Simplified Design

REINFORCED CONCRETE BUILDINGS OF MODERATE SIZE AND HEIGHT

SECOND EDITION

PCA PORTLAND CEMENT ASSOCIATION

Publication List

Book Contents

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REINFORCED CONCRETE BUILDINGS OF MODERATE SIZE AND HEIGHT

SECOND EDITION

Edited by David A. Fanella and S. K. Ghosh

PORTLAND CEMENT ASSOCIATION

An organization of cement manufacturers to improve and extend the uses of portland cement and concrete through scientific research, engineering field work, and market development.

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This publication was prepared by the Portland Cement Association for the purpose of suggesting possible ways of reducing design time in applying the provisions contained in the ACI 318-89 (Revised 1992) *Building Code Requirements for Reinforced Concrete.*

Simplified design procedures stated and illustrated throughout this publication are subject to limitations of applicability. While such limitations of applicability are, to a significant extent, set forth in the text of this publication, no attempt has been made to state each and every possible limitation of applicability. Therefore, this publication is intended for use by professional personnel who are competent to evaluate the information presented herein and who are willing to accept responsibility for its proper application.

This publication is intended SOLELY for use by PROFESSIONAL PERSONNEL who are competent to evaluate the significance and limitations of the information provided herein, and who will accept total responsibility for the application of this information. The Portland Cement Association DISCLAIMS any and all RESPONSIBILITY and LIABILITY for the accuracy of the application of the information contained in this publication to the full extent permitted by law.

About the editors: David A. Fanella is Coordinating Structural Engineer and S. K. Ghosh is Director, Engineered Structures and Codes, Portland Cement Association, Skokie, Illinois.

Foreword

The Building Code Requirements for Reinforced Concrete (ACI 318) is an authoritative document often adopted and referenced as a design and construction standard in state and municipal building codes around the country as well as in the specifications of several federal agencies, its provisions thus becoming law. Whether ACI 318 is enforced as part of building regulations or is otherwise utilized as a voluntary consensus standard, design professionals use this standard almost exclusively as the basis for the proper design and construction of buildings of reinforced concrete.

The ACI 318 standard applies to all types of building uses; structures of all heights ranging from the very tall high-rises down to single-story buildings; facilities with large areas as well as those of nominal size; buildings having complex shapes and those primarily designed as uncomplicated boxes; and buildings requiring structurally intricate or innovative framing systems in contrast to those of more conventional or traditional systems of construction. The general provisions developed to encompass all these extremes of building design and construction tend to make the application of ACI 318 complex and time consuming. However, this need not necessarily be the case, particularly in the design of reinforced concrete buildings of moderate size and height, as is demonstrated in this publication.

This book has been written as a timesaving aid for use by experienced professionals who consistently seek ways to simplify design procedures.

New to this second edition is a section in Chapter 1 on preliminary design. Guidelines and design aids are provided to help in choosing an economical floor system, and to obtain preliminary sizes for the beams, joists, columns, shearwalls, and footings.

Throughout the chapters, new design aids have been included that should save significant amounts of time. One such set of design aids is given in Chapter 3 for beams subjected to torsional loading: all required torsion reinforcement can easily be obtained via four charts. Also included in Chapter 3 are new design aids that can be used to obtain the required shear reinforcement for beams.

Chapter 5, which covers the simplified design of columns, has been significantly revised to better reflect the current ACI 318 provisions. New to this chapter are two equations that can produce a simplified interaction diagram for a section subjected to uniaxial load and bending moment.

In some of the example problems, the results obtained from the simplified design methods are compared to those obtained from PCA computer programs. These comparisons readily show that the simplified methods yield satisfactory results within the stated limitations.

Design professionals reading and working with the material presented in this book are encouraged to send in their comments to PCA, together with any suggestions for further design simplifications. PCA would also be grateful to any reader who would bring any errors or inconsistencies to our attention. Any suggestion for improvement is always genuinely welcome.

Skokie, Illinois
July, 1993

S. K. Ghosh

Acknowledgments

The first edition of this manual was prepared under the direction of Gerald B. Neville, formerly Manager of Structural Codes, PCA, and now with the International Conference of Building Officials. He provided technical assistance to the individual authors when needed, and also served as coordinator and editor.

The various chapters of the first edition were individually authored as follows:

Chapter 1—Gerald B. Neville

Chapter 2—Randal C. Cronin, formerly Senior Regional Structural Engineer, Eastern Region, PCA

Chapter 3—Gerald B. Neville

Chapter 4—S. K. Ghosh

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Chapter 6—Arnold H. Bock, formerly Senior Regional Structural Engineer, South Central Region, PCA

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Chapter 9—William R. Anthony, formerly Manager, Market Development, Concrete Construction Division, The CECO Corporation.

Chapter 10—James P. Barris, formerly Director, Codes and Standards Department, PCA.

Although the individual authors' names are no longer separately identified with the various chapters, their initial work is carried over into this edition. Sincere gratitude is expressed to each and every one of them.

Joseph "Jim" Messersmith, Jr., Coordinating Manager, Regional Code Services, PCA, provided significant input towards revisions for this second edition. Aphrodite Lisa Lehocky of the Engineered Structures and Codes staff at PCA produced this complex manuscript, including its many tables and figures, on the desk-top publishing system. Diane Vanderlinde of PCA's Office Services staff is responsible for the word-processing of the entire manuscript. The contributions of these three individuals are gratefully acknowledged.

David A. Fanella, the co-editor of this volume, is largely responsible for the significant enhancements from the first to the second edition. Without his dedicated efforts, the timely publication of this manual in its present form would have been impossible.

S. K. Ghosh

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Chapter 1

A Simplified Design Approach

1.1 THE BUILDING UNIVERSE

There is a little doubt that the construction of a very tall building, a large domed arena, or any other prominent megastructure attracts the interest of a great number of structural engineers around the country. The construction of such facilities usually represents the highest level of sophistication in structural design and often introduces daring new concepts and structural innovations as well as improvements in construction techniques.

Many structural engineers have the desire to become professionally involved in the design of such distinctive buildings during their careers. However, very few projects of this prestigious caliber are built in any given year. Truly, the building universe consists largely of low-rise and small-area buildings. Figure 1-1 shows the percentage of building floor area constructed in 1992 in terms of different building height categories. From this it can be readily seen that the vast majority of the physical volume of construction is in the 1- to 3-story height range.

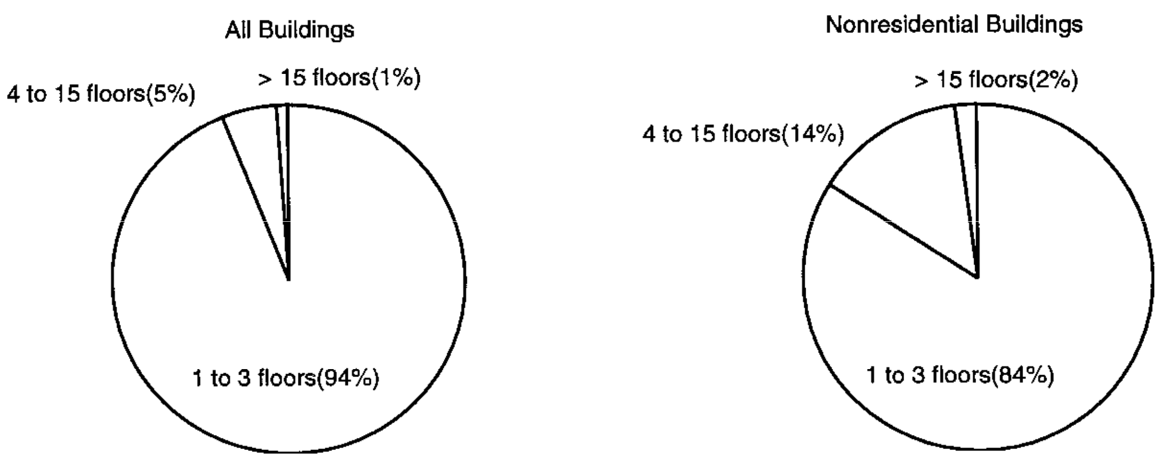

*Figure 1-1 Floor Area of Construction, 1992**

**Source: F. W. Dodge Division, McGraw-Hill Information System Company, Dodge Construction Potentials (1992).*

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In the same way, Figure 1-2 shows the percentage of nonresidential building projects constructed in various size categories. Building projects less than 15,000 sq ft dominate the building market.

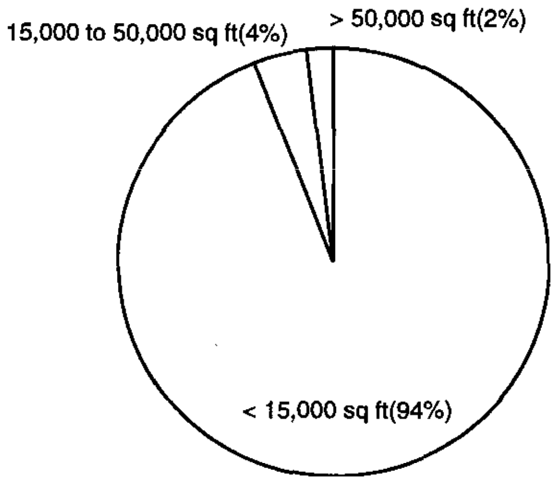


*Figure 1-2 Nonresidential Building Project Size, 1992**

When all these statistics are considered, it becomes apparent that while most engineers would like to work on prestigious and challenging high-rise buildings or other distinctive structures, it is more likely that they will be called upon to design smaller and shorter buildings.

1.2 COST EFFICIENCIES

The benefit of efficient materials use is not sought nor realized in a low-rise building to the same degree as in a high-rise facility. For instance, reducing a floor system thickness by an inch may save three feet of building height in a 36-story building and only 3 in. in a three-story building. The added design costs needed to make thorough studies in order to save the inch of floor depth may be justified by construction savings in the case of the 36-story building, but is not likely to be warranted in the design of the smaller building. As a matter of fact, the use of more material in the case of the low-rise building may sometimes enable the engineer to simplify construction features and thereby effectively reduce the overall cost of the building.

In reviewing cost studies of several nonresidential buildings, it was also noted that the cost of a building's frame and envelope represent a smaller percentage of the total building cost in low-rise buildings than in high-rise structures.

In low-rise construction, designs that seek to simplify concrete formwork will probably result in more economical construction than those that seek to optimize the use of reinforcing steel and concrete, since forming represents a significant part of the total frame costs. There is less opportunity to benefit from form repetition in a low-rise building than in a high-rise building.

Considering the responsibility of the engineer to provide a safe and cost-effective solution to the needs of the building occupant and owner, it becomes clear that, for the vast majority of buildings designed each year, there should be an extra effort made to provide for expediency of construction rather than efficiency of structural design. Often, the extra time needed to prepare the most efficient designs with respect to structural materials is not justified by building cost or performance improvements for low-rise buildings.

**Source: F. W. Dodge Division, McGraw-Hill Information System Company, Dodge Construction Potentials (1992).*

1.3 THE COMPLEX CODE

In 1956 the ACI 318 Code was printed on 73 small-size pages; by 1992, ACI 318 and 318R contained more than 330 large-size pages of Code and Commentary—a very substantial increase in the amount of printed material with which an engineer has to become familiar in order to design a concrete building.

To find the reasons for the proliferation in code design requirements of the last thirty-five years, it is useful to examine the extensive changes in the makeup of some of the buildings that required and prompted more complex code provisions.

1.3.1 Complex Structures Require Complex Designs

Advances in the technology of structural materials and new engineering procedures have resulted in the use of concrete in a new generation of more flexible structures, dramatically different from those for which the old codes were developed.

Thirty-five years ago, 3000 psi concrete was the standard in the construction industry. Today, concrete with 14,000 psi or higher strength is used for lower story columns and walls of very tall high-rise buildings. Grade 40 reinforcing steel has almost entirely been replaced by Grade 60 reinforcement.

Gradual switching in the 1963 and 1971 Codes from the Working Stress Design Method to the Strength Design Method permitted more efficient designs of the structural components of buildings. The size of structural sections (columns, beams, and slabs) became substantially smaller and utilized less reinforcement, resulting in a 20 to 25% reduction in structural frame costs.

While we have seen dramatic increases in the strength of materials and greater cost efficiencies and design innovations made possible by the use of the strength design method, we have, as a consequence, also created new and more complex problems. The downsizing of structural components has reduced overall building stiffness. A further reduction has resulted from the replacement of heavy exterior cladding and interior partitions with lightweight substitutes which generally do not contribute significantly to building stiffness. In particular, the drastic increase of stresses in the reinforcement at service loads from less than 20 ksi to more than 30 ksi has caused a significantly wider spread of flexural cracking at service loads in slabs and beams, with consequent increases in their deflections.

When structures were designed by the classical working stress approach, both strength and serviceability of the structure were ensured by limiting the stresses in the concrete and the reinforcement, in addition to imposing limits on slenderness ratios of the members. The introduction of strength design with the resulting slenderer members significantly lengthened the design process; in addition to designing for strength, a separate consideration of serviceability (deflections and cracking) became necessary.

We are now frequently dealing with bolder, larger, taller structures which are not only more complex, but also more flexible. Their structural behavior is characterized by larger deformations relative to member dimensions than we had experienced before. As a consequence, a number of effects which heretofore were considered secondary and could be neglected, now become primary considerations during the design process. In this category are changes in geometry of structures due to gravity and lateral loadings. The effects of shrinkage, creep, and temperature are also becoming significant and can no longer be neglected in tall or in long structures, because of their cumulative effects.

1.4 A SIMPLE CODE

The more complex buildings undoubtedly require more complex design procedures to produce safe and economical structures. However, when we look at the reality of the construction industry as discussed at the beginning of this chapter, it makes little sense to impose on structures of moderate size and height intricate design approaches that were developed to assure safety in highly complex structures. While the advances of the past decades have made it possible to build economical concrete structures reaching 1000 ft in height, the makeup of low-rise buildings has not changed all that significantly over the years.

It is possible to write a simplified code to be applicable to both moderately sized structures and large complex structures. However, this would require a technical conservatism in proportioning of members. While the cost of moderate structures would not be substantially affected by such an approach, the competitiveness of large complex structures could be severely impaired. To avoid such unnecessary penalties, and at the same time to stay within required safety limits, it is possible to extract from the complex code a simplified design approach that can be applied to specifically defined moderately sized structures. Such structures are characterized as having configurations and rigidity that eliminate sensitivity to secondary stresses and as having members proportioned with sufficient conservatism to be able to simplify complex code provisions.

1.5 PURPOSE OF SIMPLIFIED DESIGN

The purpose of this manual is to give practicing engineers some way of reducing the design time required for smaller projects, while still complying with the letter and the intent of the ACI Standard 318-89 (Revised 1992), *Building Code Requirements for Reinforced Concrete.*[1.1] The simplification of design with its attendant savings in design time result from avoiding building member proportioning details and material property selections which make it necessary to consider certain complicated provisions of the ACI Standard. These situations can often be avoided by making minor changes in the design approach. In the various chapters of this book, specific recommendations are made to accomplish this goal.

The simplified design procedures presented in this manual are an attempt to satisfy the various design considerations that need to be addressed in the structural design and detailing of primary framing members of a reinforced concrete building—by the simplest and quickest procedures possible. The simplified design material is intended for use by experienced engineers well-versed in the design principles of reinforced concrete and completely familiar with the design provisions of ACI 318. As noted above, this manual has been written solely as a design timesaver; that is, to simplify design procedures using the provisions of ACI 318 for reinforced concrete buildings of moderate size and height.

1.6 SCOPE OF SIMPLIFIED DESIGN

The simplified design approach presented in this manual should be used within the general guidelines and limitations given in this section. In addition, appropriate guidelines and limitations are given within the Chapters for proper application of specific simplifying design procedures.

- Type of Construction: Conventionally reinforced cast-in-place construction. Prestressed and precast construction are not addressed.

- Building Size: Buildings of moderate size and height with usual spans and story heights. Maximum building plan dimension should be in the range of 200 ft to 250 ft to reduce effects of shrinkage and temperature to manageable levels.[1.2] Maximum building height should be in the range of 4 to 6 stories to justify the economics of simplified design.

- Materials: Normal weight concrete* $f'_c = 4000$ psi
  Deformed reinforcing bars $f_y = 60{,}000$ psi

Both material strengths are readily available in the market place and will result in members that are durable** and perform structurally well.† One set of material parameters greatly simplifies the presentation of design aids. The 4000/60,000 strength combination is used in all simplified design expressions and design aids presented in this manual with the following exceptions: the simplified thickness design for footings and the tables for development lengths consider both $f'_c = 3000$ psi and $f'_c = 4000$ psi.

In most cases, the designer can easily modify the simplified design expressions for other material strengths. Also, welded wire fabric and lightweight concrete may be used with the simplified design procedures, with appropriate modification as required by ACI 318.

- Loadings: Design dead load, live load, and wind forces are in accordance with *American Society of Civil Engineers Minimum Design Loads for Buildings and Other Structures* (ASCE 7-88)[1.4], with reductions in live loads as permitted in ASCE 7-88. The building code having jurisdiction in the locality of construction should be consulted for any possible differences in design loads from those given in ASCE 7-88.

If resistance to earthquake-induced forces, earth or liquid pressure, impact effects, or structural effects of differential settlement, shrinkage, or temperature change need to be included in design, such effects are to be included separately, in addition to the effects of dead load, live load, and wind forces (see ACI 9.2.3 through 9.2.7). Also, effects of forces due to snow loads, rain loads (ponding), and fixed service equipment (concentrated loads) are to be considered separately where applicable (ACI 8.2). Exposed exterior columns or open structures may require consideration of temperature change effects which are beyond the scope of this manual. Additionally, the durability requirements given in ACI Chapter 4 must be considered in all cases (see Section 1.7 of this manual).

- Design Method: All simplified design procedures comply with provisions of *Building Code Requirements for Reinforced Concrete* (ACI 318-89), (Revised 1992) using appropriate load factors and strength reduction factors as specified in ACI 9.2 and 9.3. References to specific ACI Code provisions are noted (e.g., ACI 9.2 refers to ACI 318-89 (Revised 1992), Section 9.2).

1.7 BUILDING EXAMPLES

To illustrate application of the simplified design approach presented in this manual, two building examples are included. Example No. 1 is a 3-story building with one-way joist slab and column framing. Two alternate joist floor systems are considered: (1) standard pan joist and (2) wide-module joist. The building of Example No. 2 is a 5-story building with two-way flat plate and column framing. Two alternate wind-force resisting systems are considered: (1) slab and column framing with spandrel beams and (2) structural walls. In all cases, it is assumed that the members will not be exposed to freezing and thawing, deicing chemicals, and severe levels of sulfates. Therefore, a concrete compressive strength of $f'_c = 4000$ psi can be used for all members. ACI Chapter 4 should be consulted if one or more of these aspects must be considered. In some cases, f'_c must be larger than 4000 psi to achieve adequate durability.

**Carbonate aggregate has been assumed for purposes of fire resistance.*

***This applies to members which are not exposed to 1) freezing and thawing in a moist condition, 2) deicing chemicals and 3) severe levels of sulfates (see ACI Chapter 4).*

†A cost analysis has shown that for gravity loads, concrete floor systems with $f'_c = 4000$ psi are more economical than ones with higher concrete strengths.[1.3]

To illustrate simplified design, typical structural members of the two buildings (beams, slabs, columns, walls, and footings) are designed by the simplified procedures presented in the various chapters of this manual. Guidelines for determining preliminary member sizes and required fire resistance are given in Section 1.8.

1.7.1 BUILDING NO. 1—3-STORY PAN JOIST CONSTRUCTION

(1) Floor system: one-way joist slab
Alternate (1)—standard pan joists
Alternate (2)—wide-module joists

(2) Wind-force resisting system: beam and column framing

(3) Load data: roof LL = 12 psf
DL = 105 psf (assume 95 psf joists and beams + 10 psf roofing and misc.)

floors LL = 60 psf
DL = 130 psf (assume 100 psf joists and beams + 20 psf partitions + 10 psf ceiling and misc.)

(4) Preliminary sizing:
Columns interior = 18 × 18 in.
exterior = 16 × 16 in.

Width of spandrel beams = 20 in.
Width of interior beams = 36 in.

(5) Fire resistance requirements:
floors: Alternate (1)—1 hour
Alternate (2)—2 hours*
roof: 1 hour
columns: 1 hour **

Figure 1-3 shows the plan and elevation of Building #1.

**In some cases, floors may be serving as an "occupancy separation" and may require a higher rating based on building type of construction. For example, there may be a mercantile or parking garage on the lowest floor.*
***Columns supporting two hour rated floor, as in Alternate (2), are required to have a two hour rating.*

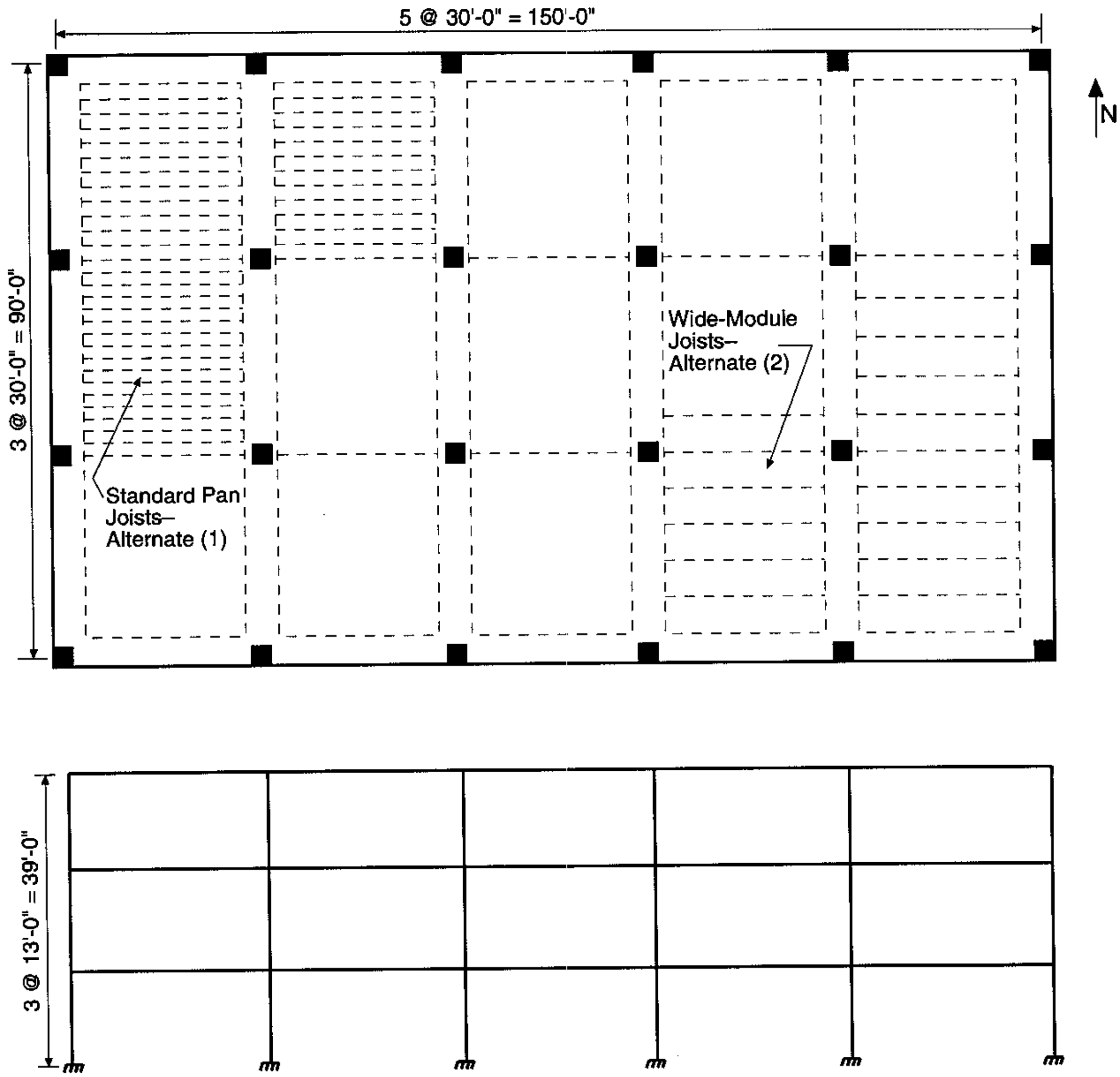


Figure 1-3 Plan and Elevation of Building #1

1.7.2 BUILDING NO. 2—5-STORY FLAT PLATE CONSTRUCTION

(1) Floor system: two-way flat plate - with spandrel beams for Alternate (1)

(2) Wind-force resisting system:
Alternate (1)—slab and column framing
Alternate (2)—structural walls

(3) Load data: roof LL = 20 psf
DL = 122 psf

floors LL = 50 psf
DL* = 142 psf (9 in. slab)
136 psf (8.5 in. slab)

(4) Preliminary sizing:
Slab (with spandrels) = 8.5 in.
Slab (without spandrels) = 9 in.

Columns interior = 16 × 16 in.
exterior = 12 × 12 in.

Spandrels = 12 × 20 in.

(5) Fire resistance requirements:

floors:	2 hours
roof:	1 hour
columns:	2 hours
shearwalls:**	2 hours

Figure 1-4 shows the plan and elevation of Building #2.

Assume 20 psf partitions + 10 psf ceiling and misc.
**Assume interior portions of walls enclose exit stairs.*

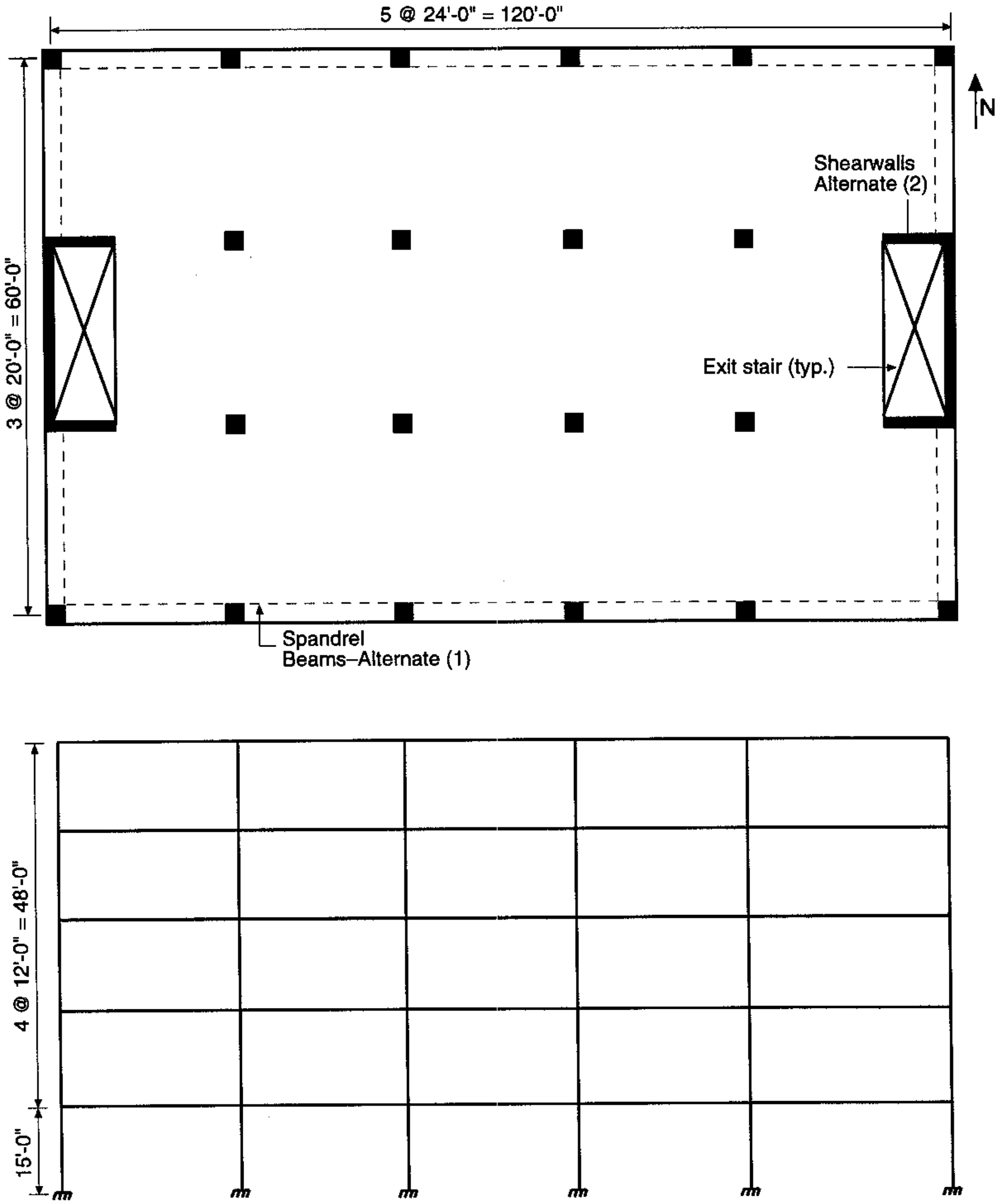


Figure 1-4 Plan and Elevation of Building #2

1.8 PRELIMINARY DESIGN

Preliminary member sizes are usually required to perform the initial frame analysis and/or to obtain initial quantities of concrete and reinforcing steel for cost estimating. Practical initial member sizes are necessary even when a computer analysis is used to determine the load effects on a structure. The guidelines for preliminary design given in the following sections are applicable to buildings of moderate size and height. These guidelines were used to obtain the preliminary sizes listed in Sections 1.7.1 and 1.7.2 for the two example buildings. Chapters 8 and 9 list additional guidelines to achieve overall economy.

1.8.1 Floor Systems

Various factors must be considered when choosing a floor system. The magnitude of the superimposed loads and the bay size (largest span length) are usually the most important variables to consider in the selection process. Fire resistance is also very important (see Section 1.8.5). Before specifying the final choice for the floor system, it is important to ensure that it has at least the minimum fire resistance rating prescribed in the governing building code.

In general, different floor systems have different economical span length ranges for a given total factored load. Also, each system has inherent advantages and disadvantages which must be considered for a particular project. Since the floor system (including its forming) accounts for a major portion of the overall cost of a structure, the type of system to be utilized must be judiciously chosen in every situation.

Figures 1-5 and 1-6 can be used as a guide in selecting a preliminary floor system with $f'_c = 4000$ psi.[1.3] A relative cost index and an economical square bay size range are presented for each of the floor systems listed. It is evident that the one-way joist system for Building #1 and the flat plate system for Building #2 are reasonable choices for the respective spans and loads. In general, an exact cost comparison should be performed to determine the most economical system for a given building.

Once a particular floor system has been chosen, preliminary sizes must be determined for the members in the system. For one-way joists and beams, deflection will usually govern. Therefore, ACI Table 9.5(a) should be used to obtain the preliminary depth of members that are not supporting or attached to partitions and other construction likely to be damaged by deflection. The width of the member can then be determined by the appropriate simplified equation given in Chapter 3. Whenever possible, available standard sizes should be specified; this size should be repeated throughout the entire structure as often as possible. For overall economy in a standard joist system, the joists and the supporting beams must have the same depth.

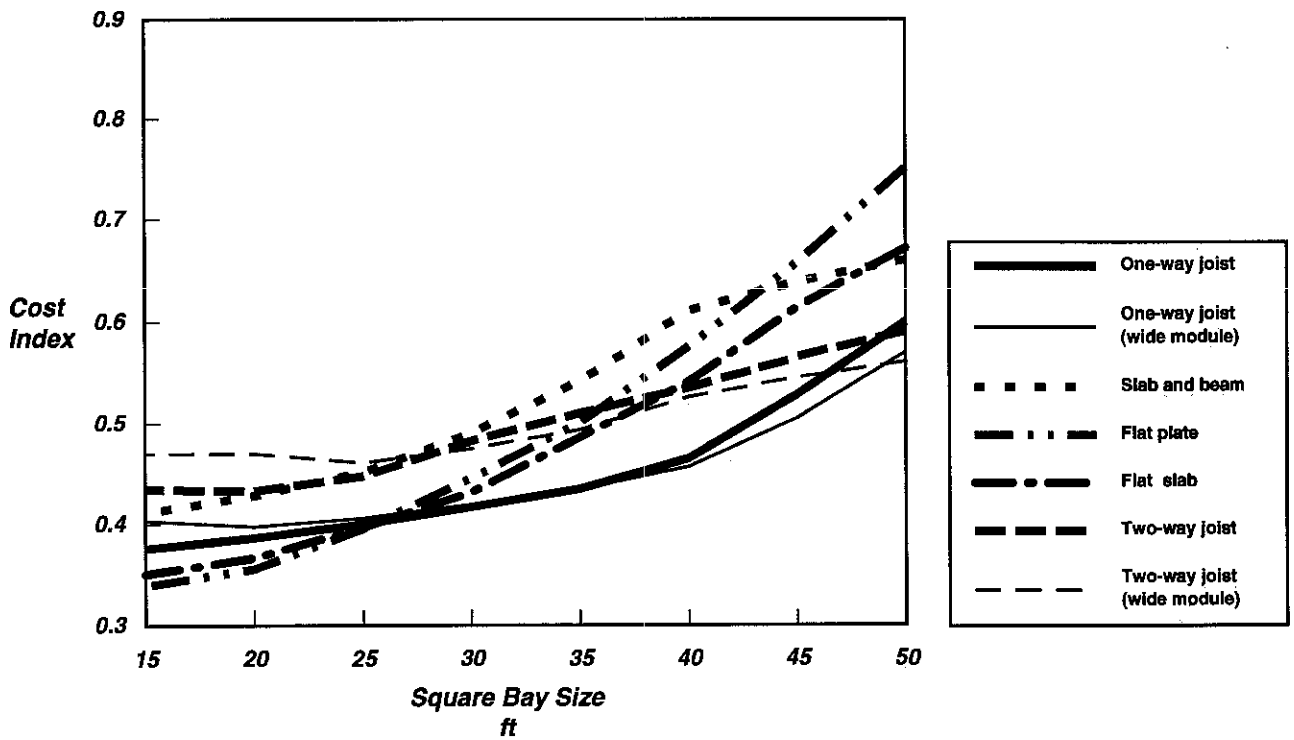
For flat plates, the thickness of the slab will almost always be governed by two-way (punching) shear. Figure 1-7 can be used to obtain a preliminary slab thickness based on two-way shear at an interior square column and f'_c = 4000 psi. For a total factored load w_u (psf) and the ratio of the tributary area of the column A to the column area c_1^2, a value of d/c_1, can be obtained from the figure. Note that d is the distance from the compression face of the slab to the centroid of the reinforcing steel. The preliminary thickness of the slab h can be determined by adding 1.25 in. to the value of d (see Chapter 4).

It is important to note that the magnitude of the unbalanced moment at an interior column is usually small. However, at an edge column, the shear stress produced by the unbalanced moment can be as large as or larger than the shear stress produced by the direct shear forces. Consequently, in most cases, the preliminary slab thickness determined from Fig. 1-7 will have to be increased in order to accommodate the additional shear stress at the edge columns. Exactly how much of an increase is required depends on numerous factors. In general, the slab thickness needs to be increased by about 15-20%; the shear stress can be checked at the edge columns after the nominal moment resistance of the column strip is determined (ACI 13.6.3.6).

Live Load = 50 psf

Superimposed Dead Load = 20 psf

f'_c = 4000 psi



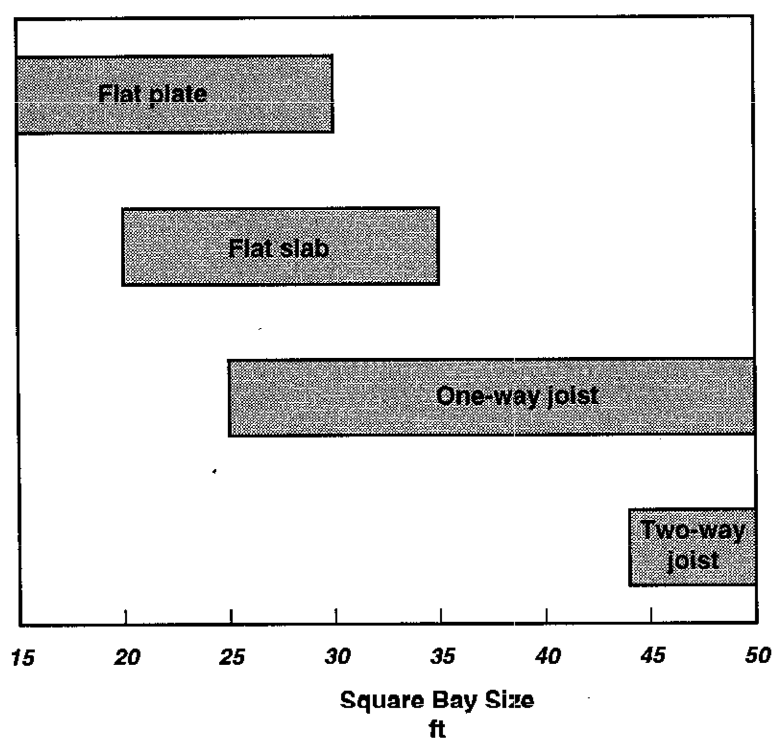
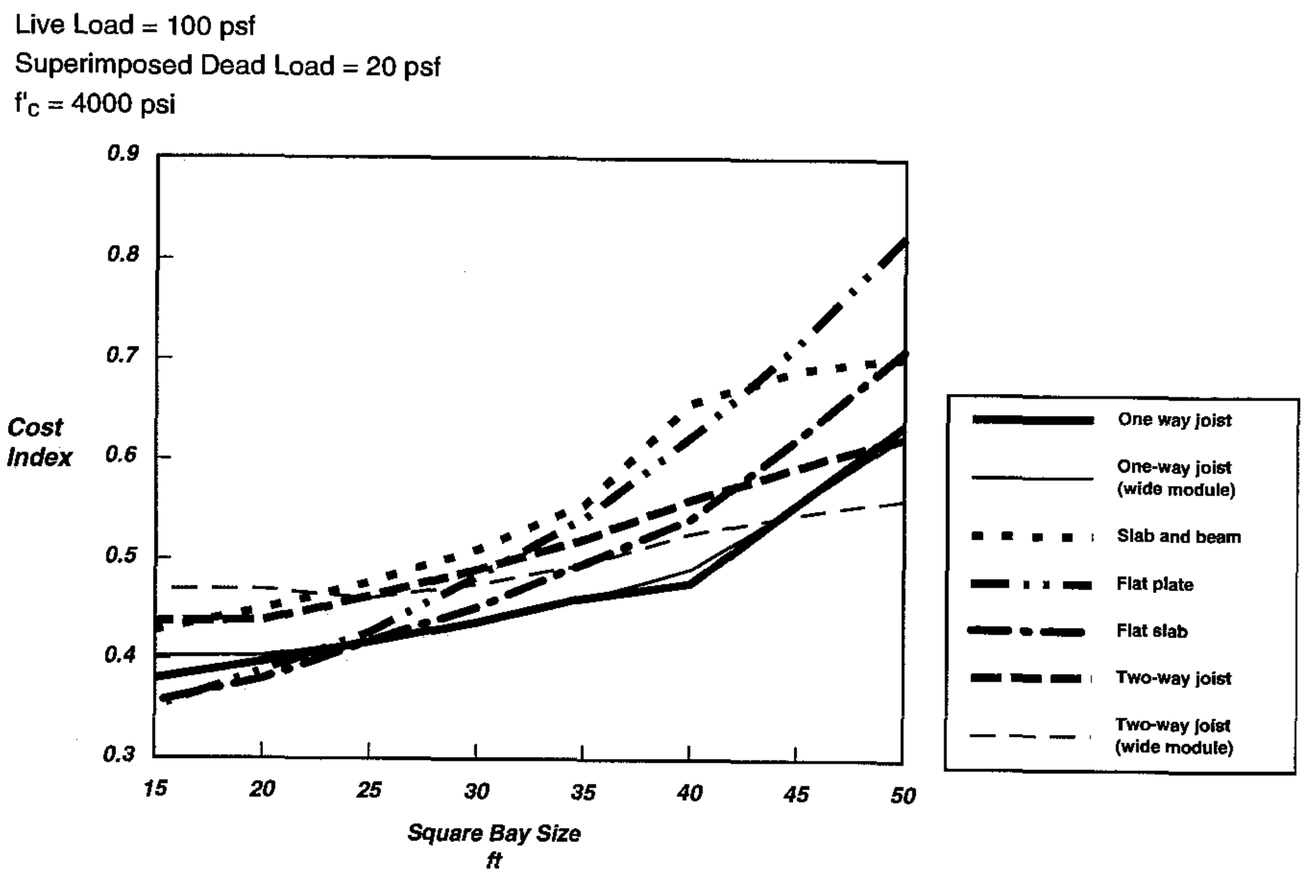


Figure 1-5 Cost Comparison of Various Floor Systems, Live Load = 50 psf[1.3]



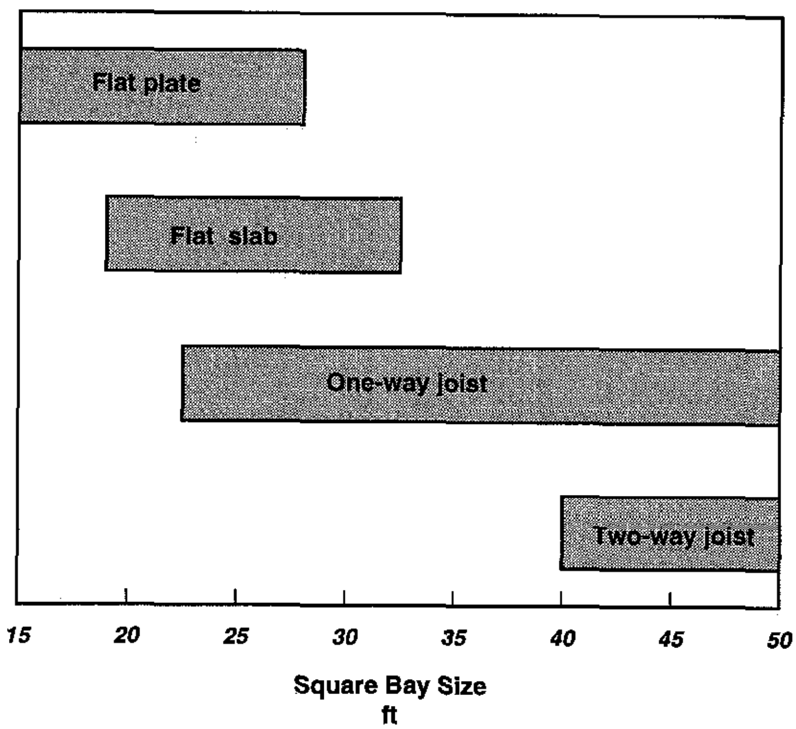


Figure 1-6 Cost Comparison for Various Floor Systems, Live Load = 100 psf[1.3]

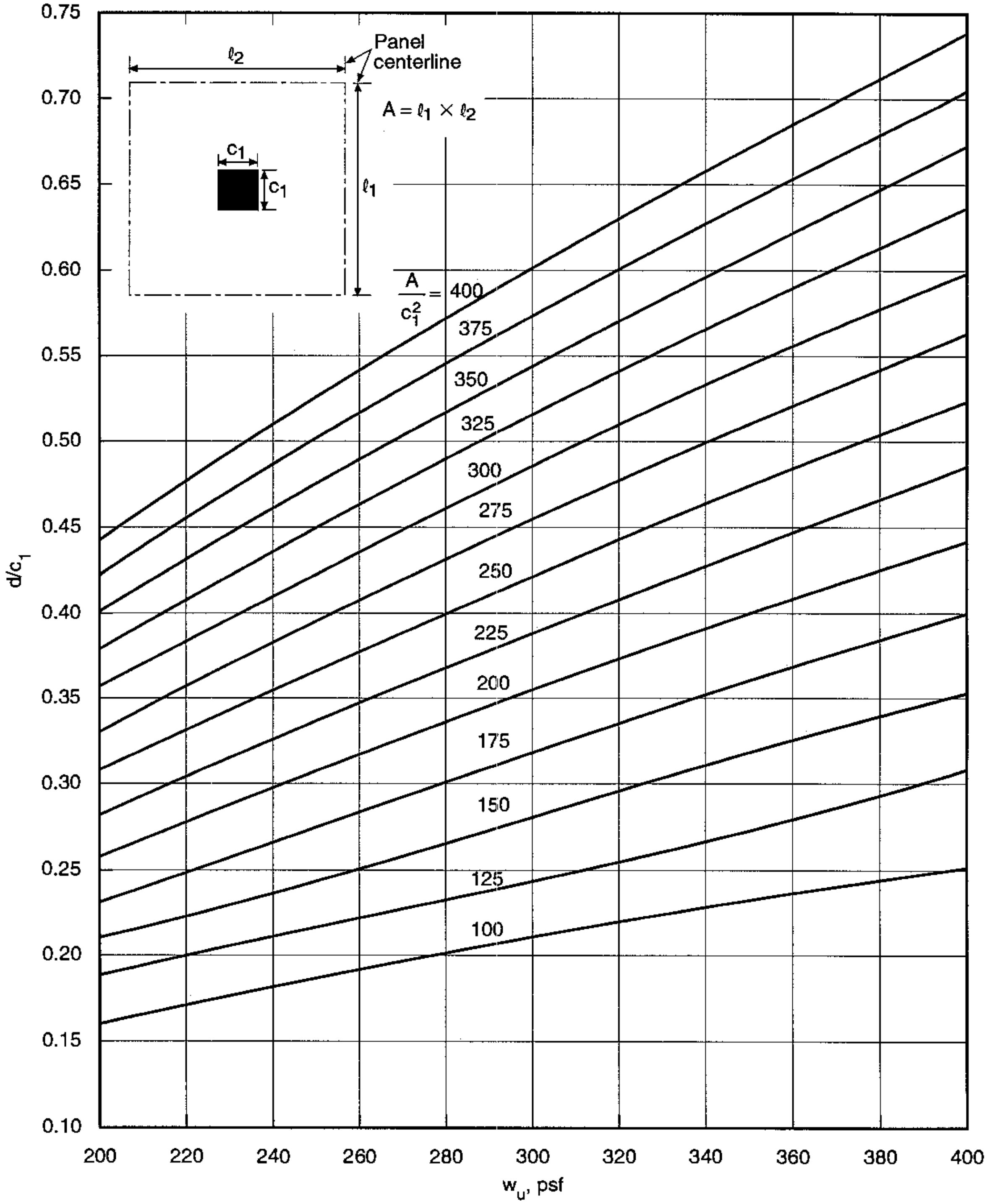


Figure 1-7 Preliminary Design Chart for Slab Thickness Based on Two-Way Shear at an Interior Square Column (f_c' = 4000 psi)

When increasing the overall slab thickness is not possible or feasible, drop panels can be provided at the column locations where two-way shear is critical. Chapter 4 gives ACI 318 provisions for minimum drop panel dimensions.

In all cases, the slab thickness must be larger than the applicable minimum thickness given in ACI 9.5.3. Figure 4-3 may be used to determine the minimum thickness as a function of the clear span ℓ_n for the various two-way systems shown.

1.8.2 Columns

For overall economy, the dimensions of a column should be determined for the load effects in the lowest story of the structure and should remain constant for the entire height of the building; only the amounts of reinforcement should vary with respect to height.* The most economical columns usually have reinforcement ratios in the range of 1-2%. In general, it is more efficient to increase the column size than to increase the amount of reinforcement.

Columns in a frame that is braced by shearwalls (no sidesway) are subjected to gravity loads only. Initial column sizes may be obtained from design aids such as the one given in Fig. 5-1: assuming a reinforcement ratio in the range of 1-2%, a square column size can be determined for the total factored axial load P_u in the lowest story. Once an initial size is obtained, it should be determined if the effects of slenderness need to be considered. If feasible, the size of the column should be increased so as to be able to neglect slenderness effects.

When a frame is not braced by shearwalls (sidesway), the columns must be designed for the combined effects of gravity and wind loads. In this case, a preliminary size can be obtained for a column in the lowest level from Fig. 5-1 assuming that the column carries gravity loads only. The size can be chosen based on 1% reinforcement in the column; in this way, when wind loads are considered, the area of steel can usually be increased without having to change the column size. The design charts given in Figs. 5-16 through 5-23 may also be used to determine the required column size and reinforcement for a given combination of factored axial loads and moments. Note that slenderness effects can have a significant influence on the amount of reinforcement that is required for a column in an unbraced frame; for this reason, the overall column size should be increased (if possible) to minimize the effects of slenderness.

1.8.3 Shearwalls

For buildings of moderate size and height, a practical range for the thickness of shearwalls is 8 to 10 in. The required thickness will depend on the length of the wall, the height of the building, and the tributary wind area of the wall. In most cases, minimum amounts of vertical and horizontal reinforcement are sufficient for both shear and moment resistance.

In the preliminary design stage, the shearwalls should be symmetrically located in the plan (if possible) so that torsional effects on the structure due to wind loads are minimized.

1.8.4 Footings

The required footing sizes can usually be obtained in a straightforward manner. In general, the base area is determined by dividing the total service (unfactored) loads from the column by the allowable (safe) soil pressure. In buildings without shearwalls, the maximum pressure due to the combination of gravity and wind loads must also be checked. The required thickness may be obtained for either a reinforced or a plain footing by using the appropriate simplified equation given in Chapter 7.

**In tall buildings, the concrete compressive strength usually varies along the height as well.*

1.8.5 Fire Resistance

To insure adequate resistance to fire, minimum thickness and cover requirements are specified in building codes as a function of the required fire resistance rating. Two hours is a typical rating for most members; however, the local building code should be consulted for the ratings which apply to a specific project.

Member sizes that are necessary for structural requirements will usually satisfy the minimum requirements for fire resistance as well (see Tables 10-1 and 10-2). Also, the minimum cover requirements specified in ACI 7.7 will provide at least a three hour fire resistance rating for restrained floor members and columns (see Tables 10-3, 10-4, and 10-6).

It is important to check the fire resistance of a member immediately after a preliminary size has been obtained based on structural requirements. Checking the fire resistance during the preliminary design stage eliminates the possibility of having to completely redesign the member (or members) later on.

In the examples that appear in the subsequent chapters, the applicable fire resistance requirements tabulated in Chapter 10 are checked for all members immediately after the preliminary sizes are obtained. The required fire resistance ratings for both example buildings are listed in Section 1.7.

References

1.1 *Building Code Requirements for Reinforced Concrete (ACI 318-89) (Revised 1992) and Commentary—ACI 318R-89 (Revised 1992)*, American Concrete Institute, Detroit, Michigan, 1992.

1.2 *Building Movements and Joints*, EB086, Portland Cement Association, Skokie, Illinois, 1982, 64 pp.

1.3 *Concrete Floor Systems–Guide to Estimating and Economizing,* SP041, Portland Cement Association, Skokie, Illinois, 1990, 33 pp.

1.4 *American Society of Civil Engineers Minimum Design Loads for Buildings and Other Structures*, ASCE 7-88, American Society of Civil Engineers, New York, N.Y., 1990, 94 pp.

Chapter 2

Simplified Frame Analysis

2.1 INTRODUCTION

The final design of the structural components in a building frame is based on maximum moment, shear, axial load, torsion and/or other load effects, as generally determined by an elastic frame analysis (ACI 8.3). For building frames of moderate size and height, preliminary and final designs will often be combined. Preliminary sizing of members, prior to analysis, may be based on designer experience, design aids, or simplified sizing expressions suggested in this manual.

Analysis of a structural frame or other continuous construction is usually the most difficult part of the total design. For gravity load analysis of continuous one-way systems (beams and slabs), the approximate moments and shears given by ACI 8.3.3 are satisfactory within the span and loading limitations stated. For cases when ACI 8.3.3 is not applicable, a two-cycle moment distribution method is accurate enough. The speed and accuracy of the method can greatly simplify the gravity load analysis of building frames with usual types of construction, spans, and story heights. The method isolates one floor at a time and assumes that the far ends of the upper and lower columns are fixed. This simplifying assumption is permitted by ACI 8.8.3.

For lateral load analysis of an unbraced frame, the Portal Method may be used. It offers a direct solution for the moments and shears in the beams (or slabs) and columns, without having to know the member sizes or stiffnesses.

The simplified methods presented in this chapter for gravity load analysis and lateral wind load analysis are considered to provide sufficiently accurate results for buildings of moderate size and height. However, determination of load effects by computer analysis or other design aids are equally applicable for use with the simplified design procedures presented in subsequent chapters of this manual. For example, PCA-Frame is a general purpose structural analysis program for two- and three-dimensional structures subject to static loads which will output shears, moments, and axial forces for any combination of gravity and/or lateral loads.[2.1]

2.2 LOADING

2.2.1 Service Loads

The first step in the frame analysis is the determination of design (service) loads and wind forces as called for in the general building code under which the project is to be designed and constructed. For the purposes of this manual, design live loads (and permissible reductions in live loads) and wind loads are based on *Minimum Design Loads for Buildings and Other Structures*, ASCE 7-88[2.2]. References to specific ASCE Standard requirements are noted (ASCE 4.2 refers to ASCE 7-88, Section 4.2). For a specific project, however, the governing general building code should be consulted for any variances from ASCE 7-88. Design for earthquake-induced load effects is not considered in this manual.

Design dead loads include member self-weight, weight of fixed service equipment (plumbing, electrical, etc.) and, where applicable, weight of built-in partitions. The latter may be accounted for by an equivalent uniform load of not less than 20 psf, although this is not specifically defined in the ASCE Standard (see ASCE Commentary Section 3.4 for special considerations).

Design live loads will depend on the intended use and occupancy of the portion or portions of the building being designed. Live loads include loads due to movable objects and movable partitions temporarily supported by the building during maintenance. In ASCE Table 2, uniformly distributed live loads range from 40 psf for residential use to 250 psf for heavy manufacturing and warehouse storage. Portions of buildings, such as library floors and file rooms, require substantially heavier live loads. Live loads on a roof include maintenance equipment, workers, and materials. Also, snow loads, ponding of water, and special features, such as landscaping, must be included where applicable.

Occasionally, concentrated live loads must be included; however, they are more likely to affect individual supporting members and usually will not be included in the frame analysis (see ASCE 4.3).

Design wind loads are usually given in the general building code having jurisdiction. For both example buildings here, the calculation of wind loads is based on the procedure presented in ASCE 6.4.2.

2.2.1.1 Example: Calculation of Wind Loads - Building #2

For illustration of the ASCE procedure, wind load calculations for the main wind-force resisting system of Building #2 (5-story flat plate) are summarized below.

Wind-force resisting system:

Alternate (1) - Slab and column framing with spandrel beams
Alternate (2) - Structural walls

(1) Wind load data

Assume building located in Midwest in flat open terrain

Basic wind speed V = 80 mph	ASCE Fig. 1
Building exposure = C, open terrain	ASCE 6.5.3
Importance factor I = 1.0	ASCE Tables 1 & 5
Gust response factor G_h = 1.20 (h = 63 ft)	ASCE Table 8

Wall pressure coefficients C_p: ASCE Fig. 2

Windward—both directions $C_p = 0.8$
Leeward—E-W direction $C_p = 0.3$ ($L/B = 120/60 = 2$)
N-S direction $C_p = 0.5$ ($L/B = 60/120 = 0.5$)

Maximum velocity pressure
exposure coefficient $K_h = 1.24$ ASCE Table 6

(2) Design wind pressures in the N-S direction

Height above ground level (ft)	K_z	q_z (psf)	Windward $q_zG_hC_p$	Leeward $q_hG_hC_p$	Total p_z (psf)
60-70	1.24	20.3	19.5	12.2	31.7
50-60	1.19	19.5	18.7	12.2	30.9
40-50	1.13	18.5	17.8	12.2	30.0
30-40	1.06	17.4	16.7	12.2	28.9
25-30	0.98	16.1	15.5	12.2	27.7
20-25	0.93	15.2	14.6	12.2	26.8
15-20	0.87	14.3	13.7	12.2	25.9
0-15	0.80	13.1	12.6	12.2	24.8

Sample calculations for 0-15 ft height range:

Design wind pressure p_z = (windward) + (leeward) ASCE Table 4

$$p_z = (q_zG_hC_p) + (q_hG_hC_p)^*$$
$$p_z = 12.6 + 12.2 = 24.8 \text{ psf}$$

where $q_z = 0.00256\ K_z(IV)^2 = 0.00256 \times 0.80\ (1 \times 80)^2 = 13.1$ psf ASCE Eq. (3)
$q_zG_hC_p = 13.1 \times 1.20 \times 0.8 = 12.6$ psf (windward pressure)

and $q_h = 0.00256\ K_h(IV)^2 = 0.00256 \times 1.24\ (1 \times 80)^2 = 20.3$ psf
$q_hG_hC_p = 20.3 \times 1.20 \times 0.5 = 12.2$ psf (leeward pressure uniform for full height of building)

(3) Wind loads in the N-S direction

Using the design wind pressures p_z, assumed uniform over the incremental heights above ground level, the following equivalent wind loads are calculated at each floor level:

<u>Alternate (1)-Slab and column framing</u>
Interior frame (24 ft bay width)

roof = $[(31.7 \times 3) + (30.9 \times 3)](24/1000) = 4.51$ kips
4th = $[(30.9 \times 6) + (30.0 \times 6)](24/1000) = 8.78$ kips
3rd = $[(30.0 \times 6) + (28.9 \times 6)](24/1000) = 8.44$ kips
2nd = $[(28.9 \times 3) + (27.7 \times 5) + (26.8 \times 4)](24/1000) = 7.96$ kips
1st = $[(26.8 \times 1) + (25.9 \times 5) + (24.8 \times 7.5)](24/1000) = 8.20$ kips

<u>Alternate (2)-Structural walls</u>
Total for entire building (120 ft width)

roof = $4.51(120/24) = 22.6$ kips
4th = $8.78(120/24) = 43.9$ kips

**Internal pressures acting on windward and leeward walls cancel.*

3rd = 8.44(120/24) = 42.2 kips
2nd = 7.96(120/24) = 39.8 kips
1st = 8.20(120/24) = 41.0 kips

(4) Wind loads in the E-W direction

Using the same procedure as for the N-S direction, the following wind loads are obtained for the E-W direction:

Alternate (1)-Slab and column framing
Interior frame (20 ft bay width)

roof = 3.17 kips
4th = 6.13 kips
3rd = 5.89 kips
2nd = 5.47 kips
1st = 5.52 kips

Alternate (2)-Structural walls
Total for entire building (60 ft width)

roof = 9.51 kips
4th = 18.4 kips
3rd = 17.7 kips
2nd = 16.4 kips
1st = 16.6 kips

(5) The above wind load calculations assume a uniform design wind pressure p_z over the incremental heights above ground level as tabulated in ASCE Table 6, i.e., 0-15, 15-20, 20-25, etc. Alternatively, wind load calculations can be considerably simplified, with results equally valid (especially for low-to-moderate height buildings), by computing design wind pressures at each floor level and assuming uniform pressure between midstory heights above and below the floor level under consideration. For one- and two-story buildings, a design wind pressure computed at the roof level and assumed uniform over full building height would also seem accurate enough.

Recalculate the wind loads in the N-S direction using design wind pressures computed at each floor level:

Story height above ground level (ft)	K_z	q_z (psf)	Windward $q_zG_hC_p$	Leeward $q_hG_hC_p$	Total p_z (psf)
roof—63	1.21	19.8	19.0	11.9	30.9
4th—51	1.14	18.7	18.0	11.9	29.9
3rd—39	1.05	17.2	16.5	11.9	28.4
2nd—27	0.95	15.6	15.0	11.9	26.9
1st—15	0.80	13.1	12.6	11.9	24.5

where $q_h = 0.00256\ K_h(IV)^2 = 0.00256 \times 1.21\ (1 \times 80)^2 = 19.8$ psf
$q_hG_hC_p = 19.8 \times 1.20 \times 0.5 = 11.9$ psf

Alternate (1)-Slab and column framing
Interior frame (24 ft bay width)

roof = 30.9 × 6 × 24 = 4.45 kips
4th = 29.9 × 12 × 24 = 8.61 kips

3rd $= 28.4 \times 12 \times 24 = 8.18$ kips
2nd $= 26.9 \times 12 \times 24 = 7.75$ kips
1st $= 24.5 \times 13.5 \times 24 = 7.94$ kips

2.2.1.2 Example: Calculation of Wind Loads - Building #1

Wind load calculations for the main wind-force resisting system of Building #1 (3-story pan joist framing) are summarized below.

Wind-force resisting system: Beam and column framing

(1) Wind load data

Assume the building is located along the hurricane oceanline in flat open terrain.

Basic wind speed V = 110 mph	ASCE Fig. 1
Building exposure = C, open terrain	ASCE 6.5.3
Importance factor I = 1.05	ASCE Tables 1 & 5
Gust response factor $G_h = 1.23$ (h = 39 ft)	ASCE Table 8
Wall pressure coefficients C_p:	ASCE Fig. 2

Windward—both directions $C_p = 0.8$
Leeward— E-W direction $C_p = 0.37$ (L/B = 150/90 = 1.67)
N-S direction $C_p = 0.5$ (L/B = 90/150 = 0.6)

Maximum velocity pressure exposure coefficient $K_h = 1.05$ ASCE Table 6

(2) Summary of wind loads

N-S & E-W directions (conservatively use N-S wind loads in both directions):
Interior frame (30 ft bay width)

Roof = 11.2 kips
2nd = 20.9 kips
1st = 19.1 kips

Note: The above loads were determined using design wind pressures computed at each floor level.

2.2.2 Live Load Reduction for Columns, Beams, and Slabs

Most general building codes permit a reduction in live load for design of columns, beams and slabs to account for the probability that the total floor area "influencing" the load on a member may not be fully loaded simultaneously. Traditionally, the reduced amount of live load for which a member must be designed has been based on a tributary floor area supported by that member. According to ASCE 7-88, the magnitude of live load reduction is based on an influence area rather than a tributary area (see Commentary Section 4.8.1). For example, for an interior column, the influence area is the total floor area of the four surrounding bays (four times the traditional tributary area). For an edge column, the two adjacent bays "influence" the load effects on the column, and a corner column has an influence area of one bay. For interior beams, the influence area consists of the two adjacent panels, while for the peripheral beam it is only one panel. For two-way slabs, the influence area is equal to the panel area.

The reduced live load L per square foot of floor area supported by columns, beams, and two-way slabs having an influence area of more than 400 sq ft is:

$$L = L_o \ (0.25 + \frac{15}{\sqrt{A_I}}) \qquad \text{ASCE Eq. (1)}$$

where L_o is the unreduced design live load per square foot, and A_I is the influence area as described above. The reduced live load cannot be taken less than 50% for members supporting one floor, nor less than 40% of the unit live load L_o otherwise. For limitations on live load reduction, see ASCE 4.8.2.

Using the above expression for reduced live load, values of the reduction multiplier as a function of influence area are given in Table 2-1.

Table 2-1 Reduction Multiplier (RM) for Live Load = $(0.25 + 15/\sqrt{A_I})$

Influence Area A_I (ft^2)	RM	Influence Area A_I (ft^2)	RM
400[a]	1.000	5600	0.450
800	0.780	6000	0.444
1200	0.683	6400	0.438
1600	0.625	6800	0.432
2000	0.585	7200	0.427
2400	0.556	7600	0.422
2800	0.533	8000	0.418
3200	0.515	8400	0.414
3600	0.500[b]	8800	0.410
4000	0.487	9200	0.406
4400	0.476	9600	0.403
4800	0.467	10000	0.400[c]
5200	0.458		

[a]No live load reduction is permitted for influence area less than 400 sq ft.

[b]Maximum reduction permitted for members supporting one floor only.

[c]Maximum absolute reduction.

The live load reduction multiplier for beams and two-way slabs having an influence area of more than 400 sq ft ranges from 1.0 to 0.5. For influence areas on these members exceeding 3600 sq ft, the reduction multiplier of 0.5 remains constant.

The live load reduction multiplier for columns in multistory buildings ranges from 1.0 to 0.4 for cumulative influence areas between 400 and 10,000 sq ft. For influence areas on columns exceeding 10,000 sq ft, the reduction multiplier of 0.4 remains constant.

The above discussion on permissible reduction of live loads is based on ASCE 4.8. The governing general building code should be consulted for any difference in amount of reduction and type of members that may be designed for a reduced live load.

2.2.2.1 Example: Live Load Reductions for Building #2

For illustration, typical influence areas for the columns and the end shear walls of Building #2 (5-story flat plate) are shown in Fig. 2-1. Corresponding live load reduction multipliers are listed in Table 2-2.

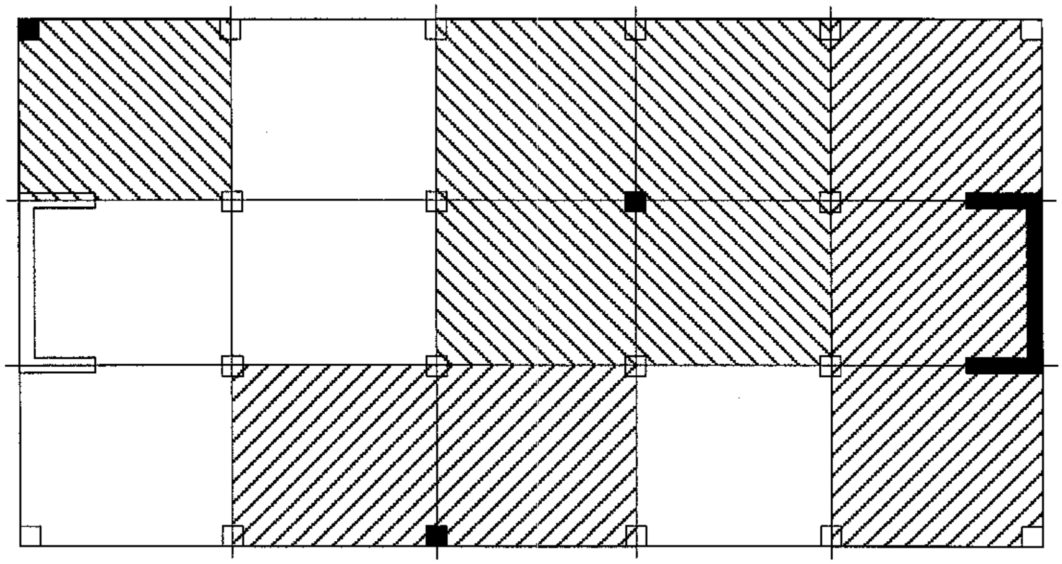


Figure 2-1 Typical Influence Areas, Building #2

Table 2-2 Reduction Multiplier (RM) for Live Loads, Building #2

Story	Interior Columns		Edge Columns		Corner Columns		End Shear Wall Units	
	A_I (ft^2)	RM	A_I (ft^2)	RM	A_I (ft^2)	RM	A_I (ft^2)	RM
5 (roof)		*		*		*		*
4	1920	0.59	960	0.73	480	0.94	1440	0.65
3	3840	0.49	1920	0.59	960	0.73	2880	0.53
2	5760	0.45	2880	0.53	1440	0.65	4320	0.48
1	7680	0.42	3840	0.49	1920	0.59	5760	0.45

*No reduction permitted for roof live load (ASCE 4.8.2); the roof should not be included in the influence areas of the floors below.

For the interior columns, the reduced live load is $L = 0.42\ L_o$ at the first story (A_I = 4 bay areas × 4 stories = 20 × 24 × 4 × 4 = 7680 sq ft). The two-way slab may be designed with an RM = 0.94 (A_I = 480 sq ft for one bay area). Shear strength around the interior columns is designed for an RM = 0.59 (A_I = 1920 sq ft for 4 bay areas), and around an edge column for an RM = 0.73 (A_I = 960 sq ft for 2 bay areas). Spandrel beams could be designed for an RM = 0.94 (one bay area). If the floor system were a two-way slab with beams between columns, the interior beams would qualify for an RM = 0.73 (2 bay areas).

2.2.3 Factored Loads

The strength method of design, using factored loads to proportion members, is used exclusively in this manual. The design (service) loads must be increased by specified load factors (ACI 9.2), and factored loads must be combined in load combinations depending on the types of loads being considered.

For design of beams and slabs, the factored load combination used most often is:

$$U = 1.4D + 1.7L \qquad \text{ACI Eq. (9-1)}$$

For a frame analysis with live load applied only to a portion of the structure, i.e., alternate spans (ACI 8.9), the factored loads to be applied would be computed separately using the appropriate load factor for each load. However, for approximate methods of analysis (such as the approximate moment and shear expressions of ACI

8.3.3), where live load can be assumed to be applied over the entire structure, it may be expedient to use a composite load factor, C, where U = C(D + L). An exact value of C can be computed for any combination of dead load and live load as:

$$C = \frac{1.4D + 1.7L}{D + L} = \frac{1.4 + 1.7(L/D)}{1 + L/D}$$

Table 2-3 gives composite load factors for various types of floor systems and building occupancies. A composite load factor may be taken directly from the table and interpolated, if necessary, for any usual L/D ratio. Alternately, a single value of C = 1.6 could be used without much error. An overdesign of about 9% for low L/D ratios and an underdesign of 1.6% for L/D = 3 would result. Since the majority of building designs will not involve L/D ratios in the very low range or approaching the value of 3.0, use of the single value of C = 1.6 for all designs can provide an effective simplification.

Table 2-3 Composite Load Factors, C

Floor System	D (psf)	Use	L (psf)	L/D	C	Approx. C	% diff.
9" Flat plate	120	roof	30	0.25	1.46	1.6	9.6
3' Waffle (25' span)	120	resid.	40	0.33	1.47	1.6	8.8
5' Waffle (50' span)	175	office	70	0.40	1.49	1.6	7.4
5" Flat plate	70	roof	35	0.50	1.50	1.6	6.7
5" Flat plate	60	resid.	40	0.67	1.52	1.6	5.3
5' Waffle (25' span)	100	office	100	1.00	1.55	1.6	3.2
5' Waffle (30' span)	173	indust.	250	1.45	1.58	1.6	1.3
3' Waffle (20' span)	125	indust.	250	2.00	1.60	1.6	-
Joists (15' × 25')	100	indust.	250	2.50	1.61	1.6	-0.6
5' Waffle (25' span)	133	library	400	3.00	1.63	1.6	-1.6

There is one final consideration when using factored loads to proportion members. The designer has the choice of multiplying the service loads by the load factors before computing the factored load effects (moments, shears, etc.), or computing the effects from the service loads and then multiplying the effects by the load factors. For example, in the computation of bending moment for dead and live loads [U = 1.4D + 1.7L or U = 1.6 (D+L)], the designer may (1) determine $w_u = 1.4\, w_d + 1.7\, w_\ell$ and then compute the factored moments using w_u; or (2) compute the dead and live load moments using service loads and then determine the factored moments as $M_u = 1.4M_d + 1.7\, M_\ell$. Both analysis procedures yield the same answer. It is important to note that the second alternative is much more general than the first; thus, it is more suitable to computer analysis, especially when more than one load combination must be investigated.

2.3 FRAME ANALYSIS BY COEFFICIENTS

The ACI Code provides a simplified method of analysis for both one-way construction (ACI 8.3.3) and two-way construction (ACI 13.6). Both simplified methods yield moments and shears based on coefficients. Each method will give satisfactory results within the span and loading limitations stated. The direct design method for two-way slabs is discussed in Chapter 4.

2.3.1 Continuous Beams and One-Way Slabs

When beams and one-way slabs are part of a frame or continuous construction, ACI 8.3.3 provides approximate moment and shear coefficients for gravity load analysis. The approximate coefficients may be used as long as all of the conditions illustrated in Fig. 2-2 are satisfied: (1) There must be two or more spans, approximately equal in length, with the longer of two adjacent spans not exceeding the shorter by more than 20 percent; (2) loads must

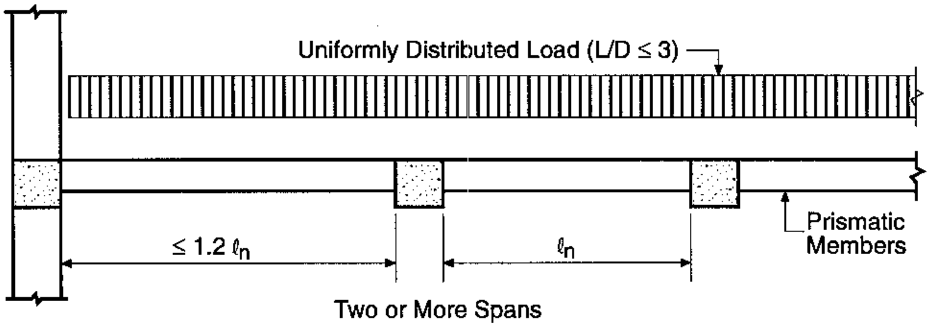


Figure 2-2 Conditions for Analysis by Coefficients (ACI 8.3.3)

be uniformly distributed, with the service live load not more than 3 times the dead load (L/D ≤ 3); and (3) members must have uniform cross section throughout the span. Also, no redistribution of moments is permitted (ACI 8.4). The moment coefficients defined in ACI 8.3.3 are shown in Figs. 2-3 through 2-6. In all cases, the shear in end span members at the interior support is taken equal to $1.15w_u\ell_n/2$. The shear at all other supports is $w_u\ell_n/2$ (see Fig. 2-7). w_u is the combined factored load for dead and live loads, $w_u = 1.4w_d + 1.7w_\ell$. For beams, w_u is the uniformly distributed load per unit length of beam (plf), and the coefficients yield total moments and shears on the beam. For one-way slabs, w_u is the uniformly distributed load per unit area of slab (psf), and the moments and shears are for slab strips one foot in width. The span length ℓ_n is defined as the clear span of the beam or slab. For negative moment at a support with unequal adjacent spans, ℓ_n is the average of the adjacent clear spans. Support moments and shears are at the faces of supports.

2.3.2 Example: Frame Analysis by Coefficients

Determine factored moments and shears for the joists of the standard pan joist floor system of Building #1 (Alternate (1)) using the approximate moment and shear coefficients of ACI 8.3.3. Joists are spaced at 3 ft on centers.

(1) Data: Width of spandrel beam = 20 in.
Width of interior beams = 36 in.
Floors: LL = 60 psf
DL = 130 psf
w_u = 1.4 (130) + 1.7 (60) = 284 psf × 3 ft = 852 plf

(2) Factored moments and shears using the coefficients from Figs. 2-3, 2-4, and 2-7 are summarized in Fig. 2-8.

2.4 FRAME ANALYSIS BY ANALYTICAL METHODS

For continuous beams and one-way slabs not meeting the limitations of ACI 8.3.3 for analysis by coefficients, an elastic frame analysis must be used. Approximate methods of frame analysis are permitted by ACI 8.3.2 for "usual" types of buildings. Simplifying assumptions on member stiffnesses, span lengths, and arrangement of live load are given in ACI 8.6 through 8.9.

2.4.1 Stiffness

The relative stiffnesses of frame members must be established regardless of the analytical method used. Any reasonable consistent procedure for determining stiffnesses of columns, walls, beams, and slabs is permitted by ACI 8.6.

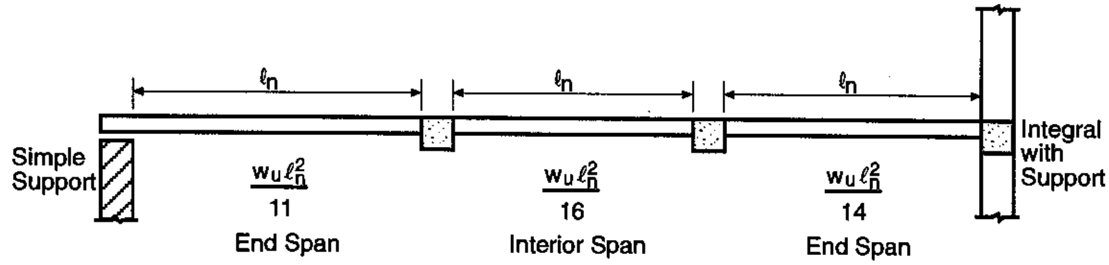


Figure 2-3 Positive Moments—All Cases

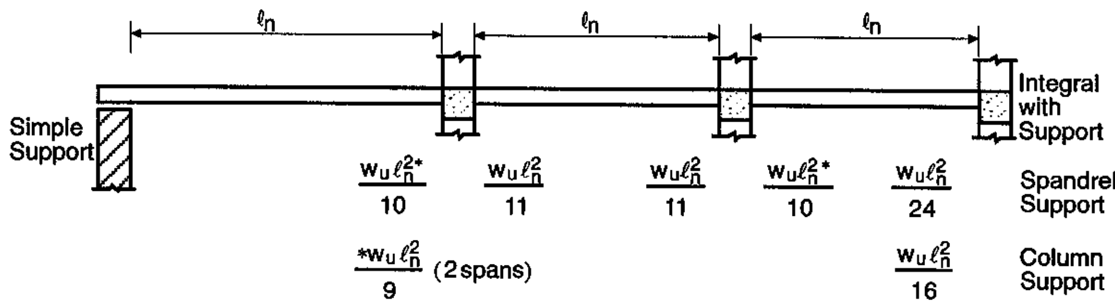


Figure 2-4 Negative Moments—Beams and Slabs

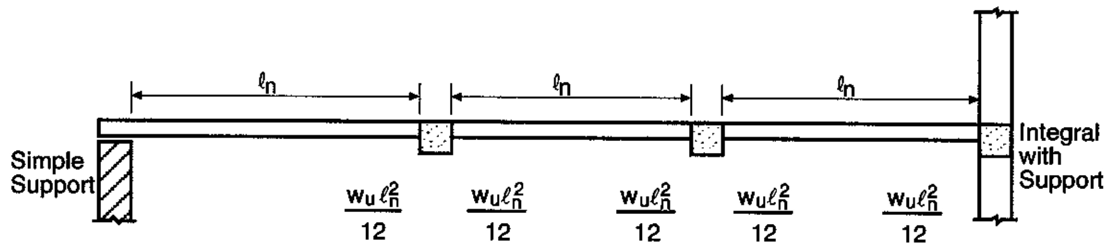


Figure 2-5 Negative Moments—Slabs with spans ≤ 10 ft

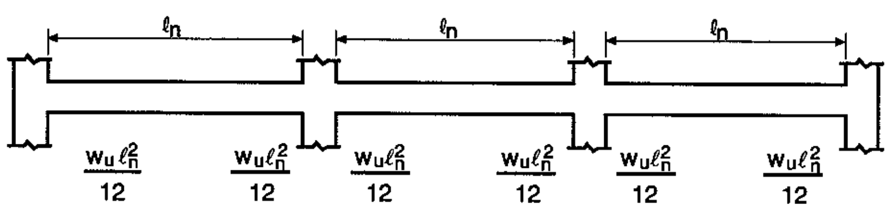


Figure 2-6 Negative Moments—Beams with Stiff Columns ($\Sigma K_c/\Sigma K_b > 8$)

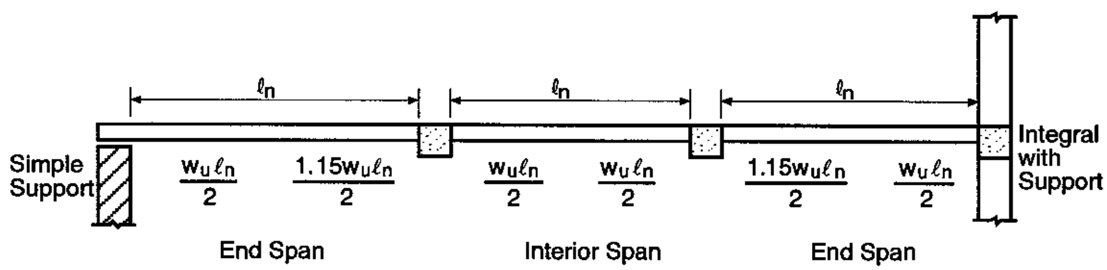
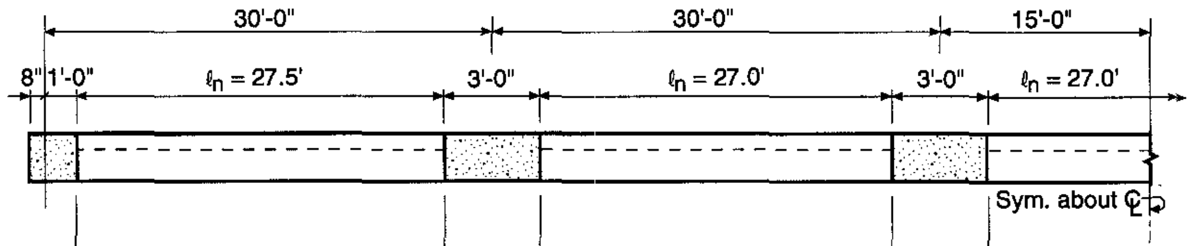


Figure 2-7 End Shears—All Cases



Total Load	$w_u\ell_n = 0.85 \times 27.5 = 23.4$ kips		$0.85 \times 27.0 = 23.0$ kips		$0.85 \times 27.0 = 23.0$ kips
Coefficient from Fig. 2-7	1/2	1.15/2	1/2	1/2	1/2
Shear V_u	11.7 kips	13.5 kips	11.5 kips	11.5 kips	11.5 kips
$w_u\ell_n^2$	643.5 ft-kips		621.0 ft-kips		621.0 ft-kips
Coefficient from Fig. 2-3	1/14		1/16		1/16
Pos. M_u	46.0 ft-kips		38.8 ft-kips		38.8 ft-kips
ℓ_n	27.5 ft	27.25 ft*	27.25 ft*	27.0 ft	27.0 ft
$w_u\ell_n^2$	643.5 ft-kips	631.2 ft-kips	631.2 ft-kips	621.0 ft-kips	621.0 ft-kips
Coefficient from Fig. 2-4	1/24	1/10	1/11	1/11	1/11
Neg. M_u	26.8 ft-kips	63.1 ft-kips	57.4 ft-kips	56.5 ft-kips	56.5 ft-kips

*Average of adjacent clear spans

Figure 2-8 Factored Moments and Shears for the Joist Floor System of Building #1 (Alternate (1))

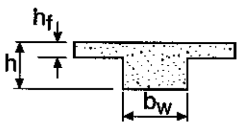
The selection of stiffness factors will be considerably simplified by the use of Tables 2-4 and 2-5. The stiffness factors are based on gross section properties (neglecting any reinforcement) and should yield satisfactory results for buildings within the size and height range addressed in this manual. In most cases where an analytical procedure is required, stiffness of T-beam sections will be needed. The relative stiffness values K given in Table 2-4 allow for the effect of the flange by doubling the moment of inertia of the web section (b_wh). For values of h_f/h between 0.2 and 0.4, the multiplier of 2 corresponds closely to a flange width equal to six times the web width. This is considered a reasonable allowance for most T-beams.[2.3] For rectangular beam sections, the tabulated values should be divided by 2. Table 2-5 gives relative stiffness values K for column sections. It should be noted that column stiffness is quite sensitive to changes in column size. The initial judicious selection of column size and uniformity from floor to floor is, therefore, critical in minimizing the need for successive analyses.

As is customary for ordinary building frames, torsional stiffness of transverse beams is not considered in the analysis. For those unusual cases where equilibrium torsion is involved, a more exact procedure may be necessary.

2.4.2 Arrangement of Live Load

According to ACI 8.9.1, it is permissible to assume that for gravity load analysis, the live load is applied only to the floor or roof under consideration, with the far ends of the columns assumed fixed. In the usual case where the

Table 2-4 Beam Stiffness Factors



Moment of inertia, excluding overhanging flanges:

$$I = \frac{b_w h^3}{12} \qquad K^* = \frac{2I}{10\ell}$$

Moment of inertia of T-section ≅ 2I

Values of K for T-beams

h	b_w	I	Span of beam, ℓ (ft)** 8	10	12	14	16	20	24	30
	6	256	6	5	4	4	3	3	2	2
	8	341	9	7	6	5	4	3	3	2
	10	427	11	9	7	6	5	4	4	3
8	11 1/2	491	12	10	8	7	6	5	4	3
	13	555	14	11	9	8	7	6	5	4
	15	640	16	13	11	9	8	6	5	4
	17	725	18	15	12	10	9	7	6	5
	19	811	20	16	14	12	10	8	7	5
	6	500	13	10	8	7	6	5	4	3
	8	667	17	13	11	10	8	7	6	4
	10	833	21	17	14	12	10	8	7	6
10	11 1/2	958	24	19	16	14	12	10	8	6
	13	1083	27	22	18	15	14	11	9	7
	15	1250	31	25	21	18	16	13	10	8
	17	1417	35	28	24	20	18	14	12	9
	19	1583	40	32	26	23	20	16	13	11
	6	864	22	17	14	12	11	9	7	6
	8	1152	29	23	19	16	14	12	10	8
	10	1440	36	29	24	21	18	14	12	10
12	11 1/2	1656	41	33	28	24	21	17	14	11
	13	1872	47	37	31	27	23	19	16	12
	15	2160	54	43	36	31	27	22	18	14
	17	2448	61	49	41	35	31	25	20	16
	19	2736	68	55	46	39	34	27	23	18
	6	1372	34	27	23	20	17	14	11	9
	8	1829	46	37	30	26	23	18	15	12
	10	2287	57	46	38	33	29	23	19	15
14	11 1/2	2630	66	53	44	38	33	26	22	18
	13	2973	74	59	50	42	37	30	25	20
	15	3430	66	69	57	49	43	34	29	23
	17	3887	97	78	65	56	49	39	32	26
	19	4345	109	87	72	62	54	43	36	29
	6	2048	51	41	34	29	26	20	17	14
	6	2731	68	55	46	39	34	27	23	18
	10	3413	85	68	57	49	43	34	28	23
16	11 1/2	3925	98	79	65	56	49	39	33	26
	13	4437	111	89	74	63	55	44	37	30
	15	5120	128	102	85	73	64	51	43	34
	17	5603	145	116	97	83	73	56	48	39
	19	6485	162	130	108	93	81	65	54	43
	6	2916	73	58	49	42	36	29	24	19
	8	3888	97	78	65	56	49	39	32	26
	10	4860	122	97	81	69	61	49	41	32
18	11 1/2	5589	140	112	93	80	70	56	47	37
	13	6318	158	126	105	90	79	63	53	42
	15	7290	182	146	122	104	91	73	61	49
	17	8262	207	165	138	118	103	83	69	55
	19	9234	231	185	154	132	115	92	77	62
	6	4000	100	80	67	57	50	40	33	27
	8	5333	133	107	89	76	67	53	44	38
	10	6667	167	133	111	95	83	67	56	44
20	11 1/2	7667	192	153	128	110	96	77	64	51
	13	8667	217	173	144	124	108	87	72	58
	15	10000	250	200	167	143	125	100	83	67
	17	11333	283	227	189	162	142	113	94	76
	19	12667	317	253	211	181	158	127	106	84
	6	5324	133	106	89	76	67	53	44	36
	6	7099	177	142	118	101	89	71	59	47
	10	8873	222	177	148	127	111	89	74	59
22	11 1/2	10204	255	204	170	146	128	102	85	68
	13	11535	288	231	192	165	144	115	96	77
	15	13310	333	266	222	190	166	133	111	89
	17	15085	377	302	251	215	189	151	126	101
	19	16859	421	337	281	241	211	169	141	112

h	b_w	I	Span of beam, ℓ (ft)** 8	10	12	14	16	20	24	30
	8	9216	230	185	155	130	115	90	75	60
	10	11520	290	230	190	165	145	115	95	75
	11 1/2	13248	330	265	220	190	185	130	110	90
24	13	14976	375	300	250	215	185	150	125	100
	15	17280	430	345	290	245	215	175	145	115
	17	19584	490	390	325	280	245	195	165	130
	19	21888	545	440	365	315	275	220	180	145
	21	24192	605	485	405	345	300	240	200	160
	8	11717	295	235	195	165	145	115	100	80
	10	14647	365	295	245	210	185	145	120	100
	11 1/2	16844	420	335	280	240	210	170	140	110
26	13	19041	475	380	315	270	240	190	160	125
	15	21970	550	440	365	315	275	220	185	145
	17	24899	620	500	415	355	310	250	205	165
	19	27892	695	555	465	400	350	280	230	185
	21	30758	770	615	515	440	385	310	255	205
	8	14635	365	295	245	210	185	145	120	100
	10	18293	455	365	305	260	230	185	150	120
	11 1/2	21037	525	420	350	300	265	210	175	140
28	13	23781	595	475	395	340	295	240	200	160
	15	27440	685	550	455	390	345	275	230	185
	17	31099	775	620	520	445	390	310	260	205
	19	34757	870	695	580	495	435	350	290	230
	21	38416	960	770	640	550	480	385	320	255
	8	18000	450	360	300	255	225	180	150	120
	10	22500	565	450	375	320	280	225	190	150
	11 1/2	25875	645	520	430	370	325	260	215	175
30	13	29250	730	585	490	420	365	295	245	195
	15	33750	845	675	565	480	420	340	280	225
	17	38250	955	765	640	545	480	385	320	255
	19	42750	1070	855	715	610	535	430	355	285
	21	47250	1180	945	790	675	590	475	395	315
	8	31104	780	620	520	445	390	310	260	205
	10	38880	970	780	650	555	485	390	325	260
	11 1/2	44712	1120	895	745	640	560	445	375	300
36	13	50544	1260	1010	840	720	630	505	420	335
	15	58320	1460	1170	970	835	730	585	485	390
	17	66096	1650	1320	1100	945	825	660	550	440
	19	73872	1850	1480	1230	1060	925	740	615	490
	21	81648	2040	1630	1360	1170	1020	815	680	545
	8	49392	1230	990	825	705	615	495	410	330
	10	61740	1540	1230	1030	880	770	615	515	410
	11 1/2	71001	1780	1420	1180	1010	890	710	590	475
42	13	80262	2010	1610	1340	1150	1000	805	670	535
	15	92610	2320	1850	1540	1320	1160	925	770	615
	17	104958	2620	2100	1750	1500	1310	1050	875	700
	19	117306	2930	2350	1950	1680	1470	1170	975	780
	21	129654	3240	2590	2160	1850	1620	1300	1080	865
	8	73728	1840	1470	1230	1050	920	735	615	490
	10	92180	2300	1840	1540	1320	1150	920	770	615
	11 1/2	105984	2650	2120	1770	1510	1320	1060	885	705
46	13	119808	3000	2400	2000	1710	1500	1200	1000	800
	15	138240	3460	2760	2300	1970	1730	1380	1150	920
	15	156672	3920	3130	2610	2240	1960	1570	1310	1040
	19	175104	4380	3500	2920	2500	2190	1750	1460	1170
	21	193536	4840	3870	3230	2760	2420	1940	1610	1290
	8	104976	2620	2100	1750	1500	1310	1050	875	700
	10	131220	3280	2620	2190	1880	1640	1310	1090	875
	11 1/2	150903	3770	3020	2510	2160	1890	1510	1260	1010
54	13	170586	4260	3410	2840	2440	2130	1710	1420	1140
	15	196830	4920	3490	3280	2810	2460	1970	1640	1310
	17	223074	5580	4460	3720	3190	2790	2230	1860	1490
	19	249318	6230	4990	4160	3560	3120	2490	2080	1660
	21	275562	6890	5510	4590	3940	3440	2760	2300	1840

*Coefficient 10 introduced to reduce magnitude of relative stiffness values

**Center-to-center distance between supports

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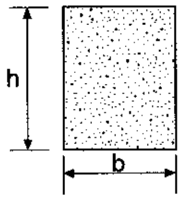
Table 2-5 Column Stiffness Factors



$$I = \frac{bh^3}{12} \qquad K^* = \frac{I}{10\ell_c}$$

Values of K for columns

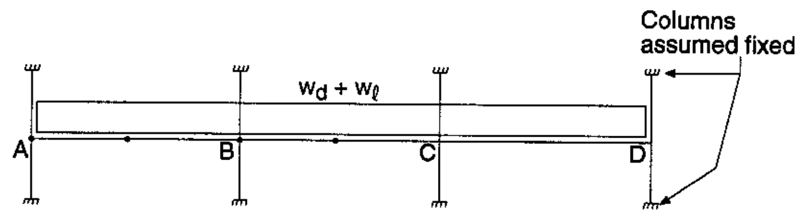
h	b	I	Height of column, ℓ_c (ft)**								h	b	I	Height of column, ℓ_c (ft)**							
			8	9	10	11	12	14	16	20				8	9	10	11	12	14	16	20
	10	427	5	5	4	4	4	3	3	2		12	13824	175	155	140	125	115	100	85	70
	12	512	6	6	5	5	4	4	3	3		14	16128	200	180	160	145	135	115	100	80
	14	597	7	7	6	5	5	4	4	3		18	20736	260	230	205	190	175	150	130	105
8	18	766	10	9	8	7	6	5	5	4	24	22	25344	315	280	255	230	210	180	160	125
	22	939	12	10	9	9	8	7	6	5		26	29952	375	335	300	270	250	215	185	150
	26	1109	14	12	11	10	9	8	7	6		30	34580	430	385	345	315	290	245	215	175
	30	1280	16	14	13	12	11	9	8	6		36	41472	520	460	415	375	345	295	260	205
	36	1536	19	17	15	14	13	11	10	8		42	48384	605	540	485	440	405	345	300	240
	10	833	10	9	8	8	7	6	5	4		12	17576	220	195	175	160	145	125	110	90
	12	1000	13	11	10	9	8	7	6	5		14	20505	255	230	205	185	170	145	130	105
	14	1167	15	13	12	11	10	8	7	6		18	26364	330	295	265	240	220	190	165	130
10	18	1500	19	17	15	14	13	11	9	8	26	22	32223	405	360	320	295	270	230	200	160
	22	1833	23	20	18	17	15	13	11	9		26	38081	475	425	380	345	315	270	240	190
	26	2167	27	24	22	20	18	16	14	11		30	43940	550	490	440	400	365	315	275	220
	30	2500	31	28	25	23	21	16	16	13		36	52728	660	565	525	480	440	375	330	265
	36	3000	38	33	30	27	25	21	19	15		42	61516	770	685	615	560	515	440	385	310
	10	1440	18	16	14	13	12	10	9	7		12	21952	275	245	220	200	185	155	135	110
	12	1728	22	19	17	16	14	12	11	9		14	25611	320	285	255	235	215	185	160	130
	14	2016	25	22	20	18	17	14	13	10		18	32928	410	365	330	300	275	235	205	185
12	16	2592	32	29	26	24	22	19	16	13	28	22	40245	505	445	400	365	335	285	250	200
	22	3168	40	35	32	29	26	23	20	16		26	47563	595	530	475	430	395	340	295	240
	26	3744	47	42	37	34	31	27	23	19		30	54880	685	610	550	500	455	390	345	275
	30	4320	54	46	43	39	36	31	27	22		36	65656	825	730	660	600	550	470	410	330
	36	5184	65	58	52	47	43	37	32	26		42	76832	960	855	770	700	640	550	480	385
	10	2287	29	25	23	21	19	16	14	11		12	27000	340	300	270	245	225	195	170	135
	12	2744	34	30	27	25	23	20	17	14		14	31500	395	350	315	285	265	225	195	160
	14	3201	40	36	32	29	27	23	20	16		18	40500	505	450	405	370	340	290	255	205
14	18	4116	51	46	41	37	34	29	26	21	30	22	49500	620	550	495	450	415	355	310	250
	22	5031	63	56	50	46	42	36	31	25		26	58500	730	650	585	530	490	420	365	295
	26	5945	74	66	59	54	50	42	37	30		30	67500	845	750	675	615	565	480	420	340
	30	6860	66	76	69	62	57	49	43	34		36	81000	1010	900	810	735	675	580	505	405
	36	8232	103	91	82	75	69	59	51	41		42	94500	1180	1050	945	860	790	675	590	475
	10	3413	43	38	34	31	28	24	21	17		12	32768	410	365	330	300	275	235	205	165
	12	4096	51	46	41	37	34	29	26	20		14	38229	480	425	380	350	320	275	240	190
	14	4779	60	53	48	43	40	34	30	24		18	49152	615	545	490	445	410	350	305	245
16	18	6144	77	68	61	56	51	44	38	31	32	22	60075	750	670	600	545	500	430	375	600
	22	7509	94	83	75	68	63	54	47	38		26	70997	885	790	710	645	590	505	445	355
	26	8875	111	99	89	81	74	63	55	44		30	81920	1020	910	820	745	685	585	510	410
	30	10240	128	114	102	93	85	73	64	51		36	96304	1230	1090	985	895	820	700	815	490
	36	12288	154	137	123	112	102	88	77	61		42	114688	1430	1270	1150	1040	955	820	715	575
	10	4860	61	54	49	44	41	35	30	24		12	39304	490	435	395	355	330	280	245	195
	12	5832	73	65	58	53	49	42	36	29		14	45855	575	510	460	415	380	330	285	230
	14	6604	85	76	68	62	57	49	43	34		18	58956	735	655	590	535	490	420	370	295
18	18	8748	109	97	87	80	73	62	55	44	34	22	72057	900	800	720	655	600	515	450	360
	22	10692	134	119	107	97	89	76	67	53		26	85159	1060	945	850	775	710	610	530	425
	26	12636	158	140	126	115	105	90	79	63		30	98260	1230	1090	985	895	820	700	615	490
	30	14580	182	162	146	133	122	104	91	73		36	117912	1470	1310	1180	1070	980	840	735	590
	36	17496	219	194	175	159	146	125	109	87		42	137564	1720	1530	1380	1250	1150	985	860	690
	10	6667	83	74	67	61	56	48	42	33		12	46656	585	520	465	425	390	335	290	235
	12	8000	100	89	80	73	67	57	50	40		14	54432	680	605	545	495	455	390	340	270
	14	9333	117	104	93	85	78	67	58	47		18	69984	875	780	700	635	585	500	435	350
20	18	12000	150	133	120	109	100	86	75	60	36	22	85536	1070	950	855	780	715	610	535	430
	22	14667	183	163	147	133	122	105	92	73		28	101088	1260	1120	1010	920	840	720	630	505
	26	17333	217	193	173	158	144	124	108	87		30	116640	1460	1300	1170	1060	970	835	730	585
	30	20000	250	222	200	182	167	143	125	100		36	139968	1750	1560	1400	1270	1170	1000	875	700
	36	24000	300	267	240	218	200	171	150	120		42	163296	2040	1810	1630	1480	1360	1170	1020	815
	10	8873	111	99	89	81	74	63	55	44		12	54872	685	610	550	500	460	390	345	275
	12	10648	133	118	106	97	89	76	67	53		14	64017	800	710	640	580	535	455	400	320
	14	12422	155	138	124	113	104	89	78	62		18	82308	1030	915	825	750	685	590	515	410
22	18	15972	200	177	160	145	133	114	100	80	38	22	100599	1260	1120	1010	915	840	720	630	505
	22	19521	244	217	195	177	163	139	122	98		26	118869	1490	1320	1190	1080	990	850	745	595
	26	23071	288	256	231	210	192	165	144	115		30	137180	1710	1520	1370	1250	1140	980	855	685
	30	26620	333	296	266	242	222	190	166	133		36	164618	2060	1830	1650	1500	1370	1180	1030	825
	36	31944	399	355	319	290	266	228	200	160		42	192052	2400	2130	1920	1750	1600	1370	1200	960

*Coefficient 10 introduced to reduce magnitude of relative stiffness values

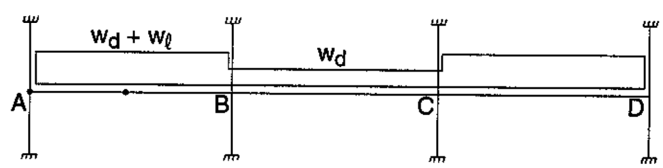
**Center-to-center distance between supports exact loading pattern is not known, the most demanding sets of design forces must be investigated. Figure 2-9 illustrates the loading patterns that should be considered for a three-span frame. In general,

(a) When the service live load does not exceed three-quarters of the service dead load ($L/D \leq 3/4$), consider only loading pattern (1) with full live load on all spans for maximum positive and negative moments.

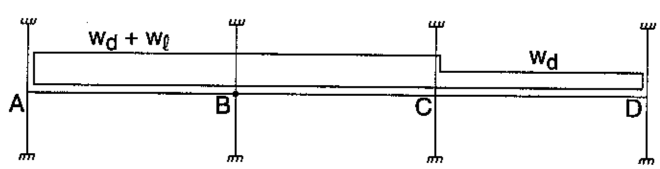
(b) When the service live-to-dead load ratio exceeds three-quarters, loading patterns (2) through (4) need to be considered to determine all maximum moments.



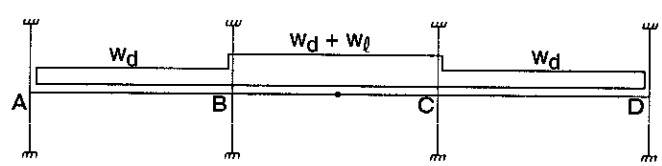
(1) Loading pattern for moments in all spans ($L/D \leq 3/4$)



(2) Loading pattern for negative moment at support A and positive moment in span AB



(3) Loading pattern for negative moment at support B

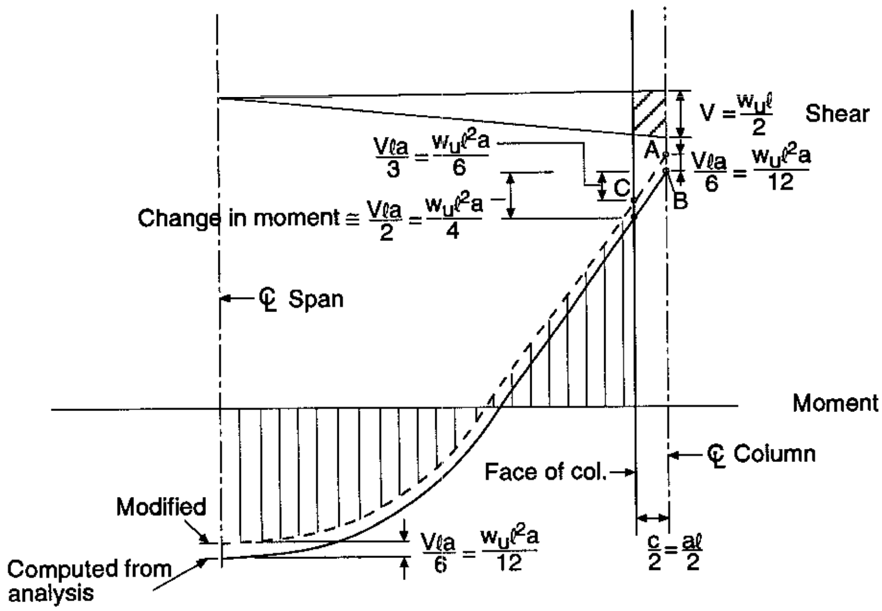


(4) Loading pattern for positive moment in span BC

Figure 2-9 Partial Frame Analysis for Gravity Loading

2.4.3 Design Moments

When determining moments in frames or continuous construction, the span length shall be taken as the distance center-to-center of supports (ACI 8.7.2). Moments at faces of supports may be used for member design purposes (ACI 8.7.3). Reference 2.3 provides a simple procedure for reducing the centerline moments to face of support moments, which includes a correction for the increased end moments in the beam due to the restraining effect of the column between face and centerline of support. Figure 2-10 illustrates this correction. For beams and slabs subjected to uniform loads, negative moments from the frame analysis can be reduced by $w_u\ell^2a/6$. A companion reduction in the positive moment of $w_u\ell^2a/12$ can also be made.



(A) = Theoretical ℄ moment including stiffening effect of column support
(B) = Computed ℄ moment ignoring stiffening effect of column support
(C) = Modified moment at face of column

w_u = uniformly distributed factored load (plf)
ℓ = span length center-to-center of supports
c = width of column support
a = c/ℓ

Figure 2-10 Correction Factors for Span Moments[2.3]

2.4.4 Two-Cycle Moment Distribution Analysis for Gravity Loading

Reference 2.3 presents a simplified two-cycle method of moment distribution for ordinary building frames. The method meets the requirements for an elastic analysis called for in ACI 8.3 with the simplifying assumptions of ACI 8.6 through 8.9.

The speed and accuracy of the two-cycle method will be of great assistance to designers. For an in-depth discussion of the principles involved, the reader is directed to Reference 2.3.

2.5 COLUMNS

In general, columns must be designed to resist the axial loads and maximum moments from the combination of gravity and lateral loading.

For interior columns supporting two-way construction, the maximum column moments due to gravity loading can be obtained by using ACI Eq. (13-4) (unless a general analysis is made to evaluate gravity load moments from alternate span loading). With the same dead load on adjacent spans, this equation can be written in the following form:

$$M_u = 0.07[w_d(\ell_n^2 - \ell_n'^2) + 0.5w_\ell \ell_n^2]\ell_2$$

where:

w_d = uniformly distributed factored dead load, psf
w_ℓ = uniformly distributed factored live load (including any live load reduction; see Section 2.2.2), psf
ℓ_n = clear span length of longer adjacent span, ft
ℓ_n' = clear span length of shorter adjacent span, ft
ℓ_2 = length of span transverse to ℓ_n and ℓ_n', measured center-to-center of supports, ft

For equal adjacent spans, this equation further reduces to:

$$M_u = 0.07(0.5w_\ell \ell_n^2)\ell_2 = 0.035w_\ell \ell_n^2 \ell_2$$

The factored moment M_u can then be distributed to the columns above and below the floor in proportion to their stiffnesses. Since the columns will usually have the same cross-sectional area above and below the floor under consideration, the moment will be distributed according to the column lengths.

2.6 LATERAL (WIND) LOAD ANALYSIS

For frames without shear walls, the lateral load effects of wind must be resisted by the "unbraced" frame. For low-to-moderate height buildings, wind analysis of an unbraced frame can be performed by either of two simplified methods: the Portal Method or the Joint Coefficient Method. Both methods can be considered to satisfy the elastic frame analysis requirements of the code (ACI 8.3). The two methods differ in overall approach. The Portal Method considers a vertical slice through the entire building along each row of column lines. The method is well suited to the range of building size and height considered in this manual, particularly to buildings with a regular rectangular floor plan. The Joint Coefficient Method considers a horizontal slice through the entire building, one floor at a time. The method can accommodate irregular floor plans, and provision is made to adjust for a wind loading that is eccentric to the centroid of all joint coefficients (centroid of resistance). The Joint Coefficient Method considers member stiffnesses, whereas the Portal Method does not.

The Portal Method is presented in this manual because of its simplicity and its intended application to buildings of regular shape. If a building of irregular floor plan is encountered, the designer is directed to Reference 2.3 for details of the Joint Coefficient Method.

2.6.1 Portal Method

The Portal Method considers a two-dimensional frame consisting of a line of columns and their connecting horizontal members (slab-beams), with each frame extending the full height of the building. The frame is considered to be a series of portal units. Each portal unit consists of two story-high columns with connecting slab-beams. Points of contraflexure are assumed at mid-length of beams and mid-height of columns. Figure 2-11 illustrates the portal unit concept applied to the top story of a building frame, with each portal unit shown separated (but acting together).

The wind load W is divided equally between the three portal units. Consequently, the shear in the interior columns is twice that in the exterior columns. In general, the magnitude of shear in the exterior column is W/2n, and in an interior column it is W/n, where n is the number of bays. For the case shown with equal spans, axial load occurs only in the exterior columns since the combined tension and compression due to the portal effect results in zero axial load in the interior columns. Under the assumptions of this method, however, a frame configuration with unequal spans will have axial load in those columns between the unequal spans, as well as in the exterior columns. The general terms for axial load in the exterior columns in a frame of n bays with unequal spans is:

$$\frac{Wh}{2n\ell_1} \text{ and } \frac{Wh}{2n\ell_n},\ \ell_n = \text{length of bay n}$$

The axial load in the first interior column is:

$$\frac{Wh}{2n\ell_1} - \frac{Wh}{2n\ell_2}$$

and, in the second interior column:

$$\frac{Wh}{2n\ell_2} - \frac{Wh}{2n\ell_3}$$

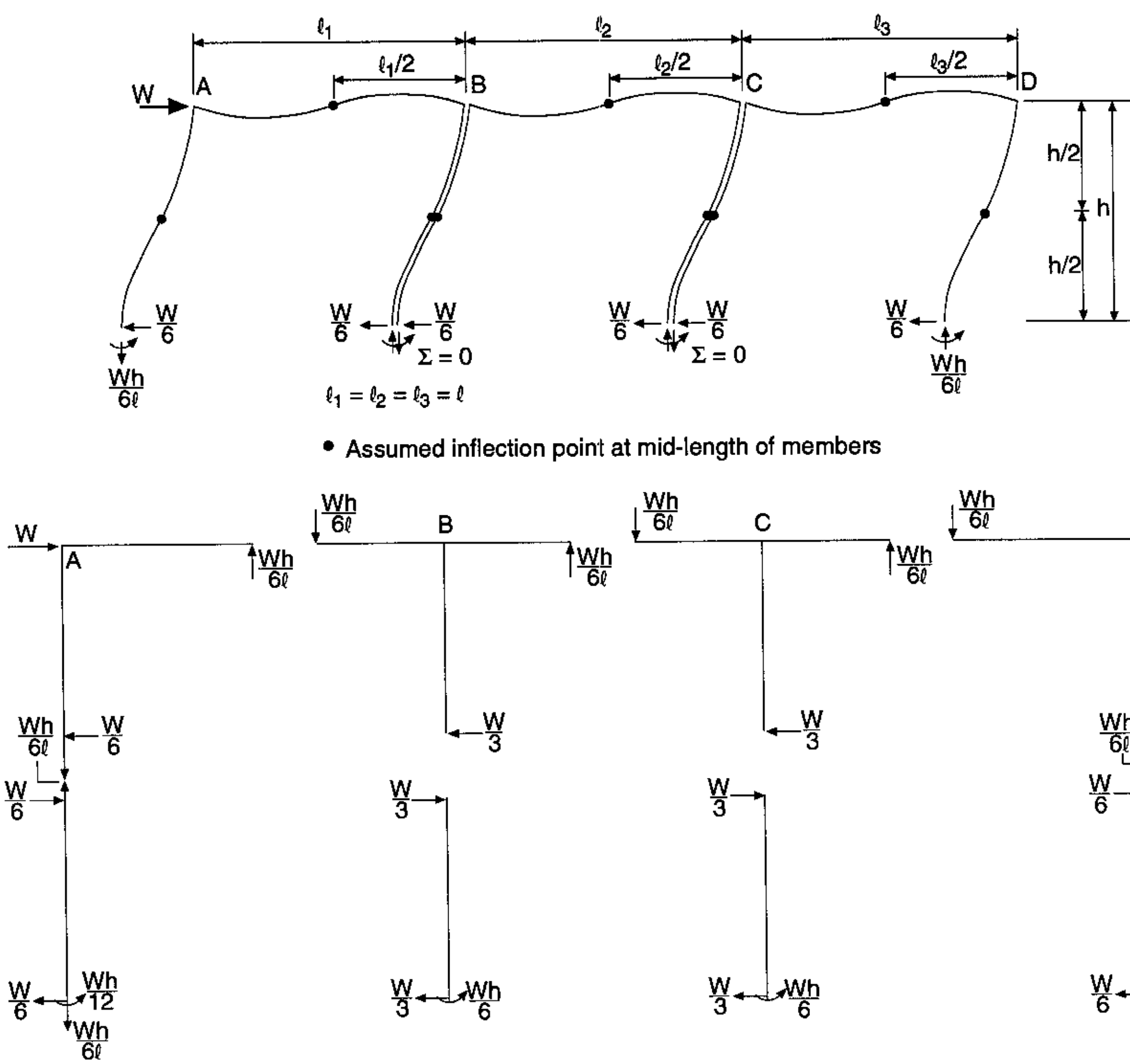


Figure 2-11 Portal Method

Column moments are determined by multiplying the column shear with one-half the column height. Thus, for joint B in Fig. 2-11, the column moment is (W/3) (h/2) = Wh/6. The column moment Wh/6 must be balanced by equal moments in beams BA and BC, as shown in Fig. 2-12.

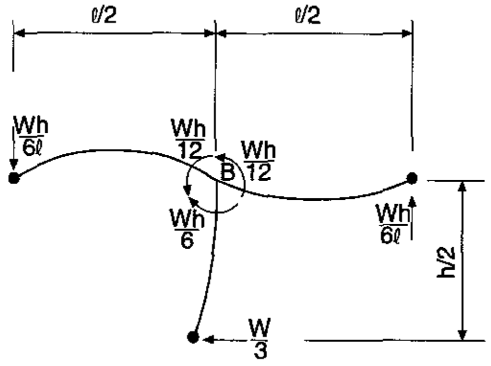


Figure 2-12 Joint Detail

Note that the balancing moment is divided equally between the horizontal members without considering their relative stiffnesses. The shear in beam AB or BC is determined by dividing the beam end moment by one-half the beam length, (Wh/12)/(ℓ/2) = Wh/6ℓ.

The process is continued throughout the frame taking into account the story shear at each floor level.

2.6.2 Examples: Wind Load Analyses for Buildings #1 and #2

For Building #1, determine the moments, shears, and axial forces using the Portal Method for an interior frame resulting from wind loads acting in the N-S direction. The wind loads are determined in Section 2.2.1.2.

Moments, shears, and axial forces are shown directly on the frame diagram in Fig. 2-13. The values can be easily determined by using the following procedure:

(1) Determine the shear forces in the columns:

For the exterior columns:

3rd story: V = 11.2 kips/6 = 1.87 kips
2nd story: V = (11.2 kips + 20.9 kips)/6 = 5.35 kips
1st story: V = (11.2 kips + 20.9 kips + 19.1 kips)/6 = 8.53 kips

The shear forces in the interior columns are twice those in the exterior columns.

(2) Determine the axial loads in the columns:

For the exterior columns, the axial loads can be obtained by summing moments about the column inflection points at each level. For example, for the 2nd story columns:

$\Sigma M = 0$: 11.2 (13 + 6.5) + 20.9 (6.5) - P (90) = 0
P = 3.94 kips

For this frame, the axial forces in the interior columns are zero.

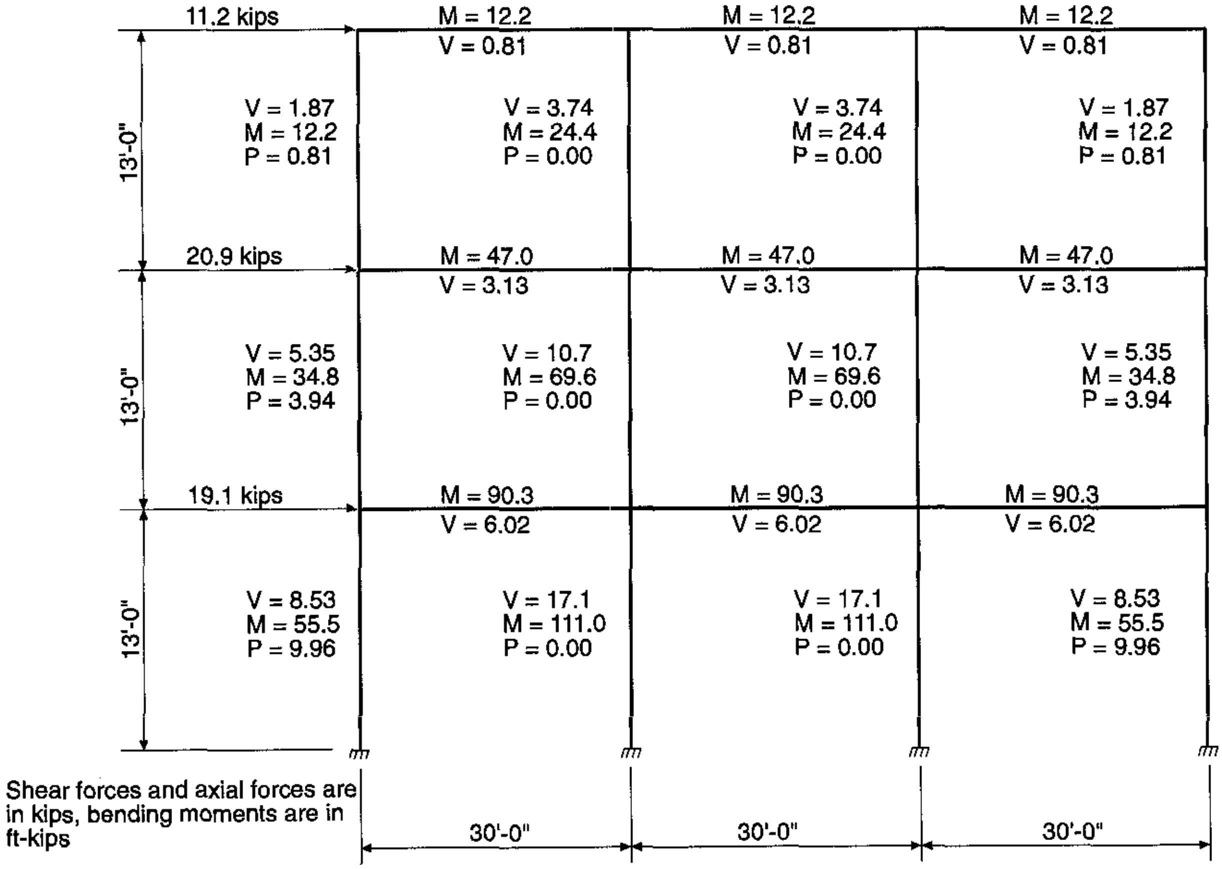


Figure 2-13 Shears, Moments, and Axial Forces Resulting from Wind Loads for an Interior Frame of Building #1 in the N-S Direction, using the Portal Method

(3) Determine the moments in the columns:

The moments can be obtained by multiplying the column shear force by one-half of the column length.

For example, for an exterior column in the 2nd story:

$$M = 5.35(13/2) = 34.8 \text{ ft-kips}$$

(4) Determine the shears and the moments in the beams:

These quantities can be obtained by satisfying equilibrium at each joint. Free-body diagrams for the 2nd story are shown in Fig. 2-14.

As a final check, sum moments about the base of the frame:

$$\Sigma M = 0\text{:}\ \ 11.2(39) + 20.9(26) + 19.1(13) - 9.96(90) - 2(55.5 + 111.0) = 0 \qquad \text{(checks)}$$

In a similar manner, the wind load analyses for an interior frame of Building #2 (5-story flat plate), in both the N-S and E-W directions are shown in Figs. 2-15 and 2-16, respectively. The wind loads are determined in Section 2.2.1.1.

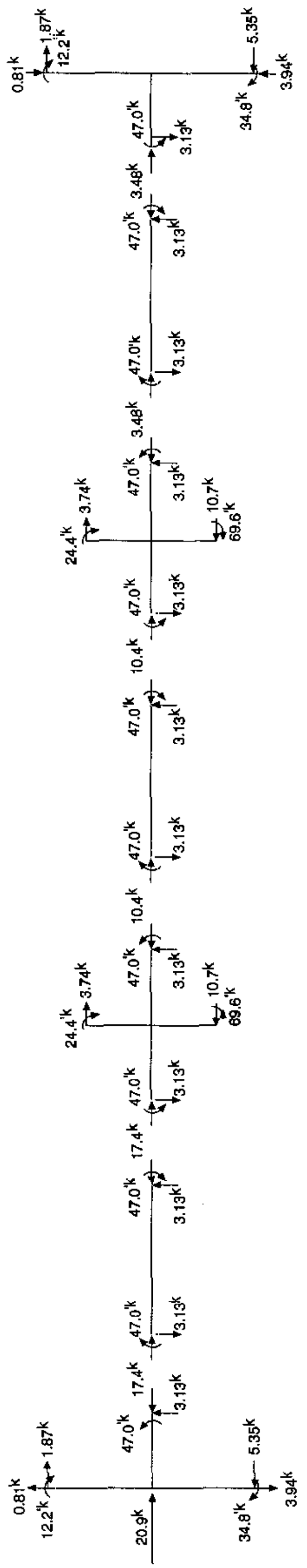


Figure 2-14 Shear Forces, Axial Forces, and Bending Moments at 2nd Story of Building #1

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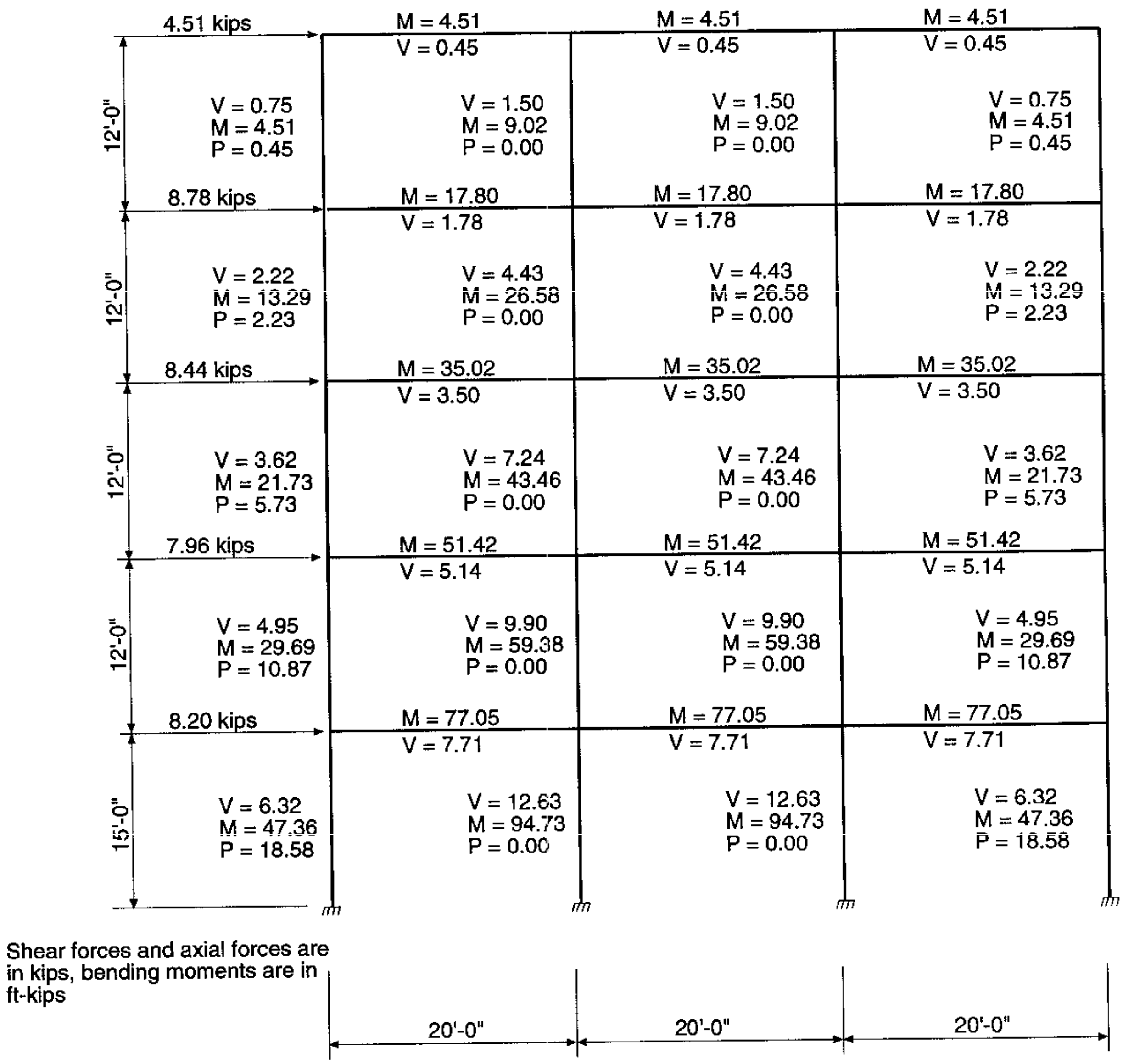

Figure 2-15 Shears, Moments, and Axial Forces Resulting from Wind Loads for an Interior Frame of Building #2 in the N-S Direction, using the Portal Method

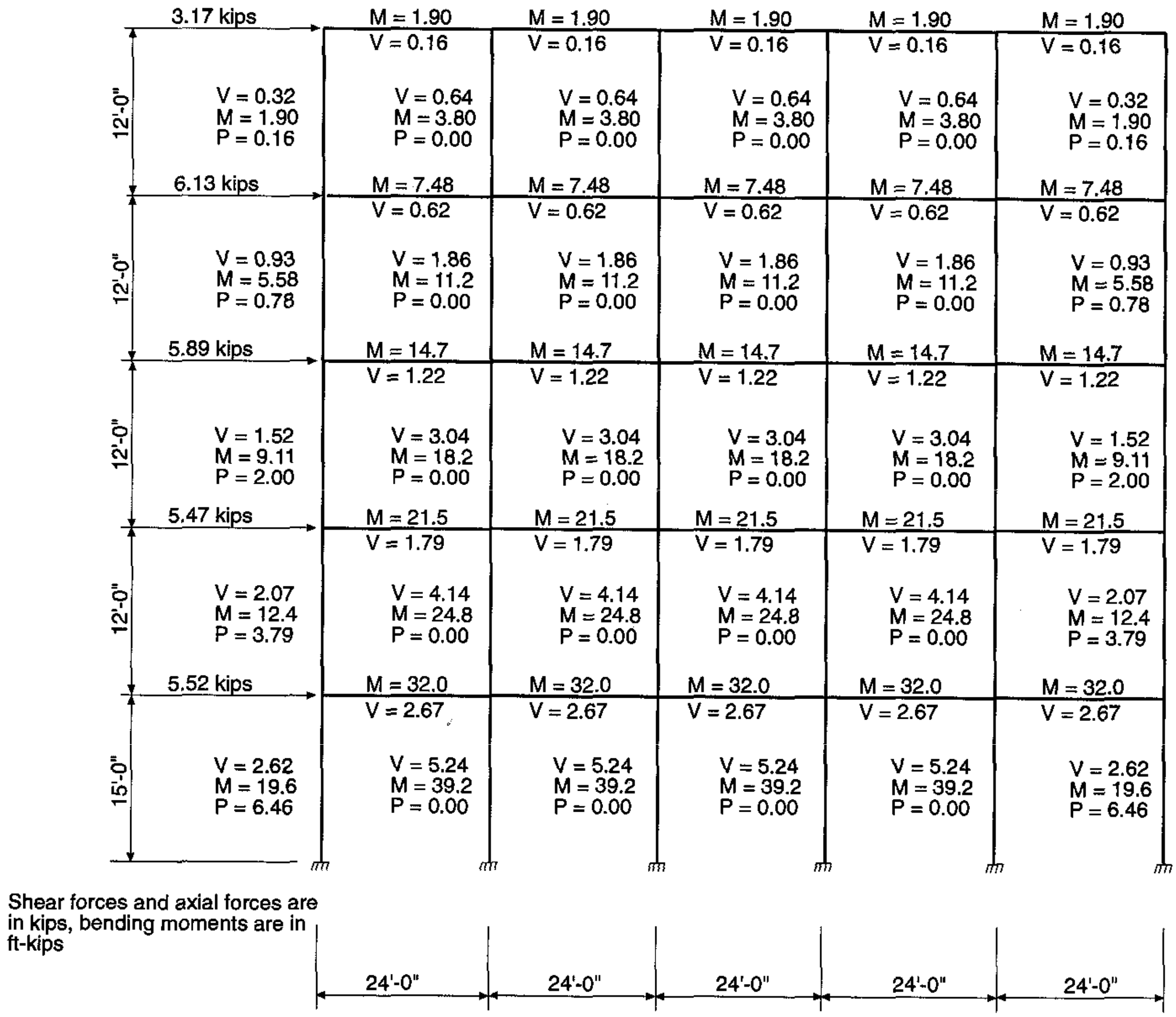

Figure 2-16 Shears, Moments, and Axial Forces Resulting from Wind Loads for an Interior Frame of Building #2 in the E-W Direction, using the Portal Method

For comparison purposes, the three frames shown in Figs. 2-13, 2-15, and 2-16 were analyzed using PCA-Frame.[2.1] Figure 2-17 shows the results for an interior frame of Building #1 in the N-S direction*. The following member sizes were used in the analysis:

Columns interior = 18 × 18 in.
exterior = 16 × 16 in.

Beams = 36 × 19.5 in.

To account for cracking, the moment of inertia of the beams was taken at 50% of the gross value.

**Maximum values are shown for each member.*

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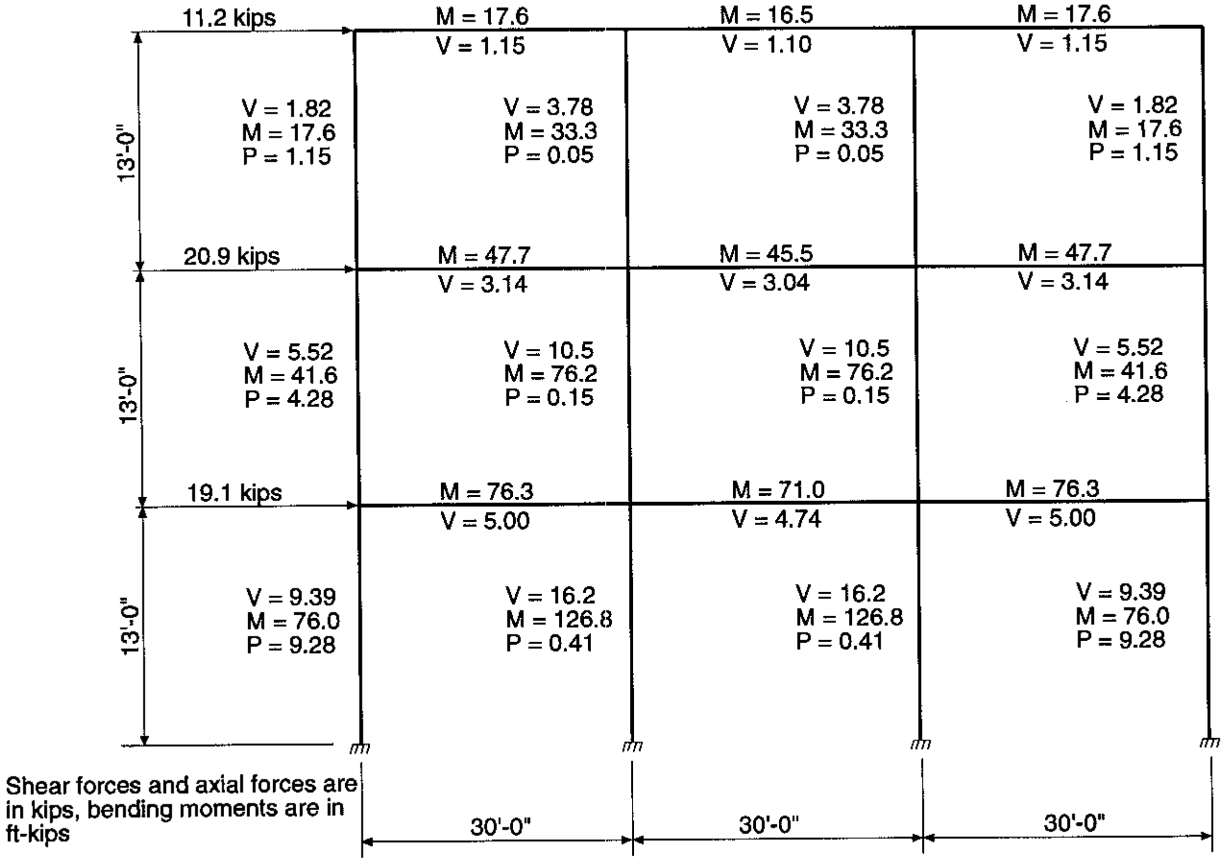

Figure 2-17 Shears, Moments, and Axial Forces Resulting from Wind Loads for an Interior Frame of Building #1 in the N-S Direction, using PCA-Frame.

Similarly, Figs. 2-18 and 2-19 show the results for an interior frame of Building #2 with wind loads in the N-S and E-W directions, respectively. In both cases, the following member sizes were used:

Columns	interior = 16 × 16 in. exterior = 12 × 12 in.
Slab	width = 120 in. (width of column strip) thickness = 9 in.

As was the case for the beams in Building #1, 50% of the gross moment of inertia of the slab was used to account for flexural cracking.

Throughout the remaining chapters, the results obtained from the Portal Method will be used when designing the members.

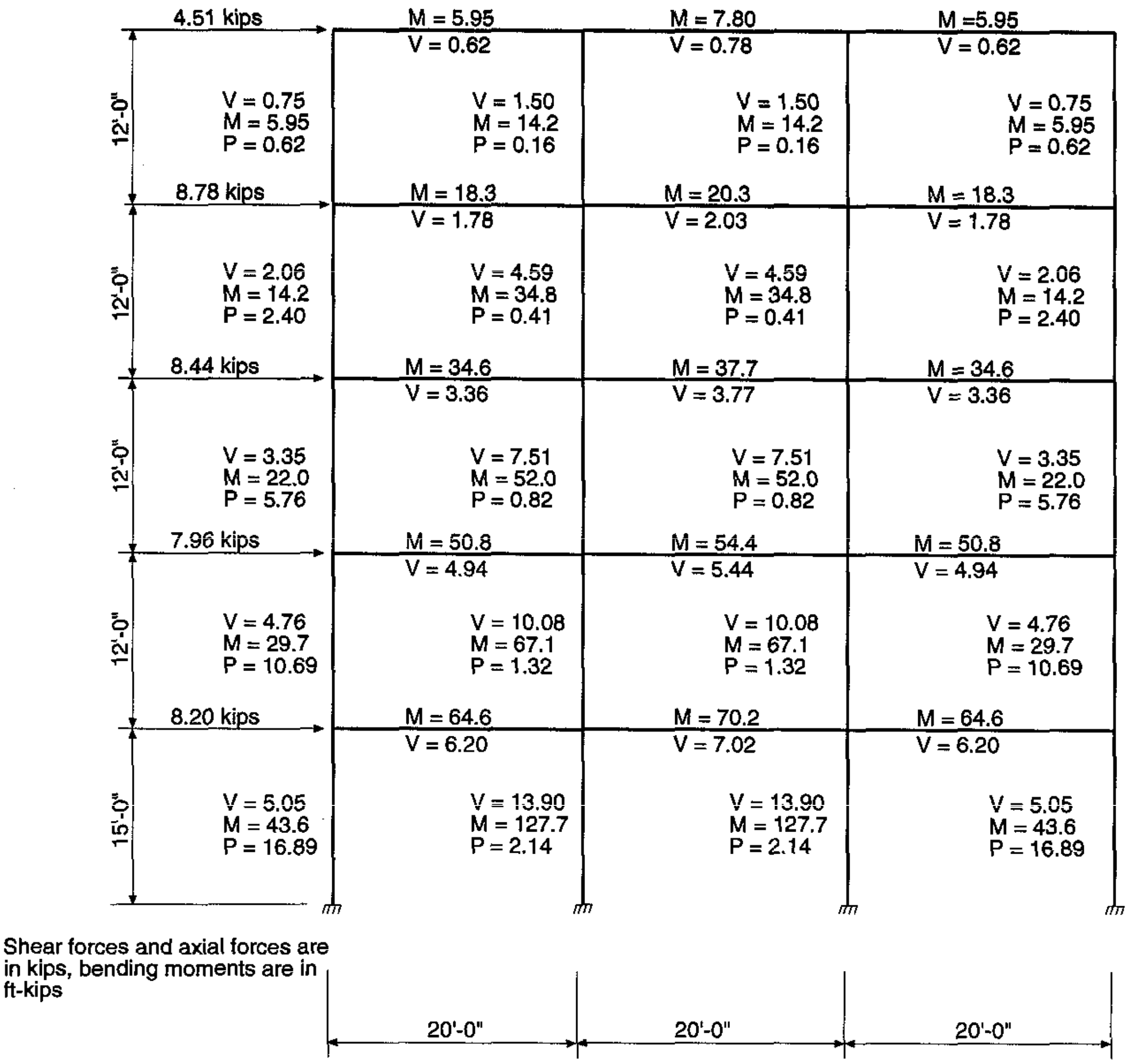

Figure 2-18 Shears, Moments, and Axial Forces Resulting from Wind Loads for an Interior Frame of Building #2 in the N-S Direction, using PCA-Frame.

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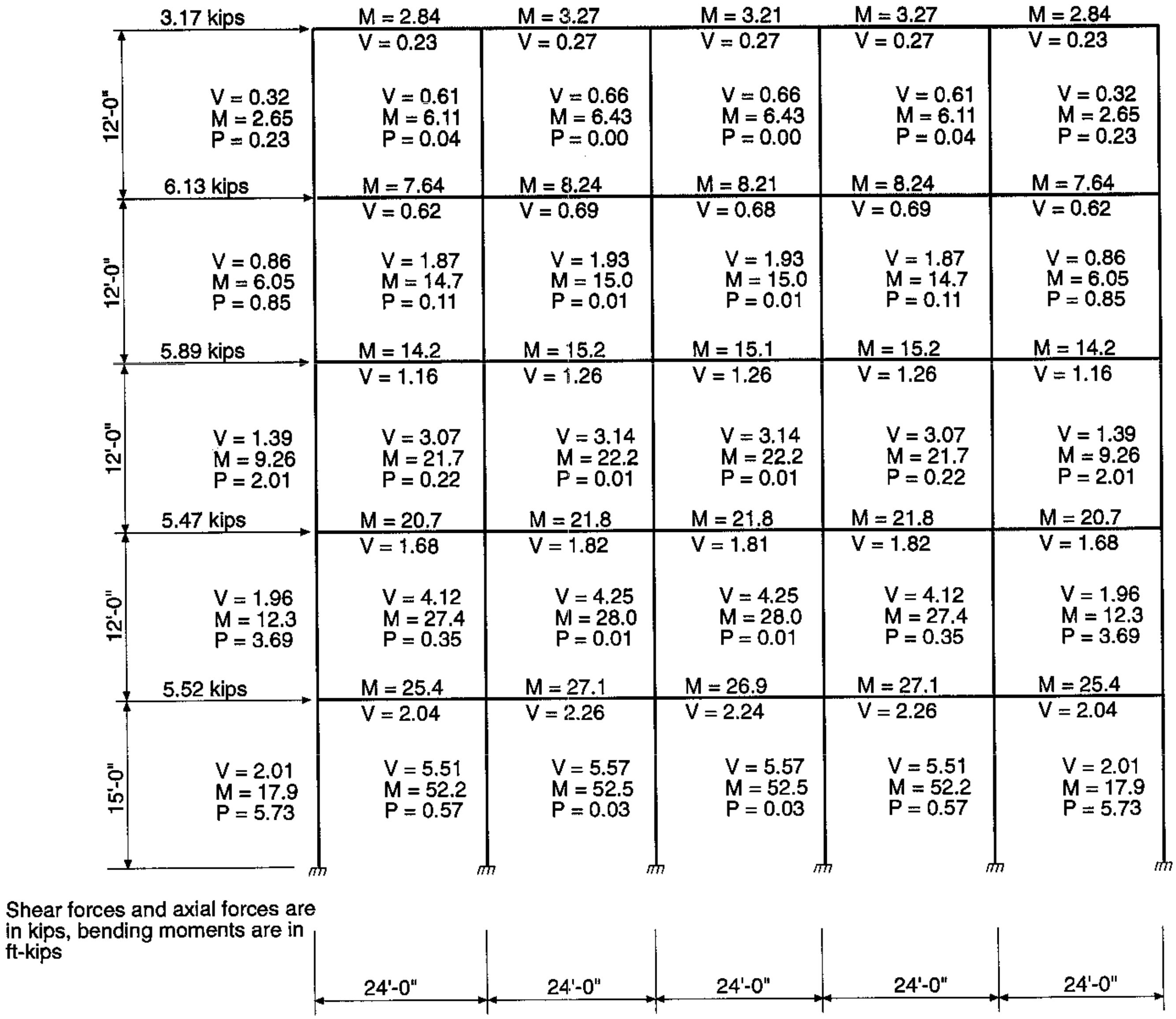

Figure 2-19 Shears, Moments, and Axial Forces Resulting from Wind Loads for an Interior Frame of Building #2 in the E-W Direction, using PCA-Frame

References

2.1 *PCA-Frame —Three Dimensional Static Analysis of Structures,* Portland Cement Association, Skokie, IL., 1992.

2.2 *American Society of Civil Engineers Minimum Design Loads for Buildings and Other Structures,* ASCE 7-88, American Society of Civil Engineers, New York, 1990, 94 pp.

2.3 *Continuity in Concrete Building Frames,* Portland Cement Association, Skokie, EB033, 1959, 56 pp.

Chapter 3

Simplified Design for Beams and Slabs

3.1 INTRODUCTION

The simplified design approach for proportioning beams and slabs (floor and roof members) is based in part on published articles,[3.1–3.3] and in part on simplified design aid material published by CRSI.[3.4, 3.7] Additional data for design simplification are added where necessary to provide the designer with a total simplified design approach for beam and slab members. The design conditions that need to be considered for proportioning the beams and slabs are presented in the order generally used in the design process.

The simplified design procedures comply with the ACI 318-89 (Revised 1992) code requirements for both member strength and member serviceability. The simplified methods will produce slightly more conservative designs within the limitations noted. All coefficients are based on the Strength Design Method, using appropriate load factors and strength reduction factors specified in ACI 318. Where simplified design requires consideration of material strengths, 4000 psi concrete and Grade 60 reinforcement are used. The designer can easily modify the data for other material strengths.

The following data are valid for reinforced concrete flexural members with $f'_c = 4000$ psi and $f_y = 60{,}000$ psi:

modulus of elasticity for concrete	E_c	= 3,600,000 psi	(ACI 8.5.1)
modulus of elasticity for rebars	E_s	= 29,000,000 psi	(ACI 8.5.2)
balanced reinforcement ratio	ρ_b	= 0.0285	(ACI 10.3.2)
minimum reinforcement ratio (beams, joists)	ρ_{min}	= 0.0033	(ACI 10.5.1)
minimum reinforcement ratio (slabs)	ρ_{min}	= 0.0018	(ACI 7.12.2)
maximum reinforcement ratio	ρ_{max}	$= 0.75\rho_b = 0.0214$	(ACI 10.3.3)

3.2 DEPTH SELECTION FOR CONTROL OF DEFLECTIONS

Deflection of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified in ACI Table 9.5(a). Table 9.5(a) may be simplified to four values as shown in Table 3-1. The quantity ℓ_n is the clear span length for cast-in-place beam and slab construction. For design convenience, minimum thicknesses for the four conditions are plotted in Fig. 3-1.

Table 3-1 Minimum Thickness for Beams and One-Way Slabs

Beams and One-way Slabs	Minimum h
Simple Span Beams or Joists*	ℓ_n /16
Continuous Beams or Joists	ℓ_n /18.5
Simple Span Slabs*	ℓ_n /20
Continuous Slabs	ℓ_n /24

*Minimum thickness for cantilevers can be considered equal to twice that for a simple span.

Deflections are not likely to cause problems when overall member thickness meets or exceeds these values for uniform loads commonly used in the design of buildings. The values are based primarily on experience and are not intended to apply in special cases where beam or slab spans may be subject to heavily distributed loads or concentrated loads. Also, they are not intended to apply to members supporting or attached to nonstructural elements likely to be damaged by deflections (ACI 9.5.2.1). For roof beams and slabs, the values are intended for roofs subjected to normal snow or construction live loads only, and with minimal water ponding or drifting (snow) problems.

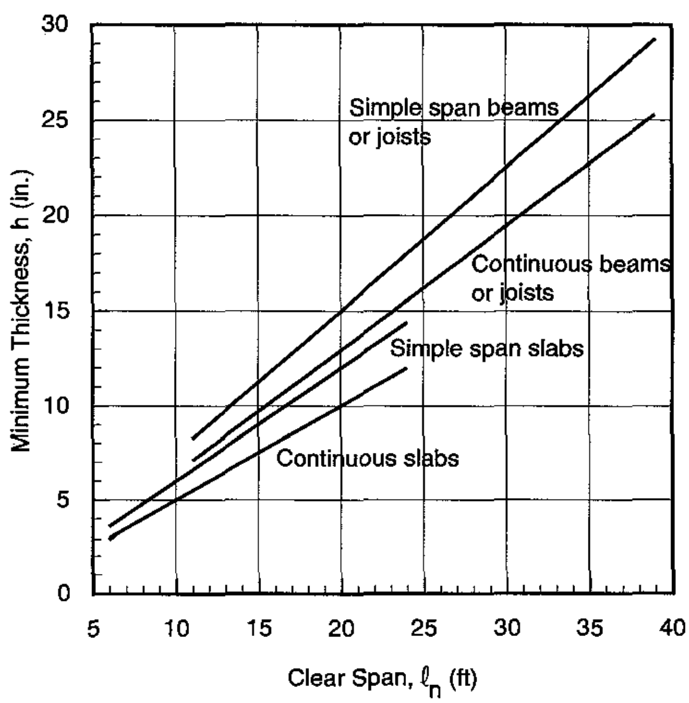

Figure 3-1 Minimum Thicknesses for Beams and One-Way Slabs

Prudent choice of steel percentage can also minimize deflection problems. Members will usually be of sufficient size, so that deflections will be within acceptable limits, when the tension reinforcement ratio ρ used in the positive moment regions does not exceed approximately one-half of the maximum value permitted ($\rho \cong 0.5\rho_{max}$). For f'_c = 4000 psi and f_y = 60,000 psi, one-half of ρ_{max} is approximately one percent ($\rho \cong 0.01$).

Depth selection for control of deflections of two-way slabs is given in Chapter 4.

As a guide, the effective depth d can be calculated as follows:

For beams with one layer of bars	d = h - (≅ 2.5 in.)
For joists and slabs	d = h - (≅ 1.25 in.)

3.3 MEMBER SIZING FOR MOMENT STRENGTH

A simplified sizing equation can be derived using the strength design data developed in Chapter 10 of Reference 3.5. For our selected materials (f'_c = 4000 psi and f_y = 60,000 psi), the balanced reinforcement ratio $\rho_b = 0.0285$. As noted above, deflection problems are rarely encountered with beams having a reinforcement ratio ρ equal to about one-half of the maximum permitted.

Set $\rho = 0.5\rho_{max} = 0.5(0.75\rho_b) = 0.375\rho_b = 0.375(0.0285) = 0.0107$

$$M_u = \phi A_s f_y (d - a/2) = \phi \rho b d f_y (d - a/2)$$

$$a = A_s f_y / 0.85 f'_c b = \rho d f_y / 0.85 f'_c$$

$$M_u / \phi b d^2 = \rho f_y \left(1 - \frac{0.5 \rho f_y}{0.85 f'_c}\right) = R_n$$

$$R_n = \rho f_y \left(1 - \frac{0.5 \rho f_y}{0.85 f'_c}\right)$$

$$= 0.0107 \times 60{,}000 \ \left[\left(1 - \frac{0.5 \times 0.0107 \times 60{,}000}{0.85 \times 4000}\right)\right]$$

$$= 581 \text{ psi}$$

$$bd^2_{reqd} = \frac{M_u}{\varphi R_n} = \frac{M_u \times 12 \times 1000}{0.9 \times 581} = 22.9 M_u$$

For simplicity, set $bd^2_{reqd} = 20M_u$.

For f'_c = 4000 psi and f_y = 60,000 psi:

$$bd^2(\rho \cong 0.5\rho_{max}) = 20M_u{}^*$$

A similar sizing equation can be derived for other material strengths.

With factored moments M_u and effective depth d known, the required beam width b is easily determined using the sizing equation $bd^2 = 20M_u$. When frame moments vary, b is usually determined for the member which has the largest M_u; for economy, this width may be used for all similar members in the frame. Since slabs are designed by using a 1-ft strip (b = 12 in.), the sizing equation can be used to check the initial depth selected for slabs; it simplifies to $d = 1.3\sqrt{M_u}$.

**Note: The above sizing equation is in mixed units: M_u is in ft-kips and b and d are in inches.*

If the depth determined for control of deflections is shallower than desired, a larger depth may be selected with a corresponding width b determined from the above sizing equation. Actually, any combination of b and d could be determined from the sizing equation with the only restriction being that the final depth selected must be greater than that required for deflection control (Table 3-1).

It is important to note that for minimum beam size with maximum reinforcement ($\rho = 0.75\rho_b, = 0.0214$), the sizing equation becomes $bd^2_{min} = 13M_u$.

3.3.1 Notes on Member Sizing for Economy

- Use whole inches for overall beam dimensions; slabs may be specified in ½-in. increments.
- Use beam widths in multiples of 2 or 3 inches, such as 10, 12, 14, 16, 18, etc.
- Use constant beam size from span to span and vary reinforcement as required.
- Use wide flat beams (same depth as joist system) rather than narrow deep beams.
- Use beam width equal to or greater than the column width.
- Use uniform width and depth of beams throughout the building.

See also Chapter 9 for design considerations for economical formwork.

3.4 DESIGN FOR MOMENT REINFORCEMENT

A simplified equation for the area of tension steel A_s can be derived using the strength design data developed in Chapter 10 of Reference 3.5. An approximate linear relationship between R_n and ρ can be described by an equation in the form $M_n/bd^2 = \rho$ (constant), which readily converts to $A_s = M_u/\phi d$(constant). This linear equation for A_s is reasonably accurate up to about two-thirds of the maximum ρ. For $f'_c = 4000$ psi and $\rho = ⅔\,\rho_{max}$, the constant for the linear approximation is:

$$\frac{f_y}{12{,}000^*}\left[1-\frac{0.5\rho f_y}{0.85f'_c}\right]=\frac{60{,}000}{12{,}000}\left[1-\frac{0.5(2/3\times 0.0214)(60)}{0.85\times 4}\right]=4.37$$

Therefore,

$$A_s=\frac{M_u}{\phi d(\text{constant})}=\frac{M_u}{0.9\times 4.37\times d}=\frac{M_u}{3.93d}\cong\frac{M_u}{4d}$$

For $f'_c = 4000$ psi and $f_y = 60{,}000$ psi:

$$A_s=\frac{M_u}{4d}^{**}$$

For all values of $\rho < ⅔\,\rho_{max}$, the simplified A_s equation is slightly conservative. The maximum deviation in A_s is less than ± 10% at the minimum and maximum permitted tension steel ratios.[3.6] For members with reinforcement ratios in the range of approximately 1% to 1.5%, the error is less than 3%.

**To convert M_u from ft-kips to in.-kips.*
***Note: This equation is in mixed units: M_u is in ft-kips, d is in in. and A_s is in sq in.*

The simplified A_s equation is applicable for rectangular cross sections with tension reinforcement only. Members proportioned with reinforcement in the range of 1% to 1.5% will be well within the code maximum of $0.75\rho_b$ (ACI 10.3.3) for singly reinforced members. For positive moment reinforcement in flanged floor beams, A_s is usually computed for a rectangular compression zone; rarely will A_s be computed for a T-shaped compression zone.

The depth of the rectangular compression zone, a, is given by:

$$a = \frac{A_s f_y}{0.85 f'_c b_e}$$

where b_e = effective width of slab as a T-beam flange (ACI 8.10).

The flexural member is designed as a rectangular section whenever $h_f \geq a$ where h_f is the thickness of the slab (i.e., flange thickness).

3.5 REINFORCING BAR DETAILS

The minimum and maximum number of reinforcing bars permitted in a given cross section is a function of cover and spacing requirements given in ACI 7.6.1 and 3.3.2 (minimum spacing for concrete placement), ACI 7.7.1 (minimum cover for protection of reinforcement), and ACI 10.6 (maximum spacing for control of flexural cracking). Tables 3-2 and 3-3 give the minimum and maximum number of bars in a single layer for beams of various widths; selection of bars within these limits will provide automatic code conformance with the cover and spacing requirements.

Table 3-2 Minimum Number of Bars in a Single Layer (ACI 10.6)*

Bar Size	INTERIOR EXPOSURE (z = 175 kips/in.) Beam width, b_w (in.)										
	10	12	14	16	18	20	22	24	26	28	30
#5	1	2	2	2	2	2	3	3	3	3	3
#6	1	2	2	2	2	2	3	3	3	3	3
#7	2	2	2	2	2	3	3	3	3	3	4
#8	2	2	2	2	2	3	3	3	3	4	4
#9	2	2	2	2	3	3	3	3	3	4	4
#10	2	2	2	2	3	3	3	3	4	4	4
#11	2	2	2	3	3	3	3	4	4	4	4

Bar Size	EXTERIOR EXPOSURE (z = 145 kips/in.) Beam width, b_w (in.)										
	10	12	14	16	18	20	22	24	26	28	30
#5	2	2	3	3	3	4	4	4	5	5	5
#6	2	3	3	3	4	4	4	5	5	5	6
#7	2	3	3	3	4	4	4	5	5	6	6
#8	2	3	3	4	4	4	5	5	5	6	6
#9	**	3	3	4	4	5	5	5	6	6	7
#10	**	3	3	4	4	5	5	6	6	6	7
#11	**	3	**	4	5	5	5	6	6	7	7

*Values based on a cover of 2 in. to the main flexural reinforcement (see Fig. 3-2).
**Minimum number required for crack control is greater than maximum permitted based on clear spacing.

The values in Table 3-2 are based on a cover of 2 in. to the main flexural reinforcement (i.e., 1.5 in. clear cover to the stirrups plus the diameter of a #4 stirrup). In general, the following equations can be used to determine the minimum number of bars n in a single layer for any situation (see Fig. 3-2):

- for interior exposure:

$$n = \frac{b_w(c_s + d_s + 0.5d_b)^2}{57.4}$$

- for exterior exposure:

$$n = \frac{b_w(c_s + d_s + 0.5d_b)^2}{32.7}$$

Table 3-3 Maximum Number of Bars in a Single Layer

Bar Size	Maximum size coarse aggregate—3/4 in. Beam width, b_w (in.)										
	10	12	14	16	18	20	22	24	26	28	30
#5	3	5	6	7	8	10	11	12	13	15	16
#6	3	4	6	7	8	9	10	11	12	14	15
#7	3	4	5	6	7	8	9	10	11	12	13
#8	3	4	5	6	7	8	9	10	11	12	13
#9	2	3	4	5	6	7	8	8	9	10	11
#10	2	3	4	4	5	6	7	8	8	9	10
#11	2	3	3	4	5	5	6	7	8	8	9

Bar Size	Maximum size coarse aggregate—1 in. Beam width, b_w (in.)										
	10	12	14	16	18	20	22	24	26	28	30
#5	3	4	5	6	7	8	9	10	11	12	13
#6	3	4	5	6	7	8	9	10	10	11	12
#7	2	3	4	5	6	7	8	9	10	10	11
#8	2	3	4	5	6	7	7	8	9	10	11
#9	2	3	4	5	5	6	7	8	9	9	10
#10	2	3	4	4	5	6	7	7	8	9	10
#11	2	3	3	4	5	5	6	7	8	8	9

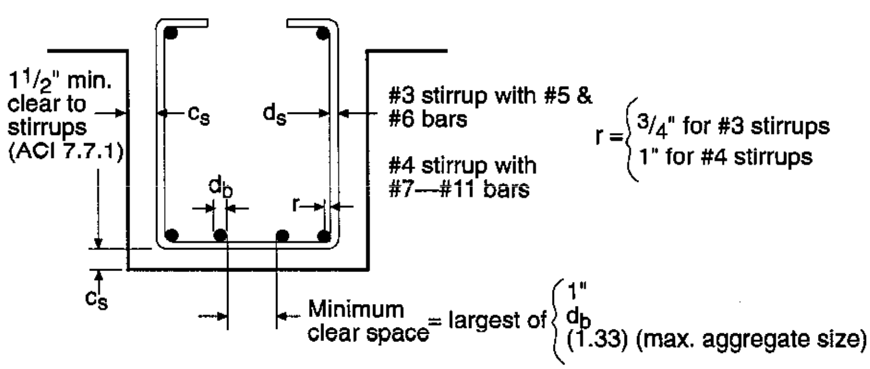

Figure 3-2 Cover and Spacing Requirements for Tables 3-2 and 3-3

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where b_w = beam width, in.
c_s = clear cover to stirrup, in.
d_s = diameter of stirrup, in.
d_b = diameter of main flexural bar, in.

The values obtained from the above equations should be rounded up to the next whole number.

The values in Table 3-3 can be determined from the following equation:

$$n = 1 + \frac{b_w - 2(c_s + d_s + r)}{(\text{minimum clear space}) + d_b}$$

where

$$r = \begin{cases} \frac{3}{4} \text{ in. for \#3 stirrups} \\ 1 \text{ in. for \#4 stirrups} \end{cases} \quad \text{(ACI 7.2.2)}$$

The minimum clear space between bars is defined in Fig. 3-2. The above equation can be used to determine the maximum number of bars in any general case; computed values should be rounded down to the next whole number.

For one-way slabs, maximum reinforcement spacings (measured center-to-center of parallel bars) to satisfy crack control requirements are given in Table 3-4. Suggested temperature reinforcement for one-way floor and roof slabs is given in Table 3-5. The provided area of reinforcement (per foot width of slab) satisfies ACI 7.12.2. Bar spacing must not exceed 5h or 18 in. (where h = thickness of slab). The same area of reinforcement also applies for minimum moment reinforcement in one-way slabs (ACI 10.5.3) at a maximum spacing of 3h or 18 in. (ACI 7.6.5). As noted in Chapter 4, this same minimum area of steel applies for flexural reinforcement in each direction for two-way floor and roof slabs; in this case, the maximum spacing is 2h or 18 in. (ACI 13.4).

As an aid to designers, reinforcing bar data are presented in Tables 3-6 and 3-7.

See Chapter 8, Section 8.2, for notes on reinforcement selection and placement for economy.

Table 3-4 Maximum Bar Spacing in One-Way Slabs for Crack Control (in.)*

Bar Size	Exterior Exposure (z = 129 kips/in.)				Interior Exposure (z = 156 kips/in.)			
	Cover (in.)				Cover (in.)			
	3/4	1	1-1/2	2	3/4	1	1-1/2	2
#4	--	14.7	7.5	4.5	--	--	13.3	8.0
#5	--	13.4	7.0	4.3	--	--	12.4	7.6
#6	--	12.2	6.5	4.1	--	--	11.6	7.2
#7	16.3	11.1	6.1	3.9	--	--	10.8	6.8
#8	14.7	10.2	5.8	3.7	--	--	10.2	6.5
#9	13.3	9.4	5.4	3.5	--	16.6	9.6	6.2
#10	12.0	8.6	5.0	3.3	--	15.2	8.9	5.9
#11	10.9	7.9	4.7	3.1	--	14.0	8.4	5.6

*Valid for $f_s = 0.6f_y$ = 36 ksi, and single layer of reinforcement. Spacing should not exceed 3 times slab thickness nor 18 in. (ACI 7.6.5). No value indicates spacing greater than 18 in.

Table 3-5 Temperature Reinforcement for One-Way Slabs

Slab Thickness h (in.)	A_s (req'd)* (in.²/ft)	Suggested Reinforcement**
3-½	0.08	#3@16
4	0.09	#3@15
4-½	0.10	#3@13
5	0.11	#3@12
5-½	0.12	#4@18
6	0.13	#4@18
6-½	0.14	#4@17
7	0.15	#4@16
7-½	0.16	#4@15
8	0.17	#4@14
8-½	0.18	#4@13
9	0.19	#4@12
9-½	0.21	#5@18
10	0.22	#5@17

*A_s = 0.0018bh = 0.022h (ACI 7.12.2).
**For minimum moment reinforcement, bar spacing must not exceed 3h or 18 in. (ACI 7.6.5). For 3 ½ in. slab, use #3@10 in.; for 4 in. slab, use #3@12 in.; for 5 ½ in. slab, use #3@11 in. or #4@16 in.

Table 3-6 Total Areas of Bars—A_s(in.²)

Bar size	Bar diameter (in.)	Number of bars							
		1	2	3	4	5	6	7	8
#3	0.375	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88
#4	0.500	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60
#5	0.625	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48
#6	0.750	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52
#7	0.875	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80
#8	1.000	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32
#9	1.128	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00
#10	1.270	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16
#11	1.410	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48

Table 3-7 Areas of Bars per Foot Width of Slab—A_s (in.²/ft)

Bar size	Bar spacing (in.)												
	6	7	8	9	10	11	12	13	14	15	16	17	18
#3	0.22	0.19	0.17	0.15	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.07
#4	0.40	0.34	0.30	0.27	0.24	0.22	0.20	0.18	0.17	0.16	0.15	0.14	0.13
#5	0.62	0.53	0.46	0.41	0.37	0.34	0.31	0.29	0.27	0.25	0.23	0.22	0.21
#6	0.88	0.75	0.66	0.59	0.53	0.48	0.44	0.41	0.38	0.35	0.33	0.31	0.29
#7	1.20	1.03	0.90	0.80	0.72	0.65	0.60	0.55	0.51	0.48	0.45	0.42	0.40
#8	1.58	1.35	1.18	1.05	0.95	0.86	0.79	0.73	0.68	0.63	0.59	0.56	0.53
#9	2.00	1.71	1.50	1.33	1.20	1.09	1.00	0.92	0.86	0.80	0.75	0.71	0.67
#10	2.54	2.18	1.91	1.69	1.52	1.39	1.27	1.17	1.09	1.02	0.95	0.90	0.85
#11	3.12	2.67	2.34	2.08	1.87	1.70	1.56	1.44	1.34	1.25	1.17	1.10	1.04

3.6 DESIGN FOR SHEAR REINFORCEMENT

In accordance with ACI Eq. (11-2), the total shear strength is the sum of two components: shear strength provided by concrete (ϕV_c) and shear strength provided by shear reinforcement (ϕV_s). Thus, at any section of the member,

$V_u \le \phi V_c + \phi V_s$. Using the simplest of the code equations for shear strength, specific values can be assigned to the two resisting components for a given set of material parameters and a specific cross section. Table 3-8 summarizes ACI 318 provisions for shear design.

Table 3-8 ACI Provisions for Shear Design*

		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \le \phi 4\sqrt{f'_c}\, b_w d$
				$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$, $\phi = 0.85$ (ACI 11.3.1.1)
**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)
†A practical limit for minimum spacing is d/4
††Maximum spacing based on minimum shear reinforcement (= $A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

The selection and spacing of stirrups can be simplified if the spacing is expressed as a function of the effective depth d (see Reference 3.3). According to ACI 11.5.4.1 and ACI 11.5.4.3, the practical limits of stirrup spacing vary from s = d/2 to s = d/4, since spacing closer than d/4 is not economical. With one intermediate spacing at d/3, the calculation and selection of stirrup spacing is greatly simplified. Using the three standard stirrup spacings noted above (d/2, d/3, and d/4), a specific value of ϕV_s can be derived for each stirrup size and spacing as follows:

For vertical stirrups:

$$\phi V_s = \frac{\phi A_v f_y d}{s} \qquad \text{ACI Eq. (11.17)}$$

By substituting d/n for s (where n = 2, 3, or 4), the above equation can be rewritten as:

$$\phi V_s = \phi A_v f_y n$$

Thus, for #3 U-stirrups @ s = d/2 with f_y = 60,000 psi and $\phi = 0.85$:

$$\phi V_s = 0.85(0.22)60 \times 2 = 22.44 \text{ kips, say 22 kips}$$

The values of ϕV_s given in Table 3-9 may be used to select shear reinforcement with Grade 60 rebars.

Table 3-9 Values of ϕV_s (f_y = 60 ksi)*

s	#3 U-stirrups	#4 U-stirrups	#5 U-stirrups
d/2	22 kips	40 kips	63 kips
d/3	33 kips	61 kips	94 kips
d/4	45 kips	81 kips	126 kips

*Valid for stirrups with 2 legs (double the tabulated values for 4 legs, etc.)

It should be noted that these values of ϕV_s are not dependent on the member size nor on the concrete strength. The following design values are valid for f'_c = 4000 psi:

$$\text{Maximum}(\phi V_c + \phi V_s) = \phi 10\sqrt{f'_c}\, b_w d = 0.54\, b_w d \qquad \text{(ACI 11.5.6.8)}$$

$$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d = 0.11\, b_w d \qquad \text{(ACI 11.3.1.1)}$$

$$\phi V_c / 2 = \phi\sqrt{f'_c}\, b_w d = 0.055\, b_w d \qquad \text{(ACI 11.5.5.1)}$$

Joists defined by ACI 8.11:

$$\phi V_c = \phi 2.2\sqrt{f'_c}\, b_w d = 0.12\, b_w d \qquad \text{(ACI 8.11.8)}$$

In the above equations, b_w and d are in inches and the resulting shears are in kips.

The design charts in Figs. 3-3 through 3-5 offer another simplified method for shear design. By entering the charts with values of d and $\phi V_s = V_u - \phi V_c$ for the member at the section under consideration, the required stirrup spacings can be obtained by locating the first line above the point of intersection of d and ϕV_s. Values for spacings not shown can be interpolated from the charts if desired. Also given in the charts are values for the minimum practical beam widths b_w that correspond to the maximum allowable $\phi V_s = \phi 8\sqrt{f'_c}\, b_w d$ for each given spacing s; any member which has at least this minimum b_w will be adequate to carry the maximum applied V_u. Fig. 3-6 can also be used to quickly determine if the dimensions of a given section are adequate: any member with an applied V_u which is less than the applicable $V_{u(max)}$ can carry this shear without having to increase the values of b_w and/or d. Once the adequacy of the cross-section has been verified, the stirrup spacing can be established by using Figs. 3-3 through 3-5. This spacing must then be checked for compliance with all maximum spacing criteria.

3.6.1 Example: Design for Shear Reinforcement

The example shown in Fig. 3-7 illustrates the simple procedure for selecting stirrups using design values for V_c and V_s.

(1) Design data: $f'_c = 4000$ psi, $f_y = 60{,}000$ psi, $w_u = 8$ kips/ft.

(2) Calculations:

V_u @ column centerline:	$w_u \ell / 2 = 8 \times 24 / 2 = 96.0$ kips
V_u @ face of support:	96 - 1.17(8) = 86.6 kips
V_u @ d from support face (critical section):	86.6 - 2(8) = 70.6 kips
$(\phi V_c + \phi V_s)_{max}$:	$0.54\, b_w d = 0.54(12)(24) = 155.5$ kips
ϕV_c:	$0.11\, b_w d = 0.11(12)(24) = 31.7$ kips
$\phi V_c/2$:	$0.055\, b_w d = 0.055(12)(24) = 15.8$ kips

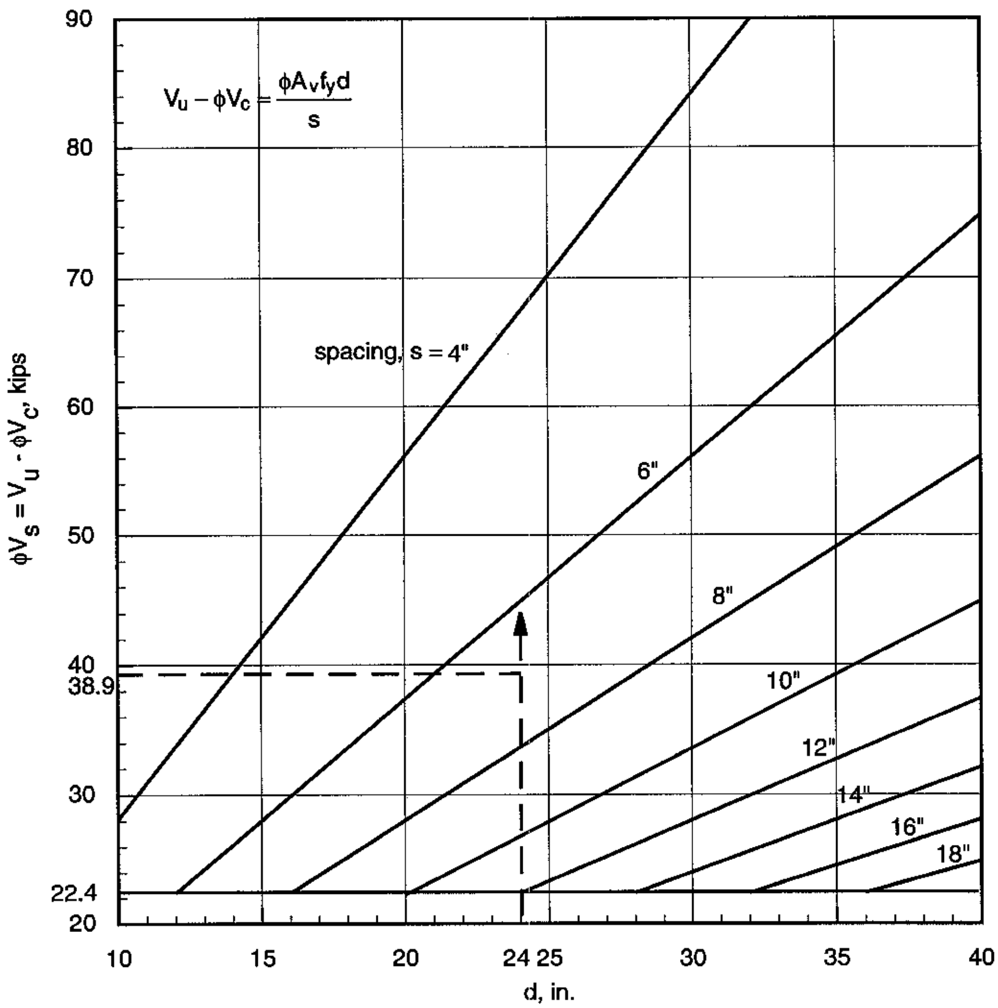

Figure 3-3 Design Chart for Stirrup Spacing, #3 U-Stirrups (2 legs, f_y = 60 ksi)[*,**]

*Horizontal line indicates ϕV_s for $s = d/2$.

**Minimum b_w corresponding to $\phi V_s = \phi 8\sqrt{f'_c}\, b_w d$ is less than 8 in. for all s

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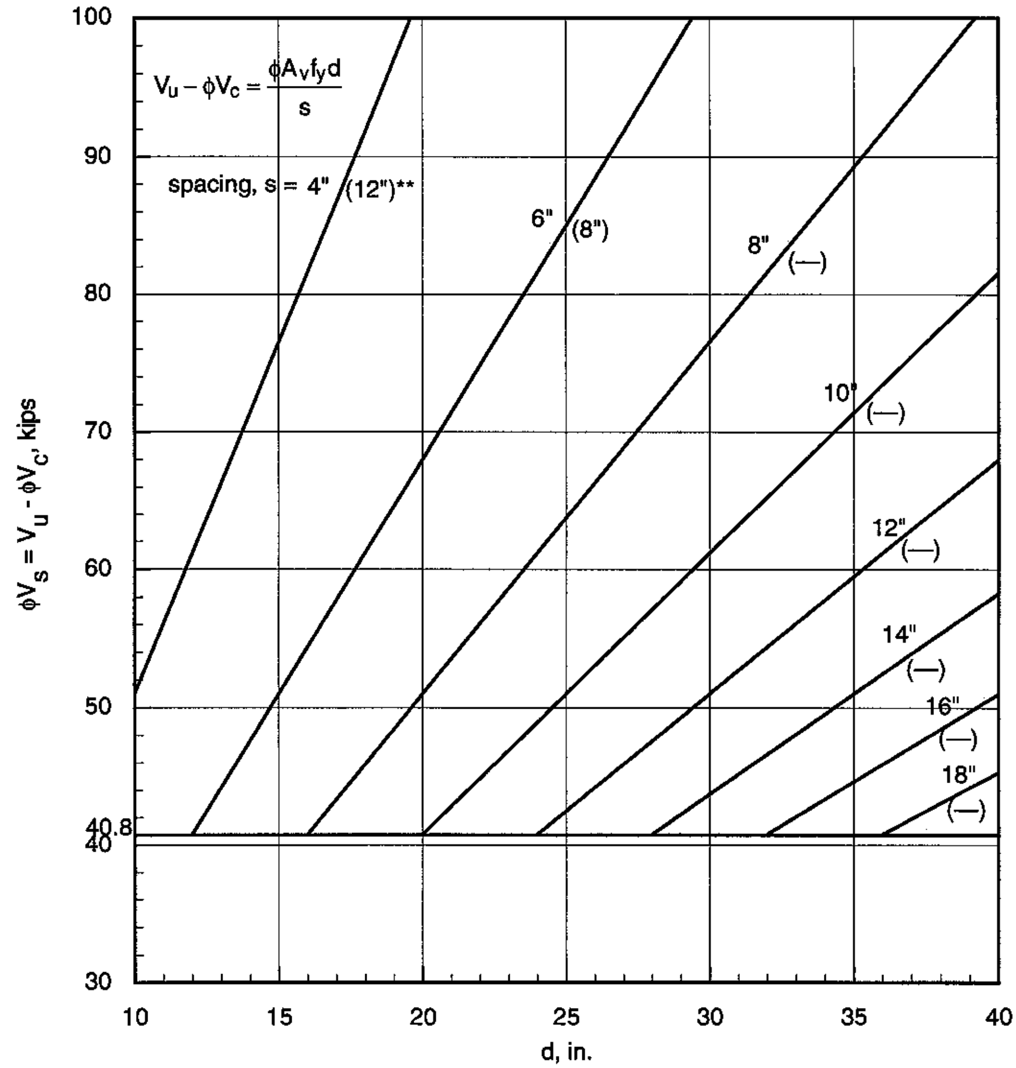

*Figure 3-4 Design Chart for Stirrup Spacing, #4 U-Stirrups (2 legs, f_y = 60 ksi)**, ***

**Horizontal line indicates ϕV_s for $s = d/2$.*

***Values in ( ) indicate minimum practical b_w corresponding to $\phi V_s = \phi 8\sqrt{f'_c}\, b_w d$ for given s.*

(-) Indicates minimum b_w corresponding to $\phi V_s = \phi 8\sqrt{f'_c}\, b_w d$ is less than 8 in. for given s.

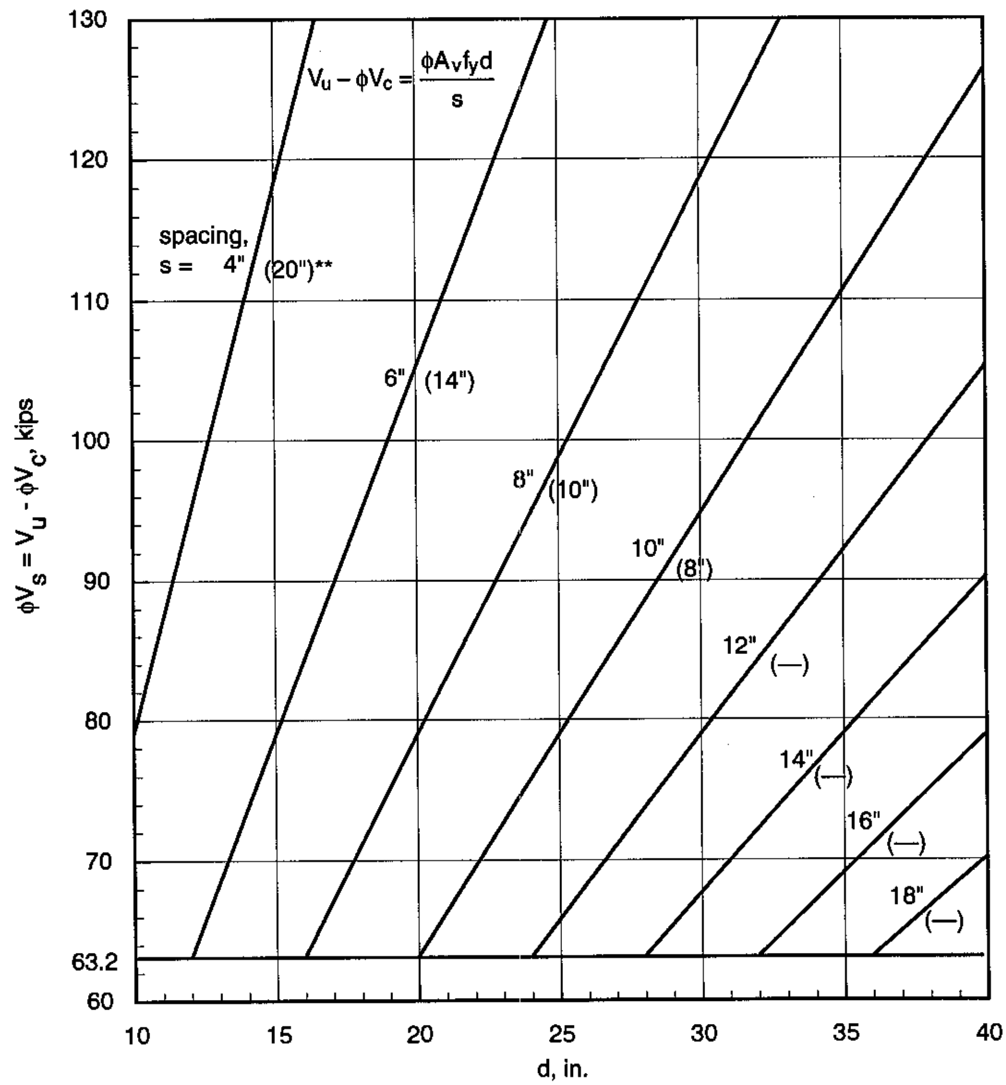

Figure 3-5 Design Chart for Stirrup Spacing, #5-U Stirrups (2 legs, f_y = 60 ksi)[*,**]

**Horizontal line indicates ϕV_s for s = d/2.*

***Values in ( ) indicate minimum practical b_w corresponding to $\phi V_s = \phi 8\sqrt{f_c'}\, b_w d$ for given s.*

(-) Indicates minimum b_w corresponding to $\phi V_s = \phi 8\sqrt{f_c'}\, b_w d$ is less than 8 in. for given s.

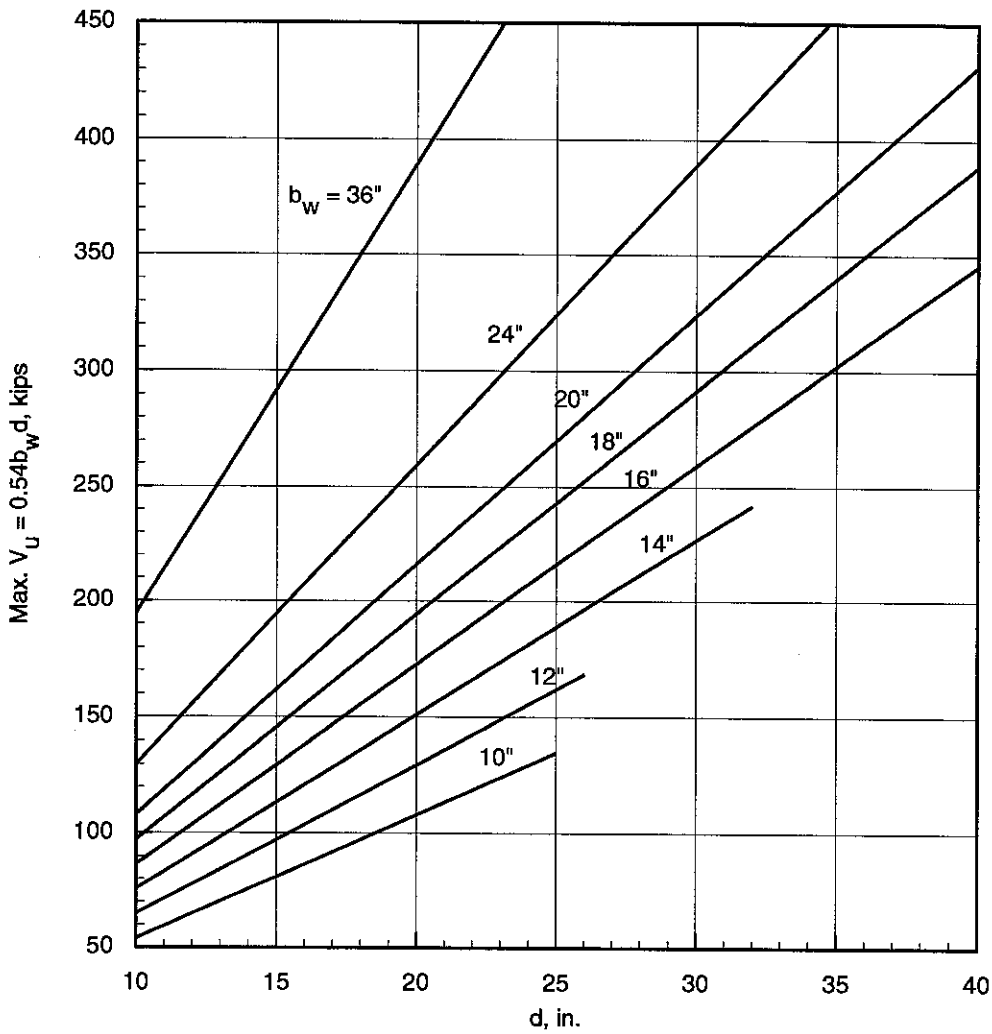

Figure 3-6 Design Chart for Maximum Allowable Shear Force ($f_c' = 4000$ psi)

(3) Beam size is adequate for shear strength, since 155.5 kips > 70.6 kips (also see Fig. 3-6). ϕV_s (required) = 70.6 - 31.7 = 38.9 kips. From Table 3-9, #4 @ d/2 = 12 in. is adequate for full length where stirrups are required since ϕV_s = 40 kips > 38.9 kips. Length over which stirrups are required is (86.6 - 15.8)/8 = 8.85 ft from support face.

Check maximum stirrup spacing:

$$\phi 4\sqrt{f_c'}\, b_w d = 0.22 b_w d = 63.4 \text{ kips}$$

Since ϕV_s = 38.9 kips < 63.4 kips, the maximum spacing is the least of the following:

$$s_{max} = \begin{cases} d/2 = 12 \text{ in. (governs)} \\ 24 \text{ in.} \\ A_v f_y / 50 b_w = 40 \text{ in.} \end{cases}$$

Use 10-#4 U-stirrups at 12 in. at each end of beam.

The problem may also be solved graphically as shown in Fig. 3-7. ϕV_s for #3 stirrups at d/2, d/3, and d/4 are scaled vertically from ϕV_c. The horizontal intersection of the ϕV_s values (22 kips, 33 kips, and 45 kips) with the shear diagram automatically sets the distances where the #3 stirrups should be spaced at d/2, d/3, and d/4. The exact numerical values for these horizontal distances are calculated as follows (although scaling from the sketch is close enough for practical design):

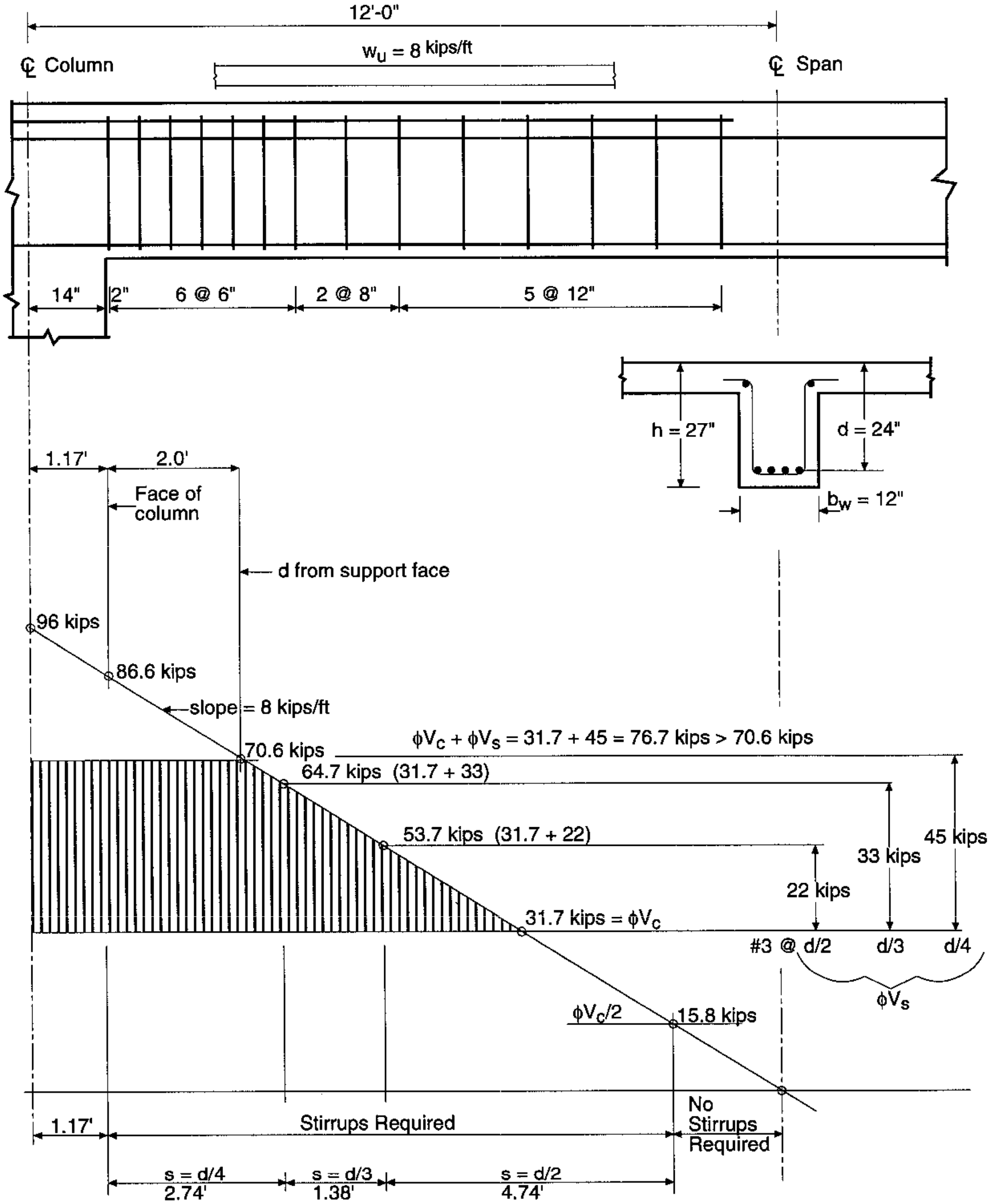

Figure 3-7 Simplified Method for Stirrup Spacing (Example 3.6.1)

#3 @ d/4 = 6 in.:	(86.6 - 64.7)/8 = 2.74 ft (32.9 in.)	use 6 @ 6 in.
@ d/3 = 8 in.:	(64.7 - 53.7)/8 = 1.38 ft (16.6 in.)	use 2@ 8 in.
@ d/2 = 12 in.:	(53.7 - 15.8)/8 = 4.74 ft (56.9 in.)	use 5 @ 12 in.

A more practical solution may be to eliminate the 2 @ 8 in. and use 9 @ 6 in. and 5 @ 12 in.

As an alternative, determine the required spacing of the #3 U-stirrups at the critical section using Fig. 3-3. Enter the chart with d = 24 in. and ϕV_s = 38.9 kips. The point representing this combination is shown in the design chart. The line immediately above this point corresponds to a spacing of s = 6 in. which is exactly what was obtained using the previous simplified method.

3.6.2 Selection of Stirrups for Economy

Selection of stirrup size and spacing for overall cost savings requires consideration of both design time and fabrication and placing costs. An exact solution with an intricate stirrup layout closely following the variation in the shear diagram is not a cost-effective solution. Design time is more cost-effective when a quick, more conservative analysis is utilized. Small stirrup sizes at close spacings require disproportionately high costs in labor for fabrication and placement. Minimum cost solutions for simple placing should be limited to three spacings: the first stirrup located at 2 in. from the face of the support (as a minimum clearance), an intermediate spacing, and finally, a maximum spacing at the code limit of d/2. Larger size stirrups at wider spacings are more cost-effective (e.g., using #4 for #3 at double spacing, and #5 and #4 at 1.5 spacing) if it is possible to use them within the spacing limitations of d/2 and d/4. For the example shown in Fig. 3-7, the 10-#4@12 in. is the more cost effective solution.

In order to adequately develop the stirrups, the following requirements must all be satisfied (ACI 12.13): (1) stirrups shall be carried as close to the compression and tension surfaces of the member as cover requirements permit, (2) for #5 stirrups and smaller, a standard stirrup hook (as defined in ACI 7.1.3) shall be provided around longitudinal reinforcement, and (3) each bend in the continuous portion of the stirrup must enclose a longitudinal bar. To allow for bend radii at corners of U stirrups, the minimum beam widths given in Table 3-10 should be provided.

Table 3-10 Minimum Beam Widths for Stirrups

Stirrup size	Minimum beam width (b_w)
#3	10 in.
#4	12 in.
#5	14 in.

Note that either the #3 or the #4 stirrup in the example of Fig. 3-7 can be placed in the 12 in. wide beam.

3.7 DESIGN FOR TORSION

For simplified torsion design of spandrel beams, where the torsional loading is from an integral slab, two options are possible:

(1) Size the spandrel beams so that torsion effects can be neglected (ACI 11.6.1) or,

(2) Provide torsion reinforcement for an assumed maximum torsional moment (ACI 11.6.3).

All ACI provisions for torsion design are summarized in Table 3-11. This table should be used with Table 3-8 for the usual cases of members subjected to shear and torsion (see Section 3.7.2).

Table 3-11 ACI Provisions for Torsion Design

		$T_u \le \phi 0.5\sqrt{f'_c}\,\Sigma x^2 y$	$\phi 0.5\sqrt{f'_c}\,\Sigma x^2 y < T_u \le \phi T_c^*$	$T_u > \phi T_c$
Required area of stirrups, A_t**		None	$\frac{25 b_w s^{***}}{f_y}$	$\frac{(T_u - \phi T_c)s}{\phi \alpha_t x_1 y_1 f_y}$
Closed stirrup spacing, s	Required	—	$\frac{A_t f_y}{25 b_w}$ †	$\frac{\phi A_t \alpha_t x_1 y_1 f_y}{(T_u - \phi T_c)}$
	Maximum†† (ACI 11.6.8.1)	—	$\frac{x_1 + y_1}{4}$ or 12 in.	$\frac{x_1 + y_1}{4}$ or 12 in.
	Recommended Minimum	—	—	4 in.
Required area of longitudinal steel, A_ℓ (ACI 11.6.9.3)		None	Larger of eqn. (11-24) or eqn. (11-25)	Larger of eqn. (11-24) or eqn. (11-25)
Maximum spacing of longitudinal steel (ACI 11.6.8.2)		—	12 in.	12 in.

*$T_c = 0.8\sqrt{f'_c}\,\Sigma x^2 y / \sqrt{1 + (0.4V_u / C_t T_u)^2}$ (ACI 11.6.6.1)

**A_t = area of one leg of a closed stirrup; $f_y \le 60$ ksi (ACI 11.6.7.4)

***This is applicable when $V_u \le \phi V_c/2$. When $V_u > \phi V_c/2$, minimum reinforcement is $A_v + 2A_t = 50 b_w s / f_y$ (ACI 11.5.5.5).

†This is applicable when $V_u \le \phi V_c/2$. When $V_u > \phi V_c/2$, required spacing is $(A_v + 2A_t) f_y / 50 b_w$ (ACI 11.5.5.5).

††Maximum spacing based on minimum torsion reinforcement must also be considered.

3.7.1 Beam Sizing to Neglect Torsion

A simplified sizing equation to neglect torsion effects can be derived based on the limiting factored torsional moment $T_u = \phi(0.5\sqrt{f'_c}\,\Sigma x^2 y)$. For a spandrel beam integral with a slab, torsional loading may be taken as uniformly distributed along the spandrel (ACI 11.6.3.2), with the torsional loading approximated as $t_u = 2T_u/\ell$ per unit length of spandrel beam (see Fig. 3-8). Set the maximum torque T_u equal to the limiting torque:

$$t_u = \frac{2\phi}{\ell}(0.5\sqrt{f'_c}\,\Sigma x^2 y)$$

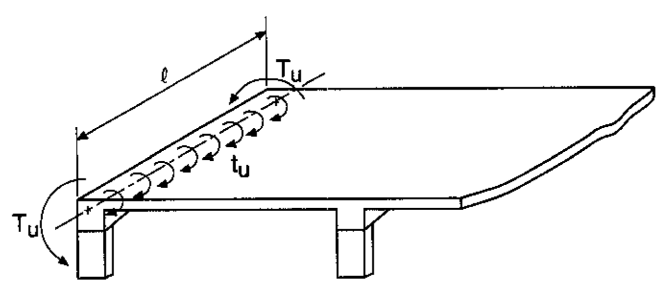

Figure 3-8 Torsional Loading on a Spandrel Beam

For a two-way flat plate or flat slab with spandrel beams, the total exterior negative slab moment is equal to $0.30M_o$ (ACI 13.6.3.3), where $M_o = w_u \ell_2 \ell_n^2/8$ (ACI Eq. 13-3); also see Chapter 4, Table 4-3. For a two-way system, the torsional moment transferred to the spandrel per unit length of beam can be approximated as $t_u = 0.30M_o/\ell$. Thus, by equating this t_u with the limiting value above, the required torsional section properties to neglect torsion can be expressed as:

$$\Sigma x^2 y = \frac{0.30M_o}{2\phi(0.5\sqrt{f'_c})}$$

For a spandrel beam section of width b and overall depth h, as shown in Fig. 3-9, the minimum size of the spandrel beam to neglect torsion reduces to:

$$b^2 h \text{ or } h^2 b = \frac{0.30M_o}{2\phi(0.5\sqrt{f'_c})} - 3h_s^3$$

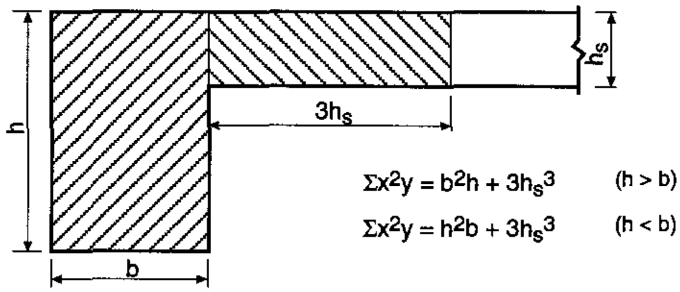

Figure 3-9 Torsional Section Properties

For our selected concrete strength $f'_c = 4000$ psi, this equation simplifies to:

$$b^2 h \text{ or } h^2 b = 67M_o - 3h_s^3$$

For a one-way slab or one-way joists with spandrel beam support, the exterior negative slab moment is equal to $w_u \ell_n^2/24$ (ACI 8.3.3). This negative moment can also be expressed as $0.33M_o$ where M_o = total static span moment $= w_u \ell_n^2/8$. Thus, for a one-way system with spandrel beams, sizing to neglect torsion reduces to:

$$b^2 h \text{ or } h^2 b = 74M_o - 3h_s^3$$

The above sizing equations are in mixed units: M_o is in foot-kips and b and h are in inches.

Architectural or economic considerations may dictate a smaller spandrel size than that required to neglect torsion effects. For a specific floor framing system, both architectural and economic aspects of a larger beam size to neglect torsion versus a smaller beam size with torsion reinforcement (additional closed stirrups at close spacing combined with longitudinal bars) needs to be evaluated. If a smaller spandrel with torsion reinforcement is a more appropriate choice, Section 3.7.2 provides a simple method for the design of the torsion reinforcement.

3.7.1.1 Example: Beam Sizing to Neglect Torsion

Determine a spandrel beam size to neglect torsion effects for Building #2, Alternate (1)—slab and column framing with spandrel beams.

For N-S spandrels: $\ell_2 = 20$ ft
$\ell_1 = 24$ ft
$\ell_n = 24 - (12 + 16)/(2 \times 12) = 22.83$ ft
$w_u = 1.4(136) + 1.7(50) = 275$ psf

$M_o = w_u \ell_2 \ell_n^2/8 = 0.275 \times 20 \times 22.83^2/8 = 358.8$ ft-kips

b^2h or h^2b(required) $= 67M_o - 3h_s^3 = 67(358.8) - 3(8.5)^3 = 22{,}197$ in.^{3}

Some possible combinations of b and h that will satisfy this equation are tabulated below.

b (in.)	h(required) (in.)	possible selection b x h (in. x in.)
12	154.1	
14	113.3	
16	86.7	
18	68.5	
20	55.5	20 x 56
22	45.9	22 x 46
24	38.5	24 x 38
26	32.8	26 x 34
28	28.3	28 x 28
30	27.2	30 x 28
32	26.3	32 x 26
34	25.6	
36	24.8	

Clearly, large beam sizes are required to neglect torsion effects. It would probably be advantageous to select a smaller beam and provide torsion reinforcement.

3.7.2 Simplified Design for Torsion Reinforcement

When required, torsion reinforcement must consist of a combination of closed stirrups and longitudinal bars (ACI 11.6.7).

For spandrel beams built integrally with a floor slab system (where reduction of torsional loading can occur due to redistribution of internal forces), a maximum torsional moment equal to $\phi(4\sqrt{f'_c}\,\Sigma x^2y/3)$ may be assumed (ACI 11.6.3).

Using this assumption, simplified design charts for torsion reinforcement can be developed. Figure 3-10 gives the required area of closed stirrups per inch, A_t/s, as a function of b_w and h. The figure is based on ACI Eqs. (11-22) and (11-23), assuming #3 closed stirrups with 1 ½ in. clear cover all around and a 5 in. slab thickness (minimum permitted per ACI 9.5.3).* The shear stress V_u/b_wd in Eq. (11-22) was set equal to $8\sqrt{f'_c}$, i.e., the maximum allowed value.** Maximizing V_u/b_wd results in minimizing T_c, and thus, maximizing A_t/s. Consequently, this figure will yield conservative values of A_t/s when the actual V_u/b_wd is less than the maximum. The values of A_t/s in Fig. 3-10 can be easily adjusted for any V_u/b_wd ratio by using Fig. 3-11 and the following equation:

**Using larger stirrup sizes will have a negligible influence on the required A_t/s. Larger slab thicknesses would require somewhat larger values of A_t/s; however, this increase becomes small for larger beams.*

***In order to satisfy ACI 11.6.9.4 (i.e., $T_s \leq 4T_c$), the section must be large enough so that $V_u/b_wd \leq 8\sqrt{f'_c}$ when $T_u = \phi 4\sqrt{f'_c}\,\Sigma x^2y/3$ (see ACI Eqs. (11-20) through (11-22)).*

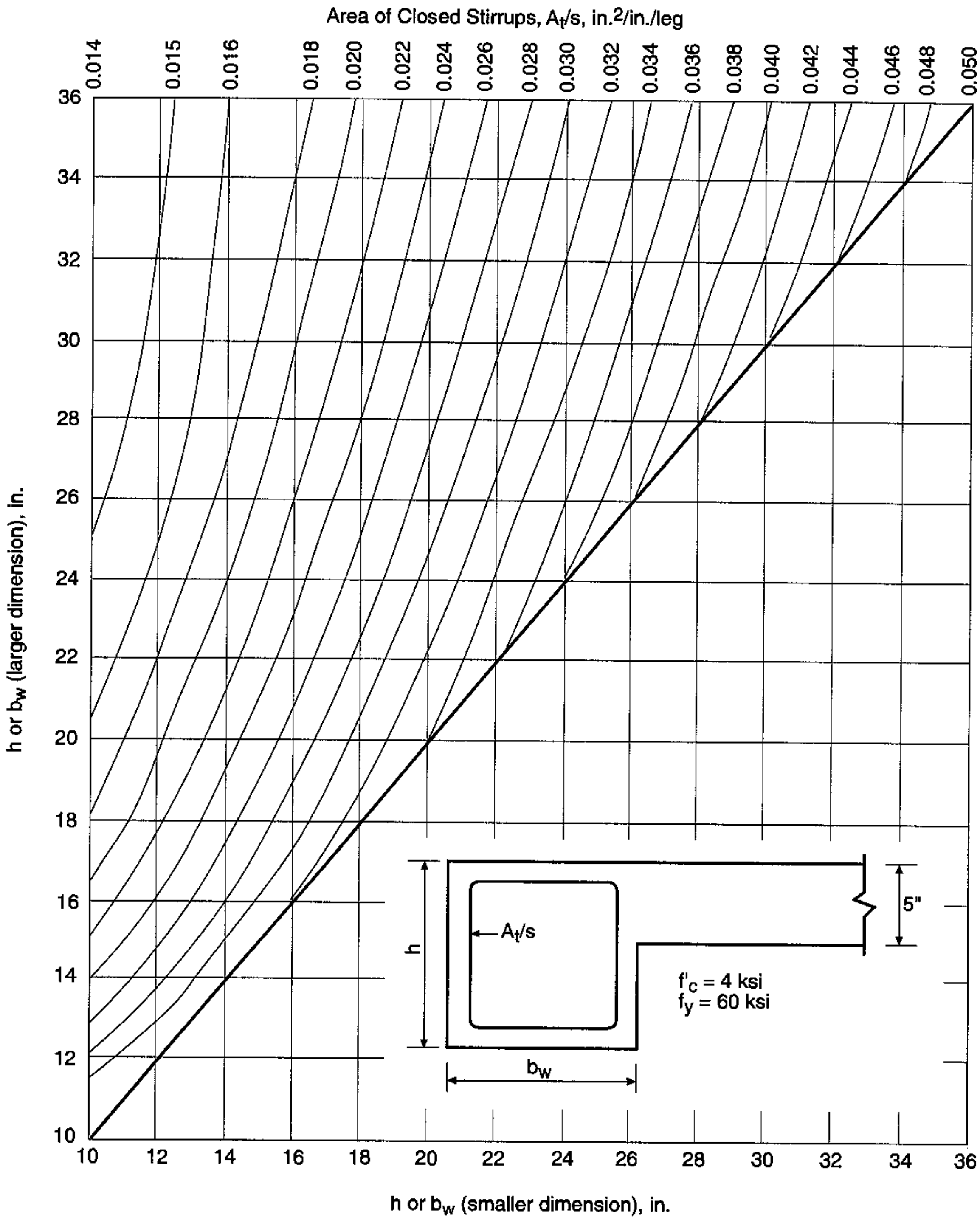

Figure 3-10 Torsion Reinforcement (A_t/s) for Spandrel Beams

$$\frac{A_t}{s} = \frac{(\text{coefficient in Fig. 3-11})}{0.9} \times \left(\frac{A_t}{s} \text{ from Fig. 3-10}\right)$$

Figure 3-12 gives the required longitudinal reinforcement A_ℓ, based on ACI Eqs. (11-24) and (11-25), again assuming #3 stirrups, 1 ½ in. cover, and a 5 in. slab thickness. To obtain the maximum A_ℓ, the shear stress V_u/b_wd was set equal to the minimum value of $\phi\sqrt{f'_c}$ in Eqs. (11-24) and (11-25). Although this figure yields

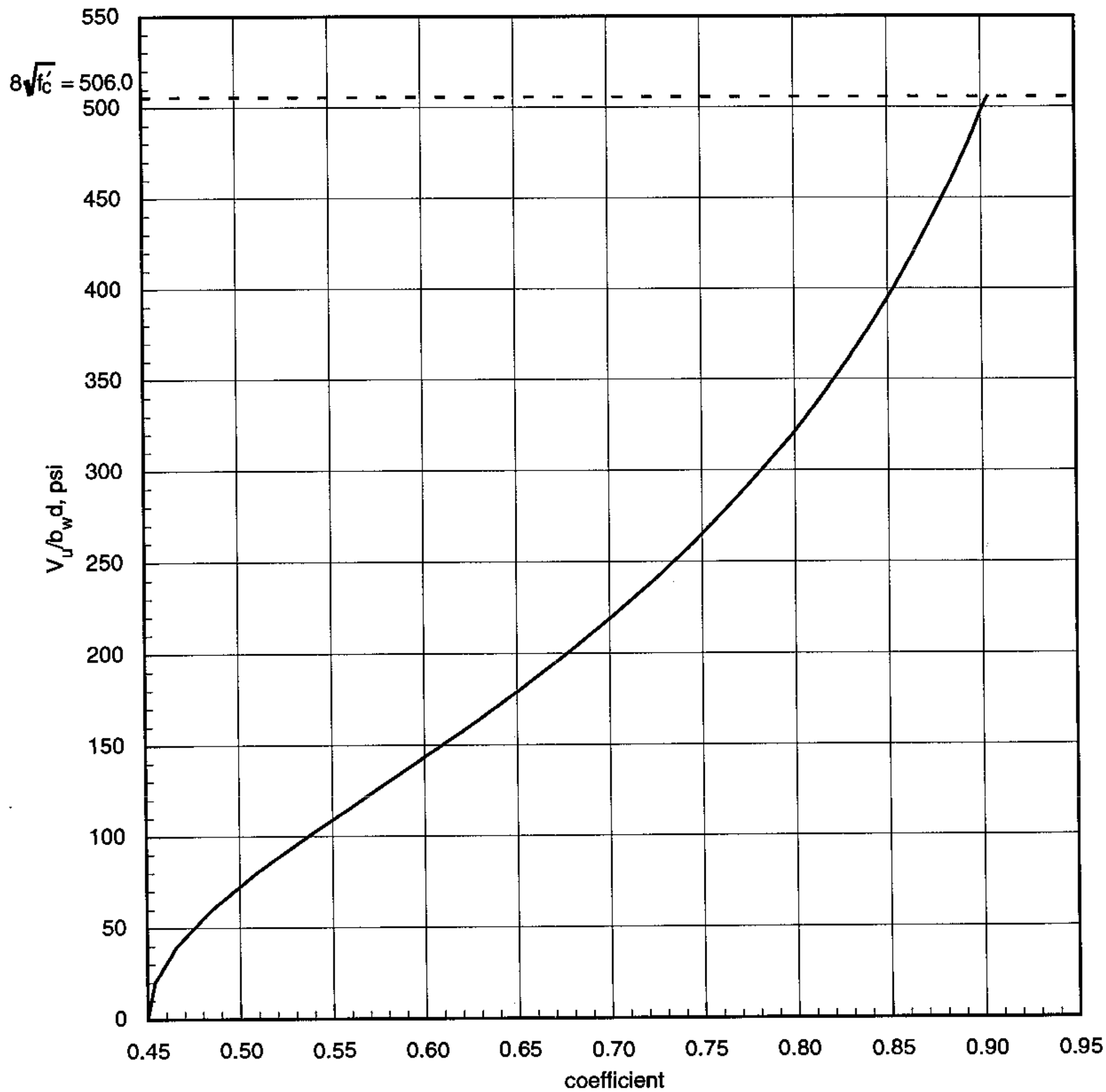

Figure 3-11 Coefficients for Determining A_t/s for any V_u/b_wd

conservative values for A_ℓ, the required reinforcement may not be excessive, since floor systems with slabs thicker than 5 in. would require larger values of A_ℓ.

For torsional loading from a uniformly loaded slab to a spandrel beam, the torsional moment variation along the spandrel will be approximately linear from a maximum at the critical section ($T_u = \phi 4\sqrt{f'_c}\,\Sigma x^2 y/3$) to zero at midspan. Accordingly, the required area of closed stirrups obtained from the above equation and Figs. 3-10 and 3-11 may be reduced linearly toward midspan, but not to less than the minimum amount required by ACI 11.5.5.5.

For members subjected to flexure, shear, and torsion, the reinforcement required for torsion is to be combined with that required for all other forces (ACI 11.6.7). In particular, the total required area per leg and spacing for the closed stirrups is:

$$\frac{\Sigma A_{v,t}}{s} = \frac{A_v}{2s} + \frac{A_t}{s}$$

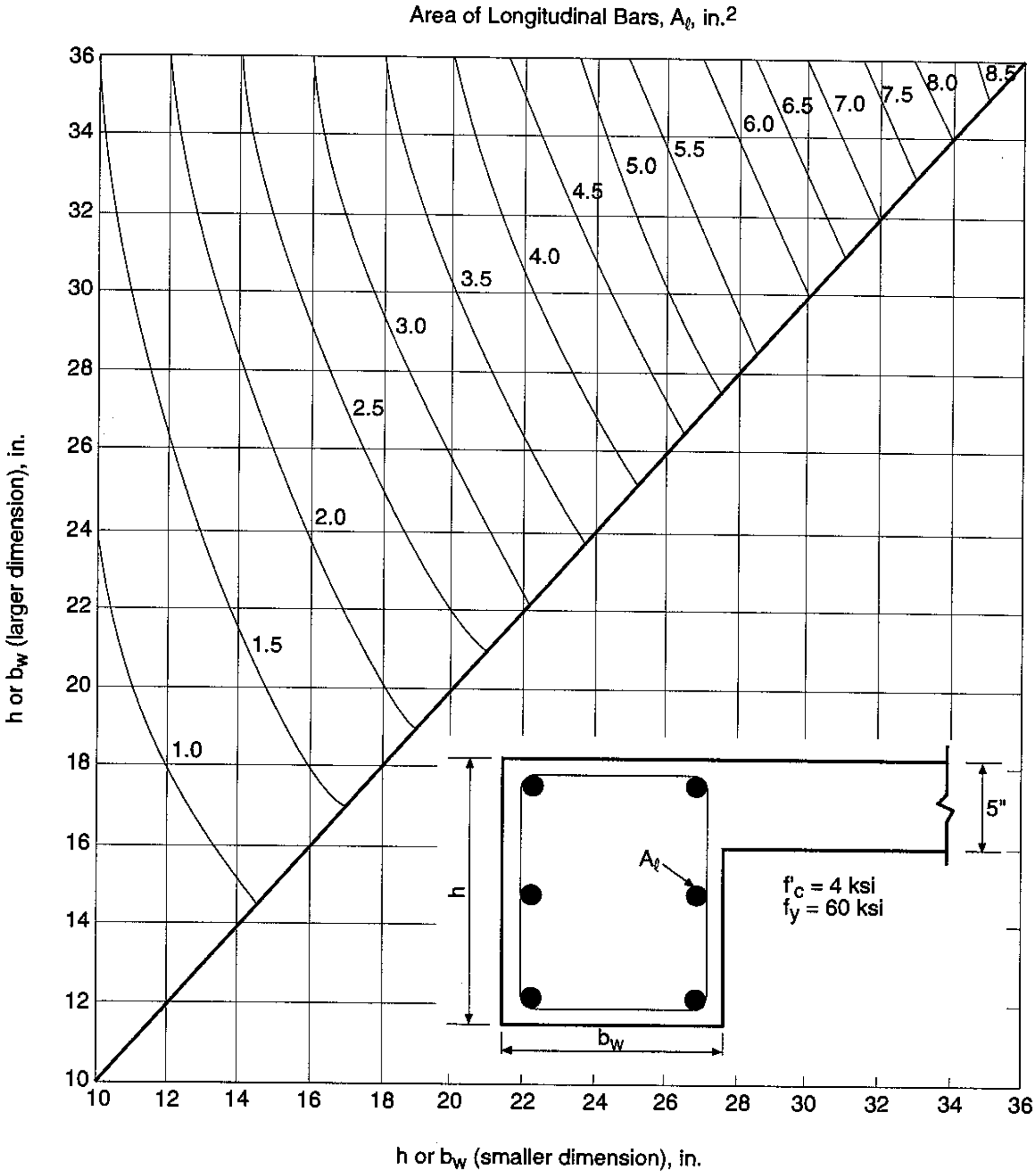

Figure 3-12 Torsion Reinforcement (A_ℓ) for Spandrel Beams

Note that A_t refers to one leg of a closed stirrup while A_v refers to two legs. Assuming as before that $T_u = \phi 4\sqrt{f'_c}\,\Sigma x^2y/3$, the quantity A_t/s can be determined using Figs. 3-10 and 3-11, and $A_v/2s$ can be obtained from Fig. 3-13. The curves in Fig. 3-13 are based on ACI Eq. (11-5) which is valid whenever $T_u > \phi 0.5\sqrt{f'_c}\,\Sigma x^2y$. All maximum spacing criteria should be checked. Additionally, the longitudinal torsional reinforcement is to be combined with that required for flexure.

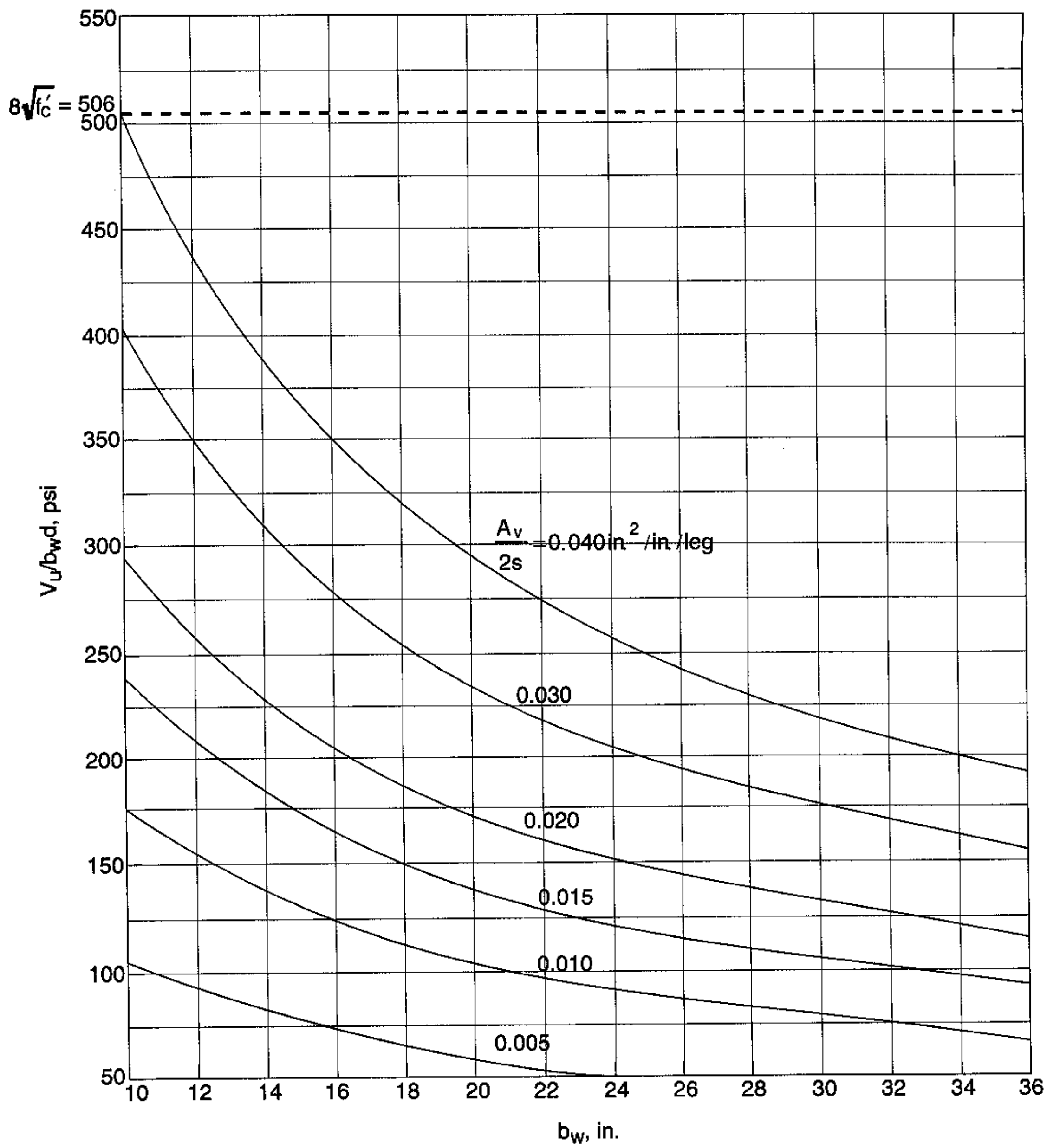

Figure 3-13 Required Shear Reinforcement ($A_v/2s$), Including Torsional Effects

3.7.2.1 Example: Design for Torsion Reinforcement

Determine the required combined shear and torsion reinforcement for the E-W spandrel beams of Building #2, Alternate (1) - slab and column framing with spandrel beams.

For E-W spandrels:

spandrel size = 12 × 20 in.
d = 20 - 2.5 = 17.5 in. = 1.46 ft
ℓ_n = 24 - (12/12) = 23.0 ft
beam weight = 1.4 (12 × 20 × 0.150/144) = 0.35 kips/ft

w_u from slab = 1.4(136) + 1.7(50) = 275 psf

Tributary load to spandrel (½ panel width) = 275(20/2)		= 2.75 kips/ft
	beam	= <u>0.35 kips/ft</u>
	Total w_u	= 3.10 kips/ft

Check for fire resistance:

From Table 10-4, for a restrained beam with a width greater than 10 in., ¾ in. cover is required for two hour fire resistance rating. That is less than the minimum required cover in ACI 7.7; therefore, section is adequate for fire resistance.

(1) Determine V_u/b_wd and check if section is adequate to carry the torsion

V_u at face of support = $w_u\ell_n/2$ = (3.10 × 23)/2 = 35.7 kips
V_u at distance d from support = 35.7 - 3.10(1.46) = 31.2 kips
V_u/b_wd = 31,200/(12 × 17.5) = 148.6 psi
Since 148.6 psi < $8\sqrt{4000}$ = 506 psi, the beam is large enough to carry the torsion.

(2) Determine A_t/s from Figs. 3-10 and 3-11

From Fig. 3-10 with b_w = 12 in. and h = 20 in., $A_t/s \cong 0.020$ in.²/in./leg.
From Fig. 3-11 with V_u/b_wd = 148.6 psi, the coefficient ≅ 0.61.

Therefore,

$$\text{required } \frac{A_t}{s} = \frac{0.61}{0.9}(0.020) = 0.014 \text{ in.}^2/\text{in.}/\text{leg}$$

(3) Determine $A_v/2s$ from Fig. 3-13

With V_u/b_wd = 148.6 psi and b_w = 12 in., required $A_v/2s \cong 0.009$ in.²/in./leg.

(4) Determine total stirrup area required for shear and torsion

$$A_t/s + A_v/2s = 0.014 + 0.009 = 0.023 \text{ in.}^2/\text{in./leg}$$

Check minimum stirrup area (ACI 11.5.5.5):

$$A_v/2s + A_t/s = 25b_w/f_y = 25 \times 12/60{,}000 = 0.005 \text{ in.}^2/\text{in./leg} < 0.023 \text{ in.}^2/\text{in./leg} \quad \text{O.K.}$$

Required spacing of closed stirrups:

For a #3 stirrup (A_b = 0.11 in.²), required spacing $s = \dfrac{0.11}{0.023} = 4.8$ in.

For a #4 stirrup (A_b = 0.20 in.²), required spacing $s = \dfrac{0.20}{0.023} = 8.7$ in.

(5) Check maximum spacing (ACI 11.6.8.1 and 11.5.4.1)

For #3 stirrups: $x_1 = 12 - 3.375 = 8.625$ in.
$y_1 = 20 - 3.375 = 16.625$ in.
$(x_1 + y_1)/4 = (8.625 + 16.625)/4 = 6.31$ in. < 12 in.

For #4 stirrups: $x_1 = 12 - 3.5 = 8.5$ in.
$y_1 = 20 - 3.375 = 16.625$ in.
$(x_1 + y_1)/4 = (8.5 + 16.5)/4 = 6.25$ in. < 12 in.

$d/2 = 17.5/2 = 8.75$ in.

Use #4 closed stirrups @ 6 in. on center.

(6) Determine the required area of longitudinal bars, A_ℓ, from Fig. 3-12

For $h = 20$ in. and $b_w = 12$ in.: $A_\ell \cong 1.10$ in.^{2}

Place the longitudinal bars around perimeter of the closed stirrups, spaced not more than 12 in. apart, and locate one longitudinal bar in each corner of the closed stirrups (ACI 11.6.8.2). For the 20 in. deep beam, one bar is required at mid-depth on each side face, with ⅓ of the total A_ℓ required at top, mid-depth, and bottom of the closed stirrups. $A_\ell/3 = 1.1/3 = 0.37$ in.^{2}. Use 2-#4 bars at mid-depth (one on each face). Longitudinal bars required at top and bottom may be combined with the flexural reinforcement.

Details for the shear and torsion reinforcement (at the support) are shown in Fig. 3-14. The closed stirrups may be reduced toward midspan, but not less than the minimum amount required by ACI 11.5.5.5. The two-piece closed stirrup detail shown in the figure is recommended for spandrel beams (see Section 3.7.3).

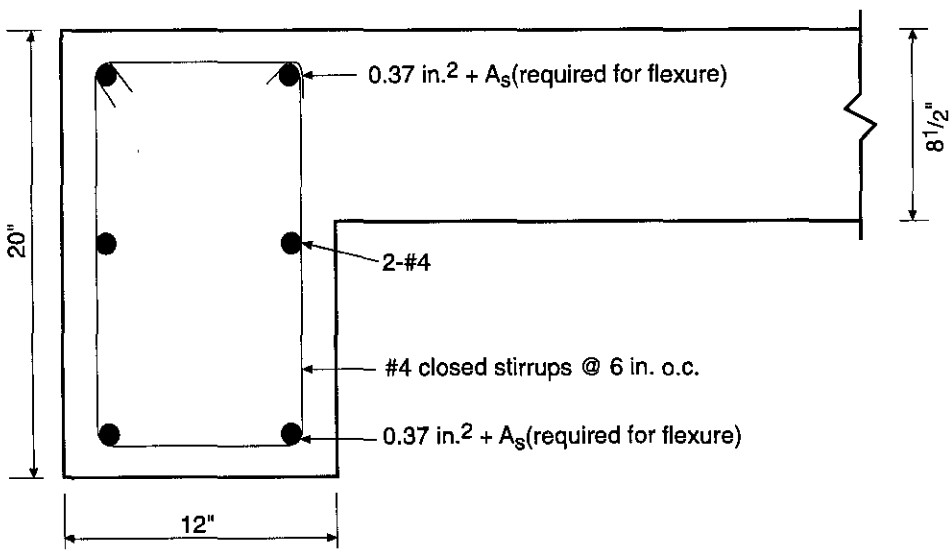

Figure 3-14 Required Shear and Torsion Reinforcement (Example 3.7.2.1)

3.7.3 Closed Stirrup Details

The requirements for closed stirrups are given in ACI 7.11.3 and are depicted in Fig. 3-15.

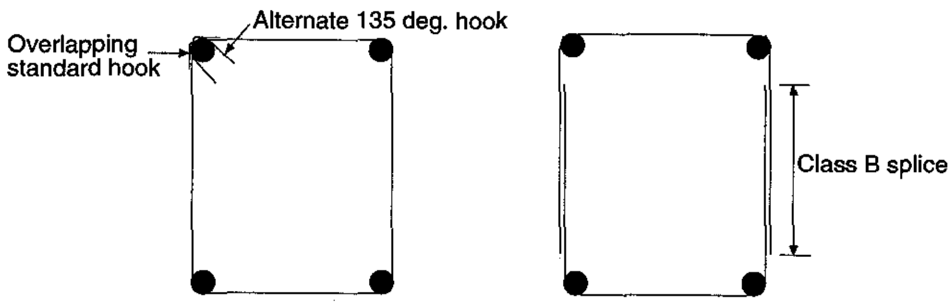

Figure 3-15 Code Definition of Closed Tie or Stirrup

The one-piece closed stirrup with overlapping end hooks is generally not practical for placement in the field. Also, neither of these closed stirrups is considered effective for members subject to high torsional loading. Even though spandrel beams are usually subjected to low torsional loading from an integral slab, it is recommended that the two-piece closed stirrup detail shown in Fig. 3-16 be used.[3.8]

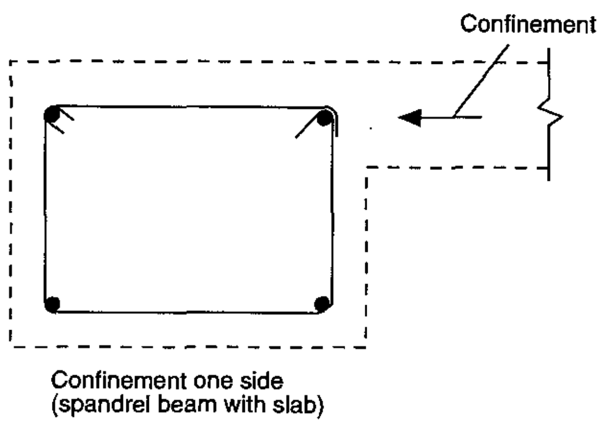

Figure 3-16 Recommended Two-Piece Closed Stirrup Detail[3.8]

3.8 EXAMPLES: SIMPLIFIED DESIGN FOR BEAMS AND SLABS

The following three examples illustrate use of the simplified design data presented in Chapter 3 for proportioning beams and slabs. Typical floor members for the one-way joist floor system of Building #1 are designed.

3.8.1 Example: Design of Standard Pan Joists for Alternate (1) Floor System (Building #1)

(1) Data: f'_c = 4000 psi (normal weight concrete, carbonate aggregate)
f_y = 60,000 psi

Floors: LL = 60 psf
DL = 130 psf (assumed total for joists and beams + partitions + ceiling & misc.)

Required fire resistance rating = 1 hour

Floor system - Alternate (1): 30-in. wide standard pan joists
width of spandrel beams = 20 in.
width of interior beams = 36 in.

Note: The special provisions for standard joist construction in ACI 8.11 apply to the pan joist floor system of Alternate (1) .

(2) Determine the factored shears and moments using the approximate coefficients of ACI 8.3.3.

Factored shears and moments for the joists of Alternate (1) are determined in Chapter 2, Section 2.3.2. Results are summarized as follows:

$$w_u = [1.4(130) + 1.7(60) = 284 \text{ psf}] \times 3 \text{ ft} = 852 \text{ plf}$$

Note: All shear and negative moment values are at face of supporting beams.

V_u @ spandrel beams	= 11.7 kips
V_u @ first interior beams	= 13.5 kips
$-M_u$ @ spandrel beams	= 26.8 ft-kips
$+M_u$ @ end spans	= 46.0 ft-kips
$-M_u$ @ first interior beams	= 63.1 ft-kips
$+M_u$ @ interior spans	= 38.8 ft-kips
$-M_u$ @ interior beams	= 56.5 ft-kips

(3) Preliminary size of joist rib and slab thickness

From Table 3-1: depth of joist $h = \ell_n/18.5 = (27.5 \times 12)/18.5 = 17.8$ in.

where ℓ_n (end span) = 30 - 1.0 - 1.5 = 27.5 ft (governs)
ℓ_n (interior span) = 30 - 3 = 27.0 ft

From Table 10-1, required slab thickness = 3.2 in. for 1-hour fire resistance rating. Also, from ACI 8.11.6.1:

Minimum slab thickness $\geq 30/12 = 2.5$ in. > 2.0 in.

Try 16 in. pan forms* + 3½ in. slab

h = 19.5 in. > 17.8 in. O.K.
slab thickness = 3.5 in. > 3.2 in. O.K.

(4) Determine width of joist rib

(a) Code minimum (ACI 8.11.2):

$b_w \geq 16/3.5 = 4.6$ in. > 4.0 in.

(b) For moment strength:

$$b_w = \frac{20M_u}{d^2} = \frac{20(63.1)}{18.25^2} = 3.8 \text{ in.}$$

where d = 19.5 - 1.25 = 18.25 in. = 1.52 ft

See Table 9-3 for standard form dimensions for one-way joists.

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Check for fire resistance: from Table 10-4 for restrained members, the required cover for a fire resistance rating of 1 hr = ¾ in. for joists.

(c) For shear strength:

V_u @ distance d from support face = 13.5 - 0.85(1.52) = 12.2 kips
$\phi V_c = 1.1(0.11 b_w d) = 0.12 b_w d$*
$\therefore b_w = 12.2/(0.12 \times 18.25) = 5.6$ in.

Use 6 in. wide joists (see Table 9-3 and Fig. 3-17).

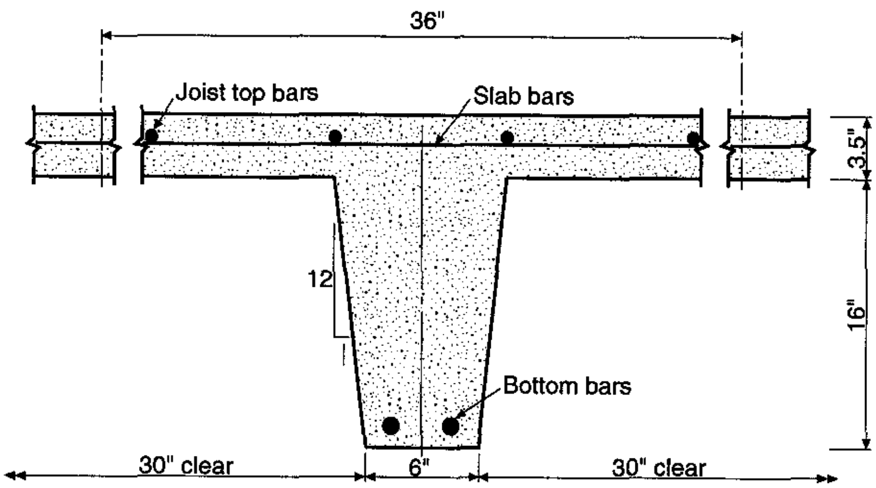


Figure 3-17 Joist Section (Example 3.8.1)

(5) Determine Moment Reinforcement

(a) Top bars at spandrel beams:

$$A_s = \frac{M_u}{4d} = \frac{26.8}{4(18.25)} = 0.37 \text{ in.}^2$$

Distribute bars uniformly in top slab:

$$A_s = 0.37/3 = 0.122 \text{ in.}^2/\text{ft}$$

Maximum bar spacing for an interior exposure and ¾ in. cover will be controlled by provisions of ACI 7.6.5 (see Table 3-4):

$$s_{max} = 3h = 3(3.5) = 10.5 \text{ in.} < 18 \text{ in.}$$

From Table 3-7: Use #3 @ 10 in. ($A_s = 0.13$ in.^{2}/ft)

(b) Bottom bars in end spans:

$$A_s = 46.0/4(18.25) = 0.63 \text{ in.}^2$$

Check rectangular section behavior:

For standard joist ribs conforming to ACI 8.11, a 10% greater shear strength ϕV_c is allowed. Also, minimum shear reinforcement is not required (see ACI 11.5.5).

$a = A_s f_y/0.85 f'_c b_e = (0.63 \times 60)/(0.85 \times 4 \times 36) = 0.31 \text{ in.} < 3.5 \text{ in.}$ O.K.

From Table 3-6: Use 2-#5 ($A_s = 0.62 \text{ in.}^2$)

Check $\rho = A_s/b_w d = 0.62/(6 \times 18.25) = 0.0057 > \rho_{min} = 0.0033$ O.K.

(c) Top bars at first interior beams:

$$A_s = [63.1/4(18.25) = 0.86 \text{ in.}^2]/3 = 0.29 \text{ in.}^2/\text{ft}$$

From Table 3-7, with $s_{max} = 10.5$ in.:

Use #5 @ 10 in. ($A_s = 0.37 \text{ in.}^2/\text{ft}$)

(d) Bottom bars in interior spans:

$$A_s = 38.8/4(18.25) = 0.53 \text{ in.}^2$$

From Table 3-6: Use 2-#5 ($A_s = 0.62 \text{ in.}^2$)

(e) Top bars at interior beams:

$$A_s = [56.5/4(18.25) = 0.77 \text{ in.}^2]/3 = 0.26 \text{ in.}^2/\text{ft}$$

From Table 3-7, with $s_{max} = 10.5$ in.:

Use #5 @ 10 in. ($A_s = 0.37 \text{ in.}^2/\text{ft}$)

(f) Slab reinforcement normal to ribs (ACI 8.11.6.2):

Slab reinforcement is often located at mid-depth of slab to resist both positive and negative moments.

Use $M_u = w_u \ell_n^2/12$ (see Fig. 2-5)

$= 0.21(2.5)^2/12 = 0.11$ ft-kips

where $w_u = 1.4(44 + 30) + 1.7(60) = 206 \text{ psf} = 0.21 \text{ kips/ft}$

$\ell_n = 30 \text{ in.} = 2.5 \text{ ft}$ (ignore rib taper)

With bars on slab centerline, $d = 3.5/2 = 1.75$ in.

$A_s = 0.11/4(1.75) = 0.02 \text{ in.}^2/\text{ft}$ (but not less than required temperature reinforcement.)

From Table 3-5, for a 3 ½ in. slab: Use #3 @ 16 in.

Note: For slab reinforcement normal to ribs, space bars per ACI 7.12.2.2 at 5h or 18 in.

Check for fire resistance: From Table 10-3, required cover for fire resistance rating of 1 hour = ¾ in. O.K.

(6) Reinforcement details shown in Fig. 3-18 are determined directly from Fig. 8-5.* Note that one of the #5 bars at the bottom must be continuous or be spliced over the support with a Class A tension splice, and be terminated with a standard hook at the non-continuous supports for structural integrity (ACI 7.13.2.1).

**The bar cut-off points shown in Fig. 8-5 are recommended for one-way joist construction. The reader may consider determining actual bar lengths using the provisions in ACI 12.10.*

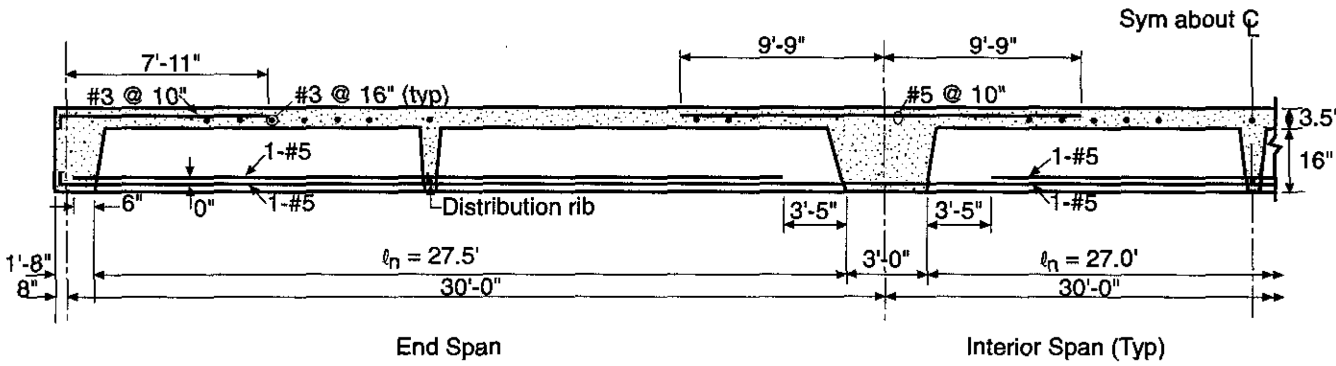


Figure 3-18 Reinforcement Details for 30 in. Standard Joist Floor System [Building #1—Alternate (1)]

(7) Distribution Ribs

The ACI Code does not require lateral supports or load distribution ribs in joist construction. Reference 3.7 suggests that 4 in. wide distribution ribs with at least one #4 bar continuous top and bottom be used to equalize deflections and to ensure that the effect of concentrated or unequal loads is resisted by a number of joist ribs. The following spacing for distribution ribs is also recommended:

- None in spans less than 20 ft
- One near the center of spans 20 to 30 ft
- Two near the third points of spans greater than 30 ft

Provide one distribution rib at center of each span as shown in Fig. 3-19.

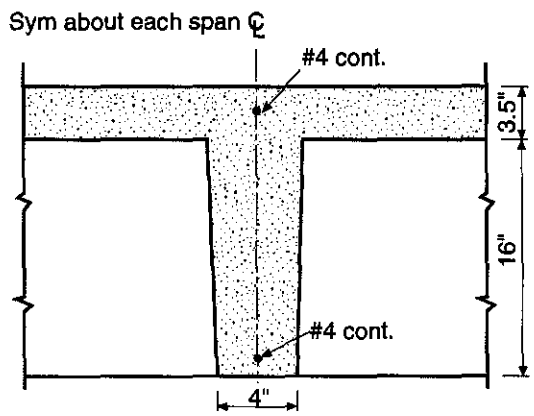


Figure 3-19 Distribution Rib

3.8.2 Example: Design of Wide-Module Joists for Alternate (2) Floor System (Building #1)

(1) Data: f'_c = 4000 psi (normal weight concrete, carbonate aggregate)
f_y = 60,000 psi

Floor: LL = 60 psf
DL = 130 psf (assumed total for beams and slab + partitions + ceiling & misc.)

Fire resistance rating: assume 2 hours.

Floor system - Alternate (2): Assume joists on 6 ft -3 in. centers (standard forms*)
width of spandrel beams = 20 in.
width of interior beams = 36 in.

Note: The provisions for standard joist construction in ACI 8.11 do not apply for the wide-module joist system (clear spacing between joists > 30 in.). Wide-module joists are designed as beams and one-way slabs (ACI 8.11.4).

(2) Determine factored shears and moments using the approximate coefficients of ACI 8.3.3 (see Figs. 2-3, 2-4, and 2-7).

$$w_u = 1.4(130) + 1.7(60)^{**} = 284 \text{ psf} \times 6.25 = 1775 \text{ plf, say } 1800 \text{ plf}$$

Note: All shear and negative moment values are at face of supporting beams.

V_u @ spandrel beams	$= 1.8(27.5)/2 = 24.8$ kips
V_u @ first interior beams	$= 1.15(24.8) = 28.5$ kips
V_u @ interior beams	$= 1.8\ (27)/2 = 24.3$ kips
$-M_u$ @ spandrel beams	$= 1.8(27.5)^2/24 = 56.7$ ft-kips
$+M_u$ @ end spans	$= 1.8\ (27.5)^2/14 = 97.2$ ft-kips
$-M_u$ @ first interior beams	$= 1.8\ (27.25)^2/10 = 133.7$ ft-kips
$+M_u$ @ interior spans	$= 1.8(27)^2/16 = 82.0$ ft-kips
$-M_u$ @ interior beams	$= 1.8(27)^2/11 = 119.3$ ft-kips

(3) Preliminary size of joists (beams) and slab thickness

From Table 3-1: depth of joist $h = \ell_n/18.5 = (27.5 \times 12)/18.5 = 17.8$ in.

where ℓ_n (end span) = 30 - 1.0 - 1.5 = 27.5 ft (governs)
ℓ_n (interior span) = 30 - 3 = 27.0 ft

Check for fire resistance: from Table 10-1, required slab thickness = 4.6 in. for 2-hour rating.

Try 16 in. pan forms + 4-½ in. slab

$$h = 20.5 \text{ in.} > 17.8 \text{ in.} \quad \text{O.K.}$$

(4) Determine width of joist rib

(a) For moment strength:

$$b_w = \frac{20M_u}{d^2} = \frac{20(133.7)}{18.0^2} = 8.25 \text{ in.}$$

where d = 20.5 - 2.5 = 18.0 in. = 1.50 ft

Check for fire resistance: for joists designed as beams, minimum cover per ACI 7.7 = 1.5 in. From Table 10-4, the required cover for restrained beams for a fire resistance rating of 2 hr = ¾ in. < 1.5 in. O.K.

**See Table 9-3 for standard form dimensions.*
***No live load reduction permitted: $A_I = 12.5 \times 30 = 375$ sq ft < 400 sq ft (see Table 2-1).*

(b) For shear strength:

V_u @ distance d from support face = 28.5 - 1.8(1.50) = 25.8 kips

$$\phi V_c/2 = 0.055\ b_w d^*$$

$$b_w = 25.8/(0.055 \times 18.0) = 26.1 \text{ in.}$$

Use 9 in.-wide joist (standard width) and provide stirrups where required. A typical cross-section of the joist is shown in Fig. 3-20.

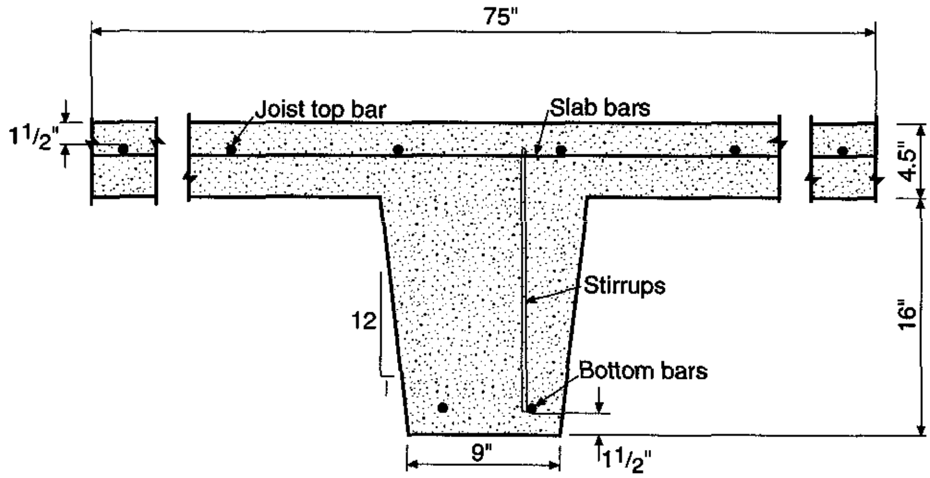


Figure 3-20 Joist Section

(5) Determine Moment Reinforcement

(a) Top bars at spandrel beams:

$$A_s = \frac{M_u}{4d} = \frac{56.7}{4(18.0)} = 0.79 \text{ in.}^2$$

Distribute bars uniformly in top slab according to ACI 10.6.6.
Effective flange width: (30 × 12)/10 = 36 in. (governs)
6.25 × 12 = 75 in.
9 + 2(8 × 4.5) = 81 in.

$$A_s = 0.79/3 = 0.26 \text{ in.}^2\text{/ft}$$

From Table 3-4, maximum bar spacing (interior exposure and 1-½ in. cover) for #4 bars is 13.3 in. Also,

$$s_{max} = 3h = 3(4.5) = 13.5 \text{ in.} < 18 \text{ in.}$$

From Table 3-7: Use #4 @ 9 in. (A_s = 0.27 in.^{2}/ft)

(b) Bottom bars in end spans:

$$A_s = 97.2/4(18.0) = 1.35 \text{ in.}^2$$

$$a = A_s f_y/0.85 f'_c b_e = (1.35 \times 60)/(0.85 \times 4 \times 36) = 0.66 \text{ in.} < 4.5 \text{ in. O.K.}$$

**For joists designed as beams, the 10% increase in ϕV_c is not permitted. Also, minimum shear reinforcement is required when $V_u > \phi V_c/2$.*

From Table 3-6: Use 2-#8 ($A_s = 1.58$ in.^{2})

Check $\rho = A_s/b_w d = 1.58/(9 \times 18.0) = 0.0098 > \rho_{min} = 0.0033$ O.K.

The 2-#8 bars satisfy all requirements for minimum and maximum number of bars in a single layer.

(c) Top bars at first interior beams:

$$A_s = [133.7/4(18.0) = 1.86 \text{ in.}^2]/3 \text{ ft} = 0.62 \text{ in.}^2/\text{ft}$$

From Table 3-7, assuming #7 bars ($s_{max} = 10.8$ in. from Table 3-4),

Use #7 @ 10 in. ($A_s = 0.72$ in.^{2}/ft)

(d) Bottom bars in interior spans:

$$A_s = 82.0/4(18.0) = 1.14 \text{ in.}^2$$

From Table 3-6, use 2-#7 ($A_s = 1.20$ in.^{2})

(e) Top bars at interior beams:

$A_s = [119.3/4(18) = 1.66 \text{ in.}^2]\ /3 \text{ ft} = 0.55 \text{ in.}^2/\text{ft}$

From Table 3-7, with $s_{max} = 11.6$ in.

Use #6 @ 9 in. ($A_s = 0.59$ in.^{2}/ft)

(f) Slab reinforcement normal to joists:

Use $M_u = w_u \ell_n^2/12$ (see Fig. 2-5)

$= 0.22(6.25)^2/12 = 0.72$ ft-kips

where $w_u = 1.4(56 + 30) + 1.7(60) = 222$ psf $= 0.22$ kips/ft

Place bars on slab centerline: $d = 4.5/2 = 2.25$ in.

$$A_s = 0.72/4(2.25) = 0.08 \text{ in.}^2/\text{ft}$$

(but not less than required temperature reinforcement).

From Table 3-5, for 4-½ in. slab: Use #3 @ 13 in. ($A_s = 0.10$ in.^{2}/ft)

Check for fire resistance: from Table 10-3 for restrained members, required cover for fire resistance rating of 2 hours = ¾ in. O.K.

(6) Reinforcement details shown in Fig. 3-21 are determined directly from Fig. 8-3(a).* For structural integrity (ACI 7.13.2.3), one of the #7 and #8 bars at the bottom must be spliced over the support with a Class A tension splice; the #8 bar must be terminated with a standard hook at the non-continuous supports.

(7) Design of Shear Reinforcement

**The bar cut-off points shown in Fig. 8-3(a) are recommended for beams without closed stirrups. The reader may consider determining actual bar lengths using the provisions in ACI 12.10.*

(a) End spans:

V_u at face of interior beam = 28.5 kips
V_u at distance d from support face = 28.5 - 1.8(1.50) = 25.8 kips

Use average web width for shear strength calculations

$$b_w = 9 + (20.5/12) = 10.7 \text{ in.}$$

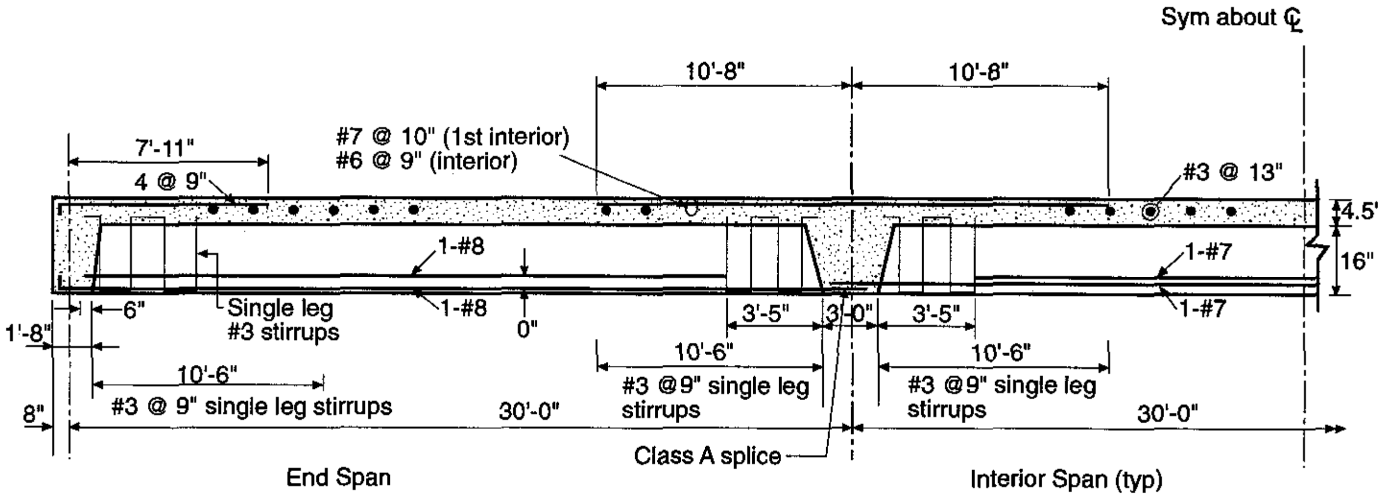


Figure 3-21 Reinforcement Details for 6 ft-3 in. Wide-Module Joist Floor System [Building #1—Alternate (2)]

$(\phi V_c + \phi V_s)_{max} = 0.54\, b_w d = 0.54(10.7)(18.0) = 104.0$ kips
$\phi V_c = 0.11\, b_w d = 0.11\,(10.7)(18.0) = 21.2$ kips
$\phi V_c/2 = 0.055\, b_w d = 10.6$ kips

Beam size is adequate for shear strength since 104.0 kips > 25.8 kips.

Since $V_u > \phi V_c$, more than minimum shear reinforcement is required. Due to the sloping face of the joist rib and the narrow widths commonly used, shear reinforcement is generally a one-legged stirrup rather than the usual two. The type commonly used is a continuous bar located near the joist centerline and bent into the configuration shown in Fig. 3-22. The stirrups are attached to the joist bottom bars.

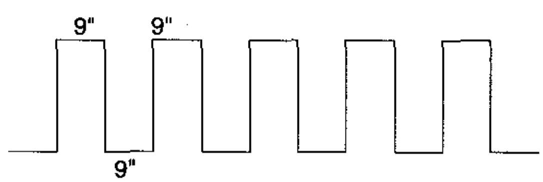


Figure 3-22 Stirrup Detail

Single leg #3 @ d/2 is adequate for full length where stirrups are required. Length over which stirrups are required: (28.5 - 10.6)/1.8 = 10.0 ft. Stirrup spacing s = d/2 = 18.0/2 = 9.0 in.

Check $A_{v(min)} = 50\, b_w s/f_y = 50 \times 10.7 \times 9.0/60{,}000 = 0.08 \text{ in.}^2$

Single leg #3 stirrup O.K.

Use 14-#3 single leg stirrups @ 9 in. Use the same stirrup detail at each end of all joists.

3.8.3 Example: Design of the Support Beams for the Standard Pan Joist Floor along a Typical N-S Interior Column Line (Building #1)

(1) Data $f'_c = 4000$ psi (carbonate aggregate)
$f_y = 60{,}000$ psi

Floors: LL = 60 psf
DL = 130 psf (assumed total for joists & beams + partitions + ceiling & misc.)

Required fire resistance rating = 1 hour (2 hours for Alternate (2)).

Preliminary member sizes
Columns interior = 18 × 18 in.
exterior = 16 × 16 in.

width of interior beams = 36 in. ≅ 2 × depth = 2 × 19.5 = 39.0 in.

The most economical solution for a pan joist floor is making the depth of the supporting beams equal to the depth of the joists. In other words, the soffits of the beams and joists should be on a common plane. This reduces formwork costs sufficiently to override the savings in materials that may be accomplished by using a deeper beam. See Chapter 9 for a discussion on design considerations for economical formwork. The beams are often made about twice as wide as they are deep. Overall joist floor depth = 16 in. + 3.5 in. = 19.5 in. Check deflection control for the 19.5 in. beam depth. From Table 3-1:

$$h = 19.5 \text{ in.} > \ell_n/18.5 = (28.58 \times 12)/18.5 = 18.5 \text{ in.} \quad \text{O.K.}$$

where ℓ_n (end span) = 30 - 0.67 - 0.75 = 28.58 ft (governs)
ℓ_n (interior span) = 30 - 1.50 = 28.50 ft

(2) Determine the factored shears and moments from the gravity loads using approximate coefficients (see Figs. 2-3, 2-4, and 2-7).

Check live load reduction. For interior beams:

$$A_I = 2(30 \times 30) = 1800 \text{ sq ft} > 400 \text{ sq ft}$$
$$L = 60(0.25 + 15/\sqrt{1800}) = 60(0.604)^* = 36.2 \text{ psf}$$
$$w_u = [1.4(130) + 1.7(36.2) = 244 \text{ psf}] \times 30 \text{ ft} = 7.3 \text{ klf}$$

Note: All shear and negative moment values are at the faces of the supporting columns.

V_u @ exterior columns	$= 7.3(28.58)/2 = 104.3$ kips
V_u @ first interior columns	$= 1.15(104.3) = 120.0$ kips
V_u @ interior columns	$= 7.3\ (28.5)/2 = 104.0$ kips
$-M_u$ @ exterior columns	$= 7.3(28.58)^2/16 = 372.7$ ft-kips
$+M_u$ @ end spans	$= 7.3(28.58)^2/14 = 425.9$ ft-kips
$-M_u$ @ first interior columns	$= 7.3(28.54)^2/10 = 594.6$ ft-kips
$+M_u$ @ interior span	$= 7.3(28.50)^2/16 = 370.6$ ft-kips

**For members supporting one floor only, maximum reduction = 0.5 (see Table 2-1).*

(3) Design of the column line beams also includes consideration of moments and shears due to wind. The wind load analysis for Building #1 is summarized in Fig. 2-13.

Note: The 0.75 factor permitted for the load combination including wind effects (ACI Eq. (9-2) is, in most cases, sufficient to accommodate the wind forces and moments, without an increase in the required beam size or reinforcement (i.e., the load combination for gravity load only will usually govern for proportioning the beams).

(4) Check beam size for moment strength

Preliminary beam size = 19.5 in. × 36 in.

For negative moment section:

$$b_w = \frac{20M_u}{d^2} = \frac{20(594.6)}{17^2} = 41.1 \text{ in.} > 36 \text{ in.}$$

where d = 19.5 - 2.5 = 17.0 in. = 1.42 ft

For positive moment section:

$$b_w = 20(425.9)/17^2 = 29.5 \text{ in.} < 36 \text{ in.}$$

Check minimum size permitted with $\rho = 0.75\ \rho_b = 0.0214$:

$$b_w = 13(594.6)/17^2 = 26.7 \text{ in.} < 36 \text{ in. O.K.}$$

Use 36 in. wide beam and provide slightly higher percentage of reinforcement ($\rho > 0.5\ \rho_{max}$) at interior columns.

Check for fire resistance: from Table 10-4, required cover for fire resistance rating of 4 hours or less = ¾ in. < provided cover. O.K.

(5) Determine flexural reinforcement for the beams at the 1st floor level

(a) Top bars at exterior columns

Check governing load combination:

• gravity load only

$$M_u = 372.7 \text{ ft-kips} \qquad \text{ACI Eq. (9-1)}$$

• gravity + wind loads:

$$M_u = 0.75\ (372.7) + 0.75(1.7 \times 90.3) = 394.7 \text{ ft-kips} \qquad \text{ACI Eq. (9-2)}$$

• also check for possible moment reversal due to wind moments:

$$M_u = 0.9(199.1) \pm 1.3(90.3) = 296.6 \text{ ft-kips, } 61.8 \text{ ft-kips} \qquad \text{ACI Eq. (9-3)}$$

where w_d = 130 × 30 = 3.9 kips/ft
M_d = $3.9(28.58)^2/16$ = 199.1 ft-kips

$$A_s = \frac{M_u}{4d} = \frac{394.7}{4(17)} = 5.80 \text{ in.}^2$$

From Table 3-6: Use 8-#8 bars (A_s = 6.32 in.^{2})

For interior exposure, minimum n = 36[1.5 + 0.5 + (1.0/2)]^2/57.4 = 4 bars < 8 O.K.

Check $\rho = A_s/bd = 6.32/(36 \times 17) = 0.0103 > \rho_{min} = 0.0033$ O.K.

(b) Bottom bars in end spans:

$$A_s = 425.9/4(17) = 6.26 \text{ in.}^2$$

Use 8-#8 bars (A_s = 6.32 in.^{2})

(c) Top bars at interior columns:

Check governing load combination:

- gravity load only:

$$M_u = 594.6 \text{ ft-kips} \qquad \text{ACI Eq. (9-1)}$$

- gravity + wind loads:

$$M_u = 0.75(594.6) + 0.75(1.7 \times 90.3) = 561.1 \text{ ft-kips} \qquad \text{ACI Eq. (9-2)}$$

A_s = 594.6/4(17) = 8.74 in.^{2}

Use 11-#8 bars (A_s = 8.70 in.^{2})

(d) Bottom bars in interior span:

$$A_s = 370.6/4(17) = 5.45 \text{ in.}^2$$

Use 7-#8 bars (A_s = 5.53 in.^{2})

(6) Reinforcement details shown in Fig. 3-23 are determined directly from Fig. 8-3(a).* Provide 2-#5 top bars within the center portion of all spans to account for any variations in required bar lengths due to wind effects.

Since the column line beams are part of the primary wind-force resisting system, ACI 12.11.2 requires at least one-fourth the positive moment reinforcement to be extended into the supporting columns and be anchored to develop full f_y at face of support. For the end spans: $A_s/4$ = 8/4 = 2 bars. Extend 2-#8 center bars anchorage distance into the supports:

- At the exterior columns, provide a 90° standard end-hook (general use). From Table 8-5, for #8 bar:

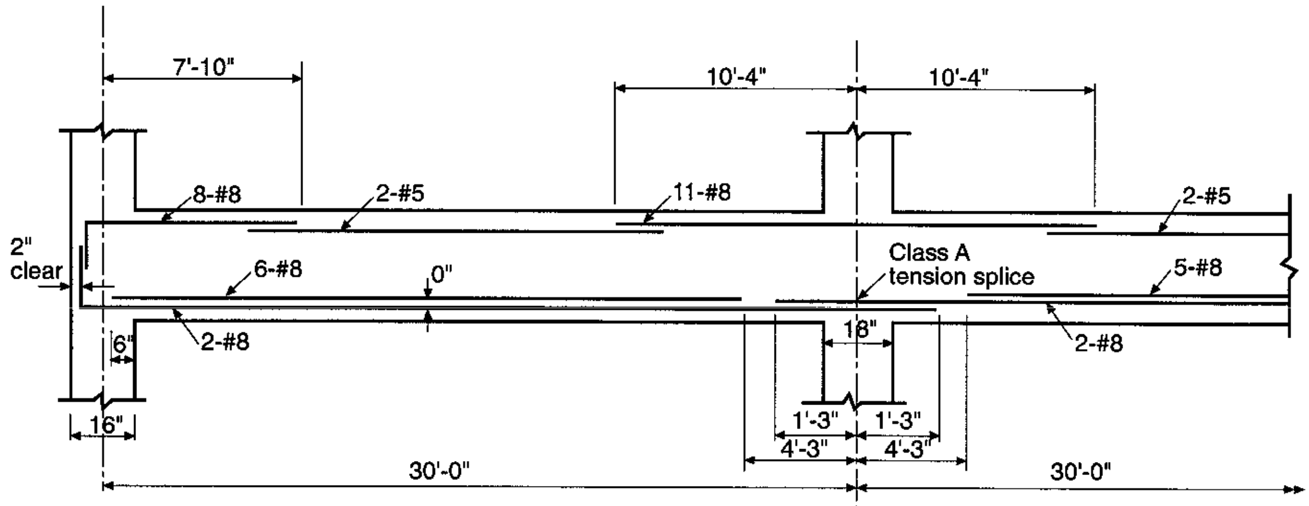
$$\ell_{dh} = 14 \text{ in.} = 16 - 2 = 14 \text{ in.} \qquad \text{O.K.}$$

- At the interior columns, provide a Class A tension splice (ACI 7.13.2.3). Clear space between #8 bars = 3.4 in. = 3.4d_b. From Table 8-2, length of splice = 1.0 × 30 = 30 in. (ACI 12.15).

(7) Design of Shear Reinforcement

**The bar cut-off points shown in Fig. 8-3(a) are recommended for beams without closed stirrups. The reader may consider determining actual bar lengths using the provisions in ACI 12.10.*

Design shear reinforcement for the end span at the interior column and use the same stirrup requirements for all three spans.



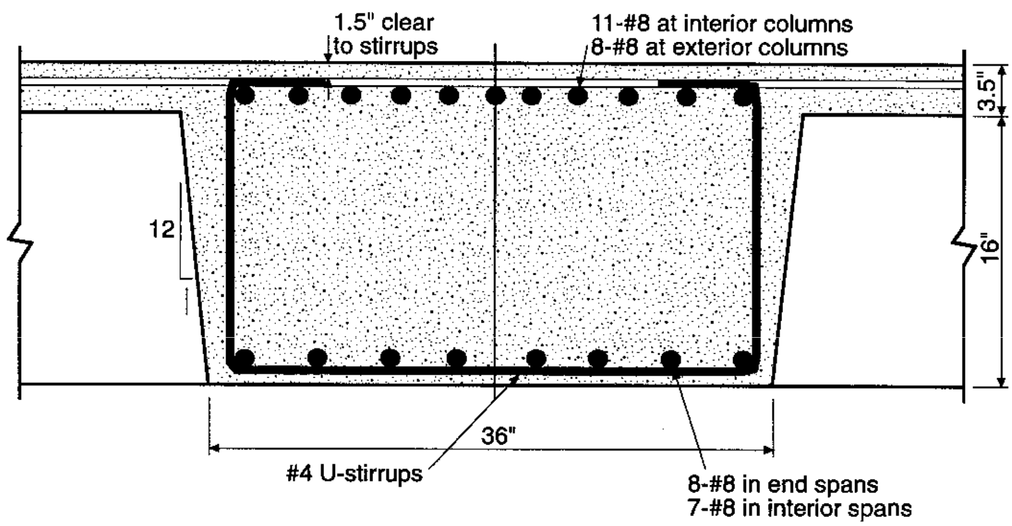


Figure 3-23 Reinforcement Details for Support Beams along N-S Interior Column Line

Check governing load combination:

- gravity load only:

$$V_u \text{ at interior column} = 120.0 \text{ kips (governs)}$$

- gravity + wind loads:

$$V_u = 0.75(120.0) + 0.75(1.7 \times 6.02) = 97.7 \text{ kips}$$

- wind only at span center:

$$V_u = 1.3(6.02) = 7.8 \text{ kips}$$

V_u @ face of column = 120.0 kips

V_u at distance d from column face = 120.0 - 7.3(1.42) = 109.6 kips

$(\phi V_c + \phi V_s)_{max} = 0.54\ b_w d = 0.54(36)17 = 330.5$ kips > 109.6 kips O.K.

$\phi V_c = 0.11\ b_w d = 0.11(36)17 = 67.3$ kips

$\phi V_c/2 = 33.7$ kips

Length over which stirrups are required: (120.0 - 33.7)/7.3 = 11.8 ft

ϕV_s (required) = 109.6 - 67.3 = 42.3 kips

Try #4 U-stirrups:

From Fig. 3-4, use #4 @ 8 in. over the entire length where stirrups are required (see Fig. 3-24).

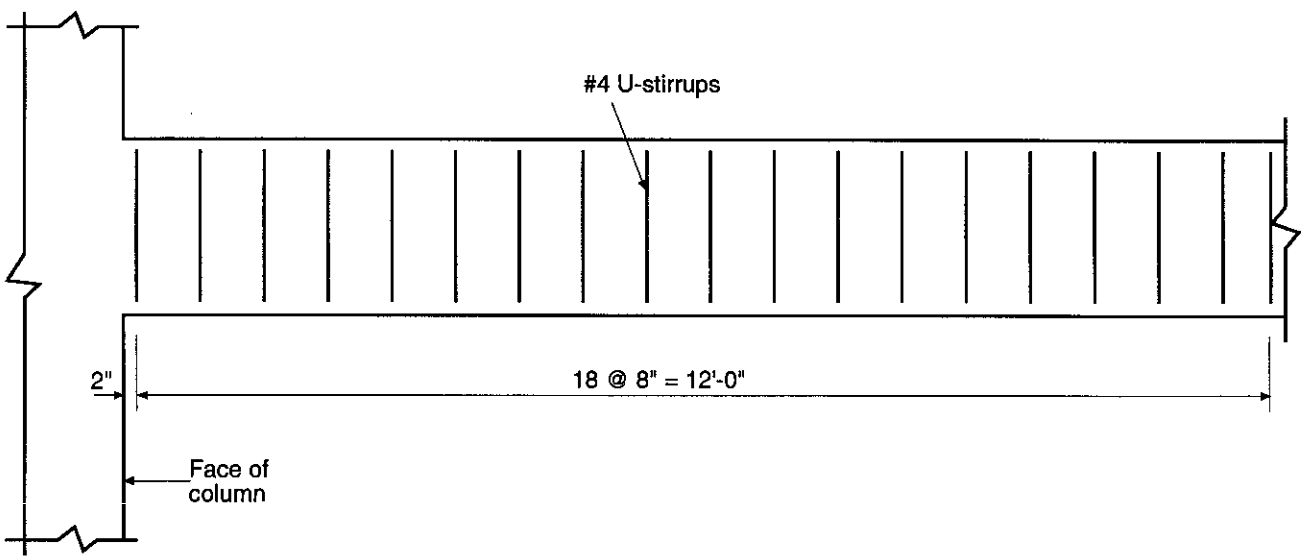


Figure 3-24 Stirrup Spacing Layout—Use same layout at each end of all beams

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3.1 Fling, Russell S., "Using ACI 318 the Easy Way", *Concrete International*, Vol. 1, No. 1, January 1979.

3.2 Pickett, Chris, *Notes on ACI 318-77, Appendix A - Notes on Simplified Design*, 3rd Edition, Portland Cement Association, Skokie, Illinois.

3.3 Rogers, Paul, "Simplified Method of Stirrup Spacing", *Concrete International*, Vol. 1, No. 1, January 1979.

3.4 *PSI - Product Services and Information*, Concrete Reinforcing Steel Institute, Schaumburg, Illinois.

(a) Bulletin 7901A *Selection of Stirrups in Flexural Members for Economy*
(b) Bulletin 7701A *Reinforcing Bars Required - Minimum vs. Maximum*
(c) Bulletin 7702A *Serviceability Requirements with Grade 60 Bars*

3.5 *Notes on ACI 318-89*, Chapter 10, "Design for Flexure", 5th Edition, EB070, Portland Cement Association, Skokie, Illinois, 1990.

3.6 *Design Handbook in Accordance with the Strength Design Method of ACI 318-89: Vol. I–Beams, Slabs, Brackets, Footings, and Pile Caps*, SP-17(91), American Concrete Institute, Detroit, 1991.

3.7 *CRSI Handbook*, Concrete Reinforcing Steel Institute, Schaumburg, Illinois, 7th Edition, 1992.

3.8 *ACI Detailing Manual - 1988*, SP-66(88), American Concrete Institute, Detroit, 1988.

Chapter 4

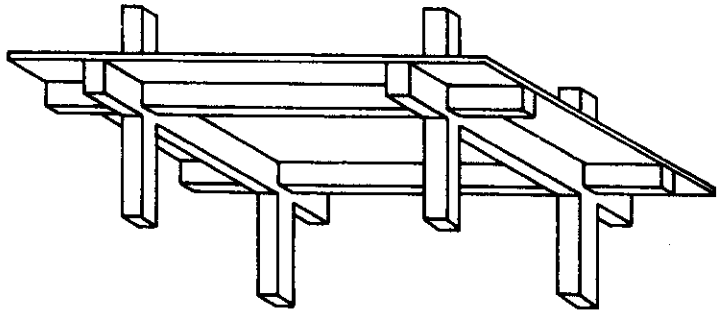
Simplified Design for Two-Way Slabs

4.1 INTRODUCTION

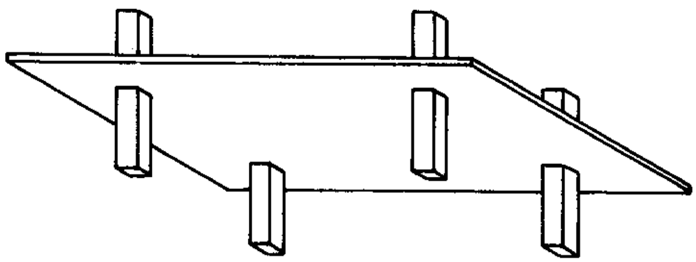
Figure 4-1 shows the various types of two-way reinforced concrete slab systems in use at the present time.

A solid slab supported on beams on all four sides [Fig. 4-1(a)] was the original slab system in reinforced concrete. With this system, if the ratio of the long to the short side of a slab panel is two or more, load transfer is predominantly by bending in the short direction and the panel essentially acts as a one-way slab. As the ratio of the sides of a slab panel approaches unity (square panel), significant load is transferred by bending in both orthogonal directions, and the panel should be treated as a two-way rather than a one-way slab.

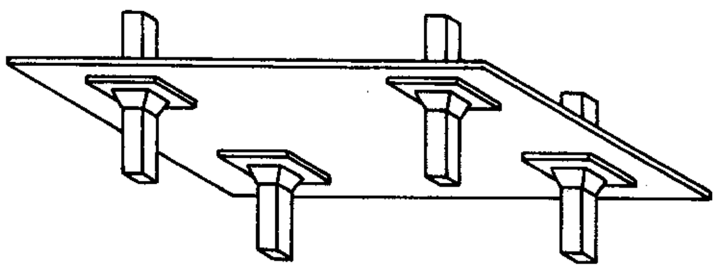
As time progressed and technology evolved, the column-line beams gradually began to disappear. The resulting slab system, consisting of solid slabs supported directly on columns, is called the flat plate [Fig. 4-1(b)]. The flat plate is very efficient and economical and is currently the most widely used slab system for multistory residential and institutional construction, such as motels, hotels, dormitories, apartment buildings, and hospitals. In comparison to other concrete floor/roof systems, flat plates can be constructed in less time and with minimum labor costs because the system utilizes the simplest possible formwork and reinforcing steel layout. The use of flat plate construction also has other significant economic advantages. For instance, because of the shallow thickness of the floor system, story heights are automatically reduced resulting in smaller overall heights of exterior walls and utility shafts, shorter floor to ceiling partitions, reductions in plumbing, sprinkler and duct risers, and a multitude of other items of construction. In cities like Washington, D.C., where the maximum height of buildings is restricted, the thin flat plate permits the construction of the maximum number of stories on a given plan area. Flat plates also provide for the most flexibility in the layout of columns, partitions, small openings, etc. Where job conditions allow direct application of the ceiling finish to the flat plate soffit, (thus eliminating the need for suspended ceilings), additional cost and construction time savings are possible as compared to other structural systems.



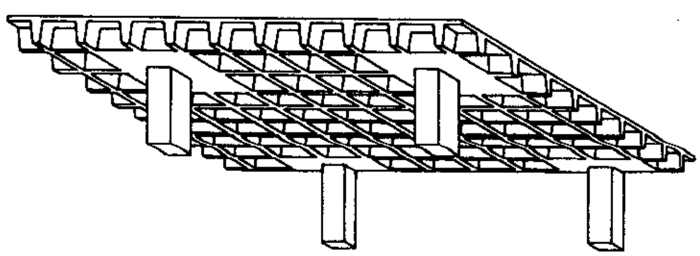
(a) Two-Way Beam-Supported Slab



(b) Flat Plate



(c) Flat Slab



(d) Waffle Slab (Two-Way Joist Slab)

Figure 4-1 Types of Two-Way Slab Systems

The principal limitation on the use of flat plate construction is imposed by shear around the columns (Section 4.4). For heavy loads or long spans, the flat plate is often thickened locally around the columns creating what are known as drop panels. When a flat plate is equipped with drop panels, it is called a flat slab [Fig. 4-1(c)]. Also, for reasons of shear around the columns, the column tops are sometimes flared, creating column capitals. For purposes of design, a column capital is part of the column, whereas a drop panel is part of the slab.

Waffle slab construction [Fig. 4-1(d)] consists of rows of concrete joists at right angles to each other with solid heads at the columns (for reasons of shear). The joists are commonly formed by using standard square dome forms. The domes are omitted around the columns to form the solid heads acting as drop panels. Waffle slab construction allows a considerable reduction in dead load as compared to conventional flat slab construction. Thus, it is particularly advantageous where the use of long span and/or heavy loads are desired without the use of deepened drop panels or support beams. The geometric shape formed by the joist ribs is often architecturally desirable.

Discussion in this chapter is limited largely to flat plates and flat slabs subjected only to gravity loads.

4.2 DEFLECTION CONTROL—MINIMUM SLAB THICKNESS

Minimum thickness/span ratios enable the designer to avoid extremely complex deflection calculations in routine designs. Deflections of two-way slab systems need not be computed if the overall slab thickness meets the minimum requirements specified in ACI 9.5.3. Minimum slab thicknesses for flat plates, flat slabs (and waffle slabs), and two-way slabs, based on the provisions in ACI 9.5.3, are summarized in Table 4-1, where ℓ_n is the clear span length in the long direction of a two-way slab panel. The tabulated values are the controlling minimum thicknesses governed by interior or exterior panels assuming a constant slab thickness for all panels making up a slab system.[4.1] Practical spandrel beam sizes will usually provide beam-to-slab stiffness ratios α greater than the minimum specified value of 0.8; if this is not the case, the spandrel beams must be ignored in computing minimum slab thickness. A standard size drop panel that would allow a 10% reduction in the minimum required thickness of a flat slab floor system is illustrated in Fig. 4-2. Note that a larger size and depth drop may be used if required for shear strength; however, a corresponding lesser slab thickness is not permitted unless deflections are computed.

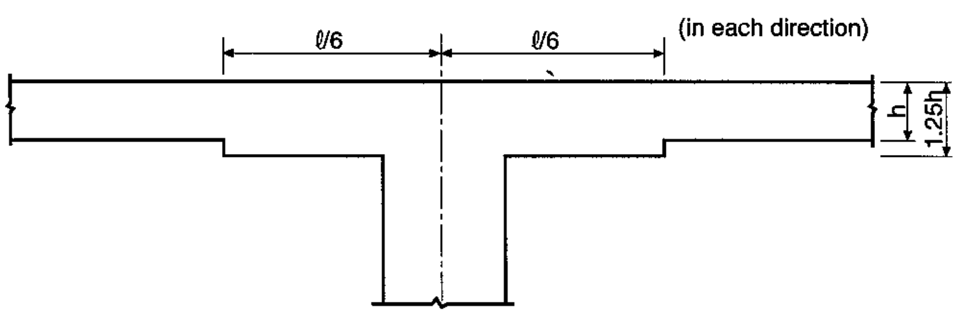


Figure 4-2 Drop Panel Details (ACI 13.4.7)

Table 4-1 gives the minimum slab thickness h based on the requirements given in ACI 9.5.3; α_m is the average value of α (ratio of flexural stiffness of beam to flexural stiffness of slab) for all beams on the edges of a panel, and β is the ratio of clear spans in long to short direction.

For design convenience, minimum thicknesses for the six types of two-way slab systems listed in Table 4-1 are plotted in Fig. 4-3.

4.3 TWO-WAY SLAB ANALYSIS BY COEFFICIENTS

For gravity loads, ACI Chapter 13 provides two analysis methods for two-way slab systems: 1) the Direct Design Method (ACI 13.6) and the Equivalent Frame Method (ACI 13.7). The Equivalent Frame Method, using member stiffnesses and complex analytical procedures, is not suitable for hand calculations. Only the Direct Design Method, using moment coefficients, will be presented in this Chapter.

Table 4-1 Minimum Thickness for Two-Way Slab Systems

Two-Way Slab System		α_m	β	Minimum h
Flat Plate	[Min. h = 5 in.]	—	≤ 2	$\ell_n/30$
Flat Plate with Spandrel Beams [1]		—	≤ 2	$\ell_n/33$
Flat Slab [2]	[Min. h = 4 in.]	—	≤ 2	$\ell_n/33$
Flat Slab [2] with Spandrel Beams [1]		—	≤ 2	$\ell_n/36$
Two-Way Beam-Supported Slab [3]		≤ 0.2	≤ 2	$\ell_n/30$
		1.0	1	$\ell_n/33$
			2	$\ell_n/36$
		≥ 2.0	1	$\ell_n/37$
			2	$\ell_n/44$
Two-Way Beam-Supported Slab [1,3]		≤ 0.2	≤ 2	$\ell_n/33$
		1.0	1	$\ell_n/36$
			2	$\ell_n/40$
		≥ 2.0	1	$\ell_n/41$
			2	$\ell_n/49$

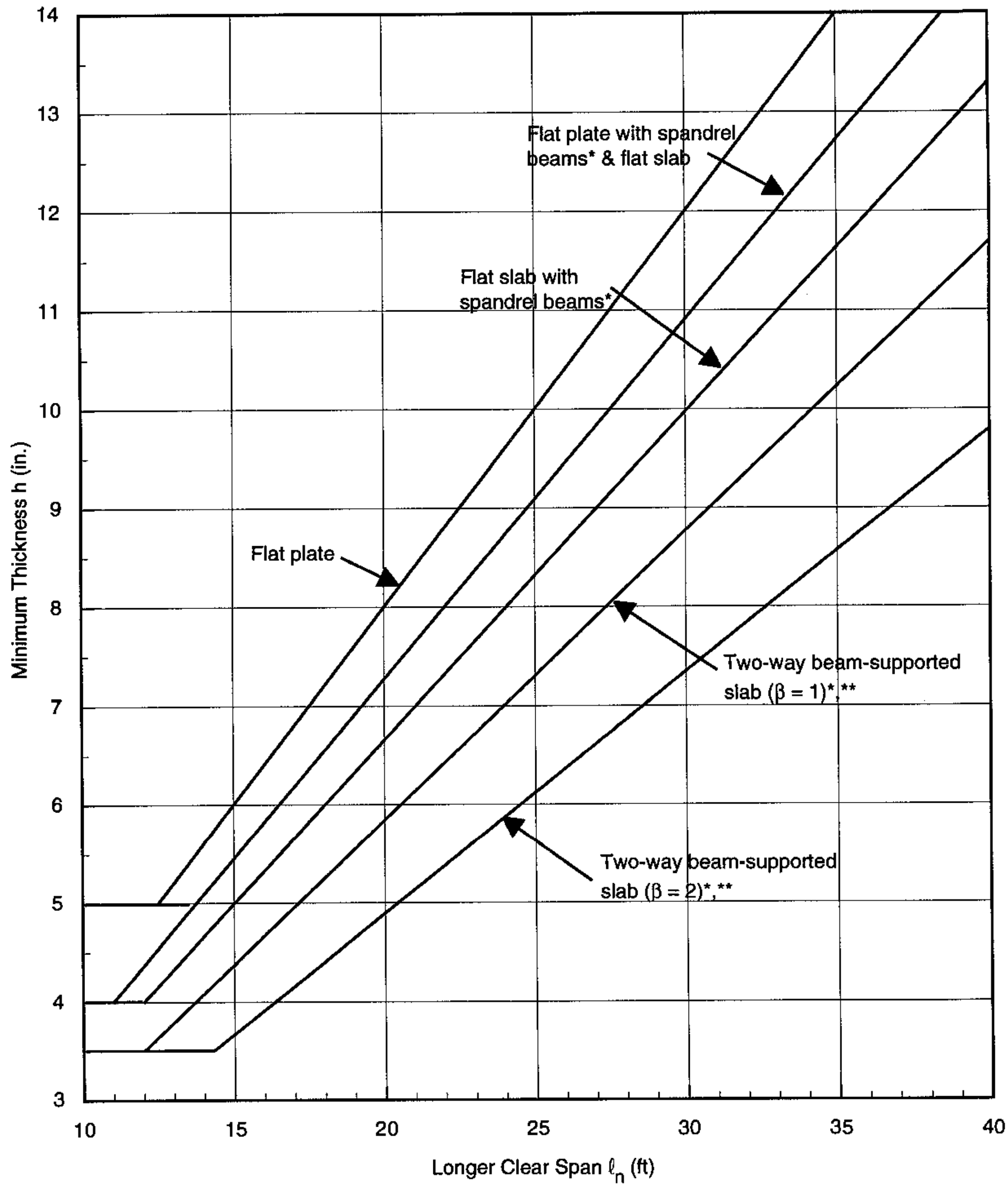
[1] Spandrel beam-to-slab stiffness ratio $\alpha \geq 0.8$ (ACI 9.5.3.3)

[2] Drop panel length $\geq \ell/3$, depth $\geq 1.25h$ (ACI 13.4.7)

[3] Min. h = 5 in. for $\alpha_m \leq 2.0$; min. h = 3.5 in. for $\alpha_m > 2.0$ (ACI 9.5.3.3)

The Direct Design Method applies when all of the conditions illustrated in Fig. 4-4 are satisfied (ACI 13.6.1):

- There must be three or more continuous spans in each direction.
- Slab panels must be rectangular with a ratio of longer to shorter span (c/c of supports) not greater than 2.
- Successive span lengths (c/c of supports) in each direction must not differ by more than one-third of the longer span.
- Columns must not be offset more than 10% of the span (in direction of offset) from either axis between centerlines of successive columns.



*Spandrel beam-to-slab stiffness ratio $\alpha \geq 0.8$

**$\alpha_m > 2.0$

Figure 4-3 Minimum slab thickness for two-way slab systems (see Table 4-1)

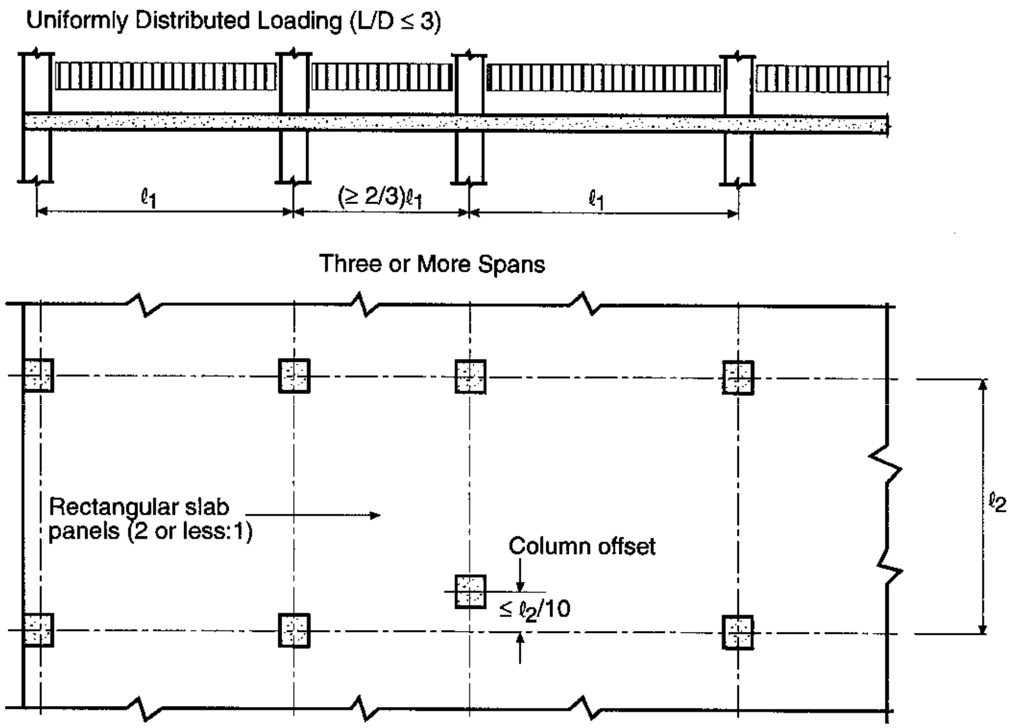


Figure 4-4 Conditions for Analysis by Coefficients

- Loads must be due to gravity only and must be uniformly distributed over the entire panel. The live load must not be more than 3 times the dead load ($L/D \le 3$). Note that if the live load exceeds one-half the dead load ($L/D > 0.5$), column-to-slab stiffness ratios must exceed the applicable values given in ACI Table 13.6.10, so that the effects of pattern loading can be neglected. The positive factored moments in panels supported by columns not meeting such minimum stiffness requirements must be magnified by a coefficient computed by ACI Eq. (13-5).

- For two-way slabs, relative stiffnesses of beams in two perpendicular directions must satisfy the minimum and maximum requirements given in ACI 13.6.1.6.

- Redistribution of moments by ACI 8.4 shall not be permitted.

In essence, the Direct Design Method is a three-step analysis procedure. The first step is the calculation of the total design moment M_o for a given panel. The second step involves the distribution of the total moment to the negative and positive moment sections. The third step involves the assignment of the negative and positive moments to the column strips and middle strips.

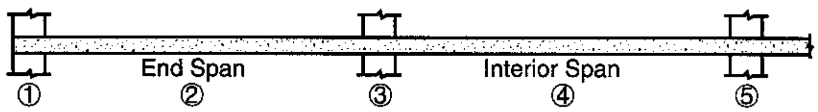
For uniform loading, the total design moment M_o for a panel is calculated by the simple static moment expression, ACI Eq. (13-3):

$$M_o = w_u \ell_2 \ell_n^2/8$$

where w_u is the factored combination of dead and live loads (psf), $w_u = 1.4\,w_d + 1.7\,w_\ell$. The clear span ℓ_n is defined in a straightforward manner for columns or other supporting elements of rectangular cross section (ACI 12.6.2.5). Note that circular or regular polygon shaped supports shall be treated as square supports with the same area (see ACI Fig. R13.6.2.5). The clear span starts at the face of support. One limitation requires that the clear span never be taken less than 65% of the span center-to-center of supports (ACI 13.6.2.5). The span ℓ_2 is simply the span transverse to ℓ_n; however, when the span adjacent and parallel to an edge is being considered, the distance from edge of slab to panel centerline is used for ℓ_2 in calculation of M_o (ACI 13.6.2.4).

Division of the total panel moment M_o into negative and positive moments, and then into column and middle strip moments, involves direct application of moment coefficients to the total moment M_o. The moment coefficients are a function of span (interior or exterior) and slab support conditions (type of two-way slab system). For design convenience, moment coefficients for typical two-way slab systems are given in Tables 4-2 through 4-6. Tables 4-2 through 4-5 apply to flat plates or flat slabs with various end support conditions. Table 4-6 applies to two-way slabs supported on beams on all four sides. Final moments for the column strip and middle strip are computed directly using the tabulated values. All coefficients were determined using the appropriate distribution factors in ACI 13.6.3 through 13.6.6.

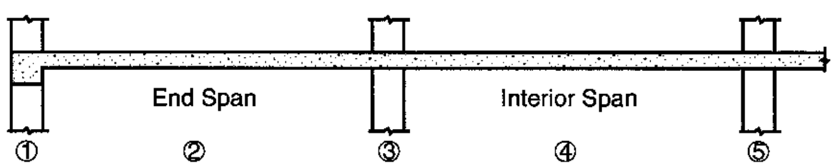
Table 4-2 Flat Plate or Flat Slab Supported Directly on Columns



Slab Moments	End Span			Interior Span	
	1 Exterior Negative	2 Positive	3 First Interior Negative	4 Positive	5 Interior Negative
Total Moment	$0.26\ M_o$	$0.52\ M_o$	$0.70\ M_o$	$0.35\ M_o$	$0.65\ M_o$
Column Strip	$0.26\ M_o$	$0.31\ M_o$	$0.53\ M_o$	$0.21\ M_o$	$0.49\ M_o$
Middle Strip	0	$0.21\ M_o$	$0.17\ M_o$	$0.14\ M_o$	$0.16\ M_o$

Note: All negative moments are at face of support.

Table 4-3 Flat Plate or Flat Slab with Spandrel Beams

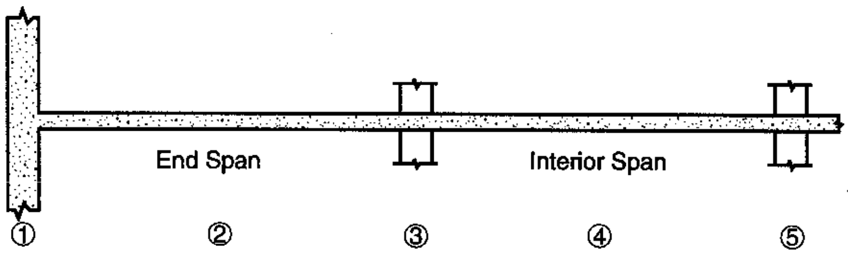


Slab Moments	End Span			Interior Span	
	1 Exterior Negative	2 Positive	3 First Interior Negative	4 Positive	5 Interior Negative
Total Moment	$0.30\ M_o$	$0.50\ M_o$	$0.70\ M_o$	$0.35\ M_o$	$0.65\ M_o$
Column Strip	$0.23\ M_o$	$0.30\ M_o$	$0.53\ M_o$	$0.21\ M_o$	$0.49\ M_o$
Middle Strip	$0.07\ M_o$	$0.20\ M_o$	$0.17\ M_o$	$0.14\ M_o$	$0.16\ M_o$

Notes: (1) All negative moments are at face of support.

(2) Torsional stiffness of spandrel beams $\beta_t \geq 2.5$. For values of β_t less than 2.5, exterior negative column strip moment increases to $(0.30 - 0.03\beta_t)\ M_o$.

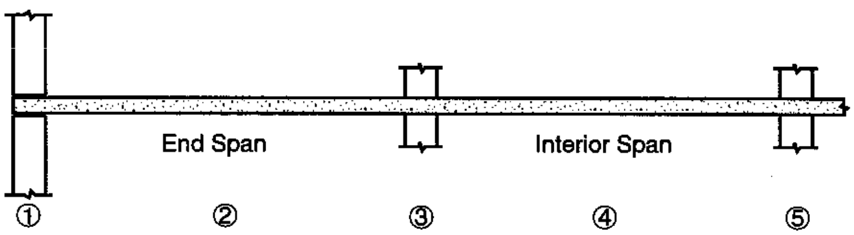
Table 4-4 Flat Plate or Flat Slab with End Span Integral with Wall



Slab Moments	End Span			Interior Span	
	1 Exterior Negative	2 Positive	3 First Interior Negative	4 Positive	5 Interior Negative
Total Moment	0.65 M_o	0.35 M_o	0.65 M_o	0.35 M_o	0.65 M_o
Column Strip	0.49 M_o	0.21 M_o	0.49 M_o	0.21 M_o	0.49 M_o
Middle Strip	0.16 M_o	0.14 M_o	0.16 M_o	0.14 M_o	0.16 M_o

Note: All negative moments are at face of support.

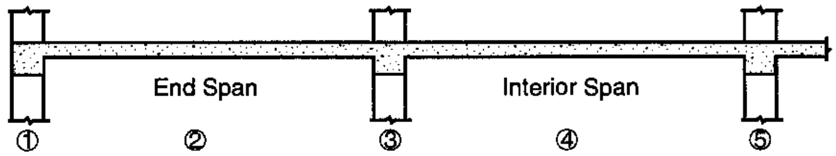
Table 4-5 Flat Plate or Flat Slab with End Span Simply Supported on Wall



Slab Moments	End Span			Interior Span	
	1 Exterior Negative	2 Positive	3 First Interior Negative	4 Positive	5 Interior Negative
Total Moment	0	0.63 M_o	0.75 M_o	0.35 M_o	0.65 M_o
Column Strip	0	0.38 M_o	0.56 M_o	0.21 M_o	0.49 M_o
Middle Strip	0	0.25 M_o	0.19 M_o	0.14 M_o	0.16 M_o

Note: All negative moments are at face of support.

Table 4-6 Two-Way Beam-Supported Slab



Span ratio	Slab Moments	End Span 1 Exterior Negative	2 Positive	3 First Interior Negative	Interior Span 4 Positive	5 Interior Negative
ℓ_2/ℓ_1	Total Moment	0.16 M_o	0.57 M_o	0.70 M_o	0.35 M_o	0.65 M_o
0.5	Column Strip Beam	0.12 M_o	0.43 M_o	0.54 M_o	0.27 M_o	0.50 M_o
	Slab	0.02 M_o	0.08 M_o	0.09 M_o	0.05 M_o	0.09 M_o
	Middle Strip	0.02 M_o	0.06 M_o	0.07 M_o	0.03 M_o	0.06 M_o
1.0	Column Strip Beam	0.10 M_o	0.37 M_o	0.45 M_o	0.22 M_o	0.42 M_o
	Slab	0.02 M_o	0.06 M_o	0.08 M_o	0.04 M_o	0.07 M_o
	Middle Strip	0.04 M_o	0.14 M_o	0.17 M_o	0.09 M_o	0.16 M_o
2.0	Column Strip Beam	0.06 M_o	0.22 M_o	0.27 M_o	0.14 M_o	0.25 M_o
	Slab	0.01 M_o	0.04 M_o	0.05 M_o	0.02 M_o	0.04 M_o
	Middle Strip	0.09 M_o	0.31 M_o	0.38 M_o	0.19 M_o	0.36 M_o

Notes: (1) Beams and slab satisfy stiffness criteria: $\alpha_1\ell_2/\ell_1 \geq 1.0$ and $\beta_t \geq 2.5$.
(2) Interpolate between values shown for different ℓ_2/ℓ_1 ratios.
(3) All negative moments are at face of support.
(4) Concentrated loads applied directly to beams must be accounted for separately.

The moment coefficients of Table 4-3 (flat plate with spandrel beams) are valid for $\beta_t \geq 2.5$. The coefficients of Table 4-6 are applicable when $\alpha_1\ell_2/\ell_1 \geq 1.0$ and $\beta_t \geq 2.5$. Many practical beam sizes will provide beam-to-slab stiffness ratios such that $\alpha_1\ell_2/\ell_1$ and β_t would be greater than these limits, allowing moment coefficients to be taken directly from the tables. However, if beams are present, the two stiffness parameters α_1 and β_t will need to be evaluated. For two-way slabs, the stiffness parameter α_1 is simply the ratio of the moments of inertia of the effective beam and slab sections in the direction of analysis, $\alpha_1 = I_b/I_s$, as illustrated in Fig. 4-5. Figures 4-6 and 4-7 can be used to determine α.

Relative stiffness provided by a spandrel beam is reflected by the parameter $\beta_t = C/2I_s$, where I_s is the moment of inertia of the effective slab section spanning in the direction of ℓ_1 and having a width equal to ℓ_2, i.e., $I_s = \ell_2h^3/12$. The constant C pertains to the torsional stiffness of the effective spandrel beam cross section. It is found by dividing the beam section into its component rectangles, each having smaller dimension x and larger dimension y, and summing the contribution of all the parts by means of the equation

$$C = \Sigma\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3} \qquad \text{ACI Eq. (13-7)}$$

The subdivision can be done in such a way as to maximize C. Figure 4-8 can be used to determine the torsional constant C.

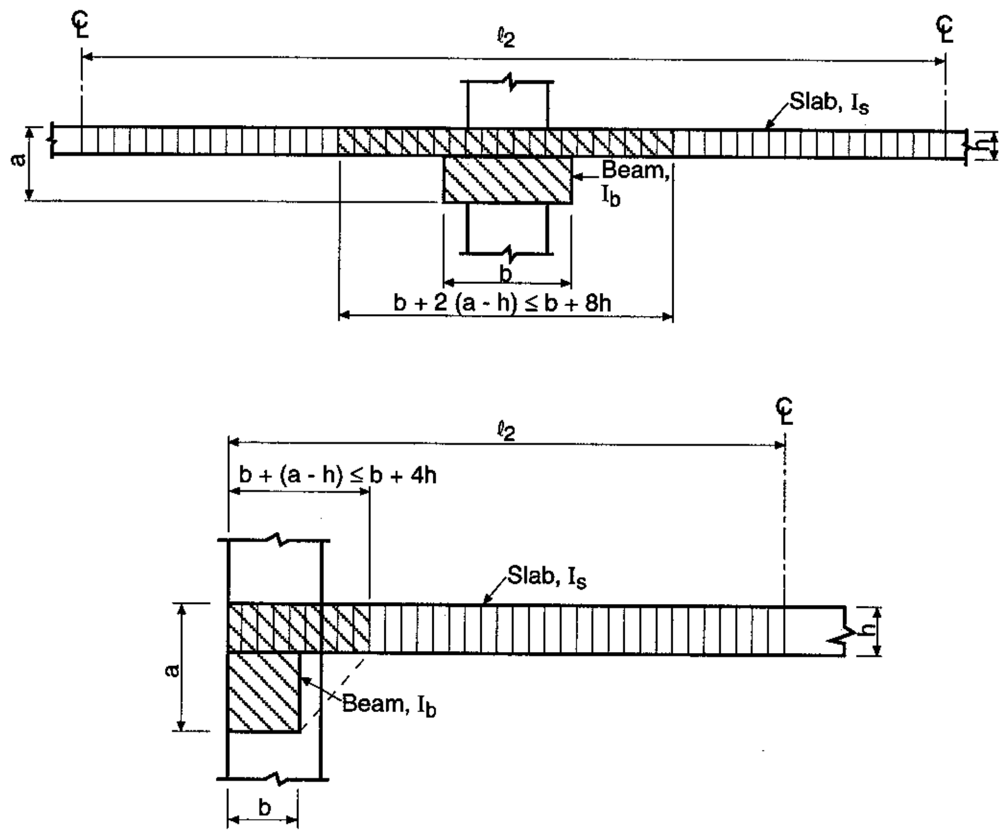
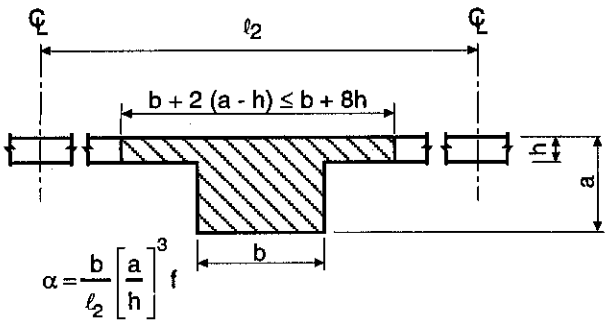


Figure 4-5 Effective Beam and Slab Sections for Stiffness Ratio α (ACI 13.2.4)



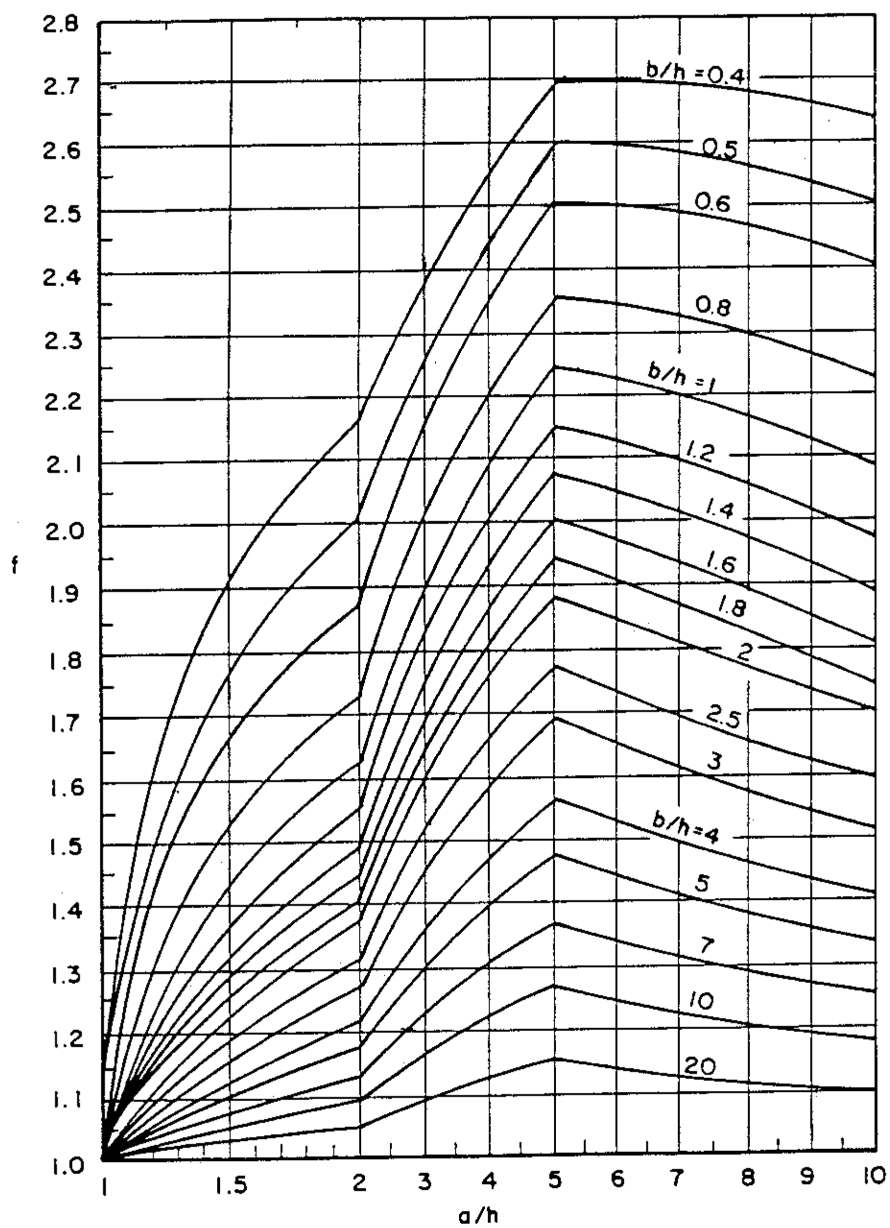


Figure 4-6 Beam to Slab Stiffness Ratio α (Interior Beams)

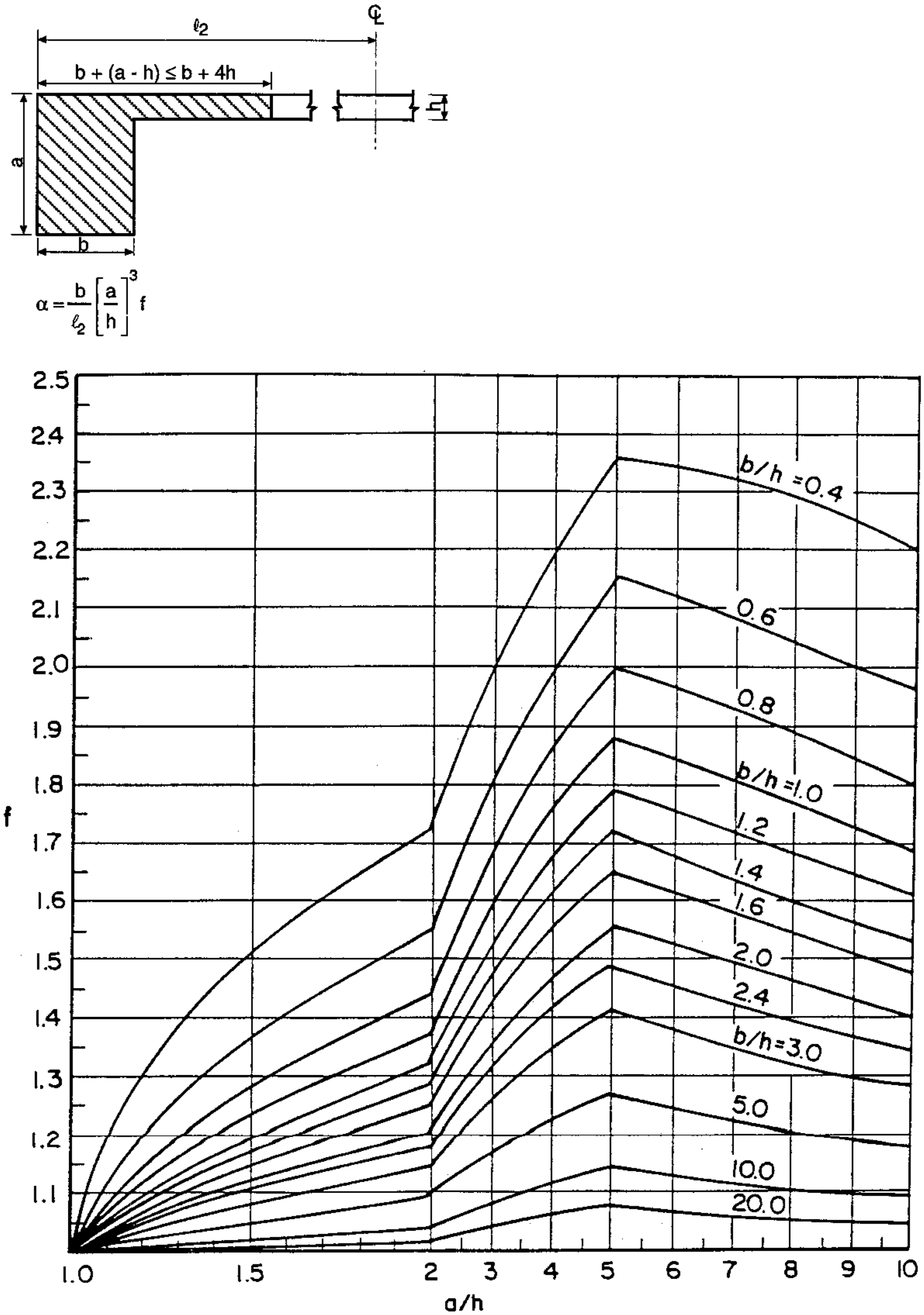
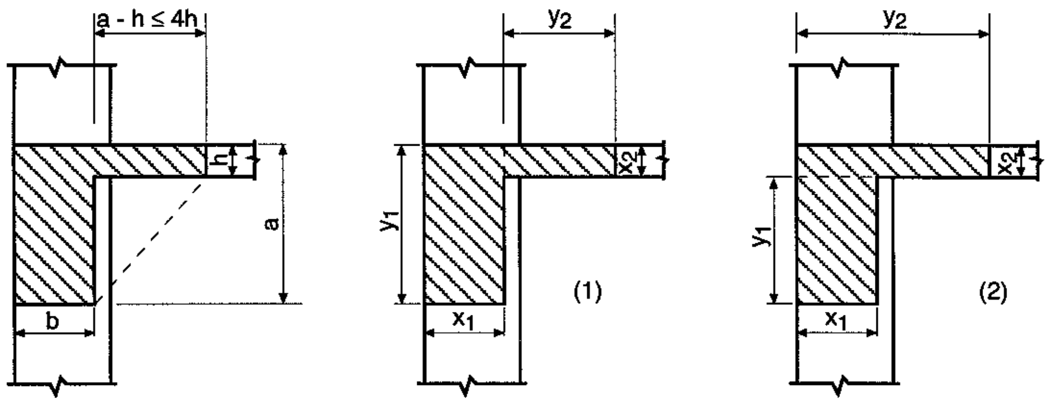


Figure 4-7 Beam to Slab Stiffness Ratio α (Spandrel Beams)



Spandrel beam (ACI 13.2.4)

Use larger value of C computed from (1) or (2)

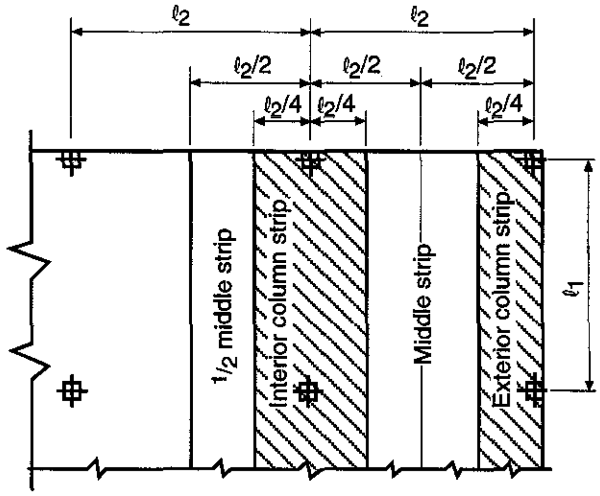
Values of torsion constant, $C = (1 - 0.63\,x/y)(x^3y/3)$

y \ x*	4	5	6	7	8	9	10	12	14	16
12	202	369	592	868	1,188	1,538	1,900	2,557	--	--
14	245	452	736	1,096	1,529	2,024	2,567	3,709	4,738	--
16	288	535	880	1,325	1,871	2,510	3,233	4,861	6,567	8,083
18	330	619	1,024	1,554	2,212	2,996	3,900	6,013	8,397	10,813
20	373	702	1,168	1,782	2,553	3,482	4,567	7,165	10,226	13,544
22	416	785	1,312	2,011	2,895	3,968	5,233	8,317	12,055	16,275
24	458	869	1,456	2,240	3,236	4,454	5,900	9,469	13,885	19,005
27	522	994	1,672	2,583	3,748	5,183	6,900	11,197	16,629	23,101
30	586	1,119	1,888	2,926	4,260	5,912	7,900	12,925	19,373	27,197
33	650	1,244	2,104	3,269	4,772	6,641	8,900	14,653	22,117	31,293
36	714	1,369	2,320	3,612	5,284	7,370	9,900	16,381	24,861	35,389
42	842	1,619	2,752	4,298	6,308	8,828	11,900	19,837	30,349	43,581
48	970	1,869	3,184	4,984	7,332	10,286	13,900	23,293	35,837	51,773
54	1,098	2,119	3,616	5,670	8,356	11,744	15,900	26,749	41,325	59,965
60	1,226	2,369	4,048	6,356	9,380	13,202	17,900	30,205	46,813	68,157

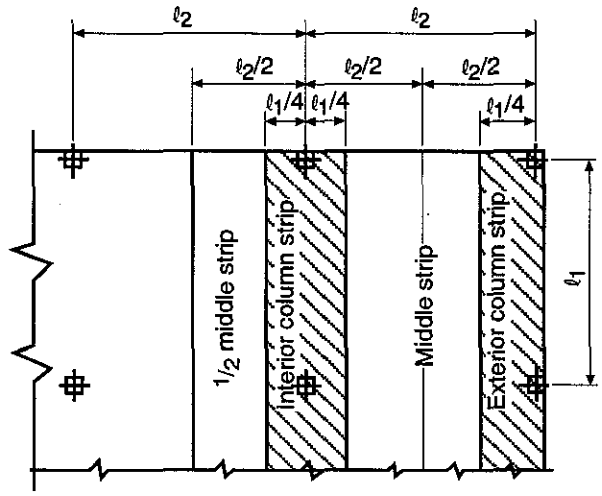
*Small side of a rectangular cross section with dimensions x and y.

Figure 4-8 Design Aid for Computing Torsional Section Constant C

The column strip and middle strip moments are distributed over an effective slab width as illustrated in Fig. 4-9. The column strip is defined as having a width equal to one-half the transverse or longitudinal span, whichever is smaller (ACI 13.2.1). The middle strip is bounded by two column strips.



(a) Column strip for $\ell_2 \leq \ell_1$



(b) Column strip for $\ell_2 > \ell_1$

Figure 4-9 Definition of Design Strips

Once the slab and beam (if any) moments are determined, design of the slab and beam sections follows the simplified design approach presented in Chapter 3. Slab reinforcement must not be less than that given in Table 3-5, with a maximum spacing of 2h or 18 in. (ACI 13.4).

4.4 SHEAR IN TWO-WAY SLAB SYSTEMS

When two-way slab systems are supported by beams or walls, the shear capacity of the slab is seldom a critical factor in design, as the shear force due to the factored loads is generally well below the capacity of the concrete.

In contrast, when two-way slabs are supported directly by columns (as in flat plates and flat slabs), shear near the columns is of critical importance. Shear strength at an exterior slab-column connection (without spandrel beams) is especially critical because the total exterior negative slab moment must be transferred directly to the column. This aspect of two-way slab design should not be taken lightly by the designer. Two-way slab systems will normally be found to be quite "forgiving" if an error in the distribution or even in the amount of flexural reinforcement is made, but there will be no forgiveness if a critical lapse occurs in providing the required shear strength.

For slab systems supported directly by columns, it is advisable at an early stage in the design to check the shear strength of the slab in the vicinity of columns as illustrated in Fig. 4-10.

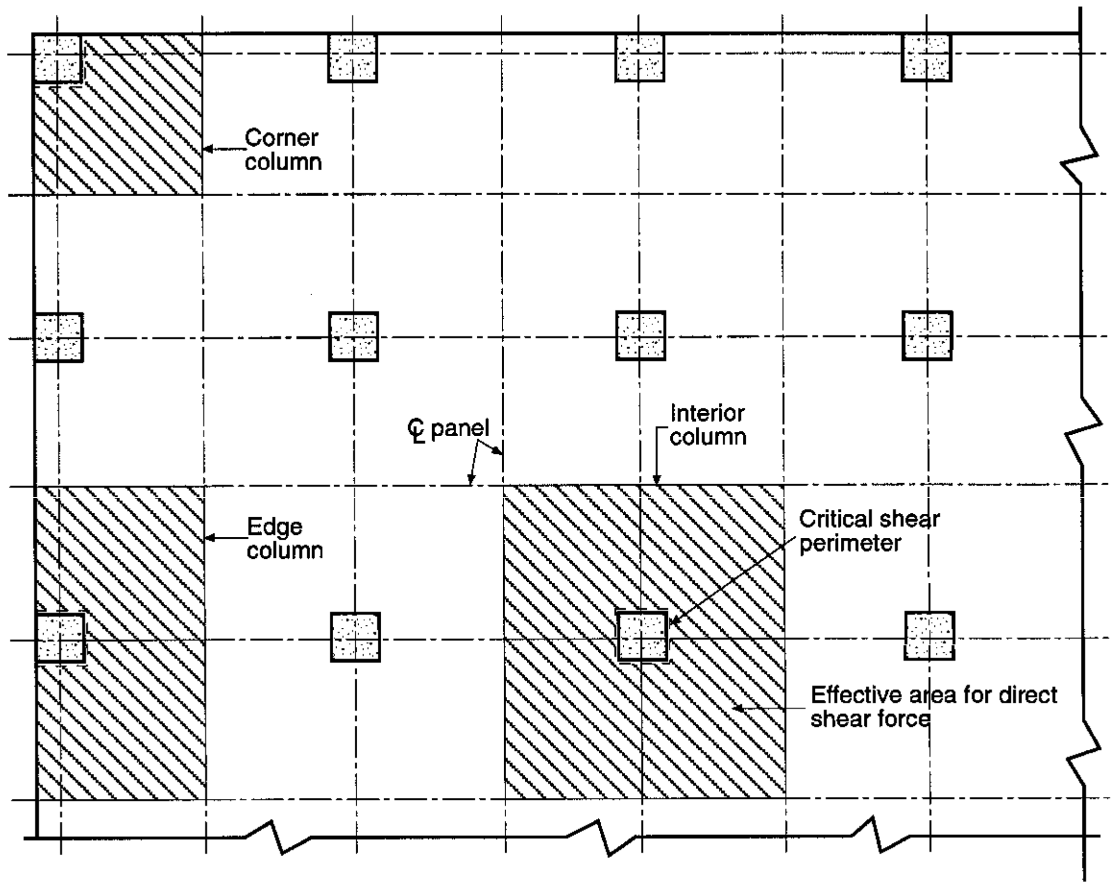
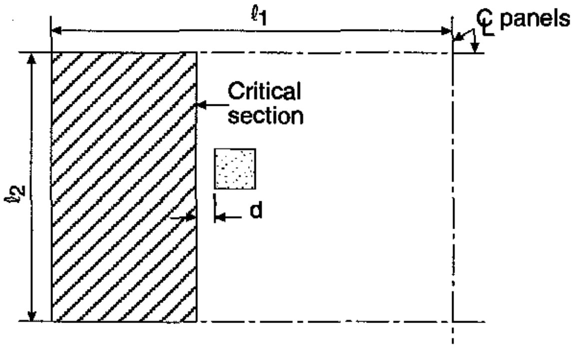


Figure 4-10 Critical Locations for Slab Shear Strength

4.4.1 Shear in Flat Plate and Flat Slab Floor Systems

Two types of shear need to be considered in the design of flat plates or flat slabs supported directly on columns. The first is the familiar one-way or beam-type shear, which may be critical in long narrow slabs. Analysis for beam shear considers the slab to act as a wide beam spanning between the columns. The critical section is taken a distance d from the face of the column. Design against beam shear consists of checking the requirement indicated in Fig. 4-11(a). Beam shear in slabs is seldom a critical factor in design, as the shear force is usually well below the shear capacity of the concrete.

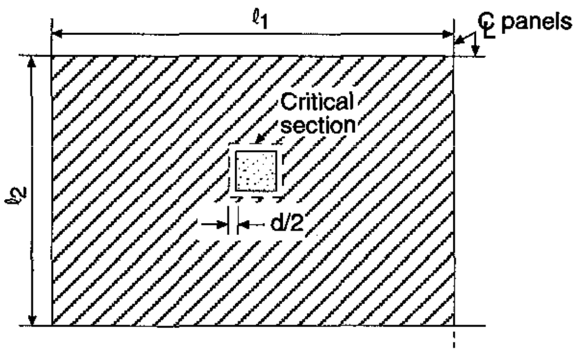
Two-way or "punching" shear is generally the more critical of the two types of shear in slab systems supported directly on columns. Punching shear considers failure along the surface of a truncated cone or pyramid around a column. The critical section is taken perpendicular to the slab at a distance d/2 from the perimeter of a column. The shear force V_u to be resisted can be easily calculated as the total factored load on the area bounded by panel centerlines around the column, less the load applied within the area defined by the critical shear perimeter (see Fig. 4-10). In the absence of a significant moment transfer from the slab to the column, design against punching shear consists of ensuring that the requirement in Fig. 4-11(b) is satisfied. Figures 4-12 through 4-14 can be used to determine ϕV_c for interior, edge and corner columns, respectively.



$$V_u \le \phi V_c$$
$$\le \phi 2 \sqrt{f'_c}\, \ell_2 d$$
$$\le 0.11\, \ell_2 d \quad (f'_c = 4000 \text{ psi})$$

where V_u is factored shear force (total factored load on shaded area). V_u is in kips and ℓ_2 and d are in inches.

(a) Beam shear



$$V_u \le \phi V_c$$

where:

$$\phi V_c = \text{least of} \begin{cases} \phi\left(2 + \dfrac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d = 0.054\left(2 + \dfrac{4}{\beta_c}\right) b_o d \\ \phi\left(\dfrac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\, b_o d = 0.054\left(\dfrac{\alpha_s d}{b_o} + 2\right) b_o d \\ \phi 4\sqrt{f'_c}\, b_o d = 0.215 b_o d \end{cases}$$

V_u = factored shear force (total factored load on shaded area), kips
b_o = perimeter of critical section, in.
β_c = long side/short side of reaction area
α_s = constant (ACI 11.12.2.1 (b))

(b) Two-way shear

Figure 4-11 Direct Shear at an Interior Slab-Column Support (see Fig. 4-10; f'_c = 4000 psi)

For practical design, only direct shear (uniformly distributed around the perimeter b_o) occurs around interior slab-column supports where no (or insignificant) moment is to be transferred from the slab to the column. Significant moments may have to be carried when unbalanced gravity loads on either side of an interior column or horizontal loading due to wind must be transferred from the slab to the column. At exterior slab-column supports, the total exterior slab moment from gravity loads (plus any wind moments) must be transferred directly to the column.

Transfer of moment between a slab and a column takes place by a combination of flexure (ACI 13.3.3) and eccentricity of shear (ACI 11.12.6). Shear due to moment transfer is assumed to act on a critical section at a distance d/2 from the face of the column, the same critical section around the column as that used for direct shear transfer [Fig. 4-11(b)]. The portion of the moment transferred by flexure is assumed to be transferred over a width of slab equal to the transverse column width c_2, plus 1.5 times the slab or drop panel thickness (1.5h) on either side of the column. Concentration of negative reinforcement is to be used to resist moment on this effective slab width. The combined shear stress due to direct shear and moment transfer often governs the design, especially at the exterior slab-columns supports.

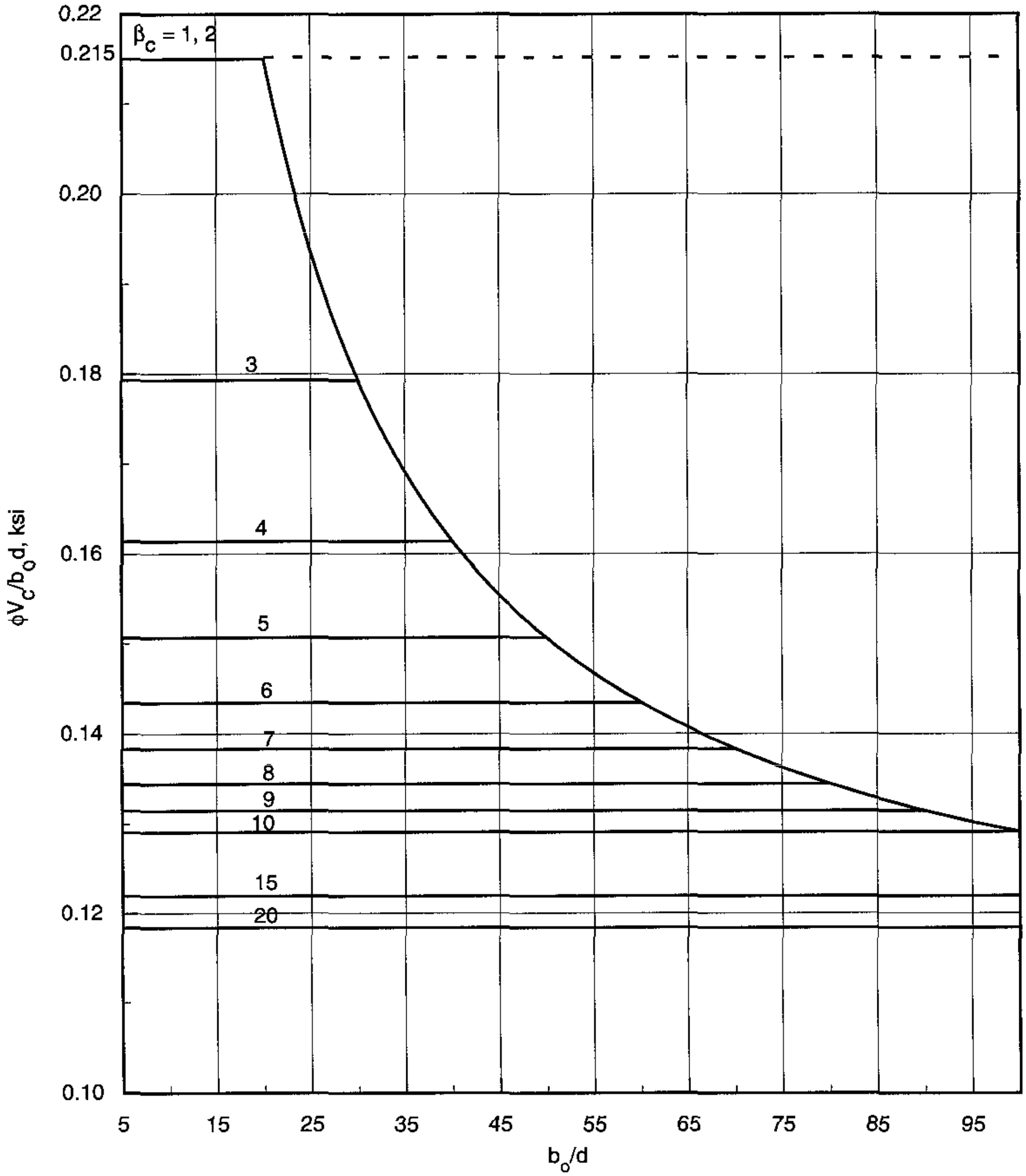


Figure 4-12 Two-Way Shear Strength of Slabs, Interior Column (α_s = 40)

The portions of the total moment to be transferred by eccentricity of shear and by flexure are given by ACI Eqs. (11-42) and (13-1), respectively. For square interior or corner columns, 40% of the moment is considered transferred by eccentricity of the shear ($\gamma_v M_u = 0.40\ M_u$), and 60% by flexure ($\gamma_f M_u = 0.60\ M_u$), where M_u is the transfer moment at the centroid of the critical section. The moment M_u at an exterior slab-column support will generally not be computed at the centroid of the critical transfer section in the frame analysis. In the Direct Design Method, moments are computed at the face of the support. Considering the approximate nature of the procedure used to evaluate the stress distribution due to moment transfer, it seems unwarranted to consider a change in moment to the transfer centroid; use of the moment values at the faces of the supports would usually be accurate enough.

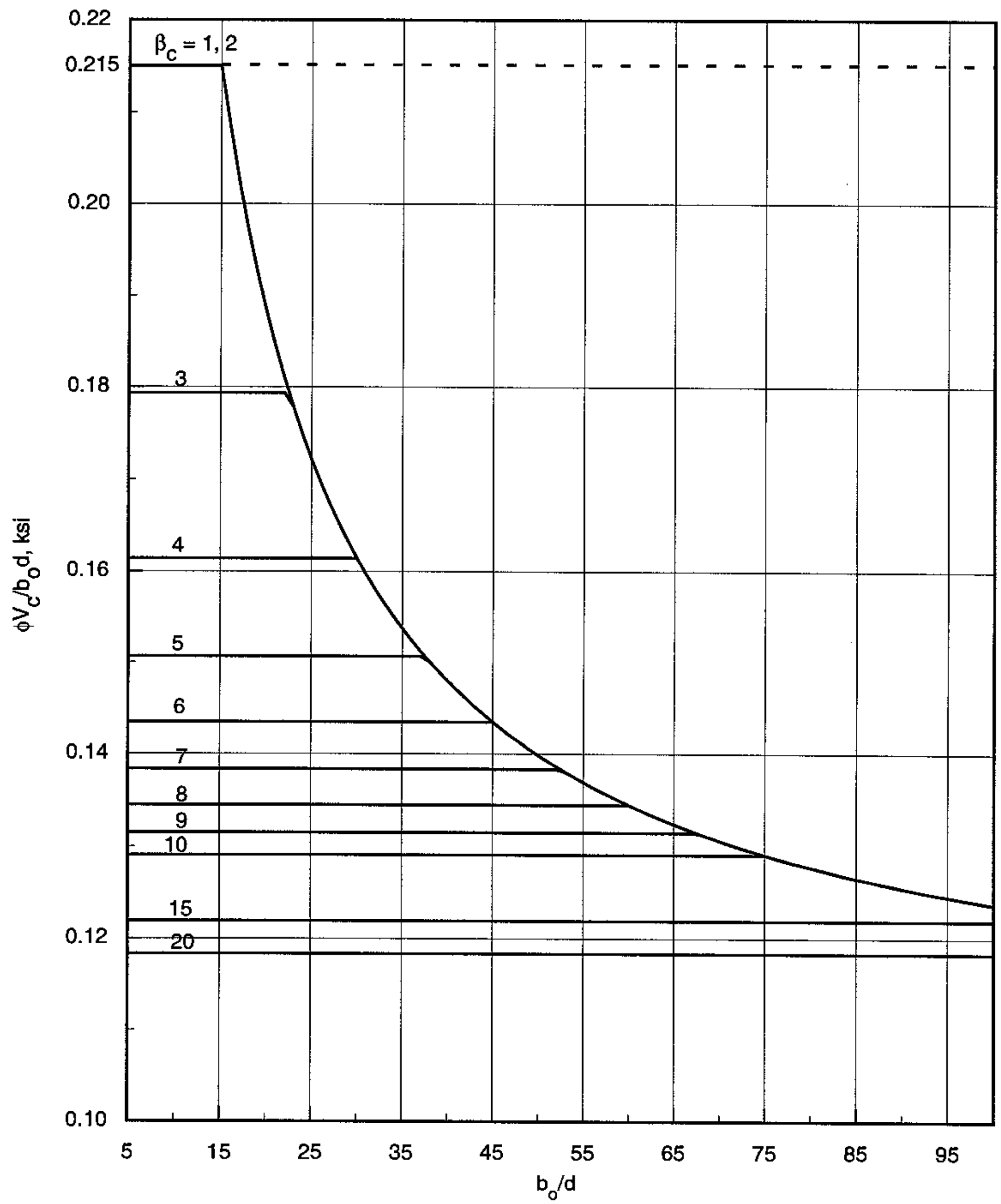


Figure 4-13 Two-Way Shear Strength of Slabs, Edge Column (α_s = 30)

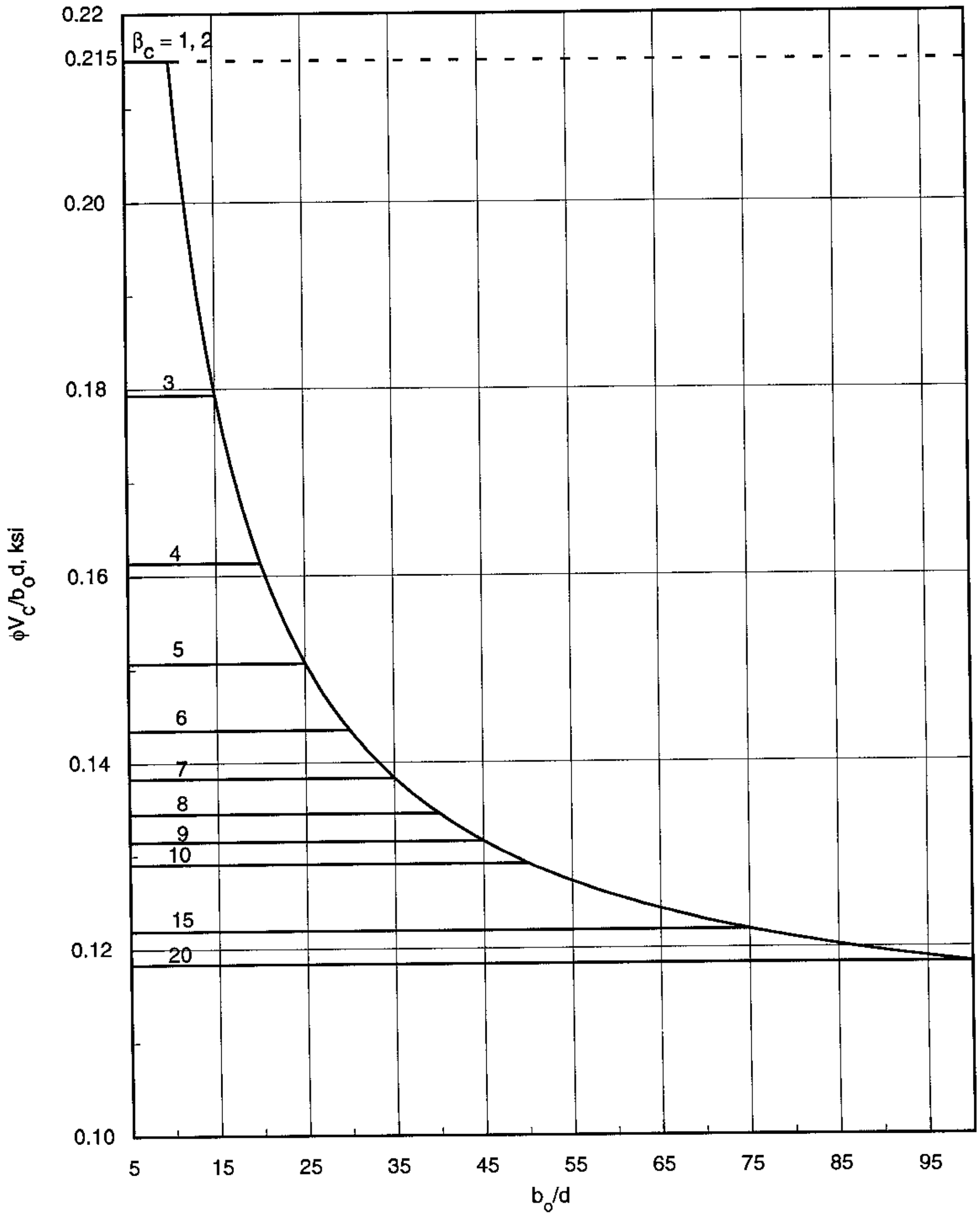


Figure 4-14 Two-Way Shear Strength of Slabs, Corner Column ($\alpha_s = 20$)

The factored shear stress on the critical transfer section is the sum of the direct shear and the shear caused by moment transfer,

$$v_u = V_u/A_c + \gamma_v M_u \, c/J$$

or

$$v_u = V_u/A_c - \gamma_v M_u \, c'/J$$

Computation of the combined shear stress involves the following properties of the critical transfer section:

A_c = area of critical section, in.^{2}

c or c' = distance from centroid of critical section to the face of section where stress is being computed, in.

J_c = property of critical section analogous to polar moment of inertia, in.^{4}

The above properties are given in terms of formulas in Tables 4-7 through 4-10 (located at the end of this chapter) for the four cases that can arise with a rectangular column section: interior column (Table 4-7), edge column with bending parallel to the edge (Table 4-8), edge column with bending perpendicular to the edge (Table 4-9), and corner column (Table 4-10). Numerical values of the above parameters for various combinations of square column sizes and slab thicknesses are also given in these tables. Properties of the critical shear transfer section for circular interior columns can be found in Reference 4.2 Note that in the case of flat slabs, two different critical sections need to be considered in punching shear calculations as shown in Fig. 4-15. Tables 4-7 through 4-10 can be used in both cases. Also, Fig. 4-16 can be used to determine γ_v and γ_f given b_1 and b_2.

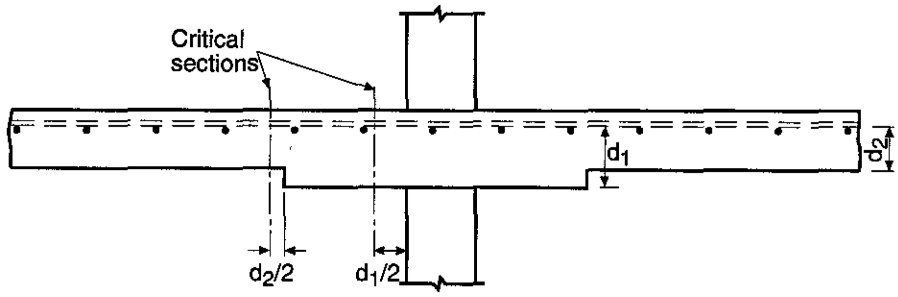


Figure 4-15 Critical Shear-Transfer Sections for Flat Slabs

Unbalanced moment transfer between slab and an edge column (without spandrel beams) requires special consideration when slabs are analyzed by the Direct Design Method for gravity load. To assure adequate shear strength when using the approximate end-span moment coefficient, the full nominal moment strength M_n provided by the column strip must be used to calculate the portion of moment transferred by eccentricity of shear ($\gamma_v M_u = \gamma_v M_n$ of column strip) according to ACI 13.6.3.6. For end spans without spandrel beams, the column strip is proportioned to resist the total exterior negative factored moment (Table 4-2). The above requirement is illustrated in Fig. 4-17. The total reinforcement provided in the column strip includes the additional reinforcement concentrated over the column to resist the fraction of unbalanced moment transferred by flexure $\gamma_f M_u = \gamma_f(0.26M_o)$, where the moment coefficient (0.26) is from Table 4-2. Application of this special design requirement is illustrated in Section 4.7.

4.5 COLUMN MOMENTS DUE TO GRAVITY LOADS

Supporting columns (and walls) must resist any negative moments transferred from the slab system. For interior columns, the approximate ACI Eq. (13-4) may be used for unbalanced moment transfer due to gravity loading, unless an analysis is made considering the effects of pattern loading and unequal adjacent spans. The transfer moment is computed directly as a function of the span length and gravity loading. For the more usual case with equal transverse and longitudinal spans, ACI Eq. (13-4) simplifies to:

$$M_u = 0.07(0.05w_\ell \ell_2 \ell_n^2) = 0.035w_\ell \ell_2 \ell_n^2$$

where w_ℓ = factored live load, psf

ℓ_2 = span length transverse to ℓ_n

ℓ_n = clear span length in direction M_u is being determined

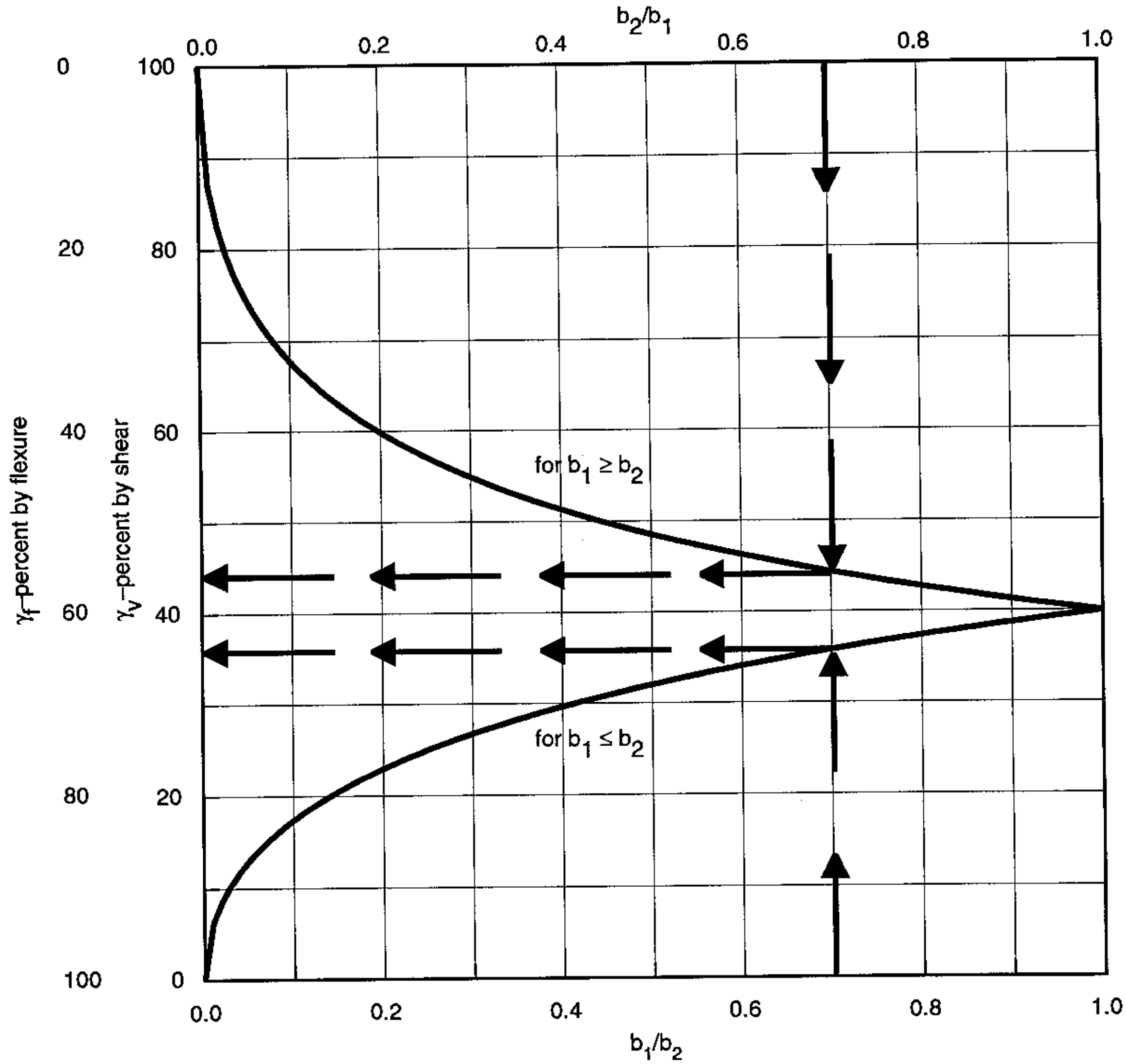


Figure 4-16 Solution to ACI Equations (11-42) and (13-1)

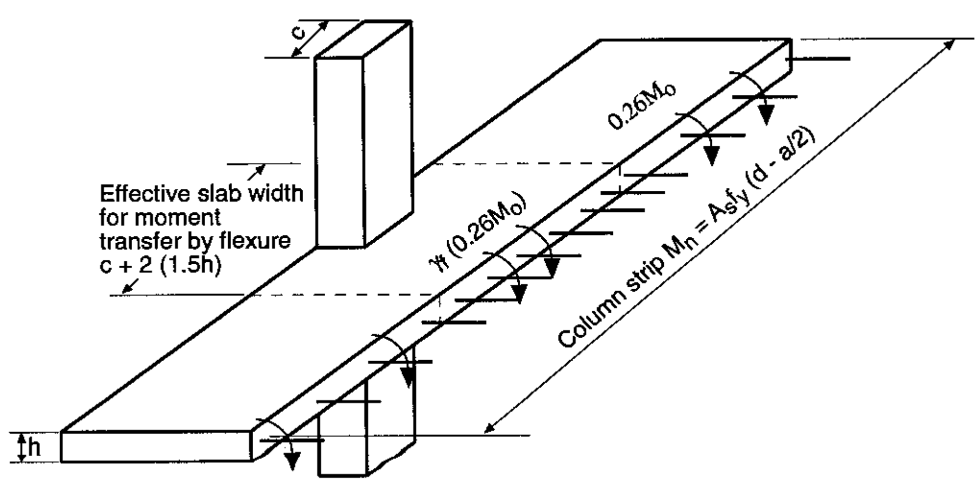


Figure 4-17 Nominal Moment Strength of Column Strip for Evaluation of $\gamma_v M_u = \gamma_v M_n$

At an exterior column, the total exterior negative moment from the slab system is transferred directly to the column. Due to the approximate nature of the moment coefficients of the Direct Design Method, it seems unwarranted to consider the change in moment from face of support to centerline of support; use of the exterior negative slab moment directly would usually be accurate enough.

Columns above and below the slab must resist portions of the support moment based on the relative column stiffnesses (generally, in proportion to column lengths above and below the slab). Again, due to the approximate nature of the moment coefficients, the refinement of considering the change in moment from centerline of slab to top or bottom of slab seems unwarranted.

4.6 REINFORCEMENT DETAILING

In computing required steel areas and selecting bar sizes, the following will ensure conformance to the Code and a practical design.

(1) Minimum reinforcement area = 0.0018 bh (b = slab width, h = total thickness) for Grade 60 bars for either top or bottom steel. These minima apply separately in each direction (ACI 13.4.1).

(2) Maximum bar spacing is 2h, but not more than 18 in. (ACI 13.4.2).

(3) Maximum top bar spacing at all interior locations subject to construction traffic should be limited. Not less than #4 @ 12 in. is recommended to provide adequate rigidity and to avoid displacement of top bars with standard bar support layouts under ordinary foot traffic.

(4) Maximum $\rho = A_s/bd$ is limited to $0.75\ \rho_b$ (ρ_b = balanced reinforcement ratio); however, $\rho_{max} \leq 0.50\ \rho_b$ is recommended to provide deformability, to avoid overly flexible systems subject to objectionable vibration or deflection, and for a practical balance to achieve overall economy of materials, construction and design time.

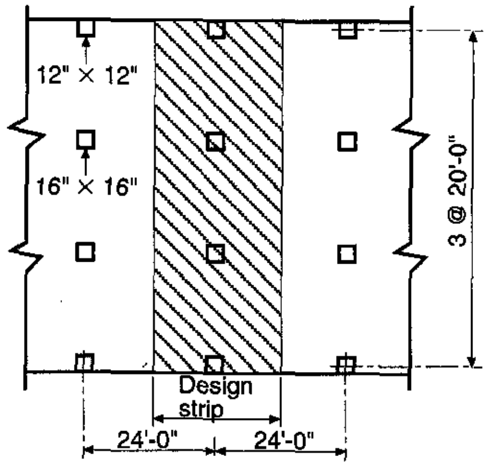
(5) Generally, the largest size of bars that will satisfy the maximum limits on spacing will provide overall economy. Critical dimensions that limit size are the thickness of slab available for hooks and the distances from the critical design sections to edges of slab.

4.7 EXAMPLES: SIMPLIFIED DESIGN FOR TWO-WAY SLABS

The following two examples illustrate use of the simplified design data presented in this chapter for the analysis and design of two-way slab systems. The two-way slab system for Building #2 is used to illustrate simplified design.

4.7.1 Example: Interior Strip (N-S Direction) of Building #2, Alternate (2)

The slab and column framing will be designed for gravity loads only; the structural walls will carry the total wind forces.



(1) Data: f'_c = 4000 psi (carbonate aggregate)
f_y = 60,000 psi

Floors: LL = 50 psf
DL = 142 psf (9 in. slab + 20 psf partitions + 10 psf ceiling & misc.)

Required fire resistance rating = 2 hours

Preliminary slab thickness:
Determine preliminary h based on two-way shear at an interior column (see Fig. 1-7).

$$w_u = 1.4(142) + 1.7(29.5^*) = 249 \text{ psf}$$
$$A = 24 \times 20 = 480 \text{ ft}^2$$
$$c_1^2 = 16 \times 16 = 256 \text{ in.}^2 = 1.8 \text{ ft}^2$$
$$A/c_1^2 = 480/1.8 = 267$$

From Fig. 1-7, required $d/c_1 \cong 0.39$

Required $d = 0.39 \times 16 = 6.24$ in.

$h = 6.24 + 1.25 = 7.49$ in.

To account for moment transfer at the edge columns, increase h by 20%.

Try preliminary h = 9 in.

(2) Check the preliminary slab thickness for deflection control and shear strength.

(a) Deflection control:

From Table 4-1 (flat plate): $h = \ell_n/30 = (22.67 \times 12)/30 = 9.07$ in.

where $\ell_n = 24 - (16/12) = 22.67$ ft

(b) Shear Strength:

From Fig. 4-11: Check two-way shear strength at interior slab-column support for h = 9 in.

**Live load reduction:* A_I *(4 panels) = 24 × 20 × 4 = 1920 sq ft*

$L = 50(0.25 + 15/\sqrt{1920}) = 29.5$ *psf*

From Table 4-7: $A_c = 736.3$ in.^{2} for 9 in. slab with 16 × 16 in. column.
$b_1 = b_2 = 2(11.88) = 23.76$ in. = 1.98 ft

$$V_u = 0.249(24 \times 20 - 1.98^2) = 118.5 \text{ kips}$$

From Fig. 4-12, with $\beta_c = 1$ and $b_o/d = 4(23.76)/(9 - 1.25) = 12.3$:

$$\phi V_c = 0.215\ A_c = 0.215(736.3) = 158.3 \text{ kips} > 118.5 \text{ kips O.K.}$$

Check for fire resistance: From Table 10-1, for fire resistance rating of 2 hours, required slab thickness = 4.6 in. < 9.0 in. O.K.

Use 9 in. slab.

(3) Check limitations for slab analysis by coefficients (ACI 13.6.1)

- 3 continuous spans in one direction, 5 in the other
- rectangular panels with long-to-short span ratio = 24/20 = 1.2 < 2
- successive span lengths in each direction are equal
- no offset columns
- LL/DL = 50/142 = 0.35 < 3
- slab system is without beams

Since all requirements are satisfied, the Direct Design Method can be used to determine the moments.

(4) Factored moments in slab (N-S direction)

(a) Total panel moment M_o:

$$M_o = w_u \ell_2 \ell_n^2/8$$
$$= 0.278 \times 24 \times 18.83^2/8 = 295.7 \text{ ft-kips}$$

where $w_u = 1.4(142) + 1.7(46.5^*) = 278$ psf

$\ell_2 = 24$ ft

ℓ_n (interior span) = 20 - 1.33 = 18.67 ft

ℓ_n (end span) = 20 - 0.67 - 0.50 = 18.83 ft

Use larger value of ℓ_n for both spans.

(b) Negative and positive factored moments:

Division of the total panel moment M_o into negative and positive moments, and then, column and middle strip moments, involves direct application of the moment coefficients in Table 4-2.

Slab Moments (ft-kips)	End Spans			Interior Span
	Exterior Negative	Positive	Interior Negative	Positive
Total Moment	76.9	153.8	207.0	103.5
Column Strip	76.9	91.7	156.7	62.1
Middle Strip	0	62.1	50.3	41.4

Note: All negative moments are at face of column.

**Live load reduction:* A_I *(1 panel) = 24 × 20 = 480 sq ft*

$L = 50(0.25 + 15/\sqrt{480}) = 46.5$ *psf*

(5) Slab Reinforcement

Required slab reinforcement is easily determined using a tabular form as follows:

Span location		M_u (ft-kips)	b^1 (in.)	d^2 (in.)	$A_s = M_u/4d$ (in.^{2})	$A_s{}^3$(min) (in.^{2})	No. of #4 Bars[4]	No. of #5 bars[4]
END SPAN								
Column Strip	Ext. Negative	76.9	120	7.75	2.48	1.94	13	8
	Positive	91.7	120	7.75	2.96	1.94	15	10
	Int. Negative	156.7	120	7.75	5.05	1.94	26	17
Middle Strip	Ext. Negative	0	168	7.75	--	2.72	14	11
	Positive	62.1	168	7.75	2.00	2.72	14	11
	Int. Negative	50.3	168	7.75	1.62	2.72	14	11
INTERIOR SPAN								
Column Strip	Positive	62.1	120	7.75	2.00	1.94	10	8
Middle Strip	Positive	41.4	168	7.75	1.34	2.72	14	11

Notes: [1]Column strip = 0.5(20 × 12) = 120 in. (see Fig. 4-9b)
Middle strip = (24 × 12) - 120 = 168 in.
[2]Use average d = 9 - 1.25 = 7.75 in.
[3]$A_{s(min)}$ = 0.0018 bh = 0.0162b
s_{max} = 2h < 18 in. = 2(9) = 18 in.
[4]Calculations:

For s_{max}: 120/18 = 6.7 spaces, say 8 bars
168/18 = 9.3 spaces, say 11 bars

For #4 bar: 2.48/0.20 = 12.4 bars
2.96/0.20 = 14.8 bars
5.05/0.20 = 25.3 bars
2.72/0.20 = 13.6 bars
2.00/0.20 = 10 bars

For #5 bars: 2.48/0.31 = 8 bars
2.96/0.31 = 9.5 bars
5.05/0.31 = 16.3 bars
2.72/0.31 = 8.8 bars < 11
2.00/0.31 = 6.5 bars < 8

(6) Check slab reinforcement at exterior column (12 × 12 in.) for moment transfer between slab and column. For a slab without spandrel beams, the total exterior negative slab moment is resisted by the column strip (i.e., M_u = 76.9 ft-kips).

Fraction transferred by flexure using ACI Eq. (13-1):

$$b_1 = 12 + (7.75/2) = 15.88 \text{ in.}$$

$b_2 = 12 + 7.75 = 19.75$ in.

From Fig. 4-16, $\gamma_f \cong 0.625$ with $b_1/b_2 = 0.8$

$M_u = 0.625\ (76.9) = 48.1$ ft-kips

$A_s = M_u/4d = 48.1/(4 \times 7.75) = 1.55$ in.^{2}

No. of #4 bars = 1.55/0.20 = 7.75 bars, say 8 bars

Must provide 8-#4 bars within an effective slab width (ACI 13.3.3.2) = $3h + c_2 = 3(9) + 12 = 39$ in.

Provide the required 8-#4 bars by concentrating 8 of the column strip bars (13-#4) within the 3 ft-3 in. slab width over the column. For symmetry, add one column strip bar to the remaining 5 bars so that 3 bars will be on each side of the 3 ft-3 in. strip. Check bar spacing:

For 8-#4 within 39 in. width: 39/8 = 4.9 in.
For 6-#4 within (120 - 39) = 81 in. width: 81/6 = 13.5 in. < 18 in. O.K.
No additional bars are required for moment transfer.

(7) Reinforcement details are shown in Figs. 4-18, 4-19, and 4-21. Bar lengths are determined directly from Fig. 8-6. Note that for structural integrity, at least two of the column strip bottom bars must be continuous or spliced at the support with Class A splices or anchored within the support (ACI 13.4.8.5).

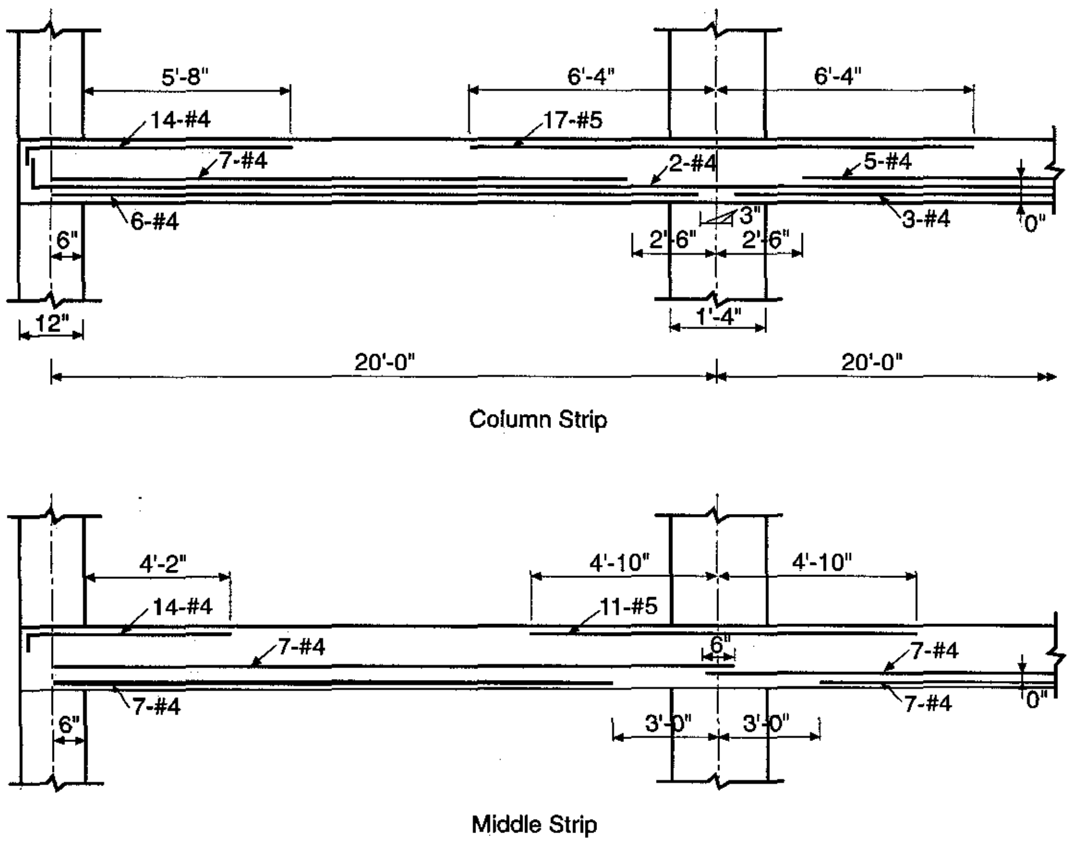


Figure 4-18 Reinforcement Details for Flat Plate of Building #2—Alternate (2) Interior Slab Panel (N-S Direction)

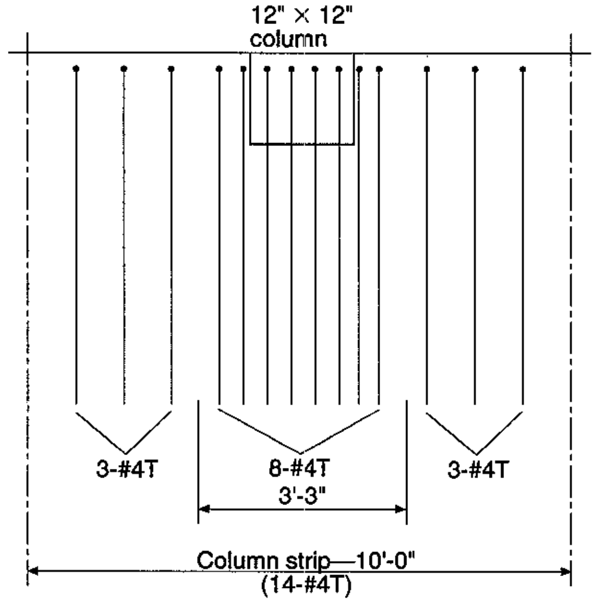


Figure 4-19 Bar Layout Detail for 14-#4 Top Bars at Exterior Columns

8) Check slab shear strength at edge column for gravity load shear and moment transfer (see Fig. 4-20).

(a) Direct shear from gravity loads:

live load reduction:

A_I(2 panels) = 24 × 20 × 2 = 960 sq ft

$L = 50(0.25 + 15\sqrt{960}) = 35.6$ psf

$w_u = 1.4(142) + 1.7(36.5) = 261$ psf

$V_u = 0.261[(24)(10.5) - (1.32)(1.65)] = 65.2$ kips

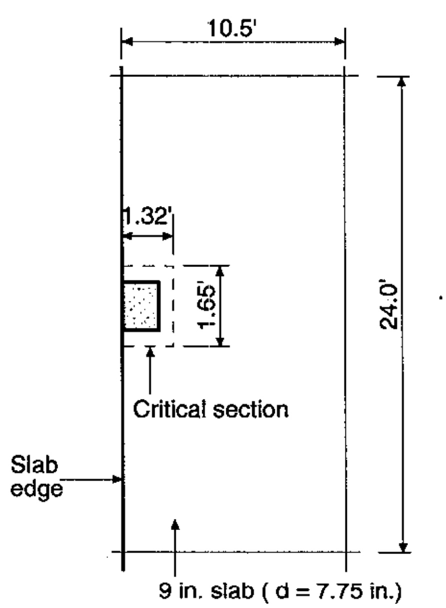


Figure 4-20 Critical Section for Edge Column

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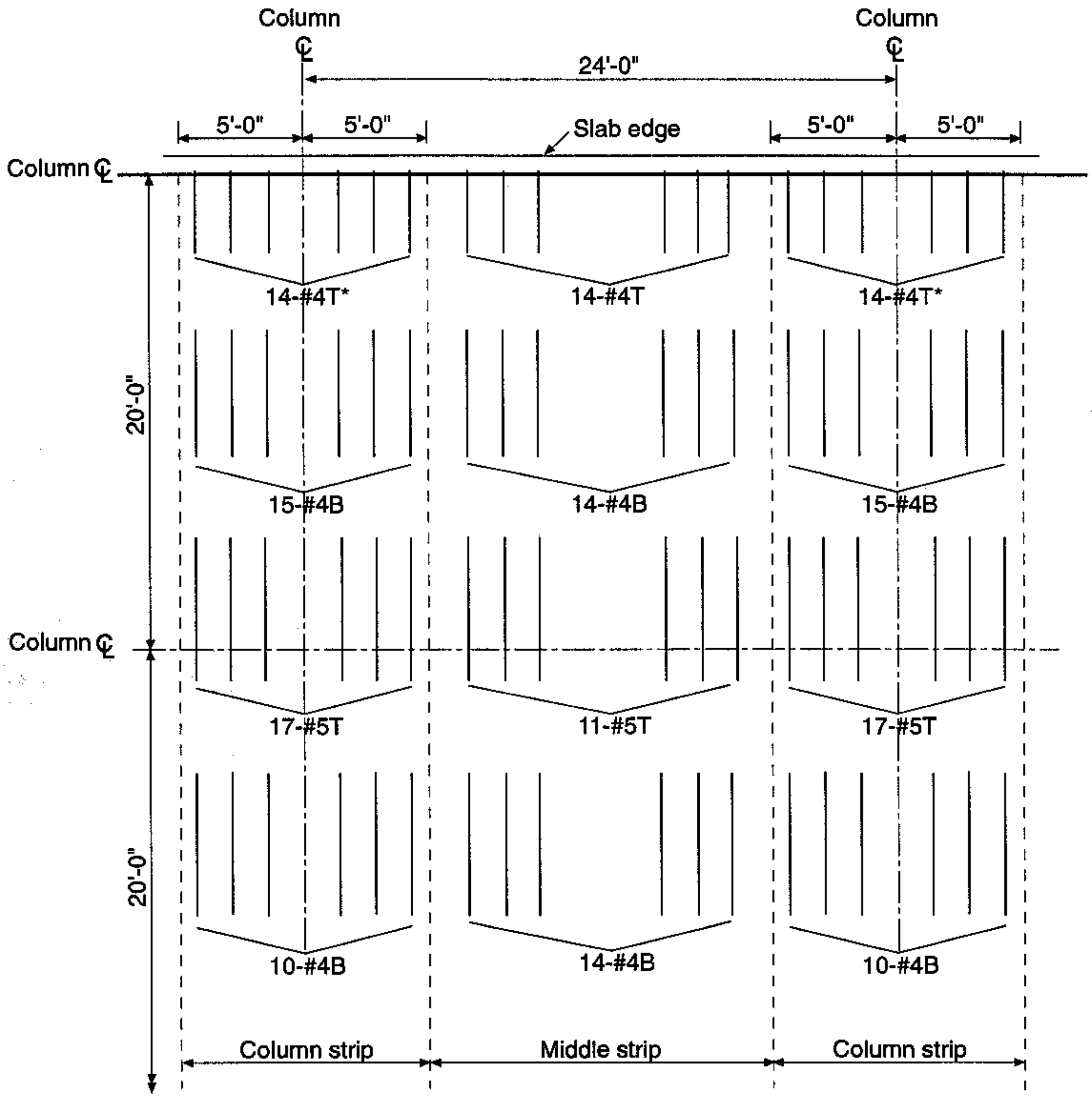


*See special bar layout detail in Fig. 4-19

Figure 4-21 Bar Layout—Space Bars Uniformly within Each Column Strip and Middle Strip

(b) Moment transfer from gravity loads:

When slab moments are determined using the approximate moment coefficients, the special provisions of ACI 13.6.3.6 apply for moment transfer between slab and an edge column. The fraction of unbalanced moment transferred by eccentricity of shear (ACI 11.12.6.1) must be based on the full column strip nominal moment strength M_n provided.

For 14-#4 column strip bars, $A_s = 14(0.20) = 2.80$ in.^{2}

Using the approximate expression for $A_s = M_u/4d$:

$$\phi M_n = M_u = A_s(4d)$$

$$M_n = \frac{2.80\,(4 \times 7.75)}{0.9} = 96.4 \text{ ft} - \text{kips}$$

(c) Combined shear stress at inside face of critical transfer section:

From Table 4-9, for 9 in. slab with 12 × 12 in. column:

$A_c = 399.1 \text{ in.}^2$
$J/c = 2523 \text{ in.}^3$

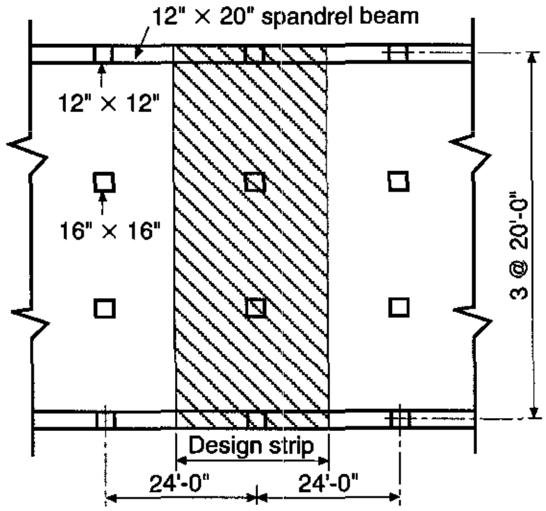
From Fig. 4-16, with $b_1/b_2 = 15.88/19.75 = 0.8$, $\gamma_v \cong 0.375$

$$\begin{aligned} v_u &= V_u/A_c + \gamma_v M_n \, c/J \\ &= (65{,}200/399.1) + (0.375 \times 96.4 \times 12{,}000/2523) \\ &= 163.4 + 171.9 = 335.3 \text{ psi} >> \phi 4\sqrt{f'_c} = 215 \text{ psi*} \end{aligned}$$

The 9 in. slab is not adequate for shear and unbalanced moment transfer at the edge columns. Increase shear strength by providing drop panels at edge columns. Calculations not shown here.

4.7.2 Example: Interior Strip (N-S Direction) of Building #2, Alternate (1)

The slab and column framing will be designed for both gravity and wind loads. Design an interior strip for the 1st-floor level (greatest wind load effects).



(1) Data: $f'_c = 4000$ psi (carbonate aggregate)
$f_y = 60{,}000$ psi

Floors: LL = 50 psf
DL = 136 psf (8½ in. slab + 20 psf partitions + 10 psf ceiling & misc.)

Preliminary sizing: Slab = 8½ in. thick
Columns interior = 16 × 16 in.
exterior = 12 × 12 in.
Spandrel beams = 12 × 20 in.

Required fire resistance rating = 2 hours

*$b_o/d = [(2 \times 15.88) + 19.75]/7.75 = 6.7$; from Fig. 4-13 with $\beta_c = 1$, $\phi V_c/b_o d = 215$ psi

(2) Determine the slab thickness for deflection control and shear strength

(a) Deflection control:

From Table 4-1 (flat plate with spandrel beams, $\alpha \geq 0.8$):

$$h = \ell_n/33 = (22.67 \times 12)/33 = 8.24 \text{ in.}$$

$$\text{where } \ell_n = 24 - (16/12) = 22.67 \text{ ft}$$

(b) Shear strength. Check shear strength for an 8½ in. slab:

With the slab and column framing designed for both gravity and wind loads, slab shear strength needs to be checked for the combination of direct shear from gravity loads plus moment transfer from wind loads. Wind load analysis for Building #2 is summarized in Fig. 2-15. Moment transfer between slab and column is greatest at the 1st-floor level where wind moment is the largest. Transfer moment (unfactored) at 1st-floor level due to wind, $M_w = 77.05 + 77.05 = 154.1$ ft-kips.

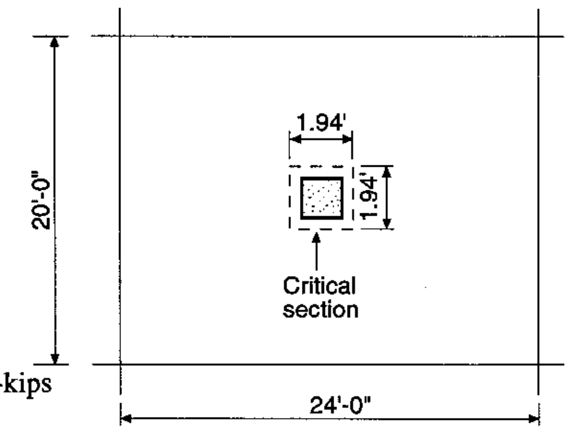
Direct shear from gravity loads:

$$w_u = 1.4(136) + 1.7(29.5^*) = 241 \text{ psf}$$

$$V_u = 0.241(24 \times 20 - 1.94^2) = 114.8 \text{ kips}$$

$$\text{where } d = 8.50 - 1.25 = 7.25 \text{ in.}$$

$$b_1 = b_2 = (16 + 7.25)/12 = 1.94 \text{ ft}$$



Gravity + wind load combination [ACI Eq. (9-2)]:

$$V_u = 0.75(114.8) = 86.1 \text{ kips}$$

$$M_u = 0.75(1.7 \times 154.1) = 196.5 \text{ ft-kips}$$

From Table 4-7, for 8½ in. slab with 16 × 16 in. columns:

$$A_c = 674.3 \text{ in.}^2$$

$$J/c = 5352 \text{ in.}^3$$

Shear stress at critical transfer section:

$$v_u = V_u/A_c + \gamma_v M_u\, c/J$$

$$= (86{,}100/674.3) + (0.4 \times 196.5 \times 12{,}000/5352)$$

$$= 127.7 + 176.2 = 303.9 \text{ psi} > \phi 4\sqrt{f_c'} = 215 \text{ psi}$$

The 8½ in. slab is not adequate for gravity plus wind load transfer at the interior columns.

Increase shear strength by providing drop panels at interior columns. Minimum slab thickness at drop panel = 1.25(8.5) = 10.63 in. (see Fig. 4-2). Dimension drop to actual lumber dimensions for economy of formwork. Try 2 ¼ in. drop (see Table 9-1).

$$h = 8.5 + 2.25 = 10.75 \text{ in.} > 10.63 \text{ in.}$$

$$d = 7.25 + 2.25 = 9.5 \text{ in.}$$

Refer to Table 4-7:

$$b_1 = b_2 = 16 + 9.5 = 25.5 \text{ in.} = 2.13 \text{ ft}$$

Live load reduction: A_I (4 panels) = 24 × 20 × 4 = 1920 sq ft

$L = 50(0.25 + 15/\sqrt{1920}) = 29.5$ psf

$A_c = 4(25.5) \times 9.5 = 969 \text{ in.}^2$

$J/c = [25.5 \times 9.5(25.5 \times 4) + 9.5^3]/3 = 8522 \text{ in.}^3$

$v_u = (86{,}100/969) + (0.4 \times 196.5 \times 12{,}000/8522)$

$\quad = 88.9 + 110.7 = 199.6 \text{ psi} < 215 \text{ psi}$ O.K.

Note that the shear stress around the drop panel is much less than the allowable stress (calculations not shown here).

With drop panels, a lesser slab thickness for deflection control is permitted. From Table 4-1 (flat slab with spandrel beams): $h = \ell_n/36 = (22.67 \times 12)/36 = 7.56$ in. Could possibly reduce slab thickness from 8½ to 8 in.; however, shear strength may not be adequate with the lesser slab thickness. For this example hold the slab thickness at 8½ in. Note that the drop panels may not be required in the upper stories where the transfer moment due to wind becomes substantially less (see Fig. 2-15).

Use 8½ in. slab with 2 ¼ in. drop panels at interior columns of 1st story floor slab. Drop panel dimensions = $\ell/3 = 24/3 = 8$ ft. Use same dimension in both directions for economy of formwork.

Check for fire resistance: From Table 10-1 for fire resistance rating of 2 hours, required slab thickness = 4.6 in. < 8.5 in. O.K.

(3) Factored moments in slab due to gravity load (N-S direction).

(a) Evaluate spandrel beam-to-slab stiffness ratio α and β_t:

Referring to Fig. 4-7:

$\ell_2 = (20 \times 12)/2 = 120$ in.

$a = 20$ in.

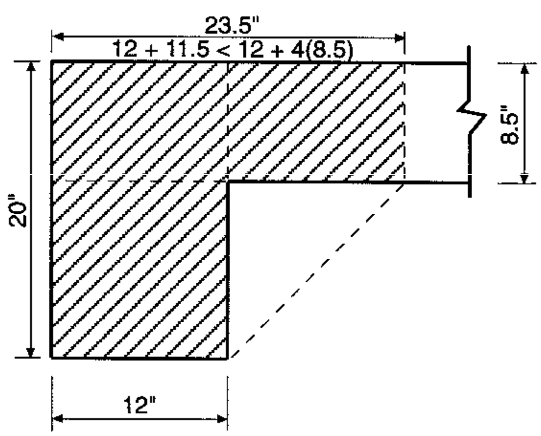
$b = 12$ in.

$h = 8.5$ in.

$a/h = 20/8.5 = 2.4$

$b/h = 12/8.5 = 1.4$

$f \cong 1.37$



$$\alpha = \frac{b}{\ell_2}(a/h)^3 f = \frac{12}{120}(2.4)^3(1.37) = 1.89 > 0.8$$

Note that the original assumption that the minimum $h = \ell_n/33$ is O.K. since $\alpha > 0.8$ (see Table 4-1).

$$\beta_t = \frac{C}{2I_s} = \frac{8425}{2(14{,}740)} = 0.29 < 2.5$$

where $I_s = (24 \times 12)(8.5)^3/12 = 14{,}740 \text{ in.}^4$
C = larger value computed for the spandrel beam section (see Fig. 4-8).

x_1 = 8.5 y_1 = 23.5 C_1 = 3714	x_2 = 11.5 y_2 = 12 C_2 = 2410	x_1 = 12 y_1 = 20 C_1 = 7165	x_2 = 8.5 y_2 = 11.5 C_2 = 1260

$\Sigma C = 3714 + 2410 = 6124$ $\Sigma C = 7165 + 1260 = 8425$ (governs)

(b) Total panel moment M_o:

$$M_o = w_u \ell_2 \ell_n^{\,2}/8$$

where $w_u = 1.4(136) + 1.7(46.5^*) = 269$ psf

$\ell_2 = 24$ ft

ℓ_n (interior span) = 20 - 1.33 = 18.67 ft

ℓ_n (end span) = 20 - 0.67 - 0.50 = 18.83 ft

Use larger value for both spans.

(c) Negative and positive factored gravity load moments:

Division of the total panel moment M_o into negative and positive moments, and then, column strip and middle strip moments involves direct application of the moment coefficients of Table 4-3. Note that the moment coefficients for the exterior negative column and middle strip moments need to be modified for β_t less than 2.5. For $\beta_t = 0.29$:

$$\text{Column strip moment} = (0.30 - 0.03 \times 0.29)M_o = 0.29M_o$$

$$\text{Middle strip moment} = 0.30M_o - 0.29M_o = 0.01M_o$$

	End Spans			Interior Span
Slab Moments (ft-kips)	Exterior Negative	Positive	Interior Negative	Positive
Total Moment	85.8	143.0	200.3	100.1
Column Strip	83.0	85.8	151.6	60.1
Middle Strip	2.8	57.2	48.7	40.0

Note: All negative moments are at faces of columns.

(4) Check negative moment sections for combined gravity plus wind load moments

(a) Exterior Negative:

Consider wind load moments resisted by column strip as defined in Fig. 4-9. Column strip width = 0.5(20 × 12) = 120 in.

gravity loads only:

$M_u = 83.0$ ft-kips ACI Eq. (9-1)

gravity + wind loads:

$M_u = 0.75(83.0) + 0.75(1.7 \times 77.05) = 160.5$ ft-kips (governs) ACI Eq. (9-2)

Also check for possible moment reversal due to wind moments:

$M_u = 0.9(42) \pm 1.3(77.05) = 137.9$ ft-kips, - 62.4 ft-kips (reversal) ACI Eq. (9-3)

where $w_d = 136$ psf

$M_d = 0.29(0.136 \times 24 \times 18.83^2/8) = 42$ ft-kips

**Live load reduction:* A_I *(one panel) = 24 × 20 = 480 sq ft*

$L = 50(0.25 + 15/\sqrt{480}) = 46.5$ *psf*

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(b) Interior Negative:

gravity loads only:

$M_u = 151.6$ ft-kips

gravity + wind loads:

$M_u = 0.75(151.6) + 0.75(1.7 \times 77.05) = 211.9$ ft-kips (governs)

and $M_u = 0.9(76.8) \pm 1.3(77.05) = 169.2$ ft-kips, - 31.1 ft-kips (reversal)

where $M_d = 42(0.53/0.29) = 76.8$ ft-kips

(5) Check slab section for moment strength

(a) At exterior negative support section :

$b = 20\ M_u/d^2 = 20 \times 160.5/7.25^2 = 61.1$ in. < 120 in. O.K.

where d = 8.5 - 1.25 = 7.25 in.

(b) At interior negative support section:

$b = 20 \times 211.9/9.50^2 = 47$ in. < 120 in. O.K.

where d = 7.25 + 2.25 = 9.50 in.

(6) Slab Reinforcement

Required slab reinforcement is easily determined using a tabular form as follows:

Span location		M_u (ft-kips)	b^1 (in.)	d^2 (in.)	$A_s = M_u/4d$ (in.^{2})	$A_s{}^3$(min) (in.^{2})	No. of #4 bars^4	No. of #5 bars^4
END SPAN								
Column Strip	Ext. Negative	160.5	120	7.25	5.53	1.84	28	18
		-62.4	120	7.25	2.15	--	11	9
	Positive	85.8	120	7.25	2.96	1.84	15	10
	Int. Negative	211.9	120	9.50	5.58	2.32	28	18
		-31.1	120	7.25	1.07	--	9	9
Middle Strip	Ext. Negative	2.8	168	7.25	0.10	2.57	13	11
	Positive	57.2	168	7.25	1.97	2.57	13	11
	Int. Negative	48.7	168	7.25	1.68	2.57	13	11
INTERIOR SPAN								
Column Strip	Positive	60.1	120	7.25	2.07	1.84	11	9
Middle Strip	Positive	40.0	168	7.25	1.38	2.57	13	11

Notes: [1]Column strip width = 0.5(20 × 12) = 120 in.

Middle strip width = (24 × 12) - 120 = 168 in.

[2]Use average d = 8.5 - 1.25 = 7.25 in.

At drop panel, d = 7.25 + 2.25 = 9.50 in. (negative only)

[3]$A_{s(min)} = 0.0018\ bh$

$s_{max} = 2h < 18$ in. = 2(8.5) = 17 in.

[4]Calculations:

For s_{max}:	120/17 = 7.1 spaces, say 9 bars
	168/17 = 9.9 spaces, say 11 bars
For #4 bars:	5.53/0.20 = 27.7 bars
	2.96/0.20 = 14.8 bars
	5.58/0.20 = 27.9 bars
	2.57/0.20 = 12.9 bars
	2.07/0.20 = 10.4 bars
	2.15/0.20 = 10.8 bars
	1.07/0.20 = 5.4 bars < 9
For #5 bars:	5.53/0.31 = 17.8 bars
	2.96/0.31 = 9.5 bars
	5.58/0.31 = 18 bars
	2.57/0.31 = 8.3 bars < 11
	2.07/0.31 = 6.7 bars < 9
	2.15/0.31 = 6.9 bars < 9
	1.07/0.31 = 3.5 bars < 9

(7) Check slab reinforcement at interior columns for moment transfer between slab and column. Shear strength of slab already checked for direct shear and moment transfer in Step (2)(b). Transfer moment (unfactored) at 1st-story due to wind, M_w = 154.1 ft-kips.

Fraction transferred by flexure using ACI Eqs. (13-1) and (9-3):

$$M_u = 0.60(1.3 \times 154.1) = 120.2 \text{ ft-kips}$$

$$A_s = M_u/4d = 120.2/(4 \times 9.50) = 3.16 \text{ in.}^2$$

For #5 bar, 3.16/0.31 = 10.2 bars, say 10-#5 bars

Must provide 10-#5 bars within an effective slab width = $3h + c_2 = 3(10.75) + 16 = 48.3$ in.

Provide the required 10-#5 bars by concentrating 10 of the column strip bars (18-#5) within the 4-ft slab width over the column. Distribute the other 8 column strip bars (4 on each side) in the remaining column strip width. Check bar spacing:

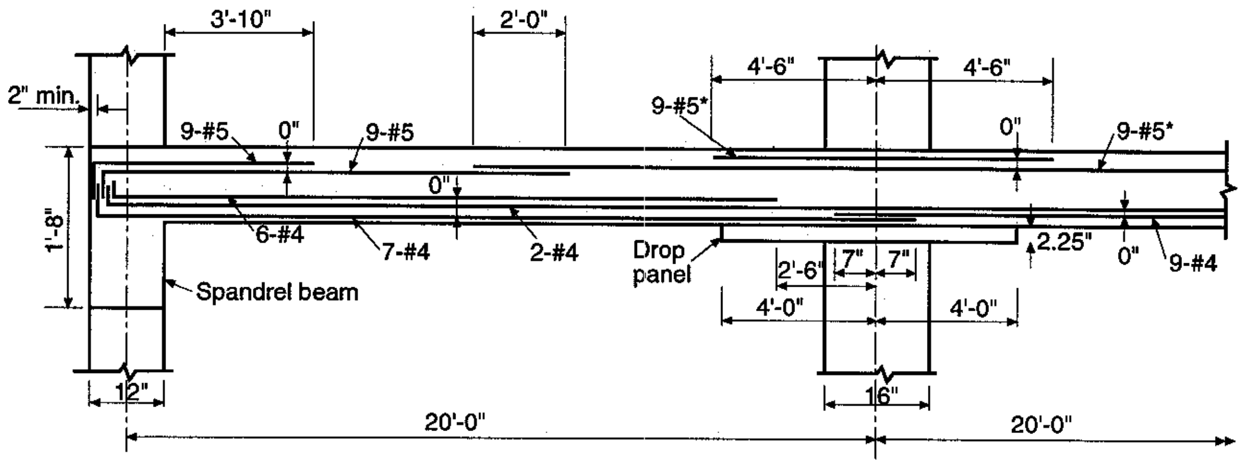
$$48/9 \text{ spaces} = \pm 5.3 \text{ in.}$$

$$(120 - 48)/7 \text{ spaces} = \pm 10.3 \text{ in.} < 17 \text{ in.} \quad \text{O.K.}$$

Reinforcement details for the interior slab are shown in Figs. 4-22 and 4-23. Bar lengths for the middle strip are taken directly from Fig. 8-6. For the column strip, the bar lengths given in Fig. 8-6 (with drop panels) need to be modified to account for wind moment effects. In lieu of a rigorous analysis to determine bar cutoffs based on a combination of gravity plus wind moment variations, provide bar length details as follows:

For bars near the top face of the slab, cut off one-half of the bars at $0.2\ell_n$ from supports and extend the remaining half the full span length, with a Class B splice near the center of span. Referring to Table 8-3, splice length = $1.3 \times 18 = 23.4$ in. ≅ 2 ft. At the exterior columns, provide a 90° standard hook with 2 in. minimum cover to edge of slab. From Table 8-5, for #5 bars, $\ell_{dh} = 9$ in. < 12 - 2 = 10 in. O.K. (For easier bar placement, alternate equal bar lengths at interior column supports.)

For the bottom bars, cut off 6-#4 in the end spans at $0.125\ell_1$ from the centerline of the interior column, and develop 7-#4 beyond the face of the column. From Table 8-3, for #4 bars, development length ℓ_d = 15.0 in. For the interior span, extend 9-#4 the full span length with ℓ_d = 15 in. beyond the face of the support. At the exterior columns, provide a 90° end-hook with 2 in. minimum cover to edge of slab for all bottom bars. At least 2 of the bottom bars in the column strip must be made continuous or be spliced at the support with Class A splices for structural integrity.



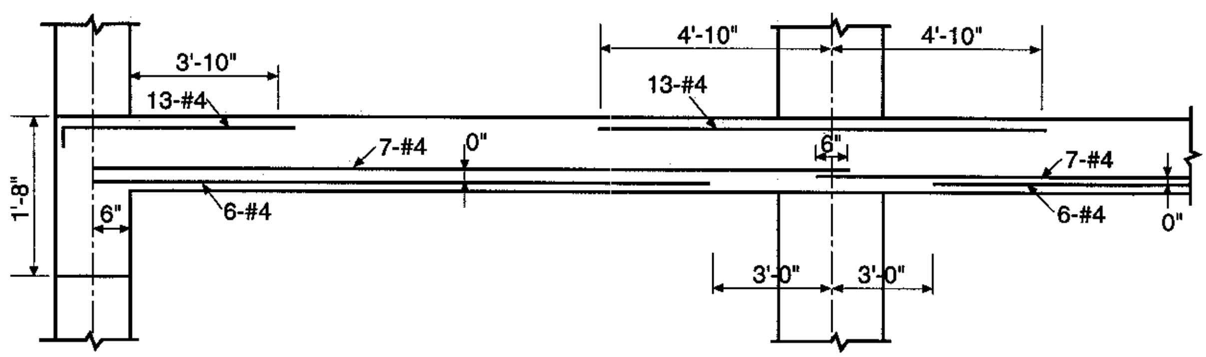


Figure 4-22 Reinforcement Details for Flat Slab of Building #2 Alternate (1)—1st Floor Interior Slab Panel (N-S Direction)

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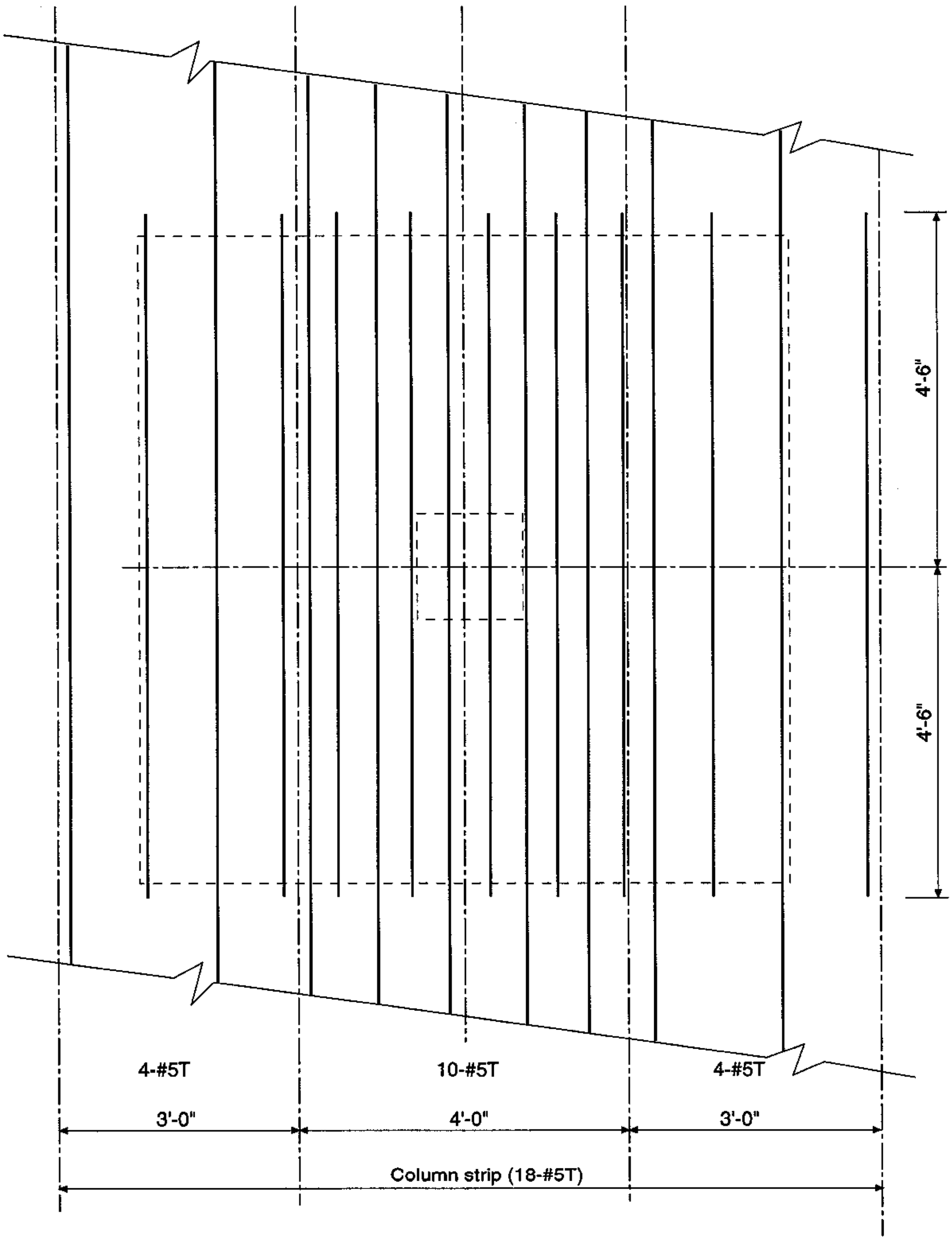

Figure 4-23 Bar Layout Detail for 18-#5 Top Bars at Interior Columns

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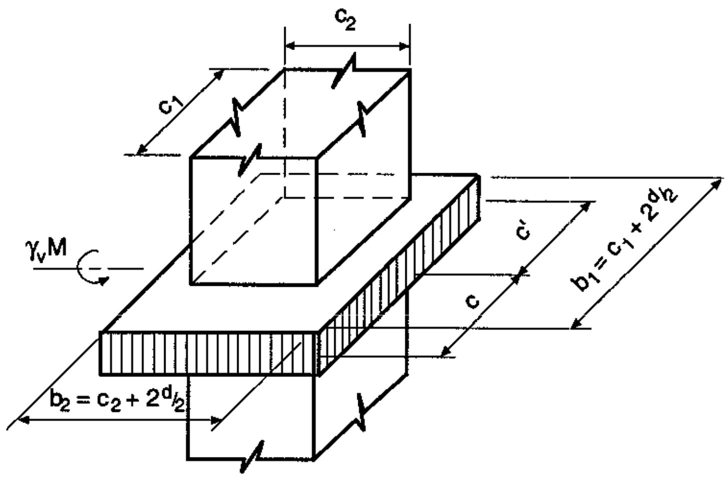
Table 4-7 Properties of Critical Transfer Section–Interior Column



Concrete area of critical section:

$A_c = 2(b_1 + b_2)d$

Modulus of critical section:

$\frac{J}{c} = \frac{J}{c'} = [b_1 d(b_1 + 3b_2) + d^3]/3$

where:

$c = c' = b_1/2$

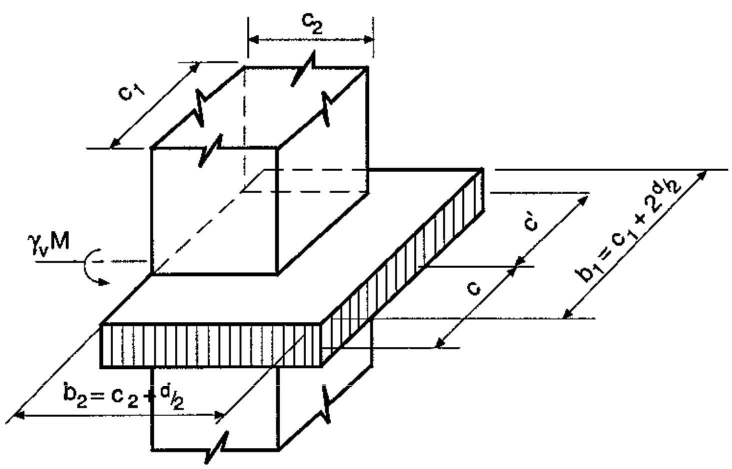
COL. SIZE	h = 5 in., d = 3-3/4 in.			h = 5-1/2 in., d = 4-1/4 in.			h = 6 in., d = 4-3/4 in.			h = 6-1/2 in., d = 5-1/4 in.		
	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.
10x10	206.3	963	6.88	242.3	1176	7.13	280.3	1414	7.38	320.3	1676	7.63
12x12	236.3	1258	7.88	276.3	1522	8.13	318.3	1813	8.38	362.3	2131	8.63
14x14	266.3	1593	8.88	310.3	1913	9.13	356.3	2262	9.38	404.3	2642	9.63
16x16	296.3	1968	9.88	344.3	2349	10.13	394.3	2763	10.38	446.3	3209	10.63
18x18	326.3	2383	10.88	378.3	2831	11.13	432.3	3314	11.38	488.3	3832	11.63
20x20	356.3	2838	11.88	412.3	3358	12.13	470.3	3915	12.38	530.3	4511	12.63
22x22	386.3	3333	12.88	446.3	3930	13.13	508.3	4568	13.38	572.3	5246	13.63
24x24	416.3	3868	13.88	480.3	4548	14.13	546.3	5271	14.38	614.3	6037	14.63

COL. SIZE	h = 7 in., d = 5-3/4 in.			h = 7-1/2 in., d = 6-1/4 in.			h = 8 in., d = 6-3/4 in.			h = 8-1/2 in., d = 7-1/4 in.		
	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.
10x10	362.3	1965	7.88	406.3	2282	8.13	452.3	2628	8.38	500.3	3003	8.63
12x12	408.3	2479	8.88	456.3	2857	9.13	506.3	3267	9.38	558.3	3709	9.63
14x14	454.3	3054	9.88	506.3	3499	10.13	560.3	3978	10.38	616.3	4492	10.63
16x16	500.3	3690	10.88	556.3	4207	11.13	614.3	4761	11.38	674.3	5352	11.63
18x18	546.3	4388	11.88	606.3	4982	12.13	668.3	5616	12.38	732.3	6290	12.63
20x20	592.3	5147	12.88	656.3	5824	13.13	722.3	6543	13.38	790.3	7305	13.63
22x22	638.3	5967	13.88	706.3	6732	14.13	776.3	7542	14.38	848.3	8397	14.63
24x24	684.3	6849	14.88	756.3	7707	15.13	830.3	8613	15.38	906.3	9567	15.63

Table 4-7 continued

COL. SIZE	h = 9 in., d = 7-3/4 in.			h = 9-1/2 in., d = 8-1/4 in.			h = 10 in., d = 8-3/4 in.		
	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.
10x10	550.3	3411	8.88	602.3	3851	9.13	656.3	4325	9.38
12x12	612.3	4186	9.88	668.3	4698	10.13	726.3	5247	10.38
14x14	674.3	5043	10.88	734.3	5633	11.13	796.3	6262	11.38
16x16	736.3	5984	11.88	800.3	6656	12.13	866.3	7370	12.38
18x18	798.3	7007	12.88	866.3	7767	13.13	936.3	8572	13.38
20x20	860.3	8112	13.88	932.3	8966	14.13	1006.3	9867	14.38
22x22	922.3	9301	14.88	998.3	10253	15.13	1076.3	11255	15.38
24x24	984.3	10572	15.88	1064.3	11628	16.13	1146.3	12737	16.38

Table 4-8 Properties of Critical Transfer Section—Edge Column—Bending Parallel to Edge



Concrete area of critical section:

$A_c = (b_1 + 2b_2)d$

Modulus of critical section:

$\frac{J}{c} = \frac{J}{c'} = [b_1 d(b_1 + 6b_2) + d^3]/6$

where:

$c = c' = b_1/2$

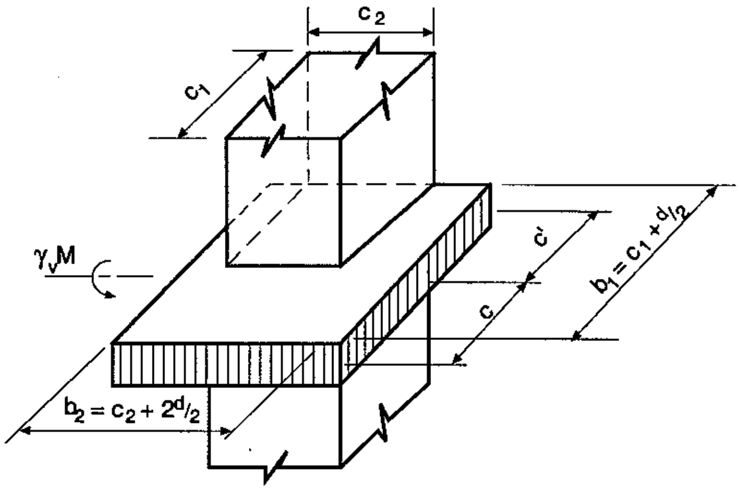
COL. SIZE	h = 5 in., d = 3-3/4 in.			h = 5-1/2 in., d = 4-1/4 in.			h = 6 in., d = 4-3/4 in.			h = 6-1/2 in., d = 5-1/4 in.		
	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.
10x10	140.6	739	6.88	163.6	891	7.13	187.6	1057	7.38	212.6	1238	7.63
12x12	163.1	983	7.88	189.1	1175	8.13	216.1	1384	8.38	244.1	1609	8.63
14x14	185.6	1262	8.88	214.6	1499	9.13	244.6	1755	9.38	275.6	2029	9.63
16x16	208.1	1576	9.88	240.1	1863	10.13	273.1	2170	10.38	307.1	2497	10.63
18x18	230.6	1926	10.88	265.6	2267	11.13	301.6	2629	11.38	338.6	3015	11.63
20x20	253.1	2310	11.88	291.1	2710	12.13	330.1	3133	12.38	370.1	3581	12.63
22x22	275.6	2729	12.88	316.6	3192	13.13	358.6	3681	13.38	401.6	4197	13.63
24x24	298.1	3183	13.88	342.1	3715	14.13	387.1	4274	14.38	433.1	4861	14.63

Table 4-8 continued

COL. SIZE	h = 7 in., d = 5-3/4 in.			h = 7-1/2 in., d = 6-1/4 in.			h = 8 in., d = 6-3/4 in.			h = 8-1/2 in., d = 7-1/4 in.		
	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.
10x10	238.6	1435	7.88	265.6	1649	8.13	322.6	2127	8.63	293.6	1879	8.38
12x12	273.1	1852	8.88	303.1	2113	9.13	366.1	2692	9.63	334.1	2393	9.38
14x14	307.6	2322	9.88	340.6	2635	10.13	409.6	3325	10.63	374.6	2969	10.38
16x16	342.1	2846	10.88	378.1	3216	11.13	453.1	4025	11.63	415.1	3609	11.38
18x18	376.6	3423	11.88	415.6	3855	12.13	496.6	4793	12.63	455.6	4311	12.38
20x20	411.1	4054	12.88	453.1	4552	13.13	540.1	5628	13.63	496.1	5077	13.38
22x22	445.6	4739	13.88	490.6	5308	14.13	583.6	6531	14.63	536.6	5905	14.38
24x24	480.1	5477	14.88	528.1	6122	15.13	627.1	7502	15.63	577.1	6797	15.38

COL. SIZE	h = 9 in., d = 7-3/4 in.			h = 9-1/2 in., d = 8-1/4 in.			h = 10 in., d = 8-3/4 in.		
	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.	A_c in.^{2}	J/c = J/c' in.^{3}	c = c' in.
10x10	352.6	2393	8.88	383.6	2678	9.13	415.6	2983	9.38
12x12	399.1	3011	9.88	433.1	3351	10.13	468.1	3713	10.38
14x14	445.6	3702	10.88	482.6	4101	11.13	520.6	4524	11.38
16x16	492.1	4464	11.88	532.1	4928	12.13	573.1	5417	12.38
18x18	538.6	5299	12.88	581.6	5832	13.13	625.6	6392	13.38
20x20	585.1	6207	13.88	613.1	6814	14.13	678.1	7449	14.38
22x22	631.6	7187	14.88	680.6	7872	15.13	730.6	8587	15.38
24x24	678.1	8239	15.88	730.1	9007	16.13	783.1	9807	16.38

Table 4-9 Properties of Critical Transfer Section—Edge Column—Bending Perpendicular to Edge



Concrete area of critical section:

$A_c = (2b_1 + b_2)d$

Modulus of critical section:

$\frac{J}{c} = [2b_1 d(b_1 + 2b_2) + d^3(2b_1 + b_2)/b_1]/6$

$\frac{J}{c'} = [2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)]/6(b_1 + b_2)$

where:

$c = b_1^2/(2b_1 + b_2)$

$c' = b_1(b_1 + b_2)/(2b_1 + b_2)$

COL. SIZE	h = 5 in., d = 3-3/4 in.					h = 5-1/2 in., d = 4-1/4 in.					h = 6 in., d = 4-3/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	140.6	612	284	3.76	8.11	163.6	738	339	3.82	8.31	187.6	878	400	3.88	8.50
12x12	163.1	815	381	4.43	9.45	189.1	973	453	4.48	9.64	216.1	1146	529	4.54	9.83
14x14	185.6	1047	494	5.09	10.78	214.6	1242	583	5.15	10.98	244.6	1453	677	5.21	11.17
16x16	208.1	1309	622	5.76	12.12	240.1	1545	730	5.81	12.31	273.1	1798	844	5.87	12.50
18x18	230.6	1602	765	6.42	13.45	265.6	1882	894	6.48	13.64	301.6	2181	1030	6.54	13.84
20x20	253.1	1924	923	7.09	14.79	291.1	2253	1075	7.15	14.98	330.1	2602	1235	7.20	15.17
22x22	275.6	2277	1095	7.76	16.12	316.6	2658	1273	7.81	16.31	358.6	3061	1459	7.87	16.51
24x24	298.1	2659	1283	8.42	17.45	342.1	3097	1488	8.48	17.65	387.1	3558	1702	8.54	17.84

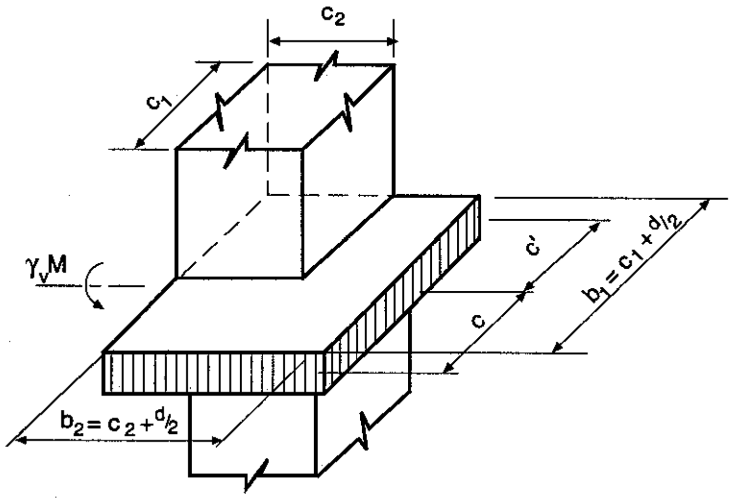
COL. SIZE	h = 6-1/2 in., d = 5-1/4 in.					h = 7 in., d = 5-3/4 in.					h = 7-1/2 in., d = 6-1/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	212.6	1030	467	3.94	8.69	238.6	1197	538	3.99	8.88	265.6	1379	616	4.05	9.07
12x12	244.1	1334	612	4.60	10.03	273.1	1537	701	4.66	10.22	303.1	1757	796	4.72	10.41
14x14	275.6	1680	779	5.26	11.36	307.6	1924	886	5.32	11.55	340.6	2185	1001	5.38	11.74
16x16	307.1	2068	966	5.93	12.70	342.1	2356	1095	5.99	12.89	378.1	2664	1231	6.05	13.08
18x18	338.6	2498	1174	6.60	14.03	376.6	2835	1326	6.65	14.22	415.6	3192	1486	6.71	14.41
20x20	370.1	2970	1404	7.26	15.36	411.1	3360	1581	7.32	15.56	453.1	3771	1766	7.38	15.75
22x22	401.6	3485	1654	7.93	16.70	445.6	3931	1858	7.98	16.89	490.6	4400	2071	8.04	17.08
24x24	433.1	4041	1926	8.59	18.03	480.1	4548	2158	8.65	18.23	528.1	5078	2401	8.71	18.42

Table 4-9 continued

COL. SIZE	h = 8 in., d = 6-3/4 in.					h = 8-1/2 in., d = 7-1/4 in.					h = 9 in., d = 7-3/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	293.6	1577	700	4.11	9.26	322.6	1792	791	4.17	9.45	352.6	2024	888	4.23	9.64
12x12	334.1	1994	898	4.78	10.60	366.1	2249	1008	4.83	10.79	399.1	2523	1124	4.89	10.98
14x14	374.6	2465	1124	5.44	11.94	409.6	2765	1253	5.50	12.13	445.6	3084	1391	5.56	12.32
16x16	415.1	2991	1376	6.10	13.27	453.1	3338	1528	6.16	13.46	492.1	3707	1689	6.22	13.65
18x18	455.6	3571	1655	6.77	14.61	496.6	3970	1832	6.83	14.80	538.6	4393	2018	6.89	14.99
20x20	496.1	4204	1961	7.43	15.94	540.1	4661	2164	7.49	16.13	585.1	5141	2378	7.55	16.33
22x22	536.6	4892	2294	8.10	17.28	583.6	5409	2526	8.16	17.47	631.6	5951	2768	8.21	17.66
24x24	577.1	5634	2654	8.76	18.61	627.1	6216	2916	8.82	18.80	678.1	6823	3190	8.88	18.99

COL. SIZE	h = 9-1/2 in., d = 8-1/4 in.					h = 10 in., d = 8-3/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	383.6	2275	992	4.29	9.83	415.6	2544	1104	4.35	10.02
12x12	433.1	2816	1248	4.95	11.17	468.1	3129	1380	5.01	11.36
14x14	482.6	3424	1537	5.62	12.51	520.6	3785	1691	5.67	12.70
16x16	532.1	4098	1858	6.28	13.85	573.1	4511	2037	6.34	14.04
18x18	581.6	4839	2213	6.94	15.18	625.6	5308	2418	7.00	15.37
20x20	631.1	5646	2601	7.61	16.52	678.1	6176	2834	7.67	16.71
22x22	680.6	6519	3021	8.27	17.85	730.6	7113	3284	8.33	18.04
24x24	730.1	7458	3474	8.94	19.19	783.1	8121	3770	9.00	19.38

Table 4-10 Properties of Critical Transfer Section—Corner Column



Concrete area of critical section:

$A_c = (b_1 + b_2)d$

Modulus of critical section:

$\frac{J}{c} = [b_1 d(b_1 + 4b_2) + d^3(b_1 + b_2)/b_1]/6$

$\frac{J}{c'} = [b_1^2 d(b_1 + 4b_2) + d^3(b_1 + b_2)]/6(b_1 + 2b_2)$

where:

$c = b_1^2/2(b_1 + b_2)$
$c' = b_1(b_1 + 2b_2)/2(b_1 + b_2)$

COL. SIZE	h = 5 in., d = 3-3/4 in.					h = 5-1/2 in., d = 4-1/4 in.					h = 6 in., d = 4-3/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	89.1	458	153	2.97	8.91	103.1	546	182	3.03	9.09	117.6	642	214	3.09	9.28
12x12	104.1	619	206	3.47	10.41	120.1	732	244	3.53	10.59	136.6	854	285	3.59	10.78
14x14	119.1	805	268	3.97	11.91	137.1	946	315	4.03	12.09	155.6	1097	366	4.09	12.28
16x16	134.1	1016	339	4.47	13.41	154.1	1189	396	4.53	13.59	174.6	1372	457	4.59	13.78
18x18	149.1	1252	417	4.97	14.91	171.1	1460	487	5.03	15.09	193.6	1679	560	5.09	15.28
20x20	164.1	1513	504	5.47	16.41	188.1	1759	586	5.53	16.59	212.6	2017	672	5.59	16.78
22x22	179.1	1799	600	5.97	17.91	205.1	2087	696	6.03	18.09	231.6	2388	796	6.09	18.28
24x24	194.1	2110	703	6.47	19.41	222.1	2443	814	6.53	19.59	250.6	2789	930	6.59	19.78

COL. SIZE	h = 6-1/2 in., d = 5-1/4 in.					h = 7 in., d = 5-3/4 in.					h = 7-1/2 in., d = 6-1/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	132.6	746	249	3.16	9.47	148.1	858	286	3.22	9.66	164.1	979	326	3.28	9.84
12x12	153.6	984	328	3.66	10.97	171.1	1124	375	3.72	11.16	189.1	1273	424	3.78	11.34
14x14	174.6	1257	419	4.16	12.47	194.1	1428	476	4.22	12.66	214.1	1609	536	4.28	12.84
16x16	195.6	1566	522	4.66	13.97	217.1	1770	590	4.72	14.16	239.1	1986	662	4.78	14.34
18x18	216.6	1909	636	5.16	15.47	240.1	2151	717	5.22	15.66	264.1	2406	802	5.28	15.84
20x20	237.6	2288	763	5.66	16.97	263.1	2571	857	5.72	17.16	289.1	2867	956	5.78	17.34
22x22	258.6	2701	900	6.16	18.47	286.1	3028	1009	6.22	18.66	314.1	3369	1123	6.28	18.84
24x24	279.6	3150	1050	6.66	19.97	309.1	3524	1175	6.72	20.16	339.1	3913	1304	6.78	20.34

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Table 4-10 continued

COL. SIZE	h = 8 in., d = 6-3/4 in.					h = 8-1/2 in., d = 7-1/4 in.					h = 9 in., d = 7-3/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	180.6	1109	370	3.34	10.03	197.6	1249	416	3.41	10.22	215.1	1398	466	3.47	10.41
12x12	207.6	1432	477	3.84	11.53	226.6	1602	534	3.91	11.72	246.1	1783	594	3.97	11.91
14x14	234.6	1801	600	4.34	13.03	255.6	2004	668	4.41	13.22	277.1	2219	740	4.47	13.41
16x16	261.6	2214	738	4.84	14.53	284.6	2454	818	4.91	14.72	308.1	2706	902	4.97	14.91
18x18	288.6	2673	891	5.34	16.03	313.6	2952	984	5.41	16.22	339.1	3246	1082	5.47	16.41
20x20	315.6	3176	1059	5.84	17.53	342.6	3499	1166	5.91	17.72	370.1	3837	1279	5.97	17.91
22x22	342.6	3724	1241	6.34	19.03	371.6	4094	1365	6.41	19.22	401.1	4479	1493	6.47	19.41
24x24	369.6	4318	1439	6.84	20.53	400.6	4738	1579	6.91	20.72	432.1	5173	1724	6.97	20.91

COL. SIZE	h = 9-1/2 in., d = 8-1/4 in.					h = 10 in., d = 8-3/4 in.				
	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.	A_c in.^{2}	J/c in.^{3}	J/c' in.^{3}	c in.	c' in.
10x10	233.1	1559	520	3.53	10.59	251.6	1730	577	3.59	10.78
12x12	266.1	1975	658	4.03	12.09	286.6	2178	726	4.09	12.28
14x14	299.1	2446	815	4.53	13.59	321.6	2685	895	4.59	13.78
16x16	332.1	2972	991	5.03	15.09	356.6	3250	1083	5.09	15.28
18x18	365.1	3553	1184	5.53	16.59	391.6	3874	1291	5.59	16.78
20x20	398.1	4189	1396	6.03	18.09	426.6	4556	1519	6.09	18.28
22x22	431.1	4879	1626	6.53	19.59	461.6	5296	1765	6.59	19.78
24x24	464.1	5625	1875	7.03	21.09	496.6	6094	2031	7.09	21.28

References

4.1 *Notes on ACI 318-89*, Chapter 8: Deflections, Fifth Edition, EB070, Portland Cement Association, Skokie, Illinois, 1990.

4.2 "Aspects of Design of Reinforced Concrete Flat Plate Slab Systems", by S. K. Ghosh, *Analysis and Design of High-Rise Concrete Buildings*, SP-97, American Concrete Institute, Detroit, Michigan, 1985, pp. 139-157.

Chapter 5

Simplified Design for Columns

5.1 INTRODUCTION

Use of high-strength materials has had a significant effect on the design of concrete columns. Increased use of high-strength concretes has resulted in columns that are smaller in size and, therefore, are more slender. Consequently, in certain situations, slenderness effects must be considered, resulting in designs that are more complicated than when these effects may be neglected.

For buildings with adequate shearwalls, columns may be designed for gravity loads only. However, in some structures—especially low-rise buildings—it may not be desirable or economical to include shearwalls. In these situations, the columns must be designed to resist both gravity and lateral loads. In either case, it is important to be able to distinguish between a column that is slender and one that is not. A simplified design procedure is outlined in this chapter, which should be applicable to most columns. Design aids are given to assist the engineer in designing columns within the limitations stated.

5.2 DESIGN CONSIDERATIONS

5.2.1 Column Size

The total loads on columns are directly proportional to the bay sizes (i.e., tributary areas). Larger bay sizes mean more load to each column and, thus, larger column sizes. Bay size is often dictated by the architectural and functional requirements of the building. Large bay sizes may be required to achieve maximum unobstructed floor space. The floor system used may also dictate the column spacing. For example, the economical use of a flat plate floor system usually requires columns that are spaced closer than those supporting a pan joist floor system. Architecturally, larger column sizes can give the impression of solidity and strength, whereas smaller columns can express slender grace. Aside from architectural considerations, it is important that the columns satisfy all applicable strength requirements of the ACI Code, and at the same time, be economical. Minimum column size and concrete cover to reinforcement may be governed by fire-resistance criteria (see Chapter 10, Tables 10-2 and 10-6).

5.2.2 Column Constructability

Columns must be sized not only for adequate strength, but also for constructability. For proper concrete placement and consolidation, the engineer must select column size and reinforcement to ensure that the reinforcement is not congested. Bar lap splices and locations of bars in beams and slabs framing into the column must be considered. Columns designed with a smaller number of larger bars usually improves constructability.

5.2.3 Column Economics

Concrete is more cost effective than reinforcement for carrying compressive axial load; thus, it is more economical to use larger column sizes with lesser amounts of reinforcement. Also, columns with a smaller number of larger bars are usually more economical than columns with a larger number of smaller bars.

Reuse of column forms from story level to story level results in significant savings. It is economically unsound to vary column size to suit the load on each story level. It is much more economical to use the same column size for the entire building height, and to vary only the longitudinal reinforcement as required. In taller buildings, the concrete strength is usually varied along the building height as well.

5.3 PRELIMINARY COLUMN SIZING

It is necessary to select a preliminary column size for cost estimating and/or frame analysis. The initial selection can be very important when considering overall design time. In general, a preliminary column size should be determined using a low percentage of reinforcement; it is then possible to provide any additional reinforcement required for the final design (including applicable slenderness effects) without having to change the column size. Columns which have reinforcement ratios in the range of 1% to 2% will usually be the most economical.

The design chart presented in Fig. 5-1, based on ACI Eq. (10-2), can be used for nonslender tied columns loaded at an eccentricity of no more than 0.1h, where h is the size of the column. Design axial load strengths $\phi P_{n(max)}$ for square column sizes from 10 in. to 24 in. with reinforcement ratios between 1 and 8% are presented. For other column sizes and shapes, and material strengths, similar design charts based on ACI Eq. (10-1) or (10-2) can be easily developed.

This design chart will provide quick estimates for a column size required to support a factored load P_u within the allowable limits of the reinforcement ratio (ACI 10.9). Using the total tributary factored load P_u for the lowest story of a multistory column stack, a column size should be selected with a low percentage of reinforcement. This will allow some leeway to increase the amount of steel for the final design, if required.

5.4 SIMPLIFIED DESIGN FOR COLUMNS

5.4.1 Simplified Design Charts

Numerous design aids and computer programs are available for determining the size and reinforcement of columns subjected to axial forces and/or moments. Tables, charts, and graphs provide design data for a wide variety of column sizes and shapes, reinforcement layouts, load eccentricities and other variables. These design aids eliminate the necessity for making complex and repetitious calculations to determine the strengths of columns, as preliminarily sized. The design aids presented in References 5.1 and 5.2 are widely used. In addition, extensive column load tables are available in the CRSI Handbook.[5.3] Each publication presents the design data in a somewhat different format; however, the accompanying text in each reference readily explains the method of use.

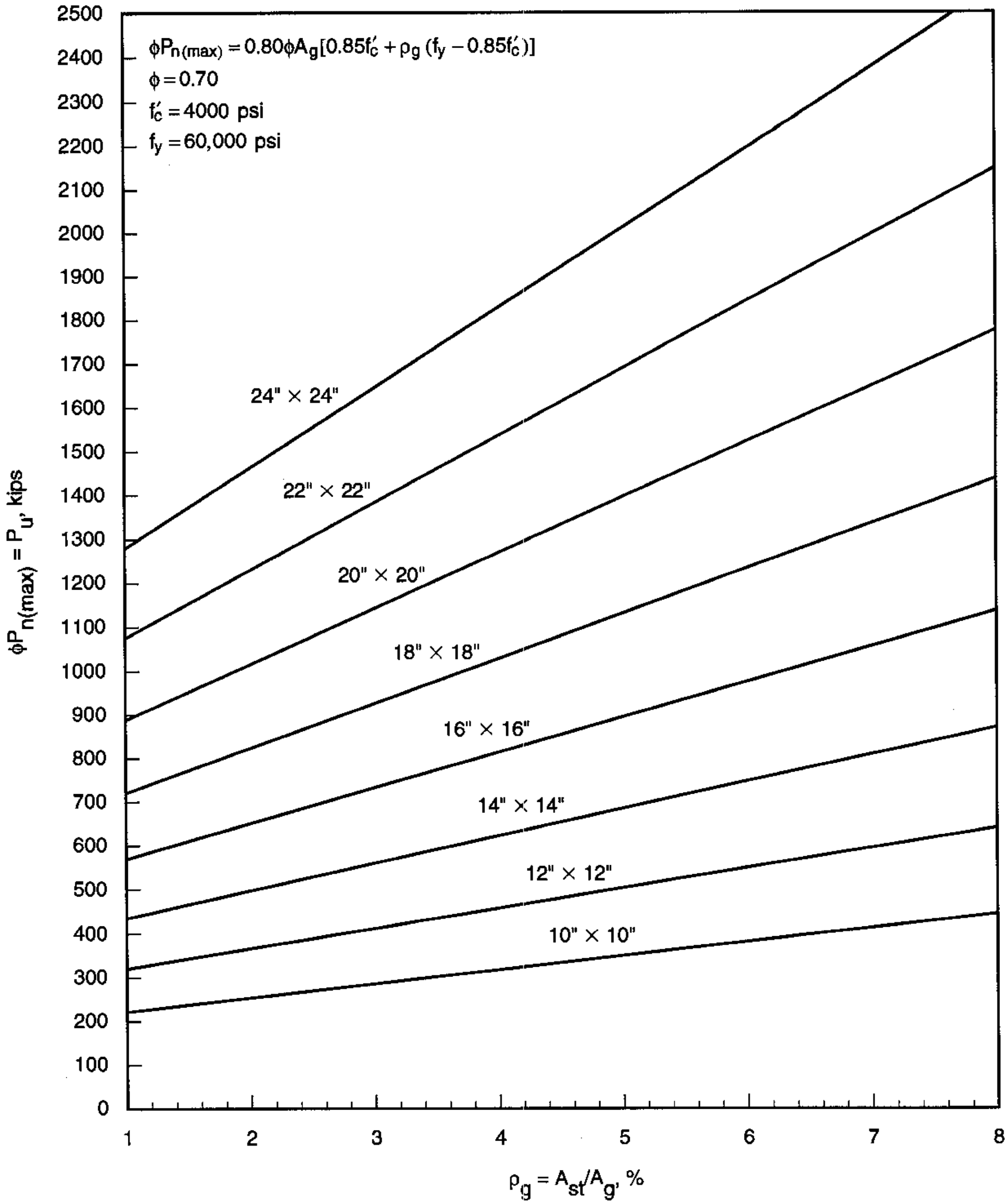


Figure 5-1 Design Chart for Nonslender, Square Tied Columns

PCA's computer program PCACOL may also be used to design or investigate rectangular or circular column sections with any reinforcement layout or pattern.[5.4] The column section may be subjected to axial compressive loads acting alone or in combination with uniaxial or biaxial bending. Slenderness effects may also be considered, if desired. PCACOL will output all critical load values and the interaction diagram (or, moment contour) for any column section.

In general, columns must be designed for the combined effects of axial load and bending moment. As noted earlier, appreciable bending moments due to wind loads may occur in the columns of buildings without shearwalls. To allow rapid selection of column size and reinforcement for a factored axial load P_u and bending moment M_u, Figs. 5-16 through 5-23 are included at the end of this chapter. All design charts are based on f'_c = 4000 psi and f_y = 60,000 psi, and are valid for square, tied, nonslender columns with symmetrical bar arrangements as shown in Fig. 5-2. The number in parentheses next to the number of reinforcing bars is the reinforcement ratio $\rho_g = A_{st}/A_g$ where A_{st} is the total area of the longitudinal bars and A_g is the gross area of the column section. A clear cover of 1.5 in. to the ties was used (ACI 7.7.1); also used was #3 ties with longitudinal bars #10 and smaller and #4 ties with #11 bars (ACI 7.10.5).

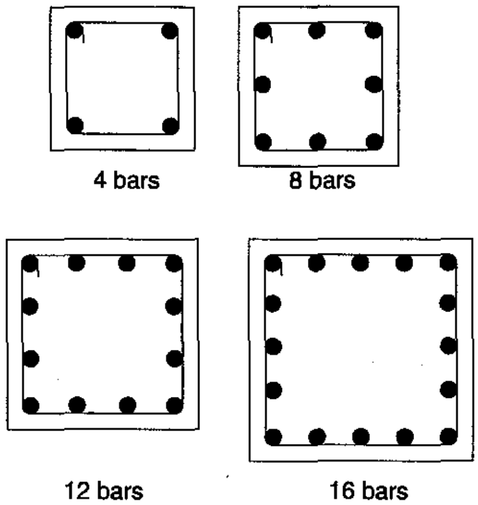


Figure 5-2 Bar Arrangements for Column Design Charts

For simplicity, each design curve is plotted with straight lines connecting a number of points corresponding to certain transition stages. In general, the transition stages are defined as follows (see Fig. 5-3):

Stage 1: Pure compression (no bending moment)

Stage 2: Stress in reinforcement closest to tension face = 0 ($f_s = 0$)

Stage 3: Stress in reinforcement closest to tension face = 0.5 f_y ($f_s = 0.5\ f_y$)

Stage 4: Balanced point; stress in reinforcement closest to tension face = f_y ($f_s = f_y$)

Stage 5: Pure bending (no axial load)

Note that Stages 2 and 3 are used to determine which type of lap splice is required for a given load combination (ACI 12.17). In particular, for load combinations falling within Zone 1, compression lap splices are allowed, since all of the bars are in compression. In Zone 2, either Class A (half or fewer of the bars spliced at one location) or Class B (more than one-half of the bars spliced at one location) tension lap splices must be used. Class B tension lap splices are required for load combinations falling within Zone 3.

Simplified equations can be derived to obtain the critical point on the design interaction diagram corresponding to each transition stage. The following equations are valid within the limitations stated above:

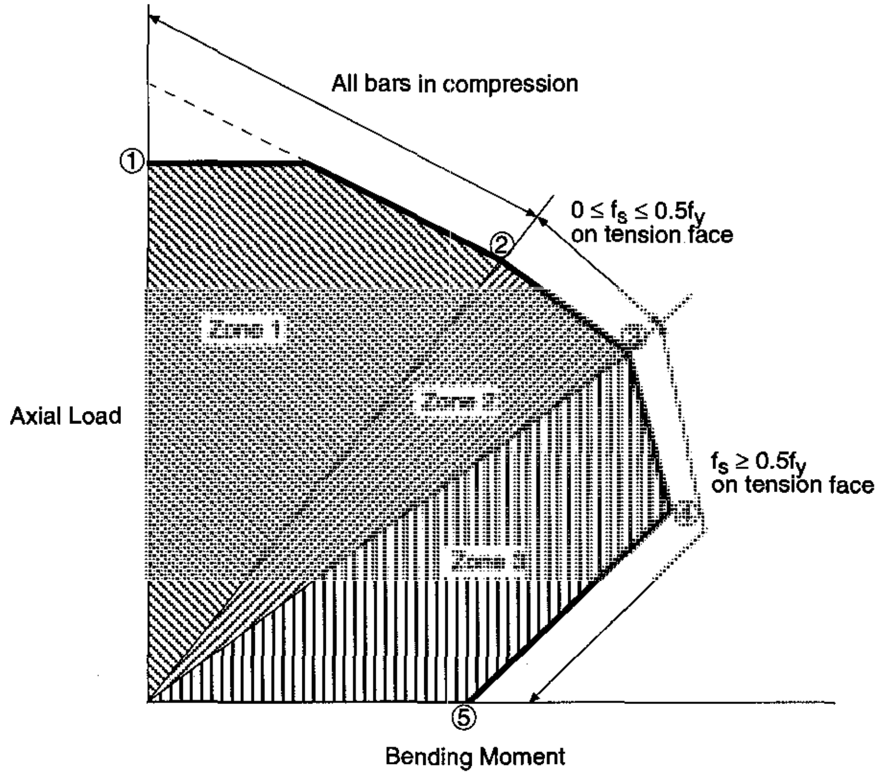


Figure 5-3 Transition Stages on Interaction Diagram

(1) Point 1 (see Fig. 5-1):

$$\phi P_{n(max)} = 0.80\phi[0.85 f'_c(A_g - A_{st}) + f_y A_{st}] \qquad \text{ACI Eq. (10-2)}$$
$$= 0.80\phi A_g[0.85 f'_c + \rho_g(f_y - 0.85 f'_c)]$$

where A_g = gross area of column, in.^{2}
A_{st} = total area of longitudinal reinforcing bars, in.^{2}
ρ_g = A_{st}/A_g
ϕ = strength reduction factor = 0.70

(2) Points 2-4 (see Fig. 5-4):

$$\phi P_n = \phi[C_1 h d_1 + 87 \sum_{i=1}^{n} A_{si}(1 - C_2 \frac{d_i}{d_1})] \qquad (5\text{-}1)$$

$$\phi M_n = \phi[0.5C_1 h d_1(h - C_3 d_1) + 87 \sum_{i=1}^{n} A_{si}(1 - C_2\frac{d_i}{d_1})(\frac{h}{2} - d_i)]/12 \qquad (5\text{-}2)$$

where h = column dimension in the direction of bending (width or depth), in.
d_1 = distance from compression face to centroid of reinforcing steel in layer 1 (layer closest to tension face), in.
d_i = distance from compression face to centroid of reinforcing steel in layer i, in.
A_{si} = total steel area in layer i, in.^{2}
n = total number of layers of reinforcement
ϕ = strength reduction factor = 0.7
C_1, C_2, C_3 = constants given in Table 5-1

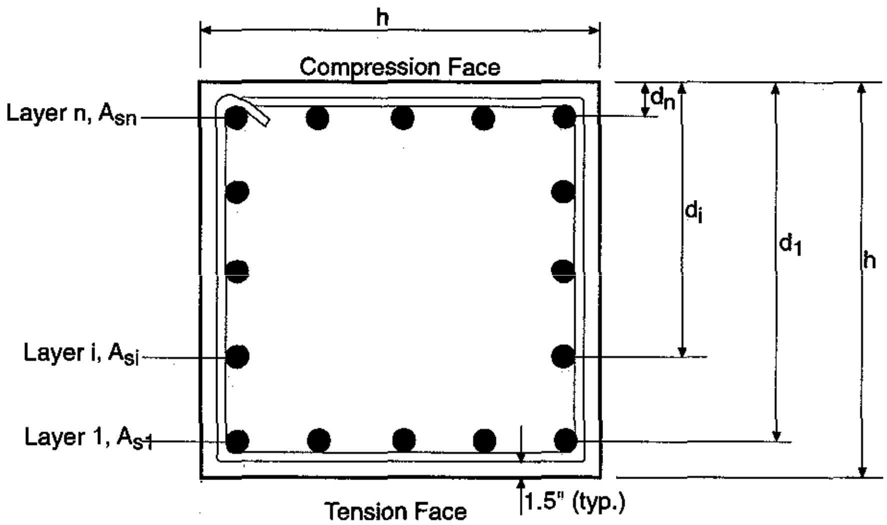


Figure 5-4 Notation for Eqs. (5-1) and (5-2)

Table 5-1 Constants for Points 2–4

Point No.	C_1	C_2	C_3
2	2.89	1.00	0.85
3	2.14	1.35	0.63
4	1.70	1.69	0.50

Values of ϕP_n obtained from the above equations are in kips and ϕM_n are in ft-kips. To ensure that the stress in the reinforcing bars is less than or equal to f_y, the quantity $(1\text{-}C_2d_i/d_1)$ must always be taken less than or equal to 60/87 = 0.69.

(3) Point 5:

For columns with 2 or 3 layers of reinforcement:

$$\phi M_n = 4A_{s1}d_1$$

For columns with 4 or 5 layers of reinforcement:

$$\phi M_n = 4(A_{s1} + A_{s2})(d_1 - \frac{s}{2})$$

where s = center to center spacing of the bars

In both equations, $\phi = 0.90$; also, A_{s1} and A_{s2} are in.^{2}, d_1 and s are in in., and ϕM_n is in ft-kips.

The simplified equations for Points 2–4 will produce values of ϕP_n and ϕM_n approximately 3% larger than the exact values (at most). The equations for Point 5 will produce conservative values of ϕM_n for the majority of cases. For columns subjected to small axial loads and large bending moments, a more precise investigation into the adequacy of the section should be made because of the approximate shape of the simplified interaction diagram in the tension-controlled region. However, for typical building columns, load combinations of this type are rarely encountered.

For a column with a larger cross-section than required for loads, a reduced effective area not less than one-half of the total area may be used to determine the minimum reinforcement and the design capacity (ACI 10.8.4).

Essentially this means that a column of sufficient size can be designed to carry the design loads, and concrete added around the designed section without having to increase the amount of longitudinal reinforcement to satisfy the minimum requirement in ACI 10.9.1. Thus, in these situations, the minimum steel percentage, based on actual gross cross-sectional area of column, may be taken less than 0.010, with a lower limit of 0.005 (the exact percentage will depend on the factored loads and the dimensions of the column). It is important to note that the additional concrete must not be considered as carrying any portion of the load, but must be considered when computing member stiffness (ACI R10.8.4).

Additional design charts for other column sizes and material strengths can obviously be developed. For rectangular or round columns, the graphs presented in Reference 5.2 may be used; these graphs are presented in a nondimensionalized format and cover an extensive range of column shapes and material strengths. Also, the CRSI Handbook[5.3] gives extensive design data for square, rectangular, and round column sections.

5.4.1.1 Example: Construction of Simplified Design Chart

To illustrate the simplified procedure for constructing column interaction diagrams, determine the points corresponding to the various transition stages for an 18 × 18 in. column reinforced with 8-#9 bars, as shown in Fig. 5-5.

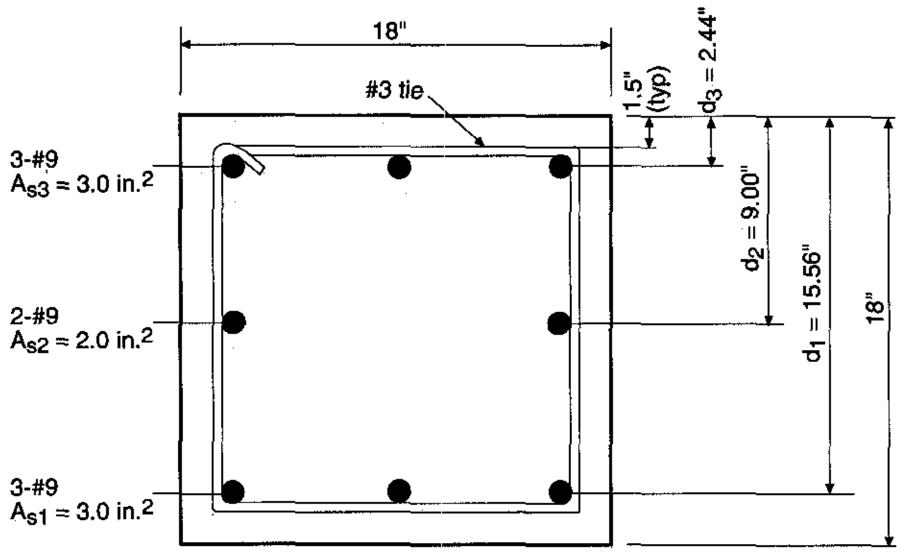


Figure 5-5 Column Cross-Section for Example Problem

(1) Point 1 (pure compression):

$$\rho_g = \frac{8.0}{18 \times 18} = 0.0247$$

$$\phi P_{n(max)} = (0.80 \times 0.70 \times 324)[(0.85 \times 4) + 0.0247(60-(0.85 \times 4))]$$
$$= 870 \text{ kips}$$

(2) Point 2 ($f_{s1} = 0$):

Using Fig. 5-5 and Table 5-1:

Layer 1: $1 - C_2 \frac{d_1}{d_1} = 1 - 1 = 0$

Layer 2: $1 - C_2\dfrac{d_2}{d_1} = 1 - 1 \times \left(\dfrac{9.00}{15.56}\right) = 0.42$

Layer 3: $1 - C_2\dfrac{d_3}{d_1} = 1 - 1 \times \left(\dfrac{2.44}{15.56}\right) = 0.84 > 0.69$

$1\text{-}C_2d_3/d_1$ being greater than 0.69 in layer 3 means that the steel in layer 3 has yielded; therefore, use $1\text{-}C_2d_3/d_1 = 0.69$.

$$\phi P_n = 0.7[C_1hd_1 + 87 \sum_{i=1}^{3} A_{si}(1 - C_2\frac{d_i}{d_1})]$$

$$= 0.7\{(2.89 \times 18 \times 15.56) + 87\ [(3 \times 0) + (2 \times 0.42) + (3 \times 0.69)]\}$$

$$= 0.7\ (809.4 + 253.2)$$

$$= 744 \text{ kips}$$

$$\phi M_n = 0.7[0.5C_1hd_1(h - C_3d_1) + 87 \sum_{i=1}^{3} A_{si}(1 - C_2\frac{d_i}{d_1})(\frac{h}{2} - d_i)]/12$$

$$= 0.7\{(0.5 \times 2.89 \times 18 \times 15.56)[18 - (0.85 \times 15.56)]$$

$$+ 87[(3.0 \times 0(9 - 15.56)) + (2.0 \times 0.42(9 - 9)) + (3.0 \times 0.69(9 - 2.44))]\}/12$$

$$= 0.7\ (1932.1 + 1181.4)/12$$

$$= 182 \text{ ft-kips}$$

(3) Point 3 ($f_{s1} = 0.5\ f_y$):

In this case, $C_1 = 2.14$, $C_2 = 1.35$, and $C_3 = 0.63$ (Table 5-1)

Layer 1: $1 - C_2\dfrac{d_1}{d_1} = 1 - 1.35 = -0.35$

Layer 2: $1 - C_2\dfrac{d_2}{d_1} = 1 - 1.35\left(\dfrac{9.00}{15.56}\right) = 0.22$

Layer 3: $1 - C_3\dfrac{d_3}{d_1} = 1 - 1.35\left(\dfrac{2.44}{15.56}\right) = 0.79 > 0.69$ Use 0.69

$$\phi P_n = 0.7\{(2.14 \times 18 \times 15.56) + 87[(3(-0.35)) + (2 \times 0.22) + (3 \times 0.69)]\}$$

$$= 0.7(599.4 + 127.0) = 508 \text{ kips}$$

$$\phi M_n = 0.7\{(0.5 \times 2.14 \times 18 \times 15.56)[18 - (0.63 \times 15.56)]$$

$$+ 87\ [(3.0(-0.35) \times (9 - 15.56)) + 0 + (3.0 \times 0.69(9 - 2.44))]\}/12$$

$$= 0.7\ (2456.6 + 1780.7)/12 = 247 \text{ ft-kips}$$

(4) Point 4 ($f_{s1} = f_y$):

In this case, $C_1 = 1.70$, $C_2 = 1.69$, $C_3 = 0.50$

Similar calculations yield the following:

$$\phi P_n = 336 \text{ kips}$$

$$\phi M_n = 280 \text{ ft-kips}$$

(5) Point 5 (pure bending):

For columns with 3 layers of reinforcement:

$$\phi M_n = 4A_{s1}d_1 = 4 \times 3.0 \times 15.56 = 187 \text{ ft-kips}$$

Each of these points, connected by straight dotted lines, is shown in Fig. 5-6. The solid line represents the exact interaction diagram determined from PCACOL. As can be seen, the simplified interaction diagram compares well with the one from PCACOL except in the region where the axial load is small and the bending moments is large; there, the simplified diagram is conservative. However, as noted earlier, typical building columns will rarely have a load combination in this region. Note that PCACOL also gives the portion of the interaction diagram for tensile axial loads (negative values of ϕP_n) and bending moments.

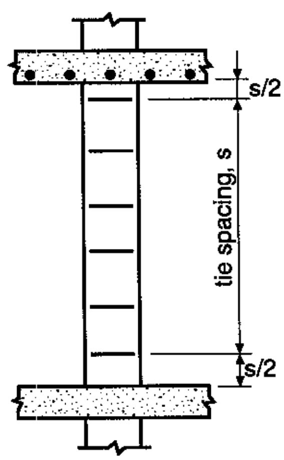
Simplified interaction diagrams for all of the other columns in Figs. 5-16 through 5-23 will compare just as well with the exact interaction diagrams; the largest discrepancies will occur in the region near pure bending only.

5.4.2 Column Ties

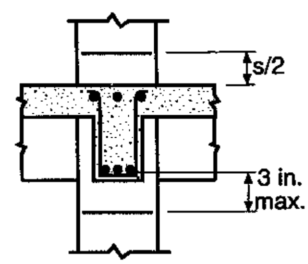
The column tie spacing requirements of ACI 7.10.5 are summarized in Table 5-2. For #10 column bars and smaller, #3 ties are required; for bars larger then #10, #4 ties must be used. Maximum tie spacing shall not exceed the lesser of 1) 16 longitudinal bar diameters, 2) 48 tie bar diameters, and 3) the least column dimension.

Table 5-2 Column Tie Spacing

Tie Size	Column Bars	Maximum Spacing* (in.)
#3	# 5	10
	# 6	12
	# 7	14
	# 8	16
	# 9	18
	#10	18
# 4	#11	22



Interior column-slab joint**



Interior column-beam joint***

*Maximum spacing not to exceed least column dimension (ACI 7.10.5.2)
**Also valid for joints with beams on less than 4 sides of the column (ACI 7.10.5.4)
***Beams on all 4 sides of the column (ACI 7.10.5.5)

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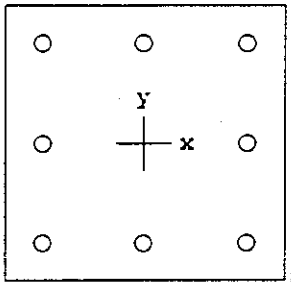


18.0 x 18.0 inch

f'_c = 4.0 ksi

f_y = 60.0 ksi

Confinement:Tied

clr cvr = 1.88 in

spacing = 5.43 in

8#9 at 2.5%

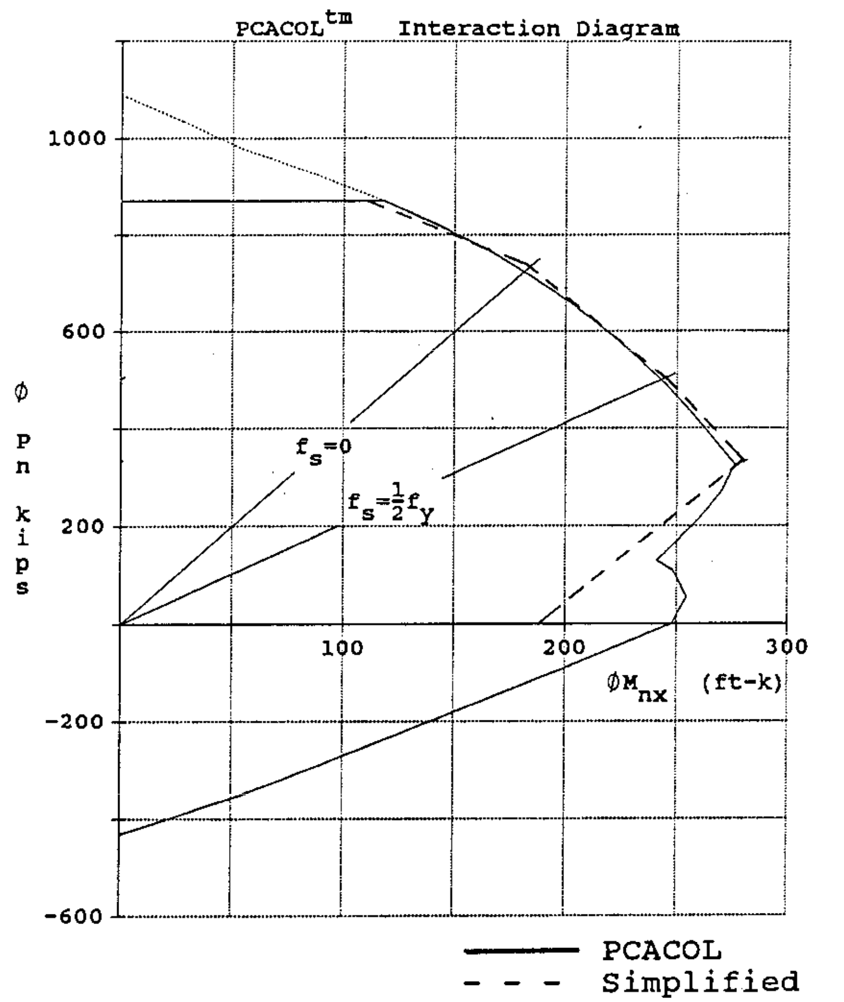
A_s = 8.00 in^2

I_x = 8748 in^4

I_y = 8748 in^4

$\bar{x}$ = 0.00 in

$\bar{y}$ = 0.00 in



Project: SIMPLIFIED DESIGN

Column Id: EX. 5.4.1.1

Engineer:

Date: Time:

Code: ACI 318-89

Version: 2.20

Slenderness not considered x-axis

File name: D:\PCACOL\DATA\SDEX5411.COL

Material Properties:

E_c = 3834 ksi ϵ_u = 0.003 in/in

f_c = 3.40 ksi E_s = 29000 ksi

β_1 = 0.85

Stress Profile : Block

Reduction: ϕ_c = 0.70 ϕ_b = 0.90

Figure 5-6 Comparison of Simplified and PCACOL Interaction Diagrams

Suggested tie details to satisfy ACI 7.10.5.3 are shown in Fig. 5-7 for the 8, 12, and 16 column bar arrangements. In any square (or rectangular) bar arrangement, the four corner bars are enclosed by a single one-piece tie (ACI 7.10.5.3). The ends of the ties are anchored by a standard 90° or 135° hook (ACI 7.1.3). It is important to alternate the position of hooks in placing successive sets of ties. For easy field erection, the intermediate bars in the 8 and 16 bar arrangements can be supported by the separate cross ties shown in Fig. 5-7. Again, it is important to alternate the position of the 90° hooked end at each successive tie location. The two-piece tie shown for the 12 bar arrangement should be lap spliced at least 1.3 times the tensile development length of the tie bar, ℓ_d, but not less than 12 in. To eliminate the supplementary ties for the 8, 12, and 16 bar arrangements, 2, 3, and 4 bar bundles at each corner may also be used; at least #4 ties are required in these cases (ACI 7.10.5.1).

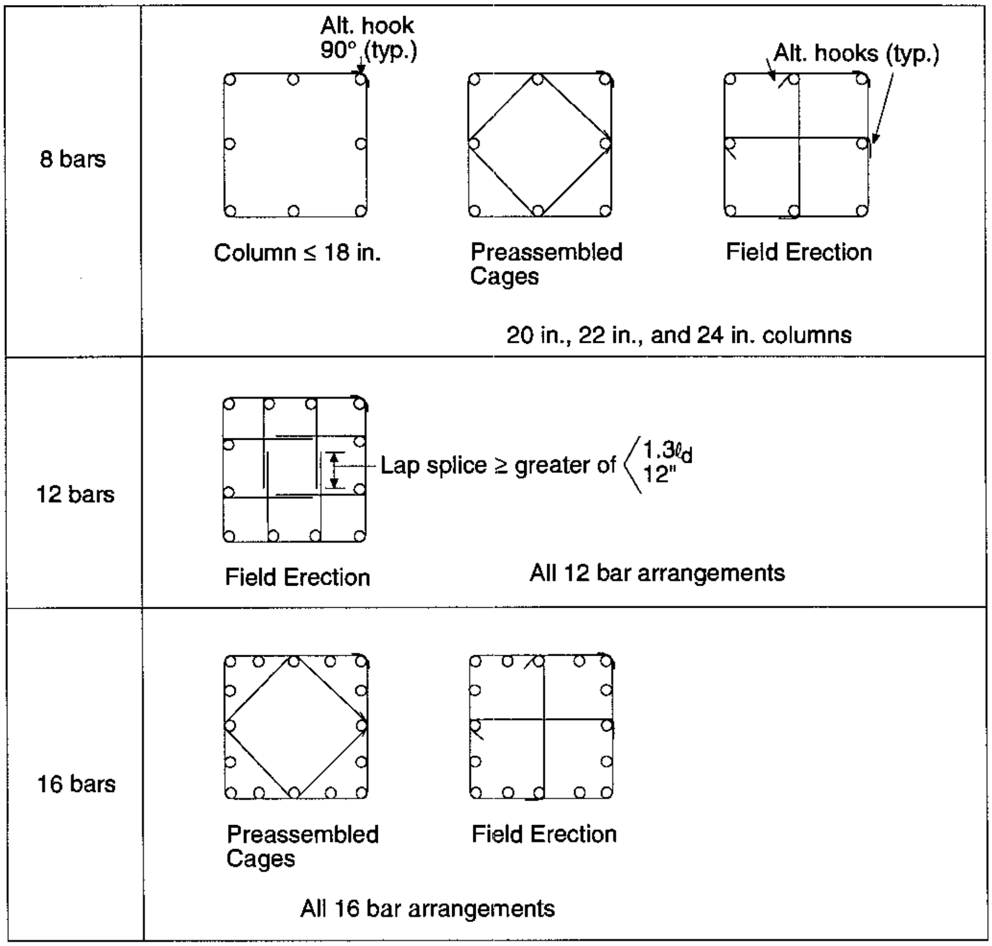


Figure 5-7 Column Tie Details

Column ties must be located not more than one-half a tie spacing above top of footing or slab in any story, and not more than one-half a tie spacing below the lowest reinforcement in the slab (or drop panel) above (see ACI 7.10.5.4 and Table 5-2). Where beams frame into a column from four sides, ties may be terminated 3 in. below the lowest beam reinforcement (ACI 7.10.5.5). Note that extra ties are required within 6 in. from points of offset bends at column splices (see ACI 7.8.1 and Chapter 8).

5.4.3 Biaxial Bending of Columns

Biaxial bending of a column occurs when the loading causes bending simultaneously about both principal axes. This problem is often encountered in the design of corner columns.

A general biaxial interaction surface is depicted in Fig. 5-8. To avoid the numerous mathematical complexities associated with the exact surface, several approximate techniques have been developed that relate the response of a column in biaxial bending to its uniaxial resistance about each principal axis (Reference 5.5 summarizes a number of these approximate methods). A conservative estimate of the nominal axial load strength can be obtained from the following (see ACI R10.3.5 and Fig. 5-9):

$$\frac{1}{\phi P_{ni}} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_o}$$

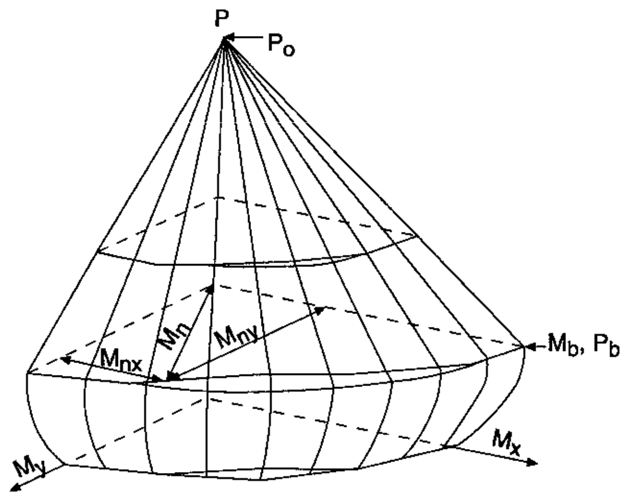
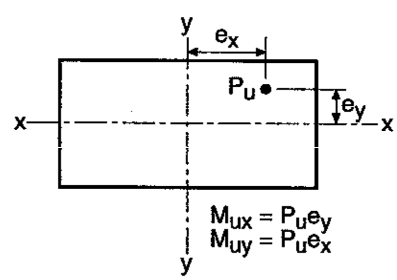


Figure 5-8 Biaxial Interaction Surface



Reinforcing bars not shown

Figure 5-9 Notation for Biaxial Loading

where P_{ni} = nominal axial load strength for a column subjected to an axial load P_u at eccentricities e_x and e_y

P_{nx} = nominal axial load strength for a column subjected to an axial load P_u at an eccentricity of e_x only

P_{ny} = nominal axial load strength for a column subjected to an axial load P_u at an eccentricity of e_y only

P_o = nominal axial load strength for a column subjected to an axial load P_u at an eccentricity of zero (i.e., $e_x = e_y = 0$)

$= 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$

The above equation can be rearranged into the following form:

$$\phi P_{ni} = \cfrac{1}{\cfrac{1}{\phi P_{nx}} + \cfrac{1}{\phi P_{ny}} - \cfrac{1}{\phi P_o}}$$

In design, $P_u \leq \phi P_{ni}$ where P_u is the factored axial load acting at eccentricities e_x and e_y. This method is most suitable when ϕP_{nx} and ϕP_{ny} are greater than the corresponding balanced axial loads; this is usually the case for typical building columns.

An iterative design process will be required when using this approximate equation for columns subjected to biaxial loading. A trial section can be obtained from Figs. 5-16 through 5-23 with the factored axial load P_u and the total factored moment taken as $M_u = M_{ux} + M_{uy}$ where $M_{ux} = P_u e_x$ and $M_{uy} = P_u e_y$. The expression for ϕP_{ni} can then be used to check if the section is adequate or not. Usually, only an adjustment in the amount of reinforcement will be required to obtain an adequate or more economical section.

5.4.3.1 Example: Simplified Design of a Column Subjected to Biaxial Loading

Determine the size and reinforcement for a corner column subjected to $P_u = 400$ kips, $M_{ux} = 50$ ft-kips, and $M_{uy} = 25$ ft-kips.

(1) Trial section

From Fig. 5-18 with $P_u = 400$ kips and $M_u = 50 + 25 = 75$ ft-kips, select a 14 × 14 in. column with 4-#8 bars.

(2) Check the column using the approximate equation

For bending about the x-axis:

$$\phi P_{nx} = 473 \text{ kips for } M_{ux} = 50 \text{ ft-kips (see Fig. 5-18)}$$

For bending about the y-axis:

$$\phi P_{ny} = 473 \text{ kips for } M_{uy} = 25 \text{ ft-kips (see Fig. 5-18)}$$

$$\phi P_o = 0.7[0.85 \times 4(14^2 - 3.16) + (60 \times 3.16)] = 592 \text{ kips}$$

$$\phi P_{ni} = \cfrac{1}{\cfrac{1}{473} + \cfrac{1}{473} - \cfrac{1}{592}} = 394 \text{ kips} \approx 400 \text{ kips}$$

Since this approximate method of analysis is known to be conservative, use a 14 × 14 in. column with 4-#8 bars.

For comparison purposes, PCACOL was used to check the adequacy of the 14 × 14 in. column with 4-#7 bars. Fig. 5-10 is the output from the program which is a plot of ϕM_{ny} versus ϕM_{nx} for $\phi P_n = P_u = 400$ kips (i.e., a horizontal slice through the interaction surface at $\phi P_n = 400$ kips). Point 1 represents the position of the applied factored moments for this example. As can be seen from the figure, the section reinforced with 4-#7 bars is adequate to carry the applied load and moments. Fig. 5-11 is also output from the PCACOL program; this vertical slice through the interaction surface also reveals the adequacy of the section. As expected, the approximate equation resulted in a more conservative amount of reinforcement (about 32% greater than the amount from PCACOL).

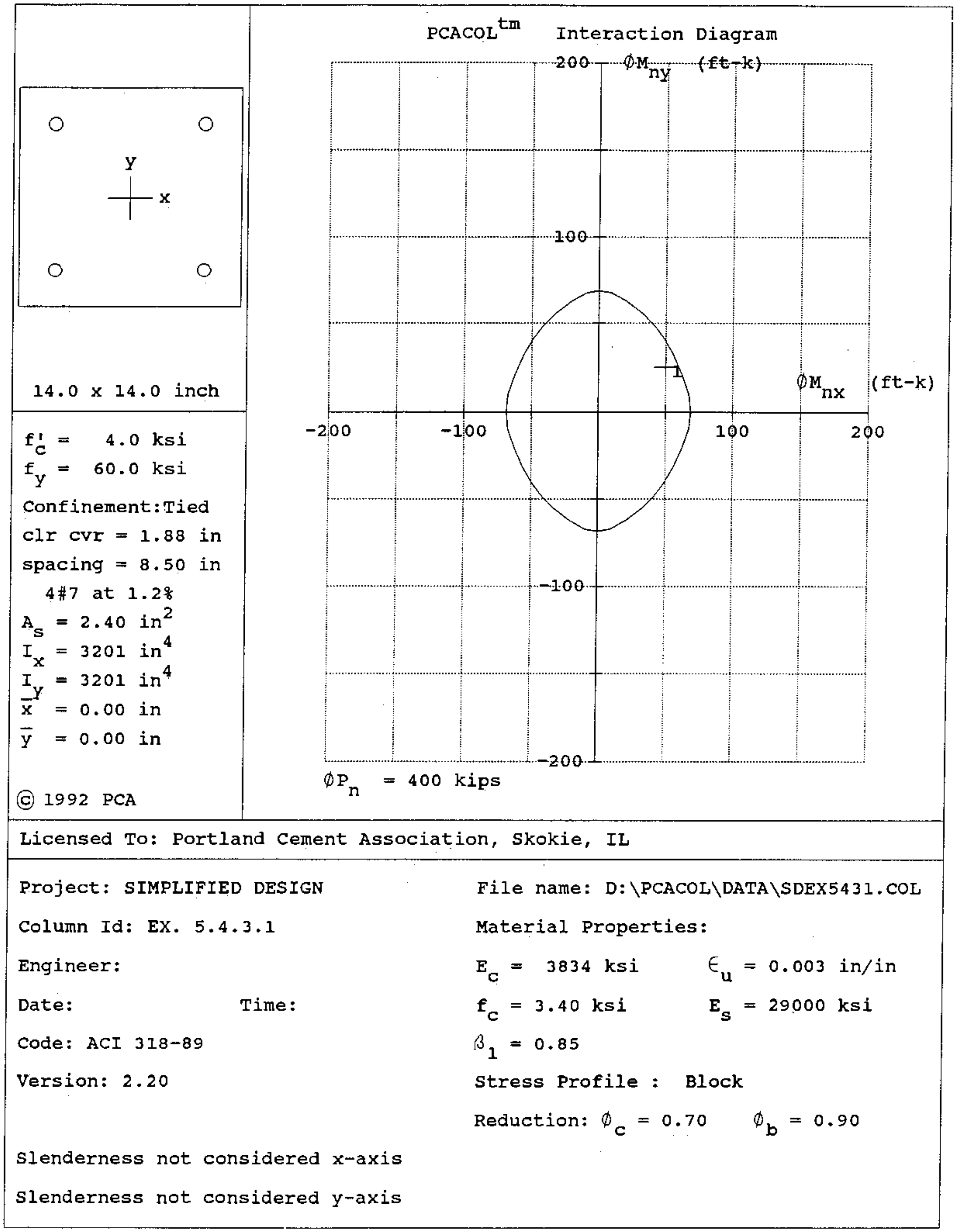
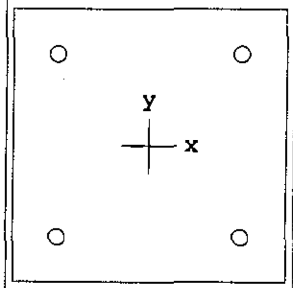
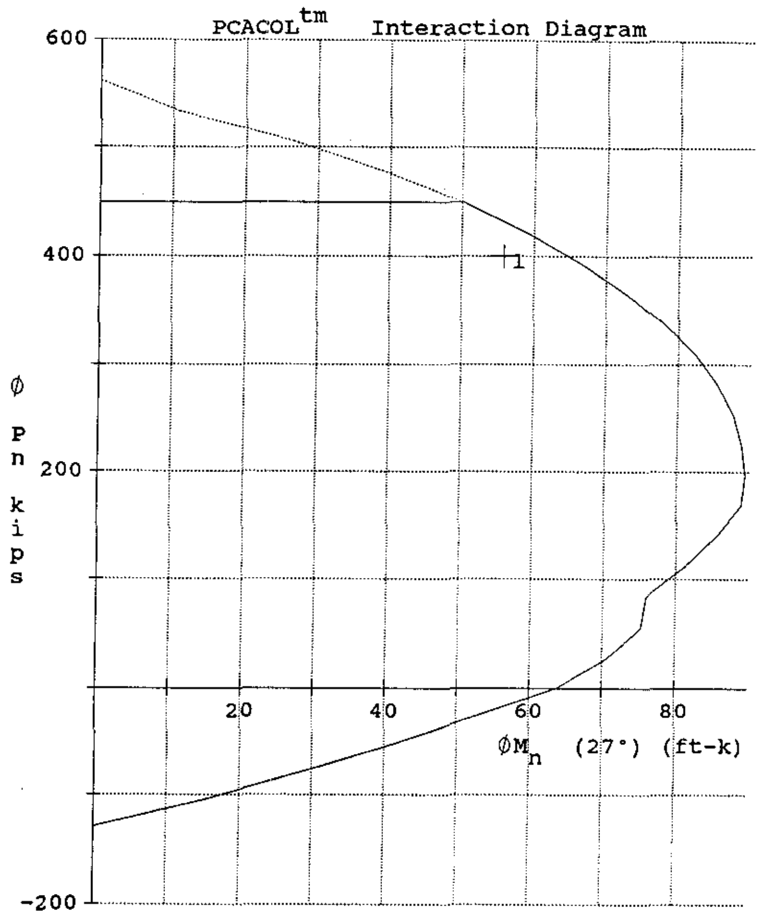


Figure 5-10 Moment Contour for 14 × 14 in. Column at ϕP_n = 400 kips



14.0 x 14.0 inch

f'_c = 4.0 ksi
f_y = 60.0 ksi
Confinement:Tied
clr cvr = 1.88 in
spacing = 8.50 in
4#7 at 1.2%
A_s = 2.40 in^2
I_x = 3201 in^4
I_y = 3201 in^4
$\bar{x}$ = 0.00 in
$\bar{y}$ = 0.00 in



Project: SIMPLIFIED DESIGN
Column Id: EX. 5.4.3.1
Engineer:
Date: Time:
Code: ACI 318-89
Version: 2.20

File name: D:\PCACOL\DATA\SDEX5431.COL
Material Properties:
E_c = 3834 ksi ϵ_u = 0.003 in/in
f_c = 3.40 ksi E_s = 29000 ksi
β_1 = 0.85
Stress Profile : Block
Reduction: ϕ_c = 0.70 ϕ_b = 0.90

Slenderness not considered x-axis
Slenderness not considered y-axis

Figure 5-11 Interaction Diagram for 14 × 14 in. Column

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5.5 COLUMN SLENDERNESS CONSIDERATIONS

5.5.1 Braced versus Unbraced Columns

When designing columns, it is important to establish whether or not the building frame is braced. There is rarely a completely braced or a completely unbraced frame. Realistically, a column within a story can be considered braced when horizontal displacements of the story do not significantly affect the moments in the column. ACI R10.11.2 gives maximum drift criteria that can be used to determine if the structure is braced or not. What constitutes adequate bracing must be left to the judgment of the engineer; in many cases, it is possible to ascertain by inspection whether or not a structure has adequate bracing.

A more approximate criterion is also given in ACI R10.11.2 to determine if columns located within a story can be considered braced:

"A compression member may be assumed braced if located in a story in which the bracing elements (shearwalls, shear trusses, or other types of lateral bracing) have a total stiffness, resisting lateral movement of a story, at least equal to six times the sum of the stiffnesses of all the columns within the story."

In framing systems where it is desirable and economical to include shearwalls, the above criteria can be used to size the walls so that the frame can be considered braced for column design (see Chapter 6, Section 6.3).

5.5.2 Minimum Sizing for Design Simplicity

Another important aspect to consider when designing columns is whether slenderness effects must be included in the design (ACI 10.10). In general, design time can be greatly reduced if 1) the building frame is adequately braced by shearwalls (per ACI R10.11.2) and 2) the columns are sized so that effects of slenderness may be neglected. The criteria for the consideration of column slenderness, as prescribed in ACI 10.11.4, are summarized in Fig. 5-12. M_{2b} is the larger factored end moment and M_{1b} is the smaller end moment; both moments, determined from an elastic frame analysis, are due to loads that result in no appreciable sidesway. The ratio M_{1b}/M_{2b} is positive if the column is bent in single curvature, negative if it is bent in double curvature. For braced columns, the effective length factor k = 1.0 (ACI 10.11.2.1).

In accordance with ACI 10.11.4.1, effects of slenderness may be neglected when braced columns are sized to satisfy the following:

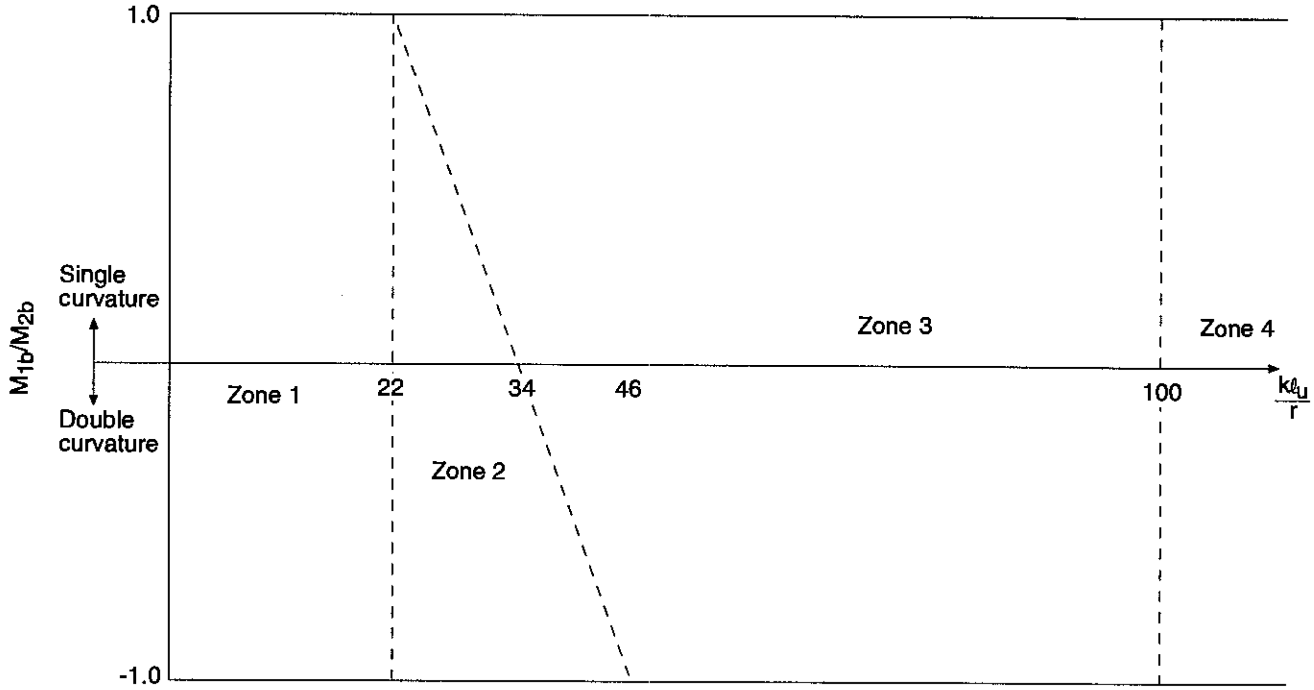
$$\frac{\ell_u}{h} \leq 14$$

where ℓ_u is the clear height between floor members and h is the column size. The above equation is valid for columns that are bent in double curvature with approximately equal end moments. It can be used for the first story columns provided the degree of fixity at the foundation is large enough.* Table 5-3 gives the maximum clear height ℓ_u for a column size that would permit slenderness to be neglected.

For an unbraced column with a column-to-beam stiffness ratio $\psi = 1$ at both ends, the effects of slenderness may be neglected when ℓ_u/h is less than 5. In this case, k = 1.3 (see the alignment chart, Fig. R10.11.2 in ACI R10.11.2).**

**For a discussion of fixity of column bases, see PCI Design Handbook-Precast and Prestressed Concrete, 4th Ed., Precast/Prestressed Concrete Institute, Chicago, IL, 1992.*

***The effective length factor k may be determined for a braced or unbraced frame using ACI Fig. R10.11.2 or using the simplified equations which are also given in ACI R10.11.2.*



Zone 1: Neglect slenderness, braced and unbraced columns
Zone 2: Neglect slenderness, braced columns
Zone 3: Consider slenderness, moment magnifier method (ACI 10.11.5)
Zone 4: Consider slenderness, PΔ analysis (ACI 10.11.4.3)

Figure 5-12 Consideration of Column Slenderness

Table 5-3 Maximum Story Heights to Neglect Slenderness—Braced Columns

Column size h (in.)	Maximum clear height ℓ_u (ft)
10	11.67
12	14.00
14	16.33
16	18.67
18	21.00
20	23.33
22	25.67
24	28.00

If the beam stiffness is reduced to one-fifth of the column stiffness at each end, then k = 2.2; consequently, slenderness effects need not be considered as long as ℓ_u/h is less than 3. As can be seen from these two examples, beam stiffnesses at the top and the bottom of a column in a structure where sidesway is not prevented will have a significant influence on the degree of slenderness of the column.

Due to the complexities involved, the design of slender columns is not considered in this manual. For a comprehensive discussion of this topic, the reader is referred to Chapter 13 of Reference 5.5. Also, PCACOL can be used to design slender columns.[5.4]

5.6 PROCEDURE FOR SIMPLIFIED COLUMN DESIGN

The following procedure is suggested for design of a multistory column stack using the simplifications and column design charts presented in this chapter. For unbraced frames with nonslender columns, both gravity and wind loads must be considered in the design. Figs. 5-16 through 5-23 can be used to determine the required reinforcement. For braced frames with shearwalls resisting the lateral loads and the columns sized so that slenderness may be neglected, only gravity loads need to be considered; the reinforcement can be selected from Figs. 5-16 through 5-23 as well.

STEP (1) LOAD DATA

(a) Gravity Loads:

Determine factored loads P_u for each floor of the column stack being considered. Include a service dead load of 4 kips per floor for column weight. Determine column moments due to gravity loads. For interior columns supporting a two-way floor system, maximum column moments may be computed by ACI Eq. (13-4) (see Chapter 4, Section 4.5). Otherwise, a general analysis is required.

(b) Lateral Loads:

Determine axial loads and moments from the wind loads for the column stack being considered.

STEP (2) LOAD COMBINATIONS

For gravity (dead + live) plus wind loading, ACI 9.2 specifies three load combinations that need to be considered:

gravity loads:

$$U = 1.4D + 1.7L \qquad \text{ACI Eq. (9-1)}$$

gravity plus wind loads:

$$U = 0.75(1.4D + 1.7L + 1.7W) \qquad \text{ACI Eq. (9-2)}$$

or

$$U = 0.9D \pm 1.3W \qquad \text{ACI Eq. (9-3)}$$

In Eq. (9-2), both full value and zero value of L must be considered (ACI 9.2.2).

Generally, for unbraced columns subjected to wind moments, Eq. (9-2) will be the controlling load combination.

STEP (3) COLUMN SIZE AND REINFORCEMENT

Determine an initial column size based on the factored axial load P_u in the first story using Fig. 5-1, and use this size for the full height of building. Note that the dimensions of the column may be preset by architectural (or other) requirements. Once a column size has been established, it should be determined if slenderness effects need to be considered (see Section 5.5). For columns with slenderness ratios larger than the limits given in ACI 10.11.4, it may be advantageous to increase the column size (if possible) so that slenderness effects may be neglected.

As noted earlier, for nonslender columns, Figs. 5-16 through 5-23 may be used to select the required amount of reinforcement for a given P_u and M_u. Ideally, a column with a reinforcement ratio in the range of 1% to 2% will result in maximum economy. Depending on the total number of stories, differences in story heights, and magnitudes of wind loads, 4% to 6% reinforcement may be required in the first story columns. If the column bars are to be lap spliced, the percentage of reinforcement should usually not exceed 4% (ACI R10.9.1). For overall economy, the amount of reinforcement can be decreased at the upper levels of the building. In taller buildings, the concrete strength is usually varied along the building height as well, with the largest f'_c used in the lower level(s).

5.7 EXAMPLES: SIMPLIFIED DESIGN FOR COLUMNS

The following examples illustrate the simplified methods presented in this chapter.

5.7.1 Example: Design of an Interior Column Stack for Building #2 Alternate (1)—Slab and Column Framing Without Structural Walls (Unbraced Frame)

f'_c = 4000 psi (carbonate aggregate)

f_y = 60,000 psi

Required fire resistance rating = 2 hours

(1) LOAD DATA

Roof: LL = 20 psf　　　Floors: LL = 50 psf

DL = 122 psf　　　DL = 136 psf (8.5 in. slab)

Calculations for the first story interior column are as follows:

(a) Total factored load*:

Factored axial loads due to gravity are summarized in Table 5-4.

Table 5-4 Interior Column Gravity Load Summary for Building #2—Alternate (1)

Floor	Dead load (psf)	Live load (psf)	Tributary area (ft^2)	Influence area, A_I (ft^2)	RM*	Reduced live load (psf)	Factored load (kips)**	Cumulative factored load (kips)
5th (roof)	122	20	480	—	1.00†	20.0	104	104
4th	136	50	480	1920	0.59	29.5	121	225
3rd	136	50	480	3840	0.49	24.5	117	342
2nd	136	50	480	5760	0.45	22.5	115	457
1st	136	50	480	7680	0.42	21.0	114	571

*RM = reduction multiplier for live load; see Table 2-2.

**Factored load = [1.4(Dead load) + 1.7(Reduced live load)] × Tributary area. Factored load includes a factored column weight = 1.4 × 4 kips = 5.6 kips per floor.

†No reduction permitted for roof live load (ASCE 4.8.2).

*Axial load from wind loads is zero (see Fig. 2-15)

(b) Factored moments:

gravity loads:* $M_u = 0.035w_\ell \ell_2 \ell_n^2 = 0.035(1.7 \times 0.05)(24)(18.83^2) = 25$ ft-kips

portion of M_u to 1st story column $= 25\left(\dfrac{12}{12+15}\right) = 11$ ft-kips

wind loads:** $M_u = 1.7(94.7) = 161$ ft-kips

(2) LOAD COMBINATIONS

For the 1st story column:

gravity loads: $P_u = 571$ kips ACI Eq. (9-1)
$M_u = 11$ ft-kips

gravity + wind loads: $P_u = 0.75(571) = 428$ kips ACI Eq. (9-2)
$M_u = 0.75(11 + 161) = 129$ ft-kips***

and

$P_u = 0.9(59 + 261 + 20) = 306$ kips ACI Eq. (9-3)

$M_u = 1.3(94.7) = 123$ ft-kips

Factored loads and moments, and load combinations, for the 2nd through 5th story columns are calculated in a similar manner, and are summarized in Table 5-5.

Table 5-5 Interior Column Load Summary for Building #2, Alternate (1)

Floor	Gravity Loads Eq. (9-1)		Gravity + Wind Loads Eq. (9-2)		Gravity + Wind Loads Eq. (9-3)	
	P_u (kips)	M_u (ft-kips)	P_u (kips)	M_u (ft-kips)	P_u (kips)	M_u (ft-kips)
5th	104	10†	78	19	57	12
4th	225	13	169	44	119	35
3rd	342	13	257	65	182	57
2nd	457	14	343	86	244	77
1st	571	11	428	129	306	123

†$M_u = 0.035(1.7 \times 0.02)(24)(18.67^2) = 10$ ft-kips.

(3) COLUMN SIZE AND REINFORCEMENT

With $P_u = 571$ kips, try a 16×16 in. column with slightly more than 1% reinforcement (see Fig. 5-1). Check for fire resistance: From Table 10-2, for a fire resistance rating of 2 hours, minimum column dimension = 10 in. < 16 in. O.K.

Determine if the columns are slender.

As noted above, a column in an unbraced frame is slender if $k\ell_u/r \geq 22$. In lieu of determining an "exact" value, estimate k to be 1.2 (a value of k less than 1.2 is usually not realistic for columns in an unbraced frame; see ACI R10.11.2)

**Moment due to dead load is small.*
***Moments due to wind loads in the N-S direction govern (see Figs. 2-15 and 2-16).*
****Maximum moment will be obtained when live load moment is included.*

For the 1st story column:

$$\frac{k\ell_u}{r} = \frac{1.2[(15 \times 12) - 8.5]}{0.3(16)} = 43 > 22$$

For the 2nd through 5th story columns:

$$\frac{k\ell_u}{r} = \frac{1.2[(12 \times 12) - 8.5]}{0.3(16)} = 34 > 22$$

Therefore, slenderness must be considered for the entire column stack. To neglect slenderness effects, the size of the column h would have to be:

$$\frac{1.2[(15 \times 12) - 8.5]}{0.3h} < 22 \rightarrow h > 31.2 \text{ in.}$$

Obviously, this column would not be practical for a building of the size considered. References 5.1, 5.2, 5.4 or 5.5 can be used to determine the required reinforcement for the 16 × 16 in. column, including slenderness effects.

Figure 5-13 shows the results from PCACOL for an interior 1st story column, including slenderness effects. Fifty percent of the gross moment of inertia of the slab column strip was used to account for cracking.* It was assumed that the column was fixed at the foundation; appropriate modifications can be made if this assumption is not true, based on the actual footing size and soil conditions. Point 1 corresponds to the load combination given in ACI Eq. (9-1). Points 2 and 3 are from ACI Eq. (9-2), with point 2 corresponding to the load combination that includes the full live load and point 3 corresponding to the case when no live load is considered. ACI Eq. (9-3)** is represented by point 4. As can be seen from the figure, 8-#10 bars are required at the 1st floor. The amount of reinforcement can decrease at higher elevations in the column stack. Note that if the column is assumed to be hinged at the base, $k\ell_u/r$ is greater than 100, and a second-order frame analysis would be required.

Check for fire resistance: From Table 10-6, for a fire resistance rating of 4 hours or less, the required cover to the main longitudinal reinforcement = 1.5 in. < provided cover = 1.875 in.   O.K.

5.7.2 Example: Design of an Interior Column Stack for Building #2 Alternate (2) - Slab and Column Framing with Structural Walls (Braced Frame)

f'_c = 4000 psi (carbonate aggregate)

f_y = 60,000 psi

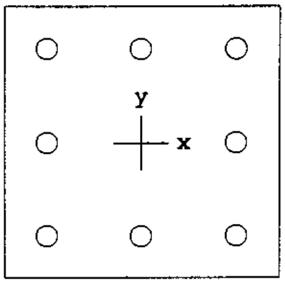
Required fire resistance rating = 2 hours

For the Alternate (2) framing, columns are designed for gravity loading only; the structural walls are designed to resist total wind loading.

(1) LOAD DATA

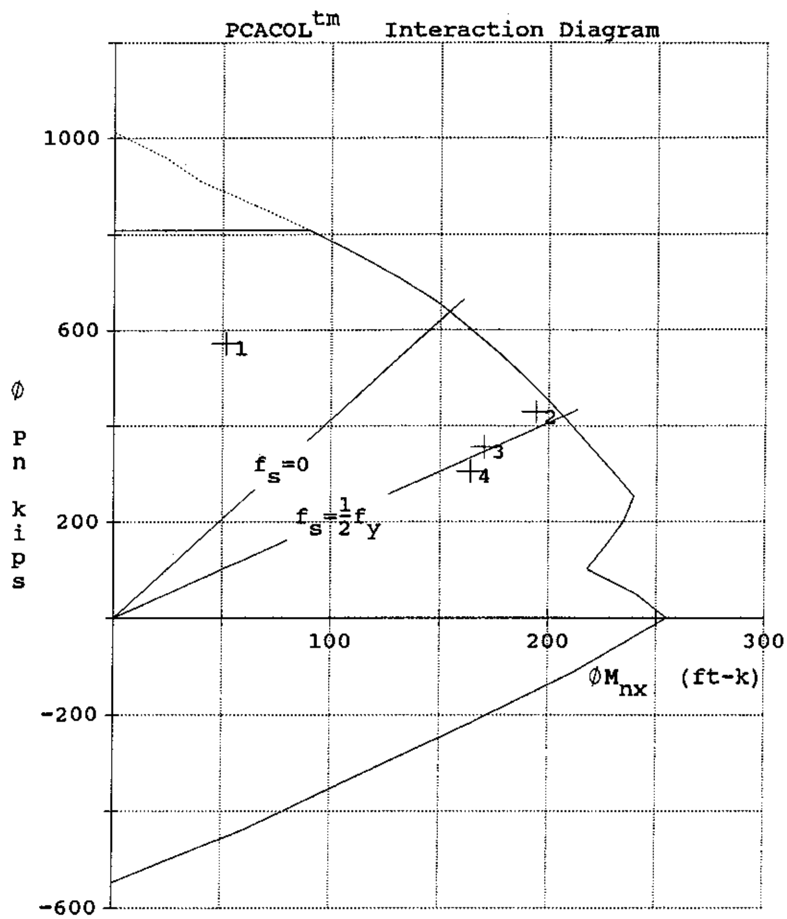
**The moments of inertia of the flexural and compression members are required in order to compute the effective length factor k of the column. ACI R10.11.2 recommends using a value of 0.5 I_g for flexural members (to account for the effect of cracking and reinforcement on relative stiffness) and I_g for compression members when computing the relative stiffness at each end of the compression member, where I_g is the gross moment of inertia of the section.*

*** U = 0.9D-1.3W*



16.0 x 16.0 inch

f'_c = 4.0 ksi
f_y = 60.0 ksi
Confinement: Tied
clr cvr = 1.88 in
spacing = 4.22 in
8#10 at 4.0%
A_s = 10.16 in^2
I_x = 5461 in^4
I_y = 5461 in^4
$\bar{x}$ = 0.00 in
$\bar{y}$ = 0.00 in



Licensed To: Portland Cement Association, Skokie, IL

Project: SIMPLIFIED DESIGN
Column Id: EXAMPLE 5.7.1
Engineer:
Date: Time:
Code: ACI 318-89
Version: 2.20

File name: D:\PCACOL\DATA\SDEX571.COL
Material Properties:
E_c = 3834 ksi ϵ_u = 0.003 in/in
f_c = 3.40 ksi E_s = 29000 ksi
β_1 = 0.85
Stress Profile : Block
Reduction: ϕ_c = 0.70 ϕ_b = 0.90

Slenderness considered X-axis k_b = 0.88 k_s = 1.53

Figure 5-13 Interaction Diagram for First Story Interior Column, Building #2, Alternate (1), Including Slenderness

Roof: LL = 20 psf

DL = 122 psf

Floors: LL = 50 psf

DL = 142 psf (9 in. slab)

Calculations for the first story interior column are as follows:

(a) Total factored load (see Table 5-6):

Table 5-6 Interior Column Gravity Load Summary for Building #2, Alternate (2)

Floor	Dead load (psf)	Live load (psf)	Tributary area (ft^2)	Influence area, A_I (ft^2)	RM*	Reduced live load (psf)	Factored load (kips)**	Cumulative factored load (kips)
5th (roof)	122	20	480	—	1.00†	20.0	104	104
4th	142	50	480	1920	0.59	29.5	125	229
3rd	142	50	480	3840	0.49	24.5	121	350
2nd	142	50	480	5760	0.45	22.5	119	469
1st	142	50	480	7680	0.42	21.0	118	587

*RM = reduction multiplier for live load; see Table 2-2.

**Factored load = [1.4(Dead load) + 1.7(Reduced live load)] × Tributary area. Factored load includes a factored column weight = 1.4 × 4 kips = 5.6 kips per floor.

†No reduction permitted for roof live load (ASCE 4.8.2).

(b) Factored gravity load moment:

$$M_u = 0.035 w_\ell \ell_2 \ell_n^2 = 0.035(1.7 \times 0.05)(24)(18.83^2) = 25 \text{ ft-kips}$$

portion of M_u to 1st story column = 25 (12/27) = 11 ft-kips

Similar calculations can be performed for the other floors.

(2) LOAD COMBINATIONS

The applicable load combination for each floor is summarized in Table 5-7. Note that only ACI Eq. (9-1) needs to be considered for columns in a braced frame.

Table 5-7 Interior Column Load Summary for Building #2, Alternate (2)

Floor	Eq. (9-1)	
	P_u (kips)	M_u (ft-kips)
5th	104	10
4th	229	13
3rd	350	13
2nd	469	14
1st	587	11

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(3) COLUMN SIZE AND REINFORCEMENT

With P_u = 587 kips, try a 16 × 16 in. column with approximately 1.3% reinforcement (see Fig. 5-1).

Check for fire resistance: From Table 10-2, for a fire resistance rating of 2 hours, minimum column dimension = 10 in. < 16 in. O.K.

Determine if the columns are slender.

Using Table 5-3, for a 16 in. column, the maximum clear story height to neglect slenderness is 18.67 ft. Since the actual clear story heights are less than this value, slenderness need not be considered for the entire column stack.

- 1st story columns:

  P_u = 587 kips, M_u = 11 ft-kips

  From Fig. 5-19, use 4-#8 bars (ρ_g = 1.23%)

- 2nd through 5th story columns:

Using 4-#8 bars for the entire column stack would not be economical. ACI 10.8.4 may be used so that the amount of reinforcement at the upper levels may be decreased. The required area of steel at each floor can be obtained from the following:

$$\text{Required } A_{st} = (\text{area of 4-\#8 bars})\left(\frac{P_u \text{ at floor level}}{588 \text{ kips}}\right)$$

where 588 kips is ϕP_n for the 16 × 16 in. column reinforced with 4-#8 bars. It is important to note that ρ_g should never be taken less than 0.5% (ACI 10.8.4). The required reinforcement for the column stack is summarized in Table 5-8.

Table 5-8 Reinforcement for Interior Column of Building #2, Alternate (2)

Floor	Required A_{st} (in^2)	Required ρ_g (%)	Reinforcement (ρ_g%)
5th	0.56	0.50*	4-#6 (0.69)
4th	1.23	0.50*	4-#6 (0.69)
3rd	1.88	0.73	4-#7 (0.94)
2nd	2.52	0.98	4-#8 (1.23)
1st	3.15	1.23	4-#8 (1.23)

*Minimum ρ_g = 0.5% (ACI 10.8.4)
Minimum $A_{st} = 0.005 \times 16^2 = 1.28$ in.^{2}

Check for fire resistance: From Table 10-6, for a fire resistance rating of 4 hours or less, the required cover to the main longitudinal reinforcement = 1.5 in. < provided cover. O.K.

Column ties and spacing can be selected from Table 5-2.

5.7.3 Example: Design of an Edge Column Stack (E-W Column Line) for Building #1—3-story Pan Joist Construction (Unbraced Frame)

f'_c = 4000 psi (carbonate aggregate)

f_y = 60,000 psi

Required fire resistance rating = 1 hour (2 hours for columns supporting Alternate (2) floors).

(1) LOAD DATA

Roof: LL = 12 psf

DL = 105 psf

Floors: LL = 60 psf

DL = 130 psf

Calculations for the first story column are as follows:

(a) Total factored load (see Table 5-9):

Table 5-9 Edge Column Gravity Load Summary for Building #1

Floor	Dead load (psf)	Live load (psf)	Tributary area (ft^2)	Influence area, A_I (ft^2)	RM*	Reduced live load (psf)	Factored load (kips)**	Cumulative factored load (kips)
3rd (roof)	105	12	450	—	1.00†	12.0	81	81
2nd	130	60	450	1800	0.60	36.0	115	196
1st	130	60	450	3600	0.50	30.0	110	306

*RM = reduction multiplier for live load; see Table 2-2.

**Factored load = [1.4(Dead load) + 1.7(Reduced live load)] × Tributary area. Factored load includes a factored column weight = 1.4 × 4 kips = 5.6 kips per floor.

†No reduction permitted for roof live load (ASCE 4.8.2).

(b) Factored moments in 1st story edge columns:

gravity loads: M_u = 372.7 ft-kips (see Section 3.8.3 - Step (2), M_u @ exterior columns)

portion of M_u to 1st story column = 372.7/2 = 186 ft-kips

wind loads (see Fig. 2-13):

$P_u = 1.7(10.0) = 17$ kips

$M_u = 1.7(55.5) = 94$ ft-kips

(2) LOAD COMBINATIONS

For the 1st story column:

gravity loads:

$P_u = 306$ kips

$M_u = 186$ ft-kips

ACI Eq. (9-1)

gravity + wind loads:

$$P_u = 0.75(306 + 17) = 242 \text{ kips} \qquad \text{ACI Eq. (9-2)}$$

$$M_u = 0.75(186 + 94) = 210 \text{ ft-kips}$$

and

$$P_u = 0.9(47+117+12) + 1.3(10.0) = 171 \text{ kips} \qquad \text{ACI Eq. (9-3)}$$

$$M_u = 0.9[½(0.130 \times 30)\frac{(28.58)^2}{16}] + 1.3(55.5) = 162 \text{ ft-kips}$$

Factored loads and moments, and load combinations, for the 2nd and 3rd story columns are calculated in a similar manner, and are summarized in Table 5-10.

Table 5-10 Edge Column Load Summary for Building #1

Floor	Gravity Loads		Gravity + Wind Loads			
	Eq. (9-1)		Eq. (9-2)		Eq. (9-3)	
	P_u (kips)	M_u (ft-kips)	P_u (kips)	M_u (ft-kips)	P_u (kips)	M_u (ft-kips)
3rd	81	256*	62	208	47	161
2nd	196	186	152	184	108	135
1st	306	186	243	210	171	162

*$w_u = [(1.4 \times 0.105) + (1.7 \times 0.012)]30 = 5.02$ klf
$M_u = 5.02 \times 28.58^2/16 = 256$ ft-kips

(3) COLUMN SIZE AND REINFORCEMENT

For edge columns, initial selection of column size can be determined by referring directly to the column design charts and selecting an initial size based on required moment strength. For largest $M_u = 256$ ft-kips, try a 16 × 16 in. column (see Fig. 5-19).

Check for fire resistance: From Table 10-2, for fire resistance ratings of 1 hour and 2 hours, minimum column dimensions of 8 in. and 10 in., respectively, are both less than 16 in. O.K.

Determine if the columns are slender.

Using k = 1.2, slenderness ratios for all columns:

$$\frac{k\ell_u}{r} = \frac{1.2[(13 \times 12) - 19.5]}{0.3(16)} = 34 > 22$$

Thus, all of the columns are slender. To neglect slenderness effects, the size of the column would have to be:

$$\frac{1.2[(13 \times 12) - 19.5]}{0.3h} < 22 \rightarrow h > 24.8 \text{ in.}$$

This column would probably not be practical for a building of the size considered.

Fig. 5-14 shows the results from PCACOL for a first story edge column, including slenderness effects. Fifty percent of the gross moment of inertia was used for the 36 × 19.5 in. column-line beam to account for cracking.* The column was assumed fixed at the foundation. As can be seen from the figure, 8-#10 bars are required in this case.

**The moments of inertia of the flexural and compression members are required in order to compute the effective length factor k of the column. ACI R10.11.2 recommends using a value of 0.5 I_g for flexural members (to account for the effect of cracking and reinforcement on relative stiffness) and I_g for compression members when computing the relative stiffness at each end of the compression member, where I_g is the gross moment of inertia of the section.*

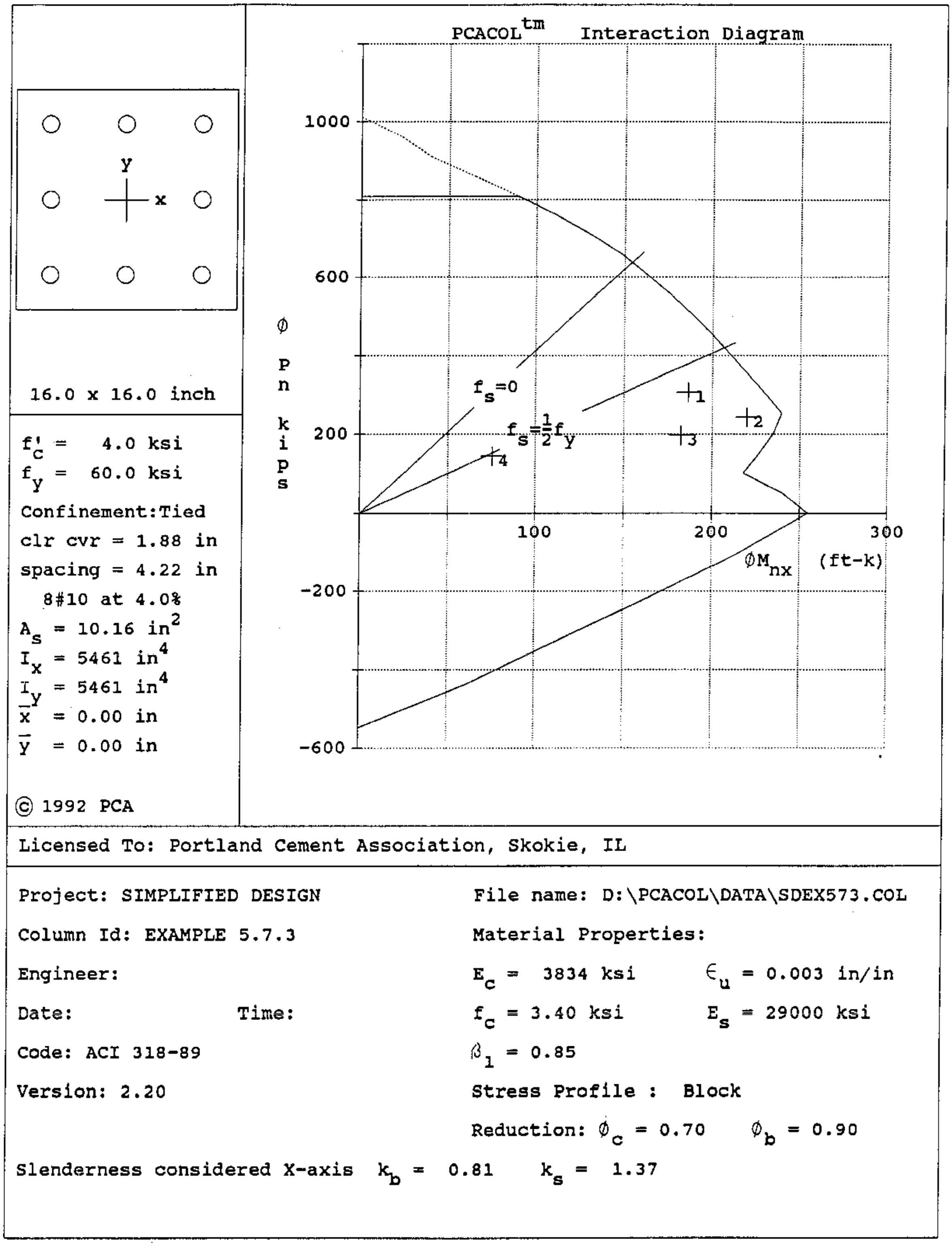

Figure 5-14 Interaction Diagram for First Story Edge Column, Building #1, Including Slenderness

Check for fire resistance: From Table 10-6, for fire resistance ratings of 4 hours or less, required cover to main longitudinal reinforcement is 1.5 in. < provided cover = 1.875 in. O.K.

5.8 COLUMN SHEAR STRENGTH

Columns in unbraced frames are required to resist the shear forces from wind loads. For members subjected to axial compression, the concrete shear strength ϕV_c is given in ACI Eq. (11-4). Fig. 5-15 can be used to obtain this quantity for the square column sizes shown. The largest bar size from the corresponding column design charts of Figs. 5-16 through 5-23 were used to compute ϕV_c (for example, for a 16 × 16 in. column, the largest bar size in Fig. 5-19 is #11).

ACI Eq. (9-3) should be used to check column shear strength:

$$U = 0.9D + 1.3W$$

$$N_u = P_u = 0.9D$$

$$V_u = 1.3W$$

If V_u is greater than ϕV_c, spacing of column ties can be reduced to provide additional shear strength ϕV_s. Using the three standard spacings given in Chapter 3, Section 3.6, the values of ϕV_s given in Table 5-11 may be used to increase column shear strength.

Table 5-11 Shear Strength Provided by Column Ties

Tie Spacing	ϕV_s - #3 ties*	ϕV_s - #4 ties*
d/2	22 kips	40 kips
d/3	33 kips	61 kips
d/4	45 kips	81 kips

*2 legs, Grade 60 bars

For low-rise buildings, column shear strength ϕV_c will usually be more than adequate to resist the shear forces from wind loads.

5.8.1 Example: Design for Column Shear Strength

Check shear strength for the 1st floor interior columns of Building #2, Alternate (1) - slab and column framing without structural walls. For wind in the N-S direction, V = 12.63 kips (see Fig. 2-15).

$$N_u = P_u = 0.9(59 + 261 + 20) = 306 \text{ kips} \quad \text{(see Example 5.7.1)}$$

$$V_u = 1.3(12.63) = 16.42 \text{ kips}$$

From Fig. 5-15, for a 16 × 16 in. column with N_u = 306 kips:

$$\phi V_c \cong 36 \text{ kips} > 16.42 \text{ kips} \quad \text{O.K.}$$

Column shear strength is adequate. With #10 column bars, use #3 column ties at 16 in. on center (least column dimension governs; see Table 5-2).

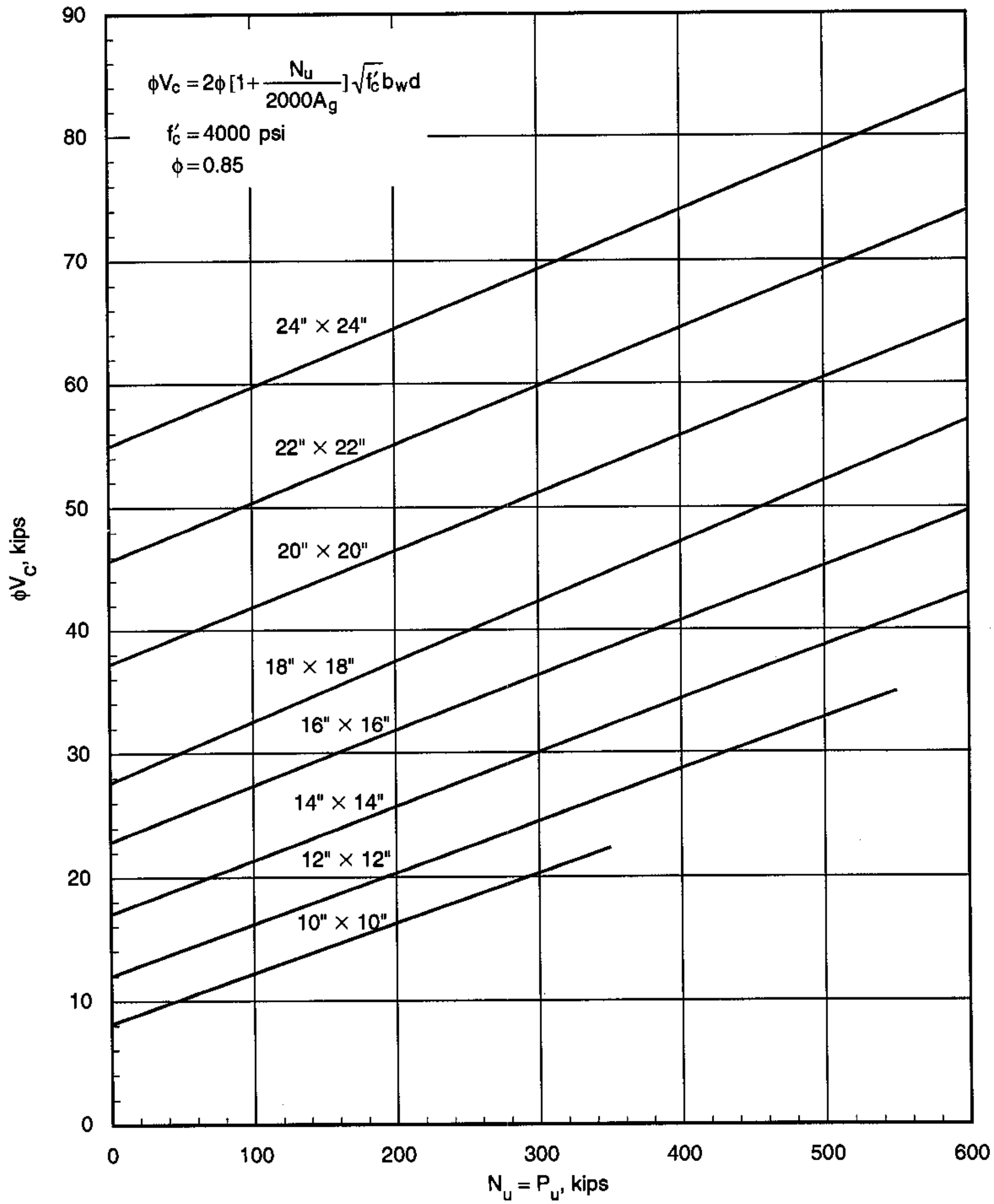

Figure 5-15 Column Shear Strength, ϕV_c

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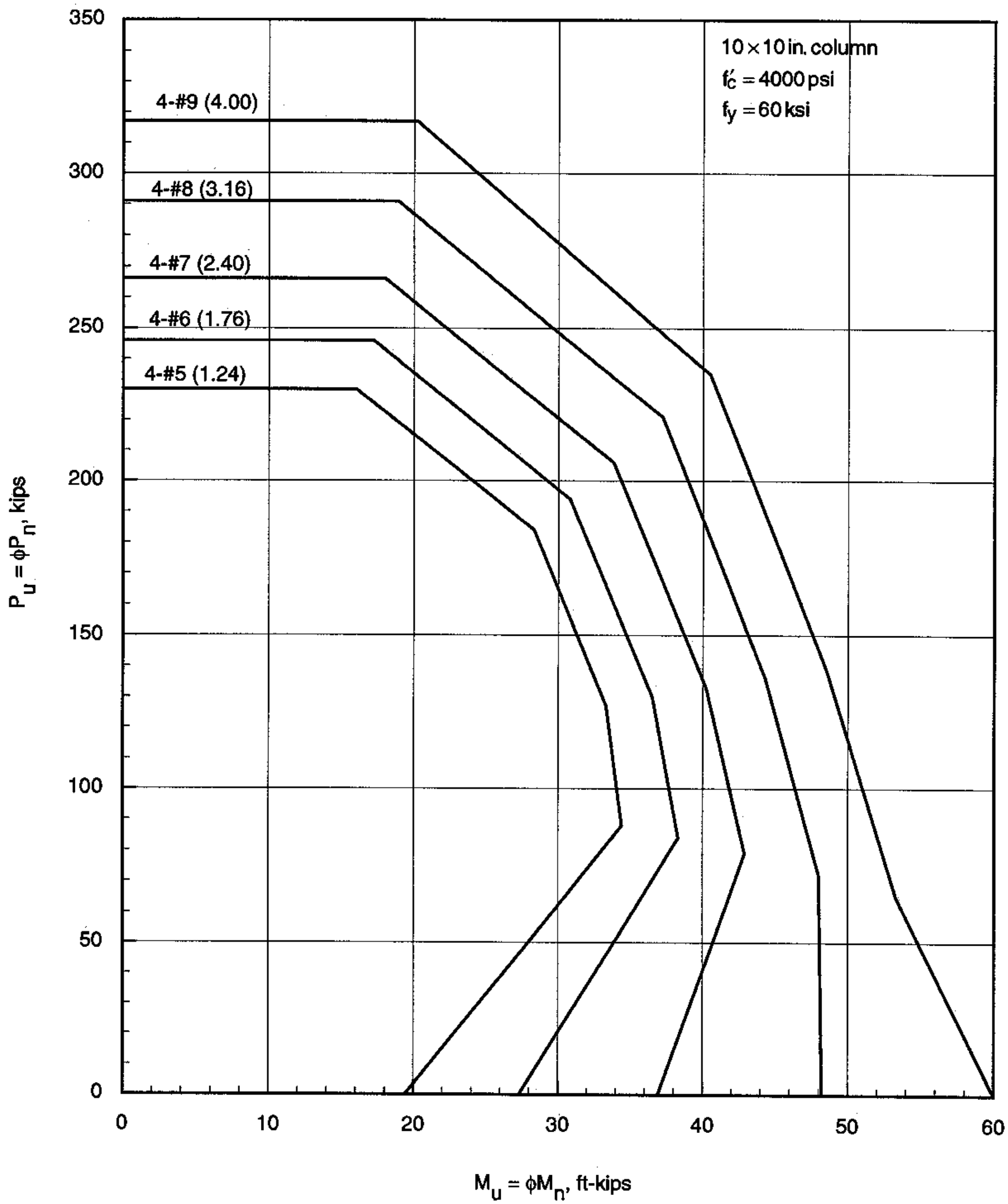

Figure 5-16 10 × 10 in. Column Design Chart

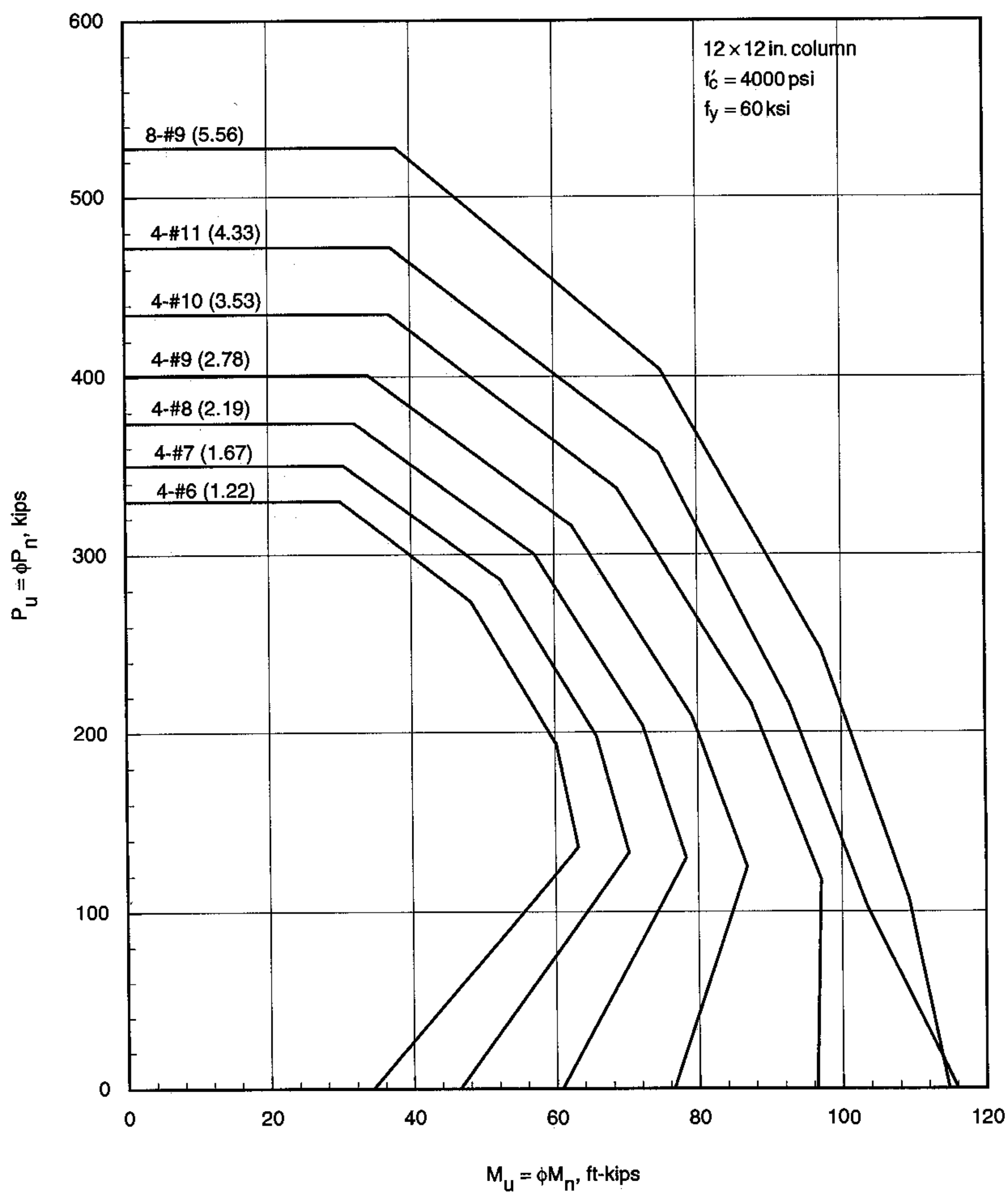

Figure 5-17 12 × 12 in. Column Design Chart

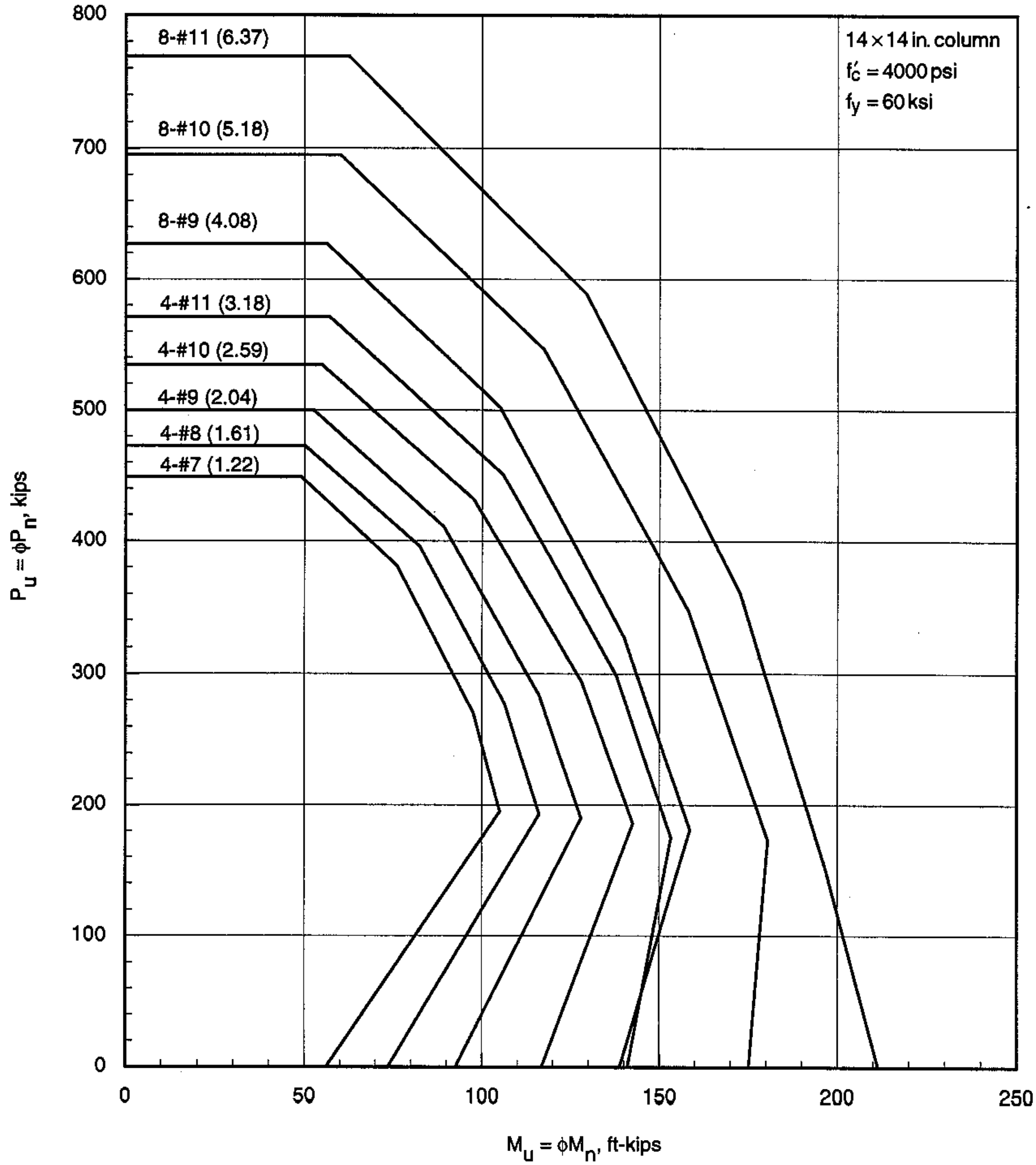

Figure 5-18 14 × 14 in. Column Design Chart

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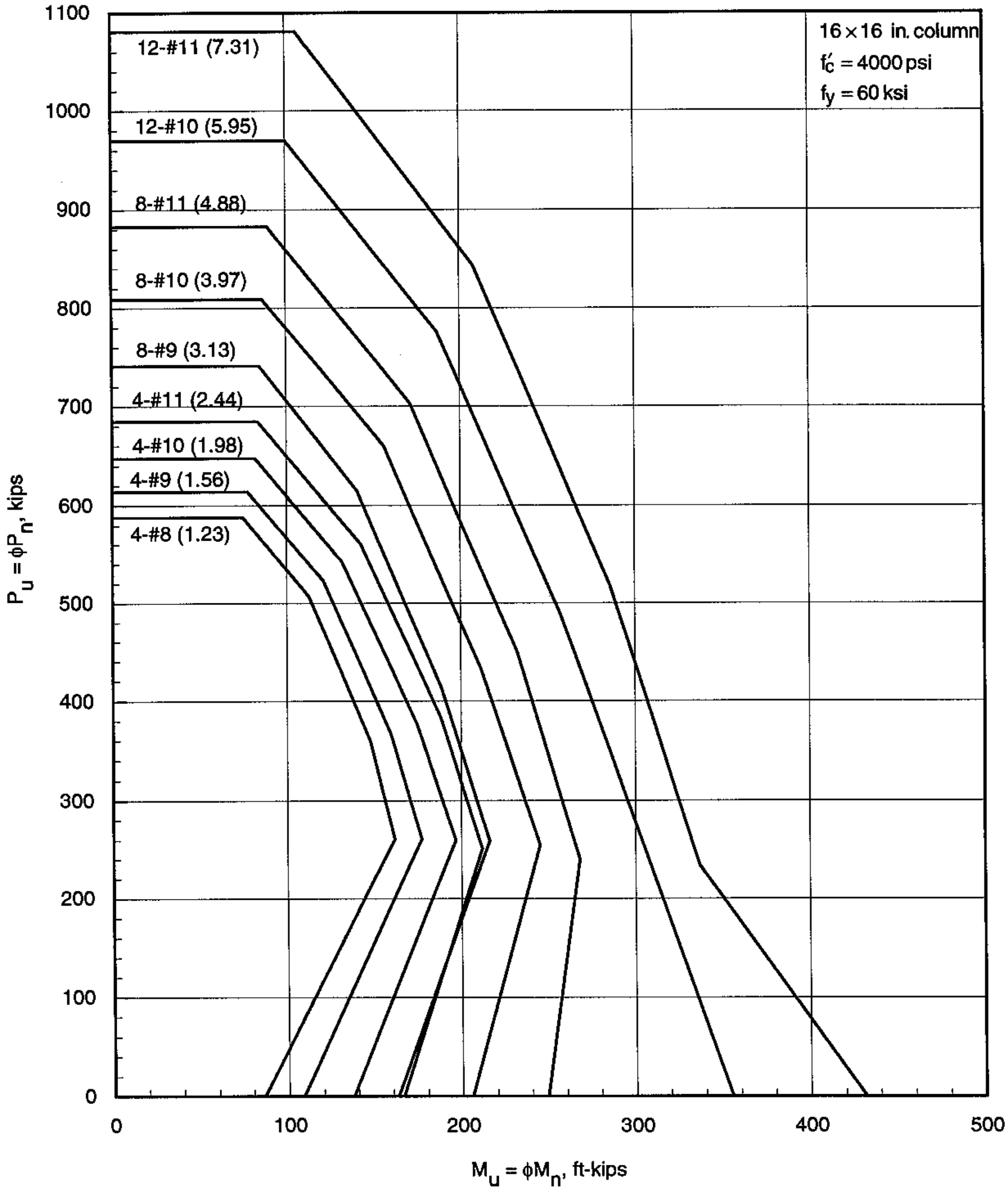

Figure 5-19 16 × 16 in. Column Design Chart

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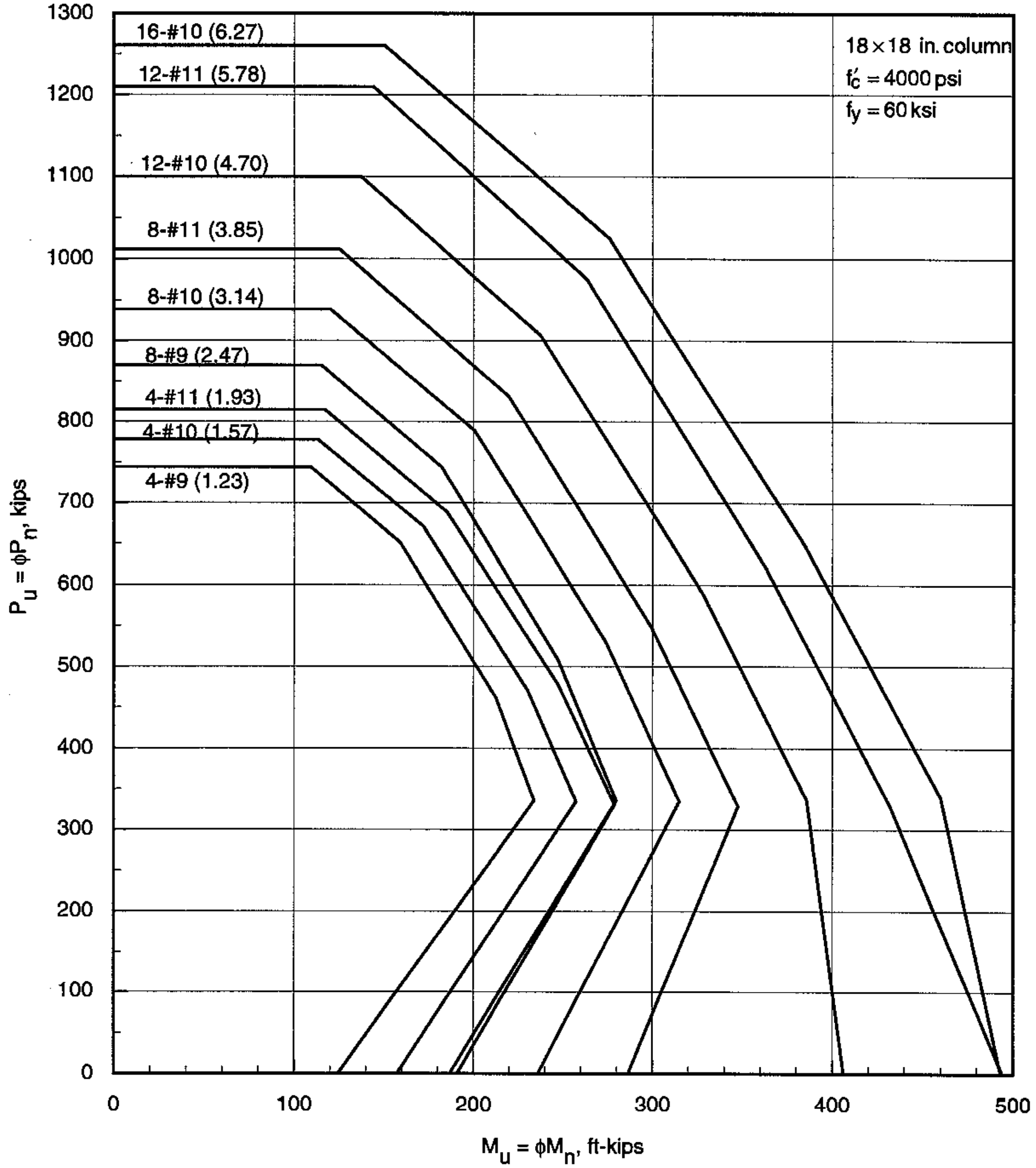

Figure 5-20 18 × 18 in. Column Design Chart

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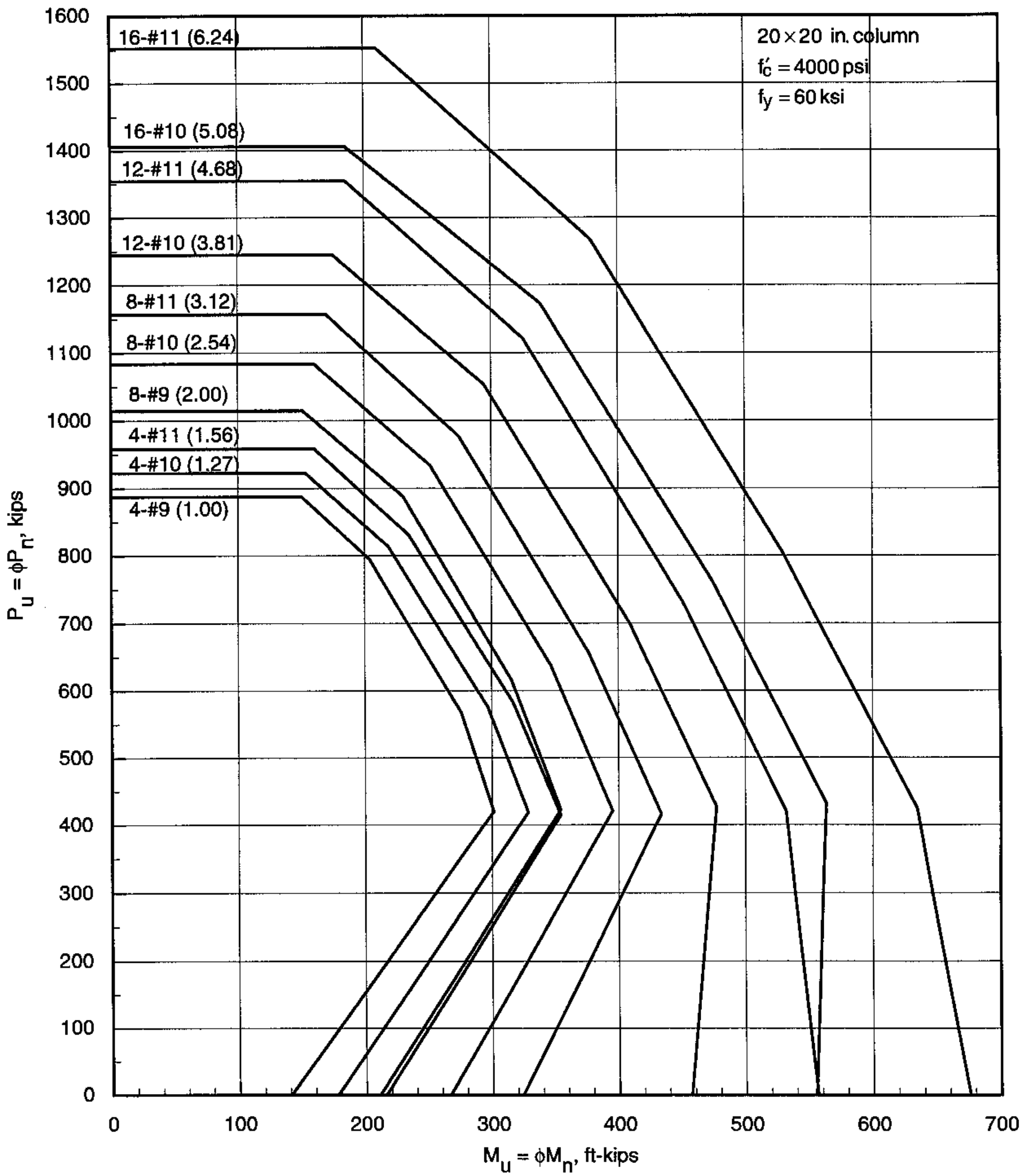

Figure 5-21 20 × 20 in. Column Design Chart

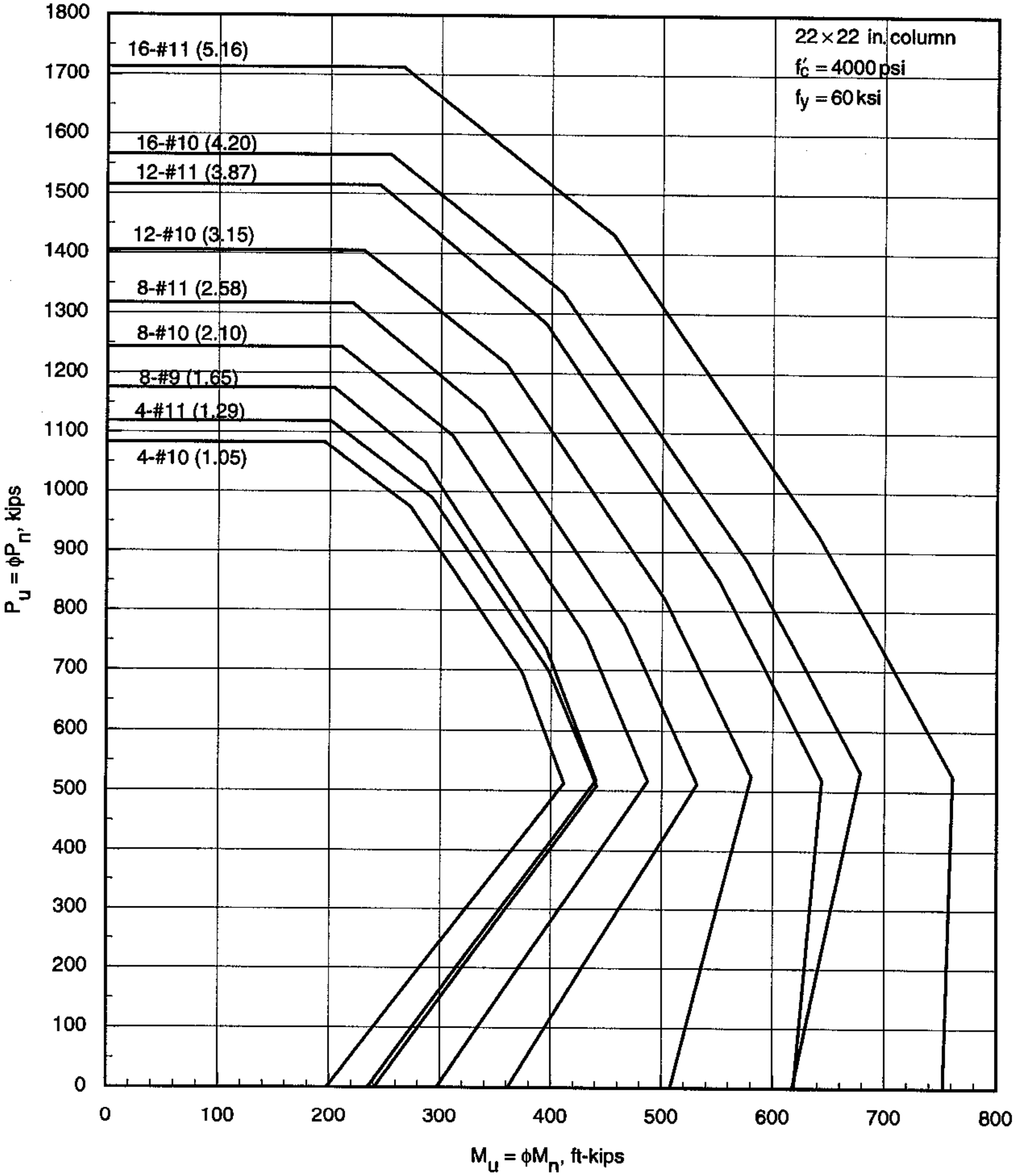

Figure 5-22 22 × 22 in. Column Design Chart

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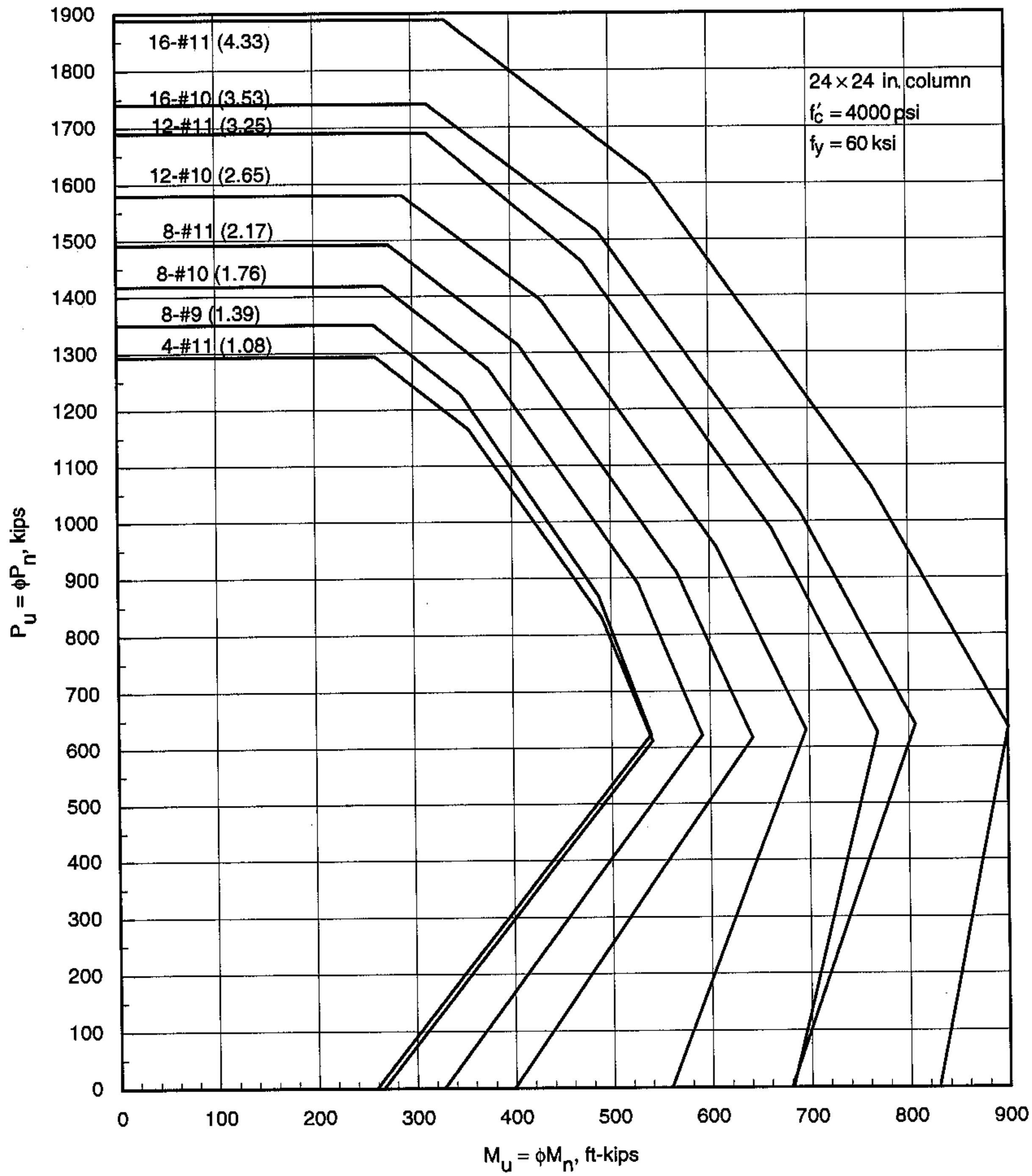

Figure 5-23 24 × 24 in. Column Design Chart

References

5.1 *Strength Design of Reinforced Concrete Columns*, Portland Cement Association, Skokie, IL, EB009, 49 pp.

5.2 *Design Handbook in Accordance with the Strength Design Method of ACI 318-89: Vol. 2–Columns*, SP17A(90).CT93, American Concrete Institute, Detroit, MI, 1990, 222 pp.

5.3 *CRSI Handbook*, Seventh Edition, Concrete Reinforcing Steel Institute, Schaumburg, IL, 1992.

5.4 *PCACOL- Strength Design of Reinforced Concrete Column Sections,* Portland Cement Association, Skokie, IL, 1992.

5.5 *Notes on ACI 318-89*, 5th Edition, EB070, Portland Cement Association, Skokie, IL, 1990.

Chapter 6

Simplified Design for Structural Walls

6.1 INTRODUCTION

For buildings in the low to moderate height range, frame action alone is usually sufficient to provide adequate resistance to lateral loads. In most cases, the members can be sized for gravity loads only; this is due to ACI Eq. (9-2) which permits the combined effects of gravity and wind loads to be reduced by a factor of 0.75. Only amounts of reinforcement may have to be increased due to the forces and moments caused by wind. Whether directly considered or not, nonstructural walls and partitions can also add to the total rigidity of a building and provide reserve capacity against lateral loads.

Structural walls or shearwalls are extremely important members in high-rise buildings. If unaided by walls, high-rise frames often could not be efficiently designed to satisfy strength requirements or to be within acceptable lateral drift limits. Since frame buildings depend primarily on the rigidity of member connections (slab-column or beam-column) for their resistance to lateral loads, they tend to be uneconomical beyond a certain height range (11-14 stories in regions of high to moderate seismicity, 15-20 stories elsewhere). To improve overall economy, structural walls are usually required in taller buildings.

If structural walls are to be incorporated into the framing system, a tentative decision needs to be made at the conceptual design stage concerning their location in plan. Most multi-story buildings are constructed with a central core area. The core usually contains, among other things, elevator hoistways, plumbing and HVAC shafts, and possibly exit stairs. In addition, there may be other exit stairs at one or more locations remote from the core area. All of these involve openings in floors which are generally required by building codes to be enclosed with walls having a fire resistance rating of one hour or two hours, depending on the number of stories connected. In general, it is possible to use such walls for structural purposes.

If at all possible, the structural walls should be located within the plan of the building so that the center of rigidity of the walls coincides with the line of action of the resultant wind loads. This will prevent torsional effects on the structure. Since concrete floor systems act as rigid horizontal diaphragms, they distribute the lateral loads to the vertical framing elements in proportion to their rigidities. The structural walls significantly stiffen the structure and reduce the amount of lateral drift. This is especially true when shearwalls are used with a flat plate floor system.

6.2 FRAME-WALL INTERACTION

The analysis and design of the structural system for a building frame of moderate height can be simplified if the structural walls are sized to carry the entire wind load. Members of the frame (columns and beams or slabs) can then be proportioned to resist the gravity loads only. Neglecting frame-wall interaction for buildings of moderate size and height will result in reasonable member sizes and overall costs. When the walls are at least six times as stiff as the columns in a given direction within a story, the frame takes only a small portion of the lateral loads. Thus, for low-rise buildings, neglecting the contribution of frame action in resisting lateral loads and assigning the total lateral load resistance to walls is an entirely reasonable assumption. In contrast, frame-wall interaction must be considered for high-rise structures where the walls have a significant effect on the frame: in the upper stories, the frame must resist more than 100% of the story shears caused by the wind loads. Thus, neglecting frame-wall interaction would not be conservative at these levels. Clearly, a more economical high-rise structure will be obtained when frame-wall interaction is considered.

With adequate wall bracing, the frame can be considered braced for column design (ACI R10.11.2). Slenderness effects can usually be neglected, except for very slender columns. Consideration of slenderness effects for braced and unbraced columns is discussed in Chapter 5, Section 5.5.

6.3 WALL SIZING FOR LATERAL BRACING

The size of openings required for stairwells and elevators will usually dictate minimum wall plan layouts. From a practical standpoint, a minimum thickness of 6 in. will be required for a wall with a single layer of reinforcement, and 10 in. for a wall with a double layer. While fire resistance requirements will seldom govern wall thickness, these building code requirements should not be overlooked. See Chapter 10 for design considerations for fire resistance. The above requirements will, in most cases, provide stiff enough walls so that the frame can be considered braced.

A simple criterion is given in ACI R10.11.2 to establish whether structural walls provide sufficient lateral bracing to qualify the frame as braced. The shearwalls must have a total stiffness at least six times the sum of the stiffnesses of all the columns in a given direction within a story:

$$I_{(walls)} \geq 6\, I_{(columns)}$$

The above criterion can be used to size the structural walls so that the frame can be considered braced.

6.3.1 Example: Wall Sizing for Braced Condition

Using the approximate criteria given in ACI R10.11.2, size the structural walls for Alternate (2) of Building #2 (5 story flat plate)*. In general, both the N-S and E-W directions must be considered. The E-W direction will be considered in this example since the moment of inertia of the walls will be less in this direction. The plan of Building #2 is shown in Fig. 6-1.

Required fire resistance rating of exit stair enclosure walls = 2 hours

For interior columns: $I = \frac{1}{12}(16^4) = 5461 \text{ in.}^4$

For edge columns: $I = \frac{1}{12}(12^4) = 1728 \text{ in.}^4$

$I_{(columns)} = 8(5461) + 12(1728) = 64{,}424 \text{ in.}^4$

$6I_{(columns)} = 386{,}544 \text{ in.}^4$

**The 5 story flat plate frame of Building #2 is certainly within the lower height range for structural wall consideration. Both architectural and economic considerations need to be evaluated to effectively conclude if structural walls need to be included in low-to-moderate height buildings.*

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Try an 8 in. wall thickness. To accommodate openings required for stairwells, provide 8 ft flanges as shown in Fig. 6-2.

From Table 10-1, for a fire resistance rating of 2 hours, required wall thickness = 4.6 in. ≤ 8 in. O.K.

E-W direction

$A_g = (248 \times 8) + (88 \times 8 \times 2) = 1984 + 1408 = 3392$ in.^{2}

$\bar{x} = [(1984 \times 4) + (1408 \times 52)]/3392 = 23.9$ in.

$I_y = [(248 \times 8^3/12) + (1984 \times 19.9^2)] + [2(8 \times 88^3/12) + (1408 \times 28.1^2)] = 2{,}816{,}665$ in.^{4}

For two walls: $I_{(walls)} = 2(2{,}816{,}665) = 5{,}663{,}330$ in.^{4} >> 386,544 in.^{4}

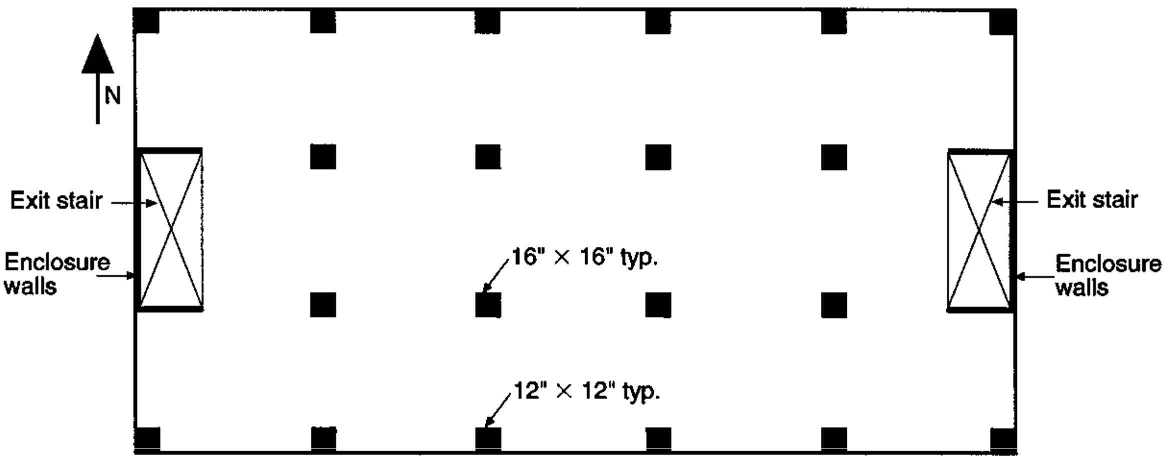

Figure 6-1 Plan of Building #2, Alternate (2)

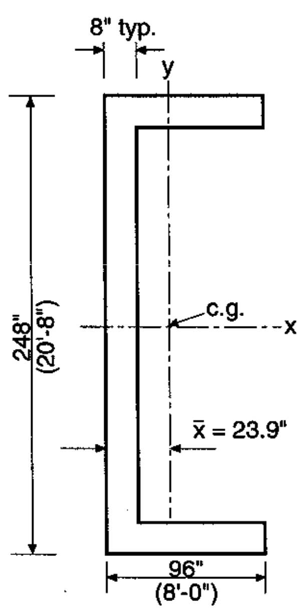

Figure 6-2 Plan View of Shearwall

Therefore, the frame can be considered braced for column design. Since the wall segments in the E-W direction provide most of the stiffness in this direction, the 8 ft length provided for the stairwell enclosure is more than adequate.

6.4 DESIGN FOR SHEAR

Design for horizontal shear forces (in the plane of the wall) can be critical for structural walls with small height-to-length ratios (i.e., walls in low-rise buildings). Special provisions for walls are given in ACI 11.10. In addition to shear, the flexural strength of the wall must also be considered (see Section 6.5).

Walls with minimum amounts of vertical and horizontal reinforcement are usually the most economical. If much more than the minimum amount of reinforcement is required to resist the factored shear forces, a change in wall size (length or thickness) should be considered. The amounts of vertical and horizontal reinforcement required for shear depends on the magnitude of the factored shear force, V_u:

(1) When the factored shear force is less than or equal to one-half the shear strength provided by concrete ($V_u \leq \phi V_c/2$), minimum wall reinforcement according to ACI 14.3 must be provided. For walls subjected to axial compressive forces, ϕV_c may be taken as $\phi 2\sqrt{f'_c}\,hd$ where h is the thickness of the wall, $d = 0.8\ell_w$ (ACI 11.10.4), and ℓ_w is the length of the wall (ACI 11.10.5)*. Suggested vertical and horizontal reinforcement for this situation is given in Table 6-1.

Table 6-1 Minimum Wall Reinforcement ($V_u \leq \phi V_c/2$)

Wall Thickness h (in.)	Vertical		Horizontal	
	Minimum A_s[a] (in.²/ft)	Suggested Reinforcement	Minimum A_s[b] (in.²/ft)	Suggested Reinforcement
6	0.09	#3 @ 15	0.14	#4 @ 16
8	0.12	#3 @ 11	0.19	#4 @ 12
10	0.14	#4 @ 16	0.24	#5 @ 15
12	0.17	#3 @ 15[c]	0.29	#4 @ 16[c]

[a]Minimum A_s/ft of wall = 0.0012(12)h = 0.0144h for #5 bars and less (ACI 14.3.2)
[b]Minimum A_s/ft of wall = 0.0020(12)h = 0.0240h for #5 bars and less (ACI 14.3.3)
[c]Two layers of reinforcement are required (ACI 14.3.4)

(2) When the design shear force is more than one-half the shear strength provided by concrete ($V_u > \phi V_c/2$), minimum shear reinforcement according to ACI 11.10.9 must be provided. Suggested reinforcement (both vertical and horizontal) for this situation is given in Table 6-2.

(3) When the design shear force exceeds the concrete shear strength ($V_u > \phi V_c$), horizontal shear reinforcement must be provided according to ACI Eq. (11-34). Note that the vertical and horizontal reinforcement must not be less than that given in Table 6-2.

Using the same approach as in Section 3.6 for beams, design for required horizontal shear reinforcement in walls when $V_u > \phi V_c$ can be simplified by obtaining specific values for the design shear strength ϕV_s provided by the horizontal reinforcement. As noted above, ACI Eq. (11-34) must be used to obtain ϕV_s:

*ϕV_c *may also be computed by ACI Eqs. (11-32) and (11-33).*

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Table 6-2 Minimum Wall Reinforcement ($\phi V_c/2 < V_u \leq \phi V_c$)

Wall Thickness h (in.)	Vertical and Horizontal	
	Minimum A_s[a] (in.^{2}/ft)	Suggested Reinforcement
6	0.18	#4 @ 13
8	0.24	#4 @ 10
10	0.30	#5 @ 12
12	0.36	#4 @ 13[b]

[a]Minimum A_s/ft of wall = 0.0025(12)h = 0.03h (ACI 11.10.9)
[b]Two layers of reinforcement are required (ACI 14.3.4)

$$\phi V_s = \phi \frac{A_v f_y d}{s_2}$$

where A_v is the total area of the horizontal shear reinforcement within a distance s_2, $\phi = 0.85$, $f_y = 60{,}000$ psi, and $d = 0.8\ell_w$ (ACI 11.10.4). For a wall which is reinforced with #4 bars at 12 in. in a single layer, ϕV_s becomes:

$$\phi V_s = 0.85 \times 0.20 \times 60 \times (0.8 \times 12\ell_w)/12 = 8.2\ell_w \text{ kips}$$

where ℓ_w is the horizontal length of wall in feet.

Table 6-3 gives values of ϕV_s per foot length of wall based on various horizontal bar sizes and spacings.

Table 6-3 Shear Strength ϕV_s Provided by Horizontal Shear Reinforcement*

Bar Spacing s_2 (in.)	ϕV_s (kips/ft length of wall)			
	#3	#4	#5	#6
6	9.0	16.3	25.3	35.9
7	7.7	14.0	21.7	30.8
8	6.7	12.2	19.0	26.9
9	6.0	10.9	16.9	23.9
10	5.4	9.8	15.2	21.5
11	4.9	8.9	13.8	19.6
12	4.5	8.2	12.7	18.0
13	4.1	7.5	11.7	16.6
14	3.9	7.0	10.8	15.4
15	3.6	6.5	10.1	14.4
16	3.4	6.1	9.5	13.5
17	3.2	5.8	8.9	12.7
18	3.0	5.4	8.4	12.0

*Values of ϕV_s are for walls with a single layer of reinforcement. Tabulated values can be doubled for walls with two layers.

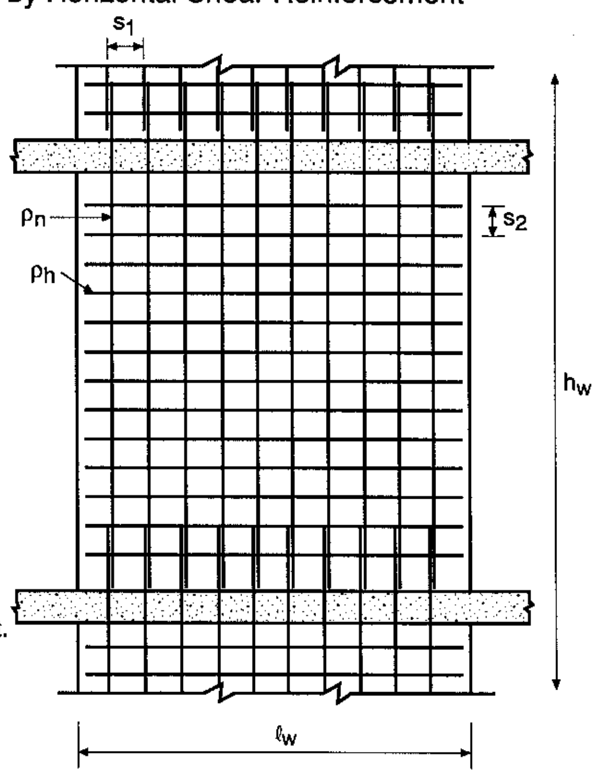


Table 6-4 gives values of $\phi V_c = \phi 2\sqrt{f'_c}\,h(0.8\ell_w)$ and limiting values of $\phi V_n = \phi V_c + \phi V_s = \phi 10\sqrt{f'_c}\,h(0.8\ell_w)$, both expressed in kips per foot length of wall.

Table 6-4 Design Values of ϕV_c and Maximum Allowable ϕV_n

Wall Thickness h (in.)	ϕV_c (kips/ft length of wall)	Max. ϕV_n (kips/ft length of wall)
6	6.2	31.0
8	8.3	41.3
10	10.3	51.6
12	12.4	61.9

The required amount of vertical shear reinforcement is given by ACI Eq. (11-35):

$$\rho_n = 0.0025 + 0.5(2.5 - h_w/\ell_w)(\rho_h - 0.0025)$$

where h_w = total height of wall

$\rho_n = A_{vn}/s_1h$

$\rho_h = A_{vh}/s_2h$

When the wall height-to-length ratio h_w/ℓ_w is less than 0.5, the amount of vertical reinforcement is equal to the amount of horizontal reinforcement (ACI 11.10.9.4). When h_w/ℓ_w is greater than 2.5, only the minimum amount of vertical reinforcement given in Table 6-2 is required.

6.4.1 Example: Design for Shear

To illustrate the simplified methods described above, determine the required shear reinforcement for the wall shown in Fig. 6-3. The service shear force from the wind loading is 200 kips. Assume total height of wall from base to top is 20 ft.

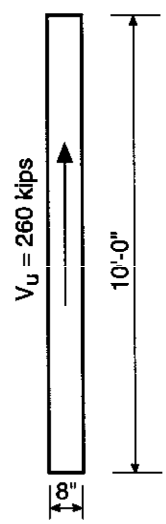


Figure 6-3 Plan View of Shearwall

(1) Determine factored shear force. Use ACI Eq. (9-3) for wind loads only

$$V_u = 1.3(200) = 260 \text{ kips}$$

(2) Determine ϕV_c and maximum allowable ϕV_n

From Table 6-4: $\phi V_c = 8.3 \times 10 = 83$ kips

$\phi V_n = 41.3 \times 10 = 413$ kips

Wall cross section is adequate ($V_u < \phi V_n$); however, shear reinforcement as determined from ACI Eq. (11-34) must be provided ($V_u > \phi V_c$).

(3) Determine required horizontal shear reinforcement

$$\phi V_s = V_u - \phi V_c = 260 - 83 = 177 \text{ kips}$$

$$\phi V_s = 177/10 = 17.7 \text{ kips/ft length of wall}$$

Select horizontal bars from Table 6-3:

For #5 @ 8 in., $\phi V_s = 19.0$ kips/ft > 17.7 kips/ft O.K.

$s_{max} = 18$ in. > 8 in. O.K.

Use #5 @ 8 in. horizontal reinforcement

Note: Use of minimum shear reinforcement for an 8 in. wall thickness is not adequate:

#4 @ 10 in. (Table 6-2) provides $\phi V_s = 9.8$ kips/ft only (Table 6-3).

(4) Determine required vertical shear reinforcement

$$\begin{aligned}\rho_n &= 0.0025 + 0.5(2.5 - h_w/\ell_w)(\rho_h - 0.0025)\\ &= 0.0025 + 0.5(2.5 - 2)(0.0048 - 0.0025)\\ &= 0.0031\end{aligned}$$

where $h_w/\ell_w = 20/10 = 2$

$\rho_h = A_{vh}/s_2h = 0.31/(8 \times 8) = 0.0048$

Required $A_{vn}/s_1 = \rho_n h = 0.0031 \times 8 = 0.0248$ in.^{2}/in.

For #5 bars: $s_1 = 0.31/0.0248 = 12.5$ in. < 18 in. O.K.

Use #5 @ 12 in. vertical reinforcement.

6.4.2 Example: Design for Shear

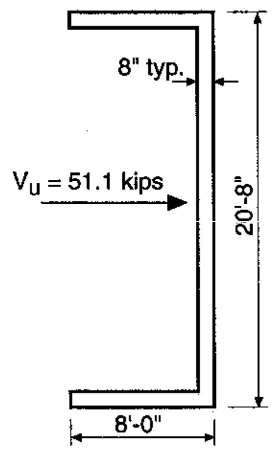
For Alternate (2) of Building #2 (5-story flat plate), select shear reinforcement for the two shearwalls. Assume that the total wind forces are resisted by the walls, with slab-column framing resisting gravity loads only.

(1) E-W direction

Total shear force at base of building (see Chapter 2, Section 2.2.1.1):

$$V = 9.5 + 18.4 + 17.7 + 16.4 + 16.6 = 78.6 \text{ kips}$$

For each shearwall, V = 78.6/2 = 39.3 kips



Factored shear force (use ACI Eq. (9-3) for wind load only):

$$V_u = 1.3(39.3) = 51.1 \text{ kips}$$

For the E-W direction, assume that the shear force is resisted by the two 8 ft flange segments only. For each segment:

$$\phi V_c = 8.3 \times 8 = 66.4 \text{ kips} \qquad \text{(see Table 6-4)}$$

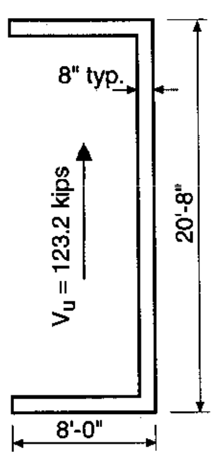
Since V_u for each 8 ft segment = 51.1/2 = 25.6 kips which is less than $\phi V_c/2$ = 66.4/2 = 33.2 kips, provide minimum wall reinforcement from Table 6-1. For 8 in. wall, use #4 @ 12 in. horizontal reinforcement and #3 @ 11 in. vertical reinforcement.

(2) N-S direction

Total shear force at base of building (see Chapter 2, Section 2.2.1.1):

$$V = 22.6 + 43.9 + 42.2 + 39.8 + 41.0 = 189.5 \text{ kips}$$

For each shearwall, V = 189.5/2 = 94.8 kips



Factored shear force:

$$V_u = 1.3(94.8) = 123.2 \text{ kips}$$

For the N-S direction, assume that the shear force is resisted by the 20 ft-8 in. web segment only. From Table 6-4:

$$\phi V_c = 8.3 \times 20.67 = 171.6 \text{ kips}$$

Since $\phi V_c/2 = 85.8$ kips < $V_u = 123.2$ kips < $\phi V_c = 171.6$ kips, provide minimum shear reinforcement from Table 6-2. For 8 in. wall, use #4 @ 10 in. horizontal as well as vertical reinforcement.

(3) Check shear strength in 2nd story in the N-S direction

$$V_u = 1.3(22.6 + 43.9 + 42.2 + 39.8)/2 = 96.5 \text{ kips}$$

The minimum shear reinforcement given in Table 6-2 is still required in the 2nd story since $\phi V_c/2 = 85.8$ kips < $V_u = 96.5$ kips < $\phi V_c = 171.6$ kips. For the 3rd story and above, the minimum wall reinforcement given in Table 6-1 can be used for all wall segments (V_u @ 3rd story = 70.7 kips < $\phi V_c/2 = 85.8$ kips). For horizontal reinforcement, use #4 @ 12 in., and for vertical reinforcement, use #3 @ 11 in.

(4) Summary of Reinforcement

Vertical bars: Use #4 @ 10 in. for 1st and 2nd stories*

#3 @ 10 in. for 3rd through 5th stories**

Horizontal bars: Use #4 @ 10 in. for 1st and 2nd stories

#4 @ 12 in. for 3rd through 5th stories

6.5 DESIGN FOR FLEXURE

For buildings of moderate height, walls with uniform cross-sections and uniformly distributed vertical and horizontal reinforcement are usually the most economical. Concentration of reinforcement at the extreme ends of a wall (or wall segment) is usually not required. Uniform distribution of the vertical wall reinforcement required for shear will usually provide adequate moment strength as well. Minimum amounts of reinforcement will usually be sufficient for both shear and moment requirements.

In general, walls that are subjected to axial load or combined flexure and axial load need to be designed as compression members according to the provisions given in ACI Chapter 10 (also see Chapter 5).*** For rectangular shearwalls containing uniformly distributed vertical reinforcement and subjected to an axial load smaller than that producing balanced failure, the following approximate equation can be used to determine the nominal moment capacity of the wall[6.1] (see Fig. 6-4):

$$\phi M_n = \phi[0.5A_{st}f_y\ell_w(1 + \frac{P_u}{A_{st}f_y})(1 - \frac{c}{\ell_w})]$$

where A_{st} = total area of vertical reinforcement, in.^{2}

ℓ_w = horizontal length of wall, in.

P_u = factored axial compressive load, kips

f_y = yield strength of reinforcement = 60 ksi

*For moment strength, #6 @ 10 in. are required in the 8 ft. wall segments within the first story (see Example 6.5.1).
**Spacing of vertical bars reduced from 11 in. to 10 in. so that the bars in the 3rd story can be spliced with the bars in the 2nd story.
***In particular, ACI 10.2, 10.3, 10.10, 10.11, 10.12, and 10.15 are applicable for walls.

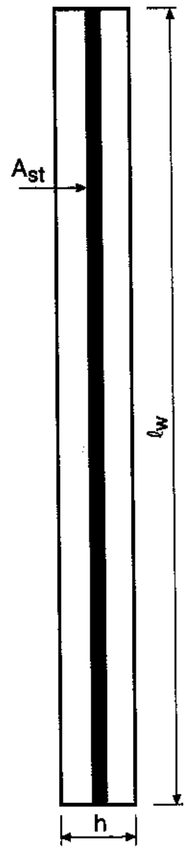


Figure 6-4 Plan View of Shearwall for Approximate Nominal Moment Capacity

$$\frac{c}{\ell_w} = \frac{\omega + \alpha}{2\omega + 0.85\beta_1}, \text{ where } \beta_1 = 0.85 \text{ for } f'_c = 4000 \text{ psi}$$

$$\omega = \left(\frac{A_{st}}{\ell_w h}\right)\frac{f_y}{f'_c}$$

$$\alpha = \frac{P_u}{\ell_w h f'_c}$$

h = thickness of wall, in.

ϕ = 0.90 (strength primarily controlled by flexure with low axial load)

Note that this equation should apply in a majority of cases since the wall axial loads are usually small.

6.5.1 Example: Design for Flexure

For Alternate (2) of Building #2 (5-story flat plate), determine the required amount of moment reinforcement for the two shearwalls. Assume that the 8 ft wall segments resist the wind moments in the E-W direction and the 20 ft-8 in. wall segments resist the wind moments in the N-S direction.

Roof: DL = 122 psf

Floors: DL = 142 psf

(1) Factored loads and load combinations

When evaluating moment strength, the load combination given in ACI Eq. (9-3) will govern.

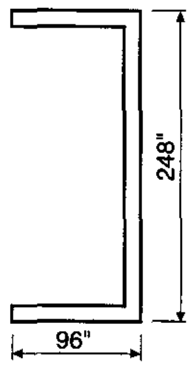
$U = 0.9D + 1.3W$

(a) Dead load at first floor level:

Tributary floor area = $12 \times 40 = 480$ sq ft/story

Wall dead load = $(0.150 \times 3392)/144 = 3.53$ kips/ft of wall height (see Sect. 6.3.1)

$P_u = 0.9[(0.122 \times 480) + (0.142 \times 480 \times 4) + (3.53 \times 63)] = 498$ kips



Proportion total P_u between wall segments:

2–8 ft segments:	$2 \times 96 =$	192 in.	192/440 = 0.44
1–20 ft-8 in. segment:		248 in.	248/440 = 0.56

For 2–8 ft segments: $P_u = 0.44(498) = 219$ kips
1–20 ft-8 in. segment: $P_u = 0.56(498) = 279$ kips

(b) Wind moments at first floor level:

From wind load analysis (see Chapter 2, Section 2.2.1.1):

E-W direction:

$$M_u = 1.3\,[(9.5 \times 63) + (18.4 \times 51) + (17.7 \times 39) + (16.4 \times 27) + (16.6 \times 15)]/2$$
$$= 1898 \text{ ft-kips/shearwall}$$

N-S direction:

$$M_u = 1.3[(22.6 \times 63) + (43.9 \times 51) + (42.2 \times 39) + (39.8 \times 27) + (41 \times 15)]/2$$
$$= 4549 \text{ ft-kips/shearwall}$$

(c) Values of P_u and M_u for the 2nd and 3rd floor levels are obtained in a similar manner:

For 2nd floor level: 2–8 ft segments: $P_u = 171$ kips
1–20 ft-8 in. segment: $P_u = 218$ kips

E-W direction: $M_u = 1131$ ft-kips/shearwall
N-S direction: $M_u = 2701$ ft-kips/shearwall

For 3rd floor level: 2–8 ft segment: $P_u = 128$ kips
1–20 ft-8 in. segment: $P_u = 162$ kips

E-W direction: $M_u = 648$ ft-kips/shearwall
N-S direction: $M_u = 1543$ ft-kips/shearwall

(2) Design for flexure in E-W direction

Initially check moment strength based on the required vertical shear reinforcement #4 @ 10 in. (see Example 6.4.2).

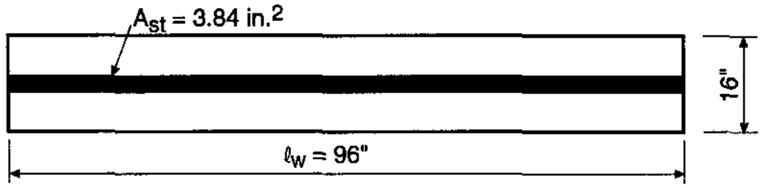
(a) For 2–8 ft wall segments at first floor level:

P_u = 219 kips

M_u = 1898 ft-kips

ℓ_w = 96 in.

combined h = 2(8) = 16 in.



For #4 @ 10 in. (2 wall segments):

$$A_{st} = 2 \times 0.24 \times 8 = 3.84 \text{ in.}^2$$

$$\omega = \left(\frac{A_{st}}{\ell_w h}\right)\frac{f_y}{f_c'} = \left(\frac{3.84}{96 \times 16}\right)\frac{60}{4} = 0.038$$

$$\alpha = \frac{P_u}{\ell_w h f_c'} = \frac{219}{96 \times 16 \times 4} = 0.036$$

$$\frac{c}{\ell_w} = \frac{\omega + \alpha}{2\omega + (0.85 \times 0.85)} = \frac{0.038 + 0.036}{2(0.038) + 0.72} = 0.093$$

$$M_n = 0.5 A_{st} f_y \ell_w \left(1 + \frac{P_u}{A_{st} f_y}\right)\left(1 - \frac{c}{\ell_w}\right)$$

$$= 0.5 \times 3.84 \times 60 \times 96\left(1 + \frac{219}{3.84 \times 60}\right)(1 - 0.093)/12 = 1630 \text{ ft} - \text{kips}$$

ϕM_n = 0.9(1630) = 1467 ft-kips < M_u = 1898 ft-kips N.G.

#4 @ 10 in. is not adequate for moment strength in the E-W direction at the first story level.

Moment strength for #5 @ 10 in. is ϕM_n = 1835 ft-kips which is also less than M_u (calculations not shown here).

Try #6 @ 10 in.:

$$A_{st} = 2 \times 0.53 \times 8 = 8.48 \text{ in.}^2$$

$$\omega = \left(\frac{8.48}{96 \times 16}\right)\frac{60}{4} = 0.083$$

$$\frac{c}{\ell_w} = \frac{0.083 + 0.036}{2(0.083) + 0.72} = 0.134$$

$$M_n = 0.5 \times 8.48 \times 60 \times 96(1 + \frac{219}{8.48 \times 60})(1 - 0.134)/12 = 2521 \text{ ft} - \text{kips}$$

$\phi M_n = 0.9(2521) = 2269$ ft-kips $> M_u = 1898$ ft-kips O.K.

(b) For 2–8 ft wall segments at 2nd floor level:

$P_u = 171$ kips

$M_u = 1131$ ft-kips

Check #4 @ 10 in.:

$A_{st} = 3.84 \text{ in.}^2$

$\omega = 0.038$

$$\alpha = \frac{171}{96 \times 16 \times 4} = 0.028$$

$$\frac{c}{\ell_w} = \frac{0.038 + 0.028}{2(0.038) + 0.72} = 0.083$$

$$M_n = 0.5 \times 3.84 \times 60 \times 96(1 + \frac{171}{3.84 \times 60})(1 - 0.083)/12 = 1472 \text{ ft} - \text{kips}$$

$\phi M_n = 0.9(1472) = 1325$ ft-kips $> M_u = 1131$ ft-kips O.K.

#4 @ 10 in. (required shear reinforcement) is adequate for moment strength above the first floor.

(c) For 2–8 ft wall segments at 3rd floor level:

$P_u = 128$ kips

$M_u = 648$ ft-kips

Check #3 @ 10 in. (required shear reinforcement above 2nd floor):

$A_{st} = 2 \times 0.13 \times 8 = 2.08 \text{ in.}^2$

$$\omega = \left(\frac{2.08}{96 \times 16}\right)\frac{60}{4} = 0.020$$

$$\alpha = \frac{128}{96 \times 16 \times 4} = 0.021$$

$$\frac{c}{\ell_w} = \frac{0.020 + 0.021}{2(0.020) + 0.72} = 0.054$$

$$M_n = 0.5 \times 2.08 \times 60 \times 96(1 + \frac{128}{2.08 \times 60})(1 - 0.054)/12 = 957 \text{ ft} - \text{kips}$$

$\phi M_n = 0.9(957) = 861$ ft-kips $> M_u = 640$ ft-kips O.K.

#3 @ 10 in. (required shear reinforcement) is adequate for moment strength above the 2nd floor.

(3) Design for flexure in N-S direction

Initially check moment strength for required vertical shear reinforcement #4 @ 10 in. (see Example 6.4.2)

(a) For 1–20 ft-8 in. wall segment at first floor level:

P_u = 279 kips

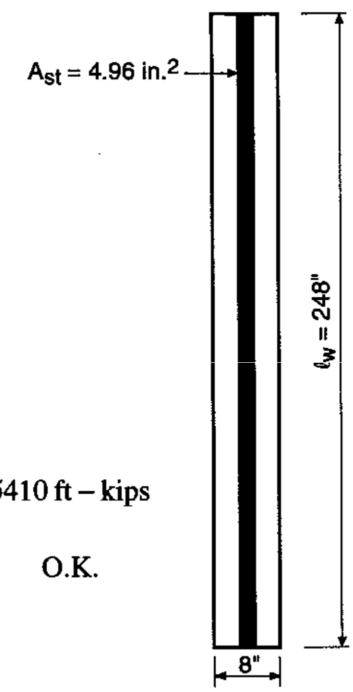
M_u = 4549 ft-kips

ℓ_w = 248 in.

h = 8 in.

For #4 @ 10 in.:

$A_{st} = 0.24 \times 20.67 = 4.96 \text{ in.}^2$



$$\omega = \left(\frac{4.96}{248 \times 8}\right)\frac{60}{4} = 0.038$$

$$\alpha = \frac{279}{248 \times 8 \times 4} = 0.035$$

$$\frac{c}{\ell_w} = \frac{0.038 + 0.035}{2(0.038) + 0.72} = 0.092$$

$$M_n = 0.5 \times 4.96 \times 60 \times 248(1 + \frac{279}{4.96 \times 60})(1 - 0.092)/12 = 5410 \text{ ft} - \text{kips}$$

ϕM_n = 0.9(5410) = 4869 ft-kips > M_u = 4549 ft-kips O.K.

(b) For 1–20 ft-8 in. wall segment at 3rd floor level:

P_u = 162 kips

M_u = 1543 ft-kips

Check #3 @ 10 in. (required shear reinforcement above 2nd floor):

$A_{st} = 0.13 \times 20.67 = 2.69 \text{ in.}^2$

$$\omega = \left(\frac{2.69}{248 \times 8}\right)\frac{60}{4} = 0.020$$

$$\alpha = \frac{162}{248 \times 8 \times 4} = 0.020$$

$$\frac{c}{\ell_w} = \frac{0.020 + 0.020}{2(0.020) + 0.72} = 0.053$$

$$M_n = 0.5 \times 2.69 \times 60 \times 248(1 + \frac{162}{2.69 \times 60})(1 - 0.053)/12 = 3165 \text{ ft} - \text{kips}$$

$$\phi M_n = 0.9(3165) = 2848 \text{ ft-kips} > M_u = 1543 \text{ ft-kips} \qquad \text{O.K.}$$

The required shear reinforcement for the 20 ft-8 in. wall segments is adequate for moment strength for full height of building.

(4) Summary

Required shear reinforcement determined in Example 6.4.2 can be used for the flexural reinforcement except for the 8 ft wall segments within the lst floor where #6 @ 10 in. are required (see Fig. 6-5).

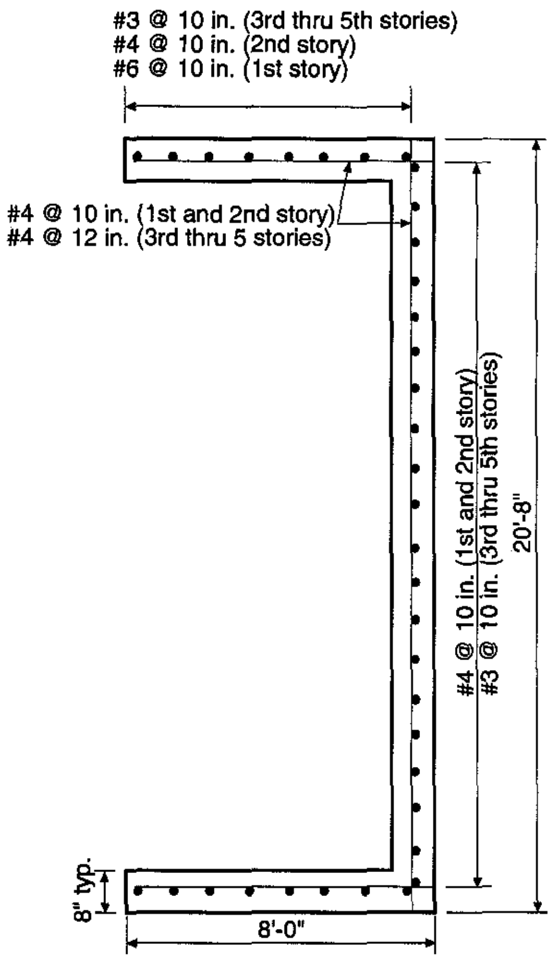
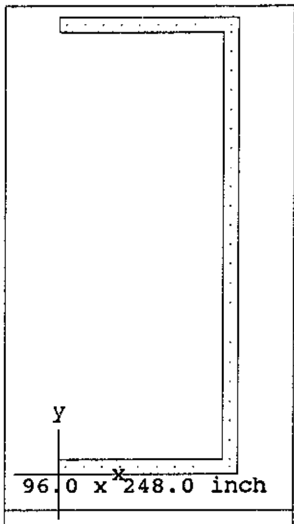
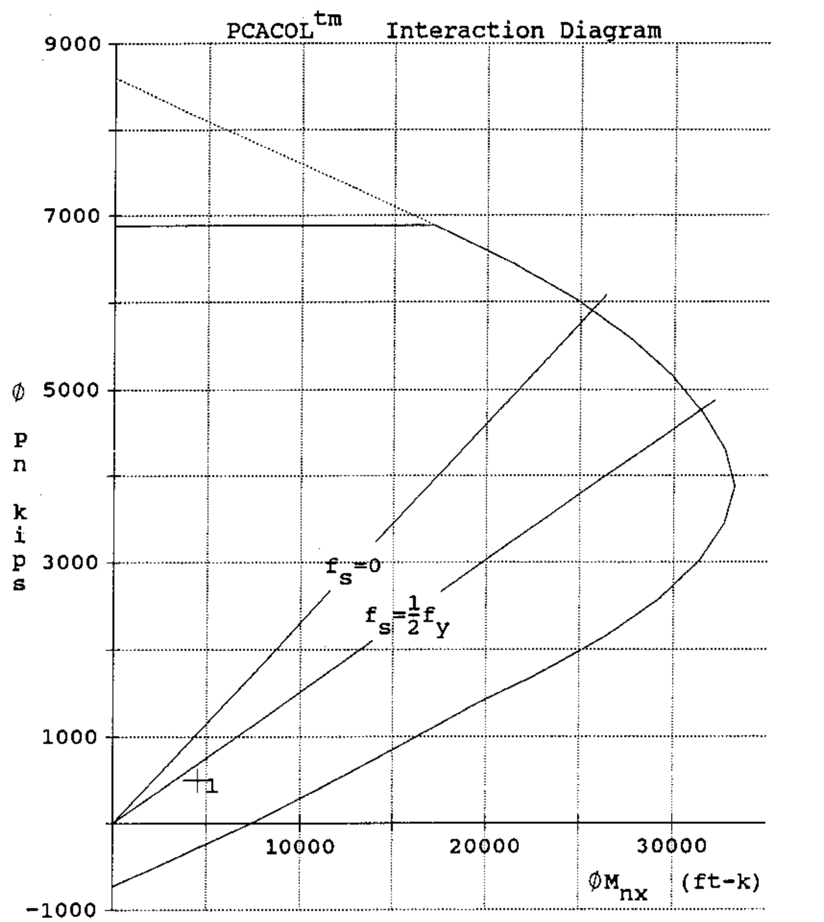


Figure 6-5 Required Reinforcement for Shearwall in Building #2

For comparison purposes, the shearwall was input into PCACOL, using the add-on module IRRCOL which enables the user to investigate any irregularly shaped reinforced concrete column.[6.2] For the reinforcement shown in Fig. 6-5 at the 1st story level, the shearwall was analyzed for the combined factored axial load (due to the dead loads) and moments (due to the wind loads) about each principal axis. The results are shown for the x and y axes in Figs. 6-6 and 6-7, respectively. As expected, the load combination point (represented by point 1 in the figures) is in the lower region of the interaction diagram, with the applied axial load well below the balanced point. Since PCACOL uses the entire cross-section when computing the moment capacity (and not only certain segments as was done in the steps above), the results based on the reinforcement from the approximate analysis will be conservative. It can be shown that the flexural strength of the wall would be adequate if the #6 @ 10 in. were replaced with #5 @ 10 in. in both 8 ft segments.



f'_c = 4.0 ksi
f_y = 60.0 ksi
Confinement:Tied
clr cvr = NA
spacing = 3.37 in
43 bars at 0.4%
A_s = 13.40 in^2
I_x = 30451368 in^4
I_y = 2816662 in^4
$\bar{x}$ = 72.08 in
$\bar{y}$ = 124.00 in



Licensed To: Portland Cement Association, Skokie, IL

Project: SIMPLIFIED DESIGN
Column Id: EX. 6.5.1
Engineer:
Date: Time:
Code: ACI 318-89
Version: 2.20

File name: D:\PCACOL\DATA\SDEX651.COL
Material Properties:
E_c = 3834 ksi ϵ_u = 0.003 in/in
f_c = 3.40 ksi E_s = 29000 ksi
β_1 = 0.85
Stress Profile : Block
Reduction: ϕ_c = 0.70 ϕ_b = 0.90

Slenderness not considered x-axis

Figure 6-6 Interaction Diagram for Shearwall Bending About the X-axis

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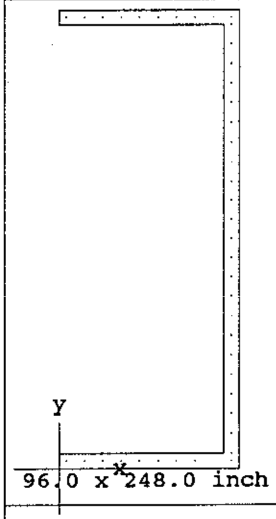
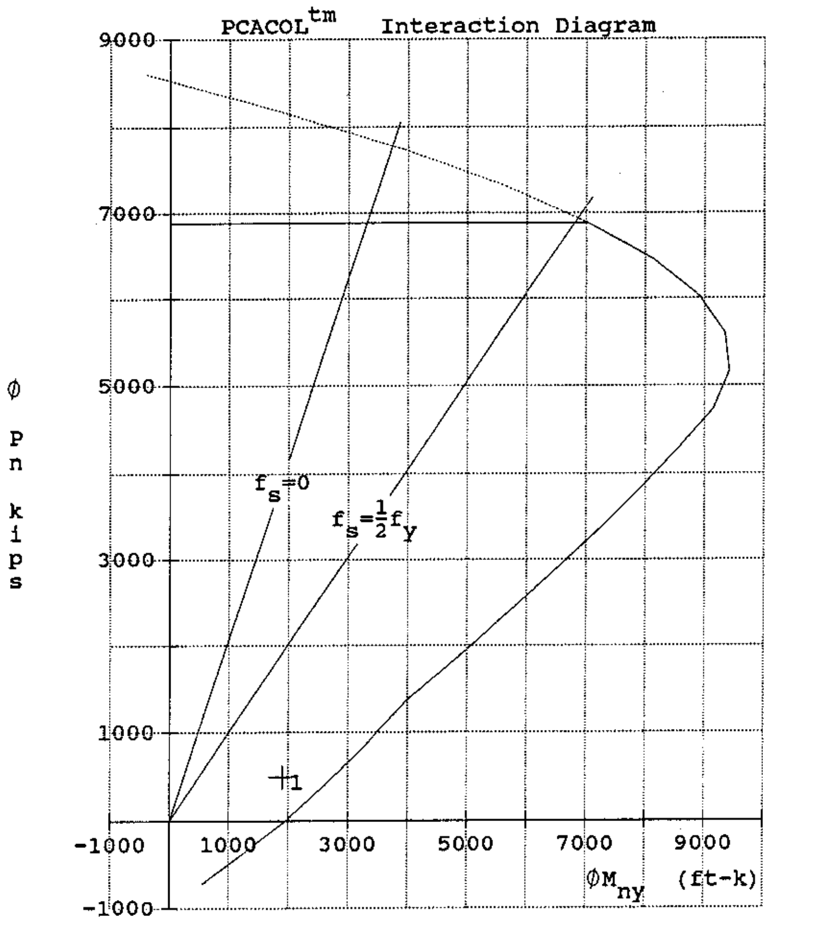


f'_c = 4.0 ksi
f_y = 60.0 ksi
Confinement:Tied
clr cvr = NA
spacing = 3.37 in
43 bars at 0.4%
A_s = 13.40 in^2
I_x = 30451368 in^4
I_y = 2816662 in^4
$\bar{x}$ = 72.08 in
$\bar{y}$ = 124.00 in



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Column Id: EX. 6.5.1
Engineer:
Date: Time:
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Version: 2.20

File name: D:\PCACOL\DATA\SDEX651.COL
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E_c = 3834 ksi ϵ_u = 0.003 in/in
f_c = 3.40 ksi E_s = 29000 ksi
β_1 = 0.85
Stress Profile : Block
Reduction: ϕ_c = 0.70 ϕ_b = 0.90

Slenderness not considered y-axis

Figure 6-7 Interaction Diagram for Shearwall Bending About the Y-axis

References

6.1 Cardenas, A.E., Hanson, J.M., Corley, W.G., Hognestad, E., "Design Provisions for Shearwalls," *Journal of the American Concrete Institute*, Vol. 70, No. 3, March 1973, pp. 221-230.

6.2 *PCACOL - Strength Design of Reinforced Concrete Column Sections and IRRCOL - Irregular Column Sections Module*, Portland Cement Association, Skokie, IL, 1992.

Chapter 7

Simplified Design for Footings

7.1 INTRODUCTION

A simplified method for design of spread footings is presented that can be used to obtain required footing thickness with a one-step design equation based on minimum footing reinforcement. Also included are simplified methods for shear, footing dowels, and horizontal load transfer at the base of a column. A simplified one-step thickness design equation for plain concrete footings is also given. The discussion will be limited to the use of individual square footings supporting square (or circular) columns and subject to uniform soil pressure.

The design methods presented are intended to address the usual design conditions for footings of low-to-moderate height buildings. Footings that are subjected to uplift or overturning are beyond the scope of the simplified method.

A concrete strength of $f'_c = 3000$ psi is the most common and economical choice for footings. Higher strength concrete can be used where footing depth or weight must be minimized, but savings in concrete volume do not usually offset the higher unit price of such concrete.

In certain situations, data are presented for both 3000 and 4000 psi concrete strengths. Also, all of the design equations and data are based on Grade 60 bars which are the standard grade recommended for overall economy.

7.2 PLAIN VERSUS REINFORCED FOOTINGS

Reinforced footings are often used in smaller buildings without considering plain footings. Many factors need to be considered when comparing the two alternatives, the most important being economic considerations. Among the other factors are soil type, job-site conditions, and building size (loads to be transferred). The choice between using reinforcement or not involves a trade-off between the amounts of concrete and steel. The current market prices of concrete and reinforcement are important decision-making parameters. If plain footings can save considerable construction time, then the cost of the extra concrete may be justified. Also, local building codes should be consulted to determine if plain footings are allowed in certain situations.

For a given project, both plain and reinforced footings can be quickly proportioned by the simplified methods in this chapters and an overall cost comparison made (including both material and construction costs). For the same loading conditions, the thickness of a plain footing will be about twice that of a reinforced footing with minimum reinforcement (see Section 7.8).

7.3 SOIL PRESSURE

Soil pressures are usually obtained from a geotechnical engineer or set by local building codes. In cities where experience and tests have established the allowable (safe) bearing pressures of various soils, local building codes may be consulted to determine the bearing capacities to be used in design. In the absence of such information or for conditions where the nature of the soils is unknown, borings or load tests should be made. For larger buildings, borings or load tests should always be made. Table 7-1 lists approximate bearing capacities for some typical foundation materials. The values are averaged from a number of building codes.*

Table 7-1 Average Bearing Capacities of Various Foundation Beds

Soil	Bearing Capacity, q_a (ksf)
Alluvial soil	≤ 1
Soft clay	2
Firm clay	4
Wet sand	4
Sand and clay mixed	4
Fine dry sand (compact)	6
Hard clay	8
Coarse dry sand (compact)	8
Sand and gravel mixed (compact)	10
Gravel (compact)	12
Soft rock	16
Hard pan or hard shale	20
Medium rock	30
Hard rock	80

In general, the base area of the footing is determined using unfactored loads and allowable soil pressures (ACI 15.2.2), while the footing thickness and reinforcement are obtained using factored loads (ACI 15.2.1). Since column design is based on factored loads, it is usually convenient to increase the allowable soil pressure q_a by the composite load factor of 1.6 (see Chapter 2) and use factored loads for the total footing design. As was shown in Chapter 2, use of the composite load factor is accurate enough for ordinary buildings.

7.4 SURCHARGE

In cases where the top of the footing is appreciably below grade (for example, below the frost line) allowances need to be made for the weight of soil on top of the footing. In general, an allowance of 100 pcf is adequate for soil surcharge, unless wet packed conditions exist that warrant a higher value (say 130 pcf). Total surcharge (or overburden) above base of footing can include the loads from a slab on grade, the soil surcharge, and the footing weight.

**The values in Table 7-1 should be used as a guide only. Local soil conditions can result in bearing capacities that are different from the ones listed.*

7.5 ONE-STEP THICKNESS DESIGN FOR REINFORCED FOOTINGS

A simplified footing thickness equation can be derived for individual footings with minimum reinforcement using the strength design data developed in Reference 7.1. The following derivation is valid for $f'_c = 3000$ psi, $f_y = 60{,}000$ psi, and a minimum reinforcement ratio of 0.0018 (ACI 10.5.3).

$$\text{Set } \rho = 0.0018 \times 1.11 = 0.002^*$$

$$R_n = \rho f_y \left(1 - \frac{0.5\rho f_y}{0.85 f'_c}\right)$$

$$= 0.002 \times 60{,}000 \left(1 - \frac{0.5 \times 0.002 \times 60{,}000}{0.85 \times 3000}\right)$$

$$= 117.2 \text{ psi}$$

For a 1 ft wide design strip:

$$d^2_{reqd} = \frac{M_u}{\phi R_n} = \frac{M_u \times 1000}{0.9 \times 117.2} = 9.48 M_u$$

where M_u is in ft-kips.

Referring to Fig. 7-1, the factored moment M_u at the face of the column (or wall) is (ACI 15.4.2):

$$M_u = q_u \left(\frac{c^2}{2}\right) = \frac{P_u}{A_f}\left(\frac{c^2}{2}\right)$$

where c is the largest footing projection from face of column (or wall). Substituting M_u into the equation for d^2_{reqd} results in the following:

$$d^2_{reqd} = 4.74 q_u c^2 = 4.74 \frac{P_u c^2}{A_f}$$

$$d_{reqd} = 2.2\sqrt{q_u c^2} = 2.2c\sqrt{\frac{P_u}{A_f}}$$

The above equation is in mixed units: P_u is in kips, c is in feet, A_f is in square feet, and d is in inches.

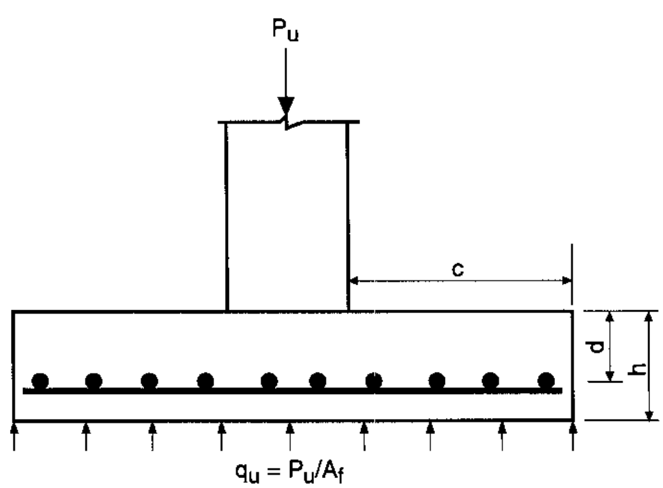

Figure 7-1 Reinforced Footing

**The minimum value of ρ is multiplied by 1.11 to account for the ratio of effective depth d to overall thickness h, assumed as d/h ≅ 0.9.*

The one-step thickness equation derived above is applicable for both square and rectangular footings (using largest value of c) and wall footings. Since f_y has a larger influence on d then does f'_c, the simplified equation can be used for other concrete strengths without a substantial loss in accuracy. As shown in Fig. 7-1, this derivation assumes uniform soil pressure at the bottom of the footing; for footings subject to axial load plus moment, an equivalent uniform soil pressure can be used.

According to ACI 11.12, the shear strength of footings in the vicinity of the column must be checked for both one-way (wide-beam) action and two-way action. Fig. 7-2 illustrates the tributary areas and critical sections for a square column supported by a square footing.

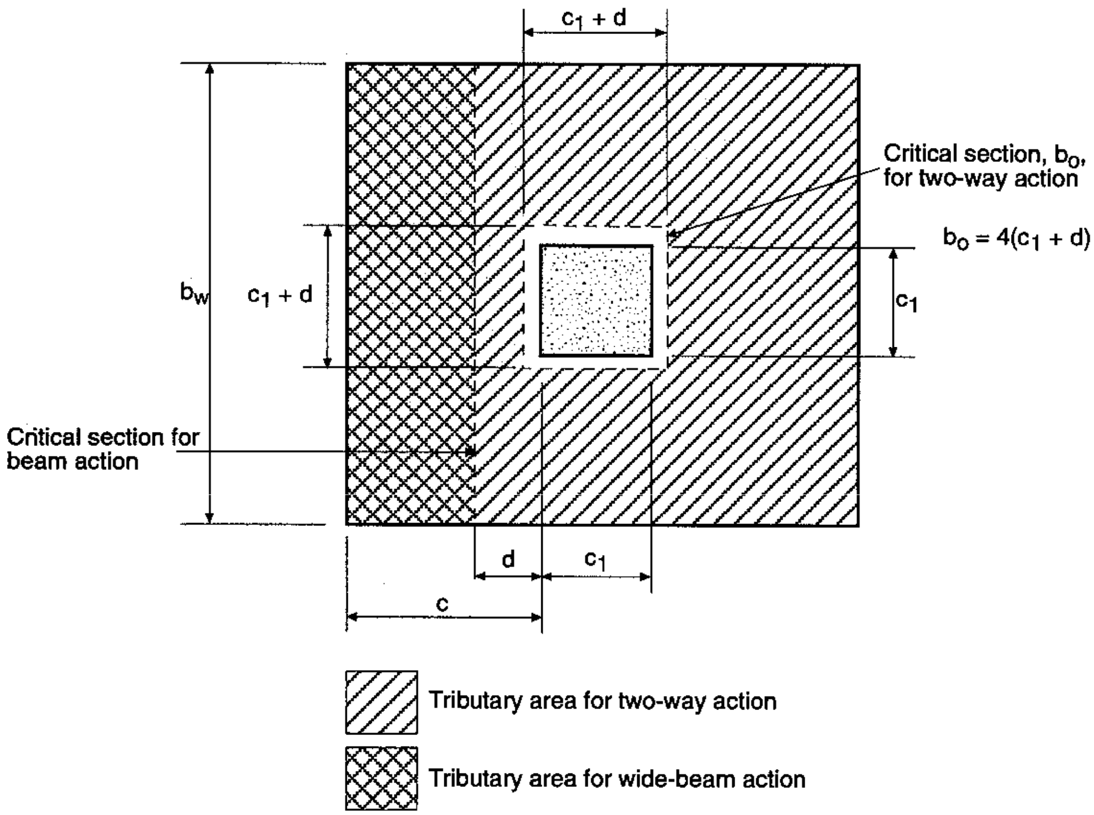

Figure 7-2 Tributary Areas and Critical Sections for Shear in Footings

For wide-beam action:

$$V_u \leq 2\phi\sqrt{f'_c}\,b_w d$$

where b_w is the width of the footing, and V_u is the factored shear on the critical section (at a distance d from the face of the column). In general,

$$V_u = q_u b_w (c - d)$$

The minimum depth d can be obtained from the following equation:

$$\frac{d}{c} = \frac{q_u}{q_u + 2\phi\sqrt{f'_c}}$$

This equation is shown graphically in Fig. 7-3 for f'_c = 3000 psi.

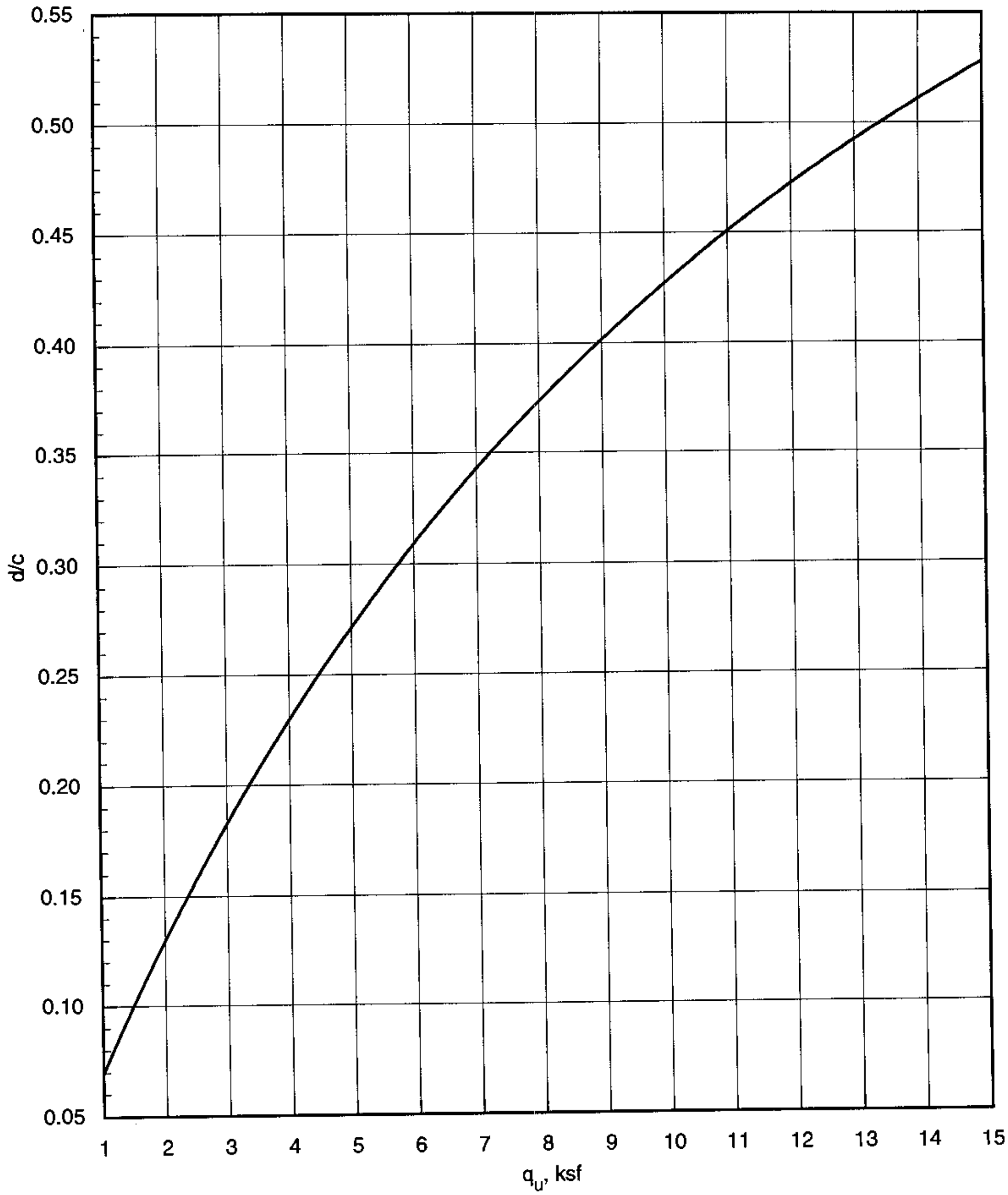

Figure 7-3 Minimum d for Wide-Beam Action

For a footing supporting a square column, the two-way shear strength will be the lesser of the values of V_c obtained from ACI Eqs. (11-37) and (11-38). Eq. (11.37) will rarely govern since the aspect ratio b_o/d will usually be considerably less than the limiting value to reduce the shear strength below $4\sqrt{f'_c}\,b_o d$*. Therefore, for two-way action,

$$V_u \leq 4\phi\sqrt{f'_c}\,b_o d$$

For square interior columns, Eq. (11-37) will govern when $d/c_1 \leq 0.25$.

where, for a square column, the perimeter of the critical section is $b_o = 4(c_1 + d)$. The factored shear V_u on the critical section (at d/2 from the face of the column) can be expressed as:

$$V_u = q_u[A_f - (c_1 + d)^2]$$

where A_f is the area of the footing. The minimum d for two-way shear can be obtained from the following equation:

$$\left(\frac{q_u}{4} + \phi v_c\right)d^2 + \left(\frac{q_u}{2} + \phi v_c\right)c_1 d - \frac{q_u}{4}(A_f - A_c) = 0$$

where A_c = area of the column = c_1^2 and $v_c = 4\sqrt{f'_c}$. Fig. 7-4 can be used to determine d for footings with f'_c = 3000 psi: given q_u and A_f/A_c, the minimum value of d/c_1 can be read from the vertical axis.

Square footings which are designed based on minimum flexural reinforcement will rarely encounter any one-way or two-way shear problems when supporting square columns. For other footing and column shapes, shear strength will more likely control the footing thickness. In any case, it is important to ensure that shear strength of the footing is not exceeded.

7.5.1 Procedure for Simplified Footing Design

(1) Determine base area of footing A_f from service loads (unfactored loads) and allowable (safe) soil pressure q_a determined for the site soil conditions and in accordance with the local building code.

$$A_f = \frac{D + L + W + \text{surcharge (if any)}}{q_a}$$

Using a composite load factor of 1.6, the above equation can be rewritten as

$$A_f = \frac{P_u}{1.6 q_a}$$

The following equations for P_u usually govern:

$$P_u = 1.4D + 1.7L \qquad \text{ACI Eq. (9-1)}$$

$$P_u = 0.75(1.4D + 1.7L + 1.7W) \qquad \text{ACI Eq. (9-2)}$$

(2) Determine required footing thickness h from one-step thickness equation:

$$h = d + 4 \text{ in.*}$$

$$h = 2.2c\sqrt{\frac{P_u}{A_f}} + 4 \text{ in.} \geq 10 \text{ in.}$$

where P_u = factored column load, kips

A_f = base area of footing, sq ft

c = greatest distance from face of column to edge of footing, ft**

h = overall thickness of footing, in.

---

*3 in. cover (ACI 7.7.1) + 1 bar diameter (≅ 1 in.) = 4 in.

**For circular columns, c = distance from the face of an imaginary square column with the same area (ACI 15.3) to edge of footing.

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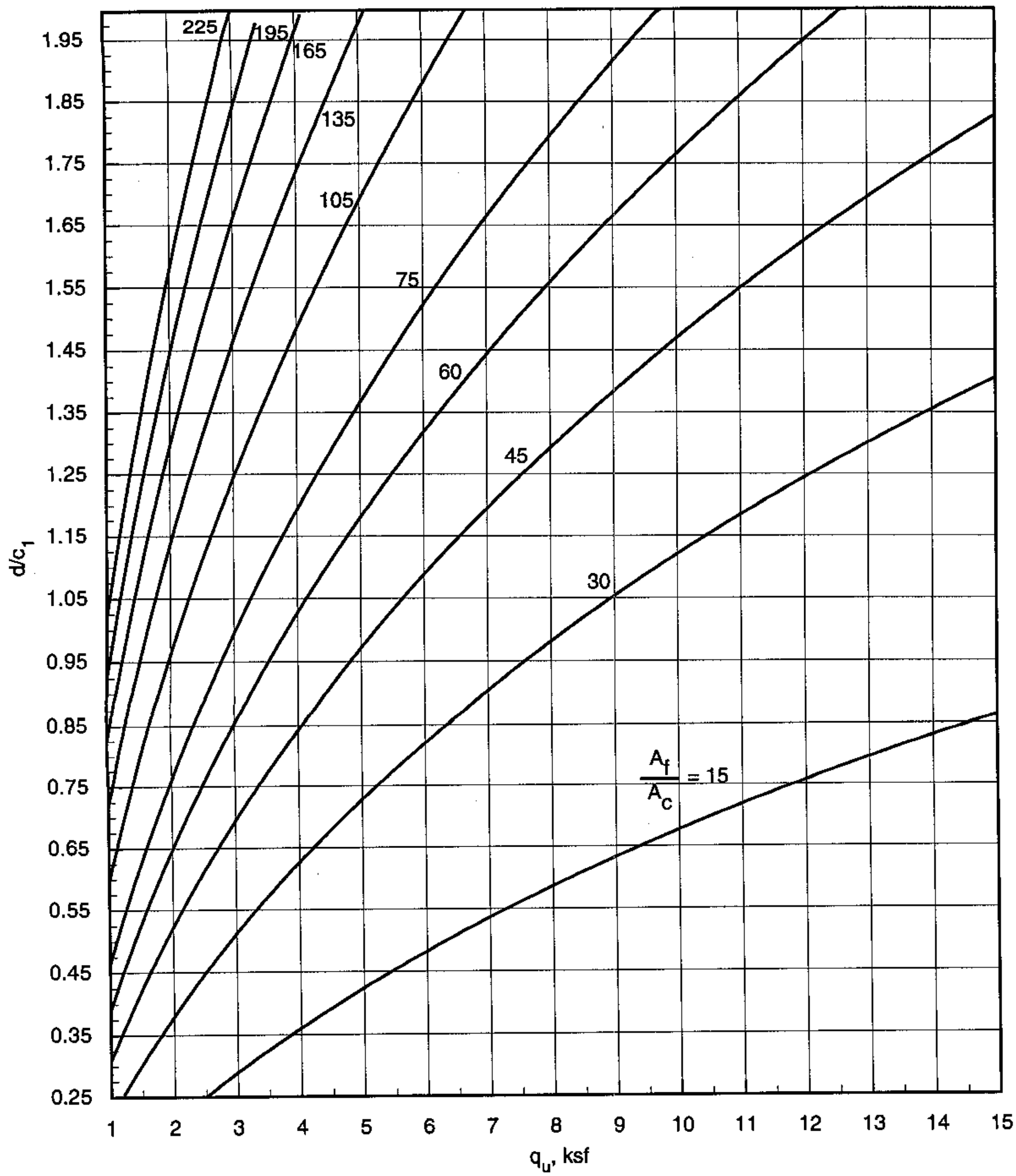

Figure 7-4 Minimum d for Two-Way Action

(3) Determine minimum d for wide-beam action and two-way action from Figs. 7-3 and 7-4, respectively. Use the larger d obtained from the two figures, and compare it to the one obtained in step (2). In general, the value of d determined in step (2) will govern. Note that it is permissible to treat circular columns as square columns with the same cross-sectional area (ACI 15.3).

(4) Determine reinforcement:

$A_s = 0.0018\ bh$

A_s per foot width of footing:

$$A_s = 0.022h \text{ (in.}^2\text{/ft)}$$

Select bar size and spacing from Table 3-7. Note that the maximum bar spacing is 18 in. (ACI 7.6.5). Also, the provisions in ACI 10.6.4, which cover the maximum bar spacing for crack control, does not apply to footings.

The size and spacing of the reinforcement must be chosen so that the bars can become fully developed. The bars must extend at least a distance ℓ_d from each face of the column, where ℓ_d is the tension development length of the bars (ACI 15.6). In every situation, the following conditions must be satisfied (see Fig. 7-5):

$$L \geq 2\ell_d + c_1 + 6 \text{ in.}$$

where L is the width of the footing and c_1 is the width of the column.

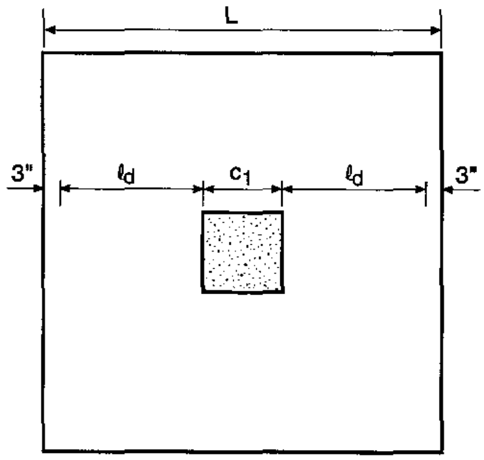

Figure 7-5 Available Development Length for Footing Reinforcement

All of the spacing and cover criteria depicted in Fig. 7-6 are usually satisfied in typical situations; therefore, ℓ_d can be computed from the following (ACI 12.2):

$$\ell_d = \text{greater of} \begin{cases} 0.8\ell_{db} \\ \dfrac{0.03 d_b f_y}{\sqrt{f'_c}} \end{cases}$$

where ℓ_{db} = basic tension development length, in.

$= 0.04 A_b f_y / \sqrt{f'_c}$ (ACI 12.2.2)

A_b = area of an individual bar, in.^{2}

d_b = diameter of a bar, in.

Values of ℓ_d for $f'_c = 3000$ psi and $f'_c = 4000$ psi are given in Table 7-2. In cases where the spacing and/or cover are less than those given in Fig. 7-6, a more detailed analysis using the appropriate modification factors in ACI 12.2 must be performed to obtain ℓ_d.

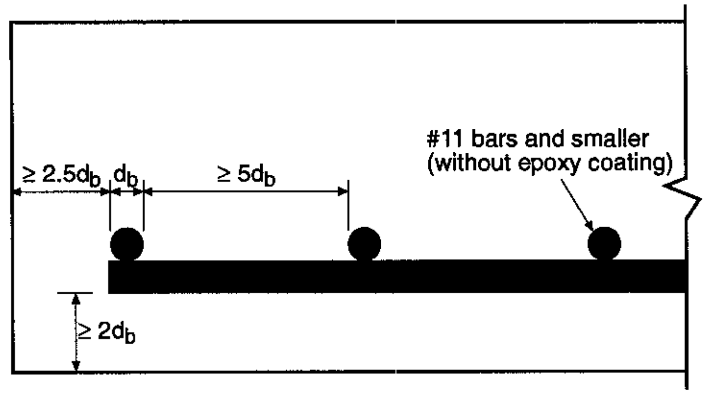

Figure 7-6 Typical Spacing and Cover of Reinforcement in Footings

Table 7-2 Minimum Development Length ℓ_d for Flexural Reinforcement in Footings (Grade 60)*

Bar Size	Development length, ℓ_d (in.)	
	f'_c = 3000 psi	f'_c = 4000 psi
#4	17	15
#5	21	18
#6	25	22
#7	29	25
#8	33	29
#9	38	33
#10	45	39
#11	55	48

*Values of ℓ_d are based on the spacing and cover criteria depicted in Fig. 7-6.

7.6 FOOTING DOWELS

7.6.1 Vertical Force Transfer at Base of Column

The following discussion addresses footing dowels designed to transfer compression forces only. Dowels required to transfer tensile forces created by moments, uplift, or other causes must be transferred to the footings entirely by reinforcement (ACI 15.8.1.2).

Compression forces must be transferred by bearing on concrete and by reinforcement (if required). Bearing strength must be adequate for both column concrete and footing concrete. For the usual case of a footing with a total area considerably larger than the column area, bearing on column concrete will always govern until f'_c of the column concrete exceeds twice that of the footing concrete (ACI 10.15.1). For concrete strength $f'_c = 4000$ psi, the allowable bearing force ϕP_{nb} (in kips) on the column concrete is equal to $\phi P_{nb} = 2.38\ A_g$, where A_g is the gross area of the column in square inches. Values of ϕP_{nb} are listed in Table 7-3 for the column sizes given in Chapter 5.

When the factored column load P_u exceeds the concrete bearing capacity ϕP_{nb}, the excess compression must be transferred to the footing by reinforcement (extended column bars or dowels; see ACI 15.8.2). Total area of reinforcement across the interface cannot be less than 0.5% of the column cross-sectional area (see Table 7-3). For the case when dowel bars are used, it is recommended that at least 4 dowels (one in each corner of the column) be provided.

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Table 7-3 Bearing Capacity and Minimum Area of Reinforcement Across Interface

Column Size	ϕP_{nb} (kips)	Min. area of reinforcement* (in.^{2})
10x10	238	0.50
12x12	343	0.72
14x14	467	0.98
16x16	609	1.28
18x18	771	1.62
20x20	952	2.00
22x22	1152	2.42
24x24	1371	2.88

*Minimum area of reinforcement = $0.005A_g$ (ACI 15.8.2.1)

Figure 7-7 shows the minimum dowel embedment lengths into the footing and column. The dowels must extend into the footing a compression development length of $\ell_{db} = 0.02d_bf_y/\sqrt{f_c'}$, but not less than $0.0003d_bf_y$, where d_b is the diameter of the dowel bar (ACI 12.3.2).* Table 7-4 gives the minimum values of ℓ_{db} for concrete with f_c' = 3000 psi and f_c' = 4000 psi.** The dowel bars are usually extended down to the level of the flexural steel of the footing and hooked 90° as shown in Fig. 7-7. The hooks are tied to the flexural steel to hold the dowels in place. It is important to note that the bent portions of the dowels cannot be considered effective for developing the bars in compression. In general, the following condition must be satisfied when hooked dowels are used:

$$h \geq \ell_{db} + r + d_{bd} + 2d_{bf} + 3 \text{ in.}$$

where r = minimum radius of dowel bar bend (ACI Table 7.2), in.

d_{bd} = diameter of dowel, in.

d_{bf} = diameter of flexural steel, in.

Table 7-4 Minimum Compression Development Length for Grade 60 Bars†

Bar Size	Development length (in.)	
	f_c' = 3000 psi	f_c' = 4000 psi
#4	11	10
#5	14	12
#6	17	15
#7	20	17
#8	22	19
#9	25	22
#10	28	25
#11	31	27

†Tabulated values may be reduced by applicable modification factors in ACI 12.3.3

For straight dowels, the minimum footing thickness h must be ℓ_{db} + 3 in.

In certain cases, the thickness of the footing must be increased in order to accommodate the dowels. If this is not possible, a greater number of smaller dowels can be used.

**The compression development length may be reduced by the applicable factor given in ACI 12.3.3.*

***ℓ_{db} can conservatively be taken as 22 d_b for all concrete with $f_c' \geq 3000$ psi.*

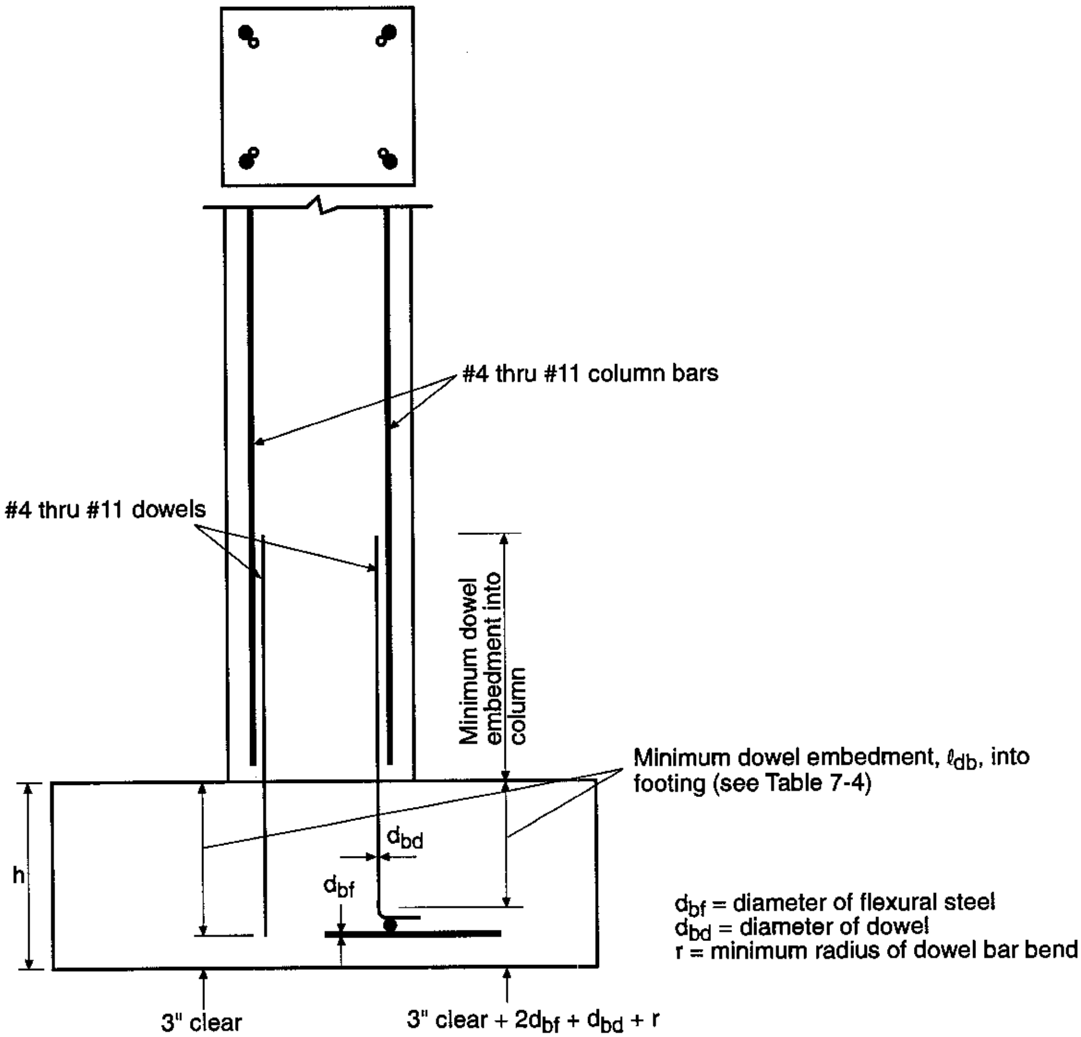

Figure 7-7 Footing Dowels

For the usual case of dowel bars which are smaller in diameter than the column bars, the minimum dowel embedment length into the column must be the larger of the compression development length of the column bar (Table 7-4) or the compression lap splice length of the dowel bar (ACI 12.16.2). The splice length is 0.0005 f_y d_b = 30 d_b for Grade 60 reinforcement, where d_b is the diameter of the dowel bar (ACI 12.16.1). Table 7-5 gives the required splice length for the bar sizes listed. Note that the embedment length into the column is 30 d_b when the dowels are the same size as the column bars.

7.6.2 Horizontal Force Transfer at Base of Column

Footing dowels may be required to transfer lateral forces from the base of the column to the footing. The shear-friction provisions of ACI 11.7 can be used to check the horizontal load-transfer strength of the footing dowel (ACI 15.8.1.4). The dowels need only be adequate for the more severe of the horizontal or vertical transfer conditions. The required area of footing dowels A_{vf} to transfer a horizontal force V_u is computed directly from ACI Eq. (11-26):

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

Table 7-5 Minimum Compression Lap Splice Length for Grade 60 Bars*

Bar Size	Lap Splice Length (in.)
#4	15
#5	19
#6	23
#7	26
#8	30
#9	34
#10	38
#11	42

* $f'_c \geq 3000$ psi

where μ = 0.6 for concrete placed against hardened concrete not intentionally roughened

= 1.0 for concrete placed against hardened concrete with the surface intentionally roughened as specified in ACI 11.7.9.

$\phi = 0.85$

The horizontal force V_u to be transferred cannot exceed $\phi(0.2 f'_c A_c)$ in pounds where A_c is gross area of column (ACI 11.7.5). For a column concrete strength $f'_c = 4000$ psi, this maximum force is equal to $\phi(800\ A_c)$.

Dowels required to transfer horizontal force must have full tensile anchorage into the footing and into the column (ACI 11.7.8). The values given in Table 8-1 and 8-5 can be modified by the appropriate factors given in ACI 12.2.

7.7 EXAMPLE: REINFORCED FOOTING DESIGN

Design footings for the interior columns of Building #2 (5-story flat plate). Assume base of footings located 5 ft below ground level floor slab (see Fig. 7-8). Permissible soil pressure $q_a = 4.5$ ksf.

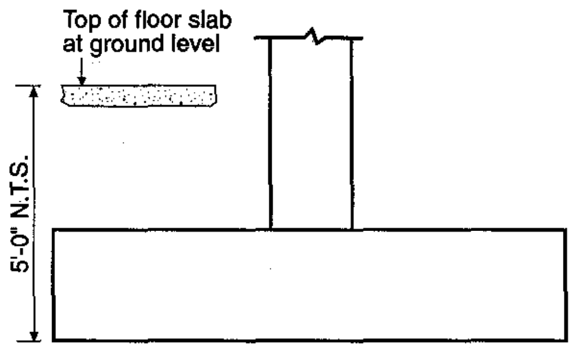

Figure 7-8 Interior Footing for Building #2

(1) Design Data:

Service surcharge = 50 psf

Assume weight of soil and concrete above footing base = 130 pcf

Interior columns: 16 in. × 16 in. (see Examples 5.7.1 and 5.7.2)

4-#8 bars (braced frame)

8-#10 bars (unbraced frame)

$$f'_c = 4000 \text{ psi (column)}$$

$$f'_c = 3000 \text{ psi (footing)}$$

(2) Load combinations

(a) gravity loads: $P_u = 587$ kips (Alternate (2))

$M_u = 11$ ft-kips

(b) gravity + wind loads: $P_u = 428$ kips (Alternate (1))

$M_u = 194$ ft-kips*

(3) Base area of footing

Determine footing base area for gravity loads only, then check footing size for gravity plus wind loads.

Total weight of surcharge = (0.130 × 5) + 0.05 = 0.70 ksf

Net permissible soil pressure = 4.5 - 0.70 = 3.8 ksf

$$A_f = \frac{P_u}{1.6q_a} = \frac{587}{1.6(3.8)} = 96.6 \text{ sq ft}^{**}$$

Use 9 ft-10 in. × 9 ft-10 in. square footing ($A_f = 96.7$ sq ft)

Check gravity plus wind loading for 9 ft-10 in. × 9 ft-10 in. footing:

$$A_f = 96.7 \text{ sq ft}$$

$$S_f = bh^2/6 = 9.83^3/6 = 158.3 \text{ ft}^3$$

$$q_u = \frac{P_u}{A_f} + \frac{M_u}{S_f} = \frac{428}{96.7} + \frac{194}{158.3} = 4.43 + 1.23 = 5.66 \text{ ksf}$$

$$< 1.6(3.8) = 6.08 \text{ ksf} \quad \text{O.K.}$$

Gravity load governs since

$$q_u\,(\text{gravity}) = \frac{587}{96.7} = 6.07 \text{ ksf} > q_u\,(\text{gravity + wind}) = 5.66 \text{ ksf}$$

(4) Footing thickness

Footing projection = c = [(9.83 - 16/12)]/2 = 4.25 ft

$$h = 2.2c\sqrt{\frac{P_u}{A_f}} + 4 \text{ in.} = 2.2(4.25)\sqrt{\frac{587}{96.7}} + 4 = 23.0 + 4 = 27.0 \text{ in.} > 10 \text{ in.} \quad \text{O.K.}$$

Try h = 27 in. (2 ft-3 in.)

**For the unbraced frame, the total moment transferred to the footing is equal to the magnified column moment (see Fig. 5-13 where point 2 represents the load combination for this case).*

***Neglect small moment due to gravity loads.*

Check if the footing thickness is adequate for shear:

$$d \cong 27 - 4 = 23 \text{ in.}$$

For wide-beam shear, use Fig. 7-3. With $q_u = 6.07$ ksf, read $d/c \cong 0.31$. Therefore, the minimum d is

$$d = 0.31 \times 4.25 = 1.32 \text{ ft} = 15.8 \text{ in.} < 23 \text{ in.} \qquad \text{O.K.}$$

Use Fig. 7-4 for two-way shear:

$$\frac{A_f}{A_c} = \frac{96.7}{(16^2/144)} = 54.4$$

Interpolating between $A_f/A_c = 45$ and 60, read $d/c_1 \cong 1.25$ for $q_u = 6.07$ ksf. The minimum d for two-way shear is:

$$d = 1.25 \times 16 = 20.0 \text{ in.} < 23 \text{ in.} \qquad \text{O.K.}$$

Therefore, the 27 in. footing depth (d = 23 in.) is adequate for flexure and shear.

(5) Footing reinforcement

$$A_s = 0.022\,h = 0.022(27) = 0.59 \text{ in.}^2\text{/ft}$$

Try #7 @ 12 in. ($A_s = 0.60$ in.^{2}/ft; see Table 3-7)

Determine the development length of the #7 bars (see Fig. 7-6):

$$\text{cover} = 3 \text{ in.} > 2d_b = 2 \times 0.875 = 1.8 \text{ in.}$$

$$\text{side cover} = 3 \text{ in.} > 2.5 \times 0.875 = 2.2 \text{ in.}$$

$$\text{clear spacing} = 12 - 0.875 = 11.1 \text{ in.} > 5 \times 0.875 = 4.4 \text{ in.}$$

Since all of the cover and spacing criteria given in Fig. 7-6 are satisfied, Table 7-2 can be used to determine the minimum development length.

For $f'_c = 3000$ psi: $\ell_d = 29$ in.

Check available development length:

$$L = 9.83 \times 12 = 118 \text{ in.} > (2 \times 29) + 16 + 6 = 80 \text{ in.} \qquad \text{O.K.}$$

Total bars required:

$$\frac{118 - 6}{12} = 9.33 \text{ spaces}$$

Use 10-#7, 9 ft-4 in. long (each way)*

(6) Footing dowels

Footing dowel requirements are different for braced and unbraced frames. For the unbraced frame, with wind moment transferred to the base of the column, all of the tensile forces produced by the moment must be transferred to the footing by dowels. The number and size of dowel bars will depend on the tension development length of the hooked end of the dowel and the thickness of the footing. The dowel bars must also be fully developed for tension in the column.

**13-#6 or 8-#8 would also be adequate.*

For the braced frame, subjected to gravity loads only, dowel requirements are determined as follows:*

(a) For 16 × 16 in. column (Table 7-3):

$$\phi P_{nb} = 609 \text{ kips}$$

Minimum dowel area = 1.28 in.^{2}

Since $\phi P_{nb} > P_u = 587$ kips, bearing on concrete alone is adequate for transfer of compressive force.

Use 4-#6 dowels (A_s = 1.76 in.^{2})

(b) Embedment into footing (Table 7-4):

For straight dowel bars,

$h \geq \ell_{db} + 3$ in.

$\ell_{db} = 17$ in. for #6 dowels with $f'_c = 3000$ psi

$h = 27$ in. $> 17 + 3 = 20$ in. O.K.

For hooked dowel bars,

$h \geq \ell_{db} + r + d_{bd} + 2d_{bf} + 3$ in.

$r = 3d_{bd} = 3 \times 0.75 = 2.25$ in. (ACI Table 7.2)

$h = 27$ in. $> 17 + 2.25 + 0.75 + (2 \times 0.875) + 3 = 24.75$ in. O.K.

(c) Embedment into column:

The minimum dowel embedment length into the column must be the larger of the following:

- compression development length of #8 column bars ($f'_c = 4000$ psi) = 19 in. (Table 7-4)
- compression lap splice length of #6 dowel bars = 23 in. (Table 7-5) (governs)

For #6 hooked dowels, the total length of the dowels is

$$23 + [27 - 3 - (2 \times 0.875)] = 45.25 \text{ in.}$$

Use 4-#6 dowels × 3 ft-10 in.

Figure 7-9 shows the reinforcement details for the footing in the braced frame.

7.8 ONE-STEP THICKNESS DESIGN FOR PLAIN FOOTINGS

Depending on the magnitude of the loads and the soil conditions, plain concrete footings may be an economical alternative to reinforced concrete footings. Structural plain concrete members are designed according to ACI Standard 318.1.[7.2] For plain concrete, the maximum permissible flexural tension stress under factored load conditions is $5\phi\sqrt{f'_c}$ (ACI 318.1, Section 6.2.1). With $\phi = 0.65$ (ACI 318.1, Section 6.2.2), the permissible tension stress f_t is

For $f'_c = 3000$ psi: $f_t = 178$ psi

For $f'_c = 4000$ psi: $f_t = 206$ psi

The horizontal forces produced by the gravity loads in the first story columns are negligible; thus, dowels required for vertical load transfer will be adequate for horizontal load transfer as well.

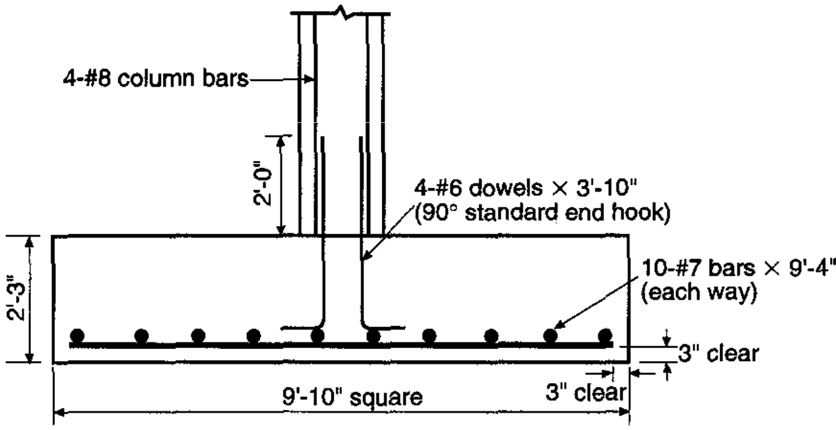

Figure 7-9 Reinforcement Details for Interior Column in Building #2 (Braced Frame)

A simplified one-step thickness design equation can be derived as follows (see Fig. 7-10):

$$M_u \leq M_n = f_t S$$

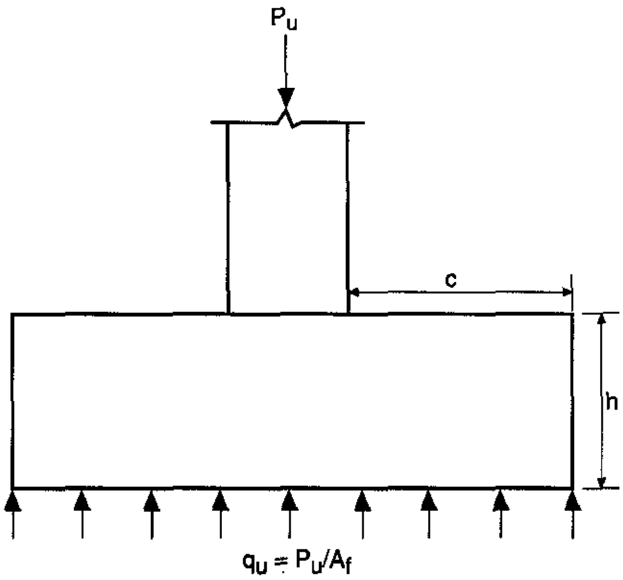

Figure 7-10 Plain Concrete Footing

For a one-foot design strip:

$$q_u = \left(\frac{c^2}{2}\right) \leq f_t \left(\frac{h^2}{6}\right)$$

$$h^2_{reqd} = 3q_u \frac{c^2}{f_t} = \frac{3P_u c^2}{A_f f_t}$$

To allow for unevenness of excavation and for some contamination of the concrete adjacent to the soil, an additional 2 in. in overall thickness is required for plain concrete footings (ACI 318.1, Section 6.3.5); thus,

$$\text{For } f'_c = 3000 \text{ psi:} \quad h = 4.1c\sqrt{\frac{P_u}{A_f}} + 2 \text{ in.}$$

$$\text{For } f'_c = 4000 \text{ psi:} \quad h = 3.8c\sqrt{\frac{P_u}{A_f}} + 2 \text{ in.}$$

The above footing thickness equations are in mixed units:

P_u = factored column load, kips
A_f = base area of footing, sq ft
c = greatest distance from face of column to edge of footing, ft (ACI 318.1, Section 7.2.5)
h = overall thickness of footing, in. ≥ 8 in. (ACI 318.1, Section 7.2.4)

Thickness of plain concrete footings will be controlled by flexural strength rather than shear strength for the usual proportions of plain concrete footings. Shear rarely will control. For those cases where shear may cause concern, the allowable stresses are given in ACI 318.1, Section 6.2.1 and the maximum factored stresses are given in ACI 318.1, Section 7.2.6.

7.8.1 Example: Plain Concrete Footing Design

For the interior columns of Building #2, (Braced Frame), design a plain concrete footing.

From Example 7.7:
A_f = 9 ft-10 in. × 9 ft-10 in. = 96.7 sq ft
P_u = 587 kips
c = footing projection = 4.25 ft

For f'_c = 3000 psi:

$$h = 4.1c\sqrt{\frac{P_u}{A_f}} + 2 \text{ in.} = 4.1(4.25)\sqrt{\frac{587}{96.7}} + 2 = 42.9 + 2 = 44.9 \text{ in.}$$

Bearing on column:

Allowable bearing load $= 0.85\phi f'_c A_g$, $\phi = 0.65$ ACI 318.1, Section 6.2.1
$= 0.85 \times 0.65 \times 4 \times 16^2 = 566 \text{ kips} \cong P_u = 587 \text{ kips}$

Figure 7-11 illustrates the footing for this case.

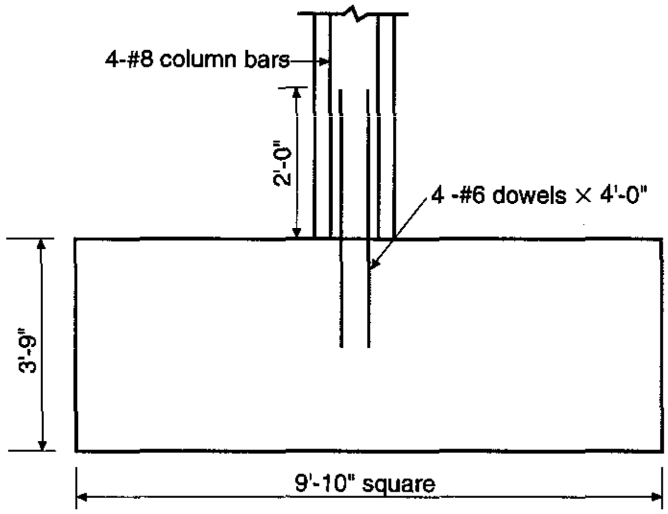

Figure 7-11 Plain Concrete Footing for Interior Column of Building #2 (Braced Frame)

References

7.1 *Notes on ACI-89*, Chapter 10, Design for Flexure, 5th Edition, EB070, Portland Cement Association, Skokie, IL, 1990.

7.2 *Building Code Requirements for Structural Plain Concrete*, ACI 318.1-89 (Revised 1992), American Concrete Institute, Detroit, 1992, 14 pp.

Chapter 8

Structural Detailing of Reinforcement for Economy

8.1 INTRODUCTION

Structurally sound details and proper bar arrangements are vital to the satisfactory performance of reinforced concrete structures. The details and bar arrangements should be practical, buildable, and cost-effective.

Ideally, the economics of reinforced concrete should be viewed in the broad perspective, considering all facets in the execution of a project. While it may be important to strive for savings in materials, many engineers often tend to focus too much on material savings rather than on designing for construction efficiencies. No doubt savings in material quantities should result from a highly refined "custom design" for each structural member in a building. However, such a savings in materials might be false economy if significantly higher construction costs are incurred in building the custom-designed members.

Trade-offs should be considered in order to minimize the total cost of construction, including the total in-place cost of reinforcement. Savings in reinforcement weight can be traded-off for savings in fabrication, placing, and inspection for overall economy.

8.2 DESIGN CONSIDERATIONS FOR REINFORCEMENT ECONOMY

The following notes on reinforcement selection and placement will usually provide for overall economy and may minimize costly project delays and job stoppages:

(1) First and foremost, show clear and complete reinforcement details and bar arrangements in the Contract Documents. This issue is addressed in Section 1.1 of *Details and Detailing of Concrete Reinforcement (ACI 315-80) (Revised 1986)*[8.1]: "...the responsibility of the Engineer is to furnish a clear statement of design requirements; the responsibility of the [Reinforcing Steel] Detailer is to carry out these requirements."

ACI 318 further emphasizes that the designer is responsible for the size and location of all reinforcement and the types, locations, and lengths of splices of reinforcement (ACI 1.2.1 and 12.14.1).

(2) Use Grade 60 reinforcing bars. Grade 60 bars are the most widely used and are readily available in all sizes up to and including #11; #14 and #18 bars are not generally inventoried in regular stock. Also, bar sizes smaller than #6 generally cost more per pound and require more placing labor per pound of reinforcement.

(3) Use straight bars only in flexural members. Straight bars are regarded as standard in the industry. Truss (bent) bars are undesirable from a fabrication and placing standpoint, and structurally unsound where stress reversals occur.

(4) In beams, specify bars in single layers only. Use one bar size for reinforcement on one face at a given span location. In slabs, space reinforcement in whole inches, but not at less than a 6-in. spacing.

(5) Use largest bar sizes possible for the longitudinal reinforcement in columns. Use of larger bar sizes and fewer bars in other structural members will be restricted by code requirements for development of reinforcement, limits on maximum spacings, and distribution of flexural reinforcement.

(6) Use or specify fewest possible bar sizes for a project.

(7) Stirrups are typically the smaller bar sizes, which usually result in the highest total in-place cots of reinforcement per ton. For overall economy and to minimize congestion of reinforcement, specify the largest stirrup bar size (fewest number of stirrups) and the fewest variations in spacing. Stirrups which are spaced at the maximum allowable spacing are usually the most economical.

(8) When closed stirrups are required, specify two-piece closed types to facilitate placing.

(9) Fit and clearance of reinforcing bars warrant special attention by the Engineer. At beam-column joints, arrangement of column bars must provide enough space or spaces to permit passage of beam bars. Bar details should be properly prepared and reconciled before the bars are fabricated and delivered to the job site. Member connections are far too important to require indiscriminate adjustments in the field to facilitate bar placing.

(10) Use or specify standard reinforcing bar details and practices:

- Standard end hooks (ACI 7.1). Note that the tension development length provisions in ACI 12.5 are only applicable for standard hooks conforming to ACI 7.1.

- Typical bar bends (see ACI 7.2 and Fig. 6 in Ref. 8.1).

- Standard fabricating tolerances (Fig. 4 in Ref. 8.1). More restrictive tolerances must be indicated by the Engineer in the Contract Documents.

- Tolerances for placing reinforcing bars (ACI 7.5). More restrictive tolerances must be indicated by the Engineer in the Contract Documents.

Care must be exercised in specifying more restrictive tolerances for fabricating and placing reinforcing bars. More restrictive fabricating tolerances are limited by the capabilities of shop fabrication equipment. Fabricating and placing tolerances must be coordinated. Tolerances for the formwork must also be considered and coordinated.

(11) Never permit field welding of crossing reinforcing bars for assembly of reinforcement ("tack" welding, "spot" welding, etc.). Tie wire will do the job without harm to the bars.

(12) Avoid manual arc-welded splices of reinforcing bars in the field wherever possible, particularly for smaller projects.

(13) A frequently occurring construction problem is having to make field corrections to reinforcing bars partially embedded in hardened concrete. Such "job stoppers" usually result from errors in placing or fabrication, accidental bending caused by construction equipment, or a design change. Field bending of bars partially embedded in concrete is not permitted except if such bending is shown on the design drawings or authorized by the Engineer (ACI 7.3.2). ACI R7.3 offers guidance on this subject. Further guidance on bending and straightening of reinforcing bars is given in Reference 8.2.

8.3 REINFORCING BARS

Billet-steel reinforcing bars conforming to ASTM A 615, Grade 60, are the most widely used type and grade in the United States. Combining the Strength Design Method with Grade 60 bars results in maximum overall economy. This design practice has made Grade 60 reinforcing bars the standard grade. The current edition of ASTM A 615 reflects this practice as only bar sizes #3 through #6 in Grade 40 are included in the specification. Also listed are Grade 75 bars in sizes #11, #14, and #18 only. These larger bar sizes in Grade 75 are usually used in columns made of high strength concrete in high-rise buildings. The combination of high strength concrete and Grade 75 bars may result in smaller column sizes, and, thus, more rentable space, especially in the lower levels of a building. It is important to note that Grade 75 bars may not be readily available in all areas of the country; also, as mentioned above, #14 and #18 bars are not commonly available in distributors' stock. ACI 3.5.3.2 permits the use of Grade 75 bars provided that they meet all the requirements listed in that section (also see ACI 9.4).

When important or extensive welding is required, or when more bendability and controlled ductility are required (as in seismic construction*), use of low-alloy reinforcing bars conforming to ASTM A 706 should be considered. Note that the specification covers only Grade 60 bars. Local availability should be investigated before specifying A 706 bars.

8.3.1 Coated Reinforcing Bars

Zinc-coated (galvanized) and epoxy-coated reinforcing bars are used increasingly for corrosion-protection in reinforced concrete structures. An example of a structure that might use coated bars is a parking garage where vehicles track in deicing salts.

Zinc-coated (galvanized) reinforcing bars must conform to ASTM A 767; also, the reinforcement to be coated must conform to one of the specifications listed in ACI 3.5.3.1. Bars are usually fabricated before galvanizing. In these cases, the minimum finished bend diameters given in Table 2a of ASTM A 767 must be specified. ASTM A 767 has two classes of coating weights. Class I (3.5 oz./sq ft of surface) is normally specified for general construction. ASTM A 767 contains three supplementary requirements: S1, S2, and S3. S1 requires sheared ends to be coated with a zinc-rich formulation. When bars are fabricated after galvanizing, S2 requires that the damaged coating be repaired with a zinc-rich formulation. If ASTM A 615 billet-steel bars are being supplied, S3 requires that a silicon analysis of each heat of steel be provided. It is recommended that S1 and S2 be specified when fabrication after galvanization includes cutting and bending. S2 should be specified when fabrication after galvanization includes only bending.

ACI 21.2.5.1 specifically requires reinforcing bars complying with ASTM A 706 to be used in frame members and in wall boundary elements subjected to seismic forces. Note that ASTM A 615 Grade 40 and Grade 60 bars are also allowed if they meet all of the requirements in the section.

Uncoated reinforcing steel (or any other embedded metal dissimilar to zinc) should not be permitted in the same concrete element with galvanized bars, nor in close proximity to galvanized bars, except as part of a cathodic-protection system. Galvanized bars should not be coupled to uncoated bars.

Epoxy-coated reinforcing bars must conform to ASTM A 775, and the reinforcement to be coated must conform to one of the specifications listed in ACI 3.5.3.1. The film thickness of the coating after curing shall be 5 to 12 mils (0.13 to 0.30 mm). Also, there shall not be more than an average of two holidays (pinholes not discernible to the unaided eye) per linear foot of the coated bar.

Proper use of ASTM A 767 and A 775 requires the inclusion of provisions in the project specifications for the following items:

- Compatible tie wire, bar supports, support bars, and spreader bars in walls.
- Repair of damaged coating after completion of welding (splices) or installation of mechanical connections.
- Repair of damaged coating after completion of field corrections, when field bending of coated bars partially embedded in concrete is permitted.
- Minimizing damage to coated bars during handling, shipment, and placing operations; also, limits on permissible coating damage and, when required, repair of damaged coating.

Reference 8.3 contains suggested provisions for the preceding items for epoxy-coated reinforcing bars.

8.4 DEVELOPMENT OF REINFORCING BARS

8.4.1 Introduction

The fundamental requirement for development (or anchorage) of reinforcing bars is that a reinforcing bar must be embedded in concrete a sufficient distance on each side of a critical section to develop the peak tension or compression stress in the bar at the section. The development length concept in ACI 318 is based on the attainable average bond stress over the length of embedment of the reinforcement. Standard end hooks or mechanical devices may also be used for anchorage of reinforcing bars, except that hooks are effective for developing bars in tension only (ACI 12.1.1).

8.4.2 Development of Straight Bars in Tension

Tension development length ℓ_d for deformed bars includes consideration of a number of modification factors that either increase or decrease the "basic" development length ℓ_{db} (ACI 12.2.2).

Values for ℓ_{db} are given in Table 8-1 for Grade 60 bars embedded in normal weight concrete with f'_c = 3000 psi and 4000 psi.

In general, the tension development length ℓ_d is obtained by multiplying ℓ_{db} by the applicable modification factors relating to 1) bar spacing, amount of cover and enclosing transverse reinforcement (ACI 12.2.3), 2) top reinforcement (ACI 12.2.4.1), 3) lightweight aggregate concrete (ACI 12.2.4.2), and 4) epoxy-coated reinforcement (ACI 12.2.4.3).* Note that the development length may be reduced when the provisions for excess reinforcement given in ACI 12.2.5 are satisfied. Also, ℓ_d must never be taken less than 12 in. (ACI 12.2.1).

**No special development requirements are given for galvanized bars.*

Table 8-1 Basic Tension Development Lengths ℓ_{db} for Grade 60 Bars (in.)

Bar size	f'_c, psi 3000	f'_c, psi 4000
#3	4.8	4.2
#4	8.8	7.6
#5	13.6	11.8
#6	19.3	16.7
#7	26.3	22.8
#8	34.6	30.0
#9	43.8	38.0
#10	55.7	48.2
#11	68.4	59.2

Values of the tension development length ℓ_d are given in Table 8-2 for Grade 60 reinforcing bars in beams or columns, and for bars in the inner layer of slab or wall reinforcement. The values of ℓ_d for beam or column bars are based on 1) minimum cover requirements specified in ACI 7.7.1 and 2) minimum transverse reinforcement specified in ACI 12.2.3.1(a). Table 8-3 lists ℓ_d for all other reinforcing bars not covered in Table 8-2. The values in both tables are based on bars that are not epoxy-coated and on normal weight concrete with $f'_c = 4000$ psi.† To obtain ℓ_d for top bars (horizontal bars with more than 12 in. of concrete cast in one lift below the bars), the tabulated values must be multiplied by 1.3 (ACI 12.2.4.1). The cover and clear spacing referred to in the tables are depicted in Fig. 8-1.

Table 8-2 Tension Development Lengths ℓ_d for Grade 60 Bars in Beams or Columns, and Inner Layer of Slab or Wall Reinforcement (in.)*,**

	Concrete cover, c					
	$c \le d_b$		$c > d_b$			
	Clear spacing, s		Clear spacing, s			
Bar size	$s < 5d_b$	$s \ge 5d_b$***	$s \le 2d_b$	$2d_b < s < 3d_b$	$3d_b \le s < 5d_b$	$s \ge 5d_b$***
#3	12	12	12	12	12	12
#4	16	15	16	15	15	15
#5	24	19	24	18	18	18
#6	34	27	34	24	22	22
#7	46	37	46	32	25	25
#8	60	48	60	42	30	29
#9	76	61	76	54	38	33
#10	97	78	97	68	49	39
#11	119	95	119	83	60	48

*Values based on bars which are not epoxy-coated and on normal weight concrete with $f'_c = 4000$ psi. For top bars, multiply tabulated values by 1.3.

**Values for bars in beams or columns are based on 1) minimum cover requirements specified in ACI 7.7.1, and 2) minimum transverse reinforcement specified in ACI 12.2.3.1(a).

***Side cover $\ge 2.5\ d_b$

As can be seen from the tables, very long development lengths are required for the larger bar sizes, especially when the cover is less than or equal to d_b and the clear spacing is less than or equal to $2d_b$. These development lengths can be shortened if 1) the cover can be increased to more than one bar diameter and 2) the clear spacing can be increased to more than two bar diameters. For beams and columns with the minimum cover specified in ACI 7.7.1, enclosing the bars along the development length with at least the transverse reinforcement specified in ACI

† *Values of ℓ_d for footings with $f'_c = 3000$ psi are given in Chapter 7, Table 7-2.*

Table 8-3 Tension Development Lengths ℓ_d for All Other Grade 60 Bars (in.)*

Bar size	Concrete cover, c								
	$c \le d_b$		$d_b < c < 2d_b$			$c \ge 2d_b$			
	Clear spacing, s		Clear spacing, s			Clear spacing, s			
	$s < 5d_b$	$s \ge 5d_b$**	$s \le 2d_b$	$2d_b < s < 5d_b$	$s \ge 5d_b$**	$s \le 2d_b$	$2d_b < s < 3d_b$	$3d_b \le s < 5d_b$	$s \ge 5d_b$**
#3	12	12	12	12	12	12	12	12	12
#4	16	15	16	15	15	16	15	15	15
#5	24	19	24	18	18	24	18	18	18
#6	34	27	34	24	22	34	24	22	22
#7	46	37	46	32	26	46	32	25	25
#8	60	48	60	42	34	60	42	30	29
#9	76	61	76	54	43	76	54	38	33
#10	97	78	97	68	54	97	68	49	39
#11	119	95	119	83	67	119	83	60	48

*Values based on bars which are not epoxy-coated and on normal weight concrete with f'_c = 4000 psi. For top bars, multiply tabulated values by 1.3.

**Side cover ≥ $2.5d_b$

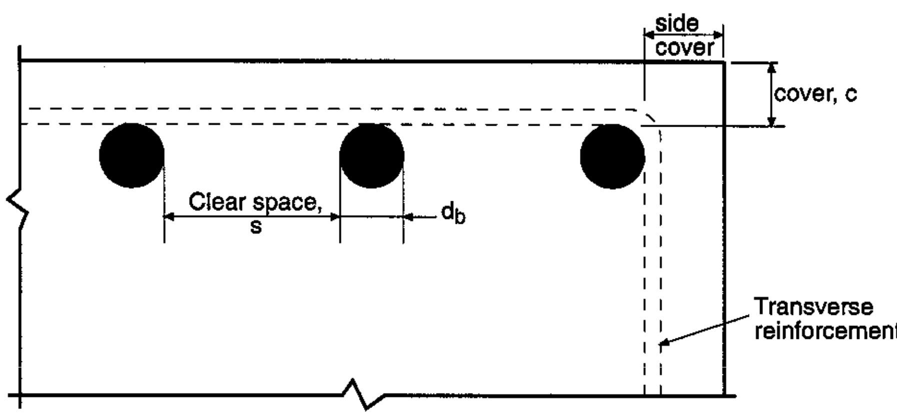

Figure 8-1 Cover and Clear Spacing of the Reinforcement

12.2.3.1(b) will result in significantly shorter development lengths (the development lengths can be reduced by as much as 50%). Table 8-4 can be used to select the required transverse reinforcement per inch (A_{tr}/s) to satisfy ACI 12.2.3.1(b).

Table 8-4 Transverse Reinforcement $A_{tr}/s = Nd_b/40$

Bar size	Number of bars being developed, N					
	N = 2		N = 3		N = 4	
	A_{tr}/s (in.²/in.)	Stirrups or ties	A_{tr}/s (in.²/in.)	Stirrups or ties	A_{tr}/s (in.²/in.)	Stirrups or ties
#5	0.031	#4 @ 12	0.047	#4 @ 8	0.063	#4 @ 6
#6	0.038	#4 @ 10	0.056	#4 @ 7	0.075	#4 @ 5
#7	0.044	#4 @ 9	0.066	#4 @ 6	0.088	#5 @ 7
#8	0.050	#4 @ 8	0.075	#4 @ 5	0.100	#5 @ 6
#9	0.056	#4 @ 7	0.085	#5 @ 7	0.113	#5 @ 5
#10	0.064	#4 @ 6	0.095	#5 @ 6	0.127	#5 @ 4
#11	0.071	#4 @5	0.106	#5 @5	0.141	#6 @6

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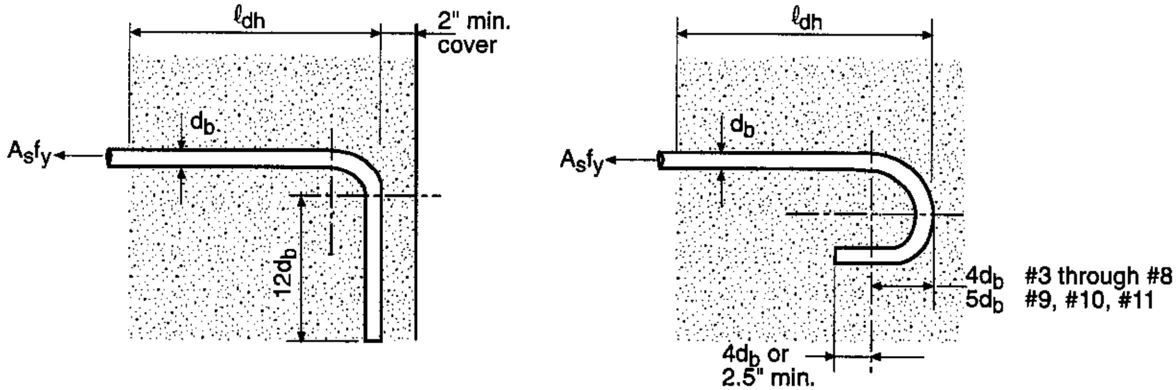
8.4.3 Development of Hooked Bars in Tension

Development length ℓ_{dh} for deformed bars terminating in a standard hook (ACI 12.5) are given in Table 8-5 for Grade 60 bars. As in the case of straight bar development, hooked bar development length ℓ_{dh} includes consideration of modification factors that either increase or decrease the "basic" hooked bar development length ℓ_{hb} given in ACI 12.5.2. When applicable, the modification factors in ACI 12.5.3.2 and 12.5.3.3 can provide significantly shorter hooked bar development lengths. These reduction factors account for the favorable confinement conditions provided by increased concrete cover and/or transverse ties or stirrups, which resist splitting of the concrete.

Table 8-5 Minimum Development Lengths ℓ_{dh} for Grade 60 Bars with Standard End Hooks (in.)*



Standard 90° hook

Standard 180° hook

	General use: • Side cover ≥ 2½ in. • End cover (90° hooks) ≥ 2 in.		Special confinement: • Side cover ≥ 2½ in. • End cover (90° hooks) ≥ 2 in. • Ties or stirrups spaced ≤ $3d_b$	
Bar size	f'_c = 3000 psi	f'_c = 4000 psi	f'_c = 3000 psi	f'_c = 4000 psi
#3	6	6	6	6
#4	8	7	7	6
#5	10	9	8	7
#6	12	10	10	8
#7	14	12	11	10
#8	16	14	13	11
#9	18	15	14	12
#10	20	17	16	14
#11	22	19	18	15

*Values based on normal weight concrete.

The general use development lengths given in Table 8-5 are applicable for end hooks with side cover normal to plane of hook of not less than 2-½ in. and end cover (90° hooks only) of not less than 2 in. For these cases, $\ell_{dh} = 0.7\ell_{hb}$, but not less than $8d_b$ or 6 in. (ACI 12.5.1). For hooked bar anchorage in beam-column joints, the hooked beam bars are usually placed inside the vertical column bars, with side cover greater than the 2-½-in. minimum required for application of the 0.7 reduction factor. Also, for 90° end hooks with hook extension located inside the column ties, the 2-in. minimum end cover will usually be satisfied to permit the 0.7 reduction factor.

The special confinement condition given in Table 8-5 includes the additional 0.8 reduction factor for confining ties or stirrups (ACI 12.5.3.3). In this case, $\ell_{dh} = (0.7 \times 0.8)\ \ell_{hb}$, but not less than $8d_b$ or 6 in.

Where development for full f_y is not specifically required, the tabulated values of ℓ_{dh} in Table 8-5 may be further reduced for excess reinforcement (ACI 12.5.3.4). As noted above, ℓ_{dh} must not be less than $8d_b$ or 6 in.

ACI 12.5.4 provides additional requirements for hooked bars terminating at the discontinuous end of members (ends of simply supported beams, free end of cantilevers, and ends of members framing into a joint where the member does not extend beyond the joint). If the full strength of the hooked bar must be developed, and if both the side cover and the top (or bottom) cover over the hook is less than 2-½ in., closed ties or stirrups spaced at $3d_b$ maximum are required along the full development length ℓ_{dh}. The reduction factor in ACI 12.5.3.3 must not be used in this case. At discontinuous ends of slabs with confinement provided by the slab continuous on both sides normal to the plane of the hook, the requirements in ACI 12.5.4 for confining ties or stirrups do not apply.

8.4.4 Development of Bars in Compression

Compression development lengths (ACI 12.3) for Grade 60 bars are given in Table 8-6. The values may be reduced by the applicable factors in ACI 12.3.3. Note that the minimum development length is 8 in.

Table 8-6 Minimum Compression Development Lengths ℓ_d for Grade 60 Bars (in.)

Bar size	f'_c = 3000 psi	f'_c = 4000 psi
#3	9	8
#4	11	10
#5	14	12
#6	17	15
#7	20	17
#8	22	19
#9	25	22
#10	28	25
#11	31	27

8.5 SPLICES OF REINFORCING BARS

Three methods are used for splicing reinforcing bars: 1) lap splices, 2) welded splices, and 3) mechanical connections. The lap splice is usually the most economical splice. When lap splices cause congestion or field placing problems, mechanical connections or welded splices should be considered. The location of construction joints, provision for future construction, and the particular method of construction may also make lap splices impractical. In columns, lapped offset bars may need to be located inside the bars above to reduce reinforcement congestion; this can reduce the moment capacity of the column section at the lapped splice location because of the reduction in the effective depth. When the amount of vertical reinforcement is greater than 4%, and/or when large factored moments are present, use of butt splices—either mechanical connections or welded splices—should be considered in order to reduce congestion and to provide for greater nominal moment strength of the column section at the splice locations.

Bars in flexural members may be spliced by noncontact lap splices (ACI 12.14.2.3); however, contact lap splices are preferred since the bars are tied and are less likely to displace when the concrete is placed.

Welded splices generally require the most expensive field labor. For projects of all sizes, manual arc-welded splices will usually be the most costly method of splicing due to the costs of inspection.

Mechanical connections are made with proprietary splice devices. Performance information and test data should be obtained directly from the manufacturers. Basic information about mechanical connections and the types of proprietary splice devices currently is available from Reference 8.4. Practical information on splicing and recommendations for the design and detailing of splices are given in Reference 8.5.

8.5.1 Tension Lap Splices

Tension lap splices are classified as Class A or Class B (ACI 12.15.1). The minimum lap length for a Class A splice is $1.0\ell_d$, and for a Class B splice it is $1.3\ell_d$, where ℓ_d is the tension development length of the bars. As was shown in Section 8.4.2, ℓ_d is obtained by multiplying the basic development length ℓ_{db} by the applicable modification factors in ACI 12.2. The factor in ACI 12.2.5 for excess reinforcement must not be used, since the splice classifications already reflect any excess reinforcement at the splice location.

The minimum lap lengths for Class A splices can be obtained from Table 8-2 or 8-3. For Class B splices, the minimum lap lengths are determined by multiplying the values from Tables 8-2 or 8-3 by 1.3. The effective clear spacings between splices bars is illustrated in Fig. 8-2. For staggered splices in slabs or walls, the effective clear spacing is the distance between adjacent spliced bars less the diameters of any intermediate unspliced bars (Fig. 8-2a). The clear spacing to be used for splices in columns with offset bars and for beam bar splices are shown in Figs. 8-2b and 8-2c, respectively.

In general, tension lap splices must be Class B except that Class A splices are allowed when both of the following conditions are met: 1) the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice and 2) one-half or less of the total reinforcement is spliced within the required lap length (ACI 12.15.2). Essentially, Class A splices may be used at locations where the tensile stress is small. It is very important to specify which class of tension splice is to be used, and to show clear and complete details of the splice in the Contract Documents.

8.5.2 Compression Lap Splices

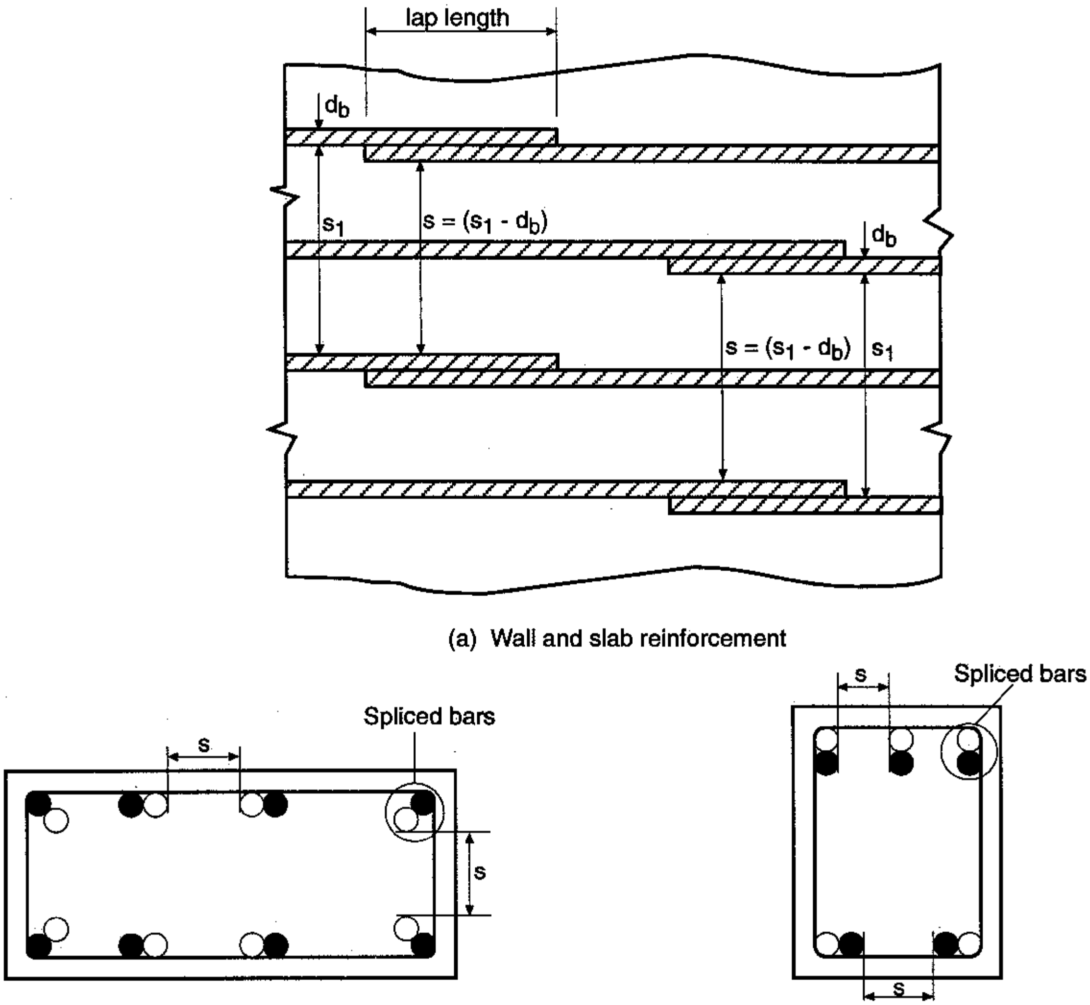
Minimum lengths for compression lap splices (ACI 12.16.1) for Grade 60 bars in normal weight concrete are given in Table 8-7. The values apply for all concrete strengths greater than or equal to 3000 psi. For Grade 60 bars, the minimum lap length is 30 d_b but not less than 12 in. When bars of different size are lap spliced, the splice length shall be the larger of 1) development length of larger bar, or 2) splice length of smaller bar (ACI 12.16.2). For columns, the lap splice lengths may be reduced by a factor of 0.83 when the splice is enclosed throughout its length by ties specified in ACI 12.17.2.4. The 12 in. minimum lap length also applies.

8.6 DEVELOPMENT OF FLEXURAL REINFORCEMENT

8.6.1 Introduction

The requirements for development of flexural reinforcement are given in ACI 12.10, 12.11, and 12.12. These sections include provisions for:

- Bar extensions beyond points where reinforcement is no longer required to resist flexure.



(b) Column with offset corner bars

(b) Beam bar splices

Figure 8-2 Effective Clear Spacing of Spliced Bars

Table 8-7 Minimum Compression Lap Splice Lengths for Grade 60 Bars*

Bar size	Minimum lap length (in.)
#3	12
#4	15
#5	19
#6	23
#7	26
#8	30
#9	34
#10	38
#11	42

* $f'_c \geq 3000$ psi

- Termination of flexural reinforcement in tension zones.
- Minimum amount and length of embedment of positive moment reinforcement into supports.
- Limits on bar sizes for positive moment reinforcement at simple supports and at points of inflection.
- Amount and length of embedment of negative moment reinforcement beyond points of inflection.

Many of the specific requirements are interdependent, resulting in increased design time when the provisions are considered separately. To save design time and costs, recommended bar details should be used. As was discussed earlier in this chapter, there is potential overall savings in fabrication, placing, and inspection costs when recommended bar details are used.

8.6.2 Recommended Bar Details

Recommended bar details for continuous beams, one-way slabs, one-way joist construction, and two-way slabs (without beams) are given in Figs. 8-3 through 8-6. Similar details can be found in References 8.1 and 8.6. The figures may be used to obtain bar lengths for members subjected to uniformly distributed gravity loads only; adequate bar lengths must be determined by analysis for members subjected to lateral loads. Additionally, Figs. 8-3 through 8-5 are valid for beams, one-way slabs, and one-way joists that may be designed by the approximate method given in ACI 8.3.3.* Fig. 8-6 can be used to determine the bar lengths for two-way slabs without beams.**

8.7 SPECIAL BAR DETAILS AT SLAB-TO-COLUMN CONNECTIONS

When two-way slabs are supported directly by columns (as in flat plates and flat slabs), transfer of moment between slab and column takes place by a combination of flexure and eccentricity of shear (see Chapter 4, Section 4.4.1). The portion of the unbalanced moment transferred by flexure is assumed to be transferred over a width of slab equal to the column width c plus 1.5 times the slab thickness h on either side of the column. For edge and interior columns, the effective slab width is (c+3h), and for corner columns it is (c+1.5h). An adequate amount of negative slab reinforcement is required in this effective slab width to resist the portion of the unbalanced moment transferred by flexure (ACI 13.3.3.3). In some cases, additional reinforcement must be concentrated over the column to increase the nominal moment resistance of the section. Note that minimum bar spacing requirements must be satisfied at all locations in the slab (ACI 13.4.2). Based on recommendations in Reference 8.7, examples of typical details at edge and corner columns are shown in Figs. 8-7 and 8-8.

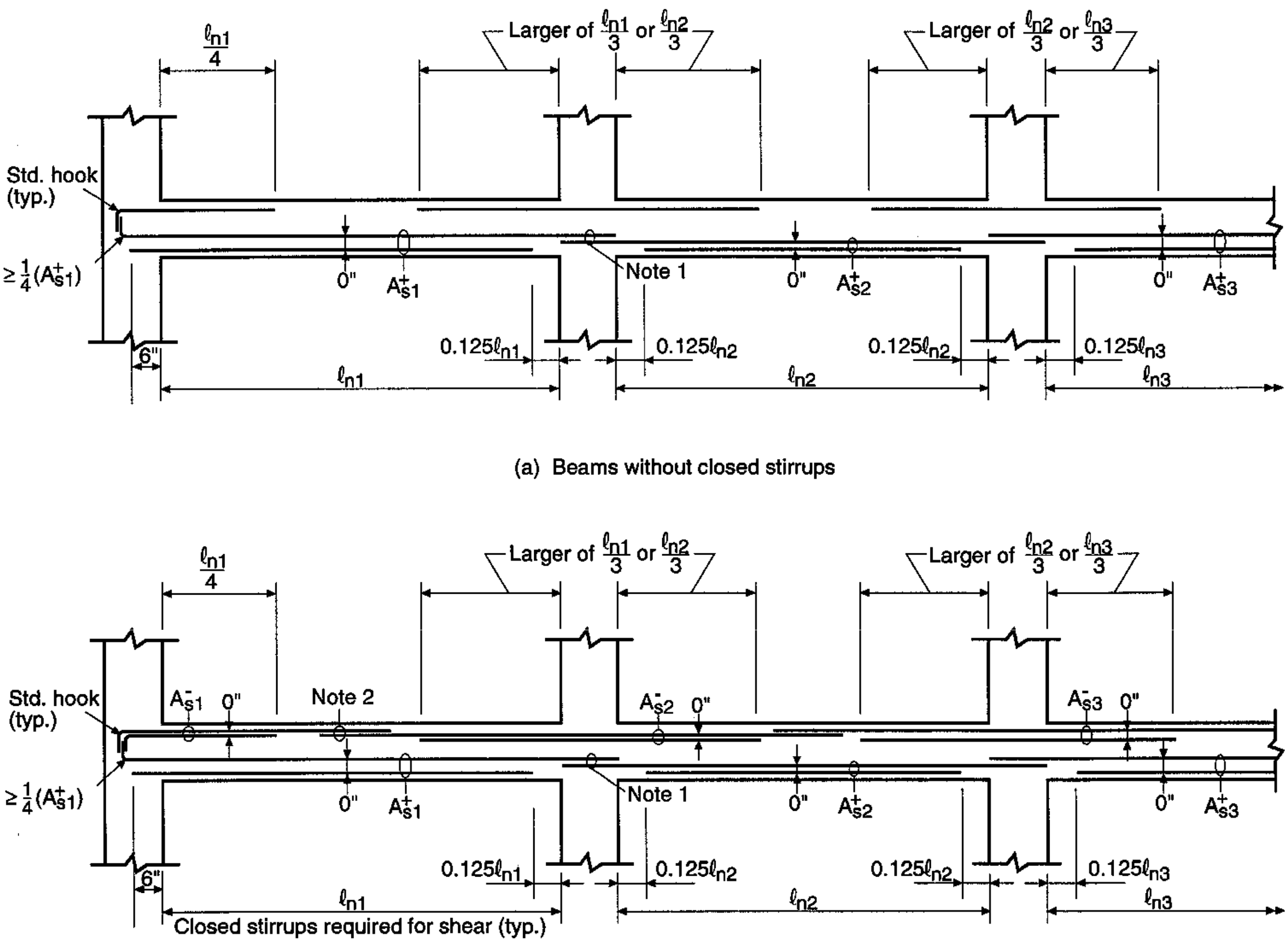
8.8 SPECIAL SPLICE REQUIREMENTS FOR COLUMNS

8.8.1 Construction and Placing Considerations

For columns in multistory buildings, one-story high preassembled reinforcement cages are usually used. It is common practice to locate the splices for the vertical column bars just above the floor level. In certain situations, it may be advantageous to use two-story high cages since this will reduce the number of splices and, for lap splices, will reduce the amount of reinforcing steel. However, it is important to note that two-story high cages are difficult to brace; the required guy wires or projecting bars may interfere with other construction operations such as the movement of cranes for transporting equipment and material. Also, it is more difficult and time-consuming to

**Under normal conditions, the bar lengths given in Figs. 8-3 through 8-5 will be satisfactory. However, for special conditions, a more detailed analysis will be required. In any situation, it is the responsibility of the engineer to ensure that adequate bar lengths are provided.*

***To reduce placing and inspection time, all of the top bars in the column strip of a two-way slab system can have the same length at a particular location (either $0.30\ell_n$ for flat plates or $0.33\ell_n$ for flat slabs), instead of the two different lengths shown in Figs. 8-6(a) and 8-6(c).*



(a) Beams without closed stirrups

(b) Perimeter beams

Notes: (1) Larger of $1/4(A_{s1}^{+})$ or $1/4(A_{s2}^{+})$ continuous or spliced with Class A splices (ACI 7.13.2.2 and 7.13.2.3)
(2) Larger of $1/6\ (A_{s1}^{-})$ or $1/6(A_{s2}^{-})$ continuous or spliced with Class A splices (ACI 7.13.2.2)

Figure 8-3 Recommended Bar Details for Beams

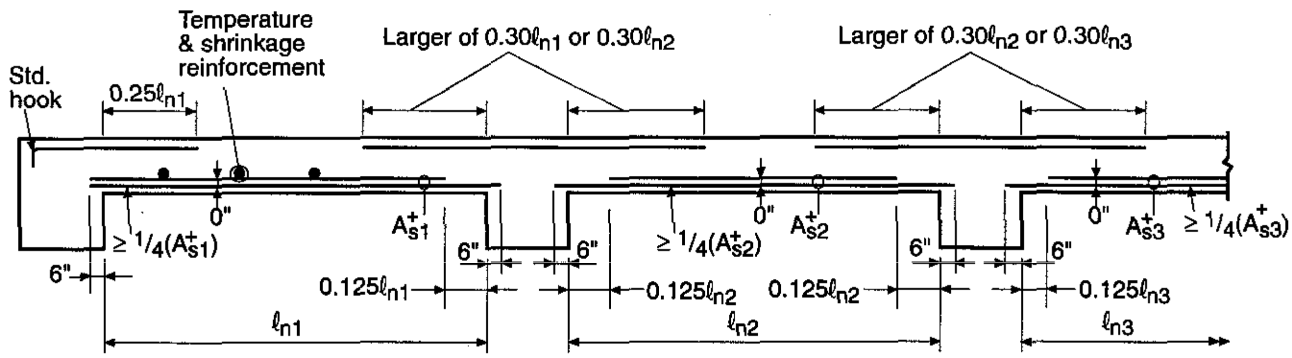


Figure 8-4 Recommended Bar Details for One-Way Slabs

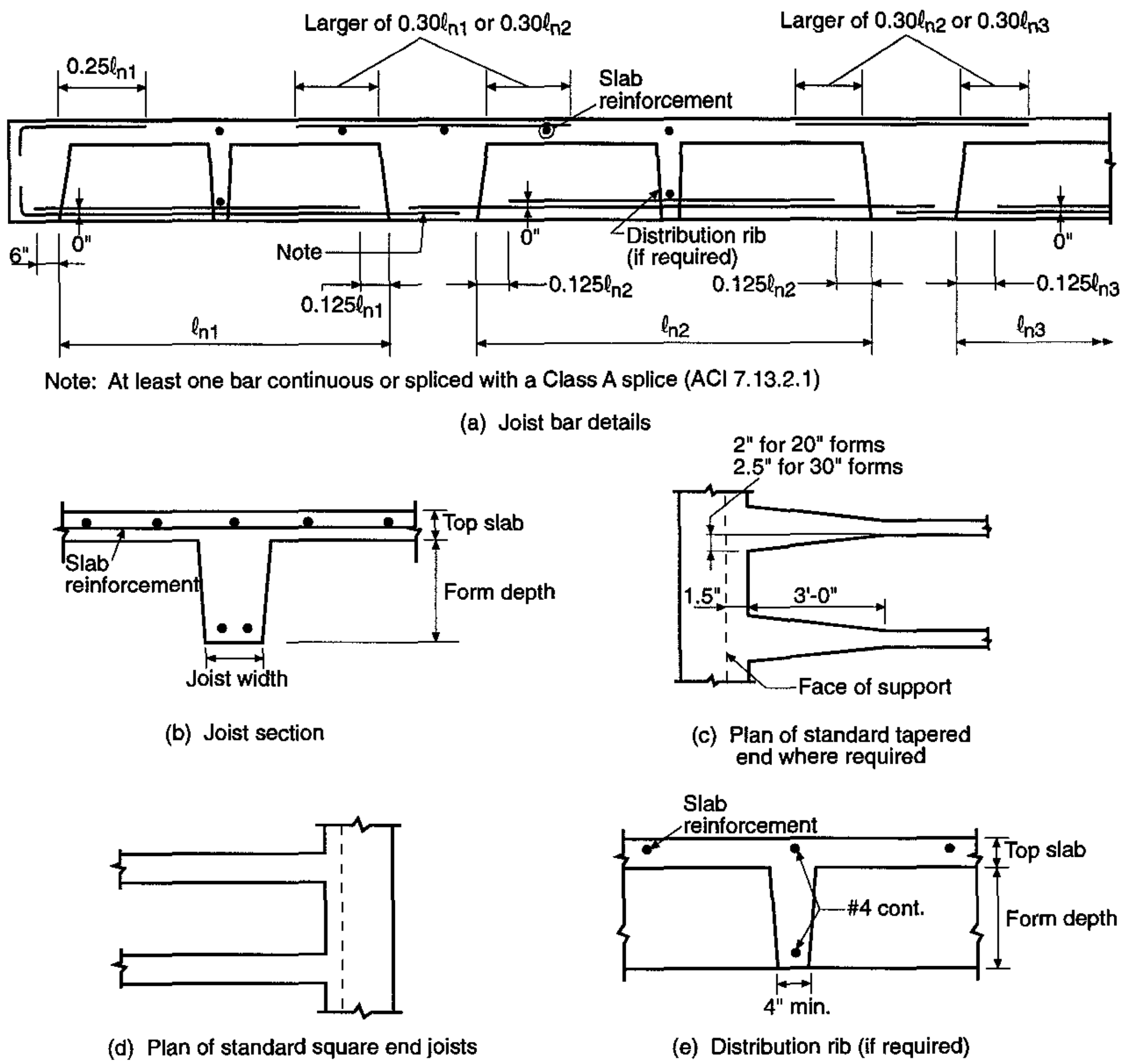
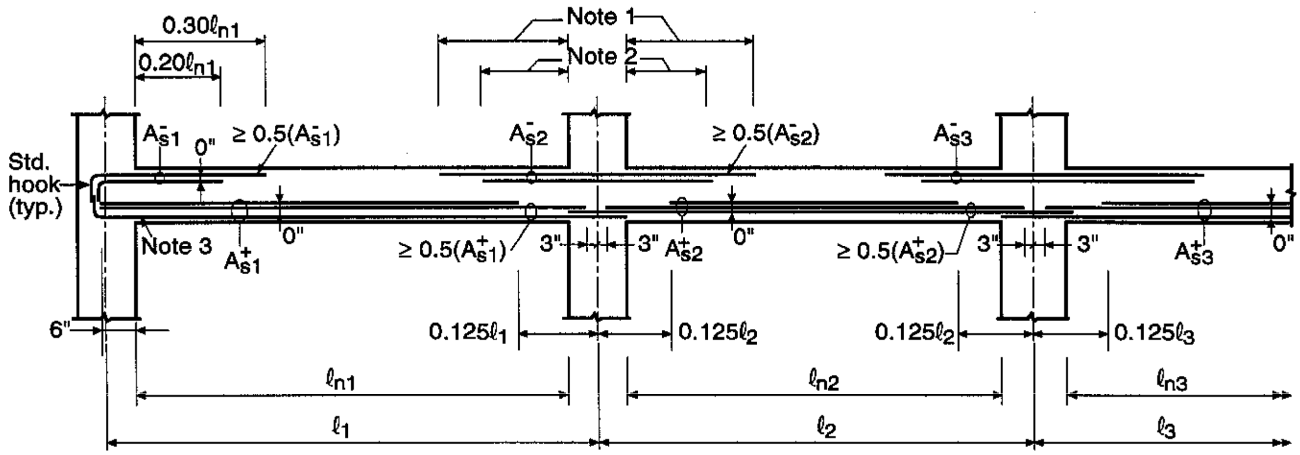
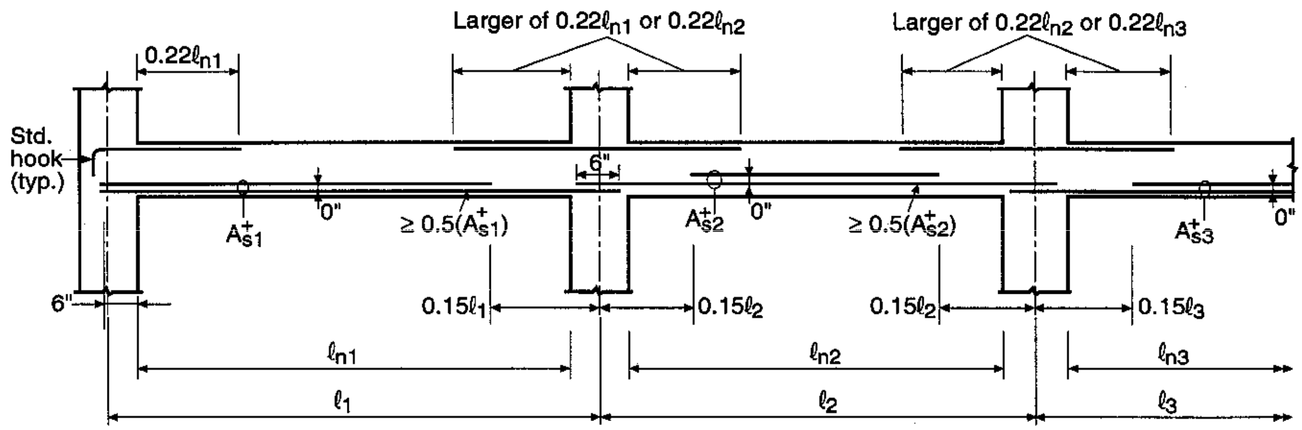


Figure 8-5 Recommended Bar Details for One-Way Joist Construction

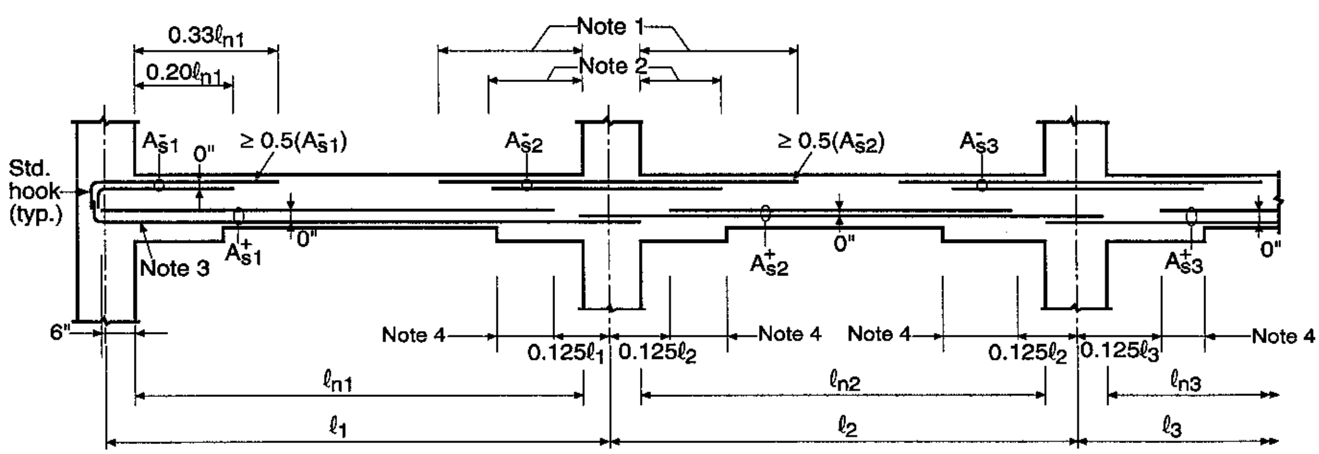


Notes: (1) Larger of $0.30\ell_{n1}$ or $0.30\ell_{n2}$
(2) Larger of $0.20\ell_{n1}$ or $0.20\ell_{n2}$
(3) At least two bars continuous or anchored per ACI 13.4.8.5

(a) Column strip



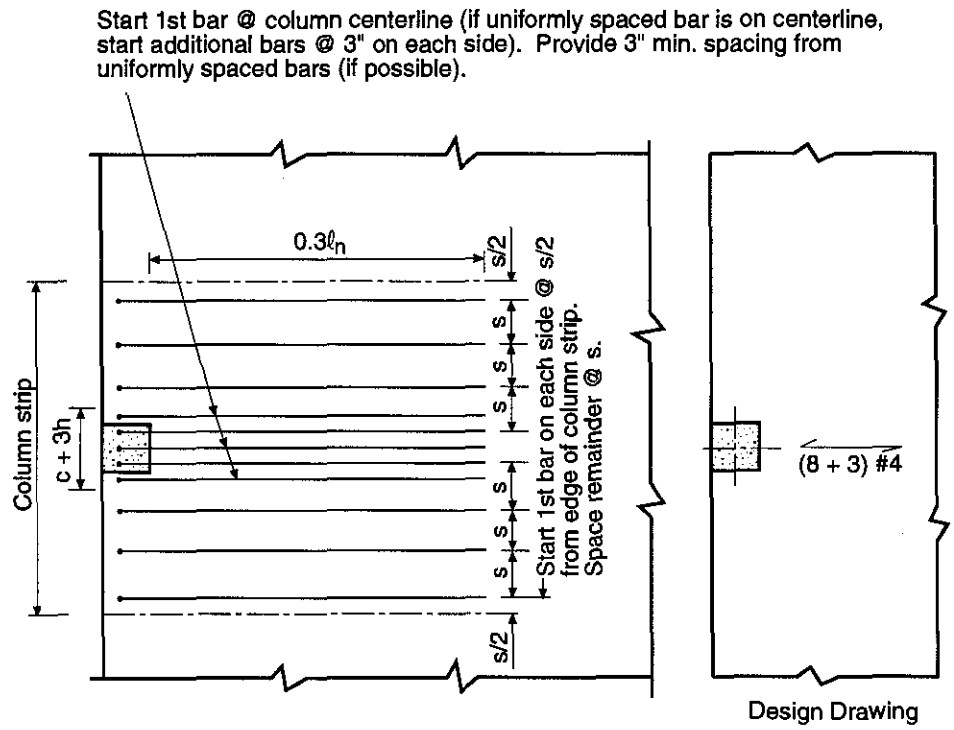
(b) Middle strip



Notes: (1) Larger of $0.33\ell_{n1}$ or $0.33\ell_{n2}$
(2) Larger of $0.20\ell_{n1}$ or $0.20\ell_{n2}$
(3) At least two bars continuous or anchored per ACI 13.4.8.5
(4) 24 bar diameters or 12 in. min. (all bars)

(c) Column strip—with drop panels

Figure 8-6 Recommended Bar Details for Two-Way Slabs (Without Beams)



Notes: (1) Maximum spacing s = 2 × slab thickness ≤ 18 in. (ACI 13.4.2)
(2) Where additional top bars are required, show the total number of bars on the design drawing as (8 + 3) #4 where 8 indicates the number of uniformly spaced bars and 3 indicates the number of additonal bars.

Figure 8-7 Example of a Typical Detail for Top Bars at Edge Columns (Flat Plate)

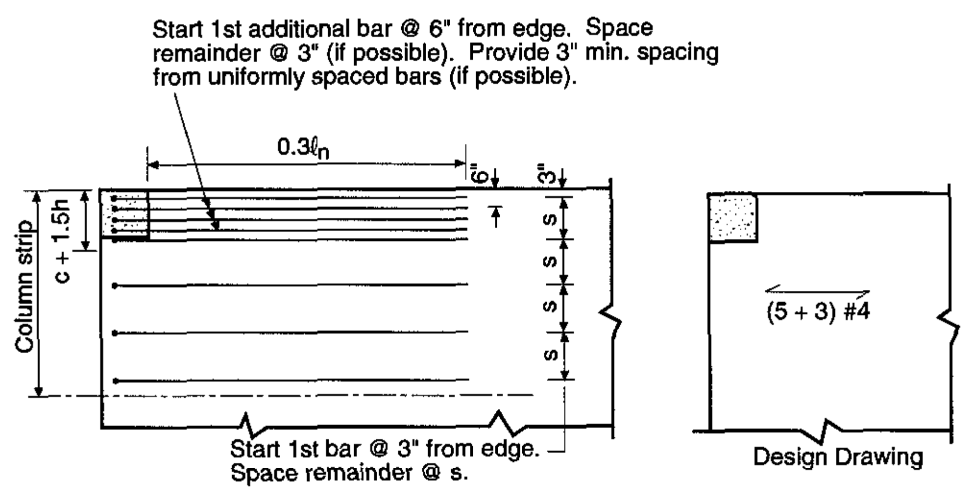


Figure 8-8 Example of a Typical Detail for Top Bars at Corner Columns (Flat Plate)

place the beam or girder bars at the intermediate floor level since they have to be threaded through the column steel. These two reasons alone are usually more than sufficient to offset any expected savings in steel that can be obtained by using two-story high cages. Thus, one-story high cages are usually preferred.

8.8.2 Design Considerations

Special provisions for column splices are given in ACI 12.17. In general, column splices must satisfy requirements for all load combinations for the column. For example, column design will frequently be governed by the gravity load combination (all bars in compression). However, the load combination which includes wind loads may produce tensile stresses in some of the bars. In this situation, a tension splice is required even though the load combination governing the column design did not produce any tensile stresses.

When the bar stress due to factored loads is compressive, lap splices, butt welded splices, mechanical connections, and end-bearing splices are permitted. Table 8-7 may be used to determine the minimum compression lap splice lengths for Grade 60 bars. Note that these lap splice lengths may be multiplied by 0.83 for columns with the minimum effective area of ties (throughout the splice length) given in ACI 12.17.2.4. In no case shall the lap length be less than 12 in. Welded splices and mechanical connectors must meet the requirements of ACI 12.14.3.3 and 12.14.3.4, respectively. A full welded splice which is designed to develop in tension at least 1.25 A_bf_y (A_b = area of bar) will be adequate for compression as well. A full mechanical connection must develop in compression (or tension) at least 1.25 A_bf_y. End-bearing splices transfer the compressive stresses by bearing of square cut ends of the bars held in concentric contact by a suitable device (ACI 12.16.4). These types of splices may be used provided the splices are staggered or additional bars are provided at splice locations (see ACI 12.17.4 and the following discussion).

A minimum tensile strength is required for all compression splices. A compression lap splice with a length greater than or equal to the minimum value given in ACI 12.16.1 has a tensile strength of at least 0.25 A_bf_y. As noted above, full welded splices and full mechanical connectors develop at least 1.25 A_bf_y in tension. For end-bearing splices, the continuing bars on each face of the column must have a tensile strength of 0.25 A_sf_y where A_s is the total area of steel on the face of the column. This implies that not more than three-quarters of the bars can be spliced on each face of the column at any one location. Consequently, to ensure minimum tensile strength, end-bearing splices must be staggered or additional bars must be added if more than three-quarters of the bars are to be spliced at any one location.

Lap splices, welded splices, and mechanical connections are permitted when the bar stress is tensile; end-bearing splices must not be used (ACI 12.16.4.1). According to ACI 12.14.3, full welded splices and full mechanical connections must develop in tension at least 1.25A_bf_y. When the bar stress on the tension face of the column is less than or equal to 0.5 f_y, lap splices must be Class B if more than one-half of the bars are spliced at any section, or Class A if half or fewer of the bars are spliced and alternate splices are staggered by the tension development length ℓ_d (ACI 12.17.2). Class B splices must be used when the bar stress is greater than 0.5 f_y.

Lap splice requirements for columns are illustrated in Fig. 8-9. For factored load combinations in Zone 1, all column bars are in compression. In Zone 2, the bar stress f_s on the tension face of the column varies from zero to 0.5 f_y in tension. For load combinations in Zone 3, f_s is greater than 0.5 f_y. The load combination that produces the greatest tensile stress in the bars will determine which type of lap splice is to be used. Load-moment design charts (such as the ones in Figs. 5-16 through 5-23 in Chapter 5) can greatly facilitate the design of lap splices for columns.

Typical lap splice details for tied columns are shown in Fig. 8-10. Also given in the figure are the tie spacing requirements of ACI 7.8 and 7.10.5 (see Chapter 5). When a column face is offset 3 in. or more, offset bent longitudinal bars are not permitted (ACI 7.8.1.5). Instead, separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces must be provided. Typical splice details for footing dowels are given in Chapter 7, Fig. 7-7.

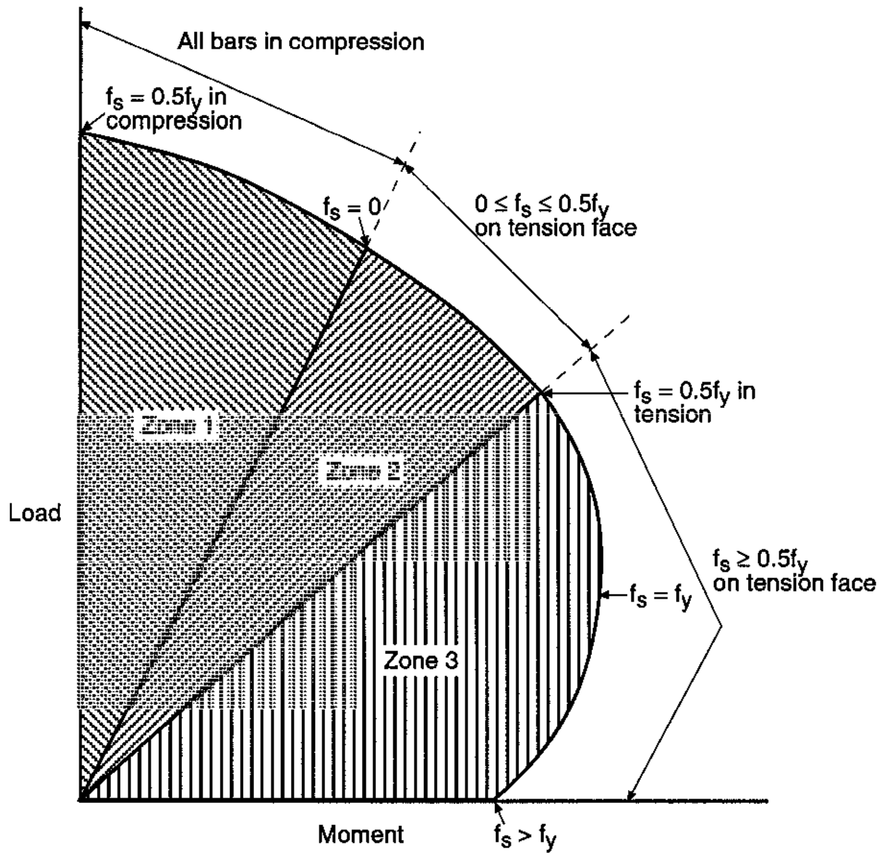


Figure 8-9 Special Splice Requirements for Columns

8.8.3 Example: Lap Splice Length for an Interior Column of Building #2, Alternate (2) Slab and Column Framing with Structural Walls (Braced Frame).

In this example, the required lap splice length will be determined for an interior column in the 2nd story; the splice will be located just above the 9 in. floor slab at the 1st level.

(1) Column Size and Reinforcement

In Example 5.7.2, a 16 × 16 in. column size was established for the entire column stack. It was determined that 4-#8 bars were required in both the 1st and 2nd floor columns.

(2) Lap Splice Length

Since the columns carry only gravity loads, all of the column bars will be in compression (Zone 1 in Fig. 8-9). Therefore, a compression lap splice is sufficient.

From Table 8-7, the minimum compression lap splice length required for the #8 bars is 30 in. In this situation, #3 ties are required @ 16 in. spacing o.c.

According to ACI 12.17.2.4, the lap splice length may be multiplied by 0.83 if ties are provided with an effective area of 0.0015hs throughout the lap splice length.

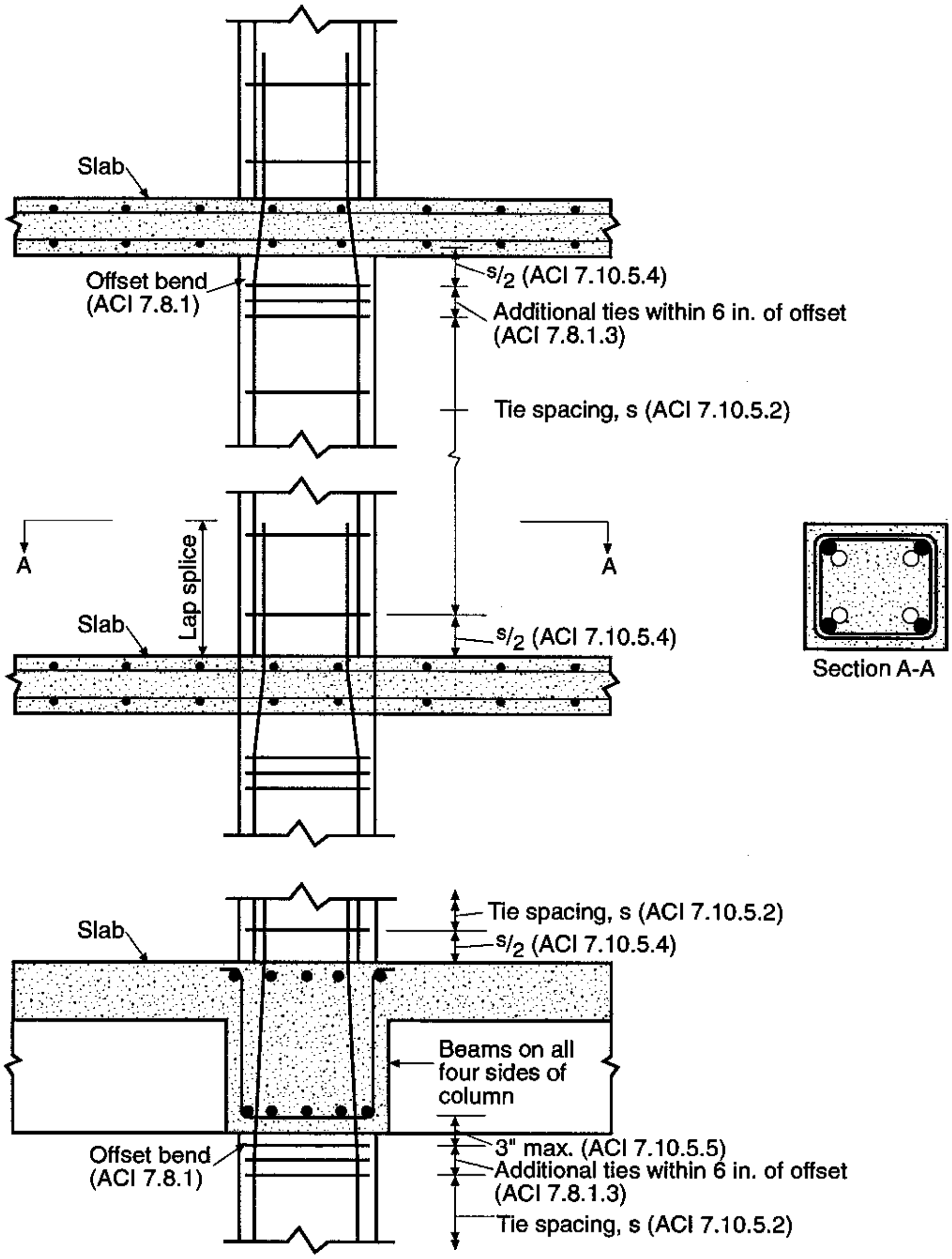


Figure 8-10 Column Splice and Tie Details

Two legs of the #3 ties are effective in each direction; thus, the required spacing s can be determined as follows:

$2 \times 0.11 = 0.22 \text{ in.}^2 = 0.0015 \times 16 \times s$

or, s = 9.2 in.

Splice length = 0.83 × 30 = 24.9 in.

Use a 2 ft-1 in. splice length with #3 ties @ 9 in. o.c. throughout the splice length.

8.8.4 Example: Lap Splice Length for an Interior Column of Building #2, Alternate (1) Slab and Column Framing Without Structural Walls (Unbraced Frame).

As in Example 8.8.3, the required lap splice length will be determined for an interior column in the 2nd story, located just above the slab at the 1st level.

(1) Load Data

In Example 5.7.1, the following load combinations were obtained for the 2nd story interior columns:

Gravity loads:	$P_u = 457$ kips	ACI Eq. (9-1)
	$M_u = 14$ ft-kips	
Gravity + Wind loads:	$P_u = 343$ kips	ACI Eq. (9-2)
	$M_u = 86$ ft-kips	
	$P_u = 244$ kips	ACI Eq. (9-3)
	$M_u = 77$ ft-kips	

(2) Column Size and Reinforcement

A 16 × 16 in. column size was established in Example 5.7.1 for the 1st story columns. This size is used for the entire column stack; the amount of reinforcement can be decreased in the upper levels. It was also shown that the 1st story columns were slender, and that 8-#10 bars were required for the factored axial loads and magnified moments at this level.

The reinforcement for the 2nd story columns can be determined using the procedure outlined in Chapter 5. As was the case for the 1st story columns, the 2nd story columns are slender (use k = 1.2; see Chapter 5):

$$\frac{k\ell_u}{r} = \frac{1.2[(12 \times 12) - 8.5]}{0.3 \times 16} = 34 > 22$$

Figure 8-11 is the output from PCACOL for the section reinforced with 8-#7 bars.* The critical load combination for the column is designated as point 2 in the figure; this corresponds to ACI Eq. (9-2).

(3) Lap Splice Length

The load combination represented by point 4 (ACI Eq. (9-3)) in Fig. 8-11 governs the type of lap splice to be used, since it is the combination that produces the greatest tensile stress f_s in the bars. Note that the load combination represented by point 2 (ACI Eq. (9-2)) which governed the design of the column does not govern the design of the splice. Since $f_s > 0.5\ f_y$ at point 4, a Class B splice must be used.

Required splice length = $1.3\ell_d$ where ℓ_d is the tension development length of the #10 bars (of the lower column).

Clear bar spacing ≅ 3.5 in. = $2.8d_b$ (see Fig. 8-2)

Cover > d_b = 1.27 in.

From Table 8-2, ℓ_d = 68 in.

**50% of the gross moment of inertia of the column strip slab was used to account for cracking.*

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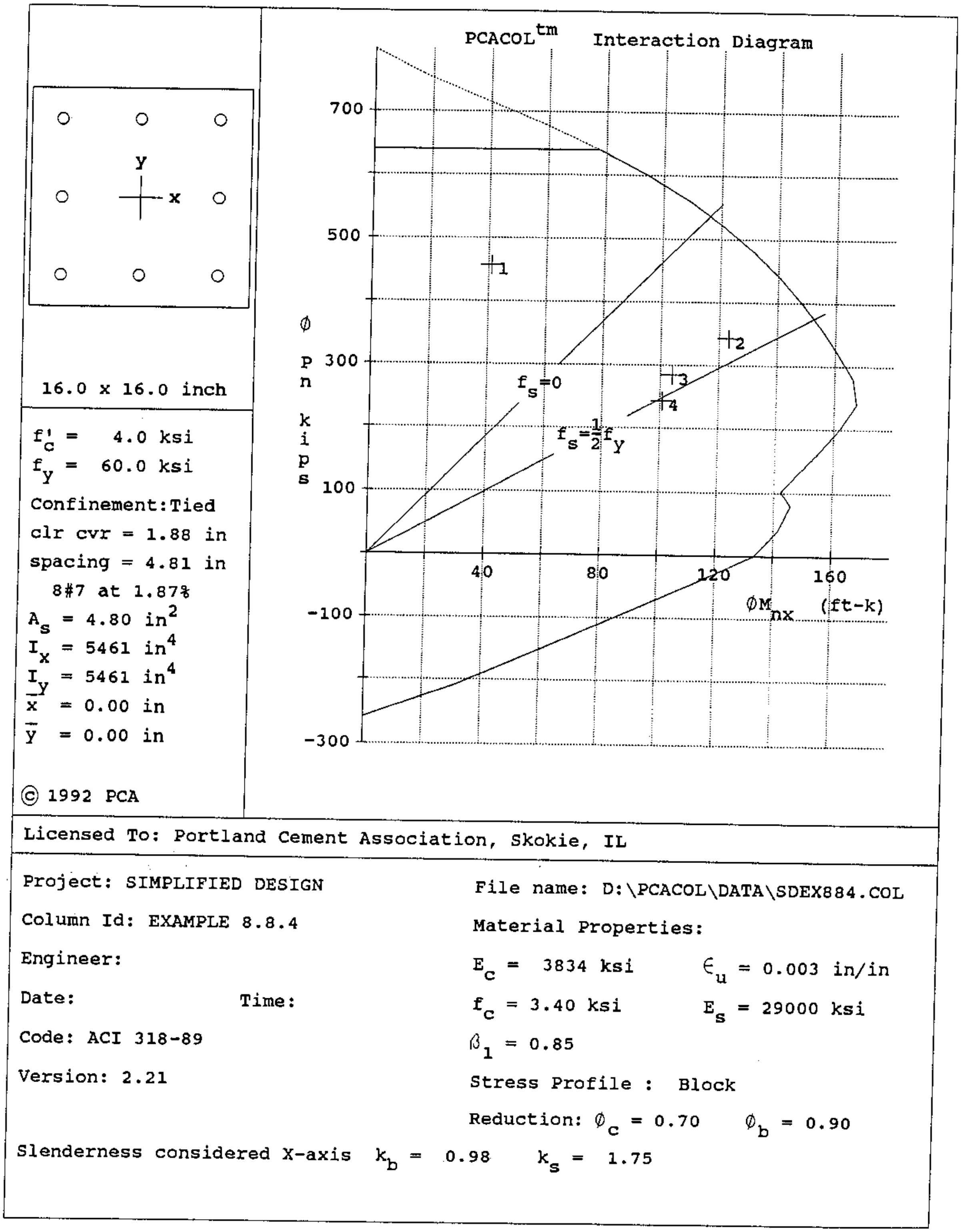

Figure 8-11 Interaction Diagram for an Interior Column in the 2nd Story of Building #2, Alternate (1)

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$1.3\ell_d = 1.3 \times 68 = 88.4$ in.

Thus, a 7 ft-5 in. splice length would be required which is more than one-half of the clear story height.

In this situation, it would be impractical to decrease the splice length by providing additional transverse reinforcement along the splice length satisfying ACI Eq. (12-1); the required transverse reinforcement per square inch A_{tr}/s to develop the 8-#10 bars is:

$$\frac{A_{tr}}{s} = \frac{8 \times 1.27}{40} = 0.25\,\text{in.}^2/\text{in.}$$

Decreasing the bar size in the 1st story columns would result in slightly smaller splice lengths; however, the reinforcement ratio would increase from 4% (8-#10) to 4.7% (12-#9). Also, labor costs would increase since more bars would have to be placed and spliced.

One possible alternative would be to increase the column size. For example, a 18 × 18 in. column would require about 8-#8 bars in the 1st story. It is important to note that changing the dimensions of the columns would change the results from the lateral load analysis, affecting all subsequent calculations; a small change, however, should not significantly alter the results.

References

8.1 *ACI Detailing Manual - 1988*, SP-66(88), American Concrete Institute, Detroit, 1988.

8.2 Stecich, J.P., Hanson, J.M., and Rice, P.F., "Bending and Straightening of Grade 60 Reinforcing Bars," *Concrete International: Design & Construction*, Vol. 6, No. 8, August 1984, pp. 14-23.

8.3 "Suggested Project Specifications Provisions for Epoxy-Coated Reinforcing Bars", *Engineering Data Report No. 19*, Concrete Reinforcing Steel Institute, Schaumburg, Illinois, 1984.

8.4 *Mechanical Connections of Reinforcing Bars*, ACI Committee 439, American Concrete Institute, Detroit, Michigan, 1991, 12 pp.

8.5 *Reinforcement: Anchorages, Lap Splices and Connections*, 3rd Edition, Concrete Reinforcing Steel Institute, Schaumburg, Illinois, 1990, 37 pp.

8.6 *CRSI Handbook*, 7th Edition, Concrete Reinforcing Steel Institute, Schaumburg, Illinois, 1992.

8.7 "Design of Reinforcement for Two-Way Slab-to-Column Frames Laterally Braced or Not Braced", *Structural Bulletin No. 9*, Concrete Reinforcing Steel Institute, Schaumburg, Illinois, June 1983.

Chapter 9

Design Considerations for Economical Formwork

9.1 INTRODUCTION

Depending on a number of factors, the cost of formwork can be as high as 60% of the total cost of a cast-in-place concrete structure. For this reason, it is extremely important to devise a structural system that will minimize the cost of formwork. Basic guidelines for achieving economical formwork are given in Reference 9.1, and are summarized in this chapter.

Formwork economy should initially be considered at the conceptual stage or the preliminary design phase of a construction project. This is the time when architectural, structural, mechanical, and electrical systems are conceived. The architect and the engineer can help reduce the cost of formwork by following certain basic principles while laying out the plans and selecting the structural framing for the building. Additional savings can usually be achieved by consulting a contractor during the initial design phases of a project.

Design professionals, after having considered several alternative structural framing systems and having determined those systems that best satisfy loading requirements as well as other design criteria, often make their final selections on the system that would have the least amount of concrete and possibly the least amount of reinforcing steel. This approach can sometimes result in a costly design. Complex structural frames and nonstandard member cross sections can complicate construction to the extent that any cost savings to be realized from the economical use of in-place (permanent) materials can be significantly offset by the higher costs of formwork. Consequently, when conducting cost evaluations of concrete structural frames, it is essential that the costs of formwork be included.

9.2 BASIC PRINCIPLES TO ACHIEVE ECONOMICAL FORMWORK

There is always the opportunity to cut costs in any structural system. The high cost of formwork relative to the costs of the other components makes it an obvious target for close examination. Three basic design principles that govern formwork economy for all site-cast concrete structures are given below.

9.2.1 Standard Forms

Since most projects do not have the budget to accommodate custom forms, basing the design on readily available standard form sizes is essential to achieve economical formwork. Also, designing for actual dimensions of standard nominal lumber will significantly cut costs. A simplified approach to formwork carpentry means less sawing, less piecing together, less waste, and less time; this results in reduced labor and material costs and fewer opportunities for error by construction workers.

9.2.2 Repetition

Whenever possible, the sizes and shapes of the concrete members should be repeated in the structure. By doing this, the forms can be reused from bay to bay and from floor to floor, resulting in maximum overall savings. The relationship between cost and changes in depth of horizontal construction is a major design consideration. By standardizing the size or, if that is not possible, by varying the width and not the depth of beams, most requirements can be met at a lowered cost, since the forms can be reused for all floors. To accommodate load and span variations, only the amount of reinforcement needs to be adjusted. Also, experience has shown that changing the depth of the concrete joist system from floor to floor because of differences in superimposed loads actually results in higher costs. Selecting different joist depths and beam sizes for each floor may result in minor savings in materials, but specifying the same depth for all floors will achieve major savings in forming costs.

9.2.3 Simplicity

In general, there are countless variables that must be evaluated, then integrated into the design of a building. Traditionally, economy has meant a time-consuming search for ways to cut back on quantity of materials. As noted previously, this approach often creates additional costs—quite the opposite effect of that intended.

An important principle in formwork design is simplicity. In light of this principle, the following questions should be considered in the preliminary design stage of any project:

1) Will custom forms be cost-effective? Usually, when standard forms are used, both labor and material costs decrease. However, custom forms can be as cost-effective as standard forms if they are required in a quantity that allows mass production.

2) Are deep beams cost-effective? As a rule, changing the beam depth to accommodate a difference in load will result in materials savings, but can add considerably to forming costs due to field crew disruptions and increased potential for field error. Wide, flat beams are more cost-effective than deep narrow beams.

3) Should beam and joist spacing be uniform or vary with load? Once again, a large number of different spacings (closer together for heavy loads, farther apart for light) can result in material savings. However, the disruption in work and the added labor costs required to form the variations may far exceed savings in materials.

4) Should column size vary with height and loading? Consistency in column size usually results in reduced labor costs, particularly in buildings of moderate height. Under some rare conditions, however, changing the column size will yield savings in materials that justify the increased labor costs required for forming.

5) Are formed surface tolerances reasonable? Section 3.4 of ACI Standard 347[9.2] provides a way of quantitatively indicating tolerances for surface variations due to forming quality. The suggested tolerances for formed cast-in-place surfaces are shown in Table 9-1 (Table 3.4 of ACI 347). The following simplified guidelines for specifying the class of formed surface will usually minimize costs: a) Class A finish should be specified for surfaces prominently exposed to public view, b) Class B finish should be specified for surfaces less prominently

exposed to public view, c) Class C finish should be specified for all noncritical or unexposed surfaces, and d) Class D finish should be specified for concealed surfaces or for surfaces where roughness is not objectionable. If a more stringent class of surface is specified than is necessary for a particular formed surface, the increase in cost may become disproportionate to the increase in quality; this is illustrated in Fig. 9-1.

Table 9-1 Permitted Irregularities in Formed Surfaces Checked with a 5-ft Template[9.1]

Type of Irregularity	Class of Surface			
	A	B	C	D
Gradual[1]	1/8 in.	1/4 in.	1/2 in.	1 in.
Abrupt[2]	1/8 in.	1/4 in.	1/4 in.	1 in.

[1]Warping, unplaneness, and similar uniform variations from planeness measured per 5-ft template length (straightedge) placed anywhere on the surface in any direction.
[2]Offsets resulting from displaced, mismatched, or misplaced forms, sheathing, or liners or from defects in forming material.

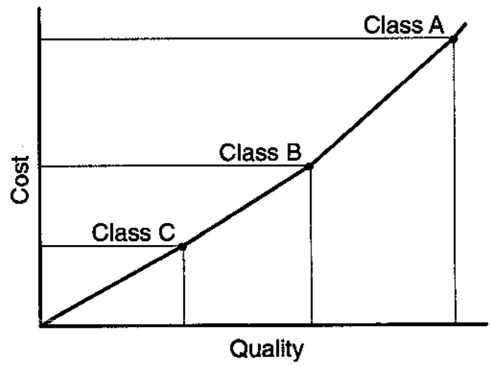

Figure 9-1 Class of Surface Versus Cost

9.3 ECONOMICAL ASPECTS OF HORIZONTAL FRAMING

Floors and the required forming are usually the largest cost component of a concrete building structure. The first step towards achieving maximum economy is selecting the most economical floor system for a given plan layout and a given set of loads. This will be discussed in more detail below. The second step is to define a regular, orderly progression of systematic shoring and reshoring. Timing the removal of the forms and requiring a minimum amount of reshoring are two factors that must be seriously considered since they can have a significant impact on the final cost.

Figures 1-5 and 1-6 show the relative costs of various floor systems as a function of bay size and superimposed load. Both figures are based on a concrete strength $f'_c = 4000$ psi. For a given set of loads, the slab system that is optimal for short spans is not necessarily optimal for longer spans. Also, for a given span, the slab system that is optimal for lighter superimposed loads is not necessarily optimal for heavier loads. Reference 9.3 provides material and cost estimating data for various floor systems. It is also very important to consider the fire resistance of the floor system in the preliminary design stage (see Chapter 10). Required fire resistance ratings can dictate the type of floor system to specify in a particular situation.

The relationship between span length, floor system, and cost may indicate one or more systems to be economical for a given project. If the system choices are equally cost-effective, then other considerations (architectural, aesthetic, etc.) may become the determining factor.

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Beyond selection of the most economical system for load and span conditions, there are general techniques that facilitate the most economical use of the chosen system.

9.3.1 Slab Systems

Whenever possible, avoid offsets and irregularities that cause a "stop and start" disruption of labor and require additional cutting (and waste) of materials (for example, breaks in soffit elevation). Depressions for terrazzo, tile, etc. should be accomplished by adding concrete to the top surface of the slab rather than maintaining a constant slab thickness and forming offsets in the bottom of the slab. Cross section (a) in Fig. 9-2 is less costly to form than cross section (b).

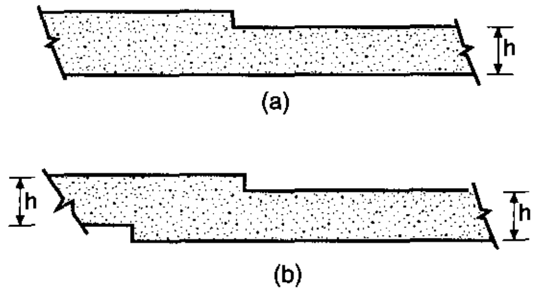


Figure 9-2 Depressions in Slabs

When drop panels are used in two-way systems, the total depth of the drop h_1 should be set equal to the actual nominal lumber dimension plus ¾ in. for plyform (see Fig. 9-3). Table 9-2 lists values for the depth h_1 based on common nominal lumber sizes. As noted above, designs which depart from standard lumber dimensions are expensive.

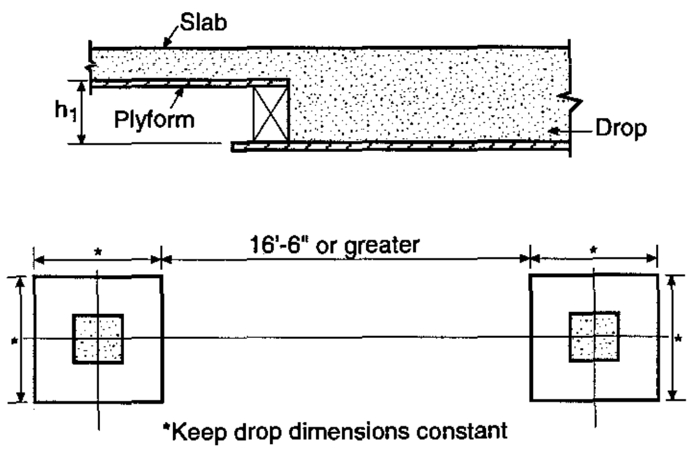


Figure 9-3 Formwork for Drop Panels

Whenever possible, a minimum 16 ft (plus 6 in. minimum clearance) spacing between drop panel edges should be used (see Fig. 9-3). Again, this permits the use of 16 ft long standard lumber without costly cutting of material. For maximum economy, the plan dimensions of the drop panel should remain constant throughout the entire project.

Table 9-2 Drop Panel Depth, h_1

Nominal lumber size	Actual lumber size (in.)	Plyform thickness (in.)	h_1 (in.)
2X	$1^1/_2$	$^3/_4$	$2^1/_4$
4X	$3^1/_2$	$^3/_4$	$4^1/_4$
6X	$5^1/_2$	$^3/_4$	$6^1/_4$
8X	$7^1/_4$	$^3/_4$	8

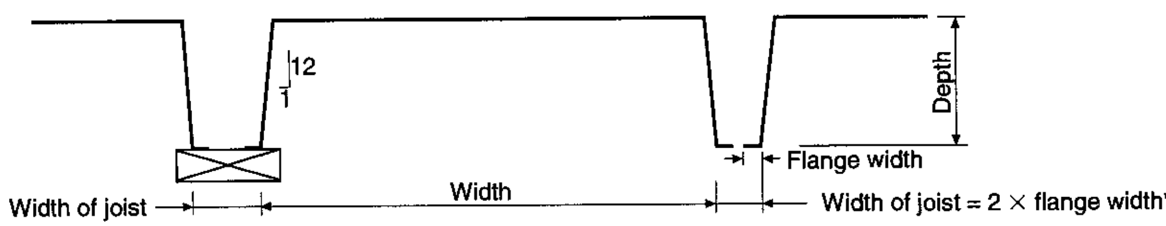
9.3.2 Joist Systems

Whenever possible, the joist depth and the spacing between joists should be based on standard form dimensions (see Table 9-3).

The joist width should conform to the values given in Table 9-3 also. Variations in width mean more time for interrupted labor, more time for accurate measurement between ribs, and more opportunities for jobsite error; all of these add to the overall cost.

Table 9-3 Standard Form Dimensions for One-Way Joist Construction (in.)

Width	Depth	Flange width	Width of joist
20	8, 10, 12	$^7/_8$, $2^1/_2$	5, 6
30	8, 10, 12, 14, 16, 20	$^7/_8$, 3	5, 6, 7
53	16, 20	$3^1/_2$	7, 8, 9, 10
66	14, 16, 20	3	6, 7, 8, 9, 10



*Applies to flange widths > $^7/_8$ in.

It is extremely cost-effective to specify a supporting beam with a depth equal to the depth of the joist. By doing this, the bottom of the entire floor system can be formed in one horizontal plane. Additionally, installation costs for utilities, partitions, and ceilings can all be reduced.

9.3.3 Beam-Supported Slab Systems

The most economical use of this relatively expensive system relies upon the principles of standardization and repetition. Of primary important is consistency in depth, and of secondary importance is consistency in width. These two concepts will mean a simplified design, less time spent interpreting plans and more time for field crews to produce.

9.4 ECONOMICAL ASPECTS OF VERTICAL FRAMING

9.4.1 Walls

Walls provide an excellent opportunity to combine multiple functions in a single element; by doing this, a more economical design is achieved. With creative layout and design, the same wall can be a fire enclosure for stair or elevator shafts, a member for vertical support, and bracing for lateral loads. Walls with rectangular cross-sections are less costly than nonrectangular walls.

9.4.2 Core Areas

Core areas for elevators, stairs, and utility shafts are required in many projects. In extreme cases, the core may require more labor than the rest of the floor. Standardizing the size and location of floor openings within the core will reduce costs. Repeating the core framing pattern on as many floors as possible will also help to minimize the overall costs.

9.4.3 Columns

Although the greatest costs in the structural frame are in the floor system, the cost of column formwork should not be overlooked. Whenever possible, use the same column dimensions for the entire height of the building. Also, use a uniform symmetrical column pattern with all of the columns having the same orientation. Planning along these general lines can yield maximum column economy as well as greater floor framing economy because of the resulting uniformity in bay sizes.

9.5 GUIDELINES FOR MEMBER SIZING

9.5.1 Beams

- For a line of continuous beams, keep the beam size constant and vary the reinforcement from span to span.
- Wide flat beams (same depth as joists) are easier to form than beams projecting below the bottom of the joists (see Fig. 9-4).

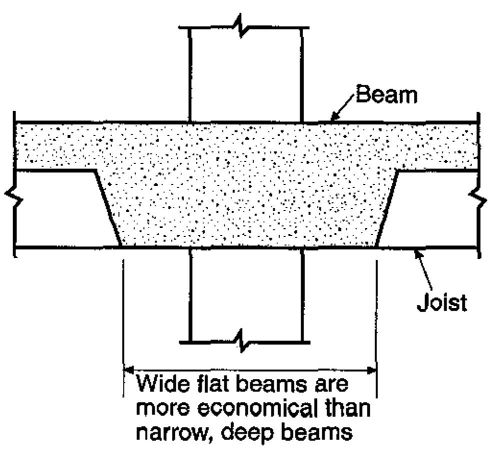


Figure 9-4 One-Way Joist Floor System

- Spandrel beams are more cost intensive than interior beams due to their location at the edge of a floor slab or at a slab opening. Fig. 9-5 lists some of the various aspects to consider when designing these members.

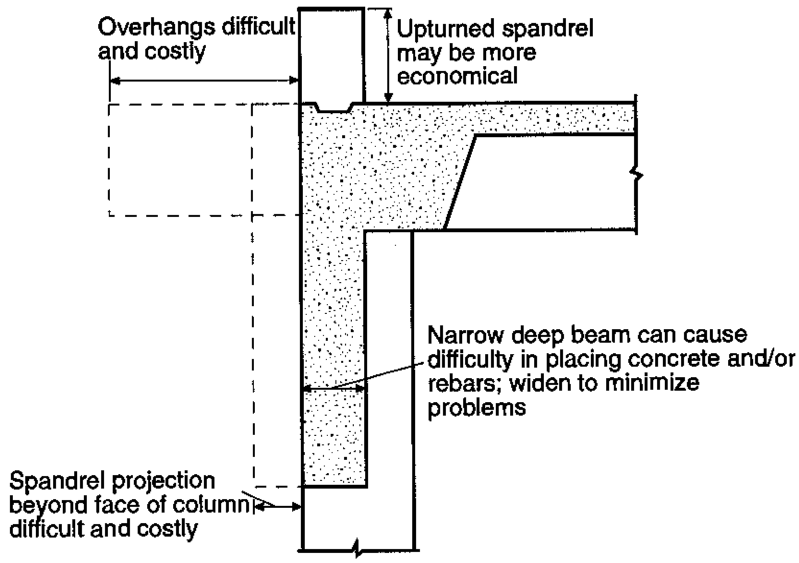


Figure 9-5 Spandrel Beams

- Beams should be as wide as, or wider than, the columns into which they frame (see Fig. 9-6). In addition to formwork economy, this also alleviates some of the reinforcement congestion at the intersection.

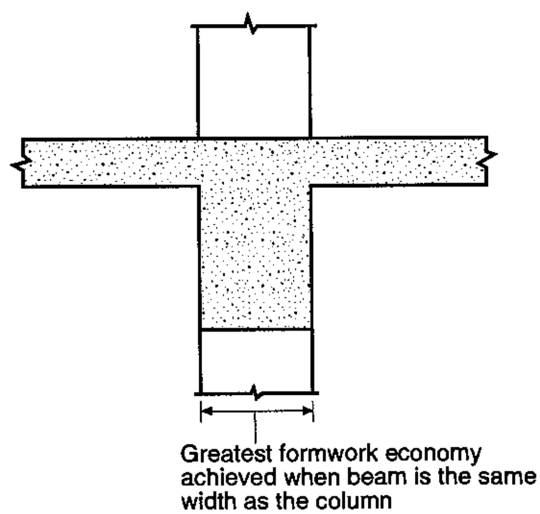


Figure 9-6 Beam-Column Intersections

- For heavy loading or long spans, a beam deeper than the joists may be required. In these situations, allow for minimum tee and lugs at sides of beams as shown in Fig. 9-7. Try to keep difference in elevation between bottom of beam and bottom of floor system in modular lumber dimensions.

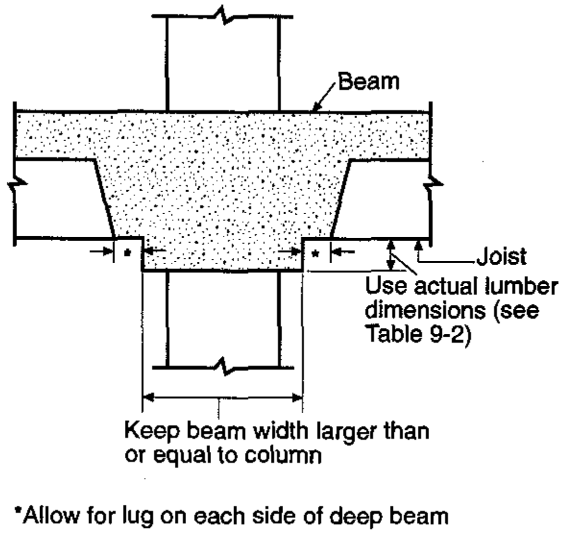


Figure 9-7 One-Way Joist Floor System with Deep Beams

9.5.2 Columns

- For maximum economy, standardize column location and orientation in a uniform pattern in both directions (see Fig. 9-8).

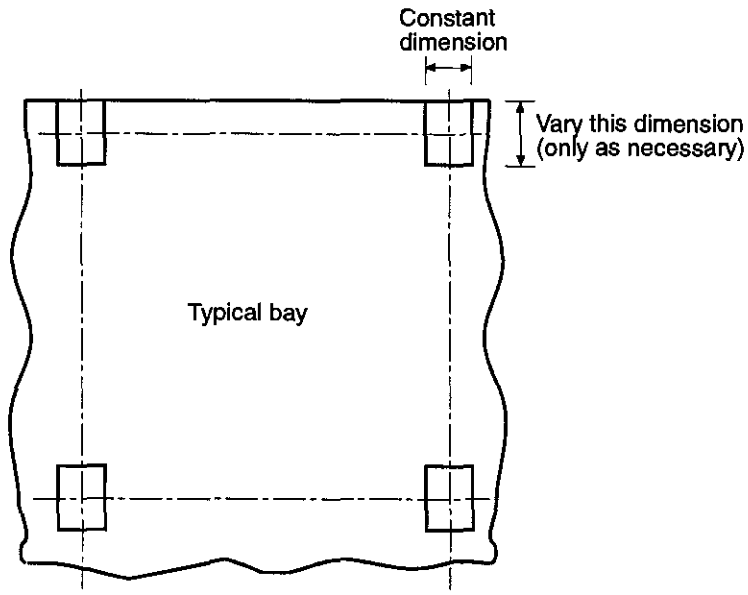


Figure 9-8 Standard Column Location and Orientation for a Typical Bay

- Columns should be kept the same size throughout the building. If size changes are necessary, they should occur in 2 in. increments, one side at a time (for example, a 22 × 22 in. column should go to a 24 × 22 in., then to a 24 × 24 in., etc.) Gang forming can possibly be used when this approach to changing column sizes is utilized. When a flying form system is used, the distance between column faces and the flying form must be held constant. Column size changes must be made parallel to the flying form.

- Use the same shape as often as possible throughout the entire building. Square or round columns are the most economical; use other shapes only when architectural requirements so dictate.

9.5.3 Walls

- Use the same wall thickness throughout a project if possible; this facilitates the reuse of equipment, ties, and hardware. In addition, this minimizes the possibilities of error in the field. In all cases, maintain sufficient wall thickness to permit proper placing and vibrating of concrete.

- Wall openings should be kept to a minimum number since they can be costly and time-consuming. A few larger openings are more cost-effective than many smaller openings. Size and location should be constant for maximum reuse of formwork.

- Brick ledges should be kept at a constant height with a minimum number of steps. Thickness as well as height should be in dimensional units of lumber, approximating as closely as possible those of the masonry to be placed. Brick ledge locations and dimensions should be detailed on the structural drawings.

- Footing elevations should be kept constant along any given wall if possible. This facilitates the use of wall gang forms from footing to footing. If footing steps are required, use the minimum number possible.

- For buildings of moderate height, pilasters can be used to transfer column loads into the foundation walls. Gang forms can be used more easily if the pilaster sides are splayed as shown in Fig. 9-9.

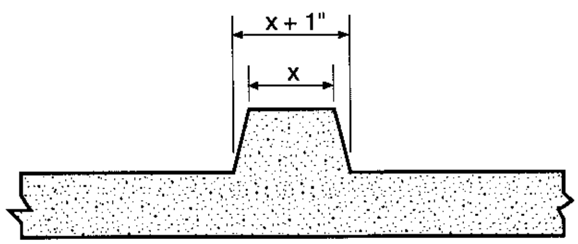


Figure 9-9 Pilasters

9.6 OVERALL STRUCTURAL ECONOMY

While it has been the primary purpose of this chapter to focus on those considerations that will significantly impact the costs of the structural system relative to formwork requirements, the 10-step process below should be followed during the preliminary and final design phases of the construction project as this will lead to overall structural economy:

1. Study the structure as a whole.

2. Prepare freehand alternative sketches comparing all likely structural framing systems.

3. Establish column locations as uniformly as possible, keeping orientation and size constant wherever possible.

4. Determine preliminary member sizes from available design aids (see Section 1.8).

5. Make cost comparisons based on sketches from Step 2 quickly, roughly, but with an adequate degree of accuracy.

6. Select the best balance between cost of structure and architectural/mechanical design considerations.

7. Distribute prints of selected framing scheme to all design and building team members to reduce unnecessary future changes.

8. Plan your building. Visualize how forms would be constructed. Where possible, keep beams and columns simple without haunches, brackets, widened ends or offsets. Standardize concrete sizes for maximum reuse of forms.

9. During final design, place most emphasis on those items having greatest financial impact on total structural frame cost.

10. Plan your specifications to minimize construction costs and time by including items such as early stripping time for formwork and acceptable tolerances for finish.

Reference 9.4 should be consulted for additional information concerning formwork.

References

9.1 *Concrete Buildings, New Formwork Perspectives*, Ceco Industries, Inc., 1985.

9.2 *Guide to Formwork for Concrete*, ACI 347R-88, American Concrete Institute, Detroit, Michigan, 1989, 33 pp.

9.3 *Concrete Floor Systems—Guide to Estimating and Economizing*, SP041, Portland Cement Association, Skokie, Illinois, 1990, 33 pp.

9.4 *Formwork for Concrete*, by M.K. Hurd (prepared under direction of ACI Committee 347, Formwork for Concrete), SP-4, 5th Ed., American Concrete Institute, Detroit, Michigan, 1989.

Chapter 10

Design Considerations for Fire Resistance

10.1 INTRODUCTION

State and municipal building codes throughout the country regulate the fire resistance of the various elements and assemblies comprising a building structure. Structural frames (columns and beams), floor and roof systems, and load bearing walls must be able to withstand the stresses and strains imposed by fully developed fires and carry their own dead loads and superimposed loads without collapse.

Fire resistance ratings required of the various elements of construction by building codes are a measure of the endurance needed to safeguard the structural stability of a building during the course of a fire and to prevent the spread of fire to other parts of the building. The determination of fire rating requirements in building codes is based on the expected fire severity (fuel loading) associated with the type of occupancy and the building height and area.

In the design of structures, building code provisions for fire resistance are sometimes overlooked and this may lead to costly mistakes. It is not uncommon, for instance, to find that a concrete slab in a waffle slab floor system may only require a 3 to 4-½ in. thickness to satisfy ACI 318 strength requirements. However, if the building code specifies a 2-hour fire resistance rating for that particular floor system, the slab thickness may need to be increased to 3-½ to 5 in., depending on type of aggregate used in the concrete. Indeed, under such circumstances and from the standpoint of economics, the fire-resistive requirements may indicate another system of construction to be more appropriate, say, a pan-joist or flat slab/plate floor system. Simply stated, structural members possessing the fire resistance prescribed in building codes may differ significantly in their dimensional requirements from those predicated only on ACI 318 strength criteria. Building officials are required to enforce the stricter provisions.

The purpose of this chapter is to make the reader aware of the importance of determining the fire resistance requirements of the governing building code before proceeding with the structural design.

The field of fire technology is highly involved and complex and it is not the intent here to deal with the chemical or physical characteristics of fire, nor with the behavior of structures in real fire situations. Rather, the goal is to present some basic information as an aid to designers in establishing those fire protection features of construction that may impact their structural design work.

The information given in this chapter is fundamental. Modern day designs, however, must deal with many combinations of materials and it is not possible here to address all the intricacies of construction. Rational methods of design for dealing with more involved fire resistance problems are available. For more comprehensive discussions on the subject of the fire resistive qualities of concrete and for calculation methods used in solving design problems related to fire integrity, the reader may consult Reference 10.1.

10.2 DEFINITIONS

Structural Concrete:

- Siliceous aggregate concrete: concrete made with normal weight aggregates consisting mainly of silica or compounds other than calcium or magnesium carbonate.

- Carbonate aggregate concrete: concrete made with aggregates consisting mainly of calcium or magnesium carbonate, e.g., limestone or dolomite.

- Sand-lightweight concrete: concrete made with a combination of expanded clay, shale, slag, or slate or sintered fly ash and natural sand. Its unit weight is generally between 105 and 120 pcf.

- Lightweight aggregate concrete: concrete made with aggregates of expanded clay, shale, slag, or slate or sintered fly ash, and weighing 85 to 115 pcf.

Insulating Concrete:

- Cellular concrete: a lightweight insulating concrete made by mixing a preformed foam with portland cement slurry and having a dry unit weight of approximately 30 pcf.

- Perlite concrete: a lightweight insulating concrete having a dry unit weight of approximately 30 pcf made with perlite concrete aggregate produced from volcanic rock that, when heated, expands to form a glass-like material or cellular structure.

- Vermiculite concrete: a lightweight insulating concrete made with vermiculite concrete aggregate, a laminated micaceous material produced by expanding the ore at high temperatures. When added to a portland cement slurry the resulting concrete has a dry unit weight of approximately 30 pcf.

Miscellaneous Insulating Materials:

- Glass fiber board: fibrous glass roof insulation consisting of inorganic glass fibers formed into rigid boards using a binder. The board has a top surface faced with asphalt and kraft reinforced with glass fibers.

- Mineral board: a rigid felted thermal insulation board consisting of either felted mineral fiber or cellular beads of expanded aggregate formed into flat rectangular units.

10.3 FIRE RESISTANCE RATINGS

10.3.1 Fire Test Standards

The fire-resistive properties of building components and structural assemblies are determined by standard fire test methods. The most widely used and nationally accepted test procedure is that developed by the American Society of Testing and Materials (ASTM). It is designated as ASTM E 119, Standard Methods of Fire Tests of Building Construction and Materials. Other accepted standards, essentially alike, include the National Fire Protection Association Standard Method No. 251; Underwriters Laboratories' U.L. 263; American National Standards Institute's ANSI A2-1; ULC-S101 from the Underwriters Laboratories of Canada; and Uniform Building Code Standard No. 43-1.

10.3.2 ASTM E 119 Test Procedure

A standard fire test is conducted by placing an assembly in a test furnace. Floor and roof specimens are exposed to controlled fire from beneath, beams from the bottom and sides, walls from one side, and columns from all sides. The temperature is raised in the furnace over a given period of time in accordance with the ASTM E 119 standard time-temperature curve shown in Fig. 10-1.

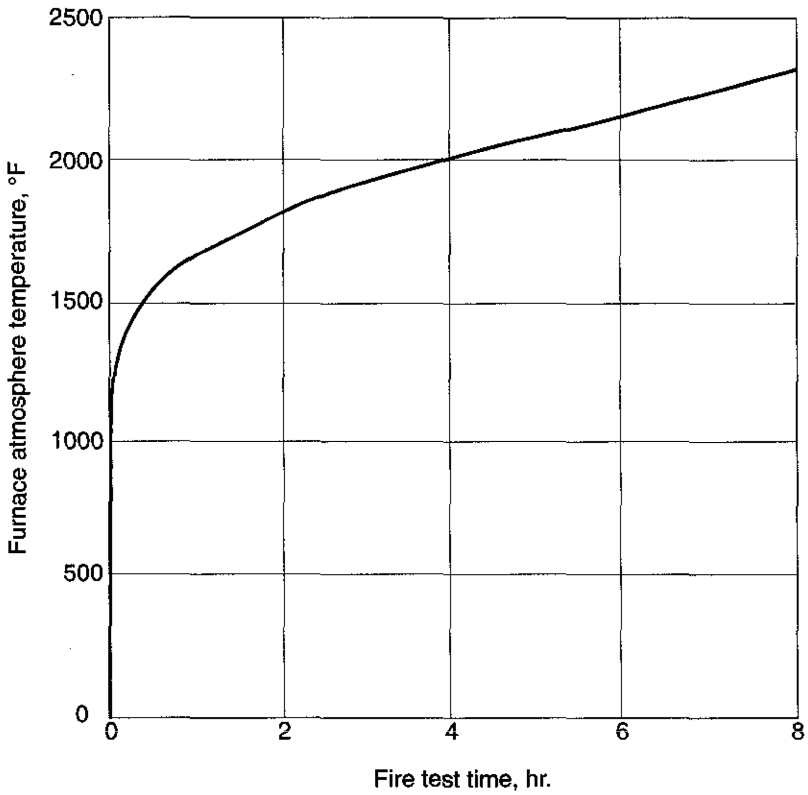


Figure 10-1 Standard Time-Temperature Relationship of Furnace Atmosphere (ASTM E 119)

This specified time-temperature relationship provides for a furnace temperature of 1000°F at five minutes from the beginning of the test, 1300°F at 10 minutes, 1700°F at one hour, 1850°F at two hours, and 2000°F at four hours. The end of the test is reached and the fire endurance of the specimen is established when any one of the following conditions first occur:

1) For walls, floors, and roof assemblies the temperature of the unexposed surface rises an average of 250°F above its initial temperature or 325°F at any location. In addition, walls achieving a rating classification of one hour or greater must withstand the impact, erosion and cooling affects of a hose stream test.

2) Cotton waste placed on the unexposed side of a wall, floor, or roof system is ignited through cracks or fissures developed in the specimen.

3) The test assembly fails to sustain the applied load.

4) For certain restrained and all unrestrained floors, roofs and beams, the reinforcing steel temperature rises to 1100°F.

Though the complete requirements of ASTM E 119 and the conditions of acceptance are much too detailed for inclusion in this chapter, experience shows that concrete floor/roof assemblies and walls usually fail by heat transmission (item 1); and columns and beams by failure to sustain the applied loads (item 3), or by beam reinforcement failing to meet the temperature criterion (item 4).

Fire rating requirements for structural assemblies may differ from code to code; therefore, it is advisable that the designer take into account the building regulations having jurisdiction over the construction rather than relying on general perceptions of accepted practice.

10.4 DESIGN CONSIDERATIONS FOR FIRE RESISTANCE

10.4.1 Properties of Concrete

Concrete is the most highly fire-resistive structural material used in construction. Nonetheless, the properties of concrete and reinforcing steel change significantly at high temperatures. Strength and the modulus of elasticity are reduced, the coefficient of expansion increases, and creep and stress relaxations are considerably higher.

Concrete strength, the main concern in uncontrolled fires, remains comparatively stable at temperatures ranging up to 900°F for some concretes and 1200°F for others. Siliceous aggregate concrete, for instance, will generally maintain its original compressive strength at temperatures up to 900°F, but can lose nearly 50% of its original strength when the concrete reaches a temperature of about 1200°F. On the other hand, carbonate aggregate and sand-lightweight concretes behave more favorably in fire, their compressive strengths remaining relatively high at temperatures up to 1400°F, and diminishing rapidly thereafter. These data reflect fire test results of specimens loaded in compression to 40% of their original compressive strength.

The temperatures stated above are the internal temperatures of the concrete and are not to be confused with the heat intensity of the exposing fire. As an example, in testing a solid carbonate aggregate slab, the ASTM standard fire exposure after 1 hour will be 1700°F, while the temperatures within the test specimen will vary throughout the section: about 1225°F at ¼ in. from the exposed surface, 950°F at ¾ in., 800°F at 1 in., and 600°F at 1-½ in.; all within the limit of strength stability.

It is to be realized that the strength loss in concrete subjected to intense fire is not uniform throughout the structural member because of the time lag required for heat penetration and the resulting temperature gradients occurring across the concrete section. The total residual strength in the member will usually provide an acceptable margin of safety.

This characteristic is even more evident in massive concrete building components such as columns and girders. Beams of normal weight concrete exposed to an ASTM E 119 fire test will, at two hours when the exposing fire is at 1850°F, have internal temperatures of about 1200°F at 1 in. inside the beam faces and less than 1000°F at 2 in. Obviously, the dimensionally larger concrete sections found in main framing systems will suffer far less net loss of strength (measured as a percentage of total cross-sectional area) than will lighter assemblies.

Because of the variable complexities and the unknowns of dealing with the structural behavior of buildings under fire as total multidimensional systems, building codes continue to specify minimum acceptable levels of fire endurance on a component by component basis—roof/floor assemblies, walls, columns, etc. It is known, for instance, that in a multi-bay building, an interior bay of a cast-in-place concrete floor system subjected to fire will be restrained in its thermal expansion by the unheated surrounding construction. Such restraint increases the structural fire endurance of the exposed assembly by placing the heated concrete in compression. The restraining forces developed are large and, under elastic behavior, would cause the concrete to exceed its original compressive strength were it not for stress relaxations that occur at high temperatures. According to information provided in Appendix X3 of ASTM E 119, cast-in-place beams and slab systems are generally considered restrained (see Table 10-5 in Section 10.4.3).

In addition to the minimum acceptable limits given in the building codes, the use of calculation methods for determining fire endurance are also accepted, depending on local code adoptions (see Reference 10.1).

10.4.2 Thickness Requirements

Test findings show that fire resistance in concrete structures will vary in relation to the type of aggregate used. The differences are shown in Tables 10-1 and 10-2.

Table 10-1 Minimum Thickness for Floor and Roof Slabs and Cast-In-Place Walls, in. (Load Bearing and Nonload-Bearing)

	Fire resistance rating				
Concrete type	1 hr.	1 1/2 hr.	2 hr.	3 hr.	4 hr.
Siliceous aggregate	3.5	4.3	5.0	6.2	7.0
Carbonate aggregate	3.2	4.0	4.6	5.7	6.6
Sand-lightweight	2.7	3.3	3.8	4.6	5.4
Lightweight	2.5	3.1	3.6	4.4	5.1

Table 10-2 Minimum Concrete Column Dimensions, in.

	Fire resistance rating				
Concrete type	1 hr.	1 1/2 hr.	2 hr.	3 hr.	4 hr.
Siliceous aggregate	8	8	10	12	14
Carbonate aggregate	8	8	10	12	12
Sand-lightweight	8	8	9	10.5	12

In studying the tables above it is readily apparent that there may be economic benefits to be gained from the selection of the type of concrete to be used in construction. The designer is encouraged to evaluate the alternatives.

10.4.3 Cover Requirements

Another factor to be considered in complying with fire-resistive requirements is the minimum thickness of concrete cover for the reinforcement. The concrete protection specified in ACI 318 for cast-in-place concrete will generally equal or exceed the minimum cover requirements shown in the following tables, but there are a few exceptions at the higher fire ratings and these should be noted.

The minimum thickness of concrete cover to the positive moment reinforcement is given in Table 10-3 for one-way or two-way slabs with flat undersurfaces.

The minimum thickness of concrete cover to the positive moment reinforcement (bottom steel) in reinforced concrete beams is shown in Table 10-4.

Table 10-3 Minimum Cover for Reinforced Concrete Floor or Roof Slabs, in.

	Restrained Slabs*				Unrestrained Slabs*			
	Fire resistance rating				Fire resistance rating			
Concrete type	1 hr.	$1^1/_2$ hr.	2 hr.	3 hr.	1 hr.	$1^1/_2$ hr.	2 hr.	3 hr.
Siliceous aggregate	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	1	$1^1/_4$
Carbonate aggregate	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$1^1/_4$
Sand-lightweight or lightweight	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$1^1/_4$

*See Table 10-5

Table 10-4 Minimum Cover to Main Reinforcing Bars in Reinforced Concrete Beams, in.
(Applicable to All Types of Structural Concrete)

		Fire resistance rating				
Restrained or unrestrained*	Beam width, in.**	1 hr.	$1^1/_2$ hr.	2 hr.	3 hr.	4 hr.
Restrained	5	$^3/_4$	$^3/_4$	$^3/_4$	1	$1^1/_4$
Restrained	7	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$
Restrained	≥ 10	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$	$^3/_4$
Unrestrained	5	$^3/_4$	1	$1^1/_4$	—	—
Unrestrained	7	$^3/_4$	$^3/_4$	$^3/_4$	$1^3/_4$	3
Unrestrained	≥ 10	$^3/_4$	$^3/_4$	$^3/_4$	1	$1^3/_4$

*See Table 10-5

**For beam widths between the tabulated values, the minimum cover can be determined by interpolation.

The minimum cover to main longitudinal reinforcement in columns is shown in Table 10-6.

10.5 MULTICOURSE FLOORS AND ROOFS

Symbols: Carb = carbonate aggregate concrete
Sil = siliceous aggregate concrete
SLW = sand-lightweight concrete

Table 10-5 Construction Classification, Restrained and Unrestrained (Table X3.1 from ASTM E 119)*

I. Wall bearing	
Single span and simply supported end spans of multiple bays:[A]	
(1) Open-web steel joists or steel beams, supporting concrete slab, precast units, or metal decking	unrestrained
(2) Concrete slabs, precast units, or metal decking	unrestrained
Interior spans of multiple bays:	
(1) Open-web steel joists, steel beams or metal decking, supporting continuous concrete slab	restrained
(2) Open-web steel joists or steel beams, supporting precast units or metal decking	unrestrained
(3) Cast-in-place concrete slab systems	restrained
(4) Precast concrete where the potential thermal expansion is resisted by adjacent construction[B]	restrained
II. Steel framing:	
(1) Steel beams welded, riveted, or bolted to the framing members	restrained
(2) All types of cast-in-place floor and roof systems (such as beam-and-slabs, flat slabs, pan joists, and waffle slabs) where the floor or roof system is secured to the framing members	restrained
(3) All types of prefabricated floor or roof systems where the structural members are secured to the framing members and the potential thermal expansion of the floor or roof system is resisted by the framing system or the adjoining floor or roof construction[B]	restrained
III. Concrete framing:	
(1) Beams securely fastened to the framing members	restrained
(2) All types of cast-in-place floor or roof systems (such as beam-and-slabs, pan joists, and waffle slabs) where the floor system is cast with the framing members	restrained
(3) Interior and exterior spans of precast systems with cast-in-place joints resulting in restraint equivalent to that which would exist in condition III (1)	restrained
(4) All types of prefabricated floor or roof systems where the structural members are secured to such systems and the potential thermal expansion of the floor or roof systems is resisted by the framing system or the adjoining floor or roof construction[B]	restrained
IV. Wood construction:	
All types	unrestrained

[A]Floor and roof systems can be considered restrained when they are tied into walls with or without tie beams, the walls being designed and detailed to resist thermal thrust from the floor or roof system.

[B]For example, resistance to potential thermal expansion is considered to be achieved when:

(1) Continuous structural concrete topping is used,

(2) The space between the ends of precast units or between the ends of units and the vertical face of supports is filled with concrete or mortar, or

(3) The space between the ends of precast units and the vertical faces of supports, or between the ends of solid or hollow core slab units does not exceed 0.25% of the length for normal weight concrete members or 0.1% of the length for structural lightweight concrete members.

Table 10-6 Minimum Cover for Reinforced Concrete Columns, in.

	Fire resistance rating				
Concrete type	1 hr.	$1^1/_2$ hr.	2 hr.	3 hr.	4 hr.
Siliceous aggregate	$1^1/_2$	$1^1/_2$	$1^1/_2$	$1^1/_2$	2
Carbonate aggregate	$1^1/_2$	$1^1/_2$	$1^1/_2$	$1^1/_2$	$1^1/_2$
Sand-lightweight	$1^1/_2$	$1^1/_2$	$1^1/_2$	$1^1/_2$	$1^1/_2$

10.5.1 Two-Course Concrete Floors

Figure 10-2 gives information on the fire resistance ratings of floors that consist of a base slab of concrete with a topping (overlay) of a different type of concrete.

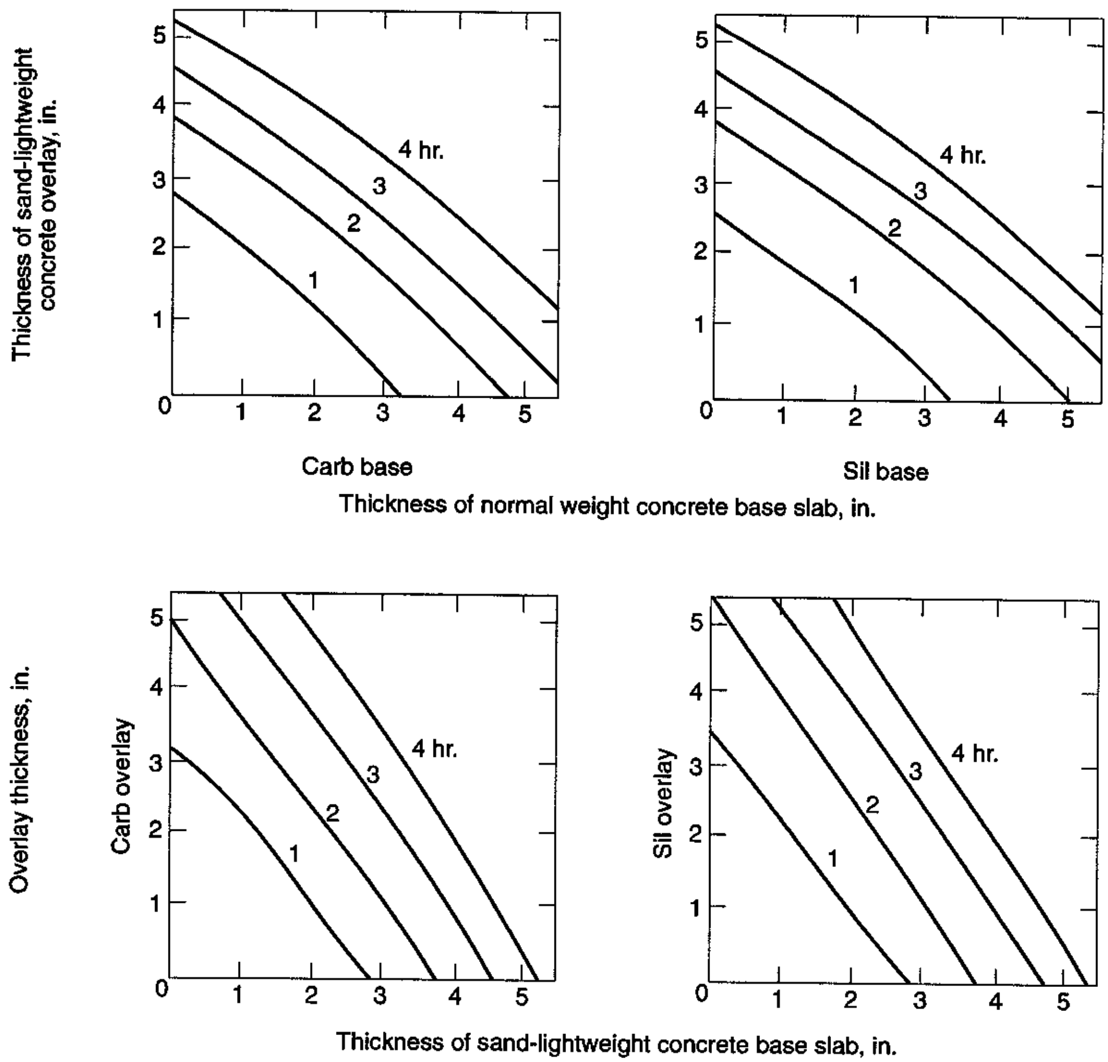


Figure 10-2 Fire Resistance Ratings for Two-Course Floor Slabs

10.5.2 Two-Course Concrete Roofs

Figure 10-3 gives information on the fire resistance ratings of roofs that consist of a base slab of concrete with a topping (overlay) of an insulating concrete; the topping does not include built-up roofing. For the transfer of heat, three-ply built-up roofing contributes 10 minutes to the fire resistance rating; thus, 10 minutes may be added to the values shown in the figure.

10.5.3 Concrete Roofs with Other Insulating Materials

Figure 10-4 gives information on the fire resistance ratings of roofs that consist of a base slab of concrete with an insulating board overlay; the overlay includes standard 3-ply built-up roofing.

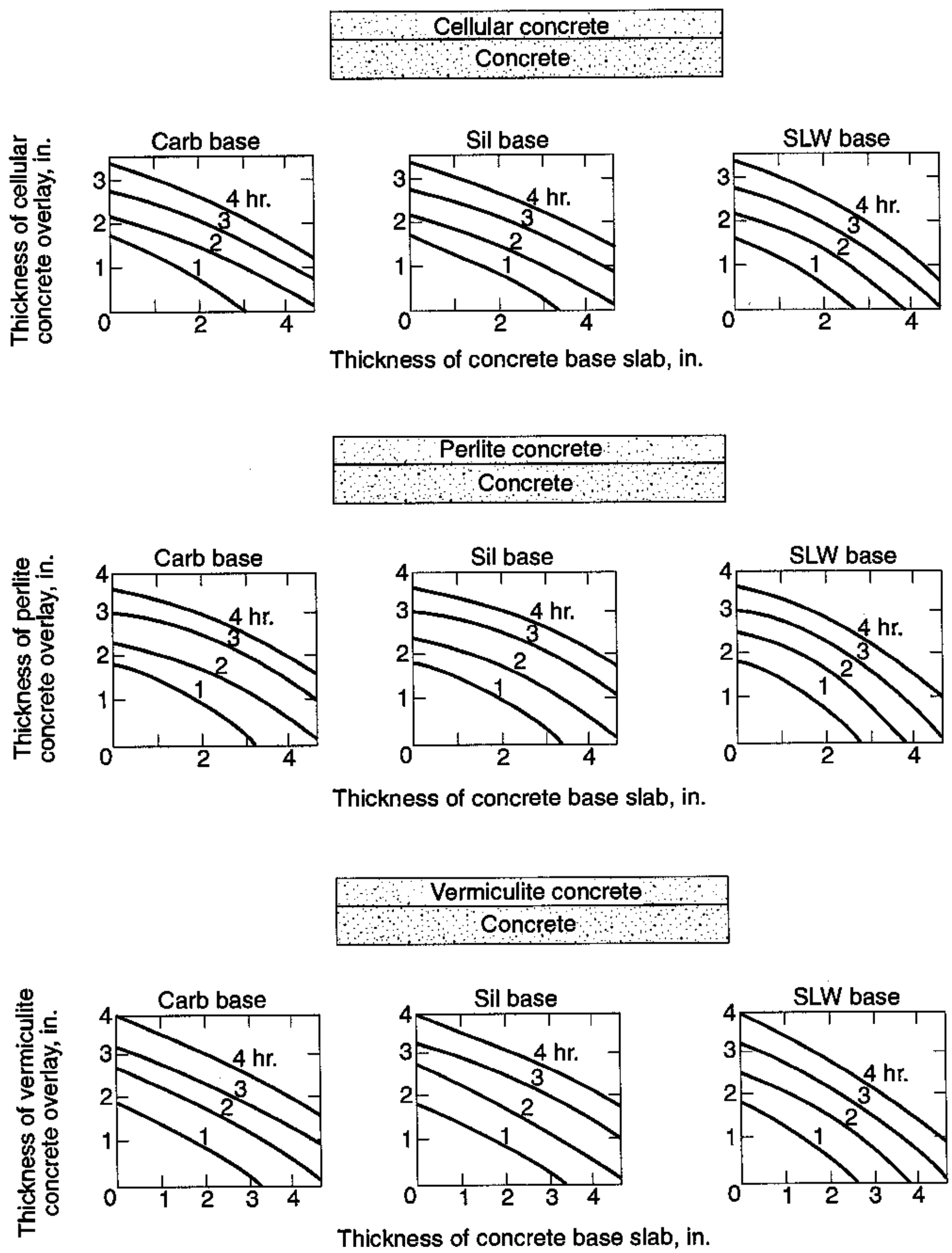


Figure 10-3 Fire Resistance Ratings for Two-Course Roof Slabs

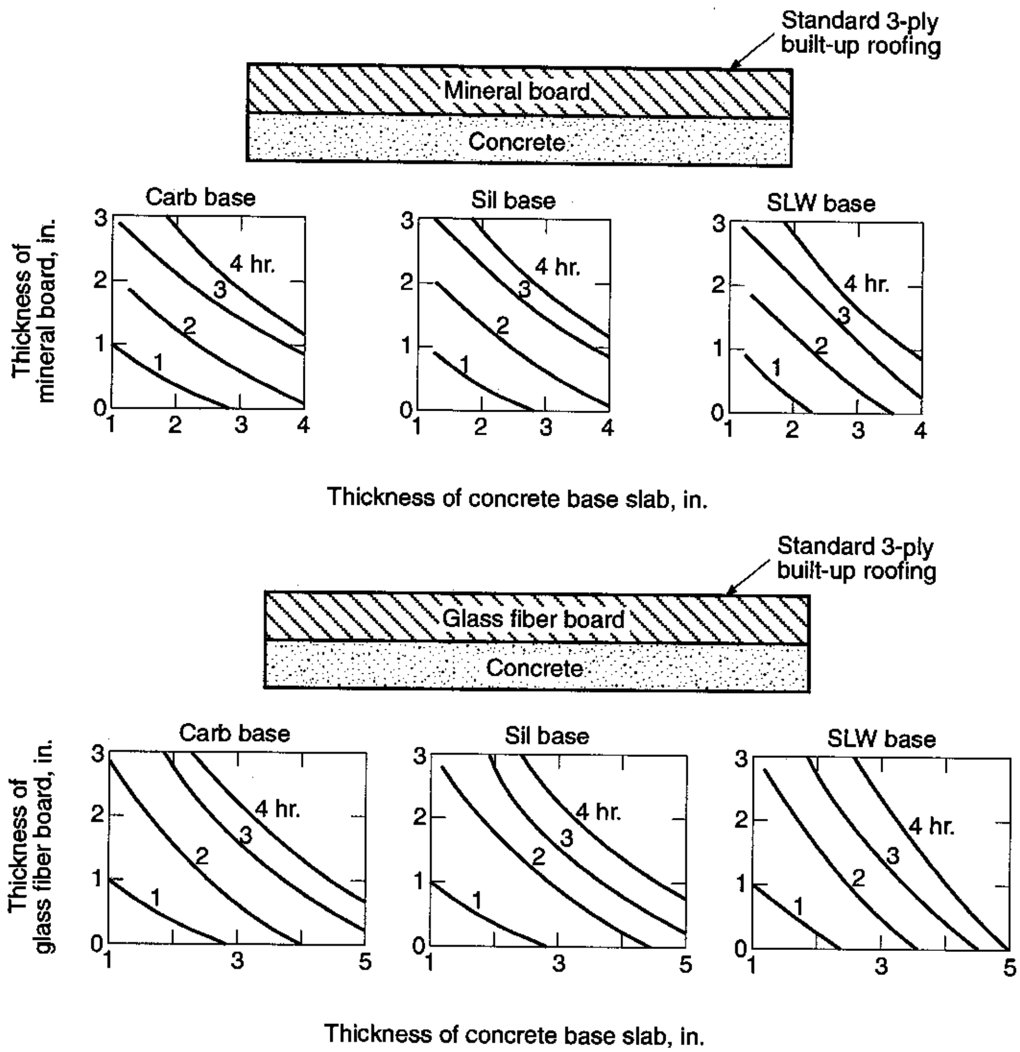
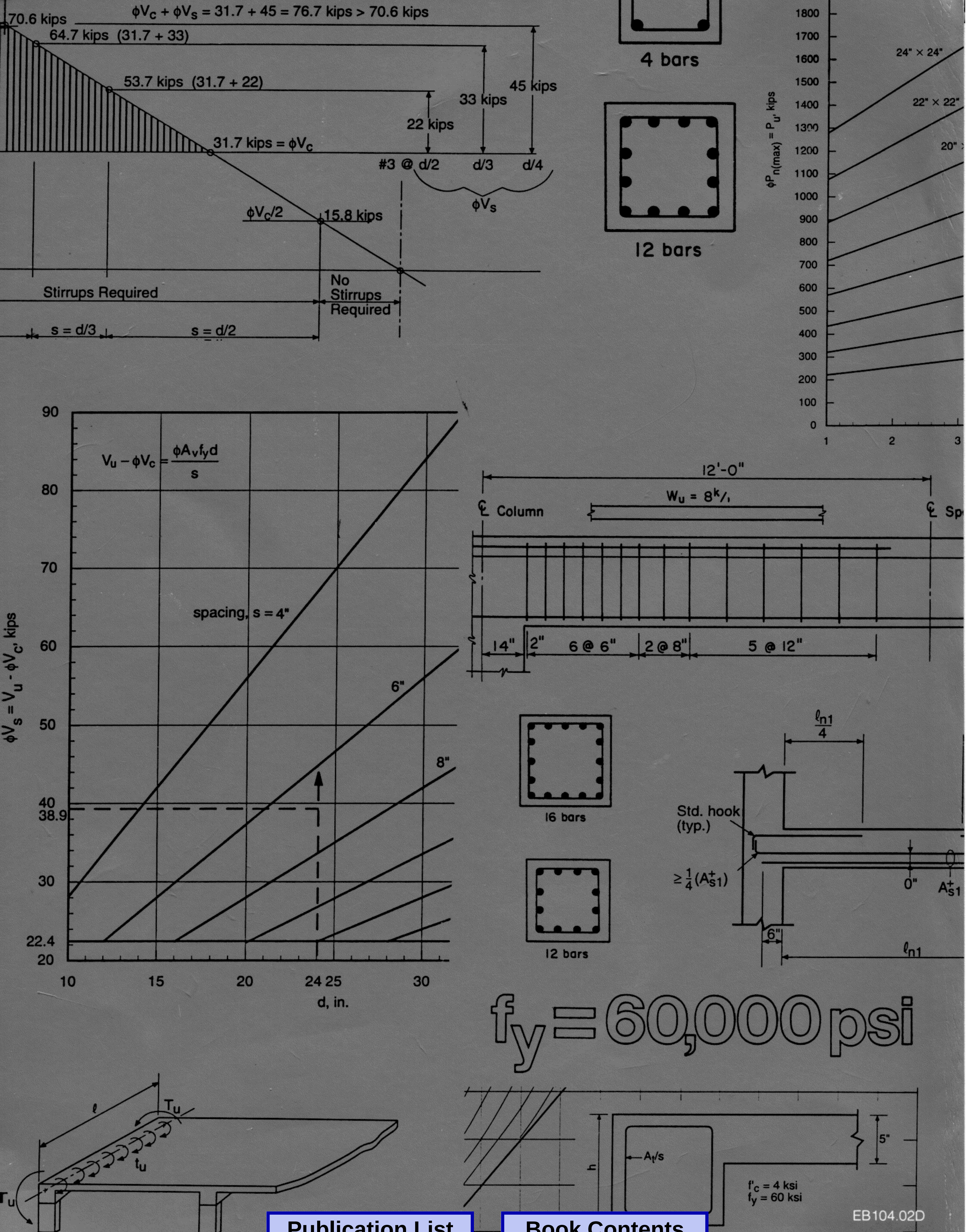


Figure 10-4 Fire Resistance Ratings for Roof Slabs With Insulating Overlays and Standard 3-Ply Built-Up Roofing

Reference

10.1 *Reinforced Concrete Fire Resistance*, Concrete Reinforcing Steel Institute, Schaumburg, Illinois, 256 pp.



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Publication List

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