE which can be constructed in polynomial time. It was also shown that it is NP-hard to

[2] For more details on potential games, see [24].

[3] Since [8] only considers unit-demand players, this is also a symmetric NCG.

construct the best or the worst PNE. In [13] it was proved that the fully mixed NE (FMNE), introduced and thoroughly studied in [22], is worse than any PNE, and any NE is at most $(6+\varepsilon)$ times worse than the FMNE, for varying players and identical parallel machines. In [21] it was shown that the FMNE is the worst possible for the case of two related machines and tasks of the same size. In [20] it was proved that the FMNE is the worst possible when the global objective is the sum of squares of loads.

[9] studies the problem of constructing a PNE from any initial configuration, of social cost at most equal to that of the initial configuration. This immediately implies the existence of a PTAS for computing a PNE of minimum social cost: first compute a configuration of social cost at most $(1+\varepsilon)$ times the social optimum ([14]), and consequently transform it into a PNE of at most the same social cost. In [7] it is also shown that even for the unrelated parallel machines case a PNE always exists, and a potential-based argument proves a convergence time (in case of integer demands) from arbitrary initial configuration to a PNE in time $\mathcal{O}\left(mW_{\text{tot}} + 4^{W_{\text{tot}}/m + w_{\max}}\right)$.

[23] studies the problem of weighted parallel-edges NCGs with player–specific costs: each allowable action of a player consists of a single resource and each player has his own private cost function for each resource. It is shown that: (1) weighted (parallel-edges network) congestion games involving only two players, or only two possible actions for all the players, or equal delay functions (and thus, equal weights), always possess a PNE; (2) even a single–commodity, 3–players, 3–actions, weighted (parallel-edges network) congestion game may not possess a PNE (using 3-wise linear delay functions).

Price of Anarchy in Congestion Games. In the seminal paper [18] the notion of coordination ratio, or price of anarchy, was introduced as a means for measuring the performance degradation due to lack of players' coordination when sharing common resources. In this work it was proved that the PoA is $3/2$ for two related parallel machines, while for m machines and players of varying demands, $\mathcal{R} = \Omega\left(\frac{\log m}{\log\log m}\right)$ and $\mathcal{R} = \mathcal{O}\left(\sqrt{m \log m}\right)$. For m identical parallel machines, [22] proved that $\mathcal{R} = \Theta\left(\frac{\log m}{\log\log m}\right)$ for the FMNE, while for the case of m identical parallel machines and players of varying demands it was shown in [17] that $\mathcal{R} = \Theta\left(\frac{\log m}{\log\log m}\right)$. In [6] it was finally shown that $\mathcal{R} = \Theta\left(\frac{\log m}{\log\log\log m}\right)$ for the general case of related machines and players of varying demands. [5] presents a thorough study of the case of general, monotone delay functions on parallel machines, with emphasis on delay functions from queuing theory. Unlike the case of linear cost functions, they show that the PoA for non-linear delay functions in general is far worse and often even unbounded.

In [26] the PoA in a multi–commodity NCG among infinitely many players, each of negligible demand, is studied. The social cost in this case is expressed by the total delay paid by the whole flow in the system. For linear resource delays, the PoA is at most $4/3$. For general, continuous, non-decreasing resource delay functions, the total delay of any Nash flow is at most equal to the total delay of an optimal flow for double flow demands. [27] proves that for this setting, it is actually the class of allowable latency functions and not the specific topology of a network that determines the PoA.

4 Unweighted Congestion Games

In this section we present some fundamental results connecting the classes of UCGs and (exact) potential games [24, 25]. Since we refer to players of identical (say, unit) weights, the players' cost functions are $\lambda^i(\varpi) \equiv \sum_{\varepsilon \in \varpi^i} d_e(x_e(\varpi))$, where, $x_e(\varpi)$ indicates the *number* of players willing to use resource e wrt configuration $\varpi \in \mathcal{P}$. The following theorem proves the strong connection of UCGs with the exact potential games.

Theorem 1 ([24, 25]). *Every UCG is an exact potential game.*

The existence of a (not necessarily exact) potential for any game in strategic form directly implies the existence of a PNE for this game. The existence of an exact potential may help (as we shall see later) the *efficient* construction of a PNE, but this is not true in general. More interestingly, Monderer and Shapley proved that every (finite) potential game is isomorphic to an UCG.

Theorem 2 ([24]). *Any finite (exact) potential game is isomorphic to an UCG.*

The size of the UCG that we use to represent a potential game is at most $(|N|+1)$ times larger than the size of the potential game. Since an UCG is itself an exact potential game, this implies an essential equivalence of exact potential and UCGs.

5 Existence and Complexity of Constructing PNE

In the present section we deal with issues concerning the existence and complexity of constructing PNE in WCGs. Our main references for this section are [8, 11, 19]. We start with some complexity issues concerning the construction of PNE in UCGs (in which a PNE always exists) and consequently we study existence and complexity issues in WCGs in general. Fix an arbitrary WCG $\Gamma = \big(N, E, (\mathcal{P}^i)_{i\in N}, (w_i)_{i\in N}, (d_e)_{e\in E}\big)$ where the w_i's denote the (positive) weights of the players.

A crucial class of problems containing the NCGs is PLS [16] (stands for *Polynomial Local Search*). This is the subclass of total functions in NP that are guaranteed to have a solution because of the fact that *"every finite directed acyclic graph has a sink"*. The problem of constructing a PNE for a WCG is in PLS, in the following cases: (1) for any UCG, since it is an exact potential game (see Theorem 1), and (2) for any weighted NCG with linear resource delays, which admits (as we shall prove in Theorem 6) a $\mathbf{b}-$potential with $b_i = \frac{1}{2w_i}, \forall i \in N$, and thus finding PNE is equivalent to finding local optima for the optimization problem with state space the action space of the game and objective the potential of the game. On the other hand, this does not imply a polynomial-time algorithm for constructing a PNE, since (as we shall see more clearly in the weighted case) the improvements in the potential can be very small and too many. Additionally, although problems in PLS admit a PTAS, this does not imply also a PTAS for finding ε-approximate PNE (approximation of the potential does not imply also approximation of each player's payoff).

5.1 Efficient Construction of PNE in UCGs

In this subsection we shall prove that for unweighted single–commodity NCGs a PNE can be constructed in polynomial time. On the other hand, even for multi–commodity NCGs it is PLS complete to construct a PNE. The main source of this subsection is the work of Fabrikant, Papadimitriou and Talwar [8].

Theorem 3 ([8]). *There is a polynomial time algorithm for finding a PNE in any unweighted single–commodity NCG.*

On the other hand, the following theorem proves that it is not that easy to construct a PNE, even in an unweighted multi–commodity NCG.

Theorem 4 ([8]). *It is PLS-complete to find a PNE in UCGs of the following types: (i) General UCGs, (ii) symmetric UCGs, and (iii) multi–commodity (unweighted) NCGs.*

5.2 Existence and Construction of PNE in WCGs

In this Section we deal with the existence and tractability of PNE in weighted network congestion games. First we show that it is not always the case that a PNE exists, even for a weighted single–commodity NCG with only linear and 2-wise linear (ie, the maximum of two linear functions) resource delays. Recall that, as discussed previously, any UCG has a PNE, for any kind of non-decreasing delays, due to the existence of an exact potential for these games. This result was independently proved by [11] and [19], based on similar constructions. In this survey we present the version of [11] due to its clarity and simplicity.

Lemma 1 ([11]). *There exist instances of weighted single–commodity NCGs with resource delays being either linear or 2-wise linear functions of the loads, for which there is no PNE.*

Consequently we show that there may exist no exact potential function for a weighted single–commodity NCG, even when the resource delays are identical to their loads. The next argument shows that Theorem 1 does not hold anymore even in this simplest case of WCGs.

Lemma 2 ([11]). *There exist weighted single–commodity NCGs which are not exact potential games, even when the resource delays are identical to their loads.*

Our next step is to focus our interest on the ℓ-layered networks with resource delays identical to their loads. We shall prove that any weighted ℓ-layered NCG with these delays admits at least one PNE, which can be computed in pseudo-polynomial time. Although we already know that even the case of weighted

ℓ-layered NCGs with delays equal to the loads cannot have any exact potential, we will next show that $\Phi(\varpi) \equiv \sum_{e\in E}[\theta_e(\varpi)]^2$ is a **b**-potential for such a game and some positive n-vector **b**, assuring the existence of a PNE.

Theorem 5 ([11]). *For any weighted ℓ-layered NCG with resource delays equal to their loads, at least one PNE exists and can be computed in pseudo-polynomial time.*

A recent improvement, based essentially on the same technique as above, generalizes the last result to the case of arbitrary multi–commodity NCGs with linear resource delays:

Theorem 6 ([12]). *For any weighted multi–commodity NCG with linear resource delays, at least one PNE exists and can be computed in pseudo-polynomial time.*

6 Congestion Games with Player Specific Payoffs

In [23] Milchtaich studies a variant of the classical UCGs, where the resource delay functions are not universal, but player–specific. In particular, there is again a set E of shared resources and a set N of (unit–demand) players with their action sets $(\mathcal{P}_i)_{i\in N}$, but rather than having a single delay $d_e : \mathbb{R}_{\geqslant 0} \mapsto \mathbb{R}$ per resource $e \in E$, there is actually a different utility function[4] per player $i \in N$ and resource $e \in E$, $U_e^i : \mathbb{R}_{\geqslant 0} \mapsto \mathbb{R}$ determining the payoff of player i for using resource e, given a configuration $\varpi \in \mathcal{P}$ and the load $\theta_e(\varpi)$ induced by it on that resource.

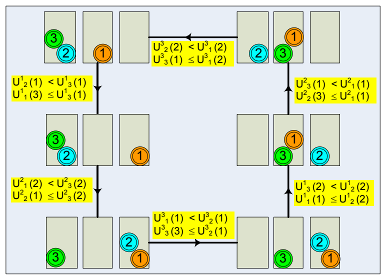

Fig. 2. A 3–players, 3–actions UCG with a best–reply cycle

On the other hand, Milchtaich makes two simplifying (yet crucial) assumptions: (1) Each player may choose only one resource from a pool E of resources (shared to all the players) for his service (ie, this is modeled as the *parallel-links* model of Koutsoupias and Papadimitriou [18]), and (2) the received payoff is *monotonically non-increasing* with the number of players selecting it. Although they do not always admit a potential, these games always possess a PNE.

In [23] Milchtaich proved that UCGs on parallel links with player–specific payoffs, involving only two strategies, possess the Finite Improvement Property (FIP). It is also rather straightforward that any 2–players UCG on parallel links with player–specific payoffs possesses the Finite Best Reply improvement path Property (FBRP). Milchtaich also gave an example of an UCG on three parallel links with three players, for which there is a best–reply cycle, although there is a

[4] In this case we refer to the **utility** of a player, that each player wishes to *maximize* rather than *minimize*. Eg, this is the negative of a player's private cost function on a resource.

PNE (see Figure 2). In this example, we only determine the necessary conditions on the (player–specific) payoff functions of the players for the existence of the best-response cycle (see figure). It is easily verified that this system of inequalities is feasible, and that configurations $(3, 1, 2)$ and $(2, 3, 1)$ are PNE for the game.

Theorem 7 ([23]). *Every UCG on parallel links with player–specific, non-increasing payoffs of the resources, possesses a PNE.*

The UCGs on parallel links and with player–specific payoffs are **weakly acyclic** games, in the sense that from any initial configuration $\varpi(0)$ of the players, there is at least one best–reply improvement path connecting it to a PNE. Of course, this does not exclude the existence of best–reply cycles (see example of Figure 2). But, it is easily shown that when the deviations of the players occur sequentially and in each step the next deviator is chosen randomly (among the potential deviators) to a randomly chosen best–reply resource, then this path will converge almost surely to a PNE in finite time.

6.1 Players with Distinct Weights

Milchtaich proposed a generalization of his variant of congestion games, by allowing the players to have distinct weights, denoted by a weight vector $\mathbf{w} = (w_1, w_2, \ldots, w_n) \in \mathbb{R}^n_{>0}$. In that case, the (player–specific) payoff of each player on a resource $e \in E$ depends on the load $\theta_e(\varpi) \equiv \sum_{i:e\in\varpi^i} w_i$, rather than the number of players willing to use it. For the case of WCGs on parallel links with player specific payoffs, it is easy to verify (in a similar fashion as for the unweighted case) that: (1) If there are only two available strategies then FIP holds; (2) if there are only two players then FBRP holds; (3) for the special case of resource specific payoffs, FIP holds. On the other hand, there exists a 3–players, 3–actions game that possesses no PNE. For example, see the instance shown in Figure 3, where the three players have essentially two strategies each (a "LEFT" and a "RIGHT" strategy) to choose from, while their third strategies give them strictly minimal payoffs and can never be chosen by selfish moves. The rationale of this game is that, in principle, player 1 would like to avoid using the same link as player 3, which in turn would like to avoid using the same link as player 2, which would finally want to avoid player 1.

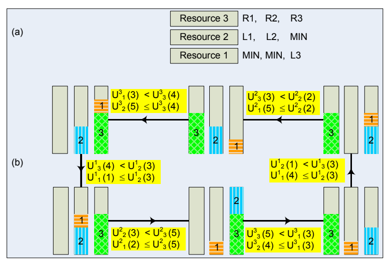

Fig. 3. A 3–players WCG (from [23]) on three parallel links with player–specific payoffs, having no PNE. (a) The LEFT-RIGHT strategies of the players. (b) The best–reply cycle.

The inequalities shown in Figure 3(b) demonstrate the necessary conditions for the existence of a best–reply cycle among 6 configurations of the players. It is easy to verify also that any other configuration has either at least one MIN strategy for some player (and in that case this player wants to defect) or is one of $(2, 2, 1), (3, 3, 3)$. The only thing that remains to assure, is that $(2, 3, 1)$ is strictly better for player 2 than $(2,2,1)$ (ie, player 2 would like to avoid player 1) and that $(3,3,1)$ is better for player 3 than $(3,3,3)$ (ie, player 3 would like to avoid player 2). The feasibility of the whole system of inequalities can be trivially checked to hold, and thus this game cannot have any PNE since there is no sink in its Dynamics graph.

7 The Price of Anarchy of WCGs

In this section we focus our interest on weighted ℓ-layered NCGs where the resource delays are identical to their loads. Our source for this section is [11]. This case comprises a highly non-trivial generalization of the well–known model of selfish routing of atomic (ie, indivisible) traffic demands via identical parallel channels [18]. The main reason why we focus on this specific category of resource delays is that there exist instances of unweighted layered NCGs that have unbounded PoA even if we only allow linear resource delays. In [26–p. 256] an example is given where the PoA is indeed unbounded (see Figure 4). This example is easily converted into an ℓ-layered network. The resource delay functions used are either constant, or M/M/1-like (ie, of the form $\frac{1}{c-x}$) delay functions. However, we can be equally bad even with linear resource delay functions: Observe the following example depicted in Figure 5. Two players, each having a unit of traffic demand from s to t, choose their selected paths selfishly. The edge delays are shown above them in the usual way. We assume that $a \gg b \gg 1 \geqslant c$. It is easy to see that the configuration (sCBt,sADt) is a PNE of social cost $2+b$ while the optimum configuration is (sABt,sCDt) the (optimum) social cost of which is $2+c$. Thus, $\mathcal{R} = \frac{b+2}{c+2}$.

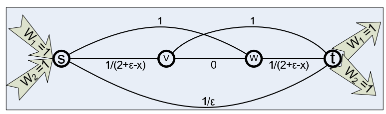

Fig. 4. A single-source NCG without a PNE ([26])

In the following, we restrict our attention to ℓ-layered networks whose resource delays are equal to their loads. The main tool is to interpret a strategies profile as a flow in the underlying network, and then upper bound the blowup of the expected maximum load due to statistical conflict of the players' traffic demands on edges of the network.

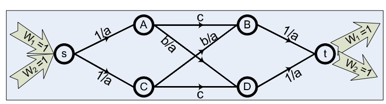

Fig. 5. An ℓ-layered network with linear resource delays and unbounded PoA [11]

Theorem 8 ([11]). *The PoA of any ℓ-layered NCG with resource delays equal to their loads, is at most* $8\,\mathrm{e}\left(\frac{\log m}{\log\log m}+1\right)$.

A recent development which is complementary to the last theorem is the following which we state without a proof:

Theorem 9 ([12]). *The PoA of any unweighted, single–commodity NCG with resource delays* $(d_e(x) = a_e \cdot x, a_e \geqslant 0)_{e\in E}$*, is at most* $24\,\mathrm{e}\left(\frac{\log m}{\log\log m}+1\right)$.

8 The Pure Price of Anarchy in Congestion Games

In this last section we overview some recent advances in the *Pure Price of Anarchy* (PPoA) of WCGs, that is, the worst-case ratio of the social cost of a PNE over the social optimum of the game.

The case of linear resource delays has been extensively studied in the literature. The PPoA wrt the total latency objective has been proved that it is $\frac{3+\sqrt{5}}{2}$, even for weighted multi–commodity NCGs [3, 4]. This result is also extended to the case of mixed equilibria. For the special case of identical players it has been proved (independently by the papers [3, 4]) that the PPoA drops down to 5/2. When considering identical users and single–commodity NCGs, the PPoA is again 5/2 wrt the maximum latency objective, but explodes to $\Theta(\sqrt{n})$ for multi–commodity NCGs ([4]). Earlier it was implicitly proved by [11] that the PPoA of any WCG on a layered network with resource delays identical to the congestion, is at most 3.

9 Conclusions

In this survey we have presented some of the most significant advances concerning the atomic (mainly network) congestion games literature. We have focused on issues dealing with existence of PNE, construction of an arbitrary PNE when such equilibria exist, as well as the PoA for many broad subclasses of NCGs.

We highlighted the significance of allowing distinguishable players (ie, players with different action sets, or with different traffic demands, or both) and established some kind of "equivalence" between games with unit-demand players on arbitrary networks with delays equal to the loads and games with players of varying demands on layered networks.

Still, there remain many unresolved questions. The most important question is a complete characterization of the subclasses of games having PNE and admitting polynomial time algorithms for constructing them, in the case of general networks.

Additionally, a rather recent trend deals with the network-design perspective of congestion games. For these games [1, 2, 15] the measure of performance is the **price of stability**, ie, the ratio of the *best* NE over the social optimum of the game, trying to capture the notion of the gap between solutions proposed by the network designer to the players and the social optimum of the game (which may be an unstable state for the players). This seems to be a rather intriguing and very interesting (complementary to the one presented here) approach of congestion games, in which there are still many open questions.

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Heuristic Approaches to Service Level Agreements in Packet Networks

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Abstract. Real-time multimedia applications on the Internet such as video and audio streaming, video conferencing, and online collaborations are expected to become increasingly popular. In order to guarantee effective support of these applications, we must be able to provide Quality of Service (QoS) guarantees such as network bandwidth and end-to-end delay, by incorporating session routing and resource reservation in admission control. In this work, we present the Utility Model, which provides an optimal allocation of resources for admission control while meeting the QoS requirements of admitted users' sessions. We describe previous heuristics to solve the Utility Model. These heuristics, though, are not suitable for larger networks due to their computation complexity, resulting in real-time decision-making and scalability issues. We are proposing a new concept to solve the Utility Model problem using a modified version of the Multicommodity Flow algorithm. This heuristic has improved computational complexity, and hence is capable of solving the Utility Model problem in lower overall execution time. This implies that admission control, which requires real-time solution of the Model, can be extended to larger networks.

1 Introduction

Over the last decade, there has been a dramatic increase in Internet traffic, and this growth is expected to continue. Network applications such as multimedia need efficient traffic management. However, the carriage of such real-time, bandwidth-demanding flows implies specific performance standards or Quality of Service (QoS) requirements; high bandwidth and low latency being chief among them [8]. For instance, to route multimedia traffic such as interactive video over the Internet, one requires a path with high bandwidth and low latency. Therefore, finding the most suitable path for an application is of primary importance in the admission control process.

In addition to these challenges, an Internet Service Provider or ISP must react quickly to the rapidly changing patterns of usage within a network [2], in order to minimize costs, while at the same time being able to honor QoS guarantees made to the customers. An ISP should not overbook its resources, which may result in QoS degradation. Also, an ISP should not purchase excessive amounts of these resources from the facilities-based carrier who owns them, in order to maintain a cost-effective

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operation. A rigorous model for admission control and resource allocation is therefore imperative for an ISP to stay competitive in today's marketplace.

Section 2 introduces the Utility Model, which is a mathematical formulation for the mapping of users' Service Level Agreement (SLA) requests to the system's resources required. Based on Dijkstra's K-shortest path algorithm, previous heuristics proposed to solve the Utility Model problem are described. These heuristics, though, do not work well for large networks due to their computation complexity. This is our motivation to propose a new heuristic, based on the Multicommodity Flow problem as described in Section 3, to improve computation efficiency and scalability. Performance analysis and comparison to previous heuristics are presented in Section 4.

2 The Utility Model

The Utility Model performs optimal allocation of resources of a server or network, while meeting the QoS requirements of users' sessions [9]. This is achieved by handling the decision-making process of selecting suitable SLAs for admission effectively and efficiently. A *session* is an entity that requires resources at the granularity level the Model operates. Each session has a number of possible *QoS levels*. For each QoS level, there is an associated *utility*, which is usually a *bid price*. The model allows the optimization of the total utility summed over all admitted sessions.

An SLA is a contract between an ISP and a customer. It is simply an agreement to provide a certain level of service, for a certain time, and at a given price. Customers submit proposed SLAs to network admission controllers to gain access. The carrier or ISP, on the other hand, wants to do *admission control*: to admit a subset of the SLAs on offer, each at the QoS level which would maximize total revenue, while fully respecting all terms and conditions (QOS guarantees)[1] of both newly-accepted and previously-accepted SLAs.

These concepts are summarized in Figure 1, where each customer session, *i,* specifies one or more generic QoS levels, Gold, Silver or Bronze. Each level is then mapped to a session utility (price), and a set of session resources (bandwidth and latency). These QoS-utility and QoS-resource mappings have been well documented in the literature and will not be discussed here. Assuming such mappings exist and are available, the goal then is to decide which section and at which level to admit so as to maximize the total utility gained without violating the resource constraints.

It should be noted that the Utility Model maps neatly to a simple model of a packet-switched network. In this case, the problem is to decide whether a new or changed traffic flow should be admitted, and if so, which path it should use. The problem may readily be split into two distinct sub-problems: (i) Deciding which level to admit from each SLA so that the total Utility (revenue) is maximized; (a *null level,* which requires zero resources and offers zero utility, is the default, hence, the selection of the null level of QoS represents denial of admission for the SLA); and (ii) Given the resource constraints (for simplicity, only bandwidth and latency of each

[1] In fact, given finite carrier resources and potentially unbounded service requests, the carrier or ISP *must* do admission control, if any credible attempt to guarantee QoS is to be made.

link in the network are considered here), finding a feasible path for the specified level of SLA in the network.

The two sub-problems are inter-related. In order to reserve resources–link bandwidths–for a session, we have to know which links' bandwidths are available. That is, admission of a session must be done with respect to a particular path through the network, a path on which the necessary unallocated bandwidth is available on each link.

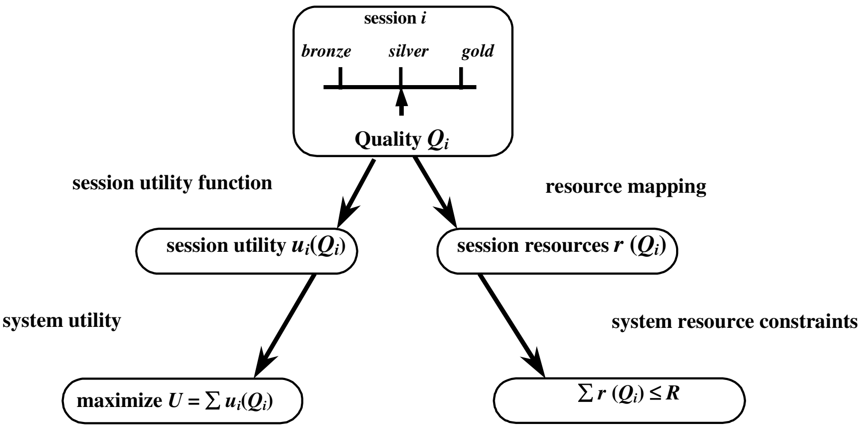


Fig. 1. Relations among qualities, utilities and resources

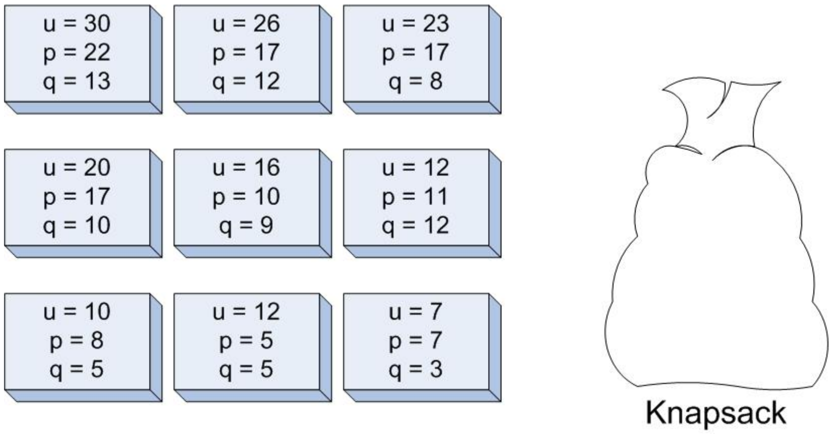
Previous methods of solving the Utility Model have mapped it to a Multi-choice Multi-dimension 0-1 Knapsack Problem (MMKP) [9]. Each SLA (with its multiple levels of QoS) is mapped onto a pile of stones. $S_1, S_2, S_3, \ldots. S_n$. Each stone of the pile has multi-dimensional volume $v_1(S), v_2(S), \ldots\ldots v_n(S)$. In this case, each pile of stones represents a session S_n. and each individual stone represents that session at a particular level of QoS (bronze, silver or gold stones). Each dimension of a stone's volume represents the session's requirement for one of the network's resources, in this case the bandwidth required by the session on a particular link of the network. The weight of each stone represents the utility offered for the session at that QoS level. It is desired to select one QoS level S'_m from each session (one stone from each pile of stones), such that the total weight (revenue or utility) of the knapsack is maximized,

$$MAX \sum V(S'_m)$$

while ensuring that none of the resources are overused, i.e., the total resource consumption is less than the total resource constraint R_n.

$$\sum v_n(S'_m) < R_n$$

Figure 2 illustrates this mapping of the Utility Model to the MMKP.



Pick at most one item from each stack in order to maximize **U =** $\sum u$**,**

subject to resource constraints: $\sum p \leq 39$, $\sum q \leq 35$

Fig. 2. Multi-choice Multi-dimensional 0-1 Knapsack Problem (MMKP)

2.1 Previous Heuristic Solutions of the Utility Model

In this section, two previously methods to solve the Utility Model are described: Khan's heuristic (HEU) and Watson's heuristic [9]. Both are based on mapping the model onto an MMKP. These heuristics begin by using Dijkstra's K-shortest path algorithm [6] to find a list of candidate paths of the network for an SLA. That is, before an SLA is admitted at a particular level of QoS, Dijkstra's algorithm is applied to find a list of paths from the source to the destination node.

Dijkstra's K-shortest path is a well-known algorithm for finding the shortest paths in graphs, from a given vertex x to all (n-1) other vertices. Starting from a source node x, the algorithm iterates and identifies a newly discovered node v for which the shortest path from x to v is known. A set of nodes S is maintained for the currenlty known shortest paths from v, and this set grows by one vertex in each iteration. In each iteration, the edge (u,v) is identified, where $u \in S$ and $v \in V - S$ such that :

$$dist(x,u) + weight(u,v) = \min_{(u',v') \in E} dist(x,u') + weight(u',v')$$

This edge (u,v) is added to a *shortest path tree*, which root is node x and describes all the shortest paths from x. By the last iteration, we have a list of shortest paths from the source node to the destination node. The execution time complexity for Dijkstra's algorithm is O(kmn), where k is the number of paths needed to be found from each source/destination pair, m is the number of edges, and n is the number of nodes in the graph; because the algorithm goes through each node and edge once during its traversal. However, with some simple data restructuring we can reduce the execution time to a bound of O(kn^2).

After the path finding process, the *admission process* decides which QoS level of each SLA to admit–including the null level for rejection of that SLA–based on the list of feasible paths found for each SLA.

2.1.1 Khan's Heuristic HEU

For Khan's HEU, the process assumes an empty system with no previously-admitted sessions. It first orders the QoS levels for each candidate session according to revenue, then selects the least profitable of the feasible QoS levels for each session as its initial solution. That is, for each of the SLAs (pile of stones in the MMKP), the least profitable level is selected initially as the starting level.

HEU then considers possible *upgrades* for each session. An *upgrade* is a move to a higher level of QoS, and hence to greater utility earned. HEU determines how much revenue can be gained from the upgrade. For each of the upgrades of an SLA, the list of candidate paths for that SLA has to be examined to determine if there are feasible paths to route the upgrade. If there is such a path, the upgrade is *feasible*; otherwise it is *infeasible*. This continual loop then considers possible upgrades for each session. The heuristic terminates when no more *feasible* upgrades can be made. In summary, Khan's HEU starts by picking the least profitable QoS level of each SLA and iteratively tries to improve the revenue earned by upgrading the SLAs to higher levels.

2.1.2 Watson's Heuristic Algorithm

To adapt the Utility Model to a packet switching network, Watson [15] introduced another faster but less effective (i.e., the solutions found are less optimal) heuristic for solving the Utility Model. Watson's heuristic combines the admission, path finding, routing, and upgrading processes. Similar to Khan's, in order to admit a session *(S, D)* from source to destination as requested by an SLA, Dijsktra's algorithm is applied to find possible routings for *(S, D).* After this path discovery process, Watson's heuristic then attempts to find a feasible path available to *(S, D)* for *initial admission*. If such a path exists, *(S, D)* is routed along it, as a starting point with subsequent upgrade attempts to gain more utility.

In the event that an *SLA(S, D)* wishes to increase its bandwidth by some amount (upgrading for an SLA). The current routing for *(S, D)* is checked to see whether it has enough *surplus capacity*[2] to accommodate the increased flow. If this is the case, then the current routing does not need to be changed. The heuristic simply increases the bandwidth allocation of flow *(S, D)* and decreases the free bandwidth of all links along the path to reflect the increased allocation to *(S, D)*. Otherwise, it simply reroutes flow *(S, D)* to another nearby path.

It is worthwhile to mention that the computational complexity of both Khan's HEU and Watson's heuristic effectively limit the Model's applicability to small networks. The execution time of these methods for larger networks generally exceeds the maximum acceptable for real-time admission control.

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[2] That is, if the most utilized edge amongst all the edges in the path still has enough surplus bandwidth to accommodate the bandwidth upgrade demand.

3 Solving the Utility Model Using a Multicommodity Flow Algorithm

Our new concept is based on mapping parts of the Utility Model onto an Unsplittable Flow problem [10]. This heuristic provides faster execution time than using Dijkstra's K-shortest path algorithm for the path finding process. In addition, for the admission process, important information for each SLA, such as a list of edge utilizations and edge capacities, is maintained thus further reducing the admission control time.

In order to solve the Unsplittable Flow problem, it is necessary to start with the regular Multicommodity Flow problem. *The Multicommodity Flow* problem [11] involves simultaneous shipment of multiple commodities through a network such that the total amount of flow on each edge is within the capacity of the edge. The objective is to maximize the amount of flow each commodity can be sent given limited edge capacities.

Formally, consider an undirected graph $G(V,E)$ with a positive capacity $u(v,w)$ for each edge $v,w \in E$. Consider also a set of commodities numbered *1* through *k*, where commodity *i* is specified by a source-sink pair $s_i, t_i \in V$ and a positive demand d_i from s_i to t_i. For commodity *i*, an amount proportional (as a percentage) to its demand d_i is shipped from s_i to t_i. *A Multicommodity Flow f* consists of *k* single commodity flows, one for each commodity**.**

A Concurrent Flow achieves *demand satisfaction* [11] if it ships an amount of each commodity equal to its demand from its source to its sink. It obeys the capacity constraints if no flow $f(v,w)$ on an edge $(v,w) \in E$ exceeds the capacity $u(v,w)$ of the edge. A *feasible Concurrent Flow* [11] achieves demand satisfaction while obeying the capacity constraints. Thus, the Multicommodity Flow problem with demand constraint from each commodity is called the *Concurrent Flow problem*.

A variant of the Concurrent Flow problem is the *Unsplittable Flow problem* [10]. The Unsplittable Flow problem differs from the regular Concurrent Flow problem by dedicating a single path to each commodity flow, instead of splitting the demand and distributing it over two or more paths. For the Unsplittable Flow problem, each commodity's demand is restricted to travel entirely on one single path.

Figure 3 illustrates the difference between a solution for the Unsplittable Flow problem and a solution for the Concurrent Flow problem. Considering the graph with a single commodity from node 1 to node 5 with a demand of 4 units, to reach a solution for the Concurrent Flow problem, each demand is split and sent through more than one path. Observe that there are three possible paths from node 1 to node 5. The demand of 4 units gets separated and shipped through 3 paths:

- Path 1, 2, 5.
- Path 1, 3, 5.
- Path 1, 4, 5.

Each path gets a portion of the demand. In this case, for example, if path 1, 4, 5 can carry 2 units of bandwidth, path 1, 3, 5 can carry 1 unit and path 1, 2, 5 can carry 1 unit, we can route the 4 units of the total demand successfully through 3 different paths.

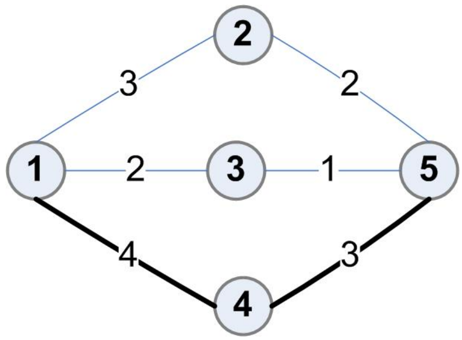

Fig. 3. An Illustration of the Concurrent Flow and Unsplittable Flow Problems

However, in an Unsplittable Flow problem, the demand sent by the commodity is restricted to one path. The solution is determined by the edge capacities and the available surplus for each edge (how much free bandwidth is left for each edge). In this example, the optimal solution is path 1, 4, 5 because the lowest edge capacity in path 1, 4, 5 is 3 units, which is the greatest of the three path choices. The solution path is highlighted.

Combining the concept of the Unsplittable Flow problem and the Concurrent Flow problem, a new heuristic for the Utility Model is developed. Observe that each commodity *i* from the Unsplittable Flow problem can represent one QoS level for one SLA in the Utility Model. Figure 4 illustrates this idea. For example, if SLA 1 (with 3 levels: bronze, silver, gold) starts from node 1 and ends at node 5, it can be mapped onto a Multicommodity Flow problem with a set of 3 commodities, all having $s_i = 1$, $t_i = 5$, but with three different values of demand (d_i).

This formulation has a drawback that three commodities have to be considered at the same time just to handle a single SLA. Since all levels within an SLA have the same source-destination pair, we can map only one of the levels of an SLA onto a commodity, and attempt upgrades or downgrades by simply changing the demand of that commodity. For example, for SLA1 in Figure 4 and the network in Figure 3, only the Gold level is treated as a commodity. The problem can then be solved by using the Unsplittable Flow algorithm and by attempting to downgrade to a lower level. In this case, path 1, 4, 5 can accommodate 3 units of bandwidth being routed through its path. Therefore, the service level for SLA1 is downgraded from the initially picked Gold level to a Silver level (with 3 units of bandwidth demand).

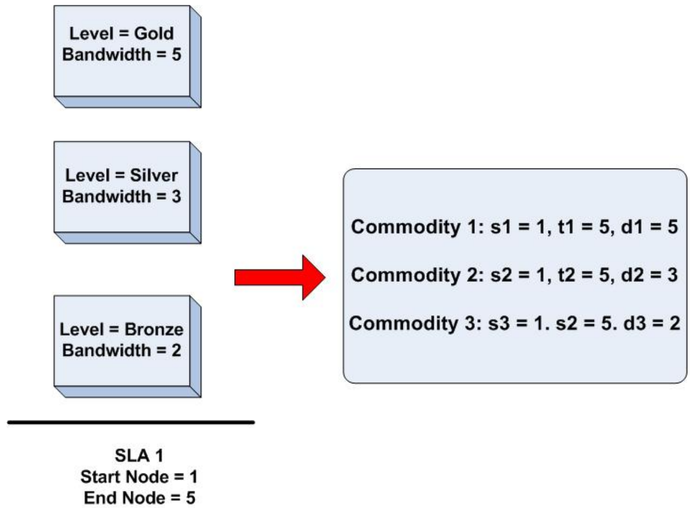

Fig. 4. Mapping a Customer SLA onto Several Commodities

4 Implementation and Performance Testing

Raghavan and Thompson [12] documented the first solution for the general Multicommodity Flow problem in 1987. Since then, many researchers have given solutions for different variations of the Multicommodity Flow and Unsplittable Flow problems. The algorithm we employed to solve the Multicommodity Flow problem is devised by Leighton et al. [11]. It is the fastest known combinatorial solution for Multicommodity Flow. This algorithm has a faster theoretical execution time than some of the best linear programming algorithms: the algorithm finds an ε*-optimal* solution in $O(k^2(\log k + \varepsilon^{-2})\log n)$ running time, where k is the number of SLAs being routed, ε is the degree of accuracy we want to reach (the error parameter), and n is the number of nodes in the graph (Recall that Dijkstra's K-shortest path algorithm's complexity is $O(kn^2)$.) Thus, by using Leighton et al.'s algorithm, the execution time of the path finding step is reduced. The difference is especially apparent in our simulation of networks with more than 30 nodes.

Table 1 shows the results of experiments by running both Dijkstra's K-shortest path algorithm and the Unsplittable Flow algorithm. Both were tested on networks of different sizes: from a 50-node, 100-edge network, to a 500-node, 2075-edge network, and for each network, batches of 20, 50 and 70 user SLAs were submitted.

From Table 1, it can be seen that for a 50-node network, the Unsplittable Flow algorithm, has a slight edge over Dijkstra's algorithm. However, as the number of network nodes and edges increases, the advantage of using the Unsplittable Flow algorithm is clearly evident. With a network of 500 nodes, the Unsplittable Flow algorithm has a 57.4% improvement over Dijkstra's algorithm. These results represent a substantial improvement for path finding in bigger networks. We have

Table 1. Execution Time Comparison Between Algorithms

(node, edges, SLAs)	Dijkstra's Algorithm	Unsplittable Flow	Percentage Improvement
Experiment 1			
(50, 100, 20)	20 ms	20 ms	0%
(50, 100, 50)	30ms	29ms	3.3%
(50, 100, 70)	40ms	35ms	12.5%
Experiment 2			
(200, 748, 20)	387ms	248ms	35.9%
(200, 748, 50)	1034ms	589ms	43.0%
(200, 748, 70)	2890ms	1571ms	45.6%
Experiment 3			
(500, 2075, 20)	3042ms	1484ms	51.2%
(500, 2075, 50)	8024ms	4710ms	41.3%
(500, 2075, 70)	10420ms	4435ms	57.4%

also performed some preliminary optimality tests by comparing solutions obtained using our heuristics to the optimal solutions using the Branch and Bound search. Our heuristics give an optimality ranging from 80% to 100%.

5 Conclusion and Future Work

The Utility Model, a mathematical formulation of network resource allocation and admission control for SLAs, is presented. Previous heuristics to solve the problem based on the mapping of the Model to an MMKP are described. In order to improve execution efficiency and scalability, a new idea to solve the Utility Model is proposed. Based on the Multicommodity Flow formulation, the new heuristic performs better than previous heuristics in many aspects. Our simulation and tests of the new heuristic were done using randomly generated network data. We intend to incorporate practical engineering concepts of real networks into our heuristic, and perform proper simulation based on several real network backbones.

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A Fixed Point Approach for the Computation of Market Equilibria

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Abstract. In proposing an open problem, Codenotti et al. [3, 5] conjectured that the *welfare adjustment scheme* can approximate the general market equilibrium by iteratively using an oracle for the Fisher's model. In this work, we analyze the scheme for a large class of market models. We show that the iterative step is in fact a Lipschitz continuous function and the residue approximation of its fixed point is a good approximation of the market equilibrium price.

1 Introduction

In this paper, we study the computation of general market equilibria. Informally, the problem considers a market model where economic agents, with initial endowments of goods, trade in the market. Each agent tries to maximize its utility function under the budget constraint. The market is at equilibrium if the supply of goods equals the demands of traders.

Arrow and Debreu [1] first established the existence of an equilibrium for market models with mild assumptions. Their proof is based on the fixed point theorem. Historically, the fixed point algorithms have been developed hand in hand with algorithms for computing general equilibrium prices. Although those algorithms are not polynomial bounded in running time, they started the study of the computation of market equilibria.

With a recent work studying the computational complexity of market equilibria [7], there has been a surge of research works of this problem in the computer science community. Up till now, various approaches are developed to solve the problem for different settings in polynomial time. The most successful approaches include [11, 10, 14, 8, 6, 4]. Codenotti, Pemmaraju and Varadarajan have a comprehensive survey [5] for this topic.

On the other hand, the fixed point problem is in general very difficult computationally, as shown by Hirsch, Papadimitriou and Vavasis [9], and Chen and Deng [2]. This is in sharp contrast of the recent development in polynomial time algorithms or polynomial time approximation algorithms for the general equilibrium problem. Therefore, there is a possibility that the fixed point problems

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derived from equilibrium computation problems have nice properties that would allow them not to be subject to the worst case analysis derived for the general fixed point problem.

Our study explores such a possibility that the fixed point problems derived in the context of general equilibrium may indeed be easy. We analyze the *welfare adjustment scheme* that may be first proposed by Jain et al [11] and later studied by Ye [14] and Codenotti et al [3]. The scheme adjusts the traders' welfare in a series of Fisher's models. The adjustment naturally induces a correspondence Φ from the space of prices to itself, and any of its fixed points is a price equilibrium. (Note that the scheme here is different from the one that was studied in the economists' society, e.g. [13, 12], although they share the same name.) We prove that Φ is a Lipschitz continuous function under certain conditions. Hence, the simplex algorithm [2] can be applied to approximate the fixed points of Φ in the residue sense. We prove that a good residue approximation of the fixed points is also a good approximation of the price equilibrium.

The paper is organized as follows: In Section 2, we will introduce the definitions in exchange market models, the welfare adjustment scheme and the induced correspondence Φ. In Section 3, we introduce some assumptions for the market model which are sufficient for Φ to be a differentiable function. We then prove the Lipschitz continuity of Φ. In Section 4, we show the equivalence between the approximation of the fixed point and the approximation of the market equilibrium. We provide our concluding remarks in the last section.

2 Model of Market Equilibrium

We consider the equilibrium problem in an exchange market which consists of n divisible goods and m traders. The traders come to the market with initial endowment of goods. The initial endowment of trader i is denoted by a vector $w_i \in \mathbb{R}^n_+$, whose j-th entry is the amount of good j held by trader i. In the paper, we assume that the total amount of each good is 1, i.e., $\sum_{i=1}^{m} w_{ij} = 1, (1 \le j \le n)$.

The behavior of each trader is determined by his preference to the bundles of goods and his budget constraint. There are two ways to describe the traders' preference, via traders' utility functions or via traders' demand functions. We will adopt both of them in the paper.

2.1 Equilibria Defined by Utility Functions

Assume each trader has a utility function $\mathbb{R}^n_+ \mapsto \mathbb{R}_+$ to present his utility for a bundle of goods. Usually, we require that the utility function $u : \mathbb{R}^n_+ \mapsto \mathbb{R}_+$ is concave, nondecreasing and $u(0) = 0$. In the market, each trader acts as both a buyer and a seller to maximize its utility. At a price $p \in \mathbb{R}^n_+$, trader i is is solving the optimization problem:

$$\max u_i(x_i) \quad s.t. \quad \langle p, x_i \rangle \le \langle p, w_i \rangle , x_{ij} \ge 0$$

Definition 1. *An equilibrium in Arrow-Debreu model of exchange economy is a price vector* $\bar{p} \in \mathbb{R}^n_+$ *and bundles of goods* $\left\{\bar{x}_i \in \mathbb{R}^n_+, i = 1,..,m\right\}$, *such that*

$$\begin{aligned} &\bar{x_i} \in argmax \left\{u_i(x_i)|x_i \geq 0, \langle x_i, \bar{p}\rangle \leq \langle w_i, \bar{p}\rangle\right\}, \quad 1 \leq i \leq m \\ &\sum_{i=1}^{m} \bar{x}_{ij} \leq 1, \quad 1 \leq j \leq n \end{aligned}$$

A special case of exchange market is the Fisher's model. In the model, the initial endowments of traders are proportional, i.e. $w_i = e_i w$ for a fixed vector $w \in \mathbb{R}^n_+$. It is equivalent to say that the traders come to the market with initial endowments of money and the goods are held by the market. The traders buy goods from the market to maximize their utility under their budget constraints. Assume trader i's money is $w_i \in \mathbb{R}_+$, then it is solving the optimization problem:

$$\max u_i(x_i) \quad s.t. \quad \langle p, x_i\rangle \leq w_i, x_{ij} \geq 0$$

We can define the equilibrium for the Fisher's model:

Definition 2. *An equilibrium in the Fisher's model is a price vector* $\bar{p} \in \mathbb{R}^n_+$ *and bundles of goods* $\left\{\bar{x}_i \in \mathbb{R}^n_+, i = 1..n\right\}$, *such that*

$$\begin{aligned} &\bar{x_i} \in argmax \left\{u_i(x_i)|x_i \geq 0, \langle x_i, \bar{p}\rangle \leq w_i\right\}, \quad 1 \leq i \leq m \\ &\sum_{i=1}^{m} \bar{x}_{ij} \leq 1, \quad 1 \leq j \leq n \\ &\sum_{j=1}^{n} p_j = \sum_{i=1}^{m} \langle p, \bar{x}_i\rangle \end{aligned}$$

Usually, computing the equilibrium in the Fisher's model is much easier than the general model.

In general, we can only approximate the equilibrium since the equilibrium point may not be a rational vector. Thus it is necessary to define the approximate equilibrium:

Definition 3. *An* ϵ*-approximate equilibrium in Arrow-Debreu model is a price vector* $\bar{p} \in \mathbb{R}^n_+$ *and bundles of goods* $\left\{\bar{x}_i \in \mathbb{R}^n_+, i = 1, ..., m\right\}$, *such that*

$$u_i(\bar{x}_i) \geq \frac{1}{1+\epsilon} max \left\{u_i(x_i)|x_i \geq 0, \langle x_i, \bar{p}\rangle \leq \langle w_i, \bar{p}\rangle\right\}, \forall i \tag{1}$$

$$\langle \bar{x}_i, \bar{p}\rangle \leq (1+\epsilon)\langle w_i, \bar{p}\rangle, \forall i \tag{2}$$

$$\sum_{i=1}^{m} \bar{x}_{ij} \leq 1+\epsilon, \forall j \tag{3}$$

2.2 Equilibria Defined by Demand Functions

In some models, it is more convenient to use demand functions to characterize the traders' preference on goods. Trader i's demand function is a map $d_i : \mathbb{R}_+ \times \mathbb{R}^n_+ \mapsto \mathbb{R}^n_+$, where $d_i(w_i, p) \in \mathbb{R}^n_+$ presents trader i's demand of goods under the price p with budget $w_i \in \mathbb{R}_+$. The aggregated demand function is defined by $D(w,p) = \sum_{i=1}^{m} d_i(w_i, p)$ and the aggregated excess demand function is defined by $Z(w,p) = D(w,p) - \mathbf{1}$.

We can define the equilibrium price in the Fisher's model by excess demand functions:

Definition 4. *In Fisher's market model, a price p is equilibrium price for the market if and only if $Z(w,p) \leq 0$ and $\langle p, Z(w,p) \rangle = 0$.*

The following proposition shows the connection between demand functions and utility functions.

Proposition 1. *If the utility function $u : \mathbb{R}^n_+ \mapsto \mathbb{R}_+$ is strictly concave, we can define demand function $d : \mathbb{R}_+ \times \mathbb{R}^n_+ \mapsto \mathbb{R}^n_+$ for them as follows:*

$$d_i(w_i, p) = argmax \left\{ u_i(x_i) \mid \langle x_i, p \rangle \leq w_i, x_i \in \mathbb{R}^n_+ \right\}$$

If u is nondecreasing, strictly concave and twice differentiable, the induced demand function d is a differentiable function.

If the demand functions are induced by utility functions, Definition 2 and Definition 4 are equivalent.

2.3 Welfare Adjustment Scheme

For a price vector $p \in \mathbb{R}^n_+$ and an exchange market M, we define a Fisher's market model M_p as follows: the goods and traders' utility functions (or demand functions) in M_p are same to M; the initial endowment of money of trader i in M_p is given by $\langle w_i, p \rangle$, where w_i is the initial endowment of goods of trader i in M.

Assume we have an oracle which can compute the equilibrium in the Fisher's model. Initially, we pick a arbitrary price p_0 from $\mathbb{R}^n_+ \setminus \{0\}$ and get the Fisher's market M_{p_0}. The oracle of Fisher's model can compute an equilibrium price p_1 for M_{p_0}. Then we move to the next Fisher's market M_{p_1} and the oracle can compute M_{p_1}'s equilibrium price p_2, and so on. Iteratively repeat the process, we may expect that the price sequence $\{p_k, \mid k = 1, 2, ...\}$ will converge to a fixed point p^*. By Def. 1 and Def. 2, the fixed point p^* is just the equilibrium price of the general model M. This iterative scheme is called *welfare adjustment scheme* in literature.

Because the Fisher equilibrium problem is usually easier to solve than the Arrow-Debreu equilibrium, the welfare adjustment scheme has been utilized in the computation of Arrow-Debreu equilibrium in practice. Unfortunately, the iteration may fall in a cycle in some special cases [14].

Although the iteration scheme may not converge in general, it do provide a connection between the Fisher's model and the general model. Now we formalize the iterative step by a correspondence $\Phi : \mathbb{R}^n_+ \mapsto 2^{\mathbb{R}^n_+}$ defined as follows:

$$\Phi(p) = \left\{ y \in \mathbb{R}^n_+ \mid p \text{ is an equilibrium price of } M_p \right\}$$

From now on, Φ is called the *welfare adjustment function* of market M. In general, it is a correspondence, i.e., a set-valued function. The correspondence Φ is hard to analyze, even in linear models (e.g., see the discussions in Ye [14]). In Section 3, we will introduce some assumptions to the market model to guarantee the uniqueness of the equilibrium price. Hence, Φ becomes a vector-valued function in that case. We will adopt Φ for a set-valued welfare adjustment function and adopt ϕ when it is vector-valued.

Example 1. We consider a market with m traders and n goods. Assume trader i's initial endowment is $w_i \in \mathbb{R}^n_+$ and his utility function is a Cobb-Douglas function: $u_i(x_i) = \prod_{j=1}^{n} x_{ij}^{\alpha_{ij}}$, where $\alpha_{ij} \geq 0$ and $\sum_{j=1}^{n} \alpha_{ij} = 1$. We can prove that in this model, ϕ is a linear function defined as:

$$\phi_i(p) = \sum_{j=1}^{n} \sum_{k=1}^{m} \alpha_{ki} w_{jk} p_j$$

Moreover, ϕ is a column-stochastic matrix. Hence the sequence $\{p_k | k = 1, 2, ...\}$ defined by $p_{i+1} = \phi(p_i)$ will exponentially converge to the unique fixed point.

3 Welfare Adjustment Function in a Large Class of Market Model

In this section, we describe the market with users' demand functions. With some reasonable assumptions to demands functions, the welfare adjustment function is vector-valued and differentiable. Furthermore, we show that the function is Lipschitz continuous with Lipschitz constant $\sqrt{n}$.

3.1 Assumptions on Demand Functions

Consider a pure exchange market consists of n divisible goods and m traders. The initial endowment is denoted by an $m \times n$ matrix W, whose i-th row W_i is the endowment of trader i. Since the amount of each good has been normalized to 1, W is a column-stochastic matrix. Assume the traders' demand functions are

$$\left\{d_i : \mathbb{R}_+ \times \mathbb{R}^n_+ \mapsto \mathbb{R}^n_+ \mid i = 1, ..., m\right\}$$

Introduce the following assumptions for demand functions:

Assumption 1. The demand functions are continuously differentiable.

Assumption 2. The demand functions are homogenous of degree zero, i.e.

$$d_i(w_i, p) = d_i(\lambda w_i, \lambda p), \quad \forall \lambda > 0$$

Assumption 3. (Strong Monotone)

$$\frac{\partial d_{i,j}}{\partial w_i} \geq 0, \qquad \lim_{p_j \to 0^+} d_{i,j}(w_i, p) = +\infty, \quad \forall j$$

Assumption 4. (Walras' law) $\langle d_i(w_i, p), p\rangle = w_i$.
Assumption 5. (Differential form of Gross Substitutability)

$$\frac{\partial d_{i,j}}{\partial p_j} \leq 0, \quad \frac{\partial d_{i,j}}{\partial p_k} > 0, \quad \forall j \text{ and } \forall k \neq j$$

Example 2. CES function has the form $u_i(x_i) = (\sum_{j=1}^{n} \alpha_{ij} x_{ij}^{\rho})^{\frac{1}{\rho}}$, $(\rho < 1)$. The demand function induced by CES utility function is:

$$d_{i,j}(w, p) = \frac{\alpha_{ij}^{\frac{1}{1-\rho}} p_j^{\frac{-1}{1-\rho}}}{\sum_{k=1}^{n} \alpha_{ik}^{\frac{1}{1-\rho}} p_k^{\frac{-\rho}{1-\rho}}} w$$

It is easy to verify that the demand function is a differentiable function on $\mathbb{R}_{++} \times \mathbb{R}^n_{++}$ and satisfy Assumption 1-5 when $0 < \rho < 1$. □

Following the notation in Section 2.3, given a price $p \in \mathbb{R}^n_+$, there is a Fisher's market model M_p, in which the budget of trader i is $w_i = \langle W_i, p\rangle$. The following proposition states the existence and uniqueness of equilibrium price in the Fisher's model:

Proposition 2. *If traders' demand functions satisfy Assumption 1 to Assumption 5, the equilibrium price $\bar{p}$ in the Fisher's model exists and is the unique root of the equation $Z(w, p) = 0$, where w is the traders' budget and $Z(w, p)$ is the aggregated excess demand function.*

3.2 Welfare Adjustment Functions in the Market

By the Assumption 1-5, the welfare adjustment function ϕ is a vector-valued function defined of $\mathbb{R}^n_{++}$, satisfying that $Z(Wp, \phi(p)) = 0$. In this subsection, we will prove the Lipschitz continuity of ϕ. For the sake of space, all the proofs are moved to the full version of this paper.

A matrix $M = (M_{ij})$ is called column-diagonal-dominated if

$$M_{ii} \geq 0, \quad M_{ij} < 0 (i \neq j), \quad \sum_{i=1}^{n} M_{ij} \geq 0, \quad \forall i, j$$

Lemma 1. *If $M = (M_{ij})$ is column-diagonal-dominated, we have*
(1) $det(M) > 0$;
(2) $0 \leq x = M^{-1}b$ if $b \geq 0$.

Lemma 2. *If the demand functions $\{d_i\}$ satisfy Assumption 1-5, the aggregated demand function $D(w, p) : \mathbb{R}^m_+ \times \mathbb{R}^n_{++} \mapsto \mathbb{R}^n_+$ has no critical point. Therefore, $D^{-1}(1) = \{(w, p) \in \mathbb{R}^m_+ \times \mathbb{R}^n_{++} \mid D(w, p) = 1\}$ is a C^1 m-dimensional manifold. For any $(w_0, p_0) \in D^{-1}(1)$, there exists an open neighborhood U of w_0 and a C^1 function $F : U \mapsto \mathbb{R}^n_+$ such that $F(w_0) = p_0$ and $D(w, F(w)) = 0$ on U.*

We list some properties of F:

Lemma 3. $\frac{\partial F_i}{\partial w_j} \geq 0$ *for all* i, j.

Lemma 4. $\sum_{j=1}^{m} \frac{\partial F_j}{\partial w_i} = 1$ *for all* $1 \leq i \leq n$.

Locally, $\phi(p) = F(Wp)$. Let Δ_n^+ denote the interior point of the simplex:

$$\Delta_n^+ = \left\{p \mid p \in \mathbb{R}_{++}^n, \langle p, 1 \rangle = 1\right\}$$

Lemma 5. *If traders' demand functions satisfy Assumption 1-5, the welfare adjustment function* ϕ *is a map from* Δ_n^+ *to itself, or equivalently*

$$\langle \phi(p), 1 \rangle = \langle p, 1 \rangle = 1, \quad \phi_j(p) > 0, \forall j$$

With this observation, we can prove the main theorem of this section:

Theorem 1. *If the traders' demand functions satisfy Assumption 1-5, the welfare adjustment function* ϕ *is differentiable and its Jacobian matrix is a column-stochastic matrix.*

Now we can prove the Lipschitz continuity of ϕ. The 2-norm of a matrix A is defined by

$$\|A\|_2 \triangleq \sup \{\|Ax\|_2 / \|x\|_2, x \neq 0\}$$

Lemma 6. D *is an* n*-dimensional convex manifold embedded in* $\mathbb{R}^n$*. The map* $F : D \mapsto D$ *is differentiable in* $int(D)$*, the interior of* D*.* J_F *is the Jacobian matrix of* F*. If* $\|J_F(x)\|_2 \leq M$ *for all* $x \in int(D)$*,* F *is Lipschitz continuous function with Lipschitz constant* M*.*

Notice that a column-stochastic matrix's 2-norm never exceeds $\sqrt{n}$, we have the following corollary by Theorem 1 and Lemma 6.

Corollary 1. *If the traders' demand functions satisfy Assumption 1-5, the welfare adjustment function* ϕ *is a Lipschitz continuous function with Lipschitz constant* $\sqrt{n}$*.*

4 Approximate the Equilibria

Now it is necessary to consider the relation between the approximation of market equilibrium and the approximation of fixed points of ϕ. We have defined the ϵ-approximate equilibrium in Section 2. The approximation of fix points of a function F can be defined in two ways. An ϵ-residual approximation of fixed points of a function F is a point $p \in \mathbb{R}^n$ such that $\|F(p) - p\| \leq \epsilon$. An ϵ-absolute approximation of a fixed point p^* is a point $p \in \mathbb{R}^n$ such that $\|p - p^*\| \leq \epsilon$. The worst computational complexity of an ϵ-absolute approximation of fixed point is infinite. Fortunately, we will prove in this section that a residual approximation of fixed points is good enough to approximate the market equilibrium price.

Let Δ_n denote the $(n-1)$-simplex embedded in $\mathbb{R}^n$:

$$\Delta_n = \left\{ x \in \mathbb{R}^n_+ | \sum_{i=1}^{n} x_i = 1, x_i \geq 0 \right\}$$

Consider an exchange economy M. From the definition of equilibrium, if $\bar{p}$ is an equilibrium price, so is $\lambda\bar{p}$ for any $\lambda > 0$. Therefore we may assume that the equilibrium price $\bar{p}$ falls in the simplex Δ_n.

Now the problem of computing the equilibrium price has been reduced to finding a fixed point p^* of the welfare adjustment function ϕ, which is proved to be Lipschitz continuous. We adopt the simplicial algorithm in Chen and Deng [2] to compute the fixed point of ϕ. For a Lipschitz continuous function, their algorithm can compute an ϵ-residual approximation of the fixed point in $O((\frac{1}{\epsilon})^{n-1})$ function evaluations.

In order to prove a good residual approximation of the fixed point is also a good approximation of the market equilibrium, the ratio between the greatest price and the smallest price is required be bounded above. The requirement can be achieved in various ways. For example, we can transform the market M to M' by adding a special trader $(m+1)$ [6]. Let $0 < \delta < 1$ be a parameter. Trader $(m+1)$'s initial endowment is $(\frac{\delta}{n}, ..., \frac{\delta}{n})$ and utility function is the Cobb-Douglas function:

$$u_{m+1}(x_{m+1}) = \prod_{j=1}^{n} x_{m+1,j}^{\alpha_{m+1,j}}$$

Let Δ_n^δ denote the region $\left\{p \in \Delta_n | p_i \geq \frac{\delta}{2n}, \forall i\right\}$ and ϕ denote the welfare adjustment function for the market M'. We can prove the following lemma.

Lemma 7. *For any $p \in \Delta_n$, $\phi(p) \in \Delta_n^\delta$.*

Thus the function ϕ is a well-defined map from Δ_n^δ to itself. Moreover, ϕ is Lipschitz continuous and has a fixed point in Δ_n^δ. Therefore M' has an equilibrium price in Δ_n^δ. Codenotti et al. [6] prove that an ϵ-approximate equilibrium for M' is an $\epsilon(1+\delta)$ equilibrium for M. Now for the transformed market M', we can prove the following lemma:

Lemma 8. *If p is an $c\epsilon$-residual approximation of the fixed point of ϕ, i.e., $\|\phi(p) - p\| \leq c\epsilon$, then p is an ϵ-approximate equilibrium of the market M'. Here*

$$c = \frac{\delta}{2n^{1.5}} \min_{1 \leq i \leq m+1} \left\{ \sum_{j=1}^{n} w_{ij} \right\}$$

Proof. For any $p \in \Delta_n^\delta$ satisfying $\|\phi(p) - p\| \leq \epsilon$, let $\bar{p}$ denote $\phi(p)$. By the construction of ϕ, there exists an allocation $\{\bar{x_i}, i = 1, 2, ..., n\}$ such that

$$\bar{x_i} = argmax \left\{ u_i(x_i) | \left\langle x_i, \bar{p} \right\rangle \leq \left\langle w_i, p \right\rangle \right\}$$

Let $\nu = \min_{1 \le i \le m+1} \left\{ \sum_{j=1}^{m} w_{ij} \right\}$. We will prove that $(\bar{p}, \bar{x_i})$ is an ϵ-approx equilibrium for M'.

$$| \langle w_i, p \rangle - \langle w_i, \bar{p} \rangle | = | \langle w_i, p - \bar{p} \rangle | \le \|w_i\| \|p - \bar{p}\|$$
$$\le \sqrt{n} \frac{\delta \nu \epsilon}{2n^{1.5}} = \frac{\delta \nu \epsilon}{2n} \le \epsilon \min \{ \langle w_i, p \rangle, \langle w_i, \bar{p} \rangle \}$$

This implies $\frac{1}{1+\epsilon} \langle w_i, \bar{p} \rangle \le \langle w_i, p \rangle \le (1+\epsilon) \langle w_i, \bar{p} \rangle$. Therefore $\langle \bar{x}, \bar{p} \rangle \le \langle w_i, p \rangle \le (1+\epsilon) \langle w_i, \bar{p} \rangle$. The condition (2) has been proved.

Let $\hat{x_i} = argmax \{ u_i(x_i) | \langle x_i, \bar{p} \rangle \le \langle w_i, \bar{p} \rangle \}$. Since u_i is non-decreasing, we can assume that:

$$\bar{x_i} = argmax \{ u_i(x_i) | \langle x_i, \bar{p} \rangle = \langle w_i, p \rangle \}$$
$$\hat{x_i} = argmax \{ u_i(x_i) | \langle x_i, \bar{p} \rangle = \langle w_i, \bar{p} \rangle \}$$

If $\langle w_i, \bar{p} \rangle \le \langle w_i, p \rangle$, we have proved the condition (1) since $u_i(\hat{x_i}) \le u_i(\bar{x_i})$. Otherwise, we have

$$u_i(\hat{x_i}) \le \frac{\langle w_i, \bar{p} \rangle}{\langle w_i, p \rangle} u_i\left(\frac{\langle w_i, p \rangle}{\langle w_i, \bar{p} \rangle} \hat{x_i}\right) \le (1+\epsilon) u_i(\bar{x_i})$$

The first inequality follows by the concavity of u_i and $u_i(0) = 0$.

The condition (3) is trivial. □

Let $\delta = \epsilon < \min \left\{ \{0.5\} \cup \left\{ \sum_j w_{ij} | i = 1, ..., m \right\} \right\}$, we reach the final theorem by Lemma 8:

Theorem 2. *The simplicial algorithm can compute an ϵ-approximate equilibrium of the general market model M in $O((\frac{1}{\epsilon})^{n-1})$ iterations.*

5 Conclusion

In this paper, we studied the welfare adjustment scheme, which can approximate the general equilibrium price by iteratively computing the Fisher's equilibrium price. We prove that the iterative step is indeed a Lipschitz continuous function whose fixed points correspond to equilibrium prices, under some reasonable conditions. The fixed point can be approximated in the residual sense with a simplicial algorithm. We show that a residual approximation of fixed point is good enough to approximate the equilibrium. Therefore, the general equilibrium problems can be reduced to the Fisher's equilibrium problem, for a large class of utility functions.

A critical problem is whether there is a polynomial time algorithm to approximate the fixed point of ϕ. In general, the answer is no. But perhaps the nice structure of the function can help us to find a better algorithm than simplicial method.

It could be interesting to extend the scheme to more market models. Any positive result will improve our knowledge to both equilibrium theory and fixed point theory.

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New Results on the Complexity of Uniformly Mixed Nash Equilibria

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Abstract. We are interested in the complexity of finding Nash equilibria with one uniformly mixed strategy (that is, equilibria in which at least one of the players plays a uniform probability distribution over some set of pure strategies). We show that, even in imitation bimatrix games, where one player has a positive payoff if he plays the same pure strategy as the opponent, deciding the existence of such an equilibrium is an NP-complete problem. We derive this result from the NP-completeness of graph-theoretical problems strictly related to this class of equilibria.

Classification: computational complexity, game theory, graph theory.

1 Introduction

Recently many lines of research tightened the connection between Theoretical Computer Science and Game Theory: tools from the latter provided to be useful to study economic aspects of the Internet such as TCP/IP congestion [7], selfish routing [13] and algorithmic mechanism design [10]. NASH is the problem of computing Nash equilibria in non-cooperative games [11]. The complexity of NASH is considered one of the most important open problem in Complexity Theory [12] and, despite various attempts, is still unknown. Even in the two player case, the best algorithm known has an exponential worst-case running time [14]. Furthermore, when one requires equilibria with simple additional properties, the problem immediately becomes NP-hard [3, 6].

Motivated by these negative results, recent studies considered the problem of computing classes of simpler equilibria, such as pure equilibria [4] and uniformly mixed equilibria [1], that are Nash equilibria in which all the strategies played with nonzero probability by a player are played with the same probability. Uniformly mixed equilibria can be viewed, in a sense, as falling between pure and mixed Nash equilibria; playing a uniformly mixed strategy is probably the simplest way of mixing pure strategies.

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In this paper we present new NP-completeness results on uniformly mixed equilibria which hold even for a constrained class of games we call *generalized imitation bimatrix games*, that extends naturally the class of the *imitation simple bimatrix games* [2, 8, 1]. Obviously, these hardness result continue to hold in the case of general games. Specifically, we show that it is NP-complete to decide if a given generalized imitation bimatrix game has a Nash equilibrium with at least one player having an uniform strategy.

We found a strong relation between this problem and the problem of deciding the existence of a regular induced subgraph, with a certain property, in a given graph, and we give NP-completeness results for some variations of this last problem. In particular, we prove that it is NP-complete to decide if, in a given graph G, there exists such an induced regular subgraph in a vertex cover V' for G, where V' is given in the input.

Furthermore, the technique we use to prove our results, based on the one presented in [1], relates a generalized imitation bimatrix game to a graph with a given structure, and the complexity properties of the game derive from the ones of the graph. This technique could be useful to prove new results based on the structural properties of the graph.

The structure of this paper is as follows. In Section 2, we give the necessary definitions and notation. In Section 3 we detail the relation between uniformly mixed equilibria and a particular class of subgraphs, then in Section 4 we present the hardness results for the graph-theoretical problems. In Section 5, we explain how the game-theoretical results follow from the graph-theoretical ones. Finally, in Section 6 we address conclusions and open problems.

2 Definitions and Notation

In this section we explain the basic game-theoretical and graph-theoretical notions that we will use and we introduce our notation.

A *bimatrix game* is specified by two $n \times n$ matrices A and B, where n is the number of *pure* strategies; we will identify the set of pure strategies with the ordered set $N = \{1, 2, \ldots, n\}$. The first player is called the *row* player and the second player is called the *column* player. If the row player plays strategy i and the column player strategy j, the payoff will be A_{ij} for the first player and B_{ij} for the second player.

A *mixed* strategy is a probability distribution over pure strategies, that is, a vector $x \in \mathbb{R}^n$ such that $\sum_i x_i = 1$ and for every $i \in N$, $x_i \geq 0$. The *support* $\text{supp}(x)$ of a mixed strategy x is the set of pure strategies i such that $x_i > 0$. When the row player plays mixed strategy x and the column player plays mixed strategy y, their expected payoffs will be, respectively, $x^t Ay$ and $x^t By$ (x^t is the transpose of vector x). A mixed strategy x will be called *uniformly mixed*, or *uniform*, if, for every $i \in \text{supp}(x)$, $x_i = 1/|\text{supp}(x)|$.

A *Nash equilibrium* [9] of the game (A, B) is a pair of mixed strategies (x, y) such that for all mixed strategies $\overline{x}$ and $\overline{y}$, $x^t Ay \geq \overline{x}^t Ay$ and $x^t By \geq x^t B\overline{y}$. A *Nash equilibrium strategy* for a player is a mixed strategy that is played in some Nash equilibrium by that player. A *uniformly mixed* Nash equilibrium is an equilibrium in which both players play uniformly mixed strategies.

We will consider, in particular, *generalized imitation* bimatrix games. A bimatrix game is an *imitation* game if the set of pure strategies available to the players is the same and one of the players, called the *imitator*, has a positive payoff if he plays the same pure strategy as the opponent and payoff 0 otherwise, so that in a Nash equilibrium (x, y) it holds that $\text{supp}(x) \subseteq \text{supp}(y)$. We will assume the row player to be the imitator. Thus, in a generalized imitation bimatrix game, the matrix A is a diagonal matrix, with positive values on the diagonal, and we will indicate it with $D = diag(d_1, d_2, \dots d_n)$.

What follows is the notation we adopt for matrices: given a $n \times n$ matrix A and two increasing sequences of integers, $\alpha = (\alpha_1, \alpha_2, \dots \alpha_h)$ and $\beta = (\beta_1, \beta_2, \dots \beta_k)$, where $1 \leq k, h \leq n$, we will denote by $A[\alpha|\beta] = A[\alpha_1, \alpha_2, \dots \alpha_h \; |\beta_1, \beta_2, \dots \beta_k]$ the $h \times k$ submatrix of A whose (i, j) entry is A_{α_i, β_j}, where $i = 1, 2, \dots, h$ and $j = 1, 2, \dots, k$. If $\alpha = \beta$ then we abbreviate $A[\alpha|\alpha]$ to $A[\alpha]$.

We now describe our graph-theoretical notation. Given a simple (no loops) undirected graph $G = (V, E)$, we will use $G(S)$ to denote the subgraph induced by the vertices in the subset $S \subseteq V$. As a shorthand for $S \cup \{v\}$ (where $v \in V$), we will write $S + v$. We will use $d_{G(S)}(x)$ to denote the degree of node $x \in S$ in the subgraph of G induced by S.

3 Nash Equilibria with a Uniform Strategy and Related Induced Subgraphs

In this section we present the relation we found between equilibria where one strategy is uniformly mixed and a particular class of induced subgraphs.

Let $G = (V, E)$ be an undirected graph. As in [1], we will say that a subset S of vertices determines a *dominant-regularity induced subgraph* (DIS) if there is a positive integer r such that

(i) $G(S)$ is r-regular;
(ii) for every $v \in V \setminus S$, the degree of v in $G(S + v)$ is at most r.

Figure 1 shows a graph G and three subgraphs: the subgraph S_1 is not a DIS because its nodes are dominated from node 5; subgraph S_2 is not a DIS because it is not regular. Subgraph S_3 is a DIS.

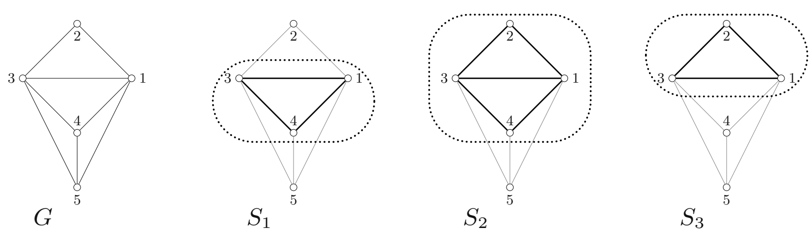

Fig. 1. Examples of subgraphs being or not being a DIS

Our contribution, about Nash equilibria, is mostly based on the following lemma.

Lemma 1. *Let* (D, B) *be a generalized imitation bimatrix game where* B *is the adjacency matrix of some undirected graph* G*, i.e., a* $(0, 1)$*-symmetric matrix. Then the uniform Nash equilibrium strategies of the row player in* (D, B) *are in one-to-one correspondence with the dominant-regularity induced subgraphs of* G*, in particular:*

– given a DIS in G, we have a corresponding not-empty set of Nash equilibria for which $\text{supp}(x)$, the support of the row player's uniform strategy, is the set of vertices of the given DIS;
– given a Nash equilibrium where the row player has a uniform strategy, we have a DIS in G induced by the set of pure strategies played by the row player.

Proof. Let $G(S)$ be a DIS with regularity r. Consider the unique uniformly mixed strategy x having support S (that is, $x_i = 1/|S|$ if $i \in S$ and $x_i = 0$ otherwise). By definition of S, we have that $|S|x^t B$ is a vector giving the degrees of every node v in the graph $G(S+v)$. But since $G(S)$ is a DIS, $x^t M$ is maximal on coordinates $i \in S$; thus, if the row player plays x, the column player has no incentive to deviate from x. So now, if the column player plays a mixed strategy y, such that $y_i = 1/d_i$ if $i \in S$ and $y_i < 1/d_i$ otherwise, then the vector of incentives for the first player is maximal on the rows in the support S, and hence (x, y) is a Nash equilibrium, for (D, B), where the row player has a uniform strategy.

For the other direction, let (x, y) be a Nash equilibrium such that x is uniformly mixed. Since the game is an imitation game, it can be easily checked that the support of x has to be included in the support of y. Let $S = \text{supp}(x)$. Since the column player has no incentive to deviate, because the two strategies are an equilibrium, we have that for every $l \in N$ and for every i in the support of y, and in particular for every $i \in S$, $(x^t B)_i \geq (x^t B)_l$. Now $|S|(x^t B)_i = \sum_{j \in S} B_{ji}$ so it follows that

$$\sum_{j \in S} B_{ji} \geq \sum_{j \in S} B_{jl}$$

But this last quantity is exactly the degree of l in $G(S+l)$. Thus, translating the maximality condition, all the nodes corresponding to strategies in S must have the same degree and moreover this must be the greatest degree a node l can have in $G(S+l)$. This implies that the subgraph induced on S is a dominant-regularity subgraph in G.

4 NP-Hardness Results in Graph Theory

In this section we formulate our results on the complexity of finding a DIS and then we will use them, in the next section, to derive some game theory complexity results. We will assume all the graphs to be simple undirected graphs with no isolated nodes. Consider the following problem.

Problem 1. **Dominant-Regularity Induced Subgraph Included in a Given Vertex Cover (Vci-Dis)**

INSTANCE: A graph $G = (V, E)$ and a subset $V' \subseteq V$ which is an independent set for G .

QUESTION: Is there a subset $\tilde{V} \subseteq V - V'$ such that $G(\tilde{V})$ is a *dominant-regularity induced subgraph*, i.e.: i) $G(\tilde{V})$ is a r-regular graph for some r and ii) $\forall v \in V - \tilde{V}$ it holds that $d_{G(\tilde{V})}(v) \leq r$?

We prove this problem NP-complete by a reduction from the decision problem 3SAT [5], defined below.

Problem 2. **3-Satisfiability (3SAT)**

INSTANCE: A collection $C = \{c_1, c_2, \ldots, c_m\}$ of clauses on a finite set $X = \{x_1, x_2, \ldots, x_n\}$ of variables such that $|c_i| = 3$ for $1 \leq i \leq m$.

QUESTION: Is there a truth assignment for X that satisfies all the clauses in C?

We construct a graph, from a generic 3SAT instance, where we assume, wlog, that no clause contains both a variable and its negation, and we use this graph in all of the reductions below. In particular we map a generic given instance of 3SAT, i.e. a set of variables U, where $|U| = n$, and a set of clauses C, where $|C| = m$, to a graph $G(V, E)$ in the following way:

- we start from an empty set V;
- for each variable $x_i \in U$ we add two nodes x_i and $\overline{x}_i$ to V and then we add an additional node x_{n+1}; we refer to this set of nodes as X;
- for each clause $c_j \in C$ we add one node c_j to V; we briefly refer to this set as C;
- we connect each node corresponding to a variable x_i with each other node in X except its opposite $\overline{x}_i$, and we connect x_{n+1} with all the other nodes;
- we connect each node c_j to all the nodes in X except the three nodes that correspond to the literals that form the clause c_j itself.

Figure 2 shows (a sketch of) a graph corresponding to a generic 3SAT instance. Now we prove that **a)** every satisfying assignment of the given 3SAT instance is mapped to a DIS included in X and with a number of vertices equal to $n + 1$, and **b)** every set $\tilde{V}$, such that $G(\tilde{V})$ is a DIS must include, for each variable x_i, either x_i or $\overline{x}_i$, but not both, plus the node x_{n+1}; in particular $|\tilde{V}| \geq n + 1$.

Proof. **a)** Consider a truth assignment for the given instance of 3SAT. Let us denote by T the subset of X that corresponds to the literals having the value `true` in this assignment. Note that $|T| = n$, since for each variable only one between x_i and $\overline{x}_i$ has the value `true`. Let us consider $\tilde{V} = T + x_{n+1}$, it is easy to verify that the graph $G(\tilde{V})$, induced on $\tilde{V}$ is a DIS because: i) $G(\tilde{V})$ is n-regular, and ii) $\forall x \in X - T$ we have $d_{G(\tilde{V})}(x) = n$ and $\forall c \in C$ we have $n - 2 \leq d_{G(\tilde{V}+c)}(c) \leq n$ because at least one (at most three) of the literals of the clause is `true`.

b) First of all notice that $\tilde{V}$ must contain the node x_{n+1}, which is connected with all the other nodes, otherwise the condition ii) in the definition of a DIS

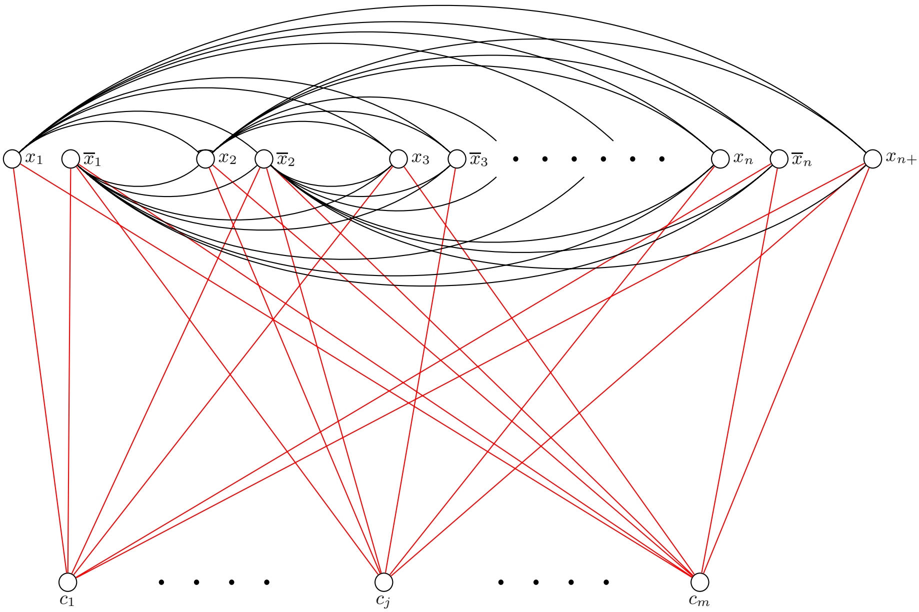

Fig. 2. Graph obtained from a generic 3SAT instance

would be violated. It follows that *at most* one, of the two nodes for each variable x_i, can be included in $\tilde{V}$. In fact if a variable x_i had both of the nodes, x_i and $\overline{x}_i$, in $\tilde{V}$ then $d_{G(\tilde{V})}(x_{n+1}) > d_{G(\tilde{V})}(x_i)$, because x_{n+1} is connected with all the nodes in $\tilde{V}$, while x_i is not, so $G(\tilde{V})$ would not be a DIS because not a regular graph. We now prove that $\tilde{V}$ must contain *at least* one of the two nodes for each variable x_i. Assume that a variable x_j is such that $x_j \notin \tilde{V}$ and $\overline{x}_j \notin \tilde{V}$. If there is a c in $C \cap \tilde{V}$, then, by the definition of DIS it follows that the graph $G(\tilde{V})$ is $d_{G(\tilde{V})}(c)$-regular; but in this case at least one between x_j and $\overline{x}_j$ is connected with c (because we assumed that no clause includes both a literal and its opposite). Wlog let it be x_j; then

$$d_{G(\tilde{V}+x_j)}(x_j) = 1 + |X \cap \tilde{V}| > d_{G(\tilde{V})}(c)$$

violating the condition ii) inthe definition of a DIS. If $C \cap \tilde{V} = \emptyset$ then the graph $G(\tilde{V})$ is $d_{G(\tilde{V})}(x_{n+1})$-regular but $d_{G(\tilde{V}+x_j)}(x_j) > d_{G(\tilde{V})}(x_{n+1})$ and there would be a violation as before. So exactly one of the nodes x_i and $\overline{x}_i$, for each variable x_i, must be included in $\tilde{V}$.

Now we are ready to prove the following results, for which each proof is based on two parts corresponding to the two directions of a polynomial time transformation: we have a polynomial transformation when a starting problem instance has positive answer if and only if the problem instance, resulting from the mapping, has a positive answer too.

Theorem 1. VCI-DIS *is* NP*-complete.*

Proof. Starting from a 3SAT instance I we construct the graph G, in the way we described above, and we set $V' = C$ obtaining an instance $\phi(I)$ of VCI-DIS. Now we prove that ϕ is indeed a reduction:

(1) Follows immediately from point **a)**.

(2) Let $\tilde{V}$ be a solution of $\phi(I)$, from point **b)** we have that $\tilde{V}$ includes exactly one node for each variable x_i, plus the node x_{n+1}, and it includes only them, since by the definition of VCI-DIS $\tilde{V} \subseteq X$, therefore $G(\tilde{V})$ is n-regular. Now for each node $c \in C$ there exists at least one $x \in \tilde{V}$ such that x and c are not connected each with the other, otherwise $d_{G(\tilde{V}+c)}(c) = n + 1$ and $\tilde{V}$ would not be a solution. This implies that each clause c, in the starting 3SAT instance, contains at least one literal belonging to the set of literals corresponding to $\tilde{V}$. Thus there is a satisfying assignment.

Theorem 2. *If in* VCI-DIS*, given an integer* $k \leq |V - V'|$*, we ask for a subset* $\tilde{V}$ *satisfying one of the following additional conditions:*

(i) $|\tilde{V}| \leq k$;
(ii) $|\tilde{V}| = k$;
(iii) $|\tilde{V}| \geq k$.

then the resulting problem is still NP*-complete.*

Proof. If we consider the instance constructed in Theorem 1 and, in addition, we set $k = n + 1$ then we still have a polynomial transformation from 3SAT:

(1) Again from point **a)**.

(2) The conditions on k do not add any constraints other than those implied by $\tilde{V} \subseteq V - V'$ together with point **b)**, so the proof is the same as point (2) in Theorem 1.

5 Game-Theoretical Results

Here we state our main results.

Theorem 3. *It is* NP*-complete to decide, given a bimatrix game, whether there is a Nash equilibrium such that at least one player has a uniformly mixed strategy.*

Proof. This is a proof by restriction, we show that this problem contains the NP-complete problem VCI-DIS. We now specify the additional restrictions to be placed on the bimatrix games we consider, then it will be easy to realize that the resulting restricted problem is equivalent to VCI-DIS.

We are interested in generalized imitation bimatrix games having the following particular form:

(1) $D = diag(1, 2, \ldots n)$.

(2) There exists an increasing sequence of integers $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_h)$ such that

$$B[\alpha] = \begin{pmatrix} 0 & n & \ldots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \ldots & 0 & (h-1)n \\ hn & 0 & \ldots & 0 \end{pmatrix}.$$

$B[\alpha]$ is a superdiagonal matrix with values increasing on the rows by multiples of n, and moreover, replacing $B[\alpha]$ with a zero matrix, the resulting matrix B' must be an adjacency matrix with an associated graph $G(V, E)$, i.e. a symmetric $(0, 1)$-matrix with no 1 on the main diagonal and at least one 1 on each row.

Now we search for a Nash equilibrium where at least one player has a uniform strategy x.

Assume that, in an equilibrium, x is played by the column player, then the row player (the imitator) will play a subset of the pure strategies played by the column player, which guarantee him the greatest expected utility. But, when x is uniformly mixed, the strategy $k = \max(\mathrm{supp}(x))$ is the only best choice for the imitator. Then the imitator plays only k, and we analyze the two possible cases: if $k \in \alpha$, say $k = \alpha_i$, then the only best choice for the column player is α_{i+1}; if $k \in N - \alpha$ (we remember that $N = \{1, 2, \ldots, n\}$ is the set of the pure strategies) then $B_{kk} = 0$, while, on row k, there exists at least one entry equal to 1 (no isolated nodes in G). In both cases the column player would not choose k, contradicting the fact that $k \in \mathrm{supp}(x)$.

It follows that there are no Nash equilibria in which the column player has a uniformly mixed strategy. So, let (x, y) be a Nash equilibrium with x uniform, we now assume that $|\mathrm{supp}(x) \cap \alpha| > 0$. Then we can take $\alpha_k = \max(\mathrm{supp}(x) \cap \alpha)$ and we note that

$$(x^t B)_l < 1 \quad \text{if} \quad l \in N - (\mathrm{supp}(x) \cap \alpha)$$

$$i \leq \frac{ni}{|\mathrm{supp}(x)|} < (x^t B)_{\alpha_{(i+1) \bmod n}} \leq \frac{ni}{|\mathrm{supp}(x)|} + 1 \leq i + 1 \quad \text{if} \quad \alpha_i \in \mathrm{supp}(x) \cap \alpha$$

thus the greatest value in $x^t B$ is on the column $\alpha_{(k+1) \bmod n} \neq \alpha_k$ (we use the modulo operator because if $k = n$ then the maximal value is on the column α_1). There is, indeed, only a best choice for the column player and it is different from the α_k strategy he plays, contradicting the hypothesis on (x, y) which must be a Nash Equilibrium, so we cannot assume $|\mathrm{supp}(x) \cap \alpha| > 0$. Therefore, in these instances, the only Nash equilibria having at least one player with a uniform strategy, are the ones in the form (x, y) with $\mathrm{supp}(x) \subseteq N - \alpha$. So, given an instance in the form we said, we actually ask if there is a Nash equilibrium (x, y) such that x is uniform and $\mathrm{supp}(x) \subseteq N - \alpha$; this restricted problem, using lemma 1, becomes: given a graph $G(V, E)$ and an independent set V' for G, is there a DIS in G with no node in V'. What we found is the VCI-DIS problem.

Theorem 4. *It is* NP*-complete to decide, given a bimatrix game, and an integer k, whether there is a Nash equilibrium such that at least one player has a uniformly mixed strategy with a support of size*

(i) at most k;
(ii) at least k;
(iii) equal to k.

Proof. We consider the instances of the restriction in the proof of the theorem 3 and, reasoning as in that proof, we actually ask for a Nash equilibrium (x, y) such that x is uniform, $\mathrm{supp}(x) \subseteq N - \alpha$ and, moreover, just one, of the three additional conditions ((i),(ii),(iii)) must be satisfied at a time. This restricted problem, using lemma 1, is translated into the version of VCI-DIS introduced in Theorem 2.

6 Open Questions and Future Work

Despite the recent efforts, the complexity of the main problem, NASH, is still an open question. In our work we presented a component design technique which can be used to construct other, hard to solve, bimatrix games, in dependence on the extent one believe NASH is NP-hard.

There is a number of open questions suggested by our work. If we focus on instances with $(0, 1)$-matrices, deciding the existence of Nash equilibria with one uniformly mixed strategy, is still an NP-complete problem?

Consider NASH restricted to the class of generalized imitation bimatrix games we introduced: is it as hard as the general NASH? Or, rather, is there a polynomial time reduction mapping NASH to this restriction?

Quasi polynomial time algorithms are known for the computation of approximate Nash equilibrium, but is it easier to find approximate Nash equilibrium for the class of games we presented?

If we take two nonnegative matrices, considering the graphs associated to their non-zero patterns, are there any structural properties that imply Nash equilibria, other than the one we said for DIS?

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Nash Equilibria in All-Optical Networks
(Extended Abstract)

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Abstract. We consider the problem of routing a number of communication requests in WDM (wavelength division multiplexing) all-optical networks from the standpoint of game theory. If we view each routing request (pair of source-target nodes) as a player, then a strategy consists of a path from the source to the target and a frequency (color). To reflect the restriction that two requests must not use the same frequency on the same edge, conflicting strategies are assigned a prohibitively high cost.

Under this formulation, we consider several natural cost functions focusing on the existence of Nash equilibria and on the complexity of recognizing and computing them.

1 Introduction

Optical networks use as physical means for transferring data lightwaves which are transmitted through optical fibers. As current technology may handle lightwaves with frequency several orders of magnitude higher than electrical signals, optical networks may reach a peer-to-peer transfer rate far greater than any electrical network. In standard optical networks however, data have to be converted from the optical to electrical form when passing through an intermediate switch and converted back to optical form for retransmission to the next station. This conversion costs in time and reduces the transfer rate to tenths of GHz. In *all-optical networks* however, the signal retains its optical form from the transmitting to the receiving end, thus achieving transfer rate of the order of tenths of THz.

To better exploit the high bandwidth of all-optical networks, all-optical network protocols are based on *wavelength-division multiplexing, WDM,* which in a sense divides the available high bandwidth in several channels. Each channel uses a different frequency (wavelength) thus allowing the simultaneous connection of several source-destination pairs through the same fiber, provided that they use different frequencies.

The efficient allocation of frequencies given a set of requests from pairs of hosts wishing to communicate, poses several interesting theoretical problems. It is common in this setting to view the network as a connected graph with its

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nodes being hosts or switches and its edges being the optical fibers that provide the actual communication. The available frequencies (i.e. different channels) of an edge are represented as different colors and by the above each edge has a palette of different colors from which a pair of communicating hosts may choose. Notice that, since we do not allow wavelength conversion, a path has to use the same color for all its edges and of course if two paths use the same edge, then they must use different colors.

The main problem therefore is the so called Routing and Path Coloring problem in which we are given a set of pairs of hosts (nodes in the graph) wishing to communicate and we are asked to provide for each pair a path and a color in such a way that no two pairs whose paths share an edge are colored the same and in addition the total number of colors used is minimized.

This problem has been shown to be NP-hard even for rings [7] but can be solved in polynomial time for chains [17] and bounded degree trees [14]. A natural way to tackle its hardness is to look for approximation algorithms. A 2-approximation algorithm is presented in [21].

Another natural problem is Maximum Routing and Path Coloring (MaxRPC). Given a graph G, a set of requests R (pairs of nodes) and a number of colors w we try to find an assignment of paths to a subset of requests $A \subseteq R$ and a coloring of these paths with different colors for overlapping paths, such that $|A|$ is maximal. The above problem is NP-hard even for rings (there is a straightforward reduction from Routing and Path Coloring). Efficient approximation algorithms for various topologies are given in [15, 2]. Other related work includes multi-fiber models [16, 12], models that use wavelength conversion [22] and also on-line algorithms for the same problems [2, 3].

Another approach that has been recently followed to study network problems is the game-theoretic one. Several researchers have study the behavior of networks in general, focusing especially in congestion problems using the powerful tools of game theory. We briefly mention here the results in [9, 18, 10].

In this paper we study all-optical WDM routing under a game theoretic approach. In our setting we view each pair of communicating nodes as a player in a non-cooperative game. The player's strategies are the different (path, color) pairs from which she may choose, first to route her communication (the path) and second, to assign a wavelength to it (the color).

Naturally, a game setup requires some *cost function* which represents the cost of a player in a state where each player has chosen some specific strategy. A state in which a player feels comfortable with her strategy, i.e. her cost does not decrease if she decides to follow another strategy, is called an *equilibrium.* J.F. Nash, in his classical paper [13], showed that every game has a mixed equilibrium but not necessarily a pure one. An important question therefore in a game setting is whether it has a pure equilibrium.

Here we consider several different natural cost functions in WDM all-optical networks as in [4, 5] and study for each one of them the following questions:

– Are there any pure Nash equilibria?
– Can we decide in P-time if a strategy profile is an equilibrium?

– Can we compute an equilibrium in polynomial time?
– If we consider a computed equilibrium as a local optimum, can we find a solution (equilibrium) of better value?

Notice that we do not yet have a general efficient algorithm for finding Nash equilibria (and, a fortiori, pure Nash equilibria). This has become a famous open problem in computational complexity theory. The reader may see [9, 19, 20] for recent results on this issue.

Giving efficient algorithms for the above questions means that in an all-optical network under some specific routing conditions, a communicating pair has efficient algorithms to decide if it is better off defecting from the current solution, or if better global solutions exist etc. Likewise, negative answers to the above problems mean that no practical algorithms for these questions exist.

Our results show that under different cost functions the complexity of recognizing a Nash equilibrium varies from NP-complete to polynomial time. Actually, we were led in redefining the cost function, in an attempt to lower the computational complexity of answering some of the above questions. Surprisingly enough, the new costs functions are also natural and lead to re-posing some of the much studied optimization problems in WDM all-optical networks.

The rest of the paper is organized as follows: In the next section we give some preliminaries and formally define the setting of the problems. In Section 3, which is divided in four subsections we study the above questions for four different cost functions. Finally, in Section 4 we summarize our results and state some open problems.

2 Preliminaries

A *network* is, for our purposes, a graph $G = (V, E)$. A *routing request* is simply a pair of nodes (s, t) with $s, t \in V$. We are given a set of M routing requests $(s_i, t_i), i = 1, \ldots, M$. We are also given *colors* which we represent as integers in the interval $C = [1..k]$, where k is the maximum number of available colors. A *solution* to the requests is a set of M paths of G, $p_i, i = 1, \ldots, M$ and a set of M colors $\chi_i, i = 1, \ldots, M$ with path p_i having as endpoints the nodes s_i and t_i. If $p_i \cap p_j \neq \emptyset$ for $i \neq j$ then $\chi_i \neq \chi_j$. This last requirement models the already stated assumption that if any two requests are routed through edge-intersecting paths, then they must use different wavelengths (colors).

A game with $n \geq 2$ players is defined by a finite set of strategies $S_i, i = 1, \ldots, n$ and n payoff functions $u_i, i = 1, \ldots, n$ one for each player, mapping $S_1 \times \ldots \times S_n$ to the integers. The elements of $S_1 \times \ldots \times S_n$ are called states. A state $s = (s_1, \ldots, s_n)$ is called pure Nash equilibrium if for every i, $u_i(s_1, \ldots, s_i, \ldots, s_n) \geq u_i(s_1, \ldots, s'_i, \ldots, s_n)$ for any $s'_i \in S_i$. A game may not have pure Nash equilibria, but Nash in [13] proved that there always exist mixed Nash equilibria (we consider as strategy any possible distribution on S_i).

We now view an all optical network G with M requests as a (non-cooperative) game in the following way.

– *Players*: The M requests $(s_i, t_i), i = 1, \ldots, M$.
– *Strategies*: Pairs $\sigma_i = (p_i, \chi_i)$ where p_i is a path from s_i to t_i and χ_i is a color (wavelength). We represent colors as integers in the interval $[1..k]$.
– *Cost*: For each player $i = 1, \ldots, M$ a cost function $g_i(\sigma_1, \ldots, \sigma_M)$.

In this paper we study the above questions under different cost functions. In all cost functions we are considering conflicting strategies are assigned a prohibitively high cost. Moreover, in all cases it is implicit that bandwidth is the main resource that needs to be carefully managed. This is reflected in the number of colors used (this is actually our first cost function that charges a path according to the number of conflicts with other colors), the color number used on a path (note that this is a different measure than the previous one and is equivalent to charging a path according to the frequency it uses, the higher the frequency the greater the cost), the most saturated link (in terms of the different paths that use it) and the highest color number on a link (this charges links, not paths that use high frequency).

3 Addressing the Questions

In this section we examine for four different cost functions the problems that we mentioned earlier.

3.1 Cost Function 1

The first cost function we consider is the number of different colors that are used along a player's path. That is, for all players their corresponding cost function is

$$g_i((p_1, x_1), \ldots, (p_M, x_M)) = \begin{cases} \infty & \text{if there is a conflict} \\ \left| \bigcup_{e \in p_i} X(e) \right| & \text{otherwise} \end{cases} \quad (1)$$

where $X(e)$ is the set of colors that are used for edge e. In other words, $|X(e)|$ is the number of different paths that use e. Under this cost function a player's possible defect is toward the direction where she has fewer conflicts with different colors.

Having defined the cost function, we first attack the problem of recognizing whether a given strategy profile, under this cost function, is a pure Nash equilibrium. We call this problem Nash Recognition:

Nash Recognition
– *Instance.* A graph G, M players (s_i, t_i) and their strategies under a given cost function g, consisting of a path and a color in $C = \{0, 1, \ldots, M\}$ such that there are no conflicts.
– *Question.* Is there an i such that nodes s_i and t_i can be connected via a path and a color, with better cost under cost function g?

For cost function 1 even recognizing a Nash equilibrium is NP-complete.

Theorem 1. Nash Recognition *under cost function 1 is NP-complete.*

Sketch. See appendix. □

3.2 Cost Function 2

In order to relax the constraints of the first function we turn our attention to the following definition of the cost:

$$g_i((p_1, \chi_1), \ldots, (p_M, \chi_M)) = \begin{cases} \infty \text{ if there is a conflict} \\ \chi_i \text{ otherwise} \end{cases} \quad (2)$$

Intuitively, under this cost function a player's possible defect is toward the direction of lower numbered colors i.e. lower wavelengths.

Under cost function 2 things are computationally easier. Pure Nash equilibria always exist and can be recognized and computed in P-time using the following greedy, online algorithm. See also theorem 1 in reference [5].

Input A graph G, M players (s_i, t_i) and a color set $\{1, \ldots, M\}$
Output A pure Nash equilibrium.

```
Find an (s_1, t_1) path and assign it color 1.
for i = 2, ..., M do
    for χ = 1, ..., M do
        Remove from G the edges belonging to paths colored χ.
        Check if there exists a path between s_i and t_i.
        if YES then use it with color χ and move to the next i.
    end
end
```

Proof. The resulting path coloring is a pure Nash equilibrium because no player can defect: player (s_1, t_1) has the minimum cost (color 1) and every player $(s_i, t_i), i = 2, \ldots, M$ gets the minimum possible color. □

Since there is a P-time algorithm to compute an equilibrium it is natural to ask for a better one. The metric we use to compare two Nash equilibria is the maximum color number they use: the smaller the maximum number, the better the equilibrium. Notice that this problem is closely related to ROUTING AND PATH COLORING, mentioned in the introduction but here part of the problem instance is a strategy that is a Nash equilibrium. However this additional fact does not lower the problem complexity (for a proof sketch see the appendix):

BETTER NASH.
- *Instance.* Graph G, k players (s_i, t_i) and a Nash equilibrium using m colors under a cost function g.
- *Question.* Is there a Nash equilibrium under cost function g using less than m colors?

Proposition 1. *Problem* BETTER NASH *under cost function 2 is NP-complete.*

3.3 Cost Function 3

The next cost function is related to the provided bandwidth:

$$g_i((p_1, \chi_1), \ldots, (p_M, \chi_M)) = \begin{cases} \infty & \text{if there is a conflict} \\ \max\{w(e) | e \in p_i\} & \text{otherwise} \end{cases} \quad (3)$$

where $w(e)$ is the number of paths that use edge e. That is, a player's defect is towards a path with lightly used edges i.e., links where a smaller portion of their bandwidth is used.

The existence of pure Nash equilibria is assured by the following theorem (see also reference [5]).

Theorem 2. *Pure Nash equilibria according to cost function 3 always exist.*

Sketch. Consider a feasible solution (valid coloring). Such a solution always exists and uses at most M colors if there are M players. We define the vector (potential) $l = (l_M, l_{M-1}, \ldots, l_i)$ where l_i is the number of players with cost equal to i. We then examine every player's strategies. If nobody can defect we are at an equilibrium point. If there is a player of cost k that defects then we have the following. Elements $l_M, l_{M-1}, \ldots, l_{k+1}$ continue having the same values, l_k is reduced and the only elements that can be increased are $l_{k-1}, l_{k-2}, \ldots, l_1$. So, a defection leads to a lexicographically better l, thus at the end we will reach an equilibrium under cost function 3. □

Proposition 2. *The question if a strategy profile is a pure Nash equilibrium can be answered in P-time.*

Proof. The proof is based on the following lemma whose proof can be found e.g. in [6].

Lemma 1. *In a weighted graph G the (s,t)-path implied by a minimum spanning tree is a min-max weighted path, i.e. a minimum weight path where we charge a path by the weight of its heavier edge.*

Consider a feasible solution in which player (s_i, t_i) uses path p_i and color χ_i. To check if the solution is an equilibrium we must examine every possible player-color combination. Examining a player k and a color χ we first remove every edge used by this color and assign each remaining edge with the weight:

$$w(e) = \left| \bigcup_{i=1,\ldots,M, i \neq k} \{\chi_i : e \in p_i\} \right| + 1. \tag{4}$$

In this way the cost of an (s,t)-path p equals $\max_{e \in p} w(e)$. This cost is minimized for the (s,t)-path implied by a minimum spanning tree, according to the above lemma. □

In addition, under cost function 3, pure Nash equilibria can not only be recognized in polynomial time, but also computed in polynomial time by the following algorithm (COMPUTE-NASH). Algorithm COMPUTE-NASH inserts one by one the players (k-loop). For each newly inserted player it tries to build a minimum spanning tree $T_k = (V_{T_k}, E_{T_k})$ using a variant of Prim's algorithm (where by V_{T_k} (resp. E_{T_k}) we denote the set of nodes (resp. edges) already included in the tree T_k). In each step of this process (j-loop, see below) the less used edge γ among those having one endpoint in V_{T_k} and the other in $V - V_{T_k}$ is selected (according to Prim's algorithm) and the tree is updated with γ. At each step the number of paths that actually use edge e is denoted by $w_{\text{use}}(e)$ while the number of trees containing e is denoted by $w(e)$. The weight of γ is then increased by one to reflect that it has now been included in the tree under construction. Then function VALIDATE is called for γ. VALIDATE examines the effect of increasing the weight of γ on the already built trees by discovering players who can defect and updating their trees.

Algorithm COMPUTE-NASH(G)
Input $G = (V, E)$, M players.
Output Nash equilibrium.

for every $e \in E$ **do** $w(e) = 0$;
for $k = 1$ **to** M **do**
    $T_k = (\{s_k\}, \emptyset)$;
    for $j = 1$ **to** $|V|$ **do**
        C: edges in the current cut $V_{T_k}, V - V_{T_k}$;
        Let γ be the edge in C with minimum w_{use};
        $w(\gamma) = w(\gamma) + 1$;
        Update T_k by adding edge γ to it;
        VALIDATE(γ);
    endfor
    Update $w_{\text{use}}(e)$ for every edge e;
endfor

Function DEFECT(γ)
Input Edge γ.
Output A player i with $\gamma \in E_{T_i}$ and who can do better by not using γ.

$C_i(e)$: the cut of G which results if we remove e from T_i;
cur($C_i(e)$): the edge with minimum w in $C_i(e)$;
Find i for which $\gamma \in T_i$ and $w(\text{cur}(C_i(\gamma))) < w(\gamma) - 1$;
if not found then set $i = 0$;
return i;

Function VALIDATE(γ)
i = DEFECT(γ);
if $i \neq 0$ **then do**
    $w(\gamma) = w(\gamma) - 1$;
    $\gamma' = \text{cur}(C_i(\gamma))$; (see function DEFECT)
    $E_{T_i} = E_{T_i} - \gamma + \gamma'$;
    $w(\gamma') = w(\gamma') + 1$;
    Update $w_{\text{use}}(e)$ for every edge e;
    VALIDATE(γ');
endif

We now have the following:

Theorem 3. *Algorithm COMPUTE-NASH correctly computes a pure Nash equilibrium under cost function 3 in polynomial time.*

Proof. The proof of correctness is based on Lemma 1. The basic idea is that of a *valid* edge: an edge e is valid if for every i

$$w(e) \geq w\left(\text{cur}(C_i(e)\right) - 1,$$

where $\text{cur}(C_i(e))$ is the minimum cost edge in the cut $C_i(e)$ of G, which results if we remove e from T_i. In other words e is valid if it's weight is at most one greater than any other edge in the cut.

The correctness follows from two conditions that are maintained throughout the k-loop:

1. (Nash-upto-k). Every tree $T_i, i = 1, \ldots, k$ is a minimum spanning tree because every edge $e \in T_i$ is valid.
2. (Nash-apart-γ). Every edge in every tree $T_i, i = 1, \ldots, k$ is valid, except (maybe) edge γ.

Consider the situation where we have completed the k-th loop and we come to player $k+1$. When we select the minimum weight (regarding the players that actually use it) edge (according to Prim's algorithm) γ and increase the weight $w(\gamma)$ it may become invalid for another tree T_i:

$$\gamma' = w(\text{cur}(C_i(\gamma))) < w(\gamma) - 1. \tag{5}$$

We point that in the above equation $w(e)$ is the number of trees that contain edge e. In the case where γ becomes invalid we replace γ with γ' in T_i, fix the weights and check whether γ' becomes invalid. This may result to a chain of calls to function VALIDATE. This chain will terminate after at most M calls because the sequence of $w(\gamma)$ decreases monotonically.

In order to complete the proof we point the following. During the construction of a tree a player reserves every edge of the tree but uses only those on her (s,t) path. To be correct we must show that the paths resulting after the construction of a tree don't change if we reduce by one the weight of an edge when it is not used by the player. So, consider an edge e which is not actually used.We check all trees:

- If a tree contains e it remains as is because e remains valid.
- If a tree does not contain e then we have two cases:
  1. The insertion of e didn't cause any calls to VALIDATE. In this case e remains valid as it returns to it's previous value.
  2. The insertion of e caused a call to VALIDATE. In this case e remains valid as it gets a value equal or greater by one than the current edge in the cut.

Regarding the complexity of the algorithm we have that an insertion of an edge results in at most M calls to VALIDATE and each of these calls needs at most $O(M)$ time due to the call to DEFECT. Finally we construct M trees, thus the time needed is $O(M^3|V|)$ where $|V|$ is the number of nodes in G. □

3.4 Cost Function 4

The last cost function is related to the bandwidth too, but from another viewpoint:

$$g_i((p_1,\chi_1),\ldots,(p_M,\chi_M)) = \begin{cases} \infty & \text{if there is a conflict} \\ \max\{W(e)|e \in p_i\} & \text{otherwise} \end{cases} \quad (6)$$

where $W(e)$ is the maximum color number that appears on edge e. That is, now an edge is charged by the highest frequency used on it. Notice that an edge may get greater weight, though it is used by only a few paths.

Proposition 3. *Pure Nash equilibria under cost function 4 always exist.*

Proof. Same as theorem 2. □

Furthermore, we have

Proposition 4. *The question if a strategy profile is a pure Nash equilibrium can be answered in polynomial time.*

Proof. Same as Proposition 2 except that the weight assigned to each edge when we consider M players with strategies $(p_i,\chi_i), i = 1\ldots M$ and examine color χ for player k is

$$w(e) = \max\left\{\{\chi\} \cup \bigcup_{i=1,\ldots,M, i\neq k} \{\chi_i : e \in p_i\}\right\}. \quad (7)$$

This way the cost of a (s,t)-path p equals $\max_{e\in p} w(e)$. This cost gets minimum over the (s,t)-path implied by a minimum spanning tree, according to Lemma 1. □

4 Conclusions and Future Work

We have examined the behavior of WDM all-optical networks under a game-theoretic approach and under several payoff functions. Our results show that the complexity of the problems of recognition and computation of pure Nash equilibria varies from polynomial to NP-complete.

As complexity theory and game theory are fastly converging, many more may be done for analyzing network problems. Some suggestions along the lines of

Table 1. Our results regarding pure Nash equilibria in all-optical networks

Function	Properties	Existence	Recognition	Computing	Find better
1	color conflicts	Open	NP-complete	Open	NP-complete
2	frequency	always	P	P	NP-complete
3	max congestion	always	P	P	Open
4	max frequency	always	P	conjectured in P	Open

this work follow: First, we may analyze WDM routing under the cost-function 4. We have done some progress along this direction: Pure Nash equilibria exist for cost-function 4, and we suspect that can be computed in P-time. Second, we may try to analyze along the same directions the multicast congestion (MC) problem [23]: we are given a network, that is an undirected graph $G = (V, E)$ and a set of subsets of S_j of V (called terminals). We are asked to find a set T_j of trees in E each spanning the respective set S_j. Let the congestion of an edge e be the number of T_j trees that contain e. How can we keep the congestion low? E.g. one may ask to keep the maximum congestion at a minimum level. Again We may try to analyze this problem by defining various analogous cost functions and considering each set of terminals as a player of a suitable non-cooperative game.

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A Proofs

A.1 Proof of Theorem 1

The problem is obviously in NP because given a path we can check in P-time the number of different colors that arise on it. To prove completeness we will use the known NP-complete problem 1-IN-3-3SAT without negation [11]:

1-IN-3-3SAT-WN
- *Instance*: A boolean formula with 3 positive literals per clause:

$$C = \bigwedge_{i=1,\ldots,m} C_i = \bigwedge_{i=1,\ldots,m} (u_{i1}, u_{i2}, u_{i3}). \tag{8}$$

- *Question*: Is there a satisfying truth assignment such that exactly one literal per clause is true?

Let C be an instance of 1-IN-3-3SAT-WN with m clauses. We construct an instance of our problem in the following way.

1. For every variable in C we introduce m new colors.
2. We introduce $m + 2$ new "universal" colors.
3. For every clause C_i we construct a graph component.

- We introduce 2 nodes x_i, y_i which we connect with 3 disjoint chain paths with $\mu_{i1}, \mu_{i2}, \mu_{i3}$ edges respectively, where $\mu_{ij}, j = 1, 2, 3$ is the total number of variable u_{ij}'s occurences in C.
- For every edge (x, y) introduced because of the 3 paths we consider 2 more nodes a, b, 2 edges (a, x) and (y, b) and a player starting at a and terminating at b.
- For every (a, b) player the instance we construct has the path (a, x, y, b) as strategy.
- Every player in a (x_i, y_i)-chain gets a different color from the set of m colors of the respective variable.

4. Every graph component connects to the next through an edge (y_i, x_{i+1}).
5. For chains that belong to the same variable we choose the same color for respective (a, b) players.
6. We introduce a new chain with $m + 1$ edges starting at node x_1 and terminating at y_m. For this chain we introduce $m+1$ (a, b) players exactly like we did in the component chains. These players use $m+1$ of the $m+2$ universal colors. The last universal color is used by a player (x_1, y_m) who follows the chain and his cost is obviously $m + 2$.

We observe that in the constructed instance every player has cost 1, except (x_i, y_i) who has cost $m + 2$ and the (a, b) players in the "long" chain who have cost equal to 2.

If there is a satisfying truth assignment for C, then every (a, b) player has obviously no better strategy. Considering player (x_1, y_m) we have that, in order to find a better strategy (with cost at most $m + 1$) we must deal with the (x_1, y_m) paths that use the component chains. Let the (x_1, y_m) path p that in every component uses the chain of the only satisfied literal. The cost of p is

$$1 + \sum_{u_i = T} \mathsf{occ}(u_i) = m + 1,$$

where $\mathsf{occ}(u_i)$ is the number of variable u_i's occurrences in the formula.

So there is a player who can change his strategy to a better cost.

For the other direction consider a situation where there is a player that can defect. Obviously this player is (x_1, y_m) (with better cost equal to $m + 1$). We construct a satisfying assignment for C in the following way. We set, in each clause, true the literal with the chain component that is used by player (x_1, y_m). Then we have the following:

- Only one literal in each clause is set true.
- The assignment is consistent. Every variable we set true charges the path with as many colors as the number of it's occurrences. So, an inconsistent assignment results in a cost greater than $m + 1$.
- The path passes through all the components, so the assignment satisfies every clause in C.

This shows that there exists a satisfying truth assignment, thus completing the proof. □

A.2 Proof of Proposition 1

The problem is obviously in NP. In order to prove completeness we will use the NP-complete problem K-DISJOINT PATHS [8].

K-DISJOINT PATHS

- *Instance.* Graph G, k pairs of nodes (s_i, t_i).
- *Question.* Is there a set of k edge disjoint paths between the nodes (s_i, t_i)?

Given an instance of K-DISJOINT PATHS, G, $(s_i, t_i), i = 1, \ldots, k$, we construct the following instance of Better Nash (G', $(s'_i, t'_i), i = 1, \ldots, 2k$):

- In order to construct G' we add in G for each (s_i, t_i), the vertices a_i, b_i and the edges (s_i, a_i), (a_i, t_i), (t_i, b_i) and (b_i, s_i).
- The new players are (s_i, t_i) and (a_i, b_i) with $i = 1, \ldots, k$.
- The equilibrium point that completes the construction uses paths (s_i, a_i, t_i) with color 1 for the (s_i, t_i) players and paths (a_i, s_i, b_i) with color 0 for the (a_i, b_i) players.

If there is a set of k disjoint paths in G then there is an equilibrium point in which every player uses color 0. In this point players (s_i, t_i) use this set of paths and players (a_i, b_i) use the paths (a_i, s_i, b_i).

On the other side, if there is a better Nash equilibrium then it uses only one color. So, every (s_i, t_i) player uses only edges of G, otherwise there is a conflict with (a_i, b_i) players. Since players (s_i, t_i) can use the same color the paths they use are edge disjoint.

We point that the above construction can be used to prove that the problem of finding the best Nash equilibrium is also NP-complete. □

Price of Anarchy, Locality Gap, and a Network Service Provider Game

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Abstract. In this paper, we define a network service provider game. We show that the price of anarchy of the defined game can be bounded by analyzing a local search heuristic for a related facility location problem called the k-facility location problem. As a result, we show that the k-facility location problem has a locality gap of 5. This result is of interest on its own. Our result gives evidence to the belief that the price of anarchy of certain games are related to analysis of local search heuristics.

1 Introduction

It is important to analyse the outcome of games involving multiple, selfish, non-cooperative agents which have a corresponding social cost. Koutsoupias and Papadimitriou [5] formulated the problem in terms of the *Nash equilibrium* attained by a set of independent, non-cooperative agents with rational behavior. In an environment in which each agent is aware of all the alternatives facing all the other agents, Nash equilibrium is a combination of choices (deterministic or randomized), one for each agent, in which, no agent has an incentive to unilaterally move away. The Nash equilibrium is known to deviate from overall optimum in many optimization scenarios. They defined *worst case equilibria* as the maximum value that the ratio of the overall optimum to the cost of a Nash Equilibrium can take over the set of all Nash equilibriums. Papadimitriou [6] called it as the *price of anarchy*. Since then, this notion has been used to analyse the efficiency of many games [7, 8, 9].

We define a network service provider game and consider the problem of bounding its price of anarchy. We first show that the price of anarchy of the game considered is the same as *locality gap* [1] of the k-facility location problem [4]. We then show that the k-facility location has a locality gap of 5. This result is of independent interest on its own. Our result gives evidence to the belief that price of anarchy and local search analysis are interrelated.

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We consider a game on a network with a distinguished node called the *root*. Some of the nodes in the network are called *clients*, denoted by C. The clients seek to obtain the root's service via a set of k service providers(SP) who may occupy one of the remaining nodes in the network, denoted by F. In a *configuration* each SP either decided to not occupy any of the nodes in F or occupies a node in F which is not occupied by any other SP. In a given configuration, each client is served by the closest SP. A client j pays to its closest SP i, an amount equal to its distance from the second closest SP (VCG cost). The revenue of an SP i is total amount paid to it by all the clients connected to i. The cost incurred by i to provide the service to its clients is the cost of connecting i to the root. The difference of the revenue and the cost incurred is the net profit of i. In order to maximize individual profits, the SPs may adopt strategies which are specified when we define the game later in the paper. A configuration of SPs is said to be in *Nash equilibrium* if none of the SPs can improve their profit using the strategies available to them. The cost of this Nash equilibrium is the total cost incurred by the SPs. The optimal cost is the minimum cost incurred by at most k SPs in serving all clients. The price of anarchy is supremum, over all Nash equilibria, the ratio of the cost of a Nash equilibrium to the optimal cost.

In the *k-facility location* problem, the input consists of a set of clients, C, and a set of facilities, F. Associated with each facility in F is a cost of opening a facility there. An integer k specifies the maximum number of open facilities. The goal is to determine the set of facilities to be opened such that the sum of opening costs and the costs of connecting each city to its closest facility is minimized. Charikar et.al. [3] gave 9.8-approximation for this problem and it was later improved to 6 in [4]. The k-facility location problem inherits features of both, the k-median and the uncapacitated facility location (UFL) problems.

In the k median problem we are permitted to open at most k facilities but there is no cost associated with opening a facility. A simple local search algorithm which swaps facilities as long as the cost of the solution reduces, has a locality gap of 5 [1]. The first constant factor approximation algorithm for the k-median problem was a $(6 + \frac{2}{3})$-approximation and was given by Charikar et.al. [3]. This ratio was improved to 6 in [4] and then to 4 in [2].

The UFL problem is a special case of the k-facility location problem when no limit is placed on the number of open facilities. Once again, a local search algorithm which at each step can add, delete or swap facilities has a locality gap of 3 [1, 2]. We show that local search with the same operations has a locality gap of 5 for the k-facility location problem. In Section 5, we exaplain why the analyses for the UFL and the k-median problem presented in [1] can not be directly adapted for the k-facility location problem.

Vetta [9] considers the social utility value of Nash Equilibrium of similar games. In this setting, the social utility value of the outcome of a game is given by a non-decreasing, submodular function and the decisions are controlled by a set of non-cooperative agents who seek to maximize their own private utility. He showed that the social utility value of a Nash Equilibrium is atleast half of the optimal social utility value. No such multiplicative bound is possible when the utility function is only submodular.

2 The k-Facility Location Problem

We are given a set of facility locations F and a set of clients C. For a facility $i \in F$, its *facility cost* $f_i \geq 0$ is the cost of *opening* that facility. We are also given distances c_{ij} between $i, j \in F \cup C$ that satisfy metric properties. The cost of connecting the client $j \in C$ to a facility $i \in F$ is given by c_{ij}. An integer k specifies the limit on the number of open facilities. The objective is to open at most k facilities in F such that the sum of the facility costs of the open facilities and the cost of serving each client by the nearest open facility is minimized.

For a subset $S \subseteq F$ of at most k facilities, let $\texttt{fac}(S) = \sum_{i \in S} f_i$ denote its facility cost, let $\texttt{serv}(S) = \sum_{j \in C} \min_{i \in S} c_{ij}$ denote its service cost, and let $\texttt{cost}(S) = \texttt{fac}(S) + \texttt{serv}(S)$ denote its total cost. We define $\texttt{cost}(\emptyset) = \infty$. Consider the following local search algorithm for the k-facility location problem. We start by opening any subset $S \subseteq F$ of at most k facilities. We then try to reduce $\texttt{cost}(S)$ iteratively by employing three local operations. For $i \in F$, we use $S - i$ and $S + i$ to denote $S \setminus \{i\}$ and $S \cup \{i\}$ respectively. The three local operations are, *delete* a facility: if there is an $i \in S$ such that $\texttt{cost}(S - i) < \texttt{cost}(S)$, then $S \leftarrow S - i$; *add* a facility: if $|S| < k$ and there is an $i \in F \setminus S$ such that $\texttt{cost}(S + i) < \texttt{cost}(S)$, then $S \leftarrow S + i$; *swap* facilities: if there is an $i \in S$ and an $i' \in F \setminus S$ such that $\texttt{cost}((S - i) + i') < \texttt{cost}(S)$, then $S \leftarrow (S - i) + i'$.

We call this a *delete-add-swap* local search algorithm. We call $S \subseteq F$ with $|S| \leq k$ a *local optimum* solution if $\texttt{cost}(S)$ cannot be reduced by doing any of the above operations. The maximum ratio of the cost of a local optimum solution to the cost of a global optimum solution is called the *locality gap* of the local search algorithm. In Section 5, we prove the following theorem.

Theorem 1. *The locality gap of the above local search algorithm is at most 5.*

3 Service Provider Game

We are given a network with a distinguished node r called *root*. Remaining nodes in the network are partitioned into a set of clients C, and a set of service locations F. The distances c_{ij} between $i, j \in F \cup C \cup \{r\}$ satisfy metric properties. Suppose that there are k SPs. Each SP is allowed to occupy at most one service location. Two SPs cannot occupy the same service location. The assignment of SPs to the service locations, say $S \subseteq F$, defines a *configuration*. It is not necessary that every provider is assigned to a location.

Consider a configuration $S \subseteq F$ with $|S| \leq k$. For an SP $i \in S$, let $N_S(i)$ be the set of clients for which i is the closest of all the SPs in S. To serve the clients in $N_S(i)$ and connect to the root r, provider i incurs an expense of $\texttt{expense}_S(i) = c_{ir} + \sum_{j \in N_S(i)} c_{ij}$. Let $\texttt{expense}(S) = \sum_{i \in S} \texttt{expense}_S(i)$ be the total expense of all the SPs in S. For a client $j \in C$, let $S_j = \min_{i \in S} c_{ij}$ be the distance of j to the closest SP in S. Then $\texttt{expense}(S) = \sum_{i \in S} c_{ir} + \sum_{j \in C} S_j$.

Each client connects to the SP closest to it, but is charged the VCG payment, which is the distance to the second closest SP. The *revenue* of an SP $i \in S$ is the total payment it receives from all the clients it serves, i.e., $\texttt{revenue}_S(i) = \sum_{j \in N_S(i)} T_j$ where $T = S - i$. The profit of an SP i is $\texttt{profit}_S(i) = \texttt{revenue}_S(i) - \texttt{expense}_S(i)$. Note that $\texttt{expense}_S(i), \texttt{revenue}_S(i)$ and $\texttt{profit}_S(i)$ are with respect to a particular configuration S.

Lemma 1. *The profit of a provider s in the configuration S is given by* $\texttt{profit}_S(s) = \texttt{expense}(S - s) - \texttt{expense}(S)$.

Proof. Let $T = S - s$.

$$\begin{aligned}
\texttt{expense}(T) - \texttt{expense}(S) &= \left(\sum_{i \in T} c_{ir} + \sum_{j \in C} T_j\right) - \left(\sum_{i \in S} c_{ir} + \sum_{j \in C} S_j\right) \\
&= \sum_{j \in C} (T_j - S_j) - c_{sr} \\
&= \sum_{j \in N_S(s)} T_j - \left(\sum_{j \in N_S(s)} S_j + c_{sr}\right) \\
&= \texttt{revenue}_S(s) - \texttt{expense}_S(s) \quad = \quad \texttt{profit}_S(s). \blacksquare
\end{aligned}$$

Each SP behaves selfishly and tries to maximize individual profit. This defines a game in which we allow only pure strategies and no two SPs can occupy the same location. The strategies of all the SPs defines a configuration. The payoff of a particular SP is equal to his profit in that configuration. A *Nash equilibrium* is a configuration so that no SP can unilaterally change his strategy and get a higher profit. The *price of anarchy* is the supremum, over all Nash equilibria, of the ratio between the cost of a Nash equilibrium and the optimum cost.

4 Connection Between Price of Anarchy and Locality Gap

There is a natural correspondence between instances of the service provider game and the k-facility location problem. The set of service locations in the game corresponds to the set of facilities in the k-facility location problem. The expense c_{ir} that an SP i incurs in connecting to the root corresponds to the facility cost f_i. The expense c_{ij} incurred in connecting to the client j is just the cost of servicing client j by facility i. So, $\texttt{expense}(S)$ which is the total expense incurred by the SPs in a configuration S is the same as $\texttt{cost}(S)$ which is the cost of solution S in the k-facility location problem. We now show that a Nash equilibrium in the service provider game corresponds to a local optimum solution in the k-facility location instance with the delete-add-swap operations.

Theorem 2. *A configuration $S \subseteq F$ is a Nash equilibrium of an instance of the service provider game if and only if S is a local optimum solution of the corresponding instance of the k-facility location problem with respect to the delete-add-swap local search.*

Proof. Let $S \subseteq F$ be a Nash equilibrium. From the definition, $\texttt{profit}_S(s) = \texttt{cost}(S - s) - \texttt{cost}(S) \geq 0$ for all $s \in S$. Therefore $\texttt{cost}(S)$ cannot be reduced by deleting a facility $s \in S$. Let $|S| < k$ and $S' = S + s$ for some $s \notin S$. Since any provider not in S did not occupy the location s, we have $\texttt{profit}_{S'}(s) \leq 0$. Therefore, from Lemma 1, we have $\texttt{cost}(S' - s) - \texttt{cost}(S') \leq 0$. Thus $\texttt{cost}(S) \leq \texttt{cost}(S + s)$. Therefore $\texttt{cost}(S)$ cannot be reduced by adding a facility $s \notin S$. Now let $S' = S - s + s'$ for some $s \in S$ and $s' \notin S$. Since the provider s does not move from location s to s', we have $\texttt{profit}_{S'}(s') \leq \texttt{profit}_S(s)$. Let $T = S - s = S' - s'$. Then we have

$$\begin{aligned}\texttt{cost}(S') - \texttt{cost}(S) &= (\texttt{cost}(T) - \texttt{cost}(S)) - (\texttt{cost}(T) - \texttt{cost}(S')) \\ &= \texttt{profit}_S(s) - \texttt{profit}_{S'}(s') \quad \geq \quad 0.\end{aligned}$$

Therefore $\texttt{cost}(S)$ cannot be reduced by swapping a pair of facilities. Thus the solution $S \subseteq F$ is indeed a local optimum solution with respect to the delete-add-swap local search. Similarly, we can show that a local optimum solution is a Nash equilibrium in our game. ■

Theorem 3. *The price of anarchy for the service provider game is at most 5.*

Proof. The proof follows from Theorems 1 and 2. ■

5 Proof of Theorem 1

The analysis uses some ideas from the analyses of local search for the k-median and the UFL problems by Arya et al. [1]. They proved that the 1-swap local search has a locality gap of 5 for the k-median problem and the delete-add-swap local search has a locality gap of 3 for the UFL problem. The k-median proof crucially uses the fact that the global optimum solution has exactly k facilities. However, for the k-facility location instance derived from the network service provider game, the global optimum may have much less than k facilities. The analysis for the UFL considers add operations irrespective of the number of facilities in the current solutions. If the k-facility location solution has exactly k facilities, we cannot add a facility to reduce its cost. Due to these reasons we cannot adapt the analyses in [1] to the k-facility location problem.

Let S denote a local optimum solution to the k-facility location problem and let O denote a global optimum solution. If $|S| < k$, then S is local optimum with respect to the addition operation as well. So the analysis for the UFL in [1] is directly applicable and we have the following lemma.

Lemma 2. *If* $|S| < k$, *then* $\texttt{cost}(S) \leq 3 \cdot \texttt{cost}(O)$.

So, in the rest of the analysis we assume that S, the local optimum has exactly k facilities implying that we can only consider swap and delete operations on S.

5.1 Notation and Preliminaries

We use s to denote a facility in S and o to denote a facility in O. The other notation is as introduced in Section 2. For a client $j \in C$, let $S_j = \min_{s \in S} c_{sj}$

and $O_j = \min_{o \in O} c_{oj}$ denote its service costs in solutions S and O respectively. For a facility $s \in S$, let $N_S(s)$ denote the set of clients served by s in the solution S and for a facility $o \in O$, let $N_O(o)$ denote the set of clients served by o in the solution O. Let $N_s^o = N_S(s) \cap N_O(o)$. A facility $s \in S$ is said to "capture" a facility $o \in O$ if $|N_s^o| > |N_O(o)|/2$. A facility $s \in S$ is called "good" if it does not capture any facility in O. It is called "1-bad", if it captures exactly one facility in O. It is called "2+bad" if it captures at least 2 facilities in O. These notions were introduced in [1].

We now define a 1-1 onto function $\pi : N_O(o) \to N_O(o)$ for each $o \in O$ as follows. First consider a facility $o \in O$ that is not captured by any facility in S. Let $M = |N_O(o)|$. Order the clients in $N_O(o)$ as $c_0, \ldots, c_{M-1}$ such that for every $s \in S$, the clients in N_s^o are consecutive, that is, there exists p, q, $0 \le p \le q \le M$ such that $N_s^o = \{c_p, \ldots, c_{q-1}\}$. Now, define $\pi(c_i) = c_j$ where $j = (i + \lfloor M/2 \rfloor)$ modulo M.

Lemma 3. *For each $s \in S$, we have $\pi(N_s^o) \cap N_s^o = \emptyset$.*

Proof. Suppose both $c_i, \pi(c_i) = c_j \in N_s^o$ for some $s \in S$. As s does not capture o, we have $|N_s^o| \le M/2$. If $j = i + \lfloor M/2 \rfloor$, then $|N_s^o| \ge j - i + 1 = \lfloor M/2 \rfloor + 1 > M/2$. If $j = i + \lfloor M/2 \rfloor - M$, then $|N_s^o| \ge i - j + 1 = M - \lfloor M/2 \rfloor + 1 > M/2$. In both cases we have a contradiction. ■

Now consider a facility $o \in O$ that is captured by a facility $s \in S$. Note that $|N_s^o| > |N_O(o) \setminus N_s^o|$. We pair each client $j \in N_O(o) \setminus N_s^o$ with a unique client $j' \in N_s^o$ and define $\pi(j) = j'$ and $\pi(j') = j$. Let $N \subseteq N_s^o$ be the subset of clients which are not paired in the above step; note that $|N| = 2|N_s^o| - |N_O(o)|$. For each $j \in N$, we define $\pi(j) = j$.

The function $\pi : N_O(o) \to N_O(o)$ satisfies the following properties for all $o \in O$.

P1. If $s \in S$ does not capture $o \in O$, then $\pi(N_s^o) \cap N_s^o = \emptyset$.
P2. If $s \in S$ captures $o \in O$ and if $((j \in N_s^o) \wedge (\pi(j) \in N_s^o))$, then $\pi(j) = j$.
P3. We have $\{j \in N_O(o) \mid \pi(j) \ne j\} = \{\pi(j) \in N_O(o) \mid \pi(j) \ne j\}$, $\forall o \in O$.

5.2 Deletes and Swaps Considered

Since S is a local optimum solution, its cost cannot be reduced by doing any deletions and swaps. Since $|S| = k$, we cannot add a facility to reduce the cost. For any $s \in S$, we have $\texttt{cost}(S - s) \ge \texttt{cost}(S)$. For any $s \in S$ and $o \in O$, we have $\texttt{cost}(S - s + o) \ge \texttt{cost}(S)$. We now carefully consider some delete and swap operations. If we delete $s \in S$, we reroute the clients in $N_S(s)$ to other facilities in $S - s$. If we swap $s \in S$ and $o \in O$, we reroute the clients in $N_S(s)$ and a subset of clients in $N_O(o)$ to the facilities in $S - s + o$. The assignment of the other clients is not changed. For each of the operations considered, we obtain an upper bound on $\texttt{cost}(S') - \texttt{cost}(S) \ge 0$ where S' is the solution obtained after the operation. We then add these inequalities to prove Theorem 1. We consider the following operations.

1. Each 1-bad facility $s \in S$ is swapped with the facility $o \in O$ that it captures. The clients $j \in N_O(o)$ are rerouted to o. The clients $j \in N_S(s) \setminus N_O(o)$ are rerouted to $s' \in S$ that serves $\pi(j)$ in S. Since s does not capture any $o' \neq o$, P1 implies that $s' \neq s$ and the rerouting is feasible. We call these swaps, **Type 1** operations.
2. Each 2+bad facility $s \in S$ is swapped with the nearest facility $o \in O$ that it captures. All the clients in $N_O(o)$ are rerouted to o. Consider a facility $o' \neq o$ captured by s. Such a facility o' is called a *far* facility. The clients $j \in N_s^{o'}$ such that $\pi(j) = j$ are rerouted to o. The remaining clients $j \in N_S(s)$ are rerouted to $s' \in S$ that serves $\pi(j)$ in S. From P1 and P2, such rerouting is feasible. We call these swaps, **Type 2** operations.
3. Let $G \subseteq S$ be the subset of facilities in S that are not swapped out in the Type 1 or Type 2 operations. Note that G is precisely the set of good facilities. Let $R \subseteq O$ be the subset of facilities in O that are not swapped-in in the Type 1 or Type 2 operations. Since $|S| = k \geq |O|$ and $|S \setminus G| = |O \setminus R|$, we have $|G| \geq |R|$. Let $G = \{s_1, \ldots, s_{|G|}\}$ and $R = \{o_1, \ldots, o_{|R|}\}$. We swap s_i with o_i for $1 \leq i \leq |R|$. For each of these swaps, we reroute the clients in $N_S(s_i) \cup N_O(o_i)$ as follows. All the clients in $N_O(o_i)$ are rerouted to o_i. The clients $j \in N_S(s_i) \setminus N_O(o_i)$ are rerouted to $s' \in S$ that serves $\pi(j)$ in S. Since s_i is a good facility, P1 implies $s' \neq s_i$ and hence this rerouting is feasible.
   We consider $|G| - |R|$ more operations as follows. For each i such that $|R| + 1 \leq i \leq |G|$, we delete s_i. After such a deletion, we reroute the clients $j \in N_S(s_i)$ to $s' \in S$ that serves $\pi(j)$ in S. Again from P1, we have $s' \neq s$ and hence this rerouting is feasible.
   We call these $|R|$ swaps and $|G| - |R|$ deletions, **Type 3** operations.
4. Let $R' \subseteq O$ be the subset of far facilities in O. Since no far facility is swapped-in in Type 1 or 2 operations, we have $R' \subseteq R$. It is no loss of generality to assume $R' = \{o_1, \ldots, o_{|R'|}\}$. Recall that $G = \{s_1, \ldots, s_{|G|}\}$ is the set of good facilities. We consider $|R'|$ swaps as follows. For each i such that $1 \leq i \leq |R'|$, we swap s_i with o_i. The clients $j \in N_O(o_i)$ such that $\pi(j) = j$ are rerouted to o_i. The clients $j \in N_S(s_i)$ are rerouted to $s' \in S$ that serves $\pi(j)$ in S. The remaining clients are not rerouted. We call these $|R'|$ swaps, **Type 4** operations.

Since S is a local optimum solution, the increase in the facility and service costs after each operation considered above is at least zero. In the sections to follow, we bound this increase due to all the 4 types of operations together. At this point, we remark that, the Type 4 operations are crucial for being able to bound the change in service cost of those clients $j \in C$ which are served by a far facility in O and $\pi(j) = j$.

5.3 Bounding the Increase in the Facility Cost

In Type 1, 2, and 3 operations, each facility in O is brought in exactly once and each facility in S is taken out exactly once. Thus in these operations the increase in the facility cost is exactly $\texttt{fac}(O) - \texttt{fac}(S)$. In Type 4 operations,

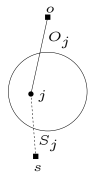

Fig. 1. Rerouting a client $j \in N_O(o)$ when o is brought in

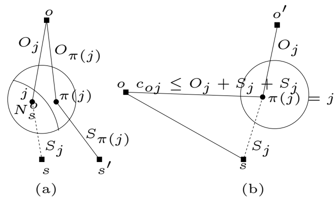

Fig. 2. Rerouting a client $j \in N_s^o$ when s is taken out and o is not brought in

each far facility is brought in exactly once and some good facilities are taken out exactly once. Thus the increase in facility cost in these operations is at most $\sum_{o:\text{far}} f_o \leq \texttt{fac}(O)$. Thus the overall increase in the facility cost is at most

$$2 \cdot \texttt{fac}(O) - \texttt{fac}(S). \tag{1}$$

5.4 Bounding the Increase in the Service Cost

We call a facility $o \in O$ a *near* facility if it is captured by some $s \in S$ and o is not a far facility.

1. A client $j \in C$ served by a near facility in O is called "white" if $\pi(j) = j$.
2. A client $j \in C$ served by a far facility in O is called "gray" if $\pi(j) = j$.
3. A client $j \in C$ is called "black" if $\pi(j) \neq j$.

Recall that a client $j \in N_s^o$ is rerouted only when either o is brought in and/or s is taken out.

Lemma 4. *Increase in the service cost of a white client $j \in C$ over all operations is atmost $O_j - S_j$.*

Proof. Let $j \in N_s^o$ where $s \in S$ is a 1-bad or 2+bad facility that captures the near facility $o \in O$. Note that we swap s with o once - either as a Type 1 operation or as a Type 2 operation. The increase in the service cost of j in this swap is $O_j - S_j$ (Figure 1). Since s or o are not considered in Type 3 or 4 operations, the client j is not rerouted in these operations. ■

Lemma 5. *Increase in the service cost of a gray client $j \in C$ over all operations is at most $3O_j - S_j$.*

Proof. Let j be served by a far facility $o' \in O$. Let s be the 2+bad facility in S that captures o'. Since $\pi(j) = j$, from P1 we have $j \in N_s^{o'}$. Let o be the closest facility in O that s captures. Note that $c_{so} \leq c_{so'}$. A gray client is not rerouted in Type 1 operations. In Type 2 operations, j is rerouted to o (Figure 2(b)) and increase in service cost of j is at most $c_{jo} - c_{js}$. Since $c_{jo} \leq c_{js} + c_{so} \leq c_{js} + c_{so'} \leq c_{js} + c_{js} + s_{jo'}$, the increase is at most $c_{js} + c_{jo'} = S_j + O_j$. In Type 3 and 4 operations, increase in the service cost of j is $O_j - S_j$ each (Figure 1). So, the increase in service cost of j over all the operations is at most $(O_j + S_j) + 2(O_j - S_j) = 3O_j - S_j$. ■

Lemma 6. *The total increase in the service cost of a black client $j \in C$ in all the 4 types of operations is at most $2D_j + O_j - S_j$ where $D_j = O_j + O_{\pi(j)} + S_{\pi(j)} - S_j$.*

Proof. Let $j \in N_s^o$ for facilities $s \in S$ and $o \in O$. We first bound the increase in the service cost of a black client j due to operations of Type 1, 2, and 3. Amongst these operations, there is exactly one swap in which o is brought in. This swap contributes $O_j - S_j$ to the increase in service cost of j. The facility s may be either deleted once or considered in a swap with $o' \in O$. If it is considered in a swap with $o' \in O$ such that $o' = o$, then the increase in service cost of client j has already been accounted for in $O_j - S_j$. If s is deleted or considered in a swap with $o' \in O$ such that $o' \neq o$, j is rerouted to s' that serves $\pi(j)$ in S and the increase in its service cost is at most $c_{js'} - c_{js} \leq c_{jo} + c_{o\pi(j)} + c_{\pi(j)s'} - c_{js} = O_j + O_{\pi(j)} + S_{\pi(j)} - S_j = D_j$ (Figure 2(a)). Thus the total increase in service cost of j due to Type 1,2, and 3 operations is at most $D_j + O_j - S_j$.

In Type 4 operations, since $\pi(j) \neq j$, the client j is rerouted only when s is (possibly) taken out. In such a case, it is rerouted to s' that serves $\pi(j)$ in S (Figure2(a)) and the increase in its service cost is again at most $c_{js'} - c_{js} \leq c_{jo} + c_{o\pi(j)} + c_{\pi(j)s'} - c_{js} = O_j + O_{\pi(j)} + S_{\pi(j)} - S_j = D_j$. Thus the total increase in the service cost of j in all 4 types of operations is $2D_j + O_j - S_j$. ■

Lemma 7. *Increase in the service cost over all the operations is atmost $5 \cdot \texttt{serv}(O) - \texttt{serv}(S)$.*

Proof. Lemmas 4,5, and 6, imply that the the total increase in the service cost of all the clients in all the 4 types of operations is at most

$$\sum_{j:\text{white}} (O_j - S_j) + \sum_{j:\text{gray}} (3O_j - S_j) + \sum_{j:\text{black}} (2D_j + O_j - S_j).$$

Now from property P3 of the function π, we have $\sum_{j:\text{black}} S_j = \sum_{j:\text{black}} S_{\pi(j)}$ and $\sum_{j:\text{black}} O_j = \sum_{j:\text{black}} O_{\pi(j)}$.

$$\sum_{j:\text{black}} D_j = \sum_{j:\text{black}} (O_j + O_{\pi(j)} + S_{\pi(j)} - S_j) \quad = \quad 2 \sum_{j:\text{black}} O_j.$$

Hence the total increase in the service cost of all the clients is at most $\sum_{j \in C}(5 \cdot O_j - S_j) = 5 \cdot \texttt{serv}(O) - \texttt{serv}(S)$. ■

5.5 Bounding the Increase in the Total Cost

From (1) and Lemma 7, it follows that the total increase in cost due to all the operations is atmost $2 \cdot \mathtt{fac}(O) - \mathtt{fac}(S) + 5 \cdot \mathtt{serv}(O) - \mathtt{serv}(S)$. As S is a local optimum, in each operation the total cost increases by a non-negative amount. Together, these imply that $cost(S) \leq 5 \cdot cost(O)$, thus proving Theorem 1. The k-median problem has a family of instances in which the ratio of the costs of local optimum to the global optimum is asymptotically equal to 5 [1]. So, our analysis of locality gap is tight.

6 Conclusions

This is the first analysis of local search for the k-facility location problem. Other approximation algorithms for the problem [3, 2, 4] can not be used to bound price of anarchy. It would be interesting to see if price of anarchy of many more games can be analysed via local search analysis.

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Network Traffic Analysis and Modeling for Games

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Abstract. A number of papers have been proposed for the purpose of analyzing the traffic data of network game and providing the models recently. The traffic characteristics of network games are different according to the game genres, and the traffic is affected by player number and the in-game behaviors. In this paper, we develop the dedicated tool *NetGame Sniffer* for measurement and analysis of network game traffic, and measure the traffic of *Quake* and *World of Warcraft*. We analyze and compare the traffic characteristics according to the number of players and user actions, and propose the game traffic models and practical uses of the models. The analysis and models of network game traffic are can be used for effective network simulation, performance evaluation and the design of network games.

1 Introduction

Network game traffic has been increased with the advance of Internet infra structure, the support of console and mobile games for network. In order to properly design and evaluate routers, protocols, and network games, we require accurate game traffic models. A better understanding of network game traffic can lead to more effective network architectures and more realistic network simulations.

Over the years a number of papers for analysis and modeling the traffic data of network games have been presented. In previous researches, game traffic was modeled by extreme, lognormal, gamma, or exponential distribution [1-5]. While the previous traffic models reflect the number of players, user actions do not considered on the models. Since game components of building, exploring and combat have an influence on the traffic of network games [6,7], in-game behaviors have to be considered on the models. And practical application of models insufficient for the network games in the previous papers.

Since the main genres of network game are FPS (First Person Shooter), RTS (Real Time Strategy), and MMORPG (Massive Multiplayer Online Role Playing Game) [8], we measure the traffic of the FPS *Quake 3* and MMORPG *World of Warcraft (WoW)*, and analyze the traffic data according to the number of players and in-game

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behaviors in this paper. For experiment, we make and use *NetGame Sniffer* [9] as a dedicated game traffic measurement and analysis. We analyze the characteristics of generated traffic by server and client in view of data rate, packet rate, packet size, and packet interarrival time. At last we present the server and client traffic models by distribution functions and propose the practical applications of the traffic models.

This paper consists of as follows, the next section explains the selected games, the experimental environment and used tool, and analyze the measured results. Section 3 present the traffic models *Quake 3* and *WoW*, and propose the practical uses of the models. Section 4 summarizes our conclusions and presents the possible future work.

2 Experimental Setup and Traffic Analysis

In this section we explain the games for experiment and the environment and tool for measurement of the game traffic. And we also analyze the data rate, packet rate, packet size, and packet interarrival time, for both server and client. For the analysis of the packet size and data rate, we consider the size of IP datagrams.

2.1 Selected Games and Experimental Setup

We choose *Quake 3* and *WoW* for our experiment. The two games are designed as a client-server application. *Quake 3* is one of the famous FPS games. Before a game start, the game server is opened by someone, either dedicated or non-dedicated mode. And UDP packets are used for exchange of update information. We consider running servers only in dedicated mode, and game map is used Q3DM17 for all tests over a set of Ethernet LANs. *WoW* as a popular MMORPG, communicate each other with TCP packets. Since we can't measure the traffic of game server, we collect the traffic data at the client. We assume that received packets of client are server traffic, and transmitted packets by client are client traffic.

All machines are Pentium 4 running Windows XP. The games played during 10 to 30 minutes, and we analyzed only in-game traffic. The experiments according to the number of players are played normally, and we divided four kinds of user action for traffic analysis about the in-game behaviors. In *Quake 3*, user actions are shooting, moving, normal play and no action, and a game is played by four players who have own action. The in-game behaviors of *WoW* are hunting the NPCs, moving, a battle with players and no play. For traffic measurement according to the user actions in *WoW*, we collected the data of two players who play with same action in the game.

For measuring network traffic, the tools such as Tcpdump, Ethereal and Sniffer are usually used. Since these ordinary tools are inconvenient for analyzing the network game traffic, we implement *NetGame Sniffer* as a dedicated game traffic measurement and analysis tool for Windows. This tool supports the packet filtering by protocol, port number, IP address and server architectures, and it shows the statistics about packet size, packet interarrival time, data rate and packet rate of each host in real time. It also generates the traffic log files both server and client separately, and shows packet statistics for each host.

2.2 Quake 3 Analysis

We analyze packet size, data rate and packet rate by the traffic transmitted from server to all clients, and interarrival time is analyzed by the packets of server to each client.

2.2.1 Data Rate and Packet Rate

The server data rate and packet rate depend on the number of players and in-game behaviors, but these rates of clients are independent on the number of players, and are dependent on the gameplays.

Table 1 and Table 2 show the data rate and the packet rate according to the number of players and four in-game behaviors. The increment of the number of client brings the linear increase of server data rate. Server transmits same number of packets to each client, but the size for each client is slightly different. As the number of players increases, server data rate becomes larger. However the mean of the client data rate remains constant to about 54 kbps. The data rates of client 1 to 3 that have player's actions are similar each other, otherwise the data rate of client 4 that have repeated death and revival is obviously low in Table 3.

Table 1. Mean / standard deviation of data rate and packet rate for the number of clients

Game	Server		Client average	
	Data rate (kbps)	Packet rate (pps)	Data rate (kbps)	Packet rate (pps)
2 players	30.8 / 4.63	40.0 / 4.55	53.8 / 6.37	88.5 / 10.40
3 players	50.5 / 6.43	60.0 / 4.61	53.9 / 4.44	89.1 / 7.23
4 players	73.0 / 9.70	79.7 / 5.19	53.8 / 4.33	88.8 / 7.22

Table 2. Mean / standard deviation of data rate and packet rate for four kinds of behavior

	Server	C1-Shooting	C2-Moving	C3-Normal	C4-No play
kbps	74.6 / 8.36	54.6 / 2.66	52.6 / 2.90	53.5 / 2.71	31.4 / 6.49
pps	79.9 / 3.88	90.3 / 4.10	90.5 / 4.23	90.8 / 4.22	59.4 / 12.27

2.2.2 Packet Size and Packet Interarrival Time

As the number of player increases, the packet size of the game server is increasing but the client packet size is similar to each other. When the game events such as death, revival, movement and attack are happened in the game, the packet size of both server and client is increasing.

Table 3 and Table 4 show the packet sizes of server and clients according to the number of clients and in-game behaviors. As the number of client increases when the number of clients is more than two, the packet size transmitted by server linearly rises about 13 bytes with an additional client. Server transmits the update packets for the information of players to all clients, so the number of sent packets by the server to each client is almost equals and the packet size is similar each other.

Table 3. Mean / standard deviation of packet size for the number of clients

	2 players	3 players	4 players	6 players
Server (byte)	91.6 / 12.30	104.9 / 16.37	119.4 / 18.09	145.6 / 19.27
Client (byte)	75.7 / 3.12	75.7 / 3.05	75.5 / 3.16	75.8 / 3.02

Table 4. Mean / standard deviation of packet size for four kinds of behaviors

	C1-Shooting	C2-Moving	C3-Normal	C4-No play
Server (byte)	97.9 / 11.86	105.2 / 13.43	105.2 / 13.66	96.4 / 12.35
Client (byte)	75.1 / 3.45	72.5 / 2.70	75.3 / 2.87	66.6 / 0.55

As the number of client increase, the distribution of server packet size shifts to the right in Fig.1-(a). The distribution of packet size of client 1 to client 3 that have the game events are variable but client 4 distribution shows similar size around 67 bytes in Fig.1-(b).

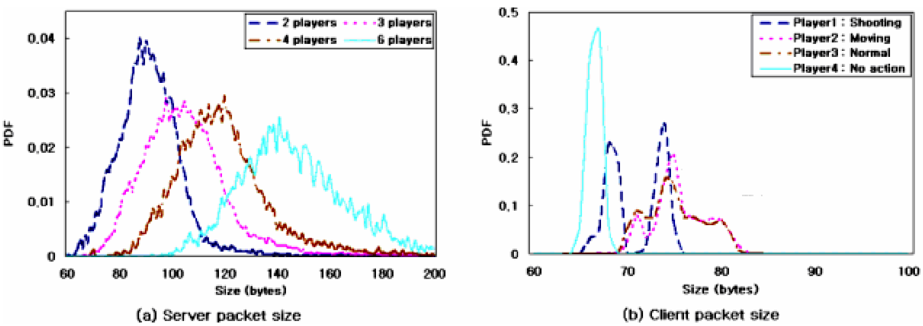

Fig. 1. PDF of packet size of server for the number of clients, and PDF of packet size of clients according to the in-game behaviors

In Table 5 and Table 6, packet interarrival time of server is independent on the number of players and in-game behaviors. The mean of server interarrival time for individual client is about 50 ms, but (a) shows two peaks around 50ms in Fig. 2.

Table 6 shows that the client interarrival time is independent on the number of players. Although the number of client increases, the mean of packet interarrival time of all clients are similar. Interarrival times of client 1 to client 3 are similar to each other but interarrival time of client 4 is longer than the others. Since client 4 doesn't have updated information except the repeated death or revival, client 4 transmits the updated packets every between 12 and 16 ms (see also Fig. 2-(b)).

Table 5. Mean / standard deviation of packet interarrival time for the number of clients

	2 players	4 players	6 players
Server (ms)	50.0 / 3.06	51.3 / 8.90	50.7 / 7.46
Client (ms)	11.3 / 0.75	14.9 / 9.52	14.2 / 7.06

Table 6. Mean / standard deviation of packet interarrival time for four kinds of behaviors

	C1-Shooting	C2-Moving	C3-Normal	C4-No play
Server (ms)	52.8 / 12.16	53.0 / 12.48	52.6 / 12.69	52.9 / 12.32
Client (ms)	12.7 / 8.27	14.4 / 9.30	13.9 / 9.78	17.6 / 9.90

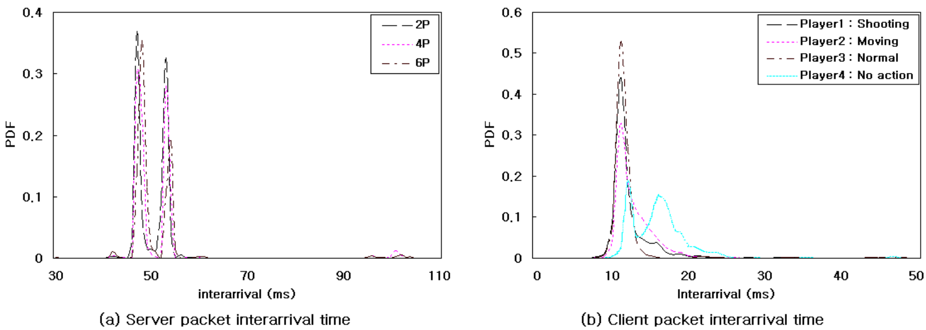

Fig. 2. PDF of packet interarrival time of server for the number of clients, and PDF of client interarrival times according to the in-game behaviors

2.3 WoW Analysis

As we can't measure directly the server traffic, we assume that server traffic is the received packets from the server on the client side, and client traffic is transmitted data from client to server. Game server transmits the packets including nearby players' information to each player [10]. Since we can't classify the nearby characters on the client side, we analyze the traffic data about the in-game behaviors only.

2.3.1 Data Rate and Packet Rate

We analyze the data rate and packet rate according as the user actions are different in Table 7. When the user action exists in the game, these rates of server and client are increasing. Although player doesn't play the game, game server communicates with the client for updated information. The client transmits more packets and data to the server after player states are changed.

Table 7. Mean / standard deviation of data rate and packet rate for in-game behaviors

Gameplay	Server to client		Client to server	
	Data rate (kbps)	Packet rate (ms)	Data rate (kbps)	Packet rate (ms)
No play	4.4 / 2.40	3.4 / 1.45	1.5 / 0.69	3.4 / 1.50
Moving	7.1 / 9.67	6.1 / 2.87	3.2 / 2.02	5.6 / 3.84
Hunting	7.9 / 3.58	7.5 / 4.13	3.7 / 1.95	7.0 / 3.16
Battle	8.4 / 5.74	6.8 / 3.04	3.4 / 2.05	6.5 / 3.32

Since server transmits all information around the client, server data rate is higher than the data rate of client. However the server transmits and receives the similar number of packets with client, because they communicate by using TCP protocol. The data rate of server and client is the highest when the player moves a long distance in the short time. While the players play continuously in the game, the client transmits more updated packets to the server.

2.3.2 Packet Size and Packet Interarrival Time

Every client transmits the packets of less than 120 bytes to the server, however server transmits the large packets of 1514 bytes. Since game pace of MMORPG is slower than FPS games, the interarrival times of server and client of *WoW* are longer then *Quake 3*.

Table 8 shows the packet size and packet interarrival time of server and client. When a player doesn't play the games, client sends the small packets of about 54 bytes and server transmits the large packets of more than 100 bytes at intervals of more than 200 ms. When the players do battle with each other, the packet size is the largest in the gameplays. Since server and client transmit more packets to each other in a hunting field, interarrival time is the shortest.

Table 8. Mean / standard deviation of packet size and interarrival time

Gameplay	Size (byte)		Interarrival (ms)	
	Server	Client	Server	Client
No play	130.5 / 54.1	54.8 / 3.2	205.7 / 93.1	212.5 / 72.9
Movement	108.6 / 54.2	70.1 / 17.8	149.3/102.4	159.8 / 104.9
Hunting	109.9 / 56.8	66.5 / 13.7	129.5 / 91.4	137.9 / 98.1
Battle	116.2 / 59.1	64.9 / 13.8	140.9 / 90.1	147.9 / 95.1

3 Network Game Traffic Modeling and Practical Uses

This section presents the traffic models of server and clients for packet size and packet interarrival time. We use empirical traffic data for modeling by the method as follow. We (1) find the available intervals consisted of more than 90 percent in all traffic data, (2) plot Probability Density Function (PDF) of observed data, and (3) estimate parameters by the least square method and represent goodness-of-fit through K-S tests and Q-Q plot for the empirical distribution against the theoretical distribution graphically, then (4) we choose the models. Server traffic means the traffic from server to clients, and client traffic is the opposite of server traffic. We also propose the practical application of these network traffic models.

3.1 Quake 3 Traffic Modeling

By comparing with the empirical data and the theoretical distributions of different kinds, we would like to suggest a lognormal distribution for packet size. The model of server packet size is lognormal(m, s) when the number of clients is n, that is two or more players because network games played with two or more players, m is shape(mean) and *s* is scale(standard deviation). The parameters *m* and *s* are formulated as follow in the available interval [60:20*n*+100]:

$$
\begin{aligned}
m(n) &= 13n + 65, \qquad n \geq 2 \\
s(n) &= 2n + 7, \qquad n \geq 2
\end{aligned}
\tag{1}
$$

In Fig. 3, (a) shows PDF of empirical distribution of server packet size and lognormal in the game of six players, and (c) shows PDF of client packet size according to the number of players. (b) and (d) describe the coincidence with empirical and lognormal distribution. When the number of players is six, we confirm that empirical and lognormal distributions are well matched for the packet size below 220 bytes through Q-Q plot. The client packets of less than 100 bytes are more than 99 percent in all packets, and the client packet size is modeled by lognormal with mean 75 and standard deviation 3 without regard to the number of players.

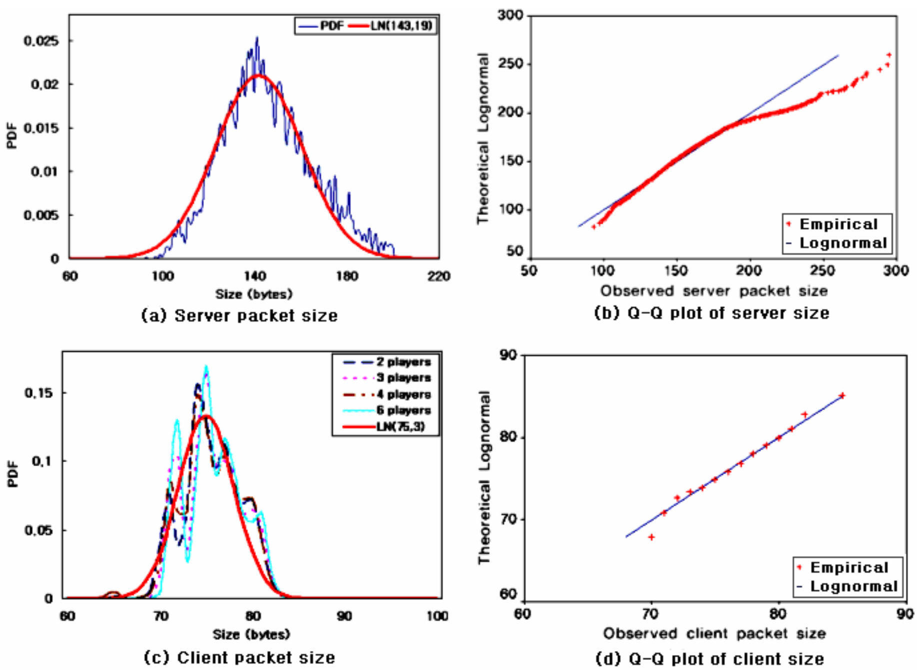

Fig. 3. Traffic models of server packet size for six players and client packet size, and Q-Q plots

Since packet interarrival times of server and clients are independent on the number of players and gameplays, the server model of interarrival time is deterministic at 50 ms. Since interarrival time of clients is similar when there are some user actions in the game, the model of interarrival time is deterministic at 11 ms.

3.2 WoW Traffic Modeling

We find the suitable distribution functions for *WoW* by the methodology for modeling. For selecting the exact models, we use continuous distribution funtions, K-S tests for goodness-of-fit, and Q-Q plot for effective interval. Table 9 shows the selected models of *WoW* about the size and interarrival time by the K-S tests. However the models of the packet size and interarrival time of the games don't exactly correspond to the empirical distributions. Through the Q-Q plots, we conform that empirical data is identical to the model on some intervals or points in fig. 4.

Table 9. Traffic Models of Starcraft and WoW

Size		Interarrival	
Server	Client	Server	Client
Exponential	Normal	Normal	Normal

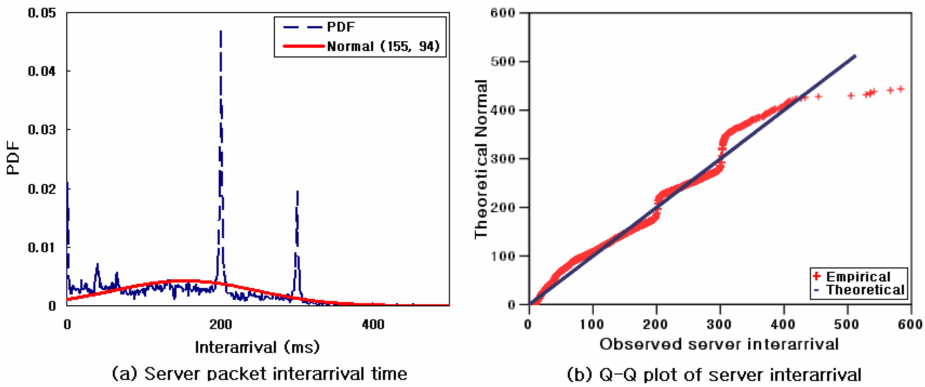

Fig. 4. Traffic models of server interarrival time and Q-Q plot

3.3 Practical Applications of Game Traffic Models

The bandwidth and the latency have to be considered on the design of network games, and the traffic models are preestimated before implementation of the game. Since the consistency of the game state is important in the multi-player network games, and the consistency is related with the interarrival time, when the interarrival period is short, consistency is well maintained but the bandwidth increases. Bandwidth and consistency are in inverse related with the interarrival time.

The game designers have to decide the packet size and interarrival time before game implementation, and game traffic models can limit the number of participants. If 100 players can participate in *Quake 3*, network bandwidth is more than 27.24 Mbps because the server data rate is 21.84 Mbps and the data rate of 100 clients is 5.4 Mbps. On the other case, if bandwidth is limited to 10 Mbps, the number of players has to be limited to 55. Although the network game is designed to different value of size and interarrival time, network traffic can be estimated by using the suggested analysis and models. If server interarrival time is 100 ms and client interarrival time is 50 ms, the total data rate is 78 kbps when the number of players is 4. These traffic models also can be applied to the design of FPS games that use the client-server architecture and UDP protocol.

Traffic analysis related to the user actions can be used for the design of the in-game events and game maps, and the performance evaluation of the games having a lot of events. When the sectors or grids are divided from the world of MMORPG, the size of sectors can be determined by the traffic analysis according to the user actions.

Traffic generator for network game simulation can be implemented by using the traffic analysis of games. The traffic analysis and models of network games can be used for the maintenance such as scalability of game servers, prediction of game traffic and router design. The game traffic models can be also used for evaluation existing networks for their suitability to support various other games. Although the user actions make a slight difference, it can be considered on the design of network game such as the structure of distributed servers.

4 Conclusions and Future Work

In this paper, we analyze the network traffic of *Quake 3* and *WoW*. For measurement and analysis of network game traffic, we make and use *NetGame Sniffer* as a dedicated tool of network games. We characterize the packet size, packet interarrival time, data rate and packet rate related to the number of players and in-game behaviors. The traffic characteristics are different according to the genre and architecture. The packet interarrival time of *Quake 3* is the shortest in the three games because the pace of FPS games is the fastest. Based on the observed traffic characteristics, we develop traffic models for server and clients. The analysis and models of network game traffic can be used for the traffic prediction and network game simulation.

In our traffic models, in-game behaviors are ignored. However game plays have to be considered on the models because user actions affect to the traffic. Since the game genres and architectures are very various in these days, the characteristics of game traffic will be different. Consequently we are planning the experiments about the other genres of network game.

Acknowledgement

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Price of Anarchy of Network Routing Games with Incomplete Information

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Abstract. We consider a class of networks where n agents need to send their traffic from a given source to a given destination over m identical, non-intersecting, and parallel links. For such networks, our interest is in computing the worst case loss in social welfare when a distributed routing scheme is used instead of a centralized one. For this, we use a noncooperative game model with price of anarchy as the index of comparison. Previous work in this area makes the complete information assumption, that is, every agent knows deterministically the amount of traffic injected by every other agent. Our work relaxes this by assuming that the amount of traffic each agent wishes to send is known to the agent itself but not to the rest of the agents; each agent has a belief about the traffic loads of all other agents, expressed in terms of a probability distribution. In this paper, we first set up a model for such network situations; the model is a noncooperative Bayesian game with incomplete information. We study the resulting games using the solution concept of *Bayesian Nash equilibrium* and a representation called the *type agent representation*. We derive an upper bound on price of anarchy for these games, assuming the total expected delay experienced by all the agents as the social cost. It turns out that these bounds are independent of the belief probability distributions of the agents. This fact, in particular, implies that the same bounds must hold for the complete information case, which is vindicated by the existing results in the literature for complete information routing games.

1 Introduction

The motivation for this paper comes from several recent papers (for example, [6], [10], [7], [2], [1]), where the authors use the index *price of anarchy* [9] to measure the worst case loss of network performance when switching from a centralized routing scheme to distributed one. This happens due to selfish behavior of non-cooperative network users when routing of the traffic is done in a distributed fashion. An important implicit assumption made by the authors in all

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these articles, while analyzing the underlying network game model, is that all the users have *complete information* about the game, including information such as how much traffic is being sent through the network by the other users. This assumption severely restricts the applicability of the model to the real world traffic networks. Very few papers in the current literature have explored the model under the incomplete information assumption. A recent paper by Martin Gairing et al [3] considers network routing games with incomplete information which is very similar to ours but they are concerned with developing a polynomial time algorithm for computing a pure Bayesian Nash equilibrium of the underlying game. [3] also gives bounds for the coordination ratio which is similar to our result on price of anarchy. In our paper, the focus is on incompleteness of information available to the users (we also use the synonym agents for users in the rest of the paper). We relax the assumption of complete information by saying that agents do not deterministically know the loads being injected by the other agents, however, having observed the traffic pattern on the network over a sufficiently long time, each agent has a belief (that is, a probability distribution) about the loads of the other agents. In this more realistic scenario, the underlying network routing game becomes a *Bayesian game of incomplete information*. It would be now interesting to study how *incompleteness* of information would affect the performance of routing schemes.

Our main results in this paper show that the bounds on price of anarchy for incomplete information routing games are independent of the belief probability distributions of the agents. This fact, in particular, implies that the same bounds must hold for complete information routing games since they are the special case of incomplete information games. In fact, the bounds we derive match with the known bounds for complete information routing games, thus validating our results.

1.1 Contributions and Outline

In this paper, our main objective is to analyze network routing games with incomplete information. We consider a special class of traffic networks where n agents need to send their traffic from a given source to a given destination over m identical, non-intersecting, and parallel links. To analyze the games that result when we relax the complete information assumption. The sequence in which we progress is as follows.

- *The Model: Bayesian Routing Game* (Section 2): We first develop a Bayesian game model of the routing game with incomplete information. Next, we present an equivalent game of complete information using the *type agent representation* [8].
- *Analysis of the Model: Bayesian Nash Equilibrium* (Section 3): We work with the type agent representation of the Bayesian routing game developed above and characterize the Bayesian-Nash equilibria of the game.
- *Bounds on Price of Anarchy* (Section 4): Next, we define the price of anarchy for Bayesian routing games and compute an upper bound for it. The bound computed turns out to be independent of the belief probability distributions of the agents.

To the best of our knowledge, this is the first time game theoretic analysis and price of anarchy are being investigated in the context of routing games with incomplete information. Our definition of social cost is *total delay experienced by all the agents*, where the delay of an individual agent is the total traffic being assigned to the link on which the agent is transmitting. This definition for social cost is the same as that employed by Roughgarden [10] and Lucking *et al* [7], but different from that employed by Koutsoupias and Papadimitriou [9].

2 Bayesian Routing Games

Consider a network in which there are m identical, non-intersecting, parallel links, $\{L^1, L^2, \ldots, L^m\}$, to carry the traffic from source S to destination D. There are n users (agents), denoted by the set $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$, who need to send traffic from S to D. We assume that the traffic injected by an agent cannot be split across the links. We also assume that each agent A_i can inject any amount of traffic load from a given finite set $\mathcal{W}_i = \{1, 2, \ldots, K\}$. We view the traffic load as an abstract quantity, however, depending upon the context, it can be measured in terms of appropriate units such as Kbps or Mbps. We shall use symbol w_i to denote the actual traffic load generated by A_i. We would like to mention here that our model is static in the sense that w_i denotes the average traffic load of the agent A_i. We assume that before the show starts, the load w_i is the private information of agent A_i and is unknown to the rest of the agents. Sticking to standard phraseology used in the context of incomplete information games (Bayesian games) [8], we prefer to call w_i as the type of agent A_i and hence $\mathcal{W}_i$ becomes the set of all possible types of agent A_i. We use the symbol $\mathcal{W} = \mathcal{W}_1 \times \mathcal{W}_2 \ldots \times \mathcal{W}_n$ to denote the set of type profiles of the agents and $\mathcal{W}_{-i} = \mathcal{W}_1 \times \ldots \times \mathcal{W}_{i-1} \times \mathcal{W}_{i+1} \ldots \times \mathcal{W}_n$ to denote the set of type profiles of agents excluding A_i. We use the symbol w and w_{-i}, respectively to denote an element of the sets $\mathcal{W}$ and $\mathcal{W}_{-i}$, respectively. $\triangle\mathcal{W}$ is the set all probability distributions over $\mathcal{W}$ and similarly $\triangle\mathcal{W}_{-i}$ is the set of all probability distributions over $\mathcal{W}_{-i}$. We also assume that each agent A_i has a belief function $p_i : \mathcal{W}_i \mapsto \triangle\mathcal{W}_{-i}$. That is, for any possible type w_i of the agent A_i, the belief function specifies the probability distribution over the set $\mathcal{W}_{-i}$, representing what agent A_i would believe about the other agents' type if its own type were w_i. The beliefs $(p_i)_{\mathcal{A}}$ are said to be *consistent* [8] iff there is some common prior distribution $P \in \triangle\mathcal{W}$ over the set of type profiles w such that each agent's belief given its type is just the conditional probability distribution that can be computed from the prior distribution by Bayes formula in following manner.

$$p_i(w_{-i}\,|w_i) = \frac{P(w_{-i}, w_i)}{\sum\limits_{s_{-i} \in \mathcal{W}_{-i}} P(s_{-i}, w_i)};\ \forall A_i \in \mathcal{A}$$

In this paper, we will stick to this assumption of consistent beliefs. Under this assumption, the probability that agent A_i's type is w_i can be given by $t_i(w_i) = \sum_{w_{-i} \in \mathcal{W}_{-i}} P(w_{-i}, w_i)$. Now consider the following problem with regard to this network. The agents are rational, selfish, and non-cooperative, and are left free to route their traffic through the network. Each agent, knowing the fact that the other agents are also doing likewise, *independently* tries to compute a strategy for routing its traffic that yields minimum expected delay. The decision problem of each agent is to choose the best link for sending its traffic through the network. We define the set of all the links as a pure strategy set of any agent A_i and denote it by $\mathcal{L}_i$. We shall be using symbol l_i to denote a particular pure strategy of agent A_i. We also define a mixed strategy for agent A_i as any valid probability distribution over the set of pure strategies. We use the symbol $\mathcal{T}_i$ to denote the set of all the mixed strategies of agent A_i, and the symbol τ_i to denote a particular mixed strategy of agent A_i, that is, $\mathcal{T}_i = \triangle\mathcal{L}_i$, and $\tau_i \in \mathcal{T}_i$. We use the symbol $\mathcal{L} = \mathcal{L}_1 \times \mathcal{L}_2 \ldots \times \mathcal{L}_n$ to denote the set of pure strategy profiles of the agents and $\mathcal{T} = \mathcal{T}_1 \times \mathcal{T}_2 \ldots \times \mathcal{T}_n$ to denote the set of mixed strategy profiles of the agents. We can define $\mathcal{L}_{-i}$ and $\mathcal{T}_{-i}$ in a similar way as earlier. We use the corresponding lowercase letters to denote individual elements of the above sets.

The payoff to an agents here is the delay experienced by the agent, which is defined as the total traffic passing through the link on which its own traffic is running. Thus, we define $u_i : \mathcal{L} \times \mathcal{W} \mapsto \Re$ as a payoff function for agent A_i, where $u_i(l, w) = \sum_{j:l_j=l_i} w_j$ is the payoff to agent A_i under the strategy profile l and traffic profile w and $u_i(\tau, w) = \sum_{l\in\mathcal{L}} \left(\prod_{A_i\in\mathcal{A}} \tau_i(l_i) \right) u_i(l, w)$ is the payoff to agent A_i under the strategy profile τ and traffic profile w. This model describes the following Bayesian game

$$\Gamma^b = \left\{(\mathcal{A}), (\mathcal{L}_i)_{A_i\in\mathcal{A}}, (\mathcal{W}_i)_{A_i\in\mathcal{A}}, (p_i)_{A_i\in\mathcal{A}}, (u_i)_{A_i\in\mathcal{A}}\right\} \tag{1}$$

Harsanyi [5], proposed a game, called the *Selten Game* to represent such games in normal form of an augmented complete information game. Myerson [8] calls such a representation *type-agent representation.*

2.1 Type Agent Representation for Bayesian Routing Games

In type-agent representation, it is assumed that there is one virtual player (or agent) for every possible type of every player (or agent) in the given Bayesian game and thus the set of agents gets augmented. In order to differentiate these agents from the actual agents (i.e. network users), we prefer to call them as *traffic agents.* Thus, for the Bayesian game Γ^b given by (1), the set of agents in the type-agent representation becomes $\mathcal{A}^t = \bigcup_{i\in\mathcal{A}} \mathcal{A}_i^t$, where $\mathcal{A}_i^t = \{A_{i1}, A_{i2}, \ldots, A_{iK}\}$ represents the set of traffic agents for agent A_i.

The pure strategy and mixed strategy sets of traffic agent A_{ij} are the same as the pure and mixed strategies set of agent A_i. That is, $\mathcal{L}_{ij} = \mathcal{L}_i$ and $\mathcal{T}_{ij} = \mathcal{T}_i$. We shall use symbols l_{ij} and τ_{ij} to denote a particular pure and mixed strategy respectively for the traffic agent A_{ij}. We use the symbol $\mathcal{L}^t = \mathcal{L}_{11} \times \mathcal{L}_{12} \ldots \times \mathcal{L}_{nK}$ to denote the set of pure strategy profiles of traffic agents and $\mathcal{T}^t = \mathcal{T}_{11} \times \mathcal{T}_{12} \times \ldots \times \mathcal{T}_{nK}$ to denote the set of mixed strategy profiles of traffic agents. Once again, $\mathcal{L}^t_{-ij}$, $\mathcal{L}^t_{-i}$, $\mathcal{T}^t_{-ij}$, and $\mathcal{T}^t_{-i}$ have their usual interpretations. Also, we shall use lowercase letters to denote individual elements of the above sets. Two other quantities that are of use are $(l^t|w)$ and $(\tau^t|w)$. The first one represents the pure strategy profile of the agents for a given pure strategy profile of the traffic agents and a given type profile of the agents. The second quantity is a mixed strategy counterpart of the first one. That is, $(l^t|w) = (l_{1w_1}, l_{2w_2}, \ldots l_{nw_n})$ and $(\tau^t|w) = (\tau_{1w_1}, \tau_{1w_2}, \ldots \tau_{nw_n})$, where $w = (w_1, \ldots, w_n)$. In the type agent representation, the payoff to any traffic agent A_{ij} is defined to be the conditionally expected payoff to agent A_i in Γ^b given that j is A_i's actual type. Formally, for any agent A_i in $\mathcal{A}$ and any type w_i, the payoff function $v_{iw_i} : \mathcal{L}^t \mapsto \Re$ in the type-agent representation is defined in the following way.

$$v_{iw_i}(l^t) = \sum_{w_{-i} \in \mathcal{W}_{-i}} p_i\left(w_{-i}|w_i\right) u_i\left((l^t|w), (w_{-i}, w_i)\right) \tag{2}$$

where $w = (w_1, \ldots, w_i, \ldots, w_n) = (w_{-i}, w_i)$. Similarly, for the mixed strategy case, the payoff is given by the following equation.

$$v_{iw_i}(\tau^t) = \sum_{l^t \in \mathcal{L}^t} \left(\prod_{A_{pq} \in \mathcal{A}^t} \tau_{pq}(l_{pq}) \right) v_{iw_i}(l^t) \tag{3}$$

Substituting the value of equation (2) in equation (3) leads to the following alternative form of $v_{iw_i}(\tau^t)$:

$$v_{iw_i}(\tau^t) = \sum_{w_{-i} \in \mathcal{W}_{-i}} p_i\left(w_{-i}|w_i\right) u_i\left(\tau^t|w, w\right) \tag{4}$$

With these definitions, the type-agent representation

$$\Gamma = \left\{ (\mathcal{A}^t), (\mathcal{L}_{ij})_{A_{ij} \in \mathcal{A}^t}, (v_{ij}(.))_{A_{ij} \in \mathcal{A}^t} \right\}$$

is indeed a complete information game in strategic form and may be viewed as a representation of the given Bayesian game.

2.2 Payoffs to Agents

Before moving to the next section, we would like to define an important quantity, namely, payoff to agent A_i in the incomplete information game. The following relations give the expected payoff to agent A_i when the pure strategy and mixed strategy profile of the traffic agents are l^t and τ^t, respectively.

$$u_i(l^t) = \sum_{w \in \mathcal{W}} P(w) u_i(l^t|w, w)$$

$$u_i(\tau^t) = \sum_{l^t \in \mathcal{L}^t} \left(\prod_{A_{pq} \in \mathcal{A}^t} \tau_{pq}(l_{pq}) \right) u_i(l^t) = \sum_{w \in \mathcal{W}} P(w) u_i(\tau^t|w, w) \tag{5}$$

The second expression for $u_i(\tau^t)$ in equation (5) can be obtained by substituting the value of $u_i(l^t)$ in the first expression of $u_i(\tau^t)$. Also, by making use of equations (2) and (4), it is very simple to get the following alternative expressions for $u_i(l^t)$ and $u_i(\tau^t)$.

$$u_i(l^t) = \sum_{w_i \in \mathcal{W}_i} t_i(w_i) v_{iw_i}(l^t); \quad u_i(\tau^t) = \sum_{w_i \in \mathcal{W}_i} t_i(w_i) v_{iw_i}(\tau^t) \tag{6}$$

This completes the definition of the type agent representation for Bayesian routing games.

3 Nash Equilibria for Bayesian Routing Games

We now carry out a game theoretic analysis of the Bayesian routing game using the solution concept of Bayesian Nash equilibrium. First we consider the Bayesian game Γ^b with a fixed type profile of the agent, that is $\mathcal{W}_i = \{w_i\}$ is a singleton set. In this situation, the game reduces to a game of complete information because the Bayesian form Γ^b and type agent representation Γ are essentially the same as $\mathcal{W}_i$ and p_i are now redundant and no more required in the Bayesian form Γ^b. Let $w = (w_1, w_2, \ldots, w_n)$ be a given traffic profile of the agents. If $\tau = (\tau_1, \ldots, \tau_i, \ldots, \tau_n) = (\tau_i, \tau_{-i})$ is a mixed strategy Nash equilibrium for this game, it can be shown that [4]:

$$\begin{aligned} u_i(\tau, w) &= M^j(\tau, w) + w_i \left(1 - \tau_i(L^j)\right) \text{ if } L^j \in S_i \\ u_i(\tau, w) &\le M^j(\tau, w) + w_i \left(1 - \tau_i(L^j)\right) \text{ if } L^j \notin S_i \end{aligned}$$

where $M^j(\tau, w)$ is the expected traffic arising on link L^j if all the agents send their traffic according to the strategy profile τ. Summing up the $u_i(\tau, w)$ values over all the m links and making use of the fact that $\sum_j M^j(\tau, w) = \sum_i w_i$, it is easy to get the following upper bound on expected payoff of agent A_i in the case of Nash equilibrium.

$$u_i(\tau, w) \le \frac{1}{m} \left\{ (m-1) w_i + \sum_i w_i \right\} \tag{7}$$

Let Γ have a Nash equilibrium τ^t with support $S^t = S_{11} \times S_{12} \times \ldots \times S_{nK}$, where $S_{ij} \subset \mathcal{L}_{ij}$ (see [8] for more details on support), then for each traffic agent A_{iw_i}, there must exist η_{iw_i} such that

$$\eta_{iw_i} = v_{iw_i}((L^j, \tau^t_{-iw_i}))\, \forall L^j \in S_i w_i; \;\; \eta_{iw_i} \le v_{iw_i}((L^j, \tau^t_{-iw_i}))\, \forall L^j \notin S_i w_i \tag{8}$$

Let Γ have a Nash equilibrium τ^t with support $S^t = S_{11} \times S_{12} \times \ldots \times S_{nK}$, where $S_{ij} \subset \mathcal{L}_{ij}$, then for each traffic agent A_{iw_i}, there must exist η_{iw_i} such that

$$\eta_{iw_i} = v_{iw_i}((L^j, \tau^t_{-iw_i}))\, \forall L^j \in S_i w_i; \; \eta_{iw_i} \le v_{iw_i}((L^j, \tau^t_{-iw_i}))\, \forall L^j \notin S_i w_i \tag{9}$$

$$\sum_{L^j \in S_{iw_i}} \tau_{iw_i}(L^j) = 1 \tag{10}$$

$$\tau_{iw_i}(L^j) = 0\, \forall L^j \notin S_{iw_i}\, ; \tau_{iw_i}(L^j) \ge 0\, \forall L^j \in S_{iw_i} \tag{11}$$

It is easy to show that η_{iw_i} that we obtain by solving the above system of equations is indeed the expected payoff of traffic agent A_{iw_i} under τ^t. Further, we can derive the following expression for the expected payoff of the agent A_i under Bayesian Nash equilibrium τ^t.

$$\sum_{w_i \in \mathcal{W}_i} t_i(w_i) u_i(\tau^t) = \sum_{w_i \in \mathcal{W}_i} t_i(w_i) v_{iw_i}(\tau^t) = \sum_{w_i \in \mathcal{W}_i} t_i(w_i) \eta_{iw_i}$$

The above relation reduces to the following relations (see [4] for details):

If $L^j \in S_{iw_i} \; \forall w_i \in \mathcal{W}_i$ then

$$u_i(\tau^t) = \sum_{w_i} t_i(w_i) \left\{ w_i + \sum_{w_{-i}} p_i(w_{-i}|w_i) M^j((\tau^t|w)_{-i}, w_{-i}) \right\}$$

otherwise

$$u_i(\tau^t) \leq \sum_{w_i} t_i(w_i) \left\{ w_i + \sum_{w_{-i}} p_i(w_{-i}|w_i) M^j((\tau^t|w)_{-i}, w_{-i}) \right\}$$

where $M^j(.)$ has its usual interpretation. We can also derive the following alternative form of the above expressions (see [4]for details):

If $L^j \in S_{iw_i} \; \forall w_i \in \mathcal{W}_i$ then

$$u_i(\tau^t) = \sum_{w_i} t_i(w_i) w_i + \sum_{w} P(w) \left\{ M^j((\tau^t|w), w) - \tau_{iw_i}(L^j) w_i \right\} \tag{12}$$

Otherwise

$$u_i(\tau^t) \leq \sum_{w_i} t_i(w_i) w_i + \sum_{w} P(w) \left\{ M^j((\tau^t|w), w) - \tau_{iw_i}(L^j) w_i \right\} \tag{13}$$

If we sum up $u_i(\tau^t)$ over all the links and make use of the above relations then it is easy to get the following bound:

$$u_i(\tau^t) \leq \frac{1}{m} \left\{ (m-1) \sum_{w_i} t_i(w_i) w_i + \sum_{w} P(w) \sum_{i} w_i \right\} \tag{14}$$

Note the similarity between the above expression and expression (7).

4 Price of Anarchy of Bayesian Routing Games

In this section, we develop a notion of price of anarchy for Bayesian routing games and derive an upper bound for it. For the sake of convenience and self-sufficiency, we discuss the case of complete information routing games.

4.1 Price of Anarchy for Complete Information Routing Games

Consider the complete information routing game $\Gamma^c = \{\mathcal{A}, (\mathcal{L}_i)_{\mathcal{A}}, (u_i(.))_{\mathcal{A}}\}$. For this game, we define the following quantities.

$$S(\tau) = \text{Social cost under mixed strategy profile } \tau = \sum_i u_i(\tau, w)$$
$$\underline{S} = \text{Optimal social cost} = \min_{\tau} S(\tau)$$
$$\underline{S}^* = \text{Social cost under the best Nash equilibrium} = \min_{\tau:\tau \text{ is a NE}} \{S(\tau)\}$$
$$\overline{S^*} = \text{Social cost under the worst Nash equilibrium} = \max_{\tau:\tau \text{ is a NE}} \{S(\tau)\}$$
$$\overline{S} = \max_{\tau} S(\tau)$$

The following inequality is a trivial consequence of the above definitions.

$$\underline{S} \leq \underline{S}^* \leq \overline{S^*} \leq \overline{S}$$

It is straightforward to see that under the centralized routing scheme, the central authority would route the agents' traffic in way that yields the social cost $\underline{S}$. However, under a distributed routing scheme, the agents would like to route their traffic in a way suggested by a Nash equilibrium strategy profile. Therefore, the worst possible scenario that may arise out of distributed routing scheme is to have social cost equal to $\overline{S^*}$. Thus, the following ratio ϕ, called as *price of anarchy for the game* Γ^c, measures the worst case loss in social welfare because of switching from centralized routing scheme to distributed routing scheme.

$$\phi = \frac{\overline{S^*}}{\underline{S}} = \frac{\text{Social cost under the worst Nash equilibrium}}{\text{Optimal social cost}}$$

In what follows we compute an upper bound for the above ratio. By making use of the upper bound for $u_i(\tau, w)$, given in (7), it is easy to show that

$$\text{If } \; m = 1 \; \text{ then } \; \overline{S^*} = \underline{S} = n \sum_i w_i \; \text{ else } \; \overline{S^*} \leq \frac{n + (m-1)}{m} \sum_i w_i \; ; \; \; \underline{S} \geq \sum_i w_i$$

The above relations result in the following theorem about bounds on price of anarchy for complete information routing games:

Theorem 1. For a complete information routing game with m identical parallel links and n users, the price of anarchy ϕ can be bounded in the following way:

$$\text{If } \; m = 1 \; \text{ then } \; \phi = 1 \quad \text{else } \; 1 \leq \phi \leq \left\{ \frac{n + (m-1)}{m} \right\}$$

Remarks:

- These bounds are not tight for the case when $1 < m$. However, for a given value of m, one can find a better approximation of $\underline{S}$ and get a tighter bound.
- Another interesting case is when $m = n$. For this case, it is easy to see that $1 \leq \phi \leq 2 - \frac{1}{m}$.
- For $m = (n-1)$, $1 \leq \phi \leq 2$.

4.2 Price of Anarchy for Bayesian Routing Games

Now we consider the Bayesian game Γ^b and define the price of anarchy for it. With regard to the type-agent representation Γ of the Bayesian game Γ^b, we define $S(\tau^t) = \sum_i u_i(\tau^t)$ to be the social cost under mixed strategy profile τ^t. The quantities $\underline{S}, \underline{S}^*, \overline{S^*}$, and $\overline{S}$ have their usual meaning. The following inequality is a direct consequence of the above definitions:

$$\underline{S} \leq \underline{S}^* \leq \overline{S^*} \leq \overline{S}$$

The price of anarchy ψ for the Bayesian game Γ^b is defined by the ratio:

$$\psi = \frac{\overline{S^*}}{\underline{S}} = \frac{\text{Social cost under the worst BN equilibrium}}{\text{Optimal social cost}}$$

In what follows, we compute an upper bound on ψ. By making use of the upper bound for $u_i(\tau^t)$, given in (14), it is easy to show that

$$\text{If } m = 1 \text{ then } \overline{S^*} = \underline{S} = n \sum_w P(w) \sum_i w_i \text{ , otherwise}$$

$$\overline{S^*} \leq \frac{(m-1) \sum_{i=1}^{n} \sum_{w_i \in \mathcal{W}_i} t_i(w_i) w_i + n \sum_w P(w) \sum_i w_i}{m} \; ; \underline{S} \geq \sum_i^n \sum_{w_i \in \mathcal{W}_i} t_i(w_i) w_i$$

We have made use of equation (12) to bound $\underline{S}$ from below when $1 < m$. The above relation results in the following bounds on price of anarchy for incomplete information routing games.

$$\text{If } m = 1 \text{ then } \psi = 1, \text{otherwise } 1 \leq \psi \leq \frac{1}{m} \left\{ (m-1) + n \frac{\sum_w P(w) \sum_i w_i}{\sum_i^n \sum_{w_i \in \mathcal{W}_i} t_i(w_i) w_i} \right\}$$

A little algebra shows that $\sum_w P(w) \sum_i w_i = \sum_i^n \sum_{w_i \in \mathcal{W}_i} t_i(w_i) w_i$. The following theorem captures the bounds on the price of anarchy for the incomplete information case.

Theorem 2. For an incomplete information routing game with m identical parallel links and n users, the price of anarchy ψ is bounded by

$$\text{If } \; m = 1 \; \text{ then } \; \psi = 1 \quad \text{ else } \; 1 \leq \psi \leq \left\{ \frac{n + (m-1)}{m} \right\}$$

Remarks:

– Note that the bounds on price of anarchy are the independent of the belief probability distributions p_i of the agents and hence the same bounds hold for complete information case as well.

– These bounds are not tight for the case when $1 < m$. However, for a given value of m, one can find a better approximation of $\underline{S}$ and get a tighter bound.
– For $m = n$, $1 \leq \psi \leq 2 - \frac{1}{m}$
– For $m = (n - 1)$, $1 \leq \psi \leq 2$.

5 Conclusions

In this paper, we have extended game theoretic analysis to network routing games with incomplete information. We have derived an upper bound on price of anarchy for such games. The results show that the bound is independent of the belief probability distributions of the agents. We believe our work is a good first step in answering the following more general question: how does distributed routing affect (improve or degrade) the actual performance when agents have probabilistic rather than deterministic information about other agents? Further investigation of this question will lead to several interesting directions for future work: (1) tighter bounds on price of anarchy; (2) price of anarchy with other metrics such as average delay, throughput, etc. (3) more general routing situations; and (4) mechanism design for evolving better routing protocols for such networks.

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General Equilibrium for Economies with Harmful Overconsumption

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Abstract. This paper studies general equilibrium models for economies with overconsumption goods. We consider two kinds of good: usual commodity (for usual consumption) and harmful overconsumption commodity. The consumption of the usual commodity always increases an agent's utility. But for the harmful overconsumption commodity, once the consumption reaches a critical point, there is disutility of consuming more. Overconsumption of this commodity is harmful. The utility function is no longer monotonically increasing when overconsumption happens. The existence of general equilibrium is solved from truncated economies and Arrow-Debreu economies with usual utility function. We provide a few examples to show general equilibria for various cases.

1 Introduction

The literature of general equilibrium considers economies with usual commodities. An agent's welfare increases when she consumes more such commodities. However, some commodities, if consumed more than one should, can be harmful. For example, taking too much meat can cause accumulation of fat inside the body and thus is harmful to one's health; too much work out is a waste of time and can even damage one's muscle. We call this type of good "overconsumption commodity". Agents' welfare decreases when they consume this commodity more than a threshold. This paper studies general equilibrium for economies with harmful overconsumption commodities. For the economy with two commodities and two agents, if there are too many endowments for overconsumption commodity, the equilibrium consumptions of the this commodity are the thresholds.

Existing general equilibrium literature focuses on usual commodities. Debreu (1959) gives an axiomatic analysis of economic equilibrium. Arrow and Debreu (1954), Debreu (1959), Debreu (1982) and so on prove the existence of general equilibrium if the preference relations satisfy standard conditions (and the utility functions are increasing and concave). However, overconsumption of some commodities may be a waste or even harmful. In this paper, we introduce wasteful overconsumption and, in particular, harmful overconsumption to the traditional general equilibrium model.

For harmful overconsumption commodities, first, we cannot claim that the more consumption of this kind of good the better for the agent. There exists a

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threshold (critical point) for the harmful overconsumption commodities. When the commodities are consumed less than the threshold, the agent's satisfaction increase with more consumption. However, when the commodities are consumed more than the threshold the agent's satisfaction decrease with more consumption. This suggests that the agent's consumptions cannot exceed the threshold for the harmful overconsumption commodities. Second, the agent's satisfaction is described by their preference relations (and utility functions). When the agent consumes less than the threshold, her utility function is increasing; but when she consumes more than the threshold her utility function is decreasing. That is, the utility function increases before the threshold and then decreases.

In this paper we assume there are two goods: one usual commodity (for usual consumption) and one harmful overconsumption commodity. The utility function is increasing for the usual commodity. For the harmful overconsumption commodity, the utility function is non-monotonic: increasing before the threshold and deceasing after the threshold. We define three types of general equilibrium. The first one is a key definition of general equilibrium for economies with harmful overconsumption good, where the utility functions are concave, increasing and then decreasing after the threshold. The second one is for economies with wasteful overconsumption good where the truncated utility functions are concave, increasing and then constant at the threshold. The third one is for economies with concave and strictly increasing utility functions, as in the traditional literature. We show that the equilibrium consumption levels for overconsumption commodity are the same for models with harmful and wasteful overconsumption. Moreover, we can find the equilibria for the first and second type of model by taking limit of the third model. We find that when the total endowments for the harmful and wasteful overconsumption commodity in the economy is less than the sum of the thresholds for the agents, each agent consumes less than her threshold. But if the total endowments is more than the sum of the thresholds, this commodity is free and thus each agent consumes at her threshold. We then use a few examples to illustrate the approach to solve for the key equilibrium with harmful overconsumption good.

To our knowledge, this is the first paper to study overconsumption commodities in the general equilibrium literature. Even though some papers (Wachtel, 1998; Brown and Cameron, 2000; Comin, 2000; Clapp, 2002; de-Geus, 2003; Dupor and Liu, 2003) use the term "overconsumption", their definition of overconsumption, to our awareness, is much different than ours. Their overconsumption means more consumption than the optimal level. But the utility functions in their papers are strictly increasing. We define overconsumption so that the utility function is non-monotonic. We then study the optimal consumption.

Section 2 presents three definitions of general equilibrium with overconsumption commodities. Section 3 shows the relations of the three equilibria. The first two equilibria are virtually identical, which can be obtained from the third equilibrium. We provide four simple examples in Section 4 for the three equilibria. Concluding remarks are in Section 5.

2 Economic Models

We consider exchange economies with two commodities: a usual one and an overconsumption one. We use X to denote the usual commodity and Y to denote the overconsumption commodity. There are I agents, indexed by $i = 1, \cdots, I$. Agent i's initial endowments are E_i and F_i for goods X and Y, with prices P and Q, respectively. Then her income is $J_i = PE_i + QF_i$. Her budget constraint is $PX_i + QY_i \leq J_i$,

$$P(X_i - E_i) + Q(Y_i - F_i) \leq 0. \tag{1}$$

The utility functions are given by $U_i(X, Y)$, which is increasing for the usual commodity X. The other properties of the utility function is discussed in the following three subsections.

2.1 Model 1 with Harmful Overconsumption Goods

The first model is our main model where the consumption of Y exceeding the threshold causes welfare loss. The utility function $U_i(X, Y)$ is increasing then decreasing for harmful overconsumption commodity Y. We assume that the utility functions are smooth and concave. Then for each overconsumption commodity Y, $\frac{\partial U_i(X,Y)}{\partial X} > 0$. There is a maximum consumption y_i for Y, which is called the threshold for agent i. Then for each usual commodity X,

$$\frac{\partial U_i(X,Y)}{\partial Y} \begin{cases} > 0, & \text{if } Y < y_i \\ = 0, & \text{if } Y = y_i \\ < 0, & \text{if } Y > y_i, \end{cases}$$

$$\frac{\partial^2 U_i(X,Y)}{\partial X^2} < 0, \qquad \frac{\partial^2 U_i(X,Y)}{\partial Y^2} < 0,$$

and

$$\frac{\partial^2 U_i(X,Y)}{\partial X^2}\frac{\partial^2 U_i(X,Y)}{\partial Y^2} - \left[\frac{\partial^2 U_i(X,Y)}{\partial X \partial Y}\right]^2 > 0.$$

We define the following general equilibrium.

Definition 1. *A general equilibrium is charactered by a price pair and a consumption system* $((\bar{P}, \bar{Q}), (\bar{X}_i, \bar{Y}_i), i = 1, \cdots, I)$ *such that the following two conditions hold:*

1. *For each* $i = 1, \cdots, I$, $(\bar{X}_i, \bar{Y}_i)$ *solves the utility optimization problem subject to budget constraint (1), given* $(\bar{P}, \bar{Q})$,

$$\begin{aligned} &\max U_i(X_i, Y_i) \\ &\text{s.t. } \bar{P}(X_i - E_i) + \bar{Q}(Y_i - F_i) \leq 0. \end{aligned}$$

2. *Market clearing conditions hold*

$$\sum_{i=1}^{I}(\bar{X}_i - E_i) = 0 \qquad \text{and} \qquad \sum_{i=1}^{I}(\bar{Y}_i - F_i) \leq 0.$$

Since the agents will not consume the harmful overconsumption commodity Y more than the threshold y_i, equilibrium consumption for Y satisfies $\bar{Y}_i \leq y_i$. Thus it is possible that inequality in budget constraints strictly holds, $\bar{P}(\bar{X}_i - E_i) + \bar{Q}(\bar{Y}_i - F_i) < 0$, and excess demand of the harmful overconsumption commodity is strictly less than zero, $\sum_{i=1}^{I}(\bar{Y}_i - F_i) < 0$.

In this paper we consider a special utility function where the two goods are separable[1] with the form

$$U_i(X_i, Y_i) = \alpha_i v_i(X_i) + V_i(Y_i), \qquad i = 1, \cdots, I \tag{2}$$

where $v_i(X)$ is strictly increasing and strictly concave and $V_i(Y)$ is increasing then decreasing and strictly concave with a maximum point y_i. That is, $v_i'(X) > 0$, $v_i''(X) < 0$,

$$V_i'(Y) \begin{cases} > 0, & \text{if } Y < y_i \\ = 0, & \text{if } Y = y_i \\ < 0, & \text{if } Y > y_i, \end{cases}$$

and $V_i''(Y) < 0$.

2.2 Model 2 with Wasteful Overconsumption Goods

In order to obtain the general equilibrium for Model 1, we introduce Model 2 where the consumption of good Y exceeding the threshold does not change the welfare. That is, the utility function remains constant once the consumption of Y reaches the threshold.

We first truncate the utility function $V_i(Y)$ at the maximum utility value as $V_{0i}(Y) = V_i(Y)1_{\{Y<y_i\}}(Y) + V_i(y_i)1_{\{y_i \leq Y\}}(Y)$, that is,

$$V_{0i}(Y) = \begin{cases} V_i(Y), & \text{if } Y < y_i \\ V_i(y_i), & \text{if } y_i \leq Y. \end{cases}$$

Then $V_{0i}(Y)$ is increasing and concave, which reaches the maximum value (at the threshold y_i) and then remains constant. That is, when $Y < y_i$, $V_{0i}'(Y) > 0$ and $V_{0i}''(Y) < 0$; when $y_i \leq Y$, $V_{0i}(Y) = V_{0i}(y_i)$ and $V_{0i}'(Y) = 0$. Thus the truncated utility function is

$$U_{0i}(X_i, Y_i) = \alpha_i v_i(X_i) + V_{0i}(Y_i), \qquad i = 1, \cdots, I. \tag{3}$$

Definition 2. *A general equilibrium is charactered by a price pair and a consumption system* $((\bar{P}_0, \bar{Q}_0),\ (\bar{X}_{0i}, \bar{Y}_{0i}), i = 1, \cdots, I)$ *such that the following two conditions hold.*

1. *For each* $i = 1, \cdots, I$, $(\bar{X}_{0i}, \bar{Y}_{0i})$ *solves the utility optimization problem subject to budget constraint (1), given* $(\bar{P}_0, \bar{Q}_0)$,

$$\begin{aligned} &\max U_{0i}(X_i, Y_i) \\ &\text{s.t. } \bar{P}_0(X_i - E_i) + \bar{Q}_0(Y_i - F_i) \leq 0 \end{aligned}$$

[1] We can study general case. But it is complicated to construct the utilities in Subsections 2.2 and 2.3.

2. Market clearing conditions hold.

$$\sum_{i=1}^{I}(\bar{X}_{0i} - E_i) = 0 \qquad \text{and} \qquad \sum_{i=1}^{I}(\bar{Y}_{0i} - F_i) \le 0$$

If agent i consumes the wasteful overconsumption commodity more than y_i, then she wastes her income without changing her utility level. Therefore the equilibrium consumption for the wasteful overconsumption commodity satisfies $\bar{Y}_i \le y_i$.

2.3 Model 3 with Usual Goods

Although utility functions in Model 2 are monotonically increasing and concave, we cannot solve the model directly. To solve it, we can approach Model 2 by Model 3 where all commodities are usual. The economy in Model 3 is in the standard framework of Arrow and Debreu (1954), Debreu (1959), and Debreu (1982). We construct traditional Arrow-Debreu economies with usual utility functions. We modify V_{0i} to a strictly increasing and strictly concave utility function $V_{\varepsilon i}(Y)$ where $V_{\varepsilon i}(Y) = V_i(Y)1_{\{Y<y_i-\varepsilon\}}(Y) + V_{\varepsilon i}^0(Y)1_{\{y_i-\varepsilon\le Y\}}(Y)$, that is,

$$V_{\varepsilon i}(Y) = \begin{cases} V_i(Y), & \text{if } \ Y < y_i - \varepsilon \\ V_{\varepsilon i}^0(Y), & \text{if } \ y_i - \varepsilon \le Y \end{cases}$$

where the form of the function $V_{\varepsilon i}^0(Y)$ can be arbitrarily chosen. For example, we can take the function $V_{\varepsilon i}^0(Y)$ of the form

$$V_{\varepsilon i}^0(Y) = A_i v_i(Y) + B_i, \qquad y_i - \varepsilon \le Y$$

such that the following two conditions hold

1. $V_i(y_i - \varepsilon) = V_{\varepsilon i}^0(y_i - \varepsilon) = A_i v_i(y_i - \varepsilon) + B_i$;
2. $V_i'(y_i - \varepsilon) = V_{\varepsilon i}^{0\prime}(y_i - \varepsilon) = A_i v_i'(y_i - \varepsilon)$.

Then

$$A_i = \frac{V_i'(y_i - \varepsilon)}{v_i'(y_i - \varepsilon)} \qquad \text{and} \qquad B_i = V_i(y_i - \varepsilon) - \frac{V_i'(y_i - \varepsilon)}{v_i'(y_i - \varepsilon)} v_i(y_i - \varepsilon)$$

and

$$V_{\varepsilon i}^0(Y) = \frac{V_i'(y_i - \varepsilon)}{v_i'(y_i - \varepsilon)}[v_i(Y) - v_i(y_i - \varepsilon)] + V_i(y_i - \varepsilon), \qquad y_i - \varepsilon \le Y.$$

Hence

$$V_{\varepsilon i}(Y) = \begin{cases} V_i(Y), & \text{if } \ Y < y_i - \varepsilon \\ \frac{V_i'(y_i-\varepsilon)}{v_i'(y_i-\varepsilon)}[v_i(Y) - v_i(y_i - \varepsilon)] + V_i(y_i - \varepsilon), & \text{if } \ y_i - \varepsilon \le Y \end{cases}$$

Thus the traditional utility function is

$$U_{\varepsilon i}(X_i, Y_i) = \alpha_i v_i(X_i) + V_{\varepsilon i}(Y_i), \qquad i = 1, \cdots, I \tag{4}$$

Definition 3. *A general equilibrium is charactered by a price pair and a consumption system* $((\bar{P}_\varepsilon, \bar{Q}_\varepsilon),\ (\bar{X}_{\varepsilon i}, \bar{Y}_{\varepsilon i}), i = 1, \cdots, I)$ *such that the following two conditions hold.*

1. *For each* $i = 1, \cdots, I$, $(\bar{X}_{\varepsilon i}, \bar{Y}_{\varepsilon i})$ *solves the utility optimization problem subject to budget constraint, given* $(\bar{P}_\varepsilon, \bar{Q}_\varepsilon)$,

$$\begin{aligned} &\max U_{\varepsilon i}(X_i, Y_i) \\ &s.t.\ \ \bar{P}_\varepsilon(X_i - E_i) + \bar{Q}_\varepsilon(Y_i - F_i) \leq 0 \end{aligned}$$

2. *Markets clear.*

$$\sum_{i=1}^{I}(\bar{X}_{\varepsilon i} - E_i) = 0 \qquad \text{and} \qquad \sum_{i=1}^{I}(\bar{Y}_{\varepsilon i} - F_i) = 0$$

The utility function $V_{\varepsilon i}(Y)$ is strictly increasing and strictly concave for $\varepsilon > 0$, then there exists a general equilibrium for this economy. For an arbitrarily small positive scalar $\varepsilon > 0$, the budget constraints $\bar{P}_\varepsilon(X_{\varepsilon i} - E_i) + \bar{Q}_\varepsilon(Y_{\varepsilon i} - F_i) = 0$ for $i = 1, 2$ are binding in the equilibrium.

3 The Relation of 3 Equilibria in Section 2

From the construction of truncated and traditional utility functions in Section 2, the utility functions in Model 2 are the limits of the utility functions in Model 3 as ε approaches to zero,

$$\lim_{\varepsilon \to 0} V_{\varepsilon i}(Y) = V_{0i}(Y) \qquad \text{and} \qquad \lim_{\varepsilon \to 0} U_{\varepsilon i}(X, Y) = U_{0i}(X, Y).$$

Then the equilibrium in Model 2 is the limit of the equilibrium in Model 3,

$$((\bar{P}_0, \bar{Q}_0), (\bar{X}_{0i}, \bar{Y}_{0i}), i = 1, \cdots, I) = \lim_{\varepsilon \to 0}((\bar{P}_\varepsilon, \bar{Q}_\varepsilon), (\bar{X}_{\varepsilon i}, \bar{Y}_{\varepsilon i}), i = 1, \cdots, I).$$

Comparing Models 1 and 2, we find the relation between the two equilibria in Models 1 and 2 is

$$((\bar{P}, \bar{Q}), (\bar{X}_i, \bar{Y}_i), i = 1, \cdots, I) = ((\bar{P}_0, \bar{Q}_0), (\bar{X}_{0i}, \bar{Y}_{0i} \wedge y_i), i = 1, \cdots, I)$$

where $\bar{Y}_{0i} \wedge y_i = \min\{\bar{Y}_{0i}, y_i\}$.

We consider a special case of 2 agents, $I = 2$. For any arbitrarily small positive scalar $\varepsilon > 0$ the equilibrium in Model 3 is $((\bar{P}_\varepsilon, \bar{Q}_\varepsilon), (\bar{X}_{\varepsilon 1}, \bar{Y}_{\varepsilon 1}), (\bar{X}_{\varepsilon 2}, \bar{Y}_{\varepsilon 2}))$. We consider the general equilibrium for the following two cases.

Case 1. $\sum_{i=1}^{2}(y_i - F_i) = (y_1 + y_2) - (F_1 + F_2) > 0$

In this case, the total endowments are less than the sum of the thresholds. Then the equilibrium in Model 3 satisfies $\bar{Y}_{\varepsilon 1} < y_1$ and $\bar{Y}_{\varepsilon 2} < y_2$ for any arbitrarily small positive scalar $\varepsilon > 0$. The equilibrium consumption is the global optimization point for the utility functions in Model 3. In the domain of $Y_1 \leq y_1$ and $Y_2 \leq y_2$, the utility functions in Model 1, 2, and 3 are the same. Therefore the three equilibria are identical, for any arbitrarily small positive scalar $\varepsilon > 0$,

$$\begin{aligned}((\bar{P}, \bar{Q}), (\bar{X}_1, \bar{Y}_1), (\bar{X}_2, \bar{Y}_2)) &= ((\bar{P}_0, \bar{Q}_0), (\bar{X}_{01}, \bar{Y}_{01}), (\bar{X}_{02}, \bar{Y}_{02})) \\ &= ((\bar{P}_\varepsilon, \bar{Q}_\varepsilon), (\bar{X}_{\varepsilon 1}, \bar{Y}_{\varepsilon 1}), (\bar{X}_{\varepsilon 2}, \bar{Y}_{\varepsilon 2})).\end{aligned}$$

and the budget constraints and market clearing conditions are binding.

Case 2. $\sum_{i=1}^{2}(y_i - F_i) = (y_1 + y_2) - (F_1 + F_2) \leq 0$

In this case, the total endowments are more than the sum of the thresholds. The equilibrium in Model 3 belongs to one and only one of the following three cases.

Case 2.1. $F_1 < y_1$ and $F_2 \geq y_2$

In the general equilibrium in Model 1, agent 2 consumes only y_2 units of harmful overconsumption commodity, and there is an excess supply of $F_2 - y_2 \geq 0$ units of harmful overconsumption commodity, which is free for agent 2. Agent 1 can consume $\bar{Y}_1 = y_1 \leq (F_1 + F_2) - y_2 = F_1 + (F_2 - y_2)$ units of harmful overconsumption commodity. In spot markets there is an excess supply of $(F_1 + F_2) - (y_1 + y_2) \geq 0$ units of harmful overconsumption commodity. Thus this commodity is free and its price, $\bar{Q}$, is 0. Therefore the two agents consume the usual commodity as many as possible, so $\bar{X}_i = E_i$ for $i = 1, 2$. Hence the equilibrium in Model 1 is

$$((\bar{P}, \bar{Q}), (\bar{X}_1, \bar{Y}_1), (\bar{X}_2, \bar{Y}_2)) = ((1, 0), (E_1, y_1), (E_2, y_2)).$$

Case 2.2. $F_1 \geq y_1$ and $F_2 < y_2$

This case is symmetric to Case 2.1. The analysis is similar except we switch between agents 1 and 2. Hence the equilibrium in Model 1 is

$$((\bar{P}, \bar{Q}), (\bar{X}_1, \bar{Y}_1), (\bar{X}_2, \bar{Y}_2)) = ((1, 0), (E_1, y_1), (E_2, y_2)).$$

Case 2.3. $F_1 \geq y_1$ and $F_2 \geq y_2$

In the general equilibrium in Model 1, agent i consumes only y_i units of harmful overconsumption commodity, and there is an excess supply $F_i - y_i \geq 0$ units of harmful overconsumption commodity, which is free for agent $i = 1, 2$, thus $\bar{Q} = 0$. In spot markets there is an excess supply of $(F_1 + F_2) - (y_1 + y_2) \geq 0$ units of harmful overconsumption commodity. Therefore the two agents consume the usual commodity as many as possible, so $\bar{X}_i = E_i$ for $i = 1, 2$. Hence the equilibrium in Model 1 is

$$((\bar{P}, \bar{Q}), (\bar{X}_1, \bar{Y}_1), (\bar{X}_2, \bar{Y}_2)) = ((1, 0), (E_1, y_1), (E_2, y_2)).$$

In a word, the equilibrium for Case 2 in Model 1 is

$$((\bar{P}, \bar{Q}), (\bar{X}_1, \bar{Y}_1), (\bar{X}_2, \bar{Y}_2)) = ((1, 0), (E_1, y_1), (E_2, y_2)).$$

We provide four examples in the next section to solve the equilibrium in Model 1 for economies with harmful overconsumption commodity from the solution in Model 2.

4 Simple Examples

We study a specific form of the utility functions,

$$v_i(X) = \frac{1}{\gamma_i} X^{\gamma_i} \qquad \textit{and} \qquad V_i(Y) = y_i^2 - (Y - y_i)^2, \qquad i = 1, 2.$$

Then the utility functions for Model 1 are

$$U_i(X_i, Y_i) = \frac{\alpha_i}{\gamma_i} X_i^{\gamma_i} + [y_i^2 - (Y_i - y_i)^2], \qquad i = 1, 2;$$

the utility functions for Model 2 are

$$U_{0i}(X_i, Y_i) = \frac{\alpha_i}{\gamma_i} X_i^{\gamma_i} + V_{0i}(Y_i), \qquad i = 1, 2$$

where

$$V_{0i}(Y) = \begin{cases} y_i^2 - (Y - y_i)^2, & \text{if } \ Y < y_i \\ y_i^2, & \text{if } \ y_i \le Y; \end{cases}$$

and the utility functions for Model 3 are

$$U_{\varepsilon i}(X_i, Y_i) = \frac{\alpha_i}{\gamma_i} X_i^{\gamma_i} + V_{\varepsilon i}(Y_i), \qquad i = 1, 2$$

where

$$V_{\varepsilon i}(Y) = \begin{cases} y_i^2 - (Y - y_i)^2, & \text{if } \ Y < y_i - \varepsilon \\ \frac{2\varepsilon}{\gamma_i}(y_i - \varepsilon)^{1-\gamma_i} Y^{\gamma_i} + (y_i - \varepsilon)\left[y_i - \frac{2-\gamma_i}{\gamma_i}\varepsilon\right], & \text{if } \ y_i - \varepsilon \le Y. \end{cases}$$

We provide the four examples for the case $\gamma_1 = \gamma_2 = \frac{1}{2}$. The characteristics of model parameters are given in Table 1. The four examples correspond to the four cases discussed in Section 3. Thresholds of harmful overconsumption commodity for the two agents are $y_1 = 10$ and $y_2 = 10$. Setting 1 : $F_1 = 4$ and $F_2 = 8$ satisfying $\sum_{i=1}^{2}(y_i - F_i) > 0$. There are three cases for Setting 2 [2]: $\sum_{i=1}^{2}(y_i - F_i) \le 0$. They are Case 1: $F_1 = 4$ and $F_2 = 16$; Case 2 : $F_1 = 12$ and $F_2 = 8$; and Case 3 : $F_1 = 12$ and $F_2 = 16$.

The four equilibria in Model 3 for the four cases are calculated in Table 2 by using GAMS codes. In Table 1, $\varepsilon = 0.01$ is small. As ε approaches zero, the limit of general equilibria in Model 3 degenerate into the general equilibria in Model 2, which is showed in Table 3. Then we have the equilibria in Model 1.

[2] If we change Case 1 to the case : $F_1 = 4$ and $F_2 > 16$, the equilibrium is the same as that in Case 1. On the other hand, if we change Case 2 to the case : $F_1 > 12$ and $F_2 = 8$, the equilibrium is the same as that in Case 2.

Table 1. Characteristics of Model Parameters

Case 1	Case 2.1	Case 2.2	Case 2.3
$\sum_{i=1}^{2}(y_i - F_i) > 0$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 > F_1$ and $y_2 \leq F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 > F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 \leq F_2$
Utility Share Parameter			
$\alpha_1 = 10$ $\alpha_2 = 10$			
Utility Power Parameter			
$\gamma_1 = \frac{1}{2}$ $\gamma_2 = \frac{1}{2}$			
Utility Adjustment Parameter			
$\varepsilon = 0.01$			
Threshold for Harmful Overconsumption Commodity			
$y_1 = 10$ $y_2 = 10$			
Initial Endowment of Usual Commodity			
$E_1 = 40$ $E_2 = 60$			
Initial Endowment of Harmful Overconsumption Commodity			
$F_1 = 4$ $F_2 = 8$	$F_1 = 4$ $F_2 = 16$	$F_1 = 12$ $F_2 = 8$	$F_1 = 12$ $F_2 = 16$

Table 2. General Equilibria for Usual Goods Model 3

Case 1	Case 2.1	Case 2.2	Case 2.3
$\sum_{i=1}^{2}(y_i - F_i) > 0$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 > F_1$ and $y_2 \leq F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 > F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 \leq F_2$
Equilibrium Price Pair			
$P = 1.000000$ $Q = 5.400561$	$P = 1.000000$ $Q = 0.015487$	$P = 1.000000$ $Q = 0.012642$	$P = 1.000000$ $Q = 0.011946$
Equilibrium Consumption of Usual Commodity			
$X_1 = 32.986531$ $X_2 = 67.013469$	$X_1 = 39.907270$ $X_2 = 60.092730$	$X_1 = 40.025180$ $X_2 = 59.974820$	$X_1 = 40.009525$ $X_2 = 59.990475$
Equilibrium Consumption of Harmful Overconsumption Commodity			
$Y_1 = 5.298589$ $Y_2 = 6.701411$	$Y_1 = 9.987742$ $Y_2 = 10.012258$	$Y_1 = 10.008162$ $Y_2 = 9.991838$	$Y_1 = 11.202667$ $Y_2 = 16.797333$
Utility Value			
$U_1 = 192.764333$ $U_2 = 252.842622$	$U_1 = 226.344256$ $U_2 = 256.039347$	$U_1 = 226.531176$ $U_2 = 254.886757$	$U_1 = 226.529625$ $U_2 = 255.025495$
Income			
$J_1 = 61.602244$ $J_2 = 103.204488$	$J_1 = 40.061947$ $J_2 = 60.247787$	$J_1 = 40.151699$ $J_2 = 60.101133$	$J_1 = 40.143356$ $J_2 = 60.191141$

5 Concluding Remarks

This paper studies economies with overconsumption commodities. We define the general equilibrium for economies with harmful overconsumption goods. By trun-

Table 3. General Equilibria for Wasteful Overconsumption Model 2

Case 1	Case 2.1	Case 2.2	Case 2.3
$\sum_{i=1}^{2}(y_i - F_i) > 0$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 > F_1$ and $y_2 \leq F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 > F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 \leq F_2$
Equilibrium Price Pair			
$P = 1.000000$	$P = 1.000000$	$P = 1.000000$	$P = 1.000000$
$Q = 5.400561$	$Q = 0.000000$	$Q = 0.000000$	$Q = 0.000000$
Equilibrium Consumption of Usual Commodity			
$X_1 = 32.986531$	$X_1 = 40.000000$	$X_1 = 40.000000$	$X_1 = 40.000000$
$X_2 = 67.013469$	$X_2 = 60.000000$	$X_2 = 60.000000$	$X_2 = 60.000000$
Equilibrium Consumption of Harmful Overconsumption Commodity			
$Y_1 = 5.298589$	$Y_1 = 10.000000$	$Y_1 = 10.000000$	$Y_1 = 11.200000$
$Y_2 = 6.701411$	$Y_2 = 10.000000$	$Y_2 = 10.000000$	$Y_2 = 16.800000$
Utility Value			
$U_1 = 192.764333$	$U_1 = 226.491106$	$U_1 = 226.491106$	$U_1 = 226.491106$
$U_2 = 252.842622$	$U_2 = 254.919334$	$U_2 = 254.919334$	$U_2 = 254.919334$
Income			
$J_1 = 61.602244$	$J_1 = 40.000000$	$J_1 = 40.000000$	$J_1 = 40.000000$
$J_2 = 103.204488$	$J_2 = 60.000000$	$J_2 = 60.000000$	$J_2 = 60.000000$

Table 4. General Equilibria for harmful Overconsumption Model 1

Case 1	Case 2.1	Case 2.2	Case 2.3
$\sum_{i=1}^{2}(y_i - F_i) > 0$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 > F_1$ and $y_2 \leq F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 > F_2$	$\sum_{i=1}^{2}(y_i - F_i) \leq 0$ $y_1 \leq F_1$ and $y_2 \leq F_2$
Equilibrium Price Pair			
$P = 1.000000$	$P = 1.000000$	$P = 1.000000$	$P = 1.000000$
$Q = 5.400561$	$Q = 0.000000$	$Q = 0.000000$	$Q = 0.000000$
Equilibrium Consumption of Usual Commodity			
$X_1 = 32.986531$	$X_1 = 40.000000$	$X_1 = 40.000000$	$X_1 = 40.000000$
$X_2 = 67.013469$	$X_2 = 60.000000$	$X_2 = 60.000000$	$X_2 = 60.000000$
Equilibrium Consumption of Harmful Overconsumption Commodity			
$Y_1 = 5.298589$	$Y_1 = 10.000000$	$Y_1 = 10.000000$	$Y_1 = 10.000000$
$Y_2 = 6.701411$	$Y_2 = 10.000000$	$Y_2 = 10.000000$	$Y_2 = 10.000000$
Utility Value			
$U_1 = 192.764333$	$U_1 = 226.491106$	$U_1 = 226.491106$	$U_1 = 226.491106$
$U_2 = 252.842622$	$U_2 = 254.919334$	$U_2 = 254.919334$	$U_2 = 254.919334$
Income			
$J_1 = 61.602244$	$J_1 = 40.000000$	$J_1 = 40.000000$	$J_1 = 40.000000$
$J_2 = 103.204488$	$J_2 = 60.000000$	$J_2 = 60.000000$	$J_2 = 60.000000$

cating the utility functions we then define the general equilibrium for economies with wasteful overconsumption goods. We can find the equilibria in these two economies by approaching them from traditional Arrow-Debreu economy with usual utility functions.

This paper considers a simple case of one usual commodity and one harmful overconsumption commodity. Both agents have harmful overconsumption utility functions. How about only one agent has a harmful overconsumption utility function (and another has a traditional utility function)? The equilibrium may be very different from this paper. Moreover, we can also consider other relating problems. For example, we can study the case where there are multiple usual commodities and multiple harmful overconsumption commodities. These are left for future research.

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Expectations, Asymmetries, and Contributions

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Abstract. This paper analyzes activity mechanism designers with. asymmetric agents of voluntary contributions to a project between several agents. He equilibrium pattern of contributions is one in which each agent makes may explain why the completion of a joint project is made step by step. We show that whenever the project is desirable, the project is completed, and in equilibrium, each agent makes large contribution. We recursive expected utility allows an agent to care about the timing of the resolution of uncertainty. Recursive expected utility achieves this flexibility by reduction of preferences are build up preferences depend on unrealized contingencies. We show that if an agent prefers resolution of uncertainty, then the forms of the most commonly used dependence, each collapsed to recursive expected utility. The agent's preferences must fail to conform to either the between be quite inconsistent in her preferences. We refers to a class of problems in which a individual, the agent, controls a decision that has consequences for many individuals with preferences. The mechanism designers, influence the agent's decisions by actions chosen under imperfect information.

JEL: C70, D83, G12.

Keywords: Asymmetric information, voluntary contributions, expectations efficiency.

1 Introduction

We conclude that the character of contributions provide a satisfactory explanation of the commonly observed contribution pattern in which agents alternate in making steps towards the completion of the transaction. We define a premium to measure of preference for think of the analogy to a risk, that sequentially consistent, the agent uses the same family of preferences to evaluate stage induced by each strategy in a decision tree. We find that and consistency with imply that deviations from expected utility are possible in he stage. Sequential consistency, restrict an agent to prefer resolution of uncertainty, and preference for resolution imply action.

Many procurement situations involve development with occurring at each stage of the process, where in each period for at most agent over time. A strategy is said to be relative to a given action if it reflects for any other action relative to the given action, that the equilibrium payoff is unique if the marginal contributions of mechanism designers to the value of the coalition. We show that in such equilibria, all mechanism designers receive their contributions as payoffs, and satisfying marginal contribution equilibria.

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We assume that the agents commitment over periods, between the payoffs received at different stages, the agent chooses the available actions for the next stage. We refer to this agenda as the agent initially determines the set of actions from which the choice of relevant policy alternatives by a mechanism designers. The equilibrium payoff to the agent depends on the degree of competition between the mechanism designers in the next stage. Since the competitiveness in turn depends on the set of available actions, he agent is able to increase her next period actions. We can be quite general in formulating the states of nature governing the transitions between states. We require that the current state depends on the previous state and the previous action by the agent. We concentrate on strategies since we want to study the effects of changes from effects created by conditioning on payoff histories. The truthful equilibrium payoffs are unique if the processes a contribution equilibrium. The structure of the paper is as follows.

Section 2 discusses mechanism asymmetries. Section 3 presents asymmetries of agents expectations.

2 Mechanism Asymmetries

Peters (2001) suppose expected utility preferences, and the set of efforts E that the agent can take is interpreted as a tariff or some policy.

The set Y is the set of feasible transfers of income from the mechanism designer to agent.

The payoff to mechanism designer $j \in \{1,\dots,n\}$ is represented by $v_j : \Pi_{k=1}^{n} \mathrm{A}_k \times E \times \Omega \to [0,1]$. For agent, payoffs are represented by the function $u : \Pi_{k=1}^{n} \mathrm{A}_k \times E \times \Omega \to [0,1]$, where the set A_j varies in applications depending on the mechanism designer's ability to write actions. The mechanism designer can make his action depend on every aspect of the agent's action, while in others he may be limited to actions that depend on some component of the agents effort, or to actions at all.

The mechanism designer's payoff is $v_j(a_j(e), e)$ while the agent's payoff is $\sum_{j=1}^{n} a_j(e) - g(e)$, where $g(e)$ is the utility associated with effort level e, the mechanism designer's payoff depends on his own action and the effort of the agent. In this environment it is natural to assume that supplier j can only write actions that specify function of quantity e_j that demand from him.

So A_j is just the set of maps from the j th component of e into the set of per unit assets, and $Q \subset \Re$ be a set of feasible asset, with each mechanism designers, and e as a vector of a components $(q_1,\dots,q_n)$ of actions that the agent makes with each of the mechanism designers, so $E = Q^n$ to the agent.

The mechanism designer condition his transfer on transactions that the agent makes with other mechanism designer, so that A_j is the set of functions $a_j : E \rightarrow Y$ that depend only on the *j* th agent for each possible quantity that the mechanism designer could have demand from to he agent.

The asset being has a random value τ whose distribution depends on the agent's information ω. The agent's payoff function when her information is ωi is given by

$$u(a_1,....,a_n,q_1,....,q_n,\omega) = E_{\tau|\omega}U\left\{\sum_{j=1}^{n} q_j\tau - \sum_{j=1}^{n} a_j(q_j)\right\}, \tag{1}$$

and the mechanism maker function is given by

$$v_j(a_1,....a_n,q_1,...,q_n,\omega) = a_j(q_j) - q_j E_{\tau|\omega}\tau. \tag{2}$$

The agents decides whether the mechanism designer's actions depend at all on the agent's effort so $\mathrm{A} = \Re^2$, when there is dependence of mechanism designers' payoffs settings. These commitments should as part of the action that their undertakes, and have output consequences that will affect the actions of the agent's competitors. For this reason it seems sensible to make allowance for this possibility in the payoff functions.

Ultimately competition among mechanism designers will be that it this competition as a offer appropriately designed menus o the agent.

It is desirable to avoid imposing additional restrictions on competition among mechanism designers, forcing them o use random mechanisms, or focussing attention on pure strategy equilibria, even though restrictions be perfectly sensible of specific applications.

The process be designing mechanisms o guide communication that the agent is bound by the mechanisms, agents communicate with the mechanism designers, possibly sending information about their types.

The mechanism designers responds to the agent's communications by taking actions, and possibly sending messages, according to the rules that they specified by their mechanism.

Sandroni (2000) show that if agent's have the rule, then the most prosperous predictions need be driven of the market. The set of states of nature is given to $T = \{1,....,L\}, L \in N$.

The true stochastic process of states of nature is given by an arbitrary probability measures defined *P* as on $(T,\Im)$. Given history $s_t \in T'$, let P_s is the defined by

$$P_{s_t}(A) = \frac{P(A_{s_t})}{P(C(S_t))},$$

for all $A \in \Im$, where A_{s_j} is the sets of all paths $s \in T$ such that $E^p(|\Im_t)(s) = (s_1,...,s_n)$.

At period an mechanism designer has initial wealth *w*. At period $t \in N_+$ she chooses a portfolio a_t that determines how the wealth is allocated among *M* assets.

A actions *a* is an $\Im_t$ such that an strategy is a actions $a = (a_t, t \in N_+)$.

The entropy of a actions is $E^p(\log(a, r_{t+1})|\Im_t)$, if consumption is a proportion $\delta \in (0,1)$ who invest

$w(a) = \delta^t \Pi_{k=1}^t r_k$.

If there is $\varepsilon > 0$, for all $t \in N_+$, then $y_k - E^p(y_k|\Im_{k-1})$, and z_k is uniformly bounded.

We have the strategy defined by

$$a_t^* = \arg\max E^p(\log(a_t r_{t+1})|\Im_t),$$

when returns are independent and states of nature follow an identical stochastic process. If agent actions under this rule and then agents holding actions with the entropy those making if all agents have correct beliefs.

An agent who has correct beliefs, which would eventually result in more wealth. At period *t*, the action of tree *m* is given by $p_{m,t}$, agent *i* 's consumption and holdings of tree *m* are given by c_t^i *and* $k_{m,t}^i$. Let w_t^i be agent *i* 's wealth

$$w_t^i = (p_{n-1,t} + 1)k_{n-1,t-1}^i + p_{n,t}k_{n,t-1}^i. \tag{3}$$

Agent *i* 's budget constraint is given by

$$c_t^i + p_{n-1,t}k_{n-1,t}^i + p_{n,t}k_{n,t}^i \le w_t^i, \quad c_t^i \ge 0, \ w_t^i \ge 0. \tag{4}$$

By agents' conditions and the market clearing condition,

$$\frac{c_t^n}{\sqrt{c_t^{n-1}}} = \frac{\lambda^{n-1}}{\lambda^n}, \quad c_t^{n-1} + c_t^n = e_t, \tag{5}$$

imply that agent's consumption may depend on the current state of nature. At period $t \in N_+$ actions are given by $e = (e_{1,t}, \ldots, e_{M,t})$, share actions are given by $p_t = (p_{1,t}, \ldots, p_{M,t})$, agents $i's$ consumption is given by c_t^i, agent *i* 's wealth is given by

$$w_t^i = (p_t + e_t)k_{t-1}^i. \tag{6}$$

We define $e = (e_t, t \in N_+)$, $p = (p_t, t \in N_+)$, $c^i = (c_t^i, t \in N_+)$, $k^i = (k_t^i, t \in N_+)$, and $w^i = (w_t^i, t \in N_+)$.

Let P and P^i be the probability measures on $(T,\Im)$ representing the probability measure and agent i 's beliefs.

We assume that markets are dynamically complete, and so agents may transfer wealth across sates of nature by actions the existing assets. At period $t \in N_+$, agent i ' s observed and anticipated budget constraints are given by

$$c^i_{t+r} + p_{t+r}k^i_{t+r} \le (p_{t+r} + e_{t+r})k^i_{t+r-1},\ w^i_{t+r} \ge 0,\ c^i_{t+r} \ge 0,\ r \in N_+ . \quad (7)$$

Market clear at period t if

$$\sum_{i=1}^{n-1} c^i_t = \sum_{m=1}^{n} e_{m,t} \ \ and \ \ \sum_{i=1}^{n-1} k^i_{n,t} = 1 ,$$

in equilibrium, agents maximize expected utility to the budget constraints and markets clear in every period.

Agent i is driven out of the market on a path $s \in T$ if agent $i' s$ wealth, if agent i is driven out of the market on s. That is, agent i makes accurate actions if the probabilities assigned by agent i become similar to the probabilities. Agent i's beliefs with eventually makes accurate, if merge with the truth.

So, agent i' s beliefs over events within many periods become to the true probabilities. The difference between is illustrated by the presented in which the probability of the state of nature, that all other states will occur in the next $n-1$ periods with probability. Agent i always makes in such that accurate next period on a path $s \in T,\ s = (s_1, ..., s_n)$, if there is $\varepsilon > 0$ $d_{n-1}(P^i_{s_is}, P_{s_i}) > \varepsilon$ for all $t \in N$.

An agent may inaccurate actions in some all periods. An agent may eventually make actions on path but always make inaccurate next period actions on another path. Some agents make accurate actions in every path $s \in T$, there exists t least gent who eventually makes accurate actions on s.

If agent i believes that a path s is much likely to occur than agent j does. They have the same factor, then agent i allocates much less wealth on s than agent j does and believe that a path s is much likely to occur than any other agent does. We provide an example in which an agent whose beliefs weakly with the truth is driven of the market although other agent eventually makes accurate next period actions.

At periods $t(k), k \in N$, agent n believes that state a will occur next period with probability 1. There were and another agent, who eventually makes accurate actions on $\bar{s}$ has arbitrarily high probability, provided that t is large enough.

3 Asymmetries of Agents Expectations

Compte and Jehiel (2003) analyzes a problem of contributions o a joint project between several agents.

Each agent i values the immediate completion of the project according to V_i, so that we allow for asymmetries between the agents. The project is completed as soon as the cumulative contribution made by agent reaches the cost *K*. We assume that neither agent can afford to complete the project alone. Let c_i^t be the amount of agent *i*' s contribution in period *t*. A history at time τ a strategy for agent *i*' s specifies the size of the agent's contribution for each history after which it is agent *i* 's turn to move.

Possible interpretation of δ is that at the end of each period, with probability $1-\delta$, in which event the contributions made up to that date *T* be the first time at which the cumulative contribution.

4 Conclusion

The proposition shows that if agents have the same factor and some agent eventually makes accurate actions, then a necessary and sufficient condition for survival is to eventually make accurate actions. We shows that only agents with survive in the market, and shows that if agents have the same inter-temporal factor and some agent eventually makes accurate next period actions, then agents who always make inaccurate next period actions are driven of the market. If agents have the same factors, and then agents which eventually make accurate actions will be driven of the market by agents, who eventually make accurate actions. The surviving agents may differ may differ across paths, and all agents who survive must be making accurate actions. Given enough data, the actions of gents over events within finitely many periods become similar but actions over some future events are different. These differences in beliefs have impact on agent's decisions. Agents with diverse preferences over risk survive wit probability if their beliefs merge with the truth. This holds the relative wealth of agents with different preferences over risk is a random variable. There are states of nature, and agent believes correctly, that they have probability.

Agents who eventually make accurate actions allocate amounts of wealth to high probability paths, and are driven of the market. Assume that the ratio of beliefs and true probabilities over states of nature in the next period is bounded away infinity.

All agents have the same inter-temporal factor, and that some agents eventually make accurate next period actions. Agents who make inaccurate next period actions are driven of the market. However, as mentioned, agents who eventually make accurate next period actions.

Although has implications concerning the efficiency of the outcome, that main interest in identifying what can induce the equilibrium contributions o be made step by step.

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A Quantile-Data Mapping Model for Value-at-Risk Based on BP and Support Vector Regression

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Abstract. A novel "Quantile predicting Return" method (Q→R) is presented to predict VaR (Value-at-Risk). In the paper, a nonlinear mapping between quintile and VaR is constructed by using neural networks instead of traditional statistical methods. Two related models are proposed. One is QDMN (Quantile-Data Mapping Network) based on BP and the other is SVR-QD (Support Vector Regression Quantile- Data mapping) based on SVM. There is no assumption for distribution in both proposed models. The operation mode and the reasonableness of measuring VaR using the two models are analyzed. QDMN and SVR-QD are applied to predict VaR in Shanghai Stock Exchange. Comparisons of the proposed methods with Monte Carlo approach are performed. Numerical experiments based on Basle traffic lights, Proportion of Failures and exception computing show that the two new models are superior to Monte Carlo approach based on a certain assumptions in accuracy.

1 Introduction

Nowadays, Value-at-Risk (VaR) has undoubtedly become one of the most important techniques for risk optimization in the field of financial engineering. It has some attractive characteristics such as simplicity, understandability and practicality and is able to summarize various risks into a real number. VaR has widespread applications in banks, non-bank institutions, and supervisory authorities.

VaR is used to estimate the potential loss of positions or portfolios held due to the reverse movement of the price over a certain period of time. VaR depends on two factors: firstly, the horizon over which the portfolio's change in value is measured and secondly the "degree of confidence" chosen by the risk manager. In 1996, Basle Committee on Banking Supervision recommended VaR as an internal solution of risk management.

There are three significant methods in measuring financial risks with VaR presently: Analytical approach (Variance-Covariance method), History simulation (HS), and Monte Carlo simulation (MC).

The most popular method of analytical approaches is Risk Metrics [1]. In the method very simple calculation is needed. Evaluating the standard deviation and the correlation coefficient between each position is the whole work, but normal

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distribution and zero-mean are two necessary assumptions for the compound return; in addition, empirical data show that the distribution of compound return has several important characteristics such as kurtosis, fat-tailedness and skewness. Generalized autoregressive conditional heteroskedastic (GARCH) method is another analytical approach used for VaR. It was proposed by Bollerslev [2] in 1986, GARCH can reflect heteroskedastic of the portfolio return to some extent, but fat-tailedness and left-skewness cannot be reflected in GARCH [3-5]. Moreover, it is discovered that the third moment of the return varies over time [5]. Other analytical methods also make distribution assumptions that always deduce the accuracy of the methods in the computational process.

HS method simply depends on a great deal of history data. So Basle Committee suggests that banks should use at least one year's history data in HS when forecasting daily VaR. However, the quantity of data is much more than a year's for high required precision in real forecasting. It is assumed that the return distribution of the whole history period is fixed, so one cannot calculate the loss outside the simulation set.

Although Monte Carlo simulation has a high accuracy, its limitations cannot be ignored. In this method, the computational procedure is very complex and the amount of calculation is very large. The precision of VaR estimation would critically depend on the number of simulation trials and the portfolio should be fully revalued in each simulation run, thereby making it time-consuming, computer-intensive and expensive compared to analytical method [6]. As for the portfolio that has thousands of instruments, it is impossible to forecast VaR with MC simulation at present.

Recent years, artificial neural networks (ANN) have been employed with success in pattern recognition, characteristic extraction, prediction and adaptive control on nonlinear systems etc. ANN also uses in finance and economics such as stock prediction, assets allocation, securities pricing and so on. Support vector machines (SVM) are a kind of machine learning technique developed in the middle of 1990s'. SVM is founded on structural risk minimization (SRM), instead of empirical risk minimization (ERM), which is commonly used in traditional neural network. SVM has been applied successfully to classification tasks [7, 8] and more recently also to regression [9]. In the articles estimating VaR using neural networks, most estimate parameters of the asset distribution with ANN. References [10] and [11] predict essential parameters with neural network, and then forecast returns with analytical approach. Reference [12] minimizes the financial risk with recursive network under certain assumptions. All articles above use neural network to estimate parameters for predicting portfolio return. They fall into the "Assumption→Parameter estimation→Return prediction" (A→P→R) trap. The A→P→R method takes redundant computing steps that may reduce the accuracy. Therefore, in this paper, Quantile-Data Mapping model based on artificial neural network (Quantile-Data Mapping Network, QDMN) and Support vector regression Quantile-Data mapping model (SVR-QD) are introduced to calculate VaR. The "Quantile→Return data" method (Q→R) is also used in the two models.

2 QDMN and SVR-QD

QDMN is a feedforward network with a single hidden layer, Back Propagation algorithm is used for training the net. In our case the transfer function in hidden layer should be a bounded, nonlinear, non-decrease one, and the output neuron transfer function may be an unbounded function. For QDMN, the two functions below are used; they are tangent sigmoid function (1) and pure linear function (2).

$$\tan sig(x) = \frac{2}{1+e^{-2x}} - 1 \qquad (-1 < \tan sig(x) < 1) \tag{1}$$

$$purelin(x) = cx \tag{2}$$

In the simplest case, QDMN is consisted with one input unit, one layer of hidden units and one output unit, the following mapping is used

$$\tilde{f}(x) = purelin(\sum_{j=1}^{Z} T_{j1} \tan sig(\omega_{1j} x + a_j) + d) \tag{3}$$

where Z is the number of the hidden units, ω_{1j} is weight between input and hidden layer, T_{j1} is the weight between the hidden and output layer. A_j and d are thresholds of the hidden and output layer.

The steps for predicting VaR using QDMN is shown in the following flow chart:

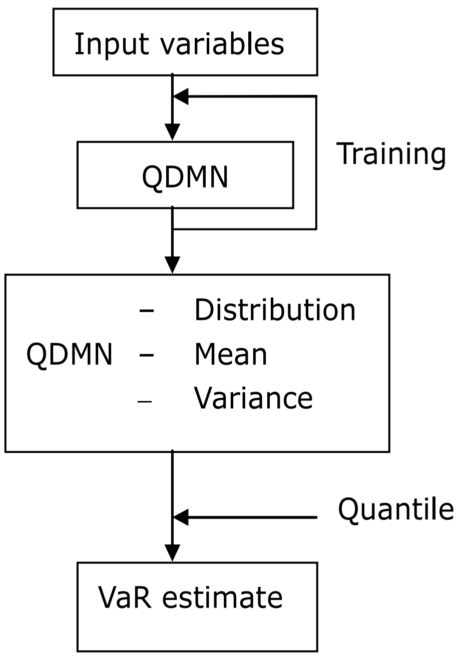

Fig. 1. Flow chart of QDMN

In SVR the basic idea is to map the data x into a high dimensional feature space F via a nonlinear mapping Φ and to do linear regression in this space [7, 8].

$$f(x) = (\omega \cdot \Phi(x)) + b \qquad \Phi : \Re \rightarrow F, \quad \omega \in F \tag{4}$$

where b is a threshold. Thus, linear regression in a high dimensional space corresponds to nonlinear regression in the low dimensional input space $\Re$. Since Φ is fixed, ω is determined from the data by minimizing the sum of the empirical risk $R_{emp}[f]$ and a complexity term $\|\omega\|^2$

$$R_{reg}[f] = R_{emp}[f] + \lambda\|w\|^2 = \sum_{i=1}^{l} C(f(x_i) - y_i) + \lambda\|w\|^2 \tag{5}$$

where l denotes the sample size. $C(.)$ is a cost function and is regularization constant. Eq.(5) can be minimized by solving a quadratic programming problem.

$$\omega = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*)\Phi(x_i) \tag{6}$$

where α_i, α_i^* denotes the solution of the aforementioned quadratic programming problem. The whole problem can be rewritten in terms of dot products in the low dimensional input space.

$$f(x) = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*)(\Phi(x_i) \cdot \Phi(x_i)) + b = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*)k(x_i, x) + b \tag{7}$$

where $k(x_1, x)=\Phi(x_1)\bullet\Phi(x)$ is a kernel function. A RBF kernel is used here.

$$k(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right) \tag{8}$$

the steps for predicting VaR using SVR-QD is similar as QDMN's. The SVR's structure is

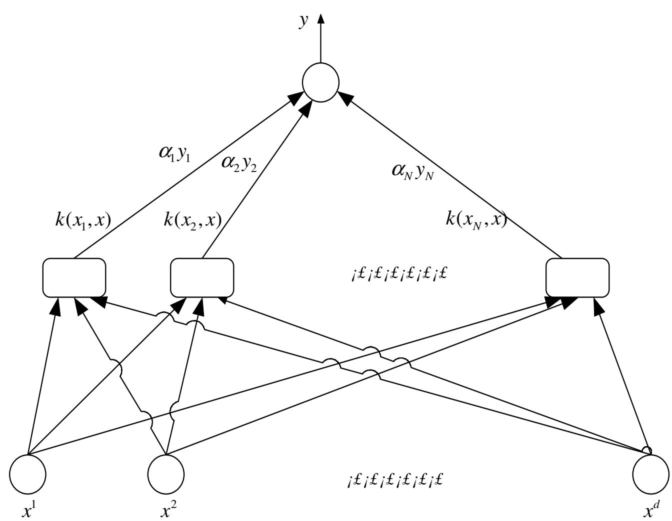

Fig. 2. SVR-QD structure

"Q→R" method can be described as follows: Let f(x) be the portfolio return distribution, $x_1, x_2, \cdots, x_n$ be the discrete points. $\Delta x = x_{i+1} - x_i$, $\varepsilon_i \in (x_i, x_{i+1}]$, (i=1, 2 …n). The following equation can be obtained according to the definition of VaR

$$1-\alpha = \int_{-\infty}^{-VaR} f(x)dx = \lim_{\Delta x \to 0} \sum_{\{\varepsilon_i | \varepsilon_i \in (x_i, x_{i+1}] \cap \varepsilon_i \le -VaR\}} f(\varepsilon_i)\Delta x \tag{9}$$

During time T, suppose the history returns $({}^{-T}r, {}^{-T+1}r, \cdots {}^{-1}r,) = \vec{R}$, when $T \to +\infty$, $T^{-1} = \Delta x$, the compound return ${}^{-k}r \approx \varepsilon_i$ ($-k \in [-T, -1]$)then (9) implies (10)

$$1-\alpha = \lim_{T \to +\infty} T^{-1} \sum_{\{{}^{-k}r | {}^{-k}r \in \vec{R} \cap {}^{-k}r \le {}^{r}VaR\}} f({}^{-k}r) = \Psi({}^{r}VaR \mid T, \vec{R}) \quad (T \to +\infty) \tag{10}$$

where ^{r}VaR denotes the logarithmic compound return rate corresponding to VaR. When T is limit, formula (10) can be considered as mapping from the compound return ^{r}VaR to confidence level α where $\vec{R}$ and T can be obtained from history data. QDMN and SVR-QD can be constructed with history data. The Q→R method mentioned above doesn't depend on any assumption and is able to enhance the accuracy of the calculation. Consider a simple example:

The recent T period history return sequence $({}^{-T}r, {}^{-T+1}r, \cdots {}^{-1}r) = \vec{R}$ is used to estimate VaR under confidence level α over one day horizon. According to the return distribution and the relationship between quantile and compound return in the definition of VaR, the compound return will change with quantile q (q=1-α , stands for the left tail probability of the density distribution). Q vary between 0 and 1. In a stable financial market, the history return will reflect the future return. ${}^{-T}r, {}^{-T+1}r, \cdots {}^{-1}r$ can be used to predict VaR. In Q→R method, the training samples of the network are composed of VaR quantile and the compound return $(q_j, {}^{-j}r)$ where $q_j \in (0, 1)$, ${}^{-j}r \in \vec{R}$. Q_j is the input, and ${}^{-j}r$ is the network teacher. Corresponding to each ${}^{-j}r$, q_j can be obtained through the following two steps: First, sort the history return ${}^{-T}r, {}^{-T+1}r, \cdots {}^{-1}r$ increasingly. Take T quantile points between 0 and 1 uniformly. The quantile points is marked as q_j (j=1…T); second, match q_j with each ${}^{-j}r$ according to their value relationship. Then T training vectors can be obtained. The quantity of the training sample and the maximum training generation relies on the required VaR confidence level. The mapping will be:

$$\Re(\alpha_j \mid T, \vec{R}) \to {}^{-j}r \tag{11}$$

Formula (11) is a more convenient form of formula (10). The training will stop when the total error $E = \sum_{k=1}^{P} e_k$ ($e_k = \sum_{l=1}^{n} | t_l^{(k)} - O_l^{(k)} |$) is less than a given positive real number η or the generation reaches its limitation.

After training, the network can be used to predict the compound return corresponding to any quantile and the return under certain VaR confidence level. Taking the quantile 1-α (such as 1-95%) as the input of the network, we can obtain the ^{r}VaR under the confidence level α, which is taken as the expected output of the network. One day horizon VaR can be estimated by formula (12) with the portfolio price P_0.

$$VaR = P_0 e^{^{r}VaR} \quad (12)$$

3 Simulation Results

The two models are used to analyze the composite index in Shanghai Stock Exchange. The data set consists of 2747 daily return covering the time from 19th, December 1991 up to 28th, February 2002.

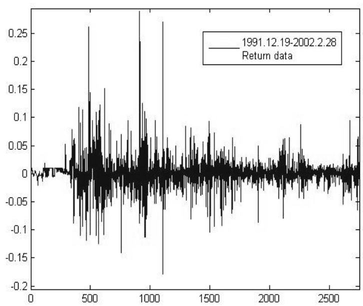

Fig. 3. Return sequence over time

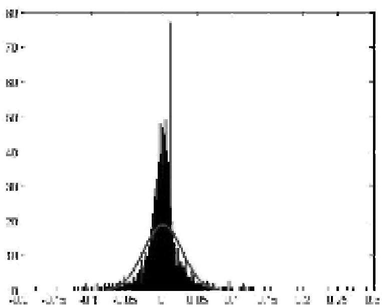

Fig. 4. Return distribution

Figure 3 shows that the return sequence over the given horizon is heteroskedastic. Figure 4 shows the kurtosis and fat-tailedness characteristics. From Figure 5 it can be seen that the normality of the distribution is denied.

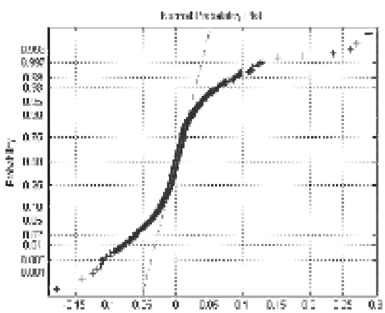

Fig. 5. Normality checkout of the return

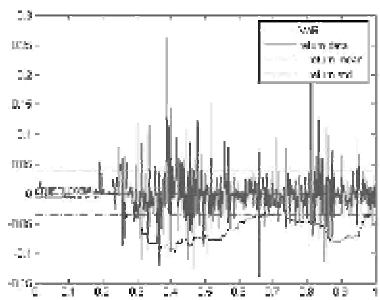

Fig. 6. Daily VaR estimated by QDMN

3.1 Estimate VaR with QDMN and SVR-QD

In QDMN, the daily VaR is forecasted under different confidence levels, different numbers of hidden layer units and different history returns. In the simulation, the maximum generation of QDMN is taken as 100, and 100 training samples are used under 90% and 99% confidence while 1000 training samples under 99.9% confidence. Simulation results are shown in Table 1, where CL means the confidence level. The total number of testing samples is 2647 under 95% and 99% confidence, while 1747 under 99.9%. The true exceptions are 132, 26 and 2, respectively, for different CLs.

Table 1. Exceptions of Daily VaR in QDMN

	CL		
N	95%(exceptions)	99%(exceptions)	99.9%(exceptions)
3	5.59%(148)	1.59%(42)	0.17%(3)
4	5.55%(147)	1.32%(34)	0.1%(2)
5	5.40%(143)	1.28%(34)	0.1%(2)
7	5.63%(149)	1.21%(32)	0.05(1)

Figure 6 shows 1000 daily VaR estimated by QDMN under 95% confidence level (1991.12.19- 1994.12.18), nearly 5% of the total return below VaR.

In SVR-QD, take σ as RBF kernel parameter, C as the bound of the Lagrange multipliers. 100 training sample under 90% and 95% confidence is used while 200 under 99% confidence.

Table 2. Exceptions of Daily VaR in SVR-QD

C	CL & σ		
	90% &0.5(exceptions)	95%&0.05	99%& 0.001
70	10.65%(282)	5.17%(137)	1.88%(48)
100	10.28%(272)	5.33%(141)	1.26%(32)
200	10.05%(266)	5.52%(146)	1.17%(30)
300	9.97%(264)	5.55%(147)	1.17%(30)

In Table 2, the total number of training samples is 100 under 90% and 95% confidence, while 200 under 99%. The true exceptions are 264, 132 and 25 under 90%, 95% and 99% confidence respectively.

3.2 Estimating VaR Using Monte Carlo Simulation

Suppose the distribution is normal, logistic, Extreme Value and lognormal respectively, 1 million samples with Monte Carlo simulation. VaR can be directly predicted from sample set. The results are shown in Table 3.

Table 3. Exceptions of daily VaR using MC

Dist/VaR	CL		
	95%(exceptions)	99%(exceptions)	99.9%(exceptions)
Normal	4.00%(110)	1.78%(49)	0.80%(22)
^{r}VaR (10^{-2})	-4.38	-6.23	-8.30
Logistic	5.72%(157)	2.58%(71)	0.80%(22)
^{r}VaR (10^{-2})	-3.49	-5.47	-8.25
Extreme Value	3.17%(87)	1.60%(44)	0.98%(27)
^{r}VaR (10^{-2})	-4.99	-6.46	-7.86
lognormal	4.22%(116)	2.33%(64)	1.02%(28)
^{r}VaR (10^{-2})	-4.19	-5.83	-7.59

In Table 3, the total number of training samples is 2747. The true exceptions are 137, 27 and 3 respectively under 95%, 99% and 99.9% confidence.

3.3 Comparison Results

Basle traffic lights [13] and Kupiec's [14] Proportion of Failures (PF)[1] are used to evaluate the proposed models. The results are shown in Table 4.

Table 4. Comparison of QDMN, SVR-QD and MC for predicting Value-at-Risk

CL/ %	VaR					
	MC /Dist	Zone/PF	QDMN/N	Zone/PF	SVM-QD	Zone/PF
95	4.2%/lognorm	G/accept	5.4%/5	G/accept	5.17%	G/accept
99	1.6%/EV	G/accept	1.21%/7	G/accept	1.17%	G/ accept
99.9	0.8%/norm	R/reject	0.1%/4|5	G/ accept	null	null

In Table 4, Dist, G and R represent distribution, Green and Red, respectively, and N is the number of units in hidden layer. From the simulation it is found that SVR-QD's convergence is very slow under 99.9% CL.

MC simulation assumes a certain distribution that cannot accurately fit the historical data in most of the running time. From Table 4, it is clearly shown that the assumption will affect the accuracy of the prediction. On the contrary, the two proposed models with Q→R method do not take any assumption. The prediction accuracy of VaR is improved significantly. QDMN is able to calculate VaR with high confidence level while SVR-QD is able to calculate VaR with more accurate. QDMN is faster than SVR-QD, while SVR-QD underestimates VaR with less probability.

4 Conclusion

History data of the financial instruments show that the distribution of return has the characteristics of kurtosis, fat-tailedness and skewness. Most analytical methods cannot

[1] Basle traffic lights were brought forward by Basle committee in 1996. It divides the results of the risk models into three zones: green, yellow and red based on binomial distribution. PF is also called Likelihood-Ratio-Test; it's a method of hypothesis testing.

deal with these problems properly, and MC method and HS method have inherent drawbacks. A neural network model QDMN and a regression method based on support vector machines, SVR-QD, are proposed in this paper to try to handle these problems. VaR can be more easily and accurately estimated by either model. The two methods are independent with distribution assumptions and encapsulate the financial parameters in the network. Simulated experiments show some useful results have been obtained as applying QDMN and SVR-QD to the forecasting of VaR. Simulations show that Q→R models perform better than the assumption models. It is able to reflect the kurtosis, fat-tailedness and skewness of the return distribution, and more accuracy can be obtained under most confidence levels with QDMN and SVR-QD.

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